Arbitrage Crashes in the Convertible Bond Market

- The Effect of Slow Moving Capital and Market Segmentation

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Abstract

We develop a pricing model for convertible bonds subject to callability and default risk. Using the data of Dick Nielsen and Rossi (2013), we find that convertible bonds were traded at a discount to their theoretical prices during the arbitrage crashes of 2005 and 2008. Furthermore, we confirm that convertible bonds were cheaper than comparable non convertible bonds during the arbitrage crashes, and that the callability option affects bond price less than the convertability option. Our results shed new light on the analysis of the arbitrage crashes of 2005 and 2008 and confirm that the matching method of Dick Nielsen and Rossi (2013) does identify cheapness of convertible bonds relative to non convertible bonds.

1 Introduction

It is often assumed that the financial market is efficient and thereby self-correcting. In such a market it is thus assumed that there is no arbitrage. When an arbitrage opportunity occurs by a price divergence of related assets it is utilized fast by arbitrageurs such that the prices converge. The arbitrage opportunity is thus closed after a very short period of time in efficient markets. It has recently been shown by Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) that arbitrage opportunities of convertible bonds in the financial crisis were present for about six months from September 2008 to March 2009 due to slow moving capital and market segmentation. When an arbitrage opportunity is present in longer periods it is called an arbitrage crash.

In this thesis we show that the convertible bond market experienced an arbitrage crash in the years of 2005 and 2008. We base our analysis on the dataset of Dick-Nielsen and Rossi (2013) and develop a pricing formula to calculate the theoretical prices of the bonds in the set. By comparing the theoretical prices with the traded prices we show that the convertible bonds were underpriced in the years of 2005 and 2008. Furthermore, we show that the convertible bonds were traded at a value lower than comparable non convertible bonds in the same periods by matching the yields to maturity as in Dick-Nielsen and Rossi (2013). We confirm the matching method as a valid method to identify arbitrage crashes.

A convertible bond is a corporate bond which the investor has the option to convert into shares of the issuer's common stock. It consists of a corporate debt obligation and an equity call option. As the convertible bond market represented approximately 500 billion dollars at the end of 2011,[1] it is a very relevant market although smaller than the markets for straight debt or equity.

Prior literature suggest that the convertible bond market experienced an arbitrage crash in the periods around 2005 and 2008. In these arbitrage crashes the convertible bonds were sold at a discount compared to their fundamental value. The present thesis is based on the two articles by Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013).

Mitchell and Pulvino (2012) develop a model to calculate theoretical prices of convertible bonds. They compare the theoretical prices with the traded prices of the convertible bonds observed in the market. Their findings indicate that the prices of the convertible bonds decrease relative to the theoretical prices in the arbitrage crashes of 2005 and 2008. Mitchell and Pulvino (2012) argue that the slow moving capital in the wake of the Lehman bankruptcy led to the arbitrage crash of 2008.

Dick-Nielsen and Rossi (2013) compare the yield to maturity of a convertible bond with the yield to maturity of a matching non convertible bond. Intuition suggests that a convertible bond should at least be equal to or worth more than a non convertible bond since the convertible bond contains an additional option which is valuable to the investor. They show that the convertible bonds were cheap relative to the non convertible bonds in the periods of 2005 and 2008. Dick-Nielsen and Rossi (2013) argue that it was not just the slow moving capital in the wake of the Lehman bankruptcy that led to the arbitrage crash of 2008 but also market segmentation. Their findings indicate that the investors bought strictly dominated non convertible bonds in the crash of 2008.
The thesis and general methodology are structured as follows.

Section 2 starts by introducing arbitrage crashes in the convertible bond mar-

[1] Dick-Nielsen and Rossi (2013)

ket. We review the practical setting of the convertible bond market and describe how the Lehman bankruptcy affected it.

In section 3 we review the two main articles this thesis is based on. We examine the different methods Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) use to show the arbitrage crashes of 2005 and 2008.

In section 4 we review the underlying theory of the paper. We introduce the bond market and define the default probability of a bond by an intensity-based model. Next we present the properties of a bond containing a call and a convert option. Based on the presented theory we develop a pricing algorithm to price the callable and convertible bonds. We finish the section by defining the yield to maturity based on the price of the bond.

The dataset and methodology are described in section 5. We describe how the data are managed to serve as input of the pricing formula. Then we implement the pricing formula and determine the fixed parameter by calibration. The section is completed by demonstrating the formula by pricing a bond from the dataset.

In section 6 we show the arbitrage crashes in the convertible bond market. The first analysis compares the theoretical prices with the traded prices and shows that the convertible bonds were underpriced in the years of 2005 and 2008. The second analysis shows that the convertible bonds were traded at a value lower than comparable non convertible bonds in the same periods by matching the yields to maturity as in Dick-Nielsen and Rossi (2013). The section is ended by comparing the two first analyses and examines the systematic differences between them.

Lastly, section 7 concludes.

Throughout the paper, references to empirical studies performed by other authors will appear. The references are intended to facilitate a comparison and a discussion of how the insights from the *Analysis* section relate to previous findings.

The *Analysis* section relies heavily on the dataset provided by Dick-Nielsen and Rossi which is the primary source of data. When any additional data are needed we collect it from the financial database Bloomberg.

Furthermore, we apply a deductive logic drawn from the theory of other authors in an attempt to provide an answer to the research questions when analyzing our results. From the premise of the theory, with our limitations in mind, we draw our conclusions and compare our results.

2 Arbitrage Crashes in the Convertible Bond Market

As mentioned in the introduction a convertible bond is a corporate bond which the investor has the option to convert into shares of the issuer's common stock. It consists of a corporate bond and a call option on equity. The benefit of convertible bonds, from the issuer's point of view, is the lower financing costs (coupons) than comparable non convertible bonds. In addition, an issuance do not dilute equity as an equity issuance would do. The benefit, from the investor's point of view, is that it is relatively easy to hedge by a short position in the underlying common stock and possibly a short straight bond depending on the moneyness of the convertible bond.

Convertible bonds are often sold at a discount to the value of their components (the straight bond and the embedded call option). The investor demands a discount due to large transaction costs, the requirement of expertise to trade such bonds and the liquidity risk associated with a smaller market. The issuer is willing to encounter the discount since it is possible to sell more quickly and at lower investment-banking fees.[2]

An arbitrage opportunity occurs when the convertible bond is cheap compared to the components that it can be hedged by. The optimal hedge strategy of a convertible bond depends on the moneyness of the bond. If the bond is in the

[2]Asness, Berger and Palazzolo (2009)

money, which is the case when the ratio between the current price of equity and the price of conversion is high, it can be hedged by shorting the stock of the issuer and readjust it dynamically. If the bond on the other hand is out of the money it contains interest rate and credit risk, which is more complicated to hedge. The interest rate risk can be hedged by shorting a straight bond and the credit risk by credit default swaps. When it is possible to hedge the most of the risk and still make a little profit, arbitrageurs often leverage the position to make the profit significant.

Arbitrage opportunities are often exploited by hedge funds, which depend on the deposits of their investors and loans from investment banks to be able to leverage their positions. When an arbitrage opportunity occurs it is usually exploited by arbitrage hedge funds who are leveraging their investments and thereby force the price discrepancies to converge. If the arbitrage opportunity comes from a shock to the market affecting the economy there is a possibility that the investors withdraw their capital and the investment banks tighten their capital constraints. In that situation the arbitrageurs can be forced to sell their securities at a discount instead of increasing their level of the cheap security. After such a shock it can be hard to obtain investment capital for a while. It is this slow movement of capital that can lead to or reinforce an arbitrage crash.

In the next section we review the arbitrage crash associated with the financial crisis in 2008.

2.1 The Financial Crisis

In the financial crisis of 2008 the hedge funds experienced a loss of debt capital and were forced to sell their assets. Since it is primarily these hedgefunds that are investing in convertible bonds, the market of these bonds is impacted in such crisis. According to the findings of Mitchell and Pulvino(2012) and Dick-Nielsen and Rossi (2013) the convertible bond market experienced an arbitrage crash in the wake of the Lehman bankruptcy.

Hedge funds that engage in arbitrage investments use significant leverage to increase the expected returns. The hedge funds obtain debt financing from prime brokerage operations of large investment banks.[3] Within the financing arrangement the prime broker requires a margin fee as collateral (also referred to as "haircut"). The more risky the security is, the higher the haircut. For small, illiquid equities haircuts up to 100% can be required. If the risk is low the hedge fund is charged with a fee that is close to the federal fund rate.[4] The prime brokers adjust this agreement on a daily basis such that changes in the haircuts can happen overnight.

If the hedge fund underperforms relative to the expected return, the prime broker is able to close the investment even though the investment still outperforms the market. Although it is a profitable investment, the prime broker forces the hedge fund to sell. To sell a good investment is not usually a problem but when several hedge funds are forced to sell there are not enough buyers. Normally, convertible arbitrageurs, such as the hedge funds, step in to buy the convertible bonds when there is a selling pressure in the market. Since these hedge funds were forced sellers the prices fell significantly to the level of fire-sales prices.

As mentioned in the last section the convertible bonds, managed by the hedge funds, are easily hedged by a position in the underlying security. The haircuts on these were therefore small and the leverage-level of hedge funds was high. To obtain cash the prime brokers put securities as collateral. In that way, they were able to obtain debt financing at rates slightly above the risk free rate if they allow the prime broker to re-lend the securities held as collateral. This practice is called rehypothecation.[5]

[3]Mitchell and Pulvino (2012)
[4]Mitchell and Pulvino (2012)
[5]Mitchell and Pulvino (2012).

In the U.S the prime brokers were able to rehypothecate 140% of the loans but in the U.K there was no restrictions on the amount that U.K prime brokers could rehypothecate. When Lehman Brothers on September 15, 2008 filed for bankruptcy rehypothecation lenders quickly started to sell securities provided as collateral by Lehman's hedge fund clients. Since Lehman's U.K broker dealer did not seek bankruptcy court protection the client's securities were suddenly worth nothing and there was no recovery. This rehypothecation lending is most prevalent prior to the crisis and were not an issue before it had this large influence on the financial crisis in 2008. The practice of rehypothecation has not disappeared now but asset managers are much more thoughtful whether to allow it or not.[6]

Lehman's bankruptcy led to the closing of the interbank market and haircuts increased from less than 1% in 2007 to 45% in the end of 2008.[7] The investment banks had enough problems financing their own balance sheets and were unable to finance their hedge funds clients. Because of that the investment banks required the hedge funds to reduce leverage. Thus the hedge funds leverage level substantially decreased in the aftermath of the Lehman bankruptcy and continued to decrease for the next few months. On average, haircuts nearly doubled and available leverage was halved for the convertible arbitrage funds during the midst of the financial crisis.[8]

Figure 2.1 displays the deleveraging of hedge funds in the aftermath of the Lehman bankruptcy. The forced selling pressure decreased the prices of the convertible bonds. Since there were no buyers in the market the convertible bonds were sold at fire-sale prices.

In the next section we review the effect of the slow movement of capital on an arbitrage crash.

[6]Wikipedia: http://lexicon.ft.com/term?term=rehypothecation.
[7]Mitchell and Pulvino (2012)
[8]Mitchell and Pulvino (2012)

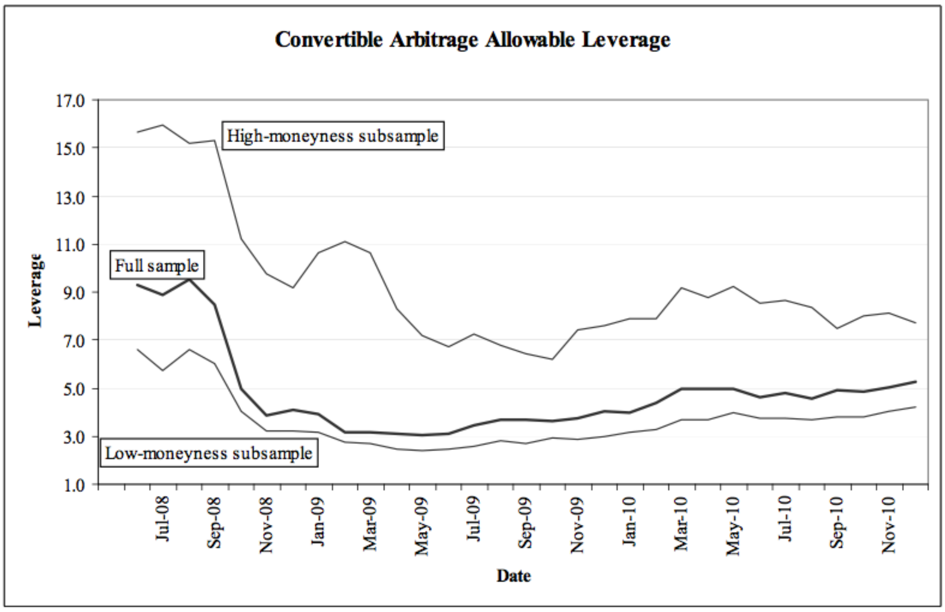


Figure 2.1: *This figure is from the article of Mitchell and Pulvino (*20012*). It displays the average monthly available leverage for a convertible arbitrage fund across six prime brokers during the period of June* 2008 *through December* 2010*. Allowable leverage levels for the full sample of convertible bonds in their dataset, and sub samples of bonds with low-moneyness (high risk) and high-moneyness (low risk) are shown.*

2.2 Slow Moving Capital

Duffie (2010) describes how asset prices can be affected by the slow movement of investment capital when a market is hit by a supply or demand shock. When investments are intermediated the effect of a shock depends on the ability of intermediaries to raise capital and thereby be able to absorb the shock. If the capacity of the intermediaries is reduced their ability to absorb shocks declines and the prices of risky assets are expected to decline. If the amount of available capital is too small to absorb the shock immediately, the effect on asset prices will then be large at first and will decrease gradually as additional capital becomes available.

Under the financial crisis in 2008, the convertible bond market was hit by a supply shock since most of the hedge funds were forced to sell out because of the tightened capital constraints required by investment banks. The sell out led to a decline in the prices which was not exploited by arbitrageurs because of the lack of available investment capital.

Mitchell, Pedersen and Pulvino (2007) address the complications of slow moving capital and the impact it had on the convertible bond market in 2005, where the market was hit by a supply shock. They state that convertible hedge funds experienced large redemptions from their investors in 2005 due to a poor investment result in 2004. The redemptions led to binding capital constraints, which caused a massive sell out of convertible bonds at a price lower than their fundamental value - a cheapness that lasted until 2006.

The above findings are not consistent with the frictionless economic paradigm where a shock to the capital of a smaller group of agents should not have a significant effect on asset prices. The new capital is supposed to flow into the market immediately and move prices back to fundamental values. The sell out of the convertible bonds indicates that if arbitrageurs are exposed to redemptions of investors a shock to the capital will in fact have an impact on asset prices. The shock can thus lead to a situation where the liquidity providers become liquidity demanders turning the market very illiquid.

Mitchell, Pedersen and Pulvino (2007) show that even multistrategy hedge funds who had the ability to quickly allocate their capital across strategies and thereby exploit the arbitrage opportunity did not increase their convertible bond investments. Some of them eventually began to increase their investments in the bonds but more than half of them actually reduced their exposure in the period after the shock. This suggests that the slow moving capital is not the only factor explaining an arbitrage crash. The existence of different investors with different investment preferences and habits suggests that market segmentation also plays a role in an arbitrage crash.

2.3 Market Segmentation

In the article by Dick-Nielsen and Rossi (2013) the prices of a convertible bond are compared with the prices of matched straight bonds. Since the convertible bond has an additional option it should be worth more than the matching non convertible bond. They show that during the convertible bond crash, investors were still buying non convertible bonds even though they could have bought the convertible bonds cheaper and thereby not only get the option part for free but also get the bond part cheaper.

When the natural buyers of convertible bonds (hedge funds and similar investors) are capital constrained and sell out the bonds instead of buying, the prices decline. If the price of convertible bonds decline considerably, such that they are cheaper than comparable non convertible bonds, it would be reasonable to see the non convertible bond investors as the new natural buyers of convertible bonds. However, these investors did not exploit the opportunity of arbitrage as they bought the strictly dominated non convertible bonds instead.

The findings of Dick-Nielsen and Rossi (2013) indicate that it is not only slow moving capital that contributes to arbitrage crashes but also market segmentation, since a group of investors were natural buyers and had the the necessary capital to invest in the convertible bonds but decided not to.

3 Slow Moving Capital or Market Segmentation

Both Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) analysed the arbitrage crashes of the convertible bond market in the period of 2005 and 2008. Their analyses are primarily concentrated on the arbitrage crash in 2008 since it was significantly larger than in 2005.

Mitchell and Pulvino (2012) examine the difference between their theoretically calculated prices and the traded prices and show that the convertible bonds were significantly underpriced in 2008. They argue that the found underpricing was a consequence of the failure of convertible arbitrage hedge funds to raise enough capital to take advantage of the relative cheapness of the convertible bonds. This challenge of raising enough capital within a short period of time is called slow moving capital.

Dick-Nielsen and Rossi (2013) show that the slow moving capital was only partially responsible for the arbitrage crash in 2008. They argue that the existence of different clienteles in the market for convertible and non convertible bonds exacerbated the underpricing of convertible bonds. They base their analysis on a comparison of convertible bonds with comparable non convertible bonds issued by the same company. They show that investors bought strictly dominated non convertible bonds instead of convertible bonds from the same issuer, a fact that suggests the bond market was segmented. A segmentation that could arise from the fact that some investors (e.g. pension funds an insurance company) were restricted to buy a certain kind of assets.

We base the empirical analysis of this thesis on the ideas of Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013). The ideas and analyses of the articles are reviewed in this section.

3.1 Data and Methodology of the Related Articles

The data set of Mitchell and Pulvino consists of weekly prices of more than 3000 convertible bonds. The bonds in the sample are traded during the period of January 1990 through December 2010 which on average is more than 400 monthly trades during the period. They calculate a theoretical price based on the weekly observations using a finite difference model in which the various embedded options of convertible bonds are accounted for. Their input estimates are the issuer's stock price, the volatility estimates of the stock, issuer credit spread estimates, and term structure of interest rates for each convertible bond.

Dick-Nielsen and Rossi use bond trading data from the enhanced TRACE database. They collect data from firms that have both a non convertible and a convertible bond issued during the sample period from July 2002 to December 2011. The bonds are chosen with maturity dates that are less than one year apart. The seniority of the non convertible bond is allowed to be worse than the seniority of the convertible bond since this do not affect any possible mispricing. They exclude bonds from the comparison if the issuer has filed for bankruptcy. The trade of the non convertible bond is required to take place within three hours of the trade of the convertible bond such that they almost trade at the same time.

The data of Dick-Nielsen and Rossi is limited compared to Mitchell and Pulvino by the fact that it only contains the convertible bonds that are traded at the same time as a similar non convertible bond. On the other hand, it contains intra day prices where the data of Mitchell and Pulvino only contains weekly prices. The analysis of this thesis is based on the data set provided by Dick-Nielsen and Rossi.

3.2 The Findings of Mithell and Pulvino (2012)

Mitchell and Pulvino (2012) show that the ability of arbitrageurs to maintain prices of similar assets at similar levels was prevented by the abrubt withdrawal of debt capital in the aftermath of the Lehman bankruptcy. They measure the relative pricing error that occurred during the financial crisis in 2008, when arbitrage hedge funds experienced a sudden loss of debt capital, causing arbitrage spreads to widen, inflicting losses and making it difficult for the funds to raise equity capital.

Based on the theory of Shleifner and Vishny (1992) they argue that since the sudden withdrawal of debt capital affected many arbitrageurs simultaneously, the convertible bonds were sold to non arbitrageurs at fire-sale prices.

They point out that the mismatch of the duration between long-term arbitrage investments and very short-term debt financing prevented the arbitrageurs from maintaining prices of similar assets at similar levels.

When prices on related assets diverge investors sell short the expensive asset and purchase the cheap asset. Normally, the market is self-correcting such that arbitrageurs close the arbitrage opportunities fast when the price on related assets are diverging. To exploit the small differences in prices on related assets the arbitrageurs leverage their investments which forces the pricing discrepancies to converge.

In the financial crisis of 2008, Mitchell and Pulvino (2012) observe arbitrage opportunities on the convertible market by showing that the convertible bonds were underpriced for a longer period. According to their findings this arbitrage crash occurred due to the lack of capital in the aftermath of the Lehman bankruptcy.

Since the arbitrageurs were forced to sell out their assets to meet the withdrawals of debt capital, they could not exploit the cheapness of the convertible bonds. The prices of the convertible bonds thereby stayed at a low level untill investors were able to raise the needed capital to invest in them.

Mitchell and Pulvino show that the traded prices, observed in the market, fell relative to their theoretically calculated prices during the financial crisis and that the theoretical prices they find are close to the traded prices in an unstressed market. They conclude that the underpricing of the convertible bonds was due to the slow movement of capital in the aftermath of the Lehman bankruptcy.

3.3 The Findings of Dick-Nielsen and Rossi (2013)

Instead of using a theoretical pricing model to examine the arbitrage crash on the convertible bond market, Dick-Nielsen and Rossi (2013) show the underpricing by matching a convertible bond with a comparable non convertible bond by the same issuer. They compare the yields to maturity to see if the spread changes through the analysed period. The spread is calculated as the difference between the yield to maturity of the convertible bond and the yield to maturity of the non convertible bond by the same issuer. Since the yield to maturity is negatively related to the price of the bond, the yield to maturity of the non convertible bond is expected to be higher than or equal to the yield to maturity of the convertible bond. When the yield of the non convertible bonds fall below the yield of the convertible bonds there exist an arbitrage opportunity. The arbitrage opportunity can be exploited by buying the convertible bond and simultaneously short the non convertible bond, capturing a positive yield difference and a free call option on the underlying stock.

