Master Thesis

Bogdana Pereboichuk
MSc in Advanced Economics and Finance
Department of Economics
Copenhagen Business School

Supervisors: Nina Lange, Department of Finance
David Skovmand, Department of Finance

MODELING OF CRUDE OIL PRICES WITH A SPECIAL EMPHASIS ON MACROECONOMIC FACTORS

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Executive Summary

This thesis aims to find out what drives price of crude oil. Since nowadays oil is rather a financial asset than industrial good its price evolves similarly to other financial assets prices. Traditional financial approach to modeling is based on assumptions about stochastic processes which allows for capturing partly random behavior of oil price changes. Besides estimating simple stochastic models (Geometric Brownian Motion (GBM) and Mean-Reversion) which are widely used for other commodities and returns modeling, influence of unique for oil factors is also assumed and tested.

A unique feature of oil market is presence of precautionary demand reflecting expectations and concerns about future need for oil. Based on the common knowledge about global significance of oil, high market power of the producers, possible political instability and also observing soundness of macroeconomic background market participants form their expectations concerning future oil necessity. If the macroeconomic background is believed to be sound – optimistic mood in the market results into high precautionary demand. At this state market is very vulnerable to any announcements, and price response is immediate and sharp.

In order to capture expectations impact on oil prices stochastic modeling is extended with factors describing macroeconomic conditions through the oil price volatility channel modeled within GARCH framework. The volatility models have better forecasting accuracy on the short horizon but produce only approximated long term expectation similarly to the simple ones.

Simple stochastic models for oil prices demonstrate that drift estimations are very uncertain but are more reliable for the GBM model. The main finding here is that crude oil price process has a drift, but it changes once in a while – it may be assumed constant but for shorter than sample time periods. This conclusion may also hold for the mean-reverting property even though the latter is not supported by the findings. But the main attention should be paid to diffusion term, estimation of which is on average the same for all models and methods. It has strong serial dependence not consistent with theoretical properties of stochastic processes.

It is reasonable to assume that serial dependence of stochastic term can be captured by heteroscedasticity modeling. GARCH models estimated reveal that oil price volatility positively responses to bond yield spread widening and depreciation of US dollar. Asymmetric property of the volatility is also documented meaning that oil prices become more volatile in occurrence of positive rather than negative shocks (the sign of the link is positive which is opposite to expected for asset returns. Typically asset prices are falling sharply in negative shocks presence, but in case of oil it may indirectly prove the precautionary demand hypothesis through the stronger impact of optimistic atmosphere rather than pessimistic). Convenience yield has a positive direct and very strong impact on oil price which is a fundamental theoretical assumption for commodity modeling consistent with theory of storage: oil price responses positively on inventories scarcity. US dollar to British pound exchange rate influences oil price also directly and this link is more stable than that in variance. Even though the obvious indicators of macroeconomic activity did not demonstrate strong influence on oil price but they apparently are not the best measure of expectations influencing the precautionary demand.

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List of Abbreviations

ACF, PACF – (Partial) Autocorrelation Function

AIC – Akaike Information Criterion

ARMA – Autoregressive Moving Average

b/d, tb/b, mb/d – barrels (thousand, million) a day

bb – billion barrels

BEKK – Baba-Engle-Kraft-Kroner

BFOE – Brent, Forties, Osenberg, Ekofisk

BIC – Bayes-Schwartz Information Criteria

BTU – British Thermal Unit

CFD – Contract for Difference

CL – Confidence Limit

EFP – Exchange Futures for Physicals

EIA – Energy Information Administration

EMM – Equivalent Martingale Measure

EOR – Enhanced Oil Recovery

E-GARCH, eGARCH – Exponential GARCH

GARCH – Generalized Autoregressive Conditional Heteroscedasticity

GARCH-M – GARCH-in-Mean

GBM – Geometric Brownian Motion

GBP – Great Britain pound

GDP – Gross Domestic Product

HQ – Hannan-Quinn information criterion

ICE – Intercontinental Exchange

IEA – International Energy Agency

iid – independently and identically distributed

LIBOR – London Interbank Offered Rate

LM-test – Lagrange-Multiplier test

log – logarithm, term used in this thesis means ‘natural logarithm’

ME, MSE, MAE – Mean (Squared, Absolute) Error

mGARCH – multivariate GARCH

MLE – Maximum Likelihood Estimation

Mtoe – Million tons of oil equivalent

NYMEX – New York Mercantile Exchange

OECD – Organization of Economic Cooperation and Development

OLS – Ordinary Least Squarres

OPEC – Organization of Petroleum Exporting Countries

OTC – Over-the-Counter

OU – Ornstein-Uhlenbeck

PDF – Probability Density Function

PRA – Price Reporting Agency

QQ-plot – quantile-quantile plot

S&P – Standard&Poors

SD – Standard Deviation

SE – Standard Error

sGARCH – standard GARCH

VAR – Vector Autoregression

VECM – Vector Error Correction Model

URR – Ultimate Recoverable Reserves

USD – United States dollar

USD/b, $/b – dollar per barrel

w.r.t – with respect to

WTI – West Texas Intermediate

Introduction

Without any doubts, from technological point of view, energy usage patterns determine foundations of the whole global economy. Any kind of physical production and transportation is completely impossible without energy. Even non-physical production (services) now is unlikely to be performed without simply electricity. Remarkably, that engineers, physics and historians often consider energy to be the primary factor input to industrial and economic development[1]. "*From an energy perspective, Industrial Revolution was essentially replacement of the muscle power of men and animals with the fossil power… Since the start of the Industrial Revolution, economic growth has been largely synonymous with increasing energy use, generally at declining real prices*." [Noreng (2007), p. 7]

The importance of energy is obvious but our understanding of its supply-demand and trading links is definitely not good enough. In macroeconomic or business cycle theories energy is either neglected or treated as an exogenous source of shocks. Therefore there is neither general explanatory theory about the role of energy in economic system nor much theory about its price formation, even though historical examples (oil shocks in the 1970s-1980s) show the reasonable necessity of such theories and much available data allow for the research to be done.

Among the energy sources oil is proved to be the most efficient concerning concentration of pure energy and convenience of usage and transportation. More than 40% of the world energy consumption is satisfied with oil (see Chart 1, Appendix 2), and in the transportation sector it, in fact, does not have other competitors: 93% of transport fuels are based on oil (see Table 1, Appendix 1). In addition to that, plastics and fibers used by practically each individual are products of petrochemical industry.

It is not difficult to deduct that crude oil prices have a huge influence on the world economy and, being highly uncertain and volatile, are a source of economic and political risks and instability. The main purpose of this thesis is trying to investigate what drives crude oil prices and which modeling method is able to give a good price forecast.

Broad literature concerning modeling of oil prices can be divided into two streams by treatment of oil: either it is an industrial product with respective supply-demand links or an underlying financial asset broadly used to price various traded derivatives. Clearly the two approaches use completely different methods to assess oil price behavior. But no matter which

[1] Noreng (2007), p.7

approach is chosen it is important to understand that oil possesses unique properties emerging from specifics of petroleum industry and oil consumption.

Oil is a physical resource limited in volumes. World oil reserves are highly concentrated in not numerous geographical locations. Half of the world oil is located on Middle East. Naturally, as in any other industry, competition level and producers' market power matter for the price formation. In addition to that, most oil producers are nationalized companies and thus supply side of the petroleum industry is often driven by political motives. This high market power feature of the industry is also greatly fueled by strategic significance of oil. Even though world dependence on oil is not that critical now as some decades ago especially with alternative energy generating methods development, but it still remains high. It is probably too early to talk about existence of economically efficient substitutes for oil. And finally, oil industry is very capital intense and thus requires huge inventories input on all levels.

Basically these specifics can be projected onto dependence of oil prices on macroeconomic foundations through expectations and precautionary demand channels. Precautionary demand concept, introduced by Kilian (2009), concerns a unique feature of oil market. Based on the common knowledge about global significance of oil, high market power of the producers and possible political instability market participants form their expectations concerning future need for oil. If the macroeconomic background is (believed to be) sound, market is in optimistic mood – thus, expectations of future oil necessity are high resulting into high precautionary demand. At this state market is very vulnerable to any announcements and price responses immediately and strongly.

So for modeling oil prices it can be beneficial to include some macroeconomic indicators in order to capture the expectations impact. However expectations formation mechanism and macroeconomic conditions soundness are hardly defined concepts. This is a big obstacle for their correct measurement and implications for modeling. Obvious macroeconomic indicators may not be relevant due to reverse causality and leading-lagged relationships with oil prices.

Oil is mostly traded on the financial markets and only a very small fraction of contracts is set for actual physical delivery. That is why it would be more efficient to model price of oil as a price of financial asset. In this thesis two simple models proposed by the financial literature are estimated. The Geometric Brownian Motion (GBM) and The Mean-Reversion models developed by Brennan, Schwartz (1985) and Schwartz (1997) are based on the assumption that oil price follows a stochastic process with known mathematical properties. The GBM model is further extended with non-constant volatility and drift term modeled within a GARCH(1,1) framework together with convenience yields and factors reflecting macroeconomic conditions.

The simple stochastic models for oil prices have similar performance: diffusion term has higher weight and outperforms the drift term. The drift estimations are very uncertain but are more reliable for the GBM model. The main finding here is that crude oil price process has a drift, but it changes over the sample period. Sometimes oil prices are strongly influenced by a time trend, sometimes not. This conclusion may also hold for the mean-reverting property even though the latter is not supported by the model estimation evidence.

Further heteroskedasticity modeling reveals that oil price volatility positively responses to bond yield spread widening and depreciation of US dollar. Asymmetric property of the volatility is also documented meaning that oil prices become more volatile in occurrence of positive rather than negative shocks (the sign of the link is positive which is opposite to expected for asset returns. Typically asset prices are falling sharply in negative shocks presence, but in case of oil it may indirectly prove the precautionary demand hypothesis through the stronger impact of optimistic atmosphere rather than pessimistic). Convenience yield has a positive direct and very strong impact on oil price which is consistent with theory of storage assumptions: oil price responses positively on inventories scarcity. US dollar to British pound exchange rate influences oil price also directly and this link is more stable than that in variance. Other macroeconomic variables do not have significant impact on oil price which can be caused either by being bad measure of expectations or by presence of lagged rather than current connections.

Concerning the models forecasting ability: simple stochastic models work roughly as good as complex extended volatility models especially for longer term predictions. But for one-month-ahead forecasts volatility models have higher precision. Volatility models also demonstrate better performance for periods when oil price are highly unstable (e.g. recent financial crisis), but they on average overestimate prices.

The rest of the thesis is organized as follows: Chapter 1 provides literature review with the main approaches and findings for crude oil price modeling. Chapter 2 contains general overview of the global petroleum industry and oil consumption in order to develop the precautionary demand hypothesis and argumentation about the macroeconomic fundamentals necessity for oil price modeling. Chapter 3 gives the theoretical framework for modeling approaches. Chapter 4 contains empirical models estimations, analysis and interpretation and ends with the forecasting ability testing of the estimated models. Concluding chapter contains discussion comments and concluding remarks.

Chapter 1
Literature Review

During recent decades oil and its prices were subjects of various scientific research, political discussions and fiction stories. There is nothing strange about massive attention to this resource since it is so deeply integrated in world economy. Oil is a national strategic resource but also used by roughly each individual household. No surprise that oil prices were never safely under control, and factors behind price changes are still not revealed with absolute confidence. Moreover, with oil significance coming onto new levels, meaning role it plays in the financial markets, new potential drivers of the prices appear which complicates their behavior understanding even more.

There are different ways to investigate oil price behavior. First, it is necessary to understand basic fundamentals of the global petroleum industry and market, and what oil price is on the background of macro- and microeconomic structures activity. It is quite dangerous to consider just oil itself separated from the rest of global economy and politics. Second, based on assumptions about state of arts in the oil industry it becomes possible to formalize it in context of world/state economy in order to check mathematically or statistically oil price dependence on different factors. Obviously each way has its advantages and disadvantages. Research without any formalization in some meaning does not have proofs: it is built on facts and history analysis, but such attempts to explain the situation are no more than reasoned guessing. Sometimes human logic may be biased by misperception or lack of information while statistical data and objective mathematics is unlikely to be affected by those. On the other hand, none of the models is able to capture complex nature of oil, its markets and industry with a lot of factors being unobserved or immeasurable. It is also quite hard to be fully confident in validity and unbiasedness of any results obtained.

This thesis is devoted to formalized research of oil price behavior which, however, would not be complete without analyzing literature on political and economic specifics of oil as a resource. It is important because before testing this or that model it is vital to know which assumptions to make and which factors' impact to emphasize.

From the first glance it can be noticed that oil price is highly politically driven due to concentration of this resource and market power of its owners. The book cited a lot in this thesis for historical and political reasoning is "Crude power" (2007) by Øystein Noreng. The author analyses state of global petroleum industry and of major nations-players on the oil market, concentration of reserves and production, global necessity for oil and history of major oil price shocks. The main conclusion is that market power and political alliances are major determinants of oil price levels. According to the author's opinion it is possible because oil is deeply strategically needed, has price inelastic demand and half of it is located at Middle East. Political events as causes of supply disruptions are considered to be major reasons of oil price shocks of 1970s-1980s. Demand shocks were also relevant for price collapses of mid 1980s and 1997.

Formalized approach includes various models for energy prices developed during decades of research. According to Most and Keles (2010) existing models can be grouped into four categories:

So-called *Fundamental (or computational) models* simulate the technical characteristics of the energy sector, especially the impact of industrial characteristics, capacities, restrictions and demand variations. Usually they were used for modeling electricity prices. Most of these approaches are based on a few internationally known and widespread models, like MARKAL (Market Allocation Model, Fishbone and Abilock, 1981), EFOM (Energy Flow Optimization Model, Finon, 1974; Van der Voort et al., 1984), MESSAGE (Model for Energy Supply System Alternatives and their General Environmental Impact, Agnew et al., 1979; Messner, 1984; Messner and Strugbegger, 2009), CEEM (Cogeneration in European Electricity Markets, Starrmann, 2001), TIMES (The Integrated MARKAL EFOM system, Remme, 2006) and PERSEUS (Program Package for Emission Reduction Strategies in Energy Use and Supply, Most, 2006; Fichtner, 1999), which was developed on the basis of EFOM. [2]

Agent-based simulations take into account interaction between different market participants. The concept includes building a behavior simulation from a player's perspective which helps to integrate aspects like strategies or imperfect information[3]. Among the agents Senstuss, Genoese (2001) simulating electricity market in Germany name consumers, utilities, renewable agents, grid operators, government agents and market operators. When assumptions about agents' behavior and contracting are made authors simulate demand and supply side of electricity production.

[2] Most, Keles (2010), p. 544
[3] Senstruss, Genoese (2001), p. 1

Further class of models is based on *Game-theoretic approaches* focusing of analyzing impact of strategic behavior[4]. Depending on the market structure (regulation versus liberalization and level of monopolization) price determining mechanisms differ.

The last type of models is the most widely used in recent literature. *Financial mathematic models* and *time-series models* tend to explain price paths just by themselves without consideration of other variables. It have been noticed that energy prices possess seasonality, mean-reversion, high volatility and spikes properties[5]. Stochastic and/or time-series modeling allows to capture these properties and to develop unconditional forecasts for prices.

In broader words, models belonging to the first three classes are structural, to the last one – non-structural or reduced form models. *Structural models* try to specify price relationship in a system of equations describing the economy and oil supply-demand links. Oil is a product of a big industry which requires huge capacity, inventories and labor inputs, to say nothing about geological and other research and environmental issues, which jointly influence cost of its production. In its turn there are also forces (including exogenous) influencing the consumption.

On the other hand, oil nowadays is a globally traded commodity, and the majority of contracts are managed without actual plans of parties to own or use it. It is not wise to not take this fact into account. Oil can be treated as underlying financial asset so prices are also behaving similarly to prices of traditional financial assets. *Financial models* usually are based on stochastic processes using certain properties of historical data.

Early models often focused on scarcity of the resource and/or on strategic behavior of OPEC. The first significant attempt to develop a structural model on oil as a scarce resource belongs to Hotelling (1931). The basic idea is that in competitive setting resource extraction depends on dynamics of interest rate since the owner has two opportunities: either extracting now and consuming/investing or leaving the extraction for the future. "*First, the static efficiency condition claims that the value of extraction is equal to the shadow value. This price component reflects the opportunity cost of using one unit of the resource today rather than tomorrow and arises only due to the fact that the supply of the resource is finite. Second, the dynamic efficiency condition states that the optimally extracted quantity adjusts such that the shadow value increases at a rate of return comparable to an alternative investment*" [Leirnert (2012), p. 1]. Today theoretical validity of the simple model is questioned but the model lies in foundations of many other more complex structural models.

[4] Most, Keles (2010), p. 545
[5] Hungtinton et al (2013), pp. 5-6

One of attempts to test the Hotelling model belongs to Lin (2009) where she uses annual data for oil price and consumption to calibrate the model for different competition settings. She finds that prior to 1973 the best fitting data model was the one assuming competitive market while afterwards till recent years the price was strongly influenced by OPEC. However the performance of the simple Hotelling model was considered to be poor.

Indirectly Hotelling rule is also tested at Leinert (2012). The paper finds that crude oil price adjusts [falls] to unexpected news about oil field discoveries. So it is proved to be sensitive to level of scarcity which can be seen as an evidence of the shadow cost component presence.

Concerning strategic behavior of the market participants resulting in oil price change one may consider Kalymon (1975) paper where OPEC export profits and domestic consumer surplus are maximized. The cartel can be viewed as one stable unity or with Saudi Arabia and Iran acting as residual producers, thus different market sharing outcomes are possible. Optimal price is found sensitive to the opportunity cost of capital, substitution cost for importers and coalition within the cartel, but not sensitive to oil reserves[6].

Separate branch of structural models is Game Theoretic models, where at least 2 players are maximizing their benefits and their actions do affect each other. This type of problems is solved with finding Nash equilibrium where both parties have no more intentions to deviate from the optimal path. Salant (1982) introduces a model combining the theory of exhaustible resources and the theory of oligopoly. In his model each producer is either one of many competitive sellers or one of several large but competing sellers (termed Cournot[7] players). A competitive player allocates their resource over the current and future years (solving the Hotelling problem) by taking the prices as given. On the other hand, Cournot players set prices taking level of production from others as given[8].

Recent structural models usually are more complex, go deeper into details or use advanced computational or estimation technics. For example Unalmis et al (2012) use theory of storage in DSGE model of US economy claiming that dynamics of storage can be an important factor influencing the short term dynamics of oil price. In the presence of oil storage, which provides intertemporal link between inventories, the market-clearing price becomes a function of availability relative to the total demand which is endogenously determined.

Even though structural models are an obvious way to model price of an industrial good (and crude oil for sure is one) but today oil has more importance as a financial asset. Moreover,

[6] Hungtinton et al (2013), p. 16
[7] Cournot game describes production quantity optimization taken price as given.
[8] Hungtinton et al (2013), p. 18

trading and price setting is performed exclusively on exchanges under the conditions these financial institutions determine. So it makes a lot of sense to consider models for financial asset pricing also relevant for crude oil price modeling.

Stochastic models[9] are based on the assumption that commodity price follows a mathematical probability law process (like Wiener process) together with or without a deterministic component. Commonly these models are based on univariate Geometric Brownian Motion or the Mean-Reverting processes. Typically current price change is believed to be depended on the current price level (Markov property) as a part of drift term and have a random component with known distributional (Gaussian) properties. In literature it is possible to find examples of stochastic models, simple or with various extensions, applied to crude oil prices analysis, forecasting and derivative pricing.

The basic theoretical GBM oil pricing model was proposed by Brennan and Schwartz (1985) and used for pricing of financial derivatives. Later Gibson and Schwartz (1990) derive a stochastic spot price model with convenience yield following a mean-reversion process. Concept of convenience yield basically explains what drives difference between spot and forward prices of the commodity: benefit of owning the commodity balanced with costs of it storage/carry. Spot price and convenience yield are assumed to follow a joint stochastic process. The authors judge about the model performance by its ability to price financial instruments linked to crude oil. They document a small overpricing error which, however, increases with maturity of contracts and emphasize the importance of convenience yields.

Schwartz (1997) compares performance of the Mean-Reversion model (one factor), a stochastic model (GBM) with convenience yield (two factors) and additionally with stochastic interest rate (three factors) which have been widely used as benchmarks in later research. The author documents evidence of mean reversion of crude oil prices finding all the parameters significant. In the two factor model oil price is modeled as GBM but with convenience yield included in the drift. The paper documents validity of the two-factor model but the drift parameter of the spot price is not always significant. Average convenience yield is found positive. Three-factor model is valid as well but adding a stochastic interest rate to the previous model does not improve the performance much while the level of complexity increases strongly.

Further research literature often uses the basic models with some extensions or complications such as stochasticity of parameters, seasonality or unexpected jumps. Special

[9] See more details about theory and modeling in Chapter 3

attention is paid to oil price volatility modeling. Financial theories and modeling tools were progressing quickly allowing for capturing different properties of energy prices and using them for sophisticated valuation of financial instruments. Theory and implications of stochastic models are well summarized in such fundamental works used in this thesis as “Energy Risk” (2007) by Dragana Pilipovic and “Stochastic modeling of electricity and related markets” (2008) by F.E. Benth, J.S. Benth and S. Koekebakker.

Indicating high instability of oil prices and their sensitivity to supply-demand shocks and news Krichene (2006) develops a jump-diffusion model adding a Poisson jump component to the basic GBM model of oil prices for the 2002-2006 period. The paper finds that oil price is influenced by both diffusion and jump components, but jump component is a dominant one. Significance of the drift term yields that oil price was strongly influenced by an upward trend. Oil prices were also modeled as Levy process to capture non-constant volatility effects. Volatility is computed either from GARCH(1,1) model or implied from observed option prices.

Meade (2010) compares the GBM and Mean-Reversion models for crude oil price forecasting and finds that both models are plausible for short time horizon (up to three months), but for longer term are not useful because of non-constant volatility and presence of jumps which is not consistent with Gaussian assumption about the random component. The paper also suggests that non-constant volatility can be well captured by a GARCH model.

Concept of convenience yield was also analyzed in Knetsch (2006). The paper proves both theoretically and empirically existence of convenience yields and their usefulness for crude oil prices forecasting. Futures prices are believed to be systematically biased predictors of future spot prices, and convenience yield is a source of this bias. Convenience yields are believed to depend on changes of inventories (Theory of storage). This hypothesis is tested and proved in Dincerler et al (2005). They find that commodity prices react positively on inventories scarcity.

The above described stochastic models by their definition are mostly univariate: oil price to some extend is determined by itself. Even if other factors are included (volatility or convenience yields) they are basically also derived from oil price paths. Contrary to financial models, structural models try to prove validity of exogenous factors determining supply and demand. Therefore, while using financial modeling approach if impact of exogenous factors is to be tested empirically then mainly time-series models are used. Basically time-series models are a somewhat modified discreet time version of continuous time stochastic models. Additionally, time series models allow for exploiting deeply lagged serial correlation or cross correlation in a multivariate case while stochastic models assume only 1-lag dependence (Markov property).

Typically for multivariate analysis Vector Autoregressive or Cointegration (VECM) models are used. For example Dees et al. (2008) find that oil prices are well explained by refinery utilization rates and OPEC spare capacity (negative link) and by expectations of future market conditions (contango) which has positive effect on prices. Bencivenga et al (2012) investigate relationship between oil prices and set of macroeconomic and financial variables (Dollar/Euro exchange rate, US interest rate, US oil imports and price of gold). All of them contribute to build a long-run equilibrium of oil price, but exchange rate and gold price are most important.

Behavior of oil prices during the resent crisis of 2007-2008 was examined by Hamilton (2009). Here a lot of attention is paid to demand elasticity measurement. The author comes to the conclusion that petroleum demand is rather income elastic than price elastic which means that in recent years changes of income were a major determinant of sharp oil price changes. Indirectly the author points onto the importance of expectations.

Continuing topic about expectations Kilian (2009) introduces and important term: precautionary demand for oil which is associated with market concerns about the availability of future oil supplies. The paper proposes structural VAR approach to examine impact of three classes of reasons influencing real oil prices: increase in precautionary demand for oil causes large and persistent increase in oil price while supply disruptions (due to politics) cause small and transitory effects. Demand for other industrial commodities has a delayed but persistent influence on oil price.

Volatility modeling is usually performed in univariate or multivariate (allows for capturing volatility spillovers from other markets) GARCH frameworks. For example Sadorsky (2006) finds that GARCH(1,1) model fits well crude oil price volatilities but other studies suggest different models from the GARCH family[10]. Empirical evidence on these models performance for crude oil price volatility is mixed.

Wang and Wu (2012) compare various univariate GARCH models and find that there is high degree in volatility persistence and the volatility is a long memory process with significant asymmetric effects. Multivariate analysis reveals significant volatility spillovers from other oil products markets (heating oil and jet fuel) to crude oil. The authors conclude that multivariate models have better performance relatively to univariate.

[10] Wang, Wu (2012), p. 2167

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Structural models strongly suggest that oil price must be sensitive to competition level of the industry, inventories, general macroeconomic dynamics and expectations. Among crucial factors are noted interest rates and exchange rates. Stochastic models are found useful for short time horizon forecasting and also are widely used to price financial instruments with crude oil as underlying. Empirical research finds evidence that the price follows the mean-reversion or classic random walk process and proves significance of convenience yields. It is also commonly found that allowing for non-constant volatility improves models' performance.

Research literature explaining crude oil price behavior is very rich in methods used for price modeling. Oil is either modeled as an industrial good taking into account strategic state of the market or as an underlying financial asset, price of which possesses unique mathematical properties. These approaches are very different in their nature: structural models tend to derive price from set of supply/demand determinants while financial approach seeks explanation in the price path itself often without considering any other factors. Despite a huge difference in assumptions and methods the both research approaches still have the same object. Therefor it can be very beneficial to combine advantages both approaches have.

Chapter 2
What is Oil and Specifics of the Oil Market

The aim of this chapter is to provide basic information about oil as a physical resource, its production, pricing and usage. Petroleum industry with its economic and political specifics is different from other industries which has a unique impact on oil price formation mechanism. The chapter ends with formulation of the precautionary demand hypothesis and a proposition to consider factors describing macroeconomic conditions as inputs to models.

2.1 Nature and origin of oil

Crude oil is a complex mixture consisting of more than 200 organic compounds, especially hydrocarbons: mostly alkenes and smaller fraction aromatics.[11]

Crude oil varies in color from nearly colorless to tar black, and in viscosity from close to that of water to almost solid. In fact, there are more than 300 different crude types produced around the world. Two the most important characteristics are density (or viscosity) and sulfur content. High-quality cruds are characterized by low density (light) and low sulfur content (sweet) and are typically more expensive than their heavy and sour counterparts: light crudes produce more high-value products, while sweet crude oils require less processing than sour. [12]

The oil and gas bearing structure is typically of porous rock such as sandstone or washed out limestone. The sand might have been laid down as desert sand dunes or seafloor. Oil and gas are formed from organic material (tiny plants and animals) deposited in early geological periods (100-200 million years ago) together with sand or silt and later transformed by high temperatures and pressure into hydrocarbons.[13] Formation of oil reservoirs requires meeting of various climatic, geophysical and historical conditions and enormous amount of time.

In 2011 proved oil reserves volume accounted for around 1473[14] billion barrels[15]. The most oil reach region is Middle East possessing 51% of total oil reserves, then – Central and South America with 16%. Europe (excluding Eastern Europe and Russia) owns only 1% of world oil

[11] Devold (2009), p. 19
[12] Dunn, Holloway (2012), p. 66
[13] Devold (2009) p. 22
[14] Source of data: EIA
[15] Barrel – a volume measure of oil or other liquids, 117.35 liters

reserves (see Chart 2, Appendix 2). On the country level the biggest oil reserves belong to Saudi Arabia and Venezuela (262.6 and 211.17 bb in 2011 respectively) together owning around one third of world oil. Among ten countries with biggest reserves are also Canada, Gulf states (Iraq, Iran, Kuwait, UAE), Russia, Libya and Nigeria (see Table 2, Appendix 1). 18 countries own reserves of more than 10 bb and 51, including Denmark, less than 1 bb.

Worldwide there more than four thousand oil fields. Most of them are relatively small with production up to 20 thousand b/d. Gigantic oil fields have daily producing capacity of more than 0.1 mb/d or reserves more than 500 mb. The world biggest oil field, Ghawar, is located in Saudi Arabia. Besides Middle East gigantic oil fields can be found in Venezuela, Mexico, Russia and US. List of fields and their estimated reserves is in Table 3, Appendix 1.

20 years ago 15 gigantic oil fields could produce more than 1 mb/d, now only 4 can produce that much[16]. Simple mathematical calculation with 1473 bb oil available and 88614 tb/d oil consumed in 2011 predicts that oil reserves will be fully exhausted in 45.5 years. But due to exploration work and new fields discoveries the estimated volume of reserves raises. For example, in 2013 compared to 2012 volume of Venezuelan oil reserve increased from 211.17 to 297.57 bb and Russian – from 60 to 80 bb. The preliminary calculations of world oil reserves change for 2012-2013 based on EIA data is increase by 117.76 bb without US dynamics (for 2011 and 2012 US volume of reserves is not reported).

2.2 Pricing of crude oil

Crude oil market both physical and financial is bigger than that of other commodities[17]: annual exports of oil exceed those of natural gas in more than four times though the production is only twice bigger. Annual turnover on the financial market of oil is three times bigger than on copper market, in four times – than on gold market and twelve times than that of natural gas (see Table 12, Appendix 1).

Almost all internationally traded oil is sold on the OTC market, which better suites heterogeneous nature of oil that requires specially tailored contracts. Around 90% of physical oil is traded under medium- and long-term contracts. Spot-trading for physical delivery is less common due to transportation costs. ‘Spot’ in this context describes more the nearest delivery: it may take up to 60 days with average of more than 10 days, which is usually longer than for other commodities (Henry Hub natural gas has next day delivery, London Metal Exchange – within

[16] Robelius (2005)
[17] Dunn, Holloway (2012), p. 65

two days). Physical crude can be purchased entering a futures contract on an organized exchange, but in fact only 1% of these contracts are settled in terms of buying physical commodity. Futures contracts specify type and quantity of oil (usually 1000 barrels) for future delivery. The two key futures contracts globally traded are NYMEX WTI (West Texas Intermediate) light sweet crude and ICE (Intercontinental Exchange) Brent oil. [18]

Since there are so many different types of crudes it is difficult to price each one of them. Pricing thus is based on a few benchmarks, notably Brent and WTI. Brent oil is produced in the North Sea and is used as a reference price for roughly two-thirds of global physical oil trade. WTI is produced in the US and accounts for two-thirds of global futures trading. These benchmarks form the base for pricing most contracts: contract-specified pricing formula nests differentials, added or subtracted from the benchmark price. Price differentials relate to factors such as the difference in quality between the contracted and benchmark oils, transportation costs and the difference in the refinery's return from refining the contracted and benchmark crudes into the various petroleum products. Oil-producing companies typically use different benchmarks for reference depending on the final destination of exported oil. [19]

Oil price benchmarks[20]

Brent was developed as a benchmark due to favorable tax regulations for oil producers in the United Kingdom in addition to the benefits of stable institutions. Moreover, ownership of Brent crude oil is well diversified (more than 15 producers), which helps to reduce individual pricing power. When the Brent benchmark was established in the mid-1980s its production was reasonably large and stable, which is an important characteristic of a benchmark – the guarantee of timely and reliable delivery. Since the Brent oil production has declined over the past decade three other North Sea crudes have been added to the benchmark basket: Forties, Oseberg and Ekofisk.

Determination of the Brent price is a quite complex process and it involves a number of different prices: Dated Brent, ICE Brent futures and Brent forwards prices.

Dated Brent is regarded as the 'spot' price. It reflects the price of a cargo of Brent crude oil which will be loaded onto a tanker at a specified date (in 10–25 days). However, very few physical trades are actually priced on an outright basis, so spot prices

[18] Dunn, Holloway (2012), p. 67
[19] Dunn, Holloway (2012), pp. 68-69
[20] From Dunn, Holloway (2012), pp. 69-73

are not directly observable. Dated Brent is then assessed by a PRA (Price Reporting Agency) using information from physical and financial markets.

Brent forwards (or 25-day BFOE) are OTC forward contracts specifying a delivery month at which the cargo will be loaded. The '25-day' means that buyers are notified of the loading dates 25 days ahead of delivery. PRAs then assess the contract-for-difference (CFD) price. CFDs are relatively short-term swaps between the floating price (Dated Brent at the time of loading) and a fixed price at a future date (Brent forward price). Taking weekly CFD values (for the next 8 weeks) and combining them with the 2nd month forward price, the PRAs can construct implied future Dated Brent prices (forward Dated Brent curve). Using this curve, implied Dated Brent prices for 10-25 days ahead can be calculated, average of which is known as the North Sea Dated Strip.

Combining this with differentials for each of the four crudes in the Brent basket gives a price for each one of them. The cheapest then becomes a final published daily quote for Dated Brent. This is typically Forties as it has the lowest quality.

Occasionally, however, there is insufficient liquidity in the Brent forward market to use this method to set daily spot. In that case, the assessment instead starts in the futures market. A synthetic Brent forward price is derived by combining the ICE Brent futures prices with 'exchange of futures for physicals' (EFPs) values. Futures contracts are settled in cash, with an option for delivery via an EFP contract. Whereas futures contracts are highly standardized, EFPs are more flexible. This allows traders converting a futures position into physical delivery.

Once Brent forward curve is derived Dated Brent is calculated as before.

The emergence of WTI as a benchmark in 1983 was also caused by the presence of secure legal and regulatory regimes in the United States. Like Brent, WTI is a light sweet crude that is available from a broad range of producers: sweet crudes from Oklahoma, New Mexico and Texas, as well as several foreign crudes. WTI oil is delivered via an extensive pipeline system (as well as by rail) to Cushing, Oklahoma.

There is only one main instrument that underlies the WTI benchmark price: the NYMEX Light Sweet Oil futures contract. It allows for physical delivery when left open at expiry, specifying 1000 barrels of WTI to be delivered to Cushing. Though the proportion of WTI futures contracts actually physically settled is very small.

Reflecting the absence of a significant forward market, the PRAs' assessed 'spot' price for WTI is determined differently to that for Dated Brent. The WTI spot is typically the most recent NYMEX WTI front-month (contract nearest to expiry) futures price in a period immediately prior to the price assessment time. At contract expiry, the PRAs' reported price reflects the new front-month futures price plus the 'cash roll' (the cost of rolling a NYMEX futures contract forward into the next month without delivering on it).

The complexity of oil pricing process makes it difficult to convince that the benchmarks reflect real demand-supply conditions rather than knowledge of financial speculators. But simultaneous movements of benchmarks differentials (WTI-Brent spread) were consistent with demand-supply changes. Till 2011 the benchmarks moved together with the spread reflecting transportation cost of Brent oil to Cushing. In recent years the difference became sufficiently large, see Chart 10, Appendix 2. One of the reasons is the increased production inflows from North Dakota and Canada to Cushing which made it difficult to remove extra oil from Cushing. Another reason of increased Brent-WTI difference is due to declines in North Sea production. [21]

2.3 Brief overview of the world petroleum industry

People were using oil for lightning purposes already thousands of years ago but the first successful oil well was drilled in 1859, North-Western Pennsylvania, by Edwin Drake which was a start to commercial production of petroleum[22]. Soon oil replaced most other fuels for motorized transport: automobile industry quickly adopted oil as a fuel and gasoline engine was sufficient to construct the first successful aircraft. Significant percentage of oil in total energy consumption (more than 20%) was already reached in the 1960s and in 1970s it increased till almost 50% (see Chart 3, Appendix 2).

Oil Supply

Oil producing companies belong to 137 national states. Middle East produces 30% of world oil compared to 51% of world reserves it owns. European oil companies produce 5% of world oil, North American – 20% (see also Chart 4, Appendix 2).

