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European Union Emission Trading Scheme: A Model for Valuation and Hedging of Emission Unit Allowances Derivatives

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Abstract

The European Union $C0_2$ Allowances (EUAs) are traded on several markets with increasing intensity. The market structure derived from the Kyoto Protocol regulations is presented in the paper. The aims of this thesis are to develop a price estimation model of European Union Emission Allowances (EUAs) and risk management methods for companies participating in the EUA market. Statistical and econometrical analysis on spot prices are performed in order to assess the main characteristics of the price dynamics that need to be incorporated in the model. A stochastic volatility model with a jump component is implemented in a VBA framework, through a Monte Carlo simulation. The model is tested for pricing and hedging against the Bluenext and European Climate Exchange (ECX) prices.

Keywords: Carbon Trading, Emission Unit Allowances (EUA), European Union Emission Trading Scheme (EU ETS), Derivatives Pricing, Hedging.

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List of Abbreviations

ADF	Augmented Dickey-Fueller
AC	Autocorrelation
AR	Autoregressive
CDM	Clean Development Mechanism
CER	Certified Emission Reductions
CO_2	Carbon Dioxide
EC	European Commission
ECX	European Climate Exchange
EEX	European Emission Exchange
ERU	Emission Reduction Unit
ETS	Emission Trading Scheme
EU	European Union
EUA	Emission Unit Allowances / European Union Allowances
GARCH	Generalized Autoregressive Conditional Heteroskedaticity
GHG	GreenHouses Gas/Gases
JI	Joint Implementation
MA	Moving Average
MSE	Mean Squared Error
NAP	National Allocation Plan
OLS	Ordinary Least Squares
OTC	Over the Counter
RMSE	Root Mean Squared Error
SSE	Sum of Squared Errors
UNFCCC	United Nation Framework Convention on Climate Change

Introduction

The European Union Emission Trading Scheme was enforced on 1st January 2005 as a primary instrument for European member states to meet their obligations under the protocol to the United Nations Framework Conference on Climate Change (UNFCCC), commonly known as the Kyoto Protocol. The protocol has been established as a response to the threat of global climate change. There is a scientific consensus that human industrial activities have caused a rise in the concentration of greenhouse gases in the atmosphere, which leads to a steady and constant increase of temperatures. It has been suggested that industrially produced gases are the main driver for temperature growth, in particular carbon dioxide (CO_2). In the last century, temperatures have risen between 0,7°C and 1°C, while it is estimated the temperatures might increase between 1,4°C to 5,8° in the next hundred years (IPCC, 2007:13). The consequences may be serious and drastic on an environmental, social, and economic level (Stern, 2006).

In response to these threats, an international political consensus emerged in the 1990s, expressing the need to control greenhouse gas (GHG) emissions. In the context of the United Nations, preferred solutions focused more on market forces rather than direct regulation, and looked to incorporate some impost which reflects the level of CO_2 emissions. The most common proposals were the introduction of carbon taxes and tradable emissions permits (Coleman, 2007). The focus of this paper is the latter solution.

Since the ratification of the Kyoto protocol, a limit on the amount of carbon emission was imposed on participating countries. For the period between 2008-2012, the European Union (EU) has agreed to reduce its emissions to 92% of the 1990 level (Mansanet et al, 2008:41). A trading scheme has therefore been established in order to provide a cost-effective and flexible instrument to help companies and parties to achieve their emission compliances. Since then, a market price has been paid by over 11.500 installations and companies within the EU for every emitting allowance needed (Bataller et al, 2006:3). The scheme involves €41 billion emission allowances and includes countries which account for 17 percent of global industrially produced CO_2 emissions (PointCarbon, 2008:23). It is so far the world's largest CO_2 emissions trading program and a fast growing market. After having shown promising results in the first phase in terms of liquidity and efficiency, the market entered its second phase in 2008. Spot trading is growing in volume, and derivatives are available and are used by participants or voluntary traders (Mansanet et al, 2006).

The purpose of this thesis is to study the market, and in particular, the most traded and currently most liquid carbon asset instrument, the EU Emission Unit Allowance (EUA). It is a contract which gives the holder the permit to cover one tonne of $C0_2$-equivalent emissions. It is a financial asset that is traded in regulated exchanges.

Despite the relevance and growth potential of the market, literature has not shown consistent results on models able to correctly identify the price dynamics of carbon assets.

The specific objective of this paper is to develop a price estimation model which reflects the main characteristic of the spot EUA price. The model has to successfully price derivatives and allow for hedging in order to reduce the risks which emerged with the establishment of the emission scheme.

A statistical and econometrical analysis is carried out to assess the main characteristics that drive the price dynamics of spot EUA. It appears necessary to include stochastic volatility and jumps component in the model in order for the model to fit the non-normal and fat-tailed historical distribution. Currently available option prices are also considered so that market expectations are included as well. The simulation is performed in a Monte Carlo framework, as it gives flexibility to incorporate all the spot price features, and allows for hedging purposes. Some variance reduction and acceleration techniques are adopted to improve the computational efficiency of the Monte Carlo simulation.

The paper is organised as follow: the first chapter describes the EU Emission Trading Scheme in the context of the Kyoto Protocol framework. It explains how the scheme works, who the participants in the market for emission permits are. Furthermore, it defines the EUA which is the object of the analysis. The features of the trading scheme that influence the pricing process are highlighted. In the second chapter, the methodology, the goal of the paper, problem statement, and literature review are presented. The third Chapter contains an analysis of spot and futures prices necessary to delineate the main characteristics of the price dynamics. Based on these findings, a model is described and implemented, with the purpose of pricing and hedging Emission Allowances in the fourth chapter. The results are commented on to assess the performance of the model. Finally conclusions and suggestions for future research end the paper.

CHAPTER 1 – The European Emission Trading Scheme

The purpose of this section is to introduce the European Emission Trading Allowances (EUAs). These are financial instruments traded on several regulated exchanges which belong to the European Emission Trading Scheme (ETS). It is firstly necessary to briefly introduce the Kyoto Protocol in order to delineate the main characteristics of the scheme. This is the because the historical and normative background lead to the emergence of the ETS. The agreements that followed the Kyoto Protocol have established three interconnected mechanisms in Europe (Ellerman & Joskow, 2008:1). The ETS itself is a consequence of one of the three mechanisms, and will be described in more depth since it is more relevant for the analysis of this paper, with regards to the characteristics affecting EUAs price determination and pricing in general. The section regarding the ETS will therefore include a presentation of the participants in the markets, the exchanges, volumes, and most important the financial instruments, or carbon assets, which are available for trading at the moment in the exchanges.

1.1. Kyoto and Short history of Emission Trading

The Protocol to the United Nations Framework Conference on Climate Change (UNFCCC), commonly known as the Kyoto Protocol, was adopted during the third session of the Conference of the Parties in Kyoto, Japan, on 11th December 1997. The protocol established a legally binding obligation for developed countries to reduce Greenhouse Gases (GHGs). The protocol was subject to ratification, acceptance, approval or accession by the Parties of the Convention. It was effectively entered into force only on 16th February 2005 when Russia ratified it, allowing to finally reach the quota of 55 countries with 55% of total CO_2 emissions on 1990 levels (Mansanet-Bataller & Pardo, 2008:3).

With the ratification of the Kyoto Protocol, the countries in Annex I[1] committed themselves to reduce GHG emissions by 5.2% from 1990 levels in the period 2008-2012. The gases considered in the regulation are carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons (HFCs), perfluorocarbons (PFCs), and sulphur hexafluoride (SF_6). However a CO_2 equivalent unit measure has been devised in order to synthetically show the climate warming potential of the different kinds of gases (Mansanet-Bataller & Pardo, 2008:3.4).

CO_2 has therefore been taken as the reference gas, among others, for assessing the reduction targets as well as the quantities in the specifications of the financial contracts for allowances. A similar target of 5% has been assigned to Annex B countries[2]. It is important to notice that among Annex I countries, the EU-15 has been treated as one entity. As consequence, the emission reduction target of 8% assigned to the EU-15 for the period has to be distributed among its members through an allocation mechanism (UNFCC, 2005:23). This mechanism will be described later in this chapter.

Even if the Kyoto Protocol regards only one commitment phase for the period 2008-2012, there are currently discussions for extending the emission reduction efforts for further years. The conventions in Montreal in 2005 and Bali in 2007 has already shown the willingness of the participants to create another agreement for the period after Kyoto Protocol. The crucial UN Climate Change conference scheduled in Copenhagen in December 2009 is attiring great attention and expectations among the public, the authorities and the experts. Nicholas Stern[3] wanted to give great emphasis to the need of global action for emission reductions by defining the meeting as the "*most important gathering since the Second World War II*" (Thomson, 2008). The meeting will conclude two years of international talks. The most important players are expected to show their commitment to extend the efforts after the Kyoto period, ending in 2012. The European

[1] Annex I countries represent 40 industrialized countries and economies in transition. The complete list is in appendix A.
[2] Annex B countries are: Australia, Bulgaria, Canada, Croatia, Czech Republic, Estonia, EU-15, Hungary, Iceland, Japan, Latvia, Liechtenstein, Lithuania, Monaco, New Zealand, Norway, Poland, Romania, Russian Federation, Slovakia, Slovenia, Switzerland, Ukraine, US (Kyoto Protocol not ratified).
[3] Stern is the author of the Stern Review, a milestone scientific paper on the evidence of emission effects on global climate change. It is a widely known and debated study.

Union wants to maintain its role of global leader in environmental issues, by implementing the so called 20-20-20 plan[4] (Ellerman & Joskow, 2008; PointCarbon.com).

In order to achieve the targets, Annex I countries have to implement climate and environmental policies, that include measures to mitigate the effects of their activities on climate change. Such measures include an enhancement of energetic efficiency, sustainable agriculture, renewable energy sources, and reducing emissions from transportation. Parties can also offset their emissions through reforestation or by removing carbon from the atmosphere through so called carbon sinks[5] (Mansanet-Bataller,2008:6).

In addition to these measures Annex I countries can implement three flexibility mechanisms which have been established in the Kyoto framework with the purpose of facilitating and reducing the costs of emission reductions. The three mechanisms are namely the Joint Implementation mechanism (JI), the Clean Development mechanism (CDM), and International Emissions Trading (IET). The ETS finds its background in the IET (UNFCC, 2005). The mechanisms will be described in the following sections.

1.2. Flexibility Mechanisms

The Joint Implementation mechanism (JI) involves the possibility for any Annex I country to invest in emission reduction projects (referred to as Joint Implementation Projects) in another Annex I country as an alternative to reducing emissions domestically. In return, a JI project awards credits called Emission Reduction Units (ERUs), which can be used to meet domestic emission compliances. Hence the JI mechanism provides a flexible alternative to reducing emissions domestically, because it allows parties take advantage of investing in countries with lower conversion costs (UNFCC; 2005:31).

[4] The EU is committed to reducing its overall emissions to at least 20% below 1990 levels by 2020, and is ready to scale up this reduction to as much as 30% under a new global climate change agreement when other developed countries make comparable efforts. It has also set for itself the target of increasing the share of renewable energy source to 20% by 2020.

[5] GHG reduction projects can be converted into Removal Units (RMUs) and Verified Emissions Reductions (VERs).

The Clean Development Mechanism (CDM) refers to the possibility for parties-to invest in non-Annex I countries. The goal is to facilitate sustainable development in non-Annex I countries. At the same time, it provides another flexible means for Annex I countries to achieve their reduction targets. Annex I countries receive credits called Certified Emission Reductions (CERs) by investing in activities that reduce emissions in developing countries as an alternative to more expensive emission reduction projects in their own countries. CDM proposals have to be approved by the Executive Committee of the CDM Board for projects. (UNFCC; 2005:29).

Even if developing countries (non-Annex I countries) are not included in any compliance of the Kyoto Protocol, the reduction of GHG in those countries is essential for the overall goal of GHG reduction, since their emissions account for about 65% of global emissions and grow with economic development (Kapoor & Ambrosi, 2008).

The third flexible mechanism of the Kyoto Protocol is the International Emissions Trading (IET). The IET allows Annex I parties to purchase Assigned Amount Units (AAUs) from other Annex I parties. Parties can therefore trade units, and take advantage of the difference between reduction costs and unit prices. All types of units can be used in order to comply with their obligations: (i.e. ERUs, CERs) (UNFCCC, 2003:32). In addition to CERs and ERUs, other types of units are available and can eventually be traded (i.e. Assigned Amount Units (AAUs), Removal Units (RMUs), Verified Emissions Reductions (VERs)). A description of these instruments goes beyond the goals of this paper and will be demarcated.

The ETS is an additional mechanism allowed by the UNFCCC, and it is related to the IET. It will be described in more detail in the next section.

1.3. The European Emission Trading Scheme

The EU 15 member states have agreed on a common emission reduction of 8.6% from 1990 levels in the period 2008-2012. This target has been then redistributed to each members (Mansanet-Bataler & Pardo, 2008:3). Table 1.1 shows the reduction for each country.

The configuration of the system has been established following debates on the possible weakening of competitiveness for European companies. The goal of a harmonized climate policy was also challenged by conflicting national interests. The debate also focused on issues such as which economic sectors to include, mandatory or voluntary participation, and baseline-and-credit against a cap-and-trade approach (Watanabe & Robinson, 2005:12).

The ETS was finally established through binding legislation proposed by the European Commission (EC) and approved by the EU member states and the European Parliament. Two directives concerning the ETS has been enforced. Directive 2003/87/EC was approved in October 2003. One year later the so-called Linking Directive (Directive 2004/101/EC) was approved. It connects the JI and CDM to the ETS[6].

The main purpose of an ETS is to achieve cost-effective and economically sensible reductions in GHG emissions. Additionally, it provides a price for emission units that can be taken into consideration by companies for investments or business planning, and to trade the emissions within and across trading platforms (Coleman, 2007; Ellerman & Buchner, 2008).

Phase I of the ETS was launched on 1st January 2005. The main purpose of Phase I was to create the necessary technical infrastructure for trading, and to allow the operators gain knowledge about how the market functions. Phase II started in 1st January 2008 together with the Kyoto Protocol commitment taking force, and will terminate on the 31st December 2012. A Phase III is planned for the period 2013-2020. (Ellerman & Buchner, 2008:23)

[6] Under the Linking Directive installations and operators the opportunity to use the emission credits accrued from the JI and CDM project activities to comply with their own emissions limits.

EU-15 Countries	CO2 Emissions (in Mt)	Reduction Requirement (in %)
Austria	55,1	13,0
Belgium	106,3	7,5
Denmark	52,8	21,0
Finland	53,2	0,0
France	354,1	0,0
Germany	943,0	21,0
Greece	71,1	-25,0
Ireland	29,7	-13,0
Italy	390,8	6,5
Netherlands	152,9	6,0
Portugal	39,0	-27,0
Spain	203,8	-15,0
Sweden	50,6	-4,0
UK	569,1	12,5
EU (Total)	**3071,5**	**8,60**

Table 1.1 – Emission Reductions Targets for EU-15 Countries[7].

At the moment the directive covers more than 11.500 installations across Europe, that are obliged to possess an emission permit for their operations, as well as to surrender Emission Units Allowances (EUAs) corresponding to the installation's CO2 emissions after every year of operations. Each EUA is equal to one tonne of CO2 and can be freely traded between the installations covered by the ETS (Mansanet-Bataller et al, 2006:3).

The installations are required to surrender emission allowances corresponding to their emissions in the previous year before 30th April. This process is called "surrendering" of allowances. Surrendered allowances are cancelled every 30th June. For every tonne of emissions that is not covered by an allowance a company will have to pay a penalty of €40 in the first phase and €100 thereafter. However payment of the penalty does not exempt from surrendering the necessary excess emissions the following year. The European Commission publishes data regarding verified emissions and allowances surrendering on May 15th (UNFCCC, 2005:38-39).

[7] All tables and figures in this chapter are based on data from Reuters, PointCarbon and Newcarbon websites.

The European Emission Trading Scheme (EU ETS) has been established as a cap and trade system, in which the total amount of emissions for a period of time is fixed. The emitting allowances and credits cannot exceed the cap, therefore limiting total emissions existing in the system to a certain level. A National Allocation Plan (NAP) has to be submitted 18 months before the start of a Phase; the European Commission has 3 months to decide upon approval or rejection. The NAP defines the national cap which is a share of the overall European target level. Consequently, in each country the targets is allocated bewteen the trading and non trading sectors. The trading sector can receive allowances through a free allocation process or through auction[8]. These proportions can change from country to country. The participants in the trading sector therefore have the possibility to trade emission allowances and credits from those who have allowances in excess. However, the cap is not necessarily fixed since more permits (CERs and ERUs) can be introduced in the trading system (Mansanet-Bataller & Pardo, 2007: 4-6).

