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Abstract

Under existing welfare arrangements, an increase in life expectancy may pose a serious threat to fiscal sustainability, and it may have dramatic effects on the intergenerational distribution of welfare. This paper finds that such effects may be countered through a policy which links the retirement age to changes in life expectancy.

Keywords: Fiscal Policy, Longevity Adjustment, Ageing, Pensions, Welfare Reform.
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1 Introduction

It is well-known that changes in the age structure of the population can have dramatic consequences for public finances. For example, under a pay-as-you-go (PAYG) scheme, changing demographics may lead to substantial swings in either taxes or welfare services. While neither of these outcomes might be politically acceptable, the issue arises as to how a welfare system can be maintained which is not only fiscally sustainable, but which also offers a fair distribution profile across different generations.

To address this challenge, several countries are now operating a medium-term fiscal strategy of public debt reduction. This should help countries avoid substantial cuts in existing welfare arrangements, yet without having to raise taxes, when the projected increase in the demographic dependency ratio becomes more severe. By developing the capacity to absorb demographic changes with stable taxes, a country would not only enjoy the efficiency gains associated with tax smoothing (Barro, 1979), but it would also be better protected against adverse inter-generational distribution effects.[1]

Provided that the underlying shock - in this case a rise in the demographic dependency burden - is of a non-permanent kind, a saving (or pre-funding) strategy can be seen as a sensible way of smoothing a fiscal adjustment. Therefore, it is an important question whether the projected increase in the demographic dependency burden constitutes a temporary or a permanent shock. While changing demographics may originate from shocks to fertility, mortality and/or migration, it seems as if it is changes in mortality which tend to generate the most significant effects on the dependency ratio (Andersen, Jensen and Pedersen, 2005). Basically, this is because changes in future life expectancy impact almost exclusively on the retired part of the population, whereas changes in fertility and migration affect individuals in all phases of life. This observation clearly assigns a critical role to changes in life expectancy.

Due to, e.g., advances within medical science, changes in life styles, more flexible working conditions etc, the projected increases in longevity may well be of a more permanent nature. If this is the case, the rise in the (old-age)

[1] However, uncertainty is inherent in demographic projections, and experience has shown that they may be prone to radical changes, even within relatively short periods of time.

dependency ratio could continue over a much longer period than typically envisaged, and a smoothing policy is not really what is called for. Also, evidence suggests that the required amount of pre-funding is very sensitive to changes in key economic variables, especially the interest rate and the rate of productivity growth, and this lack of robustness implies that there may be a need for frequent changes in benefits and/or contribution rules. However, this would not be consistent with the main objective of keeping these rules stable across different generations.

With existing age limits for eligibility to early retirement and pensions, a longer lifetime will clearly lead to an increase in the proportion of life where an individual is a net-recipient of welfare arrangements. However, since one would expect an increase in life expectancy of future generations to be associated with higher levels of welfare, the question could be raised whether current generations should contribute to the financing hereof, as a pre-funding strategy would imply. Rather, an obvious alternative to pre-funding would be to let benefits and/or contribution rules depend on changes in life expectancy. For example, by letting the age limits for retirement and pension follow expected lifetime, it is indeed possible to condition both benefits and contributions on expected lifetime.

Ideally, by introducing such a scheme the age limits become systematically linked to the generation to which an individual belongs. Since current demographic projections imply that future generations can expect longer lifetimes, it follows that those age limits would be higher for younger than for older generations. Generations expecting to live longer would therefore be active in the labour market longer, and a better balance is ensured between the number of years a person is a net-contributor and a net-recipient, respectively. Adjustments to the system should be made at intervals - say, every 10 years. When adjustments to changes in projections of expected lifetime are made well in advance of the pension age, it follows that the risk arising from unanticipated changes is smoothed over current and future generations.

In order to establish how a policy rule of longevity adjustment would affect intergenerational welfare distribution, as well as some broader macroeconomic effects, we need a formal model with overlapping generations, optimising individual behaviour and demographic shocks. We have in this paper formulated such an analytical framework by adapting, and then extending, a model first suggested by Bohn (1998, 2001). We set out the basic structure of that model in Section 2. Next, in Section 3, some macroeconomic responses to demographic shocks are derived. Section 4 introduces the policy reform of longevity adjustment. The focus here is on the potential of this initiative, compared to tax policy adjustments, in order to redistribute welfare across generations. Finally, Section 5 concludes and offers some suggestions for future research.

2 The model

This section presents the basics of our analytical framework, which is a model with overlapping generations (OLG) in the spirit of Diamond (1965) and extented by stochastic features in line with Bohn (2001). We first outline the demographic details, next household behaviour, and finally resources and the pension system.

2.1 Demographics

Individuals live for three periods: as children, as workers and as retirees, respectively. The number of children born in period t is denoted by N_t^c, where $N_t^c = b_t N_t^w$, and where $b_t > 0$ is the birth rate, and N_t^w is the number of workers living in period t. Adults work during period t (inelastic labour supply), and they are retired during period $t+1$. All individuals in each cohort are assumed to be identical.

An increase in the birth rate clearly expands the labour force: the higher is b_t, the more children are born, and the larger is next period's labour force. But the size of the labour force is also determined by the probability of surviving from childhood into the working period, μ_{1t}, as well as by the length of the working period, χ_t. If χ_t increases, for instance, the children now entering as adults into the working period must work for a longer period of time, and therefore the *effective* growth rate of the labour force increases. Thus, the effective growth rate of the labour force is given by $N_{t+1}^w / N_t^w = \mu_{1t+1}\chi_{t+1}b_t$. If the probability of survival is 1, and with a standard working period normalised at 1 (i.e., $\mu_{1t} = \chi_t = 1$), the growth rate of the labour force is b_t.

