Empirical Comparison of Alternative Option Pricing Models

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Andreas and Joachim

Abstract

This thesis examines the improvement in the pricing of the Swedish OMXS30 index options with deterministic volatility models and stochastic volatility models, by using Excel VBA. To answer our problem statement

- *Do option pricing models which incorporate the volatility smile perform better than BS (Black-Scholes) empirically using option prices from OMXS30?*

we compare empirical performances of five alternative option pricing models: (1) The classic Black-Scholes using the volatility of index returns for the last 30 trading days and fitted volatility, (2) Practitioner Black-Scholes that fits the implied volatility surface, (3) Gram-Charlier which incorporate skewness and kurtosis, (4) Heston's continuous-time stochastic volatility model and (5) Heston and Nandi's GARCH model. The alternative option pricing models are compared to the Black-Scholes models as benchmark.

We find that none of the models can fully approximate the market, but they can however improve the pricing errors significantly. Both Practitioner Black-Scholes and Heston outperform the benchmarks and other models in terms of effectiveness for in-sample and out-of-sample pricing as they are better to fit and forecast the volatility smile. The pricing errors show a pattern of being highest for out-of-the-money options and decreases as we move to in-the-money options for all models. For delta hedging, only Practitioner Black-Scholes are able to outperform Black-Scholes, but the difference in performance is marginal. The thesis concludes that models that are able to incorporate the volatility smile and mitigate the maturity bias improve the ability of pricing the options. As for hedging, these model parameters are of minor importance as Practitioner Black-Scholes barely outperforms Black-Scholes.

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Chapter 1 Introduction

Motivation and Objective

Throughout our education in economics and finance, the pricing of assets has been a major topic in financial and capital market theory. The introduction of Black & Scholes model (BS) has always been an interesting subject as this was the first option pricing model taught in class. However, our curiosity sparked after a class in "Derivatives and Risk Management Techniques" at UNSW, Australia where BS was criticized for its empirically deficiencies. Our motivation has been to seek other option pricing models that are able to perform better and circumvent the documented biases BS has been empirically criticized for. To our surprise, there is an extensive number of articles and working papers on alternative option pricing models, while the articles that compare these models have no consensus on a superior model. As we find the chosen models more theoretically attractive to BS, we want to see which models that empirically can perform better than BS as a benchmark. In addition, most papers have used large indices such as S&P 500 and FTSE100 to compare the models. As the recent event of the financial crisis in 2007-2008, we have decided to use a sample of option prices after the crisis and on a smaller index, namely the Swedish OMXS30.

Problem statement

As indicated earlier, our problem statement is as follows

- *Do option pricing models which incorporate the volatility smile perform better than BS empirically using option prices from OMXS30?*

Our objective for this thesis is to investigate and compare performance alternative option pricing models empirically on the Swedish OMXS30 in terms of modeling and forecasting the option prices, as well as delta hedging. Since there have been several papers that show weakness of the BS model and incremental contribution of alternative option pricing models, we want to contribute as a study that compare existing models.

Structure

The structure of the thesis is as follows. We start with a literature review of the research done prior to this thesis and explain how the thesis fit in the option model picture. Chapter 2 introduces the option basics to let the reader get a grasp of the concept of option pricing. We continue with financial series characteristics and investigate the volatility smile, where it stems from and why this is important in option pricing. Afterwards, we explain two important concepts, namely Brownian motion and risk-neutral valuation to better understand the models presented in Chapter 3. We describe the different models with practicality, thus the mathematical aspect is held to a minimum. The focus is on understanding the model and their differences, rather than the mathematical derivation. The models are presented in this order: Black-Scholes (1973), Practitioner Black Scholes (Dumas et al. (1998), Christoffersen & Jacobs (2003)) that fits the implied volatility surface, Gram-Charlier (2004) which incorporates skewness and kurtosis, Heston's (1993) continuous-time stochastic volatility model and Heston and Nandi's (2000) GARCH type discrete model. In Chapter 4 the data from OMXS30 is presented with statistical properties of the call option and followed by data calibration in Chapter 5 where we try to find parameters that can reproduce the market data efficiently. In this chapter we try to effectively and consistently implement parameter calibration from currently traded options. We use Excel VBA to calibrate the model as we want this thesis to be practical and implementable by others. After the calibration we focus on the pricing and hedging performance of the different models in Chapter 6 and compare them with BS as a benchmark. The emphasis will be on in-sample, out-of-sample pricing performance and delta hedging errors. At last, we conclude which models produce the best results empirically and which should be implemented.

Limitations

In this thesis we have to our full capacity tried to be consistent and as thorough as possible. Due to time constraints and computational limitations, a few simplifying assumptions was made. The assumptions are made to the best of our intentions and it is important to point this out and explain their implications.

1. The Non-Synchronous Bias. We do not have time-stamped data option prices and therefore exposed to risk that option price and index value may be recorded at different times. If there is a jump in the index shortly after the option price is registered there would be introduced some biases. We try to mitigate these errors by filtering the options.

2. We assume that all markets are perfect. The transaction costs do not affect the option price, and all the investors are rational and the market is arbitrage-free.
3. We have attempted to incorporate the dividends by using the Datastream calculated dividend yield. This may be an ad hoc approximation and does not reflect the expected dividend payout. Since the "futures" market was not reliable enough we assume the Datastream calculated dividend yield as a sufficient approximation.
4. Due to the computational burden, only call options are investigated. The results may have been different if the data set included both puts and calls. Still, the put-call parity should ensure some consistency.
5. Another aspect to consider is whether the different loss functions implemented can affect the results, as the Practitioner Black-Scholes is calibrated with a different loss function than the rest.
6. The last point to consider is the use of Excel VBA, if it is powerful enough to handle the advanced models. Due to its practicality we chose to implement all the models by using the software. It can be interesting to compare the results with other programs such as Matlab, C++ and R.[1]

Literature Review

The history of option dates back to Louis Bachelier (1900) who first applied the concept of Brownian motion in pricing of stocks. His contribution is the first paper to use advanced mathematics and became the norm in the study of finance. Prior to the Black & Scholes model, options were priced by discounting expected payoff. A major breakthrough was provided by the paper of Black & Scholes (1973) when they published their article on option pricing. The formula revolutionized the pricing and boosted options trading. The model set a new benchmark in history of finance.

Although the formula became an important tool for pricing options, its empirical deficiencies and its restrictive assumptions have motivated development of more advanced models. After the US market crash in October 1987, it is well known that implied volatilities appeared to differ across exercise prices (Rubinstein, 1994). Similar patterns have also been documented in the U.K., German and Japanese index option markets by Genmill & Kamiyama (1997). The assumption that asset returns follow a log-normal distribution and the volatility is constant throughout the life of options is mainly responsible for deviation between model and market prices (Cont, 2001). According to BS, all options with the same

[1] http://www.mathworks.se/products/matlab/, http://www.r-project.org/

maturity should have the same implied volatility. However, options which are in-the-money or -out-of-the-money seem to have higher implied volatilities. The magnitude of these violations cannot be explained by market imperfections. The pattern known as "volatility smile" suggests that the Black-Scholes model tends to misprice these options.

Several studies have suggested extensions of the Black & Scholes model to account for the volatility smile and other empirical violations. These extensions can loosely be grouped into two main approaches: deterministic volatility models and stochastic volatility models. Deterministic volatility models are based on the framework that volatility is determined by variables observable in the market, whereas stochastic volatility is based on the framework that volatility itself is stochastic, and the parameters are not directly observable in the market (Stein & Stein, 1991).

According to Buraschi & Jackwerth (2001) deterministic volatility models are attractive for several reasons. First, derivative securities can be priced by no-arbitrage without resorting to full-blown general equilibrium models and without the need to estimate risk premia, thus markets are dynamically complete in these models. Second, the models can potentially capture some empirical regularities, such as time-varying volatility, correlation between volatility and returns of the underlying asset and volatility clustering. Third, the models enable us to fit the smile exactly by calibrating the volatility surface from options on the underlying asset. Conversely, a concern of the practical use of these models is that the improved static pricing performance might be obtained at the cost of overfitting. In the deterministic volatility family, Constant Elasticity of Variance (CEV) model of Cox (1976), the Deterministic Volatility Functions (DVF's) of Dumas, Fleming & Whaley (1998) and Gram-Charlier model of Backus, Foresi & Wu (2004) managed to gain popularity.

By using models that incorporate stochastic interest rates and jump processes, many researchers argue that moneyness, maturity and interest biases stem from the constant variance assumption of the Black-Scholes model (Johnson & Shanno, 1987; Chesney & Scott, 1989; Hull & White, 1987; Wiggins, 1987; Melino & Turnbull, 1990). Consequently, stochastic volatility option pricing models have been developed to allow for the impact of changing volatility on option prices. In the stochastic family, model of Hull & White (1987), Heston (1993) and Heston and Nandi GARCH (2000) are the most popular. The Heston model is the continuous-time volatility model which models the square of the volatility process with mean-reverting dynamics, allowing for changes in the underlying asset price to be contemporaneously correlated with the changes in the volatility process. Heston and Nandi model uses an autoregressive structure of the GARCH process to capture empirical appearances like volatility clustering, leptokurtic return distributions and leverage effects.[2] The two latter models were chosen due to closed form solutions.

This thesis fills two gaps. First, this thesis considers improvements over BS by allowing deterministic and stochastic volatility models when pricing OMXS30 index options. We compare alternative volatility option pricing models with the simple, yet still valuable Black-Scholes, by using Excel VBA and SAS Enterprise. We have chosen 2 versions of the standard Black-Scholes model, Practitioner Black-Scholes, Gram-Charlier, Heston and Heston Nandi GARCH for this thesis. Although many researchers examined the performance of alternative models in major markets such as S&P 500 and FTSE 100, to our knowledge, no study has investigated their performance in a small market like the Swedish Stock Exchange. The dataset is fairly new and consists of call options on OMXS30 from 1st of June 2011 to 31st of May 2012. Second, while there are lacking studies that compare alternative groups of option pricing models, there are several studies that compare the incremental contribution of the stochastic volatility or more sophisticated models like jump diffusion models. This thesis will therefore contribute as a study that empirically compares alternative option pricing models.

[2] Leptokurtic is a distribution with a high peak, a thin midrange and fatter tails compared to the normal distribution.

Chapter 2 Option Pricing

Option Basics

"A derivative can be defined as a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. Very often the variables underlying derivatives are the prices of the assets. A stock option, for example, is a derivate whose value is dependent on the price of the stock."

John C. Hull (2009, p.1)

Options can be written on many types of financial assets like stocks, currencies, interest rates and stock indices. They can also be used to speculate in the underlying asset or to hedge a given exposure. In addition, options can be used to leverage the investment compared to investing directly in the asset.

Options are traded both on standardized exchanges and over-the-counter (OTC) from one party to another. Although these transactions are not disclosed to the general public, the OTC is by far the biggest market according to volume of trading and often deals with higher values than their standardized counterpart. The advantage of using the OTC market is that terms of a contract do not have to be those specified by an exchange and market participants are free to negotiate any mutually attractive deal. This indicates that options are a vastly used instrument throughout the financial world.

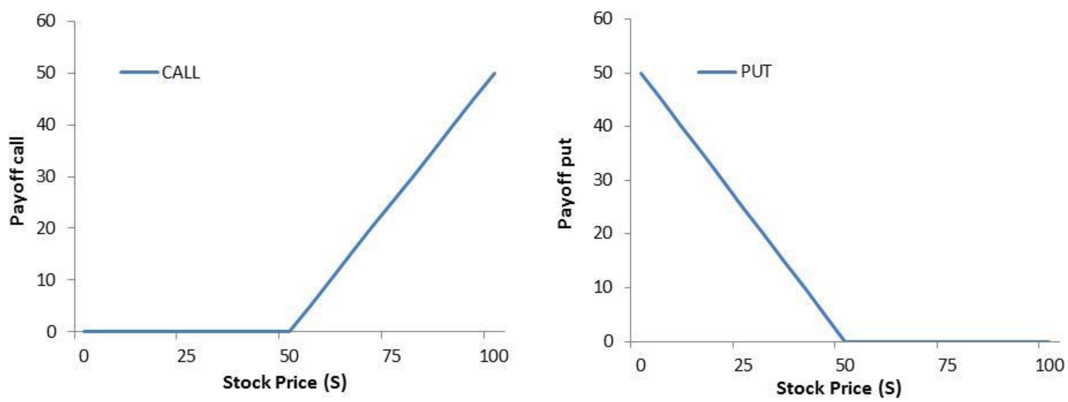
There exist many types of options, but the most common and traded on many exchanges are the European options, where the value of the option is given by the difference between the stock price and the exercise price at a pre-specified point in time. In comparison, an American option can be exercised throughout the life of the option. Other options often referred to as exotics can have many forms, one type is the Asia option where the value of the option is given by the average payoff during the life of the option.

$$Call\ = Max[S_T - K, 0] \qquad (2.1)$$

$$Put\ = Max[K - S_T.0] \qquad (2.2)$$

The two types of options: call and put options. A call option gives the holder the right to buy the underlying asset for a certain price at a certain day, while a put option gives the holder the right to sell the underlying asset at a certain day for a certain price (Hull, 2009). A call option benefits from a stock price greater than the strike price and a put option is only valuable when the stock is less than strike at maturity. The value of a call and put at maturity is given by equation 2.1 and 2.2. The payoff is illustrated by figure 1 for both calls and puts. The value of an option can never be negative and accordingly reduce the downward risk compared to investing directly in the asset.

