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Abstract

This paper analyzes the strong comovement between real stock and nominal bond yields at generational frequencies. Using a stochastic overlapping generations model with cash-in-advance constraints, we show that the simulated life-cycle patterns in savings behavior make both real stock and nominal bond yields comove with the changing population age structure. These persistent comovements account for the equilibrium relation between stock and bond markets. A stochastic Fisher decomposition of nominal bond yields reveals that, while having a moderate effect on both the inflation risk premium and expected inflation, demographic changes affect nominal yields mainly through real bond yields. Using both U.S. data and a cross-country panel, we find empirical support for these theoretical predictions. Finally, we show that the strength of the demographic effect on real yields explains cross-country differences in the comovement between stock and bond markets, while alternative demographic channels fail to explain such cross-country heterogeneity.

Keywords: demographics, financial yields, OLG, inflation risk premium.

JEL classification: E27, E31, E44, G11, G12.

1 Introduction

Yields on aggregate U.S. stock and government bond markets follow surprisingly similar paths in the post-war period, in particular post-Bretton Woods until the Great Recession (e.g., Bekaert and Engstrom, 2010; Maio, 2013). This evidence has led to valuation models, e.g., the Fed model, that rely on relative pricing of stock and bond markets. However, it is difficult to reconcile within standard macro-finance models (Duffee, 2018b) and does not extend to other countries, see Figure 1(Asness, 2003; Estrada, 2009).

Insert Figure 1 here

Another striking observation is that yields on the aggregate stock market are positively correlated with inflation (Wei, 2010). This is also puzzling, since the stock market representing real assets should be a good hedge against inflation. Behavioral explanations such as the "inflation illusion" (Modigliani and Cohn, 1979; Campbell and Vuolteenaho, 2004; Feinman, 2005), risk-based stories (Brandt and Wang, 2003; Bekaert and Engstrom, 2010), and business cycle shocks (Burkhardt and Hasseltoft, 2012; Campbell et al., 2014; Ermolov, 2015; Song, 2017) have been proposed to reconcile these two pieces of evidence, but there is no consensus. In this paper, we propose an alternative explanation.

Insert Figure 2 here

Panel A (Panel B) of Figure 2 plots the 20-year correlation between stock and 10-year nominal bond yields (inflation) over more than a century. Several observations are striking: regardless of the stock yield measure (dividend or cyclically adjusted earnings yield), the stock-bond yield correlation is highly persistent and switches sign between nearly one (pre-WWII and post-Bretton Woods) and close to minus one (Bretton Woods and, to a lesser extent, post-Great Recession). The stock yield-inflation correlation is slightly less persistent in the pre-Bretton Woods period, but behaves similarly later in the sample. These low-frequency correlations are unlikely to be driven solely by business cycle fluctuations (Campbell et al., 2017; Hasseltoft, 2009, 2012; Song, 2017; Duffee, 2018b). In contrast with the earlier literature that focuses on business-cycle frequencies, the key focus of our paper is to analyze the persistent correlations at a generational frequency.

The U.S. population age structure features twenty-year boom and bust cycles (see Figure 3). We conjecture that the population pyramid, characterized by the proportion of the middle-aged to young population, the MY ratio, drives the persistent component of financial yields. We develop a stochastic overlapping generations (OLG) model in which we introduce money through cash-in-advance constraints, in order to investigate how the demographic structure affects equity yields, inflation, and the three components of nominal bond yields: real bond yields, expected inflation, and the inflation risk premium. Two earlier papers analyze the ability of the MY ratio to forecast stock returns and bond yields, without modeling inflation. Favero et al. (2011) show a strong empirical link between dividend yield persistence and demographic fluctuations within the Campbell and Shiller (1988) framework. Favero et al. (2016), on the other hand, develop a no-arbitrage affine term structure model based on the assumption that the slow mean-reverting component of the (real) spot rate is driven by demographic fluctuations. In contrast to these papers, our proposed OLG model allows us to examine the underlying mechanisms to analyze all the components of the nominal bond yield in a single framework so as to understand low-frequency stock-bond correlation.

Insert Figure 3 here

Our model provides several testable predictions: not only are stock yields and real bond yields negatively correlated with the MY ratio (Geanakoplos et al., 2004), so are inflation and inflation risk premium. Demographic changes affect nominal yields mainly through real bond yields, and hence, on average, nominal bond yields and real stock yields comove positively at generational frequencies. However, it is possible that this comovement becomes negative in a few states of the world, depending on monetary regime switches and income shocks. Based on the demographic effects on yields and the inflation risk premium implied by the model, we also test whether the predicted future patterns of the MY ratio can improve return predictability within a present value framework.

The middle-age to young ratio as an empirical proxy for the change in the U.S. age structure is motivated by an OLG model developed by Geanakoplos et al. (2004, henceforth GMQ). We develop a monetary version of the stochastic GMQ model. Our model shows that the age structure of the U.S. population affects not only the real returns of financial assets but also the aggregate price level, inflation, and the inflation risk premium. The equilibrium relation between financial yields is robust to the presence of monetary shocks that capture monetary regime shifts. Over the life-cycle individuals facing a hump-shaped income stream save when middle aged, which in turn increases real asset prices causing a negative correlation between the MY ratio and the real yields. Middle-aged workers being more productive, a large MY ratio fosters aggregate real production and aggregate real income. As economic activity grows, money demand goes up, which leads to a reduction in the aggregate price level to equilibrate the money market. Therefore, the price level is inversely related to the MY ratio. In the model, the volatility of output and income is increasing in the proportion of young individuals as in Jaimovich and Siu (2009). Since this volatility generates a higher risk of facing low consumption growth together with high inflation, it increases the risk of holding nominal bonds. When the MY ratio is low, the model predicts that the inflation risk premium is relatively high. As a whole, the population age structure impacts all three components of nominal yields. Isolating and quantifying the demographic effect on these components is crucial for understanding the comovement between financial yields.

Empirically testing the impact of demographics on financial variables and inflation is particularly challenging due to their highly persistent nature. The fact that long time series of real bond yields are not directly observable adds a further challenge. We address the latter problem by following two separate estimation strategies. First, over the long time series, we estimate a large set of inflation forecasting models to generate inflation expectations, and we derive real bond yields by subtracting model-generated inflation expectations from nominal bond yields. We demonstrate the sensitivity of our results to different model specifications. Second, for the post-Bretton Woods sample, we explicitly take into account the inflation risk premium by using survey-based inflation expectations, and data from the inflation-indexed bond market, extended using models based on the term structure of survey-based inflation forecasts (Chernov and Mueller, 2012). To obtain the inflation risk premium, we subtract real bond yields (net of the liquidity premium) and inflation expectations from the nominal bond yields.

We test all model predictions using both Pearson correlation with bootstrapped p-values and Müller and Watson (2018) methodology. The latter allows inference using highly persistent variables to test the low-frequency correlation between key variables in our OLG model. Since the model is calibrated using U.S. data, we first show our results for the U.S. sample before extending the analysis to a cross-country panel. Our results support the theoretical predictions listed above. As a general pattern, the MY

ratio correlates negatively with nominal bond yields, real bond yields, inflation, and the inflation risk premium, although the significance varies across samples and model specifications. Moreover, relying on a stochastic Fisher decomposition of nominal bond yields, we maintain that the MY ratio affects nominal bond yields mainly through its effect on real bond yields, since the demographic effect on both the inflation risk premium and expected inflation is moderate. Consequently, both real stock and nominal bond yields comove with the changing population age structure and hence, correlate positively. Also, this correlation switches sign in a few sub-periods, in line with our model predictions. Our cross-country analysis shows that cross-sectional differences in stock-bond yield correlation are mainly explained by the differences in the magnitude of the demographic effect on real yields.[1] Moreover, we test the predictability of stock market excess returns by modifying the present-value relation suggested by Maio (2013). We incorporate information on the real stock yields, nominal bond yields and predictable future demographic fluctuations. We show that nominal bond yields together with future demographic information improve stock return forecasting ability compared to earlier studies (Favero et al., 2011; Maio, 2013). However, the forecasting ability appears to be mainly channeled through the link between the MY ratio and the level of bond yields, in line with Favero et al. (2016), rather than through the link between the MY ratio and the inflation risk premium.

Although stock and bonds are the two main asset classes considered in long-term portfolio allocation (e.g., Bali et al., 2009; Levy, 2015),[2] the existing literature mainly focuses on the comovement between stock and bond returns at business cycle frequencies. A growing body of literature focuses on the joint dynamics of stock and bond markets (Baele et al., 2010; Burkhardt and Hasseltoft, 2012; Campbell et al., 2017, 2014; Ermolov, 2015; Koijen et al., 2017; Lettau and Wachter, 2011; Hasseltoft, 2012; Song, 2017). For instance, Campbell et al. (2017) develop a model based on four state variables to explain the covariance between stock and bond returns, and find that stock-bond covariance is driven by the covariance between nominal variables and the real economy. Koijen et al. (2017) propose a arbitrage-free stochastic discount factor (SDF) model where the pricing factors are motivated by a permanent/transitory decomposition of the pricing

[1] In a recent paper, Bekaert and Ermolov (2019) show that real yields play the major role in explaining cross-country comovement in nominal yields.

[2] An article ("How Much Stock Should You Own in Retirement?") published on 3 Feb 2014 in the *Wall Street Journal* discusses the asset allocation problem from a long-term perspective.

kernel. Song (2017) develops a model that incorporates monetary policy aggressiveness and macroeconomic shocks, and thereby explains the sign switch in stock-bond return correlation. However, none of these papers consider low-frequency time-series variation in demographics as the source of a persistent component.

The impact of demographic fluctuations on real yields and inflation has been recently discussed in the literature. Carvalho et al. (2016) show the different channels through which demographic changes can affect real interest rates. They focus on the increase in longevity and the reduction in population growth. While the former reduces real rates via increased saving for retirement, the latter has counteracting effects, leading to an overall reduction in real rates. Aksoy et al. (2015) analyze the effects of demographic changes on macroeconomic variables, and show that the proportion of the dependent population (young and old) has a negative effect on real rates. Also, inflation seems to correlate positively with the share of dependents, that is, young and old individuals in the economy (Aksoy et al., 2015; Juselius and Takats, 2015; Juselius and Takáts, 2018), with net savers (Lindh and Malmberg, 2000), and with the growth rate of working age population (Bobeica et al., 2017), but correlates negatively with the old-age dependency ratio (Broniatowska, 2017). We deviate from these two strands of literature along several dimensions. First, because our focus is on the composition of the workforce (young versus middle-aged workers), and its time variation (Geanakoplos et al., 2004; Feyrer, 2007), none of those demographic channels are present in our model. Indeed, these demographic factors exhibit strong time trends, while our model is built upon a stationary population pyramid. Second, we are not focusing on the demographic effect on the real interest rate or inflation in isolation. Instead, we propose a unified analysis to investigate how the demographic structure affects all the components of nominal yields. The decomposition of the demographic effect enables us to identify the dominant channel, which is the real yield channel. Finally, we find that the alternative demographic factors put forward in this literature fail to explain the cross-country heterogeneity in stock-bond yield comovement.

The remainder of the paper is organized as follows. Section 2 introduces the monetary OLG model. Section 3 presents three theoretical predictions about the relation between the population age structure and financial markets. Section 4 tests these predictions empirically and provides evidence that a demographic factor drives the long-run component of financial yields. Section 5 concludes.

2 A Stochastic Monetary Exchange Economy

We develop a stochastic model of a monetary exchange economy in order to show the mechanisms through which the population age structure affects real returns, inflation, the inflation risk premium, and nominal yields. The stochastic feature of the model introduces an equity premium and an inflation risk premium and demonstrates the robustness of the yield correlation under different inflation regimes.

2.1 Model

2.1.1 Overview

We develop a stochastic 3-period OLG model of a monetary exchange economy. We extend the stochastic model developed by GMQ (2004) to a monetary economy by introducing a Clower (1967)-type cash-in-advance constraint. Each period lasts 20 years. Young and middle-aged individuals supply labor inelastically and receive labor income, while retired individuals live off their savings. The superscripts y, m and r indicate the individual's respective life stages: young, middle aged and retired. This life-cycle portfolio behavior, as described by Bakshi and Chen (1994), plays an important role in determining equilibrium asset prices. Two types of financial instruments, bonds and stocks, are available and allow agents to redistribute income over time. We assume that in odd (even) periods, a large (small) cohort enters the economy, so that in every odd (even) period, the demographic structure is (N,n,N) ((n,N,n)). We focus on medium- to long-run demographic fluctuations, abstracting from short-run and business cycle frequencies.

2.1.2 Stochastic Stream of Wages and Dividends

Following GMQ, we introduce random shocks to wages and dividends to circumvent the substitutability between bonds and stocks. This assumption enables us to analyze the impact of the age structure on stock prices and risk premium in a framework that incorporates the risks that individuals face when planning their life-time consumption. Labor and production plans yield real wages $w_{j,s} = (w^y_{j,s}, w^m_{j,s})$ and real dividends $d_{j,s}$, respectively, in each period j, $j = \{odd, even\}$, and income state s. Income shocks are such that both wages and dividends can take low or high values: $w_j = \{w^L_j, w^H_j\} = \{(w^{y,L}_j, w^{m,L}_j), (w^{y,H}_j, w^{m,H}_j)\}$ and $d_j = \{d^H, d^L\}$. Therefore, the stochastic income structure features four income states denoted by $s = \{s_1, s_2, s_3, s_4\}$, where $s_1 = (w_j^H, d^H)$, $s_2 = (w_j^H, d^L)$, $s_3 = (w_j^L, d^H)$, and $s_4 = (w_j^L, d^L)$. The stochastic wage structure $w_{j,s} = (w_{j,s}^y, w_{j,s}^m)$ reflects the higher productivity of middle-aged workers compared to young workers, as we assume that $w_{j,s}^y < w_{j,s}^m$ in any demographic structure j and income state s. Moreover, each individual faces a stream of wages ($w_{j,s}^y$, $w_{j+1,s+1}^m$, 0) that is concave over her life time: $w_{j,s}^y < w_{j+1,s+1}^m$ in any period j and $j+1$ and in any income state s and $s+1$.

2.1.3 The Role of Money

In our setting, the essential role of money is that of a medium of payments. We build on the cash-in-advance setting proposed by Bénassy (2005). We assume that, in each period, each individual possesses an income composed of her labor income, for working individuals, and the financial returns of previous savings, if taken. Then, the bond and stock markets open, and each individual decides upon her financial investment. The rest of her income is kept in the form of money and constitutes the individual's money demand. This money holding is eventually traded against the consumption good. As a result, agents face a within-period cash-in-advance constraint that embodies the assumption that money is the only means of purchasing the consumption good. Consequently, individuals hold money in each of their three periods of life, irrespectively of being a borrower or saver. Because it does not pay interest, money is a dominated asset that is entirely consumed during each period. In other words, bonds and equities are the only instruments that are carried across periods to smooth consumption over time. Consequently, money holdings are more closely related to consumption expenditures than to savings, a feature that matches empirical regularities (Handa, 2002). This feature is also in line with the periodicity of the model. Indeed, given that each period lasts 20 years, it is reasonable to assume that money is not carried over time to allow consumption deferral over 20 years. Such a cash-in-advance constraint, as introduced by Lucas (1982),[3] presents the following advantages. First, it isolates the money demand functions from the specific choice of utility functions, an issue that prevails in money in the utility function models. Second, different from models that feature both money and bonds as stores of value, we obtain a monetary equilibrium without relying on additional assumptions regarding demographic change or monetary policy that affect the return of money. Finally, as

[3] This cash-in-advance constraint also relates to that proposed by Artus (1995) and Heer et al. (2011).

argued by Heer et al. (2011), cash-in-advance constraints are useful in explaining the heterogeneity of money holdings across different age groups.

2.1.4 Monetary Regimes

Since monetary regimes differ in their success at establishing a credible framework to control inflation over the last century (Bordo and Haubrich, 2008; D'Agostino and Surico, 2012; Filipova et al., 2014; Meltzer, 1986), we assume that monetary policy makers pursue a time-varying inflation target. In such a long-run setting where changes in the stock of money lead to changes in the price level, this assumption translates into a time-varying adjustment M_g^S, where the subscript $g = \{g_1, g_2, g_3, g_3\}$ represents the four states of money supply. In this setting, which is similar in spirit to the exogenous monetary policy rule presented in Song (2017), expectations about future inflation are either low -g_1- (corresponding to the Mixed Regime and QE periods), or medium -g_2- (as during the Pre-Fed, Gold Standard, Bretton Woods and Great Moderation periods), or high -g_3- (Pegged Regime), or very high -g_4- (Great Inflation). For the sake of simplicity, the monetary regimes are independent. With this exogenous structure of money supply, we implicitly assume that money supply did not react to demographic fluctuations. We justify this assumption of exogeneity by providing evidence that the Fed did not adjust money supply in response to changes in inflation and the output gap that were triggered by changes in the demographic structure (see Appendix A).

2.1.5 Individuals

The utility function features constant relative risk aversion and is intertemporally additive. Therefore, a young individual born in period j, $j = \{odd, even\}$, and income state s maximizes $U(c^y_{j,s}) + \beta U(c^m_{j+1,s+1}) + \beta^2 U(c^r_{j+2,s+2})$, where $\{c^y_{j,s}, c^m_{j+1,s+1}, c^r_{j+2,s+2}\}$ is her real consumption stream over the three life periods. Let $q_{j,s}$ and $q^e_{j,s}$ be the real bond price and real stock price in period j and income state s, respectively. In Appendix B, we develop an equivalent model that posits the nominal price of a corresponding nominal bond that promises to pay $1/P_{j+1,s+1,g+1}$ units of consumption at time $j+1$, where $P_{j+1,s+1,g+1}$ is the price of the consumption good at time $j+1$. In the same appendix, we explicitly link the real and the nominal interest rate on the bond through the Fisher equation. $(zb^y_{j,s}, ze^y_{j,s}, zb^m_{j+1,s+1}, ze^m_{j+1,s+1})$ represent the real asset holdings (bonds and stocks) of an individual born in period j and income state s. The real borrowing constraints of a young individual born in period j and income state s are:

$$
\begin{aligned}
c^y_{j,s} + q_{j,s} z b^y_{j,s} + q^e_{j,s} z e^y_{j,s} &= w^y_{j,s} \\
c^m_{j+1,s+1} + q_{j+1,s+1} z b^m_{j+1,s+1} + q^e_{j+1,s+1} z e^m_{j+1,s+1} &= w^m_{j+1,s+1} + z b^y_{j,s} + (q^e_{j+1,s+1} + d_{s+1}) z e^y_{j,s} \\
c^r_{j,s+2} &= z b^m_{j+1,s+1} + (q^e_{j,s+2} + d_{s+2}) z e^m_{j+1,s+1}
\end{aligned}
$$

where $j + 2 = j$ by the cyclicity of the demographic structure.