[21] Dunn, Holloway (2012), p.73
[22] Devold (2009), p. 5

Among oil producers the biggest are Saudi Arabia, US and Russia producing together more than 10 mb/d followed by China and Canada. 38 countries, including Norway, have production of more than 1 mb/d (see Table 4, Appendix 1). US and China are among the biggest oil producers but together they also account for more than one third of net imported volume of oil (see Table 5, Appenix 1).

State-own companies in the world oil industry prevail private even though it may seem the opposite. In total national companies account for 60% of global oil production and for 80% of reserves whereas the five largest publicly traded companies – Exon Mobil, British Petroleum, Chevron, Royal Dutch Shell and Total – account for 2-3% of global oil production each and for only 3% of reserves together. World biggest oil companies are Saudi Aramco (12.1% of global production) and National Iranian Oil Company (5.2%)[23], see also Table 6, Appendix 1.

"*Today oil and gas are produced in almost every part of the world, from the small 100 barrels a day private wells to the large 4000-barels wells; in shallow 20m deep reservoirs to 3000m deep wells in more than 2km of water; from 10 thousand dollar onshore wells to 10 billion dollar offshore developments*" [Devold (2009), p.7]

Worldwide there exist thousands of wells of different capacities onshore. Wells are connected to a gas oil separation plant. From the plant the product is sent with pipelines or tanks. With high oil prices lately and limited volume of conventional reserves it became economically reasonable to exploit new potential oil sources. Those include heavy crude tar sands (heavy bitumen) and oil shale. Unconventional oil sources may triple available oil reserves.

Apart for oil extraction overland, sea offshore developments are also in active use. They may be fixed or floating production systems usually with platforms/tanks on water: shallow water complexes (in up to 100m deep water), gravity based structures (100-500m deep), compliant towers (500-1000m deep), floating systems (up to 3000m deep); or deep sea bottom systems with pipes to the shore or on-water platforms[24].

Oil production process involves the following stages: Well-drilling and flowing the resource to the surface; Gathering; Separation (gas, oil, water and waste); Storage and transportation[25]. The production process scheme is illustrated in the Appendix 2, Chart 4.

Pipelines are the safest means of liquids transportation. Total world oil and refinery products (excludes natural gas, gas liquids and petroleum gases liquids) pipes length is 774 126

[23] Dunn, Holloway (2012), p. 66
[24] Devold (2009), pp. 10-13
[25] Devold (2009), p. 13

km. The biggest pipeline network belongs to US and Canada with 43% of world total followed by Russia with 12% (see Chart 5, Appendix 2 and Tables 7 and 8, Appendix 1).

Oil Usage

Downstream of the petroleum industry is oil refinery processing crude oil to other useful products. Oil consists of mainly hydrocarbon molecules which vary in size and mass and can be transformed into other substances. When oil is distillated different substances separate from each other: lighter liquids are liquefied petroleum gases (propane and butane), kerosene (jet fuel), gasoline (base for automobile petrol) and naphtha (used in petrochemical industry to produce dyes, synthetic detergents and plastics); middle distillates: heating oil and diesel; then heavy fuel oils, lubricates, bitumen (asphalt) and waxes.[26] Approximate percentage of petroleum products output from a barrel of oil is presented in Chart 6, Appendix 2.

Oil products are mostly consumed in transportation sector – 61.5%, and in petrochemical industry – 17.1% (see Chart 7, Appendix 2).

Petroleum consumption is present in every single (217 in total) country and territory in the world. Canada and US together consume 26% of world oil (20.9 mb/d); they are closely followed by China and Japan (15 mb/d). European region consumes 16% of the petroleum produced and Middle East – 9% (shares of world petroleum consumption can be found in Chart 8, Appendix 2 and Table 9, Appendix 1).

Current dynamics of oil industry[27]

Year 2012 was somewhat transitory for global economy. Global GDP demonstrated only 3% growth in 2012 in comparison to 4.9% in 2010 and 3.6% in 2011. The lack of performance was mainly driven by developed countries but emerging countries were demonstrating slowdown as well. World oil demand forecast for 2012 was initially optimistic. The 2012 growth of oil demand was 0.8 mb/d while forecasted – 1.3 mb/d but the picture is very different for OECD and non-OECD countries.

US oil consumption was declining for already two years in a row (-0.3 mb/d in 2011 and -0.2 mb/d in 2012). Factors behind the shrink of demand are relatively high oil prices, substitution of oil by natural gas, weak industrial production and fiscal issues

[26] Sources: BP and Trencome
[27] From OPEC 2012 Annual Report

affecting countries economic performance. Negative trend of oil consumption in Europe continued in 2012 demonstrating decline in demand by 0.5 mb/d. Unsolved debt problems combined with restrictive policies and taxation of petroleum consumption led to reducing of the demand. Decrease of the demand in 2012 could have been even stronger if it was not for cold winters that year.

The strongest growth of demand was observed in China (0.3 mb/d). Part of this increase was due to stronger economic growth, high refinery throughputs and cold weather. Other Asia (including Japan) oil demand grew strongly (0.5 mb/d), with India as a biggest contributor with its booming petrochemical sector. Energy issues in East-Asia such as electricity shutdowns in India and nuclear trouble of Japan and South Korea supported oil demand growth.

The supply of oil on the world market was on an upswing in 2012, increasing by 2.2 mb/d compared to only 1.0 mb/d growth in 2011. The total world oil supply averaged 89.8 mb/d, with OPEC's crude share standing at around 34.7%.

OPEC oil supply, averaging 31.1 mb/d in 2012, increased mainly due to production from Libya and Saudi Arabia followed by Iraq, Kuwait and UAE despite sharp production drop by Iran. Non-OPEC supply averaged 53.0 mb/d in 2012. Growth was driven by the increase from the US, followed by Canada, Colombia, and Russia, while supply from Norway, South Sudan and Sudan, Syria and the United Kingdom declined.

US experienced the highest growth in oil supply among all non-OPEC countries, supported by a surge in tight oil production from shale development areas. Maintenance and unplanned shutdowns as well as a natural decline at mature producing areas heavily impacted the UK's oil supply. The same factors influenced Norway's oil supply, leading to a sharp drop of 0.3 mb/d in OECD Europe's oil supply in 2012 over the previous year. Non-OECD oil production decreased in 2012 by 0.5 mb/d, averaging 12.1 mb/d. Africa experiencing the largest production drop, followed by the Middle East, Other Asia and Latin America. The supply of Sudanese oil experienced the largest decline among all non-OPEC countries because of political issues. Oil supply from both Russia and China increased due to the new projects developments.

For 2012 oil supply and demand balance see Table 11, Appendix 1.

Average crude oil price in 2012 was $109.45/b for OPEC reference basket and $94.2/b for WTI. OPEC reference basket improved by $1.85/b over the previous year. Besides encouraging economic data from the US and China, together with speculative *activities in the crude oil future markets, contributed notably to the rise in overall crude oil prices in the 1st quarter averaging $117,49/b. Supply glitches in European (North Sea) and East African (Sudan) crude oil production were also a factor. In the 2nd quarter the basked weekend significantly till below $100/b and averaging $108/b in response to gloomy demand and oversupply of oil. In the 3rd quarter price grew after the drop but the average was kept at $107 and the price remained quite steady till the end of the year not differing much from $110/b.*

Specifics of the petroleum industry

Oil reserves, production and usage facts analysis is important to determine peculiarities petroleum industry has which influence supply-demand patterns and, naturally, price. Among them the following can be distinguished: 1) limited and concentrated physical volume; 2) high costs and risks of the production; 3) competitive situation; 4) specifics of consumption and availability of substitutes.

1) Oil is a finite resource and its recovery requires a huge amount of time. True, the ‘doomsday’ peak-oil predictions, when oil is going to finish, were promising disastrous consequences for the humanity. M. King Hubbert predicted a peak in US oil production in 1970s. Since then, plenty of exploration work was done and new fields discovered, so the world’s doomsday was postponed till 2000s. However, now our planet was examined fully and there is no space left for large undiscovered oil fields[28] which means that new oil will either come from hardly reachable not-completely explored places (like very deep ocean or Antarctic), or from very small fields, or from less efficient sources (unconventional oil). All three alternatives require much additional investments and it will make oil even more expensive (see Chart 9, Appendix 2 for illustration).

2) Petroleum production is an extremely complex process. Time, used for exploitation of an oil field from initial exploration to exhaustion of the resource, is very long – 50 years of more. Exploration and development is always a subject to huge uncertainty and costs: even in geologically promising regions plenty of dry holes can be drilled before the discovery of an oil reservoir. The industry is very capital-intense and thus requires enormous investments in equipment and infrastructure to say nothing about cost of research and geological exploration.

[28] Deffeyes (2009)

Cost of exploration and field development varies with scope of the project and region and it can account for couple of hundred million dollars. It is obvious that price of oil should be sensitive to capacity utilization and inventories levels.

3) If an oil reservoir is exploited by several independent competitive operators it influences greatly supply characteristics. If there is no a mutual agreement between producers or public regulation the decision before them is timing of extraction. In order to maximize their profits the operators will try to push the oil production until the marginal cost equals ongoing price. Marginal costs here are a sum of the two components: marginal direct cost and *marginal user cost*. Marginal user cost reflects limited capacity of the oil reservoir: unit extracted now cannot be extracted in future thus some future income is sacrificed; on the other hand, unit not extracted now can be extracted by a neighboring operator. Obviously, the higher future prices (the lower future extraction costs) expected the more beneficial it is to delay extraction, but competitive side of the field exploiting may completely offset benefits of timing. If so, then only marginal direct costs determine production volumes. That has two effects: first, supply becomes quite inelastic and thus sensitive to fluctuations of demand; second, such production setting does not allow using the resource with all possible efficiency (not only in financial meaning for the producers, but rather in physical: waste and recovery properties of oil fields are neglected). [29]

Oil industry is a natural oligopoly which means that a desired for consumers perfectly competitive setting may not necessary make everybody better off. Nature of the industry makes it reasonably inefficient to allocate production in perfect-competitors' hands even though it makes oil more expensive.

A factor supporting higher than marginal costs price of oil is concentration of market power in oligopolistic setting. For the OPEC members, especially those of Middle East, decisions about supply and investments to the industry are determined by the governments in order to meet revenue targets. Clearly, it often has political rather than economic reasoning. For some countries oil rents are the only source of income: in Iraq oil rent accounts for 77.7% of the country's GDP (see also Table 10, Appendix 1 for more details). Since the producers are interested in economic rents capture, they tend to restrict output to keep the prices well above their production costs. For non-OPEC producers prices and costs are the major determinants of supply decisions. Thus, limitation of low-cost oil supply (by Middle Eastern governments) opens the market to higher-cost oil (e.g. Alaska or North Sea) and gives incentives for investments

[29] McDonald (1965), p. 26

elsewhere[30]. This issue actually blurs out concentration feature of oil industry because the price is basically competitive but ends up on much higher than [lowest possible] marginal costs level.

4) Finally, the oil importance is a major factor to determine demand, its elasticity and thus severance of price shocks. If the world is highly dependent on oil – it is bad news for those who want to see oil prices stabilized. So far oil remains the most used energy source. It successfully competes with coal because it is cleaner and has higher energy content. Comparing to natural gas oil has an advantage of simpler transportation and storage even if it is less clean. But in general, alternatives to oil do exist.

In energy generating sector natural gas becomes more economically preferred. The same concerns heating purposes. Oil with natural gas liquefying technologies even looses its transportation advantages. Moreover, natural gas is easier accessible and more geographically dispersed than oil (but in sum its reserves are relatively less abundant than those of oil: in 2013 proven natural gas reserves account for 6824.7 trillion cubic feet[31] [7016 quadrillion BTU] while oil reserves in 2013 are 1640.4 bb[32] [9514 quadrillion BTU[33]]). For electricity generation oil is not that important anymore: only 5% of electricity is generated from oil sources and the importance and popularity of the renewables (green energy sources) constantly increases.

In the transportation sector, however, oil remains dominant if not exclusive. More than 60% of world oil produced is consumed in the form of transport fuels. Jet fuel, gasoline, diesel and bunker fuel are used by absolute majority of transportation means with hardly any alternative. There is no need to say about significance of transport sector, but in addition to that it has highly income elastic demand, meaning that economic growth would imply growth in fuels consumption.[34]

Another industry based on oil inputs is petrochemicals producing plastics, synthetic fibers and different chemical. For an outside observer it is hard to judge about possibility to use other resources instead of oil as an input, but even if such possibility exists, the substitution for sure requires capital investments and research expenditures to change the technology of production.

So, shortly saying, alternatives to oil exist, and its importance relatively decreases over time. Even if in some sectors oil still remains dominant obviously technological progress will not leave the state of affairs unchanged: the most efficient economic solution will be found given

[30] Noreng (2007), p. 103

[31] Source of data: EIA 2013 (US reserves data for 2011)

[32] Source of data: EIA 2013 (US reserves data for 2011)

[33] BTU (British Thermal Unit): 1 cubic foot of natural gas = 1028 BTU, 1 barrel of crude oil = 5 800 000 BTU with current technologies of energy production.

[34] Noreng (2007), p. 31

increasing cost of oil consumption. But the latter may take decades. Changing equipment to use other inputs/fuels instead of oil is a very costly process even if those inputs are already invented. For now world dependence on oil is much less than in 1970s, but it is still far from disappearance.

2.4 Political aspect of oil

It cannot be left without notice that about a half of proven oil reserves is located at Middle East and four fifths are controlled by OPEC cartel. Organization of Petroleum Exporting Countries was founded in September 1960 by Iran, Iraq, Kuwait, Saudi Arabia and Venezuela. Later Algeria, Ecuador, Gabon, Indonesia, Libya, Nigeria, Qatar and the UAE also joined, but Ecuador and Gabon since left the organization. Since 1990, Iraq is not taking part in the negotiations over prices and quotas. OPEC members own about 80% of the world proven oil reserves and supply about 40% of world crude oil.[35]

Inside OPEC the relationship between members is intrinsically conflictual because of different interests concerning oil prices and market shares. This leads to continuous bargaining and sometimes ends up in compromises often triggered by threats. Relatively low-cost producers (Saudi Arabia and Kuwait) are a potential economic threat to the others since they are able to increase supply and lower prices. Iraq and Iran have strong military forces which threatens other countries. Abusing these ‘advantages’ is likely to cause mutual, not just one-sided, damage.[36] However, the availability to threaten must meet not only capacity constraints but also budgetary needs. Since the most countries sell only oil or natural gas to the outside world the space for maneuver with oil prices is not that large.

Originally OPEC was established to defend the oil exporters’ interests against international oil industry. It pursued this policy in the 1960s. In 1974-75 under the shock of the first major oil crisis OPEC overtook control of oil supplies and tried to stabilize prices. After 1982 organization acted as a classic cartel, seeking agreement over market shares and prices.[37]

[35] Noreng (2007), p. 133
[36] Noreng (2007), p. 113
[37] Noreng (2007), pp. 134-136

“*The concentration of oil supplies in the Middle East means that oil prices are necessarily unstable, unless there should be robust agreement on oil prices or major oil exporters should choose not to price oil far above their marginal cost of extraction. Neither seems likely*.” [Noreng (2007), p. 6].

Political aspect of oil price behavior in fact is a matter of *common knowledge*. There exists a massive cluster of literature written about politics and oil relationships and no matter whom to ask but for the majority of people political determination of oil prices would be obvious. This superficial statement may not be true though, but it would be probably right to say that price of this commodity does not follow the same economic laws as the others.

Comparative analysis of oil price behavior together with history of political events may help to reveal if politics is a crucial factor of the price determination. Since 1970 several times due to different political occasions oil supplies were shifted, leading then to significant price changes. It may be considered as evidence to supply-side price setting. But looking deeper in the details reveals that demand behavior is actually a primary determinant of the world oil price formation.

Following historical analysis of Ø.Noreng (2007)[38]:

The first oil price shock of 1973 was essentially the result of reduction of oil output by Saudi Arabia, Kuwait and United Arab Emirates in October-December that year. The action was triggered by Egypt-Israel war. Iraq did not participate in these actions, but simultaneously reduced oil production as well. Net result was reduction of oil supply from the Persian Gulf by 15%. Naturally, oil prices jumped. By February 1974 action was called off (the conflict ended), supply volumes returned back, but price still remained about four times higher than year before. The action took market by surprise and created huge psychological effects. The oil shock was followed by severe recession of 1974-75 in oil-importing countries. Economic growth together with oil demand restored in late 1970s.

Next price shock started in January 1979 when market was in panic because of output shrunk in Iran due to workers strike and Iranian Revolution later. Oil price jumped up regardless supply increasing from Iraq and Kuwait. Net supply decrease was moderate but oil spot price in late 1979 was twice higher than at the end of the previous

[38] Noreng (2007), pp. 20-21

year and continued to grow. Moreover, in September 1980 Iraq attacked Iran which led to reductions as well. The result of the shock was again severe recession in oil-importing countries, but this time it led to restructuring of the economy towards light industry, services and high technology. Demand was reduced, and in addition to that supply from other sources increased, not least – the North Sea.

Prices started to decline at the beginning of 1981 even though the Gulf states were cutting output to defend high prices. But the effort was unsuccessful. By 1985 real oil price almost halved after the 1980 peak. Saudi Arabia and Kuwait were losing revenues and market shares and they alerted other OPEC countries about changing the strategy – towards keeping market shares rather than high prices. The Middle East output was increased to stimulate demand, and the price halved again, so at the end of 1986 total Gulf supply was cut again to the beginning on 1986 level. The prices, however, did not raise much. Remember that during 1980s Western economies were restructured towards more energy saving technology. Low oil prices could not lead to reverse – revival of heavy industry, but they influenced consumer behavior slowing the process of energy saving.

The next oil shock, of August 1990, was triggered by Iraqi occupation of Kuwait. Within a month price doubled even though the reduced output was essentially replaced with oil from Nigeria and Venezuela. This time oil shock was not a cause of recession in Western world: US entered the recession by summer 1990 and Europe and Japan were demonstrating lower economic growth.

Another major event was oil price collapse of 1998. Within a year, in autumn 1998 prices halved and reached their historical minimum in real terms – below before 1973-raise level. It had several causes: first, East-Asian crisis weakened oil demand and created a pessimistic mood in the oil market. Second, demand was further weakened because of mild winter in the US and Europe. Third, expecting high demand, Venezuela raised output in 1997. And Iraq finally returned to the market. Low oil prices caused budgetary and balance of payments problems in most oil-exporting countries.

In 1999 the OPEC agreement (supported by most Gulf countries (except Iraq), Venezuela, Mexico and Norway) and buoyant demand caused oil prices to triple.

Experience shows that timing is an important factor in the interaction between oil market and the world economy. Magnitude of oil price changes is bigger when world economy is booming (without recession tendencies in 1990 oil price increase would have been stronger). World economy is more vulnerable to oil shocks when it was booming for several years.

Historical analysis of oil price behavior during the recent decade is well summarized by Hamilton (2009)[39]:

> *After 2001 till the middle of 2007 oil prices showed steady increase and within this period they tripled. One reason of it was stagnating oil production* [see Chart 11, Appendix 2]*. Oil production was not growing either because of depletion (US and North Sea) or due to political interests (may be the case of Saudi Arabia).*
>
> *On the other hand, demand for oil contrary to supply was growing strongly, especially from the newly industrialized countries. Chinese oil consumption in 2007 was 870 tb/d higher than in 2005.*
>
> *In 2007-2008 oil price within a year increased from $92/b to $145/b: clearly it is impossible to attribute that the reason of that was economic growth faster than expected or production gains more modest than anticipated. The big surprise of the first half of 2008 was that even $100 expensive oil is not able to prevent global quantity demanded from increasing above 85.5 mb/d and that only 85.5 mb/d is going to be available. This could have happen if market participants were not much aware of the massive economic deterioration just ahead. Obviously they were not. US economic growth was slowing down, but it was not believed to enter into the true recession.*
>
> *A huge spike of 2007-2008 transformed to an even more dramatic price collapse, when oil price dropped till $40 a barrel. Again fall of economic activity alone was not sufficient to explain the magnitude of price decline. The shift in elasticity of demand could be the case: low short-run elasticity may explain the prices move up in the first half of 2008, but higher intermediate-run elasticity – the move down as consumers made delayed adjustments to earlier price increases.*

So what are the important insights of this section? No doubts that oil price is influenced by several factors and considering only supply or demand determinants alone will never be sufficient to explain the price movements. Firstly, it may seem that oil has strong politically driven price formation due to political interests and power of the producers. Supply cuts were truly the reasons of the majority of oil shocks, but they were though just one-moment events, and

[39] Hamilton (2009), p. 226

politics is definitely not something able to explain ‘routine’ price change. Secondly, sometimes no matter of political actions price did not move in an expected direction (e.g. 1985 price collapse or 1997 crisis). Oil demand in its turn is not always following the same pattern as global industrial demand so dynamics of economic activity cannot be a major determinant of oil prices. Thirdly, oil is a strategic resource, so no matter of prices demand is hard to influence because energy is a not something consumers can easily substitute or deny buying. World dependence on oil is a crucial demand determining factor.

These three insights lead to an important conclusion: oil price does respond to macroeconomic fundamentals through the expectations and *precautionary demand* [40] channels in addition to normal demand for oil among other industrial goods. Given *common knowledge* about strategic significance of oil and political turbulence general state of macroeconomic conditions determines unique for oil expectations. “*An increase in precautionary demand for crude oil causes an immediate, persistent, and large increase in the real price of crude oil; an increase in aggregate demand for all industrial commodities causes a somewhat delayed, but sustained, increase in the real price of oil that is also substantial; and crude oil production disruptions cause a small and transitory increase in the real price of oil within the first year*.” [Kilian (2009), p. 1053].

Panic of 1973 and 1979 as a consequence of political actions on the Middle East deepened the prices shocks. Absence of precautionary demand (because the economy was restructuring) in early 1980s led to the price collapse in 1986 despite active actions of suppliers to prevent it. In 1997 calm demand and low expectations led to price decline of 1998. Sound macroeconomic indicators[41] of 2000s supported energy demand growth till 2008 even though recession had already started. When that was finally realized by the market in the second half of 2008 the price collapsed more than it could have been rationally supposed to. Therefore politics without high precautionary demand is not relevant. Political events are often *triggers* of oil price shifts, but not *causes*.

Precautionary demand and expectations are largely determined by strategic significance of oil (its limited volumes, broad usage and lack of efficient substitutes) and soundness of macroeconomic background. Market power and competition apart for having direct impact on prices together with production costs also influence expectations’ patterns. Basically petroleum

[40] “Precautionary demand is associated with market concerns about the availability of future oil supplies” Kilian (2009), p. 1053

[41] Hamilton (2009), p.233

industry is an oligopoly, but significant level of competition is also present: there are relatively many producers and OPEC market share in oil production is not that big. Moreover, climate inside OPEC is conflictual which, is an obstacle for monopolized price controlling. So the direct impact of oligopolistic by nature but competitive in fact industry is much higher than marginal costs prices since relatively high-cost oil is actually available on the market as the low-cost producers postpone their extraction serving political interests (so called *oil market paradox* [42]). Impact of competition setting on expectations is more obvious: conflicts on Middle East or other significant news, given the common knowledge about concentration of the oil industry, are able to trigger sudden price jumps and panic which deepens the shocks.

Now the question of the above listed factors connection in a big picture has to be answered. It is reasonable to assume that strategic significance of oil is constant (and relatively high) over a sufficiently long time horizon: availability of oil and technologies of its usage change slowly. While macroeconomic background remains important at each point of time. Obviously, high business activity and economic growth increase demand for oil (among other industrial and consumer goods) and create optimistic atmosphere on the market. The problem of this nice situation is that precautionary demand also increases and makes expectations more vulnerable to even small disturbances. When the economy is booming for several years the precautionary demand becomes very high and then if sudden bad news is announced – oil prices jump fuelled greatly by panic on the market. Recession itself does not cause huge sudden changes of prices, demand for oil is moderate and the same holds for precautionary demand, it can even go down causing price collapses while *expectation* (realizing) of recession can easily have unpredictable consequences.

So among many factors contributing to oil price evolution macroeconomic conditions and expectations play not the least role. It is very hard to understand fully their formation mechanism to say nothing about formalizing it. Actually it is even difficult to define what sound macroeconomic conditions are and how they are best measured. Obvious macroeconomic indicators may be not relevant: first, since rather expectations than current objective situation matter then indicators may better be considered with some lags. Second, oil, being still strategically important, has a strong impact on the economy which creates reverse causality effect. In addition to that, oil price is changing faster than the majority of macroeconomic variables and thus is rather a leading indicator. So integrating macroeconomic determinants into oil price models is not an easy task from both theoretical and empirical considerations.

[42] Noreng (2007), p. 108

Chapter 3
Theoretical and Empirical Modeling

Literature Review in Chapter 1 contains an overview of the variety of theoretical models, mathematic and econometric tools used to explain energy prices movements. The existing formalized concepts depend on how researchers perceive oil and what are the assumptions made about its price. This chapter gives an overview of different modeling technics within their theoretical frameworks and implications for crude oil price explanation.

3.1 Structural models

Structural approach to modeling takes as a core economic theories about objections, constraints and behavior of market participants. Individual characteristics are not tested, just combined into a unified system of the economy. Usually it is assumed that all agents have deterministic objection functions and optimize them to choose the best strategy available[43].

Aggregated demand is usually modeled through price and income elasticities. Demand is usually assumed to be a log-linear function of GDP and oil price adjusted for technical progress. In recent studies (e.g Dées et al., 2007) exchange rates and domestic prices are also considered in the demand model. [44]

Supply side in oil sector is typically distinguished for OPEC and non-OPEC producers. Non-OPEC producers typically believed to be small, price-taking firms, while OPEC – enjoying high degree of market power. Non-OPEC firms optimize their production according to current price set with constant or asymmetric elasticity sometimes adjusted to geological data and exploration. Concerning OPEC, it is assumed to be a price-setter and market leader optimizing profits or following other decision rules e.g. Target capacity utilization (set price first, then fill the gap between demand and non-OPEC production).[45]

Assumptions about agents' behavior are made either by simulating and comparing different scenarios of actions and possible outcomes or searching for econometric fit to actual results. Typically behavior is assumed to be rational, market – with perfect information.

[43] Huntington et al (2013), p. 7
[44] Huntington et al (2013), p. 8
[45] Huntington et al (2013), p. 8-9

The first example of a structural model belongs to Hotelling (1931), which was further broadly used, developed and tested empirically. It is assumed that resource owners make the time-extraction decision maximizing total discounted cash-in-flows (revenues) from extraction subject to constraint about size of the stock available. Maximization problem solves into price/interest rate relationship[46]:

$$p_t = p_0 e^{rt}$$

where p_t is a current price of the resource (oil), p_0 – initial price and r is interest rate. p_0 is determined from supply-demand equality assuming the form of demand function.

It is the simplest model and was aimed to prove resource price sensitivity to its scarcity. But empirical research did not find much evidence in support of the theory.

Structural modeling approach is a logical way to find how an optimal price should be determined. But it is quite easy to notice weaknesses it has. First, the equations and links are based on heavy theory and assumptions. At the same time even the most complex models are a very simplified version of the reality thus cannot capture influences of all factors.

Structural models treat oil as an industrial product while nowadays it has much more importance as a financial asset. Here, physical oil trade volume, which is 10 times less than its volume on the financial market, speaks for itself (see Table 12, Appendix 2). That is why it would be more correct and efficient to model oil price like a price of financial asset.

Unconditional on other factors models (non-structural) are also safer in terms of explanation power – if the answer about behavior of prices is sought internally, in the price path itself, there is no need to consider complex theories about oil supply and demand and their determinants. Since such theories, honestly speaking, do not even exist structural models are from the beginning to the end based on assumptions and authors' beliefs. Non-structural modeling does not require naming those factors – it is enough to know that they are already incorporated in the past distribution of prices. The question to which extend past price is able to explain the current behavior, however, still remains open.

[46] Leinert (2012), p. 4

3.2 Financial models

To summarize, the main differences between the structural and financial models are three fold. First, most structural models explicitly specify oil demand and supply, whereas financial models mainly focus on oil price and its mathematical properties. Second, estimates of the structural models have economic interpretation and thus can be used for policy analysis, while the parameters in the financial models have much less economic meaning and are mostly used for the purpose of forecasting, derivatives pricing or risk management. Third, financial models use data with higher frequencies, like monthly, weekly or daily, while structural models usually use yearly or quarterly data[47].

3.2.1 Stochastic models

In mathematical finance traditional models are based on stochastic processes. "*Stochastic process is a variable that evolves over time in a way that is at least in part random… It is defined as probability law for the time evolution*" [Dixit, Pindyck (1994), p. 60]. In other words prices of financial assets are believed to have random behavior fully or partly with some known mathematical and distributional properties. Random (or stochastic) behavior is usually driven by Brownian motion (or Wiener process), *B(t)*. The most frequently used in literature basic model for price dynamics is a Brennan, Schwartz (1985) Geometric Brownian Motion model also referred as drift diffusion process[48]:

$$S(t) = S(0)\exp(\mu t + \sigma B(t)) \tag{3.2.1}$$

where current price is denoted *S(t)*, *S(0)* is initial known value of price, μ and σ are drift and volatility (diffusion) parameters to be estimated. Equation (3.2.1) can be expressed in stochastic differential equation form[49]:

$$dX_t = \mu dt + \sigma dB_t \tag{3.2.2}$$

with X_t as a variable of interest typically represented by asset log price and dB_t being an increment to Brownian motion.

Brownian motion *B(t)* is a Markovian stochastic process with independent, stationary and normally distributed increments (is iid). The increment to Brownian motion $dB(t) = B(t) - B(s)$ is distributed normally: $dB(t) \sim N(0, t - s)$. For continuous time the difference between

[47] Huntington et al (2013), p. 28
[48] Benth et al (2008), p. 19
[49] Ait-Sahalia (2006), p. 1

time points $t - s \equiv dt$ which is infinitely small. The implication is that log-returns (log prices change over time interval Δt) $\log S(t + \Delta t) - \log S(t)$ are also independent, stationary and normally distributed[50]. Thus the price follows a lognormal distribution with mean and variance[51]:

$$E_0[S_T] = S_0 e^{\mu T} \tag{3.2.3}$$

$$\text{Var}_0[S_T] = S_0^2 2^{2\mu T}(e^{\sigma^2 T} - 1) \tag{3.2.4}$$

This benchmark model is valid under the following assumptions:[52]

1. X_t is a Markov process, meaning – a no-memory process. Past realizations and their distributions do not influence current change of the process.
2. X_t is a diffusion.
3. In this case it has constant parameters.
4. X_t is observed without error.
5. For discrete time observations of X are sampled on equal time intervals $\Delta t = \frac{T}{N}$.

The assumptions are relaxed in more complex models allowing for non-constant parameters, market microstructure noise, additional factors or other properties capture.

A frequently used generalization of GBM is exponential Levy process, *L(t)* component, which allows capturing spikes behavior[53]:

$$S(t) = S(0)\exp(L(t)) \tag{3.2.5}$$

Due to skewness of empirical distribution of crude oil returns Krichene (2006) suggests to model prices as diffusion-jump process instead of a pure diffusion process[54]:

$$\frac{dS_t}{S_t} = \alpha dt + \sigma dB_t + (\exp J_t - 1)dN_t \tag{3.2.6}$$

where $dB_t \sim N(0, dt)$ is Wiener process increment and N_t is a Poisson jump counter with $prob(\Delta N_t = 1) = \lambda dt$; $prob(\Delta N_t = 0) = 1 - \lambda dt$. Price behavior in case of abnormal information is $S_t = \exp(J_t) S_{t-dt}$ and J_t is a normally distributed jump size.

An important contribution to commodities price stochastic modeling was made by Schwartz (1997) and Gibson and Shwartz (1990):

1) *One-factor model*. Commodity price follows a mean-reversion process of Ornstein-Uhlenbeck type[55]. In a log form:

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[50] Benth et al (2008), p. 20
[51] Dixit, Pindyck (1994), pp. 72-73
[52] Ait-Sahalia (2006), pp. 1-2
[53] Benth et al (2008), p. 20
[54] Krichene (2006), p. 10

$$dX = \kappa(\alpha - X) + \sigma dz \quad (3.2.7)$$

$X = \ln S$ is an oil log price; a drift parameter $\alpha = \mu - \frac{\sigma}{2\kappa}$ is a long term mean level of log price and dz is an increment to Wiener process. Parameter κ measures the speed of adjustment of log price to the long run mean α. Thus the log price follows a normal distribution with mean and variance[56]:

$$E_0[X_T] = e^{-\kappa T} X_0 + (1 - e^{-\kappa T})\alpha \quad (3.2.8)$$

$$\mathrm{Var}_0[X_T] = \frac{\sigma^2(1 - e^{-2\kappa T})}{2\kappa} \quad (3.2.9)$$

Mean-Reversion property can be reasoned with the fact that oil producing has costs and price cannot go less than mean marginal costs and thus tend to converge to it.

2) *Two-factor model*. The second factor added to commodity price is instantaneous convenience yield δ. Convenience yield can be described as benefit of owning the commodity but not the financial contract on it balanced with cost of storage. Net convenience is already proven to drive the relationship between futures and spot prices. Theory of storage posits an inverse relationship between level of inventories and net convenience[57].

Price and convenience yield are assumed to follow joint stochastic process[58]:

$$dS = (\mu - \delta)S + \sigma_1 S dz_1 \quad (3.2.10)$$

$$d\delta = \kappa(\alpha - \delta) + \sigma_2 S dz_2 \quad (3.2.11)$$

where $dz_1 dz_2 = \rho dt$ allows for correlation of random components.

3) *The three-factor model* with instantaneous interest rate as the third factor. Interest rate assumed to follow the mean-reversion process (Vasicek model). Random components are again assumed to be correlated.

It is important to notice that the models use Brownian motion under equivalent martingale measure which differs from the one under physical measure by a (constant) market price of risk. This feature is heavily used in the applications for risk-neutral asset and derivatives pricing based on commodities prices properties.

The above models can also be extended by adding deterministic part (like seasonality function) or other factors, allowing for stochasticity of parameters, adding jump processes etc.

To simulate price path it is necessary to solve stochastic differential equations describing a price change process (e.g. 3.2.2 or 3.2.5). They are usually solved by applying *Ito's Lemma*. Ito's

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[55] Schwartz (1997), p. 926
[56] Schwartz (1997), p. 926
[57] Gibson, Schwartz (1990), p. 959
[58] Schwartz (1997), p. 927

formula can be seen as a stochastic version of Taylor's expansion at order 2 for a diffusion process of X with $f(t,x)$ being a twice differentiable function[59]:

$$f(t,X_t) = f(0,X_0) + \int_0^t f_t(u,X_u)du + \int_0^t f_x(u,X_u)dX_u + \frac{1}{2}\int_0^t f_{xx}(u,X_u)(dX_u)^2 \quad (3.2.12)$$

where X_0 is an initial realization of the process, $f_t(\cdot)$ represents the first derivative of $f(t,X_t)$ with respect to *t*, $f_x(\cdot)$ the first derivative w.r.t. *x* and $f_{xx}(\cdot)$ the second derivative w.r.t. to *x*.

It can easily be seen that function $f(t,X_t)$ can be interpreted as a commodity price under the assumption that its log price X_t follows a diffusion process.

Estimation of a stochastic model parameters however is not an easy task. First of all, many variables are not directly observable at the market and are to be estimated from variables we have (e.g. convenience yield is estimated from observed forward prices). Another issue is continuous nature of stochastic processes and discrete nature of available data.

The traditional models in mathematical finance belong to semi-martingale class of models. The reason for this is existence of equivalent martingale (local) measure, being probability measure equivalent to objective probability, and such that discounted price dynamics is a (local) martingale.[60] Existence of EMM (or risk-neutral probabilities) leads to complete market without arbitrage.

