Master's Thesis
MSc in Advanced Economics and Finance
Copenhagen Business School
Department of Economics
Supervisor: Prof. Chandler Lutz

Learning, Expectations, and Endogenous Business Cycles

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August 2013

No. of Pages: 70
Characters: 123,154

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Executive Summary

We show that business cycles can emerge and proliferate endogenously in the economy due to the way economic agents learn, form their expectations, and make decisions regarding savings and production for future periods. There are no exogenous shocks of any kind to productivity or any other fundamental parameters of the economy, in contrast to Real Business Cycle models. To our knowledge this thesis is the first attempt to formally introduce adaptive learning and expectation errors as an autonomous source of endogenous business cycles.

We develop a simple, growth-less macroeconomic model, in which agents do not have perfect foresight, learn adaptively to form expectations, and solve limited inter-temporal optimization models. The theoretical possibility of cycles largely arises from the nonlinearity of the actual law of motion of price, in particular from the fact that agents always overpredict (underpredict) future prices when they are higher (lower) than equilibrium level. Even though the main version of the model is based on households having a simple logarithmic utility function, we also show that the results hold when a more generic Hyperbolic Absolute Risk Aversion utility function is chosen. Money stock is neutral in the long run in either case.

We conduct simulations in models with agents having both simple logarithmic and HARA utility functions. Following Thomas Sargent (1993), we assume agents to be "rational econometricians" using various econometric adaptive learning tools: Auto ARIMA, VAR and AR(2) models. In all simulations, output and other economic variables indeed display cyclical fluctuations around their equilibrium levels.

Both converging and diverging cycles may be obtained in simulations with Auto ARIMA models, while the VAR learning tool leads to diverging fluctuations in the majority of cases, suggesting that making agents consider several variables increases instability, at least in our setting. It is also observed that higher frequency of model switching is usually accompanied with increasing amplitude of cycles, suggesting the hypothesis that economic crises may happen when agents make drastic revisions of their beliefs about how the economy works. Only converging cycles can be obtained with AR(2), however in this case the economy may get trapped in a so called "false equilibrium", with output way below or above the true equilibrium level. Even though this is not formally an equilibrium, the convergence towards the true one is so slow that exogenous shocks may be needed to move the economy back on track. This result is in line with the Keynesian view that the economy may remain in a depressed state for quite a long period of time, and active government intervention may be required to speed up the recovery.

Within the developed framework we analyze whether active monetary policy (i.e. changes in money stock) can be used for stabilization purposes. It turns out that in the simple case, when agents have logarithmic utility function, shifts in money supply can have real effects on the economy only if they are unexpected by agents, or if future price expectations are not adjusted exactly proportionally to the announced monetary interventions. We also show that the second case is not sustainable within the adaptive learning environment, so that monetary policy may become ineffective in the long run when, and if, learning is complete.

We prove, however, that monetary interventions always have real effects in the short run in the setting with a more generic HARA utility function. Still, it is highly questionable whether the central bank is able to accurately assess the consequences of its own actions, as that would require it knowing precisely the actual law of motion of the economy, current market's expectations, and agents' reaction to news about the upcoming monetary interventions, which, moreover, can change over time.

Contents

Chapter 1

INTRODUCTION

> *"Prosperity ends in a crisis. The error of optimism dies in the crisis, but in dying it gives birth to an error of pessimism. This new error is born, not an infant, but a giant; for an industrial boom has necessarily been a period of strong emotional excitement, and an excited man passes from one form of excitement to another more rapidly than he passes to quiescence. Under the new error, business is unduly depressed."*
>
> Arthur Cecil Pigou[1]

Is the business cycle an optimal response to exogenous fluctuations of fundamental parameters of the economy? Or is it the consequence of a series of market imperfections, such as price or wage stickiness?

When it comes to explaining business cycles, even though it is reasonable to think that both approaches of current mainstream economics carry some truth with them, still the above suggested interpretations seem to leave us with a rather vague feeling of dissatisfaction, even when presented in combination. In particular, while it is certainly true that the structure of the economy has an effect on the fluctuation of its own output, we argue not only that this is not the whole story, but most importantly that it is not even the main story. In fact, the above

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[1] Pigou, Arthur C. (1927) *Industrial Fluctuations*, Macmillan.

quotation describes very well the central idea behind this work: the belief that it is the way people look at the future and learn from the past that shapes their decisions in such a way that allows cycles to emerge.

It is a general fact that the future consequences of our present decisions depend on a large set of factors; we all know it and this is why we all try to predict the most likely future states while making present choices. However, in the last few decades, the literature in economics has given increasing importance to the structural side of the economy, seemingly leaving behind the empirical study on how individuals form their expectations for the future, as well as the theoretical study on how this factor affects the functioning of the economy. To a large extent, this kind of problem was taken away in the sake of determinacy, particularly with the aid of the so-called rational expectations revolution.

Taking away this problem has indeed solved major formal issues. However, we argue that it might have been a well-hidden source of substantial difficulties, such as those contemporaneous scholars seem to have in explaining and consequently predicting economic fluctuations. For this reason, the present work is our first attempt to renew the awareness about the fundamental role of expectations in our economic system, particularly concerning the business cycles.

The idea that expectations may play a central role in causing economic fluctuations is, of course, not new, and was probably first explicitly suggested by Pigou (1927), later evolving into the notion of 'animal spirits' in Keynes's *General Theory*. However, to our knowledge, this idea remained exclusively qualitative during that period, and no formal model was built to describe how expectations are actually formed and evolve over time.

The rational expectations revolution, while causing the rapid advancement of sophisticated models, guaranteed hard times to anybody willing to study endogenous fluctuations, and eventually led to the development of the Real Business Cycles (RBC) theory. It seems that after the introduction of RBC, economists focused all their attention on extending Dynamic Stochastic General Equilibrium (DSGE) models to improve their fit to the observed data, usually through implementing various rigidities and imperfections. Most importantly, being busy with polishing RBC models, economists seemed to forget the possibility that the economy may experience business cycles without any shocks to fundamentals, or, actually, without any shocks at all. Our motivation is to demonstrate exactly that.

The starting point of our journey is to diverge from DSGE modeling. Acknowledging that rational expectations are in fact an equilibrium computational concept (Evans and Honkapohja, 2001) that need not necessarily correctly represent the dynamics of the real economy, we aimed, following Sargent (1993), to inhabit our theoretical model with human beings and switch off the 'God mode' explicitly present in every DSGE model. We should also stress, however, that we did not end up in some kind of agent-based modeling, another extreme, and built a simple, compact and solvable macroeconomic model.

The remaining of this work is structured as follows: in chapter 2 we review some of the most relevant contributions in the theory of business cycles, contextualizing our work within them; in chapter 3 we describe and solve our model; chapter 4 is reserved to the presentation of simulations, together with a discussion on their outcomes; in chapter 5 we introduce and apply methods for the analysis of the limiting behavior of the economy; in chapter 6 we develop a discussion on the effectiveness of monetary policy within our setting; finally, chapter 7 concludes.

Chapter 2

REVIEW OF BUSINESS CYCLE THEORIES

The aim of this chapter is to provide the reader with an overview of the history of business cycle theory; however, a comprehensive and exhaustive survey of the topic would easily require an entire book, and it is therefore outside of our purposes. Instead, we will introduce few selected theories, reserving particular emphasis to the positioning of our work within the literature, so to underline the relevance of our contribution.

2.1 Theories from the 19th Century

Business cycles did not receive particular attention from the classical economists of the 19th century such as Smith, Say or Ricardo, who firmly believed in the ability of the capitalistic economy to naturally gravitate towards a state of equilibrium and, in absence of exogenous shocks, to remain in such state once it has been reached. Mainstream economics regarded fluctuations in the economy as a fact of secondary importance, bound to disappear in the long-run. Following this line of thought, classical economists focused their analysis on the long run behavior of the economy and on the identification of its so-called natural state of equilibrium.

Say's law of market is arguably one of the results that most closely expresses the view of the mainstream of that time. According to Say (1803), there is no possibility of aggregate overproduction because agents use the income obtained from selling produced goods for consumption; or in other words, supply is the source of its own demand. As a direct implication, note that, since underconsumption is in principle impossible, every theory of crises based on such a thing should be regarded as inexact.

In light of these considerations, it is perhaps not surprising that, to find the first theories of cycles, one has to look at the heterodox economics of the period; more precisely, the very first and yet still partial and unstructured treatment of fluctuations can be found in Sismondi (1819). In his book *Nouveaux Principes d'Economie Politique*, Sismondi suggests a crisis theory based on overproduction, where crises are presented as a direct consequence of the complexity and the lack of centralized planning inherent to the capitalistic economy. As Mitchell (1927) effectively summarizes, Sismondi introduces at least four major arguments in favor of the possibility of crises in the capitalistic economy. From our perspective, we should note that one of these arguments is built on the belief in the presence of a economy-wide coordination problem. In particular, Sismondi observes that firms in the market face a complex and heterogenous mass of consumers, whose characteristics quite often remain unknown to the firms themselves. In fact, firms are ultimately left with market price as the only observable variable to use as a guide in their production decisions. According to Sismondi, this lack of information on the characteristics of consumers (and competitors) is a potential source of non-optimality in the production decisions, that can generate booms and crises over time.

It is important to stress the role of price expectations in Sismondi's explanation of crises. To use Richard Hyse's words:

> *"Sismondi starts with the basic assumption that the ease of consumption this year - whether the output was sold at expected prices - is the basis for production decisions for the next year in the same way that ease of consumption last year determined*

> *the production decisions of this year.*"[1]

In this sense, one could claim that arguments in the same spirit of the one on which our theory is built, have been suggested since the beginnings of the 19th century. However, at the same time one should note that, while Sismondi's intuition on the effect of expectations on economic activity was certainly brilliant for his time, nonetheless his analysis remained purely qualitative, reducing most of the times to simple postulations rather than logical or mathematical derivations, leaving space to unclear dynamics between expectations and the occurrence of crises.

In spite of the theory in Sismondi (1819), this type of planning and coordination problem was rather overlooked by the literature, which instead gave more space to purely underconsumptionist theories such as that of Malthus (1836). At the same time, in the mainstream the conviction remained strong that crises could arise only in response to exogenous shocks. For instance, Ricardo (1817) recognizes the possibility of crises and overproduction in spite of the obvious contradiction with the law of market. In fact, Ricardo overcomes this apparent inconsistency by considering the possibility of exogenous events (e.g. wars), which could change the natural state of the economy, forcing it into a period of adaptation. During this period, whose length varies according to the level of capital and labor specialization in the country, the economy (or at least some sectors of it) is expected to face a crisis.

It is clear that Ricardo's theory does not leave space for endogenously generated crises in the capitalistic economy, but again this should not come as a surprise. As long as we believe in the existence of a state in which the economic system is somehow naturally bound to remain, finding arguments in favor of endogenous fluctuations that do not come at the expense of consistency is probably better described as an art, rather than a science. In fact, in this respect, one could argue that Ricardo laid out the fundamental idea on which, after more than a

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[1] J. C. Sismonde Di Sismondi (1991) *New Principles of Political Economy: Of Wealth and Its Relation to Population* (R. Hyse, Trans. and Ed.) Transaction Publishers. (Original work published 1819.)

century, Kydland and Prescott (1982) build their Real Business Cycle Theory, i.e. cycles are the result of optimal responses to exogenous shocks to economic fundamentals. However, let us leave a detailed treatment of this theory and its extensions for section 2.5.

So far, we have reviewed theories of crisis elaborated at best under the acknowledgement that the capitalistic economy can indeed experience periods of severe recession over time. However, note that in the literature there was little awareness of the so-called cyclical behavior of the economy. In fact, once we leave the domain of crisis theory, the first structured treatise on the business cycle is found in Juglar (1862). Not only Juglar is among the first to acknowledge the presence of irregular fluctuations in the economy, but he also tries to provide an endogenous explanation of this phenomenon, in contrast to Ricardo's position. He suggests a theory of cycles based on over-investment and excessive confidence, where he divides the cycle into three phases: prosperity, crisis and liquidation.

Juglar's cycle theory is particularly relevant to the present work because of the central role of agents' confidence in it; and even though the way this confidence is built and destroyed seems to remain a rather intuitive idea for Juglar, it is clear that expectations, seen as a powerful investment driver, play a primary role in boosting the prosperity phase of the economy and in triggering the fall of it once the crisis phase is approached.

Most importantly, Juglar was probably the first, but not the last, to theorize on the presence of a structural relationship between individuals' confidence and aggregate investment. In fact, as we will see in section 2.2 and 2.3, this concept was reiterated by distinguished authors such as Pigou (1927) and Keynes (1936). Moreover, with this respect, even though the dynamics of capital markets are not fully treated in our theory, in section 3.3 we will observe how, under basic assumptions, savings behave as a function of agents' confidence expressed as expectations on next period prices.

However, let us now move to the 20th century and use the following sections to examine some of the main business cycle theories developed by the major schools of the century.

2.2 Neoclassical Economics

A first broad classification of neoclassical business cycle theories can be done by differentiating between monetary and real approaches. While the former strive to attribute the presence of fluctuations in GDP to purely nominal factors (e.g. the elasticity of money supply), the latter find explanations of cycles in the fundamental structure of the economy and in the dynamics of agents' behavior. In this section, we are going to review examples of each type, but let us start by introducing some common factors for most neoclassical theories.

Over-investment is one of the most commonly accepted theoretical explanations of the business cycle among neoclassical economists; a typical framework would be to consider an economy with two sectors, one producing capital goods and the other producing consumer goods. It is a well known empirical fact[2] that the sensitivity of investment to the business cycle is significantly higher than that of consumption; in other words, the capital goods sector tends to grow quickly during periods of prosperity and to fall sharply during crises, while the activity of the consumer goods sector follows a smoother path across the phases of the cycle. According to this neoclassical over-investment framework, this empirical fact is evidence of serious imbalances in the development of the two sectors over time. The general concept is that in periods of prosperity the capital goods sector becomes over-developed relative to the consumer goods sector; this imbalance is not sustainable over time and the result is the beginning of a period of adjustment, causing a downturn in the economy. The disagreements usually come as to the reason why such over-investment arises and whether it is a natural feature of the economy, perhaps even beneficial in the long-run, or not.

