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Master's Thesis

Liquidity and stock returns: Evidence from Denmark

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Executive summary

This master's thesis examines the relationship between liquidity and stock returns theoretically and empirically.

This thesis considers stock liquidity as a scale rather than a level. The main hypothesis of the empirical study of this thesis is that stock returns are decreasing in liquidity. This hypothesis is backed by a rigorous, theoretical model that will be derived. The hypothesis can, however, be explained by intuition - if investors are to hold stocks that cannot easily be sold without changing the price or incurring other costs, they should demand compensation. This compensation is expected to come in higher returns. Thus, investors are expected to require higher returns for less liquid stocks. In addition to this, stocks with returns that are sensitive to changes in liquidity should yield higher returns to compensate the investors for this additional risk.

Previously, the liquidity-return relationship has been investigated in a number of studies. To mention a few, Amihud and Mendelson (1986), Datar, Naik, and Radcliffe (1998), Chan and Faff (2005), and Archarya and Pedersen (2005) all find clear evidence of the pricing of liquidity or liquidity risk in equity markets.

The empirical study of this thesis is based on a sample of listed Danish companies for the time period from 1987 through November 2008. Two measures of liquidity are applied - the relative bid-ask spread and the turnover rate. In a cross-sectional framework a la Fama and Macbeth (1973), the cross-sectional effect of liquidity on stock returns is studied for both measures of liquidity. Also, the cross-sectional relationship between liquidity risk and stock pricing is studied. For both studies a number of robustness checks are carried out to determine the robustness of the findings.

The empirical study provides ambiguous evidence of the pricing of liquidity and liquidity risk in Denmark. No findings are robust to changing various assumptions or calculation techniques. Overall, there is neither strong evidence of a return premium for illiquidity nor a return premium for liquidity risk. There are, however, indications of

- an annualised illiquidity return premium in the range 400 - 520bp per 100bp of increase in the relative spread,
- an annualised illiquidity return premium in the range 62 - 71bp per 100bp decrease in the turnover rate,

- a liquidity risk premium in the range 570 - 590bp per unit of relative bid-ask spread sensitivity, and
- a liquidity risk premium of approximately 1080bp per unit of turnover rate sensitivity

The findings are not robust, so they should be considered with a certain amount of criticism. However, the findings still give an indication of the relationship between liquidity and stock returns in Denmark.

1 Introduction

In relation to assets, liquidity generally relates to the ease by which an asset can be sold immediately after purchase without incurring losses of any kind. These losses could be due to price changes or various transaction costs.

The purpose of this thesis is to examine how liquidity is priced in the Danish stock market. In specific, the relationship between stock returns and liquidity will be studied based on a sample of Danish stock companies. The test methodology is in accordance with earlier studies based on U.S. data.

In the theoretical part, the different sources of illiquidity are discussed, the different measures of liquidity are presented and the theoretical relationship between liquidity and stock returns is derived in both an intuitive and a rigorous approach. The drivers of illiquidity are exogenous trading costs, demand pressure, inventory risk, asymmetric information, and search frictions. These sources of illiquidity impose costs to the investor holding assets that are less than perfectly liquid. A rational investor would therefore be expected to demand a return premium in compensation for holding assets that are less than perfectly liquid. Thus, intuitively, there should be a positive relationship between illiquidity and stock returns. Conversely, the relationship between liquidity and stock returns should be negative. The theoretical model shows that this relationship should hold in equilibrium. Also, it can be expected that the less liquid stocks will be allocated to investors with longer investment horizons (patient investors). Finally, stocks that are sensitive to liquidity should be expected to yield higher returns to investors as a compensation for the additional risk. That is, there should be liquidity risk premium in stock pricing.

Earlier studies have documented this relationship in other stock markets (mainly the U.S. stock market). In the empirical study of this thesis, econometric models similar to those of the earlier studies will be applied to data on Danish stocks. The models will be modified in various ways in expectation to reach significant and robust results. Also, a range of robustness checks will be applied to see if the findings are robust to the different assumptions underlying the computation of variables and data analysis.

The evidence from the empirical study is ambiguous. There are indications of pricing of both liquidity and liquidity risk, but the findings are not robust to a wide range of robustness checks. This

leads to the conclusion that it cannot be rejected that both liquidity and liquidity risk has an effect on stock returns in the Danish equity market, but the support for the theory is not strong.

1.1 Problem statement

The aim of this thesis is to determine the relationship between stock pricing and liquidity in Denmark. The empirical study will be based on a sample of listed Danish stocks in the period from January 1987 through November 2008. Before the empirical study is initiated, a theoretical and methodological framework will be established.

The assessment of the goal of determining the effect of liquidity in stock pricing raises a number of questions relating to liquidity and the relationship between liquidity and stock pricing. These questions are stated below.

- What is liquidity in relation to stocks?
- What should be the relationship between liquidity and the pricing of stocks?
- How should liquidity be valued?
- How can liquidity be measured or proxied?
- What does previous research imply regarding the pricing of liquidity?
- What is the relationship between liquidity and stock returns in Denmark?
- How is liquidity risk priced in Denmark?

These questions will be answered through the different parts of this thesis. The conclusion will sum up the answers and it will be explained what these answers imply regarding the relationship between stock pricing and liquidity in Denmark.

1.2 Scope

As will also be explained in detail in the theoretical part, this thesis will focus on stock liquidity and how it is priced in the Danish equity market.

This means that in the theoretical part, the concept of liquidity will only be discussed in relation to stocks and the stock markets. In the beginning of the theoretical part, liquidity in relation to other asset classes will be touched upon, but only in order to fully illustrate that liquidity is a broad concept that can be put into the context of most assets. Many of the characteristics of stock liquidity that will be presented in the theoretical part could also be valid for other asset classes, but it is not

within the scope of this paper to discuss specific details of the liquidity characteristics of other assets than stocks.

Previous research providing empirical evidence on both the liquidity and the pricing of liquidity of many different asset classes exists. For example, the pricing and importance of liquidity in bond markets have been covered extensively in previous research. In the part where previous research is reviewed, only previous research on the relationship between stock pricing and liquidity will be presented.

The empirical study will also solely focus on liquidity and its importance in stock pricing in the Danish equity market. The reporting of results, the analysis, the discussions, and the conclusions will mainly cover the implications of liquidity on stock pricing in Denmark.

The conclusion of this thesis will not conclude on the relationship between liquidity and the pricing of other asset classes than stocks.

The data analysis will cover only stocks for which information on the desired variables is available through Thomson's Datastream (described below).

1.3 Data sources

The data source for financial information on the sample of Danish stock companies is Thomson's Datastream. This is a financial information database provided by the well-known information company, Thomson Reuters¹. Thomson Datastream is, according to their web page, the world's largest financial statistical database². Access to the database is provided by the Copenhagen Business School library. Due to that fact that Thomson Reuters is a well-known and respected information company, Datastream is generally considered a reliable data source, and therefore, the data downloaded from this database is considered valid. However, when computing variables and carrying out the statistical analyses, general sanity checks have been made, and the individual stocks have been sampled and the data has been verified by comparing it to the same data from other sources. These other sources comprise the Danish financial information web page, Euroinvestor.dk, the web page for the Copenhagen stock exchange, formerly cse.dk - now omxnordicexchange.com,

¹ Before April 2008, The Thomson Corporation. In April 2008, The Thomson Corporation acquired Reuters Group Limited, and the joint company was called Thomson Reuters.

² www.datastream.com

and the Danish stock company information web page, danskaktieanalyse.dk. Especially the web page of the Copenhagen stock exchange can be considered a valid data source as it is the official source of data for stocks quoted on the Copenhagen stock exchange. The two other sites are independent and unregulated sites, which, in principle, could have false information - however, as these sites are merely used as additional error checks, they do not impose errors in the data used in the study. When checking the data downloaded from Datastream, no errors were found. Therefore, it is concluded that Thomson Datastream is a reliable data source for the empirical study in this thesis.

2 Theory

In this chapter, a theoretical framework will be set up for investigating the effects of illiquidity in the Danish equity market. First, liquidity will be introduced in a broad sense. Then, the discussion on liquidity will be narrowed in to the effect of liquidity on the pricing of common stocks. Next, two general stock pricing theories are presented; the classical stock pricing theory of Williams (1938) who in his book “Theory of investment value” presents the dividend discount model, and the Capital Asset Pricing Model of Sharpe (1964) with additional focus on some of its assumptions. In section 2.4, some intuition on the effect of illiquidity in the pricing of stocks is introduced. The intuition is followed by three sections in which a more rigorous approach to model the connection between stock returns and the costs of illiquidity is gone through. The modelling part ends with a presentation of Archaya and Pedersen’s (2005) Liquidity-Adjusted CAPM and its implications. This chapter is concluded with a short discussion of investment horizons and liquidity.

2.1 Liquidity

As defined in the introduction, liquidity refers to the ease by which an asset can be sold immediately after purchase without lowering the price and without incurring transaction costs. This means that whenever an investor considers a potential investment in an asset, she considers very thoroughly the ability to sell it again, what it will cost to trade it in the future and at what price it can be sold. These considerations relate to the liquidity of the asset, and the issues considered can affect the future cash flows of the asset. As future cash flows are affected by liquidity, it must be an important factor in asset pricing. Costly trading and possible future price reductions in case of forced sale are not pricing factors solely related to financial assets such as stocks - thus, liquidity affects the pricing of most asset classes. Damodaran (2005) describes the cost of illiquidity as the cost of buyer's remorse:

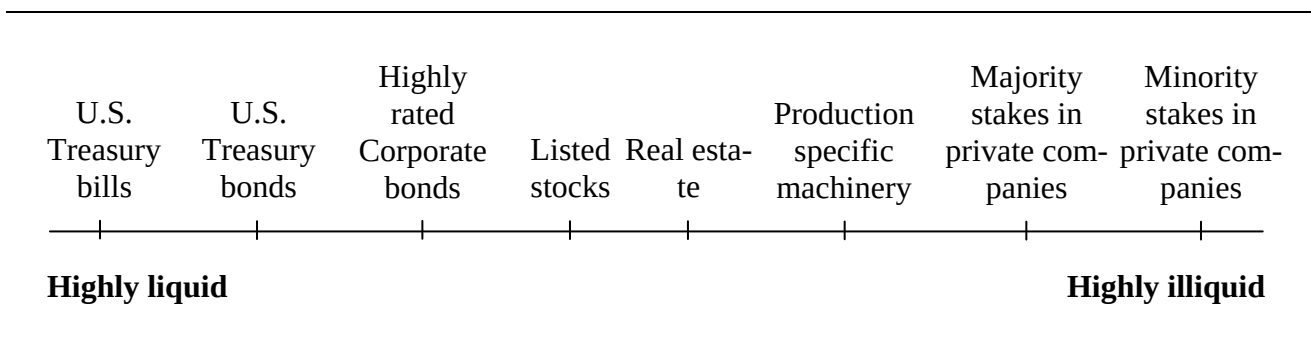
"When you buy a stock, bond, real asset or a business, you sometimes face buyer's remorse, where you want to reverse your decision and sell what you just bought. The cost of illiquidity is the cost of this remorse" Damodaran (2005)

Obviously, it matters what asset you buy. If you buy a stock in a large company publicly traded on a well-established stock exchange (e.g. the NYSE ³), you would be able to sell it again immediately

³ New York Stock Exchange

with almost no costs - you would have to pay a small fee and you would receive a slightly lower price than you paid for it - the difference between the dealer's ask and bid price (called the bid-ask spread). If you, on the other hand, buy a small private (unlisted) company, and you want to sell it again immediately, you would probably find yourself having a hard time finding a buyer. There is no market for private companies⁴ - thus, a seller needs to search the market for potential buyers and negotiate with them. This can be costly. If you were lucky enough to find a buyer immediately, you would be even luckier if this buyer wanted to pay the same price you just did - hence, there is a potential cost in having to reduce the price. When trading private companies, advisers are usually hired to help finding potential buyers, structure the selling process and negotiate. These advisers do not make do with a small fee - thus, the direct costs of trading private companies can be substantial. These examples make it very clear that the cost of buyer's remorse - the illiquidity cost - can vary considerably for different asset classes. The figure below relates liquidity to different asset classes.

Figure 2.1: The liquidity of different asset classes



Note: Inspired by Damodaran (2005)

In the figure above, listed stocks are presented as an asset class with a certain level of liquidity. In a broad sense, this holds. Compared to the other asset classes, listed stocks can be considered an asset class with *one* level of liquidity. However, liquidity may vary substantially between listed stocks.

Many of the characteristics of stock liquidity that will be presented below are very similar to the characteristics of many other financial assets (e.g. bonds). However, as the empirical investigation of liquidity later in this thesis solely covers the Danish stock market, it is most obvious to focus on stocks when presenting the characteristics and sources of liquidity.

⁴ By market is meant an organised exchange where prices are quoted

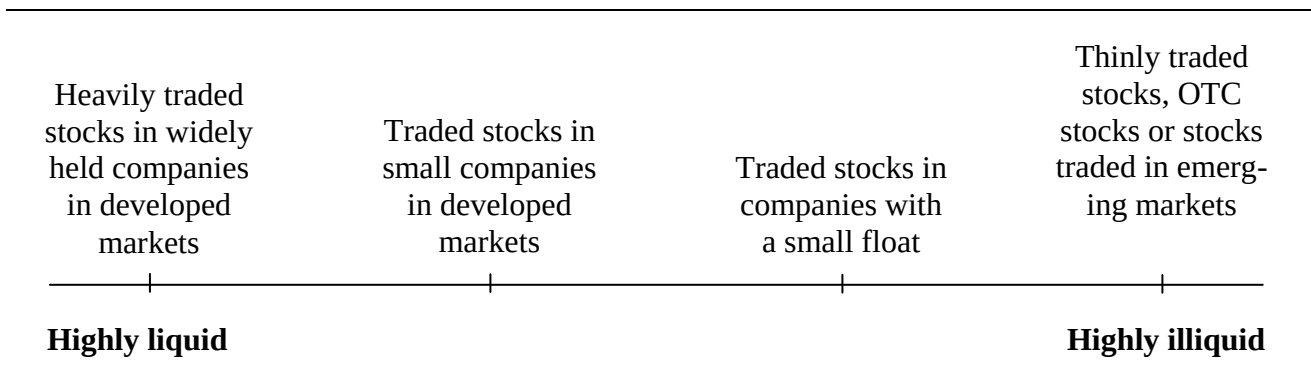
2.2 Stock liquidity

In the following, the concept of liquidity for stocks being a spectrum rather than a constant level will be discussed and the characteristics of stock liquidity will be described.

2.2.1 The stock liquidity spectrum

Below, an approximation of the liquidity of different types of stocks can be seen:

Figure 2.2: The stock liquidity spectrum



Note: Inspired by Damodaran (2005)

As can be seen, there are different levels of liquidity within stocks. The most liquid stocks are those in widely held companies in developed markets. Less liquid are stocks in companies with a small float, and even less liquid are lightly traded stocks, OTC or emerging market stocks. Stocks in unlisted companies are normally accepted to be the less liquid stocks. However, many argue that very large (e.g. market dominant) unlisted companies can be far more liquid than thinly traded stocks in small companies. The argument goes that this kind of unlisted companies will always be attractive targets due to their large market shares in their respective markets. It is not within the grasp of the present study to examine the liquidity of unlisted companies and OTC stocks. This thesis will focus solely on the liquidity of common stocks in Denmark.

The purpose of this thesis is to examine how these different degrees of liquidity affects stock pricing in the Danish equity market.

2.2.2 The characteristics of stock liquidity

When characterising stock liquidity, a good place to start would be to explain which phenomena that drive the differences in stock liquidity - what are the sources of illiquidity?

As mentioned in section 2.1, an investor considers a number of issues when deciding whether to invest in an asset. These issues are all examples of sources of illiquidity. Generally, the sources of illiquidity comprise exogenous transaction costs, demand pressure, inventory risk, asymmetric information and search frictions (Amihud, Mendelson and Pedersen (2005)). All these sources of illiquidity impose costs to the holder of the assets. These costs of illiquidity should be reflected in the asset prices, as the investors should require a compensation for holding them. Also, as the various sources of illiquidity are time-varying, so is liquidity. Thus, if investors are risk-averse they should require a compensation for holding assets with the risk attached to them that liquidity can decrease unexpectedly. The compensation for the costs and risk associated with illiquidity should be reflected in a higher expected return. Below, the different sources of liquidity are discussed in turn⁵.

Exogenous transaction costs

This explicit source of illiquidity relates to trading costs such as brokerage fees, order-processing fees and transaction taxes. These costs will have a direct influence on the profit of the trader - that is, both the seller and buyer may be affected by exogenous trading costs. As these costs represent frictions in the capital markets, they can be viewed as sources of illiquidity - they should affect the prices that investors will trade at. Also, if investors do not trade directly with each other through open market orders but instead trade with dealers (market makers), the various transaction costs will be reflected in the bid and ask prices that are quoted. That is, the market makers take into consideration their costs when quoting bid and ask prices - the spread between what they will buy for (bid price) and what they will sell for (ask price) should cover the market maker's costs. Direct transaction costs are some of these costs. The other costs for the market makers will be touched upon in the following sections.

Demand pressure – the cause of the price impact

An important source of illiquidity is the depth of the market for an asset. This is referred to as the demand pressure or the price impact. It describes the investor's possibility of selling large amounts of an asset quickly and without lowering the price. For example, if an investor is struck by a liquidity shock, she could be forced to liquidate her long position in a (less than perfectly liquid) stock. If, however, the size of the position is considerable, there is a risk that the investor would not be able to carry out the trade at the prevailing market price. The reason is that there will not necessarily be a

⁵ The sources of liquidity are presented by Amihud, Mendelson and Pedersen (2005)

buyer for the position at the market price. The investor would probably have to ask a lower price if she needs to liquidate the entire position. Hence, this large trade would move the price of the stock which is a result of the fact that the stock is less than perfectly liquid. This phenomenon is commonly referred to as demand pressure or *the price impact*. When stocks are not perfectly liquid, a large trade can cause a shock to the equilibrium between supply and demand. Thus, large orders will result in price changes when stocks are not perfectly liquid. The price change will be negative when the investor places a selling order and positive for a buying order. The smaller the price impact the more liquid the market for the stock. Part of the price impact could be informational. If one investor suddenly conducts a large trade (buy), it is possible that other investors would notice this and perceive it as a sign of this investor having some new and private information. This could place an upward pressure on the price of the stock. However, this price impact cannot be permanent - if so, it would result in speculative bubbles, and in an efficient market, the price would readjust, if it had reached unrealistically high levels.

Inventory risk

This is closely related to demand pressure. In the situation where the market for a stock is not very deep, an investor could easily find himself in a situation where there is no buyer present in the market for a certain position that needs to be liquidated immediately. Instead of waiting for a buyer to appear, the solution to this could be to sell to a market maker at her bid price. This market maker holds inventory bearing the risk that the price of the security will drop. She would want to be compensated for this risk of holding inventory, so she quotes bid and ask prices such as to ensure that the present value of the expected future losses is covered (at least). Thus the higher the inventory risk, the higher the spread between ask and bid prices - more on this will follow in section 2.3.1.

Search frictions

Another source of illiquidity closely linked to demand pressure and inventory risk is the search frictions. Search frictions arise due to the demand pressure. The search frictions are the opportunity costs and financing costs that an investor incurs when searching for a not necessarily present buyer of the stock to be sold. So first of all, there will be opportunity costs associated with waiting for a counterparty. Next, when a counterparty has been located, price negotiations begin. These negotiations may lead to a price reduction. The alternative to searching is to incur the costs of selling to a dealer – these costs comprise fees and the inventory risk that the dealer wants to be compensated for as mentioned under demand pressure and inventory risk. This kind risk is particularly distinct in over-the-counter markets.

As mentioned above, waiting for a counterpart is associated with opportunity costs. In addition to this, waiting to trade generally imposes another kind of opportunity costs to the investor. If an investor has assessed the value of a stock based on private information and finds that the stock is undervalued, waiting to buy it could result in opportunity costs for the investor, if the price of the stock increases. Thus, the opportunity costs of waiting will be the profits not received from not having bought the stock at the low price. In capital markets that are at least slightly efficient, private information will be reflected in the price rather quickly. Thus, waiting to trade on private information can be very costly.

Asymmetric information / Private information

Mentioned above was the private information. This can also impose costs in a different way - asymmetric information as a source of illiquidity relates to the fact that trading with informed counterparts can be costly. If the counterpart has private information about either the value of the company or the order flow⁶, the transaction will result in a loss. This loss can be considered an illiquidity cost. It can be expected that investors will consider the risk that the trading counterpart has superior information. This causes adverse selection - the agents with superior information want to trade. The fact that dealing with informed counterparts will result in a loss is actually what causes the illiquidity contribution from asymmetric information. Market makers face the risk of dealing with informed counterparts. The market makers obviously seek compensation for this risk. Market makers can take into consideration this risk when quoting the bidding price and asking price. This will be discussed in the next section under the bid-ask spread.

2.3 How can liquidity, illiquidity or the cost of illiquidity be measured?

Now, some of the widely used proxies for liquidity, illiquidity and the costs of illiquidity will be presented. In short, liquidity can be measured using elements of the sources of illiquidity reviewed above.

2.3.1 The bid-ask spread

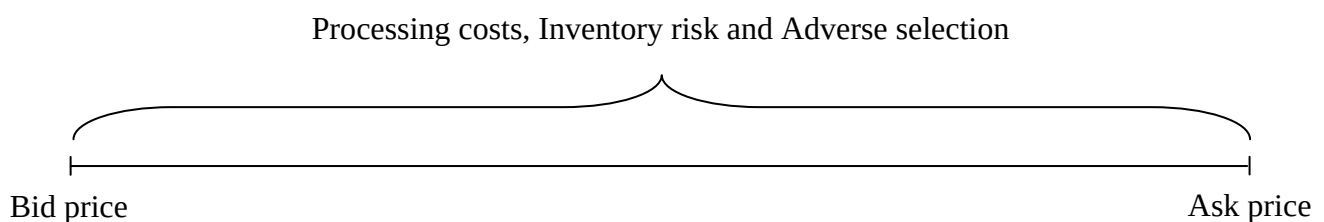
Many of the sources of illiquidity mentioned above are the drivers of the bid-ask spread. That is, the more illiquid the stock the bigger the importance of the different sources of illiquidity, and thus, the bigger the bid-ask spread. The bid-ask spread thus covers many aspects of liquidity because it is basically driven by many of the important determinants of illiquidity.

⁶ Private information about the order flow relates to the situation where an agent has private information about future large orders that are likely to affect the price of the stock. Trading on such private information should be profitable.

The bid-ask spread is the spread between the price that a stock can be sold for (the bid price) and the price it costs to purchase it (the ask price) through a market maker. This spread is a result of the fact that the dealer wants to be compensated for the processing costs (direct transaction costs), the inventory risk and the risk of dealing with informed counterparts. The dealer incurs processing costs when carrying out orders. The spread must cover these costs. The dealers or market makers quote both bid and ask prices - quoting a too high bid price or a too low ask price could result in the dealer being left with a large long (too high bid price) or short (too low ask price) position. The dealer will have to recover possible losses through the spread. Also, the dealer can adjust her position in a stock towards the optimal level by adjusting the quoted prices. Generally, as explained earlier, additional risk is imposed to the market maker due to the fact that she holds inventory that could change in value. The risk of dealing with informed investors can be referred to as an adverse selection situation - when the investors choose to place either "buy" or "sell" orders, it could be that the investors have information that the dealer does not yet have. Trading with informed counterparts can be expected to yield negative profits. Thus, to compensate for this, the dealer adjusts the bid-ask spread⁷.

The relationship between various sources of illiquidity and the bid-ask spread is summarised in the figure below:

Figure 2.3: Drivers of the bid-ask spread



2.3.2 Amihud's ILLIQ-measure

As mentioned earlier, liquidity in its true essence describes the possibility of selling large quantities of an asset immediately after purchase without changing the price. Thus, an appealing measure of

⁷ As explained in Damoradan (2005)

illiquidity is a measure of the sensitivity of prices to the traded volume. Amihud (2002) introduced such a measure which is now referred to as the Amihud ILLIQ-measure.

2.3.3 The turnover rate

A widely used proxy for liquidity is the turnover rate of a stock. It is simply the number of shares traded over a period divided by the number of shares outstanding during that period. This is an intuitive measure, as it simply states how many times the outstanding equity switched hands during a period.

2.3.4 Block trades

The earlier mentioned price impact has been investigated in studies by examining block trades - stock transactions in which a large portion of the shares in the company is traded. The studies analyse the price reaction to large block trades. The price reaction gives an indication of how liquid the stock.

2.3.5 IPOs

When there are records of share transactions prior to an initial public offering (IPO), an illiquidity discount can be estimated. The illiquidity discount can be estimated by comparing the stock price in these pre-IPO transactions with the IPO stock price. The argument is that before the IPO, the stocks can be considered illiquid and after the IPO, the stocks can be considered liquid. Thus, higher expected future liquidity should result in a higher IPO price than pre-IPO price. The percentage difference between the prices is therefore argued to be an illiquidity discount.

2.3.6 Restricted stock offerings

Many studies have examined restricted stock issues to determine illiquidity discounts. Public stock companies can achieve external financing from restricted stock issues. Restricted stocks are stocks that are restricted from being resold for usually one year after the issue. At the restricted stock offering, these restricted stocks are sold at a lower price than the traded stocks in the company. The difference between the restricted stock price and the market price of the normal stocks of the company can be considered an illiquidity discount.

2.3.7 Share classes

Many companies have more than one class of shares. It is a common phenomena that these different classes of shares trade at different prices even though they have the same cash flow rights attached

to them. If there are no differences in voting rights and no other circumstances that separate the stock classes, the price differences could be attributed to differences in liquidity. If one of the stock classes has voting rights, it complicates things - there should be a premium for voting rights. Often, the class of shares with voting rights attached to it is less heavily traded and could therefore be considered less liquid. This will result in the share price comprising both a premium for voting rights and a discount for illiquidity.

2.3.8 Longstaff's put

Longstaff (1995) modelled the value of liquidity as a put option. The basic idea is that it is valuable to be able to sell a stock. Thus, the value of liquidity could be reflected in the value of a put option on the underlying stock. The value of an option increases with the time to maturity. This translates into the main implication of his model - the costs of illiquidity increase with the length of the period that the stock cannot be sold.

2.4 Some intuition on the relationship between stock pricing and liquidity

If two companies are *exactly* identical in all aspects except for the fact that their stocks have different liquidity, the company with the less liquid stocks should be valued at a discount compared to the identical company with liquid stocks reflecting the costs of illiquidity. Let V_{IL} represent the value of the company with the illiquid stocks, V_L the value of the company with liquid stocks and C the costs of illiquidity. It then follows that

$$V_{IL} = V_L - C \Rightarrow V_{IL} < V_L \quad \text{for } C > 0$$

Expressed in expected returns, the intuition is as follows. For stocks that are less than perfectly liquid, investors will incur costs of illiquidity when liquidating a position. Rational investors would be expected to demand a return premium reflecting the expected costs of illiquidity. Thus, the required rate of return on a stock, R_{IL} , that is less than perfectly liquid should be the required rate of return on a perfectly liquid stock, R_L plus a return premium, R_C , reflecting the degree of illiquidity of the stock:

$$R_{IL} = R_L + R_C$$

The return on a perfectly liquid stock, R_L , will be a corner stone in the modelling of the theoretical relationship between stock returns and liquidity. In the following three sections, a more rigorous approach to model the relationship between the costs of illiquidity and stock returns follows.