Figure 1



Call: $Max[S_T - 50{,}0]$, *Put:* $Max[50 - S_T, 0]$

It is emphasized that the holder of the call option has the *right* to exercise the option without any obligations. As the holder of the option does not have to exercise this right, this is what distinguishes options from forwards and futures where the holder is obligated to buy or sell the underlying asset. A buyer of an option is referred to having a *long* position while sellers are referred to as having a *short* position. As figure 1 illustrates, it is evident that sellers of call options are exposed to risk as the value of a call option can in theory be indefinite.

Factors that affect option prices

- Value of the underlying asset (*S)*
- The strike price or exercise price (*K)*
- Maturity *(T-t)*
- Volatility over the underlying asset (σ)
- Interest rate (*r)*
- Dividends expected during the life of the option (*q)*

In the following table the effect of increasing the different values and how this affects the option price. For a more thorough explanation of each factor, we refer to Hull (2009, p. 202).

Table 0.1

	Stock price	Strike price	Time to maturity	Volatility	Risk free rate	Dividends
Call	+	-	?	+	+	-
Put	-	+	?	+	-	+

The table describes the effect on the price of a stock option of increasing one variable while keeping all others fixed.
+ indicates that an increase the variable causes the option price to increase, - indicates that increase in variable causes option price to decrease, ? indicates that the relationship is uncertain.[3]

For an European option, the upper limit for a call on a non-dividend stock is the stock price itself, while the upper limit for a put option is the strike K. The reason why K is the upper limit is that the stock price can never be negative and the put option is given by the following equation

$$C \leq S_0 \; and \; P \leq K \qquad (2.3)$$

The call and the put option cannot be lower than equation 2.4 and 2.5.[4] A full argument is explained in Hull (2009, p.205), but is easily derived by a simple no arbitrage argument.

$$C \geq S_0 - Ke^{-rT} \qquad (2.4)$$

$$P \geq Ke^{-rT} - S_0 \qquad (2.5)$$

The relationship between put and call options is not independent, as there exists a no-arbitrage relationship amongst them referred to as the put-call parity.

$$Put/call\; parity\colon\; Call + Ke^{-rT} = Put + Stock$$

If the call price is known, it is an easy task to calculate the price of a put option with the same strike and maturity. If this relationship does not hold, an arbitrageur can earn a risk-free profit by selling the more expensive option and create a replicating portfolio with the other instruments or vice versa.

[3] It is not clear cut whether the DTM has a positive or negative effect on option prices, but in most cases the effect is positive, the longer the maturity the more valuable is the option(European options), for American options time to maturity has always a positive effect on option prices due to early exercise.

[4] The equations are only valid for European options on a non-dividend paying stock.

The effect of dividends can be pronounced in stock options. Table 2.1 shows that the value of a call is reduced if there is an expected dividend payout during the life of the option. Similarly, the value of a put increases when there is an expected dividend payout. The special case of options on stock indices is that dividends can be paid by numerous of different firms. To adjust the index price for dividends, it is common to assume a dividend rate *q* which continuously pays out dividends. In order to incorporate the effect of dividends we can change the stock price S to account for dividends.

$$S_{Dividend\ adjusted} = S_0 e^{-qT}$$

According to Hull (2009, p. 9), the derivatives market has been outstandingly successful. One of the reasons is that the market has a great deal of liquidity, as there is usually no problem for an investor to find someone to take the opposite position of an option. This leads to that many different types of traders are attracted to the derivatives market, such as hedgers, speculators and arbitrageurs. Hedgers use derivatives to mitigate the risk exposure their position is facing, speculators use them to bet on a future direction of a market variable while arbitrageurs take offsetting positions in two or more instruments to lock in a profit.

Hedging

Hedgers use derivatives to reduce the risk that they face from potential future movements in a market variable. It is an investment that reduces the price movements of assets/liabilities and can be seen as safeguarding the assets by taking an offsetting position in another instrument. By using options, hedging is quite fascinating due to its non-linear risk, compared to the linear risk by investing directly in the asset and hedged by using futures or forwards. The problem with the non-linear risk is that there is not one single instrument that can be used to hedge the portfolio perfectly. By contrast, the option contracts provide insurance. Options offer a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favorable price movements.

In option theory there are many different risks that have to be considered. The following Taylor series expansion explains the risk that the option *f* is exposed to, referred to as the *greeks*.

$$\Delta f = \frac{df}{dS}\Delta S + \frac{df}{dt}\Delta t + \frac{df}{d\sigma_t}\Delta\sigma_t + \frac{df}{dr_t}\Delta r_t + \frac{1}{2}\frac{d^2 f}{S_t^2}(\Delta S_t)^2 + \cdots$$

where *S* is stock price, *t* is time, σ is volatility and *r* is interest rate the last term is the *gamma* of an option. The first term of the right hand side of the equation is known as *delta (Δ)* where the risk is in the change of the stock price as this will affect the option price. The second term is known as *theta* and reflects how change in time affects the option value. The third term is known as *vega* and a risk factor that represents change in volatility, while fourth term is *rho* which represents how interest rate affect option pricing. At last, *gamma* is the second derivative of *delta*, which means it is interpreted as the risk of changes in *delta*.

According to Hull (2009) to hedge the *greeks* one needs to invest in instruments that are dependent on the risk factor that is hedged. He argues that finding instruments for more complex risk is hard and can be very costly. This thesis will focus on delta hedging as it is easily implemented with only the stock index and a risk-free asset.

Delta hedging

"Delta is the rate of change of the option price with the respect to the price of the underlying asset."

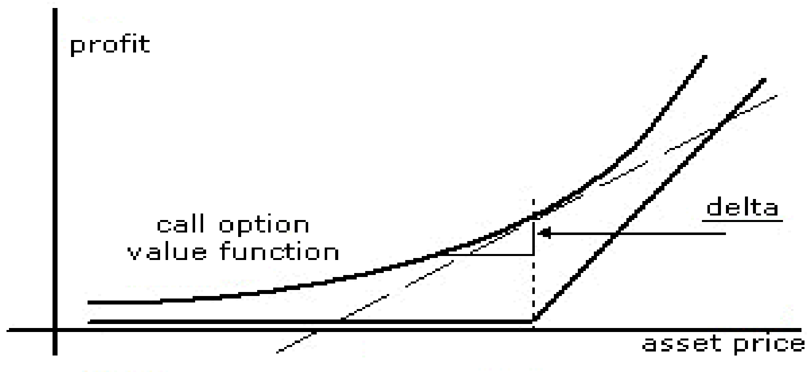
John C. Hull (2009)

Delta (Δ) is the sensitivity of the option to the price of the underlying asset, Δs = $\frac{\partial Call}{\partial S}$ and the aim of delta hedging is to keep the value of a position as close to unchanged as possible.

For a call option, Δ is a strictly positive value (negative for puts) within the range [0, 1]. Δ of zero indicates that there is no correlation between the index and the option and a value of 1 indicating a perfect relationship. This means if the Δ of a call option is 0.5, the option price changes by about 50% in the same direction of the change in the underlying asset. The strategy aims to reduce the risk in price movements in the underlying by offsetting long and short positions. Due to delta is a linear relationship calculated at a certain value, it only calculates the tangent or the slope. [5]

[5] http://www.markaz.com/DesktopDefault.aspx?tabindex=3&tab_ikey=220

Figure 2



Since the call option follows a nonlinear structure a linear hedge could not possibly be a perfect hedge as seen in figure 2. By analyzing the graph it can be seen that if the asset price has a large movement the delta hedge is less accurate. Still, delta hedging should works quite fine for reasonable changes in asset price.

A position with Δ of zero is referred to as being *delta neutral* and it is important to realize that because delta changes, the position remains delta hedged for only short period of time. For this thesis, we will focus delta neutral positions for 1 day which will be explained in Chapter 6.

Financial Times-series Characteristics

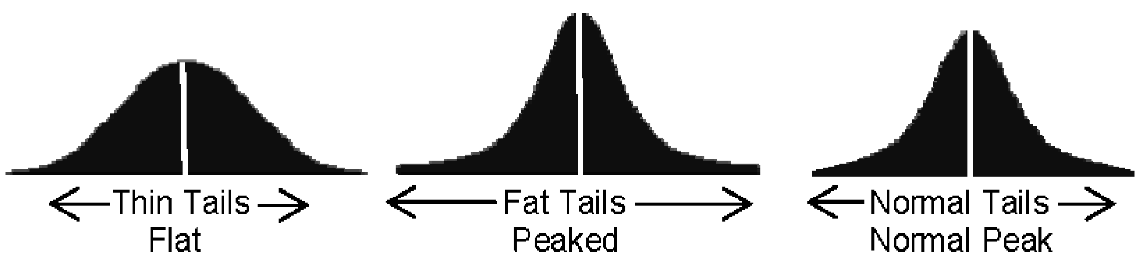
Cont (2001) did a research on stylized facts on asset returns. He found significant deviation from the normal distribution where the tails where fatter than proposed by the normal distribution, and the distribution did not seem to be symmetric. In addition, he found some persistence in volatility indicating that volatility is not constant over time.

Kurtosis, Skewness and the Leverage effect

The volatility smile is often observed in the foreign currency, interest rate options and is also essential in equity options. The volatility smile is observed when ITM and OTM options have higher implied volatilities than their ATM counterparts. According to Hull (2009, p.387), this phenomena is observed because volatility is not constant and varies over time. Another feature is that the market is not continuous, but includes jumps. This effect is closely related to the kurtosis of asset returns.

Kurtosis affects the height and width of the probability density function. The probability density function is symmetric, but is more or less "peaked" than the normal distribution. A positive kurtosis indicates a high peak, fatter tails and a thin midrange.[6] A positive kurtosis can be interpreted that fewer observations are in the intermediate range and extreme observations occur more often. A negative kurtosis indicates a flat distribution with a fat midrange and thinner tails. This indicates that returns are centered close to the mean while the probability of extreme observations is less likely compared to the normal distribution.[7]

Figure 3



A high positive kurtosis increases the probability of extreme movements in both directions compared to the normal distribution assumed in BS. When there is a higher probability of tail events, the option prices in that range are likely to increase (higher implied volatilities) and thereby create a volatility smile. The probabilities of intermediate events are less likely compared to the normal distribution and ATM options are less attractive.

[6] High peak represents many observations where returns are approximate zero.
[7] Refer to Excess kurtosis as the normal distribution has a kurtosis of 3. Figure 3 is obtained from khttp://financialplanningbodyofknowledge.net/w/index.php?title=Kurtosis

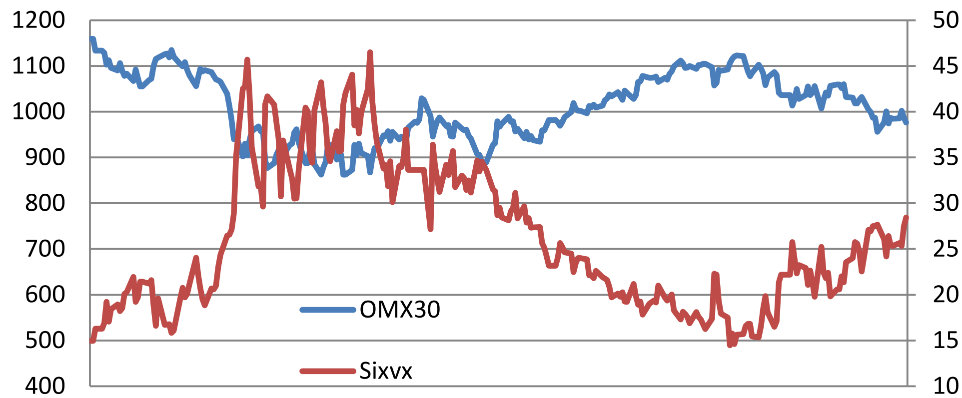
Figure 4

In equity and stock indices options there is also observed a volatility smirk (Black, 1976; Christie, 1982). The smirk is observed when implied volatility is a declining function of strike. For call options, ITM options are the most expensive while the OTM are the cheapest relatively speaking. This “anomaly” is pronounced and is observed in almost every equity market.

The smirk is created when there is a negative correlation between the stock index returns and volatility. The volatility increases when the stock drops and when the stock price rise the volatility is reduced. The justification is that when the stock falls the stock fluctuate more and the probability of extreme negative returns are more likely probability of extreme negative return is higher than the normal distribution; the ITM options should be more expensive for call options.[8] This effect is related to skewness where the probability density function is asymmetric.

