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Growth Options, Macroeconomic Conditions, and the Cross-Section of Credit Risk[☆,☆☆]

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Abstract

This paper develops a structural equilibrium model with intertemporal macroeconomic risk, incorporating the fact that firms are heterogeneous in their asset composition. Compared to firms that are mainly composed of invested assets, firms with growth options have higher costs of debt because they are more volatile and have a greater tendency to default during recession when marginal utility is high and recovery rates are low. Our model matches empirical facts regarding credit spreads, default probabilities, leverage ratios, equity premiums, and investment clustering. Importantly, it also makes predictions about the cross-section of all these features.

Keywords: Asset composition, capital structure, credit spread puzzle, equity premium, growth options, macroeconomic risk, value premium

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1. Introduction

This paper examines the impact of corporate growth options on credit spreads, equity premiums, firm value, and financial policy choices in the presence of time-varying macroeconomic conditions.

The motivation for our study derives from the empirical fact that credit risk, leverage, and equity risk premiums exhibit important cross-sectional variation. First, Davydenko and Strebulaev (2007) show that, controlling for standard credit risk factors, proxies of growth options are all positively and significantly related to credit spreads. Similarly, Molina (2005) finds that firms with a higher ratio of fixed assets to total assets have lower bond yield spreads and higher ratings. Second, firms with more growth options typically have lower leverage (see, e.g., Smith and Watts, 1992; Fama and French, 2002; Frank and Goyal, 2009). Third, value firms earn higher equity returns than growth firms (see, e.g., Fama and French, 1992). Strikingly, none of these cross-sectional properties can be explained by existing structural models of default. The reason is that these models consider firms with only invested assets, but ignore the facts that growth opportunities constitute an essential element of asset values and that firms are heterogeneous in their asset composition.[4]

We provide a model that matches these cross-sectional properties of credit risk, leverage, and equity risk premiums. In particular, we explicitly incorporate expansion options of firms into a structural model of default with macroeconomic risk. We show that heterogeneity in the composition of assets helps explain cross-sectional variation of credit spreads and leverage. Moreover, allowing firms to be heterogeneous with respect to the importance of growth options in the values of their assets explains the aggregate credit spread puzzle, not only qualitatively, but also quantitatively. Importantly, the puzzle is solved while fitting historically reported asset volatilities and default rates for realistic debt maturities. At the same time, the model matches the average equity premium and explains a significant portion of the cross-section of equity risk (the value premium).

[4]Recent research focuses on the credit spread puzzle, i.e., the fact that standard structural models of default significantly underestimate credit spreads for corporate debt (see, e.g., Elton, Gruber, Agrawal and Mann, 2001; Huang and Huang, 2003). Several papers present significant progress in solving this puzzle (see, e.g., Bhamra, Kuehn and Strebulaev, 2010a; Bhamra, Kuehn and Strebulaev, 2010b; Bhamra, Kuehn and Strebulaev, 2010c; Chen, 2010; Chen, Collin-Dufresne and Goldstein, 2009; Gomes and Schmid, 2011). However, none of these papers addresses the cross-section of credit risk.

It also generates a counter-cyclical value premium, as observed in the data. Finally, our model is consistent with aggregate and cross-sectional features of default clustering, investment spikes and busts, and recovery rates.

For our analysis, we develop a structural-equilibrium framework in the spirit of Bhamra, Kuehn and Strebulaev (2010a). Thus, we embed a pure structural model of financial decisions into a consumption-based asset pricing model with a representative agent. Our model simultaneously incorporates both intertemporal macroeconomic risk (building on work by Hackbarth, Miao and Morellec, 2006; Bhamra, Kuehn and Strebulaev, 2010b; Chen, 2010), which has been shown to be important for explaining credit spreads and leverage, as well as expansion options. Macroeconomic shocks to the growth rate and volatility of earnings as well as to the growth rate and volatility of consumption arise due to switches between two states of the economy, boom and recession. The changes in the state of the economy are modeled via a Markov chain, a standard tool to model regime switches. The representative agent has the continuous time analog of Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990; Duffie and Epstein, 1992b). Therefore, how he prices claims depends on both his risk aversion and his elasticity of intertemporal substitution. Via the market price of consumption determined by the agent's preferences, we are able to link unobservable risk-neutral probabilities used in the structural model to historical probabilities. This modeling approach allows us to study endogenously the effect of macroeconomic risk on credit spreads and optimal financing decisions.

We allow firms to have expansion options. These options are converted into invested assets when the underlying earnings process exceeds the investment boundary. We pinpoint the isolated effect of a firm's asset composition on credit risk and leverage by assuming, in the main analysis, that the exercise price of the growth option is financed through the sale of some assets in place, i.e., without additional funds being injected into the company. We also study equity-financing later in the paper. Default occurs when earnings are below the default threshold in a given regime. Shareholders maximize the value of equity by simultaneously choosing the optimal default and expansion option exercise policies. The capital structure is determined by trading off tax benefits of debt against default costs to maximize the ex-ante value of equity, i.e., the value of the firm.

The first result the model yields is that, like in other macroeconomic models, default boundaries are counter-cyclical, i.e., shareholders default earlier in recession than in boom. Thus, default is more likely during recession which, together with counter-cyclical marginal utilities and default costs, raises the costs of debt for all firms compared to a benchmark model without business cycle risk.

The central new feature of our model is that the asset composition alone matters significantly for the costs of debt. Two forces lead to the cross-sectional prediction that debt is particularly costly for firms with a high portion of expansion options in their assets' values. First, because options represent levered claims, firms with valuable growth options are more sensitive to the underlying earnings process than firms that consist of only invested assets. The volatility of the underlying earnings process would, consequently, underestimate the true default risk of growth firms. While the literature discusses this basic idea within equity-financed firms (Berk, Green and Naik, 1999; Carlson, Fisher and Giammarino, 2006), little is known about its impact on debt prices. Our structural model allows us to jointly analyze a firm's expansion policy and financial leverage. We show that the combination of these factors is critical for a full exploration of the quantitative implications of the riskiness of growth options on credit spreads.

The second driving force is that option values are more sensitive to macroeconomic regime changes than are assets in place. This higher sensitivity is, to some extent, another consequence of the idea that options represent levered claims. Importantly, an additional effect derives from the fact that the optimal exercise boundary of growth options increases in recession and decreases in boom. Intuitively, it is optimal to defer the exercise of an expansion option when the economy switches to recession, i.e., to wait for better times. Because the moneyness of growth options is regime-dependent, and because options represent levered claims, the continuation value of expansion options is more exposed to the macroeconomic state than the one of invested assets. Moreover, the changing moneyness causes expansion options to be less sensitive to the underlying development of the earnings process in recession than in boom, which reduces the value of the shareholders' option to defer default during bad times. Together, these effects amplify the counter-cyclicality of default thresholds for firms with a high portion of growth options. As marginal utility is high during bad

times, the higher tendency to default in recession causes larger credit spreads under risk-neutral pricing for firms with expansion options than for those with only invested assets.

We then investigate the quantitative performance of the model in explaining empirically observed data. The literature suggests that an average BBB-rated firm has a ten year credit spread in the range of $74 - 95$ basis points (bps). (This range is obtained by starting from the average bond yields reported in Davydenko and Strebulaev (2007) and Duffee (1998), and taking into account that around 35% of bond yields are due to non-default components.) With our main set of parameters, a model without business cycle risk produces a mere 29 bps spread for an average firm. A standard macroeconomic model with optimal default thresholds in the spirit of Bhamra, Kuehn and Strebulaev (2010b) or Chen (2010) implies a spread of 56 bps for average firms at issue that consist of only invested assets. Our estimate for the average BBB-rated US firm's asset composition is that total firm value is about 60% higher than the value of invested assets, which corresponds (approximately) to a Tobin's Q of 1.6.[5] For such a firm, we obtain a credit spread of about 66 bps when using optimal default thresholds, optimal expansion boundaries, and an earnings volatility such that the average asset volatility matches the one observed for BBB-rated firms. This spread is remarkably higher than the 39 bps our model implies for a firm with only invested assets. Note that the large difference arises even though leverage is kept constant; we only vary the characteristics of the assets themselves.

As the economy consists of a mix of firms, the result that growth firms have higher credit spreads than firms with only invested assets suggests that our model can also explain the aggregate credit spread puzzle. To evaluate this conjecture, note that when relating the implications of capital structure models for average credit spreads to empirical studies, it is crucial to take into account that such studies use aggregate data over cross-sections of firms, rather than average individual firm level data (Strebulaev, 2007). Following this line of reasoning, Bhamra, Kuehn and Strebulaev (2010b) investigate how the time evolution of the cross-sectional distribution of firms with different leverage ratios affects credit spreads and default probabilities. Building on their approach, we

[5]Market values can be higher than book values also because of off-balance sheet assets, so there is a range for the asset composition of the "typical" firm.

characterize the aggregate dynamics by simulating over time a cross-section of individual firms that is structurally similar to the empirical distribution of BBB-rated firms not only with respect to average leverage ratios but also with respect to asset composition ratios. The average ten and 20 year credit spreads of 81 and 100 basis points, respectively, from simulating this "true" cross-section in our model reflect their target credit spreads quite well. To solve the aggregate credit spread *puzzle*, a model needs to explain observed costs of debt while still matching historical default losses (given by the historical default probabilities and recovery rates), and asset volatilities. We consequently proceed by showing that the model-implied default rates and asset volatilities of BBB-rated firms are similar to the ones historically reported for realistic debt maturities.

The nature of assets, thus, has a powerful impact on costs of debt. Not surprisingly, it also affects the observed features of leverage. At initiation, we find that a firm with an average growth option optimally holds about $4-5\%$ lower leverage than one with only invested assets. Additionally, we obtain pro-cyclical optimal leverage decisions of firms, in line with Covas and Den Haan (2006) and Korteweg (2010). The reason is that the default risk is higher in recession than in boom. The negative relationship between growth options and leverage also maintains when simulating over time our model-implied true cross-section of BBB-rated firms. In this simulation, however, firms deviate from their initially optimal leverage in a way such that the aggregate market leverage of the whole sample becomes counter-cyclical, consistent with Korajczyk and Levy (2003) and Bhamra, Kuehn and Strebulaev (2010b).

We derive additional testable predictions when studying the aggregate dynamics of our model economy. Credit spreads and default rates are counter-cyclical, as reported in the literature. Next, aggregate investment patterns are strongly pro-cyclical, with investment spikes often occurring when the regime switches from recession to boom, reflecting the findings in the empirical investment literature (Barro, 1990; Cooper, Haltiwanger and Power, 1999). Our model also makes specific cross-sectional predictions. For example, realized recovery rates are lower for growth firms.

Finally, we show that the model's intuition is consistent with the literature on the value premium for equity. In the true cross-section, our model implies an annual value premium, i.e., a difference between the average value-weighted equity premium of the firms in the lowest decile of the asset

composition ratio and the premium of those in the highest decile, of 3.47%. Importantly, the model also explains the empirically reported counter-cyclical pattern of the value premium.

Our paper contributes to several streams of previous research. First, the fact that growth options are empirically strongly associated with observed leverage has, of course, also prompted other explanations. The most prominent of these additional explanations, agency, comes in two primary forms: A shareholder-bondholder conflict and a manager-shareholder conflict. Appealing to the former, Smith and Watts (1992) and Rajan and Zingales (1995) suggest that debt costs associated with shareholder-bondholder conflicts typically increase with the number of growth options available to the firm due to underinvestment (Myers, 1977) and overinvestment by way of asset substitution (Jensen, 1986; see also Sundaresan and Wang, 2007).[6] According to Leland (1998), however, optimal leverage even increases when firms can engage in asset substitution. Similarly, Parrino and Weisbach (1999) conclude that stockholder-bondholder conflicts are too limited to explain the cross-sectional variation in capital structure. Childs, Mauer and Ott (2005) show how short-term debt reduces agency costs. Hackbarth and Mauer (2010) demonstrate that the joint choice of debt priority structure and capital structure can virtually eliminate the suboptimal investment incentives of equityholders. Neither of the papers incorporates macroeconomic risk.

As for manager-shareholder conflicts, Morellec (2004) shows that agency costs of free cash flow can explain the low debt levels observed in practice, and the negative relationship between debt levels and the number of growth options; see also Barclay, Smith and Morellec (2006). Morellec, Nikolov and Schürhoff (2012) conclude that even small costs of control challenges are sufficient to explain the low-leverage puzzle. It is still a matter of debate to what extent conflicts of interest between managers and stockholders cause the empirically observed patterns. Graham (2000), for example, tests a wide set of managerial entrenchment variables and finds only "weak evidence that managerial entrenchment permits debt conservatism" (p. 1931). In any case, our model is not inconsistent with either of these views. It offers a quantitatively important reason for the cross-

[6] See Lyandres and Zhdanov (2010) for an explanation for accelerated investment that does not rely on agency.

sectional variation in leverage and credit spreads that derives solely from the nature of assets of firms.[7]

Second, at the core of our model is the notion that macroeconomic (business cycle) risk matters in powerful ways for the costs of corporate debt and financial decisions, because firms are more likely to default when doing so is costly (see, e.g., Demchuk and Gibson, 2006; Almeida and Philippon, 2007; Bhamra, Kuehn and Strebulaev, 2010b; Chen, 2010). What we add to this literature is the idea that the impact of business cycle risk depends on the asset base of a firm.

In contemporaneous and independent work, Chen and Manso (2010) set up a model similar to ours with expansion options. Their focus, however, is on the debt overhang problem, and not on explaining cross-sectional features or the credit spread puzzle – the central tasks of this paper.

Finally, our structural-equilibrium framework draws on insights from consumption-based asset pricing models (Lucas, 1978; Bansal and Yaron, 2004).

The paper proceeds as follows. In Section 2, we set up our valuation framework. We solve the model in Section 3. Section 4 discusses our parameter and firm sample choices, as well as the optimal default and expansion policies. Section 5 outlines qualitative properties of our model for the aggregate economy. We turn to the quantitative implications for BBB-rated firms in Section 6. The predictions of our model for the value premium of equity are discussed in Section 7. Section 8 concludes.

2. The model

We build a structural model with intertemporal macroeconomic risk, embedded inside a representative agent consumption-based asset pricing framework. The structural model is based on a standard continuous time model of capital structure decisions in the spirit of Mello and Parsons (1992), as extended by Hackbarth, Miao and Morellec (2006) for business cycle fluctuations.

[7] An alternative explanation for why low leverage could be optimal in the high-tech sectors is offered in Miao (2005). In his model, when a sector experiences technological growth, more competitors enter, leading to falling prices and possibly to a greater probability of default. Yet other explanations appeal to the fact that firms have the option to issue additional debt (Collin-Dufresne and Goldstein, 2001).

Additionally, we explicitly model growth opportunities. Following Bhamra, Kuehn and Strebulaev (2010b) and Chen (2010), embedding the model of capital structure into a consumption-based asset pricing model allows the valuation of corporate securities using the risk-neutral measure implied by the preferences of the representative agent.

The economy consists of N infinitely-lived firms with assets in place and possibly growth options, a large number of identical infinitely-lived households, and a government serving as a tax authority. We assume that there are two different macroeconomic states, namely boom (B) and recession (R). Formally, we define a time-homogeneous Markov chain $I_{t\geq 0}$ with state space $\{B, R\}$ and generator $Q := \begin{bmatrix} -\lambda_B & \lambda_B \\ \lambda_R & -\lambda_R \end{bmatrix}$, in which $\lambda_i \in (0,1)$ denotes the rate of leaving state i. In the main analysis, we consider $\lambda_B < \lambda_R$, as in Hackbarth, Miao and Morellec (2006).

The following properties hold: First, the probability that the chain stays in state i longer than some time $t \geq 0$ is given by $e^{-\lambda_i t}$. Second, the probability that the regime shifts from i to j during an infinitesimal time interval Δt is given by $\lambda_i \Delta t$. Third, the expected duration of regime i is $\frac{1}{\lambda_i}$, and the expected fraction of time spent in that regime is $\frac{\lambda_j}{\lambda_i + \lambda_j}$.

Aggregate output C_t follows a regime-switching geometric Brownian motion:

$$\frac{\mathrm{d}C_t}{C_t} = \theta_i \mathrm{d}t + \sigma_i^C \mathrm{d}W_t^C, \ i = B, R, \tag{1}$$

in which W_t^C is a Brownian motion independent of the Markov chain, and θ_i, σ_i^C are the regime-dependent growth-rates and volatilities of the aggregate output. In equilibrium, aggregate consumption equals aggregate output. Hence, the above specification gives rise to uncertainty about the future moments of consumption growth.

To incorporate the impact of the intertemporal distribution of consumption risk on the representative household's utility, we assume the continuous-time analog of Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which are of stochastic differential utility type (Duffie and Epstein, 1992a,b). Specifically, the utility index U_t over a consumption process C_s solves

$$U_t = \mathbb{E}^{\mathbb{P}}\left[\int_t^\infty \frac{\rho}{1-\delta} \frac{C_s^{1-\delta} - \left((1-\gamma)\, U_s\right)^{\frac{1-\delta}{1-\gamma}}}{\left((1-\gamma)\, U_s\right)^{\frac{1-\delta}{1-\gamma}} - 1} \mathrm{d}s \,|\mathcal{F}_t\right], \tag{2}$$

in which ρ is the rate of time preference, γ determines the coefficient of relative risk aversion for a timeless gamble, and $\Psi := \frac{1}{\delta}$ is the elasticity of intertemporal substitution for deterministic consumption paths.

As shown by Bhamra, Kuehn and Strebulaev (2010b) and Chen (2010), the stochastic discount factor m_t then follows the dynamics

$$\frac{\mathrm{d}m_t}{m_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + \left(e^{\kappa_i} - 1\right) \mathrm{d}M_t, \tag{3}$$

with M_t being the compensated process associated with the Markov chain, and

$$\begin{aligned} r_i &= \bar{r}_i + \lambda_i \left[\frac{\gamma - \delta}{\gamma - 1} \left(w^{-\frac{\gamma-1}{\gamma-\delta}} - 1 \right) - \left(w^{-1} - 1 \right) \right], & (4) \\ \eta_i &= \gamma \sigma_i^C, & (5) \\ \kappa_i &= (\delta - \gamma) \log \left(\frac{h_j}{h_i} \right). & (6) \end{aligned}$$

h_B, h_R solve a non-linear system of equations given in the Appendix A.1, Eq. (A-1). r_i are the regime-dependent real risk-free interest rates, composed of the interest rate if the economy stayed in regime i forever, $\bar{r}_i$, and the adjustment for possible regime switches as shown by the second term. η_i are the risk prices for systematic Brownian shocks affecting aggregate output, and κ_i is the relative jump size of the discount factor when the Markov chain leaves state i (and, consequently, $\kappa_j = \frac{1}{\kappa_i}$). The no-jump part of the interest rate, $\bar{r}_i$, is given by

$$\bar{r}_i = \rho + \delta \theta_i - \frac{1}{2} \gamma \left(1 + \delta\right) \left(\sigma_i^C\right)^2, \tag{7}$$

and

$$w := e^{\kappa_R} = e^{-\kappa_B} \tag{8}$$

measures the size of the jump in the real-state price density when the economy shifts from recession to boom (see Bhamra, Kuehn and Strebulaev, 2010b, Proposition 1).

Credit spreads are based on nominal yields and taxes are collected on nominal earnings. To link nominal to real values such as the real interest rate introduced in the previous section, we specify a stochastic price index as

$$\frac{\mathrm{d}P_t}{P_t} = \pi \mathrm{d}t + \sigma^{P,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P, \tag{9}$$

with W_t^P being a Brownian motion describing the idiosyncratic price index shock, independent of the consumption shock Brownian W_t^C and the Markov chain. π denotes the expected inflation rate, and $\sigma^{P,C} < 0, \sigma^{P,id} > 0$ are the volatilities of the stochastic price index associated with the consumption shock and the idiosyncratic price index shock, respectively. The nominal interest rate r_i^n is then given by

$$r_i^n = r_i + \pi - \sigma_P^2 - \sigma^{P,C}\eta_i, \tag{10}$$

with $\sigma_P := \sqrt{\left(\sigma^{P,C}\right)^2 + \left(\sigma^{P,id}\right)^2}$ being the total volatility of the stochastic price index.

At any point in time, the real after-tax earnings process of a firm follows

$$\frac{\mathrm{d}X_{t,real}}{X_{t,real}} = \mu_{i,real}\mathrm{d}t + \sigma_{i,real}^{X,C}\mathrm{d}W_t^C + \sigma^{X,id}\mathrm{d}W_t^X, \quad i = B, R, \tag{11}$$

in which W_t^X is a standard Brownian motion describing an idiosyncratic shock, independent of the aggregate output shock W_t^C, the consumption price index shock W_t^P, and the Markov chain. $\mu_{i,real}$ are the real regime-dependent drifts, $\sigma_{i,real}^{X,C} > 0$ the real firm-specific regime-dependent volatilities associated with the aggregate output process, and $\sigma^{X,id} > 0$ the firm-specific volatility associated with the idiosyncratic Brownian shock.

The nominal after-tax earnings process can now be written as

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i \mathrm{d}t + \sigma_i^{X,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P + \sigma^{X,id}\mathrm{d}W_t^X, \quad i = B, R, \tag{12}$$

in which $\mu_i = \mu_{i,real} + \pi + \sigma^{P,C}\sigma_{i,real}^{X,C}$ are the nominal regime-dependent drifts, and $\sigma_i^{X,C} = \sigma_{i,real}^{X,C} + \sigma^{P,C} > 0$ the nominal firm-specific regime-dependent volatilities associated with the aggregate output process. As suggested by the literature, we posit that $\sigma_B^{X,C} < \sigma_R^{X,C}$ (Ang and Bekaert, 2004).

Denote the risk-neutral measure by $\mathbb{Q}$. Note that the expected growth rates of the firm's nominal after-tax earnings under the risk-neutral measure, $\tilde{\mu}_i$, are given by

$$\tilde{\mu}_i := \mu_i - \sigma_i^{X,C}\left(\eta_i + \sigma^{P,C}\right) - \left(\sigma^{P,id}\right)^2, \tag{13}$$

and let $\tilde{\lambda}_i$ denote the risk-neutral transition intensities, determined as

$$\tilde{\lambda}_i = e^{\kappa_i}\lambda_i. \tag{14}$$

Following Chen (2010) and Bhamra, Kuehn and Strebulaev (2010b), the unlevered after-tax asset value can be written as

$$V_t = X_t y_i \quad \text{for } I_t = i, \tag{15}$$

with y_i being the price-earnings ratio in state i determined by

$$y_i^{-1} = r_i^n - \tilde{\mu}_i + \frac{\left(r_j^n - \tilde{\mu}_j\right) - \left(r_i^n - \tilde{\mu}_i\right)}{r_j^n - \tilde{\mu}_j + \tilde{p}}\tilde{p}\tilde{f}_j. \tag{16}$$

$\tilde{p} := \tilde{\lambda}_i + \tilde{\lambda}_j$ is the risk-neutral rate of news arrival, and $\left(\tilde{f}_B, \tilde{f}_R\right) = \left(\frac{\lambda_R}{\tilde{p}}, \frac{\lambda_B}{\tilde{p}}\right)$ is the long-run risk-neutral distribution. y^{-1} can be interpreted as a discount rate, in which the first two terms constitute the standard expression if the economy stayed in regime i forever, and the last term accounts for future time spent in regime j. As in Bhamra, Kuehn and Strebulaev (2010b), the price-earnings ratio in the main analysis is higher in boom than in recession, i.e., $y_B > y_R$.

