Essays in Financial Markets

Nielsen, Steen

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Contents

Chapter 1

Introduction

This dissertation adresses various issues regarding the functioning of financial markets. It consists of this introduction and five independent chapters of which the first four are empirical and the last one is theoretical. The introduction provides a brief background on some of the data used in subsequent chapters and a discussion of the main results of the dissertation. This is intended to guide the reader and does not substitute the much more detailed exposition in the following chapters.

1.1. Background

Most of the chapters in this dissertation are empirical studies of financial markets. Although some Danish financial market data had previously been collected for other purposes, it was obvious from the outset that more work was needed to supplement available data. This section provides a brief background on what data were available and what has been constructed for our purposes. The description focuses solely on the database of long-run, aggregate macroeconomic and financial market data which forms the basis of the empirical work in chapters 2, 3 and 4.[1]

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1 The database was set up by Ole Risager and myself. In this time-consuming process, we benefitted from research assistance by Ian Valsted and Michael Wieman. In chapters 3 and 4, the database is referred to as the Nielsen, Olesen and Risager database which reflects that it was updated by Jan Overgaard Olesen to include new

The starting point of many studies of the Danish stock market is the stock market index published by Statistics Denmark from around 1920. This index measures the value of equity in a comprehensive[2] set of publicly listed firms. We use the index to calculate capital gains of the market portfolio.

An ambitious study of Danish stock returns is K. Hansen (1974) which includes calculation of dividend yields. However, since Hansen does not report dividend yield for individual years we need to construct it for the entire period. Thus, we estimate market dividend yield as the capitalization-weighted average of dividend yields of a sample of about 100 firms. Our sample begins in 1922 and is selected to represent all industry categories and cover a large fraction of the entire market. In any year, coverage is between 50 and 80% of total capitalization.

Olsen and Hoffmeyer (1968) is an influential source of historical Danish interest rates. For our purposes, however, data from this source are not useful. The reason is that they average yields of bonds with different time to maturity and we are mainly interested in comparing bond and stock investments. Thus, bond yields for holding periods of fixed lengths are needed. These considerations led us to construct 1-, 5- and 10-year horizon bond yields by computing on an annual basis yields on government bonds with 1, 5 and 10 years to maturity.

Finally, data on consumer price index and population are obtained from Statistics Denmark while real consumption data originate from S.Å. Hansen (1974).

To indicate potential applications of the dataset, table 1.1 provides a few key figures for Danish stock and bond markets since the beginning of this century.

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variables and observations for 1996. Data are available to readers who are interested in checking our results, and will be publicly released in near future.

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Table 1.1. Summary Statistics of Annual Stock and Bond Returns in Denmark, 1922-95

	Stocks	Bonds[1]
Average nominal return	11.0 (22.9)	7.6 (4.5)
Covariance of real return and real per capita consumption growth	0.00124	0.00104

Notes: Standard deviations in parentheses.
1. Annual observations of annualized yields of government bond with one year to maturity. Since 1-year bond yields are not available for the first two years of the sample, the 5-year yield has been used as a proxy those two years.

It is obvious from table 1.1 that stocks yield higher average return than bonds[3] and that stocks are riskier measured by the standard deviation of annual returns. An annual return difference of this size accumulates over time to a substantial difference in portfolio value. Indeed, in 1995, the real value of a stock market investment made in the beginning of the sample was more than twice the value of a similar bond investment. Hansson (1999) finds that the Swedish market portfolio performs even better than the Danish, especially during the period from the second world war to the beginning of the 1970s. See the next section for comments on the covariances shown in table 1.1.

1.2. Asset Pricing Models

1.2.1. The consumption-based CAPM

The consumption-based capital asset pricing model, C-CAPM, was first developed by Lucas (1978) and Breeden (1979). Under the assumptions

3 Note, however, that this difference is not significant at the 5% significance level using the normal approximation as pointed out by Engsted (1999).

that (i) consumption can be modelled by a representative consumer, (ii) markets are complete, and (iii) trading costs are negligible, the model implies that investors require higher expected return on assets whose return is positively correlated with consumption growth than on assets with the opposite characteristic.

A number of empirical tests have been performed to check the validity of C-CAPM. Mehra and Prescott (1985) show that the model holds in a qualitative sense, ie., that stocks are riskier (measured by covariance of consumption growth and return) than bonds and have higher mean return, but that the equity premium is too large to be explained by reasonably parametherized standard utility functions. The problem is that both covariances are close to zero which reflects that aggregate consumption grows at a steady rate. Thus, aggregate consumption risk is so small that only a large degree of risk aversion can rationalize the size of the equity premium.

In Nielsen and Risager (1999), which is chapter 2 in this dissertation, the C-CAPM is tested with Danish data using a procedure developed by Hansen and Jagannathan (1991). We reach a similar conclusion as American studies, namely that the risk aversion required in a standard power utility function for the model to match the data appears to be too large, although in our case the level of risk aversion needed is less extreme than in the US case, see for example Hansen and Jagannathan. Table 1.1 shows that Danish annual stock returns are higher than bond returns, but also much more volatile. The return difference, however, is only around half the size of the American premium which explains why our risk aversion estimate is relatively small. Furthermore, table 1.1 illustrates that both assets like in the US covary only little with consumption growth. Considering an alternative utility function which allows for habit persistence further lowers the requirement to risk aversion. Thus, our first results weakly support the C-CAPM. However, we find that the actual price path of stocks deviate from the theoretical perfect foresight price path for long periods of time, using a framework due to Grossman and Shiller (1981)[4]. For this reason, and because of the above-mentioned evidence on risk aversion in the standard specification

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[4] See, however, the discussion in the chapter concerning the doubts about this methodology raised by Kleidon (1986).

of utility, we choose to remain skeptical about the C-CAPM. Engsted and Tanggaard (1999) reach the opposite conclusion using Danish data for the same period. However, this conclusion is probably too hasty because the study applies a money market interest rate (ie., the official discount rate of the Danish Central Bank) instead of a bond interest rate. This is problematic since there is no one-to-one relationship between money and bond market interest rates. Thus, the study by Engsted and Tanggaard does not address the reason for the large difference on bond and equity returns and it is therefore, contrary to what the authors claim, not comparable with the equity premium literature. Further evidence against the C-CAPM is presented by Nielsen and Risager (1997) who show that for horizons of 5 and 10 years the C-CAPM does not even seem to hold in a qualitative sense[5].

I consider it valuable information that the C-CAPM is also hard to reconcile with Danish data. Hence, the equity premium puzzle is not specific to the US. It suggests that aggregate data and the representative consumer assumption should be abandoned to establish an empirical link between consumption and asset returns. In reality, markets are incomplete which causes individual consumption risk to exceed aggregate risk. This was pointed out by Mehra and Prescott as a possible explanation of the large equity premium. Indeed, even though the first results in this area are discouraging, see Heaton and Lucas (1996), it seems highly necessary to build firmer understanding of individual level risk and consumption smoothing to be able to resolve the equity premium puzzle.

1.2.2. The Gordon model

Let us now turn from the general equilibrium C-CAPM to the partial equilibrium Gordon (1962) model which is designed for stock valuation. It assumes that the dividend growth rate and discount rate are constant

[5] Obviously, it is harder to obtain statistical significance at longer horizons since fewer observations are available, but it is striking that the point estimate of covariance with consumption growth in the majority of cases is smaller for stock returns than for bond yields.

and implies that the direct return of stocks (D/P) is rationally set equal to the discount rate adjusted for the growth of dividends. In chapter 4, which is joint work with Jan Overgaard Olesen, a dynamic version of the Gordon model, in which investors each period apply the static valuation rule, is fitted to Danish data. It turns out that two regimes are necessary for the model to be well-specified. Our proxy for time-varying equity premium is significant and its coefficient has the right sign in both regimes, whereas growth-adjusted interest rate is only significant in one regime where its coefficient has model-consistent sign . Furthermore, the level of real dividends and lagged D/P-ratio are found to add explanatory power (the latter is only significant in one regime). Thus, the Gordon model is useful - in certain periods more than in others - but it does not tell the whole story of stock valuation. Having an empirical model like the one in chapter 4, however, is valuable in assessing whether stocks at a given point in time are over- or undervalued compared to the past.

1.3. Behavior and predictability of stock returns

1.3.1. Behavior of returns

In addition to tests of the C-CAPM, chapter 2 also contains calculations which highlight certain interesting features of asset return data. First, stock returns appear to have been larger and more volatile toward the end of the sample (1922-95) than in the first part. This may be associated with a series of capital market liberalizations beginning with common market affiliation in 1972 and continuing through the early 1980s. The issue of returns being generated by multiple regimes is explored in further detail in chapter 3 of this dissertation. Second, chapter 2 shows that stock returns are more volatile than bond yields in the short term, whereas stock returns are *not* more volatile in the long term, see also the stimulating paper by Christiansen and Lystbæk (1994). This result can be explained by weak indications of mean reversion in stock prices which is the topic of the following section.

The second finding led us to argue that pension funds and other institutional investors with long horizons should be allowed to allocate a larger fraction of their funds to stocks than what was feasible under the and implies that the direct return of stocks (D/P) is rationally set equal to the discount rate adjusted for the growth of dividends. In chapter 4, which is joint work with Jan Overgaard Olesen, a dynamic version of the Gordon model, in which investors each period apply the static valuation rule, is fitted to Danish data. It turns out that two regimes are necessary for the model to be well-specified. Our proxy for time-varying equity premium is significant and its coefficient has the right sign in both regimes, whereas growth-adjusted interest rate is only significant in one regime where its coefficient has model-consistent sign. Furthermore, the level of real dividends and lagged D/P-ratio are found to add explanatory power (the latter is only significant in one regime). Thus, the Gordon model is useful - in certain periods more than in others - but it does not tell the whole story of stock valuation. Having an empirical model like the one in chapter 4, however, is valuable in assessing whether stocks at a given point in time are over- or undervalued compared to the past.

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The second finding led us to argue that pension funds and other institutional investors with long horizons should be allowed to allocate a larger fraction of their funds to stocks than what was feasible under the

regulatory framework in place when the first version of our paper was made public. Since then, the upper limit on the fraction of stocks has been increased by policy makers.

1.3.2. Mean reversion

Chapter 2 reports that past stock returns can be used to predict future returns, ie., stock returns display negative serial correlation. This feature is mirrored by the fact that as investment horizon, T, increases, the variance of end of period value of a stock portfolio increases to less than T times the single-period variance[6]. Similar results have been found for a number of countries using various procedures, see for example Poterba and Summers (1988) for evidence on 18 equity markets in different countries. However, results for the US are sensitive to inclusion of the 30s, which was noted by Kim, Nelson and Starz (1991). This is not the case for Denmark, cf. below.

Chapter 2 also presents average 5- and 10-year bond and stock returns and their standard deviations, see table 2.2. An important result is that standard deviations of stock returns decrease as the investment horizon increases. This is partly because the average of observations of a random variable under certain circumstances (e.g. independence) has lower standard deviation the larger the number of observations. But the decline of standard deviations is larger than implied by this effect. The remaining part of the decline is due to mean reversion. Table 1.2 quantifies the importance of mean reversion for standard deviations of real stock returns.

6 Later research by Risager (1998) has shown that correcting for small sample-bias in the variance-ratio test statistics reported in chapter 2 leaves some, although weaker, evidence of mean reversion.

Table 1.2. Standard Deviations (%) of Overlapping 1-, 5-, 10- and 20-Year Average Real Stock Returns with and without Mean Reversion.

	Standard deviation with mean reversion	Standard deviation without mean reversion
1-year	21.4	21.4
5-year	6.3	8.5
10-year	3.6	6.0
20-year	1.8	4.3

The first column corresponds to the information in tables 2.1 and 2.2. The second column, in contrast, displays results of randomly drawing a large number of returns from the sample of annual returns and calculating standard deviations of overlapping 5-, 10- and 20-year returns. Thus, the presence of negative serial correlation is ignored in the calculation of column 2. Comparing the two columns, it is obvious that mean reversion plays a role. For example, mean reversion causes the 10-year standard deviation to be only 60 per cent of its magnitude in the absence of mean reversion.

As mentioned above, chapter 3, which is joint work with Jan Overgaard Olesen, addresses the question of multiple regimes in returns by means of the Markov switching model developed by Hamilton (1990). Two regimes which are characterized by low return - low volatility and high return - high volatility, respectively, are identified. Except for a few, short episodes the period until 1972/beginning of the 1980s belonged to the former regime, whereas the latter part of the sample has been dominated by high returns and high volatility. Furthermore, a new test of mean reversion which allows for regime-shifts is applied. Over the whole sample, the evidence of mean reversion found by standard methods, such as those applied in chapter 2, is weakened by this new test which is consistent with findings of Kim and Nelson (1998) and Kim, Nelson and Starz (1998). However, in the regime, which has dominated recently, mean reversion is strong.

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So far, two alternative explanations have been suggested for the weak evidence of mean reversion stock prices. First, asset markets may be inefficient with prices deviating cyclically from fundamental value. According to this hypothesis, prices are eventually realigned with fundamentals after a long or large deviation. Second, asset markets may still be efficient: if ex ante required returns are positively autocorrelated and mean-reverting and expected dividends are independent, then an innovation to required return has a temporary effect on prices which become mean-reverting. This point was made by Lucas (1978). It has not yet been possible to establish superiority of any of these two competing hypotheses convincingly. I suspect that this is one of those questions that will have to wait a long time for an answer. The reason is that both the source of irrationality and ex ante required returns are unobserved and theoretically ambigous which means that any study of the question can be criticized for not having represented either alternative correctly. For two conflicting views on the implications of mean-reverting stock prices for market efficiency, see Poterba and Summers (1988) and Cecchetti, Lam and Mark (1990).

The implications of mean reversion for the asset allocation of a (sufficiently) risk averse investor are that the proportion of equity should (i) increase with length of horizon, and (ii) be higher (lower) when markets have been bearish (bullish) for some time. Such advice is widely accepted among practitioners but remain controversial among researchers. The controversy is solely on the strength of mean reversion because (i) and (ii) are implied by mean reversion and a certain amount of risk aversion, see Samuelson (1991).

1.3.3. Financial ratios

It has been shown that several financial statement-related variables, including price/earnings (P/E) and book-to-market (B-M), on average can be used to predict cross-sectional returns, see for example Fama and French (1992) and Lakonishok, Shleifer and Vishny (1994). Portfolio selection strategies aimed at exploiting this finding by holding stocks with low P/E, high B-M etc. are called value strategies as opposed to growth strategies. Interestingly, there is a value premium in several other countries than the US, see Fama and French (1998). Furthermore, after controlling for some of the variables mentioned above, CAPM-β has no explanatory power for returns. Thus, apparently the value premium is a robust empirical fact which poses a serious challenge to theory.

Based on a large survey of returns/earnings studies, Lev (1989) concludes that the weak contemporaneous relationship between earnings and returns is possibly due to the low information content (quality) of financial statements. Reasons for low quality include biases of accounting procedures and potential earnings manipulation by management. Hence, Lev argues that further insight into the use of financial information by investors, ie., the process of financial statement analysis, is needed.

Chapter 5 unites these two findings and provides some evidence of the potential of financial statement analysis for predicting stock returns. According to classic security analysis investors are interested in earning power rather than raw accounting earnings for determining the value of a stock, see Graham and Dodd (1934) for the definition of earning power and guidelines for deriving a measure of it from financial statements.

In chapter 5, an adjustment procedure in the spirit of Graham and Dodd is applied to accounting earnings in an attempt to estimate earning power. The most important element of the adjustment procedure is to replace accrued depreciation of tangible assets by average expenditures on new capital assets and on replacement of old assets. It is argued that the latter is more relevant to the investor than the former since this amount is not available for dividend payments. Our sample is the 20 stocks of the main Danish stock index starting in 1990. Using both a portfolio and a regression approach, the value premium is found to be positive if the investment strategy is based on adjusted earnings whereas it is insignificant when the input is unadjusted data. Hence, the value premium increases when the accounting data which are used are corrected for some of its deficiencies.

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Figure 1.1: Cumulative Value of a DKK 100 Investment

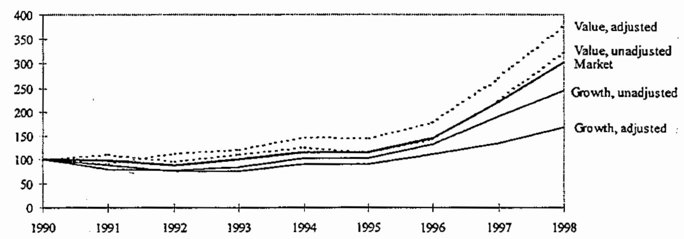

Figure 1.1, which is also found in chapter 5 as figure 5.1, illustrates how various strategies have performed since 1990. Using unadjusted earnings data, a value strategy has outperformed a growth strategy. However, the difference in performance is much larger when basing investments on the adjustment procedure put forward by Graham and Dodd. And the difference is substantial. Return on the value strategy is around 5 times as large as return on the growth strategy.

As already emphasized, these results represent preliminary work in understanding the process of financial statement analysis. There are other candidates for earnings adjustment which may increase the value premium even further. It would also be interesting to consider economically motivated adjustment of the balance sheet-related variables, like B-M, which have also been proved to possess predictive power.

Also note, that these results and results of other studies which find that asset returns are predictable, e.g. mean reversion studies, are based on historical experience. This provides an attractive setting for studying interesting features of financial markets, but conclusions of such studies do not necessarily carry over to the future.

1.4. Equity analyst forecast bias and reputation

Chapter 6 is a theoretical study which is motivated by the strong empirical finding that equity analysts are biased towards buy/hold rather than sell recommendations.

It is sometimes argued that concern about reputation causes equity analysts to reduce their forecast bias. The purpose of the chapter is to examine the scope of reputation as a disciplining force.

A simple model of the interaction of investors and equity analysts is proposed. Analysts are privately informed about the prospects of the stock market and derive profit from trading commisions. Investors are risk averse and choose their stock share in the light of analyst recommendations.

In a one-stage game, the equity analyst has a strong incentive to publish optimistic recommendations. This implies that recommendations are worthless to the investor. In a repeated game, however, analysts need to take into account that misleading investors may harm reputation and, hence, expected future profits. It is shown that analysts are less biased than in the single period game provided the analyst cares sufficiently about the future. However, we also find that concern for reputation does not completely eliminate analyst bias. Hence, reputation reduces bias but does not discipline analysts entirely.

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References

Breeden, D. (1979), An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, *Journal of Financial Economics* 7 (3), pp. 265-96.

Cecchetti, S.G., P-S Lam and N.C. Mark, Mean Reversion in Equilibrium Asset Prices, *American Economic Review* 80 (3), pp. 398-418.

Engsted, T. (1999), Comments on S. Nielsen and O. Risager: Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Engsted, T. and C. Tanggaard (1999), Risikopræmien på danske aktier, *Nationaløkonomisk Tidsskrift* 137 (2), pp. 164-77.

Fama, E.F. and K. R. French (1992), The cross-section of expected stock returns, *Journal of Finance*, vol. 47, no. 2, pp. 427-65.

Fama, E. F. and K. R. French (1998), Value versus growth: The international evidence, *Journal of Finance*, vol. 53, no. 6, pp. 1975-99.

Gordon, M.J. (1962), *Investment, financing and valuation of the Corporation*, Irwin.

Graham, B. and D. Dodd (1934), *Security Analysis*, McGraw-Hill, New York.

Grossman, S.J. and R. Shiller (1981), The Determinants of the Variability of Stock Market Prices, *American Economic Review*, 71(2), pp. 222-7.

Hamilton, J.D., 1990, Analysis of Time Series Subject to Changes in Regime, *Journal of Econometrics* 45, pp. 39-70.

K. Hansen (1974), *Om Afkastet af Danske Aktier i tiden 1900-1974*, Nordisk Fjerfabriks Jubilæumsskrift, Nordisk Fjerfabrik, 191-220.

L.P. Hansen and R. Jagannathan (1991), Implications of Security Market Data for Models of Dynamic Economies, *Journal of Political Economy*, 99, pp. 225-62.

S. Å. Hansen (1974), *Økonomisk vækst i Danmark*, Akademisk Forlag, Copenhagen.

Hansson, B. (1999), Comments on S. Nielsen and O. Risager: Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Heaton, J. and D.J. Lucas (1996), Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing, *Journal of Political Economy* 104 (3), pp. 443-87.

Kim, C.J. and C.R. Nelson (1998), Testing for mean reversion in heteroskedastic data II: Autoregression tests based on Gibbs-sampling-augmented randomization, *Journal of Empirical Finance* 5, pp. 385-96.

Kim, M.J., C.R. Nelson and R. Startz (1991), Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence, *Review of Economic Studies* 58, pp. 515-28.

Kim, M.J., C.R. Nelson and R. Startz (1998), Testing for mean reversion in heteroskedastic data based on Gibbs-sampling-augmented randomization, *Journal of Empirical Finance* 5, pp. 131-54.

Kleidon, A.W. (1986), Variance Bounds Tests and Stock Price Valuation Models, *Journal of Political Economy* 94, pp. 953-1001.

K. Hansen (1974), *Om Afkastet af Danske Aktier i tiden 1900-1974*, Nordisk Fjerfabriks Jubilæumsskrift, Nordisk Fjerfabrik, 191-220.

L.P. Hansen and R. Jagannathan (1991), Implications of Security Market Data for Models of Dynamic Economies, *Journal of Political Economy*, 99, pp. 225-62.

S. Å. Hansen (1974), *Økonomisk vækst i Danmark*, Akademisk Forlag, Copenhagen.

Hansson, B. (1999), Comments on S. Nielsen and O. Risager: Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Heaton, J. and D.J. Lucas (1996), Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing, *Journal of Political Economy* 104 (3), pp. 443-87.

Kim, C.J. and C.R. Nelson (1998), Testing for mean reversion in heteroskedastic data II: Autoregression tests based on Gibbs-sampling-augmented randomization, *Journal of Empirical Finance* 5, pp. 385-96.

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Kleidon, A.W. (1986), Variance Bounds Tests and Stock Price Valuation Models, *Journal of Political Economy* 94, pp. 953-1001.

Lakonishok, J., A. Shleifer and R. W. Vishny (1994), Contrarian investment, extrapolation, and risk, *Journal of Finance*, vol. 49, no. 5, pp. 1541-78.

Lev, B. (1989), On the Usefulness of Earnings and Earnings Research: Lessons and Directions from Two Decades of Empirical Research, *Journal of Accounting Research* 27 (Suppl.), pp. 153-201.

R. E. Jr. Lucas (1978), Asset Prices in an Exchange Economy, *Econometrica*, 46(6), pp. 1429-45.

Mehra, R. and E. C. Prescott (1985), The Equity Premium: A Puzzle, *Journal of Monetary Econmics*, 15, pp. 145-61.

Nielsen, S. and O. Risager (1997), On the Long Run Stock Return Puzzle in Denmark and the C-CAPM, *Working Paper*, Copenhagen Business School (preliminary).

Nielsen, S. and O. Risager (1999), Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Olsen, E. and E. Hoffmeyer (1968), *Dansk Pengehistorie 1914-1960*, Danmarks Nationalbank, Copenhagen.

J.M. Poterba and L.H. Summers (1988), Mean Reversion in Stock Prices - Evidence and Implications, *Journal of Financial Economics*, 22, pp. 27-59.

Risager, O. (1998), Random Walk or Mean Reversion: The Danish Stock Market Since World War I, Working Paper # 12, Economic Policy Research Unit, Copenhagen Business School.

Samuelson, P.A. (1991), Long-Run Risk Tolerance When Equity Returns Are Mean Regressing: Pseudoparadoxes and Vindication of "Businessman's Risk", in *Money, Macroeconomics, and Economic Policy*, W.C. Brainard, W.D. Nordhaus and H.W. Watts (eds), The MIT Press, pp. 181-200.

Samuelson, P.A. (1991), Long-Run Risk Tolerance When Equity Returns Are Mean Regressing: Pseudoparadoxes and Vindication of "Businessman's Risk", in *Money, Macroeconomics, and Economic Policy*, W.C. Brainard, W.D. Nordhaus and H.W. Watts (eds), The MIT Press, pp. 181-200.

Appendix

1.A. Data description[7]

1.A.1. Stock returns

The stock returns are calculated from 1922 on an annual basis; price quotes are end of december. One-year returns express the sum of capital gains over the period and dividends paid during the year as percentage of last year's price.

Capital gains

To construct capital gains the Danish Share Price Index (Totalindekset) is used. This index is published by Statistics Denmark in Statistical Yearbook, Statistiske Efterretninger and Statistisk Månedsoversigt.

The index describes the overall price development of stocks quoted at the Copenhagen Stock Exchange. The sample of companies included in computing the index has gradually expanded from around 50 companies at the beginning in 1921 to all (except mutual funds/investerings-foreninger) listed companies from 1983 onwards.

Stocks enter the index with their official price weighted in proportion to their share of overall market capitalization. Weights are changed at emissions and withdrawals from the exchange.

In the entire period, prices have been corrected to remove the effect of the timing of dividend payments. There has been a change in correction method, though. Until 1983 a standard rate (6 %) of equity value was used for expected dividends. This was changed to share-specific rates based on previous years' dividends.

From 1983 a correction has also been made in case of emissions with price discounts to previous stockholders to make return calculations reflect actual return for the pre-emission investor.

[7] This appendix is extracted from an internal data documentation which was written in collaboration with Jan Overgaard Olesen, Danmarks Nationalbank, and Ole Risager, Department of Economics, Copenhagen Business School.

To complete return calculations, information on dividends is needed. A sample of companies were chosen, see section 1.A.4 of this appendix, and dividend yields were calculated. The average dividend yield in the sample is then viewed as an approximation of market dividend yield.

The sample of companies is listed in section 1.A.4. Note that our sample in any year covers between 50 and 80 % of the total market value of the exchange ("Hovedbørsen" and later "Børs I").

For each company, dividend yield is defined as dividend paid during the calendar year divided by the stock quote at the end of the previous year. Thus, we assume, that dividend payments do not earn interest until the following December. A deduction is made in beginning-of-period price whenever a discounted emission with dividend rights in current year has taken place to capture the fact that in this case the stockholder receives dividends on a larger portfolio. The deduction is made in proportion to the theoretical price drop in response to the emission, ie., the larger the emission and the more undervalued the stock the larger the correction.

Individual companies' dividend yields are finally aggregated by using their share of total market capitalization.

1.A.2. Total stock return

Total aggregate stock return equals the sum of the two components as mentioned earlier. The total return may be underestimated due to the assumption that dividends are not reinvested within the year. This bias may be considerable, in particular until the beginning of the 1980s where the dividend yield plays an important role for the total stock return. To illustrate the bias consider the following simple example:

Over the period 1922-96, the average (arithmetic) dividend yield is 4.7% under the assumption that dividends are not reinvested within the year. The average capital gain equals 6.6%. Thus, the average total return equals 11.3%.

Case 1: Suppose dividends are paid out after 6 months. Suppose shareprices increase "linearly" such that the semiannual increase equals

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Case 1: Suppose dividends are paid out after 6 months. Suppose shareprices increase "linearly" such that the semiannual increase equals

3.25%. In case dividends are reinvested when paid out, the yield associated with dividend payments and reinvestment of these funds equals 4.85%. On top of this we have the pure capital gain equal to 6.6%. Total return is therefore 11.45% or 0.15% more than the estimate without reinvestment of dividends.

Case 2: Suppose dividends are paid out after 3 months. In this case, the quarterly growth rate in stocks equals 1.61%. The bias is 0.20%.

Due to the bias, our total return series is a conservative estimator of the return on the market portfolio of stocks. However, there may be a bias that works in the opposite direction, namely the bankruptcy bias.

Business bankruptcies were widespread in the beginning of the 1920s. The most famous case is the default of Landmandsbanken (the largest bank in Denmark at that time) in 1922. Statistics Denmark constructed two shareprice indeces; one without and one with Landmandsbanken where the latter takes into account the losses associated with the bankruptcy of Landmandsbanken. We use the latter index in the calculation of the capital gain component. On the basis of our own data we have also checked Statistics Denmarks calculation of the fall in the share price from December 1921 to December 1922 (equals 29.1%). By calculating the value-weighted fall in share prices using the 26 shares in our sample for that year and using all available information including the bankruptcy of Landmandsbanken and (partial) bankruptcy of other firms, eg. Superfos, we arrive at exactly the same estimate as Statistics Denmark.

Statistics Denmark does not, however, report how it has dealt with the bankruptcy problem subsequently. Hence, it is possible that the shareprice index is upwards biased (in case there has not been proper adjustments for business failures). Casual evidence suggests that this bias does not exceed the bias associated with the treatment of dividends when abnormal years like 1922 are disregarded.

1.A.3. Bond returns

Three series of effective rates of return on Government bonds are reported. The series represent investments end of December each year from 1921 in bonds with approximately 1, 5 and 10 years to maturity, respectively.

In cases where no bonds with the desired maturity exist, the Government bond that comes closest in maturity is chosen. Consequently, the maturity of the 1-year horizon series is typically in the range from 9 to 12 months. For certain years, in particular before 1945, it has been necessary to deviate from the specified horizon. Thus, the shortest bond used in the calculation of this series has 2½ month to maturity (1941) and the longest has almost 3 years (1973). The 5-year horizon series typically varies from 4 to 6 years. The lowest maturity is 1 year and 7 months and the highest is 10 years and 8 months, both occurring in the thirties where supply of Government bonds were exceptionally low. The typical maturity of the 10-year series is 9 to 11 years, the lowest being 6 years and 9 months (1925) and the highest 14 years and 5 months (1933).

The yields to maturity are calculated on the basis of the price of the bond on the last trading day in December, nominal interest rate and dates of coupon payment. There is taken account of the fact that in trading Danish bonds sellers are paid for accrued interest at the day where trade takes place (Vedhængende rente).

Over a long period of time, it was customary to issue bonds with some redemption each term. In these cases, expected payment streams are used.

The 1-year series is not available in 1922 and 1923. From 1960 observations in the 10-year series are from OECD.

1.A.4. Deflator

In order to deflate nominal rates, inflation in annual average of the Danish Consumer Price Index is used. The index is published by Statistics Denmark in Statistical Yearbook 1996.

The one-year returns on stocks are deflated by dividing 1 plus rate of return by 1 plus yearly inflation and subtracting 1. The return over a given calendar year is deflated by the inflation rate between the average of that year and the previous.

5 and 10 year real rates are defined analogously using 5 and 10 year inflation rates. Those are calculated as index in the sixth year divided by

respectively.

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5 and 10 year real rates are defined analogously using 5 and 10 year inflation rates. Those are calculated as index in the sixth year divided by

the index in the base year to the power of one-fifth.

1.A.5. Consumption

Annual growth rates in per capita real private sector consumption in Denmark are calculated for the period 1922 to 1995. Until 1966, calculations are based on Sv. Aa. Hansen and from 1967 on Statistics Denmark, Statistical Yearbook.

Until 1966, the point of departure is private sector consumption in annual prices. This series is available 1921-1939 and 1947-67, ie., observations are missing during the second world war.

The series is put on real terms by multiplying with the implicit deflator of total consumption which is calculated by dividing total consumption in 1929-prices with total consumption in annual prices.

From 1967, private consumption in 1980-prices is obtained from Statistics Denmark. The pre-and post-1967 series are linked by multiplying the Hansen-series with a constant, which is chosen such that applying the same procedure in 1967 would yield the index value of Statistics Denmark.

The resulting index series is divided by mean total population to get real per capita consumption. The mean is calculated by Hansen until 1970. In the remaining period, mean population has been calculated as a simple average of the population at the beginning of the year and at the beginning of the following year. Growth rates in real per capita consumption are then calculated.

During the war period, growth rates are obtained from the series of total consumption in 1929-prices over mean total population (both from Hansen). Thus, growth in real per capita private consumption is proxied by growth in real per capita total consumption.

Finally, an index of real per capita private consumption is constructed by cumulative multiplication with 1 plus annual growth rate.

1.A.6. Sample of companies

Sample of companies in the calculation of dividend series:

Banks:	Aktivbank
	Amagerbanken
	Amtssparekassen Fyn
	Andelsbanken
	C & G Banken
	Den Danske Bank
	Fyens Disconto Kasse
	Handelsbanken
	Privatbanken
	Provinsbanken
	UniDanmark
	Aarhus Privatbank
Insurance:	Alm Brand B
	Alm Brandass A
	Alm Brandass B
	Baltica
	Codan
	Københavnske Reassurance A
	Københavnske Reassurance B
	Københavnske Reassurance C
Service:	Andersen & Martini
	Sophus Berendsen A
	Sophus Berendsen B
	F L Bie
	Brdr Dahl
	D G Holding B
	Dalhoff Larsen & Horneman
	Det danske Kulkompagni
	Danske Luftfartsselskab
	Ford Motor Co
	Peder P Hedegaard
	ISS A
	ISS B
	Brdr A & O Johansen
	Jydsk Telefon

Banks:
- Aktivbank
- Amagerbanken
- Amtssparekassen Fyn
- Andelsbanken
- C & G Banken
- Den Danske Bank
- Fyens Disconto Kasse
- Handelsbanken
- Privatbanken
- Provinsbanken
- UniDanmark
- Aarhus Privatbank

Insurance:
- Alm Brand B
- Alm Brandass A
- Alm Brandass B
- Baltica
- Codan
- Københavnske Reassurance A
- Københavnske Reassurance B
- Københavnske Reassurance C

Service:
- Andersen & Martini
- Sophus Berendsen A
- Sophus Berendsen B
- F L Bie
- Brdr Dahl
- D G Holding B
- Dalhoff Larsen & Horneman
- Det danske Kulkompagni
- Danske Luftfartsselskab
- Ford Motor Co
- Peder P Hedegaard
- ISS A
- ISS B
- Brdr A & O Johansen
- Jydsk Telefon

- Korn- og Foderstofkompagniet
- Københavns Telefon
- Nesa
- C O Olesen Holding B
- Tivoli A
- Tivoli B
- Wessel og Vett C
- Th Wessel og Vett præference
- Østasiatisk Kompagni
- Østasiatisk Kompagni Holding

Shipping:
- DFDS
- D/S 1912 A
- D/S 1912 B
- D/S Bornholm
- D/S Dannebrog
- D/S Myren
- D/S Norden
- D/S Orient
- D/S Torm
- J Lauritzen
- D/S Svendborg A
- D/S Svendborg B

Industry:
- Albani A
- Albani B
- Ove Arkil
- Atlas
- Bang & Olufsen
- Bing & Grøndahl
- Burmeister & Wain Stamaktier
- Calkas A
- Calkas B
- Cheminova Holding B
- Chemitalic B
- Christiani & Nielsen B
- Coloplast B

CUBIC Modulsystem B
Dancall Radio A
Dancall Radio B
Danisco
Dansk Data Elektronik
Danske Spritfabrikker
Danske Sukkerfabrikker A
Danske Vin- og Konservesfabrikker
Forenede Bryggerier A
Forenede Bryggerier B
Forenede Bryggerier C
Forenede Papirfabrikker
Brdr Hartmann
Incentive
Kastrup Glasværk
Københavns Brødfabrikker
Nordisk Fjerfabrik A
Nordisk Fjerfabrik B
Nordisk Kabel- og Trådfabrikker
Novo Industri
C W Obel B
Royal Copenhagen A
Royal Copenhagen B
Schouw & Co A
Schouw & Co B
F L Smidt A
F L Smidt B
Superfos
Superfos præference
Thrige-Titan A
Thrige-Titan B
Aarhus Oliefabrik A
Aarhus Oliefabrik B

CUBIC Modulsystem B
Dancall Radio A
Dancall Radio B
Danisco
Dansk Data Elektronik
Danske Spritfabrikker
Danske Sukkerfabrikker A
Danske Vin- og Konservesfabrikker
Forenede Bryggerier A
Forenede Bryggerier B
Forenede Bryggerier C
Forenede Papirfabrikker
Brdr Hartmann
Incentive
Kastrup Glasværk
Københavns Brødfabrikker
Nordisk Fjerfabrik A
Nordisk Fjerfabrik B
Nordisk Kabel- og Trådfabrikker
Novo Industri
C W Obel B
Royal Copenhagen A
Royal Copenhagen B
Schouw & Co A
Schouw & Co B
F L Smidt A
F L Smidt B
Superfos
Superfos præference
Thrige-Titan A
Thrige-Titan B
Aarhus Oliefabrik A
Aarhus Oliefabrik B

1.A.7. Data

Year	Dividend yield	Nominal Stock Return	Real Stock Return	Real Consumption Index (1921=100)
1922	0.0808	-0.2098	-0.0699	105.92
1923	0.0804	0.5564	0.4941	120.89
1924	0.0668	-0.0242	-0.0794	116.45
1925	0.0715	0.1817	0.2161	109.96
1926	0.0530	-0.0198	0.1538	111.54
1927	0.0595	0.1588	0.1999	116.33
1928	0.0610	0.0283	0.0344	121.04
1929	0.0586	0.0904	0.0969	125.44
1930	0.0607	-0.0168	0.0326	132.34
1931	0.0582	-0.1584	-0.1079	134.72
1932	0.0562	0.0342	0.0412	132.30
1933	0.0528	0.3482	0.3129	137.09
1934	0.0515	0.1503	0.1069	137.19
1935	0.0506	0.0792	0.0400	133.95
1936	0.0523	0.1683	0.1543	139.83
1937	0.0532	-0.0359	-0.0694	141.54
1938	0.0602	0.0451	0.0331	144.01
1939	0.0596	-0.0002	-0.0280	146.03
1940	0.0585	0.0585	-0.1494	114.98
1941	0.0445	0.2445	0.0847	101.38
1942	0.0391	0.0562	0.0205	104.47

Year	Dividend yield	Nominal Stock Return	Real Stock Return	Real Consumption Index (1921=100)
1943	0.0348	0.1777	0.1689	103.77
1944	0.0334	0.0040	-0.0180	107.98
1945	0.0307	-0.0375	-0.0480	118.53
1946	0.0416	0.0904	0.0983	148.89
1947	0.0450	0.0062	-0.0222	152.72
1948	0.0457	-0.0511	-0.0740	151.16
1949	0.0520	0.1055	0.0795	157.80
1950	0.0515	0.0939	0.0027	168.76
1951	0.0507	-0.0225	-0.1251	168.53
1952	0.0606	0.0693	0.0462	165.21
1953	0.0623	0.0971	0.1030	168.01
1954	0.0629	0.1301	0.1090	174.93
1955	0.0705	0.2280	0.1512	174.17
1956	0.0561	0.1616	0.1063	175.25
1957	0.0526	-0.0754	-0.0862	177.72
1958	0.0551	0.2477	0.2261	180.74
1959	0.0501	0.0898	0.0674	185.76
1960	0.0434	0.0515	0.0280	196.13
1961	0.0448	0.0288	-0.0151	206.29
1962	0.0498	0.0742	0.0075	210.20
1963	0.0490	0.1522	0.0951	214.38
1964	0.0449	0.1024	0.0639	223.98

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1964	0.0449	0.1024	0.0639	223.98

Year	Dividend yield	Nominal Stock Return	Real Stock Return	Real Consumption Index (1921=100)
1965	0.0437	0.1127	0.0454	227.48
1966	0.0493	0.0171	-0.0471	232.35
1967	0.0491	-0.0509	-0.1166	240.98
1968	0.0517	0.1628	0.0769	244.00
1969	0.0496	0.0580	0.0223	258.89
1970	0.0526	-0.0466	-0.1057	265.60
1971	0.0612	0.0337	-0.0231	260.85
1972	0.0642	0.9510	0.8307	263.68
1973	0.0376	0.0376	-0.0507	274.93
1974	0.0430	-0.1697	-0.2792	266.59
1975	0.0556	0.3934	0.2712	275.23
1976	0.0437	0.0437	-0.0426	296.13
1977	0.0486	0.0385	-0.0659	298.40
1978	0.0521	-0.0091	-0.1991	299.69
1979	0.0622	-0.0030	-0.0906	303.10
1980	0.0687	0.1908	0.0600	291.59
1981	0.0635	0.4583	0.3059	284.91
1982	0.0581	0.1817	0.0728	289.10
1983	0.0385	1.1785	1.0375	296.56
1984	0.0171	-0.2025	-0.2497	306.47
1985	0.0286	0.4598	0.3937	322.25
1986	0.0202	-0.1722	-0.2011	334.94

Year	Dividend yield	Nominal Stock Return	Real Stock Return	Real Consumption Index (1921=100)
1987	0.0290	-0.0280	-0.0655	330.53
1988	0.0293	0.5238	0.4571	326.60
1989	0.0137	0.3482	0.2869	329.66
1990	0.0109	-0.1213	-0.1436	331.27
1991	0.0125	0.1331	0.1063	334.54
1992	0.0149	-0.2429	-0.2585	337.36
1993	0.0130	0.4099	0.3929	344.21
1994	0.0102	-0.0362	-0.0554	365.77
1995	0.0139	0.0626	0.0408	372.34
1996	0.0150	0.3047	0.2778	379.92

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1922	n.a.	0.0531	0.0499
1923	n.a.	0.0638	0.0562
1924	0.0692	0.0736	0.0643
1925	0.0506	0.0595	0.0586
1926	0.0728	0.0598	0.0595
1927	0.0564	0.0563	0.0570

Year	Dividend yield	Nominal Stock Return	Real Stock Return	Real Consumption Index (1921=100)
1987	0.0290	-0.0280	-0.0655	330.53
1988	0.0293	0.5238	0.4571	326.60
1989	0.0137	0.3482	0.2869	329.66
1990	0.0109	-0.1213	-0.1436	331.27
1991	0.0125	0.1331	0.1063	334.54
1992	0.0149	-0.2429	-0.2585	337.36
1993	0.0130	0.4099	0.3929	344.21
1994	0.0102	-0.0362	-0.0554	365.77
1995	0.0139	0.0626	0.0408	372.34
1996	0.0150	0.3047	0.2778	379.92

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1922	n.a.	0.0531	0.0499
1923	n.a.	0.0638	0.0562
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1925	0.0506	0.0595	0.0586
1926	0.0728	0.0598	0.0595
1927	0.0564	0.0563	0.0570

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1928	0.0506	0.0518	0.0537
1929	0.0520	0.0521	0.0534
1930	0.0445	0.0450	0.0507
1931	0.0635	0.0678	0.0620
1932	0.0368	0.0427	0.0500
1933	0.0275	0.0418	0.0450
1934	0.0333	0.0406	0.0456
1935	0.0404	0.0504	0.0505
1936	0.0454	0.0543	0.0526
1937	0.0461	0.0516	0.0512
1938	0.0373	0.0435	0.0513
1939	0.0499	0.0597	0.0597
1940	0.0107	0.0469	0.0469
1941	0.0248	0.0466	0.0459
1942	0.0190	0.0319	0.0358
1943	0.0108	0.0208	0.0312
1944	0.0108	0.0307	0.0330
1945	0.0189	0.0274	0.0280
1946	0.0142	0.0258	0.0315
1947	0.0189	0.0445	0.0205
1948	0.0428	0.0374	0.0238
1949	0.0322	0.0333	0.0442

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1950	0.0459	0.0450	0.0474
1951	0.0363	0.0560	0.0657
1952	0.0468	0.0516	0.0586
1953	0.0688	0.0510	0.0564
1954	0.0589	0.0718	0.0699
1955	0.0549	0.0668	0.0711
1956	0.0591	0.0745	0.0682
1957	0.0499	0.0675	0.0694
1958	0.0475	0.0461	0.0504
1959	0.0465	0.0609	0.0369
1960	0.0570	0.0662	0.0630
1961	0.0584	0.0720	0.0690
1962	0.0704	0.0687	0.0686
1963	0.0670	0.0627	0.0674
1964	0.0845	0.0746	0.0738
1965	0.0858	0.0875	0.0900
1966	0.1020	0.1078	0.0912
1967	0.1153	0.1137	0.0946
1968	0.0939	0.1075	0.0906
1969	0.1197	0.1000	0.1009
1970	0.1378	0.1270	0.1158
1971	0.0951	0.1009	0.1143

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1950	0.0459	0.0450	0.0474
1951	0.0363	0.0560	0.0657
1952	0.0468	0.0516	0.0586
1953	0.0688	0.0510	0.0564
1954	0.0589	0.0718	0.0699
1955	0.0549	0.0668	0.0711
1956	0.0591	0.0745	0.0682
1957	0.0499	0.0675	0.0694
1958	0.0475	0.0461	0.0504
1959	0.0465	0.0609	0.0369
1960	0.0570	0.0662	0.0630
1961	0.0584	0.0720	0.0690
1962	0.0704	0.0687	0.0686
1963	0.0670	0.0627	0.0674
1964	0.0845	0.0746	0.0738
1965	0.0858	0.0875	0.0900
1966	0.1020	0.1078	0.0912
1967	0.1153	0.1137	0.0946
1968	0.0939	0.1075	0.0906
1969	0.1197	0.1000	0.1009
1970	0.1378	0.1270	0.1158
1971	0.0951	0.1009	0.1143