Let $1/\sigma$ denote the intertemporal elasticity of substitution between consumption in any two periods. The maximization by young and middle-aged agents of their intertemporal utility functions leads to the following Euler equations that determine optimal consumption choices over time:

$$
\begin{aligned}
(c^y_{j,s})^{-\sigma} q_{j,s} &= \beta E_{j,s} (c^m_{j+1,s+1})^{-\sigma} \\
(c^m_{j,s})^{-\sigma} q_{j,s} &= \beta E_{j,s} (c^r_{j+1,s+1})^{-\sigma}
\end{aligned}
\tag{1}
$$

and

$$
\begin{aligned}
(c^y_{j,s})^{-\sigma} E_{j,s} \frac{q^e_{j,s}}{q^e_{j+1,s+1} + d_{s+1}} &= \beta E_{j,s} (c^m_{j+1,s+1})^{-\sigma} \\
(c^m_{j,s})^{-\sigma} E_{j,s} \frac{q^e_{j,s}}{q^e_{j+1,s+1} + d_{s+1}} &= \beta E_{j,s} (c^r_{j+1,s+1})^{-\sigma}
\end{aligned}
\tag{2}
$$

These equations state that individuals who are young or middle aged in period j and income state s choose to reduce their future consumption when the real cost of deferring consumption from period j to period $j+1$, $q_{j,s}$ or $E_{j,s} \frac{q^e_{j,s}}{q^e_{j+1,s+1} + d_{s+1}}$, increases or when the discount factor β decreases.

In each stage of life, the consumption good has to be paid for in cash. Because money is a dominated store of value, each individual's stream of nominal money demand $M_{j,s,g}$ equals the optimal consumption structure specified by the Euler equations times the price of the consumption good, $P_{j,s,g}$. Therefore, the within-period cash-in-advance constraints are as follows:

$$
c^y_{j,s} = \frac{M^y_{j,s,g}}{P_{j,s,g}} \qquad c^m_{j,s} = \frac{M^m_{j,s,g}}{P_{j,s,g}} \qquad c^r_{j,s} = \frac{M^r_{j,s,g}}{P_{j,s,g}}
\tag{3}
$$

2.2 Equilibrium

The economy is in a decentralized equilibrium at all times; that is, all individuals choose their consumption stream optimally (Equations (1) and (2)). Moreover, the cash-in-advance constraints (Equations (3)) must be respected in equilibrium, and the following resource constraints must be satisfied in all periods:

$$\begin{aligned} Nc^y_{o,s} + nc^m_{o,s} + Nc^r_{o,s} &= Nw^y_{j,s} + nw^m_{j,s} + d_s \\ nc^y_{e,s} + Nc^m_{e,s} + nc^r_{e,s} &= nw^y_{j,s} + Nw^m_{j,s} + d_s \end{aligned} \tag{4}$$

$$\begin{aligned} N\frac{M^y_{o,s,g}}{P_{o,s,g}} + n\frac{M^m_{o,s,g}}{P_{o,s,g}} + N\frac{M^r_{o,s,g}}{P_{o,s,g}} &= \frac{M^S_g}{P_{o,s,g}} \\ n\frac{M^y_{e,s,g}}{P_{e,s,g}} + N\frac{M^m_{e,s,g}}{P_{e,s,g}} + n\frac{M^r_{e,s,g}}{P_{e,s,g}} &= \frac{M^S_g}{P_{e,s,g}} \end{aligned} \tag{5}$$

The first two equations represent the equilibrium in the goods market, whereas the two last equations state that the money market clears in both odd and even periods.

By substituting the cash-in-advance equations into the resource constraints of the money market, the equilibrium conditions listed here can be expressed as functions of consumption levels ($c^y_{j,s}$, $c^m_{j,s}$ and $c^r_{j,s}$), asset prices ($q_{j,s}$ and $q^e_{j,s}$), saving decisions ($zb^y_{j,s}$ and $zb^m_{j,s}$) and real money supply ($\frac{M^S_g}{P_{j,s,g}}$). This means that money is neutral, that is, increases in the nominal money supply are entirely absorbed by a proportional increase in the price level and leave real activity unaffected. This explains why real variables are not indexed by the money supply state g. This feature of the model is justified in the medium to long run.

2.3 Solving the Model

Solving for the equilibrium requires identifying the four elements that constitute the state space: the population pyramid j, the state of incomes s, the state of the money supply g, and the portfolio income received by middle-aged workers, which is determined by past shocks. Note that while the population pyramid follows a deterministic path, incomes and monetary regimes are stochastic. The equilibrium is characterized as follows: i) young workers optimally choose their saving and portfolio structure, given their budget constraint when young and their expected budget constraint when middle aged; ii) middle-aged workers optimally choose their saving and portfolio structure, given their budget constraint when middle aged and their expected budget constraint when retired; iii) the bond market and the stock market clear; and iv) the asset prices that individuals expect for the following period and income state, when deciding upon their portfolio, are equal to the asset prices that clear the bond and stock markets in the following period and income state, when agents receive such portfolio income. Moreover, the savings that young workers expect to make in the following period and income state, when deciding upon their portfolio, are equal to the savings that middle-aged workers actually choose in the following period and income state, would they receive such portfolio income. The last condition assures that expectations about asset prices and saving decisions are correct.

To solve for the equilibrium, we form a grid of portfolio incomes inherited by middle-aged individuals from period $t-1$. Then, we choose initial expectation functions over asset prices and saving decisions that will be realized in $t+1$. We solve for the optimal portfolio decisions of young and middle-aged workers in t (retired individuals do not make any portfolio decision), for each point of the grid, given the expectation functions. Next, we solve for the optimal portfolio decisions of young and middle-aged workers, and therefore for the equilibrium asset prices and saving decisions in $t+1$, given the expectation functions and the portfolio income inherited by middle-aged workers from period t. The equilibrium asset prices and saving decisions are used to update the expectation functions. We repeat the algorithm until convergence.

2.4 Calibration

For the sake of comparison, we closely follow GMQ's calibration. We interpret a period as 20 years. We take $(n, N) = (52, 79)$ as the size in millions of the Great Depression (1925-1944) and Baby Boom (1945-1964) generations so that, in the model, the middle-age to young ratio MY alternates between 0.66 in even periods and 1.52 in odd periods. For comparison, we also provide the results obtained under the robustness specification $(n, N) = (69, 79)$, which represents the Baby Boom (1945-1964) and Baby Bust (1965-1984) generations.

We assume an annual discount factor of 0.97, which translates into a discount factor of 0.5 at a 20-year frequency. The value of the intertemporal elasticity of substitution is still debated.[4] We set the value of the elasticity of substitution equal to $1/4$. Robustness

[4]Papers that calibrate macroeconomic models to match growth and business cycle facts usually use values around unity. After the seminal work by Kydland and Prescott (1982), who set the substitution

checks for alternative values ($\sigma = 1; 2; 6$) show that changes in the elasticity of substitution modify only slightly the effect that the population age structure has on asset prices and does not impact the demographic effect on inflation.

Concerning incomes and dividends, we set the average wage of young and middle-aged workers over income states to 2 and 3, respectively, to match the ratio of average annual real income of middle-aged to young individuals in the U.S. The average ratio of dividends to wages is equal to 0.19 in the U.S. In the baseline specification characterized by the age structure $(n, N) = (52, 79)$, total wages in odd (even) periods are, on average across income states, equal to 314 (341), so we set the average level of dividends equal to $0.19(\frac{314+341}{2})$. In the robustness specification, the age structure is $(n, N) = (69, 79)$. Total wages in odd (even periods) are, on average across income states, equal to 365 (375), so we set the average level of dividends equal to $0.19(\frac{365+375}{2})$. To obtain the stochastic structure of wages and dividends, the average coefficient of variation of young workers' wages, middle-aged workers' wages, and dividends across odd and even periods are set to 15%, 20% and 19%, respectively (see GMQ). Additionally, we follow Jaimovich and Siu (2009), who document the negative correlation between the volatility of real GDP growth and the MY ratio,[5] and we assume dependence between the young and middle-age income coefficients of variation and the demographic structure. Specifically, we allow the income coefficients of variation to vary in odd and even periods so as to target a standard deviation of aggregate income growth that is 10% higher in odd periods than in even periods. As a result, the stochastic wage structure is $(w_o^{y,L}, w_o^{m,L}) = (1.8, 2.55)$, $(w_o^{y,H}, w_o^{m,H}) = (2.2, 3.45)$, $(w_e^{y,L}, w_e^{m,L}) = (1.6, 2.25)$, and $(w_e^{y,H}, w_e^{m,H}) = (2.4, 3.75)$. The stochastic dividend structure is given by $\{d^H, d^L\} = \{74, 50\}$ under the baseline specification $(n, N) = (52, 79)$ and $\{d^H, d^L\} = \{83, 57\}$ under the robustness specification $(n, N) = (69, 79)$. We take into account the positive correlation between wages and dividends and assign the following probabilities to each of the four income states s: $(0.4, 0.1, 0.1, 0.4)$.

elasticity to 0.66, most of the real business cycle literature has used a value close to one. Other studies, which mainly estimate Euler equations using aggregate consumption data, support lower values. Hall (1988) stands on the opposite side of the range with a value close to zero.

[5] Jaimovich and Siu (2009) show that output volatility rose in the U.S. from the early 1960s to the late 1970s, a pattern that is matched with the long-run fluctuations in the volatile-age labor force share, i.e., the share of individuals in the age ranges 15-29 and 60-64 in the 15-64 year-old labor share. Because the volatile-age labor force share is roughly the inverse of the MY ratio, this result points towards a negative correlation between output volatility and the MY ratio.

We normalize the initial price level in odd periods to one and set the money supply accordingly. The stochastic structure of the money supply is set to $g = (g_1, g_2, g_3, g_4) = (0.5\%, 2.5\%, 4\%, 6\%)$ in annualized terms, so as to match the observed average annual inflation rate over the Mixed Regime and Quantitative Easing periods (state g_1), over the Pre-Fed, Gold Standard, Bretton Woods and Great Moderation periods (state g_2), over the Pegged Regime period (state g_3), and over the Great Inflation period (state g_4). We assign the following probabilities to each of the four money supply states g: $(0.15, 0.6, 0.125, 0.125)$ to roughly match the relative length of the respective monetary regime(s) over the period 1900-2016. Because we do not know a priori how inflation expectations are formed over generational frequencies, we assume that inflation expectations only react to changes in monetary regimes and demographic fluctuations. We will assess the validity of the latter assumption in the empirical section.

3 Theoretical Predictions

In this section, we present the simulation results of the stochastic case, and we detail the static (no shocks) and deterministic (only demographic fluctuations) cases in Online Appendix A. We simulate a 100,000-period model and average the results obtained in each pyramid structure j, income state s, and money growth state g. We also report averages across states. We present the results in Tables 1 and 2 for the population age structure $(n, N) = (52, 79)$ and in Online Appendix B for the population age structure $(n, N) = (69, 79)$. Standard deviations, shown in parentheses, are small for almost all variables, which indicates that past shocks affect equilibrium values only marginally. Moreover, a paired sample t-test indicates that the average values are significantly different between odd and even periods.

3.1 Demographic Effects on Real Yields and Inflation

Individuals facing a hump-shaped income stream save when middle aged and dis-save when retired. Therefore, in odd periods, when the demographic structure is characterized by a small cohort of middle-aged individuals, aggregate saving is low and, relatedly, aggregate consumption is high. The opposite holds in even periods. The equilibrium in the goods market, and consequently in the bond and stock markets, is realized through the adjustment of the real price of financial assets. Comparing the last row of each panel of Table 2 shows that asset prices increase in even periods so as to prevent excess saving in the economy. Symmetrically, low real asset prices stimulate savings in odd periods when the MY ratio is low, and lead the asset and goods markets to clear. This explains the decrease in the annualized real interest rate by 114% and the decrease in equity yields by 50% over 20 years, on average, from odd to even periods.

While this demographic effect on asset prices is observed on average, income shocks alter the results. Indeed, high wages and dividends push individuals' demand for savings up, which makes stock prices increase and real yields fall. Inversely, stock prices are low and real yields are high when wages and dividends are low.

Insert Table 1 here

Insert Table 2 here

Using the cash-in-advance constraints, we substitute individual consumptions into money demands in the resource constraints of the goods market. Then, by embedding these resource constraints into the resource constraints of the money market, we obtain

$$P_{o,s,g} = \frac{M_g^S}{Nw_{o,s}^y + nw_{o,s}^m + d_s} \qquad P_{e,s,g} = \frac{M_g^S}{nw_{e,s}^y + Nw_{e,s}^m + d_s} \tag{6}$$

The money supply relative to aggregate real income/output determines the price level in the economy.[6] By taking logs, Equation (6) allows for a dynamic interpretation: the inflation rate is equal to the difference between the money supply growth rate and the growth rate of aggregate income/output. As economic activity grows (slows down), the demand for real cash balances increases (decreases), lowering (increasing) inflation. A similar mechanism linking real activity and inflation is put forward by Fama (1981). We take a step further and show that the level of real activity directly relates to the demographic structure. We can illustrate this relation by expressing Equations (6) as functions of MY_j, the MY ratio in period j, and $Young_j$, the number of young individuals

[6]Note that Equations (6) are special cases of the quantity theory exchange equation, in which the velocity of money is constant and equal to one. Extensions of Lucas' basic model have been provided to account for the variability of the velocity of money (see, for example Lucas, 1984; Svensson, 1985; Lucas and Stokey, 1987), but for tractability reasons, we do not introduce them into our model.

in period j. The following equation holds:

$$P_{j,s,g} = \frac{\frac{M_g^S}{Young_j}}{w_{j,s}^y + MY_j w_{j,s}^m + \frac{d_s}{Young_j}} \tag{7}$$

Middle-aged workers being more productive than young ones, a higher MY ratio implies higher aggregate productivity and hence higher aggregate real income/output. As economic activity grows, money demand increases, which leads to a decrease in the aggregate price level to sustain money market equilibrium. Therefore, the price level is inversely related to the MY ratio: prices are expected to be high (low) in odd (even) periods. The results in Table 2 confirm this prediction that the small proportion of middle-aged workers in odd periods pushes aggregate productivity down, leading to a low level of aggregate income/output and subsequently to a high price level. This mechanism generates a negative comovement between MY_j and realized inflation π_j.[7] As discussed in Section 2.1.4, this result arises because money supply does not vary with the MY ratio to offset the demographic effect on money demand and inflation.

Prediction 1: *The MY ratio correlates negatively not only with real bond yields and equity yields but also with inflation.*

Increases (decreases) in the MY ratio bring both equity yields and inflation down (up) and hence, fluctuations in the demographic structure generate comovement between equity yields and inflation, in line with the evidence in Panel B of Figure 2. Note that income shocks alter this demographic effect on inflation (Equation 6). In good income states, wages and dividends are large, and so is real output. Inflation being determined by the difference between money supply growth and the growth rate of aggregate real output, high-income states are associated with low inflation.

[7] Several empirical papers (Aksoy et al., 2015; Broniatowska, 2017; Juselius and Takats, 2015; Juselius and Takáts, 2018) document a stable and significant correlation between (trend) inflation and the share of dependents (that is, young and old individuals) in the economy, net savers (Lindh and Malmberg, 2000), and the growth rate of the working age population (Bobeica et al., 2017).

3.2 Inflation Risk Premium and Nominal Bond Yield

Because of the uncertainty introduced by wage and dividend shocks, bonds and stocks are imperfect substitutes. Our model does not include any additional channels that would generate the high equity premium that is observed in the market. Therefore, the resulting equity premium is relatively low, approximately 1%, on average. The equity premium fluctuates with the MY ratio. Since the volatility of income growth is larger in odd periods, agents investing in odd periods face greater variability in consumption growth, which makes them less tolerant of the extra risk of investing in stocks. Consequently, they demand higher compensation for taking on the greater risk in odd periods.

Using the Bekaert and Engstrom (2010) decomposition, we derive the stochastic Fisher equation and decompose the annualized yield on the nominal bond i into three components:

$$\begin{aligned} i_{j,s,g} &= r_{j,s} + E_{j,s,g}\pi_{j+1,s+1,g+1} + irp_{j,s,g} \\ \text{where } irp_{j,s,g} &= -\frac{1}{2}Var(\pi_{j+1,s+1,g+1}) + Cov(\ln\frac{(c^m_{j+1,s+1})^{-\sigma}}{(c^y_{j,s})^{-\sigma}}, \pi_{j+1,s+1,g+1}) \end{aligned} \tag{8}$$

where $r_{j,s}$ is the annualized real interest rate on bonds from period j to period $j+1$, $E_{j,s,g}\pi_{j+1,s+1,g+1}$ is the annualized expected inflation from period j to period $j+1$, $\pi_{j+1,s+1,g+1}$ denotes the realized future inflation from period j to $j+1$, and $irp_{j,s,g}$ is the inflation risk premium. See Appendix B.2 for the derivation.

Wage and dividend shocks generate unanticipated fluctuations in consumption growth and inflation. By decreasing aggregate output, adverse wage and dividend shocks increase inflation (Equation 6). Therefore, wage and dividend shocks drive consumption growth and inflation in opposite directions. This negative covariance makes nominal bonds risky and gives rise to a positive inflation risk premium that averages 0.65% across pyramid structures. This order of magnitude falls within the range of estimates proposed in previous studies: Haubrich et al. (2012) estimate the 10-year premium to average 0.44% over the period 1982-2010, Buraschi and Jiltsov (2005) find that the 10-year premium averages 0.7% over the period 1960-2000. While Ang et al. (2008) obtain a 5-year premium of 1.15% over the period 1952-2004, it reduces substantially (0.14%) in the recent period 2004-2016 according to Bekaert and Ermolov (2019).

Changes in inflation and consumption growth that stem from changes in the demographic structure are anticipated by agents and therefore are not generating any inflation risk premium. However, the demographic structure interacts with wage and dividend shocks. Indeed, following Jaimovich and Siu (2009), the variability of income growth decreases with the MY ratio. Consequently, in odd periods, agents might face lower consumption growth together with a higher inflation rate, compared to even periods. In odd periods, nominal assets are therefore more risky, and investors demand a larger premium to hold them. As a result, our estimated inflation risk premium is negatively correlated with the MY ratio (0.91% in odd periods, 0.40% in even periods).