2.5 The classic stock pricing theory and some of its assumptions

The basic idea behind the dividend based stock valuation is that the value of a stock is the present value of all future dividends⁸:

$$P_{i0} = \sum_{t=1}^{t=\infty} \frac{d_{it}}{(1 + R_i)^t}$$

The above formula is a realisation of the fact that a project should be priced at the present value of its future cash flows. In a perfect capital market, the price of a stock with a dividend payout of 100%, will be exactly equal to the present value of the expected future dividends. The above equation will be the departure of the rigorous approach to the modelling of the relationship between stock returns and the costs of illiquidity that will be presented in section 2.7.

Mentioned above is the assumption of perfect capital markets. This is basically the starting point of most theoretical capital market models. Elton, Gruber, Brown and Goetzmann (2007) summarises the idea of this assumption:

"As the physicist builds models of the movement of matter in a frictionless environment, the economist builds models where there are no institutional frictions to the movement of stock prices."

The word "*frictionless*" is really the essence of all the assumptions underlying a perfect capital market. These assumptions also form the foundation for the standard capital asset pricing model. This model will be the basis for the empirical tests in this study. Therefore, the standard capital asset pricing model will be presented in the following. Some of the assumptions underlying the model will be discussed.

2.5.1 The capital asset pricing model (CAPM)

This model is referred to as the Sharpe-Lintner-Mossin model for the general equilibrium relationship in capital markets. The relation is

$$R_i = R_F + \beta_i (R_M - R_F),$$

where

R_i = The return on asset i ,

R_F = The return on the risk free asset,

⁸ It is generally accepted that the dividend based stock pricing model developed by Williams (1938)

β_i = Beta of asset i (the sensitivity to systematic risk),

R_M = The return on the market portfolio.

This linear relationship is called the Security Market Line - a framework in which returns are explained by the sensitivity towards market risk. The relevant risk measure is defined as the market risk - that is, the exposure of the returns of a stock towards changes in the market portfolio return in excess of the risk free rate of return.

This risk is denominated by beta and is given by

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

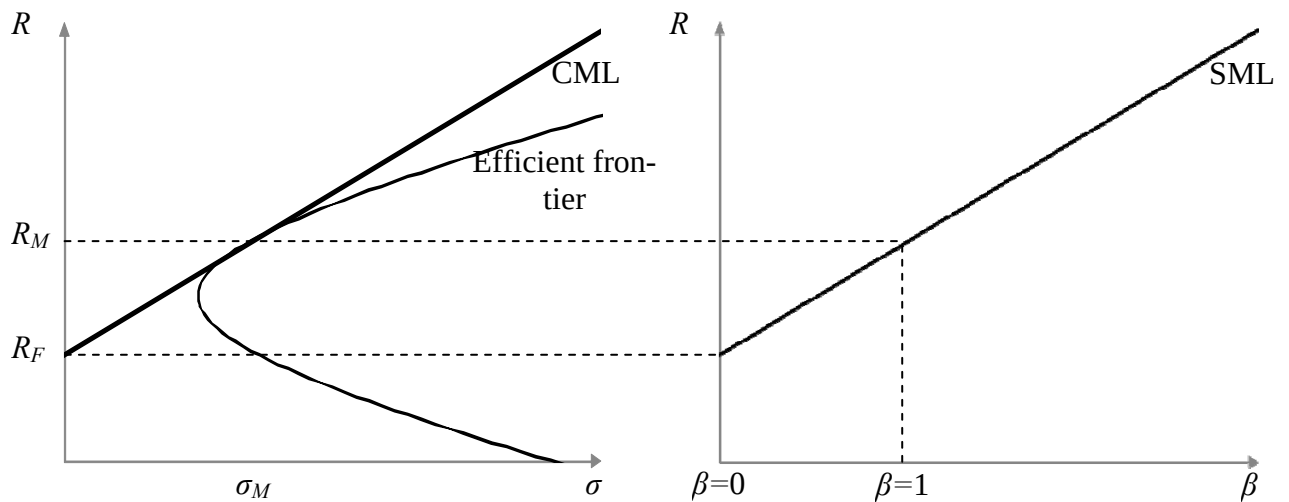
where,

σ_{iM} = the covariance between the return on the market portfolio and the return on stock i ,

σ_M^2 = the variance of the return on the market portfolio.

The model is a realisation of the efficient frontier, the tangent portfolio and some assumptions. Below, figure 2.4 summarises the connection between the efficient frontier, the risk free rate of return, the Capital Market Line (CML) and the Security Market Line (SML)

Figure 2.4: The Capital Market Line and the Security Market Line



A common denominator for the different sources of illiquidity presented in section 2.2.2 is the fact that they all represent violations of different assumptions of the capital asset pricing model. The assumptions relevant to the present study will now be presented.

As mentioned earlier, the general assumption of the CAPM is *frictionless* capital markets. First of all, this means that there are no transaction costs. This is an assumption that eliminates the most basic form of market frictions - costs of trading. Cf. section 2.2.2, costs of trading is a source of illiquidity. Some trading costs affect the bid-ask spread. Another assumption violated in relation to liquidity is the assumption that an individual cannot affect the price of a stock by trading. If all stocks are not perfectly liquid, prices *can* be sensitive to individual orders. This has been referred to earlier as the price impact. Thinly traded stocks exhibit definite signs of illiquidity in that the price impact can be considerable. That is, unlimited amounts cannot be traded without moving the price. Also, all investors are assumed to have exactly the same preferences in relation to defining the relevant horizon and the relevant portfolio selection parameters. Generally, this is associated with the assumption of homogeneity of expectations. In reality, this assumption is far from met, and within this assumption, there are elements related to liquidity of stocks. If some stocks are more liquid (or marketable) than others, the less liquid stocks would be expected to be held by investors with longer investment horizons. In other words, stocks for which the market is not very deep will be allocated to more patient investors that are willing to wait for the higher returns.

To sum up, if all stocks are not perfectly liquid, it represents a violation of the assumptions of the CAPM. Thus, when empirically testing the CAPM, one would expect to see better results if accounting for liquidity. When, in the sections to come, the theoretical relationship between stock pricing and liquidity will be derived, focus will be on returns rather than absolute prices for testing purposes. That is, the testable implications of the CAPM makes it returns the starting point for the empirical tests later in this study. As the CAPM focuses on returns rather than prices so will the modelling of stock pricing and liquidity in the following.

2.6 A single-period investment horizon model for stock pricing with illiquidity costs

The models derived in this section and in the two next sections are based on the original article by Amihud and Mendelson (1986) and the review of the article by Amihud, Mendelson and Pedersen (2005). The notation will be somewhat different to support intuition and, in addition to this, the derivation of the multiple-period investment horizon model for stock pricing with illiquidity costs will also be somewhat different and more detailed. The implications of the models will, however, be the same.

The basic assumption is a stationary equilibrium with constant prices, P_{IL} , where investors hold the illiquid stock, IL , for one period each. That is, in the first period, an investor buys stock IL at a price of P_{IL} . After one period, the investor expects to receive a dividend payment of $\bar{d}_{IL}$ and sells the stock at the price of P_{IL} to another investor that also holds the stock for one period, etc. When selling the stock, the investors incur a cost of illiquidity of C_{IL} . This cost is increasing in illiquidity. The alternative to investing in the illiquid stock, IL , would be to invest in a perfectly liquid stock yielding a return of R_L . Markets are assumed to clear. That is, the holdings of all investors in each stock must sum to 100% of the market value of the equity of each stock. Short-sales are not allowed, and the investors cannot invest more than their total wealth (that is, no lending or borrowing). Also, amounts are perfectly divisible.

For each investor, the present value of the investment in stock IL will be the expected dividend plus the selling price minus the cost of illiquidity discounted at the required rate of return on the liquid stock. In a competitive equilibrium, this value will be equal to the price of the stock:

$$P_{IL} = \frac{\bar{d}_{IL} + P_{IL} - C_{IL}}{1 + R_L}$$

by rearranging, it follows that

$$P_{IL}(1 + R_L) = \bar{d}_{IL} + P_{IL} - C_{IL} \Leftrightarrow P_{IL} + P_{IL}R_L = \bar{d}_{IL} + P_{IL} - C_{IL} \Leftrightarrow P_{IL} = \frac{\bar{d}_{IL} - C_{IL}}{R_L}$$

The last term is the present value of a perpetuity. This means that the price of the illiquid stock is given by the present value of all future expected dividends minus the present value of all future expected illiquidity costs.

The gross return on the investment in the illiquid stock must be given by

$$R_{IL} = \frac{\bar{d}_{IL} + P_{IL} - P_{IL}}{P_{IL}} \Leftrightarrow R_{IL} = \frac{\bar{d}_{IL}}{P_{IL}}$$

From this, it follows that

$$P_{IL} = \frac{R_{IL}P_{IL} - C_{IL}}{R_L} \Leftrightarrow R_L = \frac{R_{IL}P_{IL} - C_{IL}}{P_{IL}} \Leftrightarrow R_L = R_{IL} - \frac{C_{IL}}{P_{IL}} \Leftrightarrow R_{IL} = R_L + \frac{C_{IL}}{P_{IL}}$$

In the last equation, it is apparent that the required rate of return on an investment in the illiquid stock, IL , is given by the return on an investment in the perfectly liquid stock plus a premium for illiquidity represented by the illiquidity cost relative to the price of the stock.

2.7 A multiple-period investment horizon model for stock pricing with illiquidity costs

In equilibrium, the price of a stock paying a constant perpetual dividend must be the present value of the expected dividend payments plus the present value of the liquidation value. The liquidation value is given by the probability of liquidation multiplied to the price subtracted the costs of illiquidity. Hence,

$$P_{IL} = \sum_{t=1}^{\infty} \left[\bar{d}_{IL} \frac{1}{1+R_L} + \bar{d}_{IL} \frac{(1-\mu)^t}{(1+R_L)^{t+1}} \right] + \sum_{t=1}^{\infty} \left[\mu(P_{IL} - C_{IL}) \frac{1}{1+R_L} + \mu(P_{IL} - C_{IL}) \frac{(1-\mu)^t}{(1+R_L)^{t+1}} \right]$$

The intuition goes: the first dividend is considered a sure cash flow, that is, liquidation is assumed not to take place before the first dividend payment, which makes good sense. Immediately after the first dividend payment, the stock is liquidated with probability μ and a liquidation value of $P_{IL} - C_{IL}$. In the following periods, the dividend payments will roll in with probability $(1-\mu)^t$, that is, if the position has *not* yet been liquidated, the dividend payments are received. Basically, the same argument goes for the liquidation value – if the position has *not* yet been liquidated, the position will be liquidated with probability μ . By rearranging the equation above, the possibility of applying some calculus to simplify is enabled

$$P_{IL} = \sum_{t=1}^{\infty} \bar{d}_{IL} \frac{1}{1+R_L} \left[1 + \left(\frac{1-\mu}{1+R_L} \right)^t \right] + \sum_{t=1}^{\infty} \mu(P_{IL} - C_{IL}) \frac{1}{1+R_L} \left[1 + \left(\frac{1-\mu}{1+R_L} \right)^t \right]$$

Now, $\bar{d}_{IL}/(1+R_L)$ and $\mu(P_{IL} - C_{IL})$ are independent of each term in the sums, so they can be put outside the two sums

$$P_{IL} = \bar{d}_{IL} \frac{1}{1+R_L} \sum_{t=1}^{\infty} \left[1 + \left(\frac{1-\mu}{1+R_L} \right)^t \right] + \mu(P_{IL} - C_{IL}) \frac{1}{1+R_L} \sum_{t=1}^{\infty} \left[1 + \left(\frac{1-\mu}{1+R_L} \right)^t \right]$$

As each of the two sums is the sum of an infinite geometrically growing series, the following calculus applies

$$S = \sum_{n=1}^{\infty} 1 + a^n = \frac{1}{1-a},$$

and it follows that,

$$P_{IL} = \bar{d}_{IL} \frac{1}{1+R_L} \cdot \frac{1}{1-(1-\mu)/(1+R_L)} + \mu(P_{IL} - C_{IL}) \frac{1}{1+R_L} \cdot \frac{1}{1-(1-\mu)/(1+R_L)},$$

Below, this is reduced to reach a relationship between the price of a stock and its costs of illiquidity that is very intuitive

$$P_{IL} = \frac{1}{1+R_L} \cdot \frac{1}{1-(1-\mu)/(1+R_L)} [\bar{d}_{IL} + \mu(P_{IL} - C_{IL})]$$

$$P_{IL} = \frac{1}{1+R_L} \cdot \frac{1}{(1+R_L)/(1+R_L) - (1-\mu)/(1+R_L)} [\bar{d}_{IL} + \mu(P_{IL} - C_{IL})]$$

$$P_{IL} = \frac{1}{1+R_L} \cdot \frac{1}{((1+R_L) - (1-\mu))/(1+R_L)} [\bar{d}_{IL} + \mu(P_{IL} - C_{IL})]$$

$$P_{IL} = \frac{1}{1+R_L} \cdot \frac{1+R_L}{1+R_L - (1-\mu)} [\bar{d}_{IL} + \mu(P_{IL} - C_{IL})]$$

$$P_{IL} = \frac{1}{R_L + \mu} [\bar{d}_{IL} + \mu(P_{IL} - C_{IL})]$$

Rearranging this yields

$$P_{IL} (R_L + \mu) = \bar{d}_{IL} + \mu(P_{IL} - C_{IL})$$

$$R_L P_{IL} + \mu P_{IL} = \bar{d}_{IL} + \mu P_{IL} - \mu C_{IL}$$

$$R_L P_{IL} = \bar{d}_{IL} - \mu C_{IL}$$

$$P_{IL} = \frac{\bar{d}_{IL} - \mu C_{IL}}{R_L}$$

This means that the price of a stock paying a constant dividend perpetually is given by the discounted dividend payments subtracted the expected costs of illiquidity given by the expected trading intensity multiplied by the costs of trading the stock.

From this, it follows that the expected gross return on stock IL is given by

$$\frac{\bar{d}_{IL}}{P_{IL}} = R_L + \mu \frac{C_{IL}}{P_{IL}}$$

This is very intuitive as it is now obvious, that the market-observed gross return on a stock should include a premium for the expected illiquidity of that stock.

2.8 A multiple-period investment horizon model for stock pricing with illiquidity costs in a capital market where investors have heterogeneous investment horizons

Investor j aims at achieving the highest possible return when taking into account liquidity

$$\max \frac{\bar{d}_{IL} - \mu_j C_{IL}}{P_{IL}}$$

To start with, two types of investors are considered - Type 1, which is an investor with a short investment horizon, and type 2, who has a longer investment horizon. Due to the fact that different investors have different time horizons, the expected future costs of illiquidity vary and thus, the return varies between different investors. First, the return for investor type 1 is given by

$$R_1 = \frac{\bar{d}_{IL} - \mu_1 C_{IL}}{P_{IL}},$$

meaning that,

$$P_{IL} = \frac{\bar{d}_{IL} - \mu_1 C_{IL}}{R_1}$$

By investing in the same stocks, investors of type 2 will earn a higher return after correcting for costs of illiquidity due to the fact that their frequency of trade is lower than that of investor type 1. The intuition behind this is that when an investor trades less often, the costs of illiquidity will be incurred less often, and thus, the return will be higher than for the investors with a higher frequency of trade. This becomes even more obvious when looking at the return for investors of type 2 from investing in the same securities

$$R_2 = \frac{\bar{d}_{IL} - \mu_2 C_{IL}}{P_{IL}},$$

now, using the above equation for the price, it follows that

$$R_2 = \frac{\bar{d}_{IL} - \mu_2 C_{IL}}{(\bar{d}_{IL} - \mu_1 C_{IL})/R_1} \Leftrightarrow R_2 = \frac{\bar{d}_{IL} - \mu_2 C_{IL}}{\bar{d}_{IL} - \mu_1 C_{IL}} R_1.$$

Given that $\mu_1 > \mu_2$, it follows that $\bar{d}_{IL} - \mu_2 C_{IL} > \bar{d}_{IL} - \mu_1 C_{IL}$, and thus $R_2 > R_1$.

Below, the above relationship between returns for investors of different types from investing in stock IL is generalised

$$R_j = \frac{\bar{d}_{IL} - \mu_j C_{IL}}{P_{IL}}$$

As $\mu_1 > \dots > \mu_j$ it follows that $R_1 < \dots < R_j$.

To summarise, investors with longer investment horizons will obviously have lower trading frequencies and thus, will incur less costs of illiquidity and will thus realise higher returns after illiquidity costs. Amihud, Mendelson and Pedersen (2005) refer to the higher returns after illiquidity costs of the investors with longer investment horizons as “rents”.

In an efficient market, all stocks will be priced subject to the minimum required return of all investor types. In other words – the investor that has the lowest required return will pay the most for a

stock and this will set the market price for that stock (driven by competition between investors). Formally put, the gross return on all stocks will be set by the following condition:

$$E(R_{IL}) = \frac{\bar{d}_{IL}}{P_{IL}} = \min \left(R_j + \mu_j \frac{C_{IL}}{P_{IL}} \right).$$

By minimising, the relationship becomes concave.

Given the relations presented above, it follows that,

$$E(R_{IL}) = R_L + (R_j - R_L) + \mu_j \frac{C_{IL}}{P_{IL}}$$

Overall, this means that each investor's expected gross return is increasing in the amortized illiquidity cost, and, investors with longer investment horizons are compensated with a higher expected return. Amihud, Mendelson and Pedersen (2005) call the higher return a "rent" for supplying "patient capital", which is a scarce resource in the financial markets. The idea that less liquid stocks will be allocated to investors with long investment horizons is called the clientele theory. It states that the illiquidity premium is mainly due to a compensation for being patient. For more liquid securities, the primary source of the liquidity premium is the amortised illiquidity costs. This is, again, due to the clientele effect – the liquid assets will be held by investors with very short trading horizons, and thus the cost of trading takes up a large fraction of the realised return.

2.9 Archaya and Pedersen's (2005) Liquidity-Adjusted CAPM and its implications

It sounds reasonable to imagine that a stock could go from being relatively illiquid to being more liquid over time. If this is the case and illiquidity attracts a premium in equity markets, the illiquidity premium can be time-varying. Recognising the fact that liquidity can change over time, investors should demand a premium for uncertain future liquidity of the stock. In addition to this, if liquidity affects stock prices, changes in liquidity can cause changes in stock prices and this means that time-varying liquidity can affect the standard deviation of the stock price. Thus, in the same manner that the beta of a stock represents the sensitivity of the returns of the stock towards changes in the market portfolio, one could imagine a "liquidity beta" - that is, the sensitivity of the returns of a stock towards changes in liquidity. The fact that time-varying liquidity can cause stock price volatility to increase could help explaining the increased volatility in equity markets following the credit crisis that began in 2007. One of the important aspects of the credit crisis has been the decreased liquidity in the financial markets. Following this, the uncertainty in relation to future liquidity has increased

dramatically. This can contribute to the explanation of the increased volatility of stock prices. An analysis of causes and consequences of the current credit crisis is not within the scope of this thesis. Also, as the crisis is still ongoing, it may be too early to conduct a thorough analysis of this. However, the topic will be touched upon later in part 6 Data.

In the following, a liquidity-augmented capital asset pricing model will be derived following Acharya and Pedersen (2005).

The basic idea is to examine the relationship between the expected gross return on a stock and its relative illiquidity costs, the market return, and the relative market illiquidity costs. In the following, the earlier defined less than perfectly liquid stock, IL , will be replaced by stock i for simplicity.

For stock i , the expected gross return at time t is given by the dividend yield plus the relative price change of the stock

$$E(R_{i,t}) = \frac{d_{i,t}}{P_{i,t-1}} + \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \Leftrightarrow E(R_{i,t}) = \frac{d_{i,t} + P_{i,t}}{P_{i,t-1}} - 1,$$

and the relative illiquidity cost of stock i is given by

$$c_{i,t} = \frac{C_{i,t}}{P_{i,t-1}}.$$

The expected gross return on the market is determined as the sum of the dividend yields and the relative capital gains of all stocks in the market

$$E(R_{M,t}) = \frac{\sum_i S_i d_{i,t}}{\sum_i S_i P_{i,t-1}} + \frac{\sum_i S_i (P_{i,t} - P_{i,t-1})}{\sum_i S_i P_{i,t-1}} \Leftrightarrow E(R_{M,t}) = \frac{\sum_i S_i (d_{i,t} + P_{i,t})}{\sum_i S_i P_{i,t-1}} - 1,$$

and the relative illiquidity cost of the market is

$$c_{M,t} = \frac{\sum_i S_i C_{i,t}}{\sum_i S_i P_{i,t-1}}$$

Acharya and Pedersen (2005) argue that the traditional CAPM in an economy where its assumptions are fulfilled translates into a CAPM in net returns in an economy with frictions (illiquidity costs). By including the relative illiquidity costs, it follows that

$$E_t(R_{i,t+1} - c_{i,t+1}) = R_F + E_t(R_{M,t+1} - c_{M,t+1} - R_F) \frac{\text{cov}_t(R_{i,t+1} - c_{i,t+1}, R_{M,t+1} - c_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})}$$

Next, they go on to break up the covariance term into four different covariances

$$\begin{aligned}
& \frac{\text{cov}_t(R_{i,t+1} - c_{i,t+1}, R_{M,t+1} - c_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})} \\
&= \frac{\text{cov}_t(R_{i,t+1}, R_{M,t+1}) + \text{cov}_t(c_{i,t+1}, c_{M,t+1}) - \text{cov}_t(R_{i,t+1}, c_{M,t+1}) - \text{cov}_t(c_{i,t+1}, R_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})} \\
&= \frac{\text{cov}_t(R_{i,t+1}, R_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})} + \frac{\text{cov}_t(c_{i,t+1}, c_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})} - \frac{\text{cov}_t(R_{i,t+1}, c_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})} - \frac{\text{cov}_t(c_{i,t+1}, R_{M,t+1})}{\text{var}_t(R_{M,t+1} - c_{M,t+1})}
\end{aligned}$$

Each of these four terms are specified as the betas β_t , $\beta_{t,L1}$, $\beta_{t,L2}$, and $\beta_{t,L3}$, respectively. Hence, the intuition is that there are three liquidity betas that complement the market beta. By using these betas and rearranging the liquidity-augmented capital asset pricing slightly, it follows that

$$E_t(R_{i,t+1}) = R_F + E_t(c_{i,t+1}) + E_t(R_{M,t+1} - c_{M,t+1} - R_F)(\beta_t + \beta_{t,L1} - \beta_{t,L2} - \beta_{t,L3})$$

This model introduces three kinds of liquidity risk in addition to the standard market risk. As β_{L1} comprises the covariance between the liquidity of stock i and the market liquidity, it measures the extent to which the liquidity of the stock is exposed to the liquidity of the market. It can be expected that this relationship is positive – that is, when market liquidity goes down you would expect to see the liquidity of the stock decrease as well. This phenomenon is referred to as *commonality in liquidity*. β_{L2} is the liquidity risk associated with the covariance between returns of stock i and the market illiquidity costs. The intuition behind the negative sign on this beta is as follows. If the covariance between the returns of stock i and market illiquidity is high, it means that the returns of this stock are high, when market illiquidity is high. This is an attractive feature for the investors and it should make them require a lower return for that stock – hence, the relationship between expected returns and the second liquidity beta is expected to be negative. The impact of the third liquidity beta, β_{L3} , is a result of the covariance between the illiquidity costs of stock i and the market returns. The effect of this risk is expected to be negative due to the following reasoning. If the covariance between the illiquidity of stock i and the market returns is high, it means that the stock i shows increasing liquidity in a decreasing market. For investors, the value of having liquid stocks in a down market is high – thus, investors would be expected to demand a lower rate of return on a stock with this characteristic. This conclusion is very intuitive as it obviously is a nice feature for investors to have readily sellable assets in times of market downturn.

Archarya and Pedersen's (2005) model and beta measures will not be applied in the present study. However, the basic idea of their model will be used in the sense that the concept of liquidity risk will be examined. In the empirical study of this thesis, the exposure of stock portfolios towards

changes in their liquidity will be determined. Next, the premium for high liquidity sensitivity will be estimated.

2.10 Liquidity risk

Liquidity risk was brought up in the previous section as an additional risk factor that could be separated into several components. If liquidity risk is a factor of importance to investors, it should be priced. The question is whether it makes sense to speak of liquidity risk. In the previous section, a model was set up that showed that less than perfectly liquid stocks should attract a return premium that is increasing in illiquidity. It also makes sense to imagine that the liquidity of a stock could change over time. Changes in liquidity should thus cause changes in the required rate of return by the investors. If some stocks are more prone to liquidity changes than others, it follows that these stocks should be expected to yield higher returns as a compensation for the additional risk they carry.

In a CAPM setting this would involve the estimation of a liquidity sensitivity in the same manner as the sensitivity of a stock towards the excess market return is estimated (this is the beta). Next, the stocks that are highly sensitive to liquidity should yield a return premium for this additional risk. If this holds, the CAPM should include a liquidity risk premium. This will be tested later in the empirical study.

Another way to consider liquidity risk is to see liquidity as a market-wide factor. That is, instead of looking at the liquidity of each stock, one could look at the liquidity of all stocks in the market. Stocks that are more exposed to changes in the market-wide liquidity can be considered more risky and should be priced at a premium. The relationship between market-liquidity exposure and returns will also be examined in the empirical study later in the thesis. This is also one of the implications of Archarya and Pedersen's (2005) model presented above.

2.11 A brief discussion of investment horizons

In this section, the assumptions about investment horizons made when deriving the models above will be discussed.

2.11.1 What if the investment horizon is uncertain?

Earlier, sudden liquidity shocks have been mentioned. The fact that these shocks are stochastic results in the investment horizons being stochastic. The reason for this is that the stochastic liquidity

shocks make the investors liquidate (change the investment horizon) stochastically. Stochastic trading makes the illiquidity costs occur stochastically, and thus, the returns net of illiquidity costs also become stochastic. This is an additional risk that was not taken into consideration in the first model. Huang (2003) shows that stochastic net returns will increase the illiquidity premium.

2.11.2 What if the investment horizon is not given exogenously?

If investors set investment horizons reflecting optimal portfolio choices, the investment horizons are endogenously determined. In the presence of illiquidity costs, the investor faces the portfolio choice in case of a suboptimal mix of stocks of either rebalancing the portfolio and incurring illiquidity costs of trading the illiquid stocks in the portfolio or remaining passive and incurring the opportunity costs of holding a suboptimal mix. Constantinides (1986) uses this rationale to reach an equilibrium condition with a definition of the illiquidity premium as the decrease in expected return on an illiquid asset that would make the investor indifferent between the illiquid asset and a liquid asset.