There is also a probablity of surviving from the working period into the retirement period, μ_{2t}. Both survival rates, μ_{1t} and μ_{2t}, and the total lifetime, ϕ_t, comprise an expected term and an unexpected term. Specifically, $\mu_{1t} = \mu_{1t-1}^e \mu_{1t}^u$; $\mu_{2t} = \mu_{2t-1}^e \mu_{2t}^u$; and $\phi_t = \phi_{t-1}^e \phi_t^u$ where $\mu_{1t}, \mu_{2t} \in (0,1)$ and $\phi_t \in (0,2)$. The survival probabilities at the individual level are assumed to equal the aggregate survival rate, and their stochastic terms, together with the stochastic components of ϕ^e_{t-1}, ϕ^u_t, b_t and χ_t, are assumed to be identically and independently distributed.

The total adult lifetime equals the sum of the length of the working period and the retirement period. Thus, individuals initially have a total life time endowment, and if the length of the working period increases, then the length of the retirement period must decrease proportionally, as figure 1 illustrates.

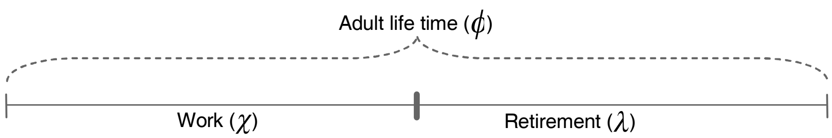

Figure 1. Adult lifetime, work, and retirement

Formally, the length of retirement period, λ, is residually determined from $\lambda_t = \phi_t \mu_{2t} - \chi_{t-1} = \phi^e_{t-1}\phi^u_t \mu^e_{2t-1}\mu^u_{2t} - \chi_{t-1}$, and is seen to be conditional on survival into old age, where $\chi_t, \lambda_t \in (0,1)$. Increases in the *total* lifetime could also be envisaged. Clearly, if the length of the working period is unchanged, an increase in total lifetime falls entirely on the length of life in the retirement period.

2.2 Individual optimisation and household behaviour

Parents make decisions about consumption on behalf of themselves and their children. With homothetic preferences, utility of generation t in the working period (as parents) features the following specification,

$$U^1_t = \frac{1}{1-\eta}\chi_t \left[\rho^w \left(C^w_t\right)^{1-\eta} + b_t \rho_{c(b_t)} \left(C^c_t\right)^{1-\eta}\right] \tag{1}$$

where C^w_t is parents' consumption, C^c_t is children's consumption, ρ^w and $\rho_{c(b_t)}$ are weights, and $\eta > 0$ is the inverse elasticity of intertemporal substitution.[2] Also, the length of the working period, χ_t, is assumed to enter positively, and is included in a fashion similar to how Bohn (2001) incorporates the length of retirement period in second period utiltity.[3] The factor $b_t \rho_{c(b_t)}$ is likely to be

[2]This specification of the utility function is quite standard. If the (inverse) elasticity of intertemporal substitution is larger than 1, then utility decreases if consumption increases. If $\eta = 1$, equation (1) simplifies to the log-utility function, $U_t = \chi_t \left[\rho^w \ln C^w_t + b_t \rho_{c(b_t)} \ln C^c_t\right]$, in which case an increase in consumption yields an increase in utility. The construction of the utility function with $\eta > 1$ is mainly meant to highlight the risk aversion motive.

[3]Auerbach and Hassett (2001, 2002) also incorporate the length of the retirement period.

non-decreasing in the number of children, and is assumed to be positive but less than ρ^w. There is no specific functional form imposed on $\rho_{c(b_t)}$, since in later derivations it will only appear *relative* to ρ^w in the time preference for consumption over period 1 and 2. The consumption constraint for generation t in their working period is simply $C_t^1 = C_t^w + b_t C_t^c$.

After maximising utility in period 1 subject to the consumption restriction, the optimal level of first-period consumption, C_t^1, is derived. Subsequently the indirect period-one utility function can be derived:[4]

$$U_t^1\left(C_t^1\right) = \frac{1}{1-\eta}\left[\chi_t \rho_{1(b_t)}\left(C_t^1\right)^{1-\eta}\right] \tag{2}$$

Surviving retirees are assumed to leave no bequests. In the retirement period, utility is given by:

$$U_{t+1}^2\left(C_{t+1}^2\right) = \frac{1}{1-\eta}\left[\lambda_{t+1}\left(C_{t+1}^2\right)^{1-\eta}\right] \tag{3}$$

The aggregate intertemporal utility function for the household is an additive composite of the indirect utilities in the two periods and may be written as,

$$U_t = U_t^1\left(C_t^1\right) + \rho_2 U_{t+1}^2\left(C_{t+1}^2\right) \tag{4}$$

where $\rho_2 > -1$ is the discount rate of old age consumption. Upon inserting (2) and (3) in (4), we get:

$$U_t = \frac{1}{1-\eta}\left[\chi_t \rho_{1(b_t)}\left(C_t^1\right)^{1-\eta} + \left(\phi_{t+1}\mu_{2t+1} - \chi_t\right)\rho_2\left(C_{t+1}^2\right)^{1-\eta}\right] \tag{5}$$

Finally, consumption in the two periods can be written as follows,

$$\chi_t C_t^1 = W_t\left(1-\theta_t\right) - S_t \tag{6}$$

and,

$$C_{t+1}^2 = \left[\frac{R_{t+1}}{\phi_{t+1}\mu_{2t+1} - \chi_t}\right] S_t + \beta_{t+1} W_{t+1} \tag{7}$$

where W_t, β_t, S_t, and R_t denote, respectively, wages, the replacement rate, savings, and the return to capital.

[4] The term $\rho_{1(b_t)} = \rho^w[1 + (\rho_{c(b_t)}/\rho^w)^{1/\eta} b_t]^{1-\eta} + b_t \rho_{c(b_t)} (\rho_{c(b_t)}/\rho^w)^{(1-\eta)/\eta}$ denotes the weight on first period consumption in the first period indirect utility.