Prediction 2: *The inflation risk premium is negatively correlated with the MY ratio.*

This negative correlation suggests that the decline in the volatility of aggregate income and consumption associated with the increase in the MY ratio after the early 1980s might have contributed to the observed decrease in the inflation risk premium over this period. Our analysis corroborates the finding of two studies (Buraschi and Jiltsov, 2005; Ang et al., 2008) that outline the dynamics of the inflation risk premium over time: the premium seems to have increased between 1960 and 1980, to have peaked in the early 1980s, and to have decreased over the following 20 years. Our finding also complements the explanation put forward by Song (2017), who shows that, starting in the early 1980s, inflation became less risky as the Federal Reserve shifted towards an active monetary policy by increasing the interest rates more than one-to-one with the inflation rate, leading to a decrease in the inflation risk premium. Song (2017) also finds that, from the 2000s onwards, pro-cyclical inflation shocks made nominal assets good hedges against income shocks, and the inflation risk premium turned negative. In our model, we gather from Equation (6) that inflation is always countercyclical and risky. For this reason, our model cannot generate a negative inflation risk premium observed in the recent period (Bekaert and Ermolov, 2019).

Compared to the effect on real bond yields, the demographic effects on expected inflation and the inflation risk premium are relatively moderate and of opposite sign. However, since the demographic effects on real yields and the inflation risk premium have the same sign, fluctuations in nominal yields across pyramid structures due to the real channel are amplified when the inflation risk premium is taken into account. Moreover, the effect of income shocks on real yields transmits into changes in nominal yields across income states. As a result, nominal and real yields correlate positively.[8]

3.3 The Comovement Between Bond and Stock Yields

The demographic effect in the model is such that an increase in the MY ratio, from odd to even periods, leads to a decrease in real bond returns, equity yields, and the inflation risk premium, and to an increase in expected inflation, as presented in the previous subsection. Because the demographic effect on expected inflation is moderate, changes in the demographic structure trigger positive comovement between nominal and real bond yields, as well as positive comovement between nominal bond yields and equity yields.

Prediction 3: *Because both nominal bond yields and equity yields are negatively driven by the MY ratio, the comovement between nominal bond yields and equity yields is positive at low frequencies, in most of the states.*

The model-implied correlation between nominal bond yields and equity yields is 0.81 in the model, a magnitude that is comparable to the correlation coefficients shown in Figure 1 and Figure 2.[9] However, it is important to note that while our model predicts positive comovement between nominal bond and real equity yields, wage and dividend shocks as well as changes in monetary regimes make the correlation negative in a few specific subperiods: when stochastic shocks counteract the demographic effect. To show this, we decompose the stochastic model results by demographic structure, income state, and money supply state, as shown in Table 3. First, low-income states, by curbing demand for saving, push bond and stock prices down and nominal and real yields up. As real yields are more sensitive to income shocks than are nominal yields, real yields increase from odd to even periods when the income shock effect dominates the demographic effect (for example, from state (Odd, s_1) to state $(Even, s_3)$). In this case, an increase in the

[8]When analyzing the volatility of the nominal bond yield, shocks other than changes in the demographic fluctuations have to be taken into account: wage and dividends shocks, and monetary regime changes. Controlling for the MY ratio, i.e. over time horizons shorter than 20 years, we obtain the following decomposition for the variance in bond yields: 76% is explained by the variance in real yields, 16% is explained by the variance in expected inflation, and 8% is explained by the variance in the inflation risk premium. The relative importance of expected inflation is in line with Duffee (2018a), between the range in Bekaert and Ermolov (2019) and Ang et al. (2008).

[9]The model implied correlation coefficient is quite robust to changes in the assumptions about the stochastic structure of the money supply. This evidence is available from the authors upon request.

MY ratio from odd to even periods will be associated with an increase in real yields and a decrease in nominal yields.

Insert Table 3 here

We summarize the effect of income shocks on the sign of the correlation in Figure 4, Panel A. The bottom-right quarter shows an average correlation of -0.04, indicating that, as an economy moves from a low-MY demographic structure and high-income state (s_1 or s_2) to a high-MY demographic structure and low-income state (s_3 or s_4), or vice versa, the model predicts the comovement between nominal bond yields and equity yields to be negative. This result provides a rationale for the extended period of negative correlation observed in the 1960s, a period characterized by a falling MY ratio, pushing up both nominal and real yields, and a booming economy counteracting the demographic effect on real yields.

Insert Figure 4 here

Second, the positive correlation between nominal bond yields and equity yields is also affected by money supply shocks and therefore by changes in individuals' expectations about future inflation. In states g_3 and g_4, high inflation expectations cause nominal yields to increase. The inverse occurs in states g_1 and g_2, when inflation expectations are relatively low. While both nominal and real yields decrease on average from odd to even periods, nominal yields would increase if expectations about future inflation increase simultaneously (for example, from states s_3, g_1 to states s_3, g_4), as seen in Table 3. This would lead to a temporary negative comovement between nominal and real yields. However, note that the model's ability to explain the recent period (unconventional monetary policy) is limited, since it is not designed to capture the peculiarities of each monetary regime (Song, 2017).

We summarize the effect of money supply shocks on the sign of the correlation in Figure 4, Panel B. In the upper-left corner, we observe that, as the MY ratio increases from odd to even periods, nominal bond yields and equity yields correlate negatively when the income state remains low (income and dividend state s_3 or s_4) and the inflation rate is expected to increase sharply, from g_1 to g_4. This mechanism, when reversed, sheds light upon the negative correlation between bond and stock yields observed during the QE period, as this period is characterized by a decreasing MY ratio and relatively low expectations about future inflation.

4 Empirical Evidence

4.1 Methodology

4.1.1 Data

In this section, we introduce the empirical counterparts of the key variables in the model. We use the ratio of the number of individuals aged 40-49 to the number of individuals aged 20-29 as a proxy for the model-implied MY_t. We use real time demographic projections to avoid look-ahead bias. We hand-collect projected values of the demographic variable from various past U.S. Census reports (the middle series of the most recent report available at the time of the forecast). For instance, the projected values for the period 1964-1969 are the forecasts from the report published in 1964. We use both the dividend price ratio, dy_t, and the cyclically adjusted earnings price ratio, ey_t, as a proxy for the equity yield. The long rate i_t is the nominal yield on the 10-year Treasury note. Annual inflation is denoted π_t and is computed using monthly inflation compounded to annual frequency (Welch and Goyal, 2008). We describe the estimation of long-run inflation expectations $E_t\pi_{lr}$ in Section 4.1.3. For other variables, we use annual data using last month if monthly data are available (see Appendix C for a detailed description of time series).

4.1.2 Empirical Strategy

We face several challenges to test the model predictions. First, real long-term bond yields are not observable for a long sample. Second, most of the variables, e.g., equity yields, i_t, and MY_t, are highly persistent (see Table 4) over both the long sample (1900-2016) and the recent post-Bretton Woods sample (1972-2016); hence, standard correlation measures can be misleading. Third, it is difficult to account for the inflation risk premium for a long time series since term structure data are not available.

Insert Table 4 here

Our first strategy is to extract real bond yields using a set of long-term inflation forecasting models, under the deterministic Fisher hypothesis (See Appendix B.1). Once we compute the model-dependent inflation expectations, we subtract them from the nominal yields to obtain real bond yields. Then, we test the long-run correlation between real bond yields and the demographic variable for both a long time series (1900-2016) and the post-Bretton Woods period (1972-2016). We report the sensitivity of the correlations to the model choice.

We compute the long-run correlation among variables using the Müller and Watson (2018) framework that allows inference using highly persistent variables to test the low-frequency correlation of key variables in our OLG model. The method relies on cosine functions to extract periodicities relevant at generational frequencies, that is, beyond 10 years. In particular, following Müller and Watson (2018), we set q=18 (q=6) for the long sample (post-Bretton Woods) capturing periodicities of T/9=13 (T/3=15) years. The main advantage of the framework is that one can also compute the Bayesian confidence sets that enable inference with highly persistent variables such as the equity and bond yields, and the demographic variable. For comparison, we also report Pearson correlations and bootstrapped p-values that take into account the persistence of each variable. We explain the bootstrap procedure in Online Appendix C.

For the post-Bretton Woods sample, we take into account the inflation risk premium. We use survey-based long-term (10-year) inflation expectations, extended backwards using the Kalman filter suggested by Bekaert and Engstrom (2010), as well as data from the inflation-indexed bond market (e.g., TIPS), extended using a model based on the term structure of survey-based inflation forecasts (Chernov and Mueller, 2012) to obtain real rates and the inflation risk premium. We also account for potential liquidity problems in the inflation-indexed bond markets, particularly in the early period of the TIPS market and during the Great Recession (D'Amico et al., 2018; Ermolov, 2017; Pflueger and Viceira, 2016).

We test the model predictions both in the U.S. sample and international data from 23 countries. A country is included in our sample if there are at least 30 years of data for all observable variables. Here we report the results from a balanced panel of 20 countries over the post-Bretton Woods period, and provide the results using the longest available data for each country in Online Appendix D. Finally, we test whether we can explain cross-country differences in stock-bond long-run correlations by taking into account alternative explanations, via stagflation incidents, the GDP-inflation correlation, the consumption growth-inflation correlation (Bekaert and Engstrom, 2010; Song, 2017) or other demographic channels such as population growth, life expectancy, the dependency ratio or the share of the elderly population (Aksoy et al., 2015; Carvalho et al., 2016).

4.1.3 Long-run Inflation Expectations

We estimate a large set of long-run inflation forecasting models to generate inflation expectations. As a theoretical benchmark, we first compute inflation expectations with perfect foresight, $E_t\pi_{lr}^{pf}$, that is, the average 10-year future inflation (up to 2006). Under the assumption of no inflation forecastability, we also document the naive random walk inflation forecast, $E_t\pi_{lr}^{nrw}$, that is, the current annual inflation, as well as the random walk with drift, $E_t\pi_{lr}^{rwd}$, that is, the sample average of annual inflation. In the spirit of Atkeson and Ohanian (2001), who use the past four quarters of inflation to forecast future annual inflation, we compute the moving average of the past 10 years of inflation to forecast long-run inflation, $E_t\pi_{lr}^{ao}$. Motivated by the learning literature, we also compute the discounted 10-year moving average, using the constant gain-learning parameter υ=0.987 (Cieslak and Povala, 2015). As parsimonious specifications, we consider autoregressive models, AR(p) with short lags, p={1,2}, and an ARMA(1,1) model in light of (Ang et al., 2007). We also estimate bivariate VAR(p) models, p={1,2}, including money growth, ΔM_t^S. We use the narrow definition of money, i.e., currency in circulation, for the long sample, and the broad definition of money, i.e., M2, for the post-Bretton Woods period. In order to capture the time-variation in model parameters, we estimate AR(p) and VAR(p) models with drifting coefficients and stochastic volatility (D'Agostino and Surico, 2012).

We start with an in-sample estimation, using 20 years of data. For the long sample, the in-sample period spans from 1880 to 1899, while for the post-Bretton Woods sample, the in-sample period spans from 1952 to 1971. We run both recursive and rolling window estimations to obtain long-run inflation expectations.[10] Next, we generate 10-year-ahead forecasts by iterating forward the one-step-ahead forecasts up to 10 years to compute the average inflation over the period:

$$
\begin{aligned}
\Pi_t &= \mu_t + A_t\Pi_{t-1} + \varepsilon_t \\
\widehat{\Pi}_{t+1|t} &= \widehat{\mu}_t + \widehat{A}_t\Pi_t \\
\widehat{\Pi}_{t+n|t} &= \sum_{j=1}^{n} \widehat{A}_t^{j-1}\widehat{\mu}_t + \widehat{A}_t^{n}\Pi_t
\end{aligned}
$$

where n={1,2,..,10}, μ_t is the time-varying drift, and A_t is the time-varying matrix of coefficients. In the case of AR(p), Π_t is equal to annual inflation π_t, while in the case of

[10] For brevity, we report recursive estimation results. Rolling window results are available upon request.

VAR(p), Π_t is equal to the vector $\begin{pmatrix} \pi_t \\ \Delta M_t^S \end{pmatrix}$.

Similarly, we produce long-run inflation forecasts from the ARMA(1,1) model (Ang et al., 2007):

$$\begin{aligned}
\pi_t &= \mu + \rho\pi_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t \\
\widehat{\pi}_{t+n|t} &= \frac{1}{1-\rho}\left[1 - \frac{\rho(1-\rho^n)}{(1-\rho)}\right]\mu + \frac{\rho(1-\rho^n)}{(1-\rho)}\pi_t + \frac{(1-\theta^n)}{(1-\theta)}\varepsilon_t
\end{aligned}$$

where μ is the constant term, ρ is the autoregressive coefficient, and θ is the MA coefficient. We compute the average inflation over n periods, n={1,2,..,10}.

Once we obtain all the long-run inflation forecasts, we compare the out-of-sample forecasting performance of each model and choose the long-run forecasting model with the lowest root mean square (out-of-sample) forecast error (RMSFE), taking the naive random walk model as the benchmark. This long-run inflation expectation is denoted $E_t\pi_{lr}^{oos}$. We also report $E_t\pi_{lr}^{cw}$, the long-run inflation expectation obtained from the model with the highest Clark and West (2006) test statistics, which take into account the finite sample bias in FRMSE comparison (Hubrich and West, 2010).

4.2 Demographic Effect on Bonds, Equities and Inflation

U.S. Evidence. In Panel A of Table 5, we report the long-run correlation of observable variables with MY_t, both over the long sample (1900-2016) and the post-Bretton Woods period.

Insert Table 5 here

The signs of all the long-run correlations of the MY ratio with observable variables are in line with *Prediction 1* of the model: the equity yield, measured by either dividend or cyclically adjusted earnings yield, is negatively correlated with MY over the long sample ($\rho_{lr}(dy_t, MY_t)$=-0.60) and highly and significantly correlated with MY over the recent sample ($\rho_{lr}(dy_t, MY_t)$=-0.87). As predicted by the model, the correlation of MY with both long-term nominal bond yields and realized inflation is negative, however, with wider confidence bands, especially in the long sample.

In Panel B of Table 5, we report the correlations of MY_t with the 10-year inflation expectations from various forecasting models described in Section 4.1.3. In particular, we show the correlations with perfect foresight forecasts, $E_t\pi_{lr}^{pf}$, and forecasts using the (discounted) moving average of past 10-year inflation, $E_t\pi_{lr}^{ao}$ ($E_t\pi_{lr}^{cw}$). We also report the correlations with forecasts obtained from the best forecasting models, namely $E_t\pi_{lr}^{oos}$ and $E_t\pi_{lr}^{cw}$. The results from all the remaining forecasting models are shown in the Online Appendix D.1. Over the long sample, the sign is negative for all measures except for $E_t\pi_{lr}^{oos}$, but none of the long-run correlations are significant. The negative (or lack of) correlation is in contrast with the model's prediction that the future inflation should be positively correlated with the MY ratio. The predicted positive correlation is mainly due to the cyclical nature of prices in the model, and to the fact that investors take into account the demographic structure when they build long-run inflation expectations. In reality, this is not necessarily the case. Instead, there is some evidence that inflation expectations are sluggish (e.g., Cieslak and Povala, 2015). In the post-Bretton Woods period, expectations, based on past moving average of 10-year inflation, $E_t\pi_{lr}^{ao}$ and $E_t\pi_{lr}^{cp}$, are highly significantly correlated with the MY ratio. This is however not surprising, since these measures are based on past inflation.

In the last Panel of Table 5, we show the correlations of MY_t with the real long-term bond yields obtained by subtracting the long-run inflation expectations from the nominal bond yields (under the deterministic Fisher hypothesis with no inflation risk premium). Over the long sample, the correlations are weak, except for the best forecasting models. Correlations are higher in magnitude in the post-Bretton Woods period. However, there is still considerable uncertainty about the correlations, which reflects the weak identification of unobservable real rates using forecasting models. We address this issue in Section 4.3 by collecting data from the inflation-indexed bonds market (e.g., TIPS, inflation-linked Gilts) for the recent sample.

Cross-Country Evidence. In order to provide out-of-sample evidence for *Prediction 1*, we repeat the exercise for 20 countries over the post-Bretton Woods (1972-2016) period. In Online Appendix Tables D.2-D.4, we report the results for a panel of 23 countries using the longest available data for each country. For each country, we collect dividend yield, nominal bond yield and inflation data. We run the same set of long-run inflation forecasting models to obtain real bond yields under the deterministic Fisher hypothesis. In Online Appendix Table D.5, we show, for each country, the best forecasting models according to RMSFE and Clark and West (2006) test statistics. Forecasting models vary across countries, but in many countries, modeling stochastic volatility seems to improve forecasting performance over the post-Bretton Woods period.

Table 6 shows that, as the model predicts, real interest rates are negatively correlated with the MY ratio in all countries except Japan, regardless of the model generating inflation expectations. Except Sweden and Finland, most European countries have a correlation between real interest rates and the demographic variable that is similar to the one prevailing in the United States. However, there is considerable uncertainty around the long-run correlation value. This is expected, since real interest rates are computed under the deterministic Fisher equation, thus ruling out an inflation risk premium.

Insert Table 6 here

In Table 7, we look at the long-run correlations among observable variables to verify whether *Prediction 1* holds across countries. We first note that the long-run correlation between dividend yield and MY_t is the strongest in magnitude in the U.S., where the stock market plays an important role in allocating capital over time. Apart from a few exceptions (Austria, Italy and South Africa), the correlation is negative, although mostly not significant. The long-run negative correlation between inflation and MY_t is also broadly in line with the model prediction. Therefore, the long-run correlation between inflation and the dividend yield is positive in all countries but South Africa, where there is no connection between the population age structure and the stock market.

Insert Table 7 here

4.3 Inflation Risk Premium

4.3.1 Demographic Effect on the Inflation Risk Premium

For the recent sample, we explicitly take into account the inflation premium to test *Prediction 2* of the model. We use the median values of the survey-based long-term (10-year) inflation expectations from the Survey of Professional Forecasters. However, since the data are not available for the earlier part of the sample, we extend the sample backwards, via a stable VAR, using the Kalman filter suggested by Bekaert and Engstrom (2010). For comparison, we also use the long-run inflation expectations generated by the forecasting models discussed above. We collect data from the Treasury Inflation-

Protected Securities (TIPS) market to obtain real rates. While, in principle, TIPS rates should be a good market-based proxy for the unobservable real rates, one has to take into account potential frictions in the market, which were particularly salient during the crisis period. Therefore, we compute the liquidity premium (for the period 1999-2016) following the recent literature (Bekaert and Ermolov, 2019; D'Amico et al., 2018; Ermolov, 2017; Pflueger and Viceira, 2016) and deduct this premium to obtain the real rate.[11] We extend the sample backwards using the model exploiting the information in the term structure of survey-based inflation forecasts (Chernov and Mueller, 2012). Under the stochastic Fisher Equation (B.2), we have all the data needed to compute the inflation premium.