3 Empirical evidence

In the following, selected empirical studies from different articles will be reviewed. The reviewed empirical evidence is from the studies by Amihud and Mendelson (1986), Datar, Naik and Radcliffe (1998), Chan and Faff (2005), and Archarya and Pedersen (2005). A common denominator for these articles is that they contribute with strong evidence for the effect of liquidity or liquidity risk in stock pricing. In addition to this, these articles have been chosen because they, in different ways, have contributed with something new to the field of liquidity and stock pricing. The first study reviewed is chosen because Amihud and Mendelson were the first to model a relationship between liquidity and stock returns and to provide evidence of the stated relationship. Next, Datar et al. were the first to proxy liquidity by the turnover rate and test its effect on stock returns. As the turnover rate will be applied later in the empirical study of this thesis, it seems obvious to review their empirical study and their findings. The study of Chan and Faff reveals significant findings, the methodology is very interesting, and the study is based on non-U.S. data which is relatively rare and of additional interest, as liquidity could be expected to be even more important in non-U.S. markets. Finally, another recent article has presented the most sophisticated and rigorous approach to modeling and providing evidence of the pricing of liquidity yet. The evidence from the study by Archarya and Pedersen (2005) will be presented at the end of this part.

3.1 Amihud and Mendelson (1986) - "Asset pricing and the bid-ask spread"

Amihud and Mendelson (1986) were the first to investigate the role of liquidity in asset pricing. In their empirical study, they test the hypotheses that the market-observed expected return is an increasing function of the relative bid-ask spread and that this function is concave. Their findings provide evidence of these hypotheses.

3.1.1 Methodology

They test the hypotheses following the methodology of Fama and MacBeth (1973) for cross-sectional regressions. They apply this methodology for estimating the cross-sectional relationship between return, market risk and spread for portfolios of stocks. These portfolios were formed on the basis of individual stock betas (market risk exposure) and the relative bid-ask spread of the stocks.

3.1.2 Data

The data sample for their empirical study covers the period from 1960 to 1980 and consists of stocks listed on the NYSE. Stock returns with a monthly frequency and yearly bid and ask prices were obtained.

As mentioned, the cross-sectional analysis was carried out on portfolios of stocks rather than individual stocks. They divide the data into twenty overlapping eleven-year periods. The eleven-year periods comprise a five-year beta estimation period, a five-year portfolio beta estimation period, and finally, a test period. When betas had been estimated, all stock were ranked by their relative spreads and divided into seven portfolios of equal size. Next, all stocks of each of these groups were then ranked by their betas and subdivided into seven equally sized portfolios. The betas for these portfolios were then estimated over the next five years. Finally, the relative bid-ask spread as of the beginning and end of the last year of the portfolio beta estimation period were averaged to obtain a measure of the illiquidity of each portfolio. This measure was then used in the following test period. By doing so, their model became predictive in nature. The average of the coefficients estimated in the test periods were then averaged to obtain the final results of the cross-sectional test.

Using this methodology, the return-spread relationship can be directly tested. To test the concavity of the return-spread relationship, dummy variables are used for each spread portfolio. This allows for different slope coefficients between different portfolios.

3.1.3 Findings

When examining the summary statistics for all the 49 portfolios, Amihud and Mendelson found two things. First of all, the returns seemed to be increasing in the spread and, secondly, the increase in return was declining when moving to higher spread portfolios.

Through the cross-sectional tests, they find that a 1% increase in the relative bid-ask spread is associated with a 0.211% increase in the monthly risk-adjusted excess return. Also, they find that the slope coefficients of the spreads are positive and generally decreasing in the spread. This means that the results imply that there is an increasing and concave connection between returns and spreads.

To test the robustness of the results and to find out whether the findings stem from the "small firm effect" rather than illiquidity, all models were re-estimated including a size variable (the natural log of the market value of the equity of the companies). This reason for examining this is that previous

research has provided evidence that stocks of small firms yield higher returns, and also, that stocks of small firm typically are traded at higher relative bid-ask spreads. Thus, the identified spread premium could, in fact, be a small firm premium. After having included the size variable, the effects of the beta and spread of the stock portfolios were still significant, and the size variable had no effect. Thus, their findings are robust and not due to the size effect.

3.2 Datar, Naik and Radcliffe (1998) - "Liquidity and stock returns: An alternative test"

Datar et al. tested the role of liquidity in stock pricing using a new proxy for liquidity - the turnover rate. This rate is given by the number of shares traded as a fraction of the number of shares outstanding. They basically apply the same methodological framework as Amihud and Mendelson (1986) but with the addition of the book-to-market ratio of the stocks. An important difference between this study and most other empirical studies of stock returns is that the analysis is based on individual stocks rather than portfolios of stocks.

3.2.1 Methodology

The econometrical framework is the Litzenberger and Ramaswamy (1979) refinement of the Fama and Macbeth methodology. As mentioned, the analyses are not based on stock portfolios but individual stocks. This could be rather problematic when it comes to estimating betas as the betas would be estimated with a relatively high level of white noise due to potential measurement errors. They deal with this problem in a rather untraditional way - they form portfolios and assign portfolio betas to all stocks within each portfolio.

3.2.2 Data

For the empirical study, they consider monthly frequency data for all stocks of non-financial companies on the NYSE from July 1962 through December 1991.

They calculate the turnover rate for each period as the average number shares traded over the preceding three months divided by the number of shares outstanding as of the period. They exclude (for three months) stocks for which the number of shares outstanding has changed during the preceding three months. They trim the data set by discarding the stocks with 1% lowest and highest turnover rates to avoid extreme observations.

As mentioned, they assign portfolio betas to all stocks within each portfolio. The way this is done is that they form portfolios following the approach of Amihud and Mendelson (1986) (liquidity and beta portfolios), they calculate the betas of these portfolios of stocks and then assign the portfolio betas to each stock in the corresponding portfolio. This is a highly unorthodox way to compute stock betas as in that way, stocks can be assigned betas that are relatively different from their "true" betas. On the face of it, one would not expect this to yield significant findings but the method does remove a lot of variation that basically is white noise. It is still valid, though, that this way of assigning betas to stock can be criticised.

3.2.3 Findings

First of all, they find that there is a significantly negative relationship between liquidity and stock returns. This is in accordance with theory - less liquid stocks should yield higher returns to compensate for the higher degree of illiquidity. Thus, a stock with a low turnover rate should yield a return premium. This is what their findings show. They also provide evidence that the effect of the turnover rate on stock returns is robust to the presence of the control variables the natural log of firm size (market value of equity) and book-to-market value. They also conduct various other robustness checks - they run the analysis on the untrimmed data set, they calculate the turnover rate in different ways and they divide the sample into two sub-periods. Their conclusion regarding the effect of the turnover rate on stock returns remains essentially unchanged - that is, stock returns are negatively related to the turnover rate. Therefore, the identified relationship seems robust. Their findings imply that, across stocks, a 1% decrease in the in the turnover rate should result in a higher return of 4.5bp

3.3 Chan and Faff (2005) - "Asset pricing and the illiquidity premium"

Taking departure in the Fama and French (1993) three-factor model, Chan and Faff (2005) investigates the role of liquidity in stock pricing by adding the return on a mimicking liquidity portfolio to the model. Liquidity is proxied by the share turnover rate. They test the four-factor model for over-identifying restrictions and reject this - hence, they find support for adding a liquidity factor to the Fama and French (1993) three-factor model.

3.3.1 Methodology

Just as the approach of Amihud and Mendelson (1986), the dependent variables of their analysis are excess returns of portfolios of stocks rather than individual stocks. These portfolios are based on size, book-to-market and liquidity. The independent/explanatory variables in their study are mimicking portfolios. This approach is known from the influential study of Fama and French (1992).

Following a mimicking portfolio approach means to form different kinds of portfolios to replicate effects of different factors that could explain returns. This idea follows the no-arbitrage arguments - the returns of risky investments should be possible to replicate by investing in assets that as a whole has the same expected future cash flows. The mimicking portfolios represent portfolios where the investor mimics risk by taking long and short positions in other assets.

3.3.2 Data

Their study employs monthly data for the period from 1989 through 1998 for listed Australian companies as of 2005. By only including the stocks of the companies active as of the time of the analysis, the well-known survivorship bias becomes a serious issue. The effect of the bias in examining the effect of liquidity in stock pricing could be that the effect becomes exaggerated. The reason is that small, illiquid stocks yielding highly negative returns until their death are excluded from the sample because they are no longer active. Only the surviving companies are left - small, illiquid stocks that has not died must be really well-performing and thus, the illiquid stocks included are only the high-return stocks - the downside of the high-return (that many of these companies do not make it) is excluded. Thus, the estimated return premium for less liquid stocks will be exaggerated.

Apart from this, it is very interesting to see a study that is not based on U.S. data due to the fact that many of the researchers behind the previous studies based on U.S. data point out that NYSE-stocks are generally very liquid and that the effect of liquidity could be expected to be much bigger in foreign stock markets.

The stock portfolios were formed in a trivariate way to reach 27 portfolios.

The mimicking portfolios for size (SMB) and book-to-market (HML) were formed in the exact same way as Fama and French (1993).

The mimicking portfolio for the liquidity factor was denominated IMV (for Illiquid Minus Very liquid). That is, the return on a portfolio of a long position in highly illiquid stocks and a short position in very liquid stocks. By expectation, the return on such a portfolio should be positive. It is calculated as the simple average return of all the six portfolios of the less liquid stocks minus the simple average return of the six most liquid stock portfolios.

3.3.3 Findings

Their empirical analysis reveals several interesting findings relating to the pricing of liquidity. The majority of the liquidity betas estimated are statistically significant, meaning that the share turnover seems to have an effect on stock returns. In addition to this, they find that there is a tendency towards less liquid stock portfolios having significantly positive liquidity betas and the more liquid stock portfolios having significantly negative liquidity betas. The main result of their study is that they find support for adding the liquidity factor to the Fama and French (1993) model. They identify an annualised turnover rate risk premium of more than 20%. Their findings are robust and provide strong evidence of the pricing of liquidity in the Australian equity market.

3.4 Archarya and Pedersen (2005) - *"Asset pricing with liquidity risk"*

Archarya and Pedersen's (2005) liquidity-adjusted CAPM was presented in the theoretical part of this thesis. It basically follows from this model that the expected return on a stock depends on its expected liquidity, the covariance between its own return and market liquidity, the covariance between its own liquidity and market liquidity, and the covariance between its own liquidity and market returns. This model is by far the most sophisticated approach to modelling the liquidity risk and return relationship yet. It enables the possibility of understanding the different sources of liquidity risk and their effect on stock returns. Using Amihud's (2002) ILLIQ-measure, they conduct an empirical test of this model which will be reviewed in the following.

3.4.1 Methodology

Their model is tested in five steps

- Illiquidity estimation: The measure of illiquidity for each stock is calculated
- Portfolio formation: 25 portfolios based in turn on illiquidity, illiquidity variation, size and book-to-market values of the individual stocks (that is 25 portfolios for each variable)
- Illiquidity innovation estimation
- Estimation and analysis of the liquidity betas: This is done using the portfolio illiquidity innovations and returns
- Cross-sectional regressions

3.4.2 Data

The data sample for the empirical study consists of daily return and volume data for all common stocks listed on the NYSE and AMEX for the period from July 1962 to through 1999.

First, Amihud's ILLIQ-measure is calculated for each stock on a daily basis and a market portfolio is formed for each month. Next, 25 portfolios are formed for each of the variables. That is, 25 portfolios are formed on the basis of illiquidity, 25 portfolios based on illiquidity variation, 25 portfolios based on size and book-to-market. All portfolios are value-weighted, but as a robustness check, the analyses were also carried out using equal weights. For all these portfolios, the innovations in liquidity were calculated. Next, the four liquidity betas (the different kinds of liquidity risk) were calculated, and, finally, the cross-sectional regressions were run.

3.4.3 Findings

Regarding the descriptive statistics of the liquidity risk (the liquidity betas), the authors found that relatively illiquid stocks generally had high volatility of returns, low turnover, a low market value of the equity and, most importantly, a high liquidity risk. This means that when markets turn illiquid, these stock become even more illiquid and their sensitivity towards market liquidity of the portfolio returns is high. This is an indication of the "flight-to-liquidity" phenomenon. This will also be discussed later in this thesis.

Through the cross-sectional tests the authors find strong and robust evidence of a pricing of both the level of liquidity and the liquidity risk. They estimate that the total effect of liquidity risk is 1.1% (annualised) and that the annualised return effect of the level of the expected liquidity is 3.5%. This sums to an overall effect of expected illiquidity and liquidity risk of 4.6% per year.

4 Empirical methodology

In this part, the econometric methodology for the empirical study will be presented. First, the Fama-MacBeth (1973) methodology will be presented briefly to establish a framework for carrying out the empirical tests. Next, this framework is expanded to being a CAPM including additional explanatory variables (so, actually, it becomes an APT model).

4.1.1 Fama-MacBeth (1973)

A cross-sectional multiple regression analysis will be carried out. The starting point is the methodology of Fama and MacBeth (1973). They developed the cross-sectional regression approach for testing the CAPM hypothesising that stock betas explain the variation in expected stock returns. Following a two-step procedure, they first estimate the betas for the individual stocks, and then, for each time period, t , regressions of the individual stock returns on their betas are run. The intercept and the slope coefficient are estimated as the means of the intercepts and slope coefficients, respectively, from the cross-sectional regressions.

After having estimated the betas for all stocks, the first step is to run the regression

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_i + \varepsilon_{it}$$

for each time period, t .

In the above regression, R_{it} is the excess return on stock i at time t over the risk free rate of return and β_i is the beta of stock i estimated using the standard CAPM. This will give a time series of gammas for each time period in the data sample (consisting of T time periods).

The second step of the Fama-MacBeth procedure is to estimate the two gammas, γ_0 and γ_1 by calculating the simple average of all the gammas for each time period. That is, γ_0 and γ_1 are given by

$$\hat{\gamma}_j = \frac{1}{T} \sum_{t=1}^T \hat{\gamma}_{jt}, \quad j = 0, 1.$$

The variance of the parameter estimates is given by

$$\hat{\sigma}_{\hat{\gamma}_j}^2 = \frac{1}{T(T-1)} \sum_{t=1}^T (\hat{\gamma}_{jt} - \hat{\gamma}_j)^2, \quad j = 0, 1,$$

and

$$\hat{\sigma}_{ij}^2 = \frac{1}{T(T-1)} \sum_{t=1}^T (\hat{\gamma}_{jt} - \hat{\gamma}_j)^2, \quad j = 0, 1.$$

The implications of this model are the standard CAPM predictions - the expected value of γ_0 should be zero and γ_1 , the implied market risk premium, should be significantly larger than zero. This is tested by applying the standard t-test

$$t(\hat{\gamma}_j) = \frac{\hat{\gamma}_j}{\hat{\sigma}_{\gamma_j}}, \quad j = 0, 1.$$

Errors-in-variables imposed by betas

When using the Fama-MacBeth methodology for testing the CAPM, an errors-in-variables problem is introduced. The CAPM is tested on the basis of betas. These betas are, however, not market observed variables that can just be applied like any other exogenous variable. You can say that the betas used are not the true betas. The betas are estimated. The fact that the betas are estimated on basis of the data sample rather than observed in the market introduces an errors-in-variables problem. There are two ways to minimise the effect of this problem - the formation of portfolios (the Fama and MacBeth (1973) solution) and to adjust the variance of the final estimates explicitly. In this thesis, both of these methods will be applied to deal with the bias. The explicit adjustment to the final estimates of the model is known as the Shanken (1992) correction⁹:

$$\hat{\sigma}_{ij}^{2*} = \hat{\sigma}_{ij}^2 \left(1 + \frac{(\hat{\mu}_m - \hat{\gamma}_0)^2}{\hat{\sigma}_m^2} \right)$$

This correction of the variance of the estimated coefficients will be carried out in order to eliminate the errors-in-variables in the t-statistic¹⁰.

The goodness of fit measures (R^2) will be calculated in accordance with Jagannathan and Wang (1996):

$$R_C^2 = \frac{\sigma_{C, \bar{R}i}^2 - \sigma_{C, \bar{\varepsilon}i}^2}{\sigma_{C, Ri}^2}$$

where

$$\bar{\varepsilon}_i = \text{Average residual for portfolio } i,$$

⁹ The Shanken-correction by Shanken (1992) is a modification of a correction proposed by Litzenberger and Ramaswamy (1979)

¹⁰ As explained in Campbell, Lo and MacKinlay (1997))

$\sigma_{C,\bar{R}i}^2$ = The cross-sectional variance of the average portfolio return,

$\sigma_{C,\bar{\varepsilon}i}^2$ = The cross-sectional variance of the average portfolio residual,

4.1.2 Using the Fama-MacBeth procedure to detect the effect of liquidity on stock returns

The above econometrical framework can easily be expanded to include more factors in stock pricing than just the market risk premium. By doing so, the regressions are no-longer simple CAPM regressions but rather APT regressions¹¹.

As mentioned, Amihud and Mendelson (1986) used the Fama-Macbeth (1973) procedure to study the relationship between the relative bid-ask spread and stock returns. To examine whether less liquid stocks attract return premiums in the Danish equity market, this study will apply a methodology similar to that of Amihud and Mendelson (1986). In specific, a four-factor CAPM is proposed consisting of three risk factors and a measure of liquidity. In that way, the direct effect of liquidity on stock returns will be determined. The risk factors will be those of the Fama and French (1993) three factor CAPM where, in addition to the excess return on the market portfolio, the returns of an SMB¹² and an HML¹³ portfolio, respectively, are to explain the cross-section of stock returns. A detailed description of the SMB and the HML portfolios as well as the measure of liquidity follows in part 6 Data.

In the following, the description of the estimation will be on the basis of portfolios of stocks rather than individual stocks as will be explained later. A thorough description of the portfolio formation process can be found in the part 6 Data.

The three factor sensitivities are estimated in the following OLS regression framework

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$$

for each portfolio i for each time period t .

Next, these portfolio parameter estimates / factor sensitivities (β_i , s_i , and h_i) are used in the first step OLS regressions of the Fama-MacBeth methodology together with the liquidity of each portfolio i calculated as the average of the liquidity proxy over the preceding period ($t-1$).

¹¹ APT is short for Arbitrage Pricing Theory, Ross (1976)

¹² SMB = "Small Minus Big", the return on a portfolio which is long in small stocks and short in large stocks

¹³ HML = High Minus Low", the return on a portfolio which is long in high book-to-market stocks and short in low book-to-market stocks

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}s_i + \gamma_{3t}h_i + \gamma_{4t}LIQ_i + u_{it}$$

for each time period, t . It should be noticed that the use of the liquidity of each portfolio for the period preceding the test period makes the model predictive in nature.

The first gamma, γ_0 , does not theoretically represent anything and should thus be zero as explained above. The next three gammas (γ_1 to γ_3) represent risk premiums and should therefore be positive while the fifth gamma, γ_4 , represents the cross-sectional effect of liquidity on the excess returns. The expected sign of the liquidity effect depends on the measure of liquidity used. If the measure is a proxy for liquidity (e.g. the turnover rate), the sign should be negative due to the fact that the less liquid the portfolio, the higher the required return. If the applied measure is a proxy for illiquidity (e.g. the relative bid-ask spread), the sign should be positive, implying that the more illiquid the higher the required return.

The second step of the Fama-Macbeth procedure can now be undertaken to obtain estimates of each gamma by averaging the time series of each parameter estimate as defined above. Also, the variances and t-statistics are calculated as presented above, and the Shanken correction is carried out to obtain variances that are unbiased.

4.1.3 Using the Fama-MacBeth procedure to detect the effect of liquidity risk in stock pricing

In addition to examining the direct effect of liquidity on stock returns, this thesis will also comprise an empirical analysis of the relationship between liquidity risk and stock returns.

This study hypothesises that the cross-section of stock returns should be explained by four factors - the Fama and French (1993) three factor CAPM added a liquidity factor. The analysis will still be based on portfolios of stocks rather than individual stocks.

The regression for the estimation of the four factor sensitivities is

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{Mt} - R_{ft}) + s_iSMB_t + h_iHML_t + l_iLIQ_{it} + \varepsilon_{it}$$

for each portfolio i at each time t . Notice that, contrary to above, the liquidity measure is already included in the estimation of portfolio characteristics. Thus, now the sensitivity towards changes in the liquidity of a portfolio will be used in the first-step regressions of Fama-Macbeth. As the liquidity measure now is transformed into a sensitivity rather than a level, the analysis will now regard liquidity a risk factor.

Next, these portfolio parameter estimates / factor sensitivities (β_i , s_i , h_i , and l_i) are used in the first step regressions of the Fama-MacBeth methodology

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}s_i + \gamma_{3t}h_i + \gamma_{4t}l_i + u_{it}$$

for each time period, t .

The regression parameters are risk premiums that follow the risk exposure of the portfolio. Thus, portfolios with higher sensitivity towards one or more of the four factors will be expected to yield higher returns due to the risk associated with this exposure. This is easy to see by looking at the standard CAPM - here, stocks that are more sensitive to changes in the market returns (high beta stocks) are expected to yield higher returns. The same accounts for other risk factors than the market portfolio.

Finally, as described above, the factor returns (gammas) are estimated as the average of the time series of each parameter estimate as defined above. The variances of the parameter estimates and the t-statistics are calculated as presented above. The errors-in-variables problem described above is also valid for other factor sensitivities than the estimated beta. Therefore, the Shanken correction is also applied to the variance of the three additional parameter estimates.

4.1.4 A comment on residual analysis

To perform an analysis of the residuals to check whether the assumptions of the models and the assumptions for t-tests are fulfilled in the cross-sectional frameworks described above, it would be necessary to perform the analysis for each time-point regression. This would mean that for each model, 143 analyses would have to be made. Instead, an approximated analysis of residuals could be applied. The simple average of all estimated coefficients for the entire test period could be calculated and then applied as portfolio characteristics that should explain the cross-section of portfolio mean returns. That is, for each portfolio, the average return is calculated and regressed on the average characteristics. From this simple regression the residuals could be calculated and analysed. This can be regarded an approximation of the residual analysis of the model, and it will be applied for all tested models.

5 Hypotheses

After having introduced the main econometric frameworks that will be applied in the empirical study later, some hypotheses regarding the estimated gammas in the regressions above will now be stated. As the empirical analysis comprise two different cross-sectional tests - investigating both the direct relationship between liquidity and stock returns and the pricing of liquidity risk, the statement of hypotheses is split into two separate parts.

5.1.1 The cross-sectional relationship between liquidity and stock returns

This was basically explained above. As the model is stated in excess returns, the intercept term, γ_0 , should be zero as mentioned above. If the model had been stated in just returns, the intercept term should be equal to the risk free rate of return. If the intercept turns out to be significantly different from zero, it represents a violation of the model and it indicates that there is something left for the model to explain. Cf. above, the model is given by

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}S_i + \gamma_{3t}h_i + \gamma_{4t}LIQ_i + u_{it}.$$

Therefore, the null hypothesis is:

$$H_0 : \gamma_0 = 0.$$

Against the alternative hypothesis that the intercept term is significantly different from zero:

$$H_A : \gamma_0 \neq 0.$$

This means that the significance of the intercept term is tested against a two-sided alternative.

The next three gammas are risk premiums in that they denote the additional return that will be required for each unit of the factor sensitivity (risk). The more sensitive the return on a portfolio is to changes in some exogenous variables, the more return will be required. That is, when the sensitivity of the excess returns of a portfolio towards either the excess market return or the returns of the SMB or HML portfolios increase, the excess return is expected to increase. Thus, the null hypothesis is that the return premium due to these risk factors is significantly higher than zero:

$$H_0 : \gamma_j > 0, j = 1, \dots, 3.$$

Against the alternative hypothesis that the risk premiums are *not* significantly larger than zero:

$$H_A : \gamma_j \leq 0, j = 1, \dots, 3.$$

Hence, as theory predicts that the three factor sensitivities attract premiums (risk premiums) the implications of the three gammas will be tested in one-sided tests.

Finally, and most importantly in the present context, the last gamma, as explained, represents the effect of either liquidity or illiquidity on stock returns, depending on the measure used. The relationship between illiquidity and stock returns are expected to be positive as predicted by theory, and the relationship between liquidity and stock returns should be negative.

For measures of illiquidity, the hypothesis for gamma four is

$$H_0 : \gamma_4 > 0 ,$$

against the alternative hypothesis that the effect of illiquidity on stock returns is *not* significantly larger than zero

$$H_A : \gamma_4 \leq 0 ,$$

For measures of liquidity, the hypothesis for gamma four is

$$H_0 : \gamma_4 < 0 ,$$

against the alternative hypothesis that the effect of liquidity on stock returns is *not* significantly negative:

$$H_A : \gamma_4 \geq 0 ,$$

As theory predicts a certain relationship between liquidity or illiquidity and stock returns, the applied t-tests will be one-sided.

5.1.2 The pricing of liquidity risk

Cf. the previous section, the model is given by

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}s_i + \gamma_{3t}h_i + \gamma_{4t}l_i + u_{it} .$$

Regarding the intercept term, the hypothesis is the same as above:

$$H_0 : \gamma_0 = 0 ,$$

against the alternative hypothesis that the intercept term is significantly different from zero:

$$H_A : \gamma_0 \neq 0 .$$

Again, the significance of the intercept term is tested against a two-sided alternative.

The other four gammas, γ_1 , γ_2 , γ_3 and γ_4 , all represent risk premiums in that they denote the additional return that will be required for each unit of the factor sensitivity (risk). The more sensitive the return on a portfolio is to changes in some exogenous variables, the more return will be required. That is, when the sensitivity of the excess returns of a portfolio towards either the excess market

return, the return on the SMB or HML portfolios or the liquidity of the portfolio, the excess return is expected to increase. Thus, the null hypothesis is that the return premium due to these factors is significantly higher than zero:

$$H_0 : \gamma_j > 0, j = 1, \dots, 4,$$

against the alternative hypothesis that the risk premiums are *not* significantly larger than zero:

$$H_A : \gamma_j \leq 0, j = 1, \dots, 4.$$

Hence, as theory predicts that the four factor sensitivities attract risk premiums the implications of the four gammas will be tested in one-sided tests.

6 Data

In this part, the data sample will be described and the computation of variables will be presented. First, the data sample and its practical limitations will be gone through. Expecting to obtain too much white noise if testing the model on individual stocks and to minimise the errors-in-variables in the beta estimation, the tests of the empirical study will be based on portfolios of stocks rather than individual stocks. Following the data presentation will be a description of the derivation of variables. Next, the beta estimation process, the portfolio formation process and finally, the cross-sectional testing process will be described.

6.1 Description of data

6.1.1 Stock data

The data set used in this thesis covers all non-financial listed companies in Denmark from January 1, 1991 through November, 2008. With a monthly frequency, data on the traded volume, the number of shares outstanding, the total return index, market-to-book ratio, size (market capitalisation), and the bid and ask prices has been collected from Thomson's Datastream. By including all companies and not just those active as of November, 2008, the effect of the well-known survivorship bias is reduced. Had only the companies active as of November 2008 been included, the return premium for illiquidity could have been overstated. The reason originates from the small firm effect. That is, stocks of smaller firms tend to yield higher returns. It seems reasonable to expect that the more liquid stocks are the stocks of large firms. Also, it seems reasonable to expect that smaller firms are more likely to go bankrupt than large firms for example due to the fact that larger firms have easier access to capital. If there is a higher fraction of small firms going bankrupt than large firms, leaving out stocks of bankrupt companies would thus mean to exclude many cases of highly negative returns for the small firms. This would exaggerate the expected return on small firm stocks. Only the primary quote has been included – that is, for example, OTC and foreign quotes are not included. The data set has, however, not been limited to the major securities – this means that all the different classes of shares listed for a company has been included. The argument for doing so is that one of the share classes often is much less liquid than the other. Thus, it is interesting to use these stocks as well because return differences between the stocks could be expected and therefore, they could contribute considerably to providing support for the pricing of liquidity.