The ETS is a hybrid system regarding the allocation of the emission allowances (Kruger et al, 2007:115). It is a combination of a centralized and a decentralized systems. In fact, the EC, acting as the central administrator, determines the structure, the regulations, the industrial sectors in the market, the participants (installations whose capacity and output exceeds a certain threshold), and the kind of GHGs which are considered in the scheme. On the other hand, each member country proposes the NAP to the EC, defining the emission cap levels in the system. The member states are also in charge of monitoring and reporting the compliance of the national emissions, and they can choose how to allocate allowances and whether emission permits can be banked across phases (UNFCCC, 2005).

The efficiency of the allocation procedure, which gives more weight towards free allocation or “grandfathering”, is debated by scholars. Critics have, in particular, pointed to the “windfall profits” effect as a distorting subsidy to some sectors acting in the system. Facilities can include in their output prices the market value of permits. Even if correct from a financial point of view, the behaviour is contested as it passes on to the customer a

[8] In I phase at most 5% of allowances could be allocated through auction. For Phase II the limit increased to 10%.

cost which has not been paid. This is seen as an consequence of insufficient competition. Alternative configurations have been proposed, such as significant increase of share of permits auctioned (Ellerman & Buchner, 2007:85-86)

At the moment, the ETS Directives include in the scheme only the following economic activities and sectors: combustion plants, oil refineries, coke ovens, iron and steel plants, and factories making cement, glass, lime, brick, ceramics, pulp and paper. Transportation, chemical, and aluminum sectors were left out due to heavy lobbying activities. The possibility of extending the Directives towards other sectors, in particular to the aviation sector, is currently under discussion (Coleman, 2007). The sectors included in the Directives are defined as trading sectors, and they must be distinguished from non trading sectors (i.e. residential and transportation), which are eligible to participate in the scheme only a voluntary basis. In any case, the member countries are responsible for the overall compliance towards the emission cap they have been assigned.

1.4. Banking and Borrowing Limitation

An important feature of the EU ETS regards the regulations on banking or borrowing allowances across years and phases. (Ellerman & Joskow, 2008:39).

There is no restriction on banking or borrowing of allowances within the same phase. From a pricing point of view, this means that spot allowances are continuously traded within the same Phase.

Allowances are issued annually but they can be used to cover emissions in any year within the same trading period. Banking is allowed since un-surrendered allowances are still valid for compliance in the next years of the same phase.

Moreover, installations can cover eventual shortages with allowances issued for the next year. This is possible since each year's issuance of allowances occurs at the end of

February, while allowances must be surrendered two months later, in April. This process effectively allows year-ahead borrowing within the trading period (UNFCCC, 2005:38-39).

On the other hand, banking or borrowing allowances across phases is not allowed. This applies for both Phase I (2005-2007) and Phase II (2008-2012). Even though the EC Directive does not explicitly prohibit inter-phase banking and borrowing, no member state has decided to implement the feature so far (Convery & Redmond, 2007:96).

The rationale of the prohibition was intended to prevent any compliance failures during the trial period from spilling over into the second trading period. Such would complicate the achievement of the EU's commitments under the Kyoto Protocol. The ultimate goal of the system is to pursue emission reductions in the short term and to ensure that Europe meets Kyoto obligations. On the opposite, the limitation has made the trial period self-contained and is one of the characteristics of the system that have attracted more criticisms from scholars. Critics of the limitation advocate that creates a distortion in the price development, it decreases the cost-efficiency of the system, and it makes pricing and risk management more difficult (Schleich et al, 2006).

For the Phase II and Phase III, unrestricted inter-period banking, but not borrowing, will be allowed (Ellerman & Joskow, 2008:40).

1.5. Carbon Assets trading

Each emitting company is responsible for ensuring to hold a sufficient amount of EUAs to offset their emissions every year. The companies have the flexibility to comply with their targets, by implementing internal emission abatement, or external strategies by participating in emission trading or in JI, CDM projects. (UNFCC, 2005:34).

The legal framework of the EU ETS does not regulate how and where the trading of carbon assets takes place. Installations with commitments may trade assets directly with each other, buy or sell them via a broker, bank or other market intermediary, or even use the organized markets, namely exchanges, to trade assets (Convery & Redmond, 2007:97-98). The analysis of this paper focuses on financial products traded in regulated exchanges.

There are a number of different allowances and credits available for trading today. $C0_2$ equivalent units can be traded in the exchanges under different forms, as presented in the previous section. Table 1.2 summarizes the instruments available for spot trading in the current state of the market: AAU, CER, ERU, EUA. A common feature is that the underlying value for every typology of carbon asset is one tonne of $C0_2$-equivalent emissions. (PointCarbpn, 2008).

Carbon Assets Typologies	**Description**
European Union Allowance (EUA)	Unit issued to installation under the EU ETS
Certified Emission Reduction (CER)	Unit of emissions reductions created through CDM projects
Emission Reduction Unit (ERU)	Unit of emissions reductions created through JI projects
Assignes Amount Unit (AAU)	Units that are issued to Annex I parties. They determine how much the Party is allowed to emit
Removal Unit (RMU)	Unit of emissions reduction created through carbon sinking projects.
Joint Implementation (JI)	Units issued for emission reduction projects developed in non-Annex I countries.

Table 1.2 – Carbon Assets Typologies.

The most commonly traded instruments are the EUAs. In 2009 Q1, they accounted for 84,2% of all ETS transactions. Additionally, primary CERs and secondary CERs[9] are also tradable and can be converted into EUAs. The difference in price is due to the degree of uncertainty regarding the final approval for CERs. Primary and Secondary CERs accounted for respectively 2,3% and 9%. The other instruments have only been introduced in the market in Q4 2008 and exhibit low trading activities. Figure 1.1 and Table 1.3 show the

[9] Primary CER are related to projects that have not received approval.

weights, volumes and change on annual basis of the different instruments as spot transactions (PointCarbon, 2008;).

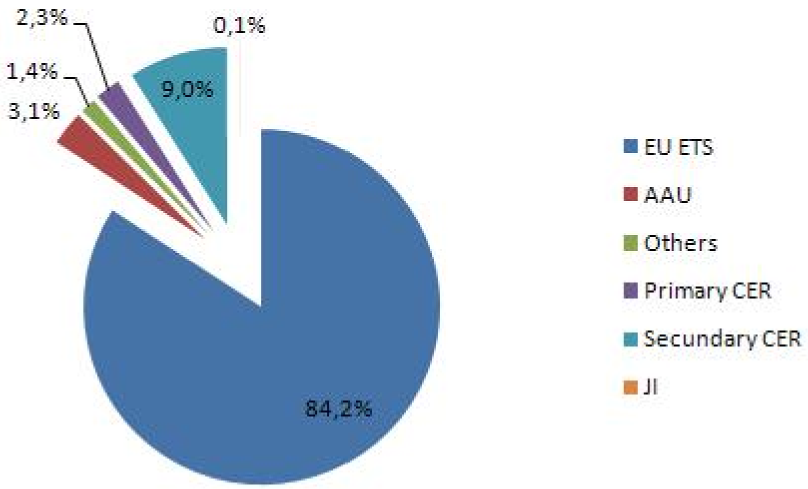

Figure 1.1 – ETS Market Share by Instrument

Although carbon assets can be traded as spot financial instruments with immediate delivery, a significant derivatives market with Futures and Futures options have developed throughout Europe since trading started in 2005. At the current state of the market, the following basic financial structures are available in regulated exchanges: immediate (spot) settlements, forward contracts, futures contracts, and option settlements.

Carbon Market Assets	**Value (million USD) in Q1 2009**	**% Change from Q1 2008**
EU ETS	23.781	35%
AAU	885	N/A
Others	250	N/A
Total Allowances	**24.917**	**41%**
Primary CER	636	-59%
Secondary CER	2.536	106%
JI	15	-99%
Others	133	-40%
Total project based assets	**3.321**	**-21%**
Total Market	**28.238**	**29%**

Table 1.3 – ETS Market Size and Change of volumes

Spot Contracts offer physical delivery taking place between 24 and 48 hours from the time of execution. Commonly, as in BlueNext (Bluenext.eu) and Nord Pool (Nordpool.com), the minimum size of the contract is 1000 tonnes of CO2-equivalent. Other exchanges allow for lower minimum amounts. The minimum tick in all cases is €0.01.

A Futures contract gives the holder the right and the obligation to buy or sell a certain underlying instrument at a certain date in the future, at a defined price. The underlying of EUA Futures Contracts can be EUAs or CERs. They are a standardized contract in all the specifications. The expiry dates are the last trading day of December, from 2009 to 2014 for EUAs Futures and from 2009 to 2012 for CERs Futures. The delivery is usually physical. Expiry date is the last Monday of the contract month, or the penultimate if that is not an available business day. Forward contracts also are available, where the maturity and the underlying amount are not fixed but decided by the parties. Specifications for amounts and ticks are similar to spot contracts (Pointcarbon.com; ECX.eu)

Options are contracts whereby one party (the holder or buyer) has the right, but not the obligation, to exercise the contract on a future date (the exercise date or expiry). The other party (the writer or seller) has the obligation to honor the specified feature of the contract. Since the option gives the buyer a right and the seller an obligation, the buyer receives the option premium.

Within EU ETS it has been possible to trade options from 13th October 2006 in ECX[10]. The exchange offers European-style options[11] with underlying EUA and CER Futures. The contract months are December 2009 to 2012. Strike price ranges from 1€ to 55€, with 1€ intervals. The expiry date is three trading days before the expiration of the underling Future contract. Both parties of a futures contract must exercise the contract (buy or sell) on the settlement date.

More sophisticated products are going to be introduced in the market. Specifically it will be possible to trade Spreads on EUAs and CERs, by buying or selling the implied price

[10] www.exc.eu

[11] The Buyer of an option can execute the option only on the expiration date.

difference between EUA and CER (for both spot and futures contracts), and Strips, by buying or selling simultaneously Futures contracts from all available maturities (spot and futures) (Bluenext.eu).

1.6. Regulated exchanges for emission allowances

The biggest change since the beginning of Phase I in January 2005 concerns the presence of a significant number of intermediaries. In fact, in the beginning there were only seven brokers operating in the market. By August 2006, national exchanges started to emerge as regulated marketplaces that trade EUAs and other carbon assets. The benefit of higher liquidity of the instruments and security on operations helped to increase trade volumes, which has expanded at a considerable pace. Figure 1.2 shows the market growth from 2005 (Phase I) to 2008 (Phase II). From 2005 to 2006 market size grew 182% in value, 106% in 2007, and 88% in 2008. Forecasts for 2009 predicts market growth to slow down to 1% compared to values of the previous year, since prices and volumes of transactions will likely be affected by global recession (Point Carbon.com, NewCarbonFinance.com).

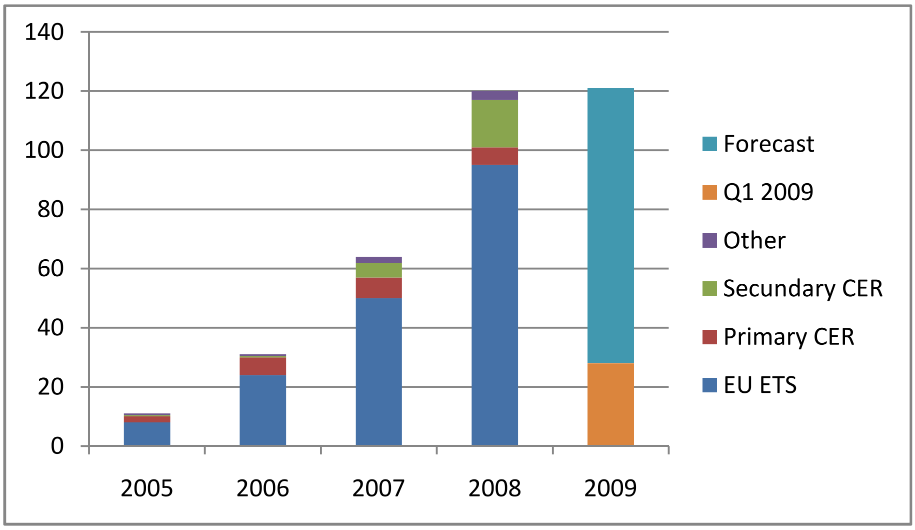

Figure 1.2 Carbon market growth for 2005 - 2009 in USD bn.

All the exchanges offer electronic trades and provide standardized versions of the products on the underlying allowances. The leading exchanges in terms of volumes are: the European Climate Exchange (ECX London), Nordpool (Oslo), Bluenext (Paris), the European Energy Exchange (EEX, Leipzig), the Green Exchange (Nymex, NewYork), and the Energy Exchange Austria (EEA, Wien) (PointCarbon, 2008).

For Phase II, ECX and Bluenext are the leading exchanges regarding respectively Futures and Spot Transactions. In 2008, for Spot trading Bluenext has accounted for about 80%, while Nordpool reached 17%. Concerning the Futures segment, ECX is by far the most liquid market, accounting for almost 98% of the market. Figure 1.3 shows how the trades for all ETS products have mainly concentrated in Bluenext and ECX in Phase II. The two exchanges have reached 92% share of all trades in Q1 2009 (Pointcarbon.com).

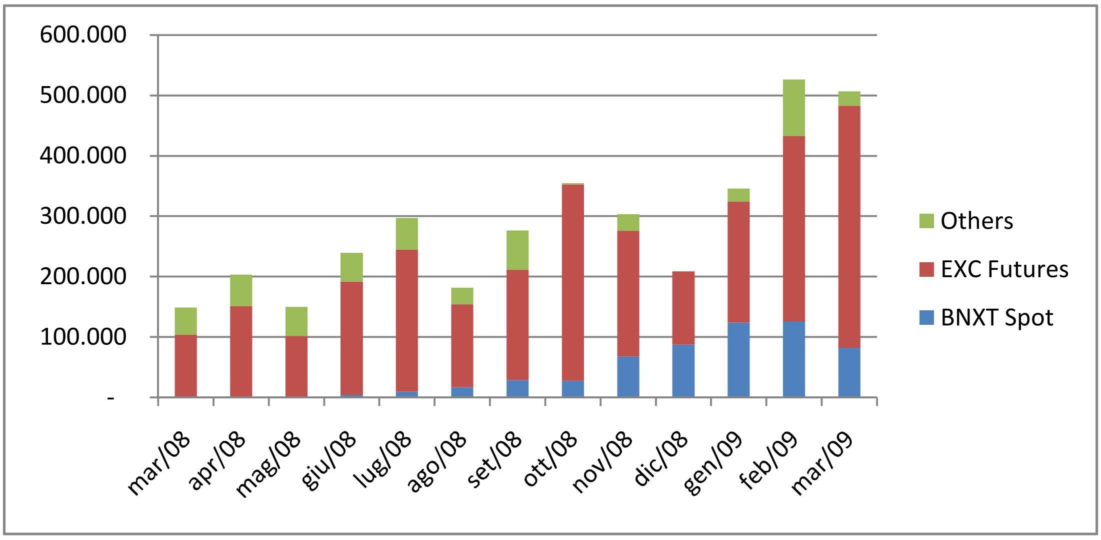

Figure 1.3 – $C0_2$ equivallent Kton Volumes for all exchanged instruments in EU ETS – Phase II

1.7. Market Participants

In order to participate in one of the markets, it is necessary to register a transaction account. Each member state monitors its own registry which reflects the balance of allowances for each participant. In Europe, single accounts are currently linked through the

Community Independent Transaction Log (CITL). The role of the CITL is to monitor and verify the correctness of every transaction. The registries are planned to be linked under a unique registry, the ITL starting from April 2009. From that moment, it will be possible to convert JIs and CDMs into EUAs to reach the emission compliances (UNFCCC, 2005:39).

The trading is a purely electronic system and as allowances are reflected in CITL registries. In order to participate in organized emissions allowance markets, it is necessary to create an account in the market where the transactions will take place. Every natural and legal person is authorized to open an account and participate in the emissions market. For this reason, not only installations with emission reduction compliances, but also individuals, companies, and institution can participate in the trading markets on a voluntary basis. (Mansanet-Bataller & Pardo, 2008:27).

A survey by Point Carbon (Point Carbon, 2008) among a sample of ETS market operators, has indicated that not all operators are pursuing compliance through trading. The research has showed that about 33% of operators work in companies regulated by the ETS, 32% are CER developers, 15% work in Financial or Banking Institutions, 3% work in Governmental Organizations, 3% in companies with emission compliances outside the ETS and 14% have other non specified purposes. Moreover, according to the World Bank the most active market participants are large power companies, banks and investment funds (Kapoor & Ambrosi, 2008)

However, the ETS regulatory framework have introduced several changes in the business environments. Thus, companies throughout the EU must contend with new responsibilities and obligations related to their emissions and climate change. The EU ETS has introduced several new financial assets and instruments that the operators need to manage and whose value has to be monitored. In general, the scheme has increased the level of uncertainty and risk, which therefore has to be addressed with proper financial tools. The increased activity and volumes will lead to higher demand for risk management instruments, for valuation and hedging purposes.

