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THE POWER OF RENEGOTIATION AND MONITORING IN SOFTWARE OUTSOURCING: SUBSTITUTES OR COMPLEMENTS?

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ABSTRACT

Monitoring and contract *renegotiation* are two common solutions for addressing information asymmetry and uncertainty between a client and a vendor of software outsourcing services. Monitoring is mostly applied in time-and-materials contracts, as a basis for inspecting and reimbursing the vendor's efforts in system development. Renegotiation, by contrast, is deployed in fixed-price and time-and-materials contracts to mitigate the loss of surplus from uncertainty after system development. We investigate the interaction between monitoring and renegotiation and examine the corresponding contract choice problem. We find that the client benefits from renegotiation based on two effects: an *uncertainty-resolution effect* and a *post-development incentive effect*, which incentivizes the vendor to exert additional effort in system development. Monitoring does not resolve uncertainty, although it does encourage the vendor to exert additional effort, a *pre-development incentive effect*. Our analysis shows that the choice of renegotiation or monitoring depends on the interactions of the above effects, which are moderated by the renegotiation cost, monitoring cost, and bargaining power in renegotiation. When renegotiation cost is low: if the client has high bargaining power and low monitoring cost, monitoring and renegotiation are *complements* and both are selected; otherwise, the two instruments are *substitutes* and contract renegotiation is preferred. When renegotiation cost is high: monitoring substitutes for renegotiation and the client only chooses monitoring if the cost to do it is low; or else neither is used. Overall, this research shows that four appropriate contract strategies should be used under somewhat different circumstances. We further analyze the impacts of some other key aspects of software outsourcing and extend the base model to address two alternative situations to show the robustness of our findings. The results apply to a range of *software reliability growth models*, including when machine learning or cloud computing are used.

Keywords: Software outsourcing, software reliability, monitoring, renegotiation, incentives, incomplete contract.

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1. INTRODUCTION

Software outsourcing has grown tremendously over the last two decades (Liang et al. 2016b). ReportLinker's (2020) analysts forecasted the global IT outsourcing market to have a compound annual growth rate (CAGR) of 5% from 2020 to 2024. According to KPMG's (2018) report, in 2017, 727 IT outsourcing contracts worth US$137.2 billion were signed worldwide, and fixed-price and time-and-materials contracts contributed over 49% and 2% of total IT business process outsourcing (BPO) deal value, respectively.[1] With so much money at stake, software outsourcing needs to be cost-effective for an organization to compete well in its markets.

The software outsourcing process has various general activities, including contracting, development, renegotiation, testing, and maintenance. Practitioners recognize that outsourcing involves various information asymmetries and uncertainties that introduce challenges in its management. They include, for example: how to estimate a vendor's effort in system development (Dey et al. 2010); the volatility of software code as business needs change (Krishnan et al. 2004); and the ongoing transformation of the IT landscape that affects systems (Moreno 2017). Monitoring and contract renegotiation are two common instruments that organizations use to address information asymmetry and uncertainty in software outsourcing. *Monitoring* is applied in time-and-materials contracts to inspect and reimburse a vendor for its efforts. *Renegotiation* occurs when two parties revise their initial contract, and it can be deployed in both fixed-price and time-and-materials contracts to mitigate the loss of valuable surplus when uncertainty arises after development. See Appendix A for a glossary of terms in this article.

Given their prevalence in industry, we focus on these two types of contracts: fixed-price and time-and-materials contracts (Gopal and Sivaramakrishnan 2008, Dey et al. 2010, Korotia 2017). A *fixed-price contract* consists of a pre-determined payment for system development, testing, and maintenance services from the vendor. By contrast, a *time-and-materials contract* includes an extra fee or reimbursement beyond the payment for services obtained based on the vendor's effort.[2] Considering the non-contractibility of effort in a time-and-materials contract, the client needs to monitor the vendor's effort to determine the appropriate compensation for what it has done.[3,4]

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[1] The analysis and findings presented in KPMG's report are based on data from IDC's contract database (www.idc.com).

[2] The essential difference between a fixed-price contract and a time-and-materials contract is whether to monitor the vendor's effort. In other words, a time-and-materials contract is a more general contract since the client needs to decide on the monitoring level. If it is zero, then a time-and-material contract will degenerate into a fixed-price contract, which is a *corner-point solution* of the more general time-and-materials contract.

[3] We wish to acknowledge Thomas Weber, who encouraged us to think of contract types as *corner-points* across a spectrum of possible solutions in view of information asymmetry and uncertainty. The suggestion was to derive the types of contracts that either do or could exist under different market circumstances. This idea is interesting, but we determined it would be more practical, in light of industry practices, to understand what we observe in industry in greater depth. Thus, we identified industries, settings and firms to motivate our choices through examples. The wider-spectrum approach is left for future research.

[4]The business press discusses other contract types in addition to the two main contracts. One is a *target-cost contract*, which is reimbursable when the scope of the project is relatively uncertain, and the exact costs are not easily estimated when the contract is

After two parties engage in an outsourcing process with a fixed-price or a time-and-materials contract, they often renegotiate the initial contract terms once the system has been developed because of the realization of uncertainties.[5] According to Gartner, approximately 75% of all existing outsourcing relationships are renegotiated during their lifetime (James 2017). For example, the Kansas Department of Health and Environment signed a fixed-price contract with Accenture, and after the system was developed, they extended the contract for five years to test and debug the system (Marso 2016). Also, the British Columbia, Canada Ministry of Health signed a time-and-materials contract with IBM and revised the contractual terms around time-to-completion and defect remediation. It required IBM to resolve system bugs during testing, resulting in cost over-runs (Auditor General of British Columbia 2015). Renegotiation of testing time addresses the uncertainties in system development and affects the vendor's effort backwardly, since testing time is determined by the quality of the effort expected to be made. Further, testing time is a decision in software outsourcing to balance the trade-offs between testing and maintenance costs and between the value of the system and bug-led disutility. If testing time is short, critical bugs may remain undiscovered, resulting in high maintenance costs and high disutility from bugs during system use. However, prolonged testing typically leads to high testing costs and system release delays, reducing value.

Mathur (2016) suggests that a fixed-price contract with renegotiation is appropriate, and the client will most often use renegotiation only. However, the Scottish Police Authority decided to terminate its fixed-price contract with Accenture within a year after renegotiating testing time (Evenstad 2016). So, renegotiation is not effective for all clients, and nobody likes to renegotiate with a vendor (Shared Services and Outsourcing Network 2012). Thus, time-and-materials contracts are popular: they reduce the likelihood of renegotiation (Knoll 2016), implying that monitoring substitutes for renegotiation. Yet, after signing a time-and-materials contract, the British Columbia, Canada Ministry of Health nevertheless renegotiated with IBM on its initiative and the cost over-runs that developed. A summary of examples with different contract forms motivating is provided in Table 1.

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signed (Grela and Jaworski 2020). The client must be willing to bear the risk associated with the cost uncertainty and any changes in project scope. Another is a *capped-budget accelerated-bonus contract* (Baird 2016). This is a fixed-price contract with an embedded bonus for the vendor to achieve additional payback that is beneficial for both sides, although completion may be ahead of schedule or somewhat below the targeted cost. Finally, a *dedicated-team contract* calls for staff members of the outsourcing vendor to join and collaborate with a client's staff, which manages the overall process. This is seen in complex projects, and when new software methods are being brought into a firm.

[5] In general, the client initiates renegotiation by sharing the renegotiation surplus with the vendor (Flinders 2012, de Novoa 2015, Saad 2017). For example, in the software outsourcing contract between the British Columbia, Canada Ministry of Health and IBM, the Ministry expected to pay US$16.2 million for IBM's implementation services at the beginning, but after renegotiation, the payment rose to US$73.5 million for a much greater amount of work (Auditor General of British Columbia 2015).

Table 1. Examples of Software Outsourcing Contracts

Client	Vendor	Contract Form	Renegotiation	Citation
Kansas Department of Health and Environment	Accenture	Fixed-price contract	Testing time	Marso (2016)
Scottish Police Authority	Accenture	Fixed-price contract	Testing time	Evenstad (2016)
British Columbia, Canada Ministry of Health	IBM	Time-and-materials contract	Testing time	Auditor General of British Columbia (2015)

Software outsourcing thus creates challenges for the client. Should it monitor the vendor's effort in development, renegotiate testing time after that, or both? Which contract type should it sign when initiating the outsourcing process? To address these questions, we build a multi-stage model in which a client outsources a customized system from an IT services vendor. It enabled us to investigate the interaction between the usage of monitoring and renegotiation, and to evaluate contract choice. Our analysis shows that renegotiation generates *direct and indirect uncertainty-resolution effects*, making testing time efficient and increasing social welfare. The direct uncertainty-resolution effect comes from the same effort as the case without renegotiation, and the indirect uncertainty-resolution effect stems from the additional effort compared to the non-renegotiation case. With the indirect uncertainty-resolution effect, when the client's required *system complexity* for the customized system is moderate, renegotiation can incentivize the vendor to exert more effort in system development. This can be regarded as the *post-development incentive* of renegotiation, and it increases with the vendor's *bargaining power* in renegotiation.[6] Monitoring stimulates the vendor's effort, which can be regarded as a *pre-development incentive* because monitoring is determined when the parties sign a time-and-materials contract.

When the costs of monitoring and renegotiation are low, a vendor with low bargaining power is not affected by the post-development incentive because of the low revenue share generated by renegotiation. The client adopts monitoring at the same time for the pre-development incentive to create the indirect uncertainty-resolution effect of renegotiation. Thus, monitoring and renegotiation are complements in this scenario, and the client selects a time-and-materials contract with renegotiation.[7] This is different from prior studies (Benaroch et al. 2016, Gopal and Koka 2010), which show that monitoring reduces the possibility of opportunistic renegotiation. If the vendor has high bargaining power, the post-development incentive is sufficiently strong to increase the vendor's effort, and renegotiation substitutes for monitoring.

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[6] The definition of *client bargaining power* (Porter 1979) is the power a client can exert on a vendor so the latter will offer higher quality products and better services at lower prices. Similarly, *vendor bargaining power* is the pressure a vendor can create on a client by upping prices, and adjusting quality, availability and delivery times. Bargaining power arises in outsourcing and other contracting when an established vendor's risk for earning revenue is less than the risk a client would face if it were to lose the vendor as a partner (de Fontenay and Gans 2008). When the vendor offers access to essential assets for the client to create value in its business (Han et al. 2008) – a classic example in the economics of property rights (Hart and Moore 1990). When the client's contract is small, the vendor has many clients to spread risk. When the client is large relative to the vendor, it may have a natural advantage in bargaining power. So, the vendor must rely on a contract to drive revenue, especially new vendors with fewer clients.

[7] *Complementarity* means the simultaneous use of the solutions strengthens the benefits of one; *substitution*, in contrast, is the simultaneous use of the solutions weakens their joint benefits (Tiwana 2010).

Thus, the client selects a fixed-price contract with renegotiation. In the earlier example, Accenture's bargaining power was large because the Kansas Department of Health and Environment was locked into a partnership that promised Accenture US$200 million in revenue. So, the Kansas Department of Health and Environment chose to sign a fixed-price contract with Accenture based on the breadth of commitments.

When monitoring has a high cost and renegotiation is inexpensive, even if the vendor has low bargaining power, renegotiation can substitute for monitoring. This is because the indirect uncertainty-resolution effect does not cause the pre-development incentive to cover the cost of monitoring and the client adopts renegotiation only for the direct uncertainty-resolution effect. Another example is related to the bargaining power of Fujitsu, which was lower than that of Whitbread PLC because the latter had other potential vendors it could choose (Hadfield 2005). Yet Whitbread PLC chose Fujitsu to provide IT services and extended its contract duration because Fujitsu agreed to fixed annual pricing, allowing Whitbread to be its outsourcing budget would be sufficient, with no need for extra monitoring cost (Fujitsu 2004, 2009).

When renegotiation is expensive, renegotiation is not adopted by the client. For example, if the State of Michigan and Hewlett-Packard (HP) extended their contract duration, then half of Michigan's IT projects over US$15 million would have run 45% over budget (Bort 2015). Michigan refused to renegotiate with HP and instead reached a US$13 million settlement (Gerstein 2017). Meanwhile, if the benefit from the pre-development incentive of monitoring dominated its cost, the client would have adopted monitoring as a substitute for renegotiation to incent vendor effort. If monitoring cost had been higher than its benefit, the client would have used neither monitoring nor renegotiation and selected a no-negotiation fixed contract.

We further explore how the vendor's development effort and the client's profit change with some key aspects of software outsourcing. For example, a vendor is more likely to exert lower effort when system complexity is greater or the bug rate is higher, and more likely to exert higher effort when system lifetime is longer. The client's profit decreases in system complexity and bug rate but increases in system lifetime. We extend the base model so the discrete levels of development effort become continuous and examine how our results change when renegotiation cost endogenously depends on the renegotiation process. We also show that the base model's findings hold qualitatively.

The rest of the paper is organized as follows. We first discuss relevant literature, and then lay out the details of an optimization model to analyze the most often-used fixed-price and time-and-materials contracts. Then, our analysis probes contract choices, based on the interaction between monitoring and renegotiation, which also discusses how we are able to arrive at theoretically meaningful and managerially valid contract strategies. We further analyze the impacts of some other aspects of software outsourcing and extend the base model to two other situations to show the robustness of our findings. We conclude with theoretical and managerial contributions and strategy conjectures built on our theory perspectives.

2. LITERATURE AND THEORY

2.1. Software and IT Outsourcing Contracts

This research is mainly related to the literature stream on software and IT outsourcing contracts. A summary of our findings related to papers in this stream is provided in Table 2. A large body of research in this stream focuses on contract choice. Fixed-price and time-and-materials contracts are commonly-used contract forms in practice (Gopal and Koka 2010, Mani et al. 2012), though the former is most common by a wide margin (KPMG 2018). Compared with a fixed-price contract, a time-and-materials contract entails higher monitoring costs (Bajari and Tadelis 2001, Roels et al. 2010). As system complexity under development increases though, the client prefers a time-and-materials contract (Gefen et al. 2008) because monitoring transforms private information about vendor effort into public information (Liang 2016a).

Gopal and Sivaramakrishnan (2008) examined data on 93 offshore projects and analyzed the different preferences between fixed-price and time-and-materials contracts from the vendor's perspective. Dey et al. (2010) presented a contract-theoretic model that incorporates the quality of a developed system, the timeliness of delivery and the post-delivery software to compare the client's profit for different contract forms. Gopal and Koka (2010) studied data collected from 100 software projects and investigated how different incentive structures inherent in fixed-price and time-and-materials contracts influence the quality provided by the vendor in software development outsourcing. Roels et al. (2010) studied IT outsourcing services and analyzed how contract form choice is driven by verifiability of the client's and the vender's effort levels. Bhattacharya et al. (2018) considered collaborative services and verifiability of the parties' effort but focused more on how task modularity influences effectiveness of single-versus multi-sourcing.

We study incomplete software outsourcing contracts because they involve unforeseen contingencies (Che and Hausch 1999), non-contractible investments and behavior (Susarla 2012), and unmeasurable performance (Fitoussi and Gurbaxani 2012). Bhattacharya et al. (2014) studied contract incompleteness and compared client and vendor profits for time-and-materials and profit-sharing contracts. They focused on negative effects of renegotiation on outsourcing and proposed an option-based contract robust to renegotiation. We focus on the positive effects of renegotiation and show it may incentivize development effort.

Table 2. Relevant Literature on Software and IT Outsourcing Contracts

Research Angles		Paper	Monitoring	Renegotiation	Major Findings
Contract Choice	Fixed-price (FP) vs. Time-and-materials (TM)	Gopal and Sivaramakrishnan (2008)	√	×	• Vendor prefers a fixed-price contract for larger, longer projects with larger teams, to secure larger information rent. • Vendor prefers a time-and-materials contract when the risk of employee attrition from the project team is high.
	FP vs. TM vs. Performance-based vs. Profit-sharing	Dey et al. (2010)	√	×	• Fixed-price contract is suitable for simple projects. • Time-and-materials contract suitable for complex projects with low monitoring costs.
	FP vs. TM	Gopal and Koka (2010)	√	×	• In time-and-materials contracts, clients increase monitoring because it results in improved services. • Fixed-price contracts enable vendors to leverage their capability and derive higher returns from software quality.
	FP vs. TM vs. Performance-based	Roels et al. (2010)	√	×	• Fixed-price contracts contingent on performance are preferred when service output is sensitive to vendor effort. • Time-and-materials contracts are optimal when output is sensitive to client effort.
	TM vs. Profit-sharing	Bhatta-charya et al. (2014)	√	√	• Compared with time-and-materials contracts, profit-sharing contracts can induce optimal effort from clients / vendors when cost of monitoring the verifiable outcome is low. • Option-based contracts robust to renegotiation which leads to hold-up problems.
	Single vs. multi-sourcing	Bhatta-charya et al. (2018)	×	×	• When tasks are modular, multi- dominates single-sourcing. • When tasks are integrated, sourcing choice depends on trade-offs among alignment between performance / project revenue, verifiability of project revenue, and moral hazard.
Incomplete Contract	Transaction cost	Richmond et al. (1992)	×	√	• Outsourcing provides incentives for vendors to sustain specific investments and promote future cost reduction. • Even if clients and vendors share renegotiation surplus, outsourcing can lead to a higher value for clients compared to a salaried internal development team.
	Low balling	Whang (1995)	×	×	• Vendors sell customized software below their marginal cost because of the learning effect and software code reusability. • For clients, directly picking a vendor and signing a contract with property rights sharing can improve the alignment of vendor incentives, compared with bidding auction.
	Renegotiation	Benaroch et al. (2010)	×	√	• Increased demand uncertainty ups clients' will to backsource. • Vendor can modulate client's tendency to outsource and increase likelihood of greater profitability, by varying usage-based subscription fee per IT service unit outsourced.
	Transaction cost	Benaroch et al. (2016)	√	√	• Contract type / extensiveness are mechanisms for saving transaction costs arising under different circumstances. • Preference for time-and-materials contracts counteracts effect of certain transaction attributes on contract extensiveness, and cancels it with transaction uncertainty.
	Asset transfer	Chang et al. (2017)	√	√	• Asset transfers affect contract design, as with inclusion of clauses that protect clients and vendors. • Outsourcing objectives are more likely met when contracts have compensation mechanisms to support asset transfer.
	Ethics	Anand and Goyal (2019)	×	√	• Renegotiation is a myopic strategy to non-ethical clients. • Ethical clients can use IP sharing to insulate against worst effect of incomplete contracts and incomplete information.
Contract Choice and Incomplete Contract	Fixed-price vs. Time-and-materials & renegotiation	Our work	√	√	• Monitoring and renegotiation are substitute or complement, with renegotiation / monitoring cost, and bargaining power. • Four contract strategies are available for clients, determined by the interaction of monitoring and renegotiation.

Contract incompleteness and renegotiation are also investigated for software and IT outsourcing contracts. Contract renegotiation has been studied in Economics, and the classical conclusion is that the hold-up problem occurs when the parties can take unilateral actions after signing a contract. The investing party fears expropriation of investment benefits by its contract partner in renegotiation (Che and Hausch 1999), leading to underinvestment (Maskin and Moore 1999). In contrast, we show that testing time renegotiation may offer the vendor additional incentive to invest in making more service-related effort.

For software and IT outsourcing contracts, previous literature investigated incomplete contracts from different perspectives, such as transaction cost, low-balling, renegotiation, asset transfer, and ethics. Richmond et al. (1992) considered the contract incompleteness and examined the impact of outsourcing on the IT developer's effort and the transaction cost of outsourcing by comparing it with using an internal development team. Whang (1995) studied whether the benefits of declining development costs are passed on to the client in the form of lower prices when vendors bid strategically – called *low-balling*. He suggested that directly signing a property rights-sharing contract with a vendor may dominate bidding in an auction. Benaroch et al. (2010) modeled the implications of a backsourcing renegotiation contract option. They distilled cost and value effects for the client and the vendor and computed the fair compensation and value for them, by pricing the option at contract initiation. Benaroch et al. (2016) later focused on the ex ante and ex post transaction costs balance for incomplete contracts and examined contract design choices in terms of transaction and relational attributes. Chang et al. (2017) studied how asset transfer, based on property rights theory, affects the client's contract design and the vendor's investment incentive, and provided a guide to clients on IT outsourcing. Finally, Anand and Goyal (2019) built a dynamic model that integrated incomplete contracts and analyzed how ethics, reputation effects, and IP sharing drive IT outsourcing.

We investigate the interplay between monitoring and renegotiation rather than just incomplete contracts in general in the present research – a clear difference. Most studies related to transaction cost and agency theory have emphasized the effect of monitoring to prevent vendor opportunism (Benaroch et al. 2016, Gopal and Koka 2010). Instead, we show that monitoring and opportunistic renegotiation (Aron et al. 2005) can complement each other under certain conditions to incentivize vendor effort in different ways.

2.2. Software Reliability and Bug Identification

Another stream is *software reliability*: how a system will perform without bugs over a period. The most commonly-used reliability prediction model is the *Goel-Okumoto non-homogeneous Poisson process model* (the G-O model hereafter) (Goel and Okumoto 1979).[8] Using this, Pham and Zhang (1999) analyzed reliability cost for optimal testing time. Jiang et al. (2012) separated testing stop time from system release

[8] It is based on standard assumptions. All bugs in a piece of software are typically independent for any failure detection procedure. The likelihood of the count of future bugs detected is a function of the number of current and past bugs. Those that get detected are assumed to be fixed prior to the next testing event. And, when a software error occurs, an effort is made to fix the bug, so no new errors like it will occur.

time, and considered testing that continues during system operation, as with post-release testing. August and Niculescu (2013) examined the software demand impact on post-release testing. Also, Jiang et al. (2017) considered reliability and market benefits and derived the optimal testing time and number of testers.

An unstated assumption in these studies is that the system will be developed completely. This is not always true though, as the business press and some of our examples suggest (Mezak 2018). Others ignored that the vendor must exert essential effort for system development. For example, it must ensure due diligence for the technology choices to be made and avoid the usual pitfalls of coding errors. Thus, we consider the development and testing stages. After the vendor develops the system, the parties may wish to renegotiate how much testing time should be in the contract before committing to it.

3. MODELING SOFTWARE OUTSOURCING CONTRACT DECISIONS

We now construct a decision model with testing time renegotiation. We present two base cases with the sum of the expected payoffs of the parties maximized under renegotiation and non-renegotiation – the *first-best solution.*[9] Comparing the cases, we obtain the impact of testing time renegotiation on outsourcing.

3.1. Model Description

We consider a *client* ("Client" hereafter) contracts for IT services from a *vendor* ("Vendor" hereafter) for three stages: development, testing, and maintenance. Table 3 presents our modeling notation. In the *system development stage*, the Vendor develops a customized system for the Client based on its system complexity needs. We use Υ to denote system complexity, for example, the size of the system's codebase or the number of modules and functions (August and Niculescu 2013). The Vendor makes development effort, including identification of the Client's desired system complexity, and planning, designing and programming the code for the system, to improve system reliability. As in Yamada (2014), we use the expected number of software bugs to indicate the system reliability level. The more effort the Vendor makes, the fewer the number of bugs and the higher the system reliability becomes. We assume two levels of effort e, high (e_H) and low (e_L), for the Vendor to choose from. The decision is the Vendor's private information, and the cost of effort e is $\frac{e}{c}$, where c represents the Vendor's capability in the development stage.

[9] *First-best* refers to the best the principal can do to achieve both optimal allocative and distributive efficiency, if she knows the agents' preferences over labor and income and the incentive compatibility is not imposed (Bolton and Dewatripont 2005).

Table 3. Modeling Notation and Technical Definitions

Notation	Definition	Comments
S	Social surplus	S represents the social surplus.
C	Client	C represents the Client as a subscript (but cost when it is not a subscript).
V	Vendor	V represents the Vendor.
CS	Customized system	CS is a customized system; the Client outsources it from a Vendor.
$N_{CS}(e)$	Expected bugs in system	Reliability of system post development; common knowledge to parties.
$e \in \{e_L, e_H\}$	Vendor's development effort (low=L, high=H), $0 < e_L < e_H$	Development effort tied to reliability of system's expected bugs.
$\hat{e} \in \{e_L, e_H\}$	Vendor's reported effort (low=L, high=H), $0 < e_L < e_H$	Vendor's reported effort determines the payment made by Client in a time-and-materials contract.
$\varepsilon \sim U[-\sigma, \sigma]$	Uncertainty about outcome of Vendor's development effort	Uniformly distributed on $[-\sigma, \sigma]$; $-\sigma, \sigma$ are min, max ε values ($\sigma > 0$).
T	System's lifetime	Time duration when system developed until withdrawal of maintenance, including testing and maintenance time.
$t \in [0, T)$	Initial testing time	Before system developed, Vendor negotiates testing time with Client.
$\tilde{t} \in [0, T)$	Renegotiated testing time	After system developed, uncertainty of Vendor's development effort is resolved, so both can renegotiate initial testing time to a new testing time.
τ	Time during testing	It can be any time which satisfies $\tau \in [t, T]$.
$\lambda \in (0,1)$	Bug failure rate during testing	Reflects the Vendor's testing effectiveness.
$\widetilde{N}_{CS}(e, t)$	Expected number of bugs in system after testing time t	Bugs of customized system decrease when testing time is increased.
$\widetilde{N}_{CS}(e, T)$	Expected number of bugs in system after system lifetime T	Bugs of customized system decrease when system lifetime is increased.
$c > 0$	Vendor development capability	When exerting the same effort, a Vendor with higher capability consumes less cost in system development.
$a > 0$	Cost of fixing one bug in testing	Average cost of fixing one bug when bug fix costs are low.
$b > 0$	Added cost of fixing one bug in maintenance after testing	Average cost of fixing one bug in later stage, when bug fix costs are high, and financial impacts on Client's business operations may occur.
$K(c) > 0$	Cost of testing per unit time	Decreases with Vendor's development capability ($\partial K(c)/\partial c < 0$).
TC	Vendor's total cost	Vendor's total costs include development cost, testing cost, and bug-fix costs during testing and maintenance.
$C_R \geq 0$	Client's cost of renegotiation	Client and Vendor share the renegotiation cost, but this yields similar results as when Client bears whole renegotiation cost. (C is not a subscript.)
RS	Renegotiation surplus	Without renegotiation cost, value from renegotiation after development.
RB	Renegotiation benefit	Without renegotiation cost, the expected difference of social surplus between the renegotiation and non-renegotiation cases.
U	Client's total utility	Increases with system complexity and system use, and decreases with expected number of software bugs during maintenance.
$\Upsilon > 0$	System complexity	Includes size of system's codebase, number of modules and functions and so on. Max value of system per unit time when zero bugs are guaranteed.
$B(c) > 0$	Bug rate	Decreases with Vendor's development capability ($\partial B(c)/\partial c < 0$).
$\delta > 0$	Client's sensitivity to bugs in customized system	If Client is more sensitive to bugs, the same expected number of bugs in a customized system will lead to higher disutility in use.
$\alpha \in [0, 1]$	Vendor's power in renegotiation	Portion of incremental surplus the Vendor attains in renegotiation.
$\beta \in (0, 1)$	Effectiveness of Vendor effort	Restriction $0 < \beta < 1$ signifies a decreasing return of Vendor's effort and it becomes increasingly difficult to reduce the expected bugs.
$P \geq 0$	Initial payment, Vendor svcs	Set at beginning of software outsourcing for Vendor's development, testing and maintenance services.
$\tilde{P} \geq 0$	Ex post payment, Vendor svcs	Generated after renegotiation, including initial payment P and Vendor's profit from renegotiation.
$r \geq 0$	Reimbursement for unit effort in time-and-materials contract	Reimbursement for Vendor effort per unit in time-and-materials contract.
$\phi \in [0, 1]$	Monitoring policy of Client	Probability that Client finds if Vendor reports its true effort by monitoring documents and development activities.
$w \geq 0$	Cost of monitoring	Per unit cost of monitoring Vendor's development docs and process.
$s > 0$	Penalty for misreported effort	Cost of reputation loss and subsequent future business loss of the Vendor.