[8] The volatility smirk is created when there are both kurtosis and skewness

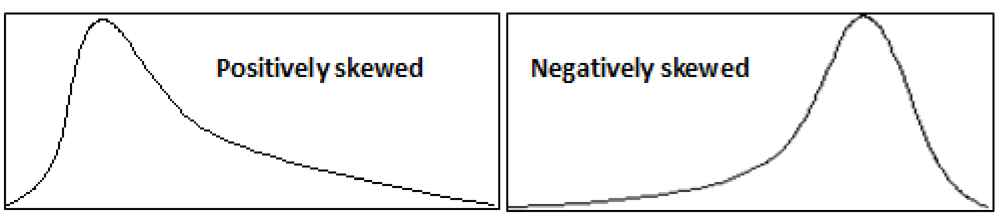
Figure 5



The graph illustrates price movement of OMXS30 and SIXVX in period 1.6.2001 to 31.5.2012.The correlation between stock index returns and volatility as measured by SIXVS is highly negative, indicating the leverage effect.[9]

Skewness tends to push one tail out and the other tail in. A negative (positive) skew indicates that the left tail (right) is longer (fatter) than the right (left) tail. The interpretation is extreme negative (positive) events happen more frequently than the positive (negative) ones. It is skewness that creates the volatility smirk seen in figure 3, this anomaly is often observed in equity markets.

Figure 6



Although there is no consensus of the explanation behind this anomaly, researchers and academics have different theories. Black (1976) and Christie (1982) argue for the "leverage effect". When the stock price drops, the $\frac{EQUITY}{DEBT}$ falls and in standard corporate finance theory this suggests more risky equity and therefore a higher volatility of equity (Modigliani, 1958). This is also true when the stock price soars and $\frac{EQUITY}{DEBT}$ increases and the volatility of equity should decrease. Other papers such as Hasanhodzic & Lo (2011), Figlewski & Wang (2006) argue that this is not a leverage effect, but rather a "down market"

[9] SIXVX Volatility Index is an indicator of forward price risk in the Swedish equity market. It is derived from prices of standardized OMXS30 index options with an average of 30 days to expiration.

effect. A hypothesis is that investors get concerned and thereby increase volatility. This implies that people are willing to pay a lot to buy put options to safe their positions. Therefore the OTM for put options are more expensive than ATM and ITM options.[10]

The put-call-parity ensures that the volatility smile/smirk is the same for both puts and calls (Hull, 2009, p. 381). There is an economic interpretation to why this is the case. ITM call options have higher implied volatilities because investors use options to leverage their position. Since deep ITM options fluctuate approximate the same as the index, investors can increase their return on investment by using the leverage incorporated in options. For put options deep OTM can be used as a protective insurance for a market turndown. Since deep OTM put options is a “cheap” insurance, investors use these options as downside protection and their implied volatilities increase.

A last note in this chapter is whether the smile/smirk is as pronounced for longer dated options. It is found that that longer dated maturities tend to have less smile effect (Duque & Lopes, 2000).[11] As stated in the beginning of this section the impact of jumps and non-constant volatility creates a volatility smile. Hull argues that the effect of non-constant volatility has larger percentage effect on prices, but less effect on the percentage change on implied volatilities. The effect of jumps has less effect on both prices and implied volatilities on longer dated options. In conclusion longed dated options should have less pronounced smiles compared to shorter-dated options. This effect is referred to as the maturity bias and is an important factor in the Swedish option market.

Volatility Clustering

A clustering effect is often observed in financial time series when stock returns are dependent. In other words, large changes in returns tend to be followed by large movements in returns. The same effect is seen on small changes in assets returns tend to be followed by small changes. This anomaly results in volatility clusters where volatility seem to group together at certain time periods. An example of this effect is seen in figure 7.

[10] Important to keep in mind that ITM for calls is the same as OTM for puts when studying the volatility smile.
[11] A reason could be that over time no extreme jumps would affect options prices, and returns will on average follow the normal distribution.

Figure 7

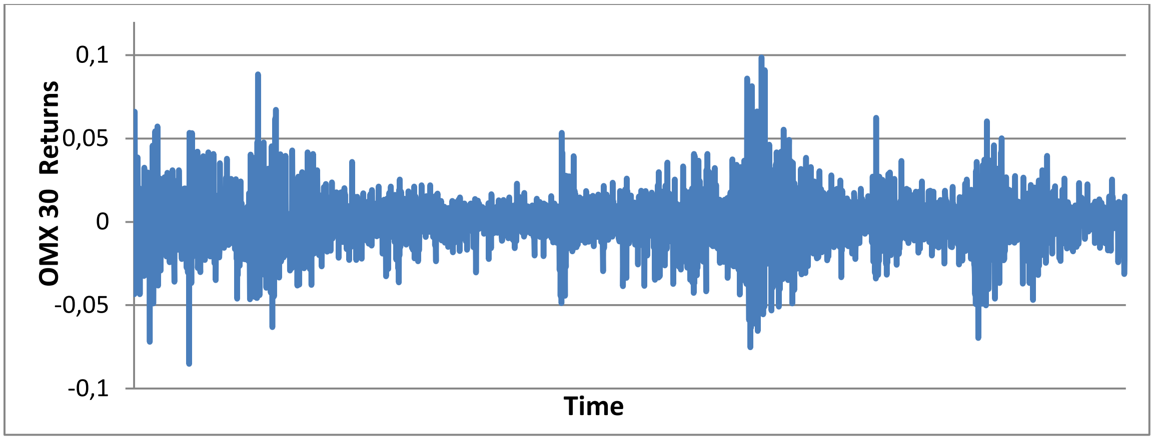


Figure 7 illustrates returns of the Swedish OMXS30 from 1.1.2001-26.06.2013

In the simple BS model the returns are often assumed to be independent and no correlation between the returns. To investigate this assumption one has to graph the correlation between returns and its lags.[12]

In figure 8 (left graph) this is illustrated by using the Swedish OMXS30. By analyzing this property it is easy to conclude that asset returns are not correlated. There seem to be no relationship between returns and its lags as the correlation fluctuate around zero and seem to appear randomly. As stated in by Cont (2001) the absolute value of returns or squared returns show another interesting property. It seems like the absolute returns do in fact show sign of autocorrelation indicating that returns are not independent. As seen in figure 8 (right graph) the absolute returns and its lags seem very dependent, significant and slow decaying function of lags. It is this dependence that creates the volatility clustering. This indicates a time varying persistent volatility where today's absolute returns are correlated with past absolute returns. To conclude the statistical properties indicate that volatility clustering is a factor in the Swedish OMXS30.

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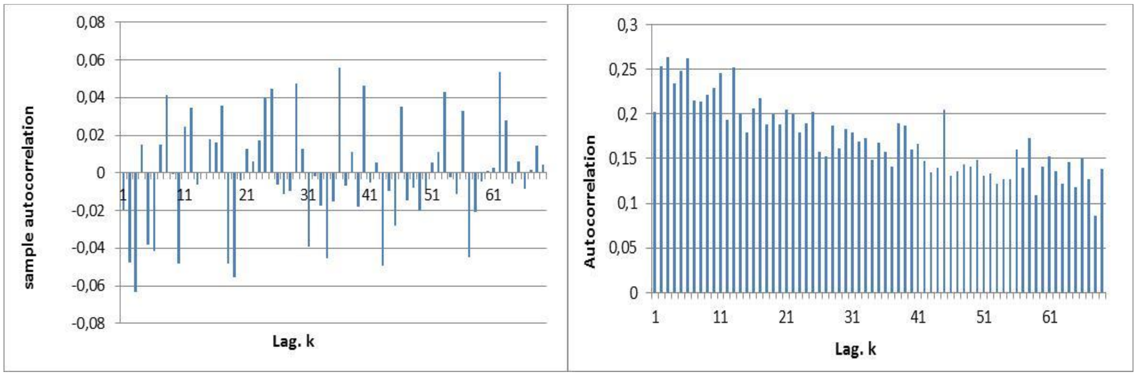
[12] $r_k = \frac{\sum_{t=1}^{n-k}(x_t-\bar{x})(x_{t+k}-\bar{x})}{\sum_{t=1}^{n-1}(x_t-\bar{x})^2} = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$, autocovariance/sample variance

Figure 8



The left graph represents the correlation between returns at time t and returns at time t-q, the right graph is similar except that returns are absolute. The graph indicates that absolute returns are correlated and there exists persistence in volatility. Figure 8 illustrates returns from 1.1.2001 to 26.06.2013

Two Important Concepts in Option Pricing

Brownian motion

In many option pricing models it is assumed that the stock price follows a diffusion process with a stochastic component. The Black-Scholes model assumes that the stock process is affected by random stochastic shocks and cannot be predicted.[13] This conceptual property is achieved by adding a Brownian motion which is a stochastic process that evolves randomly over time. It is used in Black-Scholes, Gram-Charlier, Heston & Nandi GARCH model to include randomness, while in the Heston model this process also determines stochastic volatility.

To better understand the Brownian motion and the effect of stochastic shocks an example is presented by a analyzing the diffusion process in BS.

$$\frac{\Delta S}{S} = \mu dt + \sigma dW_t \qquad (2.6)$$

In equation 2.6 W_t measures the randomness of the process while μ is the expected return, S is stock price and σ is the yearly volatility. If we assume that the expected return is zero, the only factor that determines the stock process is the Brownian motion.

$$dW_t = \varepsilon\sqrt{dt}$$

The ε follows a normal distribution and has a mean of zero and a variance rate of 1 yearly.

[13] The stock has the Markov property where historical values cannot explain the development of the stock price.

Figure 9

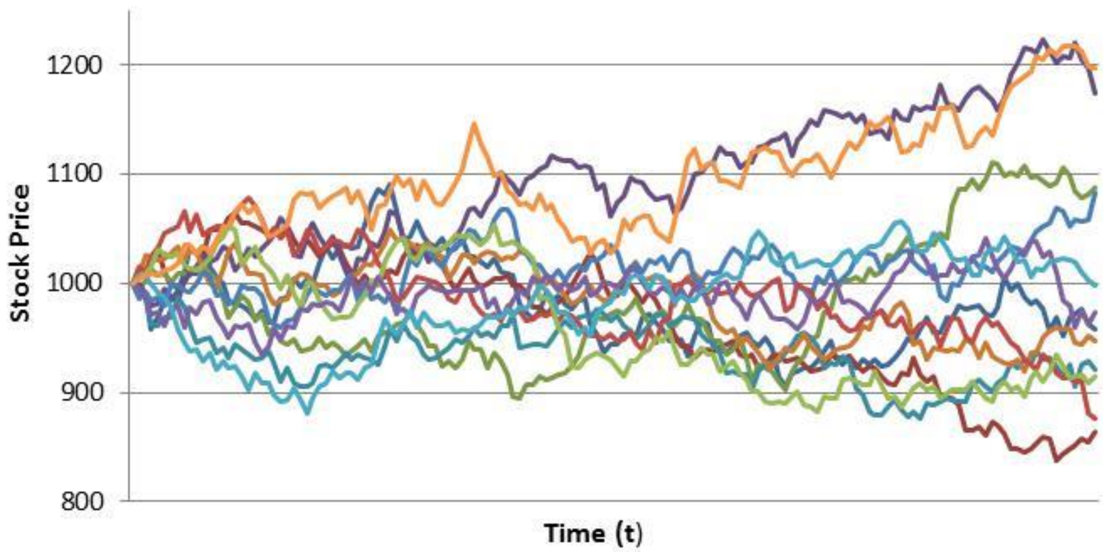


Figure 9 illustrates 12 simulated paths for S_0=1000, $\varepsilon \sim N(0,1), \Delta t = 0{,}01, \mu = 0, n = 150$.

The effect of this random variable is seen in figure 9, where many different random processes are modeled. There is not supposed to be any sign of a path and the terminal value at time *T* is based on a series of random shocks.

Risk neutral valuation

Risk-neutral valuation is a valuable tool for the analysis of derivatives and is commonly used when pricing options. According to Hull (2009), in a risk-neutral world all investors are indifferent to risk and only require the risk-free rate to invest. Investors require no compensation for risk and therefore the expected return can be substituted with the known risk free-rate. By using this principle, all the variables that enter models are without risk preference and the pricing of options becomes easier. This general principle in option pricing is known as *risk-neutral valuation.*

The fascinating and surprising fact is that the solution is valid in a world with risk-averse investors as well. The argument is in a risk-averse world the expected growth rate in the stock price changes and the discount rate that must be used for any payoffs from the derivative changes. These two effects offset each other perfectly and the price is valid in the risk-neutral world as well as in the real world. According to Hull (2009, p. 291) risk-free valuation is correct when the risk free rate is constant.