2.3 Resources and social security

Output is produced with capital and labour, according to the following Cobb-Douglas technology,

$$Y_t = K_t^{\alpha} \left(A_t N_t^w\right)^{1-\alpha} \tag{8}$$

where K_t is capital, A_t is productivity and α is the capital share. Productivity follows a stochastic trend, governed by $A_t = (1 + a_t) A_{t-1}$, where a_t is the growth rate (identically and independently distributed). Each individual supplies one unit of labour. The resource constraint of the economy is then

$$Y_t + (1 - \delta) K_t = \chi_t N_t^w C_t^1 + \left(\phi_t \mu_{2t} - \chi_{t-1}\right) N_{t-1}^w C_t^2 + K_{t+1} \tag{9}$$

where δ is the rate of depreciation of the capital stock. Individuals' savings will be next period's capital stock, i.e.:

$$K_{t+1} = N_t^w S_t \tag{10}$$

The public sector operates a pay-as-you-go (PAYG) social security system

$$\lambda_t N_{t-1}^w \beta_t W_t = \theta_t W_t N_t^w \tag{11}$$

where β (θ) denotes the benefit (contribution) rate. Equation (11) may feature both defined benefits (DB) and defined contribution (DC) schemes. For example, in a DB system the replacement rate is fixed, and the contribution rate varies. Solving for the endogenous contribution rate in a DB system, and using that $\lambda_t = \phi_t \mu_{2t} - \chi_{t-1}$ and $N_t^w / N_{t-1}^w = 1 + n_t^w$, yields

$$\theta_t = \beta_t \left[\frac{\phi_t \mu_{2t} - \chi_{t-1}}{1 + n_t^w}\right] \tag{12}$$

Evidently, with the replacement rate held fixed, an increase in the population growth rate and an increase in the retirement age leads to a lower contribution rate. Similarly, an increase in the length of the retirement period and an increase in the probability of surving into the retirement period call for a higher contribution rate.

Finally, it should be mentioned that in order to obtain a solution on the balanced growth path we state the model in effective units of labour such that, for example, $\left(C^1/A\right)_t \equiv c_t^1$; $\left(C^2/A\right)_t \equiv c_t^2$; $(W/A)_t \equiv w_t$; $K_t/(A_{t-1}N_{t-1}^w) \equiv k_{t-1}$ and $y_t = \left[k_{t-1}/\left((1 + a_t)\left(1 + n_t^w\right)\right)\right]^{\alpha}$.

This completes the presentation of the baseline OLG model.

3 Equilibrium and responses to shocks

Before introducing the policy reform framework, this section offers some illustrations of how the OLG model works when the existing pension system is designed either as DC or DB, and when various shocks hit the economy. As a starting point, we briefly outline the solution technique.

3.1 The solution technique

The model is first transformed into log-deviations from the steady state, and then the method of undetermined coefficients is used to obtain an analytical solution for the recursive equilibrium. Adopting this analytical approach, the nonlinear OLG model is replaced by a log-linearised approximate OLG model with variables stated in terms of percentage deviations from the steady state.[5]

The log-linearised law of motion of any endogenous variable, $\widehat{x}_t$, as a function of the endogenous state variable, $\widehat{k}_{t-1}$, and any exogenous state variable, $\widehat{z}_t$, may be conjectured as follows:

$$\widehat{x}_t = \pi_{xk}\widehat{k}_{t-1} + \sum_{z \in Z} \pi_{xz}\widehat{z}_t \tag{13}$$

where

$$\widehat{x}_t = \left\{ \widehat{k}_t, \widehat{c}_t^1, \widehat{c}_t^2, \widehat{w}_t, \widehat{R}_t, \widehat{\theta}_t \right\}$$

and

$$Z = \left\{ \widehat{\chi}_{t-1}, \widehat{\chi}_t, \widehat{a}_t, \widehat{b}_{t-1}, \widehat{b}_t, \widehat{\mu}^e_{1t-1}, \widehat{\mu}^e_{1t}, \widehat{\mu}^u_{1t}, \widehat{\phi}^e_{t-1}, \widehat{\phi}^e_t, \widehat{\phi}^u_t, \widehat{\mu}^e_{2t-1}, \widehat{\mu}^e_{2t}, \widehat{\mu}^u_{2t} \right\}$$

where π_{xk} and π_{xz} denote, respectively, the elasticity of $\widehat{x}$ with respect to the endogenous state variable, $\widehat{k}$, and the elasticity of $\widehat{x}$ with respect to the exogenous state variables, $\widehat{z}$. For example, the linear recursive equilibrium law of motion for the capital stock can be written as:

$$\begin{aligned}
\widehat{k}_t \quad = \quad & \pi_{kk}\widehat{k}_{t-1} + \pi_{k\chi 1}\widehat{\chi}_{t-1} + \pi_{k\chi}\widehat{\chi}_t + \pi_{ka}\widehat{a}_t + \pi_{kb1}\widehat{b}_{t-1} \\
& + \pi_{kb}\widehat{b}_t + \pi_{k\mu 1e1}\widehat{\mu}^e_{1t-1} + \pi_{k\mu 1e}\widehat{\mu}^e_{1t} + \pi_{k\mu 1u}\widehat{\mu}^u_{1t} + \pi_{k\phi e1}\widehat{\phi}^e_{t-1} \\
& + \pi_{k\phi e}\widehat{\phi}^e_t + \pi_{k\phi u}\widehat{\phi}^u_t + \pi_{k\mu 2e1}\widehat{\mu}^e_{2t-1} + \pi_{k\mu 2e}\widehat{\mu}^e_{2t} + \pi_{k\mu 2u}\widehat{\mu}^u_{2t}.
\end{aligned}$$

[5] The method is inspired by Campbell (1994), Uhlig (1999) and Bohn (1998, 2001, 2003), and it has become standard practice in the context of Real Business Cycle (RBC) models. However, in relation to stochastic OLG models this technique has, to our knowledge, not been clearly documented elsewhere.