- **Assumption 1 (Expected Number of Bugs):** *Given system complexity Υ, the Vendor's development capability c and effort e, at the end of development, the expected bugs for the system are $N_{CS}(e)$, where:*[10]

$$\underbrace{N_{CS}(e)}_{\substack{\text{expected number of bugs}\\ \text{for customized system}}} = \underbrace{\Upsilon B(c)}_{\substack{\text{initial expected}\\ \text{number of bugs}}} - \underbrace{e^{\beta}}_{\substack{\text{effort}\\ \text{performance}}} + \underbrace{\varepsilon}_{\text{uncertainty}} . \quad (1)$$

In Equation (1), $\Upsilon B(c)$ denotes the initial expected number of bugs, increasing in system complexity Υ and bug rate $B(c)$, where $B(c)$ represents the average number of bugs per thousand lines of code (August and Niculescu 2013) and decreases in the Vendor's development capability c $(\partial B(c)/\partial c < 0)$. Here, e^{β} reflects the performance of Vendor's effort on the reliability of the system. Following the single-factor Cobb-Douglas function, β is the effectiveness of Vendor effort for decreasing the expected number of bugs (Hu et al. 1998). We further assume that $\beta \in (0,1)$ because it is increasingly difficult to reduce the expected number of bugs (Dey et al. 2010, Parker and van Alstyne 2018). The uncertainty ε of the outcome of the Vendor's development effort is uniformly distributed in $[-\sigma, \sigma]$, where $-\sigma$ and σ are minimum and maximum values of ε. The uncertainty is realized when the Vendor finishes development, and the parties observe the updated expected number of bugs, $N_{CS}(e)$, in the system.

- **Assumption 2 (Existence of Bugs):** $\Upsilon B(c) - e_H^{\beta} - \sigma > 0$.

Assumption 2 implies that the minimum expected number of bugs for the system, based on the initial expectation $\Upsilon B(c)$, is positive for the Vendor at any capability, even with high effort e_H and the most favorable realization of uncertainty ε. Thus, bugs still can be detected in testing and maintenance stages. Also, given system complexity Υ, development time is constant and fixed (Ghoshal et al. 2017, Li et al. 2017). To focus on testing time renegotiation impacts, we normalize development time to 0.

After the Vendor has finished development, uncertainty ε for expected bugs $N_{CS}(e)$ is realized, and the *system testing stage* begins. We denote the time from the beginning of testing to the end of maintenance as the *lifetime of the system* T (Ji et al. 2011, August and Niculescu 2013). The Vendor spends time t $(t < T)$[11] to detect bugs with a cost of $K(c)t$, where $K(c)$ is testing cost per unit time which decreases in the Vendor's development capability $(\partial K(c)/\partial c < 0)$. Testing is a *non-homogeneous Poisson process* (Roy et al. 2015), and we assume it has two properties (Jiang et al. 2012):

- **Assumption 3 (The Memoryless Property of Bug Detection):** *The detection of each bug in a system is independent of the detection of others, and the total bug-detection rate at any time is proportional to the number of undetected bugs at that time.*

Assumption 3 implies the probability a bug will be detected by time t is $F(t) = 1 - \exp(-\lambda t)$, where λ

[10] The expected number of bugs, $N_{CS}(e)$, of the system, and the remainder after testing $\widetilde{N}_{CS}(e,t)$ and maintenance $\widetilde{N}_{CS}(e,T)$ are assumed to be common knowledge. The IS literature has widely adopted this reliability modeling (Yamada 2014). In practice, experienced developers can estimate the bugs via *source lines of code* (SLOC) (Mayer 2012), and repository objects used.

[11] We assume that system lifetime $T > \frac{1}{\lambda}\ln\left(\frac{\lambda b N_{CS}(e)}{K(c)}\right)$ to ensure the optimal testing time t has interior solutions.

is the bug failure rate. This *memoryless property* is common in software reliability growth models that use math to characterize software bugs. See Appendix B for additional details on how software models handle bugs. We assume perfect debugging for this analysis:

- **Assumption 4 (Perfect Debugging):** *A detected bug can be fixed without causing more errors.*

Debugging is imperfect in practice though, the cumulative number of bugs detected at any time is expected to be the same with models that have perfect or imperfect debugging (Ohba and Chou 1989). Assumptions 3 and 4 are from the G-O model, which is widely adopted in software engineering (Yamada 2014, Roy et al. 2015) and IS (Jiang et al. 2012, August and Niculescu 2013).[12] According to the G-O model, the expected number of bugs remaining, $\widetilde{N}_{CS}(e,t)$, after testing time t will be:

$$\underbrace{\widetilde{N}_{CS}(e,t)}_{\substack{\text{expected number of bugs remaining}\\ \text{in customized system after testing time } t}} = \underbrace{N_{CS}(e)}_{\substack{\text{total expected number of}\\ \text{bugs in customized system}}} \exp\big(-\underbrace{\lambda}_{\substack{\text{failure rate}\\ \text{of each bug}}} \cdot \underbrace{t}_{\substack{\text{testing}\\ \text{time}}}\big). \quad (2)$$

In testing, the cost of fixing bugs is approximated by $a \cdot \left(N_{cs}(e) - \widetilde{N}_{CS}(e,t)\right)$ for the Vendor, where a is the cost of fixing each bug and $\left(N_{CS}(e) - \widetilde{N}_{CS}(e,t)\right)$ is the expected number of bugs detected. When the Vendor finishes testing, the system will be given to the Client and put into operation. The Vendor maintains it after for the time $(T-t)$. In the *system maintenance stage*, maintenance cost is incurred by the Vendor when failures occur during operation. They include the direct cost of identifying and fixing bugs, the downtime loss of revenue, and other costs. The cost for a bug-fix during maintenance is higher than during testing, if the bug was detected during testing.[13] Based on this, total maintenance cost is $(a + b) \cdot \left(\widetilde{N}_{CS}(e,t) - \widetilde{N}_{CS}(e,T)\right)$ for the Vendor, where b is the incremental cost for each bug in the maintenance stage compared with the testing stage, and $\left(\widetilde{N}_{CS}(e,t) - \widetilde{N}_{CS}(e,T)\right)$ is the expected number of bugs.

Based on these assumptions, the expected total cost $TC(e,t)$ for the Vendor includes four parts:

$$\underbrace{TC(e,t)}_{\substack{\text{total cost of}\\ \text{the Vendor}}} = \underbrace{\frac{e}{c}}_{\substack{\text{develpment}\\ \text{cost}}} + \underbrace{K(c)t}_{\substack{\text{testing}\\ \text{cost}}} + \underbrace{a \cdot \left(N_{CS}(e) - \widetilde{N}_{CS}(e,t)\right)}_{\substack{\text{bug-fix cost}\\ \text{during testing}}} + \underbrace{(a+b) \cdot \left(\widetilde{N}_{CS}(e,t) - \widetilde{N}_{CS}(e,T)\right)}_{\substack{\text{bug-fix cost}\\ \text{during maintenance}}}. \quad (3)$$

They are added without discounting, although they are incurred at different times. The parameters (a, b, c, and $K(c)$) are assumed to be accounted in the usual discounting. Rewriting Equation (3) via Equation (2), where $\widetilde{N}_{CS}(e,t) = N_{CS}(e) \cdot \exp(-\lambda t)$ with λ indicating bug detection intensity per unit time, yields:

$$TC(e,t) = \frac{e}{c} + K(c)t + aN_{CS}(e) \cdot \left(1 - \exp(-\lambda t)\right) + (a+b)N_{CS}(e) \cdot \left(\exp(-\lambda t) - \exp(-\lambda T)\right). \quad (4)$$

Equation (4) can be rewritten as:

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[12] Online Appendix assesses assumptions for fixed-price and time-and-materials contracts with machine learning bug detection.

[13] IBM estimates a bug costs US$1,500 to fix when testing, but US$10,000 in maintenance (McPeak 2017).

$$\underbrace{TC(e,t)}_{\substack{\text{total cost of}\\ \text{the Vendor}}} = \underbrace{\frac{e}{c}}_{\substack{\text{develpment}\\ \text{cost}}} + \underbrace{K(c)t}_{\substack{\text{testing}\\ \text{cost}}} + \underbrace{aN_{CS}(e)\cdot\big(1-\exp(-\lambda T)\big)}_{\substack{\text{(I) bug-fix cost if all the bugs of system}\\ \text{lifetime were detected during testing}}} + \underbrace{bN_{CS}(e)\cdot\big(\exp(-\lambda t)-\exp(-\lambda T)\big)}_{\substack{\text{(II) additional bug-fix cost in maintenance}\\ \text{if some bugs were not detected during testing}}}. \quad (5)$$

In Term (I) of Equation (5), $N_{CS}(e)\cdot\big(1-\exp(-\lambda T)\big)$ represents the total expected number of bugs during system lifetime and thus Term (I) is the bug-fix cost if all the likely bugs over a system's lifetime were detected during testing. In Term (II), $N_{CS}(e)\cdot\big(\exp(-\lambda t)-\exp(-\lambda T)\big)$ represents the expected number of bugs that were not detected during testing but occur during maintenance. Term (II) thus denotes the added cost for bug-fix in maintenance compared with testing.

The Client uses the system during time period $[t,T]$, and at any time $\tau \in [t,T]$, the expected number of the remaining bugs is $\widetilde{N}_{CS}(e,\tau) = N_{CS}(e)\cdot\exp(-\lambda\tau)$, which can cause problems for the Client (Jiang et al. 2012). We assume the *bug disutility rate* at time τ is $\delta\widetilde{N}_{CS}(e,\tau)$, with δ the Client's sensitivity to bugs. The Client's disutility from the expected number of bugs during period $[t,T]$ is:

$$\int_t^T \delta\widetilde{N}_{CS}(e,\tau)\mathrm{d}\tau = \frac{\delta}{\lambda}N_{CS}(e)\cdot\big(\exp(-\lambda t)-\exp(-\lambda T)\big). \quad (6)$$

- **Assumption 5 (Client's Utility):** The Client's utility from the customized system is

$$\underbrace{U(e,t)}_{\substack{\text{utility of}\\ \text{the Client}}} = \underbrace{\Upsilon\cdot(T-t)}_{\substack{\text{value of}\\ \text{the system}}} - \underbrace{\frac{\delta}{\lambda}N_{CS}(e)\cdot\big(\exp(-\lambda t)-\exp(-\lambda T)\big)}_{\substack{\text{disutility to be incurred by the expected}\\ \text{number of bugs during maintenance}}}. \quad (7)$$

Assumption 5 shows that the Client's utility $U(e,t)$ increases with system complexity Υ and system use time $(T-t)$, and decreases with the expected number of bugs during maintenance.

- **Assumption 6 (Testing Profit Ratio):** $\frac{\lambda b}{K(c)} > \frac{\delta}{\Upsilon}$.

Here, testing profit ratio is measured as the marginal benefit of system testing divided by its marginal cost. Assumption 6 ensures that the Vendor's testing profit ratio is higher than the Client's.[14]

- **Assumption 7 (Marginal Benefit and Cost during Testing):** $(\delta+\lambda b)N_{CS}(e) > \Upsilon + K(c)$.

Assumption 7 for software outsourcing implies that the marginal benefit of system testing $(\delta+\lambda b)N_{CS}(e)$ is greater than its marginal cost $\Upsilon + K(c)$.[15]

For outsourcing, our model permits a Client to use a fixed-price or time-and-materials contract (Menon

[14] For the Vendor, $\frac{\partial(-TC(e,t))}{\partial t} = -K(c) + \lambda bN_{CS}(e)\cdot\exp(-\lambda t)$. On the cost side, $K(c)$ represents the *marginal cost of testing*. On the benefit side, $\lambda bN_{CS}(e)$ represents the *marginal benefit of testing* because $\lambda N_{CS}(e)$ is the *expected bug detection rate* at time 0, and b is the *individual bug detection-driven cost saving*. Similarly, for the Client, $\frac{\partial U(e,t)}{\partial t} = -\Upsilon + \delta N_{CS}(e)\cdot\exp(-\lambda t)$, thus Υ and $\delta N_{CS}(e)$ are the Client's *marginal cost and benefit cost of testing* (Jiang et al. 2012).

[15] $\frac{\partial(U(e,t)-TC(e,t))}{\partial t} = -(K(c)+\Upsilon) + (\delta+\lambda b)N_{CS}(e)\cdot\exp(-\lambda t)$. On the cost side, $(K(c)+\Upsilon)$ represents the *marginal cost of testing*. On the benefit side, $(\delta+\lambda b)N_{CS}(e)$ represents the *marginal benefit of testing* because $\lambda N_{CS}(e)$ is the *expected bug detection rate* at time 0, and $\left(\frac{\delta}{\lambda}+b\right)$ is the *individual bug detection-driven cost saving* (Jiang et al. 2012).

2018).[16] In the former, a pre-determined price P_{FP} is paid to the Vendor for development, testing, and maintenance services, including contractible testing time. In the latter, beyond the pre-determined price P_{TM} for services, the Client also pays for the Vendor's development effort: $P_{TM} + r\hat{e}$. Here, $\hat{e} \in \{e_L, e_H\}$ is the effort the Vendor reports to the Client, and r is the reimbursement for per effort. For any non-contractible effort the Vendor makes, the Client must monitor the Vendor to verify its reported effort $\hat{e}$.[17] The Client's monitoring policy $\phi \in [0,1]$ corresponds to the probability that the Client finds out if the Vendor reports its real effort. A higher value of ϕ indicates the Client monitors more development documents and processes. When $\phi = 1$, the Client monitors the entire process and knows the Vendor's true effort. When $\phi = 0$, the Vendor is free to misreport. A Client incurs monitoring cost $w\phi$ with per unit cost w (Dey et al. 2010). If it finds the Vendor has inflated its effort, the latter will pay a penalty $(\hat{e} - e)^+ s$, where $(x)^+ = \max\{0, x\}$. This is a *reputation loss cost* affecting future business.

At the beginning of an outsourcing relationship, the Vendor negotiates the project duration with the Client, including testing time. After development and uncertainty ε of the outcome of the Vendor's development effort are resolved, the testing time determined by the Vendor before development may not be optimal. Thus, by updating the expected number of bugs for the system $N_{CS}(e)$ after development, the Client and the Vendor are prompted to renegotiate initial testing time t to a more effective renegotiated testing time $\tilde{t}$ to generate more value (social surplus). Regardless of renegotiation cost, we define the incremental surplus generated by renegotiation as renegotiation surplus $RS(\tilde{t})$. If renegotiation occurs, the Client and the Vendor split the renegotiation surplus with proportions $(1 - \alpha)$ and α, which represent their relative bargaining power (Che and Hausch 1999, Bolton and Dewatripont 2005, Guo and Iyer 2013), and the ex post prices for the Vendor's services in the fixed-price and time-and-materials contracts become $\tilde{P}_{FP} = P_{FP} + \alpha \cdot RS(\tilde{t})$ and $\tilde{P}_{TM} = P_{TM} + \alpha \cdot RS(\tilde{t})$, respectively. Renegotiation incurs cost C_R that the Client bears.[18] Event timing for a *fixed-price* (FP) or *time-and-materials* (TM) *contract* in Figure 1.

[16] The prevalence of standardized tools, platforms, and metrics has made the development process quite mature (Synodinos 2012, Vollmer 2020, Wohlmuth 2020), which has reduced the barriers to market entry (Chang 2012). As a result, in the software outsourcing market, a large number of global vendors have been competing fiercely with many established firms for software outsourcing contracts (McFarlan and Delacey 2004) and power related to service pricing usually rests with the Clients (Dey et al. 2010, Roels et al. 2010, Bhattacharya et al. 2014, Cezar et al. 2014).

[17] Software development is complex and estimating the Vendor's effort based on bugs after development is hard. For example, the Vendor's development ideas may come from iterative refinement, but the Client only observes the final outcome (Dermdaly 2009).

[18] We also investigate the case in which the Client and the Vendor split the renegotiation cost C_R based on their respective bargaining power, $1 - \alpha$ and α, and our main results still hold. Our analysis is included in the Online Appendix.

Figure 1. Timing of Events

In **FP**: Vendor decides testing time t and input effort e. In **TM**: Vendor decides testing time t, input effort e, and reported effort $\hat{e}$.

Vendor debugs system during testing according to t or $\tilde{t}$, delivers it to Client, and maintains it

Stage 0 (Contract Choice) — Stage 1 (System Development) — Stage 2 (Renegotiation) — Stage 3 (System Testing & Maintenance)

Client chooses contract type: **FP** $\{P_{FP}\}$ or **TM**

Uncertainty of Vendor's development effort has been realized, and parties may renegotiate initial testing time t to $\tilde{t}$ and split the renegotiation surplus according to their bargaining power.

Vendor and Client Actions

- **Stage 0**: Client selects contract type: if the FP contract is chosen, then Client determines $\{P_{FP}\}$; and if the TM contract is chosen, then Client determines $\{P_{TM}, r, \phi\}$.
- **Stage 1**: During system development, Vendor decides effort level e and initial testing time t in FP contract, or effort level e to develop system, reported effort level $\hat{e}$, and initial testing time t in TM contract.
- **Stage 2:** Development is finished and the expected number of bugs $N_{CS}(e)$ is updated due to the realization of the uncertainty of Vendor's effort. Parties may renegotiate testing time t to a new one $\tilde{t}$, and split the renegotiation surplus according to their bargaining power. If parties keep the initial testing time, then $\tilde{t} = t$. The prices for Vendor's services in FP and TM contracts change correspondingly.
- **Stage 3**: Related to testing time t or $\tilde{t}$, Vendor detects and fixes system bugs. Then, the system is delivered to Client, and Vendor offers system maintenance service.

3.2. First-Best Solution without Renegotiation Cost

We next solve for optimal development effort and testing time by distilling what is socially optimal for a Vendor to do. This involves summing the maximum expected payoffs of the two parties, the first-best solution. To investigate the impact of testing time renegotiation, we ignore the renegotiation cost ($C_R = 0$) and consider two cases. The first is the first-best solution with renegotiation (FBR), where the initial testing time is revised after development. The second is the first-best solution without renegotiation (FBN), where the initial testing time remains unchanged.

In the FBR case, when there is no renegotiation cost, after system development the *renegotiation surplus* $RS(\tilde{t})$ is:

$$\underbrace{RS(\tilde{t})}_{\text{renegotiation surplus}} = \underbrace{\left(U(e,\tilde{t}) - TC(e,\tilde{t})\right)}_{\text{social surplus if testing time is } \tilde{t}} - \underbrace{\left(U(e,t) - TC(e,t)\right)}_{\text{social surplus if testing time is } t} . \tag{8}$$

Based on Equations (5) and (7), Equation (8) can be rewritten as:

$$\begin{aligned}\underbrace{RS(\tilde{t})}_{\text{renegotiation surplus}} = & \left[\underbrace{\Upsilon t + \tfrac{\delta}{\lambda}\cdot\left(\Upsilon B(c) - e^{\beta} + \varepsilon\right)\cdot \exp(-\lambda t)}_{\text{Client disutility if testing time is } t} + \underbrace{K(c)t + b\cdot\left(\Upsilon B(c) - e^{\beta} + \varepsilon\right)\cdot \exp(-\lambda t)}_{\text{Vendor testing and added bug-fix costs if testing time is } t}\right] \\ & - \left[\underbrace{\Upsilon \tilde{t} + \tfrac{\delta}{\lambda}\cdot\left(\Upsilon B(c) - e^{\beta} + \varepsilon\right)\cdot \exp(-\lambda \tilde{t})}_{\text{Client disutility if testing time is } \tilde{t}} + \underbrace{K(c)\tilde{t} + b\cdot\left(\Upsilon B(c) - e^{\beta} + \varepsilon\right)\cdot \exp(-\lambda \tilde{t})}_{\text{Vendor testing and added bug-fix costs if testing time is } \tilde{t}}\right].\end{aligned}$$

(9)

Maximizing renegotiation surplus $RS(\tilde{t})$ in Equation (9) over $\tilde{t}$, we obtain the optimal renegotiated testing time:

$$\underbrace{\tilde{t}^*}_{\substack{\text{optimal renegotiated}\\ \text{testing time}}} = \frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot(\Upsilon B(c)-e^{\beta}+\varepsilon)}{\Upsilon+K(c)}. \quad (10)$$

Equation (10) suggests that optimal renegotiated testing time decreases with Vendor effort, $\partial\tilde{t}^*(e)/\partial e<0$. The reason is that lower effort in the development stage is likely to generate a higher expected number of bugs $N_{CS}(e)$ ex post, so more testing time is needed. By determining the optimal renegotiated testing time after system development, renegotiation resolves the uncertain outcome of the expected number of bugs. We refer to it as the *uncertainty-resolution effect of renegotiation,* $UR(e)$. It can be measured at stage 0 as :

$$\underbrace{UR(e)}_{\text{uncertainty-resolution effect}} = \underbrace{E_{\varepsilon}[RS(\tilde{t}^*)]}_{\text{expected renegotiation surplus}}, \quad (11)$$

where $E_{\varepsilon}[\cdot]$ represents the expectation over random variable ε.

- **Lemma 1 (Positive Uncertainty-Resolution Effect).** *The uncertainty-resolution effect of renegotiation is positive,* $UR(e)>0$.

Lemma 1 suggests that the uncertainty-resolution effect contributes to the increase of social surplus. Further, with the optimal renegotiated testing time $\tilde{t}^*$, we can obtain social surplus in the initial contracting stage under each case, where superscripts *FBR* and *FBN* are first-best solutions with and without renegotiation, respectively, and subscript *S* is social surplus:

$$\pi_S^{FBR}(e,t)=E_{\varepsilon}\left[\underbrace{U(e,t)}_{\text{utility of Client}} - \underbrace{TC(e,t)}_{\text{total cost of Vendor}} + \underbrace{RS(\tilde{t}^*)}_{\text{renegotiation surplus}}\right]; \quad (12)$$

$$\pi_S^{FBN}(e,t)=E_{\varepsilon}\left[\underbrace{U(e,t)}_{\text{utility of Client}} - \underbrace{TC(e,t)}_{\text{total cost of Vendor}}\right]. \quad (13)$$

By solving the problems $\max_{e,t}\pi_S^{FBR}$ and $\max_{e,t}\pi_S^{FBN}$, we have the first-best solution on effort and testing time with and without renegotiation. Table C1 in Appendix C provides the details on the regions, optimal testing times and development effort levels, and the regions are divided off the thresholds for system complexity. With the optimal decisions, there is an expected difference between the renegotiation and the non-renegotiation social surplus levels, and the *renegotiation benefit* is given by:

$$\underbrace{RB^{FB}}_{\substack{\text{renegotiation benefit}\\ \text{in first-best solution}}} = \underbrace{\pi_S^{FBR}(e^*_{FBR},t^*_{FBR})}_{\substack{\text{expected social surplus}\\ \text{under FBR case}}} - \underbrace{\pi_S^{FBN}(e^*_{FBN},t^*_{FBN})}_{\substack{\text{expected social surplus}\\ \text{under FBN case}}}, \quad (14)$$

where e^*_{FBR} (e^*_{FBN}) and t^*_{FBR} (t^*_{FBN}) are the optimal effort level and optimal initial testing time, respectively, under the FBR (FBN) case. Renegotiation benefit in Equation (14) can be rewritten in two parts:

$$\underbrace{RB^{FB}}_{\substack{\text{renegotiation benefit}\\ \text{in first-best solution}}} = \underbrace{\pi_S^{FBR}(e_{FBN}^*, t_{FBN}^*) - \pi_S^{FBN}(e_{FBN}^*, t_{FBN}^*)}_{\text{(I) direct uncertainty-resolution effect}} + \underbrace{\pi_S^{FBR}(e_{FBR}^*, t_{FBR}^*) - \pi_S^{FBR}(e_{FBN}^*, t_{FBN}^*)}_{\text{(II) indirect uncertainty-resolution effect}}. \quad (15)$$

Term (I) of Equation (15) is positive, since $\pi_S^{FBR}(e_{FBN}^*, t_{FBN}^*) - \pi_S^{FBN}(e_{FBN}^*, t_{FBN}^*) = UR(e_{FBN}^*)$.We refer to Term (I) as the *direct uncertainty-resolution effect* of renegotiation under the FBR case, since the Vendor exerts the same effort as under the FBN case, e_{FBN}^*. Further, it can be shown that the uncertainty-resolution effect increases with the Vendor's effort. This is because the Client shares the incremental surplus generated by the uncertainty-resolution effect with the Vendor, and renegotiation may incentivize the Vendor to put more effort in development to enhance the uncertainty-resolution effect. We find that when the Client's system complexity is moderate, the Vendor exerts high effort in the FBR case, $e_{FBR}^* = e_H$, but low effort in the FBN case, $e_{FBN}^* = e_L$. This is the *post-development incentive* of renegotiation, since the Vendor decides the effort by anticipating that renegotiation will occur after development. Compared with the FBN case the extra effort in the FBR case incurs an additional cost, $\pi_S^{FBN}(e_L) - \pi_S^{FBN}(e_H)$, but also leads to an additional positive uncertainty-resolution effect, $UR(e_H) - UR(e_L)$. It can be verified that the positive effect is larger than the cost, i.e., $UR(e_H) - UR(e_L) - \left(\pi_S^{FBN}(e_L) - \pi_S^{FBN}(e_H)\right) > 0$. As such, Term (II) of Equation (15) is positive, where $\pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L) = UR(e_H) - UR(e_L) - \left(\pi_S^{FBN}(e_L) - \pi_S^{FBN}(e_H)\right)$. We refer to this as the *indirect uncertainty-resolution effect*.