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1972	0.1202	0.0935	0.1149
1973	0.1123	0.1079	0.1311
1974	0.1629	0.1292	0.1654
1975	0.0905	0.1093	0.1327
1976	0.1706	0.1697	0.1558
1977	0.1706	0.1733	0.1700
1978	0.1673	0.1565	0.1823
1979	0.1739	0.1785	0.1817
1980	0.1648	0.1889	0.1998
1981	0.1711	0.1926	0.2013
1982	0.1839	0.1935	0.2136
1983	0.1197	0.1254	0.1507
1984	0.1246	0.1368	0.1450
1985	0.0872	0.0939	0.1164
1986	0.0997	0.1117	0.1010
1987	0.1026	0.1009	0.1134
1988	0.0821	0.0898	0.0960
1989	0.1126	0.1056	0.0977
1990	0.1075	0.1071	0.1058
1991	0.1006	0.0908	0.0925
1992	0.1089	0.0964	0.0891
1993	0.0622	0.0571	0.0717

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1994	0.0711	0.0877	0.0794
1995	0.0464	0.0626	0.0825
1996	0.0341	0.0534	0.0710

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1922	n.a.	0.0803	0.0769
1923	n.a.	0.1015	0.0849
1924	0.0087	0.1261	0.0953
1925	0.0811	0.1158	0.0823
1926	0.2628	0.0930	0.0645
1927	0.0939	0.0832	0.0545
1928	0.0569	0.0717	0.0495
1929	0.0583	0.0624	0.0456
1930	0.0970	0.0373	0.0153
1931	0.1273	0.0451	0.0064
1932	0.0438	0.0120	-0.0092
1933	0.0007	0.0142	-0.0120
1934	-0.0057	0.0151	-0.0098

Year	1-Year Nominal Bond Yield	5-Year Nominal Bond Yield	10-Year Nominal Bond Yield
1994	0.0711	0.0877	0.0794
1995	0.0464	0.0626	0.0825
1996	0.0341	0.0534	0.0710

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1922	n.a.	0.0803	0.0769
1923	n.a.	0.1015	0.0849
1924	0.0087	0.1261	0.0953
1925	0.0811	0.1158	0.0823
1926	0.2628	0.0930	0.0645
1927	0.0939	0.0832	0.0545
1928	0.0569	0.0717	0.0495
1929	0.0583	0.0624	0.0456
1930	0.0970	0.0373	0.0153
1931	0.1273	0.0451	0.0064
1932	0.0438	0.0120	-0.0092
1933	0.0007	0.0142	-0.0120
1934	-0.0057	0.0151	-0.0098

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1935	0.0025	-0.0119	-0.0025
1936	0.0329	-0.0328	0.0014
1937	0.0099	-0.0351	0.0007
1938	0.0255	-0.0418	-0.0005
1939	0.0207	-0.0257	0.0079
1940	-0.1878	0.0034	0.0090
1941	-0.1068	0.0325	0.0107
1942	-0.0154	0.0191	0.0022
1943	0.0032	0.0048	-0.0010
1944	-0.0114	0.0143	0.0011
1945	0.0079	-0.0043	-0.0091
1946	0.0216	-0.0290	-0.0113
1947	-0.0099	-0.0100	-0.0202
1948	0.0176	-0.0109	-0.0155
1949	0.0079	-0.0138	0.0044
1950	-0.0413	0.0019	0.0140
1951	-0.0725	0.0251	0.0387
1952	0.0242	0.0228	0.0274
1953	0.0746	0.0193	0.0195
1954	0.0392	0.0390	0.0308
1955	-0.0110	0.0429	0.0322
1956	0.0086	0.0515	0.0277

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1957	0.0376	0.0338	0.0228
1958	0.0377	0.0047	-0.0022
1959	0.0249	0.0159	-0.0163
1960	0.0334	0.0129	0.0043
1961	0.0132	0.0140	0.0086
1962	0.0039	0.0094	0.0083
1963	0.0141	-0.0015	0.0033
1964	0.0466	0.0099	-0.0012
1965	0.0202	0.0217	0.0108
1966	0.0324	0.0426	0.0098
1967	0.0380	0.0498	0.0095
1968	0.0130	0.0415	0.0039
1969	0.0820	0.0125	0.0076
1970	0.0673	0.0316	0.0159
1971	0.0349	0.0017	0.0091
1972	0.0511	-0.0134	0.0063
1973	0.0176	-0.0016	0.0232
1974	0.0096	0.0276	0.0627
1975	-0.0052	0.0046	0.0376
1976	0.0738	0.0542	0.0641
1977	0.0529	0.0594	0.0844
1978	0.0613	0.0502	0.1013

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1957	0.0376	0.0338	0.0228
1958	0.0377	0.0047	-0.0022
1959	0.0249	0.0159	-0.0163
1960	0.0334	0.0129	0.0043
1961	0.0132	0.0140	0.0086
1962	0.0039	0.0094	0.0083
1963	0.0141	-0.0015	0.0033
1964	0.0466	0.0099	-0.0012
1965	0.0202	0.0217	0.0108
1966	0.0324	0.0426	0.0098
1967	0.0380	0.0498	0.0095
1968	0.0130	0.0415	0.0039
1969	0.0820	0.0125	0.0076
1970	0.0673	0.0316	0.0159
1971	0.0349	0.0017	0.0091
1972	0.0511	-0.0134	0.0063
1973	0.0176	-0.0016	0.0232
1974	0.0096	0.0276	0.0627
1975	-0.0052	0.0046	0.0376
1976	0.0738	0.0542	0.0641
1977	0.0529	0.0594	0.0844
1978	0.0613	0.0502	0.1013

Year	1-Year Real Bond Yield	5-Year Real Bond Yield	10-Year Real Bond Yield
1979	0.0708	0.0768	0.1058
1980	0.0369	0.1016	0.1330
1981	0.0486	0.1217	0.1442
1982	0.0748	0.1355	0.1647
1983	0.0473	0.0755	0.1105
1984	0.0581	0.0895	0.1095
1985	0.0379	0.0527	0.0845
1986	0.0614	0.0723	0.0712
1987	0.0601	0.0658	0.0851
1988	0.0347	0.0620	0.0709
1989	0.0619	0.0831	0.0750
1990	0.0794	0.0857	n.a.
1991	0.0746	0.0704	n.a.
1992	0.0861	0.0757	n.a.
1993	0.0493	0.0358	n.a.
1994	0.0498	0.0648	n.a.
1995	0.0249	n.a.	n.a.
1996	0.0128	n.a.	n.a.

Chapter 2

Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War[1]

with Ole Risager[2]

2.1. Introduction

The purpose of this chapter is first to characterize the return-risk characteristics of Danish stocks and bonds in the period 1920-95. On the basis of the descriptive background we analyze whether the size and development of asset returns can be explained by the Consumption-CAPM, which has become a popular asset pricing model in recent years.

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1 The paper on which this chapter is based has been presented at seminars at the MBA Programme and the Institute of Economics, Copenhagen Business School, and at the conference 'Macroeconomic Perspectives on the Danish Economy', Hornbæk, 19-20 June 1997. We wish to thank Syed M. Ahsan, Copenhagen Business School; Tom Engsted, Aarhus Business School; Lars Lund, Copenhagen Business School; Jan Overgaard Olesen, Copenhagen Business School; Bjorn Hansson, University of Lund and Paolo Pesenti, Princeton University for useful comments and suggestions. We have also benefitted from discussions with Henrik W. Mogensen, Tryg-Baltica. Finally, thanks to Ian Valsted and Michael Wieman for efficient research assistance.

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We also briefly discuss whether the existing upper limits on Danish pension funds' stock investments are reasonable in view of our results on the return-risk characteristics of Danish stocks and bonds. Throughout the chapter we focus on the economics of the stock market rather than on technicalities.

Section 2.2 begins by calculating the return on a 1-year, 5-year and 10-year investment in the market portfolio of stocks, and the associated risk as measured by the standard deviation of the portfolios. Subsequently, the chapter outlines the corresponding government bond yields and risks. Next follows a comparison of stock and bond returns - that is, a characterization of the equity premium and its development since the early 1920s. We also examine whether stock investment is more risky than bond investment, which is a commonly held view. In a Danish context, this view has been attacked by Christiansen and Lystbæk (1994). Our results show that the time horizon is the crucial factor in this issue. In simple terms, stock investments are more risky than bond investments in the short term, whereas stock investments are not more risky than bond investments in the long term. The most important reason why stocks are as safe as bonds in the long term has to do with a strong tendency for real stock returns to mean-revert. Thus, bad years in the stock market are usually followed by good years, whereas bond returns display positive autocorrelation. The results for Denmark are therefore similar to the findings for the United States reported in Siegel (1994). On the basis of stock and bond returns for the last two centuries, Siegel arrives at the conclusion that 'although stocks are certainly riskier than bonds in the short run, over the long run the returns on stocks are so stable that stocks are actually safer than either government bonds or Treasury bills'.

Section 2.3 goes on to examine whether the behaviour of stock returns can be explained by the Consumption-CAPM (see Breeden, 1979, and Lucas, 1978). In line with the majority of the papers in the literature, this paper analyzes short-run stock returns, whereas Nielsen and Risager (1997) looks at long-run stock returns. According to the Consumption-CAPM, stocks should yield a higher return than bonds if stock returns are more correlated with consumption than bond yields, because stocks in that case provide a poorer hedge against fluctuations in consumption. The predictions of this model are consistent with the

Danish data in the qualitative sense. Whether the model makes sense quantitatively is another issue, to which we return. The Consumption-CAPM (C-CAPM) is also consistent with data for the United States at the qualitative level, but the model is unable quantitatively to explain the magnitude of the United States equity premium unless it is assumed that agents are much more risk averse than what is commonly believed. That was first demonstrated by Mehra and Prescott (1985) (see also the recent survey of the so-called 'equity premium puzzle'-literature by Kocherlakota, 1996).

Tests of the Consumption-CAPM on Danish data are scarce. As far as we know, Lund and Engsted (1996) is the first paper that investigated this issue. They use the VAR technique developed by Campbell and Shiller (1988) to analyze the behaviour of short-run stock returns, and to estimate the underlying parameters (eg. the degree of risk aversion). Their estimate of the risk aversion parameter 'turn out to be of the wrong sign, but with large standard errors, so that the hypothesis of risk neutrality cannot be rejected'. In this chapter, we apply the non-parametric approach due to Hansen and Jagannathan (1991). This gives insight into the likely degree of risk aversion. Our results point attention to a degree of risk aversion that seems reasonable in the short end of the market. In spite of that, it would be premature to conclude that the theory can explain market returns. We therefore proceed to examine another aspect of the model.

Thus, in the spirit of the influential paper by Grossman and Shiller (1981), we compare the actual stock market index with the index that would apply if agents had perfect foresight and behaved in accordance with the Consumption-CAPM, using the information we have on the likely degree of risk aversion. Despite the fact that this test is informal and based on the assumption of rational expectations, it gives insight into the model's ability to explain the level and volatility of stock prices.

Section 2.4 discusses the upper limits on Danish pension funds' investments in the stock market, whereas Section 2.5 briefly summarizes the most important conclusions and implications that can be drawn from the chapter.

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Section 2.4 discusses the upper limits on Danish pension funds' investments in the stock market, whereas Section 2.5 briefly summarizes the most important conclusions and implications that can be drawn from the chapter.

2.2. Data, return definitions and view of the landscape

The stock market data are from two sources. Dividend yields are from our own sample of the listed firms on Copenhagen Stock Exchange, which covers about 70 per cent of the total market capitalization, corresponding to a total of about 100 firms. The dividend yield on the market portfolio is estimated as a weighted average of the dividend yield on each share, where the weights equal the value of each stock relative to the total market capitalization of the the firms in the sample. Capital gains are calculated on the basis of the market index published by Statistics Denmark[3]. The stock returns presented below therefore refer to the market portfolio.

Let us now introduce a few return definitions. The 1-year nominal gross return on stocks, *S1*, is defined in (2.1), and equals the dividend yield *plus* the capital gain (see list of notation below). This calculation disregards the possibility that dividends are reinvested within the year they are paid out.[4] The corresponding 1-year real return, *SR1*, seen from an investor's point of view is given in (2.2), and is only approximately equal to the more common but *less* exact definition of the real return given as the nominal return less the rate of CPI inflation.[5] The 5-year nominal (real) return equals the geometric average of the consecutive annual nominal (real) returns. The formula for the 5-year nominal return, *S5*, is thus given by (2.3). The formula for the 5-year real return, *SR5*, is defined analogously and is therefore omitted.

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3 The firms in our portfolio are not the same as those in the market index of Statistics Denmark. Thus, the indices should be viewed as estimates of market dividend yield and market index, respectively.

4 Frennberg and Hansson (1992) show that this is of very little importance for the calculation of Swedish stock returns over the period 1919-89.

5 The difference between the exact and the approximate definition can be large when returns are fairly large, which is not uncommon in stock markets.

$$SI(t) = D(t)/Q(t-1) + (Q(t)-Q(t-1))/Q(t-1) \quad (2.1)$$

$$1 + SRI(t) = (1+SI(t))C(t-1)/C(t) \quad (2.2)$$

$$(1+S5(t))^5 = (1+SI(t)) \ldots (1+S(t+4)) \quad (2.3)$$

In the above formulas, D(t) denotes dividends from time t-1 to t, Q(t) and C(t) are the stock market index and the CPI at time t, respectively.

Stock returns are compared to 5- and 10-year (annualized) government bond yields, ie., bond yields over a certain horizon are an approximation of holding period return. Thus, it is assumed that the yield curve is horizontal. In the absence of any publicly available 5-year bond yield (and 10-year yield) we have had to construct our own series using the available information on payments (amortization) streams. Because the maturity structure on the outstanding Government debt is narrow in particular in the beginning of the sample, the 5-year horizon is only approximate in the early period of the sample. The 10-year yield to maturity, *B10*, is also approximate in the early years. From 1960 and onwards we link our 10-year bond yield with OECDs series, see OECD (1996). The 5-year real bond yield, *BR5*, is proxied by,

$$(1 + BR5(t))^5 = (1+B5(t))^5(C(t-1)/C(t+4)) \quad (2.4)$$

where B5(t) is the 5-year annualized nominal bond yield. The 10-year real bond yield BR10 is defined analogously.

2.2.1. Short-term stock returns, risk and wealth effects

The movement of the annual nominal stock market return is illustrated in Figure 2.1. The figure shows that the annual nominal return has fluctuated in a relatively stable manner around a constant mean in the period 1922-82 with the high yield in 1972 as a clear outlier. In the period 1983-95 both the mean return and the variance of stock returns have increased substantially. These observations are confirmed by simple summary statistics listed in Table 2.1.

$$S1(t) = D(t)/Q(t-1) + (Q(t)-Q(t-1))/Q(t-1) \quad (2.1)$$

$$1 + SR1(t) = (1+S1(t))C(t-1)/C(t) \quad (2.2)$$

$$(1+S5(t))^5 = (1+S1(t)) \dots (1+S(t+4)) \quad (2.3)$$

In the above formulas, D(t) denotes dividends from time t-1 to t, Q(t) and C(t) are the stock market index and the CPI at time t, respectively.

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The movement of the annual nominal stock market return is illustrated in Figure 2.1. The figure shows that the annual nominal return has fluctuated in a relatively stable manner around a constant mean in the period 1922-82 with the high yield in 1972 as a clear outlier. In the period 1983-95 both the mean return and the variance of stock returns have increased substantially. These observations are confirmed by simple summary statistics listed in Table 2.1.

Fig. 2.1: Annual Nominal Stock Returns, 1922-95

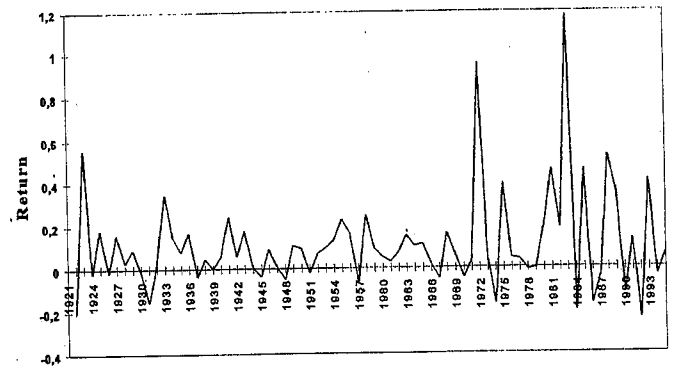

Table 2.1. Average Annual Stock Market Returns, 1922-95, Per Cent

	Nominal return	Standard deviation	Real return	Standard deviation
1922-95	11.0	22.9	6.8	21.4
1922-82	9.6	17.5	5.3	16.4
1983-95	17.8	40.0	13.4	37.1

Source: Own calculations.

The average nominal return has thus increased from 9.6 per cent in 1922-82 to 17.8 per cent in 1983-95. Figures for 1996 underscore this tendency to higher stock returns. Along with a rise in the average return, Table 2.1 also shows that the standard deviation of the annual return has more than doubled. The stock market has thus become more volatile in the short term. It is, however, interesting to note that the return-risk ratio in real terms - defined as the real mean return divided by the associated standard deviation - has increased in the period 1983-95.

The observation that returns were lower in the past is consistent with the sample evidence in K. Hansen (1974). He finds that the average annual return equals 7.6 per cent in the period 1920-74.

The corresponding real returns display roughly the same behaviour as the nominal returns and the figure is therefore omitted to save space. Spectacular returns are recorded in 1972 (83 per cent) and in 1983 (104 per cent), to which we return later. The bad years are 1931 (-11 per cent), 1940 (-15 per cent), 1974 (-28 per cent), 1984 (-25 per cent), and 1992 (-24 per cent). The sharp declines in 1974 and 1984 followed immediately after spectacular bull markets, indicating that the Danish stock market may also overreact to news - that is, display excess volatility.

Figure 2.2 presents the corresponding real stock market index, defined as the nominal index deflated by the CPI. The 1920s are characterized by an upward trend but also with considerable declines in 1922 and 1924, which in part reflects a tough monetary policy that aimed at restoring the real value of the exchange rate. Thus, the consumer price index fell by 20 per cent from 1920-4, which was accompanied by several collapses of major banks and industrial companies (see Olsen and Hoffmeyer, 1968). The Wall Street crash in 1929 is associated with a minor fall in the Danish index in 1930. The major adjustment occurs in 1931, where the index went down by 17 per cent. The stock market recovery sets in immediately after the crash, and the stock market reaches a new peak in 1936. Thereafter, the stock market displays a remarkable trendwise decline until the beginning of the 1980s. The only major interruption to this decline is 1972, which is the year when Denmark joined the EEC (now the EU). Along with this, the Copenhagen Stock Exchange was opened up for foreign investors in 1973, which in turn may have been anticipated by the market. The declining index contributed of course negatively to stock returns, but high dividend yields in this period kept them on a positive scale. Following the long period with a declining index, Figure 3.2 shows a very dramatic rise in 1983. In this year, the stock market goes up by exactly 100 per cent. In 1987, stock markets were world-wide characterized by steep declines, and the Danish index is no exception as the real stock price falls by 9 per cent, but that is immediately followed the short term. It is, however, interesting to note that the return-risk ratio in real terms - defined as the real mean return divided by the associated standard deviation - has increased in the period 1983-95.

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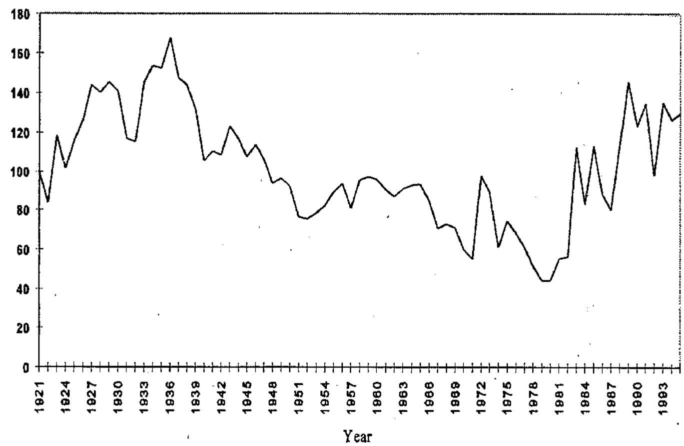
The sharp increase in the index in 1983 is the largest jump that has occurred since World War I. It is common to attribute the 1983 jump to three factors. First, there is a shift in economic policy in late 1982 towards a non-accommodation strategy with tight fiscal policy and fixed exchange rate policy as key elements (see Andersen and Risager, 1988). This change in economic policy was accompanied by a fall in the long interest rate by around 7 percentage points (see section 2.2.2). Second, a new tax on institutional investors' real bond yields was passed by Parliament in 1983 to become effective in 1984. Because stocks are not subject to this tax, the current and anticipated after tax return to equity increased sharply relative to bond investments. Third, capital market liberalizations meant that all restrictions on Danish investors foreign equity placements were lifted in 1984, whereas the Danish stock market had been open to foreigners since 1973 (see Eskesen et al., 1984). The liberalizations may have enlarged the window to the rest of the World, and because many markets experienced very high returns in these years this may have had spillover effects to the Copenhagen Stock Exchange. Note, however, that there has not been any formal attempt to test and quantify the various explanations.

The stock market jump in 1983 led to a considerable wealth effect despite the fact that the Danish stock market is relatively small by international comparison, and in particular by comparison to countries with an Anglo-Saxon financing mode. As the total capitalization-GDP ratio in 1983 was about 20 per cent, the boom was associated with a wealth increase of about 20 per cent of GDP. Measured in current Danish Kroner that amounts to about 200 billion kr. That is obviously a substantial wealth increase which, however, is partly reversed in 1984. Note also that the windfall gain is 'gross'. However, as capital gains are tax free in this period for minority shareholders, provided the holding period is at least three years, the net-wealth increase was substantial. It is an important future research topic to find out whether the sharp rise in stock prices fuelled the subsequent rise in consumption and investment (as suggested by, for instance, Tobin's q-model), or whether it just mirrored the considerable rise in consumption and sales that occurred in the period 1984-6 along with the substantial fall in interest rates in 1983. Whether the market mainly acts as a leading indicator or actually also has important spillover effects to the real economy is discussed in Poterba and Samwick (1995), for example, on the basis of data for the United States.

Fig. 2.2: Real Stock Price Index, 1921-95



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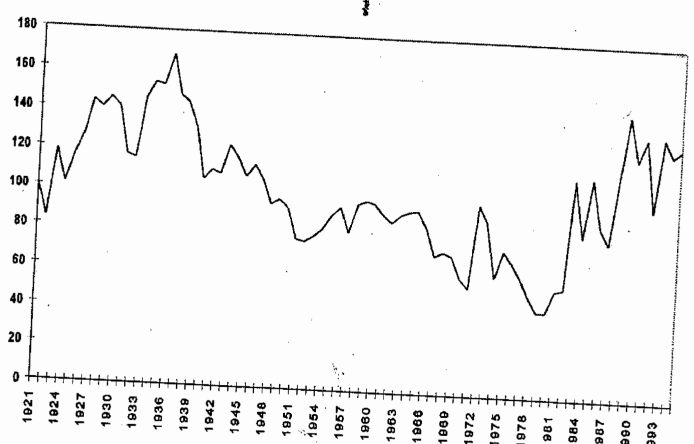


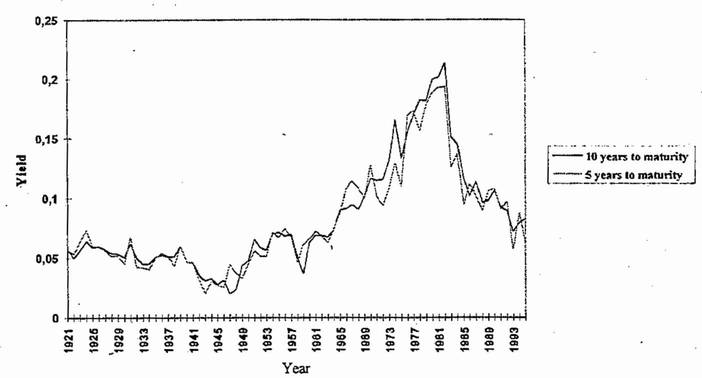
2.2.2. Long-term stock and bond investment: Return-risk characteristics.

Because stock investments often are (or should be) based on long-term considerations, this section focuses mainly on the return-risk characteristics of stocks relative to bonds in a long-term perspective, that is, over a 5- and 10-year horizon.

Our 5- and 10-year bond yields are recorded in Figure 2.3. This diagram shows that there has been very little return difference between 5- and 10-year bond investments. Moreover, bond yields appear to be highly positively correlated over time. In contrast to the Stock Market where bad(good) years quickly are followed by good(bad) years (see Figure 2.1), rising (falling) bond returns often persist over many years even through decades.

In order to compare bond and stock investment, which indeed is a complex issue, we need to make some assumptions about investor behaviour. Thus, our bond investor is assumed repeatedly to invest in government bonds. Suppose the investor chooses bonds with maturity equal to the investor's predetermined holding period. The return on a single investment is given by the yield to maturity on the bond. The

Fig. 2.3: Nominal Bond Yields, 1921-95



Source: Own calculations.

average return over the period 1922-95 is the arithmetic average of all the effective interest rates that have applied in this period. As the investor is uncertain with respect to future interest rates, such an investment strategy is risky even though the investor knows his nominal return for certain as soon as he has made his investment. If the investment was a one-shot event all nominal uncertainty of course disappears. The sort of uncertainty that stems from inflation will still be there given that the bond is a nominal claim. In sum, our perspective is an investor who repeatedly invests in nominal government bonds and hence is subject to both interest rate and inflation uncertainty. Table 2.2 shows the average bond yield over a 5- and 10-year investment horizon as well as the average 5- and 10-year stock returns (cf. (2.3)). The table shows that the average real return on a 5- and 10-year bond investment strategy equals 3.6 per cent and 3.3 per cent, respectively. The corresponding real stock returns are 5.1 per cent and 4.7 per cent, respectively. Hence, stocks yield on average a higher return than bonds.

Table 2.2. Average 5- and 10-Year Bond and Stock Returns in Per Cent, 1922-95

	Nominal return	Standard deviation	Real return	Standard deviation
5-year bond	8.0	4.2	3.6	4.2
5-year stock	9.5	7.0	5.1	6.3
10-year bond	8.3	4.6	3.3	4.4
10-year stock	9.6	4.5	4.7	3.6

Source: Own calculations.

The standard deviations of the 5- and 10-year stock returns are also presented in Table 2.2. The standard deviations decline as the average return over the period 1922-95 is the arithmetic average of all the effective interest rates that have applied in this period. As the investor is uncertain with respect to future interest rates, such an investment strategy is risky even though the investor knows his nominal return for certain as soon as he has made his investment. If the investment was a one-shot event all nominal uncertainty of course disappears. The sort of uncertainty that stems from inflation will still be there given that the bond is a nominal claim. In sum, our perspective is an investor who repeatedly invests in nominal government bonds and hence is subject to both interest rate and inflation uncertainty. Table 2.2 shows the average bond yield over a 5- and 10-year investment horizon as well as the average 5- and 10-year stock returns (cf. (2.3)). The table shows that the average real return on a 5- and 10-year bond investment strategy equals 3.6 per cent and 3.3 per cent, respectively. The corresponding real stock returns are 5.1 per cent and 4.7 per cent, respectively. Hence, stocks yield on average a higher return than bonds.

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Source: Own calculations.

The standard deviations of the 5- and 10-year stock returns are also presented in Table 2.2. The standard deviations decline as the

investment horizon increases. As the return on long-term investments is basically an 'average' of the 1-year returns, this result may simply reflect that the variance of an 'average' return declines as the number of observations increase. This measure of risk need therefore not be very informative as regards the riskiness of the portfolio - that is, as regards the the variance of the Dollar/Kroner value of the investment (see also Bodie, Kane and Marcus, 1993). Let us therefore also look at the riskiness of the portfolio value of different investment strategies.

Suppose an investor has a holding period of one year and that he invests 1 Kroner in stocks in late 1921 which gives him a certain portfolio value ultimo 1922. He repeats this 1 Kroner investment every year until 1995. By taking the (natural) logarithm of all these portfolio values and by subsequently calculating the average value and the standard deviation of these (logged) portfolio values we arrive at the first row in Table 2.3, which also distinguishes between the nominal and the real portfolio values.

Table 2.3. Return-Risk of Stock and Bond Portfolios, 1922-95

	Average (logarithmic) portfolio value (nominal)	st. dev.	Average (logarithmic) portfolio value (real)	st. dev.
<u>Stocks</u>				
1-year	0.0872	0.1814	0.0486	0.1786
5-year[1)]	0.4462	0.3104	0.2382	0.3029
10-year[2)]	0.9052	0.4249	0.4481	0.3565
<u>Bonds</u>				
5-year[1)]	0.3818	0.2013	0.1738	0.2075
10-year[2)]	0.7678	0.4671	0.3107	0.4361

Notes:
1. We have calculated average portfolio values and standard deviations for non-overlapping investments starting in 1921, 1922, 1923, 1924 and 1925. Figures in the table are averages of these numbers.
2. Same procedure as for 5-year investments. Starting years are 1921,...,1930.

Suppose the holding period is five years. The investment in 1921 accumulates therefore to a certain amount in 1926. At the end of 1926, a new investment is undertaken and associated with this a new portfolio value is recorded in 1931 and so forth. Out of this sequence we calculate the average portfolio value in logs as well as the standard deviation, see the second row in Table 2.3. Following the same procedure, Table 2.3 also records the relevant statistics associated with a 10-year non-overlapping investment strategy. The results show that a 10-year investment is more risky than a 1-year investment in the sense that the standard deviation of the portfolio value is higher. By calculating the variances it is easy to see that the variance of a 10-year investment is not 10 times as high as the variance of a 1-year investment. If the returns had been independently distributed over time, the variance would have been proportional to the investment horizon as shown below. Thus, suppose that the log of 1 plus the return ln(1+RR1) is independent and normal distributed with mean μ and variance σ^2. In this case, the portfolio value of a 1 Kroner investment over T periods equals PV= (1+RR1(1))(1+RR1(2))...(1+RR1(T)), whereas the log value equals ln(PV)=ln(1+RR1(1)) + ln(1+RR1(2)) + ...+ ln(1+RR1(T)). Hence, the mean portfolio value equals $E(\ln(PV))=T\mu$, and the variance $Var(\ln(PV))=T\sigma^2$, which is linear in time. The fact that the portfolio variance in the Danish case increases less than proportionally with the investment horizon has profound implications for the optimal portfolio strategy (see below).

Consider next a non-overlapping 5-year bond strategy. By assuming that 1 Kroner is invested in 1921, 1926, 1931 etc. we may calculate the average portfolio value in logs and the standard deviation (see Table 2.3). The same statistics are reported for a 10-year investment strategy.

By comparing the stock and bond strategy two results stand out. First, stock investments yield on average a higher return than bond investments. Second, a 10-year stock investment strategy has been less risky than a 10-year bond investment strategy.[6] The portfolio strategy implication is that stocks are for the long run, whereas bonds are more

6

Despite the importance of this result, it should be mentioned that the statistical significance is unclear (and will remain unclear for many years) as there are obviously too few 10-year periods.

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for the short run. Put differently, a long-term investor should have a higher proportion of stocks in his portfolio than a short-term investor provided these investors are sufficiently risk averse. Thus, stocks should play an important role for young investors who save for retirement purposes (see also section 2.4).

Why is it that stocks are more risky than bonds in the short term but not more risky in the long-term? To get a rough answer to this question we briefly examine the time-series characteristics of stock and bond returns. Consider the autoregressive properties of the annual real stock return. The estimated AR(4) process is given below

$$\ln(1+RR1)_t = \underset{(0.026)}{0.070} - \underset{(0.123)}{0.282}\ln(1+RR1)_{t-1} - \underset{(0.128)}{0.090}\ln(1+RR1)_{t-2} - \underset{(0.126)}{0.057}\ln(1+RR1)_{t-3} - \underset{(0.124)}{0.083}\ln(1+RR1)_{t-4}, \quad (2.5)$$

$R^2 = 0.08$ (1926-95)

where numbers in brackets are standard errors. Our earlier findings indicate strong evidence against the independence assumption insofar as the variance of the portfolios increases less than proportionally with the investment horizon, and this conclusion is further supported by the above regression insofar as the results indicate strong negative correlation in the returns. By gradually deleting those variables that appear to be insignificant[7] we arrive at the next regression equation,

$$\ln(1+RR1)_t = \underset{(0.021)}{0.064} - \underset{(0.113)}{0.286}\ln(1+RR1)_{t-1}, \quad R^2 = 0.08 \text{ (1923-95)} \quad (2.6)$$

which is a simple AR(1) process. The strong negative correlation in the real returns shows that bad years are often followed by good years, and vice versa. The tendency for real returns to mean-revert can also be verified by using the so-called 'variance-ratio test' applied in Poterba

7 Real returns are only normal distributed when we omit observations for 1972 and 1983, but in this case we also get strong negative autocorrelation.

and Summers (1988). This test exploits the fact that if the market follows a random walk such that returns are independent then the return variance should be proportional to the return horizon, as noted also in the earlier discussion. Risager (1998) shows that variance-ratio tests applied to Danish data lend strong support to the mean reversion hypothesis, and hence provide evidence against the random walk hypothesis. Furthermore, mean reversion is a stronger phenomenon in Denmark than in the markets examined in Poterba and Summers (1988). To get further insight into the long-term relationship between risk and return in the stock market, it is instructive to calculate the expected future portfolio value when the return exhibits negative first-order autocorrelation and the variance of the portfolio. Not surprisingly, the mean value of the portfolio grows linearly with time, whereas it can be shown that the variance is growing with a lower speed. Hence, the risk-return ratio declines as the investment horizon increases. It is due to this element of time diversification that stocks can be said to become less risky over time. Bonds behave differently, as indicated also by Figure 2.3. Without going into details, bond yields tend to exhibit positive serial correlation. This can be verified not only by examining the two bond yields presented in Figure 2.3, but also by looking at short interest rates which, unfortunately, are available for only the last 20 years or so[8]. The different time-series properties of the two assets are the most important explanation why stocks are less risky in the long-term, notwithstanding that stocks are much more risky in the short term.

2.2.3. More on the equity premium

We have already established that stocks on average yield a higher return than bonds. This return difference shows up in a considerable difference between bond and stock portfolios, as the following example will show. Suppose an investor had put 100.000 Kroner into bonds in late 1921 (and subsequently reinvested the coupon and the principal when paid out) whereas another had invested the same amount in shares (and

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Further evidence is presented in the Appendix to this chapter, where serial correlations of a 1-year bond yield are reported.

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subsequently reinvested the received payments). The real value of these investments in 1995 are 1.660.000 Kroner and 3.640.000 Kroner, respectively, using the annual returns and the CPI. Thus, the real value of a stock portfolio is more than twice the value of a bond portfolio. Moreover, it is quite likely that our calculation exaggerates the attractiveness of bond investments insofar as we have used the 5-year interest rate as a proxy for the missing 1-year interest rate.

The above calculation shows that stocks clearly have outperformed bonds over long historical periods. That is not the same as to say that stocks always yield higher returns than bonds. Table 2.4 summarizes the equity premium for the entire sample and for the two sub-periods 1922-82 and 1983-93. Over the whole sample the 5- and 10-year equity premium is around 1.5 per cent. In the period 1983-95 the premium is negative. Thus, although the return on stocks is high in this period as noted earlier, the yield on government bonds is even higher.

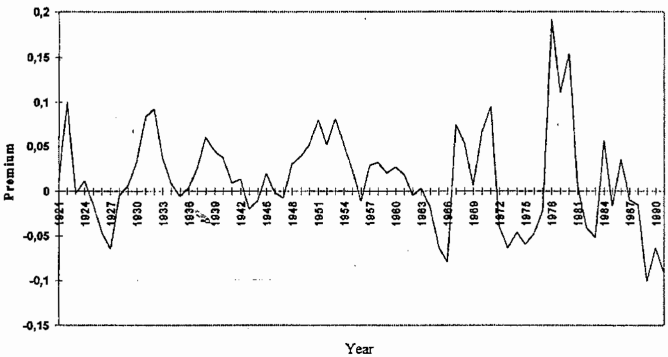
Table 2.4. Equity Premium for the sample

	Nominal
5-year investment 1922-95	1.5
5-year investment 1922-82	2.0
5-year investment 1983-95	-2.1
10-year investment 1922-95[1]	1.5
10-year investment 1922-82	1.9
10-year investment 1983-95[2]	-6.1

Notes:
1. The reason that the 10-year premium in this table is slightly different from the one implied by Table 3.2 is that in the present table only bond yields with matching stock returns are included.
2. The premium calculation is based only on three 10-year yields.
Source: Own calculations.

The 1983-95 period is not unique (see Figure 2.4, which displays the 5-year equity premium over a long historical period). Figure 2.4 shows that the premium displays a cyclical behaviour - that is, the premium varies between positive and negative values. There are seven periods in the data, where the equity premium has been negative for more than one year. The equity premium is also negative towards the end of the sample and hence also in 1991. By using the 1996 return data, it can be shown that the equity premium for 1992 is slightly positive.

Figure 2.4: 5-Year Nominal Equity Premium, 1921-91



The Danish premium is lower than the 6 per cent real premium for the United States estimated for the period 1889-1978 by Mehra and Prescott (1985). Note, however, that the premium in Mehra and Prescott (1985) is between 1-year stock returns and T-Bills. In spite of this, there is little doubt that the average historical Danish premium is below the American premium. Thus, a comparison of the 1-year Danish stock return with the 5-year Government bond yield (in the absence of 1-year bond yields) gives rise to a premium of about 3 per cent. The Danish premium is also below the premium in the United Kingdom. Engsted (1996) shows that the UK premium is close to 10 per cent in the period 1919-87. Historical stock returns in Sweden are quite similar to the Danish returns (see Frennberg and Hansson, 1992) . In the absence of a well functioning Swedish bond market back in time it is harder to compare equity premia between Copenhagen and Stockholm.

The above analysis has been concerned with the return-risk that the premium displays a cyclical behaviour - that is, the premium varies between positive and negative values. There are seven periods in the data, where the equity premium has been negative for more than one year. The equity premium is also negative towards the end of the sample and hence also in 1991. By using the 1996 return data, it can be shown that the equity premium for 1992 is slightly positive.

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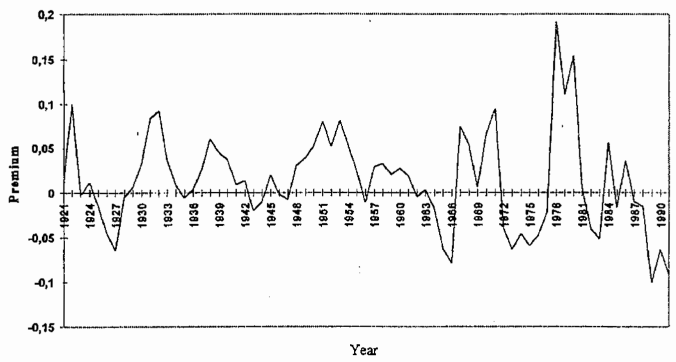

The Danish premium is lower than the 6 per cent real premium for the United States estimated for the period 1889-1978 by Mehra and Prescott (1985). Note, however, that the premium in Mehra and Prescott (1985) is between 1-year stock returns and T-Bills. In spite of this, there is little doubt that the average historical Danish premium is below the American premium. Thus, a comparison of the 1-year Danish stock return with the 5-year Government bond yield (in the absence of 1-year bond yields) gives rise to a premium of about 3 per cent. The Danish premium is also below the premium in the United Kingdom. Engsted (1996) shows that the UK premium is close to 10 per cent in the period 1919-87. Historical stock returns in Sweden are quite similar to the Danish returns (see Frennberg and Hansson, 1992) . In the absence of a well functioning Swedish bond market back in time it is harder to compare equity premia between Copenhagen and Stockholm.

The above analysis has been concerned with the return-risk

relationship on stocks and bonds in isolation. From a macroeconomic point of view it is of interest to know whether stocks are a poorer hedge against consumption fluctuations than bonds. If that is the case, the Consumption-CAPM asserts that stocks should yield a higher return than bonds. Below, we therefore relate the fluctuations in stock and bond returns to private consumption. The theory testing in this chapter is entirely concerned with the short end of the market; Nielsen and Risager (1997) pits the Consumption-CAPM against the long-run returns. The sample covariance matrix between the 1-year real returns on stocks and bonds and real private consumption growth (per capita) is as shown in Table 2.5.[9]

Table 2.5 shows that stock returns in the short term covary more with consumption growth than do bond returns. At the qualitative level, the Consumption-CAPM is therefore consistent with the data. Hence, it is of interest to test the model in a more rigorous way.

Table 2.5. Variance-Covariance Between Annual Returns, Yields, and Consumption Growth Rate, 1922-95

	Real annual stock return	Real annual bond yield	Real consumption growth rate
Real annual stock return	0.04558		
Real annual bond yield	0.00037	0.00285	
Real consumption growth rate	0.00124	0.00095	0.00275

Source: Own calculations.

9 We assume that the 5-year bond yield can be used as a proxy for the 1-year yield because the latter was not available to us when this chapter was completed. After publication of the chapter, we have constructed a 1-year government bond yield from 1924-96 as described in Appendix 1. On request of the ph.d.-committee, Appendix 2 employs the 1-year bond yield to check whether our use of the proxy is critical to the results. The conclusion of this analysis is that the results of the chapter are confirmed.

2.3. Asset returns and the Consumption-CAPM

This section briefly outlines the consumption-based asset pricing model without going into detail with respect to the underpinnings of the model. However, it is important to mention in advance that the fundamental pricing equation derived below can be obtained also in a somewhat less restrictive set-up (see Kocherlakota, 1996, for example). The most simple version of the model is based on the following three assumptions. First, individuals can be represented by a representative agent with well defined preferences. Second, asset markets are complete. Third, transaction costs are negligible. Under these assumptions, the following equation must always be fulfilled,

$$mu(t)Q(t) = (1+\theta)^{-1}E\left[mu\ (t+1)(Q(t+1)\ +\ D(t+1))\ |I(t)\right] \qquad (2.7)$$

where mu is marginal utility, Q is an asset price or vector of prices, θ is the constant subjective rate of time preference, D is dividend, and I is the information set. The left hand side (l.h.s.) is the increase in utility that occurs if the investor sells his asset and increases consumption at time t, whereas the right hand side (r.h.s.) is the discounted expected loss in utility from the fall in consumption due to not having the asset and obtaining the associated payoff at t+1. In equilibrium, the utility gain must equal the loss. If, for example, the l.h.s. exceeds the r.h.s., the investor reduces his asset holdings in t, which reduces the price $Q(t)$ and this will continue until there is equilibrium. By assuming $mu(t)>0$ we have,

$$\begin{aligned} Q(t) &= E\left[\frac{(1+\theta)^{-1}mu(t+1)}{mu(t)}\ (Q(t+1)+D(t+1))|I(t)\right] \\ &= E\left[m(t+1)(Q(t+1)+D(t+1))|I(t)\right] \end{aligned} \qquad (2.8)$$

where $(1+\theta)^{-1}mu(t+1)/mu(t)\equiv m(t+1)$ is the intertemporal marginal rate of substitution. Equation (2.8) says that the asset price in t equals the discounted expected value of the asset price and payoff in t+1, where the

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From the fundamental pricing equation (2.8) it is straightforward to get the following implication,

$$\begin{aligned} 1 &= E\left[m(t+1)\frac{Q(t+1)+D(t+1)}{Q(t)} \mid I(t)\right] \\ &= E\left[m(t+1)(1+R(t+1)) \mid I(t)\right] \end{aligned} \tag{2.9}$$

where R is the return. Because

$E(m(t+1)(1+R(t+1))=Em(t+1)E(1+R(t+1))+cov(m(t+1)(1+R(t+1))$ we have[10],

$$E(1+R(t+1)) = \frac{1-cov(m(t+1)(1+R(t+1))}{E(m(t+1))}$$

If $m(t+1)$ and $1+R(t+1)$ are negatively correlated (when high consumption growth goes hand in hand with a high return R, and low consumption growth goes hand in hand with a low return R), then the asset is risky in the sense that it provides a poor hedge against consumption fluctuations. The mean return $ER(t+1)$ must thus be relatively high as noted earlier when we discussed stylized facts for Denmark.