The data selection process was as follows. All stocks with no information on the return have been deleted from the sample (120 deleted, n=437). Also, due to the fact all portfolios will be value-weighted; all stocks with missing information on the market value of the equity have been deleted. In the sample, there were a few observations which were unidentifiable as specific company stocks and were thus regarded errors and deleted (3 deleted, n=434). Finally, Companies in the financial sector removed (141 deleted, n=293).

6.1.2 The risk-free rate of return

The one-month CIBOR will be used as a proxy for the risk free rate of return. The one-month maturity has been chosen to match the frequency of the data. Data for the one-month CIBOR is available from July 1988. Before that the official discount rate of the National Bank of Denmark has been used. The rates are quoted in annual terms, so for the analysis, the rates are transformed into monthly terms:

$$R_{f,month} = (1 + R_{f,year})^{1/12} - 1$$

6.2 Derivation of the explanatory variables

In the empirical methodology part, a four-factor CAPM was presented to determine the relationship between liquidity and stock returns: the excess return on the market portfolio, the two Fama-French factors, and a measure of liquidity. These four factors and their derivation will now be discussed in turn.

6.2.1 The market portfolio

The market portfolio is proxied as a value-weighted portfolio of all stocks with a known Total Return Index in the data set.

Value-weights or equal weights?

The main convention used when calculating portfolio returns in this study will be value-weighting. Thus, for example, the return on the market portfolio is determined as the value weighted returns of all individual stocks in the data sample. The returns of the different test portfolios (described in sections 6.3.2 and 6.3.4) are calculated using the same convention as the market portfolio to ensure consistency. Also, the liquidity measures of the portfolios are calculated using the same averaging methodology as for the returns. For example, the relative spreads of the different test portfolios are calculated as the value-weighted averages of the relative spreads of all the individual stocks in the

portfolios. However, many previous studies have applied the equal weighting methodology when calculating portfolio returns and liquidity (e.g. Amihud and Mendelson (1986), Chordia et al. (2001) and Amihud (2002)). The argument for doing so is that this is a way to compensate for the large fraction of the portfolios' wealth that stems from large companies. To test the robustness of the findings of the empirical study, the analysis will also be done using equally weighted portfolios.

6.2.2 The Fama and French (1993) factors

The three Fama and French (1993) factors are the excess market return, the return on a portfolio which is long in small stocks and short in large stocks, and the return on a portfolio which is long in high book-to-market stocks and short in low book-to-market stocks.

A very unfortunate thing about Datastream is that the available information on the book value of the equity for Danish stock companies is limited. This means that the number of companies for which the book-to-market variable is available but for the latest ten years. As the sample size is decided by the lowest common denominator it obviously means that it will be small for some years.

Another problem arising from this is that the Fama and French (1993) portfolios are formed in a very rigorous way. The stocks are divided into the size and the book-to-market portfolios completely independently. Then, after all stocks for each year have been classified as either small or big and high, medium or low book-to-market stocks, the six portfolios are formed on the basis of the intersections of the different classifications. This procedure is a very efficient way to remove possible bias from the data. The reason is that the portfolios are formed orthogonally - the stocks are assigned to portfolios based on one factor while controlling for the other. The problem is that, for relatively small data samples, this approach could result in some of the intersections between the different classifications being empty. For example, it is common that there are many stocks of small companies and few stocks of large companies. If this is the case, and in addition to this there are only a few stocks that have been classified as high book-to-market stocks, it is likely that none of the stocks of the big companies have been classified as high book-to-market stocks.

An examination of the six underlying portfolios reveals that there are many periods where the different portfolios are not defined. That is, they contain no stocks. The calculation of the SMB and the HML portfolios becomes somewhat biased when one or more of the six portfolios are not included as intended. The result is that, strictly speaking, the SMB and HML portfolios are not good estimates of the true portfolios. Another and far more serious problem is that there are actually some

periods in which neither SMB nor HML can be calculated at all. Of course, SMB and HML could be calculated if the missing excess returns of the portfolios were set to zero, but this is probably not the reality the investors face in the capital markets. Thus, this would not result in good estimates of the returns of the two portfolios. A solution to this problem could be to form the SMB and HML portfolios in a slightly different way. Instead of dividing the stocks into the “small” and “big” portfolios using the 50%-fractile, a different approach could be used. As of October 2006, the Nasdaq OMX defined shares as “Large Cap”¹⁴ if the market value of the company’s equity is more than EUR 1bn (approximately DKK 7.5bn). From the raw data set including the financial sector, it follows that the total equity market value of all Danish companies as of October 2006 was approximately DKK 1,216.2bn, and 31 companies had a market value of the equity of DKK 7.5bn. The total market value of the equity of these 31 companies was 1,014.2bn. Thus, using this definition, the market value of the equity of large companies comprise $1,014.2/1,216.2 = 83\%$ of the total equity market value of listed Danish companies. For simplicity, this fraction could be set to 75%. Assuming that this gives a reasonable picture of the distribution of small and big companies would imply that, for each year, all stocks could be divided into “small” and “big” defining the stocks in the low 25%-fractile as “small”.

Next, the problem that arises when for example none of the stocks in the B portfolio are high book-to-market stocks (“H”) and thus causes an empty B/H portfolio must be dealt with. That can be done by dividing the stocks *within* each portfolio S and B into low, medium and high book-to-market stocks. This ensures portfolios of more even size, which is a good thing, statistically. Theoretically, it is, however, an aggravation. This is due to the fact that the original SMB and HML portfolios were designed to measure the effect on expected returns of the return difference between small and big firms while minimising the influence on size of book-to-market values, and the effect of the return difference between high and low book-to-market firms while minimising the effect of size on book-to-market values. The new way to construct the SMB and HML described above does not do exactly this, but it may function as a proxy.

6.2.3 The measures of liquidity

This thesis will apply two proxies for liquidity. The first is actually a direct measure of illiquidity - the bid-ask spread. The relative bid-ask spread will be used as an explanatory variable in the study. Next, the turnover rate of the stocks will be used as a proxy for liquidity. The turnover rate does not

¹⁴ The term “Large Cap” is an abbreviation widely used in finance lingo for “Large market Capitalisation” which is means large market value of the equity, calculated as the market price of a stock multiplied by the number of stocks

measure either liquidity or illiquidity, but it may serve as a good proxy for liquidity in that the trading activity of stocks gives a signal of the depth. As either of these measures must be part of the time series regressions, where the sensitivities of the excess returns of each portfolio to the various explanatory variables are determined, the starting point of the data for the explanatory variables will be decided by the starting point of the liquidity data. Datastream does not provide information on the traded volume for stocks before April 1988, and as for the bid-ask spread no information is available before October 1991. This means that the SMB and the HML portfolios need not be formed before 1988. The market portfolio, however, must go back further five years so that the beta of each stock can be determined for the beta portfolio formation.

The relative bid-ask spread

As mentioned, the liquidity measure used by Amihud and Mendelson (1986) is the relative bid-ask spread. It is calculated as the absolute spread divided by the average of the bid and ask prices

$$S_{it} = \frac{|P_{Bid,it} - P_{Ask,it}|}{0.5(P_{Bid,it} + P_{Ask,it})}$$

The turnover rate

Following the ideas of for example Datar, Naik and Radcliffe (1998), the turnover rate of a stock will be used as a proxy for its liquidity. The turnover rate is given by

$$TR_{it} = \frac{1}{3} \sum_{j=1}^3 \frac{Volume_{i,t-j}}{Shares_{it}}$$

Datar et al. (1998) define the turnover rate as the average number of shares traded over a certain period divided by the number of shares outstanding at the end of the period. This definition could be somewhat problematic as the number of shares outstanding often changes during the fiscal year. At least, changes are normal at the end of a fiscal year. The traded number of shares can be affected by changes in the number of shares outstanding. The number of shares traded during a month should always be compared to the number of shares outstanding in the same month. Datar et al. avoid this problem by excluding companies where changes in the underlying number of outstanding shares have changed during the period for some months. However, this reduces the sample size from time to time. In a study with a relatively small sample an alternative solution is called for. If an average over some previous months is wanted, it should be an average of the turnover rates instead. That is, for each month, the turnover rate is calculated, and then, for each time period, the turnover rate is defined as the average of the turnover rates for the preceding three months. This should give a picture of how liquid the market would *perceive* the stock to be at the time of the analysis.

6.3 Beta estimation, portfolio parameter estimation and testing

Following the methodology of Amihud and Mendelson (1986), the sample is divided into overlapping periods of eleven years as shown in figure 6.1 at the end of this section. Each period then consists of a five-year beta estimation period, a five-year period of portfolio formation and estimation of the parameters of these portfolios, and finally, one year of cross-sectional testing.

6.3.1 The beta estimation period

The betas are calculated using 5 years of data (60 months). This means that stocks with less than 5 years of return data available are excluded from the sample. The beta coefficients for all stocks were estimated through time series OLS regressions of the basic CAPM relation

$$R_{jt} - R_{ft} = \hat{\alpha}_j + \hat{\beta}_j (R_{Mt} - R_{ft}) + \varepsilon_{jt}, \quad t = 1, \dots, 60,$$

where

R_{jt} = the return on stock j in month t ,

R_{ft} = the return on the risk free asset in t ,

R_{Mt} = the return on the market portfolio in month t ,

$\hat{\beta}_j$ = the estimated beta for stock j .

6.3.2 The portfolio parameter estimation period

The two different methodologies applied, makes the portfolio parameter estimation differ slightly. When detecting the effect of liquidity on stock returns, the liquidity measure does not enter into the factor sensitivity regressions as it does when detecting the pricing of liquidity risk. Therefore, this subsection is split into two parts.

Portfolio parameter estimation for the detection of the effect of liquidity on stock returns

At the beginning of each year of each portfolio parameter estimation period, the all stocks in the sample are classified into portfolios based on liquidity and beta. A detailed description of the portfolio formation follows later. During the portfolio formation period, the parameters (factor sensitivities) of each portfolio are estimated

$$R_{it} - R_{ft} = \alpha_i + \hat{\beta}_i (R_{Mt} - R_{ft}) + \hat{s}_i SMB_t + \hat{h}_i HML_t + \varepsilon_i,$$

where

R_{it} = the return on portfolio i in month t ,

R_{ft} = the return on the risk free asset in t ,

R_{Mt} = the return on the market portfolio in month t ,

SMB_t = the return on the SMB portfolio in month t ,

HML_t = the return on the HML portfolio in month t ,

$\hat{\beta}_i$ = the estimated beta for portfolio i ,

$\hat{s}_i$ = the estimated sensitivity on the excess return on portfolio i to the return on the SMB portfolio.

$\hat{h}_i$ = the estimated sensitivity of the excess return on portfolio i to the return on the HML portfolio

These factor sensitivities will enter the first step of the Fama and Macbeth (1973) regressions to control for other factors that could explain the cross-section of stock returns.

Portfolio parameter estimation when investigating the pricing of liquidity risk

As mentioned above, the stock will be allocated into portfolios based on their liquidity and beta. To analyse the effect of the liquidity risk in stock pricing, the liquidity of each portfolio will be included already in the portfolio parameter estimation period to estimate the sensitivity of each portfolio towards changes in its level of liquidity.

$$R_{it} - R_{ft} = \alpha_i + \hat{\beta}_i(R_{Mt} - R_{ft}) + \hat{s}_iSMB_t + \hat{h}_iHML_t + \hat{l}_iLIQ_{it} + \varepsilon_i,$$

where

R_{it} = the return on portfolio i in month t ,

R_{ft} = the return on the risk free asset in t ,

R_{Mt} = the return on the market portfolio in month t ,

SMB_t = the return on the SMB portfolio in month t ,

HML_t = the return on the HML portfolio in month t ,

LIQ_{it} = the liquidity on portfolio i in month t ,

$\hat{\beta}_i$ = the estimated beta for portfolio i ,

$\hat{s}_i$ = the estimated sensitivity of the excess return on portfolio i to the return on the SMB portfolio,

$\hat{h}_i$ = the estimated sensitivity of the excess return on portfolio i to the return on the HML portfolio,

$\hat{l}_i$ = the estimated sensitivity of the excess return on portfolio i to the liquidity on portfolio i .

The four factor sensitivities are estimated at the end of each year using the previous five years of data (60 observations). In that way, the factor sensitivities are available for the start of each test period, and the model becomes predictive in nature.

To test the effect of market liquidity risk, the value-weighted average liquidity of all stocks could enter the above regression instead of the liquidity of each individual portfolio of stocks. In that way, the sensitivity of individual stocks towards the market liquidity could be estimated.

6.3.3 The test period

After having estimated the factor sensitivities for each portfolio for each portfolio parameter estimation period, the first step of the Fama and MacBeth (1973) methodology can be initiated - the actual cross-sectional regressions described in the empirical methodology part.

Again, as two different methodologies will be applied, this section is divided into two parts.

Testing the cross-sectional influence of liquidity on stock returns

For each month of each test period (12 months), the five gammas are estimated in the cross-sectional regressions:

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_{it} + u_{it}$$

As explained previously (see section 4 Empirical Methodology).

This will yield a time series of 12 observations for each parameter for each test period. This will result in a time series of 143 observations (11 years of 12 months plus 11 months in 2008) for each of the five gammas. The last gamma represents the effect of liquidity on the portfolio returns.

The second step of the Fama-MacBeth procedure can now be applied to these time series by averaging each time series of gamma estimates as explained under 4 Empirical methodology.

Testing the cross-sectional effect of liquidity risk on stock returns

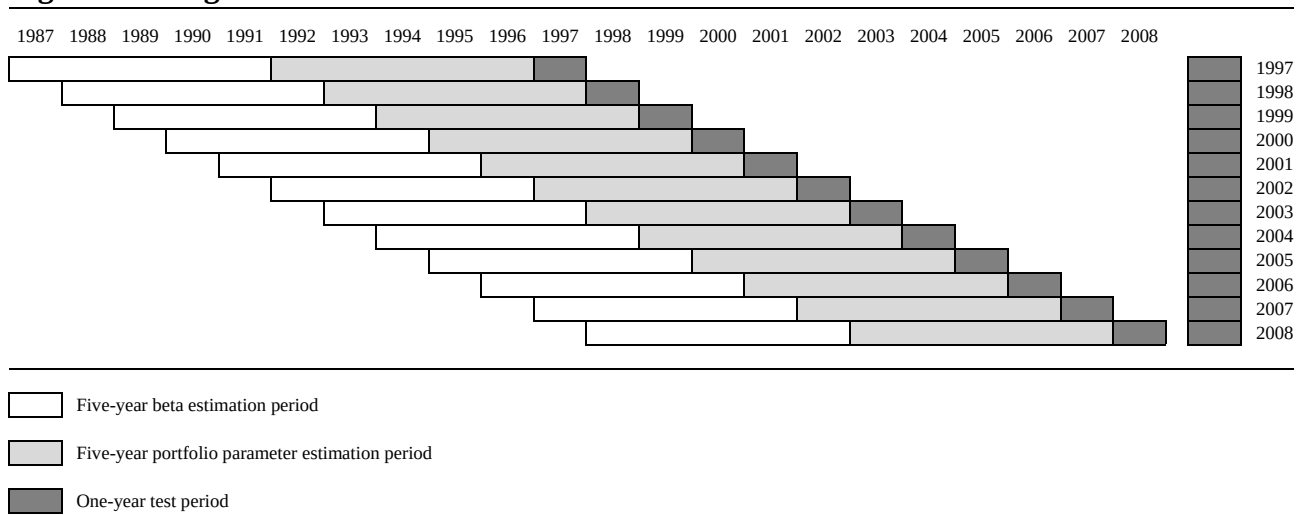
For each month of each test period, the five gammas are estimated in the cross-sectional regressions:

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

This will yield a time series of 12 observations for each parameter for each test period. This will result in a time series of 143 observations (11 years of 12 months plus 11 months in 2008) for each of the five gammas. As explained earlier, the gammas 1-4 are estimated risk premiums. The last gamma is thus the premium for liquidity risk.

Below, figure 6.1 summarises the data processing procedure conducted in the empirical study.

Figure 6.1: Regression overview



6.3.4 Portfolios

To accommodate for the white noise associated with estimating betas and other factor sensitivities for individual stocks, the empirical study will take its departure in portfolios of stocks. To see if results and conclusions are robust, these portfolios will be formed in various ways.

First, a methodology similar to that of Amihud and Mendelson (1986) will be applied. In their approach, 7x7 liquidity and beta portfolios are formed. In the present study, however, the size of the data sample (number of stocks) limits the possibility of dividing the sample into that many portfolios. Instead, 3x3 liquidity and beta portfolios will be formed. First, for each year, all stocks are ranked by their liquidity and divided into three equal groups. Next, for each liquidity group, the stocks are ranked by their betas and sub-divided into another three equal groups. This means that there will be nine portfolios of equal size.

As a test of robustness, recognising that beta might be the factor that is least precise and adds most white noise to the estimation, a portfolio formation approach focusing solely on betas will be pur-

sued. 10 beta portfolios of equal size will be formed, and the cross-sectional relation between returns and spreads, and returns and the turnover rate, respectively, will be examined.

6.3.5 Will the current credit crunch have an effect on the analysis?

The recent financial turmoil can create some white noise in the data set. For example, the concept of "flight to liquidity" can be an issue. This is, among other things, what has happened during the recent credit crisis - investors seek liquid assets. If investors demand more liquid assets - for example relatively liquid stocks, the price of such stocks could be expected to increase - at least, they would not fall as much as the less liquid stocks. If liquid stock prices increase at the same time as the prices of less liquid stocks fall, a study of the relationship between liquidity and stock returns will result in a return premium for more liquid stocks. This is the opposite of what theory predicts.

This can be expanded to cover stock liquidity sensitivity. A stock that is very sensitive to liquidity will be avoided by investors in times of liquidity crises. Thus even though this kind of liquidity-sensitive stock should attract a return premium for higher risk, the identified return will most likely be negative in times of crises, and thus it will seem as if the stock "offers" a return discount in times when liquidity is wanted. Thus, if the goal is to estimate the magnitude of the pricing of liquidity risk, the returns in a "normal market" should be observed.

To find out whether the inclusion of data after the beginning of the current credit crises has disturbed the picture of the pricing of liquidity and liquidity risk, the main analysis has also been done for a data set excluding the period since the beginning of the subprime crisis in August 2007. These results will be presented in appendix and discussed the section where robustness tests are performed.

7 Results

Before the results of the main models are presented, the results of tests of the simple CAPM and Fama and French's (1993) three-factor CAPM will be presented to give an overview of the implications of the data sample.

The empirical study will cover two investigations. As explained earlier, the cross-sectional relationship between liquidity and stock returns in Denmark will be analysed and, in addition to this, the effect of liquidity risk on stock returns in Denmark will be determined. Due to the fact that these two studies investigate two different phenomena, the presentation and analysis of the results is split into two separate parts.

As two different measures of liquidity will be applied - the relative bid-ask spread and the turnover rate - the presentation and analysis of the results is divided into two subsections.

Next, to check the robustness of the findings, all studies have been carried out changing various assumptions. The findings of these robustness checks will be reported in the end of this part. Finally, all findings will be summarised to reach some conclusions relating to the different models analysed.

7.1 The standard CAPM and the three-factor CAPM

Before adding the liquidity factor to the three-factor CAPM to examine the pricing of liquidity, the simple CAPM and the three-factor CAPM (by Fama and French 1993) will be tested in the same cross-sectional framework. The idea is to give an overview of the extent to which the current data sample and methodology gives rise to evidence of the standard CAPM and the three-factor CAPM. The results will also be shown for the beta portfolios even though the beta portfolios are just meant for checking the robustness of the results. This is done because this part only should give a brief overview, and therefore the analysis of the CAPM and the three-factor CAPM will be of ad-hoc nature.

7.1.1 The standard CAPM

The standard CAPM as presented earlier has been tested in the same cross-sectional framework as presented earlier. The results are summarised in the table below. As two different measures of liquidity were used in the 3x3 liquidity and beta portfolio formation process, the CAPM tests for these portfolios have been done for both of the measures.

Table 7.1: Cross-sectional regressions for the standard CAPM for the value-weighted 3x3 portfolios and the value-weighted beta portfolios. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + u_{it}$$

Where $\hat{\beta}_i$ is defined in section 4 Methodology. For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$
<i>(a) 3x3 spread and beta portfolios</i>		
Estimate	0.0185***	-0.0096*
Std. error	0.0054	0.0060
<i>(b) 3x3 Turnover rate and beta portfolios</i>		
Estimate	0.0201***	-0.0092*
Std. error	0.0061	0.0070
<i>(c) Beta portfolios</i>		
Estimate	0.0137**	-0.0053
Std. error	0.0060	0.0067

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

There are strong indications that there is something that the model does not capture. The reason is that the intercept term, which should be zero, is significantly larger than zero in each case. The next implication of the results shown above is that the market risk premium is significantly negative in two of the estimations. This is the direct opposite of what theory predicts - it indicates that higher market risk exposure of a portfolio would attract a discount. This makes no economical sense - thus, based on the current sample and the chosen methodologies for estimation, portfolio formation and testing, these findings indicate that the CAPM does not hold. Obviously, this is not clear cut evidence against the Capital Asset Pricing Model - it is merely an indication that given the current set of assumptions, the data sample and methodology do not provide evidence that it holds.

7.1.2 The Fama and French (1993) three-factor CAPM

Below, the results for the analyses of the Fama and French (1993) three factor CAPM can be found.

Table 7.2: Cross-sectional regressions for the Fama and French three-factor CAPM for the value-weighted 3x3 portfolios and the value-weighted beta portfolios. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology. For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$
<i>(a) 3x3 spread and beta portfolios</i>				
Estimate	0.0112	-0.0051	0.0134*	0.0058
Std. error	0.0076	0.0076	0.0090	0.0093
<i>(b) 3x3 Turnover rate and beta portfolios</i>				
Estimate	0.0155*	-0.0055	0.0104	-0.0059
Std. error	0.0084	0.0088	0.0109	0.0120
<i>(c) Beta portfolios</i>				
Estimate	0.0109	-0.0025	0.0076	-0.0054
Std. error	0.0082	0.0079	0.0075	0.0072

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

The results presented above generally do not imply much about the pricing of the three risk factors. The only significant finding (at a 10% level only) is that the sensitivity of a portfolio towards changes in the SMB portfolio is priced at an annualised premium of 17.3%¹⁵ per unit of sensitivity.

¹⁵ $(1+0.0134)^{12}-1=0.173$

This is in accordance with theory, but the premium seems overstated. All other parameters are insignificant the other estimations.

Generally, the results of the cross-sectional tests of the CAPM and the three-factor CAPM imply that the proposed models lack explanatory power. There are many findings that indicate the direct opposite of what theory would predict. Despite this, as mentioned in the analysis of the standard CAPM, the CAPM could still be a valid model for explaining equity market movements - just not under the present circumstances.

7.2 The cross-sectional relationship between liquidity and stock returns

7.2.1 The relative bid-ask spread as the measure of liquidity

To give an indication of the cross-sectional relationship between the relative spread and the excess returns of the portfolios, table 7.3 summarises the excess return and the relative spread estimated for the entire period for the 3x3 spread and beta portfolios.

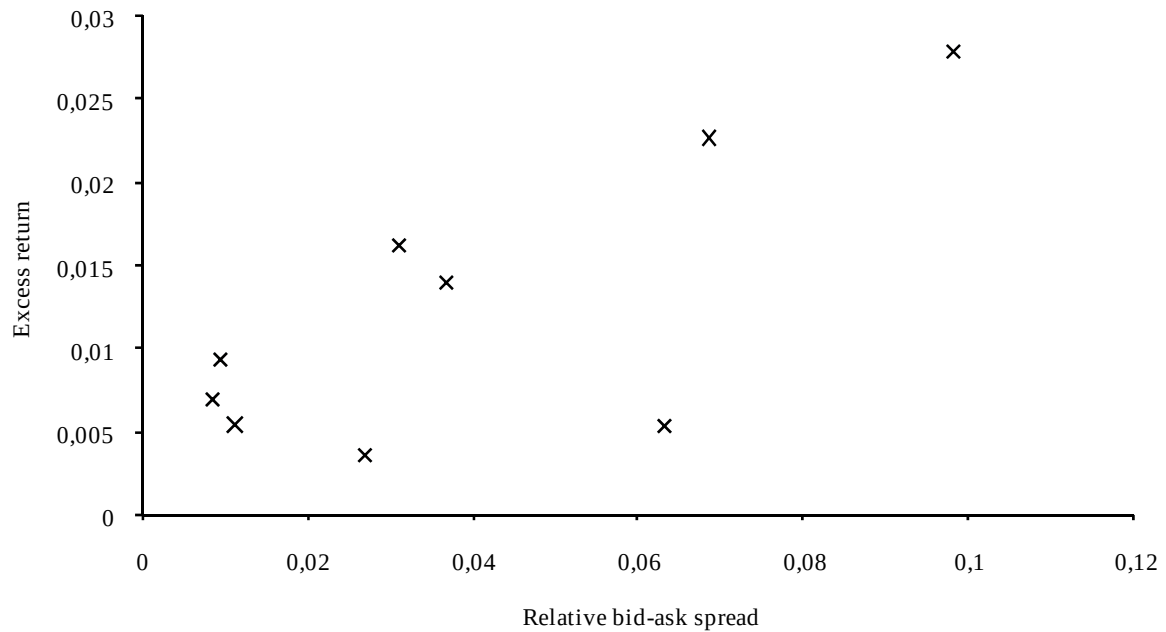
Table 7.3: Excess returns and relative spreads for the different portfolios estimated for the entire period

<i>Spread portfolio</i>	<i>L</i>			<i>M</i>			<i>H</i>		
<i>Beta portfolio</i>	<i>L</i>	<i>M</i>	<i>H</i>	<i>L</i>	<i>M</i>	<i>H</i>	<i>L</i>	<i>M</i>	<i>H</i>
$R_i - R_f$	0.0053	0.0068	0.0092	0.0139	0.0162	0.0035	0.0278	0.0226	0.0052
Spread	0.0112	0.0085	0.0094	0.0367	0.0310	0.0270	0.0980	0.0685	0.0631

Without controlling for other variables, the above table gives a slight indication of a positive cross-sectional relationship between the excess returns and the relative spread. Two of the three high-spread portfolios have yielded substantially higher returns than all other portfolios. In addition to this, the three low-spread portfolios have yielded returns in the lower end. On the other hand the three portfolios showing the lowest excess returns are placed in each of the liquidity groups so the indications are not clear-cut. Overall, the excess returns seem to be increasing in the spread. This indicates that investors demand a higher return for holding less liquid stocks.

The return-spread relationship indicated by the portfolios is shown in figure 7.1 below

Figure 7.1: XY-plot of average portfolio excess returns against the average portfolio relative spreads estimated over the period 1992-2008M1 for the 3x3 spread and beta portfolios



The plot shows indications of excess returns being positively related to the relative spread. There are two obvious outliers. If these two were discarded, there would be a nice linear relationship. One could even argue that the relationship shows indications of the concavity-characteristic that Amihud and Mendelson (1986) proposed.