The model is log-linearised around the steady state and solved using the method of undetermined coefficients. The equations which characterise the equilibrium of the model are the following:

$$\begin{aligned} 0 &= \Lambda_1\widehat{k}_{t-1} - \Lambda_5\widehat{k}_t - \Lambda_3\widehat{c}_t^1 - \Lambda_4\widehat{c}_t^2 + \Lambda_4\widehat{\chi}_{t-1} - \Lambda_{14}\widehat{\chi}_t - \Lambda_1\widehat{a}_t \\ &\quad -\Lambda_2\widehat{\mu}_{1t-1}^e - \Lambda_2\widehat{\mu}_{1t}^u - \Lambda_2\widehat{b}_{t-1} - \Lambda_4\widehat{\phi}_{t-1}^e - \Lambda_4\widehat{\phi}_t^u - \Lambda_4\widehat{\mu}_{2t-1}^e - \Lambda_4\widehat{\mu}_{2t}^u \end{aligned} \tag{14}$$

$$\begin{aligned} 0 &= -\Lambda_7\widehat{c}_{t+1}^2 + \Lambda_9\widehat{w}_{t+1} + \Lambda_{12}\widehat{R}_{t+1} + \Lambda_{12}\widehat{w}_t + \Lambda_{12}\widehat{\chi}_t - \Lambda_8\widehat{\theta}_t \\ &\quad -\Lambda_{12}\widehat{a}_t - \Lambda_{12}\widehat{\phi}_t^e - \Lambda_{12}\widehat{\phi}_{t+1}^u - \Lambda_{12}\widehat{\mu}_{2t}^e - \Lambda_{12}\widehat{\mu}_{2t+1}^u \end{aligned} \tag{15}$$

$$0 = \widehat{R}_{t+1} + \Lambda_6\widehat{c}_t^1 - \Lambda_6\widehat{c}_{t+1}^2 - \pi_{\rho_1(b)}\widehat{b}_t \tag{16}$$

$$\widehat{R}_t = -\Lambda_{10}\widehat{k}_{t-1} + \Lambda_{10}\widehat{a}_t + \Lambda_{10}\widehat{\mu}_{1t-1}^e + \Lambda_{10}\widehat{\mu}_{1t}^u + \Lambda_{10}\widehat{b}_{t-1} + \Lambda_{10}\widehat{\chi}_t \tag{17}$$

$$\widehat{w}_t = \Lambda_{11}\widehat{k}_{t-1} - \Lambda_{11}\widehat{a}_t - \Lambda_{11}\widehat{\mu}_{1t-1}^e - \Lambda_{11}\widehat{\mu}_{1t}^u - \Lambda_{11}\widehat{b}_{t-1} - \Lambda_{11}\widehat{\chi}_t \tag{18}$$

$$\widehat{\theta}_t = \widehat{\phi}_{t-1}^e + \widehat{\phi}_t^u + \widehat{\mu}_{2t-1}^e + \widehat{\mu}_{2t}^u - \widehat{\chi}_{t-1} - \widehat{\chi}_t - \widehat{\mu}_{1t-1}^e - \widehat{\mu}_{1t}^u - \widehat{b}_{t-1} \tag{19}$$

where (14) is the resource constraint; (15) is second period consumption; (16) is the Euler equation; (17) is the return to capital; (18) is the wage rate; and (19) is the pension contribution rate. The Λ's denote the relevant coefficients, which comprise combinations of steady state variables.[6]

The next step is to present (14) to (19) in accord with (13) where the elasticities in the law of motion appear explicitly. It turns out that the analytical elasticities have fairly complex expressions, and to facilitate the interpretation we therefore also report numerical elasticities. This involves calibrating the model using what we believe are realistic parameter values, as shown in table 1, and simulating the model using a Matlab routine (available upon request).

[6] Further details on the specific derivations conducted in this paper using the method of undetermined coefficients are found in a separate note which is available upon request. For a more general presentation of solution techniques relevant to stochastic OLG models, the reader is referred to Jørgensen (2006).

Parameter	*Calibration*	*Interpretation*
α	1/3	Capital share in output
δ	1	Rate of depreciation of capital
β	0.3 (0.1379)	Replacement rate (payroll taxes)
a	0.35	Productivity growth rate
$1/\eta$	1/3	Elasticity of intertemporal substitution
s	0.2	Savings rate
ϕ	1.3	Length of adult life
χ	0.8	Length of working period
λ	N.A.	Length of retirement period (residual)
b	0.1	Growth rate of the number of children
μ_1	0.95	Probability of surviving into working period
μ_2	1	Probability of surviving into retirement period
$\pi_{\rho_{1(b)}}$	0	Elasticity of the weight of 1st period consumption in utility with respect to current birth rate

Table 1. Calibration of the model

3.2 Economic effects of demographic shocks

As to the effects of demographic shocks on key economic variables, we focus on shocks to fertility, $\widehat{b}$, and to longevity, $\widehat{\phi}$, as well as to the retirement age, $\widehat{\chi}$.[7] The relevant economic effects include consumption possibilities for workers, $\widehat{c}^1$, and retirees, $\widehat{c}^2$, respectively, and the capital stock, $\widehat{k}$.

A shock to the birth rate A fall in the size of the working-age population, as currently experienced by several OECD countries (UN, 2004), may originate from a negative shock to the birth rate in an earlier period. The effects of a shock to the lagged birth rate, $\widehat{b}_{t-1}$, may be stated in terms of the relevant analytical and numerical elasticities, see table 2. Note that we are assuming a negative shock, so the elasticities in table 2 must be interpreted with the opposite sign.