According to the Austrian Business Cycle Theory, particularly as exposed by Mises (1912) and Hayek (1931), the cause of over-investment has to be found in the central bank's inflationary monetary policy. Indeed

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[2] See e.g. Hansen (1985) and Prescott (1986) for empirical evidence.

such a policy, characterized by a high monetary base, generally tends to increase the overall money supply, ultimately raising the availability of credit. As the supply of credit is high, ceteris paribus, the interest rate (i.e. the price of credit) is low and investment is incentivized. Moreover, because of the artificially low interest rate, entrepreneurs tend to undertake a relatively higher number of long-term projects, normally located into the capital goods sector. In fact, as they now tend to discount the future income stream from all projects with low interest rates, the apparent relative profitability of longer projects increases. That is, artificially low interest rates modify the absolute and relative valuations of projects by entrepreneurs, causing an increase in investment particularly in the capital goods sector. However, in the Austrians' view, the equilibrium interest rate is ultimately determined by people's time preferences, i.e. by their current decision between consumption and savings. Such preferences are a fundamental of the economic system and they are not altered by monetary policies. Thus, once the excessive supply of money shifts from indebted firms to people (through wages, rent and interests), the latter start to reestablish the equilibrium allocation of their income, decreasing savings and increasing consumption. This is when the unsustainability of the previous level of investment becomes clear and the economy experiences a downturn.

We shall note that the Austrian school's explanation of the business cycle has a marked flavor of exogeneity, as it is clear that this theory is built upon the belief that the economy would stabilize in absence of external shocks. Consistently with this point, the Austrians conclude that most of (if not all) external interventions should be avoided to ensure the stability and the efficiency of the economy.

One can find a remarkably diverging theory of over-investment in Schumpeter (1912, 1939), who interprets the business cycle as a unavoidable process that is intrinsically linked to economic growth. In Schumpeter's approach, business cycles represent the necessary adjustments for the economy to move from one static equilibrium to a new one, characterized by higher output per capita. The engine of growth and the trigger factor of fluctuations are both identified in the innovational activity of entrepreneurs, which is experienced in a wave-like form, as a few innovators are enough to prompt the herd behavior of followers. That is, innovations tend to come in clusters, laying the foundations for the manifestation of cycles. In fact, they push the economic system far away from the neighborhood of equilibrium, triggering the spontaneous reaction of agents, that drives the economy toward its new natural state. However, this adjustment is neither immediate nor immediately exact, as the economy is likely to overshoot, missing the new equilibrium in both directions multiple times and experiencing several fluctuations before reaching a new stability.

While it is evident that innovation represents the core of Schumpeter's explanation of cycles, focusing on a more marginal aspect of this theory, the careful reader might even note similarities between the endogenous process of adjustment described above and the dynamics of the model that will be introduced in chapter 3. Obviously, we do not intend to go as far as suggesting an expectations-based view of cycles in Schumpeter's analysis, as that would simply be misleading. However, there seems to be an acknowledgement that as soon as the economy is moved out of the equilibrium, the adjustment process that follows is rather lengthy and complicated, leading to errors of both signs. In this respect, the main difference between the argument in Schumpeter and the one in the present work is that, while the former claims that in absence of 'shocks' (e.g. waves of innovation) the economy will eventually reach its equilibrium, we argue that in fact this is not necessarily the case.

The fundamental idea upon which we build this latter assertion as well as the core of our theoretical work is probably best identified in the theory of the English neoclassical economist Arthur C. Pigou. In particular, the analysis of Pigou (1927) emphasizes for the first time the role of agents' expectations as the main factor through which cycles are generated. The basic idea works as follows: businessmen (i.e. firms) need to form some expectations on the future state of the economy so to take decisions regarding both their short-run operations and long-term investments. The economic system is complex, the state of the market is dynamically and discontinuously changing over time, and since firms have only a limited amount of information, their predictions are likely to be wrong. More precisely, according to Pigou, such predictions are going to be wrong *systemically* in the same direction because of the contagious nature of business opinion and the strong interdependencies among firms; i.e. not only errors do not cancel out at the aggregate level, but the mistakes of a few agents can influence the predictions of the majority if, for instance, those agents are believed to possess the best information.

Given the limited set of information available to agents, expectations are likely to be driven by what Pigou calls impulses. One can see these impulses as the discovery of new information or the occurrence of particular real, monetary, or even psychological circumstances. However, while they certainly play an important role in Pigou's pluralistic business cycle theory, it would be wrong to consider impulses as the essential prerequisite for fluctuations. In fact, the interpretation of expectations as an autonomously destabilizing process is characteristic of Pigou's thought. The idea that the economy can easily and quickly move from one period of great over-optimism to one of strong over-pessimism is clearly presented as a primary source of cycles.

Building on Pigou's theoretical work, in chapter 3 we are going to formalize the role of expectations in a simple model. We will provide evidence in favor of the fact that the dynamics of expectations alone can indeed be a sufficient element for fluctuations to arise and proliferate in the economy. More precisely, as long as the agents do not know exactly the structure and the dynamics of the system, the way they form their expectations and make their decisions can generate persistent fluctuations even while keeping constant the fundamentals and the equilibrium level of a simple economy, i.e. without the introduction of any impulse.

2.3 Keynesian Economics

In *The General Theory of Employment, Interest and Money*, Keynes suggests a theory of the business cycle based both on psychology and short-term economic analysis. According to Keynes, in the short-run, it is aggregate effective demand that determines the level of income, output and employment, and it is because of changes in aggregate demand that cycles occur. As aggregate demand consists of consumption and investment, it is the latter that is considered the primary factor responsible for the occurrence of fluctuations.

Investment is a function of the interest rate and the expected rate of return on capital, or, in Keynes' words, 'the marginal efficiency of capital'. Particular attention is reserved to this latter variable, which is supposed to be subject to cyclical and sudden variations.

The fundamental idea is that one cannot explain investment decisions using theories of rational choice. More specifically, there seems to be no reason to assume that agents will form their expectations on the rate of return of capital in a rational way. The implication is that most of the times expectations will not be correct and will generate instability. In Keynes' words:

> *"Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits – a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities."*[3]

That is, Keynes's observation is that there is a set of psychological

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[3] Keynes, John M. (1936) *The General Theory of Employment, Interest and Money*, Macmillan, London, pp.161-162

factors, what he calls ‘animal spirits’, that drives human behavior, resulting in systematic irrational decision-making. Most importantly we shall note that, even though it would probably be imprecise to reduce the whole concept of animal spirits to the only dynamics of expectations formation, that is certainly one way through which it is supposed to influence the economic outcome.

Furthermore, it is interesting to observe that not only Keynes underlines the irrelevance of mathematical expectations in the dynamics of agents’ behavior, but he also seems to suggest a rather unstructured and instinctive approach characterizing the methods of expectations formation. In comparison with this observation, our setup in chapter 4 will introduce a compromising view, where agents are assumed to act as rational econometricians, being unaware of the exact structure of the economy they live in and yet trying to forecast future prices with rational approaches, given the limited set of information they dispose of.

While we shall avoid digging further into the technical details of the Keynesian cycle, it is important to mention that, building on *The General Theory*, the so-called neo-Keynesian school suggested new explanations of the business cycle for the most part using only crude theories of investment. In particular, the interested reader can find illustrious examples in several versions of the multiplier-accelerator model such as in Samuelson (1939) and Hicks (1950).

However, the peculiarity of the neo-Keynesian approach is that, while paying particular attention to the development of an effective neoclassical synthesis of Keynes’s theory, it seems to forget the psychological part of it, which nonetheless plays a major role in the original approach. This, we argue, comes at a considerable loss of explanatory power. Fortunately, as we will have occasion to note in section 2.6, this gap has recently started to be filled by some authors from the new Keynesian camp, as well as the behavioral field. In fact, the present work serves also as our first effort in that direction.

2.4 New Classical Economics

New classical economics was developed starting from the 1970s as an alternative to the Keynesian approach. This theory is strongly characterized by its insistence on the importance of providing solid microfoundations as the basis for macroeconomic results. To do so, new classical economists introduce two fundamental assumptions in their models. First, all agents are optimizers; i.e. given a set of variables that they observe (e.g. prices, wages, interest rate, etc.), individuals make the best possible decision for their own interest. Second, all agents forecast the future using rational expectations.

From the technical point of view, the assumption of rational expectations, first introduced by Muth (1961), provides economic theorists with a powerful modeling tool, bringing the art of making formal predictions to a whole new level. However, we shall note, the use of rational expectations cannot be reduced to a mere technical expedient. In fact, from a theoretical point of view, it consists of a strong assumption on the behavior of individuals, which, as we know, is at the very basis of any economic system. Thus, we argue, one should be extremely careful about the implications of such an assumption within each specific settings, before claiming in favor of the generality of obtained results.[4]

However, this problematic has not stopped new classical authors from applying rational expectations to economic modeling, sometimes even with interesting and certainly famous results. An illustrious example can be found in Lucas (1976), whose critique undermines the validity of policy advice derived from large-scale macroeconometric models, such as those in the original Keynesian tradition. In fact, Lucas observes that the structure of an econometric model is the result of optimal decision rules of economic agents, but such decision rules are a multivariate function of several variables, including those factors through which economic policies are usually implemented, e.g. the money supply. Thus, if we change one or more of such variables, predictions based on

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[4] The interested reader can refer to Sargent (1993) and Evans and Honkapohja (2001, 2009) for a thorough analysis on the possibility to justify rational expectations as the limiting behavior of agents within an adaptive learning environment.

the assumption of constant decision rules, will reveal wrong.

Furthermore, following the well-known Lucas (1972)'s result on the neutrality of money, Sargent and Wallace (1975) develop the so-called 'policy ineffectiveness proposition'. According to this proposition, if public authorities try to use deterministic economic policies aimed at having countercyclical effects, the result will be an increased amount of noise in the economy without any effect on its average performance. That is to say, the central bank cannot systemically and effectively use monetary policy to boost employment and output. Thus, note that if the policy ineffectiveness proposition actually did hold in reality, then the role of central banks as economic stabilizers would be extremely reduced.

In fact, to provide further evidence on this matter, in chapter 6 we use our model to investigate the validity of the money neutrality result as in Lucas (1972). We will show that money neutrality holds only in the long run, while in the short term its validity is neither obvious nor general outside the rational expectations framework. This, in turn, seems to leave reasonable space for the exploitation of monetary policy.

In terms of pure cycle theory, the main new classical contribution has been the development of the equilibrium business cycle theory (EBCT), whose key and innovative aspect is the interpretation of the business cycle as an equilibrium phenomenon, rather than a disequilibrium event. Clearly, as it was already mentioned in section 2.1, there is at least an intuitive contrast between the concept of equilibrium and that of fluctuation, which makes the building of equilibrium models of endogenous business cycles a rather difficult task, particularly in (almost) perfect foresight settings such as those characterized by rational expectations.

As a consequence of the technical difficulty to generating equilibrium models of endogenous fluctuations, new classical economists have introduced different kinds of exogenous shocks into their artificial economies. Shocks to aggregate demand usually consist of unexpected changes in monetary or fiscal policy such as those in Lucas (1973, 1975), Barro (1980) and Brunner et al. (1983). Shocks to the supply side typically consist of exogenous variations in productivity and are at the base of the Real Business Cycle Theory (RBCT).

2.5 Real Business Cycle Theory

Real business cycle models, as first developed by Kydland and Prescott (1982), are characterized by the introduction of technological shocks as the main source of fluctuations within an otherwise stable dynamic general equilibrium economy. Proponents of the RBCT[5] oppose the view that monetary factors and eventual market failures have a decisive role in the determination of the business cycle.

In particular, in its most striking result, RBCT implies that fluctuations are not caused by any kind of market failure; instead, they are the consequence of optimal responses to exogenous shocks to real variables. That is, conditional on different realizations of the technology parameter, crises and booms become simply desirable events, during which the economy holds a constrained Pareto-efficient allocation of resources.

A direct implication of this result is that a policy of *laissez-faire* is in fact the optimal outcome in terms of the typical expected total welfare maximization problem. However, we shall note, not only this conclusion seems highly counterintuitive, but it also appears rather unsatisfying from the perspective of public authorities. This is the case in the sense that the government and the central bank are pronounced, at best, completely ineffective against the mighty power of random shocks, especially in view of the extremely high level of sophistication characterizing all the individuals in the economy.[6]

In fact, even if one was willing to blindly accept the belief that the fundamental source of fluctuations is indeed a series of exogenous technological shocks, we argue, it is the presence of such extremely sophisticated individuals, introduced through the rational expectations

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[5] The seminal references include, among others, Black (1982); Long and Plosser (1983); and Prescott (1986).

[6] Let us clarify this point further for the skeptical reader. Given any empirically observed state of the economy, *ceteris paribus*, the government and the central bank would clearly prefer (i) that such a state was *not* pareto-optimal and (ii) that they were able to affect it, so that a superior state could be achieved. In this sense the RBCT's result is highly unsatisfying for public authorities.

hypothesis (REH), that remains the strongest, most controversial and yet probably the most tolerated feature of RBC models. Indeed, it is essentially only thanks to the REH that one can argue in favor of such an outstanding efficiency of free markets, ruling out any possibility of beneficial external intervention.

Furthermore, we shall also note that, for RBC models to properly match empirical observations, they have to rely on large and persistent shocks, which in turn are not explainable on empirical grounds. This issue is clearly exposed in the analysis of Cogley and Nason (1995), who show that standard RBC models are characterized by weak internal propagation mechanisms. In fact, they observe that the persistence of fluctuations in this kind of models is almost exclusively due to the Solow residual, which is basically an exogenous component. However, in response to this kind of criticism, there have been several attempts of finding better propagation mechanisms, for instance by introducing labor market frictions such as in Mortensen and Pissarides (1994), Merz (1995) and Andolfatto (1996).

2.6 Recent Developments

Recent extensions of baseline RBC models have found an interesting solution to the critique of Cogley and Nason (1995) by considering the introduction of adaptive learning. In this kind of work, the empirical fit of a standard RBC model with rational expectations is usually compared to that of an identical model with adaptive learning. Cellarier (2008) and Huang et al. (2009), among others, provide clear evidence in favor of a better fit for the latter case; that is to say, the introduction of a learning environment seems to strengthen the internal propagation mechanisms of standard RBC models, significantly improving their empirical performance.

It would appear that one can find similar results by moving even further away from the REH, with the introduction of structural learning. In this case, the additional assumption is that agents have no more than an incomplete model of the economy, and they try to estimate unknown structural features by using historical data. Williams (2003) and Eusepi and Preston (2011) follow this kind of approach, documenting an even greater effect of adaptive learning as an endogenous source of fluctuations, compared to more standard learning environments.[7]

From a somewhat different perspective, Milani (2011) makes use of available survey data on economic expectations, together with a small scale new Keynesian model, to provide empirical evidence on the role of expectations as drivers of the business cycle. His analysis seems to confirm the importance of unexplained expectation shocks, interpreted as waves of undue optimism and pessimism, in explaining economic fluctuations. Theoretically, a similar result is obtained by Jaimovich and Rebelo (2007), who examine behavioral theories within a standard growth model, finding that expectation shocks tend to increase the volatility of cycles in their artificial economy.

