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Boom, Anette

*Document Version*  
Final published version

*Publication date:*  
2007

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*Citation for published version (APA):*  
Boom, A. (2007). *Vertically integrated Firms' Investments in Electricity Generating Capacities*. Department of Economics. Copenhagen Business School.

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**Copenhagen  
Business School**  
HANDELSHØJSKOLEN

# **Department of Economics**

Copenhagen Business School

**Working paper 3-2007**

## **VERTICALLY INTEGRATED FIRM S INVESTMENTS IN ELECTRICITY GENERATING CAPACITIES.**

**Anette Boom**

# Vertically Integrated Firms' Investments in Electricity Generating Capacities\*

Anette Boom<sup>†</sup>  
Copenhagen Business School

Revised October 2007

## Abstract

We compare investments in generating capacities of an integrated monopolist with the aggregate investments of two vertically integrated competing firms. The firms invest in their capacity and fix the retail price while electricity demand is uncertain. The wholesale price is determined in a unit price auction where the firms know the level of demand when they bid their capacities. Total capacities can be larger or smaller with a duopoly than with a monopoly. If the two firms select the Pareto dominant equilibrium, then the retail price is always higher and the social welfare lower in the duopoly case.

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\*This paper is a thoroughly revised and extended version of Boom (2003).

<sup>†</sup>I thank Christopher Xitco, Marco Haan, Mathias Erlei and seminar participants at the Norwegian School of Economics and Business Administration in Bergen, at the *Workshop on the Economics of Information and Network Industries* 2003 in Kiel, at the 2004 meeting of the *Industrieökonomischer Ausschuss* in Koblenz and at the 2004 conferences of the *Verein für Socialpolitik* in Dresden and of the *EARIE* in Berlin for helpful comments as well as the Social Science Research Center Berlin (WZB) for a productive atmosphere. Financial Support by the German Science Foundation (DFG) through the SFB/TR 15 "Governance and the Efficiency of Economic Systems" is gratefully acknowledged.

# 1 Introduction

In many industrialised countries the market for electricity has been liberalised. Whereas the introduction of competition was rather successful in, for example, the Scandinavian Countries and in England and Wales, others, like California and New Zealand, experienced major crises with an explosion of wholesale prices and black-outs.<sup>1</sup> The focus of this paper is, however, not to figure out what went wrong when California and New Zealand opened their markets for competition,<sup>2</sup> but rather to investigate, whether the incentives to invest in generating capacity can be suboptimal under competition compared to a monopoly market.

Why should this be the case? The electricity market like many other markets is characterised by an uncertain demand. Electricity can, however, not be stored. All competing firms must use the same distribution network, and the inflows and outflows of electricity into this network have to be balanced at each point in time. If the balance cannot be preserved, then the network collapses and none of the firms can sell electricity anymore. This creates externalities that might be better internalised by a monopolist than by competing firms.<sup>3</sup> On the other hand, the monopolist tends to produce less than is socially efficient and would therefore need and build fewer generating capacities.

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<sup>1</sup>See e.g. *The Economist* from March 7, 1998, p. 46, and from February 10, 2001, and for New Zealand *Modern Power Systems* from August 2001, p. 11.

<sup>2</sup>For analyses of the Californian electricity crisis see, e.g., Joskow (2001), Borenstein *et al.* (2002), Borenstein (2002), Wilson (2002) and Borenstein *et al.* (2006).

<sup>3</sup>Joskow and Tirole (2004a) also refer to these externalities during uncontrolled black-outs.

If we had perfect competition in the electricity market, electricity prices should equal marginal costs. Under these circumstances von der Fehr and Harbord (1997) as well as Castro-Rodriguez *et al.* (2001) show that, from a social welfare point of view, firms build suboptimal low levels of generating capacity. They also prove that firms invest more in their generating capacity, if the spot market price exceeds marginal costs at a fixed margin. In addition von der Fehr and Harbord (1997) endogenise the spot market price of electricity for an inelastic demand that is ex ante uncertain in the capacity decision stage, and known, when the firms bid their capacity in the auction.<sup>4</sup> They conclude that the firms under-invest in capacity as long as the distribution of the uncertain inelastic demand is concave, meaning skewed to the lower end of the distribution. But neither of the two compares the market outcome under competition with the one generated by a monopoly.

Contrary to von der Fehr and Harbord (1997), we consider domestic consumers with an ex ante elastic demand for electricity. They can, however, not respond to price signals from the wholesale market. In the first stage two firms invest in generating capacity. The firms are vertically integrated into the retail sector. Then consumers sign a retail contract with one of the two integrated firms in order to be delivered with electricity. The firms guarantee their retail customers a certain retail price at which the consumers can buy as much electricity as they want to, as long as there is no black-out.<sup>5</sup> The consumers choose the firm with the lowest retail price. Then nature chooses

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<sup>4</sup>The auction is a unit price auction à la von der Fehr and Harbord (1993).

<sup>5</sup>Contrary to Joskow and Tirole (2004a) we abstract from two-part tariffs for the sake of simplicity and from state contingent rationing rules which the firms fix in advance together with the retail price for their clients. The latter do not play a major role in the retail competition for domestic consumers.

the level of the demand shock. After observing demand, firms bid prices in the wholesale market in order to get the right to supply their capacity to the network. The wholesale market is modelled as a unit price auction à la von der Fehr and Harbord (1997) and (1993).<sup>6</sup> Since the two firms commit to retail prices before the auction takes place, demand is inelastic in the auction as in von der Fehr and Harbord (1997) despite being ex ante elastic.

The fact that consumers cannot instantaneously respond to price signals is due to the imperfect metering technology that is used by most residential customers. This technology does not register how much electricity they consume at a given point in time, and cannot communicate current market prices. In a companion paper Boom (2002) I abstract from this problem and assume that consumers can instantaneously respond to market prices and can therefore directly participate in the spot market for electricity.<sup>7</sup>

Before the electricity markets were liberalized they were often characterized by vertically integrated regional monopolies. These firms often survived the liberalization process and are now competing in the electricity wholesale markets as well as in the retail market.<sup>8</sup> How the vertical integration of firms affects the competition on the wholesale and the retail market has hardly

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<sup>6</sup>Green and Newbery (1992) based on Klemperer and Meyer (1989) suggested an alternative approach where firms bid differentiable supply functions instead of step function as in von der Fehr and Harbord (1997) and (1993).

<sup>7</sup>Both papers were inspired by the proponents of a better metering technology who argue that it reduces peak demands and improves the performance of liberalised electricity markets. See Borenstein (2002), Faruqui *et al.* (2001) as well as Borenstein and Holland (2005). The latter show under which circumstances increasing the number of price-responsive consumers improves social welfare in a perfectly competitive electricity market. Their approach has been further extended by Joskow and Tirole (2004b).

<sup>8</sup>A high degree of vertical integration of electricity generating firms into the retail sector can for example be observed in France, Spain and Germany. See e.g. Bergman *et al.* (1999) and European Commission (2001).

been analyzed up to now. The studies which analyse the effects of vertical integration on the wholesale prices usually consider markets with regulated or predetermined retail prices.<sup>9</sup> Contrary to them, we also endogenize the retail prices and explicitly focus on the interaction between the wholesale and the retail market, when vertically integrated firms with market power compete on the two related markets.

It turns out that the vertically integrated monopolist might install a smaller or larger capacity than the two vertically integrated competitors together at not too high capacity costs. If we focus, however, on subgame perfect Nash equilibria that are not Pareto dominated, then the two duopoly firms invest more than the monopolist. The social welfare is, nevertheless, always higher under a monopoly than in the competitive setting, because the monopoly price is always lower than the duopoly price.<sup>10</sup>

## 2 The Model

There are two firms  $j = A, B$  which generate electricity and are vertically integrated into the retail sector. They face a mass of electricity consumers that is normalised to one. Consumers suffer from demand shocks. They have a quasi-linear utility function such that their surplus function is given by

$$V(x; \varepsilon, r) = U(x, \varepsilon) - rx = x - \varepsilon - \frac{(x - \varepsilon)^2}{2} - rx, \quad (1)$$

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<sup>9</sup>See e.g. Kühn and Machado (2004), Bushnell *et al.* (2005), Mansur (2007) and Bal-dursson and von der Fehr (2007)

<sup>10</sup>These results contrast with those of the companion paper, Boom (2002), where the monopoly nearly always invests less and the social welfare is always improved by competition.

for a representative period where  $x$  is the consumed electricity,  $r$  is the retail price paid per unit for electricity, and  $\varepsilon$  is the demand shock. It hits all the consumers alike and is uniformly distributed on the interval  $[0, 1]$ . These shocks should not be mistaken for the volatility of demand during a single day which is somehow foreseeable. They refer to events like a hot summer in California or a cold winter in the north of Europe. The demand for electricity in the representative period can be derived from maximising  $V(x; \varepsilon, r)$  with respect to  $x$  and results in

$$x(r, \varepsilon) = \max\{1 + \varepsilon - r, 0\}. \quad (2)$$

Note that the single consumer's demand has no weight in the total demand. Thus, he cannot influence the balance of supply and demand on the grid and would therefore always accept the lowest retail price offered. If the offered retail prices are identical, he signs each of the two retail contracts with equal probability.

The two firms are risk neutral and maximize their (expected) profits. The variable costs of generating electricity is assumed to be constant and, for the sake of simplicity, equal to zero for both firms. Thus, the costs of firm  $j$  consist only of the costs of capacity which are assumed to be:

$$C(k_j) = zk_j \quad (3)$$

where  $z$  is a constant unit cost of capacity and  $k_j$  the generation capacity installed by firm  $j$ . Firms decide on their capacity  $k_j$  and on their retail price



offer  $r_j$  before they know the level of demand in the representative period. When they bid their capacity in the electricity wholesale market, the demand shock is realised and the retail price is fixed. Therefore the market demand is known for sure and does not respond to changes in the wholesale price.

The wholesale market price of electricity is determined in a unit price auction of the type introduced by von der Fehr and Harbord (1997) and (1993). Such an auction was at the heart of the Electricity Pool in England and Wales before the reform in 2001, and still is in place in other liberalised markets like, e.g., the Nord Pool in Scandinavia or the Spanish wholesale market.<sup>11</sup> For the sake of simplicity firms have to bid a price  $p_j$  at which they are willing to supply their whole generating capacity.<sup>12</sup> The auctioneer must secure the balance of supply and demand on the grid if possible.<sup>13</sup> Therefore he orders the bids according to their prices and determines the marginal bid that is just necessary to equal supply and demand. The price of the marginal bid is the spot market price that is paid to all the generators for each unit of capacity that is actually dispatched on the grid.<sup>14</sup> The capacity of the supplier that has bid below the marginal price is dispatched completely, whereas the marginal supplier is only allowed to deliver that amount of electricity necessary to balance supply and demand.<sup>15</sup>

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<sup>11</sup>See Bergman *et al.* (1999).

<sup>12</sup>Thus, we do not consider the problem of strategic capacity withholding in order to raise the auction price. See Crampes and Crete (2005) and Le Coq (2002) for such analyses.

