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Private Equity

A Value Adding Investment for a Private Investor?

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Executive Summary

Studies of private equity performance often face a major problem because of lack of reliable market data and liquidity. This paper addresses these issues by examining listed private equities (LPE) as a proxy for private equity. This allows utilization of standardized performance measures to investigate the performance of this asset class. Hypothesis testing is the chosen tool to answer the research questions. The hypotheses are tested by three performance measures in terms of risk and return. The sample period ranges from 1998 to 2011 and is consequently divided into several sub-periods. Moreover, this paper is conducted in two research parts.

Research Part I compares performance of LPE to the global stock market. An international sample of 108 vehicles is identified. After imposing liquidity constraints, a liquid sample of 77 vehicles are used to construct two alternative indices. Based on the data sample and selected performance measures, 67 percent of occasions support the hypothesis that LPE outperforms the global stock market, in terms of risk and return. Other findings are that larger vehicles do not outperform smaller in this market segment and better performing funds did not seem to consistently outperform other funds. LPE indices tend to underperform in bust periods but outperform in all other periods of the sample.

Research Part II addresses LPE capabilities in a mixed-asset portfolio. Portfolio capabilities are addressed by comparing an opportunity set of an Initial portfolio consisting of; stocks, bonds and gold, to identical portfolio which is extended with the addition of LPE. The performance measures conclude that 60 percent of occasions support the hypothesis that LPE is able to improve portfolio performance. Moreover, an equally-weighted portfolio and a tangent portfolio support that the LPE portfolio provides additional benefits on top of the Initial portfolio while a minimum-variance portfolio did not.

Main conclusions are that, based on the data sample and selected performance measures used in this research, private equity outperforms the global stock market in general and improves a portfolio performance in particular.

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1 Introduction

Alternative investments offer different performance characteristics than conventional asset classes, e.g. public equity and bonds. However, they are rarely publicly traded and thus relatively illiquid which makes them uncommon in the scope of the private investor. Private equity investing provides several advantages over traditional investments and, as a result, has become a popular alternative investment in the recent past.

In terms of the private equity market, this alternative asset class has experienced rapid growth since its initiation around 25 years ago with global funds under management totaling USD 2.5Tn at the end of 2009 (TheCityUK, 2010). Consequently, it has received considerable attention from financial media and within the academic research field. Investors attraction to this asset class originates primarily from a widely held belief of superior past performance; Cochrane (2005), Ljungqvist and Richardson (2003) and Lossen (2006), are among studies that corroborate these beliefs. This success is believed to be generated by top-level managers pursuing value-adding investment strategies and superior governance structures (Wright et al., 2007). In addition, it is often assumed that private equity investments have relatively low correlation to the public equity market (Ibbotson Associates (2007), and Hatch & Wainwright (2003)). Heterogeneous (i.e. low correlation) asset classes gain attention from investors that seek diversification of their portfolios. Therefore, alternative investments are often considered attractive component with ability to generate diversification effects.

In line with this rapid growth, demand for accurate information on market prices for these funds has augmented. Similarly, there is increased need for thorough empirical investigation on corresponding risk and return characteristics. Since private equity is by definition a private vehicle, their assets are usually opaque to investors and challenges emerge from the low accessibility and little transparency of the data used for performance measures. As a consequence, previous empirical results vary considerably. This is discussed further in the following literature review. The main reason for such mismatch is the fact that evaluations in traditional studies are based on book values, not market values (Zimmermann et al. 2004).

The lack of widely accepted benchmarks of private equity creates challenges for investors to understand corresponding risk and return characteristics. The few available benchmarks all face the standard problem of how to measure the true performance of private equity. However, an alternative way of gaining access to private equity has recently emerged. It is accessible by all investors and makes it possible to apply traditional evaluation measurements

(Ibbotson Associates, 2007). Consequently, this research analyzes the risk and return of listed private equity (LPE) vehicles. LPEs now constitute an attractive and rapidly growing market segment within the overall private equity market. LPE has established itself as an adequate proxy for traditional private equity as the underlying business structure is the same; the fund itself is only listed on publicly traded stock exchanges. The advantage of this market segment is the readily available market prices and other publicly available information, regulated by authorities, which enables straightforward and more reliable performance measures.

Investors in the private equity market are typically institutional, professional and wealthy investors due to the need of large capital commitments[1]. Such investors have large pool of capital and can handle the illiquid nature of this alternative asset class. Consequently, private investors generally do not have the possibility to invest directly in private equity. This fact evoked our interest of analyzing this asset class from a different perspective, namely a private investor's perspective.

Accordingly, there are two rationales for utilizing LPEs as a data source in this research. Firstly, due to several problems of unreliable data and unavailable market prices for traditional private equity funds. Secondly, due to the practical purpose from the private investor's perspective.

[1] A minimum amount of investment usually exceeds USD 1M.

2 Literature Review

There are mixed views on the performance of private equity compared to general equities in the existing literature. Due to the many challenges facing this asset class, appropriate evaluation methods are numerous and results vary accordingly. The findings range from considerable outperformance to substantial underperformance, compared to general equities. There are several reasons for this misinterpretation and they are discussed in this chapter and analyzed further when the fundamentals of private equity are scrutinized in Chapter 5.

In this paper, traditional private equity refers to funds that are not listed on stock exchanges. A simplified overview of the different types of studies on private equity is presented in Table 2.1. The chapter begins with an overview on the studies of traditional private equity whereas the latter part of this chapter includes coverage on the scarce literature on listed private equity (LPE), which is used for the empirical research of this thesis.

Table 2.1: Three broad sources of information for evaluating private equity performance.

	Type of Return	Description	Data Used
Traditional Private Equity	Returns of Underlying Companies	• Gross returns to Limited Partners • Includes management fees and carried interest	Directly observed valuation events, such as: • New financing rounds • IPOs • Acquisitions • Liquidation
	Returns to Limited Partners	• Net returns to Limited Partners • Includes General Partners estimates of Remaining Value	IRR, PI, PME calculated from: • Cash flows • Changes in Net Asset Values (NAV)
Listed Private Equity	Returns of Publicly Traded Securities	• Market prices on listed private companies whose business is private equity investing	

Source: Adapted from Conroy and Harris (2007)

2.1 Traditional Private Equity studies

In traditional private equity, outside investors such as pension and endowments funds are called Limited Partners (LPs). These investors commit capital to private equity funds, which are managed and controlled by General Partners (GPs). GPs seek out investments, which can either be specialized as venture capital (VC) or buyout (BO) investments depending on the strategy type of the fund. After identifying an investment opportunity, the GP "calls" required

capital from its LPs[2]. These investments make up the private equity fund's portfolio of companies. Limited partners receive a return on their investment that is net of management fees. Yearly management fees are typically 2 percent and GPs receive additionally carried interests, which typically are 20 percent of profits. (Conroy and Harris, 2007).

This ownership and management structure is the most common arrangement for traditional private equity. Vast majority of studies on private equity performance analyze the funds by this structure where researches are based on cash flows, changes in net asset values and direct observed valuations. Classical performance measures for these studies include e.g. internal rate of return (IRR), public market equivalent (PME) and profitability Index (PI). Data used by these studies are usually gathered from third-party data providers such as Thomson Venture Economics, Center of Private Equity Research (Cepres) and Private Equity Intelligence Ltd. (Preqin). These databases collect data over widespread of sources for both general and limited partners[3].

Existing literature on the performance of traditional private equity can be divided into two sets of studies. The first and most widespread set of studies analyze gross-of-fees performance of GPs individual investments. The second set analyzes the cash flows from the private equity funds to LPs, which includes all relevant fee payments. Therefore, they measure the net performance of investing in private equity funds.

2.1.1 Performance of GPs Investments: Gross-of-fees studies

The most comprehensive set of studies on individual projects are those by Woodward and Hall (2003), Hwang, Quigley and Woodward (2005), Cochrane (2005).

Woodward and Hall (2003) and Hwang, Quigley and Woodward (2005) calculate venture capital indices from different sources and calculate the correlation between the index and public stock-market indices. They use data from valuation events, such as IPOs, acquisitions or liquidation. The value weighted continuously rebalanced index of Woodward and Hall (2003) generates excess return of 71 monthly basis points compared to NASDAQ, which correspond to geometric abnormal returns of 8.5 percent per year. Standard deviation of their study is 68 basis points which correspond to 4.9 percent per year. With more comprehensive datasets, the index calculated by Hwang, Quigley and Woodward (2005) conversely reported similar returns as S&P 500 and a relatively low beta of 0.6.

[2] Overview of private equity structures, types, and strategies are covered in more detail in Chapter 5.
[3] Access to these databases often includes a costly subscription.

With similar observation but different approach, Cochrane (2005) assumes that the change in company's valuation follows logarithmic capital asset pricing model (CAPM). Using maximum likelihood approach to correct for selection bias, alpha and beta of log-CAPM are estimated for the time period 1987-2000. He reports an average adjusted arithmetic return of 59 percent with standard deviation of 107 percent per year. Systematic risk (CAPM beta) is 1.9 by using S&P 500 as market basis, and corresponding alpha is reported 32 percent per year. Logarithmic returns are however 15 percent with standard deviation of 89 percent per year. On the basis of such results, Cochrane observes that 'these puzzles' are not exceptional, due to the nature of venture capitals investments. He furthermore points out that small NASDAQ stocks have similarly large returns and volatility during the same period.

Phalippou and Gottschalg (2009) regard the main drawback of these studies that they only observe successful investments, thus excluding non-surviving vehicles.

2.1.2 Fund-level Studies: Net-of-fees studies

Second set of traditional private equity studies investigate on a fund level rather than individual investments the funds make. They examine the cash flow stream to investors, which includes fee payments, carried interests, and other fees that may follow. Cash flows are more likely to reflect both successful and unsuccessful investments, in contrast to observed valuations, thus they should highly reduce the selection bias.

Jones and Rhodes-Kropf (2003) argue that by holding an undiversified portfolio of illiquid assets, funds should be compensated for their total risk not just idiosyncratic components. They test principal-agent problems, in large sample of 1,245 funds, and state that the problem arises with idiosyncratic risk. They report positive but not significant alpha using assumptions that quarterly residual values reflect lagged market values and that returns follow the Fama-French three-factor model. They find beta of all funds to be 1.05, thus not much different to the benchmark. However, the difference between betas of venture capitals and buyouts varies considerably, calculating 1.80 and 0.65 respectively. This they state is consistent to the conventional wisdom that buyouts typically invest in companies with steady cash flows, contrary to venture capitals.

Ljungqvist and Richardson (2003) analyze the behavior of GPs investments and evaluate determinants of capital distribution. Their data sample is relatively small (73 international funds) but it is very precise in terms of unlikelihood of missing cash flows and in addition each of them can be traced to an investment or management fee payment. They document

average IRRs of 19.8 percent for equally weighted fund returns and 18.1 percent when returns are weighted by capital committed, net of fees. Corresponding volatility of the IRRs' is around 22 percent. The results correspond to excess returns of 5.7 percent and 8.1 percent compared to S&P 500 index over the same period. In addition, they assess the funds systematic risk by assigning each of the portfolio companies with one of 48 industry betas provided by Fama and French (1997) and calculate value-weighted beta for each fund. Average systematic risk is reported to be 1.09 in all funds.

In a sample of 746 funds, Kaplan and Schoar (2005) measure performance by IRR and PME, which is used to compare private equity investments to public market benchmarks. They focus on persistence of the fund's performance and document that follow-up funds outperform on average relative to other funds, if previous fund initiated by the GP did outperform. Their conclusions are that investment in private equity funds slightly outperforms S&P 500, net of fees. They report average IRR weighted by committed capital of 18 percent with standard deviation of 26 percent and equally weighted IRR of 17 percent with standard deviation of 31 percent. If the cash flows are replicated using returns of S&P 500 during the same period, PME is 0.96 when funds are equally weighted and 1.05 when they are value weighted[4].

Lossen (2006) analyzes the impact of internal diversification within a private equity's financial stages, industries and countries. He reports average IRR of 50.2 percent gross of fees and standard deviation of 31.7 percent. Lossen furthermore uses methodology from Kaplan and Schoar (2005) and calculates public market equivalent (PME). He reports that if the cash flows are replicated using returns of MSCI World Index, during the years 1979 to 2005, this PME is 3.075, which indicates superior return of private equity investments compared to public stock market.

In a more recent study, Phalippou and Gottschalg (2009) argue that a large part of private equity performance reported in industry associations and prior research is driven by inflated accounting values of ongoing investments and bias sample toward better performing funds. They demonstrate how using average IRRs biases performance upwards and thus use alternative measure, PI. After correcting for sample bias and overstated accounting values, they find that average gross-of-fees performance outperforms S&P 500 by 3.8 percent but

[4] PME takes the timing of all cash inflows and outflows into account and can be seen as buying shares of a public market index (when capital is called by a PE fund) and selling shares (when distributions are made). Cash flows are thus perfectly replicated by an investment in the benchmark. Public market return equals 1.00.

net-of-fees underperforms by -3 percent. After adjusting for risk they find the underperformance rises, to -6 percent per year[5]. This indicates the large influence that fees have on actual LPs performances.

These studies of traditional private equity performance often show high returns of private equity investments, but risk is usually relatively high as well. Performance studies often suffer from methodology challenges such as selection biases and difficulties related to returns derived from cash flows and net asset values.

2.1.3 Mixed Asset Portfolio Studies

There are two studies on traditional private equity which enlightened us of an alternative aspect on private equity performance; specifically, they assess private equity performance in a mixed-asset portfolio. However, there exists relative little guidance in the literature regarding private equity's portfolio capabilities.

Schmidt (2006) measures returns of a portfolio based on cash flows from 3,620 private equity investments made by 123 funds. He simulates a portfolio composed of private equity and stocks and finds optimal asset allocations to each asset, under the assumption of non-constrained investor. With respect to minimum-variance and maximum performance asset allocations, he finds that weighting in private equity varies between 3 and 65 percent. He concludes that an investor can reduce overall risk of a portfolio by combining private equity and stocks. The optimum allocation in private equity stated by Schmidt varies on a wide spread and does not clarify the question on optimal allocation to private equity in a portfolio.

In a study by Quigley and Woodward (2003), they include analysis of an optimal allocation among T-bills, stocks, long-term bonds and venture capital. In non-constrained risky portfolio, investors short-sell T-bills and allocate 20 to 50 percent of wealth in venture capital and 200 to 300 percent in stocks. In more realistic portfolio, where short sale is not permitted, investors allocate 10 to 15 percent of wealth to venture capital. They conclude that including venture capital in a portfolio could increase returns at the same levels of risk by less than one percentage point.

[5] They state that assumption of beta as 1 is likely to overstate performance and thus use "industry/size-matched cost-of-capital".

2.2 Listed Private Equity studies

Although several studies focus on the traditional private equity structure, there is only limited academic research on risk and return characteristics of listed private equity (LPE).

Zimmermann et al. (2005) cover 287 listed vehicles in the time period 1986 to 2003.[6] After applying liquidity constraints they investigate a sample of 114 instruments which they use to construct three different indices; one value weighted and two equally weighted with different rebalancing frequency. They calculate risk and return statistics as well as Sharpe ratios, Jensen's alpha, and autocorrelation coefficients. Their only fully rebalanced strategy, equally weighted fully rebalanced index (EW-RB), generates highest average annual return corresponding to 15.99 percent over the whole period, while the other two indices have returns around 5 percent. This deviation demonstrates the dramatic impact that rebalancing has in this market segment, deriving from the exceptional growth in number of listings experienced in the recent past. Sharpe ratio is also highest for the EW-RB index, measuring 0.57, while the other two are close to zero. They moreover address three potential biases and suggest correction that results from thin-trading, large bid-ask spreads and sample selection. Correcting for autocorrelation, caused by thin trading, leads to a lower Sharpe ratio of EW-RB index, shifting from 0.57 to 0.33. Correcting for the cost induced by a large bid-ask spread reduces mean returns and Sharpe ratios even further. The resulting annual return bias calculates 8.33 percent for the EW-RB index, however still outperforming the buy-and-hold index. Despite the potential selection bias in their sample, caused by the difficulty identifying non-surviving vehicles, the correction for sample selection did not lower annual mean returns of any of the indices, surprisingly it slightly increased.

Lahr and Herschke (2009) build their work on Zimmermann et al. (2005) and identify 446 listed private equity vehicles from the period 1986 to 2008. After imposing liquidity constraints they investigate stock performance of 274 vehicles. They classify private equity by different organization structure and find that performance differs strongly depending on their organizational form. Lahr and Herschke find that market risk is higher for internally managed vehicles than externally managed ones. They construct three alternative indices; one value weighted and two equally weighted indices. One of the equally weighted indices is adjusted for transaction cost by subtracting half of the average bid-ask spread from the index, which correspond to the second bias correction made by Zimmermann et al. (2005).

[6] They furthermore split the period into two sub-periods, the first representing the boom market until 2000 and second representing the bust market after 2000.

Furthermore, they address the issue caused by autocorrelation structure and international scope of LPEs by employing a combination of international CAPM and a Dimson regression. The Value weighted and equally weighted indices show a Dimson beta of 1.7 and 1.2 respectively. They find no excess return on value weighted index but significant 7.5 percent annual excess return on the equally weighted index, by measures of Jensen's alpha.

In a slightly different approach Bergmann et al. (2009) compare traditional Private Equity to Listed Private Equity vehicles with the use of Public Market Equivalent (PME) approach, which they say is sensible and useful measure of traditional Private Equity funds relative to a public market alternative. They conclude that an investment in LPE exhibit equivalent characteristics as investment in traditional private equity. They furthermore discuss additional features of LPE, such as flexible investment horizon and increased liquidity deriving from quotation on a public stock exchange, which is very attractable feature from a private investor perspective.

2.3 Summary of Literature Review

All of the previously discussed studies provide important insight into several different aspects of the private equity universe. However they vary considerably from alternative methods and measures. In order to simplify and formulate enhanced comparison, the focal empirical results, valuable for this study, are summarized in Table 2.2. The studies of Woodward and Hall (2003), Cochrane (2005), Ljungqvist and Richardson (2003) and Lossen (2006) all find private equity performances superior to the market. Three studies did not find significant difference to the market and Phalippou and Gottschalg (2009) criticized previous methods used to evaluate true private equity performance. They criticize that many earlier studies do not focus on total performance of private equity but rather on specific aspects of the investments (e.g. performance persistence and principal agent problem). Their findings are that private equity underperforms the S&P500 index net-of-fees.

The results of the interesting new market of LPE also vary somewhat similar to the traditional private equity studies. The main problem with LPE estimates is how highly sensitive they are to liquidity issues and therefore both Zimmermann et al. (2005) and Lahr and Herschke (2009) utilized liquidity constraints in order to neutralize that problem and obtain reliable results. They are also confounded by concerns of biases which they address and correct for. Note that the presented figures in Table 2.2 are pre bias-correction results. Both studies present a significantly positive alpha for their equally weighted indices while value weighted

indices are not dissimilar to the market. These findings suggest that the larger LPE vehicles do not seem to perform better than smaller ones.

Table 2.2: Summary of empirical results in selected previous studies.

The table contains sample period and size for each study and relevant performance measures. G.O.F refers to gross-of-fees studies. ~0 denotes no significant difference to corresponding benchmark. Beta may vary between single-factor models, Fama-French Three factor models and a Dimsons Beta. For PME, the public market return equals 1.00. Standard deviations are presented in parentheses.

Traditional Private Equity Studies: **Performance of GPs Investments**	Sample Period	Sample Size	Abnormal Return	Beta	Alpha
Woodward and Hall (2003)	1980-2004	-	-	-	8,5%
Hwang, Quigley and Woodward (2005)	1987-2003	15.583	~ 0	0,6	1%
Cochrane (2005)	1987-2000	7.765	43%	1,9	32%

Traditional Private Equity Studies: **Fund-level studies**	Sample Period	Sample Size	Abnormal return	Beta	PME	IRR (% p.a.)		PI
						Equally-Weighted	Value-Weighted	
Jones and Rhodes-Kropf (2003)	1969-2002	1.245	~ 0	1,05	-	16.38% (46%)	9.18% (39%)	
Ljungqvist and Richardson (2003)	1981-1993	73	5% to 8%	1,09	-	19.8% (22%)	18.1% (22%)	
Phalippou and Gottschalg (2009)	1980-1993	852	-3% to -6%	1,3	-	12,13%	12,22%	0.96 to 0.99
Kaplan and Schoar (2005)	1980-2001	746	~ 0	-	0.96 to1.05	18% (26%)	17% (31%)	
Lossen (2006) [G.O.F.]	1979-1998	134	-	-	3,075	50.2% (32%)	-	

Listed Private Equity Studies	Sample Period	Sample Size	Equally-Weighted			Value-Weighted		
			Beta	Alpha	SR	Beta	Alpha	SR
Zimmermann et al (2005)	1986 - 2003	114	0,6	10,18%	0,57	1,2	-1,20%	~0
Lahr and Herschke (2009	1986 - 2008	274	1,2	7,50%	0,13	1,7	~ 0	0,049

Source: Adopted from Flemming (2010)

Studies that are not specifically mentioned in the summary table are also worth noting. The mixed assets studies of Quigley and Woodward (2003) allocate 10 to 50 percent of its portfolio to private equity and Schmidt (2006) allocates 3 to 65 percent. This fairly large deviation is not very informative guidance on how to allocate your private equity share in a mixed-asset portfolio. Furthermore they do not give significant results on private equity performance in a mixed-asset portfolio. Additionally, the portfolio in Schmidt (2006) only holds two asset classes, however interesting is to investigate how private equity would perform in more diversified portfolio including other asset classes as well (i.e. bonds and commodities). The last study, by Bergman et al. (2009), gives very informative and interesting conclusions by concluding the similarity of traditional private equity and LPEs.

This examination on previous literature regarding private equity gradually built up an opening for our contribution to the literature. Most valuable and straightforward work is towards the least documented fields; the recently developed LPE market and private equity performance in a mixed asset portfolio.

3 Research Question and Scope

3.1 Research Question

The aim of this paper is to determine whether a private investor should consider private equity as an appropriate value adding investment. An intrinsic component of private equity is its private nature. Evaluations are therefore heavily depended on self-reported book values that may differ in accounting standards, which makes standard performance methods not directly applicable. Accordingly, illiquidity arises for investors, which is major restriction to this asset class. Therefore, heterogeneous opinions were documented in existing literature concerning private equity's performance, in regards to risk and return. The objective of this thesis is to provide an empirical analysis whose validity is not limited by either the lack of liquidity or data.

By addressing the topics above, the research question answered in this thesis is:

> **How does private equity perform in terms of risk and return, in comparison to global stocks?**

The abovementioned research question is further extended by adding the following sub-question:

> **How does private equity affect portfolio performance, for a private investor, when added to a portfolio consisting of stocks, bonds and gold?**

The contribution in this thesis is of a quantitative analysis, based on an index construction and three asset allocation strategies which are evaluated by performance measures. The limitations are elaborated in the scope.

3.2 Scope

The focus throughout this thesis is applicability, which influences the boundaries of our research. The investor's view in this thesis is from a private investor's perspective. This is viable since investing in this asset class has not been easily accessible for a private investor. Furthermore, to be able to generalize our findings to a broad group of audience we do not limited this work to a specific country. In addition, to make a valid analysis of this asset class the findings are not influenced by short term market cycles.

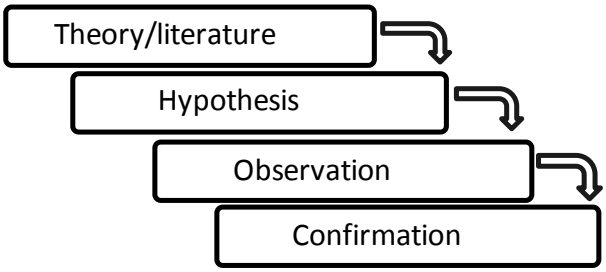
The thesis utilizes listed private equity (LPE) as data source to represent private equity. The findings of Bergman et al. (2009) support the validity of using LPEs as a proxy for traditional private equity. To be not limited by the lack of liquidity, several company specific factors are gathered from Thomson Datastream 5.0, which designates tradability of each vehicle. Subsequently, liquidity constraints are applied to the data sample before the index is constructed.

Three asset allocation models are applied in the portfolio part, a simple equal weighting portfolio and the other two deriving from the mean-variance framework published by Harry Markowitz in 1952. Even though there are shortcomings in these models, and more advanced models do exist, the simplicity and intuition makes them most appropriate models for a private investor to seek answer for the sub-research question. The mean-variance framework suggests optimal allocation to each asset. However, the portfolio part is not intended to recommend an optimal portfolio composition but rather to analyze the dynamics of different asset allocation models. That makes the possibility of ranking corresponding performance measures in order to address whether private equity improves the portfolio performance. A discussion of the underlying fundamentals of risk aversion and utility theory is excluded; it is assumed that the reader has sufficient understanding on the subject and its relationship to the portfolio theory.

The research is a quantitative investment analysis and therefore does not include any social and macroeconomic examination. Such investigation does not fall within the scope of the thesis. In order to make the contributions as general as possible, taxes and transaction costs are not included. Taxes are investor and country specific and transaction costs are subject to several factors and are therefore difficult to generalize. This thesis is therefore structured in order for the corresponding findings to be useful to a broader group of investors, since individual specific factors are excluded.

4 Methodology

A deductive research approach is chosen as the appropriate method to answer the research question. Deductive research implies that existing literature forms the starting point from which hypothesis are deduced from. They are intended to generate answers to the research questions through empirical data testing. Then, based on these findings, the hypothesis is either rejected or not. The process of deductive research is illustrated in Figure 4.1.



Source: Social Research Methods (2006)

Figure 4.1: Deductive Research Method.

The research questions are modified into the hypotheses statements that are tested. That is, the research questions are reduced to a definite statement about the value of a quantity; it therefore becomes a statistically testable hypothesis. This is done for the tools and concepts of hypothesis testing to address the research questions. In other words, statistical inference is utilized to make judgment about private equity. Statistical inference is the process of using a small sample as base for a judgment on a larger population (DeFusco, McLeavey, Pinto, & Runkle, 2007).

4.1 Hypothesis

The following hypotheses are put forward to test the research questions:

H_0: Private equity outperforms the global stock market, in terms of risk and return.

H_1: Private equity does not outperform the global stock market, in terms of risk and return.

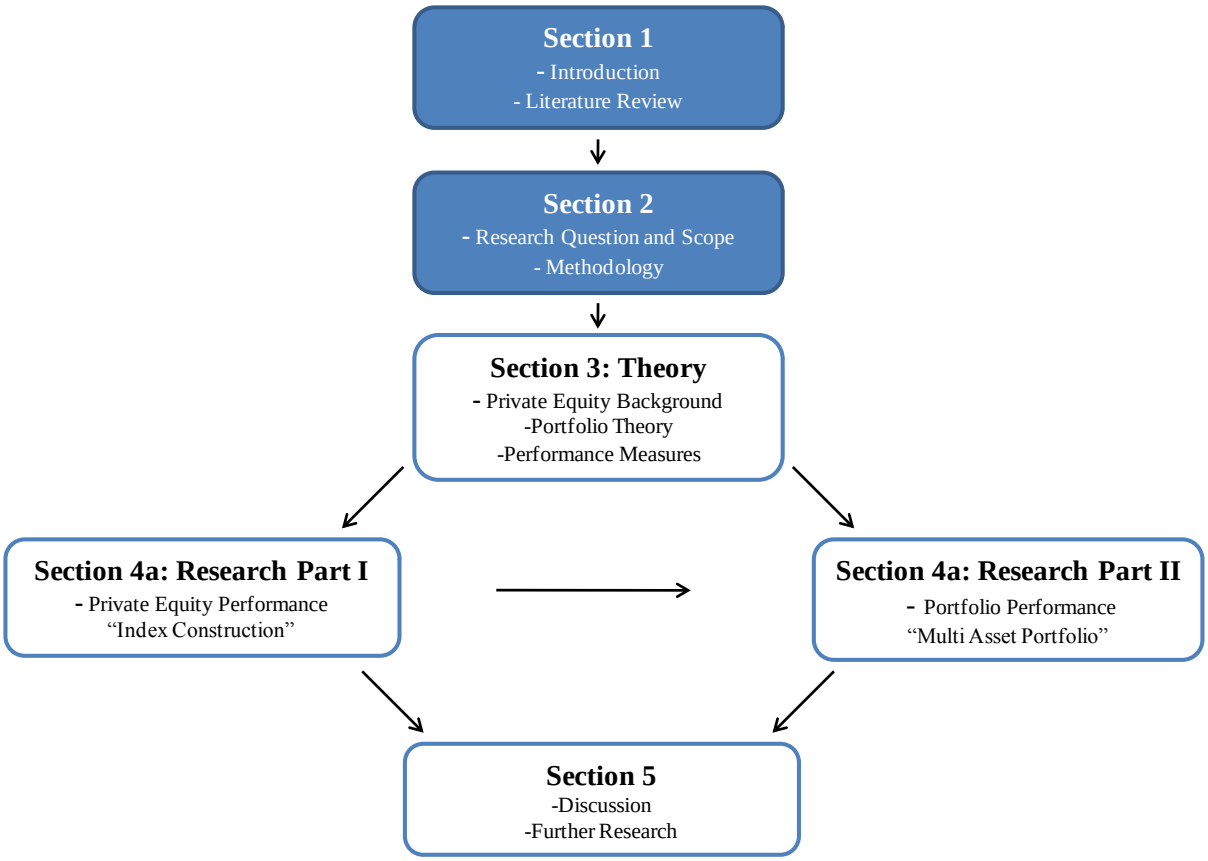
H_0: Private equity improves portfolio performance, consisting of stocks, bonds and gold, in terms of risk and return.