The following insights may be reported. First, the difference between DB and DC systems is captured by the elasticity of the contribution rate with

[7] Much attention is devoted to the length of the retirement period, λ, which is derived residually ($\lambda_t = \phi_t \mu_{2t} - \chi_{t-1}$). Since we calibrate χ with 0.8, ϕ with 1.3, and μ_2 with 1, then λ becomes 0.5.

respect to a lower birth rate: In the pure DC system, $\pi_{\theta b1}$ is 0 by definition, while in the DB system it is equal to 1.

Policy coeff.	*DB*	*DC*	*Analytical elasticity*
$\pi_{\theta b1}$	1	0	
Effect on:			
π_{c2b1}	0.3659	0.5057	$\frac{(\Lambda_8\pi_{\theta b1}-\Lambda_{12}\pi_{wb1})(\Lambda_3\Lambda_{13}\pi_{Rk}-\Lambda_3\pi_{c2k}-\Lambda_5)}{(\Lambda_9\pi_{wk}-\Lambda_7\pi_{c2k}+\Lambda_{12}\pi_{Rk})\Lambda_4} - \frac{\Lambda_2}{\Lambda_4}$
π_{c1b1}	-0.1018	-0.1981	$(\pi_{c2k} - \Lambda_{13}\pi_{Rk})\,\pi_{kb1}$
π_{kb1}	-0.1421	-0.2764	$\frac{\Lambda_4\pi_{c2b1}+\Lambda_2}{\Lambda_3\Lambda_{13}\pi_{Rk}-\Lambda_3\pi_{c2k}-\Lambda_5}$

Table 2. Economic effects of a fertility shock

Second, a lower workforce has a negative effect on the consumption possibilities of retirees ($\pi_{c2b1} = -0.3659$). The reason is that retirees earn two types of income: interest earnings on capital, and pension benefits. Since the latter are fixed in a DB system, the only source of change to the consumption of retirees stems from interest fluctuations which generate changes in rent earnings on the capital stock owned by the retirees. This shows that the endogenous response of factor prices plays a critical role: when the number of workers falls, there is a rise in the capital-labour ratio, and the real rate of return to capital falls and hence the capital income accruing to retirees will fall.

Third, the effect on workers' consumption possibilities, π_{c1b1}, is more complicated. On the one hand, with a fall in the number of workers, each worker has to pay more taxes to finance the fixed benefits to retirees, which impacts negatively on the consumption of workers. On the other hand, workers get higher wages because of the higher capital-labour ratio. The net effect is thus ambiguous. With the parameter values in table 1, the "factor price effect" is seen to dominate the "fiscal effect". In fact, we can state the necessary condition under which the factor price effect dominates the fiscal effect. By log-linearising workers' income, $c_t^1 = w_t\,(1-\theta_t)\,/\chi_t$, around the steady state, we get $\widehat{c}_t^1 = \widehat{w}_t - \frac{\theta}{1-\theta}\widehat{\theta}_t - \widehat{\chi}_t$, and we can then insert the law of motion, where the elasticities have already been derived, for wages and contributions (see equations (18) and (19)) to get $\widehat{c}_t^1 = \left[-\alpha\widehat{b}_{t-1}\right] - \frac{\theta}{1-\theta}\left[-\widehat{b}_{t-1}\right] - \widehat{\chi}_t$. Focusing on a shock to lagged fertility of -1%, we obtain: $\widehat{c}_t^1 = \alpha - \frac{\theta}{1-\theta}$. So, provided that $\alpha > \frac{\theta}{1-\theta}$ the factor price effect dominates the fiscal effect.[8]

If the pension system is of the DC type, wages still increase due to the higher capital-labour ratio, yielding a positive effect on their consumption. Workers pay fixed contributions to pensions, so there is no negative "fiscal effect" and the impact on consumption is unambiguously positive. However, the total amount available to retirees is lower, and the replacement rate falls, which naturally reduces consumption of retirees. The interest earnings will still fall, due to the lower return on capital caused by the higher capital-labour ratio. Thus, retirees will lose in terms of both types of their income. There is a stronger transfer of risk between generations in the DB system compared to the DC system, as reflected by the fact that the difference between π_{c1b1} and π_{c2b1} is smaller in a DB than in a DC system, see table 2.

A shock to expected future longevity A shock of this kind will not impact on the consumption of current retirees, because the capital-labour ratio does not change. Since the contribution rate is not a function of the expected future longevity (only of the current longevity, $\widehat{\phi}^{e}_{t-1}$), taxes cannot be used as a policy instrument to reallocate risk in relation to this shock.[9] Current workers, however, anticipate a longer lifetime as retirees. They would therefore begin to save more, by giving up current consumption ($\pi_{c1\phi e} = -0.2225$), and the capital stock would increase ($\pi_{k\phi e} = 0.8900$), see table 3.

Policy coeff.	*DB*	*DC*	*Analytical elasticity*
$\pi_{\theta\phi e}$	N.A.	N.A.	
Effect on:			
$\pi_{c2\phi e}$	0	0	N.A.
$\pi_{c1\phi e}$	-0.2225	-0.2587	$\pi_{c2\phi e1} - \Lambda_{13}\pi_{R\phi e1} + (\pi_{c2k} - \Lambda_{13}\pi_{Rk})\,\pi_{k\phi e}$
$\pi_{k\phi e}$	0.8900	1.0346	$\frac{\Lambda_3\pi_{c2\phi e1} - \Lambda_3\Lambda_{13}\pi_{R\phi e1} + \Lambda_4\pi_{c2\phi e}}{\Lambda_3\Lambda_{13}\pi_{Rk} - \Lambda_3\pi_{c2k} - \Lambda_5}$

Table 3. Economic effects of a longevity shock.