Finally, note that the volatility of the earnings process in regime i is

$$\tilde{\sigma}_i = \sqrt{\left(\sigma_i^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2}. \tag{17}$$

The expansion option of the firm is modeled as an American call option on the earnings. Specifically, at any time $\bar{t}$, the firm can pay exercise costs K to achieve additional future after-tax earnings of sX_t for all $t \geq \bar{t}$ for some factor $s > 0$. We assume that if a firm exercises its expansion option, the option is converted into assets in place, such that the firm consists of only invested assets. The exercise of the growth option is assumed to be irreversible. At default, bondholders

recover not only a fraction of the assets in place, but also a fraction of the option's value. Intuitively, the option can be exercised independently of the considered firm.

For the financing of investment, we present two variants. In the main analysis, we wish to abstract away from the effect of fund injections by debt- or equityholders to pay the exercise price, and instead to isolate the effect of growth options in the value of firms' assets on corporate securities. Therefore, we first assume that, at exercise, the firm pays the exercise costs K of the option by selling a part of its assets in place.[8] In detail, while exercising the option at time $\bar{t}$ entitles the firm to total future after-tax earnings of $(s+1)X_t$ for all $t \geq \bar{t}$, financing the exercise costs requires to sell a fraction $\frac{K}{X_{\bar{t}} y_{\bar{i}}}$ of these earnings, in which $\bar{i}$ is the realized state of the economy at the time of exercise. Hence, the total after-tax earnings of the firm at any point in time after exercise correspond to $\left((s+1) - \frac{K}{X_{\bar{t}} y_{\bar{i}}}\right) X_t$. Second, we also consider equity financing of the exercise costs K.

The critical measure to capture the relative importance of a firm's expansion option in the value of its assets is the *asset composition ratio*. We define it as the value of the firm, divided by the value of invested assets. Certainly, the value of the firm does not only contain the value of the invested assets and the expansion option, but also the value of the tax shield and bankruptcy costs. Nevertheless, we use this measure because the direct empirical analog of the asset composition ratio is Tobin's Q. Furthermore, the impact of the tax shield and bankruptcy costs on the ratio is relatively small.

Corporate taxes are paid at a constant rate τ, and full offsets of corporate losses are allowed. In our framework, firms are leveraged because debt allows it to shield part of its income from taxation. Once debt has been issued, a firm pays a total coupon c at each moment in time. Following the standard in the literature, we assume that firms finance coupons by injecting funds. At any point in time, shareholders have the option to default on their debt obligations, as well as the possibility to exercise an expansion option. Default is triggered when shareholders are no longer willing to inject

[8] Indirect financing by selling assets often occurs, e.g., when acquirers divest part of target companies' assets following takeovers (Bhagat, Shleifer and Vishny, 1990; Kaplan and Weisbach, 1992). Of course, the model simplifies in that in reality, firms have different types of assets.

additional equity capital to meet net debt service requirements (Leland, 1998). If default occurs, the firm is immediately liquidated and bondholders receive the unlevered asset value less default costs, reflecting the absolute priority of debt claims. The default costs in regime i are assumed to be a fraction $1-\alpha_i$ of the unlevered asset value at default, with $\alpha_i \in (0,1]$. We suppose that recovery rates are lower in recession, i.e., $\alpha_R < \alpha_B$ (Frye, 2000). The incentive to issue debt is limited due to the possibility of costly financial distress.

Equityholders face the following decisions: First, once debt has been issued, they select the default and expansion policies that maximize equity value. Hence, both expansion and default are chosen endogenously. Second, they determine the optimal capital structure by choosing the coupon level that maximizes the value of the firm. The model does not allow restructuring of debt neither when the option is exercised nor at endogenous restructuring points. The main reason is that expansion opportunities preclude a scaling feature of the model solution.[9]

The main text presents the model and its solution for infinite debt maturity. We also solve and use the case of finite debt maturity, in which we consider the stationary environment of Leland (1998): The firm issues debt with a constant principal p and a constant total coupon c paid at each moment in time. A fraction m of the total debt is continuously rolled over. In particular, the firm continuously retires outstanding debt principal at rate mp and replaces it with new debt vintages of identical coupon and principal. Finite maturity debt is, therefore, characterized by the tuple (c, m, p). This setup leads to a time-homogeneous setting. Throughout the paper, it is assumed that debt is issued at par.

3. Model solution

We solve the model by backward induction. First, the value of the growth option for given expansion policies is derived. Then, for given corporate policies and capital structure, we proceed

[9]The scaling property states that, conditional on the current regime of the economy, the optimal coupon, the optimal default thresholds, the investment boundaries, as well as the values of debt and equity at restructuring points are all homogeneous of degree one in earnings. When assuming dynamic capital structure adjustments, the absence of a scaling property impedes not only closed-form results, but also the application of numerical solution methods with backward induction.

with the valuation of corporate securities for a firm that consists not only of assets in place, but also holds an expansion option. Finally, we obtain the expansion and default policies that simultaneously maximize the value of equity, as well as the capital structure that maximizes the firm value.

As in Hackbarth, Miao and Morellec (2006), we assume that the optimal strategies are of regime-dependent threshold type in X (for a formal proof in the case of expansion thresholds only, see Guo and Zhang, 2004). Precisely, suppose that $\hat{D}_i$ and D_i are the default thresholds in regime $i = B, R$ of a firm with only invested assets, and of a firm with both invested assets and a growth option, respectively. X_i denotes the exercise boundary of the growth option in regime $i = B, R$. In what follows, we present the case in which $D_B < D_R$, $X_B < X_R$ and $\hat{D}_B < \hat{D}_R$, i.e., the boundaries are lower in boom for both expansion and default (before and after expansion).[10] Finally, we presume that $\max\left\{D_R, \hat{D}_R\right\} < X_B$, i.e., we are interested in firms that exercise their expansion option with a positive probability, and we exclude the possibility of immediate default after expansion. The optimal default and expansion policies for relevant parameter regions satisfy this ordering.

3.1. The value of the growth option

Denote the value functions of the growth option in regime B and R by $G_B(X)$ and $G_R(X)$, respectively. The following proposition states the value of a growth option subject to regime switches.

Proposition 1. *For any given pair of exercise boundaries $[X_B, X_R]$, the value of the growth option in regime i is given by*

$$G_i(X) = \begin{cases} \bar{A}_{i3}X^{\gamma_3} + \bar{A}_{i4}X^{\gamma_4} & X < X_B, \quad i = B, R \\ \bar{C}_1 X^{\beta_1^R} + \bar{C}_2 X^{\beta_2^R} + \tilde{\lambda}_R \frac{sy_B X}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} - \tilde{\lambda}_R \frac{K}{r_R^n + \tilde{\lambda}_R} & X_B \le X < X_R, \quad i = R \\ sXy_i - K & X \ge X_i, \quad i = B, R, \end{cases} \tag{18}$$

in which $\gamma_k, k = 3, 4$, are the positive roots of the quartic equation

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B, \tag{19}$$

[10] Note that we can assume without loss of generality that $D_B < D_R$ (if not, interchange the names of the regimes). The case $D_B < D_R, \hat{D}_B < \hat{D}_R$, and $X_B > X_R$, (i.e., the exercise boundary in recession is lower than the one in boom) can be solved by analogous techniques.

and β_k^R, $k = 1, 2$, *are given by*

$$\beta_{1,2}^R = \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}}. \tag{20}$$

$\bar{A}_{Rk}$, $k = 3, 4$, *is a multiple of* $\bar{A}_{Bk}$ *with the factor*

$$\bar{l}_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1)). \tag{21}$$

$\left[\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1, \bar{C}_2\right]$ *solve a linear system given in Appendix A.2.*

Proof. See Appendix A.2. □

The functional form of the solution (18) is analogous to the one presented in Guo and Zhang (2004). For each regime i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise it is optimal to wait (option continuation region). We remark that, similar to the occurrence of default, there are two possible ways of triggering the exercise of the expansion option: Either when the idiosyncratic shock X reaches the exercise boundary X_i in a given regime, or when the regime switches from recession to boom and X lies between X_B and X_R.

In the option continuation region, the solution (18) reflects the changes in value that occur either when the idiosyncratic shock reaches a boundary, or when the regime switches. Proposition 1 shows that in the region $X < X_B$, i.e., the case in which the option is in the continuation region in both boom and recession, these value changes are captured by two terms. When the option is in the continuation region in recession only, i.e., $X_B \leq X < X_R$, the solution exhibits four terms. These four terms reflect the value changes when leaving this region due to hitting a boundary, either X_R from below or X_B from above, or due to a regime-switching induced exercise of the option. In the option exercise region, $X \geq X_i$, the firm obtains earnings of sX by investing K.

Proposition 1 determines the value of the growth option for any given pair of exercise boundaries X_B and X_R. In the full model solution, we derive option values for optimal exercise boundaries of equityholders in both levered and unlevered firms. In unlevered firms, the optimal exercise boundaries are denoted X_B^{unlev} and X_R^{unlev}, respectively. They are determined by smooth pasting conditions at the option exercise boundary. For ease of notation, we denote the unlevered value of

the growth option by G_i^{unlev}, i.e., $G_i^{unlev}(X) = G_i\left(X \left| X_B^{unlev}, X_R^{unlev}\right.\right)$. Appendix A.2 states the complete set of boundary conditions for the unlevered option value and presents the solution.

3.2. Firms with invested assets and expansion options

In this section, we derive the value of corporate securities of a general firm, as well as the default and expansion thresholds selected by shareholders.

After exercise, a firm consists of only invested assets, endowed with the initially determined optimal coupon level. The post-exercise value of corporate securities influences their pre-exercise value. As the default policy is an ex-post policy, the optimal default thresholds after exercise correspond to the ones of a firm with only invested assets. That is, equityholders optimally adapt their default policy upon expansion. Debtholders anticipate this change. Let $\hat{d}_i(X)$ denote the value of corporate debt of a firm with only invested assets, and $d_i(X)$ the value of debt of a firm with invested assets and an expansion option in regime $i = B, R$. The solution for $\hat{d}_i(X)$ can be found in Appendix A.3, the derivation being analogous to Hackbarth, Miao and Morellec (2006). The following proposition states the value of infinite maturity debt of a firm with invested assets and an expansion option.

Proposition 2. *For any given set of default and exercise boundaries $[D_B, D_R, X_B, X_R]$, the value of infinite maturity debt in regime i is given by*

$$d_i(X) = \begin{cases} \alpha_i\left(Xy_i + G_i^{unlev}(X)\right) & X \le D_i, \quad i = B, R, \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & \\ \quad + \tilde{\lambda}_B \frac{\alpha_R y_R}{r_B^n - \tilde{\mu}_B + \tilde{\lambda}_B} X + \frac{c}{r_B^n + \tilde{\lambda}_B} & D_B < X \le D_R, \quad i = B \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + \frac{c}{r_i^P} & D_R < X \le X_B, \quad i = B, R \\ B_1 X^{\beta_1^R} + B_2 X^{\beta_2^R} + Z(X) + \tilde{\lambda}_R \frac{c}{r_i^P\left(r_R^n + \tilde{\lambda}_R\right)} + \frac{c}{r_R^n + \tilde{\lambda}_R} & X_B < X \le X_R, \quad i = R \\ \hat{d}_i\left(\bar{s}X - \frac{K}{y_i}\right) & X > X_i, \quad i = B, R. \end{cases} \tag{22}$$

G_i^{unlev} denotes the unlevered option value in regime i (see Proposition 1), and

$$\beta_{1,2}^i = \frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2}\right)^2 + \frac{2\left(r_i^n + \tilde{\lambda}_i\right)}{\tilde{\sigma}_i^2}} \tag{23}$$

$$C_5 = \alpha_R \frac{\bar{l}_3}{l_3} \bar{A}_{B3}^{unlev} \tag{24}$$

$$C_6 = \alpha_R \frac{\bar{l}_4}{l_4} \bar{A}_{B4}^{unlev}. \tag{25}$$

$\gamma_k, k = 1, 2, 3, 4$ *are the roots of the quartic equation*

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B. \tag{26}$$

A_{Rk}, $k = 1, 2, 3, 4$, *is a multiple of* A_{Bk} *with the factor*

$$l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1)), \tag{27}$$

and r_i^p *is the perpetual risk-free rate given by*

$$r_i^p = r_i + \frac{r_j - r_i}{\tilde{p} + r_j}\tilde{p}\tilde{f}_j, \tag{28}$$

in which $\tilde{p} = \tilde{\lambda}_1 + \tilde{\lambda}_2$ *is the risk-neutral rate of news arrival, and* $\left(\tilde{f}_B, \tilde{f}_R\right) = \left(\frac{\lambda_R}{\tilde{p}}, \frac{\lambda_B}{\tilde{p}}\right)$ *the long-run risk-neutral distribution. The function* $Z(X)$ *is given by*

$$Z(X) = \tilde{\lambda}_R \sum_{i,k=1,2} \frac{2(-1)^{i+1}\bar{s}^{\gamma_k}\hat{A}_{Bk}}{\tilde{\sigma}_R^2\left(\beta_2^R - \beta_1^R\right)\left(\gamma_k - \beta_i^R\right)} X^{\gamma_k}{}_2F_1\left(-\gamma_k, \beta_i^R, \beta_i^R - \gamma_k + 1; -\frac{K}{\bar{s}Xy_B}\right), \tag{29}$$

in which ${}_2F_1$ *is Gauss' hyperbolic function.* $\hat{d}_i(\cdot)$ *denotes the value of debt of a firm with only invested assets.* $[A_{B1}, A_{B2}, A_{B3}, A_{B4}, C_1, C_2, B_1, B_2]$ *solve a linear system given in Appendix A.4.*

Proof. See Appendix A.4. □

In each regime, the firm faces three different regions depending on the value of X: Below the default threshold, i.e., $X \leq D_i$, the firm is in the default region where it defaults immediately, and debtholders receive a fraction α_i of the total asset value. The firm is in the continuation region if X is between the default threshold and the exercise boundary, i.e., $D_i < X \leq X_i$. Finally, the exercise region is reached if $X > X_i$, i.e., X is above the exercise boundary.

In the continuation region, the value of corporate debt is determined by three components. The first component is the value of a risk-free claim to the perpetual stream of coupon. The second and third components reflect the changes in the value of debt that occur either due to the idiosyncratic shock reaching a boundary, or due to a regime switch. Proposition 2 shows that for the region $D_R < X \leq X_B$, i.e., when the firm is in the continuation region in both boom and recession, the solution consists of five terms. The value of the risk-free claim to the coupon is given by the last term. The coupon is discounted by the perpetual risk-free rate r_i^p that takes into account

the expected future time spent in each regime. The first four terms capture the changes in value due to the idiosyncratic shock X hitting a region boundary, or due to a change of regime. When $D_B < X \leq D_R$, i.e., the firm is in the continuation region only in boom, the solution consists of six terms. The last term is the value of the risk-free claim to the coupon. Here, the discount rate is given by the nominal interest rate in boom, r_B^n, increased by $\tilde{\lambda}_B$ to reflect the possibility of a regime switch to recession. The first five terms capture the changes in debt value that occur when the idiosyncratic shock reaches a boundary, or when the regime switches to recession. For the region $X_B < X \leq X_R$, i.e., when the firm is in the continuation region only in recession, the solution consists of five terms. The last term is the value of a risk-free perpetual claim to the coupon. To account for a possible regime switch to boom, the discount rate is here given by the interest rate in recession, r_R^n, increased by $\tilde{\lambda}_R$. The remaining four terms capture the value changes due to reaching a region boundary, either X_B from above or X_R from below, or due to a regime switch to boom triggering immediate option exercise.[11]

The following remark shows how to express the value of finite maturity debt, tax benefits, and bankruptcy costs using Proposition 2. Let $t_i(X), b_i(X)$ denote the value of the tax shield and bankruptcy costs of a firm with both assets in place and an expansion option in regime $i = B, R$, respectively, and $\hat{t}_i(X), \hat{b}_i(X)$ the corresponding value functions of a firm with only invested assets.

Remark 1. *(i) The value of finite maturity debt with principal p and a fraction m of debt continuously rolled over is given by (22) in Proposition 2, in which c and r_i^n are replaced by $c + mp$ and $r_n^i + m$, respectively, and $\hat{d}_i$ is replaced by the value of finite maturity debt of a firm with only invested assets.*

(ii) The value of the tax shield is given by (22) in Proposition 2, in which c and α_i are replaced by $c\tau$ and 0, respectively, and $\hat{d}_i$ in the last line of (22) is replaced by $\hat{t}$.

(iii) The value of bankruptcy costs is given by (22) in Proposition 2, in which c and α_i are replaced by 0 and $1 - \alpha_i$, respectively, and $\hat{d}_i$ in the last line of (22) is replaced by $\hat{b}$.

Proof. See Appendix A.4. □

[11] Since the exercise of the option is financed by selling assets in place, the debt value after exercise is not homogeneous in X. The function $Z(X)$ captures this non-homogeneity after exercise, and can, therefore, not be simplified to a finite sum.

Next, the total firm value f_i in regime $i = B, R$ is given by the value of assets in place $y_i X$, plus the value of the expansion option $G_i(X)$ and the value of tax benefits from debt $t_i(X)$, less the value of default costs $b_i(X)$, i.e.,

$$f_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X). \tag{30}$$

As the total firm value equals the sum of debt and equity values, the equity value $e_i(X)$, $i = B, R$, can, hence, be written as

$$e_i(X) = f_i(X) - d_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X) - d_i(X). \tag{31}$$

Equityholders select the default and investment policies that maximize the value of equity ex-post. Denote these policies by D_i^* and X_i^*, respectively. Formally, the default policy that maximizes the equity value is determined by postulating that the first derivative of the equity value has to be zero at the default boundary in each regime. Simultaneously, optimality of the option exercise boundaries is achieved by equating the first derivative of the equity value at the exercise boundary with the first derivative of the equity value of a firm with only invested assets after expansion, evaluated at the corresponding earnings in both regimes. These four optimality conditions are smooth-pasting conditions for equity at the respective boundaries:

$$\begin{cases} e'_B(D_B^*) &= 0 \\ e'_R(D_R^*) &= 0 \\ e'_B(X_B^*) &= \hat{e}'_B((s+1)X_B^* - \frac{K}{y_B}) \\ e'_R(X_R^*) &= \hat{e}'_R((s+1)X_R^* - \frac{K}{y_R}). \end{cases} \tag{32}$$

We solve this system numerically.

For each coupon level c, debtholders evaluate debt at issuance anticipating the ex-post optimal default and expansion decisions of shareholders. As debt-issue proceeds accrue to shareholders, the latter do not only care about the value of equity, but also about the value of debt. Hence, the optimal capital structure is determined ex-ante by the coupon level c^* that maximizes the value of equity and debt, i.e., the value of the firm. Denote by $f_i^*(X)$ the firm value given optimal ex-post

default and expansion thresholds as determined by the System (32). The ex-ante optimal coupon of this firm solves

$$c_i^* := \text{argmax}_c f_i^*(X). \tag{33}$$

As indicated in Eq. (33), the optimal initial capital structure depends on the current regime.

4. Results

This section summarizes the model results for individual firms. Section 4.1 presents the parameter choice. We describe the firm sample in Section 4.2. Next, Section 4.3 discusses the properties of the expansion option. Section 4.4, finally, analyzes the optimal default policies of individual firms with different portions of the expansion options' value in the overall value of assets.

4.1. Choice of parameters

Table 1 summarizes our parameter choice. Panel A shows the firm characteristics that are selected to roughly reflect a typical BBB-rated S&P 500 firm.[12] We start with an initial value of the idiosyncratic after-tax earnings X of 10. While this value is arbitrary, neither credit spreads nor optimal leverage ratios depend on this choice. As is standard in the literature, we set the tax advantage of debt to $\tau = 0.15$ (Hackbarth, Miao and Morellec, 2006). Bhamra, Kuehn and Strebulaev (2010b) estimate growth rates and systematic volatilities of nominal earnings in a two regime model. Their estimates are similar to those obtained by other authors who jointly estimate consumption and dividends with a state-dependent drift and volatility (e.g., Bonomo and Garcia, 1996). Hence, the real earnings growth rates ($\mu_{i,real}$) and volatilities ($\sigma_{i,real}^{X,C}$) are chosen such that the nominal growth rates and systematic volatilities correspond to their empirical counterparts. Note that the relation $\sigma_B^{X,C} = 0.0834 < 0.1334 = \sigma_R^{X,C}$ captures the observation in Ang and Bekaert (2004) that asset volatilities are lower in boom than in recession.

Following Acharya, Bharath and Srinivasan (2007), we assume that recovery rates fall during recession. They report that recovery in a distressed state of the industry is lower than the recovery

[12] Our qualitative results do not depend on the ratings of firms.

in a healthy state of the industry by up to 20 cents on a dollar. The reason can be financial constraints that industry peers of defaulted firms face as proposed by the fire-sales or the industry-equilibrium theory of Shleifer and Vishny (1992), or time-varying market frictions such as adverse selection. We choose recovery rates as $\alpha_B = 0.7$ and $\alpha_R = 0.5$, respectively, which matches the 20 cents on a dollar difference in Acharya, Bharath and Srinivasan (2007), and is close to the standard of 0.6 used in the literature (Hackbarth, Miao and Morellec, 2006; Chen, 2010). Our qualitative results are insensitive to the choice of α_i as long as $\alpha_B > \alpha_R$.

Panel B shows the parameters we use to capture growth options. We select an exercise price of $K = 310$. The choice of a relatively high K is motivated by our intention to investigate firms that do not exercise their expansion option immediately. The scale parameter s for a typical firm is calibrated such that the asset composition ratio at initiation given optimal financing equals the average Tobin's Q of 1.6 in our sample of BBB-rated firms. In particular, s is set to $s = 1.89$ for firms initiated in boom, and to $s = 2.05$ for firms initiated in recession. To analyze growth firms with a larger (smaller) portion of option values in the overall value of their assets, we will later use higher (lower) scale parameters at initiation.