During the financial crisis of 2008, Dick-Nielsen and Rossi (2013) show that some of the convertible bonds were traded at yields significantly above the comparable non convertible bonds. In figure 3.1 we have plotted the yield spreads based on the yield to maturities found by Dick-Nielsen and Rossi. The yield spread is determined by subtracting the yield to maturity of a convertible bond from yield to maturity of the comparable non convertible bond.

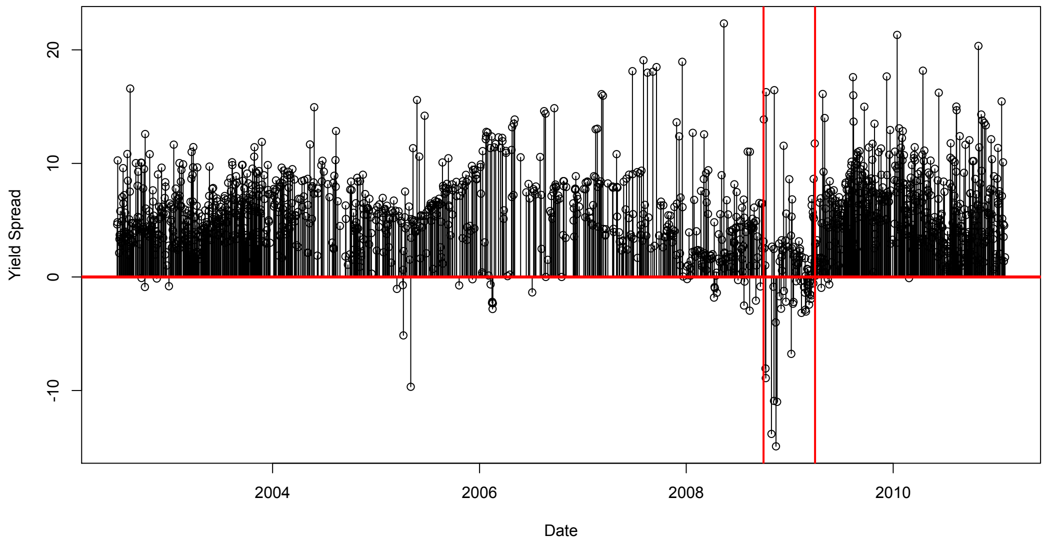


Figure 3.1: *The figure shows the yield spread between the yield to maturity on the convertible bonds and non convertible bonds (negative when the yield to maturity is higher for the convertible bond). The Arbitrage crash from the fall of* 2008 *and into* 2009 *(*2008/09/15 *to* 2009/03/15*) is marked by the verticle red lines.*

In an unstressed market the yield to maturity of the convertible bonds should be lower than the yield to maturity of the non convertible bonds since the convertible bonds include the embedded call option on equity. Figure 3.1 thus shows that the convertible bond marked were stressed in the period of 2008, experiencing an arbitrage crash.

The vertical lines in Figure 3.1 show the period from the fall of 2008 to March of 2009. Based on the comparison of the yields we see that the arbitrage crash continued into early 2009. This suggests that it was not just an extremely short-term phenomenon as an arbitrage opportunity often is expected to be.

In Figure 3.1 the differences of the yields are generally positive which means the convertible bonds are sold at prices higher than the non convertible bonds, which is expected. There are some negative yield spreads in the arbitrage crash of 2005 and otherwise only a few negative observations. In the arbitrage crash of 2008 the yield spreads became more negative and more frequent indicating a significantly larger arbitrage crash.

The method of Dick-Nielsen and Rossi indicates that the arbitrage crashes were not only due to the lack of capital. Since the strictly dominated non convertible bond was bought, they argue, that the liquidity was not the entire problem. The investors that bought the non convertible bonds should have sold them to finance the buying of the convertible bonds and thereby exploiting the arbitrage opportunity. The buying volume of the non convertible bonds did not vanish during the crisis of 2008. It fell to a lower level but it was still present. When the convertible bonds decline enough the non convertible bond investors would be the obvious new convertible bond investors. But these investors did not take the opportunity to do so. Instead, they bought strictly dominated non convertible bonds. The existence of a significant buying volume of dominated non convertible bonds is a clear indication of market segmentation.

3.4 The Relation Between the Articles and This Thesis

Overall, the articles agree on the arbitrage crashes in the convertible bond market. They both find that it is significantly larger in the financial crisis but though present in 2005 as well. They agree on the fact that it was initiated by cash withdrawal of investors and prime brokers which led to a slow movement of capital, but Dick-Nielsen and Rossi furthermore argue that it was exacerbated by a market segmentation in the bond market.

In this thesis we analyse the arbitrage crash in the convertible bond market by using both a pricing method as in Mithell and Pulvino (2012) and a yield matching method as in Dick-Nielsen and Rossi (2013). We can thereby examine if the two methods lead to the same conclusion. Furthermore, we can verify if the results of the articles can be confirmed in our setting.

4 Theory

In order to examine the arbitrage crashes in the convertible bond market we develop a pricing algorithm to determine the value of a convertible bond. In this section we explain the convertible and callable bond market which is the theoretical background for the pricing algorithm.

First we define the bond market in general. We review the assumptions made in order to work with the bond market theoretically and describe the pricing of a bond under a risk neutral measure, Q. Then, we define the default probability by an intensity-based model and show how the bond is priced in such a set-up. Thereafter we review the different possibilities of recovery and argue why we assume recovery of face value.

Once we have defined the bond market and the default probability of the bond, we present the properties of a bond which is callable and convertible. We end the theoretical background with a backward recursion problem which we use in the valuation of a convertible bond. Later on we implement the pricing algorithm in `R`.

4.1 The Bond Market

A bond is a financial security where an investor lends an amount of money to an entity for a defined period of time. Coupon bonds pay the investor coupons throughout the life of the bond and the face value at maturity, whereas a zero-coupon bond only pays the face value at maturity. Zero-coupon investors earn the difference between the price of the bond and the amount received at maturity. Zero-coupon bonds are thus purchased at a discount to the face value of the bond, where the price of a coupon-paying depends on the difference between the effective yield and coupon. This thesis is based on coupon paying bonds but the results easily apply for zero-coupon bonds as well.

As mentioned earlier our focus in this thesis is on option-embedded corporate bonds where the issuer has the option to call the bond and the investor has the option to convert the bond. Such a bond is called a callable and convertible bond.

A convertible bond is a bond that gives the investor the right, but not the obligation, to exchange it for a pre-specified fraction of shares of the issuing firm. In effect it is a straight bond and a call option on the issuers stock which combines the downside protection of a straight bond with the upside potential of a stock.

Since convertible bond issuers often retain the right to call the bond, a convertible bond is often callable too, why we thus focus on those corporate bonds with optionality on each side of the bond contract.

In this section, we define the bond market, the riskless rate of return, and the P-measure. Thereafter we define the risk-neutral Q-measure, find the price of a bond under the Q-measure.

In the end of the section we shortly review the structural approach for pricing defaultable bonds by Merton(1974) and explain its complications when the issuing firm has a complex balance sheet.

4.1.1 Assumptions in the Bond Market

A coupon bond with maturity date T promise the investor of the bond the face value, F, to be paid at time T and a stream of coupon payments in the interval $[0;T]$. The deterministic coupon, c_i, is received at time $T_i, i = 1, ..., n$ such that the total value of the coupons payed at time t can be expressed as $C_t = \sum_i c_i 1_{t \geq T_i}$.

We define the price of a bond as the present value of the bond's future cash flow and is in this section denoted as $\Pi(t,T)$.

To guarantee the existence of a sufficiently rich and regular bond market, we assume the following:

- There exist a frictionless market for bonds with maturity T for every $T > 0$. Frictionless means that there are no transaction costs, that the trading in asset does not affect prices, and that there is unlimited access to short and fractional positions.

- The relation $\Pi(t,t) = 1$ holds for all t to avoid arbitrage. Otherwise it would be possible to get an instantaneous gain by buying a bond when $\Pi(t,t) < 1$ or shortselling a bond when $\Pi(t,t) > 1$.

- For each fixed t, the bond price $\Pi(t,T)$ is differentiable with respect to time of maturity, T.

- There exists a short rate, $(r_t)_t$, which is a function of state variables, $r_t = f(t, X_t)$, where X_t is a (possibly multidimensional) stochastic process. We are then able to define a money account, which is a security, M, with price dynamics

  $$dM_t = r_t M_t dt,$$

  where the rate of return equals the short rate

  $$\frac{dM_t}{M_t} = r_t dt,$$

  and the time t value is given by

  $$M_t = e^{\int_0^t r_u du}.$$

In addition, we assume that the probability space $(\Omega, \mathcal{F}, P)$, where P is the physical or data generating measure, is given, and that there exists a standard Wiener-process $(W_t)_t$ which generates the filtration $\mathbb{F}_t$.

4.1.2 Bond Pricing under the Risk Neutral Measure

When pricing under the P measure, the price is assumed to depend on the risk preferences of each individual investor. This assumption is realistic but not very easy to implement theoretically.

If we assume that all agents are risk-neutral we are able to price bonds with the help of a risk-neutral probability measure, Q. Q is a probability measure under which we expect the current value of all financial assets at time t to be equal to the future pay-off of the asset discounted at the short rate, given the information structure available at time t, $\mathbb{F}_t$. The Q measure is an equivalent martingale measure, which means that it is equivalent to the original P measure, and that all discounted price processes are martingales with respect to $\mathbb{F}_t$ under Q.

The Q measure plays an important role in determining whether or not a market is arbitrage free and complete. The first fundamental theorem of finance states that the model is arbitrage free if and only if there exist a risk-neutral probability measure Q. If the measure, Q, in addition, is unique, then the second fundamental theorem of finance states that the market is complete.[9]

If there exists such a measure, $Q \sim P$, then the time t price of any security is given by

$$Price(t) = E_t^Q \left[\frac{\text{Future payoffs}}{\text{Discounting with } r_t} \right].$$

where E^Q denotes the expectation under Q.

Under the Q measure the price of a default free coupon-paying bond which pays a face value, F, at maturity is equal to the expected discounted value of the face value and the coupon payments under the Q measure.

$$\Pi(0,T) = E^Q \left[e^{-\int_0^T r_u du} \cdot F + e^{-\int_0^{T_i} r_u du} \cdot C_t \right]. \tag{4.1}$$

[9]Björk 2009

When pricing bonds in continuous time, using Itô's lemma is a big advantage. It states that in continuous models, the structure of a stochastic differential equation with a drift term and a normally distributed (noise) term is preserved even at non-linear transformation as long as the transformation is two times differentiable. It is thus possible to solve the conditional expectation (4.1) as a partial differential equation and thereby determine the bond price.

When pricing bonds with default risk the approach by Merton,[10] where corporate debt is seen as a contingent claim whose pay-off depends on the total assets of the firm, is often used. In that case default is triggered when the assets fall below some boundary such as the face value of the debt. When the capital structure only consist of equity and a zero-coupon bond, the pay-off of the bond is then a function of the assets available at the bonds maturity date given by

$$\text{Payoff} = min(F, V_T),$$

where F is the face value and V_T is the total value of assets at maturity.

The approach is more applicable when the issuing firm has a simple balance sheet but that is not very often the case. Capital structures are often more complicated consisting of bonds of different types and maturities, covenants, call features, convertibility options, prioritization, and other instruments that are very complicated to model. Instead it can be convenient to use an intensity-based model where default risk is defined in terms of an intensity. In the next section we review such an intensity-based model and show how it is used to define the price of a defaultable coupon bond.

[10] Merton 1974

4.2 Intensity Modelling

In the following we model default risk in terms of an intensity-based model, where the default event is characterized exogenously by a jump process with an intensity process, λ, for the default jump time, τ. This is done in the setting of a doubly stochastic Poisson process known as the Cox process. One of the advantages of a jump process is that it takes event risk into account, such as a default, which causes the stock to take a sudden jump up or down.

After modelling the probability of default in an intensity framework we assume the probability of default, λ_t, is a function of the equity price. It is modelled such that the probability of default increases when the equity price decreases.

We assume the equity price follows a geometric Brownian motion and evolves with a mean of $r + \lambda_t$ where r is the risk free rate which is assumed to be constant throughout the life of the bond. In the end of the section we define the default risk of the bond by an intensity parameter, which is used to adjust the rate of return of the bond.

4.2.1 Default Intensity

In this section we define the time of default by a single jump time constructed through a Cox process. We assume a probability space $(\Omega, \mathcal{F}, Q)$, where Q is the risk-neutral measure, is given. We define X_t as a process of state variables in R^d on the probability space and λ as a non-negative measurable function $\lambda : R^d \to R$. We will then construct a jump process, N_t, with a $\mathcal{F}_t$-intensity given by $\lambda(X_t)$ and define the first jump time, τ, of the process as the time of default. Since default is assumed only to happen once, we only focus on the first jump time of the process and have that

$$N_t = \begin{cases} 0 & \text{for } \tau > t \\ 1 & \text{for } \tau \leq t. \end{cases}$$

Now we let $\mathcal{G}_t = \sigma\{X_s : 0 \leq s \leq t\}$ denote the filtration generated by X such that it contains the information about the state variable development until time t. Let $\mathcal{H}_t = \sigma\{N_s : 0 \leq s \leq t\}$ denote the filtration generated by N such that it contains the information about the development of the jump process until time t. We then have that $\mathcal{F}_t = \mathcal{G}_t \wedge \mathcal{H}_t$ contains information about the development of both X and N.

If E_1 is a random exponential distributed variable which is independent of $\mathcal{G}_t$ and has a mean value of 1, the time of default, τ, can then be defined as[11]

$$\tau = \inf\{t : \int_0^t \lambda(X_s)ds \geq E_1\}.$$

The conditional probability of no default is then given by

$$\begin{aligned} Q(\tau > T \mid \mathcal{G}_T) &= Q\left(\int_0^T \lambda(X_s)ds < E_1 \mid \mathcal{G}_T\right) \\ &= 1 - Q\left(\int_0^T \lambda(X_s)ds \geq E_1 \mid \mathcal{G}_T\right) \\ &= 1 - \left(1 - e^{-\int_0^T \lambda(X_s)ds}\right) \\ &= e^{-\int_0^T \lambda(X_s)ds}, \end{aligned}$$

since $e^{-\int_0^T \lambda(X_s)ds}$ is known when $\mathcal{G}_T$ is known.

[11] Lando 2004, chapter 5.3

4.2.2 Bond Pricing with Intensity

To be able to see how the default intensity affects bond pricing, we will compare the price of a default-free zero-coupon bond with a principal of 1 at time zero,

$$ZCB(0,T) = E\left[e^{-\int_0^T r(X_s)ds}\right],$$

to the price of a defaultable bond with zero recovery and a principal of 1 at time zero,

$$\begin{aligned} B(0,T) &= E\left[e^{-\int_0^T r(X_s)ds} 1_{\tau > T}\right] \\ &= E\left[E\left[e^{-\int_0^T r(X_s)ds} 1_{\tau > T} \mid \mathcal{G}_T\right]\right]. \end{aligned}$$

When we condition on $\mathcal{G}_T$ the expression $\exp\left(-\int_0^T r(X_s)ds\right)$ is known and we get that

$$\begin{aligned} B(0,T) &= E\left[e^{-\int_0^T r(X_s)ds} E\left[1_{\tau > T} \mid \mathcal{G}_T\right]\right] \\ &= E\left[e^{-\int_0^T r(X_s)ds} Q(\tau > T \mid \mathcal{G}_T)\right] \\ &= E\left[e^{-\int_0^T r(X_s)ds} e^{-\int_0^T \lambda(X_s)ds}\right] \\ &= E\left[e^{-\int_0^T (r+\lambda)(X_s)ds}\right]. \end{aligned}$$

The short rate is thus replaced by the intensity adjusted rate $(r+\lambda)(X_s)$ when pricing a defaultable bond with zero recovery.

In this thesis we want to be able to price a defaultable bond with principal F and a coupon payment of $C_t = \sum_i c_i 1_{t \geq T_i}$ which can be done by modifying the general example such that

$$\begin{aligned} B(0,T) &= E\left[e^{-\int_0^T r(X_s)ds} F \cdot 1_{\tau>T} + \sum_i e^{-\int_0^{T_i} r(X_s)ds} c_i 1_{\tau>t\geq T_i}\right] \\ &= E\left[e^{-\int_0^T (r+\lambda)(X_s)ds} \cdot F + \sum_i e^{-\int_0^{T_i} (r+\lambda)(X_s)ds} \cdot c_i 1_{t\geq T_i}\right]. \end{aligned}$$

We are now able to price a defaultable bond with coupon payments where the risk of default is defined by the intensity process, λ.

When pricing a defaultable bond the choice of recovery assumption also plays an important role. In the next subsection we review the recovery assumption and the impact it has on the bond price.

4.2.3 Bond Pricing with Recovery

When default occurs the recovery of the bond can be given in several different ways. It can have zero recovery, it can have a recovery based on the face value of the bond, or it can have a recovery based on the market value of the bond.

Assuming that the bond has recovery of market value, the investor receives a fraction, $(1-L)$, of the market value of the bond just before the time of default. If we consider the price process of the defaultable bond, B, it is said to have a recovery of market value, $(1-L)$, at the time of default, τ, if the amount recovered is given by

$$h(\tau) = (1-L)B(\tau-,T) \text{ for } \tau \leq T,$$

where $B(\tau-,T)$ is the value of the bond just before the event of default and $L \in (0,1]$ is the loss rate.

Assuming no coupons the defaultable bond price is then given by[12]

$$B(0,T) = E\left[e^{-\int_0^T (r+L\lambda)(X_s)ds} \cdot F\right].$$

[12] Lando 2004, chapter 5.6

The intensity adjusted rate is thus downgraded with the loss rate, L, when pricing a defaultable bond with recovery of market value.

The assumption of recovery of market value is convenient to use in a continuous time setting as it will lead to a closed-form expression. In our setting, however, we will use a discrete binomial tree approach. In that case it will be convenient to assume recovery of face value since it is in fact the closest to real practice.

Assuming that the bond has recovery of face value, the investor receives a fraction, $(1 - L)$, of face value. This type of recovery assures that debt with the same priority is assigned a fractional recovery depending on the outstanding notional amount - not on maturity or coupon. The price of the defaultable bond with no coupons is in this case given by

$$B(0,T) \;=\; e^{-(r+\lambda)T}F + (1 - e^{-\lambda T})e^{-rT}F(1 - L).$$

We have now modelled the probability of default as an intensity based model and defined the recovery of default. In this thesis we will define the default probability as a function of the equity price.

4.2.4 Default Probability as a Function of the Equity Price

In this section we model the risk-neutral default intensity, λ, as a function of the equity price of the issuer, S_t, depending on time. The probability of default is set to increase when the equity price goes down and is low when the equity price is high. We define the probability of default by[13]

$$\lambda_t = \frac{\gamma}{S_t}$$

where γ is a constant, positive parameter, $\gamma > 0$. In Figure 4.1 the link between the default intensity, λ_t, and the equity price, S_t, is shown.

[13] Duffie and Singleton (2003)

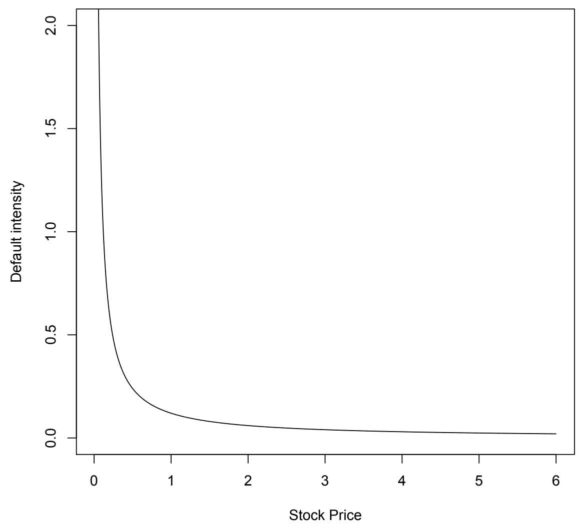


Figure 4.1: *The figure shows the default intensity,* $\lambda_t = \frac{\gamma}{S_t}$*, where* γ *is set to* 0.12.

The parameter, γ, is in this case chosen to be 0.12.[14] The value of γ is not of importance here, since it is only used to show the boundary conditions of λ. The figure shows that the default intensity goes to zero as the equity price goes to infinity and that it goes to infinity when the equity price goes to zero.

We notice that this assumption of the default probability is not true in case of a stock split. A stock split decreases the stock price and thus would increases the probability of default. The probability of default does not increase in case of a stock split. The issue could be avoided if the default probability was based on the total market value of the equity. We though accept the assumption of the default probability to depend on the stock price to limit the amount of inputs in our model.

We assume that the stock price follows a geometric Brownian motion and satisfies

$$dS_t = (r + \lambda_t)S_t dt + S_t \sigma dW_t$$

[14] Duffie and Singleton (2003) also choose $\gamma = 0.12$ in their example of showing the boundary conditions of λ

where W_t is a Wiener-process under the Q-measure. The stock evolves with a mean of $r + \lambda_t$ such that the usual stock price risk-neutral mean rate of return, r, is elevated by the risk neutral default intensity, λ_t. The equity price thus captures the risk of default.

Duffie and Singleton (2003) show that the development of the interest rate does not affect the price of the callable and convertible bond for typical parameter choices. They show that the volatility level of the interest rate has to be very high before it affects the price of the convertible and callable bond. They show that at more common lower levels of interest rate volatility the price is more influenced by equity risk than by interest rate risk. Since the interest rate volatility plays a small role we assume that the interest rate is a constant throughout the life of the bond.