Chapter 3 Theory of the Models

Black-Scholes

In modern financial theory, the groundbreaking article from Fisher Black and Myron Scholes in 1973 is one of the most important contributions to option model pricing.[14] Together with Robert Merton, the well-known Black-Scholes (or Black-Scholes-Merton) model was developed and has made a huge influence on how traders price and hedge options. With a restrictive set of assumptions, Black-Scholes (BS) formula calculates the price for a European *plain vanilla option*. [15] We will not put emphasis of the derivation of the model as BS is well-known and hardly needs introduction. However, by modifying the model we can take dividend yield into account and the value of a call option can be calculated as

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

where d_1 and d_2 equals

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+(r-q+0{,}5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \qquad (3.1)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right)+(r-q-0{,}5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \qquad (3.2)$$

The restricted assumptions include constant risk free rate, constant volatility and the same for all maturities. *S* is the underlying stock price, *K* is strike price, *r* is risk free rate, *q* is dividend yield, *σ* is volatility and *T-t* is time to maturity. As the model assumes the asset price follows geometric Brownian motion with constant volatility, numerous articles have been criticizing the restrictive assumptions. Rubinstein (1994) showed that BS implied volatilities tend to differ across exercise prices and time to expiration on the S&P 500 index option market.[16] In this thesis we will apply 2 different versions of the BS model, *BS30* which use the returns from the 30 last trading to estimate historical volatility and *BS*

[14] Black, F., Scholes, M. (1973), "The pricing of Options & Corporate Liabilities"
[15] A vanilla option is a normal call or put option that has standardized terms and no special or unusual features.
[16] Rubinstein (1994)

which fits the volatility to minimize the %RMSE. The calibration of the models will be explained in Chapter 5.

Practitioner Black-Scholes

"if the volatility of a stock changes over time,
the option formulas that assume a constant volatility are wrong"

Fisher Black (1976)

As mentioned in in the literature review many researchers that have documented the volatility smile. The shape of the smile has motivated researchers to model implied volatility as a quadratic function of moneyness and maturity. To circumvent the assumption of constant volatility Dumas, Fleming & Whaley (1998) introduces an ad-hoc model with a deterministic volatility function (DVF) approach to model implied volatility.[17] The authors show that the BS model leads to the largest valuation errors, consistent with the notion that volatility is not constant across moneyness and maturity. This model was later referred to as the Practitioner Black-Scholes (PBS) model by Christoffersen & Jacobs (2004), which is a simple way to price options based on implied volatilities and the BS pricing formula. To implement the model, a series of BS implied volatilities is required to run multiple regressions under ordinary least squares (OLS) by using 4 steps.[18]

1. Use cross-section of option prices with a variety of strike price and time to maturity to obtain a set of BS implied volatilities.
2. Choose a deterministic volatility function and estimate its parameters.
3. For a given strike price and maturity, obtain the volatility as the fitted value of the volatility function in step 2.
4. Obtain the option price using BS formula, using the fitted volatility from step 3 and the same strike price and maturity.

[17] Dumas, Fleming and Whaley (1998) "Implied Volatility Functions: Empirical Tests"
[18] OLS is a method for estimating the unknown parameters in a linear regression model.

In this thesis, we will use the following DVF functions

$$DVF\ 1: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2$$

$$DVF\ 2: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2 + a_3T + a_4KT$$

$$DVF\ 3: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2 + a_3T + a_4T^2 + a_5KT$$

where σ_{iv} is the BS implied volatility, K is strike price and T is time to maturity. A minimum value of the local volatility is imposed to prevent negative values. According to Dumas et al. (1998) the DVF models improves the valuation errors compared to the ordinary BS model. DVF 1 attempts to capture variation in volatility attributable to asset price by having a quadratic function of moneyness. a_0 is the intercept, a_1 is the linear function of K while a_2 is the quadratic function of K. The parabola opens upward if a_2 is positive or downwards if a_2 is negative. The value of $|a_2|$affects the curve of the parabola: the parabola opens wider when $|a_2|$ is less than 1 while it opens narrower when $|a_2|$ is greater than 1. Thus for DVF 1 to capture a volatility smile, we expect a negative a_1 and a positive a_2.

DVF 2 and 3 capture additional variation attributable to time. DVF 2 has two more parameters than DVF 1, *T* and the interaction coefficient *KT.* According to earlier empirical research, the implied volatility differs for same strike prices with different maturities. The options with the nearest expiration date have a higher implied volatility. The interaction term, *KT* means that the effect of moneyness on volatility is different for different values of *T*, time to maturity. Therefore the unique effect of moneyness is not limited to a_1 and $a_{2,}$ but also depends on the values of a_4 and time to maturity. As *T* is a linear function, different values of a_3 will change the placement of the fitted volatility smile holding the other coefficients constant. In other words, DVF 2 can better fit volatility smiles for different maturities. The maturity bias of the option pricing is meant to be captured by a_3 and a_4 and consequently, we expect DVF 2 to perform better than DVF 1.

In additional to *T* and the interaction term*,* DVF 3 has the quadratic function of time to maturity, T^2*.* The effect of the coefficient of T^2 is more prominent for options with longer maturities and is expected to further decrease the maturity bias of options. These quadratic forms of volatility functions are chosen in part because the BS implied volatilities for options tend to have a parabolic shape. Because DVF 3 has the most parameters to fit the volatility smile, we expect DVF 3 to have the best in-sample fit among the

DVF functions. The DVF functions can also be estimated using more flexible nonparametric methods such as kernel regressions or splines, but is avoided due to overparameterization.[19]

By using S&P 500 index options, Dumas et al. (1998) conclude in their paper that even with fairly parsimonious models of the volatility process, they achieve almost a perfect fit of observed option prices.[20] The DVF models' prediction errors grow larger as the volatility function specifications become less parsimonious when predicting option prices a week ahead. In particular, specifications that include a time parameter do worst of all, indicating that the time variable is an important cause of overfitting at estimation stage. And at last, they found that hedge ratios determined by BS appear to be more reliable than those obtained from DVF models.

Gram-Charlier

The Gram-Charlier model (GC) was developed by Backus, Foresi &Wu (2004) and is not restricted by the log normal distribution assumption. They implement a Gram-Charlier expansion up to the fourth moment which allows the model to incorporate skewness and kurtosis.[21] This model does not allow for changing volatility as assumed in the BS model.

$$\gamma_1 = E\left[\left(\frac{R_{t+1}-\mu}{\sigma}\right)^3\right] \qquad \gamma_2 = E\left[\left(\frac{R_{t+1}-\mu}{\sigma}\right)^4\right] - 3$$

Here equation γ_1 represents the T period skewness and equation γ_2 represents the T period excess kurtosis. When the authors derive the model, they calculate the BS price first and then adjust for excess kurtosis and skewness as seen below.

The price of a call option according to the GC is shown to be approximately

$$C_{GC} \approx S_t \mathrm{N}(d) - Ke^{-rT}\mathrm{N}(d-\sigma_T) + S_t N(d)\sigma_T\left[\frac{\gamma_{1T}}{3!}(2\sigma_T - d) - \frac{\gamma_{2T}}{4!}(1-d^2+3d\sigma_T-3\sigma_T^2)\right] \quad (3.3)$$

[19] See Ait-Sahalia & Lo (1998): "Nonparametric estimation of state-price densities implicit in financial asset prices"
[20] Parsimony principle is to keep the regression model as simple as possible (Gujarati & Porter, 2009, p. 42)
[21] The Gram-Charlier expansion was pioneered by Jarrow & Rudd (1972)

$$d = \frac{\log\left(\frac{S_t}{K}\right) + T(r-q) + \sigma_T^2/2}{\sigma_T}$$

A nice feature of the GC model is its consistency. By setting y_1 and y_2 to zero in the bracket of equation (3.3) the only remaining function is the traditional BS formula. We refer to Chapter 1 for more thorough explanation of skewness and kurtosis.

The effect of introducing skewness and kurtosis

In order to analyze the effect of skewness and kurtosis, it is interesting to see the effect on the volatility smile/smirk. The values that are used are chosen arbitrary, but indicate the effect of the different parameters.

Figure 10

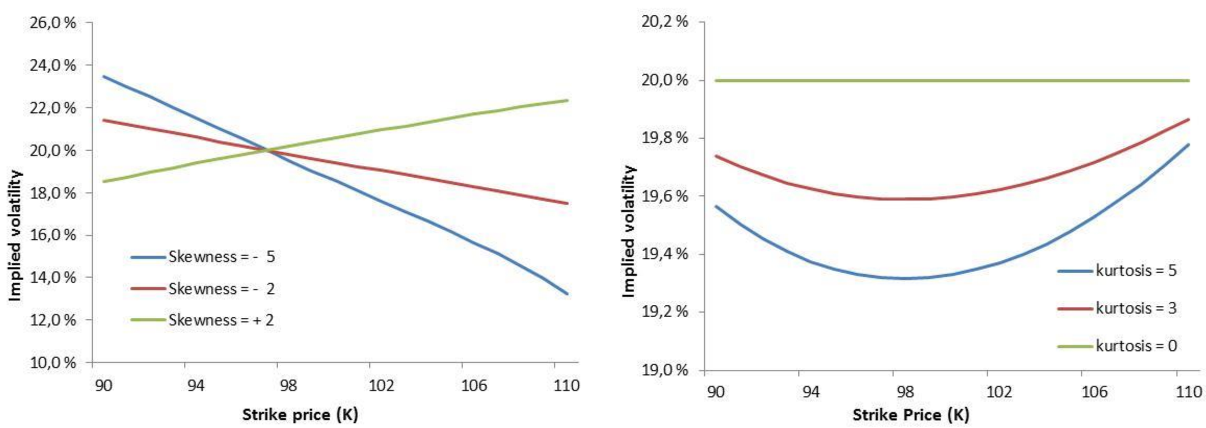


Figure 10 illustrates skewness and kurtosis by using default values of 0 ,rf = 0, yearly volatility =0,20, dividend =0, T=0,5. The figure shows the effects the volatility smile for arbitrarily chosen values of skewness and kurtosis.

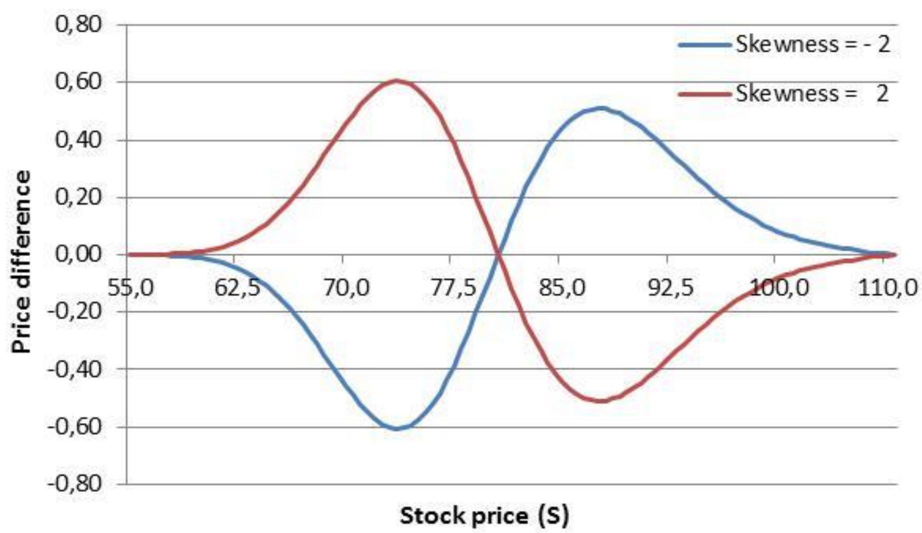
It is evident from the figure 10 (left) that the GC model is able to create the volatility skew by introducing skewness. When the skewness parameter is negative, the slope is a downward function of strike and this is consistent with the leverage effect. On the contrary, if skewness is positive a forward skew is observed. There is a theoretical justification (leverage effect) for negative skewness since this creates a downward slope as seen figure 10 (left), it is expected to detect negative values for this parameter when fitting the model to market prices.

The effect of kurtosis is seen in the figure 10 (right) and it is obvious that a positive kurtosis allows for the well-known volatility smile. The larger the parameter value the more pronounced is the volatility smile with low ATM volatilities and high implied volatilities for ITM and OTM options. On the other hand, if kurtosis is negative the smile will be reversed. In order to fit the theoretical volatility smile in the GC model, we expect the skewness to be negative and the kurtosis to be positive.

Comparison of BS and GC when introducing skewness

To further analyze the effect on skewness it is valuable to compare the price difference between GC and the BS model when skewness is adjusted. In the following graph the price difference between BS and GC is graphed with two different values of skewness, specifically skew = -2, skew = 2.

Figure 11



The graph represents the relative price difference between BS and GC when adding skewness. The horizontal numbers reflect the spot price S. The kurtosis parameter is set to zero to emphasize the effect of skewness. It is Important to keep in mind that the right side of the figure represents in-the-money options for call options.

When there is a negative skewness, the GC assigns a higher price for in the money options and lower prices for OTM options. This is illustrated by the blue line in figure 11. The opposite is true for a positive skewness, where the OTM prices are priced higher than BS and ITM are priced lower compared to the BS model. If the skewness parameter is zero there is no price difference between the models assuming that kurtosis is set to zero.

Restrictions in the Gram-Charlier

The GC model uses a Taylor series expansion approximation to calculate the call price.[22] This approximation can restrict the model. Backus, Foresi, & Wu (2004) studied this phenomenon in their article and found evidence that the absolute value of skewness calculated from this expansion underestimates the true skewness. They also found evidence that kurtosis is underestimated for larger values. They argue that the GC model may perform poorly for some combinations of skewness and kurtosis and could lead to a wrongly specified model. Another article by Jondeau & Rockinger (2001) did a research on the identification procedure and also discovered that the skewness and kurtosis in this model are not stable and tend to differ from their true values. It seems like the GC model is able to capture the effects of skewness and kurtosis, but can suffer from a wrongly specified model and it still assumes constant volatility.

