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Pension reform and wealth inequality: Theory and evidence[☆]

Torben M. Andersen [a], Joydeep Bhattacharya [b], Anna Grodecka-Messi [c], Katja Mann [d,*]

[a] University of Aarhus, Denmark
[b] Iowa State University, United States of America
[c] Sveriges Riksbank, Sweden
[d] Copenhagen Business School, Denmark

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ABSTRACT

A growing literature explores reasons for rising wealth inequality, but is mostly silent on the role of pension systems despite their well-understood influence on life-cycle savings. This paper develops a simple life-cycle model to lay bare the primary theoretical mechanisms connecting pension systems, asset accumulation, and the wealth distribution. Mandated fully-funded plans transform individuals with lower incomes, often characterized as low savers, into asset owners, and may also imply a more equal wealth distribution than pay-as-you-go-based systems. To test the empirical validity of these predictions, the paper explores a pension reform in Denmark, a country that witnessed declining wealth inequality over the last decades. In a calibrated life-cycle model employing unique register data, the Danish pension reform emerges as a key factor explaining the downward trend in wealth inequality.

1. Introduction

In recent decades, wealth inequality has displayed an upward trend in many countries (Zucman, 2019). The list of contributory factors includes increasing inequality in labor earnings, intergenerational transfers, heterogeneity in preferences and investment returns, and the role of different asset holdings across the wealth distribution.[1] Doubtless, these drivers are important. Nevertheless, somewhat surprisingly, the extant literature has been mostly silent on the role of the pension system and pension reforms in explaining wealth inequality, especially among the retired. This paper is an attempt to fill this void. It uses Denmark as a laboratory because (a) it has bucked the trend among OECD countries: wealth inequality (as we document below) has gone down, and (b) it has witnessed major pension reforms in recent decades.

Prototype pension systems are either a pay-as-you-go (PAYG) or a fully-funded (FF) scheme. In a typical PAYG scheme, the working generations contribute, and the receipts are distributed forthwith among current retirees. In a typical FF scheme, workers

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* Correspondence to: Department of Economics, Copenhagen Business School, Porcelænshaven 16A, 2000 Frederiksberg, Denmark

E-mail addresses: tandersen@econ.au.dk (T.M. Andersen), joydeep@iastate.edu (J. Bhattacharya), anna.grodecka.messi@riksbank.se (A. Grodecka-Messi), kma.eco@cbs.dk (K. Mann).

[1] Prominent, recent examples include (Piketty, 2014), De Nardi and Fella (2017), Hubmer et al. (2020), Fagereng et al. (2020), Bach et al. (2020) and Garbinti et al. (2021), among others.

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contribute to their individual account, and those savings, managed professionally by investment firms, are invested in the market, delivering a return post retirement. In recent decades, a transition to a multi-pillar architecture with an increased role for private pension funds and less dominance of PAYG public pensions, is underway in almost all OECD countries (OECD, 2019). This paper studies how such a transition impacts wealth inequality.

A transition from a mainly PAYG-based to a FF (or a hybrid) system has complex implications for the distribution of wealth. First, a PAYG scheme is part of an inter-generational social compact in which the state does *not* accumulate assets in anticipation of future pension claims but taxes the *then* working population to fund the pensions of the retired generation (Barr and Diamond, 2006). In contrast, a FF scheme is a method of accumulating financial assets on behalf of a participant. Put differently, a FF scheme involves an explicit wealth position whereas a PAYG scheme does not. Second, irrespective of the pension scheme, pension contributions and benefits enter the intertemporal budget constraint of agents and influence their retirement wealth-accumulation decisions. It is well established theoretically that both a PAYG and a mandated FF scheme will crowd out voluntary retirement saving (Andersen and Bhattacharya, 2011, 2021). For a PAYG scheme, as benefit generosity rises, the mass of people not saving actively for their own retirement rises. Overall, total retirement saving (or *retirement wealth*) declines. By contrast, in a FF scheme, the explicit saving mandate converts low income, possibly hand-to-mouth people, into savers; consequently, total retirement saving *increases*. Third, the return to participation in each scheme (and the assets held) are fundamentally different. In an economy that is not growing, the implicit rate of return offered by a PAYG scheme is the population growth rate; in contrast, the FF scheme offers the same return as the market, which, under dynamic efficiency, exceeds the population growth rate. Finally, while a PAYG scheme is usually re-distributive (transferring resources both within and across generations), a prototype FF scheme is not.

When the aforediscussed four mechanisms are taken in unison, it is readily apparent that a transition from a mainly PAYG-based to a FF (or a hybrid) system has complex implications for the distribution of wealth. We develop a simple two-period life-cycle model that lays bare these mechanisms connecting pension systems, asset accumulation and inequality. The upshot is low-and middle-income households participate in financial markets through their pension funds; at the same time, redistribution via the pension system gets muted with ascendancy of the FF scheme. These channels entangle the level *and* the distribution of wealth. When one adds to these mechanisms those that exist in the real world – means-testing, "bequeathability", and others – it is clear simple models cannot offer a definitive answer to the question of how a transition from a mainly PAYG system to a FF (or a hybrid) pension system influences wealth inequality.

To this end, we next turn to a life-cycle model calibrated to Denmark. Why Denmark? First, unlike most other advanced economies, Denmark has experienced a downward trend in wealth inequality over the last decades. Fig. 1 displays the evolution of the wealth Gini coefficient between 1986–2017 (panel a) and the shares of wealth held by the top 10% and bottom 50% of the distribution (panel b).[2] Second, over the last three decades, Denmark has undergone a major pension reform, diminishing the dependency on the PAYG segment of the welfare state while amplifying the prominence of occupational fully-funded, defined contribution schemes. As Fig. 2 shows, contribution rates for private sector employees have increased from near zero in 1992 to an average of about 12 percent from 2009 onward. Furthermore, the Danish administrative wealth registry data is of high quality and permits the direct observation of the market value of various wealth components, including pension wealth, for the entire population. This advantageous feature spares researchers from many complexities encountered when working in other settings, such as France (Garbinti et al., 2021).

We introduce a set of fresh empirical insights on wealth inequality in Denmark and the significant role played by pensions. Inequality metrics that cover various age brackets inherently show wealth disparities due to the distinct life-cycle stages experienced by the age groups. However, this effect dissipates when examining specific age cohorts. Therefore, in addition to presenting figures for the entire population, we focus our attention on the 60–69-year-old cohort as they approach retirement, where pensions emerge as their most substantial asset. We then develop a structural life-cycle model, calibrated to pre-transition Denmark, that is used to quantify the impact of the pension transition on Danish near-retirement wealth inequality. The model features a small open economy with long-lived overlapping generations, risky labor income, preference heterogeneity, uncertain lifespans and a borrowing constraint. We include preference heterogeneity because prior work, e.g. Hendricks (2007), Suen (2014), and Carroll et al. (2017), find discount factor-heterogeneity to be salient in the evolution of wealth[3] – also, unlike other drivers of wealth inequality such as bequest inequality or return heterogeneity, it is likely invariant to policy changes.[4]

Our findings suggest that the shift in the pension system in Denmark played a substantial role in the reduction of wealth inequality from 1992 to 2017. Therefore, a transition from a PAYG system to a multi-pillar structure has the potential to decrease wealth inequality. The model simulation closely aligns with the empirical data, accounting for nearly all observed trends. The decline in wealth inequality is primarily driven by increased savings among lower-income households.

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[2] We follow the by-now standard definitions used in Roine and Waldenström (2015), Alvaredo et al. (2018), Zucman (2019), and others. Wealth measure used in this paper corresponds to net wealth (assets–liabilities) without consumer durables including private pension assets net of taxes and is presented at the normalized (equal-split) household level.

[3] In this context, a valuable recent contribution is (Epper et al., 2020). They collate data from incentivized preference-elicitation experiments with Danish administrative data for roughly 3600 mid-life Danes to document time-discounting heterogeneity and its association with the wealth distribution. Charles and Hurst (2003) find significant parent–child similarities in savings behavior, indicative of preference-instilling behavior by parents. More recently, Cronqvist and Siegel (2015) using the Swedish Twin Registry data report 39% of the cross-sectional variation in wealth accumulated up to retirement is explained by genetic variation which includes preferences.

[4] Krusell and Smith (1998) were among the first to add heterogeneity in discount rates to a Bewley model. They find that small differences in patience across agents can create large heterogeneity in wealth.

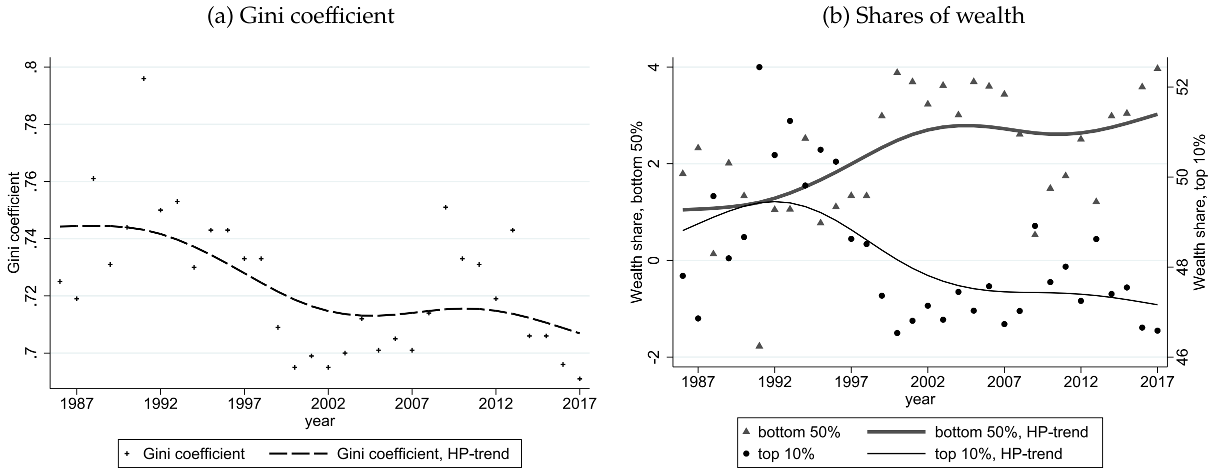

Fig. 1. Measures of wealth inequality for Denmark, 1986–2017.
Note: Wealth corresponds to net wealth without consumer durables including private pension assets net of taxes and is presented at the normalized (equal-split) household level. Self-employed and households with more than two adults are excluded. For the HP-filter, $\lambda = 300$ is used.

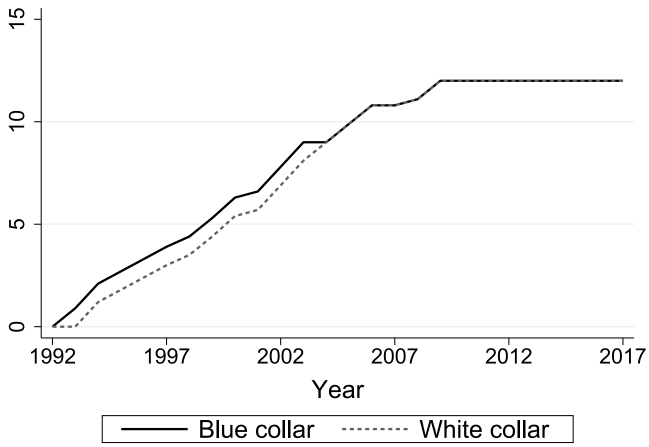

Fig. 2. Contribution rates for occupational labor market pensions in Denmark, 1992–2017.
Note: Contribution rates apply to DI/CO collective agreements. Since 2009, contribution rates have been identical for blue and white collar workers.
Data source: Finansministeriet (2017a).

In this context, a few remarks on the wealth definition are in order. In the main part of the paper, wealth constitutes "assets and liabilities in a narrow economic sense – comprising items that have an economic value and are subject to ownership rights" – see OECD (2013, p.54).[5] This is consistent with definitions used in national accounting and ensures aggregate wealth is consistent with national account numbers on aggregate savings. It is also the relevant wealth concept when analyzing who owns the capital stock and is entitled to its return. Our definition aligns with most of the international wealth inequality literature (see e.g., Roine and Waldenström, 2015; Saez and Zucman, 2016; Zucman, 2019). Broader measures of wealth could include the value of public pensions with the aim of measuring "consumption potential". For example, Greenwald et al. (2021) distinguish between financial wealth (wealth according to our definition) and total wealth, the sum of financial and human wealth (including PAYG pensions). The issue of what constitutes wealth in the context of pensions goes back to Feldstein (1976), who shows how marketable wealth, excluding social security, is more unevenly distributed than wealth including the present value of future pension benefits. More recently, Catherine et al. (2020) show that, contrary to popular views about widening U.S. wealth inequality over recent decades, once social security "wealth" is incorporated, wealth inequality has not moved much.

In evaluating consumption potential, a critical factor to consider is whether pensions can be front-loaded, which is contingent upon their eligibility as collateral for borrowing. In the United States, earnings-based PAYG payouts may be used as collateral

[5] In contrast, claims on social security are uncertain since the government can change the basis on which the entitlements are determined, e.g. in response to demographic aging.

and enjoy protection from creditors in cases of bankruptcy. However, this differs in Denmark. In Denmark, public pensions are not earnings-based; instead, they are designed to mitigate poverty and are safeguarded under bankruptcy law. On the contrary, occupational pensions are not protected by bankruptcy law (the income flow can be seized), so including them as part of wealth is relevant. Consequently, while a broader definition of wealth may be warranted in other nations, a more narrow definition (involving financial wealth) is deemed more appropriate for our specific focus on wealth and its distribution. Nevertheless, recognizing the growing interest in a broader wealth measure, in the final part of the paper we have undertaken a separate analysis wherein we estimate PAYG pensions for Danish households. Including the means-tested PAYG wealth in the extended wealth measure counteracts the decline in wealth inequality observed over time, leaving the Gini coefficient for all households between 1992 and 2017 roughly unchanged. For households close to retirement, bringing PAYG wealth into the picture more than halves the Gini coefficient compared to the benchmark net wealth measure, highlighting the redistributive role of social security for this age group. Over time, the extended wealth measure suggests an increase in wealth inequality for the retirees. Nonetheless, it is still true that the wealth holdings of households at the bottom of the wealth distribution increase, supporting the main channel through which the considered pension reform affected the wealth distribution.