Insert Table 8 here

U.S. Evidence. In Panel A of Table 8, we show the long-run correlation of the demographic variable with the real interest rate from the TIPS market and the inflation risk premium (irp_t) computed using different models of long-run inflation expectations These include the moving average of past 10-year inflation (irp_t^{ao}), and the best forecasting models chosen according to RMSFE and Clark and West (2006). The last column in Panel A shows the correlation of MY_t with the inflation risk premium obtained via long-run inflation expectations from the Survey of Professional Forecasters, irp_t^{sur}. The last row of the panel shows the average real interest rate ($\overline{r_t}$) and the average inflation risk premia ($\overline{irp_t}$) over the post-Bretton Woods period. We first note that explicitly taking into account the inflation risk premium reduces the magnitude of the correlation between MY_t and the real interest rate. However, and more importantly, regardless of how the inflation risk premium is obtained, its correlation with the MY ratio is negative (with the exception of irp_t^{ao}), which is in line with *Prediction 2*, and significant in most cases. The magnitude of the premium varies between 0.63% (irp_t^{sur}) and 2.03% (irp_t^{oos}). The model prediction of 0.65% lies within this range, and very close to the inflation premium obtained using survey forecasts.

Cross-Country Evidence. Because the sample is limited by the data from the inflation-indexed bond market, we can repeat the analysis only for Australia and the U.K. over the period 1985-2016. Following Ermolov (2017), we control for the liquidity

[11] We also take into account the deflation protection premium which plays a minor role (D'Amico et al., 2018; Ermolov, 2017). We provide the details of the computations in Online Appendix E.

premium from 1996 (2007) onward in the U.K. (AU) inflation-indexed bond market. Panels B and C of Table 8 show that the long-run correlation between MY_t and the real interest rate remains negative and of relatively high magnitude for both countries. While the average risk premium differs between Australia (negative, except for irp_t^{sur}) and the U.K. (positive),[12] the long-run correlations with the MY ratio are similar to the U.S. evidence.

In Table E.5 of the Online Appendix, we extend the analysis for a large cross-section of countries with longest available data for each country (e.g., starting from 1900 in the U.S.). Since we do not have a direct proxy for the inflation risk premium obtained from the inflation-indexed bond markets, we follow Bekaert and Engstrom (2010) and assume that the inflation risk premium is a function of inflation uncertainty measured by the absolute value of forecast errors of long-run (10-year) inflation forecasting models. Despite the weaker identification in the long sample, the proxy for irp_t is negatively correlated with the MY ratio in almost all the countries, albeit relatively low in magnitude.

Overall, these results suggest that the decomposition of the nominal interest rate is important for identifying the channels through which demographic changes affect the nominal bond yields. However, since both the real interest rate and the inflation risk premium co-move negatively with the demographic variable, the decomposition under the deterministic Fisher hypothesis is a valid approximation for the real/nominal decomposition at generational frequency. This will turn out to be useful in the cross-country analysis of stock-bond correlation, since the inflation-indexed bond market data are not available for many countries.

4.3.2 Return Predictability, Is it there?

Present value models provide an ideal environment to test whether an equilibrium relation between equity and bond yields exists. Earlier studies proposing valuation models show equity return predictability using either bond yields (Lander et al., 1997; Asness, 2003), a demographic variable (Favero et al., 2011), or yield spreads (Maio, 2013). For example, Favero et al. (2011) establish the empirical link between the slowly evolving mean in the log dividend-price ratio and MY_t. Using the decomposition of the log-dividend price ratio

[12] For Australia, our irp_t estimates are lower then the estimates proposed by Moore (2016). This is mainly due to our higher inflation forecasts. In fact, both the random walk model and survey forecasts produce, on average, positive irp_t.

within the dynamic dividend growth model (Campbell and Shiller, 1988), they show that current demographic information is useful in generating accurate forecasts for real stock market returns but not for future changes in dividends. However, in their setting, the future projections of the demographic variable are not exploited for forecasting excess market returns.

The present value relation of Campbell and Shiller (1988), shown in Online Appendix F, explains the link between the stock yield and future returns. Moreover, it justifies the use of the dividend yield as a predictor of future market returns (e.g., Ang and Bekaert, 2007). Under the Pure Expectations Hypothesis, hence ruling out the term premium, Maio (2013) shows that the yield gap between the log equity yield and scaled bond yield is better predictor of the equity premium. In contrast, in a setting characterized by a time-varying term premium, we investigate whether a modified yield gap that incorporates information on future demographic changes can predict excess stock returns via either the inflation risk premium and/or the expectations on distant future risk-free rates, a term included in the present-value relation.

Modified Yield Gap. Based on our OLG model prediction (*Prediction 2*) and the evidence in the previous section, we test whether the long-run correlation between MY_t and the time-varying inflation risk premium can be used for return forecasting. To the extent that the future demographic fluctuations improve our inference on future risk premia (over the maturity of the long-term bond yield) and/or distant future level of risk-free rates (beyond the maturity of the bond), a model including both the yield gap and projections of MY_t should improve the forecasting accuracy for market excess returns, given the lack of dividend growth forecastability (e.g., Cochrane, 2008). We construct a modified version of the yield gap (see Online Appendix F for the derivation):[13]

$$\begin{aligned} yg_t &\equiv dp_t - n \times i_t \\ ygd_t &\equiv yg_t + E_t(MY_t^{n+h}) \end{aligned}$$

where dp_t is the log dividend price ratio, n is the maturity of the long-term bond, h is the forecast horizon, and $E_t(MY_t^{n+h})$ represents the projections of the average MY_t over the maturity of the bond plus h periods beyond. We hypothesize that a modified version of yield gap variable (ygd_t) or a bivariate forecasting model including both the yield gap

[13] In Online Appendix G we discuss the link between the yield gap and MY_t.

(yg_t) and projections of MY_t improve upon forecasting models for market excess returns (e.g., Favero et al., 2011; Maio, 2013). In Table 9, we test this claim by estimating long-run (h= 1 year, 5 years, 10 years) return predictability regressions and we conduct a pseudo out-of-sample forecasting exercise. In line with earlier analysis, we set the forecast period as the post-Bretton Woods period (1972-2016). Univariate models include the log dividend yield (dy_t), the yield gap (yg_t) and the modified yield gap (ygd_t). The bivariate model includes both the yield gap (yg_t) and $E_t(MY_t^{n+h})$. As a benchmark, we also report the bivariate model including the log dividend yield (dy_t) and MY_t (Favero et al., 2011). The dependent variable is the cumulative excess stock market (S&P500) returns. The coefficient estimates are based on the entire sample 1900-2016. The reported p-values, in parentheses, and the asterisks are based on the IVX approach (Kostakis et al., 2015). In square brackets we also report the p-values obtained from a bootstrap exercise, which accounts for the persistence of predictor variables and imposes the joint null hypothesis of no predictability (Maio, 2013). The last four columns report the IVX Joint Wald test (full sample), adjusted R^2_{adj} (full sample), the out-of-sample coefficient of determination R^2_{OS} (Campbell and Thompson, 2008), and MSE-adjusted Clark and West (2007) test over the forecast period (1972-2016).[14]

Insert Table 9 here

In univariate models, the significance of the log dividend yield and yield gap variables depends on the forecast horizon, and the out-of-sample R^2_{OS} shows that the forecasting performance of either does not improve upon a simple model based on historical averages. In contrast, the modified yield gap is significant in all forecasting models, but the out-of-sample R^2_{OS} remains negative although CW test statistics hint at the forecasting ability, at least for longer horizons. Importantly, the only model that produces positive out-of-sample R^2_{OS} is the bivariate model that includes the yield gap and future projections of MY_t. Based on all the statistics, and regardless of the horizon, its forecasting performance is superior to the model with the dividend yield and current MY_t. In Online Appendix F.1, F.2 and F.3, we show that the predictability evidence is robust to including alternative control variables (e.g., $term_t$, $default_t$, cay_t) and generates economic value (Campbell and Thompson, 2008; Maio, 2013). For example, a trading strategy with short-sale constraints

[14] In univarite (bivariate) models the restricted model is the historical average (univariate model).

that extracts signals from the modified yield gap generates a Sharpe ratio up to 0.96 (over the 1972-2016 forecast period) dominating a buy-hold strategy with a Sharpe ratio of 0.68.

Note that if the predictability of excess returns were mainly channeled through the inflation risk premium, the model's prediction (*Prediction 2*) would cause the sign of the coefficient on MY_t to be negative. However, the positive and highly significant coefficient suggests that the ability of the MY ratio to predict the level of the future risk-free rates dominates its predictability of the future inflation risk premium. Indeed, in Tables F.4 and F.5 of the Online Appendix, we decompose the market excess returns into nominal returns and risk-free rates. We show that the forecasting ability is mainly due to the equilibrium relation between future MY ratios and the level of future risk-free rates. This channel, which is explicitly shown in Equation 6 of the Online Appendix F, has been exploited in a bond yield forecasting setting using an affine term-structure model by Favero et al. (2016), though without taking into account (expected) inflation. This result is somewhat in contrast with the equity premium forecasting evidence using MY ratio (Favero et al., 2011) documented in earlier sample.

We also note that the forecasting evidence is limited to the U.S. market. In fact, when we replicate the same out-of-sample exercise with U.K. and Australian stock market excess returns (see Online Appendix Tables F.6-F.7), it does not reveal any forecasting ability. This is not surprising given that the model is calibrated to U.S. data.[15]

4.4 Comovement: Stock and Bond Yields

4.4.1 Equilibrium Relation: Fed Model

Can we believe in a valuation model that relies on a (rational) mechanism that ties stock and bond markets? Several earlier papers try to address this question. For example, Bekaert and Engstrom (2010) suggest a channel where expected inflation coincides with periods of high uncertainty and risk aversion, hence rationalize the strong comovement between stock and bond yields, that is, the Fed model (e.g., Asness, 2003; Lander et al., 1997; Maio, 2013). Some papers explain the comovement by incorporating other business cycle shocks into the models (Burkhardt and Hasseltoft, 2012; Campbell et al., 2014; Ermolov, 2015; Song, 2017). Maio (2013), on the other hand, exploits the yield gap

[15]In this section, we do not focus on bond risk premium predictability, because a variance decomposition of the modified yield gap suggests that the predictability of the bond risk premium is limited, especially in the recent forecast sample (see Online Appendix Table F.8).

between stock and bond yields and shows strong predictability of stock returns. However, there are still some concerns about the validity of the Fed model: i) while it is conceivable that short-term (e.g., one-year) inflation expectations are counter-cyclical, it is less clear whether a similar cyclical pattern holds for long-term inflation expectations, ii) it works perfectly in some subsamples, but less so during the Bretton Woods and Global Financial Crisis period (e.g., Asness, 2003; Hasseltoft, 2009), ii) there is no robust evidence on stock-bond yield comovement across countries (Estrada, 2009). None of these papers focus on the persistent component of the comovement and provide a long time series evidence from a large cross-section of countries.

In this section, we first test the Fed model using annual data over a century. In particular, we project stock yields (proxied either by the dividend price ratio or the cyclically-adjusted earnings price ratio) on the long-term (10-year) nominal bond yield and we control for the relative stock-bond volatility (Asness, 2003) as the benchmark valuation model. Then, we augment the model controlling for demographic fluctuations via MY_t. In further specifications, we augment the baseline model with the demographic variable and several other controls. We consider several supply-side variables for the stock, bond and money markets. We also include time-varying habit-based risk aversion (Campbell and Cochrane, 1999) as a control variable.

Insert Table 10 here

In all specifications, MY_t enters significantly with a negative sign, and improves the adjusted R^2, suggesting that MY_t captures the equilibrium relation between real stock and nominal bond yields as our model predicts. In our model, we assume the supply side of the stock and bond markets does not respond to demographic fluctuations and the demographic effect prevails through the demand channel. In our empirical analysis, we control for supply side variables for stock, bond and money markets. The results remain similar once we control for supply-side variables, supporting the idea that demographics mainly effect stock-bond yield comovement through the demand channel. Finally, in the last specification, the significance of the demographic variable still persists once we control for time-varying risk aversion. Overall, this evidence suggests that the omitted demographic component plays an important role in determining the long-run relation between equity and bond yields.

4.4.2 Cross-Country Evidence

In this subsection, we test the model *Prediction 3* that the persistent comovement between financial yields reflects the time variation in population age structure and its impact through the real vs. nominal channel. We acknowledge that the real channel may incorporate an inflation risk premium, but we refrain from such a decomposition due to the lack of detailed data necessary to estimate inflation risk premia for a large set of countries. Nevertheless, since we find empirical evidence for the model's prediction that both real interest rates and the inflation risk premium are negatively affected by the MY ratio in the previous section, we believe that this composition is less crucial to test the real vs. nominal channel.

In the last column of Table 7, we report the long-run correlations between dividend yield and long-term nominal bond yield for a cross-section of 20 countries (see the Online Appendix Table D.4 for the unbalanced panel). Similar to the evidence by Estrada (2009), the comovement varies substantially across countries. In some countries, the long-run correlation is positive and highly significant (e.g., Belgium, Denmark, South Korea, the Netherlands, the U.K., and the U.S.), while we do not observe such correlation in other countries. We also note that, in those countries where the stock-bond yield correlation is high, MY_t has a negative effect on both dividend and nominal bond yields, as the model predicts, albeit with high uncertainty affecting the correlation values. The importance of stock markets as a channel for aggregate savings varies substantially across countries. This heterogeneity is likely to reflect different stock market participation patterns (Giannetti and Koskinen, 2010). In fact, the U.S. is the only country where the MY ratio has a very strong impact on the dividend yield.

While Table 7 is informative of the magnitude of the correlations, it does not allow us to infer through which component of the nominal yield the population age structure affects stock-bond yield comovement. To this end, we decompose the long-term bond yields and investigate the cross-country differences in stock-bond yield correlation. In particular, we test the model *Prediction 3*, as we explore whether the real channel (real interest rate plus the inflation risk premium) or the nominal channel (expected inflation) plays the dominant role in explaining yield comovement. First, we proceed with an analysis similar to the one suggested by Bekaert and Engstrom (2010). In Figure 5, we plot the cross-sectional stock-bond yield long-run correlations (Müller and Watson, 2018) against the demographic effect on the real component of the nominal bond yields using different specifications to forecast long-term inflation expectations under the deterministic Fisher hypothesis. The negative relationship is strong and consistent; the downward slopes observed across panels indicate that, regardless of the model choice, the stronger the effect of the demographic variable on the real channel, the stronger is the long-run correlation between stock and bond yields.[16]

Insert Figure 5 here

4.4.3 Cross-sectional Regressions

Next, we test the cross-country evidence by estimating robust cross-sectional regressions. The univariate regression results are shown in Panels A-C of Table 11. In all specifications, the dependent variable is $\rho_{lr}(dy_{j,t}, i_{j,t})$, the median long-run correlation (Müller and Watson, 2018) between the dividend yield, $dy_{j,t}$, and the long term nominal bond yield, $i_{j,t}$, in country j (n=20). In Panel A, the independent variables are the long-run correlations between the MY ratio and real yields obtained from different inflation forecasting models, while in Panel B, the independent variables are the long-run correlations between the MY ratio and inflation expectations obtained from different inflation forecasting models.

Insert Table 11 here

In Panel A, we note that the real channel is highly significant, with an $R^2_{adj.}$ varying between 21% and 40%, regardless of the specification used to obtain real rates. The importance of the real channel persists once we control for the dividend yield and MY ratio correlation (Panel D). In principle, this effect might be operating either through the real yields or through the inflation risk premium, but this empirical test cannot disentangle these two channels. However, Panel B clearly shows that the nominal channel does not explain the cross-country differences in stock-bond yield correlation, except in the case where future inflation is correlated with $MY_{j,t}$. In fact, untabulated univariate regressions suggest that the long-run correlation between realized inflation and the MY ratio seems to be a significant factor; however, the significance disappears once we include the correlations between the MY ratio and real yields.

[16] In Online Appendix Figure H.1., we plot the same figure with the Pearson correlations.

Business Cycle. Panel C of Table 11 shows the validation results with alternative business cycle variables discussed in the recent literature (Bekaert and Engstrom, 2010; Burkhardt and Hasseltoft, 2012; Song, 2017): the percentage of observations during which the country experiences stagflation, that is, recession (two consecutive quarters of negative real GDP growth) and high inflation (more than 10% annualized inflation per quarter), $stag_j^{perc}$; the country-specific time-series mean of the interaction between inflation and recession, $\overline{\pi_{j,t} * rec_{j,t}}$; the long-run correlation between annual real GDP growth and inflation, $\rho_{lr}(\Delta gdp_{j,t}, \pi_{j,t})$; and the long-run correlation between annual real consumption growth and inflation, $\rho_{lr}(\Delta cons_{j,t}, \pi_{j,t})$.

The results show that none of these alternative variables can capture the persistent comovement of stock and bond yields. Clearly, this evidence does not rule out earlier explanations based on business cycle forces at play. On the whole, the effects of a time-varying age structure on financial markets vary substantially across countries. However, the demographic effect operating through the real channel provides a consistent explanation for the joint path (and lack thereof) of stock and bond yields.

Other Demographic Changes. Before we conclude, we also test whether alternative demographic channels can explain the cross-country stock-bond yield comovement. We consider demographic variables from other studies (Aksoy et al., 2015; Carvalho et al., 2016) to explain the secular decline in real rates: annual population growth, life expectancy at birth, the dependency ratio (the ratio of population aged 0-24 and 65+ per working population aged 25-64) and the elderly population share (population aged 65+ over total population) in each country. Shown in Table 12, the univariate cross-sectional robust regression results indicate that none of the alternative demographic channels can consistently explain the cross-country variation in stock-bond yield comovement.

Insert Table 12 here

5 Conclusion

This paper documents the role of changing population age structure on stock and bond yields. The net demand for financial assets by certain age groups provides important information on the aggregate demand for financial assets as the population structure changes. Thus, this paper suggests a channel through which demography shapes the puzzling time-series behavior of both key financial variables and provides an economic rationale for the comovement of stock yields and nominal bond yields by introducing money in an OLG model. The decomposition of the nominal bond yields reveal that the real channel via real bond yields and inflation risk premium play the primary role in explaining stock-bond yield correlation. Clearly, the demographic channel in this study cannot explain all the time variation in these variables, but the first-order effects of the population age structure on financial markets are too important to be dismissed.