1.8. Results from ETS Phase I

The price of the EUA started to increase steadily already in the early 2005 and during the summer it peaked to over 30 € per tonne. When it stayed on the level of 20-25 € for the whole winter and next spring it became clear that the forecasts and estimates for the EUA price had been too low. Most of the predictions of the price level in the first period ranged around 5- 20 € (PointCarbon, 2008)[12].

Fundamental price determinants, such as electricity prices and with fuel prices, and especially the switching cost from coal to gas, were investigated closely, but their ability to explain the price changes was still quite poor. The high price was seen as an indicator of scarcity and of a strong demand of the allowances. The market was said to be short (i.e. demand was supposed to be greater than supply) (PointCarbon, 2008).

However, as the information of the true emissions was published for the first time in the end of April 2006, the market reacted rapidly. The price dropped within a couple of days from almost 30 € to below 10 € per tonne. Most of the member states and sectors within the EU ETS in fact had excess supply of allowances. After the initial market reaction, the price slowly recovered to the level of 15-20 € and stayed there until autumn 2006.

The publication of the 2006 emission and allowance data did not cause any further dramatic price change. However it showed that most sectors and member states had allowances in excess to their needs. This fact became slowly evident for the whole market and the price for the first period allowances started to go towards zero. The price decline was not dramatic but steady. In February 2007, the price for EUA Phase I dropped below €1 and never recovered from that level. It might be plausible to believe that if market operators had more experience in the workings of the scheme, the price would have dropped drastically towards a zero level (PointCarbon).

[12] This section is based on news feed from PointCarbon and Reuters Community websites.

It is well recognized among scholars and operators that initial allocation has a major role in the success and efficacy of the market. An over allocation in Phase II would put a serious threat to the credibility of the scheme (e.g. Ellerman & Buchner, 2006).

1.9. Intra-Phases and Inter-Phases Futures

Emission allowance futures in the EU ETS are primarily traded in the Dutch European Climate Exchange (ECX). ECX does not provide EUA spot trading and uses Bluenext spot prices as a reference for futures.

An important consideration about the condition of the Futures market is needed, as Futures trading accounts for most of the transactions and besides derivatives are written on EUA Futures prices. The issue has an implication on the pricing methodology, as spot prices from Bluenext will be used in the paper to derive Futures and Options prices from ECX.

Emission Unit Allowances can be seen as production factors for companies participating in the ETS. Their price is determined by expected market scarcity established by current demand and supply. In order to cope with normal stochastic fluctuations in both production and consumption, participants will hold a certain level of inventories, to reduce the costs of adjusting production overtime or to avoid shortfall. In order to avoid uncertainty in the prices and the transaction cost for having to make additional transactions and/or undo previous transactions, an operator can either store spot allowances, or enter in a Futures (or Forward) position. To the extent that there is uncertainty and positive transaction costs, allowances of different maturities will not show the same price today (Paolella & Taschini, 2006).

The benefit that accrues from holding a stock of allowances is called the convenience yield. The value of the convenience yield determines the state of the Futures term structure and provides information about market expectations of inventories levels and consequently price development (Uhrig-Homburg & Wagner, 2007:3).

The Futures market is usually described as being in four possible states (Pyndick, 2001). The market exhibits backwardation when futures price $F_{t,T}$ (price in time t, with maturity or delivery in T with $T > t$) is less or equal the current spot price S_t. It is defined as normal backwardation if $F_{t,T}$ is less or equal to the expected spot price in T, $E(S_T)$. On the contrary, contango usually denotes a situation where futures price is higher than the spot price, and normal contango similarly defines the market condition in which $F_{t,T} \geq E(S_T)$. A table 1.4 summarizes the four possible states.

Futures Market Situations	
Backwardation	$F_{t,T} \leq S_T$
Normal Backwardation	$F_{t,T} \leq S_T e^{r(T-t)}$
Contango	$F_{t,T} \geq S_T$
Normal Contango	$F_{t,T} \geq S_T e^{r(T-t)}$

Table 1.4 – Backwardation and Contango

The literature on the state of futures prices is rich, but at the moment, it has not yet been consistently applied to the Emissions market[13]. There is no clear consensus about the reasons which drive the market towards one of the above conditions, nor on the actual situation of the market (see e.g Daskalakis et al, 2008; Milunovic & Joyeux, 2007; Paolella & Taschini, 2006). They appear to be in contango (Borak, 2007:23), as operators believe the demand for Phase II instruments will rise, and that the market is not in over-allocation as in Phase I (PointCarbon).

Various approaches are possible to determine term structure by using alternative model specifications for the convenience yield term. For what concerns this paper, it will be

[13] See section 2.2 for a review of existing literature.

illustrated that a cost and carry model with zero convenience yield explains successfully intra-phases spot-futures relationship. That is a consequence of the banking regulation, and it is consistent to the characteristics of the instruments. In fact a spot EUA can be easily stored until the end of the period, therefore there is no clear advantage to hold a long Futures position rather than a long spot position (Uhrig-Homburg & Wagner, 2007:6). Chapter three will show that spot and intra phases futures are cointegrated (they have a stable long term correlation) and the convenience yield is insignificant.

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In this section the market for carbon emission has been introduced and described in its essential characteristics. The problem statement of the paper focuses on the European Emission Trading Allowance, spot and derivatives. Therefore It has been necessary to define those instruments in the context of the market where the instruments are traded. The background and foundation of the emission markets rely on the Kyoto Protocol agreement, as the ETS it is a consequence of the flexibility mechanism allowed by the UNFCCC agreement. Some carbon assets have been created, and regulated exchanges allow for them to be traded among operators and investors. The new and growing market therefore has brought some challenges and opportunities, that have to be addressed with valid risk management tools. In this chapter, some specific characteristics of the scheme have been highlighted as the relevant ones for the methodology that will be applied. In the next section the problem statement will be described and the methodology to address it will be cleared out. Some of the characteristics of the market will affect the choice of methods and some assumptions behind pricing.

Chapter 2 – Problem Formulation and Methodology

In this section, the problem statement of the paper will be defined. The background and motivations that lead to the research question will be firstly introduced. Hence the main problem will be presented in an open form, which will be expanded into sub-problems. Therefore, the theoretical framework to address the analytical issues, will be introduced in order to delineate the overall structure of the work. The problem definition and limitations are going to be made explicit, and finally a literature review from previous studies will be included.

2.1. Motivations and interest in the topic

The background and motivation of the thesis come from the fact that the emission trading scheme is a young and growing market. Volumes are increasing and investors and operators are attracted by the opportunities created by new market. It is also attiring interest from newspapers, magazines and the public, even if it is not always well understood. The attention of the public opinion and of researchers has been more oriented towards the economics and efficiency of the trading system, and on pure environmental and ethical issues.

This paper instead focuses more on the financial side of the problem: the advent of the ETS has introduced some new challenges as companies are now required to employ monitor and control systems, to develop knowledge on future market trends, and incorporate costs of emission allowances in their risk management practices. In order to deal successfully with the challenges posed by the ETS, firms need to thoroughly understand its mechanisms, grasp its orientation and perspectives. Companies that participate in ETS on a compulsory or voluntary basis must address issues related to the financial instruments that have been created by the new regulatory system.

The study of this market is challenging and exciting as it embodies quantitative and energetic issues at the same time. More important, there is a lack of academic research on the specific issue of the dynamic of the price of EUAs and of pricing of carbon derivatives. The current literature is sparse, and rarely addresses the topic directly. So far, a model able to fit the historical data and at the same time useful for hedging or speculation purposes has not been clearly indicated. Moreover, the emission markets have not been consistently studied from an investment theory point of view[14].

This thesis aims to fill some gaps in the studies from an asset pricing approach, and gives some possible directions for possible studies. As the issue is complex and can be addresses with different perspectives and methodologies, it is not a comprehensive study. However, it suggests some solutions and models that can be used or modified by future students or practitioners.

2.2. Object of the paper

The goal of this paper is to study the market for Emission Unit Allowances. The purpose is to define and implement a stochastic model on the price of EUAs in order to price and hedge spot and futures positions on the allowances. The model must be flexible enough to incorporate the main characteristics of the price dynamic, but still be parsimonious in the number of parameters because of the lack of a long series of historical data.

2.3. Problem Statement

The main problem statement is **to develop a model for valuation and hedging of derivatives instruments on Emission Unit Allowances**. The study will show the characteristics and the assumption which have to be taken into consideration in a model.

[14] Investment theory is intended as the body of knowledge used to support the decision-making process of choosing investments for various purposes. It includes for example portfolio theory, the Capital Asset Pricing Model, Arbitrage Pricing Theory, and the Efficient market hypothesis.

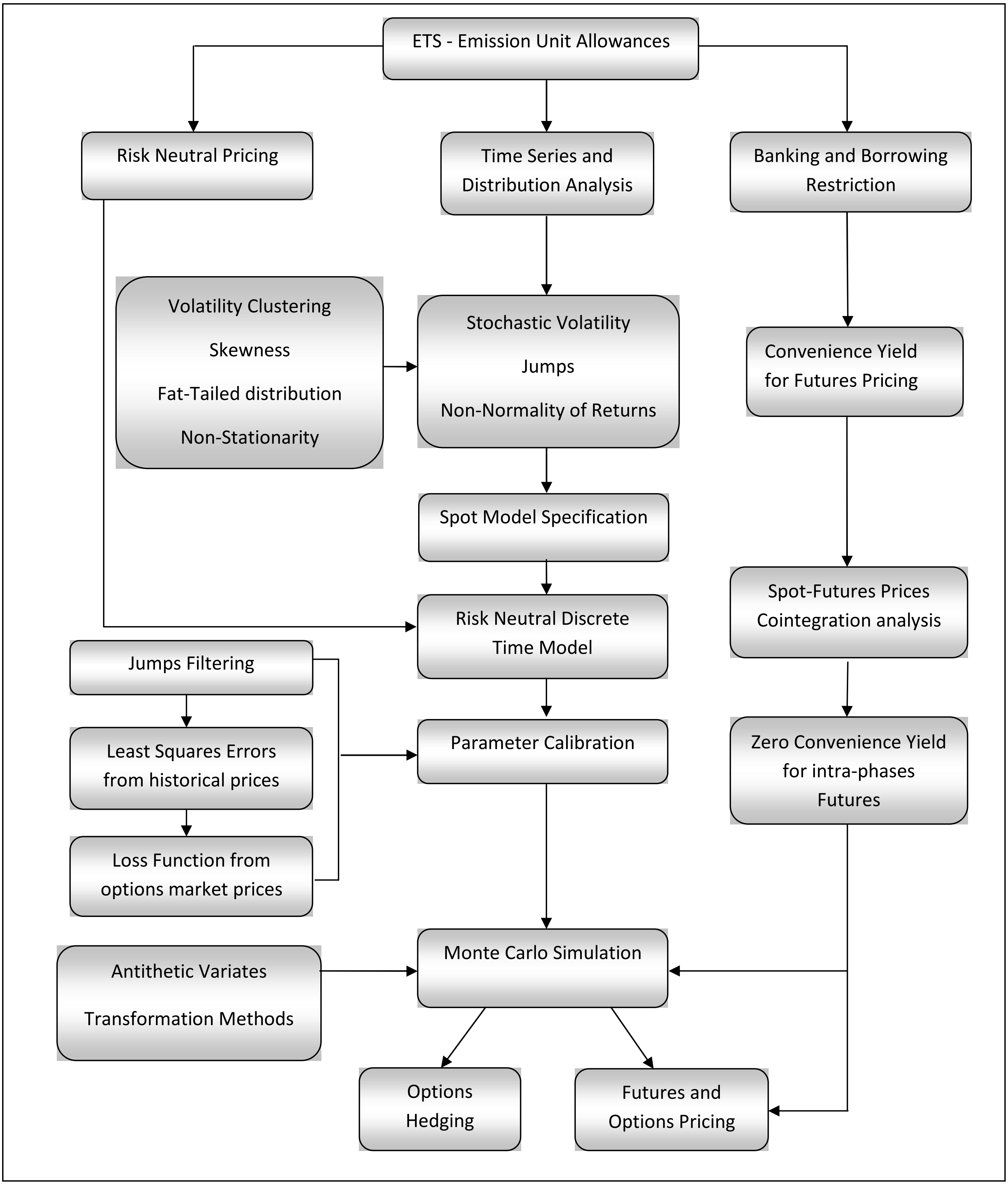

Figure 2.1 – Thesis Structure

The research question therefore concerns how to identify a valid stochastic equation which reflects the feature of the historical and market data of EUAs, and that can be applied for pricing and hedging purposes.

The main research problem is approached through the sub-problems explained below. Figure 2.1 gives a visual overview of how the model was developed.

Sub Problem 1: How to choose a valid model for EUA prices?

Following a methodology consistent with (Eydeland & Wolyniec, 2006:118; Geman, 2005:49; Pilipovic, 2007:53) the modeling process consists of the following steps:

- Indentify market qualitative and quantitative properties in the historical data. Perform a time series analysis and distribution analysis on the time series.
- Find the most appropriate mathematical structure for the spot price S, given the properties observed in the historical set of data. The choice of the process should lead to a probability distribution of the random variable $S(t)$ which reflects the empirical moments and other observed features of the historical distribution, and be consistent with the dynamic of the process (the properties of the change of S between two dates).
- Once chosen a process, estimate parameters from market data. Liquid markets and clean data are necessary in order to have the right information from which to calculate the parameters.
- Find a benchmark to evaluate the performance of the model
- Check the performance of the selected model. Evaluate the in-the-sample fit of the model and hedging results.

The quantitative model therefore has to be consistent with the characteristics of the drivers of the underlying. These drivers has to be included in the model with the constraint to be practically implemented.

A statistical descriptive analysis will be carried out in chapter three, with a time series and distribution investigation of the historical data. The first moments of the distribution and the autocorrelations coefficients will be computed for the series of prices and returns.

Formal testing will be performed in order to assess the presence of common features for financial data, namely the non-normality of returns, volatility clustering, presence of spikes, and mean reversion. These hypothesis will be tested through some statistical and econometrical tests:

- Distribution Tests
- Variance analysis
- Unit Root or Stationarity test of the time series

Further details will be included in chapter three. The methodology is coherent with previous literature as (e.g. Koop, 2006:223; Eydeland, 2006:68; Daskalakis et al, 2008:6; Campbell et al, 1997:64; Pilipovic, 2007:71).

Sub Problem 2: Which model adequately reflects historical data characteristics?

Spot models are usually implemented because of their reliability on the description of the evolution of the price. They are versatile and relatively easy to incorporate characteristics of the historical series. However, one main drawback of the models is the difficulty in parameter estimation, given the number of parameters and the small number of observations in a non mature market as the ETS (Cartea & Figureoa, 2005).

The contribution of this paper is to incorporate a stochastic volatility structure for the volatility diffusion term in order to reflect better the volatility clustering. A similar approach has been used in the electric markets which are characterized by volatility clustering as well (Bierbrauer et al, 2007; Escribano et al 2002,). The stochastic volatility Heston model (1993) has been adopted here, with the addition of a jump component.

The overall configuration is still flexible and easy to adapt to the purpose of building a hedging model and not only a pricing model. However, given the number of parameters, and the unavailability of closed form solutions, it has been decided to implement a Monte Carlo Simulation to determine the spot price dynamic.

Sub Problem 3: Are conditions for risk neutral pricing valid?

The model has to rely on correct theoretical framework. Derivative pricing relies on application of a risk neutral measure, resulting from the assumption that the current value of all financial assets is equal to the expected value of the future payoff of the asset discounted at the risk-free rate.

Financial assets prices depend on the risk they are bearing, as investors expect a reward for the uncertainty of future price outcomes. Expected values have therefore to be adjusted with the risk involved. However, under a complete market[15] and a no arbitrage assumption, it is possible to apply an alternative solution. It is possible, in fact, to adjust the probabilities of futures outcomes, and then discounting future expected values at a risk free rate. (Eydeland & Wolyniec, 2007:141).

In other words, if real probabilities are taken, then each assets should be discounted at a different rate expressing the difference in risk. Alternatively the risk neutral approach allows for using the same risk free rate to discount the expected payoff. Mathematically, it is possible to transform the measure to an equivalent martingale measure only if there are no arbitrage opportunities. If the markets are complete, then the equivalent measure is unique (Geman, 2006:35; Eydeland & Wolyniec, 2007).

[15] In complete markets all possible future outcomes can be replicated by existing assets. One example is the put-call parity (Hull, 2009).