Most prior work on the drivers of wealth inequality is focused on the U.S. experience. De Nardi and Fella (2017) survey the different determinants: transmission of bequests, entrepreneurship, medical expense risk, preference heterogeneity, rich earnings processes and heterogeneity in rates of return. Studying these through the lens of a Bewley–Huggett–Aiyagari model, they find bequests, entrepreneurship and medical-expense risk to be particularly important in the U.S. Hubmer et al. (2020), in a similar model set-up with heterogeneous, infinitely-lived agents, find that the drop in tax progressivity in the U.S. since the 1970s is the most important driver of the increase in wealth inequality. Hendricks (2007) show that discount rate heterogeneity can increase the Gini coefficient of wealth to levels close to the data and can explain the large wealth inequality observed among households with similar lifetime earnings. These findings have recently been confirmed by Carroll et al. (2017). Benhabib et al. (2019) estimate a macroeconomic model of the distribution of wealth in the U.S. with special emphasis on matching the upper tail. The extant literature is, however, mostly silent on the role played by the pension system. An exception is (Kaymak and Poschke, 2016), who introduce dynastic elements into a Bewley model and find that the expansion of the social security system in the U.S. contributed to increased wealth inequality through its impact on savings of low and middle income groups. It is noteworthy that the mechanisms highlighted in U.S.-focused research are limited in their ability to explain wealth inequality in Denmark. For one, medical expenses in Denmark are mostly covered by the state, and tax reforms have not played the same role as in the U.S. Also, wealth inequality in Denmark, like in Sweden (Domeij and Klein, 2002), is driven mainly by the large fraction of households with zero or near-zero net wealth rather than by high wealth of the top 1% or top 0.1%. Indeed, Jakobsen et al. (2020) document that, in years 1980–2012, the wealth shares of the top 1%, and top 0.1% in Denmark have stayed relatively stable compared to the U.S.

Our paper also relates to the large empirical literature documenting an incomplete crowding-out effect of pensions on private non-pension wealth. Most recent papers focus on social security, i.e. the expected future stream of PAYG income (Gale, 1998; Attanasio and Rohwedder, 2003; Attanasio and Brugiavini, 2003; Alessie et al., 2013; Lachowska and Myck, 2018). An exception is (Engelhardt and Kumar, 2011) who use a comprehensive measure of both public and employer-provided pension plans in the U.S. Chetty et al. (2014) consider the impact of defined-contribution employer pension plans in Denmark. Some papers document evidence that crowding-out increases with income (Engelhardt and Kumar, 2011) or education (Gale, 1998; Alessie et al., 2013; Lachowska and Myck, 2018). This mechanism is also present in our calibrated model, where we observe the richer (and higher-income) households to offset more than poorer ones due to different distance to the constraint. On average, the offset in our model is 0.442 kroner, i.e. a one-kroner increase in mandatory pension savings decreases voluntary savings of households by 0.442 kroner.

Our pursuit gains added significance within the backdrop of diminishing labor shares and escalating returns on capital. As hinted by Piketty (2014), there exist self-reinforcing dynamics wherein the affluent population saves more, amasses greater wealth, and consequently appropriates a larger share of capital returns. He has compellingly advocated for wealth taxation, albeit without delving into the role of pensions or pension reform.

The paper is organized as follows. Section 2 reviews main elements of the transition in the Danish pension system. Section 3 presents the basic analytical model and develops intuition for the different implications of both pension schemes for wealth accumulation and its distribution. Section 4 provides new stylized facts on Danish wealth inequality over the period 1986–2017 and the role of pension assets in the wealth distribution. Section 5 develops a rich life-cycle model calibrated to Denmark that allows us to quantify the importance of the pension reform for observed decrease in wealth in Denmark. In Section 6 we explore the implications of the broader wealth measure incorporating social security. Finally, Section 7 offers a few concluding remarks.

2. The Danish pension system

Over the last decades, the Danish pension system has undergone a major change from a mainly defined-benefit (DB) PAYG scheme towards a hybrid scheme with an important role for defined contributions (DC) funded occupational pensions (see Andersen, 2021). Unlike the U.S. and most other developed countries' social security system, the DB scheme in Denmark is tax financed and offers an universal entitlement – a flat-rate pension (common to all) and a means-tested supplement – to any resident reaching the statutory retirement age.[6] Prior to 1993, supplementary occupational pension schemes only existed for particular groups, primarily in the public sector and for the highly educated. In the late 1980s, an agreement was reached to broaden the coverage of occupational

[6] Offers additional details on the PAYG scheme.

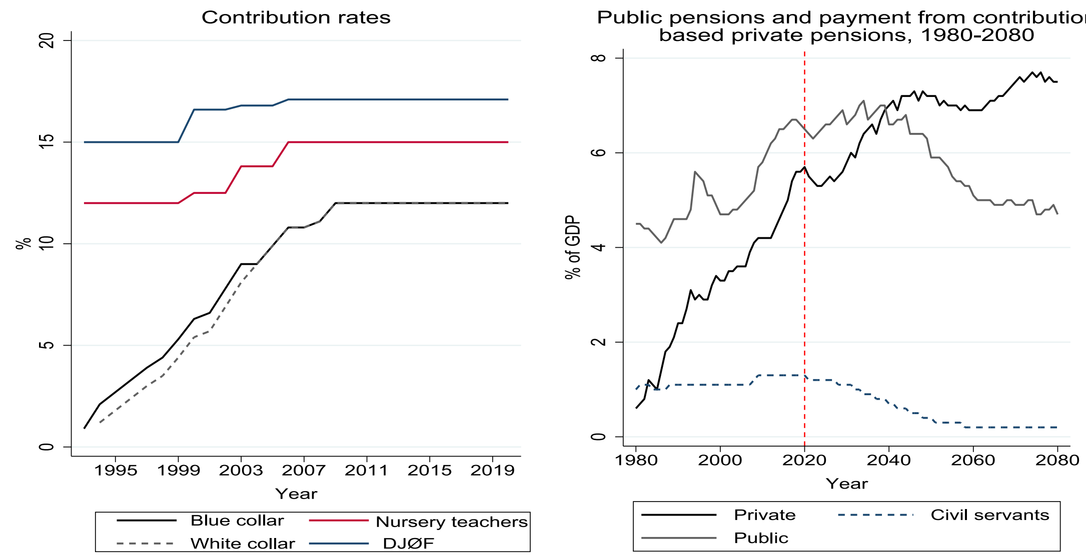

Fig. 3. Contribution rates, occupational labor market pensions, and total pension expenditures for public sector and contribution based pension funds.
Note: Contribution rates apply to DI/CO collective agreements. Since 2009 contribution rates have been identical for blue and white collar workers. Expenditures: data 2015 onwards are projected expenditures. Civil servants pensions are gradually phased out and of minor interest for this analysis.
Data source: Finansministeriet (2017a).

pensions to almost the entire labor market. These DC occupational pensions arose out of a concern that private savings were too low and while public pension amounts may forestall poverty among the retired, they may not be enough to support decent living standards (pension adequacy) since private retirement saving levels were too low. Importantly, the alterations made to the Danish pension system were not designed with the specific goal of diminishing wealth inequality.

Between 1993–2009, contribution rates increased substantially for private sector workers from close to zero to a level of about 12 to 15 percent in most occupations. Fig. 3(a) shows the evolution of contribution rates as shown in the Introduction, but supplemented by information of two groups of employees with traditionally higher contribution rates, nursery teachers (public sector) and lawyers and economists (DJØF). The increasingly dominant presence of FF pensions is also seen clearly from Fig. 3(b). The figure shows pension payments from contribution-based pension funds as a share of GDP; the increasing trend until about 2045 reflects the maturation of the scheme, as it still takes several decades for contributions at the politically desired level to apply to an entire work career and pensions benefits for the entire pension period to be received based on such contributions. By the middle of this century, the Danish system will be a hybrid with public PAYG-pensions mainly serving distributional objectives, and the occupational FF-pensions, insurance and replacement objectives, in accordance with the recommendations by the (World Bank, 1994).

Occupational pensions are widespread, and for the larger part determined in collective agreements and thus mandatory for the individual. The occupational pension funds are organized based on skill-groups, and have independent boards appointed by unions and employer organizations. The pension funds invest retirement savings in a wide range of risky and safe financial assets including liquid and non-liquids assets such as real estate. Some pension funds allow their clients an active decision regarding the riskiness of the portfolio, but most individuals stay with the default option. Between 2001–2019, the real annual return on retirement savings plans (subtracting taxes) was 3.7% (OECD, 2020). This means that the performance of these funds has been comparable to the market.

The coverage of the DC scheme is almost universal, covering about 85% of the Danish working population. Technically, these schemes are considered 'voluntary' because they result from collective agreements, but participation is mandatory for the individual worker. In addition, where bargained pensions do not apply, there are often occupational pension arrangements at the firm level. This creates a so-called residual group with no coverage which includes the self-employed and some individuals with a marginal attachment to the labor market. However, being quasi-mandatory, the Danish system differs from DC schemes in the U.S., like IRAs, which are voluntary.

Partly due to the expansion of FF schemes, Denmark has seen a steep increase in the aggregate household savings rate from near 5% in the 1980s to 18% in 2019. Consequently, substantial pension funds have been accumulated. Pension assets have been rising from around 50% of GDP in the 1980s to more than 200% of GDP in 2021, the highest among OECD countries (OECD, 2020).

The Danish system delivers replacement rates among the highest in the OECD and EU, see OECD (2019) and European Commission (2018a,b). The system is strong on poverty alleviation, it is financially viable and fiscally sustainable, which is also reflected in Denmark consistently being ranked at the top in the Melbourne Mercer Global Pension Index.[7]

[7] The index is based on sub-indices for adequacy, sustainability and integrity, and includes in total 40 indicators, see https://info.mercer.com/rs/521-DEV-513/images/MMGPI%202019%20Full %20Report.pdf.

3. Pension systems, saving and the retirement wealth distribution – the basics

We develop a simple, tractable model to understand several fundamental channels by which pension systems, PAYG or FF, influence variables such as saving, consumption, wealth, and inequality. To that end, we study a two-period, overlapping-generations model of a small open economy with no population growth, productive activity, risk, or bequests. Our objective is simply to elucidate the four principal mechanisms that interconnect pension systems, asset accumulation, and inequality, as highlighted in the introduction. For clarity's sake, many real-world features (means-testing, precautionary saving, etc.) are missing in the analytical model, but will appear in the calibrated model further below.

It is helpful to recall the four channels to facilitate a better understanding of the model below. First, a FF scheme accumulates financial assets on behalf of a participant while the PAYG scheme does not — retirees are paid by transferring resources from the working population in Denmark. Second, irrespective of the pension scheme, contributions, and benefits enter the intertemporal budget constraint of agents and influence their decisions. Third, absent population growth, the implicit gross rate of return on a PAYG scheme is 1, while the FF scheme offers the gross market return, $R > 1$ (under assumed dynamic efficiency). Finally, while a PAYG scheme is usually redistributive (transferring resources within and across generations), a FF scheme is not.

3.1. Primitives

New-born agents are indexed by i. They receive an endowment or earned income, w_i, when young (no endowment when old), and save for their retirement using a single asset with gross return R.

Preferences are given by $\Omega_i = u(c_i^y) + \beta_i u(c_i^o)$, where c_i^y and c_i^o is consumption of agent i when young and old respectively, $u(\cdot)$ has standard properties, and β_i is the subjective discount factor. Assume $\beta_i \in \left[\underline{\beta}, \overline{\beta}\right]$, with a cumulative distribution function $\mathcal{G}(\beta)$ and associated density function $g(\beta)$; also, $\beta_i < \beta_j$ for $i < j$, that is, the lower the index, the more impatient the agent.

While income and patience influence the distribution of savings, and hence wealth, they work in qualitatively similar ways. As such, we proceed assuming only one source of heterogeneity – in β – analytically the simplest. In Appendix B.1 we analyze the case of wage heterogeneity. The calibrated model will feature uncorrelated simultaneous heterogeneity in w and β.

3.2. Laissez faire

In a world without any policy intervention, i.e., **laissez faire** (LF), let s_i denote voluntary retirement saving by agent i; then, $c_i^y = w - s_i$, $c_i^o = Rs_i$, and $\left(c_i^y, c_i^o\right) \in \mathfrak{R}^2_{++}$. Denote optimal saving by $s^{LF} \equiv s^{LF}(\beta_i)$; then

$$-u_{c^y}(w - s^{LF}(\beta_i)) + R\beta_i u_{c^o}(Rs^{LF}(\beta_i)) \equiv 0.$$

First note, in the absence of income in the second period, standard assumptions on preferences imply everyone saves: $s^{LF}(\beta_i) > 0$ $\forall \beta_i$. Second, optimal retirement saving rises with patience $\frac{\partial s^{LF}(\beta_i)}{\partial \beta_i} > 0$ because of a stronger desire to smooth lifecycle consumption. Third, the distribution of patience induces a distribution in retirement wealth, χ_i with $\chi^{LF}(\beta_i) \equiv Rs^{LF}(\beta_i)$. In this context, notice $\frac{\partial s^{LF}}{\partial \beta_i} > 0 \Leftrightarrow \frac{\partial \chi^{LF}}{\partial \beta_i} > 0$, implying high-$\beta_i$ people have more retirement wealth.