The basics of the bond market is now set. In the next section we focus on the bonds containing an option to convert the bond in to the underlying stock.

4.3 Callable and Convertible Bonds

In this section we describe the characteristics of a bond when it is convertible and callable. When a bond is convertible the investor is able to convert the bond into a given number of shares. The value of the convertible bond at maturity is the larger of the face value and the value of the given number of shares. A convertible bond can additionally be callable. When a bond is callable the issuer is able to redeem the debt at a predetermined call price, $\overline{C}$.

We assume that the investor prefers to keep the convertible bond until maturity due to the upside potential of the underlying stock. If the bond is called by the issuer before maturity, however, the investor convert the bond if the conversion value is higher than the call price. The issuer has incentive to call the bond whenever the call price is lower than the current value of the bond. The decision to call are though affected by the ability and the price of issuing new debt.

We assume the probability of a call decision is based on an intensity process, Y_t. The probability of a call depends on the excess of the current price of the bond, C_t over the value if called, $max(kS_t, \overline{C})$. When the cost of calling the bond is larger than the current price of the bond there will be no incentive to call the bond.

Convertible Bonds

When a bond is convertible the investor of the bond has the right to convert the bond into a given number of shares of equity. Since the investor at any time has the right to convert the bond into shares of equity both credit and equity risk play a role.

The number of shares received upon conversion is the conversion ratio. The conversion ratio depends on the conversion price and the face value and is given by

$$Conversion\ Ratio = \frac{Face\ Value}{Conversion\ Price}$$

where the conversion price is the nominal price per share (in terms of the face value) at which the conversion takes place. We let k denote the conversion ratio, which is positive, $k < 0$.

The equity price is denoted S_t and the value of the convertible bond at maturity, given that default has not occured before the time T, is

$$V_T = max\left[F, kS_T\right].$$

The value of the convertible bond at maturity is depicted in Figure 4.2. The value of the convertible bond is the face value, F, until the equity price times the conversion ratio exceeds the face value.

Figure 4.2: *The figure shows the payoff of a convertible bond as a function of the underlying equity price at the maturity date, T, of the bond.*

The option to convert increases the price of the bond. The investor is protected from a decrease in the firm's value and can benefit from an increase in the stock value since the investor is a potential stockholder. The investor cannot benefit from both the dividend and the coupon of the bond. Thus, if the bond is converted in the middle of a period, the stock starts to pay out dividends in the next period. In this thesis we assume that the stock does not pay dividends. When the stock does not pay dividends the investor keeps the option to convert until maturity because of the upside potential of the stock.[15] As investor, you should either convert later if it is profitable due to an increase in stock price or obtain the face value of the bond if the stock has decreased such that it is not profitable. Because of the time value the investor keeps the bond as long as possible and only convert at maturity.

[15]An option's premium is comprised of two components: its intrinsic value and its time value. The intrinsic value is the difference between the price of the underlying security and the strike price of the option. Any premium that is in excess of the option's intrinsic value is referred to as its time value. (http://www.investopedia.com/terms/t/timevalue.asp).

As we shall see later, when the bond is also callable, the investor can be forced to convert the bond before maturity if the issuer chooses to call.

When the bond is converted the number of shares of equity outstanding is increased. Each share is then worth less than before. Thus each share is a smaller fraction of the total market value of equity at conversion. We will in this thesis ignore the risk of dilution.

As mentioned before a convertible bond sometimes contains an option for the issuer to call. The issuer thus has the option to redeem the debt at a predetermined call price. The characteristics of a callable bond are reviewed in the next section.

Callable Bonds

A bond is callable if the issuer of the bond has the opportunity to redeem the bond at a given time by paying the predetermined call price, $\overline{C}$, of the bond.

The issuer is not always able to call the bond at any time. In practice callable bonds are contractually restricted. The bonds cannot be called before the first call date. For example, a 5 no-call 1 is a 5-year bond which can be called 1 year after issuance. In that case the first call date is 1 year from issuance.

If we assume there exists only one liability on the issuers side, the call options are specified by a standard rational-exercise American option pricing model. In a rational-exercise pricing model we have decision nodes for every period. In every decision node the bond is worth the minimum of the call price and the present value of the bond, C_t, expressed by

$$V_t = min(C_t, \overline{C}).$$

The issuer exercises the bond to minimize the market value of the liability. In the next section we argue, that the call decision is not always based on the perfect market assumption and that the issuer does not always react instantly to the opportunity to call.

Delay of a Call Decision

In the rational-exercise model, described above, it is assumed that the bond is called whenever there is no contractual restriction against calling and whenever the price of the bond is higher than the call price. In practice, however, the decision to call can be delayed under certain circumstances. The decision to delay the call of the bond can be affected by the tax shields that the issuer obtains by holding debt. In addition, the decision to call are as mentioned affected by the ability and the price of issuing new debt to finance the redemption. A company that is cash-constrained are not able to eliminate a current liability even though it would lower the cost of the total liability.

Thus, call decisions are not entirely based on the perfect-market assumption of minimizing the market value of the bond. This i also shown in Duffie and Singleton (2003). They show that the issuer delay a call decision even though the excess conversion value is reached.

The call decision is, however, assumed to depend on an intensity based model such that the probability of a call decision is larger when the current price exceeds the value of calling.

The amount by which the callable and convertible bond price exceeds the call price is called the safety premium. If the convertible bond is far into the money and is called by the issuer the bond is likely to be converted by the investor after the call decision. When the convertible bond is in the money means that the option is worth exercising. The issuer of the bond delays a call decision as long as the safety premium is large. This scenario is seen when the bond is near the money and the equity volatility is significant.

Call Decision as Intensity

As mentioned above we assume that the probability to call is based on an intensity model. It depends on the excess of the current price of the callable and convertible bond, C_t, over the value of the call, which is the larger of the conversion value and the call price, $max(kS_t, \overline{C})$. The likelihood of a call at time t is thus assumed to depend on

$$Y_t = \frac{C_t}{max(kS_t, \overline{C})} - 1$$

If the value of calling the bond is larger than the current price of the bond there is no incentive to call the bond and $Y_t \leq 0$. If $Y_t > 0$ there is an incentive to call the bond. The value of calling the bond will exceed the current price of the bond.

When $Y_t < 0$ the probability of call is assumed to be zero. When Y_t is positive we assume that the probability of a call is ρY_t where ρ is a positive call speed parameter.[16] The value of ρ determines the probability of the issuer to call when the price of the bond exceeds the value of the call. In a perfect market setting the issuer would call instantly whenever the $Y_t > 0$.

The option to call the bond decreases the value of the bond since the option is exercised when it is most profitable for the issuer and thus least profitable for the investor.

4.3.1 Pricing of Callable and Convertible Bonds

A callable and convertible bond is a bond that both have options for the issuer and the investor. The issuer can at any time, if not contractually restricted, redeem the bond at a predetermined call price and the investor can at any time convert the bond into a number of shares. The option to convert is the strongest option [17] since the investor has the opportunity to convert after the bond is called by the

[16] ρ is calibrated in section 5.2.4 on the dataset from Dick-Nielsen and Rossi (2013). The optimal value of ρ is 9 since it is the value that makes the theoretical model fit the traded prices the best in the control period.

[17] Duffie and Singleton (2003)

issuer. The investor utilize the option to convert when the value of conversion is higher than the call price.

The decisions of the issuer and the investor whether to exercise their options or not can be set up as a decision tree. First, the issuer decide whether to call or not. Thereafter, the investor choose to convert or not. The decision tree are pictured in Figure 4.3.

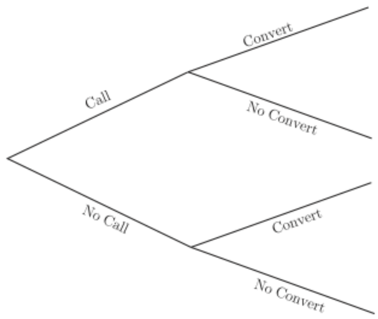


Figure 4.3: *The figure shows the decision nodes where the issuer and the investor decides whether to call or not and whether to convert or not respectively*

As seen in Figure 4.3 the issuer need to assess the value of the call decision at each node to be able to decide whether to call or not.

In the standard contracts of convertible bonds the investor has 30 days after the bond is called to decide whether to convert or not. In these 30 days after the call announcement the conversion value, kS_t, changes with the movements in the stock price. The investor thus has an potentially upside gain in the 30 days. In this thesis we ignore the effect of the 30-days-safety premium. We assume that the investor choose whether to convert or not just after the call announcement.

If the investor are in need of the liquidity there is a risk associated with converting the bond instead of accepting the call price. The conversion value is not certain to be realised when selling the stock as the sale of the stock can lead to a decrease in the stock price if the position in the stock is illiquid.

The price of a convertible and callable bond are thus decreased by the call option and increased by the option to convert.

In order to substantiate the theory of the underpricing of the convertible bonds during the crisis we calculate a theoretical price of a callable and convertible bond. In this section we derive a pricing algorithm to determine the price of a callable and convertible bond.

Since the options to call and convert the bond are American call options we start by reviewing the binomial option pricing model with backward recursive pricing. Thus, our pricing algorithm is based on the expectation of the evolution of the equity price.

Backward Recursive Pricing

To price the options to call and convert the bond we use the binomial options pricing model. This model uses a discrete-time model to find the present value using expectation of the evolution in the underlying asset. The model is used to value American options, which are exercisable at any time like the options to call and convert the bond is assumed to be in this thesis.

A binomial lattice model values the option for a number of time steps between the start date and the expiration date. Each node in the lattice is an evolution of the underlying asset. The valuation is performed iteratively, meaning that it starts by the final nodes and works backwards through the tree towards the first node which is the date of the valuation.

In this thesis the equity price is assumed to move either up or down in the time interval between the nodes. This change in the equity price is assumed to be a specific factor, u or d where u is the upward change in the equity price and d is the downward change. By definition, $u \geq 1$ and $0 < d \leq 1$. In the next period the equity price is given by uS_t or dS_t. This upward and downward change occurs with the risk neutral possibilities p and $1 - p$, respectively.

Before considering the risk of default the backward recursive pricing algorithm is given by

$$B(S_{t-1}) = e^{-r}\left[pB(uS_t) + (1-p)B(dS_t)\right]$$

where the price, B, at time $t-1$ depends on the expected evolution of the equity price at time t.

Since we do not consider default risk, the risk neutral rate of return on equity has to equals the risk-free rate, r. The parameters r, p, u and d of the model are restricted such that[18]

$$e^r = pu + (1-p)d.$$

When the event of default is accounted for, a third branch for default is added. The risk neutral one period default probability is $1 - e^{-\lambda_{t-1}}$ and the probability of an up return and a down return is thus multiplied by the probability of survival. The default adjusted up and down return branches thus have the probabilities $e^{-\lambda_{t-1}}p$ and $e^{-\lambda_{t-1}}(1-p)$, respectively. Figure 4.4 shows the incorporating of the event of default into the binomial pricing model.

The length of the time periods is 1 and the parameters are again restricted and calibrated such that the risk neutral rate of return equals the risk free rate. For periods of length 1 the calibration of the parameters is

$$e^r = e^{-\lambda_{t-1}}pu + e^{-\lambda_{t-1}}(1-p)d + (1 - e^{-\lambda_{t-1}})\hat{S}$$

where $\hat{S}$ is the recovery value of equity at default. The recovery value of equity is often 0 which also is assumed in this thesis. The derivative price is now calculated by the derivative pricing algorithm

$$B(S_{t-1}) = e^{-(r+\lambda_{t-1})}\left[pB(S_t u) + (1-p)B(S_t d)\right] + e^{-r}(1 - e^{-\lambda_{t-1}})(1-L)F$$

where $(1-L)F$ is recovery of the bond at default.

[18] Duffie and Singleton (2003)

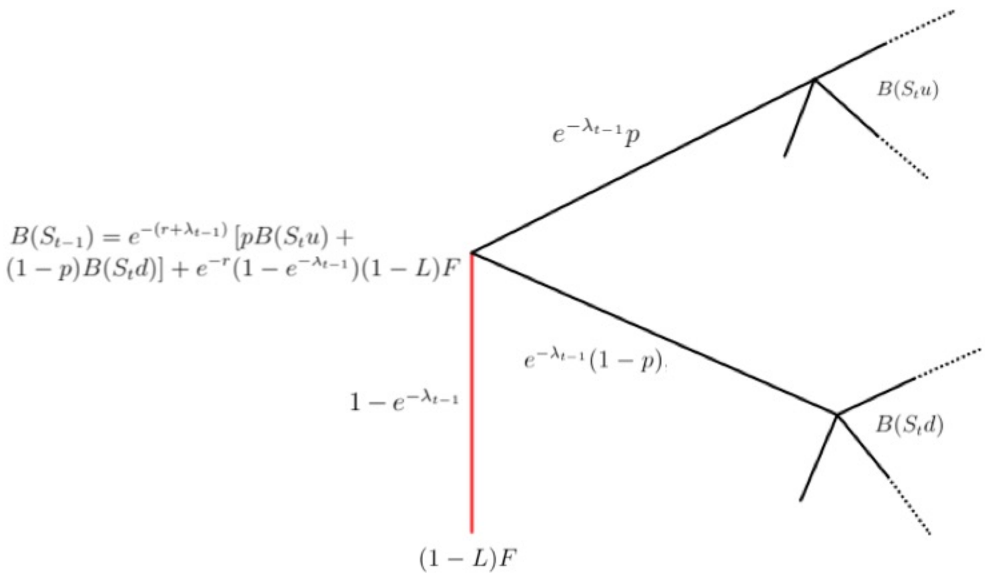


Figure 4.4: *The figure shows the equity derivative algorithm with default and recovery.*

We can approach continuous time by looking at a very short interval, Δ, between the nodes. The default branch then has the risk neutral probability $1-e^{-\lambda_t}\Delta$ for the period of the length Δ.

Pricing Algorithm

At each time t the price in the next period with length Δ is calculated. The present value of the bond is the risk neutral expected discounted value of the bond in the next time period, $t+\Delta$. If the call and convert options are not considered yet and assume that the bond has not defaulted at time t, the price of the bond is given by

$$B(S_t, r, t) = e^{-[r_t+\lambda_t]\Delta} E_t^Q \left[B(S_{t+\Delta}, r, t+\Delta)\right] + (1 - e^{\lambda_t \Delta})e^{-r\Delta}(1 - L)F \qquad (4.2)$$

where E_t^Q denotes the risk neutral expectation under the Q-measure at time t. $(1 - L_d)$ is, as mentioned earlier the recovery at default.

When we introduce the options to call and convert into the pricing algorithm (4.2) the pricing algorithm of a callable and convertible bond, CCB, is given by

$$CCB(S_t, r, t) = C_t + e^{-\rho Y_t \Delta} B(S_t, r, t) + (1 - e^{-\rho Y_t \Delta}) max(kS_t, \overline{C}) \tag{4.3}$$

where C_t is the coupon payments at time t if there is any. The second term is the risk-neutral probability of no call in the period between time t and time $t + \Delta$ multiplied by the market value of the bond before considering the call and convert options, (4.2), if the bond is not called in the next period. The final term is the risk neutral probability of the bond to be called multiplied by the value of the bond if called, which is the larger of the conversion value, kS_t and the call price $\overline{C}$.

We have now determined the price of a callable and convertible bond at a certain period based on the value in the following period.

When comparing a convertible bond with a non convertible bond it is not possible to compare the prices of the bonds if the value of the coupons and the time to maturity differ. We thus determine an expression of the yield to maturity that takes these differences in to account.

4.3.2 Yield to Maturity

In this section we determine the yield to maturity of a bond based on the price of the bond. In the calculation of the yield to maturity the bond's current price, the face value, coupons and time to maturity are taken into account. Some of these parameters are seen to differ between the convertible bond and the non convertible bond in our dataset.

The yield to maturity is the effective yield that satisfies that the expected present value of the future cash flows equals the price of the bond

$$CCB_t = \sum_{s=t+1}^{T} \frac{1}{(1+ytm)^s}(Future\ Cashflow)$$

where ytm is the yield to maturity. The effective yield is an expected yield, which is only realised when the bond is held to maturity without defaulting.

In the next section we will implement the pricing algorithm and the yield to maturity in `R`.

5 Data and Methodology

In the following we describe the data, our methodology and the assumptions we have made to be able to price the bonds as precisely as possible without ending up with a too complex pricing formula.

We start out by describing our data sources and how the data are processed. Then we explain how we construct our pricing formula by using a binomial tree approach where the probability parameters are found by moment matching. We show how the formula is implemented in `R` and use the pricing formula to calibrate the call speed parameter based on a control period. We finish the section with a short example where we calculate the price and the yield to maturity of a bond in the dataset and compare them to the actual traded values.

5.1 Data

The bond trading data used in the analysis of this thesis is from the dataset generated by Jens Dick-Nielsen and Marco Rossi. Their dataset is from the enhanced TRACE database which reports real-time over-the-counter (OTC) corporate bond trades. The database contains more than 99% OTC trades of the US corporate bond market. The bond characteristics are from Mergent Fixed Income Securities Database (FISD) which covers more than $140,000$ corporate, U.S. agency and U.S treasury debt.

The sample period is from July 2002 to December 2011. The sample therefore contains both the credit crisis of 2005, described by Mitchell, Pedersen and Pulvino (2007), and the arbitrage crash in the fall of 2008. The sample consists convertible and non convertible bonds with and without the option of the issuer to call.

In order to compare convertible bonds and non convertible bonds in the analysis a convertible and non convertible bond by the same issuer are paired in the dataset. The pair of bonds is chosen such that the non convertible bond is traded at least within three hours of the trade of the matched convertible bond and such that the maturity dates are less than one year apart, if the closest maturity date is before December 2012, and less than two years apart, otherwise. The seniority of the convertible bond is required to be at least as high as the seniority of the non convertible bond. The non convertible bonds are allowed to have worse senority since it would make the yields of the non convertible bond higher and thus make the result of underpricing even stronger. Since the bonds are issued by the same company the underlying stock and its volatility are the same.

To be able to calculate a theoretical price of the bonds we add data for a few more parameters to the dataset by Dick-Nielsen and Rossi. We add the risk free interest rate at the trade date of the bonds, the call price at which the issuer is able to call the bond, the recovery rate in case of default and the issuer's probability of default.

We use observed values of the risk free rate on every trading day of the bonds. The risk free rate is held constant throughout the life of the bond. We find the risk free rate from Saint Louis Federal Reserve Economic Data (FRED) as the 4-Week Treasury Bill: Secondary Market Rate (DTB4WK) with daily observations. So if the bond is traded on 02/03/2004 we look up the 4-Week Treasury Bill on that date and hold the rate constant until the maturity of the bond.

Since the different bonds can be called at different call prices, we find the call prices of each cusip from Bloomberg. A cusip is an identification number assigned to all stocks and registered bonds.[19] As mentioned earlier if the bond is callable, in practice it can only be called on specific dates and to different call prices depending on the dates. We assumed in this thesis that the call price is constant to maturity but to get the most realistic call price we obtain all the specific call prices of the different call dates of each cusip. Then when a bond is traded we use the next call price after that date. This call price is then held constant. Our model is therefore not able to deal with changing call prices during the life of the bond but it is close to a correct call price. If a bond is traded after the latest possible call date we use the latest available call price.

We use Moddy's rating of the bonds to determine the default probability and the recovery rate of the bonds. In the dataset the rating of the issuer is listed in a rating code from $1-28$. In the article of Dick-Nielsen and Rossi (2013), $TableA$ shows the converting from the rating code to Moody's Rating.

The article Corporate Default and Recovery Rates, $1920-2010$, Exhibit 35, from Moody's investor Service (2011) shows the Average Cumulative Issuer-Weighted Global Default Rates, $1983-2010$. We see that the default probabilities depend on both the rating and the time to maturity of the bond.[20] We thus convert the rating to default probablities based on the time to maturity of the bonds.

We could also have determined the probability of default based on credit spreads. We though think that it could be misleading because the CDS during the crisis were less liquid. We also believe that using Moody's matches the method of Dick-Nielsen and Rossi better, why we choose the Moody's default probabilities. Otherwise we would have one more external parameter that could influence the findings of our analasis in relation to Dick-Nielsen and Rossi (2013).

Table 5.1 shows the converting from the rating codes to the recovery rates in case of default. In the article Corporate Default and Recovery Rates, $1920-2010$, from Moody's investor Service (2011), Exhibit 22 shows the recovery rates based

[19]http://www.investopedia.com/terms/c/cusipnumber.asp
[20]The chart from Moody's Investor Service (2011) is shown in Appendix9.1.

Rating Code	Moody's Rating	Recovery Rate
1	Aaa	60.00%
2	Aa	37.24%
3	Aa	37.24%
4	Aa	37.24%
5	A	31.77%
6	A	31.77%
7	A	31.77%
8	Baa	41.47%
9	Baa	41.47%
10	Baa	41.47%
11	Ba	47.11%
12	Ba	47.11%
13	Ba	47.11%
14	B	37.90%
15	B	37.90%
16	B	37.90%
17	Caa	35.50%
18	Caa	35.50%
19	Caa	35.50%
20	Ca	35.50%
21	C	35.50%

Table 5.1: *Recovery Rates. This table shows the converting from the rating codes used in Dick-Nielsen and Rossi's dataset to Moody's Rating and from the Moody's rating to the recovery rates.*

on the rating of the issuer. Exhibit 22 shows the average senior unsecured recovery rates and we use the ones observed within one year.[21]

Our dataset initially consists of 33, 220 observations. On some of the traded bonds the stock prices are not specified so we reduce our sample such that it only consists of observations where there exist a stock price. We have the same issue with missing volatilities on some of the observations and unspecified coupons. In Dick-Nielsen and Rossi's dataset there is a rating which by Moody's is NR, Not Rated. We exclude these observations as well which leads to a dataset consisting

[21] In the report of Moody's Investor Service (2011) there existes non observable defaults of a issuer with a rating A. We therefore set the recovery rate to 60% because it then is higher than lower rated issuers recovery rate.