The formal test of the cross-sectional relationship is carried out as described in the methodology part. Below, the results of the cross-sectional analysis of the relative spread and stock returns are shown in table 7.4.

Table 7.4: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and LIQ_i is the relative bid-ask spread of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0071	0.0016	0.0185*	-0.0098	0.0598
Std. error	0.0087	0.0072	0.0118	0.0098	0.2103
$R^2 = 0.6487$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

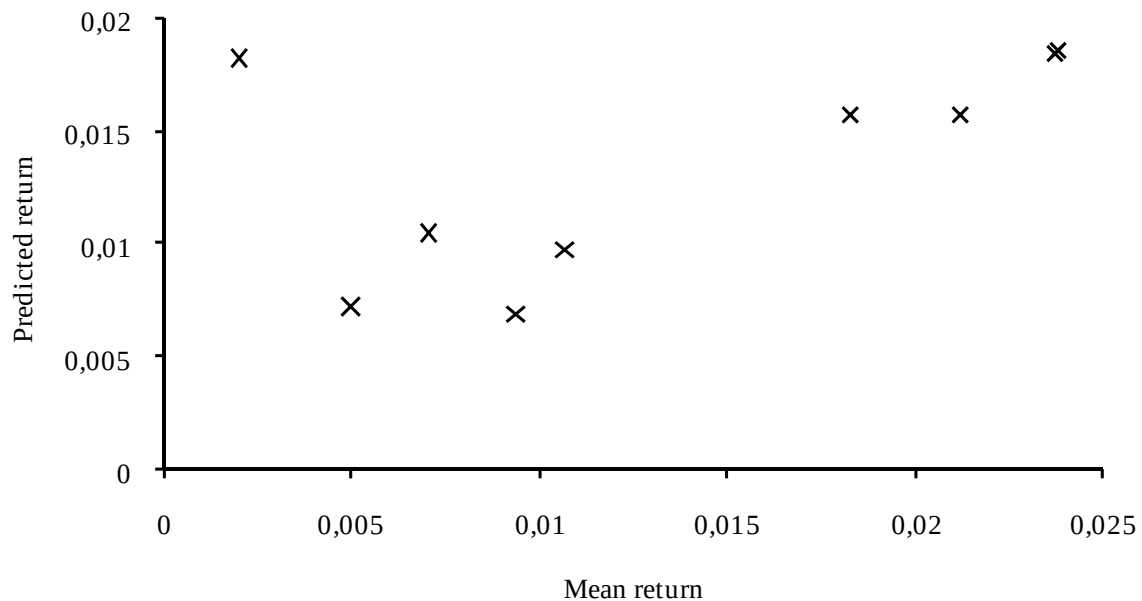
*** Indicates statistical significance at a 0.01 level

These results indicate that liquidity is not an important factor in explaining the cross-section of stock portfolio returns. Coefficient for the relative bid-ask spread, (denominated LIQ), is not significantly larger than zero at any interesting level. This indicates that the relative bid-ask spread does not significantly help explain the cross-sectional variance of the 3x3 spread and beta portfolio returns. The only significant parameter estimate is the risk premium for the SMB factor. It should be noticed that the premium is only significant at a 10% level. Based on these stock portfolios and explanatory variables, there is no evidence of a positive relationship between stock returns and illiquidity in the Danish equity market. The robustness of this finding will be tested later.

To give an overview of the empirical fit of the model, an approximation is called for - otherwise, an analysis would be necessary for each point in time (that would be 143 months). Instead, one could use the simple average of all the estimated coefficients for the test period and interpret these as general portfolio characteristics. Then, these average coefficients could be applied in a simple cross-sectional regression where the average returns of the portfolios over the test period are regressed against these portfolio characteristics. This model can be regarded a rough approximation of the model reported above. The predictions of this model can then be plotted against the average returns

of the portfolios. This is done below. If the model fits perfectly, the plot would be a 45-degree line from (0,0).

Figure 7.2: XY-plot of approximated predictions of the model against the mean returns for the entire test period 1997-2008M11 for the 3x3 spread and beta portfolios



There is a tendency of clustering and an outlier. Otherwise, the fit 45-degree line would have been acceptable. On the face of it, it seems as if one observation is missing but this is not the case - two of the observations are almost identical.

In appendix B, a residual analysis of the approximated model can be found. It should be noted that the number of observations used in the residual analysis is only nine - there is one residual for each portfolio. This means that the residual analysis generally should be considered statistically weak. This aside, the residual analysis reveal that there does not seem to be violations of the general assumptions of the estimation and testing. The Jarque-Bera test cannot reject that the residuals are normally distributed, the Durbin-Watson test of autocorrelation does not indicate that autocorrelation is present in the residuals, White's test of homoskedasticity is not rejected, thus no heteroskedasticity seems present, and finally there are no indications of severe multicollinearity.

7.2.2 The turnover rate as the measure of liquidity

Below, table 7.5 gives an overview of the relationship between the turnover rate and the excess returns of the portfolios. The excess returns and the liquidity are estimated for the entire period for the 3x3 liquidity and beta portfolios.

Table 7.5: Average excess return and turnover rate for the 3x3 turnover rate and beta portfolios. Estimated over the time period 1992-2008M11

<i>Turnover portfolio</i>	<i>L</i>			<i>M</i>			<i>H</i>		
<i>Beta portfolio</i>	<i>L</i>	<i>M</i>	<i>H</i>	<i>L</i>	<i>M</i>	<i>H</i>	<i>L</i>	<i>M</i>	<i>H</i>
$R_i - R_f$	0.0245	0.0208	0.0039	0.0163	0.0098	0.0085	0.0070	0.0122	0.0117
Turnover rate	0.0068	0.0101	0.0073	0.0342	0.0367	0.0375	0.5148	0.3242	0.2179

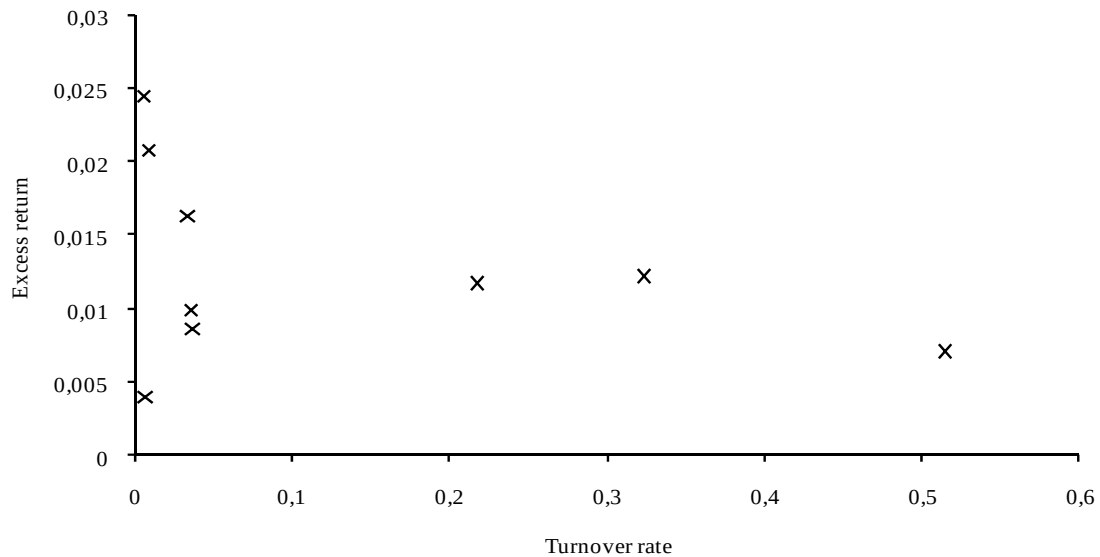
It can be seen that the excess returns of two out of three low liquidity (low turnover rate) portfolios are higher than the excess returns of the other portfolios. There is generally an indication of excess returns being higher for the portfolios of stocks that are less frequently traded. The average excess return on the low liquidity portfolios is approximately 21.6%¹⁶, annualised - this is 8.5 percentage points higher than the annualised excess return on the high liquidity portfolios of approximately 13.1%¹⁷. Thus, there are signs that the cross-section of excess returns is negatively related to the turnover rate as predicted by theory. It is worth noticing, though, that the second least liquid portfolio has yielded the lowest return of all portfolios. It is findings like this that makes the picture less clear.

¹⁶ $(1+0.0164)^{12-1} = 0.2156$

¹⁷ $(1+0.0103)^{12-1} = 0.1308$

As was done in the previous section where liquidity was proxied by the relative spread, the above indicative analysis is summarised in a figure showing XY-plots of the data in the two tables above:

Figure 7.3: XY-plot of average portfolio excess returns against the average portfolio turnover rate estimated over the period 1992-2008M1 for the 3x3 spread and beta portfolios



If a couple of outliers were removed, this plot would give an indication that the excess returns are decreasing in the turnover rate. The picture is not very clear however.

To establish whether there is a significantly negative relationship between liquidity (proxied by the turnover rate) and stock returns, the formal test described in the methodology part has been carried out. The results can be viewed in table 7.6 below.

Table 7.6: The direct cross-sectional effect of the turnover rate of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and LIQ_i is the turnover rate of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0206*	-0.0114	0.0087	-0.0112	-0.0124
Std. error	0.0107	0.0100	0.0114	0.0203	0.0393
$R^2 = 0.5249$					

* Indicates statistical significance at a 0.1 level

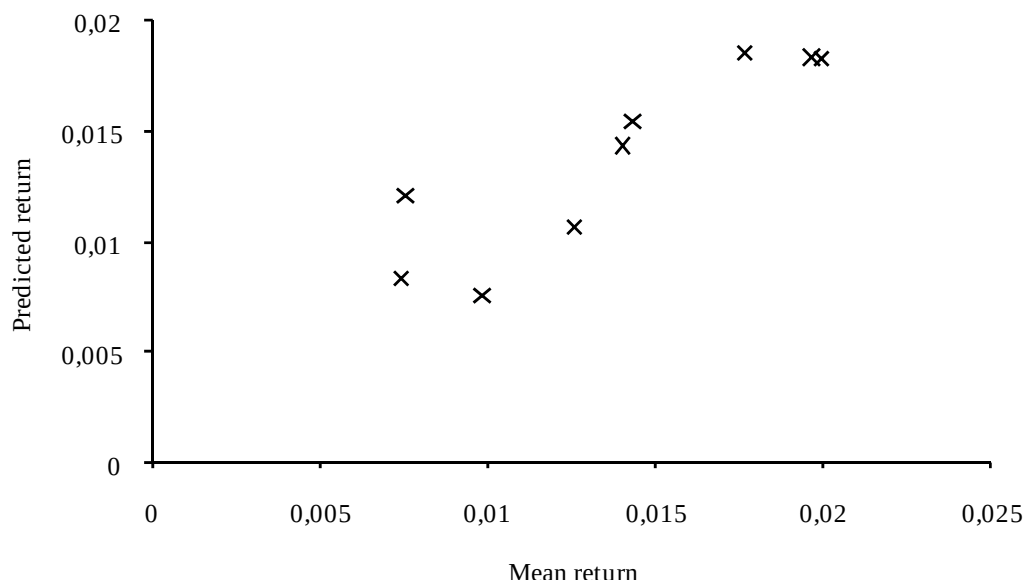
** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

As was the case when the relative bid-ask spread was used to proxy illiquidity, the turnover rate is not found to significantly explain the cross-sectional variance of the portfolios. The parameter estimate is not significantly lower than zero. According to theory, this should have been the case. The only coefficient slightly significant in the results above is the intercept term which should have been zero if theory held. So even though the graphical inspection showed signs of a negative relationship between the turnover rate and the portfolio returns, it does not seem to be significant - at least not in the presence of the other explanatory variables. Based on the 3x3 turnover rate and beta value-weighted portfolios, there is no evidence of a significantly negative relationship between liquidity (proxied by the turnover rate) and stock portfolio returns. The robustness of this finding will be tested in a number of robustness checks in the last part of the following section.

As for the analysis of the spread, an approximated model has been set up for producing predictions and enabling residual analysis. The predictions of this model are plotted against the average returns of the portfolios. This can be seen below.

Figure 7.4: XY-plot of approximated predictions of the model against the mean returns for the entire test period 1997-2008M11 for the 3x3 turnover rate and beta portfolios



The approximated residual analysis shown in appendix B reveals that there does not seem to be violations of the general assumptions of the estimation and testing. The Jarque-Bera test does not reject that the residuals are normally distributed, the Durbin-Watson test of autocorrelation does not provide evidence of autocorrelation in the residuals, White's test of homoskedasticity is not rejected, so there does not seem to be presence of heteroskedasticity, and the condition index does not indicate that there are severe signs of multicollinearity.

7.3 Robustness tests of the liquidity-return relationship

For all regressions discussed in the following section, approximated residual analyses have been done and the results can be found in appendix B.

7.3.1 Forming portfolios on the basis of stock betas only

The relative bid-ask spread as the measure of liquidity

In table A.43 and figure A.1 in appendix A an overview and plot, respectively, of the returns and spreads of the different portfolios are provided. There seems to be a positive relationship between the excess returns and the relative spreads.

In the table below, the cross-sectional test of the three factor CAPM added the relative bid-ask spread for the beta portfolios is reported.

Table 7.7: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and LIQ_i is the relative bid-ask spread of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0125	-0.0047	-0.0057	-0.0105	0.4254**
Std. error	0.0102	0.0094	0.0140	0.0091	0.2207
$R^2 = 0.4199$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

In opposite to the tests in the previous section, the coefficient for the relative spread is now significantly larger than zero indicating a positive relationship between illiquidity and expected returns. This finding indicates that if the relative spread increases by one (that is, 100%p), the required return increases by 42.5%p monthly. It would make more sense to state this differently - if the relative spread increases by 100bp, the required return increases by 42.54bp - an annualised premium of about 520bp¹⁸. It must be said that this finding is not robust to the portfolio formation criteria. In the previous section, no evidence was provided by the 3x3 spread and beta portfolios. To see if the finding is altered by excluding the Fama-French factors, the table below show the results of a simplified model for the beta portfolios.

¹⁸ $(1+0.004254)^{12-1} = 5.2\%$

Table 7.8: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$ is defined in section 4 Methodology, and LIQ_i is the relative bid-ask spread of portfolio i .

For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0060	0.0003	0.3312**
Std. error	0.0074	0.0066	0.1661
$R^2 = 0.3872$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Apparently, the illiquidity premium estimated above is not sensitive to the presence of the Fama-French factors. The annualised premium is estimated to 400bp per 100bp of increase in the relative spread.

The turnover rate as the measure of liquidity

The average excess returns and the average turnover rates of the beta portfolios are summarised in table A.44 and figure A.2 in appendix A. There are signs that the returns are decreasing in the turnover rate. The beta portfolio with the lowest turnover rate has yielded the highest excess return which is in line with theory, but again, the next-to-least liquid stock portfolio has yielded the lowest excess returns across all the portfolios, which is puzzling. Below, the result of the cross-sectional regression of the proposed model is reported.

Table 7.9: The direct cross-sectional effect of the turnover rate of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and LIQ_i is the turnover rate of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0161	-0.0032	0.0025	-0.0065	-0.0590*
Std. error	0.0101	0.0082	0.0103	0.0096	0.0411
$R^2 = 0.4335$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Again, the beta portfolios provide support for theory that was not found when the 3x3 liquidity and beta portfolios were analysed. No significant relationship between the turnover rate and the excess returns were found when the 3x3 beta portfolios were analysed. As the coefficient for the turnover rate is now significantly negative, the earlier finding that the turnover rate had no effect on returns was not robust to the portfolio formation technique. Though only significant at a 10% level, the above results indicate that, when the turnover rate increase by one¹⁹, the required return should decrease by 590bp per month. A change of one in the turnover rate seems drastic, though. It would be more intuitive to state this in terms of basis points of changes in the turnover rate. The results imply that, if the turnover rate of a stock decreases by 100bp, the return demanded by investors would increase by 5.9bp per month - an annualised premium of 71bp²⁰.

¹⁹ That is, 100%, as the turnover rate expresses the number of times all shares were traded - one is thus 100%

²⁰ $(1+0.059/100)^{12}-1 = 0.0071$

In the table below, the results of the simplified model where the Fama-French factors have been removed are shown.

Table 7.10: The direct cross-sectional effect of the turnover rate of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

Where $\hat{\beta}_i$ is defined in section 4 Methodology, and LIQ_i is the turnover rate of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0161**	-0.0034	-0.0514**
Std. error	0.0063	0.0068	0.0248
$R^2 = 0.2697$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

The coefficient for liquidity now becomes even more significant. It is now significantly below zero at a 5% level. The indicated annualised illiquidity premium is 62bp per 100bp of decrease in the turnover rate.

7.3.2 Leaving out the Fama-French factors

Now, to see if the conclusions of the analyses of 3x3 liquidity and beta portfolios are altered by excluding the Fama-French factors from the regression, tables A.1 and A.2 in appendix A presents the results of the simplified models. Neither of the tables provides any evidence of the pricing of liquidity, nor are there indications of the pricing of market risk - the estimated market risk premium is not significantly larger than zero in either of the two reported models. The only significant parameter is the intercept in table A.2, which indicate that something is missing from the model. The intercept was also significant in the full model (table 7.6).

As the conclusions regarding the pricing of liquidity have remained unchanged in the restricted models, the finding that, for the 3x3 liquidity and beta portfolios, there is no sign of pricing of liquidity, is robust to the presence of the Fama-French factors.

7.3.3 Equal weights

As mentioned in the discussion about the weighting convention used when calculating portfolio returns and liquidity, there are some problems with the value-weighting convention. As many researchers previously have pointed out in similar studies to this one, the value-weighting will, in the case that a number of small stocks have been excluded from the sample, result in large companies being assigned too high weights. In this study, stocks for which information on either the market value, the bid-ask spread or the traded volume have been excluded from the sample. Often, stocks with a low level of available information are relatively illiquid stocks of very small companies. Thus, the results presented in the previous sections could be biased in that the weights assigned to the large companies may have been too high. The implication of overweighting the large cap stocks could be a dilution of the true effect of illiquidity costs or liquidity risk in stock pricing. The reason is that if the large, normally relatively liquid stocks are assigned too high weights, the illiquid stocks will be assigned too low weights and thus, their effect on stock pricing will be undermined. Theory predicts that illiquid stocks are expected to yield higher returns, and if these higher returns are assigned too low weights, their effect will be crowded out by the (presumably) lower returns of the large (liquid) stocks. To compensate for this possible bias, equal weighting of returns and liquidity can be applied. One should keep in mind, though, that this could overestimate the effect of liquidity on stock returns - the reason is that equal weights could undermine the large stocks' fraction of investors' wealth, and then the argumentation is basically the same as above (just in the opposite direction).

In the following, the analyses from the previous sections carried out using the equal weights when forming portfolios will be commented. All tables can be found in appendix A.

The relative bid-ask spread as the measure of liquidity

The results of the analyses of the equally weighted 3x3 spread and beta portfolios can be found in tables A.3 and A.4 in appendix A. The conclusions reached when value-weights were applied when forming the 3x3 portfolios are unchanged - no coefficients are significant regardless of whether the Fama-French factors are included. Thus, the finding that, there are no indications of the pricing of liquidity based on the 3x3 portfolios is robust to the averaging convention applied.

The results of the analyses of the equally weighted beta portfolios are provided in tables A.5 and A.6. For the full model (table A.5), the coefficient for the spread is now insignificant, but when the Fama-French factors are excluded, the coefficient becomes slightly significant. The coefficient is

economically small compared to the earlier findings and other research. Generally, the coefficient for the relative bid-ask spread for the beta portfolios should therefore be considered sensitive to the weighting convention chosen in the portfolio formation process.

The turnover rate as the measure of liquidity

Tables A.7 and A.8 in appendix A show the results of the analyses from the previous sections when assigning equal weights to the stocks in the 3x3 turnover rate and beta portfolios. Changing the weighting of the stocks does not alter the conclusion reached previously - there are no signs of significant pricing of liquidity proxied by the turnover rate. This holds both when the Fama-French factors are included and excluded. The conclusion that the 3x3 turnover rate and beta portfolios reveal no significantly negative cross-sectional relationship between the turnover rate and the excess returns of the portfolios is robust to the weighting applied in the portfolios.

As for the equally weighted beta portfolios, the results are shown in appendix tables A.9 and A.10. Both when including and excluding the Fama-French factors, there is no significantly negative relationship between the turnover rates and the excess returns of the portfolios. This finding is contrary to what was found when the stocks were value-weighted in the beta portfolios. Therefore, the premiums estimated for increasing illiquidity for the value-weighted beta portfolios must be considered sensitive to the weighting convention.

7.3.4 Excluding the period from the start of the sub-prime crisis

In the appendix tables A.11 to A.18, the results of the analysis of the 3x3 liquidity and beta portfolios and the beta portfolios excluding the period from August 2007 to November 2008 (rest of the sample). The reason for excluding the period is explained in the data part, but in short, there is a risk that this period messes up the overall picture. When excluding the sub-prime crisis, there are three cases where the effect of liquidity identified earlier disappears. This is surprising in the sense that the sub-prime crisis were expected to have made the picture less clear, and it was expected that "flight to liquidity" would have the opposite effect on findings. Apparently, this is not what drives the effect of liquidity. What could help explain this finding is that in periods of financial turmoil, many extreme observations occur and the effect of liquidity parameters become very important. This can cause extremely high slope coefficients that prove to be influential on the overall average. It can be concluded that some of the findings relating to the direct effect of liquidity on stock prices are sensitive to the presence of the sub-prime crisis in the data set.

7.4 The cross-sectional relationship between liquidity risk and stock returns

7.4.1 Liquidity as a stock characteristic

The relative bid-ask spread as the measure of liquidity

In table 7.11 below, the results of the 3x3 liquidity and beta portfolios approach to the four-factor CAPM where liquidity is measured as the relative spread can be found.

Table 7.11: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the relative bid-ask spread sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0042	0.0012	0.0158	0.0102	-0.0038
Std. error	0.0146	0.0129	0.0143	0.0138	0.0048
$R^2 = 0.4054$					

* Indicates statistical significance at a 0.1 level

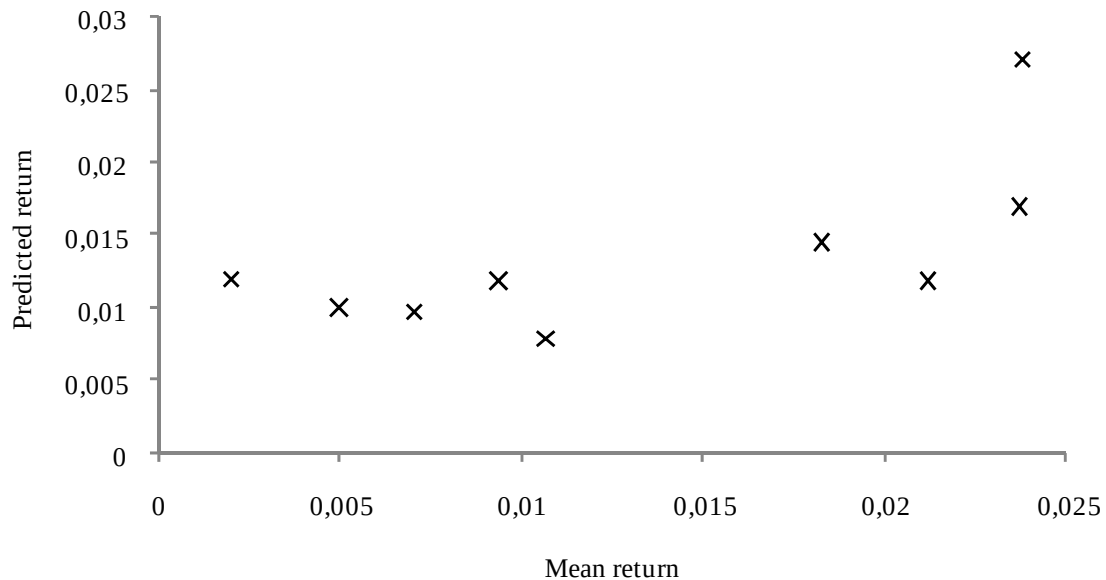
** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

As no statistical significance is found for any of the factors, these results do not support the four-factor CAPM. As the results are not significant at any interesting level, nothing can be concluded regarding the risk factor return premiums and, in particular, nothing can be concluded about the premium for liquidity risk.

As was done in the previous section, the predictions of the approximated model are plotted against the mean returns of the portfolios. This can be viewed in the figure below.

Figure 7.5: XY-plot of average portfolio excess returns against the average portfolio relative spreads estimated over the period 1992-2008M11 for the beta portfolios



The plot supports the poor results - the empirical fit seems far from perfect. The approximated residual analysis, the results can be found in appendix B. The analysis of the residuals does not provide any evidence of severe violations of the assumptions of the model.

The turnover rate as the measure of liquidity

Table 7.12 summarises the cross-sectional regressions of the model, where liquidity is proxied as the turnover rate defined as described in part 6 Data and stocks have been divided into portfolios based on liquidity and beta as explained earlier.

Table 7.12: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the turnover rate sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0224**	-0.0139	0.0086	-0.0030	0.0086*
Std. error	0.0104	0.0113	0.0119	0.0125	0.0057
$R^2 = 0.3521$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

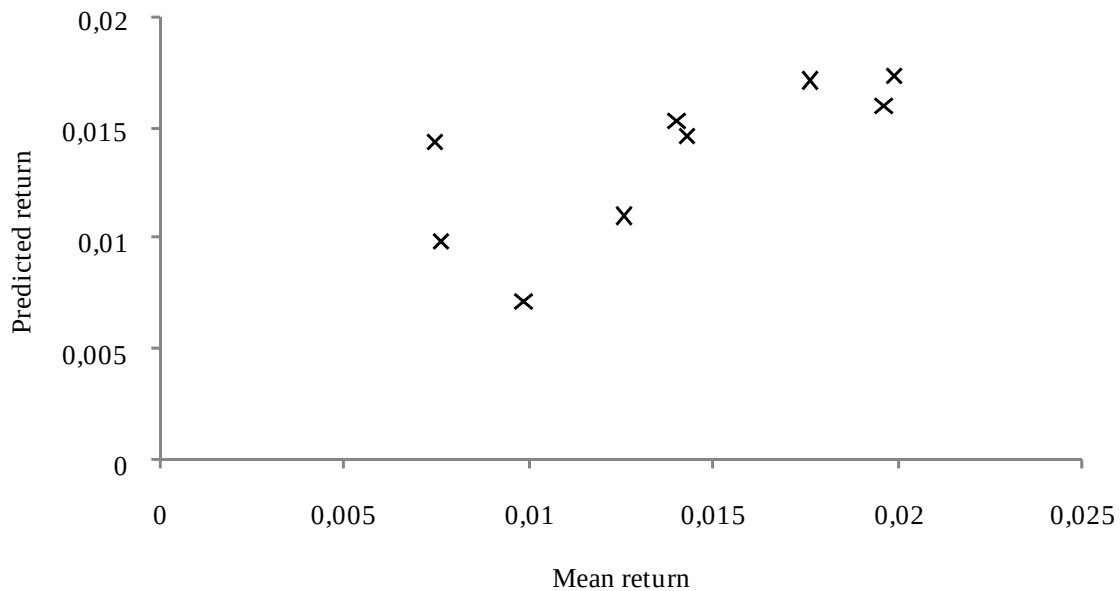
*** Indicates statistical significance at a 0.01 level

Deriving from these results are two findings. There are pricing factors that the model seems to fail to capture - as was the case in one of the spread cross-sectional analyses, the intercept is significantly different from zero at a 5% level. Apart from this, there seems to be a pricing of liquidity risk measured by the sensitivity of the excess returns of a portfolio towards the turnover rate of that portfolio. There is an indication of a liquidity sensitivity return premium of approximately 1080bp per year for each unit of turnover rate sensitivity²¹. It should be noticed, though, that this finding is only significant at a 10% level. Therefore, the conclusion relating to the estimated return premium for liquidity sensitivity from this model should be considered and applied with a certain degree of caution.