Our results have important implications for long-term investors with stylized portfolio choice. If changes in the population age structure are a common source of variation both for stock and bond markets, then keeping a substantial portion of a retirement portfolio in local stock and bond markets might not be a good idea for diversification. Finally, it implies that excluding a country's population age structure from the information set may harm an investor who considers international markets for long-term investment.

Appendix A Money Supply Rule

In Section 2.1.4, we introduce the assumption that the central bank does not adjust money supply in response to changes in inflation and in the output gap that are triggered by changes in the demographic structure. To justify this assumption, we first estimate the following money supply rule which mirrors the Taylor rule:

$$\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \epsilon_t$$

as introduced by Chowdhury and Schabert (2008). μ_t represents the growth rate of non-borrowed reserves, $E_t\pi_{t+n}$ is the expected inflation rate in $t+n$, y_t is real output, and y_t^* is time-varying potential output. The data are quarterly time series taken from the St. Louis Fed's FRED database. The growth rate of non-borrowed reserves is constructed as the annual log difference in non-borrowed reserves. The inflation rate is the compounded annual rate of change in the CPI index from time t to $t+n$.[17] The output gap is the percentage gap between actual and potential output.

The results presented in Table A.1 show that, over the entire period and the pre-crisis period, money supply did not significantly react to inflation, a result that is in line with the existing literature (Chowdhury and Schabert, 2008; Sargent and Surico, 2011) and with the history of the Fed's monetary policy strategy (Meulendyke, 1998). We also split the sample into two sub-periods: the pre-Volker period (1961Q1-1979Q2) and the post-Volker period (1982Q4-2013Q1). The results suggest the absence of a consistent money supply feedback to inflation over these two sub-periods. Looking at the entire period, the pre-crisis period, and the two subperiods, the results also indicate that the Fed targeted money supply to stabilize output.

Next, we test for the reaction of money supply to changes in the MY ratio, directly or indirectly through inflation and the output gap. We add the MY ratio as a control variable in our money supply rule:

$$\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \mu_{MY} MY_t + \epsilon_t$$

The estimates of the regression coefficients of inflation and the output gap are affected

[17] The use of the GDP deflator instead of the CPI does not alter the results significantly. Moreover, the results are shown for $n = 1$, and robustness checks indicate that the results are not affected by a change in the horizon ($n = 4$). These robustness checks are available from the authors upon request.

only slightly, and the estimated coefficient of the MY ratio does not significantly differ from zero. This result indicates that the central bank does not systematically adjust the money supply to offset inflationary and expansionary effects of the MY ratio. For this reason, we assume that money supply growth is exogenous.

Table A.1: GMM-Estimation of the Money Supply Rule

Baseline model: $\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \epsilon_t$
Baseline model + control: $\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \mu_{MY}MY_t + \epsilon_t$

	Whole sample 1961q1-2013q1		Pre-crisis period 1961q1-2007q4		Pre-Volker period 1961q1-1979q2		Post-Volker period 1982q4-2013q1	
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
ρ	0.770**	0.861**	0.846**	0.985**	0.766**	0.724**	0.833**	0.668**
	(14.89)	(20.31)	(16.73)	(19.61)	(11.71)	(9.98)	(13.81)	(11.57)
μ_π	0.087*	0.040	0.056	0.100	0.066	0.067	−0.061	0.285
	(1.78)	(0.60)	(1.21)	(1.62)	(1.14)	(1.28)	(-0.35)	(0.81)
μ_y	−0.521**	−0.398**	−0.368**	−0.241*	−0.204	−0.262*	−0.650*	−1.259**
	(-3.39)	(-3.27)	(-2.87)	(-2.07)	(-1.26)	(-2.63)	(-2.61)	(-3.82)
μ_{MY}		0.347		−0.612		0.189		−0.973
		(0.74)		(-1.52)		(0.45)		(-0.92)
$Adj.R^2$	0.69	0.68	0.67	0.64	0.52	0.52	0.70	0.69
J	0.419	0.194	0.308	0.168	0.553	0.739	0.398	0.594

The set of instruments includes four lags of money supply growth, inflation and the output gap, as well as four lags of the MY ratio in specifications that include the MY ratio. Standard errors are in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987), where the bandwidth has been selected following the procedure described in Newey and West (1994). We test the overidentifying restrictions of our model specification and report the p-value of the Hansen's J-statistics. In columns (a), (c), (e) and (g), we estimate our baseline model. In columns (b), (d), (f) and (h), we add the MY ratio as a control variable.

Appendix B Nominal Bond Price and Fisher Equation

In this section, we simplify the time subscript for clarity reasons and use $t = \{j, s, g\}$. Q_t and i_t are respectively the nominal bond price and the nominal interest rate on the bond, with $1+i_t=\frac{1}{Q_t}$. P_t is the price of the consumption good. $\{c_t^y, c_{t+1}^m, c_{t+2}^r\}$ represents the real consumption stream over the three life periods. $(zb_t^y, ze_t^y, zb_{t+1}^m, ze_{t+1}^m)$ represent the real asset holdings (bonds and stocks) of an individual born in period t.

The real borrowing constraints of a young individual born in period t write

$$\begin{aligned}
c_t^y+\frac{Q_t}{P_t}zb_t^y+q_{j,s}^e ze_{j,s}^y&=w_t^y\\
c_{t+1}^m+\frac{Q_{t+1}}{P_{t+1}}zb_{t+1}^m+q_{t+1}^e ze_{t+1}^m&=w_{t+1}^m+\frac{zb_t^y}{P_{t+1}}+(q_{t+1}^e+d_{t+1})ze_t^y\\
c_{t+2}^r&=\frac{zb_{t+1}^m}{P_{t+2}}+(q_{t+2}^e+d_{t+2})ze_{t+1}^m
\end{aligned}$$

In such a framework, the Euler equations are

$$\begin{aligned}
\frac{(c_t^y)^{-\sigma}}{P_t}Q_t&=\beta E_t\frac{(c_{t+1}^m)^{-\sigma}}{P_{t+1}}\\
\frac{(c_t^m)^{-\sigma}}{P_t}Q_t&=\beta E_t\frac{(c_{t+1}^r)^{-\sigma}}{P_{t+1}}
\end{aligned}\tag{9}$$

and Equation (2) is unaffected.

B.1 Deterministic Fisher Equation

r_t denotes the real interest rate on bonds, with $1+r_t=\frac{1}{q_t}$. In absence of stochastic income and monetary regime shocks, the price and consumption levels in period $t+1$ are known in period t. Therefore, using Equations (1) and (9), we obtain $Q_t=q_t\frac{P_t}{P_{t+1}}$. It follows that

$$\begin{aligned}
\ln Q_t&=\ln(q_t\frac{P_t}{P_{t+1}})\\
\ln\frac{1}{1+i_t}&=\ln\frac{1}{1+r_t}+\ln\frac{P_t}{P_{t+1}}\\
-\ln(1+i_t)&\approx-\ln(1+r_t)-\pi_{t+1}\\
i_t&\approx r_t+\pi_{t+1}
\end{aligned}$$

B.2 Stochastic Fisher Equation

If X_t and Y_t are two log-normal random variables, we have

$$\begin{aligned}
\ln E(X_t) &= E(\ln X_t) + \frac{1}{2}Var(\ln X_t), \text{ and} \\
\ln E(X_t Y_t) &= \ln E(X_t) + \ln E(Y_t) + Cov(\ln X_t, \ln Y_t)
\end{aligned}$$

We assume that $\frac{P_t}{P_{t+1}}$ and the ratio of marginal utilities are jointly log-normal distributed. Applying the aforementioned rules on the first of the two Euler equations (9) (it is equivalent to use the second of the Euler equation), we obtain

$$\begin{aligned}
\ln Q_t &= \ln E(\beta \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}} \frac{P_t}{P_{t+1}}) \\
\ln Q_t &= \ln E(\beta \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}) + \ln E(\frac{P_t}{P_{t+1}}) + Cov(\ln \beta \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}, \ln \frac{P_t}{P_{t+1}})
\end{aligned}$$

Plugging in Equation (1), we get

$$\begin{aligned}
\ln Q_t &= \ln q_t + E(\ln \frac{P_t}{P_{t+1}}) + \frac{1}{2}Var(\ln \frac{P_t}{P_{t+1}}) + Cov(\ln \beta + \ln \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}, \ln \frac{P_t}{P_{t+1}}) \\
\ln(\frac{1}{1+i_t}) &\approx \ln(\frac{1}{1+r_t}) - E(\pi_{t+1}) + \frac{1}{2}Var(-\pi_{t+1}) + Cov(\ln \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}, -\pi_{t+1}) \\
-\ln(1+i_t) &\approx -\ln(1+r_t) - E(\pi_{t+1}) + \frac{1}{2}Var(\pi_{t+1}) - Cov(\ln \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}, \pi_{t+1}) \\
i_t &\approx r_t + E(\pi_{t+1}) - \frac{1}{2}Var(\pi_{t+1}) + Cov(\ln \frac{(c^m_{t+1})^{-\sigma}}{(c^y_t)^{-\sigma}}, \pi_{t+1})
\end{aligned}$$

Appendix C Description of Time Series

Equity market data: S&P 500 index yearly prices, 1900-2016 (December observations), are from Welch and Goyal (2008). Dividends (Earnings) are twelve-month moving sums of dividends (earnings) paid on the S&P 500 index. Dividend yield is defined as the ratio of one-year trailing dividends to the one-year lagged equity market index (S&P500). We collect cyclically adjusted earnings yield data, that is, the ratio of the ten-year moving average of earnings to the equity market index, collected from Robert Shiller's website, 1900-2016.

Inflation: We collect the monthly CPI index from Global Financial Data for the period 1900m1-2016m12. Following Welch and Goyal (2008), we compute the annual inflation, by computing the monthly inflation and compound to obtain annual inflation: $\pi_m = \frac{CPI_m}{CPI_{m-1}}$, $\pi_a = (\pi_1 * \pi_2 ... * \pi_{12}) - 1$

Bond yields: Long-term nominal government (real) bond yields are 10-year (inflation indexed) Treasury note yields obtained from Global Financial Data.

Demographic Variable: The U.S. annual population estimates series are collected from U.S. Census Bureau and the sample covers estimates from 1900-2050. The middle-aged to young ratio, MY_t, is calculated as the ratio of the age group 40-49 to age group 20-29. Past MY_t projections for the period 1950-2016 are hand collected from various past Census reports collected from the U.S. Census Bureau's website.

Money growth: For the long sample 1900-2016, we compute December-to-December money growth using narrow money data (currency in circulation) from Global Financial Data. For 1972-2016, we use the broad money measure from the Jorda-Schularick-Taylor macrohistory database updated using OECD M3 data.

International database: Cross-country stock and bond yields are collected from Global Financial Data up to 2016. Stock yield is the dividend yield to the benchmark index, and bond yield is the 10-year constant maturity government bond yield. International MY_t estimates for the period 1950-2016 are from United Nations World Population Prospects available at https://esa.un.org/unpd/wpp/DataQuery/. We collect narrow money data (currency in circulation) from Global Financial Data and compute annual money growth from (December to December) for the long sample 1900-2016. For the recent sample (1972-2016), we use the broad money measure (except for Austria, South Korea, Malaysia and South Africa) from the Jorda-Schularick-Taylor macrohistory database updated using M3 data from OECD.

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Table 1: Stochastic Model - Consumption and Savings Decisions

Odd	$c^y_{o,s}$	$c^m_{o,s}$	$c^r_{o,s}$	*Bond holding*$^y_{o,s}$	*Equity holding*$^y_{o,s}$	*Bond holding*$^m_{o,s}$	*Equity holding*$^m_{o,s}$
$s_1(w^H_o, d^H)$	1.91 (0.01)	2.29 (0.06)	1.99 (0.05)	−0.64 (0.04)	0.92 (0.03)	0.97 (0.06)	0.33 (0.03)
$s_2(w^H_o, d^L)$	1.89 (0.01)	2.19 (0.05)	1.77 (0.05)	−0.57 (0.03)	0.88 (0.02)	0.87 (0.05)	0.30 (0.02)
$s_3(w^L_o, d^H)$	1.62 (0.01)	1.77 (0.06)	1.63 (0.05)	−0.28 (0.02)	0.46 (0.01)	0.42 (0.03)	0.14 (0.01)
$s_4(w^L_o, d^L)$	1.60 (0.00)	1.65 (0.05)	1.42 (0.04)	−0.24 (0.01)	0.44 (0.01)	0.36 (0.02)	0.11 (0.01)
Average across s states	1.76 (0.01)	1.97 (0.03)	1.70 (0.03)	−0.43 (0.02)	0.68 (0.01)	0.66 (0.03)	0.22 (0.01)
Even	$c^y_{e,s}$	$c^m_{e,s}$	$c^r_{e,s}$	*Bond holding*$^y_{e,s}$	*Equity holding*$^y_{e,s}$	*Bond holding*$^m_{e,s}$	*Equity holding*$^m_{e,s}$
$s_1(w^H_e, d^H)$	2.99 (0.05)	2.46 (0.05)	2.78 (0.13)	−3.36 (0.27)	2.76 (0.21)	2.21 (0.18)	2.38 (0.20)
$s_2(w^H_e, d^L)$	2.89 (0.05)	2.37 (0.05)	2.56 (0.12)	−2.91 (0.22)	2.42 (0.17)	1.91 (0.15)	2.05 (0.16)
$s_3(w^L_e, d^H)$	1.86 (0.03)	1.75 (0.04)	1.91 (0.09)	−0.67 (0.06)	0.41 (0.03)	0.44 (0.04)	0.53 (0.05)
$s_4(w^L_e, d^L)$	1.76 (0.02)	1.60 (0.04)	1.79 (0.08)	−0.50 (0.04)	0.34 (0.02)	0.33 (0.03)	0.38 (0.03)
Average across s states	2.37 (0.02)	2.03 (0.03)	2.27 (0.06)	−1.90 (0.11)	1.52 (0.09)	1.25 (0.07)	1.36 (0.08)

This table presents the simulation results of the stochastic model calibrated to the population age structure of (n,N)=(52,79). The subscripts o and e represent the demographic structure $\{odd, even\}$. $s = \{s_1, s_2, s_3, s_4\}$ represents the four wage and dividend states (see Section 2.1.2). The superscripts y, m, and r indicate the individual's respective life stages: young, middle aged and retired. Individual consumption is denoted by c. Bond holding is equal to $q_{j,s}zb^y_{j,s}$, and equity holding is equal to $q^e_{j,s}ze^y_{j,s}$.

Table 2: Stochastic Model - Stock and Bond Yields, and Inflation

Odd	$r_{o,s}$	$q^e_{o,s}$	$ey_{o,s}$	$rp_{o,s}$	$i_{o,s,g}$	$irp_{o,s,g}$ *Av. across g states*	$\pi_{e,s,g}$ *Av. across g states*	$E_{o,s,g}\pi_{e,s+1,g+1}$ *Av. across g states*
$s_1(w^H_o, d^H)$	2.31% (0.00)	90.21 (3.62)	0.08 (0.00)	1.60% (0.00)	6.46% (0.01)	0.92% (0.01)	2.41% (0.02)	3.24% (0.01)
$s_2(w^H_o, d^L)$	2.75% (0.00)	84.84 (2.86)	0.06 (0.00)	1.60% (0.00)	6.65% (0.01)	0.92% (0.01)	2.72% (0.02)	2.98% (0.01)
$s_3(w^L_o, d^H)$	6.47% (0.00)	43.37 (1.56)	0.17 (0.01)	1.66% (0.00)	9.57% (0.01)	0.91% (0.01)	3.42% (0.02)	2.19% (0.02)
$s_4(w^L_o, d^L)$	7.10% (0.00)	40.27 (1.11)	0.12 (0.00)	1.66% (0.00)	9.87% (0.01)	0.91% (0.00)	3.81% (0.02)	1.86% (0.02)
Average across s states	4.69% (0.00)	65.02 (1.55)	0.11 (0.00)	1.63% (0.00)	8.15% (0.01)	0.91% (0.01)	3.10% (0.01)	2.56% (0.01)
Even	$r_{e,s}$	$q^e_{e,s}$	$ey_{e,s}$	$rp_{e,s}$	$i_{e,s,g}$	$irp_{e,s,g}$ *Av. across g states*	$\pi_{o,s,g}$ *Av. across g states*	$E_{e,s,g}\pi_{o,s+1,g+1}$ *Av. across g states*
$s_1(w^H_e, d^H)$	−5.33% (0.00)	332.00 (23.13)	0.02 (0.00)	0.58% (0.00)	−0.72% (0.01)	0.40% (0.01)	1.41% (0.02)	4.22% (0.01)
$s_2(w^H_e, d^L)$	−4.65% (0.00)	287.81 (18.29)	0.02 (0.00)	0.58% (0.00)	0.25% (0.02)	0.40% (0.01)	1.63% (0.02)	3.99% (0.01)
$s_3(w^L_e, d^H)$	2.63% (0.00)	63.47 (6.25)	0.11 (0.01)	0.61% (0.00)	5.32% (0.01)	0.41% (0.01)	3.36% (0.02)	2.30% (0.02)
$s_4(w^L_e, d^L)$	4.16% (0.00)	47.65 (4.39)	0.10 (0.01)	0.62% (0.00)	6.48% (0.01)	0.40% (0.01)	3.75% (0.02)	1.92% (0.02)
Average across s states	−0.67% (0.00)	186.99 (11.20)	0.06 (0.00)	0.60% (0.00)	2.81% (0.01)	0.40% (0.01)	2.56% (0.01)	3.08% (0.01)

This table presents the simulation results of the stochastic model calibrated to the population age structure of (n,N)=(52,79). The subscripts o and e represent the demographic structure $j = \{odd, even\}$. $s = \{s_1, s_2, s_3, s_4\}$ represents the four wage and dividend states (see Section 2.1.2). g denotes the monetary regimes. $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bonds from period j to period $j+1$, respectively. q^e_j is the real stock price in period j. $ey_{j,s}$ refers to the annualized earnings yield on stocks and is defined as $ey_{j,s} = 2*(d_s/20)/q^e_{j,s}$. $rp_{j,s}$ is the annualized risk premium defined as $rp_{j,s} = average((\frac{q^e_{j+1,s+1}+d_{s+1}}{q^e_{j,s}})^{\frac{1}{20}} - 1 - r_{j,s})$. $irp_{j,s,g}$ is the inflation risk premium as defined by Equation (8). $\pi_{j,s,g}$ is the annualized inflation rate from period $j-1$ to period j. $E_{j,s,g}\pi_{j+1,s+1,g+1}$ is annualized expected inflation, that is, the expected inflation rate from period j to period $j+1$.