[8] The value of θ is derived through calibration for $\beta = 0.3$ such that $\frac{\theta}{1-\theta} = 0.16$. With a value of $\alpha = 1/3$, the pension system must be relatively large (with contribution rates above 30% before this result is overturned.

[9] However, the retirement age, $\widehat{\chi}_t$, could be used, as we will discuss in the next section.

In the DB system the increase in the expected future longevity will require higher contributions in the future, while in the DC system contributions will not vary and current workers will have to save more, since the risk is not shared with future workers. Concequently, the DC system will again magnify the results of the DB system.

A shock to the retirement age A positive shock to the retirement age means that an individual must work for a longer time horizon, and the length of the retirement period is residually lowered. Like a shock to lagged fertility, this affects the labour force and hence the capital-labour ratio. With a shorter retirement period, the need for savings would fall, and a higher level of consumption is available to workers. However, consumption in the now longer working period would now be spread over more sub-periods. In a DB system, the net effect on savings is negative ($\pi_{k\chi} = -1.0345$), see table 4.

Policy coeff.	*DB*	*DC*	*Analytical elasticity*
$\pi_{\theta\chi}$	-1	0	
Effect on:			
$\pi_{c2\chi}$	0.3659	0.5057	$\frac{\left(\Lambda_8\pi_{\theta\chi}-\Lambda_{12}\pi_{w\chi}\right)(\Lambda_3\Lambda_{13}\pi_{Rk}-\Lambda_3\pi_{c2k}-\Lambda_5)}{(\Lambda_9\pi_{wk}-\Lambda_7\pi_{c2k}+\Lambda_{12}\pi_{Rk})\Lambda_4} - \frac{\Lambda_2}{\Lambda_4}$
$\pi_{c1\chi}$	0.1207	0.0606	$\pi_{c2\chi1} - \Lambda_{13}\pi_{R\chi1} + \left(\pi_{c2k} - \Lambda_{13}\pi_{Rk}\right)\pi_{k\chi}$
$\pi_{k\chi}$	-1.0321	-1.3111	$\frac{\Lambda_3\pi_{c2\chi1}-\Lambda_3\Lambda_{13}\pi_{R\chi1}+\Lambda_4\pi_{c2\chi}+\Lambda_2}{\Lambda_3\Lambda_{13}\pi_{Rk}-\Lambda_3\pi_{c2k}-\Lambda_5}$

Table 4. Economic effects of delayed retirement.

The capital-labour ratio falls since the labour force has increased. This produces lower wages and a higher return on savings, as reflected in more consumption available to retirees ($\pi_{c2\chi} = 0.3659$). Lower wages would also in itself cause a fall in workers' consumption. However, each worker now needs to pay less contributions to the DB pension system, which points to increase workers' consumption. With the chosen parameter values, the net effect is positive ($\pi_{c1\chi} = 0.1207$). Clearly, with fixed contributions, as in a DC system, retirees gain even more. This is so because of the higher return on the capital stock, but also because contributions are fixed and there are more sub-periods for taxation ($\pi_{c2\chi} = 0.5057$). And since the workers pay contributions over a longer period, their gains are smaller in the DC system compared to the DB system.

While several other types of shocks could be studied, there is already enough material to make the point that within a very *passive* policy framework, such as a public pension system operating on a period-by-period basis, demographic shocks may lead to highly unequal distributions of consumption possibilities across generations. For that reason it is worth considering an alternative more *active* policy adjustment in order to achieve more equitable outcomes following demographic shocks. This is the aim of the next section.

4 Evaluating alternative policy rules

4.1 Performance criteria

When studying the effects of demographic shocks we have seen that two main forces are operating: first, the (endogenous) factor price effect and, second, the fiscal effect originating from the pension system. The latter constitutes the (passive) policy rule which plays a major role for how the welfare effects of demographic shocks are distributed across generations. In general, it was found that the fiscal effects were not sufficient to counteract the factor price effects and, consequently, workers and retirees were exposed differently to the demographic shocks.[10]

In order to evaluate the social desirability of the results obtained in section 3, we would want to compare those results to a socially optimal allocation derived from a social welfare function. If the optimal allocation differs from the allocation found in the previous section, we may need to consider redistributional policies. This immediately raises the question as to how the social

[10]From the specification of the pension system in (11) it is clear that the contribution rate, θ, will change if the lagged fertility rate, b_1, changes. The magnitude of this change will be in equal proportion to the size of the shock to lagged fertility, and this is accounted for in the equivalent log-linearised equation (19) in terms of $\pi_{\theta b1}$. Therefore, the automatic (passiv) change in contributions, and thus in the "fiscal effect", $\left(\frac{\theta}{1-\theta}\right)$, of $\widehat{b}_{t-1} = \pi_{\theta b1} = 1\%$. We have shown that the "factor price effect" dominates the "fiscal effect" for a reasonable size of the pension system ($\beta = 0.3$ and thus $\theta = 0.1379$). This is the key fiscal mechanism that has the potential to redistribute income across generations living in the same period. As a result, the higher (more active) is $\pi_{\theta b1}$ the stronger is the fiscal effect, and the more likely is it that the fiscal effect will offset the factor price effects - and hence that risks will be shared more equally across generations.

welfare function should be formulated.