<sup>13</sup>Transmission constraints are not considered here, although they might interact with constraints in the generating capacity. See Wilson (2002) for insights into this problem and for the analysis of isolated transmission constraints Borenstein *et al.* (2000), Joskow and Tirole (2000) and Léautier (2001).

<sup>14</sup>This differs the analysis here from simple Bertrand competition with capacity constraints as in Kreps and Scheinkman (1983) where the undercutting firm receives only its own price per unit sold even if its capacity is too low to serve all the customers.

<sup>15</sup>According to Wilson (2002) we assume an integrated system because participation in

Since in our framework demand does not respond to changes in the wholesale price and since the total amount of installed capacities can also not be influenced by the wholesale price, the auctioneer may also fail to find a price that balances supply and demand in the market. Then a black-out occurs in the representative period. In reality firms compete not only once in a representative period but repeatedly on the wholesale and on the retail market. We abstract from the repeated nature of both retail and wholesale markets in order to simplify the analysis and to avoid running into issues of collusion.<sup>16</sup> Therefore a black-out in our representative period can be interpreted as an inadequate supply of electricity in the system. We assume that no firm can sell and deliver electricity, and all the firms realise zero profits. Thus, we also abstract from any sort of rationing by the auctioneer or the generators of electricity.<sup>17</sup> This is done in order to maximize the punishment for the firms if their aggregate capacity is too small, thus also maximizing the incentive to install capacity.

If total capacities are sufficient to satisfy demand, the auctioneer accepts only price offers that do not exceed the maximum price level  $\bar{p}$  that ensures zero profits for the net-buyer in the auction. Alternatively, we could have assumed that firms declare bankruptcy and neither supply electricity nor demand it on behalf of their customers on the wholesale market if doing so generates losses. This would establish the same maximum price. Consumers

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the auction is compulsory if a generating firm wants to sell electricity.

<sup>16</sup>This has been analysed by Fabra (2003) and by Dechenaux and Kovenock (2005) for the wholesale market.

<sup>17</sup>See Joskow and Tirole (2004a) for an analysis of a market where retailers propose not only prices to consumers but also rationing rules which they want to apply. Although in reality residential consumers are sometimes rationed, the rationing rules are usually not spelled out in any sort of contract with their retailers.

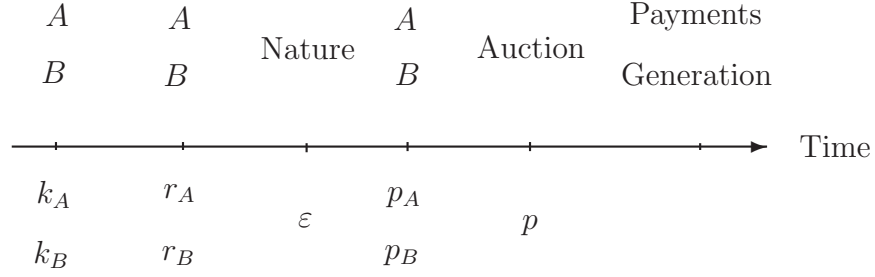


Figure 1: The Timing of the Game

would nevertheless extract electricity from the grid. This triggers most likely a black-out due to the unavailable capacity of the bankrupt provider, and even if not, the still available net-supplier will not receive any payments from the bankrupt firm anymore.

The timing of the game is depicted in figure 1. The game proceeds as follows:

1. The two generating firms simultaneously choose their respective capacity  $k_j$  with  $j = A, B$ .
2. The firms simultaneously set their retail price  $r_j$ . The consumers subscribe to the firm with the lowest price or, subscribe to each firm with probability one half, if both prices are identical.<sup>18</sup>
3. Nature determines the demand shock  $\varepsilon$ .
4. Both firms bid a price  $p_j$  for their whole capacity  $k_j$  in the wholesale market.
5. The auctioneer determines the market clearing price  $p$ , if this is pos-

<sup>18</sup>Committing to a retail price is not unusual in reality. Green (2003) reports, for example, that in Britain retail prices are typically set for a year.

sible, and which generator is allowed to deliver which amount of electricity to the grid.

6. If supply and demand cannot be balanced, a black-out occurs. The consumers are not served and do not pay anything to the generators. If the balance on the wholesale market can be achieved, the consumers are served and pay the retail price for each unit of their demand to the firm to which they subscribed. The firms have to pay the wholesale price for each unit of electricity that their retail customers required and receive the wholesale price for each unit of electricity that they were allowed to dispatch on the grid.

It is assumed that the two firms select a Pareto dominant Nash equilibrium if there are multiple Nash equilibria in one of the stages.

### **3 The Benchmark Cases**

As benchmark case we analyse how much an integrated monopolist and a social planner would invest in generating capacity, if he had to choose the capacity and to fix the retail price  $r$  before the uncertainty of demand is resolved.

### 3.1 The Monopolist's Capacity Choice

Note that with a monopolist a wholesale market does not exist. The monopolist's profit function is given by:

$$\pi_m(r, k) = \begin{cases} \int_{\max\{r-1, 0\}}^1 r(1 + \varepsilon - r)d\varepsilon - zk & \text{if } r \geq 2 - k, \\ \int_{\max\{r-1, 0\}}^{k-1+r} r(1 + \varepsilon - r)d\varepsilon - zk & \text{if } \max\{0, 1 - k\} \leq r < 2 - k, \\ -zk & \text{if } r < \max\{0, 1 - k\}. \end{cases}$$

The first line in the equation corresponds to a high capacity  $k$  and/or a high retail price  $r$  which ensure that the monopolist can serve any possible demand. With intermediate levels of  $k$  and  $r$  black-outs cannot be prevented for high demand shocks. For a low capacity  $k$  and/or a low retail price  $r$  a black-out always occurs because the monopolist can never cover demand.

From the monopolist's profit function the optimal retail price for a given generating capacity can be calculated.

**Lemma 1** *For a given generating capacity  $k$ , the monopolist's profit maximising retail price is  $r^*(k) = \max\{2 - k, 3/4\}$ .*

**Proof:** See appendix A. ■

The monopolist sets always a price that ensures no electricity outages for all possible demand shocks  $\varepsilon$ . For lower generating capacities ( $0 \leq k < 5/4$ ) the retail price just avoids the black-out in the highest demand state and is therefore decreasing in the generating capacity. As soon as the generating capacity is large enough ( $k \geq 5/4$ ) to satisfy even the largest possible demand

at the monopoly price for the expected demand, the monopolist chooses this price and sticks to it.

The monopolist's capacity choice is given in proposition 1.

**Proposition 1** *The monopolist chooses the generation capacity*

$$k_m^* = \begin{cases} \frac{5}{4} - \frac{z}{2} & \text{if } z \leq \frac{1}{2} \\ 0 & \text{if } z > \frac{1}{2}. \end{cases}$$

*The retail price in equilibrium is  $r_m^* = 3/4 + z/2$  as long as the investment in generation capacity is positive, meaning  $z \leq \frac{1}{2}$ .*

**Proof:** See appendix A. ■

The monopolist's investment decreases in the capacity cost  $z$ , but is discontinuous at  $z = \frac{1}{2}$  because even optimal capacity levels and an optimal retail price would result in negative profits. The retail price increases in the capacity cost  $z$ .

### 3.2 The Social Planner's Capacity Choice

The social planner maximizes the sum of the expected consumer's and producer's surplus. The latter is given by

$$W(r, k) = \begin{cases} \int_0^1 U(x(r, \varepsilon), \varepsilon) d\varepsilon - zk & \text{if } r \geq 2 - k \\ \int_0^{k-1+r} U(x(r, \varepsilon), \varepsilon) d\varepsilon \\ \quad + \int_{k-1+r}^1 U(0, \varepsilon) - zk & \text{if } 2 - k > r \geq \max\{1 - k, 0\} \\ \int_0^1 U(0, \varepsilon) - zk & \text{if } 1 - k > r \geq 0 \end{cases} \quad (4)$$

where  $U(\cdot)$  corresponds to the consumers' gross surplus function defined in equation (1) and  $x(r, \varepsilon)$  to the consumers' state dependant demand given in equation (2). From the social welfare function the socially optimal retail price for a given generating capacity can be calculated.

**Lemma 2** *For a given generating capacity  $k$ , the social welfare maximising retail price is  $r^{**}(k) = \max\{2 - k, 0\}$ .*

**Proof:** See appendix B. ■

The social planner always sets a price that ensures no electricity outages for all possible demand shocks  $\varepsilon$ . For lower generating capacities ( $0 \leq k < 2$ ) the retail price just avoids the black-out in the highest demand state and is therefore decreasing in the generating capacity. As soon as the generating capacity is large enough ( $k \geq 2$ ) to satisfy even the largest possible demand, the social planner chooses a retail price which coincides with the marginal production cost of 0.

The social planner's capacity choice is given in proposition 2.

**Proposition 2** *The social planner chooses the generation capacity*

$$k_s^{**} = \begin{cases} 2 - z & \text{if } z \leq 2 - \sqrt{\frac{5}{3}} \\ 0 & \text{if } z > 2 - \sqrt{\frac{5}{3}}. \end{cases}$$

*The retail price in equilibrium is  $r_s^{**} = z$  as long as the investment in generation capacity is positive, meaning  $z \leq 2 - \sqrt{\frac{5}{3}}$ .*

**Proof:** See appendix B. ■

The social planner's investment decreases in the capacity cost  $z$ , but is discontinuous at  $z = 2 - \sqrt{\frac{5}{3}}$  because even optimal capacity levels and an optimal retail price would result in a social welfare below the level achieved without any electricity consumption and capacity investment. Given the socially optimal capacity, the optimal retail price coincides with the capacity cost  $z$ .

## 4 Investments in Generating Capacity with Two Competing Firms

Here we derive the subgame perfect equilibrium of the game described in section 2 by backward induction. We start with the analysis of the wholesale price for any retail price and capacity level of the two firms. Then we determine the retail price for any level of installed capacity when the two firms anticipate the resulting equilibrium on the wholesale market. Finally the



investment decisions of the two firms are analysed, given that the integrated firms anticipate the consequences of their decision for the retail competition and for the equilibrium on the wholesale market.