There are several ways to test Eq. (2.8). Below, we outline the visual Grossman and Shiller (1981) test and the more recent Hansen and Jagannathan (1991) method.

[10] We have used the law of iterated expectations.

2.3.1. Informal tests of stock price volatility a la Grossman and Shiller

Do stock prices vary too much relative to the prediction of the Consumption-CAPM-model? In Grossman and Shiller (1981) this question is addressed (in an informal way) by comparing actual stock price volatility with the volatility implied by a perfect foresight version of (2.8) solved forward in time. Besides this, this method of course also allows one to compare the level of the actual index with the index under perfect foresight.

The starting point is to solve (2.8) forward in time by recursive substitution (see (2.10)). The perfect foresight price Q^* is obtained by discarding the expectations operator (see (2.11)). Thus, we calculate the price that obtains if agents had perfect foresight with respect to the sequence of dividends $D(t+1),....,D(T)$ and the terminal price $Q(T)$, using an appropriate sequence of intertemporal marginal rate of substitution (IMRS) parameters $m(t+1),...,m(T)$ (see below).

$$Q(t) = E\left[\sum_{j=1}^{T-t}(1+\theta)^{-j}\frac{mu(t+j)D(t+j)}{mu(t)}+(1+\theta)^{-(T-t)}\frac{mu(T)Q(T)}{mu(t)}\,\middle|\,I(t)\right] \quad (2.10)$$

$$Q^*(t) = \sum_{j=1}^{T-t}(1+\theta)^{-j}\frac{mu(t+j)D(t+j)}{mu(t)}+(1+\theta)^{-(T-t)}\frac{mu(T)Q(T)}{mu(t)} \quad (2.11)$$

The IMRS $(1+\theta)^{-1}mu(t+1)/mu(t)\equiv m(t+1)$ is obtained by using a specific parametrization of the utility function (see Section 2.3.2). Thus, with a specific utility function we can estimate the marginal utilities using our consumption data. In the expression for IMRS, the parameter θ also enters. The latter is estimated by substituting the (sample) mean values of $mu(t+1)/mu(t)$ and $1+R(t+1)$ into (2.9). In the calculation of $mu(t+1)/mu(t)$, using a specific utility function, we will usually also have to take a stand on the degree of risk aversion since that affects the

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marginal utilities. This piece of information is obtained from the Hansen-Jagannathan method. Before we turn to that, we note that $Q^* = Q + U$, where the (expectations) error term U must be uncorrelated with the current price, which implies that $\mathrm{Var}(Q^*) > \mathrm{Var}(Q)$. Thus, the perfect foresight price should be more volatile than the actual price.

2.3.2. Hansen-Jagannathan bounds for the intertemporal marginal rate of substitution

According to the pricing formula (2.8), the asset price equals the expectation of the product of the payoff, $Q(t+1) + D(t+1)$, and the IMRS, $m(t+1)$.

The Hansen-Jagannathan method determines first the IMRS (mean) and its standard deviation that is consistent with asset market data - that is, consistent with (2.8). That is sometimes referred to as the 'admissible mean' and 'standard deviation' of the IMRS. The second step is to calculate the mean and standard deviation of the IMRS implied by a particular utility function for a representative agent. Next, we compare the mean and standard deviation of the IMRS implied by asset returns with the mean and standard deviation implied by the utility function in order to check whether the particular utility/consumption-based asset pricing model falls within the admissible region.

By taking unconditional expectations of the pricing formula (2.8) we get,

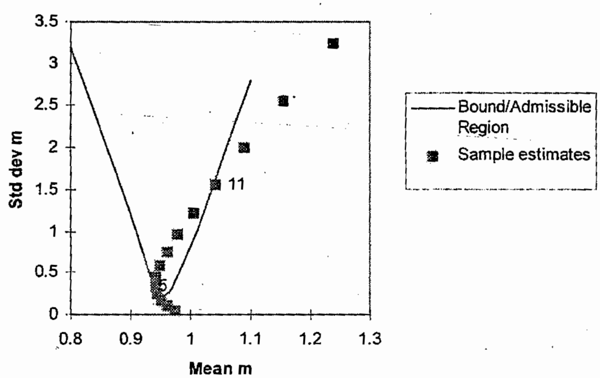
$$EQ(t) = E\left[m(t+1)(Q(t+1) + D(t+1))\right] \quad (2.12)$$

On the basis of (2.12) one can then derive the admissible region for the mean and standard deviation of the IMRS (see Hansen and Jagannathan, 1991). The standard deviation σ_m is given as,

$$\sigma_m = \left((EQ-E(Q+D)Em)\textstyle\sum_{Q+D}^{-1}(EQ-E(Q+D)Em)\right)^{1/2} \quad (2.13)$$

where Σ_{Q+D} is the covariance matrix of asset returns. Under risk neutrality the discount factor is constant - that is, the IMRS is constant, implying that $\sigma_m = 0$. Hence, the mean of all asset prices is strictly proportional to the mean of asset payoffs, where the factor of proportionality equals the constant *Em*. The standard deviation of the IMRS can therefore be thought of as the quadratic form in the deviations of the average prices from their risk neutral prices. Moreover, large deviations from risk neutrality imply large volatility of the IMRS. From (2.13) we then derive the relationship between *Em* and σ_m, that is, the admissible region consistent with asset market data. Figure 2.5 illustrates such an admissible region.

Fig. 2.5: Bounds on Moments of m and Sample Estimates



Source: Own calculations.

A common utility function in this area of research is the one with constant relative risk aversion (see (2.14)), which produces the IMRS given by (2.15).

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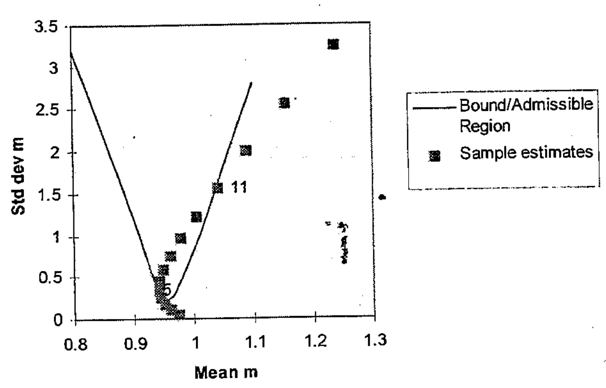

Source: Own calculations.

A common utility function in this area of research is the one with constant relative risk aversion (see (2.14)), which produces the IMRS given by (2.15).

$$u(Z) = (Z^{1-\alpha}-1)/(1-\alpha) \quad (2.14)$$

$$m(t+1) = (1+\theta)^{-1}((Z_t)/Z_{t+1}))^{\alpha} \quad (2.15)$$

where Z denotes consumption. Given data for consumption and guess values for the risk aversion coefficient α and $(1+\theta)^{-1}$, we obtain a time series for $m(t+1)$.[11] We then calculate the mean and standard deviation of the IMRS consistent with the utility function. This procedure is repeated for different values of α. By plotting the pair of mean and standard deviations associated with different risk aversion parameters into Figure 2.5 we may check whether the particular consumption-based asset pricing falls within the admissible region. Notice, that this gives information about the admissible values of the risk aversion parameter.

2.3.3. Estimated risk aversion and intertemporal marginal rate of substitution

Figure 2.5 shows the estimated bounds on combinations of the mean IMRS and its standard deviation using our asset return data (cf. (2.13)). The mean IMRS and its standard deviation consistent with the CRRA utility function is shown as the dotted line, assuming that the rate of time preference is 1 per cent (that is $(1+\theta)^{-1} = 0.99$). As the relative risk aversion parameter α increases, Em first declines but increases at a later stage. The standard deviation is always increasing. For a risk aversion parameter $\alpha = 5$, the dotted line is in the admissible region - that is, the consumption-based asset pricing model appears to be consistent with asset returns. Figure 2.5 also shows that Em is in the interval (0.9-1.0), which seems to be the interval where most would expect to find the mean value of the IMRS. The standard deviation is high but below 0.5, suggesting that the IMRS shows a great deal of variability (see also below).

In case $(1+\theta)^{-1} = 0.97$, the risk aversion parameter α will have to

11 Data is real consumption per capita obtained from Danmarks Statistik and S.Å. Hansen (1974).

equal 3.7 in order for the model to be consistent with market data. A risk aversion parameter of that size may be too high to be credible (see Table 2.6).

Table 2.6. What amount, X, would make a person with CRRA utility indifferent between participating in a gamble with equal probabilities of receiving 50 and 100, and receiving X with probability 1?

X	α
70.711	1
63.246	3
58.566	5
53.991	10
51.858	20
51.209	30

Source: Mankiw and Zeldes (1991).

So far, we have assumed that the utility function is time-separable, implying that individuals are not subject to any form of habit formation. In reality, individuals with a high consumption level today may have strong preferences for maintaining that level in the future. The idea that consumers tend to get spoiled is modelled by the habit persistence utility function, see Constantinides (1990) and the survey by Kocherlakota (1996). Formally, a high level of consumption today increases the marginal utility of consumption in the future which therefore pulls in the direction of a high consumption level also in the future. Similarly, when income falls consumers are reluctant in adjusting consumption first - that is, there is a ratchet effect. Habit persistence tends in general to increase the IMRS and its standard deviation, see Constantinides (1990). Hence, a lower value of the risk aversion parameter is required in order for the consumption asset pricing model to be consistent with market prices. Our results show that the risk aversion parameter may be as low as 2 equal 3.7 in order for the model to be consistent with market data. A risk aversion parameter of that size may be too high to be credible (see Table 2.6).

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both in the case where $(1+\theta)^{-1} = 0.99$, and where $(1+\theta)^{-1} = 0.97$.[12]

As the degree of risk aversion seems to be at a level that does not seem to be too unrealistic, it is of interest to explore further aspects of the consumption-based asset pricing theory.

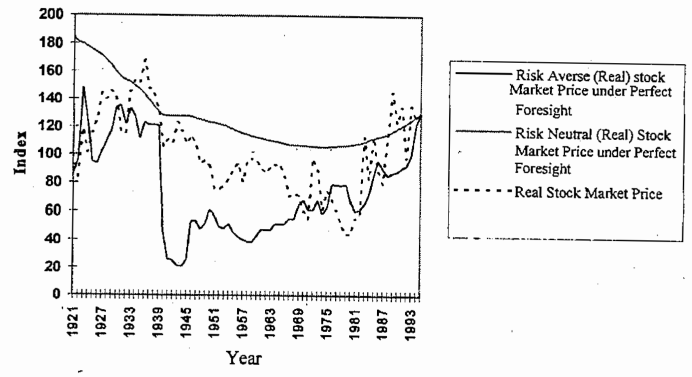
2.3.4. On the level and variability of stock prices since 1920

What would stock prices be if agents have perfect foresight and behave according to (2.11)? Figure 2.6 plots the actual index Q and the real perfect foresight index Q^*, under the assumptions of constant relative risk aversion, $(1+\theta)^{-1} = 0.99$, and $\alpha = 3.5$, where the relative risk aversion parameter is in the neighborhood of our estimated value.

The perfect foresight index traces the market index well until 1933. Thereafter, Q^* starts to fall whereas Q rises until 1936. The biggest decline in Q^* occurs in 1940. Thus, Q^* falls from 121 in 1939 to 47 in 1940. Q only declines from 131 in 1939 to 106 in 1940. Hence, Q is more than twice as high as Q^* in 1940.

It is of interest to find out why Q^* drops by so much in 1940. Recall that Q^* equals the discounted dividends, where the discount rates are the

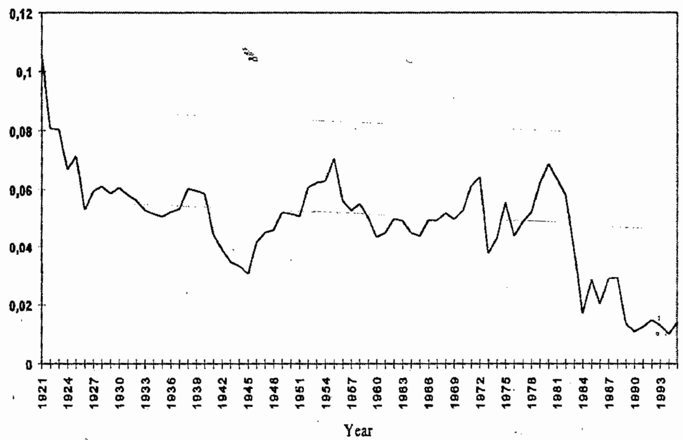
Figure 2.6: Perfect Foresight Stock Price Decomposition, 1921-95



12 We use the utility function suggested by Constantinides (1990).

IMRS. Because consumption falls very significantly in 1940, owing to the outbreak of World War II, marginal utility increases substantially. That leads to a significant reduction in the IMRS, which in part accounts for the large fall in Q^*. Put differently, the outbreak of war and the sharp fall in consumption leads to a large increase in agents' subjective discount rate. Our calculations show that the latter is high though slightly below 20 per cent during the war, where the subjective discount rate is the internal interest rate implied by the IMRS from 1940 and onwards.[13] Besides the increase in the real interest rate, there is also a fall in the dividend yield (see Figure 2.7).

Figure 2.7: Dividend Yield in Per Cent, 1921-95



It is possible to show that it is the rise in the discount rate that is the most important explanation of the enormous fall in Q^*. Thus, by calculating the perfect foresight path under the assumption of a constant

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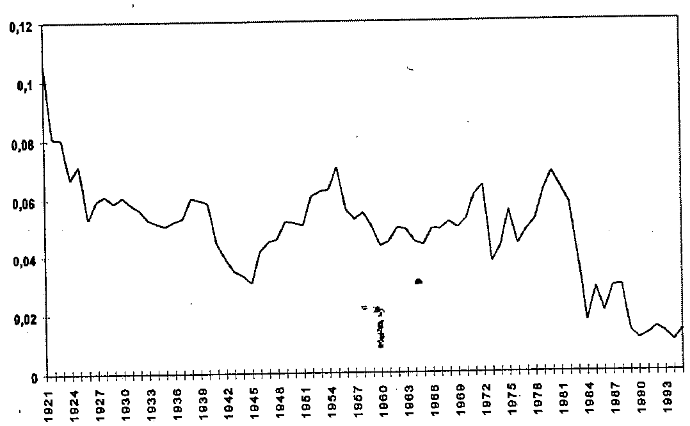

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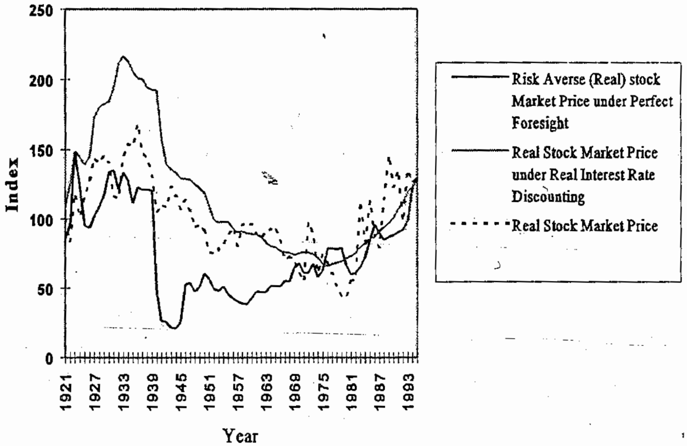
discount rate, we get the risk neutral price line also shown in Figure 2.6. The risk neutral price line displays only a tiny fall in 1940. Hence, it is mainly the rise in the discount rate that explains the jump in Q^* in 1940.

Insofar as Q falls only relatively little it is plausible that part of the divergence between Q and Q^* reflects a discrepancy between the discount rate used in the market and the discount rate implied by the Consumption-CAPM - that is, a failure of the model to produce a realistic discount rate. This need not be entirely due to the theory; it may in part be due to a relatively imprecise measure of private agents' consumption. Moreover, the discrepancy between Q and Q^* may of course also reflect uncertainty and expectations errors. It is important to stress that the divergence between Q and Q^* does not allow us automatically to conclude that the theory is invalid. Simulations by Kleidon (1986) show that if stock prices are non-stationary (follow a geometric random walk) and by construction are consistent with rational valuation models like ours, then it is still possible to have deviations between Q and Q^* of the magnitude shown in Figure 2.6, even though the underlying data generating process is the theory model. In this context it is, however, important to recall that our previous results reject the random walk hypothesis, as does the earlier Danish study by Jennergren and Toft-Nielsen (1977). Moreover, the large divergence in 1940, which is mainly due to a large discrepancy between the market and the model discount rate, does indicate that in order to get a better understanding of the stock market we need to get a better understanding of how the market discounts the future dividends.

It is interesting to note that the problem with tracing the level of the stock market does not carry over to its variability. Thus, the variances of the rate of change of Q and Q^* are almost identical. This is surprising in view of the findings for the United States (see Grossman and Shiller, 1981, and Kleidon, 1986). Moreover, it indicates that the market over the entire period 1922-95 has not been excessively volatile. This conclusion is underscored by the fact that the sample variance of the perfect foresight index may be severely downward biased (see Flavin, 1983).

It is natural to investigate the implications of replacing the theory-based IMRS with the actual real interest rates as discount factors. Figure 2.8 shows the index when the actual real rate is used in the discounting as well as the actual index and the Consumption-CAPM index also shown in Figure 2.6. It appears that the index based on the actual real interest rate as the discount rate does a slightly better job in tracing the level of the stock market, whereas the Consumption-CAPM index is more in line with reality when it comes to explaining stock market volatility.

Figure 2.8: Perfect Foresight Stock Prices under C-CAPM and Real Interest Rate Discounting, 1921-95



2.4. Policy implications

As mentioned earlier, Danish pension funds and life insurance companies are subject to an upper limit on their stock investments. The current regulations state that only 40 per cent of these institutional investors' pension liabilities can be held in stocks. The Wage Earner's Fund (Lønmodtagernes Dyrtidsfond) and the Supplementary Pension Fund (ATP) are subject to even tougher regulations, as the upper limit for them is 35 per cent.

The main reason for the quantitative regulation is the perception that stocks are more risky than other assets. This view is, for instance, as well as the actual index and the Consumption-CAPM index also shown in Figure 2.6. It appears that the index based on the actual real interest rate as the discount rate does a slightly better job in tracing the level of the stock market, whereas the Consumption-CAPM index is more in line with reality when it comes to explaining stock market volatility.

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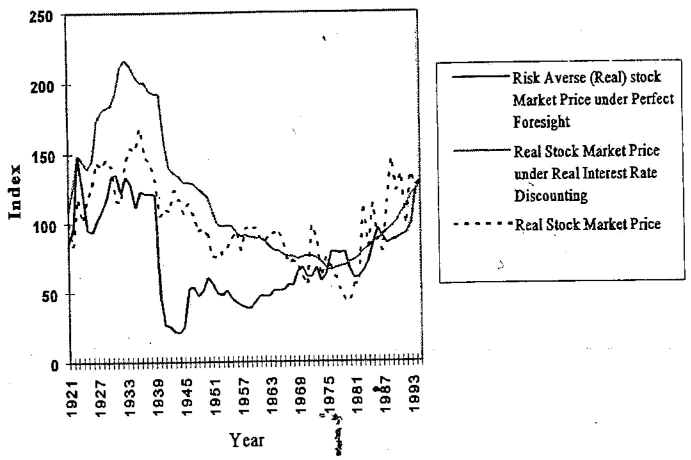

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The main reason for the quantitative regulation is the perception that stocks are more risky than other assets. This view is, for instance,

expressed very clearly in 1981 when the Government proposed to raise the limit that applied in 1981. In the text of the law, stocks are characterized as 'risky assets' as opposed to bonds, and in comments attached to the proposal it is argued that:

> from the savers' point of view, variation in rates of return due to changes in business profitability implies a risk of receiving less return than could have been obtained elsewhere, even if the investment is viewed over a long time span (our translation).

Another reason for the quantitative regulation may be various governments' need to finance their budget deficits - that is, to be able to issue government bonds at a favourable price. This argument applies, of course, only under imperfect international capital mobility, and may therefore be of less importance nowadays. A third concern is political and has to do with the fear of Fund's socialism.

As documented earlier, stocks are not more risky than bonds provided the portfolio is sufficiently diversified and provided the investment horizon is sufficiently long. Hence, the 'risk argument' is based on false premises. There is therefore a need to reconsider the present regulatory framework even though it is only the Wage Earner's Fund that effectively has been constrained by the upper limit. One reason why other investors (for example, insurance companies) have not hit the roof so far is that they are also constrained by certain minimum yield requirements, which distort their portfolios towards assets with low short-term risk (eg. bonds).

There are several potential advantages of liberalizing the investment regime. First, by encouraging pension funds and other investors to allocate a larger proportion of their funds to stocks, savers can obtain a higher expected return on their investments without incurring a larger risk. Of course, this assumes that the equity premium will apply also in the future and that pension funds are not able perfectly to match bond maturity with liabilities. Simple calculations suggest that the gains of increasing the proportion of stocks in the portfolios are substantial. Second, firms will get easier access to capital, which may encourage investment and growth. Finally, as bond markets are nowadays highly integrated internationally, a further softening of the regulatory framework may not affect governments' ability to finance deficits.

Besides the 40 per cent regulation, there is also a 20 per cent upper limit on the holdings of assets denominated in other currencies than liabilities. Given that the liabilities are in Danish Kroner, this means that these investors can only allocate 20 per cent of their funds to foreign assets. The possibility of having a sufficiently internationally diversified portfolio may therefore be severely restricted. This may lead to portfolios that are inefficiently biased towards home assets. The home-bias issue is outside the scope of this chapter, but we plan to return to this issue in future work.

2.5. Conclusions

The main achievement of this chapter has been to calculate and report the return-risk properties of Danish stocks and bonds over the historical period 1922-95 using our own database. Thus, we have reported the return-risk characteristics of 1-, 5- and 10-year stock investments, and the yield-risk characteristics of 5- and 10-year Government bond investments. In the subsequent theoretical sections, we have tested whether the behaviour of short-run stock returns can be explained by the Consumption-CAPM using the non-parametric test by Hansen and Jagannathan (1991) and the informal test by Grossman and Shiller (1981).

Our results show that stocks yield a higher average return than bonds and that short-run stock investments are much more risky than bond investments, whereas long-run stock investments are less risky. This chapter's explanation of this apparently provocative and paradoxical finding, reported also in Christiansen and Lystbæk (1994) and Siegel (1994), is the mean-reversion property of real stock returns. Thus, bad years in the market are usually more than offset by the good years that follow later, whereas bond yields display positive autocorrelation. Because the existing regulatory framework for institutional investors is based on the premise that stocks are more risky than bonds, we have argued that these return-risk results call for a further softening of the quantitative regulations, if not a complete abandoment, simply because pension funds and other institutional investors are able to pursue long-framework may not affect governments' ability to finance deficits.

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run strategies with highly diversified portfolios. We have also argued that such a policy change will be close to a Pareto improvement insofar as both pensioners and firms listed on the stock market will benefit from such a liberalization.

Our tests of the consumption-based asset pricing model yield mixed results. The estimate of the risk aversion parameter under constant relative risk aversion and in particular under habit formation is in the plausible range. However, as the level of the implied perfect foresight stock market index is very far from the actual index from the late 1930s and until the beginning of the 1970s, we have doubts about the validity of the underlying theory even though Kleidon (1988) has shown that this type of information may be too soft to reject the theory. However, Nielsen and Risager (1997) also reject the consumption-based view of stock prices and this is indeed also the typical finding in the international literature. It seems to us that the key issue is to get a better understanding of the discounting process insofar as dividends explain only a tiny fraction of stock market volatility. We plan to return to this issue in future work.

References

Andersen, T.M. and O. Risager (1988), Stabilization Policies, Credibility, and Interest Rate Determination in a Small Open Economy, *European Economic Review*, 32, pp. 669-79.

Bodie, Z., A. Kane and A. J. Marcus (1993), *Investments*, (Boston: Irwin).

Breeden, D. (1979), An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, *Journal of Financial Economics*, 7(3), pp. 265-96.

Campbell, J.Y. and R. J. Shiller (1988), The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies*, 1, pp. 195-228.

Christiansen, J. and B. Lystbæk (1994), Afkast og Risiko På Aktier og Obligationer 1915-1993, *Finans/Invest* 10-13.

Constantinides, G.M. (1990), Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98 (3), pp. 519-43.

Deaton, A. (1992), *Understanding Consumption,*: Clarendon Press, Oxford, UK.

Engsted, T. (1996), Evaluating the Consumption - Capital Asset Pricing Model using Hansen-Jagannathan Bounds, Evidence from the UK, *Working Paper,* Aarhus Business School.

Eskesen, L., F. D. Jensen, E. Rygner and O. Zacchi (1984), *Finansielle Institutioner og Markeder,* Jurist- og Økonomforbundets Forlag, Copenhagen.

Flavin, M.A. (1983), Excess Volatility in the Financial Markets, *Journal of Political Economy*, 91, pp. 929-56.

Frennberg, P. and B. Hansson (1992), Computation of a Monthly Index for Swedish Stock Returns 1919-1989, *Scandinavian Economic History Review*, 1, pp. 3-27.

Grossman, S.J. and R. Shiller (1981), The Determinants of the Variability of Stock Market Prices, *American Economic Review*, 71(2), pp. 222-7.

Hansen, K. (1974), *Om Afkastet af Danske Aktier i tiden 1900-1974*, Nordisk Fjerfabriks Jubilæumsskrift, Nordisk Fjerfabrik, 191-220.

Campbell, J.Y. and R. J. Shiller (1988), The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies*, 1, pp. 195-228.

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Constantinides, G.M. (1990), Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98 (3), pp. 519-43.

Deaton, A. (1992), *Understanding Consumption,*: Clarendon Press, Oxford, UK.

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Grossman, S.J. and R. Shiller (1981), The Determinants of the Variability of Stock Market Prices, *American Economic Review*, 71(2), pp. 222-7.

Hansen, K. (1974), *Om Afkastet af Danske Aktier i tiden 1900-1974*, Nordisk Fjerfabriks Jubilæumsskrift, Nordisk Fjerfabrik, 191-220.

Hansen, L.P. and R. Jagannathan (1991), Implications of Security Market Data for Models of Dynamic Economies, *Journal of Political Economy*, 99, pp. 225-62.

Hansen, S.Å. (1974), *Økonomisk vækst i Danmark*, Akademisk Forlag, Copenhagen.

Jennergren, L.P. and P. Toft-Nielsen (1977), An investigation of random walks in the Danish stock market, *Nationaløkonomisk Tidsskrift,* pp. 254-69.

Kleidon, A.W. (1986), Variance Bounds Tests and Stock Price Valuation Models, *Journal of Political Economy*, 94, pp. 953-1001.

Kocherlakota, N.R. (1996), The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature*, 42-71.

Lucas, R.E. Jr. (1978), Asset Prices in an Exchange Economy, *Econometrica*, 46(6), pp. 1429-45.
Lund, J. and T. Engsted (1996), GMM and Present Value Tests of the C-CAPM: Evidence From the Danish, German, Swedish and UK Stock Markets, *Journal of International Money and Finance*, 15, pp. 497-521.

Mankiw, N.G. and S. P. Zeldes (1991), The Consumption of Stockholders and Non-Stockholders, *Journal of Financial Economics*, 29, pp. 97-112.

Mehra, R. and E. C. Prescott (1985), The Equity Premium: A Puzzle, *Journal of Monetary Economics*, 15, pp. 145-61.

Nielsen, S. and O. Risager (1997), On the Long Run Stock Return Puzzle in Denmark and the C-CAPM, *Working Paper,* Copenhagen Business School (preliminary).

OECD (1996), *Statistical Compendium*, CD-Rom.

Olsen, E. and E. Hoffmeyer (1968), *Dansk Pengehistorie 1914-1960*, Danmarks Nationalbank, Copenhagen.

Pagan, A. (1996), The Econometrics of Financial Markets, *Journal of Empirical Finance*, 3, pp.15-102.

Poterba, J.M. and A.A. Samwick (1995), Stock Ownership Patterns, Stock Market Fluctuations, and Consumption, *Brookings Papers on Economic Activity*, 2, pp. 295-372.

Poterba, J.M. and L.H. Summers (1988), Mean Reversion in Stock Prices - Evidence and Implications, *Journal of Financial Economics*, 22, pp. 27-59.

Risager, O. (1998), Random walk or mean reversion: The Danish stock market since World War I, Working paper #12, Economic Policy Research Unit, Copenhagen Business School.

Siegel, J.J. (1994), *Stocks For The Long Run*, Irwin, New York.

Olsen, E. and E. Hoffmeyer (1968), *Dansk Pengehistorie 1914-1960*, Danmarks Nationalbank, Copenhagen.

Pagan, A. (1996), The Econometrics of Financial Markets, *Journal of Empirical Finance*, 3, pp.15-102.

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Appendix

2.A. Serial correlation of 1-year yield

The 1-year real bond yield has a first order serial correlation of +0.53 (sample 1924-95) whereas serial correlation for real stock returns is -0.25 (sample 1924-95). This confirms the impression from figure 2.3 and the positive serial correlation of short interest rates mentioned on p. 50, namely that bond yields, contrary to stock returns, are positively serially correlated.

2.B. Variance-covariance with stock return and consumption

Below is the equivalent of table 2.5 based on the 1-year yield instead of the proxy:

Table A.2.5. Variance-Covariance Between Annual Returns, 1-Year Yields, and Consumption Growth Rate, 1922-95

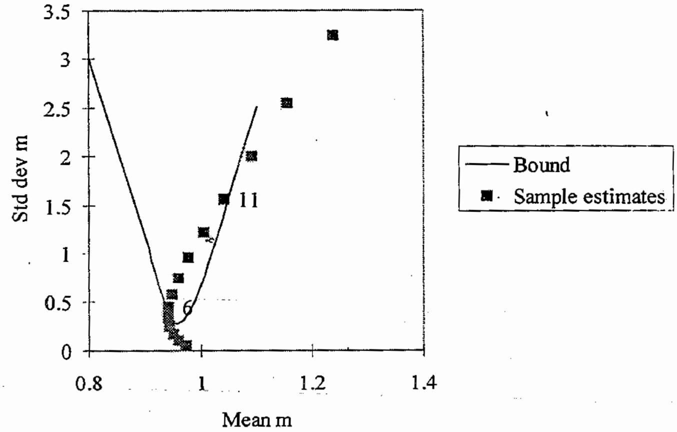
	Real annual stock return	Real annual bond yield[1]	Real consumption growth rate
Real annual stock return	0.04558		
Real annual bond yield	0.00053	0.00334	
Real consumption growth rate	0.00124	0.00104	0.00275

Note:
1. Since 1-year bond yields are not available for the first two years of the sample, the 5-year yield has been used as a proxy those two years.
Source: Own calculations.

The numbers are very similar to those in table 2.5. Hence, the qualitative conclusions from the table remain unchanged.

2.C. Hansen-Jagannathan bounds

Fig. A.2.5: Bounds on Moments of m and Sample Estimates using 1-Year Bond Yield



When the 1-year bond yield is being used, figure 2.5 changes only slightly:

With $(1+\theta)^{-1}=0.99$, our estimate of α changes from 5 to 6. In case $(1+\theta)^{-1}=0.97$, the estimate changes from 3.7 to 7. Thus, risk aversion estimates are only slightly greater than in section 2.3. Hence, our conclusion against C-CAPM are strengthened.

Under habit persistence, risk aversion estimates do not change.

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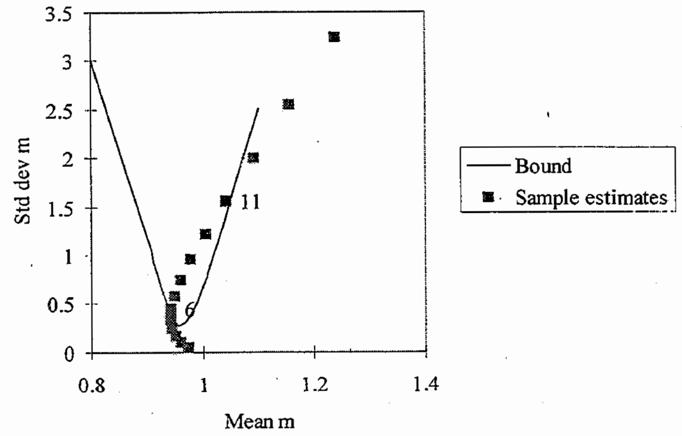

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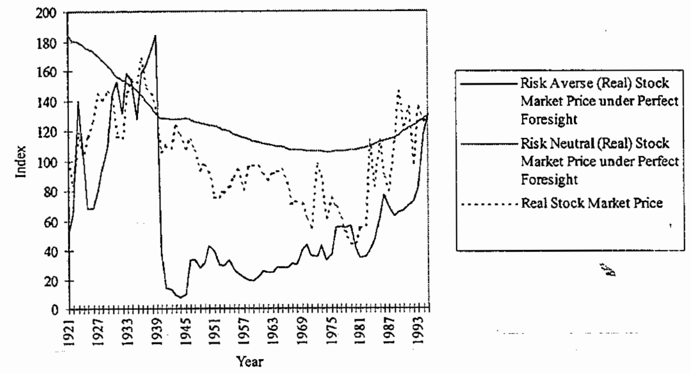
Under habit persistence, risk aversion estimates do not change.

2.D. Perfect foresight price path

The revised risk aversion estimates (α=6) causes a minor adjustment of figure 2.6:

Fig. A.2.6: Perfect Foresight Stock Price Decomposition with a=6, 1921-95

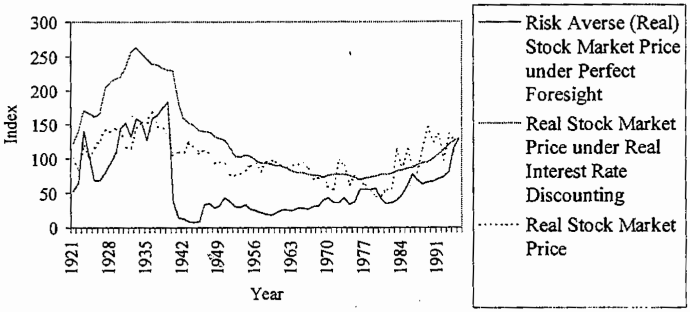


The swings of the perfect foresight price path increase relative to figure 2.6, but the conclusions do not change. Thus, the perfect foresight price deviates substantially from actual price.

2.E. Real interest rate discounting

Using the 1-year yield for interest rate discounting and the risk averse perfect foresight price from Appendix 2.D, we obtain:

Figure A.2.8: Perfect Foresight Stock Prices under C-CAPM and Real Interest Rate Discounting, 1921-95



It seems, like in section 2.3, that real interest rate discounting is superior in capturing the level of stock prices whereas variability is better explained by the perfect foresight price.

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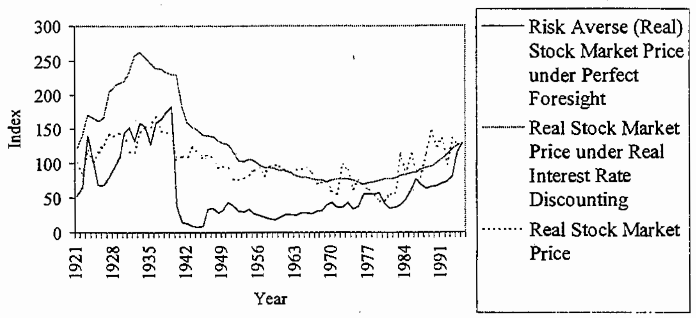

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Chapter 3

Regime-Switching Stock Returns and Mean Reversion[1]

with Jan Overgaard Olesen[2]

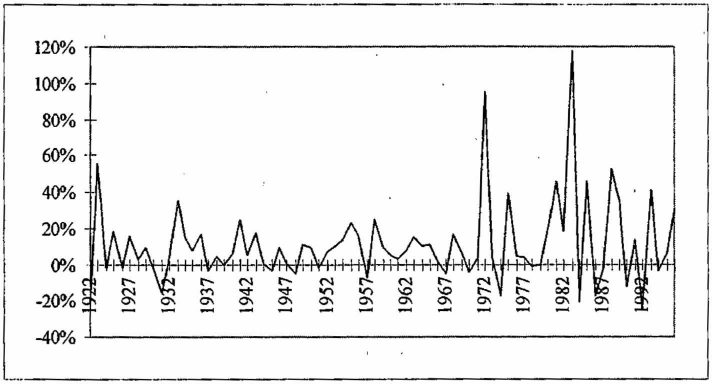
3.1. Introduction

A plot of Danish stock returns over time suggests that returns were low and relatively stable from the 1920s until the beginning of the 1970s whereas the period since then has been characterized by higher average return and more volatility:

1 We thank participants in the workshop "Stock Market Economics" at Copenhagen Business School, May 1999, for useful comments.

2 Danmarks Nationalbank.

Figure 3.1. Annual Nominal Stock Returns in Denmark 1922-96



Note: Market portfolio of stocks listed at the Copenhagen Stock Exchange. Data are from the Nielsen, Olesen and Risager (1999) database.

This observation was also made on an informal basis by Nielsen and Risager (1999)[3]. In this chapter, we fit a time series model to the nominal return data which allows for the presence of more than one regime. This provides for a formal analysis of whether there have been several regimes and when changes of regime occurred. Furthermore, this approach enables us to test the hypotheses that mean return and volatility are higher in one regime than in the other. Identification of multiple regimes is important for understanding the time series properties of stock returns and may, in particular, be valuable for forecasting purposes.

The plot also indicates that annual stock returns display negative serial correlation (most obviously in the latter part of the sample), ie., that stock prices mean-revert. This question was first raised by Fama and French (1988) and Poterba and Summers (1988) and has been examined for Denmark by Risager (1998). These papers all report weak evidence

3

However, they view the return in 1972 as an outlier and conclude that the change of regime takes place in 1983.

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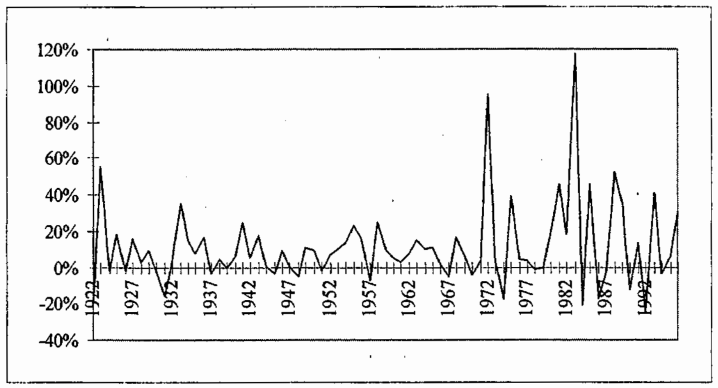

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of mean reversion[4]. The present chapter provides an alternative test of this issue within the framework of the regime-switching model. Thus, our approach leads to a mean reversion test which allows for multiple regimes in the return process.

Our procedure takes into account the specific pattern of heteroskedasticity, ie., regime shifts in volatility level, identified by the regime-switching model. There are two related papers by Kim and Nelson (1998) and Kim, Nelson and Startz (1998) in which a similar model for returns is estimated. They standardize returns by estimated volatility and calculate variance ratio and autoregression tests for standardized returns. Our approach, on the other hand, is a parametric test of negative serial correlation which directly utilizes estimates obtained for the regime-switching model.

Furthermore, the chapter provides new evidence about the extent to which serial correlation differs across regimes, ie., whether the visual impression, that negative serial correlation is stronger in the latter part of the sample, is correct. In order to apply the tests we calculate analytical expressions for unconditional and state-specific means, variances and serial correlations for the regime-switching model with an autoregressive term.

The following section fits a regime-switching model to our return data. Section 3.3 derives analytical means and variances of the model and tests hypotheses. Similarly, serial correlation and implications for mean reversion is considered in section 3.4. Finally, section 3.5 concludes.

3.2. Estimating a regime-switching model for returns

Given the apparent change in behavior of Danish stock returns we are led to estimate a model which accounts for stochastic changes in regime. We employ a two-state version of the model developed by Hamilton (1990). According to this model there is an unobserved state variable, s_t, which takes on the values 0 or 1. The state variable is assumed to follow

4 The former paper analyzes real return, the second real and excess return, and the latter real and nominal return. In the present chapter, we examine nominal returns.

a Markov chain, ie., the transition probabilities satisfy $p_{00} \equiv P(s_t=0 | s_{t-1}=0) = P(s_t=0 | s_0=i_0,...,s_{t-2}=i_{t-2},s_{t-1}=0)$ and $p_{11} \equiv P(s_t=1 | s_{t-1}=1) = P(s_t=1 | s_0=i_0,...,s_{t-2}=i_{t-2},s_{t-1}=1)$ for any sequence i_0, ..., i_{t-2} and any t. The observed stock return depends on the state variable:

$$R_t = \mu_0 + (\mu_1-\mu_0)s_t + \phi_0 R_{t-1} + (\phi_1-\phi_0)s_t R_{t-1} + \sigma_0\epsilon_t + (\sigma_1-\sigma_0)s_t\epsilon_t , \tag{3.1}$$

where ϵ_t ~ n.i.d. (0,1).

Thus,

$$R_t | s_t=0 = \mu_0 + \phi_0 R_{t-1} + \sigma_0\epsilon_t \tag{3.2}$$

and

$$R_t | s_t=1 = \mu_1 + \phi_1 R_{t-1} + \sigma_1\epsilon_t . \tag{3.3}$$

Note, that this version of the model allows for distinct μ's and σ's, and that an autoregressive term is included in each state.

The parameter vector is estimated by numerically maximizing the log likelihood function described in Hamilton (1994), section 22.4. The algorithm used to evaluate the log likelihood has two other interesting byproducts. First, it is possible to evaluate the probability that a given observation was generated by, say, state 0 conditional on information available at that time (filtered probabilities), ie., current and past stock returns. This provides insight about timing of regime changes. Second, the algorithm generates one-period-ahead probabilities which can be used to construct return forecasts.

Estimating the model described above does not immediately give satisfactory results. The main problem is that the estimate of one of the transition probabilities is at a corner, $\hat{p}_{00}=0$, and that the estimate of the autoregressive term in state 0 is above 1, $\hat{\phi}_0=1.59$. Both of these estimates thus violate the assumptions under which specification tests proposed in Hamilton (1996) are derived. Hence, the distribution of test a Markov chain, ie., the transition probabilities satisfy $p_{00} \equiv P(s_t=0 \mid s_{t-1}=0)=P(s_t=0 \mid s_0=i_0,\ldots,s_{t-2}=i_{t-2},s_{t-1}=0)$ and $p_{11} \equiv P(s_t=1 \mid s_{t-1}=1)=P(s_t=1 \mid s_0=i_0,\ldots,s_{t-2}=i_{t-2},s_{t-1}=1)$ for any sequence $i_0, \ldots, i_{t-2}$ and any t. The observed stock return depends on the state variable:

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statistics is unknown. However, informal diagnostic tests of standardized residuals of the three-state model suggests that the three-state model suffers from autocorrelation in the error term, cf. Appendix A. In this formulation, the filtered probabilities conditional on information available at time t only assign three observations to state 0, namely 1923, 1972 and 1983 which all represent years with extraordinary returns (cf. figure 3.1). Thus, state 0 may be viewed as a state which picks up outliers whereas state 1 is the ordinary state.

To pursue the question of whether there exist two states in addition to the outlier state we estimate a three-state version of the model. This results in an outlier state for 1972 and 1983 and two ordinary states for the remaining observations. The ordinary regimes have low return-low volatility and high return-high volatility, respectively, and the timing of regimes is in line with what we anticipated from looking at data. However, transition probabilities and the autoregressive term of the outlier state cause the same problem as above.

To be able to perform the Hamilton (1996) specification tests of the model and given the indication of misspecification revealed by residual-based tests we therefore choose to introduce dummies for 1972 and 1983 in the two-state model. The two dummy variables have zeroes every year except in 1972 and 1983, respectively, where the value is 1. They are added to equation (3.1) as two additional variables with potentially distinct coefficients in the two states to allow maximum flexibility. Thus, the resulting model is:

$$R_t = \mu_0+(\mu_1-\mu_0)s_t+\mu_0^{72}D72_t+(\mu_1^{72}-\mu_0^{72})s_t D72_t+\mu_0^{83}D83_t+(\mu_1^{83}-\mu_0^{83})s_t D83_t+\phi_0 R_{t-1}+(\phi_1-\phi_0)s_t R_{t-1}+\sigma_0\epsilon_t+(\sigma_1-\sigma_0)s_t\epsilon_t \quad ; \tag{3.1'}$$

where $s_t \in \{0,1\}$ and $\epsilon_t \sim$ nid (0,1). μ_0^{72} and μ_0^{83} are the coefficients to the dummy variables in state 0, and likewise for state 1.[5]

The fundamental difference between the three-state and the dummy model is the assumption of the latter that 1972 and 1983 are abnormal and non-recurring events which can be ignored while fitting a model for the remaining observations. On the other hand, the three-state model views 1972 and 1983 as belonging to a separate, extreme state which there is a (small) positive probability of returning to.

