Investment strategies for pension funds

Defined contribution versus defined benefit

Master thesis

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Abstract

This paper investigates the pension sector in Norway, a hot topic among economists in Norway for the recent years. We look at different asset allocation strategies for pension funds and our results indicate that the current investment strategies underperform alternative investment strategies suggested in academic literature.

We compare the defined benefit and the defined contribution pension scheme and find that, given today's low defined contribution rates, the defined contribution scheme will not perform equivalent pension payments to the defined benefit scheme, neither in terms of size or risk.

These findings have two important implications; firstly, pension funds should reconsider their investment strategies and secondly, the defined contribution rates are on average too low for pension holders under the defined contribution scheme to be equivalent to the defined benefit scheme.

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Chapter 1

Introduction

1.1 Motivation

Throughout our degree, we have obtained valuable insights to economic thinking and acknowledged models within finance. In our master thesis we wanted to challenge ourselves with a topic where we had limited knowledge from our studies to learn more within a field we wanted to know more about. We looked into the Norwegian market to investigate what we felt could be a relevant topic that we had yet not covered extensively in our study. We both had an interest to learn more about pensions and how pension holdings are managed, because it is relevant both for us as individuals and for the society as a whole. By looking into pensions, the variety of possible angles to approach the paper was huge. Pension is as well considered a "hot topic" among economists in Norway, because the future challenges are huge and changes within the pension sector is still expected to be made [NOU 2004:I]. In order to be up-to-date on this discussion, we felt that writing our thesis about this topic would be a great opportunity for us to get additional insights.

We were pleased with writing about an applied topic that requires a discussion of the relationship between financial calculations and individual behaviour. In addition, we got to apply several of the topics we have found the most interesting throughout our study. We felt that pensions provided us an opportunity to be creative in the design of our paper and gave us an opportunity to apply many of the theories that we have learned throughout our studies at AEF in a more practical context.

1.2 Background

In 2006, the Norwegian government introduced the law of mandatory occupational pension for all workers in Norway. A large proportion of the workforce in Norway was only covered by the public retirement pension that was introduced to secure a minimum pension for all pensioners in 1967 [NOU 2004:I]. While most employers in the public sector are covered with what is called a defined benefit pension scheme, most private firms turned to a defined contribution scheme after the introduction of mandatory occupational pensions in 2006. We will elaborate in detail how these pension schemes work later in this paper.

The financial crisis in 2007/2008 caused turmoil in the financial markets and was of major concern for pension savers under the defined contribution scheme, as well as for pension funds investing capital for defined benefit pension savers. Many pension funds who managed pension capital for defined benefit savers had promised a yearly return they could no longer maintain, and most of these pension funds had to take on major losses to cover up for their promised return. Because a defined benefit scheme guarantees the employer an amount of its salary at retirement, most pension savers under this scheme was relatively unaffected by the disruptions in the financial markets. The story is quite different for pension savers under the defined contribution scheme.

The defined contribution pension scheme makes every individual pension owner an investor of his or her own pension capital. This means that each individual is responsible for his or her return on the pension capital. To help pension holders manage their pension savings, professional Life Insurance Companies(LIC) offer pension saving services. This allows individuals to invest in professionally diversified portfolios which, for most individuals, yield a higher return than what they could achieve themselves. The LICs contract themselves to companies and manage the pension savings for all employers working in that particular company. LICs offer different pension saving strategies to individuals, and it is ultimately the individuals decision which pension saving strategy they want to invest in.

Throughout the past 10 years, lots of research have been done on pension funds and pension saving strategies. Academics have criticized the LICs for their investment strategies, arguing that they are suboptimal. They refer to what they call *"the glidepath illusion"*, which they define as an illusion that pension holders should reduce their stock proportion the closer they get to retirement in order to reduce risk[Estrada, 2013; Arnott, 2012]. Instead, they suggest a higher stock proportion throughout the investment period. In today's low-yield environment, we believe that this critique may be even more relevant.

1.3 Problem statement

Even though the market conditions have changed after the financial crisis in 2007/2008, the principles of the investment strategies for LICs are still the same. Most pension savers still invest in these investment strategies, despite empirical results stating that these strategies have been suboptimal historically [Estrada, 2013]. In this paper, we want to investigate investment strategies for LICs to test whether these strategies are suboptimal if we make predictions about the future returns on chosen asset classes. By forecasting chosen variables, we will test strategies that have been discussed in the literature to compare our results with the results of those who have back-tested identical strategies. In addition, we want to include an empirical strategy suggested by one of the biggest LICs in Norway, Storebrand, to see how this strategy will perform against the theoretical strategies proposed in the academic literature. Our main research question is therefore:

How do the investment strategy of Storebrand perform against alternative investment strategies proposed in academic literature for the next 65 years, given our forecasted variables?

In order to get an understanding of the pension level obtained in each defined contribution strategy, we believe it is interesting to compare the defined benefit and the defined contribution pension schemes in terms of risk and expected pension payments. While the defined contribution scheme is not ment to be a perfect substitute for the defined benefit scheme, we believe pensions in the public and the private sector should be relatively similar as the wage differences are small [Statisk sentralbyrå, a]. Because of a higher risk for pension savers using the defined contribution scheme, we want to investigate if they are compensated for that risk. In addition, we want to test at what defined contribution rates the different investment strategies under the defined contribution scheme performs similar pension payments to the defined benefit scheme for an individual starting their pension savings at the age of 25 today and retire at age 67. Our sub research question is therefore:

Does the defined contribution pension scheme offer a satisfactory pension with regards to level and risk, compared with the defined benefit scheme?

Chapter 2

The Norwegian Pension Sector

In this chapter, we will look at the purpose and development of pensions in Norway, before we in the following sections will go more detailed into how the pension system in Norway is constructed. Finally, we will briefly introduce those who are managing the pension capital for pension savers.

2.1 Purpose of a pension system

By law, Norwegians are allowed to retire at the earliest from age 62, while most people retire at age 67. For some professions, other rules apply. The purpose of a pension system is to secure all pensioners a minimum income during their years as a retiree. The pension system is built to reward those who have been working most of their lives, but also to support those who, for some reason, have not been able to work due to different circumstances. A pension system facilitate consumption equalization by forcing individuals to save money for their retirement period. We will in the next section elaborate on the pension sector in Norway.

2.2 The Norwegian Pension Sector

The Norwegian pension system can be divided into three pillars. We will in the following elaborate on these.

2.2.1 Pillar I: Public Retirement Pension

Norway introduced what is today known as "Folketrygden" in 1967, a social security with the purpose of securing economic and social safety for all individuals living in Norway [NOU2004:1]. This social security is covering up for most disabilities that prevent individuals from working. The largest liability of Folketrygden is the public retirement pension, a pension introduced to secure all workers in Norway with a minimum income after retirement.

The public retirement pension is a guaranteed pension for all individuals who have been living or working in Norway for at least 3 years and paid into the social security scheme. The insurance is funded by the government, primarily through taxes. The purpose of a public retirement pension is that every individual is secured a minimal living standard, disregarding previous work experience. The size of the public retirement pension is adjusted to be "reasonable" with regards to each individual's income during their working years. In section 6.1 we will elaborate on how the pension is calculated when we simulate a public retirement pension for our pension scheme comparison. The pension is equalization based, meaning that this year's pension expenses are covered with this year's tax payments from the work force. The steadily increase in life expectancy [Statisk Sentralbyrå, b] increases the cost of the public retirement pension. We will elaborate more on this issue in the following section, where we discuss the defined benefit pension scheme.

2.2.2 Pillar II: Occupational Pension

In 2006, the Norwegian government introduced the law of mandatory occupational pensions for all workers in Norway [Lov om tjenestepensjon]. The background for the introduction was a demand for better pension conditions for employees in the private sector. Most firms in the private sector turned to the defined contribution scheme for their employees. We will in the following explain these pension schemes and explain why defined contribution was attractive for firms.

The occupational pension is based fully on previous work experience and is separated into two types of pensions; the defined benefit scheme and the defined contribution scheme. We will in the following go through these two schemes.

Defined Benefit

A defined benefit scheme offers the holder a guaranteed pension payment each month, calculated as a fraction of the salary at the time of retirement. In the public sector for a worker with full pension rights, the minimum pension with a defined benefit scheme is $\frac{2}{3}$ of the end salary, either on a gross level included the public retirement pension or as a net level in addition to the public retirement pension [Finans Norge, 2010, a]. In the private sector $\frac{2}{3}$ of end salary is considered the target, rather than a minimum. Defined benefit schemes are therefore slightly different for employees in the public and the private sector.

Under a defined benefit scheme, individual pension holders are not exposed to falling returns in the financial markets, because they are guaranteed an amount of their end salary independent of market movements. Financial instability over time may indirectly affect them through lower wage increases, but not directly. Under a defined benefit scheme, the employer will have to make continuous pension payments to a LIC in accordance with working years, age and salary. This means that the pension payments under this scheme vary from one year to another, causing an undesired uncertainty for the employer.

LICs offer firms using a defined benefit scheme what is called a *base rate*, a guaranteed return on their pension payments [Store norske leksikon, a]. The base rate is currently 2%[Finans Norge, 2014, b], meaning that pension funds guarantee for at least this return yearly for pensions issued after 1st of January 2015. After the financial crisis, several pension funds struggled with fulfilling their guaranteed base rate because old pension liabilities are tied up to a historically higher base rate. One of the largest pension funds in Norway, Storebrand, states in a report from 2013 that 50% of their base rate guarantees are at 4% and 35% are at 3%, meaning that old guarantees are still a major cost for them [Storebrand, 2013].

Defined Contribution

In a defined contribution pension scheme, the employee and the employer decide on a yearly rate of the salary that is being paid into a LIC. We define this as the *defined contribution rate*. In Norway, there is an interval for this rate where a minimum contribution rate of 2% of the salary has to be paid into a LIC or a private pension saving (Pensjonskasse) each year. For salaries up to 7.1G the maximum contribution rate is 7%, while for salaries between 7.1G and 12G, additional 18.1% can be added, totalling 25.1% [Regjeringen, 2013]. "G" is the denotation of the Norwegian basic income, an amount of NOK 88 370 from 01.04.2014 [NAV, 2014] and we will in the rest of the paper only refer to "G" when we refer to the Norwegian basic income.

The purpose of the defined contribution scheme was to introduce a new pension scheme that was attractive for both employers and employees. In this scheme, the employer make payments monthly, but the individual is held responsible for managing their own pension capital. The individual is usually provided the right to choose between different pension plans offered by the LIC that manage the pension capital for the employees of the firm he or she works in. Individuals are provided with professionally diversified portfolios where they can decide the allocation of high risk and low risk assets in their own portfolio. After the financial crisis, the financial markets have been volatile and investments have become more insecure. Low economic growth have gradually reduced the interest rates, and today's return potential on bond investments is limited. This is one important reason why we believe it is interesting to test the investment strategies looking into the future. However, increased risk in investments are not without costs for LICs. The introduction of the Solvency II directive caused furhter guidelines for insurance companies, and we will in the following briefly discuss the impact it has for pension companies.

Solvency II

Solvency II is a regulatory framework introduced for regulating insurance companies, including LICs. The Solvency II directive is a complicated and extensive framework and we will only touch into the part of the framework that affects LICs to understand the impact it has for pension companies.

Solvency II applies for members of the European Union, but The Financial Supervisory Authority of Norway has decided that Norwegian companies have to adjust to most of the regulations [Finanstilsynet.no]. The directive builds on a pillar system that addresses capital requirements, governance and increased transparency [European Commission, 2014].

The criteria for capital requirements have become stricter under the Solvency II regulations. The directive has raised the minimum capital requirements, meaning that all LICs need a larger capital buffer that can absorb potential losses. It has introduced more risk factors that insurance companies need to take into the equation when they calculate their Solvency Capital Ratio, a ratio that needs to be above a certain threshold given in the directive. This

Solvency Capital Ratio is directly influenced by the base rate previously mentioned, because pension liabilities are one of the major factors in the Solvency Capital Ratio equation. Because of these new regulations, it has become more costly for insurance companies to take on higher risk, and risky investments needs to be supported by adding more loss absorbing capital into the company.

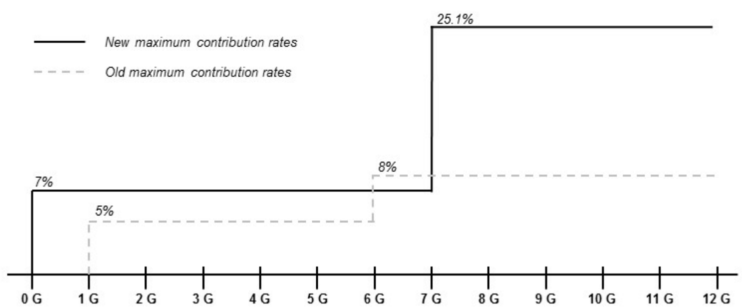
Finally, the Solvency II directive demands more transparency around firm investments, which should give incentives for improved corporate governance in the insurance companies.

Development of the defined contribution rates

Until 1st of January 2014 the defined contribution rate was limited to a minimum of 2% and a maximum of 5% between 1G and 6G, while the maximum contribution rate above 5G until 12G was 8%. After 1st of January 2014, the maximum contribution rates were changed to a minimum of 2% and a maximum of 7% between 0G and 7.1G, while income between 7.1G and 12G now have a maximum contribution rate of 25.1% [Finansdepartementet, 2013a]. For income above this level, no additional contributions can be made.

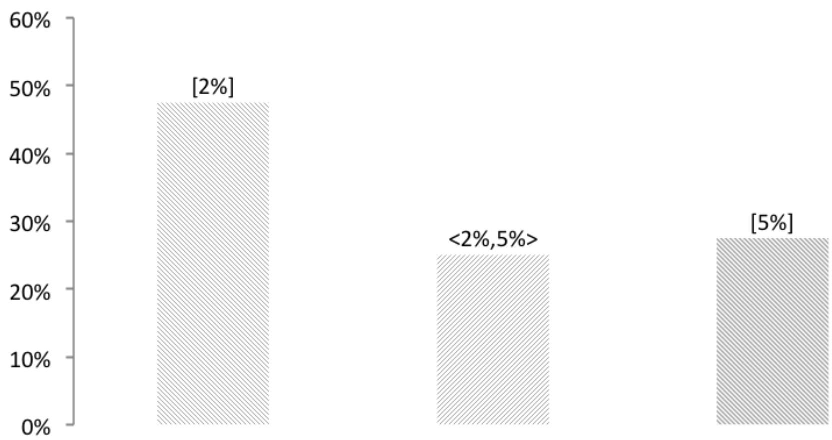
The new and old regulations for contribution rates are illustrated in figure 2.1, where the grey dotted line illustrate the old regulations and the black solid line illustrates the new regulations.

Figure 2.1: New and old maximum contribution rates



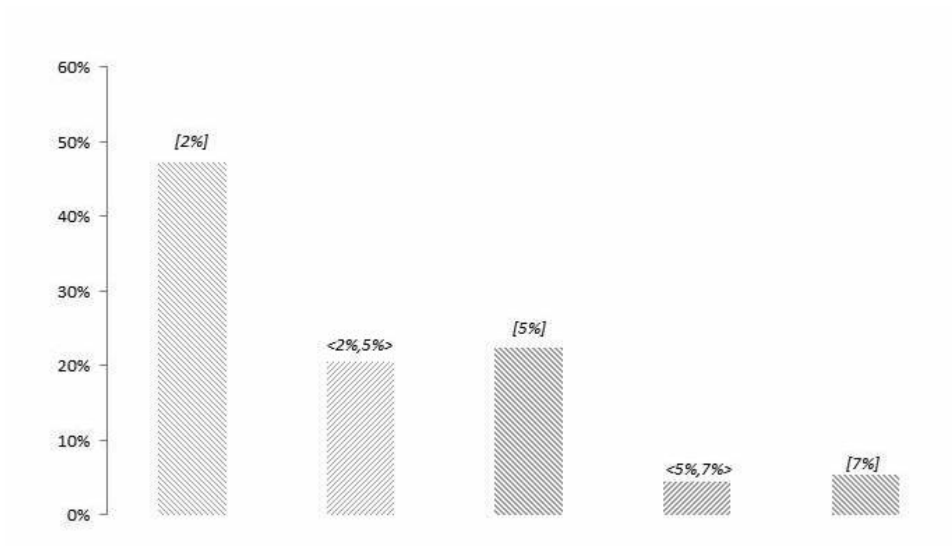
We have chosen to look at contribution rates for income between 0G and 7.1G, because we want to focus the paper around the average income individual, rather than high-income individual. Figure 2.2, illustrates the allocation of individuals on the different contribution rates under the old system [Johansen and Nordbye, 2014].

Figure 2.2: Contribution rates - old system



From the 1st of January 2017, all pension holders with a defined contribution scheme must follow the new defined contribution rates [Regjeringen, 2013]. The pension holders that have already changed to the new system, is illustrated in figure 2.3 [Mørk and Nordbye, 2015].

Figure 2.3: Contribution rates - new system



In the new regulations, companies can start pension payments from the pension holder's first earned Norwegian Krone (NOK), while in the old regulations, pension contributions started after the first earned G. The figure above include contribution rates both for individuals who receive pension payments from their first earned Norwegian Krone and individuals who receives pension payments starting after earning more than 1G. Like in figure 2.2 we still observe that the largest proportion of pension holders have a low defined contribution rate. Data recieved from Finans Norge on defined contribution rates can be found in the appendix.

2.2.3 Pillar III: Individual Pension Savings

Individual pension savings (IPS) is a long-term saving where individuals have the right to invest an amount regulated by the law of IPS. Today, individuals have the right to invest NOK 15 000 each year and deduct taxes from the invested amount[Gjensidige.no]. They do not have to pay any taxes for interest income or holding the money, but they have to hold the invested amount until pension age. IPS is not widespread in Norway, possibly due to the limitations of the invested amount each year.

2.3 Life Insurance Companies

We will briefly mention those who are responsible for managing the pension funds, namely the Life Insurance Companies.

A Life Insurance Company (LIC) is a company that is specialized within investing life insurances and pensions. For the purpose of this paper, we will focus on pensions. LICs are larger in scale than private pension funds (with the exempt of Statens Pensjonskasse in Norway, allocating the pensions for most of the employees in the public sector), both in terms of pension capital to invest and the amount of employees working on optimizing the risk-return relationship for the pension holders. LICs offer standardized investment alternatives for all their customers, and their pension investment alternatives are easily accessible for all workers. LICs investments are regulated by the Fiancial Supervisory Authority (FSA) in Norway, where risk reduction is emphasized on the cost of potential return.

Chapter 3

Theoretical Background

We will in this section elaborate around relevant theories and frameworks from the academic literature that we have used both for calculations and discussions throughout our papers.

This chapter will start by introducing modern portfolio theory and the underlying foundations for this. Even though our thesis do not specifically deal with all of the financial calculations presented in the section, we believe it's important to understand some of the considerations that must be accounted for by pension holders and pension funds. Thereafter, we introduce the concept behavioural finance where we look into individual risk aversion that is highly relevant for pension savers.

3.1 Modern Portfolio Theory

In this section we will go into the basics of modern portfolio theory and explain how funds theoretically should be invested according to this framework.

Henry Markowitz made portfolio theory a frequently discussed topic when he first published his article, *Portfolio Selection* in 1952 [Markowitz, 1952] and thereafter when he gave out his book, *Portfolio Selection: Efficient Diversification of Investments* in 1959 [Markowitz, 1959]. In both publications Markowitz presented what have been one of the cornerstones in modern portfolio theory ever since; the relationship between risk and return. Higher risk should correspond to higher expected return and vice versa. This build on the foundation that, for a given level of risk, return should be maximized and the other way around. For a given level of return, risk should be minimized. Combining this for a set of portfolios with different levels of risk we obtain a set of optimal portfolios, this is called the *efficient frontier* and is illustrated in figure 3.1.

Central for the concept of the efficient portfolios is that stocks should not be selected individually, but instead together. By selecting stocks together investors can obtain diversification effects that will reduce the risk of the portfolio [Evans and Archer, 1968; Statman, 1987] because of their negative correlation. By increasing the number of securities, the non-systematic risk of the securities are diversified away, and systematic-risk which is risk that not can be diversified away, is the only risk that still exist [Wanger and Lau, 1971].

Markovitz added another asset class into his model, the risk-free asset. By adding the risk-free asset, the efficient frontier is still the same, but the figure include an extra line called the Capital Allocation line. The Capital Allocation line graph all possible combinations of risky and risk-free assets [Bodie et al., 2011], starting from the point were all funds are invested in the risk-free assets and then gradually increasing the proportion invested in risky assets. At the point where the Capital Allocation line is tangent with the efficient frontier, we find the tangency portfolio where all capital is invested in risky assets. Moving further up from this point means that we add leverage to our portfolio, i.e. increasing the proportion of risky assets to account for more than our capital. To make this possible, it is necessary to short risk-free assets, meaning that we lend money to invest in risky assets. Moving along the Capital Allocation line, the expected return can be explained by equation 3.1.

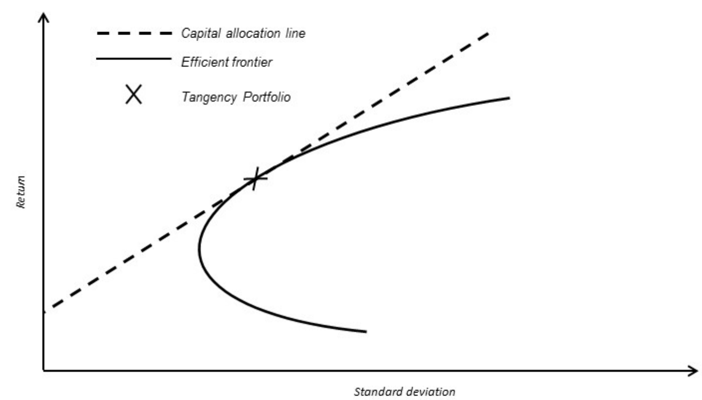
$$E(R_{cal}) = r_f + \sigma_c \times \frac{E(R_p) - r_f}{\sigma_p} \tag{3.1}$$

σ_p is the risk of the portfolio when all capital is invested in risky assets, while σ_c is the risk of the combined portfolio of risky and the risk-free assets. We can both increase and decrease the risk of the combined portfolio by changing the weights in the risky and risk-free assets, as given by equation 3.2

$$\sigma_c = (1 - w_{rf}) \times \sigma_p \tag{3.2}$$

At the point where the efficient frontier and the capital allocation line is tangent, σ_c is equal σ_p. To increase the expected return it's necessary that $\sigma_c > \sigma_p$. By equation 3.2 its therefore necessary to have a negative weight in the risk-free asset, or what we referred to as shorting or lending money. Similar for the other way around, if $\sigma_c < \sigma_p$ then the expected return will be lower and we will have a positive weight in the risk-free asset.

Figure 3.1: Capital allocation line



We have chosen to look away from the option of shorting, and therefore only consider portfolios that starts from were the capital allocation line intercept with the vertical axis and until its tangent with the efficient frontier. This is under the assumption that we consider our bond investment as risk-free, even though it practically is a low-risk option. If we do not consider our bond investments as risk-free, we will theoretically lie somewhere at the efficient frontier. The reason why we looked away from shorting is that we believe individuals are sufficiently risk averse to not bet more in risky assets than their actual pension. We therefore argue that leveraging pension holdings will be taking excessive risk and therefore exclude this possibility in our analysis.

We will in the following section look into how behavioral finance explains the impact of individual psychology on investment decisions and the allocation of the different asset classes.

3.2 Behavioral Finance

The field of behavioural finance evolved as a criticism to the theory of "The Economic Man" in the late 1950s, but had its breakthrough in 1979 with the *Prospect Theory* developed by Kahneman and Tversky [Kahneman and Tversky, 1979]. Behavioral finance academics such as Kahneman & Klein [Kahneman & Klein, 2009] rejects the theories of individuals making fully rational decisions and claims that bounded awareness and bounded rationality limits our minds to make irrational decisions.

Individual risk preferences are an important element in understanding how individuals prefer to invest their wealth. Even though excessive risk taking within pension could become an issue for the society as well as for the individual, ultimately it is the individuals own decision how they want to invest their wealth. We will briefly go through the elements of risk aversion on an individual level and one of the most acknowledged theories within behavioral finance, the Prospect Theory.

Individual Risk Preferences

To understand an individual's preference toward risk, we start by introducing the individual utility functions first presented by Daniel Bernoulli from his paper *Exposition of a New Theory on the Measurement of Risk* first published in 1738 [digitally published in Bernoulli, 1954]. The paper presents individual utility curves as functions based on certain parameters that determine the individual utility. We only shed light on the intuition of the theory in this paper, and refer the reader to the mentioned paper for mathematical evidence. The risk utility curve is concave, meaning that most individuals are sensitive to the element of risk. What differ in the utility curves are each individual's preferences toward risk. In risk theory, we separate between risk averse, risk neutral and risk seeking individuals.

Risk averse individuals have an inherent resistance to risk and will always prefer the safest option. To provide an example that can be used for all individuals, we can use the example of a lottery. The lottery offers the following; with certainty you can receive NOK 50 before the lottery, or you can either win NOK 100 or end up with nothing. The likelihood of each outcome is both fifty percent, so that the expected value of the lottery is NOK 50. Risk averse individuals will always choose the certain 50 NOK. In fact, the risk averse individual may even have a certainty equivalent lower than the expected value of NOK 50, and might therefore be willing to accept a certain amount lower than the offered NOK 50. The risk neutral individual would be indifferent between the two options. Because the expected value of the lottery is the same as the certain alternative, the individual would consider these two options equal. The risk seeking individual would prefer the lottery. A risk seeking individual might even chose the lottery if the expected value was less than the certain alternative as long as the best outcome of the lottery is better than the certain alternative.

This discussion is relevant for pension holders because their risk preferences may be an important determinant for their investment strategy decision and their choice of withdrawal rate, a concept we introduce later on in this paper.

Prospect Theory

While the utility curves of Bernoulli explains individual risk preferences based on certain parameters, they fail to take into account the reference point of the individual. Because we expect pension savers to be different in terms of income, inherited wealth and so on, this reference point is essential to understand their preferences toward risk. The use of a reference point is one of the most important elements of the Prospect Theory introduced by Daniel Kahneman and Amos Tversky [Kahneman and Tversky, 1979]. Kahneman explains our perception to be reference dependent:*"that the perceived attributes of a focal stimulus reflect the contrast between that stimulus and a context of prior and concurrent stimuli"* [Kahneman, 2003, p.1454].