- **Lemma 2 (Positive Renegotiation Benefit).** *Without renegotiation cost, in the first-best solution, the Client always benefits from renegotiation,* $RB^{FB} > 0$.

Lemma 2 suggests that the direct uncertainty-resolution effect from effort e_{FBN}^* and the indirect uncertainty-resolution effect from extra effort $(e_{FBR}^* - e_{FBN}^*)$ (if any) together comprise the positive renegotiation benefit, RB^{FB}. This result is consistent with Susarla (2012), who empirically showed that renegotiation enables surplus enhancement.

4. ANALYSIS OF CONTRACTS

We next examine Client–Vendor decisions under fixed-price and time-and-materials contracts with or without renegotiation, and obtain the impacts of renegotiation on their attractiveness for use and also on the contract terms.

4.1. Fixed-Price Contract

Under a fixed-price contract, the Client first determines the initial payment P_{FP} for software services at the contracting stage. Observing the payment, the Vendor decides the initial testing time t and exerts effort e to build the system during the development stage. In the case of a fixed-price contract with renegotiation (FPR), after system development the renegotiation stage occurs. The two parties revise initial testing time t to renegotiated testing time $\tilde{t}$, and share the renegotiation surplus $RS(\tilde{t})$ via their bargaining power. The Client incurs renegotiation cost C_R. Then the Vendor offers system testing and maintenance services to the Client. Thus, in the FPR case, given the initial fixed payment P_{FP}, the expected profit for the Vendor in the development stage is as follows, where subscript V represents the Vendor,

$$\pi_V^{FPR}(e,t) = E_\varepsilon\left[\underbrace{P_{FP}}_{\text{initial payment}} - \underbrace{TC(e,t)}_{\text{total cost of Vendor}} + \underbrace{\alpha \cdot RS(\tilde{t}^*)}_{\text{profit from renegotiation}}\right]. \tag{16}$$

In the case of a fixed-price contract without renegotiation (FPN), after system development, the Vendor tests the system for the initial amount of testing time t. Thus, Vendor profit in the development stage is:

$$\pi_V^{FPN}(e,t) = E_\varepsilon\left[\underbrace{P_{FP}}_{\text{payment}} - \underbrace{TC(e,t)}_{\text{total cost of Vendor}}\right]. \tag{17}$$

In the contracting stage, anticipating the Vendor's best response, e^* and t^*, the Client determines the initial payment, P_{FP}, to maximize its own profit in the FPR and FPN cases, where subscript C represents the Client:

$$\pi_C^{FPR}(P_{FP}) = E_\varepsilon\left[\underbrace{U(e^*,t^*)}_{\text{utility of Client}} - \underbrace{P_{FP}}_{\text{initial payment}} + \underbrace{(1-\alpha)\cdot RS(\tilde{t}^*) - C_R}_{\text{profit from renegotiation}}\right]; \tag{18}$$

$$\pi_C^{FPN}(P_{FP}) = E_\varepsilon\left[\underbrace{U(e^*,t^*)}_{\text{utility of Client}} - \underbrace{P_{FP}}_{\text{payment}}\right]. \tag{19}$$

Assuming the reservation profit of the Vendor is 0, the *individual rationality* (IR) constraints for the Vendor are $\pi_V^{FPR}(e^*,t^*) \geq 0$ and $\pi_V^{FPN}(e^*,t^*) \geq 0$ in the two cases, respectively. They guarantee the Vendor a minimum expected profit to accept the contract. According to Equations (16) to (19), the optimal development effort levels e_{FPR}^* and e_{FPN}^*, and optimal initial testing times t_{FPR}^* and t_{FPN}^* under the FPR and FPN cases in the fixed-price contract can be obtained. See Table C2 in Appendix C for the optimal initial testing times and development effort levels, where the regions are divided off the thresholds for the Vendor's bargaining power and system complexity.

Note that higher system complexity Υ leads to a greater expected number of bugs $N_{CS}(e)$, thus we observe the optimal initial testing times in both the FPN and FPR cases increase with Υ, as Appendix C, Table C2 shows. In the FPN case, from $\frac{\partial^2 \pi_V^{FPN}(e,t)}{\partial t \partial e} < 0$, initial testing time is inversely related to the Vendor's effort, thus lower (higher) system complexity leads the Vendor to exert high (low) effort. In the FPR case, according to Equation (10), optimal renegotiated testing time $\tilde{t}^*$ is inversely related to the optimal Vendor's effort e_{FPR}^* as well. When the Vendor has low bargaining power, low (high) initial and renegotiated testing times stemming from low (high) system complexity lead the Vendor to exert high (low) effort; and when the Vendor has high bargaining power, low (high) renegotiated testing time causes it to exert high (low) effort.

Like the first-best solution, when system complexity is moderate ($\Upsilon_3 \leq \Upsilon < \min\{\Upsilon_4, \Upsilon_5\}$ in Table C2, where Υ_3, Υ_4 and Υ_5 are the thresholds), the Vendor's optimal effort for the FPR case is higher than that for the FPN case by the post-development incentive of renegotiation, leading to a positive indirect uncertainty-resolution effect. The thresholds Υ_4 and Υ_5 both increase with Vendor bargaining power α, implying that the post-development incentive increases with the Vendor's bargaining power. The intuition is that when the Vendor has higher bargaining power, it attains a larger share of renegotiation surplus and is incentivized to exert more effort to improve the total renegotiation surplus – if system complexity is moderate.

4.2. Time-and-Materials Contract

In a time-and-materials contract, the Client first determines the initial fixed payment P_{TM}, per unit effort reimbursement r (for Vendor reported effort), and monitoring policy ϕ. Then, the Vendor decides its input effort e, reported effort $\hat{e}$, and initial testing time t. After development, an updated expected number of system bugs $N_{CS}(e)$ is recognized. The parties renegotiate testing time t to $\tilde{t}$ when renegotiation happens (TMR). Thus, given contract terms $\{P_{TM}, r, \phi\}$ in the TMR case, the expected Vendor profit in the development stage is:

$$\pi_V^{TMR}(e,\hat{e},t) = E_\varepsilon\left[\underbrace{(P_{TM} + r\hat{e})}_{\text{initial payment}} - \underbrace{TC(e,t)}_{\substack{\text{total cost}\\\text{of Vendor}}} - \underbrace{\phi s \cdot (\hat{e} - e)^+}_{\substack{\text{penalty for}\\\text{misreporting}}} + \underbrace{\alpha \cdot RS(\tilde{t}^*)}_{\substack{\text{profit from}\\\text{renegotiation}}}\right]. \quad (20)$$

In the time-and-materials contract without renegotiation (TMN), the parties commit to testing time t, and the Vendor's profit is:

$$\pi_V^{TMN}(e,\hat{e},t) = E_\varepsilon\left[\underbrace{(P_{TM} + r\hat{e})}_{\text{payment}} - \underbrace{TC(e,t)}_{\substack{\text{total cost}\\\text{of Vendor}}} - \underbrace{\phi s \cdot (\hat{e} - e)^+}_{\substack{\text{penalty for}\\\text{misreporting}}}\right]. \quad (21)$$

With the Vendor's best response $(e^*, \hat{e}^*, t^*)$, the Client determines $\{P_{TM}, r, \phi\}$ to maximize profit in the TMR and TMN cases:[19]

$$\pi_C^{TMR}(P_{TM}, r, \phi) = E_\varepsilon\left[\underbrace{U(e^*, t^*)}_{\text{utility of Client}} - \underbrace{(P_{TM} + r\hat{e}^*)}_{\text{initial payment}} - \underbrace{w\phi}_{\text{monitoring cost}} + \underbrace{(1-\alpha) \cdot RS(\tilde{t}^*) - C_R}_{\text{profit from renegotiation}}\right]; \quad (22)$$

$$\pi_C^{TMN}(P_{TM}, r, \phi) = E_\varepsilon\left[\underbrace{U(e^*, t^*)}_{\text{utility of Client}} - \underbrace{(P_{TM} + r\hat{e}^*)}_{\text{payment}} - \underbrace{w\phi}_{\text{monitoring cost}}\right]. \quad (23)$$

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[19] The penalty for Vendor misreporting, $\phi s \cdot (\hat{e} - e)^+$, is its reputation loss and future business loss costs, but this does not affect Client profit because the latter will not obtain this result in practice.

Beyond the *individuality rationality* (IR) constraints for the Vendor, $\pi_V^{TMR}(e^*, \hat{e}^*, t^*) \geq 0$ and $\pi_V^{TMN}(e^*, \hat{e}^*, t^*) \geq 0$, in the TMR and TMN cases, the Client faces *incentive compatibility* (IC) constraints: they are $\pi_V^{TMR}(e^*, \hat{e}^* = e^*, t^*) \geq \pi_V^{TMR}(e^*, \hat{e}^* \neq e^*, t^*)$ and $\pi_V^{TMN}(e^*, \hat{e}^* = e^*, t^*) \geq \pi_V^{TMN}(e^*, \hat{e}^* \neq e^*, t^*)$, and ensure that the Vendor reveals its true effort. With Equations (20) to (23), optimal Client monitoring policies, Vendor efforts, and initial testing time under the TMR and TMN cases can be obtained, as shown in Table C3 of Appendix C, where the regions are divided off the thresholds for the Vendor's bargaining power and system complexity.

In Table C3, we also observe that, when system complexity Υ is too low or too high, the Client's monitoring policies under the TMR and TMN cases become 0 ($\phi_{TMR}^* = 0$ and $\phi_{TMN}^* = 0$). This implies the time-and-materials contract degenerates into the fixed-price contract. The intuition is that when system complexity Υ is too low, under either contract (TM or FP) the Vendor exerts the effort of the first-best solution. When system complexity Υ is too high, the Vendor does not exert high development effort under either contract. While, because the time-and-materials contract includes an effort reimbursement, the Vendor intends to misreport its effort. To deter this, the Client should adopt a high-cost monitoring policy. Thus, it will no longer employ effort reimbursement and monitoring.

Further, compared to the fixed-price contract, when system complexity satisfies the condition $\Upsilon_3 \leq \Upsilon < \Upsilon_6$ (Υ_3 and Υ_6 are the thresholds in Table C3), the Vendor exerts high development effort in the time-and-materials contract without renegotiation (TMN) but low effort in the fixed-price contract without renegotiation (FPN), as shown in the details of Appendix C, Table C3. This implies that the Client can incentivize the Vendor's effort in the time-and-materials contract via the *pre-development incentive* of monitoring. Similar to the first-best solution, as above, when system complexity is moderate, ($\Upsilon_6 \leq \Upsilon < \min\{\Upsilon_7, \Upsilon_8\}$ in Table C3), the Vendor exerts high effort in the TMR case but low effort in the TMN case because of the post-development incentive of renegotiation. In the TMR case, the positive direct uncertainty-resolution effect and positive indirect uncertainty-resolution effect (if any) constitute the positive aggregate renegotiation benefit.

4.3. The Comparison between Initial and Renegotiated Testing Time

In this subsection, we consider the possible renegotiation in the FPR and TMR cases and analyze the impacts of renegotiation on testing time and the payments.

For the testing time, it can be verified that the expected renegotiated testing time $E_\varepsilon[\tilde{t}^*]$ is shorter (longer) than the optimal initial testing times in both the FPR and TMR cases if the Vendor has low (high) bargaining power. The intuition is that when deciding the initial testing time, the Vendor aims to maximize its own profit in Equations (16) and (20). However, when deciding the renegotiated testing time, the Vendor and the Client maximize the total returns (social surplus). Shorter testing time implies a higher-value customized system for the Client, $\Upsilon \cdot (T - t)$, but it leaves a greater expected number of bugs in the system after testing, $N_{CS}(e) \cdot \exp(-\lambda t)$, and higher maintenance cost for the Vendor. Thus, if the Vendor has low bargaining power, the renegotiated testing time is expected to be shorter than the optimal initial one. Meanwhile, if the Vendor has high bargaining power, with the high revenue share of the renegotiation surplus, it sets the initial testing time at 0 and then renegotiates a higher testing time with the Client, which reduces the Vendor's initial payoff but substantially increases the renegotiation surplus. On the other hand, the realized renegotiated testing time $\tilde{t}^*$ in the renegotiation stage also depends on the outcome of Vendor's effort in system development. It may be longer (shorter) than the optimal initial testing time when the outcome is unfavorable (favorable) after system development.

For the payment, it can be verified that the ex post $\tilde{P}_{FP}$ and $\tilde{P}_{TM}$ in both the FPR and TMR cases increase after renegotiation, compared with initial prices P_{FP} and P_{TM}. This is because, if there is no renegotiation, the Client reimburses the Vendor for the cost $TC(e,t)$ based on the initial testing time. However, the ex post prices $\tilde{P}_{FP}$ and $\tilde{P}_{TM}$ are comprised of the initial prices P_{FP} and P_{TM} and the share of renegotiation surplus $\alpha \cdot RS(\tilde{t}^*)$ stemming from the renegotiated testing time. According to Lemma 1, renegotiation surplus is always positive, thus the Vendor obtains higher payments after renegotiation.

5. INTERACTION BETWEEN MONITORING AND RENEGOTIATION

Since some Clients choose to renegotiate with their Vendor while others do not, renegotiation is a realistic choice. We found earlier that monitoring and renegotiation can incentivize a Vendor's effort. The difference between a fixed-price and a time-and-materials contract is whether to monitor Vendor effort. Thus, monitoring is a realistic choice too. This led us to investigate the interaction between monitoring and renegotiation and the appropriate contract for the Client.[20] We analyze contract choice when: monitoring and renegotiation are costless; one incurs cost but not the other (and vice versa); and both incur costs.

5.1. Costless Monitoring and Costless Renegotiation

We first consider negligible monitoring and renegotiation costs, $w = 0$ and $C_R = 0$. By comparing the Client's profits in the FPR, FPN, TMR, and TMN cases, we can derive the Client's choice for monitoring and renegotiation. For this, we define two thresholds for the Vendor's bargaining power, α_1 and α_2, with $\alpha_1 < \alpha_2$. The Online Appendix provides additional details. Client choice is given by:

- **Proposition 1 (Client Contract Choice for Costless Monitoring and Costless Renegotiation).** *When monitoring and renegotiation are costless,* $w = 0$ *and* $C_R = 0$:

  (i) If $\alpha_1 < \alpha < \alpha_2$ *, the Client chooses both monitoring and renegotiation (TMR), which complement each other.*

  (ii) Otherwise, the Client chooses renegotiation only (FPR) and it substitutes for monitoring.

We use *complements* and *substitutes* for the relationship between monitoring and renegotiation. When

[20] Following the earlier example, the Kansas Department of Health and Environment renegotiated with Accenture, while the State of Michigan declined to do so with HP.

they are complements, their simultaneous use increases the benefits of the other; in contrast, when they are substitutes, their simultaneous use decreases the benefits from the other (Tiwana 2010). Recall that renegotiation benefits exist in both fixed-price and time-and-materials contracts. When renegotiation is costless, the Client always chooses renegotiation to increase its profit. If the Vendor's bargaining power becomes low in Proposition 1(*i*), ($\alpha_1 < \alpha < \alpha_2$), the post-development incentive is limited in the fixed-price contract. Note that the positive indirect uncertainty-resolution effect stems from the additional effort compared to the non-renegotiation case. Thus, the Client adopts monitoring as a complement to incentivize Vendor effort, and a time-and-materials contract with renegotiation (TMR) is preferred.

If the Vendor has high bargaining power, $\alpha_2 \leq \alpha \leq 1$, the post-development incentive is strong, so the Vendor exerts high effort, and the Client does not use monitoring. Thus, renegotiation substitutes for monitoring, and the Client prefers a fixed-price contract with renegotiation (FPR). For example, NASA and Google partnered to sign a fixed-price contract worth US$10 million with D-Wave for quantum computing systems (Knapp 2013) and extended it two years later (Harris 2015). D-Wave had higher bargaining power in contracting since it was the only commercial supplier of quantum computers (Alto 2017).

Note that the threshold $\alpha_1 \geq 0$ weakly increases with system complexity, and when the complexity is high, $\alpha_1 > 0$. In this case, for the region $0 \leq \alpha \leq \alpha_1$, the Vendor exerts low effort, and the pre- and post-development incentives do not work. Further, the time-and-materials contract degenerates into a fixed-price contract, and the Client uses renegotiation to benefit from the direct uncertainty-resolution effect only.

5.2. Costly Monitoring and Costless Renegotiation

What about when monitoring policy ϕ incurs cost $w\phi$ with no renegotiation costs, so $C_R = 0$? We define a threshold for per unit cost of monitoring $\hat{w}$. (Detailed expositions in the Online Appendix.)

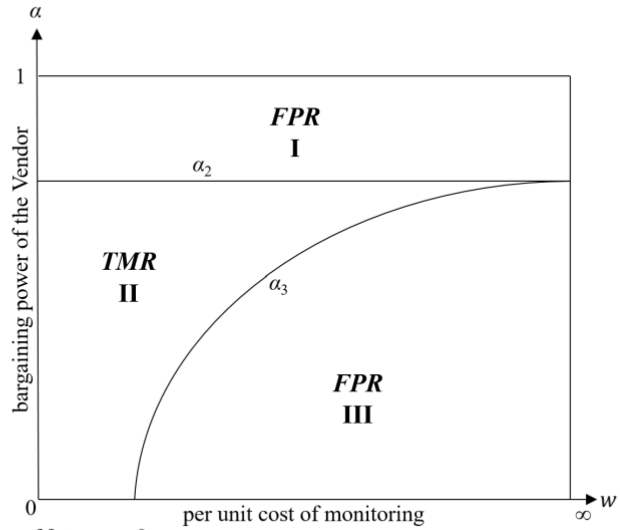
- **Proposition 2 (Client Contract Choice for Costly Monitoring and Costless Renegotiation).** *When monitoring policy ϕ incurs cost $w\phi$ ($w > 0$) and renegotiation is costless ($C_R = 0$), the Client's choice is determined in one of two ways:*

  (i) If $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \hat{w}$, the Client selects monitoring and renegotiation (*TMR*) (*complementary elements*).

  (ii) Otherwise, the Client chooses renegotiation only (*FPR*)*, and it substitutes for monitoring.*

We illustrate the results of Proposition 2 when $\alpha_1 = 0$ in Figure 2. In Region II, for Proposition 2(*i*), where per unit cost of monitoring is low and the Vendor has low bargaining power, the post-development incentive is limited by the low renegotiation surplus share of the Vendor in the fixed-price contract. Recall that the indirect uncertainty-resolution effect of renegotiation stems from added Vendor effort compared with the non-renegotiation case. Thus, the Client chooses to adopt monitoring for the pre-development incentive to create the indirect uncertainty-resolution effect of renegotiation. A time-and-materials contract with renegotiation (TMR) is preferred, and the two contract elements are complementary.

Figure 2. Client Contract Choice under Costly Monitoring and Costless Renegotiation



Note. $\alpha_1 = 0$.

Regions I and III depict the results of Proposition 2(*ii*). In Region I, the post-development incentive from renegotiation is sufficient because of the Vendor's high bargaining power, and it applies high effort to develop the system. Thus, the Client adopts renegotiation only (FPR), and that substitutes for monitoring. When per unit cost of monitoring is high and the Vendor has low bargaining power in Region III, the indirect uncertainty-resolution effect of renegotiation is not strong enough to enhance the benefit from the pre-development incentive of monitoring, which is dominated by monitoring cost. So, the Client chooses renegotiation only (FPR) consistent with Benaroch et al. (2016). This shows that renegotiation substitutes for monitoring when it is costly. To illustrate, General Motors (GM) has other Vendors and an IT services subsidiary, EDS (Barkholz 2010), implying HP's bargaining power is lower than GM's. Because of the high monitoring cost (Savitz 2013), GM and HP inked a fixed-price contract and renewed it four years later (Barkholz 2010).

Similar to costless monitoring and renegotiation, with costly monitoring, when $\alpha_1 > 0$, in the region $0 \leq \alpha \leq \alpha_1$, the Vendor never exerts high effort because of high system complexity, making the pre- and post-development incentives invalid. Thus, the Client adopts a fixed-price contract with renegotiation (FPR) and reaps the benefit from the direct uncertainty-resolution effect of renegotiation only.

5.3. Costless Monitoring and Costly Renegotiation

What about when monitoring cost is negligible, $w = 0$, and renegotiation incurs a cost, $C_R > 0$? For this, we define another threshold for system complexity Υ_9, discussed in the Online Appendix also.

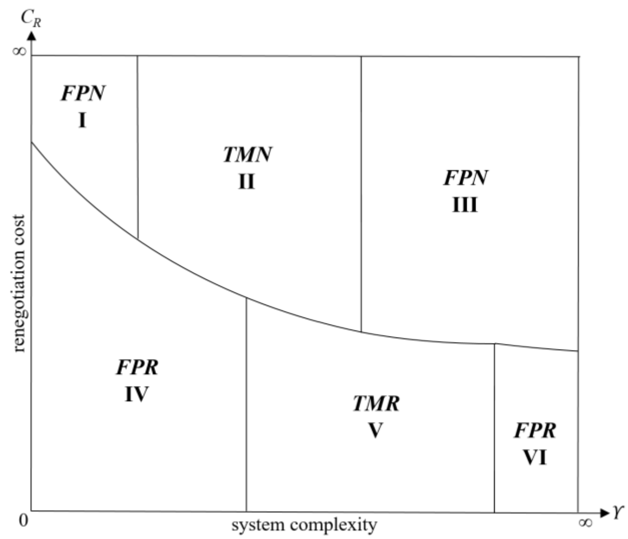
- **Proposition 3 (Client Contract Choice for Costless Monitoring and Costly Renegotiation).** *When monitoring is costless* ($w = 0$) *and renegotiation has cost* $C_R > 0$, *the Client's choice is:*

  (i) For $C_R < \min\{RB^{FP}, RB^{TM}\}$, *(a) if* $\alpha_1 < \alpha < \alpha_2$, *it chooses both monitoring and renegotiation (TMR), which complement each other; and (b) otherwise, renegotiation substitutes for monitoring and it chooses renegotiation only (FPR).*

(ii) For $C_R \geq \min\{RB^{FP}, RB^{TM}\}$*, (a) if* $\Upsilon_3 \leq \Upsilon < \Upsilon_9$*, it uses monitoring only (TMN), which substitutes for renegotiation; and (b) otherwise, it uses neither (FPN).*

The results of Proposition 3 are shown in Figure 3.

Figure 3. Client Contract Choice under Costless Monitoring and Costly Renegotiation



When renegotiation cost is low, in Proposition 3(i) the Client always chooses renegotiation for development uncertainties. But whether the Client uses monitoring depends on the Vendor's bargaining power in renegotiation. If it has low power, $\alpha_1 < \alpha < \alpha_2$, in Proposition 3($i.a$) (moderate system complexity in Region V), the post-development incentive is limited. The Client chooses monitoring for the pre-development incentive to obtain the indirect uncertainty-resolution effect of renegotiation. Thus, monitoring and renegotiation are complements, and the Client selects a time-and-materials contract with renegotiation (TMR).

If the Vendor has high bargaining power, $\alpha_2 \leq \alpha \leq 1$, in Proposition 3($i.b$), the post-development incentive is strong enough to incentivize it to exert high effort. Thus, in Region IV renegotiation substitutes for monitoring, and the Client selects a fixed-price contract with renegotiation (FPR). Similar to Propositions 1 and 2, when $\alpha_1 > 0$ (with high system complexity in Region VI), in the region $0 \leq \alpha \leq \alpha_1$, the pre- and post-development incentives do not work. So, the Client selects a fixed-price contract with renegotiation (FPR) for the benefit from the direct uncertainty-resolution effect only.

When renegotiation cost is high, as in Proposition 3(ii), the renegotiation benefit is dominated by its cost and the Client does not select renegotiation. Thus, the Client decides whether to adopt monitoring. If it has moderate system complexity as in Proposition 3($ii.a$), monitoring substitutes for renegotiation to incentivize more Vendor effort, and the Client adopts a time-and-materials contract without renegotiation (TMN) (Region II). If system complexity is low (Region I), the Vendor exerts high effort. Thus, the Client does not need to incentivize the Vendor, and a fixed-price contract only (FPN) works. If system complexity is high (Region III), the Vendor exerts low effort and the pre-development incentive does not work. Thus, the Client uses neither monitoring nor renegotiation (FPN).