Heston

The Heston model is the most popular stochastic model and is commonly used in the industry. Heston incorporates stochastic volatility and therefore relaxes the assumption of constant volatility in the BS model. The popularity of this model compared to other stochastic models is its closed form solution and its ability to reproduce a volatility smile observed in market data.

The underlying asset or stock index is assumed to follow the following process

$$dS_t = \mu S_t \, dt + \sqrt{(vt\ )}\, \mathrm{S_t dW_t^S}$$

$$dv_t \ = \kappa(\theta - v_t)dt + \sigma\sqrt{(vt\ )}\mathrm{dW_t^v}$$

$$\langle dW_t^s : dW_t^v \rangle = \ \rho dt$$

[22] Taylor series expansion is an infinite sum of the functions derivatives.

The first diffusion process is identical to the BS model.[23] The main contribution of this model is that the volatility is stochastic. This is achieved by adding another diffusion process that ensures random volatility. The Heston model converges to the Black & Scholes model when υ_t is constant.[24]

There are some interesting features of this second equation. The first term is similar to the Cox, Ingersoll & Ross' interest rate model (1985), where the interest rates are mean reverting. Heston uses the same concept with mean reverting volatility. The current variance (υ_t) has at some point converged to the long term volatility theta (θ). Even though the current variance is high/low today, it must be some underlying dynamic that pulls the volatility to a long run average. The parameter kappa (κ) is the mean reverting parameter and determines how fast the current variance converges to the long run mean. The second term is the volatility of volatility parameter (σ) and specifies the magnitude of the stochastic shock. It is multiplied by a different Wiener process which allows the volatility of the model to be stochastic. The two Wiener processes are correlated with a parameter Rho (ρ), which ensures that volatility and the stock index returns are correlated. The effect of these parameters will be discussed shortly.

In order to avoid negative variance Albrecher, Mayer, Schoutens, & Tistaert (2007) suggest the Feller condition to be fulfilled. This condition ensures that stochastic shocks are not large enough to create negative variance as the mean reverting parameter κ and long term variance θ pulls the volatility back.

$$2\kappa\theta > \sigma^2$$

In order to price the model we apply a risk neutral valuation. This is the same procedure as we presented in Chapter 1. The expected value of the stock index is replaced with the risk free rate as the investors are assumed to be indifferent to risk and only require the risk free rate. In the Heston model additional risk arises from the uncertainty of stochastic volatility. As mentioned earlier, investors are always assumed to be risk neutral and the stochastic risk parameter is set to zero. This is standard practice in option pricing and allows for pricing options in a consistent way.

Prior to Heston groundbreaking article from 1993, all stochastic models had to be computed by simulating different stock paths. Heston derives a closed form solution which drastically reduced the computational burden and made the stochastic models a powerful tool.

[23] The only difference is that variance is a square root process; this ensures that only positive numbers can enter the first diffusion process.
[24] Sigma needs to be approximate zero, as a value of zero will disrupt the calculations in the Heston model.

The value of a call option is

$$C = SP_1 - Ke^{-r(T-t)}P_2$$

According to Gatheral (2006) the interpretation of P_1 and P_2 is as follows. P_1 is the pseudo expectations of the final index level given that the option is in the money, while P_2 is the pseudo probability of exercising the option. Finding these probabilities is a complicated task and requires the use of characteristic functions.[25] In addition, the calculations require a numerical integration of complex numbers which is quite a burdensome task. The full formula is given in the Appendix C.

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{e^{-i\phi ln(k)}f_j}{i\phi}\right]d\phi$$

The calculation of the Heston option price is quite complicated and contains two complex integrals. There are other ways to calculate the integrals, as one could merge the two integrals into one to increase the computational speed. Another procedure is to use the fundamental transform introduced by Lewis (2000). We choose to implement the original integral as this makes the parameter interpretation more intuitive.

Analysis of the Heston parameters and the effect on the volatility smile

The effect of adding stochastic volatility makes the model quite flexible. By changing the different parameters it is quite easy to create a volatility smile. In this section the focus is on the properties of the different parameters and how the parameters affect the volatility smile.

[25] If we have knowledge about the characteristic functions we can find the probability density function of the stochastic variable using an Inverse Fourier transform. This is often easier than working directly with the density functions. This approach can be used since the characteristic functions depend on the same state variables as the probabilities P. In P_j for j(0,1), The *i* is the complex number $\sqrt{-1}$. In the integral it is only the real part of this function and is solved by using a numerical integration called trapezoidal integration. Due to the fact that the integrals converge quickly it is not necessary to integrate to more than to a 100. The derivations are not showed but a proof can be found in Heston (1993).

Figure 12

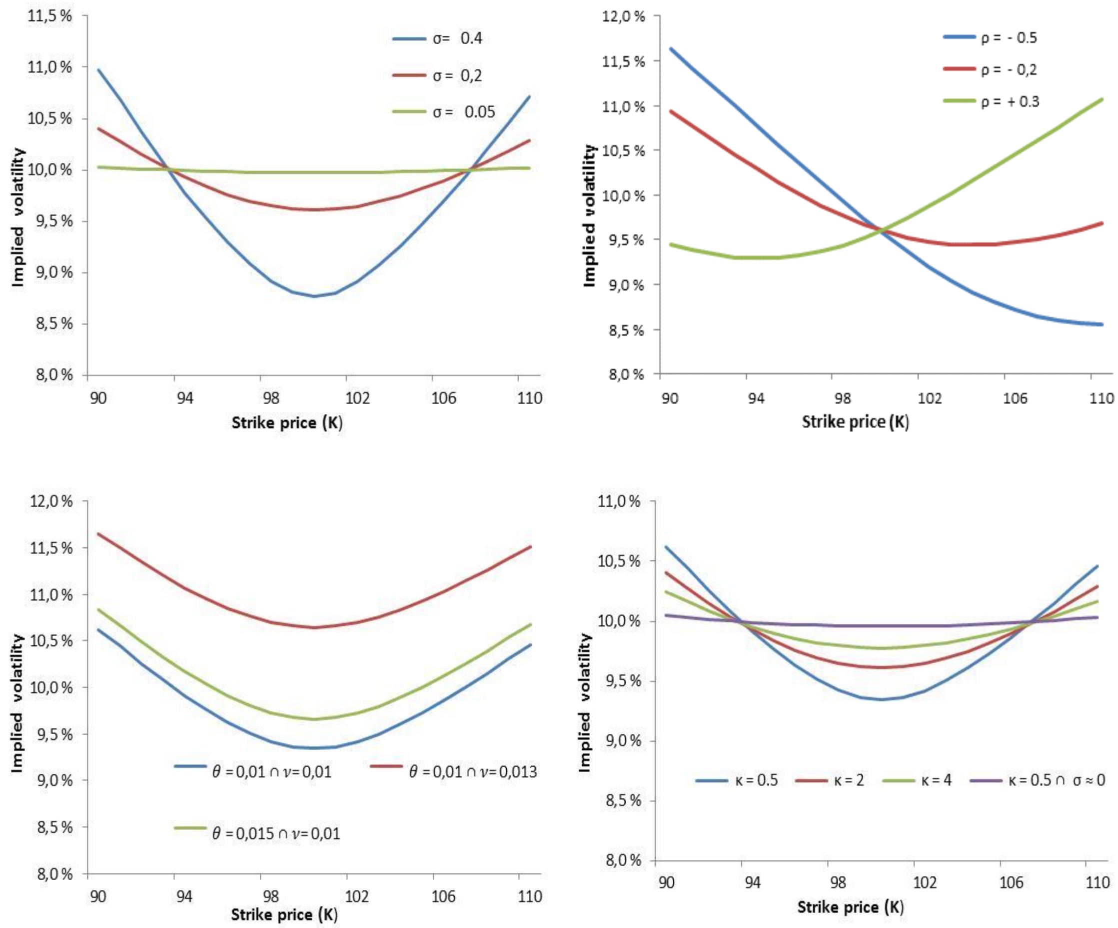


Figure 12 illustrates the effects of the parameters by using default values: S_0=100, ρ = 0, σ = 0,20, θ and $v_t = 0{,}01$, T-t= 0.5

The σ adds kurtosis in the density function and is the parameter that creates the smile. The σ allows for higher prices for ITM and OTM traded options The higher the σ the more pronounced is the smile. In contrast, if σ ≈ zero the smile effect disappears and the model converges to the BS model.

In order to create the equity smirk that is observed in options on equity indices, the skewness parameter ρ is essential. A negative ρ ensures an equity smirk, with low OTM implied volatilities. A positive ρ will have the opposite effect as seen in the graph. We would always expect the ρ to be negative if we observe a downward sloping smile.

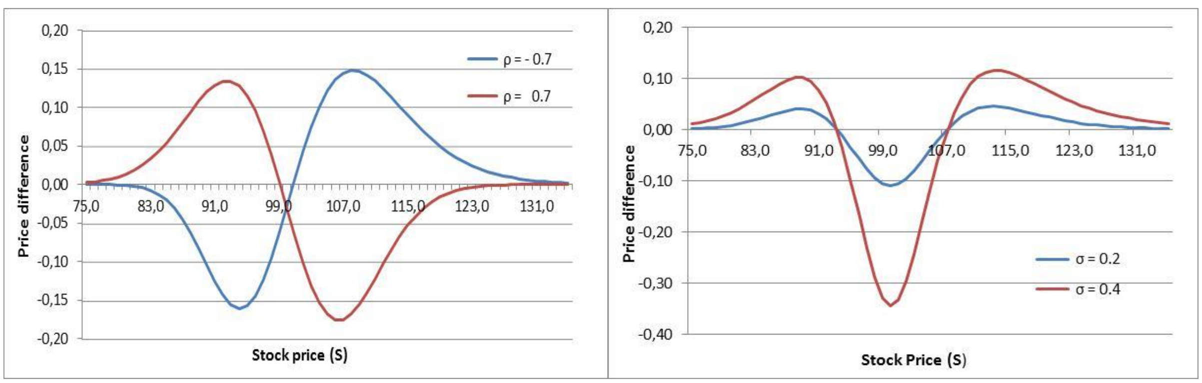
v_t *and* θ: The current variance and the long term variance has no pronounced effect on the smile, but lesser or increases the curve as seen in picture. It seems that the v_t has a more significant effect on the variance.

κ is the mean reverting parameter and also has a significant effect on the smile. A higher κ will reduce the smile effect while a low kappa will make the smile "pointy" The κ is not a smile parameter in itself, but when smile is already created by σ, the κ amplify the smile.

It is interesting to see how the value of ρ affects the price difference between the Heston and BS model. Figure 13 (left) displays the price difference for different values of ρ. When ρ is negative the OTM prices are less expensive compared to the BS model and the opposite is true for ITM options. A negative ρ decrease the probability of large positive shocks and thereby OTM options become cheaper. The ITM options are more expensive than BS model due to the fact that negative skewness increases the left tail of the distribution and therefore the ITM call prices increases relatively to the BS model.[26]

The effects of changing the ρ in the Heston model are similar to the effects of changing the skewness parameter in the GC model.

Figure 13



*The graphs represent the price difference between the Heston model and BS. The first graph shows the price difference when changing the skewness parameter **ρ**, when **σ** is zero. The second graph represents the price difference when changing **σ**, when **ρ** is zero.*

The effect of changing volatility of volatility is symmetrical for both ITM and OTM options. The effect of a positive σ is that ITM and OTM options are priced higher in the Heston model while ATM are priced lower. The rationale is that higher kurtosis gives fatter tails and higher probability of extreme values which surge the ITM and OTM options compared to BS and lowers the prices of ATM options. The larger values of σ, the more pronounced is the price difference.

[26] It can be more intuitive if puts are analyzed, as increased probability of negative shocks would lead to higher put prices. The put-call parity ensures that the implied volatility needs to be approximate the same for puts and calls.

Heston Nandi GARCH

GARCH (1.1)

In order to better understand the Heston and Nandi GARCH model, it is essential to understand the Generalized Autoregressive Conditional Heteroskedasticity process as GARCH models are often used to estimate and predict variance. In general, GARCH models try to predict today's variance based on lagged values of return and variance on the underlying asset. The main difference between GARCH and unconditional variance is that GARCH models emphasize the more recent variance and returns.[27] The effect of past observations is reflected in the parameters α and β, as in the following equation

$$\sigma_{t+1}^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t+1-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t+1-j}^2$$

The α in the equation reflects how much of past returns explain tomorrows variance, while β decides how much of the previous variance that predicts tomorrows variance. The number of lagged values of (p,q) is in principle indefinite, but in this thesis the focus will be on the p=1, q=1 process

$$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \beta \sigma_t^2$$

The α emphasize of much of today's return will explain the variance the following day, while β is how much of current variance that explain t+1 variance. ω is an intercept variable and the larger the omega the higher the variance. GARCH modeling is an effective way to eliminate autocorrelation as defined in Chapter 1 and therefore an effective way to handle the problem of persistence in volatility.