Some have even tried to maintain the REH, while adding other exogenous impulses to the traditional technological shock. For instance, Beaudry and Portier (2004) introduce exogenous imperfect information signals that allow their economy to experience recessions even in absence of technological regress. Nonetheless, the unconvincing aspect of this approach remains its essential reliance on exogenous factors.

In the last few decades there have also been remarkable developments in the literature on sunspot equilibria (cf. Woodford, 1990; Howitt and McAfee, 1992; and Benhabib and Farmer, 1999) and in that on self-fulfilling expectations (cf. Grandmont, 1985; and Wen, 2001).[8] With respect of their approach, these two strands of literature are somewhat similar in that they both attempt to explain business cycles by relying on the existence of equilibria in which expectations drive individuals' behavior in a way that causes those same expectations to be fulfilled. In some cases (e.g. Farmer and Guo, 1994) this kind of approach has even been motivated in view of Pigou's theory of over-optimism and over-pessimism, as well as the Keynesian concept of animal spirits. However, we argue, this interpretation is rather misleading as it misses the point

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[7] Nonetheless, even in structural learning frameworks as those mentioned above, the technological shock is maintained as the fundamental source of instability.

[8] In fact it is not unusual to see these two literatures overlapping with each other.

that it seems to be the *erroneous* nature of people's expectations that drives business cycles according to both such theories.

Finally, we shall mention the presence of a rather young literature making use of agent-based modeling (cf. Paul, 2003; Dosi et al., 2006; and Lengnick, 2011). Agent-based models allow the treatment of extremely complex economies, generally with a large set of heterogenous agents and events that take place with different periodicity. This economies are usually claimed to be very realistic, and even though one cannot mathematically solve such complicated models, it is possible to obtain interesting output by running simulations. On the one hand, these models seem to show that it is indeed possible for a real economy to experience purely endogenous business cycles; but on the other hand, agent-based models are so complicated that one cannot really identify the dynamics behind their fluctuations.

In fact, in spite of the remarkable research effort in this area, to our knowledge, so far, nobody has ever developed a model of disequilibrium business cycles that is simple enough to be formally analyzed and properly understood (i.e. diverging from the agent-based approach), and that can generate persistent fluctuations without having to resort to any exogenous element. This, in fact, is the purpose of the following chapter.

Chapter 3

MODEL

3.1 Setup

Here we describe a very simple model of growth-less economy with N identical firms and H identical households. Firms produce homogeneous output that is used both as consumer and capital good. There is constant money stock M in the economy, and velocity of money is 1, so it holds that $Y_tP_t = M$. Firms use two production factors: capital and labor. In each period, capital, which equals real investments from the previous period, is being fully utilized. Firms plan their output and employment at the beginning of every period, and then stick to their decisions.

Each period one household disposes of M/H in cash, which is the sum of its labor income and savings from the previous period, implying:

$$M = HS_{t-1} + w_tNL_t \tag{3.1}$$

where w_t is nominal wage and L_t is employment per firm in period t. Thus, because of this restriction, there is a negative relation between employment and nominal wage. While seemingly counterintuitive, this result does not constitute a problem. Because of the minimal number of moving parts in the model, correlations and dependencies between existing variables are illustratively much higher than in real economy. Since higher employment leads to higher output and lower price, it makes perfect sense for the nominal wage to go down, following the

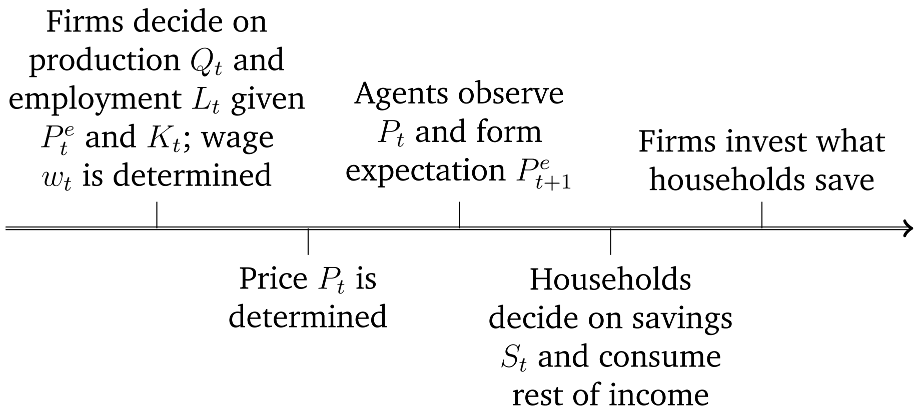

Figure 3.1: Timeline in Period t

price level.

There is no nominal interest rate in the economy. Still, households save part of their nominal income from various considerations, and each period they are kind enough to borrow their savings for firms to make investments. We assume that firms do not decide over capital and passively invest what households save.[1] This also ensures that the amount of money M is spent to purchase aggregate output. Since firms' cash inflow and outflow in each period equal exactly M, their retained profits are 0.

Figure 3.1 shows the timing of events in the model for each period t. First, each firm plans its production Q_t and decides on how much labor L_t it will employ given the available amount of capital (real savings from previous period per firm: $K_t = \frac{HS_{t-1}}{NP_{t-1}}$) and current wage (determined simultaneously). We assume that aggregate labor demand never exceeds labor supply, so that firms can always hire the desired amount of labor. Next, price in period t is determined from the quantity equation of money: $P_t = \frac{M}{Y_t}$ with $Y_t = NQ_t$. Households and firms observe P_t and form their expectation of P_{t+1}.[2] Having price expectations in place, households decide on how to split their nominal income between consumption and saving. Firms passively invest what households save,

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[1] While this is obviously a strong assumption for any particular firm, it makes sense at the macro level, especially in a interest rate free environment.

[2] In this simple model we explicitly assume that households and firms have the same expectations.

i.e. buy the good produced in the economy to use it as capital in the next period.

Having described the timing, we proceed by expressing and solving mathematically firms' and households' problems, assuming particular functional forms of the utility and production functions.

3.2 Firms and Economy Output

We assume that firms are price takers, i.e. they perceive themselves to be too small to influence market price with output. Furthermore, firms are assumed to have Cobb-Douglas production function, with technology A being constant over time.

Therefore, each firm's maximization problem at the beginning of period t is:

$$\max_{L_t} \left\{ P_t^e A K_t^\alpha L_t^\beta - w_t L_t \right\}. \tag{3.2}$$

One important remark should be made regarding this maximization problem. In particular, note that firms are maximizing their nominal income rather than the real one; moreover, expected price, which represents also the expected aggregate price level, enters the same problem that is solved to make decisions regarding real employment and output. Nonetheless, even though these properties may seem to contradict the widespread view that an increase in the general price level should not stimulate firms to produce more, as their real profits do not change, the specification in (3.2) is not problematic in this sense. Think of a firm, that perceives itself to be too small to alter price or wage by its decisions, making plans for the next period. The only thing that this firm can do is to maximize its nominal profit, as it automatically maximizes its real profit under given wage and expected price levels. To see that, divide (3.2) by P_t^e to obtain the equivalent maximization problem in real terms:

$$\max_{L_t} \left\{ A K_t^\alpha L_t^\beta - \frac{w_t}{P_t^e} L_t \right\},$$

where $\frac{w_t}{P_t^e}$ is nothing else than expected real wage. The first order condition yields an expression for the firm's decision L_t:

$$L_t = \left(\frac{\beta P_t^e A K_t^\alpha}{w_t} \right)^{\frac{1}{1-\beta}} . \tag{3.3}$$

So, each firm would be willing to hire more when wage is lower, expected price is higher, and the firm has larger capital stock, which increases the marginal productivity of labor

Since the nominal constraint (3.1) should be satisfied, we can substitute (3.3) into (3.1) and solve for nominal wage:

$$w_t = \left(\frac{N}{M - HS_{t-1}} \right)^{\frac{1-\beta}{\beta}} \left(\beta P_t^e A K_t^\alpha \right)^{\frac{1}{\beta}} . \tag{3.4}$$

Quite naturally, nominal wage positively depends on expected price level and available capital. Combining (3.3) and (3.4) we can get an expression for the actual amount of labor that each firm will hire:

$$L_t = \left(\frac{M - HS_{t-1}}{N \beta P_t^e A K_t^\alpha} \right)^{\frac{1}{\beta}} . \tag{3.5}$$

Strikingly, after the elimination of nominal wage, hired amount labor now depends negatively on expected price and available capital! This has quite a straightforward explanation, however: when price expectations and capital increase, the negative effect of wage on employment obviously surpasses the expected revenue benefit. To clarify this point even further, note that obviously firms still make decisions using (3.3). Instead, equation (3.5) should be regarded as the macroeconomic result, coming from mechanisms that are not observable by any particular firm.

Substituting (3.5) into the production function, we obtain the actual production per firm:

$$Q_t = \frac{M - HS_{t-1}}{N \beta P_t^e},$$

so that aggregate real output in the economy is:

$$Y_t = \frac{M - HS_{t-1}}{\beta P_t^e}. \tag{3.6}$$

As in the case of employment, aggregate output negatively depends on expected price. Even though exaggerated due to the simplicity of the model, this macroeconomic result is consistent with empirical evidence that price in fact is countercyclical (Kydland and Prescott, 1982).

Another observation is that output in this simple model does not depend on capital. The root of this result is the absence of interest rate and firms not deciding over investments. Treating available capital as constant, firms choose the amount of labor that equates marginal labor productivity with real wage. And since both marginal productivity and wage directly depend on capital, capital cancels out when computing actual output. This, however, is also consistent with the empirical observation that output has insignificant correlation with capital over the business cycle (Kydland and Prescott, 1982).

Note that since S_{t-1} is a function of past and expected prices as shown in the next section, price level is the only source of dynamics in this model.

3.3 Households and Savings

We will ignore the labor supply decision for now[3] and focus on the consumption-savings decision. For simplicity, we assume that households use logarithmic utility function to value current real consumption and real savings expressed in expected purchasing power next period. As we show in section 3.6, this simplifying assumption does neither affect nor cause the main features and results of the model, which are also valid under the most generic Hyperbolic Absolute Risk Aversion (HARA) utility function (algebra becomes cumbersome though).

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[3] As we have already assumed, labor supply always exceeds demand. Since the wage is also determined through firms' decisions and nominal constraint (3.1), we only need labor supply to calculate the unemployment rate for illustrative purposes, and it does not have any effect on the dynamics of the economy in this model.

Therefore, household's maximization problem[4] in period t becomes:

$$\max_{S_t}\left\{\ln\left(\frac{I_t - S_t}{P_t}\right) + \delta\ln\left(\frac{S_t}{P_{t+1}^e} + C\right)\right\}. \tag{3.7}$$

Here I_t and S_t are nominal income and savings of each household in period t; $0 < \delta \leq 1$ is the weight of the savings part in the utility, so that the weight on the consumption part is normalized to be 1; δ can also be seen as a subjective rate of time preference. $C > 0$ is a constant required to reduce marginal utility of savings; naturally, households expect to earn some income next period, that would bring marginal utility of savings down, but for simplicity we avoid modeling income expectations explicitly. Taking first order condition and solving for S_t, we obtain:

$$S_t = \frac{\delta}{1+\delta} I_t - \frac{C}{1+\delta} P_{t+1}^e. \tag{3.8}$$

Nominal savings depend positively on current nominal income, but negatively on expected price level next period. So, with simple logarithmic utility, only the substitution effect is at work, while the income effect of expected relative price change is absent. Indeed, a household saves more as expected real interest rate[5], i.e. $\frac{P_t}{P_{t+1}^e} - 1$, increases.[6] As shown in section 3.6, in case of HARA utility function, both effects are explicitly present in the savings function, and the substitution effect

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[4] It is further assumed that utility function is specified and being maximized at the household level. While it is a rather unconventional approach, it may represent reality better, since members of a household are expected to care about each other and derive utility from making others feel better. For example, members that are able to work, when deciding on their labor supply, will think not only about their own trade-off between utility of higher consumption and disutility of effort, but also of the fact that they have to take care of other members of their households that are unable to work. Similarly, those who are staying at home obtain utility from other members spending more time with them instead of working. Even though we have not proven it formally, we believe that one utility function may better represent this complicated set of synergies than the sum of individual utility functions.

[5] Remember that there is no contracted nominal interest rate in the model. Still, following Grandmont (1985), decrease in future price can be seen as real income, as if households were paid a real interest rate (which can, obviously, also be negative if price level increases).

[6] Some empirical evidence for this can be found, for example, in the overview by Elmendorf (1996).

needs not necessarily dominate the income effect any more. However, as mentioned already, this does not affect the main findings of the paper.

3.4 Equilibrium Price Level and Output

In the model described above, expected price level determines output, which in turn determines actual price through the quantity equation of money. Note that the nominal income of each household is predetermined to be $I_t = \frac{M}{H}$. Substituting it in (3.8), putting the result in (3.6), and finally combining it with the quantity equation of money, we get the actual law of motion for price:

$$\begin{aligned} P_t &= \frac{M}{Y_t} \\ &= \frac{\beta M}{M - HS_{t-1}} P_t^e \\ &= \frac{\beta M(1+\delta)}{M + HCP_t^e} P_t^e. \end{aligned} \tag{3.9}$$

Let us define equilibrium in the model as the state in which the price level and output reach some fixed values P^* and Y^* and stay constant over time. Then it should also be the case that economic agents form correct expectations $P^{e*} = P^*$; if this was not the case, agents would adjust their expectations for the next period causing the price not being constant over time.

Call $D(P_t^e) = \frac{\beta M(1+\delta)}{M+HCP_t^e}$ the price expectation multiplier, which itself is a decreasing function of P_t^e. It is straightforward to see that when D is greater than 1, economic agents underpredict price; when $D < 1$, they overpredict the price; finally, when $D = 1$, economic agents form correct expectations of price.[7]

After substituting P^* in (3.9) instead of all price variables, it is straightforward to see that equilibrium price in the model would be that satisfying $D(P^*) = 1$. Indeed, this condition guarantees that agents form correct expectations, and because of that, following a widely accepted

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[7] The careful reader might have already noted that an economically less relevant case of equilibrium is that with $P^{e*} = P^* = 0$.

terminology, this equilibrium can be characterized as the Rational Expectations Equilibrium (REE). Still, note that our model in general is not built on the assumption of rational expectations. Solving for equilibrium price yields:

$$P^* = \frac{M(\beta(1+\delta)-1)}{HC}. \tag{3.10}$$

So, the equilibrium price is proportional to money stock: any change in money stock would cause exactly the same percentage change in the equilibrium price level. Now we can also compute the equilibrium output in the economy:

$$Y^* = \frac{HC}{\beta(1+\delta)-1}. \tag{3.11}$$

A first important observation from (3.11) is that equilibrium output is independent from the money stock. This is consistent with the mainstream view that monetary policy has no real effect and results only in inflation in the long run. Still, there could be room for stabilizing monetary policy in the short run, and this question will be extensively analyzed later. A second observation is that equilibrium output is proportional to population, so that if population growth were introduced, it would cause equilibrium output to grow at the same pace. Even though we are leaving economic growth out of analysis for now, this observation is very important for a sanity check.