#### 4.1 The Wholesale Market

If two firms compete, a wholesale market exists. Both firms committed in an earlier stage to a retail price and they observe the demand shock  $\varepsilon$ . Therefore total market demand is fixed and given by:

$$d(r_A, r_B, \varepsilon) = x(\min\{r_A, r_B\}, \varepsilon)$$

where  $x(\cdot)$  is defined in equation (2). The demand of firm  $j$ 's retail customers depends on its retail price and on the retail price of its rival. It is given by:

$$d_j(r_j, r_h, \varepsilon) = \begin{cases} x(r_j, \varepsilon) & \text{if } r_j < r_h, \\ \frac{1}{2}x(r_j, \varepsilon) & \text{if } r_j = r_h, \\ 0 & \text{if } r_j > r_h, \end{cases} \quad \text{with } j, h \in \{A, B\}, j \neq h. \quad (5)$$

In principle we can distinguish three different situations illustrated in figure 2. First, total capacity might be smaller than the market demand ( $d(r_j, r_h, \varepsilon) > k_j + k_h$ ). Then the auctioneer would not find any auction price that balances demand and supply and there would be a black-out; firms' profits would be 0. Note that the investments in generation capacity are sunk at this stage. Second, if total capacity is sufficient to serve the market demand

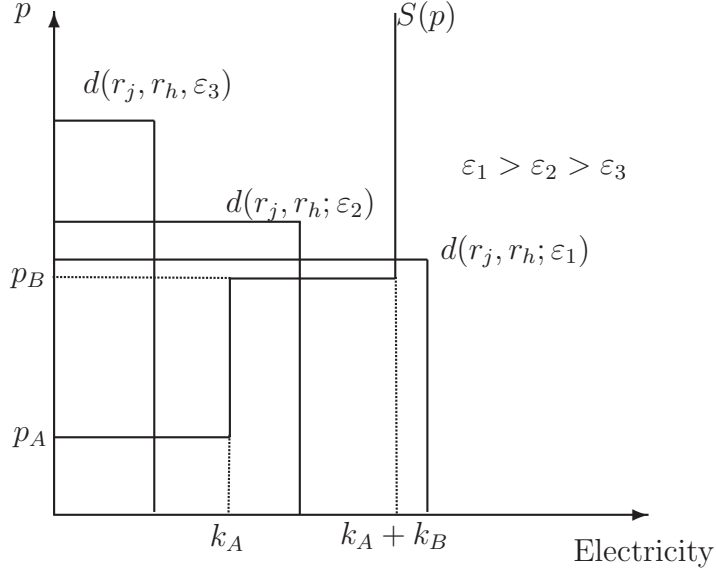


Figure 2: The Wholesale Market

( $d(r_j, r_h, \varepsilon) \leq k_j + k_h$ ), then the auction price is

$$p(p_j, p_h, \varepsilon) = \left\{ \begin{array}{ll} p_j & \text{if } d(r_j, r_h, \varepsilon) > k_h, \\ p_h & \text{if } d(r_j, r_h, \varepsilon) \leq k_h, \end{array} \right\} \text{ and } p_j \geq p_h. \quad (6)$$

In the first case (see the first line of (6)) the capacity of the firm with the lower bid is not sufficient to satisfy total demand if, for example,  $\varepsilon = \varepsilon_2$  in figure 2. Therefore the auctioneer sets the market price according to the higher bid  $p_j$  and dispatches the generating capacities of both firms. In the second case (see the second line of (6)) is the low-bidding firm's capacity large enough to satisfy the market demand as for  $\varepsilon_1$  in figure 2. Only this firm's capacity is dispatched and the market price coincides with the lower bid. How much of a firm's capacity is dispatched depends on its bid and

coincides with

$$y_j(p_j, p_h, \varepsilon) = \begin{cases} \min\{k_j, d(r_j, r_h, \varepsilon)\} & \text{if } p_j < p_h, \\ \frac{1}{2} \min\{k_j, d(r_j, r_h, \varepsilon)\} \\ \quad + \frac{1}{2} \max\{d(r_j, r_h, \varepsilon) - k_h, 0\} & \text{if } p_j = p_h, \\ \max\{d(r_j, r_h, \varepsilon) - k_h, 0\} & \text{if } p_j > p_h, \end{cases} \quad (7)$$

given  $d(r_j, r_h, \varepsilon) \leq k_j + k_h$ . Firm  $j$ 's profit in terms of its own bid price  $p_j$  and its rival's bid price  $p_h$  is

$$f_j(p_j, p_h, r_j, r_h, \varepsilon) = r_j d_j(r_j, r_h, \varepsilon) + p(p_j, p_h, \varepsilon)(y_j(p_j, p_h, \varepsilon) - d_j(r_j, r_h, \varepsilon)).$$

The firm collects all the revenues from its retail customers, has to pay the wholesale market price for each unit of electricity which its retail customers consume and earns the wholesale price on each unit of electricity it is allowed to generate and to feed into the grid. The analysis of the firms' best responses in bid prices is presented in detail in appendix C and results in lemma 3.

**Lemma 3** *If both firms have installed enough generation capacity to satisfy the demand of their retail customers ( $k_j \geq d_j(r_j, r_h)$  for  $j = A, B$ ) then the bid prices and the auction price satisfy  $p_A = p_B = p(p_A, p_B, \varepsilon) = 0$  and each firm's profit net of capacity costs in equilibrium is  $f_j(r_j, r_h, \varepsilon) = r_j d_j(r_j, r_h, \varepsilon)$  for  $j = A, B$ .*

*If the total capacities installed are sufficient to satisfy the market demand ( $k_A + k_B \geq d(r_j, r_h, \varepsilon)$ ), but if one firm  $j$ , cannot satisfy the demand of its retail customers ( $k_j < d_j(r_j, r_h)$ ), then the bid prices satisfy  $p_h = \bar{p}(r_j, r_h, \varepsilon) =$*

$p(p_j, p_h, \varepsilon)$  and  $p_j \leq \hat{p}(r_j, r_h, \varepsilon) < \bar{p}(r_j, r_h, \varepsilon)$  with

$$\bar{p}(r_j, r_h, \varepsilon) = \frac{r_j d_j(r_j, r_h, \varepsilon)}{d_j(r_j, r_h, \varepsilon) - k_j} \text{ and}$$

$$\hat{p}(r_j, r_h, \varepsilon) = \frac{r_j d_j(r_j, r_h, \varepsilon)}{\min\{k_h, d(r_j, r_h, \varepsilon)\} - d_h(r_j, r_h, \varepsilon)}.$$

Profits are  $f_j(r_j, r_h, \varepsilon) = 0$  and  $f_h(r_j, r_h, \varepsilon) = r_j d_j(r_j, r_h, \varepsilon) + r_h d_h(r_j, r_h, \varepsilon)$ .

If total capacities fall short of the market demand ( $k_A + k_B \geq d(r_j, r_h, \varepsilon)$ ), then no market clearing price exists and the system collapses. Both firms realise zero profits.

**Proof:** See appendix C. ■

Given that both firms can serve the demand of their retail customers, each firm prefers to undercut its rival during the auction. Thus, the Nash equilibrium results in a zero auction price and each firm's profit is limited to the revenues earned from its retail customers. Now consider a situation where one firm cannot serve the demand of its retail customers, and the other cannot only serve its own customers, but can also make up for its rival's deficit. The deficit firm cannot avoid becoming a net payer during the auction. It can, however, minimize its net demand position by undercutting. The firm with the generation surplus is always a net supplier of electricity in this situation. The unique Nash equilibrium is then characterised by the surplus firm always bidding the maximum price. The deficit firm undercuts sufficiently, so that the surplus firm has no incentive to undercut itself. The deficit firm cannot realise a positive profit anymore, whereas the surplus firm can appropriate all the potential rents in the market.

## 4.2 The Retail Price Competition

In principle each firm can use three strategies in the retail price competition: It can undercut its rival which results in all consumers signing a contract with this firm (see equation (5)). It can offer the same retail price as its rival, thus gaining half of the consumers and, thus, also half of the market demand, or it can request a higher price which means no consumer subscribes to the firm. Undercutting yields the following expected profit:

$$\pi_j(r_j, r_h) = \begin{cases} \int_{\max\{0, r_j-1\}}^{\min\{1, k_j+r_j-1\}} r_j x(r_j, \varepsilon) d\varepsilon & \text{if } \max\{1 - k_j, 0\} \leq r_j < r_h, \\ 0 & \text{if } 0 \leq r_j < \min\{1 - k_j, r_h\}. \end{cases} \quad (8)$$

Firm  $j$  can only realise positive profits if the demand shock is, on the one hand small enough that it can serve the demand of its own costumers, and, on the other hand, large enough that the market demand is positive.

If firm  $j$  sets the same retail price as its rival, then its expected profit depends on the relative capacities of the two firms. If the considered firm  $j$  has a smaller capacity than its rival, the structure of its expected profit is the same as in the undercutting case, except that the firm realises only half of the market demand:

$$\pi_j \mid_{r_j=r_h} = \begin{cases} \int_{\max\{0, r_h-1\}}^{\min\{1, 2k_j+r_h-1\}} \frac{r_h x(r_h, \varepsilon)}{2} d\varepsilon & \text{if } r_h \geq \max\{1 - 2k_j, 0\} \\ 0 & \text{if } 0 \leq r_h < 1 - 2k_j \end{cases} \quad (9)$$

for  $k_j \leq k_h$ . If firm  $j$  has, however, a larger capacity than firm  $h$ , then it correctly anticipates that it can appropriate all its rival's rents in the

wholesale market, if firm  $h$  is not able to serve its own consumers and if firm  $j$ 's surplus of capacity above the demand of its retail customers makes up for firm  $h$ 's deficit. For  $k_j > k_h$  the expected profit is

$$\pi_j \mid_{r_j=r_h} = \begin{cases} \int_{\max\{0, r_h-1\}}^1 \frac{r_h x(r_h, \varepsilon)}{2} d\varepsilon & \text{if } r_h \geq 2 - 2k_h, \\ \int_{\max\{0, r_h-1\}}^{2k_h+r_h-1} \frac{r_h x(r_h, \varepsilon)}{2} d\varepsilon + \\ \int_{2k_h+r_h-1}^{\min\{k_h+k_j+r_h-1, 1\}} r_h x(r_h, \varepsilon) d\varepsilon & \text{if } \max\{1 - 2k_h, 0\} \\ < r_h < 2 - 2k_h, \\ \int_0^{\min\{k_h+k_j+r_h-1, 1\}} r_h x(r_h, \varepsilon) d\varepsilon & \text{if } \max\{1 - k_j - k_h, 0\} \\ < r_h < 1 - 2k_h, \\ 0 & \text{if } 0 \leq r_h < 1 - k_j - k_h. \end{cases} \quad (10)$$

If firm  $j$  sets a higher retail price than its rival, no consumer subscribes to firm  $j$ . It can, however, earn positive revenues when its rival cannot serve the demand of its retail customers at its price  $r_h$  and firm  $j$ 's capacity is large enough to step in. Then firm  $j$  can again appropriate the whole rent via the wholesale auction. Thus, the expected profit is:

$$\bar{\pi}_j(r_j, r_h) = \begin{cases} 0 & \text{if } r_j > r_h \geq 2 - k_h, \\ \int_{\max\{0, r_h-1+k_h\}}^{\min\{1, r_h-1+k_h+k_j\}} r_h x(r_h, \varepsilon) d\varepsilon & \text{if } \max\{1 - k_h - k_j, 0\} \\ < r_h < \min\{r_j, 2 - k_h\}, \\ 0 & \text{if } 0 \leq r_h < \min\{r_j, \\ & 1 - k_h - k_j\}. \end{cases} \quad (11)$$

From the analysis of the firms' profit functions one can derive each firm's best response in retail prices. This is done in appendix D in detail. If the sum of both firms generation capacities is rather large, both firms will always want to undercut each other as in the usual Bertrand competition without any capacity constraints and strategic considerations concerning the wholesale market. The resulting Nash equilibrium is  $r_h = r_j = 0$  and yields zero profits for both firms.