Derivatives pricing methods are based on some assumption about the market that have to be verified within the characteristics of the ETS. A typical assumption often include that markets are efficient and arbitrage free (Pilipovic, 2007; Geman, 2006:35). Some authors have already verified the efficiency and the no arbitrage condition in the ETS market (e.g. Seifert & Wagner, 2006:23; Daskalakis & Markellos, 2008). Their results are taken as an assumption in the pricing methodology of this paper. Furthermore, derivative instruments and their underlying have to be continuously traded through the duration of the contract so that a perfectly hedged portfolio can be constructed. In efficient markets, such a combination of the derivative and its underlying leaves no space for arbitrage opportunities and bears no risk, therefore giving as return only a risk free rate (Hull, 2009; Eydeland & Wolyniec, 2007). The no arbitrage condition allows for the stochastic equation of the model to be expressed in risk neutral probability measures.

As shown in section 1.9 and given previous studies, it is reasonable to assume that for intra-phases instruments the conditions are respected. Under the no arbitrage assumption, Futures and Spot prices must converge at maturity date to avoid arbitrage opportunities (Eydelend & Wolyniec, 2007:21). Moreover, in section 3.9 it will be verified that Spot and Futures prices follow the same process, so that a spot price model can be used to price derivatives written on Futures.

Sub Problem 4: Which method can be applied to implement the model?

It is possible to indentify three broad classes of methods to price derivatives and measure their risk (Broadie & Glasserman, 1998:173):

- Analytical Formulas
- Deterministic numerical methods (including binomial trees and partial differential equations)
- Monte Carlo simulations

As the complexity of the derivative increase, simulation becomes the only method which is general enough to capture fully the complexity of the instrument. Consequently, on the base of the chosen model, a Monte Carlo method will be implemented to price and hedge the financial products. The method presents many advantages. It is straightforward to comprehend for any kind of user. It offers a flexible and easy implementation, and it is still very powerful and has a broad application range: it can be virtually applied to any kind of derivatives, and it allows for several hedging strategies easily. It is a popular method as it allows for pricing even if analytical solutions for the stochastic model are not available (Joskow, 2007:307). Furthermore, it is a powerful tool for derivatives written on more than one assets (Haugh, 2007:345). It will implemented in Excel and Visual Basic for Application (VBA) framework[16].

However, the drawback for the Monte Carlo Simulation robustness is that it can be very slow and computationally expensive. In its basic form, it is necessary to run a great amount of iterations before the simulation converge to market values. Therefore solutions to accelerate the convergence can be used. Variance reduction techniques are going to be used, namely antithetic variables, control variates and quasi random sampling (Broadie & Glasserman, 1998:183). One disadvantage with these variance reduction techniques is that they have to be specially designed to each problem. An alternative approach for convergence improvements is to change the choice of sequence generator. Quasi-Monte Carlo methods for example use quasi-random (a.k.a. low-discrepancy) sequences instead of pseudo-random (Clewlow & Strickland, 1998:123).

In the paper antithetic variate and Box-Muller transformation with polar rejection are the techniques adopted for generating random numbers. (Clewlow & Strickland, 1998:81; Haugh, 2007:358; Broadie, M., & Glasserman, 1998:183). They allow to increment the speed and accuracy without having to increase the number of iterations in the simulation. They will be explained more in details in section 4.4 and 4.5.

[16] The programming codes are included in the Appendices and in the enclosed Excel files.

Sub Problem 5: How can the performance of the model be tested?

The outcome of the model will be tested with an in-the-sample and out-sample comparison. The simulation will be run in the period which is covered by the historical data. In this way, it will be possible to assess whether the model is able to reflect the historical characteristics of the time series. Furthermore, the simulation will be tested on its capability to price options, using market data which are available at the moment of the analysis. The closed form solutions from Black (1976) model will be used as a benchmark for comparison and evaluation purposes.

Unfortunately the hedging results will not be tested, as it is not possible to have historical data on prices of options for all the range of prices. As will be showed in chapter four, the main idea behind hedging using the Greeks value is to continuously rebalance an option portfolio according to price or parameters changes. However, ECX provides only the premiums of effectively traded options in the past, without showing the premiums for all the prices range. The absence of data does not allow to assess the performance of the proposed strategies. Therefore, only an example of an hedging strategy will be illustrated, without having the possibility to assess its efficacy on available data.

2.4. Limitations and definition of the study

The approach of this study is purely financial. Even though it is necessary to introduce the market characteristics, it is not in the scope of the thesis to address economical issues related to the most effective or efficient structure of the ETS.

The pricing and hedging methodology will be applied to the characteristics of the market in Phase II. Regulatory economics has been addressed extensively by previous studies (e.g. Ellerman & Joskow, 2008; Egenhofter, 2007). A short summary of the existing literature will be included in the latter of this section.

The thesis does not aim to explicitly analyse the consequences of the allowance allocation process, even though that has repercussions on the price itself. Likewise, the banking and borrowing prohibition will not be examined in its economical rationale and foundation, nor for repercussion on market efficiency.

However, the restriction has a serious consequence on the assumptions for pricing financial instrument that has to be made explicit. In fact, in order to have risk neutral pricing, both the derivative and the underlying have to be continuously traded throughout the life of the derivative contract (Hull, 2009; Black, 1976). On the other hand, inter-phases Futures can be traded during phase II and phase III, but underlying spot allowances from different phases cannot. The restriction on banking means that emission allowances become worthless at the end of each phase, hence an inter-phase derivative is essentially written on an asset that is not tradable during the whole life of the underlying contract (Daskalakis et al, 2008:4).

The restriction put a serious doubt on the applicability of the theoretical framework of the cost-and-carry pricing for futures and the Black framework for derivatives (see Uhrig-Homburg & Wagner, 2007:21). For inter-phase assets it should therefore be necessary to adopt alternative models, as for example equilibrium models (Daskalakis et al, 2008) or multi-factors models (Lucia and Schwartz, 1997) .

Nevertheless, this paper addresses exclusively intra-phases contracts, so that the assumption for cost-and-carry can be preserved. In any case, the assumption will be tested in order to ensure its applicability. A cointegration analysis on the spot and futures price processes will be carry out in the next section to demonstrate the validity of a zero or constant convenience yield relation (Daskalakis et al, 2008; Milunovic & Joyeux, 2007, Koop, 2006).

The financial product that will be treated in the paper is the Emission Unit Allowance (EUA), as it is so far the most liquid and the one traded with more volumes. Likewise, Bluenext for spot contracts and ECX for Futures and Options will be taken as the exchanges with more liquid contracts. Consequently, the assumption of continuously traded assets is preserved. The price is examined during the period 26^{th} February 2008 – 8^{th} May 2009, forming 304 observations.

2.5. Previous studies and Literature Review

Relevant academic works on Carbon Trading have been so far sparse. Some research on the homologous Sulfur Dioxide Emission market (SO_2) in the U.S. have been focusing on derivatives instrument pricing, but since the market is organized differently from the EU ETS, it is not possible to make comparisons (Paolella & Taschini, 2006).

A higher attention has instead been put towards the economics of the ETS, in order to assess whether the market will effectively lead towards a reduction of the emission, or if it will provide the necessary flexibility and cost abatement for reaching Kyoto Protocol compliances. (Coleman, 2007; Egenhofer, 2007; Kruger et al, 2007; Veith et al, 2008). However an analysis on the best structure for the emission market is not the object of this paper. What is relevant for derivatives valuation is the implication of the emission allowance banking prohibition for financial markets (e.g., Cason & Gangadharan, 2006; Rubin, 1996; Schennach, 2000; Godal & Klaasen, 2006; Schleich et al., 2006) as it has been shown previously in the section 1.4.

From a financial point of view, studies are not numerous, and an extensive and consistent analysis of the market is still needed. Conclusive results on the best pricing model applied to derivatives is not currently present. The analysis that have been made still concern Phase I , sometimes even at its early stages.

Paolella and Taschini (2008) have taken an econometrical analysis of the spot market founding that the unconditional tails can be described by a Pareto distribution. They propose a GARCH structure to model the heteroskedastic (changing volatility) dynamics of the returns. Benz and Trück (2008) suggested the use of Markov switching model and AR-GARCH models for stochastic modeling because of the presence of different stages in the price development and in volatility behaviour. Seifert et al (2008) presented a stochastic equilibrium model reflecting stylized features of the EU ETS.

With regards to Futures modeling, Uhrig-Homburg and Wagner (2007) adopted a cost-of carry pricing mechanism for Futures expiring within the first phase of the market. However, Borak et al. (2007) showed that futures that are written within the first phase and expire within the second, have significant convenience yields. Milunovic and Joyeux, (2007) found evidence of departure from the cost-of-carry model, while Daskalakis et al (2008) suggested to model inter-phases Futures with stochastic convenience yield. Currently there is no clear consensus on the situation in Futures markets, and different approaches have been proposed.

About market efficiency, Daskalakis and Markellos (2008) found inconsistent evidence with the weak form of market efficiency, and they explained their founding with the immaturity of the EU ETS and to the restrictions imposed on short-selling and on banking. Similar findings from Milunovic and Joyeux (2007) conclude for efficiency of the market in the long run. However the analysis were performed in early stages of Phase I, and they may be not applicable for Phase II.

Other studies have focused on price discovery and on the major factors which might drive the EUA prices. EUA prices seems to be influenced by energy prices (e.g. Bunn & Fezzi 2007; Benz & Klar, 2008), oil prices (Bataller et al, 2006), unanticipated weather conditions (Alberola et al, 2008). Furthermore there might be a strong linkage between electricity prices (and electricity prices premia) and carbon assets prices (Daskalakis & Markellos, 2007).

Generally, big spikes in both directions seemed to be linked to oil markets, economic expectations and energy sector. Previous studies have tried to identify major drivers for EUA prices, which appear to be energy prices (e.g. Bunn & Fezzi 2007; Benz & Klar, 2008), oil prices (Bataller et al, 2006), unanticipated weather conditions (Alberola et al, 2008), among all. Furthermore, given that the power sector is the main operator in the emission scheme, it seems to be a strong linkage between electricity prices (and electricity prices premia) and CO prices (Daskalakis, Markellos, 2007).

Because all currently available studies belong to Phase I, further updated studies on ETS Phase II prices are needed. This paper aims to contribute in reducing this knowledge gap.

2.6. Structure of the thesis

In chapter one the market structure has been introduced. It was necessary to understand how the Emission Trading Scheme works, and to define the financial instruments that are going to be priced. In chapter three a statistical and econometrical analysis will be carried out, in order to highlight the main features to be included in the model (volatility clustering, jumps, and non normal distribution). At the same time, a Cointegration analysis on Spot and Futures relationship will be performed to justify formally assumptions in the pricing methodology. In chapter four the model will be presented more in depth. Parameters are going to be calibrated based on the historical time series and on market data. A Monte Carlo simulation will then be employed and the results will be discuss to assess the performance of the model.

Chapter 3 – Evidence from the Spot Market

In this section the spot price dynamics will be analysed in order to identify the main characteristics on the series. In fact, consistently with the methodology defined in chapter one, before choosing a pricing model, it is necessary to perform a statistical analysis on the series to identify the features of the price dynamics.

In particular, the data set will be here presented, followed by an commentary on the range of the price, the spikes happened in the observed period, and the volatility. Hence a standard statistical analysis will describe the first four moments of the distribution of the prices. Moreover the autocorrelation of the series will be presented. Based on the initial finding, formal tests will be carry out: normality tests (Jarque-Bera and QQ-plot), and stationarity tests (Augmented Dickey-Fuller), and heteroskedasticity for volatility.

3.1. Dataset

Spot trading in the ETS is mainly performed in through the two largest exchange platforms, the Bluenext based in Paris, and the Nordic Nord Pool. However, as showed in chapter 1, Bluenext is by far the most liquid market, and it is therefore chosen as the object of analysis. It had a daily average volume of 2,72 Mil tonnes in the observed period (with a record volume of 14,31 Mil tonnes on 24/02/2009). Trading for EU Spot EUA Phase 2 started for on 26/02/2008.

It has been shown in previous researches (see e.g. Milunovic & Joyeux, 2007; Daskalakis & Markellos, 2008b) that prices in different European markets are highly correlated, and the discrepancy among markets is lower than transaction costs. This leaves a narrow space for arbitrages and it is therefore a signal for liquid and mature markets[17]. The efficiency of the market is a basic assumption for a no-arbitrage condition, necessary for derivative pricing and to apply the risk-neutral measure in the stochastic equation in chapter four.

The dataset for Phase II Emission Allowances comprises of closing prices from 26/02/ 2008 to 07/05/2009, totaling 306 observations. The dynamics of the price and the log-returns[18] is shown in Figure 3.1 and 3.2. Descriptive statistics are summarized in Table 3.1. The historical prices have been obtained directly from the Bluenext exchange website[19]. All the analysis as performed on the Excel framework and are included in the file of the enclosed CD.

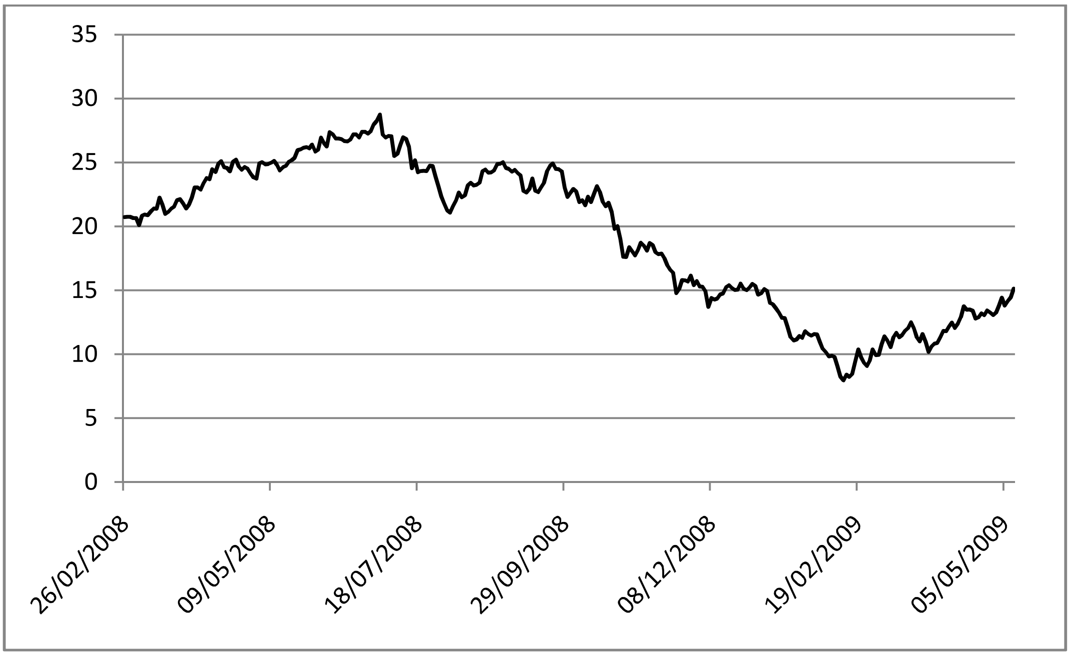

Figure 3.1 – Bluenext Spot EUA Phase II Closing Prices – (26/02/2008 – 07/05/2009)

[17] According to Daskalis and Markellos (2008b) the transaction cost to operate simultaneously on two trading platforms is on average less than 0,03€. For minimum volumes trades transaction cost is 0,04€.
[18] LogReturns are computes as the natural logarithm of the price ratio, $Ln(P_t/P_{t-1})$.
[19] www.bluenext.eu

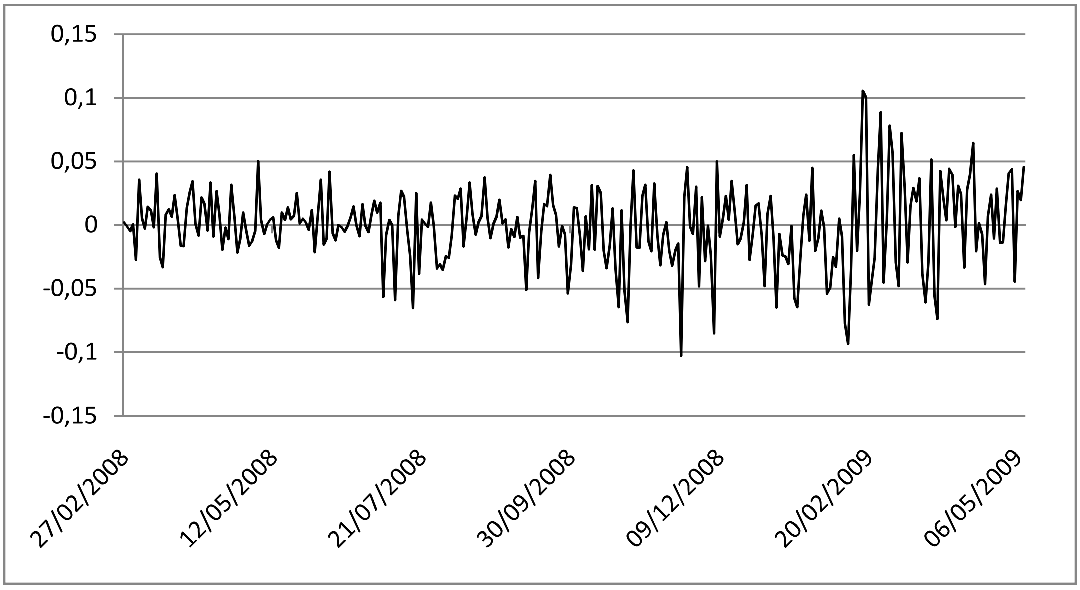

Figure 3.2 – Bluenext Spot EUA Phase II LogReturns – (26/02/2008 – 07/05/2009)

3.2. Descriptive Statistics

Data ranged from a minimum of € 7,96 (12/02/2009) to a maximum value of € 28,73 (01/07/2008). The record low came with the market turmoil that took place in February 2009, as an effect of plunging energy prices. On the contrary, the maximum was a consequences of high oil prices, with pressured on energy prices. In June 2008, transactions had reached the record level (€240 mil in value) (PointCarbon). The average price for the period has been €19,26, while the mean return has been -0,0010.

