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A Tax Optimality Index

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By

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Abstract: This paper introduces an index of tax optimality that measures the distance of some current tax structure from the optimal tax structure in the presence of public goods. In doing so, we derive a $[0, 1]$ number that reveals immediately how far the current tax configuration is from the optimal one and, thereby, the degree of efficiency of a tax system. We call this number the Tax Optimality Index. We show how the basic method can be altered in order to derive a revenue equivalent uniform tax, which measures the size of the public sector. A numerical example is used to illustrate the method developed.

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1. Introduction

The present paper contributes to the theory of measuring the efficiency costs of taxes. To address this issue the standard approach in the public finance literature is to use the excess burden of taxation, which measures the efficiency cost of taxes *in money terms*. In this paper, we propose a *non-money metric* aggregate that contains normative information about the current level of taxes in a quite intuitive way. We argue that this measure is easy to use for comparisons between countries and over time.

Despite its wide appeal, the excess burden measure has some well-known defects in its application. Quoting Auerbach (1985), the excess burden of taxation is defined *"as the amount that is lost in excess of what the government collects. Unfortunately, while this definition makes intuitive sense, it is too vague to permit a single interpretation"* (p.67). The problem with excess burden is that it represents a money metric aggregate. Moreover, a money metric equivalent of the tax distortion can be derived both by using an equivalent variation and a compensating variation method. Thus, while in Mohring (1971) the excess burden is the amount in excess of taxes being collected that the consumer would give up in exchange for the removal of all taxes (an equivalent variation calculation), in Diamond and McFadden (1974) the excess burden is the amount that the government must supply to the consumer to allow her to maintain the initial level of utility (a compensating variation calculation). The results, as we know, differ. More importantly, and independently of which method we use, comparisons of different countries' excess burden should be done with care as issues, such as using a purchasing power parity corrected exchange rate, are involved (see Neary, 2004).

In the present paper we suggest a measure of tax efficiency that is not expressed in money terms and is therefore easy to use for international and inter-temporal comparisons. Building on distance function techniques recently utilized by Anderson and Neary (1996), we propose an index that measures the welfare burden of a given tax configuration as its distance from optimal taxes (where the welfare burden is taken to be zero). We call it the *Tax Optimality Index* (TOI). Our measure has an immediate, and intuitive, interpretation: for example, a TOI equal to 0.6 implies that current taxes are 60% efficient, or, in other words, that a 40% reduction of optimal tax rates will reduce welfare to the level that exists at the current tax rates. Being a distance measure of tax efficiency, it is not expressed in money terms and can be directly compared between different countries and/or time periods.

In addition to the TOI, we also propose a *Revenue Equivalent Uniform Tax* (REUT) measure to address an issue frequently considered in public economics, viz. the uniform tax rate that keeps the provision of public goods unchanged. This corresponds to a reform of taxes that changes the non-uniform initial tax structure to a uniform one that yields the same amount of tax revenues and thus the same provision of public goods (a flat-tax type of reform). The resulting uniform tax rate is a convenient and easily interpreted measure of the size of the public sector.

The rest of the paper proceeds as follows. The model of a small open economy with public good provision and distortionary taxation is presented in section 2. Section 3 contains the definition of the tax optimality index, its properties and its interpretation. The derivation of the revenue-equivalent average tax is presented in section 4. To illustrate the techniques used, a numerical example is presented in section 5. Finally, section 6 concludes.

2. A Small Open Economy with Public Good Production

Consider a small open economy that faces fixed world prices on goods that it trades with the rest of the world. Assume that the number of traded goods is $1+M$, with M being the number of non-numeraire goods. The world prices of these non-numeraire goods are denoted by the vector p.[1]

The government raises revenues by taxing consumption.[2] Denote t as the $(M \times 1)$ vector of ad-valorem taxes, which create a wedge between world and producer prices p and consumer prices $q = (1 + t) \cdot p$.[3] The tax revenues finance the production of a public good g that is returned to consumers free of charge.

We assume the existence of a representative agent that achieves utility level u by consuming private and public goods and raising income through its (fixed) factor supply. The consumer decisions are characterized by the expenditure function $e(q, g, u)$, which denotes the minimum expenditure needed to achieve utility level u, given consumer prices $q = (1+t) \cdot p$ and a level g of the public good.[4] Standard properties of this function (see Dixit and Norman, 1980, Woodland, 1982, and Cornes,

[1] We use the first of the traded goods as a numeraire, its price being normalised to unity. In addition, it is assumed, without loss of generality, that this good is not taxed. Unless otherwise mentioned, all vectors are taken to be column vectors and the transpose of a vector x is denoted by x'.

[2] Extending the model to include both income taxes and consumption taxes is straightforward and therefore delegated to the appendix. Similarly, taxes and subsidies on international trade can easily be incorporated into the model.

[3] The notation "$\cdot$" denotes the horizontal product of two vectors; if $z = x \cdot y$ then $z_i = x_i y_i$. In the expression for q, 1 is a vector of ones.

[4] The unit price of the numeraire good is also an argument of the expenditure function, but it is supressed for simplicity. We do the same to the revenue function with respect to both the price of the numeraire good and the (fixed) factor endowments vector.

1992) imply that $e_q \equiv \partial e/\partial q$ is the vector of compensated demand functions for the private goods, $-e_g$ is the marginal willingness to pay for the public good and e_u is the inverse of the marginal utility of income.[5]

Let the restricted revenue function $r(p, g)$ denote the value of total income generated in the private sector given producer prices p and the level of the provision of the public good g. The gradient of this function with respect to producer prices ($r_p \equiv \partial r/\partial p$) gives the vector of domestic supplies of private goods and r_g is the supply shadow price of the public good ($-r_g > 0$ is the unit cost of producing the public good). It is assumed that the production technology exhibits constant returns to scale technology, implying that $r_{gg} \equiv \partial^2 r/\partial g^2 = 0$.