$\alpha + \beta$ represent the persistence of volatility and is restricted to be less than 1 in order to keep the process mean reverting, and the variance to be stationary. The closer $\alpha + \beta$ is to zero the stronger the mean reverting effect, while there is more persistence in the volatility when it is close to 1. A non-stationary series where $\alpha + \beta > 1$ is not realistic since this implies that variance can grow indefinite, which is highly unlikely in the market.

The Heston Nandi GARCH model

In the Heston Nandi option pricing model we assume the GARCH process from GARCH(1.1). In the article by Heston and Nandi, they prove that this model converge to the stochastic Heston model presented

[27] $\sigma = s = \sqrt{\frac{1}{n-1}\sum_{t01}^{n}(r_t - r_{average})^2}$ in fact unconditional standard deviation, but squaring this value gives the unconditional variance.

earlier when the time difference between observations converge to zero. The attractiveness of this model compared to other GARCH models is that Heston Nandi has a closed form solution which simplifies calculation. A nice feature of the GARCH model is that volatility can be obtained by looking at historical values instead of implying variance from other traded options, while still it still can create volatility smile implied by options.

The model is driven by this set of equations:

$$r_{t+1} = r + \lambda\sigma_{t+1}^2 + \sigma_{t+1}z_{t+1}$$

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha(z_t - \gamma\sigma_t)^2$$

where r_{t+1} is the logarithmic return of the stock index ($\ln(S_{t+1}/S_t)$), σ_t^2 is the condition variance, r is the risk-free rate, z_t is the error term distributed as a standard normal variable ($z_t \sim N(0,1)$) and $\lambda, \omega, \alpha, \beta, \gamma$ are model parameters.

This is quite similar to the simple GARCH model. In addition Lambda (λ) represents the level of risk and is multiplied by the conditional variance. The interpretation is that the risk parameter is linearly related to the variance of the stock and an investor requires higher returns when adding risk. The last term is quite familiar and has the same properties as Heston and BS model, where the value is dependent on a normally distributed shock. The most interesting difference is that the squared return is rt^2 is replaced by $(z_t - \gamma\sigma_t)^2$, and cannot be directly interpreted as lagged returns as in the simple GARCH (1,1).

The risk neutral version is represented in the article by Heston and Nandi (2000) and the equations look like;

$$r_{t+1} = r + \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}z_{t+1}^*$$

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha(z_t^* - \gamma^*\sigma_t)^2$$

Proposition 1 of Heston Nandi ensures that the risk neutral process has similar properties as the real process. The difference is that λ is replaced by $\frac{1}{2}$ and γ is replaced with $\gamma^* = \gamma + \lambda + \frac{1}{2}$.

The price of a call option has the same general formula as the Heston model, and value of the call is calculated by finding the probabilities P_1 and P_2.

$$Call_{HN} = S_t P_1 - Ke^{-rT} P_2$$

The calculation of the probabilities is quite similar to the Heston model and also includes integration of complex numbers.

$$P_1 = \frac{1}{2} + \frac{e^{-rT}}{\pi S_t} \int_0^{\infty} Re\left[\frac{K^{-i\phi} f^*(i\phi + 1)}{i\phi}\right] d\phi$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re\left[\frac{K^{-i\phi} f^*(i\phi)}{i\phi}\right] d\phi$$

To find the GARCH variance at t+1, one has to work recursively back from time t+T. In order for the model to work, a starting value of the variance needs to be estimated. Following Heston and Nandi the unconditional variance is set as a starting parameter.

Another important feature is that the process will be mean reverting if $\beta + \alpha\gamma^2 < 1$, similar to the GARCH (1,1)

Interpretation of the different parameters

ω determines the height of the variance and can be interpreted as an intercept. The value has similar interpretation as the ω in the simple GARCH(1,1). A higher value of omega will increase the height of the volatility smile, but will not produce a volatility smile/skew.

Heston and Nandi show that α represent the kurtosis and a positive α creates the volatility smile. The higher the α the more pronounced is the smile. The value of the α is a bit different from the normal GARCH(1,1) where α just represent the lagged return to explain the t+1 variance.

γ is the skewness parameter and measures the correlation between variance and the log return of the stock index. It is similar to the ρ of the Heston model. According to the Heston & Nandi, the relation is as follows:

$$Cov_{t+1}[\sigma_{t+1}^2 \colon \log(S_t)] = -2\alpha\gamma\sigma_t^2$$

It states when α and γ are positive, the correlation between variance and spot return is negative creating a volatility smirk/leverage effect similar to effect we analyzed in Chapter 1.

The last parameter β is another parameter that determines how much the previous GARCH variance will affect t+1 variance. The higher the beta the bigger the effect of lagged GARCH values. The β determines how fast the variance returns to its long term variance and the larger the beta the more clustering effect is seen in the data.[28]

[28] If α and β is zero, the process coincides with the discrete time geometric Brownian motion, which is the stochastic process for the asset price in the discrete time Black-Scholes model.

Chapter 4 Data Description

Introduction to the OMX 30 Index

“OMXS30 is the third most traded domestic index derivative in Europe and continues to be the world’s most used Swedish index”

Magdalena Hartman, NASDAQ OMX Group.

The Stockholm Stock Exchange was founded in 1893 and located in Stockholm, Sweden. In 1998, it was acquired by OMX and later on merged operations with the Helsinki Stock Exchange. OMX launched a virtual Nordic Exchange in 2006 by listing Swedish, Danish and Finnish companies in the same market place, which required common rules, trading hours and trading system for the joint market. By 2008 NASDAQ and OMX merged into NASDAQ OMX Group and is the world’s largest exchange company.[29]

The OMXS30 is a capitalization-weighted price index and consists of the 30 most-traded stocks in the Swedish stock market, which guarantees that all underlying stocks included are liquid. The index was listed in 1986 at a base-value of 125 and a 1:4 split was made in 1998. As of 31st of May 2012 the Swedish Stock Exchange had a market capitalization of 3.489 trillion while OMXS30 had 3.01 trillion SEK.[30] By design, the index tracks the Swedish stock market and act as underlying for financial products such as options, futures, exchange-traded and mutual funds. The composition of stock in the index is semi-annual reviewed and made effective on the first trading day of January and July every year. However, the composition remained unchanged since 1st of July 2009 (prior to our sample period) and consists of the same stocks as of today (3rd of June 2013).

Due to several factors, we found the choice of OMXS30 index interesting. Since there have been a high number of empirical research on option pricing models using S&P500 index and similar large indices, it is interesting to see how well-known models perform in a smaller market. Even though OMXS30 is smaller in size than S&P 500, precautions were made to make sure the index and options are liquid to represent the true price.

[29] www.nasdaqomxnordic.com
[30] Source: Main Market Total Equity Trading 1205 and from Datastream Index Market Capital

See the table below for a comparison the Nordic stock exchanges. It is clear that the Swedish Stock Exchange has highest market capitalization, number of firms in the respective market and number of traded shares.[31]

Table 0.1

May 2012			
Stock Exchange	**In SEK**	**Listed Companies**	**Traded Shares**
Stockholm	3.489 trillion	256	141 168 527
Copenhagen	1.382 trillion	175	18 261 600
Iceland	1.511 trillion	8	12 746 826
Helsinki	0.983 trillion	123	64 188 188

The table presents a comparison of the Nordic Stock Exchanges.

For all the indices that represent their respective markets, OMXS 30 is the index with the highest turnover compared to all of those indices traded in the Nordic countries.

Table 0.2

31st of May 2012	
Index	**Turnover**
OMXS 30	12 727 883 360
OMXC C20	3 836 914 126
OMXC C20 CAP	3 630 232 780
OMX Nordic 40	1 995 958 424
OMX Helsinki 25	469 556 445
OMX Iceland 6 ISK	92 143 722

Turnover is quoted in local currency.[32]

The contract size is index value multiplied with a 100 of local currency (SEK for OMXS30 and DKK for OMXC 20 CAP), and expires the 3rd Friday of the expiration month.[33] Of the Nordic indices presented, we find that OMXS 30 is an appropriate index for this empirical analysis of option pricing models.

[31] Main market Total Equity Trading 1205
[32] www.nasdaqomxnordic.com
[33] www.nordic.nasdaqomxtrader.com/trading/optionsfutures/Product_information/Index_Options/

Sample Description

Our sample period includes 1st of June 2011 through 31st of May 2012. The models used for this thesis requires a collection of historical data of daily bid-ask quotes of call options, closing index price, open interest and volume traded. Our analysis focuses on plain vanilla call options. We do not find this a fundamental limitation of our study as Jiang & Oomen (2001) states that using only European call options can provide insights into general risks of derivatives because of the following reasons: first, risks of put options are similar to those of call options based on put-call parity (see Chapter 2) and put-call symmetry (Carr, Ellis & Gupta (1998)). Second, adjusting for early exercise premium, American option prices can be reckoned from European option prices. Third, numerous exotic derivatives can be created from portfolios of plain vanilla call and put options.

The information is collected from Thomson Reuters Datastream 5.1, available at CBS. Datastream is one of the world's largest databases for financial and economic information. It collects data from a number of other information providers and contains more than two million financial instruments, securities and indicators for over 175 countries in 60 markets.[34] The Bloomberg Terminal is another alternative to collect the required information, however because both databases use the same source for OMXS30, Datastream was chosen out of convenience.

We use midprices as a proxy for the true market price of an option, Datastream-calculated dividend yield and STIBOR as risk-free rate. In the following paragraphs, we will explain the reason behind the use of these proxies.

Midprice

The bid price represents the highest price a buyer is willing to pay for a security, while the ask price is the lowest offered price a seller is willing to receive for the security. Bid-ask prices are therefore "quotes" that buyer and seller are willing to do a deal, however these recorded prices are not the actual transaction price of the security and may not seem a good proxy for the market price. Nonetheless, the true market price has to be in the bid-ask spread as no one are willing to sell below the bid price or buy higher than the ask price. On the other hand, "last price" represents the last transaction price recorded before the market close. The advantage is that the recorded price represents a fair value of the option at the time of the transaction. However, the time of the last transaction may differ from the "true" market price, especially if the security is not traded actively. Since the recorded price is not time-stamped, it can

[34] www.datastream.com

be affected by a non-synchronous bias. This means that the option price may not be recorded at the same time as the index and therefore be a poor proxy for the true market price. Another problem that may occur is that if there were no recorded transaction price, then the midprice is quoted by Datastream.[35] Consequently, by using last price the bid-ask quote may be included in the data set either way.

In our analysis, we first adopted "last price" as a proxy and we found that for some trading days last price was outside of the bid-ask spread which may be caused of the non-synchronous bias. A call option could be more expensive than a call option with a lower strike price. As these errors distort the option pricing models, we assume the mid-price is the best representative for the true market price which is the average of the bid and ask price. According to Dumas et al. (1998), by using midprice rather than last price reduces noise in the cross-sectional estimation of the volatility function.

Dividend yield and closing index price

By using the cost-of-carry condition of futures, the risk-free rate discounted futures price can be used as the current value of the index. Since expected dividend payments are implicitly discounted, no explicit dividend correction is needed. However, because Datastream cannot provide satisfactory data (only settlement price) on the futures price and futures are not actively traded as the matching maturity options starts to trade, futures prices are not used.

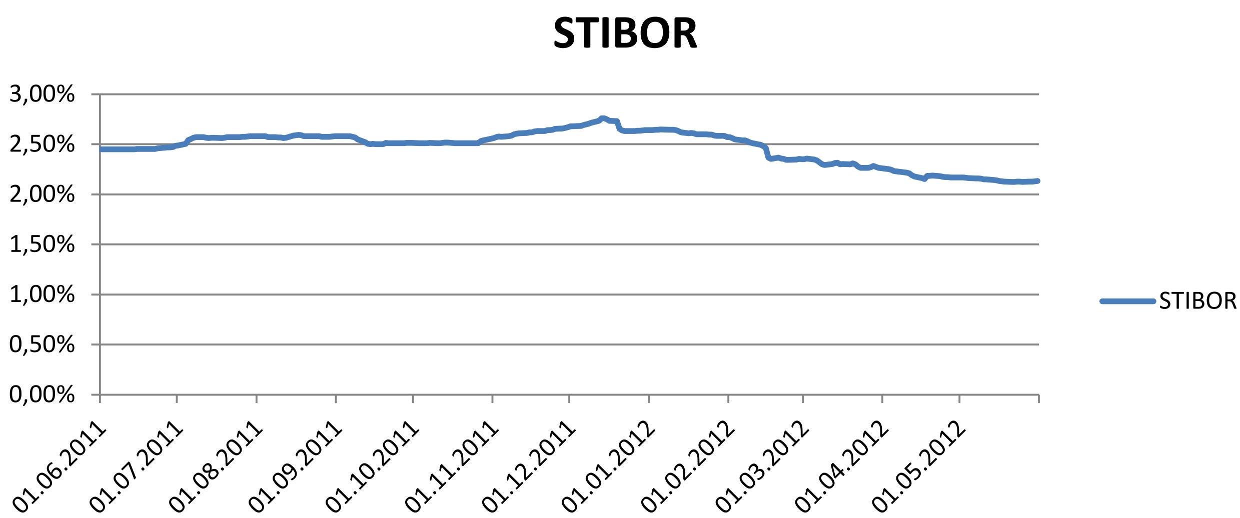
On the other hand, Datastream do calculate daily dividend yield of the index that are approved by NASDAQ OMX. Since no or low liquidity may introduce misrepresented futures prices, we use the closing index price and take into account the dividend yield.