Panel C, finally, lists the variables describing the underlying economy. The rate of leaving regime i (λ_i), the consumption growth rates (θ_i), and the consumption growth volatilities σ_i^C are estimated in Bhamra, Kuehn and Strebulaev (2010b). We take the same values for comparability. In the described economy, the expected duration of regime B (R) is 3.68 (2.03) years, and the average fraction of time spent in regime B (R) is 64% (36%). The inflation parameters are estimated using the price index for personal consumption expenditures from the Bureau of Economic Analysis from 1947 to 2005. We obtain an expected inflation rate (π) of 0.0342, a volatility of the price index of 0.0137, and a correlation between the price index and real non-durables plus service consumption expenditures of -0.2575. These parameters imply a systematic price index volatility of $\sigma^{P,C} = -0.0035$ and an idiosyncratic price index volatility of $\sigma^{P,id} = 0.0132$.

The annualized rate of time preference, ρ, is 0.015, the relative risk aversion, γ, is equal to 10 and the elasticity of intertemporal substitution, Ψ, is set to 1.5. This parameter choice is commonly used in the literature (Bansal and Yaron, 2004; Chen, 2010).

Our choice of parameters implies that real interest rates are $r_B = 0.0416$ and $r_R = 0.0227$ in the baseline specification. The relative decline in the value of invested assets following a shift from boom to recession is equal to 12.61%, which is similar to the one assumed in Hackbarth, Miao and Morellec (2006).

INSERT TABLE 1 HERE

4.2. Firm sample

Balance sheet and ratings data are collected over the period from 1995 to 2008 from Compustat. We use data for BBB-rated firms. We calculate the quasi-market leverage of a firm as the ratio of book debt to the sum of book debt and market value of equity. Tobin's Q is defined as total assets plus the market value of equity minus the book value of equity divided by total assets.[13] We delete financial and utility firms from the sample. For each firm, we calculate the average of the leverage ratios and Tobin's Qs over the observation period. Next, we cut extreme values of both average leverage and Tobin's Q at 1% to avoid that our results are driven by outliers. Our sample then consists of 717 distinct firms. Fig. 1 plots the resulting data points. For the entire sample of BBB-rated firms, the mean leverage is 41.83%, and the mean Tobin's Q (asset composition ratio) is 1.59.

INSERT FIG. 1 HERE

4.3. Properties of the expansion option

To understand the implications of our model for credit spreads, it is instructive to first consider some properties of the expansion option.

[13] In these definitions, we follow, e.g., Baker and Wurgler (2002), Fama and French (2002) and Daines, Gow and Larcker (2010). Book debt is total assets (item 6, AT) minus book equity. Book equity is total assets minus total liabilities (item 181, LT) minus preferred stock (item 10, $PSTKL$, replaced by item 56 when missing, $PSTKRV$) plus deferred taxes (item 35, $TXDITC$) plus convertible debt (item 79, $DCVT$). The market value of equity is given by the closing price (item 24, $PRCC\_F$) times the number of common shares outstanding (item 25, $CSHO$).

Fig. 2 depicts the equity value maximizing exercise policy of the expansion option in a typical firm initiated in boom. Recall that the expansion policy is simultaneously determined with the default policy.

INSERT FIG. 2 HERE

The area above the dashed line is the exercise region in recession, and the area below the dashed line corresponds to the continuation region. In boom, the regions are defined analogously with respect to the solid line. The graph is drawn for optimal leverage. Exercising the option at time $\bar{t}$ entitles the firm to total future after-tax earnings of $(s+1)X_t$ for all $t \geq \bar{t}$. As expected, the endogenous exercise boundaries decrease with s. For example, consider initiation in boom: With a scale parameter of $s = 1.89$ (that induces an asset composition ratio of 1.6 at initiation), the corresponding optimal option exercise boundaries are $X_B^* = 18.26$ and $X_R^* = 19.55$. Setting s to 2.73 creates a growth firm with an asset composition ratio of 2.2, and optimal option exercise boundaries of $X_B^* = 12.88$ and $X_R^* = 13.90$, respectively. Importantly, Fig. 2 also shows that the expansion option is exercised at lower levels of the idiosyncratic earnings X in boom than in recession. Intuitively, the main reason is that the value of the option of waiting is higher in recession due to the potential switch to boom with a higher valuation of earnings.[14] The same qualitative option value properties also hold at non-optimal leverage levels.

Fig. 3 plots the value of the expansion option as a function of the after-tax earnings X, using jointly optimal expansion and default policies.

INSERT FIG. 3 HERE

Obviously, the option's value is affected by the current regime. When the asset value jumps due to a regime switch, so does the value of the option. Critically, relative value changes of expansion options are higher than relative value changes of assets in place when the regime switches. The

[14]The regime dependent volatilities and default thresholds also affect optimal exercise boundaries in boom and recession. We find that the valuation of earnings is the dominating effect for reasonable parameter values.

reasons are that options represent levered claims, and that the endogenous exercise boundary is higher in recession than in boom, as shown in Fig. 2.[15]

Additionally, Fig. 3 shows that both option value functions are convex, but the value function in boom is steeper than the one in recession. Therefore, the expansion option's value is less sensitive to the underlying earnings in recession than in boom. Intuitively, the exercise boundary increases and the earnings' drift decreases in recession, which drives options out-of-the money. As a consequence, an expansion option represents a less levered claim in bad times. While in recession the volatility of X is higher, the sensitivity of a growth option's value to changes in the earnings is lower. As discussed in the next section, this lower sensitivity attenuates the increase in the equityholder's default option due to a higher volatility of X during recession.

4.4. Optimal default policy

This section explains how the optimal default policy is affected by the presence of growth options in the value of firms' assets. To keep the intuition tractable, we do not comment on the (minor) impact of the exercise boundaries on default thresholds, which arises due to the simultaneous optimization of the expansion and default policy.

For all firms – those with and those without an expansion option – the optimal default policy is determined by recognizing that, at any point, shareholders can either make coupon payments and retain their claim together with the option to default, or forfeit the firm in exchange for the waiver of debt obligations. When the economy shifts from boom to recession, the present value of future after-tax earnings declines mainly because firm earnings have a lower drift, and because they become both more volatile and more correlated with the market. This present value decline reduces the continuation value (the expected value from keeping the firm alive) for equityholders, inducing them to default earlier (at higher earnings levels) in recession. We refer to this effect as the *value effect*. On the other hand, a high earnings volatility in recession makes the option to default more valuable, which defers default in bad times. This is the *volatility effect*. As in the

[15] Relative value changes are determined in Appendix A.2. In untabulated results, we confirm numerically that the relative value changes are indeed higher for expansion options than for the underlying assets in place for plausible parameter values.

models for invested assets of Bhamra, Kuehn and Strebulaev (2010b) and Chen (2010), the value effect usually dominates the volatility effect, generating higher default thresholds in recession, i.e., leading to counter-cyclical default thresholds. Counter-cyclical default thresholds together with a high volatility in bad times imply counter-cyclical default probabilities, consistent with empirical evidence (Chava and Jarrow, 2004; Vassalou and Xing, 2004). Additionally, default losses are empirically reported to be higher in recession because many firms experience poor performances during such times. Combined with higher marginal utilities in bad times, these mechanisms raise the present value of expected default losses for bondholders which leads to higher credit spreads and lower optimal leverage ratios than in standard contingent claim models.

Fig. 4 draws the equity value maximizing default policy of levered firms initiated in boom. The graph shows default thresholds for a range of asset composition ratios. Leverage is held constant at 41.83%.[16] The solid line shows the default threshold in boom, and the dashed line the one in recession. For example, for a firm with only invested assets the optimal default thresholds are $D_B^* = 2.45$ and $D_R^* = 2.69$. For an average firm with an asset composition ratio of 1.6 they are $D_B^* = 3.19$ and $D_R^* = 3.55$, and for a growth firm with an asset composition ratio of 2.2 they are $D_B^* = 3.52$ and $D_R^* = 3.94$. In the no-default region above the line corresponding to a given regime, the continuation value for equityholders exceeds the default value and it is optimal for shareholders to keep injecting funds into the firm.

INSERT FIG. 4 HERE

Two points from Fig. 4 are particularly noteworthy. First, the optimal default thresholds increase as the asset composition ratio increases, inducing a higher default probability. This finding evolves from the observation that growth options represent levered claims, which are relatively more sensitive than invested assets to a given decrease in X. Second, while all firms are more likely to default in recession than in boom, the increasing distance between D_B^* and D_R^* for larger asset composition ratios indicates that the counter-cyclicality of default boundaries is particularly

[16] When the scale parameter is changed but the coupon is left constant, default thresholds are not directly comparable. The reason is that the total asset value increases with s for every X. Considering constant leverage assures that the considered coupon changes consistently with the increase in the total asset value when we alter s.

pronounced for growth firms. The reason is that due to the higher relative value change of growth options upon a regime switch, the value effect is stronger for a firm with a high asset composition ratio. Additionally, because options represent less levered claims in recession than in boom, the increase in the equityholders' default option - due to the higher volatility of X when the regime switches to recession - is attenuated for growth firms. In other words, the volatility effect, which tends to decrease the distance between the default thresholds, is weaker for firms with larger expansion options.

5. Aggregate dynamics of leverage, asset composition, investment and defaults

To validate our structural equilibrium framework with intertemporal macroeconomic risk and investment, we analyze the dynamic properties of our model-implied economy. In this section, we qualitatively compare the aggregate predictions for the entire economy to empirically reported capital structure, investment, and default patterns.

5.1. Simulation

We generate a dynamic economy of average firms implied by our model. We consider 1,000 identical firms with infinite debt maturity. Initially, each firm's after-tax earnings are $X = 10$, and the option scale parameter is assumed to be $s = 1.89$ if the firm's initial regime is boom, and $s = 2.05$ otherwise. These choices of s imply an asset composition ratio of 1.6 in both states at initiation, given optimal leverage. Firms receive the same macroeconomic and inflation shocks, but experience different idiosyncratic shocks. Each firm observes its current earnings as well as the current regime on a monthly basis and behaves optimally: If the current earnings are below the corresponding regime-dependent default threshold, the firm defaults immediately; if the current earnings are above the corresponding regime-depending option exercise boundary, the firm exercises its expansion option; otherwise, the firm takes no action.

In our model, firms have a growth option, which can only be exercised once. To maintain a balanced sample of firms, and to avoid that the average asset composition ratio is systematically trending towards the one of a firm with only invested assets when we simulate the economy over

time, we exogenously introduce new firms. In particular, we substitute each defaulted or exercised firm by a new firm whose growth option is still intact. New firms have initial after-tax earnings of $X = 10$, and an option scale parameter s according to the current regime as described above.

To ensure convergence to the long-run steady state, we first simulate the economy for 100 years. The starting period for the reported results is the final period of the first 100 years of simulation. Next, we simulate the model for 200 years and present the aggregate dynamics.

5.2. Results

We start by discussing the cyclicality of leverage. Hackbarth, Miao and Morellec (2006) generate counter-cyclical optimal leverage ratios in their macroeconomic model. As in our framework, the optimal coupon rate of debt initiated in boom exceeds the one in recession. At the same time, the value of assets is greater in boom. The second effect dominates the first, generating the counter-cyclicality in optimal leverage. We additionally incorporate the empirical fact that asset volatility is regime-dependent. Because the latter decreases in boom and increases in recession, our optimal coupon rate varies more than in Hackbarth, Miao and Morellec (2006) when the regime changes. With this extension, the change in the value of optimal debt dominates the change in the value of assets, generating pro-cyclical optimal leverage ratios for realistic parameter values, in line with Covas and Den Haan (2006) and Korteweg (2010). Fig. 5 plots the simulated market leverage in the economy. Shaded areas represent recessions. Even though our optimal initial leverage ratios are pro-cyclical, the simulated time series shows that actual aggregated market leverage is counter-cyclical. The reason is that when firms are stuck with the debt issued at initiation, the equity value declines more than the debt value during recessions, which tends to increase leverage in bad times. This prediction conforms to Korajczyk and Levy (2003) who show that unconstrained firms' leverage ratios vary counter-cyclically.

INSERT FIG. 5 HERE

Fig. 6 shows the time series of the aggregate asset composition ratio in the simulated economy. As expansion options are more sensitive to the underlying stochastic processes than invested assets, the ratio behaves pro-cyclically, as reported in the literature.

INSERT FIG. 6 HERE

We investigate aggregate default rates in Fig. 7. Simulated default rates are counter-cyclical, consistent with the empirical fact that most defaults occur during economic recessions. Additionally, the graph shows several spikes in default rates that occur right at the time when the economy enters into a recession, consistent with the empirical evidence in Duffie, Saita and Wang (2007) and Das, Duffie, Kapadia and Saita (2007) (see, e.g., around years 50 and 90). Recall that defaults can occur because either the idiosyncratic earnings reach the default threshold in a given regime, or due to a change of the macroeconomic regime from boom to recession. The clustered default waves occur due to an increase in firms' default thresholds upon such a regime change. All firms between the two thresholds default simultaneously when the regime switches to recession, even though their earnings do not exhibit instantaneous regime-induced changes. After such waves of default, the default frequency tends to remain high during recessions.

As a refinement of this general result, we expect that the tendency to default during recession should be particularly pronounced for firms with high expansion options. This prediction is suggested by the fact that the degree of counter-cyclicality of default thresholds is positively related to the initial asset composition ratio. We investigate the propensity to default during recession in a dynamic, simulation-based setting by counting default rates of two separate aggregate economies. The first one is designed as above, consisting of firms with both assets in place and growth options, such that the asset composition ratio at initiation is 1.6. The second setting consists of firms with only invested assets. To construct a number of cross-sectional distributions of firms, we first simulate 20 dynamic economies for ten years. Using each economy obtained at the end of the first ten years, the default rates in both regimes are observed for 50 subsequent simulations of the following 20 years, resulting in a total of 1,000 simulations. The average percentage of defaults that occurs during recession is then calculated.[17] We find that in the first economy, on average, 75.41%, 76.79%, and 77.66% of total defaults of firms with assets in place and growth opportunities

[17]The distance to default in the aggregate economy of firms with only invested assets is trending over time. The reason is that firms that default are replaced, but there are no option exercises after which well performing firms could be replaced. Consequently, we do not compare absolute default rates of the two economies, but rather the fraction of defaults occurring in each regime.

occur during recession over five, ten, and 20 years, respectively. In the economy in which firms only have invested assets, the corresponding numbers are considerably smaller at 66.40%, 71.66%, and 73.71%, respectively.

This finding is also related to the observation that, on average, growth firms have lower recovery rates than value firms (Cantor and Varma, 2005). The standard argument offered by Shleifer and Vishny (1992) is that growth firms as potential buyers of growth assets have little cash relative to the value of assets. Hence, they are likely to be themselves credit constrained when other growth firms sell their assets upon default, which lowers recovery rates. Our model delivers an alternative explanation: We show that growth options in the value of firms' assets create a propensity to default during recession, when recovery rates are low.

INSERT FIG. 7 HERE

A significant literature suggests that business cycle shocks common to all firms play a crucial role in explaining aggregate investment. In particular, there is evidence that aggregate investment is characterized by both episodes of very intense investment activity and periods of very low investment activity (Doms and Dunne, 1998; Oivind and Schiantarelli, 2003). Moreover, aggregate investment and the probability of investment spikes are strongly pro-cyclical (Barro, 1990; Cooper, Haltiwanger and Power, 1999). Our model reflects these features. First, when the regime switches from recession to boom, firms in the region between the two investment boundaries exercise their expansion option simultaneously by investing K. Fig. 8 shows that investment spikes often occur upon such regime switches (see for example around year 35, or year 60). After these spikes, simulated investment rates tend to remain high during boom due to the positive drift of the earnings. Hence, we observe pro-cyclical investment spikes followed by higher investment activity during booms. At the other end, investment activity often dries out when the economy switches from boom to recession, because the optimal exercise boundary jumps up and the expected earnings' drift turns negative. Our model also predicts that observed investment waves should be mainly driven by firms with high expansion options.

INSERT FIG. 8 HERE

Finally, we plot simulated average credit spreads in Fig. 9. Credit spreads are calculated as $(c/d_i(X)) - r_i^p$, in which r_i^p is the perpetual risk-free rate defined in (28). Consistent with the empirical literature (Fama and French, 1989), we find counter-cyclical credit spreads. When the economy stays in boom, credit spreads tend to decline as distances to default increase due to the positive expected drift of the earnings and the lower default threshold. Conversely, in recession, credit spreads rise as distances to default tend to decline and the volatility increases.

INSERT FIG. 9 HERE

6. Quantitative implications and empirical predictions

In this section, we discuss the quantitative implications and empirical predictions of our model. The attention is restricted to BBB-rated firms since it has been argued that the pricing of very high-grade investment firms is dominated by factors other than credit risk such as liquidity risk or a tax component (Longstaff, Mithal and Neis, 2005; De Jong and Joost, 2006). We start by determining target observed average credit spreads. Duffee (1998) estimates an average yield spread in the industrial sector between BBB-rated bonds and Treasury yields of 198, 148, and 149 bps for bonds with a mean maturity of 21 years (long), 8.9 years (medium), and 4.7 years (short), respectively. Davydenko and Strebulaev (2007) report somewhat lower spreads of 143 bps for bonds with $15-30$ years (long), 115 bps for $7-15$ years (medium), and 115 bps for $1-7$ years maturity, respectively.[18] From these spreads, we subtract 35.5% to reflect the results in Longstaff, Mithal and Neis (2005) and Han and Zhou (2011) who find non-default components in BBB bond yields of 29% and 42%, respectively. We arrive at a plausible target range of around 92 to 128 bps for long maturities, 74 to 95 bps for medium maturities, and 74 to 96 bps for short maturities.[19] Panel A in Table 2 tabulates these target credit spread ranges. In Panel B, we also report empirical default rates of BBB-rated debt over five, ten, and 20 years from Moody's (2010).

[18] The estimates of short and medium maturities in Huang and Huang (2003) are higher because of the embedded call options in the corporate bond sample and the inclusion of two recessions with high spreads.

[19] We recalculate target ranges by subtracting the absolute non-default component for BBB firms of 61.8 bps reported in Han and Zhou (2011), or by subtracting the 29% reported in Longstaff, Mithal and Neis (2005) for an earlier sample period. Our model's performance does not depend on the exact definition of targets.

INSERT TABLE 2 HERE

We discuss the implications of our model for credit spreads and leverage along two dimensions. First, we follow the traditional way of investigating a typical individual firm. Second, we implement an approach similar to the one proposed by Bhamra, Kuehn and Strebulaev (2010b) in which credit spreads and leverage ratios are calculated as cross-sectional averages based on a simulation of the empirical distribution of BBB-rated individual firms.

6.1. Credit spreads

6.1.1. Typical firm with endogenous default boundary

Credit spreads for various models on newly issued corporate debt are calculated in Table 3 for five (short), ten (medium), and 20 (long) years maturity.[20] We follow the standard approach in structural models by calibrating the idiosyncratic earnings volatility such that the total asset volatility is approximately 25% in each model, the average asset volatility of firms with outstanding rated corporate debt (Schaefer and Strebulaev, 2008). Additionally, we fix leverage at the average ratio of 41.83% in our BBB-firm sample.

Importantly, the default boundaries and expansion thresholds are assumed to be chosen optimally by equityholders, as we are interested in whether our model can generate both realistic prices of corporate claims and realistic endogenous default and expansion rates. Specifying default boundaries exogenously such that a model's actual default probabilities match the data (as done in Chen, Collin-Dufresne and Goldstein (2009) or Huang and Huang (2003)) not only substantially dilutes the value of the option to default, but also distorts the value of the expansion option because the latter depends on the default policy.

It is well-known that structural models of default typically generate credit spreads that are too low compared to their empirical counterpart. To illustrate this point, we first analyze the model without business cycle risk in Panel A of Table 3. The expected drifts and systematic volatilities of earnings and consumption are set equal to their unconditional means. Panel A shows credit

[20] The value of a finite maturity risk-free bond is given in Appendix A.4, Formula (A-74), in which c is replaced by $c + mp$ and r_i^n is replaced by $r_i^n + m$.

spreads for different maturities of the standard structural model of Leland (1998). The empirical target credit spreads in Table 2 are about five times larger for the short maturity, and about three times larger for the medium and long term than those predicted by the structural model.

INSERT TABLE 3 HERE

Bhamra, Kuehn and Strebulaev (2010b) and Chen (2010) derive structural multi-regime models for typical firms that consist of only invested assets. We closely replicate their approach for an average firm within a two-regime model. To match the asset volatility of 25%, the idiosyncratic earnings volatility is set to $\sigma^{X,id} = 0.21$. Panel B reports unconditional credit spreads, calculated as a weighted average of the state-dependent credit spreads, in which the weights correspond to the long-run distribution of the Markov chain. For comparability to our setting with expansion options, the results without debt restructuring are presented. While the credit spreads for typical firms of 35, 56, and 78 bps for five, ten, and 20 years maturity, respectively, are clearly higher than in the one regime model, they are still considerably below their targets.[21]

Next, we investigate our model with expansion options for a typical BBB-rated firm. Note that for a given idiosyncratic earnings volatility, firms with different asset composition ratios have different total asset volatilities due the inherent leverage of their expansion option. Moreover, a firm's asset volatility is not constant over time, as its option's moneyness changes when X moves towards or away from the exercise boundary. To obtain the average volatility for a certain rating class, the standard approach in the literature is to average the calculated asset volatilities over all firms with the same rating (Vassalou and Xing, 2004; Duan, 1994; Schaefer and Strebulaev, 2008). We calibrate the idiosyncratic volatility $\sigma^{X,id}$ to the empirically reported average asset volatility of 0.25: Given an idiosyncratic volatility $\sigma^{X,id}$, we simulate model-implied samples of BBB-rated firms over ten years, and calculate the resulting average asset volatility. (Details on the simulation can be found in Appendix A.5.1.) The calibration yields $\sigma^{X,id} = 0.168$, which ensures that the

[21] Bhamra, Kuehn and Strebulaev (2010b) use higher recovery rates, lower leverage and do not model the impact of principal repayments on default thresholds, which results in marginally lower credit spreads in their static case. Chen (2010) obtains larger ten year credit spreads in a model with nine states and a dynamic capital structure, but uses higher leverage, and a cash flow volatility that induces a much higher asset volatility than empirically observed.

average asset volatility of our simulated BBB-rated firms with expansion options corresponds to its empirical counterpart.[22]

Panel C of Table 3 shows the resulting credit spreads for typical firms. Several aspects are noteworthy about these results. Our model increases the unconditional credit spreads of an average firm for five, ten, and 20 years from 18 bps to 45 bps (+150%), from 29 bps to 66 bps (+128%), and from 41 bps to 84 bps (+105%), respectively, compared to the one regime model in Panel A. To understand this large effect, recall first that macroeconomic models generate larger credit spreads than one regime models because recessions are times of high marginal utility, so that default losses that occur during these times will affect investors more. An important economic implication is that the average duration of bad times in the risk-neutral world is longer than in the actual world. Since the representative agent uses risk-neutral and not actual probabilities to account for risk and to compute prices, credit spreads are larger and the agent behaves more conservatively than historical default losses imply. Second, if firms have a higher tendency to default in recession, this discrepancy will increase due to the higher risk premium. Our model shows that because of the strong sensitivity of option values to regime switches, and because they are less sensitive to the underlying earnings during recession, the counter-cyclicality of default thresholds is more pronounced for firms with larger growth options. The resulting stronger counter-cyclicality of the default probability of growth firms thus drives up their credit spreads. As can be seen in row 2 of Panel C, the credit spreads for an average firm, consisting of both invested assets and growth options, are 45 bps, 66 bps, and 84 bps for debt maturities of five, ten, and 20 years, respectively. This is, respectively, 29%, 18%, and 8% higher than the credit spreads of an average firm in the standard macroeconomic model with only invested assets.[23]

[22] We also repeat this exercise with different specifications, such as alternative simulation length and debt maturity. The resulting idiosyncratic volatilities are fairly insensitive to these variations. An alternative approach is to calibrate the idiosyncratic volatility to the cumulative default probability of BBB-rated firms (Chen, 2010). This procedure, however, usually leads to asset volatilities that are higher than the ones empirically observed.