In describing the equilibrium of this economy we use the private sector's net expenditure function E defined as

$$E((1+t) \cdot p,\ g,\ u) = e((1+t) \cdot p,\ g,\ u) - r(p,\ g). \tag{1}$$

Thus, net expenditure is the consumer's expenditure on private goods minus the revenue earned by the production sector from the sale of private goods. As is well known, the price gradient $E_p = e_p - r_p$ is the vector of utility-compensated excess demands. The derivative $-E_g = -e_g - (-r_g)$ denotes the wedge between the marginal willingness to pay for the public good ($-e_g > 0$) and the marginal cost of producing it ($-r_g > 0$). Clearly, if this wedge is positive (negative) an extra unit of the public

[5] The term e_g represents the reduction in expenditure on the private goods as a result of an extra unit consumption of the public good, holding utility level constant. In that sense, e_g is the shadow demand price of the public good and $-e_g(> 0)$ is then the marginal willingness to pay for the public good.

good will increase (decrease) welfare.[6]

The equilibrium of our economy is described by the following two equations:

$$E((1+t)\cdot p,\ g,\ u) = -r_g(p,\ g)g \tag{2}$$

$$(t\cdot p)' e_p = -r_g(p,\ g)g. \tag{3}$$

The first equation is the private sector's budget constraint expressed in domestic prices. It demands that the extra money consumers spent on private goods (extra in the sense that is in addition to the income that they earn by working in the private sector) comes from working in the public sector ($-r_g g$ is the total cost of producing g units of the public good, which, under the assumption of constant returns to scale, is equal to total income generated in that sector). The second equation is the public sector budget constraint, equating tax revenues and total public sector costs. Given world prices p and the tax vector t, these two equations simultaneously determine the level of utility u and the level of provision of public goods g.

The public sector budget constraint (3) may be solved for the quantity of the public good g as a function of t and u, i.e. $g = g(t,\ u)$. Substituting this solution into equation (2) and re-writing the private sector budget constraint as the balance of trade function $B(t,\ u)$, we obtain:

$$B(t,\ u) \equiv E((1+t)\cdot p, g(t,\ u),\ u) + r_g(p,\ g(t,\ u))g(t,\ u) = 0. \tag{4}$$

[6] The condition $-E_g = 0$ (i.e. $-e_g = -r_g$, or $MRS = MRT$) is the so-called Samuelson rule for optimal public good provision in a closed economy without distortionary taxation. This rule does not apply here as we consider a small open economy with distortionary taxation.

Equation (4) represents the general equilibrium budget constraint for the economy, making sure that the public good market is in equilibrium and that consumers and the government cannot spend more money than they earn. It is expressed in terms of the tax rates and the level of utility. The utility level that satisfies equation (4) is given by the indirect utility function expressed as $u = U(t)$.

3. The Tax Optimality Index

Having expressed the economy's equilibrium with a single compact equation (4), we now construct a measure of the efficiency of the tax structure in the presence of public goods.

Suppose that we observe a country with tax rates and utility given by (t^1, u^1) and a level of public good provision given by $g^1 = g(t^1, u^1)$. Let (t^0, u^0) be the welfare maximizing choice of taxes and corresponding utility level and let the corresponding optimal public good provision be $g^0 = g(t^0, u^0)$. This welfare optimal solution is obtained by maximizing the indirect utility function $U(t)$ with respect to the tax vector t, yielding solution t^0. The objective is to obtain a measure of how well the observed tax-public good situation compares with the optimal tax-public good solution.

With these preliminaries in hand, we can now define the Tax Optimality Index (TOI) as the distance function

$$T\left(t^0, u^1\right) \equiv \min\left\{\delta : B(\delta\ t^0,\ u^1) = 0,\ \delta > 0\right\}. \tag{5}$$

The solution δ^0 to this problem determines a tax vector $\widetilde{t}^1 \equiv \delta^0 t^0$ that yields the reference utility level u^1. This new tax vector has the property that it is a contraction of the optimal tax vector t^0 towards the origin and thus lies on the ray from the origin to t^0 in tax space. The solution δ^0 to the minimization problem in (5) is the scaling factor by which the optimal tax vector is contracted. Thus, the Tax Optimality Index $T\left(t^0, u^1\right)$ is the proportion of the optimal tax vector that achieves the same level of welfare u^1 as achieved by the observed tax vector t^1.[7]

If the economy is already at the welfare optimum, then $t^1 = t^0$ and $u^1 = u^0$ and so the index takes a value of unity. If the country is not at the welfare optimum, its level of utility is $u^1 < u^0$ and so a proportionally smaller vector of tax rates than t^0 will allow the country to maintain its level of utility u^1.[8] Our index will therefore be less than unity. In an extreme case, the observed tax vector is $t^1 = 0$ and there is a zero provision of public goods $g^1 = 0$. In this case, the solution to the above problem is $\delta^0 = 0$ and so the tax optimality index takes the value of zero. Thus, in short, the tax optimality index $T\left(t^0, u^1\right)$ ranges from zero to unity. Zero indicates that there is, effectively, no public sector in the observed situation. Unity indicates that the government's choice of taxation rates (and hence the provision of public goods) is optimal. In between, a higher index indicates greater proximity to the optimum. We can refer to this index, therefore, as a *Tax Optimality Index (TOI)*.

[7] This definition of the TOI is expressed as a distance function in tax space. Anderson and Neary (1996) use a distance function in commodity price space to derive measures of the trade restrictiveness of tariffs, but their measure can also be expressed in tax space. Chau et.al (2003) use distance functions in quantity space to derive measures of economic inefficiency of tariffs. These authors provide also a comparison between their efficiency measure and the coefficient of resource utilisation (Debreu, 1951), the open economy index of deadweight loss (Diewert, 1985), and the well-known equivalent variation measure.