If the aggregate generation capacity in the market is smaller, one can distinguish the symmetric case with  $k_A = k_B$  and the asymmetric case with  $k_A \neq k_B$ . Let us first consider the asymmetric, but not too asymmetric case with  $k_j > k_h$  and  $k_h \geq \frac{k_j-1}{2}$ . Then the best response of firm  $j$  with the larger capacity would be characterised by undercutting as long as  $r_h > \hat{r}$ . It is characterised by  $r_j > r_h$  for  $\max\{0, 1 - 2k_h\} < r_h \leq \hat{r}$ , by  $r_j \geq r_h$  for  $\max\{0, 1 - k_h - k_j\} \leq r_h \leq \max\{0, 1 - 2k_h\}$ , and by indifference for  $0 \leq r_h < 1 - k_h - k_j$ , because firm  $j$ 's profit is then zero no matter which retail price it chooses. Firm  $h$  with the smaller capacity undercuts if  $r_j > r'_j$ , and chooses the same price as the larger provider  $j$  if  $r''_j \leq r_j \leq r'_j$ . It sets  $r_h > r_j$  if  $\max\{0, 1 - k_j - k_h\} \leq r_j < r''_j$  and is indifferent because of zero profits for  $r_j = 0$  or  $0 \leq r_j < 1 - k_j - k_h$ . The critical retail prices  $\hat{r}$ ,  $r'_j$  and  $r''_j$  depend on the two firms' capacity levels and are defined in appendix D. One can show, however, that  $r''_j < \hat{r} < r'_j$  always holds. The Nash equilibrium is again  $r_h = r_j = 0$  for  $k_h + k_j \geq 1$  and  $r_h < 1 - k_h - k_j$  as well as  $r_j < 1 - k_h - k_j$  for  $k_h + k_j < 1$ . In both cases the firms realise zero profits. Now consider the very asymmetric case with  $k_h < \frac{k_j-1}{2}$ . Firm  $j$  can still serve

the whole market for any possible demand shock  $\varepsilon$  at prices where firm  $h$  cannot even serve half of the market when they split the market at  $r_h = r_j$ . Again firm  $h$  undercuts if  $r_j > r'_j$ , it sets  $r_h = r_j$ , if  $1 - 2k_h \leq r_j \leq r'_j$ , and it is indifferent between undercutting, setting  $r_h = r_j$  and  $r_h > r_j$ , if  $2 - k_j \leq r_j < 1 - 2k_h$ , because firm  $h$  would realise zero profits anyway. For  $0 < r_j < 2 - k_j$  firm  $h$  sets  $r_h > r_j$  and can earn positive profits again, because firm  $j$  is no longer able to serve any demand possible. For  $r_j = 0$  firm  $h$  is indifferent between all  $r_h \geq 0$ , because profits are again zero. Firm  $j$ 's best response does not change in principle. The Nash equilibrium with  $r_h = r_j = 0$  still exists, but there are other Nash equilibria where firm  $h$  realises zero and firm  $j$  positive profits. In these equilibria firm  $j$  sets  $r_j \in [2 - k_j, 1 - 2k_h]$ , where firm  $h$  realises zero profits no matter which price it sets, and firm  $h$  undercuts or sets  $r_h > r_j$  if  $r_j = \max\{3/4, 2 - k_j\}$ . The results for asymmetric generation capacities of the two firms are summarised in the following lemma:

**Lemma 4** *If the two firms differ in their generation capacity ( $k_A \neq k_B$ ), then there are no Nash equilibria in retail prices where both firms realise positive profits. The Nash equilibrium in retail prices with  $r_A^* = r_B^* = 0$  always exists. For  $k_A + k_B < 1$  there are also multiple Nash equilibria with  $r_A^* < 1 - k_A - k_B$  and  $r_B^* < 1 - k_A - k_B$ . All these equilibria result in zero profits for both firms.*

*For  $k_h < (k_j - 1)/2$  with  $j, h = A, B$  and  $j \neq h$  there are, in addition, multiple Nash equilibria with  $\max\{r_h^*, 2 - k_j\} \leq r_j^* \leq \max\{1 - 2k_h, 3/4\}$ . If  $k_h < \min\{1/8, (k_j - 1)/2\}$  holds, then there is an additional equilibrium with*



either  $r_j^* = 3/4 < r_h^*$  for  $k_j > 5/4$  or  $r_j^* = 2 - k_j \leq r_h^*$  for  $1 < k_j \leq 5/4$ . The low capacity firm  $h$  realises zero profits in all these additional Nash equilibria and firm  $j$  with the larger capacity:

$$\pi_j(r_j^*, r_h^*) = \min\{r_j^*, r_h^*\} \left( \frac{3}{2} - \min\{r_j^*, r_h^*\} \right) > 0.$$

For  $k_A \neq k_B$  there are no other Nash equilibria in retail prices.

**Proof:** See appendix D . ■

If the two firms are symmetric in their generation capacities ( $k_A = k_B$ ) and the capacities are not too large, both firms undercut, as long as the rivals price exceeds  $\hat{r}$ . When the rival sets its price at  $\hat{r}$ , each firm is indifferent between setting a higher price and the same price as the rival. If the rival's price is below  $\hat{r}$ , but not smaller than  $1 - k_A - k_B$ , then both firms want to set a higher price than their rival. If the rival sets a retail price of zero or a price below  $1 - k_A - k_B$ , then the considered firm is indifferent between undercutting, setting a higher or the same price, because all three options result in zero profits. Thus, the same Nash equilibrium or equilibria exist(s) as in the asymmetric case with zero equilibrium profits. In addition there is a Nash equilibrium with  $r_A = r_B = \hat{r} > \max\{0, 1 - k_A - k_B\}$ , where both firms realise positive profits in equilibrium. Our results concerning the competition in retail prices with symmetric generation capacities are summarised in lemma 5.

**Lemma 5** *If the two firms have the same generation capacity  $k_A = k_B = k$  then there is always a Nash equilibrium in retail prices with  $r_A = r_B = 0$ .*

For  $k < 1/2$  there are also multiple Nash equilibria with  $r_A < 1 - 2k$  and  $r_B < 1 - 2k$ . All these equilibria result in zero profits for both firms. For  $k < \sqrt{5/2}$ , there is an additional Nash equilibrium with

$$r_A^* = r_B^* = \begin{cases} 2 - \sqrt{2}k & \text{if } 0 \leq k < \frac{1}{\sqrt{2}}, \\ \frac{1}{2} (3 - \sqrt{4k^2 - 1}) & \text{if } \frac{1}{\sqrt{2}} \leq k < \sqrt{\frac{5}{2}}, \end{cases}$$

where the expected equilibrium profits are

$$\pi_A(r_A^*, r_B^*) = \pi_B(r_A^*, r_B^*) = \begin{cases} k^2 \left(1 - \frac{k}{\sqrt{2}}\right) & \text{if } 0 \leq k < \frac{1}{\sqrt{2}}, \\ \frac{1}{8} (1 - 4k^2 + 3\sqrt{4k^2 - 1}) & \text{if } \frac{1}{\sqrt{2}} \leq k < \sqrt{\frac{5}{2}}. \end{cases}$$

**Proof:** See appendix D . ■

In the following section we assume that the two firms select for given capacities the equilibrium that Pareto dominates all the other possible equilibria in retail prices. Which equilibrium is Pareto dominant for which combination of capacities is illustrated in figure 3. If both firms choose symmetric capacities with  $k_A = k_B = k < \sqrt{5/2}$  (see the straight line in figure 3 which cuts into  $\mathcal{A}$  and  $\mathcal{B}$ ), the Pareto dominant Nash equilibrium results in the retail prices  $r_A^* = r_B^* > 0$  given in Lemma 5. With modestly asymmetric capacities (area  $\mathcal{A}$  in figure 3) the unique Nash equilibrium in retail prices is  $r_h = r_j = 0$ . With asymmetric capacities and  $k_h < \max\{(k_j - 1)/2, 1/8\}$  (area  $\mathcal{C}$  in figure 3) or  $k_j < \max\{(k_h - 1)/2, 1/8\}$  (area  $\mathcal{C}'$  in figure 3) the Pareto dominant

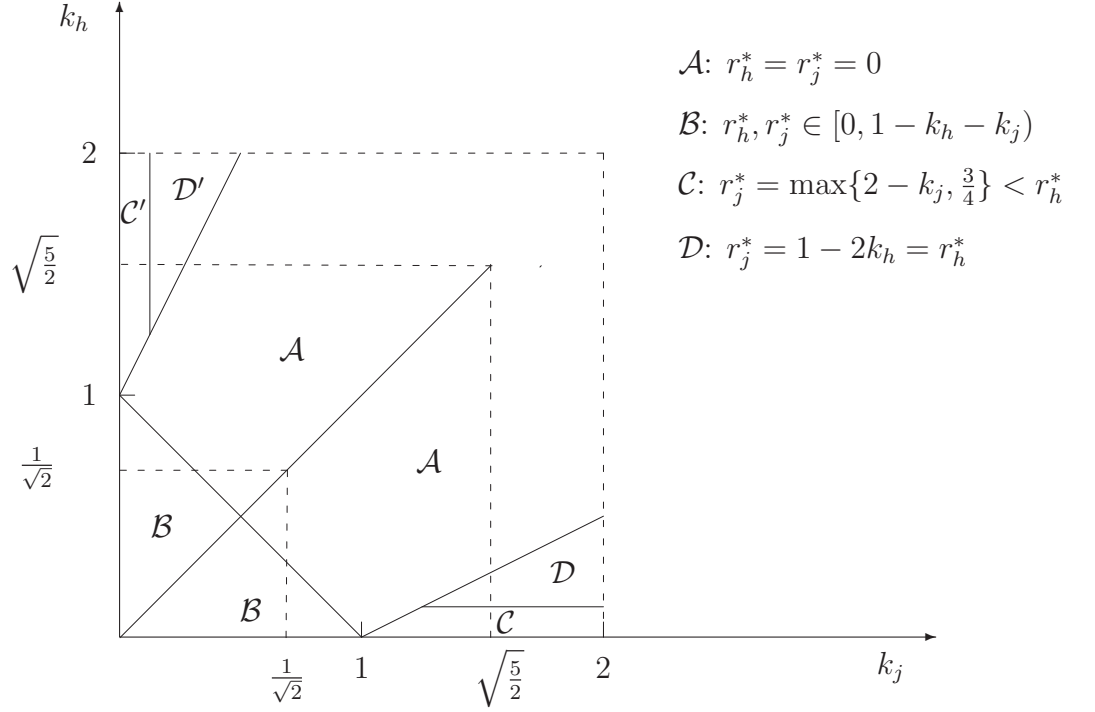


Figure 3: Pareto Dominant Nash Equilibria in Retail Prices

Nash equilibrium is characterised by  $r_j^* = \max\{3/4, 2 - k_j\} < r_h^*$  in the first case and by  $r_h^* = \max\{3/4, 2 - k_h\} < r_j^*$  in the second. It implies that the firm with the large capacity chooses the monopoly retail price without being undercut by the low capacity firm. If the two firms' capacities satisfy  $1/8 \leq k_h < (k_j - 1)/2$  or  $1/8 \leq k_j < (k_h - 1)/2$  (area  $\mathcal{D}$  and  $\mathcal{D}'$  in figure 3), then the firms set identical retail prices  $r_h^* = r_j^* = 1 - 2k_h$  in the Pareto dominant Nash equilibrium, but the low capacity firm can never serve its customers and therefore realizes zero profits. For capacities which satisfy  $k_j + k_h < 1$  (area  $\mathcal{B}$ ) all the equilibria imply always a black-out. They are pay-off equivalent because both firms realise zero profits.