Table 5.2: *Bond Characteristics*

	Mean for convertible bonds	Mean for non convertible bonds
Coupon	0.042	0.074
Callable	0.193	0.334
Rating	Ba	Ba

This table presents bond characteristics for our sample of the convertible and non convertible bonds.

of 24, 234 traded pairs.

Some of the bond characteristics in the final dataset is shown in Table 5.2. The rating of the convertible and the non convertible bonds is on average equal. The coupons of the convertible bonds are nearly half the size of the coupons of the non convertible bonds which we expected since the convertible bonds include the embedded option.

The callability of the bonds is a binary variable indicating whether the bond is callable or not. We see that more of the non convertible bonds are callable than of the convertible bonds. When we compare the yields the result can be misleading since the callability decreases the prices and thus increases the yields.

The dataset is limited by the fact that it only contains the convertible bonds that are traded at the same time as a similar non convertible bond. There is a risk of not including all trades of the convertible bonds in the sample period. On the other hand, it contains intra day prices which add accuracy to the analysis.

5.1.1 Determining the default intensity γ

To implement the pricing formula 5.2 we need to determine the values of the default intensity, λ, and the call speed parameter, ρ. To be able to derive the default intensity, $\lambda_t = \frac{\gamma}{S_t}$, we have to determine an expression for γ. When default is modeled by an intensity-based model as in this thesis, the probability of default is given by

$$PD = 1 - e^{-\frac{\gamma}{S_0}}. \tag{5.1}$$

Since the rating of the bonds is known we can find the probability of default in the table in Appendix9.1, where Moody's rating is converted to a default probability. We then isolate γ in equation (5.1) to get the expression

$$\gamma = -ln(1 - PD)S_0.$$

Once we have developed the program for the pricing formula, we will calibrate ρ such that our found prices matches the traded prices as good as possible in the control period.

5.2 Derivative Pricing with a Binomial Tree Approach

In section 6.1 we derived the time t value of the callable and convertible bond based on the risk-neutral expected value of the bond in the next time period as

$$CCB(S_t, r_t, t) = C_t + e^{-\rho Y_t(t)\Delta} B(S_t, r_t, t) + (1 - e^{-\rho Y_t(t)\Delta}) max(aS_t, \overline{C}), \tag{5.2}$$

where $Y_t = 0$ when the bond is not callable.

The bond price can as mentioned be solved by backward recursion from the maturity date. This can be done with the models developed by Cox, Ross and Rubinstein (The CRR model), Jarrow and Rudd (The JR model), or by Tian (The TIAN model). The CRR model is constructed on a discrete-time lattice and the evolution of the price converges weakly to a lognormal diffusion when the time between two events goes to zero. The JR Model is an extension of the CRR model, which accounts for the local drift term. It is constructed such that the first two moments of the discrete and continuous processes match. The TIAN Model is yet another extension which matches discrete and continuous local moments up to third order. The main difference between the three models is their definition of the upward and downward change, u and d, and thereby the definition of the risk neutral probability, p.

It has been proven by Leisen and Reimer (1996) that the order of convergence in pricing options for all three methods is equal to one, which means the three models are equivalent. We thus get approximately the same price with all three models. We base our pricing model on an adapted version of the JR model.

In the following section we review a binomial tree approach to value derivative instruments on lognormally distributed asset prices. First we describe the binomial tree pricing process in general. Thereafter we review the JR model of option pricing including the use of moment matching, and finally we adapt the process to the case where a bond is callable and convertible and thus implement expression (5.2).

5.2.1 The Binomial Tree Pricing Process

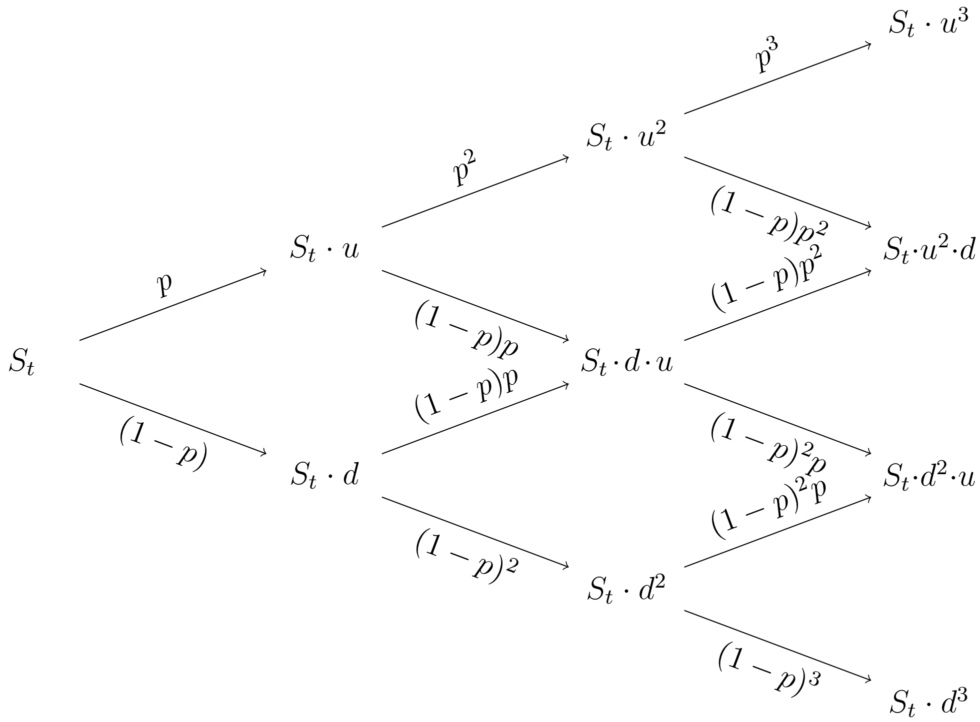
The binomial tree process is a three step process. First the expected development of the stock is generated in a binomial tree. Then the option value is calculated in each of the final nodes such that the option price at the valuation date, time t, can be found by backwards recursion.

The expected development of the stock is generated by forward recursion starting at the valuation date and working forward to the expiration date. As mentioned before the stock is at each step expected to move up by the factor u with the risk neutral probability, p, and down by the factor d with $1 - p$.

If the number of upward steps is denoted by n_u, the expected value of the stock after n steps in each node is then calculated by

$$S_{t+n}^{n_u} = S_t \cdot u^{n_u} d^{n-n_u}$$

as shown in the figure below.



At the expiration date, time T, the investor can either choose to receive the face value, F, or the conversion ration, k, times the value of the stock, S_T. The investor chooses the largest of the two options based on the value of stock. The convertible bond can thus be seen as an option on equity and the time T payoff can be expressed by

$$CCB_T = max(F, k \cdot S_T). \tag{5.3}$$

When the values at the final nodes are determined, we work backwards through the nodes to find the value of the bond at the valuation date, t. Since the value today is given by the future pay off discounted by the risk free rate under the risk neutral measure, we calculate the value at each node by

$$CCB_{t,i} = e^{-r\Delta t} \left(pCCB_{t+\Delta t,i+1} + (1-p)CCB_{t+\Delta t,i} \right), \tag{5.4}$$

where $CCB_{t,i}$ is the value of the bond for the i^{th} node at time t.
In the next section we review the previous mentioned Jarrow-Ruud model of option pricing to determine the value of the risk neutral probability, p.

5.2.2 Option Price Modelling by Moment Matching

As mentioned earlier we determine the value of the bond by the Jarrow-Ruud model which uses the moment matching technique to determine the discrete distribution of the change in the stock price. In this section we show how this technique is used to value derivative instruments on lognormal distributed asset prices needed to value the option on equity.

In a continuous time setting we assume the stock, S, under Q follows the Geometric Brownian Motion process

$$dS_t = (r + \lambda)Sdt + \sigma S dW_t,$$

where r is the instantaneous risk-free interest rate and σ is the instantaneous volatility of the stock price. If we set $X = \ln(S)$ we get by Itô's lemma that

$$dX_t = \left((r + \lambda) - \frac{\sigma^2}{2}\right) dt + \sigma dW_t = \alpha dt + \sigma dW_t$$

such that X follows a generalized Wiener process for $(0, t)$. The variable $\hat{X} = X_{t+\Delta t} - X_t = \ln\left(\frac{S_{t+\Delta t}}{S_t}\right)$ is then normally distributed with mean $\alpha \cdot \Delta t$ and volatility $\sigma^2 \cdot \Delta t$. a short period of time, Δt.

In a discrete time setting the stock moves from S_t to $S_{t+\Delta t} = S_t \cdot \hat{x}_j$ in a short period of time, Δt, for $j = 1, \ldots, n$, where $\hat{x}_j$ is the proportional change in the value of S on the time interval Δt and n is the number of possible changes in the value. In our model we set $n = 2$, where the change is either up or down.

The discrete distribution of the change in the lognormal stock price $\hat{X}$ in Δt is then given by

$$Q(\hat{X} = \hat{x}_j) = p_j,$$

where Q is the risk-neutral probability measure. The first moment of the discrete distribution is given by

$$M_1 = \sum_{j=1}^{n} p_j \hat{x}_j,$$

and the centered kth order moment by

$$M_k = \sum_{j=1}^{n} p_j \left(\hat{x}_j - m\right)^k = \sum_{j=1}^{n} p_j z_j^k,$$

where z_j is the distance from p_j to the mean value.

In a continuous model we have that the first central moment, $\tilde{m}_1$, is the mean of $\hat{X}$ and that the second, $\tilde{m}_2$ is the variance of $\hat{X}$.

The moment matching technique is applied by matching the central moments of the discrete distribution by those of the continuous distribution. The first moments of the distributions are matched by setting

$$M_1 = \sum_{j=1}^{n} p_j \hat{x}_j = E(\hat{X}) = \tilde{m}_1.$$

and the kth moment by

$$M_k = \sum_{j=1}^{n} p_j z_j^k = \tilde{m}_k,$$

We match the discrete distribution with the underlying distribution by solving the non linear system

$$\tilde{m}_k = \sum_{j=1}^{n} p_j z_j^k = m_k^X, \quad k = 0, \ldots, N$$

with respect to the unknown parameters p_j and z_j, $j = 1, \ldots, n$, where m_k^X is the kth order central moment of the underlying continuous distribution and N is the number of moments that are matched.

To be able to solve the system and thereby define the multinomial lattice we need to have $n+1$ equations, which will be the case when the number of possible changes, n, equals the number of moments matched, N. The condition that the probabilities sum to one is our first equation and the rest of the equations are obtained by matching the first n central moments of the discrete distribution with the first n central moments of the underlying continuous distribution.

In our setting the moment matching is applied with $n = 2$ thus the two first moments are matched. In that case the model is a two jump process and thereby a binomial model. The stock can either go up from S_0 to uS_0 or down to dS_0, where $u > e^{rt}$ to avoid arbitrage and $d < 1$. In this binomial setting the variable $\hat{X}$ has the discrete distribution

$$\hat{X} = \begin{cases} \ln(u), & \text{with risk neutral probability } p \\ \ln(d), & \text{with risk neutral probability } 1-p \end{cases}$$

The first central moment of the continuous underlying process is $E(\hat{X}) = \alpha \cdot \Delta t$ and the second central moment is $VAR(\hat{X}) = \sigma^2 \cdot \Delta t$. This gives us the following system

$$p \cdot ln(u) + (1-p) \cdot ln(d) = \alpha \Delta t$$
$$p(1-p)(ln(u) - ln(d))^2 = \sigma^2 \Delta t$$

of two equations with three unknowns u, d and p. If we set $w_1 = \frac{ln(u)-\alpha\Delta t}{\sigma\sqrt{\Delta t}}$ and $w_2 = \frac{ln(d)-\alpha\Delta t}{\sigma\sqrt{\Delta t}}$ and add the constraint that the third moment equals zero for the normal distribution, we get the following system

$$pw_1 + (1-p)w_2 = 0$$
$$p(w_1)^2 + (1-p)(w_2)^2 = 1$$
$$p(w_1)^3 + (1-p)(w_2)^3 = 0$$

of three equations with three unknowns. The unique solution is then $p = \frac{1}{2}$, $w_1 = 1$ and $w_2 = -1$ which gives us the up and down parameters as[22]

$$u = e^{\alpha\Delta t + \sigma\sqrt{\Delta t}}$$
$$d = e^{\alpha\Delta t - \sigma\sqrt{\Delta t}}.$$

We are now able to use the developed expressions for p, u and d to calculate the expected value of the stock and thereby the expected value of the bond as shown in the previous subsection.

In the next section we develop the pricing formula in the program `R` and show how it is used to price a callable and convertible bond.

5.2.3 Callable and Convertible Bond Pricing in R

We follow the three step process explained in section 5.2.1 to find the expected value of a callable and convertible bond and implement the model in the program `R`.[23] We define the function `CCBond` as a function of the inputs

```
CCBond <- function(S0, FV, ttm, r, sigma, L, k, CallPrice,
 rho, c, PD, CallYN).
```

`S0` is the stock price at the valuation date, `FV` is the face value of the bond, `ttm` is the time to maturity of the bond declared in years, `r` is the risk free interest

[22] Jarrow-Ruud (1983)
[23] The entire and coherent code is presented in the Appendix.

rate, `sigma` is the volatility of the stock price, `L` is the loss rate at default, `k` is the conversion ratio, `CallPrice` is the cost of calling the bond before maturity, `rho` is the annualized call speed parameter, `c` is the coupon payment of the bond, `PD` is the probability of default and `CallYN` is whether the bond is callable or not.

In the first step we generate the expected development of the stock, `S`, and thereby the default intensity, `lambda`, by an iteration based on the parameters u and d. In `R` this is done by

```
S[1,1] <- S0
lambda[1,1] <- gamma/S[1,1]

for ( j in 2:n){
for ( i in 1:j ){
if(i==j){
  S[i,j]=S[i-1,j-1]*d*exp(lambda[i-1,j-1]*dt)
  lambda[i,j] <- gamma/S[i,j]
} else {
  if(i<j){
    S[i,j]=S[i,j-1]*u*exp(lambda[i,j-1]*dt)
    lambda[i,j] <- gamma/S[i,j]
  } else {
    S[i,j] = 0
    lambda[i,j] <- 0
  }
}}}
```

In the next step the option value of the callable and convertible bond is calculated in each of the final nodes based on the expected values of the stock at the expiration date. Equation (5.3) is then implemented in `R` by

```
CCValue[i,j] <- max(FV,k*S[i,j])
```

In the next step we work backward through all the nodes in the tree by implementing equation (5.4) and equation (5.2) in R in the iteration.

We use a j loop which is moving backwards through time and an i loop which is moving through the nodes at each time step.

```
for ( j in seq(from=n, to=1, by=-1) ){
for ( i in 1:j ){
```

At first, the value of the bond without the call and conversion feature is derived based on the expected value of the bonds in the subsequent notes.

```
NoCall[i,j] <- exp(-r*dt)*(
exp(-lambda[i,j]*dt)*(p*CCValue[i,j+1] +
(1-p)*CCValue[i+1,j+1]) + (1-exp(-lambda[i,j]*dt))*(1-L)*FV
)
```

The coupon payments are as mentioned earlier defined by

$$C_t = \sum_i e^{-(r+\lambda_d(t))T_i} \cdot c_i 1_{t \geq T_i}.$$

where c_i is the coupon payment at time T_i. For the simplicity we asume the coupon to be payed in every step corresponding to each month of the bond's life. The coupon payment in each step is derived in R by

```
Coupon <- exp(-(r+lambda[i,j])*dt)*c*FV*(ttm/n)
```

where `ttm/n` is the size of the steps which corresponds to a month.

The call and conversion feature are then derived by

```
if (CallYN == 1){

Call[i,j] <- max(k*S[i,j],CallPrice)
Y[i,j] <- (NoCall[i,j]/Call[i,j])-1
if (Y[i,j]<0) { Y[i,j] = 0 }

} else {

Call[i,j] <- 0
Y[i,j] <- 0
}
```

where `Call[i,j]` is the value the issuer of the bond receives if the bond is called. If the value of conversion, `k*S[i,j]`, is higher than the callprice, the investor will convert the bond after it is called[24]. The value that the issuer receives by calling is thus the maximum of the call price and the conversion value.

Finally, the total value of the bond in each step is found by adding the value of the coupon, the value of the bond without call and conversion features and the seperate value of the call and conversion feature.

```
CCValue[i,j] <- Coupon + exp(-(rho*Y[i,j])*dt)* NoCall[i,j] +
(1-exp(-(rho*Y[i,j]*dt)))* Call[i,j].
```

The bond price at the valuation date is equal to the value of the element in the first row and the first column of the `CCValue` matrix.

[24]Remember, that there typically is a 30 days period after the call, where the investor can convert the bond to the underlying stock

5.2.4 Determining the value of ρ

As mentioned, the second of the two unknown parameters in the pricing formula is the call speed parameter, ρ. We choose the value of ρ such that the calculated prices are as close as possible to the traded prices in the control period from January 2006 until December 2007. We therefore define the sum of squared errors by

$$SSE = \sum_{i=1}^{n} (\text{PriceTheo}_i - \text{PriceTraded}_i)^2$$

where i represents each trade in dataset from January 2006 until December 2007, n is number of trades in the period, PriceTheo is the theoretical calculated price and PriceTraded is the actual traded price.

To determine the value of ρ we calculate the sum of squared errors for $\rho \in (1; 21)$ as our calculations suggest the call speed parameter to be in that interval. The result is shown in figure (5.1).

The figure shows the sum of squared errors has a minimum for $\rho = 9$. This indicates that the model fits the prices in the control period best for that value of ρ. With foundation in this optimization we set $\rho = 9$ in the analysis of this thesis.

5.2.5 Deriving the Yields of Callable and Convertible bonds

The effective yield, ytm, is derived by finding the specific yield that satisfies the price of the bond to be equal to the discounted future cash flows. The future cash flows of a convertible bond are either equal to coupons plus face value or coupons plus conversion price, depending on which value that is the highest. The yield should therefore satisfy

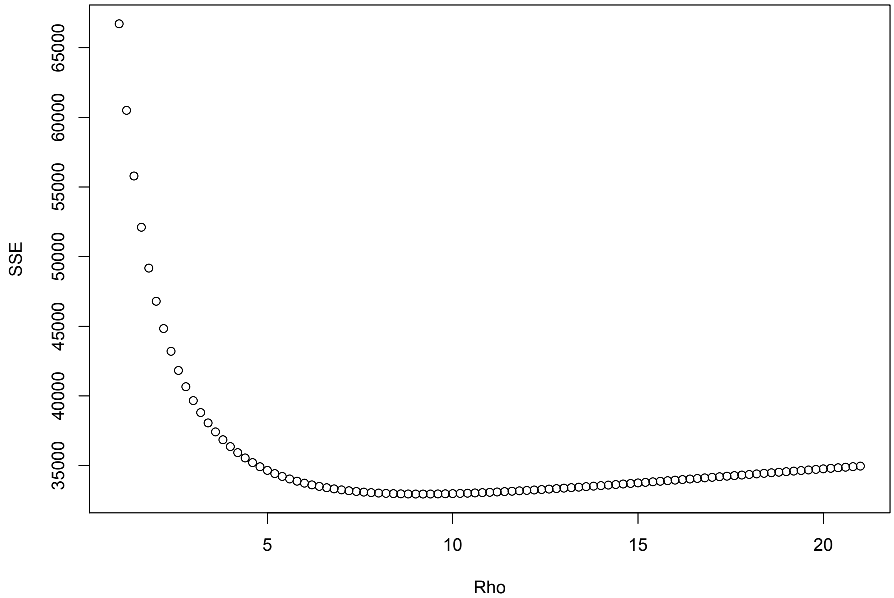


Figure 5.1: *The figure shows the sum of squared errors as a function of the parameter,* $\rho \in (1; 21)$.

$$CCB_t = \sum_{s=t+1}^{T} \frac{1}{(1+ytm)^s} (Future\ Cashflow)$$

where n is the number of coupon payments.

We determine the effective yield in `R` by solving the equation above for ytm. This is done in the function `ytm`, with inputs: face value, time to maturity in years, yearly coupon rate, conversion ratio, stock price and the theoretical calculated price. The function is defined by

```
ytm <- function(face, price, ttm.y, coupon.rate.y){


ttm.y <- max(ttm.y,1/12)
coupon.y <- coupon.rate.y*face


## Monthly coupon payments and then a payment of face at maturity
cashflowsdates <- seq(1/12, ttm.y, by=1/12)
```

```
cashflows <- c(rep(coupon.y/12, length(cashflowsdates)-1),
face+(coupon.y/12))
## Solve for the internal rate of return (x) that makes the discounted
cash flows equal to the price
uniroot(function(x) sum(cashflows/((1+x)^cashflowsdates))-price,
interval=c(-0.99, 0.99))$root
}
```

where the `cashflows` are the annual coupon payments and the payment of either the face value or the conversion price at maturity and where the uniroot function solves for the *ytm* that makes the discounted cash flows equal to the calculated price.

In the last section we use the `ytm` function on one of the trades in our dataset as an example and compare the found *ytm* with the yield in the dataset.