²¹ $(1+0.0086)^{12}-1 = 10.8\%$

Below the approximated predictions of the model have been plotted against the average excess returns of the portfolios as has been done for the earlier models.

Figure 7.8: XY-plot of average portfolio excess returns against the average portfolio relative spreads estimated over the period 1992-2008M11 for the beta portfolios



Again, the approximation of the model does not provide a very good empirical fit even though the results were positive. In the approximated residual analysis in appendix B, there are no indications of serious model assumption violations.

7.4.2 Liquidity as a market characteristic

As presented in the empirical methodology part of this thesis, the pricing of market liquidity risk can be assessed by averaging the liquidity of all stocks in the market and then uses this market liquidity measure in the cross-sectional regressions. In this way the premium for market liquidity risk of a portfolio can be estimated. In the following, liquidity will be considered a market-wide variable that can have an effect of the pricing of stocks. As hypothesised earlier, stock portfolios that are very sensitive to market liquidity should be considered more risky by investors and should thus yield higher returns. The results of the cross-sectional tests of this hypothesis follow below.

The value-weighted market relative bid-ask spread as the measure of liquidity

Below, the result of the cross-sectional analysis of the effect of market liquidity risk on returns is provided.

Table 7.13: The cross-sectional effect of the market relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the market relative bid-ask spread sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0075	-0.0012	0.0064	0.0119	-0.0015
Std. error	0.0132	0.0125	0.0113	0.0156	0.0024
$R^2 = 0.2822$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

As no significant risk premiums are found, nothing can be concluded about the pricing of market liquidity risk or any of the other three risk factors based on the 3x3 spread and beta portfolios.

The value-weighted market turnover rate as the measure of liquidity

To find out whether the sensitivity of portfolio returns towards market liquidity measured as the value-weighted average of the turnover rate of all stocks, the cross-sectional analysis has been done using the turnover rate as the measure of liquidity. The results of this analysis can be found in the table below.

Table 7.14: The cross-sectional effect of the market turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the market turnover rate sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0076	0.0021	0.0169	0.0192	-0.0190
Std. error	0.0121	0.0119	0.0132	0.0167	0.0305
$R^2 = 0.2790$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Again, no significant premiums for any of the risk factors are found. Thus, based on the current sample, methodology and assumptions, the conclusion is that the portfolio return sensitivity towards changes in the market liquidity is not a priced risk factor.

Overall, no evidence of a premium for market liquidity risk has been found. In appendix A, tables A.29 to A.36 provide robustness tests of this conclusion. There is actually no evidence of the pricing of market liquidity risk using the current measures, so this conclusion seems robust. As none of the robustness checks reveal any interesting findings in relation to the identification of a market liquidity risk premium, these will not be presented and analysed in turn in the following section where the robustness of findings is tested.

7.5 Robustness checks of the liquidity risk - return relationship

In this section, the findings of a wide range of robustness checks will be described and analysed. First of all, the findings of analyses of 10 stock portfolios formed on the basis of stock betas instead of the 3x3 liquidity and beta portfolios will be presented. These findings provide evidence of a significant effect of liquidity on stock returns so the tables with results of these analyses will be pro-

vided in the text. The rest of the robustness checks show ambiguous results and only small indications of a liquidity-return relationship. Basically, these analyses indicate that the first findings of the analyses above are robust to the changed assumptions. The results of these analyses can be found in the appendices of this thesis. As mentioned above, none of the robustness checks for the analysis of market liquidity risk pricing altered the conclusion reached in the main tests. Therefore, there is no reason for going through each of the robustness checks just to state the same conclusion each time. This section will thus only cover the pricing of portfolio liquidity risk.

The statistical assumptions of all the models have been tested in the approximated residual analysis framework presented earlier. The results of these residual analyses can be found in appendix B. Generally, these residual analyses show no signs of serious violations of the assumptions of the approximated models.

7.5.1 Forming portfolios on basis of stock betas only

Now, the models for the pricing of liquidity risk will be tested for portfolios formed on the basis of individual stock betas only.

The relative bid-ask spread as the measure of liquidity

Table 7.15: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the relative bid-ask spread sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0213**	-0.0154*	0.0067	-0.0103	0.0048**
Std. error	0.0102	0.0102	0.0095	0.0096	0.0028
$R^2 = 0.4606$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

These results reveal some interesting findings. There is now an indication that liquidity risk is priced. That is, the coefficient for the portfolio sensitivity towards its liquidity is significant at a 5%-level. The sign is in accordance with expectations which indicate that there is a premium for the liquidity sensitivity. A portfolio with a liquidity sensitivity of $l = 1$, for instance, would be priced at a required return premium from liquidity sensitivity of approximately 590bp pro annum²². Even though the magnitude may not be as large as indicated by evidence of earlier studies, this finding supports the notion of the pricing of liquidity risk. Also, it should be noticed that the intercept term is significantly different than zero, and there is slightly significant market risk *discount*. These findings are not in accordance with expectations, but the liquidity risk premium is still a valid finding. To see if the finding relating to the relationship between relative bid-ask spread sensitivity of the portfolios and their returns is sensitive to the presence of the Fama and French factors, table 7.16 below, shows the results of the regressions where the Fama and French factors are left out.

Table 7.16: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$ and is defined in section 4 Methodology, and $\hat{l}_i$ is the relative bid-ask spread sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0190***	-0.0142*	0.0046**
Std. error	0.0065	0.0082	0.0022
$R^2 = 0.4538$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

The results shown in table 7.16 provide evidence of the pricing of liquidity risk measured by the relative bid-ask spread sensitivity of portfolio returns. The results indicate an annualised liquidity

²² $(1+0.0048)^{12}-1 = 0.059$. See appendix for results with more decimals

risk premium of 570bp per unit of liquidity sensitivity²³. Finally, it should be noted that the market risk premium is negative.

The turnover rate as the measure of liquidity

Table 7.17: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$, $\hat{s}_i$ and $\hat{h}_i$ are defined in section 4 Methodology, and $\hat{l}_i$ is the turnover rate sensitivity of portfolio i . For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0177**	-0.0139*	0.0110*	-0.0101*	-0.0182
Std. error	0.0089	0.0094	0.0085	0.0077	0.0197
$R^2 = 0.2634$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

There are several implications that represent evidence *against* theory. First of all, there is no evidence of pricing of liquidity risk. In addition to this, the intercept term is not zero and two of the risk premiums are negative. The only finding in accordance with theory is that there is a slightly significant premium for risk associated with portfolio return sensitivity towards the SMB portfolio. To see if the indication that liquidity risk is not priced is sensitive to the presence of the Fama and French factors, these are excluded in table 7.18 below.

²³ $(1+0.0046)^{12-1} = 5.7\%$

Table 7.18: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

The estimated gammas below are average gammas from cross-sectional regressions

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{l}_i + u_{it}$$

Where $\hat{\beta}_i$ is defined in section 4 Methodology, and $\hat{l}_i$ is the turnover rate sensitivity of portfolio i .

For a thorough description see section 4 Methodology.

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0149**	-0.0096*	0.0128
Std. error	0.0058	0.0071	0.0367
$R^2 = 0.2950$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Simplifying the regression model where liquidity is proxied by the turnover rate does not provide evidence of a significant pricing of portfolio sensitivity towards liquidity. This result indicates that the above finding that there is no significant pricing of turnover rate sensitivity for the beta portfolios. This finding is robust to the presence of the Fama and French factors.

7.5.2 Leaving out the Fama-French factors

This has already been done a few times but not for the main models. As the SMB and HML portfolios were determined in a different way than in the original study by Fama and French (1993), there is a possibility that there is too much white noise in the estimates of the returns or simply that their returns are not comparable to those of the original portfolios. These problems could result in the SMB and HML entering spuriously in the regressions. If this is the case, one can expect to obtain more significant results when the SMB and HML portfolios are absent from the regressions. Below, the results of cross-sectional regressions of a CAPM where the only additional explanatory variable is the liquidity measure are discussed.

The relative bid-ask spread as the measure of liquidity

In table A.19 in appendix A, no significant risk premiums are found, but the intercept term is highly significant indicating that the two included risk measures fail to explain something. These results confirm the first findings that indicated no pricing of liquidity risk.

The turnover rate as the measure of liquidity

Simplifying the regression models where liquidity is proxied by the turnover rate does not provide evidence of a significant pricing of portfolio sensitivity towards liquidity, cf. table A.20 in appendix A. This finding is contrary to what was found in the first tests of the full model. This indicates that the premium determined earlier is sensitive to the presence of the two Fama-French risk factors (sensitivities towards SMB and HML, respectively). This means that the identified liquidity risk premium is not robust to the way the model is stated.

7.5.3 Equal weights

The relative bid-ask spread as the measure of liquidity

The results of the analysis where the stocks in the 3x3 spread and beta portfolios have been assigned equal weights can be found in appendix A, table A.21. Only one of the coefficients is significant and there is no indication of a risk premium for liquidity sensitivity on the basis of these results. This finding is interesting in the sense that, regarding a premium for liquidity risk, this is exactly what was found when the averaging convention was value-weighting. So on the basis of the 3x3 spread and beta portfolios, the conclusion that there are no indications of significant pricing of liquidity risk is robust to the averaging convention. In table A.22 in appendix A, the results are shown for the restricted model where the Fama-French factors have been excluded. Here, no coefficients are significant, so regarding the pricing of liquidity the conclusion remains unchanged - there does not seem to be a significant pricing of liquidity risk.

Tables A.23 and A.24 in appendix A presents the results of the analysis of the equally weighted beta portfolios. In neither the unrestricted nor the restricted (Fama-French factors excluded) model, there are signs of a liquidity risk premium. Compared to when the stocks were value-weighted, this represents a change in the conclusion. When the beta portfolios were value-weighted, there were significant liquidity risk premiums in both models. This finding is apparently not robust to how the stocks are weighted in the portfolios.

The turnover rate as the measure of liquidity

The results of the analysis of the equally weighted 3x3 turnover rate and beta portfolios are provided in tables A.25 and A.26 in appendix. The unrestricted model reveals some surprising results - three out of four risk premiums are negative which means that they are in fact *discounts*. This implies that additional market risk, "SMB risk" and liquidity risk would make the investors demand lower returns, which is highly counter-intuitive. In the restricted model, neither the market risk nor the liquidity risk is priced. The fact that the liquidity risk was negatively priced in the model including the Fama-French factors is of additional interest as this is the direct opposite of what was found for the value-weighted portfolios, where a slightly significant liquidity risk premium was found. When changing the averaging technique changes the sign of the coefficient, it is a definite indication that neither finding is very robust. Therefore, one should not conclude too much on the basis of this.

As for the beta portfolios, the results of applying equal weights are shown in tables A.27 and A.28 in appendix. In the model including the Fama-French factors, there is no premium for liquidity found, but the market risk factor and the SMB risk factor both attract significant premiums. If the Fama-French factors are excluded, the market risk premium becomes significantly negative, but the liquidity risk premium becomes significantly positive. First of all, this is an indication that the first findings are not robust to the presence of the Fama-French factors. Next, the fact that the findings vary considerably generally gives an image of a relationship between the turnover rate sensitivity and returns that is ambiguous.

7.5.4 Excluding the period the start of the sub-prime crisis

In relation to the direct effect of liquidity on returns (sections 7.2 and 7.3), excluding the period from the start of the sub-prime crisis showed that some findings were sensitive to the presence of the credit crunch. As for the effect of liquidity risk, excluding the sub-prime crisis has no impact on the findings. Tables A.29 to A.36 present the results of the analyses of the effect of the portfolio sensitivity towards relative spread and the turnover rate, respectively, on returns for both the 3x3 liquidity and beta portfolios and the beta portfolios for the for data set excluding the period of the sub-prime crisis. As follows from the tables, excluding the period imposes no changes in the overall conclusion regarding the pricing of liquidity risk. None of the tables present results that, in relation to the significance and sign of the liquidity risk coefficient, are remarkably different than the full period findings.

7.6 Summary of findings

7.6.1 The effect of liquidity on stock returns

On the basis of the 3x3 liquidity and beta value-weighted portfolios, neither the relative bid-ask spread nor the turnover rate of the portfolios provided any evidence of a significant relationship between liquidity and stock returns. These findings are robust to the presence of the Fama-French factors - the no-effect finding does not change when the Fama-French factors are excluded from the analysis. Also, the findings are robust to the weighting convention - equally weighting the stocks in the portfolios does not provide evidence of a relationship between liquidity and stock returns for either measure of liquidity.

The findings are, however, not robust to the portfolio formation criteria. Value-weighted portfolios based on stock betas reveal evidence of a significant effect of liquidity on stock returns with signs being in line with theory for both the relative bid-ask spread and the turnover rate. It should be noticed that the identified relationship between liquidity and stock returns for the beta portfolios is not robust to the weighting convention. If the portfolios are formed based on stock betas and stocks are assigned equal weights, some of the evidence disappears. The relationship between the spread and returns for the beta portfolios when the Fama-French factors are excluded persists an equal weighting of the stocks in the portfolios.

7.6.2 The effect of liquidity risk on stock returns

The first tests revealed only small signs of a pricing of liquidity risk. As for the analysis where the relative bid-ask spread was the proxy for liquidity, there were no signs of a risk premium for liquidity risk. The turnover rate proxy provided evidence of a slightly significant liquidity risk premium in the returns of the 3x3 turnover rate and beta portfolios. However, this finding is not robust to the weighting convention, the portfolio formation or the presence of the Fama-French factors. As for the equally weighted beta portfolios, there was a slightly significant premium for the liquidity risk level if the Fama-French factors are excluded, but again, this was not a robust finding.

As for the bid-ask spread, the finding that there was no significant pricing of the liquidity risk was found to be robust to both the presence of the Fama-French factors and the weighting convention. However, when the portfolios were formed as value-weighted portfolios based on stock betas, significant evidence of the pricing of liquidity risk was found. The estimated premium for liquidity risk is significant regardless of the presence of the Fama-French factors. It should be noticed, though,

that the spread does not provide evidence of the pricing of liquidity risk when the stocks in the beta portfolios are equally weighted.

In conclusion about the pricing of liquidity risk in the Danish equity market it can be said that evidence is ambiguous. There are indications of liquidity risk attracting a premium, but the findings are far from robust.

The two most important sources of error in the study that can have contributed to making the results ambiguous relate to the sample size and the time period considered. The sample size is critically small, and the time period considered is relatively short.

8 Conclusion

The main purpose of this thesis was to determine the relationship between liquidity and stock returns. First, the theoretical relationship was derived. This was followed by an empirical study based on a sample of listed Danish stocks. Here, the cross-sectional relationship between liquidity and stock returns, and the cross-sectional relationship between liquidity risk and stock returns was studied.

In the theoretical part of this thesis this thesis, liquidity in relation to stocks was defined as the ease by which large quantities of a stock can be traded immediately after purchase at low cost without affecting the price.

Several theoretical models indicate both a pricing of liquidity and a pricing of liquidity risk. These models prove that rational investors would demand a higher rate of return for stocks that are less than perfectly liquid. It has also been established that, if liquidity is important for returns, then sensitivity towards liquidity is a risk that should be priced by investors. Thus, illiquidity and liquidity risk should attract premiums in equity markets.

In relation to stocks, liquidity can be proxied in a number of ways. In this thesis, various ways to measure liquidity or illiquidity has been presented. These proxies comprise the bid-ask spread, the turnover rate, Amihud's ILLIQ-measure, block trade studies, IPO studies, restricted stock offerings studies and Longstaff's put.

In the empirical study of this thesis, the relative bid-ask spread and the turnover rate was used as measures of liquidity. The relative bid-ask spread is calculated as the absolute difference between the bid and the ask price divided by the average of the two. The turnover rate was calculated as the three-month average of the monthly traded volume divided by the number of shares outstanding that month.

Empirical evidence from previous research indicates that liquidity and liquidity risk is priced in the stock markets. Since the eighties, researchers have studied the relationship between liquidity and stock pricing. Selected studies were reviewed in this thesis. Amihud and Mendelson first presented a model for the relationship between the relative bid-ask spread and stock returns. This model has become a cornerstone in the field of liquidity and stock returns, and the model derived in the theo-

retical part of this study is basically the same as Amihud and Mendelson's model. In their article they also provide evidence of a positive relationship between the relative bid-ask spread and stock returns based on a sample of US stocks.

The empirical study of this thesis applied the cross-sectional methodology of Fama and Macbeth (1973). This was done on two different ways as to examine the cross-sectional effect of liquidity on stock returns and the cross-sectional effect of liquidity risk on stock returns. To reduce white noise, the stocks in the sample were divided into 3x3 value-weighted portfolios based on the liquidity and beta of the stocks. These portfolios formed the basis of the cross-sectional analyses. The robustness of the portfolio formation criteria were tested by carrying out the analyses on basis of equally weighted portfolios as well as portfolios based on betas (both equally and value weighted). Also, as the derivation of the SMB and HML portfolios was different than the original derivation in Fama and French (1993), and thus could be showing something different than what they are ought to, they could actually enter the model spuriously. If this is the case, there might be a better case of detecting an illiquidity return premium or a liquidity risk premium when the two portfolios are excluded from the regressions. Therefore, as additional robustness checks, all analyses (including other robustness tests) were also done excluding the Fama-French factors.

In the study of the cross-sectional relationship between the relative bid-ask spreads and the returns of the 3x3 spread and beta portfolios, no indication of an effect of illiquidity on returns were found. This finding is robust to the presence of the Fama-French factors and the chosen weighting convention (using value weights does not change the conclusion). The same accounts for the study of the cross-sectional relationship between the turnover rates and the returns of the 3x3 turnover rate and beta portfolios. If, instead, portfolios are formed on the basis of individual stock betas only, evidence of a significant relationship between stock portfolio returns and the relative bid-ask spread was found, and also between portfolio returns and the turnover rate of the portfolios. This implies that the first findings of no pricing of liquidity are not robust to the portfolio formation methodology chosen. Whereas the turnover rate provides evidence of a negative relationship (as predicted by theory) for the value weighted beta portfolios, the finding is not robust to the weighting of the stocks in the portfolios - when equal weights are applied, the relationship becomes insignificant. As for the relative bid-ask spread, the positive relationship between illiquidity and portfolio returns is robust to the weighting when the Fama-French factors are left out of the analysis - otherwise not. Again, the findings are not very robust, but they represent evidence of some degree of pricing of liquidity. In the cases where a significant spread-return relationship is found, the required annual

excess return on a stock portfolio should be expected to increase by 400 to 520bp if the relative bid-ask spread increases by 100bp. When a significantly negative turnover rate-return relationship is identified, the excess return on a stock portfolio should increase by 62 to 71bp the turnover rate of that stock portfolio decreases by 100bp. As mentioned, the findings should be interpreted with a great deal of caution due to the fact that these findings are sensitive to many different assumptions.

The empirical study of the cross-sectional relationship between the relative bid-ask spread sensitivity (a proxy of illiquidity risk) of the 3x3 spread and beta portfolios and excess stock portfolio returns show no signs of a pricing of liquidity risk. That is, no liquidity risk premium is detected. This finding persists when the Fama-French factors are excluded from the analysis, and assigning equal weights to the stocks in the portfolios does not change the conclusion either. A slightly significant premium for turnover rate sensitivity for the 3x3 turnover rate and beta portfolios is identified, but this finding does not hold when the Fama-French factors are taken out of the analysis and the averaging convention is changed. Thus, the finding is not very robust. When changing the portfolio formation methodology to being based on individual stock betas only, evidence of a significant liquidity risk premium is identified for the bid-ask spread analysis. This finding persists when the Fama-French factors are excluded, but not if the averaging is changed. When proxying liquidity risk by applying the turnover rate sensitivities of the portfolios, no significant results are found for the beta portfolios. Equally weighting the stocks in the beta portfolios provides slightly significant evidence of a liquidity risk premium, but obviously, this cannot be considered a robust finding. Generally, the results of the turnover rate analysis provide poor evidence of the pricing of liquidity risk. When the spread sensitivity-return analysis yields significant risk premiums for spread sensitivity, the excess return on a stock portfolio should be expected to increase by 570 to 590bp for each unit of increase in the spread sensitivity. As for the turnover rate sensitivity, the slightly significant risk premiums indicate that the expected excess return on a stock portfolio should increase by about 1080bp per unit of increase in the turnover rate sensitivity of the stock. As mentioned and explained above, these findings should be considered merely indications. The findings are not robust, and the statistical tests and the parameter estimates are not perfectly reliable. Therefore, the results should be interpreted only as slight indications.

The closing remark of this thesis is that liquidity should have an effect in equity markets and the empirical study of the Danish equity market provides some evidence of this. It is left for further research to examine the relationship between liquidity or liquidity risk and stock returns using a sample covering a longer period of time and maybe using finer measures of liquidity - e.g. Amihud's

ILLIQ measure. It could also be interesting to see Archarya and Pedersen's (2005) model applied to Danish stock market data.

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Appendix A: Results of robustness checks

A.1 The relationship between liquidity and stock returns

Leaving out the Fama-French factors

Table A.1: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0109	-0.0041	0.1193
Std. error	0.0077	0.0072	0.1124
$R^2 = 0.3356$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.2: The direct cross-sectional effect of the turnover rate of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0202***	-0.0088	-0.0220
Std. error	0.0063	0.0071	0.0262
$R^2 = 0.6288$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Equal weights

Table A.3: The direct cross-sectional effect of the relative bid-ask spread of the equally weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0029	-0.0017	-0.0071	0.0078	0.0572
Std. error	0.0063	0.0063	0.0093	0.0158	0.0670
$R^2 = 0.7620$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.4: The direct cross-sectional effect of the relative bid-ask spread of the equally weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0019	0.0016	0.0602
Std. error	0.0065	0.0072	0.0512
$R^2 = 0.2889$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.5: The direct cross-sectional effect of the relative bid-ask spread of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0032	-0.0005	-0.0014	0.0009	0.0706
Std. error	0.0086	0.0065	0.0153	0.0153	0.0696
$R^2 = 0.4318$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.6: The direct cross-sectional effect of the relative bid-ask spread of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0071	-0.0048	0.0867*
Std. error	0.0065	0.0048	0.0628
$R^2 = 0.3086$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.7: The direct cross-sectional effect of the turnover rate of the equally weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0064	0.0011	-0.0187*	0.0116	-0.0329
Std. error	0.0058	0.0057	0.0124	0.0183	0.0267
$R^2 = 0.8053$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.8: The direct cross-sectional effect of the turnover rate of the equally weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0080	-0.0023	-0.0020
Std. error	0.0057	0.0051	0.0177
$R^2 = 0.0583$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.9: The direct cross-sectional effect of the turnover rate of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	-0.0030	0.0124**	0.0133**	0.0056	0.0060
Std. error	0.0068	0.0064	0.0065	0.0083	0.0404
$R^2 = 0.7482$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.10: The direct cross-sectional effect of the turnover rate of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0109*	-0.0040	0.0043
Std. error	0.0055	0.0044	0.0304
$R^2 = 0.2616$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Excluding the period the start of the sub-prime crisis

Table A.11: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0178**	-0.0031	0.0207*	-0.0080	-0.0616
Std. error	0.0088	0.0073	0.0130	0.0103	0.2229

$R^2 = 0.6184$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.12: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0205**	-0.0078	0.0326
Std. error	0.0080	0.0073	0.1128

$R^2 = 0.2409$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.13: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0234**	-0.0082	-0.0024	-0.0078	0.1341
Std. error	0.0107	0.0099	0.0159	0.0099	0.1797

$R^2 = 0.3781$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.14: The direct cross-sectional effect of the relative bid-ask spread of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M12

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0171**	-0.0035	0.0546
Std. error	0.0071	0.0068	0.0847

$R^2 = 0.2966$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.15: The direct cross-sectional effect of the turnover rate of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0236**	-0.0079	0.0103	-0.0008	-0.0152
Std. error	0.0104	0.0090	0.0128	0.0123	0.0349
$R^2 = 0.4386$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.16: The direct cross-sectional effect of the turnover rate of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0262***	-0.0077	-0.0387*
Std. error	0.0066	0.0070	0.0285
$R^2 = 0.5553$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.17: The direct cross-sectional effect of the turnover rate of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0201*	-0.0034	0.0057	-0.0036	-0.0387
Std. error	0.0107	0.0081	0.0114	0.0102	0.0413

$R^2 = 0.3798$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.18: The direct cross-sectional effect of the turnover rate of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 LIQ_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0215***	-0.0044	-0.0368*
Std. error	0.0066	0.0071	0.0212

$R^2 = 0.1895$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

A.2 The relationship between liquidity risk and stock returns

Leaving out the Fama-French factors

Table A.19: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0165***	-0.0078	-0.0024
Std. error	0.0061	0.0067	0.0021
$R^2 = 0.0997$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.20: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0213***	-0.0108*	0.0040
Std. error	0.0067	0.0081	0.0044
$R^2 = 0.5003$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Equal weights

Table A.21: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the equally weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$		
Estimate	0.0055	-0.0005	-0.0064	0.0377*	-0.0020
Std. error	0.0078	0.0071	0.0108	0.0250	0.0044
$R^2 = 0.8631$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.22: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the equally weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0032	0.0020	0.0005
Std. error	0.0060	0.0053	0.0029
$R^2 = 0.4961$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.23: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$					
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0080	-0.0039	-0.0088	-0.0019	0.0013
Std. error	0.0086	0.0081	0.0132	0.0173	0.0079
$R^2 = 0.5489$					

* Indicates statistical significance at a 0.1 level
 ** Indicates statistical significance at a 0.05 level
 *** Indicates statistical significance at a 0.01 level

Table A.24: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0162***	-0.0111**	-0.0042
Std. error	0.0059	0.0059	0.0071
$R^2 = 0.3666$			

* Indicates statistical significance at a 0.1 level
 ** Indicates statistical significance at a 0.05 level
 *** Indicates statistical significance at a 0.01 level

Table A.25: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the equally weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0133**	-0.0095*	-0.0366**	0.0198	-0.0137*
Std. error	0.0065	0.0069	0.0169	0.0170	0.0103
$R^2 = 0.9189$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.26: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the equally weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0139*	-0.0085	0.0006
Std. error	0.0077	0.0083	0.0086
$R^2 = 0.4874$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.27: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	-0.0016	0.0114**	0.0122**	0.0059	-0.0131
Std. error	0.0074	0.0073	0.0062	0.0086	0.0209
$R^2 = 0.8008$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.28: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the equally weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0181***	-0.0134**	0.0412*
Std. error	0.0061	0.0065	0.0300
$R^2 = 0.3080$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Excluding the period from the start of the sub-prime crisis