[23] We cannot directly compare the results for invested assets in Panel C to the ones for average firms in Panel B, even though the latter consist of only invested assets. The reason is that in our model, the idiosyncratic volatility is calibrated such that the asset volatility of the entire sample of BBB-rated firms matches 0.25, whereas in Panel B, $\sigma^{X,id}$ is chosen such that firms with only invested assets have an asset volatility of 0.25.

Besides the fact that they generate too low credit spreads, another problem of existing structural models is that the implied term structure of credit spreads at initiation is much steeper than its empirical counterpart for a typical firm. The reason is that the implied spreads are particularly low at the short end. Most existing studies with macroeconomic models use the default thresholds of infinite maturity debt (that is, debt without principal repayments) to numerically calculate the risk-neutral default probability for each maturity. As the credit risk literature identifies firms' debt maturity as an important determinant of credit risk (Gopalan, Song and Yerramilli, 2010; He and Xiong, 2011), we endogenously derive optimal default thresholds also for finite debt maturity following the approach of Leland (1998). Due to the continuous principal repayments, these thresholds are considerably higher for short maturities than for infinite debt, resulting in larger credit spreads at the short end. The resulting term structure of credit spreads for an average firm in Panel A, B, and C is consequently flatter and, hence, closer to the shape observed in target spreads than when using default thresholds of infinite maturity debt.[24]

The rows in Panel C of Table 3 identify the cross-sectional relationship between the asset composition ratio and credit risk. To tease out the effect of growth options on credit spreads, we vary the asset composition ratio by altering s. As raising s increases the value of the expansion option, we simultaneously adapt the coupon to maintain a constant leverage of 41.83%.[25] This exercise shows that the asset base of the firm is an important driver of credit risk, implying a positive relationship between the portion of growth options in the value of a firm's assets and the costs of debt. In particular, altering the asset composition ratio of a firm from 1 to 2.2 increases credit spreads by about 56% to 96%, depending on the debt maturity. This effect is remarkable given that we solely vary the assets' characteristics. It arises for two reasons in our model. First, because options are levered, and due to the endogenous investment boundary, expansion options are

[24] We use default boundaries for the appropriate debt maturities in both Panels B and C to highlight the pure effect of expansion options on credit spreads.

[25] Alternatively, changing both s and K to alter the asset composition qualitatively retains the aggregate and cross-sectional predictions. Holding s constant while only varying K implies large decreases in the option exercise boundaries for relatively small increases of the asset composition ratio. In the extreme, a firm with a very low K will exercise its expansion option almost immediately; in essence, credit spreads then virtually mirror those of a firm with only invested assets, diluting the model's cross-sectional predictions. Note also that any variation in K changes the costs of investment. By only varying s, we instead avoid that our results are driven by different sizes of the expected financing in case of equity-financed investment costs.

more sensitive to the underlying uncertainty, and, hence, more volatile. This higher volatility drives up the default probability of growth firms. Second, a higher portion of the expansion option's value in the overall asset value of a firm induces a higher counter-cyclicality of the default probability, which raises expected default costs. The higher default probability and larger default costs both increase the costs of debt for growth firms.

Note that while firms with growth options generally have a higher credit spread than firms with only invested assets (ceteris paribus), credit risk is concave in the asset composition ratio. This concavity occurs because firms with a larger asset composition ratio are closer to their exercise boundary, where credit spreads also reflect that the asset volatility and the counter-cyclicality of the default thresholds will decrease when a firm exercises its expansion option.

Our model rationalizes empirical properties of the cross-section of credit risk. For example, Davydenko and Strebulaev (2007) find that market-to-book asset values, the ratio of research and development expenses to total investment expenditure, and one minus the ratio of net property, plant, and equipment to total assets are all significantly and positively related to credit spreads (Table VI on p. 2652). Similarly, Molina (2005) shows that firms with a higher ratio of fixed assets to total assets have lower bond yield spreads and higher ratings (Table II on p. 1438). This evidence implies that, empirically, even after controlling for most factors relevant to credit risk in standard structural models, credit spreads are higher for growth firms. Hence, while an average firm with valuable growth options exhibits, for example, a different tax advantage of debt or payout ratio than a firm that only consists of invested assets, simple variation of such input parameters would not explain these findings. What is needed to address the aggregate puzzle and the mentioned cross-sectional evidence is a model that generates higher explained credit risk than standard models for a given level of input parameters. Our model delivers this result.

6.1.2. True cross-section

The previous section calculates credit spreads of a typical individual firm, which is consistent with the historically observed average input parameters of firms in the same rating class of which the individual firm is representative.

In this section, inspired by the work of Bhamra, Kuehn and Strebulaev (2010b), we employ a simulation approach to capture the dynamics of the cross-sectional distribution of firm characteristics. The central insight of our approach is that BBB-rated firms are very different with respect to both leverage and asset composition ratios, and that credit spreads and default rates are highly non-linear in these characteristics. Moreover, the previous section considers credit spreads solely at debt issuance points, when the principal corresponds to the market value of debt. The majority of empirically reported spreads are, however, based on observations made at times when debt is not being issued. To capture the impact of these issues, it is important to calculate credit spreads and default rates for a simulated sample of firms that matches the observed empirical distribution, i.e., the true observed cross-section of BBB-rated companies. The resulting average of simulated credit spreads can then be compared to the empirical average credit spread. Simultaneously, the approach allows us to verify whether the default probabilities implied by our model correspond to the reported historical default probabilities of BBB-rated firms.

To obtain the implications of the true cross-section of BBB-rated firms, we start by generating a distribution of firms implied by the model. In particular, we set up a grid of optimally leveraged firms with scale parameters s ranging from zero up to the largest possible value such that the option is not exercised immediately. The step size is 0.05, and 50 identical firms are considered for each value of the option scale parameter. Earnings paths of all firms are then simulated forward over ten years, resulting in a model-implied economy populated by more than $3,000$ firms. This economy has a broad range of leverage ratios and asset composition ratios.

In a second step, we match our historical distribution of BBB-rated firms with its model-implied counterpart. For each observation in the average empirical cross-section, we select the firm in our model-implied economy with the minimum distance regarding the percentage deviation from the target average market leverage and asset composition ratio. The matching is generally very accurate. Considering a debt maturity of ten years yields an average Euclidean distance of 0.0648, with the 85%-quantile being 0.0865.[26] That is, on average, only 15% of the firms are matched

[26] Other debt maturities yield virtually identical results for the matching accuracy.

with the root of the sum of the squared percentage deviations being larger than 8.65%.[27] Note that while our initial model-implied economy potentially contains firms with different ratings, the described matching procedure allows us to construct a cross-sectional distribution of model-implied firms that closely reflects its empirical BBB-rated counterpart.

Next, earnings paths of the 717 matched BBB-model-firms are simulated forward for 20 years on a monthly basis. This simulation is repeated 50 times.

The outcome of both the matching and the forward simulation of the matched sample also depends on the particular realizations of the idiosyncratic shocks and the states of the economy in the first simulation step. Hence, to explore the distributional properties of our results, the entire procedure is conducted 20 times, which results in a total of $1,000$ simulations. Details on the simulation are given in Appendix A.5.2.

Panel D of Table 3 summarizes the results. The average credit spreads, calculated during five years after the matching, are 57 bps for five years, 81 bps for ten years, and 100 bps for 20 years.[28] Hence, our model closely matches the historical levels reported in Table 2 for ten and 20 years. Five year credit spreads are somewhat lower than their target. We also measure the cyclicality of credit spreads. Average ten year credit spreads, for example, are 59 basis points during boom, and 115 during recession. As expected, they are strongly counter-cyclical.

Importantly, average credit spreads for the simulated true cross-section are considerably higher than the ones of a typical firm at initiation. There are two reasons for this result. First, some firms will be near default, and credit spreads are convex in the distance to default. Second, the market value of debt corresponds to the principal at initiation. In practice, however, firms are not at initiation most of the time. The actual market value of debt will, therefore, often underestimate

[27] The market leverage is matched with an average distance of 0.0248. The average percentage distance of the asset composition ratio of 0.0549 is larger. This number is driven by a few firms with unusually high asset composition ratios. As they would optimally exercise their expansion option immediately in our model, these firms are matched with model firms with a somewhat lower asset composition ratio. We expect a minor impact of this limitation on our results, because firms with unusually high asset composition ratios also have very low leverages, and, hence, are not driving our average credit spreads.

[28] We follow Bhamra, Kuehn and Strebulaev (2010b) in measuring average credit spreads over a five year period. During longer periods, many firms could deviate substantially from the initial average distribution, and would, therefore, not be BBB-rated anymore.

the burden from the principal repayments, and especially so for firms approaching their default boundary. The reason is that the market value can hardly go beyond the principal as it is bounded above by the value of risk-free debt, but can easily reach values below the principal when earnings deteriorate. Our simulation of the true cross-section captures these asymmetric deviations over time, resulting in higher average credit spreads than those of firms observed at initiation. Compared to Bhamra, Kuehn and Strebulaev (2010b), the additional credit spreads generated from simulating the true cross-section are lower, because we do not incorporate debt restructuring.

To verify whether our model generates default rates corresponding to the empirically reported default frequencies for realistic debt maturities, we also count cumulative default rates in the simulated true cross-section. The model-implied average and median cumulative default rates over several years are reported in each Panel of Table 4. Panel A presents default rates over five, ten, and 20 years from simulations with firms issuing infinite maturity debt. Panels B, C, and D show default rates from simulations with firms issuing finite maturity debt. Due to the principal repayments, default thresholds of firms with finite maturity debt are considerably higher than those of firms with infinite maturity debt. Note that simulated credit spreads are consistent with a range of realized ex-post default rates, as observed default rates vary depending on a particular realization of good and bad states. Therefore, we also report the 25% and 75% percentiles of the distribution.

Empirically, Datta, Iskandar-Datta and Patel (2000) report a mean maturity of IPO bonds of 12 years, Guedes and Opler (1996) obtain an average maturity of 12.2 years for seasoned debt offers, and Davydenko and Strebulaev (2007) measure a mean time to maturity of BBB-bonds in the industrial sector of 9.51 years. Panel C of Table 4 shows that when assuming that firms have a debt maturity of ten years, our model-implied median default rates over five, ten, and 20 years are very close to the historical default probabilities observed from 1920 to 2009 reported in Table 2. Hence, for a realistic debt maturity, our median economy is consistent with historical default frequencies of BBB-rated firms. The average default rates are somewhat larger than their targets due to a few realizations with long sojourn times in recession, resulting in high default rates.[29]

[29]The standard deviation of the sojourn times generated by Markov chains is quite large. In our model, long sojourn times in recession cause high default rates for some sample paths. As default rates are non-linear in the distance to default, long sojourn times in boom do not counterbalance the high rates in recession.

Panels A and D show that while the generated rates tend to be too low in Panel A, but too large in Panel D, historical default frequencies still fall within the 25% to 75% range of model-implied median default rates for most years.

The large difference between Panel A and D in both average and median default rates illustrates that debt maturities and the associated default thresholds have an important effect on model-implied default rates. It is, therefore, important to incorporate a realistic debt maturity when calibrating models with endogenous default thresholds.

INSERT TABLE 4 HERE

In sum, our results demonstrate that the average credit spreads implied by our model for the true cross-section are simultaneously consistent with historically observed average asset volatilities, and, especially for typical debt maturities, with default rates reported for BBB-rated firms.

6.2. Leverage

This section analyzes the features of leverage ratios resulting from our model. We first investigate how growth options affect the initial choice of optimal leverage in our model. At initiation, a firm consisting of only invested assets has an optimal leverage that is between four and five percentage points higher than the one of a typical firm with an asset composition ratio of 1.6 for all debt maturities.[30] The reason is that a higher asset composition ratio increases the default probability, particularly so in recessions in which default losses are larger and harder to bear. Due to the resulting higher costs of debt, firms with growth options optimally select lower initial leverage.

As argued by Bhamra, Kuehn and Strebulaev (2010a), however, it can be misleading to make quantitative statements simply based on optimal leverage at issue. Hence, we investigate the leverage ratios of our true cross-section of BBB-rated firms simulated over five years after matching. For the main analysis, the debt maturity is assumed to be ten years.

[30]The difference depends on the initial regime and the debt maturity. For example, with infinite debt maturity, the difference in optimal initial leverage between a firm with only invested assets and a firm with an asset composition ratio of 1.6 is 4% if the firms are initiated in boom. (The optimal leverage ratios in this case are 45.4% and 41.4%, respectively.) For firms initiated in recession, the difference is 4.4% (= 44.2% minus 39.8%).

INSERT TABLE 5 HERE

Panel A in Table 5 shows that the average leverage is 40.89%, which is, naturally, close to the average of 41.83% of our BBB-rated firm sample used for the matching. (The average leverage is 40.57%, 40.93%, and 41.45% for five years, 20 years, and infinite debt maturity, respectively.)

In Panel B, we compare leverage ratios in boom and recession. While optimal leverage is pro-cyclical at initiation, it is counter-cyclical over time for the true cross-section of BBB-rated firms. In particular, the average leverage is 36.94% in boom, and 46.20% in recession. The reason is that the market value of equity is more sensitive to regime switches than the market value of debt, making leverage counter-cyclical. This mechanism dominates the optimally pro-cyclical leverage choice at initiation for our typical firms. The result mirrors the property we previously established for the aggregate economy, and confirms that it holds also when matching to real empirical samples.

Finally, Panel C investigates the relationship between growth options and market leverage. Regressing the average leverage of each firm on its average asset composition ratio in our empirical BBB-rated firm sample yields a coefficient of -0.165. We conduct the same regressions with the averages of asset composition ratios and leverage ratios from each of the $1,000$ simulations of the true cross-section. The average coefficient from this regression is -0.184, close to its empirical counterpart. Hence, the observed magnitude of the negative relationship between growth options and market leverage is preserved during the simulation.

Our qualitative finding for the cross-sectional relationship between growth options and leverage is widely accepted (Bradley, Jarrell and Kim, 1984; Barclay, Smith and Morellec, 2006; Johnson, 2003; Rajan and Zingales, 1995). Consistent with the literature, the coefficient is robustly negative. Moreover, its quantitative size, implied by the 25% and 75% quantiles, is comparable to the one in empirical studies. Fama and French (2002), for example, obtain a coefficient of -0.096 in their regression of market leverage on a similar ratio of asset composition and standard controls, and Johnson (2003) finds that increasing the asset composition ratio by one decreases leverage by around 7.8 percentage points in a pooled regression.

6.3. Robustness

In this section, we discuss the robustness of the results to variations in the critical input parameters. Additionally, we also show how our predictions are affected if we assume that the expansion is financed by issuing equity instead of selling assets.

To analyze the impact of preferences on our results, we show ten year credit spreads and the simulated average leverage for $\gamma = 7.5$ in the second column of Table 6, a value which is also sometimes used in the literature (Bansal and Yaron, 2004; Chen, 2010). All other parameters are kept constant at their baseline levels from Table 1. The debt maturity is assumed to be ten years.

INSERT TABLE 6 HERE

Lower risk aversion induces a smaller demand for precautionary savings, which increases the real risk-free rate. At the same time, it raises the risk-neutral earnings drift, because risk prices for systematic Brownian shocks (η_i) decrease. Both mechanisms reduce the default probability, leading to the lower credit spreads and slightly lower leverage.

In column 3 of Table 6, we investigate the impact of the exercise costs on credit spreads and leverage. As we are mainly interested in firms with intact expansion options, we present the results for K equal to 350, i.e., a higher K than in the baseline case. (Lowering K induces many growth firms to exercise their expansion option almost immediately.) Generally, credit spreads and the average leverage are very similar to the ones of our baseline specification. For high asset composition ratios, such as 2.2, credit spreads at initiation slightly increase because a higher K induces a larger distance to the optimal exercise boundary compared to the baseline specification. This increase in credit spreads from the larger distance arises because close to the exercise boundary, credit spreads also reflect the fact that the firm will imminently be converted into a firm with only invested assets, and, hence, with lower credit risk. When simulating the true cross section, the impact of increasing K from 310 to 350 on the average credit spread is below one basis point.

Finally, we also analyze in column 4 of Table 6 the case in which the exercise price of the expansion option (K) is financed by issuing additional equity instead of selling assets. Appendix

A.6 presents the solution for the value of corporate debt. New equity decreases the leverage after exercise and, hence, lowers credit risk. As firms with a high asset composition ratio are close to the endogenous exercise boundary where new equity-financing occurs, credit spreads are strongly reduced for typical growth firms compared to the benchmark model. In the simulation of the true cross-section, however, the effect is relatively small because most firms have a large distance to the exercise boundary. Especially those firms that contribute the most to the average credit spread, i.e., distressed firms, are particularly far away from the exercise boundary. Additionally, Panel B shows that the average leverage is only marginally affected.

The result for typical growth firms in column 4 shows that close to firms' exercise boundaries, credit spreads are driven by the expected new financing upon investment, and do not primarily reflect the nature of assets. This insight validates our focus on asset-financing rather than on equity-financing of growth option exercises to analyze the isolated impact of the asset composition on credit risk and corporate policy choices.

We conclude that while alternative specifications and settings can have an impact on the quantitative results, our qualitative aggregate and cross-sectional predictions are robust.

7. Equity value premium

In this section, we investigate the value premium for equity implied by our model, i.e., the difference in the equity risk premium between value and growth firms. As in the analysis of credit spreads, we show that considering the true cross-section is crucial when exploring the quantitative implications of our model.

The following proposition presents the instantaneous equity risk premium, defined as the expected difference between the instantaneous yield on corporate equity and the yield on the corresponding risk-free security.

Proposition 3. *The nominal instantaneous equity risk premium* $ep_i(X)$ *of a firm is given by*

$$ep_i(X) = \frac{e_i'(X)X}{e_i(X)}\gamma\sigma_i^{X,C}\sigma_i^C + \frac{e_i'(X)X}{e_i(X)}(\sigma_i^{X,C}\sigma^{P,C} + (\sigma^{P,id})^2) - \lambda_i(\frac{e_j(X)}{e_i(X)} - 1)(e^{\kappa_i} - 1). \tag{34}$$

Proof. See Appendix A.7. □

The first term of the equity risk premium comes from the compensation for the systematic volatility of stock returns caused by Brownian shocks. The second term accounts for the fact that the equity premium is calculated in nominal terms. The last term is the jump risk premium, in which $(\frac{e_j(X)}{e_i(X)} - 1)$ is the volatility of stock returns that is caused by Poisson shocks.

In Panel A of Table 7, we investigate the initial, instantaneous value premium for our cross-section of matched BBB-rated firms. We report yearly equity premiums. At matching, firms are sorted into ten portfolios based on their asset composition ratio.[31] Portfolio one contains all firms in the lowest asset composition ratio decile (value firms), and portfolio ten the ones in the highest decile (growth firms).[32] The average equity premium for each portfolio is calculated as the equity value-weighted average of the instantaneous equity premiums of the matched firms in the corresponding portfolio. The panel reports average equity premiums of 20 matchings after the corresponding pre-matching simulations. The pattern across the portfolios at matching is in accordance with the positive relationship between the book-to-market ratio and the equity returns reported in the literature.

INSERT TABLE 7 HERE

Because growth options are levered claims, growth firms are more volatile, which induces a larger equity risk premium than for value firms. At the same time, however, growth firms hold lower levels of debt. The average leverage in portfolio one, for example, is 61.99%, and the one in portfolio ten is 28.82%. Consistent with the empirical literature (e.g., Bhandari (1988), Fama and French (1992), and Gomes and Schmid (2010)), financial leverage increases the equity risk premium in our model. The effect of leverage dominates the impact of the volatility such that value firms have a higher equity risk premium than growth firms. The yearly premiums in the lowest and highest asset composition ratio deciles are 6.79% and 5.65%, respectively. The value premium calculated as the difference between these two premiums is 1.14%.

[31] We do not sort based on the market-to-book equity ratio for two reasons. First, the asset composition ratio unambiguously identifies value and growth firms in our model. Second, using the market-to-book equity ratio requires to define model-implied book asset values, and book values of debt. There is, however, no unique definition of book values in our model, and different definitions influence the sorting.

[32] The average asset composition ratio in the value portfolio is 0.99, and 2.08 in the growth portfolio.

Panel B analyzes the impact of the dynamics of the true cross-section of firms on the value premium. To this end, we simulate future earnings paths for each firm in our initial cross-section of matched firms over five years. The procedure is analogous to the one in the simulation approach for credit spreads and leverages. However, we do not consider firms that have already exercised their growth option. The reason is that, as our model does not incorporate new debt financing, the equity premium of exercised firms is very small due to the low leverage after option exercise. Additionally, exercised firms have very large equity values. Hence, including them causes a heavy downward bias of the equity premium when applying equity-weighting in the calculation of the average equity premium.[33] The average value-weighted equity premium in our entire simulated true cross-section of BBB-rated firms is 5.69% per year, consistent with the average equity premium reported in the literature (Campbell, Lo and MacKinley, 1997; Gomes and Schmid, 2010). Following the sorting procedure proposed in Fama and French (1992), simulated firms are then sorted into ten different portfolios at the beginning of each simulated year. We measure the average value-weighted equity premium of each portfolio during the subsequent year. The first line in Panel B shows that the resulting value premium of 3.47%, given by the equity premium of portfolio one minus the equity premium of portfolio ten, is much larger than in Panel A. The value premium calculated as the difference between the top and bottom portfolio quintiles is 1.45%. The second and third lines of Panel B report the average leverage and asset composition ratios of each portfolio in the simulated true cross-section.