5.2.6 Example: Pricing a Callable and Convertible Bond

The pricing formula can now be used to price a callable and convertible bond. As an example we will price the Steel Dynamics Inc bond which was traded at August 16 2010. It was traded at a price of $111.98. The bond had a yearly coupon payment of 5.13%, and it matured after approximately 3 years and 10 months. It had the option to be converted in to the the Class A common stock of the company which had a price of $14.32 and a volatility of 2.49% with a conversion ratio of 5.70. The risk free interest rate is 0.15%[25] and the rating of the bond is Ba which gives a default probability of 9.07% and a recovery rate of 47.11%[26].

Using these inputs in the `CCBond` function gives us the price

```
> CCBond(S0= 14.32, FV= 100, ttm= 3.83, r= 0.0015,
sigma= 0.0249, L=1-0.4711, k= 5.70, CallPrice= 0,
```

[25] Saint Louis Federal Reserve Economic Data (FRED)

[26] Based on Exhibit 27 in Corporate Default and Recovery Rates, 1920 – 2010

```
 rho=9, c= 0.0513, PD= 0.0907, CallYN= 0)

[1] 112.42
```

which is close to the traded price \$111.98.

The parameters of the bond can also be used as inputs in the `ytm` function developed in the last section to get the effective yield of the bond. For the bond the yield to maturity based on the traded price is 1.83%. Using the inputs from before we get

```
> ytm(face = 100, price = 112.42, ttm.y = 3.83,
 coupon.rate.y = 0.0513, k = 5.70, S = 14.32,
 r = 0.0015)

[1] 0.0172
```

which again is close to the effective yield of 1.83% which is observed in the dataset.

The fact that both our theoretical calculated values is close to the observed values indicates that the model gives a price close to the real price and that the parameters are well chosen. In the next section we show that the price accuracy generalizes to a large amount of the traded convertible bonds in certain periods.

This method can now be used to price callable and convertible bonds and thereby examine if they were underpriced in the periods of 2005 and 2008.

6 Analysis

In the fall of 2008 when the financial market collapsed the convertible arbitrage hedge funds realized large losses. Dick-Nielsen and Rossi (2013) and Mitchell and Pulvino (2012) show the arbitrage crash in the convertible bond market in the wake of the forced deleveraging. In this section we analyse the arbitrage crashes based on the pricing formula developed in the previous section.

We start the analysis by comparing the theoretical prices of the convertible bonds with the traded prices of the convertible bonds. We show that the pricing model price the convertible bonds relatively close to the prices at which the bonds are traded in certain periods. Furthermore, we show that the traded prices fall relative to the theorical prices in the period of 2008 indicating an arbitrage crash.

In the second part of the analysis, we compare the yields of the convertible bond with yields of similar non convertible bonds and show that the convertible bonds were traded at a value lower than comparable non convertible bonds in the periods of 2005 and 2008.

We finish the section by showing that the matching of bonds method is more stringent than the price comparing method. The pricing method verifies the existence of an arbitrage opportunity for almost all the trades that are identified to be arbitrage opportunities in the matching method.

6.1 Comparing Traded Prices and Theoretical Prices

The pricing model developed in the previous section is used to calculate the theoretical prices of the traded convertible bonds. We are thus able to analyse the difference between the theoretical calculated prices and the actual traded prices. In an unstressed market we expect the theoretical prices and the traded prices to be equal if the model reflects the actual traded prices. Furthermore, we expect to see a decline in the traded prices relative to the theoretical prices in the periods of the arbitrage crashes.

Both Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) claim that the convertible bonds were sold at fire-sale prices in the wake of the Lehman bankruptcy in September 2008. Mitchell, Pedersen and Pulvino (2007) showed that the demand for convertible bonds fell in 2005 resulting in very cheap convertible bonds. We expect to verify these statements through our model by showing a difference between the theoretical and the traded prices in 2005 and 2008.

Figure 6.1 shows the traded prices and the theoretically calculated prices of the convertible bonds. The theoretical and traded prices seem to be equal when the market is unstressed market which verifies our model based on the applied data set. In the wake of the Lehman bankruptcy the theoretical calculated prices are higher than the traded prices. This indicates that the convertible bonds were sold at a discount compared to their fundamental value. There is a tendency of higher theoretical prices compared to the traded prices in 2005 but it is not as clear as in 2008. Before, after, and between the two underpriced periods the model seems to be able to calculate theoretical prices which are closer to the actual traded prices.

In order to get a better graphical overview of the result, we have calculated the one month moving average theoretical and traded prices. The result is shown in Figure 6.2.

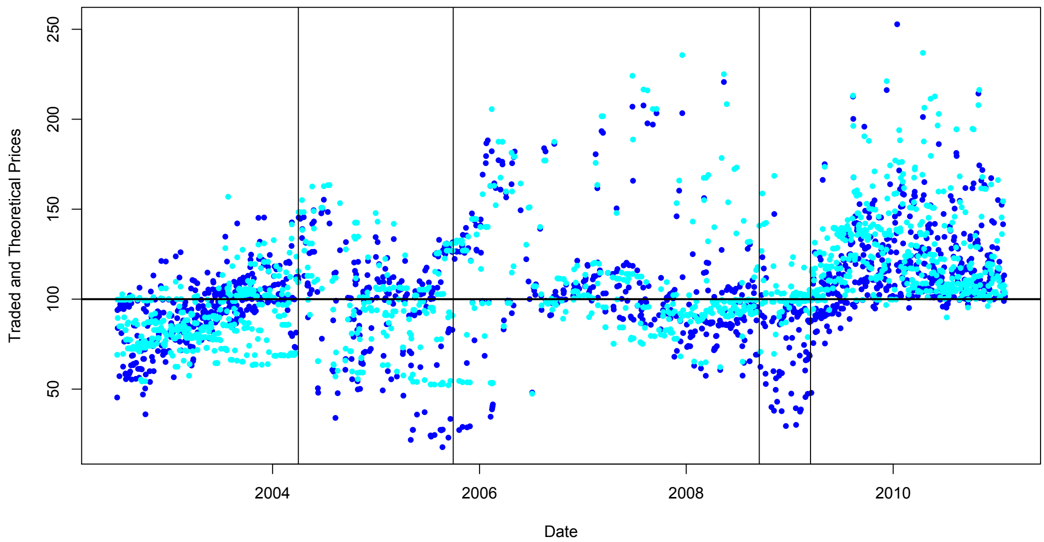


Figure 6.1: *This figure shows the theoretical prices, calculated from the model, and the traded prices observed in the market. The theoretical prices are shown as the turquoise dots and the traded prices are shown as the blue dots. The Arbitrage crash in the fall of* 2008*, more precise the period from* 15/09/08 *to* 15/03/09*, and the crash of* 2005*,* 01/04/2004 *to* 01/09/2005*, are marked by the verticle lines.*

Figure 6.2 confirms the conclusion of Figure 6.1. We see that the difference in the prices in 2005 is limited compared to the situation in 2008. The figure shows that the model detects a significant price difference in the spring of 2008 with a relative cheap traded price compared to the theoretical price. We though still see a positive correlation between the prices in the period. Just after the Lehman bankruptcy the traded prices fell relatively to the theoretical prices which resulted in a negative correlated development in the prices. In the period of 2002 to 2011 the biggest price difference turns out to be just after the Lehman bankruptcy in September 2008. The convertible bonds stayed very cheap for a period of 6 months which indicates that the convertible bond market experienced an arbitrage crash in the period. Since the model identify the periods of arbitrage crashes it can be used as a tool to determine if the traded bonds are underpriced.

After the arbitrage crash in 2008 we expected the market to normalize such that the price difference diminished but the underpricing seemed to continue far

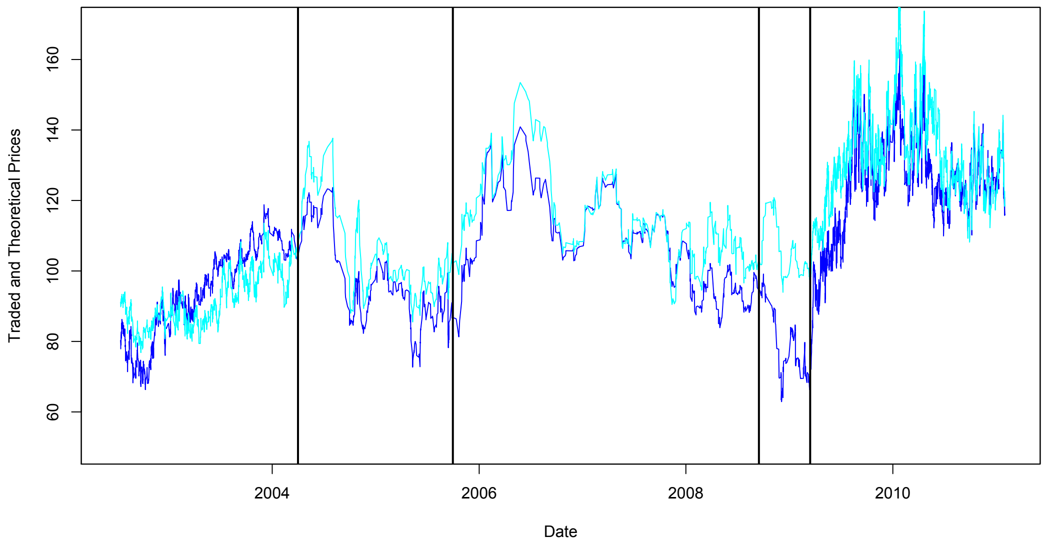


Figure 6.2: *This figure shows the one month moving average of the theoretical prices and the traded prices. The theoretical prices are shown as the turquoise line and the traded prices are shown as the blue line. The Arbitrage crash in the fall of* 2008*, more precise the period from* 15/09/08 *to* 15/03/09*, and the crash of* 2005*,* 01/04/2004 *to* 01/09/2005*, are marked by the verticle lines.*

into 2009. The prices fluctuated in the period indicating a very volatile market. In figure 6.3 we zoom in on the period of March 2009 to January 2011. We see that the traded prices primarily follow the theoretical prices even though there is a small difference in the values in the beginning of the period and around June 2010. The correlation between the theoretical prices and the traded prices for this period is 0.91 which indicates that the traded prices follow the theoretical prices all in all.

For the entire period of our data set, we notice that the traded prices follow the theoretical prices except for the period just after the Lehman bankruptcy. Thus, it is expected that the theoretical prices and the traded prices are highly correlated. The correlation between the prices based on the entire data set is 0.89, which is described as a strongly correlation.[27]

[27] A correlation greater that 0.8 is generally described as strong, whereas a correlation less than 0.5 is generally described as weak. (http://mathbits.com/MathBits/TISection/Statistics2/correlation.htm)

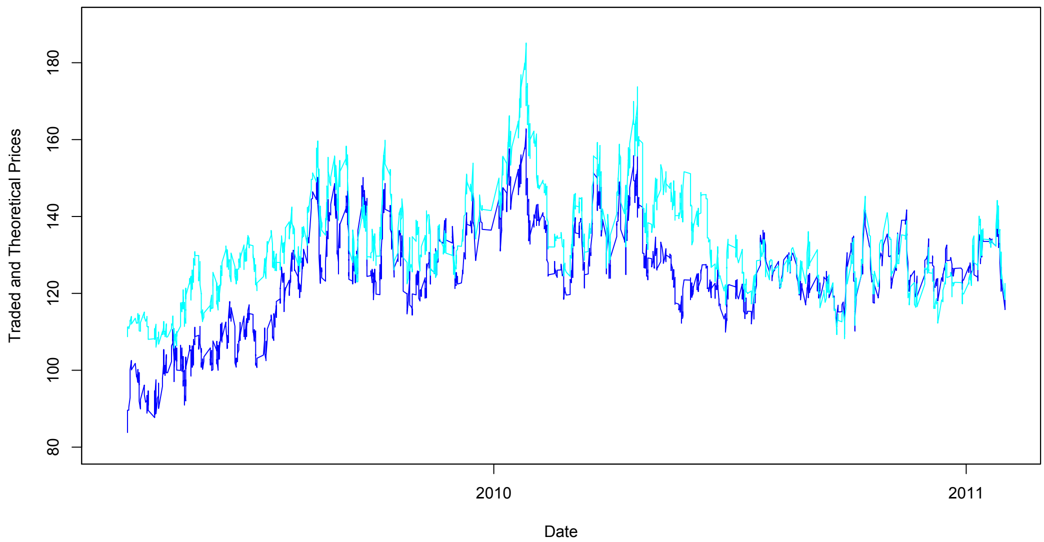


Figure 6.3: *This figure shows the one month moving average of the theoretical prices and the traded prices in the period after the crash of* 2008*. The theoretical prices are shown as the turquoise line and the traded prices are shown as the blue line. The figure shows that the traded and the theoretical prices follows approximately and the correlation between the prices seems to be high.*

6.1.1 Linear Relationship Between Traded and Theoretical Prices

The strong correlation indicates the existence of a positive linear relationship between the traded prices and the theoretical prices such that

$$Traded\ Price = \alpha + \beta\, Theoretical\ Price + \epsilon.$$

Figure 6.4 shows the theoretical prices plotted against the traded prices. When $\alpha = 0$ and $\beta = 1$ the plot of the theoretical prices against the traded prices all lie on a straight line with origin in zero. The theoretical prices are then completely correlated with the traded prices and the regression line would thus be able to explain all of the variation. The bigger difference between the line and the points in the scatterplot, the less it is able to explain. The figure shows that the regression line is not able to pass through every point in the scatterplot but we do expect a low α and a β close to one because of the high observed correlation.

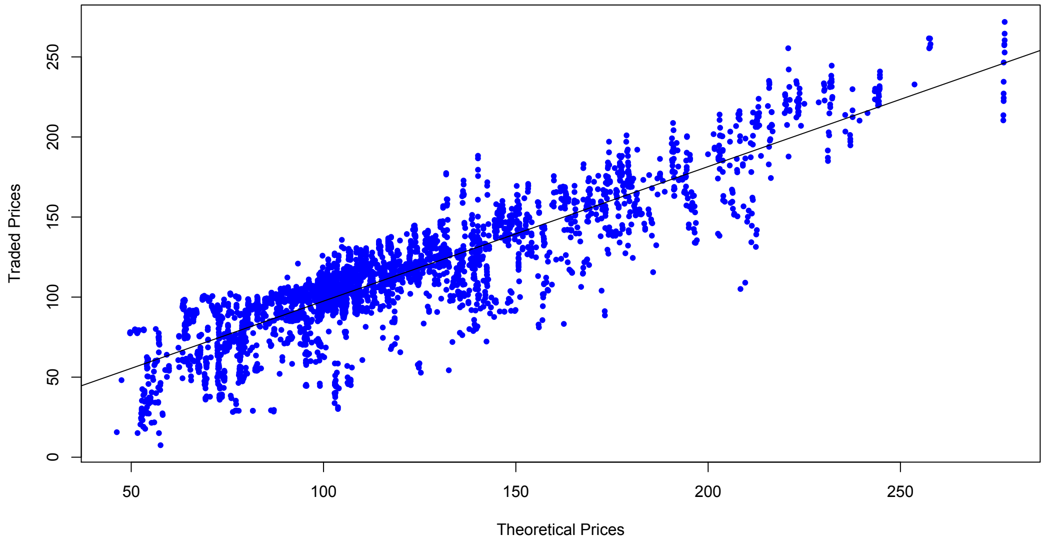


Figure 6.4: *This figure represents the relation between the theoretical prices and the traded prices. The line represents the linear regression equation with interception* 13.51 *and slope* 0.84.

From the regression analysis we get the value of $\alpha = 13.51$ which is relatively low and $\beta = 0.84$ which is close to one. A β close to one indicates that our model does not calculate systematically wrong. The regression line is

$$\textit{Traded Price} = 13.51 + 0.84 \cdot \textit{Theoretical Price} + \epsilon. \tag{6.1}$$

The coefficient of determination, R^2, explains the proportion of the fluctuation of the theoretical prices that is predictable from the traded prices. It denotes the strength of the linear association between the traded prices and the theoretical prices.

The coefficient of determination of our linear regression equation is 79%. This means that 79% of the total variation in the theoretical prices can be explained by the linear relationship between the traded prices and the theoretical prices in equation (6.1.1). The other 21% remains unexplained. Since the coefficient of determination is high the regression line, Equation 6.1.1, represents the data very well.

6.1.2 Excluding Credit-Sensitive Bonds

With reference to Mitchell and Pulvino (2012), which uses a comparative model to ours, they reduce their dataset only including bonds with moneyness larger than 0.63 in order to reduce estimation errors. Applying this assumption to our model results in a better correlation between the theoretical calculated prices and the actual traded prices.

In the computation of the theoretical prices we use an estimation of the default probability from Moody's Investor Service (2011). When a bond is out of the money it is more sensitive to the default probability than a bond in the money since an out of the money bond is more credit sensitive than equity sensitive. To reduce the sensitivity of the estimate of the default probability we exclude bonds that are credit sensitive bonds, defined as bonds with a moneyness less than 0.625.

To define whether the convertible bonds are credit-sensitive or equity-sensitive we use the median of the moneyness.[28] The median moneyness of our dataset is 0.625 and we therefore define convertible bonds with moneyness less than 0.625 as credit-sensitive bonds and convertible bonds with moneyness larger than 0.625 as equity-sensitive bonds.

We define moneyness as

$$\text{Moneyness} = \text{Stock price}/\text{Conversion price}.$$

If a convertible bond is in the money it has a moneyness larger than 1 and a convertible bond that is out of the money has a moneyness less than 1.

[28] The same method used by Mitchell and Pulvino (2012).

Moneyness describes the intrinsic value of an option in its current state.[29] It tells whether the exercising for the option holders would lead to a profit or non profit. A typically traded convertible bond has a moneyness of less than 1.[30]

The average moneyness of our dataset is 0.69 which indicates that the convertible bonds were typically traded out of the money. When we examine the observations in the period of September 15 2008 to March 15 2009 we see that the average moneyness decreases to 0.51. This means that our observations indicate that bonds were traded more out of the money in the financial crisis. According to the analysis of Mitchell and Pulvino (2012) the average moneyness decreased in the financial crisis. This difference from our model could be due to the difference in the actual analysed convertible bonds. In our analysis we use intra day prices of convertible bonds that have a comparable non convertible bond, see Section 5.1 where the data of Mitchell and Pulvino (2012) only contains weekly prices. Intuitively, we expect the average moneyness of the bonds traded in the crisis to decrease since the stock prices decreased in the crisis and they are the only variable parameter in the expression of the moneyness.

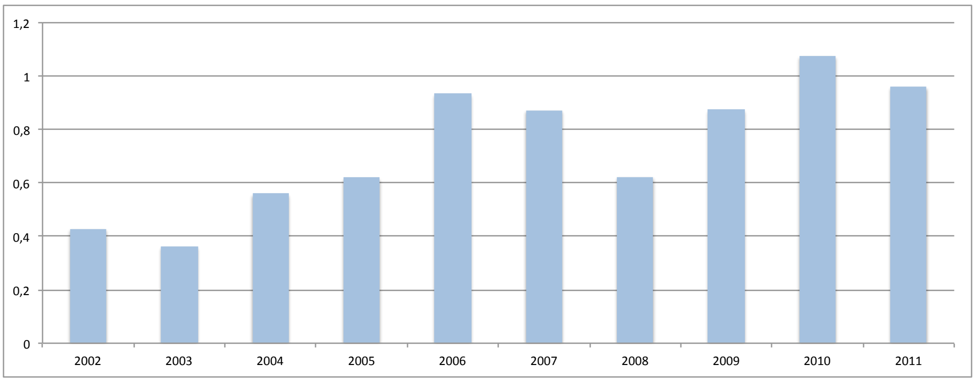


Figure 6.5: *This figure presents the average moneyness of the traded bonds of our dataset.*

The average moneyness of the years in our dataset is shown in Figure 6.5. The highest average moneyness is seen to be in the years of 2006 and 2010 where our

[29]http://www.investopedia.com/terms/m/moneyness.asp

[30]According to Mitchell and Pulvino (2012) several convertible bonds were traded in the money in the financial crisis of 2008.

model fits the traded prices well.

When we exclude bonds with a moneyness less than 0.625 our model shows an increased correlation between the theoretical and traded prices. The correlation between the prices is now 0.91 thus higher than the correlation of 0.89 before. The coefficient of de termination is also increased from 79% to 82%. The correlation between the theoretical and trade prices decreases when using bonds with moneyness less than 0.625. The correlation between the bonds is 0.39 when we only include credit-sensitive bonds. The theoretical and traded prices are shown in Figure 6.6 only including equity-sensitive bonds.