Our choice of the two-state dummy model is motivated by the fact that there are solid economic reasons for treating these years as special. In 1972 Denmark decided to join the EEC and agreed to allow foreign ownership of Danish stocks. In 1983 nominal interest rates were dramatically reduced as a result of the adoption of a fixed exchange rate policy and further capital market liberalizations, and a new pension fund tax was introduced on bond yields only. These events are potential explanations of the outstanding stock returns of these particular years.

The following estimates are obtained for the two-state model with dummies[6]:

5 The likelihood function is identical to the one presented in Hamilton (1994), p. 692, where the elements in η_t are (using the notation of this paper):

$$\frac{1}{\sqrt{2\pi}\sigma_i}\exp\left\{\frac{-(R_t-\mu_i-\mu_i^{72}D72_t-\mu_i^{83}D83_t-\phi_i R_{t-1})^2}{2\sigma_i^2}\right\} \quad , \quad i=0,1$$

6 Parameter estimates of the two-state dummy model are similar to estimates of the three-state model, cf. Appendix 3.A.

dummy variables in state 0, and likewise for state 1.[5]

The fundamental difference between the three-state and the dummy model is the assumption of the latter that 1972 and 1983 are abnormal and non-recurring events which can be ignored while fitting a model for the remaining observations. On the other hand, the three-state model views 1972 and 1983 as belonging to a separate, extreme state which there is a (small) positive probability of returning to.

Our choice of the two-state dummy model is motivated by the fact that there are solid economic reasons for treating these years as special. In 1972 Denmark decided to join the EEC and agreed to allow foreign ownership of Danish stocks. In 1983 nominal interest rates were dramatically reduced as a result of the adoption of a fixed exchange rate policy and further capital market liberalizations, and a new pension fund tax was introduced on bond yields only. These events are potential explanations of the outstanding stock returns of these particular years.

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6 Parameter estimates of the two-state dummy model are similar to estimates of the three-state model, cf. Appendix 3.A.

Table 3.1. Two-state Model with Dummies for 1972 and 1983, Sample 1923-96

μ_0	0.0601 (0.0244)	p_{00}	0.8497 (0.1430)
μ_1	0.1802 (0.0461)	p_{11}	0.8304 (0.1400)
ϕ_0	- 0.0446 (0.0955)	μ_0^{72}	0.8925 (0.0825)
ϕ_1	- 0.3297 (0.1256)	μ_1^{72}	0.7819 (0.5881)
σ_0^2	0.0056 (0.0030)	μ_0^{83}	1.1265 (0.2028)
σ_1^2	0.0385 (0.0126)	μ_1^{83}	1.0582 (0.2150)

Note: Standard errors in parantheses estimated by second derivatives of log likelihood. μ's with superscripts 72 and 83 refer to coefficients to dummy variables.

Point estimates of μ and σ are smaller in state 0 than in state 1, and as we are going to see in section 3.3 a non-trivial implication of table 3.1 is that state 0 is the low return-low volatility state whereas state 1 is characterized by high return and high volatility. ϕ_0 is insignificant but we choose to keep it for use in the next section. Finally, to determine whether the regimes are statistically different we may for example test a hypothesis that the μ's are equal across states. A Wald test rejects this hypothesis (p-value is 0.0244) which confirms that there are 2 distinct regimes.

Note also, that the problem of corner solutions is avoided and that both AR-terms are numerically less than 1. Hence, specification tests suggested by Hamilton (1996) may be applied.

Table 3.2. Specification Tests.

White tests, $\chi^2(4)$		
Autocorrelation	0.7832	(0.9403)
ARCH	6.4781	(0.1677)
Markov property	0.3786	(0.9835)
Lagrange multiplier tests, $\chi^2(1)$		
Autocorrelation in regime 0	0.2250	(0.6394)
Autocorrelation in regime 1	0.0116	(0.9151)
Autocorrelation across regimes	0.5266	(0.4717)
ARCH in regime 0	1.3324	(0.2547)
ARCH in regime 1	0.1414	(0.7079)
ARCH across regimes	0.9079	(0.3454)

Note: P-values in parentheses. Large sample tests of Hamilton (1996).

The tests show that the residuals of (3.1') fulfil the white noise requirements, ie., they are serially uncorrelated and homoskedastic (no ARCH), both within and across regimes. Furthermore, the Markov property of the transition probabilities cannot be rejected, ie., the probabilities of the future state outcome are determined exclusively by the most recent state realization.

The model clearly passes all specification tests at the conventional significance level using large sample distributions. Using the small sample corrections suggested by Hamilton (1996) leads to even clearer acceptance of the model. Furthermore, informal diagnostic tests confirm that standardized residuals is white noise, cf. Appendix 3.B.

We are now ready to analyze the timing of regimes. Figure 3.2 shows the filtered probabilities, ie., the probability that observation t belongs to state 0 given the information on current and past stock returns available at time t.

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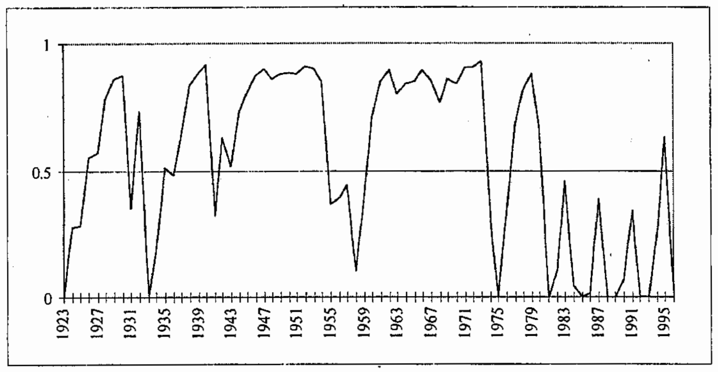
We are now ready to analyze the timing of regimes. Figure 3.2 shows the filtered probabilities, ie., the probability that observation t belongs to state 0 given the information on current and past stock returns available at time t.

Figure 3.2. Probability that Observation t is in State 0 given Information Available.

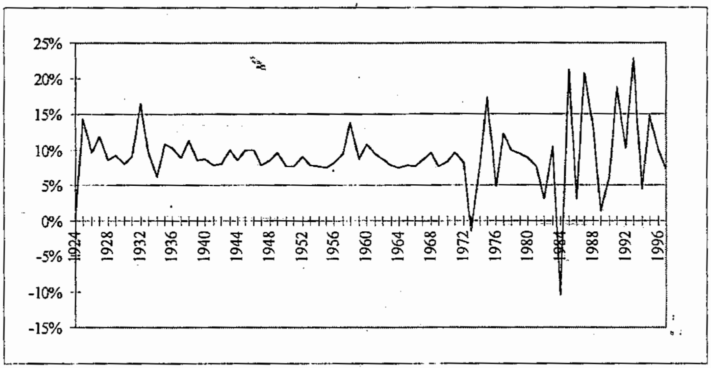


This confirms that after a long period of state 0 dominance state 1 has recently become more frequent. Except for a few, short episodes, returns were in the low return-low volatility state with probability greater than one half until 1973. The exceptions are in the beginning of the 20s which was a period of financial distress in Danish financial and industrial companies, the beginning of the 30s which covers both the decline and recovery in the wake of the Wall Street crash, and the latter half of the 50s which marks the beginning of a long business cycle boom. All the episodes occur in periods of volatile stock returns, cf. figure 3.1. Since 1973, and especially during the 80s and 90s, the high return-high volatility regime has dominated. One possible explanation is that liberalization has made the Danish stock market more vulnerable to foreign volatility.[7] A similar argument is made by Sellin (1996) in relation to a recent Swedish liberalization.

7 Although the Danish stock market is formally opened to foreigners around 1972, foreign holding of Danish stocks does not accelerate until the beginning of the 1980s, cf. Eskesen et al. (1984). This explanation is consistent with the observation that a persistent regime-shift seems to take place in the beginning of the 1980s.

Figure 3.3 shows the return forecast of the model[8] for time t given information available at t-1. Assuming that market participants know the return process, we may interpret the model forecast as a measure of market expectations at time t-1 about time t return. We see that the market almost always expected returns within the 5 - 15 per cent per year range in the long period from 1924 to 1972. Since then, and in particular since 1981, market expectations have been extremely volatile and, in fact, more often outside the 5 - 15% range than inside. This reflects that returns have been more volatile in the latter part of the sample and that current returns affect forecasted returns significantly in the state which dominates towards the end.

Figure 3.3. Model's Return Forecast at t-1



8

We have excluded the dummy terms in forming forecasts which is natural since the necessity of dummies could not have been anticipated. The forecast is calculated by:

$$E(R_t|\Omega_{t-1}) = P(s_t=0|\Omega_{t-1})(\mu_0+\phi_0 R_{t-1})+P(s_t=1|\Omega_{t-1})(\mu_1+\phi_1 R_{t-1})$$

where the probabilities are one-period ahead probabilities, cf. Hamilton (1994), section 22.4.

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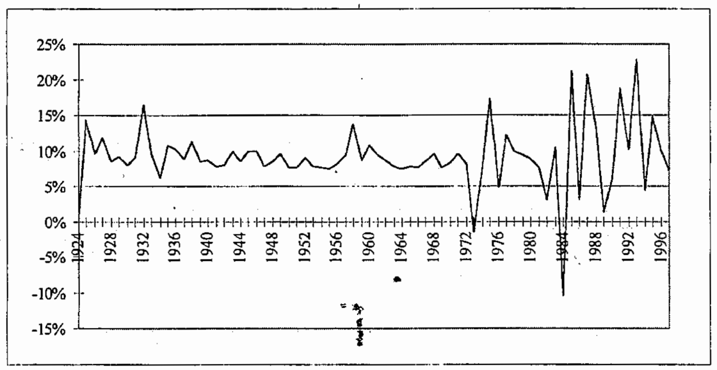

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where the probabilities are one-period ahead probabilities, cf. Hamilton (1994), section 22.4.

3.3. Means and variances of the two states

In this section, we calculate means and variances of the return process estimated in the previous section. Both unconditional and conditional means and variances are calculated. We consider an 'ordinary' year, ie., the dummy terms are ignored.

The calculations in sections 3.3 and 3.4 are complicated by the presence of the AR-term and have to our knowledge not been presented elsewhere. It is important to include the AR-term for two reasons. First, table 3.1 shows that the AR-term is statistically significant. Hence, a model without this component would be misspecified and mean and variance calculations would be invalid. Second, in section 3.4, we suggest an alternative test for mean reversion in returns which, basically, tests the significance of the AR-term.

3.3.1 Means

The unconditional mean of model (3.1) is:

$$\begin{aligned} E(R_t) &= P(s_t=0)E(R_t|s_t=0) + P(s_t=1)E(R_t|s_t=1) \\ &= \pi_0(\mu_0+\phi_0 E(R_{t-1}|s_t=0)) + \pi_1(\mu_1+\phi_1 E(R_{t-1}|s_t=1)) , \qquad (3.4) \end{aligned}$$

where $\pi_0 \equiv P(s_t=0)=(1-p_{11})/(2-p_{00}-p_{11})$ and $\pi_1 \equiv P(s_t=1)=1-\pi_0$ are unconditional (ergodic) probabilities of being in the particular state, cf. Hamilton (1994). Note, that the mean depends on expected return in the previous period conditional on the current state[9]:

$$\begin{aligned} E(R_{t-1}|s_t=1) &= P(s_{t-1}=0|s_t=1)E(R_{t-1}|s_{t-1}=0) + \\ &\quad P(s_{t-1}=1|s_t=1)E(R_{t-1}|s_{t-1}=1) \\ &= pE(R_{t-1}|s_{t-1}=0) + (1-p)E(R_{t-1}|s_{t-1}=1) , \qquad (3.5) \end{aligned}$$

9 This is derived in Appendix 3.C.

where $p \equiv P(s_{t-1}=0/s_t=1)$ is the probability that the state variable in the previous period was in state 0 given it currently is 1 which can be interpreted as an 'inverse' transition probability. Using Bayes' rule it can be shown that:

$$
\begin{aligned}
p &= \frac{\pi_0 p_{01}}{\pi_0 p_{01} + \pi_1 p_{11}} \\
&= p_{10}
\end{aligned}
\tag{3.6}
$$

Thus, the inverse transition probability equals the ordinary transition probability.

Assuming covariance stationarity, ie., that means and autocovariances are constant over time, the dating on the right hand side of (3.5) may be changed:

$$E(R_{t-1}|s_t=1) = pE(R_t|s_t=0) + (1-p)E(R_t|s_t=1) \ . \tag{3.7}$$

Similarly,

$$E(R_{t-1}|s_t=0) = qE(R_t|s_t=0) + (1-q)E(R_t|s_t=1) \ , \tag{3.8}$$

where $q \equiv P(s_{t-1}=0/s_t=0)$ is another inverse transition probability. Using Bayes' rule it can be shown that:

$$
\begin{aligned}
q &= \frac{\pi_0 p_{00}}{\pi_0 p_{00} + \pi_1 p_{10}} \\
&= p_{00}
\end{aligned}
\tag{3.9}
$$

(3.7) and (3.8) can be inserted in:

$$E(R_t|s_t=0) = \mu_0 + \phi_0 E(R_{t-1}|s_t=0) \tag{3.10}$$

where $p \equiv P(s_{t-1}=0/s_t=1)$ is the probability that the state variable in the previous period was in state 0 given it currently is 1 which can be interpreted as an 'inverse' transition probability. Using Bayes' rule it can be shown that:

$$p = \frac{\pi_0 p_{01}}{\pi_0 p_{01} + \pi_1 p_{11}} = p_{10} \tag{3.6}$$

Thus, the inverse transition probability equals the ordinary transition probability.

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where $q \equiv P(s_{t-1}=0/s_t=0)$ is another inverse transition probability. Using Bayes' rule it can be shown that:

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(3.7) and (3.8) can be inserted in:

$$E(R_t|s_t=0) = \mu_0 + \phi_0 E(R_{t-1}|s_t=0) \tag{3.10}$$

$$E(R_t|s_t=1) = \mu_1 + \phi_1 E(R_{t-1}|s_t=1) \tag{3.11}$$

(derived from (3.1)) to get two equations in two unknowns. The solutions are[10]:

$$E(R_t|s_t=1) = \frac{A}{B}$$
$$E(R_t|s_t=0) = \frac{\phi_0 (1-q) A + \mu_0 B}{(1-\phi_0 q) B} , \tag{3.12}$$

where $A \equiv \mu_1 - \phi_0 q \mu_1 + \phi_1 p \mu_0$ and $B \equiv 1-\phi\, q-\phi\, (1-p)-\phi\, (p-q)$. Finally, insert (3.12) in (3.4) to get the unconditional mean.

$E(R_t|s_t=i)$ is the expected return in state i. It depends not only on the parameters of state i but also on the parameters of the alternative state. This is due to the AR-terms in returns which force us to consider expected return, and hence the value of the unobserved state variable, in the previous period to form expectations about returns in this period. For example, if $\phi_1>0$ and $p>0$, state 1 expected return increases in μ_0 since there is some probability, p, that the state variable was 0 in the previous period in which case μ_0 affects expected return last period which, in turn, affects expected return in the present period via the positive AR-term in state 1 (ϕ_1).

Given the analytical means in (3.4) and (3.12) we are able to estimate:

10 Assuming $B \neq 0$.

Table 3.3. Unconditional and Conditional Means

$E(R_t)$	0.0955
	(0.0177)
$E(R_t \mid s_t=0)$	0.0570
	(0.0211)
$E(R_t \mid s_t=1)$	0.1390
	(0.0330)
Wald test, H_0: $E(R_t \mid s_t=0)=E(R_t \mid s_t=1)$	3.9806
	[0.0460]

Note: Each of the means are calculated as a function, $f(\theta)$, (cf. (4) and (12)) of the estimated parameter vector, $\theta=[\hat\mu_0, \hat\mu_1, \hat\mu_0^{72}, \hat\mu_1^{72}, \hat\mu_0^{83}, \hat\mu_1^{83}, \hat\Phi_0, \hat\Phi_1, \hat\sigma_0^2, \hat\sigma_1^2, \hat p_{00}, \hat p_{11}, \hat p\ ]'$. Standard errors in parentheses are calculated as: $Std(f(\theta))=[\hat J' Var(\theta) \hat J]^{1/2}$, where $J=[\partial f/\partial\theta]$. The restriction being tested has been reformulated as $g(\theta)=0$, and the test statistic is calculated as: $W=g(\theta)'[\hat J' Var(\theta)\hat J]^{-1} g(\theta)$, where $J=[\partial g/\partial\theta]$. W is asymptotically χ^2 with degrees of freedom equal to number of restrictions (ie., 1).
P-value in square brackets.

The estimated unconditional expected return is 9.5% per year which is close to the simple average[11] of 9.1%. State 0 expected return is estimated to 5.7% per year whereas state 1 has an expected return of 13.9%. The Wald test just rejects (at 5% significance) the hypothesis that means are equal in favor of the alternative that means are different.[12] Thus, we are justified in saying that regime 0 has lower expected return than regime 1.

[11] From 1923 to 1996 excluding 1972 and 1983.

[12] In addition to the Wald test, we have performed a Likelihood Ratio test of the same hypothesis which has a p-value of 0.0643 leading to acceptance of the hypothesis at 5%. We have more confidence in the Wald test, however, since filtered probabilities change completely under the restriction which in our opinion makes the test hard to interpret. Possibly, the existence of multiple local maxima of the unrestricted likelihood function reduce the power of Likelihood Ratio tests.

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3.3.2 Variances

Unconditional variance is:

$$Var(R_t) = E(R_t^2) - E(R_t)^2 \tag{3.13}$$

Consider,

$$\begin{aligned} E(R_t^2) &= P(s_t=0)E(R_t^2 \mid s_t=0) + P(s_t=1)E(R_t^2 \mid s_t=1) \\ &= \pi_0(\mu_0^2+2\mu_0\phi_0E(R_{t-1} \mid s_t=0)+\phi_0^2E(R_{t-1}^2 \mid s_t=0)+\sigma_0^2)+ \\ &\quad \pi_1(\mu_1^2+2\mu_1\phi_1E(R_{t-1} \mid s_t=1)+\phi_1^2E(R_{t-1}^2 \mid s_t=1)+\sigma_1^2) \end{aligned} \tag{3.14}$$

using the model, that is (3.2) and (3.3).

In this expression, we have that

$$\begin{aligned} E(R_{t-1}^2 \mid s_t=0) &= qE(R_{t-1}^2 \mid s_{t-1}=0) + (1-q)E(R_{t-1}^2 \mid s_{t-1}=1) \\ E(R_{t-1}^2 \mid s_t=1) &= pE(R_{t-1}^2 \mid s_{t-1}=0) + (1-p)E(R_{t-1}^2 \mid s_{t-1}=1) \end{aligned} \tag{3.15}$$

Assuming covariance-stationarity, we need to solve[13]

$$\begin{aligned} E(R_t^2 \mid s_t=0) &= E[(\mu_0+\phi_0R_{t-1}+\sigma_0\epsilon_0)(\mu_0+\phi_0R_{t-1}+\sigma_0\epsilon_0)' \mid s_t=0] \\ &= \mu_0^2+2\mu_0\phi_0E(R_{t-1} \mid s_t=0)+\phi_0^2(qE(R_t^2 \mid s_t=0)+ \\ &\quad (1-q)E(R_t^2 \mid s_t=1))+\sigma_0^2 \end{aligned} \tag{3.16}$$

and a similar expression for $E(R_t^2 \mid s_t=1)$ to obtain $E(R_t^2)$. The solutions for $E(R_t^2 \mid s_t=i)$ are in appendix 3.D. Subtracting the squared means derived earlier gives expressions for unconditional and conditional variances.

Unconditional and conditional variances can now be estimated:

[13] $E(R_{t-1} \mid s_t=0)$ is known from section 3.3.1.

Table 3.4. Unconditional and Conditional Variances

$Var(R_t)$	0.0246
	(0.0074)
$Var(R_t \mid s_t=0)$	0.0056
	(0.0030)
$Var(R_t \mid s_t=1)$	0.0425
	(0.0143)
Wald test, H_0:$Var(R_t \mid s_t=0)=Var(R_t \mid s_t=1)$	7.7977
	[0.0052]

Note: See note to table 3.3 where f now relates to the variance formulae derived above. Standard errors in parentheses and p-value in square brackets.

The estimated unconditional standard error of annual returns is 15.7% which corresponds to the sample standard error of 16.4%.[14] State 0 standard deviation is 7.5% whereas state 1 standard deviation is 20.6%. The hypothesis that conditional variances are equal is strongly rejected with a p-value of less than 1 per cent.[15] Hence, volatility is lower in state 0 than in state 1.

Finally, a Wald test rejects the joint hypothesis that both means and variances are equal across states (p-value 0.0013).

3.4. Serial correlation: Test for mean reversion

The question of whether stock prices are mean-reverting has received a lot of attention since the papers by Fama and French (1988) and Poterba and Summers (1988). A number of studies have produced evidence of mean reversion using variance-ratio and autoregression tests, see Risager (1998) for an analysis of the Danish return data. In this section, we

14 From 1923 to 1996 excluding 1972 and 1983.

15 A Likelihood Ratio test of the hypothesis has a p-value of 0.1193 leading to acceptance of H_0. However, the test is not easily interpretable, cf. footnote 11.

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$Var(R_t)$	0.0246
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$Var(R_t \mid s_t=1)$	0.0425
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Wald test, H_0: $Var(R_t \mid s_t=0)=Var(R_t \mid s_t=1)$	7.7977
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provide evidence based on an alternative test procedure which has the important feature that it explicitly allows for regime-shifts in the return process.

We choose to focus attention on first order serial correlation. Specification tests in table 3.2 and Appendix 3.B show no sign of autocorrelation in the error term, so any higher order serial correlation is due to first order serial correlation. We calculate the analytical first order serial correlations of the two-state Markov switching model, see appendix 3.E. Then we obtain point estimates and standard errors:

Table 3.5. Unconditional and Conditional First Order Serial Correlation

$Corr(R_t,R_{t-1})$	-0.1993
	(0.1104)
$Corr(R_t,R_{t-1} \mid s_t=0)$	0.0297
	(0.2482)
$Corr(R_t,R_{t-1} \mid s_t=1)$	-0.3340
	(0.1214)
Wald test, H_0: $Corr(R_t,R_{t-1})=0$	3.2567
	[0.0711]
Wald test, H_0: $Corr(R_t,R_{t-1} \mid s_t=0)=0$	0.0143
	[0.9048]
Wald test, H_0: $Corr(R_t,R_{t-1} \mid s_t=1)=0$	7.5669
	[0.0059]

Note: See note to table 3.3 where f relates to serial correlation formulae displayed in Appendix 3.E. Standard errors in parentheses and p-value in square brackets.

Our estimate of first order serial correlation across regimes is -0.2 which is significantly less than zero at 10% significance level but cannot be rejected to be zero at the 5% level. Hence, there is weak evidence of mean reversion in nominal stock returns which is consistent with findings of others.[16]

16 Risager (1998) finds slightly more support for mean reversion in real than in nominal returns which indicates that the p-value would be slightly less than 7.11% if

Interestingly, the same hypothesis has a p-value of 0.0042 in a standard one-regime AR 1-specification with dummies for 1972 and 1983 and using OLS standard errors which leads to clear acceptance of mean reversion[17]. Hence, allowing for multiple regimes results in much less support for mean reversion than standard methods. This finding is consistent with the results of Kim and Nelson (1998) who also conclude that accounting for the specific pattern of heteroskedasticity found in the data weakens the evidence of mean reversion.[18]

Thus, it is important to take account of heteroskedasticity when making inference about mean reversion, in particular, since p-values are close to the conventional significance level even small changes may have large qualitative importance for conclusions. Although OLS gives consistent estimates of coefficients, a procedure which allows for heteroskedasticity (of the correct form) leads to more efficient inference. Moreover, usual OLS estimates of variances including coefficient standard errors are biased. Heteroskedasticity consistent standard errors (such as White) improve inference asymptotically, but may have problems in small samples. For example, in our case, using White standard errors only increases the p-value to 0.0064, whereas we found a p-value of around 7%.

Our regime-switching model includes the standard one-regime model as a special case, and, hence, is more general. Therefore, we have more confidence in results of the regime-switching model. We interpret the conflicting inference as evidence of weaknesses of the standard approach.

Our analysis highlights two important points. First, in the presence of multiple persistent regimes which have the feature that some but not all regimes exhibit mean reversion, it is important to have observations from each regime in order to draw correct inference. In the case of

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our analysis were applied to real returns.

17

The estimated coefficient to lagged returns is -0.235 with a t-statistic of -2.957. These results are similar to the findings in Nielsen and Risager (1999) and Risager (1998).

18

A similar conclusion has been found for the variance-ratio test by Kim, Nelson and Startz (1998).

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nominal Danish stock returns it is particularly important to have enough observations after the beginning of the 80s to be able to detect two regimes. This parallels the socalled peso problem encountered in the exchange rate literature, see e.g. Evans (1996), ie., in order to identify a process with rare, discrete events, a large sample is needed.[19] Second, there are two sources to negative serial correlation if the true return generating process is regime-switching. First of all, a negative autoregressive term creates mean reversion as in the usual one-state AR case. But, even if the autoregressive term is zero in both states serial correlation may be different from zero just because the process shifts between states (assuming these have different means).

Within our framework, we are able to distinguish serial correlation of the two states. As table 3.5 shows, our estimate of serial correlation is only negative in state 1. In fact, only in state 1 is serial correlation significantly different from zero. Hence, we conclude that the weak evidence of mean reversion presented in table 3.5 is (mainly) a result of serial correlation in the high return-high volatility state which has dominated the most recent decades. This is in contrast to results for the US which indicate that mean reversion was stronger before World War II than after, see Kim, Nelson and Startz (1991) and Kim and Nelson (1998).

The evidence of mean reversion parallels the findings in Risager (1998). Using standard autoregressive and variance-ratio tests, he finds weak support of the mean reversion hypothesis. Furthermore, the paper suggests splitting the sample into subsamples. This analysis indicates that mean reversion has been stronger in the most recent part of the sample, that is, since the 1970s. This conclusion is consistent with the results of the regime-switching model in the present paper.

Given the strong presence of mean reversion in recent years, what should we expect for the future? This basically depends on whether one believes that the current regime is absorbing or not. From a purely statistical point of view, there is a probability of returning to the no-mean-reversion state which implies that unconditional serial correlation

19 We conjecture that since our model is constructed to identify regime-shifts, it will stand a better chance of solving peso problems and lead to more reliable inference on mean reversion in small samples.

is the right measure, thus suggesting only weak support for mean reversion. From an economic point of view, however, it is essential to focus on the underlying factors which cause regime changes and, in particular, to analyze whether all the variables causing the most recent regime-shift are reversible. It is perhaps not likely that the liberalizations, which we argue led to the latest transition to high volatility, will be reversed within a foreseeable future. However, other factors, such as a decrease of US stock market volatility, may be able to cause a return to low volatility. In other words, we use capital market liberalizations as one (of several) component to explain the latest transition to high volatility but do not view deliberalization as necessary for a return to the low volatility regime. Hence, economic considerations have ambigous implications for the question of mean reversion.

3.5. Conclusion

We have estimated a well-specified two-state regime-switching model for Danish stock returns. The model identifies two regimes which have low return-low volatility and high return-high volatility, respectively. The low return-low volatility regime dominated, except in a few, short episodes, until the beginning of the 70s whereas the 80s and 90s have been characterized by high return and high volatility.

We propose an alternative test of mean reversion which allows for multiple regimes with potentially different constant and autoregressive terms and different volatility. Using this test procedure we find mean reversion at 10% but not at 5% significance level. This is weaker evidence than produced by the standard method of testing for significance of the AR-term in a one-regime autoregressive model. Furthermore, when analyzing contributions of the two regimes, we find that the indication of mean reversion is due to the recent high return-high volatility regime only.

The regime-switching model has also been applied by Kim and Nelson (1998) and Kim, Nelson and Startz (1998) on stock returns using US data. Our approach differs by allowing for an autoregressive term and by incorporating regime-shifts in the mean. Both features are shown to be relevant for Danish data.

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References

Eskesen, L., F.D. Jensen, E. Rygner and O. Zacchi (1984), *Finansielle institutioner og markeder*, Jurist- og økonomforbundets forlag, Copenhagen

Evans, M.D.D. (1996), Peso Problems: Their Theoretical and Empirical Implications, in **Statistical Methods in Finance, Handbook of Statistics** vol. 14, eds. G.S. Maddala and C.R. Rao, Elsevier, North-Holland.

Fama, E.F. and K.R. French (1988), Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96, no. 2, pp. 246-73.

Nielsen, S. and O. Risager (1999), Macroeconomic Perspectives on Stock and Bond Investments in Denmark since World War I, in *Macroeconomic Perspectives on the Danish Economy*, eds. T.M. Andersen, S.E.H. Jensen and O. Risager, MacMillan.

Hamilton, J.D. (1990), Analysis of Time Series Subject to Changes in Regime, *Journal of Econometrics* 45, pp. 39-70.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.

Hamilton, J.D. (1996), Specification Testing in Markov-Switching Time-Series Models, *Journal of Econometrics* 70, pp. 127-57.

Kim, C.J. and C.R. Nelson (1998), Testing for mean reversion in heteroskedastic data II: Autoregression tests based on Gibbs-sampling-augmented randomization, *Journal of Empirical Finance* 5, pp. 385-96.

Kim, M.J., C.R. Nelson and R. Startz (1991), Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence, *Review of Economic Studies* 58, pp. 515-28.

Kim, C.J., C.R. Nelson and R. Startz (1998), Testing for mean reversion in heteroskedastic data based on Gibbs-sampling-augmented randomization, *Journal of Empirical Finance* 5, pp. 131-54.

Nielsen, S. and O. Risager (1999), Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Nielsen, S., J.O. Olesen and O. Risager (1999), Macroeconomic Database, Economics Department, Copenhagen Business School.

Poterba, J.M. and L.H. Summers (1988), Mean Reversion in Stock Prices, *Journal of Financial Economics* 22, pp. 27-59.

Risager, O. (1998), Random Walk or Mean Reversion: The Danish Stock Market Since World War I, Working Paper # 12, Economic Policy Research Unit, Copenhagen Business School.

Sellin, P. (1996), Inviting Excess Volatility? Opening Up a Small Stock Market to International Investors, Scandinavian Journal of Economics 98 (4), pp. 603-12.

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Nielsen, S. and O. Risager (1999), Macroeconomic Perspectives on Stock and Bond Investments in Denmark since the First World War, in *Macroeconomic Perspectives on the Danish Economy*, T.M. Andersen, S.E.H. Jensen and O. Risager (eds.), MacMillan, London.

Nielsen, S., J.O. Olesen and O. Risager (1999), Macroeconomic Database, Economics Department, Copenhagen Business School.

Poterba, J.M. and L.H. Summers (1988), Mean Reversion in Stock Prices, *Journal of Financial Economics* 22, pp. 27-59.

Risager, O. (1998), Random Walk or Mean Reversion: The Danish Stock Market Since World War I, Working Paper # 12, Economic Policy Research Unit, Copenhagen Business School.

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Appendix

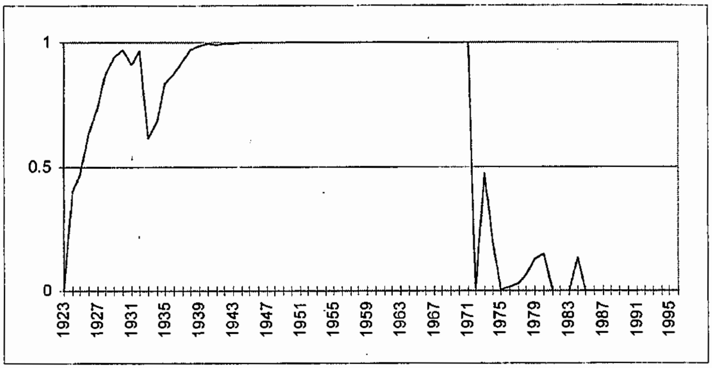
3.A. Three-state model

Parameter Estimates

μ_0	0.0781 (0.0167)	p_{00}	0.9703 (0.0228)
μ_1	0.1614 (0.0493)	p_{01}	0.0000
μ_2	0.8923 (0.0021)	p_{02}	0.0297
ϕ_0	- 0.0888 (0.1163)	p_{10}	0.0000
ϕ_1	- 0.2616 (0.1234)	p_{11}	0.9328 (0.0495)
ϕ_2	1.5922 (0.0127)	p_{12}	0.0672
σ_0^2	0.0091 (0.0019)	p_{20}	0.2741
σ_1^2	0.0440 (0.0130)	p_{21}	0.7259
σ_2^2	0.0000 (0.0000)	p_{22}	0.0000

Note:
Standard errors in parentheses estimated by second derivatives of log likelihood. Omitted standard errors cannot be calculated due to corner solutions.

Filtered Probabilities for State 0



The outlier state has filtered probabilities close to 1 in 1923, 1972 and 1983 and zero otherwise.

All point estimates of the three-state model are within one standard deviation of the two-state dummy model estimates. The main difference is that the regimes are estimated to be more persistent in the three-state model. This has the implication that inference about the state and the timing of regime shifts is much clearer than in the two-state model. Another difference between the models is that the three-state model assigns some probability to the event that s_t returns to the outlier state.

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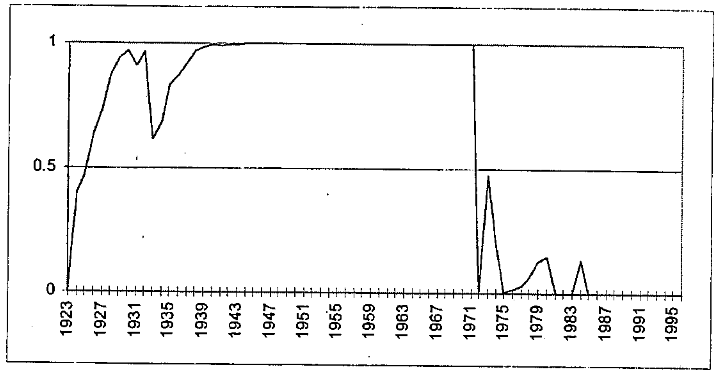

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Diagnostic tests of standardized residuals:[20]

	Test statistic	P-value
AR(1)	0.0000	0.9967
AR(2)	3.2539	0.0445 *
AR(3)	2.1393	0.1030
AR(4)	1.853	0.1286
AR(5)	1.5575	0.1839
AR(6)	1.5812	0.1663
AR(7)	1.3459	0.2433
AR(8)	1.2564	0.2818
ARCH(1)	0.0661	0.7978
Normality	0.9729	0.6148

3.B. Analysis of Standardized Residuals of 2-State Dummy Model

Standardized residuals are calculated as the difference between actual and fitted return divided by conditional standard deviation, ie., the square root of (derived in chapter 4):

$$Var(R_t|\Omega_{t-1}) = P(s_t=0|\Omega_t)\sigma_0^2 + P(s_t=1|\Omega_t)\sigma_1^2 + P(s_t=0|\Omega_t)P(s_t=1|\Omega_t)\left(E(R_t|\{\Omega_{t-1},s_t=0\}) - E(R_t|\{\Omega_{t-1},s_t=1\})\right)^2$$

where Ω_t contains information about current and past stock returns. Fitted returns are:

20 Standardized residuals are calculated as in Appendix 3.B except for the extra state.

$$\hat{R}_t = P(s_t{=}0|\Omega_T)(\mu_0+\mu_0^{72}D72_t+\mu_0^{83}D83_t+\phi_0 R_{t-1})+$$
$$P(s_t{=}1|\Omega_T)(\mu_1+\mu_1^{72}D72_t+\mu_1^{83}D83_t+\phi_1 R_{t-1})$$

which is conditioned on information on past stock returns and uses filtered probabilities for each state (that is, probabilities conditioned on Ω_T which includes all available stock returns of the sample). The standardized residuals are estimates of ϵ_t in (3.1').

The standardized residuals have been tested for autocorrelation from lag 1 to 8, ARCH and normality:

	Test statistic	P-value
AR(1)	0.0390	0.8440
AR(2)	1.5828	0.2126
AR(3)	1.2332	0.3042
AR(4)	0.9278	0.4530
AR(5)	0.8111	0.5458
AR(6)	0.7359	0.6225
AR(7)	0.6219	0.7360
AR(8)	0.6334	0.7468
ARCH(1)	1.6207	0.2071
Normality	0.4076	0.8156

The following plot confirms that the standardized residuals are well-behaved:

$$\hat{R}_t = P(s_t=0|\Omega_T)(\mu_0+\mu_0^{72}D72_t+\mu_0^{83}D83_t+\phi_0 R_{t-1})+$$
$$P(s_t=1|\Omega_T)(\mu_1+\mu_1^{72}D72_t+\mu_1^{83}D83_t+\phi_1 R_{t-1})$$

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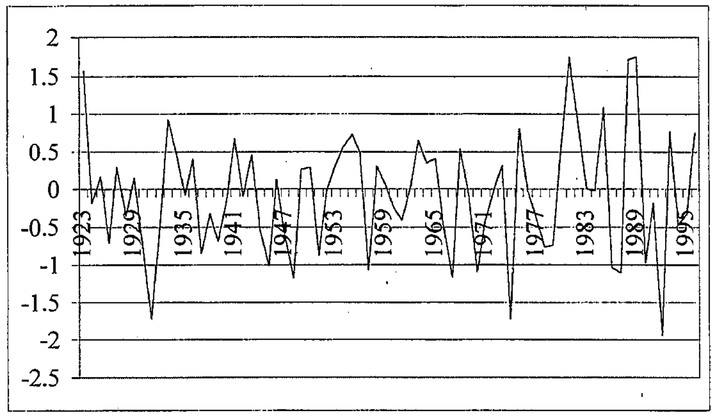
The following plot confirms that the standardized residuals are well-behaved:

Standardized Residuals



3.C. Derivation of (3.5)

$$E(R_{t-1}|s_t=1)=\int R_{t-1}f(R_{t-1}|s_t=1)dR_{t-1}$$
$$=\int R_{t-1}\sum_{j=0}^{1}f(R_{t-1},s_{t-1}=j|s_t=1)dR_{t-1}$$
$$=\int R_{t-1}\sum_{j=0}^{1}f(R_{t-1}|s_{t-1}=j,s_t=1)P(s_{t-1}=j|s_t=1)dR_{t-1}$$
$$=\int R_{t-1}\sum_{j=0}^{1}f(R_{t-1}|s_{t-1}=j)P(s_{t-1}=j|s_t=1)dR_{t-1}$$
$$=\sum_{j=0}^{1}\int R_{t-1}f(R_{t-1}|s_{t-1}=j)P(s_{t-1}=j|s_t=1)dR_{t-1}$$
$$=\sum_{j=0}^{1}P(s_{t-1}=j|s_t=1)\int R_{t-1}f(R_{t-1}|s_{t-1}=j)dR_{t-1}$$
$$=\sum_{j=0}^{1}P(s_{t-1}=j|s_t=1)E(R_{t-1}|s_{t-1}=j)$$
$$=P(s_{t-1}=0|s_t=1)E(R_{t-1}|s_{t-1}=0)+P(s_{t-1}=0|s_t=1)E(R_{t-1}|s_{t-1}=0)$$

3.D. Solutions for $E(R_t^2|s_t=i)$

$$E(R_t^2|s_t=0) = \frac{CD + \phi_0^2(1-q)[(1-\phi_0^2q)E + \phi_1^2pD]}{(1-\phi_0^2q)C}$$

$$E(R_t^2|s_t=1) = \frac{(1-\phi_0^2q)E + \phi_1^2pD}{C},$$

where

$$\begin{aligned} C &= 1-\phi_0^2q-\phi_0^2\phi_1^2p(1-q)-\phi_1^2(1-p)+\phi_0^2\phi_1^2(1-p)q \\ D &= \mu_0^2+2\mu_0\phi_0E(R_{t-1}|s_t=0)+\sigma_0^2 \\ E &= \mu_1^2+2\mu_1\phi_1E(R_{t-1}|s_t=1)+\sigma_1^2 \end{aligned}$$

3.E. Serial Correlation

Unconditional first order serial correlation is defined as (assuming covariance stationarity):

$$Corr(R_t,R_{t-1}) = \frac{Covar(R_t,R_{t-1})}{Var(R_t)}$$

Thus, we need:

$$\begin{aligned} E(R_tR_{t-1}) = {} & \pi_0(q[\mu_0^2+\mu_0\phi_0E(R_{t-1}|s_t=0)+\mu_0\phi_0E(R_t|s_t=0)] + \\ & (1-q)[\mu_0\mu_1+\mu_0\phi_1E(R_{t-1}|s_t=1)+\mu_1\phi_0E(R_t|s_t=1)]) + \\ & \pi_1(p[\mu_0\mu_1+\mu_1\phi_0E(R_{t-1}|s_t=0)+\mu_0\phi_1E(R_t|s_t=0)] + \\ & (1-p)[\mu_1^2+\mu_1\phi_1E(R_{t-1}|s_t=1)+\mu_1\phi_1E(R_t|s_t=1)]) + \\ & (\pi_0q\phi_0+\pi_1p\phi_1)(\phi_0E(R_tR_{t-1}|s_t=0)+\sigma_0^2) + \\ & (\pi_0(1-q)\phi_0+\pi_1(1-p)\phi_1)(\phi_1E(R_tR_{t-1}|s_t=1)+\sigma_1^2) \end{aligned}$$

Hence, we must solve

3.D. Solutions for $E(R_t^2|s_t=i)$

$$E(R_t^2|s_t=0) = \frac{CD + \phi_0^2(1-q)[(1-\phi_0^2 q)E + \phi_1^2 pD]}{(1-\phi_0^2 q)C}$$

$$E(R_t^2|s_t=1) = \frac{(1-\phi_0^2 q)E + \phi_1^2 pD}{C},$$

where

$$\begin{aligned} C &= 1-\phi_0^2 q-\phi_0^2\phi_1^2 p(1-q)-\phi_1^2(1-p)+\phi_0^2\phi_1^2(1-p)q \\ D &= \mu_0^2+2\mu_0\phi_0 E(R_{t-1}|s_t=0)+\sigma_0^2 \\ E &= \mu_1^2+2\mu_1\phi_1 E(R_{t-1}|s_t=1)+\sigma_1^2 \end{aligned}$$

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Hence, we must solve

$$\begin{aligned} E(R_tR_{t-1}|s_t=0) = {} & q(\mu_0^2+\mu_0\phi_0 E(R_{t-1}|s_t=0)+\mu_0\phi_0 E(R_t|s_t=0)+ \\ & \phi_0^2 E(R_tR_{t-1}|s_t=0)+\phi_0\sigma_0^2)+(1-q)(\mu_0\mu_1+\mu_0\phi_1 E(R_{t-1}|s_t=1)+ \\ & \mu_1\phi_0 E(R_t|s_t=1)+\phi_0\phi_1 E(R_tR_{t-1}|s_t=1)+\phi_0\sigma_1^2) \end{aligned}$$

and a similar expression for $E(R_tR_{t-1}|s_t=1)$. The solutions are:

$$E(R_tR_{t-1}|s_t=0) = \frac{CF + (1-q)\phi_0\phi_1[(1-\phi_0^2 q)G + p\phi_0\phi_1 F]}{(1-\phi_0^2 q)C}$$

$$E(R_tR_{t-1}|s_t=1) = \frac{(1-\phi_0^2 q)G + p\phi_0\phi_1 F}{C},$$

where

$$\begin{aligned} F = {} & q\mu_0^2+(1-q)\mu_0\mu_1+q\mu_0\phi_0 E(R_{t-1}|s_t=0)+(1-q)\mu_0\phi_1 E(R_{t-1}|s_t=1)+ \\ & q\mu_0\phi_0 E(R_t|s_t=0)+(1-q)\mu_1\phi_0 E(R_t|s_t=1)+q\phi_0\sigma_0^2+(1-q)\phi_0\sigma_1^2 \\ G = {} & p\mu_0\mu_1+(1-p)\mu_1^2+p\mu_1\phi_0 E(R_{t-1}|s_t=0)+(1-p)\mu_1\phi_1 E(R_{t-1}|s_t=1)+ \\ & p\mu_0\phi_1 E(R_t|s_t=0)+(1-p)\mu_1\phi_1 E(R_t|s_t=1)+p\phi_1\sigma_0^2+(1-p)\phi_1\sigma_1^2 \end{aligned}$$

Inserting these solutions and the results from the previous sections above gives $E(R_tR_{t-1})$. Subtract $E(R_t)^2$ to obtain $Covar(R_t,R_{t-1})$. Similarly for state dependent covariances.