Specifically, we assume that aggregate social welfare can be measured as:

$$W_t = E\left\{\sum_{t=-1}^{\infty} \Phi_t N_t^w U_t\right\} = \left\{\Phi_{t-1} N_{t-1}^w U_{t-1} + \Phi_t N_t^w U_t\right\} \tag{20}$$

where Φ is the weight on the utility of a given generation. The problem of the policy maker is now to maximise (20), subject to the resource constraint in (9) and the lifetime utility function, as given by (5). Assuming that all generations are weighted equally ($\Phi_{t-1} = \Phi_t$), the efficiency condition (in log-deviations from the steady state) may be stated as follows:

$$\widehat{c}_t^1 = \widehat{c}_t^2 + \left(\pi_{\rho 1(b)}/\eta\right) \widehat{b}_t \tag{21}$$

where the factor $\rho_{1\left(\widehat{b}_t\right)}$ is assumed to be log-linearised as $\pi_{\rho_{1(b)}} \widehat{b}_t$, and where $\pi_{\rho_{1(b)}}$ is the elasticity of the weight of first period consumption in utility with respect to the current birth rate.[11] If there is no shock to the current birth rate, (21) states that the condition for an *efficient risk sharing* (Bohn, 2001) is that the percentage change in the consumption of workers equals the percentage change in consumption of retirees. The key implication of a policy reform is thus a redistribution of income. If the equilibrium generated by a passive policy rule in the presence of demographic shocks is characterised by unequal changes in different generations' consumption possibilities, then the government should "correct" this outcome, by redistributing income from workers to retirees (or vice versa) up to the point where both generations bear the burden (or share the gains) of shocks in equal proportions.

Intergenerational redistribution could either be achieved through changes in the contribution rate (or the benefit rate) of the pension system, *or* through structural (or labour market) reforms by changes in the retirement age. The potential of the latter has only been explored to a very limited extent compared to the former. Yet, from a policy perspective, it would be of interest to consider an alternative way of coping with demographic changes rather than through adjustments to the contribution and/or benefit *rates* of the public PAYG pension system. More promising, however, might be reforms which affect the *number* of retirees or workers, or the lifetime labour supply of each worker, through the introduction of a link between longevity changes and the retirement age.

[11] The derivation of this optimality condition is documented in a separate note, which is available upon request.

The shock we consider is a "composite shock", taking the form of a negative shock to lagged fertility and a positive shock to expected future longevity. Our motivation for focusing on these shocks is their empirical relevance: the fall in the lagged fertility rate reminds us of the so-called "baby bust" phenomenon in the 1970-80s, and an expectation about increased longevity has become common ground among social and medical scientists. Indeed, due to changes in life-style and advances in medical science, people are expected to live longer in the future (UN, 2005).

4.2 Results

Taxation as policy instrument Basically, the idea is to solve for the response of the contribution rate which ensures an efficient allocation of risk for the composite shock in accord with (21). Since taxes cannot respond to a shock to $\widehat{\phi}_t^e$, but only to $\widehat{b}_{t-1}$, we choose to denote the efficient response of taxes for the composite shock by $\pi^*_{\theta b1}$. Leaving out the details of the derivation, we find:

$$\pi^*_{\theta b1} = \frac{\pi_{c2\phi e} - \pi_{c1\phi e}}{\Omega_7 - \Omega_9}\widehat{\phi}_t^e/\widehat{b}_{t-1} + \frac{\Omega_8 - \Omega_{10}}{\Omega_7 - \Omega_9}\pi_{wb1} - \frac{\Omega_{11}}{\Omega_7 - \Omega_9} \tag{22}$$

where the $\Omega's$ comprise steady state variables and other elasticities.[12] With the parameter values reported in table 1, the numerical elasticity of the contribution rate with respect to the composite amounts to $\pi^*_{\theta b1} = -2.04$. Recall that the empirically relevant shock to lagged fertility is negative and to expected future longevity is positive. Thus, an optimal risk sharing implies an increase in the tax rate of about 2%.

It would be interesting to compare the effects obtained with a passive rule to the results obtained with an active response. With the above parameter values, we get $\widehat{c}_t^1 = \pi_{c1b1}\widehat{b}_{t-1} + \pi_{c1\phi e}\widehat{\phi}_t^e = -0.12$ and $\widehat{c}_t^2 = \pi_{c2b1}\widehat{b}_{t-1} + \pi_{c2\phi e}\widehat{\phi}_t^e = -0.37$. In words, workers' consumption stand to decrease by about 0.12% while that of retirees is expected to decrease by 0.37%. The pension contribution rate is designed to automatically respond to the 1%-shocks by also increasing proportionally by 1%, but this is not enough to ensure equal sharing of the risks associated with the shocks. As shown, we find that the tax rate must increase by about 2 percentage points, in order to transfer enough income from workers to retirees to achieve efficient risk sharing across generations. When this active fiscal policy is adopted both generations bear the burdens exactly

[12] We again refer to a separate note (available upon request) for the details.

in equal proportions by a decrease in consumption of about 0.22% for both workers and retirees (i.e. $\widehat{c}_t^1 = \widehat{c}_t^2 = -0.22$). Thus, when the contribution rate is used to guarantee an efficient risk sharing, there is a *net* welfare loss on the part of both generations. Against that, it would be of interest to see if a better outcome could be achieved, and this is the objective of the remainder of the paper.

4.2.1 Retirement age as policy instrument

From an analytical perspective, it is not really straightforward how a change in the retirement age should be conceptualized.[13] An increase in the length of the working period can, in principle, be thought of in two different ways. First, workers could simply choose to work more years for some exogenous reason(s), or, second, it could be the result of a government policy designed such that the age limit for eligibility to pension benefits of a representative worker is postponed. In the latter case we would assume that people are induced to work for this extra sub-period of what in any case is "working life". In this analysis it is assumed that people do not want to cover their expenses out of savings in this extra sub-period, which previously was part of the retirement period. Instead, they decide to work this extra sub-period until they become recipients of ordinary pension benefits.

Within the formal framework set out above, this discussion is captured by the parameter $\widehat{\chi}_t$, which denotes "the change in the length of the working period from its steady state value". For example, one can think of $\widehat{\chi}_t$ as an exogenous shock to the supply of labour such as a change in people's perception of work which is not derived from the model structure. It is also possible to think of $\widehat{\chi}_t$ as a policy variable, which the policy maker can change as part of, say, a labour market reform.