In other words, our reference point is based on *"the stage before this stage"*. He criticizes Bernoulli's utility functions for considering utility by the reference point of the final state, rather than the current state, and therefore does not take into account the short-term emotions affected by changes in wealth. Further, he states that Bernoulli's utility functions do not maximize the utility of the outcomes as they are actually experienced.

The Prospect Theory states that individuals are more risk averse towards gains and more risk seeking towards losses [Kahneman and Tversky, 1979]. This is relevant for individual pension savers who may be either wealthy or poor at retirement age. While Prospect Theory claims individuals to be risk seeking toward losses, this seems less intuitive in terms of pension saving. If the expected pension is already low, it seems hard to believe that individuals will chose to go *"all-in"* with their pension, but rather sit tight on the little they have. On the other hand, wealthy individuals may be willing to take on higher risk because they can afford the losses.

The Prospect Theory and the utility functions of Bernoulli both provide interesting perspectives on individual's preferences toward risk in general. We will not be making any assumptions about risk preferences in our analysis, but point out where it is relevant that differences in risk aversion may influence what each individual may find the most attractive strategy to invest in and what withdrawal rate they will prefer.

Chapter 4

Investment Strategies

This chapter will deal with investment strategies for LICs. First, we will introduce the concept of asset allocation. Thereafter, we will look at what investment strategies are mostly used by LICs today. We will present some critics from the literature against these strategies that argues today's investment practice is suboptimal. We will separate between the accumulation phase where pension savings occur and the retirement phase where withdrawals are being made, before we end the chapter with presenting the investment strategies we will test in our paper.

4.1 Asset Allocation

Asset allocation can be described as the allocation of different asset classes in a portfolio, such as stocks, bonds and cash. Asset allocation is closely related to portfolio theory and mean-variance optimization that we discussed in section 3.1. We can separate asset allocation into tactical asset allocation and strategic asset allocation. Strategic asset allocation is known as the proportion of broad asset classes held in a portfolio designed to provide an investor an appropriate risk/return over a longer period of time [UBS, 2009]. Tactical asset allocation is known as near-market activity where you search for short-term market anomalies that can be exploited through active trading in that period of time. Tactical asset allocation can therefore be interpreted as temporary deviations from the long-term strategic goal to achieve additional returns.[UBS, 2009]

We look at investment strategies for pension holders that invest for a long time horizon. Strategic asset allocation is therefore of relevance for this paper, where individual differences toward factors such as goals, risk tolerance and investment horizon determine their preferred allocation. For pension holders, age has historically been an important determinant for the asset allocation, typically reducing the risk of the portfolio the older the individual get. We will discuss our strategy testing in light of several of these factors.

4.2 Investment Strategies

In this section we will first review the traditional investment strategy for pension capital that is commonly used today, before we present some of the critics that have been raised against the strategy. Thereafter, we will look at alternative strategies that have been proposed. We have separated between investment strategies that actively change asset allocation until retirement and strategies that changes asset allocation throughout the retirement period as well as before retirement. We will discuss the differences of these two types of strategies and elaborate on the concepts of the accumulation and retirement period.

4.2.1 Pension Strategies Today

The defined contribution pension scheme makes every holder of pension capital an investor of their own pension savings. The LICs are designed to solve an issue for these investors; to construct a professionally diversified portfolio at a low cost. LICs offer individuals and companies investment alternatives from a menu of options, where the pension holder can choose between investment strategies offered based on their risk preferences.

Historically, LICs have adjusted the risk of the pension holders as a function of year to retirement. They recommend an investment profile holding a high stock proportion in the beginning of the investment period and reduce this proportion in favour of bond investments as retirement approaches. This reallocation from stocks to bond is ment to reduce the risk of losing pension capital when the individual is closing up to retirement and have limited options to cover up these losses [Estrada, 2013].

In Norway, Storebrand are together with DNB Livforsiking, Nordea Livforsikring and KLP the main LICs with a market share of more than 84% [Finans Norge, 2015, c]. They all offer pension saving strategies that gradually decrease the proportion of stocks when retirement is closing up. The individual is free to choose the initial risk they want to take on by choosing between investment strategies that start with different stock proportions. Since all LICs follow the same principles we have chosen to use Storebrand's recommended investment strategy as an empirical portfolio we can test against the more theoretical strategies. We use this strategy as a benchmark for today's empirical portfolios, and discuss LICs in general based on the results that this strategy provides us. We will present this strategy in more detail later in this section.

4.2.2 Critics of the Traditional Pension Saving Strategy

From a risk perspective, it seems intuitive to reduce the stock proportion when retirement is closing up, because pension holders want to be protected against downside risk. However, the strategy of reducing stock proportion as a function of age, named a "lifecycle strategy" in academic papers, have been criticized by academics.

Arnott[Arnott, 2012] argue that investors should care more about their accumulated capital when they retire, rather than risk at specific points in time. This is, among others, also argued for by Estrada [Estrada, 2013]. Both Arnott and Estrada test the traditional lifecycle investment strategy against a mirroring strategy where they invest the traditional investment strategy the other way around; starting with a low proportion of stocks and increase the proportion as a function of age. Estrada [Estrada, 2013] presents in his article *The Glidepath Illusion: An International Perspective* evidence from 19 countries and two regions that the mirroring strategy outperforms the traditional investment strategy using historical data. While both Arnott [Arnott, 2012] and Estrada [Estrada, 2013] states that they do not necessarily recommend mirroring strategies, they still argue that they are a better alternative than what is being offered today. We will look more into the mirroring strategies presented by Estrada and Arnott when we present our tested strategies in section 4.2.5 and 4.2.6.

In his paper *Life-cycle portfolios as government policy* from 2005, Robert Shiller is critical to the traditional "rule of thumb" strategy, where approximately the composition of stocks in the portfolio equals to 100 minus the age of the pension holder [Shiller, 2005]. This can be characterized as a traditional lifecycle strategy as the investment strategy is a function of age and the risk of the portfolio is gradually reduced. Shiller's main criticism against traditional lifecycle strategies is that people are invested heavily in stocks in younger years when the capital base is low and barely invested in stocks when the capital is large in the older years. This is an important observation we must keep in mind when testing investment strategies, because while the time weighted average in asset allocation might be identical in some of the strategies, the capital weighted average differ widely.

Paul Samuelson has also questioned traditional lifecycle strategies. Samuelson has published several papers related to the topic and in his first paper *Lifetime portfolio selection by dynamic stochastic programming*, he argued for an unchanged proportion of stocks in the portfolio throughout the lifetime of the investment [Samuelson, 1969], namely the 50/50 strategy that we will return to in section 4.2.5. Samuelson argued that investor's asset allocation should not depend on their holding period, which is an important argument for the traditional lifecycle strategy.

Samuelson's theory only hold under a set of specific assumptions and is not applicable to the real world. In fact, Samuelson himself disagreed with some of his assumptions in a later paper that he wrote with Bodie and Merton [Bodie et al., 1992]. An important point noted by Samuelson in his first paper related to the topic is that investing for many periods does not itself introduce additional tolerance for risk at early stages in life, contradictory to what traditional lifecycle strategies implies.

Even though traditional lifecycle strategies have been criticized in recent time and alternative strategies have been proposed, it's important to note that traditional lifecycle strategies are the preferred strategy for LICs and also have support from the academic literature. Bodie, Merton and Samuelson argue that younger people can invest with higher risk in early years since they have the opportunity to work harder if faced with lower returns to cover up their losses, supporting the investment strategy of the major LICs [Bodie et al., 1992].

4.2.3 Accumulation and retirement phase

Investing your pension savings is not only important during the working years; investment strategies in the retirement period prove to be an important determinant for the total pension. We have split the investment period into two phases; the accumulation period and the retirement period. The accumulation period is the period where pension savings are being earned, that is, when pension contributions are being paid by the employer. In this period, the defined contribution rate together with the returns determines the value of the total pension holdings. In the retirement period, the size of the pension withdrawals together with the return on the pension holdings will determine how sustainable the pension holdings are. The accumulation and retirement period is illustrated in the figure 4.1 below.

Figure 4.1: Accumulation and retirement period

4.2.4 Investment strategies in the retirement period

When Estrada [Estrada, 2013] and Arnott [Arnott,2012] tested investment strategies, they assumed a constant asset allocation after retirement. The literature provides different views on how pension capital should be invested after retirement, and we will in this section look at some of them.

Pfau and Kitces argue that the proportion of stocks should increase throughout the retirement period [Kitces and Pfau, 2014], while Estrada on the other hand argue that the stock proportion should decrease throughout retirement [Estrada, 2014]. Combining Estrada's findings in his article from 2013[Estrada, 2013] where he tested an increase in the stock proportion throughout the accumulation period, with his finding in his 2014 paper[Estrada, 2014], where he suggested a decrease in stock proportion throughout the retirement period, he introduced what he called an inverted U-shaped strategy. Pfau and Kitces [Kitces and Pfau, 2014] on the other hand suggested decreasing the proportion of stocks in the accumulation period and increasing the proportion of stocks in the retirement period. This is what we later will refer to as an U-shaped strategy. These strategies will be presented graphically in section 4.2.6.

We have extended the investment strategy and its mirroring strategy, and similar linear strategies with different initial stock proportions to include the retirement period as well. This way, we get to compare these strategies if they stop the asset allocation at the retirement age or if they keep changing the asset allocation until the end date of the pension withdrawals.

4.2.5 Glide-to strategies

Glide-to strategies are investment strategies that change the proportion invested in different asset classes in the accumulation period, but hold a constant asset allocation in the retirement period [Estrada, 2013].

We have tested eight Glide-to strategies where four of these strategies are traditional lifecycle strategies for LICs, namely 100/0, 80/20, 60/40 and Storebrand's recommended investment strategy. The strategies are named after their initial and end proportion in stocks and bonds, e.g. 60/40 is 60% in stocks and 40% in bonds. We have included three mirroring strategies, the 40/60, 20/80 and 0/100. Finally, we have one strategy with constant weights during the entire investment period, the 50/50 strategy.

Table 4.1: Stock/bond allocation for Glide-to strategies

Initial allocation stock/bonds	Allocation in the retirement period
100/0	0/100
80/20	20/80
60/40	40/60
50/50	50/50
40/60	60/40
20/80	80/20
0/100	100/0

In six of the portfolios the asset allocation are linearly changed, which can be seen in figure 4.2 and 4.3. The linear change is in accordance with what have been done in previous research by Estrada and Arnott [Estrada, 2013; Arnott, 2012]. The 50/50 portfolio holds a constant proportion, while Storebrand's strategy [Storebrand, 2015] gradually change in accordance with the description in table 4.2.

Table 4.2: Storebrand's recommended saving profile

Age	20-42	42-45	45-50	50-52	52-55	55-60	60-65	65-67	67>
Proportion in stocks	80%	76%	68%	65%	54%	35%	23%	20%	20%

The initial allocation in stocks is high in the beginning and gradually reduced, before the proportion becomes constant in the retirement period.

Figure 4.2 illustrates three of the traditional lifecycle strategies of LICs, the 100/0, 80/20 and 60/40 strategy. The asset allocation is held constant in the retirement period. Depending on the weights we can observe that portfolio strategies starting with an initially higher proportion of stocks change the allocation faster.

Figure 4.2: Traditional lifecycle strategies

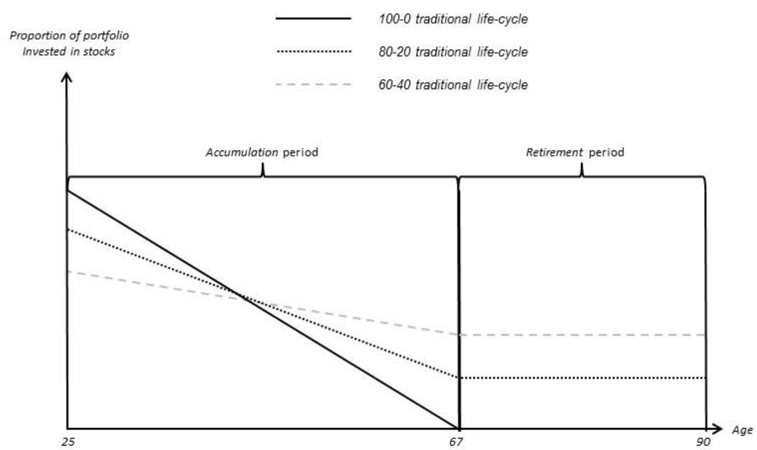
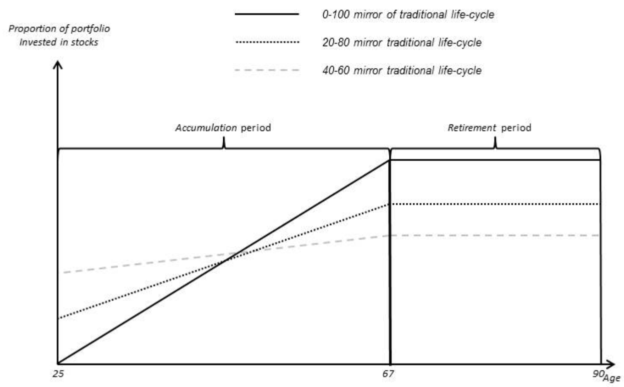


Figure 4.3 illustrates three mirror strategies of traditional lifecycle strategies, starting out with initial weights opposite of the traditional investment strategies described above. Again the allocation in the retirement period is held constant, whereas the strategies that initially have the lowest proportion of stocks change the asset allocation faster.

Figure 4.3: Mirror of traditional lifecycle strategies



4.2.6 Glide-through strategies

Glide-through strategies differ from Glide-to strategies in the way that they change the asset allocation in the retirement period and doesn't hold it constant from the age of retirement [Estrada,2013].

We have divided our Glide-through strategies into three categories. The first six strategies in table 4.3 follow a linear change in allocation from the start of the investment period until the end of retirement. The next three strategies are characterized by a U-shape, first reducing the proportion of stocks until retirement and then increasing it again in the retirement period. The third category is the inverse U-shape, which is the mirroring strategy of the U-shaped strategies. In total we have 12 Glide-through strategies and table 4.3 show the initial allocation, allocation at retirement and allocation at the end of retirement period for all strategies

Table 4.3: Allocation in Glide-through strategies

Initial allocation stocks-bonds	Allocation at the age of retirement	Allocation end of retirement
100/0	35/65	100/0
80/20	41/59	80/20
60/40	47/53	60/40
40/60	53/47	40/60
20/80	59/41	20/80
0/100	65/35	0/100
100/0	0/100	100/0
80/20	20/80	80/20
60/40	40/60	60/40
40/60	60/40	40/60
20/80	80/20	20/80
0/100	100/0	0/100

The linear glide through strategies is illustrated in figure 4.4 and 4.5, while the U-shaped and the inverse U-shaped strategies can be seen in figure 4.6 and 4.7.

Figure 4.4: Allocation in lifecycle Glide-through strategies

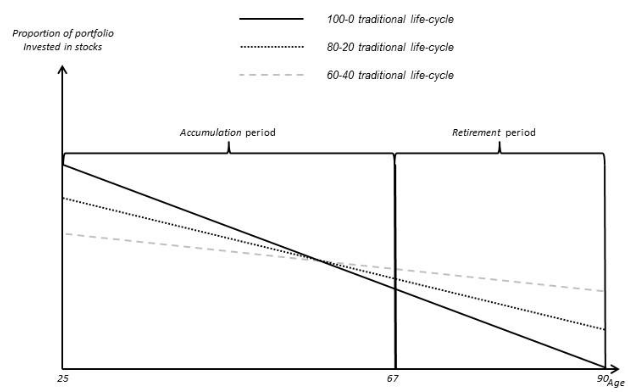


Figure 4.5: Allocation in mirroring Glide-through strategies

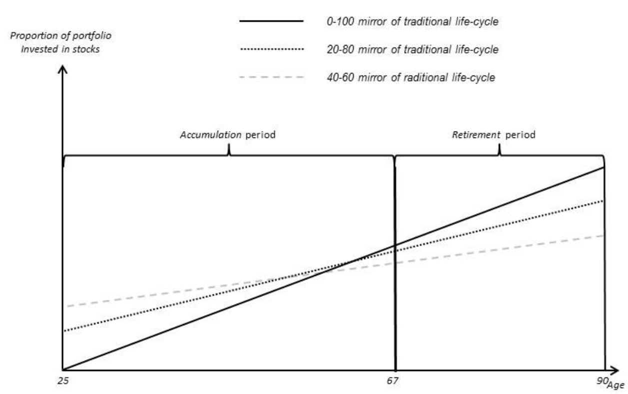


Figure 4.6: Allocation in U-shaped Glide-through strategies

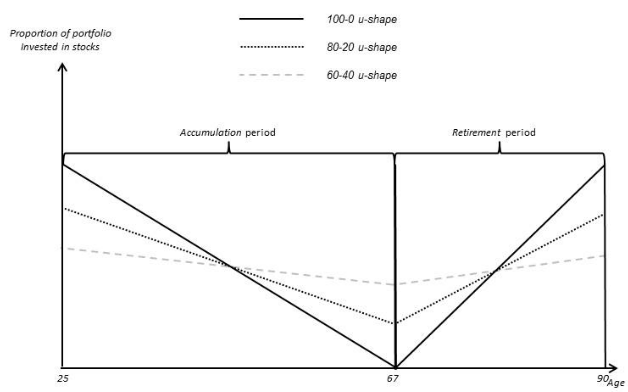
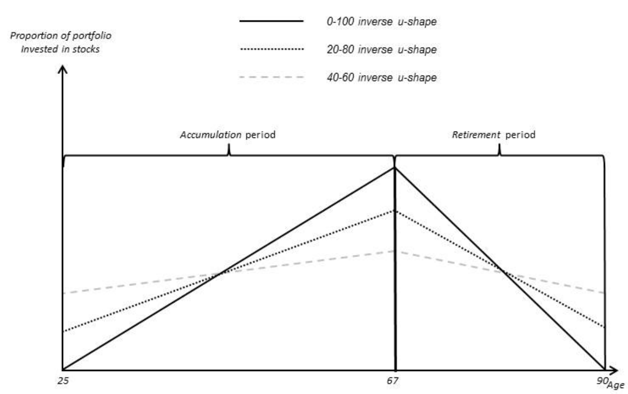


Figure 4.7: Allocation in inverse U-shaped Glide-through strategies



4.2.7 CPPI - An Alternative Strategy

Similar to Estrada [Estrada, 2013] and Arnott [Arnott, 2012], we have assumed linear changes in the asset allocation in our investment strategies. While most portfolios offered by LICs are not fully linear, many of them follow almost the same pattern, such as Storebrand's strategy that we presented earlier. We believe using linear changes was a simple, but realistic way to capture the differences in the investment strategies.

We are aware of other potential allocation strategies, such as Constant Proportion Portfolio Insurance (CPPI). The purpose of CPPI is to obtain both downside protection and keep a desired upside potential [Black & Perold, 1992]. From the same paper, Black and Perold describes CPPI as a strategy that "*invest a constant multiple of the cushion in risky assets up to the borrowing limit, where the cushion is the difference between wealth and a specified floor*" [Black and Perold, 1992, p. 404]. The benefit of this strategy is the individual's right to choose a floor that determines what level of losses on the portfolio the individual is willing to take before reallocating the proportion of risky assets. A similar approach was introduced by Lewis and Okunev [Lewis and Okunev, 2007], who had a value-at-risk perspective. Looking away from transaction costs, Black and Perold describes the CPPI strategy as an equivalent to investing in perpetual American call options, and with a higher expected return than what could be obtained under a stop-loss strategy such as the one proposed by Lewis and Okunev [Lewis and Okunev, 2007].

We believe the nature of the CPPI strategy is attractive for pension savers because of its limited downside risk. The strategy itself is complex and requires substantial work and testing. For the scope of this paper, we have stuck with linear strategies because testing both would be excessive, but we believe testing the CPPI strategy could be an interesting topic for future research.

Chapter 5

Data

Stock & Watson [Stock & Watson, 2012] stated that VAR-models should be kept as small as possible, especially for forecasting purposes where large VAR-models often generate imprecise coefficients. We decided to create *one* VAR-model for all our variables of interest. We will in the following shortly present the variables we have used.

We have used the U.S. 10 year Government bond, the changes in U.S. Consumer Price Index, U.S. Unemployment rate, the changes in the S&P500 index and the S&P500 dividend yield. All data was retrieved from Robert Shiller's database with data from February 1950 to February 2015 [Shiller, 2015]. We chose to use U.S. data because there are more historical data available for U.S. data than for European data. This allowed us to run our OLS regressions with longer time series.

U.S. 10 Year Government bond

We used U.S. 10 year Government bond as our low risk bond investment alternative for pension funds. Pension savers typically have a long term investment horizon and we therefore believe that the U.S 10 year Government bond seemed like a good alternative. We have used the bond rates to simulate future bond rate movements, and thereafter computed the return on the bonds. We will elaborate on how these returns were measured in section 6.3. We have re-written the data to monthly data by formula

$$r_m = (1 + r_y)^{(1/12)} - 1 \tag{5.1}$$

Figure 5.1: U.S. 10 Year Government bond

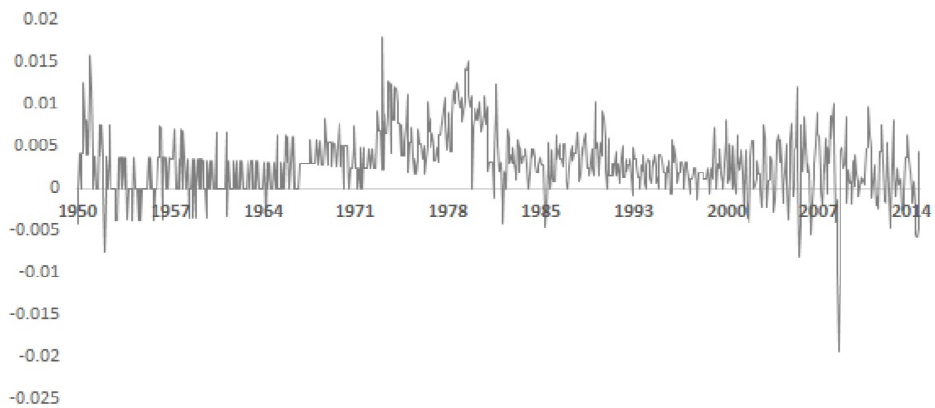
Changes in the U.S. Consumer Price Index

We have used the changes in the U.S Consumer Price Index (CPI) as our measurement of inflation. We have included inflation so that we could adjust our pension portfolios with an inflation that is interdependent with our other estimated variables. In addition, inflation is relevant for predicting some of our other included variables. We have re-written our historical data from

CPI to CPI changes with the formula

$$\triangle CPI = log\frac{(CPI_t)}{(CPI_{t-1})} \tag{5.2}$$

Figure 5.2: Monthly CPI



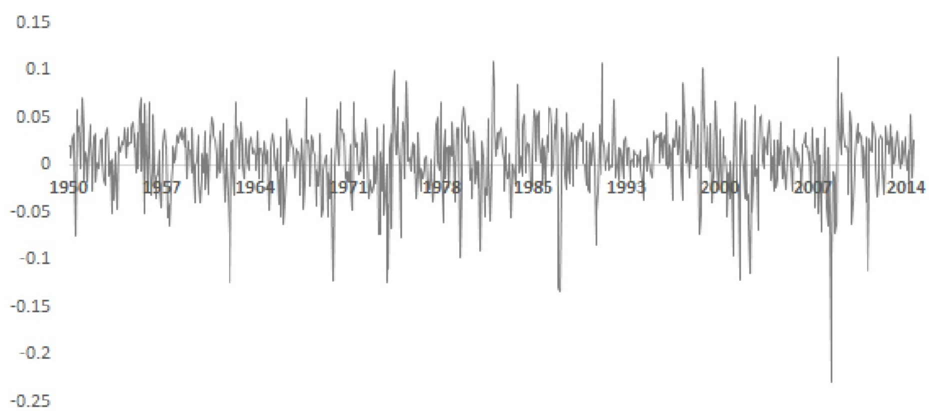
S&P 500 returns

We have used returns on the S&P500 index as our measure for capital gains on stock investments. The S&P500 index is a good proxy for a diversified portfolio, so we believe it is a good estimate for our purpose. The S&P500 index captures the market movements for many of the largest companies in the world and reflects a realistic view on expected returns in the stock market.

We found our monthly returns on the S&P500 index with the formula

$$S\&P500returns = log\frac{(S\&P500_t)}{(S\&P500_{t-1})} \tag{5.3}$$

Figure 5.3: Monthly S&P 500 Returns



S&P500 Dividend Yield

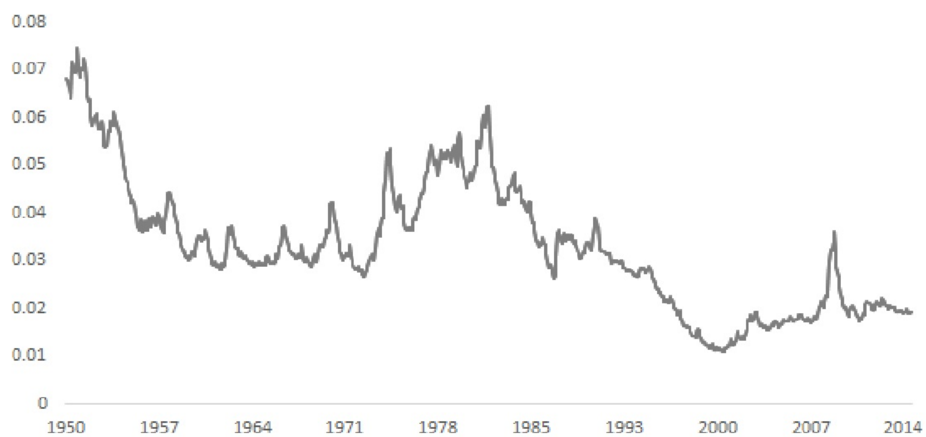
The S&P500 dividend yield is considered a good predictor of future developments on the S&P500 index, because dividend yields are correlated with future returns on the index. This is supported in the literature [Shiller, 1984; Campbell, 1987;Fama & French, 1987]. By forecasting the dividend yield on the S&P500 index, we could capture returns from both stock price changes and dividend yields of the stocks.

We have added the estimated dividend yield in the model to find our total return for stocks. We are aware that actual total return also incorporate other factors such as right offerings and other distributions realized over a period of time, but including capital gains and dividends capture most of the total return.