Chapter 4

Modeling the Dividend-Price Ratio: The Role of Economic Fundamentals using a Regime-Switching Approach[1]

with Jan Overgaard Olesen[2]

4.1. Introduction

In empirical finance the dividend-price ratio, defined as the ratio between a given periods dividend payments per share and the end-of-period stock price per share, is often - explicitly or implicitly - used as an indicator of whether stock prices are (too) high or (too) low. For instance, Campbell and Shiller (1998) report a very gloomy prediction for the US stock market based on the fact that the dividend-price ratio has fallen far below its historical mean, suggesting an overvalued stock market. Fama and French (1988) and Hodrick (1992) are other examples of the numerous studies that use dividend-price to forecast future stock returns, see also the survey in Campbell, Lo and MacKinlay (1997,

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chapter 7).

However, according to standard finance theory one should expect time variation in the dividend-price ratio as a result of changes in the underlying economic fundamentals, in particular changes in the (ex ante) real interest rate and the risk premium on stocks (relative to bonds). Hence, it is crucial to consider the economic fundamentals when using the dividend-price ratio to judge whether stocks are fairly valued or not. For this purpose we need an economic model for dividend-price. This is the topic of the present chapter. Motivated by a Gordon growth type model which is modified to incorporate a time-varying discount rate, we formulate an empirical model for the dividend-price ratio using a real interest rate proxy, a proxy for the risk premium and the level of real dividend payments as explanatory variables. The real interest rate and risk premium proxies together capture the effects from the time-varying discount rate while the inclusion of real dividends allows for the possibility that innovations in dividends are reflected less than proportionately in stock prices. We also include lagged dividend-price in the model to allow for slow adjustment in the dividend-price process.

The economic model is estimated for the aggregate Danish stock market, using annual observations for the period 1927-1996. All variables turn out to be significant with the right signs and a reasonably good fit is obtained. However, the model suffers from structural breaks as the coefficients to the explanatory variables are highly unstable. This suggests that we have omitted an important (or several important) fundamental variable(s). In the Danish case a possible explanation for a structural break is a change in investor taxation as of 1983, i.e., the introduction of a separate pension fund tax on bond investments, cf. below, affecting the relative profitability of stock investments. Modifying the economic model in order to take account of the omitted variable is obviously the ideal solution in such a situation. However, in practice this may not always be realistic or even possible because the omitted variable may be difficult (or impossible) to identify and, subsequently, quantify. When modeling the effects of investor taxes in a heterogeneous tax system as the Danish where taxes differ significantly among investor groups, it is essential to correctly identify the 'marginal investor', defined as the stock holder having lowest willingness-to-pay, at every single point of time. However, the 'marginal investor' is unobservable and hence the inclusion of investor taxes in the model is difficult. In the case of the new pension fund tax, matters are, moreover, complicated by the gradual implementation of the tax.

In this chapter, we take a 'short-cut' by estimating the economic model using the two-state regime-switching approach of Hamilton (1990). We consider this approach to be a practical tool of incorporating and indirectly modeling the omitted factor(s) which give rise to the structural breaks that we encounter in the one-regime specification, without having to explicitly model those factors. The regime-switching approach is based on the assumption that the economic model differs across (a finite number of) distinct regimes, whose timing is governed by an exogenous, discrete (and latent) state-variable. This means that the type of omitted factors which we can capture by this approach are the more persistent factors that relate to the 'economic environment' of the model and that result in the outcome of distinct regimes over time with distinct economic models. Such factors often relate to the institutional or policy framework of the economy, leading to distinct policy or institutional regimes over time, and are typically also the factors that are difficult to model. We find in our case that the regimes identified by the regime-switching approach are highly persistent which is consistent with the interpretation that the omitted factor(s) represents changes in the economic environment rather than being a further temporary explanatory variable. In particular, we conjecture that the identified regimes may be given the interpretation of different tax policy regimes.

Beyond providing a practical modeling tool, we also consider the analysis based on the regime-switching approach to be a useful step in identifying the possible omitted factor(s) because the results provide valuable insight regarding the timing of regime-shifts, without being conditioned on apriori information. Hence, the regime-switching model lets the data determine if and when regime shifts occur. This information can consequently be used to identify candidates for omitted factors by examining relevant institutional or policy changes around these dates of regime-shifts[3].

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3 Note that the presence of distinct regimes in the model for dividend-price makes it even harder to judge the level of stock prices from an inspection of dividend-price alone.

The regime-switching approach of Hamilton (1990) has previously been used in the empirical literature to model asset pricing in situations where the pricing process changes over time e.g. due to shifts in the process governing economic fundamentals (for instance as a result of policy regime shifts), shifts in the predominance of different investor types over time or changes in the institutional set up or taxation rules of relevance for the stock market. The importance of regime-shifts in the pricing process has recently been emphasized for the US stock market by Driffill and Sola (1998) who motivate shifts in the pricing process with regime-shifts in the underlying process for dividends, cf. the discussion at the end of this chapter. The possible influence of different investor types with different investment rules has been examined for the currency market by Vigfusson (1996), who assumes that the market on a high-frequency (daily) basis shifts between being driven by chartists and fundamentalists. In the context of the stock market, a potential motivation for time differences in investment and, hence, pricing rules could be that the market misprices stocks in high-inflation regimes by using nominal rather than real interest rates, whereas investors may price stocks more correctly in low-inflation regimes, cf. Modigliani and Cohn (1979), who argue that US stocks were mispriced (undervalued) in the high-inflation regime of the 1970s. In such a setting we should apriori expect the regimes identified by the regime-switching approach to be identical to different inflationary regimes. In this chapter we do not attempt at formally explaining the regime shifts but the working hypothesis motivating the use of the regime-switching approach is that the regime shifts are related to (persistent) changes in the 'economic environment', leading to (persistent) shifts in the economic model linking dividend-price to the economic fundamentals. We think that changes in investor taxation is a prime candidate but institutional changes or changes in the processes for the economic fundamentals leading to changes in expectations formation and hence the economic model[4] may do as well. In any case, a closer examination of the causes

4 To illustrate this point, consider a change in the process for the real interest rate leading to increased short-run volatility. This may imply that investors put less emphasis on the current level of the real interest rate when forming expectations about the future 'long-run, average real interest rate', which is the relevant measure for the pricing of stocks. The implication is a change in the economic model with a smaller

underlying the regime shifts would be interesting but this is left for future work.

Results from estimating the two-state regime-switching model with economic fundamentals show that all the fundamentals variables including the real interest rate and the risk premium are highly significant in at least one regime. Hence, we have succeeded in modeling a time-varying discount rate, here decomposed into a time-varying real interest rate and a time-varying risk premium for stocks, that is significant in explaining dividend-price empirically. This is an innovation compared to the existing empirical literature where the discount rate is either assumed to be fixed or not quantified directly (no closed-form measure) when modeling the behavior of the dividend-price ratio or, more generally, stock prices, cf. e.g. Driffill and Sola (1998), Froot and Obstfeld (1991) and Campbell, Lo and MacKinley (1997, chapter 7). Our model is not perfect in terms of misspecification tests but passes at a 5% significance level, is stable over time and provides a rather good fit to dividend-price. Moreover, results show that two regimes are both necessary and sufficient to remove the structural breaks from the underlying economic model. The model clearly identifies 3 distinct subperiods over which the regimes reign (1927-1949, 1950-1985 and 1986-1991), thereby providing valuable insight which can be used as a basis for future work on inferring the possible causes of the two distinct regimes.

The outline of the chapter is as follows. In section 4.2 we formulate an operational empirical model, based on a simple, ad-hoc theoretical framework which is derived from the standard Gordon growth model by allowing for a time-varying discount rate. The data is reviewed in section 4.3. In section 4.4 we first estimate the economic model under the assumption that only one regime applies, i.e., assuming that the model is stable over the entire sample. In section 4.5 we estimate the regime-switching model allowing for 2 distinct regimes over the sample. Section 4.6 finally concludes the chapter.

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4.2. The empirical model

In formulating the empirical model, we take as a starting point the textbook 'Gordon growth model' for the price of a stock with a constant discount rate and constant expected dividend-growth, see e.g. Gordon (1962) or Campbell, Lo and MacKinlay (1997). We modify Gordon's model in a rather simple way to allow for time variation in the discount rate, reflecting time variation in both the real interest rate and the risk premium on stocks. The resulting theoretical framework is ad hoc but allows us to formulate an operational empirical model with specific candidates for economic variables that may explain dividend-price. The theoretical model can be given the interpretation that market participants at each point in time price stocks according to the constant-discount-rate/constant-dividend-growth Gordon model, i.e., as if the discount rate and dividend-growth were in fact constant, while using the prevailing levels for nominal bond returns, expected nominal dividend growth and the risk premium on stocks as inputs[5].

Thus, let equilibrium in stock and bond markets at each point in time t be described by a no-arbitrage relation stating that the expected (nominal) return on stocks $E_t[S_{t+1}]$ from time t to t+1 should be equal to the corresponding (nominal) return on bonds B_t augmented by a risk premium γ_t on stocks relative to bonds:

$$E_t[S_{t+1}] = B_t + \gamma_t \tag{4.1}$$

We take B_t to be the yield-to-maturity on a one-period bond so that it is predetermined and known as of time t.

The return on stocks is given as the sum of capital gains and

[5] Campbell and Shiller (1988) have generalized Gordon's growth model to take account of a stochastic, time-varying discount rate, the so-called 'dynamic Gordon growth model'. However, their model is - at least in its general version - not as operational as the one we set up. In particular, the Campbell and Shiller (1998) model does not entail a closed-form expression for the time-varying discount rate. Our assumptions on expectations formation imply that stocks can be priced within the original Gordon model despite the fact that the discount rate (and dividend growth) vary over time.

dividend yield:

$$E_t[S_{t+1}] = \frac{E_t[P_{t+1}] - P_t}{P_t} + \frac{E_t[D_{t+1}]}{P_t} \tag{4.2}$$

where P_t is the *ex dividend* price per share as of time t (i.e., at the beginning of period t+1) while D_{t+1} is the dividend payment per share paid during period t+1.

Even though B_t and γ_t are allowed to vary stochastically over time, we shall assume that market participants only form point estimates when forming expectations wrt. future bond returns and risk premia, i.e., 'Certainty Equivalence' is assumed to apply. Moreover, we assume that market participants expect bond returns and risk premia to be constant over time, so that any innovations in the two factors are viewed as being permanent. These assumptions - while clearly restrictive in a theoretical setting - allow us to set up an empirically tractable model. Thus, under the additional Gordon assumption of constant expected dividend growth, (4.1) can be solved by forward recursion to give the following no-bubble solution for the dividend-price ratio (assuming that $R_t+\gamma_t > 0$):

$$\frac{D_t}{P_t} = \frac{R_t + \gamma_t}{1+G_t} \approx R_t + \gamma_t \quad , \quad where \quad R_t \equiv B_t - G_t \tag{4.3}$$

G_t is the expected nominal growth in dividends per share as of time t. G_t is also allowed to vary over time. According to (4.3), the dividend-price ratio is in equilibrium equal to the sum of the (ex ante) growth-adjusted real interest rate $R_t \equiv B_t$-G_t and the risk premium on stocks γ_t. (4.3) resembles the solution of the standard (constant discount rate) Gordon growth model with the main difference being the allowed variation in the real interest rate and the risk premium and thereby the appropriate discount rate (the sum of the two).

Based on (4.3) we set up the empirical model:

$$\frac{D_t}{P_t} = \beta_0 + \beta_1 R_t + \beta_2 \gamma_t + \beta_3 DR_t + \beta_4 \frac{D_{t-1}}{P_{t-1}} + \varepsilon_t \tag{4.4}$$

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where ϵ_t is the residual of the equation. We have augmented the empirical model with the lagged dividend-price ratio $(D/P)_{t-1}$ and the log-level of real dividends per share DR_t as further potential explanatory variables. The introduction of the former allows for slow or partial adjustment in the dividend-price ratio so that (4.3) (or rather the long-run solution to (4.4)) is thought of as a model for the long run, providing us with an equilibrium relation to which dividend-price adjusts in the long run. The introduction of DR_t allows for the possibility that real stock prices may react more or less than proportional to innovations in real dividend payments. According to (4.3), the relation between real stock prices and real dividends should be proportional as the dividend-price ratio is unaffected by innovations in dividends. The reason is that market participants expect any innovation in current dividends to be permanent under the Gordon constant-dividend-growth setting. However, this may not be the case empirically[6]. Froot and Obstfeld (1991) and Driffill and Sola (1998) also include real dividends in their models for price-dividend with the motivation that the real dividend component captures the possibility of 'intrinsic bubbles' in stock prices, i.e., rational bubbles that depend on fundamental variables. As standard in econometric work, we allow for a constant term in (4.4), even though not strictly implied by the theoretical model. Hence, we intend to explain the variations in rather than the actual levels of the dividend-price ratio.[7]

The challenge facing (4.4) is the fact that the real interest rate R_t and the risk premium γ_t are unobservable. We therefore have to use suitable proxies for these two variables, cf. below.

4.3. The data

The data are depicted in Figures 4.1-4.4. The source database is Nielsen, Olesen and Risager (1999) which comprises data for the Danish stock and bond markets. Stock market data relate to the aggregate market level

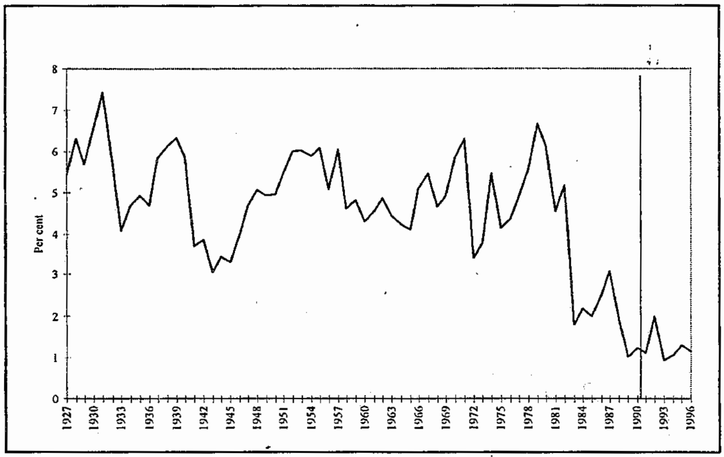
6 We use real dividends rather than nominal dividends because the dividend-price ratio is a real variable.

7 Note that according to the constant-discount-rate Gordon model, $\beta_1=\beta_2=\beta_3=\beta_4=0$.

of all Danish firms listed at the Copenhagen Stock Exchange. The market index by Statistics Denmark is used for stock prices while dividend payments are estimated from a large sample of firms, cf. Nielsen, Olesen and Risager (1999) for further details. Bond data relate to the markets for government bonds. All observations are annual. The empirical analysis in the following sections uses the sample period 1927-1991 which is the longest available sample for all variables.

Figure 4.1 shows the Dividend-Price Ratio over the period 1927-1996. The plot suggests a cyclical component in the ratio with large and often persistent deviations from its sample mean in particular in the first half of the period. For instance, stock prices seem to have been persistently low compared to dividends in the first half of the 1950s while stock prices were high during World War II. In relative terms the ratio is often subject to large year-by-year changes where in particular the drop in the ratio from 5.2 pct. in 1982 to 1.8 pct. in 1983 (a decrease of 65 pct. in relative terms) attracts attention. This drop in dividend-price which is a result of capital gains on stocks of 114 pct. that year coincides with at least two important events. First of all, there was a major shift in economic policy as a new conservative-liberal government came into

Figure 4.1. Dividend-Price Ratio, 1927-96



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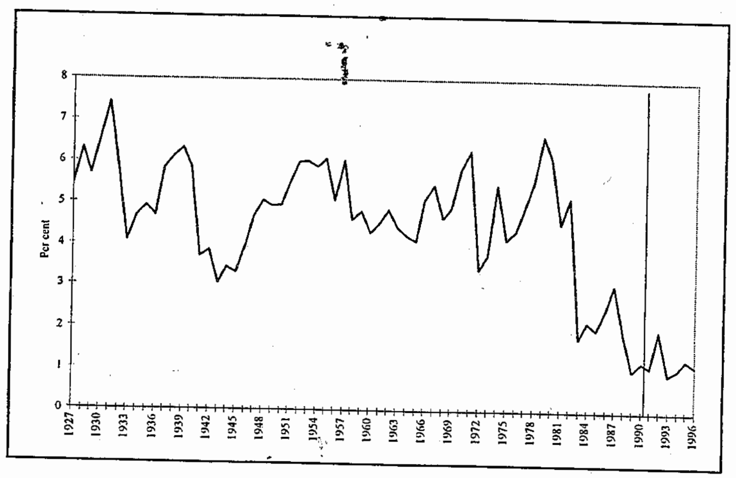

office in October 1982, emphasizing tight economic policies including a fixed exchange rate policy. Second, a new tax was introduced on the returns on pensions funds' bond holdings while the returns on stocks were exempted from taxation[8]. This ceteris paribus gave pension funds an incentive to invest more in stocks and less in bonds. It can be noted that the dividend-price ratio has been at a historically low level since 1983. The post-1983 average is 1.7 pct. which compares to an average of 5.1 pct. over the years before 1983. This low level is a key issue in understanding what drives the dividend-price ratio and it is in particular of interest to know whether the persistent low level can be explained by economic fundamentals or whether it marks a new regime compared to the pre-1983 history.

The proxy that we use for the latent real interest rate R_t is plotted in Figure 4.2. The real interest rate as of time t is constructed as the 5 year yield-to-maturity on government bonds at time t minus the realized-growth in nominal dividends over the (corresponding) 5 year period following time t. The proxy is therefore an *ex post (or perfect foresight)* growth-adjusted real interest rate. Adjustment for inflation and real growth is done wrt. (actual) growth in nominal dividend payments, the relevant measure according to the theoretical framework of section 4.2. Because of the forward-looking nature of the real interest rate proxy we loose 5 observations towards the end of the sample period so that the effective sample for the empirical analysis becomes 1927-1991[9].

As evident from Figure 4.2, the real interest rate proxy is highly volatile. The fluctuations are mainly driven by the variation in the

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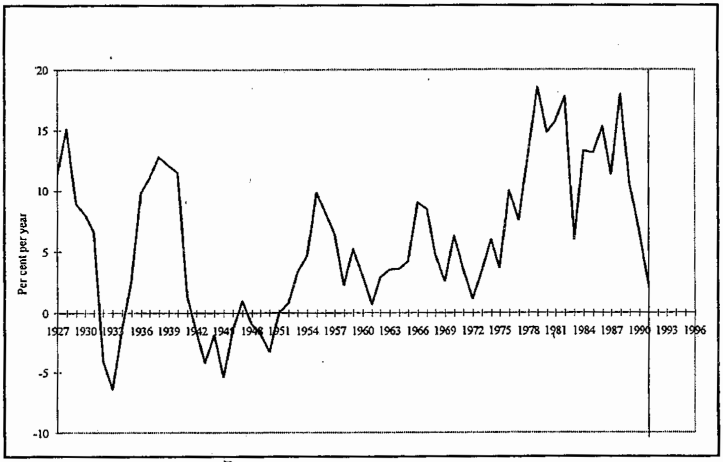
8 The new tax was decided in 1983 and came into effect as of Jan 1 1984. Because pension savings before 1984 were exempted from taxation, the tax was phased in gradually.

9 On grounds of (lack of) data availability we had to choose between the 1, 5 and 10 year horizons. We excluded the 10 year horizon because it would imply a loss of too many observations towards the end of the sample. We excluded the 1 year horizon because the resulting 1-year proxy turned out to be a very 'noisy' measure with large year-to-year variability and no explanatory power wrt. the dividend-price ratio. It can also be argued that 1 year is too short a maturity to be of relevance for the pricing of stocks.

Figure 4.2. Growth-Adjusted Real Interest Rate, 1927-91



dividend growth part of the proxy, whereas the 5-year bond return is relatively stable throughout most of the sample. For instance, the low levels of the real interest rate in the 1940s is due to high future dividend growth that is not accompanied by higher nominal bond returns. Due to non-credible economic policy-making amongst other things, the Danish economy experienced very high nominal and real interest rates towards the end of the 1970s and in the beginning of the 1980s. Nominal interest rates declined following the new policy regime as of late 1982, but nominal dividend growth declined likewise, sustaining the high real interest rate level until the end of the 1980s.

We also need a proxy for the risk premium on stocks γ_t. For this purpose we draw on Olesen and Risager (1999) who examine whether the Danish premium on stocks defined as the excess of stock returns over bond returns can be predicted from a set of possible predictor variables such as the dividend price ratio, dividend yield, bond returns, lagged equity premia etc. They conclude that the 5-year premium on stocks is predictable from the dividend yield, the 5-year bond return and past 1-year equity premia, see Olesen and Risager (1999) for details. This predicted or fitted 5 year premium can in an efficient markets framework be interpreted as an estimate of the risk premium on stocks

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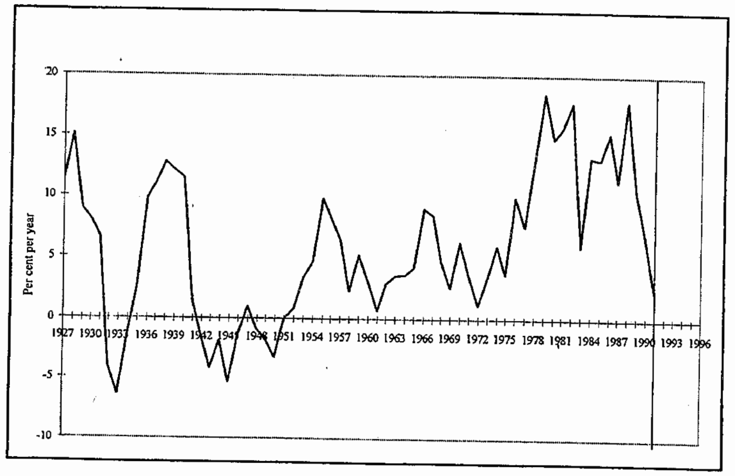

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relative to bonds. However, Olesen and Risager (1999) use the dividend yield as a predictor, and the dividend yield comes close to the dividend-price ratio variable. In terms of (4.4) one could therefore possibly argue that using the fitted premium in Olesen and Risager (1999) as the risk premium proxy γ_t we would basically be explaining the dividend-price ratio with a variable that comes close the ratio itself, the dividend yield.

In order to be immune to this critique, we have therefore estimated a predictor model without the dividend yield and the dividend-price ratio as potential predictor variables. The resulting model is (standard errors in parentheses)[10]:

$$\hat{PR5}_t = \underset{(0.727)}{2.804} - \underset{(0.023)}{0.113} * PR1_{t-1} - \underset{(0.037)}{0.106} * PR1_{t-2} - \underset{(0.030)}{0.093} * PR1_{t-3} \quad (4.5)$$

where $PR1_t$ and $PR5_t$ are the equity premia, calculated as the simple difference between stock and bond returns, over the 1 year, respectively 5 year holding period starting at time t. $\hat{PR5}_t$ is the 5-year premium predicted or fitted from the model. According to (4.5) the 5-year premium on stocks can be predicted from the preceding 3 years of 1-year equity premia[11]. It in fact turns out the prediction from (4.5) comes close to that of Olesen and Risager (1999) in particular wrt. the significant

10

Following the approach of Olesen and Risager (1999), (4.5) is estimated in a 'general-to-specific' manner by first estimating a full model where the 5-year premium is regressed on all potential predictor variables (bond returns, term structure components, past 1-year equity premia), after which insignificant predictors are removed successively, using a 5% significance level. (4.5) is the resulting parsimonious model. All parameters are estimated by OLS while Newey-West standard errors which are consistent to heteroskedasticity and serial correlation in the disturbance term up to lag 5 are used as standard errors of the coefficient estimates. The sample is the available period 1927-1992, using overlapping observations. (4.5) explains 36% (=R^2) of the variation in the actual 5-year equity premium. The residual has a standard deviation of 4.5%. Notice that we differ from Olesen and Risager (1999) by using absolute rather than logarithmic returns.

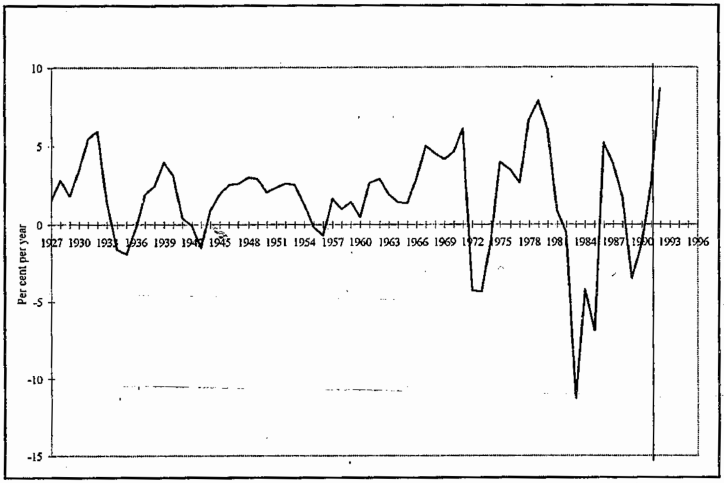
11

It should be noted that these past 1-year equity premia do not overlap with the future 5 year investment horizon and therefore do only contain historical information.

movements and turning points[12]. The prediction in (4.5) is used as the proxy for the risk premium, i.e., $\gamma_t \equiv \hat{PR5}_t$. Note that from (4.1) the risk premium γ_t should actually be equal to the predicted premium on stocks so that the proxy chosen is consistent with the theoretical framework.

γ_t is plotted in Figure 4.3.

Figure 4.3. Risk Premium on Stocks, 1927-1992



The risk premium proxy also turns out be highly volatile, in particular towards the end of the sample. Note the large drop in the risk premium in the beginning of the 1980s which partially coincides with the shift in the economic policy regime, cf. above. The large negative

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12

See the 5-year model in Olesen and Risager (1999). A model similar to (4.5) can actually be found in Olesen and Risager (1999) as one of the 'single-variable' models for the 5 year horizon, see their Table 2 (entry 7). The two models differ, however, because Olesen and Risager (1999) use logarithmic returns (log to one plus returns), where we use simple returns in compliance with the theoretical model.

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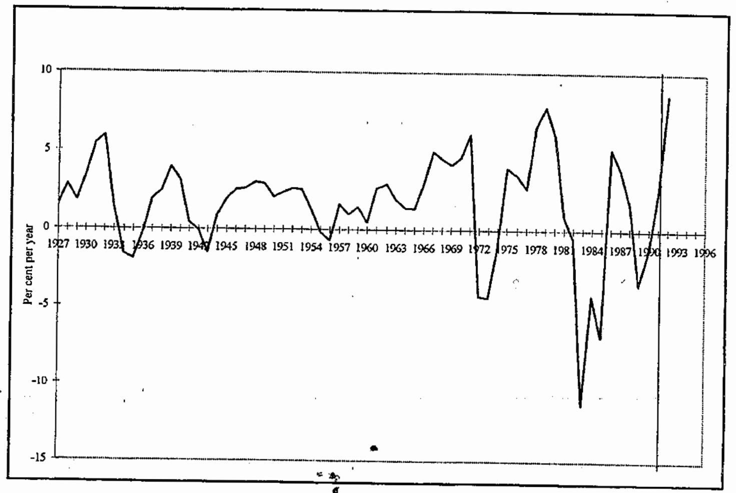

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risk premia in the years 1983-1985 may at least partially be explained by the introduction of the new pension fund tax on bond returns, which induces pension funds to demand a smaller 'risk' premium before-tax on stocks relative to bonds. To see this, note that in presence of the pension fund tax the no-arbitrage relation between stock and bond returns changes from (4.1) to

$$E_t[S_{t+1}] = (1-\tau)B_t + \gamma_t^* \qquad (4.1')$$

assuming that a pension fund is the representative (marginal) investor. τ is here the pension fund tax on bond returns, $(1-\tau)B_t$ is the bond return after tax and γ_t^* denotes the 'pure' after-tax risk premium on stocks. By (4.1) and (4.1'), the before-tax (risk) premium on stocks γ_t is related to the after-tax premium as $\gamma_t = -\tau B_t + \gamma_t^*$, so that the introduction of the pension fund tax ceteris paribus lowers the before-tax premium. For sufficiently high bond returns B_t - and bond returns were still high in the years 1983-1985 - the premium may even become negative[13 14]. Note

13 The term 'risk premium' is not entirely adequate for γ_t in presence of the pension fund tax as γ_t both captures the actual or 'true' risk premium γ_t^* and the distortionary tax effect $-\tau B_t$.

14 Using (4.1') instead of (4.1), the with-tax solution for the dividend-price ratio becomes:

$$\frac{D_t}{P_t} \approx (1-\tau)B_t - G_t + \gamma_t^* = B_t - G_t + \gamma_t \qquad (4.3')$$

where the final equation follows from the relationship between γ_t and γ_t^*.(4.3') is actually identical to the without-tax solution in (4.3). Thus, in terms of the theoretical framework of section 4.2 the introduction of the pension fund tax does not change *the structural equation* for dividend-price, the reason being that we in the equation use the before-tax 'risk premium' γ_t which fully incorporates the stock price effects of the new tax. Note, however, that the pension fund tax - ceteris paribus - lowers the level of dividend-price by lowering γ_t. Moreover, it is crucial for the result that the (real) interest rate and the risk premium have the same quantitative effects on dividend-price. Thus, allowing for taxes in the empirical model (4.4) (replacing R_t and γ_t with the after-tax real interest rate $(1-\tau)B_t - G_t$ and the 'true' risk premium, γ^*, respectively, and that we could in principle construct a proxy for the 'pure' risk premium γ_t^* if we knew the relevant tax rate τ for each year in the sample. However, constructing data for τ is complicated both by interim arrangements for the pension fund tax and by the fact that we need to know the relevant but latent 'marginal investor'. We therefore use the 'before-tax' proxy γ_t.

As evident from figures 4.2 and 4.3 both the real interest rate proxy and the risk premium proxy turn out to be negative in some of the years and also the sum of the two proxies turn out to be negative occasionally. The latter obviously does not make sense in terms of the theoretical framework of section 4.2 which requires the sum $R_t+\gamma_t$ to be strictly positive in order to result in a well-defined (finite) forward-looking stock price solution. The estimation results in the following sections show that we should not confine ourselves that strictly to the theoretical model. In particular the results suggest that market participants - in contrast with the theory - expect a significant degree of 'mean reversion' in the real interest rate and the risk premium so that negative values for the current real interest rate and the current risk premium may be perfectly valid because it is expected to be a temporary phenomenon. In terms of the empirical model (4.4), what matters is the variation of the real interest rate and the risk premium proxies (in which we may have more confidence) rather than the actual levels as we have (as standard) included a constant term.

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rewriting) we get

$$\frac{D_t}{P_t} = \beta_0+\beta_1 R_t+\beta_2\gamma_t+(\beta_2-\beta_1)\tau B_t+\beta_3 DR_t+\beta_4\frac{D_{t-1}}{P_{t-1}}+\epsilon_t \qquad (4.4')$$

The pension fund tax leaves the structural model unchanged iff the coefficients β_1 and β_2 are identical. If the coefficients differ, a further explanatory variable τB_t capturing the tax distortion is introduced into the model. As the estimation results show, the latter is in fact the case empirically and we should apriori expect a regime-shift in the empirical model (as the extra variable is not included). To conclude, in a more general (theoretical and empirical) setting than (4.3) we cannot be sure that the structural model for dividend-price will be unaffected by the pension fund tax and the question of whether the model survives the introduction of the tax basically becomes an empirical issue.

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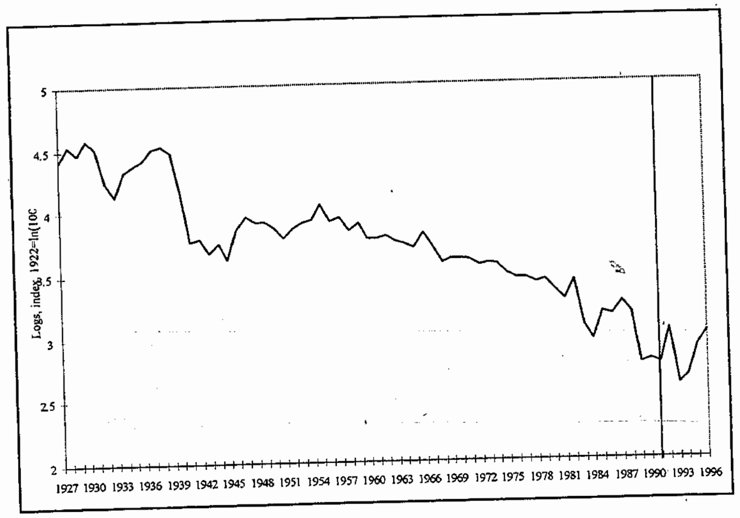
Finally, Figure 4.4 depicts the log-level of real dividend payments. Dividends show some turbulence in the beginning and towards the end of the sample but have otherwise shown a steady declining trend.

Figure 4.4. (Log)-Real Dividend Payments Per Share, 1927-96



4.4. Results using a one-regime approach

Column 2 in Table 4.1 shows the results from estimating (4.4) over the whole sample, assuming that only one regime prevails. The estimations are done by the Maximum Likelihood (ML) method under the assumption that the disturbance term of (4.4) is normal and independently distributed with homoskedastic variance (ϵ_t ~ Nid(0,σ^2)). The ML coefficient estimates correspond to those that would be obtained by OLS.

Using the ML standard errors, all coefficients are highly significant. The real interest rate and the risk premium have the expected positive

Table 4.1. Maximum Likelihood Estimates and Specification Testing: Models with and without Regime-Switching

		One-Regime Model	Regime-Switching Model	
Parameter estimates			Regime #1	Regime #2
Constant term	β_0	-2.7186 **	-5.8118 **	-4.8685 **
		(0.6976)	(0.6552)	(1.615)
Real interest rate	β_1	0.0345 **	0.0172	0.0725 **
		(0.0127)	(0.0112)	(0.0195)
Risk premium	β_2	0.1444 **	0.1399 **	0.1535 **
		(0.0250)	(0.0418)	(0.0242)
Real dividends	β_3	1.2848 **	2.3701 **	2.1473 **
		(0.2202)	(0.2414)	(0.4386)
Lagged D/P	β_4	0.4433 **	0.1143	0.2456 **
		(0.0728)	(0.0876)	(0.0913)
Variance	σ^2	0.3719	0.1255	0.2296
		(0.0652)	(0.0366)	(0.0525)
Transition probability	p_{ii}	—	0.9740	0.9626
			(0.0291)	(0.0295)
Ergodic probability		—	0.5901	0.4099
Log-likelihood		-60.0815	-43.7682	
AIC		132.2	115.5	
HQ		137.3	127.6	
SC		145.2	146.0	
White specification test [1]				
Autocorrelation	F(4,51)	2.737 (0.103)	2.333 (0.068)	
ARCH	F(4,51)	1.970 (0.166)	1.834 (0.137)	
Markov specification	F(4,51)	—	2.199 (0.082)	
LM specification test [1]				
Autocorr. regime #1	F(1,51)	—	0.529 (0.470)	
Autocorr. regime #2	F(1,51)	—	2.659 (0.109)	
Autocorrelation	F(1,51)	2.732 (0.104)	1.249 (0.269)	
ARCH regime #1	F(1,51)	—	2.493 (0.121)	
ARCH regime #2	F(1,51)	—	4.844 (0.032) *	
ARCH	F(1,51)	1.913 (0.172)	0.287 (0.595)	
Standardized residuals [1) 2)]				
AR(1)	F(1,63)	3.106 (0.083)	0.863 (0.356)	
AR(3)	F(3,61)	5.519 (0.002) **	2.402 (0.076)	
AR(5)	F(5,59)	4.543 (0.002) **	1.403 (0.237)	
Normality	$\chi^2(2)$	2.810 (0.245)	3.775 (0.151)	
Andrews test for structural break [3]		23.009 **	8.964 *	

Note: Asymptotic standard errors of parameter estimates shown in parentheses, based on second derivatives of log likelihood function. A '*' shows significance at the 5% level, '**' at 1% level. The Akaike, Schwarz and Hannan-Quinn model selection criteria are calculated as: $AIC=-2l+2k$, $HQ=-2l+2ln(ln(T))k$, and $SC=-2l+kln(T)$, where l is the log-likehood value, k is the number of freely estimated parameters and T is the number of observations.

1) Test distributions apply to regime-switching model. For one-regime model, White and Lagrange Multiplier (LM) tests are distributed F(1,59). Tests are small-sample approximations based on the F-distribution, as suggested by Hamilton (1996). Critical significance levels in parentheses. The White and LM tests are described in Hamilton (1996).

2) For regime-switching model, the serial correlation (AR) tests are standard LM specification tests applied to a regression of the standardized residuals on a constant term. For one-regime model, standard LM tests on the regression equation. Normality test by Doornik and Hansen (1994).

3) Asymptotic critical test values are 8.85 (5% significance level) and 12.35 (1%), see Andrews (1993).

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effects on the dividend-price ratio. The magnitudes are, however, less than predicted by theory. This applies both to the 'short run effects' (coefficients of 0.0345 and 0.1444, respectively) and the 'long run effects' (0.062 and 0.259, respectively) where we take account of the apparent slow adjustment in the dividend-price process, cf. below[15]. According to the theoretical framework of section 4.2 we should have a coefficient of one for both variables and this is far higher than the point estimates and what the uncertainty of the coefficient estimates allows for. The result is no surprise when inspecting the data plots in Figures 4.1-4.3. The variation intervals for the real interest rate and the risk premium are much larger than for the dividend-price ratio, implying that the effects will be less than one if the former turn out to be significant. This suggests that market participants do not expect innovations in the two variables to be permanent, as assumed in the theoretical framework, but that they on the contrary expect some significant degree of 'mean-reversion' in the real interest rate and the risk premium, implying that the current levels (or rather deviations in the current levels from the two variables' means) receive less importance[16]. The 'mean-reversion' feature seems perfectly reasonable from the time series behavior of the two variables, cf. figures 4.2 and 4.3[17].

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15 By dividing through by one minus the autoregressive coefficient of 0.4433, the 'long-run equilibrium model' becomes (ignoring the residual term):

$$\frac{D_t}{P_t} = -4.883 + 0.062\ R_t + 0.259\ \gamma_t + 2.308\ DR_t$$

16 In the context of the theoretical framework, one could interpret $\beta_1 R_t$ and $\beta_2 \gamma_t$ (rather than R_t and γ_t) as the relevant real interest rate and the relevant risk premium, respectively, defined as the market participants' expected 'long-run average' real interest rate and risk premium, where the latter two are the relevant measures for the pricing of stocks.

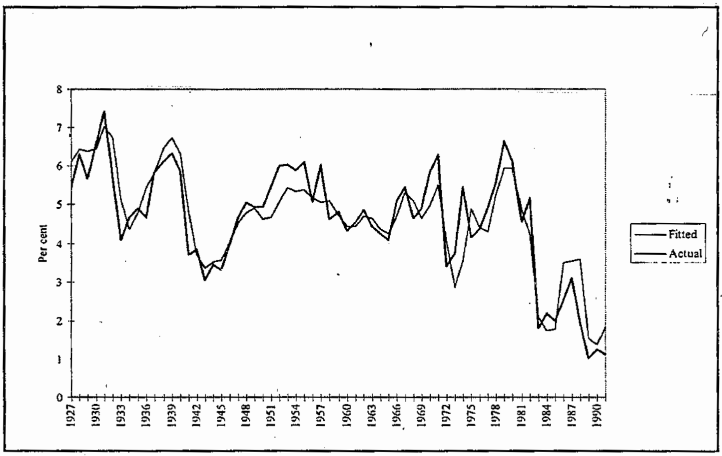
17 Of course, the result could also suggest that the proxies used for the real interest rate and the risk premium are poor (too volatile). However, the high significance of the

Real dividends also have a significant effect on dividend-price. The effect is positive, implying that an increase in real dividends gives rise to a less than proportional increase in real stock prices. This is, again, in conflict with theory and suggests that market participants do not view innovations in dividends as being permanent either (as assumed in the theoretical framework), but expect some degree of 'mean reversion' in dividends.

Finally, the significance of lagged dividend-price indicates slow or partial adjustment in the dividend-price process.

Figure 4.5 illustrates the fit of the model. The one-regime model seems to work reasonably well and is in particular able to track the significant drop in dividend-price in 1983. There are, however, also episodes of systematic under- or overvaluation of dividend-price, cf. for instance the periods 1946-1956 and 1985-1991.

Figure 4.5. Dividend-Price Ratio: Actual and Fitted One-Regime Model



The model passes the White and LM specification tests for serial correlation (of lag 1) and heteroskedasticity (ARCH) at conventional

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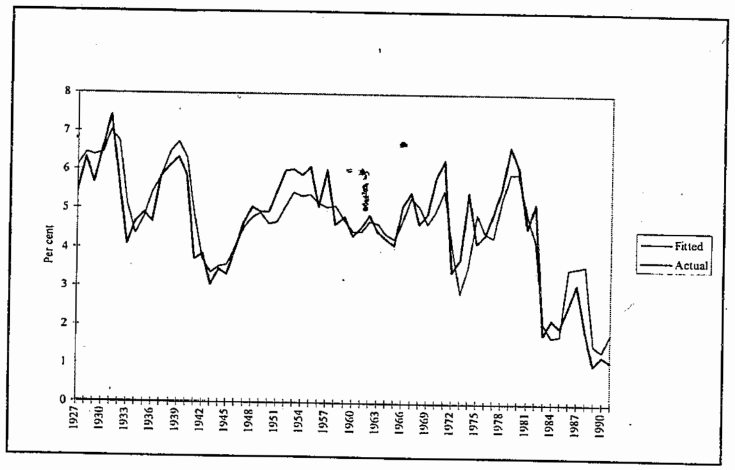

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significance levels, see the bottom half of Table 4.1[18]. There is, however, strong evidence of serial correlation at higher lags (AR(3) and AR(5)) leading to a rejection of the model. Note that the documented serial correlation implies that the coefficient estimates are inconsistent, given the presence of the lagged dependent dividend-price as a regressor. The coefficients should therefore be interpreted with caution.

Another severe problem with the model is that is highly unstable over time. Figures 4.6-4.10 show recursive estimates of the model coefficients including 95% confidence bands, obtained by recursive least squares. With the exception of the risk premium, the coefficients are very unstable and there is strong indication of structural breaks in the model taking place both in the beginning and towards the end of the sample.

The apparent instability of the model can be further documented by formal testing. The Andrews test, see Table 4.1, allows one to perform a test for structural break without having to pre-specify a candidate time for a breakpoint, see Andrews (1993) and Hamilton (1996) for details. The Andrews test procedure basically performs a LM test for a shift in the mean for each time point in the sample, excluding the first 15% and the last 15% of the observations. One then chooses the observation with the highest LM test value and compares with critical test values, as tabulated in Andrews (1993). The evidence for the one-regime model is a clear indication of a (at least one) structural break in the sample with a test value of 23 compared to critical values of 8.85 (5% significance level) and 12.35 (1%) (where the latter seems most appropriate in small samples, cf. Hamilton (1996)). The highest test value is attained for the year 1947[19].

To conclude, the estimation results suggest that we have found some economic fundamentals that have power in explaining the variability of the dividend-price ratio, including the large drop in dividend-price in

18 The tests are documented in Hamilton (1996). We use the suggested small-sample versions of the tests whereby the asymptotic test is transformed to a small-sample test based on the F-distribution. The tests for serial correlation are tests for AR(1) in the disturbance term.

19 The individual LM test statistics for each observation in the sample are reported in the Appendix to this chapter.

Figures 4.6-4.10. Recursive Parameter Estimates for One-Regime Model
Recursive point estimates (bold line) and 95% confidence band limits, 1942-1991. Sample start in 1927. Recursive least squares.