Table 3: Stochastic Model - Stock and Bond Yields across States

Odd	g	$r_{o,s}$	$q^e_{o,s}$	$r^e{}_{o,s}$	$i_{o,s,g}$		$irp_{o,s,g}$		$E_{o,s,g}\pi_{e,s+1,g+1}$	
s_1	g_1	2.31%	90.21	3.88%	4.16%	6.46%	0.90%	0.92%	0.94%	3.24%
	g_2				6.14%		0.92%		2.91%	
	g_3				7.61%		0.90%		4.39%	
	g_4				9.67%		0.94%		6.42%	
s_2	g_1	2.75%	84.84	4.34%	4.33%	6.65%	0.97%	0.92%	0.60%	2.98%
	g_2				6.30%		0.92%		2.63%	
	g_3				7.78%		0.90%		4.14%	
	g_4				9.78%		0.90%		6.11%	
s_3	g_1	6.47%	43.37	8.18%	7.24%	9.57%	0.90%	0.91%	−0.13%	2.19%
	g_2				9.25%		0.92%		1.87%	
	g_3				10.73%		0.90%		3.36%	
	g_4				12.76%		0.90%		5.40%	
s_4	g_1	7.10%	40.26	8.76%	7.52%	9.87%	0.90%	0.91%	−0.48%	1.86%
	g_2				9.55%		0.91%		1.54%	
	g_3				11.03%		0.90%		3.03%	
	g_4				13.03%		0.90%		5.03%	
Average		4.69%	65.01	6.31%		8.15%		0.91%		2.56%
Even	g	$r_{e,s}$	$q^e_{e,s}$	$r^e_{e,s}$	$i_{e,s,g}$		$irp_{e,s,g}$		$E_{e,s,g}\pi_{o,s+1,g+1}$	
s_1	g_1	−5.33%	332.00	−4.76%	−3.01%	−0.72%	0.42%	0.40%	1.90%	4.22%
	g_2				−1.03%		0.39%		3.91%	
	g_3				0.47%		0.39%		5.41%	
	g_4				2.45%		0.39%		7.40%	
s_2	g_1	−4.65%	287.81	−4.06%	−2.61%	−0.25%	0.39%	0.41%	1.65%	3.99%
	g_2				−0.59%		0.40%		3.66%	
	g_3				0.97%		0.47%		5.15%	
	g_4				2.91%		0.39%		7.16%	
s_3	g_1	2.63%	63.47	3.25%	3.04%	5.32%	0.45%	0.40%	−0.05%	2.30%
	g_2				4.97%		0.39%		1.95%	
	g_3				6.46%		0.39%		3.46%	
	g_4				8.47%		0.39%		5.47%	
s_4	g_1	4.16%	47.64	4.79%	4.14%	6.48%	0.39%	0.40%	−0.42%	1.92%
	g_2				6.14%		0.40%		1.58%	
	g_3				7.63%		0.39%		3.08%	
	g_4				9.65%		0.41%		5.08%	
Average		−0.67%	186.99	−0.07%		2.81%		0.40%		3.08%

This table presents the simulation results of the stochastic model calibrated to the population age structure of (n,N)=(52,79). The subscripts o and e represent the demographic structure $j = \{odd, even\}$. $s = \{s_1, s_2, s_3, s_4\}$ represents the four wage and dividend states (see Section 2.1.2). $g = \{g_1, g_2, g_3, g_3\}$ represents the four states of money supply (see Section 2.1.4). $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bonds from period j to period $j+1$, respectively. $q^e_{j,s}$ and $r^e_{j,s}$ are the real stock price and real interest rate on stocks, respectively. $irp_{j,s,g}$ is the inflation risk premium as defined by Equation (8). $E_{j,s,g}\pi_{j+1,s+1,g+1}$ is annualized expected inflation, that is, the expected inflation rate from period j to period $j+1$.

Table 4: Data Summary Statistics

Panel A. Long Sample	mean	std.dev.	skew.	kurt.	min	max	AC(1)
dy_t	0.04	0.02	0.09	2.53	0.01	0.09	0.901
ey_t	0.07	0.03	1.30	5.21	0.02	0.21	0.869
i_t	0.05	0.03	1.46	4.95	0.02	0.14	0.934
π_t	0.03	0.05	0.65	6.00	-0.11	0.20	0.548
MY_t	0.79	0.18	0.43	1.97	0.56	1.16	0.982
Panel B. Post-Bretton Woods	**mean**	**std.dev.**	**skew.**	**kurt.**	**min**	**max**	**AC(1)**
dy_t	0.03	0.01	0.50	2.08	0.01	0.06	0.926
ey_t	0.06	0.03	0.75	2.21	0.02	0.13	0.906
i_t	0.06	0.03	0.47	2.70	0.02	0.14	0.887
π_t	0.04	0.03	1.54	4.82	0.00	0.13	0.743
MY_t	0.87	0.22	-0.14	1.41	0.57	1.16	0.985

This table presents the descriptive statistics of the U.S. observable variables: dividend yield (dy_t), that is, annual dividend divided by lagged price of the S&P 500 index; the cyclically adjusted earnings-price ratio (ey_t) obtained from Robert Shiller's website; the 10-year U.S. nominal bond yield (i_t, p.a.); annual inflation (π_t); and middle aged-young ratio MY_t. The last column reports the first-order autocorrelations. Panel A shows the summary statistics over the long sample, 1900-2016, while Panel B shows the summary statistics over the post-Bretton Woods sample, 1972-2016. Annual data.

Table 5: Long-run Correlations with the MY Ratio

	Long Sample (1900-2016)		Post-Bretton Woods (1972-2016)	
	$\rho_{lr}(x_t, MY_t)$	$\rho(x_t, MY_t)$	$\rho_{lr}(x_t, MY_t)$	$\rho(x_t, MY_t)$
Panel A. Observables				
dy_t	-0.597^{*}	$-0.614^{\star\star\star}$	-0.865^{***}	$-0.917^{\star\star\star}$
	$(-0.800,-0.273)$	(<0.000)	$(-0.980,-0.604)$	(<0.000)
ey_t	-0.408^{**}	$-0.567^{\star\star}$	-0.756^{***}	$-0.885^{\star\star\star}$
	$(-0.762,-0.184)$	(<0.000)	$(-0.956,-0.443)$	(<0.000)
i_t	-0.331	-0.233	-0.513^{*}	$-0.783^{\star\star}$
	$(-0.638,0.100)$	(<0.012)	$(-0.862,-0.199)$	(<0.000)
π_t	-0.273^{*}	-0.174	-0.414^{*}	$-0.632^{\star\star}$
	$(-0.497,-0.001)$	(<0.060)	$(-0.862,-0.013)$	(<0.000)
Panel B. Expected Inflation				
$E_t\pi_{lr}^{pf}$	-0.226	-0.118	-0.030	$-0.581^{\star\star\star}$
	$(-0.401,0.291)$	(0.225)	$(-0.664,0.537)$	(<0.000)
$E_t\pi_{lr}^{ao}$	-0.273	-0.194	-0.807^{***}	$-0.929^{\star\star\star}$
	$(-0.477,0.291)$	(0.036)	$(-0.956,-0.502)$	(<0.000)
$E_t\pi_{lr}^{cp}$	-0.273	-0.235	-0.670^{**}	$-0.884^{\star\star\star}$
	$(-0.495,0.291)$	(0.011)	$(-0.935,-0.331)$	(<0.000)
$E_t\pi_{lr}^{oos}$	0.226	$0.525^{\star\star}$	-0.477^{*}	$-0.707^{\star\star}$
	$(-0.226,0.664)$	(<0.000)	$(-0.885,-0.100)$	(<0.000)
$E_t\pi_{lr}^{cw}$	-0.001	0.295	-0.350	$-0.606^{\star}$
	$(-0.273,0.604)$	(0.001)	$(-0.834,0.011)$	(<0.000)
Panel C. Real Rates				
r_t^{pf}	-0.103	-0.048	-0.408^{*}	-0.430
	$(-0.386,0.291)$	(0.620)	$(-0.716,-0.030)$	(0.010)
r_t^{ao}	-0.001	-0.050	-0.209	-0.418
	$(-0.462,0.269)$	(0.595)	$(-0.664,0.162)$	(0.004)
r_t^{cp}	-0.003	-0.048	-0.340	$-0.556^{\star}$
	$(-0.462,0.269)$	(0.609)	$(-0.762,0.001)$	(<0.000)
r_t^{oos}	-0.474^{*}	$-0.432^{\star}$	-0.480^{*}	$-0.721^{\star}$
	$(-0.716,-0.100)$	(<0.000)	$(-0.834,-0.131)$	(<0.000)
r_t^{cw}	-0.401^{*}	$-0.402^{\star}$	-0.500^{*}	$-0.755^{\star\star}$
	$(-0.716,-0.226)$	(<0.000)	$(-0.862,-0.178)$	(<0.000)

Panel A reports the correlation between the MY ratio and the following observable variables: dividend yield (dy_t), the earnings yield (ey_t) proxied by cyclically adjusted earnings price ratio, the 10-year nominal bond yield (i_t), and annual inflation (π_t). Panel B shows the correlation of MY_t with different long-run (10-year) inflation expectations obtained by estimating different inflation forecasting models: $E_t\pi_{lr}^{pf}$ is the inflation expectation with perfect foresight, that is, the average 10-year future inflation, up to 2006; $E_t\pi_{lr}^{ao}$ ($E_t\pi_{lr}^{cp}$) is the inflation forecast obtained using the (discounted) average past 10-year inflation; $E_t\pi_{lr}^{oos}$ is 10-year average inflation forecast obtained from the forecasting model with the lowest RMSFE; and $E_t\pi_{lr}^{cw}$ is 10-year average inflation forecast obtained from the forecasting model with the highest Clark and West (2006) test statistics. Panel C reports the correlations between the MY ratio and the corresponding real interest rates calculated by subtracting the long-run inflation expectations from the nominal bond yield (i_t). The long-run correlation, ρ_{lr}, is the median of the posterior obtained using the Müller and Watson (2018) framework, with 67% confidence set in parentheses. For the long sample (post-Bretton Woods sample), we set q=18 (q=6), which captures periodicities longer than 13 (15) years. Asterisks *,** and *** denote significance according to 67, 90 and 95 percent confidence set, respectively. We also report Pearson's correlation ρ (p-values using Student's t-distribution in parentheses). Stars *, ** and *** show significance at 10, 5 and 1 percent based on bootstrapped p-values that account for the persistence of each variable. Annual data.

Table 6: International Evidence: Real Rates

Countries	$\rho_{lr}(r_t^{pf}, MY_t)$ $\rho(r_t^{pf}, MY_t)$	$\rho_{lr}(r_t^{ao}, MY_t)$ $\rho(r_t^{ao}, MY_t)$	$\rho_{lr}(r_t^{oos}, MY_t)$ $\rho(r_t^{oos}, MY_t)$	$\rho_{lr}(r_t^{cw}, MY_t)$ $\rho(r_t^{cw}, MY_t)$
AT	-0.744^{***} $(-0.760^{\star\star})$	-0.321 $(-0.622^{\star})$	-0.456 $(-0.757^{\star\star})$	-0.477 $(-0.799^{\star\star})$
AU	-0.273 (-0.180)	0.008 (-0.148)	-0.438^{*} $(-0.666^{\star})$	-0.337^{*} (-0.513)
BE	-0.597^{*} $(-0.696^{\star\star\star})$	-0.296 (-0.547)	-0.594^{*} $(-0.812^{\star\star})$	-0.709^{**} $(-0.847^{\star\star})$
CA	-0.273 (-0.216)	0.106 (-0.226)	-0.178 (-0.276)	-0.477^{*} $(-0.791^{\star\star})$
CH	-0.052 (0.141)	0.027 (0.144)	-0.445^{*} $(-0.770^{\star\star})$	0.102 (0.106)
DE	-0.734^{**} $(-0.669^{\star\star})$	-0.421^{*} $(-0.643^{\star})$	-0.379 $(-0.718^{\star\star\star})$	-0.456^{*} $(-0.767^{\star\star\star})$
DK	-0.200 (-0.486)	-0.438^{*} $(-0.740^{\star\star})$	-0.474^{*} $(-0.844^{\star\star\star})$	-0.474^{*} $(-0.844^{\star\star\star})$
ES	-0.539^{*} $(-0.728^{\star\star\star})$	-0.013 (-0.041)	-0.304 $(-0.502^{\star})$	-0.317 $(-0.410^{\star\star})$
FI	0.013 (0.154)	0.302 (0.438)	-0.206 (-0.463)	-0.226 $(-0.509^{\star})$
FR	-0.401^{*} $(-0.450^{\star\star\star})$	-0.036 (-0.304)	-0.890^{***} $(-0.908^{\star\star\star})$	-0.911^{***} $(-0.934^{\star\star\star})$
IT	-0.198 (-0.370)	-0.027 (-0.140)	-0.178 (-0.261)	-0.282 (-0.666)
JP	0.463^{*} (0.461)	0.184 (0.299)	-0.168 (-0.434)	0.082 (0.125)
KR	-0.319 $(-0.544^{\star})$	-0.321^{*} $(-0.566^{\star})$	-0.493^{*} $(-0.810^{\star\star})$	-0.321^{*} $(-0.566^{\star})$
MY	-0.269 $(-0.602^{\star\star})$	-0.292 (-0.539)	-0.418 $(-0.730^{\star\star\star})$	-0.319 $(-0.475^{\star\star})$
NL	-0.317 $(-0.573^{\star\star})$	-0.045 (-0.323)	-0.474 $(-0.846^{\star\star})$	-0.477 $(-0.856^{\star\star\star})$
NO	-0.130 (-0.019)	-0.034 (-0.183)	0.001 (-0.011)	-0.477^{*} $(-0.809^{\star\star})$
SE	0.319 (0.479)	0.386 (0.012)	0.377 (-0.232)	-0.036 $(-0.534^{\star\star})$
UK	-0.321^{*} $(-0.508^{\star\star})$	-0.003 (-0.103)	-0.448 $(-0.809^{\star\star\star})$	-0.493 $(-0.875^{\star\star\star})$
US	-0.408^{*} (-0.430)	-0.209 (-0.418)	-0.480^{*} $(-0.721^{\star})$	-0.500^{*} $(-0.754^{\star\star})$
ZA	0.001 (0.094)	0.630^{*} $(0.396^{\star\star})$	0.304 (0.313)	-0.421^{*} $(-0.652^{\star\star})$

The table shows the median long-run correlations, ρ_{lr}, and the Pearson correlations, ρ, (in parentheses) between each country's MY ratio and the real interest rate that is obtained by estimating different inflation forecasting models: r_t^{pf} is the real interest rate obtained by assuming perfect foresight for inflation expectations, that is, using the average 10-year future inflation (up to 2006); r_t^{ao} is obtained by using average past 10-year inflation for inflation forecasts (Atkeson and Ohanian, 2001); r_t^{oos} is obtained by using the best inflation forecast based on RMSFE; and r_t^{cw} is obtained by using the best inflation forecast based on the Clark and West (2006) test statistics. The long-run correlation is the median of the posterior obtained using the Müller and Watson (2018) framework. Asterisks *,** and *** denote significance according to 67, 90 and 95 percent confidence interval, respectively. Statistical significance of Pearson correlations is assessed based on bootstrapped p-values that account for the persistence of each variable. Stars *, ** and *** show significance at 10, 5 and 1 percent, respectively. Post-Bretton Woods period (1972-2016). Annual data.

Table 7: International Evidence: Observables

Countries	$\rho_{lr}(dy_t, MY_t)$ $\rho(dy_t,MY_t)$	$\rho_{lr}(\pi_t, MY_t)$ $\rho(\pi_t,MY_t)$	$\rho_{lr}(\pi_t, dy_t)$ $\rho(\pi_t,dy_t)$	$\rho_{lr}(i_t, MY_t)$ $\rho(i_t,MY_t)$	$\rho_{lr}(i_t, dy_t)$ $\rho(i_t,dy_t)$
AT	0.027 (−0.032)	−0.129 (−0.452***)	0.421* (0.442**)	−0.460 (−0.808***)	0.184 (0.254)
AU	−0.445* (−0.625**)	−0.480 (−0.760**)	0.545* (0.764***)	−0.502* (−0.791**)	−0.001 (0.325)
BE	−0.281 (−0.447*)	−0.013 (−0.474)	0.500* (0.578***)	−0.653** (−0.863***)	0.651** (0.557**)
CA	−0.571* (−0.771**)	−0.480* (−0.769**)	0.513* (0.750***)	−0.477* (−0.783**)	0.209 (−0.458)
CH	−0.209 (−0.320)	−0.493* (−0.650**)	0.103 (0.334*)	−0.460* (−0.784**)	−0.102 (0.086)
DE	−0.234 (−0.435)	−0.184 (−0.400***)	0.448* (0.583***)	−0.418 (−0.746**)	0.273 (0.549**)
DK	−0.447* (−0.717**)	−0.477* (−0.770***)	0.951*** (0.837***)	−0.493* (−0.863***)	0.653** (0.750**)
ES	−0.020 (−0.146)	−0.212 (−0.433)	0.480* (0.597**)	−0.413* (−0.686*)	0.630* (0.681**)
FI	−0.502* (−0.629**)	−0.651** (−0.818**)	0.385* (0.527**)	−0.230 (−0.533*)	0.178 (0.265)
FR	−0.317 (−0.560*)	−0.445* (−0.748**)	0.892*** (0.856***)	−0.889*** (−0.932***)	0.597* (0.687***)
IT	0.377* (0.411*)	−0.103 (−0.504)	0.042 (−0.008)	−0.317 (−0.704*)	−0.050 (−0.144)
JP	−0.158 (−0.144)	−0.412* (−0.514*)	0.445* (0.452*)	−0.226 (−0.530*)	0.011 (−0.046)
KR	−0.103 (−0.478)	−0.255 (−0.478*)	0.841*** (0.844***)	−0.477* (−0.830**)	0.709*** (0.826***)
MY	−0.013 (0.055)	−0.162 (−0.321**)	0.054 (0.078)	−0.401 (−0.762**)	−0.226 (−0.070)
NL	−0.319 (−0.725**)	−0.226 (−0.558***)	0.445* (0.658**)	−0.448 (−0.847**)	0.653** (0.741***)
NO	−0.273 (−0.220)	−0.750*** (−0.844***)	0.480* (0.424**)	−0.462* (−0.793**)	0.045 (0.051)
SE	−0.255 (−0.327)	−0.477 (−0.766**)	0.333 (0.288)	−0.013 (−0.515*)	−0.070 (−0.169)
UK	−0.350 (−0.592***)	−0.168 (−0.569)	0.911*** (0.783***)	−0.493 (−0.878***)	0.648** (0.779***)
US	−0.865*** (−0.917***)	−0.414* (−0.632**)	0.712*** (0.746***)	−0.513* (−0.783**)	0.667** (0.787***)
ZA	0.000 (−0.211)	−0.776*** (−0.760***)	−0.013 (0.311)	−0.408 (−0.633**)	−0.480 (−0.092)

The table shows the median long-run correlation, ρ_{lr}, and the Pearson correlations, ρ, (in parentheses) between each country's MY ratio and the following variables: the dividend yield (dy_t), annual inflation (π_t), and the long-term (10-year) nominal bond yield (i_t). The fourth column reports the correlation between the dividend yield (dy_t) and annual inflation (π_t), while the last column shows the correlation between the dividend yield (dy_t) and nominal bond yield (i_t). The long-run correlation is the median of the posterior obtained using the Müller and Watson (2018) framework. Asterisks *,** and *** denote significance according to 67, 90 and 95 percent confidence set, respectively. Statistical significance of Pearson correlations is assessed based on bootstrapped p-values that account for the persistence of each variable. Stars *, ** and *** show significance at 10, 5 and 1 percent, respectively. Post-Bretton Woods period (1972-2016). Annual data.