We have re-written the dividend yields to monthly yields with the formula

$$Divyield_m = (1 + Divyield_y)^{(1/12)} - 1 \tag{5.4}$$

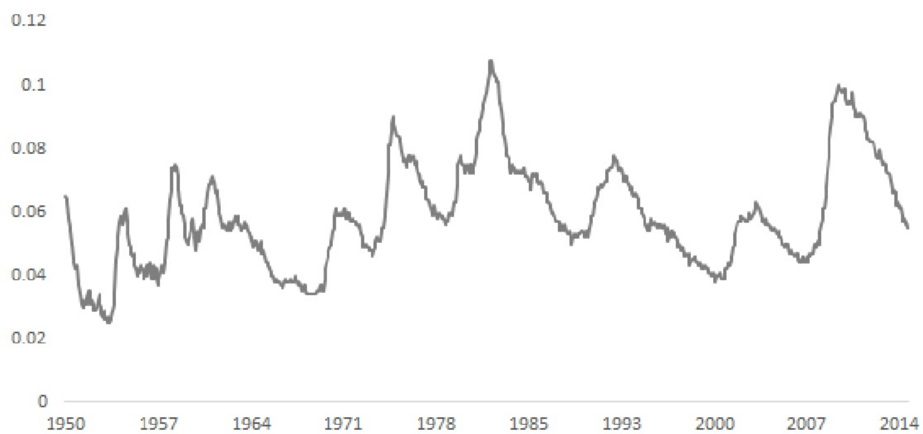
Figure 5.4: S&P 500 Dividend Yield



The U.S. Unemployment rate

The U.S unemployment rate is closely monitored by the financial markets as the impact of the measure can be substantial. Household spending accounts for more than $\frac{2}{3}$ of the U.S. economy's total output and a higher unemployment rate causes lower household income. Lower household income causes lower spending which among other things could affect inflation and stock and bond markets [Baumohl, 2012]. From an economic perspective the unemployment rate is therefore a relevant economic indicator and natural to include as one of the variables in our VAR-model

Figure 5.5: U.S. Unemployment rate



Chapter 6

Methodology

We want to investigate investment strategies given today's market conditions to draw conclusions on how they perform with our forecasted data. We have made 1000 simulations of forecasted economic data to test all our strategies on a large sample. We will in this section elaborate on what data we have used to test the investment strategies and how we have done the testing. Because we simulate data that impact the wage development, we have simulated 1000 different wage scenarios in accordance with our other simulations. We have used the same 1000 simulations for all investment strategies to make them comparable. We will in the methodology section elaborate all computations we have done to get our final results by using one simulated path, and we have repeated this method for all the remaining simulations. In addition, we wanted to compare the defined contribution pension scheme with the defined benefit scheme to see how the choice of investment strategy and how different defined contribution rates affect the comparison.

In both these tests, we have been looking into the pension savings for an individual aged 25, who starts his or her pension saving today with an initial income of NOK 350 000. The individual starts his or her pension payments from the first earned NOK, and has a real wage increase of 1% yearly. We will elaborate further on the assumptions throughout the methodology section. We calculate the pension strategy for 65 years until the individual is 90 years old. We have therefore included a risk margin that takes into consideration that people might get older than the life expectancy of about 85 years for this generation [Statistisk Sentralbyrå, b].

Firstly, we will go through how we have calculated the public retirement pension so that we could add this to the monthly occupational pension withdrawals from our strategy testing. This way, we can directly compare the monthly payments from the defined contribution scheme with the defined benefit scheme.

Secondly, we will elaborate on the VAR-model that we have used to simulate our forecasted data.

Finally, we will go through how we have computed the portfolio returns for the different portfolios ,how we have calculated the monthly occupational pension withdrawals and how the strategies are measured.

6.1 Public Retirement Pension

In order to compare the outcome of a defined contribution scheme with a defined benefit scheme, we have used the formulas that NAV, the responsible institution for calculating the public retirement pension in Norway, is using to calculate the public retirement pension. Calculating the public retirement pension is a complex process, because it is based on several parameters, such as years of work, sick days, military service and special circumstances such as leaving work to take care of children with special needs and so on. In our calculations, we have used 42 years as working years, just as we do in the strategy testing. We have assumed no sick days, no military service or other circumstances that will affect the pension either positively or negatively. The calculations are solely based on work income, nominal wage increase and inflation adjustments. The details of these adjustments can be found in the Norwegian law Folketrygdloven §20-18[Folketrygdloven].

We assume that the comparable individual using the defined benefit scheme is a public pension holder, with a claim of $\frac{2}{3}$ of the end salary on a gross level, including the public retirement pension.

Following Folketrygdloven §20-4 [Folketrygdloven], the pension is calculated yearly and accounts for 18.1% of the yearly income. The public retirement pension is upward limited to 18.1% of income up to 7.1G with no additions for income above this level. Following Folketrygdloven §20-18[Folketrygdloven], the pension deposits are adjusted yearly for the nominal wage increase, but generate no additional returns. When the individual starts withdrawing pension, the portfolio is adjusted with the nominal wage increase minus 0.75% yearly in an additional adjustment, mainly to control for inflation. Further elaboration can be found in Folketrygdloven §20-18[Folketrygdloven].

We have used a slightly simplified equation to calculate the estimated public retirement pension. We have calculated *one* public retirement payment that we use for all our comparisons. We have assumed a 1% real wage increase during the working years and no adjustments in the retirement period so that nominal wage increases and inflation balance each other out perfectly. We believe the best way to compare the two different schemes is to assume equality, meaning that the public retirement pension should be close to identical for both. Our opinion is therefore that using the same public retirement pension is sufficiently accurate. We start by the equation for total pension savings (TPS).

$$TPS = \sum_{i=1}^{42} Income\ up\ to\ 7.1G_i \cdot 18.1\% \tag{6.1}$$

Income is yearly adjusted with a real wage increase of 1% and G is adjusted by the government yearly. The yearly adjustment of G has historically been higher than 1%, meaning that we do not expect our calculated average salary to ever exceed the cap of 7.1G. The equation used by NAV for calculating the monthly pension withdrawals (MPW) is given by

$$MPW = \frac{TPS}{(Life\ Expectancy - Age)} \cdot \frac{1}{12} \tag{6.2}$$

The life expectancy is computed yearly based on death ratios of older generations, more details on the computations can be obtained in Folketrygdloven §20-13. We have been testing the strategy for people living until they are 90 years old, so we substitute the life expectancy with this age in equation 6.2. The last term is adjusting yearly pension withdrawals to monthly withdrawals.

6.2 Econometric modelling of return

To create the forecasts of our desired variables, we decided to use a VAR-model as it seemed to serve our purpose well. We wanted to design a robust model that was simple and yet managed to capture the historical movements that we could use to predict the future. We will in the following go through, step-by-step, the approach we used to design our model. In the result section, we will provide sanity checks to confirm that our forecasts seem reasonable. Firstly, we will elaborate on the econometric theory we have used for our modelling.

6.2.1 Econometric Theory

In this section, we will present the econometric framework we have used when we have created our simulated forecasts. First, we will introduce the basic concepts of the Ordinary Least Square(OLS) methods. Thereafter, we will elaborate on Vector Auto Regression, which is the framework we have used for forecasting. Finally, we will discuss several econometric concepts we have used when we have created our forecasts.

Ordinary Least Squares

When looking into statistical data, we want to find a relationship that captures the movement of our observed data, often expressed as an equation. One of the most common ways to choose how to draw this equation for linear regression models is by using what is called an Ordinary Least Squares (OLS) method. OLS minimizes the total squared estimation errors to find the equation that fits our observations best [Stock & Watson,2012]. When we use the OLS method in linear regressions we find the true intercept, α, and the true coefficients, β_i, by equation 6.3 and 6.4.

$$\alpha = \bar{Y} - \hat{\beta}_1 \bar{X}_1 - \hat{\beta}_2 \bar{X}_2 - \ldots - \hat{\beta}_k \bar{X}_k \tag{6.3}$$

$$\beta_i = (X^T X)^{-1} X^T Y = \left(\sum x_i x_i^T\right)^{-1} \left(\sum x_i y_i\right) \tag{6.4}$$

The OLS method serves as an underlying framework for several statistical tools, such as the vector auto regression that we will return to in the following. Because the vector auto regression build on the OLS assumptions, we will briefly go through these assumptions in this section.

OLS builds on three assumptions that must hold for the OLS method to provide reliable results. First the distribution of the error term, ϵ, must be independent of the X variable and have a mean of zero. This means that the error term on average equals to zero, so that the positive and negative error terms will balance each other over time, and the expected value of the beta coefficients is therefore correct. In addition, it states that no dependent variables in a regression can be correlated to the error term. The assumption can be summarized with the following equation

$$E(u_i|X_i) = 0 \tag{6.5}$$

The second OLS assumption states that $(X_i, Y_i), i = 1, \ldots, n$, are independently and identically distributed [Stock & Watson, 2012]. That the sample is identical means that all random variables have the same probability distribution. Independent distribution means that the sample must be drawn randomly, otherwise the sample may be biased. Dependent variables is often observed in time series, because time series data tend to be correlated with each other from one period to the next.

The third OLS assumption states that large outliers are unlikely. This means that observations far outside the usual range of the data are unlikely [Stock & Watson, 2012]. Large outliers can make the OLS results biased because large outliers can make us overestimate or underestimate the impact of obtained coefficients.

Finally the fourth OLS assumption states no perfect multicollinearity. This means that none of the independent variables included in the regression are a linear combination of another indendent variable in the same regression.

Vector auto regression

To create our forecasts, we have used a vector auto regression (VAR) model as our statistical tool. Sims [Sims, 1980] state that VARs provide a coherent and credible approach to forecasting, allowing us to make a model based on an acknowledged framework. Campbell, Chacko, Rodriguez and Viceira have used this framework in their paper on strategic asset allocation [Campbell et al., 2004]. We will in this section present the concept of a VAR-model.

VAR is a popular method to find empirical evidence of how variables react to various exogenous impulses [Iacoviello, 2011]. A VAR is a set of k time-series regressions, where the regressors are lagged values of all k series [Stock & Watson, 2001]. Stock & Watson provides a simple, yet accurate description of the VAR model: "*A VAR with k time series variables consists of k equations, one for each of the variables where the regressors in all equations are lagged values of all the variables*" [Stock & Watson, 2012, p.674]. The coefficients in the VAR is found by estimating each equation individually using the OLS method. For each dependent variable Y, the coefficients are obtained by the regression

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \vdots \\ \hat{\alpha}_k \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{1_1} & \hat{\beta}_{2_1} & \dots & \hat{\beta}_{k_1} \\ \hat{\beta}_{2_1} & \hat{\beta}_{2_2} & \dots & \hat{\beta}_{k_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{1_k} & \hat{\beta}_{2_k} & \dots & \hat{\beta}_{k_k} \end{bmatrix} + \begin{bmatrix} \hat{\epsilon}_{1_t} \\ \hat{\epsilon}_{2_t} \\ \vdots \\ \hat{\epsilon}_{3_t} \end{bmatrix} \tag{6.6}
$$

For the first forecast period, α's and β's are those retrieved from equation 6.3 and 6.4 and the VAR will look like

$$
\begin{bmatrix} \hat{Y}_{1_{t+1}} \\ \hat{Y}_{2_{t+1}} \\ \vdots \\ \hat{Y}_{k_{t+1}} \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \vdots \\ \hat{\alpha}_k \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{1_1} & \hat{\beta}_{2_1} & \dots & \hat{\beta}_{k_1} \\ \hat{\beta}_{1_2} & \hat{\beta}_{2_2} & \dots & \hat{\beta}_{k_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{1_k} & \hat{\beta}_{2_k} & \dots & \hat{\beta}_{k_k} \end{bmatrix} \cdot \begin{bmatrix} Y_{1_t} \\ Y_{2_t} \\ \vdots \\ Y_{k_t} \end{bmatrix} + \begin{bmatrix} \hat{\epsilon}_{1_{t+1}} \\ \hat{\epsilon}_{2_{t+1}} \\ \vdots \\ \hat{\epsilon}_{k_{t+1}} \end{bmatrix} \tag{6.7}
$$

The left side of the equation mark represent the forecast for period *t+1*, the first period after the last observed period. On the right side of the equation mark, the α matrix symbolize the intercepts, the β matrix symbolize the coefficients of the different variables, the Y_t matrix symbolize last periods observations and the ϵ symbolize the error term which is changing randomly each period following the OLS assumption number one. For the next period the VAR will look like

$$
\begin{bmatrix} \hat{Y}_{1_{t+2}} \\ \hat{Y}_{2_{t+2}} \\ \vdots \\ \hat{Y}_{k_{t+2}} \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \vdots \\ \hat{\alpha}_k \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{1_1} & \hat{\beta}_{2_1} & \dots & \hat{\beta}_{k_1} \\ \hat{\beta}_{1_2} & \hat{\beta}_{2_2} & \dots & \hat{\beta}_{k_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{1_k} & \hat{\beta}_{2_k} & \dots & \hat{\beta}_{k_k} \end{bmatrix} \cdot \begin{bmatrix} \hat{Y}_{1_{t+1}} \\ \hat{Y}_{2_{t+1}} \\ \vdots \\ \hat{Y}_{k_{t+1}} \end{bmatrix} + \begin{bmatrix} \hat{\epsilon}_{1_{t+2}} \\ \hat{\epsilon}_{2_{t+2}} \\ \vdots \\ \hat{\epsilon}_{3_{t+2}} \end{bmatrix} \tag{6.8}
$$

For the second forecast period, the same coefficients are being used, while the previous forecasted period is multiplied with the estimated coefficients, and with a new random error term.

A VAR model estimates coefficients proportionally to the number of variables included in the VAR model. Stock & Watson [Stock & Watson, 2012] suggest that the number of variables in a VAR model should be kept small to reduce the probability of estimation errors that can result in a reduction in accuracy in forecasts. The number of variables included in a VAR model should therefore be limited to those of high relevance to minimize the inaccuracy of the estimated coefficients.

The number of lag lengths in a VAR model is determined either by using an F-test on the lags to test their statistical significance or by using either the Bayesian Information Criteria(BIC) or the Akaike Information Criteria(AIC) for VAR models. In our model, we have used the two information criteria and we will therefore elaborate on these two in the following section.

Estimating the quality of the model

Working with time series requires testing of the model to identify what information is captured by the model and what is lost. In time series models, it is possible to include lagged variables back to the first observation of the variable, and tests needs to be made to determine how many lagged variables justifies to be included. There are two commonly used formulas for testing this issue, the Akaike Information Criterion and the Bayesian Information Criterion.

Akaike Information Criterion (AIC) The AIC is a formula for measuring p, the number of lags to include in a time series model. The AIC formula for VAR models is written

$$AIC(p) = ln\Big[det\Big(\hat{\sum}_u\Big)\Big] + k(kp+1)\frac{2}{T} \tag{6.9}$$

In this equation, p is the number of lags included, $det \sum$ is the determinant of the covariance matrix generated by the residuals from the OLS equations and T is the number of observations. Using the AIC, you search for the number of lags that returns the lowest AIC, which is the point where adding a lag is not compensated sufficiently in the reduction of the sum squared residuals to weigh up for the increase in the last term of the equation.

Bayesian Information Criterion (BIC)

The BIC, also known as Schwarz information criterion (SIC/SBC) is another formula used for estimating the number of lags to include in the regression model. The BIC formula is written

$$BIC(p) = ln\Big[det\Big(\hat{\sum}_u\Big)\Big] + k(kp+1)\frac{ln(T)}{T} \tag{6.10}$$

The coefficients used are the same as in the AIC formula, but the last term is slightly changed. The last term in equation 6.10, $\frac{ln(T)}{T}$ is larger than the last term in equation 6.9, $\frac{2}{T}$, meaning that the BIC criterion is more restrictive than the AIC to adding an extra lag into the regression. When we look for the optimal number of lags, we would usually look at both these measurements simultaneously to trade-off the number of lags to include. If there is a reason to believe the BIC is too conservative, more weight will be put on the AIC measurement.

Stationarity

In a time series analysis, we use past data to quantify historical relationships. In order to forecast based on historical data, we need to known that the future is similar to the past. Stationary data mean that historical relationships can be generalized to the future [Stock & Watson, 2012]. When a time serie is not stationary, we can no longer draw forecasts based on historical data, because we do not expect the history to reflect the future. Non-stationary data can contain a trend, which Stock & Watson [Stock & Watson, 2012, p.588] define as "*a persistent long-term movement of a variable over time. A time series variable fluctuates around its trend.*". The trend can either be deterministic, meaning that it is non-random, or it can be stochastic, meaning that we do not know how and when it will occur. A stochastic trend is known as a *unit root*.

If a time series contains a unit root, the first OLS assumption is violated as it relies on the stochastic process to be stationary. The OLS can then cause invalid estimates. In case of a unit root, one should apply the difference operator to the series and test whether the unit root is removed from the time series. We will not elaborate on the mathematical calculations behind the difference operator in this paper.

One of the most common ways of testing for stationarity in time series is using the Dickey Fuller test introduced by David Dickey and Wayne Fuller in their paper *Distribution of the estimators for autoregressive time series with a unit root* from 1979 [Dickey and Fuller, 1979]. The test is working with the hypothesis that the data contains a unit root, and we test to either keep or reject this hypothesis. Stock & Watson [Stock & Watson, 2012] report the following critical values in the Dickey-Fuller statistic

Table 6.1: Critical values for Dickey Fuller testing

Critical values	10%	5%	1%
Intercept only	-2.57	-2.86	-3.43
Intercept and time trend	-3.12	-3.41	-3.96

We do not elaborate on the mathematics in this testing and will use statistical software to perform these tests in our methodology section.

Partial Autocorrelation

In the previous section, we discussed how data may not be stationary and therefore cause imprecise estimates when we use the OLS method. A common problem with time series data is that they are often correlated. For example, interest rates in one period are highly correlated with interest rates in the next period. One way of reading out correlation in data is to test for partial autocorrelation in the residuals of our dataset.

A partial autocorrelation is the amount of correlation between a variable and a lag of itself that is not explained by correlations at lower-order lags [Nau, 2015]. The partial autocorrelation captures the additional correlation between one lag and the next. This means that we can always interpret the actual correlation between two lags no matter what lags we are looking at, opposite to the regular autocorrelation function which may capture correlation between previous lags in following estimations. For example, the partial autocorrelation between the lag of period *t-1* and *t-2* will only capture the correlation between these two lags, while the autocorrelation may as well capture the correlation between period t and *t-1*. We will use the partial autocorrelation to interpret our residuals when we are testing our time series data.

6.2.2 Creating the VAR-model

Designing a VAR-model contains substantial econometric testing. We will start by discussing variables we have included in our VAR-model. Thereafter, we will elaborate on the stationarity of our data and how many lags we included in the model. We will end the section with the steps for computing the forecasts and error terms.

OLS regressions

We ran our OLS regressions using these data and used the t-statistics to interpret the significance of each variable. We used one lagged coefficient on all our included variables in the OLS regression, and we will return to the decision of lags included in later sections.

We believe that the historical government bond rates might overestimate the future rates based on the development in the financial markets during the past years. In order to be able to adjust for these expectations, we have demeaned all our data before running the OLS regressions, and thereafter added the mean to our simulations. This simple process allowed us to test how the bond rates would move if we believe the historical average is higher than what we expect to be the average in the future, because we could add a mean lower than the historical mean to our dataset after simulating demeaned paths. We elaborate around the concept of demeaning data in our appendix. When we demean our data, the interpretation of the intercept changes. Because all our intercepts naturally came close to zero, we have dropped them out of our estimations.

We ran the following OLS regressions

$$\begin{bmatrix} S\&P500 \\ Divyield \\ CPI \\ Bond \\ Unemp \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_{S\&P500} \\ \hat{\alpha}_{Divyield} \\ \vdots \\ \hat{\alpha}_{Unemp} \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{1,1} S\&P500_{t-1} & \hat{\beta}_{2,1} Divyield_{t-1} & \cdots & \hat{\beta}_{5,1} Unemp_{t-1} \\ \hat{\beta}_{1,2} S\&P500_{t-1} & \hat{\beta}_{2,2} Divyield_{t-1} & \cdots & \hat{\beta}_{5,2} Unemp_{t-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{1_5} S\&P500_{t-1} & \hat{\beta}_{2_5} Divyield_{t-1} & \cdots & \hat{\beta}_{5,5} Unemp_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1_t} \\ \epsilon_{2_t} \\ \vdots \\ \epsilon_{5_t} \end{bmatrix} \tag{6.11}$$

S&P500 is the S&P500 returns, Divyield is the dividend yield of the S&P500, CPI is the changes in the Consumer Price Index, bond is the 10 Year U.S. Government bond rate and Unemp is the U.S Unemployment rate. We will use these abbreviations throughout the paper.

We have used monthly observations 65 years back as our dependent variable in each regression and 65 years of first lagged variables as explanatory variables, all data was demeaned.

In table 6.2, we report the coefficients and t-statistics in brackets from the OLS regression where the *t-1* variables are the explanatory variables regressed at the dependent variable in the top line

Table 6.2: Coefficients and t-statistics

Dependent/Explanatory	S&P 500	Dividend Yield	CPI	10Y Treasury	Uneployment
S&P500$_{t-1}$	0.22099	-0.00057	0.00404	0.00076	-0.00037
	(6.36368)	(-5.56137)	(1.32210)	(3.70908)	(-2.33088)
Dividend Yield$_{t-1}$	3.50726	0.98720	0.27635	-0.00331	0.00678
	(2.99424)	(284.90844)	(2.67704)	(-0.47521)	(1.25927)
CPI$_{t-1}$	-0.91281	0.00343	0.45714	0.01105	0.00124
	(-2.49686)	(3.17263)	(14.18885)	(5.08162)	(0.73918)
10Y Treasury$_{t-1}$	-0.85482	0.00306	0.30405	0.99094	0.00414
	(-1.32106)	(1.60159)	(5.33184)	(257.31717)	(1.39344)
Unemployment$_{t-1}$	2.74915	-0.00849	-0.14944	-0.00525	0.99122
	(2.75865)	(-2.88096)	(-1.70153)	(-0.88435)	(216.30183)

We concluded that all included variables had statistical significance on at least two of our variables of interest. The t-statistics on some of our variables seem odd, especially the lagged values on the dependent variable itself. We acknowledge that our model has some econometric weaknesses that we were unable to adjust for. In financial time series such as stock returns, the variance change over time and we experience heteroskedasticity. Heteroskedasticity makes the standard errors under OLS biased, meaning that the t-statistics may be biased [Stock & Watson, 2012]. Plotted residuals can be found in the appendix. We believe that all the included variables made sense from an economic perspective and we found that the model gave robust results. This view will be supported by our sanity checks in chapter 7.

Stationarity testing

We have used the statistical software program SAS in order to run a test on autocorrelation in the different data. We performed Dickey Fuller tests on all our data, testing for time trends and obtained the following τ values

Table 6.3: Dickey Fuller tests

	S&P500	Divyield	CPI	Bond	Unemp
Time trend	No	Yes	No	No	Yes
τ value	-18.02	-2.75	-11.47	-1.73	-2.22
Stationary	Yes	No	Yes	No	No

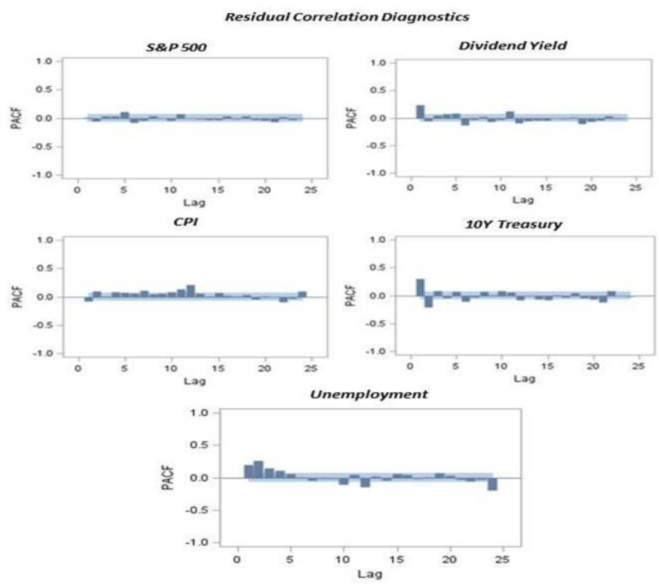
We conclude that we do not have fully stationary data, but as long as our model produce robust results, we are satisfied.

Included lags

An important decision is how many lags of the variables should be included to find the most accurate regression. We used both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to test the number of lags that should be included in our models. We also looked into the residuals to check for lags that may contain valuable information.

The residuals showed significant spikes at various levels up to the 25th lag, which generated the lowest AIC/BIC values. Adding 25 lags of several variables would have caused an enormous model and we believe there should not be any valuable information to retrieve from more than 2 year old data that is not reflected in more recent data. We wanted to keep the model small, and the AIC/BIC values for less than 10 lags, suggested that we should use only one lag. We therefore decided to stick with one lag for all variables.

Figure 6.1: PACF on the residuals for our included variables



The error term

To create the error term, ϵ, for our forecast, we have used the residuals from the OLS regressions to create a covariance matrix for the residuals. From the covariance matrix, we used Monte Carlo simulations and Cholesky decomposition to generate our residuals. The use of Cholesky decomposition and Monte Carlo simulations allowed us to have error terms that are linked to the historical variance of the variables, yet they are randomly distributed. For an explanation of Monte Carlo and Cholesky decomposition we refer to the appendix.

The error term for each period is computed

$$\begin{bmatrix} \epsilon_{S\&P500} \\ \epsilon_{Div} \\ \epsilon_{CPI} \\ \epsilon_{Bond} \\ \epsilon_{Unemp} \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & Cholesky\ matrix & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} \tag{6.12}$$

Where the Z-matrix is the matrix of randomly generated numbers using Monte Carlo simulations with a normal distribution, N(0,1).

Creating the paths

For the first forecasting period, *t+1*, we multiplied the beta coefficients obtained from the OLS regressions with the last demeaned historical observations and added the error term. We used the following matrix computation for our first forecast

$$\begin{bmatrix} \widehat{S\&P500}_{t+1} \\ \widehat{Div}_{t+1} \\ \widehat{CPI}_{t+1} \\ \widehat{Bond}_{t+1} \\ \widehat{Unemp}_{t+1} \end{bmatrix} = \begin{bmatrix} \hat{\beta}_{S\&P500_1} & \hat{\beta}_{Div_1} & \cdots & \hat{\beta}_{Unemp_1} \\ \hat{\beta}_{S\&P500_2} & \hat{\beta}_{Div_2} & \cdots & \hat{\beta}_{Unemp_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{S\&P500_5} & \hat{\beta}_{Div_5} & \cdots & \hat{\beta}_{Unemp_5} \end{bmatrix} \cdot \begin{bmatrix} S\&P500_t \\ Div_t \\ \vdots \\ Unemp_t \end{bmatrix} + \begin{bmatrix} \epsilon_{S\&P500_t+1} \\ \epsilon_{Divyield_t+1} \\ \vdots \\ \epsilon_{Unemp_t+1} \end{bmatrix} \tag{6.13}$$

Like mentioned, we dropped the intercept because of the demeaned variables that caused the intercept to be approximately zero.