3.5 Possibility of Fluctuations

The possibility of fluctuations driven by adaptive learning and expectations arises from the fact that the *actual law of motion* (ALM) function (3.9) is nonlinear in expected price. The intuitive explanation goes as follows. Economic agents know neither the ALM nor the equilibrium values of price and output. Moreover, as shown above, they always overestimate (underestimate) future prices when they are higher (lower) than P^*. The existence of expectation errors causes them to learn, i.e. to update their tools used for forecasting to get more precise predictions.

If, for example, agents keep overpredicting price for a while, they will eventually revise their forecasting tool to generate lower predictions, and vice versa. However, as they approach equilibrium from either side, not knowing what the equilibrium level is, they need not necessarily stop there and may enter a zone where the sign of forecast errors reverses. After that, agents start to revise their expectation tool in the opposite direction, and so on and so forth. Cycles that thereby arise remind those in Pigou's opening quotation: errors of optimism alternate with errors of pessimism. Note that these cycles need not necessarily be decaying. Conducted simulations show that they may in fact diverge, depending on the model fundamentals and learning rules, as shown in chapter 4.

It is also important to mention that it need not necessarily be the case that agents do not know where the equilibrium is. It is sufficient that they perceive themselves to be too small to affect market price and that they are unable to cooperate to reach the equilibrium together. If they realize that equilibrium is not going to happen, they simply want to get the most precise forecast of future price to plan production and savings.

3.6 Model with Hyperbolic Utility Function

In this section we show that the main results of the model preserve when we assume that households use the Hyperbolic Absolute Risk Aversion (HARA) utility function, and therefore do not rely on the simplifying assumption of logarithmic utility. The uninterested reader may proceed directly to chapter 4.

Analyzing the model with HARA utility function is, in fact, just one step short of a generic analysis, which is outside the scope of this thesis and left for future work. The HARA utility function presented in its standard form

$$U(W) = \frac{1-\gamma}{\gamma}\left(\frac{\alpha W}{1-\gamma} + b\right)^{\gamma};\ \alpha > 0,\ \frac{\alpha W}{1-\gamma} + b > 0 \tag{3.12}$$

nests almost all special cases used in the literature: linear utility when $\gamma = 1$; quadratic utility when $\gamma = 2$; constant absolute risk aversion (CARA) exponential utility function if $b = 1$ and $\gamma \to -\infty$; and power utility function if $\gamma < 1$ and $\alpha = 1 - \gamma$, which in turn nests constant relative risk aversion (CRRA) utility function ($b = 0$) and logarithmic utility ($\gamma \to 0$). [8]

Preserving the assumptions from section 3.3 that households only optimize over two periods and do not forecast their incomes, household's maximization problem with HARA utility function becomes:

$$\max_{S_t} \left\{ \frac{1-\gamma}{\gamma} \left(\frac{\alpha(I_t - S_t)}{(1-\gamma)P_t} + b_c \right)^{\gamma} + \delta \frac{1-\gamma}{\gamma} \left(\frac{\alpha S_t}{(1-\gamma)P^e_{t+1}} + b_s \right)^{\gamma} \right\}. \tag{3.13}$$

Analogously to the simple case with logarithmic utility, we claim that it should be the case that $b_s > b_c$ when $\gamma < 1$, and $b_s < b_c$ when $\gamma > 1$, as, given all other parameters of utility items equal, we should account for the fact that households *will* receive income in period $t + 1$, so that marginal utility from future consumption of current savings should be adjusted downwards. Taking the first order condition and solving for savings, we obtain:

$$S_t = \frac{\delta^{\frac{1}{1-\gamma}} I_t + \frac{(1-\gamma)}{\alpha} \left[b_c \delta^{\frac{1}{1-\gamma}} P_t - b_s \left(\frac{P^e_{t+1}}{P_t} \right)^{\frac{\gamma}{1-\gamma}} P^e_{t+1} \right]}{\left(\frac{P^e_{t+1}}{P_t} \right)^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}}}. \tag{3.14}$$

Let us have a close look at equation (3.14). It simplifies exactly to (3.8) when $\alpha = 1 - \gamma$, $\gamma \to 0$, and $b_c = 0$. Next, we will eliminate these specific parameter restrictions one by one and see what new characteristics each of them brings to the savings function.

First, if we make $b_c \neq 0$, current price P_t will start having effect on savings. So that savings now depend not only on *expected* inflation, but also on *observed* one. If $b_s > b_c > 0$, then savings positively depend on current price level and negatively on expected inflation, i.e. the

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[8] An overview of utility functions can be found in many textbooks on financial economics, e.g. in Cuthbertson and Nitzsche (2004)

substitution effect is at work. Indeed, in this case savings correlate positively with expected real interest rate $\frac{P_t}{P^e_{t+1}} - 1$. If, on the other hand, $b_c < b_s < 0$, then only the *income* effect plays a role.

Secondly, eliminating the condition of $\gamma \to 0$ leads to the savings function becoming nonlinear in prices. Whether savings depend positively or negatively on current and expected prices is now ambiguous. We postpone thorough analysis of this issue until we get to the ALM function. Finally, easing $\alpha = 1 - \gamma$ further adjusts the impact of prices in the numerator of (3.14), even though it appears to be the least important effect.

As before, we obtain the actual law of motion of price by substituting (3.14) into (3.6), and combining the resulting equation for output with the quantity equation of money:

$$P_t = \frac{\beta M \left(\left(\frac{P^e_t}{P_{t-1}} \right)^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}} \right)}{M \left(\frac{P^e_t}{P_{t-1}} \right)^{\frac{\gamma}{1-\gamma}} + H \frac{(1-\gamma)}{\alpha} \left[b_s \left(\frac{P^e_t}{P_{t-1}} \right)^{\frac{\gamma}{1-\gamma}} P^e_t - b_c \delta^{\frac{1}{1-\gamma}} P_{t-1} \right]} P^e_t, \tag{3.15}$$

where the price expectation multiplier $D(P_{t-1}, P^e_t)$ can be defined as the fraction in front of P^e_t.

The derivation of equilibrium price and output is analogous to that in section 3.4: for equilibrium price to be consistent with learning and expectations formation, it should be the case that $D(P^*, P^*) = 1$. The obtained equations are:

$$P^* = \frac{\alpha M \left(\beta + \beta \delta^{\frac{1}{1-\gamma}} - 1 \right)}{H(1-\gamma) \left(b_s - b_c \delta^{\frac{1}{1-\gamma}} \right)}, \tag{3.16}$$

$$Y^* = \frac{H(1-\gamma) \left(b_s - b_c \delta^{\frac{1}{1-\gamma}} \right)}{\alpha \left(\beta + \beta \delta^{\frac{1}{1-\gamma}} - 1 \right)}. \tag{3.17}$$

Even though equations (3.16) and (3.17) are a bit more complicated than (3.10) and (3.11), they preserve the most important messages of the latter. Firstly, equilibrium price is proportional to money stock M. And secondly, equilibrium output is proportional to population and independent from the money stock, so that money stock is still neutral in the long run.

Having said that, it is now important to make several comments about the possibility of cycles in this setting. As discussed in section 3.5, cycles may emerge because of nonlinearity of the actual law of motion, particularly from the fact that agents always overestimate (underestimate) future prices when they are higher (lower) than P^*. So that, in Pigou's terms, errors of optimism will be forced to turn into errors of pessimism, and vice versa.

Therefore, the necessary condition for fluctuations to emerge is that the price expectation multiplier D should be lower than 1 when prices are higher than equilibrium level and greater than 1 when prices are below the equilibrium. In fact, this is also a sufficient condition for the equilibrium to be stable, as if that was not the case, the economy would theoretically either shrink or diverge. Note, however, that this condition is not sufficient to observe cycles; whether fluctuations will actually emerge a great deal depends on how agents learn and form their expectations. We address this question in chapter 4.

However, what is not clear from the above paragraph is *which prices* should be compared with equilibrium level: expectation P_t^e or previous realization P_{t-1}? A somewhat inaccurate, but very simplifying and intuitive answer would be *both*. When what might be called the "general level of prices", i.e. both realized and expected prices, is high, then obviously D should be smaller than 1, and vice versa.

What complicates this simplistic view is that $D(P_{t-1}, P_t^e)$ is nonlinear in both prices. It may well happen also that while realized price from the previous period is still above (below) P^*, expected price falls below (above) the equilibrium level. Moreover, the analysis is greatly complicated also by the fact that the ratio of prices matters: the sensitivity of D to P_t^e, for example, will a great deal depend on P_{t-1}. A quick look at the first derivatives $D'_{P_{t-1}}$ and $D'_{P_t^e}$ obtained in *Mathematica* confirms that further formal analysis would be an extremely challenging task taking a lot of time and paper space, so we decided to leave it of out of the scope of this thesis. The rationale behind this decision is best expressed by Willem Buiter's words: "a privately and socially costly waste of time and other resources" (Buiter, 2009). Indeed, taking into account the simplicity of the underlying assumptions of our model, there is no much value added, if any, from deriving very precise and cumbersome requirements for parameters in particular functional forms, except, maybe, demonstrating our strong mastery of algebra.

We proceed instead, just to give an example, with a fairly simple and intuitive case, in which we can reduce the analysis of behavior of the function of two variables to a single-variable function. In particular, let us assume that agents have a very short memory and always expect that currently observed price will be the same next period, i.e. $P_t^e = P_{t-1} = \overline{P_t}$. Then the "general level of prices" mentioned above becomes a single variable. However, we should bring the reader's attention to the fact that this simplification leads to the omission of the impact of expected real interest rate, or expected relative price change, on the savings decision. Expected real interest rate under this assumption is simply always 0. Having said that, let us have a look at the resulting equation for the price expectation multiplier:

$$D(\overline{P_t}) = \frac{\beta M \left(1 + \delta^{\frac{1}{1-\gamma}}\right)}{M + H\frac{(1-\gamma)}{\alpha}\left(b_s - b_c \delta^{\frac{1}{1-\gamma}}\right)\overline{P_t}} \tag{3.18}$$

Since this function should be decreasing in $\overline{P_t}$ for equilibrium to be stable and to allow fluctuations to emerge theoretically,[9] it should hold that $\frac{(1-\gamma)}{\alpha}\left(b_s - b_c \delta^{\frac{1}{1-\gamma}}\right) > 0$, which is equivalent to $C > 0$ in the simple case with logarithmic utility. Given that $\alpha > 0$, it translates into the requirement that (i) $b_s > b_c \delta^{\frac{1}{1-\gamma}}$ when $\gamma < 1$, and (ii) $b_s < b_c \delta^{\frac{1}{1-\gamma}}$ when $\gamma > 1$.

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[9] In fact, as will be discussed in chapter 4, fluctuations will not emerge if expectations are formed using an AR(1) model, which is exactly the case here.

Chapter 4

SIMULATION RESULTS

4.1 Economic Agents as Rational Econometricians

In the previous chapter we have described the general setup of our model, i.e. how the economy works and responds to actions of firms and households, in particular to changes in their expectations. Now it is time to say something about how these expectations are formed.

One of our aims was to make agents behave more realistically than in the DSGE models. On the other hand we did not want to engage into incorporating various documented behavioral patterns in the model since (i) it is hard to distinguish which of them are the most important for economic fluctuations and (ii) there is also a risk of ending up with a very cumbersome model where it is not crystal clear how results are obtained. While acknowledging that the incorporation of findings from behavioral economics could become a very powerful strand of research and future work, we instead decided to maintain the assumption of rational economic agents, but to make their rationality *bounded*.

Among the variety of possible ways to introduce bounded rationality, we found the approach described by Thomas Sargent in his book *Bounded Rationality in Macroeconomics* (1993) to be the most promising. Rather than knowing the whole underlying model of the economy, i.e. the actual law of motion, forming correct mathematical expectations of future outcomes which may not come true only because of exogenous shocks, and solving infinite horizon inter-temporal optimization problems, as usually the inhabitants of DSGE models do, boundedly rational agents instead are only left with the possibility to observe available economic data, treat it *rationally*, produce forecasts over finite horizons, and, therefore, solve limited optimization problems. Based on this description Sargent has developed a type of agents which he himself called *'rational econometricians'*.

The idea is that agents should deal with available data as econometricians would: try to build and estimate as good an econometric model as possible. While Sargent goes further in his book and discusses how e.g. AI could be used for this purpose, we stick to the simplest cases. Essentially, agents in our simulations estimate pre-specified econometric models using available data, make forecasts based on those models, and make economic decisions as described in chapter 3. This approach is also often referred to in the literature as *adaptive learning*, even though the latter one is a broader concept.

It is important to notice that while Sargent and the bulk of literature[1] on adaptive learning and expectations usually study models and attempt to find the conditions under which adaptive learning converges to *rational expectations equilibrium* (REE), we consider adaptive learning as a source of potential instability in our setting.

Let us now turn directly to the simulations.

4.2 Auto ARIMA in the Simple Logarithmic Utility Case

In what follows we assume that economic agents try to model price as a time series without considering the possibility that other economic variables may be useful in forecasting it. A short discussion is needed here. On the one hand this assumption can be justified by the fact that, by design, all variables in the model are functions of realized and

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[1] see e.g. Evans and Honkapohja (2001)

expected price, and therefore there is no mechanical causality from e.g. unemployment rate to future price. On the other hand, if observed economic variables affect price expectations of economic agents, they effectively impact the actual price realizations. Moreover, observing the co-movements in various economic variables may help firms and households to learn faster how the economy works, and make the system more stable. A larger number of variables can also cause confusion and become an additional source of instability. In short, the consequences of relaxing the above-mentioned assumption are ambiguous and depend on the particular learning tool that agents use.

We start with probably the most interesting and, at the same time, hardest to study formally learning tool: Auto ARIMA model. The idea is that every period agents select the best (according to some chosen criterion) ARIMA model based on the available price data and use it for forecasting. In this way we allow agents not only to update the coefficients in their econometric model, but to change model specification as well. Once again, this may either make the learning process faster and the economy more stable, or become an additional source of instability. Also, switching models in favor of seemingly better ones may actually not be far from real life behavior.