H_1: Private equity does not improve portfolio performance, consisting of stocks, bonds and gold, in terms of risk and return.

Since both risk and return are measurable components, the hypotheses enable performance of quantitative testing. By comparing private equity performance to a global stock market benchmark, the first hypothesis can either be rejected or not. Furthermore, by comparing two portfolios, i.e. one containing private equity and the other one not, the second hypothesis can either be rejected or not. To test the hypotheses, quantitative investment analysis tools are applied. After the hypothesis has been tested there should be a clearer evidence of whether the pre-defined perception is true. However, the conclusion always stops short of certainty and need to be approached with a certain sense of reservation (DeFusco et al., 2007). The following sub-chapter describes the structure of the thesis.

4.2 Thesis Structure

The thesis consists of twelve chapters, which are divided into five sections. The following coverage briefly describes previous sections where more extensive explanations are provided of the remaining sections. Overview of the thesis structure is illustrated in Figure 4.2.



Source: Authors contribution.

Figure 4.2: Overview of the thesis structure.

Section 1 began with an introduction to the subjects of the thesis. It also documented previous literature findings that formed the motivation for creating the thesis at hand. Section 2 described the process which formulated the research question and presented the scope and the methodology of the thesis.

Section 3: Theory

Section 3 includes analysis of theoretical fundamentals which are a necessary precursor for the two research parts. Firstly, it includes an insight into the background of the private equity markets, both the traditional and the LPE market segments and compares the main fundamentals. Secondly, it contains the portfolio theory with coverage of risk and return characteristics of risky assets which are essential to constructing a portfolio of mixed assets (Research part II). Finally, it introduces the measures to evaluate performances, which are utilized in both research parts.

Section 4a: Research Part I

To test the first hypothesis, Research Part I is committed to the construction of an LPE index. The initial data sample is derived from well known intermediaries within the industry[7]. The sample therefore represents most acknowledged and liquid private equity vehicles listed on worldwide stock exchanges. A persistency analysis of individual LPE vehicles is performed, consistent to the study by Kaplan and Schoar (2005). This provides an intuitive way to get insight into the functionality of this market segment.

When measuring the performance of an asset class, it is not very intuitive to compare an individual security to an index benchmark due to the risk of selection bias. Accordingly, it would not be possible to generalize corresponding findings to an asset class as a whole; the findings would rather solely apply to the specific security in question. A more sensible approach is to analyze an index which can serve as a possible representative of the asset class. Therefore, this research part constructs and investigates an index. In order to generate more valid results, liquidity constraints are applied to the data sample prior to the index construction.

Two indices are constructed as this achieves a comparable analysis and more informative results. For the first index, the weights are determined by the relative market capitalization of

[7] Intermediaries are LPX, S&P LPE and LPEQ and description of them can be seen in Chapter 8.2.

the individual vehicles. Since this is a value weighted index it naturally allocates excessive weights to larger vehicles. For the second index, the weights are equally allocated to all vehicles. To be able to quantitatively test the hypothesis, performance measures are evaluated and accordingly provide evidence regarding the performance of private equity, in comparison to global stocks. The work of this research part is used as the proxy for private equity in Research Part II.

Section 4b: Research part II

To test the second hypothesis, Research Part II addresses portfolio capabilities of private equity in a mixed asset portfolio. To make a comparative analysis two portfolios are constructed. Firstly, an Initial portfolio is created, which is utilized as a benchmark. The Initial portfolio represents the general asset classes that investors commonly include in their portfolio; it includes stocks, bonds and gold. The opportunity set of this Initial portfolio is then extended with the opportunity of investment in private equity. The second portfolio is constructed in the same way as the Initial portfolio, but with the addition of private equity.

The role of private equity in a mixed asset portfolio is studied with three different strategic asset allocation strategies. The first asset allocation strategy is a simple equal weighted portfolio (1/N), which completely neglects all historical performance characteristics. The second two allocation strategies are mean-variance optimization strategies. The first being the minimum-variance strategy, which as its name implies, minimizes the variance of the portfolio. The final asset allocation determines the optimal tangent portfolio of a combination of risky assets and a risk-free rate. These two portfolios are used in creating the efficient frontier, which according to Black (1972), are the only two portfolios needed to establish the whole efficient frontier. These portfolios are constructed both with and without constraints on short-selling. Furthermore, the portfolios are created for different sub-periods. Consequently, these strategies are compared with accompanying performance measures to conclude whether the hypothesis is rejected or not.

5 Private Equity Background

This chapter covers the background and fundamentals of private equity. The breakdown is in-line with the previous literature review, where the first part will cover background on traditional private equity whereas the latter part will introduce the listed private equity market. This chapter then concludes with a comparison of those two with an analysis of the advantages and disadvantages.

5.1 Traditional Private Equity

5.1.1 Definition

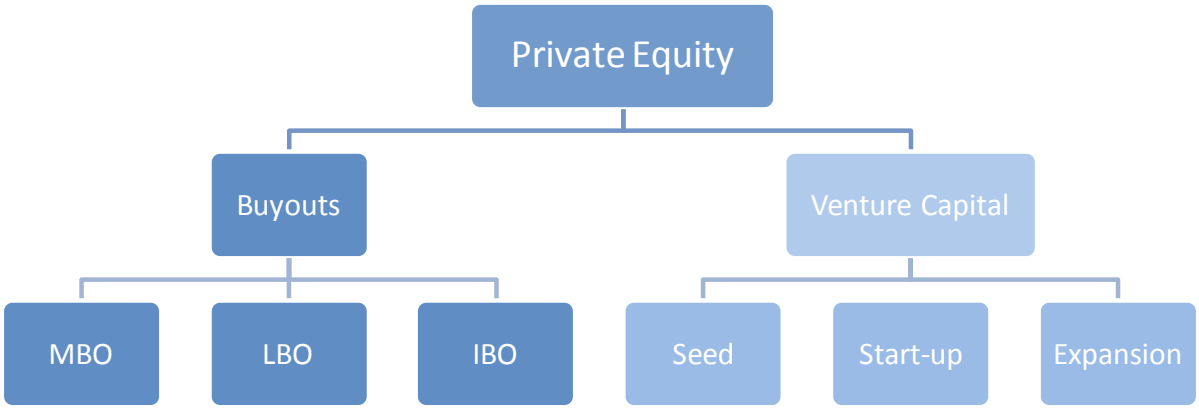
As peculiar as it seems, the question "what is private equity?" is not so easily answered. People usually have an instinct for what private equity investment is, but it is growing increasingly difficult to classify the exact parameters of this asset class because classifications alter between individuals and geographies.

In Europe, this asset class as a whole is called *private equity* and it is broadly sub-divided into *buyouts* (BO) and *venture capital* (VC). However, while the sub-division holds in the United States, the terminology is used differently. In the US, the asset class as a whole is usually called *VC,* where *BO* is referred to as *private equity* (Guy Fraser-Sampson, 2007). This mismatch between classifications is originated by very diverse business nature of the two of them. Since the funds in the data sample used in this research are both categorized as BOs and VCs, the 'European' terminology is utilized, where private equity is a collective umbrella term of both VCs and BOs.

In this thesis, the same definition is used to define private equity as, for example, Lahr and Herschke (2009) employ, or as *professionally managed equity investments in securities of non-public companies*.

5.1.2 Forms of Private Equity

Private equity firms raise money mainly for two types of funds; venture capital and buyouts. Other forms of investment styles can be realized and there exists a great deal of overlap in definitions, subsequent recitation follows definitions, for example, in report published by OECD. Summary of the main forms of private equity with corresponding sub-categories are illustrated in Figure 5.1.



Source: Adopted from Lahr and Herschke (2009) and Wright et al., 2007.

Figure 5.1: Forms of private equity.

5.1.2.1 Buyouts

Buyout can be defined when a controlling stake of over 50 percent in an existing business changes ownership and new independent legal entity is created (Wright et al., 2007). When private equity firms pursue buyout strategy, they generally become active investors by taking place in the company's board and monitor the behavior of the management by specifying detailed reporting requirements. Buyouts can take number of forms. The classic buyout has always been the management buyout (MBO), where the existing management, or a group of employees, acquires substantial proportion of a company or a business unit of the company[8]. The acquisition of the business from the parent company is normally supported by a private equity firm or a bank. However the scale of these transactions is often relatively small compare to other forms. In contrast, leverage buyout (LBO) is distinguished as buyout which is not initiated by a management team, either internal or external, and transactions are in relatively large scales. LBO usually involves an acquisition of a publicly-quoted firm or a large division of a group by a LBO association (Guy Fraser-Sampson, 2007).[9] In case of public listing, the company is unlisted and the resulting private company is typically controlled by board of directors representing the LBO association. The private equity firms acquires significant equity stake in a transactions that generally include high leverage as the name suggests. In somewhat similar, but with less use of leverage, an investor-led buyout

[8] Additional variations of MBO are i.e. MBI and MEBO. MBI is simply a MBO with the management team being outsiders; it involves greater risk since incoming management does not have benefits of inside knowledge about the company. MEBO is extension of MBO where employees create a management-employee buyout, this may occur when it's important to tie in a specific human capital.

[9] Due to large transactions it may be convenient to think of LBO equivalent to an industrial acquisition where the acquirer often a consortium of buyout firms.

(IBOs) typically involves an acquisition of a whole company or a division of a larger group led by a private equity firm. Value creation may be gained via development of the company, not just through financial efficiency improvements. The private equity firm usually either retains existing management to run the company or brings in new management to run it (Wright et al., 2007). This previously mentioned recitation is not exhaustive. To a great extent, practitioners probably do not heavily depend on where their current deal falls in spectrum.

In terms of private equity transactions, the largest share of buyouts is derived from family firms, where there is no obvious family successor. Secondary buyout, where private equity firm sells the company to another private equity firm, has become increasingly popular activity in the recent past. Public-to-private (PTP) deals, where companies are de-listed from stock exchange, have gained the most attention, with RJR Nabisco deal probably the most high profile LBO of all time, with value of USD 30.2bn in 1989[10]. PTP buyouts are normally larger in market value which is driven by large amount of capital invested rather than number of deals completed.

5.1.2.2 Venture Capital

Venture capital (VC) can be defined as "Professional equity co-invested with the entrepreneur to fund an early-stage (seed and start-up) or expansion venture." (EVCA, n.d.). Venture capital funds provide financing to high growth potential companies that are in early stage of development and often not in a position to access public equity market or secure traditional debt financing (Gogineni and Megginson 2010). Unlike buyouts, which are classified broadly by the type and size of the deal, most VCs are similar in their form. Apart from periods of irrational exuberance, variations in their size tend to be function of stage. Thus it is useful to categorize venture capitals by sector and stage.

Venture capital is sub-divided into following stages according to the point of development which the company has reached at the time it needs financing:

[10] This was the world's largest buyout until 2006 when HCA surpassed the Nabisco deal with USD 32.1bn deal (in nominal value).

- Seed stage refers to provision of mostly mezzanine capital (e.g. convertible stocks) to a company, which has not yet been established[11]. Often investments in research and development (R&D) or prototype development.
- Start-up stage refers to company in a life-cycle where its product or service has not yet been sold commercially. Thus investment is needed for marketing and product developments. Firms in this stage are not yet making profit and thus cannot finance their development from operational cash flows only.
- Expansion stage refers to companies which are already producing and selling a product and are going for their first expansion. (Lahr & Herschke, 2009).

VCs therefore invest in companies that have not shown prior track record or currently generating revenue stream. This is contrary to BOs that invest in companies with established cash-flow stream. As a result, VC is generally riskier than BO and accordingly, requires higher expected returns.

5.1.3 Ownership Structure

> Private equity funds are created in order for investors to delegate investment management to professionals, fully dedicated to this task, who can theoretically identify the best opportunities, negotiate the best deals and help companies grow.
>
> Demaria, 2010, p. 45-46.

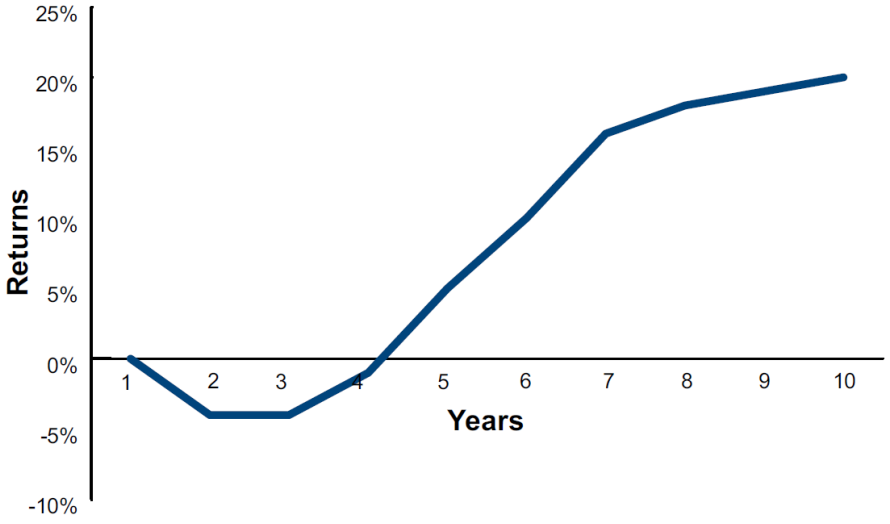
Private equity firms usually use limited partnership, as the legal form of their funds, to finance their investments. Funding is raised from institutional investors, such as banks, pension funds, endowments or other professional investors which are referred to as *limited partners* (LP). Generally, first round financing is raised at the time these funds are initiated and then additional funding is raised when investments are made. Private equity funds are managed by professional fund managers who serve as *general partners* (GP). GPs manage the portfolio companies and are in charge of identifying, completing and realizing the investments. The structure of this partnership agreement between LPs and GPs includes covenants that describe the 'rules of behaviors'[12]. After agreement is made, LPs usually do not intervene in GPs investment decisions. (Bottazzi, 2010).

[11] Mezzanine is a hybrid of debt and equity financing. It is a debt capital which can be converted, by the lender, to equity in the company if the loan is not fully paid in time. Hence it works as collateral.
[12] For further detail on terms and conditions of these covenants see Fleming (2010).

Compensation structure of private equity funds, paid to GPs by LPs, typical amounts to yearly management fee of 1 to 3 percent and 20 percent carried interest of profits. However, other incentive fees exist but they are inconsistent and vary by funds.[13] The use of limited partnership and covenants is considered efficient structure for mitigating the principal agent problems in the private equity investment model. Furthermore, limited partnerships facilitate long-term investment focus by GPs, which is driven by performance incentives.

5.1.4 Investment Horizon and the J-Curve

Limited partnership usually has a finite horizon, commonly about 7-12 years, which may be extended for few years to allow GPs to seek liquidity. The term of vintage years is used for investment horizons of private equity funds. The relationship between vintage years and returns of private equity funds has a unique shape, known as J-curve. The J-curve demonstrates the tendency of private equity funds to deliver negative returns in early years and profits typically in the 4th or 5th year of ownership, as illustrated in Figure 5.2.



Source: Phillips, Hager & North Investment Management Ltd. (2008).

Figure 5.2: J-Curve

As with wines, vintage year refers to year of inception, which in this coherence is the year of which the fund is closed to investors. The J-Curve describes typical return distribution of private equity funds. In the initial years, the returns are negative due to several factors; management fees are charged and early underperformance is often written down. It is not until the fund starts selling its investments (portfolio companies), which typically occurs mid-

[13] For further insight on different fees structure see Metrick & Yasuda (2010).

way through the fund's life cycle, that the fund begins to realize profits and generating payouts to LPs. (Phillips, Hager & North Investment Management Ltd., 2008).

To familiarize this time component to the research scope, this information elaborated that illiquidity is generated because LPs investments need to be locked for fixed time horizon and minimum capital committed is often very large. Furthermore because of this distinctive time horizon, evaluating short term performance of private equity funds faces challenges. Further information explaining the model of private equity, financial structure, and value creation along with figure of the historical waves of private equity over last decades can be found in Appendix C.

5.2 Listed Private Equity

The large amount needed and the long time horizon contributes to private equity investments being mainly the domain of large institutional investors. Listed private equity (LPE) could be the solution for investors that want to invest in this asset class but does not have required capital or the time needed. LPE provides shareholders an immediate exposure to a diversified private equity portfolio (Bergmann et al. 2009).

There are numerous reasons that hinder disclosure by private equity firms. However, listed private equity companies increasingly exhibit efforts to provide more information to their shareholders, both current and potential. Bergmann et al. (2009) show that the typical features for private equity, such as investments styles, financing styles and other important characteristics are shared between the unlisted and the listed private equity universe.

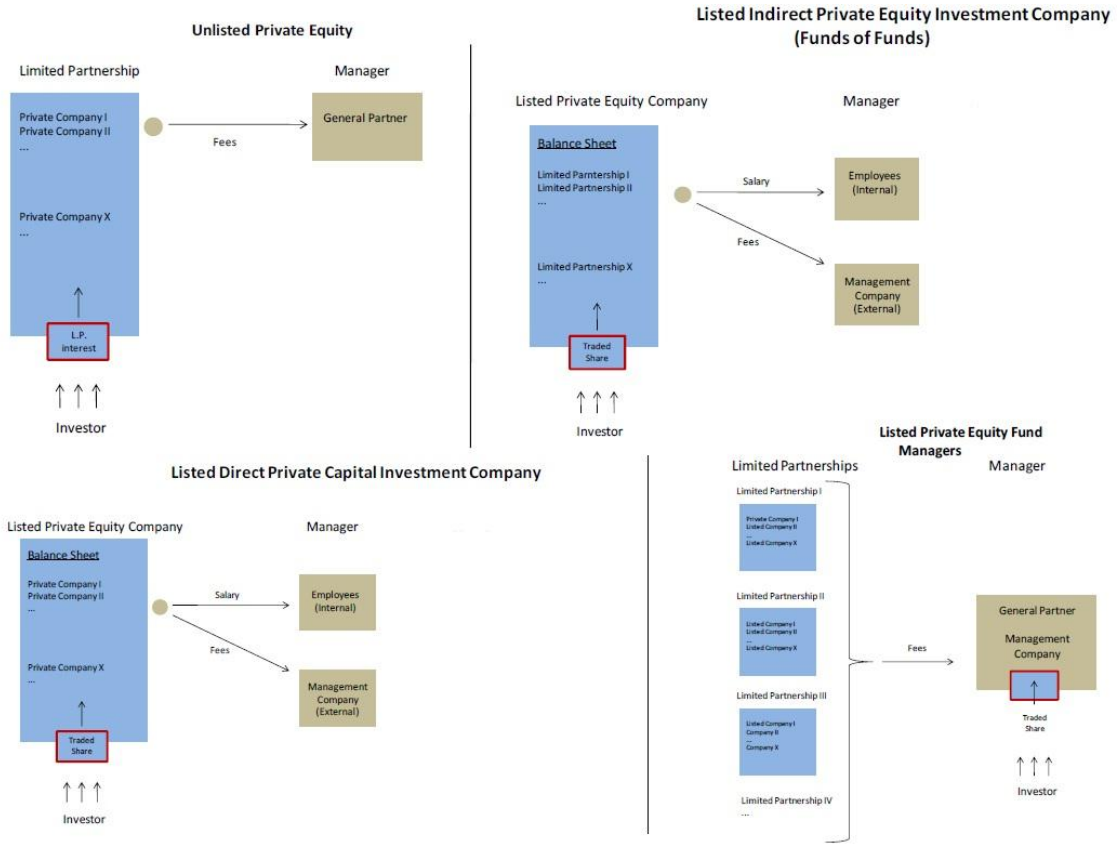
5.2.1 Definition

There are several synonyms for this asset class mentioned in the literature; listed private equity, publicly traded private equity, quoted private equity and liquid private equity are among few names that are used. The most common name used for this asset class today is listed private equity (LPE). The term private equity does not mean that the investing company itself needs to be private. LPEs are publicly traded vehicles on stock exchanges that invest capital in privately held companies. Whether or not the private equity company itself is listed does not influence its core business, which is investing in unlisted companies. The definition of LPE from an investor's perspective followed in this thesis is, to get exposure to private equity directly or indirectly, by investing in publicly traded vehicle. Having an organized market to buy and sell private equity makes this asset class more liquid. Since there is no need to commit capital for a long period of time, this can be seen as an appropriate exposure to private equity for a private investor.

5.2.2 Organizational Forms

Investment and financing styles are shared between both traditional (unlisted) and listed private equity. However, when it comes to organizational structures there are some differences. Unlisted private equity is mainly structured as a limited partnership, while the listed companies have more complex structures. Bergmann et al. (2009) propose the following categorization, which is described in more details in Figure 5.3; listed direct private equity investment companies, listed direct private mezzanine capital investment companies, listed indirect private equity investment companies, and listed private equity fund managers.

The first two present many similarities in organizational structure and are therefore combined into listed direct private capital companies.



Source: Bergmann et al. (2009)

Figure 5.3 Organizational Structure.

5.2.2.1 Listed Direct Private Capital Companies

The term 'direct' indicates that the company invests directly in the underlying companies and not through limited partnerships. LPs are therefore excluded in this structure, meaning that the traditional structure between LPs and GPs does not hold. By buying shares in companies in this category, the investor gets a diversified portfolio of private companies that are directly held by the listed vehicle.

5.2.2.2 Listed Indirect Private Equity Investment Companies (Funds of funds)

Here the term 'indirect' indicates that the company does not invest directly in private equity but does so indirectly by investing in one or more limited partnerships. This category could be described as 'actively managed traditional private equity' since the LP and GP relationship does still hold. The advantage over a traditional fund is the diversified and permanent exposure to the private equity asset class. By being diversified indicates the ability to invest

in more than one LPs and permanent refers to that it has no finite life span but proceeds from older vintage funds reinvested in new funds.

5.2.2.3 Listed Private Equity Fund Managers

Listed fund managers have typically no direct or indirect exposure to private companies. This structure has the traditional private equity structure except that the investor buys an interest in the GP but not in limited partnership. Therefore the relationship between LPs and GPs does still hold where the GP is the listed vehicle. The revenue flow is only generated by the fees collected by the GPs.

5.3 LPE versus Traditional Private Equity

To summarize the main differences between listed private equity and traditional private equity, Table 5.1 provides a simplified overview.

	Listed Private Equity (LPE)	**Traditional Private Equity**
Lifetime	Unlimited lifetime	Generally fixed for 7-10 years
Fees	Generally lower than LPs, returns carry the underlying MGMT cost	Generally MGMT fee of 1- 3% and 20% carry interests of profits.
Price	Opportunity to buy at a discount	Initially invest at net asset value
Liquidity	Liquid, shares can be freely bought and sold on public market	Illiquid, limited and costly secondary market
Investments	Generally well diversified	Generally well focused
Minimum investment size	No minimum size of investment	Large minimum size of investment, generally above USD 1 million
Proceeds	Realisation proceeds usually reinvested, although some listed funds return capital (e.g. dividends)	Realisation proceeds are returned to investors
Disclosure	Reports and accounts meets minimum standards	Current investors receive detailed information on the investments
Shareholder rights	Shareholder democracy and strong corporate governance	Investors rights limited to changing the manager but few other rights

Source: Adopted from LPEQ

Table 5.1 Listed Private Equity (LPE) versus Traditional Private Equity.

Investor can get an exposure to private equity either by buying traditional or listed private equity. In the former, the investor faces high minimum size of committed capital and normally a fixed time horizon. In the latter, the investor faces no minimum capital and can exits his investments when suitable. One major difference regards the fee structure; in traditional private equity, fees are quite high but they are generally much lower for the LPEs, where management cost is inhered in the price. Another difference derives from when capital of a LPE vehicle has been raised; it is recycled from one deal to another. This option is not

available through traditional limited partnerships, where the relationship has pre-defined horizon. Traditional private equity measures return by internal rate of return (IRR) without the consideration for time. But because listed companies are priced daily, investment returns are time weighted. Furthermore, an investor within the traditional segment seeking diversification on his investment, by sectors or geographies, is forced to invest in a several funds since each fund is generally focused. However, that requires an extremely large amount of capital. Diversification in the LPE universe is simply achieved by buying a stock portfolio of LPE companies.

5.3.1.1 Summary of the Private Equity Markets

Traditional private equity investment generates direct access to managers which can be attractive for large, sophisticated investors that are comfortable with the illiquidity component. However, LPE can be a solution for some of the drawbacks of traditional private equity. Institutional investors that are not allowed to invest in private companies, now have the possibility to invest in private equity through listed vehicles.

Investing in LPE is similar to investing in regular stocks where shares can be traded at investor's discretion. There is no minimum investment needed, except the minimum investment corresponding to a single share. Accordingly, diversification can be obtained with minimum investments. As a requirement from the stock exchange, a listed company needs to publish all news that could potentially impact the share price. This substantially improves the transparency for a listed company. However, transparency in traditional private equity is also rather high for the LPs, whereby it is low for outside investors. This transparency is due to the considerable amount of information regarding the fund's underlying portfolio companies that the funds provide LPs, which they do not make available for outside investors.

By going public, company enhances its name recognition with greater publicity and has the opportunity to establish itself as an international brand. Furthermore and maybe more valuable, it can also increase credibility with investors. An unsuccessful company might not be able to fulfill the necessary conditions needed to be listed on exchanges or generate enough interest for its stock. With the availability of market prices, the performance characteristics of LPE vehicles can be analyzed with classic valuation methods, which are not applicable for traditional funds. Performance measurements are therefore not limited to information from the fund management itself. LPE can be traded on daily basis if there is sufficient bid-ask volume. An investor can therefore liquidate his investment whenever he

desires, given that there is someone willing to purchase the share. There are, of course, some drawbacks of investing in LPE. Illiquidity is still one of the main drawbacks which might be generated from dual factors. LPE vehicles have relatively small market capitalization and many LPE investors are long-term investors, which can be reflected in large bid-ask spreads and infrequent trading. Accordingly, if a large block-holder wants to sell his share he might need to sell it at a discount. Furthermore the rights of a shareholder in publicly listed companies are mainly restricted to participating in the annual general meetings, reading annual reports and monitoring share prices.

6 Portfolio Theory

Portfolio theory is an integral part within modern finance and this chapter introduces this concept for two reasons: It introduces the fundamentals to risk and returns characteristics for one or more securities, which is essential for the evaluations in both research parts. Furthermore, it prepares the reader for Research Part II, which assesses private equity in a mixed asset portfolio.