5.4. Costly Monitoring and Costly Renegotiation

We now examine the scenario where both monitoring and renegotiation are costly. The Client's choice for using monitoring and renegotiation is summarized by:

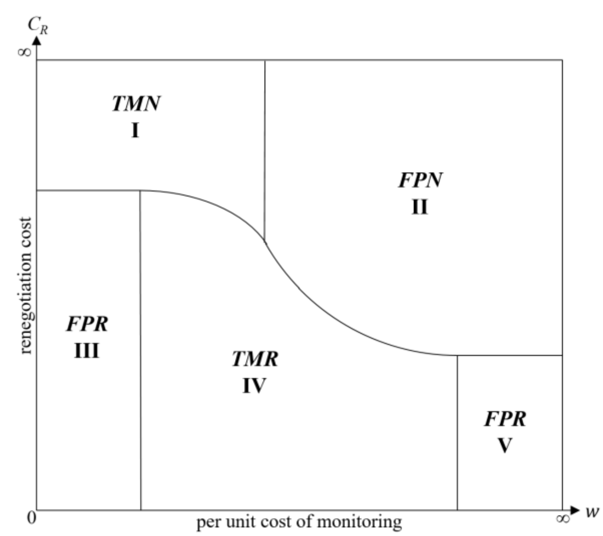
- **Proposition 4 (Client Contract Choice under Costly Monitoring and Costly Renegotiation).** *When monitoring and renegotiation are costly ($w > 0, C_R > 0$), the Client's choice is:*

  (i) For $C_R < \max\{RB^{FP}, RB^{TM}\}$, (a) if $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \hat{w}$, then it chooses both monitoring and renegotiation (TMR), which complement each other; and (b) otherwise, renegotiation substitutes for monitoring and it uses renegotiation only (FPR).

  (ii) For $C_R \geq \min\{RB^{FP}, RB^{TM}\}$, (a) if $\Upsilon_3 \leq \Upsilon < \Upsilon_6$, it uses monitoring only (TMN), which substitutes for renegotiation; and (b) otherwise, it uses neither one (FPN).

Figure 4 shows results for Proposition 4 when system complexity is moderate. Naturally, when renegotiation cost is low, the Client always prefers it in Proposition 4(*i*), and when the cost is high, renegotiation is not used in Proposition 4(*ii*). With low renegotiation cost, similar to Proposition 2, Proposition 4(*i.a*) shows that if the Vendor has low bargaining power and per unit cost of monitoring is low, monitoring and renegotiation are complements, and the Client selects a time-and-materials contract with renegotiation (TMR) (Region IV, Figure 4). Otherwise, in Proposition 4(*i.b*) (Regions III and V), renegotiation substitutes for monitoring, and the Client selects a fixed-price contract with renegotiation (FPR). In Region III of Figure 4, the post-development incentive from renegotiation is sufficient because of the Vendor's high bargaining power (like Region I, Figure 2). Thus, the Client adopts renegotiation only (FPR). In Region V of Figure 4, the benefit from the pre-development incentive of monitoring is dominated by its cost (similar to Region III, Figure 2). So, the Client adopts renegotiation only (FPR).

Figure 4. Client Contract Choice under Costly Monitoring and Costly Renegotiation



Note. $\Upsilon_3 \leq \Upsilon < \min\{\Upsilon_7, \Upsilon_8\}$.

With high renegotiation cost, similar to Proposition 3(*ii*), renegotiation is not used and the Client's decision is whether to adopt monitoring. If the benefit from the pre-development incentive of monitoring dominates its cost, in Proposition 4(*ii.a*) (Region I, Figure 4), the Client adopts monitoring to incent Vendor effort and selects a time-and-materials contract without renegotiation (TMN). Yet, if monitoring cost is higher than its benefit, Proposition 4(*ii.b*), the Client uses neither monitoring nor renegotiation and selects a fixed-price contract without renegotiation (FPN) (Region II, Figure 4).

Further, when per unit cost of monitoring becomes lower, the Client prefers to adopt both monitoring and renegotiation with larger renegotiation costs. The reason for such adoption is that when the Client uses monitoring to stimulate high Vendor effort with lower cost, the indirect uncertainty-resolution effect of renegotiation will be created. The Client benefits more from renegotiation, generating a higher threshold for the renegotiation cost that determines whether renegotiation is chosen. For example, the U.S. Department of Defense (DoD) built a partnership with SAP, resulting in low monitoring cost. This further led DoD to sign a time-and-materials contract with SAP for enterprise resource planning and financial programs (Hoover 2016), and the contract was renewed within the budget (Culclasure and Neff 2016).

6. VENDOR EFFORT AND CLIENT PROFIT CHANGES: SENSITIVITY ANALYSIS RESULTS

We now analyze how Vendor effort and Client profit change with the model parameters. We assess only some elements though: system complexity Υ, bug rate $B(c)$ and system lifetime T.

6.1. System Complexity and Bug Rate

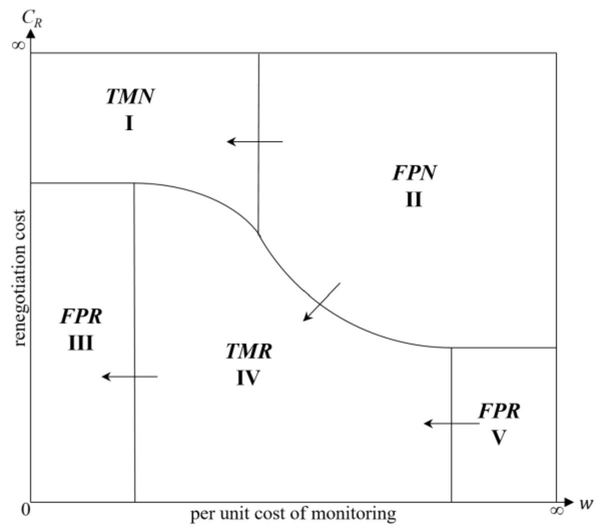
Clients have various system complexity needs Υ, and Vendors differ in development capability leading to diverse bug rates $B(c)$. We assess how system complexity and the bug rate affect Client and Vendor.

- **Corollary 1 (Impacts of System Complexity and Bug Rate on Vendor Effort and Client Profit).** *The Vendor is more likely to exert low effort when system complexity Υ is greater or the bug rate $B(c)$ is higher, and Client profit decreases in Υ and $B(c)$.*

It can be verified that greater system complexity or a higher bug rate leads to a greater expected number of bugs in the system, resulting in longer initial testing time in both the fixed-price and time-and-materials contracts. Since the optimal initial testing time is inversely related to the Vendor's effort, it is more likely to exert low effort with increasing system complexity and bug rate. Further, the marginal benefit of the Vendor's effort from the renegotiation surplus decreases in both system complexity and bug rate. Thus, the increased system complexity or bug rate also weakens the post-development incentive of renegotiation and further reduces the Vendor's desire to exert high effort in system development.

According to Proposition 4, the Client's profit decreases in the expected number of bugs. Thus, the Client obtains lower profit when system complexity or the bug rate increases. Further, with Corollary 1, we observe that when system complexity is moderate, the region where the Client selects a fixed-price contract without renegotiation (FPN) is enlarged by the increased system complexity or bug rate, as shown in Figure 5. The reason is that the direct and indirect uncertainty-resolution effects decrease with the expected number of bugs, which increase, in turn, with system complexity and bug rate. Recall that the indirect uncertainty-resolution effect generates post-development incentive of renegotiation and strengthens the pre-development incentive of monitoring. Thus, the increased system complexity or bug rate reduces the effort incentives of renegotiation and monitoring, yielding a low likelihood of selecting monitoring and renegotiation.

Figure 5. Impacts of System Complexity and Bug Rate on Client Contract Choice



Note. $Y_3 \leq Y < \min\{Y_7, Y_8\}$ and "⟵" implies the direction that increasing Y or $B(c)$ pushes the region line to move into.

6.2. System Lifetime

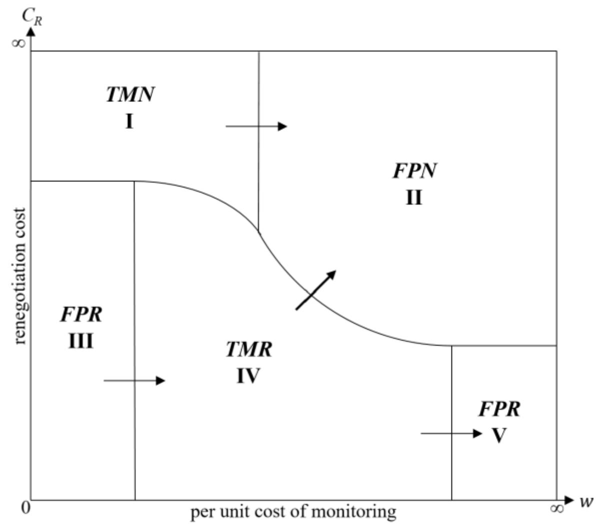
A customized system's lifetime T plays an important role in outsourcing (Ji et al. 2011, August and Niculescu 2013), and it includes the testing and maintenance times in our model. The following corollary shows how system lifetime affects the Client's and Vendor's decisions:

- **Corollary 2 (Impacts of System Lifetime on Vendor Effort and Client Profit).** *The Vendor is more likely to exert high effort when system lifetime* T *is longer, and Client profit increases in* T.

Corollary 2 first suggests that increased system lifetime incentivizes the Vendor to exert high effort in development. The intuition is that, with a longer system lifetime, more bugs may occur in both the fixed-price and time-and-materials contracts, generating higher bug-fix costs for the Vendor. To reduce them, the Vendor makes high effort in development to decrease their expected number. Meanwhile, the Client benefits from the decreased bugs resulting from Vendor's high effort after it has finished development. In addition, longer system lifetime increases the value of the system, which benefits the Client as well.

Based on Corollary 2, we further observe that when system complexity is moderate, the regions in which the Client selects renegotiation are enlarged by the increased system lifetime, as illustrated in Figure 6. The reason is that according to Lemma 2, the increased Vendor effort caused by the longer system lifetime creates the indirect uncertainty-resolution effect and increases the renegotiation benefit in the fixed-price and time-and-materials contracts. Thus, the Client is more likely to select renegotiation when system lifetime increases.

Figure 6. Impacts of System Lifetime on Client Contract Choice



Note. $Y_3 \leq Y < \min\{Y_7, Y_8\}$ and "⟶" implies the direction that increasing T pushes the region line to move into.

7. EXTENSIONS

Our base model considers two effort levels, e_H and e_L, that the Vendor chooses to make in system development. We now extend discrete effort to continuous effort, and first check how the Client's and the Vendor's decisions change. Then, we examine how our main results are affected when renegotiation cost is endogenous and depends on the renegotiation process.

7.1. Continuous Development Effort

Compared with the base model, we allow that the Vendor exerts effort $e (e \geq 0)$ to decrease the expected number of bugs in system development. By solving the Vendor's and the Client's problems with continuous development effort, we prove that, similar to the base model, renegotiation generates direct and indirect uncertainty-resolution effects (both are positive) and a post-development incentive, and monitoring generates a pre-development incentive. We next summarize the Client's contract choice when both monitoring and renegotiation are costly. For this, we define a function for per unit cost of monitoring $\Omega(w)$, two thresholds for per unit cost of monitoring w_{E1} and w_{E2}, and two thresholds for renegotiation cost C_R^{E1} and C_R^{E2}. The Online Appendix offers additional details of the modeling derivation for this.

- **Proposition 5 (Client Contract Choice under Continuous Development Effort).** *When monitoring and renegotiation are costly ($w > 0, C_R > 0$), the Client's choice is:*

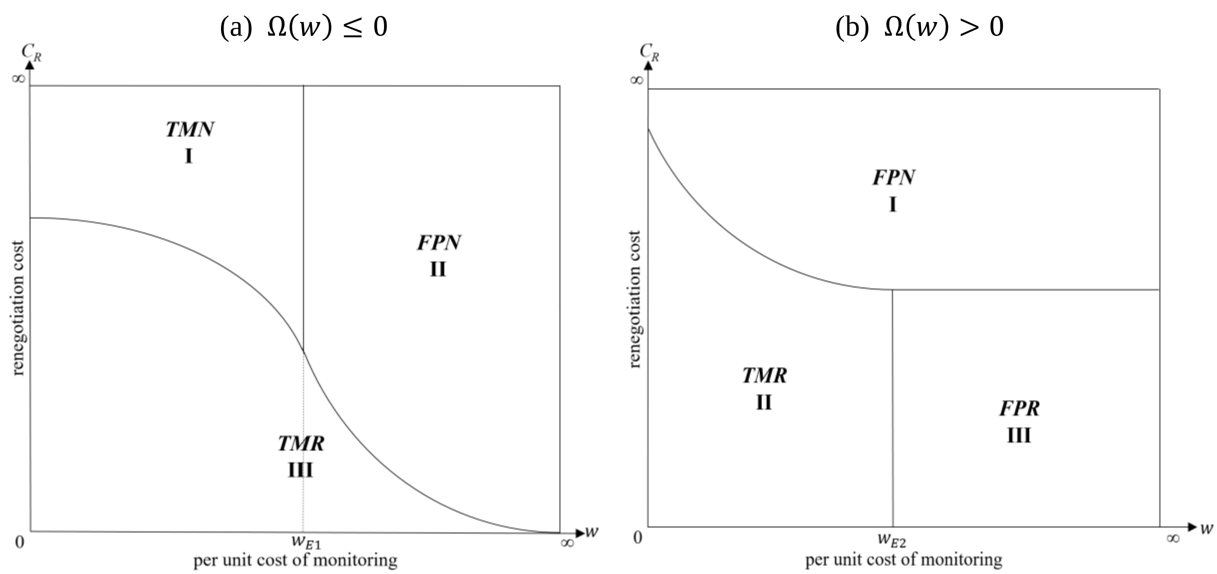
  (i) For $\Omega(w) \leq 0$, (a) if $C_R < C_R^{E1}$, the Client chooses both monitoring and renegotiation (TMR); (b) if $C_R \geq C_R^{E1}$ and $0 < w \leq w_{E1}$, it chooses monitoring only (TMN); and (c) otherwise, it uses neither one (FPN).

(ii) For $\Omega(w) > 0$, (a) if $C_R < \max\{C_R^{E1}, C_R^{E2}\}$ and $0 < w \leq w_{E2}$, the Client chooses both monitoring and renegotiation (TMR); (b) if $C_R < \max\{C_R^{E1}, C_R^{E2}\}$ and $w > w_{E2}$, it chooses renegotiation only (FPR); and (c) otherwise, it uses neither (FPN).

Figure 7 illustrates Proposition 5 when $\Omega(w) \leq 0$ and $\Omega(w) > 0$. When $\Omega(w) \leq 0$, with low renegotiation cost (Region III, Figure 7a), Proposition 5 ($i.a$) shows that the Client chooses both monitoring and renegotiation (TMR), and the two are complements. With high renegotiation cost, renegotiation becomes inefficient and the Client decides whether to adopt monitoring. If per unit cost of monitoring is low (Region I, Figure 7a), Proposition 5(*i.b*) indicates the Client chooses time-and-materials contract without renegotiation (TMN). If per unit cost of monitoring is high (Region II, Figure 7a), the pre-development incentive of monitoring is dominated by its cost. Thus, the Client selects the fixed-price contract without renegotiation (FPN) in Proposition 5(*i.c*).

When $\Omega(w) > 0$ (implying $w > w_{E1}$), the benefit from the pre-development incentive of monitoring is dominated by its cost if only monitoring is adopted. With high renegotiation cost (Region I, Figure 7b), Proposition 5(*ii.c*) states the Client selects neither monitoring nor renegotiation (FPN). With low renegotiation cost, the Client always chooses renegotiation for its post-development incentive and direct and indirect uncertainty-resolution effects. At the same time, if per unit cost of monitoring w is less than w_{E2} (Region II, Figure 7b), the indirect uncertainty-resolution effect of renegotiation enhances the pre-development incentive of monitoring, and they dominate the monitoring cost. Thus, the Client chooses the time-and-materials contract with renegotiation (TMR), as in Proposition 5(*ii.a*). If per unit cost of monitoring is high (Region III, Figure 7b), Proposition 5(*ii.b*) shows that the Client chooses fixed-price contract with renegotiation (FPR) because the benefit of monitoring is less than its cost.

Figure 7. Client Contract Choice under Continuous Development Effort



7.2. Endogenous Renegotiation Cost

In the base model, we assume the cost of renegotiation is independent of its outcome. Previous literature also has treated renegotiation cost as an endogenous variable linked to renegotiation surplus (Guasch et al. 2006). We now examine when renegotiation cost is proportional to renegotiation surplus and how our results are affected. In contrast to the base model, we assume that the renegotiation cost becomes $\zeta \cdot RS(\tilde{t})$, where $\zeta \geq 0$. Then, when the renegotiation process begins, the Vendor and the Client split the renegotiation profit $(1-\zeta) \cdot RS(\tilde{t})$ according to their bargaining power α and $1-\alpha$. Solving the Vendor's and the Client's problems with endogenous renegotiation cost, we can derive the Client's optimal contract. We find that the base model's results keep qualitatively. For this, we define a threshold for the Vendor's bargaining power α_{E1}, two thresholds for per unit cost of monitoring w_{E3} and w_{E4}. See the Online Appendix for details.

- **Proposition 6 (Client Contract Choice under Endogenous Renegotiation Cost).** *When renegotiation cost is proportional to renegotiation surplus ($C_R = \zeta \cdot RS(\tilde{t})$), the Client's choice is:*

  (i) For $0 < \zeta < 1$, if (a) $0 < w \leq w_{E3}$, or (b) $w_{E3} < w < w_{E4}$ and $\alpha_{E1} < \alpha \leq 1$, it selects both monitoring and renegotiation (TMR); and if (c) $w_{E3} < w < w_{E4}$ and $0 < \alpha \leq \alpha_{E1}$, or (d) $w \geq w_{E4}$, it chooses renegotiation only (FPR).

  (ii) For $\zeta \geq 1$, if (a) $0 < w < w_{E1}$, it chooses monitoring only (TMN); and (b) if $w \geq w_{E1}$, it uses neither (FPN).

According to Proposition 6(*i*), when $0 < \zeta < 1$, implying the renegotiation cost is lower than the renegotiation surplus (similar to when renegotiation cost C_R is low in our base model), the Client always chooses renegotiation. Proposition 6(*i.a*) (Proposition 6(*i.d*)) shows that, if per unit cost of monitoring is low (high), the Client selects (does not select) monitoring, and adopts a time-and-materials (fixed-price) contract with renegotiation. If per unit cost of monitoring is moderate and the Vendor has high bargaining power, in Proposition 6(*i.b*), the increased post-development incentive of renegotiation enhances the indirect uncertainty-resolution effect and further increases the benefit from the pre-development incentive. Thus, the Client chooses both monitoring and renegotiation (TMR). If per unit cost of monitoring is moderate and the Vendor has low bargaining power, Proposition 6(*i.c*) shows that the Client chooses renegotiation (FPR) only because the indirect uncertainty-resolution effect is not strong enough to allow the pre-development incentive to cover the cost of monitoring.

When $\zeta \geq 1$, the renegotiation cost is higher than the renegotiation surplus (similar again to when renegotiation cost C_R is high in our base model), renegotiation is not chosen and the Client decides to select monitoring instead. Proposition 6(*ii.a*) shows that the Client chooses the time-and-materials contract without renegotiation (TMN) with low monitoring cost, and Proposition 6(*ii.b*) suggests that it selects the fixed-price contract without renegotiation (FPN) with high monitoring cost.

8. DISCUSSION

We conclude with contributions and insights obtained for a client's software outsourcing contract strategy. We also share our thoughts on related applications of the modeling ideas.

8.1. Contributions

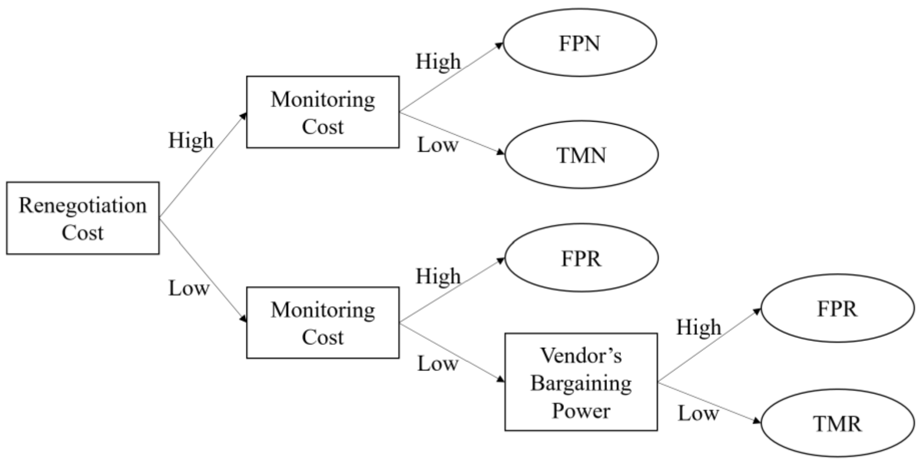
Our primary contribution is to provide novel insights about the way monitoring and opportunistic renegotiation may be complements, counter to the prevailing wisdom that monitoring prevents the vendor's ex post opportunism. An important reason for this is that we identify the positive direct and indirect uncertainty-resolution effects generated by renegotiation to resolve uncertainty about system development and make testing time efficient ex post. The indirect uncertainty-resolution effect stems from the added effort compared with the non-renegotiation case. Since monitoring generates a pre-development incentive to stimulate vendor effort, the client may adopt monitoring as a complement to incentivize vendor effort, resulting in the indirect uncertainty-resolution effect. This occurs only when the vendor has low bargaining power, and monitoring and renegotiation costs are low though.

A related insight is that monitoring and renegotiation are substitutes: they both stimulate the vendor's effort. Counter to the conclusion that contract theory research has drawn, renegotiation results in underinvestment by the investing party that fears expropriation of its benefits by its contracting partner in the process (Maskin and Moore 1999). This, again, is the familiar hold-up problem. We demonstrate that testing time renegotiation incentivizes a vendor to invest in more development effort because the positive indirect uncertainty-resolution effect results from extra effort compared with the non-renegotiation case, and the vendor attains renegotiation surplus according to its bargaining power. To stimulate the indirect uncertainty-resolution effect and obtain more benefits from renegotiation, the vendor has an incentive to exert more effort in development. This is a post-development incentive which increases in the vendor's bargaining power.

Another contribution of our work is that, as the client contemplates which contract form works better – a fixed-price or time-and-materials contract – and whether to renegotiate with the vendor after system development, the optimal contract strategy is determined by the interaction between monitoring and renegotiation. They are further moderated by the renegotiation cost, monitoring cost, and the two parties' bargaining power in renegotiation. These insights are far-reaching for practice.

With low renegotiation cost, the client always adopts renegotiation. If the Vendor has low bargaining power and per unit cost of monitoring is low, monitoring and renegotiation are complements, and the client selects a time-and-materials contract with renegotiation. Otherwise, renegotiation substitutes for monitoring, and the client chooses a fixed-price contract with renegotiation. With high renegotiation cost, renegotiation should not be chosen, and the client decides whether to adopt monitoring. If the per unit cost of monitoring is low, monitoring substitutes for renegotiation to incentivize the vendor's effort, and the client adopts a time-and-materials contract without renegotiation. Otherwise, the client does not use monitoring or renegotiation and instead goes with a fixed-price contract without renegotiation. A decision tree summarizes our results for the client's contract choice, as Figure 8 shows.

Figure 8. A Decision Tree for Client Contract Choice



We further find that the vendor is more likely to exert low effort when system complexity is greater or the bug rate gets higher, and to exert high effort when system lifetime lengthens. Moreover, the client's profit decreases in system complexity and bug rate and increases in system lifetime. The main findings in the base model hold qualitatively in two situations where we consider continuous effort and the renegotiation cost endogenously depends on the renegotiation process.

Our analysis related to bug detection is founded on the G-O model, which is valid and offers an appropriate way to handle bug detection in the presence of new technologies, including automated bug detection with machine learning (ML) and cloud computing. The Online Appendix offers further discussion of the G-O model assumptions related to this. Thus, our results can be applied in large-scale system implementation projects and *software-as-a-service* (SaaS) settings with cloud-based configurations.

8.2. Limitations

In closing, we share some thoughts about the limitations of this research. First, investigating the effect of testing time renegotiation on the value of the vendor's private information about testing efficiency is valuable in management science terms. We assume the client knows the vendor's testing efficiency and the software failure rate for each bug in our model. Testing efficiency may be the vendor's private information, yet it affects the parties' renegotiation of testing time. Since the client can observe the performance of effort and renegotiate the initial contract ex post, the value of information for the vendor's testing efficiency may decrease. In contrast, renegotiation may amplify the effect of the vendor's private information on reducing the client's profit, although it increases the social surplus. Thus, the value of the vendor's private information on testing efficiency may increase when the client chooses renegotiation.

The client and the vendor may engage in renegotiation for other reasons. For example, if the client has new requirements for the complexity of a customized system, or if the client knows that the vendor's cost has changed, they may wish to renegotiate with each other. Thus, investigating the extent to which the usage of renegotiation results from different types of uncertainty is worthwhile. Finally, in practice, the client can outsource a system involving more than one vendor. The effect of vendor competition on the client's usage of monitoring and renegotiation is a worthwhile direction for additional research as a result.