3.3. PAYG pension system

Consider a stylized PAYG pension system in which each young agent faces a contribution, τ^P, and the proceeds are transferred forthwith to each of the existing retired as a lump-sum pension (b). Given a fixed population, the budget of the scheme is simply $\tau^P = b$. Notice, this implies the redistributive role of PAYG pensions is absent — it re-emerges under w-heterogeneity as discussed in Appendix B.1. Assume households cannot borrow using future pension benefits as collateral, that is, voluntary retirement savings must be non-negative, $s_i \geq 0$ (Andersen and Bhattacharya, 2011, 2021).

The agent budget constraints are reformulated as: $c_i^y = w - \tau^P - s_i$, $c_i^o = Rs_i + b$. Let $s_i^P \equiv s^P(\beta_i, \tau^P)$ denote optimal voluntary retirement saving (the P refers to the PAYG regime). Agents take τ^P and b as given. Clearly

$$-u_{c^y}(w - \tau^P - s^P(\beta_i, \tau^P)) + R\beta_i u_{c^o}(Rs^P(\beta_i, \tau^P) + b) \equiv 0,$$

and $s^P(\beta_i, \tau^P) \geq 0$ implies young agents are not permitted to borrow against b.

It follows that the equilibrium response of saving for non-constrained agents (after imposing $\tau^P = b$) satisfies $\partial s^P(\beta_i, \tau^P)/\partial \tau^P < 0$ implying the public pension crowds out voluntary retirement savings. Consequently, some people save nothing on their own ("zero saving corner") once a critical level of public pension payout is reached — see Andersen and Bhattacharya (2011, 2021). This, in no way, precludes agent i from saving for other reasons (such as precautionary or liquidity reasons that are absent here, but included in the calibrated model). Define a critical level of patience, $\beta^P\left(\tau^P\right)$, corresponding to the zero saving corner, $s^P = 0$, by $u_{c^y}(w - \tau^P) - R\beta^P u_{c^o}(\tau^P) \equiv 0$, where $\frac{\partial \beta^P(\tau^P)}{\partial \tau^P} > 0$. It follows that voluntary retirement saving for person i is

$$s^P\left(\tau^P\right) = \begin{cases} 0 \text{ for } \beta_i \leq \beta^P(\tau^P) \text{ (passive)} \\ > 0 \text{ for } \beta_i > \beta^P(\tau^P) \text{ (active)} \end{cases}.$$

For future use, we term those with zero voluntary savings, $s^P\left(\tau^P\right) = 0$, "passive" savers: they do not respond to an increase in pension generosity. The rest are "active" savers. Note also, $\partial s^P\left(\beta_i, \tau^P\right)/\partial \beta_i > 0$ for $\beta_i > \beta^P(\tau^P)$, meaning active savers with higher

β save more than their lower β counterparts. It can be shown that a more generous pension unambiguously *decreases* total savings – intuitively, as b rises, the cut-off, $\beta^P(\tau^P)$, rises, raising the mass of passive savers who do not save voluntarily. Additionally, active savers cut their private retirement saving. As for retirement wealth: since agents cannot borrow against b,

$$\chi^P(\beta_i,\tau^P) = \begin{cases} 0 \text{ for } \beta_i \le \beta^P \\ Rs^P(\beta_i,\tau^P) \text{ for } \beta_i > \beta^P \end{cases}$$

A quick comparison with laissez faire is in order. Under our assumptions, every agent in laissez faire privately saves a positive amount for retirement, $s^{LF} > 0 \ \forall \ i$. Under a PAYG system, passive savers save nothing, $s^P = 0 < s^{LF}$ $(\beta_i \le \beta^P(\tau^P))$ – active savers' savings are lower than in laissez faire. Overall, $s^P < s^{LF} \ \forall \ i$. Passive savers have positive retirement wealth in laissez faire; the same agents have none under the PAYG program. Active savers have positive but lower retirement wealth under the PAYG scheme. As a consequence wealth is lower, and more unequally distributed, in the PAYG case compared to laissez faire.[8]

3.4. FF pension system

Next, consider a fully-funded (FF) pension scheme which *mandates* the young to contribute τ^F to a personal pension account earning R per unit receivable when old. Clearly, there is no redistribution in this case.

Here, $c_i^y = w - \tau^F - s_i$, $c_i^o = Rs_i + R\tau^F$, and own interior retirement saving (the F refers to the FF regime), $s^F\left(\beta_i,\tau^F\right)$ is determined by

$$-u_{c^y}(w - \tau^F - s_i^F) + R\beta_i u_{c^o}(R\left(s_i^F + \tau^F\right)) \equiv 0.$$

Notice, in such a scheme, the household perceives a link between contributions made, τ^F, and pensions received, $R\tau^F$. Although the contribution is mandated, the account is personal. The capital, including the return, accrues solely to the individual, and hence is included in the computation of retirement wealth — given by $R\left(s_i^F + \tau^F\right)$.

As with PAYG payouts, continue to assume agents cannot borrow against their retirement savings $\left(s^F\left(\beta_i,\tau^F\right) > 0\right)$ held in the FF accounts. If $s^F\left(\beta_i,\tau^F\right) > 0$, it follows $\partial s^F\left(\beta_i,\tau^F\right)/\partial\tau^F = -1$: increasing the mandated contribution to the FF scheme crowds out voluntary saving one-for-one since the return on the two forms of retirement saving, mandated and voluntary, are the same — total retirement saving (mandated plus voluntary) does not budge. The mandate is, thus, ineffective in affecting total saving if $s^F > 0$; an effect arises only for those for whom the borrowing constraint binds. Define a cutoff $\beta^F \equiv \beta^F\left(\tau^F\right)$ by $-u_{c^y}(w-\tau^F)+R\beta^F u_{c^o}(R\tau^F) \equiv 0$. Then, voluntary retirement saving is

$$s^F\left(\beta_i,\tau^F\right) = \begin{cases} 0 \text{ for } \beta_i \le \beta^F(\tau^F) \text{ (passive)} \\ > 0 \text{ for } \beta_i > \beta^F(\tau^F) \text{ (active)} \end{cases},$$

and total (mandated plus voluntary) retirement saving is

$$s^F + \tau^F = \begin{cases} \tau^F \text{ for } \beta_i \le \beta^F(\tau^F) \\ s^{LF}\left(\beta_i\right) \text{ for } \beta_i > \beta^F(\tau^F) \end{cases},$$

It can be shown that aggregate retirement saving rises with an increase in the mandate. Also, $\frac{\partial\beta^F}{\partial\tau^F} > 0$. Finally, under a FF system, individual retirement wealth is given by

$$\chi^F\left(\beta_i,\tau^F\right) = \begin{cases} R\tau^F \text{ for } \beta_i \le \beta^F(\tau^F) \\ R\left[s^F\left(\beta_i,\tau^F\right) + \tau^F\right] = Rs^{LF}(\beta_i) \text{ for } \beta_i > \beta^F(\tau^F) \end{cases}.$$

Active savers under FF accrue the same voluntary retirement wealth as in laissez faire. Passive savers accumulate mandated wealth equal to $R\tau^F$ as compared to $Rs^{LF}\left(\beta_i\right)$. If $\tau^F > s^{LF}(\underline{\beta})$, where $s^{LF}(\underline{\beta})$ is the LF retirement saving of the least patient agent, then agents with $\underline{\beta} \le \beta_i \le \beta^F$ hold more wealth under FF than under LF. It follows that the FF scheme *decreases* retirement-wealth inequality relative to laissez faire: high-β agents (active savers) save the same as in LF, whereas low-β agents (passive savers) save more.

3.5. Comparison: the pension scheme and its effect on wealth and its distribution

Next, we compare the PAYG and the FF schemes assuming $\tau^P = \tau^F = \tau$. The split of households into active and passive savers depends on the cutoffs, β^F and β^P. The cut-off value is higher in FF case: $\beta^F > \beta^P$, that is, there are – evaluated for the same contributions – more passive households under the FF scheme than the PAYG scheme. This is because the FF scheme offers a higher explicit return (R) than the implicit return (here = 1) offered by the PAYG scheme, reducing the need to save.[9]

[8] Define the wealth share for agent i in laissez faire by $\phi^{LF}(\beta_i) \equiv \frac{s^{LF}(\beta_i)}{S^{LF}(\cdot)}$, and with a PAYG pension scheme $\phi^P(\beta_i,\tau^P) \equiv \frac{s^P(\beta_i,\tau^P)}{S^P(\tau^P)}$. Then the conclusion (that wealth is lower and more unequally distributed in the PAYG case compared to laissez faire) follows using that $\int_{\underline{\beta}}^{\overline{\beta}} \phi^P(\beta_i,\tau^P)g(\beta)\,d\beta = \int_{\underline{\beta}}^{\beta^P(\tau^P)} \phi^P(\beta_i,\tau^P)g(\beta)\,d\beta + \int_{\beta^P(\tau^P)}^{\overline{\beta}} \phi^P(\beta_i,\tau^P)g(\beta)\,d\beta = 1$ and $\int_{\underline{\beta}}^{\overline{\beta}} \phi^{LF}(\beta_i) = \int_{\underline{\beta}}^{\beta^P(\tau^P)} \phi^{LF}(\beta_i)g(\beta)\,d\beta + \int_{\beta^P(\tau^P)}^{\overline{\beta}} \phi^{LF}(\beta_i)g(\beta)\,d\beta = 1$. Since $\int_{\underline{\beta}}^{\beta^P(\tau^P)} \phi^P(\beta_i,\tau^P)g(\beta)\,d\beta = 0$ and $\int_{\beta^P(\tau^P)}^{\overline{\beta}} \phi^{LF}(\beta_i)g(\beta)\,d\beta > 0$, it follows that $\int_{\beta^P(\tau^P)}^{\overline{\beta}} \phi^P(\beta_i,\tau^P)g(\beta)\,d\beta > \int_{\beta^P(\tau^P)}^{\overline{\beta}} \phi^{LF}(\beta_i)g(\beta)\,d\beta$.

[9] Since $u_{c^y}(w-\tau) - R\beta^F(\tau)u_{c^o}(R\tau) \equiv 0$, and $u_{c^y}(w-\tau) - R\beta^P(\tau)u_{c^o}(\tau) \equiv 0$, and $R > 1$, it follows $\beta^F > \beta^P$.

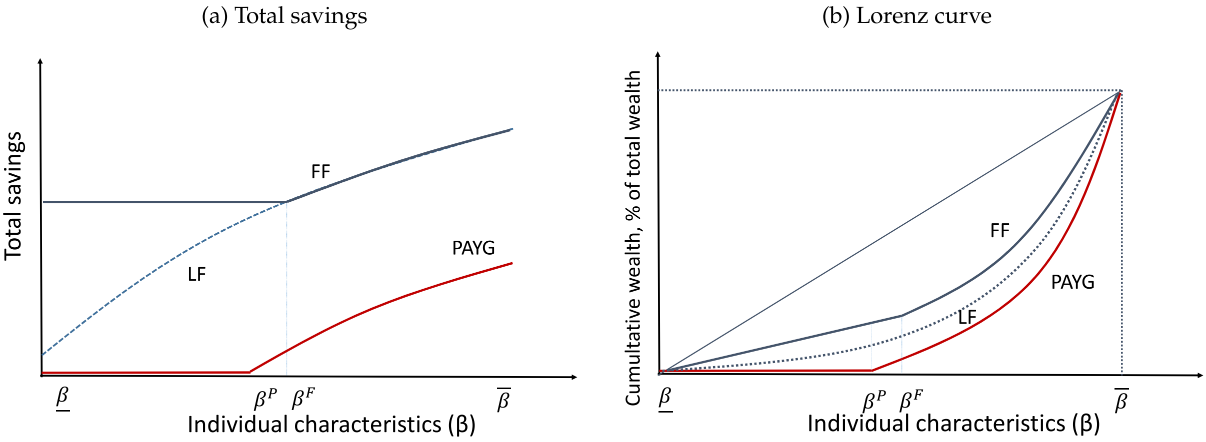

Fig. 4. Savings and Lorenz curves.

Proposition 1. *If $\tau > s^{LF}(\underline{\beta})$, then the ranking of retirement wealth under LF, FF, and PAYG schemes for person i is given by*

$$\chi^F(\beta_i,\tau) > \chi^{LF}(\beta_i) > \chi^P(\beta_i,\tau) = 0 \text{ for } \underline{\beta} \le \beta_i \le \beta^P(\tau) \text{ (P-passive, F-passive)},$$
$$\chi^F(\beta_i,\tau) > \chi^{LF}(\beta_i) > \chi^P(\beta_i,\tau) > 0 \text{ for } \beta^P(\tau) < \beta_i \le \beta^F(\tau) \text{ (P-active, F-passive)},$$
$$\chi^F(\beta_i,\tau) = \chi^{LF}(\beta_i) > \chi^P(\beta_i,\tau) > 0 \text{ for } \beta^F(\tau) < \beta_i < \overline{\beta} \text{ (P-active, F-active)}.$$

Fig. 4(a) shows individual savings by β_i under the three different regimes. The Lorenz curve is illustrated in Fig. 4(b). The pension system affects both the level of wealth and its distribution. Under PAYG, total wealth decreases, the wealth share for low β-types decreases (compared to both the LF and the FF case) because they save nothing voluntarily. However, for high β-types, the comparison of wealth shares is ambiguous: they have lower savings due to the crowding out, but total savings are also lower (both the denominator and nominator in the ratio change.) In general, the respective Lorenz curves may cross for high levels of β. However, one thing is clear: the wealth share of "low savers" (here, low patience) under FF is higher compared to both laissez faire and PAYG.[10]

3.6. Discussion

The analytical model discussed above was strictly barebones, intended solely to develop intuition in a clean, simple setup. That leaves several caveats which must be recorded.