After the first forecast, we used the same β coefficients and multiplied them with our first forecast and added a new random error term ϵ. The forecast in period t+2 then becomes

$$\begin{bmatrix} \widehat{S\&P500}_{t+2} \\ \widehat{Div}_{t+2} \\ \widehat{CPI}_{t+2} \\ \widehat{Bond}_{t+2} \\ \widehat{Unemp}_{t+2} \end{bmatrix} = \begin{bmatrix} \hat{\beta}_{S\&P500_1} & \hat{\beta}_{Div_1} & \cdots & \hat{\beta}_{Unemp_1} \\ \hat{\beta}_{S\&P500_2} & \hat{\beta}_{Div_2} & \cdots & \hat{\beta}_{Unemp_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{S\&P500_5} & \hat{\beta}_{Div_5} & \cdots & \hat{\beta}_{Unemp_5} \end{bmatrix} \cdot \begin{bmatrix} \widehat{S\&P500}_{t+1} \\ \widehat{Div}_{t+1} \\ \vdots \\ \widehat{Unemp}_{t+1} \end{bmatrix} + \begin{bmatrix} \epsilon_{S\&P500_t+2} \\ \epsilon_{Div_t+2} \\ \vdots \\ \epsilon_{Unemp_t+2} \end{bmatrix} \tag{6.14}$$

This procedure is again used for the rest of the forecasts, using an iterative approach where one period is simulated at a time.

6.2.3 Limitations

The model is designed using historical data to generate coefficients that are being used for our simulated forecast. This mean that we capture economic shocks and possibly breaks in our dataset that we cannot replicate in a forecast. Our simulated data are reverting to the added mean, and we will therefore not see any shocks where bond rates or S&P500 returns drift away to unknown levels. If we look at this from a historical perspective, it seems unlikely that we will not see any shocks in the period of 65 years, even if we cannot say anything with certainty. In the end, no one really knows how the economy will develop the next 65 years, but the limitations of the model must be kept in mind when we do our strategy testing. Assuming no shocks either way is a strong assumption that can cause us to be either too optimistic or pessimistic about certain strategies.

6.2.4 Adjusting the Government bond rate

When we created the VAR-model to forecast the movements of the bond rates, these rates were based on historical data. We believe that using historical data for bonds might bias our results upward, as today's interest rates are historically low. The European Central Bank is still pushing quantitative eases to release some of the pressure of several economies, meaning that we can not assume that the economic situation has turned. Experts such as Krämer [Krämer, 2015] and the J.P Morgan Asset Management department [J.P. Morgan, 2014] suggests changes in investment strategies as a result of historical low yields, supporting our view that the low yields may last. While U.S. 10 Year Government Bond rates in the middle of the 1970s reached close to 16% annually, the rate of mid June 2015 is just above 2% [Bloomberg, 2015].

The historical average bond rate for the past 65 years is 5.87%, and we have in addition added an average of 4% to test the impact of adding a lower mean. This number is not academic, but not unrealistic and serves the purpose of testing what will happen if the future average bond rates will be lower than the historical average. A lower bond average will lead to more modest returns on bond investments, which we expect would influence our remarks on the performance of the tested investment strategies.

We will in our result section refer to our simulations where we have used the historical average as Simulation I and the simulations where we have added the 4% mean as Simulation II.

6.3 Testing of Investment Strategies

We have used the two main types of investment strategies discussed in the investment strategy section, the Glide-to and the Glide-through strategies. Firstly, we will go through how we have computed bond returns. Then we will go through the computation of the portfolio returns, and finally we will elaborate on the computation of our reported monthly withdrawals and the assumptions we have made when we have designed the pension contributions and pension withdrawals.

Generating bond returns

For the simplicity of calculating bond returns, we assume investing in zero-coupon bonds only. These bonds pay no coupons before the face value of the bond is repaid at maturity unless e.g. the company or government issuing the bonds run bankrupt. Since there are no interest payments before maturity for zero-coupon bonds, we can calculate the returns on these bonds with ease. Zero-coupon bonds sell with a discount to face value as long as the interest rates are positive and at a premium when they are negative. The change in interest rates from one period to the next can be seen as the return on the bond investment. We can write the value of a zero-coupon bond with equation 6.15

$$Zero\ coupon\ value_t = \frac{Face\ Value}{(1+r_t)^n} \tag{6.15}$$

Where n is the number of periods until maturity and r_t is the monthly bond rate.

From equation 6.15, we see that higher interest rates mean higher discount rates. This will reduce the value of the zero-coupon bond. This works similarly the other way around, a lower discount rate will increase the value of the zero-coupon bond through a lower discount rate. We can therefore observe that low interest rates limit the upside potential of bond investments. The value of the bond will as well depend on the discount period. In our testing, we will consider U.S. 10 Year Government zero-coupon bonds. Because our interest rate forecasts are monthly, we will discount monthly in 120 periods to find the price of the zero-coupon each month. We will rebalance our portfolio monthly and calculate the returns in monthly intervals. If we denote the price of the zero-coupon in period t, P_t, the price is calculated as in 6.16

$$P_t = \frac{Face\ Value}{(1+r_t)^{120}} \tag{6.16}$$

Here, r_t is the monthly bond rate. To find the price in the next period P_{t+1}, we subtract one period from the discount period. We therefore discount with 119 periods to find P_{t+1} as in equation 6.17

$$P_{t+1} = \frac{Face\ value}{(1+r_{t+1})^{119}} \tag{6.17}$$

The return of the bond, r_b, is in each period then calculated as in equation 6.18

$$r_b = \frac{P_t}{P_{t-1}} - 1 \tag{6.18}$$

Designing wages

We have chosen to simulate individual wage paths that are adjusted by the inflations we have forecasted in our VAR-model. We have assumed an initial yearly wage of NOK 350 000 for our individual. With an initial yearly wage of NOK 350 000, an individual will end up with approximately today's average salary in Norway at the age of 67 using real wage increase [Statistisk Sentralbyrå, a]. To calculate the initial monthly wage, W_t, we have used equation 6.19

$$W_t = \frac{Initial\ yearly\ wage}{12} \tag{6.19}$$

To calculate the next period's monthly wage, we have used equation 6.20.

$$W_{t+1} = W_t \cdot (1 + i_t + Real\ wage\ increase_t) \tag{6.20}$$

Both inflation and real wage increases are calculated in monthly terms. By calculating changes in wages monthly, we could capture the differences in inflation better, even if we are aware that wages are usually adjusted once a year only.

We have assumed the real wage increase in Norway to be 1% yearly throughout the accumulation period. For the last decades, Norway has been in a special situation taking into account the economic impact the oil sector has had on the rest of the economy and lower oil prices will result in lower real wage increases in the future [Cappelen et al., 2014]. We believe it is better to take on a conservative approach rather than a too optimistic approach, and have therefore decided to reduce the expectations for the real wage increase for the future. Hansen and Skoglund have calculated the real wage increase on a yearly basis in the period from 1960-2002 to be approximately 2% [Hansen and Skoglund, 2012]. We have decided to use a real wage increase estimate for our simulated period to be 1% yearly, given today's economic situation in Norway.

Calculting pension contributions and withdrawals

In previous literature, researchers have presented their investment strategy results either as a lump sum terminal value of the pension portfolio or as annuities for an expected retirement period. Arnott argues that annuities should be used as they reflect the purchasing power the pension owner will have in each period, rather than the terminal value of the portfolio [Arnott, 2012]. We wanted to investigate both what investment strategy performs the best given our measurement criteria and also how the defined contribution scheme compared with the defined benefit scheme. Because we could calculate the average monthly withdrawal for a defined benefit holder, reporting our results as monthly pension withdrawals served that purpose better than a terminal portfolio value. Instead of using annuities, we used a modified method to calculate monthly withdrawals so that we could adjust for our simulated returns each period, rather than using the same returns for each period as is being done using an annuity formula. We will return to our modified method later in this section.

We define pension payments from the employer to the pension savers portfolio as *pension contributions* and monthly withdrawals under retirement as *pension withdrawals*.

Monthly contributions

We have calculated the value of the pension portfolio by using monthly pension contributions, monthly returns and monthly inflations. Monthly returns are calculated periodically by equation 6.21, which take the weights in stocks and bonds into account

$$R_p = w_s \cdot (r_s + div\ yield) + (1 - w_s) \cdot r_b \tag{6.21}$$

Monthly pension contribution in period t is calculated in equation 6.22.

$$Pc_t = Monthly\ income_t \cdot Defined\ contribution\ rate_t \tag{6.22}$$

The same equation is used for all other periods as well, where monthly income can change in each period, while we have assumed a constant defined contribution rate throughout the accumulation period, i.e. if we start out with a 2% defined contribution rate we will keep this the whole accumulation period.

The pension contributions have been adjusted for return and inflation in each period, so that the pension holdings, Ph, in period t+1 is calculated with equation 6.23

$$Ph_{t+1} = \frac{(1 + r_t) \cdot Ph_t + Pc_{t+1}}{(1 + i_t)} \tag{6.23}$$

Which accounts for all other periods than the first period, where the portfolio value is simply

$$Ph_t = Pc_t \tag{6.24}$$

We assumed that the pension contribution is done at the start of each period, and we have reported the pension holdings in the beginning of each period using equation 6.23. Ph_{t+1} is therefore the value of the pension holdings in the beginning of period *t+1*. We have sensitivity tested all investment strategies with defined contribution rates in the interval 2-7%. We only report results for a 2% defined contribution rate in the result chapter and we refer to the appendix for results for defined contribution rate of 3%-7%.

Monthly withdrawals

We calculated the pension holdings as described in equation 6.23 up until including the age of 67. After the age of 67, the pension withdrawals start. We continued to invest the portfolio after retirement and therefore multiplied returns with the pension holdings and adjusted for inflation. For simplicity, we assumed the retirement age to be the start of a new period, where period *tr* represents the first period after retirement. The pension holdings in period *tr+1* after the first monthly withdrawal was calculated with equation 6.25, where Ph_{Retire} is the pension holdings and mw_t is the monthly withdrawal.

$$Ph_{Retire_{tr+1}} = \frac{(PhRetire_{tr} - mw_{tr}) \cdot (1 + Rp_{tr})}{(1 + i_{tr})} \tag{6.25}$$

We have used the same equation for all periods after *tr+1*.

Our monthly withdrawals are calculated by dividing the pension holdings by the number of periods left to retirement. For the first period of retirement, period *tr*, the monthly withdrawals is calculated by equation 6.26.

$$Monthly\ withdrawal_{tr} = \frac{Pension\ holdings\ retirement_{tr}}{Periods\ to\ retirement_{tr}} \tag{6.26}$$

This equation have been used for all periods after the first period.

We have different returns and inflations in each period from our simulations, so the size of the monthly withdrawals will be slightly different each period. This means that the withdrawals may change from time to time, but not by large amounts. We have reported expected withdrawals from each simulation by taking the average of all the withdrawals, and we believe that the average serves as a good indicator of the expected withdrawals for each simulation path because of the low variance in the monthly amounts. The reported withdrawals is calculated with equation 6.27.

$$Reported\ withdrawals = \frac{\sum_{i=1}^{276} Monthly\ withdrawals_i}{276} \tag{6.27}$$

where 276 is monthly observations for 23 years.

We have simulated 1000 monthly withdrawal averages in each strategy based on equation 6.27. From these simulations, we have reported several measurement criteria that we will return to later.

6.3.1 Measurement Criteria

In this section, we will summarize the measurement criteria we have used to analyze the different investment strategies. Most of the criteria have been retrieved from Estrada [Estrada, 2013], but we have as well introduced some measurements from Blanchet [Blanchet, 2007] that we find highly relevant, namely the Success Ratio and the Success-to-Variability Ratio. In addition, we introduce the concept withdrawal rate that we have used to interpret our results in a different manner. We will in the following introduce these concepts.

Estrada's criteria

We summarize all the criteria used by Estrada [Estrada, 2013] that we have used as well. Each measurement is described below.

Mean Withdrawal: reports the average pension withdrawal for each strategy.

Median: minimum pension withdrawal a pension holder will revience with 50% certainty for each strategy

Maximum Value: reports the best possible pension withdrawal

Minimum Value: reports the worst possible pension withdrawal

Standard Deviation: reports the volatility of the pension withdrawal

VAR 10%: Reports 10% VAR for pension withdrawals

Average 10% worst scenarios: Allows us to adjust for potential downside outliers in all strategies

Top 10% best scenarios: Allows us to compare the spread on the best possible scenarios

<**than bonds:** Calculate the probability of the strategy to perform worse than only investing in bonds the whole investment period

The Success Ratio

The Success Ratio introduced by Blanchet [Blanchet, 2007] quantify the probability of the pension holdings running out before the end of the retirement period. The end of the retirement period is undefined as we don't know how long we will live, so it is usually calculated based on the current life expectancy. Equation 6.28 show how the Success Ratio is calculated.

$$SR = \frac{\sum_{i=1}^{n=1000} Successful\,portfolios_i}{Numbers\,of\,portfolios} \tag{6.28}$$

If a portfolio turns out to be successful, i.e. if it doesn't have a negative value at the end of retirement, the portfolio will be considered successful and get a value of one. On the other hand, if the portfolio have a negative value at the end of retirement it will be considered unsuccessful and get a value of zero. Whether a portfolio turns out to be successfull or not is highly dependent on the withdrawal rate that we will introduce below.

Withdrawal rate (wr) is defined by Bengen as the percentage of the pension holdings that is withdrawn from the pension savings each year [Bengen, 2004] Mathematically it can expressed as in equation 6.29.

$$Pension\ withdrawals = wr \cdot pension\ holdings\ at\ retirement \tag{6.29}$$

The rate of withdrawal was first introduced by Bengen [Bengen, 2004] who brought the withdrawal rate up to discussion. The main finding in Bengen's article lead to what is called the "4%-rule", namely that a portfolio never obtained a negative value faster than 33 years using a 4% withdrawal rate on a portfolio allocated 50% in stocks and 50% in bonds. Others findings were that portfolios with a 3% withdrawal rate never obtained a negative value in 50 years. Exceeding the "4%-rule" lead to what Bengen defined as a risky because the pension holdings in many cases were depleted way faster than the end of retirement.

Since Bengen's research, a broad set of literature have developed around the topic. Among the literature we find Pfau, Finke and Blanchett [Finke et al., 2013] who questioned Bengen's "4%-rule" validity given the current low-yield environment and whether the rule can be considered a safe withdrawal rate anymore.

The rate of withdrawal can be considered a trade-off between increasing the pension withdrawals today versus the risk of running out of pension holdings (considered an unsuccessfull portfolio) and thereby live solely on the public retirement pension and other savings. We relate this decision to the individiual risk preferences that we discussed in section 3.2.

We have sensitivity tested withdrawal rates (wr) in the interval from 3-10%. When we test withdrawal rates, the pension withdrawal (pw) is equal in each period, found with equation 6.30 at the time of retirement.

$$pw = pension\ holding\ at\ retirement \cdot wr \tag{6.30}$$

The pension holdings (ph) in the next period, which is time t in the retirement plus one period (tr+1), is given by equation 6.31. Pension withdrawals are equal in each period, as given in equation 6.29, and therefore have no subscript. In addition, both return and inflation for the relevant period is also taken into account. For period two in retirement (tr+2), the calculations will be the same.

$$ph_{tr+1} = \frac{(ph_{tr} - pw) \cdot (1 + r_{tr})}{(1 + i_{tr})} \tag{6.31}$$

This will continue until the last period in the retirement. Whether the portfolio is successful or not will depend on whether the pension holdings in the last period is positive or not. This will of course highly depend on the withdrawal rate.

The Success-To-Variability Ratio

By extending the Success Ratio, we get to the Success-to-Variability Ratio which is an extension of the Success Ratio. It can in many ways be compared with the Sharpe Ratio, first introduced by Sharpe in 1966 [Sharpe, 1966].

The Sharpe Ratio is measured as excess return over the risk of the portfolio, measured in standard deviations with the formula

$$Sharpe\ Ratio = \frac{E[R_m] - rf}{\sigma_p} \tag{6.32}$$

The ratio measures the return per unit of risk. Instead of using excess return in the numerator we instead use the Success Ratio, but keep standard deviation in the denominator. Standard deviation here is still the volatility of the portfolio. This result is the Success-to-Variability Ratio given in equation 6.33. By using the Success-To-Variability ratio, we obtain a risk-adjusted score for each strategy that allows us to rank them based on both the likelihood of success and the underlying risk of the portfolio [Blanchet, 2007].

$$Success\ to\ Variability\ Ratio = \frac{Success\ Ratio}{\sigma_p} \tag{6.33}$$

This can potentially change the picture of which strategy rank highest. An investment strategy with a high Success Ratio, but at the same time with a high standard deviation will be ranked lower based on the Success-to-Varibility Ratio measure compared to the Success Ratio measure. However, there are drawbacks with this measurement. Even though standard deviation is a commonly used risk measure, the deviations capture the upside volatility as well. Therefore, other risk definitions, such as semideviation that can measure downside risk, will as well be a powerful risk measure. Still, the concept of Success-to-Varibility Ratio can be useful because it both measure the probability of success and the underlying risk of the portfolio.

Even though we don't use semideviations in our calculation we will briefly present it below since it can give additional depth to our discussion when we present our results.

Using Dimson-Marsh-Staunton 110 year dataset, Estrada [Estrade, 2011] looked into the returns and volatilities in 19 different countries using annualized returns. He primarily looked at the spread of the highest and lowest return for both stocks and bonds, and the standard deviation of each asset class. His results proposed that, over a short-term period, stocks are without question riskier than bonds in terms of volatility and spread. However, over a 10-year holding period, his results states that the annualized volatility of stocks (6.5%) was, on average almost the same as that of bonds (6.3%) by using semideviation. Semideviation with respect to a benchmarked return is a measure of downside risk, known as the square root of semivariance [Estrada, 2006], which is found by the formula:

$$Semideviation_B = \sqrt{\frac{1}{T} \cdot \sum_{t=1}^{T} [Min(R_t - B), 0]} \tag{6.34}$$

Where R is return, and B is a benchmarked return.

For a longer period, the annualized volatility of stocks was in fact, in most countries and on average, lower than that of bonds and the spread was just slightly higher. His findings supported the view that over long term, bonds will rarely outperform stocks, but also the fact that in the long term, bonds have higher downside potential. It is important to emphasize that there will always be a chance that a holder of a portfolio can have bad luck and face several losses that can have dramatic consequences, but the odds will be with the portfolio holder holding a high proportion stocks in the long term.

The cumulative returns were measured using the same data, but have some different interpretations. Measuring cumulative returns keep the conclusions that stocks are more risky than bonds in the short-term, but Estrada

[Estrada, 2011] argues that cumulative stock returns have an increasing volatility rather than decreasing, as of annualized returns. He finds that holding a portfolio for no more than 5 years, the semideviation of stocks is lower than that of bonds. He also finds that the shortfall for stocks peaks at 5 years, meaning that the downside is being reduced after holding the portfolio for more than 5 years. This can be interpreted so that the increased volatility in the cumulative returns over time represents uncertain upside potential rather than downside potential. In other words, Estrada states that our fear of high risk (measured in volatility) is only rational when the risk involves a downside potential.

The results for SR and SV will be presented in an own section in chapter 7. These measurements differ from the other measurements that we presented in the beginning of this section, since the monthly withdrawal is kept constant for both SR and SV compared to the other measurement were the monthly withdrawals are calculated using equation 6.26.

6.3.2 Assumptions

We have done some exceptions in the model that we need to address. We will briefly discuss how leaving out taxes, trading costs and gender inequality would have affected our results.

Taxes

Taxes serve the purpose of smoothing out income differences. Including taxes would have reduced the differences between our best scenarios and worst scenarios, because in the best scenarios, the pension owner would pay a higher tax because of a higher income. Taxes for pension savings are calculated differently from regular income tax. Tax deductions for those with pension withdrawals is one of the elements that would have increased the complexity of our calculations substantially. We therefore limit ourselves to keep in mind that taxes would have reduced the differences in the scenarios.

Trading costs

We have ignored trading costs, where we include management fees under this expression. Like taxes, trading costs and management fees would have adjusted our monthly withdrawals down, but they would have been identical for all strategies. We are rebalancing all our investment portfolios periodically, so the trading costs would have been similar for our Glide-to and Glide-through strategies. There would therefore be no changes in the interpretation of what strategy we believe is more attractive, but only changes in the monthly withdrawal levels.

Gender inequalities

We have not separated between genders, even though females are expected to live slightly longer than men. There are two main reasons why we felt this would be of less relevance in this study; first of all, the defined contribution pension is based purely on individual pension contributions, meaning that, in opposite to the public retirement pension, this part of the pension are not covered by the tax payers' bill. There will be no discriminating benefit for females, because they will simply have to allocate their pension capital over a longer period than men. Secondly, we consider the difference in life expectancy between men and women not large enough to make a difference in the choice of strategy. After all, we have decided to use a security margin of about 5 years for men and 3 years for women, so we believe that there is no more need for any adjustments on this issue.

Chapter 7

Results

In this section, we will go through the results from our simulations. We will start by providing the calculated public retirement pension. Secondly, we provide sanity checks where we compare our simulated variables with the historical observations we have from the past 65 years. Finally, we present the findings from our investment strategy testing, where we make general comments about how the different strategies perform on our chosen criteria. We will draw some comparisons between the defined contribution scheme and the defined benefit scheme using our findings from the first section.

7.1 Calculation of the Public Retirement Pension

We have used the framework from section 6.1 to calculate a public retirement pension for an individual under the assumptions we made, as is listed in the table below

Initial income	NOK 350 000
Yearly real wage increase	1%
Working years	42 years
Retirement age	67
Calculated living years	90 years

The estimated public retirement pension for our individual is NOK 11 975 per month.

7.2 Sanity Check for Forecasted Variables

Below, we will present sanity checks for our forecasted variables to see how our simulations move, compared to how they have moved historically. While we emphasize that historical movements not necessarily reflect future movements, they will be our best estimate to compare with. The figures will show how five different randomly selected estimations paths move compared to the historical movements, using the past 65 years as historical movements to compare with the same time length that we have forecasted.

In each figure, the black solid line represent the historical movements of the variable, while the black dotted lines are the 95% confidence interval(CI), both the upper and lower bound. The five randomly selected estimation paths are solid grey lines.

We have calculated the CI for all our plotted data using the formula

$$CI_t = Observation_t \pm 1.96 * \sigma_{historical\ data} \tag{7.1}$$

Where the $\pm$ determines the upper and lower CI, the σ is the standard deviation of the variable of interest based on historical volatility and 1.96 is the critical value for a 95% CI under the normal distribution, i.e. 95% of the observations should lie within ± 1.96 standard deviations, plus the relevant observation. We use the same equation for all periods where only the historical observation change in the equation.

Below we can observe all the simulated variables and the historical values with CI.

Figure 7.1: Historical and simulated S&P500 returns

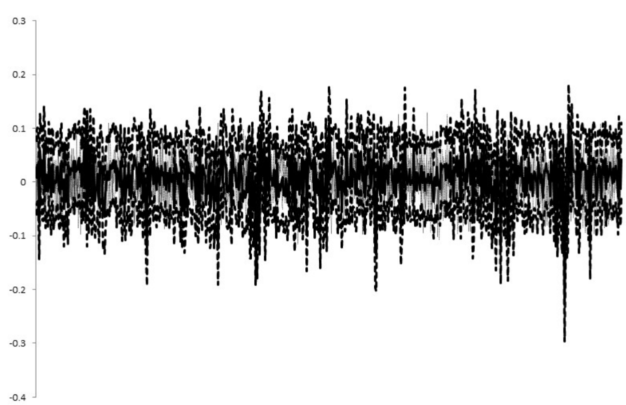


Figure 7.2: Historical and simulated dividend yield

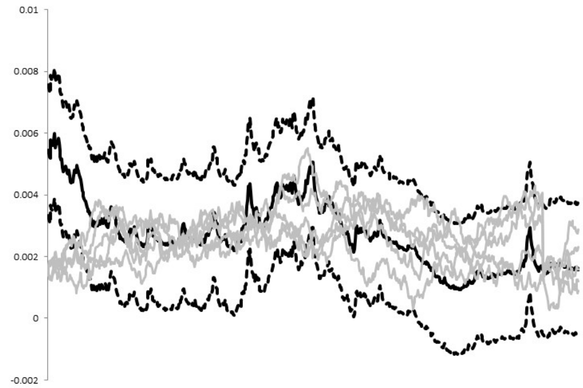


Figure 7.3: Historical and simulated bond returns

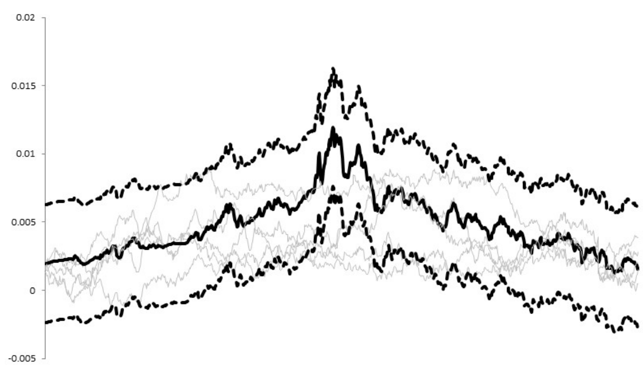


Figure 7.4: Historical and simulated changes in CPI

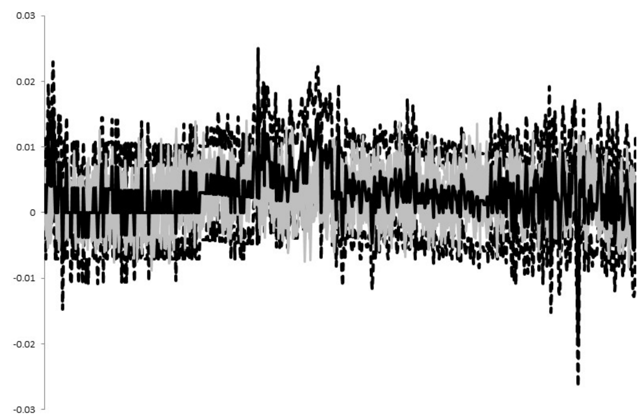
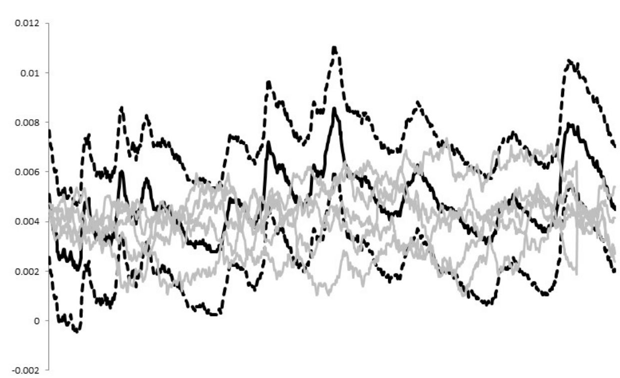


Figure 7.5: Historical and simulated unemployment



All our variables of interest moves quite similarly to the historical. We believe our model have created robust results that have captured the historical pattern of the data well. We are therefore under the impression that our model has created sufficient accuracy to test our forecasts for different investment strategies. Both figure 7.1 and figure 7.4 produce figures that is a bit difficult to read, due to the frequent movements.