To conduct the simulations we use R programming language. For ARIMA model selection the Bayesian information criterion (BIC) is chosen and *auto.arima()* function from package *forecast* is used. Each period, after the price expectation is formed and aggregate output Y is determined, the latter is rounded to three digits after the decimal point to prevent small and unobserved differences in the series to affect the model selection. We take the logarithm of price data before feeding them into *auto.arima()*.[2] The number of households and firms is 100 and 10 respectively, and money stock M equals 100. Technology A is unity and does not change over time. We assume constant return to scale so that $\alpha + \beta = 1$. Each simulation starts with the initial price

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[2] There are two reasons for this: (i) taking the logarithm of prices prevents forecasts from being negative; (ii) if agents take the first difference of logged price data (which *auto.arima()* can suggest), then they will be working with inflation rates, and it makes economic sense.

expectation 1% above P^*, and then evolves as a purely deterministic process, without any kind of exogenous shocks. After the very first period agents have only one observation of price, which is therefore their best guess for the second period, and which is exactly produced by *auto.arima()*. But as more price data becomes available, agents build more sophisticated ARIMA models.

A quite expectable question that may be asked by the careful reader is whether the fact that we start our simulations off the equilibrium can be seen as an external shock. Our view is that there is absolutely no reason why agents should know and expect the equilibrium price level at the "beginning of history." If they are lucky enough to guess P^* right away, then indeed no fluctuations will emerge. But this situation is no more likely than them starting at $1.01 \cdot P^*$ or at any other level.

Figure 4.1a plots the output Y[3] from a simulation with labor production share $\beta = 0.6$ and subjective discount rate $\delta = 0.7$. All other parameters are computed so that equilibrium output Y^* equals 100 and $P^* = 1$. The vertical grey lines indicate points where agents switch their models. The first and most important observation is that output indeed fluctuates around its equilibrium level (horizontal line). Secondly, observed cycles are of varying length and amplitude: something that economists usually fail to achieve in standard RBC models.

The amplitude of output peaks roughly in the middle of the plot, when Y drops by around 4% of its equilibrium level. An interesting fact is that it is preceded by several drastic revisions of the models: in period 159 agents switch from ARMA(2,5) to AR(1); then, in period 160 they switch to ARMA(2,2); and in period 163 they choose ARMA(3,1), which they maintain until the end of the simulation. This observation may suggest an interesting hypothesis that major revisions of the agents' beliefs about how the economy functions may cause economic crises.

Figure 4.1b demonstrates that cycles can be also diverging. With $\beta = 0.55$ and $\delta = 0.95$, the amplitude of output fluctuations is constantly increasing. Note that the frequency of switching between model specifi-

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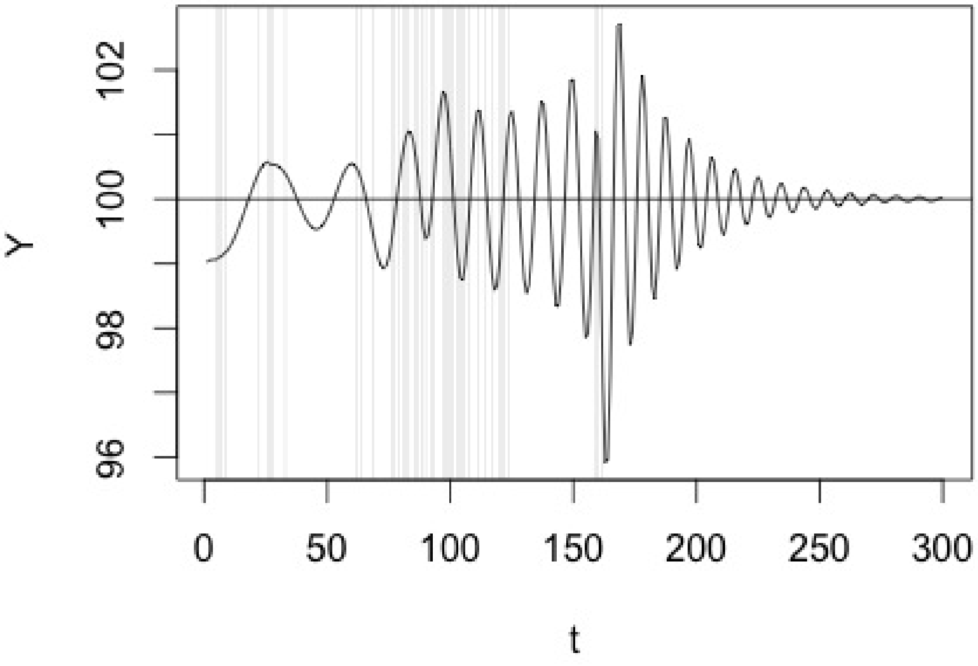
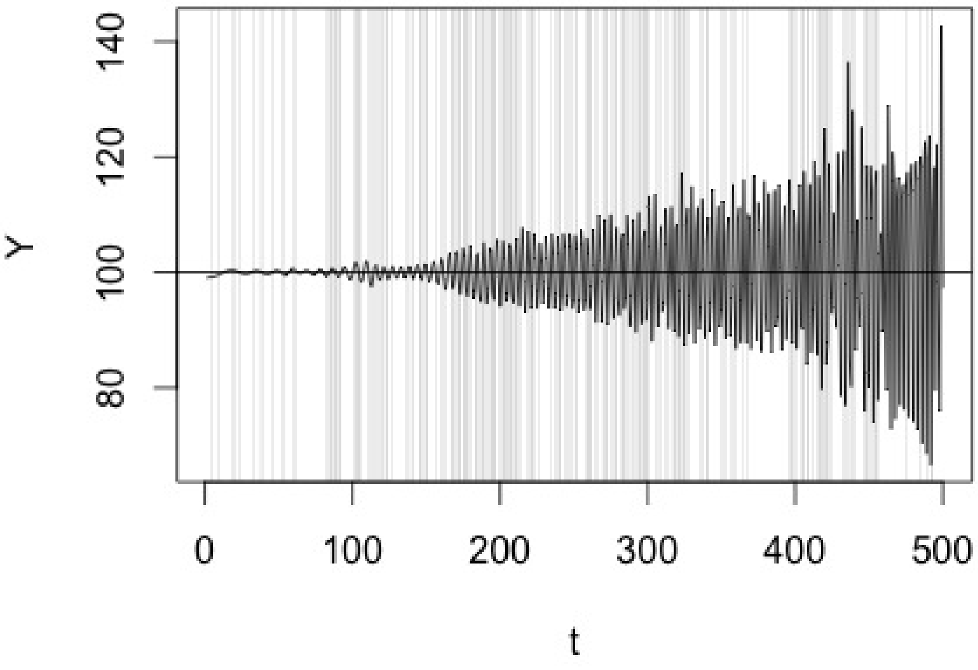
[3] The plot of prices always mirrors that of output because of the quantity equation of money together with fixed M.



(a) $\beta = 0.6$, $\delta = 0.7$



(b) $\beta = 0.55$, $\delta = 0.95$

Figure 4.1: Cycles in Simple Logarithmic Case

cations is much higher here than on figure 4.1a. This supports the claim that model switching may also be a serious source of instability.

It should be mentioned that in some cases of seemingly diverging simulations, convergence towards equilibrium was observed in the long run, while in other cases authors observed diverging oscillations with any simulation horizon.[4] Formal analysis of this issue is needed, and in chapter 5 we will introduce a possible approach for future work on this line.

4.3 Unemployment

It was assumed in the model setup that households' labor supply does not matter when employment, wage, production and prices are determined; firms are always able to hire the required amount of labor and wage is determined through labor demand and nominal constraint (3.1). However, if we wanted to show the behavior of unemployment in our model, it would not be sufficient to just show the actual employment, as it reflects only firms decisions. It could well happen that households' labor supply is not constant and varies as economic conditions change.

To obtain labor supply in the economy, we modify the household's maximization problem (3.7) to incorporate utility from leisure:[5]

$$\max_{S_t, l_t} \left\{ \ln \left(\frac{w_t Z l_t + S_{t-1} - S_t}{P_t} \right) + \delta \ln \left(\frac{S_t}{P^e_{t+1}} + C \right) + \mu \ln \left(Z(1 - l_t) \right) \right\}. \tag{4.1}$$

Here Z is the size of one household, e.g. the number of people, l_t is the fraction of employed individuals in each household,[6] and μ is a coefficient representing the importance of leisure for households. In (4.1) households *perceive* that they can increase their nominal income

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[4] Though it was not proved that these indeed diverge forever.

[5] We continue with our assumption that utility being maximized at the household level.

[6] For example, it would be a normal situation when a household of $Z = 7$ people consists of 4 elders, 2 working-age persons and one child. Then one would expect $l_t = 2/7$ under normal economic conditions. l_t can also be seen in a more traditional way: as a fraction of the individual's time allocated to work.

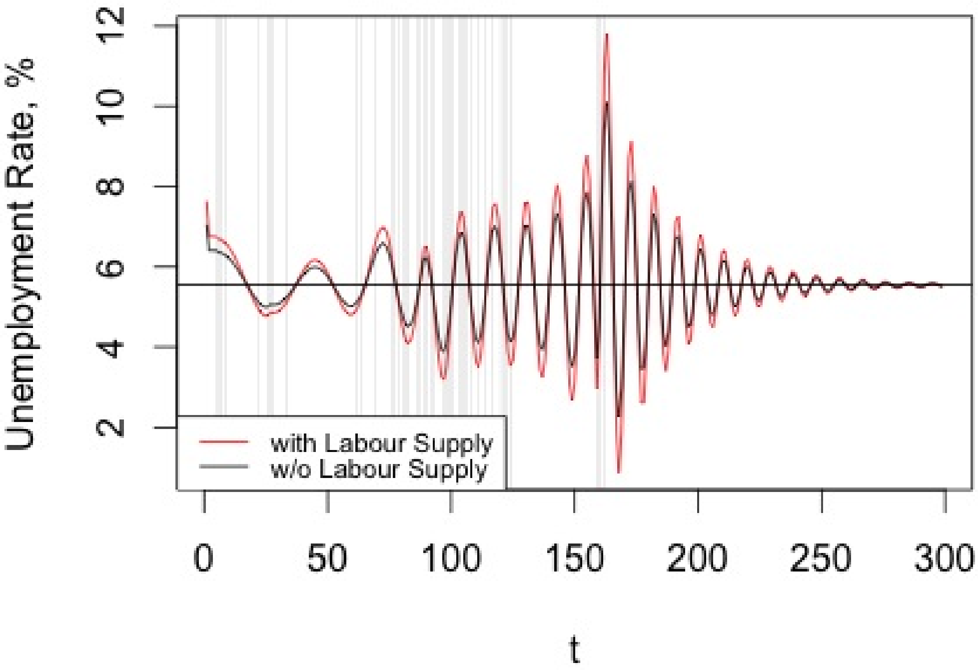


Figure 4.2: Umeployment Rate: $\mu = 2.65$

by working more through $w_t Z l_t$, even though in the end employment is decided by firms and households only decide over savings as in (3.7). Taking first order conditions, solving for Zl_t, and multiplying by the number of households H, we obtain total labor supply in the economy:

$$L_t^S = \frac{H}{1+\mu+\delta}\left(Z(1+\delta) - \mu\frac{S_{t-1} + CP_{t+1}^e}{w_t}\right). \tag{4.2}$$

Labor supply (i) is proportional to the number of households; (ii) depends negatively on savings from the previous period, as households have less incentive to increase their nominal income; (iii) depends negatively on expected price next period, as it reflects the fact that households will be able to buy less for the same amount of labor provided; (iv) depends positively on wage w_t. Let us now turn to simulation.

Figure 4.2 plots the unemployment rate from the simulation with the same parameters as on figure 4.1a in section 4.2. The black line shows what the unemployment rate would be if households did not decide on how much labor they would like to supply, i.e. L^S is constant at its equilibrium level; while the red line shows the behavior of unemployment taking into account changes in labor supply. What we see on the plot, namely that the red line is more volatile than the black one, proves the hypothesis that changes in labor demand from firms tell only half of the story, and that the actual unemployment rate is more volatile, since households are willing to work more during recessions and less during economic booms.

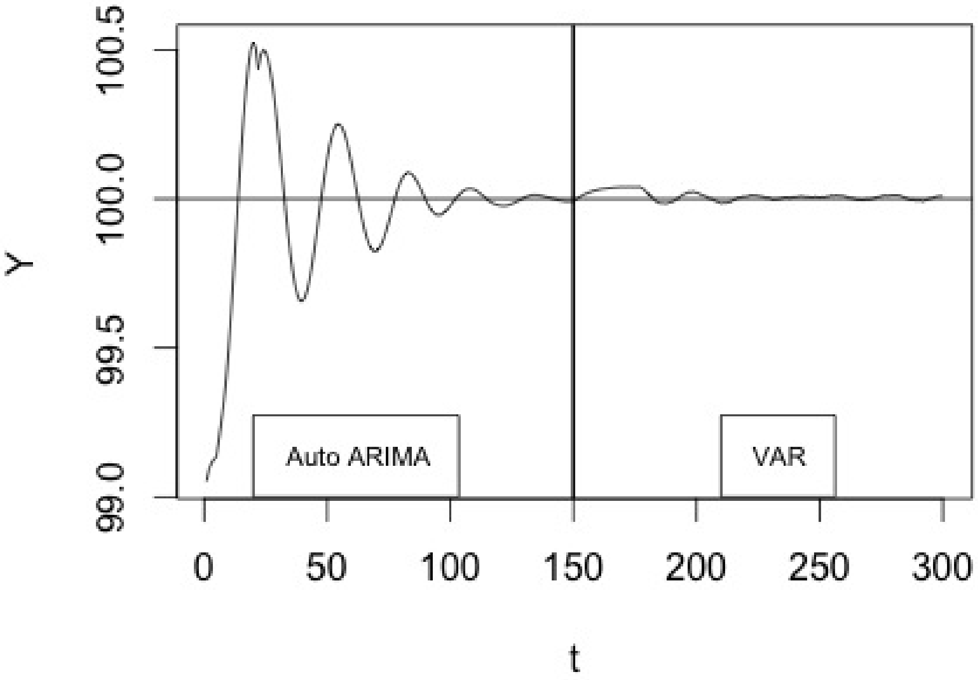
It is worth mentioning also that the effect of labor supply may be understated in the above simulation due to the fact that we use logarithmic utility, which is usually associated with “myopish” behavior. We suspect that actual unemployment may be even more volatile in alternative utility specifications, and studying unemployment under generic functional forms is worth effort in the future.