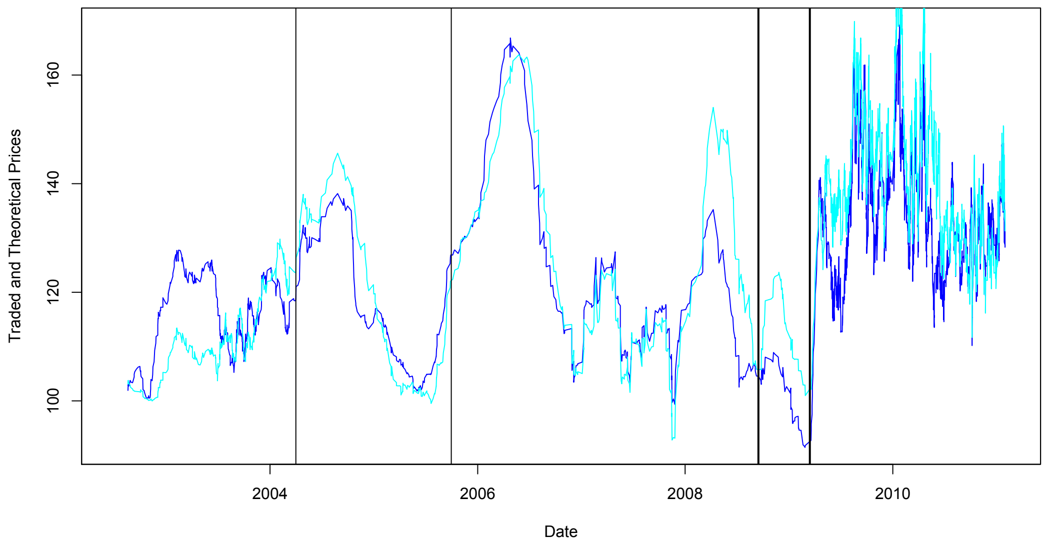


Figure 6.6: *This figure shows the one month moving average of the theoretical prices and the traded prices. The theoretical prices are shown as the turquoise line and the traded prices are shown as the blue line. The Arbitrage crash in the fall of* 2008*, more precise the period from* 15/09/08 *to* 15/03/09*, and the crash of* 2005, 01/04/2004 *to* 01/09/2005*, are marked by the verticle lines. Only convertible bonds that have a moneyness larger than* 0.625 *is included.*

Figure 6.6 shows a minor underpricing in the period corresponding to the arbitrage crash of 2005 which is similar to Figure 6.2. Again the underpricing of 2008 starts before the Lehman bankruptcy. However, the increased difference between the prices after the Lehman bankruptcy, that we saw in Figure 6.2, is now reduced and show a continued positive correlation between the price developments. Since the correlation after excluding credit-sensitive bonds is close to the correlation on the entire dataset and as since the exclusion halves the number of observations in our dataset we choose not to exclude credit-sensitive bonds. We choose to base our analysis on the whole dataset.

6.1.3 The Cheapness of Convertible Bonds

The cheapness of the bonds is now examined to see how cheap the bonds were during the arbitrage crashes. We define the cheapness/richness measure as

$$Cheapness/Richness = \frac{TradedPrice - TheoreticalPrice}{TheoreticalPrice}.$$

The convertible bonds are thus cheap if the traded prices are lower than the theoretical prices and rich otherwise. Figure 6.7 displays the cheapness/richness of the convertible bonds in the data set. If our data set of theoretical prices represents the market well we should expect to see a cheapness close to zero from the beginning of our sample in 2002 until the end of 2011 with the exception of the two arbitrage crashes. In the two arbitrage crashes the figure is expected to show an increase in the cheapness (being more negative).

Figure 6.7 shows the cheapness/richness primarily varies between -10% to 10%. On average, the convertible bonds were traded at prices 0.02% lower than theoretical prices in the sample. Thus the average cheapness is close to zero. The cheapness meassure ranges from nearly 20% rich in 2003 to about 38% cheap in the arbitrage crash of 2008. The convertible bonds were cheap in both of the two arbitrage crashes in 2005 and 2008. The convertible bonds already began to cheapen in the beginning of 2008 but the maximum level of the cheapness is first reached on the crash. When the crash dissolved the cheapness of the bonds decreased, although they kept cheap until the end of 2009. The same conclusion as in Figure 6.2 is thus repeated here. The cheapness measure fluctuates a lot in

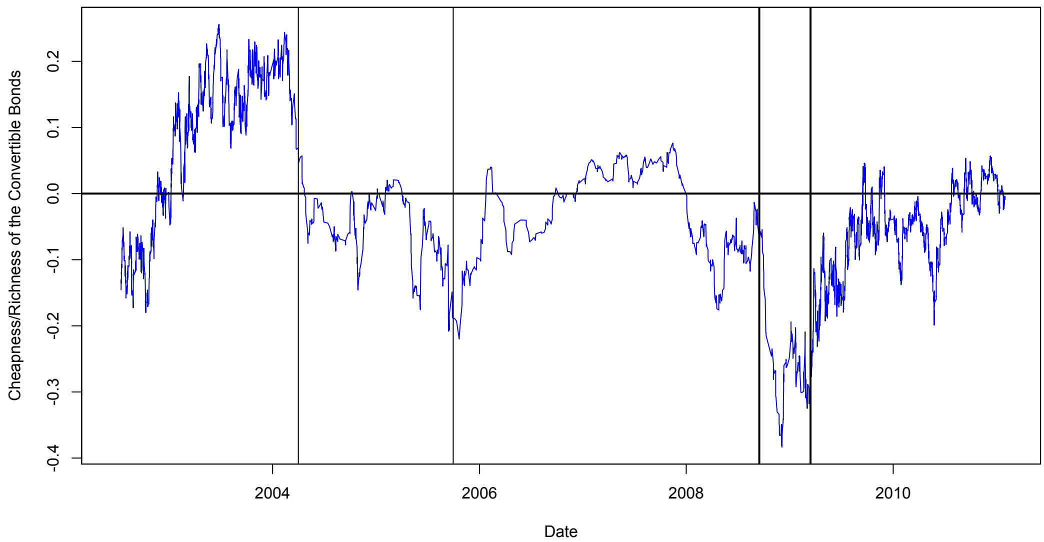


Figure 6.7: *This figure shows the cheapness of the convertible bonds. The cheapness/richness is defined as* $\frac{TradedPrice - TheoreticalPrice}{TheoreticalPrice}$ *and the one month moving average of the meassure is shown. The Arbitrage crash in the fall of* 2008*, more precise the period from* 15/09/08 *to* 15/03/09*, and the crash of* 2005*,* 01/04/2004 *to* 01/09/2005*, are marked by the vertical lines.*

the beginning and in the end of the sample indicating high volatility levels in the equity markets.

Figure 6.7 shows the convertible bond market experienced an arbitrage crash in 2005, which Mitchell, Pedersen and Pulvino (2007) argue was a result of fund-of-funds and other large institutional investors redeemed the investments because of low returns. However, we see that the crash of 2008 is bigger than the crash of 2005.

6.1.4 Testing for Underpricing

As mentioned earlier the linear regression explains 79% of the variation in the traded and theoretical prices. In this section we test if we are able to assume equal means in the two distributions of the prices. If we can accept equal means in the traded prices and the theoretical prices in the unstressed periods and reject

Period	P-value	Mean of Traded Prices	Mean of Theoretical Prices
Control Period	0.79	114.66	113.76
Crash of 2008	2.513e-14	75.96	103.21
Crash of 2005	0.12	95.06	99.54
The Entire Period	3.65e-08	110.22	115.15

Table 6.1: *This table presents the results of the T-test. The hypothesis of the means being equal is tested in different periods of the dataset.*

the hypothesis in the crashes of 2005 and 2008 we can confirm the mispricing of the convertible bonds. We set up the following hypothesis

$$H_0 : \mu_{TheoreticalPrice} = \mu_{TradedPrice}. \tag{6.2}$$

We test if the means are equal on a 95%-confidence interval, which is the case if and only if the null hypothesis is accepted on a 5% significance level.

The results of the T-tests are shown in Table 6.1. The p-value, the mean of the traded prices, and the theoretical prices are specified in the table. Because of the high coefficient of determination in the regression we expect the hypothesis of equal means over the entire data set to be accepted.

The mean of the traded prices over the entire period is 110.22 where as the mean of the theoretical prices is 115.15. Our model does on average price the bonds a little higher than the traded prices. We reject the hypothesis of the means being equal on a 95% confidence interval since the p-value$<$0.05. This states that the theoretical prices are not close enough to the traded prices over the entire period.

Next, we test if the null hypothesis can be accepted in the control period of 2006 and 2007 which is expected based on Figure 6.1. The mean of the traded prices is 114.66 and the mean of the theoretical prices is 113.76. There is a little difference in the mean values but as seen in Table 6.1 the result of the T-test shows a p-value = 0.79. The hypothesis of the mean values being the same on a 95% confidence interval is thus accepted. This result shows that model is capable of pricing the convertible bonds in the control period. Furthermore, the correlation between the prices in this period is 0.95 which also emphasize the theoretical and traded prices being equal.

From Figure 6.2 it is seen, in the crisis of 2008, that the traded prices fell relatively to the theoretical prices. We therefore test the hypothesis of the means being equal with the expectation of a rejection confirming the arbitrage crash. The mean of the theoretical prices in this period is 103.21 and the mean of the traded prices is 75.96. The traded prices are thus much lower than the theoretical prices. The p-value is, as expected, less than 0.05 which confirms that the difference is significant. The hypothesis of the means being equal is therefore rejected on a 95% confindece interval. Our theoretical prices do not at all match the traded prices in the crisis why we according to our model verify the existence of an underpricing of the convertible bonds in the wake of the Lehman bankruptcy.

Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) observed an arbitrage crash in the period of April 2004 to September 2005. This crash is not really observable in this setting as seen in Table 6.1. The hypothesis of equal mean values in the two prices is accepted. Our result shows that the arbitrage crash in 2008 was much more significant.

6.1.5 Extreme Underpricing

In the last section we showed the existence of underpricing in certain periods. We now illustrate the extreme case of underpricing where the traded price of the convertible bond is lower than the conversion value of the bond.

We consider the American Tower Corp convertible bond issued in August 2004, which matures in August 2012, and pays an annual coupon of 3%. On October 9., 2008 an amount of $345,000 face value was traded at $164.08 per $100 face value.

On the same date we calculate a theoretical price of \$186.85 which indicates that the bond was sold at a 12.19% discount to the theoretical value.

The conversion value of a convertible bond is given by

$$\text{Conversion value} = \text{Stock price} \cdot \text{Conversion ratio.}$$

As the conversion ratio of the American Tower Corp bond is 4.88 and the stock price on October 9., 2008 was \$35.97, the conversion value of the bond was $4.88 \cdot \$35.97 = \$175.53$ per \$100 face value. This indicates that instead of selling the convertible bond for \$164.08, the investor could have converted the bond and received \$175.53 by selling the shares. So the seller of the bond gave away a valuable option for free.

The choice of selling the bond at a discount can be explained in three ways. First, by converting the bond into shares the investor gives up accrued interest since the last coupon. Secondly, the bond can not be converted immediately. It normally takes up to a month for the conversion to be processed. It is thus not possible to sell the acquired shares immediately. The investor might therefore prefer to sell the bond at a discount if the liquidity is needed immediately. At last, it can be explained by the liquidity risk of the stock relative to cash. If the cash is needed the selling of the stock can have a market impact leading to a lower pay out than indicated.

The sale of the American Tower Corp convertible bond at a discount both to the theoretical value and even to the conversion value indicates that the liquidity was needed immediately by the investor. It also indicates that none of the first-best-buyers had the financial resources to buy the bond even at its conversion value.

To examine if the American Tower Corp example generalizes to a larger sample we define an extreme case of underpricing by a trade where the conversion value is higher than the traded price.

If we set

$$\text{Extreme} = \begin{cases} 1, & \text{if Conversion Value} > \text{Traded price} \\ 0, & \text{if Conversion Value} \leq \text{Traded price} \end{cases}$$

we can determine the volume weighted extremeness of underpricing by multiplying the volume of the trade of the convertible bond by the extreme parameter for the trade. We determine the volume weighted extremeness of underpricing for 2002 until 2011 and plot the result in figure 6.8.

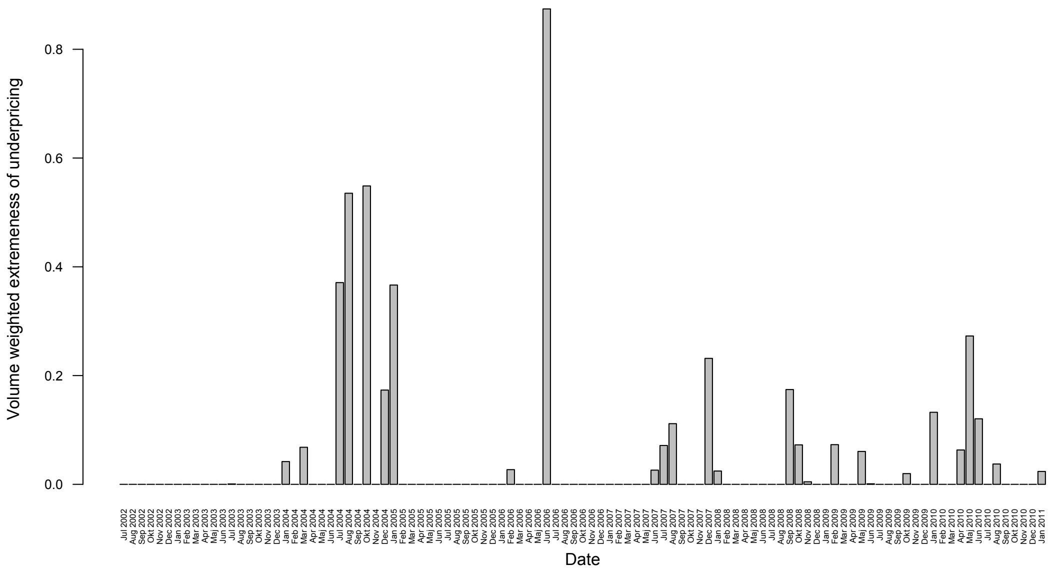


Figure 6.8: *The figure shows the volume weighted extremeness of underpricing for the period* 2002Q3-2011Q1.

The figure shows that the the American Tower Corp example do generalizes to a larger sample. The volume weighted extremeness exceeds 20% in several months and even exceeds 50% in a few months.

The figure do not show exactly the image we expected based on the previous section. We would expect the largest concentration of extreme cases to be in the financial crisis of 2008 since we showed the largest underpricing in that period. Instead we see, that the overweight of extreme cases are in the period of the first crash.

The findings of this section suggest that the underpricing of the convertible bonds were on an extreme level for certain trades in the arbitrage crashes. It reveals that it was possible to buy a convertible bond and convert it immediately to get an amount of the common stock worth more than the traded value.

6.2 Matching Convertible and Non Convertible Bonds

In the last section we showed that the convertible bonds were underpriced in the periods of June 2004 to July 2005 and September 2008 to March 2009 by comparing our theoretically found prices with the actual traded prices. In this section we compare the convertible bonds with similar non convertible bonds to examine if the convertible bonds are cheap relative to the non convertible bonds.

6.2.1 Yield Spread

As explained earlier in the thesis we expect a convertible bond to be at least as valuable as a comparable non convertible bond as it can be seen as a non convertible bond plus an option with a minimum value of zero. When a convertible bond is cheaper than a comparable non convertible bond it is not rational to invest in the non convertible bond at all. If the convertible bond is bought instead, the investor gets the option part for free and the bond part cheaper. The analysis of this section shows that strictly dominated non convertible bonds were traded instead of the comparable convertible bond. As mentioned in section 2.3 this suggests that the convertible bond market was segmented.

In order to examine the value of convertible bonds relative to non convertible bonds we pair a convertible and a non convertible bond by the same issuer and with identical values for the stock price, face value and stock price volatility. However, the value of the coupon, call price, whether they are callable or not, the rating of the bond, and the time to maturity can differ between the compared bonds. The pair of bonds is chosen such that the non convertible bond is traded at least within three hours of the trade of the matched convertible bond.

Since the paired bonds have different time to maturity and coupon payments we are not able to compare the bonds directly by their price. Instead, we derive the yield to maturity based on the price of the pair and use it for comparison. By doing so we take both time to maturity and coupon payments into account.

Since the yield of a bond is negatively dependent on the price of the bond, we expect the yield of the convertible bond to be lower than or equal to the yield of the comparable straight bond. In a normal state of the economy we thus expect the following relationship

$$\begin{aligned} price_C \geq price_{NC} \quad &\Longleftrightarrow \quad ytm_C \leq ytm_{NC} \\ &\Longleftrightarrow \quad ytm_{NC} - ytm_C \geq 0 \\ &\Longleftrightarrow \quad YS \geq 0. \end{aligned}$$

where YS is the yield spread between the convertible bond and the comparable convertible bond.

In the following two sections we examine the yield spread in a theoretical and a real world setting. First we determine the yields of the convertible and non convertible bonds based on theoretical prices. The spread between them is then examined in order to determine a relation in the theoretical setting. Thereafter we determine the yields of the convertible and non convertible bonds based on the actual traded prices.

6.2.2 Yield Spread based on Theoretical Prices

In this section the yield to maturity, ytm, is based on the theoretical prices of the convertible bonds derived in the last section. The theoretical prices of the comparable non convertible bonds are calculated by a simple adjustment in the pricing formula of the convertible bond[31]. Based on these prices we determine ytm_{NC}. By subtracting the yield of the convertible bond from the yield of the non convertible bond we get the yield spread based on the theoretical prices.

In figure 6.9 the yield spreads based on the theoretical calculated prices of the matched pairs are plotted for the period of July 2002 to January 2011.

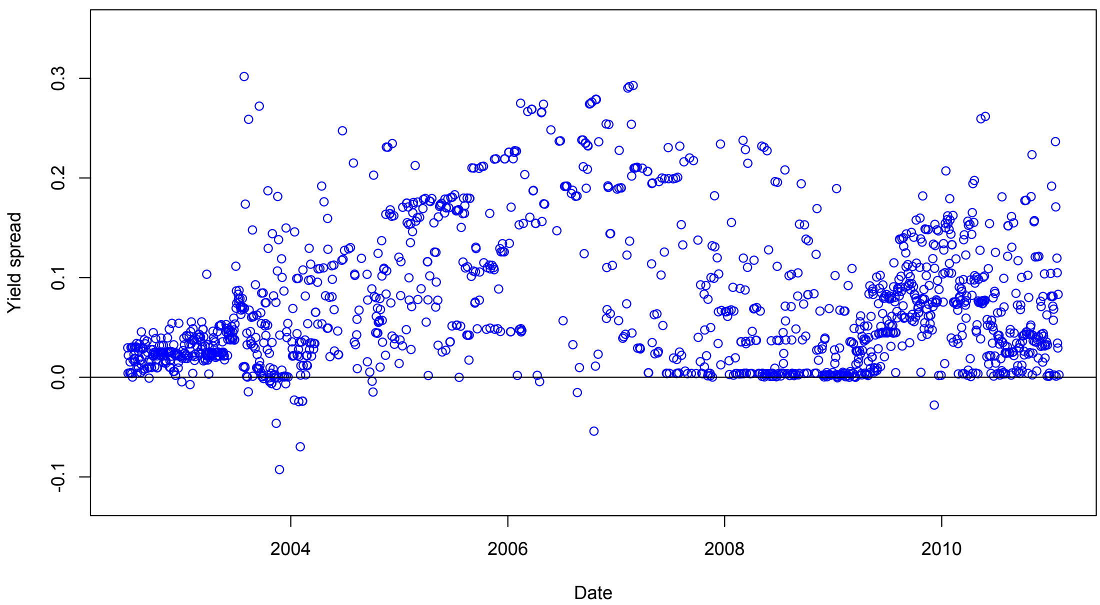


Figure 6.9: *The figure shows the yield spread based on the theoretical determined price of the matched pairs (positive when the convertible bond has the lowest ytm) for the period of July 2002 to January 2011. One extreme observation with a spread higher than 0.38 is removed to get a clearer image of the differences.*

Since these yield spreads are based on our theoretically found prices we expect the yield spread to be higher than or equal to zero throughout the period as a direct consequence of the extra option of the convertible bond. Figure 6.9 shows a clearly overweight of positive yield spreads throughout the analysed period. We though see a smaller amount of negative spreads primarily in the beginning of the period. Most of the observations seem to be extremely close to zero though.

[31]R code is presented in Appendix A

To examine the frequency of the negative yield spreads we define a violation as a pair of bonds with a negative yield spread.

If we set

$$\text{Violation} = \begin{cases} 1, & \text{if yield spread} < 0 \\ 0, & \text{if yield spread} \geq 0 \end{cases}$$

we can determine the volume weighted violation by multiplying the volume of the trade of the convertible bond by the violation parameter for the trade. We determine the volume weighted violation for the period of 2002 to 2011 on a monthly basis and plot the result in figure 6.10.

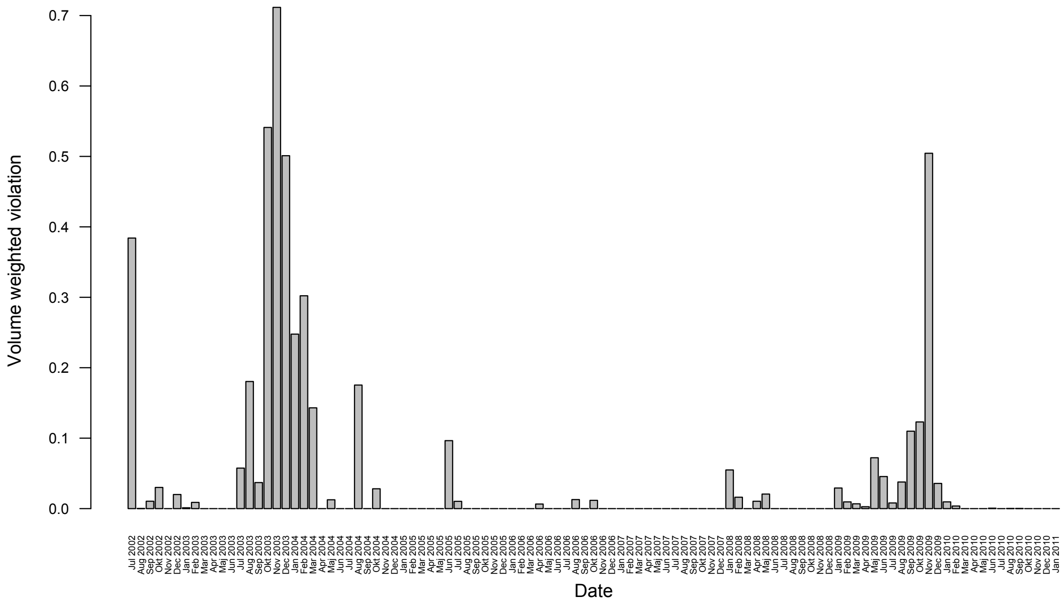


Figure 6.10: *The figure shows the volume weighted frequency of violations which are defined by the pair of bonds which have a negative yield spread. The yield spreads are based on the theoretical prices for the period of July 2002 to January 2011.*

Figure 6.10 shows a minor frequency of violations in the beginning and in the end of the period. The major part of the period has a very low an often non existence of violations. Furthermore, we notice that figure 6.9 showed that the observed violations were small especially in the end of the period. Our findings are therefore in accordance with our expectation that a convertible bond should always be worth more than a comparable non convertible bond.