7.3 Performance of Strategies - Simulation I

In this section, we will present the results of our investment strategy testing. We will start with presenting results for glide-to and glide-through strategies for our Simulation I forecasts where we have added the historical bond mean to our forecast. Thereafter we will present our testing of the success ratio and the success-to-variability ratio for each strategy. In the next section, we will present the same results for Simulation II. All our presented tables are the results when we have used a constant defined contribution rate of 2%. We chose to present these tables because most defined contribution pensions have a defined contribution rate of 2%, refering to the statistics about defined contribution rates that we presented in chapter 2. Results for defined contribution rates (DCR) between 3%-7% can be found in the appendix.

Results glide to strategies

We have tested all eight glide-to strategies presented in section 4.2.5 and present the results in table 7.1

Table 7.1: Simulation I - glide to strategies

Strategies	0/100	20/80	40/60	50/50	60/40	80/20	100/0	Stb*
Mean withdrawals	17996	15449	13185	12156	11193	9455	7952	10740
Median	15271	13268	11411	10442	9653	8131	6773	9208
Min	2970	2895	2788	2706	2373	1784	1243	2199
Max	98733	79057	65775	59724	54072	43947	36291	45155
Sd	10547	8524	7102	6557	6096	5363	4799	5860
VAR 10%	8453	7397	6453	5909	5337	4297	3444	5111
Av bottom 10%	6727	6051	5269	4844	4409	3586	2826	4206
D90	31062	25966	22082	20217	18583	16170	14021	18090
<than bonds	2.2%	4.3%	7.8%	11.5%	15.9%	28.9%	42.2%	18.9%

∗ Note: Storebrand's strategy

Our results from these eight investment strategies are similar to the ones presented by Estrada [Estrada, 2013]. The 0/100 strategy outperforms the other strategies in all terms. We see that all mean withdrawals are higher than the medians, meaning that less than 50% will receive the mean withdrawal. In other words, only in "lucky" scenarios can the pension saver expect to get the mean withdrawal. The positive skewness of the mean also implies a higher upside potential than downside potential for the pension saver. An interesting observation is that the median of the 0/100 strategy is higher than the mean withdrawal of 6 of the other strategies, the only exception is the 20/80 strategy. Saving in the 0/100 strategy therefore provides a minimum value with 50% certainty that is *higher* than the minimum value for "lucky' scenarios in other strategies. While the standard deviation is higher for the 0/100 strategy, it seems like a desirable volatility since the strategy score higher on all measurements criteria and therefore indicate upside potential.

The difference in the worst possible scenario, the minimum pension withdrawal, is 139% for the best and worst strategy. This percentage difference remains about constant for the average of the bottom 10% (138%), meaning that the differences in absolute values are stable in the worst scenarios. There seems to be no reason to believe that the more risky 0/100 strategy end up with any scenarios that are significantly worse than in other strategies, in fact the results show the opposite.

If we look at the best scenarios, the difference between the best and worst strategy using the 90th decile is 121%, which is still quite substantial. The difference in the maximum pension withdrawal between the best and worst strategy is 172%, which is even larger than for the 90th decile. This can be due to an existing opportunity for extreme outliers in the best scenarios, which seemed to be less prevalent for the worst scenarios.

We have tested each strategy against fully investing in bonds, which we consider the investment strategy with the lowest possible risk. We did this to determine the probability of ending up with a larger monthly mean withdrawal than you would by investing in any of the tested strategies. What might be a surprising result is that by looking at the empirical portfolio of Storebrand, the likelihood of performing worse than investing in low risk bonds only is almost 19%. For other strategies, such as the 80/20, the likelihood is even higher, while the 0/100 strategy only perform worse in 2.2% of the scenarios. We will return to the compensation for risk in the discussion section.

For the ease of our comparison between the defined contribution and the defined benefit scheme, we have used the same assumptions for wage development for an individual holding a defined benefit pension as we did for an individual using a defined contribution pension. With an average yearly real wage increase of 1% and an initial yearly salary of NOK 350 000, the salary at retirement age is NOK 531 500. By dividing this on twelve, and thereafter multiplying by $\frac{2}{3}$, we find the pension that is claimed through a defined benefit system on a monthly basis to be NOK 29 532. Because the defined benefit scheme is the combination of both Pillar I public retirement insurance and Pillar II pension, we need to subtract the public retirement pension from this amount to compare with the monthly withdrawals from our tested strategies. In section 7.1, we found the public retirement insurance to be NOK 11 975 for an identical individual with a defined contribution scheme. We assume that these individuals have the right to claim identical public retirement pensions, meaning that the target number for our portfolio annuities is NOK 17 557 (NOK 29532 - NOK 11 975).

Table 7.2 illustrates at what defined contribution rates the different investment strategies reach this target number for both the median and the mean withdrawal

Table 7.2: Sensitivity analysis of DC rates compared to DB

	2%	3%	4%	5%	6%	7%
0/100	*	**	**	**	**	**
20/80	-	**	**	**	**	**
40/60	-	*	**	**	**	**
50/50	-	*	**	**	**	**
60/40	-	-	**	**	**	**
80/20	-	-	*	**	**	**
100/0	-	-	-	*	**	**
Storebrand	-	-	**	**	**	**

− Mean withdrawal & median <17 557
* Mean withdrawal >17 557 & median <17 557
** Mean withdrawal & median >17 557

Most strategies reach the target number for both the mean withdrawal and the median at a defined contribution rate of 4%. With a 2% defined contribution rate, which is the one mostly applied, only the 0/100 strategy reach this number and only for the mean withdrawal. This means that none of the strategies offer *at least* the target amount with a 50% certainty when we have a defined contribution rate of 2%, even though these individuals carry a higher risk. Only at a defined contribution rate of 6% all strategies reach the target for both mean and median withdrawal.

Results from glide-through strategies

In our testing of glide-through strategies, we have tested 12 different portfolios introduced in section 4.2.6 Table 7.3 and 7.4 illustrate the results for the glide-through testing

Table 7.3: Simulation I glide-through strategies, part 1/2

Strategies	100/0	80/20	60/40	40/60	20/80	0/100
Mean withdrawal	11391	11692	12000	12314	12632	12954
Median	9944	10104	10318	10589	10938	11128
Min	2462	2563	2671	2606	2391	2198
Max	49910	53673	57653	61848	66247	70839
Sd	6051	6210	6425	6705	7058	7495
VAR 10%	5558	5671	5797	5986	5958	5906
Av bottom 10%	4605	4724	4814	4860	4868	4837
D90	18986	19395	19970	20670	21240	22460
<than bonds	14.7%	13.5%	11.9%	10.7%	10.8%	10.7%

Table 7.4: Simulation I glide-through strategies, part 2/2

Strategies	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean withdrawal	9271	10351	11529	12809	14194	15688
Median	7753	8798	9937	11008	12297	13442
Min	1744	2101	2505	2741	2798	2836
Max	43151	49396	56157	63411	71121	79239
Sd	5530	5865	6295	6856	7588	8534
VAR 10%	4068	4746	5495	6268	7003	7601
Av bottom 10%	3310	3928	4539	5140	5675	6125
D90	16025	17463	19075	21590	24229	26613
<than bonds	30.2%	23.0%	15.2%	8.8%	5.9%	4.3%

Looking at the mean withdrawals, the U-shaped strategies outperforms the other strategies. An interesting result is that for more conservative portfolios that hold a higher bond proportion the older the pension holder gets, the linear glide through strategies perform better than the U-shaped. For more risky portfolios with a higher equity proportion the older the pension holder gets, the result is opposite. The same trend is clear for glide through strategies as for glide to strategies; those ending up with a higher stock allocation the older you get outperform the more conservative strategies. This illustrates the argument of Shiller [Shiller, 2005] who state that the equity weighted stock proportion is too low today. Those strategies that end up with a high proportion of stocks is more invested in stocks than in bonds using equity weights, in contrast to the conservative strategies. The minimum mean withdrawals for the conservative strategies are smaller than those for more "risky" strategies, which again illustrate the upside potential in the "risky" strategies. This observation is interesting, because the conservative strategies are constructed to avoid individuals ending up short on their pensions, and based on these findings, and the findings of Estrada [Estrada, 2013], these strategies do not seem to serve that purpose. Rather, they end up with no possibilities for obtaining the upside potential, but a higher risk of ending up with a low pension.

The spreads in the glide through strategies are significantly smaller than for the glide to strategies. This is not unexpected, because the differences in the stock-bond allocation are bigger for glide to strategies. The differences between the largest and smallest minimum monthly withdrawal for glide through strategies are 62.6%. Also in these strategies, the differences is increasing when you look at the VAR 10% (86.8%) and the average of the bottom 10% scenarios (85.05%), and quite stable for these two criteria. In other words, not only is the worst outcome substantially worse, but the differences increases even for the worst 10% scenarios. When we look at the best scenarios using the 90th decile, the difference is 66.07% between the 0/100U and the 100/0U, which is slightly larger than the spread for the worst scenarios. The differences in the best scenario between the top and worst performing strategy is 183.6%. Relatively, it seems like the difference between worst possible scenarios is higher in glide through than in the glide to strategies, while glide to strategies provides bigger differences in the best possible scenarios.

Looking at the probabilities of producing worse returns than investing in bonds only, we find similar patterns as for the glide to strategies. The best performing strategies have the lowest probability of obtaining smaller monthly withdrawals than investing in bonds only. For the best performing strategy, the 0/100 U-shaped, the probability is 4.3%. This is approximately twice the risk of the 0/100 Glide-to strategy(2.2%). In our discussion, we will look into these numbers and try to make some intuitive comments that might influence our view on these results.

Finally, we can note that none of the glide-through strategies reach the level of the highest performing glide to strategies in terms of mean withdrawal and median, meaning that if we look simply at returns, glide-through strategies seems less favourable.

Below is a similar comparison between the defined benefit and defined contribution scheme that we provided for the glide to strategies.

Table 7.5: Sensitivity analysis of DC rates compared to DB

	2%	3%	4%	5%	6%	7%
0/100	-	*	**	**	**	**
20/80	-	*	**	**	**	**
40/60	-	*	**	**	**	**
60/40	-	*	**	**	**	**
80/20	-	-	**	**	**	**
100/0	-	-	**	**	**	**
0/100 U	-	*	**	**	**	**
20/80 U	-	*	**	**	**	**
40/60 U	-	*	**	**	**	**
60/40 U	-	-	**	**	**	**
80/20 U	-	-	**	**	**	**
100/0 U	-	-	*	**	**	**

− Mean withdrawals & median <17 557
∗ Mean withdrawal >17 557 & median <17 557
∗∗ Mean withdrawal & median >17 557

For glide-through strategies, none of them reach the target level of NOK 17 557 with a defined contribution rate of 2% and only some of the strategies reach the mean withdrawal at 4%. Almost all strategies reach the level, both for the mean withdrawal and the median with a rate of 4%. While both conditions are fulfilled for higher defined contribution rates.

7.3.1 Success ratios

We have introduced withdrawal rates as an alternative measure of the risk of the different strategies. We have tested different withdrawal rates for our strategies and provide the results below.

Success ratios for glide-to strategies

The figures below present the success ratios (SR) and the success-to-variability (SV) for the different withdrawal rates for glide-to strategies. Both figure 7.6 and figure 7.7 work in the same way, the pillar to the left for each strategy is a 3% withdrawal rate the next one to the right is a 4% withdrawal rate and so on until a withdrawal rate of 10%. The exact numbers for figure 7.6 and figure 7.7 can be found in the appendix.

Figure 7.6: SR

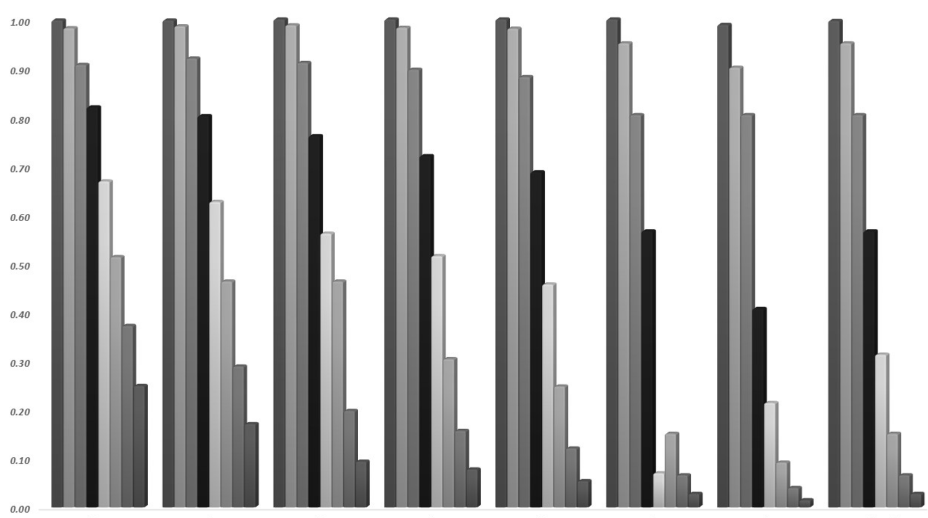


Figure 7.7: SV

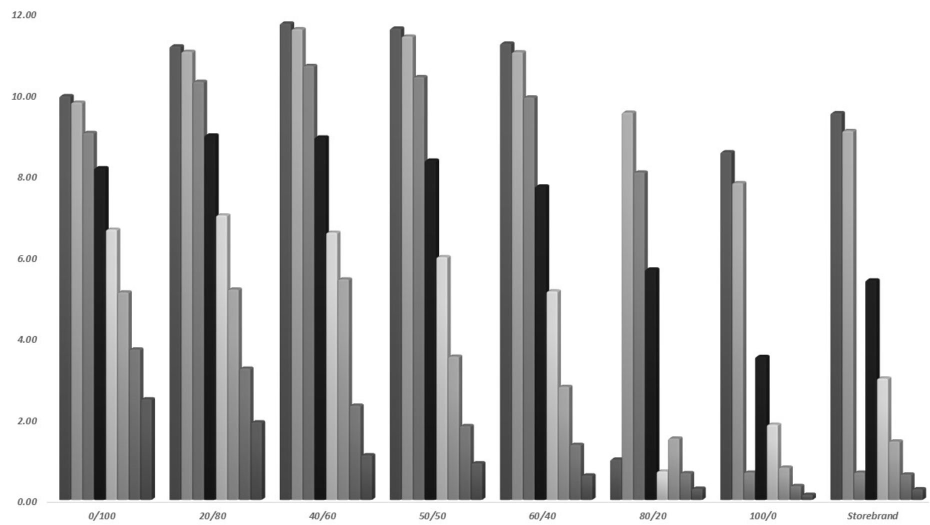


Figure 7.6 and 7.7 provides us some interesting results that both supports and contradicts our findings using the other measurement criteria. A withdrawal test what rate of the portfolio value we can subtract each year without running out of pension holdings before the last period. It is in this case easier to use individual risk preferences to determine what strategy is more attractive for each individual. Our previously presented results showed that the 0/100 portfolio provided us the highest results on all measurement criteria, so there was no ambiguity in our results for the glide to strategies which strategy that performed best.

Looking first at SR the picture seem to be the same, the 0/100 have a bit higher SR than the rest of the strategies for all withdrawal rates. All strategies almost offer a 3% guaranteed withdrawal rate, while some of the strategies start to fail at a 4% withdrawal rate. The 100/0 for instance have a SR of 0.90 at a 4% withdrawal rate which means that 100 of the 1000 simulations run out of pension holdings before the last period. Increasing the withdrawal rate more will naturally lead to a fall in SR for all strategies and the risk of running out of pension holdings before retirement is naturally increasing with the withdrawal rate.

Its important to be aware of that the numbers are relative, and the actual monthly withdrawal may be hugely different for the different strategies, depending on the terminal wealth for each strategy. As this is reflected in the results for the strategies we have chosen to not report withdrawal here and refer to equation 6.30 for how the monthly pension withdrawal is calculated.

Relating SR to behavioural finance that we presented in section 3.2 it will be natural to observe the following. Risk averse individuals will prefer safety and therefore strategies that offers higher SR may be more attractive than the 0/100 strategy, which "only" has a success ratio of 99%. We should keep in mind that the 0/100 strategy in most cases will pay out substantially larger monthly payments than those strategies with a 3% withdrawal "guarantee", and most individuals will most likely be willing to accept the 1% risk for a higher monthly payment. If we move to slightly less risk averse individuals, we can see that the same portfolios that performed best are becoming better and better compared to the others as we increase the withdrawal rate. At a 5% withdrawal rate, the 0/100 strategy is again superior in terms of SR and this sustains for all withdrawal rates higher than 5%.

Having considered SR, its good to look at SV to get another performance measure and a more nuanced view. While 0/100 provided the highest SR for all strategies, this change with SV. In fact its actually the 40/60 strategy that delivers the highest SV and the 0/100 strategy is actually also beaten by the 20/80, 50/50 and 60/40 strategy. In the in it would be each individual's choice to determine how far they are willing to go to maximize their pension, but we can conclude that the success ratio and success-to-variability ratio put some doubt into the clear conclusion that the 0/100 strategy is the undoubtedly best portfolio.

Success ratios glide-through

Figure 7.8: SR

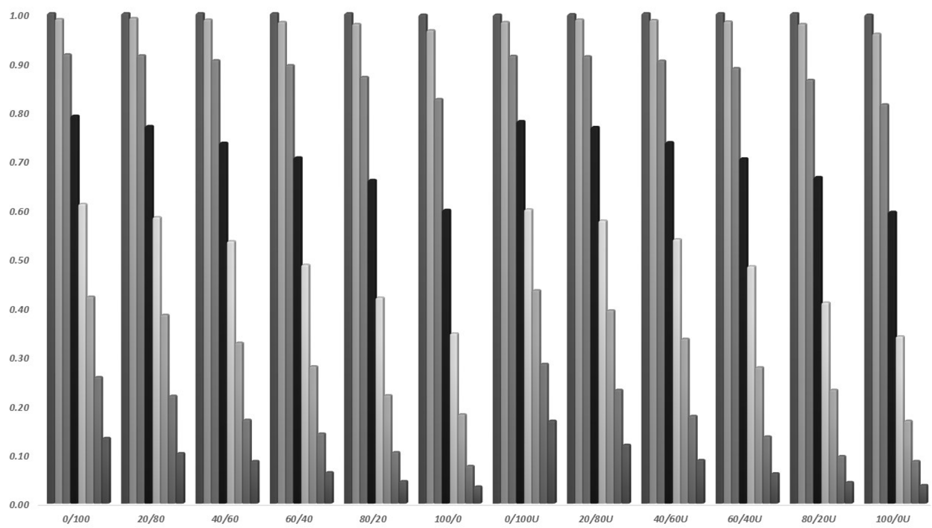
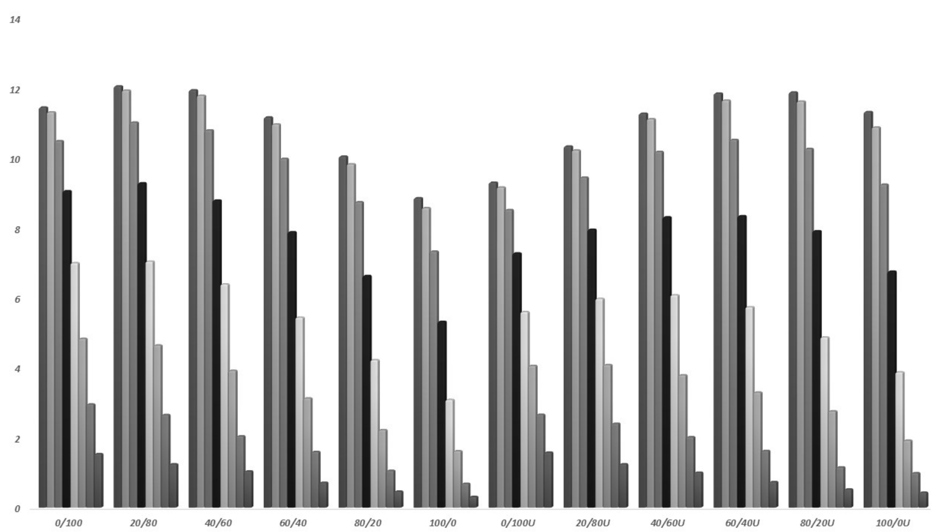


Figure 7.9: SV



We can observe the same for figure 7.8 as we did for figure 7.6. Most of the strategies provide a guarantee of 3% in withdrawal rate, even if they are not the top performing strategies. Again, it is a matter of taste for the individual. SV disfavours the 0/100 U-shaped strategy despite our conclusion in the previous section where it was the best performing strategy. However, SV may not be an appropriate measure if the volatility is limited to upside potential but still standard deviation is common measured used for risk. The SV result clearly states how different measurement ratios can provide different results, and the need for several measures to make qualified interpretations.

Again we see that a strategy such as the 0/100 glide through strategy provides highest SR up to 8% withdrawal rate where it drops slightly below the 0/100 U-shaped strategy. The monthly pension payments on average are 37.7% higher for the 0/100 U-shaped than for the 0/100, and we expect these differences to be of importance as well as the SR when individuals make their decision of how to invest their pension funds. Again we can observe that strategies that delivered the highest SR, like 0/100 and 0/100 U not deliver highest on SV. Looking at SV its 20/80 who obtain the highest ratio.

7.4 Performance of Strategies - Simulation II

In this section, we will provide the same analysis as we did in section 7.3 with our other Simulation data where we have added a lower mean to the bond rates, which we discussed in section 6.2.4. We will mainly focus on the changes from using Simulation I, and not discuss implications that are identical to the Simluation I results.

Results from Glide-to strategies

Table 7.6: Results from Glide-to strategies from Simulation II

Strategies	0/100	20/80	40/60	50/50	60/40	80/20	100/0	Stb*
Mean withdrawal	15972	12737	10256	9236	8337	6840	5663	8137
Median	13697	11238	9238	8240	7338	5687	4546	6757
Min	3464	3397	2368	1980	1660	1181	848	1425
Max	88483	48459	47834	49960	51938	55341	57848	74826
Sd	8662	6106	4906	4602	4413	4230	4160	5349
VAR 10%	7933	6643	5260	4651	4066	2972	2140	3411
Av bottom 10%	6462	5639	4504	3909	3341	2388	1656	2734
D90	26511	20831	16673	15107	13743	11854	10200	14176
<than bonds	0.3%	0.8%	2.2%	4.6%	9.7%	23.9%	42.4%	15.9%

∗ Note: Storebrand's portfolio

All our results follow identical pattern for Simulation II as Simulation I. This result is not particularly surprising, as the best performing strategies still hold the highest equity stake at the end period, and our equity returns have not been adjusted. The worst performing strategies have a larger bond stake at the end of the saving period, and these returns have become even worse.

All measures have dropped in absolute levels, but we can find some rather interesting changes in differences between best and worst scenarios. The difference of the highest and lowest mean withdrawal for the Simulation I data is 126%, while it is 82% in Simulation II. The median differences in Simulation I was 125%, while Simulation II the difference is 201%, a drastic increase. On the contrary, the differences in maximum monthly withdrawals is 172% in Simulation I, but only in Simulation II 53%. We would therefore believe that if we expect bond rates to remain lower in the future, the differences between the strategies in the worst scenarios would increase, while differences in the best scenarios will decrease.

We can see that the probability of generating lower returns than fully investing in bonds is decreasing for almost all portfolios, especially for our top performing portfolios. While the probabilities for generating less than the bond investments has dropped by 86.36% and 83.39% for the 0/100 strategy and the 20/80 strategy respectively, the 80-20 strategy has only dropped 17.3% and the 100/0 strategy has actually increased risk of performing worse. The empirical strategy from Storebrand drops with only 15.87% in the same measure, and the 50/50 strategy drops by 60% in comparison. If we expect future bond rates to stay low, strategy design should probably be re-investigated.

Table 7.7 summarizes the required defined contribution rates for achieving a pension money equivalent both in terms of mean withdrawal and median to the defined benefit scheme. Compared to Simulation I the analysis change slightly as actually none of the defined contribution rates fullfill the requirements for mean withdrawal and median. The 0/100 fullfill the requirements already from a 3% defined contribution rate.