Figure 4.6 Constant Term

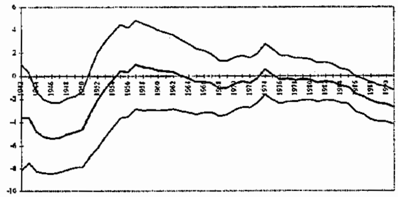


Figure 4.7 Real Interest Rate

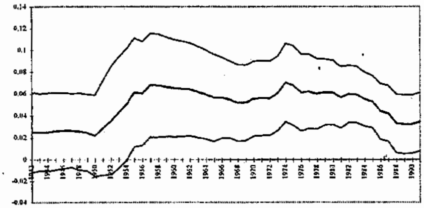


Figure 4.8 Risk Premium

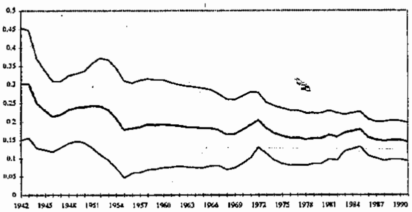


Figure 4.9 Real Dividends

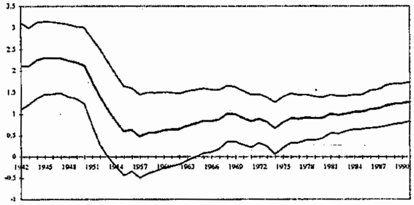
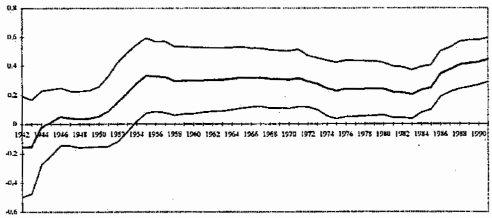


Figure 4.10 Lagged Dividend-Price Ratio



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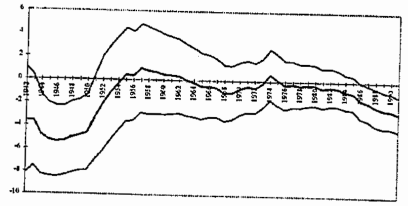


Figure 4.7 Real Interest Rate

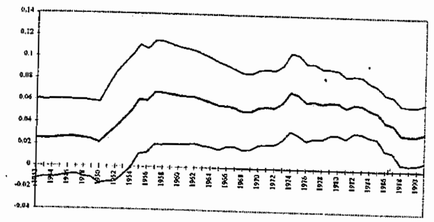


Figure 4.8 Risk Premium

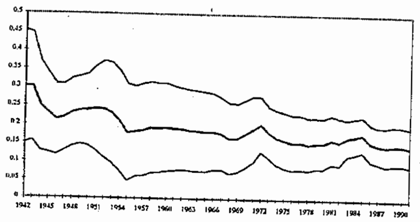


Figure 4.9 Real Dividends

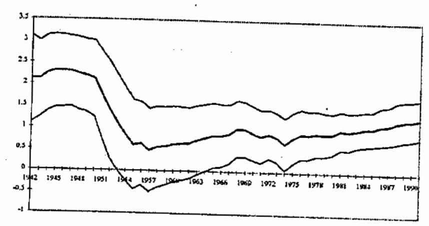
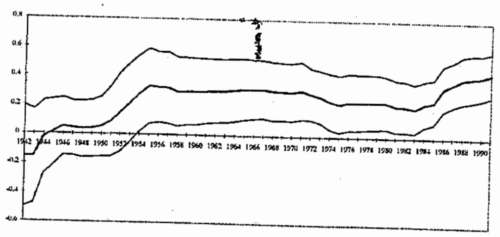


Figure 4.10 Lagged Dividend-Price Ratio



1983. There are, however, specification problems with the model and there is in particular strong evidence that the one-regime model is unstable over time, suggesting that more than one regime applies over the sample period.

4.5. Results using a regime-switching approach

Motivated by the analysis of section 4.4 and in particular the apparent instability of the structural model (4.4) over the sample period, we now estimate a model that allows for more than one regime. A regime is here defined as a sub-period (or several sub-periods) over which (4.4) is stable, meaning that the coefficients of the different economic factors (including the constant term) and the explanatory power of the model (as measured by the residual variance) are constant. A regime shift takes place whenever the underlying structural framework (4.4) for dividend-price changes either because of a change in the importance of the different fundamentals or because of a change in the part of the volatility in dividend-price that is not explained by the model. We use the Markovian regime-shifting model developed by Hamilton (1990). This approach has the advantage of letting the data - as opposed to apriori information - determine whether there are more than one regime and - if affirmative - when the regime shifts take place. In order to keep the model as simple as possible we only allow for two regimes from the outset and subsequently test whether two regimes are sufficient to eliminate the apparent structural breaks in (4.4).

Under the regime-shifting approach the economy can at each point of time be in one of two possible states, as indexed by an unobservable state-variable s_t which takes on the values 1 or 2[20]. Each regime is described by a distinct model for the dividend-price ratio:

20 For a detailed outline of the regime-switching model including the statistical foundations we refer to Hamilton (1990), Hamilton (1996) or the textbook exposition in Hamilton (1994). Numerous applications of the model can be found, including those in Driffill and Sola (1998), Engel and Hamilton (1990) and Hamilton and Lin (1996).

$$\frac{D_t}{P_t} = \beta_0(s_t)+\beta_1(s_t)R_t+\beta_2(s_t)\gamma_t+\beta_3(s_t)DR_t+\beta_4(s_t)\frac{D_{t-1}}{P_{t-1}}+\sigma(s_t)\epsilon_t, \quad s_t=1,2 \tag{4.6}$$

where the parameters depend on the prevailing state s_t. Note that (4.6) is identical to (4.4) except for the state-dependence so that the underlying economic framework is fundamentally unchanged. The crucial difference in (4.6) is that we here operate with (possibly) two distinct models which differ wrt. parameters, i.e., the coefficients (including the constant term) $\beta_i(s_t)$ and the residual variance $\sigma(s_t)^2$.

What model applies at a given point of time is governed by the state-variable s_t. s_t is stochastic and is assumed to follow a (2-state) Markov Chain with constant transition probabilities p_{ij}, defined as the probability of being in state or regime j in period t conditional of having been in state i in period t-1, i.e., $p_{ij}\equiv Pr\{s_t=j|s_{t-1}=i\}$ $(i,j=1,2)$. s_t is by assumption independent of the residual term ϵ_t of (4.6) across all time periods, so that the state-process is 'purely' exogenous to the dynamics of dividend-price.

Under the assumption that ϵ_t is independent standard normal (ϵ_t~Nid(0,1)), we can estimate (4.6) by Maximum Likelihood, see e.g. Hamilton (1994, section 22). The results are shown in columns 3 and 4 in Table 4.1[21].

First of all we note that all coefficients have the expected signs. In regime 2 all coefficients can be shown to be significant at the 1 per cent significance level, whereas the real interest rate and lagged dividend-

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The maximum likelihood estimation is done under the assumption that the state probabilities of the initial observation is given by the ergodic probabilities. Estimation of the initial probabilities does, however, not change the results. The computations are done with the BFGS algorithm in GAUSS using a variety of different starting values for the algorithm. We identify more than one local maximum (approximately 5) depending on the starting values and, moreover, encounter a singularity problem of the likelihood function, that is, for certain starting values the likelihood becomes 'large' without convergence as one of the regime-dependent variances goes to zero. The results of Table 4.1 apply to the local maximum with the highest likelihood. This choice is consistent with Kiefer (1978) who in the context of the mixed-distribution model - where a global maximum does not exist - shows that there is a bounded local maximum of the likelihood function (with variances being positive) that exhibits the usual maximum likelihood properties of being consistent and asymptotically efficient.

$$\frac{D_t}{P_t} = \beta_0(s_t)+\beta_1(s_t)R_t+\beta_2(s_t)\gamma_t+\beta_3(s_t)DR_t+\beta_4(s_t)\frac{D_{t-1}}{P_{t-1}}+\sigma(s_t)\epsilon_t,\ s_t=1,2 \quad (4.6)$$

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First of all we note that all coefficients have the expected signs. In regime 2 all coefficients can be shown to be significant at the 1 per cent significance level, whereas the real interest rate and lagged dividend-price turn out to be insignificant in regime 1[22]. The two remaining factors (risk premium and real dividends) are highly significant in regime 1 also. In fact the coefficient estimates for the risk premium and real dividends are quite close across regimes. This suggests that we have two (regime-dependent) underlying models for dividend-price, one in which there is partial adjustment in dividend-price and where both the real interest rate, the risk premium and real dividends matter (regime 2), and one in which there is immediate adjustment and where only the risk premium and real dividends are important (regime 1). Thus, the real interest rate is only important in one of the regimes (regime 2). As the estimated residual variance is somewhat higher in regime 2 than in regime 1, the uncertainty attached to the model's fit is largest in the former regime (despite having more significant factors)[23].

The presence of an autoregressive term in the dividend-price model - reflecting partial adjustment - means that the impact of the various economic fundamentals is somewhat higher in the long than in the short run. This difference between the long and the short run is most pronounced for regime 2 where the autoregressive term has the highest coefficient and the adjustment to long run equilibrium therefore is the slowest one[24]. The long run equilibrium relations can be calculated from Table 4.1 (dividing through by one minus the autoregressive coefficient

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The maximum likelihood estimation is done under the assumption that the state probabilities of the initial observation is given by the ergodic probabilities. Estimation of the initial probabilities does, however, not change the results. The computations are done with the BFGS algorithm in GAUSS using a variety of different starting values for the algorithm. We identify more than one local maximum (approximately 5) depending on the starting values and, moreover, encounter a singularity problem of the likelihood function, that is, for certain starting values the likelihood becomes 'large' without convergence as one of the regime-dependent variances goes to zero. The results of Table 4.1 apply to the local maximum with the highest likelihood. This choice is consistent with Kiefer (1978) who in the context of the mixed-distribution model - where a global maximum does not exist - shows that there is a bounded local maximum of the likelihood function (with variances being positive) that exhibits the usual maximum likelihood properties of being consistent and asymptotically efficient.

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A Likelihood Ratio test with two degrees of freedom of the joint hypothesis that the interest rate and lagged dividend-price are insignificant in regime 1 gives a test value of 3.4 (critical significance value of 18.4 per cent) leading to acceptance of the hypothesis at conventional significance levels. We have decided to keep the two variables in the model because the resulting parsimonious model fails the specification tests.

23

From a probabilistic inference, cf. below, we can estimate regime 2 to have reigned over the period 1950-1985. Using the model's overall fit, the coefficient of determination over this sub-period is 81%. This is considerably lower than the 96% over the remaining periods 1927-1949 and 1986-1991 (regime 1), indicating a lower explanatory power for the model in regime 2. Over the whole sample the coefficient of determination is 91%.

24

The adjustment in regime 2 is actually fast as nearly 80% of the adjustment happens within the first year of a shock, compared to nearly 90% for regime 1.

and ignoring the error term)[25]:

$$\frac{D_t}{P_t} = -6.562 + 0.019R_t + 0.158\gamma_t + 2.676DR_t \quad (regime\ 1)$$
$$\frac{D_t}{P_t} = -6.319 + 0.094R_t + 0.199\gamma_t + 2.787DR_t \quad (regime\ 2) \tag{4.7}$$

The main difference between the two regimes lies in the real interest rate impact which, cf. above, is insignificant in regime 1. The impact of the risk premium is also somewhat higher in regime 2, whereas the coefficients to real dividends (and the constant terms) are almost equal across regimes. Note again that the model of regime 2 is the one with highest uncertainty.

As was also the case for the one-regime model, both the real interest rate and the risk premium have smaller effects than expected from the theoretical framework of section 4.2 (less than one). In regime 2 an increase in the real interest rate by 1 percentage point is expected to increase the dividend-price ratio in the long run by approximately 0.09 percentage point, i.e., an impact of around 10% in absolute levels. The impact of the risk premium is somewhat higher as an increase in the risk premium of 1 percentage point gives rise to an increase in the expected long-run dividend-price ratio by 0.16 (0.20) percentage points in regime 1 (2), i.e., an impact of approximately 20% in absolute levels in both regimes. One possible explanation for the impacts being less than one-for-one is, again, that the shocks to the real interest rate and the risk premium are expected by the market participants to be transitory to a

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As shown in chapter 3, the computation of the regime-dependent mean, $E[(D/P)_t\,|s_t]$, is highly complicated when allowing for an autoregressive dependent term. (4.7) should therefore correctly be interpreted as the expected dividend-price ratio conditional on being in regime 1 and 2, respectively, *both in the current and previous period*, i.e., $E[(D/P)_t\,|s_t{=}s_{t-1}{=}i]$ (i=1,2). It turns out, however, that this mean actually comes close to that of $E[(D/P)_t\,|s_t]$ whenever the regimes are relatively persistent, which is the case for our model.

and ignoring the error term)[25]:

$$\frac{D_t}{P_t} = -6.562 + 0.019R_t + 0.158\gamma_t + 2.676DR_t \quad (regime\ 1)$$
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significant degree[26].

The level of real dividends has a significant positive impact on dividend-price so that stock prices appear to 'under-react' to shocks to dividends, as compared to theory. A prime candidate for explaining this feature of 'under-reaction' is again that shocks to dividends are expected to be transitory to some extent. Because we measure dividends in log-levels, the coefficients can be interpreted as 'semi-elasticities', so that a *1 per cent* increase in real dividends implies an expected increase in dividend-price by approx. *0.03 percentage points* in the long run.

The fit of the regime-switching model is depicted in Figure 4.11, while Figure 4.12 shows the standardized residual, calculated as the difference between actual and fitted dividend-price and standardized by the fitted standard error. Both the fit and the residual are calculated using the filtered probabilities for the state s_t. The fitted (or expected) dividend-price ratio is calculated across regimes as[27]:

$$E\left[\frac{D_t}{P_t}\right] = E\left[\frac{D_t}{P_t} \mid s_t=1\right] p_1^f + E\left[\frac{D_t}{P_t} \mid s_t=2\right] p_2^f \qquad (4.8)$$

$p_i^f \equiv Pr(s_t = i \mid I_T)$ (i =1, 2) is the filtered probability of state i at time t, conditional on the information set I_T which contains all available information on observables (including dividend-price) in the sample, cf. Hamilton (1994), chapter 22. The state-conditioned means $E[.\mid s_t]$ follow immediately from (4.6), using the fact that the residual term has a wero mean. The variance of dividend-price around its fitted value,

26

Note that for the real interest rate variable this may in particular be true in regime 1 (1927-1949 and 1986-1991) where the real interest rate is subject to very large fluctuations, cf. Figure 4.2, implying that a relatively large portion of the variation in the current level of the real interest rate is transitory. This could possibly explain the low and insignificant effect of the real interest rate on dividend-price in regime 1.

27

All moments in (4.8) and (4.9) and the following derivations are conditioned on the information set containing the past and current levels of the explanatory variables (including lagged dividend-price) as of period t (omitted for notational convenience).

$\text{Var}(D_t/P_t) \equiv E[D_t/P_t - E(D_t/P_t)]^2$, can be derived by using a formula similar to (4.8) for the second moment $E(D_t/P_t)^2$ and exploiting the fact that $E((D_t/P_t)^2|s_t) \equiv \sigma(s_t)^2 + E((D_t/P_t)|s_t))^2$ (by the definition of variances). Subtracting the term $(E(D_t/P_t))^2$ (this follows from (4.8)) then gives the variance. The result is:

$$Var\left(\frac{D_t}{P_t}\right) = p_1^f\sigma(1)^2 + p_2^f\sigma(2)^2 + p_1^f p_2^f\left(E\left[\frac{D_t}{P_t}|s_t=1\right] - E\left[\frac{D_t}{P_t}|s_t=2\right]\right)^2 \qquad (4.9)$$

The uncertainty of dividend-price is a result of both the unknown error term (captured by the first two terms in (4.9)) and the uncertainty arising from the fact that the state is unknown and the state-dependent means differ (the last term in (4.9)). The standardized residual which is a point estimate of the error term, ϵ_t in (4.6) can, finally, be calculated as the difference between actual and fitted dividend-price, divided by the standard error of dividend-price (the square root of (4.9)).

Figures 4.11 and 4.12 show that the model captures the significant movements of dividend-price over most of the sample and in particular performs well in the beginning and towards the end of the sample. Like the one-regime model, the regime-switching model tracks the significant fall in dividend-price in 1983. There are, however, also less appealing features. The 1974 observation seems to be an outlier and - potentially more seriously - there are two sub-periods (1947-1955 and 1958-1968) over which the model systematically under-estimates, respectively over-estimates, actual dividend-price.

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Figure 4.11. Dividend-Price Ratio: Actual and Fitted Regime-Switching Model

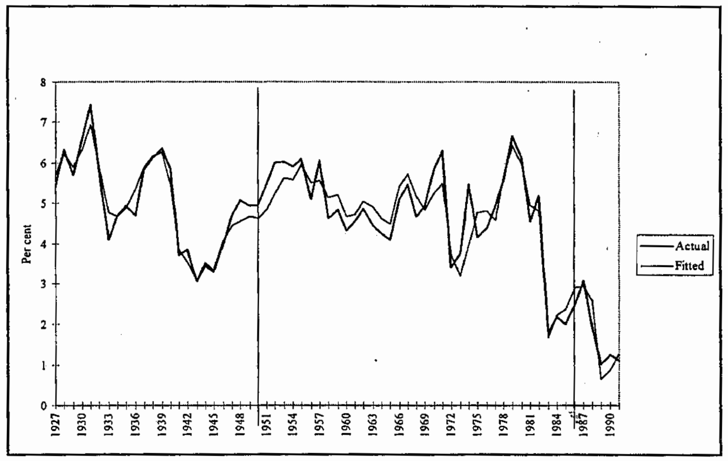
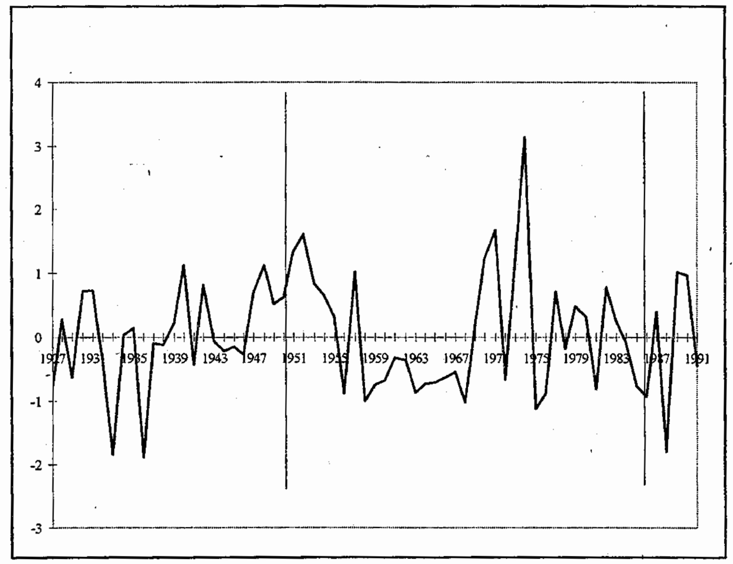


Figure 4.12. Standardized Residuals, Regime Switching Model



Specification tests, cf. Table 4.1, reveal no misspecification at the conventional 5 per cent significance level, expect for the LM test for ARCH over regime 2. However, using an alternative small-sample correction to that used in Table 4.1, cf. Hamilton (1996), the test for ARCH over regime 2 is (just) passed[28]. Note that the tests for serial correlation including the tests for serial correlation in the two regimes are passed so that the apparent systematic 'under-' and 'over-estimation' noted from figures 4.11 and 4.12 is not deemed significant by the specification tests. Also note that the Andrews test for structural break is at its 5% significance level. Hamilton (1996) suggests that a 1% significance level is used for this test in small samples due to the test being 'over-sized', in which case the test is passed with a comfortable margin[29].

We conclude that the regime-switching model is overall well-specified, even though the model seems 'biased' over two sub-periods in the 1940s/1950s and 1960s, respectively. These 'problematic' sub-periods turn out to be concentrated in regime 2 exclusively, cf. below. The model clearly performs better in regime 1. The Andrews test suggests that two regimes are sufficient in order to remove the apparent structural breaks in the one-regime model. No further regimes therefore seem needed[30].

According to the estimated transition probabilities in Table 4.1, both regimes are highly persistent with the probability of continuing in a given regime being 96-97%. The state variable s_t is unobservable, but it

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28

The specification tests used in Table 4.1 are based on an asymptotically valid LR test. Hamilton (1996) suggests two possible small sample corrections: either to perform a transformation of the tests and use a small sample version based on the F-distribution (that used in Table 4.1), or to use a 1% significance level for the asymptotic χ^2-tests. According to Monte Carlo simulations, both help in correcting for an 'over-size' of the specification tests in small samples. For the test for ARCH over regime 2 the LR statistic which is asymptotically χ^2-distributed with 1 degree of freedom gives a test value of 6.17. The critical significance level is 1.3%, i.e., slightly above the 1% level.

29

The individual LM statistics used in the Andrews test are reported in the Appendix to this chapter. The test value is attained for the year 1969.

30

The model's 'bias' over sub-periods could possibly be avoided by introducing a further regime, though.

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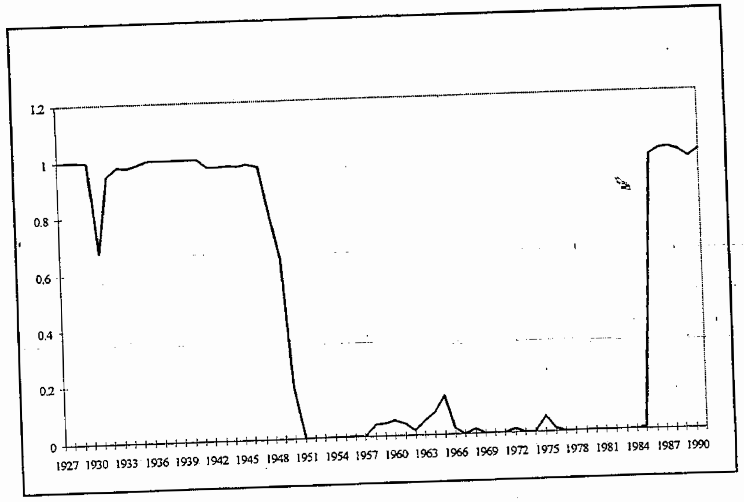
is possible from the estimated transition probabilities and the estimated regime-dependent models to draw a probabilistic inference about the state at a given point of time. This inference is expressed by the 'filtered state probability' defined as the probability of being in, say, state 1 at a given time t, conditional on all information about observables (dividend-price and the economic fundamentals) up to and including time t. The estimated filtered probabilities for the model of Table 4.1, expressed as the probability of being in regime 1, are shown in Figure 4.13. This plot

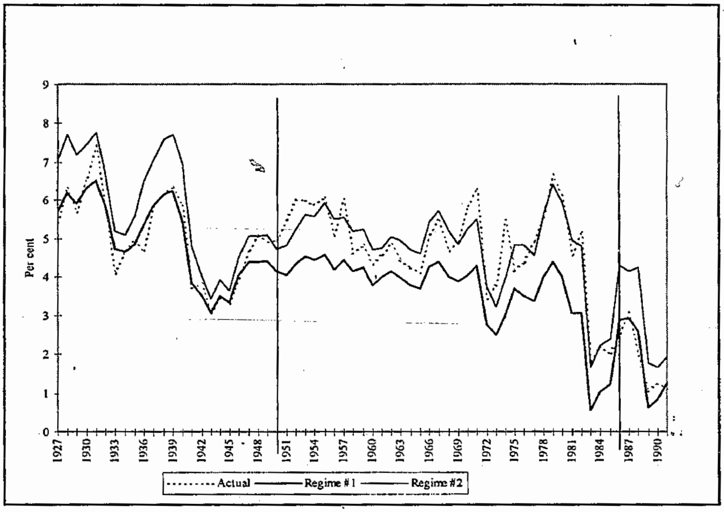
Figure 4.13. Filtered Probabilities for Regime 1



confirms that the regimes are highly persistent. It furthermore gives a very clear inference about the state variable, suggesting that we can divide the whole sample period into 3 distinct sub-periods (using the 50% probability as the dividing line between sub-periods): 1927-1949, where the model of regime 1 governed the dividend-price process; 1950-1985 (regime 2), and 1986-1991 (regime 1). Over the whole sample period regime 2 has been the most frequent one. Note that the identification of regimes corresponds quite well with the recursive plots of figures 4.6-4.10 which at an informal level suggests that there are two regime-shifts over the sample, one in the beginning and one towards the end.

A further understanding of the two regimes can be facilitated by inspecting Figure 4.14 which shows the fit of each of the two regime-dependent models together with the actual dividend-price ratio. It is evident that the model of regime 1 (where only the risk premium and

Figure 4.14. Dividend-Price Ratio: Actual and Regime-Dependent Predictions of Regime-Switching Model



real dividends matter) systematically predicts a lower dividend-price ratio than that of regime 2 over the whole sample period (with the difference being 1.1 percentage points on average over the sample). This suggests that regime 1 (2) is one with low (high) dividend-price and - correspondingly - high (low) stock prices, given and correcting for the underlying economic fundamentals. The recent period from 1986 could therefore be interpreted as one with high stock prices and the regime identification suggests that this period in fact resembles that of the beginning of the sample period, 1927-1949. The long period 1950-1985 of figures 4.6-4.10 which at an informal level suggests that there are two regime-shifts over the sample, one in the beginning and one towards the end.

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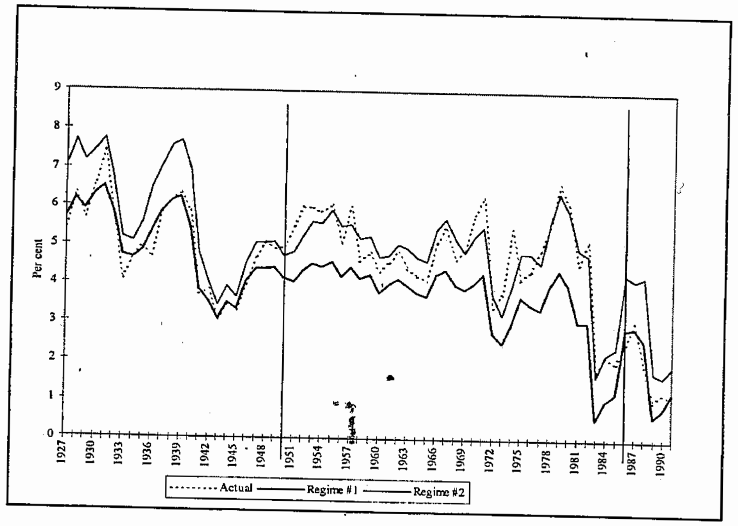

real dividends matter) systematically predicts a lower dividend-price ratio than that of regime 2 over the whole sample period (with the difference being 1.1 percentage points on average over the sample). This suggests that regime 1 (2) is one with low (high) dividend-price and - correspondingly - high (low) stock prices, given and correcting for the underlying economic fundamentals. The recent period from 1986 could therefore be interpreted as one with high stock prices and the regime identification suggests that this period in fact resembles that of the beginning of the sample period, 1927-1949. The long period 1950-1985

has on the other hand been one with relatively low stock prices.

The regime-switching model leads to the conclusion that regime shifts took place in the economic model in 1950 and 1986. The evidence that a regime-shift (towards a lower dividend-price ratio) should have taken place in the 1980s seems plausible given the large changes in the Danish economic environment in that period, with the implementation of a new economic policy and the introduction of a new pension fund tax, cf. section 4.3. The timing of the regime-shift (1986) may on the other hand come as a slight surprise, at least at first sight, given that the large adjustment in dividend-price as well as the structural changes took place already in 1983. Thus, it is interesting to note that the significant fall in dividend-price (and the underlying increase in stock prices) in 1983 can be explained by the economic model without referring to a regime-shift. From the data plots in Figures 4.2-4.4 and the coefficients for the prevailing regime 2 it emerges that the prime factor in explaining this fall in dividend-price is the huge drop in the 'risk premium' by about 11 percentage point that year which in itself gives rise to a fall in dividend-price by 1.7 percentage points. Recall from section 4.3 that this large drop in the premium is partially motivated by the introduction of the new pension fund tax, so that this particular variable incorporates one of the big structural changes in 1983. A fall in the real interest rate (by nearly 12 percentage points) and real dividends (by 30 percent) also contribute with an estimated expected impact of 0.6-0.9 percentage points each. Instead of 1983 the regime-shift first takes place in 1986. In terms of the model, this regime shift is needed in order to explain why the dividend-price ratio remains low despite a reversal in the real interest rate, the risk premium and real dividends to levels close to those prevailing before 1983. Note that the gradual phasing in of the pension fund tax is consistent with this 'lagged' regime-shift[31]. Note also that the timing of the regime-shift in the 1980s highlights the importance of taking due account of underlying economic fundamentals when trying to understand the dividend-price ratio, and in particular when

[31] It is of interest that expected dividend-price would have been low (compared to the historical average) after 1986 also in regime 2, i.e., even without a regime-shift (even though not low enough). Thus, the economic fundamentals by themselves predict a low dividend-price as the reversal to the levels before 1983 is not complete.

determining whether a regime shift has taken place or not. Finally, note that the pension fund tax affects dividend-price both through the before-tax 'risk premium' γ_t (leading to lower γ_t and, hence, lower dividend-price) and, potentially, by inducing the shift to the 'low dividend-price' regime as of 1986[32].

The result that a regime-shift also took place in 1950 and that the pre-1950 regime should resemble that of the post-1986 regime is somewhat harder to explain and a closer examination is needed.

It is evident from Table 4.1 that the allowance for regime-shifts significantly alters the estimated coefficients. The one-regime model does not come close to any of the regime-dependent models and we in particular encounter differences for real dividends and lagged dividend-price, whereas the estimate for the risk premium effect comes closer to that of the one-regime model[33]. The regime-switching model is better than the one-regime model in terms of fit (as measured by log-likelihood value or the estimated residual variance) which is no surprise as the regime-switching model contains more parameters. However, even after correcting for the number of parameters the regime-switching model seems superior. Table 4.1 shows the values for three information criteria, which are often used as the basis for model selection: the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ) and the Schwarz criterion (SC). According to the first two criteria, the regime-switching model is the preferable one, while the SC does not give a clear answer.

There are two further and more evident reasons for choosing the regime-switching model. First of all, the allowance for two regimes solves a clear problem with a structural break in the one-regime model. Second, within the context of the regime-switching model a one-regime model is only valid if the two regime-dependent models do not differ in any significant way. This hypothesis can be put to a formal test by (for

32

Cf. note 12. The regime-shift basically suggests that the inclusion of the pension fund tax in the γ_t-construction does not sufficiently account for the effects of this tax on dividend-price.

33

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instance) testing whether all coefficients (including the constant term) are identical across the two regimes[34]. The Likelihood Ratio test with 5 degrees of freedom of this hypothesis gives a test statistic of 29.7 corresponding to a critical significance level of 0.00 pct., i.e., a clear rejection. Thus, there are two distinct regimes in the data. The evidence is therefore strongly in favor of the regime-switching model[35].

4.6. Conclusion

We have estimated a model with economic fundamentals for the dividend-price ratio. The results show that our proxies for the growth-adjusted real interest rate and the risk premium on stocks are significant in modeling dividend-price. This identification of a time-varying discount rate which is useful for empirical modeling is the main contribution of this paper. The existing empirical literature on modeling stock price behavior often ignores the time variation in the discount rate by assuming it to be constant. The estimated coefficients of the real interest rate and the risk premium are significantly less than one, the value predicted by a Gordon-type theoretical model where all innovations in the two variables are expected by the market participants to be permanent. This suggests that innovations are partially transitory. It also turns out that lagged dividend-price and the level of real dividends are important explanatory variables, the former capturing slow or partial adjustment in the dividend-price process. The significance of real dividends also indicate that shocks to the dividend process are viewed - at least to some extent - to be transitory.

34 As noted by Hamilton (1990), we cannot perform a LR test of the more adequate hypothesis that all parameters (including the variances) are identical across regimes, because the asymptotic information matrix becomes singular under the null, which is a violation of one of the standard regularity conditions underlying the LR test.

35 The specification tests based on the standardized residuals, cf. Table 4.1, also suggest that the regime-switching model corrects for a problem with serial correlation at higher lag orders. However, one should be careful with these tests for the regime-switching model as they are based on the residuals of a model which includes the lagged dependent variable.

We estimate the economic model using both a one-regime and a regime-switching approach. The latter is used to account for non-modeled changes in the (exogenous) 'economic environment', leading to structural changes in the fundamental economic model. The results show that it is important to allow for more than one regime over the sample period in order to avoid structural breaks in the model for dividend-price. Two regimes seem to suffice. The regimes correspond to two distinct sub-models which differ wrt. the relative importance of the economic fundamentals, both in terms of short run dynamics and long run equilibrium effects. The main difference is that the real interest rate is only significant in one of the regimes. In that regime lagged dividend-price also turns out to be insignificant, suggesting immediate adjustment in dividend-price.

Over the sample period the two regimes differ wrt. the level of dividend-price as one of the regimes gives a systematically higher level for dividend-price (corresponding to lower stock prices) as the other. One way to interpret the two regimes is therefore to view and distinguish them as a 'low-dividend-price' regime (high stock prices), respectively a 'high-dividend-price' regime (low stock prices), where 'high' and 'low' is used in the context of the economic model which takes due account of the underlying economic fundamentals. The results clearly identify 3 distinct sub-periods in the sample (1927-1949, 1950-1985 and 1986-1991) over which the regimes (sub-models) apply. The 'low-dividend-price'-regime applies to the first and third sub-period, the 'high-dividend-price'-regime to the second. The fact that dividend-price is 'low' - and stock prices correspondingly are 'high' - after 1986 could possibly be explained by the gradual phasing in of a new separate tax on pension funds' bond holdings, initiated in 1983. The evidence that a regime-shift also took place around 1950 and that the early pre-1950 regime should resemble that of the late post-1986 regime may be somewhat more puzzling. Whether changes in investor taxation also can motivate the latter regime-shift is an open question.

Related literature is Driffill and Sola (1998) who estimate a 2-state regime-switching model for the US stock market over the period 1900-1987, using the price-dividend ratio as the endogenous variable. They argue for the presence of 2 states within the context of the standard (constant discount rate/constant dividend-growth) Gordon model due to

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2 distinct states in the underlying dividend process, one of low-growth/high-variance and one of high-growth/low-variance. These two states result in two different fundamental solutions for the price-dividend ratio. The main result in Driffill and Sola (1998) - besides the evidence of two states being present in the processes for dividends and price-dividend - is that the allowance for two regimes leads to a significant improvement of the model, in particular in terms of fit. Driffill and Sola (1998) also test for the presence of intrinsic bubbles in stock prices, as originally proposed by Froot and Obstfeld (1991), by allowing for the level of real dividends to explain price-dividend. Even though they cannot formally reject the presence of intrinsic bubbles, they conclude based on explanatory power that the inclusion of intrinsic bubbles is not important when one first allows for different regimes.

Our analysis differs from that of Driffill and Sola (1998) by using economic fundamentals, in particular, a time-varying real interest rate and a time-varying risk premium, in explaining dividend-price, whereas Driffill and Sola (1998) focus exclusively on the regime-switching element, assuming a constant discount rate as in the standard Gordon model[36]. Our approach is 'ad hoc' compared to Driffill and Sola (1998) who have a firm theoretical foundation for the presence of distinct states in the pricing process, based on the presence of distinct states in the underlying dividend process. The significance of real dividends in our analysis could - as in Driffill and Sola (1998) and Froot and Obstfeld (1991) - be suggestive of intrinsic bubbles in stock prices. However, this conclusion is only valid if certain restrictions on the parameters of the dividends and price processes are fulfilled, cf. Driffill and Sola (1998) and Froot and Obstfeld (1991). These have not been tested in the present chapter.

The model that we set up fits dividend-price well, is overall well-specified using a strict 5% significance level and does in particular work well in regime 1 (the periods 1927-1949 and 1986-1991). The model is, however, not satisfactory over sub-periods in the middle of the sample where we encounter a systematic tendency to 'under-', respectively

36 It should be noted that the approach of Driffill and Sola (1998) is not applicable for Denmark as there is no evidence of distinct states in the Danish dividend process which is crucial to their approach.

‘over-estimate’ dividend-price. These ‘problematic’ sub-periods are concentrated in regime 2 (1950-1985). The latter less appealing feature is obviously a point where the model could be improved. Even though two regimes formally suffice according to the Andrews test, one possibility would be to allow for 3 regimes as the ‘problematic’ sub-periods could be suggestive of a third regime applying here. One should be aware, though, that allowance for a third regime lowers the degrees of freedom by 10 by increasing the number of parameters correspondingly, and, furthermore, that practical problems may be encountered in performing maximum likelihood estimation with that many parameters.

The regime-switching model identifies regime-shifts in 1950 and 1986. An obvious but also challenging issue for future research is to identify the causes of these regime shifts and, if possible, incorporate these factors formally in the model, leading to a stable (one-regime) economic model. We have conjectured that the introduction of the pension fund tax on bond returns is a possible explanation for the regime-shift in 1986. By incorporating taxation in the economic model, the validity of this conjecture can be tested. Moreover, it would allow us to test whether changes in taxation also can account for the regime-shift in 1950. If so (and taxation is the sole explanation for the regime-shifts), the incorporation of taxation should remove the structural breaks, implying that a one-regime extended model should be stable.

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References

Andrews, D.W.K. (1993), Tests for Parameter Instability and Structural Change with Unknown Change Point, *Econometrica*, vol. 61, no. 4, pp. 821-856.

Campbell, J.Y., A.W. Lo and A.C. MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press.

Campbell, J.Y. and R.J. Shiller (1988), The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors, *Review of Financial Studies*, vol. 1, pp. 195-228.

evt. Campbell, J.Y. and R.J. Shiller (1998), Valuation Ratios and the Long-Run Stock Market Outlook. Ratios are extraordinarily bearish, *Journal of Portfolio Management*, Winter 1998, pp. 1-26.

Driffill, J. and M. Sola (1998), Intrinsic Bubbles and Regime-Switching, *Journal of Monetary Economics*, vol. 42, pp. 357-373.

Engel, C. and J.D. Hamilton (1990), Long Swings in the Dollar: Are They in the Data and Do Markets Know It?, *American Economic Review*, vol. 80, no. 4, pp. 689-713.

Fama, E. and K. French (1988), Dividend Yields and Expected Stock Returns, *Journal of Financial Economics*, vol. 22, pp. 3-27.

Froot, K.A. and M. Obstfeld (1991), Intrinsic Bubbles: The Case of Stock Prices, *American Economic Review*, vol. 81, no. 5, pp. 1189-1214.

Gordon, M. (1962), *The Investment, Financing, and Valuation of the Corporation*, Irwin, Homewood.

Hamilton, J.D. (1990), Analysis of Time Series Subject to Changes in Regime, *Journal of Econometrics*, vol. 45, pp. 39-70.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press.

Hamilton, J.D. (1996), Specification Testing in Markov-Switching Time-Series Models, *Journal of Econometrics*, vol. 70, pp. 127-157.

Hamilton, J.D. and G. Lin (1996), Stock Market Volatility and The Business Cycle, *Journal of Applied Econometrics*, vol. 11, pp. 573-593.

Hodrick, R. (1992), Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, *Review of Financial Studies*, vol. 5, pp. 357-386.

Kiefer, N.M. (1978), Discrete Parameter Variation: Efficient Estimation of a Switching Regression Model, *Econometrica*, vol. 46, pp. 427-434.

Modigliani, F. and R.A. Cohn (1979), Inflation, Rational Valuation and the Market, *Financial Analysts Journal*, pp. 24-44.

Nielsen, S., J.O. Olesen and O. Risager (1999), Macroeconomic Database, Economics Department, Copenhagen Business School.

Olesen, J.O. and O. Risager (1999), On the Predictability of the Danish Equity Premium, Working Paper, Economics Department, Copenhagen Business School (forthcoming)

Vigfusson, R. (1996), Switching between Chartists and Fundamentalists: A Markov Regime-Switching Approach, Bank of Canada, Working Paper 96-1.

Hamilton, J.D. (1996), Specification Testing in Markov-Switching Time-Series Models, *Journal of Econometrics*, vol. 70, pp. 127-157.

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Appendix

Figure 4.A.1. Individual LM Statistics used in Andrews Test One-Regime Model, 1928-91

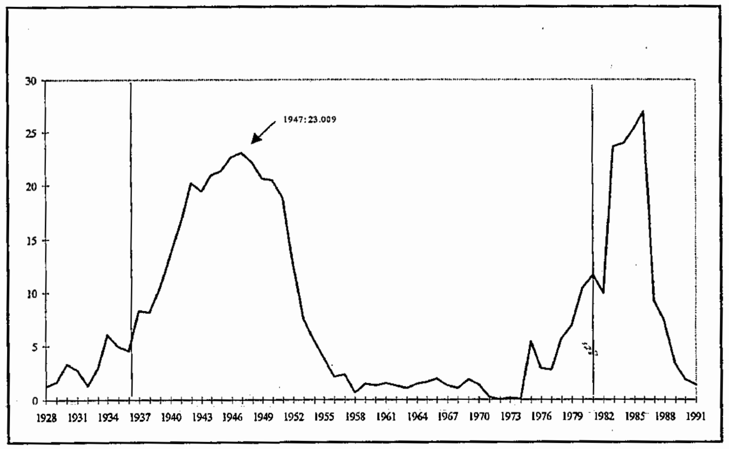
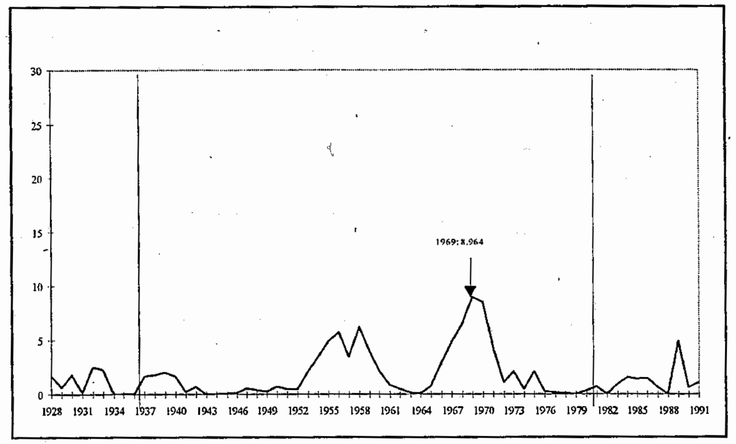


Figure 4.A.2. Individual LM Statistics used in Andrews Test Regime-Switching Model, 1928-91



Chapter 5

Earnings Adjustment and Stock Return Predictability[1]

5.1. Introduction

This chapter shows that the premium on *value* strategies of buying stocks which are priced low relative to their value by some accounting measure increases when the accounting data which are used to determine value are corrected for some of its deficiencies.

Accounting data are produced for auditing, internal control and, to some extent, decision-making within the company. It is generally accepted that caution must be exercised when using accounting data for an economic analysis, and that adjustment may need to be made.

We focus on one particular measure of value, namely earnings. It has been demonstrated by, for example, Fama and French (1992) and Lakonishok, Shleifer and Vishny (1994) (henceforth LSV) that (among other variables) price to earnings, P/E, on average predicts returns. An interpretation of this is that current earnings forecast future earnings/ability to pay dividends, and, thus, that stocks with a low price relative to earnings can be considered cheap[2]. However, raw reported

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earnings might not be the optimal measure to use. This chapter argues that from an investor's point of view, value is better measured by adjusted earnings.

We realize that adjustment of earnings data is controversial for at least two reasons. First, reported earnings aggregates many events which are individually unidentifiable and may ideally require individual adjustment. Therefore, adjustment of reported earnings cannot be expected fully to reveal economic value. However, usually annual reports are the most detailed information available to investors. Hence, we believe that our focus on adjustment of reported earnings is justified by the fact that this is what many practical investment decisions (at most) rely on. Second, a major challenge is to determine which adjustment procedure to use. In reality, the interpretation of financial statements is highly subjective. Analysts follow different procedures and, hence, generally arrive at different perceptions. We aim at describing these variations by an adjustment procedure which is believed to be representative of procedures actually employed. The representative procedure is obtained by adapting some of the basic ideas of the classical Graham and Dodd (1934) textbook on security analysis. This is a standard reference which is still published and generally acknowledged to have had major impact on security analysis. Since the Graham-Dodd methods presumably have been widely applied for many years they appear to be a reasonable basis for describing how the average investor values different stocks.