Table A.29: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0087	0.0012	0.0181*	0.0067	-0.0032
Std. error	0.0142	0.0125	0.0138	0.0127	0.0047

$R^2 = 0.3691$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.30: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0227***	-0.0087	-0.0024
Std. error	0.0066	0.0074	0.0024

$R^2 = 0.1476$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.31: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$				
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0253**	-0.0144*	0.0122	-0.0013	0.0045*
Std. error	0.0109	0.0101	0.0103	0.0091	0.0031
$R^2 = 0.4588$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.32: The cross-sectional effect of the relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

	$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$		
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0234***	-0.0123	0.0040**
Std. error	0.0068	0.0076	0.0023
$R^2 = 0.4077$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.33: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0262**	-0.0116	0.0095	0.0035	0.0096*
Std. error	0.0114	0.0120	0.0130	0.0125	0.0064
$R^2 = 0.2820$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.34: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0271***	-0.0111*	0.0042
Std. error	0.0071	0.0082	0.0050
$R^2 = 0.3454$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.35: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0242**	-0.0155*	0.0138*	-0.0083	-0.0200
Std. error	0.0096	0.0097	0.0095	0.0083	0.0224
$R^2 = 0.3698$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.36: The cross-sectional effect of the turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risks. 1997M1-2007M7

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0217***	-0.0115*	0.0142
Std. error	0.0061	0.0073	0.0418
$R^2 = 0.1947$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Market liquidity risk

Table A.37: The cross-sectional effect of the market relative bid-ask spread sensitivity of the excess returns of the value-weighted 3x3 spread and beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0112	-0.0038	-0.0009
Std. error	0.0078	0.0079	0.0015

$R^2 = 0.3167$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.38: The cross-sectional effect of the market relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0076	0.0007	0.0066	-0.0074	-0.0008
Std. error	0.0087	0.0085	0.0079	0.0076	0.0007

$R^2 = 0.3847$

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.39: The cross-sectional effect of the market relative bid-ask spread sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0099	-0.0010	-0.0010*
Std. error	0.0068	0.0074	0.0006
$R^2 = 0.1536$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.40: The cross-sectional effect of the market turnover rate sensitivity of the excess returns of the value-weighted 3x3 turnover rate and beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0196	-0.0087	-0.0124
Std. error	0.0084**	0.0080	0.0317
$R^2 = 0.5867$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.41: The cross-sectional effect of the market turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk, and the effect of the Fama-French risk factors. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{s}_i + \gamma_3 \hat{h}_i + \gamma_4 \hat{l}_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
Estimate	0.0093	-0.0010	0.0079	-0.0097	0.0130
Std. error	0.0091	0.0085	0.0080	0.0081	0.0163
$R^2 = 0.1796$					

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.42: The cross-sectional effect of the market turnover rate sensitivity of the excess returns of the value-weighted beta portfolios on portfolio excess returns while controlling for market risk. 1997M1-2008M11

$$R_{it} - R_{ft} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 l_i + u_{it}$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Estimate	0.0111*	-0.0032	0.0116
Std. error	0.0058	0.0064	0.0133
$R^2 = 0.1528$			

* Indicates statistical significance at a 0.1 level

** Indicates statistical significance at a 0.05 level

*** Indicates statistical significance at a 0.01 level

Table A.43: Excess returns and relative spreads for th different portfolios estimated for the entire period, beta portfolios

<i>Beta portfolio</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>High</i>
$R_i - R_f$	0.0265	0.0184	0.0174	0.0069	0.0115	0.0081	0.0123	0.0064	0.0106	0.0055
Spread	0.0779	0.0479	0.0342	0.0308	0.0190	0.0206	0.0170	0.0145	0.0116	0.0117

Table A.44: Average excess return and turnover rate for the beta portfolios. Estimated over the time period 1992-2008M11

<i>Beta portfolio</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>High</i>
$R_i - R_f$	0.0271	0.0183	0.0174	0.0069	0.0113	0.0081	0.0123	0.0066	0.0107	0.0055
Turnover rate	0.0681	0.2749	0.1112	0.2267	0.1986	0.2779	0.1338	0.1864	0.1828	0.1036

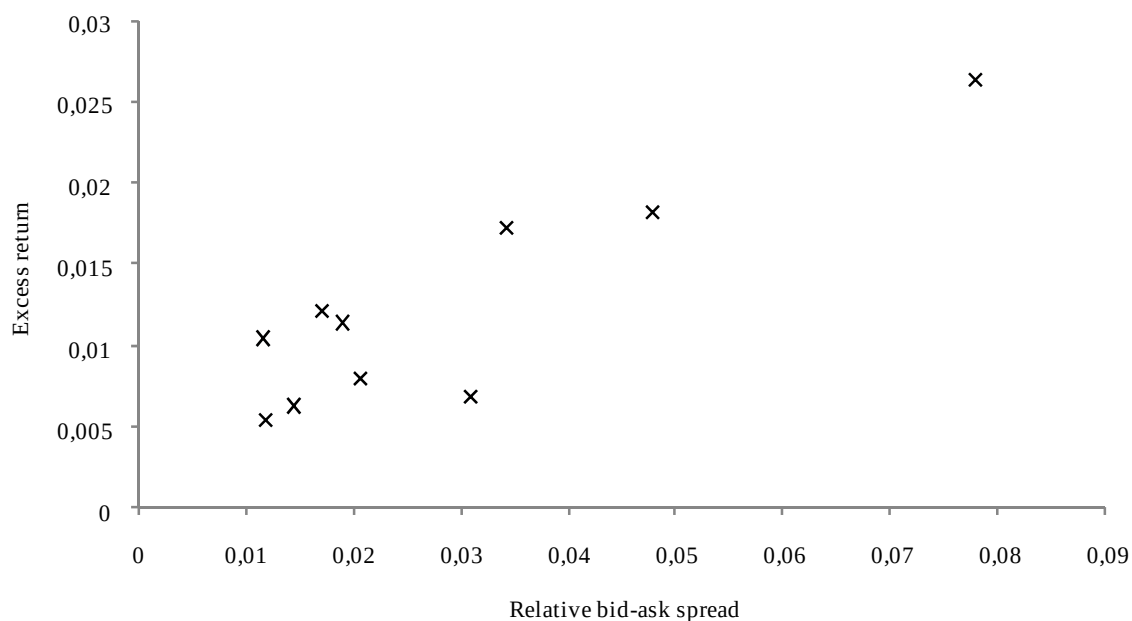
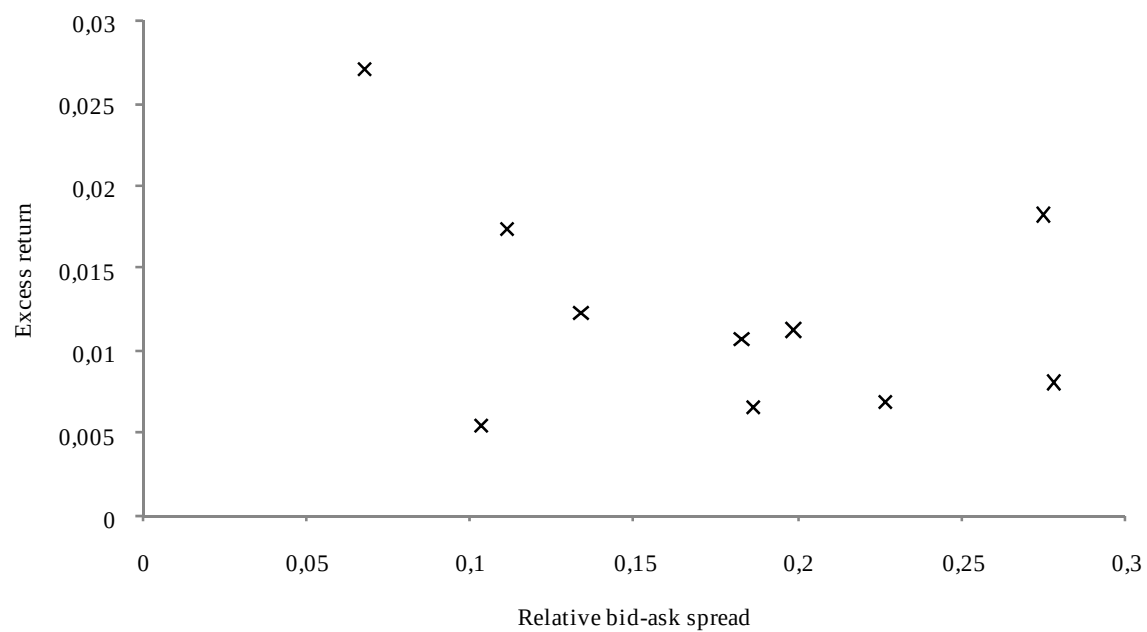
Figure A.1: XY-plot of average portfolio excess returns against the average portfolio relative spreads estimated over the period 1992-2008M11 for the beta portfolios

Figure A.2: XY-plot of average portfolio excess returns against the average portfolio relative spreads estimated over the period 1992-2008M1 for the beta portfolios



Appendix B: Residual analysis

B.1 The relationship between liquidity and stock returns

Approximated residual analysis for the model presented in table 7.4 [3x3 spread AM]

Ordinary Least Squares Estimates			
SSE	0.00037856	DFE	4
MSE	0.0000946	Root MSE	0.00973
SBC	-54.16013	AIC	-55.146253
Regress R-Square	0.3305	Total R-Square	0.3305
Normal Test	4.4091	Pr > ChiSq	0.1103
Durbin-Watson	1.3497		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	2.82	0.9454

Collinearity Diagnostics							
Number	Eigenvalue	Condition	Proportion of Variation				
		Index	Intercept	Beta	s	h	l
1	3.86907	1.00000	0.00356	0.00429	0.00262	0.01836	0.00189
2	0.72450	2.31091	0.02301	0.02101	0.04380	0.00221	0.00247
3	0.34083	3.36927	0.01291	0.00233	0.00990	0.88672	0.00680
4	0.04645	9.12651	0.54909	0.96682	0.01714	0.09260	0.01589
5	0.01915	14.21286	0.41142	0.00556	0.92654	0.00011039	0.97294

Approximated residual analysis for the model presented in table 7.6 [3x3 TR AM]

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	7.72	0.5622

Ordinary Least Squares Estimates			
SSE	0.00003634	DFE	4
MSE	9.08445E-6	Root MSE	0.00301
SBC	-75.25187	AIC	-76.237993
Regress R-Square	0.8004	Total R-Square	0.8004

Ordinary Least Squares Estimates			
Normal Test	1.1034	Pr > ChiSq	0.5760
Durbin-Watson	3.0450		

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.08189	1.00000	0.00238	0.00283	0.02166	0.01780	0.02868
2	0.93745	1.81315	0.00000166	0.00009888	0.05163	0.58790	0.06750
3	0.68592	2.11969	0.00003432	0.00004603	0.30414	0.00001304	0.40969
4	0.28149	3.30886	0.01173	0.02694	0.32930	0.39428	0.45101
5	0.01326	15.24737	0.98585	0.97008	0.29328	4.852963E-7	0.04312

Approximated residual analysis for the model presented in table 7.8 [beta spread AM]

Ordinary Least Squares Estimates			
SSE	0.00012987	DFE	5
MSE	0.0000260	Root MSE	0.00510
SBC	-72.62404	AIC	-74.136966
Regress R-Square	0.5108	Total R-Square	0.5108
Normal Test	0.5463	Pr > ChiSq	0.7610
Durbin-Watson	2.7204		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.40	0.4012

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.63405	1.00000	0.00184	0.00217	0.00283	0.00676	0.00325
2	1.02996	1.87839	0.00017619	0.00063307	0.00468	0.54310	0.00315
3	0.28980	3.54116	0.01591	0.03383	0.02814	0.38904	0.03071
4	0.03341	10.42935	0.09897	0.08556	0.47106	0.03317	0.64772
5	0.01279	16.85944	0.88310	0.87781	0.49329	0.02792	0.31517

Approximated residual analysis for the model presented in table 7.9 [beta spread AM simple]

Ordinary Least Squares Estimates			
SSE	0.00014472	DFE	7
MSE	0.0000207	Root MSE	0.00455
SBC	-76.146565	AIC	-77.05432
Regress R-Square	0.4549	Total R-Square	0.4549
Normal Test	0.7926	Pr > ChiSq	0.6728
Durbin-Watson	2.6562		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	5.43	0.3662

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	l
1	2.62734	1.00000	0.00568	0.00859	0.02430
2	0.34828	2.74658	0.00320	0.06465	0.39704
3	0.02438	10.38188	0.99112	0.92676	0.57866

Approximated residual analysis for the model presented in table 7.11 [beta TR AM]

Ordinary Least Squares Estimates			
SSE	0.00004868	DFE	5
MSE	9.73503E-6	Root MSE	0.00312
SBC	-82.437576	AIC	-83.950501
Regress R-Square	0.8162	Total R-Square	0.8162
Normal Test	2.0275	Pr > ChiSq	0.3629
Durbin-Watson	2.5117		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.45	0.3968

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.54040	1.00000	0.00159	0.00264	0.01295	0.00646	0.01304
2	0.99126	1.88987	8.085436E-7	0.0001334	0.02974	0.54728	0.00276
3	0.32483	3.30141	0.00002226	0.0005755	0.38534	0.07827	0.26497
4	0.13043	5.21007	0.01359	0.10656	0.24241	0.31254	0.49634
5	0.01308	16.45026	0.98480	0.89010	0.32961	0.05545	0.22289

Approximated residual analysis for the model presented in table 7.12 [beta TR AM simple]

Ordinary Least Squares Estimates			
SSE	0.0000967	DFE	7
MSE	0.0000138	Root MSE	0.00372
SBC	-80.177837	AIC	-81.085592
Regress R-Square	0.6348	Total R-Square	0.6348
Normal Test	0.4665	Pr > ChiSq	0.7920
Durbin-Watson	2.6541		

Test of First and Second Moment Specification		
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DF	Chi-Square	Pr > ChiSq
5	5.10	0.4043

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.75942	1.00000	0.00979	0.01262	0.02794
2	0.19316	3.77963	0.02538	0.15052	0.82886
3	0.04742	7.62820	0.96483	0.83686	0.14320

Approximated residual analysis for the model presented in table A.1 [3x3 spread AM simple]

Ordinary Least Squares Estimates			
SSE	0.00044103	DFE	6
MSE	0.0000735	Root MSE	0.00857
SBC	-57.180081	AIC	-57.771755
Regress R-Square	0.2200	Total R-Square	0.2200
Normal Test	0.9701	Pr > ChiSq	0.6157
Durbin-Watson	1.5985		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.10	0.6848

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.59072	1.00000	0.00802	0.01108	0.03323
2	0.37598	2.62498	0.00556	0.06820	0.52283
3	0.03330	8.81993	0.98642	0.92073	0.44393

Approximated residual analysis for the model presented in table A.2 [3x3 TR AM simple]

Ordinary Least Squares Estimates			
SSE	0.00007018	DFE	6
MSE	0.0000117	Root MSE	0.00342
SBC	-73.722228	AIC	-74.313902
Regress R-Square	0.3912	Total R-Square	0.3912

Ordinary Least Squares Estimates			
Normal Test	1.6724	Pr > ChiSq	0.4333
Durbin-Watson	2.4725		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	2.59	0.7633

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.47343	1.00000	0.00523	0.00499	0.06001
2	0.50953	2.20325	0.01031	0.00650	0.87045
3	0.01703	12.05023	0.98446	0.98851	0.06954

Approximated residual analysis for the model presented in table A.3 [3x3 spread AM equal]

Ordinary Least Squares Estimates			
SSE	0.00002975	DFE	4
MSE	7.43816E-6	Root MSE	0.00273
SBC	-77.051339	AIC	-78.037462
Regress R-Square	0.7993	Total R-Square	0.7993
Normal Test	0.3649	Pr > ChiSq	0.8332
Durbin-Watson	1.2378		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	6.79	0.5598

Collinearity Diagnostics							
Num- ber	Eigenva- lue	Condi- tion Index	Inter- cept	Proportion of Variation			
				Beta	s	h	I
1	3.06097	1.00000	0.00347	0.00349	0.00002388	0.00737	0.00432
2	1.09790	1.66974	0.00118	0.00024446	0.20238	0.00011006	0.00243
3	0.79179	1.96619	0.00976	0.00290	0.00344	0.14995	0.00046967

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
4	0.02855	10.35516	0.05056	0.17757	0.70101	0.62743	0.98343
5	0.02079	12.13321	0.93502	0.81581	0.09315	0.21514	0.00935

Approximated residual analysis for the model presented in table A.4 [3x3 spread AM simple equal]

Ordinary Least Squares Estimates			
SSE	0.00011014	DFE	6
MSE	0.0000184	Root MSE	0.00428
SBC	-69.666426	AIC	-70.258099
Regress R-Square	0.2571	Total R-Square	0.2571
Normal Test	0.7744	Pr > ChiSq	0.6790
Durbin-Watson	2.5674		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.98	0.5522

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	l
1	2.72096	1.00000	0.00393	0.00327	0.03200
2	0.26504	3.20412	0.02569	0.00865	0.75404
3	0.01400	13.94073	0.97038	0.98808	0.21396

Approximated residual analysis for the model presented in table A.5 [beta spread AM equal]

Ordinary Least Squares Estimates			
SSE	0.00008494	DFE	5
MSE	0.0000170	Root MSE	0.00412
SBC	-76.869961	AIC	-78.382886
Regress R-Square	0.4955	Total R-Square	0.4955
Normal Test	0.1925	Pr > ChiSq	0.9082
Durbin-Watson	2.3570		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	4.46	0.8788

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.93168	1.00000	0.00098754	0.00132	0.00308	0.00000788	0.00451
2	1.57750	1.36324	0.00015315	0.00011149	0.09852	0.08076	0.00022357
3	0.45618	2.53508	0.00003523	0.00019767	0.41966	0.21627	0.00517
4	0.02941	9.98390	0.02245	0.09940	0.02611	0.69847	0.82799
5	0.00523	23.66985	0.97637	0.89897	0.45262	0.00449	0.16210

Approximated residual analysis for the model presented in table A.6 [beta spread AM simple equal]

Ordinary Least Squares Estimates			
SSE	0.00010271	DFE	7
MSE	0.0000147	Root MSE	0.00383
SBC	-79.575392	AIC	-80.483148
Regress R-Square	0.3899	Total R-Square	0.3899
Normal Test	1.2060	Pr > ChiSq	0.5472
Durbin-Watson	2.6659		

Test of First and Second Moment Specification		
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DF	Chi-Square	Pr > ChiSq
5	4.04	0.5433

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	L
1	2.87882	1.00000	0.00270	0.00304	0.01698
2	0.10857	5.14934	0.02614	0.04806	0.94406
3	0.01261	15.10788	0.97116	0.94890	0.03896

Approximated residual analysis for the model presented in table A.7 [3x3 TR AM equal]

Ordinary Least Squares Estimates			
SSE	0.00007015	DFE	4
MSE	0.0000175	Root MSE	0.00419
SBC	-69.332237	AIC	-70.31836
Regress R-Square	0.3915	Total R-Square	0.3915
Normal Test	0.7832	Pr > ChiSq	0.6760
Durbin-Watson	2.3915		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	5.88	0.6603

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.47291	1.00000	0.00387	0.00319	0.0001339	0.0000808	0.02666
2	1.48631	1.28988	0.0004102	0.0003642	0.13481	0.15703	0.01742
3	0.83223	1.72379	0.0007882	0.0006317	0.05665	0.64103	0.04632
4	0.19700	3.54297	0.01991	0.00295	0.55043	0.14982	0.63255
5	0.01154	14.63619	0.97502	0.99287	0.25798	0.05204	0.27705

Approximated residual analysis for the model presented in table A.8 [3x3 TR AM simple equal]

Ordinary Least Squares Estimates			
SSE	0.00007018	DFE	6
MSE	0.0000117	Root MSE	0.00342
SBC	-73.722228	AIC	-74.313902
Regress R-Square	0.3912	Total R-Square	0.3912
Normal Test	1.6724	Pr > ChiSq	0.4333
Durbin-Watson	2.4725		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	2.59	0.7633

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	L
1	2.47343	1.00000	0.00523	0.00499	0.06001
2	0.50953	2.20325	0.01031	0.00650	0.87045
3	0.01703	12.05023	0.98446	0.98851	0.06954

Approximated residual analysis for the model presented in table A.9 [beta TR AM equal]

Ordinary Least Squares Estimates			
SSE	0.00002256	DFE	5
MSE	4.51156E-6	Root MSE	0.00212
SBC	-90.128442	AIC	-91.641367
Regress R-Square	0.8918	Total R-Square	0.8918
Normal Test	1.8078	Pr > ChiSq	0.4050
Durbin-Watson	2.0647		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	6.23	0.7168

Collinearity Diagnostics						
Num- ber	Eigenva- lue	Condi- tion Index	Proportion of Variation			
			Intercept	Beta	s	h
						l

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.14831	1.00000	0.00126	0.00131	0.02406	0.00578	0.02557
2	1.14906	1.65526	0.00000694	0.00004485	0.13268	0.36693	0.00025350
3	0.37652	2.89165	0.00061225	0.00003416	0.66224	0.35293	0.25664
4	0.31922	3.14047	0.00729	0.00978	0.13663	0.08176	0.51229
5	0.00689	21.37941	0.99083	0.98883	0.04439	0.19260	0.20525

Approximated residual analysis for the model presented in table A.10 [beta TR AM simple equal]

Ordinary Least Squares Estimates			
SSE	0.00016224	DFE	7
MSE	0.0000232	Root MSE	0.00481
SBC	-75.00366	AIC	-75.911415
Regress R-Square	0.2220	Total R-Square	0.2220
Normal Test	0.5490	Pr > ChiSq	0.7600
Durbin-Watson	2.5850		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.96	0.5552

Collinearity Diagnostics						
Number	Eigenvalue	Condition Index	Proportion of Variation			
			Intercept	Beta		l
1	2.67953	1.00000	0.00335	0.00342	0.04499	
2	0.30745	2.95220	0.01143	0.01295	0.95056	
3	0.01302	14.34544	0.98521	0.98363	0.00445	

Approximated residual analysis for the model presented in table A.11

Ordinary Least Squares Estimates			
SSE	0.0004849	DFE	4
MSE	0.0001212	Root MSE	0.01101
SBC	-51.93207	AIC	-52.918193

Ordinary Least Squares Estimates			
Regress R-Square	0.2297	Total R-Square	0.2297
Normal Test	1.4415	Pr > ChiSq	0.4864
Durbin-Watson	0.9782		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	5.12	0.7451

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	4.00638	1.00000	0.00364	0.00385	0.00141	0.01614	0.00101
2	0.65081	2.48112	0.02513	0.01860	0.02650	0.00327	0.00256
3	0.28333	3.76034	0.01981	0.01383	0.00171	0.95080	0.00337
4	0.04845	9.09317	0.67272	0.81886	0.00007789	0.00644	0.00068594
5	0.01102	19.06289	0.27871	0.14486	0.97031	0.02336	0.99238

Approximated residual analysis for the model presented in table A.12

Ordinary Least Squares Estimates			
SSE	0.00053325	DFE	6
MSE	0.0000889	Root MSE	0.00943
SBC	-55.471138	AIC	-56.062812
Regress R-Square	0.1529	Total R-Square	0.1529
Normal Test	0.7399	Pr > ChiSq	0.6908
Durbin-Watson	1.2215		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.34	0.5018

Collinearity Diagnostics						
Number	Eigenvalue	Condition Index	Proportion of Variation			
			Intercept	Beta		l
1	2.58994	1.00000	0.00891	0.01219	0.03509	
2	0.37296	2.63520	0.00640	0.07527	0.55538	
3	0.03710	8.35499	0.98470	0.91254	0.40953	

Approximated residual analysis for the model presented in table A.13

Ordinary Least Squares Estimates			
SSE	0.00010115	DFE	5
MSE	0.0000202	Root MSE	0.00450
SBC	-75.123173	AIC	-76.636098
Regress R-Square	0.6947	Total R-Square	0.6947
Normal Test	2.2793	Pr > ChiSq	0.3199
Durbin-Watson	2.6609		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	7.46	0.5893

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	L
1	2.93312	1.00000	0.00276	0.00364	0.01984	0.02506	0.00800
2	1.00459	1.70872	0.00005368	0.00006215	0.03887	0.05405	0.57956
3	0.81835	1.89319	0.00030582	0.00007619	0.02051	0.71700	0.13939
4	0.22960	3.57418	0.00929	0.04033	0.59372	0.20207	0.14402
5	0.01433	14.30649	0.98759	0.95589	0.32706	0.00182	0.12903

Approximated residual analysis for the model presented in table A.14

Ordinary Least Squares Estimates			
SSE	0.0002049	DFE	7
MSE	0.0000293	Root MSE	0.00541
SBC	-72.669	AIC	-73.576755
Regress R-Square	0.3816	Total R-Square	0.3816
Normal Test	0.6481	Pr > ChiSq	0.7232
Durbin-Watson	2.5847		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	7.23	0.2041

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.62265	1.00000	0.00600	0.00899	0.02533
2	0.35169	2.73079	0.00341	0.06641	0.41100
3	0.02566	10.10972	0.99059	0.92460	0.56367

Approximated residual analysis for the model presented in table A.15

Ordinary Least Squares Estimates			
SSE	0.00004159	DFE	4
MSE	0.0000104	Root MSE	0.00322
SBC	-74.037171	AIC	-75.023294
Regress R-Square	0.8340	Total R-Square	0.8340
Normal Test	0.8795	Pr > ChiSq	0.6442
Durbin-Watson	3.1891		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	4.61	0.7980

Collinearity Diagnostics							
Num- ber	Eigenva- lue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	I
1	3.08448	1.00000	0.00276	0.00332	0.02296	0.01920	0.02879
2	0.90791	1.84319	0.00001757	0.00004127	0.07197	0.63849	0.04180
3	0.70200	2.09615	0.00006195	8.383745E-9	0.26242	0.00398	0.47229
4	0.29005	3.26101	0.01280	0.03181	0.40339	0.33614	0.39773
5	0.01555	14.08394	0.98436	0.96483	0.23926	0.00218	0.05939

Approximated residual analysis for the model presented in table A.16

Ordinary Least Squares Estimates			
SSE	0.00010323	DFE	6
MSE	0.0000172	Root MSE	0.00415
SBC	-70.249419	AIC	-70.841093
Regress R-Square	0.5880	Total R-Square	0.5880
Normal Test	0.4643	Pr > ChiSq	0.7928
Durbin-Watson	2.4150		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	6.28	0.2797

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	2.44076	1.00000	0.01726	0.01762	0.06612
2	0.49907	2.21148	0.02649	0.03305	0.92888
3	0.06017	6.36905	0.95624	0.94933	0.00500

Approximated residual analysis for the model presented in table A.17

Ordinary Least Squares Estimates			
SSE	0.00006616	DFE	5
MSE	0.0000132	Root MSE	0.00364
SBC	-79.36872	AIC	-80.881646
Regress R-Square	0.7997	Total R-Square	0.7997
Normal Test	0.6485	Pr > ChiSq	0.7231
Durbin-Watson	2.7257		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.58	0.3851