	Prices	LogReturns
N. Obs.	304	303
Mean	19,26	-0,0010
Median	21,31	0,0000
Maximum	28,73	0,1055
Minimum	7,96	-0,1029
St. Deviation	5,8252	0,0305
Skewness	0,3094	-0,1078
Kurtosis	1,6398	4,1009
AC(1)	0,9963	0,1416
AC(2)	0,9920	-0,1685
AC(3)	0,9886	0,0563
AC(4)	0,9849	0,1752
AC(5)	0,9801	0,0120

Table 3.1 – Descriptive Statistics for Bluenext Phase II Closing Prices

The two biggest spikes occurred on 18/02/2009 and 19/02/2009 with upside jumps of about 11% and 10%. The reason is probably to assign to a market correction, after the all time low of € 7,96 which took place the week before (NewCarbonFinance). The lowest negative spike instead happened on 20/11/2008, when some pessimistic reports about deeper-than-expected recession came out, dragging down energy prices, and likewise carbon price.

Generally big spikes in both directions seems to be linked to oil markets, economic expectations and energy sector. Previous studies have tried to identify major drivers for EUA prices, which appear to be energy prices (e.g. Bunn & Fezzi 2007; Benz & Klar, 2008), oil prices (Bataller et al, 2006), unanticipated weather conditions (Alberola et al, 2008), among all. Furthermore, given that the power sector is the main operator in the emission scheme, it seems to be a strong linkage between electricity prices (and electricity prices premia) and CO prices (Daskalakis & Markellos, 2007). Nonetheless more updated studies on Phase II prices are needed.

3.3. Volatility Analysis

The jumps are the main reason for the high level of historical volatility. The price historical standard deviation[20] has been of 5,8252, giving an annualized volatility of 92,31. For price returns instead the standard deviation has been 0,0304, with an annualized value of 0,4835.

By analyzing the volatility of the returns, it seems that the series is characterized by heteroskedasticity. By looking at a 20-days rolling volatility, it appears that volatility level is not constant through the observed period, but it fluctuates significantly. Following Jorion (2007), an estimate for the volatility is constructed through a moving average of the

[20] The variance is defined as the population variance. $\frac{1}{N}\sum_{t=1}^{N}[r_t - E(r_t)]^2$. Population standard deviation is the square root of variance. The choice of $\frac{1}{N}$ and not $\frac{1}{N-1}$ has been taken in consistency with all other Excel functions. Implicitly the choice lead to the assumption that historical data represent the real distribution.

returns. The rolling volatility estimate is deliberately made only on returns instead of returns around the mean, as for very short periods that is a negligible bias (Jorion, 2007:222):

$$MA(\sigma_{20,t}) = \sqrt{\frac{1}{20}\sum_{i=1}^{20} r_{t-1}^{2}}$$

Figure 3.3 shows the graph for the $MA(\sigma_{20,t})$, and it seems clear that the volatility fluctuacted drasticly over time (from a minimum of 0,010791 to a maximum value of 0,062226) compared to the historical volatility for all the period of 0,0304. This can be a sign of volatility clustering or heteroskedasticity in the series.

A formal analysis of volatility can be carried out with an Auto-Regressive model with one lag, AR(1), as suggested by Koop (2003:212). The technique simply constructs an AR(1) equation of the squared deviations from the mean of the returns series.

$$Y_t = \alpha + \phi Y_{t-1} + e_t$$

The value ϕ is related to the autocorrelation of the series, and to its stationarity. In fact, if $\phi = 1$ the variable Y in the model is non stationary and has a unit root. Any other values, $|\phi| < 1$, implies stationarity[21].

If Y has a unit root, its autocorrelation will be close to one and will decrease slowly as lags increase. In this case, the series will have a long memory, meaning that a shock will have a long enduring effect on the price levels. On the other hand, a stationary time series does not have a long memory, and it might exhibit mean reversion. Unit root series exhibit trend behaviour. If Y has a unit root, then the differentiated series[22], ΔY, will be stationary, and Y will be a difference stationary series.

[21] The hypothesis $|\phi| > 1$ means that the series exhibits explosive behaviour over time, and it is commonly not considered in financial analysis (Koop, 2007).
[22] $\Delta Y = Y_t - Y_{t-1}$

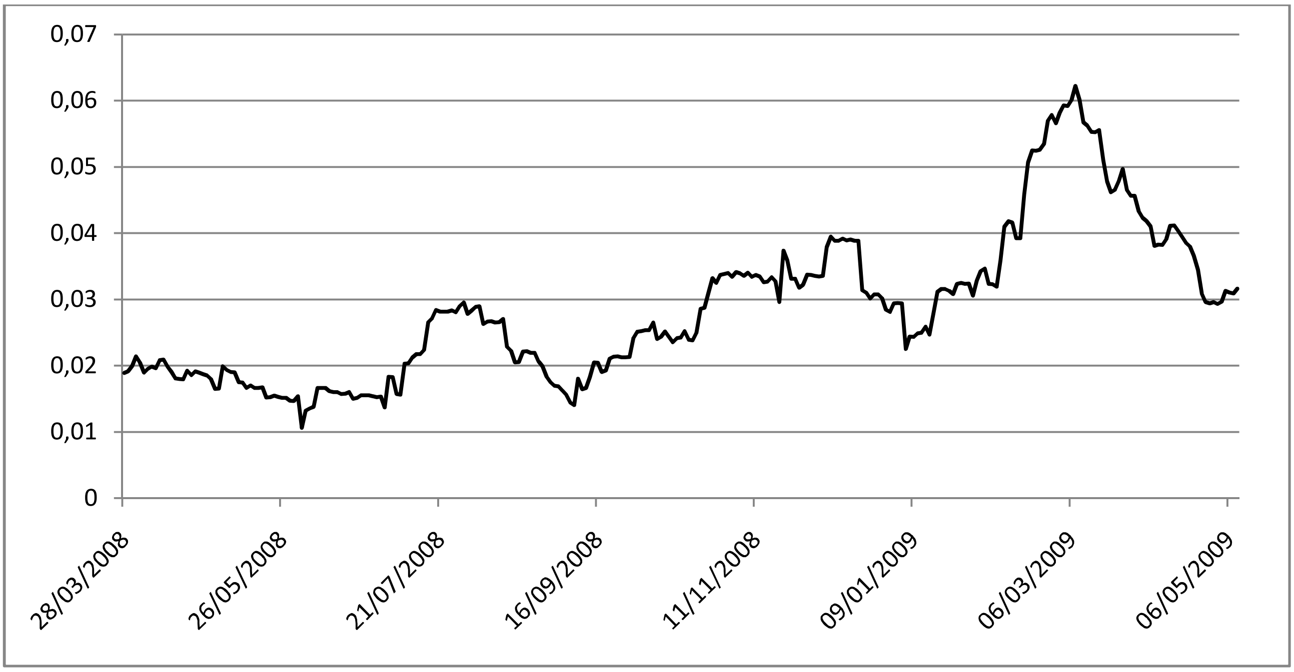

Figure 3.3 – 20 days Returns Rolling Volatility

The results of the test show that there is a significant explanatory power of the daily volatility for next day level. Even though the R^2 has is low (0,09), the regression and the coefficients are statistically significant at 95% and 99% levels. The average previous day volatility explains 0,30 of the next day volatility. The conclusion is that volatility seems to possess a stationary behaviour. This implies that the series is characterized by periods maintaining high volatility, and periods with relatively low dispersion. The volatility clustering feature should be therefore captured by a pricing model which allows for heteroskedasticity.

3.4. Skewness and kurtosis

As for the skewness and kurtosis values, they imply that the LogReturns distribution is negatively skewed and leptokurtic. In fact, the negative skewness suggests that the distribution has a long left tail, hence generating large negative values. Similarly the Pearson's coefficient of skewness is approximately -0,01 and consequently suggesting a negatively skewed distribution. Figure 3.4 represents the frequency distribution for

LogReturns. However the Pearson value is relatively low, meaning that the distribution is approximately symmetric (Middleton, 2000:41).

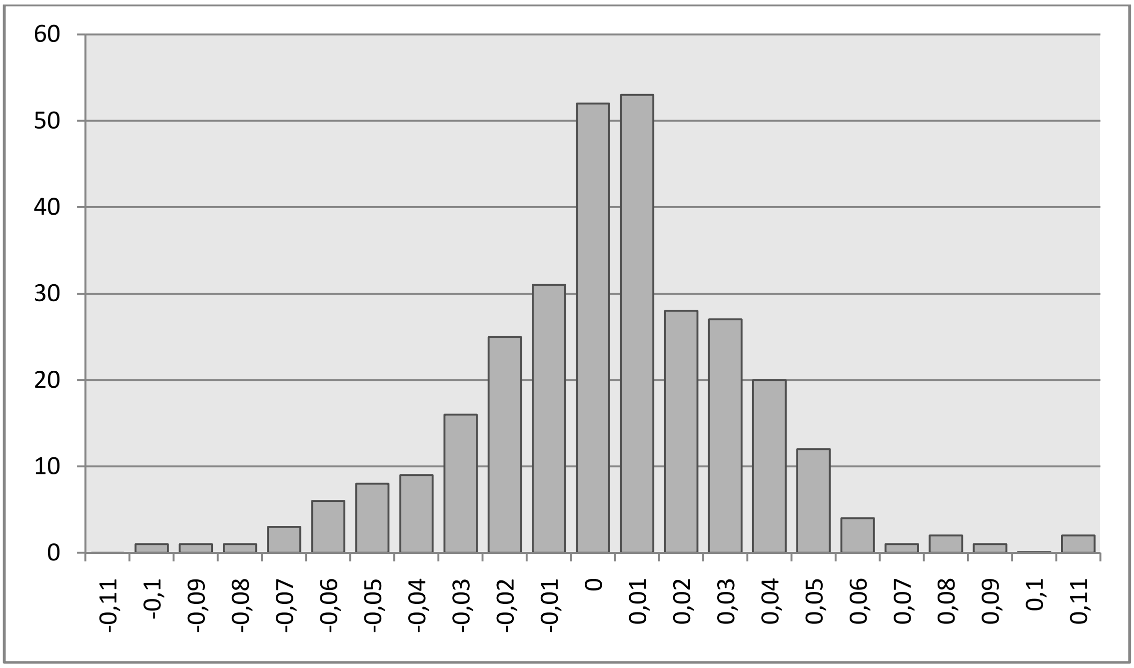

Figure 3.4 – LogReturns Distribution

The value for kurtosis, which is higher than three, indicates that tails decay less quickly compared to a normal distribution (the standardized value for the kurtosis is higher than the standard normal value), therefore implying the distribution has fat tails.

Information deriving from kurtosis and skewness will be combined in the Jarque-Bera test, which will be performed the next section.

3.5. Distribution Analysis

The Jarque-Bera test is performed in order to assess the null hypothesis of normality of the returns series (Jorion, 2007:97). The statistics is constructed as:

$$JB = T\left(\frac{\hat{\gamma}^2}{6} + \frac{(\hat{\delta} - 3)^2}{24}\right)$$

Where $\hat{\gamma}$ and $\hat{\delta}$ are the estimate of skewness and kurtosis that have been showed in the previous paragraph. The statistics takes the values of 28,28 and 15,88 for respective prices and LogReturns series, therefore rejecting the null hypothesis of normality of the distribution for both. The values are in fact higher than the cutoff point of 95% and 99% levels of confidence.

The next step is to employ a QQ-plot to demonstrate further the departure from the normal distribution for the log-returns series. The chart is showed in Figure 3.5. The Q-Q test compares the actual probabilities of the random variable with the expected probabilities of a normal distribution. If the variable is normally distributes, then the QQ-plot looks like a straight diagonal line (Pilipovic, 2007:81; Jackson, 2001:60).

Here the chart is built as a scatter chart, with the returns series standardized values plotted against theoretical normal values. The standardized values, the Z-scores, are computed as:

$$Z_i = \frac{R_i - \bar{R}}{SD_R}$$

With R_i being the LogReturns ordered in ascending order, $\bar{R}$ and SD_R the logreturns average and standard deviations. If the distribution of the returns of normal, then it should lay on a straight line in the chart. However, it appears from Figure 3.5 that the actual distribution departs from normal distribution as the tails have higher values. That implies that the observed distribution is presenting fat tails, where extreme values are more likely to appear than in a normal distribution.

The distribution analysis therefore leads one to reject the assumption of normal distribution. A different distribution must be considered for the stochastic model to take into consideration the fat tails of the historical distribution.

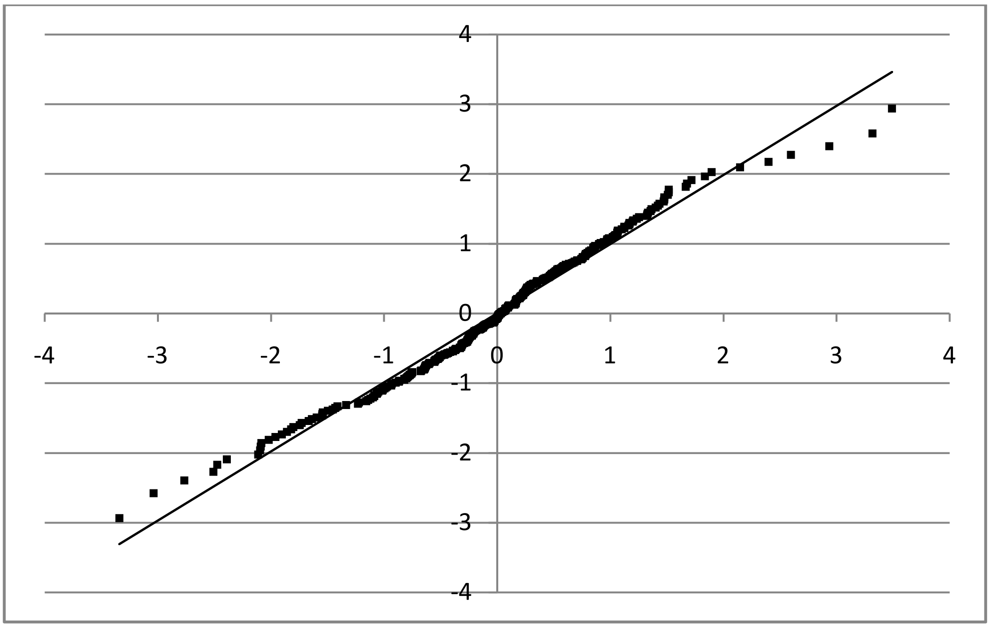

Figure 3.5 – QQ plot chart – Returns z-scores (y-axis) against theoretical normal scores (x-axis)

3.6. Autocorrelations

The autocorrelation coefficients show a high persistency. The correlations coefficients, with lags from 1 to 5 days, are close to 1 in value and decrease very slowly in their levels. This may be a signal for a non stationary series. (Campbell et all, 1997:44). A Unit Toot test is then performed to assess whether the price series has stationary properties.