Adjusting earnings turns out to improve predictability of P/E in the sense of increasing the value premium. However, we do not claim that this is the only possible adjustment of earnings. Indeed, further research may establish superiority of other procedures.

Furthermore, to be able to focus attention properly on the issue of adjusted versus unadjusted accounting data, we choose one of the financial ratios that have been suggested to predict stock returns, namely the price/earnings ratio. It would be equally interesting to consider similar adjustment of the balance sheet-related ratios with predictive power, but such an analysis is beyond this chapter.

Section 5.2 describes the adjustment procedure, and section 5.3

French (1993).

presents data. The ability of the two competing price/earnings measures (adjusted and unadjusted) to predict stock returns is evaluated in section 5.4. Finally, section 5.5 concludes.

5.2. Adjustment procedure

To an investor, the value of a share equals the discounted sum of expected future dividends:

$$V_t = E_t \left[\sum_{j=1}^{\infty} \frac{1}{(1+k)^j} D_{t+j}\right], \tag{5.1}$$

where k is the (constant) market capitalization rate of the stock, D_t is dividends in period t, and E_t denotes expectations based on available information at t.

The Graham and Dodd financial statement analysis focuses on the concept of earning power. It is understood to be that amount of income which, in the future, may be distributed to shareholders without weakening the position of the company. Earning power is more relevant to investors than accounting earnings because it measures what dividends the firm is able to pay in the future. Hence, earning power, EP, may be substituted for dividends in (5.1):

$$V_t = E_t \left[\sum_{j=1}^{\infty} \frac{1}{(1+k)^j} EP_{t+j}\right], \tag{5.2}$$

(5.2) is often expressed in terms of free cash flow instead of earning power where free cash flow is defined as net revenue not reinvested in the firm. Thus, earning power may be seen as an estimate of future free cash flow.

Assume that earning power grows with a constant rate, g, in which case (5.2) simplifies to:

$$V_t = \frac{(1+g)EP_t}{k-g} \tag{5.3}$$

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Notice that value is proportional to earning power given the assumption of a constant discount factor. This justifies comparing the ratio of share price to earning power per share to its historical average to determine whether the share is currently undervalued.

Earning power is derived through an adjustment of accounting earnings. Graham and Dodd discuss three elements which may cause the two earnings measures to be different, i.e., create the need for adjustment of reported earnings (p. 353):

1. Nonrecurrent profits and losses.
2. Operations of subsidiaries or affiliates.
3. Reserves.

5.2.1. Nonrecurring items

Obviously, the use of past earnings is only relevant in this context as a basis for predicting future earnings. Since nonrecurring items by definition are irrelevant for future earnings, they should not be included in earnings. This observation seems to be more general than the distinction between accounting earnings and earning power. Hence, we choose to exclude nonrecurring items from both adjusted and unadjusted earnings.

Kinney and Trezevant (1997) document the use of nonrecurring items to smooth earnings. The fact that management uses its discretion to determine the size and magnitude of these items is another reason for leaving them out when considering earning power.

As a general rule, nonrecurring items recorded as such in annual reports have been deducted. In addition, a few companies in our sample report profits on sale of assets as part of ordinary operations even though such activity does not seem to be an integral part of business. Those profits are also considered nonrecurring and are therefore subtracted.

5.2.2. Subsidiaries

The activities of subsidiaries are taken into account by using consolidated income statements. No adjustments have therefore been recorded under this heading.

5.2.3. Reserves

Graham and Dodd recommend that a particularly critical analysis be applied to depreciation of tangible assets. Depreciations are to a large degree subject to judgements with respect to determining initial book value and expected lifetime of the asset in question, and the choice of depreciation scheme has significant implications for resulting depreciation charges, and, hence, earnings.

From an investor's point of view, expenditures on new capital assets and on replacement of old assets is the minimum amount of depreciation which must be allowed for, since this amount is not available for dividend payments. Furthermore, Graham and Dodd argue that it is not necessary to accumulate additional reserves to replace worn-out assets. The reason for this is that assets are typically replaced due to obsolescence rather than wearing out, and the risk of obsolescence is a business risk of a more general nature which should be reflected in the discount factor used in valuing the company. It does not affect earning power. Hence, the amount of expenditures on tangibles is a more appropriate allowance for depreciations than the company's depreciation charges.

Now, additions to tangibles may vary substantially from year to year. Therefore, Graham and Dodd suggest averaging expenditures over previous years. Furthermore, to obtain a reasonable measure of average expenditures we have in a few cases excluded exceptional values[3]. This average is called expended depreciation, and it replaces the original depreciation charges in the adjusted series[4].

It may seem that our measure of earning power potentially is

3

The expenditure of a given year is exceptional if it results in average expenditures exceeding the following year's expenditure by more than 200 per cent.

4

The cumulative average is restarted in a few cases where a major change in business conditions (like a merger or sale/purchase of a significant subsidiary) occurs during the sample period.

ted income statements. No adjustments have therefore been recorded under this heading.

5.2.3. Reserves

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It may seem that our measure of earning power potentially is underestimated in cases where companies aim at higher future earnings through a period of heavy investments. This is probably a valid point against our adjustment procedure in cases of young and upcoming firms. However, as described in section 5.3 our sample only includes established companies. Furthermore, we reduce the problem by averaging over several years and by excluding exceptional values.

5.2.4. Other adjustments

Finally, earnings per share are adjusted for changes in capitalization. Specifically, on two occasions companies in our sample have issued convertible bonds. The adjustment procedure treats such cases as if the equity (into which the loan eventually can be converted) were issued in the first place. If the company had chosen the equity form of finance, it would have been relieved from interest payments, and, therefore, actual interest payments are added to earnings during the lifetime of the bonds. Graham and Dodd describe this procedure in chapter 39.

The earnings figure is profits net of company taxes. Any tax effects of adjustments are ignored.

Table 5.1: Adjustment of Earnings

	Profits net of company taxes
-	Nonrecurring items
=	**Unadjusted earnings**
+	Depreciations and amortizations of tangible assets charged
-	Expended depreciation
+	Interest payment on convertible bonds
=	**Adjusted earnings**

5.3. Sample

The sample portfolio consists of the 20 blue chip stocks which were in

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4 The cumulative average is restarted in a few cases where a major change in business conditions (like a merger or sale/purchase of a significant subsidiary) occurs during the sample period.

the Danish KFX index by mid-1998[5]. Thus, although the actual KFX portfolio has changed over time (the portfolio is revised annually according to volume of trade at the time) our index calculations at any time include those of the 20 stocks for which data are available. By focusing on the same set of stocks consistency is maximized.

Choosing the end-of-period rather than the beginning-of-period index portfolio introduces a potential survivor bias in average returns. The reason is that firms which fail during the sample period drop out of the index and, hence, are excluded from our analysis by construction. Thus, returns are on average higher for our portfolio than for a portfolio of firms randomly selected at the beginning of the sample period. However, since the bias applies to both the value and glamour portfolios, a comparison of returns to the two strategies is admissible.

There are several reasons for limiting the analysis to large firms only. From a practical point of view, the larger firms typically publish more information in annual reports and, therefore, allow more sophisticated security analysis to be carried out. The careful study of annual reports and the number of calculations necessary for adjustment also make it impractical to analyze such a large number of firms as is standard in the literature[6].

Prices, dividends, and earnings are recorded for each of these shares for as long a period as possible. The adjustment procedure requires detailed data from annual reports (for example, net expenditure on tangible assets), and in most cases it is the lack of publication of these data that prevents taking the analysis further backwards in time. Only observations with both adjusted and unadjusted earnings available are included.

The first year for which earnings data is available ranges from 1985 to 1993 with only one share at each of these extremes. The typical

5

One stock was added to the KFX shortly before mid-1998 as a result of the demerger of one of the previous KFX stocks. This new stock is not considered here since no data obviously are available prior to the demerger.

6

Other advantages of concentrating on larger stocks are that the measurement problems suggested by Ball, Kothari and Shanken (1995) do not arise, and that trading strategies for blue chips are more attractive to investors, in particular to institutional investors.

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starting years are 1990 and 1991 (5 and 4 shares respectively) and the average starting time is 89.8. More than half of the shares have their starting year in 1990 or prior.

Prices are the official closing quotes on the last day of June as published by the Copenhagen Stock Exchange. The corresponding earnings and adjustments are calculated on the basis of annual reports which are publicly available at the end of June. The use of annual data as opposed to data at higher frequency reduces the significance of transaction costs.

Prices, dividends, and earnings per share (adjusted and unadjusted) are adjusted for stock splits, dividends, and offers by a procedure analogous to the one employed by the Center for Research in Security Prices, CRSP, University of Chicago.

The adjustment is of considerable magnitude: Across all shares and times in the sample average adjusted earnings amounts to approximately half of average unadjusted earnings. Thus, P/Es are generally much higher with adjusted than unadjusted earnings. However, the level of adjusted earnings as compared to unadjusted has no direct implications on the analysis. More importantly, the correlation between adjusted and unadjusted earnings is as high as 0.95. That is, in this sense the adjustment has only minor significance.

Past earnings is used to forecast the future earnings potential of the company which is not allowed to be negative. Thus, if the most recent earnings is negative, a missing value is recorded for P/E.

It is commonly known that average P/Es differ among industries. This is, for example, noted by Rutterford (1993), p. 147. She points out that on October 1991, the average P/E ratio for firms in the British Electricity industry is 8.94 compared to 17.91 in the Store industry. This large difference may be due to larger growth potential and/or less uncertainty in the Store industry. This fact implies that if we were to use absolute P/E ratios as the basis of our portfolio formation, certain industries would be overrepresented in particular portfolios, e.g. Electricity firms would tend to be allocated to the value portfolio. In order to avoid such a systematic pattern in the portfolios, we choose to form portfolios based on P/E relative to the historical average of P/Es rather than P/E itself. A corollary of this is that one observation is lost for each share, since no historical average is available for the first P/E.

5.4. Results

The earnings data derived from the procedure outlined in the previous section are used to calculate P/E-ratios. This section presents the results on predictability of stock returns on the basis of P/E-ratios.

Two approaches are used to evaluate predictability. The first approach is to form two portfolios each year which consist of high (i.e., growth) and low (i.e., value) relative P/E stocks, respectively, and to compare raw returns the following year on these portfolios. The second approach is to perform cross-sectional regressions of return on relative P/E, and compute coefficient averages over time.

5.4.1 Portfolio approach

Based on the value of price to the most recent earnings at the end of June relative to its average in previous years stocks are grouped in two portfolios each year. A stock is included in the low (high) P/E portfolio if its current P/E is less than (greater than or equal to) its historical average P/E, i.e., if its relative P/E is less than (greater than or equal to) 1.

Stocks are weighted in proportion to market capitalization. The first portfolio is formed in June 1990, and the last is formed in 1997. The number of shares to choose from increases from 5 (2) in 1990 to 19 (18) in 1997 in case of unadjusted (adjusted) relative P/E's.

Average returns to the portfolios are summarized in table 5.2:

Table 5.2: Average Portfolio Returns

	Unadjusted	Adjusted
High relative P/E	0.137	0.080
Low relative P/E	0.180	0.196

The figures in the first column correspond to the first row of panel C, table I in LSV in which average one-year returns on ten portfolios sorted on E/P are presented. The differences in construction are that LSV

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a) have more portfolios, b) use P/E rather than relative P/E, and c) use equal weights. In spite of the differences, the results are very similar. The simple average of the returns on LSV portfolios 1 - 5 (high P/E) is 13.1 per cent, and the average return on portfolios 6 - 10 (low P/E) is 17.2 per cent. Thus, the first column of table 5.2 shows that the result of LSV (that P/E predict returns) also holds for the Danish data used here. That the value premium is not specific to American stock markets is also demonstrated by Fama and French (1998).

The same pattern is present when using the adjusted data. And what is even more interesting in our context is that the average return difference between the two portfolios is more than doubled by using the adjustment. This is due to both an increase in return to the value strategy and a decrease in return to the growth strategy. Thus, it seems that adjusted P/Es have more predictive power than the conventional unadjusted P/Es.

Table 5.3 shows the return difference between the low and high relative P/E strategies using unadjusted and adjusted data. With adjusted data only few and small negative values are recorded. Furthermore, in this case the t-statistic tells that the average difference is significantly different from zero at 10% significance level.

Table 5.3: Return Differences between Value and Growth Strategies

	Unadjusted	Adjusted
1990	0.328	0.023
1991	-0.116	0.413
1992	0.093	0.080
1993	-0.119	0.011
1994	-0.090	-0.018
1995	-0.033	-0.031
1996	0.102	0.309
1997	0.178	0.141
Average	0.043	0.116
t-statistic	0.757	2.011

The large difference between the two columns of table 5.3 reflects that the earnings adjustment causes many stocks to change portfolio. On average, almost half of the stocks change portfolio when going from unadjusted to adjusted data. In that sense, the earnings adjustment has major impact on the success of a contrarian investment strategy on the Danish stock market.

Figure 5.1 illustrates the cumulative value over time from 1990 of a DKK 100 investment in each of the portfolios and in the market which is here defined as the stocks in our sample. Clearly, the return difference between value and growth strategies increases when using adjusted data. The value premium in terms of cumulative annual return increases from 4 to 11 per cent.

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Figure 5.1: Cumulative Value of a DKK 100 Investment

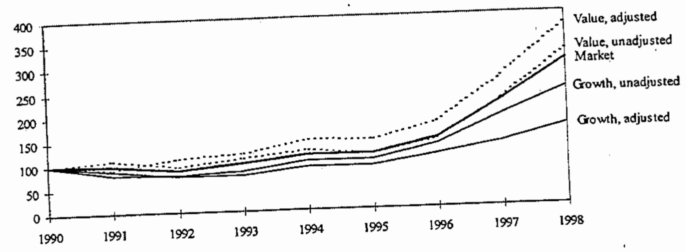

Table 5.4 reports three popular risk measures for each strategy:

Table 5.4: Risk measures, 1990-98

	Sharpe ratio	Average market cap (billion DKK)	Average β
Growth, unadjusted	0.29	7.3	0.8260
Value, unadjusted	0.43	9.5	0.7154
Growth, adjusted	0.03	4.8	0.6063
Value, adjusted	0.59	8.5	0.9346

The first column shows Sharpe ratios (ie., mean excess return divided by standard deviation where the risk-free rate is the average 1-year government bond yield over the sample, see chapter 1). The Sharpe ratio is higher for the value than for the growth strategy both using adjusted and unadjusted earnings. Furthermore, in both cases, the Sharpe ratio is higher on a value strategy than the market Sharpe ratio which is

0.19.[7] Thus, this measure does not suggest that the value premium is due to risk. The second column provides average market capitalizations. In studies of US stock markets, small size has been associated with high risk, see Fama and French (1993). There is no similar Danish study. Table 5.4 shows that the average stock is larger in the value portfolio than in the growth portfolio. Hence, the value premium is not due to small size risk. The final column reports average portfolio CAPM-βs. this involves estimating β (ie., covariance of security and total Danish stock market returns divided by variance of market returns) for all stocks at each portfolio formation date using data for the previous 60 months. If we first consider the case of unadjusted data, the growth strategy has slightly greater β than the value strategy and, hence, the growth strategy is riskier according to the β-measure. This confirms the result from columns 1 and 2 that the value strategy does not seem to be riskier than the growth strategy. However, when strategies are implemented using adjusted data, the value strategy seems to be riskier in terms of higher β. Although the difference of βs appears to be small, this finding offers some support to the view that the value premium is due to risk.

5.4.2 Regression approach

Each year annual returns of the sample of companies are regressed on relative P/E's which are known at the beginning of the return period. The first regression is returns from June 1991 to June 1992 on relative P/E's known in 1991.

Average R^2s and average slope coefficients from the cross-sectional regressions, and a time series t-statistic are shown in table 5.4. The t-statistic is calculated in accordance with Fama and McBeth (1973) and LSV and is defined as the average slope coefficient divided by its standard deviation.

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7 This is estimated using the Nielsen, Olesen and Risager database stock returns for the same period.

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Table 5.5: Average Results of Return - Relative P/E Regressions

	Unadjusted	Adjusted
Average R^2	0.159	0.114
Average slope coefficient	-0.077	-0.089
Time series t-statistic	-0.809	-3.758

First, note that the estimated coefficients are negative which confirms the above result that high relative P/E on average is followed by low return and that low relative P/E predicts high return.

The higher average R^2 in regressions with unadjusted data is partly due to two years in which the estimated coefficient is positive which indicates that unadjusted regressions are less stable than adjusted ones in this sample. This is also evidenced by a t-statistic which is more than four times as high with adjusted as with unadjusted data. In fact, the coefficient is insignificant if unadjusted data are used. This confirms the portfolio approach result that the use of adjusted earnings data gives more precise predictions for returns than unadjusted earnings.

The same qualitative patterns are present in pooled OLS and GLS regressions although coefficients are not significant in those cases. However, we follow the existing literature in focusing on the results of the portfolio approach and the regression approach described in this section.

5.5. Conclusion

From the study of Danish blue chip stocks presented in this chapter we can conclude that a strategy of investing in stocks with historically low price/earnings has a higher return than the opposite strategy. This result is analogous to conclusions from similar studies of American and international data.

Furthermore, it seems natural to expect that investors exploit that part of the information contained in financial statements which is beyond reported earnings. This chapter has analyzed whether a more detailed analysis of annual reports adds to the return on contrarian strategies proposed by Lakonishok, Shleifer and Vishny (1994) and others, and an affirmative answer is suggested: The relatively simple adjustment rules derived from Graham and Dodd (1934) tend to strengthen predictability of stock returns. We showed that the value premium is not due to risk as measured by Sharpe ratio and average market capitalization but that CAPM-β may provide a risk-based explanation in the case of adjusted earnings data.

This chapter has provided some initial evidence on the role of adjusted earnings. Further research may establish superiority of more sophisticated adjustment procedures. The conclusion of this chapter also raises another questions: Can a similar adjustment be devised for balance sheet variables (like book-to-market value) which also have been suggested to have predictive power? Further research on the issue of accounting data adjustment will probably be useful for understanding the effects at play and their causes.

References:

Ball, R., S. P. Kothari and J. Shanken (1995), Problems in measuring portfolio performance: An application to contrarian investment strategies, *Journal of Financial Economics*, vol. 38, no. 1, pp. 79-107.

Daniel, K. and S. Titman (1997), Evidence on the characteristics of cross-sectional variation in common stock returns, *Journal of Finance*, vol. 52, pp. 1-33.

Fama, E. F. (1998), Market efficiency, long-term returns, and behavioral finance, *Journal of Financial Economics*, vol. 49, pp. 283-306.

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Fama, E. F. and K. R. French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, vol. 33, pp. 3-56.

Fama, E. F. and K. R. French (1998), Value versus growth: The international evidence, *Journal of Finance*, vol. 53, no. 6, pp. 1975-99.

Fama, E. F., and J. D. MacBeth (1973), Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy*, vol. 81, pp. 607-36.

Graham, B. and D. Dodd (1934), *Security Analysis*, McGraw-Hill, New York.

Kinney, M. and R. Trezevant (1997), The use of special items to manage earnings and perceptions, *Journal of Financial Statement Analysis*, Vol. 3, no. 1, pp. 45-53.

Lakonishok, J., A. Shleifer and R. W. Vishny (1994), Contrarian investment, extrapolation, and risk, *Journal of Finance*, vol. 49, no. 5, pp. 1541-78.

Rutterford, J. (1993), *Ordinary shares*, 2nd edition, MacMillan.

Chapter 6

The Impact of Reputation on Analysts' Forecast Bias

6.1. Introduction

This chapter examines the scope of reputation as a disciplining force on the forecast bias of equity analysts. Numerous empirical studies document that analysts issue overly optimistic stock recommendations, including McNichols and O'Brien (1997) and Womack (1996). Frequently, it is argued that reputational considerations may induce a reduction or even a complete elimination of this bias, see for example Hansen and Sarin (1998) and Heitner (1991). The purpose of the present chapter is to explore the impact of reputation in that respect.

The literature provides four explanations of analysts' forecast bias. First, the brokerage or bank in which the analyst is employed may potentially lose investment bank business due to unfavorable stock reports. The hypothesis that such considerations affect stock recommendations is supported by Lin and McNichols (1998) who find that analysts who have underwriting relationships with the firm under evaluation issue more favorable growth forecasts and stronger buy recommendations than unaffiliated analysts. Second, analysts are dependent on access to management information which may be cut off as a result of pessimistic forecasts. Lim (2000) shows that analysts from smaller brokerage firms tend to be more biased than analysts from larger firms. Assuming that analysts in larger firms have easier access to information from other sources, this result suggests that small firm analysts need to be biased to maintain management access. Krishnan and

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Sivaramakrishnan (1999) develop a model of analyst bias based on relationship with management. Third, optimistic forecasts generate more trading commission than pessimistic forecasts because sell recommendations only apply to current stockholders and short-sell investors whereas buy recommendations appeal to all potential investors. The analysis in Hayes (1998) uses trading commissions to explain positive bias. Fourth, security analysts may irrationally overreact to recent data. This explanation was proposed by De Bondt and Thaler (1990)[1].

This chapter provides a simple model of the interaction of investors and privately informed equity analysts who are motivated by trading commissions; thus our analysis draws on the third explanation above. Analysts collect and process information about firms' future prospects and transmit information to uninformed investors. Analysts may be biased towards buy recommendations because of their incentive to generate trading commision. Investors, on the other hand, will realize the profit motive and adjust their beliefs accordingly. Our modeling of reputation is an application of the approach developed by Benabou and Laroque (1992) and Kreps and Wilson (1982) where investors are uncertain about analyst type. The contribution of this chapter is to apply the reputation model to equity analysts and merge it with a standard asset allocation problem of risk averse investors.

We first analyze a single period game. In this case, there is a strong incentive to distort information since analysts need not worry about future reputation. The low information content of stock reports induces investors to care less about recommendations when making portfolio decisions.

In a repeated game, however, analysts need to consider the fact that misleading investors may harm reputation and, hence, expected future profits. The chapter analyzes under which conditions analysts are less biased in a repeated than in a single period game. We furthermore address the question whether reputational concerns can eliminate analysts' bias completely.

1 De Bondt and Thaler (1990) acknowledge the profit motive of brokerage houses but conclude that "forecasted changes are simply too extreme to be considered rational".

In a related paper, Chammanur and Fulghieri (1994) study the role of reputation for investment banks' choice of effort in their screening of candidates for initial public offering. They find that in the absence of reputational considerations, investment banks have an incentive to save resources in the screening process and tell investors that the issuing firm has growth potential. In a repeated game, however, the investment bank realizes the potential harm to reputation caused by such behavior and, hence, chooses to extend its effort in searching for valuable firms. The paper concludes that concern for reputation increases the information content of IPO marketing material. The present chapter focuses on recommendations of stocks on secondary markets as opposed to IPOs. Therefore, it is natural in our context to model the asset allocation decision explicitly, ie., to allow for risk aversion.

Hayes (1998) also considers the effect of trading commissions for forecast bias. She finds that since buy recommendations are more profitable, analysts will only gather information on stocks which are expected to perform well. Therefore, reports for these stocks will be more accurate than reports for other stocks. The paper does not address the role of reputation.

Section 6.2 describes the model. Section 6.3 contains the analysis of the single period game whereas the repeated game is examined in section 6.4. Finally, section 6.5 concludes.

6.2. Model

6.2.1. Players and timing

There is a large number of private investors who are assumed to be homogeneous. The representative private investor is endowed with initial exogenous wealth and must choose in which form to save wealth in order to be able to consume at the end of period. The options are the risk-free asset and stocks. Investors are assumed to be risk averse.

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We assume that there is only one equity analyst. This allows us to study the analyst-investor relationship without being distracted by strategic interactions between analysts.

Assume there is one stock which has a random value at the end of the period. In the beginning of period, the equity analyst receives a noisy signal about the value of the stock. The analyst signals expected end-of-period value of the stock to private investors. After receiving the expected value signal, investors decide which fraction of wealth to allocate to stocks. They can only buy stock through the analyst.

The equity analyst receives a trading commission which is a constant, exogenously determined fraction of the amount of capital allocated to the analyst.[2] The objective of the analyst is to maximize profits accruing from trading commissions.

The price of the stock is determined after investors make their asset allocation decision as the market clearing value.

6.2.1.1. The stock

The end-of-period value of the stock is random. The average value is either high, θ^H, or low, θ^L. We say that state of nature of the stock is $s \in S = \{L,H\}$. The probability of these two states of nature is one half each. The value, θ, is normally distributed with mean θ^s and variance $\sigma_\theta^2 \in \mathbf{R}_{++}$.

Although this is not strictly ensured by the normal distribution, one should think of stock value as being positive. This is a reasonable approximation if the means are large relative to the standard deviation.

At the beginning of each period the equity analyst receives a signal, s, about the average value of the stock. The parameters of the distributions are known to all players.

6.2.1.2. The equity analyst

After observing the signal in the beginning of the period, the analyst chooses expected stock value announcement, θ^m, to maximize expected profit:

[2] The commission is supposed to capture the essence of a fee structure which is often more complex in reality. Furthermore, as section 6.3 demonstrates it is consistent with the empirical finding that stock reports are positively biased.

$$\max_{\theta^m} E[\phi n \gamma^e W_0], \tag{6.1}$$

where $\phi \in (0,1)$ is the exogenous share of investments which the equity analyst keeps as trading commission, $n \in \mathbf{N}_{++}$ is the number of investors, γ^e is the fraction of private investor wealth expected to be allocated to the analyst by the representative investor and W_0 is beginning-of-period private investor wealth. Since investors know the distribution of θ, $\theta^m \in \{\theta^L, \theta^H\}$.

The equity analyst receives a signal and issues an announcement, θ^m. Based on the empirical evidence, which tells us that announcements are positively biased, we restrict the message response to high signals to being optimistic, that is $\theta^m(H) = \theta^H$. This forces us to consider only those equilibria that are interesting from an empirical perspective. Message response to low signals, on the other hand, is allowed to be in the probability measures on the set of possible announcements, ie., the analyst is free to choose a strategy of lying about a low signal with probability q and telling the truth with probability 1-q.

Thus, an equilibrium strategy for the equity analyst is an announcement rule:

$$m(\theta^m(s)) = \begin{cases} \theta^H \ \textit{if } s=H \\ \theta^H \ \textit{with probability } q \ \textit{if } s=L \\ \theta_L \ \textit{with probability } 1-q \ \textit{if } s=L \end{cases} \tag{6.2}$$

Hence, any strategy, $m(\cdot)$, is fully characterized by q. The investor is said to be unbiased if $q = 0$, and biased if $q > 0$.

6.2.1.3. Private investors

Investors can invest in a risk-free asset or the stock.

The representative investor chooses the portfolio share of the stock, γ. The objective is to maximize expected value of an exponential utility

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function defined over end-of-period wealth:

$$\max_{\gamma} E[-\exp[-AW] | \theta^m] \ , \tag{6.3}$$

where $A > 0$ is constant absolute risk aversion. This utility function is widely used, see for example Grossman and Stiglitz (1980).

The investor uses his prior on stock value and the message received from the equity analyst to form a posterior belief about the value of the stock. The investor also forms a belief about the price of the stock, p^e. In forming this belief, the investor ignores his effect on price through γ, ie., the investor is assumed to be price taker.

We model reputation like in Benabou and Laroque (1992) and Kreps and Wilson (1982). The investor is uncertain about whether the analyst is biased. With probability ρ, the analyst is assumed to send unbiased messages, ie., q=0, and with probability $1-\rho$ the analyst is assumed to choose $q \in [0,1]$ in order to maximize profits as described above. The purpose of this set-up is to determine the equilibrium behavior of a profit-maximizing analyst given some prior reputation, ρ. For example, we consider whether reputation affects the analyst's incentive to manipulate financial markets.

The private investor is assumed to follow a pure strategy of choosing $\gamma:\{\theta^L,\theta^H\} \rightarrow (0,1)$. A representative investor needs to hold an amount of each asset which is the motivation for assuming $\gamma \in (0,1)$.

6.2.2. Prices

The price of the risk-free asset is set to 1.

The number of outstanding shares is normalized to 1. The demand is $n\gamma W_0(1-\phi)/p$. Hence, the market clearing price is:

$$p = n\gamma W_0(1-\phi) \tag{6.4}$$

Notice that the assumption that $\gamma \in (0,1)$ ensures nonpositive prices. It is furthermore assumed that investors' price expectations are rational, ie., that p^e equals the equilibrium price.

6.2.3. Equilibrium concept

Investors observe the message sent by the analyst before choosing their action. Thus, the game is dynamic. On the other hand, investors do not have access to the signal on stock value which makes the game one of incomplete information.

We are going to use perfect bayesian equilibrium as our solution concept. This is standard in dynamic games of incomplete information. According to this concept, an equilibrium consists of a set of strategies which are optimal for each agent given beliefs, and beliefs which are consistent with strategies. In our particular case, this implies that private investors whenever possible form beliefs about stock value through bayesian updating based on analyst messages while taking the strategy of the equity analyst into consideration. Beliefs are then used to choose the optimal portfolio share of the stock.

The equity analyst, on the other hand, considers the effect on beliefs, and, hence, on investment, of his choice of expected value announcement.

6.2.4. Discussion

We assume that investors only have access to stock through equity analysts. This is not entirely realistic, but presumably a large number of transactions are carried out by banks, brokerages etc. The assumption allows us to skip rationalizing the existence of equity analysts.

6.3. Single period game

This section considers whether there exist single period perfect bayesian equilibria in which the analyst's strategy is as described in (6.2). Thus, we need to characterize equilibrium private investor beliefs about liquidating value, i.e., beliefs which are consistent with analyst ie., that p^e equals the equilibrium price.

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strategy. Furthermore, we need to describe optimal investor strategy, and to ensure that the analyst strategy is optimal given the stipulated private investor behavior.

6.3.1. Private investor beliefs

The investor updates prior beliefs on stock value using the message received from the analyst. In case the message is high, the analyst may have received either a high or a low signal. The prior probability of a low signal is ½. The investor believes there is a $1-\rho$ probability that the analyst is biased and the strategy of a biased analyst is to announce a high message with probability q when the signal is low. Thus, the possibility that the signal is low receives a weight of $\frac{1}{2}(1-\rho)q$. This weight increases with analyst bias and decreases with analyst reputation. Alternatively, the signal is high. This has prior probability ½. In this case, both analyst types transmit a high signal. Thus, the weight ½ is attached to the possibility that the signal is high. Hence, after having received a high message, the investor believes that with probability $(1-\rho)q/(1+q+q\rho)$ stock value is normal distributed around θ^L and with probability $1/(1+q+q\rho)$ stock value is normal distributed around θ^H. That is, the investor's posterior beliefs after a high message are distributed as a mixture of two normal distributions. This is formally derived in Appendix 6.A. Notice that unless the analyst has a reputation at its lower limit (ρ=0) and systematically lies (q=1), a high message will lead investors to believe that a high signal is more likely than a low. Thus, if reputation is positive, a high signal will make investors update prior beliefs.

In case the message is low, however, the investor realizes that the signal must be low too. Thus, posterior beliefs after a low message are normal distributed around θ^L. This is also derived in Appendix 6.A.

6.3.2. Optimal private investor strategy

Having derived beliefs, we may calculate expected utility of the private investor. Expected utility depends on whether the received message is optimistic, $\theta^m = \theta^H$, or pessimistic, $\theta^m = \theta^L$. Beginning with the latter case, expected utility is:

$$\begin{aligned} E[u|\theta^L] &= E[-\exp[-AW]|\theta^L] \\ &= \int_{\theta\in\mathbb{R}} -\exp[-AW(\theta)]b(\theta(\theta^L))\, d\theta \end{aligned}$$

It is shown in Appendix 6.B that this is equivalent to:

$$E[u|\theta^L] =$$
$$-\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2-A(1-\gamma)(1+r_f)W_0]$$

The first order condition for the investor's maximization problem thus is:

$$-A\frac{\theta^L}{p^e}(1-\phi)W_0+A^2\frac{1}{(p^e)^2}(1-\phi)^2\sigma_\theta^2W_0^2\gamma+A(1+r_f)W_0 = 0$$

Hence, the optimal stock share given a pessimistic message is:

$$\gamma = \frac{(\theta^L-(1+r_f)p^e/(1-\phi))p^e}{A(1-\phi)W_0\sigma_\theta^2} \qquad (6.5)$$

This is the standard solution to the asset allocation decision of risk averse investors when the value of the risky asset is normal distributed. For example, (6.5) may be rewritten in terms of the representative agent's demand for the stock:

$$\gamma W_0(1-\phi)/p^e = \frac{\theta^L-(1+r_f)p^e/(1-\phi)}{A\sigma_\theta^2}$$

which is identical to equation (8) in Grossman and Stiglitz (1980) except investor. Expected utility depends on whether the received message is optimistic, $\theta^m = \theta^H$, or pessimistic, $\theta^m = \theta^L$. Beginning with the latter case, expected utility is:

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which is identical to equation (8) in Grossman and Stiglitz (1980) except

for the trading commission which is not present in their model. Demand for stock depends positively on the expected value given a low signal. On the other hand, demand depends negatively on trading commission, risk aversion, volatility of stock value, risk-free return and the expected price of the stock. That demand is independent of wealth for a given price is a standard feature of constant absolute risk aversion utility functions in combination with normal distributed stock value. When the message is pessimistic, investors are certain that the signal is low. Therefore, the asset allocation decision is made independently of analyst reputation and bias.

Let us return to (6.5) and insert the rational expectations assumption, $p^e = p = n\gamma W_0(1-\phi)$, to obtain the optimal stock share given a pessimistic message:

$$\gamma = \frac{\theta^L n\gamma-(1+r_f)n^2\gamma^2W_0}{A\sigma_\theta^2}$$

This is equivalent to:

$$\gamma^P = \frac{\theta^L n-A\sigma_\theta^2}{(1+r_f)W_0n^2} \tag{6.6}$$

Since we wish to limit attention to $\gamma^P > 0$ cases only, (6.6) implies a parameter restriction, namely that $\theta^L > A\sigma_\theta^2/n$. Furthermore, the restriction that $\gamma^P < 1$ is equivalent to $\theta^L < A\sigma_\theta^2/n+(1+r_f)W_0n$.

In a similar manner, it is possible to calculate expected utility when the message is optimistic. However, this calculation is complicated by the fact that investors' belief about stock value is a mixture of two normal distributions rather than a normal distribution. Therefore, calculations have been put in Appendix 6.C.

Appendix 6.D derives the first order condition. Again using the rational expectations assumption, we obtain the following optimal stock share when the message is optimistic:

$$\gamma^O = \frac{\theta^L n - A\sigma_\theta^2}{(1+r_f)W_0 n^2} + \frac{(\theta^H - \theta^L)\exp[-\frac{A}{n}\theta^H]}{((1-\rho)q\exp[-\frac{A}{n}\theta^L] + \exp[-\frac{A}{n}\theta^H])(1+r_f)W_0 n}$$

(6.7)

$\gamma^O > \gamma^P$ since $\theta^H > \theta^L$ by definition which implies that the second term in (6.7) is positive. Thus, high messages attract investors into stocks despite the possibility that high messages are due to analyst distortion rather than high signals. This is because investors who receive a high message believe a high signal to be more likely than a low as discussed in section 6.3.1. On the other hand, if the message is low, investors know for sure that the signal is low.

When the message is optimistic, investors consider reputation and bias when choosing their asset allocation. First, when the analyst's reputation improves, high signals become more credible which make investors increase their stock share (ie., $\partial\gamma^O / \partial \rho > 0$). The exception is when both types of analyst are unbiased (ie., $q = 0$). In that case, reputation does not matter for asset allocation because analysts are identical. Second, optimal stock share is decreasing in analyst's bias since an increase in bias makes messages less credible (ie., $\partial\gamma^O / \partial q < 0$). However, in the special case where investors are certain that the analyst is unbiased, the degree of bias for a biased analyst becomes irrelevant. The derivatives are in Appendix 6.E.

Since γ^O is greater than γ^P, it is also greater than 0 under the parameter restriction imposed above. Thus, we only need to ensure that $\gamma^O < 1$. This is done by realizing that the largest possible γ^O is obtained when $\rho = 1$ and $q = 0$, cf. the derivatives above. Thus, we need:

$$\frac{\theta^L n - A\sigma_\theta^2}{(1+r_f)W_0 n^2} + \frac{\theta^H - \theta^L}{(1+r_f)W_0 n} < 1$$

$\Updownarrow$

$$\theta^H < \frac{A}{n}\sigma_\theta^2 + (1+r_f)W_0 n$$

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$$\Updownarrow$$

$$\theta^H < \frac{A}{n}\sigma_\theta^2 + (1+r_f)W_0 n$$

Since $\theta^L < \theta^L$ by definition, this parameter restriction implies $\theta^L < A\sigma_\theta^2/n + (1+r_f)W_0 n$ which was imposed in association with the solution for γ^P. Thus, the necessary parameter restrictions to ensure that $\gamma^P \in (0,1)$ and $\gamma^O \in (0,1)$ are:

$$\theta^L > \frac{A}{n}\sigma_\theta^2$$

$$\theta^H < \frac{A}{n}\sigma_\theta^2 + (1+r_f)W_0 n$$

6.3.3. Optimal equity analyst strategy

The equity analyst chooses to send the message which maximizes expected profit. When the message is pessimistic, expected profit is:

$$E[\pi^P] = \phi n \gamma^P W_0 \tag{6.8}$$

Similarly, when the message is optimistic:

$$E[\pi^P] = \phi n \gamma^O W_0 \tag{6.9}$$

Since $\gamma^O > \gamma^P$, expected profits are maximized by sending optimistic messages. This implies that it is not optimal for the analyst to randomize messages in response to low signals. Instead, the analyst optimally chooses systematically to distort low signals, ie., q=1. Hence, in the equilibrium of the single period game, no information is transmitted from analyst to investors.

6.3.4. Discussion

In the Benabou and Laroque (1992) model, there are one journalist and two kinds of investors, namely rational investors and noise traders. The journalist receives a noisy signal on the value of financial assets and may send a message to rational investors. The journalist may be honest about signals or opportunistic in which case messages are used to maximize expected utility. Notice that in contrast to the present chapter, the opportunistic journalist distorts both good and bad signals. Thus, the journalist is unbiased.

Our single period game equilibrium is similar to the result in Benabou and Laroque (1992) that the opportunistic journalist always lies. These results correspond to the fact that the investment bank in Chammanur and Fulghieri (1994) markets equities which it has only imperfectly evaluated.

6.4. Repeated game

In the single period game, the incentive for the equity analyst to distort information is so strong that profit-maximizing analysts never reveal low signals. However, in a repeated game, the analyst is also concerned with future profits, and in this section we analyze whether reputational considerations can motivate the analyst to tell the truth.

Investors will use the track record of analysts to update reputation. Deviation of actual stock value from message indicates to the investor that the analyst distorted the signal, for example if the message was high and the actual value turned out to be low. However, since the analyst's information is noisy, the deviation could just be bad luck. Hence, investors never know for certain that information was distorted.

In period t, investors' prior probability that the analyst is of the unbiased type is denoted by ρ_t. We assume that investors live one period. Each generation has initial endowment W_0 and learns the updated reputation from the previous generation.[3]

The equity analyst has an infinite horizon. The objective of the analyst is assumed to be maximization of the discounted sum of expected future profit. The discount factor, $\delta \in [0,1]$, is assumed

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constant.

Investors' equilibrium behavior is identical to their behavior in the single period game as described by (6.6) and (6.7) with ρ replaced by ρ_t. The analyst, on the other hand, now needs to consider future as well as current effects of her announcements; telling a lie increases current profit but harms reputation and, hence, decreases expected future profits.

Let us consider an analyst who has initial reputation ρ_0, follows a strategy characterized by q and receives a low signal. It is useful to calculate the expected value of telling the truth, $V^t(q,\rho_0)$, and the expected value of telling a lie, $V^l(q,\rho_0)$. A comparison of $V^t(\cdot)$ and $V^l(\cdot)$ facilitates characterization of the equilibrium. For example, if $V^t(\cdot)$ is greater than $V^l(\cdot)$, the analyst prefers to tell the truth. In that case, $q > 0$ will not be optimal.

The decision of whether to lie or tell the truth determines expected future reputation. Let $\rho_1^t, \rho_2^t, \ldots$ be the sequence of expected reputations following a true message and $\rho_1^l, \rho_2^l, \ldots$ the sequence of expected reputations following a lie. Then the expected value of telling the truth is:

$$V^t(q,\rho_0) = \pi^P + \delta\left\{\frac{1}{2}(1+q)\pi^O(q,\rho_1^t) + \frac{1}{2}(1-q)\pi^P(q,\rho_1^t)\right\} + \delta^2\left\{\frac{1}{2}(1+q)\pi^O(q,\rho_2^t) + \frac{1}{2}(1-q)\pi^P(q,\rho_2^t)\right\} + \ldots \tag{6.10}$$

The expected value of lying is:

$$V^l(q,\rho_0) = \pi^O(q,\rho_0) + \delta\left\{\frac{1}{2}(1+q)\pi^O(q,\rho_1^l) + \frac{1}{2}(1-q)\pi^P(q,\rho_1^l)\right\} + \delta^2\left\{\frac{1}{2}(1+q)\pi^O(q,\rho_2^l) + \frac{1}{2}(1-q)\pi^P(q,\rho_2^l)\right\} + \ldots \tag{6.11}$$

6.4.1. Strategy of unbiased announcements, q=0

Let us first consider the strategy of unbiased announcements, ie., q=0. In that case, we know from the single period game that reputation is irrelevant since analyst types are identical, ie., $\pi^O(0,x)-\pi^O(0,y)=0$,

$\forall x,y \in [0,1]$.

Hence,

$$
\begin{aligned}
&V^t(0,\rho_0) - V^l(0,\rho_0)\\
&= \pi^P - \pi^O(0,\rho_0) + \delta\left\{\frac{1}{2}(\pi^O(0,\rho_1^t) - \pi^O(0,\rho_1^l)) + \frac{1}{2}(\pi^P(0,\rho_1^t) - \pi^P(0,\rho_1^l))\right\} + ..\\
&= \pi^P - \pi^O(0,\rho_0)\\
&< 0
\end{aligned}
$$

This implies that it is preferable for the analyst to lie rather than follow the unbiased strategy. Thus, like in the single period game, the analyst always lies with positive probability in the repeated game. This result is also consistent with Proposition 3 in Benabou and Laroque (1992) and with the empirical evidence mentioned in the introduction.

6.4.2. Strategy of systematic distortion, q=1

Next, let us consider the opposite strategy of always distorting information, ie., q=1. Appendix 6.F shows that when q=1, $\rho_1^t = \rho_2^t = ... = 1$. Furthermore, $\rho_1^l = c\rho_0/(1-\rho_0+c)$,

$$
\rho_2^l = \frac{\rho_1^l}{2}\left[\frac{1}{1-c\rho_1^l+c} + \frac{c}{1-\rho_1^l+c}\right]
$$

$$
\rho_3^l = \frac{\rho_2^l}{2}\left[\frac{1}{1-c\rho_2^l+c} + \frac{c}{1-\rho_2^l+c}\right]
$$

and so forth where $c = exp[-(\theta^H-\theta^L)^2/(2\sigma_\theta^2)]$. Telling the truth has a favorable impact on reputation in all future periods, ie., $\rho_t^t \geq \rho_t^l$, $\forall t$.

Appendix 6.F also computes the effects on expected reputation in the following period of telling the truth and lying, respectively. This analysis shows, firstly, that better prior reputation improves future reputation because the latter is a revision of the former. Secondly, when the analyst sends a low message, investors believe that the analyst is either profit-

$\forall x,y \in [0,1]$.

Hence,

$$V^t(0,\rho_0)-V^l(0,\rho_0)$$
$$= \pi^P-\pi^O(0,\rho_0)+\delta\left\{\frac{1}{2}(\pi^O(0,\rho_1^t)-\pi^O(0,\rho_1^t))+\frac{1}{2}(\pi^P(0,\rho_1^t)-\pi^P(0,\rho_1^t))\right\}+..$$
$$= \pi^P-\pi^O(0,\rho_0)$$
$$< 0$$

This implies that it is preferable for the analyst to lie rather than follow the unbiased strategy. Thus, like in the single period game, the analyst always lies with positive probability in the repeated game. This result is also consistent with Proposition 3 in Benabou and Laroque (1992) and with the empirical evidence mentioned in the introduction.