Spot market data exists on daily (or hourly) basis – is discrete in other words. Thus we cannot use notation *S(t)* for *any* time *t*, but only for observed time points. Let $\tilde{S}(t)$ denote instantaneous but unobserved spot price process of the commodity (for *any* time *t*) which is price of the commodity within delivery interval *[t, t+dt)*. What we do observe is a price *on* specific delivery time points t_i but not what between them. Entering a spot contract will cost[61]:

$$\int_{t_i}^{t_{i+1}} \tilde{S}(u)du \quad (3.2.9)$$

The natural assumption then is that the discreet spot price S_i is a best prediction of the above given the information set $\mathcal{F}_{t_i}$ prior to the delivery[62]:

$$S_i = E\left[\int_{t_i}^{t_{i+1}} \tilde{S}(u)du \,|\mathcal{F}_{t_i}\right] \quad (3.2.10)$$

Equation (3.2.10) also means that discrete price S_i contains all market information before the delivery, but not during time of it (measurability property).

[59] Iacus (2008), p. 38
[60] Benth et al (2008), p. 22
[61] Benth et al (2008), p. 23
[62] Benth et al (2008), p. 23

Approximating the integral and using the convention that time is measured in units (hours, days) with the same difference between the neighboring ones so that ($t_{i+1} - t_i = 1$):

$$S_i \approx E\left[\tilde{S}(t_i)|\mathcal{F}_{t_i}\right] = \tilde{S}(t_i) \tag{3.2.11}$$

This argues in favor of discrete process elements being observations of the continuous process. This assumption is always made in the literature when estimated stochastic models with market data[63].

3.2.2 Time series models

For empirical research on oil prices various econometric tools allow for direct discreet data modeling. Time-series models provide a framework for analyzing dynamic structure of the series. A benchmark univariate model for (price) series X_t is an Autoregressive Moving Average model (ARMA) with lags p for autoregression and q for past random shocks impact:

$$X_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i X_{t-1} - \sum_{j=1}^{q} \theta_j a_{t-j} + a_t$$

where φ_i and $\theta_i$are the parameters and $a_t \sim N(0, \sigma^2)$ is a white noise series[64].

For an adequate modeling data used must be weakly stationary, which means that both mean of X_t and covariance between X_t and X_{t-l} must be time invariant[65].

In multivariate setting usually Vector Autoregressive (VAR) framework is used. System of equations in the matrix form for each of the components of multivariate time series $\boldsymbol{r_t}$[66]:

$$\boldsymbol{r_t} = \boldsymbol{\varphi_0} + \sum_{i=1}^{p} \Phi_i \boldsymbol{r_{t-i}} + \boldsymbol{a_t}$$

where $\boldsymbol{\varphi_0}$ is *(k×1)* vector of constants, Φ_i is *(k×k)* matrix of coefficients, *k* – total number of variables (components), *p* – number of lags (order of VAR).

Vector form can also be used for Multivariate Moving Average or ARMA models (VMA and VARMA).

However series for modeling must be now weekly stationary and not co-integrated. In univariate models non-stationarity is handled via differencing the data. But in multivariate setting if series are co-integrated it may lead to a strong bias of the results. Two series are co-integrated if they both are non-stationary and share a common stochastic trend. Since this trend

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63 Benth et al (2008), p. 24
64 Tsay (2010), p. 66
65 Tsay (2010), p. 30
66 Tsay (2010), p. 403

is unobservable there is no way to explain any joint impact of these series on each other or other variables. Only one unit-root will be reflected in vector framework instead of the two[67]. Co-integration exists if a linear combination of the two series is stationary. In case of co-integrated series in econometrics Vector Error Correction model form (VECM) of VAR is frequently used:

$$\Delta r_t = \mu_t + \Pi r_{t-1} + \sum_{i=1}^{p-1} \Phi_i^* \Delta r_{t-i+1} + a_t$$

Πr_{t-1} is an error correction term. If there are no co-integrated variables $\Pi=0$ (rank Π = 0) and ECM form reduces to VAR(p-1) for Δr_t series. For $m < k$ co-integrated series rank of $\Pi = m$. And Φ_i can be recovered as[68]:

$$\Phi_1 = I + \Pi + \Phi_1^*$$

$$\Phi_i = \Phi_i^* - \Phi_{i-1}^*$$

Structural VAR or VECM models are broadly used in the literature to test empirically relationship between oil prices and different exogenous factors e.g Exchange rates – Bencivenga et al (2012), capacity utilization – Dees et al (2008) or supply/demand shocks – Kilian (2009).

3.2.3 Volatility models

Both stochastic and time-series model can be extended assuming time-variant variance. Here conditional standard deviation of asset return is referred as volatility. Volatility is an important factor for asset pricing and trading even though it is not directly observable. The most usable methods to estimate volatility are either estimation of implied volatility from observed option prices (e.g. using Black-Scholes formula and solving for standard deviation) or choosing a 'rolling window' and simply calculate variance of asset return/prices.

Even though volatility is unobserved, but it has some commonly seen characteristics[69]: first, there exist volatility clusters (volatility may be high for certain periods and low for others); second, volatility evolves continuously through time and jumps are rare; third, it does not diverge to infinity, or, statistically speaking, it is stationary; fourth, volatility seems to react differently on big price increases and drops – asymmetric effect. These properties must be taken into account for development of modeling approaches.

The most usable family of volatility models is Generalized Autoregressive Conditional Heteroscedasticity models (GARCH). The basic idea of these models is that a random shock to

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[67] Tsay (2010), p. 430
[68] Tsay (2010), p. 430
[69] Tsay (2010), p. 111

asset return[70] (a_t) is serially uncorrelated but dependent. This dependence can be expressed with a simple quadratic function of its lagged values[71] (autoregression). In other words an unobserved shock usually referred as residual in econometric models contains conditional volatility which can be modeled and thus reduce the uncertainty of the model.

GARCH model of order *(m, s)* has the following specification applicable also for energy prices modeling[72]:

$$r_t = \mu + a_t \tag{3.2.12}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{3.2.13}$$

Equation (3.2.12) is mean equation for asset return r_t with a (constant) drift parameter μ where a residual $a_t = \sigma_t \epsilon_t$ (ϵ_t is iid and ~*N(0,1)*). Variance equation (3.2.13) with the following constraints implying that variance of the return σ_t^2 is finite:

$\alpha_0 > 0,\ \alpha_i \geq 0, \beta_i \geq 0, \sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1.$

Family of GARCH models is quite broad and includes different types of functional form specifications (e.g. exponential GARCH, threshold GARCH, non-linear GARCH etc.) Both mean and variance equations can be extended with exogenous regressors to examine impact of other factors. But it is important here not to mix these GARCH models with multivariate volatility models. Multivariate volatility models (mGARCH or BEKK) examine volatility spillovers between different assets. They have some similarities with vector autoregressive framework described in the previous section.

Empirical evidence[73] for applying GARCH models to energy markets is quite mixed. Sadorsky (2006) for instance, finds that the standard GARCH(1,1) model fits well for crude oil and unleaded gasoline volatilities. But other papers argue that none of the GARCH models can outperform all of the others. Moreover, univariate volatility models do not capture relationships and influences between different markets (energy and financial assets markets or markets for different energy commodities). Wang, Wu (2012) find that multivariate GARCH models have better performance than univariate for asset price forecasting referring to crude oil and refinery products volatility spillovers[74].

[70] Asset return specification is also applicable to energy spot prices. Usually "return" is interpreted as difference of log spot prices
[71] Tsay (2010), p. 116
[72] Tsay (2010), p. 132
[73] Wang, Wu (2012), pp. 2167-2168
[74] Wang, Wu (2012), p. 2179

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In the above sections a brief overview of the most common ways to model energy prices is given. The question, however, is how one can investigate impact (if any) of macroeconomic variables on crude oil prices and integrate it into theoretical modeling framework. Chapter 2 of this thesis proposes hypothesis that oil price due to precautionary demand existence depends much on macroeconomic expectations. Expectations in their turn among other factors do depend on soundness of macroeconomic conditions. And it is still true that due to high capital intensity in the industry oil price shall be sensitive to inventories.

3.3 Macroeconomic factors for oil price modeling

One of the factors determining oil precautionary demand discussed above is soundness of macroeconomic conditions. It is very hard though to define the latter and track the impact it has on expectations and the oil demand.

Obviously a measure of economic activity or income would be a good candidate to proxy for both overall industrial demand (including oil) and general state of the economy: terms ‘boom’ and ‘recession’ usually concern GDP as a formal reference. But here is important to remember that stochastic models deal with data of high frequencies while national accounts data typically has maximum quarterly frequency. Therefore an alternative way to measure industrial demand growth is Manufacturing Index for US production which is available on monthly basis. An often taken economic activity measure is index of stock prices movements, e.g. Standart&Poors Index for 500 biggest US companies. Considering the data for US economy makes sense because besides being the world biggest economy and political power US also is the world biggest oil consumer. Moreover, data on crude oil price in this thesis for empirical research used is WTI – oil produced and priced in US.

Other obvious variables determining economic background are exchange rate and interest rate. It makes sense to use US dollar exchange rate to other strong world currencies. First choice would be Euro but its history is not that long so US dollar to British pound exchange rate is taken. World oil price is expressed in US dollars per barrel so the currency dynamics has an obvious direct impact on oil. Interest rate in its turn appears a lot in both structural and stochastic models. As a measure of interest rate I take the benchmark for most interest rates calculations – LIBOR based on US dollar.

Risk is also an important feature of general macroeconomic conditions state. It can be well measured by spread between riskless and risky bonds. Intuitively the higher is overall risk in the economy the higher is risk premium. Safe and risky asset yields do change with different speed in response to risk – thus if the yield spread widens it reflects growth of the general riskiness of the economy. A good example of this spread is yield spread of corporate bonds rated Baa and Aaa by Moody's.

Finally, a good assumption to make is that oil price moves partly the same as other commodities. That is why price of gold movements may nest information common to all commodities markets.

Chapter 4
Empirical Results and Analysis of Crude Oil Price Behavior

This chapter contains the stochastic models for oil prices estimation results. Starting from simple Mean-Reversion and GBM models, conditional heteroskedasticity modeling necessity is further revealed. A classic GBM model thus is extended with a non-constant volatility assumption. The model also includes macroeconomic factors and convenience yields. The chapter ends with analyzing forecasting ability of the estimated models.

4.1 Data

First consideration to take into account about crude oil prices is numerous crudes types existence. Thus the question is which product price to choose. A good candidate here would be one of the two existing global benchmarks for crude oil prices: Brent (Intercontinental Exchange) or WTI (NYMEX). All other crudes' and petroleum products' prices are based on these benchmarks and contracted differentials due to physical characteristics of the products. Second issue is a notion of a current oil price. Spot prices are not observable but are calculated from futures prices (Dated Brent or WTI spot price) for daily, weekly or monthly frequencies. Typically spot price means a price of the nearest delivery. In this thesis data on WTI spot oil price for monthly frequency is used. Sample available at EIA database is quite big and starts from 1946 but for this analysis oil price sample covers 1980 to 2013 (398 observations). Monthly frequency is chosen in order to add other macroeconomic variables to analysis. Due to limited availability of data for other factors most of the analysis is done based on sample 1986-2013 including 326 observations.

Oil prices demonstrated quite stable behavior during 1980-2004 ranging between $15-40/b. Historical minimum is fixed in December 1998 with $11/b price, historical maximum – $36/b in October 1990. After 2004 prices demonstrated exponential growth peaking on $133/b in June of 2008, but then dropped sharply till $39/b in February 2009. At recent years oil prices were extremely volatile ranging between $40-110/b.

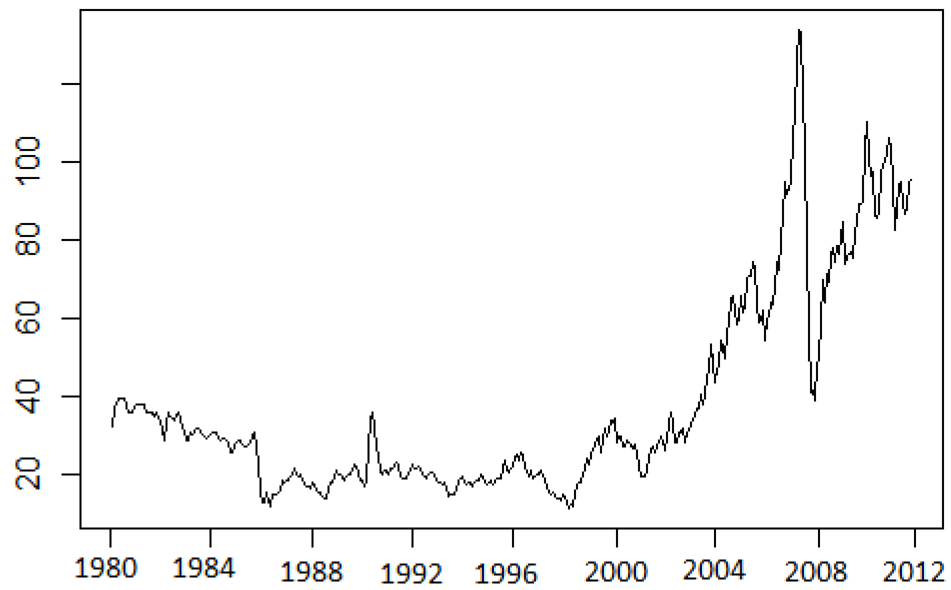

Chart 1. 1980-2013 crude oil price dynamics, *USD per barrel*

To calculate convenience yield WTI crude oil futures for 1, 2 and 3 months maturities for 1986-2013 period are used. The futures prices demonstrate the same dynamics as crude oil 'spot' price. On average during sample period spot price is below futures by $0.002/b for 1-month contracts, $0.08/b for 2-months and $0.12/b for 3-months contract maturities.

For convenience yield calculation data on interest rate is also needed. Interest rate here is measured as 1-month LIBOR based on US dollar for 1986-2013. Since oil price is expressed in US dollars interest rate measure is also consistent to take connected to US currency and LIBOR is a good benchmark since it is often used by governments as well as private banks to conduct their policies. Chart 12, Appendix 2 shows the dynamics of the interest rate. During the sample period interest rate demonstrated clustering behavior: sharp jumps or drops were followed by relatively stable 'plateau' dynamics during several years. Maximum interest rate was fixed at 10% in 1989 while minimum of 0.18% is quite recent – July 2011. Time trend for interest rate is negative and current dynamics remains very low and very stable.

As exogenous macroeconomics factors to measure soundness of economic conditions Standard&Poors500 Stock price index (as a measure of general business activity), US Industrial Production: Manufacturing index (as measure of industrial demand), Moody's Corporate Bonds Baa-Aaa Yield Spread (as a measure of riskiness of markets) and US dollar to British pound exchange rate (a key macroeconomic variable) are taken. All data series have monthly frequency and capture period of 1986-2013. Charts 13-16, Appendix 2 demonstrate dynamics of the indicators respectively.

S&P500 index had stable exponential growth till the end of 1990s. After that it had cyclical falls and rises every 4-6 years ranging between 660-1500 points. Recent dynamics was very volatile and demonstrated an upward sloping trend. Industrial production index is calculated as percentage of 2007 US production. It has similar dynamics to the Stock price index but is smoother. After the drop to 80 in 2009 the industrial production is growing back till 2007 level. Exchange rate of dollar to pound was highly unstable and moved unsystematically but in a relatively narrow interval. US dollar was the most depreciated (2.07 US/GBP) in November 2007 and the most appreciated (1.402 USD/GBP) in June 2001. Current dynamics of exchange rate is relatively stable and witnessing in favor of more expensive US currency.

To capture connection of crude oil price to other commodities prices price of gold is also included to the analysis. Its dynamics over time is shown at Chart 17, Appendix 2. Gold price of around $370/ounce was very stable till 2004. Since then till the present a sharply growing trend is observed: gold price more than tripled in 8 years and still remains high.

Table 13, Appendix 1 contains descriptive statistics for all variables.

Data analysis, modeling and estimations are performed in R statistical software.

4.2 Stochastic models estimations

4.2.1 The Mean-Reversion model

The mean-reversion, or Schwartz-1997 one-factor model, allows for capturing stochastic behavior of oil prices. It is based on the assumption that logarithm of oil price reverts to its long-term mean (see also section 3.2.1). An economic argument to justify a mean-reversion property is that in the long-run due to supply-demand equilibrium prices tend to revert to, roughly saying, marginal costs, which include production costs, inventory capacities and competition effects.

The model for log price consists of the two components: deterministic drift term and a stochastic random component with known distribution properties. Volatility of oil prices plays the key role in this setting as well. Stochastic models with Markov property (no memory processes) do not use previous price paths to predict current price change – only current price levels.

Basically if crude oil price follows the mean-reversion process, theoretical assumptions hold and the parameters are estimated correctly it would be possible to predict accurately current price changes. To estimate the parameters I use Kalman filter method carefully described in Schwartz, (1997) paper[75].

Oil log price path is described by the theoretical stochastic differential equation in continuous time of the form (identical to equation (3.2.7)):

$$dX_t = \kappa(\alpha - X_t)dt + \sigma dW_t \tag{4.2.1}$$

where S_t is crude oil price at time point *t*; $X_t = \log(S_t)$ is logarithm of oil price at time *t*; parameters κ and α are constants and note speed of adjustment to the mean and long-term mean level for oil log price respectively. σ is a volatility term of log prices and is assumed constant in this setting. dt is a time increment which is infinitely approaching 0: in continuous time the distance between neighboring time points is equivalent to zero. The last term captures the stochastic behavior of the process: $dW_t \sim N(0, dt)$ is an increment to Brownian motion.

Transformation of a theoretical continuous-time model into a relevant discrete-time one will allow using historical oil price data in order to estimate the parameters. With rearranging terms equation (4.2.1) has the following empirical representation which can be estimated using linear regression methods (OLS):

$$X_t = c_t + Q_t X_{t-1} + \eta_t \tag{4.2.2}$$

where X_t denotes oil log price at time *t*, a term corresponding to the time difference dt with monthly frequency of discrete historical data is $\Delta t = \frac{1}{12}$, $c_t = \kappa\alpha\Delta t$ is the regression intercept term, the coefficient $Q_t = 1 - \kappa\Delta t$, and the residual $\eta_t \sim N(0, \sigma^2\Delta t)$. In this case *c* and *Q* are assumed to be constant. OLS estimation output is in Table 14, Appendix 1.

The Mean-Reversion model parameters now can be calculated $\kappa = \frac{1-Q}{\Delta t} = 0.06096$, $\alpha = \frac{c}{\kappa\Delta t} = 3.975984$, knowing that $\Delta t = 1/12$ and having estimates for *c* and *Q*. Remembering that the residual incorporates a random (Brownian motion) component and a constant volatility term and knowing the theoretical distribution properties of the residual: $\eta_t \sim N(0, \sigma^2\Delta t)$ it is possible to estimate the volatility parameter σ from the variance of the residual: $\mathrm{var}(\eta_t) = \sigma^2\Delta t$ thus $\sigma = \sqrt{\frac{\mathrm{var}(\eta_t)}{\Delta t}} = 0.2817828$.

The estimation yields reasonable values of the parameters. In Schwartz (1997) the estimation of the one factor model for crude oil results in kappa from 0.099 to 0.694, alpha around 2.8-3 and sigma from 0.129 to 0.334[76]. Kappa and sigma are close to the present estimation, but the long run mean is too big which can easily be explained by 15 more years of

[75] Schwartz (1997), p. 932

[76] Schwartz (1997), p. 937

oil price data used in this research. α estimated is close to the current oil log price mean (3.4459). But it is important to notice that the intercept c is statistically insignificant so estimation of mean-reversion level (α) is also not statistically significant.

Having the parameters' values one must be able to simulate oil log price having initial price, and it must be very close to the historical price path. Applying *Ito's Lemma* mentioned in Section 3.2.1, equation (3.2.12), the stochastic differentiated process for log price change (4.2.1) can be transformed into the form for the log price[77]:

$$\ln(S_t) = (1 - e^{-\kappa\Delta t})\alpha + e^{-\kappa\Delta t}\ln(S_{t-1}) + \sigma Z\sqrt{\frac{1-\exp(-2\kappa\Delta)}{2k}} \qquad (4.2.3)$$

where $\ln(S_t)$ is a natural logarithm of the crude oil price at time t; κ, α and σ are constants estimated by OLS and $\Delta t = \frac{1}{12}$. Z is a set of iid standard normal variables: $Z{\sim}N(0,1)$.

Knowing the constants and distribution of Z it is easy to simulate oil price. But when the simulation is performed it reveals that simulated paths never are systematically close to the historical prices: they can even follow a wrong time trend. Each simulation is individual and alone cannot be considered relevant for analysis or forecasting. The drift term should be sufficiently large to outperform the randomness of the process, but apparently deterministic component is relatively weak and thus oil price process does not demonstrate strong mean-reversion.

Three reasons can lead to this result: either oil prices do not follow a mean-reversion process at all, or the estimated parameters are wrong, or the true random component has different distribution properties than theoretical.

Parameters validity simulation checks

If the model is specified correctly the parameters should be recovered from the simulations in the same way as from the historical data. The following procedure is applied to check for this:

1) Simulate several price paths with estimated 'true' parameters and the initial oil log price using equation (4.2.7);
2) Treat simulations as historical data and run OLS procedure (Kalman filter, equation (4.2.2) again to estimate new parameters;
3) Check if the returned parameters are roughly the same as the 'true' ones.

[77] Tao Lin (2007), p. 20

Example of simulated price paths can be seen on Chart 18, Appendix 2. Returned parameters κ, α and σ estimated by the OLS procedure treating simulations as historical prices are in Table 15, Appendix 1.

Remembering that the 'true' values are $\kappa = 0.06096$, $\alpha = 3.975984$, and $\sigma = 0.2817828$ for OLS estimation it is clear that among the returned parameters only sigma is close to the 'true' – 0.2761 on average. Alpha average estimation, 3.5305, is close to historical mean of 3.4456 even though with huge standard deviation (1.38); kappa varies a lot and unsystematically.

Alternatively to parameters' estimation by OLS method using (4.2.2) it also possible to do the simulations check with a help of the Maximum Likelihood Estimation (MLE). This method is used to find unknown distribution parameters which give the desired output maximum probability of happening. Based on the assumption that conditional distribution of returns (from oil prices) is normal with known mean and variance one can use probability density function for normal distribution to form a likelihood function.

Normal distribution $x \sim N(\mu, \vartheta^2)$ has a probability density function (PDF)[78]:

$$f(x|\mu, \vartheta^2) = \frac{1}{\sqrt{2\pi}\vartheta} \exp\left(-\frac{(x-\mu)^2}{2\vartheta^2}\right) \tag{4.2.4}$$

Likelihood function is simply a product of conditional densities[79]:

$$L(\mu, \vartheta^2|x_t) = \prod_{t=1}^{T} f(x_t|\mu, \vartheta^2) = f(x_1 \dots x_T|\mu, \vartheta^2) = \left(\frac{1}{2\pi\vartheta^2}\right)^{\frac{T}{2}} \exp\left(-\frac{\sum_{t=1}^{T}(x_t-\mu)^2}{2\vartheta^2}\right)$$

For simplicity it is commonly taken in logs:

$$l(\mu, \vartheta^2|x_t) = -\frac{T}{2}\ln(2\pi\vartheta^2) - \frac{\sum_{t=1}^{T}(x_t-\mu)^2}{2\vartheta^2} \tag{4.2.5}$$

Typically to find distribution parameters μ and ϑ^2 function (4.2.5) has to be maximized.

Oil log price $X_t = \ln S_t$ follows the Mean-Reversion process as in equation (4.2.1) with unknown parameters (κ, α, σ). Distribution of X_t is normal with mean and variance equivalent to (μ, ϑ^2) from (4.2.4). Thus for discrete-time sampling conditional distribution is[80]:

$$X_t|X_{t-1} \sim N\left(\alpha(1-e^{-k\Delta t}) + e^{-k\Delta t}X_{t-1}, \frac{\sigma^2(1-e^{-2k\Delta t})}{2k}\right) \tag{4.2.6}$$

Substitution of expressions for mean and variance of oil log price from (4.2.6) instead of (μ, ϑ^2) in the theoretical log likelihood function (4.2.5) yields the final log likelihood function to be maximized with respect to κ, α and σ having series X_t – simulated oil log prices (*T=397* since 1 observation drops out):

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[78] Greene (2002), p. 66
[79] Greene (2002), p. 125
[80] Phillips, Yu (2008), p. 4

$$l(\kappa,\alpha,\sigma|X_t) = -\frac{T}{2}\left(\ln 2\pi + \ln\left(\frac{\sigma^2(1-e^{-2k\Delta t})}{2k}\right)\right) - \frac{\kappa}{\sigma^2}\frac{\sum_{t=1}^{T}(X_t - \alpha - e^{-k\Delta t}(X_{t-1}-\alpha))^2}{(1-e^{-2k\Delta t})}$$

The returned parameters estimated from several simulations by MLE method are reported in Table 16, Appendix 1.

First thing coming out for interpretation is that the MLE parameters of the Mean-Reversion model for historical log prices are identical to those estimated by OLS: $\kappa = 0.062055$, $\alpha = 3.96354$, $\sigma = 0.282151$ from MLE versus $\kappa = 0.06096$, $\alpha = 3.975984$, and $\sigma = 0.2817828$ from OLS which is a good though obvious sign. In general, findings of the MLE simulation check are consistent with OLS simulation check exercised above. It is easy to see that MLE estimation returns only sigma close to the 'true' (0.27754). Drift parameter κ returned on average is much bigger and has huge uncertainty. Situation with mean reversion level (alpha) is a little bit better (3.43) which is close to the OLS returned estimate but still quite far from the 'true'.

Comparing parameters returned from simulations with the estimated from historical prices (true) it is easy to see that the 'true' drift parameters can be considered as correct, but estimation of them has big uncertainty. Only the volatility parameter is very stable no matter what estimation method to use for actual or simulated prices. This may mean that either there is not much support of the mean-reversion property or that the parameters are non-constant and changing over time.

Additional, the problem of simulations irrelevance may be caused by failing the assumptions about residual distribution properties. Another nice check if the estimated parameters are correct is to use equation (4.2.7) with the known parameters and initial oil log price, but with estimated from empirical regression residual instead of theoretical Z ('standardized' residual $\frac{\eta_t}{\sigma\sqrt{\Delta t}}$ from the regression (4.2.2). Apparently parameters were estimated correctly since the simulated price path is very close to the actual what can be seen at Chart 19, Appendix 2.

4.2.2 The Geometric Brownian Motion model

The uncertainty of drift estimation for crude oil prices discovered in the previous subsection can mean that oil price does not follow the mean-reversion process. The random component has the biggest weight and the volatility parameter is of high importance. That is why it is may be true that the price process is that of random walk or classic Geometrical Brownian

Motion (GBM). GBM has similar form and properties than the Mean-Reversion model. Again the model has a deterministic drift term and a stochastic component. For the oil price standard representation of the GBM model[81] corresponds to a stochastic differential equation (3.2.2):

$$dS_t/S_t = \mu dt + \sigma dW_t \tag{4.2.7}$$

where S_t is an oil price at time *t*, μ is constant drift parameter, σ – constant volatility parameter, dt is an infinitely approaching 0 time difference between time points *t* and *t-1* and the last term involves random $dW_t \sim N(0, dt)$ increment to Brownian motion process.

The logic here is the same as before: knowing the parameters and the true distribution properties of the random component it would be possible to forecast oil price.

One can estimate the parameters μ and σ using historical data for oil prices following the procedure described in Tsay (2010)[82], remembering also that the time difference for data with monthly frequency is $\Delta t = \frac{1}{12}$:

1) Define return as a difference of log prices: $r_t = \ln(S_t) - \ln(S_{t-1})$ with mean $\bar{r} = \frac{\sum_{t=1}^{n} r_t}{n} = 0.002710326$ $\left(s.e._{\bar{r}} = \frac{s_r}{\sqrt{N}} = 0.004085265\right)$ and variance $s_r^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2 = 0.006625687$ $\left(s.e._{s_r^2} = \sqrt{\frac{2s_r^4}{N-1}} = 0.0004708671\right)$;

   It is not difficult to notice that standard error of return sample mean is so high that the estimate cannot be considered significant. Variance parameter is significant.

2) Remembering that for GBM process $r_t = d\ln S_t \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t;\ \sigma^2 \Delta t\right)$ therefor

$$\mu_r = \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t = \bar{r} \tag{4.2.8}$$

$$\sigma_r^2 = \sigma^2 \Delta t = s_r^2 \tag{4.2.9}$$

3) So knowing the return mean $\bar{r}$ and variance s_r^2 from 1) and solving (4.2.8) and (4.2.9) for μ and σ yields:

   $\sigma = \frac{s_r}{\sqrt{\Delta t}} = 0.2819721$ and $\mu = \frac{\bar{r}}{\Delta t} + \frac{\sigma^2}{2} = 0.072278$, which however is statistically insignificant.

   There is an alternative way to estimate the parameters: OLS method for the empirical equation got from rearranging the (4.2.7):

$$\frac{dS_t}{S_t} = c + \eta_t \tag{4.2.10}$$

[81] Tsay (2010), p. 294
[82] Tsay (2010), p. 295

where the regression intercept $c = \mu\Delta t$ is a drift term and the residual is $\eta_t = \sigma Z\sqrt{\Delta t} \sim N(0, \sigma^2\Delta t)$. Estimation of regression coefficients is in Table 17, Appendix 1.

Thus the GBM parameters are $\mu = \frac{c}{dt} = 0.072036$ and $\sigma = \sqrt{\frac{\mathrm{var}(\eta_t)}{\Delta t}} = 0.2812751$ which are almost the same as estimated with the previous method with the drift term not significant again.

For GBM stochastic process the differential equation for price change (4.2.2) applying *Ito's Lemma* equation (3.2.12) transforms into a path of the oil prices with the known parameters and a random standard normally distributed set of variables (Z)[83] in the log form:

$$\ln(S_t) = \ln(S_{t-1}) + \left(\mu - \tfrac{1}{2}\sigma^2\right)\Delta t + \sigma Z\sqrt{\Delta t} \qquad (4.2.11)$$

The above equation means that using the distribution on Z (assumed standard normal) a simulated price path should be close to the historical. However simulations in the GBM model similarly to those of the Mean-Reversion model are highly random and individually are not relevant for consideration. But if the model is specified correctly the parameters must be recovered from simulations in the same way as from the historical time series.

Parameters validity simulation checks

The procedure of simulations check is familiar: simulate several price paths using equation (4.2.11) and initial oil log price, then treating them as historical data estimate the parameters either with OLS or MLE methods. Example of simulation paths can be found in Chart 20, Appendix 2.

Table 18, Appendix 1 contains returned parameters from some simulations estimated by OLS method. Comparing the 'true' parameters $\sigma = 0.2819721$ and $\mu = 0.072278$ with the returned ones it is easy to see that sigma again is returned close to the 'true' – averaging in 0.2844. The same can on average be said about the drift, which is returned as 0.0627, though the standard deviation is still big – 0.04242.

Similarly to the subsection before parameters can be checked with the help of MLE method. Procedure of deriving a likelihood function is the same but it is necessary to remember than oil price which follows a drift diffusion process now has log normal distribution:

[83] From Tsay (2010), p. 294 rearranging terms and remembering that dw_t is an increment to Wiener process so $dw_t \sim N(0, \Delta t)$ thus it can be replaced with $\sqrt{\Delta t}Z_t$ where $Z_t \sim N(0,1)$ and iid.

$x \sim LN(\theta, \vartheta^2)$. But taking oil log prices into consideration allows using exactly the same likelihood function for normal density as (4.2.5)

Using mean and variance parameters for oil price drift diffusion process[84]:

$$S_t|S_{t-1} \sim LN\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \ln S_{t-1}, \sigma^2 \Delta t\right) \tag{4.2.12}$$

and substituting them into (4.2.5) instead of (θ, ϑ^2)[85] yields into the final log likelihood function for maximization with respect to μ and σ:

$$l(\mu, \sigma|S_t) = -\frac{T}{2}(\ln 2\pi + \ln(\sigma^2 \Delta t)) - \frac{\sum_{t=1}^{T}\left(\ln S_t - \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t - \ln S_{t-1}\right)}{2\sigma^2 \Delta t}$$

Returned parameters can be seen in Table 19, Appendix 1.

This time MLE yields similar to the other methods results. Again, MLE estimation of the parameters for actual oil prices is identical to the above methods output ($\mu = 0.072036$, $\sigma = 0.2812751$ before and $\mu = 0.072058$, $\sigma = 0.281616$ now). The returned from simulations drift parameter is 0.04 which is relatively close to the 'true' and returned sigma is very close as well – 0.28035.

Even though the estimated GBM drift parameter is not statistically significant but the simulations checks return a quite close estimate with much less uncertainty than in the Mean-Reversion model. Apparently the oil price modeling as GBM would be more efficient compared to its modeling as Mean-Reverting process, but it is still not enough to judge which of the models has better performance.

If the GBM parameters returned from simulations are more or less the same as estimated it means that oil price does follow the GBM process but using theoretical standard normal random component is just guessing. If one uses empirical (standartized) residual Z from (4.2.10) instead of a theoretically simulated in equation (4.2.11) the picture shows that simulated price is very close to the actual (Chart 21, Appendix 2).

This naturally leads to the conclusion that the random component and price volatility have the major influence outperforming the drift term. But the distribution of the random component needs some investigation. Theoretically, a set of normally identically and independently distributed (iid) with mean 0 and variance 1 variables must work. Therefor it might be interesting to check whether these properties belong to the real set of variables $\hat{Z}$ extracted from historical

[84] Phillips, Yu (2008), p. 4

[85] To avoid confusion theoretical distribution mean parameter is θ instead of μ in (4.2.4) because in this section μ corresponds to GBM drift parameter

prices. From (4.2.11) using actual oil log prices $\ln(S_t)$ and estimated 'true' parameters μ and σ actual random set $\hat{Z}$ is:

$$\hat{Z}_t = \frac{\ln(S_t) - \ln(S_{t-1}) - \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t}{\sigma\sqrt{\Delta t}} \tag{4.2.13}$$

Basically $\hat{Z}$ estimated in equation (4.2.13) is the same as the 'standardized' residual $\frac{\eta_t}{\sigma\sqrt{\Delta t}}$ got from the empirical regression (4.2.10), which was used above in order to check the validity of the parameters.

If one looks at the distributional properties of $\hat{Z}$ it may look standard normal at the first glance: $E[\hat{Z}] = -3.3796 \cdot 10^{-17} \equiv 0$, $\mathrm{var}(\hat{Z}) = 1$ and probability density function has the usual bell-shape form (see Chart 22, Appendix 2).

But this is not enough to conclude that the distribution is standard normal. Conducted tests for normality, Shapiro-Wilk and Jarque-Bera, reject the Null hypothesis about normality.

Test	Null-hypothesis	Statistics	p-value
Shapiro-Wilk	Normality	W = 0.9589	4.27e-09
Jarque-Bera	Normality	X-squared = 206.2874	<2.2e-16

Table 1. Result of normality tests for the empirical residual

Moreover, shape of the QQ-plot (normal versus actual quantiles) confirms this conclusion demonstrating heavy tails of the empirical distribution (See Chart 23, Appendix 2).

Therefore the residual series is not normally distributed. The first logical guess to explain this deviation comes from definition of the standard normal distribution: in addition to required mean-variance properties series must be distributed identically and independently. Thus there should not be any correlation between the components. Tests for autocorrelation confirm this guess proving the presence of serial dependence in the empirical residual. Autocorrelation function (ACF) has significant lags 1, 6 and some higher (See Chart 24, Appendix 2).

Test	Null-hypothesis	Statistics	p-value
Box-Pierce	No serial correlation	X-squared = 32.7758	1.034e-08
Ljung-Box	No serial correlation	X-squared = 33.0241	9.102e-09

Table 2. Tests for serial correlation in the empirical residual results

This result may witness in favor of presence of conditional hederoskedasticity: since the volatility of oil prices in the above models was assumed constant and it is estimated from the variance of the residual it make sense to believe that the serial dependence property of volatility stays back in the residual. Stochastic and time variant volatility properties have also been documented in the literature, e.g. Krichene (2006) or Meade (2010).

4.2.3 Generalized Autoregressive Conditional Heteroshedasticity modeling

It can be true that serial dependence of the residual term comes from serial dependence of oil price volatility. However, elimination of conditional heteroscedasticity may not necessary guarantee correction of the random term empirical distribution towards theoretical standard normal. It is fairly likely that other than volatility unobserved (maybe even immeasurable) factors hidden in the residual influence oil price evolution.