[8] There will also exist another set of taxes that is higher than t^0 that can maintain the same level of utility. The index then will be greater than unity, as optimal taxes will have to be inflated. Our definiton, and thus convention, is to consider only the lower than t_0 taxes (hence the min in equation (5)).

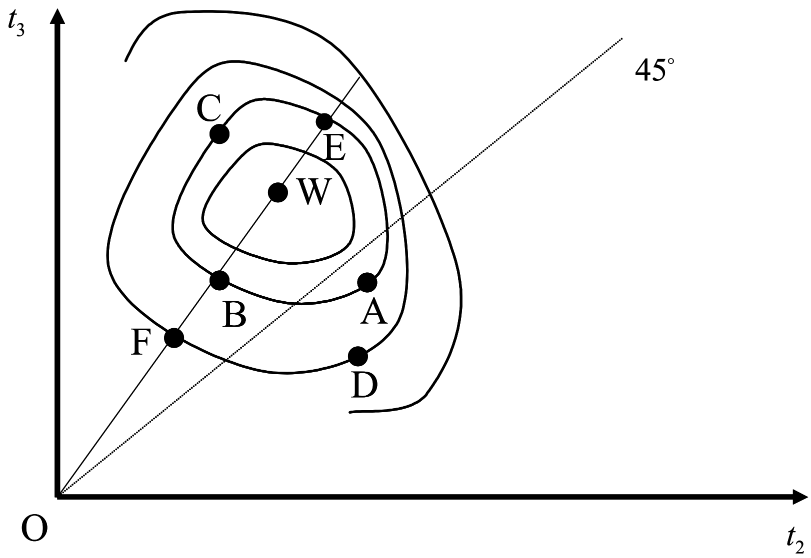

Figure 1: The Tax Optimality Index

The index is illustrated in Figure 1. The figure depicts iso-utility contours in a three (two taxable) good small open economy with public good provision. Since we assume without loss of generality that one good (the numeraire) is not taxed, the index may be illustrates in the two-dimensional tax space (t_2, t_3). Point W represents the optimal tax situation where non-zero taxes finance the production of public goods, while point A is the assumed current tax situation for the economy. The contours (indifference curves) show the sets of tax rates that yield various levels of utility, point W being on the highest feasible indifference curve.

A proportional contraction of the optimal taxes given by W yield tax vectors on the ray passing through the origin and W. In particular, one such deflation of the optimal taxes takes us to point B in Figure 1, which produces the same level of utility as at the current tax equilibrium given by point A. The tax optimality index for the current tax equilibrium is therefore given by the ratio $TOI = OB/OW$. This tax vector B, as defined above, is (a) a uniform contraction of the optimal tax vector and (b) yields the same level of utility as the initial tax situation. It should be noted that the level of provision of the public good will generally be different at points W, A and B, but these differences have no bearing on the tax optimality index TOI, which focuses on welfare alone.[9] The TOI index in based on welfare comparisons; points B and A are welfare equivalent and yield lower welfare than point W.

If the initial tax situation is given by point C, the same point B is obtained and so the index takes the same value as for situation A. This is as it should be; even though they may produce different levels of the public good, both have the same utility and so they are equal in a welfare sense. At initial situation D, on the other hand, the level of utility is lower than at A and C and so the tax optimality index will also be lower. Tax configuration D is further from the welfare optimum W than are A and C in the sense that it is on a lower indifference curve. Its tax optimality index is given by OF/OW, which is lower than A's index. In summary, our TOI measures the distance of any initial tax vector from the optimal tax vector, distance being measured along the ray OW. This distance, relative to the distance OW, accurately ranks initial tax situations according to their levels of utility relative to the optimal utility point W. Thus, we measure the true welfare cost of taxes as the distance from the optimal tax structure.[10]

[9] It is possible, of course, for the level of provision of public goods to be the same at points A and W. The level of utility at A will be lower than at W by choosing a sub-optimal tax vector to finance the public sector.

[10] As we have mentioned, the techniques we use are based on the work of Anderson and Neary (1996), whose method measures the welfare-preserving uniform tax when optimal taxes are zero. However, when optimal taxes are not zero, such a measure may not exist. To see this, consider Figure 1: clearly the 45 line (which depicts uniform taxes) may not intersect with the iso-utility contour corresponding to a current tax configuration. The only welfare-preserving tax that we can always find, is the tax that lies on the line that connects the origin with the optimal tax configuration.

Note that there can be two scalars that will do the job: if the current tariff structure is described by point A, the optimal tariff rates can be both proportionally increased and decreased to get the same utility level (points E and B, respectively). Our convention is to consider only points on the OW line (that is, only point B is considered) and then measure the TOI by the ratio OB/OW.

The distance BW measures (in tax space) the loss of welfare associated with initial point A compared to the best attainable welfare at W. The ratio BW/OW can therefore be interpreted as a *Tax Inefficiency Index (TII)*. Of course, the two indices are related by the equation $TII = 1 - TOI$.

Some of the properties of the TOI are apparent from the above discussion. The main properties are brought together as follows:

1. The TOI has the range $[0, 1]$. If an economy has optimal choices of both public goods provision and commodity tax rates, then $TOI = 1$. If an economy has sub-optimal choices of either public goods provision or commodity tax rates, then $TOI < 1$.

2. The TOI is monotonically increasing in utility. Thus, higher values for TOI indicate higher welfare; same values for TOI indicate the same level of welfare.

3. The TOI is homogeneous of degree zero in world prices, and hence independent of the choice of numeraire.

The first two properties are easily proved and follow from the definition of the

The position of this welfare-preserving tax on this line is exactly what our Tax Optimality Index measures.

TOI. The final property follows from the homogeneity properties of the revenue and expenditure functions.[11]

These properties make the TOI particularly appropriate for measuring the optimality or, conversely, inefficiency of tax structures and for undertaking international or inter-temporal comparisons. Property 2 states that the TOI and the level of welfare are positively and uniquely related. Thus, in comparing various alternative tax/public good situations for a given economy the TOI is a perfectly accurate measure of welfare.[12] For example, index calculations of $TOI^1 = 0.9$ and $TOI^2 = 0.8$ indicate that situation 1 is more efficient than situation 2 and that $u^1 > u^2$.