First, as stated, heterogeneity in β removes the distributive function of PAYG schemes. With income (w) heterogeneity, the PAYG system is redistributive: households contribute according to their income but receive a transfer based on the economy's mean income. Hence, low-w households are net gainers and high-w households are net-contributors to the scheme. In Appendix B.1, we show that the results in the section above continue to hold under w-heterogeneity.

Second, the model considered the two pension systems in isolation while in practice they often coexist — the PAYG arm targets old-age poverty prevention (redistribution) and the FF arm targets consumption smoothing (replacement rates) over the life-cycle. In the Danish case, some cohorts started facing FF mandates even as they were paying into a PAYG scheme (which was left mostly unchanged). In Appendix B.2 we show that introducing a FF scheme atop a pre-existing PAYG scheme generates qualitative results similar to increasing the FF pension analyzed in the analytical model. Clearly, other pensions reforms – PAYG pensions are reduced and FF pensions increased – may give different results.

Next, we explore the factual implications of transitioning from mostly PAYG to a hybrid pension system with a more dominant role of the FF scheme by examining the Danish data in Section 4. We document that the shift in the pension system was accompanied by a considerable decrease in wealth inequality among Danish households. The structural model in Section 5 confirms that the pension reform played a dominant role in generating the observed pattern of wealth inequality.

4. Stylized facts on wealth and its distribution in Denmark

4.1. Data sources and definitions

Several datasets from *Statistics Denmark* are merged to obtain an annual longitudinal panel with information on wealth, earnings, household composition, etc., for all Danish households. The databases and variables used in our analysis are described in Appendix

[10] Compared to LF, the wealth share also unambiguously decreases for "high savers" (here, high β).

C. We pay particular attention to years 1992 and 2017. 1992 is a benchmark, pre-transition year and is chosen because from 1993, contribution rates for private-sector employees started to increase, see Fig. 2(a). 2017 is the last year in our sample, and contribution rates have reached their permanent level this year. From 2014 on, *Statistics Denmark* provides a new database on households' assets and liabilities. This database allows for a straightforward construction of various net and gross wealth measures, including person-specific pension assets for the period 2014–2017. Before 2014, wealth was subject to slightly other definitions coming from a different dataset. Individual pension asset data are available only from 2014, and pension assets for previous years are imputed as described in later parts of this section.

Throughout, net wealth is defined to include real and financial assets (encompassing pension assets net of taxes but not consumer durables such as cars) minus liabilities. This definition follows a vast international literature as described in Appendix C.2 that discusses wealth measures used in other studies. Wealth is considered from the households' perspective, as savings decisions are often made at the household, and not the individual level. The sample is restricted to individuals above twenty years of age, excluding those above that age who live with their parents. We aggregate household wealth by considering assets owned by each household member and create a normalized (equal-split) household measure. The aggregate household wealth is divided between adult members of the household. Moreover, we attribute specific household head characteristics, such as age, to the entire household. The household head is defined as the man in mixed-sex households. We keep the original household head definition provided by *Statistics Denmark* in the case of same-sex households.[11] The data excludes self-employed people because they do not pay into the mandatory pension scheme. Therefore, their savings are not comparable to the rest of the population. It also excludes households with more than two adult members due to the difficulty of attributing wealth items correctly in the case of households with more than two adults.[12] Henceforth, when using the term household, we refer to the equal-split household (adult individual) measure, which is also the basis for the calibration of our life-cycle model.

4.2. The role of pension assets in the asset portfolio of Danish households

We start by presenting some basic facts using the most recent and reliable data on wealth holdings of Danish households. Fig. 5 presents cross-sectional wealth holdings of Danish households in 2017. Compared to 1992, we observe a large increase in the level of wealth, consistent with the theoretical predictions, see Figure C.3 in Appendix C.3 and Table 3. Here and in the following graphs, numbers are presented in constant 2015 DKK. Panel (a) visualizes the asset and debt holdings of the mean and panel (b) of the median household.[13] For the purpose of this graph, the Danish households are grouped into ten-year age bins, where the first bin covers households with a head between 20 and 29 years old, etc., and we restrict the sample to households whose head is younger than 100 years. The life-cycle pattern of asset and debt holdings across age groups among Danish households resembles the one known from other countries (see Cowell et al., 2012, for a comparative study).

Younger households typically have relatively modest net wealth, as they simultaneously accumulate assets and incur debt, particularly until they reach their 40s. The majority of this debt is typically related to housing, primarily in the form of mortgages. The peak of assets and net wealth is generally observed among households in the 60–69 age bracket. Post-retirement, households tend to retain their wealth holdings, including housing assets, albeit to a somewhat lesser extent. Pension assets typically reach their pinnacle for households in the 60–69 age range, following which most households transition into retirement and commence drawing on their pension wealth. Even among households in the oldest age category, a significant amount of assets is typically retained.[14] A comparison between the mean and median values indicates that the distribution is right-skewed, as the median household in the age group 90–99 holds relatively less net wealth than the mean household at the end of the lifetime. It is also visible that for the median household in 2017, private pension assets play a dominant role in their asset portfolio, particularly near retirement age.

The role of pension assets in the asset portfolio may be visualized with Lorenz curves for wealth presented in Fig. 6. The left side of the graph presents accumulated net wealth shares held by each wealth decile including (solid line) and excluding (dashed line) pension assets for the entire population in 2017. Analogous Lorenz curves are presented for the group of 60–69 year olds on the right hand side of the graph. We focus on that group for several reasons. The Gini coefficient across all age groups displays wealth inequality mainly because age groups are in different phases of the life-cycle. This effect is eliminated by considering specific age-groups, and 60–69 group is relevant since people start to retire in their 60s in Denmark. Moreover, private pension assets peak for households in this age group. If we do not include pension assets in the calculation, accumulated net wealth is negative for the first six (all age groups) and four (age group 60–69) wealth deciles, respectively. When private pension assets are included, both curves approach the 45-degree line, indicating lower wealth inequality as compared to the measure without pension assets.

Fig. 6 thus shows how important pension assets are for the wealth distribution in Denmark, but also highlights differences across age groups.[15] Some of this is because they were subject to different pension policies over the last decades.

[11] Alternatively, we could define the household head as the primary earner. This alternative definition resulted in similar wealth distributions at the household level as in the current paper.

[12] The overall conclusions are the same when considering all households, see Appendix C.4 and Andersen et al. (2022).

[13] Due to the data restrictions from Denmark Statistics, the 'median' graph is based on the average asset and debt holdings of 100 households in a given age group with net wealth closest to the median value.

[14] This pattern is also present when focusing on households without children (see Figure C.4), which suggests that it is not driven – or at least not exclusively – by a bequest motive.

[15] Apart from age, one can also look at asset holdings from the respective of net wealth deciles. Figure C.5 in Appendix C.3 illustrates that private pension assets dominate the portfolios of the lowest-ranked households. The role of private pension assets declines with the increase in net wealth decile and is the lowest for the most wealy Danish households. Top wealth deciles have higher voluntary savings reflected in holdings of stocks. Housing is one of the two (aside from private pensions) most important asset categories for all households, but plays a relatively minor role for the top percentiles.

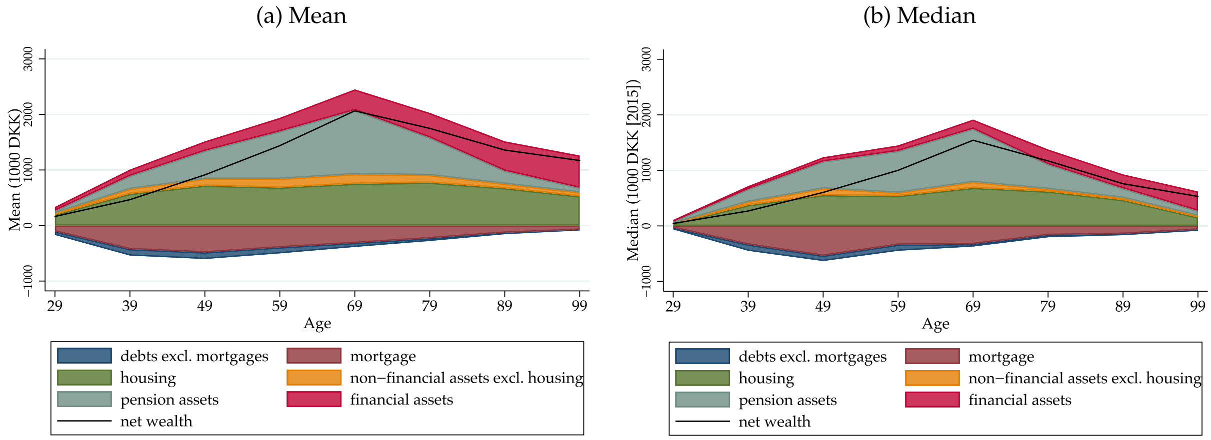

Fig. 5. Cross-sectional wealth holdings of Danish households, 2017.
Note: Wealth is presented at the normalized (equal-split) household level. Self-employed and households with more than two adults are excluded.

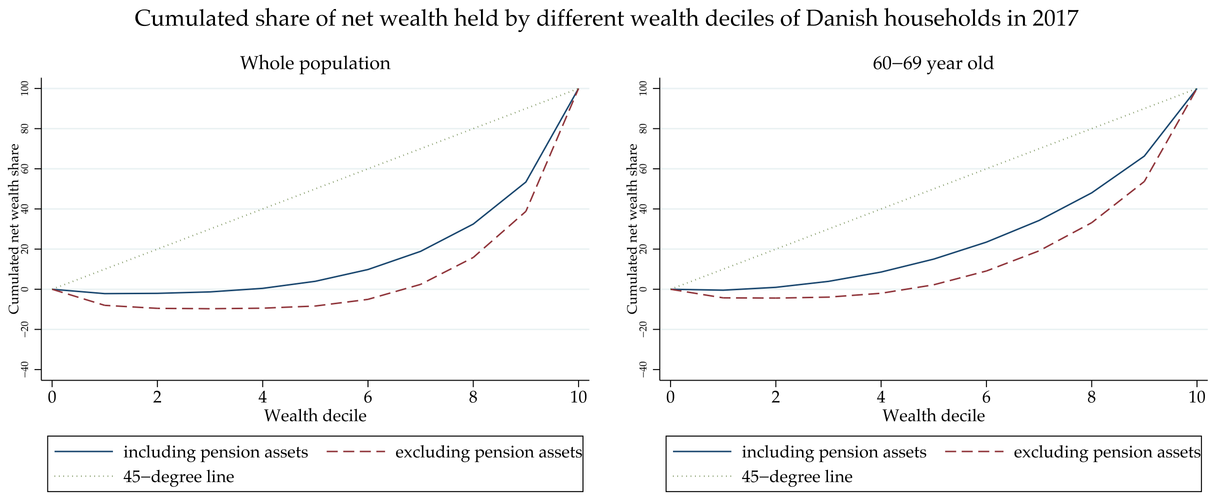

Fig. 6. Lorenz curve for net wealth, 2017.
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. Self-employed and households with more than two adults are excluded.

4.3. Long-run trends in Danish wealth inequality

To look at long-run trends in net wealth, we impute pension assets for those years in the sample where we lack this data.[16] In our imputation, we slightly modify the method used in Jakobsen et al. (2020) — aggregate private pension assets are attributed to individual agents based on their share of income and contributions to private pensions. Details on the imputation are available in Appendix C.1.1.

We start by comparing shares of net wealth (including private pension assets) held by different deciles between 1986–2017. As shown in Fig. 1(b) in the introduction, the share held by the top 10% of the distribution is decreasing over time.[17] The bottom 50% of the Danish households hold net wealth near zero or even negative in some periods. In the last years, the share of wealth held by these households is increasing. Our analytical model suggests a PAYG to FF transition increases the wealth of “low savers”. Fig. 7 presents the evolution of net wealth shares held by the first five wealth deciles of households in 1986–2017. The dashed vertical

[16] Figure C.6 in Appendix C.3 shows the overlap of our imputed series with the data in 2014–2017. Our imputation matches the aggregate wealth shares held by specific wealth deciles quite well.

[17] Even though our paper does not aim to understand the wealth held by the most wealthy, the evolution of the top 1 and 0.1% shares over time is shown in Appendix C.3 in Figure C.7. These shares, too, have decreased substantially compared to the early 1990s. Our imputation matches the top wealth shares well. Figure C.8 in Appendix C.3 shows the overlap between the imputed and combined series in years 2014–2017. Our conclusions hold once we include self-employed households in the sample.