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Appendix A. A Glossary of Managerial Terms for the Customized Software Outsourcing Context

Term	Definition
Backsourcing renegotiation	The conditions of client business warrant redress from the vendor, so the former can expend less cost and bring development effort in-house (backsourcing), by adjusting the contract to allow this.
Client bargaining power	Bargaining power client can exert on vendor so it offers higher-quality products, better services at lower prices.
Development stage	Stage of the software development life cycle in which design and coding occur.
First-best solution	Contract solution that can be achieved if details of the client's required system complexity and vendor's capabilities are known to both sides.
Fixed-price contract	Software development contracts that consist of a pre-determined payment for development, testing and maintenance services from the vendor.
Hold-up problem	Vendor perceives the risk of expropriation of its investment benefits from its software by the client when contract renegotiation occurs, and vice versa.
Incentive compatibility	Client and the vendor are both able to obtain the best outcomes when they act according to their preferences.
Maintenance	The purpose of the last stage is to modify the software product after delivery to correct bugs, and to improve performance or other attributes.
Monitoring	Used to gauge how well the cost involved in software development is in step with the budgetary constraints of the client under different types of contracts. It also is the process of inspecting and reimbursing the service vendor's non-contractible effort in system development.
Opportunistic renegotiation	Renegotiation between a client and a vendor when one is weakened by some events in its business, making it susceptible to negotiation pressure and more adverse contract terms.
Pre-develop. incentive	An incentive for the vendor when the client decides on the monitoring policy prior to when the vendor begins making development effort.
Post-develop. incentive	An incentive for the vendor, created when the client and vendor engage in post-development renegotiation, affecting the value exchange they settled on when the initial contract was signed.
Renegotiation	Bilateral interaction between the client and the vendor, in which the client attempts to mitigate the loss of surplus from uncertainty about system completion and performance after development occurs.
Renegotiation-proof	Describes a software contract is not subject to renegotiation after it is agreed upon by the client and the vendor because the costs for changing terms will be too high for either party to bear.
Renegotiation surplus	Value that can be split between a client and its vendor, when the two parties renegotiate a contract.
Software bugs	An error, flaw, failure, or fault in a computer program or system that causes it to violate at least one of its functional or non-functional requirements.
System complexity	What the client requires from a system's complexity to address its managerial and operational uses, including the size of codebase, the number of modules and interfaces, and the extent of functionality.
Testing	First part of the testing and maintenance stage, whose purpose is to detect software failures that arise due to coding bugs. Since testing never reveals all of the problems that are present, it is typically undertaken based on hypotheses about why failures may occur, against a testing mechanism, to detect the bug.
Time-and materials contract	Software development contracts that consist of an extra fee beyond the payment for services obtained, based on the vendor's effort.
Uncertainty-resolution	An effect that arises from having the client and the vendor renegotiate their contract terms, such that the client will resolve some uncertainties it faces about the vendor's development effort that is observed.
Vendor bargaining power	Pressure the vendor can bring to bear on a client by upping prices, adjusting software quality, and controlling availability and delivery times, by leveraging its competitive position.

Appendix B. How Software Reliability Growth Models (SRGM) Represent Bugs at Time t via $N(t)$

The G-O model (Goel and Okumoto 1979) is the most commonly used model among the SRGMs. Two types use *stochastic SRGMs* and *deterministic SRGMs*, as shown in Table B1. The difference between them is the assumption about $N(t)$, the cumulative number of software bugs detected in time interval $(0, t]$. In stochastic SRGMs, $N(t)$ is a random variable, while in deterministic SRGMs, $N(t)$ is based on curves with specific functional forms.

Table B1. Deterministic and Stochastic Software Growth and Reliability Models

Type	SGRM Name		*N*(*t*)	Comments
Stochastic SRGMs	Non-homo-geneous Poisson process (NHPP)	Exponential (G-O Model)	$N(t) = a(1 - e^{-bt})$	Software failure occurs with a constant bug-detection rate at an arbitrary time.
		Modified exponential	$N(t) = a\sum_{i=1}^{2} p_i(1 - e^{-b_i t})$	Bug-detection difficulty during testing is considered.
		Delayed S-shaped	$N(t) = a[1 - (1 + bt)e^{-bt}]$	Bug-detection: failure-detection, bug-isolation process.
		Inflection S-shaped	$N(t) = \frac{a(1 - e^{-bt})}{(1 + ce^{-bt})}$	Failure occurs with mutually-dependent detected bugs.
		Testing effort-dependent	$N(T) = a\left[1 - e^{-rW(t)}\right]$ $(W(t) = \alpha\left(1 - e^{-\beta t^m}\right))$	Time-dependent behavior of testing effort and cumulative detected bugs both considered.
		Testing domain-dependent	$N(t) = a\left[1 - \frac{1}{v - b}(ve^{-bt} - be^{-vt})\right]$	Focus: software functions are influenced by past test cases.
		Log Poisson execution time	$N(t) = \frac{1}{\theta} ln(\lambda_0 \theta t + 1)$	Exponentially-decreasing failure model with cumulative bugs.
	Markovian software reliability model (MSRM)		Uses a counting process, $N(t) \geq 0$, with a random variable for time-interval, $S_{i,n}(i \leq n)$, distributed as $G_{i,n}(t)$ for $S_{i,n}$.	Failure time-interval between events is exponentially-distributed.
Deterministic SRGMs	Logistic curve model		$N(t) = \frac{a}{1 + me^{-\alpha t}}$	Regression models.
	Gompertz curve model		$N(t) = a\beta^{(\alpha^t)}$	

Appendix C. Optimal Decisions in the FBN, FBR, FPN, FPR, TMN and TMR Cases

Table C1. Optimal Decisions for First-Best Solution

Case	Region	Initial Testing Time	Effort Level
FBN	$0 < \Upsilon < \Upsilon_1$	$\frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{\Upsilon+K(c)}$	e_H
	$\Upsilon \geq \Upsilon_1$	$\frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{\Upsilon+K(c)}$	e_L
FBR	$0 < \Upsilon < \Upsilon_2$	$[0, T)$	e_H
	$\Upsilon \geq \Upsilon_2$	$[0, T)$	e_L

Note. Detailed expositions of Υ_1 and Υ_2 $(\Upsilon_1 < \Upsilon_2)$ are available. (See the Online Appendix.)

Table C2. Optimal Decisions for Fixed-Price Contract

Case	Region		Initial Testing Time	Effort Level
FPN	$0 < \Upsilon < \Upsilon_3$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{K(c)}$	e_H
	$\Upsilon \geq \Upsilon_3$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{K(c)}$	e_L
FPR	$0 < \alpha < \hat{\alpha}$	$0 < \Upsilon < \Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H
		$\Upsilon \geq \Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_L
	$\hat{\alpha} \leq \alpha < 1$	$0 < \Upsilon < \Upsilon_5$	0	e_H
		$\Upsilon \geq \Upsilon_5$	0	e_L

Note. Expositions of $\hat{\alpha}$, Υ_3, Υ_4, Υ_5, for $\Upsilon_3 < \min\{\Upsilon_4, \Upsilon_5\}$, are also available. (See Online Appendix.)

Table C3. Optimal Decisions for Time-and-Materials Contract

Case	Region		Initial Testing Time	Effort Level	Monitoring Policy
TMN	$0 < \Upsilon < \Upsilon_3$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{K(c)}$	e_H	0
	$\Upsilon_3 \leq \Upsilon < \Upsilon_6\}$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{K(c)}$	e_H	$\min\{\frac{r_1}{s}, 1\}$
	$\Upsilon \geq \Upsilon_6$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{K(c)}$	e_L	0
FPR	$0 < \alpha < \hat{\alpha}$	$0 < \Upsilon < \Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H	0
		$\Upsilon_4 \leq \Upsilon < \Upsilon_7$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H	$\min\{\frac{r_2}{s}, 1\}$
		$\Upsilon \geq \Upsilon_7$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_L	0
	$\hat{\alpha} \leq \alpha < 1$	$0 < \Upsilon < \Upsilon_5$	0	e_H	0
		$\Upsilon_5 \leq \Upsilon < \Upsilon_8$	0	e_H	$\min\{\frac{r_3}{s}, 1\}$
		$\Upsilon \geq \Upsilon_8$	0	e_L	0

Note. Detailed expositions for Υ_6, Υ_7, and Υ_8 and r_1, r_2, r_3 are available. (See the Online Appendix.)

Online Appendix to ISR Paper:

THE POWER OF RENEGOTIATION AND MONITORING IN SOFTWARE OUTSOURCING: SUBSTITUTES OR COMPLEMENTS?

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Appendix 1. Goel-Okumoto (G-O) Model Assumptions When ML Is Used to Detect Bugs

The memoryless property of software bug detection. An issue for emerging software bug detection practices is whether an assumption of the Goel-Okumoto (1979) approach remains valid and yields an appropriate way to handle model bug detection in the presence of new technologies, including automated bug detection and machine learning (ML)-based bug detection. The G-O model is parsimonious and yields quantitative measures for the performance of a software system. When the available data are in the form of time between bugs or the number of bugs in given time intervals, the model's parameters can be estimated. Concerns about validity pertain to Assumption 3 (The Memoryless Property of Bug Detection), based on a non-homogeneous stochastic Poisson process for bug detection, which has a memoryless property. In the G-O model, $N(t)$, the expected cumulative number of software bugs detected during the interval $(0, t]$ is $N(t) = a \cdot (1 - e^{-bt})$. Here, a and b are the expected number of initial bugs and the bug-detection rate.

Applications and a question. This model has been widely used in software testing. Arora et al. (2006) built a model for undetected bugs at testing time t and showed it satisfies the properties of their more inclusive approach. But is it suitable for assessing contract choices, given the current practices, including alpha / beta testing and ML bug detection?

Alpha and beta testing. Software testing is used to find bugs in a system, and ~60 different types exist (Testing Excellence 2019). *Alpha testing* is a form of internal testing at the developer, not the client site. *Beta testing* involves user acceptance testing of *beta versions* of software released to beta-testers outside the developer's site. They test to identify if software has bugs. Beta versions are shared to increase feedback from future users. This serves to deliver value quickly – even for indefinite periods, as with *perpetual beta software*.

ML testing. As a newer software testing technique used for alpha and beta testing, ML tools are able to learn from data. Malhotra (2015) reviews ways to do ML bug identification. They allow future bugs to be detected based on a tool's ability to learn how to perform a task better based on past bugs. However, this process enables future bug detection to involve interdependencies, and so no longer can be an entirely memoryless stochastic process.

Use of ML bug detection results in less time to find a bug, increasing the failure rate that occurs in *software reliability growth models* (SRGM). In the G-O model, the ML effect increases the failure rate λ, which is exogenous in our model. ML can: (1) discover bug associations from past data; and (2) classify software components as defect-prone (Song et al. 2011). ML appears to increase the efficiency of reliability testing but not change the bugs remaining in a system. We have identified evidence that ML modifies the memoryless property of bugs though. Our work is based estimating the remaining bugs in a system, a more direct testing approach that promotes effective use of limited resources (Savitz 2013).

Other approaches. The G-O model can be applied to beta- and ML-testing, and Jiang et al. (2017) established a beta-testing model with it. They made a new assumption though: that *public beta-testers evaluate beta-test software independently*. The expected number of detected bugs at the end of public beta-testing time t is given by $\theta(t) = k \cdot (1 - e^{-\lambda Z t})$, where k is the initial bugs, λ is the detection rate, and Z is the number of beta-testers at time t.

Roy et al. (2015) proposed an *artificial neural network* (ANN) based on the G-O model. They used $f(x) = 1 - e^{-x}$ as the *activation function* in the hidden layer of the NN, and a *linear activation function*, $g(x) = x$ in the output layer. They included $y(t) = w_{23}(1 - e^{-w_{12}t})$, the form of the mean value function for software bugs used by the G-O model. Subscripts 1, 2, and 3 are the input, hidden, and output layers of the ANN. Their model computes the weighted sum of the input signals x_j in the presence of bias θ_i, and passes the sum through the activation function f_i that processes the input signals and generates the output y_j of the ANN overall.

Conclusion. From this assessment of the related literature, we conclude that our approach, which continues to using the G-O model, will not be greatly affected by the new technologies that have become available in industry during the past 10 years. Thus, modeling software bugs, as we have, remains appropriate.

Appendix 2. Technical Proofs

Note: For the modeling notation and definitions, the reader should refer to Table 3 in the paper.

Proof of Lemma 1 (Positive Uncertainty-Resolution Effect).

According to Equation (9), we can obtain the first-order and second-order derivatives of $RS(\tilde{t})$ with respect to $\tilde{t}$ as follows,

$$\begin{cases} \frac{\partial RS(\tilde{t})}{\partial \tilde{t}} = -\big(\Upsilon + K(c)\big) + (\delta + \lambda b) \cdot \big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\exp(-\lambda \tilde{t}) \\ \frac{\partial^2 RS(\tilde{t})}{\partial \tilde{t}^2} = -\lambda \cdot (\delta + \lambda b) \cdot \big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\exp(-\lambda \tilde{t}) \end{cases}.$$

Note that $\frac{\partial^2 RS(\tilde{t})}{\partial \tilde{t}^2} < 0$, so the optimal renegotiated testing time can be derived through $\frac{\partial RS(\tilde{t})}{\partial \tilde{t}} = 0$, and we have $\tilde{t}^* = \frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot(\Upsilon B(c)-e^{\beta}+\varepsilon)}{\Upsilon+K(c)}$.

Then, substituting $\tilde{t}^*$ into $RS(\tilde{t})$ yields

$$\begin{aligned} E_{\varepsilon}[RS^*] = {} & \big(\Upsilon + K(c)\big)t + \left(\frac{\delta}{\lambda} + b\right) \cdot \big(\Upsilon B(c) - e^{\beta}\big)\exp(-\lambda t) - \frac{\Upsilon+K(c)}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)} \\ & -\frac{\Upsilon+K(c)}{2\sigma\lambda}\cdot\Big[\big(\Upsilon B(c) - e^{\beta} + \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} + \sigma\big) - \big(\Upsilon B(c) - e^{\beta} - \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} - \sigma\big)\Big]. \end{aligned}$$

The first-order and second-order derivatives of $E_{\varepsilon}[RS^*]$ with respect to t as follows,

$$\begin{cases} \frac{\partial E_{\varepsilon}[RS^*]}{\partial t} = \big(\Upsilon + K(c)\big) - (\delta + \lambda b) \cdot \big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\exp(-\lambda t) \\ \frac{\partial^2 E_{\varepsilon}[RS^*]}{\partial t^2} = \lambda \cdot (\delta + \lambda b) \cdot \big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\exp(-\lambda t) \end{cases}$$

Note that $\frac{\partial^2 E_{\varepsilon}[RS^*]}{\partial t^2} > 0$, so the minimum value of $E_{\varepsilon}[RS^*]$ can be derived through $\frac{\partial E_{\varepsilon}[RS^*]}{\partial t} = 0$. By substituting the solution $t = \frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot(\Upsilon B(c)-e^{\beta})}{\Upsilon+K(c)}$ of the equation $\frac{\partial E_{\varepsilon}[RS^*]}{\partial t} = 0$ into $E_{\varepsilon}[RS^*]$, the minimum value of $E_{\varepsilon}[RS^*]$ is

$$E_{\varepsilon}[RS^*] = \frac{\Upsilon+K(c)}{\lambda} \cdot \left\{1 + \ln\big(\Upsilon B(c) - e^{\beta}\big) - \frac{1}{2\sigma} \cdot \begin{bmatrix} \big(\Upsilon B(c) - e^{\beta} + \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} + \sigma\big) \\ -\big(\Upsilon B(c) - e^{\beta} - \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} - \sigma\big) \end{bmatrix}\right\}.$$

By Hadamard's inequality, we have $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\big[-1 - \ln\big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\big]\mathrm{d}\varepsilon > -1 - \ln\big(\Upsilon B(c) - e^{\beta}\big)$, where $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\big[-1 - \ln\big(\Upsilon B(c) - e^{\beta} + \varepsilon\big)\big]\mathrm{d}\varepsilon = -\frac{1}{2\sigma} \cdot \begin{bmatrix} \big(\Upsilon B(c) - e^{\beta} + \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} + \sigma\big) \\ -\big(\Upsilon B(c) - e^{\beta} - \sigma\big)\ln\big(\Upsilon B(c) - e^{\beta} - \sigma\big) \end{bmatrix}$. Thus, $E_{\varepsilon}[RS^*] > 0$ always holds, implying the positive uncertainty-resolution effect $UR(e)$. ■

Proof of Lemma 2 (Positive Renegotiation Benefit).

We first derive the optimal decisions in the FBN and FBR cases showed in Table C1 in the paper, and then obtain positive renegotiation benefit by comparing the two cases.[1]

Table C1. Optimal Decisions for First-Best Solution

Case	Region	Initial Testing Time	Effort Level
FBN	$0<\Upsilon<\Upsilon_1$	$\frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{\Upsilon+K(c)}$	e_H
	$\Upsilon\geq\Upsilon_1$	$\frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{\Upsilon+K(c)}$	e_L
FBR	$0<\Upsilon<\Upsilon_2$	$[0,T)$	e_H
	$\Upsilon\geq\Upsilon_2$	$[0,T)$	e_L

(1) The First-Best Solution without Renegotiation Case (FBN). According to Equations (1), (5) and (7), Equation (13) can be written as

$$\pi_S^{FBN}(e,t)=\Upsilon\cdot(T-t)-\frac{\delta}{\lambda}\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(\exp(-\lambda t)-\exp(-\lambda T)\right)-\frac{e}{c}-K(c)t$$
$$-a\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)-\ b\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(\exp(-\lambda t)-\exp(-\lambda T)\right).\quad\text{(A1)}$$

With discrete effort level e and continuous initial testing time t, we first obtain the optimal initial testing time t_{FBN}^{*} related to the different effort level e (e_H or e_L), and then determine the optimal effort level e_{FBN}^{*} by comparing the corresponding social surplus $\pi_S^{FBN}(e_H)$ and $\pi_S^{FBN}(e_L)$.

First, according to Equation (A1), the first-order and second-order derivatives of $\pi_S^{FBN}(e,t)$ with respect to t for a given e are

$$\begin{cases}\frac{\partial\pi_S^{FBN}(e,t)}{\partial t}=-\left(\Upsilon+K(c)\right)+(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda t)\\ \frac{\partial^2\pi_S^{FBN}(e,t)}{\partial t^2}=-\lambda\cdot(\delta+\lambda b)\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda t)\end{cases}.$$

Note that $\frac{\partial^2\pi_S^{FBN}(e,t)}{\partial t^2}<0$, so the optimal initial testing time is derived by $\frac{\partial\pi_S^{FBN}(e,t)}{\partial t}=0$ and we have $t^{*}=\frac{1}{\lambda}\ln\frac{(\delta+\lambda b)\cdot(\Upsilon B(c)-e^{\beta})}{\Upsilon+K(c)}$. Substituting t^{*} into Equation (A1) yields

$$\pi_S^{FBN}(e)=\Upsilon T-\frac{\Upsilon+K(c)}{\lambda}\ln\frac{(\delta+\lambda b)\cdot(\Upsilon B(c)-e^{\beta})}{\Upsilon+K(c)}-\frac{\Upsilon+K(c)}{\lambda}+\frac{\delta+\lambda b}{\lambda}\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda T)$$
$$-\frac{e}{c}-a\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right).\quad\text{(A2)}$$

Based on Equation (A2), define the function $\Delta\Pi_S^{FBN}=\pi_S^{FBN}(e_H)-\pi_S^{FBN}(e_L)$, and $\Delta\Pi_S^{FBN}$ can be written as

$$\Delta\Pi_S^{FBN}=\frac{\Upsilon+K(c)}{\lambda}\ln\left(1+\frac{e_H^{\beta}-e_L^{\beta}}{\Upsilon B(c)-e_H^{\beta}}\right)-\frac{\delta+\lambda b}{\lambda}\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\exp(-\lambda T)$$
$$-\frac{e_H-e_L}{c}+a\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right).\quad\text{(A3)}$$

[1] We redemonstrate Tables C1, C2 and C3 in the Online Appendix, in order to facilitate reading.

When $\Upsilon \to 0$, based on Assumption 2, we have $\left(\Upsilon B(c) - e_H^{\beta}\right) \to 0$ and $\ln\left(1 + \frac{e_H^{\beta} - e_L^{\beta}}{\Upsilon B(c) - e_H^{\beta}}\right) \to +\infty$. Thus, $\lim_{\Upsilon \to 0} \Delta\Pi_S^{FBN} > 0$. When $\Upsilon \to +\infty$, by L'Hospital's rule, we obtain

$$\lim_{\Upsilon \to +\infty} \Delta\Pi_S^{FBN} = \frac{K(c)\cdot\left(e_H^{\beta} - e_L^{\beta}\right)}{\lambda B(c)} - \frac{\delta + \lambda b}{\lambda} \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\exp(-\lambda T)$$
$$- \frac{e_H - e_L}{c} + a \cdot \left(e_H^{\beta} - e_L^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right).$$

Besides, based on Assumption 7, $\Upsilon \to +\infty$ leads to $(\delta + \lambda b) \to +\infty$, so it can be verified that $\lim_{\Upsilon \to +\infty} \Delta\Pi_S^{FBN} < 0$. Further, according to Equation (A3), we have

$$\frac{\partial \Delta\Pi_S^{FBN}}{\partial \Upsilon} = \frac{1}{\lambda}\ln\left(\frac{\Upsilon B(c) - e_L^{\beta}}{\Upsilon B(c) - e_H^{\beta}}\right) - \frac{\Upsilon + K(c)}{\lambda} \cdot \frac{B(c)\cdot\left(e_H^{\beta} - e_L^{\beta}\right)}{\left(\Upsilon B(c) - e_H^{\beta}\right)\cdot\left(\Upsilon B(c) - e_L^{\beta}\right)}.$$

By Hadamard's inequality, we have $\frac{1}{e_H^{\beta} - e_L^{\beta}} \int_{e_L^{\beta}}^{e_H^{\beta}} \frac{1}{\Upsilon B(c) - e} \mathrm{d}e < \frac{1}{2} \cdot \left(\frac{1}{\Upsilon B(c) - e_H^{\beta}} + \frac{1}{\Upsilon B(c) - e_L^{\beta}}\right)$, where $\frac{1}{e_H^{\beta} - e_L^{\beta}} \int_{e_L^{\beta}}^{e_H^{\beta}} \frac{1}{\Upsilon B(c) - e} \mathrm{d}e = \ln\left(\frac{\Upsilon B(c) - e_L^{\beta}}{\Upsilon B(c) - e_H^{\beta}}\right)$. In addition, it can be derived that $\frac{1}{2} \cdot \left(\frac{1}{\Upsilon B(c) - e_H^{\beta}} + \frac{1}{\Upsilon B(c) - e_L^{\beta}}\right) < \frac{(\Upsilon + K(c))\cdot B(c)\cdot\left(e_H^{\beta} - e_L^{\beta}\right)}{\left(\Upsilon B(c) - e_H^{\beta}\right)\cdot\left(\Upsilon B(c) - e_L^{\beta}\right)}$, so that $\frac{\partial \Delta\Pi_S^{FBN}}{\partial \Upsilon} < 0$. Thus, there is a unique interior solution $\Upsilon^* = \Upsilon_1$ for the equation $\Delta\Pi_S^{FBN} = 0$. That is, $\Upsilon^* = \Upsilon_1$ is the unique interior solution for the equation $\pi_S^{FBN}(e_H) - \pi_S^{FBN}(e_L) = 0$.

(2) The First-Best Solution with Renegotiation Case (FBR). According to (1), (5), (7) and (10), Equation (12) can be written as

$$\pi_S^{FBR}(e) = \Upsilon T - \frac{\Upsilon + K(c)}{\lambda}\ln\frac{\delta + \lambda b}{\Upsilon + K(c)} + \frac{\delta + \lambda b}{\lambda} \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda T) - \frac{e}{c}$$
$$-a \cdot \left(\Upsilon B(c) - e^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right) - \frac{\Upsilon + K(c)}{2\sigma\lambda} \cdot \begin{bmatrix} \left(\Upsilon B(c) - e^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} - \sigma\right) \end{bmatrix} \tag{A4}$$

First, according to Equation (A4), $\pi_S^{FBR}(e)$ is independent with t, so that the optimal initial testing time t_{FBR}^* can be any value in $[0, T)$. Then, we compare the social surplus $\pi_S^{FBR}(e_H)$ and $\pi_S^{FBR}(e_L)$ to obtain the optimal effort level e_{FBR}^*. Define the function $\Delta\Pi_S^{FBR} = \pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L)$, and $\Delta\Pi_S^{FBR}$ can be written as

$$\Delta\Pi_S^{FBR} = -\frac{\delta + \lambda b}{\lambda} \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\exp(-\lambda T) - \frac{e_H - e_L}{c} + a \cdot \left(e_H^{\beta} - e_L^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right)$$
$$-\frac{\Upsilon + K(c)}{2\sigma\lambda} \cdot \begin{bmatrix} \left(\Upsilon B(c) - e_H^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e_H^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e_H^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e_H^{\beta} - \sigma\right) \end{bmatrix}$$

$$+\frac{\Upsilon+K(c)}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\end{bmatrix}. \tag{A5}$$

By L'Hospital's rule, we have

$$\lim_{\Upsilon\to 0}\Delta\Pi_S^{FBR}=-\frac{\delta+\lambda b}{\lambda}\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\exp(-\lambda T)+a\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)$$
$$-\frac{e_H-e_L}{c}+\frac{B(c)}{2\sigma\lambda}\ln\left(1+\frac{2\sigma\cdot\left(e_H^{\beta}-e_L^{\beta}\right)}{\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\cdot\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)}\right).$$

When $\Upsilon\to 0$, based on Assumption 2, we obtain $\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\to 0$, implying $\ln\left(1+\frac{2\sigma\cdot\left(e_H^{\beta}-e_L^{\beta}\right)}{\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\cdot\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)}\right)\to+\infty$. Thus, $\lim_{\Upsilon\to 0}\Delta\Pi_S^{FBR}>0$. When $\Upsilon\to+\infty$, similar to the proving process of the **FBN** case, $\lim_{\Upsilon\to+\infty}\Delta\Pi_S^{FBR}<0$. Further, according to Equation (A5), we have

$$\frac{\partial\Delta\Pi_S^{FBR}}{\partial\Upsilon}=-\frac{1}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\end{bmatrix}-\frac{(\Upsilon+K(c))B(c)}{2\sigma\lambda}\ln\frac{\Upsilon B(c)-e_H^{\beta}+\sigma}{\Upsilon B(c)-e_H^{\beta}-\sigma}$$
$$+\frac{1}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\end{bmatrix}+\frac{(\Upsilon+K(c))B(c)}{2\sigma\lambda}\ln\frac{\Upsilon B(c)-e_L^{\beta}+\sigma}{\Upsilon B(c)-e_L^{\beta}-\sigma}.$$

Define another function

$$f_1(x)=\begin{bmatrix}(\Upsilon B(c)-x+\sigma)\ln(\Upsilon B(c)-x+\sigma)\\-(\Upsilon B(c)-x-\sigma)\ln(\Upsilon B(c)-x-\sigma)\end{bmatrix}+\left(\Upsilon+K(c)\right)B(c)\cdot\ln\frac{\Upsilon B(c)-x+\sigma}{\Upsilon B(c)-x-\sigma}.$$

The first-order derivative of $f_1(x)$ with respect to x is

$$\frac{\partial f_1(x)}{\partial x}=2\sigma\cdot\left\{\frac{\Upsilon B(c)}{(\Upsilon B(c)-x+\sigma)\cdot(\Upsilon B(c)-x-\sigma)}-\frac{1}{2\sigma}\cdot\left[\ln(\Upsilon B(c)-x+\sigma)-\ln(\Upsilon B(c)-x-\sigma)\right]\right\}$$
$$+\frac{2\sigma\Upsilon B(c)}{(\Upsilon B(c)-x+\sigma)\cdot(\Upsilon B(c)-x-\sigma)}.$$

By Hadamard's inequality, $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\frac{1}{\Upsilon B(c)-x+\varepsilon}\mathrm{d}\varepsilon<\frac{1}{2}\cdot\left(\frac{1}{\Upsilon B(c)-x+\sigma}+\frac{1}{\Upsilon B(c)-x-\sigma}\right)$, where $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\frac{1}{\Upsilon B(c)-x+\varepsilon}\mathrm{d}\varepsilon=\frac{1}{2\sigma}\cdot\left[\ln(\Upsilon B(c)-x+\sigma)-\ln(\Upsilon B(c)-x-\sigma)\right]$. In addition, it can be derived that $\frac{1}{2}\cdot\left(\frac{1}{\Upsilon B(c)-x+\sigma}+\frac{1}{\Upsilon B(c)-x-\sigma}\right)<\frac{\Upsilon B(c)}{(\Upsilon B(c)-x+\sigma)\cdot(\Upsilon B(c)-x-\sigma)}$, so that $\frac{\partial f_1(x)}{\partial x}>0$ and $f_1\left(e_H^{\beta}\right)>f_1\left(e_L^{\beta}\right)$, implying $\frac{\partial\Delta\Pi_S^{FBR}}{\partial\Upsilon}<0$. Thus, there is a unique interior solution $\Upsilon^*=\Upsilon_2$ for the equation $\Delta\Pi_S^{FBR}=0$. That is, $\Upsilon^*=\Upsilon_2$ is the unique interior solution for the equation $\pi_S^{FBR}(e_H)-\pi_S^{FBR}(e_L)=0$. The optimal effort level and initial testing time are derived and shown in Table C1.