6.1 Modern Portfolio Theory (MPT)

The work by the Nobel laureate Harry Markowitz in his paper "Portfolio Selection", published in the *Journal of finance* 1952, is one of the milestones in the modern financial theory. His ideas regarding quantitative management of portfolios, today popularly known as the mean-variance framework, built the foundations of modern portfolio theory.

The pioneer work by Markowitz quantifies the benefits of diversification and assesses the opportunity for improving the performance of a portfolio by combining assets. Essential to MPT is its quantification of the risk parameters of investments (as measured by variance and standard deviation of returns) and the relationship between risk and return, with assumption that investor must be compensated for his perceived risk. The portfolio theory departs from traditional security analysis by focusing on portfolio as a whole, or as Harry Markowitz (1959) stated:

> *A portfolio analysis starts with information concerning individual securities. It ends with conclusions concerning portfolios as a whole. The purpose of the analysis is to find portfolios which best meet the objectives of the investor.*

Reasons for Markowitz's success are that his mean-variance framework is not very complicated and therefore relatively easy to implement. Today, financial models, which are based on these same principles, are being reinvented and incorporated into new findings (Fabozzi, 2007)[14].

[14] i.e. by Markowitz's co-Nobel-laureates William Sharpe, with his contribution to CAPM, and Merton Miller, in his work on CMT Derivatives and Financial engineering.

In its simplest form, mean-variance criterion can be stated as follows: Portfolio A dominates B if

$$E(r_A) \geq E(r_B) \tag{6.1}$$

and

$$\sigma_A \leq \sigma_B \tag{6.2}$$

where

$E(r_A)$ is the expected return of a portfolio A

$E(r_B)$ is the expected return of a portfolio B

σ_A is the standard deviation of a portfolio A

σ_B is the standard deviation of a portfolio B

and at least one inequality is strict (Bodie, Kane and Marcus, 2009).

When constructing portfolios in practice, the portfolio choice problem can be separated into two independent tasks, which is often referred to as two-fund separation (Fabozzi, 2007):

1) Optimal risky portfolio: How the risky portion of the investment is distributed between individual risky assets.[15] This task is purely technical.
2) Asset allocation of the complete portfolio: The allocation of investor's wealth between risk-free and risky assets. This task depends on personal preferences (risk aversion).

6.2 Risk and Return of a Single Risky Asset

In brief, the mean-variance framework builds on constructing a combination of risky assets; hence it is essential to examine the necessary characteristics. Since stocks are well-recognized source of relatively risky assets, it is used for demonstration on characteristics of risky asset. The future return of a stock is not known with certainty; therefore the notation of expected return is frequently used, which is given by

[15] The term 'risky assets' denotes that the assets are not risk-free.

$$E(r) = \sum_{i=1}^{I} p_i r_i \tag{6.3}$$

and the corresponding variance of a single stock is[16]

$$\sigma^2 = \sum_{i=1}^{I} p_i (r_i - E(r))^2 \tag{6.4}$$

where

p_i is the probability of obtaining r_i

r_i is the return of incident i

i is the number of possible outcomes

Variance measures expected values of squared deviation. All differences are squared before probabilities are calculated since differences can both be positive and negative and thus tend to cancel each other out. Standard deviation is simply the square root of the variance and is designated by σ. Standard deviation is more useful risk measure as it measures the average deviation of returns from its mean and is expressed as following:

$$\sigma = \sqrt{\sum_{i=1}^{I} P_i (r_i - E(r))^2} \tag{6.5}$$

Portfolio theory explains the potential of risk reduction from holding a portfolio of assets that do not move in perfect unison (Elton et al., 2007). Therefore, when measuring affects of combined assets it is necessary to take other measures than risk and return into consideration. These measures quantify the degree to which returns of two securities move together. Corresponding information can be obtained by calculating the covariance between two securities

$$\sigma_{12} = \sum_{i=1}^{I} P_i \, (r_{1i} - E(r_1))(r_{2i} - E(r_2)) \tag{6.6}$$

[16] Using historical data with N observations, variance is calculated: $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{N}(R_i - \overline{r})^2$

where r_{1i} and r_{2i} are the return possibilities for stock 1 and 2, respectively. The probability weighed average summarizes the average tendency for how two variables co-vary across scenarios (Bodie, Kane and Marcus, 2009).

By scrutinizing the covariance formula one can see that covariance between stock 1 and 2 must be the same as the covariance between stocks 2 and 1:

$$\sigma_{12} = \sigma_{21} \tag{6.7}$$

For many purposes it is useful to standardize the covariance. Dividing the covariance of the two assets by the standard deviation of each asset produces a variable with the same properties as the covariance but on the scale of -1 to 1. This measure is called the correlation coefficient:

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} \tag{6.8}$$

A correlation of 1 signifies that the two assets are perfectly positive correlated, meaning that they have perfect linear relationship in same direction simultaneously. Contrary, correlation of -1 signifies that the two assets are perfectly negative correlated, meaning that they have perfect linear relationship in opposite direction simultaneously. As correlation approaches zero the less of a relationship the two assets have, and correlation of zero means that the two assets are uncorrelated.

6.3 Risk and Return of a Portfolio

The simplest form of a portfolio contains two assets and for simplicity reasons following notations begins by demonstrating with two terms. Expected return of a portfolio is exactly the average of the mean returns of two assets; hence it includes the weighting of each asset and is relative easy measure

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) \tag{6.9}$$

where $w_1$and w_2 are the fraction of the portfolio held in asset 1 and 2, respectively.

When generalizing for N numbers of risky assets, expected return of a portfolio is given by:

$$E(r_P) = \sum_{i=1}^{N} w_i \, E(r_i) \tag{6.10}$$

While expected return of the portfolio is simple-weighted average of the expected returns on individual assets, the same is not true for the risk of a portfolio. The risk of a combination of assets is not a weighted average of the individual asset risks, it depends on how returns of individual assets tend to move together when some assets give positive return and other give negative return. The variance of the return on the portfolio is equal to

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12} \tag{6.11}$$

with standard deviation of portfolio consisting two stocks then being:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}} \tag{6.12}$$

When generalizing for N number of assets, the variance of a portfolio is given by:

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=1}^{N} w_iw_j\sigma_{ij} \tag{6.13}$$

Variance of a portfolio is therefore a weighted sum of covariances of individual assets (Bodie, Kane and Marcus, 2009). After having examined the relationship between covariance and correlation one can see that in the extreme case when two assets are perfectly positively correlated the standard deviation is not reduced by adding them together as a portfolio, i.e. there is no diversification benefit. Conversely, when two assets are perfectly negatively correlated then it should theoretically be possible to find a combination where the portfolio would have zero standard deviation. This demonstrates the power of diversification and indicates how strong the effect of correlation has on portfolio performance. In other words, diversification benefits can be achieved and total portfolio risk can be reduced by holding assets that have correlation below 1.

6.3.1 Matrix Notations

For practical purpose, the following matrix notations are included, which are extension of previously mentioned portfolio characteristics. These notations are directly utilized, by the use of Excel, in Research Part II when the mixed asset portfolios are constructed. Matrix notation greatly simplifies the writing of the portfolio calculations. In general case of N number of assets, where each of which has column vectors of expected returns R and portfolio composition x, relevant matrices are

$$R = \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \tag{6.14}$$

Where each x_i represents the proportion of the portfolio invested in risky asset *i* and total assets weights sum to one

$$\sum_{i=1}^{N} x_i = 1$$

According to Benninga (2008) the matrix notation of portfolio variance is the most economical representation. Furthermore, it is easy to implement for large portfolios. In this representation, *S* corresponds to the *N x N* variance-covariance matrix:

$$S = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \cdots & \sigma_{N1} \\ \sigma_{12} & & \ddots & \sigma_{N2} \\ \vdots & & & \vdots \\ \sigma_{1N} & \sigma_{1N} & \cdots & \sigma_{NN} \end{bmatrix} \tag{6.15}$$

The variance-covariance matrix is computed with a VBA code, which can be seen in Appendix J. Expected return of the portfolio is then calculated by the products of x and *R*

$$E(r_P) = x^T R \tag{6.16}$$

and corresponding portfolio variance is calculated

$$\sigma_P^2 = x^T S x \tag{6.17}$$

where *T* indicates a transposed vector.

7 Performance Measures

This chapter introduces relevant measured that are utilized to compare the performances in both research parts. Before the selected measures are applied, each one is defined and analyzed. The measures that will be utilized are; the Sharpe ratio, Jensen's alpha and risk-adjusted performance (often referred to as M – squared). Both the Sharpe ratio and the M – squared compare total risk, which capture both systematic and unsystematic risk. The Jensen's alpha measure however assesses the unsystematic risk.

To be able to calculate these performance measures, a relevant risk-free rate is required. The risk-free rate is the minimum rate of return the investor expects to receive in the market because he will not accept additional risk unless the potential return is higher. Treasury bills (T-bills) are commonly viewed as 'the' risk-free asset since their short term nature should make their values less sensitive to changes in interest rates (Bodie, Kane and Marcus, 2009). However, in a long term perspective this picture might alter. Although their standard deviation is low over short-term intervals, their percentage basis can significantly increase as the time period lengthens, making them effectively an increasingly risky asset as the investment time horizon grows (Elton et al., 2007). Therefore, in practice there is no such thing as a risk-free rate but short-term government notes are usually considered most appropriate.

The risk-free rate used for calculations of the performance measures are 3-month Treasury bills derived from Bank of England. The average rate of return is utilized since the short term interest rates fluctuate considerably over the sample period; accordingly they should be more appropriate to represent the risk-free rate (See risk-free rate graph in Appendix D). Consequently, when portfolios are structured for sub-periods the average risk-free rate for corresponding period is used, not the last day's rate.

7.1 Benchmark

To utilize the performance measures and effectively make a comparative analysis, an appropriate benchmark is required. A benchmark can represent whatever is relevant for comparison, whether it is a geographic comparison, asset class comparison or comparing individual assets within the same asset class.

7.1.1 Benchmark for Research Part I

To be able to measure how private equity performs compared to the market, a relevant benchmark needs to be a good representative of the general equity market. Since the investor‘s scope of this thesis was a global investor, this benchmark must contain global equities. With this in mind, the MSCI World Index was chosen to be the appropriate benchmark.

The MSCI World Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed markets. The MSCI World Index has been calculated since 1969 and is often considered a good representative for the global stock market as it consists of 24 developed country indices and 1,500 companies[17].

7.1.2 Benchmark for Research Part II

To be able to measure how private equity performs in a mixed asset portfolio, in terms of risk and return, a comparison needs to be done of two portfolios where all things equal, one contains the opportunity to invest in private equity while the other does not. The benchmark is therefore referred to as the Initial portfolio, which represents the general asset classes that private investors commonly include in their portfolio; stocks, bonds and gold.

7.2 The Sharpe Ratio

Investors face a tradeoff between risk and reward. In 1966 William Sharpe introduced a ratio he called *Reward to Variability*; this ratio would ultimately come to be known as the Sharpe Ratio. This ratio is used as a measure and ranks funds performances. Jack L. Treynor had introduced his portfolio performance measurement, the Treynor Ratio, a year earlier and Sharpe extended his work. The difference between the two ratios is that the Sharpe ratio maximizes expected return of a fund in relation to total risk while the Treynor ratio maximizes a company return in relation to its specific risk.

The Sharpe Ratio is calculated by

$$SR = \frac{\overline{r_p} - \overline{r_f}}{\sigma_p} \tag{7.1}$$

where $\overline{r_p}$ is the average return on portfolio p and $\overline{r_f}$ is the average risk free rate and σ_p is the standard deviation of the returns (Bodie, Kane, & Marcus, 2009). All things being equal, the

[17] Following countries make up the index: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

higher the Sharpe ratio the better the performance of the fund, in terms of average return to perceived risk. The Sharpe ratio in itself is not very informative; it is only useful when comparing funds or portfolios to a benchmark.

7.2.1 Significant Test of the Sharpe Ratio

The Sharpe ratio is a good tool to rank the performance of funds and portfolios. However, without measures of significance or confidence interval, the inferential value of this ranking is limited.

The method that is utilized was presented by Jobson and Korkie (1981) and later corrected by Memmel (2003, cited in Opdyke, 2007). Since the two Sharpe ratios cannot be considered independent the distribution needs to be asymptotical. Hence, Z test statistics are utilized to perform the test; they are asymptotically distributed as a standard normal

$$Z = \frac{SR_A - SR_B}{\sqrt{\theta}} \tag{7.2}$$

where the difference of the Sharpe ratios is divided with the asymptotic standard deviation. The asymptotic variance of the difference of the Sharpe ratios is calculated by

$$\theta = \frac{1}{T-M} * \left(2 - 2\rho_{A,B} + \frac{1}{2}\left(SR_A^2 + SR_B^2 - 2SR_A SR_B \rho_{A,B}^2\right)\right) \tag{7.3}$$

where SR_A and SR_B are the Sharpe ratios, $T - M$ is the sample size and $\rho_{A,B}$ is the correlation of the returns.

The following null hypothesis is put forward: H_0: $SR_A - SR_B = 0$. To test whether the hypothesis can be rejected or not, probability values (p-value) are computed from the Z statistics. They are compared to a relevant critical value of a two-tailed test, representing 5 percent level of significance. If the hypothesis is rejected then the differences of the Sharpe ratios are statistically different from zero and it is possible to conclude that the index or the portfolio truly performs better than the benchmark.

7.3 Jensen's Alpha

Michael Jensen (1967) derived a measurement of portfolio performance that is risk adjusted. This measure, known as Jensen's Alpha, estimates how much of the fund's return is due to the manager's forecasting ability. The Jensen's alpha measurement is built from the following asset pricing model formula:

$$\tilde{r}_{jt} - \overline{r_f} = \beta_j * \left(\tilde{r}_{Mt} - r_{ft}\right) + \tilde{e}_{jt} \tag{7.4}$$

The term on the left represents the risk premium earned by portfolio *j*. If the asset pricing model is valid then this premium is equal to $\beta_j * \left(\tilde{r}_{Mt} - r_{ft}\right)$ plus the error term. This equation is good estimator of the systematic risk by an unmanaged portfolio or a single security, however it does not inform anything about the manager's forecasting ability. Allowing for such forecasting ability can be done by simply not letting the regression pass through the origin. In other words, the existence of a non-zero constant is allowed by using following equation

$$\tilde{r}_{jt} - \overline{r_f} = \alpha_j + \beta_j * \left(\tilde{r}_{Mt} - \overline{r_f}\right) + \tilde{u}_{jt} \tag{7.5}$$

where the new error term $\tilde{u}_{jt}$ will now have $E\left(\tilde{u}_{jt}\right) = 0$, and should be serially independent. The Alpha represents the average incremental rate of return on the portfolio per unit time, which is solely due to the manager's ability to forecast future security prices. If a positive alpha is observed, then the manager has an ability to predict market movements in security prices. Conversely, a negative alpha indicates no forecasting ability from the manager and the return does not exceed the benchmark. A naïve buy-and-hold strategy can be expected to yield a zero intercept.

In order to make a deduction whether the alpha is statistically different from zero, a measure of standard error of the estimate is needed. The theory on ordinary least square regression (OLS) provides an estimate of the dispersion of the sampling distribution of the alpha. The distribution of the alpha is by a student *t* distribution with $n_j - 2$ degrees of freedom. This provides the information needed to make the deductions regarding the statistical significance of the estimated performance measure (Jensen, 1967).

7.4 Risk-Adjusted Performance (M^2)

When a fund outperforms its benchmark on the basis of total return, one should ask if that performance was achieved only by taken on more risk. Modigliani and Modigliani (1997) propose a measure of the performance by any managed portfolio against a relevant unmanaged market portfolio. This method makes the performance comparison only after the portfolio return has been properly adjusted for risk. Risk adjusted performance (RAP), is usually referred to as M – squared and is calculated in the following way

$$RAP = \left(\frac{\sigma_m}{\sigma_i}\right) * \left(r_i - r_f\right) + r_f$$

$$= \left(\frac{r_i - r_f}{\sigma_i} * \sigma_m\right) + r_f \qquad (7.6)$$

which is essentially the Sharpe ratio scaled by the standard deviation of the market, or a benchmark portfolio. The risk adjusted performance of a fund can now be compared and ranked in the same manner as the Sharpe ratio. By comparing M^2, the fund can be identified, which achieves the highest risk adjusted return, at any given risk level (Scholz and Wilkens, 2005).

According to Scholz and Wilkens (2005) the differential return can be calculated in alternative way:

$$Differential\ return = \left(\frac{r_i - r_f}{\sigma_i} - \frac{r_{benchmark} - r_f}{\sigma_{benchmark}}\right) * \sigma_{benchmark}$$

$$= \left(Sharpe_p - Sharpe_{benchmark}\right) * \sigma_{benchmark} \qquad (7.7)$$

On the basis of risk-adjusted performance, fund *p* outperforms the benchmark whenever the differential return is positive. Since Sharpe ratios are also calculated in this thesis, the most informative way is to calculate M^2 by this equation.

Since the M^2 is measured in basis points, a differential return can be calculated by subtracting two M^2 measures, as stated in the following:

$$Differential\ Return = RAP_i - RAP_M \qquad (7.8)$$

Here is where the strength of the M^2 comes through, as it can be used both for ranking and in calculating differential returns in basis points. Therefore it is easily understandable for all novice private investors.

8 Research Part I: Private Equity Performance

8.1 Introduction

This chapter tests the first hypothesis; *private equity outperforms the global stock market, in terms of risk and return,* which was put forward in Section 2. Private equity is frequently assumed to provide attractive risk-adjusted returns and furthermore to have relative low correlations with stocks and even lower compared with bonds. As mentioned in the literature review, these findings are widely questioned in the academic studies. A comprehensive sample of listed private equity (LPE) vehicles are investigated and analyzed. The obvious advantage of LPE is the availability of market data, which enables straightforward measure of risk and return characteristics. While performance of traditional private equities is not easily measurable, these advantages of LPE results in more reliable performance measures. Therefore LPE is used as an appropriate proxy to represent the private equity market segment as a whole.

8.2 Data Providers

There are several intermediaries available that offer an overview of LPE companies which were developed to measure the performance of this sector. They are dispersed around the globe and furthermore divided into varying sub-samples of the LPE universe. Collectively, they represent a widespread of the LPE universe but they are not always accessible for a private investor to trade directly.

8.2.1.1 LPX Index Family

LPX GmbH was first to publish a LPE index and is probably the most known LPE index. The LPX index series was launched in 2004 and a family of indices has been developed since inception. The index family consists of global indices, regional indices and style indices. In order for a company to be considered as a possible constituent in an index, at least 50 percent of its business must be in the area of private equity. The companies considered for an index have to fulfill five liquidity constraints as well. These constraints consist of; a maximum bid-ask spreads ranging from 1.5 to 4.0 percent. An average minimum market capitalization of EUR 20–150m, an average minimum daily trading volume (measured relative to the market capitalization) is 0.03 to 0.10 percent, minimum trade continuity of 75 to 98 percent, and a minimum number of price observations of 150. Rebalancing of the index is done twice a year. (LPX Group, 2010).

8.2.1.2 S&P LPE Index

The Standard & Poor's Listed Private Equity Index consists of 30 large, liquid private equity stocks that are traded on exchanges in North–America, Europe and the Asia Pacific. They must meet constraints related to exposure, frequency of investments, size and liquidity. The constituents considered are all listed companies in an S&P database CapitalIQ which have private equity terms in their business description. Standard & Poor's reviews the business description and documents that are publicly available, and allocate an exposure score of 1, 0.5 and 0. Only companies scoring 1 or 0.5 are eligible for inclusion, other companies are not exposed enough to the private equity business.

Furthermore, constituents have to meet certain liquidity constraints; a minimum market capitalization of USD 150m, a three month average daily trading value of USD 500,000, traded on a developed market exchanges and on average at least 10,000 shares daily for the preceding twelve months. Constituent's weight is hence liquidity driven to meet the need for high basket liquidity. Further constraints regard the weighting of individual constituents and their sums. No single stock can exceed a weight greater than 7.5 percent. The sum of the weights for those constituents with weighs more than 4.0 percent must be less than 36 percent. If there are companies in the index with an exposure score of 0.5, the sum of these companies' weights cannot exceed 15 percent. (Standard and Poor's, 2009).

8.2.1.3 LPE_Q

Listed Private Equity (LPE_Q) is a non-profit association of LPE companies that are listed on the London Stock Exchange and other major European exchanges. There are 20 members in the association with combined market capitalization of EUR 8bn. LPE_Q was formed in 2006 and began as a sector marketing initiative. Its mission is to improve levels of knowledge and understanding regarding listed private equity. They intend to do this by commissioning and publishing regular independent research. LPE_Q's members are dedicated to increasing transparency in the private equity universe and raising awareness of the important role that private equity plays in the economy. The transparency and awareness is reached by improving understanding of how private equity funds invest and create value in their investments. (Listed Private Equity (LPE_Q), n.d.).

8.3 Data

8.3.1 Base Sample Analysis

Vehicles are classified as LPE if the underlying business is private equity investments, but the funds themselves are traded on a stock exchange. Companies which do not fundamentally invest in private equity are therefore not part of the sample, e.g. investment banks, holding companies and similar financials. The instruments have diverse underlying structure within private equity business and include vehicles that invest in private equity either directly, i.e. investing in private companies, or indirectly, i.e. through private funds.

By gathering data from LPX, S&P LPE index and LPEQ, a total of 115 vehicles were identified. Several companies overlap between these intermediaries but all of them include some exclusive vehicles. Out of the total sample of 115 vehicles, relevant data was available for 108 vehicles. Therefore seven instruments were deleted from the sample at this point meaning that these 108 vehicles are representative of the base sample. The required data was collected by the use of Datastream 5.0 where weekly observations for the period 1 January 1998 to 9 March 2011 were extracted. Table 8.1 denotes how many vehicles were listed each year throughout the observation period. The years 2006 and 2007 stand out in respect to frequent listings, counting for 16 and 19 new listings respectively. Alternatively, the year 2002 stands out with no new vehicle being listed.

The table displays how many occurance of listings take place each year of the sample period.

Years of listings	Number of Companies	Percentages
Prior to 1998	35	32%
1998	2	2%
1999	5	5%
2000	4	4%
2001	6	6%
2002	0	0%
2003	2	2%
2004	5	5%
2005	7	6%
2006	16	15%
2007	19	18%
2008	3	3%
2009	1	1%
2010	3	3%
Total	108	100%

Table 8.1 Number of listings over the sample period

At the beginning of the sample period, only 35 companies were listed, which exhibits the substantial growth in this market segment. Figure 8.1 illustrates how the accumulative listing steadily increases over the sample period.

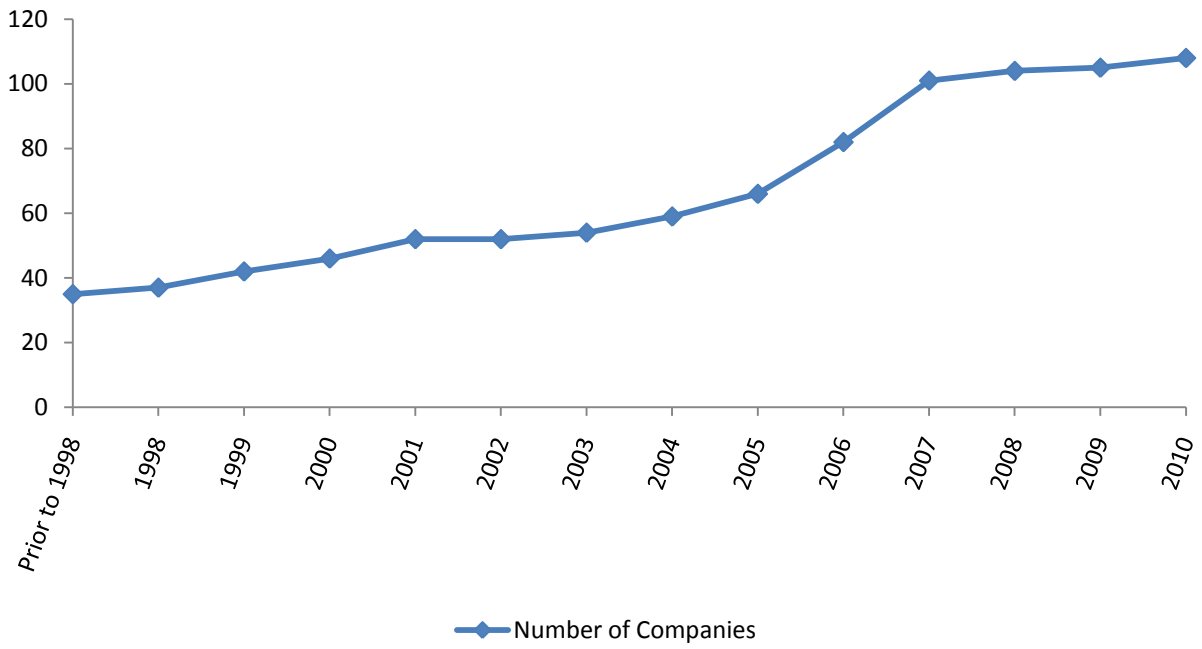


Figure 8.1 Accumulative listings of vehicles

When splitting the vehicles by their LPE structure (presented in Chapter 5.2.2), the majority of the companies are *Listed direct private capital companies* with 80 companies that fall into this category. 24 companies out of the base sample fall into *Listed indirect private equity investment companies (funds of funds)* category and only four companies are *Listed private equity fund managers*. This division is illustrated in Figure 8.2. Therefore it can be concluded that the vast majority of our sample invest directly in the underlying companies and not through limited partnerships.

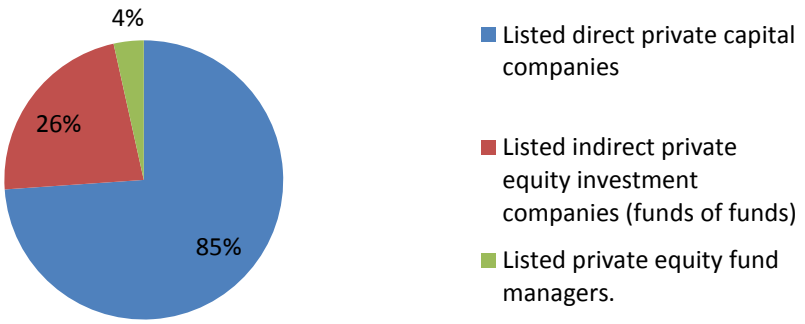


Figure 8.2 Organizational Structure of LPE

With respect to geographical distribution and market capitalization, the distribution into continents and countries is displayed in Table 8.2. The vehicles are globally dispersed where Europe contains the largest portion where almost 70 percent of the companies are listed. One-fourth of the companies are listed in N-America and remaining 6 percent of the sample is listed in other continents. With respect to individual countries, most of them are listed in the UK (34%), which is mainly due to tax alleviation, according to Zimmermann et al. (2005). The US also contains large portion (24%) while the remaining companies are well dispersed among other countries.

Table 8.2: Regional characteristics of the vehicles.