Risk-free rate

For the option pricing models, a risk free rate is needed. According to Hull (2009, p. 75), it is natural to assume Treasury bills and Treasury bonds as the correct benchmark for risk free rates. In contrast, traders regard the LIBOR (London Interbank Offered Rate) rate as their opportunity cost of capital and usually use LIBOR rates as short-term risk-free rates. Traders argue that Treasury rates are too low to be used as risk-free rates because of regulatory requirements and favorable tax treatments. The interbank rate is approximately equal to the short-term borrowing rate of an AA-rated company and thus a small chance of default. We follow the same procedure as practitioners and use the interbank rate as the risk-free rate.

[35] See "Last price" definition of Datastream Navigator

Figure 14



The graph illustrates the STIBOR rate movements for the whole sample period.

Because OMXS30 is traded in Swedish currency, it is rational to use the Swedish Interbank rate. The 3-month Stockholm Interbank Offered Rate (STIBOR) is provided by Central Bank of Sweden and is used as a proxy for the risk-free rate.

In theory, one should match the maturity of the risk free rate with the remaining days to expiration for options. . However, to circumvent this, we interpolate the maturity from the 3-month STIBOR rate as we believe the interest rate is of minor importance for short-dated options.

Filtering the data

The following rules are applied to filter data needed for the empirical test.

1. Following Bakshi, Cao, & Chen (1997), all observations were checked for lower boundary condition

$$C \geq S_t - Ke^{-rT}$$

   Option prices which do not satisfy the lower boundary restriction are excluded.
2. Since very deep ITM or very deep OTM options are less actively traded, their price quotes may not generally reflect "true" option value. Moneyness (S_t/K) greater (less) than 1,10 (0,90) are

therefore excluded. Within this interval, only actively traded options are included in the sample which means there have to be traded volume and open interest for each strike price. The moneyness interval from 0.90 to 1.10 captures the majority of the options traded, however those options which are outside of this interval are not traded daily and the price quotes are generally not supported by actual trades.

3. Options with less than 3 days and more than 70 days to expiration may induce liquidity-related biases. While options with 3 days maturity are highly sensitive to price-volatility bias, options with more than 70 days to expiration are typically not traded and are therefore rejected.
4. Prices lower than 0.50 are not included to mitigate the impact of price discreteness as the errors on these prices will relatively have a huge impact.
5. Swedish public holidays are included as a trading day when the raw data is gathered from Datastream. Because there are no trading activity (therefore no change in prices), these observations are rejected.
6. Days with missing observations and with implied volatility less than 1% or greater than 100% are deleted, following the approach of Bakshi & Kapadia (2003). Traded options which do not have recorded bid and ask-prices are excluded from the sample. As we are using mid-price as a proxy for the "true" market price, both bid and ask quotes must be recorded. Options with implied volatility as mentioned above may arise from a non-synchronous bias and therefore distorts the models. Consequently, these observations are omitted.

The final set of remaining data amounts to 4849 of total 8768 traded call options, which is 55% of all traded options. For the purpose of this thesis we divided option data into six categories of moneyness: very deep out-of-the-money (very DOTM) if the moneyness is less than 0.94 (<0.94), deep out-of-the-money (DOTM) if moneyness is between 0.94 and 0.97, out-of-the-money (OTM) if the moneyness is between 0.97 and 1.00, in-the-money (ITM) between 1.00 and 1.03, deep-in-the-money (DITM) if the moneyness is between 1.03 and 1.06 and very deep in-the-money (very DITM) if moneyness is greater than 1.06. The dataset is also divided according to maturity. The following segmentations is used, $T < 20$, $20 \leq T < 40$ and $T \geq 40$, the maturity is divided according to trading days.

Table 0.3
OMXS30 Options Data

T < 20							
S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**All**
Option Price	3,22	6,16	14,28	29,35	49,75	76,23	19,05
Imp.Vol	27,73 %	25,24 %	26,03 %	27,86 %	31,43 %	35,39 %	27,25 %
Observations	247	370	433	381	222	114	1767

20 ≤ T < 40							
S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**All**
Option Price	6,97	14,51	26,02	40,79	61,24	85,94	25,96
Imp.Vol	23,45 %	23,80 %	25,26 %	26,53 %	29,99 %	32,18 %	25,30 %
Observations	426	437	402	287	139	65	1756

T ≥ 40							
S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**All**
Option Price	13,32	21,96	35,09	48,84	69,75	94,58	28,36
Imp.Vol	23,32 %	23,02 %	24,24 %	24,60 %	27,95 %	30,16 %	23,88 %
Observations	428	351	311	162	55	15	1322

This table reports the average option price, implied volatilities by inverting the BS formula and sum of observations separately for each moneyness and maturity category. Moneyness is defined as S/K, where S denotes the spot index price, K denotes strike price and T denotes days to expiration of the option.

Table 4.3 describes certain sample properties of the OMXS30 option prices used in this thesis. Summary statistics are reported for the option price and the total number of observations, according to each moneyness and maturity category. We can see that option prices increase when we move from DOTM to DITM and the liquidity of OMXS30 is concentrated in the options with short (less than 20 days to maturity) and medium-term maturity (between 20 and 39 days to maturity). For all maturities, a volatility smile can be observed across the moneyness and maturity categories where it is most prominent for options with the nearest contract expiry. These findings of clear moneyness-related and maturity-related biases associated with the BS are consistent with those in the existing literature (e.g., Bates (1996b)).

Therefore, any acceptable alternative to the BS model must show an ability to properly price non-ATM options, especially short-term OTM calls. Since there is pattern where prices are the lowest for OTM options, we expect the percentage errors to be higher for DOTM options across all maturities.

Chapter 5 Calibration of the Models

"The price to pay for more realistic models is the increased complexity of model calibration. Often, the estimation method become as crucial as the model itself"

Jacquier.E (2000)

As expressed in the above statement the calibration of the models is crucial to get good and robust results. The importance of correct calibration increases with the complexity and the number of parameters to be estimated. It is easy to calibrate a single parameter model like volatility in BS, but it can be complicated to calibrate advanced models like Heston and GARCH.[36] The results may be spurious if the calibration is erroneous and the result can be misleading. In this section the focus is on how to calibrate the models by looking at best practice as well as calibrating the models in Excel VBA.

The problem of perfect calibration may be regarded as an almost impossible task. Obtaining the true unbiased parameters that are by nature unobservable seems like rigorous task. There are two ways to obtain the unobservable parameters. The first way is to look at historical values and infer the parameters from past observations. The second way is to infer the parameters from the current cross-sectional option prices.

1. Historical values:

Historical data only reflect past information and only include the backward-looking information rather than future expectations. There can be significant differences between expected values and past realizations. Another unfavorable feature is that risk premiums are hard to obtain from historical values. In contrast, historical values are easily available and do not require implying information from other instruments.

2. Cross-sectional data:

Chernov & Ghysels (2000) propose a way to use forward-looking information as well as mitigating the problem of pricing risk. They suggest using current option prices and from this infer the parameters that reflect forward looking information. In this thesis the focus will be on obtaining the parameters from a

[36] GARCH is from now referred to as the Heston Nandi GARCH(1,1)

cross-section of currently traded option prices. The only exception is the GARCH and BS models, which use both historical and cross-sectional data.

Since there is no guarantee that market makers/traders are in fact rational investors, there is an ill-posed problem to obtain the parameters form currently traded options. One has to assume that the market is close to perfect and all information is known and priced into the options. This method also requires that option prices are consistent and "noise" from the data is negligible (Rebonato, 2003).[37]

In order to use traded options to calibrate the parameters the models need to have an analytical or semi-analytical solution. The reason is that parameters are obtained by inverting the prices that minimizes the difference between market and model with the use of an objective function. This strategy is common in literature and should yield a good parameter fit if done correctly.

Objective Function

"The choice of loss function is key because it implicitly assumes a particular error structure"

Christoffersen & Jacobs (2004)

The objective function or a loss function is a common procedure to solve complex problem like finding parameters from the traded options. In recent research, there has been different use of objective functions. There are in general three general common loss functions. Each of them has its strengths and weaknesses

1. $\$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (P_i^{market} - P_i^{model})^2}$

The dollar Root Mean Squared Errors minimize the dollar amount between the market prices and the model prices. It emphasizes the raw difference between them and emphasize expensive in the money options. This approach is quite common and is used in many research papers like Bakshi, Cao & Chen (1997), Heston & Nandi (2000) and Singh (2013).

2. $\%RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}((P_i^{market} - P_i^{model})/P_i^{market})^2}$

[37] Noise could be inaccurate data recording, a high bid-ask spread and illiquidity

This loss function emphasizes inexpensive OTM option as percentage errors between small values become crucial. The better in-sample for the cheap OTM options may be on the expense of ITM and long term options. This procedure has been used by Kim & Kim (2004) on Korean KOSPI 200 option index market and Su, Chen, & Huang (2010). Jacquier & Jarrow (2000) argue that percentage error can be the theoretical correct loss function.

3. IVRMSE $= \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\sigma_i^{market} - \sigma_i^{model})^2}$

The last loss function focuses on minimizing the market implied volatility and model implied volatility. This procedure gives approximate equal weights to each option, since calculating the implied volatilities will not produce great differences between ITM options and OTM options. The problem with this loss function is that it can be quite computational demanding.

There is no common consensus on which object function that performs best and different approaches have been used. Heston (1993), Bakshi et al. (1997), Heston & Nandi (2000) have used the $RMSE while others (Kim & Kim, 2004) have used the %RMSE. Christoffersen & Jacobs (2004) argue that the loss function is an issue of pragmatism and the purpose of the study/industry should determine the choice. He argues that the loss function should have a connection with the evaluation of the model. This has not necessarily been the case in recent research. If one chooses the $RMSE, the evaluation criteria should be based on the dollar difference between model and market. Since the evaluation criterion of this paper is based largely on percentage errors, the natural candidate is the %RMSE objective function. In the rest of the thesis %RMSE is used to estimate the parameters from information on the market prices, if not stated otherwise.[38] Consequently, we will put more emphasis on MAPE (Mean Absolute Percentage Errors) for pricing errors which is explained in Chapter 6.

$$\%RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} Wi((P_i^{market} - P_i^{model})/P_i^{market})^2}$$

The objective functions are flexible and if one wish to emphasize liquid options, ATM options or short term options this can be achieved by applying different weights to different options. For the rest of the thesis all of the options will have equal weights (w_i=1).

[38] In Appendix A we have performed a comparison of the loss function $RMSE and the %RMSE for the Heston model for July and February.

Practical implementation

To estimate new parameter sets, every sample day is not directly consistent with the fact that the parameters in theory are supposed to be slow moving. An article by Mikhailov & Nögel (2003) states that the parameters are in fact non-stationary and fluctuate over time. This indicates that in order to get the correct market outlook models should be calibrated daily.

Three consecutive months of option data is used in the calibration. Consider if a model is being calibrated for 1st of January. Options that expires in January, February and March is used to calibrate the model parameters, given they have passed the filtering rules mentioned in Chapter 4.

It has been proven that volatility is higher when the assets are traded. The trader tend to ignore days when the exchange is not open and only use trading days when calculating time to maturity (Hull, 2009). Thus to incorporate this affect trading days are used as days to maturity.

In the analysis we have put effort in giving every model a level playing field and every model is calibrated to current market data. In addition to this, the BS model is tested using historical values.

Calibrating with using Excel VBA

In order to solve the complex task of calibrating models we need a statistical software package that is able to minimize the objective function with a model with up to up to 5 different parameters. The problem is complex and finding the global minimum can be difficult.

To find a global minimum one needs advanced search algorithms which is very time consuming.[39] To be more practical we have decided to use Excel VBA to obtain the model parameters. This is generalized reduced gradient non-linear searcher and is used for smooth convex problems. Since the advanced models often follow non-smooth functions, this optimizer may only produce a local minimum.[40] The choice of Excel VBA is because we want this to be a practical and applicable approach to option pricing, as most people have this software package available.

It is argued that local minima can give less accurate results. On the other hand, it is found in Mikhailov & Nögel (2003) that Excel can provide robust and reliable results if used right.