Empirically, the yearly value premium is between 6.29% and 12.55% when comparing the top and bottom book-to-market deciles (Fama and French, 2002; Patton and Timmermann, 2010; Ang and Kristensen, 2011). Gomes and Schmid (2010) report 7.19% based on portfolio quintiles. Hence, the results in Panel B show that our model explains about 28% to 55% of the value premium for deciles. For quintiles, about 20% are explained.

The value premium is higher in Panel B than in Panel A because the equity risk premium is an increasing and convex function of the leverage ratio. Hence, whenever the economy switches to

[33]The average ten year credit spread when omitting exercised firms is 98 bps, which is even higher than the model predicted credit spread in the main case (see Panel D of Table 3).

recession in the dynamic simulation, the equity risk premium of value firms with an initially larger leverage increases, on average, more than the one of growth firms with an initially lower leverage. Empirically, Choi (2010) confirms that a further leverage increase of already highly leveraged value firms during times with large risk premiums contributes to higher value premiums. Consistent with our simulation results, he argues that the joint dynamics of asset values and leverage drive, at least partially, the value premium.[34]

A direct consequence of this dynamic source of the value premium is that it is strongly counter-cyclical. In the simulation of the true cross-section of matched firms over five years, the yearly value premium based on portfolios sorted by asset composition ratio deciles is, on average, 8.74% in recession, and 0.78% in boom. Based on quintiles, the value premium is 3.52% in recession, and 0.41% in boom. Our result is consistent with the growing body of literature that shows that value firms are particularly risky in bad times. For example, Petkova and Zhang (2005) and Chen, Petkova and Zhang (2008) find that the value effect is empirically much stronger in bad times than in good periods.

In sum, our analysis shows that by simply exploring the cross-sectional dynamics of firms with endogenous default and investment decisions a significant portion of the value premium and its counter-cyclical pattern can be explained.

8. Conclusion

It is now well-accepted that macroeconomic risk is central for understanding credit risk and capital structure choices. Specifically, defaults are more likely during recession, when they are particularly costly and harder to bear. This counter-cyclicality increases the costs of debt for all firms. But to explain the cross-sectional variation in apparently excessive costs of debt, we need variation inside the firm. This paper formalizes the role of one particularly important aspect of this heterogeneity, the asset composition of firms. It is not surprising that in principle the asset

[34]The finding that value stocks have higher returns than growth stocks has prompted many other explanations. For example, Choi (2010) shows that fixed operating costs can generate a value premium. Similarly, Zhang (2005) argues that asymmetric adjustment costs change the underlying business risk of value firms.

composition can be important for optimal capital structure. After all, economists have devoted much effort to understanding the difference between value and growth firms in terms of their financial structure, starting with Myers (1977) and Jensen (1986). Little was known, however, about the quantitative importance of this factor and its relation with macroeconomic risk.

The present structural equilibrium model allows us to jointly analyze a firm's expansion policy and financial leverage in the presence of macroeconomic risk. We demonstrate that incorporating the combination of these factors goes a long way towards explaining the empirically observed cross-sectional variation in costs of debt, leverage, and equity risk premiums. Our model implies that companies with a high portion of expansion options tend to be riskier in general, and, at the same time, particularly sensitive to macroeconomic risk. They are not only more volatile (because growth options represent levered claims), but also have a higher propensity to default in bad times than firms with a low portion of expansion options. Thus, the default probability and its counter-cyclicality are higher the greater the ratio of expansion options to total assets. Together with higher marginal utility of the representative agent in recession, this relation (exacerbated by costly liquidation in recession) implies higher costs of debt and more important endogenous shadow costs of leverage for firms with growth options than for those with only invested assets. Thus, our findings explain why the credit spread puzzle is empirically more pronounced for growth firms, and why growth firms hold less debt even after controlling for standard determinants of credit risk. Moreover, because the economy is made up of a cross-sectional mix of firms, the model accounts, in quantitatively fairly accurate ways, for the average credit spread puzzle. The model also yields a counter-cyclical value premium for equity, consistent with the data.

We have studied one type of real options of firms, namely, growth options. However, firms have a wide and varying range of options, including abandonment and shut-down options. A model incorporating these options could, therefore, yield further cross-sectional predictions.

While recent research has made important progress in enhancing our understanding of average credit risk, the cross-section of credit risk has not received sufficient attention. Analyzing it empirically is, fortunately, quite feasible. Liquid credit default swap quotes are now widely available on a firm-by-firm basis, allowing researchers to investigate specific relationships between firm-specific

characteristics such as growth options and credit spreads. Our paper also provides a theoretical basis that can guide empirical research in this direction.

9. Figures

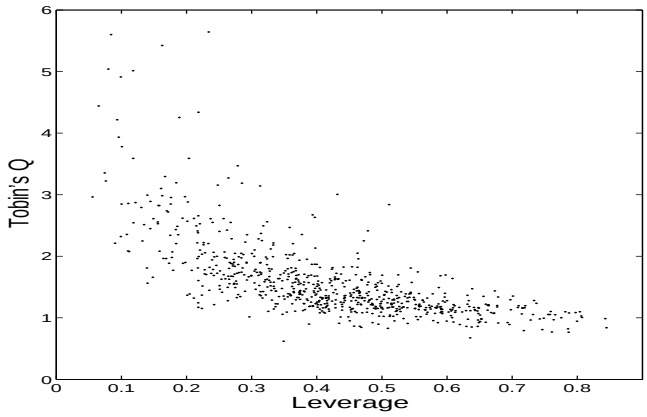

Figure 1. *Cross-Section of BBB-Rated Firms.* This scatterplot shows the average leverage and Tobin's Q for each observed BBB-rated firm over the period from 1995 to 2008.

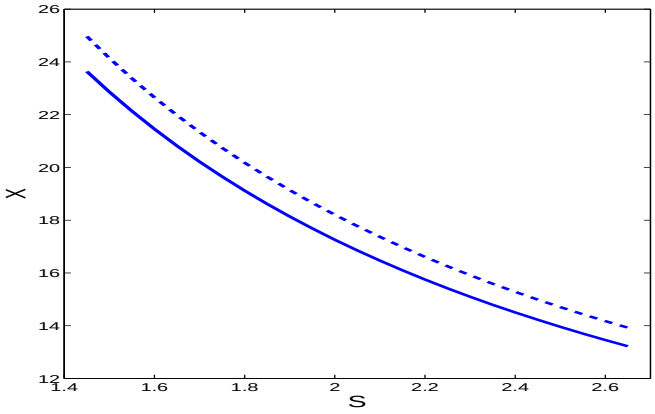

Figure 2. *Optimal Exercise Boundary.* The solid line shows the optimal exercise boundary in boom for a range of scale parameters s. The dashed line represents the corresponding exercise boundary in recession. The graph is drawn for optimal leverage with infinite debt maturity. The baseline parameter specification from Table 1 is used.

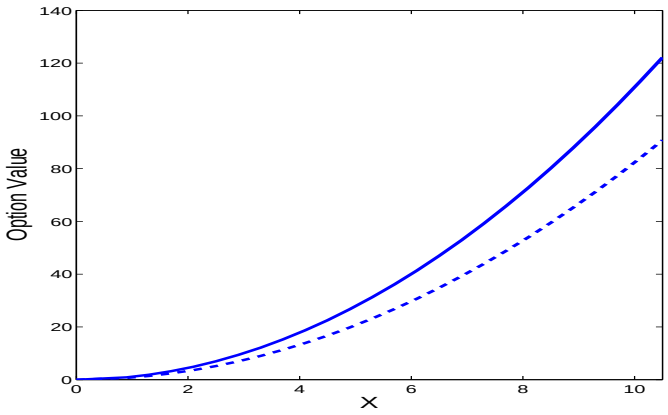

Figure 3. *Option Values.* The solid line represents the value of the expansion option in boom for a range of starting earnings between 0 and 10. The dashed line shows the corresponding values of the same option in recession. The graph is drawn for optimal leverage with infinite debt maturity. The baseline parameter specification from Table 1 is used.

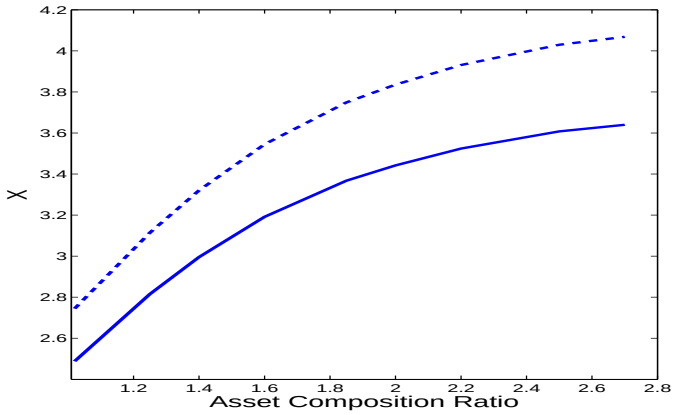

Figure 4. *Default Policy and Asset Composition.* The solid line represents the default threshold in boom for a range of asset composition ratios. The dashed line shows the default threshold in recession. The graph is drawn for constant leverage (41.83%) at each point. Debt maturity is assumed to be infinite. The baseline parameter specification from Table 1 is used, with s being varied to generate the desired asset composition ratio.

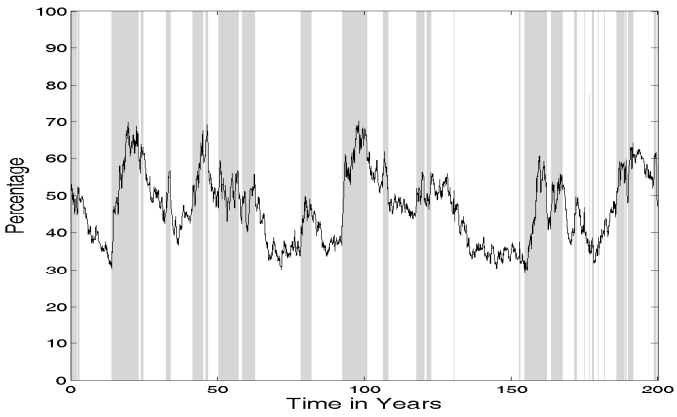

Figure 5. *Time Series of Market Leverage.* The solid line shows the aggregate market leverage of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

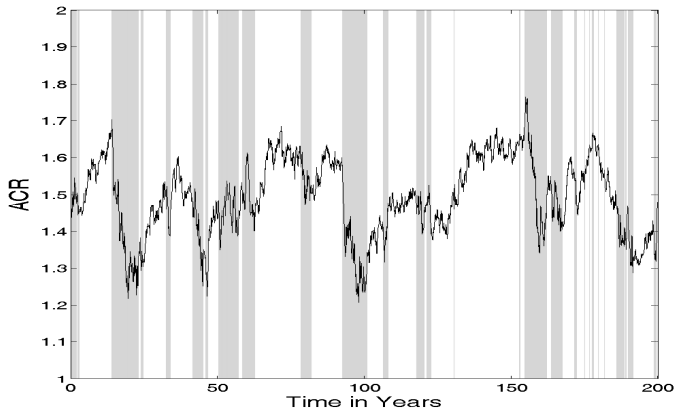

Figure 6. *Time Series of ACR.* The solid line shows the aggregate asset composition ratio of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

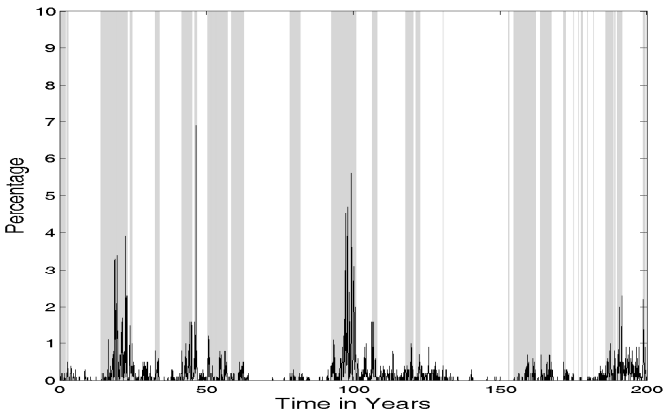

Figure 7. *Monthly Default Rates.* The solid line shows the percentage of firms that default during a given month in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

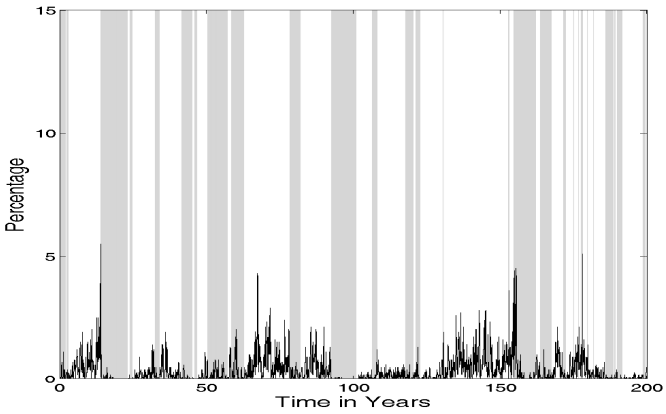

Figure 8. *Monthly Expansion Rates.* The solid line shows the percentage of firms that exercise their expansion options during a given month in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

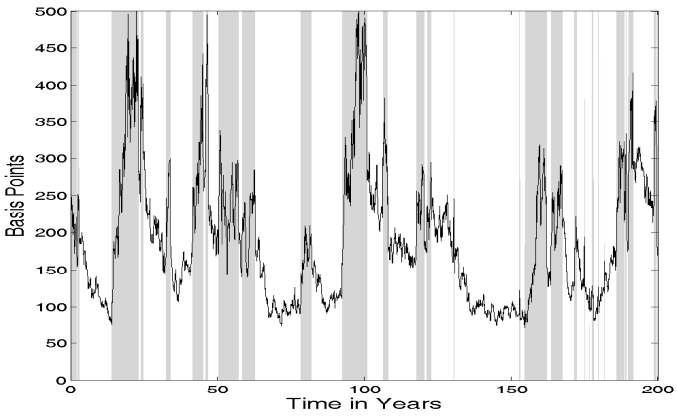

Figure 9. *Time Series of Credit Spread.* The solid line shows the average credit spread of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

10. Tables

Table 1
Baseline Parameter Choice

This table describes our baseline scenario. Panel A contains the annualized parameters of a typical BBB-rated S&P 500 firm. Panels B and C show our parameter choice for the expansion option and the macro economy, respectively. The asset composition ratio (ACR) is the value of the firm, divided by the value of the invested assets.

Parameter	Boom	Recession
Panel A. Firm Characteristics		
Initial Value of After-Tax Earnings (X)	10	10
Tax Advantage of Debt (τ)	0.15	0.15
Nominal Earnings Growth Rate (μ_i)	0.0782	-0.0401
Systematic Earnings Volatility ($\sigma_i^{X,C}$)	0.0834	0.1334
Recovery Rate (α_i)	0.7	0.5
Panel B. Expansion Option Parameters of a Typical Firm (ACR=1.6)		
Exercise Price (K)	310	310
Scale Parameter if Initiated in Boom (s)	1.89	
Scale parameter if Initiated in Recession (s)	2.05	
Panel C. Economy		
Rate of Leaving Regime i (λ_i)	0.2718	0.4928
Consumption Growth Rate (θ_i)	0.042	0.0141
Consumption Growth Volatility (σ_i^C)	0.0094	0.0114
Expected Inflation Rate (π)	0.0342	0.0342
Systematic Price Index Volatility ($\sigma^{P,C}$)	-0.0035	-0.0035
Idiosyncratic Price Index Volatility ($\sigma^{P,id}$)	0.0132	0.0132
Rate of time preference (ρ)	0.015	0.015
Relative Risk Aversion (γ)	10	10
Elasticity of Intertemporal Substitution (Ψ)	1.5	1.5

Table 2
Target Credit Spreads and Default Probabilities

This table lists our target credit spreads and default probabilities. Panel A reports annualized target average credit spreads for various debt maturities. They are calculated as the BBB-rated bond minus treasury yields of Davydenko and Strebulaev (2007) and Duffee (1998), net of a 35.5% non-default component. Credit spreads are quoted in basis points. Panel B reports average cumulative issuer-weighted default rates in percent for BBB-debt over five, ten, and 20 years for US firms (Moody's, 2010).

Panel A: Target Credit Spreads (in basis points)			
Debt Maturity	Short	Medium	Long
Davydenko and Strebulaev (2007)	74	74	92
Duffee (1998)	96	95	128
Panel B: Historical BBB Default Probabilities (in percent)			
Years	5	10	20
1920-2009	3.136	7.213	13.684
1970-2009	1.926	4.851	12.327

Table 3
Implications for Credit Spreads

This table demonstrates the implications of our model for credit spreads of BBB-rated firms. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Parameters are taken from Table 1, and the leverage is set equal to 41.83%. In the one regime model, parameters are chosen to match their unconditional mean. The standard two regime model is adapted from Bhamra, Kuehn and Strebulaev (2010b). Annualized credit spreads for various debt maturities are calculated as the coupon divided by the debt value, minus the yield on an otherwise identical riskfree bond. They are quoted in basis points. Credit spreads of typical firms in Panels B and C are obtained by weighting the credit spreads in boom and recession by the average expected time spent in each regime, respectively. Panel D contains the average credit spreads of our simulated true cross-section of BBB-rated firms.

Debt Maturity (Years)	5	10	20
Panel A: One Regime Model			
Average Firm	18	29	41
Panel B: Standard Two Regime Model With Only Invested Assets			
Average Firm	35	56	78
Panel C: Two Regime Model With Expansion Option			
Invested Assets (ACR=1)	24	39	55
Average Firm (ACR=1.6)	45	66	84
Growth Firm (ACR=2.2)	47	69	86
Panel D: Two Regime Model With True Cross-Section			
Average Credit Spread	57	81	100

Table 4
Implications for Default Rates

This table shows the simulated cumulative default rates in percent of our true cross-section of BBB-rated firms. Panels A to D vary the underlying debt maturity used to calculate the default thresholds in our model.

Years	5	10	20
Panel A: Infinite Debt Maturity			
Average Default Rates	2.24	6.54	13.76
Median Default Rates	0.84	3.07	9.34
25% Quantile of Default Rates	0.42	1.12	3.35
75% Quantile of Default Rates	2.37	9.07	19.80
Panel B: 20 Years Debt Maturity			
Average Default Rates	4.39	10.72	18.67
Median Default Rates	1.81	6.00	13.81
25% Quantile of Default Rates	0.70	2.23	5.44
75% Quantile of Default Rates	4.88	14.92	26.57
Panel C: 10 Years Debt Maturity			
Average Default Rates	6.30	13.61	21.92
Median Default Rates	2.79	8.09	16.95
25% Quantile of Default Rates	1.12	3.49	7.11
75% Quantile of Default Rates	7.39	19.39	32.50
Panel D: 5 Years Debt Maturity			
Average Default Rates	8.05	16.32	25.09
Median Default Rates	4.04	10.60	20.22
25% Quantile of Default Rates	1.67	4.60	9.34
75% Quantile of Default Rates	10.32	23.36	36.75

Table 5
Implications for Leverage

This table demonstrates the implications of our model for the leverage features of the true cross-section of BBB-rated firms. Leverage ratios (given in percent) are calculated as the market value of debt divided by the market value of the firm. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Parameters are taken from Table 1. The debt maturity is assumed to be ten years.

Panel A: Unconditional Leverage		
Average Leverage	40.89	
Panel B: Conditional Leverage		
Regime	Boom	Recession
Average Leverage	36.94	46.20
Median Leverage	34.36	44.19
25% Quantile	22.49	29.88
75% Quantile	48.51	60.39
Panel C: Regression of Leverage on ACR		
Average Coefficient	-0.184	
Median Coefficient	-0.184	
25% Quantile	-0.268	
75% Quantile	-0.096	

Table 6

Credit Spreads and Leverage for Alternative Specifications

This table shows annualized ten year credit spreads and simulated average leverage ratios (given in percent) of BBB-rated firms for alternative specifications of our basic model. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Credit spreads are calculated as the coupon divided by the debt value, minus the yield on an otherwise identical risk-free bond. They are quoted in basis points. The altered parameter is indicated in the first line, all other parameters are taken from Table 1. Credit spreads in the first three lines of Panel A for typical firms at issue are obtained by weighting the credit spreads in boom and recession by the expected times spent in each regime, respectively. The leverage is set equal to 41.83% to generate the credit spreads of typical firms. The last row in Panel A contains average credit spreads of our simulated true cross-section of BBB-rated firms. Panel B shows simulated average leverage ratios for BBB-rated firms. The debt maturity is assumed to be ten years.

Specification	$\gamma = 7.5$	$K = 350$	Equity Financing
Panel A: 10 Year Credit Spreads			
Invested Assets (ACR=1)	33	39	39
Average Firm (ACR=1.6)	53	67	65
Growth Firm (ACR=2.2)	56	72	58
True Cross-Section	68	81	77
Panel B: Unconditional Leverage			
Average Leverage	41.10	41.22	41.14

Table 7

Equity Premiums of Portfolios Formed on the Asset Composition Ratio

This table shows the yearly equity premiums for the ten different portfolios based on the deciles of the asset composition ratio (ACR). The equity premium for each portfolio is calculated as the equity value-weighted average of the premiums of all firms within the corresponding portfolio. It is reported as the average yearly premium expressed in percent. The leverage for each portfolio (given in percent) is obtained by averaging over firms' individual leverage in the corresponding portfolio, in which the individual leverage is calculated as the ratio of the market value of debt to total firm value. The asset composition ratio (ACR) is defined as the portfolio average of the firm values divided by the values of the invested assets. The debt maturity is assumed to be ten years.