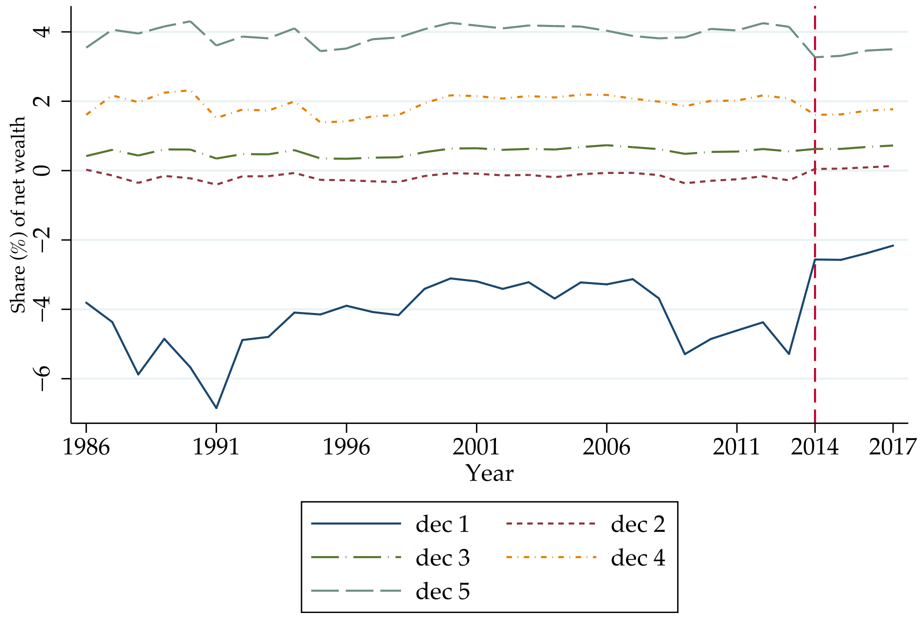

Fig. 7. Share of net wealth held by the lowest wealth deciles of Danish households in 1986–2017.
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members from the sample.

line in 2014 separates our imputed measure from the most recent data using the new wealth dataset. Evidently, the increase in the wealth share of the bottom 50% – Fig. 1(b) – is mostly driven by the lowest three wealth deciles, in particular the lowest one. In this sense, our model predictions find support in the data.[18]

This development is even more evident when considering different wealth inequality measures for 1992 and 2017 as documented in Table 1. The table shows numbers for the whole population and corresponding numbers for households where the household head is in the age group 60–69. The Gini coefficient for our sample in 1992 was 0.75 and 0.69 in 2017 (the trend over time is visualized in Fig. 1(a) in the introduction). For households with the household head near retirement, the drop in the Gini coefficient was even more notable, by 18%. Alternative measures of wealth inequality such as the general entropy index (2), as well as ratios less sensitive to values close to zero (P80/P20 and P75/P25) indicate an even more pronounced decline in wealth inequality in Denmark between 1992 and 2017.[19]

In sum, the developments are in accordance with our theoretical predictions. A larger role for FF pension should both increase overall saving and wealth accumulation, as well as wealth shares held by the poorest part of the population. The transition towards a more funded pension scheme was accompanied by a notable decrease in Danish wealth inequality across different measures. Equipped with these new stylized facts about wealth inequality and asset accumulation of Danish households over the life-cycle, we proceed with the presentation of a life-cycle model which offers a lens with which to view these massive changes.

5. A quantitative life-cycle model of the Danish economy

We formalize the transition in the Danish pension system in an otherwise standard life-cycle model wherein overlapping generations of long-lived agents live through work and retirement phases and save optimally over their life-cycle. Inequality between agents within a cohort is generated in two ways: via heterogeneous time preferences and via transitory and permanent shocks to labor income. The model features uncertain lifespan and time-varying demographics. All factor prices are assumed exogenous capturing the small, open economy status of the Danish economy.

To quantify the effect of the Danish pension reform on the wealth distribution and life-cycle savings, we run annual simulations between two steady states: 1970 and 2100 (by which time, the pension system has fully matured). The model is calibrated to the Danish economy in 1992, the year prior to the pension reform. Outcomes for individual wealth and its distribution in 2017 are then compared to the data.

[18] Our conclusions hold once we include self-employed households in the sample. See Appendix C.4 and Andersen et al. (2022).

[19] The ratio P80/P20 shows the ratio of household wealth at the 80th percentile divided by wealth at the 20th percentile, and P75/P25 shows the ratio of household wealth at the 75th percentile to that at the 25th percentile.

Table 1
Wealth inequality measures in Denmark, 1992 vs 2017.

	All households	Head 60–69 years old
	Gini coefficient	
1992	0.75	0.63
2017	0.69	0.51
% Δ 1992–2017	−8%	−18%
	GE(2)	
1992	11.23	2.42
2017	3.73	0.06
% Δ 1992–2017	−67%	−64%
	P80/P20	
1992	121.78	35.93
2017	6.57	2.82
% Δ 1992–2017	−95%	−92%
	P75/P25	
1992	29.12	17.64
2017	19.01	4.91
% Δ 1992–2017	−35%	−72%

Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members from the sample.

5.1. Model set-up

Demographics

Time is discrete and denoted by $t = 1, 2, \ldots, \infty$. At each date t, agents are born at age $n = N^b$ and start work immediately. They may live to a maximum age of N^d and survival until this point is stochastic: $\delta_{n,t}$ is the probability with which an agent of age $n-1$ survives until age n in year t. Agents retire at a fixed age of N^r. In the following, we drop the time index for the ease of notation.

Preferences

Agents have expected lifetime utility

$$\Omega_{0,i} = \mathbb{E}\left[\sum_{n=N^b}^{N^d} \beta_i^{n-N^b} \tilde{\delta}_n \frac{C_{i,n}^{1-\vartheta}}{1-\vartheta}\right], \tag{1}$$

where $C_{i,n}$ is consumption of agent i at age n, ϑ is the CRRA parameter, and $\tilde{\delta}_n$ is the cumulative survival probability between periods N^b and n, $\tilde{\delta}_n = \prod_{m=N^b}^{n} \delta_m$, which is 0 at age $n = N^d$.

Time preferences are idiosyncratic (but fixed over an agent's lifetime), with β_i denoting the individual-specific discount factor. Heterogeneity in β_i not only represents heterogeneity in patience, but is also a catch-all for other factors that differ across people, e.g. differences in investment opportunities, tax schedules, risk aversion or income growth expectations. We model heterogeneity in time preferences by assuming a uniform distribution for $\beta_i \in \left[\underline{\beta}, \bar{\beta}\right]$, from which agents draw at the beginning of their lives.

Labor income process

All labor income is generated exogenously. As is common in the life-cycle literature (going back to Zeldes, 1989 and Carroll, 1992, for example), we write labor income (after taxes and contributions) for agent i of age n as

$$Y_{i,n} = P_{i,n}\epsilon_{i,n} \qquad \text{if} \quad n < N^r, \tag{2}$$

with $P_{i,n}$, a permanent income component, defined as

$$P_{i,n} = G_n P_{i,n-1}\eta_{i,n}. \tag{3}$$

G_n is an age-specific and time-varying component of income, $\epsilon_{i,n}$ is an idiosyncratic, transitory income shock with $\ln \epsilon_{i,n} \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and $\eta_{i,n}$ is a permanent income shock with $\ln \eta_{i,n} \sim \mathcal{N}(0, \sigma_\eta^2)$, uncorrelated with $\epsilon_{i,n}$. In this way, the labor income process generates both precautionary and life-cycle saving consumption smoothing saving motives.

Pension system

Reflecting the reality in Denmark, as described in Section 2, the PAYG pillar consists of a flat-rate scheme, providing a basic amount to everybody, and two means-tested supplements, which can be collapsed into one without loss of generality. Denoting FF pension annuities by $\tilde{Y}_{i,n}^{FF}$, PAYG income is

$$\tilde{Y}_{i,n}^{PG} = \theta_0 + \max\left\{0, \theta_1 - \max\left(\kappa\left(\tilde{Y}_{i,n}^{FF}\right), 0\right)\right\} \quad \text{if} \quad n \geq N^r, \tag{4}$$

where θ_0 and θ_1 are DKK values of the flat-rate scheme and the maximum amount to be obtained under the means-tested scheme, respectively. Both are time-dependent. $\kappa(\tilde{Y}_{i,n}^{FF})$ is a positive function of the labor market pension income introducing a negative relationship between private and public pension payouts. We abstract from public pension contributions. The Danish pension system is tax-financed, so the public pension scheme itself does not have to adhere to a balanced budget. Defining $Y_{i,n}$ as after-tax income, changes in the financing costs of the PAYG pension system along the transition path will show up as a change in the time-varying element, G_n, which we estimate.

Agents contribute a fraction $\pi_{i,n}$ of their labor income into the mandatory FF pension scheme, so that $(1-\pi_{i,n})Y_{i,n}$ is disposable income net of taxes and contributions. As labor supply is exogenous, so are pension contributions. We specify $\pi_{i,n}$ as $\pi_{i,n} = \bar{\pi} + \pi(Y_{i,n})$ where $\bar{\pi}$ is a positive trend, the essence of the pension reform, and $\pi(Y_{i,n}) > 0$ captures the empirical observation that contribution rates are higher for occupations with higher salaries. Pension assets are administered by pension funds and cannot be accessed pre-retirement. Contributions are invested in the financial market and earn an exogenous gross market rate of return R. It follows, just before retiring, agents hold end-of-period pension assets

$$LW_{i,N^r-1} = \sum_{n=N^b}^{N^r-1} \pi_{i,n} Y_{i,n} R^{N^r-n} \tag{5}$$

FF pensions are paid out as annuities.[20] The annuity paid out at the beginning of each age $n \geq N^r$ is

$$\tilde{Y}_i^{FF} = LW_{i,N^r} \left(\sum_{m=N^r}^{N^d} \frac{\prod_{k=m}^{N^d} \delta_k}{\mathbb{E}(R^{m+1-N^r})} \right)^{-1}. \tag{6}$$

The annuity payment equals the accumulated pension wealth at the point of retirement adjusted for the probability of surviving until each age and inversely discounted at rate R. Note the risk diversification of individual mortality implied by such life-annuitization schemes. The FF pension wealth of agents dying prematurely gets implicitly added to the accounts of the survivors within the same cohort ensuring payment as long as the saver is alive.

Private saving

In addition to mandatory pension savings, agents can decide to accumulate private savings. They will do so for precautionary reasons and to smooth consumption over their life-cycle. We denote end-of-period accumulated voluntary savings by $A_{i,n}$. Households invest into a single, safe asset, which yields a return of R, the same as the mandatory pension savings. Although empirically relevant, we do not explicitly model housing. Housing is an illiquid asset, and most housing in Denmark is owner-occupied (of course, not all households are home-owners) indicating it is held not just for investment reasons but also for consumption purposes. This is why it is often excluded in models of life-cycle savings decisions (see e.g., De Nardi and Fella, 2017). Additionally, because borrowing takes place mainly through mortgages, we abstract from it by introducing a zero-borrowing constraint, $A_{i,n} \geq 0$.

The budget constraint of a working-age individual therefore reads

$$C_{i,n} + A_{i,n} \leq A_{i,n-1}R + (1-\pi_{i,n})Y_{i,n} \qquad \text{if} \quad n < N^r. \tag{7}$$

Retired agents receive an annuity out of their funded pension wealth as well as public pensions. The budget constraint is

$$C_{i,n} + A_{i,n} \leq A_{i,n-1}R + \tilde{Y}_{i,n}^{PG} + \tilde{Y}_{i,n}^{FF} \qquad \text{if} \quad n \geq N^r. \tag{8}$$

Agents choose their voluntary private asset holdings $A_{i,n}$ such as to maximize lifetime utility Eq. (1) subject to Eqs. (7) and (8). The solution to the optimization problem can be expressed by the Euler equation

$$C_{i,n}^{-\vartheta} \geq \beta_i \delta_{n+1} R\, \mathbb{E}\left[C_{i,n+1}^{-\vartheta}\right] \tag{9}$$

which holds with equality when the borrowing constraint is non-binding, i.e., $A_{i,n} > 0$. Appendix D.1 provides a recursive formulation and defines the equilibrium.

Dying agents leave accidental bequests. These are completely taxed by the government to fund activities outside the model. In a model extension with bequest motive in Section D.4, we also introduce intentional bequests, which get allocated to the young agents.

5.2. Calibration and simulation

The phasing-in of the fully funded pensions is modeled as a transition from an initial, pre-transition steady state (1970) to a new, post-transition steady state (2100). This allows us to capture time trends in labor income in the 1970s and 1980s, important for achieving a good model fit. The long transition period reflects that the mandated pension system will take full effect only when all living individuals have contributed at the permanent level of contribution rates throughout their entire life cycle and draw on these savings. We are most interested in 1992, for which the calibration targets Gini coefficient of the 60–69 year olds and the peak life-cycle total assets, and 2017, the post-reform year.

[20] In reality, a smaller part of the pensions is paid out as periodic installments and lump sum payments, but we abstract from these to keep the model simple. As we are interested mainly in the wealth of the near-retirees, the payout mode is of little consequence to our results.

The model is solved using endogenous gridpoint method (Carroll, 2006). We simulate the transition between the two steady states by performing annual simulations (see Appendix D.2 for details). Agents are assumed to have perfect foresight and know the entire future path of pension contribution rates. To generate a distribution, the model is simulated for 22 million independent draws of income shocks.

Calibration

Labor income process: We apply the methodology of Cocco et al. (2005), described in detail in Appendix C.5, and use registry data from 1987–2017. We regress the logarithm of equal-split after-tax labor income on age fixed effects and a set of demographic control variables, and fit a fifth-order polynomial. A time trend is included to capture aggregate income growth. The error structure of the labor income process is determined by a variance decomposition (Carroll and Samwick, 1997).