Table 8: Inflation Risk Premium

	r_t	irp_t^{ao}	irp_t^{oos}	irp_t^{cw}	irp_t^{sur}
Panel A. US (1972-2016)					
$\rho_{lr}(x_t, MY_t)$	-0.304	0.131	-0.638^{**}	-0.530^{*}	-0.304
$(\rho(x_t, MY_t))$	(-0.514^{***})	(0.075)	(-0.761^{***})	(-0.727^{***})	(-0.298^{**})
$\overline{x_t}$	1.98%	0.76%	2.03%	1.50%	0.63%
Panel B. AU (1985-2016)					
$\rho_{lr}(x_t, MY_t)$	-0.282	0.036	-0.777^{***}	-0.757^{***}	-0.653^{**}
$(\rho(x_t, MY_t))$	(-0.577^{***})	(0.213)	(-0.816^{***})	(-0.799^{***})	(-0.679^{***})
$\overline{x_t}$	3.37%	-0.47%	-0.69%	-1.00%	0.21%
Panel C. UK (1985-2016)					
$\rho_{lr}(x_t, MY_t)$	-0.321	0.462^{*}	-0.273	-0.714^{***}	-0.706^{**}
$(\rho(x_t, MY_t))$	(-0.776^{***})	(0.703^{***})	(-0.429^{**})	(-0.867^{***})	(-0.901^{***})
$\overline{x_t}$	1.93%	0.19%	0.19%	1.01%	1.63%

The table shows the median long-run correlations, ρ_{lr}, and the Pearson correlations, ρ, (in parentheses) between the MY ratio and both the real interest rate, r_t, obtained from the inflation-indexed bond market (net of liquidity premium) and the inflation risk premium (irp_t) obtained by using different models for long-run inflation expectations. The last column in each panel shows the correlation of MY_t with the inflation risk premium (irp_t^{sur}) obtained via long-run inflation survey expectations. The last row of each panel shows both the average real interest rate ($\overline{r_t}$) and the average inflation risk premium ($\overline{irp_t}$) over the sample period. Panel A shows the results obtained with the U.S. data over the post-Bretton Woods period. Panels B and C repeat the analysis for Australia and the U.K., respectively, for the period 1985-2016 (the sample is limited by the data from the TIPS market). The long-run correlation is the median of the posterior obtained using the Müller and Watson (2018) framework. Asterisks *,** and *** denote significance according to 67, 90 and 95 percent confidence set, respectively. Statistical significance of Pearson correlations is assessed based on bootstrapped p-values that account for the persistence of each variable. Stars *, ** and *** show significance at 10, 5 and 1 percent, respectively. Annual data.

Table 9: Out-of-Sample Forecasts

Excess Stock Returns: $xret_{t,t+h} = \alpha_0 + \alpha_1 x_t + \varepsilon_{t,t+h}$

Panel A. h=1	dy_t	yg_t	ygd_t	$MY_t/E_t(MY_t^{n+h})$	$Wald$	R^2_{adj}	R^2_{OS}	CW
$xret_{t,t+1}$	0.062* (0.10)[0.10]				2.77*	0.01	−0.08	0.44
$xret_{t,t+1}$		0.055* (0.07)[0.04]			3.29*	0.02	−0.07	0.73
$xret_{t,t+1}$			0.087** (0.01)[0.00]		6.15**	0.04	−0.01	1.13
$xret_{t,t+1}$	0.175*** (0.00)[0.01]			0.368** (0.03)[0.01]	8.23**	0.06	0.01	2.03**
$xret_{t,t+1}$		0.101*** (0.01)[0.00]		0.321** (0.05)[0.00]	8.62**	0.07	0.06	2.29**
Panel B. h=5	dy_t	yg_t	ygd_t	$MY_t/E_t(MY_t^{n+h})$	$Wald$	R^2_{adj}	R^2_{OS}	CW
$xret_{t,t+5}$	0.045** (0.03)[0.00]				4.85**	0.05	−0.23	1.20
$xret_{t,t+5}$		0.045 (0.14)[0.00]			1.83	0.10	−0.40	1.34*
$xret_{t,t+5}$			0.072*** (0.01)[0.00]		6.63**	0.20	−0.27	2.05**
$xret_{t,t+5}$	0.103*** (0.03)[0.00]			0.189*** (0.00)[0.00]	22.27***	0.14	−0.22	1.44*
$xret_{t,t+5}$		0.085*** (0.00)[0.00]		0.301*** (0.00)[0.00]	29.87***	0.38	0.23	4.04***
Panel C. h=10	dy_t	yg_t	ygd_t	$MY_t/E_t(MY_t^{n+h})$	$Wald$	R^2_{adj}	R^2_{OS}	CW
$xret_{t,t+10}$	0.051** (0.04)[0.00]				4.41**	0.18	−0.25	2.26**
$xret_{t,t+10}$		0.035** (0.03)[0.00]			4.84**	0.14	−0.70	1.20
$xret_{t,t+10}$			0.055*** (0.00)[0.00]		13.39***	0.30	−0.54	1.72**
$xret_{t,t+10}$	0.084*** (0.00)[0.00]			0.108*** (0.00)[0.00]	45.86***	0.25	−0.32	0.50
$xret_{t,t+10}$		0.067*** (0.00)[0.00]		0.277*** (0.00)[0.00]	39.03***	0.64	0.31	4.91***

This table reports the results of long-run (1, 5, 10 years) excess stock return forecasting regressions based on univariate and bivariate models. Univariate models are based on the log dividend yield (dy_t), the yield gap (yg_t), and the augmented yield gap that includes expectations of distant future middle-aged to young ratio $E_t(MY_t^{n+h})$. In the univariate model, the coefficient of $E_t(MY_t^{n+h})$ is restricted to one. The unrestricted bivariate model includes both the yield gap, that is, log dividend price ratio minus n times the log of 10-year nominal bond yield (n=10), and $E_t(MY_t^{n+h})$. As a benchmark, we also report the bivariate model including the log dividend yield and MY_t (Favero et al., 2011). The dependent variable is the cumulative excess stock market (S&P500) returns. The coefficient estimates and in-sample R^2_{adj} are based on the full sample (1900-2016). The reported p-values (in parentheses) are based on the IVX approach (Kostakis et al., 2015). In square brackets we also report the p-values obtained from a bootstrap exercise that accounts for the persistence of predictor variables and imposes the joint null hypothesis of no predictability of returns (Maio, 2013). The last three columns report the IVX Joint Wald test (full sample), the out-of-sample coefficient of determination R^2_{OS}, and the CW test statistics (Clark and West, 2007) for the in-sample period (1900-1971) and the forecast period (1972-2016). Asterisks *,**, and *** show significance at 10, 5 and 1 percent, respectively. Annual data.

Table 10: FED Model

FED Model Specifications						
Panel A. dp_t	$const$	i_t	MY_t	σ_t^{EB}	$ctrl_t$	R^2_{adj}
(1) Fed Model + σ_t^{EB}	-0.031^{***} (−2.70)	0.354^{***} (4.41)		0.017^{***} (6.81)		0.42
(2) Model (1) + MY_t	0.037 (1.54)	0.117 (1.10)	-0.045^{***} (−2.76)	0.011^{***} (3.55)		0.56
(2) + $ctrl_t = es_t$	0.065^{***} (3.73)	−0.005 (−0.06)	-0.065^{***} (−6.72)	0.009^{***} (4.04)	0.002 (0.04)	0.66
(2) + $ctrl_t = bs_t$	0.042 (1.96)	0.096 (0.93)	-0.052^{***} (−3.41)	0.010^{***} (3.29)	0.010 (0.86)	0.55
(2) + $ctrl_t = ms_t$	0.014 (0.56)	0.152 (1.62)	-0.044^{***} (−3.73)	0.010^{***} (3.29)	0.039 (1.39)	0.59
(2) + $ctrl_t = ra_t$	0.039 (1.77)	0.116 (1.10)	-0.046^{***} (−3.00)	0.011^{***} (3.63)	−0.010 (−0.29)	0.56
Panel B. ep_t	$const$	i_t	MY_t	σ_t^{EB}	$ctrl_t$	R^2_{adj}
(1) Fed Model + σ_t^{EB}	-0.067^{**} (−2.21)	0.998^{***} (4.76)		0.028^{***} (3.58)		0.36
(2) Model (1) + MY_t	0.031 (0.76)	0.660^{***} (3.06)	-0.065^{**} (−2.08)	0.019^{***} (2.71)		0.44
(2) + $ctrl_t = es_t$	0.099^{***} (3.75)	0.323^{**} (2.26)	-0.103^{***} (−5.65)	0.013^{***} 5.08)	-0.184^{**} (2.06)	0.74
(2) + $ctrl_t = bs_t$	0.033 (0.91)	0.693^{***} (3.20)	-0.084^{***} (−3.75)	0.019^{**} (2.45)	0.033 (1.62)	0.45
(2) + $ctrl_t = ms_t$	−0.001 (−0.02)	0.708^{***} (3.77)	-0.062^{**} (−2.31)	0.018^{**} (2.16)	0.053 (0.65)	0.45
(2) + $ctrl_t = ra_t$	0.046 (1.35)	0.656^{***} (3.16)	-0.068^{**} (−2.60)	0.018^{**} (2.55)	−0.084 (−1.21)	0.46

This table reports the estimates of the Fed model that posits a long-run relation between equity yields, proxied either by the dividend price ratio (Panel A) or the cyclically-adjusted earnings yield (Panel B), and long-term nominal bond yields, and controls for the relative stock-bond volatility (baseline model 1). Model 2 is the augmented version of Model 1 and includes the MY ratio. Further controls include es_t, net equity expansion (twelve month moving sums of net issues, Welch and Goyal (2008)) over total NYSE market capitalization; bs_t, the bond supply measured by government debt over GDP (1900-2012); ms_t, the money supply (M2) over GDP; and ra_t, the time-varying habit-based risk aversion proxied by the surplus ratio, that is, real personal consumption relative to its 10-year moving average. Relative stock-bond volatility is the logarithm of the ratio of the realized volatilities of stock and bond markets. Bond volatility is measured as the standard deviation of monthly observations using a 10-year rolling window. Stock volatility is obtained from daily stock returns (Welch and Goyal, 2008), that is, square root of svar (sum of squared daily returns on S&P500). The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987), where the bandwidth has been selected following the procedure described in Newey and West (1994). Asterisks *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. The last column reports adjusted R^2_{adj}. Annual data. Sample 1900-2016.

Table 11: Cross-Country Regressions

Specification: $\rho_{lr,j}(dy_{j,t}, i_{j,t}) = \alpha_0 + \alpha_1 x_j + \varepsilon_j$				
Panel A. Real Rates	$\rho_{lr,j}(r_{j,t}^{pf}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{ao}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{oos}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{cw}, MY_{j,t})$
coef.	-0.555^{**}	-0.764^{***}	-0.826^{***}	-0.797^{**}
(z-stat)	(-2.308)	(-2.773)	(-3.509)	(-2.505)
$R^2_{adj.}$	0.21	0.27	0.40	0.22
Panel B. Exp. Inflation	$\rho_{lr,j}(E_t\pi_{lr,j}^{pf}, MY_{j,t})$	$\rho_{lr,j}(E_t\pi_{lr,j}^{ao}, MY_{j,t})$	$\rho_{lr,j}(E_t\pi_{lr,j}^{oos}, MY_{j,t})$	$\rho_{lr,j}(E_t\pi_{lr,j}^{cw}, MY_{j,t})$
coef.	0.674^{**}	-0.192	0.532	-0.281
(z-stat)	(2.370)	(-0.463)	(1.161)	(-1.180)
$R^2_{adj.}$	0.25	-0.04	0.02	0.03
Panel C. Business Cycle	$stag_j^{perc}$	$\overline{\pi_{j,t} * rec_{j,t}}$	$\rho_{lr,j}(\Delta gdp_{j,t}, \pi_{j,t})$	$\rho_{lr,j}(\Delta cons_{j,t}, \pi_{j,t})$
coef.	-0.063^{*}	-0.273	0.158	-0.075
(z-stat)	(-1.757)	(-1.256)	(0.396)	(-0.416)
$R^2_{adj.}$	0.12	0.04	0.01	-0.04
Panel D. Real Rates (Biv.)	$\rho_{lr,j}(r_{j,t}^{pf}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{ao}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{oos}, MY_{j,t})$	$\rho_{lr,j}(r_{j,t}^{cw}, MY_{j,t})$
coef.	-0.729^{***}	-0.732^{***}	-0.757^{***}	-0.682^{**}
(z-stat)	(-3.161)	(-2.882)	(-3.220)	(-2.220)
coef.$(\rho_{lr,j}(dy_{j,t}, MY_{j,t}))$	-0.570^{**}	-0.520^{*}	-0.396	-0.423
(z-stat)	(-2.166)	(-1.950)	(-1.512)	(-1.519)
$R^2_{adj.}$	0.36	0.38	0.44	0.27

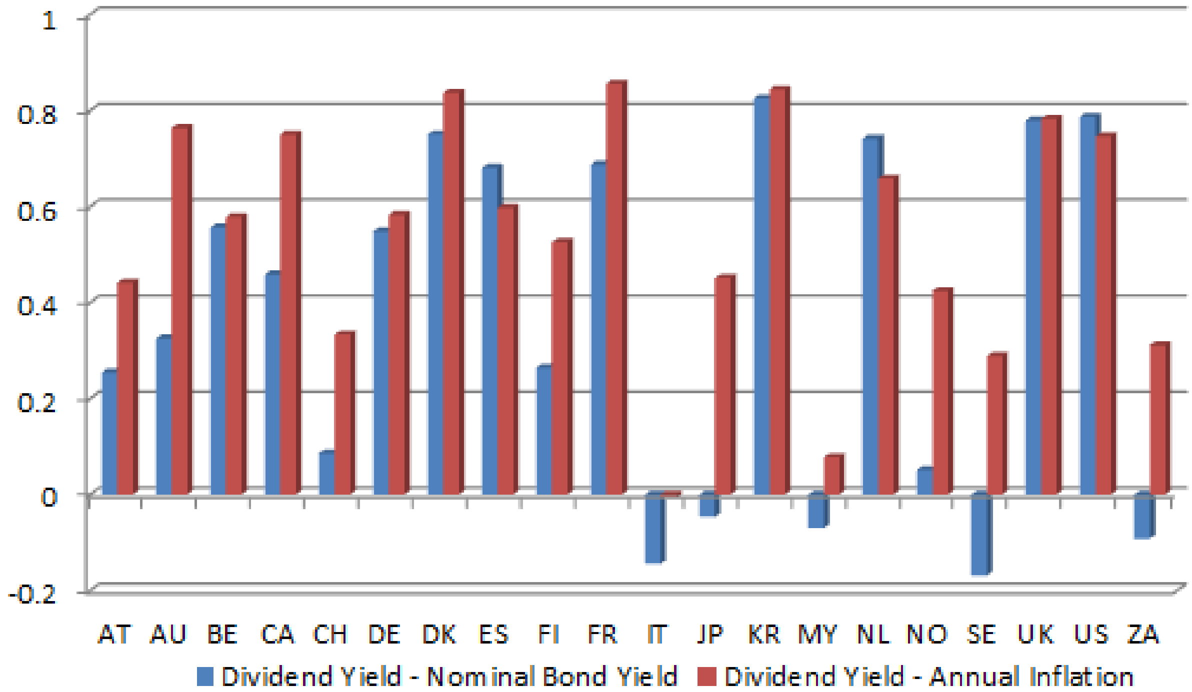
The table reports the robust cross-sectional regression results. In all specifications, the dependent variable is $\rho_{lr,j}(dy_{j,t}, i_{j,t})$, the median long-run correlation from Müller and Watson (2018) between the dividend yield, $dy_{j,t}$, and the long term nominal bond yield, $i_{j,t}$, in country j (n=20), over the period 1972-2016 (except for the specifications that include variables with perfect foresight (1972-2006) and $\Delta cons_{j,t}$ (1984-2016), n=15). x_j represents the independent variables. In Panel A, the independent variables are the long-run correlations between $MY_{j,t}$ and the real interest rates obtained from different inflation forecasting models: $r_{j,t}^{pf}$ is the real interest rate obtained by assuming perfect foresight for inflation expectations, that is, using the average 10-year future inflation (up to 2006); $r_{j,t}^{ao}$ is obtained by using average past 10-year inflation for inflation forecasts (Atkeson and Ohanian, 2001); $r_{j,t}^{oos}$ is obtained by using the best inflation forecast based on RMSFE; and $r_{j,t}^{cw}$ is obtained by using the best inflation forecast based on the Clark and West (2006) test statistics. In Panel B, the independent variables are the long-run correlations between $MY_{j,t}$ and the inflation expectations obtained from different inflation forecasting models. Panel C shows the validity results with alternative variables: $stag_j^{perc}$, the percentage of observations during which country j experiences stagflation, that is, recession (two consecutive quarters of negative real GDP growth) and high inflation (more than 10% annualized inflation per quarter); $\overline{\pi_{j,t} * rec_{j,t}}$, the country-specific time-series mean of the interaction between inflation and recession; $\rho_{lr,j}(\Delta gdp_{j,t}, \pi_{j,t})$, the long-run correlation between annual real GDP growth and inflation; and $\rho_{lr,j}(\Delta cons_{j,t}, \pi_{j,t})$, the long-run correlation between annual real consumption growth and inflation. In panel C, n=19, as Malaysia is excluded. Panel D reports the bivariate cross-sectional regressions where the independent variables are the long-run correlations between $MY_{j,t}$ and the real interest rates obtained from different inflation forecasting models, controlling for the long-run correlation between dividend yield $dy_{j,t}$ and the demographic variable $MY_{j,t}$. The reported z-statistics are based on the robust regression using bisquare weighting function. Asterisks *, ** and *** indicate significance at 10, 5 and 1 percent levels, respectively. n is the number of countries in each specification. The last row of each panel reports the OLS adjusted R^2.