Since volatility is not directly observed historical data about it does not exist. That means that whatever volatility model is estimated it is not possible to check if it actually matches reality. So the only way possible is usage of modeled volatility in the pricing model and evaluating how well it works.

The easiest way to handle serial correlation is some kind of an autoregression model, in volatility case it is Generalized Autoregressive Conditional Heteroscedasticity model (GARCH). Since volatility is unobserved, GARCH models work with series potentially nesting volatility – residuals or disturbances. The residual from GBM model for instance (before estimating sigma $\eta_t = \sigma Z\sqrt{\Delta t} \sim N(0, \sigma^2 \Delta t)$) is a perfect candidate. Moreover, here I use GARCH framework in order to extend a stochastic model for oil price (GBM) with time variant volatility. GBM is chosen over the Mean-Reversion model because it seems more reasonable due to drift non-importance of the latter and a smaller number of parameters. The models can also be further extended with additional explanatory factors, e.g. macroeconomic variables in drift or volatility relationships.

To start the modeling of volatility with GARCH model one should first test for presence of ARCH effects. For this purpose sometimes it is enough just to look at the autocorrelation function of the residuals squared (Chart 25, Appendix 2) – under the assumption that volatility (σ^2) is a part of the residual squared η^2. ARCH effects are present up to 6 lags meaning that for monthly oil prices volatility correlates with its own values up to 6 months before. The visual interpretation is not always reliable so it is better to conduct additional formal tests. For testing presence of ARCH effects typically a Lagrange Multiplier test is used. Null hypothesis of the test is absence of ARCH effects thus p-value of 0.007 for 6 lags allows to reject the Null on 1% confidence level.

Test	N of lags	Null-hypothesis	Statistics	p-value
ARCH-LM	6	No ARCH effects	Chi-squared = 17.5302	0.00752

Table 3. ARCH-effects test result

In proven presence of conditional heteroskedasticity it is possible to proceed with GARCH modeling. According to the partial ACF (PACF) function serial correlation is present for 6 lags so the ARCH of order 6 model should be built.

Standard ARCH/GARCH model consists of two equations. The first, mean equation, describes dynamics of the main argument of research – like asset returns or, in this case, price of crude oil. An essential component of the mean equation is variance, usually – standard deviation (σ) as part of the residual. Volatility (σ or σ^2) explicitly can also be present in the mean equation (e.g. in GARCH-in-Mean class of models). The second equation of the model is a variance equation typically describing and autoregressive nature of the volatility. Usually it looks similarly to Moving-Amerage (MA) or Autoregressive Moving-Average (ARMA) model but with the notice that volatility is a part of the random disturbances (see Sections 3.2.2 and 3.2.3).

The ARCH(6) model has the following formal representation[86]:

Mean equation: $r_t = \mu_t + a_t$ (4.2.14)

where the residual/disturbances $a_t = \sigma_t \epsilon_t$ with ϵ_t being standard normal. From now on the notation a_t instead of η_t for residual is used; r_t is a time series of the 'main argument' (returns or prices), and μ_t is its time-varying [or can be assumed constant] mean or drift term.

Variance equation: $\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \alpha_3 a_{t-3}^2 + \alpha_4 a_{t-4}^2 + \alpha_5 a_{t-5}^2 + \alpha_6 a_{t-6}^2$ describes dependence of the volatility at time t (σ_t^2) on lagged residual terms (a_{t-i}^2). Here similarity to MA(6) model can be noticed, but this model is actually autoregressive if recon that the residual contains volatility. ω and α_i are the model parameters.

In order to adopt GARCH representation to the GBM model base but with time-varying volatility and constant or time varying drift term ARCH/GARCH theoretical mean equation (4.2.14) can be expressed as:

$$\frac{dS_t}{S_t} = \mu_S dt + \sqrt{dt}\sigma_{S,t}\epsilon_t \quad (4.2.15)$$

being a combination of equations (4.2.7) and (4.2.14) so that $\frac{dS_t}{S_t} = r_t$ equivalent to the 'main argument', $\mu_S dt = \mu$ is a drift term and the rest $\sqrt{dt}\sigma_{S,t}\epsilon_t = a_t = \sigma_t \epsilon_t$ being the residual term, dt is a time difference as usual. Dynamics of $\sigma_{S,t}$ squared is described by a typical GARCH

[86] Tsay (2010), p. 119 and p. 116

variance equation (3.2.13). Having GARCH-GBM functional form it is very important to remember that estimate of μ in the GARCH should be divided by dt in order to get a GBM drift term μ_S, and also that GARCH model simulates not interesting for us volatility of oil price ($\sigma_{S,t}$) but $\sigma = \sigma_{S,t}\sqrt{dt}$ and thus has to be divided by $\sqrt{dt}$.

Estimation of ARCH(6) suggests that intercept and lags 1 and 5 for shocks are significant, see the output at Table 20, Appendix 1.

Translating the results into GBM framework: the drift parameter of the GBM $\mu_S = \frac{\mu}{dt} = 0.116844$ and significant on the 5% level, which is also relatively close to the one estimated by the GBM model in the Section 4.2.2 (0.072278). Volatility (σ_t^2) development has also a constant drift (omega) of 0.002436 and significant 1 and 5 months lags of serial dependence.

To check if the model reached its goal – eliminating of conditional heteroscedasticity from our empirical residual – one should perform an ARCH-LM test for the new standardized residuals. Standardized residual $\epsilon_t = \frac{a_t}{\sigma_t}$ where a_t is a residual of the estimated mean equation (4.2.14) of the model (in GBM words it is η_t residual) and σ_t is a ARCH/GARCH simulated volatility (square root of it). Theoretically ϵ_t should be standard normal and, naturally, without autocorrelations. The test output of 0.9807 suggests that there are no ARCH-effects in the standardized residuals squared (accepting the Null) so the ARCH(6) model is valid.

Test	N of lags	Null-hypothesis	Statistics	p-value
ARCH-LM	10	No ARCH effects	Chi-squared = 3.0313	0.9807

Table 4. ARCH effects test result, ARCH(6) model

ARCH(6) model produces the volatility ($\sigma_{S,t}$, Chart 26, Appendix 2) fit with the mean 0.2795233 which is fairly close to the estimations of the parameter σ in both GBM and Mean-reversion models in sections above.

Even though ARCH specification is a relatively easy model for understanding but it often requires a lot of parameters to be estimated: 8 in this case. Often Generalized ARCH is considered to be more efficient – less parameters present and the volatility serial dependences are captured in the general term. Moreover GARCH modeling is very developed and a lot of modifications are available to exploit this or that assumption about prices/returns behavior.

GARCH specification has more similarities to the ARMA model. Usually GARCH of order (1,1) is used because the models of higher orders typically become too complex and rarely outperform the basic one. The construction of the GARCH model is the same than that of ARCH[87]: *mean equation* is identical to (4.2.14): $r_t = \mu_t + a_t$. But *variance equation* becomes:

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{4.2.16}$$

where time-variant volatility σ_t^2 depends on past residual and volatility with lag 1 month; ω, α_1 and β_1 are the parameters.

If one uses the GBM specification the mean equation is identical to (4.2.15) with GARCH(1,1) formal relationship for variance (4.2.16):

$\frac{dS_t}{S_t} = \mu_S dt + \sqrt{dt} a_t$ where $a_t = \sigma_{S,t}\epsilon_t$, $\epsilon_t \sim N(0,1)$; S_t is crude oil price at time t, $\mu_S dt$ is a [constant] drift term with time difference $dt \equiv \Delta t = \frac{1}{12}$; and $\sigma_{S,t}^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ for volatility. GARCH(1,1) model estimation produces the output with all parameters highly statistically significant.

Parameter	Estimate	Standard error	t-value	p-value
mu ($\boldsymbol{\mu_S dt}$)	0.009899	0.004335	2.2833	0.022411 *
omega ($\boldsymbol{\omega}$)	0.001629	0.000589	2.7670	0.005657 **
alpha1	0.248615	0.075980	3.2721	0.001067 **
beta1	0.529210	0.115561	4.5795	0.000005 ***

Table 5. GARCH(1,1) estimation results

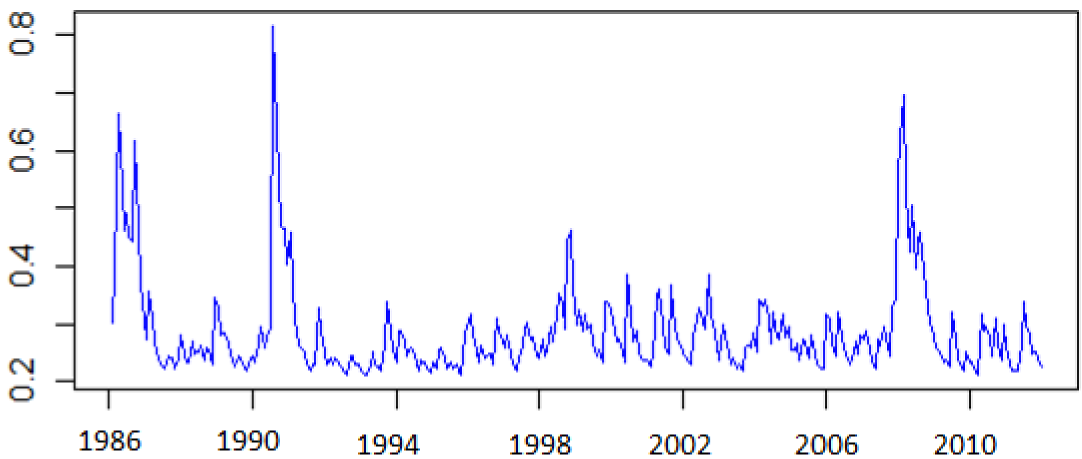

Chart 2. GARCH(1,1) volatility fit, $\sigma_{S,t}$

[87] Tsay (2010), p. 132

The GBM drift parameter $\mu_S = \frac{\mu}{dt} = 0.118788$. Volatility has significant constant intercept 0.001629 and both parameters alpha and beta. GARCH(1,1) fit of volatility[88] ($\sigma_{S,t}$) with mean equal to 0.2873869 which is quite consistent with the previous estimations

ARCH-LM test for standardized residuals suggests that there are no ARCH-effects so the model is valid.

Test	N of lags	Null-hypothesis	Statistics	p-value
ARCH-LM	10	No ARCH effects	Chi-squared = 4.4298	0.9259

Table 6. ARCH-effects test result, GARCH(1,1)

To check the validity of GARCH parameters I use equation (4.2.11) for the oil log price in the GBM form, but inserting the drift parameter (μ_S) and non-constant volatility ($\sigma_{S,t}$) simulated by GARCH as in previous sections. Using the empirical standardized residual ($\hat{\epsilon}_t$ similarly to $\hat{Z}_t$) instead of the theoretical demonstrates (Chart 27, Appendix 2) that modeled log price path is very close to the actual prices of crude oil which witnesses in favor of parameters validity.

On the first glance ARCH(6) model seems to have better predictive power due to higher log likelihood (375.7258 vs 357.1389). But model likelihood always increases with the number of parameters included. The majority of parameters in ARCH model are not significant. The ARCH volatility fit is noisier than that of GARCH, but the mean sigma is less than the true[89]. Residual analysis is useful to compare the models.

Residuals of ARCH/GARCH process are $a_t = \sigma_t \epsilon_t$, or $\epsilon_t = \frac{a_t}{\sigma_t}$ is a standardized residual. Residual $\epsilon_t \sim N(0,1)$ is standard normal in theory. ARCH-LM test examines if standardized residual *squared* has serial correlation which would reveal that conditional heteroskedasticity analysis is not yet done. I compare distributional properties of the standardized residuals from ARCH(6) and GARCH(1,1) models (see Chart 28, Appendix 2) – if they are close to that of the standard normal distribution: mean and variance should equal 0 and 1 respectively, residuals should be distributed normally (by conducting Shapiro-Wilk test with Null hypothesis about normality) and there should not autocorrelation in the residuals (by conducting Box-Pierce test with Null hypothesis about absence of serial correlation).

[88] Remember to divide volatility from GARCH fit by $\sqrt{\Delta t}$

[89] Assumption about the "true" volatility is made based on volatility estimations in the models, considered in earlier sections (4.2.1 and 4.2.2)

Model	Mean(ϵ_t)	Var(ϵ_t)	Shapiro-Wilk test (p-value)	Box-Pierce test (p-value)
ARCH(6)	-0.04767307	1.077821	0.00209	0.0003423
GARCH(1,1)	-0.0240361	1.018764	0.0006685	0.0004851

Table 7. Comparative residual analysis for ARCH(6) and GARCH(1,1) models

Mean and variance of the standardized residuals for both models are close to theoretical, but normality test does not allow to accept the Null with p-values 0.00209 and 0.0006685 for ARCH(6) and GARCH(1,1) respectively. There is also a small serial correlation in the standardized residuals (rejecting the Null of Box-Pierce test) of both models (not in residuals *squared* – that would mean presence of ARCH effects). So the residuals are not distributed standard normally and both of them are a little bit skewed. But the mean-variance indexes are better for the GARCH model.

GARCH (1,1) model due to its parameters and performance efficiency is preferred over ARCH(6). Moreover various complications can be applied to the GARCH and massive literature supports this base model validity for crude oil volatility (e.g. Sadorsky (2006)

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GARCH(1,1) model for volatility (in the GBM framework) can also be extended with external explanatory variables. In the following subsections macroeconomic variables and also convenience yields are included to the GARCH(1,1) model in both mean and variance equations. Thus the basic Geometric Brownian Motion model gets extensions namely through the *time-varying drift* term, received from the mean equation of the GARCH with exogenous factors, and *non-constant volatility*, estimated in the variance equation of the model.

4.2.4 Inclusion of the macroeconomic factors

In order to include external variables into the mean and variance equations the GARCH(1,1)-GBM equations (4.2.15) and (4.2.16) are to be transformed as:

$$\frac{dS_t}{S_t} = \mu_S dt + \varphi X_t + a_t \tag{4.2.17}$$

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \tau Y_t \tag{4.2.18}$$

where S_t is a crude oil price at time t, μ_S is a GBM drift parameter, $a_t = \sigma_{S,t}\epsilon_t$ is a residual with $\epsilon_t \sim N(0,1)$, $dt \equiv \Delta t = \frac{1}{12}$, σ_t^2 is oil price volatility at time t with usual GARCH(1,1) dependence; X_t and Y_t are matrices for external variables and can contain one or many variables. Parameters $\mu_S, \varphi, \omega, \alpha_1, \beta_1, \tau$ are constant.

To investigate which macroeconomic variables have the influence on oil price and its volatility I estimate the equations (4.2.17) and (4.2.18) with different factors (one factor at a turn). It results into Baa-Aaa bond yield spread and USD/GBP exchange rate being significant in the GARCH volatility equation and only exchange rate in the mean equation. The output of the models with additional factors in volatility, just basic GARCH(1,1) and the models with additional factors in mean is in the Table 21, Appendix 1.

GARCH models with additional factors in the *volatility equations* all have significant drift terms (μ_S) which is estimated in a relatively small range from 0.114396 to 0.121692. Moreover, in these models (with one exception) estimated parameters alpha and beta are highly significant: alpha varies from 0.213497 to 0.259104, beta – from 0.524025 to 0.536061. The exception is the GARCH model with bond yield spread in variance. Here alpha is close to the others, but beta is estimated 0 and not significant. Apparently this factor – spread between Baa-Aaa corporate bonds yields – overtakes the significance of the past oil price volatility parameter (beta).

Among macroeconomic factors coefficients of exchange rate and yield spread are positive and highly significant, coefficients of other variables are positive but estimates are very close to 0 and not significant. However coefficients for Industrial index and S&P500 demonstrate 'nearly' significance – if 15% level existed. All the models are valid according to the ARCH-LM test demonstrating no further ARCH effects.

The models with additional factors in *mean* have the drift term estimates not significant. It becomes significant only in the model with interest rate. Magnitude of the GARCH coefficients is similar to that of just above class of models: alpha varies from 0.254314 to 0.265787 and beta from 0.49149 to 0.538853. These coefficients are highly significant for all models. External variables are not significant except exchange rate, which has a positive and significant on 10% level coefficient. These models have no ARCH effects in the standardized residuals as well.

The models estimation and reporting coefficients is an easy task but it is basically useless without relevant interpretation of results. First observation coming out is importance of the macroeconomic factors in general for crude oil price and its volatility. In fact they do not make a big difference into the oil price modeled by the basic GARCH: the parameters are not that sensitive to changes of the model, which means that basic relationships of the volatility (its dependence on itself and the random disturbances) are very stable and cannot be influenced by something exogenous (at least by the variables used in this thesis). The volatility fits of different models show similar dynamics, same frequency of spikes and relatively close average values. Even though the influence of the macroeconomic factors is small but it cannot be neglected.

Apparently *price of gold* has no impact on oil price at all, neither directly nor through volatility. This result may seem surprising because commodity markets are believed to be interrelated and crude oil together with other commodities is deeply integrated into the global financial system. But in fact it is quite likely that oil price moves faster than a lot of other industrial, financial and macroeconomic indexes. Thus, oil is rather a determinant than determined here.

Interest rate included to the components of GARCH model almost does not influence the parameters. It does not make any difference for the oil price volatility relationship and is also insignificant in the mean equation. It is quite a surprising result because interest rate importance is emphasized by a lot of theories. Even not taking into consideration Hotelling model, where interest rate is a key price determinant, it is widely used for stochastic modeling purposes (first as an input to convenience yield[90], second – in Schwartz 3-factor model), but there was not found strong supportive evidence about significance of interest rate.

Industrial index and *S&P500* would be the obvious determinants of oil prices. Since oil is an industrial commodity and used only for production of other industrial or consumption goods the state of manufacturing sector could represent demand for oil. S&P500 index often is a measure of economic activity and general mood on the global markets. Consistent with conclusions of Chapter 2 macroeconomic background through the expectations should have a strong influence on oil price. These two factors, however, do not influence oil price volatility at all, but have some weak impact on the current prices of oil directly. This result does not mean that macroeconomic background is not important to consider for oil price modeling but it may prompt that the *current* state of it is not that useful. And moreover it is always an open question how to measure soundness of the macroeconomic background because higher activity may also reveal overoptimism which is quite dangerous for the economy. Positive signs of the coefficients for both factors in the mean equations should be understood as increasing of the indexes (higher economic activity and more optimistic markets) make oil prices grow faster. Increase of S&P500 index by 10 points will make oil price grow by 0.0013%[91], today with oil price around $95/b it

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[90] See details in section 4.2.5

[91] $\frac{dS_t}{S_t}$ grows by 0.00015, but assuming functional form of equation (4.2.11) $\Delta \ln\ S_t = (0.000015 * (sp_{t-1} + 10) - 0.000015 * sp_{t-1})dt = 0.00015 * dt = 0.0000125.$ $\Delta \ln S_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = 0.0000125 => \frac{S_t}{S_{t-1}} = \exp(0.0000125) = 1.000013$

will imply growing by 0.1 cents/b holding other conditions equal. If industrial index grows by 1 point oil price grows by 0.0032%[92] or by 0.3 cents/b for $95/b expensive oil.

Exchange rate and bond yield spread have the strongest impact on both oil price and its volatility. Moreover, *yield spread* changes crucially the volatility relationship: one-month lagged volatility can apparently be substituted with current yield spread. The usefulness of this result may not be obvious: it is rather past values of oil volatility that determine current bond yield spread than the other way around, but the important part is that oil price volatility is not observed while information about spread between bond yields is easily accessible. This result has also a strong economic support. Difference in yields of safe (Aaa) and relatively risky (Baa) bonds is a reflection of the market riskiness. If corporations/governments are rated Aaa they are generally not believed to be subject to risks contrary to the Baa bonds issuers. Therefore, if the yield spread widens markets become less stable and volatile. It concerns oil market too: volatility of oil prices increases with the wider spread. If the yield spread (expressed in percentage) increases by 1 percentage point volatility (σ^2) increases by 7 percentage points[93] holding all other conditions equal.

Exchange rate has a positive significant sign in the variance so if US dollar depreciates by 10 cents to 1 British pound volatility of oil prices (σ^2) increases by 0.116 percentage points[94]. Concerning direct influence of the 10 cents USD depreciation (significance in the mean) it leads to 0.0427% increase of oil price[95] or 4 cents for $95/b oil. The positive link between dollar depreciation (but towards Euro) and oil prices was also documented by Bencivenga et al (2012)[96]. Exchange rate is a key macroeconomic variable influencing oil prices both nominally – since the latter is measured in US dollars per barrel, and qualitatively – dollar depreciation may witness in favor of the biggest oil consumer economic growth and also international trade (exports) increase, and thus their oil demand.

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92 $\frac{dS_t}{S_t}$ grows by 0.000385, but using equation (4.2.11) $\Delta \ln\ S_t = (0.000385 * (ind_{t-1} + 1) - 0.000385 * ind_{t-1})dt = 0.000385 * dt = 0.000032$. $\Delta \ln S_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = 0.000032 => \frac{S_t}{S_{t-1}} = \exp(0.000032) = 1.000032$

93 $\sigma^2 dt$ increases by 0.0056, thus σ^2 increases by $\frac{0.0056}{dt} = 0.07$

94 $\sigma^2 dt$ increases by 0.000977*0.1, thus σ^2 increases by $\frac{0.0000977}{dt} = 0.00116$

95 $\frac{dS_t}{S_t}$ grows by 0.00513, but using equation (4.2.11) $\Delta \ln\ S_t = (0.0513 * (ex_{t-1} + 0.1) - 0.0513 * ex_{t-1})dt = 0.00513 * dt = 0.00043$. $\Delta \ln S_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = 0.00043 => \frac{S_t}{S_{t-1}} = \exp(0.000032) = 1.000427$

96 Bencivenga et al (2012), p. 236

4.2.5 Alternative GARCH specifications

Due to complexity of asset returns behavior and volatility relationships simple GARCH model is not always able to explain accurately the nature of conditional variance. Therefor the family of extended and modified models has been proposed to capture different properties of the volatility.

Exponential GARCH (eGARCH)[97] may be useful to overcome some weaknesses of standard GARCH: Log conditional variance allows for relaxing positiveness constraints on coefficients, and asymmetric response of the model to positive and negative shocks is enabled. The *mean equation* here is the same as in standard GARCH, equation (4.2.14), but *variance equation* takes the following functional form:

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|a_{t-1}| + \gamma a_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2)$$

with $\ln(\sigma_t^2)$ being a natural logarithm of volatility at time t, a_t disturbances/residual term and $\alpha_0, \alpha_1, \gamma, \beta_1$ as parameters. Parameter γ signifies the leverage effect of past innovations and is expected to be negative in real applications. For negative gamma volatility is more sensitive (increases) to negative shocks.

GARCH-in-mean (GARCH-M)[98] model is usually taken into consideration if return is assumed to be dependent on its own volatility. This condition is likely to hold for energy prices behavior. *Variance equation* (4.2.16) is unchanged, but the *mean equation* of the standard GARCH (4.2.14) is extended with an additional term:

$$r_t = \mu + c\sigma_t^2 + a_t$$

where r_t denotes return/price at time t, σ_t^2 its time-varying volatility and a_t the residual. $\mu, c, \alpha_0, \alpha_1, \beta_1$ are the parameters. Parameter c is often referred as risk premium: positive value of the parameter means that return responses positively to volatility increase. GARCH-M model would also imply serial correlation of returns driven by serial dependence of volatility. This property is also observed for crude oil price returns if run Box-Pierce test.

Return	Box-Pierce Chi-squared (null: no serial correlation)	p-value
$\boldsymbol{r_t = dS_t/S_t}$	38.5122	5.441e-10
$\boldsymbol{r_t = d\ln S_t}$	32.7758	1.034e-08

Table 8. Result of test for oil returns serial dependence

[97] Tsay (2010), p. 143
[98] Tsay (2010), p. 142

These two modified GARCH models are likely to improve oil prices conditional heteroscedasticity modeling. Even though they are complexions of the basic model but still are easy to understand. Asymmetric response of volatility to shocks is a quite reasonable extension: oil price volatility behaves differently for positive and negative random disturbances. The assumption about volatility in mean is obvious to be made for any risky asset.

The models estimation together with significant macroeconomic factors (exchange rate and bond yield spread) in mean and variance produces the output shown in Table 22, Appendix 1.

Macroeconomic factors have similar performance and role in these models that in the basic GARCH(1,1). In GARCH-M they even have the same coefficients thus interpretation remains identical to that of Section 4.2.4. In the exponential GARCH the coefficients magnitudes are different but the principle of interpretation remains the same. Exchange rate in the mean has exactly the same performance for both modified and basic models. But exchange rate in the variance becomes insignificant for the E-GARCH model: apparently it becomes redundant when the volatility asymmetry response is modeled directly. Yield spread overtakes significance of betas as before and has an enormous influence in the eGARCH model meaning that if asymmetric volatility response to shocks is allowed it is sensitive even more to the general riskiness of the markets.

Interesting part starts with the interpretation of models specific parameters. GARCH-M parameter c, synonymous to the risk premium influence, is never significant (but positive as expected). From this result I would not make the obvious conclusion that oil price change does not depend on its volatility. It is far more likely that the price response is unsystematic: oil prices change in an unpredictable direction and magnitude because of volatility shifts. Since c is insignificant, performance of the GARCH-M model does not differ from performance of the standard GARCH(1,1) model.

Contrary to expectations, asymmetry parameter gamma in the E-GARCH model has a positive sign. It implies that crude oil price becomes more volatile when the positive shocks occur rather than negative. But actually it is quite a logical outcome if the model is built for price change not for asset returns as usual. Occurrence of the negative shock (a_t) makes a price change (dS_t) smaller which basically means that oil price grows slower (decrease in volatility) comparing to the case of positive shock. However this conclusion also holds for price falls – if the magnitude of the negative shock is sufficiently big. The explanation may come from the differences in behavior of oil returns and other financial assets returns: apparently market for oil becomes overexcited (implying higher volatility of prices) in presence of positive shocks contrary to other financial returns. Interesting to notice that E-GARCH volatility output is less extreme than volatility of the benchmark model: Chart 29, Appendix 2.

4.2.6 Convenience yields inclusion

Another factor usually considered theoretically valid to add to commodities price models is convenience yield. It accounts for benefits from possibility of usage the resource balanced with carrying/storage costs which basically drives differences between spot and futures prices. It can easily be seen that convenience yield also allows for an additional channel of macroeconomic factors influence. Since oil industry has high capital intensity the price must be sensitive to level of inventories. Convenience yield contains inventories impact on the oil price because, obviously, it is a storable commodity. Inventories level in its turn can be seen as determined by macroeconomic conditions. It is an explicit representation of oil supply and decisions about changes in it are driven by the same motives as about state of supply.

Basically convenience yield approach uses futures oil prices to predict spot price. It is also known as ‘futures market hypothesis’, which, however, is both theoretically and empirically questioned. Basically, h-step ahead spot forecast is viewed as market price of h-periods ahead delivery (futures price), therefore it is believed that futures prices are unbiased and efficient predictors of respective future spots. Empirical evidence against this hypothesis is controversial, but there exists also theoretical argumentation. Crude oil is the means of production, and oil reserves and extraction technologies allow for cost minimization smoothing. Due to this fact even if arbitrage arises it does not mean that oil-processing companies will sell inventories and go long with oil futures, because they will occasionally enable price speculation. Therefor there always exist wedge between futures price and cost of carry [convenience yield] which is an ultimate source of bias between futures and future spot predictions relationships. So convenience yields are not just a theoretical assumption to be aware of, but they are of strong significance for prices, especially for crude oil market. [99]

Marginal convenience yield can be adequately approximated with mean-reversion process (Gibson, Schwartz (1990), but other research proves that this result is not significant or model is misspecified (Knetsch (2006).

[99] Knetsch (2006), pp. 1-2

Measurement of convenience yields is usually a solution of equation (4.2.19) for convenience yield δ_t observing futures prices $F(t,T)$ for different maturities T, spot price at time t, $S(t)$, and non-constant interest rate r_t [100]:

$$\delta_t = r_t - \frac{1}{T}\log\left(\frac{F(t,T)}{S(t)}\right) \tag{4.2.19}$$

Interest rate is assumed non-constant (actual LIBOR data are used). It makes sense to use real interest rate (even though it should not make a big difference for convenience yield estimation) due to the fact that sample period covers 30 years of data and some long term relationship may be present. As futures prices data crude oil futures for 3 maturities are used. Equation (4.2.19) reflects that futures price is a respectively discounted current price and the convenience yield is a part of a discount factor.

Taking the average of three convenience yields series for different maturities gives the true series of convenience yield (see Chart 30, Appendix 2). Descriptive statistics can be found in Table 23, Appendix 1.

Extending the GARCH(1,1)-GBM model mean equation with exogenous factors (4.2.17) and with current convenience yield δ_t can be expressed as:

$$\frac{dS_t}{S_t} = \mu_S dt + \theta\delta_t + \varphi X_t + \sqrt{dt}a_t \tag{4.2.20}$$

While the variance equation (4.2.18) remains the same: $\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \tau Y_t$

In other words this model is extended with time-varying drift term (due to non-constant convenience yield and macrofactors) and time-varying volatility version of the GBM model.

Estimation of the above GARCH models described by equations (4.2.20) and (4.2.18) with convenience yields in the mean equation and significant macroeconomic factors (exchange rate and bond yield spread) in mean or variance output is presented in Table 24, Appendix 1.

Again exchange rate in mean of the model with convenience yield has almost the same coefficient than in the model without. Yield spread in variance behaves identically to that in the benchmark model. Exchange rate in the volatility equation, however, has less importance when convenience yield is included.

Significance of convenience yield for oil price models (three models estimated in this section) is always present. The positive sign obtained means that change of log price increases with convenience yield which is consistent with economic argumentation witnessing in favor of theory of storage. Convenience yield is closely related to inventory capacity available. Naturally,

[100] From Gibson, Schwartz (1990), p. 963

oil price increases with inventories withdrawals since storage becomes more costly. "*Up to 42% of the variation in convenience yields can be explained by inventory*" [Dinceler et al (2005), p.3] and convenience yield is negatively related to inventory.

Convenience yield is positive when futures price is below spot[101] – thus it is beneficial to own the commodity now rather than sell it. Negative convenience yield means that storage costs are negligible and there are opportunities for arbitrage: selling forward since the futures price is above spot. So negative convenience yield can be associated with high storage capacity available which easily translates into positive convenience yield-spot oil price relationship. This result was also documented in Cassarus and Collin-Dufresne (2004).[102]

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So far in the above sections basic GARCH(1,1) with or without additional variables, such as macroeconomic factors and convenience yields, GARCH-in-mean and exponential GARCH models with macroeconomic factors were estimated and interpreted. Remarkably that among the macroeconomic factors only USD/GBP exchange rate and the Baa-Aaa corporate bonds yield spread were significant for modeling crude oil prices through the volatility channel or directly. Indexes S&P500 and Industrial Manufacturing showed a very weak significance, but the result is still worth considering. Performance of convenience yield is always significant and economically reasoned. Impact of yield spread is quite stable: it almost does not change when different extensions are modeled. The same concerns exchange rate influence on oil prices directly while its influence on the volatility changes a lot and becomes negligible when asymmetric response of the volatility to positive and negative shocks is allowed.

Models estimation and interpretation is logically to finish with comparisons of the models, choosing the most efficient ones and testing whether they match the real behavior of oil prices

4.2.7 Volatility model choice

Now the task is to choose the most efficient from the bunch of GARCH models estimated in Sections 4.2.3-4.2.6. For this purpose it is useful to compare various information criteria and tests for different models:

- *Log likelihood* for model parameters when maximized allows comparing models explicitly. Adding new variables, even not significant, always increases likelihood, but if they are too

[101] Can easily be seen from data
[102] Dincerler et al (2005), p. 2

many it can lead to a problem of overfitting. Therefor different information criteria are useful for consideration because they give penalty for number of the parameters.

- Akaike information criterion[103]: $AIC = \frac{-2}{T} l + \frac{2}{T} k$, where l is log likelihood, T is sample size, k – number of parameters. The smaller is AIC the bigger is increased likelihood gain versus 'additional' likelihood coming from more parameters.
- Schwartz-Bayesian information criterion: $BIC = \frac{-2}{T} l + \frac{\ln(T)}{T} k$. Interpretation is similar to AIC.
- Hannan-Quinn information criterion[104]: $HQ = \frac{-2}{T} l + \frac{(\ln(T))}{T} 2k$ should also be minimized.
- *Nyblom stability test* shows if the parameters' estimates are jointly stable for the sample used.
- *Pearson goodness-to-fit chi-squared test* examines if observed frequency distribution is equal to theoretical (null hypothesis).

In the Table 25, Appendix 1 a comparison of the models using above described criteria is performed. The first cohort includes the benchmark model GARCH (1,1) without any additional factors. The second cohort compares the models of different functional specifications (standard GARCH, exponential GARCH and GARCH-in-mean models) with the macroeconomic variables found significant (USD/GBP exchange rate and bond yield spread). Third cohort compares the models adding convenience yield to the mean equation.

Comparing the models from the second cohort among GARCH functional forms standard GARCH(1,1) and GARCH-in-mean work the best. Exponential GARCH is also valid, but according to all criteria it has weaker performance. The likelihood and information criteria for the eGARCH model are even worse than for the benchmark model without any additional factors.

The benchmark model performance is improved if external significant regressors are added. Yield spread in the volatility works the best for all model forms: models with it have higher likelihood and lower information criteria comparing to their counterparts of the same functional form. Moreover, such models are the only to have significant joint parameters stability (*Nyblom stability test*). Exchange rate has better results in the mean than in the volatility equations.

Including convenience yield to the mean equation significantly improves the models' performance. Remarkably that adding this factor insures the joint parameter stability for all the models compared. In the third cohort only sGARCH and GARCH-M models are compared

[103] Tsay (2010), p. 48

[104] Bierens (2006), p. 1

because of the eGARCH weakness revealed during the previous cohort of models comparison. GARCH-M models have higher log likelihood but also higher information criteria than that of sGARCH. And again models with yield spread in variance outperform the others.

Goodness-to-fit test is satisfactory for all models.

According to different criteria the following models should be chosen:

Maximum likelihood – sGARCH with convenience yield in mean and yield spread in variance has almost the same likelihood as GARCH-M model with the same regressors. However the GARCH-in-mean coefficient is not significant, so then there is not enough reasons to use this model;

AIC minimized, BIC minimized and *HQ minimized* – sGARCH with convenience yield in mean and yield spread in variance;

So the best specifications to choose are the standard GARCH with convenience yield in mean and yield spread or exchange rate in mean or variance (since the models are so different it is better to consider all alternatives).