FF pensions: We limit the contribution rates to take a value between 0 and 0.18 to reflect the reality in Denmark. For $\bar{\pi}$, we use average contribution rates for the individuals in our benchmark sample. Between 1992 and 2009, the contribution rate in the data increases from 5.2% to 10.9%. We calibrate $\pi(Y_{i,n})$ to match peak FF pension assets in 1992.[21]

PAYG pensions: The parameters θ_0, θ_1 and $\kappa(\tilde{Y}^{FF}_{i,n})$ are calibrated to match PAYG replacement rates over time from Finansministeriet (2017b). The Danish Ministry of Finance documents a hump shape for replacement rates between 1960 and the 2000s, which we approximate by fitting a third-order polynomial for θ_1.

Time preferences: The uniform distribution of β_i is approximated by an equi-distant grid following (Carroll et al., 2017). Our grid is defined as

$$[b-3d \quad b-2d \quad b-d \quad b \quad b+d \quad b+2d \quad b+3d]$$

We assume an equal probability mass at each grid point and calibrate b and d jointly to match two data moments for our benchmark sample in 1992: the peak of non-pension cross-sectional assets of a representative household, where we set income shocks to their expected values and average across β-types, and the Gini coefficient for the 60–69 year olds. This results in a discount factor $\beta_i \in [0.9189, 0.9891]$. The dispersion is slightly larger than in other macroeconomic papers (e.g., Krusell and Smith, 1998, Carroll et al., 2017 and Hendricks, 2007) for two reasons. First, we consider a life-cycle framework wherein a larger difference in patience is typically needed to generate wealth inequality compared to models with infinitely-lived agents. Second, our agents are ex-ante identical in their labor income trajectories. While the income shocks they face in their lives will render the agents different ex-post, this mechanism generates less income inequality than models where agents start with innately different levels of permanent income. For example, Carroll et al. (2017) consider three educational groups with different income trajectories. For the data on Denmark, a specification with homogeneous initial permanent income is more suitable as it results in a good match for the income Gini coefficient. The behavioral literature tends to find quite large differences in discount factors, closer to our values – see e.g., Harrison et al. (2002), Andersen et al. (2014), and Epper et al. (2020) for estimates for Denmark.

Demographics: In addition to using external sources for survival probabilities, we use the actual cohort size by year from the Danish registry data (benchmark sample) for the calculation of distributional measures. This has the advantage of reflecting migration patterns and making the model results directly comparable with the data.

The retirement age, N^r, is set to 65. This has been the statutory retirement age between 2004–2018. Prior to that, it was 67, and from 2019, it is gradually increasing again. Given that the actual retirement age tends to be below the official one, 65 matches the data well.

Other parameters The coefficient of relative risk aversion ϑ is set to 0.65 based on an estimate by Andersen et al. (2014) for Denmark. This relatively low number may be justified by people expecting to rely more on home production post retirement. Denmark is also characterized by a generous welfare system, which implies that Danes do not need to precaution against negative health or unemployment shocks to the same extent as e.g. in the United States. The return on assets R corresponds to the real return on retirement savings plans during 2001–2019 (OECD, 2020) subtracting income tax and tax on returns.

Table 2 summarizes the parameter values.

Goodness of fit

Table 3 provides a comparison between empirical moments and the predicted values from the model. For the targeted moments, the fit between model-implied moments and data is very good. We match both the moments pertaining to the average or representative agent – peak pension and non-pension assets – and the Gini coefficient of the group closest to retirement, the 60–69 year olds, very well.

In the middle part of the table, we assess the performance of our model on a range of non-targeted moments for 1992. Our model should closely match these, as they capture important features of the Danish economy. Considering these moments thus provides an out-of-sample test of the structure imposed by our model and the values of the parameters we have estimated. The model performs well on the Gini coefficient for the full population, which the model only slightly overestimates. It also comes close to matching the Gini coefficient for labor income, which is calculated on the working population. Being well below the wealth Gini coefficient, our results confirm existing estimates for other countries (see Kaymak et al., 2022 and references therein) and highlights the importance

[21] Note, aggregate income growth implies average pension contributions increase by much more than what is implied by the change in $\bar{\pi}$. Prior to the pension reform, pension contributions are zero for a sizable share of individuals.

Table 2
Parameter values.

Symbol	Value	Description	Source
Parameters estimated from register data			
G_n	polynomial[a]	Labor income, deterministic process	Register data
σ_η	0.010	var. of permanent income shocks	Register data
σ_ϵ	0.063	var. of transitory income shocks	Register data
Calibrated parameters			
$\pi(Y_{i,n,t})$	$-0.112 + 0.00035Y_{i,n,t}$	Income-dependent FF pension contr.	Calibrated
θ_0	83.92	Flatrate PAYG payment	Calibrated
θ_1	polynomial[b]	Maximum means-tested PAYG amount	Calibrated
$\kappa(\tilde{Y}^{FF}_{i,n,t})$	$0.309\left(\tilde{Y}^{FF}_{i,n,t} - 0.14\right)$	Income-dependent PAYG pensions	
b	0.954	Midpoint of β-distribution	Calibrated
d	0.0117	Dispersion of β-distribution	Calibrated
Parameters from external sources			
$\delta_{n,t}$		Survival probabilities	UNPOP
N_r	65	Retirement age	OECD (2019)
ϑ	0.65	CRRA parameter	Andersen et al. (2014)
R	1.037	return on assets	OECD (2020)
Other parameters			
N^b	20	Age at birth	
N^d	99	Maximum lifetime	

Note: Monetary values refer to thousands of 2015 DKK.

[a] See App. C.5.

[b] $66.68 - \frac{3}{21875}s^3 + \frac{199}{4375}s^2 - \frac{359}{175}s$, where s is 1 in 1971 and linearly increases over the transition years.

of introducing preference heterogeneity as an additional dimension along which households differ. Overall, the model is able to provide a good approximation to the data in 1992.

The bottom part of the table shows the performance of our model on untargeted moments for 2017. Note that here, we do not necessarily match the data closely, since the only element of change between 1992 and 2017 in our model is the pension reform. Our aim is to assess to what extent the pension reform can explain the changes in the data. Contradictions between model and data simply mean that other time-varying elements, which we do not capture, have also played a role.

The Gini coefficients, as discussed in more detail below, display a declining trend in the model that closely mirrors the trend observed in the data. While the model slightly underestimates the decrease in the Gini coefficient for individuals aged 60–69, it predicts a lower Gini coefficient for the entire population compared to the actual data. An examination of asset holdings in 2017 across different age groups reveals that both pension and non-pension assets are lower in the model when compared to real-world data. However, the proportion of pension assets aligns quite well and slightly exceeds 50% in both the data and the model. This suggests that the model accurately captures the empirical observation that, in 2017, pension assets had grown significantly in importance within individuals' portfolios compared to 1992, when this share was just below one-fourth. The income Gini coefficient is matched satisfactorily in 2017, but again is much lower than the wealth Gini coefficient. The slight decline between 1992 and 2017 in the model is due to aggregate income growth.

5.3. Quantifying the effects of pension reform on savings and wealth

In the following, we start by presenting the model predictions for the wealth distribution over time. To determine how much of the change in wealth inequality can be attributed to the pension reform – rather than, for example, aggregate income growth – several counterfactual analyses are carried out. Then, we explore the economic mechanism driving the results, before discussing the implications of the pension reform for consumption inequality. Model results for life-cycle savings and income profiles of the representative agent can be found Appendix D.3.

Wealth distribution

Fig. 8 shows the change in the Gini coefficient between 1992 and 2017, comparing data and model. We are plotting a HP-filtered version of the data since the model does not incorporate high-frequency movements. The model predicts a decline of almost ten percentage points in the Gini coefficient of the 60–69 year olds, close to the data decline. The remaining decrease of the Gini coefficient in the data may, e.g., be due to short- or medium-run macroeconomic fluctuations in the Danish economy, which our model does not consider. The model slightly overpredicts the decrease in the Gini coefficient for the total population. One reason could be the changing income distribution and an increase in the variance of income shocks. This has been documented for the U.S. (Heathcote et al., 2010), and also seems to be the case in Denmark, as documented in Table 3.

Table 3
Calibration fit.

Moment	Model	Data
Targeted, 1992		
Wealth Gini coefficient, 60–69 year olds	0.626	0.626
Peak cross-section non-pension assets	458	458
Peak cross-section pension assets	132	132
Share of pension assets in total peak assets	0.224	0.224
Untargeted, 1992		
Wealth Gini coefficient, full population	0.772	0.750
Income Gini coefficient	0.344	0.285
Untargeted, 2017		
Wealth Gini coefficient, 60–69 year olds	0.537	0.511
Wealth Gini coefficient, full population	0.642	0.691
Peak cross-section non-pension assets	675	875
Peak cross-section pension assets	718	1,154
Share of pension assets in total peak assets	0.515	0.569
Income Gini coefficient	0.331	0.301

Note: Monetary values refer to thousands of 2015 DKK.

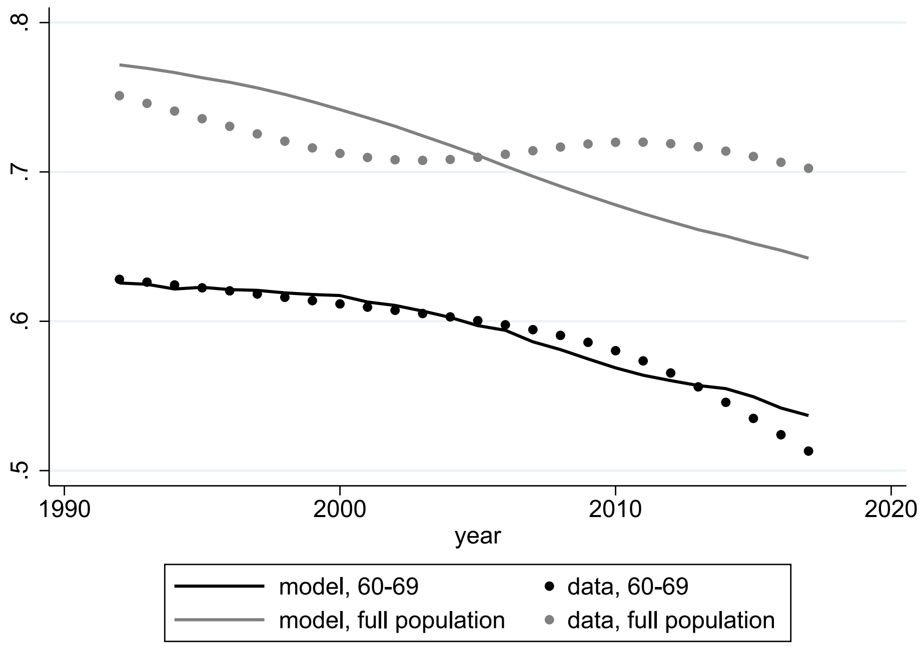

Fig. 8. Gini coefficient over time — data vs. model. Note: We apply the HP-filter to the data series 1992–2017. $\lambda = 100$ is used.

The Gini coefficient is known for being sensitive to the tails of the distribution. To paint a more comprehensive picture of the changing wealth distribution, we therefore present Lorenz curves in Fig. 9. Additional inequality measures are presented in Appendix Table D.1. In the model as well as in the data, the wealth distribution across wealth and income deciles becomes more equal. In Fig. 9(a), although the levels are slightly different (driven by negative wealth in the lowest deciles of the empirical distribution), the model predicts the correct shift: the share of wealth held by the lowest deciles increases over time. Fig. 9(b) shows generally a good fit between model and data. The figure highlights that it is mostly households at the bottom of the income distribution whose wealth increases over time.

Counterfactual analyses

The pension reform impacted wealth accumulation by mandating individuals to save more for retirement. At the same time, the PAYG pillar became more means-tested, which has potentially also played a role for voluntary savings behavior. This has happened not through a change in policy, but though a general increase in income levels, which implies that fewer people became eligible. Additionally, factors not related to the pension reform, such as a growth in aggregate income, took place. To highlight the importance of the pension reform and to understand the role of the different pension reform elements, we carry out counterfactual analyses in which we mute these elements one by one. Otherwise the model stays unchanged. The counterfactual model worlds are identical

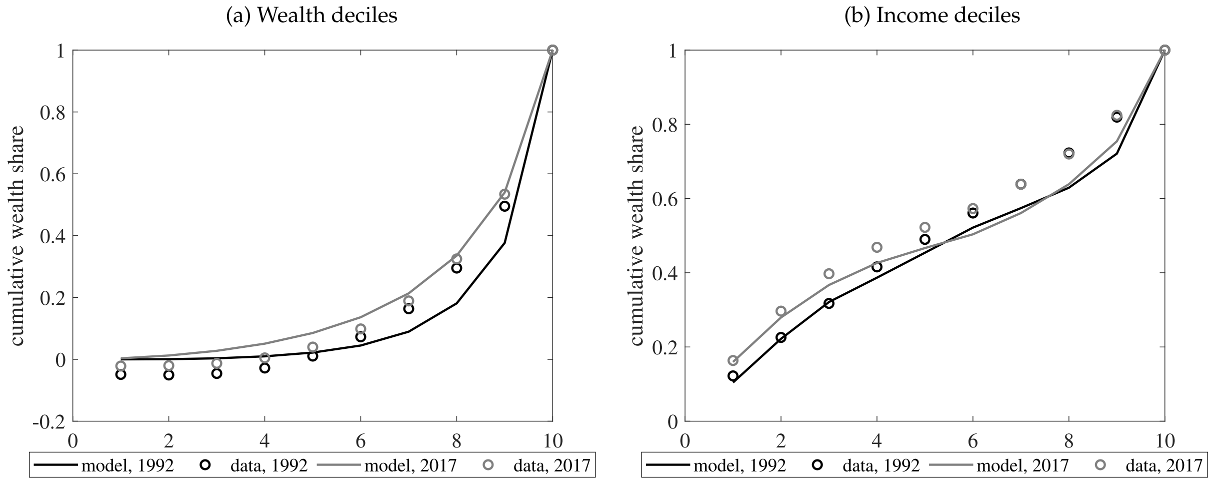

Fig. 9. Wealth distribution.