The slightly negative spreads could be due to the differences between the compared bonds. As mentioned in the last section the compared bonds can have different coupon payments, option to call, call price, rating and time to maturity. The differences in coupon payments and time to maturity are accounted for in the determination of the yields. The option to call at a certain price and the rating, however, are not accounted for in our examination.

The call option parameter in our set is set to 1 if the bond is callable and 0 if it is not callable. The mean value of the call option thus denotes the percentage of bonds that are callable in set. The option of the issuer to call the bond has a negative value to the investor and therefore decreases the value of the bond. Since the yield of the bond is negatively dependent on the price of the bond the yield increases when such a call option is present.

The rating of the bond determines the default probability and the recovery rate at default. In table 5.1 in section 5.1 we display how our rating parameter are linked to the recovery rate. In Appendix 9.1 the link between the rating, time to maturity and the probability of default is shown. The rating code in our dataset is defined such that the lowest number, 1, corresponds to the best of Moody's Rating, Aaa. A better rating leads to a lower probability of default and a higher recovery rate which in both cases will lead to a higher price of the bond. A higher rating code in our dataset thus increases the yield of the bond.

In order to determine if the differences between the paired bonds leads to the observed violations we split the data set by the violation parameter. The first dataset then consist of all the pairs with a yield spread higher than or equal to zero and the second set consist of all the pairs with a negative yield spread. The first dataset consists of 4265 observations and the second of 211 observations. The

Dataset 1: The pairs with a yield spread higher than or equal to zero

	Convertible bond		Non convertible bond	
	Call option	Rating	Call option	Rating
Minimum	0.00	4.00	0.00	4.00
Median	0.00	11.00	0.00	11.00
Mean	0.24	11.86	0.33	11.87
Max	1.00	21.00	1.00	21.00

Dataset 2: The pairs with a negative yield spread

	Convertible bond		Non convertible bond	
	Call option	Rating	Call option	Rating
Minimum	0.00	4.00	0.00	4.00
Median	0.00	17.00	0.00	15.00
Mean	0.10	14.81	0.57	13.51
Max	1.00	18.00	1.00	18.00

Table 6.2: *Summary statistics for the paired bonds. This table shows the summary statistics for the call option and the rating of the paired bonds splitted in two data sets. Dataset 1 consist of all the pairs with a yield spread higher than or equal to zero and dataset 2 consist of all the pairs with a negative yield spread.*

summary statistics of the two data sets are shown in table 6.2.

The table shows that the mean of the call option is higher for non convertible bonds in the set with violation pairs. In the set of non violated pairs the mean of the call option are close for the two bonds. Since the percentage of call options for the convertible bonds in the violated set is low, the hypothesis that the call option causes the violations is rejected.

The ratings of the two bonds are very close in both of the sets leading to a rejection of the hypothesis that the rating of the bonds causes the violations.

The existence of a systematic error are not present for the variables used to price the bonds. The violations could then be due to trade specifics as the volume of the trade and the traded volume of the specific bonds on the day of the trade.

Our analysis mainly reinforce the assumption that a convertible bond is at least as valuable as a comparable non convertible bond. The assumption is only violated for a small amount of the traded bonds and the few violations are very close to zero.

In the next section we examine the yield spread between a convertible and non convertible bond based on the actual traded prices. The spread between them is then compared to the ones in this section to examine if the convertible bonds were traded at a discount to the theoretical price.

6.2.3 Yield Spread based on Traded Prices

The yield to maturity is now determined based on the actual traded prices of the convertible and non convertible bonds. We can thereby determine the yield spread based on the traded prices of the matched pair and examine if the convertible bonds were underpriced in the periods of June 2004 to July 2005 and September 2008 to March 2009.

In figure 6.11 the yield spread based on the traded price of the matched pairs is plotted for the period July 2002 till January 2011.

In the last section we found that the yield spread should be larger than or equal to zero in a theoretical setting. The convertible bond is thus underpriced if it is traded at a price leading to a negative yield spread.

Figure 6.11 shows a predominant amount of positive yield spreads. We though see some negative yield spread throughout the periods of 2005 to 2006 and 2008 to 2009. In these periods the ytm of the non convertible bond is lower than the ytm of the convertible bond for some of the traded pairs. This indicates the existence of underpricing in the convertible bond market.

We notice that the yield spread seems to be more negative in the period 2008-2009, which was the period of the financial crisis. It also seems to be the period with the largest overweight of negative yield spreads. The analysis suggests that That indicates that the convertible bonds market was experiencing an arbitrage crash in that period.

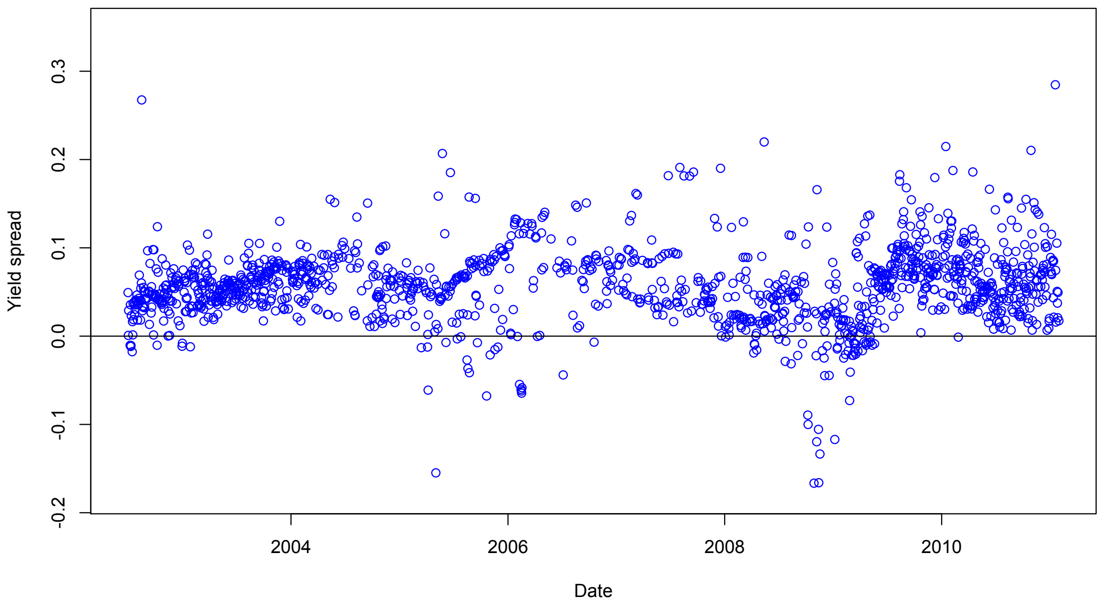


Figure 6.11: *The figure shows the yield spread based on the traded price of the matched pairs (positive when the convertible bond has the lowest ytm) for the period of July 2002 to January 2011.One extreme observation with a spread higher than 0.8 is removed to get a clearer image of the differences.*

The violation is as mentioned above defined as a traded pair of bonds with a negative yield spread. The volume weighted violations throughout the analysed period based on the actual traded prices is plotted in figure 6.12.

Figure 6.12 gives a clearer image of the violations in the analysed period. Based on the findings of Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) we expected to see violations in the period around 2005 and 2008. This is also the case in our analysis of the convertible bonds. The highest concentration of violation is seen in the financial crisis but the violations are also present in the months of the first expected crash period.

We also see a smaller amount of violations both in the beginning and the end of the period. In section 6.1 we showed large price fluctuations for both of these periods. The violations could therefore be due to a high market volatility.

To examine if there is a connection between the volatility and the violations

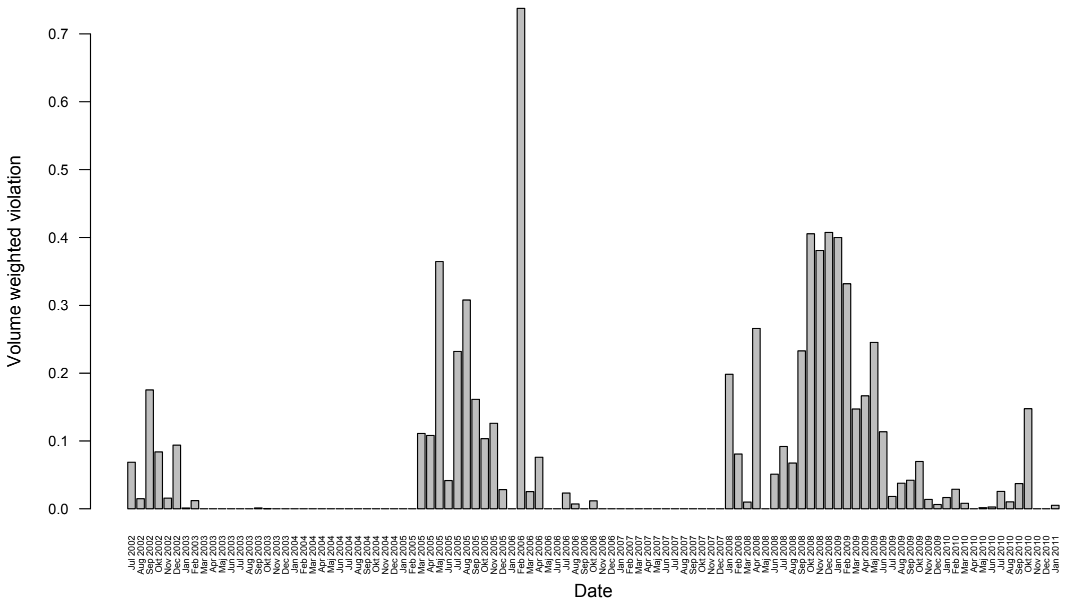


Figure 6.12: *The figure shows the volume weighted frequency of violations which are defined by the pair of bonds which have a negative yield spread. The yield spreads are based on the actual traded prices for the period of July 2002 to January 2011.*

we depict the volatility for the period of the analysis. Since a convertible bond is equity sensitive the volatility is defined as the standard deviation of the largest American stock index, S&P500.

In figure 6.13 the volatility in the returns of S&P500 is shown for the analysed period.

The figure shows a clear correlation between the market volatility and the violations shown in figure 6.12. The largest negative spreads are in the period with the highest volatility and the periods of violations are of the same length as the periods with higher volatility. The only exception is the arbitrage crash in 2005. As mentioned earlier the convertible hedge funds were performing poorly in the period before the first crash, which led the investors to draw their money out. The underpricing we see in the period around 2005 is therefore induced by the investor withdrawals in the convertible bond hedge funds and not the market volatility.

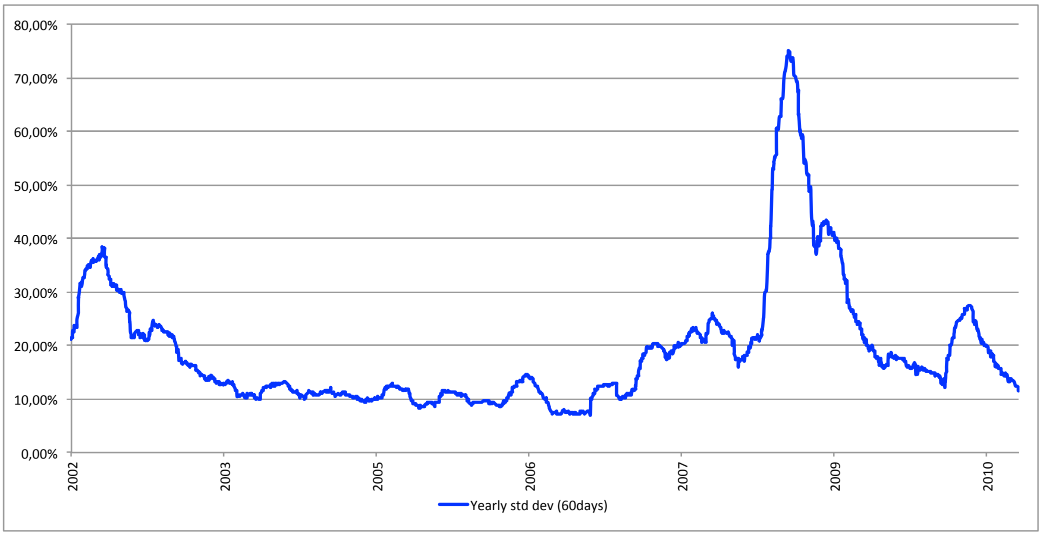


Figure 6.13: *The figure shows the yearly standard deviation of the daily returns of S& P500, based on a 60-days rolling window, in the period of July 2002 to January 2011.*

In the next section examine the two methods, that are used to show underpricing in this paper. We compare the results found by matching in this section with the results found by price comparison in section 6.1.

6.3 Agreements in the Analysis of Arbitrage Crashes in the Convertible Bond Market

In this section we examine if the matching method show an arbitrage opportunity for the same trades that we found was traded at a lower price than the theoretical. In section 6.1 we showed that the convertible bonds were underpriced in certain periods as they were traded at a lower prices than the theoretical calculated prices. In the previous section we showed that the convertible bonds were traded at a lower value relative to the comparable non convertible bonds for certain periods.

To be able to compare the two methods we set the arbitrage measure for each of them as follows.

The pricing method:

$$\begin{cases} \text{Traded price - Theoretical price} > 0 & \text{No arbitrage} \\ \text{Traded price - Theoretical price} < 0 & \text{Arbitrage} \end{cases}$$

The matching method:

$$\begin{cases} \text{Yield spread} > 0 & \text{No arbitrage} \\ \text{Yield spread} < 0 & \text{Arbitrage} \end{cases}$$

were the yield spread is based on the traded prices.

Figure 6.14 shows the yield spread between the matched bonds plotted against the difference between the traded and the theoretical price of the convertible bonds.

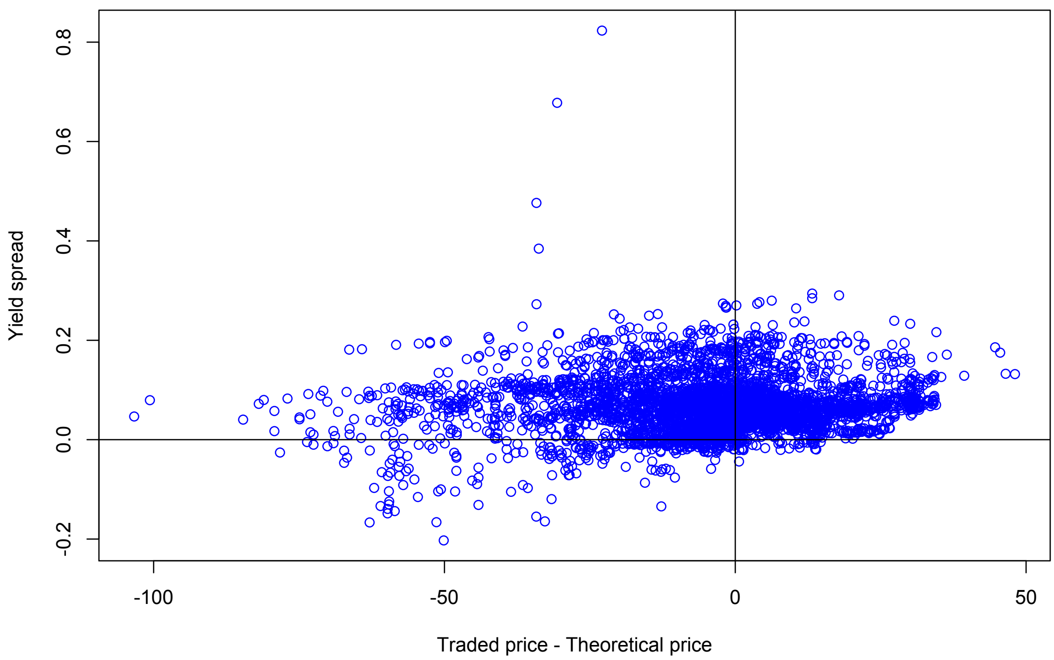


Figure 6.14: *The figure shows the yield spread based on the traded price of the matched pairs (positive when the convertible bond has the lowest ytm) plotted against the difference between the traded and the theoretical price of the convertible bonds.*

The lower left area in the figure represents the trades that are determined to be arbitrage opportunities in both methods. The upper right area represents the trades that do not contain an arbitrage opportunity according to both of the methods. If the two methods completely agreed on arbitrage opportunities all the observations would be in these areas.

The figure shows that the largest amount of observations is in the upper left area. These trades are underpriced according to the pricing method but they do not contain an arbitrage opportunity according to the matching method. We find that 51.41% of the observations are in this area. This suggests that the requirements of arbitrage opportunities are more stringent for the matching method than the pricing method.

We find that 7.15% of the observations are in the area where both methods agree on arbitrage and 40.62% is in the area where they agree on no arbitrage. The two methods thus agree on approximately half of the observations.

For the trades in the lower left area the matching method concludes arbitrage opportunities whereas the pricing method does not result in underpricing of the convertible bonds. In this area, the matching method of Dick Nielsen and Rossi (2013) identify cheapness of convertible bonds relative to straight bonds that are not present in the pricing method. This area only contains 0.82% of the observations though.

To determine whether the differences in the results of the methods are systematic we examine if the parameters of the model collectively have an effect on whether the methods agree or not. We define the response parameter, RE, by

$$\text{RE} = \begin{cases} 1, & \text{if the methods agree} \\ 0, & \text{if the methods do not agree.} \end{cases}$$

The parameters with the largest impact on RE are found by a multiple logistic regression in `R`. The results are shown in Table 6.3.

When the value of the Estimate is positive (negative), the probability of agreement between the methods increases (decreases) when the parameter increases.

	Coefficient	z value	Pr(>| z|)
(Intercept)	6.079994	13.879	$< 2e - 16$
sigma	-9.504247	-5.295	$1.19e - 07$
NewCallYN	9.674345	5.107	$3.27e - 07$
NewCallYNnc	15.330673	3.512	0.000444
S0	-0.023623	-6.719	$1.83e - 11$
ttm	-0.292724	-12.033	$< 2e - 16$
r	10.263043	2.934	0.003345
recovery	7.304408	2.776	0.005511
recoverync	-18.926128	-7.353	$1.93e - 13$
k	-0.049640	-7.572	$3.68e - 14$
CallPrice	-0.079607	-4.267	$1.98e - 05$
CallPricenc	-0.151288	-3.589	0.000332
c	16.362088	5.392	$6.99e - 08$
MoodysNY	2.199420	2.578	0.009938
MoodysNYnc	-2.016352	-2.617	0.008865
Moneyness	-0.726890	-6.949	3.68e-12

Table 6.3: *Coefficients of the multiple linear regression. The table shows the result of the logistic regression in R. The rating of the bonds, the time to maturity and the coupon payments of the non convertible bond have been removed as they were not significant.*

The table shows that the time to maturity of the convertible bond has the highest numerical value and thereby the largest impact on the agreement between the methods. Since the coefficient of the parameter is negative the time to maturity increases the probability of disagreement between the methods. In addition, the volatility, the stock price, the conversion rate, the moneyness of the convertible bond and the recovery of the non convertible bonds affect the agreement of the methods negatively with a relatively large impact.

The largest positive impacts on the agreement are the call parameter and the coupon payment of the convertible bond. The probability of agreement between the methods is thereby increased when the convertible bond is callable and the coupon payments increase.

Overall, the regression shows that the methods primarily disagree for longer time to maturities of the convertible bond. As seen in Figure 6.14 the disagreement mainly consists of the trades that are underpriced according to the pricing method but do not contain an arbitrage opportunity according to the matching method. This indicates that the matching method is more stringent than the pricing method. The pricing method thereby verifies, that there exists an arbitrage opportunity for almost all the trades that are identified to be arbitrage opportunities in the matching method. This suggests that it is possible to identify arbitrage opportunities without having a theoretical pricing model. We though notice that the matching of the yields do not capture all the arbitrage opportunities found by the pricing method.

When Dick-Nielsen and Rossi (2013) match the convertible and non convertible bonds they do not account for the option to call the bond, which possibly shorten the life of the bond. They argue that the callability affects the price of the bond less than the convertibility. In the next section we examine if this assumption is valid for the observations in our data set.

6.4 The Significance of the Callability

In this section we verify the assumption made by Dick-Nielsen and Rossi (2013) that the callability affects the price less than the convertibility. In the previous section we showed that the callability increased the probability of agreement why we expect to verify the assumption. Ingersoll (1977) shows that a convertible bond which is not callable is worth more than a bond that is both callable and convertible, when everyone acts rational. This means that the call feature decreases the price of the bond and the convertible feature increases the price of the bond. Dick-Nielsen and Rossi (2013) argue that since the proportion of callable bonds is higher among non convertible bonds this should make violations much harder to find, given that callability generally increases yields.