Vehicles are divided into continents and countries

Region/ Country	Number of Companies	Percentage	Market Value (mGBP)	Percentage	Mean Market Value (mGBP)	Median Market Value (mGBP)
Americas	**27**	**25.00%**	**27,128**	**46.01%**	**1,005**	**292**
US	26	24.07%	24,513	41.57%	943	292
Canada	1	0.93%	2,615	4.44%	2,615	2,615
Asia/Pacific	**5**	**4.63%**	**1,455**	**2.47%**	**291**	**169**
Australia	1	0.93%	29	0.05%	29	29
Hong Kong	1	0.93%	194	0.33%	194	194
Japan	2	1.85%	1,064	1.80%	532	532
Singapore	1	0.93%	169	0.29%	169	169
Europe	**75**	**69.44%**	**30,103**	**51.05%**	**401**	**154**
Austria	1	0.93%	31	0.05%	31	31
Belgium	1	0.93%	813	1.38%	813	813
Denmark	1	0.93%	70	0.12%	70	70
Finland	2	1.85%	147	0.25%	73	73
France	4	3.70%	6,113	10.37%	1,528	1,427
Germany	7	6.48%	1,095	1.86%	156	191
Greece	1	0.93%	213	0.36%	213	213
Ireland	1	0.93%	71	0.12%	71	71
Italy	2	1.85%	472	0.80%	236	236
Netherlands	4	3.70%	3,793	6.43%	948	623
Spain	1	0.93%	118	0.20%	118	118
Sweden	5	4.63%	3,501	5.94%	700	291
Switzerland	8	7.41%	3,821	6.48%	478	92
UK	37	34.26%	9,845	16.70%	266	122
Rest of the World	**1**	**0.93%**	**278**	**0.47%**	**278**	**278**
South Africa	1	0.93%	278	0.47%	278	278
Total	**108**	**100%**	**58,964**	**100%**		

In further examination of corrasponding sizes of the vehicles, the market capitalization of the vehicles varies considerably. Furthermore, it is apparent that eventhough Europe which counts for almost 70 percent of the companies counts for only about 50 percent of the market value. While the Americas shifts from having 25 percent of the companies to having 46 percent of the market value. As for the individual countries, UK which represents roughly 34 percent of the vehicles in the base sample represents only 16.7 percent of the market value. This is not true for all European countries, where France, Netherlands and Sweeden contain relatively large companies. However, the largest companies are located in the US where 24 percent of vehicles actually count for more than 40 percent of the market value. This shift between quantaties of companies and their sizes is more clearly demonstrated in Figure 8.3.

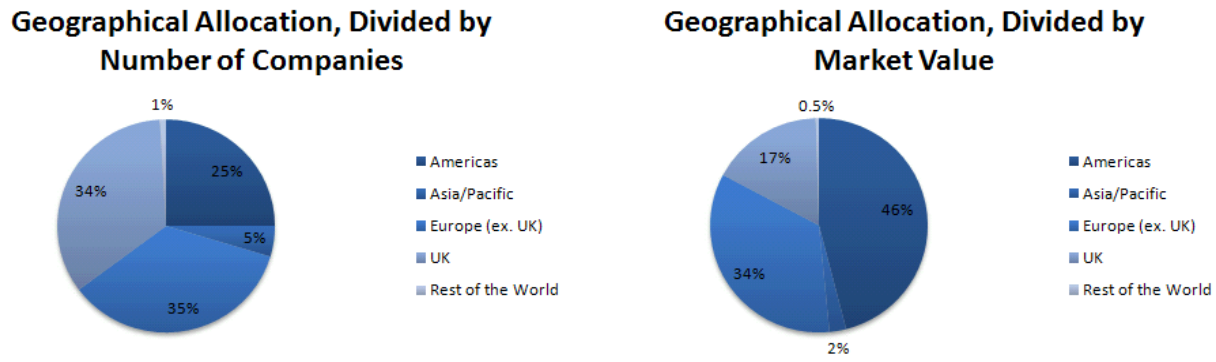


Figure 8.3: Geographical Difference in terms of Quantity and Size

The time period was chosen with respect to being sufficiently long enough for statistical significance and furthermore to include miscellaneous market cycles from optimistic and pessimistic views of investors. This is done to not obtain biased results deriving from a bull or a bear phase only. The years from 1998 to 2011 contains both the phases referred to as the *IT bubble* in 2001 and the recent crisis in 2008, including corresponding prior bull phases. Consequently, the period is additionally divided into boom and bust sub-periods, to assess private equity performance pattern in different cycles. Bergmann et al. (2009) divide their period into four sub-periods and these periods were used as a base for the boom-bust periods in this thesis. These periods match high and low points of the benchmark stock market index (MSCI) in our data, with occasional mismatch of only few days. Therefore, this split is also suitable in our research. Additionally, the last period, counting after most recent low point, is referred to as post crisis period. Table 8.3 denotes the corresponding division split of the five sub-periods.

Table 8.3: The division of the sub-periods.

	Sub-Periods		
Boom 1	January 1998	-	February 2000
Bust 1	March 2000	-	March 2003
Boom 2	April 2003	-	June 2007
Bust 2	July 2007	-	December 2008
Post Crisis	January 2009	-	March 2011

We chose not to increase the sample period further back than 1998 since we wanted our initial number of companies to be a representative of well diversified sample. Statman (1987) argued that over 30 securities are sufficient to have well diverse sample. However, if we had gone further back, as e.g. Zimmermann et al. (2004 & 2005) who begin with only eight companies, we could potentially have a bias in regards to too small sample.

8.3.2 Liquidity Constraints

In order for the LPE index to be comparable to the benchmark, a set of liquidity constraints are applied to the sample. Practically all private equity indices utilize one or more constraints on liquidity which can be measured by various market characteristics. To be considered for inclusion in our index, vehicles must meet all of the following criteria:

Minimum number of price observations: 30 weekly price observations are required to ensure accuracy of parameter estimates.

Minimum average market capitalization: An average market capitalization of minimum 20 Million GBP.

Minimum trading activity: To ensure a minimum trading activity, the relative trading volume must be at least 0.2 percent per week, defined by:

$$Relative\ volume_t = \frac{Trading\ volume_t * Price_t}{Market\ value_t} \qquad (8.1)$$

Trade continuity: Trade continuity is measured by the percentage of weeks where at least one transaction takes place. It is set to be at least 85 percent.

Maximum average bid-ask spread: To ensure not too large spread between bid-price and ask-price, a maximum spread is set to 5 percent. The bid-ask spread is defined by

$$Spread_t = \frac{Ask_t - Bid_t}{MidPrice_t} \qquad (8.2)$$

where the mid-price is defined as the arithmetic average of the ask and bid quotes.

For calculations of these liquidity constraints, following data was gathered; price (P), market capitalization (MV), turnover by volume (VO), price-bid (PB) and price-ask (PA).[18]

Relevant criteria utilized in this thesis are based on the constraints used by Zimmermann et al. (2005) which state that their level of constraints is chosen a bit arbitrarily. Their types of constraints are considered sound and valid but as they state, their levels can be questioned. By increasing the level of constraints will result in excluding further illiquid vehicles. The research by Zimmermann et al. is made 2005, which is prior to the IPO surge experienced in the years 2006 and 2007. In their latest work, the making of LPX, those levels of constraints

[18] Abbreviations in parentheses are the data-type names in Datastream.

have been intensified[19]. Since LPX's initiation, the awareness of LPE market has become more efficient and accordingly the liquidity problem becomes less severe. Accordingly, in this research the levels of constraints are also intensified and in some way chosen a bit arbitrarily. Zimmermann et al. (2005) allow non-trading in 85 percent of instances in their research and this level is re-evaluated. Minimum continuity of trade has been 'turned around' with minimum trading must exceeding 85 percent of instances, coherence with 80-98 percent range in LPX's latest modification. The increased level of constraints was mainly done to minimize the data biases which are experienced and corrected for in the studies by Zimmermann et al. (2005), and Lahr & Herschke 2009.

For comparison, Table 8.4 denotes the number of vehicles in the liquid sample under the two alternative levels of liquidity constraints.

Table 8.4 Number of vehicles under alternative liquidity constraint levels.

This table demonstrates the changes in number of vehicles under the constraints levels utilized by Zimmermann et al. (2004 & 2005) and Lahr & Herschke (2009), and our intensified level of constraints. 'Minimum market capitalization' is denominated in GBPm.

	Liquidity constraints					
	Minimum of weekly observation	Minimum average market capitalization	Minimum relative trading volume	Minimum continuity of trade	Maximum average bid-ask spread	Number of Vehicles
Zimmermann et al. (2004 & 2005) & Lahr & Herschke (2009)	30	2	0.10%	15%	20%	99
Our Thesis	30	20	0.20%	85%	5%	77

99 vehicles would constitute our liquid sample that fulfills the constraints used by Zimmermann et al. (2005), and Lahr & Herschke (2009). With the intensified level of constraints 77 vehicles fulfill the constraints, resulting in roughly 30 percent of vehicles excluded from the base sample.

In sensitivity analysis of the constraints, which can be seen in Appendix E, it is apparent that the bid-ask spread is the most sensitive constraint. When the spread was decreased from 5 percent to 3 percent the number of vehicles dropped from 77 to 66 and when it was increased from 5 percent to 7 percent the number of vehicles rose from 77 to 82. This stricter bid-ask spread is regarded to minimize the bid-ask bias experienced the other studies. When other constraints were altered the change was much less dramatic, apart from the large modification of the minimum continuity of trade which is the second most sensitive constraint.

[19] LPX make 11 indices which differ in levels of liquidity constraints. The interval of the constraints are following; a minimum of 150 price observations, minimum market value in the range of 20-150M EUR, minimum trading volume in the range of 0.03-0.1%, minimum continuity of trade in the range of 75-98% and maximum bid-ask spread between 1.5-4%. (LPX Group, September 2010)

Accordingly, there is evidence that illiquidity is still present, even in the listed private equity (LPEs) segment.

According to Lahr & Herschke (2009), the benefit of increasing the constraints is that only most liquid vehicles are qualified, but the disadvantage is increased selection bias toward better performing funds. The potential compromises of this bias are assessed by performance persistency analysis in the following sub-chapter. However, it is thought sensible to intensify the levels with regards to LPX's latest modifications in mind and the sensitivity analysis of the constraints.

8.3.3 Performance Persistence

In order to analyze performance persistency of individual vehicles, a percentile analysis of all 77 vehicles are calculated. By a measure of yearly returns, vehicles are ranked into quartiles representing an interval of 25 percent each. Resulting quartiles are thus classified into top and bottom ranking, and consequent quartiles in-between. The period is divided both on yearly basis and into pre-defined sub-periods as Table 8.5 denotes.

Table 8.5: Quartile analysis.

This table presents the top and bottom quartiles of the liquid sample of 77 vehicles. Percentages denote the lowest rate of yearly return for a correspondent quartile.

	Top Quartile	-	-	**Bottom Quartile**
1998	17.66%	4.19%	-8.71%	-69.41%
1999	72.87%	30.18%	17.75%	-121.60%
2000	22.98%	-0.13%	-30.08%	-219.54%
2001	4.42%	-16.29%	-31.29%	-161.08%
2002	-3.38%	-20.92%	-34.93%	-93.38%
2003	36.92%	21.27%	5.98%	-110.84%
2004	31.87%	17.51%	4.13%	-49.11%
2005	33.01%	25.85%	11.53%	-22.54%
2006	27.11%	17.68%	7.41%	-73.87%
2007	7.89%	-5.30%	-21.18%	-52.00%
2008	-36.75%	-63.29%	-97.77%	-202.71%
2009	73.61%	42.19%	23.27%	-54.56%
2010	38.51%	24.73%	12.41%	-59.28%
Boom period 1	59.64%	33.93%	18.44%	-33.43%
Bust period 1	-8.21%	-16.55%	-41.45%	-92.40%
Boom period 2	31.15%	20.76%	9.19%	-77.63%
Bust period 2	-33.63%	-50.07%	-75.05%	-152.80%
Post Crisis	55.42%	33.49%	19.23%	-36.97%

To be positioned in relevant quartile, a company needs to have at least the return presented in corresponding column. This categorization can be a very useful statistical device in assessing the best performing companies; however it does not state the spread of the observations in within corresponding quartile. As can be seen from the table, to be eligible for the top quartile, the returns need to be positive in all years except for the 'hangover' years in 2002 and 2008. Bear in mind, it only marks the bottom return of the quartile, not the top return. An indication of high volatility in this market is well notable around the previous mentioned years and furthermore by examining the sub-periods, which fluctuate severely in coherence to market cycles.

A summary statistics of the best performing vehicles, i.e. vehicles that are most frequent in the top quartile, can be seen in Table 8.6. (Ranking of all individual vehicles are presented in Appendix F) Nine vehicles are five years, or more often, out of the total 13 in the top quartile. It is however interesting to note that in corresponding analysis of how frequent these same vehicles appear in the bottom half, seven out of those nine vehicles are equally or more often in the bottom half of the sample.

Table 8.6: Performance ranking of the most frequent vehicles in the top quartile.

This table shows the ranking of the top nine vehicles, on yearly basis. The number 1 indicates a ranking in the top quartile while the number 4 indicates a ranking in the bottom quartile.

Index Constituents	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Frequency in the top quartile	Frequency in the bottom half
BURE EQUITY	2	4	1	2	4	4	1	2	1	1	1	3	4	5	5
CHINA MRCH.CHINA DINV.	4	3	3	1	1	1	4	4	1	1	4	1	4	6	7
JAPAN ASIA INVESTMENT	3	1	4	4	1	1	1	1	4	2	4	2	2	5	5
ONEX	1	1	3	1	3	4	2	4	1	1	2	3	2	5	5
PANTHEON INTL.PARTS.	3	2	1	3	1	4	3	1	3	2	4	1	1	5	6
RATOS 'B'	3	3	2	1	1	3	2	1	1	1	1	3	2	6	4
SAFEGD.SCIENTIFICS	2	1	4	1	4	1	4	4	1	3	3	1	1	6	6
WENDEL	1	3	2	2	4	1	1	1	1	3	3	2	1	6	4
MCG CAP.				1	3	1	4	4	1	4	4	1	1	5	5

This frequent jumping, between top and bottom, indicates that the past is not always a good indicator of the future and persistent performance is therefore not certain in this market segment. This indicates that the potential bias towards better performing funds in our sample, as was posed in last sub-chapter, will not be a severe problem.

These findings demonstrate a great inconsistency of performance in this market segment, which is contrary to previous studies by Kaplan and Schoar (2005). This difference might be explained by the alternative dataset used in these studies. Kaplan and Schoar perform their study on traditional private equity investments which might elucidate one explanation of the

difference between traditional private equity and LPEs. That is, when vehicles are listed on public stock exchanges they show similar symptoms to traditional stocks, i.e. they follow some kind of random walk.

This inconsistency between the LPE market and the traditional PE market represents that when investing in the LPE market segment, it seems more suitable to invest in an index rather than relying on picking individual vehicles. As often regarded true for traditional private equities (see Fraser-Sampson, 2007). This finding is suitable for private investor's perspective, which might not have enough resource to be able to successfully pick the best performing vehicles.

8.3.4 Distribution of the Data

8.3.4.1 Normality

Most financial models rely on the assumption that the data is normally distributed. It is an important assumption as it makes risk calculations more straightforward. To be able to apply the performance measures and asset allocation strategies, it is paramount to assume that the data is normally distributed. However, this normality assumption is rarely met, as historical returns tend to have fat tails. (Jorion, 2007).

The density function is symmetrical around the mean in the normal distribution where the Probability Density Function (PDF), mean and variance are defined as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)\right]^2} \tag{8.3}$$

$$\mu = E[X]$$

$$\sigma^2 = var[X]$$

The normal distribution can be fully described by the first two moments, mean and variance, that is $N(\mu,\sigma^2)$. The normal distribution is called standard normal distribution when it is tabulated with mean zero and variance unity, that is, $N(0,1)$. As mentioned before, there are many financial models that are based on the assumption that returns are normally distributed. If the investor chooses his investment based on mean-variance analysis alone, he is ignoring the higher moments, the third and fourth moments referred as the skewness and kurtosis.

8.3.4.2 Higher Moments (Skewness and Kurtosis)

The mean and the standard deviation are the only parameters necessary to fully describe the normal distribution. To analyze a time series further it is useful to evaluate two other moments. The distribution's skewness is a third moment and describes how much the distribution deviates from symmetry. The skewness of a normal distribution is S = 0, because the normal distribution is completely symmetric. Skewness is defined as:

$$S = \frac{\left\{\int_{-\infty}^{+\infty}[x - E(X)]^3 f(x)dx\right\}}{\sigma^3} \tag{8.4}$$

Negative skewness indicates that the left tail is longer than the right tale and therefore generates larger negative values than positive. The opposite is true for a positive skewness (Jorion, 2007).

Kurtosis is the fourth moment and it describes how quickly (or slowly) the tails decay. The kurtosis of a normal distribution is K = 3. If the coefficient is greater than 3 then the tails decay slower than the normal distribution which indicates greater likelihood of larger values, either positive or negative. The kurtosis is defined as:

$$K = \frac{\left\{\int_{-\infty}^{+\infty}[x - E(X)]^4 f(x)dx\right\}}{\sigma^4} \tag{8.5}$$

Since a high kurtosis, or leptokurtic, indicates that there is a greater chance of obtaining large values, a risk-averse investor will prefer a low kurtosis.

These two moments can easily be utilized to validate whether the distribution is close to being normally distributed. If the time series are normally distributed then skewness and kurtosis will not be an issue. According to Scott & Horvath (1980), risk-averse investors prefer high mean and low variance, not both low mean and variance. Consequently, they will also prefer a low kurtosis and a positive skew rather than no skew or negative skew. If the data is not normally distributed then the validity of the performance measures are compromised. Therefore a test of the normality assumption is performed for our data.

8.3.4.3 Test for Normality and Higher Moments

Normality can be detected both graphically and numerically. Graphically, it is performed by examining a histogram of the distribution; however, it is not fully conclusive to merely rely on a graphical inspection. To be certain, a numerical test needs to be performed. There are several numerical tests that check for the normality of the distribution. The most widely used

is the Jarque-Bera test, which uses the higher moments of the distribution to test for normality. The Jarque-Bera test of normality is an asymptotic (or large-sample) test.

The Jarque-Bera (JB) test is defined as follows

$$JB = n\left(\frac{S^2}{6} + \frac{K^2}{24}\right) \sim \chi^2(2) \tag{8.6}$$

where *n* is the sample size, *S* is the skewness coefficient and *K* is the kurtosis coefficient (Gujarati and Porter, 2009).

The JB follows a chi-square distribution with two degrees of freedom. The JB is a test of joint hypothesis, Skewness = 0 and Kurtosis = 3, to assess whether the values depart from the norm. This test was performed with SAS Enterprise Guide. The null-hypothesis is that the series are normally distributed. The results are that this hypothesis can be rejected for all 77 vehicles, at the 0.05 significance level.[20] Accordingly, none of them are found to be normally distributed.

These findings are no coincident as financial series are known to be non-normally distributed. This test was however utilized in order to state this fact and furthermore to acknowledge it as we proceed. Accordingly, normality assumption is assumed and the other two moments therefore ignored. Risk and return characteristics are assumed sufficient to test the corresponding hypotheses, even though they might implicitly be biased estimators. There are ways to tackle this problem but they furthermore may contain other faults that are not part of the scope of this research.

8.4 Index Construction

This chapter presents construction principles of the indices required for investigation of risk and return characteristics of the liquid LPE sample. Performance measurements of a certain asset class are usually based on indices, which can be structured as value weighted (e.g. S&P 500), price weighted (e.g. Dow Jones Industrial Average) or equally weighted[21]. There are a couple of issues that need to be addressed before constructing an index for this market segment. First, the number of listings in the over the sample period steadily increases as was illustrated in Figure 8.1. The liquid sample begins with 32 vehicles which demonstrates that rebalancing, whenever new listing takes place, is exceptional important. Second, the sample

[20] The JB statistics and corresponding P-values for all available vehicles are presented in Appendix B.
[21] A price weighted index assumes that investor is equally likely to buy a share of any stock, irrelevant of the underlying business.

is heterogeneous with respect to the market capitalization of the individual vehicles, as was notable in Table 8.2. In order to capture the difference between vehicles of diverse market values, two types of indices are constructed; a value weighted index and an equally weighted index. Their performance is then investigated in a comparative analysis.

8.4.1 Value-weighted Index (VW)

For the first index, the weights of the constituents are determined by their relative market capitalization. The value of this index at time *t* is calculated by

$$I_t = I_{t-1} \sum_{i=1}^{n} \frac{M_{it}}{M_t} * \frac{(P_{it} + D_{it})Adj_{it}}{P_{it-1}} \quad (8.7)$$

where

I_t is the value of the index at time *t*

M_{it} is the market value of vehicle *i* at time *t* [22]

M_t is the sum of the market values of all the vehicles in the index at time *t*

P_{it} is the price of the vehicle *i* at time *t*

D_{it} is the dividend of the vehicle *i* at time *t*

Adj_{it} is an adjustment factor in case of corporate action, such as stock split or capital increase.

In order to not investigate historical corporate actions of each vehicle, Datastream provides a total return index which takes these actions into consideration and, according to Lahr & Herschke (2009), the last term is a ratio between two return indices on two consecutive days. It follows that

$$\frac{R_{it}}{R_{it-1}} = \frac{(p_{it} + D_{it})Adj_{it}}{p_{it-1}} \quad (8.8)$$

where R_{it} is a return index for vehicle *i* at time *t* and R_{it-1} is a return index for vehicle *i* at time *t-1*. By combining equations 8.7 and 8.8, the following equation is used to construct the value weighted index:

$$I_t = I_{t-1} \sum_{i=1}^{n} \frac{M_{it}}{M_t} * \frac{R_{it}}{R_{it-1}} \quad (8.9)$$

[22] M_{it} is calculated by shares outstanding (NOSH) x Price (P).

The index is rebalanced weekly and new listings are considered at the first rebalancing date after listing occurred.

8.4.2 Equally-weighted Index (EW)

For the second index, an equal weight is allocated to all constituents and to maintain the weights, rebalancing is done on weekly basis. Since the value weighted index allocates most of its weights to only a few companies, this index provides a good comparison as to whether larger constituents perform better than smaller ones. The value of the index at time *t* is calculated by:

$$I_t = I_{t-1} * \frac{1}{n} * \sum_{i=1}^{n} \frac{(P_{it} + D_{it})Adj_{it}}{P_{it-1}} \tag{8.10}$$

where I_t is the value of the index at time *t*, *n* is the number of vehicles in the index at time *t*. P_{it} and D_{it} are the price and the dividend of the vehicle *i* at time *t* and Adj_{it} is an adjustment factor in case of a stock split or capital increase. After applying the correction made by Lahr & Herschke (2009) the equally weighted index is calculated as:

$$I_t = I_{t-1} * \frac{1}{n} * \sum_{i=1}^{n} \frac{R_{it}}{R_{it-1}} \tag{8.11}$$

As with the value weighted index, new listings are considered at the first rebalancing date after their listing.

8.5 Empirical Results on Private Equity Performance

Price performance of the two LPE indices and the benchmark stock index is illustrated in Figure 8.4. All indices are rebased at the price of 1000 in the beginning, as if equal amount of capital is invested in each index at the beginning of the sample period. By graphical inspection, both LPE indices seem to perform better than the market in general, and moreover exhibit more volatility.

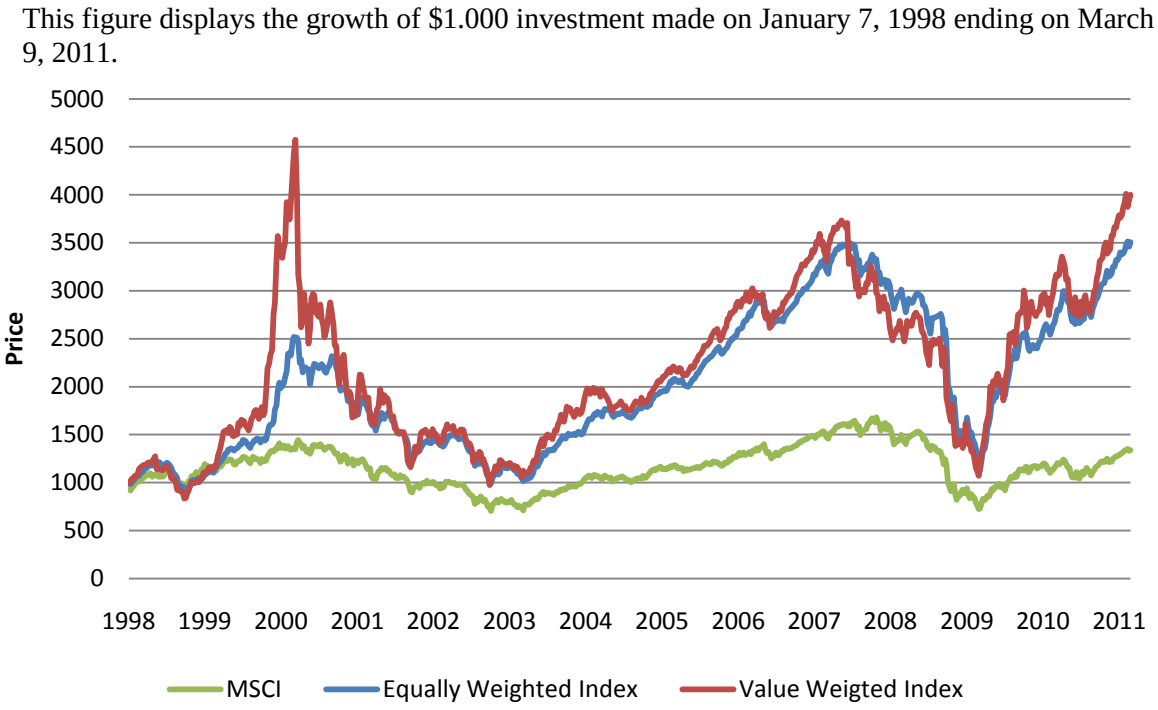


Figure 8.4: Performance of the Indices.

In a comparative analysis between large and small vehicles, the composition of the value-weighted (VW) index is 68.51 percent of constituents are made up by only the ten largest vehicles. On the other end, 38 companies out of the total of 77 (or about 50 percent of the constituents) make up only 7.23 percent of the index, based on market capitalization.[23]

Since value weighted index allocates excessive weights to few companies but equally weighted (EW) index allocates equal weights, the deviation between them gives implication of performance between small and large vehicles. By a graphical inspection, the price deviations of the two LPE indices are relatively similar, indicating that larger vehicles do not perform better than smaller ones in this market segment.

[23] The composition of the index can be found in Appendix H.

8.5.1 Descriptive Results

More comprehensive information is presented in Table 8.7 where risk and return characteristics, among other statistical measurements, are revealed for the three indices. The table presents weekly observations of the three indices over the whole time period[24]. All time series refer to continuously compounded rate of returns, computed by

$$\ln\left(\frac{P_t}{P_{t-1}}\right) \tag{8.12}$$

Table 8.7: Descriptive Statistics of Weekly Returns.

As the research is based on weekly observations Min and Max figures must be denominated on same frequency-level. As a result, the risk and return characteristics in this table are reported on weekly basis. Skewness, kurtosis and correlations are not in percentage. Correlation are measure of values compared to the MSCI index.

	Min	Median	Mean	Max	SD	Skewness	Kurtosis	Correlation
MSCI	-16.55%	0.25%	0.05%	9.27%	2.49%	-0.75	3.99	1.00
Equally Weighted Index	-19.64%	0.41%	0.18%	10.83%	2.65%	-0.91	6.50	0.79
Value Weigted Index	-23.69%	0.45%	0.20%	14.86%	4.18%	-0.54	3.74	0.76

The returns on VW index have largest deviation in negative and positive returns measured by the min and max statistics. Correspondingly, the volatility of returns is highest for VW index (4.18%). This can be explained by the lack of diversification benefits of the VW index because of heavy weight on few vehicles. The EW index has slightly lower return than VW, but significantly lower risk measure (2.65% versus 4.18%). The risk measure of EW index is somewhat similar to that of MSCI, which however provides significantly lower returns (0.05%) than both of the LPE indices.

When analyzing the relationship between the statistics of the average it is apparent that mean returns are lower than median returns for all indices. This result might indicate that few vehicles that experience extreme low rate of return affect lower mean, as outliers impact mean more than they do on median. This is contrary to e.g. Schmidt (2006), which experience few extremely well-performing funds that cause abnormal high average returns. This deviation is proportionally larger for MSCI than for both LPE indices as the median for MSCI is five times higher than the mean but for both LPE indices the median is roughly more than double the size of the mean. Since MSCI has lower fluctuation between extreme negative and positive returns, this suggests that there are few extremely large positive returns that are able to pull up the mean.

[24] Corresponding yearly risk and return measures are presented in Table 8.8.