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$$\rho_2^l = \frac{\rho_1^l}{2}\left[\frac{1}{1-c\rho_1^l+c}+\frac{c}{1-\rho_1^l+c}\right]$$
$$\rho_3^l = \frac{\rho_2^l}{2}\left[\frac{1}{1-c\rho_2^l+c}+\frac{c}{1-\rho_2^l+c}\right]$$

and so forth where $c = exp[-(\theta^H-\theta^L)^2/(2\sigma_\theta^2)]$. Telling the truth has a favorable impact on reputation in all future periods, ie., $\rho_t^t \ge \rho_t^l$, $\forall t$.

Appendix 6.F also computes the effects on expected reputation in the following period of telling the truth and lying, respectively. This analysis shows, firstly, that better prior reputation improves future reputation because the latter is a revision of the former. Secondly, when the analyst sends a low message, investors believe that the analyst is either profit-

maximizing and telling the truth or unbiased. The more the profit-maximizer tends to lie, the more likely it becomes that the analyst is unbiased. Therefore, expected future reputation after telling the truth increases with bias, q. On the other hand, when the analyst sends a high message, this may reflect that the signal is high or that the analyst lies about a low signal. The more biased the analyst is, the more weight attached to the latter scenario and, hence, the lower expected reputation when telling a lie. Thirdly, the more precise the signal is (ie., the lower c), the less likely it is that inaccurate messages are just due to bad luck, which implies that reputation is harmed more by lying. On the other hand, precision does not affect reputation after telling the truth since in that case investors know that the signal is low and that both types of analyst are subject to the same degree of chance.

Appendix 6.G shows that

$$V^t(1,\rho_0)-V^l(1,\rho_0)=\frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\left[-K_0+\delta K_1+\delta^2K_2+\delta^3K_3+...\right]$$

where $K_0 = \dfrac{1}{(1-\rho_0)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}$, and

$$K_t = \frac{(\rho_t^t-\rho_t^l)\exp[-\frac{A}{n}\theta^L]}{((1-\rho_t^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])((1-\rho_t^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}.$$

for t=1,2, ...

Since $(\phi(\theta^H - \theta^L)\exp[-A\theta^H/n])/(1+r_f) > 0$, the sign of $V^t(1, \rho_0) - V^l(1, \rho_0)$ is determined by the sign of the expression in squared brackets. Notice that K_0 and K_t are positive. The first term in squared brackets is negative and reflects the current loss from telling the truth rather than lying. The remaining terms are positive and express expected future gains associated with higher reputation.

Let us examine under which circumstances the strategy of always lying is inoptimal in the repeated game, ie., $V^t(1, \rho_0) - V^l(1, \rho_0) > 0$. First of all, when the analyst does not care about the future at all, that is δ=0, the expression is negative. Then the analyst gains more from lying than from telling the truth which is consistent with q=1. Thus, when the analyst is extremely impatient, it is indeed an equilibrium strategy for the analyst systematically to distort all low signals. Furthermore, it is a unique equilibrium which was shown in section 6.3. The derivative of $V^t(1, \rho_0)$ - $V^l(1, \rho_0)$ with respect to δ equals

$$\frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\left[K_1+2\delta K_2+...\right]$$

which is positive. Thus, $V^t(1, \rho_0)$ - $V^l(1, \rho_0)$ is maximized at δ=1. Therefore, it suffices to consider $V^t(1, \rho_0)$ - $V^l(1, \rho_0)$ at δ=1 to determine whether it ever becomes positive. Thus, when δ=1, we have:

$$V^t(1,\rho_0)-V^l(1,\rho_0) = \frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\left[-K_0+K_1+K_2+...\right] \tag{6.8}$$

Appendix 6.H shows that $K_t > K_{t-1}$ for t=2,3, ... iff reputation is expected to decline monotonically after a lie, ie., $\rho^l_{t-1} > \rho^l_t$. Appendix 6.I demonstrates that the latter inequality is fulfilled on [0,1). This implies that the smallest element in the sum K_1, K_2, ... is K_1. Thus,

$$\sum_{t=1}^{\infty} K_t > \sum_{t=1}^{\infty} K_1$$

$K_1 > 0$ when $\rho^l_1 < 1$. Hence, the infinite sum diverges (see theorem 9.4, p. 213 in Protter and Morrey, 1991) which implies that $V^t(1,\rho_0)$ - $V^l(1,\rho_0) > 0$ when $\delta = 1$.

Since $V^t(1,\rho_0)$ - $V^l(1,\rho_0)$ is negative at $\delta = 0$ and positive at $\delta = 1$ and the derivative with respect to δ is positive, there is a δ* such that $V^t(1,\rho_0)$ - $V^l(1,\rho_0) = 0$. That is, if $\delta < \delta^*$, the analyst cares only little about the future and, therefore, it will be an equilibrium strategy for her to systematically mislead investors. If, on the other hand, the analyst is sufficiently patient, ie., $\delta > \delta^*$, the future reputation loss will induce the than from telling the truth which is consistent with q=1. Thus, when the analyst is extremely impatient, it is indeed an equilibrium strategy for the analyst systematically to distort all low signals. Furthermore, it is a unique equilibrium which was shown in section 6.3. The derivative of $V^t(1, \rho_0)$ - $V^l(1, \rho_0)$ with respect to δ equals

$$\frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\left[K_1+2\delta K_2+...\right]$$

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$$V^t(1,\rho_0)-V^l(1,\rho_0) = \frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\left[-K_0+K_1+K_2+...\right] \quad (6.8)$$

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Since $V^t(1,\rho_0)$ - $V^l(1,\rho_0)$ is negative at δ = 0 and positive at δ = 1 and the derivative with respect to δ is positive, there is a δ* such that $V^t(1,\rho_0)$ - $V^l(1,\rho_0) = 0$. That is, if δ < δ*, the analyst cares only little about the future and, therefore, it will be an equilibrium strategy for her to systematically mislead investors. If, on the other hand, the analyst is sufficiently patient, ie., δ > δ*, the future reputation loss will induce the

analyst to refrain from the strategy of lying with certainty.

Chammanur and Fulghieri (1994) reach a somewhat stronger conclusion, namely that the presence of reputation unambigously reduces investment banks' bias. This may reflect that the investment bank in their model is assumed to have δ = 1.

6.5. Conclusion

The chapter examines analyst forecast bias by merging a model of equity analyst reputation with a standard asset allocation problem. We show that short-sighted profit-maximizing equity analysts generate maximum trading commission by systematically misleading investors when prospects of the equity market are poor. This behavior is not necessarily optimal in a repeated game. Thus, an analyst who cares sufficiently about the future trades off current profit for higher expected future reputation by publishing less biased stock recommendations than the myopic analyst. However, concern for reputation does not completely eliminate distortion of information. Hence, the equilibrium of the model is consistent with empirical findings.

References

Benabou, R. and G. Laroque (1992), Using privileged information to manipulate markets: Insiders, gurus, and credibility, *Quarterly Journal of Economics*, pp. 921-58.

Chammanur, T.J. and P. Fulghieri (1994), Investment bank reputation, information production, and financial intermediation, *Journal of Finance* 49 (1), pp. 57-79.

De Bondt, W.F.M. and R.H. Thaler (1990), Do security analysts overreact?, *American Economic Review* 80 (2), pp. 52-57.

Grossman, S.J. and J.E. Stiglitz (1980), On the impossibility of informationally efficient markets, *American Economic Review* 70 (3), pp. 393-408.

Hansen, R.S. and A. Sarin (1998), Are analysts overoptimistic around seasoned equity offerings?, working paper, Santa Clara University.

Hayes, R.M. (1998), The impact of trading commission incentives on analysts' stock coverage decisions and earnings forecasts, *Journal of Accounting Research* 36 (2), pp. 299-320.

Heitner, P. (1991), Isn't it time to measure analysts' track records?, *Financial Analysts Journal*, pp. 5-6.

Kreps, D.M. and R. Wilson (1982), Reputation and imperfect information, *Journal of Economic Theory* 27, pp. 253-279.

Krishnan, M. and K. Sivaramakrishnan (1999), A theory of analyst forecast bias, working paper, Rutgers University.

Lim, T. (2000), Rationality and analysts' forecast bias, forthcoming in *Journal of Finance*.

References

Benabou, R. and G. Laroque (1992), Using privileged information to manipulate markets: Insiders, gurus, and credibility, *Quarterly Journal of Economics*, pp. 921-58.

Chammanur, T.J. and P. Fulghieri (1994), Investment bank reputation, information production, and financial intermediation, *Journal of Finance* 49 (1), pp. 57-79.

De Bondt, W.F.M. and R.H. Thaler (1990), Do security analysts overreact?, *American Economic Review* 80 (2), pp. 52-57.

Grossman, S.J. and J.E. Stiglitz (1980), On the impossibility of informationally efficient markets, *American Economic Review* 70 (3), pp. 393-408.

Hansen, R.S. and A. Sarin (1998), Are analysts overoptimistic around seasoned equity offerings?, working paper, Santa Clara University.

Hayes, R.M. (1998), The impact of trading commission incentives on analysts' stock coverage decisions and earnings forecasts, *Journal of Accounting Research* 36 (2), pp. 299-320.

Heitner, P. (1991), Isn't it time to measure analysts' track records?, *Financial Analysts Journal*, pp. 5-6.

Kreps, D.M. and R. Wilson (1982), Reputation and imperfect information, *Journal of Economic Theory* 27, pp. 253-279.

Krishnan, M. and K. Sivaramakrishnan (1999), A theory of analyst forecast bias, working paper, Rutgers University.

Lim, T. (2000), Rationality and analysts' forecast bias, forthcoming in *Journal of Finance*.

Lin, H. and M.F. McNichols (1998), Underwriting relationships, analysts' earnings forecasts and investment recommendations, *Journal of Accounting and Economics* 25, pp. 101-127.

McNichols, M. and P.C. O'Brien (1997), Self-selection and analyst coverage, *Journal of Accounting Research* 35 (suppl.), pp. 167-199.

Protter, M.H. and C.B. Morrey (1991), *A first course in real analysis*, 2nd edition, Springer-Verlag, New York.

Womack, K.L. (1996), Do brokerage analysts' recommendations have investment value?, *Journal of Finance* 51 (1), pp. 137-167.

Appendix

6.A Private investor beliefs

The private investor updates prior beliefs about stock value according to the following Bayesian scheme:

$$b(\theta(\theta^m)) = \frac{f_\theta(\theta) f_{\theta^m|\theta}(\theta^m|\theta)}{\int_{\theta\in\mathbb{R}} f_{\theta^m|\theta}(\theta^m|\theta) f_\theta(\theta)\, d\theta} \tag{6.A.1}$$

Let us consider,

$$\begin{aligned} f_{\theta^m|\theta}(\theta^m|\theta) &= \frac{f_{\theta^m,\theta}(\theta^m,\theta)}{f_\theta(\theta)} \\ &= \frac{\int_{s\in S} f_{\theta^m,\theta|s}(\theta^m,\theta|s) f_s(s)\, ds}{f_\theta(\theta)} \\ &= \frac{\int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)\, ds}{f_\theta(\theta)} \end{aligned}$$

Inserting this in (6.A.1) yields

$$b(\theta(\theta^m)) = \frac{\int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)\, ds}{\int_{\theta\in\mathbb{R}} \int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)\, ds \; d\theta}$$

Consider the posterior belief after having received a positive message:

$$\begin{aligned} b(\theta(\theta^H)) &= \frac{(1-\rho) q f_{\theta|s}(\theta|L)\frac{1}{2} + (\rho\cdot 1 + (1-\rho)1) f_{\theta|s}(\theta|H)\frac{1}{2}}{\int_{\theta\in\mathbb{R}} (1-\rho) q f_{\theta|s}(\theta|L)\frac{1}{2} + (\rho\cdot 1 + (1-\rho)1) f_{\theta|s}(\theta|H)\frac{1}{2} d\theta} \\ &= \frac{(1-\rho) q f_{\theta|s}(\theta|L) + f_{\theta|s}(\theta|H)}{1+q-q\rho} \end{aligned}$$

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Let us consider,

$$\begin{aligned} f_{\theta^m|\theta}(\theta^m|\theta) &= \frac{f_{\theta^m,\theta}(\theta^m,\theta)}{f_\theta(\theta)} \\ &= \frac{\int_{s\in S} f_{\theta^m,\theta|s}(\theta^m,\theta|s) f_s(s)ds}{f_\theta(\theta)} \\ &= \frac{\int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)ds}{f_\theta(\theta)} \end{aligned}$$

Inserting this in (6.A.1) yields

$$b(\theta(\theta^m)) = \frac{\int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)ds}{\int_{\theta\in\mathbb{R}}\int_{s\in S} f_{\theta^m|s}(\theta^m|s) f_{\theta|s}(\theta|s) f_s(s)ds\ d\theta}$$

Consider the posterior belief after having received a positive message:

$$\begin{aligned} b(\theta(\theta^H)) &= \frac{(1-\rho)q f_{\theta|s}(\theta|L)\frac{1}{2} + (\rho\cdot 1 + (1-\rho)1) f_{\theta|s}(\theta|H)\frac{1}{2}}{\int_{\theta\in\mathbb{R}} (1-\rho)q f_{\theta|s}(\theta|L)\frac{1}{2} + (\rho\cdot 1 + (1-\rho)1) f_{\theta|s}(\theta|H)\frac{1}{2}d\theta} \\ &= \frac{(1-\rho)q f_{\theta|s}(\theta|L) + f_{\theta|s}(\theta|H)}{1+q-q\rho} \end{aligned}$$

Similarly,

$$\begin{aligned} b(\theta(\theta^L)) &= \frac{(\rho+(1-\rho)(1-q)) f_{\theta|s}(\theta|L)\frac{1}{2}}{\int_{\theta\in\mathbb{R}} (\rho+(1-\rho)(1-q)) f_{\theta|s}(\theta|L)\frac{1}{2}d\theta} \\ &= f_{\theta|s}(\theta|L) \end{aligned}$$

The description of investors' beliefs is completed by noting that $f_{\theta|s}$ is normal with mean θ^s and variance σ_θ^2.

6.B Expected utility given low signal

$$\begin{aligned} & E[u|\theta^L] \\ &= E[-\exp[-AW]|\theta^L] \\ &= \int_{\theta\in\mathbb{R}} -\exp[-AW(\theta)] b(\theta(\theta^L))d\theta \\ &= \int_{\theta\in\mathbb{R}} -\exp[-A(\gamma\frac{\theta}{p^e}(1-\phi)+(1-\gamma)(1+r_f))W_0]\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^L)^2}{2\sigma_\theta^2}]d\theta \\ &= -\exp[-A(1-\gamma)(1+r_f)W_0]\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L)}{2\sigma_\theta^2}]d\theta \\ &= -\exp[-A(1-\gamma)(1+r_f)W_0]\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\times \\ & \exp[\frac{\begin{array}{c}(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L-2A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0\sigma_\theta^2 \\ +A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2\sigma_\theta^4 W_0^2+2A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0\sigma_\theta^2-A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2\sigma_\theta^4 W_0^2)\end{array}}{2\sigma_\theta^2}]d\theta \\ &= -\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2 W_0^2\sigma_\theta^2-A(1-\gamma)(1+r_f)W_0]\times \\ & \int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^L+A\gamma\frac{1}{p^e}(1-\phi)W_0\sigma_\theta^2)^2}{2\sigma_\theta^2}]d\theta \\ &= -\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2 W_0^2\sigma_\theta^2-A(1-\gamma)(1+r_f)W_0] \end{aligned}$$

since the integrand in the penultimate line is the density of a normal random variable with mean

$$\theta^L - A\gamma\frac{1}{p^e}(1-\phi)W_0\sigma_\theta^2$$

and variance σ_θ^2 which integrates to 1.

6.C Expected utility given high message

$$\begin{aligned}
&E[u|\theta^H] \\
&= E[-\exp[-AW]|\theta^L] \\
&= \int_{\theta\in\mathbb{R}} -\exp[-AW(\theta)]b(\theta(\theta^H))d\theta \\
&= \int_{\theta\in\mathbb{R}} -\exp[-A(\gamma\frac{\theta}{p^e}(1-\phi)+(1-\gamma)(1+r_f))W_0] \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^L)^2}{2\sigma_\theta^2}] \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^H)^2}{2\sigma_\theta^2}]\right\}d\theta \\
&= -\exp[-A(1-\gamma)(1+r_f)W_0]\times \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L)}{2\sigma_\theta^2}]d\theta \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L)}{2\sigma_\theta^2}]d\theta\right\} \\
&= -\exp[-A(1-\gamma)(1+r_f)W_0]\times \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2] \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\exp[-A\gamma\frac{\theta^H}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2]\right\}
\end{aligned}$$

where the integrals are treated like in Appendix 6.B.

random variable with mean

$$\theta^L - A\gamma\frac{1}{p^e}(1-\phi)W_0\sigma_\theta^2$$

and variance σ_θ^2 which integrates to 1.

6.C Expected utility given high message

$$\begin{aligned}
&E[u|\theta^H] \\
&= E[-\exp[-AW]|\theta^L] \\
&= \int_{\theta\in\mathbb{R}} -\exp[-AW(\theta)]b(\theta(\theta^H))d\theta \\
&= \int_{\theta\in\mathbb{R}} -\exp[-A(\gamma\frac{\theta}{p^e}(1-\phi)+(1-\gamma)(1+r_f))W_0] \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^L)^2}{2\sigma_\theta^2}] \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{-(\theta-\theta^H)^2}{2\sigma_\theta^2}]\right\}d\theta \\
&= -\exp[-A(1-\gamma)(1+r_f)W_0]\times \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L)}{2\sigma_\theta^2}]d\theta \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\int_{\theta\in\mathbb{R}}\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[\frac{(-A\gamma\frac{\theta}{p^e}(1-\phi)W_0 2\sigma_\theta^2-\theta^2-(\theta^L)^2+2\theta\theta^L)}{2\sigma_\theta^2}]d\theta\right\} \\
&= -\exp[-A(1-\gamma)(1+r_f)W_0]\times \\
&\quad \left\{\frac{(1-\rho)q}{1+q-q\rho}\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2] \right.\\
&\quad \left. +\frac{1}{1+q-q\rho}\exp[-A\gamma\frac{\theta^H}{p^e}(1-\phi)W_0+\frac{1}{2}A^2\gamma^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2]\right\}
\end{aligned}$$

where the integrals are treated like in Appendix 6.B.

6.D First order condition for choice of γ given high message

Differentiating expected utility derived in appendix 6.C with respect to γ yields the following first order condition:

$$\begin{aligned}
&(1-\rho)q\left((A(1+r_f)W_0-A\frac{\theta^L}{p^e}(1-\phi)W_0+A^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2\gamma\right)\times \\
&\exp[-A\gamma\frac{\theta^L}{p^e}(1-\phi)W_0] \\
&+\left(A(1+r_f)W_0-A\frac{\theta^H}{p^e}(1-\phi)W_0+A^2\frac{1}{(p^e)^2}(1-\phi)^2W_0^2\sigma_\theta^2\gamma\right)\times \\
&\exp[-A\gamma\frac{\theta^H}{p^e}(1-\phi)W_0] = 0
\end{aligned}$$

Due to rational expectations, this is equivalent to:

$$\begin{aligned}
&(1-\rho)q\left((1+r_f)W_0-\frac{\theta^L}{n\gamma}+\frac{A\sigma_\theta^2}{n^2\gamma}\right)\exp[-\frac{A}{n}\theta^L] \\
&+\left((1+r_f)W_0-\frac{\theta^H}{n\gamma}+\frac{A\sigma_\theta^2}{n^2\gamma}\right)\exp[-\frac{A}{n}\theta^H] = 0 \\
&\Updownarrow \\
&(1-\rho)q\left(\frac{\theta^L n-A\sigma_\theta^2}{n^2\gamma}\right)\exp[-\frac{A}{n}\theta^L]+\left(\frac{\theta^H n-A\sigma_\theta^2}{n^2\gamma}\right)\exp[-\frac{A}{n}\theta^H] \\
&=\left((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]\right)(1+r_f)W_0 \\
&\Updownarrow \\
&\gamma = \frac{(1-\rho)q(\theta^L n-A\sigma_\theta^2)\exp[-\frac{A}{n}\theta^L]+(\theta^H n-A\sigma_\theta^2)\exp[-\frac{A}{n}\theta^H]}{n^2((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1+r_f)W_0} \\
&= \frac{(1-\rho)q(\theta^L n-A\sigma_\theta^2)\exp[-\frac{A}{n}\theta^L]+(\theta^H n-\theta^L n+\theta^L n-A\sigma_\theta^2)\exp[-\frac{A}{n}\theta^H]}{n^2((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1+r_f)W_0} \\
&= \frac{\theta^L n-A\sigma_\theta^2}{(1+r_f)W_0 n^2}+\frac{(\theta^H-\theta^L)n\exp[-\frac{A}{n}\theta^H]}{n^2((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1+r_f)W_0}
\end{aligned}$$

6.E Derivatives of optimal stock share wrt. ρ and q

Recall (6.7):

$$\gamma^O = \frac{\theta^L n - A\sigma_\theta^2}{(1+r_f)W_0 n^2} + \frac{(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1+r_f)W_0 n}$$

Hence,

$$\frac{\partial\gamma^O}{\partial\rho} = \frac{(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]q\exp[-\frac{A}{n}\theta^L]}{(1+r_f)W_0 n((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])^2}$$

$$\frac{\partial\gamma^O}{\partial q} = \frac{-(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H](1-\rho)\exp[-\frac{A}{n}\theta^L]}{(1+r_f)W_0 n((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])^2}$$

6.F Expected future reputation

Let us denote the analyst's type by a. Then $a \in \{B, U\}$ where B and U represent biased and unbiased, respectively.

After each period, the representative investor observes the actual stock value. This is compared with the analyst's message to update prior reputation, ρ_t. The following Bayesian scheme is applied for the posterior belief about analyst type:

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Recall (6.7):

$$\gamma^O = \frac{\theta^L n - A\sigma_\theta^2}{(1+r_f)W_0 n^2} + \frac{(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1+r_f)W_0 n}$$

Hence,

$$\frac{\partial\gamma^O}{\partial\rho} = \frac{(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]q\exp[-\frac{A}{n}\theta^L]}{(1+r_f)W_0 n((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])^2}$$

$$\frac{\partial\gamma^O}{\partial q} = \frac{-(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H](1-\rho)\exp[-\frac{A}{n}\theta^L]}{(1+r_f)W_0 n((1-\rho)q\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])^2}$$

6.F Expected future reputation

Let us denote the analyst's type by a. Then $a \in \{B, U\}$ where B and U represent biased and unbiased, respectively.

After each period, the representative investor observes the actual stock value. This is compared with the analyst's message to update prior reputation, ρ_t. The following Bayesian scheme is applied for the posterior belief about analyst type:

$$\mu(a(\theta^m,\theta)) = \frac{f_a(a)\, f_{\theta^m,\theta|a}(\theta^m,\theta|a)}{\int_{a\in\{B,U\}} f_{\theta^m,\theta|a}(\theta^m,\theta|a) f_a(a)da}$$

$$= \frac{f_a(a)\int_{s\in S} f_{\theta^m,\theta|s,a}(\theta^m,\theta|s,a)\, f_s(s)\; ds}{\int_{a\in\{B,U\}}\int_{s\in S} f_{\theta^m,\theta|s,a}(\theta^m,\theta|s,a)\, f_s(s)\; ds\, f_a\,(a)da}$$

$$= \frac{f_a(a)\int_{s\in S} f_{\theta^m|s,a}(\theta^m|s,a)\, f_{\theta|s}(\theta|s)\, f_s(s)\; ds}{\int_{a\in\{B,U\}}\int_{s\in S} f_{\theta^m|s,a}(\theta^m|s,a)\, f_{\theta|s}(\theta|s)\, f_s(s)\; ds\, f_a\,(a)da}$$

The latter equality is due to the independence of θ from θ^m and a. Thus, posterior reputation after a high message is:

$$\rho_{t+1}(\theta^H,\theta)$$

$$= \frac{\rho_t\,(0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})}{\left((q\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})(1-\rho_t) + (0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})\rho_t\right)}$$

$$= \frac{\rho_t\,\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]}{(1-\rho_t)q\,exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]+\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]}$$

Similarly,

$$\rho_{t+1}(\theta^L,\theta)$$

$$= \frac{\rho_t \left(1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2}\right)}{\left(\begin{array}{l}((1-q)\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})(1-\rho_t) \\ +(1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})\rho_t\end{array}\right)}$$

$$= \frac{\rho_t \exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]}{((1-\rho_t)(1-q)+\rho_t)\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]}$$

$$= \frac{\rho_t}{1-q+\rho_t q}$$

If the analyst has a low signal, she expects a stock value of θ^L. We may now insert this in the derivations above to compute expected future reputation after lying or telling the truth in the current period.

Insert θ^L for θ in the expression immediately above to get:

$$\rho_1^t = \rho_1(\theta^L,\theta^L) = \frac{\rho_0}{1-q+\rho_0 q}$$

Notice that $\rho_1^t>0$ when $\rho_0>0$. Furthermore, telling the truth improves reputation, ie., $\rho_1^t>\rho_0$, iff

$$\frac{\rho_0}{1-q+\rho_0 q}>\rho_0$$

$$\Updownarrow$$

$$(1-\rho_0)q>0$$

which is true when $\rho_0<1$ and $q>0$. That is, it is not possible to improve reputation further if it is already at its maximum or the analyst is known not to lie.

Next period, the analyst expects to get a low signal with probability one half and to lie with probability q. Thus,

$$\rho_{t+1}(\theta^L,\theta)$$

$$= \frac{\rho_t\ (1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})}{\left(((1-q)\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})(1-\rho_t) + (1\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]\frac{1}{2}+0\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-(\theta-\theta^H)^2/(2\sigma_\theta^2)]\frac{1}{2})\rho_t\right)}$$

$$= \frac{\rho_t\ \exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]}{((1-\rho_t)(1-q)+\rho_t)\exp[-(\theta-\theta^L)^2/(2\sigma_\theta^2)]}$$

$$= \frac{\rho_t}{1-q+\rho_t q}$$

If the analyst has a low signal, she expects a stock value of θ^L. We may now insert this in the derivations above to compute expected future reputation after lying or telling the truth in the current period.

Insert θ^L for θ in the expression immediately above to get:

$$\rho_1^t = \rho_1(\theta^L,\theta^L) = \frac{\rho_0}{1-q+\rho_0 q}$$

Notice that $\rho_1^t>0$ when $\rho_0>0$. Furthermore, telling the truth improves reputation, ie., $\rho_1^t>\rho_0$, iff

$$\frac{\rho_0}{1-q+\rho_0 q}>\rho_0$$

$$\Updownarrow$$

$$(1-\rho_0)q>0$$

which is true when $\rho_0<1$ and $q>0$. That is, it is not possible to improve reputation further if it is already at its maximum or the analyst is known not to lie.

Next period, the analyst expects to get a low signal with probability one half and to lie with probability q. Thus,

$$\rho_2^t = \frac{1}{2}(q\rho_2(\theta^H,\theta^L)+(1-q)\rho_2(\theta^L,\theta^L)+\frac{1}{2}\rho_2(\theta^H,\theta^H)$$

$$= \frac{\rho_1^t}{2}\left[\frac{1}{(1-\rho_1^t)qc+1}+\frac{qc}{(1-\rho_1^t)q+c}+\frac{1-q}{1-q+\rho_1^t q}\right]$$

where $c=exp[-(\theta^H-\theta^L)^2/(2\sigma_\theta^2)] \in (0,1]$.

It is possible to obtain expected reputations further into the future by repeating the final step, ie.,

$$\rho_t^t = \frac{\rho_{t-1}^t}{2}\left[\frac{1}{(1-\rho_{t-1}^t)qc+1}+\frac{qc}{(1-\rho_{t-1}^t)q+c}+\frac{1-q}{1-q+\rho_{t-1}^t q}\right]$$

Let us then consider expected reputation after a lie:

$$\rho_1^l = \rho_1(\theta^H,\theta^L) = \frac{\rho_0 c}{(1-\rho_0)q+c}$$

Notice that a lie is expected to worsen reputation, ie., $\rho_1^l<\rho_0$, iff

$$\frac{c\rho_0}{(1-\rho_0)q+c}<\rho_0$$

$$\Updownarrow$$

$$(1-\rho_0)>0$$

which is the case when $\rho_0<1$ and $q>0$. Thus, $\rho^{\,l} \leq_1 \rho'$. Expected reputations further into the future are obtained like above, ie.,

$$\rho_t^l = \frac{\rho_{t-1}^l}{2}\left[\frac{1}{(1-\rho_{t-1}^l)qc+1}+\frac{qc}{(1-\rho_{t-1}^l)q+c}+\frac{1-q}{1-q+\rho_{t-1}^l q}\right]$$

From the expressions for ρ_1^l and $\rho_1{}'$ the following derivatives are derived:

$$\frac{\partial \rho_1^t}{\partial c} = 0$$

$$\frac{\partial \rho_1^t}{\partial q} = \frac{\rho_0(1-\rho_0)}{(1-q+\rho_0 q)^2} > 0$$

$$\frac{\partial \rho_1^t}{\partial \rho_0} = \frac{1-q+\rho_0 q-\rho_0 q}{(1-q+\rho_0 q)^2} > 0$$

$$\frac{\partial \rho_1^l}{\partial c} = \frac{\rho_0((1-\rho_0)q+c)-\rho_0 c}{((1-\rho_0)q+c)^2} > 0$$

$$\frac{\partial \rho_1^l}{\partial q} = \frac{-\rho_0 c(1-\rho_0)}{((1-\rho_0)q+c)^2} < 0$$

$$\frac{\partial \rho_1^l}{\partial \rho_0} = \frac{c((1-\rho_0)q+c)-\rho_0 c}{((1-\rho_0)q+c)^2} = \frac{c+cq-c\rho_0 q-\rho_0 c}{((1-\rho_0)q+c)^2} > 0$$

derived:

$$\frac{\partial \rho_1^t}{\partial c} = 0$$

$$\frac{\partial \rho_1^t}{\partial q} = \frac{\rho_0(1-\rho_0)}{(1-q+\rho_0 q)^2} > 0$$

$$\frac{\partial \rho_1^t}{\partial \rho_0} = \frac{1-q+\rho_0 q-\rho_0 q}{(1-q+\rho_0 q)^2} > 0$$

$$\frac{\partial \rho_1^l}{\partial c} = \frac{\rho_0((1-\rho_0)q+c)-\rho_0 c}{((1-\rho_0)q+c)^2} > 0$$

$$\frac{\partial \rho_1^l}{\partial q} = \frac{-\rho_0 c(1-\rho_0)}{((1-\rho_0)q+c)^2} < 0$$

$$\frac{\partial \rho_1^l}{\partial \rho_0} = \frac{c((1-\rho_0)q+c)-\rho_0 c}{((1-\rho_0)q+c)^2} = \frac{c+cq-c\rho_0 q-\rho_0 c}{((1-\rho_0)q+c)^2} > 0$$

Appendix 6.G $V^t(1,\rho_0)$-$V^l(1,\rho_0)$

$$V^t(1,\rho_0)-V^l(1,\rho_0)$$

$$=\pi^P-\pi^O(1,\rho_0)+\delta\left\{\pi^O(1,\rho_1^t)-\pi^O(1,\rho_1^l)\right\}+\delta^2\left\{\pi^O(1,\rho_2^t)-\pi^O(1,\rho_2^l)\right\}+\ldots$$

$$=\phi n W_0\left[\gamma^P-\gamma^O(1,\rho_0)+\delta\left\{\gamma^O(1,\rho_1^t)-\gamma^O(1,\rho_1^l)\right\}+\delta^2\left\{\gamma^O(1,\rho_2^t)-\gamma^O(1,\rho_2^l)\right\}+\ldots\right]$$

$$=\frac{\phi}{1+r_f}\left[\frac{-(\theta^H-\theta^L)\exp[-A\theta^H/n]}{(1-\rho_0)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right.$$
$$+\delta\left(\frac{(\theta^H-\theta^L)\exp[-A\theta^H/n]}{(1-\rho_1^t)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}-\frac{(\theta^H-\theta^L)\exp[-A\theta^H/n]}{(1-\rho_1^l)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right)$$
$$\left.+\delta^2\left(\frac{(\theta^H-\theta^L)\exp[-A\theta^H/n]}{(1-\rho_2^t)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}-\frac{(\theta^H-\theta^L)\exp[-A\theta^H/n]}{(1-\rho_2^l)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right)+\ldots\right]$$

$$=\frac{\phi(\theta^H-\theta^L)\exp[-A\theta^H/n]}{1+r_f}\left[\frac{-1}{(1-\rho_0)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right.$$
$$+\delta\left(\frac{1}{(1-\rho_1^t)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}-\frac{1}{(1-\rho_1^l)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right)$$
$$\left.+\delta^2\left(\frac{1}{(1-\rho_2^t)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}-\frac{1}{(1-\rho_2^l)\exp[-A\theta^L/n]+\exp[-A\theta^H/n]}\right)+\ldots\right]$$

$$
\begin{aligned}
&= \frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\Bigg[\frac{-1}{(1-\rho_0)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]} \\
&+\delta\Bigg(\frac{(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])} \\
&-\frac{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg) \\
&+\delta^2\Bigg(\frac{(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])} \\
&-\frac{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg) \\
&+\ldots\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\Bigg[\frac{-1}{(1-\rho_0)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]} \\
&+\delta\Bigg(\frac{(\rho_1^t-\rho_1^l)\exp[-\frac{A}{n}\theta^L]}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg) \\
&+\delta^2\Bigg(\frac{(\rho_2^t-\rho_2^l)\exp[-\frac{A}{n}\theta^L]}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])} \\
&+\ldots\Bigg]
\end{aligned}
$$

$$=\frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\Bigg[\frac{-1}{(1-\rho_0)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}$$

$$+\delta\Bigg(\frac{(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}$$

$$-\frac{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg)$$

$$+\delta^2\Bigg(\frac{(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}$$

$$-\frac{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg)$$

$$+\ldots\Bigg]$$

$$=\frac{\phi(\theta^H-\theta^L)\exp[-\frac{A}{n}\theta^H]}{1+r_f}\Bigg[\frac{-1}{(1-\rho_0)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H]}$$

$$+\delta\Bigg(\frac{(\rho_1^t-\rho_1^l)\exp[-\frac{A}{n}\theta^L]}{((1-\rho_1^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_1^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}\Bigg)$$

$$+\delta^2\Bigg(\frac{(\rho_2^t-\rho_2^l)\exp[-\frac{A}{n}\theta^L]}{((1-\rho_2^t)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])(1-\rho_2^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}$$

$$+\ldots\Bigg]$$

6.H $K_t > K_{t-1}$ for $t=2,3,\ldots$

$K_t > K_{t-1}$ for $t=2,3,\ldots$ iff

$$\frac{(1-\rho_t^l)\exp[-\frac{A}{n}\theta^L]}{\exp[-\frac{A}{n}\theta^H]((1-\rho_t^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])} > \frac{(1-\rho_{t-1}^l)\exp[-\frac{A}{n}\theta^L]}{\exp[-\frac{A}{n}\theta^H]((1-\rho_{t-1}^l)\exp[-\frac{A}{n}\theta^L]+\exp[-\frac{A}{n}\theta^H])}$$

$$\Updownarrow$$

$$(1-\rho_t^l)(1-\rho_{t-1}^l)\exp[-\tfrac{A}{n}\theta^L]+(1-rh_t^l)\exp[-\tfrac{A}{n}\theta^H] >$$
$$(1-\rho_t^l)(1-\rho_{t-1}^l)\exp[-\tfrac{A}{n}\theta^L]+(1-\rho_{t-1}^l)\exp[-\tfrac{A}{n}\theta^H]$$

$$\Updownarrow$$

$$(\rho_{t-1}^l-\rho_t^l)\exp[-\tfrac{A}{n}\theta^H] > 0$$

$$\Updownarrow$$

$$\rho_{t-1}^l > \rho_t^l$$

6.I $\rho^l_{t-1} > \rho^l_t$ for t=2,3, ...

$\rho^l_{t-1} > \rho^l_t$ for t=2,3, ... iff

$$\rho^l_{t-1} > \frac{\rho^l_{t-1}}{2}\left[\frac{1}{1-c\rho^l_{t-1}+c}+\frac{c}{1-\rho^l_{t-1}+c}\right]$$

$$\Updownarrow$$

$$2 > \frac{1}{1-c\rho^l_{t-1}+c}+\frac{c}{1-\rho^l_{t-1}+c}$$

$$\Updownarrow$$

$$2-\frac{c}{1-\rho^l_{t-1}+c} > \frac{1}{1-c\rho^l_{t-1}+c}$$

$$\Updownarrow$$

$$\frac{2-2\rho^l_{t-1}+c}{1-\rho^l_{t-1}+c} > \frac{1}{1-c\rho^l_{t-1}+c}$$

$$\Updownarrow$$

$$2-2\rho^l_{t-1}+c-2c\rho^l_{t-1}+2c(\rho^l_{t-1})^2-c^2\rho^l_{t-1}+2c-2c\rho^l_{t-1}+c^2 > 1-\rho^l_{t-1}+c$$

$$\Updownarrow$$

$$2c(\rho^l_{t-1})^2-(1+4c+c^2)\rho^l_{t-1}+1+2c+c^2 > 0$$

To find the roots of this second order polynomial, we need the discriminant,

$$\begin{aligned} D &= (1+4c+c^2)^2-8c(1+2c+c^2) \\ &= 1+4c+c^2+4c+16c^2+4c^3+c^2+4c^3+c^4-8c-16c^2-8c^3 \\ &= 1+2c^2+c^4 \\ &=(1+c^2)^2 \end{aligned}$$

Thus, the roots are:

$$\rho^l_{t-1} = \frac{1+4c+c^2\pm(1+c^2)}{4c} = 1,\ \frac{1+2c+c^2}{2c}$$

Since both roots are greater than or equal to 1, the polynomial is greater than 0 on [0,1).

6.I $\rho^l_{t-1} > \rho^l_t$ for t=2,3, ...

$\rho^l_{t-1} > \rho^l_t$ for t=2,3, ... iff

$$\rho^l_{t-1} > \frac{\rho^l_{t-1}}{2}\left[\frac{1}{1-c\rho^l_{t-1}+c}+\frac{c}{1-\rho^l_{t-1}+c}\right]$$

$\Updownarrow$

$$2 > \frac{1}{1-c\rho^l_{t-1}+c}+\frac{c}{1-\rho^l_{t-1}+c}$$

$\Updownarrow$

$$2-\frac{c}{1-\rho^l_{t-1}+c} > \frac{1}{1-c\rho^l_{t-1}+c}$$

$\Updownarrow$

$$\frac{2-2\rho^l_{t-1}+c}{1-\rho^l_{t-1}+c} > \frac{1}{1-c\rho^l_{t-1}+c}$$

$\Updownarrow$

$$2-2\rho^l_{t-1}+c-2c\rho^l_{t-1}+2c(\rho^l_{t-1})^2-c^2\rho^l_{t-1}+2c-2c\rho^l_{t-1}+c^2 > 1-\rho^l_{t-1}+c$$

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Chapter 7

Summary in Danish

Afhandlingen består af en introduktion og fem selvstændige kapitler, hvoraf de fire første er empiriske mens det sidste er teoretisk. Alle kapitler omhandler aspekter ved finansielle markeder.

Introduktionen indledes med en beskrivelse af databasen, som er grundlaget for kapitlerne 2, 3 og 4. Derefter beskrives indholdet af kapitlerne og den beslægtede litteratur.

Nielsen og Risager (1999), som er kapitel 2 i afhandlingen, analyserer afkast og risiko ved danske 1-, 5- og 10-årige aktie- og statsobligationsinvesteringer i perioden 1922-95. Resultaterne viser, at markedsporteføljen af aktier giver et højere gennemsnitligt afkast end obligationer, at aktier er meget mere risikable end obligationer på kort sigt, men at aktier ikke er mere risikable end obligationer på langt sigt. Sidstnævnte resultat antyder, at pensionskasser og andre institutionelle investorer bør have mulighed for at placere en større andel af formuen i aktier end den nuværende (medio 1997) lovgivning tillader. I kapitlet testes endvidere den forbrugsbaserede CAPM med kortsigts-afkast ved anvendelse af metoder udviklet af Hansen og Jagannathan samt Grossman og Shiller.

I kapitel 3, der er forfattet i samarbejde med Jan Overgaard Olesen, estimeres en velspecificeret regime-skift-model for danske aktieafkast. Modellen identificerer to regimer, der har henholdsvis lavt afkast-lav volatilitet og højt afkast-høj volatilitet. Førstnævnte regime dominerede med undtagelse af få og korte episoder indtil begyndelsen af 1970'erne, hvorimod 1980'erne og 1990'erne har været kendetegnet ved højt afkast og høj volatilitet. Vi foreslår et nyt test for mean reversion, der tillader multiple regimer med potentielt forskellige konstant- og autoregressionsled og forskellig fejlledsvarians. Ved anvendelse af dette test finder vi mean reversion på 10% men ikke på 5% signifikansniveau, hvilket giver svagere støtte til mean reversion-hypotesen end hvis sædvanlige tests anvendes. Ved at analysere bidragene fra de enkelte regimer finder vi desuden at indikationen på mean reversion udelukkende skyldes det seneste regime med højt afkast-høj volatilitet.

I kapitel 4, der ligeledes er forfattet i samarbejde med Jan Overgaard Olesen, estimeres en teoretisk model for udbytte/pris-forholdet for markedsporteføljen af danske aktier i perioden 1927-96. De fundamentale variable er tidsvarierende mål for vækstjusteret realrente og risikopræmien på aktier, hvorved der tillades for en tidsvarierende diskonteringsfaktor. Niveauet for reale udbytter og det laggede udbytte/pris-forhold, der medtages for at muliggøre træghed i tilpasningen, inkluderes ligeledes som forklarende variable. Resultaterne i kapitlet viser, at det er vigtigt at tillade for mere end et regime for at undgå strukturelle brud i modellen gennem stikprøveperioden. Ved anvendelse af en regimeskift-metode til modellering af uforklarede ændringer i de økonomiske vilkår finder vi, at de fundamentale variable er signifikante i mindst et regime. En vigtig forskel på regimerne er, at realrenten kun er signifikant i det ene regime. Endvidere kan et af regimerne fortolkes som lavt udbytte/pris-forhold, mens det andet er kendetegnet ved højt udbytte/pris, efter at have kontrolleret for de fundamentale variable. De estimerede regimer er meget persistente og modellen inddeler klart stikprøven i 3 delperioder, nemlig 1927-49, 1950-85 og 1986-91.

Kapitel 5 viser, at præmien til en såkaldt værdi-strategi, som består i at investere i aktier der er værdisat lavt relativt til virksomhedens overskud, er større hvis de anvendte regnskabstal korrigeres for bedre at udtrykke virksomhedens økonomiske værdi. Proceduren til korrektion af overskuddet bygger på det klassiske bidrag af Graham og Dodd (1934), der til investeringsbrug anbefaler indtjeningskraft (earning power) snarere end bogført overskud. Det vigtigste element i denne procedure består i at erstatte afskrivninger på anlægsaktiver med gennemsnitlige udgifter til nye anlægsaktiver og udskiftning/vedligeholdelse af ekstisterende aktiver. Dette beløb hævdes at have mere relevans for investor, da det ikke er til rådighed for udbyttebetaling. Kapitlets stikprøve er de 20 aktier i KFX-indekset i perioden fra 1990. Såvel en portefølje- som en regressionsmetode tages i anvendelse for at vise, at værdipræmien er positiv hvis investeringsstrategien baseres på korrigeret overskud, mens den er insignifikant hvis ukorrigeret overskud anvendes.

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Kapitel 6 analyserer samspillet mellem investorer og aktieanalytikere og dets konsekvenser for informationsindholdet af priser på værdipapirer. Rådgivning fra institutioner og deres ansatte påvirkes af profit- og/eller aflønningsinteresser. Empirisk forskning har vist, at rådgivningen har tendens til at være for optimistisk. Kapitlet modellerer en situation, hvor en aktieanalytiker har kortsigtet interesse i at publicere optimistiske afkastforventninger for at tiltrække investeringer. Udover det korte sigt kan fordrejning af information føre til tab af omdømme, hvilket kan skade profitten. Det vises, at modellen, som bevidst er udformet til at være konsistent med den nævnte empiriske tendens, ikke har en-periode ligevægte, hvor priserne på værdipapirer afspejler analytikerens information. Derimod findes sådanne ligevægte i det gentagne spil, forudsat at analytikeren bekymrer sig tilstrækkeligt om fremtidig profit.

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