According to our pricing algorithm of the convertible bond with the call feature we see that the call feature reduces the price. To validate the feature in our data set we do a comparison of the features of a convertible bond and a comparable non convertible bond. The analysis on the yields to maturity is carried out as in the previous section. We compare the yield to maturity of a callable and convertible bond with the yield to maturity of a non callable and non convertible bond.

If the convert feature increases the price more than the call feature decreases the price of the bond, as Ingersoll (1977) shows, then a bond with both features should be worth more than a bond with non of the features. We therefore choose our comparable pairs of the dataset where the non convertible bond is non callable and the convertible bond is callable.

If we assume that the price of a callable and convertible bond is higher than the price of a non callable and convertible bond then the yield to maturity of the callable and convertible bond should be less than the yield to maturity of the non callable straight bond.

$$ytm_{CCB} < ytm_{NCCB}$$

where ytm_{CCB} is the yield to maturity of the callable and convertible bond and ytm_{NCCB} is the yield to maturity of a non convertible bond with no call feature.

If the call option affects the price of the bond less than the convert option on our data set, then the spread between the yield to maturity of a convertible bond with no call feature and the yield to maturity of the callable and convertible bond has to be positive.

$$ytm_{NCCB} - ytm_{CCB} > 0.$$

When we plot the spread based on our data set we expect most of the observations to be positive.

Figure 6.15 shows the difference between the yields to maturity of the non convertible bond with no call feature and the callable and convertible bond. We see that almost every observation, 95%, are positive verifying the assumption that

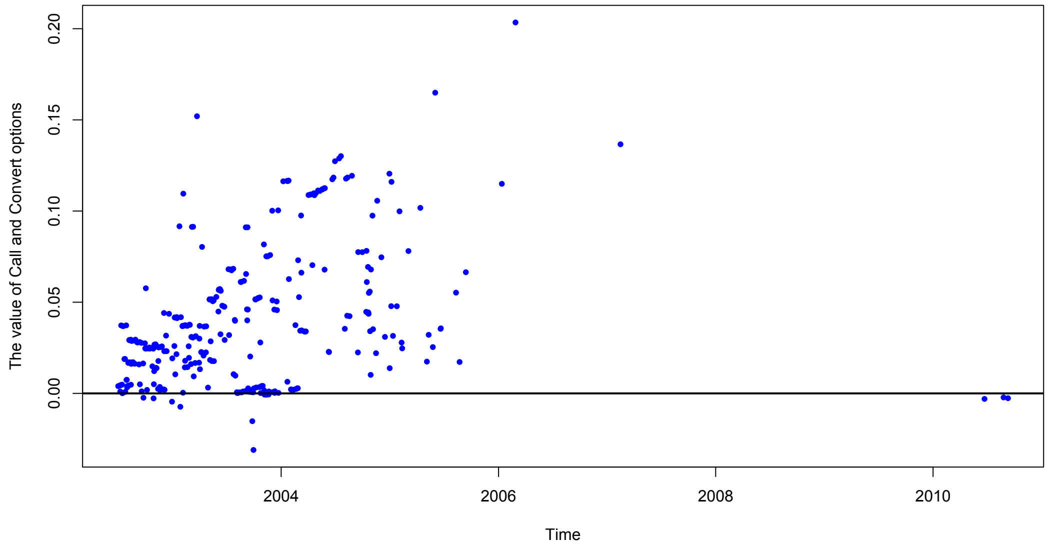


Figure 6.15: *This figure shows the difference between the yields to maturity of the non callable and convertible bond and the callable and convertible bond.*

the callability option affects the bond price less than the convertability option. This supports the assumption that ignoring the callability does not affect the conclusion of the paper of Dick-Nielsen and Rossi (2012). We though notice, that this analysis is made based on a small data set, which makes the verification a bit weaker.

7 Conclusion

The empirical analysis of this thesis add to the existing literature of arbitrage crashes in the convertible bond market. We base the analysis of the thesis on the ideas of Mitchell and Pulvino (2012) and Dick-Nielsen and Rossi (2013) who argue that the arbitrage crashes was initiated by cash withdrawal of investors and prime brokers leading to a slow movement of capital. Dick-Nielsen and Rossi furthermore argue that it was exacerbated by a market segmentation in the bond market.

We analyse the arbitrage crash in the convertible bond market by using both a pricing method as in Mithell and Pulvino (2012) and a yield matching method as in Dick-Nielsen and Rossi (2013). Both methods leads to the conclusion that the convertible bond market experienced a significant arbitrage crash in 2008 and a smaller one in 2005.

We develop a pricing model for convertible bonds subject to callability and default risk. Using the data of Dick Nielsen and Rossi (2013) as input to the model we analyse the arbitrage crashes with several different approaches.

By comparing the theoretically calculated prices with the traded prices we show that the convertible bonds were underpriced in the years of 2005 and 2008. We show that the hypothesis of equality in the mean values of theoretical and traded prices can be accepted on a 95% confidence interval in the period of 2006 to 2007. We thus verify the validity of our model in an unstressed market. On the contrary, the same hypothesis is rejected in the period of 2008.

We define a measure of cheapness and show that the convertible bonds were cheap in both 2005 and 2008. Additionally, we show that some of the convertible bonds were sold at prices lower than their conversion value. Consequently, investors could gain profit by buying the convertible bond and immediately convert it into shares of equity.

In the second part of the analysis, we turn to yields to maturity instead of prices. We show that the convertible bonds were traded at a value lower than comparable non convertible bonds in both periods by matching the yields to maturity as in Dick-Nielsen and Rossi (2013). We argue that the existence of a significant buying volume of strictly dominated non convertible bonds is a clear indication of market segmentation in the bond market.

Furthermore, we show that the matching method is more stringent than the pricing method. The pricing method verifies the existence of an arbitrage opportunity for almost all the trades that are identified to be arbitrage opportunities in the matching method. This suggests that it is possible to identify arbitrage opportunities without having a theoretical pricing model. We though notice that the matching of the yields do not capture all the arbitrage opportunities found by the pricing method.

Conclusively, all our analysis point to the fact that the the convertible bond market experienced an arbitrage crash in the periods of 2005 and 2008. The results shed new light on the analysis of arbitrage crashes in the convertible bond market by combinig and comparing the existing methods.

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9 Appendix

9.1 Default Probability based on Rating and Time to Maturity

EXHIBIT 35

Average Cumulative Issuer-Weighted Global Default Rates, 1983-2010*

Rating	1	2	3	4	5	6	7	8	9	10
Aaa	0	0.016	0.016	0.048	0.086	0.132	0.182	0.186	0.186	0.186
Aa	0.023	0.066	0.116	0.202	0.291	0.351	0.388	0.419	0.447	0.501
A	0.062	0.2	0.414	0.623	0.853	1.099	1.371	1.677	1.969	2.216
Baa	0.202	0.561	0.998	1.501	2.06	2.636	3.175	3.71	4.26	4.89
Ba	1.197	3.437	6.183	9.067	11.51	13.757	15.76	17.679	19.526	21.337
B	4.466	10.524	16.526	21.774	26.524	31.034	35.301	39.032	42.312	45.194
Caa	15.529	27.592	37.251	45.146	51.803	56.26	59.232	62.759	67.199	73.035
Ca-C	38.739	50.58	59.678	66.353	71.652	73.385	75.92	78.884	78.884	78.884
Inv Grade	0.095	0.274	0.508	0.769	1.054	1.343	1.622	1.907	2.185	2.467
Spec Grade	4.944	10.195	15.233	19.671	23.477	26.82	29.79	32.433	34.804	39.967
All rated	1.819	3.717	5.485	6.988	8.241	9.303	10.212	11.006	11.706	12.344

Rating	11	12	13	14	15	16	17	18	19	20
Aaa	0.186	0.186	0.186	0.186	0.186	0.186	0.186	0.186	0.186	0.186
Aa	0.586	0.722	0.869	0.993	1.126	1.262	1.445	1.763	2.268	2.754
A	2.449	2.673	2.934	3.241	3.633	4.125	4.762	5.519	6.104	6.641
Baa	5.541	6.225	7.079	8.004	8.881	9.845	10.738	11.492	12.165	12.72
Ba	23.033	24.843	26.653	28.663	30.722	32.449	33.992	35.325	37.036	38.372
B	47.76	50.361	52.884	55.42	57.456	58.903	60.602	62.768	64.315	65.936
Caa	77.147	77.369	77.369	77.369	79.539	83.436	84.472	84.472	84.472	84.472
Ca-C	78.884	78.884	78.884	78.884	78.884					
Inv Grade	2.75	3.045	3.394	3.768	4.167	4.627	5.14	5.703	6.222	6.688
Spec Grade	38.877	40.781	42.631	44.574	46.411	47.908	49.294	50.636	52.066	53.292
All rated	12.918	13.48	14.056	14.653	15.245	15.829	16.435	17.074	17.688	18.235

*Data in percent

Figure 9.1: *This table represents the link between the rating and time to maturity to the probability of default. The Table is from the article of Corporate Default and Recovery Rates, 1920-2010, from MOODY'S INVESTOR SERVICE (2011).*

9.2 R code

```
########################
# The pricing formula #
########################

CCBond <- function(S0, FV, ttm, r, sigma, L, k, CallPrice, rho, c, PD,
    CallYN)
{

# Defines the timeinterval and length

dt <-  1/12
n <- ttm/dt+1
n <- ceiling(n)

# Define the parameters

u <- exp( (r-(sigma^2)/2)*dt+sigma*sqrt(dt) )
d <- exp( (r-(sigma^2)/2)*dt-sigma*sqrt(dt) )
p <- 1/2

#u <- exp(sigma*sqrt(dt) )
#d <- 1/u
#p <- (exp(r*dt)-d)/(u-d)

gamma <- -log(1-PD)*S0
#print(gamma)

# Defining the stock and the default intensity

S <- lambda <- rep(0, times=n*n)
dim(S) <- c(n,n)
dim(lambda) <- c(n,n)
#dim(gamma) <- c(n,n)

S[1,1] <- S0
lambda[1,1] <- gamma/S[1,1]

for ( j in 2:n){
```

```
for ( i in 1:j ){
  if(i==j){
    S[i,j]=S[i-1,j-1]*d*exp(lambda[i-1,j-1]*dt) ##KORREKTION med lambda
    #gamma[i,j]<- -log(1-PD)*S[i,j]*dt
    lambda[i,j] <- gamma/S[i,j]
  } else {
    if(i<j){
      S[i,j]=S[i,j-1]*u*exp(lambda[i,j-1]*dt) ##KORREKTION med lambda
      #gamma[i,j]<- -log(1-PD)*S[i,j]*dt
      lambda[i,j] <- gamma/S[i,j]
    } else {
      S[i,j] = 0
      lambda[i,j] <- 0
    }
  }
}
}

# Calculating the value of the callable and convertible Bond in each node

CCValue <- NoCall <- Y <- Call <- rep(0, times=n*n)

dim(CCValue) <- c(n,n)
dim(NoCall) <- c(n,n)
dim(Y) <- c(n,n)
dim(Call) <- c(n,n)


for ( j in seq(from=n, to=1, by=-1) ){
for ( i in 1:j ){
  if (j==n){

    CCValue[i,j] <- max(FV,k*S[i,j])
    } else {
    NoCall[i,j]<- exp(-r*dt)*(
                        exp(-lambda[i,j]*dt)*(p*CCValue[i,j+1] + (1-p)*
                            CCValue[i+1,j+1]) + (1-exp(-lambda[i,j]*dt))*(1-
                            L)*FV
                        )
    Coupon <- exp(-(r+lambda[i,j])*dt)*c*FV*(ttm/n)

```

```
    if (CallYN == 1){
    Call[i,j] <- max(k*S[i,j],CallPrice)
    Y[i,j] <- (NoCall[i,j]/Call[i,j])-1
    #print(Y[i,j])
    if (Y[i,j]<0) { Y[i,j] = 0 }
    } else {
    Call[i,j] <- 0
    Y[i,j] <- 0
    }
    CCValue[i,j] <- Coupon + exp(-(rho*Y[i,j])*dt)* NoCall[i,j] + (1-exp
        (-(rho*Y[i,j]*dt)))* Call[i,j]
  }
}
}
CCValue[1,1]
}

##############################################
# The pricing formula WITHOUT CONVERTABILITY #
##############################################

NonConvBond <- function(S0, FV, ttm, r, sigma, L, CallPrice, rho, c, PD,
    CallYN)
{

# Defines the timeinterval and length

dt <-  1/12
n <- ttm/dt+1
n <- ceiling(n)

# Define the parameters

u <- exp( (r-(sigma^2)/2)*dt+sigma*sqrt(dt) )
d <- exp( (r-(sigma^2)/2)*dt-sigma*sqrt(dt) )
p <- 1/2

#u <- exp(sigma*sqrt(dt) )
#d <- 1/u
#p <- (exp(r*dt)-d)/(u-d)

```

```
gamma <- -log(1-PD)*S0
#print(gamma)

S <- lambda <- rep(0, times=n*n)
dim(S) <- c(n,n)
dim(lambda) <- c(n,n)
#dim(gamma) <- c(n,n)

S[1,1] <- S0
#gamma[1,1] <- -log(1-PD)*S[1,1]
lambda[1,1] <- gamma/S[1,1]

for ( j in 2:n){
for ( i in 1:j ){
  if(i==j){
    S[i,j]=S[i-1,j-1]*d*exp(lambda[i-1,j-1]*dt) ##KORREKTION med lambda
    #gamma[i,j]<- -log(1-PD)*S[i,j]*dt
    lambda[i,j] <- gamma/S[i,j]
  } else {
    if(i<j){
      S[i,j]=S[i,j-1]*u*exp(lambda[i,j-1]*dt) ##KORREKTION med lambda
      #gamma[i,j]<- -log(1-PD)*S[i,j]*dt
      lambda[i,j] <- gamma/S[i,j]
    } else {
      S[i,j] = 0
      lambda[i,j] <- 0
    }
  }
}
}

# Calculating the Value of the Callable and Convertible Bond in each node

CCValue <- NoCall <- Y <- Call <- rep(0, times=n*n)

dim(CCValue) <- c(n,n)
dim(NoCall) <- c(n,n)
dim(Y) <- c(n,n)
dim(Call) <- c(n,n)


```

```r
for ( j in seq(from=n, to=1, by=-1) ){
for ( i in 1:j ){
  if (j==n){

    CCValue[i,j] <- FV
  } else {
    NoCall[i,j]<- exp(-r*dt)*(
                        exp(-lambda[i,j]*dt)*(p*CCValue[i,j+1] + (1-p)*CCValue[i+1,j+1]) + (1-exp(-lambda[i,j]*dt))*(1-L)*FV
                        )

    Coupon <- exp(-(r+lambda[i,j])*dt)*c*FV*(ttm/n)

    if (CallYN == 1){
    Call[i,j] <- CallPrice
    Y[i,j] <- (NoCall[i,j]/Call[i,j])-1
    #print(Y[i,j])
    if (Y[i,j]<0) { Y[i,j] = 0 }
    } else {
    Call[i,j] <- 0
    Y[i,j] <- 0
    }
    CCValue[i,j] <- Coupon + exp(-(rho*Y[i,j])*dt)* NoCall[i,j] + (1-exp(-(rho*Y[i,j]*dt)))* Call[i,j]
  }
}
}
CCValue[1,1]
}

######################
# Yields to Maturity #
######################
ytm <- function(face, price, ttm.y, coupon.rate.y){

  ttm.y <- max(ttm.y,1/12)
  coupon.y <- coupon.rate.y*face

  ## Annual coupon payments and then a payment of face at maturity
  cashflowsdates <- seq(1/12, ttm.y, by=1/12)
```

```
  cashflows <- c(rep(coupon.y/12, length(cashflowsdates)-1), face+(coupon
      .y/12))
# print(cashflows)
  ## Solve for the internal rate of return (x) that makes the discounted
      cash flows equal to the price
  uniroot(function(x) sum(cashflows/((1+x)^cashflowsdates))-price,
      interval=c(-0.99, 0.99))$root
}


##################
# Retrieves data #
##################
Data<-read.csv2("/Users/Kia/Dropbox/Speciale/Data/DataM.csv", header =
    TRUE, sep=";",dec='.')
Data <- na.omit(Data)
Data$TradeDate <- as.Date(Data$TradeDate, "%d/%m/%y")

DataMean <- aggregate(cbind(TradedPrice,TradedPrice_nc,ytm,ytm_nc) ~
    Company + TradeDate + Cusip + S0 + FV + ttm + sigma + k + c + Rating +
     r + MoodysNY + CallPrice + NewCallYN + Moneyness + cusip_nc + ttm_nc
    + coupon_nc + Rating_nc + MoodysNY_nc + CallPrice_nc + NewCallYN_nc +
    recovery + recovery_nc, Data,  mean)
DataSum <- aggregate(Vol ~ Company + TradeDate + Cusip + S0 + FV + ttm +
    sigma + k + c + Rating + r + MoodysNY + CallPrice + NewCallYN +
    Moneyness + cusip_nc + ttm_nc + coupon_nc + Rating_nc + MoodysNY_nc +
    CallPrice_nc + NewCallYN_nc + recovery + recovery_nc, Data, FUN =
    function(x){sum(as.numeric(x))})

Data2 <- merge(DataMean,DataSum, by=c("Company","TradeDate","Cusip","S0",
    "FV","ttm","sigma","k","c","Rating","r","MoodysNY","CallPrice","
    NewCallYN","Moneyness","cusip_nc","ttm_nc","coupon_nc","Rating_nc","
    MoodysNY_nc","CallPrice_nc","NewCallYN_nc","recovery","recovery_nc"),
    all.x=TRUE)

Data2<-DataMean
nrow(Data)
head(Data)
head(Data2)
nrow(DataSum)
summary(Data2)
```

```r

v2 <- aggregate( v2 ~ c1 + c2, data = df, mean)

summary(Data2)
nrow(Data2)

#################################
# Deriving prices for all dates #
#################################

rownr <- nrow(Data2)
Price <- Yield <- rep(0, times=rownr)

#PRICES CONVERTIBLE BONDS
for ( i in 1:rownr){

Price[i] <- CCBond(S0= Data2$S0[i], FV= Data2$FV[i], ttm= Data2$ttm[i], r = Data2$r[i], sigma= Data2$sigma[i], L=(1-Data2$recovery[i]), k= Data2$k[i], CallPrice= Data2$CallPrice[i], rho=9, c= Data2$c[i], PD= Data2$MoodysNY[i], CallYN= Data2$NewCallYN[i])
}

Data2$TheoPrice <- Price

#PRICES NON CONVERTIBLE BONDS
for ( i in 1:rownr){

Price[i] <- NonConvBond(S0= Data2$S0[i], FV= Data2$FV[i], ttm= Data2$ttm_nc[i], r= Data2$r[i], sigma= Data2$sigma[i], L=(1-Data2$recovery_nc[i]), CallPrice= Data2$CallPrice_nc[i], rho=9, c= Data2$coupon_nc[i], PD= Data2$MoodysNY_nc[i], CallYN= Data2$NewCallYN_nc[i])
}

Data2$TheoPrice_nc <- Price

#################################
# Deriving Yields for all dates #
#################################

#Yields of Theoretical Prices
for ( i in 1:rownr){
```

```

Yield[i] <- ytm(face=Data2$FV[i], price=Data2$TheoPrice[i], ttm.y=Data2$
    ttm[i], coupon.rate.y=Data2$c[i])
}

Data2$Yield <- Yield

for ( i in 1:rownr){

Yield[i] <- ytm(face=Data2$FV[i], price=Data2$TheoPrice_nc[i], ttm.y=
    Data2$ttm_nc[i], coupon.rate.y=Data2$coupon_nc[i])
}

Data2$Yield_nc <- Yield

#Yields of Traded Prices
for ( i in 1:rownr){

Yield[i] <- ytm(face=Data2$FV[i], price=Data2$TradedPrice[i], ttm.y=Data2
    $ttm[i], coupon.rate.y=Data2$c[i])
}

Data2$YieldTraded <- Yield

for ( i in 1:rownr){

Yield[i] <- ytm(face=Data2$FV[i], price=Data2$TradedPrice_nc[i], ttm.y=
    Data2$ttm_nc[i], coupon.rate.y=Data2$coupon_nc[i])
}

Data2$YieldTraded_nc <- Yield

##################
# Control period #
##################

DataControl <- Data2[ which(Data2$TradeDate>as.Date("2006-01-01") & Data2
    $TradeDate<as.Date("2007-12-31")), ]

Data2<-Data
###################
```

```
# Calibrating rho #
###################

rownr <- nrow(DataControl)
Price <- Diff <- DiffSQ <- rep(0, times=rownr)
SSE <- rep(0, times=101)
DiffMatrix <- rep(0, times=rownr*101)
DiffMatrix <- matrix(DiffMatrix,rownr,101)

a<-0
for ( j in seq(from=1, to=55, by=0.5) ){
  print(j)
  a<-a+1
for ( i in 1:rownr){

Price[i] <- CCBond(S0= DataControl$S0[i], FV= DataControl$FV[i], ttm=
    DataControl$ttm[i], r= DataControl$r[i], sigma= DataControl$sigma[i],
    L=(1-DataControl$recovery[i]), k= DataControl$k[i], CallPrice=
    DataControl$CallPrice[i], rho=j+0.05, c= DataControl$c[i], PD=
    DataControl$MoodysNY[i], CallYN= DataControl$NewCallYN[i])
}

Diff <- Price - DataControl$TradedPrice
DiffMatrix[,a]<- Diff
DiffSQ <- Diff*Diff
SSE[a]<-sum(DiffSQ)
print(SSE[a])
}

plot(seq(from=1, to=11, by=0.1),SSE)
```

R script used in the analysis.