Portfolio	1	2	3	4	5	6	7	8	9	10
Panel A: Initial Cross-Section										
Equity Premium	6.79	6.07	6.29	5.73	5.71	5.37	5.37	5.36	5.25	5.65
Panel B: Simulated True Cross-Section										
Equity Premium	9.48	6.74	6.36	5.95	5.83	5.94	5.71	5.72	5.73	6.01
Leverage	65.70	56.05	52.17	47.83	44.63	41.50	38.83	36.30	33.78	33.85
ACR	0.96	1.07	1.15	1.22	1.30	1.38	1.48	1.58	1.71	1.92

A. Appendix

A.1. The stochastic discount factor

Solving the Bellman equation associated with the consumption problem of the representative agent, it can be shown that the stochastic discount factor m_t follows the dynamics (3) (see Bhamra, Kuehn and Strebulaev, 2010b; Chen, 2010). The parameters h_B, h_R solve

$$0 = \rho\frac{1-\gamma}{1-\delta}h_i^{\delta-\gamma} + \left((1-\gamma)\theta_i - \frac{1}{2}\gamma(1-\gamma)\left(\sigma_i^C\right)^2 - \rho\frac{1-\gamma}{1-\delta}\right)h_i^{1-\gamma} + \lambda_i\left(h_j^{1-\gamma} - h_i^{1-\gamma}\right). \tag{A-1}$$

One-regime model. To isolate the effect of business cycle risk, we also consider the model with only one economic regime. The dynamics of the stochastic discount factor then read

$$\frac{\mathrm{d}m_t}{m_t} = -r\mathrm{d}t - \eta\mathrm{d}W_t^C. \tag{A-2}$$

The real interest rate r and the risk price η are given by

$$r = \bar{r} = \rho + \delta\theta - \frac{1}{2}\gamma(1+\delta)\left(\sigma_i^C\right)^2, \tag{A-3}$$

$$\eta = \gamma\sigma^C. \tag{A-4}$$

The nominal interest rate is calculated as

$$r^n = r + \pi - \sigma_P^2 - \sigma^{P,C}\eta, \tag{A-5}$$

and the expected growth rate is given by

$$\tilde{\mu} = \mu - \sigma^{X,C}\left(\eta + \sigma^{P,C}\right) - \left(\sigma^{P,id}\right)^2. \tag{A-6}$$

The earnings-price ratio simplifies to

$$y^{-1} = r^n - \tilde{\mu}, \tag{A-7}$$

and the total earnings volatility is

$$\tilde{\sigma} = \sqrt{\left(\sigma^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2}. \tag{A-8}$$

A.2. The value of the growth option

Proof of Proposition 1. For each regime i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise it is optimal to wait (option continuation region). This structure results in the following system of ODEs for the value function:

For $0 \leq X < X_B$:

$$\begin{cases} r_B^n G_B(X) = \tilde{\mu}_B X G_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 G_B''(X) + \tilde{\lambda}_B\left(G_R(X) - G_B(X)\right) \\ r_R^n G_R(X) = \tilde{\mu}_R X G_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 G_R''(X) + \tilde{\lambda}_R\left(G_B(X) - G_R(X)\right). \end{cases} \tag{A-9}$$

For $X_B \leq X < X_R$:

$$\begin{cases} G_B(X) = sXy_B - K \\ r_R^n G_R(X) = \tilde{\mu}_R X G_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 G_R''(X) + \tilde{\lambda}_R\left(sXy_B - K - G_R(X)\right). \end{cases} \tag{A-10}$$

For $X \geq X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ G_R(X) &= sXy_R - K. \end{cases} \tag{A-11}$$

Whenever the process X is in the option continuation region, which corresponds to System (A-9) and the second equation of (A-10), the required rate of return r_i^n (left-hand side) must be equal to the realized rate of return (right-hand side). The latter is obtained by Ito's lemma for regime switches. Here, the last term accounts for a possible jump in the value of the growth option due to a regime switch. It is calculated as the instantaneous probability of a regime shift, $\tilde{\lambda}_B$ or $\tilde{\lambda}_R$, times the associated change in the value of the option. The first equation of (A-10) and the System (A-11) state the payoff of the option at exercise, since the process is in the option exercise region in these cases. The boundary conditions are given by:

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = B, R \tag{A-12}$$

$$\lim_{X \searrow X_B} G_R(X) = \lim_{X \nearrow X_B} G_R(X) \tag{A-13}$$

$$\lim_{X \searrow X_B} G_R'(X) = \lim_{X \nearrow X_B} G_R'(X) \tag{A-14}$$

$$\lim_{X \nearrow X_R} G_R(X) = sX_R y_R - K \tag{A-15}$$

$$\lim_{X \nearrow X_B} G_B(X) = sX_B y_B - K. \tag{A-16}$$

Condition (A-12) ensures that the option value goes to zero as earnings approach zero. Conditions (A-13) and (A-14) are the value-matching and smooth-pasting conditions of the value function in recession at the exercise boundary in boom. The remaining conditions (A-15)-(A-16) are the value-matching conditions at the exercise boundaries in boom and recession, respectively.

The functional form of the solution is given by

$$G_i(X) = \begin{cases} \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4} & 0 \leq X < X_B, \quad i = B, R \\ \bar{C}_1 X^{\beta_1^R} + \bar{C}_2 X^{\beta_2^R} + \bar{C}_3 X + \bar{C}_4 & X_B \leq X < X_R, \quad i = R \\ sXy_i - K & X \geq X_i \quad i = B, R, \end{cases} \tag{A-17}$$

in which $\bar{A}_{B3}, \bar{A}_{B4}, \bar{A}_{R1}, \bar{A}_{R2}, \bar{C}_1, \bar{C}_2, \bar{C}_3, \bar{C}_4, \gamma_3, \gamma_4, \beta_1^R$, and β_2^R are real-valued parameters to be determined.

We first consider the region $0 \leq X < X_B$, and plug the functional form $G_i(X) = \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}$ into both equations of (A-9). Comparison of coefficients yields that $\bar{A}_{Bk}$ is a multiple of $\bar{A}_{Rk}$, $k = 3, 4$, with the factor $\bar{l}_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \gamma_k - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_k(\gamma_k - 1))$, i.e., $\bar{A}_{Rk} = \bar{l}_k \bar{A}_{Bk}$. Using this relation and comparing coefficients, we find that γ_3 and γ_4 correspond to the positive roots of the quartic equation

$$\left(\tilde{\mu}_R \gamma + \frac{1}{2}\tilde{\sigma}_R^2 \gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R \tilde{\lambda}_B. \tag{A-18}$$

The reason for taking the positive roots is given by boundary condition (A-12).

Next, we consider the region $X_B \leq X < X_R$. Plugging the functional form $G_R(X) = \bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X + \bar{C}_4$ into the second equation of (A-10), we find by comparison of coefficients that

$$\begin{aligned} \beta_{1,2}^R &= \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}} \\ \bar{C}_3 &= \tilde{\lambda}_R \frac{sy_B}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} \\ \bar{C}_4 &= -\tilde{\lambda}_R \frac{K}{r_R^n + \tilde{\lambda}_R}. \end{aligned} \tag{A-19}$$

The remaining unknown parameters are $\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1$ and $\bar{C}_2$. Plugging the functional form (A-17) into conditions (A-13)-(A-16) yields

$$\begin{aligned}
\bar{C}_1 X_B^{\beta_1^R} + \bar{C}_2 X_B^{\beta_2^R} + \bar{C}_3 X_B + \bar{C}_4 &= \bar{l}_3 \bar{A}_{B3} X_B^{\gamma_3} + \bar{l}_4 \bar{A}_{B4} X_B^{\gamma_4} && \text{(A-20)}\\
\bar{C}_1 \beta_1^R X_B^{\beta_1^R} + \bar{C}_2 \beta_2^R X_B^{\beta_2^R} + \bar{C}_3 X_B &= \bar{l}_3 \bar{A}_{B3} \gamma_3 X_B^{\gamma_3} + \bar{l}_4 \gamma_4 \bar{A}_{B4} X_B^{\gamma_4} && \text{(A-21)}\\
\bar{C}_1 X_R^{\beta_1^R} + \bar{C}_2 X_R^{\beta_2^R} + \bar{C}_3 X_R + \bar{C}_4 &= s y_R X_R - K && \text{(A-22)}\\
\bar{A}_{B3} X_B^{\gamma_3} + \bar{A}_{B4} X_B^{\gamma_4} &= s y_B X_B - K. && \text{(A-23)}
\end{aligned}$$

This four-dimensional system is linear in its four unknowns $\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1$ and $\bar{C}_2$. We define the matrices

$$\begin{aligned}
\bar{M} &:= \begin{bmatrix} \bar{l}_3 X_B^{\gamma_3} & \bar{l}_4 X_B^{\gamma_4} & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ \bar{l}_3 \gamma_3 X_B^{\gamma_3} & \bar{l}_4 \gamma_4 X_B^{\gamma_4} & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \\ X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 \end{bmatrix} \\
\bar{b} &:= \begin{bmatrix} \bar{C}_3 X_B + \bar{C}_4 \\ \bar{C}_3 X_B \\ -\bar{C}_3 X_R - \bar{C}_4 + s y_R X_R - K \\ s y_B X_B - K \end{bmatrix},
\end{aligned}$$

such that $\bar{M} \begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{b}$. Hence, the solution to the remaining unknowns is given by

$$\begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{M}^{-1} \bar{b}. \tag{A-24}$$

□

Relative price change sensitivity. The relative price change sensitivity is

$$\frac{G_i'(X)}{G_i(X)} = \begin{cases} \frac{\gamma_3 \bar{A}_{i3} X^{\gamma_3 - 1} + \bar{A}_{i4} \gamma_4 X^{\gamma_4 - 1}}{\bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}} & X < X_B, \quad i = B, R \\ \frac{\bar{C}_1 \beta_1 X^{\beta_1 - 1} + \bar{C}_2 \beta_2 X^{\beta_2 - 1} + \bar{C}_3}{\bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X + \bar{C}_4} & X_B \le X < X_R, \quad i = R \\ \frac{s y_i}{s y_i X - K} & X \ge X_i \quad i = B, R. \end{cases} \tag{A-25}$$

The unlevered value of the growth option. The unlevered value of the growth option can be calculated by imposing the smooth-pasting boundary conditions at option exercise:

$$\begin{aligned}
\lim_{X \nearrow X_R^{unlev}} G_R^{unlev'}(X) &= s y_R && \text{(A-26)}\\
\lim_{X \nearrow X_B^{unlev}} G_B^{unlev'}(X) &= s y_B. && \text{(A-27)}
\end{aligned}$$

The solution method is analogous to the one for the levered option value up to and including (A-19). Then, System (A-20)-(A-23) is augmented by the two equations corresponding to the additional boundary conditions:

$$\begin{aligned}
\bar{C}_1^{unlev} \beta_1^R \left(X_R^{unlev}\right)^{\beta_1^R - 1} + \bar{C}_2^{unlev} \beta_2^R \left(X_R^{unlev}\right)^{\beta_2^R - 1} + \bar{C}_3 &= s y_R && \text{(A-28)}\\
\bar{A}_{B3}^{unlev} \gamma_3 \left(X_B^{unlev}\right)^{\gamma_3 - 1} + \bar{A}_{B4}^{unlev} \gamma_4 \left(X_B^{unlev}\right)^{\gamma_4 - 1} &= s y_B. && \text{(A-29)}
\end{aligned}$$

The full system is six-dimensional with the six unknowns $\bar{A}_{B3}^{unlev}, \bar{A}_{B4}^{unlev}, \bar{C}_1^{unlev}, \bar{C}_2^{unlev}$ X_B^{unlev}, and X_R^{unlev}, linear in the first four unknowns, and non-linear in the last two unknowns. It is solved numerically.

One-regime model. Denote the investment boundary by X_1. The system to solve is given by:

$$\begin{array}{rcll} r^n G(X) & = & \tilde{\mu} X G'(X) + \frac{\tilde{\sigma}^2}{2} X^2 G''(X) & X < X_1 \\ G(X) & = & sXy - K & X \geq X_1. \end{array} \tag{A-30}$$

The boundary conditions are given by a value matching condition and the fact that the option must become worthless when the earnings approach zero:

$$\lim_{X \searrow 0} G(X) = 0 \tag{A-31}$$

$$\lim_{X \nearrow X_1} G(X) = syX_1 - K \tag{A-32}$$

The functional form of the solution is

$$G(X) = \begin{cases} \bar{A} X^{\beta_1} & X < X_1 \\ sXy - K & X \geq X_1, \end{cases} \tag{A-33}$$

in which $\bar{A}$ and β_1 are real-valued parameters to be determined. It is straightforward to show that

$$\beta_1 = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r^n}{\tilde{\sigma}^2}} \tag{A-34}$$

$$\bar{A} = (syX_1 - K) X_1^{-\beta_1}. \tag{A-35}$$

The relative price change sensitivity of the option is

$$\frac{G'(X)}{G(X)} = \begin{cases} \frac{\beta_1}{X} & X < X_1 \\ \frac{sy}{syX - K} & X \geq X_1. \end{cases} \tag{A-36}$$

The unlevered value of the option satisfies the additional smooth-pasting condition

$$\lim_{X \nearrow X_1^{unlev}} G^{unlev'}(X) = sy. \tag{A-37}$$

A.3. Firms with only invested assets

The solution for the values of corporate securities is based on Hackbarth, Miao and Morellec (2006).

The valuation of corporate debt. Without loss of generality, we consider the case in which the default boundary in boom is lower than in recession, i.e., $\hat{D}_B < \hat{D}_R$. An investor holding corporate debt requires an instantaneous return equal to the risk-free rate r_i^n. Once the firm defaults, debtholders receive a fraction α_i of the asset value Xy_i. The required rate of return on debt must be equal to the realized rate of return plus the coupon proceeds from debt. Therefore, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:
For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{d}_B(X) = \alpha_B X y_B \\ \hat{d}_R(X) = \alpha_R X y_R. \end{cases} \tag{A-38}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} r_B^n \hat{d}_B(X) = c + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\alpha_R X y_R - \hat{d}_B(X)\right) \\ \hat{d}_R(X) = \alpha_R X y_R. \end{cases} \tag{A-39}$$

For $X > \hat{D}_R$:

$$\left\{ \begin{array}{rcl} r_B^n \hat{d}_B(X) & = & c + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\hat{d}_R(X) - \hat{d}_B(X) \right) \\ r_R^n \hat{d}_R(X) & = & c + \tilde{\mu}_R X \hat{d}'_R(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 \hat{d}''_R(X) + \tilde{\lambda}_R \left(\hat{d}_B(X) - \hat{d}_R(X) \right). \end{array} \right. \tag{A-40}$$

The boundary conditions read

$$\lim_{X \to \infty} \frac{\hat{d}_i(X)}{X} < \infty, \quad i = B, R \tag{A-41}$$

$$\lim_{X \searrow \hat{D}_R} \hat{d}_B(X) = \lim_{X \nearrow \hat{D}_R} \hat{d}_B(X) \tag{A-42}$$

$$\lim_{X \searrow \hat{D}_R} \hat{d}'_B(X) = \lim_{X \nearrow \hat{D}_R} \hat{d}'_B(X) \tag{A-43}$$

$$\lim_{X \searrow \hat{D}_B} \hat{d}_B(X) = \alpha_B D_B y_B \tag{A-44}$$

$$\lim_{X \searrow \hat{D}_B} \hat{d}_R(X) = \alpha_R D_R y_R. \tag{A-45}$$

Condition (A-41) is the no-bubbles condition. The remaining boundary conditions are the value-matching conditions (A-42), (A-44), and (A-45), and the smooth-pasting condition at the higher default threshold $\hat{D}_R$ for the debt function in boom $\hat{d}_B(\cdot)$, Eq. (A-43). As debtholders do not choose the optimal default thresholds, there are no smooth-pasting conditions at default to be considered. The functional form of the solution is

$$\hat{d}_i(X) = \left\{ \begin{array}{lll} \alpha_i X y_i & X \leq \hat{D}_i & i = B, R \\ \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4 & \hat{D}_B < X \leq \hat{D}_R, & i = B \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_{i5} & X > \hat{D}_R, & i = B, R, \end{array} \right. \tag{A-46}$$

in which $\hat{A}_{B1}, \hat{A}_{B2}, \hat{A}_{R1}, \hat{A}_{R2}, A_{B5}, A_{R5}, \hat{C}_1, \hat{C}_2, C_3, C_4, \gamma_1, \gamma_2, \beta_1^B$, and β_2^B are real-valued parameters to be determined.

We first consider the region $X > \hat{D}_R$, and use the standard approach of plugging the functional form $\hat{d}_i(X) = \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_{i5}$ into both equations of (A-40). Comparing coefficients and solving the resulting two-dimensional system of equations for A_{i5}, we find that

$$A_{i5} = \frac{c\left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j} = \frac{c}{r_i^p}, \tag{A-47}$$

and that $\hat{A}_{Bk}$ is always a multiple of $\hat{A}_{Rk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \gamma_k - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_k(\gamma_k - 1))$, i.e., $\hat{A}_{Rk} = l_k \hat{A}_{Bk}$. Using these results and comparing coefficients again, we obtain that γ_1 and γ_2 are the negative roots of the quartic equation

$$\left(\tilde{\mu}_R \gamma + \frac{1}{2}\tilde{\sigma}_R^2 \gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R \tilde{\lambda}_B. \tag{A-48}$$

The reason for taking the negative roots is the no-bubbles condition for debt stated in (A-41).

Next, we consider the region $\hat{D}_B \leq X \leq \hat{D}_R$. Plugging the functional form $d_B(X) = \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4$ into the first equation of (A-39), we find by comparison of coefficients that

$$\begin{aligned} \beta_{1,2}^B &= \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2\left(r_B^n + \tilde{\lambda}_B\right)}{\tilde{\sigma}_B^2}} \\ C_3 &= \frac{\tilde{\lambda}_B \alpha_R y_R}{r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B} \\ C_4 &= \frac{c}{r_B^n + \tilde{\lambda}_B}. \end{aligned} \tag{A-49}$$

The remaining unknown parameters are $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$. We plug the functional form (A-46) into conditions (A-42)-(A-45), and obtain a four-dimensional linear system in the four unknowns $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$:

$$\begin{aligned} \hat{A}_{B1}\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_{B5} &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\ \hat{A}_{B1}\gamma_1\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\gamma_2\hat{D}_R^{\gamma_2} &= \hat{C}_1\beta_1^B\hat{D}_R^{\beta_1^B} + \hat{C}_2\beta_2^B\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R \\ \alpha_B\hat{D}_B y_B &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\ l_1\hat{A}_{B1}\hat{D}_R^{\gamma_1} + l_2\hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_{R5} &= \alpha_R\hat{D}_R y_R. \end{aligned} \tag{A-50}$$

We define the matrices

$$\hat{M} := \begin{bmatrix} \hat{D}_R^{\gamma_1} & \hat{D}_R^{\gamma_2} & -\hat{D}_R^{\beta_1^B} & -\hat{D}_R^{\beta_2^B} \\ \gamma_1\hat{D}_R^{\gamma_1} & \gamma_2\hat{D}_R^{\gamma_2} & -\beta_1^B\hat{D}_R^{\beta_1^B} & -\beta_2^B\hat{D}_R^{\beta_2^B} \\ 0 & 0 & \hat{D}_R^{\beta_1^B} & \hat{D}_R^{\beta_2^B} \\ l_1\hat{D}_R^{\gamma_1} & l_2\hat{D}_R^{\gamma_2} & 0 & 0 \end{bmatrix}$$

$$\hat{b} := \begin{bmatrix} C_3\hat{D}_R + C_4 - A_{B5} \\ C_3\hat{D}_R \\ \alpha_B\hat{D}_B y_B - C_3\hat{D}_R - C_4 \\ \alpha_R\hat{D}_R y_R - A_{R5} \end{bmatrix},$$

such that $\hat{M}\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{b}$. Hence, the solution of the unknowns is given by

$$\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{M}^{-1}\hat{b}. \tag{A-51}$$

Default policy. The value of equity is calculated as firm value minus the value of debt. The firm value consists of the value of assets in place plus the value of the option and the tax shield minus default costs. Once debt has been issued, managers select the ex-post default policy that maximizes the value of equity. Formally, the default policy is determined by equating the first derivative of the equity value to zero at the corresponding default boundary:

$$\begin{cases} \hat{e}_B'(\hat{D}_B^*) &= 0 \\ \hat{e}_R'(\hat{D}_R^*) &= 0. \end{cases} \tag{A-52}$$

We solve this problem numerically.

Capital structure. Denote by $\hat{f}_i^*(X)$ the firm value of a firm with only invested assets, given optimal ex-post default thresholds. The ex-ante optimal coupon of a firm solves

$$\hat{c}^* := \text{argmax}_{\hat{c}} \hat{f}_i^*(X). \tag{A-53}$$

One-regime model. Let $\hat{D}_1$ be the default threshold. Note that for a risk-neutral agent, the model corresponds to the one of Leland (1994). Equations (A-5)-(A-8) provide the parameters used in the setup and solution of the one-regime model. Postulating that the required return must be equal to the expected realized return plus the proceeds from debt, we find the following system:

$$\begin{array}{rcll} r^n \hat{d}(X) & = & c + \tilde{\mu} X \hat{d}'(X) + \frac{\tilde{\sigma}^2}{2} X^2 \hat{d}''(X) & X > \hat{D} \\ \hat{d}(X) & = & \alpha X y & X \leq \hat{D}. \end{array} \tag{A-54}$$

The boundary conditions are the no-bubbles condition, as well as value-matching at default:

$$\begin{array}{rcl} \lim\limits_{X \to \infty} \dfrac{\hat{d}(X)}{X} & < & \infty \\ \lim\limits_{X \searrow \hat{D}} \hat{d}\,(X) & = & \alpha y \hat{D}. \end{array} \tag{A-55}$$

The functional form of the solution is

$$\hat{d}(X) = \begin{cases} \alpha y X & X < \hat{D} \\ \hat{B} X^{\beta_2} + A_5 & X \geq \hat{D}, \end{cases} \tag{A-56}$$

in which $\hat{B}$ and β_2 are real-valued parameters. It is straightforward to show that

$$A_5 = \frac{c}{r} \tag{A-57}$$

$$\beta_2 = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r^n}{\tilde{\sigma}^2}} \tag{A-58}$$

$$\hat{B} = \left(\alpha y \hat{D} - \frac{c}{r^n}\right) \hat{D}^{-\beta_2}. \tag{A-59}$$

The default policy and capital structure can be determined analogously to the two-regime model.

A.4. Firms with invested assets and an expansion option

As in the main text, we consider the case $D_B < D_R$, $\hat{D}_B < \hat{D}_R$, and $X_R > X_B$. We present a constructive proof for the valuation of corporate debt.