### 4.3 The Firms' Investments in Generation Capacity

The firms anticipate the resulting retail prices and the prices on the wholesale market when they decide on their generation capacities. For a very low capacity of its rival, a firm can either choose a very large capacity and ensure itself monopoly revenues, or at least restricted monopoly revenues, or it can choose the same generation capacity as its rival in order to generate positive revenues. If the firm chooses a smaller or only a modestly larger capacity than its rival, it cannot earn positive profits. Firm  $j$ 's profit is:

$$\Pi_j(k_j, k_h) = \begin{cases} -zk_j & \text{if } k_j < k_h, \\ k_j^2 \left(1 - \frac{k_j}{\sqrt{2}}\right) - zk_j & \text{if } k_j = k_h, \\ -zk_j & \text{if } k_h < k_j \leq 2k_h + 1, \\ (2 - k_j) \left(k_j - \frac{1}{2}\right) - zk_j & \text{if } 2k_h + 1 < k_j \leq \frac{5}{4}, \\ \frac{9}{16} - zk_j & \text{if } k_j > \frac{5}{4}, \end{cases} \quad (12)$$

for  $0 \leq k_h < \frac{1}{8}$ . If the rival's capacity is larger, but still small, the firm can no longer earn monopoly revenues, but for some small levels of  $k_h$ , still positive revenues, if it invests in a very much larger capacity than its rival. Its profit

is:

$$\Pi_j(k_j, k_h) = \begin{cases} -zk_j & \text{if } k_j < k_h, \\ k_j^2 \left(1 - \frac{k_j}{\sqrt{2}}\right) - zk_j & \text{if } k_j = k_h, \\ -zk_j & \text{if } k_h < k_j \leq 2k_h + 1, \\ \max \left\{ (1 - 2k_h) \left(\frac{1}{2} + 2k_h\right), 0 \right\} - zk_j & \text{if } 2k_h + 1 < k_j, \end{cases} \quad (13)$$

for  $\frac{1}{8} \leq k_h < \frac{1}{\sqrt{2}}$ . If the rival chooses intermediate levels of capacities, firm  $j$ 's revenues from investing in the same level of capacity changes and monopolization is no option any more. Firm  $j$ 's profit is:

$$\Pi_j(k_j, k_h) = \begin{cases} -zk_j & \text{if } k_j < k_h, \\ \frac{1}{8} \left(1 - 4k_j^2 + 3\sqrt{4k_j^2 - 1}\right) - zk_j & \text{if } k_j = k_h, \\ -zk_j & \text{if } k_h < k_j, \end{cases} \quad (14)$$

for  $\frac{1}{\sqrt{2}} \leq k_h < \sqrt{\frac{5}{2}}$ . If the firm's rival chooses very large capacities, then firm  $j$  can no longer earn positive revenues independent of its own capacity level. Firm  $j$ 's profit is:

$$\Pi_j(k_j, k_h) = -zk_j \quad (15)$$

for  $k_h > \sqrt{\frac{5}{2}}$ . From these profit functions one can derive each firm's best response function in generation capacity and the resulting subgame perfect Nash equilibria. This is done in detail in appendix E. Since both firms are symmetric, the best response functions for the two firms are also symmetric

and depend on the level of the capacity costs  $z$ . For very low levels of the rivals capacity  $k_h$  firm  $j$  can always monopolize the market by choosing the same capacity as the monopolist. This is always the best response as long as the capacity cost is low enough to ensure a positive monopoly profit. If the rival's capacity increases, then firm  $j$  can still monopolize the market, but must increase its own capacity beyond the optimal monopoly level. Thus, firm  $j$ 's profit from monopolization decreases when the rival's capacity increases. At a certain threshold of the rival's capacity  $k_h$  it is optimal for firm  $j$  to drastically reduce its capacity and to switch either to a symmetric capacity choice, where both firms share the market, or to zero capacity when monopolization as well as sharing yields negative profits, depending on the level of the capacity costs  $z$ . For very large capacities of the rival it is always optimal for firm  $j$  to install no capacity. The best responses of the two firms and the resulting equilibria are illustrated in figure 4 for different levels of capacity costs .

With low capacity costs there are multiple subgame perfect Nash equilibria in which the firms choose identical capacities and share the market. For intermediate levels of capacity costs there are still multiple Nash equilibria where the firms choose identical capacities. In addition there are, however, two equilibria where one of the two firms chooses the monopoly capacity and the other does not invest. For high levels of capacity costs the sharing equilibria vanish. Only the two subgame perfect Nash equilibria, where one firm monopolizes the market, do still exist. They disappear when the capacity costs are so high that even a monopolist can not generate positive profits in expectation. Our results concerning the investments of the two competing

firms in generation capacities are summarised in proposition 3, where we use

$$\underline{k} \equiv \frac{1}{\sqrt{2}} \left( 1 - \sqrt{1 - 2z\sqrt{2}} \right), \quad (16)$$

which defines the smallest capacity that ensures zero profit, when both firms choose the same capacity,

$$\bar{k} \equiv \left\{ k \in \left[ \frac{1}{\sqrt{2}}, \sqrt{\frac{5}{2}} \right] \mid \frac{1 - 4k^2 + 3\sqrt{4k^2 - 1}}{8} - zk = 0 \right\}, \quad (17)$$

which defines the largest capacity that ensures zero profit, when both firms choose the same capacity, and

$$\begin{aligned} \tilde{k} \equiv \left\{ k \in \left[ 0, \frac{1}{2} \right] \mid k^2 \left( 1 - \frac{k}{\sqrt{2}} \right) - zk = \right. \\ \left. (1 - 2k) \left( \frac{1}{2} + 2k \right) - z(2k + 1) \right\}, \end{aligned} \quad (18)$$

which defines the capacity where the necessary capacity to monopolize is as profitable as choosing an identical capacity as one's rival.

**Proposition 3** *In the subgame perfect Nash equilibria the two competing firms choose identical generation capacities with*

$$k_A^* = k_B^* = k_d^* \in [\tilde{k}, \bar{k}], \text{ if the capacity costs satisfy } 0 \leq z < 0.2118.$$

*They choose either identical generation capacities with*

$$k_A^* = k_B^* = k_d^* \in [\min\{\tilde{k}, \underline{k}\}, \bar{k}] \text{ or asymmetric capacities with}$$

$$k_j^* = k_m^* \text{ and } k_h^* = 0, \quad j, h = A, B, \quad j \neq h, \text{ if } 0.2118 \leq z < \frac{1}{2\sqrt{2}},$$

where  $k_m^*$  is the monopoly capacity defined in proposition 1.

The firms choose only asymmetric capacities with

$$k_j^* = k_m^* \text{ and } k_h^* = 0, \quad j, h = A, B, \quad j \neq h \text{ if } \frac{1}{2\sqrt{2}} \leq z < \frac{1}{2}.$$

For  $z \geq \frac{1}{2}$  both firms do not invest in generation capacity in equilibrium.

The retail price in the equilibria with identical positive capacities is

$$r_d^* = \begin{cases} 2 - \sqrt{2}k_d^* & \text{if } \min\{\tilde{k}, \underline{k}\} \leq k_d^* < \frac{1}{\sqrt{2}}, \\ \frac{1}{2} \left( 3 - \sqrt{4(k_d^*)^2 - 1} \right) & \text{if } \frac{1}{\sqrt{2}} \leq k_d^* < \bar{k}. \end{cases}$$

Whereas the retail price coincides with the monopoly price  $r_m^*$ , given in proposition 1, if the firms choose asymmetric capacities in equilibrium such that one firm monopolizes the market.

**Proof:** See Appendix E. ■

Note that regardless of whether the firms play a duopoly equilibrium with  $k_A = k_B \in [\min\{\tilde{k}, \underline{k}\}, \bar{k}]$  or a monopoly equilibrium, no black-outs occur independent of the demand shock  $\varepsilon$ .

In the following we focus mainly on subgame perfect equilibria, which are not Pareto dominated by other equilibria. By comparing each firm's profits in the different equilibria described in proposition 3 one can conclude:

**Corollary 1** *If the capacity cost satisfies  $0 \leq z < 0.2118$ , then there is a unique Pareto dominant subgame perfect Nash equilibrium where both firms*



choose  $\hat{k}$ , which is defined in (19). For  $0.2118 \leq z < \frac{1}{2\sqrt{2}}$  the subgame perfect equilibria that are not Pareto dominated are either characterised by both firms choosing  $\hat{k}$  or by one firm choosing  $k_m^*$  and the other installing zero capacities. If  $\frac{1}{2\sqrt{2}} \leq z < \frac{1}{2}$  holds, then there are two equilibria, which are not Pareto dominated, with one firm choosing  $k_m^*$  and the other installing zero capacities. For  $z \geq \frac{1}{2}$  there exists only one equilibrium where both firms do not invest in capacities.

The capacity  $\hat{k}$  is the capacity that maximises each firm's profit, if both firms choose the same level of capacity:

$$\hat{k} = \operatorname{argmax}_{k \in [\frac{1}{\sqrt{2}}, \bar{k}]} \left\{ \frac{1 - 4k^2 + 3\sqrt{4k^2 - 1}}{8} - zk \right\}. \quad (19)$$

For intermediate levels of capacity costs the equilibrium with  $k_A = k_B = \hat{k}$  Pareto dominates all the other equilibria with  $k_A = k_B \in [\min\{\tilde{k}, \underline{k}\}, \bar{k}]$ , but not the two monopoly equilibria with  $k_A = k_m^*$  and  $k_B = 0$  and  $k_B = k_m^*$  and  $k_A = 0$ . There the monopolist realises always a higher profit than each single firm in the duopoly equilibrium whereas the firm without any generation capacity has zero profits instead of positive profits in the duopoly equilibrium with  $k_A = k_B = \hat{k}$ . Thus, even if we consider only Pareto dominant subgame perfect Nash equilibria, uniqueness cannot be achieved.

## 5 Comparison of the Two Market Structures

In Figure 5 the total capacities in the monopoly and duopoly cases as well as the socially optimal capacity are outlined. The limits of the total capacities

in the competitive equilibria, as well as the total capacity in the Pareto dominant competitive equilibrium and the total capacity in the monopolistic equilibrium in the duopoly case are painted in blue. The capacity installed in the monopoly case is characterised by a red line and the socially optimal capacity is drawn in green. It is obvious that together the duopolists might invest more or less than the monopolist or the social planner. If we assume, however, that the two firms select a subgame perfect equilibrium that is not Pareto dominated then we arrive at proposition 4.