Table 4
Gini coefficient, percentage point change.

		60–69 yrs	All
(a)	Full model	−8.9	−13.0
(b)	No pension reform: no trend in $\bar{\pi}_t$	−4.1	−5.9
(c)	No income-dependent pension contributions	−7.1	−10.6
(d)	No change in PAYG rate	−8.6	−13.2

Note: Percentage point change in Gini coefficient in the baseline model and counterfactual models. Each counterfactual model is identical to the baseline model until 1992, from which point onward it enters a different transition path and converges to a different steady state.

to the baseline model up until 1992, from which point onward they enter a different transition path and converge to a different steady state. Agents are fully aware of the model world that they live in.

Table 4 compares changes in the Gini coefficient between 1992 and 2017 in each counterfactual scenario to the change in the base case considered above.

In (b) we keep $\bar{\pi}$ at its value of 1992 (5.2%), muting the trend growth of contribution rates over time. This is the most relevant scenario because it assumes that the pension reform has not taken place. However, it does not necessarily imply that individual pension contribution rates stay the same as in 1992. Because of changes in aggregate income, the income-dependent element of the pension contributions in $\pi(Y_{i,n})$ still increases, which means many agents will pay higher contributions. This additional pension channel is taken into account in scenario (c), where we make the pension contributions income-independent, but keep the growth of $\bar{\pi}$ as in the baseline version. In (d) we fix the PAYG replacement rate at its 1992 value. This value is considerably higher than the 2017 value due to the role of means-testing (see Figure D.2).

Shutting down the pension reform in scenario (b) has the biggest impact on wealth inequality both for the full population and the 60–65 year olds. The Gini coefficient declines by less than half the value of the baseline scenario. Conversely, more than half of the decline in wealth inequality is directly attributable to the pension reform. In contrast, when muting the income-dependent part of FF pensions, the reduction in the Gini coefficient is similar to the full model, so this element of the pension system by itself has a more limited effect on inequality.

Fixing PAYG replacement rates delivers results that are even more similar to the baseline scenario. Not allowing for changes in the PAYG rates even leads to a larger decline in wealth inequality for the full population. On the one hand, agents have less desire to save because the pension system is more generous. On the other hand, the means-tested element is muted, which seems to be the dominant driver for the full population.

Next to the different pension-related mechanisms, part of the decline in wealth inequality in the model is driven by aggregate income growth. As everyone receives a higher income, this results in a level shift in wealth and therefore, rich and poor individuals become relatively more similar.

Economic mechanism

Why did the pension reform lead to lower wealth inequality? The mechanisms at work in the calibrated model capture the essence of the analytical model in Section 3: Poor and impatient agents are mandated to save (through the pension system) a larger share of their labor income than before the reform. Since it is not possible to borrow, they cannot offset their high pension contributions — they are pushed into the zero savings corner. In contrast, patient agents are able to fully offset the increase in pension savings

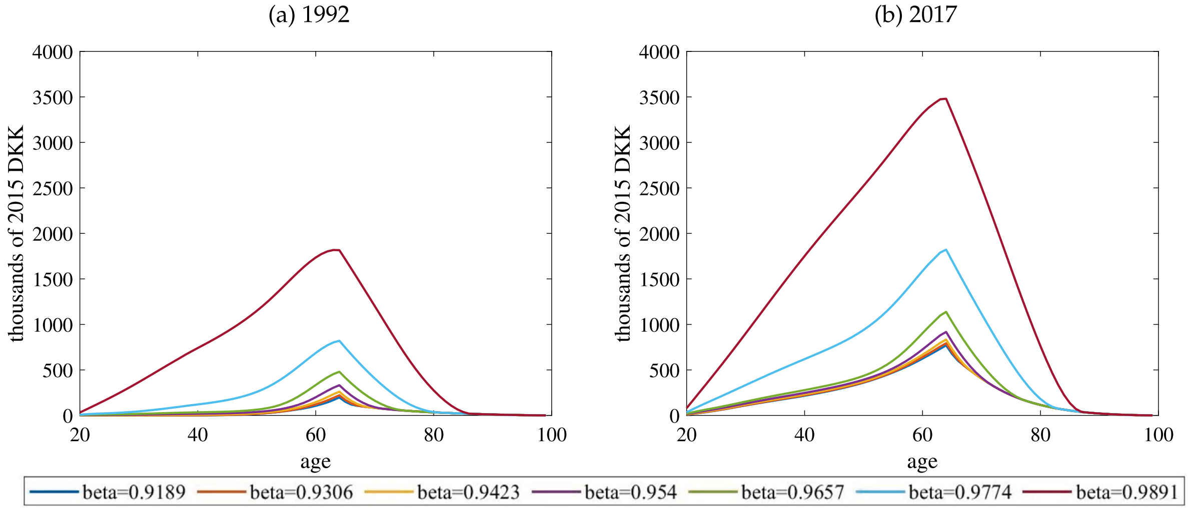

Fig. 10. Cross-sectional asset holdings, by β type.

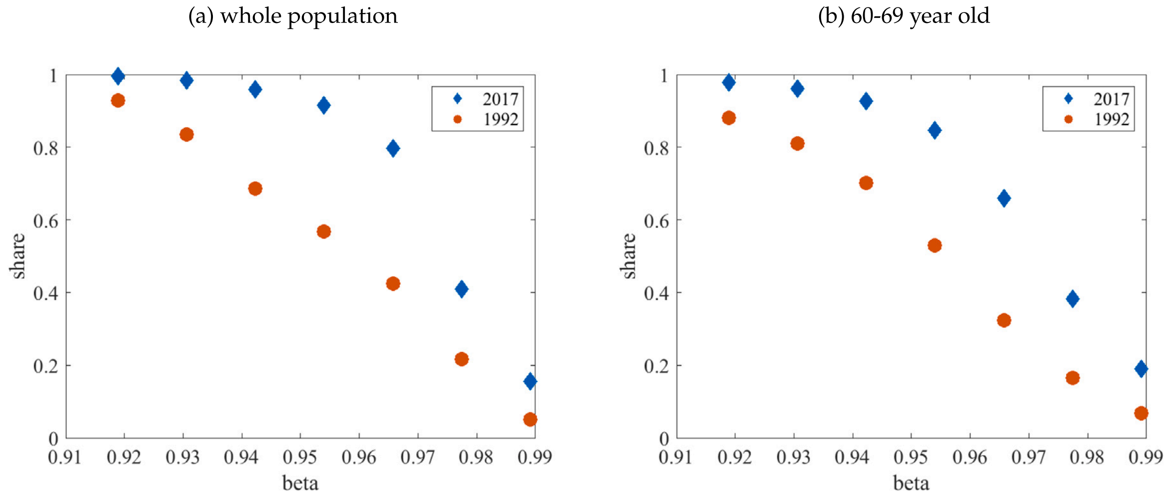

Fig. 11. Portfolio share of pension assets.

by lowering their voluntary savings. The structural model generalizes the analytical model by allowing for precautionary saving. Additional model elements such as the increases in aggregate labor income and longevity, and a less generous PAYG system (since eligibility declined with growing incomes even though eligibility criteria did not change) in the structural model imply that agents aim for a higher overall level of savings in 2017 than in 1992.

Fig. 10 shows total asset holdings by discount factor. Not surprisingly, there is large variation across β-types. However, the dispersion decreases over time. In 1992, pre-retirement wealth is almost 15-times higher for the most patient agents compared to the most impatient ones. In 2017, they only hold about five times more wealth. This finding aligns with the results from the stylized theoretical model discussed above: when shifting from a mainly PAYGO to a hybrid system, overall savings in the economy increase and the wealth share of impatient agents becomes larger.

Fig. 11 shows the share of pension assets in total assets for the whole population (panel (a)) and for the 60–69 year olds (panel (b)), comparing 1992 and 2017. Across all β-types, the reform manifests via an increase in the portfolio share of pension assets. Where the groups differ, however, is in the extent to which they are able to offset the mandatory pension savings by lowering voluntary savings. The lowest β-type is always at the zero savings corner. This group holds (almost) zero voluntary savings both in 1992 and in 2017. Being forced to save more through the reform, the discrepancy between desired and actual savings gets larger over time. In contrast, the groups with $\beta \in [0.93, 0.97]$ are not at the zero savings corner prior to the reform, but pushed in this direction in the course of the reform. The two most patient β-types are far from the zero-savings corner both in 1992 and 2017 and are therefore able to offset the increase in mandatory savings. Comparing panels (a) and (b), the effect on the full population is more pronounced than for the 60–69 year olds. Towards retirement, all β-types choose to accumulate some additional retirement

Table 5
Offset in the calibrated model.

	β_1	β_2	β_3	β_4	β_5	β_6	β_7
offset	−0.141	−0.183	−0.253	−0.389	−0.590	−0.726	−0.812

The offset rate is calculated as the difference in voluntary savings relative to the difference in mandatory savings between baseline and counterfactual.

Table 6
Gini coefficients for consumption.

	60–69yrs	All
1992	0.242	0.283
2017	0.242	0.264

savings, so they are less affected by the reform than other age groups.[22] Due to the precautionary savings motive, all agents choose to hold a small amount of voluntary savings. If we define the set of constrained agents as those for whom the portfolio share of pension assets is more than 90%, their share is 16% of the working-age population in 1992 and 38% in 2017, so their share more than doubles.

We can calculate the extent of offset by comparing voluntary and mandatory savings in 2017 as presented in Figs. 10 and 11 with counterfactual simulations as in scenario (b) in the previous section. This comparison allows us to assess how much additional savings agents have in the post-reform baseline model compared to a hypothetical scenario in which the pension reform had not taken place (but otherwise, the world would look exactly the same as in the baseline). Table 5 calculates the offset for each β type at age 64, at the peak of life-cycle savings.

The most impatient group barely offsets: in response to a one-kroner increase in mandatory savings, they decrease voluntary savings by 0.141 kroner. In contrast, the most patient group decreases voluntary savings by 0.812 kroner. On average, offset is 0.442 kroner. These numbers are in line with the literature estimating the crowding-out effect of pensions on non-pension wealth. It has been documented that most people do not fully offset increases in mandatory pension savings. Chetty et al. (2014) find that only about 15% of savers respond to an increase in employer contributions to retirement accounts by reducing voluntary savings, whereas the rest remains passive. Papers considering changes to PAYG pension claims also find that although households respond to reductions in social security entitlements, crowding-out is much less than one-for-one (Gale, 1998; Attanasio and Rohwedder, 2003; Attanasio and Brugiavini, 2003; Alessie et al., 2013; Lachowska and Myck, 2018). Our simulation results are also in line with the finding by Engelhardt and Kumar (2011) and Chetty et al. (2014) that crowding-out increases with income or wealth.

Consumption inequality

Wealth does not directly correspond to consumption, and wealth inequality does not necessarily translate into consumption or welfare inequality. With the aim of protecting old-age consumption, the pension reform required certain households to save more than they would otherwise do — thereby reducing their consumption possibilities at young ages. Appendix Figure D.4 shows that while all parts of the population saw an increase in consumption between 1992 and 2017 due to aggregate income growth, consumption increased more strongly for retirees than for working-age individuals. This provides evidence that the pension reform induced individuals to shift consumption towards later in life. It is beyond the scope of this paper to analyze the welfare rationale for the changes in the Danish pension system and relate it to the theoretical literature pointing to various reasons why individuals might “undersave“ and thus the need for intervention in pension savings. Andersen and Bhattacharya (2021), for example, consider the welfare case for mandated pensions in a model with present-biased preferences. Welfare implications of the transition in the pension system are an interesting question for future research to explore, in particular which groups have benefited the most from the changes in the pension system. Here, we focus on how the pension reform has affected the distribution of consumption across households. Table 6 shows consumption Gini coefficients for the full population and the group of 60–69 year-old agents. The level of consumption inequality is generally lower than that of wealth inequality, which is due to consumption smoothing over the life-cycle. For the group of 60–69 year olds, the pension reform left the consumption Gini coefficient virtually unchanged, whereas it slightly declined for the full population. Appendix Table D.2 shows that the decline is universal to all age groups. Thus, the pension reform has not increased consumption inequality within or across age groups.

6. Considering a broader wealth measure

In the definition of wealth, we have so far followed the OECD (2013) and focused on assets and liabilities that have economic value and are subject to ownership rights. This definition excludes any kind of government transfers, among them PAYG pensions. However, it is important to recognize that PAYG pensions also provide resources for retirement. In this section, we therefore consider

[22] Across the working-age cohorts, the largest impact of the reform is on the group of 40–49 year olds. On the one hand, they face high contribution rates (whereas younger age groups often fall below the income threshold), while on the other hand they have not yet reached the peak of their earnings and would like to borrow for consumption-smoothing purposes. The majority of agents in this age group is at or around the zero-savings corner.