Proof of Proposition 2. For brevity of notation, define $\bar{s} := s + 1$. An investor holding corporate debt requires an instantaneous return equal to the nominal risk-free rate r_i^n. Hence, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:
For $0 \leq X \leq D_B$:

$$\begin{cases} d_B\,(X) & = & \alpha_B \left(X y_B + G_B^{unlev}\,(X)\right) \\ d_R\,(X) & = & \alpha_R \left(X y_R + G_R^{unlev}\,(X)\right). \end{cases} \tag{A-60}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n d_B\,(X) & = & c + \tilde{\mu}_B X d_B'\,(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''\,(X) \\ & & + \tilde{\lambda}_B \left(\alpha_R \left(X y_R + G_R^{unlev}\,(X)\right) - d_B\,(X)\right) \\ d_R\,(X) & = & \alpha_R \left(X y_R + G_R^{unlev}\,(X)\right). \end{cases} \tag{A-61}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n d_B\,(X) & = & c + \tilde{\mu}_B X d_B'\,(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''\,(X) + \tilde{\lambda}_B \left(d_R\,(X) - d_B\,(X)\right) \\ r_R^n d_R\,(X) & = & c + \tilde{\mu}_R X d_R'\,(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''\,(X) + \tilde{\lambda}_R \left(d_B\,(X) - d_R\,(X)\right). \end{cases} \tag{A-62}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ r^n d_R(X) &= c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R\left(\hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right). \end{cases} \tag{A-63}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-64}$$

In System (A-60), the firm is in the default region in both boom and recession. In this region, debtholders receive $\alpha_i\left(Xy_i + G_i^{unlev}(X)\right)$ at default. As the default boundary in boom is lower than the one in recession, System (A-61) corresponds to the firm being in the continuation region in boom, and in the default region in recession. For the continuation region in boom, the left-hand side of the first equation is the rate of return required by investors for holding corporate debt for one unit of time. The right-hand side is the realized rate of return, computed by Ito's lemma as the expected change in the value of debt plus the coupon payment c. The last term captures the possible jump in the value of debt in case of a regime switch, which triggers immediate default. Similarly, equations (A-62) describe the case in which the firm is in the continuation region in both boom and recession. The next system, (A-63), deals with the case in which the firm is in the exercise region in boom, and in the continuation region in recession. After exercising the option, the firm owns total assets in place with value $Xy_i + sXy_i - K$, reflecting the notion that the exercise costs of the growth option are financed by selling assets. The value of debt must then be equal to the value of debt of a firm with only invested assets, i.e., $d_B(X) = \hat{d}_B((s+1)X - \frac{K}{y_B})$, which is the first equation in (A-63). The second equation in this case is obtained by the same approach as in (A-62), in which the last term captures the fact that a regime switch from recession to boom triggers immediate exercise of the expansion option. Finally, equations (A-64) describe the case in which the firm is in the exercise region in both boom and recession. The system is subject to the following boundary conditions:

$$\lim_{X \searrow D_R} d_B(X) = \lim_{X \nearrow D_R} d_B(X) \tag{A-65}$$

$$\lim_{X \searrow D_R} d_B'(X) = \lim_{X \nearrow D_R} d_B'(X) \tag{A-66}$$

$$\lim_{X \searrow D_B} d_B(X) = \alpha_B\left(D_B y_B + G_B^{unlev}(D_B)\right) \tag{A-67}$$

$$\lim_{X \searrow D_R} d_R(X) = \alpha_R\left(D_R y_R + G_R^{unlev}(D_R)\right) \tag{A-68}$$

$$\lim_{X \searrow X_B} d_R(X) = \lim_{X \nearrow X_B} d_R(X) \tag{A-69}$$

$$\lim_{X \searrow X_B} d_R'(X) = \lim_{X \nearrow X_B} d_R'(X) \tag{A-70}$$

$$\lim_{X \nearrow X_B} d_B(X) = \hat{d}_B\left(\bar{s}X_B - \frac{K}{y_B}\right) \tag{A-71}$$

$$\lim_{X \nearrow X_R} d_R(X_R) = \hat{d}_R\left(\bar{s}X_R - \frac{K}{y_R}\right). \tag{A-72}$$

(A-65) and (A-66) are the value-matching and smooth-pasting conditions for the debt value in boom at the default boundary in recession. Similarly, (A-69) and (A-70) are the corresponding conditions for the debt value in recession at the option exercise boundary in boom. (A-67) and (A-68) are the value-matching conditions at the default thresholds, and (A-71) and (A-72) are the value-matching conditions at the option exercise boundaries. The default thresholds and option exercise boundaries are chosen by equityholders,

and, hence, we do not have the corresponding smooth-pasting conditions for debt. To solve this system, we start with the functional form of the solution:

$$d_i(X) = \begin{cases} \alpha_i \left(X y_i + G_i^{unlev}(X) \right) & X \leq D_i & i = B, R, \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_3 X + C_4 & D_B < X \leq D_R, & i = B \\ \quad + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & & \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} & D_R < X \leq X_B, & i = B, R \\ \quad + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + A_{i5} & & \\ B_1 X^{\beta_1^R} + B_2 X^{\beta_2^R} + Z(X) + B_4 & X_B < X \leq X_R, & i = R \\ \hat{d}_i \left(\bar{s} X - \frac{K}{y_i} \right) & X > X_i, & i = B, R, \end{cases} \tag{A-73}$$

in which $A_{B1}, A_{B2}, A_{R1}, A_{R2}, C_1, C_2, C_3, C_4, C_5, C_6, B_1, B_2, B_4, \beta_1^B, \beta_2^B, \beta_1^R, \beta_2^R, \gamma_1, \gamma_2, \gamma_3$, and γ_4 are real-valued parameters to be determined (or to be confirmed). $Z(X)$, as stated in the sixth line of (A-73), can be expressed in closed form using Gauss' hypergeometric function. It will be given explicitly in the following calculations.

We first consider the region $D_R < X \leq X_B$. Plugging the functional form $d_i(X) = A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + A_{i5}$ into both equations of (A-62) and comparing coefficients, we find that

$$A_{i5} = \frac{c \left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j \right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j} = \frac{c}{r_i^p}. \tag{A-74}$$

As before, A_{Bk} is always a multiple of A_{Rk}, $k = 1, \ldots, 4$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \gamma_k - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_k(\gamma_k - 1))$, i.e., $A_{Rk} = l_k A_{Bk}$. Using this relation and comparing coefficients, we find that $\gamma_1, \gamma_2, \gamma_3$, and γ_4 correspond to the roots of the quartic Eq. (A-48), which is given by

$$\left(\tilde{\mu}_R \gamma + \frac{1}{2}\tilde{\sigma}_R^2 \gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n \right) \left(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n \right) = \tilde{\lambda}_R \tilde{\lambda}_B. \tag{A-75}$$

By arguments of Guo (2001), this quartic equation always has four distinct real roots, two of them being negative, and two positive. The value of debt in both regimes will be subject to boundary conditions from both below (default) and above (exercise of expansion option). To meet all boundary conditions, we need four terms with the corresponding factors A_{ik} as well as exponents γ_k, which requires usage of all four roots of (A-75). The no-bubbles condition is already implemented in the value function $\hat{d}_i$ of a firm with only invested assets and, hence, does not need to be imposed again. The unknown parameters for this region are $A_{Bk},\ k = 1, \ldots, 4$.

Next, we consider the region $D_B \leq X \leq D_R$. Plugging the functional form $d_B(X) = C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4}$ into the second equation of (A-61), we find by comparison of coefficients that

$$\beta_{1,2}^B = \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \right)^2 + \frac{2\left(r_B^n + \tilde{\lambda}_B \right)}{\tilde{\sigma}_B^2}} \tag{A-76}$$

$$C_3 = \tilde{\lambda}_B \frac{\alpha_R y_R}{r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B} \tag{A-77}$$

$$C_4 = \frac{c}{r_B^n + \tilde{\lambda}_B} \tag{A-78}$$

$$C_5 = \alpha_R \frac{\bar{l}_3}{l_3} \bar{A}_{B3}^{unlev} \tag{A-79}$$

$$C_6 = \alpha_R \frac{\bar{l}_4}{l_4} \bar{A}_{B4}^{unlev}. \tag{A-80}$$

The unknown parameters left for this region are C_1 and C_2.

Finally, consider the region $X_B < X \leq X_R$. The corresponding differential equation for $i = R$ is (see (A-63)):

$$\left(r_R^n + \tilde{\lambda}_R\right) d_R(X) = c + \tilde{\mu}_R X d'_R(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d''_R(X) + \tilde{\lambda}_R \hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-81}$$

To solve this inhomogeneous differential equation, we use a standard approach by first finding a fundamental system of solutions of the homogeneous differential equation, and then calculating the solution of the inhomogeneous equation as the sum of the solutions of the homogeneous equation and a particular solution of the inhomogeneous equation (Polyanin and Zaitsev, 2003, pages 21-23).

(A-81) is equivalent to

$$X^2 d''_R(X) + \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2} X d'_R(X) - \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2} d_R(X) = -\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-82}$$

Therefore, the corresponding homogeneous differential equation is

$$X^2 d''_R(X) + \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2} X d'_R(X) - \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2} d_R(X) = 0. \tag{A-83}$$

A fundamental system of solutions is given by $\{z_1, z_2\}$, with

$$\begin{aligned} z_1 &:= X^{\beta_1^R}, \\ z_2 &:= X^{\beta_2^R}, \end{aligned}$$

and

$$\beta_{1,2}^R = \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}}. \tag{A-84}$$

These solutions can be calculated by plugging the functional form into the homogeneous ODE (A-83), and solving for $\beta_{1,2}^R$.

For notational convenience, we now define $f_2 := X^2$, $f_1 := \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2} X$, $f_0 := -\frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}$, and

$$g(X) := -\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-85}$$

These notations allow to write the ODE (A-82) as

$$f_2 d''_R(X) + f_1 d'_R(X) + f_0 d_R(X) = g(X). \tag{A-86}$$

The general solution of this inhomogeneous ODE is given by

$$d_R(X) = B_1 z_1 + B_2 z_2 + z_2 \underbrace{\int z_1 \frac{g}{f_2} \frac{\mathrm{d}X}{W}}_{=:I_1(X)} - z_1 \underbrace{\int z_2 \frac{g}{f_2} \frac{\mathrm{d}X}{W}}_{=:I_2(X)}, \tag{A-87}$$

in which $W = z_1 z_2' - z_2 z_1'$ is the Wronskian determinant, and B_1 and B_2 are coefficients (see e.g. Polyanin and Zaitsev (2003), page 22, (7)). The first two terms of Eq. A-87 are a linear combination of the solutions of the homogeneous ODE, and the last two terms are a particular solution of the inhomogeneous ODE.

We start by calculating the Wronskian determinant

$$\begin{aligned} W &= z_1 z_2' - z_2 z_1' \\ &= \beta_2^R X^{\beta_1^R} X^{\beta_2^R - 1} - \beta_1^R X^{\beta_1^R - 1} X^{\beta_2^R} \\ &= \left(\beta_2^R - \beta_1^R\right) X^{\beta_1^R + \beta_2^R - 1}. \end{aligned} \tag{A-88}$$

Hence, the integral $I_1(X)$ is given by

$$\begin{aligned} I_1(X) &= \int z_1 \frac{g}{f_2} \frac{\mathrm{d}X}{W} \\ &= \int X^{\beta_1^R} X^{-2} \frac{1}{\beta_2^R - \beta_1^R} X^{1-\beta_1^R-\beta_2^R} g(X) \mathrm{d}X \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int X^{-1-\beta_2^R} g(X) \mathrm{d}X \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int X^{-1-\beta_2^R} \left(-\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \hat{d}_B \left(\bar{s} X - \frac{K}{y_B} \right) \right) \mathrm{d}X \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int X^{-1-\beta_2^R} \left(-\frac{2c}{\tilde{\sigma}_R^2} \right. \\ &\quad \left. - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \left\{ \hat{A}_{B1} \left(\bar{s} X - \frac{K}{y_B} \right)^{\gamma_1} + \hat{A}_{B2} \left(\bar{s} X - \frac{K}{y_B} \right)^{\gamma_2} + A_{B5} \right\} \right) \mathrm{d}X \\ &= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R - \beta_1^R\right) \tilde{\sigma}_R^2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s} X - \frac{K}{y_B} \right)^{\gamma_1} \mathrm{d}X}_{=: I_{11}(X)} \\ &\quad - \frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R - \beta_1^R\right) \tilde{\sigma}_R^2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s} X - \frac{K}{y_B} \right)^{\gamma_2} \mathrm{d}X}_{=: I_{12}(X)} \\ &\quad + \frac{2 A_{B5} \left(\tilde{\lambda}_R + r_R^n \right)}{\left(\beta_2^R - \beta_1^R\right) \beta_2^R \tilde{\sigma}_R^2} X^{-\beta_2^R}. \end{aligned} \tag{A-89, A-90}$$

We use the definition of the function $g(X)$, see (A-85), and the solution of the debt value of a firm with only invested assets $\hat{d}_R(\cdot)$, see (A-46).

The integrals $I_{11}(X)$ and $I_{12}(X)$ can be evaluated immediately with standard computer algebra packages. Alternatively, using the integral representation of Gauss' hypergeometric function ${}_2F_1(\cdot,\cdot,\cdot;\cdot)$, we can write the closed-form solution of the integrals as

$$I_{11}(X) = \frac{1}{\gamma_1 - \beta_2^R} \bar{s}^{\gamma_1} X^{\gamma_1 - \beta_2^R} {}_2F_1 \left(-\gamma_1, \beta_2^R - \gamma_1, \beta_2^R - \gamma_1 + 1; -\frac{K}{\bar{s} X y_B} \right), \tag{A-91}$$

$$I_{12}(X) = \frac{1}{\gamma_2 - \beta_2^R} \bar{s}^{\gamma_2} X^{\gamma_2 - \beta_2^R} {}_2F_1 \left(-\gamma_2, \beta_2^R - \gamma_2, \beta_2^R - \gamma_2 + 1; -\frac{K}{\bar{s} X y_B} \right). \tag{A-92}$$

Plugging the solutions (A-91) and (A-92) into the expression for the integral I_1, (A-90) yields

$$\begin{aligned} I_1(X) &= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_1-\beta_2^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_2^R}{}_2F_1\left(-\gamma_1,\beta_2^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad -\frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_2-\beta_2^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_2^R}{}_2F_1\left(-\gamma_2,\beta_2^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad +\frac{2A_{B5}\left(\tilde{\lambda}_R+r_R^n\right)}{\left(\beta_2^R-\beta_1^R\right)\beta_2^R\tilde{\sigma}_R^2}X^{-\beta_2^R}. \end{aligned} \tag{A-93}$$

Similarly, we find for the second integral $I_2(X)$:

$$\begin{aligned} I_2(X) &= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_1-\beta_1^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_1^R}{}_2F_1\left(-\gamma_1,\beta_1^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad -\frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_2-\beta_1^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_1^R}{}_2F_1\left(-\gamma_2,\beta_1^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad +\frac{2A_{B5}\left(\tilde{\lambda}_R+r_R^n\right)}{\left(\beta_2^R-\beta_1^R\right)\beta_1^R\tilde{\sigma}_R^2}X^{-\beta_1^R}. \end{aligned} \tag{A-94}$$

Plugging (A-93) and (A-94) into (A-87) and simplifying, we finally obtain the solution

$$d_R(X) = B_1X^{\beta_1^R} + B_2X^{\beta_2^R} + Z(X) + B_4, \tag{A-95}$$

with

$$Z(X) = \sum_{i,k=1,2}\frac{2(-1)^{i+1}\tilde{\lambda}_R\bar{s}^{\gamma_k}\hat{A}_{Bk}}{\tilde{\sigma}_R^2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right) \tag{A-96}$$

$$B_4 = \tilde{\lambda}_R\frac{c}{r_i^p\left(r_R^n+\tilde{\lambda}_R\right)}+\frac{c}{r_R^n+\tilde{\lambda}_R}, \tag{A-97}$$

for some parameters B_1 and B_2 determined by the boundary conditions. The first derivative $Z'(X)$ can be calculated as follows:

$$\begin{aligned} Z'(X) &= \frac{\mathrm{d}}{\mathrm{d}X}Z(X) \\ &= \frac{\mathrm{d}}{\mathrm{d}X}\left(X^{\beta_2^R}I_1(X)-X^{\beta_2^R}I_2(X)\right) \\ &= \beta_2^RX^{\beta_2^R-1}I_1(X)+\frac{1}{\beta_2^R-\beta_1^R}X^{\beta_2^R-1}X^{-1-\beta_1^R}g(X) \\ &\quad -\beta_1^RX^{\beta_1^R}I_2(X)-\frac{1}{\beta_2^R-\beta_1^R}X^{\beta_1^R}X^{-1-\beta_1^R}g(X) \\ &= \beta_2^RX^{\beta_2^R-1}I_1(X)-\beta_1^RX^{\beta_1^R-1}I_2(X) \\ &= \sum_{i,k=1,2}\frac{2(-1)^{i+1}\tilde{\lambda}_R\bar{s}^{\gamma_k}\hat{A}_{Bk}\beta_i^R}{\tilde{\sigma}_R^2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k-1}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right). \end{aligned} \tag{A-98}$$

To solve for the unknown parameters $A_{B1}, A_{B2}, A_{B3}, A_{B4}, C_1, C_2, B_1$ and B_2, we plug the functional form (A-73) into the system of boundary conditions (A-65) - (A-72):

$$\begin{aligned}
\sum_{k=1}^{4} A_{Bk} D_R^{\gamma_k} + A_{B5} &= C_1 D_R^{\beta_1^B} + C_2 D_R^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\
\sum_{k=1}^{4} A_{Bk} \gamma_k D_R^{\gamma_k} &= C_1 \beta_1^B D_R^{\beta_1^B} + C_2 \beta_2^B D_R^{\beta_2^B} + C_3 X + C_5 \gamma_3 X^{\gamma_3} + C_6 \gamma_4 X^{\gamma_4} \\
\alpha_B \left(D_B y_B + G_B^{unlev}(D_B) \right) &= C_1 D_B^{\beta_1^B} + C_2 D_B^{\beta_2^B} + C_3 D_B + C_4 + C_5 D_B^{\gamma_3} + C_6 D_B^{\gamma_4} \\
\sum_{k=1}^{4} l_k A_{Bk} D_R^{\gamma_k} + A_{R5} &= \alpha_R \left(D_R y_R + G_R^{unlev}(D_R) \right) \\
\sum_{k=1}^{4} l_k A_{Bk} X_B^{\gamma_k} + A_{R5} &= B_1 X_B^{\beta_1^R} + B_2 X_B^{\beta_2^R} + Z(X_B) + B_4 \\
\sum_{k=1}^{4} l_k A_{Bk} \gamma_k X_B^{\gamma_k} &= B_1 \beta_1^R X_B^{\beta_1^R} + B_2 \beta_2^R X_B^{\beta_2^R} + X_B Z'(X_B) \\
\sum_{k=1}^{4} A_{Bk} X_B^{\gamma_k} + A_{B5} &= \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B} \right) \\
B_1 X_R^{\beta_1^R} + B_2 X_R^{\beta_2^R} + Z(X_R) + B_4 &= \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R} \right).
\end{aligned} \tag{A-99}$$

Using matrix notation, we write

$$M := \begin{bmatrix}
D_R^{\gamma_1} & D_R^{\gamma_2} & D_R^{\gamma_3} & D_R^{\gamma_4} & -D_R^{\beta_1^B} & -D_R^{\beta_2^B} & 0 & 0 \\
\gamma_1 D_R^{\gamma_1} & \gamma_2 D_R^{\gamma_2} & \gamma_3 D_R^{\gamma_3} & \gamma_4 D_R^{\gamma_4} & -\beta_1^B D_R^{\beta_1^B} & -\beta_2^B D_R^{\beta_2^B} & 0 & 0 \\
0 & 0 & 0 & 0 & D_B^{\beta_1^B} & D_B^{\beta_2^B} & 0 & 0 \\
l_1 D_R^{\gamma_1} & l_2 D_R^{\gamma_2} & l_3 D_R^{\gamma_3} & l_4 D_R^{\gamma_4} & 0 & 0 & 0 & 0 \\
l_1 X_B^{\gamma_1} & l_2 X_B^{\gamma_2} & l_3 X_B^{\gamma_3} & l_4 X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\
l_1 \gamma_1 X_B^{\gamma_1} & l_2 \gamma_2 X_B^{\gamma_2} & l_3 \gamma_3 X_B^{\gamma_3} & l_4 \gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\
X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R}
\end{bmatrix}$$

$$b := \begin{bmatrix}
-A_{B5} + C_3 D_R + C_4 + C_5 D_R^{\gamma_1} + C_6 D_R^{\gamma_2} \\
C_3 D_R + \gamma_1 C_5 D_R^{\gamma_1} + \gamma_2 C_6 D_R^{\gamma_2} \\
-C_3 D_B - C_4 - C_5 D_B^{\gamma_3} - C_6 D_B^{\gamma_4} + \alpha_B \left(D_B y_B + G_B^{unlev}(D_B) \right) \\
-A_{R5} + \alpha_R \left(D_R y_R + G_R^{unlev}(D_R) \right) \\
-A_{R5} + Z\left(X_B\right) + B_4 \\
X_B Z'\left(X_B\right) \\
-A_{B5} + \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B} \right) \\
-Z\left(X_R\right) + B_4 + \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R} \right)
\end{bmatrix}.$$

Thus, the solution to the remaining unknowns is given by

$$\begin{bmatrix} A_{B1} & A_{B2} & A_{B3} & A_{B4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b. \tag{A-100}$$

□

Proof of Remark 1.

(i) In our framework, debt characteristics (c, m, p) are chosen and fixed at initiation. This setting allows us to calculate closed-form solutions for the values of corporate securities of firms with both invested assets and growth options, even with finite maturity debt. For given debt characteristics (c, m, p), the value of finite maturity debt satisfies the following system of ODEs:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) &= \alpha_B\left(Xy_B + G_B^{unlev}(X)\right) \\ d_R(X) &= \alpha_R\left(Xy_R + G_R^{unlev}(X)\right). \end{cases} \tag{A-101}$$

For $D_B < X \leq D_R$:

$$\begin{cases} (r_B^n + m)\, d_B(X) &= c + mp + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) \\ & \quad + \tilde{\lambda}_B\left(\alpha_R\left(Xy_R + G_R^{unlev}(X)\right) - d_B(X)\right) \\ d_R(X) &= \alpha_R\left(Xy_R + G_R^{unlev}(X)\right). \end{cases} \tag{A-102}$$

For $D_R < X < X_B$:

$$\begin{cases} (r_B^n + m)\, d_B(X) &= c + mp + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) \\ & \quad + \tilde{\lambda}_B\left(d_R(X) - d_B(X)\right) \\ (r_R^n + m)\, d_R(X) &= c + mp + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) \\ & \quad + \tilde{\lambda}_R\left(d_B(X) - d_R(X)\right). \end{cases} \tag{A-103}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ (r_R^n + m)\, d_R(X) &= c + mp + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) \\ & \quad + \tilde{\lambda}_R\left(\hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right). \end{cases} \tag{A-104}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-105}$$

$\hat{d}_i(\cdot)$ denotes the value of debt of a firm with only invested assets with the same principal, coupon, and debt maturity. The solution of $\hat{d}_i$ is given in Hackbarth, Miao and Morellec (2006). It corresponds to the value of infinite maturity debt of a firm with only invested assets with a coupon $c + mp$ and interest rates $r_i^n + m$. The boundary conditions for System (A-101)-(A-105) are the same as in the case of infinite maturity debt, see (A-65). Comparing this System (A-101)-(A-105) for finite maturity debt to the corresponding System (A-60)-(A-64) for infinite maturity debt, we conclude that for given debt characteristics (c, m, p), the value of finite maturity debt corresponds to the value of infinite maturity debt with a coupon $c + mp$ and nominal interest rates $r_i^n + m$. Hence, the value of finite maturity debt is given by the corresponding Formula (22) in Proposition 2.