(3) Comparing FBR with FBN. According to the optimal decisions for the FBN and FBR cases, the first-order derivative of $E_\varepsilon[RS^*]$ with respect to e for given optimal initial testing times t^*_{FBN} and t^*_{FBR} is

$$\frac{\partial E_\varepsilon[RS^*]}{\partial e} = \frac{\beta\cdot(\Upsilon+K(c))e^{-(1-\beta)}}{\lambda}\cdot\left\{\frac{1}{2\sigma}\cdot\left[\ln\left(\Upsilon B(c)-e^\beta+\sigma\right)-\ln\left(\Upsilon B(c)-e^\beta-\sigma\right)\right]-\frac{1}{\Upsilon B(c)-e^\beta}\right\}.$$

By Hadamard's inequality, $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\frac{1}{\Upsilon B(c)-e^\beta+\varepsilon}\mathrm{d}\varepsilon > \frac{1}{\Upsilon B(c)-e^\beta}$, where $\frac{1}{2\sigma}\int_{-\sigma}^{\sigma}\frac{1}{\Upsilon B(c)-e^\beta+\varepsilon}\mathrm{d}\varepsilon = \frac{1}{2\sigma}\cdot\left[\ln\left(\Upsilon B(c)-e^\beta+\sigma\right)-\ln\left(\Upsilon B(c)-e^\beta-\sigma\right)\right]$. Thus, $\frac{\partial E_\varepsilon[RS^*]}{\partial e}>0$ always holds, implying that the positive uncertainty-resolution effect increases in the Vendor's development effort. Substituting $\Upsilon=\Upsilon_1$ into $\Delta\Pi_S^{FBR}$, it can be derived that $\Delta\Pi_S^{FBR}>0$, implying $\Upsilon_1<\Upsilon_2$.

Then, according to Lemma 1, $E_\varepsilon[RS^*]>0$, we have $\pi_S^{FBR}(e^*_{FBN}\ ,t^*_{FBN}) > \pi_S^{FBN}(e^*_{FBN},t^*_{FBN})$. With optimal decisions for the FBR case, we have $\pi_S^{FBR}(e^*_{FBR},t^*_{FBR}) \geq \pi_S^{FBR}(e^*_{FBN}\ ,t^*_{FBN})$. Thus, $\pi_S^{FBR}(e^*_{FBR},t^*_{FBR}) > \pi_S^{FBN}(e^*_{FBN},t^*_{FBN})$, implying positive renegotiation benefit, $RB^{FB}>0$. ■

Proof of Optimal Decisions for Fixed-Price Contract.

We first derive the optimal decisions in the FPN and FPR cases showed in Table C2 in the paper, and then compare the two cases to obtain the impacts of renegotiation on the fixed-price contract.

Table C2. Optimal Decisions for Fixed-Price Contract

Case	Region		Initial Testing Time	Effort Level
FPN	$0<\Upsilon<\Upsilon_3$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^\beta\right)}{K(c)}$	e_H
	$\Upsilon\geq\Upsilon_3$		$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_L^\beta\right)}{K(c)}$	e_L
FPR	$0<\alpha<\hat{\alpha}$	$0<\Upsilon<\Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^\beta\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H
		$\Upsilon\geq\Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_L^\beta\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_L
	$\hat{\alpha}\leq\alpha<1$	$0<\Upsilon<\Upsilon_5$	0	e_H
		$\Upsilon\geq\Upsilon_5$	0	e_L

(1) The Fixed-Price Contract without Renegotiation Case (FPN). Using backward induction, we first consider the Vendor's problem. According to Equations (1) and (5), Equation (17) can be written as

$$\pi_V^{FPN}(e,t) = P_{FP}-\frac{e}{c}-K(c)t-a\cdot\left(\Upsilon B(c)-e^\beta\right)\cdot\left(1-\exp(-\lambda T)\right)$$
$$-\ b\cdot\left(\Upsilon B(c)-e^\beta\right)\cdot\left(\exp(-\lambda t)-\exp(-\lambda T)\right). \qquad \text{(A6)}$$

With discrete effort level e and continuous initial testing time t, after deriving the optimal initial testing time t^*_{FPN} related to the different effort level e (e_H or e_L), we compare corresponding Vendor revenue $\pi_V^{FPN}(e_H)$ and $\pi_V^{FPN}(e_L)$ to obtain the optimal effort level e^*_{FPN}.

From Equation (A6), the first-order and second-order derivatives of $\pi_V^{FPN}(e,t)$ with respect to t for a given e are

$$\begin{cases}\frac{\partial \pi_V^{FPN}(e,t)}{\partial t} = -K(c) + \lambda b \cdot \left(\varUpsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \\ \frac{\partial^2 \pi_V^{FPN}(e,t)}{\partial t^2} = -\lambda^2 b \cdot \left(\varUpsilon B(c) - e^{\beta}\right)\exp(-\lambda t)\end{cases}.$$

Note that $\frac{\partial^2 \pi_V^{FPN}(e,t)}{\partial t^2} < 0$, so the optimal initial testing can be derived through $\frac{\partial \pi_V^{FPN}(e,t)}{\partial t} = 0$, and $t^* = \frac{1}{\lambda}\ln\frac{\lambda b \cdot (\varUpsilon B(c) - e^{\beta})}{K(c)}$. Then, substituting t^* into Equation (A6) yields

$$\pi_V^{FPN}(e) = P_{FP} - \frac{e}{c} - \frac{K(c)}{\lambda}\ln\frac{\lambda b \cdot (\varUpsilon B(c) - e^{\beta})}{K(c)} - a \cdot \left(\varUpsilon B(c) - e^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right)$$
$$- \frac{K(c)}{\lambda} + b \cdot \left(\varUpsilon B(c) - e^{\beta}\right)\exp(-\lambda T). \quad \text{(A7)}$$

According to Equation (A7), define the function $\Delta\Pi_V^{FPN} = \pi_V^{FPN}(e_H) - \pi_V^{FPN}(e_L)$, and $\Delta\Pi_V^{FPN}$ can be written as

$$\Delta\Pi_V^{FPN} = -\frac{e_H - e_L}{c} + \frac{K(c)}{\lambda}\ln\left(\frac{\varUpsilon B(c) - e_L^{\beta}}{\varUpsilon B(c) - e_H^{\beta}}\right) + a \cdot \left(e_H^{\beta} - e_L^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right) - b \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\exp(-\lambda T). \quad \text{(A8)}$$

Solving the equation $\Delta\Pi_V^{FPN} = 0$, the solution $\varUpsilon^* = \varUpsilon_3$ is derived, where $\varUpsilon_3 = \frac{1}{B(c)} \cdot \left\{\frac{e_H^{\beta} - e_L^{\beta}}{\exp\left(\frac{\lambda}{K(c)} \cdot \left\{\frac{e_H - e_L}{c} + [b\exp(-\lambda T) - a \cdot (1 - \exp(-\lambda T))] \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\right\}\right) - 1} + e_H^{\beta}\right\}$. Besides, according to Equation (A8), the first-order derivative of $\Delta\Pi_V^{FPN}$ with respect to is $\varUpsilon$ $\frac{\partial \Delta\Pi_V^{FPN}}{\partial \varUpsilon} = -\frac{K(c)B(c) \cdot \left(e_H^{\beta} - e_L^{\beta}\right)}{\lambda \cdot \left(\varUpsilon B(c) - e_H^{\beta}\right) \cdot \left(\varUpsilon B(c) - e_L^{\beta}\right)}$, and $\frac{\partial \Delta\Pi_V^{FPN}}{\partial \varUpsilon} < 0$. Thus, when $0 < \varUpsilon < \varUpsilon_3$, we have $\Delta\Pi_V^{FPN} > 0$, implying $\pi_V^{FPN}(e_H) > \pi_V^{FPN}(e_L)$; and when $\varUpsilon \geq \varUpsilon_3$, we have $\Delta\Pi_V^{FPN} \leq 0$, implying $\pi_V^{FPN}(e_H) \leq \pi_V^{FPN}(e_L)$.

Next, in the Client's contracting problem, the Client's profit decreases in the payment P_{FP}. Thus, when the individual rationality (IR) constraint is binding, the Client achieves maximal profit, and the optimal payment P_{FP}^* can be derived.

Further, we compare the value of thresholds $\varUpsilon_1$ in the FBN case and $\varUpsilon_3$ in the FPN case. Substituting $\varUpsilon = \varUpsilon_3$ and $\Delta\Pi_V^{FPN} = 0$ into $\Delta\Pi_S^{FBN}$ yields

$$\Delta\Pi_S^{FBN}(\varUpsilon_3) = \frac{1}{\lambda} \cdot \left[\left(\varUpsilon_3 \ln\left(\varUpsilon_3 B(c) - e_L^{\beta}\right) + \delta e_L^{\beta}\exp(-\lambda T)\right) - \left(\varUpsilon_3 \ln\left(\varUpsilon_3 B(c) - e_H^{\beta}\right) + \delta e_H^{\beta}\exp(-\lambda T)\right)\right].$$

Define another function $f_2(x) = \varUpsilon_3 \ln(\varUpsilon_3 B(c) - x) + \delta x \cdot \exp(-\lambda T)$. The first-order derivative of $f_2(x)$ with respect to x is $\frac{\partial f_2(x)}{\partial x} = -\frac{\varUpsilon_3}{\varUpsilon_3 B(c) - x} + \delta\exp(-\lambda T)$. Since $\delta < \frac{\lambda b \varUpsilon}{K(c)}$, it can be derived that $\frac{\partial f_2(x)}{\partial x} < 0$ and $f_2\left(e_H^{\beta}\right) < f_2\left(e_L^{\beta}\right)$, implying $\Delta\Pi_S^{FBN}(\varUpsilon_3) > 0$ and $\varUpsilon_1 > \varUpsilon_3$.

(2) The Fixed-Price Contract with Renegotiation Case (FPR).

Similar to the proving process of the FPN case, using backward induction, we first consider the Vendor's problem. According to Equations (1), (5) and (10), Equation (16) can be written as

$$\begin{aligned}\pi_V^{FPR}(e,t) = & P_{FP} - [(1-\alpha)K(c) - \alpha\Upsilon]t - \frac{[(1-\alpha)\lambda b - \alpha\delta]}{\lambda}\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \\ & -\frac{e}{c} - a\cdot\left(\Upsilon B(c) - e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right) + b\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda T) \\ & -\frac{\alpha\cdot(\Upsilon + K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon + K(c)} - \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\end{bmatrix}. \quad \text{(A9)}\end{aligned}$$

From Equation (A9), the first-order and second-order derivatives of $\pi_V^{FPR}(e,t)$ with respect to t for a given e are

$$\begin{cases}\frac{\partial \pi_V^{FPR}(e,t)}{\partial t} = -[(1-\alpha)K(c) - \alpha\Upsilon] + [(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \\ \frac{\partial^2 \pi_V^{FPR}(e,t)}{\partial t^2} = -\lambda\cdot[(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t)\end{cases}.$$

Define $\hat{\alpha} = \frac{\lambda b}{\lambda b + \delta}$, and we have the solutions of t for a given e as follows.

***(a)* When $\boldsymbol{0 < \alpha < \hat{\alpha}}$**, note that $\frac{\partial^2 \pi_V^{FPR}(e,t)}{\partial t^2} < 0$, so the optimal initial testing can be derived through $\frac{\partial \pi_V^{FPR}(e,t)}{\partial t} = 0$, and $t^* = \frac{1}{\lambda}\ln\frac{[(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c) - e^{\beta}\right)}{(1-\alpha)K(c) - \alpha\Upsilon}$. In addition, it can be verified that $t^* > E_{\varepsilon}[\tilde{t}^*]$. Then, substituting t^* into Equation (A9) yields

$$\begin{aligned}\pi_V^{FPR}(e) = & P_{FP} - \frac{(1-\alpha)K(c) - \alpha\Upsilon}{\lambda}\ln\frac{[(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c) - e^{\beta}\right)}{(1-\alpha)K(c) - \alpha\Upsilon} - \frac{(1-\alpha)K(c) - \alpha\Upsilon}{\lambda} \\ & -\frac{e}{c} - a\cdot\left(\Upsilon B(c) - e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right) + b\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda T) \\ & -\frac{\alpha\cdot(\Upsilon + K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon + K(c)} - \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\end{bmatrix}. \quad \text{(A10)}\end{aligned}$$

According to Equation (A10), define the function $\Delta\Pi_V^{FPR} = \pi_V^{FPR}(e_H) - \pi_V^{FPR}(e_L)$, and $\Delta\Pi_V^{FPR}$ can be written as

$$\begin{aligned}\Delta\Pi_V^{FPR} = & \frac{(1-\alpha)K(c) - \alpha\Upsilon}{\lambda}\ln\left(\frac{\Upsilon B(c) - e_L^{\beta}}{\Upsilon B(c) - e_H^{\beta}}\right) + a\cdot\left(e_H^{\beta} - e_L^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right) - b\cdot\left(e_H^{\beta} - e_L^{\beta}\right)\exp(-\lambda T) \\ & -\frac{e_H - e_L}{c} - \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e_H^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e_H^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e_H^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e_H^{\beta} - \sigma\right)\end{bmatrix} \\ & +\frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e_L^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e_L^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e_L^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e_L^{\beta} - \sigma\right)\end{bmatrix}. \quad \text{(A11)}\end{aligned}$$

According to Equation (A11), similar to the proving process of the **FBR** case, $\lim_{\Upsilon\to 0}\Delta\Pi_V^{FPR} > 0$ and

$\lim_{\Upsilon\to+\infty} \Delta\Pi_V^{FPR} < 0$. The first-order derivative of $\Delta\Pi_V^{FPR}$ with respect to Υ is $\frac{\partial\Delta\Pi_V^{FPR}}{\partial\Upsilon} = \frac{\partial\Delta\Pi_V^{FPN}}{\partial\Upsilon} + \alpha \cdot \left(\frac{\partial\Delta\Pi_S^{FBR}}{\partial\Upsilon} - \frac{\partial\Delta\Pi_S^{FBN}}{\partial\Upsilon}\right)$. By Hadamard's inequality, $\frac{\partial\Delta\Pi_S^{FBR}}{\partial\Upsilon} < \frac{\partial\Delta\Pi_S^{FBN}}{\partial\Upsilon}$, so that $\frac{\partial\Delta\Pi_V^{FPR}}{\partial\Upsilon} < 0$. Thus, there is a unique interior solution $\Upsilon^* = \Upsilon_4$ for the equation $\Delta\Pi_V^{FPR} = 0$. That is, $\Upsilon^* = \Upsilon_4$ is the unique interior solution for the equation $\pi_V^{FPR}(e_H) - \pi_V^{FPR}(e_L) = 0$.

***(b)* When $\hat{\alpha} \le \alpha < 1$**, note that $\frac{\partial^2\pi_V^{FPR}(e,t)}{\partial t^2} > 0$, so the optimal initial testing time can be derived by comparing $\pi_V^{FPR}(e, t = 0)$ and $\pi_V^{FPR}(e, t = T)$. Since $\pi_V^{FPR}(e, t = 0) > \pi_V^{FPR}(e, t = T)$, $t^* = 0$ is the optimal initial testing for the Vendor. In addition, it can be verified that $t^* < E_\varepsilon[\tilde{t}^*]$. Then, substituting t^* into Equation (A9) yields

$$\begin{aligned}\pi_V^{FPR}(e) = P_{FP} &- \frac{[(1-\alpha)\lambda b - \alpha\delta]}{\lambda} \cdot \left(\Upsilon B(c) - e^\beta\right) - \frac{e}{c} \\ &- a \cdot \left(\Upsilon B(c) - e^\beta\right) \cdot \left(1 - \exp(-\lambda T)\right) + b \cdot \left(\Upsilon B(c) - e^\beta\right)\exp(-\lambda T) \\ &- \frac{\alpha\cdot(\Upsilon + K(c))}{\lambda}\ln\frac{\delta + \lambda b}{\Upsilon + K(c)} - \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e^\beta + \sigma\right)\ln\left(\Upsilon B(c) - e^\beta + \sigma\right) \\ -\left(\Upsilon B(c) - e^\beta - \sigma\right)\ln\left(\Upsilon B(c) - e^\beta - \sigma\right)\end{bmatrix}. \end{aligned} \quad \text{(A12)}$$

According to Equation (A12), $\Delta\Pi_V^{FPR}$ can be written as

$$\begin{aligned}\Delta\Pi_V^{FPR} = &\frac{[(1-\alpha)\lambda b - \alpha\delta]}{\lambda} \cdot \left(e_H^\beta - e_L^\beta\right) + a \cdot \left(e_H^\beta - e_L^\beta\right) \cdot \left(1 - \exp(-\lambda T)\right) - b \cdot \left(e_H^\beta - e_L^\beta\right)\exp(-\lambda T) \\ &- \frac{e_H - e_L}{c} - \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e_H^\beta + \sigma\right)\ln\left(\Upsilon B(c) - e_H^\beta + \sigma\right) \\ -\left(\Upsilon B(c) - e_H^\beta - \sigma\right)\ln\left(\Upsilon B(c) - e_H^\beta - \sigma\right)\end{bmatrix} \\ &+ \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c) - e_L^\beta + \sigma\right)\ln\left(\Upsilon B(c) - e_L^\beta + \sigma\right) \\ -\left(\Upsilon B(c) - e_L^\beta - \sigma\right)\ln\left(\Upsilon B(c) - e_L^\beta - \sigma\right)\end{bmatrix}. \end{aligned} \quad \text{(A13)}$$

According to Equation (A13), similar to the proving process of the **FBR** case, $\lim_{\Upsilon\to 0} \Delta\Pi_V^{FPR} > 0$ and $\lim_{\Upsilon\to+\infty} \Delta\Pi_V^{FPR} < 0$. The first-order derivative of $\Delta\Pi_V^{FPR}$ with respect to Υ is $\frac{\partial\Delta\Pi_V^{FPR}}{\partial\Upsilon} = \alpha \cdot \frac{\partial\Delta\Pi_S^{FBR}}{\partial\Upsilon}$. Thus, $\frac{\partial\Delta\Pi_V^{FPR}}{\partial\Upsilon} < 0$ and there is a unique interior solution $\Upsilon^* = \Upsilon_5$ for the equation $\Delta\Pi_V^{FPR} = 0$. That is, $\Upsilon^* = \Upsilon_5$ is the unique interior solution for the equation $\pi_V^{FPR}(e_H) - \pi_V^{FPR}(e_L) = 0$.

Next, in the Client's contracting problem, the Client's profit decreases in the initial payment P_{FP}. Thus, when the individual rationality (IR) constraint is binding, the Client achieves maximal profit, and the optimal payment P_{FP}^* can be derived. The optimal effort level and initial testing time are derived and shown in Table C2.

(3) Comparing FPR with FPN. Substituting $\Upsilon = \Upsilon_3$ and $\Delta\Pi_V^{FPN} = 0$ into $\Delta\Pi_V^{FPR}$ yields $\Delta\Pi_V^{FPR}(\Upsilon_3) > 0$, implying $\Upsilon_3 < \min\{\Upsilon_4, \Upsilon_5\}$. Besides, according to the equation $\pi_V^{FPR}(e_H) - \pi_V^{FPR}(e_L) =$

0, it can be derived that $\partial \Upsilon_4(\alpha)/\partial \alpha > 0$ and $\partial \Upsilon_5(\alpha)/\partial \alpha > 0$.

For Footnote 18, if the Client and the Vendor split the renegotiation cost C_R via their bargaining powers $1-\alpha$ and α, it can be proved that all the first-order derivatives mentioned above are not affected, implying the results hold qualitatively. ■

Proof of Optimal Decisions for Time-and-Materials Contract.

We first derive the optimal decisions in the TMN and TMR cases showed in Table C3 in the paper, and then compare the two cases to obtain the impacts of renegotiation on the time-and-materials contract.

Table C3. Optimal Decisions for Time-and-Materials Contract

Case		Region	Initial Testing Time	Effort Level	Monitoring Policy
TMN		$0 < \Upsilon < \Upsilon_3$	$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{K(c)}$	e_H	0
		$\Upsilon_3 \le \Upsilon < \Upsilon_6$	$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{K(c)}$	e_H	$\min\{\frac{r_1}{s},1\}$
		$\Upsilon \ge \Upsilon_6$	$\frac{1}{\lambda}\ln\frac{\lambda b\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{K(c)}$	e_L	0
FPR	$0 < \alpha < \hat{\alpha}$	$0 < \Upsilon < \Upsilon_4$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H	0
		$\Upsilon_4 \le \Upsilon < \Upsilon_7$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_H^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_H	$\min\{\frac{r_2}{s},1\}$
		$\Upsilon \ge \Upsilon_7$	$\frac{1}{\lambda}\ln\frac{[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e_L^{\beta}\right)}{(1-\alpha)\cdot K(c)-\alpha\Upsilon}$	e_L	0
	$\hat{\alpha} \le \alpha < 1$	$0 < \Upsilon < \Upsilon_5$	0	e_H	0
		$\Upsilon_5 \le \Upsilon < \Upsilon_8$	0	e_H	$\min\{\frac{r_3}{s},1\}$
		$\Upsilon \ge \Upsilon_8$	0	e_L	0

In the TMN and TMR cases, the incentive compatibility (IC) constraints should be satisfied. This is because the Client prefers to deter the Vendor from misreporting its development effort. Thus, the Client is willing to offer an incentive-compatible contract for per unit effort reimbursement $r \le \phi s$, where the latter terms represent the probability that the Client discovers the Vendor's misreporting ϕ, and is able to enforce the penalty s. And we have the Vendor's reported effort is equal to its development effort, i.e., $\hat{e} = e$.

(1) The Time-and-Materials Contract without Renegotiation Case (TMN). Using backward induction, we first consider the Vendor's problem. According to Equations (1) and (5) and $\hat{e} = e$, Equation (21) can be written as

$$\pi_V^{TMN}(e,t) = P_{TM} + re - \frac{e}{c} - K(c)t - a\cdot\left(\Upsilon B(c) - e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)$$
$$-\ b\cdot\left(\Upsilon B(c) - e^{\beta}\right)\cdot\left(\exp(-\lambda t) - \exp(-\lambda T)\right). \quad \text{(A14)}$$

With discrete effort level e and continuous initial testing time t, after deriving the optimal initial testing time t_{TMN}^{*} related to the different effort level e (e_H or e_L), we compare corresponding Vendor revenue $\pi_V^{TMN}(e_H)$ and $\pi_V^{TMN}(e_L)$ to obtain the optimal effort level e_{TMN}^{*}.