4.4 Multivariate Models

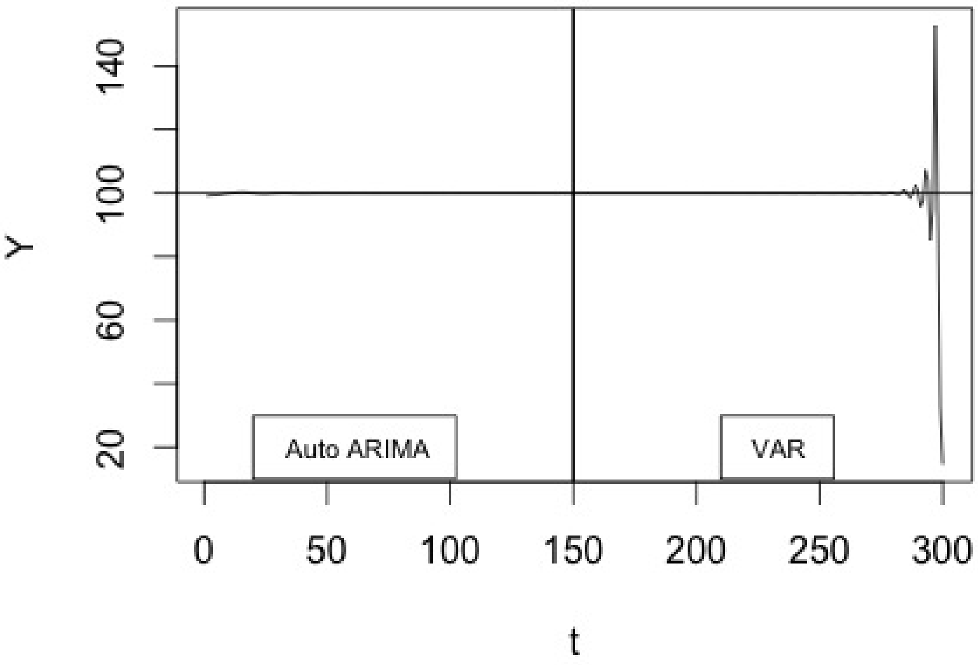
It was mentioned in section 4.2 that allowing agents to build multivariate models for predicting price has ambiguous consequences. In this section we quickly sketch the preliminary results from simulations with agents using Vector Autoregressive (VAR) models.

In the following simulations we added wage as the additional variable. As estimating VAR models requires having some dataset already, in the first half of the simulations data is generated with the *auto.arima()* function as in section 4.2, and then agents switch to VAR. We use the package *vars* in R. In every period, the function *VARselect()* is used to choose the order of VAR model, and then the function *VAR()* is used to estimate the value of the parameters of the model. As before, we take the logarithm of price and wage data before feeding them into the VAR model to prevent the generation of negative forecasts in the case of diverging cycles. Figure 4.3 shows examples of obtained simulations.

Switching to VAR models turned out to be extremely destabilizing in our simulations. Even though there were cases when, following a quickly converging to equilibrium Auto ARIMA data generating process, the VAR part exhibited negligible fluctuations around the equilibrium,



(a) $\beta = 0.55$, $\delta = 0.9$



(b) $\beta = 0.6$, $\delta = 0.95$

Figure 4.3: Cycles with Agents Using VAR Models

as shown on figure 4.3a, the overwhelming majority of simulations resulted in rapidly diverging output once agents switch to VAR. Figure 4.3b shows a typical simulation. The last few cycles have such a large amplitude that all the previous fluctuations are hardly distinguishable from the equilibrium line; also, after period 300 the function *VARselect()* reported an error, as no suitable VAR model could be found any more.

Needless to say that the above result has to be treated with caution, however, as it is obtained with the simplest simulations imaginable. It is based on the assumptions that agents know only reduced-form modeling, and are not able to develop “good” structural economic models.

4.5 Simulation with HARA Utility

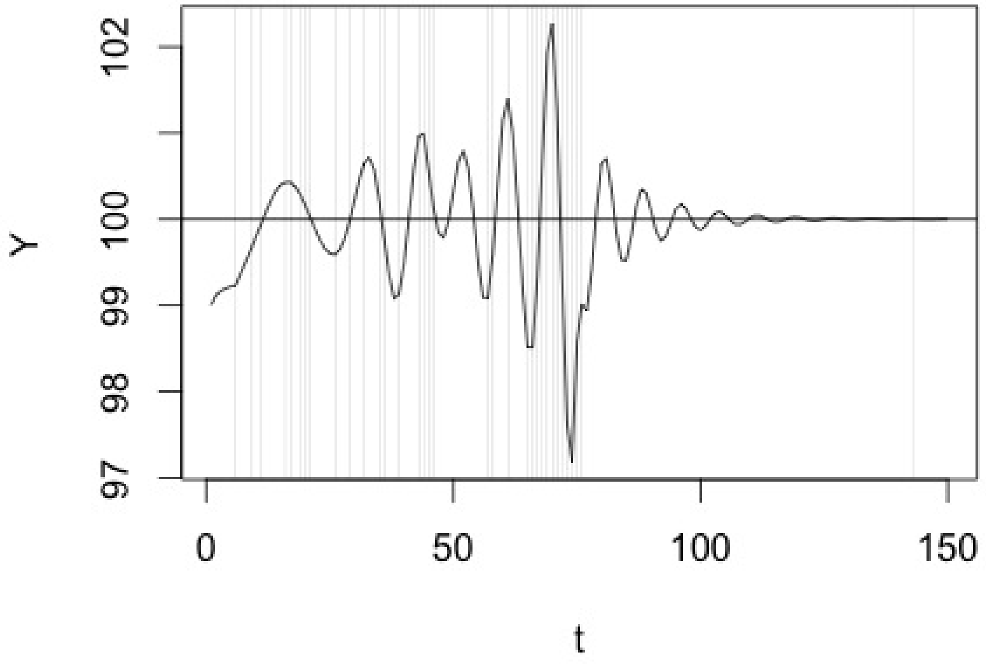
It was shown in section 3.6 that the obtained results hold theoretically when we switch from the simplifying assumption of logarithmic utility to more generic utility functions, in particular Hyperbolic Absolute Risk Aversion (HARA) utility.

Figure 4.4 shows that we can obtain both converging (figure 4.4a) and diverging (figure 4.4b) cycles when the parameters of the HARA utility function are selected so that it does not simplify to one of the popular utility function types. As before, we assume that agents use Auto ARIMA. Constant return to scale is assumed, and the parameter b_s is computed so that $P^* = 1$ and $Y^* = 100$.

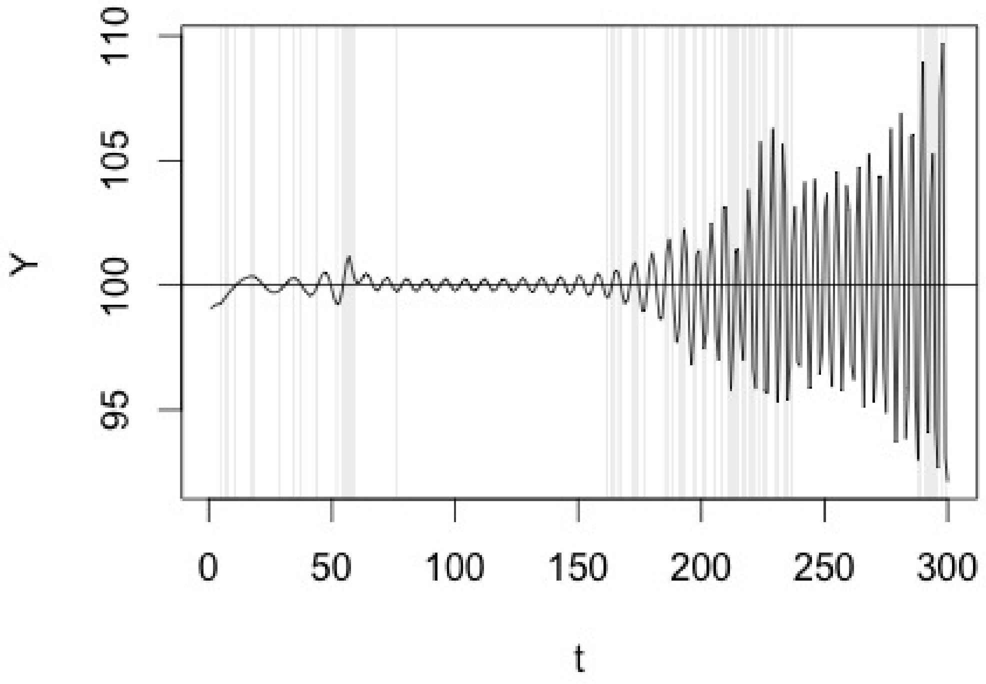
On both figure 4.4a and 4.4b we can see that the amplitude of fluctuations increases when the frequency of model switching increases. This is more evidence in support of the hypothesis that revisions by agents of their beliefs about how the economy works may trigger instability.

4.6 AR(2) and “False Equilibria”

All previous simulations were based on models with changing specifications, and therefore are very hard to study formally. Here we consider a simpler case when agents have a pre-specified model and only update



(a) $\beta = 0.6$, $\delta = 0.85$, $b_c = 0.03$,
$\gamma = -0.4$, $\alpha = 2$



(b) $\beta = 0.6$, $\delta = 0.8$, $b_c = 0.2$,
$\gamma = -1$, $\alpha = 1$

Figure 4.4: Cycles with Agents Having HARA Utility

parameters over time. Two things motivate us for this exercise. Firstly, it is itself value-adding to show that cycles can emerge and proliferate in the economy even if agents do not switch model specification; model switching is then considered only as an *additional* source of instability. Secondly, sticking to one particular model specification opens the possibility of formal analysis of how economy behaves in the long run, in particular whether it diverges or not. More analysis of this question will follow in chapter 5.

Our choice of the specification to study is the AR(2) model. It is fairly simple, yet yields business cycles and shows some unexpected and interesting behavior. Also, it seems to be the case that for fluctuations to emerge, one of the fundamental economic variables (most often output) should follow an autoregressive process at least of order 2. This is essentially achieved when agents use AR(2); all the simulations with them using AR(1) resulted in price level and output smoothly converging to their equilibrium levels. It is very interesting and worth noting that two of the most popular models of business cycles, the Multiplier-Accelerator model by P. Samuelson and the RBC model by Kydland and Prescott, in different ways yield equations in which output depends on its values in the previous 2 periods, essentially following an AR(2) process.

Figure 4.5 shows the simulation with $\beta = 0.7$ and $\delta = 0.45$, in which agents estimate the following AR(2) model:

$$P_t = \phi_0 + \phi_1 P_{t-1} + \phi_2 P_{t-2}, \tag{4.3}$$

with the initial values of parameters $\phi_0 = 0.5$, $\phi_1 = 1.6$, and $\phi_2 = -1$. These parameters are used in the first 5 periods, and after that, agents start to update them. Additionally, we introduce memory of length 300. This means that agents never use more than the 300 most recent price observations to estimate the parameters of their model.

Quite expectedly, output in the simulation exhibits cyclical fluctuations. However, as we can see, in the first 500 periods or so, it fluctuates not around its equilibrium level, but way below it, and only after cy-

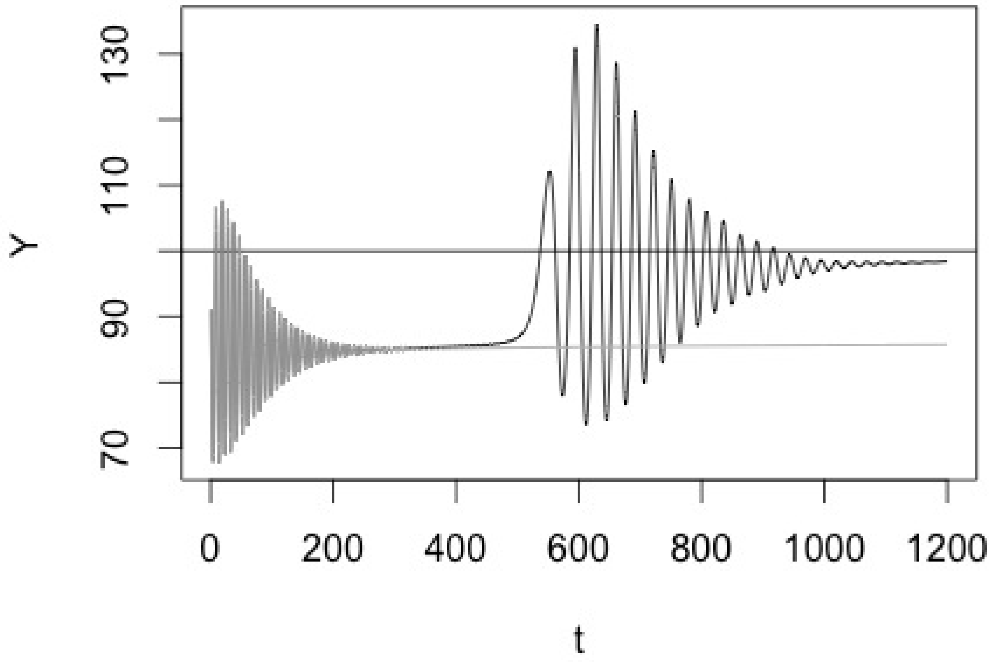


Figure 4.5: AR(2) Model with Memory and "False Equilibria": $\beta = 0.7$, $\delta = 0.45$

cles almost die out, it starts to fluctuate once again, even with higher amplitude than before, this time around a value much closer to the equilibrium level. What happens here is that memory comes to play its role, and when instability at the beginning of the simulation is "forgotten", agents suddenly realize that they are far off the equilibrium.

However, if memory was not introduced, this second wave of cycles would never emerge, and output instead would follow the light-gray line, extremely slowly converging toward the equilibrium.[7] The speed of convergence in this second case is so slow, that one can say that the economy is stuck in a "false equilibrium".[8] This notion very much reminds of Keynes's belief that the economy may end up being in a depressed state for a surprisingly long time, and therefore active government intervention is needed to "cheer up" the animal spirits. In our

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[7] This happens due to the fact that in this case the expectation multiplier $D(P_t^e)$ is only marginally different from unity.

[8] Even though this is not an equilibrium technically speaking, and, of course, not an exactly constant value over time.

simulation the animal spirits were enlivened by short memory.

Chapter 5

EXPECTATIONAL STABILITY

In this chapter we present and attempt the application of some standard methods for the analysis of the limiting behavior of stochastic systems with adaptive learning.[1] This is a highly technical part, the main purpose of which is to build a solid basis for future analysis; the uninterested reader may proceed directly to chapter 6.

Instead, for further examination of the methods introduced in this chapter, the excited reader can refer to Sargent (1993), as well as to Evans and Honkapohja (2001, 2009).