Accordingly to the fact that the mean returns are all lower than the median, all returns are fairly negatively skewed, with EW experiencing lowest skew (-0.90). This suggests that bulk of the values (including the median) lie right of the mean, meaning that left tale of the distribution is longer than the right tale. This finding represents the real-life market cycles over the observation period, which might be reflected by the cluster of bad news (compared to good news) that produces occasional but large drops in prices.

VW index returns do not seem to show additional kurtosis on top of that induced by the market. However, EW index returns show roughly 50 percent larger excess kurtosis which could be due to padded prices, especially of smaller funds according to Lahr & Herschke (2009)[25].

Correlations of both LPE indices are fairly close to the market contrary to several previous studies on traditional private equity. This may indicate bad diversification capability in a portfolio also consisting of stocks. This relationship is scrutinized further in Research Part II, of whether LPE can still improve performance of a portfolio, given this high correlation. Argument for this high correlation might be that since the private equity proxy is listed vehicles, they therefore have tendency to move in-line with the global stock market.

8.5.2 Performance Measures

After having examined the descriptive statistics of the sample, it is useful from an investor's point of view to assess whether LPE offer risk-adjusted returns. Table 8.8 presents relevant performance measures in order to compare the three indices. For practical purpose the table reports risk and returns characteristics on yearly basis and additionally divides the whole period into pre-defined sub-periods.

Analysis of return characteristics in different sub-periods shows that returns of both LPE indices are consistently higher (lower) in boom (bust) periods, compared to the market. This higher volatility is consistent with the implied risk and returns measures of all indices for the full period. More interestingly, when analyzing the sub-period characteristics, is the fact that returns of the post-crisis period have surpassed the returns experienced in the most recent boom period (Boom 2), for all indices. (This outcome is more noticeably illustrated in Appendix G). In the same context, analysis of the risk measures for those same two periods (Boom 2 & Post Crisis) indicates that while the returns have increased by only 1 percent for

[25] Padded prices are used when there is no available price for an entity within a particular week.

MSCI, the corresponding risk measure has increased by 102 percent[26]. Similarly, the LPE indices experience equivalent increased portion in relation to risk and return relationship between the same two periods. In more detail, EW returns increase by 39 percent while corresponding risk increases by 155 percent and VW returns increase by 81 percent with corresponding risk measure increasing by 114 percent. From these results it is worth noting that all indices seem to have fully recovered from last down-phase in comparison of the previous boom period, by measure of returns. In other words, the recovery of the market in general has substantially been reached, and well better. However, some analysts have suggested that there might be a pattern of double-dip in the global economy.

Table 8.8: Performance measures of all indices, divided into sub-periods.

Performance measures of the three indices, divided into sub-periods. Return and risk parameters are the same as in the summary statistic table, but measured on yealy basis.

	Mean	SD	Sharpe Ratio	Jensen's Alpha	M2	βeta
MSCI						
Full Period	2.76%	17.93%	-0.07	-	-	-
Boom period 1	17.17%	15.63%	0.73	-	-	-
Bust period 1	-17.94%	19.91%	-1.13	-	-	-
Boom period 2	16.95%	10.10%	1.25	-	-	-
Bust period 2	-35.92%	26.63%	-1.52	-	-	-
Post-crisis period	17.13%	20.42%	0.81	-	-	-
Equally Weighted Index						
Full Period	9.57%	19.08%	0.29**	7%*	6.48%	0.85
Boom period 1	39.54%	17.54%	1.93**	26%**	18.70%	0.73
Bust period 1	-26.21%	20.15%	-1.53**	-13%	-7.90%	0.78
Boom period 2	28.15%	9.11%	2.61**	15%**	13.75%	0.70
Bust period 2	-54.31%	26.75%	-2.20**	-23%*	-18.12%	0.89
Post-crisis period	39.01%	23.27%	1.65**	23%*	17.17%	0.94
Value Weigted Index						
Full Period	10.50%	30.11%	0.22**	8%	5.11%	1.27
Boom period 1	61.85%	29.03%	1.93**	43%**	18.79%	1.09
Bust period 1	-39.69%	38.86%	-1.14	-13%	-0.18%	1.38
Boom period 2	25.68%	15.43%	1.38	7%	1.36%	1.14
Bust period 2	-54.05%	33.69%	-1.74	-13%	-5.84%	1.14
Post-crisis period	46.55%	32.96%	1.40**	24%*	11.90%	1.38

*/** denotes that the number is significant at the 0.05/0.01 level

By analyzing the characteristics of the performance measures, the risk-adjusted measures confirm previous findings. The LPE indices almost always present a superior Sharpe ratio compare to the MSCI index. The MSCI has a negative Sharpe ratio for the whole period (-0.07), while both the EW and VW indices have a significant positive ratio, 0.29 and 0.22 respectively[27]. For the sub-periods the story is very much the same, the EW index regularly exceeds the ratios of VW and MSCI where the MSCI index always has the lowest ratios. This

[26] MSCI returns are 16.95% for Boom period 2 and 17.13% for Post-Crisis period, meaning an increase of 1.02%. Corresponding risk measures are 10.10% and 20.42%, respectively, calculating an increase of 102%.
[27] The Sharpe Ratio is awkward to interpret when it is negative.

total-risk measure indicates that the LPE indices generally had superior risk adjusted performance compare to the MSCI.

Findings are somewhat similar by the measure of excess return over the stock market, by the measure of Jensen's Alpha. The EW index significantly outperforms the market over the whole period by seven percent. The index significantly outperforms in both boom periods and post-crisis period but underperforms in bust periods. However, the alpha for the VW index did not turn out to be statistically significant over the full period and only two sub-periods were statistically significant. We can then conclude that the higher return of the VW index (e.g. 10.5% versus 2.76% for full period) might mainly be due to correspondingly higher risk.[28]

By analyzing M^2 for the full period, both LPE indices risk-adjust outperform the market by more than five percent. EW index outperforms the VW index by slightly more than one percentage point. When examining the sub-periods there is clear tendency toward LPE indices outperforming in boom periods but underperforming in bust periods. In comparison between the LPE indices it is apparent that EW index outperforms VW for the two boom periods and also the post crisis period but VW outperforms in both bust periods. This is contrary to the higher volatility measure of the VW index and one might be tempted to conclude that larger companies do perform better in market downturns. This might however be problematic to interpret since Sharpe ratios are negative for both bust periods.

Beta coefficient is inversely related to the market risk in a CAPM context by definition, which gives us indication whether systematic risk is similar to the market's risk. The diversification benefits by the beta coefficients are computed by a regression of the LPE indices on the market. The few large constituents of VW feature higher volatility than the EW, measuring 1.27 and 0.85 respectively. For both indices, the beta coefficient is usually higher in the bust periods compared to the boom periods. The beta coefficient for EW and VW are similar to the coefficients of Zimmerman et al. (2005).

Before making a comparative analysis to the studies by Zimmermann et al. (2005) it is worth noting again that the full time periods are not equivalent, apart from the Bust period 1 which matches their second sub-period.[29] Furthermore, only EW is comparable to their fully rebalanced EW index, not their buy-and-hold indices which include less frequent rebalancing.

[28] Full regression results can be seen in Appendix K.
[29] Their study ranges from the years 1986-2003 but ours 1998-2011.

Our EW index generates 9.57 percent average return which is not as high as their fully rebalanced EW index which generated 15.99 percent. However the standard deviations for the full periods are much alike. This lower return is according to expectations since our sample period feature two phases of downturns, which is notable by the low return of MSCI over the full sample period. For the sub-periods, our EW index in the bust period 1 outperforms their second sub-period by almost ten percentage points, computing -26.21 percent and -35.39 percent respectively. The results of the performance measures are in consistent to their study where the Sharpe ratio is somewhat lower, measuring 0.29 in our study while it is 0.57 in their study. However, after they correct for their volatility bias, their ratio drops to very similar level, or 0.33. The alpha of our study outperforms the market by seven percent but around ten percent in their study.

8.5.3 Hypothesis Testing Results for Private Equity Performance

The performance measures should present a solid and quantitative basis for a decision on whether or not the hypotheses can be rejected or not and thereby answer corresponding research question. This summary is done in the end of each research part. The results on hypothesis testing for Research Part I is presented in Table 8.9.

Table 8.9: Results on Hypothesis Testing for Reseach Part I.

The table presents whether the hypothesis can be rejected or not, for a corresponding index and period. Hypotheses that are not rejected are referred to as 'confirmed'.

Private equity outperforms the global stock market, in terms of risk and return.	Sharpe Ratio	Alpha	M^2
Equally Weigthed Index			
Full Period	Confirmed**	Confirmed*	Confirmed
Boom period 1	Confirmed**	Confirmed**	Confirmed
Bust period 1	Rejected	Rejected	Rejected
Boom period 2	Confirmed**	Confirmed**	Confirmed
Bust period 2	Rejected	Rejected	Rejected
Post-crisis period	Confirmed**	Confirmed*	Confirmed
Value Weighted Index			
Full Period	Confirmed**	Confirmed	Confirmed
Boom period 1	Confirmed**	Confirmed**	Confirmed
Bust period 1	Rejected	Rejected	Rejected
Boom period 2	Confirmed	Confirmed	Confirmed
Bust period 2	Rejected	Rejected	Rejected
Post-crisis period	Confirmed**	Confirmed*	Confirmed

*/** denotes that corresponding hypothesis is confirmed by a significance level of 0.05/0.01 respectively.

Overall, there are a total of 36 instances that examine whether the LPE indices outperform the global stock market, in terms of risk and return. The hypothesis is not rejected in 24 instances

out of the total 36, meaning an outperformance of the LPE indices in 67 percent occasions. Out of those 24 instances, 13 occasions denote that LPE significantly outperformed the market, with the remaining also including M^2 that does not contain significance testing. There is a clear tendency for the LPE indices to underperform in the bust periods but outperform in all of the other periods. The results of the hypothesis testing for both indices are always in accordance, with occasional exceptions in significance levels.

Since the majority of the hypothesis testing results cannot be rejected, it can be concluded that, based on the sample of this research, LPE does outperform the global stock market, in terms of risk and return.

8.6 Conclusion on Private Equity Performance

Had an investor been considering an LPE investment back in 1998, the somewhat arbitrarily decision of whether to invest in stocks or LPE would have made profound impact on the subsequent 12-year investment experience. While the world stock market over the sample period has provided average annual return of about two percent, the LPE indices have provided return of roughly ten percent. Although this high return is accordingly followed by higher risk, especially for the value-weighted index, the LPE indices have outperformed the stock market on risk-adjusted basis, as confirmed by the performance measurements.

On which this study is based on, the Sharpe ratios of the EW index significantly exceed the ratio found for the market and moreover deliver a positive alpha of seven percent. The VW index did also outperform, but the difference did not turn out to be statistically significant. There are two reasons that might explain this outperformance of LPE. Firstly, the frequent rebalancing of the LPE indices might result in a 'rebalancing bonus', similar to the findings of Zimmerman et al. where the fully rebalanced index outperforms the partially rebalanced index[30]. Since relatively large number of vehicles in our data sample do not provide over 10-year history, a frequent rebalancing was required for mitigation of this problem. The frequent need of rebalancing might turn out costly for investor in real-life situations and thus question the applicability. However, these procedures were necessary to obtain unbiased measures and catch the development of this immature market segment. Therefore, this should be less of a problem going forward, as this market segment matures. Secondly, negatively skewed distribution leads to underestimation of risk parameters, which might be problematic for the high skewness of the EW index. Consequently, the risk measures might be underestimated

[30] Further information regarding 'rebalancing bonus' can be seen in Bernstein & Bernstein (1997)

and indicate that the EW index might not be performing as well as it appeared. However as stated earlier, the non-normality was acknowledged and the time series need to be assumed normally distributed for applying the standard measures of risk and returns characteristics.

The price deviations of the two LPE indices are relatively similar which indicates that larger vehicles do not perform better than smaller ones in this market segment. Additionally, after having implemented performance persistency analysis like Jones and Rhodes-Kropf (2003), we did not derive the same conclusion. Better performing funds did not seem to consistently outperform other funds; rather they frequently soared from top to bottom quartiles. Since the VW index has inferior diversification benefits to the EW index, the EW index is thought more suitable to represents the LPE asset class in Research Part II.

9 Research Part II: Private Equity Portfolio Performance

After having compared LPE to the global stock market and examined relevant risk-adjusted performance measures, this chapter analyzes whether LPE improves performance of a mixed-asset portfolio. Moreover, this chapter tests the second hypothesis; *private equity improves portfolio performance, consisting of stocks, bonds and gold, in terms of risk and return*. To answer the hypothesis, this chapter begins by introducing relevant asset allocation strategies. Subsequently, the opportunity set of the investor is presented, which is followed by an analysis on their individual historical performances. Finally, the empirical results of the asset allocations are presented with corresponding comparative analysis.

9.1 Asset Allocations Strategies

Asset allocation is the strategic decisions on how the composition of a portfolio is divided to each asset. A policy recommended in a classical work made by Benjamin Graham, initially published in 1965, states that investor cannot afford to put all his funding into one basket because of uncertainties regarding the future. He should neither put all his funding in bonds despite relative low risk, nor put all his funding in stocks despite prospect of higher return. The investor should depend on constructing a portfolio and seek to maximize his return while minimizing his risk.

The intuitions of classical asset allocations strategies are compared to assess whether private equity improves the performance of a multi-asset portfolio. They do therefore not give a recommendation on the optimal strategy nor the optimal allocation to each asset, since that is outside the scope of this thesis.

9.1.1 Equally-weighted Strategy (Naïve Strategy)

The simplest strategy utilized in this research part is a naïve strategy. A naïve strategy is one in which a fraction 1/N of wealth is allocated to each of N available assets. This strategy is often included as a benchmark to other strategies and according to DeMiguel, Garlappi and Uppal (2009) due to two reasons. Firstly, it completely ignores the data and does not rely either on estimation of the returns or any optimization factors. Therefore, is easy to implement as it is unaffected by all input factors. Secondly, despite past progress in advanced technological improvements used to design optimal portfolios, many investors still prefer to utilize simple models for allocating their wealth. (DeMiguel et al., 2009).

Accordingly, this strategy is included to compare with optimization strategies. It is interesting to observe whether the optimization strategies outperform the simpler strategies or not.

9.1.2 Mean-variance Framework

In the absence of a risk-free asset, investors that employ mean-variance analysis, construct optimal portfolios of risky assets that provides best risk-reward ratio. An efficient portfolio is the portfolio of (risky) assets that gives the lowest variance of return out of all portfolios generating the same expected return. Alternatively, efficient portfolio has the highest expected return of all portfolios having the same variance. Mathematically, for a given return an efficient portfolio is the one that solves

$$\max \mathrm{E}(r_P) = \sum_{i=1}^{N} w_i \, E(r_i) \tag{9.1}$$

subject to

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$\sum_i w_i = 1$$

Or equally by minimizing risk for all given levels of return

$$\min \sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{9.2}$$

subject to

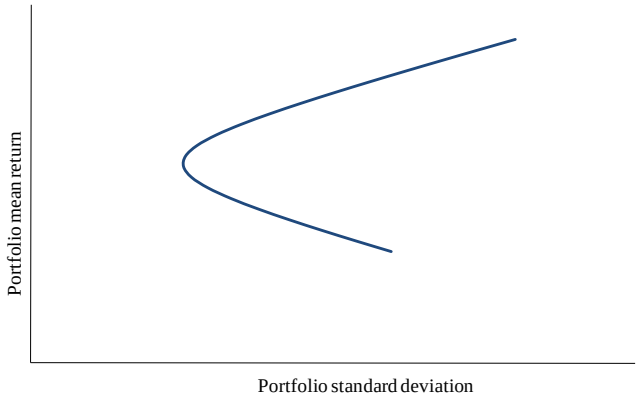
$$\mathrm{E}(r_P) = \sum_{i=1}^{N} w_i \, E(r_i)$$

$$\sum_i w_i = 1$$

where same notation is used as before; σ_i^2 indicates variance of asset *i*, r_i indicates return of asset *i* and w_i indicates weighting in asset *i*. (Benninga, 2008).

The Markowitz efficient frontier is the set of all efficient portfolios and is illustrated in Figure 9.1. There is not a possibility to obtain portfolio located above the efficient frontier and

portfolios below the frontier are achievable but they are sub-optimal. Therefore, all risk avoiding investors would hold portfolios located on the efficient frontier regardless of their specific risk tolerance.

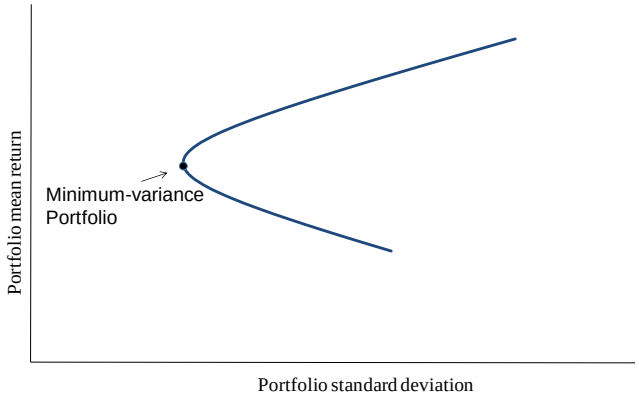


Source: Authors contribution.

Figure 9.1: Efficient frontier

9.1.2.1 Minimum-variance Portfolio

As the name suggest, the portfolio that has the lowest possible variance is the minimum-variance portfolio and it is located on the efficient frontier as illustrated in Figure 9.2. With given input data for expected return, variance and covariance, the minimum-variance portfolio can be calculated. Note that the portfolios on the efficient frontier that lie above the minimum-variance portfolio are all efficient portfolios. Any portfolio that lies below the minimum-variance portfolio is not efficient portfolio since there is a portfolio with the same risk but greater expected return positioned directly above it. In other words, the part of the efficient frontier that is lower than the minimum-variance portfolio is in fact inefficient.



Source: Authors contribution.

Figure 9.2: Minimum-variance Portfolio

Mathematically, to find the minimum level to witch portfolio variance can be held is to solve the minimization problem of portfolio variance. The portfolio variance is differentiated with respect to one of the weights (the other being $w_2 = 1 - w_1$) and setting the derivative equal to zero to obtain[31]:

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \tag{9.3}$$

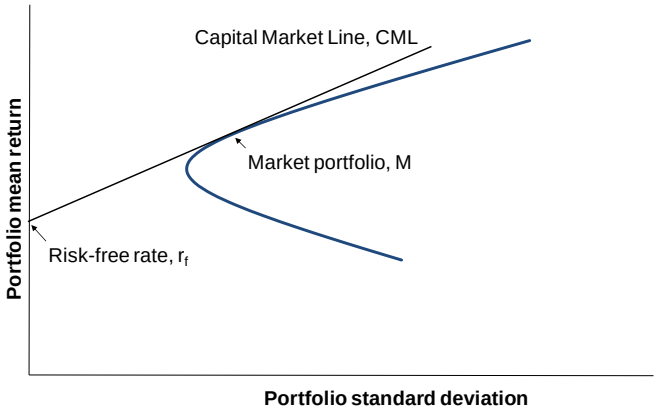
The above equation is therefore the weight of one asset that will provide minimum-variance portfolio. (Bodie, Kane and Marcus, 2009).

To implement this strategy, only estimates of the covariance matrix of asset returns are required and it completely ignores the estimate of expected returns. In that sense, this strategy cannot be considered fully optimizing strategy as it solely focuses on minimizing the portfolio risk. However, by doing that it eliminates all possible issues regarding estimation errors related to expected returns. This strategy is suitable for the most extreme risk-averse investors and is furthermore good for comparison, to see the lowest possible risk for a given efficient portfolio.

9.1.2.2 Tangent Portfolio (The Capital Market Line)

After having examined the efficient risky portfolios that create the efficient frontier, we now look into how to construct the optimal complete portfolio consisting of both risky assets and a risk-free asset. In the presence of only risky assets, the efficient frontier has a parabolic shape as previously illustrated, however as demonstrated by William Sharpe (1964), James Tobin (1958) and John Lintner (1965), the efficient set of portfolios available for investor simplifies to a linear function in the presence of risk free assets (Fabozzi, 2007). The combinations of risk-free and the risky assets lie on a straight line originating from the vertical axis' intercept at the risk-free rate (r_f) and tangent to the Market portfolio (M). The Market portfolio is the optimal portfolio of risky assets and can be calculated by maximizing the Sharpe ratio optimization problem, which is the slope of the line. This capital allocation line (i.e. alternative efficient frontier) is called the Capital Market Line (CML) and is illustrated in Figure 9.3.

[31] Two-asset portfolio is used for simplicity reasons. The formulation with more than two assets is more complicated hence the Excel function 'Solver' is used in our research, with the intuition and idea being the same.



Source: Authors contribution.

Figure 9.3: Capital Market Line

The CML depicts all the risk-return combinations available to investors, with different points along the line reached by holding the Market portfolio and lending or borrowing at the risk free rate, according to investor's degree of risk aversion. With exception of the Market portfolio, the portfolios that are combined of the market portfolio and the risk free asset (along the CML line) are superior to the portfolios on the Markowitz efficient frontier for the same level of risk. (Fabozzi, 2007).

Mathematically, the equation for CML is

$$E(r_P) = r_f + \left[\frac{E(r_M) - r_f}{\sigma_M}\right]\sigma_P \qquad (9.4)$$

where the Market portfolio has expected return of $E(r_M)$ and standard deviation of σ_M. Rational investors seek to maximize the return relative to risk; accordingly, investor will maximize the slope of the CML, which is the bracket term in the equation. This is often referred to as the *risk premium*. The portfolios left of the Market portfolio represent the combination of risk-free assets and risky assets. The portfolios right of the Market portfolio includes purchases of risky assets with borrowed funding at the risk-free rate. Such portfolios are known as *leveraged portfolios* as they include the use of borrowed funding. (Fabozzi, 2007).The CML describes the risk and return ratio of portfolios, not individual securities.

9.2 Criticism of the Mean-Variance Framework

The Mean-variance framework can be criticized, as most theoretical models, for making assumptions regarding the investor and the financial markets that might not hold in practice. One of the main assumptions in the framework is the assumption of normally distributed

returns. Financial returns are assumed to be normally distributed to be able to model reality by the use of standardized measures. However, in reality financial markets returns of equities and other asset classes are frequently observed not to be normally distributed, moreover they commonly exhibit skewness and excess kurtosis. As a result of non-normally distributed returns, the notion in the framework regarding market risk measure, such as variance, is not able to capture the entire distribution of returns and may therefore not be an adequate measure of risk. Furthermore, the variance implies that abnormal high returns are just as risky, and unwanted, as abnormal low returns.

Additionally, the framework assumes that investor's attitude towards risk can be explained by quadratic utility function. Quadratic utility implies that the only sufficient factors for investor's decision are mean and variance, even in the presence of skewness and excess kurtosis. However quadratic utility is questionable because it implies increasing absolute risk aversion and fails to capture the loss aversion, i.e. investor care more about losses than gains. Moreover, the framework assumes constant volatility and correlation over time. This assumption does not hold in reality since the systemic relationships between the underlying assets do frequently change.

Markowitz acknowledged these limitations from the beginning and suggested alternative option which could be preferable as risk measure. This measure is called semi-variance and focuses on downside risk. However, Markowitz did not utilize this methodology due to computational problems which should not be a problem nowadays. Other popular measure of downside risk is the Value at Risk, which highlights the potential loss from extreme negative returns. However, previous researches on downside risk are quite dispersed and question still remains whether downside risk measure leads to a more efficient measure than variance.

9.3 Weight Constraints

To make the analysis more reliable, constraints are applied on asset weights. These weight constraints will supply more realistic weighting scheme, which is possible for a private investor to simulate in practice.

According to Almazan, Brown, Carlson, & Chapman (2004) in a sample of over 9.000 funds, roughly 70 percent of investment managers are prohibited by authorities and strategy restrictions from short selling. Of those eligible, fewer than 10 percent actually engage in short selling, resulting interest for short selling typically counts only for 1.5 percent of market value. The fact that there is a small portion of professional investment managers that engage

in short-selling is an indication that this activity is significantly less frequent for private investors, who must comply to stricter restrictions.

There are two variants of short-selling constraints. The first is non-negative, denoting that short-selling is simply not allowed (referred to as *without short-sale*). The second variant allows short-selling, but only up to 50 percent in each asset (referred to as *with short-sale*). This limit is chosen a bit arbitrarily but mainly meant to exclude extreme short positions and furthermore to assist the excel computation of the mean-variance models, in order to achieve more reliable results.

9.4 The Opportunity Set of the Investor

In order to evaluate the performance of private equity in mixed asset portfolio, an opportunity set of available assets must be defined, from which investor chooses his investments. This sub-chapter includes description of the data used for the portfolio construction. For comparison reasons, two portfolios are constructed for each asset allocation strategy. Firstly, the focal point of our portfolio part is an initial portfolio which is used as the benchmark. The Initial portfolio represents the general asset classes that investors commonly include in their portfolios; it includes stocks, bonds and gold. The opportunity set of this Initial portfolio is then extended with the option of investment in private equity. Second portfolio is therefore constructed the same way as the Initial portfolio, with the addition of private equity. The opportunity set available for each portfolio is illustrated in Figure 9.4.

This figure displays the difference of the Initial Portfolio and the Portfolio with LPE. The weighting varies according to asset allocation strategies, the figure presents equal weighting.

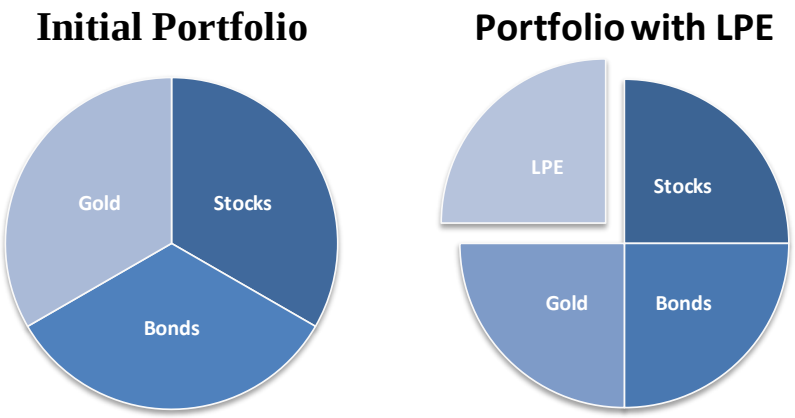


Figure 9.4: The opportunity set available for each portfolio

To represent these asset classes all proxies are in form of indices, not single securities. Indices are more suitable to represent a well diversified sample of the corresponding asset classes and therefore contain less selection bias than relying on a single security. All data for the Initial portfolio was gathered from Datastream 5.0.

9.4.1 Stock Index

The world stock index (MSCI) is used to represent stocks in the portfolio part of our research. This is the same index as was used for benchmark in Research Part I. This is done to keep consistency from previous results. As stated before, this index covers stocks from around the globe, which enables generalization of results and does not merely rely on one single market.

9.4.2 Bond Index

To represent bonds in our portfolio, a corporate bond index from Bank of America Merrill Lynch (BofA ML) is used[32]. The BofA ML Corporate Master Index tracks the performance of U.S. dollar-denominated investment-grade corporate debt. The securities are publicly issued and SEC registered. Investment-grade rating denotes that the qualifying securities must be graded A, AA or AAA based on the average rating of Fitch Ratings, Moody's and Standard & Poor's rating agencies. The bonds are therefore not categorized as high-yield bonds (i.e. junk bonds). Each security must have more than one year of remaining maturity, a fixed coupon schedule, and a minimum amount outstanding of $250 million. Index constituents are capitalization-weighted based on their current shares outstanding and the index is rebased at the last calendar day of the month. (Federal Reserve Bank of St. Louis, 2011)

There are two reasons why this index is suitable for representing bonds in this research. Firstly, this index is not supposed to represent risk-free assets, hence corporate bonds are more appropriate than short term government bonds. Corporate bonds are liquid securities as well, which should have relatively low risk (compared to equities). Secondly, a large part of the securities are global securities whereas they are issued simultaneously in the Eurobond and US domestic bond markets. This matches our pre-defined view of global investor's perspective.