Table 7.7: Sensitivity analysis of DC rates compared to DB

	2%	3%	4%	5%	6%	7%
0/100	-	**	**	**	**	**
20/80	-	*	**	**	**	**
40/60	-	-	**	**	**	**
50/50	-	-	*	**	**	**
60/40	-	-	-	**	**	**
80/20	-	-	-	-	*	**
100/0	-	-	-	-	-	*
Storebrand	-	-	-	*	**	**

− Mean withdrawals & median <17 557
∗ Mean withdrawal >17 557 & median <17 557
∗∗ Mean withdrawal & median >17 557

Glide-through strategies

Table 7.8: Results from Glide-through strategies for Simulation 1, part 1/2

Strategies	100/0	80/20	60/40	40/60	20/80	0/100
Mean withdrawal	8715	8879	9102	9385	9732	10144
Median	7477	7753	8062	8406	8696	9098
Min	1582	1716	1883	2087	2330	2616
Max	71048	61802	53654	46502	40245	34882
Sd	5315	4947	4686	4551	4568	4768
VAR 10%	3892	4203	4506	4784	5039	5331
Av bottom 10%	3135	3435	3745	4063	4331	4511
D90	14905	14758	14985	15276	15768	16584
<than bonds	11.3%	8.4%	6.1%	3.9%	2.3%	2.1%

Table 7.9: Results from Glide-through strategies for Simulation 1, part 2/2

Strategies	100/0U	80/20U	60/40U	40/60U	20/80U	0/100U
Mean withdrawal	6711	7593	8640	9887	11380	13179
Median	5584	6508	7655	8909	10264	11589
Min	1214	1592	1843	2125	2438	2785
Max	48508	49443	49908	49891	49394	70465
Sd	4136	4197	4407	4878	5761	7248
VAR 10%	2899	3608	4310	4973	5517	6003
Av bottom 10%	2308	2926	3586	4214	4712	5028
D90	11847	12776	14165	16288	18852	21965
<than bonds	26.9%	15.7%	7.5%	3.4%	1.3%	1.4%

For the glide-through strategies, we find similar results as we found for glide-to strategies. The best performing strategies from Simulation I still perform best, but the spreads have increased for worst performers and decreased for top performers. The probabilities of generating worse return than pure bond investments have decreased for all strategies, but significantly more for the best performing strategies. The poorly performing 100/0 U-shaped strategy dropped by 10.92%, while the well performing 0/100 U-shaped strategy dropped by 67.44%. The 20/80 performed even better on this measure with a drop of 77.96%. If we look at the absolute terms, we can tell that most of the more attractive strategies have a diminishing probability of performing worse than the a full bond investment, meaning that a risky investment is almost guaranteed a higher return.

All mean withdrawals and medians have dropped, meaning that a higher defined contribution rate is necessary for these strategies in order to generate the same monthly withdrawals as under the defined benefit scheme. This can be seen in table 7.10 below. Only at a defined contribution rate of 7% all strategies obtain a mean withdrawal and median higher than under the defined benefit scheme and none of the strategies obtain this under 2% and 3% defined contribution rates.

Table 7.10: Sensitivity analysis of DC rates compared to DB

	2%	3%	4%	5%	6%	7%
0/100	-	-	**	**	**	**
20/80	-	-	*	**	**	**
40/60	-	-	*	**	**	**
60/40	-	-	*	**	**	**
80/20	-	-	*	**	**	**
100/0	-	-	-	**	**	**
0/100 U	-	*	**	**	**	**
20/80 U	-	-	**	**	**	**
40/60 U	-	-	**	**	**	**
60/40 U	-	-	-	**	**	**
80/20 U	-	-	-	*	**	**
100/0 U	-	-	-	-	*	**

− Mean withdrawals & median <17 557
∗ Mean withdrawal >17 557 & median <17 557
∗∗ Mean withdrawal & median >17 557

7.4.1 Success Ratios

We will in this section do a similar analysis of the success ratio and the success-to-variability ratio as we did in section 7.3.1.

Glide-to

Figure 7.10: SR

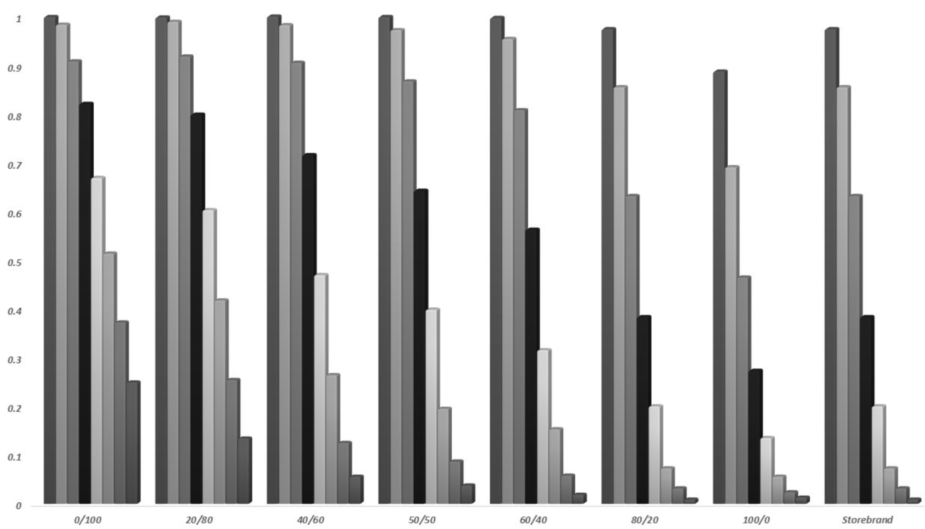
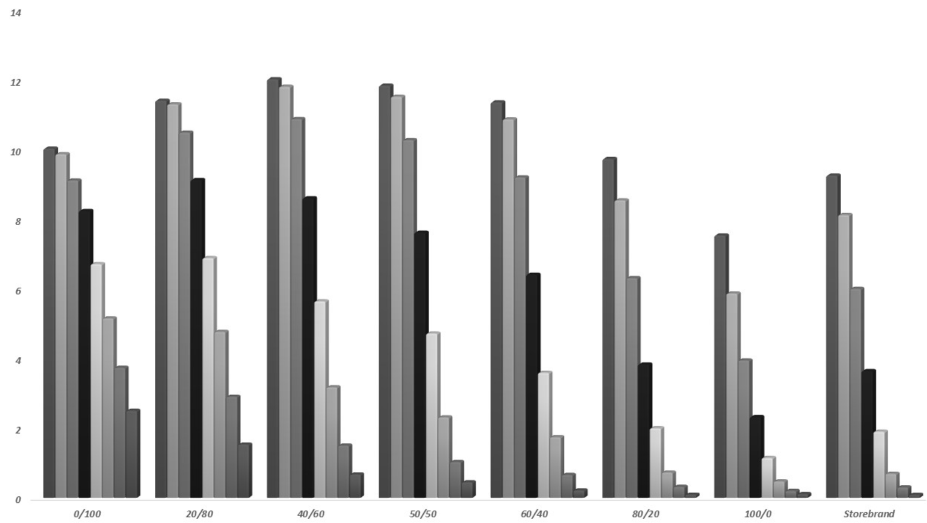


Figure 7.11: SV



We note an important difference in the SR from Simulation I. None of the strategies now offer a guarantee for a withdrawal rate even at the 3% rate, even if they are close. In other words, the less attractive strategies no longer offer the attractive option of a guaranteed sum each month, removing their only advantage over the higher performing strategies.

The 0/100 strategy become relatively even more attractive because the SR remains almost unchanged using Simulation II data, while the success ratios for all other strategies drop. In some strategies we can see the drop already at a 4% withdrawal rate, but most strategies experience a significant drop at the 5% rate compared to SR from Simluation I. In other words, the probability of being able to have a withdrawal rate of 5% or higher have drastically decreased.

Looking at SV, we can observe the same again. The strategies that performed best in terms of SR do not perform best under SV. Strategies with lower risk, like the 40/60 and 50/50 strategy provide is with the highest SV. This is a natural result to observe since strategies with higher risk is "punished" for their higher standard deviation under SV.

Glide-through

Figure 7.12: SR

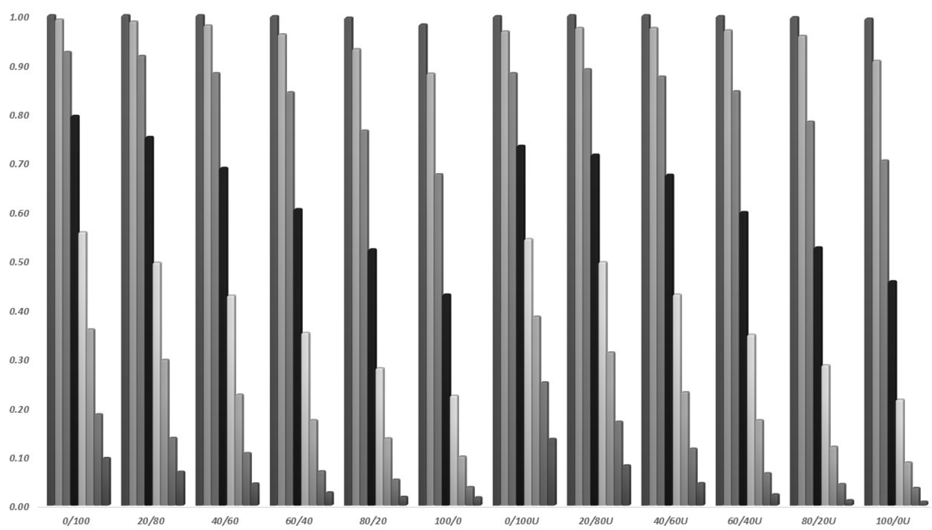
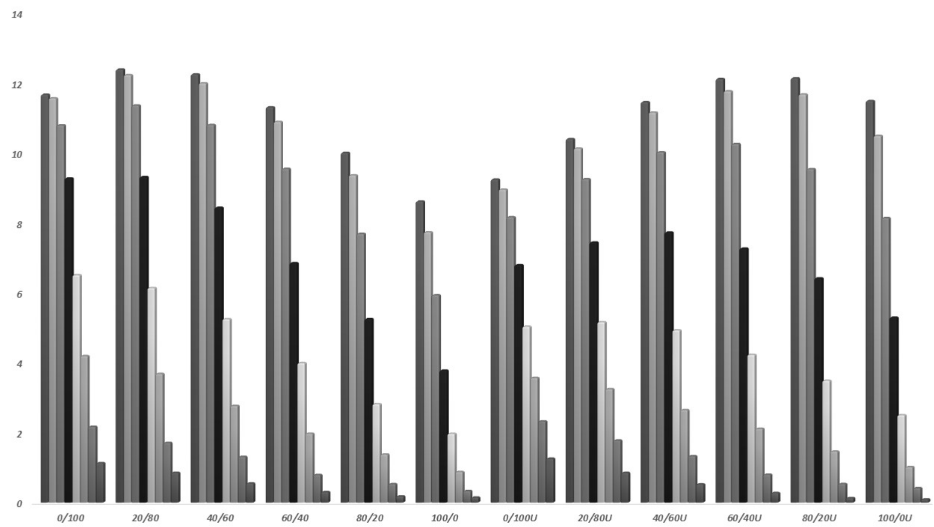


Figure 7.13: SV



Also for the glide-through strategies none of the strategies do not longer guarantee a withdrawal rate even at 3%.

While the 0/100 and the 0/100U shaped strategies had almost identical success ratios at all withdrawal rates using the Simulation I data, the 0/100 strategy now has a 4.3% higher success rate at the 5% withdrawal rate and a 6.1% higher success rate at the 6% withdrawal rate.This means that for those who choose to be risk tolerant to a certain level, the 0/100 strategy has become increasingly attractive for these individuals. On the contrary, for those individuals willing to take on higher risk, the probability of success for high withdrawal rates has been increasing for the 0/100 U-shaped strategy compared to the 0/100 strategy. Depending on the individuals risk preferences, these changes may impact their attraction to different portfolios.

For SV the results are the same as previously discussed. SV deliver higher ratios on strategies with lower risk.

Chapter 8

Discussion

In this section, we will make some comments and discuss the results from our investment strategy testing. We will try to make some arguments combining the portfolio results with our knowledge of pension saving, including both individual differences and potential social issues. We will start by discussing the different investment strategies and what determines which strategy seems more attractive. Thereafter, we will use our results to compare the defined benefit and the defined contribution scheme, and make some comments about sustainability and what we believe the future may look like.

8.1 Investment Strategy Decision

Our results provided us some clear insights; the glide-to strategy moving from zero percent stock allocation to a hundred percent stock allocation at retirement performed the highest average monthly withdrawals, the highest median value and the highest minimum value. The second best strategy, the U-shaped 0-100 glide-through performed the second best on all the same measures. Does this mean that all LICs invest our pension savings wrong today?

The results are clearly not as black and white as they look. We have previously argued for both the limitations of our model in terms of controlling for shocks in our simulation data. Secondly, we have argued that other factors are important determinants when we pick our pension saving strategy, such as a secure and stable income when we retire. If we start by looking away from the limitation in our model, both our results and the results of Estrada [Estrada, 2013] suggest that strategies with the highest stock allocation when the pension holdings is at its largest, also performs the best minimum values. This means that these strategies not only perform better when the market are in their favor, but also when the market is slow. In other words, it is even more risky to invest in the conservative stock-reducing strategies than the stock increasing strategies. If both historical back-testing and forecasting suggest the same, would we then expect these results to be guiding for future pension saving strategies?

What makes pension saving extremely difficult is the uncertainty about future market movements. In general, there are no easily recognizable patterns that can be used to predict future changes in the marked conditions. Estrada tested a period of approximately 100 years and found results favoring a mirroring strategy rather than the traditional investment strategy. We found similar results, but our results were also based on the same data when making our simulations. The possibility of a complete change in market conditions exist, so to recommend a pension saving strategy 65 years ahead seems bold. Our results are aligned with those of Estrada [Estrada, 2013], suggesting that at least a higher proportion of the pension holdings should be invested in stocks to a later stage than today.

The major concern with increasing the stock allocation at a later stage in your saving period is the possibility of a major fall in the stock market. Imagining the pension portfolio dropping by 20% just before retirement will not be a problem for the individual only, but for the entire society who will experience a generation of poor pensioners. For that generation who are maximally unlucky, the consequences will be enormous. The argument of cumulative returns discredit some of this concern, because even though the drop in the last period may be huge, they excess return generated over many years more than compensate the drop, compared to the low risk investments.

Norway offers all Norwegian citizens a minimum security with the public retirement pension. We can expect that individuals in Norway have quite different opportunities, directly related to the reference point discussed in Prospect theory. Those who make good money throughout their career will maximize their public retirement pension and will most likely afford to take on a higher risk than those who have a low income. The same accounts for those who inherits great values.

The main issue of being a pension saving individual under the defined contribution scheme is that today's market conditions suggest that taking on low risk with their pension savings seems equivalent to have a negative real return. In other words, choosing a pension saving strategy with higher risk is necessary to maintain the pension level most individuals compare to, namely the defined benefit scheme. This leads us to our next discussion, the comparison and sustainability of the defined benefit scheme.

8.2 A Comparison of the Pension Schemes

The defined benefit pension saving have been the most prevalent occupational pension saving in Norway since pension savings were first introduced. After 2006, all Norwegian workers have been legally entitled to receive occupational pensions after pressure from workers in the private sector to improve their pensions. All public employees in Norway still have the defined benefit scheme, but 2014 was the first year where there were just as many pension savers using the defined contribution scheme as the defined benefit scheme in Norway[Finans Norge, 2014, d]. An interesting discussion is how these two different saving schemes are compared with each other, and whether they *should* be substitutes.

In our result section, we could see that for a person with an initial income of NOK 350 000 and a real wage increase of 1% yearly, a defined contribution rate of 2% was not sufficient in any of the strategies to generate minimum a pension withdrawals cash equivalent to the defined benefit scheme for the with 50% certainty . At a 3% level, it was sufficient for the most risky strategies such as the 0/100 and the 20/80 strategy, but not for the empirical strategy from Storebrand. These results are alarming, because about 60% of the Norwegian defined contribution pension savers have a defined contribution rate of between two and three percent and secondly.

The defined benefit scheme is riskless for the individual saver, meaning that it would be reasonable with a higher compensation for those saving in a defined contribution scheme for taking on the additional risk. Norwegian companies have been released from the risk they used to carry, and most of them have offered pension plans that does not compare well with the defined benefit scheme. On the contrary, for those who actually offer a defined contribution rate of 7%, they will match on almost all investment strategies, even the conservative ones. It is difficult to make a statement on how much each individual should be compensated for the risk they have to take under the defined contribution scheme. The government made changes in the defined contribution scheme in 2006 to improve the conditions for pension savers, and we believe our results suggest that the discussion around the terms of this pension saving scheme is not finished.

We believe that comparing these two pension schemes may not be reasonable in the sense that a defined benefit scheme seems *too good* today. Growing pension liabilities in the public sector is a major concern in Norway today and financial newspapers argue that changes needs to be made [Dagens

Næringsliv, 2013]. The defined benefit scheme the way it is today is may be floating on borrowed time, and a direct comparison may therefore not be particularly fruitful. After all, it is important to keep in mind that the alternative of having a defined contribution pension scheme is better than the alternative of having no occupational pension at all.

We believe increased responsibility over your own pension has come to stay for Norwegians, which now have to take more control over their own pension savings. While this may have positive consequences for the general reflection around individuals overall wealth, it does not change the fact that our results present some concerns on the size of pension payments for our generation. Given our assumptions, the recommended portfolio of Storebrand will not take on sufficient risk for a pension close to the $\frac{2}{3}$ target with the defined contribution rates most savers have today. In our opinion we believe there are three ways to control for these concerns

1. *Increase the defined contribution rates*
2. *Increase the risk in the investment strategies of the pension funds*
3. *Increase private savings*

We believe a solution might be a result of all these three factors, but leave it up for discussion to find the optimal trade-off.

Chapter 9

Conclusion

Throughout this paper, we have investigated how investment strategies proposed in academic literature performs against investment strategies used by pension funds today. Our main research question was whether the investment strategy used by large pension funds today was the best performing strategy when testing for forecasted data for a period of 65 years. Our study showed that traditional pension saving strategies that reduces the stock proportion when age increases performed worse than their mirroring strategies in all terms. We have made comments about the limitations of the forecasts, because we are aware of their high uncertainty. However, we believe that are forecasts are robust and our results suggest that alternative investment strategies should be considered if the economic recession remains for a period of time. Some strategies performed worse than investing in bonds only in more than 40% of our simulations while containing way higher risk. The traditional pension saving strategy seemed to have bigger downside potential and limited upside potential, making them less attractive for investors.

We have also compared the defined contribution scheme with the defined benefit scheme in order to investigate how pension savers within each pension scheme stands against each other. We have argued that with the low defined contribution rates of most pension savers under the defined contribution scheme, these pension savers are not compensated sufficiently to call the defined contribution a fair substitute for the defined benefit. In fact, with our calculations, most pension savers under the defined contribution scheme can expect to perform worse or equal at best if they invest in the most risky investment strategies. We have mentioned three ways to balance these two pension schemes in terms of cash payments; increasing the defined contribution rates, increase the risk of the investment strategies or increase the individual pension savings.

Our results suggest suboptimal pension saving strategies and a misalignment between the two pension schemes existing in Norway today. Both our forecasting and other academics historical back-testing are only curiosities, because they offer no certainty about the future, but serves as important critics to pension funds that manage the future of millions of people. We believe it could be interesting for future research to look into the CPPI strategy to test this against our strategies. Strategies that serves to minimize downside risk while maintaining upside potential seems like an attractive option for most investors, but especially for pension savers which their life depend on not losing their pension capital.

Chapter 10

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Chapter 11

Appendix

Demeaning variables

Where the beta of the variable is unaffected even after we have subtracted the mean, while the intercept coefficient now has changed its interpretation. However, we are simply interested in the coefficient on each variable and how they predict the future of our Y. Because we centered variables in the bond model, it did not make sense to keep the intercept in the model, and therefore we dropped it.

$$Y = \alpha + \beta(X - \bar{X}) \tag{11.1}$$

$$Y = \alpha - \beta\bar{X} + \beta X \tag{11.2}$$

$$Y = \alpha^{*} + \beta X \tag{11.3}$$

$$\alpha^* = \alpha - \beta\bar{X} \tag{11.4}$$

Cholesky Decomposition

By running a VAR model, we were able to generate forecasts based on the coefficients retrieved from the regression on historical data that we treated as "true". We wanted to create 1000 different forecast periods in order to draw on a large sample in our investment strategy testing, and to get different results, we needed to simulate a random component ϵ that varied in each simulation and that used the variance of the residuals to create a realistic variation.

A general way to generate correlated random numbers with a given covariance matrix C, is done by finding a matrix u such that

$$U^T U = C \tag{11.5}$$

By finding the u-matrix, we can use the formula

$$R_C = RU \tag{11.6}$$

Where R is a vector of random normally distributed numbers and R_c equals the random correlated numbers we want to use as ϵ in our regression.

The Cholesky decomposition is used to find the u-matrix out of the covariance matrix of the residuals generated in the previous regression. From the covariance matrix

$$\begin{bmatrix} Cov_{A,A} & Cov_{A,B} & Cov_{A,C} \\ Cov_{B,A} & Cov_{B,B} & CovB, C \\ Cov_{C,A} & Cov_{C,B} & CovC, C \end{bmatrix} \tag{11.7}$$

We can use equation 11.8 and 11.9

$$Iki = \frac{Aki - \sum_{j=1}^{n} -1 \times Ikilkj}{Iii} \tag{11.8}$$

$$Ikk = \sqrt{akk - \sum_{j=1}^{k} -1 I^{2k}} \tag{11.9}$$

For k'th row and i'th column to compute the triangular Cholesky matrix. The Cholesky matrix is defined as the matrix that you can multiply with its own transpose in order to generate the original matrix, following from equation 11.5. Having designed the residuals, we could now finish the equation for the bond forecast with the equation

$$Bond_{t+1} = \alpha + \beta_1 + \beta_2 + \cdots + \beta_k + \epsilon \tag{11.10}$$

Monte Carlo Simulations

Monte Carlo simulation is a technique used to assess the impact of volatility and risk in financial forecasting models by generating random numbers under a chosen distribution. When developing a forecasting model using the Monte Carlo method, you have to make certain assumption about parameters in the model. When forecasting movements in variables, you typically look for the distributions of the numbers, the mean of the numbers and their volatility measured in standard deviation. Because the predictions you make are into the future, the best you can do is to estimate these inputs using historical data and expertise. We have assumed normal distribution of all our variables, so that we have computed our random numbers with a N (0; 1) distribution.

When using the Monte Carlo simulation, you can calculate thousands of paths, each time using different random numbers. When the simulation is complete, you have a large number of results based on random input variables that you can use to describe the probability of reaching various results in the model.

Defined contribution rates - statistics

The following data were recieved from Finans Norge on Defined Contribution rates. Companies are gradually changing towards the new rates, which will be mandatory from 01.01.2017.

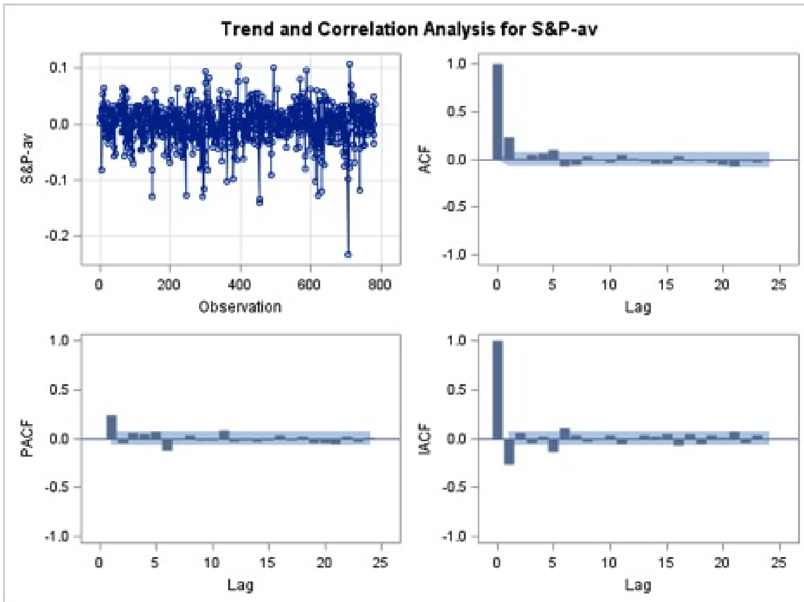
Table 11.1: Statistics - old contribution rates

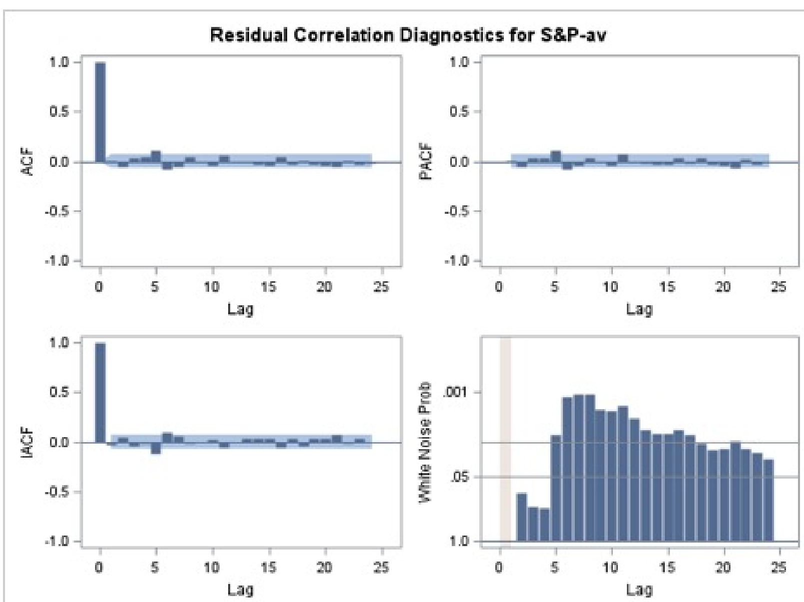
DCR	# of pensions	In percent
[2%]	462 610	47%
[2%, 5%]	244 140	25%
[5%]	268 336	28%

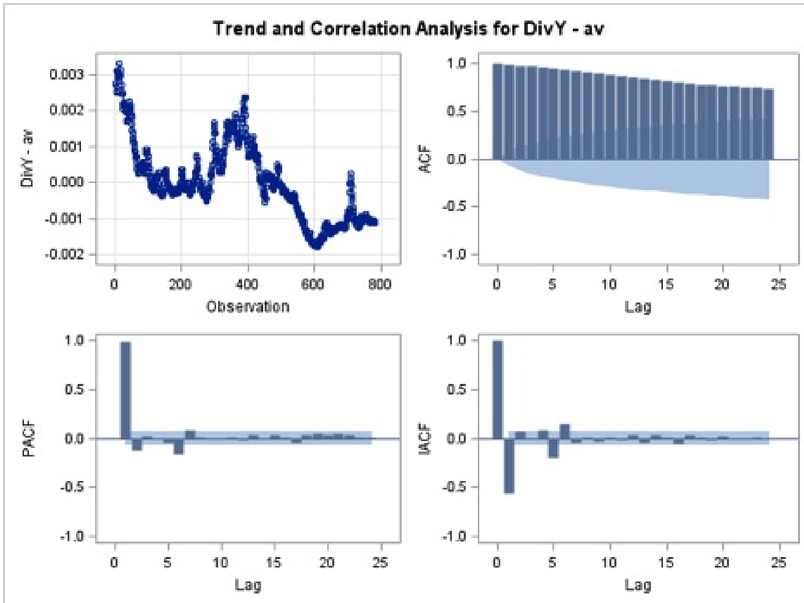
Table 11.2: Statistics - new contribution rates