Because the tax optimality index is homogeneous of degree zero in world prices (Property 3) and so is a "pure number", it can be readily used for comparisons between different countries and/or different time periods. Different countries may have different preferences and technologies and, thus, different optimal taxes and different optimal levels of public good provision. Nevertheless, comparisons on the basis of the TOI are valid, as the TOI measures the distance of the current tax structure from its optimal tax configuration. Thus, the TOI can be easily used to rank countries in terms of distance from optimality. Such rankings of the TOI generally cannot be translated into welfare rankings, of course.[13]

[11] The homogeneity of the TOI in prices follows from the homogeneity properties of the expenditure and revenue functions. If all world prices (including the numeraire's price) are doubled, the expenditure and revenue via functions e and r, double and, hence, net expenditure via E doubles. In addition, the shadow supply price $r_g(p, g)$ doubles as does tax revenues. Thus, the solution for the real variables u and g in equations (2) and (3) remain unchanged as a result of the world price inflation. Hence the balance of trade function $B(t, u)$ is homogeneous of degree zero in prices and this implies that the TOI is also homogeneous of degree zero in prices.

[12] Of course, we are dealing with the special case of a single household economy here.

[13] On the other hand, if two countries have the same technologies, preferences and endowments then a comparison of their TOIs will enable an accurate welfare comparison. A similar remark applies to a comparison of the same economy in different time periods.

When only small tax changes have occurred between time periods, we can use calculus to uncover some properties of the TOI. Totally differentiating (5) around the initial equilibrium, holding t^0 fixed, we get that $B_t'(\delta t^0, u^1)t^0 d\delta + B_u(\delta t^0, u^1)du = 0$. Total differentiation of $B(t^1, u^1) = 0$ yields $B_t'(t^1, u^1)dt + B_u(t^1, u^1)du = 0$, which may be used to solve for the change in utility consequent upon marginal changes in the initial taxes. Substituting the result into the first expression yields the relationship

$$\frac{dT_u}{T_u} = \frac{B_u(\widetilde{t}^1, u^1)}{B_u(t^1, u^1)B_t'(\widetilde{t}^1, u^1)\widetilde{t}^1} B_t'(t^1, u^1)dt, \tag{6}$$

where it is recalled that $\widetilde{t}^1 = \delta t^0$.

We can easily see from the above formula that the proportional change of the T_u deflator is related (via a scaling factor - the first term) to the weighted average of the tax changes (dt), with the weights being the marginal welfare effects of taxes (B_t) evaluated at the initial equilibrium. The structure of these weights in terms of derivatives of the revenue and expenditure functions is obtained by totally differentiating (4) to get

$$B_t' = e_p' - \frac{e_g}{r_g + (t \cdot p)' e_{pg}} \left[(p \cdot e_p)' + (t \cdot p)' e_{pp} \right]. \tag{7}$$

The term B_t' denotes the marginal effects of tax changes upon the balance of trade (4), which can be interpreted as the foreign exchange needed (zero in our case) to sustain utility u given world prices p, taxes t, and public good provision g. The change in B following a change in taxes gives the money metric measure of the resulting welfare effect.[14]

[14] Totally differention of (4) yields

$$B_u du + B_t' dt = 0,$$

The first term on the right hand side of this expression (7) for B_t' gives the "snapshot" effect of a tax change, ignoring the general equilibrium effects incorporated in the remaining term. If only this "snapshot" effect were to be taken into account, the expression for (6) would be the approximation $d\widehat{T}_u/\widehat{T}_u = -c'dt/c't$, where $c = e_p$ is the consumption vector at the initial equilibrium. This expression weights the marginal changes in the tax rates by the consumption vector and measures the (snapshot) percentage change in consumption tax revenue. While this is easy to compute and "intuitive", it ignores important general equilibrium effects. The appropriate marginal index is given by (6) and (7). This is the marginal version of our TOI.

The TOI, as defined above, compares the existing tax situation to the optimal tax situation. The optimal tax situation is one in which the economy chooses its consumption tax vector and the level of public good provision to maximize utility.[15] A variation on this tax optimality index may be defined for the situation where the level of public goods provision is not endogenously determined. Consider a tax structure that maximizes utility subject to the requirement that the level of public good provision is equal to an exogenously determined public revenue requirement. In this case, the constrained optimal tax structure maximizes utility u subject to the

where

$$B_u = e_u - \frac{e_g}{r_g + (t \cdot p)'e_{pg}}(t \cdot p)'e_{pu}$$

and B_t is given as (7). The derivative B_u denotes the welfare gain to a unit increase of the economy's purchasing power, while B_t' denotes the marginal welfare effects of tax changes. Since the balance of trade equation can be interpreted as determining the foreign exchange needed (zero in our case) to sustain utility u given world prices p, taxes t, and public good provision g, the change in B following a change in taxes gives the money metric measure of the resulting welfare effect. Clearly, and due to the existence of public goods, B_t' is not always negative and, thus, an optimal level of taxes can be derived by setting $B_t' = 0$.

[15] We should clarify here that by optimality we mean a constrained optimality where lump sum taxes do not exist.

private budget constraint (2) in which $g = g^1$ is given, that is, subject to

$$\overline{B}(t, u, g^1) \equiv E((1+t) \cdot p,\ g^1,\ u) + r_g(p,\ g^1)g^1 = 0. \tag{8}$$

Call the solution for the tax vector $\overline{t}^0$. The constrained tax optimality index (CTOI) can then be defined, analogously to the TOI as

$$\overline{T}\left(t^0, u^1, g^1\right) \equiv \min\left\{\delta : \overline{B}(\delta\ \overline{t}^0,\ u^1, g^1) = 0,\ \delta > 0\right\}. \tag{9}$$

This index measures the efficiency or optimality of the existing tax structure relative to the optimal tax structure that achieves the same public good provision outcome as the initial equilibrium. If the initial taxes are optimal for this purpose then the index is unity; if sub-optimal for this purpose, the index is less than one. This index, therefore, does not assume optimality of the public good provision choice.