[32] After the acquisition of Merril Lynch by Bank of America (BofA) in January 2009, the corporate and investment division of BofA is referred to as Bank of America Merril Lynch.

9.4.3 Gold Index

To represent gold in our portfolio, a gold index is used from the commodity index family generated by Standard & Poor and Goldman Sachs. The S&P GSCI is widely recognized in the measures of general commodities. The S&P GSCI Gold Index provides reliable and publicly available benchmark tracking the COMEX gold future. The weighting of S&P GSCI family is calculated on a world production basis and consists of physical commodities that are subject to active and liquid futures. The use of production as the weighting determinant is intended to reflect appropriate proxy for relevant commodity and furthermore preserve tradability of the index. The rebalance of the index is made once a year and may therefore not fully reflect real-time information. In order to be included in the index, futures contracts must meet several eligibility criteria, defined as general requirements and volume and weight requirements. Consequently, number of contracts that comprise the index is determined. (Standard and Poor's, 2010).

Reason for including gold in our portfolio is that for centuries, individuals have sought to possess gold as insurance against day-to-day uncertainties of general equities. It is often thought of as an alternative to stocks and bonds, therefore commonly used in portfolios for investors seeking diversification benefits.

9.4.4 LPE Index

The resulting index of the Research Part I are used to represent the private equity asset class in the portfolio. The EW index provided better diversification benefits than the VW index, which was concentrated to very few vehicles, therefore the EW index is more suitable representative for the asset class as a whole.

To sum up relevant proxies used for individual asset classes, Table 9.1 clarifies what proxy represents corresponding asset class.

Table 9.1: Asset classes and corresponding poxies

This table summerizes the asset classes and corrasponding asset class proxies used in this study.

Asset Classes	Asset Class Proxy
Stocks	MSCI
Bonds	BofA Merril Lynch Corporate Index
Gold	S&P Gold
LPE	Equally Weighted Index (From Research Part 1)

9.5 Historical Performances

After having defined the opportunity set available for the investor, following analysis is devoted on historical performances. All returns indicate continues rate of return as before. To be able to compare the asset classes, all assets were rebased at the price 1000 at the beginning of the sample period. Same time period is used as in Research Part I and the development over time is illustrated in Figure 9.5. By graphical investigation, it seems that LPE has the highest volatility that fluctuates the most. Contrary, the bond index shows relative steady increase over the whole period.

This figure displays the growth of $1.000 investment made on January 7, 1998 ending on March 9, 2011.

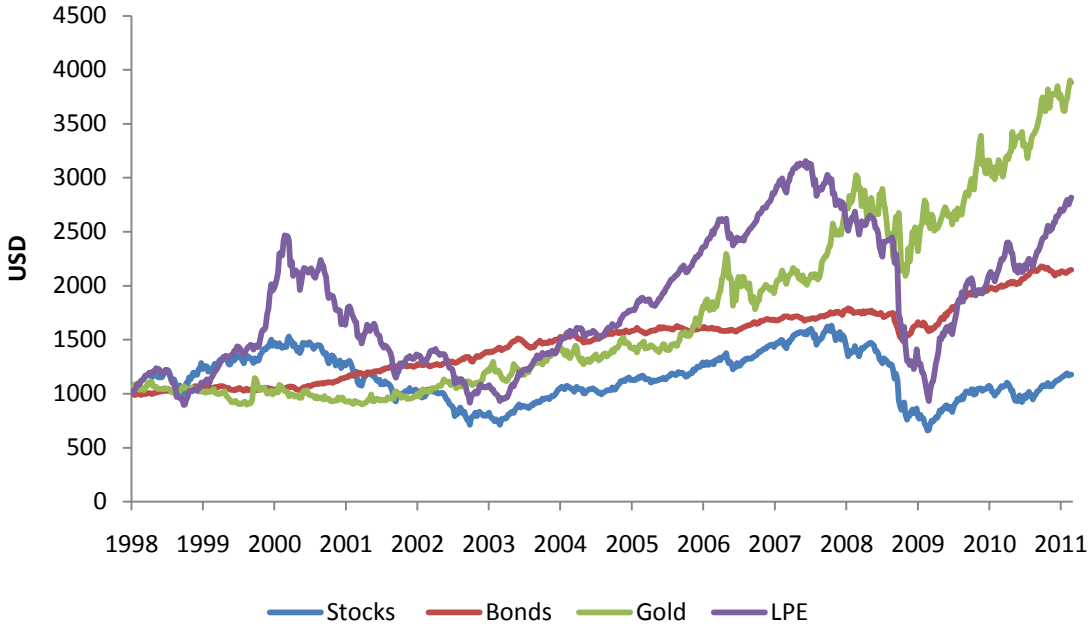


Figure 9.5: Performance of each asset class

Descriptive statistics of the data over the whole sample period is presented in Table 9.2, which includes weekly statistics of individual asset classes. For convenience the table additionally provides returns and risk parameters on yearly basis.

Table 9.2: Descriptive Statistics of the available asset classes over the sample period.

Descriptive statistics of asset classes from 1998 to 2011.

	Stocks	Bonds	Gold	LPE
Weekly Min	-16.55%	-4.20%	-13.19%	-19.64%
Weekly Max	9.27%	2.82%	12.97%	10.83%
Weekly Return	0.05%	0.11%	0.23%	0.18%
Weekly Standard Deviation	2.49%	0.72%	2.53%	2.65%
Yearly Return	2.76%	5.96%	11.88%	9.57%
Yearly Standard Deviation	17.93%	5.22%	18.26%	19.08%

The bond index shows lowest deviations in both min and max measures meaning it has lowest volatility which is supported by the lowest standard deviation, measuring 5.22 percent on yearly basis. Contrary, LPE experience the largest deviation fluctuating from weekly return of around 11 percent to a negative return of roughly 20 percent. Accordingly, the LPE has the highest standard deviation, or around 19 percent. However, both Gold and Stocks have similar high standard deviations, representing the high volatile market over the sample period. Gold and LPE provide highest return over the whole period with corresponding yearly return of around 12 and 10 percent respectively.

9.5.1 Correlation Analysis

One of the main arguments for holding multiple assets in a portfolio is to achieve diversification benefits. The correlation analysis measures the strength or degree of linear association between two variables as stated in Chapter 6, where the correlation coefficient was explained. Accordingly, following coverage measures and analyzes the correlation coefficients between individual asset classes.

As the correlation measures the strength of a relationship, this relationship must also be associated with a measure of its significance. Therefore a significant test of the correlation coefficients is included, to assess whether the correlations are statistically different from zero. This test is two-side test and two hypotheses are proposed. The null hypothesis, H_0, is that the true correlation in the population is zero ($\rho = 0$). The alternative hypothesis, H_1, is that the correlation in the population is different from zero ($\rho \neq 0$). The formula for the t-test is as follows

$$t = \frac{\rho\sqrt{(n-2)}}{\sqrt{1-\rho^2}} \tag{9.5}$$

The test statistics follows t distribution with n-2 degrees of freedom. (DeFusco et al., 2007). As the sample size is n=687, the critical value in a two-sided test at the 5 percent significance level is 1.960. Table 9.3 presents the results of the correlation analysis.

Table 9.3: Correlation of the asset classes

Correlation coefficients between individual asset classes. * indicates that corresponding correlation coefficient is significant at 5 percent significance level and ** indicates 1 percent significance level. The t-statistics can be seen in Appendix.

	Stocks	Bonds	Gold	LPE
Stocks	1			
Bonds	0,209**	1		
Gold	0,251**	0.026	1	
LPE	0,796**	0,076*	0.050	1

The significant test concludes that the null hypothesis can be rejected for four out of six correlation coefficients, meaning that corresponding coefficients are statistically different from zero. However, most of those significant coefficients are fairly low (all below 0.3 apart from one) which should be within reasonably boundaries. Additionally, the null hypothesis cannot be rejected for two of the gold coefficients. This indicates that these two correlations (between gold & bond and gold & LPE) are not statistically different from zero. Note, as stated in Chapter 6, zero correlation implies good prospects of diversification benefits.

Consequently, all assets seem to have quite good diversification prospects as most coefficients are fairly low. However, as was briefly pointed out in the Research Part I, the correlation between stocks and the LPE index is especially high. The underlying factors for this high correlation might be derived from the fact that private equity vehicles are not only driven by the underlying private equity investments, they are also exposed to market movements. There are factors in the macroeconomic environment that generate close-link between private equity and the market. The performance of stocks is known immediately while traditional private equity investments need to be computed and annualized after the sale (exit) of the portfolio companies. These sales are mainly through IPOs which can be affected in depressed market cycles, where public companies make fewer acquisitions and/or negotiate lower valuations. Additionally, private equity is affected by interest rates as they impact the conditions of financial leverage. Companies use borrowed money to make acquisitions and higher interest rates creates challenges for making the acquisitions profitable. These factors hold true for both traditional and listed private equities, while LPE

vehicles are additionally influenced by being publicly listed and therefore might show symptoms of any path of random walk.

Although there is a perception of low correlation in traditional PE studies, according to the data sample of this study, this is apparently not true for LPEs. This correlation of LPEs to stocks and bonds are relatively similar to the findings of Bergman et al. (2009), which also do their study on LPE vehicles and find correlation to global stocks to be 0.772 and global bonds to be 0.002. This high correlation to stocks might be of concern in an asset allocation perspective and indicate that LPE cannot provide diversification benefits on top of the Initial portfolio. However, the returns of LPE greatly outperform the stock returns over the sample period which might result in LPE working as a substitute-factor for stocks. Additionally the correlation of LPE to bonds and gold are very low, indicating good diversification benefits with those assets.

9.6 Empirical Results on Private Equity Portfolio Performance

The following coverage compares relative performances of the two portfolios by three different asset allocation strategies, measured by relevant performance measures. Note that the following asset allocations are not meant to evaluate which is superior; their role is to be able to compare the Initial portfolio to the LPE portfolio. The three different asset allocations therefore serve as three different variations to make comparative analysis of the two portfolios.

While the equally weighted portfolios are not exposed to short-selling the Markowitz portfolios are. Consequent results are denoted both with the limited possibility to short-sell and where short selling is not allowed at all. The performance measures of Sharpe, Jensen's Alpha and M^2 all measure performances compared to a benchmark. In this context, the Initial portfolio represents the benchmark. Therefore, higher Sharpe ratio indicates superior risk-adjusted performance to the Initial portfolio and observing positive (negative) alpha indicates that the marginal return associated with LPE investing generates superior (inferior) performance to the initial portfolio. These coefficients are measured by a statistically significance test as in previous results. Additionally, observing positive (negative) M^2 indicates by how much the risk-adjusted performance of LPE portfolio is superior (inferior) to the Initial portfolio.

9.6.1 Equally-weighted Portfolios

The uncomplicated compositions of the equal weighted portfolio and corresponding performances of the two portfolios are presented in Table 9.4 and Table 9.5. Hence, the Initial portfolio, which contains only three assets, has 33 percent weighting in each and the LPE portfolio thus has 25 percent weighting in each.

The portfolio with LPE has higher expected returns than the Initial portfolio. This higher return is moreover followed by higher standard deviation. This is correspondent to both high return and standard deviations experienced by the LPE index, as denoted in Chapter 9.5 of historical performances.

By examining the two portfolios when consequent performance measures are taken into considerations all measures indicate that the portfolio with LPE has superior performance to the initial portfolio. In comparison of the two Sharpe ratios, the portfolio with LPE has slightly higher ratio, measuring 0.30 and 0.34 respectively. This slightly higher ratio of the LPE portfolio is significant higher by the measure of Sharpe significance test. Additionally, the other two relative measures are positive, although the measure of alpha is not significant. The measure of M^2 is positive and denotes that the LPE portfolio outperforms the initial portfolio by 0.32 percent. All in all, these performance measures indicate that, in this data sample and in a portfolio with equal weights, the LPE portfolio outperforms the Initial portfolio.

Table 9.4: Asset weights in Equally-weighted portfolios

	Initial Portfolio	Portfolio with LPE*
Stocks	33%	25%
Bonds	33%	25%
Gold	33%	25%
LPE	-	25%

Table 9.5: Performance of the Equally-weighted portfolios

	Initial Portfolio	Portfolio with LPE*
Er	6.87%	7.54%
SD	9.43%	10.48%
Sharpe	0.30	0,34*
Alpha	-	1.97%
M^2	-	0.32%

Even though this strategy is simple it is a good indicator of what novice private investor actually might utilize when considering adding private equity to his portfolio. Therefore this result gives a practical insight before our examination moves on to more complex methods.

9.6.2 Minimum-variance Portfolios

The weights and performance measures of the minimum variance portfolios are presented in Table 9.6 and Table 9.7. There is hardly any difference in the composition of these portfolios. The only difference is the LPE portfolio with short-selling, which allocates slightly more to stocks than the others and has a minor short position in the LPE.

Table 9.6: Asset weights in Minimum-variance portfolios

	Initial Portfolio		Portfolio with LPE*	
	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale
Stocks	6.08%	6.08%	6.11%	6.08%
Bonds	88.30%	88.30%	88.31%	88.29%
Gold	5.62%	5.62%	5.62%	5.62%
LPE	-	-	-0.03%	0.00%

Table 9.7: Performance of Minimum-variance portfolios

	Initial Portfolio		Portfolio with LPE*	
	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale
Er	6.10%	6.10%	6.10%	6.10%
Stdev	4.92%	4.92%	4.92%	4.92%
Sharpe	0.42	0.42	0.42	0.42
Alpha	-	-	0%	0%
M^2	-	-	0%	0%

The performance measures are in line with the composition, where there is no difference in performance at all. Reason for these findings may be originated by the standard deviations of the asset classes and that this strategy totally ignores expected returns. Accordingly, bonds receive highest allocation since they had considerably lower standard deviations than the other asset classes and LPE is not allocated any weighting since it supplies the highest standard deviation. Stocks and gold are allocated small portions each, equivalent to their risk level. Therefore, it is sound to assume that minimum risk strategy does not allocate large positions to the riskiest asset.

Concluding remarks which can be observed from these findings are that investor who optimizes his portfolio performance by utilizing minimum-variance strategy does not gain additional value, on top of that provided by the Initial portfolio, by including private equity.

9.6.3 Tangent Portfolios

The compositions of the tangent portfolios, both with and without the short-selling restrictions, are displayed in Table 9.8.

Table 9.8: Asset weights in Tangent Portfolios

	Initial Portfolio		Portfolio with LPE*	
	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale
Stocks	-9.86%	0.00%	-50.00%	0.00%
Bonds	81.43%	75.12%	69.29%	65.23%
Gold	28.44%	24.88%	29.87%	22.13%
LPE	-	-	50.84%	12.64%

There are similarities between the compositions of the two portfolios. In all occurrences, the largest allocations are to bonds (ranging from 65 to 81 percent of the allocation). Roughly quarter (from 22 to 30 percent) of the composite is allocated to gold while stocks do not count for any long positions, only a short-position when short-sale is allowed, with fully short position in the LPE portfolio. It appears that LPE contribute value adding components to the portfolio as when LPE is added to the opportunity set, it is allocated slightly over 50 percent when short-selling is allowed and around 12 percent when short-selling is not allowed. Bonds are lowered by roughly more than 10 percentage points in both constraints variants and gold is lowered by around 2.5 percentage points when short-selling is not allowed and increased by two percent when it is allowed.

Table 9.9 presents the performance measures of the two tangent portfolios. When there is a possibility to short-sell the assets, the expected return on the tangent portfolio increases from 7.96 to 11.17, when LPE is added to the opportunity set of the investor. Not as dramatic surge is experienced when short-selling restrictions are imposed, but the LPE portfolio still provides higher return than the Initial portfolio with slight increase from 7.44 percent to 7.73 percent. Corresponding analysis on volatility measures is somewhat surprising. When short selling is allowed the standard deviation increases from 6.77 to 8.51 but when short-selling is not allowed the standard deviation actually slightly decreases from the Initial portfolio and the LPE portfolio. This denotes that LPE contributes diversification benefits on top of the Initial portfolio due to low correlation to bonds and gold.

Table 9.9: Performance of the Tangent Portfolios

	Initial Portfolio		Portfolio with LPE*	
	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale
Er	7.96%	7.44%	11.17%	7.73%
SD	6.77%	6.07%	8.51%	6.05%
Sharpe	0.58	0.56	0.84**	0.61**
Alpha	-	-	1,63%*	0.90%
M^2	-	-	1.74%	0.31%

In an analysis of the performance measures, the Sharpe ratio is significantly higher for the LPE tangent portfolio compared to the Initial tangent portfolio, both with and without the short-selling constraint. The ratio increases from 0.58 to 0.84 when short-selling is allowed and from 0.56 to 0.61 when short-selling is not allowed. This generates the steeper slope of the CML, providing higher return for same amount of risk. These findings conclude that the higher return of the LPE portfolio is not only due to the higher risk it generates. Moreover, the LPE portfolio seems to provide value added risk and return characteristics. A positive alpha, as previously stated, measures whether the LPE portfolio provides abnormal return to the Initial portfolio, and by how much. Both alphas for the LPE portfolio are positive, measuring 1.63 and 0.90 percent respectively, with significant measure when short-selling is allowed. This indicates that from the mean-variance allocation of tangent portfolio when short-selling is allowed, the LPE portfolio outperforms the Initial portfolio with marginal abnormal return of 1.63 percent. The measure of M^2 denotes the marginal risk-adjusted return of what the LPE portfolio outperforms the initial portfolio. By the measure of M^2, the LPE portfolios outperform the Initial portfolio both with and without the short-selling constraints, by 1.74 and 0.31 percent respectively. Overall, these findings of the performance measures, for the tangent portfolios over the full period, conclude that LPE improves the performance of the Initial portfolio, in terms of risk and return.

9.6.4 Summary of the Markowitz Portfolios

A summary of the minimum-variance and the tangent asset allocations strategies, over the whole sample period, is illustrated in Figure 9.6, with short-selling allowed.

Markowitz portfolio's results over the whole period when short selling is allowed. The shapes are explained as follows: Circles denote tangent portfolios and the square denotes minimum variance portfolio.

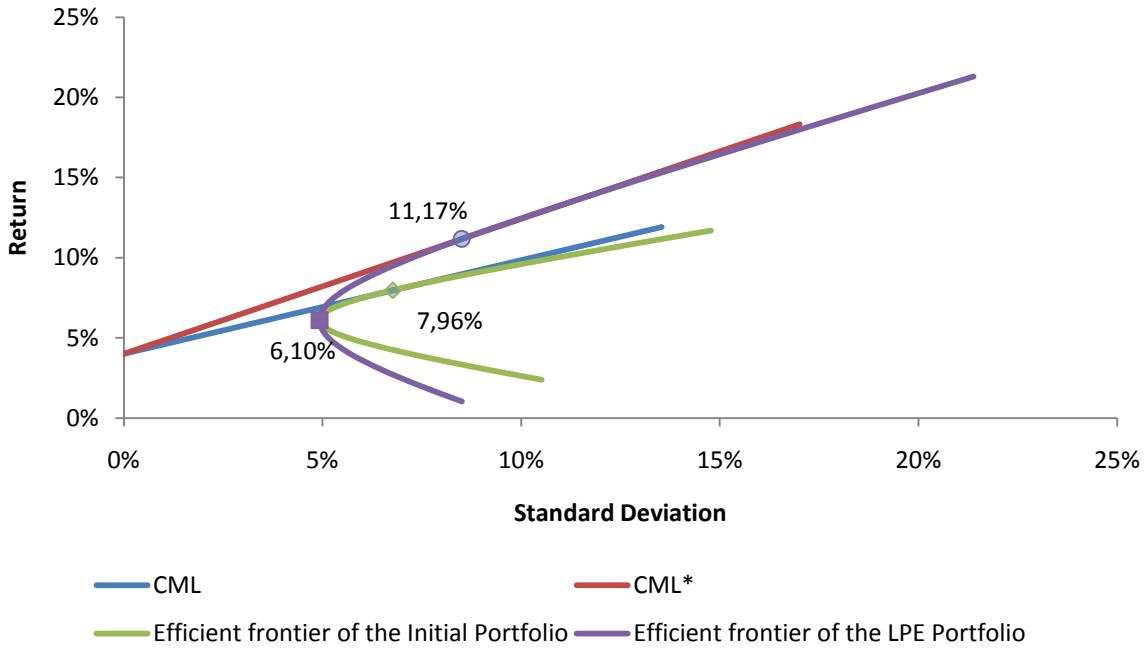


Figure 9.6: Tangent portfolios

The portfolio with LPE improves the risk and return characteristics of the Initial portfolio, which can be seen by the expansion of the efficient frontier. There are two points worth noting. Firstly, the frontier is wider with the additional opportunity set of LPE. Secondly, the slope of the CML is steeper due to higher Sharpe ratio. Those points imply higher expected return for same amount of risk, which indicates superior performance of the LPE portfolio.

9.6.5 Sub-Periods

Equivalent structure of performance of the three portfolios, when the sample period is divided into predefined sub-periods, is displayed in Table 9.10. However, it is worth noting that these are periods of large volatility which resulted in occasional unrealistic compositions of the portfolios that exposed some of the weakness of the models[33].

[33] The weighting can be seen in Appendix A.

Table 9.10: Performance Measures for sub-periods

This table includes performance measures for all portfolios, divided into all sub-periods.

		Equally Weighted		Minimum Variance Portfolio				Tangent Portfolio			
		Initial Portfolio	Portfolio with LPE*	Initial Portfolio		Portfolio with LPE*		Initial Portfolio		Portfolio with LPE*	
				With Shortsale	Without Shortsale	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale	With Shortsale	Without Shortsale
Boom 1	Er	7.96%	15.86%	3.77%	3.77%	4.59%	4.59%	27.38%	17.17%	75.87%	39.54%
	Stdev	7.95%	8.95%	4.34%	4.34%	4.32%	4.32%	26.90%	15.56%	33.66%	17.46%
	Sharpe	0.99	1,76**	0.84	0.84	1,04**	1,04**	1.01	1.10	2,25**	2,26**
	Jensen's	-	7,91%**	-	-	0,79%**	0,79%**	-	-	56,53%**	25,48%**
	M2	-	6.15%	-	-	0.84%	0.84%	-	-	33.26%	18.08%
Bust 1	Er	-1.81%	-7.91%	6.98%	6.98%	6.29%	6.29%	22.51%	9.50%	29.44%	9.50%
	Stdev	7.60%	9.47%	4.26%	4.26%	4.20%	4.20%	11.63%	4.61%	13.83%	4.61%
	Sharpe	-0.25	-0.84	1.62	1.62	1,48**	1,48**	1.93	2.04	2,12*	2.04
	Jensen's	-	-5,28%*	-	-	-0.63%	-0.63%	-	-	6.01%	0.00%
	M2	-	-4.51%	-	-	-0.61%	-0.61%	-	-	2.25%	0.00%
Boom 2	Er	12.06%	16.08%	6.17%	6.17%	7.80%	7.80%	17.67%	16.84%	31.47%	28.15%
	Stdev	7.99%	7.44%	3.89%	3.89%	3.83%	3.83%	10.60%	9.94%	9.43%	9.09%
	Sharpe	1.50	2,15**	1.56	1.56	2,01**	2,01**	1.66	1.69	3,33**	3,09**
	Jensen's	-	4,84%**	-	-	1,69%**	1,69%**	-	-	20,84%**	15,05%**
	M2	-	5.21%	-	-	1.75%	1.75%	-	-	17.68%	13.91%
Bust 2	Er	-6.20%	-18.22%	-0.20%	-0.20%	-0.83%	-0.59%	27.14%	19.34%	54.80%	19.34%
	Stdev	14.52%	15.67%	7.18%	7.18%	7.17%	7.17%	17.83%	27.61%	28.75%	27.61%
	Sharpe	-0.43	-1.17	-0.04	-0.04	-0.13	-0.09	1.52	0.70	1,90**	0.70
	Jensen's	-	-11,85%*	-	-	-9.09%	-0.39%	-	-	15,64%*	0.00%
	M2	-	-10.68%	-	-	-0.63%	-0.39%	-	-	6.88%	0.00%
Post Crisis	Er	17.00%	22.50%	13.54%	13.57%	14.49%	13.57%	13.99%	13.99%	22.04%	17.00%
	Stdev	10.71%	12.48%	5.58%	5.51%	5.72%	5.51%	5.59%	5.59%	7.71%	6.28%
	Sharpe	1.59	1,80**	2.43	2.46	2.53	2.46	2.50	2.50	2,86**	2,70**
	Jensen's	-	4.10%	-	-	0.66%	0.00%	-	-	5.51%	2.02%
	M2	-	2.31%	-	-	0.60%	0.00%	-	-	2.01%	1.15%

By analyzing the LPE portfolio performances in boom periods, all ten portfolios generate higher expected returns for the LPE portfolio compared to the Initial portfolio. Only three of them have corresponding higher standard deviations but seven achieve to lower the standard deviations. Accordingly the performance measures all indicate superior performance by the LPE portfolio, also the three LPE portfolios that have higher risks. These findings conclude that, for bull markets, LPE significantly improves the portfolio performance.

By analyzing the LPE portfolio performance in the bust periods, the findings are somewhat different. Both equally weighted portfolios and the all of the minimum-variance portfolios generate lower expected returns of the LPE portfolio compared to the initial portfolio. However the minimum-variance portfolios are able to provide slightly lower risk in all of them. In the more optimal strategies, both of the LPE tangent portfolios, when short-selling is allowed, generate higher expected returns while both of the fully constraint (non-negative) portfolios have same compositions and therefore same performance. Short-selling should be appealing approach for the Markowitz portfolios in bear markets. In these periods, this option is value adding component for both tangent-strategy portfolios, as they both generate superior returns when short-selling is allowed compare to when it is not allowed.

By analyzing the LPE portfolio performance in the post-crisis period the findings are somewhat similar to the boom periods. All the strategies provide superior return apart from the minimum-variance portfolio with full short-selling constraint, where the returns are the

same. These higher returns are all followed by accordingly higher standard deviation, which indicates high volatility of the LPE in this period.[34] The Sharpe ratios are all significance higher, apart from the minimum-variance portfolio, which have similar compositions. The measure of alpha is however never statistical significant meaning that the marginal outperformance is not large enough to be statistically different from zero. The M^2 measures all state marginal positive risk-adjusted returns, apart from one of the minimum-variance portfolio.

9.6.6 Hypothesis Testing Results for Private Equity Portfolio Performance

This sub-chapter presents results of the hypothesis testing of the performance measures, for Research Part II. A summary of these results is presented in Table 9.11, which specifies whether the hypothesis can be rejected or not for each performance measure. Furthermore, results on significance testing are also presented where relevant.

Table 9.11: Results on Hypothesis Testing for Research Part II

The table presents the results of whether the hypothesis can be rejected or not for the whole sample-period. Hypotheses that are not rejected are referred to as 'confirmed'.

Private equity improves portfolio performance, consisting of stocks, bonds and gold, in terms of risk and return.	Equally Weigthed Portfolio	Minimum-variance Portfolio	Tangent Portfolio
With Shortsale			
Sharpe Ratio	-	Rejected	Confirmed**
Alpha	-	Rejected	Confirmed*
M^2	-	Rejected	Confirmed
Without Shortsale			
Sharpe Ratio	Confirmed*	Rejected	Confirmed**
Alpha	Confirmed	Rejected	Confirmed
M^2	Confirmed	Rejected	Confirmed

*/** denotes that corresponding hypothesis is confirmed by a significance level of 0.05/0.01 respectively.

Overall, there are a total of 15 instances of performance measures that examine whether the LPE portfolio generates superior performance to the Initial portfolio, in terms of risk and return. The hypothesis is confirmed in nine instances out of the total 15, resembling improved performance in 60 percent occasions. Out of those nine instances, four portfolios were significantly improved, with the remaining also including M^2 that does not contain significance testing. Of those six instances that are rejected, all of them derive from the minimum-variance portfolio. However, the minimum-variance findings originate from unchanged composition of the two portfolios, due to LPE not providing any risk benefits and

[34] Standard deviation of LPE is 23 percent over the post-crisis period, as can be seen in Table 8.8.

the strategy completely ignores return parameters. Since the majority of the hypothesis testing results cannot be rejected, it can be concluded that based on the sample of this research, LPE is able to improve portfolio performance, in terms of risk and return.