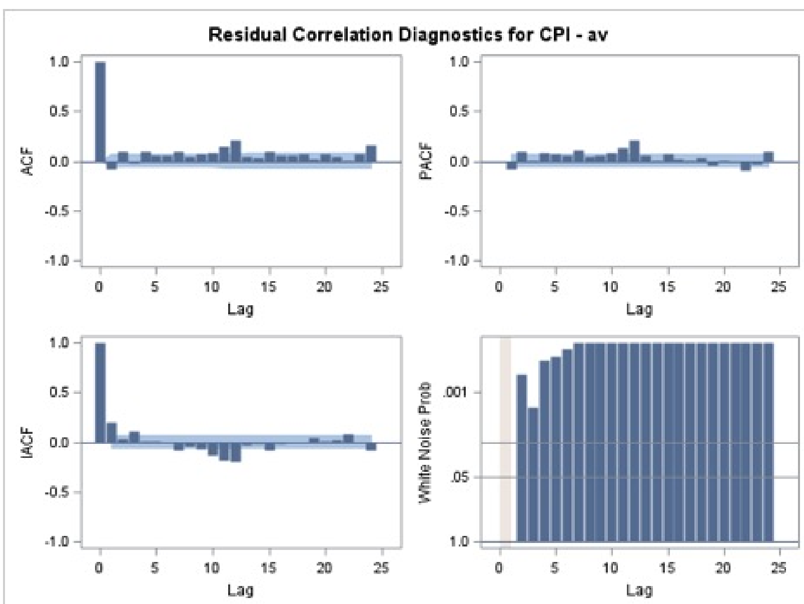
DCR	# of people	In percent
[2%]	105 831	47%
[2%, 5%]	45 758	20%
[5%]	49 885	22%
[5%, 7%]	10 047	4%
[7%]	11 884	5%

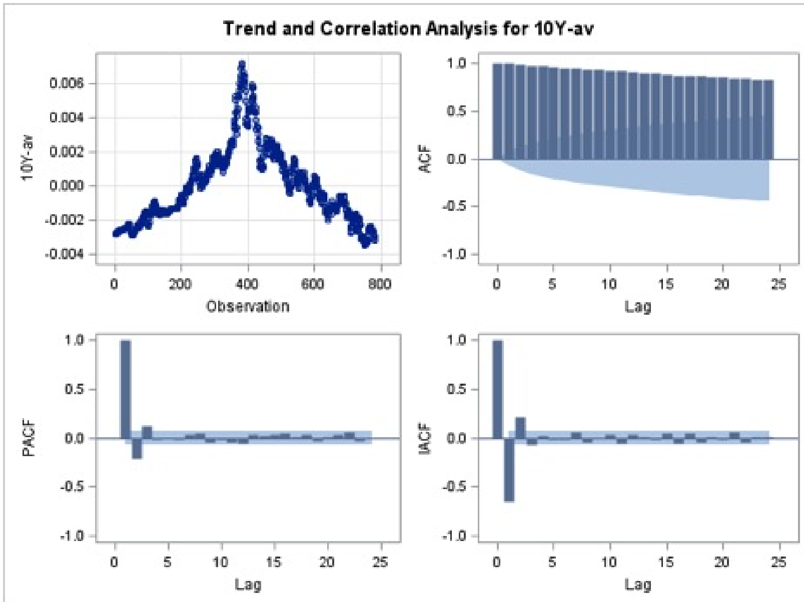
Econometric testing

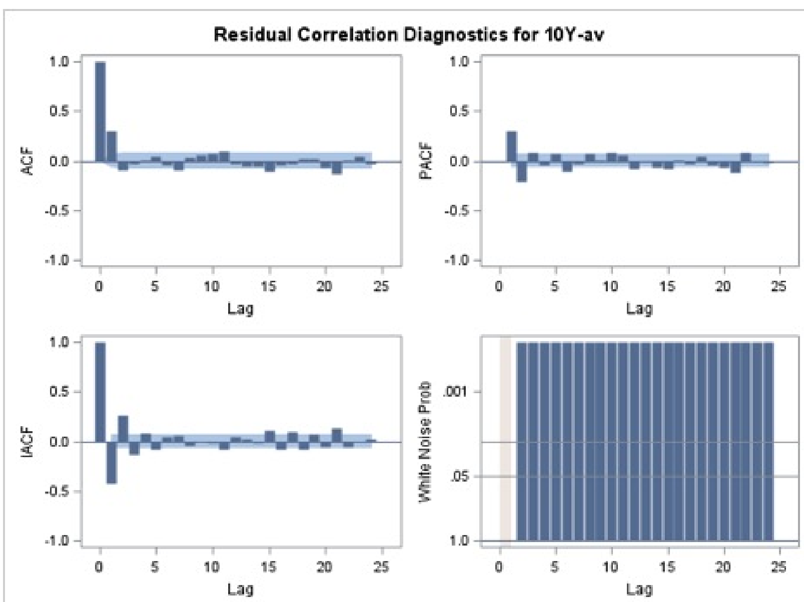


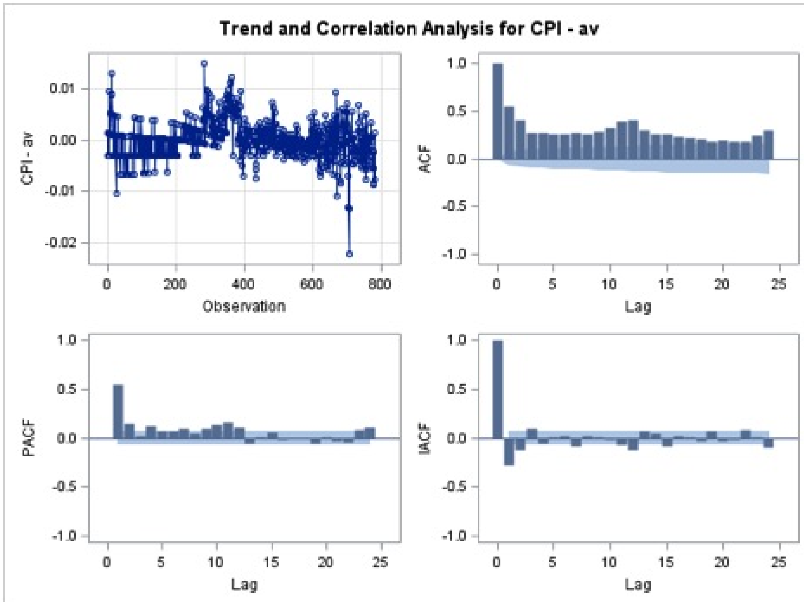


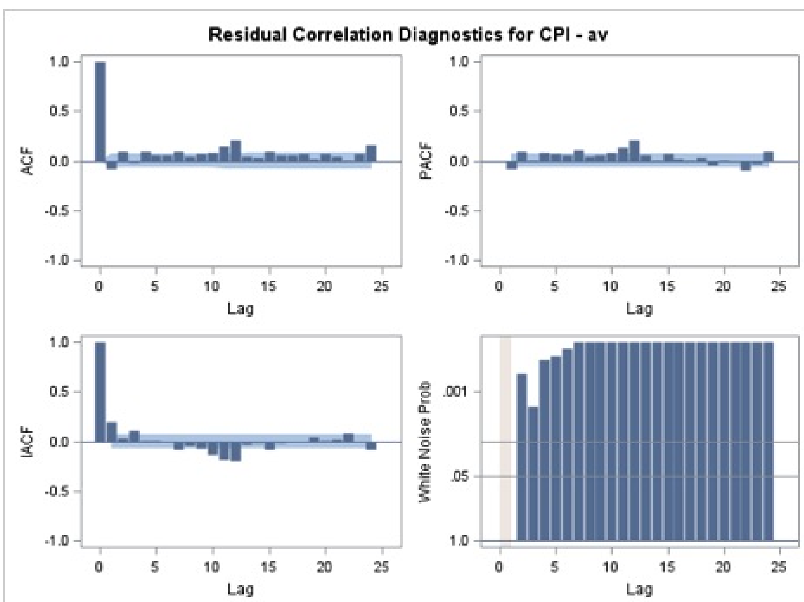

Trend and Correlation Analysis for DivY - av
DivY - av
0.003
0.002
0.001
0.000
-0.001
-0.002
0
200
400
600
800
Observation
ACF
PACF
IACF
1.0
0.5
0.0
-0.5
-1.0
0
5
10
15
20
25
Lag

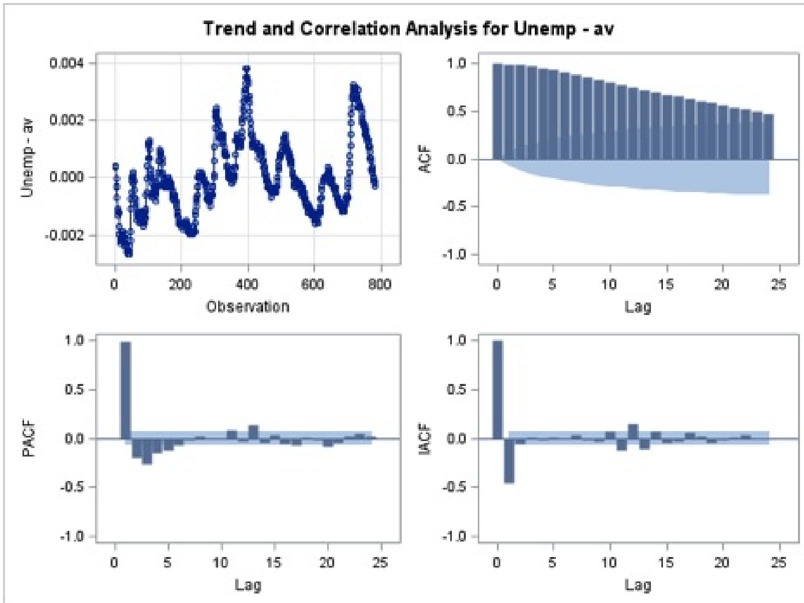

Residual Correlation Diagnostics for CPI - av
ACF
PACF
IACF
White Noise Prob
1.0
0.5
0.0
-0.5
-1.0
.001
.05
0
5
10
15
20
25
Lag


Trend and Correlation Analysis for 10Y-av
10Y-av
Observation
ACF
PACF
IACF
Lag

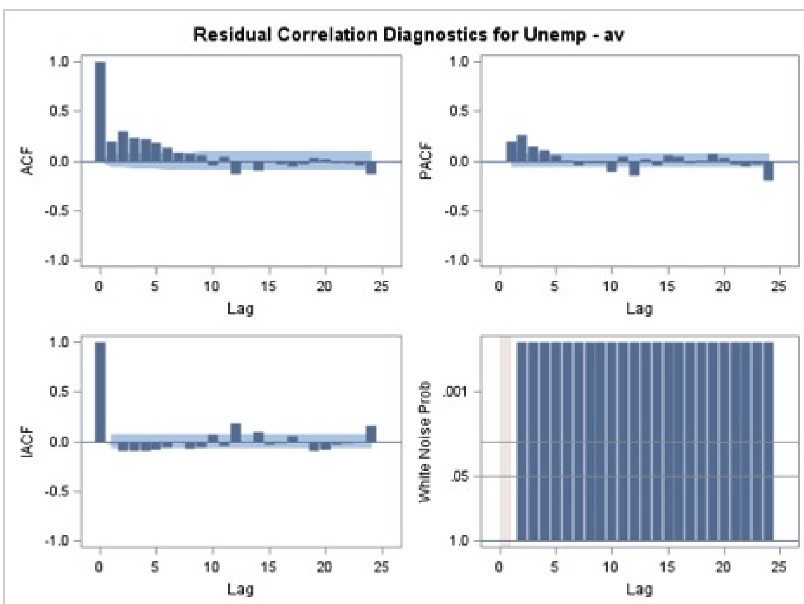

Residual Correlation Diagnostics for 10Y-av
ACF
PACF
IACF
White Noise Prob
Lag


Trend and Correlation Analysis for CPI - av
CPI - av
Observation
ACF
Lag
PACF
Lag
IACF
Lag


Residual Correlation Diagnostics for CPI - av
ACF
Lag
PACF
Lag
IACF
Lag
White Noise Prob
Lag

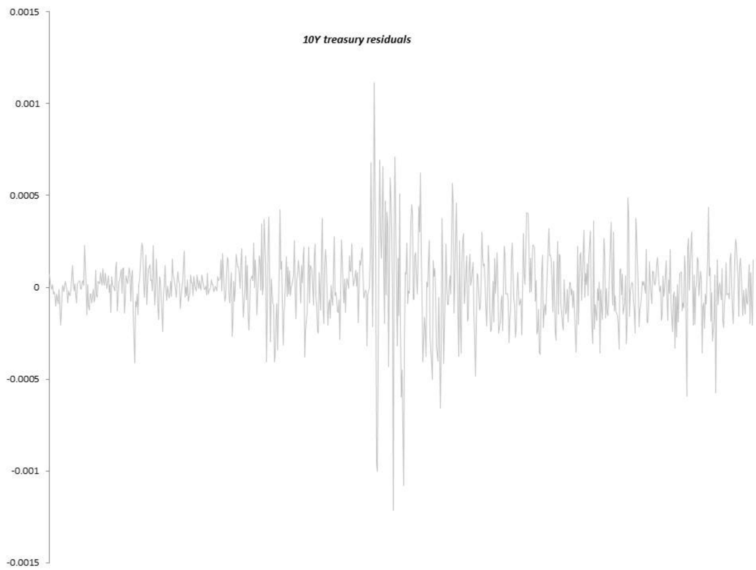


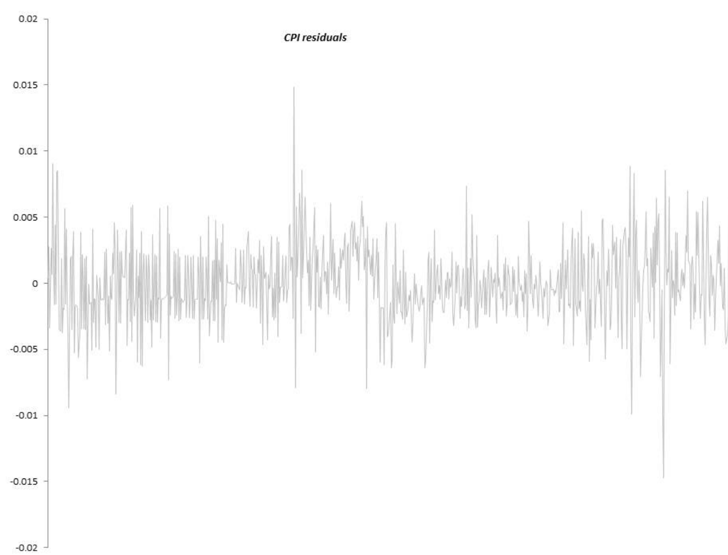
Trend and Correlation Analysis for Unemp - av
Unemp - av
0.004
0.002
0.000
-0.002
0
200
400
600
800
Observation
ACF
PACF
IACF
1.0
0.5
0.0
-0.5
-1.0
0
5
10
15
20
25
Lag

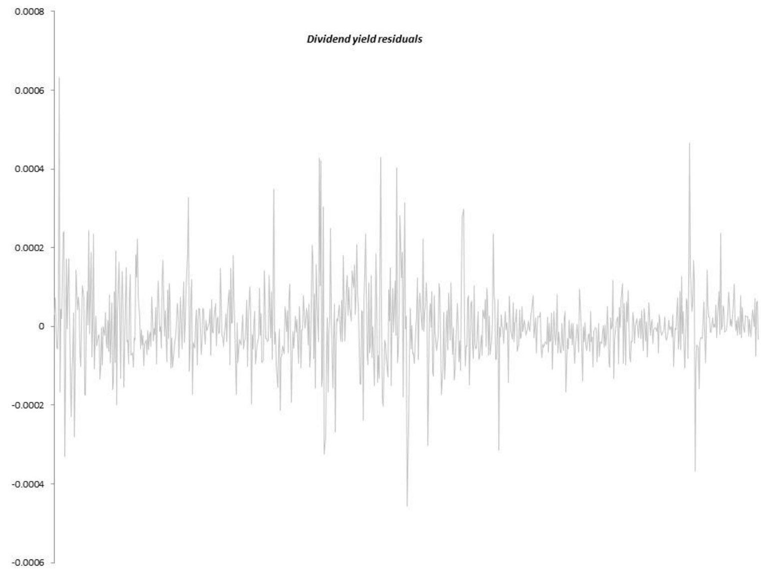


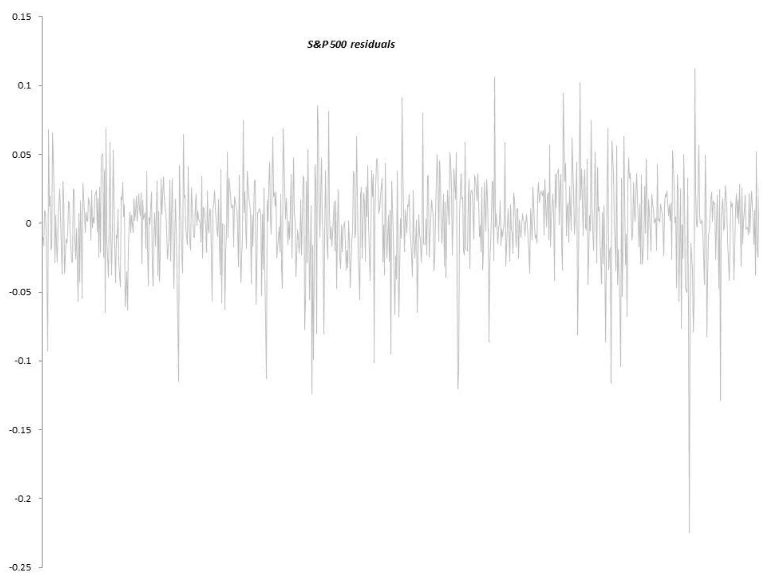
Residual Correlation Diagnostics for Unemp - av
ACF
PACF
IACF
White Noise Prob
1.0
0.5
0.0
-0.5
-1.0
.001
.05
1.0
0
5
10
15
20
25
Lag

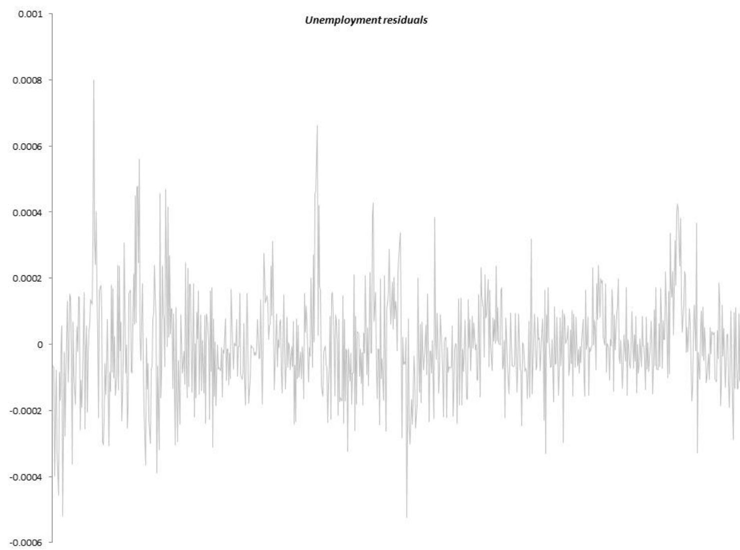
Residual plots






Dividend yield residuals
0.0008
0.0006
0.0004
0.0002
0
-0.0002
-0.0004
-0.0006


S&P 500 residuals
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Unemployment residuals
0.001
0.0008
0.0006
0.0004
0.0002
0
-0.0002
-0.0004
-0.0006

Simulation I - Glide-to with DCR from 3%-7%

Table 11.3: Results from Glide-to strategies for Simulation I - 3% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Stb
Mean	26994	23173	19778	18234	16789	14183	11929	16109
Median	22906	19902	17117	15663	14479	12196	10160	13811
Min	4455	4343	4183	4058	3560	2676	1864	3299
Max	148100	118586	98662	89586	81108	65921	54437	67733
Sd	15821	12787	10652	9835	9144	8045	7198	8790
Low 10%	12680	11096	9679	8863	8005	6445	5166	7667
Low 25%	16451	14522	12574	11555	10422	8484	6900	9892
Av D10	10091	9077	7904	7266	6614	5379	4238	6309
Av Q25	12769	11378	9796	8982	8187	6686	5349	7789
D90	46593	38949	33123	30325	27874	24255	21032	27134
Riskfree mean	9075	9075	9075	9075	9075	9075	9075	9075
LessThanRF	0.022	0.043	0.078	0.115	0.159	0.289	0.422	0.189

∗ Note: Storebrand's strategy

Table 11.4: Results from Glide-to strategies for Simulation I - 4% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	35992	30897	26370	24312	22385	18910	15905	21479
Median	30542	26536	22822	20884	19306	16261	13547	18415
Min	5940	5791	5577	5411	4746	3568	2486	4399
Max	197466	158114	131549	119448	108144	87894	72582	90310
Sd	21094	17049	14203	13113	12192	10726	9597	11720
Low 10%	16907	14795	12906	11817	10674	8594	6888	10222
Low 25%	21934	19363	16766	15406	13896	11312	9199	13189
Av D10	13455	12103	10539	9687	8818	7172	5651	8411
Av Q25	17025	15171	13061	11976	10916	8915	7132	10385
D90	62124	51932	44164	40434	37166	32340	28042	36179
Riskfree mean	12100	12100	12100	12100	12100	12100	12100	12100
LessThanRF	0.022	0.043	0.078	0.115	0.159	0.289	0.422	0.189

∗ Note: Storebrand's strategy

Table 11.5: Results from Glide-to strategies for Simulation I - 5% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	44990	38622	32963	30390	27982	23638	19881	26849
Median	38177	33170	28528	26105	24132	20327	16933	23019
Min	7425	7238	6971	6764	5933	4459	3107	5499
Max	246833	197643	164437	149310	135180	109868	90728	112888
Sd	26368	21311	17754	16391	15240	13408	11997	14650
Low 10%	21133	18494	16132	14771	13342	10742	8610	12778
Low 25%	27418	24204	20957	19258	17370	14140	11499	16486
Av D10	16818	15128	13173	12109	11023	8965	7064	10514
Av Q25	21281	18963	16326	14970	13645	11144	8915	12981
D90	77655	64915	55205	50542	46457	40426	35053	45224
Riskfree mean	15125	15125	15125	15125	15125	15125	15125	15125
LessThanRF	0.022	0.043	0.078	0.115	0.159	0.289	0.422	0.189

∗ Note: Storebrand's strategy

Table 11.6: Results from Glide-to strategies for Simulation I - 6% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	53988	46346	39555	36468	33578	28365	23857	32219
Median	45813	39804	34233	31325	28959	24392	20320	27623
Min	8910	8686	8365	8117	7119	5351	3729	6598
Max	296200	237171	197324	179173	162216	131841	108873	135465
Sd	31641	25573	21305	19670	18287	16089	14396	17580
Low 10%	25360	22192	19358	17726	16011	12890	10332	15334
Low 25%	32901	29045	25148	23109	20844	16968	13799	19784
Av D10	20182	18154	15808	14531	13227	10758	8477	12617
Av Q25	25537	22756	19592	17965	16374	13373	10697	15578
D90	93186	77898	66246	60650	55749	48511	42063	54269
Riskfree mean	18150	18150	18150	18150	18150	18150	18150	18150
LessThanRF	0.022	0.043	0.078	0.115	0.159	0.289	0.422	0.189

∗ Note: Storebrand's strategy

Table 11.7: Results from Glide-to strategies for Simulation I - 7% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	62987	54071	46148	42546	39174	33093	27834	37588
Median	53448	46438	39939	36546	33785	28457	23707	32227
Min	10395	10133	9759	9469	8306	6243	4350	7698
Max	345566	276700	230211	209035	189251	153815	127019	158043
Sd	36915	29836	24856	22948	21335	18771	16795	20510
Low 10%	29587	25891	22585	20680	18679	15039	12054	17889
Low 25%	38385	33886	29340	26961	24318	19796	16099	23081
Av D10	23546	21180	18443	16953	15432	12551	9890	14720
Av Q25	29793	26549	22857	20959	19103	15602	12480	18174
D90	108717	90881	77287	70759	65040	56596	49074	63313
Riskfree mean	21175	21175	21175	21175	21175	21175	21175	21175
LessThanRF	0.022	0.043	0.078	0.115	0.159	0.289	0.422	0.189

∗ Note: Storebrand's strategy

Simulation I - Glide-through with DCR from 3%-7%

Table 11.8: Simulation I, glide through with 3% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	17087	17538	17999	18470	18948	19431
Median	14915	15156	15477	15883	16407	16692
Min	3694	3844	4006	3910	3586	3297
Max	74865	80509	86480	92771	99371	106259
Sd	9076	9315	9638	10057	10587	11242
Low 10%	8337	8507	8695	8979	8937	8860
Low 25%	10633	10993	11304	11693	11876	11854
Av D10	6907	7086	7221	7290	7302	7256
Av Q25	8446	8697	8901	9050	9141	9158
D90	28479	29093	29956	31006	31860	33690
Riskfree mean	9075	9075	9075	9075	9075	9075
LessThanRF	0.147	0.135	0.119	0.107	0.108	0.107

∗ Note: Storebrand's strategy

Table 11.9: Simulation I, glide through with 3% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	13906	15527	17294	19213	21291	23532
Median	11630	13198	14905	16512	18445	20163
Min	2616	3151	3758	4111	4197	4254
Max	64727	74094	84235	95116	106681	118859
Sd	8296	8797	9442	10284	11383	12801
Low 10%	6102	7119	8243	9402	10505	11401
Low 25%	8220	9436	10833	12197	13561	14866
Av D10	4965	5891	6809	7710	8513	9188
Av Q25	6287	7337	8433	9530	10586	11563
D90	24037	26195	28612	32385	36344	39919
Riskfree mean	9075	9075	9075	9075	9075	9075
LessThanRF	0.302	0.23	0.152	0.088	0.059	0.043