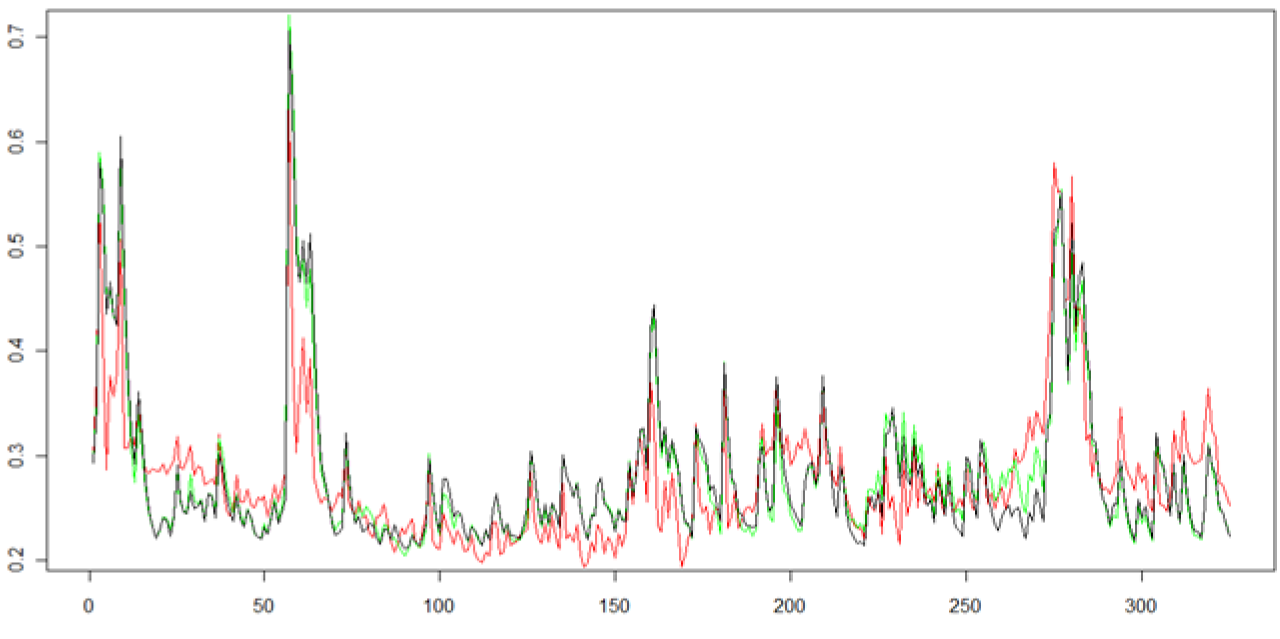

Chart 3. Volatiity fit, $\sigma_{S,t}$ of sGARCH(1,1) with convenience yield and exchange rate in mean (black), sGARCH with convenience yield in mean and yield spread in variance (red), sGARCH with convenience yield in mean and exchange rate in variance (green) models, $\sigma_{S,t}$

4.3 Models Testing and Forecasting

So far I have been describing different methods for analyzing crude oil price behavior. Among the known financial models Geometric Brownian Motion model and the Mean-Reversion (Schwartz one-factor) model were estimated and analyzed with all details. Classic simple stochastic model (GBM) was also extended with time-variant drift and volatility terms by adding convenience yields and exogenous macroeconomic variables built in the GARCH class framework. Among different GARCH models three standard GARCH(1,1) models with convenience yields, exchange rate and bond yield spread were found the best.

Estimation of models is not yet enough to claim that oil prices do follow some stochastic process or do depend on other factors. It is necessary to test how the models perform relatively to each other and real oil prices. Basically, a stochastic model forecast/fit can be characterized by two dimensions: uncertainty and accuracy. Both dimensions are considered below.

4.3.1 Uncertainty of the forecast

Contrary to deterministic models, where model fit is unique and can easily be compared to the actual data, stochastic models depend highly on a random component which always produces some uncertainty. Thus, a stochastic model can never fit perfectly to data and each simulated path is individual even if its properties are exactly the same as those of real prices. That is why it does not make sense to judge about a model performance by looking at its individual simulations but rather compare different models' expectations based on 'all' possible outcomes.

For this purpose I generated 10 thousand simulations by each of the five estimated models for the last year of sample period (2012 and 2 months of 2013). Then, expectations (means) for each model are compared with the true prices. In particular, 1-month and 12-months ahead forecasts are considered in order to evaluate the models performance.

Of course individual simulations as mentioned above are not relevant for price forecasting, but it is still true that each simulated value for the same time point is a potential future state of the world. The probability of these 'worlds' is negligibly small since at each step there are 10000 realizations for them. But it may be interesting to know what a chance of guessing the oil price would be. Here it is enough to find out how likely it is to get price of oil close to the true ±$5 per barrel for 1-month and 12-month forecasts. Thus I calculate probability of getting oil log price $\hat{S}_t^k$ for model k lying in the interval $\hat{S}_t^k \in (S_t - 0.05; S_t + 0.05)$, where S_t is actual log price assuming normal distribution properties of the simulated values for each time point (see calculation results in Table 26, Appendix 1). For example there is 45.82% chance that a random oil price value simulated by simple GBM model hits the true oil price within $10 interval one month ahead or 12.62% chance of the hit within $10 interval for 12 month forecast.

The first step to evaluate performance of different models is computing the expected values. But since the models so heavily rely on the random component that expectation (model mean) lies within an interval with some level of certainty. Mean-uncertainty can be easily reduced if one generates sufficiently large number of simulations. Simulated values are also distributed with some variance for each point of time with its confidence limits.

A usual way to find mean and variance confidence limits assuming normal distribution of the simulated values for each time horizon, $N(\mu; \sigma)$[105] is

$$z = \frac{\sqrt{n}(\bar{x}-\mu)}{s} \sim t(n-1) \tag{4.3.1}$$

$$c = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1) \tag{4.3.2}$$

where n is a sample size, $\bar{x}$ and s are observed mean and standard deviation of the true μ and σ of simulated values. z and c are the respective critical values of the (standard) normal and chi-squared distributions with *(n-1)* degrees of freedom. So for 95% certainty level mean and variance of simulated prices for each time point lie in the intervals:

$$Prob\left(-z \leq \frac{\sqrt{n}(\bar{x}-\mu)}{s} \leq z\right) = 0.95 \tag{4.3.3}$$

$$Prob\left(c_L \leq \frac{(n-1)s^2}{\sigma^2} \leq c_H\right) = 0.95 \tag{4.3.4}$$

From (4.3.3) and (4.3.4) knowing critical values of the distributions (z and c_i), sample size ($n=10\ 000$) and observing mean and standard deviation for forecasted values ($\bar{x}$ and s) one can estimate intervals, which true mean and variance (μ and σ^2) belong to. It is very easy to see that sample size when increasing reduces the width of the confidence limits which means that the expectations are more or less certain but it does not mean that the forecast itself is certain. Computed expectations of oil log prices by different models compared to the actual oil prices can be found in Charts 31-35, Appendix 2.

For the GBM model 1-month expectation for oil price is smaller than actual price by $1.43/b. 12-months forecast shows much less precision: the expectation overestimates future price by $14.08/b. Since the number of simulations is sufficiently high the uncertainty of these estimations is very small (narrow confidence limits for both mean and variance estimations).

[105] Greene (2002), p. 145

Actual oil log price 01.01.2012	GBM expectation and its 95% conf.interv.	Variance and its 95% conf.interv.	Error [USD/ barrel]	Actual oil log price 01.01.2013	GBM expectation and its 95% conf.interv.	Variance and its 95% conf.interv.	Error [USD/ barrel]
4.607567 $100.24/b	4.593144 (4.591562; 4.594727)	0.0066739 (0.006493; 0.0068628)	0.0144 [1.435]	4.480174 $88.25/b	4.628176 (4.622667; 4.633684)	0.0781574 (0.076035; 0.0803699)	-0.148 [-14.08]

Table 9. GBM model 1-month and 12-month ahead expectations for log price of oil

Apparently this model expectation captures the first part of the real oil price time trend correctly and has good short-term predictions but in longer run overestimates prices strongly

Performance of the Mean-reversion model is a little bit different: trend of expected values is closer to the time trend of real oil prices, but the 1-month forecast is less accurate. A difference between actual oil price and expected now is equivalent to $2.016/b. 12-months forecast compared to the GBM is closer to the actual price with the error of overestimation of $6.702/b.

Actual oil log price 01.01.2012	OU expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]	Actual oil log price 01.01.2013	OU expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]
4.607567	4.587252 (4.585653; 4.588851)	0.0065096 (0.006333; 0.0066938)	0.0203 [2.016]	4.480174	4.553376 (4.548044; 4.558709)	0.0743302 (0.072312; 0.0764344)	-0.0732 [-6.702]

Table 10. Mean-Reversion model 1-month and 12-month ahead expectations for log price of oil

The same forecast-testing procedure is applied to the three GARCH(1,1) models with convenience yield in mean equation, exchange rate in mean or variance and bond yield spread in variance. For these factors actual data is taken since forecasting of them is out of scope of this thesis. Volatility forecast is simulated within GARCH's with estimated parameters.

All GARCH models forecasts capture correct 1-year descending trend similarly to the Mean-Reversion model output. For GARCH(1,1) with both convenience yield and exchange rate in mean 1-month prediction error accounts for $2.41/b, but 12-month prediction error is remarkably small for this model – overestimation of just $4.566/b.

Actual oil log price 01.01.2012	G11 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]	Actual oil log price 01.01.2013	G11 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]
4.607567	4.583217 (4.581509; 4.584925)	0.0075592 (0.007354; 0.0077732)	0.0244 [2.411]	4.480174	4.530616 (4.525547 4.535685)	0.0719402 (0.069987; 0.0739767)	-0.0504 [-4.566]

Table 11. GARCH with convenience yield and exchange rate in mean model 1-month and 12-month ahead expectations for log price of oil

In GARCH(1,1) with convenience yield in mean and bond yield spread in variance for short-term a difference between actual and forecasted oil prices is $1.875/b, for long-term period estimated price exceeds actual by $7.92/b.

Actual oil log price 01.01.2012	G12 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]	Actual oil log price 01.01.2013	G12 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]
4.607567	4.588688 (4.58675; 4.590626)	0.0097775 (0.009512; 0.0100543)	0.0189 [1.875]	4.480174	4.566158 (4.560091; 4.572226)	0.0958414 (0.093239; 0.0985545)	-0.0859 [-7.924]

Table 12. GARCH with convenience yield in mean and bond yield spread in variance model 1-month and 12-month ahead expectations for log price of oil

For GARCH(1,1) with convenience yield in mean and exchange rate in variance 1-month and 12-months errors between actual and expected prices are respectively $1.649/b and -$9.318/b.

Actual oil log price 01.01.2012	G13 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]	Actual oil log price 01.01.2013	G13 expectation and its 95% conf.interv.	Variance and its 95% conf.iterv.	Error [USD/ barrel]
4.607567	4.590982 (4.589317; 4.592647)	0.0072925 (0.007095; 0.0074989)	0.0166 [1.649]	4.480174	4.580553 (4.575461; 4.585645)	0.0676301 (0.065794; 0.0695446)	-0.1004 [-9.318]

Table 13. GARCH with convenience yield in mean and exchange rate in variance model 1-month and 12-month ahead expectations for log price of oil

Among the three GARCH models with convenience yields the one with exchange rate in mean seems to be the most accurate on the long-time horizon and it predicts the closest to actual time trend. Model with bond yield spread in variance captures right trend as well, but it is less accurate and it has also wider 'possibilities' corridor. The model with exchange rate in variance has forecasting performance similar to the GBM model: close to true short-term prediction, but very bad on a longer horizon. It is interesting to notice that all GARCH models are based on the GBM theoretical specification, but they all show very different performances from the basic one. A logical conclusion here would be that adding exogenous factors (convenience yields, exchange rate and bond yield spread) improves the model forecasts for longer time horizons. Basic GARCH(1,1) without any external regressors shows the worst long-term forecast, but also the best short term prediction: the spread between actual and 1-month expected price is only \$1.1844/b compared to \$1.43/b for GBM with constant volatility) (See Chart 36, Appendix 2).

However, as mentioned above, these forecasts and their confidence limits evaluation did concern the Monte-Carlo uncertainty, which is easily reduced by increasing the number of simulations. Therefor to compare uncertainty of the models it is interesting to look at the range within which simulations can lie with 95% probability. For this purpose confidence intervals for conditional variance of models expectations are to be considered.

Defining variance of the final future oil price S_T conditional on other factors X_T as:

$$\sigma(T-t) = \sqrt{\sigma_t^2(S_T|X_T)} = \sqrt{E_t[(S_T - E_t[S_T|X_T])^2|X_T]} \quad (4.3.5)$$

Assuming Gaussian approximation allows to use properties of normal distribution $S_T|X_T \sim N(S_t + \mu(T-t), \sigma_{T-t}^2)$ where $\mu(T-t)$ denotes cumulative from time t to T drift term and σ_{T-t}^2 – cumulative variance. The confidence limits for mean, similarly to (4.3.3) but knowing relationship (4.3.5) for the conditional variance, are:

$$Prob\left(-1.96 < \frac{S_T - (S_t + \mu(T-t))}{\sigma_{T-t}} < 1.96\right) = 0.95 \quad (4.3.6)$$

Having estimates of mean and variance of the expectations for each time horizon from the simulations ($\mu(T-t)$ and σ_{T-t}) and initial value of log price (S_t, t basically is 01.12.2011) one can find confidence limits where the expectation (S_T) lies with 95% certainty for each T (in this case it is either 01.01.2012 or 01.01.2013).

The wider is the confidence limit the more uncertain are models possible outcome. The visual representation can be found in Charts 37-41, Appendix 2.

A model with the least uncertainty produced by a forecast on the short run is GBM: its 1-month forecast of oil price with 95% probability lies in the interval with width size of \$31.28/b. The with size for the GBM with non-constant volatility (benchmark GARCH(1,1) is bigger: \$33.76/b. The most uncertain appears to be GARCH(1,1) with bond yield spread in variance: the confidence limit for 1-month oil price prediction has width of \$38.46/b. For 12-month forecast the most stable now is GARCH(1,1) with exchange rate in variance, the most unstable – GARCH(1,1) with bond yield spread in variance.

For the long run confidence intervals are very wide and thus prediction of one-year ahead is very uncertain. Therefor reasonable judgment would rather prefer models succeeding for the nearest future – simple models in this case. However from efficiency perspective it is also important to have a broader outlook which is supported better by more complex or extended models.

	GBM	OU	GARCH11	GARCH12	GARCH13
one-month expectation (conf.limits)	4.591893 (4.43404; 4.749747)	4.589393 (4.430816; 4.74797)	4.585157 (4.416762; 4.753551)	4.588339 (4.394023; 4.782655)	4.590025 (4.423109; 4.756941)
12-months expectation (conf.limits)	4.617775 (4.064344; 5.171206)	4.556314 (4.017274; 5.095355)	4.530127 (4.009931; 5.050323	4.56109 (3.949969; 5.172211)	4.583504 (4.075073; 5.091934)
Size of the 1-month CL	0.315707	0.317154	0.336789	0.388632	0.333832
Size of the 12-month CL	1.106862	1.078081	1.040392	1.222242	1.016861
Mean size	0.7769737	0.7643636	0.7221072	0.8672292	0.710835

Table 14. The models expectations and their confidence limits. OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

4.3.2 Accuracy of the forecast

Models expectations computed and analyzed in the previous subsection can actually be considered as out-of-sample forecasts. Uncertainty of these forecasts was discussed as one dimension of the model testing. The second dimension is accuracy of the forecasts and their matching reality. Accuracy simply accounts for the magnitude of errors between forecasted and real prices of oil but comparing arithmetic errors between prices is not enough to judge about models predictive ability. The errors calculated, even sophisticated ones, may not be statistically significant. Thus different methods of comparing forecasts accuracies have to be used.

1) Errors of prediction

Calculating of forecast errors is the first step typically taken when assessing accuracy of forecasts. There are different types of errors which allow looking at forecasting methods from different points of view.

The simplest set of errors is an arithmetic difference between true S_t and forecasted $\hat{S}_t^k$ values of oil log prices. Thus the Mean Error for model k is an average of this set $ME^k = \frac{\sum_{t=1}^{T}(S_t - \hat{S}_t^k)}{T}$. This indicator is useful because it gives general overview of the forecast: if it overestimates or underestimates the price and average magnitude of the deviation. The Mean Error sometimes, however, is not informative. If errors are big in magnitude but proportionally with opposite signs – the Mean error is going to equal zero claiming a perfect forecast.

In such a case to investigate true average magnitude of deviations (Root of) Mean Squared Error is calculated $RMSE^k = \sqrt{\frac{\sum_{t=1}^{T}(S_t - \hat{S}_t^k)^2}{T}}$ or alternatively Mean Absolute Error $MAE^k = \frac{\sum_{t=1}^{T}|S_t - \hat{S}_t^k|}{T}$, which takes errors in modulus.

For the last year of sample all models on average underestimate oil log prices. GBM model has the highest errors in magnitude, GARCH with exchange rate in mean – the lowest. It is important to mention that all models errors are distributed normally (Shapiro-Wilk test with Null for normality), demonstrate no serial correlation on 12 lags (Box-Pierce test with Null about no serial correlation) but are highly correlated among themselves (Table 27, Appendix 1).

Model	Mean error	Root of MSE	Mean absolute error
GBM	-0.06366374	0.09963200	0.08316235
OU	-0.02356132	0.06771348	0.05123624
GARCH1	-0.01030765	0.06240371	0.04799831
GARCH2	-0.03275658	0.07424687	0.05862588
GARCH3	-0.03896656	0.07855228	0.06341341

Table 15. Errors of prediction for models' expectations, *log price*

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

2) Morgan-Newbold-Granger test[106]

Instead of just computing the errors as in previous subsection this test answers the question about their statistical significance relatively to each other and aims to find out if different forecasting methods have the same predictive ability.

Let errors of prediction at time t for the model k to be defined as: $e_t^k = S_t - \hat{S}_t^k$, where S_t is actual oil price (log price in this case) at time t and $\hat{S}_t^k$ is a respective model's k forecast. Typically tests for forecasting accuracy are based on a loss functions $g(e_t^k)$ with the error

[106] Mariano (2000), pp. 2-3

differential defined as $d(t) = g(e_t^i) - g(e_t^j)$ for two forecasting methods i and j. The models have the same predictive accuracy if the expectation of the differential $E[d(t)] = 0$.

For $x_t = e_t^i + e_t^j$ and $z_t = e_t^i + e_t^j$ the forecasting methods i and j have the same accuracy if $cov(x_t, z_t) = E\left[(e_t^i)^2 - (e_t^j)^2\right] = 0$ for all t.

The test is resulting into a rejection of the null hypothesis on 95% significance level for all pairs of models which means that the models have different forecasting accuracy and predictive power (see Table 28, Appendix 1). So the next step would be determining of the model with the best predictive accuracy among the rest.

3) Diebold-Mariano test[107]

Diebold-Mariano test compares the quality of two forecasting methods based on errors investigation.

The null-hypothesis states that the two methods have the same predictive accuracy – error differential $d(t) = 0$. So that, if the differential is statistically different from 0 (p-value of the test is smaller than 0.1) the null is rejected against the alternative: second method (horizontal axis in the table) has greater/less accuracy than the first (vertical axis) method.

Model	OU	GARCH1	GARCH2	GARCH3
GBM	0.0002753 (greater)	0.0008399 (greater)	0.0002367 (greater)	0.0001502 (greater)
OU	n/a	0.04292 (greater)	0.001365 (less)	0.001871 (less)
GARCH1	n/a	n/a	0.01052 (less)	0.008192 (less)
GARCH2	n/a	n/a	n/a	0.004474 (less)

Table 16. Diebold-Mariano test results, last year of sample, p-value reported, alternative hypothesis in brackets.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

According to Diebold-Mariano test GBM model has the worst forecasting accuracy among all models: the null is rejected against the hypothesis about greater predictive accuracy of

[107] Mariano (2000), pp. 6-7

alternative to GBM methods. GARCH(1,1) with exchange rate in mean has greater accuracy than other models, but other GARCH models have less accuracy then the Mean-Reversion model.

4) Reality match

One period forecasting comparison provided in this section may however be not informative enough if the results are not proven statistically (for other sample periods). For example, it is interested to look how the models would forecast the recent financial crisis. Standing on October 1, 2007 data point and generating a 2-year forecast[108] reveals a different picture than before.

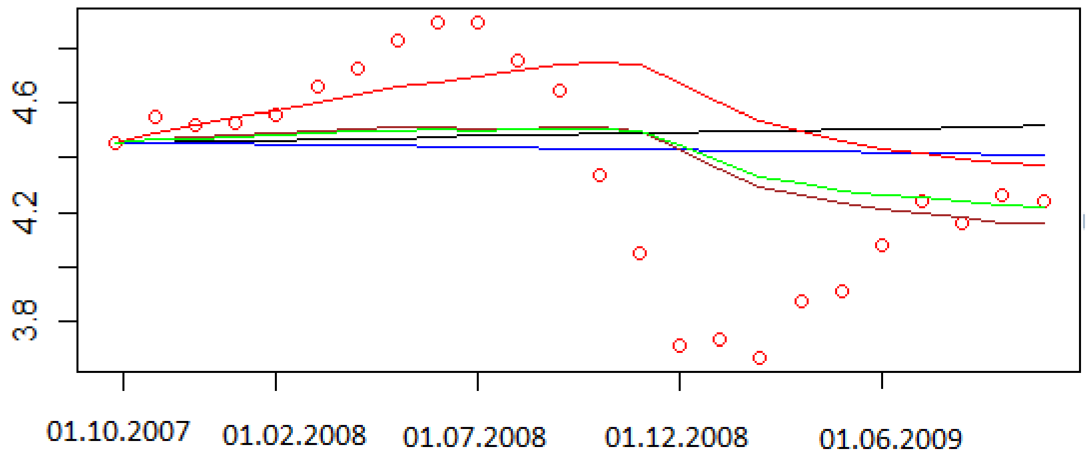

Chart 4. Financial crisis forecast, *log price*

GBM – *black*, Mean-Reversion – *blue*, GARCH with convenience yield and exchange rate in mean – *red*, GARCH with convenience yield in mean and bond yield spread in variance – *brown*, GARCH with convenience yield in mean and exchange rate in variance – *green*, actual oil log price – *red dots*.

Here it is easily seen that high oil price volatility captured by GARCH models was an essential part of the paths simulated. Notably, GBM model expectation is *always* upward sloping straight line. Even though on average oil prices do follow a growing time trend but it is not true sometimes. Mean-Reversion model is able to change the trend eventually and due to volatility capture – GARCH models as well.

The forecast of the financial crisis is the most accurate for the GARCH model with yield spread in the variance according to Diebold-Mariano test. GBM model has smaller forecasting accuracy than other models. The two GARCH models with additional factors in the variance work the best here while the one with exchange rate in the mean (which was the best for the last-sample-year forecast) has the same accuracy than the Mean-Reversion model.

[108] By term "forecast" expectation of 10000 log price paths generated by a model is meant.

Model	OU	GARCH1	GARCH2	GARCH3
GBM	0.002333 (greater)	0.5749 (greater)	0.000008 (greater)	0.000006 (greater)
OU	n/a	0.1415 (less)	0.0001256 (greater)	0.0001289 (greater)
GARCH1	n/a	n/a	0.007989 (greater)	0.01138 (greater)
GARCH2	n/a	n/a	n/a	0.001702 (less)

Table 17. Diebold-Mariano test results, financial crisis forecast, p-value reported, alternative hypothesis in brackets.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

From this analysis we can make the conclusion that simple models work not bad but at times when high volatility maters they lose at play comparing of the models which include non-constant variance.

The best way to evaluate model's performance and thus validity of forecasts is to check how it works historically. For this purpose, at each time step short-term (1-month) and long term (12-month) horizon forecasts are generated. Then, it becomes possible to compare forecast with actual data[109]. Since this exercise compares more than 300 forecasts with actual data (instead of 14-24 observations as above) it gives statistically reliable results

There is no need to say that short term predictive accuracy is very high for all models. Average error between 1-month prediction and the true (1 month ahead) oil price value is very small – maximum $0.3/b in terms of 2013 oil price. The best 1-month forecasting accuracy according to the ME criterion has a GARCH with yield spread model.

	GBM	OU	GARCH1	GARCH2	GARCH3
ME (log prices)	0.002988184	0.002891088	0.000247178	0.0000162594	0.00086832

Table 18. Mean error of forecasts, 1-month ahead, whole sample.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

[109] For 325 observations of oil price it is possible to perform 324 1-month forecasts and 313 12-month forecasts.

But according to the Diebold-Mariano test the models have more or less the same predictive accuracy. GARCH with exchange rate in variance outperforms the simple models but does not forecast better than the rest of the GARCH models. Other GARCH models at the same time do not outperform the simple ones. This result does not really allow to judge about comparative forecasting accuracy.

Model	OU	GARCH1	GARCH2	GARCH3
GBM	0.4974 (greater)	0.2077 (greater)	0.1115 (greater)	0.06988 (greater)
OU	n/a	0.2061 (greater)	0.1016 (greater)	0.06104 (greater)
GARCH1	n/a	n/a	0.2315 (greater)	0.2465 (greater)
GARCH2	n/a	n/a	n/a	0.5193 (greater)

Table 19. Diebold-Mariano test results, 1-month ahead forecasts, whole sample; p-value reported, alternative hypothesis in brackets.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

Naturally, for 12-months predictions models' forecasting power is much lower. The maximum forecasting error is now \$3.5/b for \$95/b expensive oil (2013). Comparison of different errors shows that the GBM model has the biggest errors while GARCH with exchange rate in mean – the smallest.

	Mean Error	Root of MSE	Mean absolute error
GBM	0.03709271	0.30490581	0.2367719
OU	0.03686084	0.29993272	0.23525681
GARCH1	0.00192399	0.29827098	0.23263244
GARCH2	0.00231205	0.29930559	0.24125723
GARCH3	0.01231185	0.29602392	0.23544313

Table 20. Errors of forecasts, 12-month ahead, whole sample.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

Diebold-Mariano test states that all models have the same predictive accuracy. The only statistically supported conclusion is that GARCH with exchange rate in variance outperforms GARCH with yield spread in variance.

Model	OU	GARCH1	GARCH2	GARCH3
GBM	0.2383 (greater)	0.2456 (greater)	0.2643 (greater)	0.1161 (greater)
OU	n/a	0.4306 (greater)	0.4674 (greater)	0.2665 (greater)
GARCH1	n/a	n/a	0.5695 (greater)	0.3503 (greater)
GARCH2	n/a	n/a	n/a	0.03341 (greater)

Table 21. Diebold-Mariano test results, 12-month ahead forecasts, whole sample; p-value reported, alternative hypothesis in brackets.

OU stands for Mean-Reversion model, GARCH1 for GARCH with convenience yield and exchange rate in mean, GARCH2 for GARCH with convenience yield in mean and bond yield spread in variance, GARCH3 for GARCH with convenience yield in mean and exchange rate in variance.

***

Forecasting ability of the models is quite hard to test and apparently they all are very different having pluses and minuses. Simple models are beneficial in a sense that they do not require other factors to be considered and are very straightforward. Complex (volatility) models in general can be said to have better performance, but for example for forecasting purposes it would be necessary to assume behavior of additional factors such as convenience yield and macroeconomic variables.

According to the reality match exercise GARCH models work better than simple models on short time horizon. For long term forecasting all models have the same predictive accuracy, but mean errors indexes are still better for the volatility models. Moreover volatility models capture significant fluctuations of prices, even if not perfectly, while expectation of the simple models will always be a straight line.

So on average all models have good predictive power and their performance is rather similar. Even though benefits from complex models are higher on average but it not always holds – prediction of simple models may already be enough.

Discussion and Concluding Remarks

There is no need to say that oil price behavior still remains burning research topic despite decades of scientific attention to it. Numerous sophisticated theoretical methods and empirical checks conclusions have quite mixed and controversial results thus leaving the factors driving oil price undetermined.

This research aims to find out which of available models work the best to forecast oil prices and also to test oil price dependence on macroeconomic conditions. The latter proposition comes from the precautionary demand assumption which claims that demand for oil highly depends on expectations about future need for oil driven by expected soundness of macroeconomic conditions, strategic significance for oil and common knowledge about the state of the industry with its political and economic specifics.

The first questionable issue is which modeling approach to choose, second – how measure correctly the expectations channel influencing oil price and integrate it into the models.

For this research stochastic modeling approach is chosen. In general it is easy to justify usage of stochastic models because of high frequency data availability and financialization of oil. No doubts that oil price change demonstrates partly random behavior. Moreover, this research claims that oil prices are *mainly* driven by a random component and estimation of constant drift simply cannot be assessed. It is quite likely that drift term is different for smaller periods of the sample.

But the main attention should be paid to the random component and volatility of oil prices. Since average volatility estimated has almost the same value no matter which model or method is used it is quite obvious that variance of oil prices is quite persistent. Serial dependence of the random component, however, was *not* eliminated by a conditional heteroscedasticity assumption. This would be a good starting point to continue the research: apparently GARCH modeling may not be the best way to capture conditional variance of oil prices. The biggest disadvantage of GARCH is probably correct simulation of volatility (squared), but it is impossible to assess the sign of deviation and thus whether the price goes up or down. It is also worth saying that the GARCH model is quite improved with inclusion of macroeconomic factors and especially – convenience yields. However, convenience yields theory was not considered much in this thesis so developing this concept or direct modeling of convenience yields may clarify more oil price behavior.

In general macroeconomic variables input into the stochastic models yielded consistent with the precautionary demand hypothesis results. But they though did not make a big difference for oil prices and not all of them were significant as expected. It is not really surprising due to extreme complexity of the task to incorporate the factors consistent with the precautionary demand assumption into a stochastic model. First consideration is that probably current measure of macroeconomic conditions is not able to reflect correctly state of current expectations. Second, remembering about reverse causality effect oil as a strategic resource has on global economy gives an additional argument to use lagged indicators or make more consistent assumptions about expectation formation.

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R packages used:

TSA; tSeries; car; rugarch; forecast; backtest; vars

http://www.r-project.org/

APPENDIX 1
Tables

Table 1. Total World energy consumption by sources, *million tons of oil equivalent*, 2010

World

2010

(Mtoe)

SUPPLY AND CONSUMPTION	Coal/ peat	Crude oil	Oil products	Natural gas	Nuclear	Hydro	Biofuels and waste	Other[a]	Total
…									
Industry	677.86	12.51	310.02	463.87	-	-	195.83	762.85	2 422.94
Transport[c]	3.36	0.04	2 195.89	89.06	-	-	57.56	23.91	2 369.81
Other	135.96	6.75	435.64	612.83	-	-	848.62	1 046.73	3 086.53
Non-energy use	35.97	15.05	593.93	152.40	-	-	-	-	797.35

Source: Key World Energy Statistics, The International Energy Agency, 2012.

Table 2. Countries with biggest proven oil reserves, *billion barrels*, 2011

Country	Reserves (bb)	Country	Reserves (bb)
Saudi Arabia	262.6	Russia	60
Venezuela	211.17	Libya	46.42
Canada	175.214	Nigeria	37.2
Iran	137.01	Quatar	25.38
Iraq	115	USA	23.267
Kuwait	104	China	20.35
United Arab Emirates	97.8	Brazil	12.857

Source: The International Energy Agency, 2013.

Table 3. World biggest oil fields ultimate recoverable reserves, *giga barrels*, 2005

Field Name	Country	Discovery year	Range of URR [GB]
Ghawar	Saudi Arabia	1948	66-100
Burgan Greater	Kuwait	1938	32-60
Safaniya	Saudi Arabia	1951	21-36
Bolivar Coastal	Venezuela	1917	14-36
Berri	Saudi Arabia	1964	10-25
Rumalia N&S	Iraq	1953	22
Zakum	Abu Dhabi	1964	17-21
Cantarell Complex	Mexico	1976	11-20
Manifa	Saudi Arabia	1957	17
Kirkuk	Iraq	1927	16
Gashsaran	Iran	1928	12-15
Abqaiq	Saudi Arabia	1941	10-15
Ahwaz	Iran	1958	13-15
Marun	Iran	1963	12-14
Samotlor	Russia	1961	6-14
Agha Jari	Iran	1937	6-14
Zuluf	Saudi Arabia	1965	12-14
Prudhoe Bay	Alaska	1969	13

Source: Robelius (2005)

Table 4. Top countries by oil production, *thousand barrels per day*, 2012

Country	Production (tb/d)	Country	Production (tb/d)
Saudi Arabia	11545.68	Iraq	2986.641
United States	11126.76	Mexico	2936.009
Russia	10396.97	Kuwait	2796.788
China	4416.177	Brazil	2651.939
Canada	3867.956	Nigeria	2524.143
Iran	3538.386	Venezuela	2489.242
United Arab Emirates	3213.194	Norway	1902.084

Source: The International Energy Agency, 2013

Table 5. Top oil net exporters and importers, 2010, *million tons*

Net exporters	Mt
Saudi Arabia	333
Russian Federation	246
Nigeria	129
Islamic Rep. of Iran	126
United Arab Emirates	105
Iraq	94
Venezuela	87
Angola	84
Norway	78
Mexico	71
Others	609
Total	**1 962**

Net importers	Mt
United States	513
People's Rep. of China	235
Japan	181
India	164
Korea	119
Germany	93
Italy	84
France	64
Netherlands	60
Singapore	57
Others	483
Total	**2 053**

Source: Key World Energy Statistics, The International Energy Agency, 2012.

Table 6. World biggest oil companies production, *percentage of total*

Rank	Company	Nationality	State-owned	Production 2010	Reserves End 2011
				Per cent of total	
1	Saudi Aramco	Saudi Arabia	Yes	12.1	17.4
2	NIOC	Iran	Yes	5.2	9.9
3	PdV[(a)]	Venezuela	Yes	3.6	13.9
4	Pemex	Mexico	Yes	3.5	0.7
5	CNPC	China	Yes	3.4	1.7
6	KPC	Kuwait	Yes	3.1	6.7
7	Exxon Mobil	United States	No	2.9	0.8
8	INOC	Iraq	Yes	2.9	9.4
9	BP	United Kingdom	No	2.9	0.7
10	Rosneft	Russia	75%	2.8	1.2

(a) Excludes Venezuela's oil sands; if they are included, PdV's reserves exceed those of Saudi Aramco

Source: Dunn, Holloway (2012)

Table 7. Top countries by crude oil and petroleum products pipelines length, *kilometers*, 2013

Country	Oil and petroleum products pipes length, km	Country	Oil and petroleum products pipes length, km
United States	244620*	Kazakhstan	12408
Russia	94518	United Kingdom	10350
Canada	75000*	Venezuela	10347
China	38401	Colombia	10225
India	20032	Argentina	9879
Mexico	16709	Brazil	9553
Iran	16562	Ukraine	8877

*2010 data

Source: Central Intelligence Agency, 2013

Table 8. World longest transition pipelines length, *kilometers*, 2010

Name	Location	Length (km)	Name	Location	Length (km)
Eastern Siberia Pacific Ocean (ESPO)	Russia	4700	Caspian Pipeline Consortium (CPC)	Kazakhstan-Russia	1510
Druzhba	Russia-Germany	4000	Trans-Alaska Pipeline System (TAPS)	Alaska-US	1287
Kazakhstan to China Oil Pipeline (KCP)	Kazakhsan-China	2228	Trans-Arabian Pipeline	Saudi Arabia-Syria	1214
Baku-Tbilisi-Ceyhan	Azarbaijan-Turkey	1768	Trans-Mountain Pipeline System	Canada-US	1150
Greater Nile Oil Pipeline	Sudan	1600	Capline	US	1024

Source: Enerbridge

Table 9. Top countries by petroleum consumption, *thousand barrels per day*, 2012

Country	Petroleum consumption (tb/d)	Country	Petroleum consumption (tb/d)
United States	18554.57	Germany	2388.139
China	10276.83	Canada	2292.841
Japan	4728.538	Korea, South	2268.322
India	3621.751	Mexico	2191.353
Russia	3195.474	France	1738.41
Saudi Arabia	2861.347	Iran	1709.407
Brazil	2806.936	United Kingdom	1518.825

Source: The International Energy Agency, 2013

Table 10. Oil rents as *percentage of GDP*, top countries, 2011

Country	Oil rents as % of GDP	Country	Oil rents as % of GDP
Iraq	77.69627	Nigeria	32.91653
Congo, Rep.	71.53505	Venezuela	29.97723
Saudi Arabia	55.5285	Brunei Darussalam	28.45933
Kuwait	49.86871	Kazakhstan	27.48843
Gabon	47.91358	Ecuador	25.59743
Angola	46.3445	United Arab Emirates	21.93283
Azerbaijan	41.89273	Turkmenistan	21.32879
Equatorial Guinea	41.05322	Algeria	18.97793
Oman	40.21142	Yemen	18.70817
Chad	36.75434	Russian Federation	15.42291
World	**3.07**		

Source: World Bank Data Bank, 2013

Table 11. World supply and demand balance, *million barrels per day*, 2009-2012

	2009	2010	2011	2012
World demand *(mb/d)*				
OECD	46.3	46.9	46.5	46.1
OECD Americas	23.7	24.1	24.1	23.8
OECD Europe	14.7	14.7	14.3	13.8
OECD Asia Pacific	8.0	8.1	8.1	8.5
DCs	25.6	26.5	27.2	28.0
FSU	4.0	4.2	4.3	4.4
Other Europe	0.7	0.6	0.6	0.6
China	8.3	9.0	9.4	9.7
(a) Total world demand	**84.8**	**87.2**	**88.1**	**88.9**
Non-OPEC supply *(mb/d)*				
OECD	19.8	20.0	20.2	21.0
OECD Americas	14.4	15.0	15.6	16.7
OECD Europe	4.7	4.4	4.1	3.8
OECD Asia Pacific	0.6	0.7	0.6	0.5
DCs	12.4	12.7	12.6	12.1
FSU	13.0	13.2	13.2	13.3
Other Europe	0.1	0.1	0.1	0.1
China	3.8	4.1	4.1	4.2
Processing gains	2.0	2.1	2.1	2.2
Total non-OPEC supply	**51.1**	**52.3**	**52.4**	**53.0**
OPEC NGLs + NCOs	4.3	5.0	5.4	5.7
(b) Total non-OPEC supply and OPEC NGLs + NCOs *(mb/d)*	**55.5**	**57.3**	**57.8**	**58.6**
OPEC crude oil production[1]	**28.8**	**29.2**	**29.8**	**31.1**
Total supply *(mb/d)*	**84.2**	**86.6**	**87.6**	**89.8**

Source: OPEC Annual Report, 2012

Table 12. Commodities physical and financial markets volume, *billion USD*, 2011

	Physical market[a]		Financial market (exchange-traded)	
	Annual production	Annual exports	Annual turnover	Open interest[b]
Oil	3 250	2 211	40 194	288
Natural gas	1 578	530	3 160	38
Coal	1 203	187	40	3
Iron ore	318	164	8[d]	1[d]
Rice[c]	285	22	58	1
Corn[c]	245	27	2 865	48
Wheat[c]	200	43	1 257	27
Copper	173	51[e]	13 726	93
Gold	139	156[e]	9 362	85
Soybeans[c]	119	45	6 540	70
Sugar[c]	93	32	3 614	28

(a) RBA estimates based on volumes and indicative world prices
(b) Open interest is the total dollar value of futures and options contracts outstanding that are held by market participants at the end of each month; averaged over the year
(c) Physical market data are for 2011/12 US financial year
(d) Includes exchange-traded swaps
(e) Export data are for 2010

Source: Dunn, Holloway (2012)

Table 13. Data descriptive statistics

Series	Unit	Time period	N	Min	Max	Mean	SD	Source
Crude oil price	USD per barrel	01.01.1980-01.02.2013	398	11.28	133.9	37.92	26.133	St.Louis FredDatabase with indicated source as Dow Jones& Company
Interest rate	Per-cent	01.01.1986-01.02.2013	326	0.185	10.06	4.321	2.6297	St.Louis FredDatabase with indicated source as British Bankers' Association
S&P500 stock price index	Points	01.01.1986-01.02.2013	326	103.0	1540	736.5	473.64	St.Louis FredDatabase with indicated source as Standard and Poor's
Exchange rate	USD per GBP	01.01.1986-01.02.2013	326	1.093	2.416	1.664	0.2178	St.Louis FredDatabase with indicated source as Board of Governors of the Federal Reserve System
Manufac-turing production index	Index, 2007 = 100	01.01.1986-01.02.2013	326	40.57	101.1	71.24	19.196	St.Louis FredDatabase with indicated source as Board of Governors of the Federal Reserve System

Gold price	USD per troy ounce	01.01.1986-01.02.2013	326	254.8	1814	530.6	354.97	The World Gold Council
Moody's Baa-Aaa corporate bond yield spread	Percentage points	01.01.1986-01.02.2013	326	0.550	3.380	1.125	0.4896	Calculated as difference between Moody's Baa and Aaa corporate bond yields (from St.LouisFred Data)
Oil futures prices* (3 maturities)	USD per barrel	01.01.1986-01.02.2013	326					Energy Information Administration

*See Table 23.