(ii) The value of the tax shield satisfies the following system of ODEs:

For $0 \leq X \leq D_B$:

$$\begin{cases} t_B(X) &= 0 \\ t_R(X) &= 0. \end{cases} \tag{A-106}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n t_B(X) &= c\tau + \tilde{\mu}_B X t_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 t_B''(X) + \tilde{\lambda}_B\left(0 - t_B(X)\right) \\ t_R(X) &= 0. \end{cases} \tag{A-107}$$

For $D_R < X < X_B$:

$$\left\{\begin{array}{rcl} r_B^n t_B(X) & = & c\tau + \tilde{\mu}_B X t_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 t_B''(X) + \tilde{\lambda}_B\left(t_R(X) - t_B(X)\right) \\ r_R^n t_R(X) & = & c\tau + \tilde{\mu}_R X t_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 t_R''(X) + \tilde{\lambda}_R\left(t_B(X) - t_R(X)\right). \end{array}\right. \tag{A-108}$$

For $X_B \leq X < X_R$:

$$\left\{\begin{array}{rcl} t_B(X) & = & \hat{t}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ r_R^n t_R(X) & = & c\tau + \tilde{\mu}_R X t_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 t_R''(X) \\ & & +\tilde{\lambda}_R\left(\hat{t}_B\left(\bar{s}X - \frac{K}{y_B}\right) - t_R(X)\right). \end{array}\right. \tag{A-109}$$

For $X \geq X_R$:

$$\left\{\begin{array}{rcl} t_B(X) & = & \hat{t}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ t_R(X) & = & \hat{t}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{array}\right. \tag{A-110}$$

The boundary conditions write:

$$\begin{array}{rcl} \lim\limits_{X\searrow D_R} t_B(X) & = & \lim\limits_{X\nearrow D_R} t_B(X) \\ \lim\limits_{X\searrow D_R} t_B'(X) & = & \lim\limits_{X\nearrow D_R} t_B'(X) \\ \lim\limits_{X\searrow D_B} t_B(X) & = & 0 \\ \lim\limits_{X\searrow D_R} t_R(X) & = & 0 \\ \lim\limits_{X\searrow X_B} t_R(X) & = & \lim\limits_{X\nearrow X_B} t_R(X) \\ \lim\limits_{X\searrow X_B} t_R'(X) & = & \lim\limits_{X\nearrow X_B} t_R'(X) \\ \lim\limits_{X\nearrow X_B} t_B(X) & = & \hat{t}_B\left(\bar{s}X_B - \dfrac{K}{y_B}\right) \\ \lim\limits_{X\nearrow X_R} t_R(X_R) & = & \hat{t}_R\left(\bar{s}X_R - \dfrac{K}{y_R}\right). \end{array} \tag{A-111}$$

Comparing this System (A-106)-(A-110) and its boundary conditions (A-111) to the system for infinite maturity debt, (A-60)-(A-64), and its boundary conditions (A-65) yields that the tax shield corresponds to the value of debt with a coupon of $c\tau$ and default costs of zero. The solution for the value of the tax shield is, therefore, given by the corresponding Eq. (22) in Proposition 2.

(iii) The system for bankruptcy costs is given by:
For $0 \leq X \leq D_B$:

$$\left\{\begin{array}{rcl} b_B(X) & = & (1-\alpha_B)\left(Xy_B + G_B^{unlev}(X)\right) \\ b_R(X) & = & (1-\alpha_R)\left(Xy_R + G_R^{unlev}(X)\right). \end{array}\right. \tag{A-112}$$

For $D_B < X \leq D_R$:

$$\left\{\begin{array}{rcl} r_B^n b_B(X) & = & \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) \\ & & +\tilde{\lambda}_B\left((1-\alpha_R)\left(Xy_R + G_R^{unlev}(X)\right) - b_B(X)\right) \\ b_R(X) & = & (1-\alpha_R)\left(Xy_R + G_R^{unlev}(X)\right). \end{array}\right. \tag{A-113}$$

For $D_R < X < X_B$:

$$\left\{\begin{array}{rcl} r_B^n b_B(X) & = & \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B\left(b_R(X) - b_B(X)\right) \\ r_R^n b_R(X) & = & \tilde{\mu}_R X b_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 b_R''(X) + \tilde{\lambda}_R\left(b_B(X) - b_R(X)\right). \end{array}\right. \tag{A-114}$$

For $X_B \leq X < X_R$:

$$\begin{cases} b_B(X) &= \hat{b}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ r_R^n b_R(X) &= \tilde{\mu}_R X b_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 b_R''(X) + \tilde{\lambda}_R\left(\hat{b}_B\left(\bar{s}X - \frac{K}{y_B}\right) - b_R(X)\right). \end{cases} \tag{A-115}$$

For $X \geq X_R$:

$$\begin{cases} b_B(X) &= \hat{b}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ b_R(X) &= \hat{b}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-116}$$

The system is subject to the following boundary conditions:

$$\begin{aligned} \lim_{X \searrow D_R} b_B(X) &= \lim_{X \nearrow D_R} b_B(X) \\ \lim_{X \searrow D_R} b_B'(X) &= \lim_{X \nearrow D_R} b_B'(X) \\ \lim_{X \searrow D_B} b_B(X) &= (1-\alpha_B)\left(D_B y_B + G_B^{unlev}(D_B)\right) \\ \lim_{X \searrow D_R} b_R(X) &= (1-\alpha_R)\left(D_R y_R + G_R^{unlev}(D_R)\right) \\ \lim_{X \searrow X_B} b_R(X) &= \lim_{X \nearrow X_B} b_R(X) \\ \lim_{X \searrow X_B} b_R'(X) &= \lim_{X \nearrow X_B} b_R'(X) \\ \lim_{X \nearrow X_B} b_B(X) &= \hat{b}_B\left(\bar{s}X_B - \frac{K}{y_B}\right) \\ \lim_{X \nearrow X_R} b_R(X_R) &= \hat{b}_R\left(\bar{s}X_R - \frac{K}{y_R}\right). \end{aligned} \tag{A-117}$$

This System (A-112)-(A-116) and its boundary conditions (A-117) correspond to the system for infinite maturity debt, (A-60)-(A-64), and its boundary conditions (A-65), with a coupon of zero and a recovery rate of $1-\alpha_i$. The solution for bankruptcy costs is, therefore, given by the corresponding Eq. (22) in Proposition 2.

□

One-regime model. Denote the default boundary by D_1, the firm's investment boundary by X_1, and the default boundary of a firm with only invested assets by $\hat{D}_1$. The system to solve is:

$$\begin{cases} d(X) &= \alpha\left(yX + G^{unlev}(X)\right) & X \leq D_1 \\ r^n d(X) &= c + \tilde{\mu} X d'(X) + \frac{\tilde{\sigma}^2}{2} X^2 d''(X) & D_1 < X < X_1 \\ d(X) &= \hat{d}\left(\bar{s}X - \frac{K}{y}\right) & X \geq X_1. \end{cases} \tag{A-118}$$

This system is analogous to the one of the two regime model, (A-60)-(A-64). Similarly, the boundary conditions are the value-matching conditions at default and exercise:

$$\lim_{X \searrow D_1} d(X) = \alpha\left(yD_1 + G^{unlev}(D_1)\right) \tag{A-119}$$

$$\lim_{X \nearrow X_1} d(X) = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-120}$$

The functional form of the solution is

$$d(X) = \begin{cases} \alpha\left(yX + G^{unlev}(X)\right) & X \leq D_1 \\ E_3 X^{\beta_1} + E_4 X^{\beta_2} + A_5 & D_1 < X < X_1 \\ \hat{d}\left(\bar{s}X - \frac{K}{y}\right) & X \geq X_1, \end{cases} \tag{A-121}$$

in which E_3, E_4, A_5, β_1, and β_2 are real-valued parameters to be determined (or to be confirmed). We need to solve for the region $D_1 < X < X_1$. By plugging the functional form (A-121) into the differential equation (A-118) and comparing coefficients, we find that

$$A_5 = \frac{c}{r} \tag{A-122}$$

$$\beta_{1,2} = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r}{\tilde{\sigma}^2}}. \tag{A-123}$$

B_3 and B_4 are determined by the following two-dimensional linear system defined by the corresponding boundary conditions:

$$E_3 D_1^{\beta_1} + E_4 D_1^{\beta_2} + \frac{c}{r} = \alpha\left(yD_1 + G^{unlev}(D_1)\right) \tag{A-124}$$

$$E_3 X_1^{\beta_1} + E_4 X_1^{\beta_2} + \frac{c}{r} = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-125}$$

Using matrix notation and defining

$$M_1 := \begin{bmatrix} D_1^{\beta_1} & D_1^{\beta_2} \\ X_1^{\beta_1} & X_1^{\beta_2} \end{bmatrix}$$

$$b_1 := \begin{bmatrix} \alpha\left(yD_1 + G^{unlev}(D_1)\right) - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r} \end{bmatrix},$$

we find that

$$\begin{bmatrix} E_3 & E_4 \end{bmatrix}^T = M_1^{-1} b_1 \tag{A-126}$$

$$= \frac{1}{D_1^{\beta_1} X_1^{\beta_2} - D_1^{\beta_2} X_1^{\beta_1}} \begin{bmatrix} X_1^{\beta_2} & -D_1^{\beta_2} \\ -X_1^{\beta_1} & D_1^{\beta_1} \end{bmatrix} \begin{bmatrix} \alpha\left(yD_1 + G^{unlev}(D_1)\right) - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r} \end{bmatrix}, \tag{A-127}$$

which completes the calculation of the solution.

The values of finite maturity debt, the tax shield, and bankruptcy costs can be found analogously to the two-regime model (cf. Remark 1).

A.5. Details on the simulations

A.5.1. Calibration of the idiosyncratic volatility

We calibrate the firm-level idiosyncratic volatility of our BBB sample to the empirically observed total asset volatility of 0.25. The procedure starts by simulating a model-implied economy for ten years (pre-matching simulation). Next, we match the model-implied distribution after ten years with the empirical cross-section of BBB-rated firms, and finally simulate the obtained matched sample for another ten years (post-matching simulation). The average asset volatility of the post-matching simulation is then calculated. The details of this procedure are outlined in the following paragraphs.

We consider infinite maturity debt in the pre-matching simulation for all debt maturities in the post-matching simulation. We do so to abstract away from the impact of different initial principals on the results, allowing us to analyze the pure effect of debt maturities on credit spreads in the post-matching simulation. Additionally, starting with infinite maturity debt yields initial leverage ratios (principals) close to the ones empirically reported.[35] The model-implied economy is generated as follows: Starting with a value firm ($s = 0$), we generate a range of firms by increasing the option scale parameter s by steps of 0.05, up to the largest possible value of s such that the option is not exercised immediately. At initiation, the capital structure is chosen optimally for all firms. For each option scale parameter s, 50 firms are considered, resulting in an initial sample of more than 3,000 firms. During the ten year pre-matching simulation of this initial sample, firms default and expand optimally. Defaulted firms are not replaced, and exercised firms continue as firms with only invested assets. At the end of the pre-matching simulation, we calculate the model-implied leverage and asset composition ratio for each firm, using the assumed debt maturity and the corresponding optimal boundaries. We obtain a model-implied economy of firms covering a broad range of both asset composition ratios and leverage ratios.

In the second step, we match our average historical distribution of BBB-rated firms with its model-implied counterpart. For each observation in the average historical distribution, we select the firm in our model-implied economy at the final period of the pre-matching simulation that exhibits the minimum distance regarding the percentage deviation from the target market leverage and asset composition ratio. That is, the empirical observation of a firm with leverage lev_{emp} and asset composition ratio acr_{emp} is matched with the model-implied firm with leverage lev_{mi} and asset composition ratio acr_{mi} if - given the set of all model-implied firms - it minimizes the Euclidean distance

$$\sqrt{\left(\frac{lev_{emp} - lev_{mi}}{lev_{emp}}\right)^2 + \left(\frac{acr_{emp} - acr_{mi}}{acr_{emp}}\right)^2}. \tag{A-128}$$

The final step conducts a post-matching simulation with the obtained sample of model-implied BBB-firms over ten years. For each simulation, we obtain the realized asset volatility for each firm, and calculate the resulting average asset volatility over firms. When measuring and averaging asset volatilities, we incorporate the entire initially matched BBB-sample, including the evolution of the assets of firms that default during the ten year post-matching simulation. This approach avoids a weighting bias when averaging over simulations towards firms with lower leverage and asset volatility, which have a smaller tendency to default during the post-matching simulation.

The pre-matching simulation and the subsequent matching are conducted 20 times. The initial regime is chosen according to the stationary distribution of the states. This approach also guarantees convergence to the steady-state distribution of regimes at the time of matching. For each matched sample of firms, the post-matching simulation is run 50 times. These numbers result in a total of 1,000 simulations. The procedure is conducted for different post-matching debt maturities.

A.5.2. Simulation of the true cross-section

To ensure consistency, the simulation of the true cross-section is implemented analogously to the one performed to calibrate the idiosyncratic volatility: We first simulate a model-implied distribution of firms for ten years (pre-matching simulation), and then match the model-implied distribution with the average empirical cross-section (for details, see above). The final step consists of simulating the matched sample for 20 years (post-matching simulation). We assume that firms default and exercise optimally. Defaulted

[35] An unreported robustness analysis confirms that starting with finite maturity debt in the pre-matching simulation yields similar results for the post-matching simulation. Credit spreads are slightly lower as the initial principals are smaller.

firms are not recorded after default, whereas exercised firms are maintained in the sample, and continue as firms with only invested assets. Credit spreads and leverage ratios are measured during five years after the matching: For each firm in the sample, we calculate the actual credit spread and leverage every month, and then report the average over all firms and all simulations. Default rates are observed for five, ten, and 20 years. To assess the impact of the realized regimes at initiation and at the time of matching, we present quantiles of post-matching average rates. As in the calibration of the volatility, the initial state is chosen according to the stationary distribution. The pre-matching simulation is run 20 times, and the post-matching simulation is conducted 50 times, resulting in a total of 1,000 simulations.

A.6. Financing the exercise of the growth option by issuing additional equity

We consider the case in which the exercise price K of the growth option is financed by issuing additional equity. The corresponding system of ODEs for corporate debt is:

For $0 \leq X \leq D_B$:

$$\left\{ \begin{array}{rcl} d_B(X) & = & \alpha_B \left(X y_B + G_B^{unlev}(X) \right) \\ d_R(X) & = & \alpha_R \left(X y_R + G_R^{unlev}(X) \right). \end{array} \right. \tag{A-129}$$

For $D_B < X \leq D_R$:

$$\left\{ \begin{array}{rcl} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) \\ & & + \tilde{\lambda}_B \left(\alpha_R \left(X y_R + G_R^{unlev}(X) \right) - d_B(X) \right) \\ d_R(X) & = & \alpha_R \left(X y_R + G_R^{unlev}(X) \right). \end{array} \right. \tag{A-130}$$

For $D_R < X < X_B$:

$$\left\{ \begin{array}{rcl} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(d_R(X) - d_B(X) \right) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(d_B(X) - d_R(X) \right). \end{array} \right. \tag{A-131}$$

For $X_B \leq X < X_R$:

$$\left\{ \begin{array}{rcl} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(\hat{d}_B(\bar{s}X) - d_R(X) \right). \end{array} \right. \tag{A-132}$$

For $X \geq X_R$:

$$\left\{ \begin{array}{rcl} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ d_R(X) & = & \hat{d}_R(\bar{s}X). \end{array} \right. \tag{A-133}$$

The boundary conditions read:

$$\begin{array}{rcl} \lim\limits_{X \searrow D_R} d_B(X) & = & \lim\limits_{X \nearrow D_R} d_B(X) \\ \lim\limits_{X \searrow D_R} d_B'(X) & = & \lim\limits_{X \nearrow D_R} d_B'(X) \\ \lim\limits_{X \searrow D_B} d_B(X) & = & \alpha_B \left(D_B y_B + G_B^{unlev}(D_B) \right) \\ \lim\limits_{X \searrow D_R} d_R(X) & = & \alpha_R \left(D_R y_R + G_R^{unlev}(D_R) \right) \\ \lim\limits_{X \searrow X_B} d_R(X) & = & \lim\limits_{X \nearrow X_B} d_R(X) \\ \lim\limits_{X \searrow X_B} d_R'(X) & = & \lim\limits_{X \nearrow X_B} d_R'(X) \\ \lim\limits_{X \nearrow X_B} d_B(X) & = & \hat{d}_B(\bar{s}X_B) \\ \lim\limits_{X \nearrow X_R} d_R(X_R) & = & \hat{d}_R(\bar{s}X_R). \end{array} \tag{A-134}$$

Comparing this System (A-129)-(A-133) and its boundary conditions (A-134) to System (A-60)-(A-64) with boundary conditions (A-65), we conclude that the value of debt given that the option exercise is financed by issuing additional equity corresponds to the value of debt given that the option exercise is financed by selling assets in place with an exercise price K of zero. Hence, the value of debt in case of equity financed exercise costs can be calculated by the corresponding Formula (22) in Proposition 2. In particular, using the properties of Gauss' hyperbolic function ${}_2F_1$ and the definition of $\beta_{1,2}^R$ in (23), we find that the function $Z(X)$ as stated in line 5 of (22) in Proposition 2 simplifies to

$$Z(X) = \tilde{\lambda}_R B_5 X^{\gamma_1} + \tilde{\lambda}_R B_6 X^{\gamma_2}, \tag{A-135}$$

with

$$B_5 = \frac{\bar{s}^{\gamma_1} \hat{A}_{B1}}{r_R^n - \tilde{\mu}_R \gamma_1 - \frac{1}{2}\tilde{\sigma}_R^2 \gamma_1 (\gamma_1 - 1) + \tilde{\lambda}_R}, \tag{A-136}$$

$$B_6 = \frac{\bar{s}^{\gamma_2} \hat{A}_{B2}}{r_R^n - \tilde{\mu}_R \gamma_2 - \frac{1}{2}\tilde{\sigma}_R^2 \gamma_2 (\gamma_2 - 1) + \tilde{\lambda}_R}. \tag{A-137}$$

A.7. The equity risk premium

Proof of Proposition 3. According to Bhamra, Kuehn and Strebulaev (2010a), the equity premium $ep_i(X)$ is given by

$$ep_i(X) = \mathbb{E}_t\left[\mathrm{d}R_t - r_i^n \mathrm{d}t\right] = -\mathbb{E}_t\left[\mathrm{d}R_t \frac{\mathrm{d}\pi_t^{nom}}{\pi_t^{nom}}\right], \tag{A-138}$$

with

$$R_t := \frac{\mathrm{d}e_i(X) + (1-\tau)(X-c)\,\mathrm{d}t}{e_{i-}(X)}, \tag{A-139}$$

and $i-$ denotes the left limit of the Markov chain at time t. An application of Ito's lemma shows that

$$\mathrm{d}R_t = \mu_{R,i-}(X)\,\mathrm{d}t + \sigma_{i-}^{e,C}(X)\,\mathrm{d}W_t^C + \sigma_{i-}^{e,P}(X)\,\mathrm{d}W_t^P + \sigma_{i-}^{e,X}(X)\,\mathrm{d}W_t^X + \left(\frac{e_i(X)}{e_{i-}(X)} - 1\right)\mathrm{d}M_t, \tag{A-140}$$

with

$$\mu_{R,i}(X) = \mu_{i-}\frac{e_{i-}'(X)X}{e_{i-}(X)} + \frac{1}{2}\left(\left(\sigma_{i-}^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2\right)\frac{e_{i-}''(X)X^2}{e_{i-}(X)} + \frac{(1-\tau)(X-c)}{e_{i-}(X)}, \tag{A-141}$$

$$\sigma_{i-}^{e,C}(X) = \frac{e_{i-}'(X)X}{e_{i-}(X)}\sigma_{i-}^{X,C}, \tag{A-142}$$

$$\sigma_{i-}^{P,C}(X) = \frac{e_{i-}'(X)X}{e_{i-}(X)}\sigma^{P,id}, \tag{A-143}$$

$$\sigma_{i-}^{e,X}(X) = \frac{e_{i-}'(X)X}{e_{i-}(X)}\sigma^{X,id}. \tag{A-144}$$

Next, the nominal state price density is linked to the real state price density by $\pi_t^{nom} = \frac{\pi_t^{real}}{P_t}$. Hence, using Ito's lemma, the dynamics of the nominal state price density can be written as

$$\begin{aligned}\frac{\mathrm{d}\pi_t^{nom}}{\pi_t^{nom}} &= -\left(\mu_t^{\pi} + \pi - \left(\sigma^{P,C}\right)^2 - \left(\sigma^{P,id}\right)^2 - \gamma\sigma^{P,C}\sigma_i^C\right)\mathrm{d}t \\ &\quad - \left(\gamma\sigma_i^C + \sigma^{P,C}\right)\mathrm{d}W_t^C - \sigma^{P,id}\mathrm{d}W_t^P + \left(e^{\kappa_i} - 1\right)\mathrm{d}M_t. \end{aligned} \tag{A-145}$$

Plugging A-145 into (A-138) and taking the expectation yields the equity premium

$$ep_i(X) = \frac{e_i'(X)X}{e_i(X)}\sigma_i^{X,C}\left(\gamma\sigma_i^C + \sigma^{P,C}\right) + \frac{e_i'(X)X}{e_i(X)}\left(\sigma^{P,id}\right)^2 - \lambda\left(\frac{e_j(X)}{e_i(X)} - 1\right)\left(e^{\kappa_i} - 1\right). \tag{A-146}$$

□

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