3.7. Unit Root Stationary Test

Following the literature (Escribano et al, 2002:14; Daskalakis et al, 2008:31; Campbell et al, 1997:65) in order to perform a unit root test, an Augmented Dickey-Fueller test (ADF) has been applied on the log-Prices series. The test is done on the t-statistic for the null hypothesis, with 4 lags. The statistic is the t-ratio for the ϕ coefficient of an auto-regressive model specified as:

$$\Delta y_t = \mu + \phi y_{t-1} + \sum_{j-1}^{p-1} \phi_j \Delta y_{t-j} + \mu_t$$

with p=4 and $\Delta y_t = y_t - y_{t-1}$ (with y standing alternatively for prices and log-prices). The cutoff values for this particular test cannot be taken from standard distributions. MacKinnon values are used instead, being -3.4360 at 1% level, -2.8632 at the 5% and -2.5677 at 10%. (Campbell et al, 1997; Koop, 2006:157).

Regarding the definition of the number of lags $\phi_j \Delta y_{t-j}$ included in the model, a technique suggested by Koop (2006:155) has been followed. The model is firstly specified with a high number of lags, which are tested for significance. If the coefficient for the highest lag is not significant, then another regression is run without the non significant lagged variable. The process continues until the coefficient with the highest lag is significant. Hence a similar treatment is applied to the time drift μ_t which represents a deterministic trend.

The test is performed on the t-statistic of the ϕ coefficient. The final results, being –0,88 the t-value for the ϕ coefficient higher than the cutoff values, leads one to accept the hypothesis that the series has a unit root. In the significant configuration of the Dickey Fuller test, the deterministic trend has been dropped as it was not significant. The series seems to contain a stochastic trend rather than a deterministic trend.

The results suggest that at conventional significance levels, logarithmic spot prices are non-stationary. Since EUAs are usually considered to be commodities, the result contradicts the common finding of mean reverting behaviour observed in other commodities and energy assets (Schwartz, 1997). Therefore a mean reversion feature will not be incorporated in the pricing model. The drift of the price will be modeled with a stochastic diffusion component.

3.8. Role of the Convenience Yield for Intra-phases Futures

Recalling section 1.9, Intra-Phases Futures and Spot prices can be seen as substitutes, as there is no clear advantage on holding a Futures position instead of a Spot Position on EUAs which expire within the same period.

In order to apply the pricing methodology, by which derivatives with Futures as underlying are priced with a spot price model, the assumption that spot and Futures prices follow the same stochastic process must be justified. It is an important issue, as it is necessary to justify the no arbitrage condition, and the fact that Spot and Futures Prices converge at expiry date (Eydeland & Wolyniec, 2007).

Therefore the relationship that link Spot and Futures must be analysed. In particular, it will be demonstrated here that a cost and carry model with zero convenience yield is able to describe correctly Futures prices. A cointegration analysis will show that Spot and Futures EUA instruments follow the same stochastic process. Figure 3.6 shows prices for Phase II Spot and December 2009 Futures.

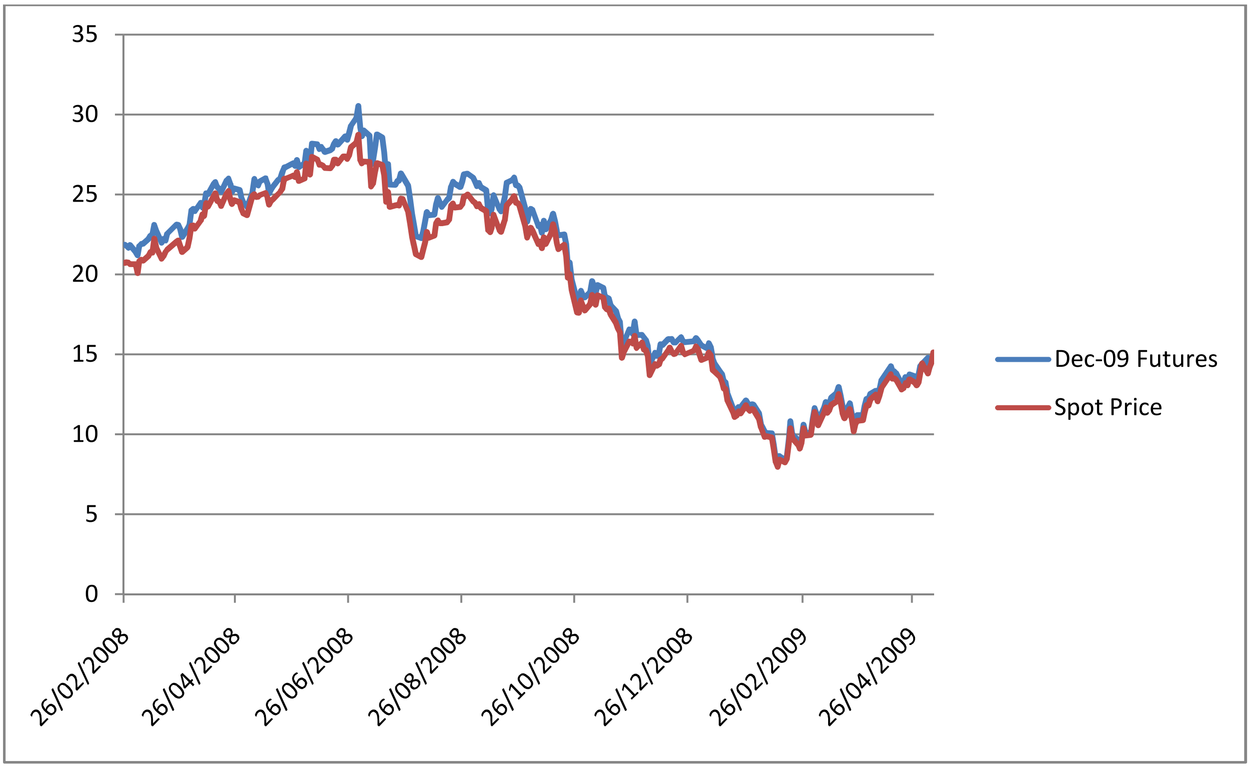

Figure 3.6 – Phase II Spot EUA and December 2009 Futures Prices – 26/02/2008 – 07/05/2009

The analysis here performed is consistent with previous literature (Milunovich & Joyeux, 2007; Koop, 2006; Brenner & Kroner, 1995). The authors propose to perform a cointegration analysis on the cost-of-carry model.

The cost-of-carry relationship can be expressed as follows:

$$F_{t,T} = S_t e^{(r_T - \delta)(T-t)}$$

With r_t being a risk free rate[23], $F_{t,T}$ the current price of a Futures expiring in $(T-t)$ days, and δ the convenience yield, as introduced in section 1.9 .

In a perfectly efficient and frictionless market, the pricing relationship so expressed should hold at every instant over a futures contract life. The above arguments suggest that the no-arbitrage condition would hold in the long-run but not necessarily in the short-term when applied to an imperfect market with frictions.

After taking natural logarithms the equation can be expressed as a cointegrating relation of the following form:

$$lnF_t = lnS_t + r_t(T-t) - \delta(T-t) + u_t$$

The test for cointegration is performed on the residual u_t . If the residulas follow a unit root process, then it is possible to accept the hypothesis that F_t and S_t are cointegrated. The unit root test is an Augmented Dickey-Fueller test over the residuals u_t. The test takes the same form as described in the previous section, with the only difference being that no deterministic trend is included (Koop, 2006:151).

The regression shows that the convenience yield coefficient is not significant, or is nul. This is consistent with the absence of any advantage in holding a Future instead of a Spot EUA within the same period.

As showed in the previous section, the ADF test must be performed on the t-statistics of the lagged variable. Here the ADF test on the residuals shows that the t-statistics (-7,983)

[23] Euribor 6-months

is more negative then the cut off values, therefore the hypothesis of unit root hypothesis is rejected and hypothsis of stationarity of the series is accepted. The results hence prove that there is cointegration between Spot and Futures.

Furthermore, Futures and Spot prices appear to be highely correlated, as S_t explains 99,9% of the future price level. The explanatory power of the risk free rate is relatively low, but still significant. The analysis so performed allows for employing Spot prices to model the dynamic of derivatives written on Futures.

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The analysis performed in chapter three have been necessary to highlight the main features which drive the price dynamics. The empirical analysis has showed that the historical distribution is not normal. Furthermore, the likelihood of extreme values is higher than a normal distribution. A fat tailed distribution is more appropriate to describe historical prices. The actual distribution seems to contain spikes in the returns. Volatility is not constant, as it shows clustering over time. The price dynamics process appears to be non-stationary, leading to a rejection of the idea of a mean reverting process, as it is common in commodities assets. A cointegration analysis allows the assumption that spot and futures prices follows the same process and are almost perfectly correlated.

The next step in the study will be to include the characteristics of the historical price in to a pricing model. In the following section, a stochastic model, able to capture the main feature empirically found, will be described and applied to pricing and hedging purposes.

Chapter 4 – Monte Carlo Simulation for the Pricing Model

In this section the model will be implement for pricing and hedging purposes. On the base of the results from the previous section the model will be presented and described. The parameters will be therefore be calibrated from historical and market data through a Loss Function technique. Consequently a Monte Carlo Simulation approach will be applied on the specified model. The performance of the simulation have been enhanced through variance reduction and acceleration technique, in order to reduce the drawbacks of Monte Carlo methodology showed in chapter two. The model will be tested on EUA options on December Futures in the ECX exchange, and commentaries on results will conclude this section.

4.1. Spot Price Model Specification

The results of the previous section have highlighted some characteristics of the historical series that has to be taken into consideration when choosing the valuation model. In the case of the Phase II EUA prices, the model has to be able to reflect the following features:

- Non normality of distribution
- Fat Tails
- Presence of Jumps
- Heteroskedasticity

The finding has lead to the definition of a stochastic process as follows:

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dW_t + J(S_t, t)dq_t$$

The showed equation is in a general form, under the real probability measure P, with S_t being the EUA spot price at time t, W_t a Wiener Process[24], $\mu(S_t, t)dt$ the drift term,

[24] A Wiener Process is a continuous-time stochastic process, having increments which are independent and identically distributed so that $\Delta W \sim N(0, \Delta t)$.

$\sigma(S_t, t)dW_t$ the diffusion coefficient, and $J(S_t, t)$ the jump amplitude component. The drift, diffusion and jump component are functions of the spot price S and time. dW_t , dq_t , and J are assumed to be independent. The Jump component is controlled by a Poisson process q_t with constant parameter, with $\Pr(dq_t = 1) = \lambda dt$ and $\Pr(dq_t = 0) = 1 - \lambda dt$.

Spot models are usually implemented because of their reliability on the description of the evolution of the price. They are versatile and relatively easy to incorporate characteristics of the historical series. However, one main drawback of the models is the difficulty in parameters estimation, given the number of parameters and the small number of observations in a non mature markets as the ETS. (Cartea, Figureoa 2005).

The model which is implemented, is the Heston (1993) model. In this model, the asset price and the asset volatility follow their own diffusion process. The choice relies on the need to incorporate volatility clustering in the stochastic equation. The Heston model, being a stochastic volatility model, allows for heteroskedasticity. Fat tails of the distribution are introduced by including the jump component. Given that the distribution appears to be almost symmetric from previous section analysis, the jumps are modeled to be symmetrical (negative and positive spikes have the same likelihood to happen). The diffusion process includes a drift term, but differently from common uses in commodities assets, no mean reversion component is used. The choice is consistent with the finding of non stationarity of the time series (Daskalakis et al, 2008). Once a shock happens, modeled through the jump component, the effect will have long-term on the level of the price. A correlation parameter links the increments in the drift and volatility terms.

The model assumes that the stock price S_t follows the diffusion equation (Heston, 1993:135):

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dz_{1,t}$$

Where μ is the drift parameter and $z_{1,t}$ a Wiener Process. The volatility instead follows the diffusion:

$$d\sqrt{v_t} = -\beta\sqrt{v_t}dt + \delta dz_{2,t}$$

With $z_{1,t}$ being a Wiener process with ρ correlation with $z_{1,t}$. Applying Ito's Lemma to the volatility equation, it is possible to simplify the process into:

$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t}dz_{2,t}$$

Where θ is the long run mean of variance, κ a mean reversion parameter and σ the volatility of volatility.

In order to price options, it is necessary to express the above equations in risk-neutral dynamics. It can be showed (Rouagh, 2007:137) that the risk neutral equations, with risk neutral parameters κ and θ ,take the forms:

$$dx_t = \left[r - \frac{1}{2}v_t\right]dt + \sqrt{v_t}dz_{1,t}^*$$

$$dv_t = \kappa[\theta - v_t]dt + \sqrt{v_t}dz_{1,t}^*$$

The model must be expressed in discrete time for simulation purposes. Discretization is a common approach, even if discretization of continuous-time stochastic differential equations introduce an estimation bias. However, bias for small sampling interval bias are negligible (Rouah, 2007:140; Broadie & Glasserman, 1998:182). Another risk of discretization is to obtain negative values for the variance: the choice to define the model with the natural logarithmic variance avoids this risk.

The final configuration of the processes that will allow simulation take the form:

$$lnS_{t+\Delta t} = lnS_t + \left(r - \frac{1}{2}v_t\right)\Delta t + \sqrt{v_t}\sqrt{\Delta t}\varepsilon_{S,t+1}$$

$$lnv_{t+\Delta t} = lnv_t + \frac{1}{v_t}\left(\kappa[\theta - v_t] - \frac{1}{2}\sigma\right)\Delta t + \sigma\frac{1}{\sqrt{v_t}}\sqrt{\Delta t}\varepsilon_{v,t+1}$$

$$\varepsilon_{v,t+1} = \rho\varepsilon_{S,t+1} + \sqrt{1-\rho^2}\varepsilon_{t+1}$$

The contribution of this paper is identify and implement a model which has not been applied before in the context of Emission Allowances Scheme. There are several advantages in adopting the model. Incorporating a stochastic volatility term structure for the volatility leads to a better representation of the volatility clustering. The correlation term allows to link the change in the volatility term to the drift component. Moreover a jump component is incorporated in order to reflect the fat tails of the historical data distribution, as it lets extreme values to appear. The parameter calibration technique also permits to combine historical and actual market data.

4.2. Parameter Calibration

Following a technique described by Bierbreuer et al (2007:3473), Cartea & Figureoa (2005:7), and Clewlow & Strickland (2000), a recursive filter is used to identify jumps in the sample distribution of daily log-returns. The filter works as an iteration which is repeated until no other jumps are found in the series. In the first run of the iteration the sample standard deviation is calculated; hence a limit value of three standard deviation is used to identify jumps: the returns exceeding the threshold are identified as jumps. Once identified, they are removed from the series and replaced by the median value[25]. The iteration loops for three times. The parameters for the spot model will be calibrated on the corrected series, expecting therefore to have a better fit on the filtered data[26]. Residuals are expected to be more symmetric, and show less kurtosis than in the original sample.

For the drift and volatility components, the parameters are obtained by minimizing the error between model and market prices. This method is consistent with historical returns and option prices (Rouah & Vainberg, 2007:275). For this particular model, the choice comes from the fact that it is not possible to obtain risk neutral parameters by using only

[25] No seasonality has been taken into consideration in this procedure, as seasonality itself is not included in the model. There are no studies currently showing EUAs price following seasonality patterns.

[26] The calculations for the parameter calibration are shown in the Excel file “Chapter 3 – Stochastic volatility and jumps parameters calibration”.

price returns. Option prices have to be used in the estimation instead of only spot price returns.

The procedure includes two steps. In the first, initial parameters are calculated by an Ordinary Least Squared Errors estimate (OLS) (Rouagh & Vainberg, 13). The technique involves the minimization of the sum of squared error (SSE) between observed data y_i and model prices $\hat{y}_i$:

$$SSE = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

Using the OLS parameters as starting values, the second step involves measuring the error between market prices of options and model prices, measured by a Loss Function (Rouagh & Vainberg, 2007:283). A Loss Function can be defined in several different configurations. Here a Root Mean Squared Error ($RMSE$) Loss Function is computed as the square root of the sample mean of squared estimation errors:

$$RMSE(\Theta) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e_i(\Theta)^2}$$

Where $e_i(\Theta) = C_i - C_i(\Theta)$ are N estimation errors, C_i are N market option prices, and $C_i(\Theta)$ are the model prices which depend on the set of paramaters Θ. The parameters of the model are the ones which minimize the $RMSE$ function.

Alternative specifications for a loss function could be adopted. Each configuration assigns different weights to options. The $RMSE$ specifically assigns more weight to in-the-money options.

The algorithm to find the minimum of the Loss Function is the Nelder-Mead Algorithm. It is a popular method to find roots of multivariate functions. (Rouagh & Vainberg, 2007:13). It converges very quickly regardless of the starting points used as inputs for the parameters[27].

4.3. Monte Carlo Simulation

Monte Carlo Simulation is inefficient on a computational level when applied in its basic form, therefore it is necessary to use control variates and deterministic sequences (or quasi-random sampling) in order to decrease the number of iteration needed and the standard error of the simulation (Haugh, 2007:356; Jackson, 2001:199; Clewlow & Strickland, 2001:87).