Fourth column includes number of observations for each time series (N); Fifth, sixth and seventh columns contain minimum, maximum and average values; Eight – Standard Deviation (SD).

Table 14. OLS estimation output for the Mean-Reversion model, log oil price

Coefficient	Estimate	Standard error	t-statistics	p-value
Intercept (*c*)	0.020198	0.023451	0.829	0.407
Log(oil(*t-1*)) (*Q*)	0.99492	0.006973	142.690	<2e-16 ***
Residual standard error	0.08145			
Multiple R-squared	0.981			
F-statistic	2.036e+04			
p-value	<2.2e-16			

Table 15. OLS simulations check for the Mean-Reversion model parameters validity

Simulation	Kappa	Alpha	Sigma
simulation #1	0.2898	4.215321	0.274859
simulation #2	0.123156	3.141577	0.2819483
simulation #3	0.31932	3.266817	0.2749998
simulation #4	0.27768	3.646067	0.2828555
simulation #5	0.16644	3.807498	0.2755484
simulation #6	0.05184	4.182407	0.2635935
simulation #7	0.07164	3.497487	0.2988802
simulation #8	0.09252	0.7713359	0.2714635
simulation #9	0.084012	6.251535	0.2596286
simulation #10	0.179652	2.525149	0.2772487
Mean/SD*	0.1656/0.09853	3.5305/1.38378	0.2761/0.0108

*Mean and Standard Deviation for the parameters recovered from simulated price paths

Table 16. MLE simulations check for the Mean-Reversion model parameters validity

Parameter	Actual oil price	Simulations						Mean/SD*
Kappa	0.062	0.3024	0.1687	0.2653	0.4358	0.4967	0.2169	0.3143/0.127
Alpha	3.96354	3.8006	3.7433	3.1306	3.1348	3.9569	2.8195	3.431/0.461
Sigma	0.28215	0.2841	0.2816	0.2664	0.2726	0.2864	0.2741	0.2754/0.008

*Mean and Standard Deviation for the parameters recovered from simulated price paths

Table 17. OLS estimation output for the GBM model, oil price

Coefficient	Estimate	Standard error	t-statistics	p-value
Intercept (c)	0.006003	0.004075	1.473	0.142

Table 18. OLS simulations check for the GBM model parameters validity

Simulation	Mu	Sigma
simulation #1	0.0143081	0.2869142
simulation #2	0.04333175	0.2662468
simulation #3	0.06747817	0.2837945
simulation #4	0.06636612	0.2839439
simulation #5	-0.00203889	0.2807939
simulation #6	0.09684771	0.2947284
simulation #7	0.1280485	0.2959468
simulation #8	0.04570845	0.2913658
simulation #9	0.1042785	0.276277
Mean/SD*	0.0627/0.04242	0.2844/0.00938

*Mean and Standard Deviation for the parameters recovered from simulated price paths

Table 19. MLE simulations check for the GBM model parameters validity

Parameter	Actual oil price	Simulations						Mean/SD*
Mu	0.072	0.0514	0.0485	0	0.0354	0.0624	0.0443	0.04035/0.022
Sigma	0.2816	0.2723	0.2794	0.2949	0.2837	0.2808	0.2709	0.28035/0.009

*Mean and Standard Deviation for the parameters recovered from simulated price paths

Table 20. ARCH(6) model for log oil prices estimation

Parameter	Estimate	Standard error	t-value	p-value
mu ($\mu_S dt$)	0.009733	0.004408	2.20783	0.027256 *
omega (ω)	0.002436	0.000652	3.73490	0.000188 ***
alpha1	0.278561	0.125877	2.21296	0.026900 *
alpha2	0	0.211931	0	1
alpha3	0.025845	0.055522	0.46550	0.641574
alpha4	0.017086	0.049502	0.34516	0.729972
alpha5	0.373667	0.147132	2.53968	0.0110955 *
alpha6	0	0.102527	0	1

Significance codes: *** – significant on 0.1% level; ** – on 1% level; * – on 5% level; . – on 10% level

Table 21. GARCH(1,1) models with exogenous explanatory variables in mean or variance equations estimation, p-value in brackets

Parameters Additional factor (X_t or Y_t)	Mu $\mu_S dt$	Omega ω	Alpha α_1	Beta β_1	Phi or Tau φ, τ	ARCH-LM test for standardized residuals (p-value)
Exchange rate in *variance*	0.009533 (0.027075) *	0 (0.998713)	0.240782 (0.001180) **	0.536061 (0.000002) ***	0.000977 (0.005009) **	0.8991
Yield spread in *variance*	0.009989 (0.022322) *	0 (0.996875)	0.213497 (0.021200) *	0.000003 (0.999986)	0.005585 (0.000233) ***	0.8750
Industrial index in *variance*	0.009726 (0.024234) *	0.000877 (0.418189)	0.259104 (0.001189) **	0.524025 (0.000005) ***	0.000010 (0.473173)	0.9258
S&P500 in *variance*	0.009792 (0.023041) *	0.001316 (0.045515) *	0.258729 (0.001135) **	0.525094 (0.000004) ***	0 (0.489356)	0.9226
Interest rate in *variance*	0.010141 (0.019968) *	0.001354 (0.020913) *	0.233950 (0.001723) **	0.535442 (0.000005) ***	0.000071 (0.360967)	0.8808
Gold price in *variance*	0.009899 (0.023040) *	0.001629 (0.015664) *	0.248615 (0.001220) **	0.529211 (0.000005) ***	0.000000 (0.999998)	0.9259

NULL	0.009899 (0.022411) *	0.001629 (0.005657) **	0.248615 (0.001067) **	0.529210 (0.000005) ***	N/A	0.9259
Exchange rate in *mean*	-0.074651 (0.11083)	0.001521 (0.003796) **	0.254314 (0.001081) **	0.538853 (0) ***	0.051281 (0.069442) .	0.8030
Yield spread in *mean*	-0.004198 (0.786386)	0.001783 (0.011651) *	0.265787 (0.001193) **	0.49149 (0.000424) ***	0.015463 (0.343218)	0.8543
Industrial index in *mean*	-0.019638 (0.348755)	0.001704 (0.00574) **	0.260057 (0.001115) **	0.508845 (0.000033) ***	0.000385 (0.149956)	0.7729
S&P500 in *mean*	-0.002811 (0.774805)	0.001715 (0.005604) **	0.256485 (0.001142) **	0.509747 (0.000032) ***	0.000015 (0.149822)	0.7765
Interest rate in *mean*	0.016853 (0.046272) *	0.001745 (0.007687) **	0.264103 (0.001436) **	0.499772 (0.000123) ***	-0.001652 (0.338588)	0.7655
Gold price in *mean*	0.006089 (0.416726)	0.001635 (0.005376) **	0.25495 (0.00111) **	0.523302 (0.000006) ***	0.000007 (0.53115)	0.8086

NULL – Benchmark GARCH(1,1) model, without additional factors

Significance codes: *** – significant on 0.1% level; ** – on 1% level; * – on 5% level; . – on 10% level

Table 22. Alternative GARCH models with exogenous factors estimation, p-value in brackets

Model	Mu	Omega	Alpha	Beta	Other coefficient	External factor	ARCH-LM test p-value
E-GARCH with ex_rate in *variance*	0.008116 (0.065029) .	-0.942691 (0.009109) **	-0.08543 (0.07982) .	0.85417 (0) ***	*gamma*: 0.278661 (0.000148) ***	*ex_rate* 0.124808 (0.37631)	0.7978
E-GARCH with yield spread in *variance*	0.009189 (0.034363) *	-6.141364 (0.002593) **	0.021614 (0.8216)	0 (1)	*gamma*: 0.232716 (0.02911) *	*yield* 1.111999 (0.00822) **	0.8318
E-GARCH with ex_rate in *mean*	-0.086968 (0.053812) .	-0.562655 (0.005503) **	-0.098038 (0.03885) *	0.888084 (0) ***	*gamma*: 0.287142 (0.000066) ***	*ex_rate* 0.057028 (0.035443) *	0.4359

GARCH-M with ex_rate in *variance*	-0.013014 (0.556615)	0 (0.997523)	0.226717 (0.00123) **	0.561235 (0) ***	*c*: 0.290755 (0.302158)	*ex_rate* 0.000911 (0.004873) **	0.8067
GARCH-M with yield spread in *variance*	0.010424 (0.362211)	0 (0.996912)	0.213712 (0.02097) *	0.000005 (0.99997)	*c*: -0.005621 (0.96821)	*Yield* 0.005582 (0.000086) ***	0.8796
GARCH-M with ex_rate in *mean*	-0.088966 (0.080671) .	0.001445 (0.004677) **	0.243737 (0.00122) **	0.557363 (0) ***	*c*: 0.186863 (0.489739)	*ex_rate* 0.051230 (0.071657) .	0.8406

Significance codes: *** – significant on 0.1% level; ** – on 1% level; * – on 5% level; **.** – on 10% level

E-GARCH denotes Exponential GARCH, GARCH-M – GARCH-in-mean models; *ex_rate* – exchange rate; *yield* – bond yield spread.

Table 23. Descriptive statistics for oil futures contracts and estimated convenience yield series

Series	Measurement unit	Period	N	Min	Max	Mean	SD
Forward, 30 days maturity	USD per barrel	01.01.1986-01.02.2013	326	11.30	134.50	39.19	28.975
Forward, 60 days maturity	USD per barrel	01.01.1986-01.02.2013	326	11.35	134.80	39.23	29.207
Forward, 90 days maturity	USD per barrel	01.01.1986-01.02.2013	326	11.49	134.90	39.24	29.398
Convenience yield	Percentage particles	01.01.1986-01.02.2013	326	-0.993	1.0712	0.066	0.2479

N – number of observations; Min – minimum value; Max – maximum value; Mean – average value; SD – standard deviation

Table 24. Estimation of the GARCH(1,1) model with convenience yield and macroeconomic factors, p-value in brackets

Model	Mu	Omega	Alpha	Beta	Theta	External (Phi or Tau)	ARCH-LM test (p-value)
c.yield in *mean*, *ex_rate* in *variance*	0.004717 (0.302294)	0 (0.998175)	0.207651 (0.00484) **	0.568231 (0) ***	0.05384 (0.003942) **	*ex_rate*, φ 0.000933 (0.003947) **	0.7339

c.yield in *mean*, *yield* in *variance*	0.004735 (0.297126)	0 (0.99588)	0.156502 (0.06853) .	0 (0.99995)	0.06432 (0.000469) ***	*yield*, φ 0.005677 (0) ***	0.9186
c.yield and *ex_rate* in *mean*	-0.090142 (0.04979) *	0.001477 (0.003527) **	0.216903 (0.0043) **	0.569245 (0) ***	0.056191 (0.002669) **	*ex_rate*, τ 0.057588 (0.036875) *	0.8011

Significance codes: *** – significant on 0.1% level; ** – on 1% level; * – on 5% level; . – on 10% level
c.yield – convenience yield; *ex_rate* – exchange rate; *yield* – bond yield spread

Table 25. Comparison of various GARCH models estimated

Model	Other (m,var)	Log lik	AIC	BIC	HQ	Nyblom	Goodness to fit
sGARCH	NULL NULL	357.1389	-2.1732	-2.1266	-2.1546	0.8954 (1.07)	0.4219
sGARCH	*ex_rate* NULL	358.7053	-2.1766	-2.1184	-2.1534	0.9757 (1.28)	0.3629
sGARCH	NULL *ex_rate*	357.5305	-2.1694	-2.1112	-2.1462	1.0389 (1.28)	0.6003
sGARCH	NULL *yield*	360.3607	-2.1868	-2.1286	-2.1636	2.1059 (1.28)	0.8554
eGARCH	*ex_rate* NULL	354.9132	-2.1472	-2.0773	-2.1193	0.7515 (1.49)	0.7899
eGARCH	NULL *ex_rate*	353.2137	-2.1367	-2.0668	-2.1088	0.7907 (1.49)	0.8108
eGARCH	NULL *yield*	358.4437	-2.1689	-2.0990	-2.1410	2.4942 (1.49)	0.2714
GARCH-M	*ex_rate* NULL	358.9473	-2.1720	-2.1021	-2.1441	0.9982 (1.49)	0.5753
GARCH-M	NULL *ex_rate*	358.062	-2.1665	-2.0967	-2.1387	1.1678 (1.49)	0.6908
GARCH-M	NULL *yield*	360.3609	-2.1807	-2.1108	-2.1528	2.1947 (1.49)	0.8940
sGARCH	*c.yield* NULL	361.2249	-2.1922	-2.1339	-2.1689	1.7207 (1.28)	0.1182
sGARCH	*c.yield* +*ex_r* NULL	363.2858	-2.1987	-2.1288	-2.1708	1.6178 (1.49)	0.2267
sGARCH	*c.yield* *ex_rate*	361.7447	-2.1892	-2.1193	-2.1613	2.0791 (1.49)	0.0655
sGARCH	*c.yield* *yield*	366.3208	-2.2174	-2.1475	-2.1895	3.6633 (1.49)	0.7147

GARCH-M	*c.yield* NULL	361.8661	-2.1899	-2.1201	-2.1621	1.6558 (1.49)	0.1314
GARCH-M	*c.yield +ex_r* NULL	363.8653	-2.1961	-2.1146	-2.1636	1.6719 (1.69)	0.7533
GARCH-M	*c.yield ex_rate*	362.8371	-2.1898	-2.1083	-2.1572	2.4306 (1.69)	0.0951
GARCH-M	*c.yield yield*	366.2979	-2.2111	-2.1296	-2.1785	3.038 (1.69)	0.2113

sGARCH – standard GARCH; E-GARCH – exponential GARCH; GARCH-M – GARCH-in-mean. NULL – no additional factors in GARCH model equations; c.yield – convenience yield; ex_rate – exchange rate; yield – bond yield spread.

First column contains GARCH specification of the model; Second describes additional factors in the mean and variance equation; Third contains log likelihood function value; Fourth-Sixth – information criteria values; Seventh – Nyblom test statistics with critical for stability on 10% level value in brackets; Eighth – Pearsons Goodness-to-Fit test p-value.

Table 26. Probabilities of getting oil price in the interval (True ± $5/b) from the models

Model	1-month forecast	12-month forecast	Joint probability
GBM	0.4582391	0.126211	0.05783482
OU	0.4494546	0.138271	0.06214654
GARCH1	0.4231992	0.1494509	0.0632475
GARCH2	0.3799397	0.1236858	0.04699315
GARCH3	0.4385613	0.1411707	0.06191201

GBM – Geometric Brownian Motion model; OU – Mean-Reversion model; GARCH1 – GARCH(1,1) model with convenience yield exchange rate in mean; GARCH2 – GARCH(1,1) model with yield spread in variance; GARCH3 – GARCH(1,1) model with exchange rate in variance.

Table 27. Errors of prediction, last year of sample forecasts

Model	Shapiro-Wilk test	p-value	Box-Pierce Test	p-value
GBM	0.9599	0.6906	12.4029	0.4139
OU	0.9763	0.9378	9.7581	0.6372
GARCH1	0.9713	0.8769	9.9719	0.6184
GARCH2	0.9759	0.9339	9.6263	0.6487
GARCH3	0.9765	0.9397	10.1888	0.5994

GBM – Geometric Brownian Motion mode; OU – Mean-Reversion model; GARCH1 – GARCH(1,1) model with convenience yield exchange rate in mean; GARCH2 – GARCH(1,1) model with yield spread in variance; GARCH3 – GARCH(1,1) model with exchange rate in variance.

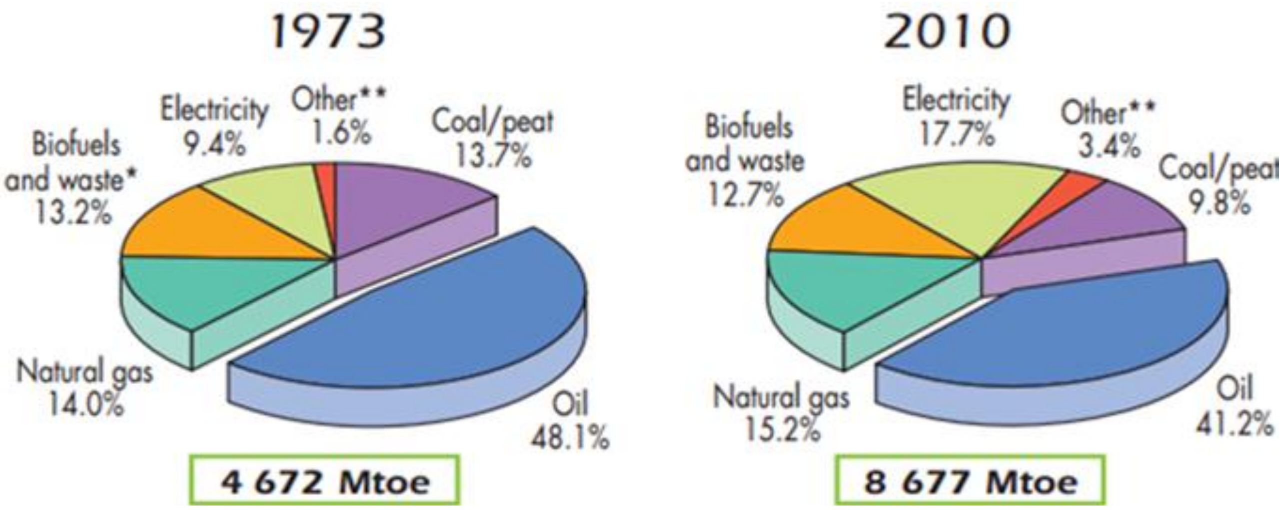
Table 28. Morgan-Granger-Newbold test results for forecasting accuracy, last year of sample

Pair of models	statistics	p-value	Pair of models	statistics	p-value
GBM-OU	4.246614	0.0008	OU-G2	-2.314284	0.0364
GBM-G1	3.698685	0.0024	OU-G3	-4.101076	0.0011
GBM-G2	4.333812	0.0007	G1-G2	-2.550107	0.0231
GBM-G3	4.116931	0.001	G1-G3	-3.208316	0.0063
OU-G1	2.393986	0.0312	G2-G3	-4.276228	0.0008

GBM – Geometric Brownian Motion mode; OU – Mean-Reversion model; G1 – GARCH(1,1) model with convenience yield exchange rate in mean; G2 – GARCH(1,1) model with yield spread in variance; G3 – GARCH(1,1) model with exchange rate in variance.

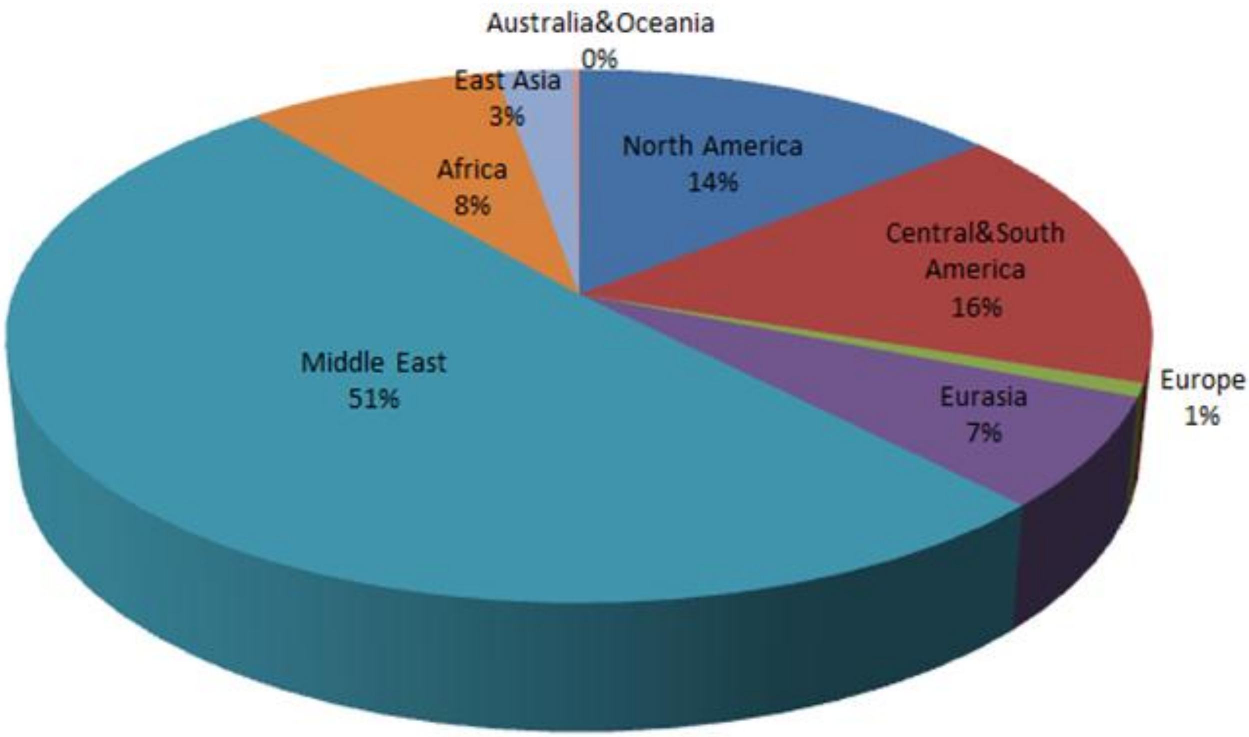
APPENDIX 2
Graphs and Charts

Chart 1. Total world energy consumption by sources, *percentage of total*



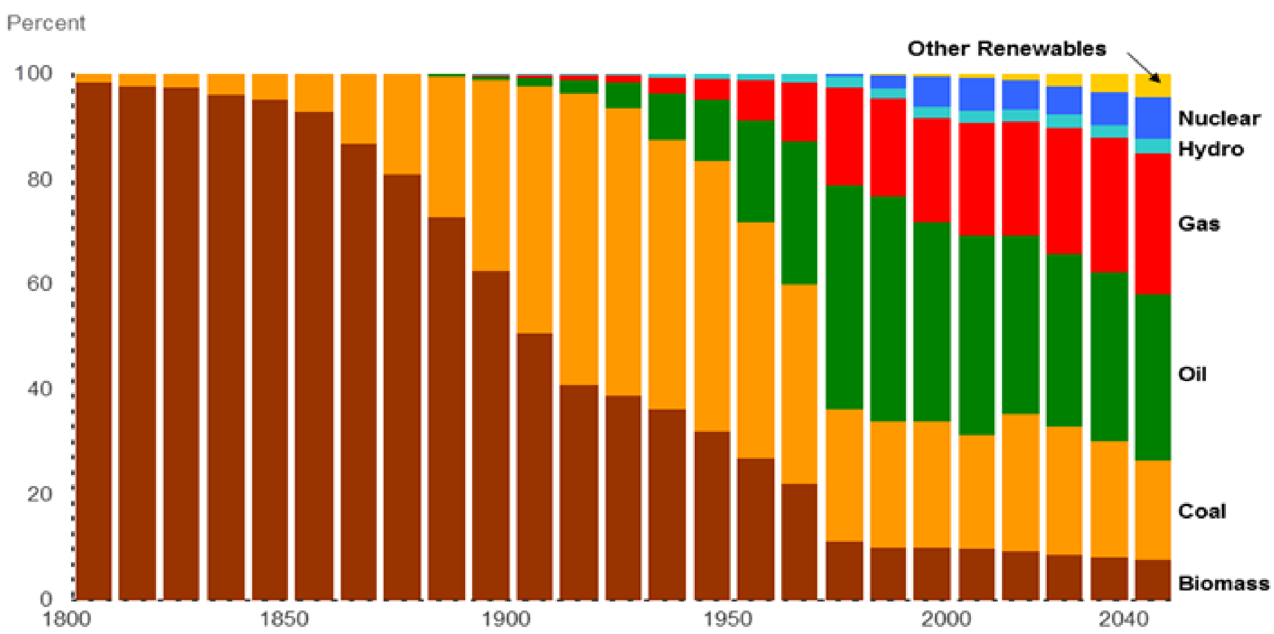
Source: Key World Energy Statistics, The International Energy Agency, 2012.

Chart 2. World proven oil reserves, *shares by region*, 2011



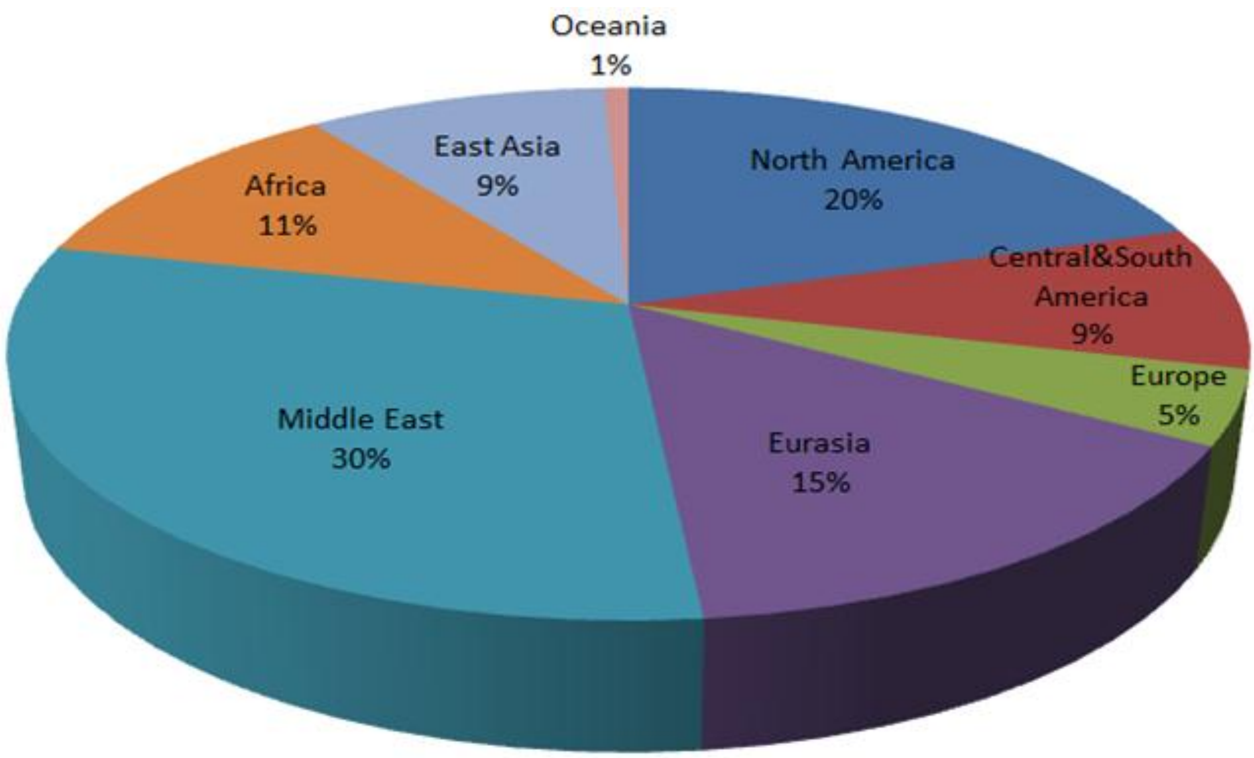
Source: The International Energy Agency, 2013

Chart 3. World energy mix by sources, *percentage of total*, 1800-2050



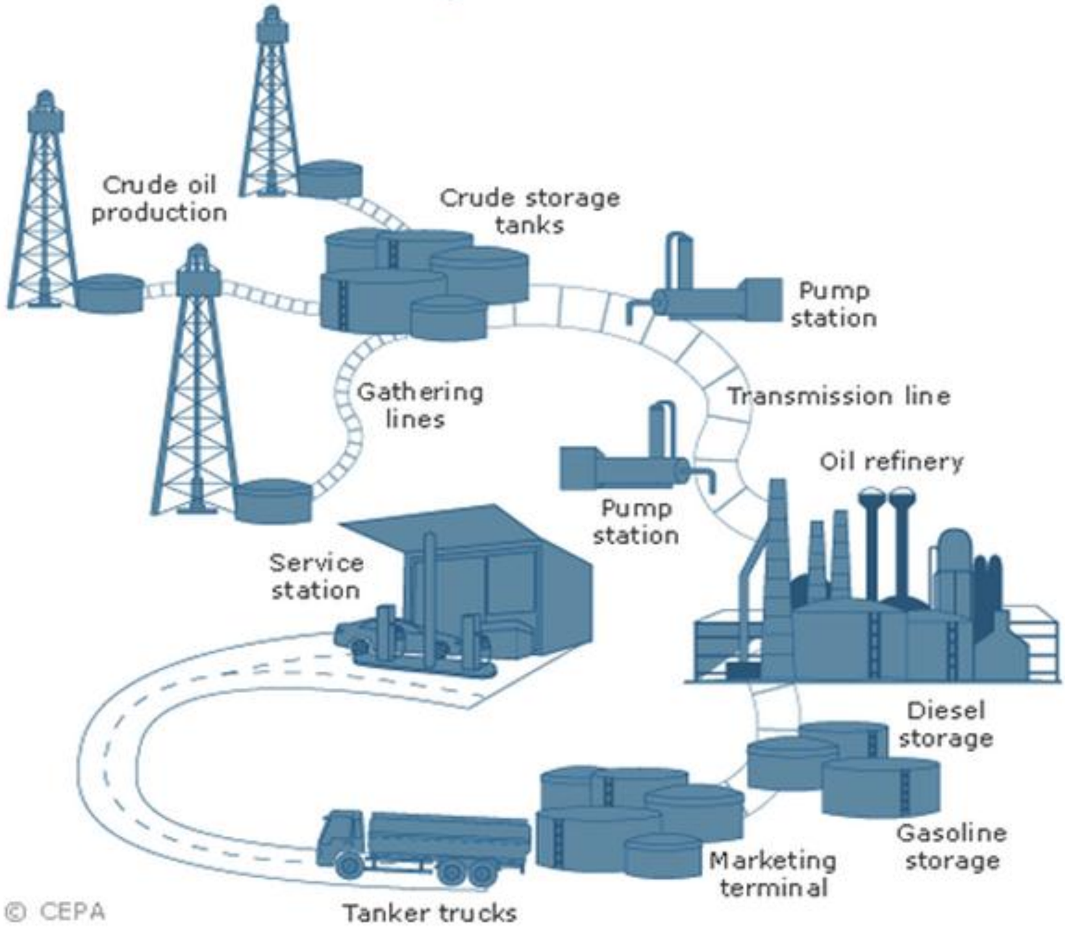
Source: Exxonmobil, 2013

Chart 4. Global oil production, *shares of total* by region, 2012



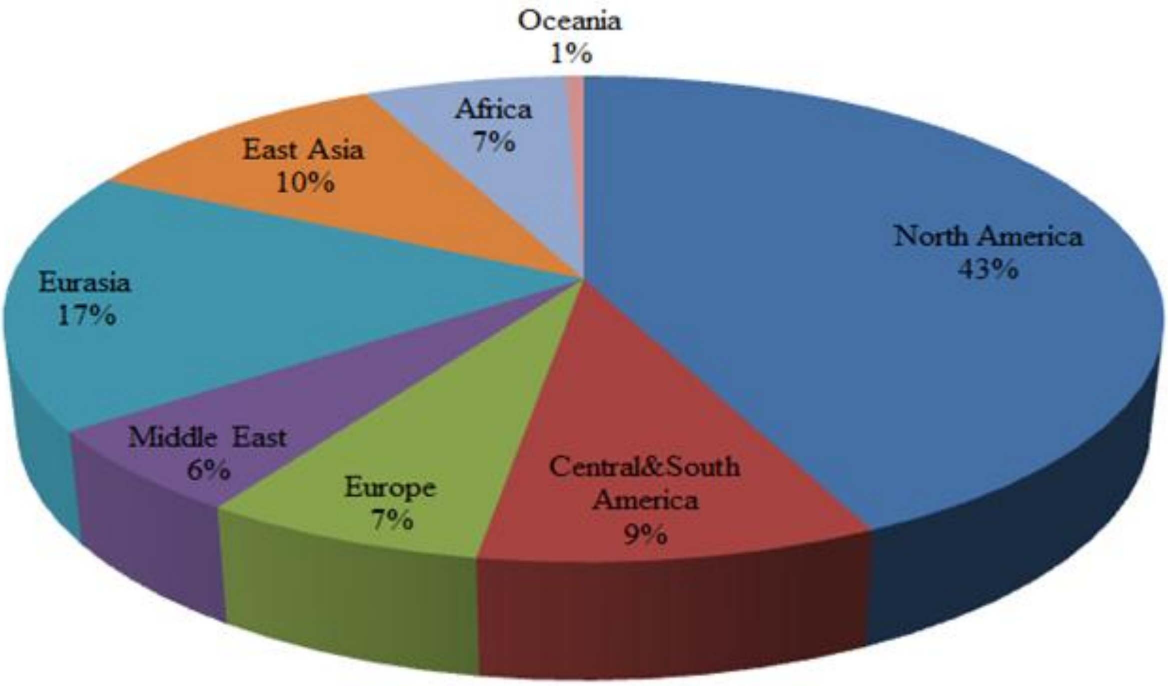
Source: The International Energy Agency, 2013