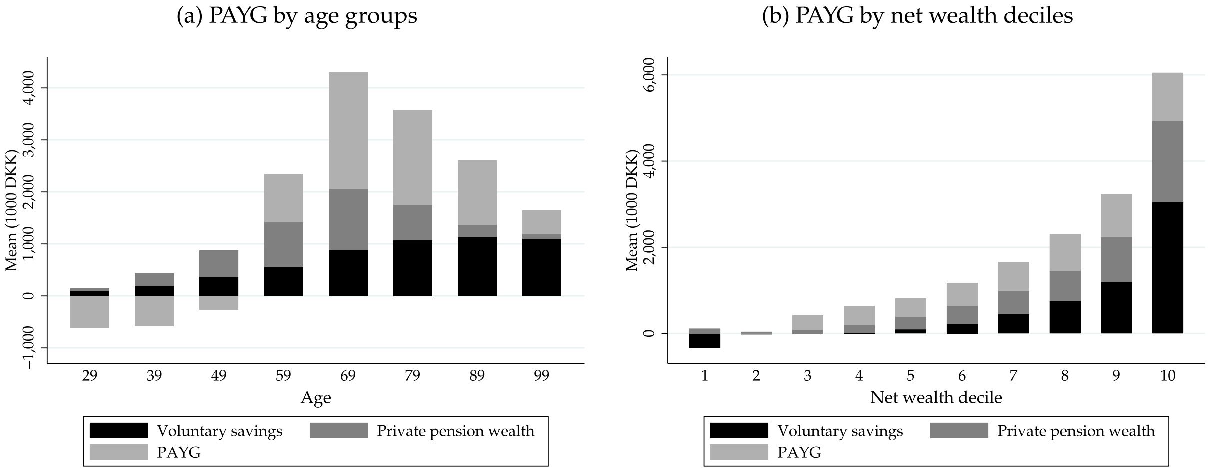

Fig. 12. The distribution of PAYG wealth in Denmark across age groups and wealth deciles, 2017.
Note: Extended wealth consists of net wealth and imputed PAYG wealth. Self-employed and households with more than two adults are excluded. For the age groups, we consider ten-year age bins, where the first bin covers households with a head between 20 and 29 years old, etc.

a broader definition of wealth, which includes the net present value (NPV) of social security. As shown by Catherine et al. (2020) for the U.S., such a broader concept may attenuate wealth inequality, as low-financial wealth households benefit disproportionately from government transfers. The question is, then, how the transition from a mainly PAYG-based system towards a hybrid system with a strong role for the funded pillar, will affect wealth inequality according to the broader measure.

In the analytical model, the overall effect of pension reform on inequality for the broad notion of wealth is ambiguous, depending on the type of heterogeneity considered. Appendix B.1 provides details on derivations for the wage-heterogeneity case and Appendix B.3 for β-heterogeneity. Be reminded that in Denmark, the PAYG scheme is means-tested (see Section 2). With the increased prominence of the FF scheme, some of erstwhile poor households get wealthier and may stop receiving the PAYG supplement, in effect receiving a lower PAYG pension. This may partially negate the effect of FF schemes on the broad wealth measure. The ambiguity of analytical results motivates the need for further empirical analysis.

To address this question empirically, we roughly follow (Catherine et al., 2020) in imputing PAYG wealth. The NPV of PAYG wealth of an individual aged n is calculated as

$$PW_{i,n} = \sum_{m=n+1}^{N^d} \left(\prod_{k=n+1}^{m} \delta_k \right) \frac{Y_{i,m}^{PG} - \tau_{i,m}}{\mathbb{E}(R^{m-n})}$$

where $Y_{i,n}^{PG}$ are individual PAYG pension payments, which are zero before retirement, and $\tau_{i,n}$ are pension contributions to be paid during working-age.

For retirees, where contributions are zero and we directly observe the pension benefits in the register data, calculating PAYG wealth is straightforward. For working-age individuals, we face several challenges. First, since the Danish pension system is funded through taxation, there are no explicit contributions. We proceed by making the assumption of a balanced government budget and posit that implicit contributions are proportionate to the total income taxes paid by individuals. Second, we do not observe the whole life-cycle path of labor earnings. Here we rely on simulations from the calibrated model. Details on the procedure are described in Appendix C.6. Note that similar to Catherine et al. (2020), our estimates are conservative since we use a simplified PAYG formula, excluding some additional benefits aimed at poverty protection that are offered to retired households in the real world.

Before discussing wealth inequality over time, we briefly describe how PAYG wealth is distributed across age groups and wealth deciles. As Fig. 12(a) shows, imputed PAYG wealth is negative for young households, as the NPV of their contributions is larger than that of the benefits. The opposite is true for old agents. Not surprisingly, imputed PAYG wealth is highest for households in the age group 60–69, around the retirement age.[23] If we consider the imputed PAYG wealth by net wealth deciles, it becomes clear that it is relatively most important for households in the middle of the net wealth distribution. In the lowest decile, mean PAYG wealth is low due to the presence of young households. At the very top are wealthy households in prime saving ages.

Table 7 compares the Gini coefficients in 1992 and 2017 for the extended wealth measure with our benchmark net wealth specification. Including PAYG wealth in the definition of wealth does not affect the level of the Gini coefficient for all households compared to our benchmark wealth definition as there are two opposing effects at work: for the working population, extended wealth inequality is higher than in our benchmark case, whereas for households close to the retirement, the Gini coefficient for the extended wealth measure is much lower, highlighting the redistributive role of social security for this age group. Over time, we

[23] Similar patterns for PAYG wealth across the age groups have been found in Catherine et al. (2020) for 1989.

Table 7
Gini coefficient in Denmark, 1992 vs 2017.

	All households		Head 60–69 years old	
	Extended wealth	Net wealth	Extended wealth	Net wealth
1992	0.75	0.75	0.19	0.63
2017	0.75	0.69	0.23	0.51
% Δ 1992–2017	0.67%	−8%	17.53%	−18%

Note: Extended wealth is a sum of net wealth and imputed PAYG wealth. We remove self-employed households and households with more than two adult members from the sample.

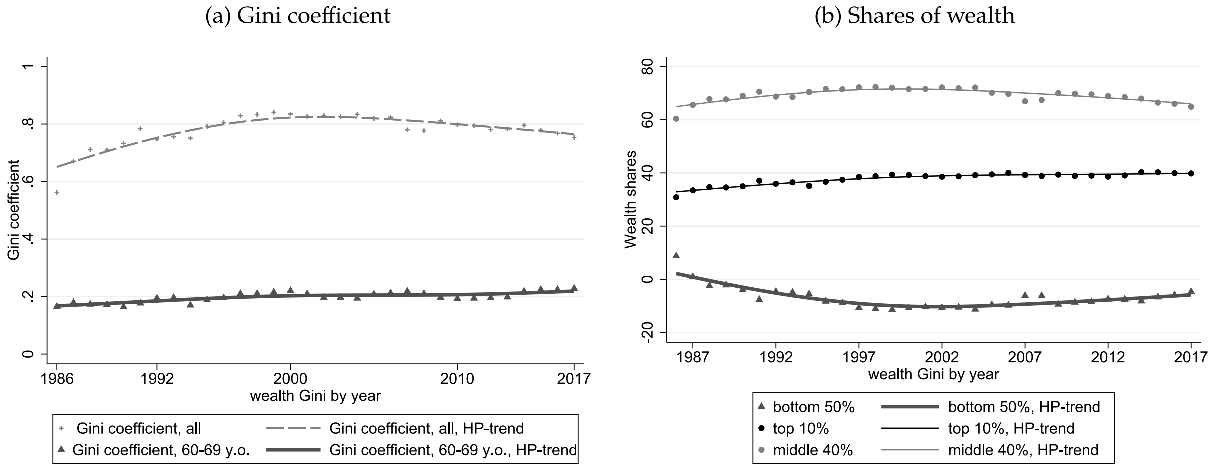

Fig. 13. Measures of extended wealth inequality for Denmark, 1986–2017.
Note: Extended wealth consists of net wealth and imputed PAYG wealth. Self-employed and households with more than two adults are excluded. For the HP-filter, $\lambda = 300$ is used.

see an increase in extended wealth inequality for households close to retirement, while for all households the change in the Gini coefficient over time is negligible.[24] Thus, adding the PAYG wealth to our benchmark net wealth measure counteracts the decrease in wealth inequality shown in the main part of the paper.

Fig. 13 visualizes the development of Gini coefficients and chosen decile shares for the extended wealth measure over time. Once we take into account PAYG wealth, the share of wealth held by the bottom 50% of population decreases compared to the conventional definition of wealth, and it shows a slight downward trend. The share of wealth held by the middle 40% of the distribution is higher than in the case of net wealth measure, but it also slightly decreases over time, while the top 10% (holding a much lower share of wealth compared to the benchmark net wealth measure) exhibits a slightly increasing trend. Overall, changes over time are small.

This conclusion is reinforced when we compare the Lorenz curves for extended wealth in 1992 and 2017, depicted in Fig. 14. We see that the poorest households (precisely, the first two deciles) became relatively richer between 1992 and 2017, even if we take into account the imputed PAYG wealth that counteracts the reform through its means-testing. This is true both for households across all age groups (panel a) and households close to retirement (panel b). Our findings for the extended wealth measure lend support to the main mechanism highlighted in our paper: namely that the pension reform increased wealth holdings of households at the bottom of the wealth distribution.

7. Conclusion

The design of the pension system has been shown to have a quantitatively significant effect on wealth accumulation and its distribution. Exploiting Danish data and a transition from a largely PAYG-pension system to a hybrid with a large weight on FF pensions, we document how this transition implies not only increased wealth accumulation but also a more equal wealth distribution.

It should be emphasized that the preceding analysis is positive because we do not attach value judgments to whether the post-reform functional distribution of income (ownership of capital) is desirable in any sense. This is not to suggest our topic is devoid of normative content. The rising inequality concerns those who correlate wealth with augmented political power and fret about the

[24] In our life-cycle model, too, the extended wealth measure suggests a decline in wealth inequality over time for the working population, but an increase for the population around the retirement age.

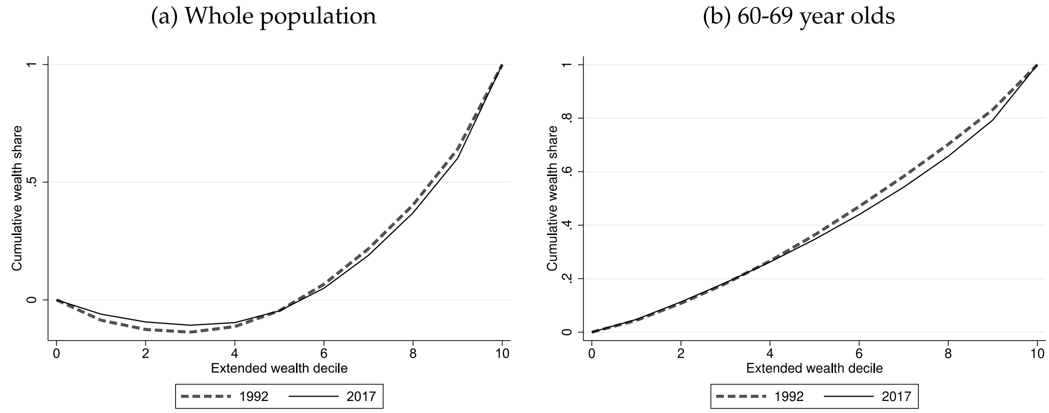

Fig. 14. Lorenz curves for extended wealth, 1992 and 2017.
Note: The graphs show cumulated share of extended wealth held by different extended wealth deciles of Danish households. Extended wealth consists of net wealth and imputed PAYG wealth. Self-employed and households with more than two adults are excluded.

impact on democratic institutions, more so if the wealth distribution is widely perceived to be unfair.[25] Others see wealth inequality as an important – again, possibly unfair – channel by which economic advantage, opportunities, and affluence are transmitted across generations.[26] To neoclassical economists like Kuznets and Solow, the issue of who owns capital is not important: along a balanced growth path, both owners of capital and labor would benefit from growth to the same degree (Piketty, 2014). Data from the 1990s onward are at odds with this "balanced growth" view: if anything, labor's share of output is falling which raises the imperative to know who owns capital.[27] After all, as the percentage of value-added in output going to owners of capital is rising, FF schemes, by allowing workers to become capital owners, may help pass on some of the benefits to the workers. Our work demonstrates how pension reform has democratized the ownership of capital thereby helping to bring down wealth inequality in Denmark. Whatever the underlying motivation, governments are concerned about the rise in wealth inequality, and intervene in ways to curb it. Many such attempts suggest the direction of the taxation of wealth. We do not enter that discussion. We merely offer the view that pension reform, already underway in many high wealth-inequality nations, may reduce wealth inequality by itself.

Data availability

The data that has been used is confidential.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2024.104746.

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[25] The worry surrounding worsening ownership of capital is for some its effect on politics. For others, there is jubilance over the emergence of a 'public shareholder class' and how 'even blue-collar workers now often have sufficient personal savings to justify investment in equity securities [through pension funds]...no longer do labor and capital constitute clearly distinct interest groups in society' (Ireland, 2005). Drucker (1976) was among the first to notice the potential for pension programs to reshape the ownership of capital. He commented on how in the late 60s and early 70s, American financial institutions, notably retirement funds were diversifying away from mostly "safe" investments, such as treasuries and bonds, and investing in public company stock. To Drucker, this meant the future "owners" of American companies would not be old-style capitalists but workers in the public and private sector (through their fiduciaries, trustees and managers of pension funds).

[26] As Kopczuk (2015) frames its "the extent to which the well-off are going to rely on work versus rely on the returns to their wealth in the future is clearly important for assessing the extent to which a society will view itself as in some way a meritocracy."

[27] Between 1991 and 2014, the labor share declined in 29 of the largest 50 economies overturning one of the enduring Kaldor stylized facts (Karabarbounis and Neiman, 2014). There is mounting evidence that the labor income share has decreased (increased) for low (high)-skilled workers (Dao et al., 2017).

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