4. The Revenue-Equivalent Uniform Tax

We now turn to the case of deriving a uniform tax that holds the provision of public goods, and thus the tax revenue, constant.[16]

For this, we use the private sector's budget constraint (2) to solve for utility u as a function of t and g, i.e. $u = u(t, g)$, and substitute the solution into the public sector's budget constraint (3). Rewriting the public sector budget constraint, we have:

$$\Pi(t, g) \equiv (t \cdot p)'\, e_p((1+t)p,\ g,\ u(t,\ g)) + r_g(p,\ g)g = 0. \tag{10}$$

[16] Anderson and Neary (2003) make a similar application in which they keep fixed the initial trade vector.

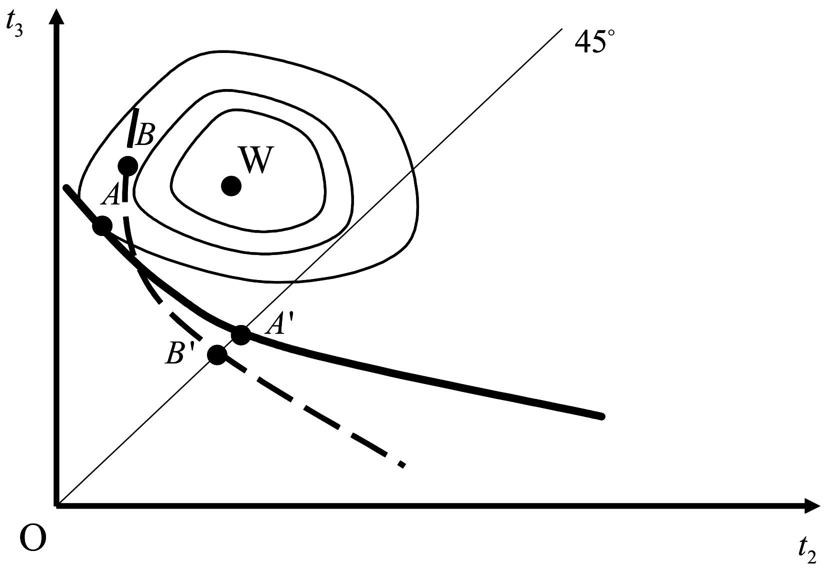

Figure 2: The Revenue Equivalent Uniform Tax

Equation (10) represents the general equilibrium public sector budget constraint for the economy. Denoting by t^1 and g^1 the current level of taxes and public good provision, we define the revenue equivalent uniform tax T_g as

$$T_g : \Pi(1 \cdot T_g,\ g^1) = 0. \tag{11}$$

According to this definition, T_g is the uniform tax rate that will yield the reference level of public good provision (g^1) for the economy. Since this uniform tax rate satisfies the general equilibrium public sector budget constraint (10), both the public and private sector budget constraints are satisfied.

Figure 2 depicts this revenue-equivalent uniform tax (REUT) rate. In the figure, point A is the initial tax rate configuration (t^1), while W is the optimal vector of taxes. The contours concentric to point W represent different levels of utility. The locus of points given by the solid curve through point A defines the set of taxes that can support the public good provision (g^1) given at the initial situation. The shape of this locus is determined by the general equilibrium public sector budget constraint, i.e. equation (10), but it will be downward sloping. This constant-g contour cuts the 45 degree line at point A'. Hence, the uniform tax (T_g) that reproduces the same level of the public good (g^1) is then given by point A'. Clearly, imposing this uniform tax rate may increase or decrease welfare (as Figure 2 is drawn, welfare falls).

We can now use this uniform tax as a measure of the size of the public sector in different time periods within a country or between different countries. Even if the shape of the constant public good locus is not the same between countries, they will all cut the 45 degree line thus allowing for such a comparison. As an example, consider a second country's tax configuration at point B with a constant public good locus given by the stippled curve, which passes through points B and B'. The second country's revenue equivalent uniform tax vector is then B', which is smaller than A', and, thus, the second country B has a smaller public good sector than the first country A. What makes the revenue equivalent uniform tariff the correct tool for comparing the two countries is, of course, the fact that it takes into account the general equilibrium effects of changing taxes and is independent of the choice of numeraire.

To put our revenue equivalent uniform tax measure into perspective, we briefly discuss a somewhat different uniform tax used in public finance, viz. the *average effective tax.* The average effective tax (also called, the tax burden) is used to measure the size of taxation of a particular activity. It is defined as the ratio of tax revenues over tax base. For the case of, say, consumption taxes, the average effective consumption tax (t_c) is defined as $t_c = \sum_i \frac{t_i p_i c_i}{C}$, where c_i is the consumption of good i, t_i is the consumption tax rate and $C = \sum_i (1 + t_i) p_i c_i$ is total value of consumption at consumer prices. Thus, the resulting scalar is constructed by using current tax rates (t_i) weighted by current activity information (c_i). Clearly, its popularity is its simplicity: reference to the national statistics of a given country provides all the information needed for its calculation.[17] However, the above method is theoretically problematic. The use of current activity information is the culprit of a classic index-number problem: the activity that is highest taxed weighs less in the index! As the tax on a particular good rises, the weight put on that good (here, its consumption level) falls. At prohibitively high levels of taxation the weight is zero and thus the constructed index underestimates the true tax burden in the economy. These are typical index number problems that could be addressed by applying index number techniques.[18] Clearly, these type of problems do not exist in the calculation of our revenue-equivalent uniform tax as it uses correct general equilibrium weights.