The value of an option can be computed as the risk neutral expectation of its discounted payoff[28]. (Clewlow & Strickland, 2001:87; Broadie & Glasserman, 1998:182). It is possible to estimate it by averaging a large number of discounted payoffs. Thus the risk neutral process for the state variables will be simulated from value in time zero to maturity T, and the value of the payoff of the instrument will be computed for this simulation. The payoff C will be discounted with the simulated short term interest rate:

$$C_{0,j} = \exp\left(-\int_0^T r_u du\right) C_{T,j}$$

If the interest rate is assumed to be constant, the above expression simplifies to:

$$C_{0,j} = \exp\left(-rT\right) C_{T,j}$$

The simulation process will be repeated M times, then calculating the average of all the outcomes:

$$\hat{C}_0 = \frac{1}{M}\sum_{i=1}^{M} C_{0,j}$$

[27] The code for the Nelder-Mead algorithm is included in the appendix.
[28] The risk free rate applied in this study is the 6 months Euribor rate.

4.4. Acceleration and Variance Reduction Methods

In order to get an accurate estimate for an option price, usually a large number of iteration is needed. However, it is possible to address this problem through the implementation of variance reduction techniques. They are very similar to techniques to hedge an option position. The methods rely on the fact that payoff of an hedged portfolio will have smaller variability than the correspondent unhedged portfolio.

The technique involves creating an hypothetical asset perfectly negatively correlated with the original asset. The antithetic variate is indeed the created asset. The main idea behind the antithetic variance reduction is to use pairs of options with same underlying price and volatility, and same random increment with opposite sign. The variance of the payoff is expected to be lower variability. (Clewlow & Strickland, 2001:96).

Just as an example, assume there are two options on asset S_1 and S_2 , which are perfectly negatively correlated. S_1 and S_2 follow the stochastic differential equations, with same r and volatility σ:

$$dS_{1,t} = rS_{1,t}dt + \sigma S_{1,t}dz_t$$

$$dS_{2,t} = rS_{2,t}dt - \sigma S_{2,t}dz_t$$

The equations express the payoff of an European call options. It is necessary only to take random sample with opposite signs ε_j and $-\varepsilon_j$ in the stochastic equation. This lead to the following simulated payoffs:

$$C_{T,j} = \max\left\{0, S\exp\left(rT + \sigma\sqrt{T}(\varepsilon_j)\right) - K\right\}$$

$$\bar{C}_{T,j} = \max\left\{0, S\exp\left(rT + \sigma\sqrt{T}(-\varepsilon_j)\right) - K\right\}$$

Subsequently the average of the two payoffs is taken as the payoff for that iteration of the simulation. The advantage of this technique is twofold: in this way a more accurate estimate of M pairs of $(C_{T,j}\ , \bar{C}_{T,j})$ is obtained rather than 2M of $C_{T,j}$, and at the same time this saves computational resources. Also the normally distributed ε are ensured to have zero mean, adding strength to the Monte Carlo simulation.

4.5. Random Numbers Generators

An important step in the simulation process is the generation of standard normal random variables. Almost all programming languages or spreadsheet provide an embedded uniform pseudo-random random number generator. The algorithm usually gives back a number between zero and an upper value, with equal probability. The number so generated is expected to appear as random when tested with statistical tests for randomness. However, some routines are more efficient than others.

Starting from a standard uniform random generator, there are many alternative techniques to convert the output into a standard normal random number. One of the most popular, due to its simplicity and execution time, is the Box-Muller transformation with polar rejection, that is an exact transformation of a pair of standard uniform random numbers into a pair if standard normal random numbers. (Benninga, 2008:760). Given two standard uniformly distributed random numbers between -1 and + 1, then if the sum of squares of these numbers is less than 1, the transformation is obtained through:

$$(x_1, x_2) = \left(rand_1 * \sqrt{\frac{-2\ln(S_1)}{S_1}}, rand_2 * \sqrt{\frac{-2\ln(S_1)}{S_1}} \right)$$

Where $S_1 = rand_1^2 + rand_2^2$.

This algorithm is very similar to the Box-Muller transformation in its basic form but avoids to calculate sin and cos, which slow down the process. It also allows for storing both the random numbers for computational efficiency.

The output of the random generator is a series of numbers which is indistinguishable from independent random variables uniformly distributed over [0,1]. But unfortunately, since they have been generated by a deterministic processor, random numbers are still not independent. However a valid random number generator has to appear as indipendent, and must have a long cycle before it starts to repeat itself. (Haugh, 2007:357).

4.6. Commentary on pricing results

Intra-Phases Futures prices are calculated through an equation with zero convenience yield. The output of the pricing methodology is estimated with an in-sample comparison with the observed Futures prices. The error from observed prices is measured by a Mean Squared Error term (MSE) computed as:

$$MSE = \frac{100}{N} \sum_{t=1}^{N} \left(\frac{F(T)_t^T - F(T)_t^A}{F(T)_t^A} \right)^2$$

Where N is the number of observations, $F(T)_t^T$ are the theoretical prices from the model, and $F(T)_t^A$ are the actual Futures prices. The period under analysis has been 22th February 2008 – 22th May 2009[29]. Based on the results from section 3.8 the theoretical Futures price has been computed as:

$$F_t = S_t e^{r(T-t)}$$

The MSE for December 2009 Futures for the in sample period time frame is 1,7011% . For comparison purposes, mean transaction costs for the period have been computed, as the fraction of cost per trade on Futures value[30]. The error of the model falls well below the

[29] The results are included in the Excel file "Chapter 3-4 Futures".
[30] ECX and Bluenext apply a fee of €2/lot on futures for low volumes.

average transaction cost of 11,2%. As a benchmark also a linear regression model has been estimated. The linear model is even superior to the exponential cost of carry, since the MSE 1,6019%. The reason of the better performance may be found in the low level of interest rates in the sample period. For higher levels in the interest rates, the exponential would fit the data better than a linear model.

The result leads to conclude that the cost of carry with zero convenience yield and storage cost model reflects correctly the dynamics of the spot price and evaluates accurately the term structure of Futures prices. The results leads to conclude that a cost of carry model with constant parameter is an adequate configuration for Intra-phases futures. The difference between Futures and spot price is just driven by the lost interest rate, and a convenience yield seems not to be present. The results are consistent with previous literature (Uhrig-Homburg & Wagner, 2007; Daskalakis et al, 2008).

In order to assess the empirical validity of the proposed model for option pricing, the results are evaluated through an in-sample and out-of-sample pricing using actual option market data from ECX and Bluenext[31]. The methodology is consistent with Eydeland & Wolyniec (2007:121), Pilipovic (2006:53), Daskalakis et al (2008:35), and Lucia & Schwartz (2001:31).

The Black model (1976) for Futures pricing is used as a benchmark for option pricing. The model gives a closed analytical solution for a call option[32] with strike price K and expiry date in T as follows:

$$c = e^{-rT}[FN(d_1) - FN(d_2)]$$

Where:

$$d_1 = \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

[31] Formulas and data are included in the Excel files "Chapter 4 – Monte Carlo call option".
[32] The analysis has been performed on call option. Similar results can be achieved on put options. They are omitted to avoid repetitions.

$d_2 = d_1 - \sigma\sqrt{T}$

The implied volatility provided by ECX for each traded option, and the Euribor rate have been used as input parameters for the drift and volatility term in the Black model.

The in-sample and out-sample analysis of the models is presented in Table 4.1 . Pricing performance has been evaluated for the aggregate sample and across three moneyness levels[33] on the basis of the Mean Root Squared Error (MRSE), defined in section 4.2.

The results suggest that intra-phase call options are more accurately priced by using the stochastic volatility with jump component model. Results from in-the-sample and out-of-sample evaluation bring to the same conclusions.

Out of sample				
	RMSE			
	Out of money	At the money	In the money	All
Augmented Heston Model	0,007	0,005	0,020	0,017
Black Model	0,030	0,073	0,089	0,069
In the sample				
	RMSE			
	Out of money	At the money	In the money	All
Augmented Heston Model	0,108	0,062	0,120	0,102
Black Model	0,082	0,124	0,091	0,121

Table 4.1 – Summary of pricing accuracy

For the out-sample analysis, the augmented Heston model is superior to the benchmark model for all moneyness levels. The RMSE in fact was 0,017 compared to the 0,069 of the Black model. The accuracy is improved by about 37%, 93% and 77% in terms of reduction of the RMSE for respectively out-of-the-money, at-the-money, and in-the-money options.

[33] Moneyness refers to the ratio K/S, strike price on spot price, to indicate if the option is in-the-money, at-the-money or out-of-the-money.

The value of the RMSE leads to appraise the model as a valid model for EUA options. In fact, since that the RMSE can be interpreted as the average error in percentage from market prices, the results imply the model produces an average error of 1,7% from observed options premiums.

However, the stochastic model tends to underperform for in-the-money options. Compared to the other levels of moneyness, RMSE for in-the-money options is four times higher, and it appears to be the main driver of the overall RMSE. This is in line with previous literature findings about in the money and out of the money options being more difficult to estimate (Bakhsi, 1997). Nevertheless the performance of the model for in-the-money is lower than out-of-the-money options.

Regarding the in sample analysis, the model provides a poorer fit. The results do not come as a surprise, since the model parameters have been calibrated to the front end option market prices. In other word, the model provides a close match to options traded the day after the sample time frame. The methodology gives more weight to actual market data than past values.

Even though the in-sample results are not as precise as for the out-of-sample period, the Heston augmented model provides better accuracy than the benchmark model. For all the sample, and for at-the-money options the RMSE is lower. On the other hand The Black model gives a better match for out-of-the-money and in-the-money levels.

Overall, the empirical evidence here presented suggests that it is worth accounting for jumps and for a stochastic volatility when pricing EUA options on futures. However, the model relies on six parameters that have to be calibrated and on full evaluation approach, compared to a parsimonious structure as in Black model, which also provides closed form solutions. Clearly the augmented Heston model is more computationally expensive.

4.7. Hedging Strategies with the Greeks

The "Greeks" are quantities representing the sensitivities of the option prices to changing in the underlying variables from which the price is dependent. The Greeks are a vital instruments in risk management and in portfolio management. In fact, the magnitude and sign of the sensitivities show how the option price will change after a change in one variable: accordingly they are used for dynamic hedging and portfolio rebalancing (Rouah & Vainberg, 2007; Clewlow & Strickland, 1998; Haugh, 2007).

Here the most commonly used five Greeks are presented. Referring to f as the price of the call or put option, the Greeks are defined as follows[34]:

- Delta - the sensitivity of the option in the price of change the underlying asset. $\Delta = \frac{\partial f}{\partial S}$.
- Gamma – the sensitivity of the delta to change in the price of the underlying asset. $\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 f}{\partial S^2}$.
- Vega[35] – the sensitivity of the option price to change in the volatility. $\upsilon = \frac{\partial f}{\partial \sigma}$.
- Rho – the sensitivity of the option to change in the risk-free rate. $\rho = \frac{\partial f}{\partial r}$.
- Theta – the sensitivity of the option to the time to maturity. $\Theta = -\frac{\partial f}{\partial T}$.

For some pricing models (e.g in the common Black-Scholes and the Black model) the Greeks formulas are available in closed forms. For more sophisticated models, however, closed forms do not exist or are difficult to achieve. Therefore it is necessary to compute an approximation (Haugh, 2007:85).

In the case of this paper, the value of the Greeks will be approximated by finite differences, by introducing a small perturbation in the value of the input variable and then calculating the change in the option price[36]. This method is generic, as it can be applied to every

[34] ∂ denotes the first derivative, therefore the change of the price of the option after the change in the value of the underlying variable.
[35] Vega is not a Greek letter. The Greek character *nu, υ,* is commonly used instead.
[36] The Greeks are computed in Excel files "Chapter 4 – Greeks".

model, and it can be also used for different approaches as Binomial or Trinomial Trees based Methods (Rouagh & Vainberg, 2007:217).

The approximations are accurate and are very close to closed form solutions values for small perturbations value. A further advantage of the method is that it is not necessary to implement a different algorithm, because it just relies on the Monte Carlo calibration with small perturbations in the initialized parameters of the model.

In a general form, the Greeks will be computed as follows:

$$\text{Greek} = \frac{\partial f}{\partial x} \approx \frac{f(x+dx)-f(x)}{dx}$$

Where x can alternatively be the spot price S (for Delta), the interest rate r (for Rho), or the volatility (for Vega), and time to maturity (with reversed sign, for Theta). dx denotes a small perturbation in x.

For Gamma, Γ, two values are needed to be computed:

$$\Gamma \approx \frac{\Delta_2 - \Delta_1}{dS} = \frac{f(S+2dS)-2f(S+dS)+f(S)}{(dS)^2}$$

Where:

$$\Delta_1 \approx \frac{f(S+dS)-f(S)}{dS}$$

$$\Delta_2 \approx \frac{f(S+2dS)-f(S+dS)}{dS}$$

Regarding the values of the perturbations, it is necessary to consider that the model is specified with daily quantities in the parameters. Namely, for Theta, the perturbation has to take a discrete value. For Vega, the volatility value σ_{t+1}^2 has to be perturbed by a daily volatility and then squared.

It is however not possible to evaluate the performance of the hedging techniques. In fact it is not possible to have prices of options for all the ranges of prices and maturities in order to validate the hedging strategies. Therefore strategies using Greeks values will be only described. Here Greeks are derived from closed form solution of the Heston (1993) model[37].

The most common uses of the Greeks are to create and maintain riskless hedges for an option portfolio or position, to determine the amount of options needed to be purchased to hedge a certain exposure, to quantify the risk and size of possible losses for an un-hedged portfolio, and to conduct sensitivity analysis on the value of an option position (Joskow, 2007; Haugh, 2007).

Delta measure the option premium change for an increase in the underlying. For calls an increase in the underlying asset creates more value, while for puts it decreases the premium. Delta for calls varies from 0 to 1, with 0 meaning the option is deeply out of the money and 1 deeply in the money. For puts the range is 0 to -1.

As an example for an application of the delta measure, a portfolio of a call and underlying over two following days is compared. On 7th may 2009 in the ECX a call option with strike price € 12 had a premium of € 3,73 when the spot price was € 14,6. If an operator was long on the call, and was willing to hedge the risk of price change, he could have sold an amount of underlying equal to the delta. The model provides a delta of -0,7608 for that particular option. Therefore, assuming the operator had an overall position of 100.000 call contracts, he could also have sold -0,7608 * €14,6 * 100.000 of underlying. The next day the spot prices moved to 14,85, and the premium of that option changed into € 3,93. The effect on the portfolio value is however minimized, as the increase in the premium (€ 0,20) is offset by the short spot position (- € 0,19). The mismatch is due to the fact that delta is calculated with an approximation method, and that not only the spot price but also other parameters have changed in the same time frame.

A delta hedging strategies involves continuous rebalancing of the portfolio over a period of time. In fact, delta is not a static value, but changes with time. An operator has to sell and

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[37] See Appendix D for the derivation of the closed form solutions.

buy at repeated intervals in order to have a significant risk reduction effect on the portfolio (Benninga, 2008:572). However Delta is a linear approximation of the change in value of the premium. Gamma hedging correct the errors due to the convexity of the portfolio respect to the underlying. repeatedly rebalanced. Such a constructed portfolio will be delta and gamma neutral. Due to lack of market data on options premiums, it is not possible to show empirically the results over a period of time.

Vega measures instead the relationship between a change in volatility and the change in premium. It is positive for long positions and negative for short. The relationship is linear. Vega value is relevant for hedging because it permits to reduce exposure to change in to volatility. Volatility parameter is one of the most important and also difficult to estimate, therefore imperfections in its calibration might lead to errors in the final output. A Vega hedging strategy can be helpful to make a portfolio insensitive to the parameter. (Wilmott, 2006:874).

Vega is an important measure for the sensibility of a position or a portfolio to change of the variability in the underling. It is taken into consideration by volatility traders, who often associate Vega with changes in the implicit volatility. It is useful to assess how the value of a un-directional strategy, as for example a straddle or strangle[38], is changing respect to change in volatility (De Weert, 2006:39). However the lack of data availability does not allow to show the use of the Vega and Greeks values in practice.

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In this Chapter a stochastic volatility model (Heston, 1993) augmented with jumps has been described and implemented. The specification of the model lead to the incorporation of the main characteristics of the spot dynamics highlighted in chapter three: volatility clustering, non normal distribution, fat tails and spikes. Parameters have been calibrated

[38] Buying or selling a call and a put with same or very close strike price.

on historical and actual market data. The model has been implemented in a Monte Carlo framework. Acceleration techniques have been incorporated to speed the random number generation process. The results have been showed with an in-sample and out-of-sample comparison. The popular Black (1976) model has been used as a benchmark. An application of the model on hedging strategies using Greeks value has also been presented.