**Proposition 4** *If the two firms in the duopoly case always select a subgame perfect equilibrium, which is not Pareto dominated by an other one, then the total capacity in the duopoly case exceeds the installed capacity of a monopolist for capacity costs that satisfy  $0 \leq z < 0.2118$ . It is the same or larger for  $0.2118 \leq z < \frac{1}{2\sqrt{2}}$ , and it is the same for  $z \geq \frac{1}{2\sqrt{2}}$ . If it is positive, it is still inefficiently low from a social welfare perspective.*

**Proof:** The statement follows from  $k_s^{**} > 2\hat{k} > k_m^*$  for all  $0 \leq z < \frac{1}{2\sqrt{2}}$  and from  $k_s^{**} > k_m^*$  for all  $\frac{1}{2\sqrt{2}} \leq z < 2 - \sqrt[5]{3}$ . ■

Proposition 4 seems to confirm common wisdom that oligopolistic firms want to produce more than a monopolist and would, therefore, also install more capacity. If we compare, however, the prices in equilibrium for those subgame perfect Nash equilibria that are not Pareto dominated, we derive proposition 5.

**Proposition 5** *If the two firms in the duopoly case select a subgame perfect equilibrium that is not Pareto dominated by an other equilibrium, then the*

retail price in the duopoly case exceeds the retail price of the monopolist for capacity costs that satisfy  $0 \leq z < 0.2118$ . The retail price is the same or higher for  $0.2118 \leq z < \frac{1}{2\sqrt{2}}$ , and it is the same for  $z \geq \frac{1}{2\sqrt{2}}$ . Monopoly and duopoly retail prices exceed the retail price in the social optimum for all capacity costs  $z$ .

**Proof:** The statement follows from substituting  $\hat{k}$  into  $r_d^*$  from proposition 3 and comparing it with  $r_m^*$  from proposition 1 and  $r_s^{**}$  from proposition 2. ■

Proposition 5 contradicts the common view that oligopolistic firms want to produce more and would therefore install larger capacities. Since the two firms would set a higher retail price than a monopolist, consumers would always consume less under the duopoly than in the monopolistic case. Thus, the higher level of installed capacities in the competitive equilibrium are mainly a consequence of strategic considerations. Larger capacities and a higher retail price ensure that a firm has a higher chance to serve the demand of its own retail customers. With a small capacity and low retail prices a firm risks losing all its profits in those cases where it can not meet the demand of its retail customers.

As long as capacities are positive in the market equilibrium consumers can always realise their consumption because black-outs do neither occur in the duopoly nor in the monopoly case for any possible demand shock. Thus, social welfare for both cases is given by:

$$W = \int_0^1 U(x(r, \varepsilon), \varepsilon) d\varepsilon - zk, \quad (20)$$

where  $U(x(r, \varepsilon), \varepsilon)$  is defined in equation (1). The higher prices that result in lower consumption levels, and the larger capacities, which only increase the capacity costs without creating any extra gain from a more secure supply in the duopoly case, do explain proposition 6.

**Proposition 6** *If the two firms in the duopoly case select always a subgame perfect equilibrium that is not Pareto dominated by any other equilibrium, then the social welfare in the duopoly case is smaller than in the case with a monopolist for capacity costs that satisfy  $0 \leq z < 0.2118$ . Social welfare is the same or smaller for  $0.2118 \leq z < \frac{1}{2\sqrt{2}}$ , and it is the same for  $z \geq \frac{1}{2\sqrt{2}}$ . The achieved social welfare in the monopoly case is for all  $0 \leq z < 2 - \sqrt{5/3}$  sub-optimal.*

**Proof:** The statement follows from substituting the relevant consumption levels  $x(r, \varepsilon)$  into the social welfare function (20) and from comparing the social welfare achieved in a monopoly with the one in the duopoly and in the socially optimal equilibrium. ■

If we take into account all possible subgame perfect equilibria in the duopoly case, retail prices can also be lower than in the monopoly case. It turns out, however, that social welfare is nearly always lower in the duopoly case than in the monopoly. The reason is that in those equilibria where the retail prices are lower, the duopoly capacities are so much higher that capacity costs outweigh the gains of the consumers from higher consumption levels.

## 6 Conclusions

Given the multiplicity of subgame perfect Nash equilibria in the duopoly case, it is not easy to derive clear cut conclusions from the analysis here. If we focus on subgame perfect Nash equilibria which are not Pareto dominated by other equilibria, then it is not a problem that competitive firms do not invest enough in generating capacity. Their investments together usually exceed the capacity installed by a monopolist. Their retail price is, however, too high which leads to a lower social welfare than realised in the monopoly case.

Thus, in the case analysed here where consumers cannot respond directly to electricity prices, but are guaranteed a certain retail price before an uncertain demand is realised, competition is bad for social welfare. This contrasts with the results in a companion paper (Boom, 2002), where consumers can directly respond to electricity price changes and can therefore take part in the electricity auction, and in a follow-up paper (Boom and Buehler, 2007), where generating firms are vertically separated from the retail firms. In both settings competition is always beneficial for social welfare.

What drives our results here is the retail price competition where prices turn out to be relatively high for the installed capacities. A large installed capacity, as well as a high retail price makes it less probable that a firm is not able to satisfy the demand of its retail customers, thus, losing all its rents during the auction.

Our analysis supports the suspicion of the European Union against electricity

generating firms that are vertically integrated.<sup>19</sup> In order to judge, however, whether the European Union's suspicion is indeed justified because of the strategic effect identified in our model, it is necessary to check the robustness of this effect under different sets of assumptions. It would be interesting, for example, to consider the same type of model if in case of an unsatisfied own retail demand the loss would be less drastic. This can be expected if there would be either more competitors or if there would be a lower regulated price cap in the auction which would punish the competitor, who is not able to meet his contracted demand, less. The analysis of a model with more competitors and a lower price cap are left to further research.

## Appendix

### A The Monopolist's Price and Capacity Choice

After integration the monopolist's profit function is

$$\pi_m(r, k) = \begin{cases} \frac{r}{2}(2-r)^2 - zk & \text{if } r > 1, \\ r\left(\frac{3}{2} - r\right) - zk & \text{if } 2 - k \leq r \leq 1, \\ \frac{r}{2}(2r - r^2 + k^2 - 1) - zk & \text{if } 0 \leq r < 2 - k, \end{cases}$$

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<sup>19</sup>The European Union ruled in its Directive 2003/54/EC concerning common rules for the internal market in electricity adopted on 26 June 2003 that electricity generating firms which are also integrated into the transmission and distribution of electricity have to be functionally disintegrated.

for  $k \geq 1$  and

$$\pi_m(r, k) = \begin{cases} \frac{r}{2}(2-r)^2 - zk & \text{if } r \geq 2-k, \\ r\frac{k^2}{2} - zk & \text{if } 1 \leq r < 2-k, \\ \frac{r}{2}(2r-r^2+k^2-1) - zk & \text{if } 1-k \leq r < 1, \\ -zk & \text{if } 0 \leq r < 1-k, \end{cases}$$

for  $0 \leq k < 1$ . Differentiating  $\pi_m(r, k)$  with respect to  $r$  yields that  $\pi_m(r, k)$  has a maximum at  $r^*(k) = \frac{3}{4}$ , as long as  $k \geq \frac{5}{4}$ , and at  $r^*(k) = 2-k$  for  $0 \leq k < \frac{5}{4}$ . Substituting these retail prices into  $\pi_m(r, k)$  yields:

$$\Pi_m(k) = \begin{cases} \frac{9}{16} - zk & \text{if } k \geq \frac{5}{4}, \\ \frac{5k}{2} - k^2 - 1 - zk & \text{if } 1 \leq k < \frac{5}{4}, \\ \frac{1}{2}k^2(2-k) - zk & \text{if } 0 \leq k < 1. \end{cases}$$

Differentiating  $\Pi_m(k)$  with respect to  $k$  yields that  $\Pi_m(k)$  has a maximum at  $k^*(z) = \frac{5}{4} - \frac{z}{2}$  for  $0 \leq z \leq \frac{1}{2}$  and at  $k^*(z) = \frac{1}{6}(4 + \sqrt{16 - 24z})$  for  $z > \frac{1}{2}$ . The maximised profit,  $\Pi_m(k^*(z))$ , is continuous and monotonously decreasing in  $z$ . It becomes negative at  $z = \frac{1}{2}$ .

## B The Social Planner's Price and Capacity Choice

After substituting  $U(x, \varepsilon)$  from (1) and  $x(r, \varepsilon)$  from (2) in the social welfare function (4) and integrating it, we obtain

$$W(r, k) = \begin{cases} \frac{2-(3-r)r^2}{3} - zk & \text{if } r \geq 2-k, \\ \frac{k^3+3rk^2-4}{6} - zk & \text{if } 2-k > r \geq 1, \\ \frac{k^3+3rk^2+(3-2r)r^2-5}{6} - zk & \text{if } 1 > r \geq 1-k, \\ -\frac{2}{3} - zk & \text{if } 1-k > r \geq 0, \end{cases} \quad (21)$$

for  $0 \leq k < 1$  and

$$W(r, k) = \begin{cases} \frac{2-(3-r)r^2}{3} - zk & \text{if } r \geq 1, \\ \frac{1-r^2}{2} - zk & \text{if } 1 > r \geq 2-k, \\ \frac{k^3+3rk^2+(3-2r)r^2-5}{6} - zk & \text{if } 2-k > r \geq 0, \end{cases} \quad (22)$$

for  $k \geq 1$ . Maximizing the social welfare with respect to the retail price results in the socially optimal retail price

$$r^{**}(k) = \max\{2 - k, 0\}.$$

Substituting the retail price in (21) and (22) yields

$$\mathcal{W}(k) = \begin{cases} \frac{1}{2} - zk & \text{if } k > 2, \\ \frac{1-(2-k)^2}{2} - zk & \text{if } 1 \leq k < 2, \\ \frac{2-(1+k)(2-k)^2}{3} - zk & \text{if } 0 \leq k < 1, \end{cases} \quad (23)$$

as the social welfare in terms of the capacity, given socially optimal retail prices. Differentiating  $\mathcal{W}(k)$  with respect to  $k$  shows that  $\mathcal{W}(k)$  has a maximum at  $k_s^{**} = 2 - z$ . The maximized welfare function  $\mathcal{W}(2 - z)$  is  $U$ -shaped in  $z$  and results in  $\mathcal{W}(2 - z) < \mathcal{W}(0) = -2/3$  for  $2 - \sqrt{5/3} < z < 2$ . For  $z \geq 2$  the social planner can never do better than choosing  $k = 0$ .