5. Calculating the TOI: A Numerical Example

In order to illustrate the use of the TOI, we present a numerical example. The model we use has four commodities - three private and one public good. As in our theoretical

[17] Due to that, the literature on measuring the average effective tax has concentrated on the details of what should be included in the nominator and denominator of the above expression (a recent overview can be found in Sørensen, 2004).

[18] The problem in its basic form is the same as the one encountered in the price index literature concerning the relation between *Paasche*, *Laspeyres*, and *Konüs* Indexes, viz. $P_L > P_K > P_P$ (see Diewert, 1981). While the former two have the advantage of using easily available information, it is only the latter that truly expresses the true cost of living. This latter index, however, presupposes knowledge of the utility function and as such it is difficult to use. The solution is to calculate the *Fisher* ideal price index, which is a combination of the *Paasche* and *Laspeyres* indexes ($P_F = [P_L P_P]^{0.5}$), and which can closely approximate the *Konüs* index. Such a development could also be done in the average effective tax literature, where the current measue underestimates the true tax burden. A similar procedure, to a different issue, is also argued by Cornes (1996).

model, the economy is a small open economy with the prices of the private goods being given by world market conditions. Commodity 1 is taken to be the numeraire.

5.1. Production. The production side of the economy is described by the revenue function $R(p, v)$, where p is now a vector of producer prices for the four goods and v is the vector of endowments. There is just one endowment in the numerical model. The revenue functional form is

$$R(p, v) = \left[\sum_{i=1}^{4} l_i p_i + \left(\sum_{i=1}^{4} b_i (p_i)^2 \right)^{0.5} \right] v = \left[l(p) + \widetilde{R}(p) \right] v,$$

where $b = (b_1, b_2, b_3, b_4) > 0$ and $l = (l_1, l_2, l_3, l_4) \leq 0$ are vectors of parameters and $v > 0$ is a scalar. This functional form is linearly homogeneous and convex in prices (assumed positive, of course). It is increasing in prices over a cone of prices; if $l = 0$ then it is increasing for all positive prices. The output supply functions are

$$y_i(p, v) = \left[l_i + \frac{b_i p_i}{\widetilde{R}(p)} \right] v.$$

The parameter vector l is introduced to allow an output to be zero. Specifically, we might want to set the output of the public good equal to zero, in which case $y_4 = 0$. If $l_4 < 0$, then there exists a shadow supply price q_4^s such that a zero output is feasible.

More generally, setting $g = y_4(p, v)$ we can solve for the shadow supply price for the public good p_4^s, which can then be eliminated from the revenue function to get the restricted revenue function that we define in section 2, i.e. $r(p_1, p_2, p_3, g, v) = R(p_1, p_2, p_3, p_4^s(p_1, p_2, p_3, g, v), v)$. We do not have to perform this solution and substitution analytically as it can be done numerically in the example.

5.2. Consumption. The expenditure function is

$$\widetilde{e}(q,u) = \sum_{i=1}^{4} k_i q_i + \left(d \prod_{i=1}^{4} (q_i)^{a_i} \right) u = k(q) + \epsilon(q)u,$$

where $a = (a_1, a_2, a_3, a_4) > 0$ and $k = (k_1, k_2, k_3, k_4) \leq 0$ are vectors of parameters, d is a scalar parameter and $u > 0$ is a scalar representing utility. The consumer prices are denoted by $q_i = (1+t_i)p_i$ and $t_4 = 0$. This functional form is linearly homogeneous and concave in domestic prices (assumed positive, of course). It is increasing in prices over a cone of prices; if $k = 0$ then it is increasing for all positive prices.

The compensated demand functions are

$$c_i(q,u) = k_i + \frac{a_i \epsilon(q)}{q_i} u,$$

which is the linear expenditure system. The parameter vector k is introduced to allow an output to be zero. Specifically, we might want to set the quantity of the public good equal to zero, in which case $c_4 = 0$. If $k_4 < 0$, then there exists a shadow demand price q_4^d such that a zero output is feasible.

More generally, setting $g = c_4(q,u)$ we can solve for the shadow demand price for the public good q_4^d, which can then be eliminated from the expenditure function to get the expenditure that we define in section 2, i.e. $e(q_1, q_2, q_3, g, u) = \widetilde{e}(q_1, q_2, q_3, q_4^d(p_1, p_2, p_3, g, u), u)$. Again, we do not have to perform this solution and substitution analytically as a numerical solution suffices.

5.3. Parameter Values and Results. The parameter values chosen are given in table 1:

Table 1: Parameter values

Revenue:	$l = \begin{bmatrix} 0 & 0 & 0 & -0.5 \end{bmatrix}'$,	$b = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}'$	
Expenditure:	$k = \begin{bmatrix} 0 & 0 & -0.15 & -0.5 \end{bmatrix}'$,	$a = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.4 \end{bmatrix}'$,	$d = 2$
Other:	$p = \begin{bmatrix} 1 & 0.7 & 0.5 \end{bmatrix}'$,	$v = 1$	

As seen from the parameter values, we treat private goods (goods 1-3) symmetrically in our revenue function specification, but asymmetry is introduced into preferences via the choice of the parameter k_3 for private good 3. Good 4, the public good, enters the expenditure function with a larger coefficient (a_4), indicating a higher marginal willingness to pay for the public good than for the private goods. Finally, we normalize the endowment of the single factor (v) to unity and we set the world prices of the private goods (p) so that the country exports the numeraire good (good 1) and imports the other two goods (goods 2 and 3) (although the trade pattern does not matter here).

Table 2 below summarizes the results of the equilibrium for the open economy at the initial or reference tax vector $t^1 = [0,\ 0.16,\ 0.21]'$ and at the optimal tax vector t^0. In the table, $t_i, i = 2, 3$ are the ad valorem consumption taxes on the non-numeraire goods, c is the consumption vector, g is the quantity of public production and TI^i

denotes the traditional tax burden indexes.