From Equation (A14), we derive

$$\begin{cases} \frac{\partial \pi_V^{TMN}(e,t)}{\partial t} = -K(c) + \lambda b \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \\ \frac{\partial^2 \pi_V^{TMN}(e,t)}{\partial t^2} = -\lambda^2 b \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \end{cases}.$$

Note that $\frac{\partial^2 \pi_V^{TMN}(e,t)}{\partial t^2} < 0$, so the optimal initial testing can be derived through $\frac{\partial \pi_V^{TMN}(e,t)}{\partial t} = 0$, and $t^* = \frac{1}{\lambda}\ln\frac{\lambda b \cdot (\Upsilon B(c) - e^{\beta})}{K(c)}$. Then, substituting t^* into Equation (A14) yields

$$\pi_V^{TMN}(e) = P_{TM} + re - \frac{K(c)}{\lambda}\ln\frac{\lambda b \cdot (\Upsilon B(c) - e^{\beta})}{K(c)} - a \cdot \left(\Upsilon B(c) - e^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right)$$
$$- \frac{e}{c} - \frac{K(c)}{\lambda} + b \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda T). \quad \text{(A15)}$$

According to Equation (A15), define the function $\Delta\Pi_V^{TMN} = \pi_V^{TMN}(e_H) - \pi_V^{TMN}(e_L)$, and $\Delta\Pi_V^{TMN}$ can be written as

$$\Delta\Pi_V^{TMN} = r \cdot (e_H - e_L) - \frac{e_H - e_L}{c} + \frac{K(c)}{\lambda}\ln\left(\frac{\Upsilon B(c) - e_L^{\beta}}{\Upsilon B(c) - e_H^{\beta}}\right)$$
$$+ a \cdot \left(e_H^{\beta} - e_L^{\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right) - b \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\exp(-\lambda T). \quad \text{(A16)}$$

Solving the equation $\Delta\Pi_V^{TMN} = 0$, the solution $\Upsilon^* = \bar{\Upsilon}_1(r)$ is derived, where $\bar{\Upsilon}_1(r)$ is a function of effort reimbursement r and $\bar{\Upsilon}_1(r) = \frac{1}{B(c)} \cdot \left\{\frac{e_H^{\beta} - e_L^{\beta}}{\exp\left(\frac{\lambda}{K(c)} \cdot \left\{\frac{e_H - e_L}{c} + \left[b\exp(-\lambda T) - a \cdot (1 - \exp(-\lambda T))\right] \cdot \left(e_H^{\beta} - e_L^{\beta}\right)\right\} - r \cdot (e_H - e_L)\right) - 1} + e_H^{\beta}\right\}$. Besides, according to Equation (A16), $\frac{\partial \Delta\Pi_V^{TMN}}{\partial \Upsilon} = \frac{\partial \Delta\Pi_V^{FPN}}{\partial \Upsilon}$, so that $\frac{\partial \Delta\Pi_V^{TMN}}{\partial \Upsilon} < 0$. Thus, when $0 < \Upsilon < \bar{\Upsilon}_1(r)$, we have $\Delta\Pi_V^{TMN} > 0$, implying $\pi_V^{TMN}(e_H) > \pi_V^{TMN}(e_L)$ and $e^* = e_H$; and when $\Upsilon \geq \bar{\Upsilon}_1(r)$, we have $\Delta\Pi_V^{TMN} \leq 0$, implying $\pi_V^{TMN}(e_H) \leq \pi_V^{TMN}(e_L)$ and $e^* = e_L$.

Next, in the Client's contracting problem, according to Equations (1) and (7), Equation (21) can be written as

$$\pi_C^{TMN}(P_{TM}, r, \phi) = \Upsilon \cdot (T - t^*) - \frac{\delta}{\lambda} \cdot \left(\Upsilon B(c) - e^{*\beta}\right) \cdot \left(\exp(-\lambda t^*) - \exp(-\lambda T)\right) - P_{TM} - re^* - w\phi. \quad \text{(A17)}$$

Based on Equation (A17), it is intuitive that the Client's profit decreases in the payment P_{TM}. Thus, when the individual rationality (IR) constraint is binding, the Client achieves maximal profit and we have

$$P_{TM}^* = -re^* + \frac{e^*}{c} + \frac{K(c)}{\lambda}\ln\frac{\lambda b \cdot \left(\Upsilon B(c) - e^{*\beta}\right)}{K(c)} + \frac{K(c)}{\lambda}$$
$$+ a \cdot \left(\Upsilon B(c) - e^{*\beta}\right) \cdot \left(1 - \exp(-\lambda T)\right) - b \cdot \left(\Upsilon B(c) - e^{*\beta}\right)\exp(-\lambda T).$$

In addition, the Client's profit decreases in the monitoring policy ϕ. Thus, when the incentive compatibility (IC) constraint is binding, the Client achieves maximal profit. According to the IC constraint, the optimal monitoring policy ϕ_N^* is r_N^*/s, subject to $0 \le \phi \le 1$. Substituting t^*, P_{TM}^* and ϕ_N^* into Equation (A17) yields

$$\pi_C^{TMN}(e) = \Upsilon T - \frac{\Upsilon+K(c)}{\lambda}\ln\frac{\lambda b\cdot(\Upsilon B(c)-e^\beta)}{\Upsilon+K(c)} - \frac{(\lambda\delta+b)K(c)}{\lambda b} + \frac{\delta+\lambda b}{\lambda}\cdot\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda T)$$
$$-\frac{e}{c} - a\cdot\left(\Upsilon B(c)-e^\beta\right)\cdot\left(1-\exp(-\lambda T)\right) - \frac{wr}{s}. \quad \text{(A18)}$$

Substitute $\Upsilon = \bar{\Upsilon}_1(r)$ into Equation (A18), and it can be derived that there is a solution $r^* = r_1$ for the equation $\pi_C^{TMN}(e_H) - \pi_C^{TMN}(e_L) = 0$. Note that $\phi_{TMN}^* = r_N^*/s$, subject to $0 \le \phi \le 1$. Thus, by comparing the value of r_1/s to 1 and 0, respectively, the optimal per unit effort reimbursement r_N^*, monitoring policy ϕ_N^*, effort level e_{TMN}^* and initial testing time t_{TMN}^* are derived. See in Table O3, where $\Upsilon_6 = \min\{\bar{\Upsilon}_1(s), \bar{\Upsilon}_1(r_1)\}$.

(2) The Time-and-Materials Contract with Renegotiation Case (TMR). Similar to the proving process of **TMN** case, using backward induction, we first consider the Vendor's problem. According to Equations (1), (5) and (10), Equation (20) can be written as

$$\pi_V^{TMR}(e,t) = P_{TM} + re - [(1-\alpha)K(c) - \alpha\Upsilon]t - \frac{[(1-\alpha)\lambda b - \alpha\delta]}{\lambda}\cdot\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda t)$$
$$-\frac{e}{c} - a\cdot\left(\Upsilon B(c)-e^\beta\right)\cdot\left(1-\exp(-\lambda T)\right) + b\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda T)$$
$$-\frac{\alpha\cdot(\Upsilon+K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)} - \frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e^\beta+\sigma\right)\ln\left(\Upsilon B(c)-e^\beta+\sigma\right)\\ -\left(\Upsilon B(c)-e^\beta-\sigma\right)\ln\left(\Upsilon B(c)-e^\beta-\sigma\right)\end{bmatrix}. \quad \text{(A19)}$$

From Equation (A19), the first-order and second-order derivatives of $\pi_V^{TMR}(e,t)$ with respect to t for a given e are

$$\begin{cases}\frac{\partial\pi_V^{TMR}(e,t)}{\partial t} = -[(1-\alpha)K(c)-\alpha\Upsilon] + [(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda t)\\ \frac{\partial^2\pi_V^{TMR}(e,t)}{\partial t^2} = -\lambda\cdot[(1-\alpha)\lambda b-\alpha\delta]\cdot\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda t)\end{cases}.$$

Define $\hat{\alpha} = \frac{\lambda b}{\lambda b+\delta}$, and the solutions of t for a given e as follows.

(*a*) When $0 < \alpha < \hat{\alpha}$, note that $\frac{\partial^2\pi_V^{TMR}(e,t)}{\partial t^2} < 0$, so the optimal initial testing can be derived through $\frac{\partial\pi_V^{TMR}(e,t)}{\partial t} = 0$, and $t^* = \frac{1}{\lambda}\ln\frac{[(1-\alpha)\lambda b-\alpha\delta]\cdot(\Upsilon B(c)-e^\beta)}{(1-\alpha)K(c)-\alpha\Upsilon}$. In addition, it can be verified that $t^* > E_\varepsilon[\tilde{t}^*]$. Then, substituting t^* into Equation (A19) yields

$$\pi_V^{TMR}(e) = P_{FP} + re - \frac{(1-\alpha)K(c)-\alpha\Upsilon}{\lambda}\ln\frac{[(1-\alpha)\lambda b-\alpha\delta]\cdot(\Upsilon B(c)-e^\beta)}{(1-\alpha)K(c)-\alpha\Upsilon} - \frac{(1-\alpha)K(c)-\alpha\Upsilon}{\lambda}$$
$$-\frac{e}{c} - a\cdot\left(\Upsilon B(c)-e^\beta\right)\cdot\left(1-\exp(-\lambda T)\right) + b\left(\Upsilon B(c)-e^\beta\right)\exp(-\lambda T)$$

$$-\frac{\alpha\cdot(\Upsilon+K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)}-\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\end{bmatrix}. \quad \text{(A20)}$$

According to Equation (A20), define the function $\Delta\Pi_V^{TMR}=\pi_V^{TMR}(e_H)-\pi_V^{TMR}(e_L)$, and $\Delta\Pi_V^{TMR}$ can be written as

$$\Delta\Pi_V^{TMR}=\frac{(1-\alpha)K(c)-\alpha\Upsilon}{\lambda}\ln\left(\frac{\Upsilon B(c)-e_L^{\beta}}{\Upsilon B(c)-e_H^{\beta}}\right)+a\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)-b\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\exp(-\lambda T)$$
$$+r\cdot(e_H-e_L)-\frac{e_H-e_L}{c}-\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\end{bmatrix}$$
$$+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\end{bmatrix}. \quad \text{(A21)}$$

According to Equation (A21), similar to the proving process of **FPR** case, we derive $\lim_{\Upsilon\to 0}\Delta\Pi_V^{TMR}>0$, $\lim_{\Upsilon\to+\infty}\Delta\Pi_V^{TMR}<0$, and $\frac{\partial\Delta\Pi_V^{TMR}}{\partial\Upsilon}<0$. Thus, there is a unique interior solution $\Upsilon^*=\bar{\Upsilon}_2(r)$ for the equation $\Delta\Pi_V^{TMR}=0$, where $\bar{\Upsilon}_2(r)$ is a function of per uint effort reimbursement r. That is, $\Upsilon^*=\bar{\Upsilon}_2(r)$ is the unique interior solution for the equation $\pi_V^{TMR}(e_H)-\pi_V^{TMR}(e_L)=0$. When $0<\Upsilon<\bar{\Upsilon}_2(r)$, we have $\pi_V^{TMR}(e_H)>\pi_V^{TMR}(e_L)$ and $e^*=e_H$; and when $\Upsilon\geq\bar{\Upsilon}_2(r)$, we have $\pi_V^{TMR}(e_H)\leq\pi_V^{TMR}(e_L)$ and $e^*=e_L$.

***(b)* When $\hat{\alpha}\leq\alpha<1$**, note that $\frac{\partial^2\pi_V^{TMR}(e,t)}{\partial t^2}>0$, so the optimal initial testing time can be derived by comparing $\pi_V^{TMR}(e,t=0)$ and $\pi_V^{TMR}(e,t=T)$. Since $\pi_V^{TMR}(e,t=0)>\pi_V^{TMR}(e,t=T)$, $t^*=0$ is the optimal initial testing for the Vendor. In addition, it can be verified that $t^*<E_\varepsilon[\tilde{t}^*]$. Then, substituting t^* into Equation (A19) yields

$$\pi_V^{TMR}(e)=P_{TM}+re-\frac{[(1-\alpha)\lambda b-\alpha\delta]}{\lambda}\cdot\left(\Upsilon B(c)-e^{\beta}\right)-\frac{e}{c}$$
$$-a\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)+b\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda T)$$
$$-\frac{\alpha\cdot(\Upsilon+K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)}-\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\end{bmatrix}. \quad \text{(A22)}$$

According to Equation (A22), $\Delta\Pi_V^{TMR}$ can be written as

$$\Delta\Pi_V^{TMR}=\frac{[(1-\alpha)\lambda b-\alpha\delta]}{\lambda}\cdot\left(e_H^{\beta}-e_L^{\beta}\right)+a\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)-b\cdot\left(e_H^{\beta}-e_L^{\beta}\right)\exp(-\lambda T)$$
$$+r\cdot(e_H-e_L)-\frac{e_H-e_L}{c}-\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_H^{\beta}-\sigma\right)\end{bmatrix}$$

$$+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e_L^{\beta}-\sigma\right)\end{bmatrix}. \tag{A23}$$

According to Equation (A23), similar to the proving process of **FPR** case, we derive $\lim_{\Upsilon\to 0}\Delta\Pi_V^{TMR}>0$, $\lim_{\Upsilon\to+\infty}\Delta\Pi_V^{TMR}<0$, and $\frac{\partial\Delta\Pi_V^{TMR}}{\partial\Upsilon}<0$. Thus, there is a unique interior solution $\Upsilon^*=\bar{\Upsilon}_3(r)$ for the equation $\Delta\Pi_V^{TMR}=0$, where $\bar{\Upsilon}_3(r)$ is a function of per uint effort reimbursement r. That is, $\Upsilon^*=\bar{\Upsilon}_3(r)$ is the unique interior solution for the equation $\pi_V^{TMR}(e_H)-\pi_V^{TMR}(e_L)=0$. When $0<\Upsilon<\bar{\Upsilon}_3(r)$, we have $\pi_V^{TMR}(e_H)>\pi_V^{TMR}(e_L)$ and $e^*=e_H$; and when $\Upsilon\geq\bar{\Upsilon}_3(r)$, we have $\pi_V^{TMR}(e_H)\leq\pi_V^{TMR}(e_L)$ and $e^*=e_L$.

Next, in the Client's contracting problem, according to Equations (1) and (7), Equation (22) can be written as

$$\pi_C^{TMR}(P_{TM},r,\phi)=\Upsilon\cdot(T-t^*)-\frac{\delta}{\lambda}\cdot\left(\Upsilon B(c)-e^{*\beta}\right)\cdot\left(\exp(-\lambda t^*)-\exp(-\lambda T)\right)$$
$$-P_{TM}-re^*-w\phi+(1-\alpha)\cdot E_\varepsilon[RS(\tilde{t}^*)]-C_R. \tag{A24}$$

According to Equation (A24), the Client's profit decreases in the initial payment P_{TM}. Thus, when the individual rationality (IR) constraint is binding, the Client achieves maximal profit and we have

$$P_{TM}^*=-re^*+[(1-\alpha)K(c)-\alpha\Upsilon]t^*+\frac{[(1-\alpha)\lambda b-\alpha\delta]}{\lambda}\cdot\left(\Upsilon B(c)-e^{*\beta}\right)\exp(-\lambda t^*)$$
$$+\frac{e^*}{c}+a\cdot\left(\Upsilon B(c)-e^{*\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)-b\cdot\left(\Upsilon B(c)-e^{*\beta}\right)\exp(-\lambda T)$$
$$+\frac{\alpha\cdot(\Upsilon+K(c))}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)}+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e^{*\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e^{*\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e^{*\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e^{*\beta}-\sigma\right)\end{bmatrix}. \tag{A25}$$

In addition, the Client's profit decreases in the monitoring policy ϕ. Thus, when the incentive compatibility (IC) constraint is binding, the Client achieves maximal profit. According to the IC constraint, the optimal monitoring policy ϕ_{TMR}^* is r_R^*/s, subject to $0\leq\phi\leq 1$. Substituting t^*, P_{TM}^* and ϕ_{TMR}^* into Equation (A25), we have

$$\pi_C^{TMR}(e)=\Upsilon T-\frac{\Upsilon+K(c)}{\lambda}\ln\frac{\delta+\lambda b}{\Upsilon+K(c)}+\frac{\delta+\lambda b}{\lambda}\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda T)-\frac{e}{c}-\frac{wr}{s}-C_R$$
$$-a\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)-\frac{\Upsilon+K(c)}{2\sigma\lambda}\cdot\begin{bmatrix}\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}+\sigma\right)\\-\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\ln\left(\Upsilon B(c)-e^{\beta}-\sigma\right)\end{bmatrix} \tag{A26}$$

Substitute $\Upsilon=\bar{\Upsilon}_2(r)$ $(\Upsilon=\bar{\Upsilon}_3(r))$ into Equation (A26), and it can be derived that there is a solution $r^*=r_2$ $(r^*=r_3)$ for the equation $\pi_C^{TMR}(e_H)-\pi_C^{TMR}(e_L)=0$. Note that $\phi_{TMR}^*=r_R^*/s$, subject to $0\leq\phi\leq 1$. Thus, by comparing the value of r_2/s (r_3/s) to 1 and 0, respectively, the optimal per unit effort reimbursement r_R^*, monitoring policy ϕ_{TMR}^*, effort level e_{TMR}^* and initial testing time t_{TMR}^* are derived. See in Table C3, where $\varUpsilon_7 = \min\{\bar{\varUpsilon}_2(s), \bar{\varUpsilon}_2(r_2)\}$ and $\varUpsilon_8 = \min\{\bar{\varUpsilon}_3(s), \bar{\varUpsilon}_3(r_3)\}$.

(3) Comparing TMR with TMN. Substituting $\varUpsilon = \varUpsilon_6$ and $\Delta\Pi_V^{TMN} = 0$ into $\Delta\Pi_V^{TMR}$, we obtain $\Delta\Pi_V^{TMR} > 0$, implying $\varUpsilon_6 < \min\{\varUpsilon_7, \varUpsilon_8\}$.

Note that for Footnote 18, if the Client and the Vendor split the renegotiation cost C_R via their bargaining powers $1 - \alpha$ and α, it can be proved that all the first-order derivatives mentioned above are not affected, implying the results hold qualitatively. ■

Proof of Proposition 1 (Client Contract Choice for Costless Monitoring and Costless Renegotiation).

Suppose that renegotiation cost $C_R = 0$ and per unit cost of monitoring $w = 0$. According to optimal decisions for fixed-price and time-and-materials contracts, renegotiation benefit RB^{FP} and RB^{TM} are positive. Thus, when $C_R = 0$, the Client attains more profit in the renegotiation case than that in the non-renegotiation case. Note that $\underline{\alpha}(\varUpsilon)$ is the inverse function of $\varUpsilon(\alpha)$, where $\varUpsilon(\alpha) = \min\{\bar{\varUpsilon}_2(s), \bar{\varUpsilon}_3(s)\}$, and the threshold for the Vendor's bargaining power $\alpha_1 = \max\{0, \underline{\alpha}(\varUpsilon)\}$; $\bar{\alpha}(\varUpsilon)$ is the inverse function of $\varUpsilon(\alpha)$, where $\varUpsilon(\alpha) = \min\{\varUpsilon_4, \varUpsilon_5\}$, and the threshold for the Vendor's bargaining power $\alpha_2 = \min\{\bar{\alpha}(\varUpsilon), 1\}$.

By comparing π_C^{FPR} and π_C^{TMR}, when (*i*) $\alpha_1 < \alpha < \alpha_2$, then $\pi_C^{FPR} < \pi_C^{TMR}$ and the Client selects TMR; and (*ii*) otherwise, $\pi_C^{FPR} = \pi_C^{TMR}$ and the Client selects FPR.

For complementarity. When $\alpha_1 < \alpha < \alpha_2$: (*i*) By comparing FPR with FPN, it can be observed that the Vendor's effort $e_{FPR}^* = e_L$ and $e_{FPN}^* = e_L$. Thus, without monitoring, the indirect uncertainty-resolution effect of renegotiation becomes invalid, $\pi_S^{FBR}(e_L) - \pi_S^{FBR}(e_L) = 0$. (*ii*) In TMR, $e_{TMR}^* = e_H$. Thus, compared with FPR, a positive indirect uncertainty-resolution effect can be verified in TMR, $\pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L) > 0$. (*iii*) By comparing TMR with TMN, we show that renegotiation increases the benefit from pre-development incentive via the uncertainty-resolution effect, $UR(e_H) > 0$. ■

Proof of Proposition 2 (Client Contract Choice for Costly Monitoring and Costless Renegotiation).

Suppose that renegotiation cost $C_R = 0$. According to optimal decisions for fixed-price and time-and-materials contracts, renegotiation benefit RB^{FP} and RB^{TM} are positive. Thus, when $C_R = 0$, the Client attains more profit in the renegotiation case than that in the non-renegotiation case. Note that $w(\varUpsilon)$ (short for $\widehat{w}$) is the inverse function of $\varUpsilon(w)$, where $\varUpsilon(w) = \min\{\bar{\varUpsilon}_2(r_2), \bar{\varUpsilon}_3(r_3)\}$. By comparing π_C^{FPR} and π_C^{TMR}, we have: (*i*) when $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \widehat{w}$, then $\pi_C^{FPR} < \pi_C^{TMR}$ and the Client selects TMR; (*ii*) otherwise, $\pi_C^{FPR} = \pi_C^{TMR}$ and the Client selects FPR.

For complementarity. When $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \widehat{w}$: (*i*) By comparing FPR with FPN, it can be observed that the Vendor's effort $e_{FPR}^* = e_L$ and $e_{FPN}^* = e_L$. Thus, without monitoring, the indirect uncertainty-resolution effect of renegotiation becomes invalid, $\pi_S^{FBR}(e_L) - \pi_S^{FBR}(e_L) = 0$. (*ii*) In TMR, $e_{TMR}^* = e_H$. Thus, compared with FPR, a positive indirect uncertainty-resolution effect can be verified in TMR, $\pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L) > 0$. (*iii*) By comparing TMR with TMN, it can be concluded that renegotiation increases the benefit from pre-development incentive via the uncertainty-resolution effect, $UR(e_H) > 0$. ■

Proof of Proposition 3 (Client Contract Choice for Costless Monitoring and Costly Renegotiation).

Suppose that per unit cost of monitoring $w = 0$. First, when $C_R < \min\{RB^{FP}, RB^{TM}\}$, the Client attains more profit in the renegotiation case than that in the non-renegotiation case. By comparing π_C^{FPR} and π_C^{TMR}, similar to Proposition 1, we have: if (*i*) $\alpha_1 < \alpha < \alpha_2$, then $\pi_C^{FPR} < \pi_C^{TMR}$ and the Client selects TMR; (*ii*) otherwise, $\pi_C^{FPR} = \pi_C^{TMR}$ and the Client selects FPR.

Second, when $C_R \geq \min\{RB^{FP}, RB^{TM}\}$, the Client attains more profit in the non-renegotiation case than that in the renegotiation case. By comparing π_C^{FPN} and π_C^{TMN}, we have: (*i*) if $\Upsilon_3 \leq \Upsilon < \Upsilon_9$, then $\pi_C^{FPN} < \pi_C^{TMN}$ and the Client selects TMN; (*ii*) otherwise, $\pi_C^{FPN} = \pi_C^{TMN}$ and the Client selects FPN. Note that $\Upsilon_9 = \min\{\bar{\Upsilon}_1(s), \bar{\Upsilon}_1(r_4)\}$ and $r^* = r_4$ is the solution for the equation $\pi_C^{TMN}(e_H) - \pi_C^{TMN}(e_L) = 0$, when $w = 0$.

For complementarity. When $C_R < \min\{RB^{FP}, RB^{TM}\}$ and $\alpha_1 < \alpha < \alpha_2$: (*i*) By comparing FPR with FPN, it can be observed that $e_{FPR}^* = e_L$ and $e_{FPN}^* = e_L$. Thus, without monitoring, the indirect uncertainty-resolution effect of renegotiation becomes invalid, $\pi_S^{FBR}(e_L) - \pi_S^{FBR}(e_L) = 0$. (*ii*) In TMR, $e_{TMR}^* = e_H$. Thus, by comparing with FPR, a positive indirect uncertainty-resolution effect can be verified in TMR, $\pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L) > 0$. (*iii*) By comparing TMR with TMN, it shows that renegotiation increases the benefit from pre-development incentive via the uncertainty-resolution effect, $UR(e_H) > 0$. ■

Proof of Proposition 4 (Client Contract Choice under Costly Monitoring and Costly Renegotiation).

First, when $C_R < \min\{RB^{FP}, RB^{TM}\}$, the Client attains more profit in the renegotiation case than that in the non-renegotiation case. By comparing π_C^{FPR} and π_C^{TMR}, similar to Proposition 2, we have : (*i*) if $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \hat{w}$, then $\pi_C^{FPR} < \pi_C^{TMR}$ and the Client selects TMR; (*ii*) otherwise, $\pi_C^{FPR} = \pi_C^{TMR}$ and the Client selects FPR.

Second, when $C_R \geq \min\{RB^{FP}, RB^{TM}\}$, the Client attains more profit in the non-renegotiation case than that in the renegotiation case. By comparing π_C^{FPN} and π_C^{TMN}, similar to Proposition 3, we have: (*i*) if $\Upsilon_3 \leq \Upsilon < \Upsilon_6$, then $\pi_C^{FPN} < \pi_C^{TMN}$ and the Client selects TMN; (*ii*) otherwise, $\pi_C^{FPN} = \pi_C^{TMN}$ and the Client selects FPN.

For complementarity. When $C_R < \min\{RB^{FP}, RB^{TM}\}$, $\alpha_1 < \alpha < \alpha_2$ and $0 < w < \hat{w}$: (*i*) By comparing FPR with FPN, we have $e^*_{FPR} = e_L$ and $e^*_{FPN} = e_L$. Thus, without monitoring, the indirect uncertainty-resolution effect of renegotiation becomes invalid, $\pi_S^{FBR}(e_L) - \pi_S^{FBR}(e_L) = 0$. (*ii*) In TMR, $e^*_{TMR} = e_H$. Thus, compared with FPR, a positive indirect uncertainty-resolution effect can be verified in TMR, $\pi_S^{FBR}(e_H) - \pi_S^{FBR}(e_L) > 0$. (*iii*) Comparing TMR with TMN yields that renegotiation increases the benefit from pre-development incentive via the uncertainty-resolution effect, $UR(e_H) > 0$. ■

Proof of Corollary 1 (Impacts of System Complexity and Bug Rate on Vendor Effort and Client Profit)

According to Equation (1), $N_{CS}(e) = \Upsilon B(c) - e^{\beta} + \varepsilon$, it is straightforward that the initial expected number of bugs $\Upsilon B(c)$ increases in system complexity Υ and bug rate $B(c)$. Further, when Υ or $B(c)$ increases, the expected number of bugs $N_{CS}(e)$ becomes greater. And from optimal decisions for fixed-price and time-and-materials contracts, the Client's profit decreases in $N_{CS}(e)$. Thus, the Client's profit decreases in both Υ and $B(c)$. Besides, when $N_{CS}(e)$ increases, it can be derived that all the solutions for the equations, $\pi_V^{FPN}(e_H) - \pi_V^{FPN}(e_L) = 0$, $\pi_V^{FPR}(e_H) - \pi_V^{FPR}(e_L) = 0$, $\pi_V^{TMN}(e_H) - \pi_V^{TMN}(e_L) = 0$ and $\pi_V^{TMR}(e_H) - \pi_V^{TMR}(e_L) = 0$, decrease. Thus, the Vendor is more likely to exert low effort when Υ or $B(c)$ increases. ■

Proof of Corollary 2 (Impacts of System Lifetime on Vendor Effort and Client Profit).