## C The Nash Equilibrium in Price Bids on the Wholesale Market.

Here it is assumed that total capacities are sufficient to satisfy the market demand ( $k_A + k_B \geq d(r_A, r_B, \varepsilon)$ ). Then, taking into account (7) each firm's profit function is given by:

$$f_j(p_j, p_h, r_j, r_h, \varepsilon) = \begin{cases} r_j d_j(r_j, r_h, \varepsilon) + p(p_j, p_h, \varepsilon) \\ \quad \cdot (\min\{k_j, d(r_j, r_h, \varepsilon)\} - d_j(r_j, r_h, \varepsilon)) & \text{if } p_j < p_h, \\ \\ r_j d_j(r_j, r_h, \varepsilon) + \frac{p(p_j, p_h, \varepsilon)}{2} \\ \quad \cdot (\min\{k_j, d(r_j, r_h, \varepsilon)\} - d_j(r_j, r_h, \varepsilon)) \\ \quad + \frac{p(p_j, p_h, \varepsilon)}{2} (\max\{d(r_j, r_h, \varepsilon) - k_h, 0\} \\ \quad - d_j(r_j, r_h, \varepsilon)) & \text{if } p_j = p_h, \\ \\ r_j d_j(r_j, r_h, \varepsilon) + p(p_j, p_h, \varepsilon) \\ \quad \cdot (\max\{d(r_j, r_h, \varepsilon) - k_h, 0\} - d_j(r_j, r_h, \varepsilon)) & \text{if } p_j > p_h. \end{cases}$$

If both firms are able to serve the demand of their retail customers, meaning  $k_j \geq d_j(r_j, r_h, \varepsilon)$  with  $j = A, B$ , then both firms want to undercut their rival because by doing so they avoid being a net payer in the auction, but become



a net receiver of payments. Therefore  $p_j = p_h = 0 = p(p_j, p_h, \varepsilon)$  is the unique Nash equilibrium. Substituting this into  $f_j(p_j, p_h, r_j, r_h, \varepsilon)$  yields  $f_j(r_j, r_h, \varepsilon)$  from lemma 3 for  $k_j \geq d_j(r_j, r_h, \varepsilon)$  with  $j = A, B$ .

Suppose now that only firm  $h$  can meet the demand of its retail customers ( $k_h \geq d_h(r_j, r_h)$ ), whereas firm  $j$  cannot ( $k_j < d_j(r_j, r_h)$ ). Then firm  $j$  is always a net payer and firm  $h$  a net receiver in the auction. Firm  $j$  minimizes its payments by always undercutting firm  $h$ . If firm  $h$  sets  $p_h > p_j$ , the auction price would be  $p(p_j, p_h, \varepsilon) = p_h$  and its optimal price bid would be  $p_h = \bar{p}(r_j, r_h, \varepsilon)$ , given in lemma 3, which ensures zero profits for firm  $j$  and the profit  $f_h(r_j, r_h, \varepsilon) = r_j d_j(r_j, r_h, \varepsilon) + r_h d_h(r_j, r_h, \varepsilon)$  for firm  $h$ . The optimal price bid from below would either be indeterminate, if both capacities were needed to satisfy market demand ( $k_h < d(r_j, r_h, \varepsilon)$ ), or it would be  $p_h = p_j - \mu$  with  $\mu \rightarrow 0$  if  $k_h \geq d(r_j, r_h, \varepsilon)$ . The auction price would in both cases satisfy  $p(p_j, p_h, \varepsilon) = p_j$  and firm  $h$ 's profit would be  $f_h(p_j, p_h, r_j, r_h, \varepsilon) = r_h d_h(r_j, r_h, \varepsilon) + p_j (\min\{k_h, d(r_j, r_h, \varepsilon)\} - d_h(r_j, r_h, \varepsilon))$ . Firm  $h$  prefers undercutting firm  $j$ 's price as long as  $p_j > \hat{p}(r_j, r_h, \varepsilon)$  holds, and  $p_h = \bar{p}(r_j, r_h, \varepsilon)$  otherwise. Thus, there are multiple Nash equilibria with  $p_j \leq \hat{p}(r_j, r_h, \varepsilon)$  and  $p_h = \bar{p}(r_j, r_h, \varepsilon)$ , and a unique auction price  $p(p_j, p_h, \varepsilon) = \bar{p}(r_j, r_h, \varepsilon)$ . Substituting the auction price into the two firms' profit functions yields  $f_j(r_j, r_h, \varepsilon)$  and  $f_h(r_j, r_h, \varepsilon)$  from lemma 3 for  $k_j < d_j(r_j, r_h)$ .

## D The Nash Equilibrium in Retail Prices.

If firm  $j$  undercuts its rival's retail price, its best response is then

$$\underline{r}_j(r_h) = \begin{cases} \max\{2 - k_j, \frac{3}{4}\} & \text{if } r_h > \max\{2 - k_j, \frac{3}{4}\} \\ r_h - \mu & \text{if } 0 \leq r_h \leq \max\{2 - k_j, \frac{3}{4}\} \end{cases} \quad (24)$$

with  $\mu \rightarrow 0$  being the smallest unit in which retail prices can be announced. If firm  $j$  sets  $r_j > r_h$ , then it is indifferent between all prices that satisfy this restriction, because its profit, given in (11), does not depend on the level of  $r_j$ . Therefore the overall best response is determined by the comparison of  $\underline{\pi}_j(\underline{r}_j(r_h), r_h)$ , derived from (8) and (24), with  $\pi_j(r_j, r_h)|_{r_j=r_h}$  from (9) or (10), respectively, and with  $\bar{\pi}_j(r_j, r_h)$  defined in (11).

Suppose that  $k_j > k_h$ . Then the overall best response of firm  $j$  for  $k_j > k_h \geq \sqrt{5 - k_j^2}$  is given by  $r_j(r_h) = \underline{r}_j(r_h)$  from (24). For  $\min\{k_j, \sqrt{5 - k_j^2}\} >$

$k_h \geq 0$  the overall best response is

$$r_j(r_h) \begin{cases} = \max \left\{ 2 - k_j, \frac{3}{4} \right\} & \text{if } r_h > \max \left\{ 2 - k_j, \frac{3}{4} \right\}, \\ = r_h - \mu & \text{if } \hat{r} < r_h \leq \max \left\{ 2 - k_j, \frac{3}{4} \right\}, \\ > r_h & \text{if } \max\{0, 1 - 2k_h\} < r_h \leq \min \left\{ \hat{r}, \right. \\ & \quad \left. \max \left\{ 2 - k_j, \frac{3}{4} \right\} \right\}, \\ \geq r_h & \text{if } \max\{0, 1 - k_j - k_h\} < r_h \leq \\ & \quad \min \left\{ 1 - 2k_h, \max \left\{ 2 - k_j, \frac{3}{4} \right\} \right\}, \\ \geq 0 & \text{if } 0 \leq r_h \leq \max\{0, 1 - k_j - k_h\}. \end{cases} \quad (25)$$

with

$$\hat{r} = \begin{cases} \frac{3 - \sqrt{2(k_h^2 + k_j^2) - 1}}{2} & \text{if } k_j \geq k_h > \min\{\sqrt{1 - k_j^2}, k_j - 1\}, \\ 1 - k_h & \text{if } 0 \leq k_h \leq k_j - 1, \\ 2 - \sqrt{k_h^2 + k_j^2} & \text{if } 0 \leq k_h \leq \min\{\sqrt{1 - k_j^2}, k_j\}. \end{cases} \quad (26)$$

Firm h's best response in retail prices is  $r_h(r_j) = \underline{r}_j(r_h)$  for  $k_j > k_h \geq \sqrt{\frac{5}{2}}$ .

For  $\min\{k_j, \sqrt{\frac{5}{2}}\} > k_h \geq \max\left\{0, \frac{k_j - 1}{2}\right\}$  firm h's best response in retail prices is given by

$$r_h(r_j) \begin{cases} = \max \left\{ 2 - k_h, \frac{3}{4} \right\} & \text{if } r_j > \max \left\{ 2 - k_h, \frac{3}{4} \right\}, \\ = r_j - \mu & \text{if } \max \left\{ 2 - k_h, \frac{3}{4} \right\} \geq r_j > r'_j, \\ = r_j & \text{if } r'_j \geq r_j > r''_j, \\ > r_j & \text{if } r''_j \geq r_j > \max\{0, 1 - k_j - k_h\}, \\ \geq 0 & \text{if } \max\{0, 1 - k_j - k_h\} \geq r_j \geq 0, \end{cases} \quad (27)$$

where the critical price  $r'_j$  for the rival with the larger capacity is defined as:

$$r'_j = \max \left\{ \frac{3 - \sqrt{4k_h^2 - 1}}{2}, 2 - \sqrt{2}k_h \right\}, \quad (28)$$

and the critical price  $r_j''$  as:

$$r_j'' = \max \left\{ \frac{3 - \sqrt{4k_j^2 - 1}}{2}, \frac{5 - \sqrt{12k_h^2 + 6k_j^2 - 2}}{3}, 0 \right\} \quad (29)$$

for  $k_j > 1$  and

$$r_j'' = \begin{cases} \max \left\{ \frac{3 - \sqrt{4k_j^2 - 1}}{2}, \frac{5 - \sqrt{12k_h^2 + 6k_j^2 - 2}}{3} \right\} & \text{if } k_j \geq k_h \geq \sqrt{\frac{1 - k_j^2}{2}}, \\ 2 - \sqrt{2k_h^2 + k_j^2} & \text{if } \sqrt{\frac{1 - k_j^2}{2}} > k_h \geq 0 \end{cases} \quad (30)$$

for  $\frac{1}{\sqrt{2}} < k_j \leq 1$  and

$$r_j'' = \max \left\{ 2 - \sqrt{2}k_j, 2 - \sqrt{2k_h^2 + k_j^2} \right\} \quad (31)$$

for  $0 \leq k_j < \frac{1}{\sqrt{2}}$ . It can be shown that  $r_j' > \hat{r} > r_j''$  holds for  $k_j > k_h \geq \frac{k_j - 1}{2}$ . Thus,  $r_h = r_j = 0$  is the only Nash equilibrium in retail prices for  $k_j > \max\{1 - k_h, k_h\}$  and  $k_h \geq \frac{k_j - 1}{2}$ . There are multiple Nash equilibria with  $r_j < 1 - k_j - k_h$  and  $r_h < 1 - k_j - k_h$  for  $1 - k_h \geq k_j > k_h \geq \frac{k_j - 1}{2}$ .

For  $\frac{k_j - 1}{2} > k_h > 0$  firm h's best response in retail prices is

$$r_h(r_j) = \begin{cases} = 2 - k_h & \text{if } r_j > 2 - k_h, \\ = r_j - \mu & \text{if } 2 - k_h \geq r_j > r_j', \\ = r_j & \text{if } r_j' \geq r_j \geq 1 - 2k_h, \\ \geq 0 & \text{if } 1 - 2k_h > r_j \geq 2 - k_j, \\ > r_j & \text{if } 2 - k_j > r_j > 0, \\ \geq 0 & \text{if } r_j = 0, \end{cases} \quad (32)$$

where  $r_j'$  is defined in (28). Thus,  $r_j \in [2 - k_j, \min\{1 - 2k_h, 3/4\}]$  and  $r_h \leq r_j$  is always an equilibrium. For  $k_h < \max\left\{\frac{k_j - 1}{2}, \frac{1}{8}\right\}$  an additional equilibrium exists with  $r_j = \max\{2 - k_j, 3/4\} < r_h$ .