Table 2: Results

Reference equilibrium	Optimal equilibrium
$t_2^1 = 0.16,\ t_3^1 = 0.21$	$t_2^0 = 0.2290,\ t_3^0 = 0.2080$
$c_1^1 = 0.4161,\ c_2^1 = 0.5124,\ c_3^1 = 0.5377$	$c_1^0 = 0.4176,\ c_2^0 = 0.4854,\ c_3^0 = 0.5414$
$g^1 = 0.1116$	$g^0 = 0.1265$
$u^1 = 1.0601$	$u^0 = 1.0609$
$TI^1 = 0.1814$	$TI^0 = 0.2197$
$TI^1/TI^0 = 0.8258$	
$REUT = 0.1812$	
$TOI = 0.7972$	

At the reference equilibrium, the tax rate on good 2 is significantly lower than the tax rate on good 3. The optimal tax solution reverses this divergence of tax rates and, due to the asymmetries in the technology and preferences between private goods, the optimal tax rates are different and are given by $t_2^0 = 0.2290$ and $t_3^0 = 0.2080$. The consumer responds by reducing consumption of the good taxed at a higher rate (good 3) and increasing consumption of the other two private goods (good 2's tax rate being lowered) compared to the reference situation. The optimal solution calls for a higher level of public good provision than at the reference equilibrium, and hence a greater tax revenue (in terms of the numeraire) is required. Of course, the level of utility is higher at the optimal solution.

Calculating the Tax Optimality Index we find that $TOI = 0.7972$. Thus, we can

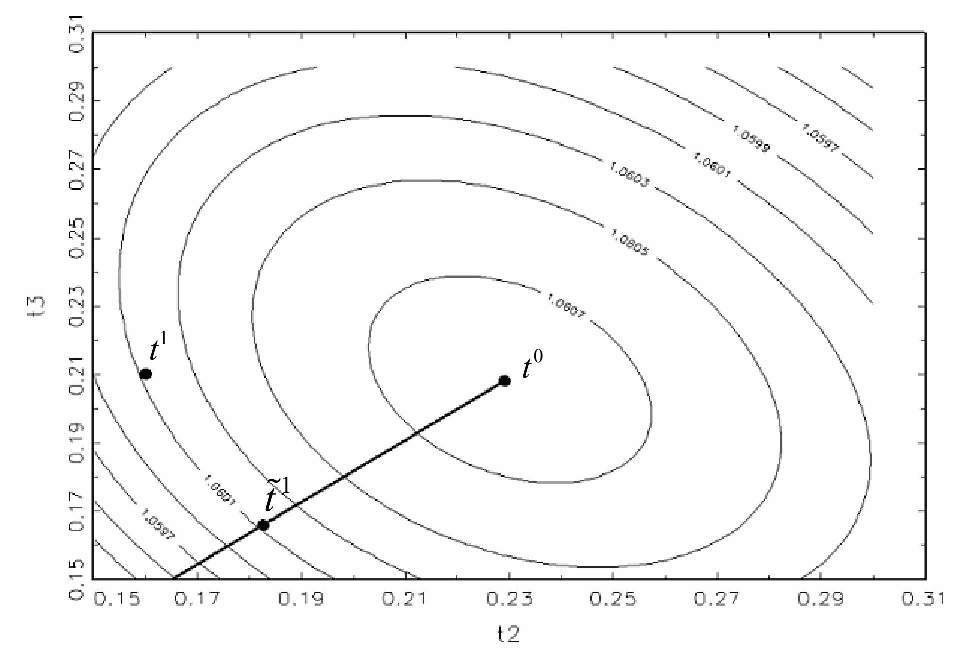

Figure 3: The Tax Optimality Index: Numerical Example

achieve the reference utility u^1 by using tax rates that are precisely 0.7972 times the optimal taxes t^0. Thus, we say that the reference point taxes t^1 are 79.7% efficient. In other words, an 20.3% proportionate reduction in the optimal tariffs would achieve the reference utility level u^1.

Figure 3 provides an illustration of the TOI in tax space. The figure shows the reference tax point t^1 , the optimal tax point t^0 and the constructed tax point $\widetilde{t}^1 = TOI * t^0$. By construction, this tax point lies on the ray through the optimal tax point.

Figure 4 illustrates the REUT rate. The figure shows the tax revenue contours in tax space along with the reference tax point, the optimal tax point and the revenue-equivalent uniform tax vector. The reference tax rates are $t_2^1 = 0.16$ and $t_3^1 = 0.21$

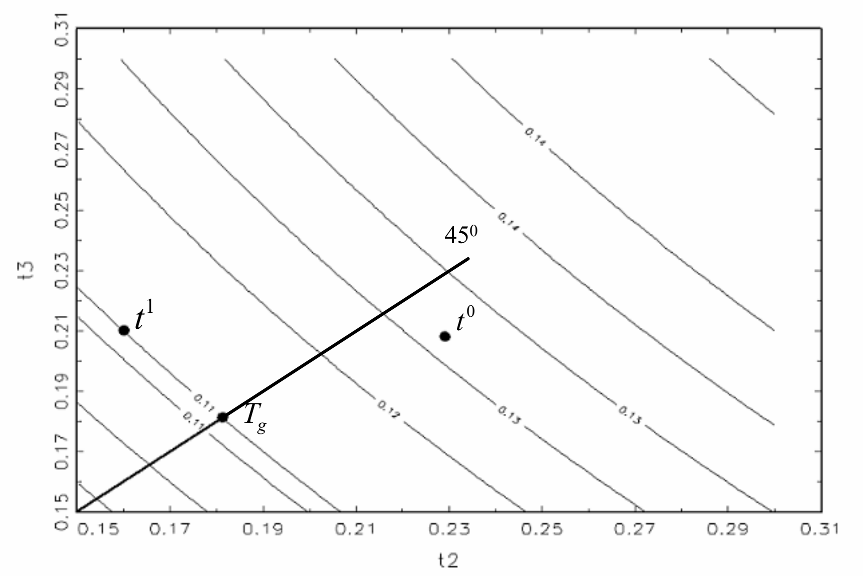

Figure 4: Revenue Equivalent Uniform Tax: Numerical Example (labels rounded to two decimal places)

and public good provision is $g^1 = 0.1165$. The same public good provision may be attained using a uniform tax rate of $T_g = 0.1812$. This provides a readily interpreted measure of the size of the public sector: the public sector size corresponds to an 18.1% uniform tax rate.