According to optimal decisions for fixed-price and time-and-materials contracts, we have $\partial^2\pi_V^{FPN}(e)/\partial e\partial T > 0$, $\partial^2\pi_V^{FPR}(e)/\partial e\partial T > 0$, $\partial^2\pi_V^{TMN}(e)/\partial e\partial T > 0$ and $\partial^2\pi_V^{TMR}(e)/\partial e\partial T > 0$. Thus, the Vendor is more likely to exert high effort when system life time T becomes longer. From Equation (1), $N_{CS}(e) = \Upsilon B(c) - e^{\beta} + \varepsilon$, when the Vendor's effort e increases, the expected number of bugs $N_{CS}(e)$ becomes less. Thus, increasing system lifetime T can increase the valuation of the system $\Upsilon \cdot (T - t)$ and decrease the expected number of bugs in customized system $N_{CS}(e)$, which adds the Client's profit. ■

Proof of Proposition 5 (Client Contract Choice under Continuous Development Effort).

The proving processes of direct and indirect uncertainty-resolution effects and a post-development incentive under continuous development effort (CDE) is similar to the proving processes of **Lemmas 1 and 2**. Then, we first derive the Vendor's and the Client's optimal decisions in the FPN, FPR, TMN and TMR cases and compare the Client's profit in each case to obtain the Client's optimal contract choice under continuous development effort.

(1) The Fixed-Price Contract without Renegotiation Case (FPN). Using backward induction, we first consider the Vendor's problem. From Equation (A6), we obtain

$$\begin{cases} \frac{\partial \pi_V^{FPN-CDE}(e,t)}{\partial e} = -\frac{1}{c} + \beta \cdot \left[a \cdot \left(1 - \exp(-\lambda T)\right) + b \cdot \left(\exp(-\lambda t) - \exp(-\lambda T)\right)\right] e^{-(1-\beta)} \\ \frac{\partial \pi_V^{FPN-CDE}(e,t)}{\partial t} = -K(c) + \lambda b \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \end{cases}. \tag{A27}$$

The Hessian matrix is given by $\mathbb{H}^{FPN-CDE} = \begin{bmatrix} \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e^2} & \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e \partial t} \\ \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t \partial e} & \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t^2} \end{bmatrix}$. According to Equations (A27), the second-order derivatives are

$$\begin{cases} \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e^2} = -\beta \cdot (1-\beta) \cdot \left[a \cdot \left(1 - \exp(-\lambda T)\right) + b \cdot \left(\exp(-\lambda t) - \exp(-\lambda T)\right)\right] e^{-(2-\beta)} \\ \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t^2} = -\lambda^2 b \cdot \left(\varUpsilon B(c) - e^{\beta}\right) \exp(-\lambda t) \\ \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e \partial t} = -\beta \lambda b e^{-(1-\beta)} \exp(-\lambda t) \\ \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t \partial e} = -\beta \lambda b e^{-(1-\beta)} \exp(-\lambda t) \end{cases}.$$

It is can be proved that $\frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e^2} < 0$, $\frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t^2} < 0$ and $\frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e^2} \cdot \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t^2} - \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial e \partial t} \cdot \frac{\partial^2 \pi_V^{FPN-CDE}(e,t)}{\partial t \partial e} > 0$, indicating the Hessian matrix $\mathbb{H}^{FPN-CDE}$ is negative definite. Thus, the optimal development effort and optimal initial testing time $(e^*_{FPN-CDE}, t^*_{FPN-CDE})$ is determined by the equations $\begin{cases} \frac{\partial \pi^{FPN-CDE}(e,t)}{\partial e} = 0 \\ \frac{\partial \pi^{FPN-CDE}(e,t)}{\partial t} = 0 \end{cases}$.

Next, in the Client's contracting problem, its profit decreases in the payment P_{FP}. Thus, when the IR constraint is binding, the Client achieves maximal profit, and the optimal payment P^*_{FP-CDE} is derived.

(2) The Fixed-Price Contract with Renegotiation Case (FPR). Using backward induction, we first consider the Vendor's problem. From Equation (A9), the first-order derivatives of $\pi_V^{FPR-CDE}(e,t)$ with respect to e and t are

$$\begin{cases} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} = -\frac{1}{c} + \frac{\beta e^{-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha) \cdot \lambda b - \alpha\delta] \cdot \exp(-\lambda t) + \lambda a \cdot \left(1 - \exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha \cdot (\varUpsilon + K(c))}{2\sigma} \cdot \ln \frac{\varUpsilon B(c) - e^{\beta} + \sigma}{\varUpsilon B(c) - e^{\beta} - \sigma} \end{Bmatrix}. \\ \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial t} = -[(1-\alpha)K(c) - \alpha \varUpsilon] + [(1-\alpha)\lambda b - \alpha\delta] \cdot \left(\varUpsilon B(c) - e^{\beta}\right) \exp(-\lambda t) \end{cases} \quad \text{(A28)}$$

(*a*) When $0 < \alpha < \hat{\alpha}$, it can be verified that $\begin{cases} \lim\limits_{e \to 0} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} > 0 \\ \lim\limits_{e \to +\infty} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} < 0 \end{cases}$ and $\begin{cases} \lim\limits_{t \to 0} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial t} > 0 \\ \lim\limits_{t \to T} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial t} < 0 \end{cases}$.

Thus, the optimal effort level and optimal initial testing time $(e^*_{FPR-CDE}, t^*_{FPR-CDE})$ can be determined by the equations $\begin{cases} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} = 0 \\ \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial t} = 0 \end{cases}$.

(*b*) When $\hat{\alpha} \leq \alpha < 1$, it can be verified that $\begin{cases} \lim\limits_{e\to 0} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} > 0 \\ \lim\limits_{e\to +\infty} \frac{\partial \pi_V^{FPR-CDE}(e,t)}{\partial e} < 0 \end{cases}$ and $\frac{\partial^2 \pi_V^{FPR-CDE}(e,t)}{\partial t^2} > 0$, $\pi_V^{FPR-CDE}(e, t=0) > \pi_V^{FPR-CDE}(e, t=T)$. Thus, $e^* = e_{FPR-CDE}^*$ is the solution for the equation $-\frac{1}{c} + \frac{\beta e^{-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha)\cdot\lambda b - \alpha\delta] + \lambda a \cdot \left(1-\exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma} \cdot \ln \frac{\Upsilon B(c) - e^{\beta} + \sigma}{\Upsilon B(c) - e^{\beta} - \sigma} \end{Bmatrix} = 0$. That is, $e_{FPR-CDE}^*$ is the optimal effort level. The optimal initial testing time $t_{FPR-CDE}^* = 0$.

Next, in the Client's contracting problem, the lower fixed payment indicates that the Client's profit is higher. Thus, when the IR constraint is binding, maximal profit can be achieved and the optimal initial payment P_{FP-CDE}^* can be derived. From the equation $-\frac{1}{c} + \frac{\beta e^{-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha)\cdot\lambda b - \alpha\delta] + \lambda a \cdot \left(1-\exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma} \cdot \ln \frac{\Upsilon B(c) - e^{\beta} + \sigma}{\Upsilon B(c) - e^{\beta} - \sigma} \end{Bmatrix} = 0$, we have $\frac{\partial e_{FPR-CDE}^*}{\partial \alpha} > 0$. This implies that the optimal Vendor's development effort $e_{FPR-CDE}^*$ increases with the Vendor's bargaining power α. Further, since $\frac{\partial \pi^{FPR-CDE}(e,t)}{\partial e}\Big|_{e=e_{FPN-CDE}^*} > 0$, $e_{FPR-CDE}^* > e_{FPN-CDE}^*$ is derived. ■

(3) The Time-and-Materials Contract without Renegotiation Case (TMN). First, similar to the discrete effort scenario (Proof of Optimal Decisions for Time-and-Materials Contract), in the TMN-CDE case, the IC constraint should be satisfied. The Client is willing to offer an incentive-compatible contract for per unit effort reimbursement $r \leq \phi s$, and the Vendor's reported effort is equal to its development effort, $\hat{e} = e$. Thus, in the Vendor's problem, the first-order derivatives of $\pi_V^{TMN-CDE}(e,t)$ with respect to e and t from Equation (A14) are

$$\begin{cases} \frac{\partial \pi_V^{TMN-CDE}(e,t)}{\partial e} = r - \frac{1}{c} + \beta \cdot \left[a \cdot \left(1-\exp(-\lambda T)\right) + b \cdot \left(\exp(-\lambda t) - \exp(-\lambda T)\right)\right] e^{-(1-\beta)} \\ \frac{\partial \pi_V^{TMN-CDE}(e,t)}{\partial t} = -K(c) + \lambda b \cdot \left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \end{cases}. \quad \text{(A29)}$$

The Hessian matrix is given by $\mathbb{H}^{TMN-CDE} = \begin{bmatrix} \frac{\partial^2 \pi_V^{TMN-CDE}(e,t)}{\partial e^2} & \frac{\partial^2 \pi_V^{TMN-CDE}(e,t)}{\partial e \partial t} \\ \frac{\partial^2 \pi_V^{TMN-CDE}(e,t)}{\partial t \partial e} & \frac{\partial^2 \pi_V^{TMN-CDE}(e,t)}{\partial t^2} \end{bmatrix}$. According to Equation (A29), the second-order derivatives of $\pi_V^{TMN-CDE}(e,t)$ are

$$\begin{cases}\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial e^2} = -\beta\cdot(1-\beta)e^{-(2-\beta)}\cdot\left[a\cdot\left(1-\exp(-\lambda T)\right)+b\cdot\left(\exp(-\lambda t)-\exp(-\lambda T)\right)\right]\\ \frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial t^2} = -\lambda^2 b\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda t)\\ \frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial e\partial t} = -\beta\lambda b e^{-(1-\beta)}\exp(-\lambda t)\\ \frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial t\partial e} = -\beta\lambda b e^{-(1-\beta)}\exp(-\lambda t)\end{cases}.$$

It is can be derived that $\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial e^2}<0$, $\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial t^2}<0$ and $\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial e^2}\cdot\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial t^2}-\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial e\partial t}\cdot\frac{\partial^2\pi_V^{TMN-CDE}(e,t)}{\partial t\partial e}>0$. Thus, the Hessian matrix $\mathbb{H}^{TMN-CDE}$ is negative definite, and the optimal development effort and initial testing time (e^*,t^*) can be determined from the equations $\begin{cases}\frac{\partial\pi^{TMN-CDE}(e,t)}{\partial e}=0\\ \frac{\partial\pi^{TMN-CDE}(e,t)}{\partial t}=0\end{cases}$.

Next, in the Client's contracting problem, the IR constraint is binding, and thus the optimal payment P^*_{TM-CDE} can be derived. The higher monitoring policy ϕ indicates the higher monitoring cost and the lower client's profit, while the IC constraint $r\le\phi s$ should be satisfied. Thus, the IC constraint is binding and $r^*_{N-CDE}=\phi^*_{TMN-CDE}s$. According to (e^*,t^*), we have

$$r^*=\frac{1}{c}-\beta\cdot\left[a\cdot\left(1-\exp(-\lambda T)\right)+\frac{K(c)}{\lambda\cdot\left(\Upsilon B(c)-e^{*\beta}\right)}-b\exp(-\lambda T)\right]e^{*-(1-\beta)}.$$

Substituting r^* into Equation (A18) yields

$$\begin{aligned}\pi_C^{TMN-CDE}(e) &= \Upsilon T-\frac{\Upsilon+K(c)}{\lambda}\ln\frac{\lambda b\cdot(\Upsilon B(c)-e^{\beta})}{\Upsilon+K(c)}-\frac{(\lambda\delta+b)K(c)}{\lambda b}-\frac{e}{c}\\ &+\frac{\delta+\lambda b}{\lambda}\cdot\left(\Upsilon B(c)-e^{\beta}\right)\exp(-\lambda T)-a\cdot\left(\Upsilon B(c)-e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right)\\ &-\frac{w}{s}\cdot\left\{\frac{1}{c}-\beta\cdot\left[a\cdot\left(1-\exp(-\lambda T)\right)+\frac{K(c)}{\lambda\cdot(\Upsilon B(c)-e^{\beta})}-b\exp(-\lambda T)\right]e^{-(1-\beta)}\right\}.\end{aligned}\quad\text{(A30)}$$

Based on the first-order condition in Equation (A30), we obtain a threshold w_{E1}, which satisfies $\frac{\partial\pi_C^{TMN-CDE}(e)}{\partial e}|_{w=w_{E1},e=e^*_{FPN-CDE}}=0$. Thus, when $w\ge w_{E1}$, we have $r^*_{N-CDE}=0$, $\phi^*_{TMN-CDE}=0$ and $e^*_{TMN-CDE}=e^*_{FPN-CDE}$. Further, when $0<w<w_{E1}$, there exists an interior point $\bar{e}$ to satisfy $\frac{\partial\pi_C^{TMN-CDE}(e)}{\partial e}|_{e=\bar{e}}=0$, and $\frac{\partial^2\pi_C^{TMN-CDE}(e)}{\partial e^2}<0$. Substituting $e=\bar{e}$ into r^* yields $r^*=\frac{1}{c}-\beta\cdot\left[a\cdot\left(1-\exp(-\lambda T)\right)+\frac{K(c)}{\lambda\cdot(\Upsilon B(c)-\bar{e}^{\beta})}-b\exp(-\lambda T)\right]\bar{e}^{-(1-\beta)}$ defined as $\bar{r}$. Thus, if $\bar{r}/s<1$, then $\phi^*_{TMN-CDE}=\bar{r}/s$, $r^*_{N-CDE}=\bar{r}$, and $e^*_{TMN-CDE}=\bar{e}$; otherwise $\phi^*_{TMN-CDE}=1$, $r^*_{N-CDE}=s$ and $e^*_{TMN-CDE}$ is the solution for the equation $s-\frac{1}{c}+\beta\cdot\left[a\cdot\left(1-\exp(-\lambda T)\right)+b\cdot\left(\exp(-\lambda t)-\exp(-\lambda T)\right)\right]e^{-(1-\beta)}=0$.

(4) The Time-and-Materials Contract with Renegotiation Case (TMR). First, similar to the **TMN-**

CDE case, under the optimal solutions, we have $r \le \phi s$ and $\hat{e} = e$. In the Vendor's problem, from Equation (A19) we obtain

$$\begin{cases} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} = r - \frac{1}{c} + \frac{\beta e^{-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha)\cdot\lambda b - \alpha\delta]\cdot\exp(-\lambda t) + \lambda a \cdot \left(1-\exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma}\cdot \ln\frac{\Upsilon B(c) - e^{\beta} + \sigma}{\Upsilon B(c) - e^{\beta} - \sigma} \end{Bmatrix} \\ \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial t} = -[(1-\alpha)K(c) - \alpha\Upsilon] + [(1-\alpha)\lambda b - \alpha\delta]\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda t) \end{cases}. \quad \text{(A31)}$$

***(a)* When $0 < \alpha < \hat{\alpha}$,** it can be verified that $\begin{cases} \lim\limits_{e\to 0} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} > 0 \\ \lim\limits_{e\to +\infty} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} < 0 \end{cases}$ and $\begin{cases} \lim\limits_{t\to 0} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial t} > 0 \\ \lim\limits_{t\to T} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial t} < 0 \end{cases}$.

Thus, the optimal effort level and optimal initial testing time (e^*, t^*) can be determined by the equations $\begin{cases} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} = 0 \\ \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial t} = 0 \end{cases}$.

***(b)* When $\hat{\alpha} \le \alpha < 1$,** it can be verified that $\begin{cases} \lim\limits_{e\to 0} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} > 0 \\ \lim\limits_{e\to +\infty} \frac{\partial \pi_V^{TMR-CDE}(e,t)}{\partial e} < 0 \end{cases}$ and $\frac{\partial^2 \pi_V^{TMR-CDE}(e,t)}{\partial t^2} > 0$, $\pi_V^{TMR-CDE}(e, t=0) > \pi_V^{TMR-CDE}(e, t=T)$. Thus, the optimal effort level e^* is the solution for the equation $r - \frac{1}{c} + \frac{\beta e^{-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha)\cdot\lambda b - \alpha\delta] + \lambda a \cdot \left(1-\exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma}\cdot \ln\frac{\Upsilon B(c) - e^{\beta} + \sigma}{\Upsilon B(c) - e^{\beta} - \sigma} \end{Bmatrix} = 0$ and the optimal initial testing time $t^* = 0$.

Next, in the Client's contracting problem, when the IR constraint is binding, it achieves maximal profit, and the optimal initial payment P_{TM-CDE}^* can be derived. The higher monitoring policy ϕ indicates higher monitoring cost and lower Client profit, while the IC constraint $r \le \phi s$ should be satisfied. Thus, the IC constraint is binding and $r_{R-CDE}^* = \phi_{TMR-CDE}^* s$. According to (e^*, t^*) we have：

$$r^* = \frac{1}{c} - \frac{\beta e^{*-(1-\beta)}}{\lambda} \cdot \begin{Bmatrix} [(1-\alpha)\cdot\lambda b - \alpha\delta]\cdot\exp(-\lambda t) + \lambda a \cdot \left(1-\exp(-\lambda T)\right) \\ -\lambda b \cdot \exp(-\lambda T) + \frac{\alpha\cdot(\Upsilon + K(c))}{2\sigma}\cdot \ln\frac{\Upsilon B(c) - e^{*\beta} + \sigma}{\Upsilon B(c) - e^{*\beta} - \sigma} \end{Bmatrix}.$$

Substituting r^* into Equation (A26) yields

$$\pi_C^{TMR-CDE}(e) = \Upsilon T - \frac{\Upsilon + K(c)}{\lambda}\ln\frac{\delta + \lambda b}{\Upsilon + K(c)} + \frac{\delta + \lambda b}{\lambda}\cdot\left(\Upsilon B(c) - e^{\beta}\right)\exp(-\lambda T) - \frac{e}{c} - C_R$$
$$-a\cdot\left(\Upsilon B(c) - e^{\beta}\right)\cdot\left(1-\exp(-\lambda T)\right) - \frac{\Upsilon + K(c)}{2\sigma\lambda}\cdot\begin{bmatrix} \left(\Upsilon B(c) - e^{\beta} + \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} + \sigma\right) \\ -\left(\Upsilon B(c) - e^{\beta} - \sigma\right)\ln\left(\Upsilon B(c) - e^{\beta} - \sigma\right) \end{bmatrix}$$

$$-\frac{w}{s}\cdot\left\{\frac{1}{c}-\frac{\beta e^{*-(1-\beta)}}{\lambda}\cdot\left\{\begin{matrix}[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\exp(-\lambda t)+\lambda a\cdot\left(1-\exp(-\lambda T)\right)\\ -\lambda b\cdot\exp(-\lambda T)+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma}\cdot\ln\frac{\Upsilon B(c)-e^{*\beta}+\sigma}{\Upsilon B(c)-e^{*\beta}-\sigma}\end{matrix}\right\}\right\}. \quad (A32)$$

Based on the first-order condition in Equation (A32), we obtain a threshold for per unit cost of monitoring w_{E2}, which satisfies $\frac{\partial\pi_C^{TMR-CDE}(e)}{\partial e}|_{w=w_{E2},e=e^*_{FPR-CDE}}=0$. Thus, when $w\geq w_{E2}$, $r^*_{R-CDE}=0$, $\phi^*_{TMR-CDE}=0$ and $e^*_{TMR-CDE}=e^*_{FPR-CDE}$. Further, when $0<w<w_{E2}$, there exists an interior point $\hat{e}$ to satisfy $\frac{\partial\pi_C^{TMR-CDE}(e)}{\partial e}|_{e=\hat{e}}=0$, and $\frac{\partial^2\pi_C^{TMR-CDE}(e)}{\partial e^2}<0$. By substituting $e=\hat{e}$ into r^* yields $r^*=-\frac{1}{c}+\frac{\beta\hat{e}^{-(1-\beta)}}{\lambda}\left\{\begin{matrix}[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\exp(-\lambda t)+\lambda a\cdot\left(1-\exp(-\lambda T)\right)\\ -\lambda b\cdot\exp(-\lambda T)+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma}\cdot\ln\frac{\Upsilon B(c)-\hat{e}^{\beta}+\sigma}{\Upsilon B(c)-\hat{e}^{\beta}-\sigma}\end{matrix}\right\}$ defined as $\hat{r}$. Thus, if $\hat{r}/s<1$, then $\phi^*_{TMR-CDE}=\hat{r}/s$, $r^*_{R-CDE}=\hat{r}$, and $e^*_{TMR-CDE}=\hat{e}$; and otherwise $\phi^*_{TMR-CDE}=1$, $r^*_{R-CDE}=s$ and $e^*_{TMR-CDE}$ is the solution for the equation

$$s-\frac{1}{c}+\frac{\beta e^{-(1-\beta)}}{\lambda}\left\{\begin{matrix}[(1-\alpha)\cdot\lambda b-\alpha\delta]\cdot\exp(-\lambda t)+\lambda a\cdot\left(1-\exp(-\lambda T)\right)\\ -\lambda b\cdot\exp(-\lambda T)+\frac{\alpha\cdot(\Upsilon+K(c))}{2\sigma}\cdot\ln\frac{\Upsilon B(c)-e^{\beta}+\sigma}{\Upsilon B(c)-e^{\beta}-\sigma}\end{matrix}\right\}=0.$$

Finally, define $\Omega(w)=-\frac{s\cdot\frac{\partial\pi^{FBR-CDE}(e)}{\partial e}|_{e=e^*_{FPR-CDE}}+w\cdot\frac{\partial^2\pi_V^{FPN-CDE}(e)}{\partial e^2}|_{e=e^*_{FPR-CDE}}}{w\cdot\frac{\partial^2\mathrm{E}_\varepsilon[RS(\tilde{t}^*)]}{\partial e^2}|_{e=e^*_{FPR-CDE}}}$, $C_R^{E1}=\left(\pi_C^{TMR-CDE}+C_R\right)-\pi_C^{TMN-CDE}$ and $C_R^{E2}=\left(\pi_C^{FPR-CDE}+C_R\right)-\pi_C^{FPN-CDE}$. By comparing the Client's expected profit $\pi_C^{FPR-CDE}$, $\pi_C^{FPN-CDE}$, $\pi_C^{TMR-CDE}$ and $\pi_C^{TMN-CDE}$, Proposition 5 is derived. ■

Proof of Proposition 6 (Client Contract Choice under Endogenous Renegotiation Cost).

According to Proposition 5, we define two thresholds for per unit cost of monitoring, $w_{E3}=-\frac{s\cdot\frac{\partial\pi_S^{FBR-ERC}(e)}{\partial e}|_{e=e^*_{FPR-ERC}}}{\frac{\partial^2\pi_V^{TMN-CDE}(e)}{\partial e^2}|_{e=e^*_{FPR-ERC}}}$ and $w_{E4}=-\frac{s\cdot\frac{\partial\pi_S^{FBR-ERC}(e)}{\partial e}|_{e=e^*_{FPR-ERC}}}{\frac{\partial^2\pi_V^{TMN-CDE}(e)}{\partial e^2}|_{e=e^*_{FPR-ERC}}+\frac{\partial^2\mathrm{E}_\varepsilon[(1-\zeta)RS(\tilde{t}^*)]}{\partial e^2}|_{e=e^*_{FPR-ERC}}}$, and further define a threshold for Vendor bargaining power, $\alpha_4=-\frac{s\cdot\frac{\partial\pi_S^{FBR-ERC}}{\partial e}|_{e=e^*_{FPR-ERC}}+w\cdot\frac{\partial^2\pi_V^{TMN-CDE}}{\partial e^2}|_{e=e^*_{FPR-ERC}}}{w\cdot\frac{\partial^2\mathrm{E}_\varepsilon[(1-\zeta)RS(\tilde{t}^*)]}{\partial e^2}|_{e=e^*_{FPR-ERC}}}$.

When $\zeta\geq 1$, the renegotiation cost is higher than the renegotiation surplus. Thus, the Client does not renegotiate testing time after system development. By comparing the Client's expected profit $\pi_C^{FPN-CDE}$ and $\pi_C^{TMN-CDE}$ in Proposition 5, the Client's contract choice can be derived.

When $0<\zeta<1$, the renegotiation cost is lower than the renegotiation surplus. Thus, the Client selects renegotiation after system development. By substituting $\alpha\cdot(1-\zeta)\cdot RS(\tilde{t})$ for $\alpha\cdot RS(\tilde{t})$ as the Vendor's renegotiation profit, and $(1-\alpha)\cdot(1-\zeta)\cdot RS(\tilde{t})$ for $[(1-\alpha)\cdot RS(\tilde{t})-C_R]$ as the Client's renegotiation profit respectively, similar to Proposition 5, we obtain the Client's optimal profit $\pi_C^{FPR-ERC}$ and $\pi_C^{TMR-ERC}$. Comparing the Client's expected profit $\pi_C^{FPR-ERC}$ and $\pi_C^{TMR-ERC}$, its contract choice can be derived. ■

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