Chart 4. The Crude Oil Delivery Scheme



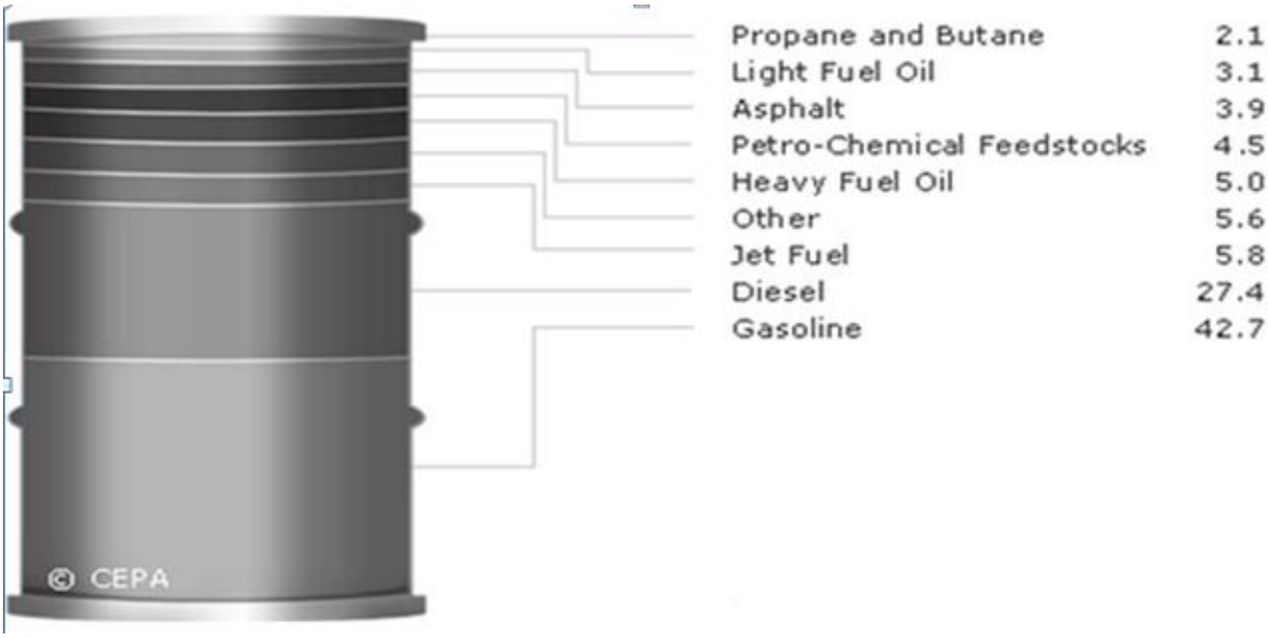
Source: Canadian Energy Pipelines Association, 2013

Chart 5. Crude oil and petroleum products pipelines location, *shares by region*, 2013



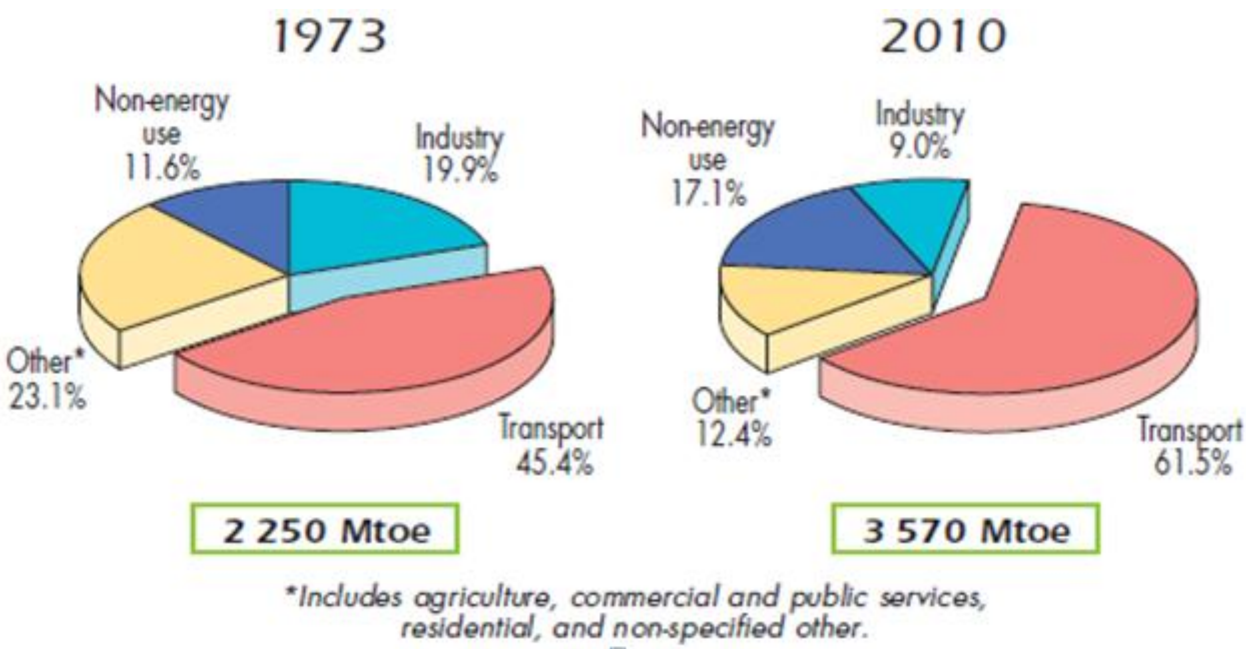
Source: Central Intelligence Agency, 2013

Chart 6. Percentage output from a barrel of oil



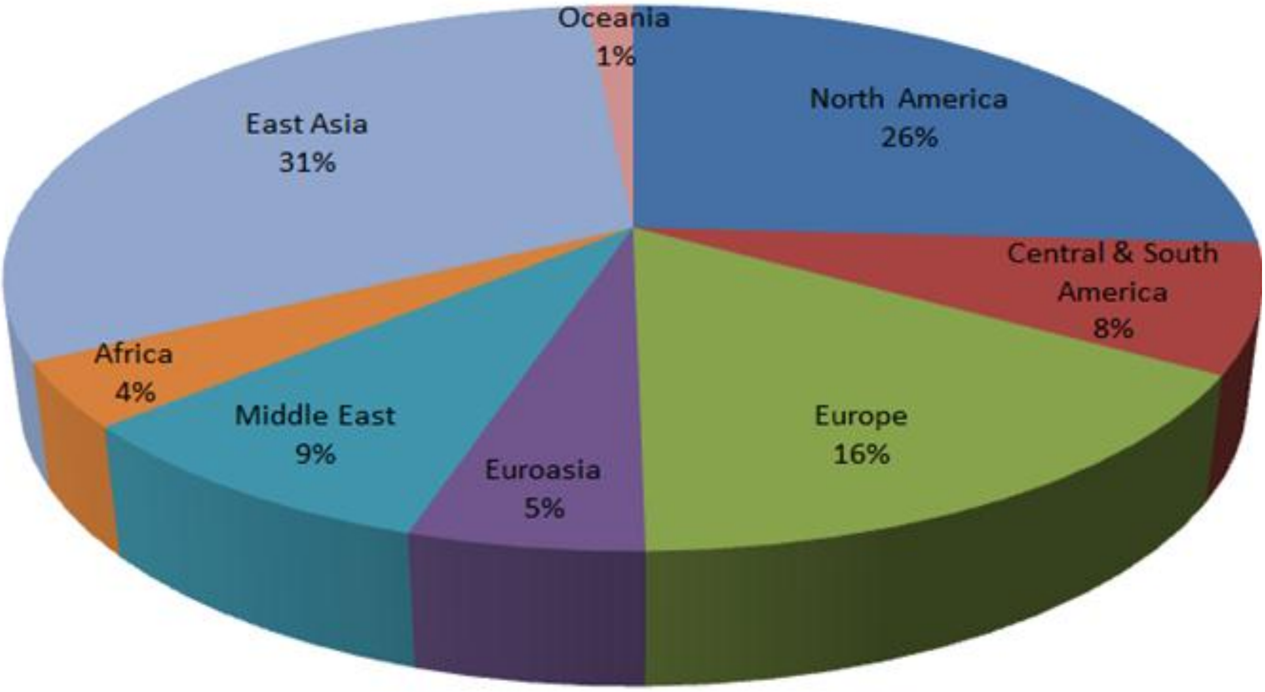
Source: Canadian Energy Pipelines Association, 2013.

Chart 7. World petroleum consumption, *shares by sector*



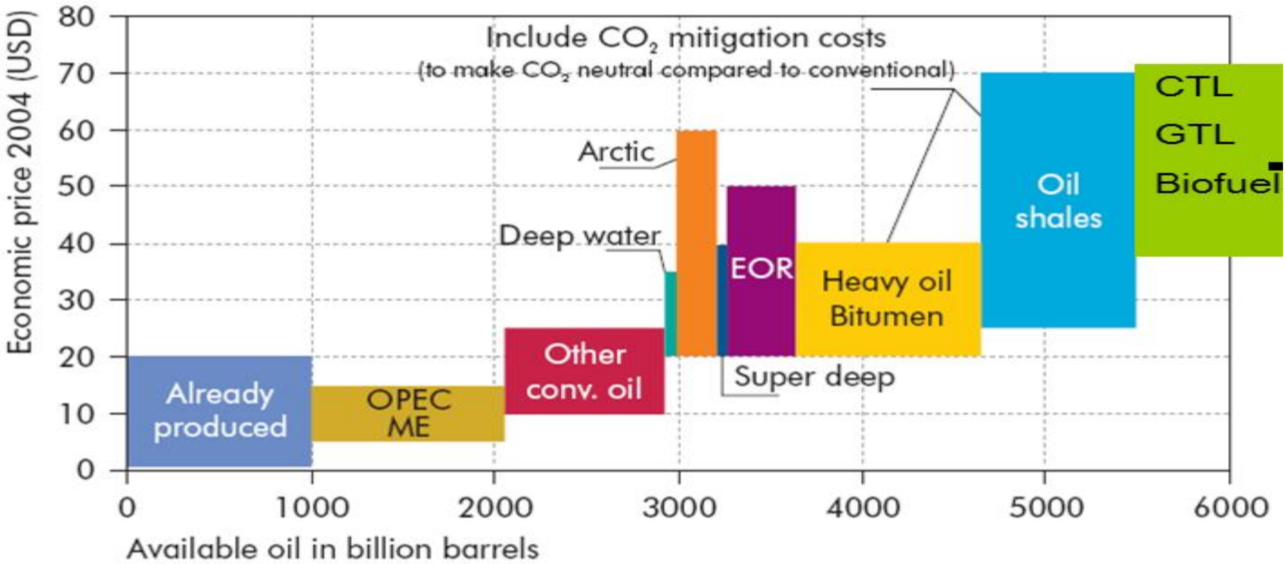
Source: Key World Energy Statistics, The International Energy Agency, 2012.

Chart 8. Global petroleum consumption by region, *percentage of total by region*, 2012



Source: The International Energy Agency, 2013

Chart 9. Economic price of oil by sources, *2004 USD per barrel*, 2005.

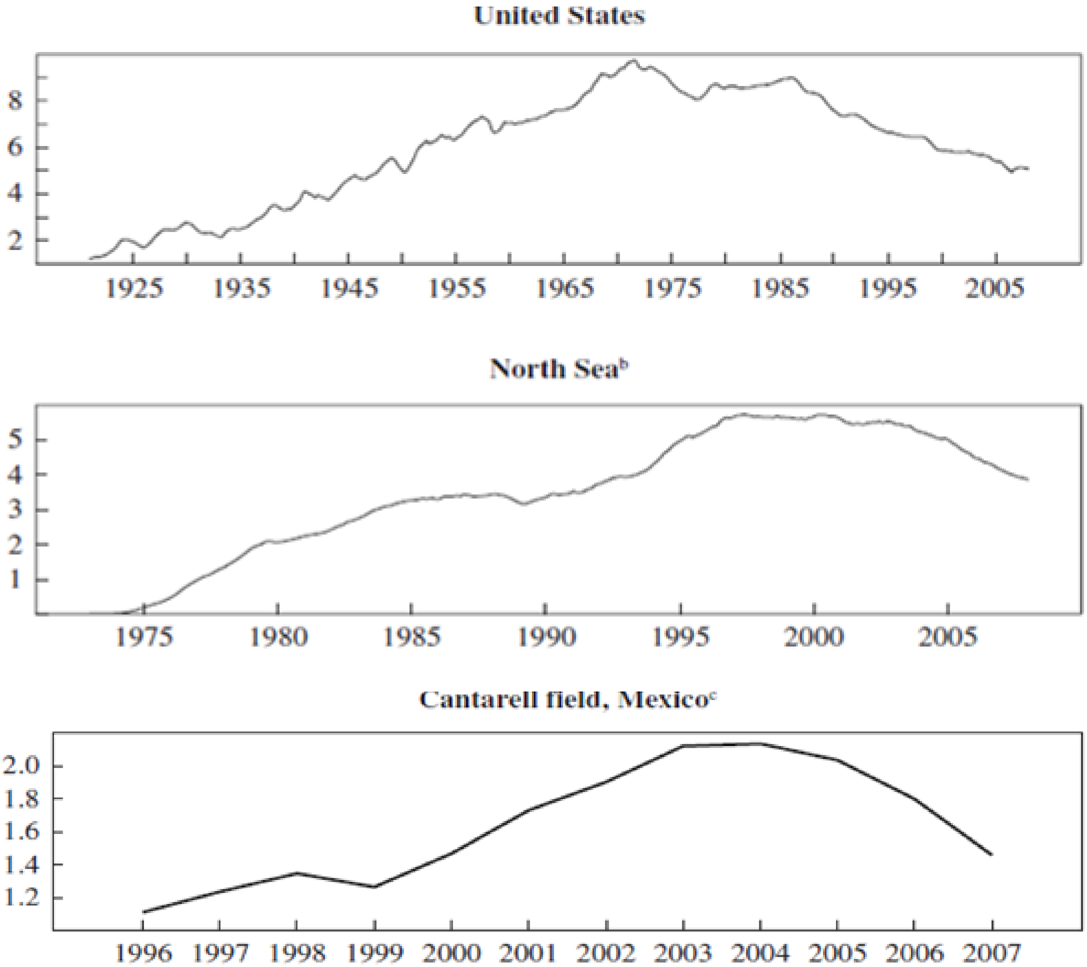


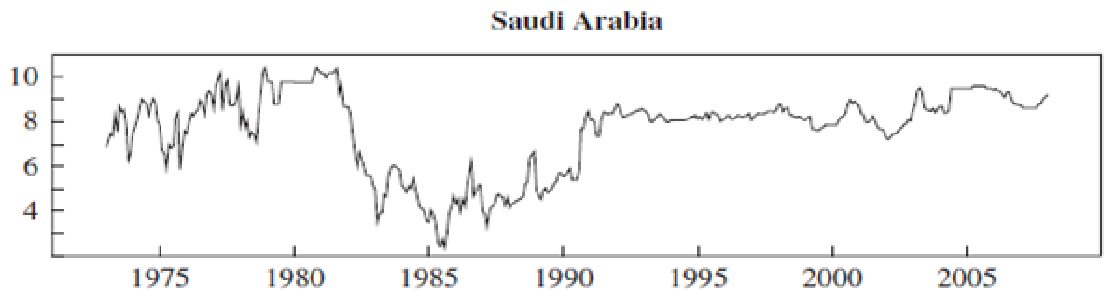
Source: International Energy Agency, 2013

Chart 10. Global oil benchmark prices, *USD per barrel*, 2012

Source: Dunn, Holloway (2012)

Chart 11. Crude oil production, selected countries, *million barrels a day*





Source: Noreng (2007)

Chart 12. 1986-2013 dynamics of interest rate, *percentage points*

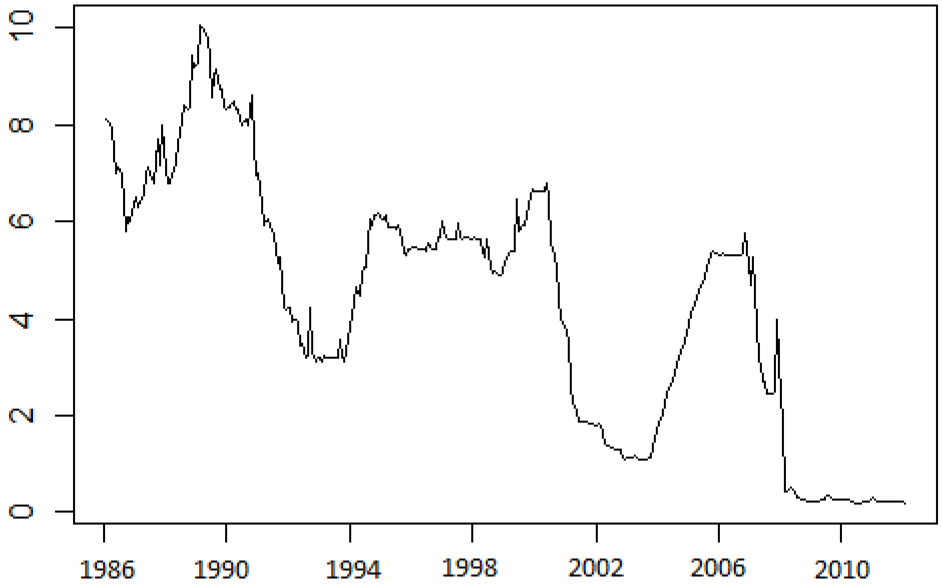


Chart 13. 1986-2013 dynamics of S&P500 stock price index, *points*

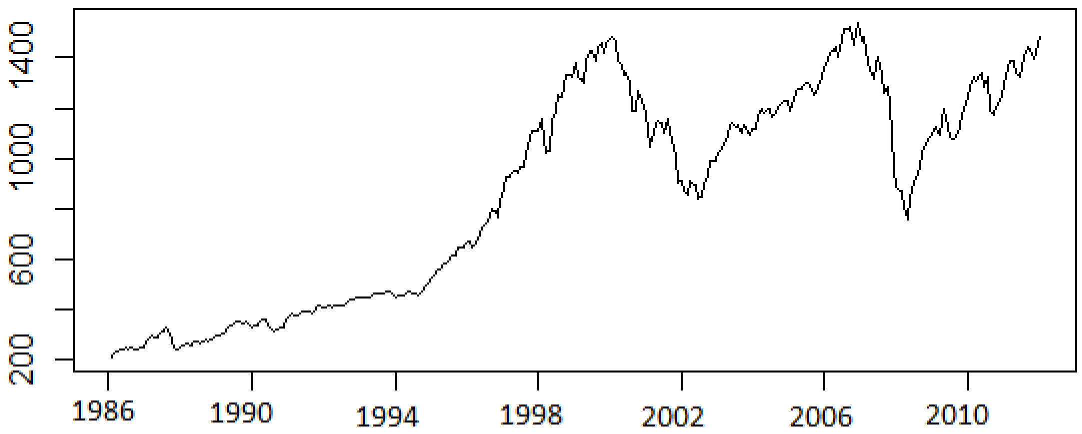


Chart 14. 1986-2013 dynamics of US industrial index, *index points*, 2007 = 100

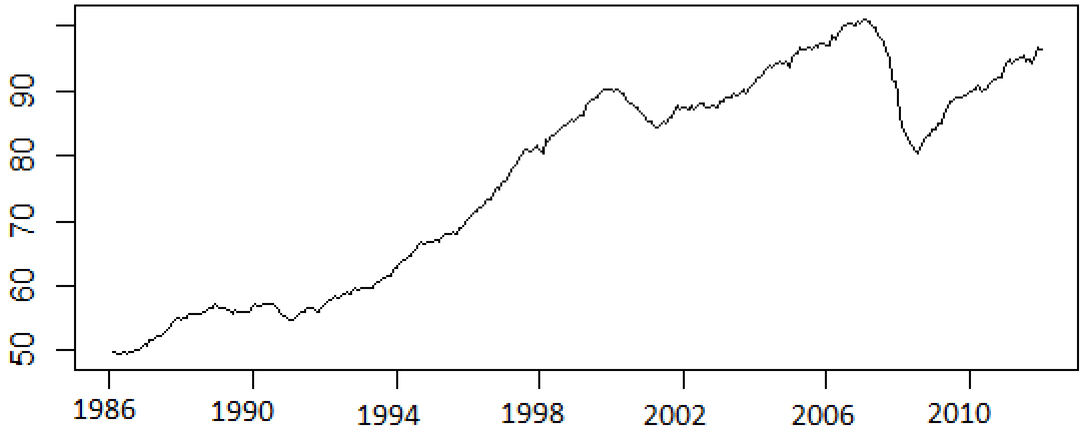


Chart 15. 1986-2013 dynamics of Moody's Baa-Aaa corporate bonds yield spread, *percentage points*

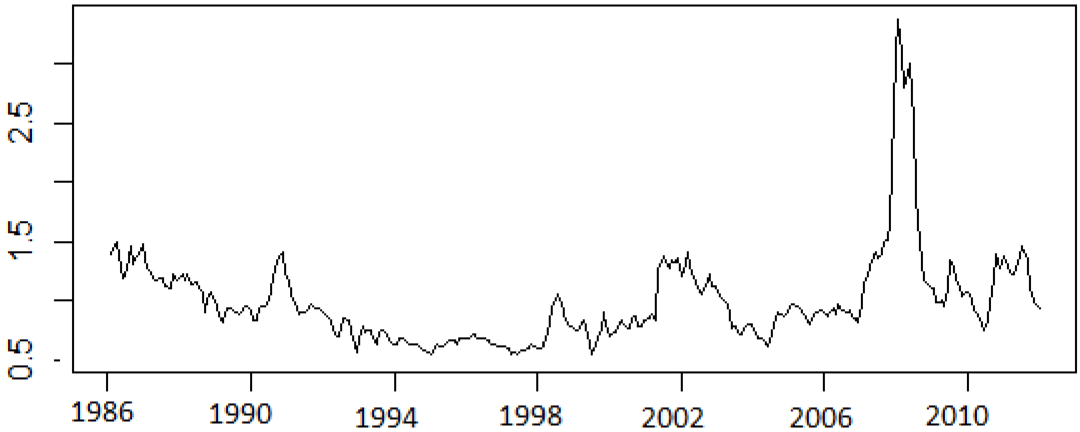


Chart 16. 1986-2013 dynamics of USD/GBP exchange rate, *USD per 1 GBP*

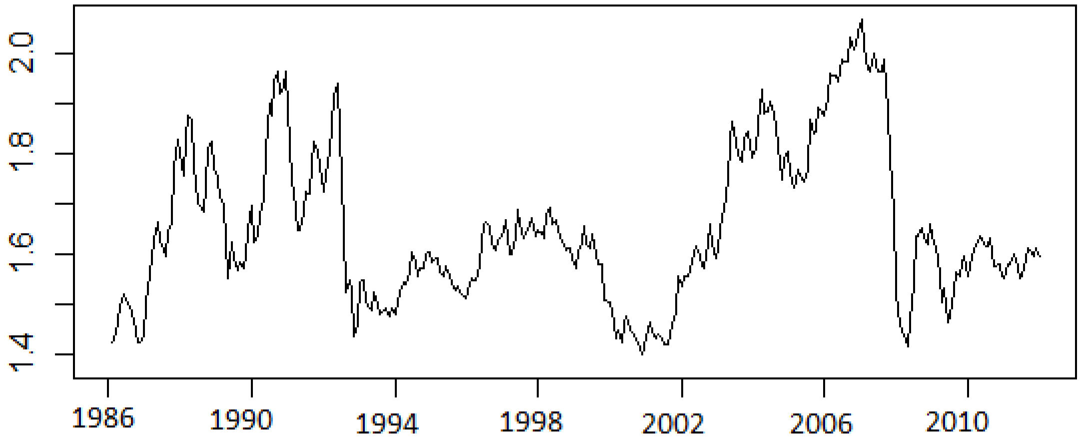


Chart 17. 1986-2013 dynamics of gold price, *USD per ounce*

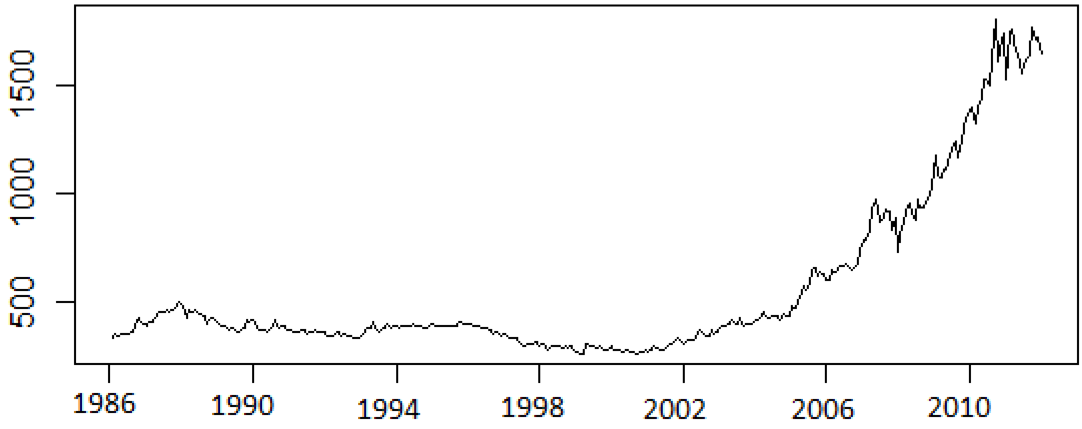
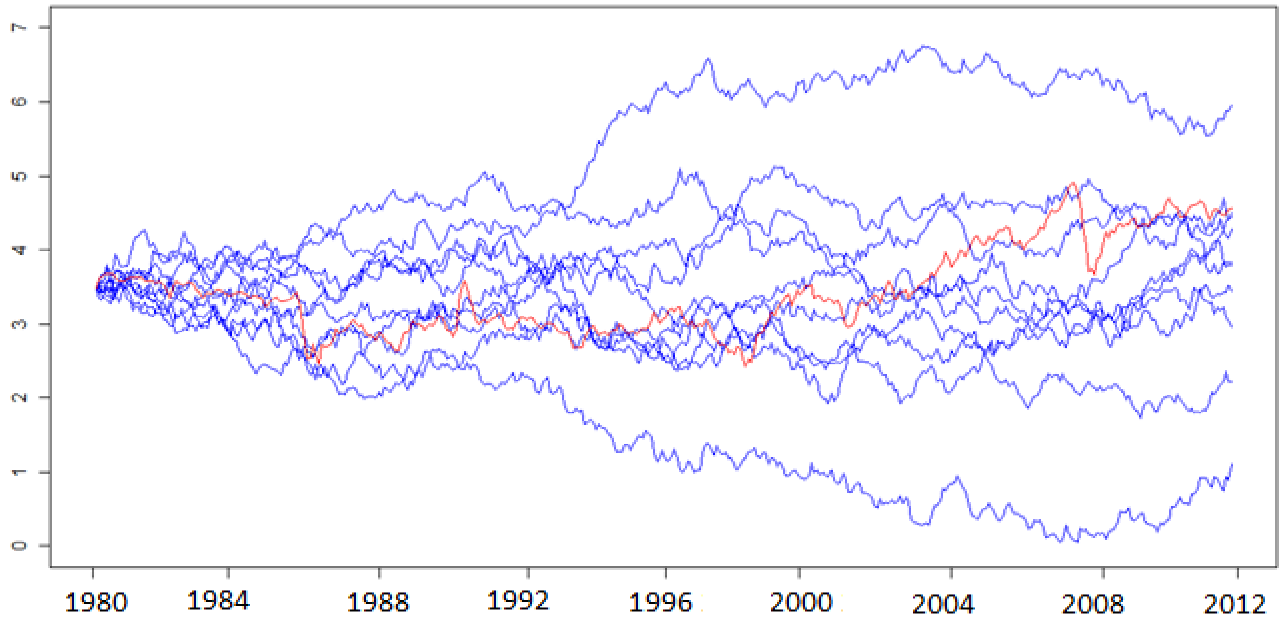
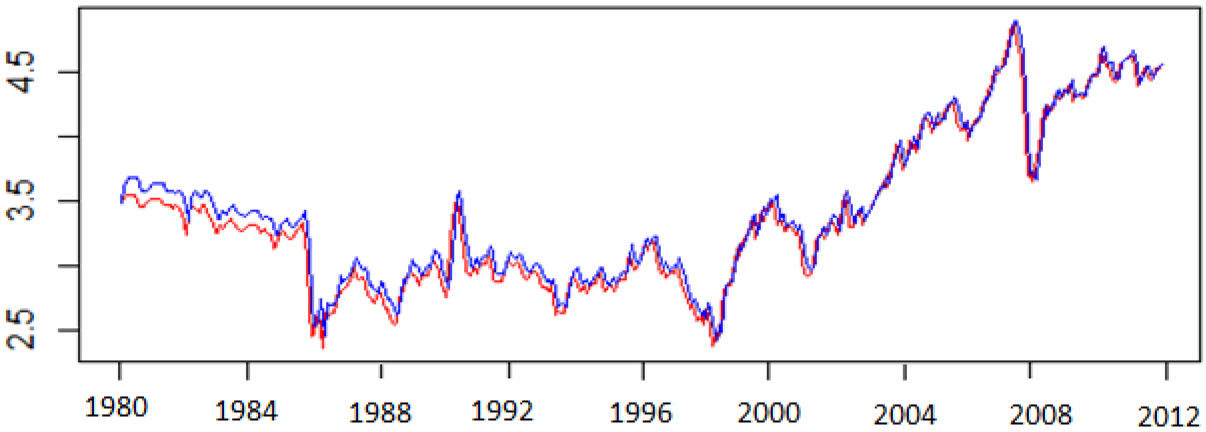


Chart 18. Simulations of oil log price paths by the Mean-Reversion model, *log price*



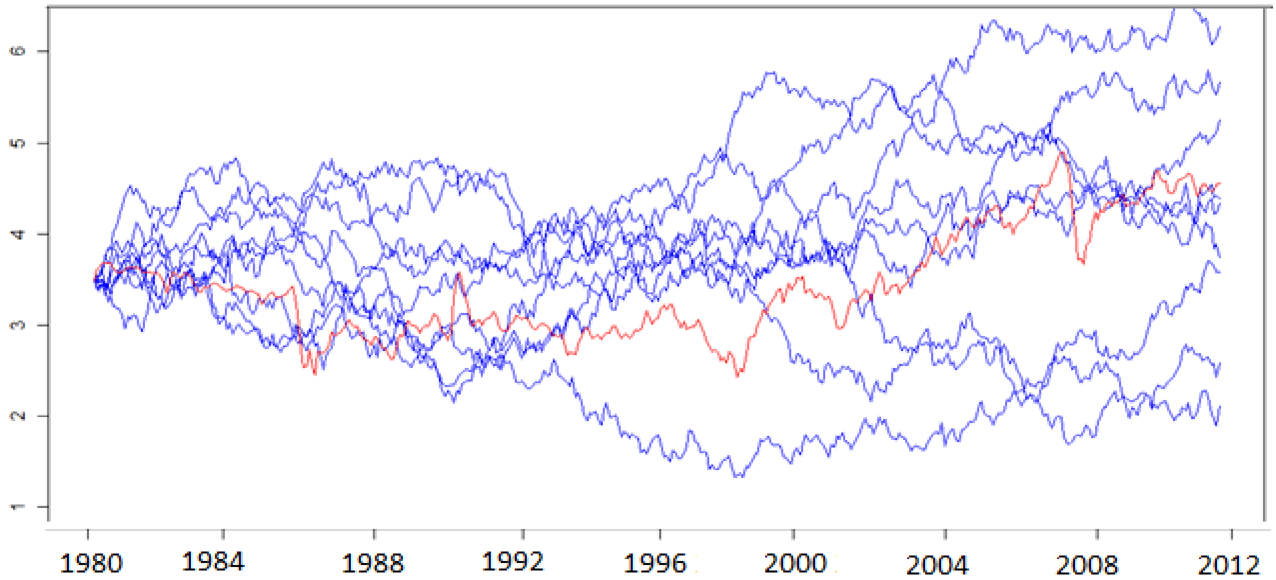
Blue – simulated log prices, red – actual oil log price.

Chart 19. Simulated oil log price path with the empirical residual, Mean-Reversion model, *log price*



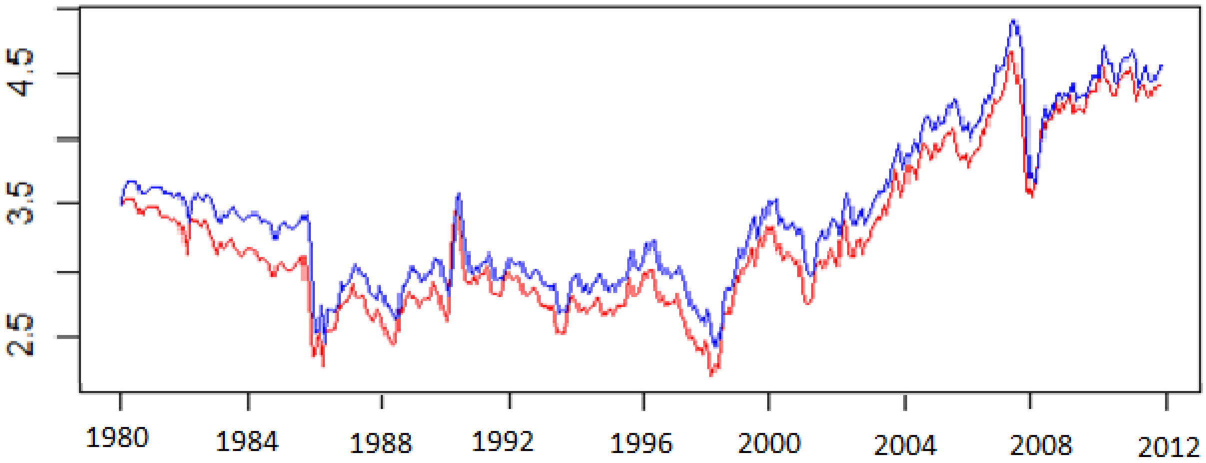
Blue – actual oil log price, red – simulation.

Chart 20. Simulations of oil log price paths by the GBM model, *log price*



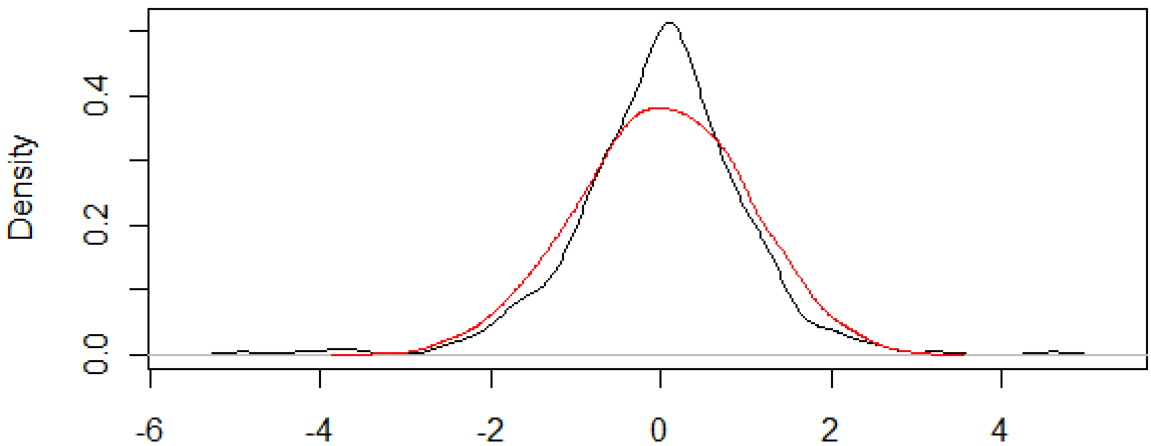
Blue – simulated log prices, red – actual oil log price.