Table 12: Alternative Demographic Channels

Specification: $\rho_{lr,j}(dy_{j,t}, i_{j,t}) = \alpha_0 + \alpha_1 x_j + \varepsilon_j$	$\rho_{lr,j}(r^{pf}_{j,t}, x_{j,t})$	$\rho_{lr,j}(r^{ao}_{j,t}, x_{j,t})$	$\rho_{lr,j}(r^{oos}_{j,t}, x_{j,t})$	$\rho_{lr,j}(r^{cw}_{j,t}, x_{j,t})$
Panel A. Population Growth				
coef.	-0.290	0.198	-0.028	-0.224
(z-stat)	(-1.203)	(0.627)	(-0.128)	(-0.894)
$R^2_{adj.}$	0.02	-0.03	-0.06	-0.01
Panel B. Life Expectancy				
coef.	-0.256	-0.353	-0.325	-0.653^{**}
(z-stat)	(-0.599)	(-0.584)	(-1.132)	(-2.324)
$R^2_{adj.}$	-0.04	-0.04	0.02	0.21
Panel C. Dependency Ratio				
coef.	0.034	0.454	0.437	0.325
(z-stat)	(0.109)	(0.989)	(1.406)	(0.974)
$R^2_{adj.}$	-0.06	-0.01	0.02	-0.01
Panel D. Share of Old (65+)				
coef.	0.155	-0.611^{*}	-0.363	-0.429
(z-stat)	(0.726)	(-1.780)	(-1.471)	(-1.437)
$R^2_{adj.}$	-0.02	0.12	0.08	0.05

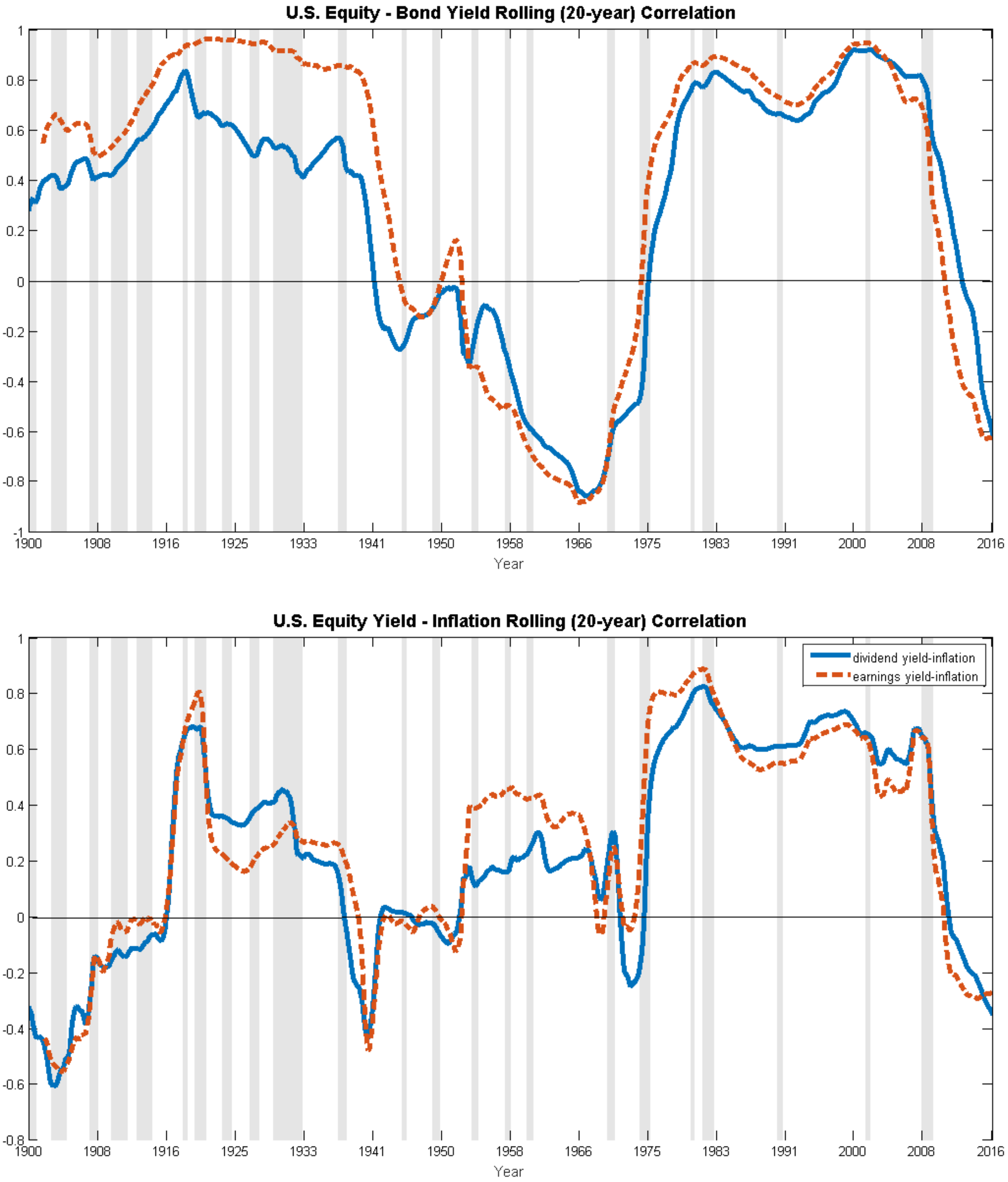
The table reports the robust univariate cross-sectional regression results. In all specifications, the dependent variable is $\rho_{lr}(dy_j, i_j)$, the median long-run correlation from Müller and Watson (2018) between the dividend yield dy_j and the long term nominal bond yield, i_j, in country j (n=20), over the period 1972-2016 (except for the specifications that include variables with perfect foresight (1972-2006)). x_j represents the independent variables. In Panel A, the independent variables are the long-run correlations between annual population growth and the real interest rates obtained from different inflation forecasting models: r^{pf}_{lr} is the real interest rate obtained by assuming perfect foresight for inflation expectations, that is, using the average 10-year future inflation (up to 2006); r^{ao}_{lr} is obtained by using the average past 10-year inflation for inflation forecasts (Atkeson and Ohanian, 2001); r^{oos}_{lr} is obtained by using the best inflation forecast based on RMSFE; and r^{cw}_{lr} is obtained by using the best inflation forecast based on the Clark and West (2006) test statistics. In Panel B, the independent variables are the long-run correlations between life expectancy at birth and the real interest rates obtained from different inflation forecasting models. In Panel C, the independent variables are the long-run correlations between the dependency ratio (that is, the share of the 25-64 population aged either 0-24 or 65+) and the real interest rates obtained from different inflation forecasting models. In Panel D, the independent variables are the long-run correlations between the share of the elderly population (that is, the share of population that is aged 65+) and the real interest rates obtained from different inflation forecasting models. The reported z-statistics are based on the robust regression using bisquare weighting function. Asterisks *, ** and *** indicate significance at 10, 5 and 1 percent levels, respectively. n is the number of countries in each specification. The last row of each panel reports the OLS adjusted R^2.

Figure 1: Correlations: International Panel



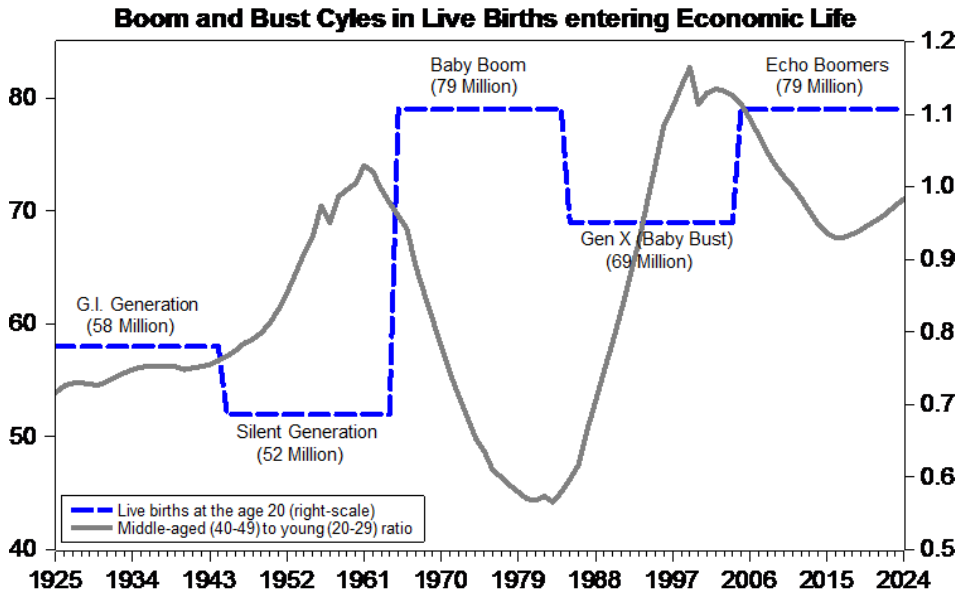
This figure plots the correlation between dividend yields and nominal bond yields (blue-bars), as well as the correlation between dividend yields and inflation (red bars), over the post-Bretton Woods sample, for a sample of 20 countries.

Figure 2: Time varying Correlations: Stocks, Bonds and Inflation



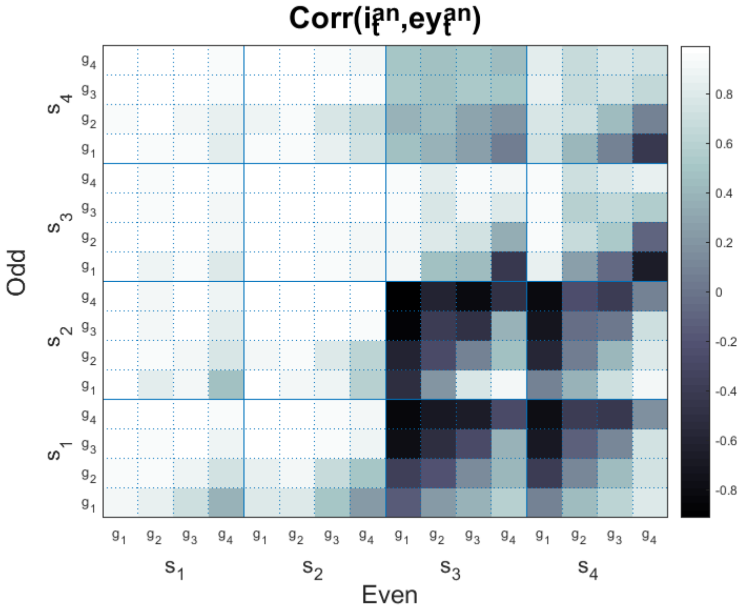
Panel A plots the 20-year (240 months) rolling correlation between dividend yields and the 10-year nominal bond yield (solid blue), and the 20-year (240 months) rolling correlation between cyclically adjusted earnings yields and bond yields (dashed red). Panel B plots the 20-year (240 months) rolling correlation between dividend yields and annual inflation (solid blue), and the 20-year (240 months) rolling correlation between cyclically adjusted earnings yields and annual inflation (dashed red). Grey shaded areas are NBER recessions. Sample 1880m1-2016m12. Monthly data.

Figure 3: Boom Bust Cycles in Live Births

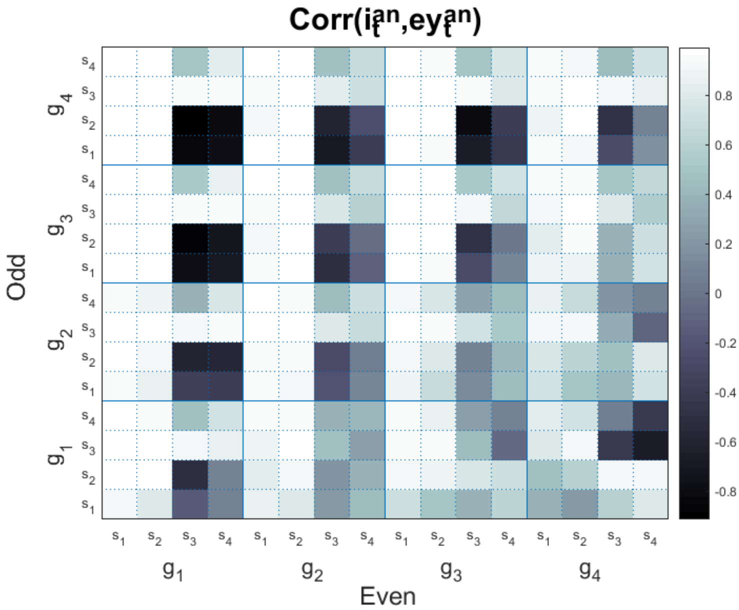


This figure plots the total number of life births (bar graph with dashed-line) at age 20 (the start of economic life) and the demographic variable, MY_t, (solid line) measured as the proportion of middle-aged (40-49) to young (20-29) population. Sample 1925-2024. Annual data.

Figure 4: Correlation between Bond and Stock Yields across States



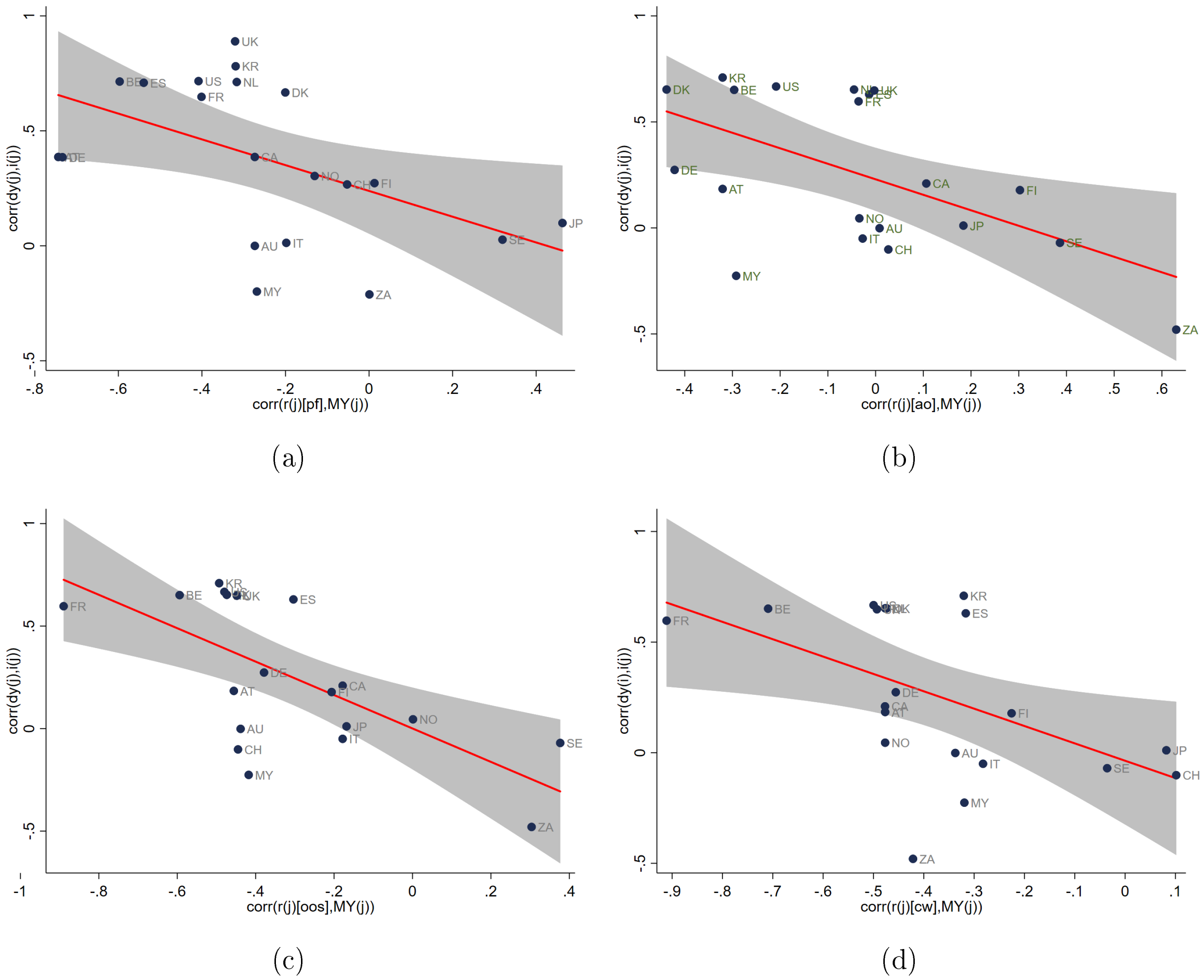
(a)



(b)

Panels A and B report the correlation between nominal bond yields and equity yields. In panel A, the correlation shows the comovement between yields from state (j,s,g) to state (j+1,s+1,g+1), where $j = \{odd, even\}$, $s = \{s_1, s_2, s_3, s_4\}$ and $g = \{g_1, g_2, g_3, g_4\}$. In panel B, the correlation shows the comovement between yields from state (j,g,s) to state (j+1,g+1,s+1).

Figure 5: Stock-Bond Yield Comovement: Real Channel



(a) (b) (c) (d)

All panels provide a scatter plot of the demographic effect on real bond yields (x-axis) and stock-bond yield correlation (y-axis). The demographic effect on real bond yields is proxied by the median of the posterior correlation obtained using the Müller and Watson (2018) framework between real interest rates and MY_t in each country. r^{pf} is the real interest rate obtained by assuming perfect foresight for inflation expectations (Panel A); r^{ao} is obtained by using the average past 10-year inflation for inflation forecasts (Panel B); r^{oos} is obtained by using the best inflation forecast based on RMSFE (Panel C); and r^{cw} is obtained by using the best model based on the Clark and West (2006) test statistics (Panel D). The panels show the regression line (red) and the 95% confidence interval (gray shaded area).