In the following we adopt the practice of treating $\widehat{\chi}_t$ as an exogenous variable, and allowing it to be used as a policy parameter. We then impose the condition for efficient risk sharing (21) on the recursive equilibrium law of motion for the consumption of workers and retirees, respectively. Solving for the optimal length of the working period in accord with (21), $\widehat{\chi}_t^*$, yields

$$\widehat{\chi}_t^* = \omega_1 \widehat{b}_{t-1} + \omega_2 \widehat{\phi}_t^e \tag{23}$$

[13] Cutler (2001), in his comment on Bohn (2001), suggests an extension of Bohn's model to incorporate the length of the working period.

where the coefficients ω_1 and ω_2 comprise steady state variables and other elasticities. Assuming the shocks to be -1% for lagged fertility and $+1\%$ for expected future longevity, this will lead to an efficient policy response of an increase in the retirement age of *exactly* **1**%. In fact, the two shocks generate dynamics which offset each other. Using the parameter values of table 1, a numerical analysis shows that retirees are affected by -0.3659 from $\widehat{b}_{t-1}$; by 0 from $\widehat{\phi}_t^e$; and by 0.3659 from $\widehat{\chi}_t$. Similarly, workers experience a response of 0.1018 from $\widehat{b}_{t-1}$; -0.2225 from $\widehat{\phi}_t^e$; and 0.1207 from $\widehat{\chi}_t$. Note that both retirees and workers experience the combination of three shocks, but the net effect is zero for both generations. Importantly, this is assuming a policy response of 1% for $\widehat{\chi}_t$. This could also be regarded from the perspective of an efficient policy reponse of $\widehat{\chi}_t$. For a shock only to $\widehat{b}_{t-1}$ then $\widehat{\chi}_t = 1.9074$, and $\widehat{\chi}_t = -0.9074$ for a shock only to $\widehat{\phi}_t^e$. By adding these effects, we get exactly the result corresponding to $\widehat{\chi}_t = 1\%$.

The intuition behind this result is as follows: A negative shock to $\widehat{b}_{t-1}$ will reduce the current labour force, and a positive shock to $\widehat{\chi}$ will offset this reduction. The other feature of the shock is in connection with the retirement period. A positive shock to $\widehat{\chi}_t$ will lead to a lower expected retirement period, while a positive $\widehat{\phi}_t^e$ will offset it. This is because the length of the retirement period is modelled to be residually determined from changes in the length of the working period and the total length of life ($\lambda_{t+1} = \phi_{t+1}\mu_{2t+1} - \chi_t$). As such, a policy response of $\widehat{\chi}_t = 1\%$ will offset all effects stemming from the two demographic shocks.

This result indicates that the retirement age is a better policy instrument than the contribution rate. Indeed, the utility effect from employing taxes would generate a utility loss for both generations of ($\widehat{c}_t^1 = \widehat{c}_t^2 = -0.2207$ given that $\pi^*_{\theta b1} = 2.0388$). When employing the retirement age as policy instrument there will not be any utility loss for either generation ($\widehat{c}_t^1 = \widehat{c}_t^2 = 0$ given that $\widehat{\chi}_t = 1\%$). This result, among other assumptions, is based on the choice of modelling, incorporating with equal weights in utility both the length of the working period and the length of the retirement period for each generation (χ_t and λ_{t+1}).

Finally, we have found that this result is robust over both a DB and a DC PAYG regime, and also in a model without any pension system. Different combinations of shocks could be analysed, but the empirically most relevant shock is the one covered in this paragraph. While much more academic work is needed in this area, it seems reasonable to suggest that an indexation scheme of retirement age relative to expected longevity, in order to ensure intergenerationally efficient risk sharing, would be a sensible policy adjustment.

5 Concluding remarks

Based on a stochastic OLG model, this paper has shown how various demographic shocks may affect the intergenerational distribution of welfare. The paper has also discussed how policy rules may be designed in order to achieve outcomes which are more equitable compared to outcomes obtained within a passive policy framework.

The novelty of the paper is a study of longevity adjustment of the retirement age as an instrument to generate efficient risk sharing in an economy faced by demographic shocks. We find that a rise in the retirement age following an increase in expected longevity may leave both workers and retirees better off. However, this is not the case within an alternative setting where taxes on wage incomes are adjusted to share risks efficiently across generations. So, the retirement age outperforms taxes as a policy instrument.

While we feel convinced that the analytical framework used in this paper offers a fruitful starting point for studying an important policy area, we are also aware of several limitations and extensions that we want to address in future research.

First, the supply of labour is assumed exogenous in our analysis, and an obvious extension would therefore be to introduce endogenous labour supply decisions by households. While this would raise a number of technical complexities, important insights could be obtained into the effects of a higher retirement age on labour supply. For example, would an increase in labour supply at the extensive margin be offset by a fall in the endogenous labour supply at the intensive margin? In any case, appropriate policy measures would need to take into account the endogeneity of labour supply.

Second, it is also feasible to endogenise other variables and solve for additional dynamics, which are absent in this paper. Future research could include endogenous human capital formation (through, e.g., education), and endogenous fertility decisions in order to address the reasons behind the negative shock to fertility in the 1970s and 1980s.

Third, while the present analysis is stated in terms of a closed economy, it would be of interest to consider an open economy perspective. For example, in the case of a small open economy, one would need to specify the structure of the OLG model differently and incorporate an exogenously determined interest rate.

Fourth, in relation to the probability distributions of the exogenous state variables, one could specify detailed distributions instead of the *i.i.d.* specification which we follow. As such, a more accurate picture of the stochastic properties of the demographics could be obtained.

Finally, to study the welfare effects of demographic shocks to the sustainability of public finances, a more detailed government sector could be incorporated into the model. Some, if not all, of the above extensions we have in mind for our future research in this area.

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