5.1 Learning as a Recursive Algorithm

In the course of this chapter, unless otherwise specified, we only consider the version of our model in which households have logarithmic utility. Moreover, we proceed by assuming that agents use an AR(2) model to forecast future prices based on past observations. This choice is justified by the fact that this model provides an effective trade-off between simplicity and usability.[2] In other words, our assumption is that the perceived law of motion (PLM) is AR(2), so that agents form their

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[1] Please note that in fact we are going to apply these methods to a deterministic system.

[2] On the one hand, the simplicity of an AR(1) model does not allow for the fluctuations to arise, making this case uninteresting from an economic perspective. On the other hand, auto ARIMA or VAR models have rather complex dynamics, making their formal analysis an extremely challenging task.

expectations fitting the following process:

$$P_t = \phi_0 + \phi_1 P_{t-1} + \phi_2 P_{t-2} + \varepsilon_t,$$

where ε_t is an i.i.d. error term with zero mean. As an immediate result, we observe that

$$P_t^e = \phi_0 + \phi_1 P_{t-1} + \phi_2 P_{t-2}.$$

Combining (3.9) with (5.1) we obtain the ALM as a function of observed prices:

$$P_t = \frac{\beta M(1+\delta)(\phi_0 + \phi_1 P_{t-1} + \phi_2 P_{t-2})}{M + HC(\phi_0 + \phi_1 P_{t-1} + \phi_2 P_{t-2})}. \tag{5.1}$$

Given a set of starting values ϕ_0, ϕ_1, ϕ_2 and observations P_i with $i = 1, 2, ..., t$, it is possible to express the learning process (i.e. the dynamics of ϕ_0, ϕ_1, ϕ_2) over time by using the recursive least squares (RLS) algorithm. The result is the following system of two equations:

$$\boldsymbol{\Phi_t} = \boldsymbol{\Phi_{t-1}} + t^{-1}\boldsymbol{R_t^{-1}}\boldsymbol{p_{t-1}}(P_t - \boldsymbol{p_{t-1}^\top}\boldsymbol{\Phi_{t-1}}) \tag{5.2}$$

$$\boldsymbol{R_t} = \boldsymbol{R_{t-1}} + t^{-1}(\boldsymbol{p_{t-1}}\boldsymbol{p_{t-1}^\top} - \boldsymbol{R_{t-1}}) \tag{5.3}$$

where $\boldsymbol{p_t} := (1, P_t, P_{t-1})^\top$, $\boldsymbol{\Phi_t} := (\phi_{0_t}, \phi_{1_t}, \phi_{2_t})^\top$ and $\boldsymbol{R_t}$ is the moment matrix for $\boldsymbol{p_t}$ using observations P_i with $i = 1, ..., t$. From (5.1), note that in fact P_t is a function of the previous period parameters $\boldsymbol{\Phi_{t-1}}$ and observations $\boldsymbol{p_{t-1}}$; that is, $P_t := f(\boldsymbol{\Phi_{t-1}}, \boldsymbol{p_{t-1}})$. Using this result in (5.2), defining $S_{t-1} := R_t$ and moving (5.3) one period ahead, we obtain:

$$\boldsymbol{\Phi_t} = \boldsymbol{\Phi_{t-1}} + t^{-1}\boldsymbol{S_{t-1}^{-1}}\boldsymbol{p_{t-1}}(f(\boldsymbol{\Phi_{t-1}}, \boldsymbol{p_{t-1}}) - \boldsymbol{p_{t-1}^\top}\boldsymbol{\Phi_{t-1}}) \tag{5.4}$$

$$\boldsymbol{S_t} = \boldsymbol{S_{t-1}} + t^{-1}\left(\frac{t}{t+1}\right)(\boldsymbol{p_t}\boldsymbol{p_t^\top} - \boldsymbol{S_{t-1}}). \tag{5.5}$$

Formally, the system (5.4)-(5.5) is a recursive algorithm fully describing agents' learning dynamics at any time in the future, for given initial values. Our aim is now to investigate if and how it is possible to use this system to understand the asymptotic properties of our model.

5.2 Theory of Stochastic Approximation

Consider a (stochastic) recursive algorithm of the form

$$\boldsymbol{\theta_t} = \boldsymbol{\theta_{t-1}} + \gamma_t Q(t, \boldsymbol{\theta_{t-1}}, \boldsymbol{x_t}(\boldsymbol{\theta_{t-1}})), \tag{5.6}$$

where $\boldsymbol{\theta_t}$ is the vector of parameter estimates, $\boldsymbol{x_t}$ is the vector of observations, and γ_t is a deterministic sequence of gains.

Following Evans and Honkapohja (2001), we know that it is possible to study the limiting behavior of any recursive algorithm in the form of (5.6), by analyzing its associated ordinary differential equation (ODE). The latter is defined as

$$\frac{d\boldsymbol{\theta}}{d\tau} = h(\boldsymbol{\theta}(\tau)), \tag{5.7}$$

where $h(\boldsymbol{\theta})$ is given by

$$h(\boldsymbol{\theta}) = \lim_{t\to\infty} E\left[Q(t, \boldsymbol{\theta}, \bar{\boldsymbol{x}}(\boldsymbol{\theta}))\right], \tag{5.8}$$

with $E[\cdot]$ denoting the mathematical expectation operator and $\bar{\boldsymbol{x}}(\boldsymbol{\theta})$ being the value of $\boldsymbol{x_t}$ obtained by holding $\boldsymbol{\theta_t}$ fixed at its limiting value, i.e. $\boldsymbol{\theta_t} = \boldsymbol{\theta}$.

Most importantly, from stochastic approximation results,[3] it is possible to establish a direct correspondence between possible limiting points of the recursive algorithm, and locally stable equilibria of the ODE (5.7).

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[3] cf. Marcet and Sargent (1989)

5.3 An Application to Our Setting

Let us attempt an application of this result to our case. First of all, note that the system (5.4)-(5.5) is indeed in the form of (5.6) with

$$\begin{aligned}\boldsymbol{\theta_t} &= \text{vec}(\boldsymbol{\Phi_t}\;\; \boldsymbol{S_t}) \\ \boldsymbol{x_t} &= (1, P_t, P_{t-1}, P_{t-2})^\top \\ \gamma_t &= t^{-1}.\end{aligned}$$

Therefore, we can use (5.4)-(5.5) to retrieve the explicit form of $Q(\cdot)$, so to study its properties. For simplicity we split the function into two components as follows:

$$Q_\Phi(t, \boldsymbol{\theta}, \bar{\boldsymbol{x}}) = \boldsymbol{S^{-1}}\bar{\boldsymbol{p}}(f(\boldsymbol{\Phi}, \bar{\boldsymbol{p}}) - \bar{\boldsymbol{p}}^\top \boldsymbol{\Phi}) \tag{5.9}$$

$$Q_S(t, \boldsymbol{\theta}, \bar{\boldsymbol{x}}) = \left(\frac{t}{t+1}\right)(\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^\top - \boldsymbol{S}). \tag{5.10}$$

We can now use (5.9) and (5.10) to find the ODEs associated to the system (5.4) - (5.5):

$$h_\Phi(\boldsymbol{\Phi}) = \lim_{t\to\infty} E\left[\boldsymbol{S^{-1}}\bar{\boldsymbol{p}}\left(\frac{\beta M(1+\delta)(\bar{\boldsymbol{p}}^\top \boldsymbol{\Phi})}{M + HC(\bar{\boldsymbol{p}}^\top \boldsymbol{\Phi})} - \bar{\boldsymbol{p}}^\top \boldsymbol{\Phi}\right)\right] \tag{5.11}$$

$$h_S(\boldsymbol{S}) = \lim_{t\to\infty} E\left[\frac{t}{t+1}(\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^\top - \boldsymbol{S})\right]. \tag{5.12}$$

First, it is important to observe that in our case everything inside the expectation operator is deterministic. That is, there is no stochastic part that might prevent us from taking expectations of each single component separately.

Second, let us call $\hat{\boldsymbol{p}} := (1, \hat{P}, \hat{P})^\top$ the mathematical expectation of $\bar{\boldsymbol{p}}$. Note that either $\hat{\boldsymbol{p}} = (1, P^*, P^*)^\top$ or $\hat{\boldsymbol{p}} = (1, 0, 0)^\top$ because for any other value of $\hat{\boldsymbol{p}}$ a fixed $\boldsymbol{\Phi}$ would not be sustainable, since the learning process would cause it to change. Therefore, it is not rational to expect any value of $\hat{\boldsymbol{p}}$ other than those two.

Furthermore, defining $\boldsymbol{W} := E[\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^\top] = \hat{\boldsymbol{p}}\hat{\boldsymbol{p}}^\top$, it is easy to see that

(5.12) becomes

$$h_S(\boldsymbol{S}) = \boldsymbol{W} - \boldsymbol{S},$$

implying the first condition for equilibria of the ODE to be $\boldsymbol{W} = \boldsymbol{S}$. Replacing this result in (5.11) and solving, we obtain

$$h_\Phi(\boldsymbol{\Phi}) = \boldsymbol{\Phi} \left(\frac{\beta M(1+\delta)}{M + HC(\hat{\boldsymbol{p}}^\top \boldsymbol{\Phi})} - 1 \right). \tag{5.13}$$

This is the core ODE defining possible limiting points of our recursive algorithm (5.4)-(5.5).

Even though formally we should speak of equilibria of the ODE in terms of $\boldsymbol{\Phi}$, it is more illustrative to present them in terms of P^e. In fact, we can see that (5.13) has two equilibria in terms of P^e. The first one corresponds to the solution of

$$\frac{\beta M(1+\delta)}{M + HC(\hat{\boldsymbol{p}}^\top \boldsymbol{\Phi})} - 1 = 0$$

and that is $P^e = P^*$, while the second one corresponds to $\boldsymbol{\Phi} = 0$ and implies $P^e = 0$.[4] Interestingly enough, we will see that the second equilibrium can be disregarded since it is locally unstable for any meaningful choice of parameters.

However, before moving to the determination of local stability, it is interesting to note that the analysis of (5.13) is indeed greatly complicated by the nonlinearity of the ALM.[5] Intuitively, this seems to be a further confirmation of the importance of this factor in our model for the provision of adequate dynamics for the generation of cycles. That is, non-linearities seem to play an important role in the functioning of the economy, suggesting that heavy reliance on linear approximations might cause a loss of important features of the economic system.

To investigate the stability of these fixed points, we derive the Jaco-

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[4] Note that indeed these equilibria correspond to the two points that were identified in chapter 3. Furthermore, please note also that technically there is an infinite number of equilibria identifiable in terms of $\boldsymbol{\Phi}$.

[5] cf. Evans and Honkapohja (2001) for cases with linear ALM.

bian matrix of the function on the right-hand side of (5.13). Defining $A := \frac{\beta M(1+\delta)}{M+HC(\hat{\boldsymbol{p}}^\top \boldsymbol{\Phi})}$ and $B := \frac{-HC}{M+HC(\hat{\boldsymbol{p}}^\top \boldsymbol{\Phi})}$, it is possible to express the Jacobian as

$$J = \begin{pmatrix} (A-1)+(AB)\phi_0 & (AB\hat{P})\phi_0 & (AB\hat{P})\phi_0 \\ (AB)\phi_1 & (A-1)+(AB\hat{P})\phi_1 & (AB\hat{P})\phi_1 \\ (AB)\phi_2 & (AB\hat{P})\phi_2 & (A-1)+(AB\hat{P})\phi_2 \end{pmatrix},$$

where either $\hat{P} = P^*$ or $\hat{P} = 0$.

From fundamental results on the stability of ODEs, we know that if all the eigenvalues of J evaluated at a fixed point have negative real parts, then that fixed point is locally asymptotically stable. Instead, if at least one eigenvalue of J evaluated at a fixed point has positive real part, then that fixed point is locally asymptotically unstable.

Finally, let us define $K := J - \lambda I$, where λ is a 1×3 vector and I is the 3×3 identity matrix. To obtain the eigenvalues of J, we need to solve $det(K) = 0$.

5.4 Convergence for the Case of $\boldsymbol{\Phi} = 0$

We are now going to consider local stability of the equilibrium of (5.13) characterized by $\boldsymbol{\Phi} = 0$. As already mentioned, this case can also be expressed as $P^e = P = 0$.

Other than a simple example, this analysis is relevant in that a credible economic model can afford neither to allow its agents to expect a zero price nor certainly to permit the economy to actually reach that level; even more so considering that in the latter case production would be infinite. That is to say, even if in reality it might be a desirable situation, we certainly do not want to model an economy where production can be infinite and everything can be free of charge.

First of all, let us note that in this case the matrix K is greatly simplified by the fact that $\phi_0 = \phi_1 = \phi_2 = 0$. Let us call K_0 the matrix resulting from such a simplification of K. Then,

$$K_0 = \begin{pmatrix} [\beta(1+\delta)-1]-\lambda & 0 & 0 \\ 0 & [\beta(1+\delta)-1]-\lambda & 0 \\ 0 & 0 & [\beta(1+\delta)-1]-\lambda \end{pmatrix},$$

implying that J has only one eigenvalue with multiplicity 3, i.e. $\lambda_1 = \lambda_2 = \lambda_3 = \beta(1+\delta) - 1$. Thus, the condition for $P^e = P = 0$ to be locally asymptotically stable becomes:

$$\begin{aligned} \beta(1+\delta) - 1 < 0 \\ \beta < \frac{1}{1+\delta}. \end{aligned}$$

However, by looking at (3.10) it is easy to note that for this condition to be verified, the equilibrium price P^* has to be negative.[6] This, in turn, implies that for any positive equilibrium price P^*, the economy will never converge to the fixed point with $\mathbf{\Phi} = 0$ and $P^e = P = 0$ unless it starts from it, which is clearly never the case in our setting. That is, for any reasonable set of parameters, our economy will never converge to an economically meaningless equilibrium solution.

One more conclusion that can be drawn from this result is that $\hat{\boldsymbol{p}} = (1, P^*, P^*)^\top$ for all the relevant cases. In fact, if $P^* > 0$ and the economy does not start with $\mathbf{\Phi} = 0$, then we have just proved that it will never converge to a point with $P^e = P = 0$. Thus, one should not mathematically expect it to do so.

5.5 Notes on Convergence for $P^e = P^* > 0$

Even though the equilibrium of (5.13) characterized by $P^e = P^* > 0$ is more cumbersome to treat analytically, we can at least simplify the matrix K in view of our latter result. In fact, by noting that we can use (3.10) to substitute for P^*, and that in this case:

1) $\hat{\boldsymbol{p}} = (1, P^*, P^*)^\top$,

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[6] This is the case because M, H and C must all be positive to make economic sense.