∗ Note: Storebrand's strategy

Table 11.10: Simulation I, glide through with 4% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	22782	23383	23999	24627	25264	25907
Median	19887	20209	20635	21178	21876	22256
Min	4925	5126	5342	5213	4781	4396
Max	99820	107345	115307	123695	132494	141678
Sd	12102	12420	12851	13410	14116	14990
Low 10%	11116	11342	11594	11973	11916	11813
Low 25%	14177	14658	15072	15591	15835	15805
Av D10	9209	9448	9629	9721	9736	9675
Av Q25	11261	11595	11868	12066	12188	12210
D90	37972	38790	39941	41341	42480	44920
Riskfree mean	12100	12100	12100	12100	12100	12100
LessThanRF	0.147	0.135	0.119	0.107	0.108	0.107

∗ Note: Storebrand's strategy

Table 11.11: Simulation I, glide through with 4% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	18541	20703	23058	25617	28388	31376
Median	15506	17597	19873	22016	24593	26884
Min	3488	4201	5011	5482	5596	5672
Max	86302	98792	112314	126821	142242	158479
Sd	11061	11729	12589	13712	15177	17068
Low 10%	8136	9493	10990	12537	14007	15201
Low 25%	10960	12581	14444	16263	18082	19821
Av D10	6620	7855	9078	10281	11350	12250
Av Q25	8383	9783	11244	12707	14114	15418
D90	32049	34927	38149	43180	48458	53226
Riskfree mean	12100	12100	12100	12100	12100	12100
LessThanRF	0.302	0.23	0.152	0.088	0.059	0.043

∗ Note: Storebrand's strategy

Table 11.12: Simulation I, glide through with 5% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	28478	29229	29999	30784	31581	32384
Median	24859	25261	25794	26472	27345	27820
Min	6156	6407	6677	6516	5976	5495
Max	124775	134182	144133	154619	165618	177098
Sd	15127	15525	16063	16762	17645	18737
Low 10%	13895	14178	14492	14966	14895	14766
Low 25%	17721	18322	18841	19488	19794	19756
Av D10	11512	11810	12036	12151	12171	12093
Av Q25	14076	14494	14835	15083	15234	15263
D90	47466	48488	49926	51676	53100	56150
Riskfree mean	15125	15125	15125	15125	15125	15125
LessThanRF	0.147	0.135	0.119	0.107	0.108	0.107

∗ Note: Storebrand's strategy

Table 11.13: Simulation I, glide through with 5% DCR part 2/2

Strategies	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	23177	25878	28823	32022	35485	39220
Median	19383	21996	24842	27519	30741	33605
Min	4361	5251	6263	6852	6995	7090
Max	107878	123490	140392	158527	177802	198099
Sd	13826	14662	15737	17140	18971	21335
Low 10%	10170	11866	13738	15671	17509	19002
Low 25%	13700	15726	18055	20329	22602	24776
Av D10	8275	9819	11348	12851	14188	15313
Av Q25	10479	12229	14055	15884	17643	19272
D90	40062	43658	47687	53974	60573	66532
Riskfree mean	15125	15125	15125	15125	15125	15125
LessThanRF	0.302	0.23	0.152	0.088	0.059	0.043

∗ Note: Storebrand's strategy

Table 11.14: Simulation I, glide through with 6% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	34173	35075	35999	36941	37897	38861
Median	29831	30313	30953	31767	32814	33384
Min	7387	7689	8013	7819	7172	6594
Max	149730	161018	172960	185543	198742	212517
Sd	18152	18630	19276	20115	21174	22485
Low 10%	16674	17013	17390	17959	17874	17719
Low 25%	21265	21987	22609	23386	23753	23708
Av D10	13814	14171	14443	14581	14605	14512
Av Q25	16891	17393	17802	18099	18281	18315
D90	56959	58185	59911	62011	63720	67380
Riskfree mean	18150	18150	18150	18150	18150	18150
LessThanRF	0.147	0.135	0.119	0.107	0.108	0.107

∗ Note: Storebrand's strategy

Table 11.15: Simulation I, glide through with 6% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	27812	31054	34587	38426	42581	47064
Median	23259	26395	29810	33023	36890	40327
Min	5233	6302	7516	8223	8394	8507
Max	129454	148188	168471	190232	213363	237718
Sd	16591	17594	18884	20568	22765	25602
Low 10%	12203	14239	16485	18805	21010	22802
Low 25%	16440	18872	21665	24395	27123	29732
Av D10	9930	11783	13618	15421	17026	18375
Av Q25	12575	14675	16865	19061	21172	23127
D90	48074	52390	57224	64769	72688	79839
Riskfree mean	18150	18150	18150	18150	18150	18150
LessThanRF	0.302	0.23	0.152	0.088	0.059	0.043

∗ Note: Storebrand's strategy

Table 11.16: Simulation I, glide through with 7% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	39869	40921	41999	43098	44213	45338
Median	34802	35365	36112	37061	38283	38948
Min	8618	8970	9348	9123	8367	7693
Max	174685	187855	201787	216467	231865	247937
Sd	21178	21735	22489	23467	24703	26232
Low 10%	19453	19849	20289	20952	20853	20672
Low 25%	24809	25651	26377	27284	27712	27659
Av D10	16116	16533	16850	17011	17039	16931
Av Q25	19706	20292	20769	21116	21328	21368
D90	66452	67883	69896	72346	74340	78610
Riskfree mean	21175	21175	21175	21175	21175	21175
LessThanRF	0.147	0.135	0.119	0.107	0.108	0.107

∗ Note: Storebrand's strategy

Table 11.17: Simulation I, glide through with 7% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	32447	36229	40352	44830	49678	54908
Median	27136	30794	34778	38527	43038	47048
Min	6105	7352	8768	9593	9793	9925
Max	151029	172886	196549	221937	248923	277338
Sd	19357	20526	22031	23996	26559	29869
Low 10%	14237	16612	19233	21939	24512	26602
Low 25%	19180	22017	25276	28461	31643	34687
Av D10	11585	13747	15887	17991	19863	21438
Av Q25	14670	17120	19676	22237	24700	26981
D90	56087	61122	66761	75564	84802	93145
Riskfree mean	21175	21175	21175	21175	21175	21175
LessThanRF	0.302	0.23	0.152	0.088	0.059	0.043

∗ Note: Storebrand's strategy

Simulation II - Glide-to strategies with DCR from 3%-7%

Table 11.18: Glide to strategies for Simulation II - 3% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	23958.36	19105	15384	13854	12505	10260	8495	12205
Median	20545	16856	13858	12361	11007	8530	6820	10136
Min	5195.484	5096	3552	2970	2491	1772	1271	2138
Max	132724.7	72688	71750	74939	77906	83011	86772	112240
Sd	12992.66	9159	7360	6902	6619	6344	6240	8023
Low 10%	11899.78	9964	7890	6976	6099	4458	3210	5116
Low 25%	15487.77	12750	10064	8978	7958	6096	4614	7170
Av D10	9693.152	8458	6757	5864	5012	3581	2484	4101
Av Q25	12105.11	10210	8154	7176	6251	4628	3366	5350
D90	39766.31	31247	25010	22660	20615	17781	15300	21265
Riskfree mean	6040.791	6041	6041	6041	6041	6041	6041	6041
LessThanRF	0.003	0.008	0.022	0.046	0.097	0.239	0.424	0.159

∗ Note: Storebrand's strategy

Table 11.19: Glide to strategies for Simulation II - 4% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	31944	25474	20512	18471	16673	13679	11327	16274
Median	27393	22475	18477	16481	14676	11374	9093	13514
Min	6927	6794	4736	3961	3321	2362	1695	2851
Max	176966	96918	95667	99919	103875	110682	115696	149653
Sd	17324	12212	9813	9203	8825	8459	8320	10698
Low 10%	15866	13285	10520	9301	8132	5944	4279	6821
Low 25%	20650	17000	13418	11971	10611	8128	6152	9560
Av D10	12924	11277	9009	7818	6683	4775	3312	5468
Av Q25	16140	13613	10873	9568	8335	6171	4488	7134
D90	53022	41662	33346	30213	27486	23708	20400	28353
Riskfree mean	8054	8054	8054	8054	8054	8054	8054	8054
LessThanRF	0.003	0.008	0.022	0.046	0.097	0.239	0.424	0.159

∗ Note: Storebrand's strategy

Table 11.20: Glide to strategies for Simulation II - 5% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	39931	31842	25640	23089	20841	17099	14158	20342
Median	34242	28094	23096	20601	18345	14217	11366	16893
Min	8659	8493	5920	4951	4151	2953	2119	3563
Max	221208	121147	119584	124899	129844	138352	144620	187066
Sd	21654	15265	12266	11504	11032	10574	10401	13372
Low 10%	19833	16607	13150	11626	10166	7430	5349	8526
Low 25%	25813	21250	16773	14964	13264	10160	7689	11949
Av D10	16155	14097	11261	9773	8353	5969	4140	6835
Av Q25	20175	17016	13591	11960	10419	7714	5609	8917
D90	66277	52078	41683	37767	34358	29635	25500	35441
Riskfree mean	10068	10068	10068	10068	10068	10068	10068	10068
LessThanRF	0.003	0.008	0.022	0.046	0.097	0.239	0.424	0.159

∗ Note: Storebrand's strategy

Table 11.21: Glide to strategies for Simulation II - 6% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	47917	38211	30768	27707	25010	20519	16990	24410
Median	41090	33713	27715	24721	22014	17061	13639	20271
Min	10391	10192	7103	5941	4981	3543	2543	4276
Max	265449	145376	143501	149879	155813	166023	173544	224479
Sd	25985	18318	14719	13805	13238	12689	12481	16046
Low 10%	23800	19928	15780	13952	12199	8916	6419	10232
Low 25%	30976	25501	20128	17957	15917	12192	9227	14339
Av D10	19386	16916	13513	11728	10024	7163	4968	8202
Av Q25	24210	20419	16309	14352	12503	9257	6731	10701
D90	79533	62494	50019	45320	41230	35562	30600	42529
Riskfree mean	12082	12082	12082	12082	12082	12082	12082	12082
LessThanRF	0.003	0.008	0.022	0.046	0.097	0.239	0.424	0.159

∗ Note: Storebrand's strategy

Table 11.22: Glide to strategies for Simulation II - 7% DCR

	0/100	20/80	40/60	50/50	60/40	82/20	100/0	Storebrand
Mean	55903	44579	35896	32325	29178	23939	19822	28479
Median	47938	39331	32334	28841	25683	19904	15913	23650
Min	12123	11890	8287	6931	5811	4134	2966	4988
Max	309691	169606	167418	174858	181782	193693	202468	261893
Sd	30316	21371	17173	16106	15444	14804	14561	18721
Low 10%	27766	23249	18410	16277	14232	10402	7489	11937
Low 25%	36138	29751	23482	20950	18569	14224	10765	16729
Av D10	22617	19736	15765	13682	11694	8356	5796	9569
Av Q25	28245	23822	19027	16744	14586	10800	7853	12484
D90	92788	72909	58356	52873	48101	41489	35700	49618
Riskfree mean	14095	14095	14095	14095	14095	14095	14095	14095
LessThanRF	0.003	0.008	0.022	0.046	0.097	0.239	0.424	0.159

∗ Note: Storebrand's strategy

Simulation II - Glide through strategies with DCR from 3%-7%

Table 11.23: Simulation II, glide through with 3% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	13072	13319	13653	14078	14597	15217
Median	11215	11629	12093	12610	13045	13648
Min	2373	2574	2825	3130	3495	3923
Max	106571	92702	80481	69752	60367	52324
Sd	7972	7420	7029	6827	6852	7152
Low 10%	5838	6304	6758	7176	7559	7996
Low 25%	7872	8361	8817	9141	9676	10062
Av D10	4702	5152	5618	6095	6497	6766
Av Q25	6015	6487	6955	7396	7810	8163
D90	22357	22137	22477	22914	23652	24875
Riskfree mean	6041	6041	6041	6041	6041	6041
LessThanRF	0.113	0.084	0.061	0.039	0.023	0.021

Table 11.24: Simulation II, glide through with 3% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	10066	11390	12960	14830	17070	19768
Median	8376	9762	11482	13364	15396	17384
Min	1821	2387	2765	3188	3658	4177
Max	72762	74164	74861	74836	74091	105698
Sd	6204	6295	6611	7317	8642	10873
Low 10%	4349	5412	6465	7460	8275	9004
Low 25%	5871	7127	8390	9589	10947	12407
Av D10	3462	4388	5379	6321	7068	7542
Av Q25	4474	5502	6633	7688	8641	9449
D90	17771	19163	21248	24431	28278	32948
Riskfree mean	6041	6041	6041	6041	6041	6041
LessThanRF	0.269	0.157	0.075	0.034	0.013	0.014

Table 11.25: Simulation II, glide through with 4% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	17430	17758	18204	18770	19463	20289
Median	14954	15505	16124	16813	17393	18197
Min	3163	3432	3767	4174	4660	5231
Max	142095	123603	107308	93003	80490	69765
Sd	10630	9893	9372	9103	9136	9536
Low 10%	7785	8405	9011	9568	10079	10662
Low 25%	10495	11148	11756	12188	12901	13416
Av D10	6270	6870	7490	8127	8662	9022
Av Q25	8020	8649	9273	9861	10414	10884
D90	29809	29516	29969	30552	31536	33167
Riskfree mean	8054	8054	8054	8054	8054	8054
LessThanRF	0.113	0.084	0.061	0.039	0.023	0.021

Table 11.26: Simulation II, glide through with 4% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	13421	15187	17280	19774	22761	26358
Median	11168	13015	15309	17819	20528	23179
Min	2428	3183	3687	4250	4877	5570
Max	97016	98885	99815	99781	98788	140931
Sd	8272	8393	8815	9756	11523	14497
Low 10%	5798	7216	8620	9946	11033	12005
Low 25%	7828	9503	11187	12785	14596	16543
Av D10	4616	5851	7172	8428	9423	10056
Av Q25	5966	7336	8844	10251	11522	12599
D90	23694	25551	28331	32575	37704	43931
Riskfree mean	8054	8054	8054	8054	8054	8054
LessThanRF	0.269	0.157	0.075	0.034	0.013	0.014

Table 11.27: Simulation II, glide through with 5% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	21787	22198	22755	23463	24329	25361
Median	18692	19381	20155	21016	21741	22746
Min	3954	4290	4708	5217	5825	6539
Max	177619	154504	134135	116254	100612	87206
Sd	13287	12366	11715	11378	11420	11920
Low 10%	9731	10506	11264	11960	12598	13327
Low 25%	13119	13935	14695	15234	16126	16771
Av D10	7837	8587	9363	10158	10828	11277
Av Q25	10025	10811	11592	12326	13017	13605
D90	37261	36895	37461	38190	39420	41459
Riskfree mean	10068	10068	10068	10068	10068	10068
LessThanRF	0.113	0.084	0.061	0.039	0.023	0.021

Table 11.28: Simulation II, glide through with 5% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	16776	18984	21600	24717	28451	32947
Median	13960	16269	19136	22274	25660	28973
Min	3035	3979	4608	5313	6096	6962
Max	121270	123607	124769	124727	123486	176163
Sd	10340	10491	11018	12195	14404	18121
Low 10%	7248	9020	10775	12433	13792	15006
Low 25%	9785	11879	13983	15981	18244	20679
Av D10	5770	7314	8965	10535	11779	12569
Av Q25	7457	9170	11055	12814	14402	15749
D90	29618	31939	35413	40719	47130	54913
Riskfree mean	10068	10068	10068	10068	10068	10068
LessThanRF	0.269	0.157	0.075	0.034	0.013	0.014

Table 11.29: Simulation II, glide through with 6% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	26144	26637	27305	28155	29195	30433
Median	22431	23258	24185	25219	26089	27295
Min	4745	5148	5650	6261	6990	7847
Max	213143	185405	160961	139505	120735	104647
Sd	15945	14840	14058	13654	13704	14304
Low 10%	11677	12608	13517	14353	15118	15992
Low 25%	15743	16722	17635	18281	19351	20125
Av D10	9405	10304	11236	12190	12993	13532
Av Q25	12030	12973	13910	14792	15621	16326
D90	44714	44273	44954	45828	47304	49751
Riskfree mean	12082	12082	12082	12082	12082	12082
LessThanRF	0.113	0.084	0.061	0.039	0.023	0.021

Table 11.30: Simulation II, glide through with 6% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	20132	22780	25920	29661	34141	39536
Median	16752	19523	22964	26728	30792	34768
Min	3641	4775	5530	6375	7315	8355
Max	145524	148328	149723	149672	148183	211396
Sd	12408	12590	13222	14634	17284	21745
Low 10%	8698	10824	12930	14920	16550	18008
Low 25%	11742	14255	16780	19177	21893	24815
Av D10	6924	8777	10758	12642	14135	15083
Av Q25	8949	11004	13266	15376	17283	18899
D90	35541	38327	42496	48863	56556	65896
Riskfree mean	12082	12082	12082	12082	12082	12082
LessThanRF	0.269	0.157	0.075	0.034	0.013	0.014

Table 11.31: Simulation II, glide through with 7% DCR part 1/2

	100/0	80/20	60/40	40/60	20/80	0/100
Mean	30502	31077	31856	32848	34061	35506
Median	26169	27134	28216	29422	30438	31844
Min	5536	6006	6591	7304	8155	9155
Max	248667	216306	187788	162755	140857	122088
Sd	18602	17313	16401	15930	15988	16687
Low 10%	13623	14709	15769	16745	17637	18658
Low 25%	18367	19509	20574	21328	22577	23479
Av D10	10972	12022	13108	14221	15159	15788
Av Q25	14034	15136	16228	17257	18224	19047
D90	52166	51652	52446	53466	55188	58042
Riskfree mean	14095	14095	14095	14095	14095	14095
LessThanRF	0.113	0.084	0.061	0.039	0.023	0.021

Table 11.32: Simulation II, glide through with 7% DCR part 2/2

	100/0 U	80/20 U	60/40 U	40/60 U	20/80 U	0/100 U
Mean	23487	26577	30240	34604	39831	46126
Median	19544	22777	26791	31183	35924	40563
Min	4248	5570	6452	7438	8534	9747
Max	169778	173049	174676	174618	172880	246629
Sd	14476	14688	15425	17074	20165	25370
Low 10%	10147	12628	15085	17406	19308	21009
Low 25%	13699	16631	19576	22373	25542	28951
Av D10	8078	10239	12552	14749	16491	17597
Av Q25	10440	12838	15478	17939	20163	22049
D90	41465	44715	49579	57006	65982	76878
Riskfree mean	14095	14095	14095	14095	14095	14095
LessThanRF	0.269	0.157	0.075	0.034	0.013	0.014

SR and SV

Table 11.33: SR and SV for glide-to and mean bond

		0/100	20/80	40/60	50/50	60/40	80/20	100/0	STB
3%	SR	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00
	SV	9.93	11.15	11.72	11.59	11.22	1.00	8.55	9.51
4%	SR	0.98	0.99	0.99	0.98	0.98	0.95	0.90	0.95
	SV	9.77	11.02	11.58	11.40	11.01	9.53	7.79	9.07
5%	SR	0,91	0,92	0,91	0,90	0,88	0,80	0,80	0,80
	SV	9.03	10.28	10.67	10.40	9.90	8.05	0.68	0.68
6%	SR	0.82	0.80	0.76	0.72	0.69	0.57	0.41	0.57
	SV	8.16	8.96	8.92	8.35	7.71	5.67	3.52	5.40
7%	SR	0.67	0.63	0.56	0.52	0.46	0.07	0.21	0.31
	SV	6.65	7.00	6.57	5.97	5.13	0.70	1.85	2.99
8%	SR	0.51	0.46	0.46	0.30	0.25	0.15	0.09	0.15
	SV	5.10	5.17	5.42	3.52	2.78	1.51	0.80	1.44
9%	SR	0.37	0.29	0.20	0.16	0.12	0.07	0.04	0.07
	SV	3.70	3.23	2.32	1.82	1.36	0.66	0.35	0.63
10%	SR	0.25	0.17	0.09	0.08	0.05	0.03	0.02	0.03
	SV	2.48	1.91	1.10	0.90	0.61	0.28	0.13	0.27

Table 11.34: SR and SV for glide-through and mean bond

		0/100	20/80	40/60	60/40	80/20	100/0
3%	SR	1.00	1.00	1.00	1.00	1.00	1.00
	SV	11.42	12.03	11.92	11.14	10.02	8.84
4%	SR	0.99	0.99	0.99	0.98	0.98	0.97
	SV	11.29	11.91	11.76	10.94	9.80	8.55
5%	SR	0.92	0.91	0.90	0.89	0.87	0.83
	SV	10.46	10.99	10.77	9.96	8.72	7.31
6%	SR	0.79	0.77	0.74	0.71	0.66	0.60
	SV	9.04	9.26	8.77	7.87	6.62	5.31
7%	SR	0.61	0.58	0.54	0.49	0.42	0.35
	SV	6.98	7.02	6.37	5.43	4.21	3.08
8%	SR	0.42	0.39	0.33	0.28	0.22	0.18
	SV	4.82	4.63	3.91	3.12	2.22	1.61
9%	SR	0.26	0.22	0.17	0.14	0.11	0.08
	SV	2.95	2.65	2.04	1.59	1.05	0.68
10%	SR	0.13	0.10	0.09	0.06	0.05	0.04
	SV	1.53	1.24	1.04	0.71	0.46	0.31

Table 11.35: SR and SV for glide-through and mean bond

		0/100U	20/80U	40/60U	60/40U	80/20U	100/0U
3%	SR	1.00	1.00	1.00	1.00	1.00	1.00
	SV	9.28	10.31	11.25	11.82	11.85	11.29
4%	SR	0.98	0.99	0.99	0.98	0.98	0.96
	SV	9.14	10.20	11.09	11.62	11.59	10.85
5%	SR	0.91	0.91	0.90	0.89	0.86	0.81
	SV	8.50	9.42	10.16	10.50	10.24	9.22
6%	SR	0.78	0.77	0.74	0.70	0.67	0.60
	SV	7.26	7.94	8.29	8.32	7.89	6.74
7%	SR	0.60	0.58	0.54	0.48	0.41	0.34
	SV	5.58	5.96	6.06	5.72	4.86	3.86
8%	SR	0.44	0.39	0.34	0.28	0.23	0.17
	SV	4.05	4.07	3.78	3.29	2.75	1.91
9%	SR	0.29	0.23	0.18	0.14	0.10	0.09
	SV	2.65	2.40	2.01	1.62	1.15	0.99
10%	SR	0.17	0.12	0.09	0.06	0.04	0.04
	SV	1.57	1.24	1.00	0.73	0.52	0.43

Table 11.36: SR and SV for glide-to and 4% bond

		0/100	20/80	40/60	50/50	60/40	80/20	100/0	STB
3%	SR	0.99	0.99	0.99	0.99	0.99	0.97.	0.88.	0.97
	SV	10.02	11.39	12.01	11.83	11.35	9.72.	7.53	9.24
4%	SR	0.98	098	0.98	0.97	0.95	0.85	0.69	0.85
	SV	9.86	11.29	11.79	11.51	10.86	8.54	5.87	8.11
5%	SR	0.91	0.92	0.90	0.86	0.80	0.63	0.46	0.63
	SV	9.108	10.48	10.87	10.26	9.19	6.30	3.94	5.99
6%	SR	0.82	0.79	0.71	0.64	0.56	0.38	0.27	0.38
	SV	8.23	9.11	8.60	7.61	6.41	3.83	2.32	3.64
7%	SR	0.67	0.6	0.47	0.4	0.32	0.2	0.14	0.2
	SV	6.71	6.88	5.64	4.72	3.6	2	1.15	1.9
8%	SR	0.51	0.42	0.26	0.20	0.15	0.07	0.06	0.07
	SV	5.15	4.77	3.18	2.31	1.74	0.73	0.48	0.69
9%	SR	0.37	0.25	0.13	0.09	0.06	0.03	0.02	0.03
	SV	3.74	2.9	1.5	1.03	0.66	0.32	0.20	0.30
10%	SR	0.25	0.13	0.06	0.04	0.02	0.01	0.01	0
	SV	2.5	1.53	0.67	0.45	0.22	0.09	0.11	0.09

Table 11.37: SR and SV for glide-through and 4% bond

		0/100	20/80	40/60	60/40	80/20	100/0
3%	SR	1	1	1	1	0.99	0.98
	SV	11.65	12.37	12.23	11.29	9.99	8.6
4%	SR	0.99	0.99	0.978	0.96	0.93	0.88
	SV	11.54	12.20	11.97	10.86	9.34	7.72
5%	SR	0.92	0.92	0.88	0.84	0.76	0.68
	SV	10.77	11.34	10.78	9.53	7.68	5.92
6%	SR	0.79	0.75	0.69	0.60	0.52	0.43
	SV	9.26	9.3	8.42	6.84	5.24	3.77
7%	SR	0.56	0.5	0.43	0.35	0.28	0.22
	SV	6.49	6.18	5.24	3.99	2.81	1.97
8%	SR	0.36	0.3	0.23	0.17	0.14	0.1
	SV	4.19	3.68	2.77	1.97	1.38	0.88
9%	SR	0.19	0.14	0.11	0.07	0.05	0.04
	SV	2.17	1.71	1.31	0.80	0.53	0.33
10%	SR	0.1	0.07	0.05	0.03	0.02	0.01
	SV	1.13	0.85	0.55	0.31	0.18	0.15

Table 11.38: SR and SV for glide-through and 4% bond

		0/100U	20/80U	40/60U	60/40U	80/20U	100/0U
3%	SR	1	1	1	1	0.99	0.99
	SV	9.22	10.38	11.43	12.1	12.12	11.47
4%	SR	0.97	0.97	0.97	0.97	0.96	0.91
	SV	8.94	10.11	11.14	11.74	11.65	10.47
5%	SR	0.88	0.89	0.87	0.84	0.78	0.7
	SV	8.15	9.24	10	10.24	9.52	8.13
6%	SR	0.73	0.72	0.67	0.6	0.53	0.46
	SV	6.78	7.43	7.71	7.25	6.4	5.28
7%	SR	0.54	0.5	0.43	0.35	0.29	0.22
	SV	5.02	5.15	4.92	4.22	3.48	2.5
8%	SR	0.39	0.31	0.23	0.17	0.12	0.09
	SV	3.56	3.24	2.64	2.11	1.46	1.02
9%	SR	0.25	0.17	0.12	0.07	0.04	0.04
	SV	2.32	1.78	1.33	0.8	0.54	0.42
10%	SR	0.14	0.08	0.05	0.02	0.01	0.01
	SV	1.26	0.85	0.53	0.28	0.13	0.09