Chart 21. Simulated oil log price with the empirical residual, GBM model, *log price*



Blue – simulated log prices, red – actual oil log price.

Chart 22. Probability density function of the empirical standardized residual $\hat{Z}$, GBM model



Black – empirical residual PDF, red – theoretical standard normal PDF.

Chart 23. QQ-plot of the empirical standardized residual $\hat{Z}$, GBM model

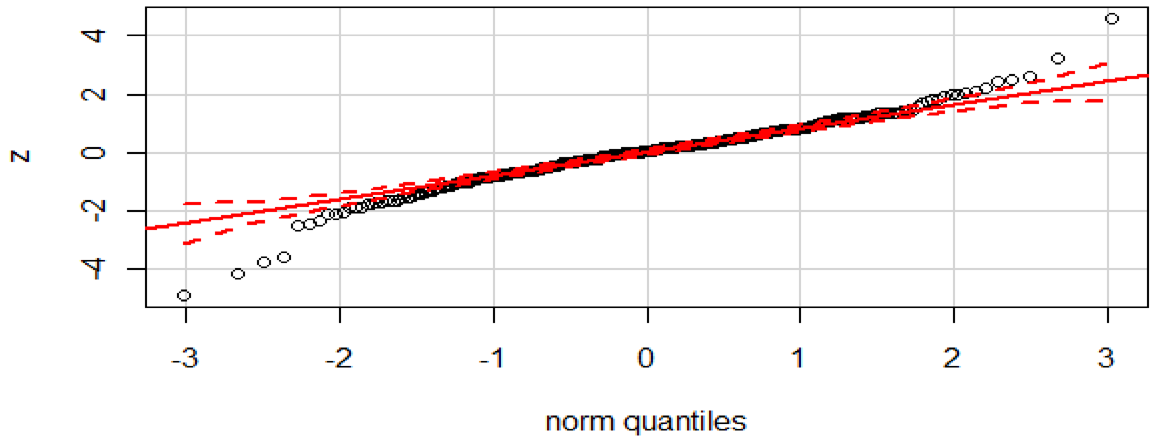


Chart 24. Autocorrelation function of the empirical standardized residual $\hat{Z}$, GBM model

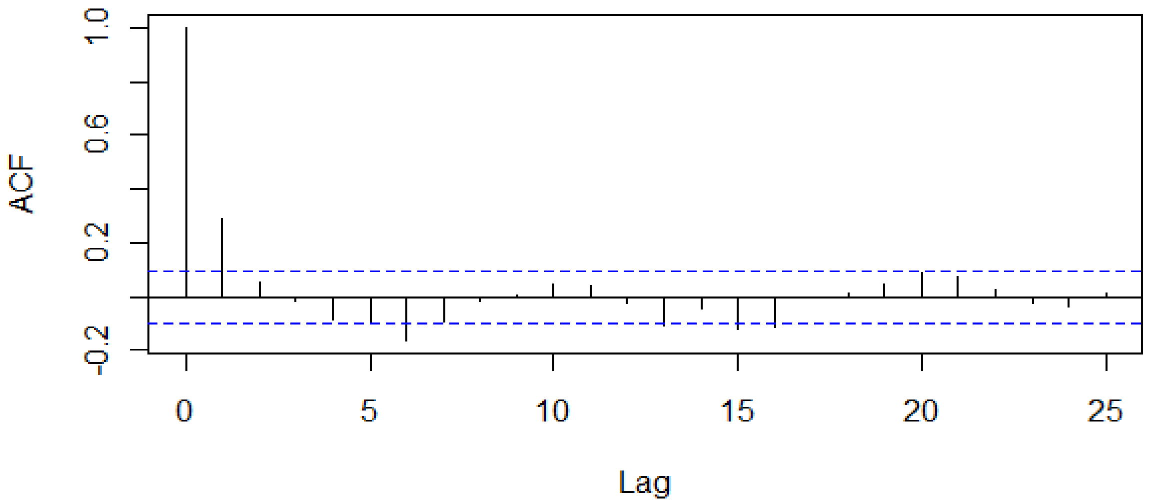


Chart 25. Autocorrelation function of the empirical residual squared η^2, GBM

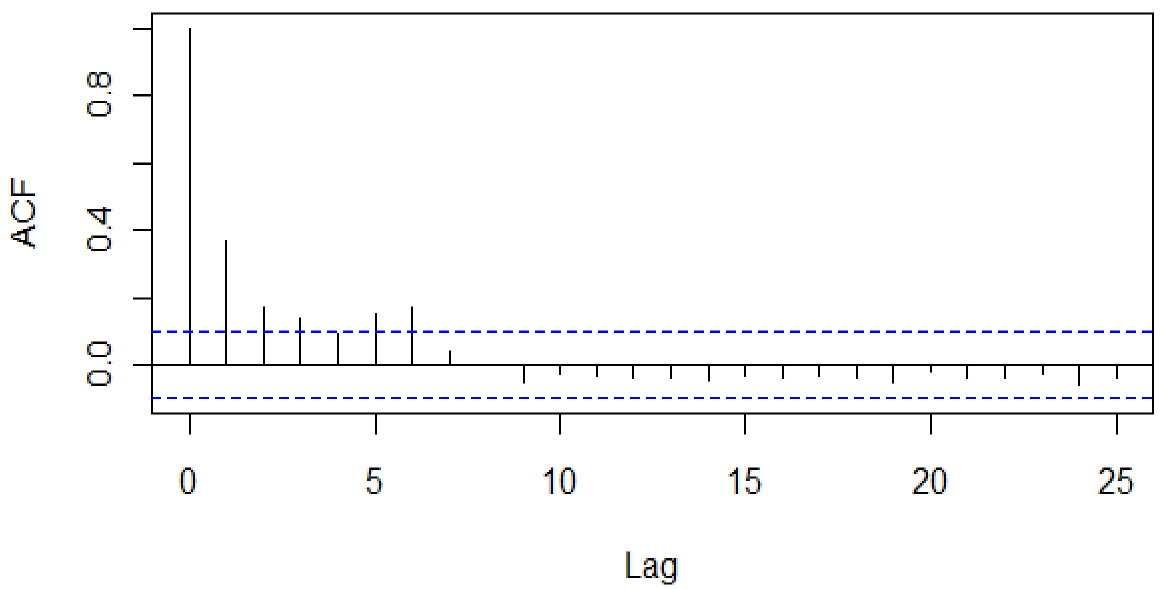


Chart 26. ARCH(6) model volatility fit, $\sigma_{S,t}$

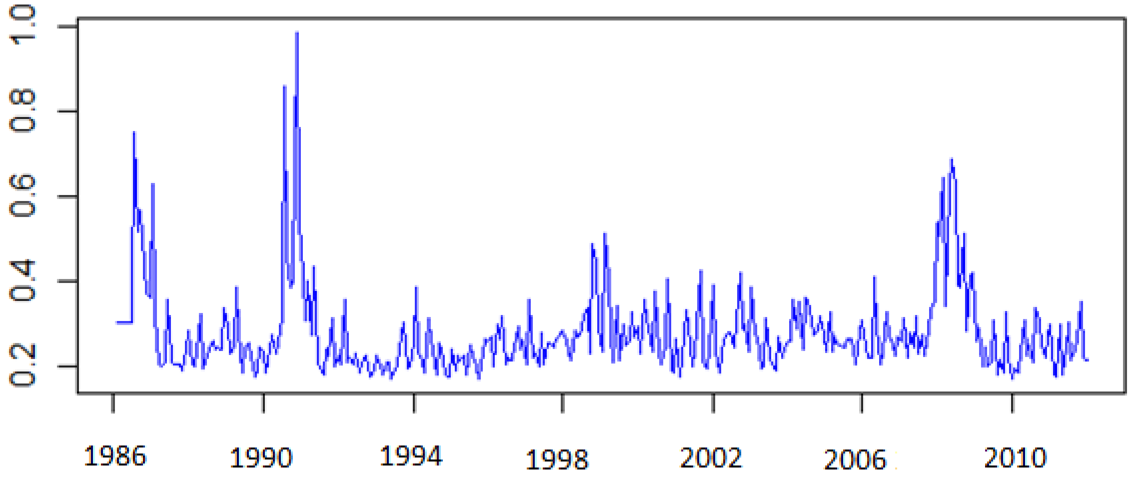
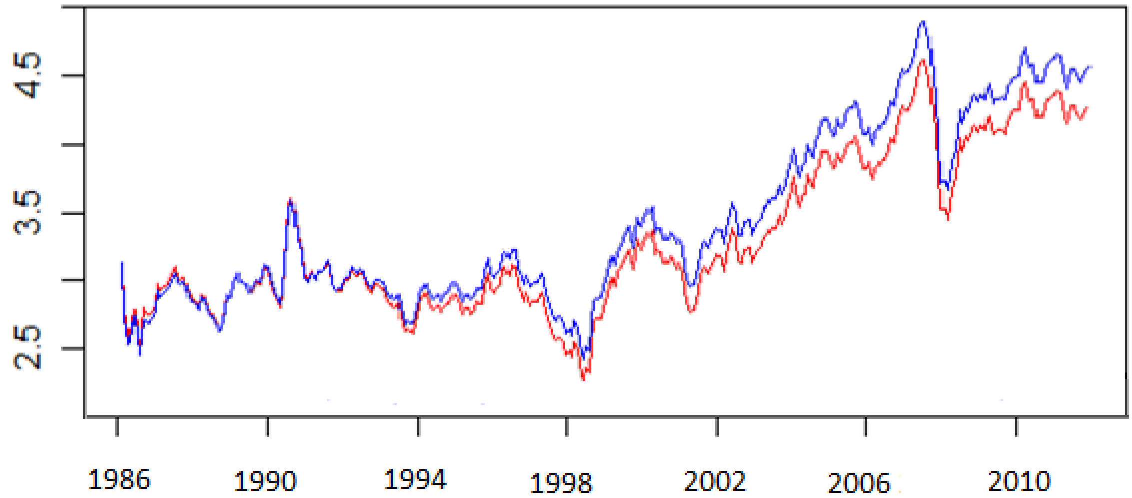
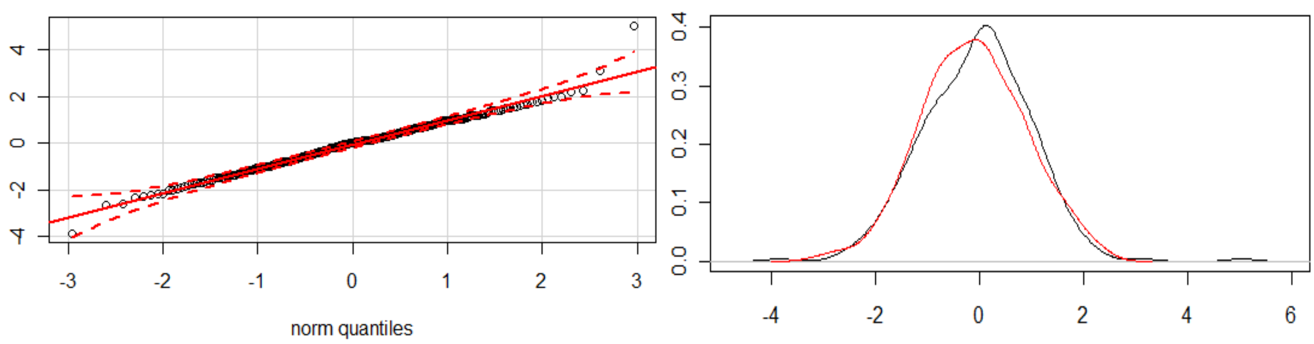


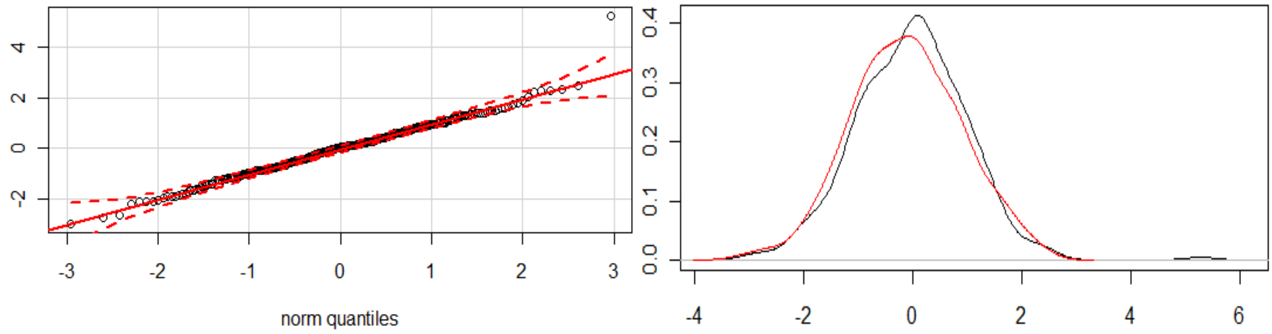
Chart 27. Simulated oil log price path with the empirical residual, GBM-GARCH(1,1), *log price*



Blue – simulated log prices, red – actual oil log price.

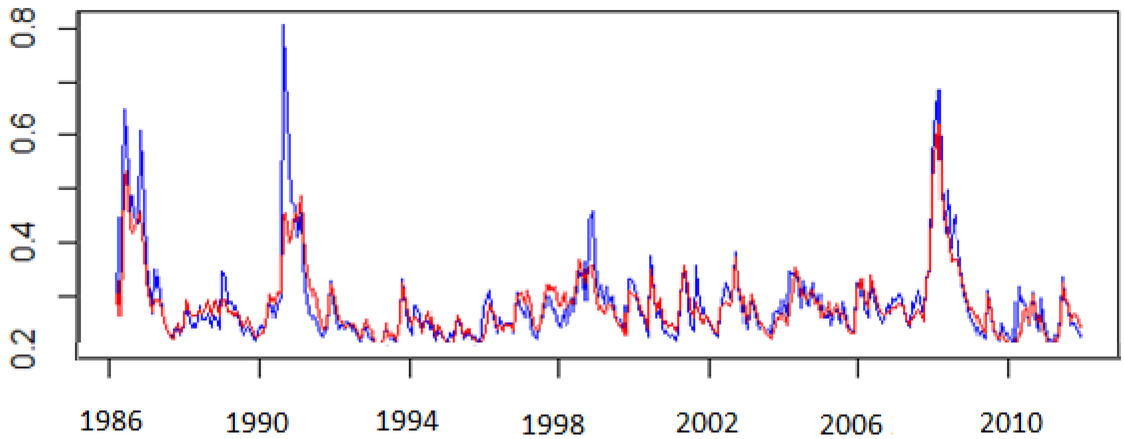
Chart 28. Comparative residual analysis (QQ plots and PDF functions) of the ARCH(6) (upper panel) and GARCH(1,1) (lower panel)





Black – empirical residual PDF, red – theoretical standard normal PDF.

Chart 29. Volatility fit of the GARCH(1,1) and E-GARCH(1,1) models, $\sigma_{S,t}$



Blue – GARCH(1,1), red – E-GARCH

Chart 30. 1986-2013 dynamics of estimated crude oil convenience yields, δ_t

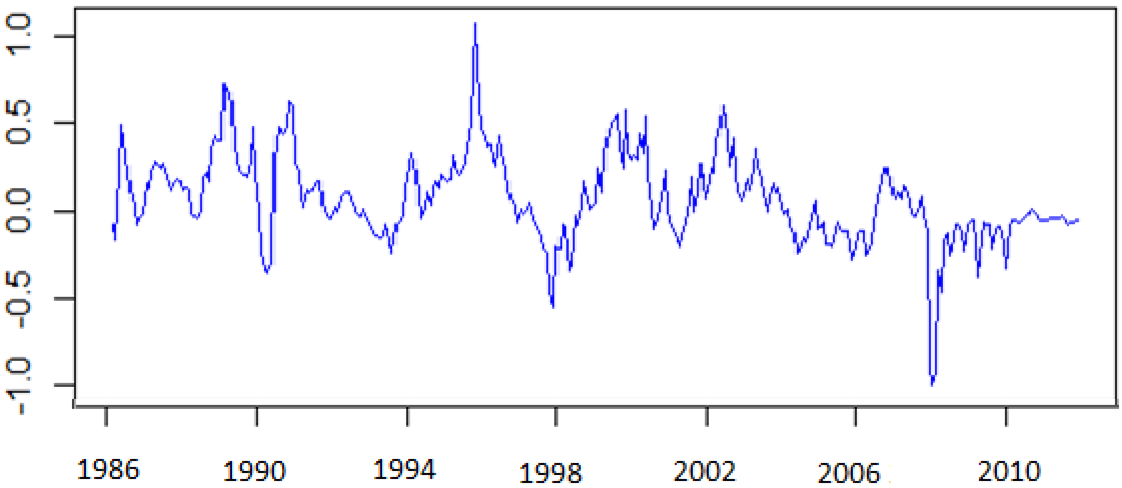
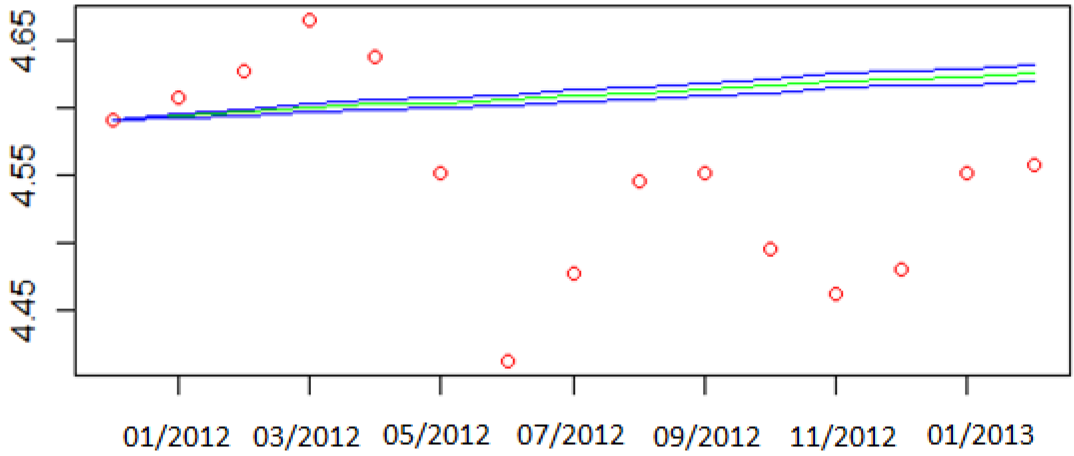
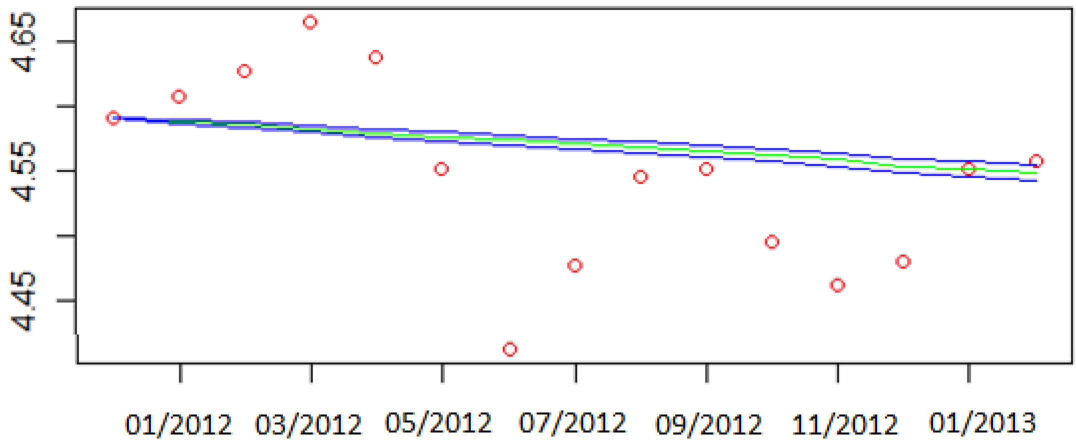


Chart 31. Out-of-sample forecast for 2012 of the oil log price as an expectation of the GBM model, *log price*



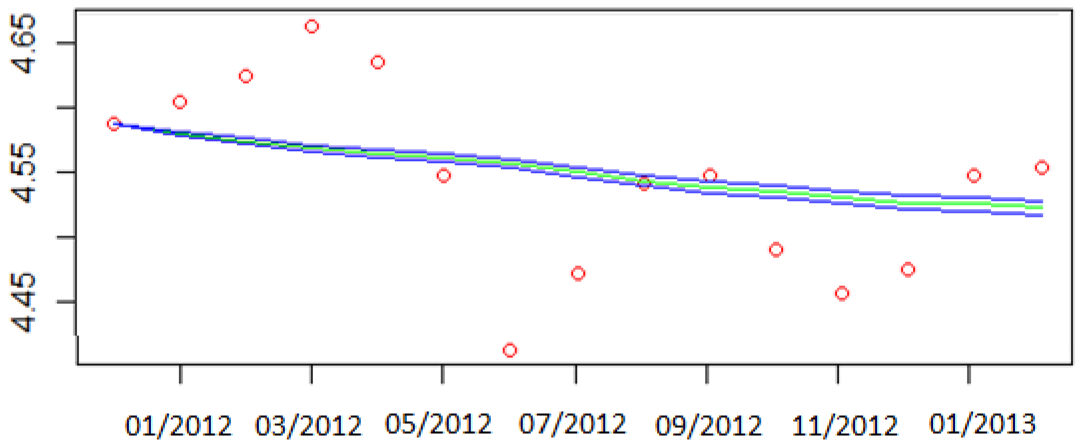
Red – actual log price of oil, green – expected values

Chart 32. Out-of-sample forecast for 2012 of the log oil price as an expectation of the Mean-Reversion model, *log price*



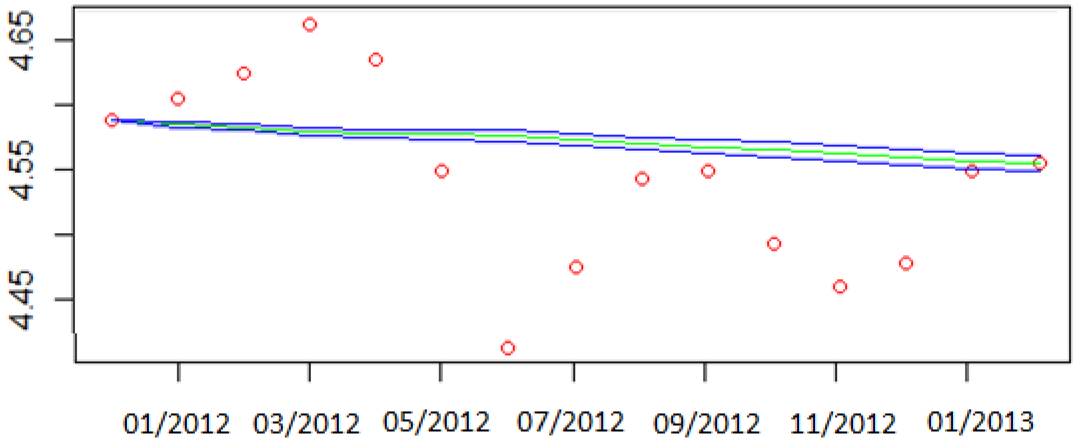
Red – actual log price of oil, green – expected values

Chart 33. Out-of-sample forecast for 2012 of the log oil price as an expectation of the GBM-GARCH model with convenience yield and exchange rate in mean, *log price*



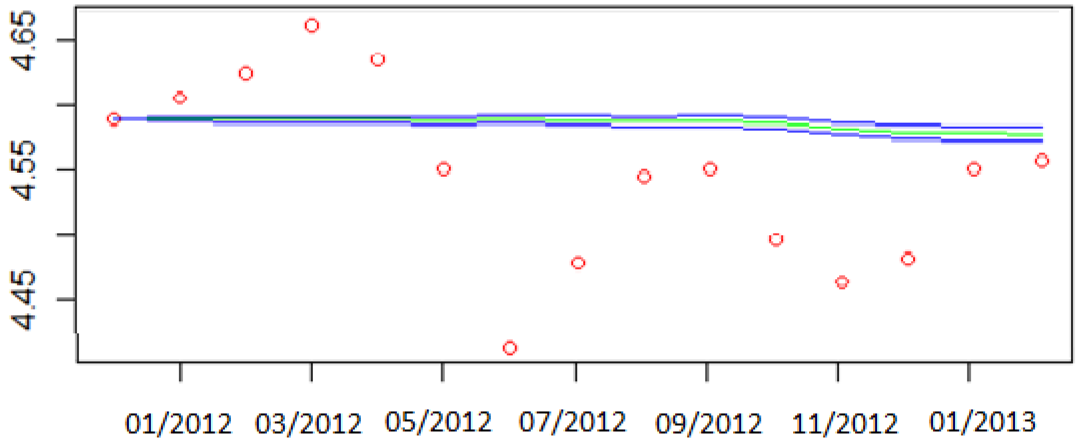
Red – actual log price of oil, green – expected values

Chart 34. Out-of-sample forecast for 2012 of the log oil price as an expectation of the GBM-GARCH model with convenience yield and bond yield spread in variance, *log price*



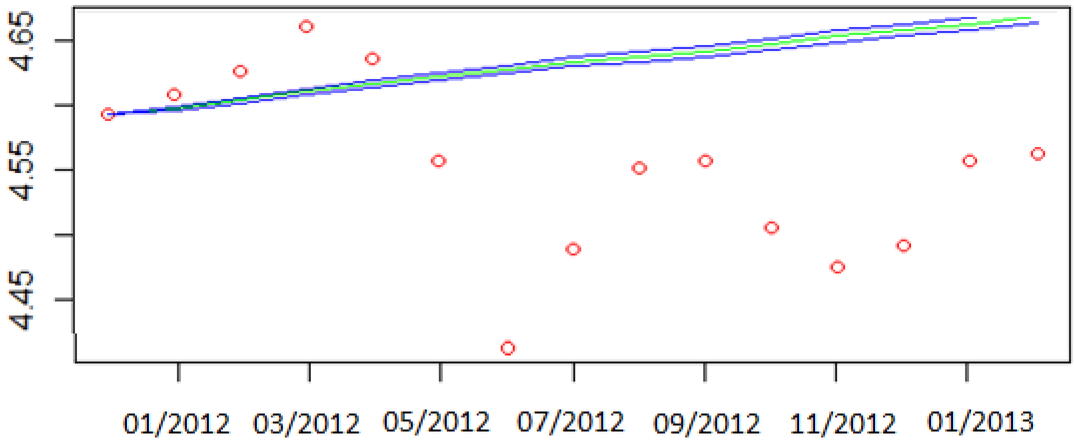
Red – actual log price of oil, green – expected values

Chart 35. Out-of-sample forecast for 2012 of the log oil price as an expectation of the GBM-GARCH model with convenience yield and exchange rate in variance, *log price*



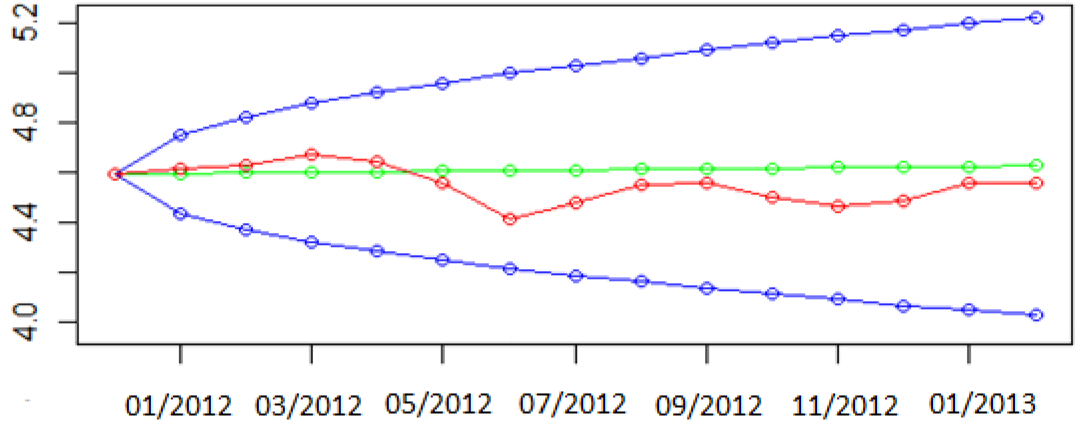
Red – actual log price of oil, green – expected values

Chart 36. Out-of-sample forecast for 2012 of the log oil price as an expectation of the basic GBM-GARCH model without any other additional factors. *log price*



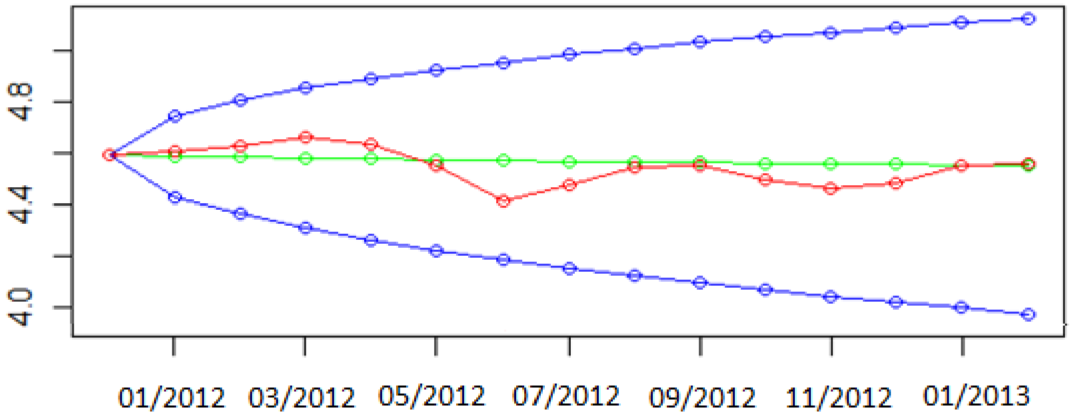
Red – actual log price of oil, green – expected values

Chart 37. Expectation for 2012 of the log oil price and its confidence limits from the GBM model, *log price*



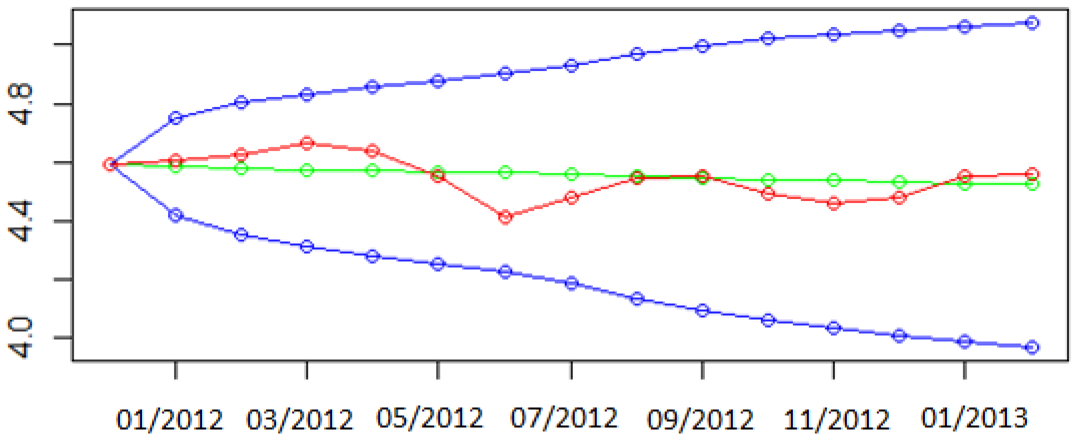
Red – actual log price of oil, green – model expectation, blue – its 95% confidence interval

Chart 38. Expectation for 2012 of the log oil price and its confidence limits from the Mean-Reversion model. *log price*



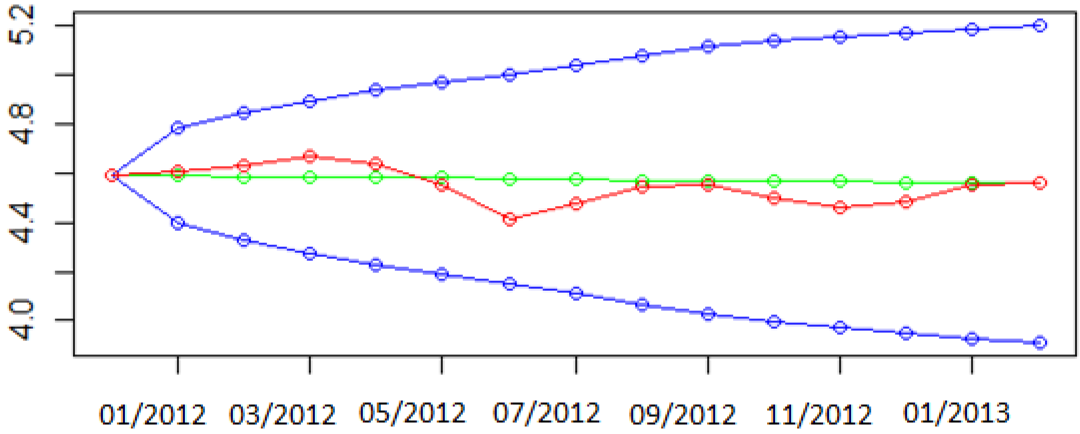
Red – actual log price of oil, green – model expectation, blue – its 95% confidence interval

Chart 39. Expectation for 2012 of the log oil price and its confidence limits from the GBM-GARCH model with convenience yield and exchange rate in mean, *log price*



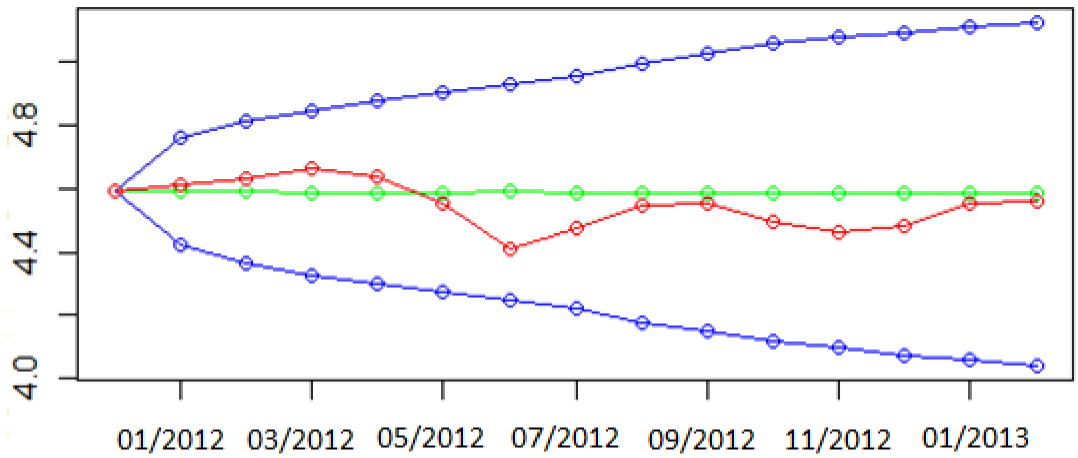
Red – actual log price of oil, green – model expectation, blue – its 95% confidence interval

Chart 40. Expectation for 2012 of the log oil price and its confidence limits from the GBM-GARCH model with convenience yield and bond yield spread in variance, *log price*



Red – actual log price of oil, green – model expectation, blue – its 95% confidence interval

Chart 41. Expectation for 2012 of the log oil price and its confidence limits from the GBM-GARCH model with convenience yield and exchange rate in variance, *log price*



Red – actual log price of oil, green – model expectation, blue – its 95% confidence interval