2) $P^e = P^*$,

3) $P^* = \phi_0 + \phi_1 P^* + \phi_2 P^*$,

one can obtain the following ‘simplified’ version of K:

$$K_{P^*} = \begin{pmatrix} (a-1)(1-\phi_1-\phi_2)-\lambda & \frac{(a-1)^2}{ba}(1-\phi_1-\phi_2) & \frac{(a-1)^2}{ba}(1-\phi_1-\phi_2) \\ (ba)\phi_1 & (a-1)\phi_1-\lambda & (a-1)\phi_1 \\ (ba)\phi_2 & (a-1)\phi_2 & (a-1)\phi_2-\lambda \end{pmatrix},$$

with $a := \frac{1}{\beta(1+\delta)}$ and $b = \frac{-HC}{M}$.[7]

Having obtained K_{P^*}, with some tedious algebra it is possible to show that in this context the eigenvalues of the Jacobian matrix are given by the solutions of

$$\begin{aligned} \lambda^3 - \lambda^2(a-1) + \lambda\Big((a-1)\phi_1\phi_2(a-1-ba)\Big) \\ -\Big((a-1)^2(1-\phi_1-\phi_2)\phi_1\phi_2(a-1-ba)\Big) = 0. \end{aligned} \tag{5.14}$$

It is certainly possible to solve analytically (5.14) by using a standard cubic formula. However, in view of the complexity of the solution, it would be hard to make economic sense out of it.

A smarter approach would be to consider the case in which $P^* = 1$, exactly as we imposed in all simulations in chapter 4. Note that since $P^* = \frac{a-1}{ba} = 1$, equation (5.14) simplifies to

$$\lambda^3 - \lambda^2(a-1) = 0, \tag{5.15}$$

and it is now easy to see that the solutions of (5.15) are

$$\begin{aligned} \lambda_1 &= (a-1) \\ \lambda_2 &= 0. \end{aligned}$$

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[7] Please note that this implies $P^* = \frac{a-1}{ba}$.

However, while $\lambda_1 < 0$ is indeed verified for all cases with $P^* > 0$,[8] the presence of a null eigenvalue complicates the analysis in the sense that it would require us to consider at least terms of second-order (i.e. the Hessian matrix) of (5.13) to formally identify the exact conditions for stability, and therefore convergence.

In line with the exemplary and introductory role of this chapter we decide not to follow that path. However, an interesting and powerful approach that could be taken in future research is the application of numerical methods in the attempt of shedding light on this kind of mathematical problems.

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[8] Obviously this is the case because we assume, as it should be done, that M, H and C are all positive.

Chapter 6

STABILIZING MONETARY POLICY

> *A central banker comes into the caffe and orders a cake. The waiter asks:*
> *– Would you like it cut into 6 or 12 pieces?*
> *The central banker replies:*
> *– 6, please, I do not feel like eating 12.*

In this chapter we will analyze the possibility of stabilizing monetary policy in our simple model. As already noted, this model is characterized by long-run money neutrality; i.e. any permanent increase in M would lead to a proportional increase in the equilibrium price P^* (see equations (3.10) and (3.16)), leaving the equilibrium level of output Y^* unaffected (see equations (3.11) and (3.17)). Still, a possibility exists that shifts in the money stock may affect output in the short-run, making stabilizing monetary policy possible and the above joke not funny any more.

6.1 Monetary Policy with Simple Logarithmic Utility

Let us first investigate whether monetary policy is effective in the setting where households have a simple logarithmic utility function. To conduct this formal analysis, we change the previously constant money stock M into M_t. The actual law of motion of price from (3.9) becomes:

$$P_t = \frac{\beta M_t(1+\delta)}{M_t + HCP_t^e} P_t^e, \tag{6.1}$$

while the ALM for output is:

$$Y_t = \frac{M_t}{\beta(1+\delta)P_t^e} + \frac{HC}{\beta(1+\delta)}. \tag{6.2}$$

Looking at equation (6.2) as a function of time, it is straightforward to note that it contains a fixed part and a variable part, with the latter depending positively on the money stock M_t and negatively on price expectation P_t^e.

The immediate observation from equation (6.2) is that monetary policy does not have any real effect even in the short run if economic agents know exactly the upcoming change in the money stock *and* adjust their initial price expectation by the same proportion; i.e. if they know that the money stock is going to increase by $x\%$ next period and they consequently increase their price expectation by exactly $x\%$. It is easy to see from (6.1) that in this case actual price will also increase by $x\%$. Even without any formal proof, it is intuitive to see therefore that such behavior of agents is consistent with learning experience: if actual price always responds to change in the money stock exactly as agents expect it to, they have no reasons to change their beliefs.

In light of this result, there are only two cases in which short-term stabilizing monetary policy can actually work in this model: (i) the monetary policy is in part unexpected, i.e. agents do not know exactly what money supply will be in the next period; (ii) agents believe that changes in the money stock, for some reason, do not imply exactly the same change in prices (at least in the short run). While the first case belongs to a rather political discussion on transparency of central banks and not much on it can be said here, we will have a closer look at the second one.

Let us investigate what happens if agents believe that a $x\%$ change in the money stock does not cause a proportional shift in price level, resulting instead in a percentage change of price equal to $\psi \cdot x\%,\ \psi > 0$. This would imply that the derivative of price expectation with respect to the money stock equals: $\frac{\partial P_t^e}{\partial M_t} = \psi \frac{P_t^e}{M_t}$. Furthermore, it is easy to verify that the derivative of actual price with respect to the money stock is:

$$\frac{\partial P_t}{\partial M_t} = \beta(1+\delta)\frac{M_t^2 \frac{\partial P_t^e}{\partial M_t} + HCP_t^{e2}}{(M + HCP_t^e)^2},$$

which, after substituting $\frac{\partial P_t^e}{\partial M_t}$, becomes:

$$\frac{\partial P_t}{\partial M_t} = \beta(1+\delta)\frac{\psi M_t + HCP_t^e}{(M + HCP_t^e)^2} P_t^e. \tag{6.3}$$

For the so called ψ-belief to be sustainable within an adaptive learning environment, it should be the case that actual price responds to changes in the money stock also with the rate ψ, implying $\frac{\partial P_t}{\partial M_t} = \psi \frac{P_t}{M_t} = \beta(1+\delta)\frac{\psi}{M + HCP_t^e} P_t^e$. However, It is straightforward to see that if $\psi < 1$ ($\psi > 1$) the 'responsiveness' of actual price is greater (smaller) than ψ. Therefore, any value of ψ other than unity is not sustainable from the learning perspective, because at some point agents would realize that their belief is not coming true and update it in the correct direction. The question is, however, whether they will converge to $\psi = 1$ or end up fluctuating around this value.

If they do converge, then the ultimate conclusion is that monetary policy may initially have short-term real effects, but it will become ineffective in the longer perspective when (and if!) learning is completed.

On the other hand, as long as agents keep changing their beliefs regarding ψ over time, stabilizing monetary policy will remain possible. However, as ψ will be changing, the same will do the responsiveness of price and output to monetary shocks according to (6.3). Therefore, even assuming that a central bank knows exactly how the economy functions, it would also need to know the market sentiment, i.e. what economic agents expect from monetary policy, to operate effectively. Same shifts in the money stock may have very different effects depending on economic situation.

6.2 Stabilizing Policy in a General Setting

The conclusions made in the previous section change even further in favor of the effectiveness of monetary policy if we consider the actual law of motion of output when agents use a more generic HARA utility function:

$$Y_t = \frac{M_t \left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} + H\frac{(1-\gamma)}{\alpha}\left[b_s\left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} P_t^e - b_c\delta^{\frac{1}{1-\gamma}}P_{t-1}\right]}{\beta P_t^e\left(\left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}}\right)} \tag{6.4}$$

Indeed, now even if agents knew exactly the upcoming shift in money stock and adjusted their expectations proportionally, these adjustments simply would not cancel out as they did in (6.2). Therefore, monetary interventions will always have real effects on the economy!

However, central bankers should refrain from celebrating too early. In fact, only if they knew precisely how the economy works, knew the exact values of all the coefficients in the model, and, moreover, knew very well agents' price expectations and how these expectations react to news about monetary policy; in other words, if they were some kind of economic gods, then indeed they would be able to conduct a very effective stabilizing monetary policy.

There is no reason to believe, however, that central bankers know significantly much more about the economy than economists at research departments of large private companies and banks. After all, they usually go to the same schools and study the same economic theories and econometric methods. The hard truth is also that private institutions usually offer higher salaries. To sum up, it is highly doubtful that central banks can assess accurately the consequences of their own actions, and therefore they should not abuse the power they have.

Having said that, we should cheer up our disillusioned central bankers and tell them that monetary policy is not going to be thrown away. As someone has said, it is better to do something imperfectly than do nothing perfectly. If the economy is obviously falling into a recession, or is already in a crisis, monetary intervention will have a real (and hopefully positive) effect, and therefore might be used. Not to mention the situation of “false equilibrium” shown in section 4.6, in which case monetary policy is one of the instruments able to move the economy back on track.

Chapter 7

DISCUSSION AND CONCLUDING REMARKS

In this concluding chapter we would like to quickly systemize our findings and point out the most interesting and relevant directions for future work. The contributions of this thesis largely belong to two major fields of economics: business cycle theory and effectiveness of stabilizing monetary policy. We will shortly describe our findings in a consecutive order, discussing possible improvements and extensions on the way.

I.

With a simple, growth-less macroeconomic model, we show theoretically that business cycles can emerge endogenously in the economy when agents do not have perfect foresight, learn adaptively to form expectations, and solve limited inter-temporal optimization models. This possibility largely arises from the nonlinearity of the actual law of motion of price, in particular from the fact that agents always overpredict (underpredict) future prices when they are higher (lower) than equilibrium level. Even though the main version of the model is based on households having simple logarithmic utility function, we also show that our results hold when a more generic Hyperbolic Absolute Risk Aversion utility function is chosen. Interestingly enough, Money stock is neutral in the long run in either case.

We see three ways in which our model should be developed further. Firstly, we believe that the most important next step would be to abandon the use of particular functional forms for the utility and production functions, and move to the analysis of a fully generic setting. A greater degree of generality is preferable in that it would potentially allow the derivation of necessary and sufficient conditions on the shape[1] of the utility and production functions for the emergence and proliferation of cycles. This mighty task however, requires more time and maturity of the research, and will appear in future publications.

Secondly, the model can be extended to become more realistic. Introducing growth, making agents forecast and optimize over longer horizons, incorporating additional important economic variables such as contracted interest rate, could all greatly enhance the usability of the model, if done in a simple and elegant way.

The third thing that could be done is deriving the same results in a completely alternative setting. Developing a "more standard, more IS-LM'y, with-Philips-Curve'y" model[2] could bring this line of research to a qualitatively new level, perhaps even making its message accessible to a more general public.

II.

We conduct simulations in models with agents having both simple logarithmic and HARA utility functions. Following Sargent (1993), we assume agents to be "rational econometricians" using various econometric adaptive learning tools: Auto ARIMA, VAR and AR(2) models. In all simulations, output and other economic variables indeed display cyclical fluctuations around their equilibrium levels, without the introduction of any kind of exogenous shocks to productivity or any other fundamental parameters of the economy, in contrast to RBC models. To our knowledge this thesis is the first attempt to formally introduce

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[1] As opposed to deriving conditions on parameters, that often becomes cumbersome when parametrization rises.

[2] A very valuable advice given by Prof. George Akerlof at the ISEO Summer School 2013, for which we are extremely grateful to him.

adaptive learning and expectation errors as an autonomous source of endogenous business cycles.

Both converging and diverging cycles may be obtained in simulations with Auto ARIMA models, while the VAR learning tool leads to diverging fluctuations in the majority of cases, suggesting that making agents consider several variables increases instability, at least in our setting. It is also observed that higher frequency of model switching is usually accompanied with increasing amplitude of cycles, suggesting the hypothesis that economic crises may happen when agents make drastic revisions of their beliefs about how the economy works. Only converging cycles could be obtained with AR(2), however in this case the economy may get trapped in a so called “false equilibrium”, with output way below or above the true equilibrium level. Even though this is not formally an equilibrium, the convergence towards the true one is so slow that exogenous shocks may be needed to move the economy back on track. This result is in line with the Keynesian view that the economy may remain in a depressed state for quite a long period of time, and active government intervention may be required to speed up the recovery.

However, in conducted simulations, the behavior of the economy has proved very sensitive to the specified learning tools, initial conditions, and parameters of the model. Therefore, more work should be done in order to achieve a better understanding of how the dynamics of the economy depend on all of these preconditions. Once again, generic analysis would be of great help here.

A major extension would be to make agents build structural economic models instead of using simple reduced-form modeling. This would require developing AI and a “rational theorist” type of agents. This direction for future research was also first outlined in Sargent (1993). A promising idea for economic historians would be to check if major economic perturbations can indeed be explained by significant revisions in economic theory.

We believe there to be a major step between running simulations and calibrating a model in order to reproduce real-world economic dynamics. In fact, not only the possibility of calibration would allow a better evaluation of the empirical fit of our model, but it could also open the possibility of using it for economic forecasting and policy analysis. This, of course, can be done only when the above mentioned extensions and improvements are already accomplished.

III.

Within the developed framework we analyze whether active monetary policy (i.e. changes in money stock) can be used for stabilization purposes. It turns out that in the simple case, when agents have logarithmic utility function, shifts in money supply can have real effects on the economy only if they are unexpected by agents, or if future price expectations are not adjusted exactly proportionally to the announced monetary interventions. We also show that the second case is not sustainable within the adaptive learning environment, so that monetary policy may become ineffective in the long run when, and if, learning is complete.

We prove, however, that monetary interventions always have real effects in the short run in the setting with a more generic HARA utility function. Still, it is highly questionable whether the central bank is able to accurately assess the consequences of its own actions, as that would require it knowing precisely the actual law of motion of the economy, current market's expectations, and agents' reaction to news about the upcoming monetary interventions, which, moreover, can change over time. In fact, there are no reasons to believe that central bankers are systematically smarter than economists at private institutions.

Nevertheless, monetary policy might be an effective instrument to move the economy back on track when it is entering a recession or stuck in a crisis. Our point is rather that central banks should not abuse the power they have and overdo what is required from them.

Furthermore, one might be worried whether monetary intervention, while initially having a positive effect on output, would not cause more economic instability and higher amplitude of business cycles later, when agents start to revise their forecasting models in view of an increased difference between their expectations and actual outcomes. Simulations of the consequences of shifts in money supply in a longer perspectives might become an important practical application of our model in the future. It would be an interesting exercise also to simulate a setting in which the central bank is trying to learn the actual law of motion and conduct stabilizing monetary policy, and compare the results with simulations where the central bank is absent.

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