It is useful to compare our REUT calculations in this numerical example with traditional public finance measures used in the literature. One such measure is the "tax burden". Calculating the average effective trade tax at the reference equilibrium as the ratio of tax revenues over tax base, gives us a $TI^1 = 0.1814$. This is not exactly the same as our REUT, indicating that the two methods are different.[19]

[19] As it is well known, the effective average tax has no normative value. Still, we could induce a normative assesment if we were: (i) calculating the average effective tax at the optimal equilibrium TI^0, and (ii) comparing TI^1 and TI^0. Step (i) gives us a $TI^0 = 0.2197$, implying a tax burden of 21.97%. Step (ii) reveals that the average reference trade tax does not differ substantially from the optimal average effective tax ($TI^1/TI^0 = 0.8258$). However, the tax burden indices were not designed to measure the optimality of the tax structure. Our TOI is designed for measuring optimality and,

6. Concluding Remarks

We have proposed a new measure of tax inefficiency, viz. the Tax Optimality Index. Its advantage is its intuitive and informative interpretation: it tells immediately how efficient are the current taxes. For example, a TOI equal to 0.8 implies that current taxes are 80% efficient, or in other words, that a 20% reduction of the optimal taxes will bring welfare down to the current (distorted) welfare level. The type of information that the index conveys can easily be compared between countries and time, and thereby it avoids a basic defect possessed by the excess burden measure of inefficiency (calculated either as an equivalent or compensating variation).

We illustrated our methods by considering a numerical example. Admittedly, the true application of our method is to take it to the data and compute TOIs for different countries and time periods. That, remains to be done.

as indicated above, gives a result $TOI = 0.7972$, indicating a somewhat bigger tax inefficiency than the one derived using relative tax burden measures.

Appendix: The Model with Indirect and Direct Taxes

In addition to the traded goods mentioned in section 2 of the paper, we now assume the existence of N non-traded goods, whose price vector w is endogenously determined by markets within the country. There are two types of non-tradeables: private goods, whose characteristics makes them non-tradeable, and factors with elastic supply and no possibility of international mobility. However, as long as both markets clear competitively and factor supplies are elastic, there is no modelling advantage in distinguishing between the two. To simplify, we assume that all N non-traded goods are factors facing an endogenous (factor) price w, while the only private goods are the M traded ones facing a fixed world price p.

The government raises revenues by taxing the consumption of private goods and by taxing factors' income. Denote t the $(M \times 1)$ vector of ad-valorem indirect taxes and τ the $(N \times 1)$ vector of ad-valorem direct taxes.

The consumer decisions are characterized by the expenditure function $e((1+t) \cdot p, (1-\tau) \cdot w, g, u)$, which denotes the minimum expenditure needed to achieve utility level u, given consumer prices $(1+t) \cdot p$, after-tax factor prices $(1-\tau) \cdot w$ and a level g of the public good.[20] Standard properties of this function imply, in addition to the properties mentioned in section 2, that e_w is the vector of factor supplies.

Let the restricted revenue function $r(p, w, g)$ to denote the value of total income generated in the private sector given producer prices p, producer factor prices w and the level of the provision of the public good g. The new property to note here is that

[20] There is arguably a tension between the assumption of a representative agent and that of many factors. However, we can easily consider the case of many agents if we assume the existence of a social welfare function that a government maximises by use of lump-sum transfers.

r_w denotes the vector of factor demands.

The net expenditure function E is defined as

$$E((1+t)p,(1-\tau)w,g,u) = e((1+t)p,(1-\tau)w,g,u) - r(p,w,g).$$

The price gradient $E_w = e_w - r_w$ is the vector of excess factor supply.

The equilibrium of our economy is described by the following three equations:

$$\begin{aligned} E((1+t)\cdot p,(1-\tau)\cdot w,g,u) &= -r_g g && \text{(A1)} \\ (t\cdot p)' e_p + (\tau\cdot w)' e_w &= -r_g g && \text{(A2)} \\ E_w &= 0. && \text{(A3)} \end{aligned}$$

The first equation is the private sector's budget constraint expressed in domestic prices. The second equation is the public sector budget constraint, equating tax revenues and total costs. Finally, the third equation ensures that the factor markets clear.

We can solve the last two equations together expressing w and g as functions of t, τ, and u, i.e. $w = w(t,\tau,u)$ and $g = g(t,\tau,u)$. Substituting these into the first equation, and rewriting the private budget constraint as a balance of trade function, we have:

$$B(t,\tau,u) \equiv E((1+t)\cdot p,(1-\tau)\cdot w(\cdot),g(\cdot),u) + r_g(p,w(\cdot),g(\cdot))g(\cdot) = 0. \qquad \text{(A4)}$$

Equation (A4) represents the general equilibrium budget constraint for the economy, making sure that the markets for the non-tradeable goods (both the factor markets and the public good market) are in equilibrium and that consumers and the government can not spent more money that they earn. We can now define the TOI as:

$$T_u\left(t^0, \tau^0, u^1\right) \equiv \min\{\delta : B(\delta t^0, \delta\tau^0, u^1) = 0\}. \tag{A5}$$

Thus, the Tax Optimality Index $T\left(t^0, \tau^0, u^1\right)$ is the proportion of the optimal tax vectors that achieves the same level of welfare u^1 as achieved by the observed tax vectors t^1, τ^1. Thus, the *TOI* developed in the paper is readily extended to this more general context.

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