9.7 Conclusion on Private Equity Portfolio Performance

Relevant asset allocation strategies were introduced in Research Part II, which began by introducing the simple equally-weighted strategy. Consequently, the mean-variance framework was introduced with demonstration on how to compute the efficient frontiers and different allocation strategies along it. Simple asset allocation strategies were chosen sufficient to test the hypothesis of whether LPE can improve portfolio performance in terms of risk and return. If the hypothesis would contain recommendation on optimal allocation to each asset, more advanced allocation strategies would be required. The mean-variance framework only uses approximation of the true risk and returns estimations. This approximation equals true estimates plus an error term. The importance of this error term is often neglected in standard portfolio theory exercises. Therefore is the equally-weighted portfolio good for comparison, as it excludes estimation factors.

A constraint on short-selling was applied to achieve more realism to the weighting scheme. This is regarded intuitive constraint due to how few private investors engage in short-selling. Additional constraints could have been utilized, as for example maximum allocation in each asset. However, since the opportunity set of the investor only contains four assets in total, including a constraint on maximum allocation to each would be too large influencing factor to the strategies. Therefore it was not regarded suitable or value adding component. Even thought additional constraints might give somewhat more practical results too many restrictions could inhibit the functionality of the models.

Both the equally-weighted strategy and the tangent portfolio confirmed that the portfolio performance was improved, by the opportunity of investing in LPE. The minimum-variance portfolios did not provide any additional benefits on top of that provided by the Initial portfolio. However, the two minimum-variance portfolios had same compositions and therefore no change was observed in their performances. Overall, nine out of 15 portfolios did not reject the hypothesis, concluding that LPE is able to improve portfolio performance, in terms of risk and return. Dividing the sample period into sub-periods provided more or less equivalent intuition regarding LPE portfolio performance in bear and bull market cycles. The findings indicate that LPE improves the performance of a portfolio in boom markets, but not

in bust markets[35]. This confirms the perception of private equity being a risky asset class and support the fact that investor should consider this investment over a long-term horizon. However, as previously stated, some of the compositions in these periods were extreme, resulting from the very volatile cycles of these periods.

After analyzing historical performance of the asset classes, the perception was that LPE might work as a substitute-factor for stocks. This is due to their high correlations and LPE providing considerable higher returns while only slightly higher risk. These perception was more and less confirmed by the tangent portfolio, which shorted at maximum limit (-50%) to stocks while allocating equally large share (50%) to LPE. Although the spread of allocation to LPE is not the main objective of this research it is still worth noting that this allocation varies from zero to 50 percent which is not unlike the findings of Schmidt (2006).

All else equal, expanding the investor's opportunity set and allocating to additional available asset classes should improve the trade-off between risk and return. Accordingly, the findings of superior performance of the LPE portfolios in this research might simply be because of diversification benefits. A way to approach and solve this problem was by the use of indices to represent each asset class, which include numerous securities and therefore should be well diversified. This problem might still be present since the change from three-asset portfolio to four-asset portfolio might simply be enough to generate superior diversification benefits. However, almost all of the strategies allocated some share to LPE which is a good indicator that LPE brings value-adding components to the Initial portfolio. Accordingly, the diversification benefits of more available asset classes are apparently not the only influencing factors.

[35] Only tangent portfolio with short selling was able to do that.

10 Discussions

The documentation of the previous literature revealed great inconsistency regarding private equity performance. Very limited research is apparent on both the new market segment of LPEs and private equity capabilities in a mixed-asset portfolio. With inspiration by the literature review, relevant research questions were put forward to assess the two limited research areas. Hypotheses were formed in order to quantitatively test the research questions which were measured by accompanying performance measures. Consequently, the necessary tools were available to conclude whether private equity generates value-adding benefits, and if so, by how much.

Private equity investing is a long-term investment by nature and therefore it is necessary to have the sample period sufficiently long enough. The time period chosen conducted previous 12 years and is prosperous of various market cycles. This period was furthermore divided into sub-sequent periods for more informative comparison on private equity investing. The aim of the thesis is applicability and to support this aim the required data represented the view from a global private investor's perspective.

The performance measures and the asset allocations models assume normality which could limit the validity of our findings. A test of the distribution of the data was conducted in Chapter 8.3.4 that concluded the time series are non-normally distributed, as is common for most financial time series. These models and measures only rely on the first two moments, mean and variance, and ignore the second two moments. However, due to the applicability and simplicity reasons, these models are the most suitable for the scope of this research. Therefore, comparison of their findings should provide correct intuitive and consequently they are the best at hand to answer relevant research questions.

The Research Part I analyzes how private equity performs in general, compared to the global stock market. In order to not limit our quantitative research to lacking of the data, we constructed our own index. In that way, the compositions are known and biases that are realized in previous researches were avoided, which enables better comparative analysis. The overall results of Research Part I are that the hypothesis test is not rejected in 24 instances out of the total 36, meaning an outperformance of the LPE indices in 67 percent occasions. While the world stock market over the full sample period provided average annual return of about two percent, the LPE indices provided return of roughly ten percent. Although this high return is accordingly followed by higher risk, especially for the value-weighted index, the

performance measures confirmed that LPE indices outperform the stock market on risk-adjusted basis. The Sharpe ratios of the equally-weighted index significantly exceed the ratio found for the market and moreover deliver a positive alpha of seven percent. The value-weighted index did also outperform, but the difference did not turn out to be statistically significant for the fairly well concentrated index. By analyzing the sub-periods the findings observed tendency that LPE indices underperform in bust periods but outperform in all other sub-periods. The price deviations of the two LPE indices are relatively similar indicating that larger vehicles do not outperform smaller in this market segment. Additionally, performance persistency analysis was conducted which did not generate the same conclusions as Jones and Rhodes-Kropf (2003). Better performing funds did not seem to consistently outperform other funds; rather they frequently soared between quartiles.

The Research Part II analyzes private equity capabilities in a mixed-asset portfolio which exposes how this asset class performs along with other major asset classes and how it correlates with them. The results of Research Part II are that both the equally-weighted portfolio and the tangent portfolio confirmed that the portfolio performance was improved, by the opportunity of investing in LPE. The minimum-variance portfolios did not provide any additional benefits on top of that provided by the Initial portfolio. Overall, nine out of 15 portfolios did not reject the hypothesis, concluding that LPE is able to improve portfolio performance, in terms of risk and return. The sub-period analysis did not give reliable weighting scheme for the all of the portfolios; therefore the substance of their conclusions was minimized. However their findings support the sub-period findings of Research Part I.

Overall, by combining the conclusions of both research parts, larger part of the hypotheses were confirmed in each research part. Therefore, it can be concluded that, based on the sample used in this research, private equity outperforms the global stock market in general, and improves a mixed-asset portfolio performance in particular, in terms of risk and return.

11 Further Research

As the LPE market recently emerged, consequently there is a need for more thorough research for this immature market segment. When this market segment has fully matured, a well recognized benchmark should be established, which could be used instead of the historical simulation of Research Part I. Consequently, it would be possible to focus on more advanced portfolio strategies that do not assume only the first two moments, but count for the non-normality of the financial data. Instead of using the mean-variance framework, an alternative way would be to utilize resampled mean-variance optimization. That framework combines Monte Carlo simulation with the traditional mean-variance framework. A Markow-switching methodology is alternative method that counts for the non-normality and measures the possibility of being in upstate and downstate market environments by regime switching. These more advanced models could assess and recommend what is the optimal allocation to LPE, in different states of market cycles.

If the perspective would not be from a private investor, a model made by Black and Litterman (1991) could be appropriate. However, this model is not without shortcomings as it relies heavily on input factors and predefined views from the investor. Consequently, the investor needs great resources to establish his views, which is only possible for a professional investor or from a professionally made research.

To make these findings even more practical, an investor who would like to bypass the first research part, and not construct his own index, could invest in exchange traded funds (ETF). To be consistent with the ideology in this thesis the ETF would need to be on worldwide private equity vehicles and they would need to be not limited by the lack of liquidity. ETF and mutual funds that track some of these indices are offer by many intermediaries such as; ALPS Fund Services, BlackRock Advisors, Deutsche Bank, Invesco, Merrill Lynch, RBS and UBS. However, since private equity was found not to be value adding investment in a downturn market cycles, an investor in the current market turmoil should possibly take precautions and make sure that the world economy has recovered from the current financial crisis.

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13 Appendices

Overview of Appendices

Appendix A. Overview of Content on the Compact Disc (CD)

File Name	Main Sheet Name	Description
Research Part I		
	Equally Weighted Index	The data and calculations for EW Index
	Value Weighted Index	The calculation of the VW Index
	Combined	The price and return of the three indices; EW, VW and the MSCI
	Return Index - Local	Component used in calculating the indices
	Shares Outstanding	Component used in calculating the value weighted index
	Price	Component used in calculating the value weighted index
	R_i	Component used in calculating the value weighted index
	M_{it} & M_t	Component used in calculating the value weighted index
	M_{it}_M_t	Component used in calculating the value weighted index
	M x R_i	Component used in calculating the value weighted index
	Tables	Tables containing results
	Performance Measures	Regression results for the indices
	Weighting of VW Index	The weights of individual constituents in the Value Weighted Index
Research Part II		
	Data	The risk free rate, the gold index and the bond index
	Combined	The price and return of each index
	Shortsales	Portfolio allocations with shortsale
	No Shortsale	Portfolio allocations without shortsale
	Periods Shortsales	Portfolio allocations for the sub-periods with shortsale
	Periods NoShortsale	Portfolio allocations for the sub-periods without shortsale
	Results	A table with the results
	Alpha	The Alpha calculated
	Alpha (periods)	The Alpha calculated for the sub-periods
	Regression	Regression results for the portfolios
	Regression (periods)	Regression results for the portfolios in each sub-period
Percentiles		
	Price	Component used in calculating percentiles
	Return	Component used in calculating percentiles
	MSCI	The MSCI World Index
	Percentiles	Calculating the percentiles
	Percentile Pos	Ranking the constituents in quartiles
	Quartiles	The result of the ranking

Appendix B. Listed Private Equity Data Sample and Normality Results

	Company Name	Region	Country	Category	Style	Focus	Data Available	Jarque-Bera Test	P-Values
1	3I Group plc	UK	UK	Direct private equity	Buyout	Diversified	yes	531	<.0001
2	Aberdeen Private Equity	UK	UK	Direct private equity	Buyout	Diversified	yes	10937	<.0001
3	Absolute Private Equity AG	Europe	Switzerland	Private equity fund of funds	Buyout	Diversified			
4	Allied Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified			
5	Altamir Amboise S.A.	Europe	France	Direct private equity	Buyout	Diversified	yes	7032	<.0001
6	Amanda Capital Plc	Europe	Finland	Private equity fund of funds	Buyout	Diversified	yes	1709338	<.0001
7	American Capital, Ltd.	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	15901	<.0001
8	Amphion Innovations plc	UK	UK	Direct private equity	Venture	IT/Health Care	yes	833	<.0001
9	AP Alternative Assets, L.P.	Europe	Netherlands	Private equity fund of funds	Buyout	Diversified			
10	APEN AG	Europe	Switzerland	Private equity fund of funds	Buyout	Diversified	yes	194405	<.0001
11	Apollo Investment Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	7735	<.0001
12	ARC Capital Holdings	UK	UK	Direct private equity	Growth	Diversified	yes	862	<.0001
13	Ares Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	1280	<.0001
14	Arques Industries AG	Europe	Germany	Direct private equity	Buyout	Diversified			<.0001
15	Aurelius AG	Europe	Germany	Direct private equity	Buyout	Diversified	yes	155	<.0001
16	Aurora Russia Limited	UK	UK	Direct private equity	Buyout	Diversified	yes	4517	<.0001
17	Better Capital Limited	UK	UK	Direct private equity	Buyout	Diversified	yes	621	<.0001
18	BlackRock Kelso Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	251	<.0001
19	BMP AG	Europe	Germany	Direct private equity	Venture	Diversified	yes	134	<.0001
20	Brait S.A.	ROW	South Africa	Direct private equity	Buyout	Diversified	yes	51222	<.0001
21	Bure Equity AB	Europe	Sweden	Direct private equity	Buyout	Diversified	yes	71038	<.0001
22	Candover Investments plc	UK	UK	Direct private equity	Buyout	Diversified	yes	26698	<.0001
23	CapMan Plc	Europe	Finland	Private equity fund manager	Buyout	Diversified	yes	278	<.0001
24	Castle Private Equity AG	Europe	Switzerland	Private equity fund of funds	Buyout	Diversified	yes	2227	<.0001
25	China Growth Opportunities Limited	UK	UK	Direct private equity	Growth	Diversified	yes	3109	<.0001
26	China Merchants China Direct Investments	Asia/Pacific	Hongkong	Direct private equity	Growth	Financials	yes	1284	<.0001
27	Cleantech Invest AG	Europe	Germany	Direct private equity	Growth	Cleantech	yes	76	<.0001
28	Compass Diversified Holdings	North America	USA	Direct private equity	Buyout	Diversified	yes	27	<.0001
29	Conversus Capital, L.P.	Europe	Netherlands	Private equity fund of funds	Buyout	Diversified	yes	276	<.0001
30	DeA Capital S.p.A.	Europe	Italy	Direct private equity	Buyout	Financials/HC	yes	5434	<.0001
31	Deutsche Beteiligungs AG	Europe	Germany	Direct private equity	Buyout	Industrials	yes	1419	<.0001
32	Dinamia Capital Privado, S.C.R., S.A.	Europe	Spain	Direct private equity	Buyout	Diversified	yes	852	<.0001
33	Dunedin Enterprise Investment Trust PLC	UK	UK	Direct private equity	Buyout	Diversified	yes	5136	<.0001
34	East Capital Explorer AB	Europe	Sweden	Private equity fund of funds	Growth	Diversified	yes	13	<.0001
35	EIH plc	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	349	<.0001
36	Electra Private Equity PLC	UK	UK	Direct private equity	Buyout	Diversified	yes	2184	<.0001
37	Equus Total Return, Inc.	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	7686	<.0001
38	Eurazeo S.A.	Europe	France	Direct private equity	Buyout	Industrials/Cons.	yes	125	<.0001
39	F&C Private Equity Trust plc B	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	4434	<.0001
40	Fifth Street Finance Corp.	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	150	<.0001
41	Fortress Investment Group LLC	North America	USA	Direct private equity	Buyout	Diversified	yes	633	<.0001
42	GIMV N.V.	Europe	Belgium	Direct private equity	Buyout	Diversified	yes	388	<.0001
43	Gladstone Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	6037	<.0001
44	Gladstone Investment Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	4286	<.0001
45	GP Investments Limited	South America	Brazil	Direct private equity	Buyout	Diversified			
46	Graphite Enterprise Trust PLC	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	4243	<.0001
47	Greenwich Loan Income Fund Ltd	UK	UK	Direct private mezzanine	Buyout	Diversified	yes	134135	<.0001
48	GSC Investment Corp.	North America	USA	Direct private mezzanine	Buyout	Diversified			
49	Harbourvest Global Private Equity	Europe	Netherlands	Private equity fund of funds	Buyout	Diversified	yes	97246	<.0001
50	Harris & Harris Group, Inc.	North America	USA	Direct private equity	Venture	Nanotechnology	yes	1223	<.0001
51	HBM BioVentures AG	Europe	Switzerland	Direct private equity	Venture	Health Care	yes	1225	<.0001
52	Heliad Equity Partners GmbH & Co. KGaA	Europe	Germany	Direct private equity	Growth	Diversified	yes	119	<.0001
53	Helikos SE	Europe	Germany	Direct private equity	Buyout	Diversified	yes	0,35	0.8404
54	Henderson Diversified Income Limited	UK	UK	Direct private mezzanine	Buyout	Diversified	yes	262	<.0001
55	Henderson PE Investment Trust plc	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	6267	<.0001
56	Hercules Technology Growth Capital, Inc.	North America	USA	Direct private mezzanine	Venture	Diversified	yes	539	<.0001
57	HgCapital Trust plc	UK	UK	Direct private equity	Buyout	Diversified	yes	307	<.0001
58	Imperial Innovations Group plc	UK	UK	Direct private equity	Venture	Diversified	yes	1086	<.0001
59	ING Private Equity Access Limited	Asia/Pacific	Australia	Private equity fund of funds	Buyout	Diversified	yes	968	<.0001
60	Ingenious Media Active Capital Limited	UK	UK	Direct private equity	Venture	Media	yes	1076	<.0001

61	Intermediate Capital Group PLC	UK	UK	Direct private mezzanine	Buyout	Diversified	yes	2538	<.0001
62	Internet Capital Group, Inc.	North America	USA	Direct private equity	Buyout	IT	yes	555	<.0001
63	IP Group plc	UK	UK	Direct private equity	Venture	Health Care	yes	4327	<.0001
64	J.P. Morgan Private Equity Limited	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	154	<.0001
65	Jafco Co., Ltd.	Asia/Pacific	Japan	Direct private equity	Venture	Diversified	yes	74	<.0001
66	Japan Asia Investment Co., Ltd.	Asia/Pacific	Japan	Direct private equity	Venture	Diversified	yes	1743	<.0001
67	JZ Capital Partners Limited	UK	UK	Direct private mezzanine	Buyout	Diversified	yes	18179	<.0001
68	k1 Ventures Limited	Asia/Pacific	Singapore	Direct private equity	Buyout	Diversified	yes	2735	<.0001
69	Kayne Anderson Energy Development	North America	USA	Direct private mezzanine	Buyout	Energy	yes	295	<.0001
70	KKR & Co. (Guernsey) L.P.	Europe	Netherlands	Private equity fund manager	Buyout	Diversified	yes	2,25	0.3246
71	Kohlberg Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	516	<.0001
72	Kubera Cross Border Fund Limited	UK	UK	Direct private equity	Growth	Diversified	yes	3182	<.0001
73	Ledstiernan AB	Europe	Sweden	Direct private equity	Venture	Diversified			
74	LMS Capital plc	UK	UK	Direct private equity	Venture	Diversified	yes	153	<.0001
75	Main Street Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	16	0.0003
76	Management & Capitali	Europe	Italy	Direct private equity	Buyout	Diversified	yes	1740	<.0001
77	Marfin Investment Group S.A.	Europe	Greece	Direct private equity	Buyout	Diversified	yes	16388	<.0001
78	MCG Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	10789	<.0001
79	Mithras Investment Trust plc	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	9252	<.0001
80	MVC Capital, Inc.	North America	USA	Direct private equity	Buyout	Diversified	yes	1281	<.0001
81	NAXS Nordic Access Buyout Fund AB	Europe	Sweden	Private equity fund of funds	Buyout	Diversified	yes	70	<.0001
82	NB Private Equity Partners Limited	Europe	Netherlands	Private equity fund of funds	Buyout	Diversified	yes	330	<.0001
83	New Value AG	Europe	Switzerland	Direct private equity	Venture	Cleantech/HC	yes	16	0.0003
84	New Venturetec AG	Europe	Switzerland	Direct private equity	Venture	Diversified	yes	442	<.0001
85	NGP Capital Resources Company	North America	USA	Direct private mezzanine	Buyout	Energy	yes	682	<.0001
86	Northern Investors Company PLC	UK	UK	Direct private equity	Buyout	Diversified	yes	7807	<.0001
87	Novestra AB	Europe	Sweden	Direct private equity	Venture	Diversified	yes	307	<.0001
88	Oakley Capital Investments Ltd.	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	417	<.0001
89	OFI Private Equity Capital SCA	Europe	France	Direct private equity	Buyout	Diversified	yes	16785	<.0001
90	Onex Corporation SV	North America	Canada	Direct private equity	Buyout	Diversified	yes	306	<.0001
91	Origo Partners PLC	UK	UK	Direct private equity	Growth	Diversified	yes	805	<.0001
92	Pantheon International Participations PLC	UK	UK	Private equity fund of funds	Balanced	Diversified	yes	7010	<.0001
93	Partners Group Holding AG	Europe	Switzerland	Private equity fund manager	Buyout	Diversified	yes	99	<.0001
94	PennantPark Investment Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	48	<.0001
95	Princess Private Equity Holding Limited	Europe	Germany	Private equity fund of funds	Buyout	Diversified	yes	155	<.0001
96	Private Equity Holding AG	Europe	Switzerland	Private equity fund of funds	Balanced	Diversified	yes	3745	<.0001
97	Private Equity Investor PLC	UK	UK	Private equity fund of funds	Venture	Diversified	yes	1786	<.0001
98	Promethean PLC	UK	UK	Direct private equity	Buyout	Diversified	yes	7656	<.0001
99	Prospect Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	1326	<.0001
100	Quorum Oil and Gas Technology Fund Ltd	UK	UK	Direct private mezzanine	Other	IT/Energy	yes	238	<.0001
101	Ratos AB	Europe	Sweden	Direct private equity	Buyout	Diversified	yes	142	<.0001
102	Safeguard Scientifics, Inc.	North America	USA	Direct private equity	Venture	Health Care/Tech	yes	143	<.0001
103	Scandinavian Private Equity AS	Europe	Denmark	Private equity fund of funds	Buyout	Diversified	yes	170	<.0001
104	shaPE Capital AG	Europe	Switzerland	Private equity fund of funds	Buyout	Diversified	yes	14718	<.0001
105	Solar Capital Ltd.	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	20	<.0001
106	Spark Ventures plc	UK	UK	Direct private equity	Venture	Diversified	yes	2822	<.0001
107	Standard Life European PE Trust PLC	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	4561	<.0001
108	SVG Capital plc	UK	UK	Private equity fund of funds	Buyout	Diversified	yes	145255	<.0001
109	Symphony International Holdings Limited	UK	UK	Direct private equity	Buyout	Diversified	yes	194	<.0001
110	The Blackstone Group L.P.	North America	USA	Private equity fund manager	Buyout	Diversified	yes	166	<.0001
111	TICC Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	3309	<.0001
112	Triangle Capital Corporation	North America	USA	Direct private mezzanine	Buyout	Diversified	yes	233	<.0001
113	TVC Holdings	Europe	Ireland	Direct private equity	Buyout	Diversified	yes	277	<.0001
114	Unternehmens Invest AG	Europe	Austria	Direct private equity	Buyout	Diversified	yes	1905	<.0001
115	Wendel S.A.	Europe	France	Direct private equity	Buyout	Industrials	yes	577	<.0001

This Appendix presents the LPE vehicles used to construct the indices. Further information is provided on geographies, underlying business and investment styles. Additionally, the results of each the normal distribution test are presented, which state that statistics of two vehicles are normal-distributed. However, their results are biased because of too few observations.

Equally Weighted Index

Jarque-Bera Test

Statistic	Value	Prob	Label
Normal Test	1284,4816	<.0001	Pr > ChiSq

Durbin-Watson Statistics

Order	DW
1	1,6017

S&P Gold Index

Jarque-Bera Test

Statistic	Value	Prob	Label
Normal Test	118,6412	<.0001	Pr > ChiSq

Durbin-Watson Statistics

Order	DW
1	0,0035

Value Weighted Index

Jarque-Bera Test

Statistic	Value	Prob	Label
Normal Test	425,1914	<.0001	Pr > ChiSq

Durbin-Watson Statistics

Order	DW
1	1,7191

BOFA ML Corporate Bond Index

Jarque-Bera Test

Statistic	Value	Prob	Label
Normal Test	21,9228	<.0001	Pr > ChiSq

Durbin-Watson Statistics

Order	DW
1	0,0012

MSCI World Index

Jarque-Bera Test

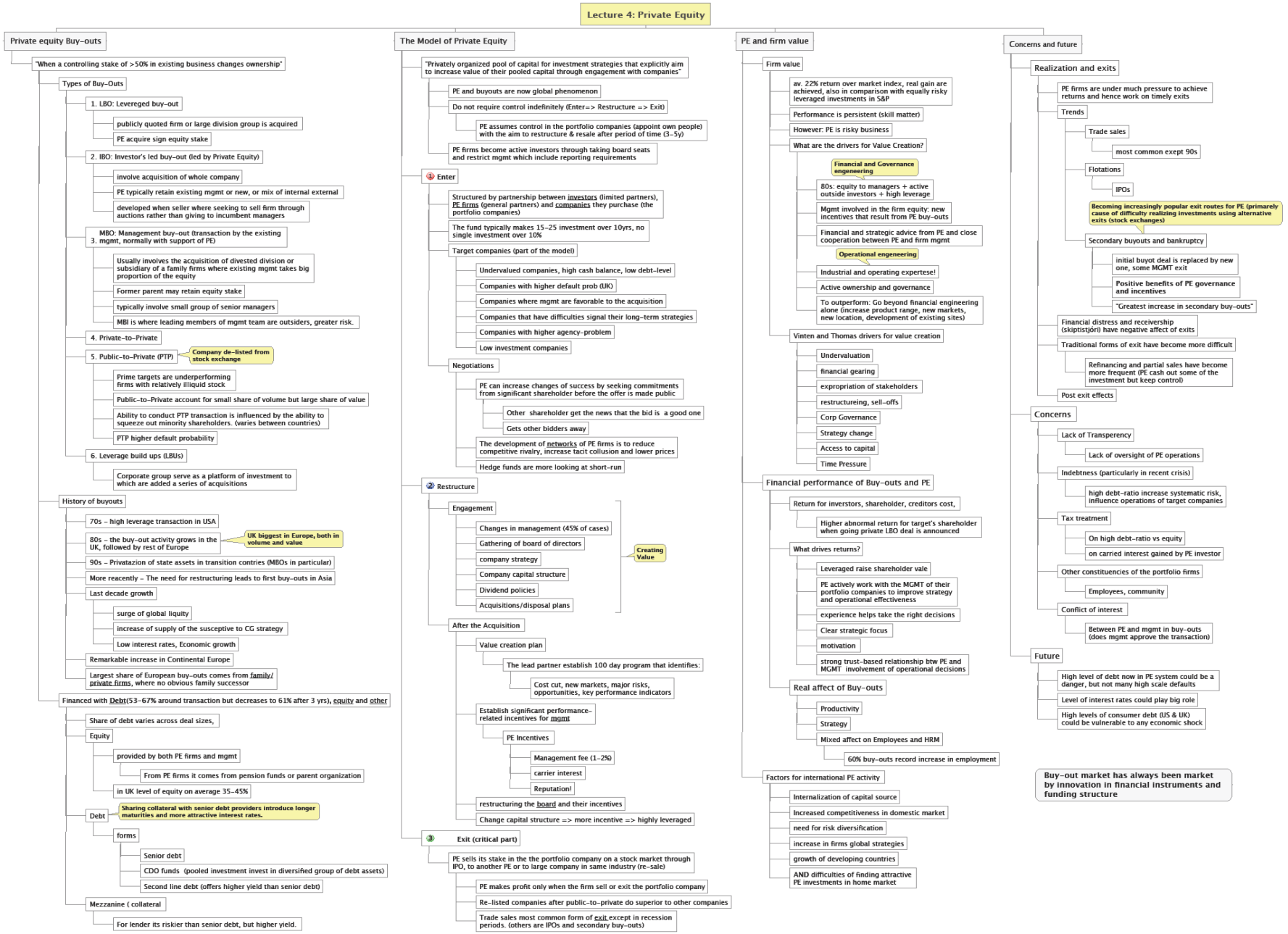
Statistic	Value	Prob	Label
Normal Test	511,6989	<.0001	Pr > ChiSq

Durbin-Watson Statistics

Order	DW
1	2,0455

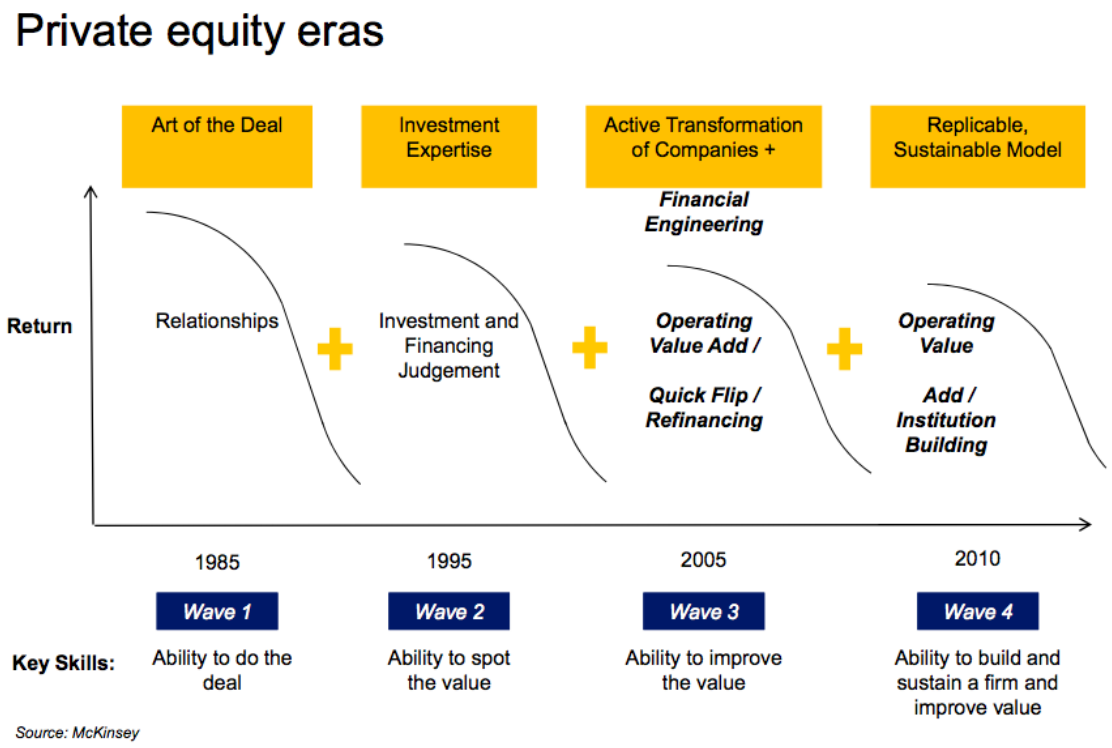
The tables above present the normal distribution tests of the LPE indices and all asset classes for Research Part II. No time series is normally distributed.