Conclusion

The purpose of the paper has been to identify and implement a model for pricing derivative instruments on Emission Unit Allowances in the context of the European Union Emission Trading Scheme. A specification with stochastic volatility and a jump component has been recognized as a valid model for options on Futures. On the other hand, a cost-of-carry model with zero convenience yield correctly reflects Futures prices. The markets under analysis were the European Climate Exchange (ECX) for Futures and Vanilla Options, and the Bluenext for Spot EUA.

The need for such models comes from the establishment of the ETS, which has bright to the creation of carbon assets, like EUAs. The framework that has lead to the establishment of the scheme has been explained. The Kyoto Protocol was ratified by the Annex I countries in order to address the rising threat of global climate change. Targets for emissions reduction have been assigned to each party. At the same time, flexibility mechanisms have been proposed to help in reaching the emission reduction compliances in a cost-efficient manner. The three mechanisms, Joint Implementation, Clean Development Mechanism, and the International Emissions Trading have been introduced. The Emission Trading Scheme is the application at European level of these instruments, and it allows parties with emission reduction targets to trade CO_2 allowances with each other. The market has been described with particular focus on its organization, the participants, the carbon assets and financial instruments, and the currently active exchanges.

The objective of this paper has been to identify and implement a valid model for pricing and hedging of Emission Unit Allowances. In order to do so, it has been necessary to highlight the main characteristics of the market that might affect the pricing methodology. Therefore an empirical analysis has been carried out on the spot price dynamic in order to recognize the main features of the historical price series.

The statistical and econometrical analysis has showed that the historical distribution diverge significantly from standard normal distribution. In fact, the time series has fat tails, and volatility clustering. Furthermore, it appears to be non stationary. It seems necessary to define a model which allows for heteroskedasticity, or changing volatility, and presence of extreme values, with a jump component.

The selected model was the Heston (1993) model, which is a stochastic volatility model. Furthermore a jump component has been added to ensure to capture the fat tails feature of the historical distribution.

Assumptions behind the pricing methodology have been made clear, in particular, regarding the banking limitations and the use of the cost and carry relationship with zero convenience yield to price options written on EUA Futures. The conditions appear to be valid for Intra-phase instruments. Spot and Futures prices show cointegration. The market in Phase II is liquid and efficient, and no arbitrage opportunity is present. A risk neutral valuation can be correctly applied.

The pricing and hedging has been carried out through a Monte Carlo Simulation. The method is flexible and it is applicable for theoretically any spot price model. Even though it is a computationally expensive technique, some acceleration methods have been employed in order to improve the iterative simulation. It is also applicable to hedging strategies by adopting approximate differences.

An evaluation using actual market data has shown the validity of the model. It is able to correctly reflect the options premium currently available in the market. The validity of the model has been benchmarked against the Black (1976) closed form solutions, with positive results. Some hedging strategies to construct portfolios with low risk have been proposed. Similarly, for Futures pricing, the specification with cost of carry and no convenience yield correctly explains the Futures prices from spot prices.

The paper has some important implication on risk management and asset management practices because companies that participate in the market need to deploy their trading practices. In addition to that, the carbon investors community is growing. The knowledge about the market instrument will help to achieve the main goal of the ETS, being to achieve emission compliance at the least possible cost. Furthermore, the ability of investors and operators to trade and hedge their position is clearly a main element for sustain an efficient and liquid market.

Future research proposals

The thesis focused on some specific instrument. However the market offers a wider range of assets, which may be investigated. Furthermore, an analysis on the correlation of the different instrument and on the spreads between them appears to be relevant. More sophisticated instrument, such as spread trading or calendar spreads, are going to be introduced soon.

The current literature is not complete and consistent, as it belongs to phase I instruments. Furthermore, their conclusions might be outdated or no longer applicable. New studies based on phase II and inter-phases instruments are necessary.

The Emission Trading Scheme can be studied also on a cross market perspective. It should be analysed the relationship between energy markets, mainly gas and electricity, and their reciprocal effects on carbon assets. Similarly, a low correlation of carbon assets with stocks can be object of analysis.

The model presented here can be improved in several ways. Better random generators could be adopted, or a closed form solution may be found and applied in order to avoid the computationally expensive Monte Carlo Simulation. The jump component can be modeled differently. Furthermore, even if the model specification appears to be valid for pricing EUA, a more parsimonious configuration could be implemented allowing for faster outcomes.

Regarding EUA prices, other models could be valid for pricing purposes. Garch-type models or regime switching model should be able to incorporate the high level of volatility and the erratic behaviour of the price development. Furthermore, as more data will be available, models with higher number and more stable parameters can be applied.

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APPENDIX A – List Annex I parties to the UNFCCC

Australia
Austria
Belarus
Belgium
Bulgaria
Canada
Croatia
Czech Republic
Denmark
Estonia
Finland
France
Germany
Greece
Hungary
Lithuania
Liechtenstein
Latvia
Japan
Italy
Ireland
Iceland
Luxemburg
Monaco
Netherlands
New Zealand
Norway
Poland
Portugal
Romania
Russian Federation
Slovakia
Slovenia
Spain
Sweden
Switzerland
Turkey
Ukraine
United Kingdom of Great Britain
United States of America

APPENDIX B – VBA Code for Monte Carlo Simulation for call option

```
Function  HestonMC(kappa,  theta,  lambda,  rho,  sigmav,  daynum,_
                   startS, r, startv, K, jmpprob, jmpamp, ITER)

Dim allS() As Double, Stock() As Double, allS2() As Double, T As_
Double, L As Double, x1 As Double, x2 As Double, jmp As Double,_
jmpprob as double, jmpamp as double

simPath = 0

ReDim allS(daynum) As Double, Stock(ITER) As Double,_
allS2(daynum) As Double

deltat = (1 / 365)

For itcount = 1 To ITER

     lnSt = Log(startS)
     lnSt2 = Log(startS)
     lnvt = Log(startv)

     curv = startv
     curS = starts
     curs2 = startS

     For daycnt = 1 To daynum

     Dim rand1, rand2, S1, S2, e1, e2 As Double

     start:    'Box-Muller Transformation

     rand1 = 2 * Rnd() - 1
     rand2 = 2 * Rnd() - 1
     S1 = rand1 ^ 2 + rand2 ^ 2

     If S1 > 1 Then GoTo start
     S2 = Sqr(-2 * Log(S1) / S1)
     e1 = rand1 * S2
     e2 = rand2 * S2


     ev = rho * e1 + Sqr(1 - rho ^ 2) * e2

     lnSt = lnSt + (r - 0.5 * curv) * deltat + Sqr(curv) *_
            Sqr(deltat) * e1

     lnSt2 = lnSt2 + (r - 0.5 * curv) * deltat + Sqr(curv)_ *
             Sqr(deltat) * (-e1)
```

```
            ' Antithetic variates, e1 and -e1

            curS = Exp(lnSt)
            curs2 = Exp(lnSt2)


    lnvt = lnvt + (kappa * (theta - curv) - lambda * curv - 0.5 *_
           sigmav) * deltat + sigmav * (1 / Sqr(curv)) *_
           Sqr(deltat) * ev

    curv = Exp(lnvt)

    allS(daycnt) = curS
    allS2(daycnt) = curs2

    jmp = Rnd()


    If jmp < jumpprob Then

               If e1 < 0.5 Then   'the sign of e1 is used to
                                  'determine whether the jump will
                                  'be positive or negative

               curS = curS + curS * jmpamp
               curs2 = curs2 + curS * jmpamp

               Else
               curS = curS + curS * (-jmpamp)
               curs2 = curs2 + curS * (-jmpamp)

               End If

    Else
           curS = curS
           curs2 = curs2
    End If

    Next daycnt

simPath = simPath + Exp((-daynum / 365) * r) *_
          Application.Max(allS(daynum) - K, 0) + Exp((-daynum /_
          365) * r) * Application.Max(allS2(daynum) - K, 0)

Next itcount

HestonMC = simPath / ITER

End Function
```

APPENDIX C – Nelder-Mead Algorithm for finding minimum of a function

The Nelder-Mead is an algorithm to find minimum of multivariate functions. The VBA function requires the function for which the minimum has to be found, and a set of starting parameters. The algorithm gives as an output the parameters which minimize the specified function. The formula is an iterative process that retains the best values of the inputs at every repetition according to five rules: reflection, expansion, outside contraction, inside contraction, shrink step rules. The Nelder-Mead code calls for a function, BubSort, to sort the values in increasing order. The codes for both functions are provided here. (Rouah & Vainberg, 2007).

```
Function NelderMead(fname As String, startParams)

Dim resMatrix() As Double

Dim x1() As Double, xn() As Double, xw() As Double, xbar() As_
Double, xr() As Double, xe() As Double, xc() As Double, xcc() As_
Double

Dim funRes() As Double, passParams() As Double

MAXFUN = 1000
TOL = 0.0000000001
rho = 1
Xi = 2
gam = 0.5
sigma = 0.5

paramnum = Application.Count(startParams)

ReDim resmat(paramnum + 1, paramnum + 1) As Double
ReDim x1(paramnum) As Double, xn(paramnum) As Double, _
xw(paramnum) As Double, xbar(paramnum) As Double, _
xr(paramnum) As Double, xe(paramnum) As Double, _
xc(paramnum) As Double, xcc(paramnum) As Double

ReDim funRes(paramnum + 1) As Double, passParams(paramnum)

For i = 1 To paramnum
    resmat(1, i + 1) = startParams(i)
Next i

resmat(1, 1) = Run(fname, startParams)

    For j = 1 To paramnum
```

```
    For i = 1 To paramnum
        If (i = j) Then
            If (startParams(i) = 0) Then
                resmat(j + 1, i + 1) = 0.05
            Else
                resmat(j + 1, i + 1) = startParams(i) * 1.05
            End If
        Else
            resmat(j + 1, i + 1) = startParams(i)
        End If
        passParams(i) = resmat(j + 1, i + 1)
    Next i

    resmat(j + 1, 1) = Run(fname, passParams)

Next j

For lnum = 1 To MAXFUN

    resmat = BubSortRows(resmat)
    If (Abs(resmat(1, 1) - resmat(paramnum + 1, 1)) < TOL)Then
        Exit For
    End If

    f1 = resmat(1, 1)

    For i = 1 To paramnum
        x1(i) = resmat(1, i + 1)
    Next i

    fn = resmat(paramnum, 1)

    For i = 1 To paramnum
        xn(i) = resmat(paramnum, i + 1)
    Next i

    fw = resmat(paramnum + 1, 1)

    For i = 1 To paramnum
        xw(i) = resmat(paramnum + 1, i + 1)
    Next i

    For i = 1 To paramnum
        xbar(i) = 0
        For j = 1 To paramnum
            xbar(i) = xbar(i) + resmat(j, i + 1)
        Next j
        xbar(i) = xbar(i) / paramnum
    Next i

    For i = 1 To paramnum
```

```
        xr(i) = xbar(i) + rho * (xbar(i) - xw(i))
    Next i

    fr = Run(fname, xr)

    shrink = 0
    If ((fr >= f1) And (fr < fn)) Then
        newpoint = xr
        newf = fr
    ElseIf (fr < f1) Then

        For i = 1 To paramnum
            xe(i) = xbar(i) + Xi * (xr(i) - xbar(i))
        Next i

        fe = Run(fname, xe)

        If (fe < fr) Then
            newpoint = xe
            newf = fe
        Else
            newpoint = xr
            newf = fr
        End If

    ElseIf (fr >= fn) Then

        If ((fr >= fn) And (fr < fw)) Then

            For i = 1 To paramnum
                xc(i) = xbar(i) + gam * (xr(i) - xbar(i))
            Next i

            fc = Run(fname, xc)

            If (fc <= fr) Then
                newpoint = xc
                newf = fc
            Else
                shrink = 1
            End If

        Else
            For i = 1 To paramnum
                xcc(i) = xbar(i) - gam * (xbar(i) - xw(i))
            Next i

            fcc = Run(fname, xcc)

            If (fcc < fw) Then
                newpoint = xcc
```

```
                newf = fcc
            Else
                shrink = 1
            End If
        End If

    End If
    If (shrink = 1) Then

        For scnt = 2 To paramnum + 1

            For i = 1 To paramnum
                resmat(scnt, i + 1) = x1(i) + sigma *_
                (resmat(scnt, i + 1) - x1(1))

               passParams(i) = resmat(scnt, i + 1)
            Next i

            resmat(scnt, 1) = Run(fname, passParams)

        Next scnt

    Else

        For i = 1 To paramnum
            resmat(paramnum + 1, i + 1) = newpoint(i)
        Next i

        resmat(paramnum + 1, 1) = newf

    End If

Next lnum

If (lnum = MAXFUN + 1) Then

    MsgBox "Maximum Iteration (" & MAXFUN & ") exceeded"

End If

resmat = BubSortRows(resmat)

For i = 1 To paramnum + 1
    funRes(i) = resmat(1, i)
Next i

funRes(1) = funRes(1)

NelderMead = Application.Transpose(funRes)

End Function
```

```
Function BubSortRows(passVec)

    Dim tmpVec() As Double, temp() As Double

    uVec = passVec
    rownum = UBound(uVec, 1)
    colnum = UBound(uVec, 2)

    ReDim tmpVec(rownum, colnum) As Double
    ReDim temp(colnum) As Double

    For i = rownum - 1 To 1 Step -1

        For j = 1 To i

            If (uVec(j, 1) > uVec(j + 1, 1)) Then

                For K = 1 To colnum
                    temp(K) = uVec(j + 1, K)
                    uVec(j + 1, K) = uVec(j, K)
                    uVec(j, K) = temp(K)
                Next K

            End If

        Next j
    Next i

    BubSortRows = uVec

End Function
```

APPENDIX D – Heston Closed Form

The Heston (1993) model can be expressed as follows (Rouagh & Vainberg, 2007:138):

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dz_{1,t}$$

$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t} dz_{2,t}$$

Where θ is the long run mean of variance, κ a mean reversion parameter and σ the volatility of volatility.

The call price in t of an option with maturity $(T - t)$ takes the following form:

$$call(S, v, t) = S_t P_1 - KP(t, T)P_2$$

Where S_t is the spot price of the asset, K the strike price, and $P(t, T)$ a discount factor. The discount factor can be computed as $e^{-r(T-t)}$.

The value of a put is given by the put-call parity:

$$put(S, v, t) = call(S, v, t) + KP(t, T) - S_t$$

The quantities P_1 and P_2 are the probabilities that the option expires in the money. They are conditional to the log of the asset price $x_t = ln[S_t] = x$, the volatility $v_t = v$ at each time t.

The risk neutral dynamics can be formulated in terms of the risk neutral parameters θ and κ is:

$$dx_t = \left[r - \frac{1}{2}v_t\right]dt + \sqrt{v_t} dz_{1,t}^*$$

$$dv_t = \kappa[\theta - v_t]dt + \sqrt{v_t} dz_{1,t}^*$$

The probabilities can be seen as risk-adjusted or risk neutral probabilities, taking the form:

$$P_j = \Pr\,(x_T \geq \ln\,(k)|x_t = x, v_t = v$$

With $j = 1,2$. The probabilities P_j can be computed as the inverted characteristic functions f_j defined below (Heston, 1993:340).

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi \ln(K)} f_j}{i\phi}\right] d\phi$$

Where:

$f_i = \exp\,(C_j + D_j v + i\phi x)$ with $v = v_t$

$$C_j = r\phi i(T-t) + \frac{\kappa\theta}{\sigma^2}\left\{\left(b_j - \rho\sigma\phi i + d_j\right)(T-t) - 2ln\left[\frac{1 - g_j e^{d_j(T-t)}}{1 - g_j}\right]\right\}$$

$$D_j = \frac{b_j - \rho\sigma\phi i + d_j}{\sigma^2}\left[\frac{1 - e^{d_j(T-t)}}{1 - g_j e^{d_j(T-t)}}\right]$$

$$g_j = \frac{b_j - \rho\sigma\phi i + d_j}{b_j - \rho\sigma\phi i - d_j}$$

$$d_j = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2\mu_j\phi i - \phi^2)}$$

$i = \sqrt{-1}$

$\mu_1 = 1/2$

$\mu_2 = -1/2$

$b_1 = \kappa + \lambda - \rho\sigma$

$b_2 = \kappa + \lambda$

$\lambda =$ price of volatility risk, or the differential between the return an asset gives and the risk free rate, normalized by the asset's volatility (Pilipovic, 2007:25).

The solution of the probabilities terms P_j are closed form solutions, but require numerical integration and operations on complex numbers (Rouagh & Vainberg, 2007:139).