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	L
1	3.55807	1.00000	0.00170	0.00281	0.01351	0.00839	0.01362
2	0.95059	1.93469	3.250344E-7	0.00002968	0.03687	0.55575	0.00271
3	0.34053	3.23246	0.00001361	0.00028299	0.36874	0.07804	0.29235
4	0.13664	5.10285	0.01426	0.11133	0.29159	0.26696	0.46695
5	0.01417	15.84622	0.98403	0.88554	0.28929	0.09085	0.22437

Approximated residual analysis for the model presented in table A.18

Ordinary Least Squares Estimates			
SSE	0.00015516	DFE	7
MSE	0.0000222	Root MSE	0.00471
SBC	-75.450031	AIC	-76.357787
Regress R-Square	0.5302	Total R-Square	0.5302
Normal Test	0.6089	Pr > ChiSq	0.7375
Durbin-Watson	2.4076		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.07	0.5395

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.74970	1.00000	0.01032	0.01292	0.02959
2	0.20099	3.69872	0.02866	0.13994	0.85186
3	0.04930	7.46797	0.96102	0.84713	0.11855

B.2 The relationship between liquidity risk and stock returns

Approximated residual analysis for the model presented in table 7.13 [3x3 spread]

Ordinary Least Squares Estimates			
SSE	0.00030283	DFE	4
MSE	0.0000757	Root MSE	0.00870
SBC	-56.168929	AIC	-57.155052
Regress R-Square	0.4644	Total R-Square	0.4644
Normal Test	0.3723	Pr > ChiSq	0.8302
Durbin-Watson	1.9356		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	7.43	0.4911

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.26699	1.00000	0.00186	0.00250	0.02039	0.03146	0.00938
2	0.83823	1.97421	7.698386E-7	0.0006799	0.07676	0.0007165	0.31437
3	0.58568	2.36180	0.00132	0.00104	0.05859	0.92254	0.01166
4	0.29679	3.31781	0.00686	0.03082	0.59067	0.04333	0.08117
5	0.01231	16.29056	0.98996	0.96495	0.25360	0.00195	0.58341

Approximated residual analysis for the model presented in table 7.14 [3x3 TR]

Ordinary Least Squares Estimates			
SSE	0.00008376	DFE	4
MSE	0.0000209	Root MSE	0.00458
SBC	-67.735729	AIC	-68.721852
Regress R-Square	0.5399	Total R-Square	0.5399
Normal Test	1.5114	Pr > ChiSq	0.4697
Durbin-Watson	2.5798		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
10	5.94	0.8204

Collinearity Diagnostics							
Num- ber	Eigenvalue	Condi- tion Index	Proportion of Variation				
			Intercept	Beta	s	h	L
1	2.71421	1.00000	0.00159	0.00187	0.01375	0.02390	0.00087179
2	1.49266	1.34847	0.00002268	0.00001336	0.02861	0.08301	0.11920
3	0.44199	2.47808	0.00258	0.00200	0.01474	0.68533	0.15441
4	0.34451	2.80686	0.00089216	0.00743	0.34279	0.00850	0.22282
5	0.00664	20.22003	0.99492	0.98870	0.60010	0.19925	0.50269

Approximated residual analysis for the model presented in table 7.15

Ordinary Least Squares Estimates			
SSE	0.00031019	DFE	4
MSE	0.0000775	Root MSE	0.00881
SBC	-55.952909	AIC	-56.939032
Regress R-Square	0.4514	Total R-Square	0.4514
Normal Test	0.2843	Pr > ChiSq	0.8675
Durbin-Watson	1.8874		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	7.76	0.4576

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.60361	1.00000	0.00292	0.00343	0.01574	0.01722	0.01199
2	0.83746	2.07437	0.00029219	0.00165	0.02037	0.29459	0.12261
3	0.37631	3.09453	0.02093	0.03964	0.16908	0.32220	0.02168
4	0.15911	4.75911	0.00026883	0.00663	0.78484	0.33746	0.64379

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
5	0.02351	12.38138	0.97559	0.94865	0.00996	0.02854	0.19993

Approximated residual analysis for the model presented in table 7.16

Ordinary Least Squares Estimates			
SSE	0.00008043	DFE	4
MSE	0.0000201	Root MSE	0.00448
SBC	-68.101029	AIC	-69.087152
Regress R-Square	0.5582	Total R-Square	0.5582
Normal Test	1.4635	Pr > ChiSq	0.4811
Durbin-Watson	2.5593		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	7.56	0.4778

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.75243	1.00000	0.00267	0.00306	0.03340	0.02444	0.00432
2	1.28731	1.46223	0.00000590	0.00008159	0.02482	0.18222	0.28078
3	0.56920	2.19901	0.00006839	0.00013453	0.32874	0.24337	0.41644
4	0.37941	2.69342	0.00614	0.01469	0.36894	0.53257	0.08552
5	0.01166	15.36584	0.99112	0.98203	0.24409	0.01740	0.21294

Approximated residual analysis for the model presented in table 7.17 [beta spread]

Ordinary Least Squares Estimates			
SSE	0.00005138	DFE	5
MSE	0.0000103	Root MSE	0.00321
SBC	-81.895949	AIC	-83.408875
Regress R-Square	0.8065	Total R-Square	0.8065
Normal Test	2.6782	Pr > ChiSq	0.2621
Durbin-Watson	3.0715		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.11	0.4276

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.91277	1.00000	0.00264	0.00356	0.01851	0.01766	0.01039
2	1.01880	1.69087	0.00001572	0.00000418	0.000005186	0.26323	0.44970
3	0.84896	1.85229	0.00012542	2.013052E-7	0.07169	0.39572	0.23020
4	0.20595	3.76076	0.00946	0.04359	0.54478	0.32208	0.21888
5	0.01352	14.67937	0.98776	0.95285	0.36497	0.00131	0.09082

Approximated residual analysis for the model presented in table 7.18 [beta spread simple]

Ordinary Least Squares Estimates			
SSE	0.00009608	DFE	7
MSE	0.0000137	Root MSE	0.00370
SBC	-80.242508	AIC	-81.150264
Regress R-Square	0.6381	Total R-Square	0.6381
Normal Test	1.0858	Pr > ChiSq	0.5811
Durbin-Watson	1.4575		

Test of First and Second Moment Specification	
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DF	Chi-Square	Pr > ChiSq
5	4.46	0.4856

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.04648	1.00000	0.02332	0.02336	0.04014
2	0.89965	1.50823	0.00404	0.00638	0.94199
3	0.05387	6.16335	0.97264	0.97026	0.01787

Approximated residual analysis for the model presented in table 7.19 [beta TR]

Ordinary Least Squares Estimates			
SSE	0.00009741	DFE	5
MSE	0.0000195	Root MSE	0.00441
SBC	-75.499756	AIC	-77.012682
Regress R-Square	0.6321	Total R-Square	0.6321
Normal Test	0.5413	Pr > ChiSq	0.7629
Durbin-Watson	2.7031		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.34	0.4069

Collinearity Diagnostics								
Number	Eigenvalue	Condition Index	Proportion of Variation					
			Intercept	Beta	s	h	I	
1	3.24117	1.00000	0.00174	0.00223	0.01132	0.01068	0.01665	
2	1.07743	1.73443	0.00080889	0.00040421	0.03104	0.25557	0.04256	
3	0.48989	2.57218	0.00526	0.00501	0.03054	0.11292	0.37575	
4	0.18025	4.24047	0.00238	0.02862	0.48957	0.46681	0.17783	
5	0.01125	16.97271	0.98981	0.96374	0.43753	0.15401	0.38721	

Approximated residual analysis for the model presented in table 7.20 [beta TR simple]

Ordinary Least Squares Estimates			
SSE	0.00014117	DFE	7
MSE	0.0000202	Root MSE	0.00449
SBC	-76.394651	AIC	-77.302406
Regress R-Square	0.4668	Total R-Square	0.4668
Normal Test	0.2032	Pr > ChiSq	0.9034
Durbin-Watson	2.4372		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.08	0.5375

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	2.33550	1.00000	0.01621	0.01531	0.06512
2	0.61571	1.94760	0.02578	0.01090	0.83979
3	0.04879	6.91901	0.95801	0.97378	0.09509

Approximated residual analysis for the model presented in table A.19 [3x3 spread simple]

Ordinary Least Squares Estimates			
SSE	0.00048687	DFE	6
MSE	0.0000811	Root MSE	0.00901
SBC	-56.290128	AIC	-56.881802
Regress R-Square	0.1389	Total R-Square	0.1389
Normal Test	0.9525	Pr > ChiSq	0.6211
Durbin-Watson	1.3943		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.77	0.5824

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	2.22067	1.00000	0.01687	0.01780	0.05560
2	0.73134	1.74253	0.00535	0.02241	0.71333
3	0.04799	6.80238	0.97778	0.95979	0.23107

Approximated residual analysis for the model presented in table A.20 [3x3 TR simple]

Ordinary Least Squares Estimates			
SSE	0.00009297	DFE	6
MSE	0.0000155	Root MSE	0.00394
SBC	-71.191444	AIC	-71.783118
Regress R-Square	0.4893	Total R-Square	0.4893
Normal Test	2.1693	Pr > ChiSq	0.3380
Durbin-Watson	2.6189		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.12	0.5323

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	1.96312	1.00000	0.02728	0.02731	0.01055
2	0.98061	1.41490	0.00205	0.00058518	0.95610
3	0.05627	5.90664	0.97067	0.97211	0.03335

Approximated residual analysis for the model presented in table A.21 [3x3 spread equal]

Ordinary Least Squares Estimates			
SSE	0.00003918	DFE	4
MSE	9.79545E-6	Root MSE	0.00313
SBC	-74.573686	AIC	-75.559809
Regress R-Square	0.7357	Total R-Square	0.7357
Normal Test	0.5651	Pr > ChiSq	0.7539
Durbin-Watson	1.3928		

Test of First and Second Moment Specification		
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DF	Chi-Square	Pr > ChiSq
8	8.98	0.3442

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.34062	1.00000	0.00346	0.00355	0.00366	0.01764	0.04247
2	1.25824	1.36390	0.00069989	0.00045341	0.28552	0.24472	0.00065308
3	1.02185	1.51346	0.00112	0.00202	0.18648	0.12987	0.21136
4	0.36822	2.52121	0.00104	0.00202	0.46323	0.59149	0.58661
5	0.01106	14.54581	0.99368	0.99196	0.06111	0.01628	0.15890

Approximated residual analysis for the model presented in table A.22 [3x3 spread simple equal]

Ordinary Least Squares Estimates			
SSE	0.000078	DFE	6
MSE	0.0000130	Root MSE	0.00361
SBC	-72.772007	AIC	-73.363681
Regress R-Square	0.4739	Total R-Square	0.4739
Normal Test	1.1570	Pr > ChiSq	0.5607
Durbin-Watson	1.6771		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	5.42	0.3663

Collinearity Diagnostics						
Number	Eigenvalue	Condition Index	Proportion of Variation			
			Intercept	Beta	L	
1	1.98342	1.00000	0.00763	0.00763	0.00027	365
2	1.00134	1.40740	0.00007231	0.00000584	0.88489	
3	0.01524	11.40939	0.99229	0.99236	0.11484	

Approximated residual analysis for the model presented in table A.23 [beta spread equal]

Ordinary Least Squares Estimates			
SSE	0.00001275	DFE	5

Ordinary Least Squares Estimates			
MSE	2.54974E-6	Root MSE	0.00160
SBC	-95.834965	AIC	-97.34789
Regress R-Square	0.9243	Total R-Square	0.9243
Normal Test	0.7773	Pr > ChiSq	0.6780
Durbin-Watson	3.3932		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	8.57	0.4779

Collinearity Diagnostics							
Num- ber	Eigenva- lue	Condi- tion Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.32568	1.00000	0.00192	0.00183	0.0002419 9	0.0106 5	0.0629 3
2	1.54785	1.22577	0.0001551 2	0.0002462 5	0.11987	0.1485 1	0.0036 6
3	0.69592	1.82808	0.00129	0.00118	0.00189	0.0275 7	0.9291 7
4	0.42491	2.33951	0.0008418 1	0.0000521 0	0.40659	0.6214 4	0.0020 0
5	0.00564	20.3104 5	0.99578	0.99669	0.47141	0.1918 3	0.0022 4

Approximated residual analysis for the model presented in table A.24 [beta spread simple equal]

Ordinary Least Squares Estimates			
SSE	0.00004578	DFE	7
MSE	6.53992E-6	Root MSE	0.00256
SBC	-87.656085	AIC	-88.563841
Regress R-Square	0.7281	Total R-Square	0.7281
Normal Test	0.7231	Pr > ChiSq	0.6966
Durbin-Watson	2.4089		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.38	0.6421

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	2.23993	1.00000	0.00469	0.00468	0.06360
2	0.74735	1.73123	0.00292	0.00280	0.93536
3	0.01273	13.26741	0.99239	0.99252	0.00105

Approximated residual analysis for the model presented in table A.25 [3x3 TR equal]

Ordinary Least Squares Estimates			
SSE	0.0000378	DFE	4
MSE	9.44887E-6	Root MSE	0.00307
SBC	-74.897889	AIC	-75.884012
Regress R-Square	0.6721	Total R-Square	0.6721
Normal Test	0.7068	Pr > ChiSq	0.7023
Durbin-Watson	3.0470		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
8	7.40	0.4938

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	L
1	2.08645	1.00000	0.00537	0.00543	0.02801	0.02512	0.00445
2	1.53082	1.16746	0.00130	0.00120	0.10537	0.15996	0.14981
3	0.85983	1.55775	0.0001970	0.0003471	0.48640	0.0003051	0.42101
4	0.50945	2.02374	0.0000133	0.0002159	0.36109	0.74410	0.31882
5	0.01347	12.44687	0.99312	0.99281	0.01913	0.07051	0.10591

Approximated residual analysis for the model presented in table A.26 [3x3 TR simple equal]

Ordinary Least Squares Estimates			
SSE	0.00006191	DFE	6
MSE	0.0000103	Root MSE	0.00321
SBC	-74.85128	AIC	-75.442953

Ordinary Least Squares Estimates			
Regress R-Square	0.4630	Total R-Square	0.4630
Normal Test	0.5315	Pr > ChiSq	0.7666
Durbin-Watson	2.9003		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.77	0.5829

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	1.98222	1.00000	0.00880	0.00880	0.00006222
2	1.00019	1.40778	7.653043E-7	0.00001470	0.98219
3	0.01759	10.61412	0.99120	0.99119	0.01775

Approximated residual analysis for the model presented in table A.27 [beta TR equal]

Ordinary Least Squares Estimates			
SSE	0.00001633	DFE	5
MSE	3.26689E-6	Root MSE	0.00181
SBC	-93.356495	AIC	-94.86942
Regress R-Square	0.9217	Total R-Square	0.9217
Normal Test	0.2919	Pr > ChiSq	0.8642
Durbin-Watson	2.4597		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.55	0.3884

Collinearity Diagnostics								
Number	Eigenvalue	Condition Index	Proportion of Variation					
			Intercept	Beta	s	h	I	
1	2.59783	1.00000	0.00248	0.00196	0.01488	0.00221	0.00814	
2	1.23663	1.44939	0.00004128	0.00007923	0.00572	0.14822	0.06654	
3	1.05082	1.57232	0.00003001	0.00005673	0.11405	0.01738	0.12369	
4	0.10712	4.92455	0.03176	0.00700	0.71521	0.53560	0.61079	
5	0.00760	18.48358	0.96568	0.99091	0.15013	0.29660	0.19084	

Approximated residual analysis for the model presented in table A.28 [beta TR simple equal]

Ordinary Least Squares Estimates			
SSE	0.0001438	DFE	7
MSE	0.0000205	Root MSE	0.00453
SBC	-76.210535	AIC	-77.11829
Regress R-Square	0.3105	Total R-Square	0.3105
Normal Test	0.8424	Pr > ChiSq	0.6563
Durbin-Watson	2.3003		
Test of First and Second Moment Specification			
DF	Chi-Square	Pr > ChiSq	
5	2.14	0.8289	

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	L
1	2.22623	1.00000	0.00451	0.00456	0.05910
2	0.76163	1.70967	0.00218	0.00303	0.88499
3	0.01214	13.53988	0.99331	0.99241	0.05591

Approximated residual analysis for the model presented in table A.29

Ordinary Least Squares Estimates			
SSE	0.00030395	DFE	4
MSE	0.0000760	Root MSE	0.00872
SBC	-56.1357	AIC	-57.121823
Regress R-Square	0.5172	Total R-Square	0.5172
Normal Test	0.8166	Pr > ChiSq	0.6648
Durbin-Watson	1.7545		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
10	8.95	0.5369

Collinearity Diagnostics						
Num- ber	Eigenvalue	Condi- tion Index	Proportion of Variation			
			Intercept	Beta	s	h

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	L
1	3.31595	1.00000	0.00213	0.00302	0.01833	0.02996	0.01150
2	0.85662	1.96748	0.00014279	0.00150	0.14631	0.07979	0.17647
3	0.54322	2.47068	0.00170	0.00095592	0.00030296	0.87750	0.11819
4	0.26934	3.50874	0.00733	0.04513	0.60678	0.01275	0.14905
5	0.01486	14.93563	0.98869	0.94940	0.22827	2.903555E-7	0.54479

Approximated residual analysis for the model presented in table A.30

Ordinary Least Squares Estimates			
SSE	0.00061128	DFE	6
MSE	0.0001019	Root MSE	0.01009
SBC	-54.242125	AIC	-54.833799
Regress R-Square	0.0290	Total R-Square	0.0290
Normal Test	0.7815	Pr > ChiSq	0.6766
Durbin-Watson	1.1240		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	7.72	0.1722

Collinearity Diagnostics						
Number	Eigenvalue	Condition Index	Proportion of Variation			
			Intercept	Beta		I
1	2.25099	1.00000	0.01723	0.01842	0.05715	
2	0.69808	1.79570	0.00621	0.02720	0.71016	
3	0.05093	6.64817	0.97657	0.95438	0.23270	

Approximated residual analysis for the model presented in table A.31

Ordinary Least Squares Estimates			
SSE	0.00010115	DFE	5
MSE	0.0000202	Root MSE	0.00450

Ordinary Least Squares Estimates			
SBC	- 75.123173	AIC	- 76.636098
Regress R-Square	0.6947	Total R-Square	0.6947
Normal Test	2.2793	Pr > ChiSq	0.3199
Durbin-Watson	2.6609		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	7.46	0.5893

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.93312	1.00000	0.00276	0.00364	0.01984	0.02506	0.00800
2	1.00459	1.70872	0.00005368	0.00006215	0.03887	0.05405	0.57956
3	0.81835	1.89319	0.00030582	0.00007619	0.02051	0.71700	0.13939
4	0.22960	3.57418	0.00929	0.04033	0.59372	0.20207	0.14402
5	0.01433	14.30649	0.98759	0.95589	0.32706	0.00182	0.12903

Approximated residual analysis for the model presented in table A.32

Ordinary Least Squares Estimates			
SSE	0.00018068	DFE	7
MSE	0.0000258	Root MSE	0.00508
SBC	- 73.927304	AIC	- 74.835059
Regress R-Square	0.4547	Total R-Square	0.4547
Normal Test	0.8826	Pr > ChiSq	0.6432
Durbin-Watson	1.3335		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.16	0.6757

Collinearity Diagnostics			
Number	Eigenvalue	Condition	Proportion of Variation

		Index	Intercept	Beta	L
1	2.07106	1.00000	0.02332	0.02334	0.04747
2	0.87325	1.54002	0.00606	0.00707	0.95025
3	0.05570	6.09788	0.97062	0.96959	0.00228

Approximated residual analysis for the model presented in table A.33

Ordinary Least Squares Estimates			
SSE	0.00015079	DFE	4
MSE	0.0000377	Root MSE	0.00614
SBC	-62.444514	AIC	-63.430636
Regress R-Square	0.3981	Total R-Square	0.3981
Normal Test	2.2319	Pr > ChiSq	0.3276
Durbin-Watson	2.5221		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	8.78	0.4575

Collinearity Diagnostics							
Num- ber	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.71075	1.00000	0.00224	0.00260	0.01885	0.02456	0.00002349
2	1.48928	1.34914	7.772225E-7	3.066792E-7	0.02977	0.10407	0.14667
3	0.44411	2.47057	0.00356	0.00252	0.00770	0.76351	0.21259
4	0.34653	2.79690	0.00149	0.01027	0.45339	0.00099088	0.22364
5	0.00933	17.04340	0.99271	0.98462	0.49029	0.10688	0.41707

Approximated residual analysis for the model presented in table A.34

Ordinary Least Squares Estimates			
SSE	0.00017201	DFE	6
MSE	0.0000287	Root MSE	0.00535
SBC	-65.653918	AIC	-66.245591
Regress R-Square	0.3134	Total R-Square	0.3134

Ordinary Least Squares Estimates			
Normal Test	4.0608	Pr > ChiSq	0.1313
Durbin-Watson	2.5502		

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	I
1	1.94104	1.00000	0.03003	0.03005	0.00148
2	0.99875	1.39409	0.00052269	0.00001150	0.97236
3	0.06021	5.67777	0.96944	0.96994	0.02616

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.53	0.4761

Approximated residual analysis for the model presented in table A.35

Ordinary Least Squares Estimates			
SSE	0.00011245	DFE	5
MSE	0.0000225	Root MSE	0.00474
SBC	-74.064491	AIC	-75.577417
Regress R-Square	0.6595	Total R-Square	0.6595
Normal Test	0.1852	Pr > ChiSq	0.9115
Durbin-Watson	2.8955		

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	I
1	3.28275	1.00000	0.00187	0.00236	0.01239	0.01148	0.01688
2	1.01645	1.79711	0.00086458	0.00055325	0.03757	0.27566	0.04013
3	0.50257	2.55576	0.00600	0.00477	0.02789	0.10286	0.38936
4	0.18582	4.20308	0.00301	0.03104	0.54077	0.41301	0.16556
5	0.01240	16.27207	0.98825	0.96128	0.38138	0.19700	0.38808

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	6.36	0.7034

Approximated residual analysis for the model presented in table A.36

Ordinary Least Squares Estimates			
SSE	0.00021173	DFE	7
MSE	0.0000302	Root MSE	0.00550
SBC	-72.341246	AIC	-73.249001
Regress R-Square	0.3589	Total R-Square	0.3589
Normal Test	0.4671	Pr > ChiSq	0.7917
Durbin-Watson	2.1415		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	5.45	0.3636

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	2.35857	1.00000	0.01659	0.01582	0.06713
2	0.58998	1.99943	0.02679	0.01383	0.87812
3	0.05145	6.77059	0.95662	0.97035	0.05475

Approximated residual analysis for the model presented in table A.37

Ordinary Least Squares Estimates			
SSE	0.00034043	DFE	6
MSE	0.0000567	Root MSE	0.00753
SBC	-59.510186	AIC	-60.101859
Regress R-Square	0.3979	Total R-Square	0.3979
Normal Test	0.1208	Pr > ChiSq	0.9414
Durbin-Watson	1.2693		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	5.53	0.3545

Collinearity Diagnostics

Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	L
1	2.45770	1.00000	0.00749	0.01020	0.02712
2	0.51499	2.18457	0.00201	0.04583	0.31950
3	0.02731	9.48595	0.99051	0.94397	0.65338

Approximated residual analysis for the model presented in table A.38

Ordinary Least Squares Estimates			
SSE	0.00011922	DFE	5
MSE	0.0000238	Root MSE	0.00488
SBC	-73.479655	AIC	-74.99258
Regress R-Square	0.5510	Total R-Square	0.5510
Normal Test	0.5593	Pr > ChiSq	0.7560
Durbin-Watson	2.7306		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	9.23	0.4167

Collinearity Diagnostics							
Num- ber	Eigenva- lue	Condi- tion Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	3.35449	1.00000	0.00130	0.00168	0.01890	0.01114	0.01151
2	0.99177	1.83911	0.00002155	0.00041911	0.03620	0.55669	0.01694
3	0.43838	2.76621	0.00200	0.01175	0.00436	0.15335	0.28498
4	0.20638	4.03162	0.00610	0.01036	0.90107	0.27880	0.09370
5	0.00897	19.33292	0.99058	0.97579	0.03947	0.00002158	0.59287

Approximated residual analysis for the model presented in table A.39

Ordinary Least Squares Estimates			
SSE	0.00018345	DFE	7
MSE	0.0000262	Root MSE	0.00512

Ordinary Least Squares Estimates			
SBC	-73.774875	AIC	-74.68263
Regress R-Square	0.3090	Total R-Square	0.3090
Normal Test	0.6883	Pr > ChiSq	0.7088
Durbin-Watson	1.8485		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.79	0.5807

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	L
1	2.51994	1.00000	0.00263	0.00412	0.01009
2	0.46999	2.31553	0.00048667	0.02445	0.12036
3	0.01007	15.82039	0.99689	0.97143	0.86955

Approximated residual analysis for the model presented in table A.40

Ordinary Least Squares Estimates			
SSE	0.00009979	DFE	6
MSE	0.0000166	Root MSE	0.00408
SBC	-70.554393	AIC	-71.146066
Regress R-Square	0.4518	Total R-Square	0.4518
Normal Test	2.3401	Pr > ChiSq	0.3103
Durbin-Watson	2.5859		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	3.49	0.6246

Collinearity Diagnostics					
Number	Eigenvalue	Condition Index	Proportion of Variation		
			Intercept	Beta	L
1	1.96727	1.00000	0.02014	0.02008	0.00919
2	0.99195	1.40827	0.00002847	0.00358	0.68306
3	0.04078	6.94594	0.97983	0.97633	0.30775

Approximated residual analysis for the model presented in table A.41

Ordinary Least Squares Estimates			
SSE	0.00010638	DFE	5
MSE	0.0000213	Root MSE	0.00461
SBC	-74.618865	AIC	-76.131791
Regress R-Square	0.5982	Total R-Square	0.5982
Normal Test	0.7754	Pr > ChiSq	0.6786
Durbin-Watson	3.0115		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
9	6.34	0.7057

Collinearity Diagnostics							
Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	Beta	s	h	l
1	2.88016	1.00000	0.00305	0.00394	0.02401	0.00884	0.00727
2	1.43164	1.41837	0.00002407	0.00020947	0.00705	0.17520	0.20596
3	0.46459	2.48984	0.00000213	0.00000613	0.12740	0.34734	0.62963
4	0.20847	3.71697	0.01094	0.04415	0.60044	0.41797	0.05119
5	0.01514	13.79219	0.98598	0.95169	0.24109	0.05065	0.10594

Approximated residual analysis for the model presented in table A.42

Ordinary Least Squares Estimates			
SSE	0.00017636	DFE	7
MSE	0.0000252	Root MSE	0.00502
SBC	-74.16918	AIC	-75.076936
Regress R-Square	0.3339	Total R-Square	0.3339
Normal Test	0.8253	Pr > ChiSq	0.6619
Durbin-Watson	1.9848		

Test of First and Second Moment Specification		
DF	Chi-Square	Pr > ChiSq
5	4.60	0.4672

Collinearity Diagnostics					
Number	Eigenvalue	Condition	Proportion of Variation		
		Index	Intercept	Beta	I
1	2.15886	1.00000	0.01546	0.01599	0.04819
2	0.80062	1.64209	0.00334	0.01452	0.69200
3	0.04052	7.29943	0.98120	0.96948	0.25981