Appendix C. The Private Equity Model and Waves



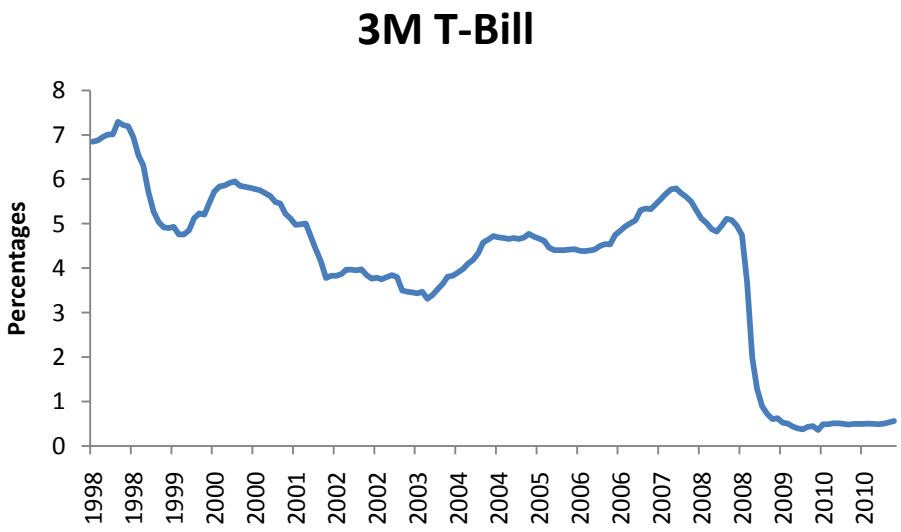
Source: Authors Contribution

Appendix C.a The Private Equity Historical Waves



In broader context the 1980s is often referred as LBO Boom, 1990s as the LBO Boom and VC Bubble and the 2000s as the Dot-com Bubble and the credit crunch.

Appendix D. Risk-Free Rate over the Sample Period



Source: Bank of England, *Statistical Interactive Database / interest & exchange rates data* (n.d.)

Appendix E. Sensitivity Analysis

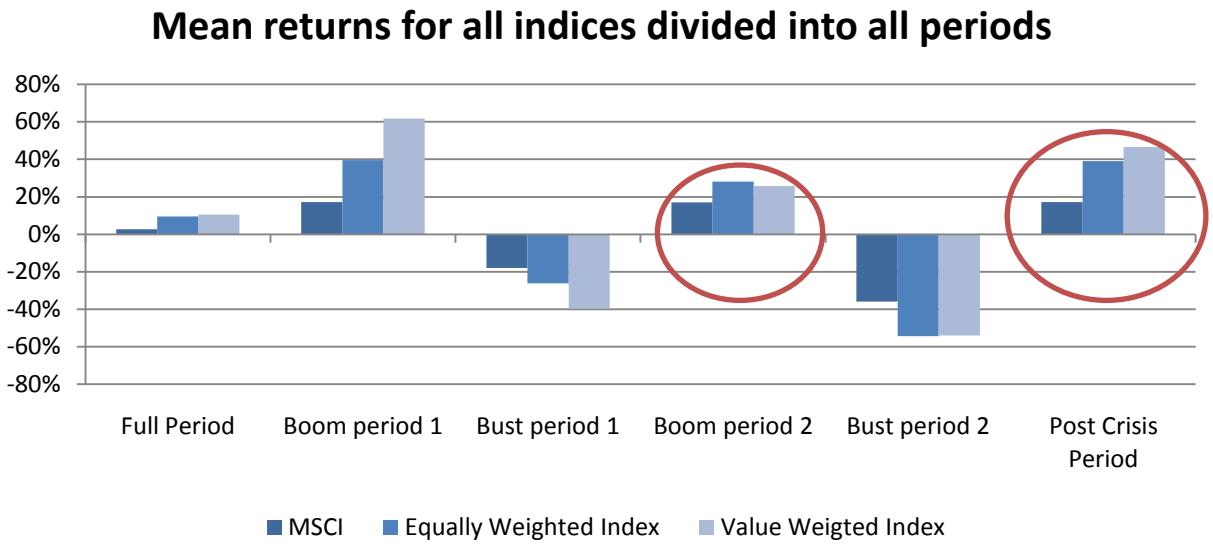
Constraints were altered by approxmitaley 40%.

Liquidity constraints						
Minimum of weekly observation	Minimum average market capitalization	Minimum relative trading volume	Minimum continuity of trade	Maximum average bid-ask spread	**Number of Vehicles**	**Total change in Vehicles**
18	20	0.20%	85%	5%	**77**	
30	20	0.20%	85%	5%	**77**	**1%**
42	20	0.20%	85%	5%	**76**	
30	12	0.20%	85%	5%	**79**	
30	20	0.20%	85%	5%	**77**	**3%**
30	28	0.20%	85%	5%	**77**	
30	20	0.12%	85%	5%	**79**	
30	20	0.20%	85%	5%	**77**	**8%**
30	20	0.30%	85%	5%	**73**	
30	20	0.20%	50%	5%	**83**	
30	20	0.20%	85%	5%	**77**	**16%**
30	20	0.20%	95%	5%	**70**	
30	20	0.20%	85%	3%	**66**	
30	20	0.20%	85%	5%	**77**	**20%**
30	20	0.20%	85%	7%	**82**	

Appendix F. Performance Persistency: Individual Vehicles Ranking

	Yearly distribution																Sub-period distribution						
Index Constituents	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Frequenzy in the top quartile	Frequenzy in the bottom quartile		Boom period 1	Bust period 1	Boom period 2	Bust period 2	Post crisis	Frequenzy in the top quartile	Frequenzy in the bottom quartile
3I GROUP	2	2	2	3	4	3	3	3	3	2	4	2	3	0	2		2	3	2	4	2	0	1
ALTAMIR AMBOISE	1	3	2	4	1	4	1	2	3	2	4	1	3	4	3		1	4	1	4	1	**3**	2
AMANDA CAPITAL	4	1	4	2	1	2	3	2	1	2	2	4	4	3	4		2	4	2	2	4	0	2
AMERICAN CAPITAL	2	3	2	1	2	2	3	4	2	4	4	1	1	3	3		3	1	3	4	1	2	1
BURE EQUITY	2	4	1	2	4	4	1	2	1	1	1	3	4	5	4		4	4	1	1	3	2	2
CANDOVER INVS.	2	2	3	2	1	3	3	2	3	2	3	3	1	2	0		3	1	2	3	2	1	0
CHINA MRCH.CHINA DINV.	4	3	3	1	1	1	4	4	1	1	4	1	4	**6**	5		4	1	1	2	2	2	1
DINAMIA CAPITAL PRIVADO	4	4	1	1	1	3	2	2	1	2	3	4	4	4	4		4	1	2	3	4	1	2
DUNEDIN ENTERPRISE	3	2	3	3	2	3	2	2	4	2	2	3	3	0	1		4	2	2	2	3	0	1
ELECTRA PRIVATE EQUITY	2	2	3	4	3	2	2	2	1	1	3	1	2	3	1		2	3	2	3	1	1	0
EQUUS TOTAL RETURN	4	4	3	2	2	2	4	3	4	4	1	4	4	1	7		4	2	4	2	4	0	3
EURAZEO	1	4	1	2	2	3	3	1	2	3	3	2	3	3	1		3	1	2	3	3	1	0
GIMV	1	4	4	4	4	2	1	3	4	2	1	4	3	3	6		3	4	2	1	4	1	2
GRAPHITE ENTERPRISE TST.	1	3	2	3	2	3	2	2	3	1	3	2	3	2	0		2	2	3	2	2	0	0
HARRIS & HARRIS GP.	4	1	4	2	1	1	1	4	4	4	2	3	4	4	6		1	3	1	3	3	2	0
HENDERSON PRIV.EQ.IT.	3	3	2	4	1	4	2	3	2	2	4	2	1	2	3		4	2	3	4	1	1	2
HG CAPITAL TRUST	2	2	2	3	3	2	1	2	2	1	1	3	3	3	0		3	1	1	1	4	**3**	1
JAFCO	4	1	4	3	2	1	4	1	4	4	1	4	3	4	6		1	3	3	2	3	1	0
JAPAN ASIA INVESTMENT	3	1	4	4	1	1	1	1	4	2	4	2	2	5	4		1	3	1	4	1	**3**	1
K1 VENTURES	1	4	3	1	3	1	2	2	2	3	2	4	4	3	3		3	3	2	1	4	1	1
NEW VENTURETEC 'B'	1	1	3	4	4	2	3	1	3	4	2	4	4	3	5		1	4	1	3	4	2	2
ONEX	1	1	3	1	3	4	2	4	1	1	2	3	2	5	2		1	2	2	2	2	1	0
PANTHEON INTL.PARTS.	3	2	1	3	1	4	3	1	3	2	4	1	1	5	2		3	1	3	3	1	2	0
RATOS 'B'	3	3	2	1	1	3	2	1	1	1	1	3	2	**6**	0		4	1	1	1	3	**3**	1
SAFEGD.SCIENTIFICS	2	1	4	1	4	1	4	4	1	3	3	1	1	**6**	4		1	4	1	3	1	**3**	1
SVG CAPITAL	3	1	1	3	1	2	2	3	3	2	4	2	1	4	1		2	1	3	4	1	2	1
WENDEL	1	3	2	2	4	1	1	1	1	3	3	2	1	**6**	1		2	2	1	3	2	1	0
JZ CAPITAL PARTNERS	4	3	3	2	3	3	3	2	4	4	3	3	2	0	3		4	3	3	3	2	0	1
CASTLE PRIVATE EQ.	3	2	1	4	4	2	4	1	3	1	4	2	1	4	4		3	4	1	4	2	1	2
PRIVATE EQUITY 'R'		2	1	4	4	4	1	3	2	1	3	4	3	3	4		1	4	1	1	3	**3**	1
BMP		4	1	4	3	1	2	2	4	2	3	4	3	2	4		2	4	1	4	3	1	2
APEN		4	1	3	3	4	2	2	4	2	4	4	3	1	5		2	2	3	4	4	0	2
PRIVATE EQ.INVESTOR			2	4	4	3	4	1	3	3	1	4	1	3	4		1	3	3	1	4	2	1
DEA CAPITAL			4	4	4	1	4	3	2	4	1	4	4	2	7		4	4	2	2	4	0	3
NOVESTRA			4	2	4	1	1	1	4	4	1	2	4	4	5		4	4	1	1	3	2	2
MVC CAPITAL			4	2	2	4	3	3	2	1	1	4	2	2	3			3	2	1	4	1	1
CAPMAN 'B'				1	3	3	1	3	3	2	4	2	2	2	1			1	2	4	2	1	1
STD.LF.EUR.PRIV.EQ.TST.				3	2	3	2	1	2	1	4	2	2	2	1			2	2	4	2	0	1
F&C PRIVATE EQUITY TST.				1	3	4	3	1	3	1	3	2	3	3	1			2	3	3	2	0	0
GLADSTONE CAPITAL				1	2	2	3	4	2	3	2	3	1	2	1			1	3	2	3	1	0
SHAPE CAPITAL				3	2	4	4	3	3	1	4	1	1	3	3			2	3	4	1	1	1
MCG CAP.				1	3	1	4	4	1	4	4	1	1	5	4			3	3	4	1	1	1
IP GROUP						4	1	4	1	3	2	3	4	2	3			1	2	2	4	1	1
TICC CAPITAL						2	4	4	4	4	2	2	1	1	4			1	4	3	1	2	1
APOLLO INV.							4	3	2	3	2	2	3	0	1				3	2	2	0	0
PROSPECT CAPITAL							4	3	3	3	1	4	4	1	3				4	1	4	1	2
ARES CAP.							1	4	2	3	2	1	2	2	1				4	2	1	1	1
NGP CAP.RES.							3	4	1	2	2	3	3	1	1				4	2	3	0	1
HELIAD EQ.PARTNERS							1	1	1	3	3	4	3	3	1				1	3	4	1	1
HERCULES TECH.GW.CAP.								4	2	2	1	2	4	1	2				4	1	3	1	1
GLADSTONE INV.								4	3	4	1	3	1	2	2				4	2	2	0	1
AURORA RUSSIA									4	2	4	1	4	1	3				4	4	1	1	2
PARTNERS GROUP HOLDING									1	1	2	2	2	2	0				1	2	2	1	0
COMPASS DIVERSIFIED									2	3	1	3	2	1	0				3	1	3	1	0
NEW VALUE 'R'									4	1	1	4	4	2	3				4	1	4	1	2
LMS CAPITAL									2	3	2	4	4	0	2				3	1	4	1	1
MANAGEMENT & CAPITALI									4	3	2	1	1	2	1				4	2	2	0	1
ARC CAPITAL HOLDINGS									3	1	4	1	2	2	1				2	3	2	0	0
KANYE ANDERSON EN.DEV.									4	1	3	1	2	2	1				4	3	1	1	1
KOHLBERG CAPITAL									1	3	4	1	1	3	1				1	4	1	2	1
KUBERA CROSS-BORDER FUND										3	2	3	3	0	0				4	2	3	0	1
FORTRESS INVESTMENT GP.										4	4	1	1	2	2				4	4	1	1	2
TRIANGLE CAPITAL										4	1	3	1	2	1				4	1	2	1	1
PENNANTPARK INVESTMENT										4	3	1	2	1	1				4	4	1	1	2
NORDIC ACS.BUYOUT FUND										4	1	3	2	1	1				4	1	3	1	1
HENDERSON DIVR.INCOME										2	2	4	4	0	2				4	1	4	1	2
BLACKROCK KELSO CAPITAL										1	1	3	2	2	0				4	1	3	1	1
BLACKSTONE GROUP										4	4	1	3	1	2				4	4	1	1	2
ABERDEEN PRIV.EQ.FD.£										3	3	4	2	0	1					3	3	0	0
NB PRIVATE EQUITY PTNS.										4	3	1	2	1	1					4	1	1	1
MAIN STREET CAPITAL										4	1	2	3	1	1					1	2	1	0
EAST CAPITAL EXPLORER										3	3	2	2	0	0					3	2	0	0
HBM BIO VENTURES											3	3	4	0	1					3	4	0	1
FIFTH STREET FINANCE											2	2	3	0	0					2	3	0	0
BETTER CAPITAL												1	4	1	1						4	0	1
SOLAR CAPITAL													1	1	0						3	0	0
KKR & COMPANY													1	1	0						1	1	0

Appendix G. Mean Returns for all indices divided into all periods



Analysis of return characteristics in different sub-periods demonstrates the post-crisis returns of the LPE indices are higher than in previous Boom period.

Appendix H. Weights of Constituents in the Value weighted Index

Largest 10 Companies by Mkt. Cap.	68,51%
Smallest 50% of companies by Mkt. Cap.	7,23%

Nr.	Company	Weights
1	BLACKSTONE GROUP	23,38%
2	WENDEL	6,03%
3	PARTNERS GROUP HOLDING	5,68%
4	3I GROUP	5,64%
5	RATOS 'B'	5,39%
6	EURAZEO	4,99%
7	ONEX	4,96%
8	KKR & COMPANY	4,37%
9	ARES CAP.	4,25%
10	AMERICAN CAPITAL	3,81%
11	APOLLO INV.	2,83%
12	JAFCO	1,83%
13	GIMV	1,54%
14	SVG CAPITAL	1,48%
15	PROSPECT CAPITAL	1,27%
16	FORTRESS INVESTMENT GP.	1,17%
17	ELECTRA PRIVATE EQUITY	1,11%
18	SOLAR CAPITAL	1,04%
19	BLACKROCK KELSO CAPITAL	0,95%
20	FIFTH STREET FINANCE	0,87%
21	COMPASS DIVERSIFIED HDG. SHS.OF I	0,77%
22	DEA CAPITAL	0,73%
23	MCG CAP.	0,59%
24	HBM BIO VENTURES	0,59%
25	HG CAPITAL TRUST	0,58%
26	BURE EQUITY	0,55%
27	EAST CAPITAL EXPLORER	0,55%
28	PENNANTPARK INVESTMENT	0,54%
29	JZ CAPITAL PARTNERS	0,52%
30	ARC CAPITAL HOLDINGS	0,50%
31	PANTHEON INTL.PARTS.	0,50%
32	SAFEGD.SCIENTIFICS	0,48%
33	HERCULES TECH.GW.CAP.	0,48%
34	NB PRIVATE EQUITY PTNS.	0,47%
35	GRAPHITE ENTERPRISE TST.	0,47%
36	STD.LF.EUR.PRIV.EQ.TST.	0,47%
37	BETTER CAPITAL	0,46%
38	TICC CAPITAL	0,46%
39	CASTLE PRIVATE EQ.	0,46%
		10 Companies

Nr.	Company	Weights
40	MAIN STREET CAPITAL	7,23%
41	ALTAMIR AMBOISE	6,80%
42	TRIANGLE CAPITAL	6,37%
43	MVC CAPITAL	5,95%
44	CHINA MRCH.CHINA DI	5,55%
45	K1 VENTURES	5,18%
46	LMS CAPITAL	4,86%
47	GLADSTONE CAPITAL	4,57%
48	CANDOVER INVS.	4,29%
49	NGP CAP.RES.	4,03%
50	IP GROUP	3,78%
51	KANYE ANDERSON EN.I	3,55%
52	DINAMIA CAPITAL PRIV	3,32%
53	CAPMAN 'B'	3,10%
54	PRIVATE EQUITY 'R'	2,88%
55	KOHLBERG CAPITAL	2,67%
56	GLADSTONE INV.	2,47%
57	F&C PRIVATE EQUITY TS	2,26%
58	HARRIS & HARRIS GP.	2,06%
59	DUNEDIN ENTERPRISE	1,87%
60	JAPAN ASIA INVESTMEI	1,68%
61	ABERDEEN PRIV.EQ.FD.	1,50%
62	MANAGEMENT & CAPIT	1,32%
63	SHAPE CAPITAL	1,16%
64	HENDERSON DIVR.INCC	1,02%
65	KUBERA CROSS-BORDEI	0,88%
66	APEN	0,78%
67	HENDERSON PRIV.EQ.IT	0,68%
68	PRIVATE EQ.INVESTOR	0,59%
69	NORDIC ACS.BUYOUT FI	0,50%
70	AURORA RUSSIA	0,41%
71	HELIAD EQ.PARTNERS	0,32%
72	NEW VALUE 'R'	0,26%
73	AMANDA CAPITAL	0,19%
74	NOVESTRA	0,13%
75	BMP	0,08%
76	EQUUS TOTAL RETURN	0,05%
77	NEW VENTURETEC 'B'	0,02%
		38 companies

Appendix I. Correlation Significance Test

Correlation significant test.
This table determines the t-statistics of the correlation coefficients in table 9.3.* indicates that the corresponding correlation coefficient is significant at 5 percent significance level, ** indicates that the corresponding correlation coefficient is significant at 1 percent significance level

	Stocks	Bonds	Gold	LPE
Stocks	-			
Bonds	5.60**	-		
Gold	6.79**	0.68	-	
LPE	34.45**	2.01*	1.31	-

Appendix J. VBA Code for Variance-Covariance Matrix.

```
Function VarCovar(rng As Range) As Variant
    Dim i As Integer
    Dim j As Integer
    Dim numCols As Integer
    numCols = rng.Columns.Count
    Dim matrix() As Double
    ReDim matrix(numCols - 1, numCols - 1)
      For i = 1 To numCols
        For j = 1 To numCols
            matrix(i - 1, j - 1) = Application.WorksheetFunction.Covar(rng.Columns(i), rng.Columns(j))
        Next j
    Next i
    VarCovar = matrix
End Function
```

Appendix K. Regression Results

Appendix K.a Equally-weighted Portfolio

SUMMARY OUTPUT

EW Bust 1 period

Regression Statistics	
Multiple R	0.76615411
R Square	0.586992121
Adjusted R Square	0.584394587
Standard Error	0.018012311
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.073317887	0.073318	225.9805486	2.41575E-32
Residual	159	0.051586493	0.000324		
Total	160	0.12490438			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	-0.002538017	0.001436169	-1.76721	0.079110939	-0.005374446	0.000298412	-0.005374446	0.000298412
X Variable 1	0.775391622	0.051580503	15.03265	2.41575E-32	0.673520325	0.877262919	0.673520325	0.877262919

SUMMARY OUTPUT

EW Boom 1 period

Regression Statistics	
Multiple R	0.648712461
R Square	0.420827857
Adjusted R Square	0.415562656
Standard Error	0.018516581
Observations	112

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.02740382	0.027404	79.92626174	1.04098E-14
Residual	110	0.037715015	0.000343		
Total	111	0.065118835			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.004943576	0.00176147	2.806506	0.005925038	0.001452757	0.008434395	0.001452757	0.008434395
X Variable 1	0.728080463	0.081439412	8.940149	1.04098E-14	0.56668667	0.889474257	0.56668667	0.889474257

SUMMARY OUTPUT

EW Post Crisis period

Regression Statistics	
Multiple R	0.826745093
R Square	0.683507449
Adjusted R Square	0.680681622
Standard Error	0.018234467
Observations	114

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.080423682	0.080424	241.8787865	9.49727E-30
Residual	112	0.03723953	0.000332		
Total	113	0.117663212			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.004353805	0.001714635	2.539203	0.012482586	0.000956476	0.007751133	0.000956476	0.007751133
X Variable 1	0.942043476	0.06057202	15.55245	9.49727E-30	0.822027787	1.062059165	0.822027787	1.062059165

SUMMARY OUTPUT

EW Bust 2 period

Regression Statistics	
Multiple R	0.886973347
R Square	0.786721718
Adjusted R Square	0.78395187
Standard Error	0.017245272
Observations	79

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.084470555	0.084471	284.0306653	1.4871E-27
Residual	77	0.022899755	0.000297		
Total	78	0.107370311			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	-0.00437371	0.001982267	-2.20642	0.030335422	-0.008320907	-0.00042651	-0.008320907	-0.000426513
X Variable 1	0.890983008	0.05286726	16.85321	1.4871E-27	0.785710845	0.996255171	0.785710845	0.996255171

SUMMARY OUTPUT

EW Boom 2 period

Regression Statistics	
Multiple R	0.772563511
R Square	0.596854378
Adjusted R Square	0.595021898
Standard Error	0.008042728
Observations	222

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.021068607	0.021069	325.708518	2.76569E-45
Residual	220	0.014230803	6.47E-05		
Total	221	0.03529941			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.002905374	0.000548296	5.29892	2.8239E-07	0.00182479	0.003985958	0.00182479	0.003985958
X Variable 1	0.697446003	0.038645242	18.0474	2.76569E-45	0.621283746	0.77360826	0.621283746	0.77360826

Appendix K.b Value Weighted Portfolio

SUMMARY OUTPUT

VW Bust 1 period

Regression Statistics	
Multiple R	0.70553753
R Square	0.497783207
Adjusted R Square	0.49462461
Standard Error	0.038308898
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.231283923	0.231283923	157.5963426	1.47983E-25
Residual	159	0.233343891	0.001467572		
Total	160	0.464627814			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	-0.002591515	0.00305447	-0.848433817	0.397471895	-0.008624081	0.003441051	-0.008624081	0.003441051
X Variable 1	1.377174134	0.109702314	12.55373819	1.47983E-25	1.160512483	1.593835786	1.160512483	1.593835786

SUMMARY OUTPUT

VW Boom 1 period

Regression Statistics	
Multiple R	0.585137804
R Square	0.34238625
Adjusted R Square	0.336407943
Standard Error	0.03265848
Observations	112

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.061084362	0.061084362	57.27144153	1.24234E-11
Residual	110	0.117323393	0.001066576		
Total	111	0.178407755			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.008298015	0.003106779	2.670938496	0.00871198	0.002141109	0.014454921	0.002141109	0.014454921
X Variable 1	1.087023285	0.143638146	7.567789739	1.24234E-11	0.802366204	1.371680367	0.802366204	1.371680367

SUMMARY OUTPUT

VW Post Crisis period

Regression Statistics	
Multiple R	0.854673933
R Square	0.730467531
Adjusted R Square	0.728060991
Standard Error	0.023837345
Observations	114

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.172473999	0.172473999	303.534353	1.14127E-33
Residual	112	0.063640532	0.000568219		
Total	113	0.236114531			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.004699912	0.002241488	2.096782166	0.038263922	0.000258691	0.009141132	0.000258691	0.009141132
X Variable 1	1.379560613	0.079183895	17.42223731	1.14127E-33	1.22266788	1.536453347	1.22266788	1.536453347

SUMMARY OUTPUT

VW Bust 2 period

Regression Statistics	
Multiple R	0.901144266
R Square	0.812060988
Adjusted R Square	0.809620222
Standard Error	0.020384163
Observations	79

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.138244601	0.138244601	332.7073791	1.12429E-29
Residual	77	0.031994584	0.000415514		
Total	78	0.170239185			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	-0.002412053	0.002343068	-1.02944203	0.306494875	-0.007077697	0.002253592	-0.007077697	0.002253592
X Variable 1	1.139831977	0.06248987	18.24026807	1.12429E-29	1.015398751	1.264265203	1.015398751	1.264265203

SUMMARY OUTPUT

VW Boom 2 period

Regression Statistics	
Multiple R	0.747758144
R Square	0.559142242
Adjusted R Square	0.557138343
Standard Error	0.014238812
Observations	222

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.056571022	0.056571022	279.0271714	5.34292E-41
Residual	220	0.04460363	0.000202744		
Total	221	0.101174652			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.001321809	0.0009707	1.361706973	0.174683187	-0.000591252	0.00323487	-0.000591252	0.00323487
X Variable 1	1.142851193	0.06841738	16.70410642	5.34292E-41	1.008013842	1.277688545	1.008013842	1.277688545