Now suppose that  $k_A = k_B = k$ , then each firm has the same best response in retail prices. If  $k > \sqrt{\frac{5}{2}}$ , then both firms' best response is given by  $\underline{r}_j(r_h)$  from equation (24) and the only possible Nash equilibrium is  $r_A = r_B = 0$ . If  $0 \leq k < \sqrt{\frac{5}{2}}$ , then each firm's best response in retail prices is given by:

$$r_j(r_h) = \begin{cases} = \max \left\{ 2 - k, \frac{3}{4} \right\} & \text{if } r_h > \max \left\{ 2 - k, \frac{3}{4} \right\}, \\ = r_h - \mu & \text{if } \max \left\{ 2 - k, \frac{3}{4} \right\} \geq r_h > \hat{r} \\ \geq r_h & \text{if } r_h = \hat{r} \\ > r_h & \text{if } \hat{r} > r_h > \max\{0, 1 - 2k\}, \\ \geq r_h & \text{if } \max\{0, 1 - 2k\} \geq r_h \geq 0, \end{cases} \quad (33)$$

with  $\hat{r}$  from (26) with  $k_j = k_h = k$ . Then again there is always a Nash equilibrium with  $r_A = r_B = 0$ . If  $k < 1/2$ , then there are also multiple Nash equilibria with  $r_A < 1 - 2k$  and  $r_B < 1 - 2k$ , which result as well in zero profits for both firms. In addition there is always a Nash equilibrium with  $r_A = r_B = \hat{r}$  and positive profits for both firms.

## E The Nash Equilibrium in Generation Capacities

From the maximization of  $\Pi_j(k_j, k_h)$  given in (12), (13), (14) and (15) with respect to  $k_j$  one can derive the best response of firm  $j$  in its generation capacity  $k_j$ . It is given by:

$$k_j(k_h) = \begin{cases} \frac{5}{4} - \frac{z}{2} & \text{if } 0 \leq k_h < \frac{1}{8} - \frac{z}{4}, \\ 1 + 2k_h & \text{if } \frac{1}{8} - \frac{z}{4} \leq k_h < \tilde{k}, \\ k_h & \text{if } \tilde{k} \leq k_h < \bar{k}, \\ 0 & \text{if } k_h \geq \bar{k}, \end{cases} \quad (34)$$

for  $0 \leq z < 0.2484$ , where  $\tilde{k}$  is defined in (18) and  $\bar{k}$  in (17), by:

$$k_j(k_h) = \begin{cases} \frac{5}{4} - \frac{z}{2} & \text{if } 0 \leq k_h < \frac{1}{8} - \frac{z}{4}, \\ 1 + 2k_h & \text{if } \frac{1}{8} - \frac{z}{4} \leq k_h < \frac{1-2z+\sqrt{9+4z(z-5)}}{8}, \\ 0 & \text{if } \frac{1-2z+\sqrt{9+4z(z-5)}}{8} \leq k_h < \underline{k}, \\ k_h & \text{if } \underline{k} \leq k_h < \bar{k}, \\ 0 & \text{if } k_h \geq \bar{k}, \end{cases} \quad (35)$$

for  $0.2484 \leq z < \frac{1}{2\sqrt{2}}$ , where  $\underline{k}$  is defined in (16), by

$$k_j(k_h) = \begin{cases} \frac{5}{4} - \frac{z}{2} & \text{if } 0 \leq k_h < \frac{1}{8} - \frac{z}{4}, \\ 1 + 2k_h & \text{if } \frac{1}{8} - \frac{z}{4} \leq k_h < \frac{1-2z+\sqrt{9+4z(z-5)}}{8}, \\ 0 & \text{if } k_h \geq \frac{1-2z+\sqrt{9+4z(z-5)}}{8}, \end{cases} \quad (36)$$

for  $\frac{1}{2\sqrt{2}} \leq z < \frac{1}{2}$  and by

$$k_j(k_h) = 0 \quad (37)$$

for  $z \geq \frac{1}{2}$ . Checking for  $k_j(k_h(k_j)) = k_j$  and  $k_h(k_j(k_h)) = k_h$  yields the subgame perfect Nash equilibria described in Proposition 3.

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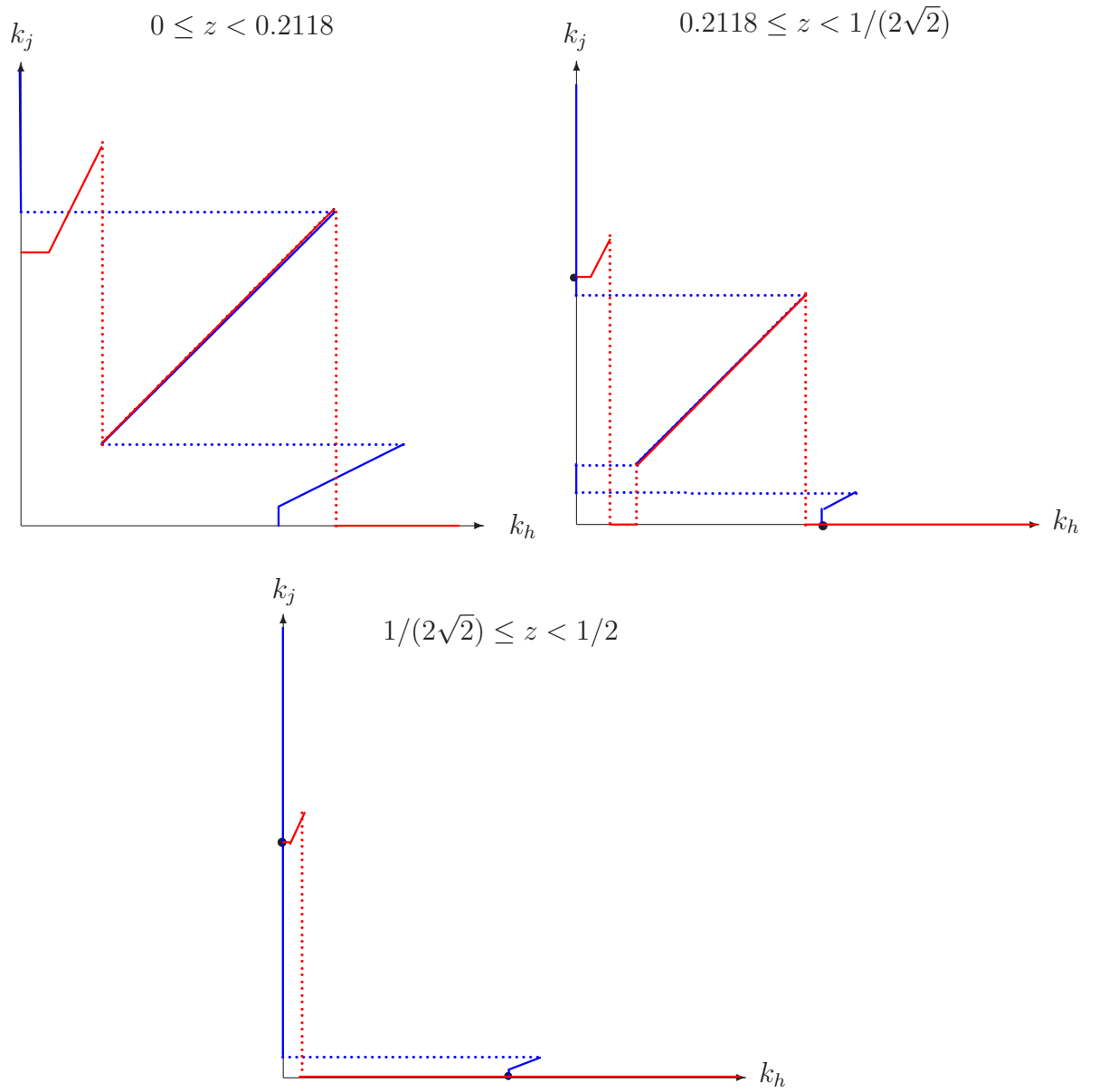


Figure 4: Nash Equilibria in Capacities

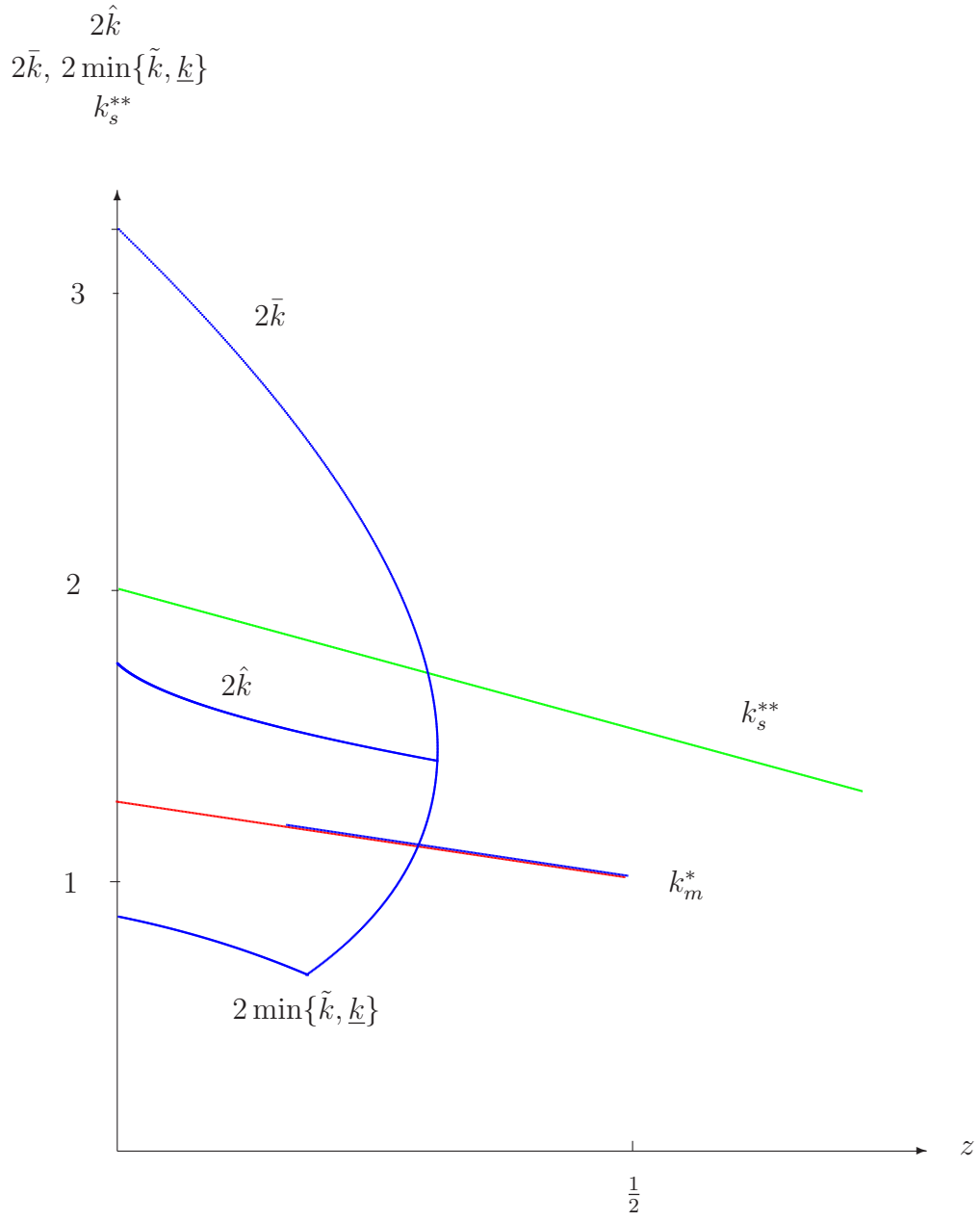


Figure 5: Total Capacities with a Duopoly, with a Monopoly and in the Social Optimum