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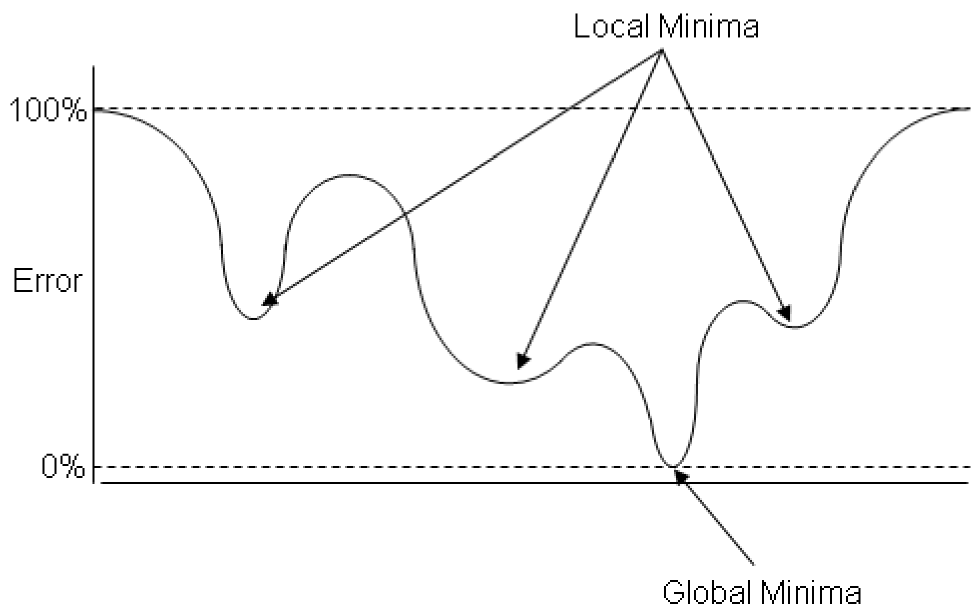
[39] An example is Adaptive Simulated Annealing (ASA) which is a Global minimum searcher developed by Lester Ingber, http://www.ingber.com. This procedure is very time demanding, making it unpractical for empirical work

[40] Calculates the "derivatives" of the parameters and then adjust the parameters which have the largest effect on the loss function. For further information we refer to: http://www.solver.com/content/basic-solver-what-solver-can-and-cannot-do

The problem that often occurs by using Excel is that the solution may only be a local minimum instead of the preferred global minimum.[41] This can affect the stationarity of the parameters since different local minima can be different from each other. Due to the local minimum constraint, the initial guess becomes important. This is especially true for complex models like Heston and GARCH, but is of minor importance for the simpler models. To mitigate this problem, we have taken precautions to acquire appropriate results in estimating the model parameters.

Figure 15



The figure illustrates an example of local minima versus global minima.[42]

[41] Local minimum searchers are also common when using other software packages, for example Matlab.
[42] Source: http://mnemstudio.org/neural-networks-multilayer-perceptron-design.htm

Estimation Procedure

Black & Scholes and Gram-Charlier

When calculating BS30 based on historical values, the returns from the 30 last trading days is used is used to estimate historical volatility. The volatility is updated every day so the historical values fit the new market condition. The rule of thumb is to use the same number of observations as the length of the option. One can argue whether we should put more emphasis on recent volatility, but as we want this so be a simple benchmark we have decided not to do so. The volatility is calculated from the formulas below with n being the last 30 trading days.

When finding the volatility for the BS and GC, one has to estimate volatility [σ] for the BS and [Volatility, Skewness and Kurtosis] for the GC.[43] We follow the strategy presented in Chapter 3 and minimize the %RMSE to obtain the parameter values. Since this is an easy task in Excel, the initial guess is of less importance and set at an arbitrary level of 20% for BS and $\langle 0{,}10; -0{,}20; 0{,}10 \rangle$ respectively for GC.

$$\text{Daily returns } r_i = ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$\text{standard deviation} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - r_{average})^2}$$

$$\text{yearly volatility } \hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

Practitioner Black-Scholes

In the PBS model, the calibration procedure looks somewhat different than the other models. PBS finds the fitted implied volatility by using Ordinary Least Squares. As mentioned in Chapter 3 of the thesis, the deterministic functions look like

[43] One has to keep in mind that the different parameters have to be in the same time period. If we use a three month kurtosis and three month skewness, the interest rate and volatility needs to be three months estimates.

$$DVF1: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2$$

$$DVF2: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2 + a_3T + a_4KT$$

$$DVF3: \quad \sigma_{iv} = a_0 + a_1K + a_2K^2 + a_3T + a_4T^2 + a_5KT$$

In order to extract the parameters in this model we calculate implied volatilities from market prices. Then the deterministic volatility function is used to create fitted volatilities. Finally, we use OLS to minimize the difference between the implied volatilities and fitted volatilities by changing the structural parameters in our model $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. Then fitted volatilities is obtained and inverted to find the correct option prices.

The estimation procedure is somewhat different from the other models as an implied loss function is used and not %RMSE. This can impact the model evaluation later on and we bear in mind this difference. Even so as found by Christoffersen & Jacobs (2004), there are other researchers that also have specified different loss functions and we believe this will not greatly alter our results . According to the authors, the industry usually use the implied volatility loss function for PBS, hence we follow the industry standard.

Heston

The stochastic volatility model is one of the more complex models that are included in the thesis. The importance of good calibration is vital to get robust results for a complex model like Heston. As stated earlier, the use of Excel is not the best program for such complicated integration and it is important with a good initial guess. Using Excel Solver for the Heston model, we followed the same VBA programming of Rouah & Vainberg (2007). The complexity of this task also results in considerable long run time for Excel and due to the computational limitations, a time constraint was set. The following rules were used in the calibration process;

1. Using the initial guess for the day.
2. Max iterations of 15 approximate or 2000 seconds.[44]
3. Used previous days calculated parameters as an initial guess the next day.

[44] If the time constraint limit was reached before the 10th iteration we continued the calibration to at least 11 iterations. A comparison on how different time/iterations restrictions affected the results are illustrated by two random examples in Appendix B.

4. When the result clearly indicated a wrongly specified parameter set the calibration that day was recalibrated with a new initial guess.

Initial guess and parameter restrictions in Heston

Excel Solver can be a satisfactory approximation if the initial guess is good. For the initial guess, we used market data to pin-point the size of the parameters

$$\langle v_t\,, \theta\; \rho, \sigma, \kappa, \lambda \rangle$$

v_t (Current variance) is approximated from the SIXVIX where currently traded ATM options indicate daily volatility. The long term variance (θ) was approximated by the averaging the yearly SIXVX.

The rest of the parameters it is less clear cut what the start values should be. Since the leverage effect seem to be a major concern in stock indices, the parameter ρ is set to -0.5. Since there are no suitable initial guess for σ, trial and errors where done to obtain a decent approximation. The κ was set to a low integer at 2 and the price of risk parameter (λ) is set to zero.[45]

Table 0.1

ρ	**κ**	**θ**	**σ**	**v**	**λ**
-0,5	2	Yearly Average SIXVX	(0,2;0,7;0.9)	Daily SIXVX	0

Each Wednesday/Thursday the initial guess was evaluated and compared to the last days parameter set. The better of these two conditions was used to calibrate the model.

In order to avoid negative variance the Feller condition has to be fulfilled $2\theta\kappa > \sigma^2$. We set this condition to be true in Excel and later evaluate whether the restriction should be included in the parameter estimation. According to Andersen (2007) and Haastrect & Pellser (2008) this condition is rarely fulfilled in the market data.

Heston & Nandi GARCH

In the Heston & Nandi model, previous research has used two different calibration methods. The first one is based on Maximum likelihood based solely on historical data. The other approach is based on cross-sectional data and extracts the current variance from historical data. In this thesis, the focus will

[45] Further restrictions: $\kappa < 15, \sigma < 5, v_T\ and\ \theta < 1$. Theta was also restricted to be larger than 0.008 to avoid long term negative volatility. These restrictions are not very restrictive as the parameters rarely “touched” their restricted boundaries.

be on the latter since all the models are tested on this basis. Another fact is that estimating the option parameters solely from historical values is not highly accurate and does not include forward looking information.

As in the Heston model we experienced irregular values when Excel tried to find a local minimum, therefore restrictions on different parameters was included. All the parameters were restricted to be positive, to avoid extreme or negative values for the variance. As Heston & Nandi (2000) we use historical values from the previous year (252 observations) to obtain conditional current variance. The starting variance 1 year prior is set to the unconditional variance of the previous year**.** This is standard practice when using Heston and Nandi GARCH(1,1) model. Heston & Nandi argue that the 1 year data should be sufficient time to revert itself to its hopefully correct variance.

After extensive hours of work the result is an estimation of 23*12 days of market data for up to 5 models each day.[46] Note that complex models like GARCH and Heston are models that are quite time consuming when estimating a parameter set, especially GARCH and a single estimation can easily be going for hours without finding an solution. Before analyzing any result one can conclude that computational burden is quite high for GARCH and Heston.

[46] Average time for Heston was about 2500 seconds and for GARCH approximate 7000 seconds.

Comments to the calibration of the models

Heston

The Feller condition was a restrictive condition for many of the calibrated days. So we decided not to implement this condition. Another fact as mentioned by Moodley (2005) is Excel has trouble handling small numbers and due to its restrictions the Feller condition was abandoned.

As stated previously the Heston model is hard to calibrate and the results obtained may be local minimum instead of global minima. Although Excel did or did not find the optimal solution for every calibration we feel quite comfortable with the result. By letting Excel find an optimal solution for a few dates without time restriction, and compared to the result found by stopping the calibration at 2000 seconds/15 iterations, did not alter the results substantially.[47] Still, there is no guarantee the preferred global minimum was found due to the limitations of Excel.

GARCH

Excel used tremendously amount time for each calibration and did not have the ability to manage the skew parameter gamma. In order to facilitate the process we decided to only calibrate the model every week and every time there was a structural break in the option data. Although this is not entirely consistent with the rest of the models, this procedure is the same as taken by Heston & Nandi (2000) and Christoffersen et al. (2006). Instead of estimating the model parameters every day, the parameters $\langle \alpha, \beta, \omega \rangle$ were updated weekly, while the historical value of log returns where updated daily. This is important to bear in mind when analyzing the results.

Another caveat is that Excel Solver did not find an appropriate value of gamma (leverage parameter), therefore every calibration day the estimation was executed by setting the gamma equal to 0, 50 or 100. Then the best fit was used to calculate in sample and out of sample results. The values 0, 50 and 100 were chosen arbitrary and seemed to be the best parameters for our sample. Although we tried to achieve a proxy for this value, we are full aware of its implications and that the result of the Heston Nandi GARCH model will be negatively biased compared to the other models.

[47] See appendix for a comparison.

Table 0.2

	HN (0)		HN (50)		HN (100)		HN (Best)
α	8,10E-06	α	1,01E-05	α	8,43E-06	α	8,85E-06
	(6,85E-06)		(1,09E-05)		(7,58E-06)		(7,09E-06)
β	0,8196	β	0,8064	β	0,7840	β	0,8027
	(0,1535)		(0,1562)		(0,1601)		(0,1522)
γ	0,5068	γ	50,50	γ	100,50	γ	64,068
	(0)		(0)		(0)		(40,428)
ω	1,24E-05	ω	1,0106E-05	ω	1,01E-05	ω	9,416E-06
	(1,88E-05)		(1,30E-05)		(1,21E-05)		(1,24E-05)
σ^2_{t+1}	1,87E-04	σ^2_{t+1}	1,82E-04	σ^2_{t+1}	1,55E-04	σ^2_{t+1}	1,69E-04
	(1,15E-04)		(1,11E-04)		(9,48E-05)		(1,09E-04)

The table reports the estimation results of the parameters. HN(Best) consists of 23% of γ=0 , 28% of γ=50 and 49% of γ=100 where best in-sample results from the most appropriate γ where used. Consequently, HN(Best) has the least in-sample errors.

GARCH parameters were obtained by testing the values $\langle 0,50,100 \rangle$ for the skewness parameter γ and the results are reported in table 5.2.[48] It seems like the different parameters do not change much although the skewness parameter is changed. The model appears to have problems adapting to market data as the parameters are less volatile compared to the other models. To compensate for the skewness parameter, we have chosen the parameters that give the best in-sample fit, which is HN (Best).

Analysis of the parameters

The parameters that represent the volatility smile have the expected sign for every model. K^2(PBS) is positive, volatility of volatility σ is positive (Heston) and the kurtosis parameter for GC is also positive. This indicates that on average every model creates a volatility smile which is consistent with theory.

Another interesting property is whether the models capture the volatility smirk. The smirk which indicates the leverage effect represented in Chapter 1. All of K (PBS), ρ (Heston) and skewness parameter in GC are negative indicating the different models capture the volatility smirk on average.

To further analyze the different models we can see what kind of volatilities the different models imply from the market. The BS implied volatility (21.6%), historical volatility (27,9%), current variance Heston (26,1%), volatility GC (27,7%), SIXVX (26,0%), It seems that models which incorporates smile/smirk effects seems to imply the same volatility, while BS Fitted is significantly lower.

[48] The only exception is the parameter λ, but this value was insignificant and had no effect on option prices.

Stationarity of the Parameters

Table 5.3 is a summary of the different parameter estimates throughout the period and it seems no model have stationary parameters. For some models the standard deviation is larger than the parameter itself. The GARCH parameters are more stationary than the other models, which is to be expected. This was also evident in the article by Kim & Kim (2004). The non-stationarity of the parameters clearly indicates that the market changes and the model have to absorb new information by changing its parameters. This is the contrary of what the theory suggests: The model parameters should be slow moving and change little throughout time. Other articles such as Kim & Kim (2004) found evidence that parameters have large standard deviations. They argue that stability of the interdependence between the parameters is far more important than focusing on the standard deviations.

Table 0.3

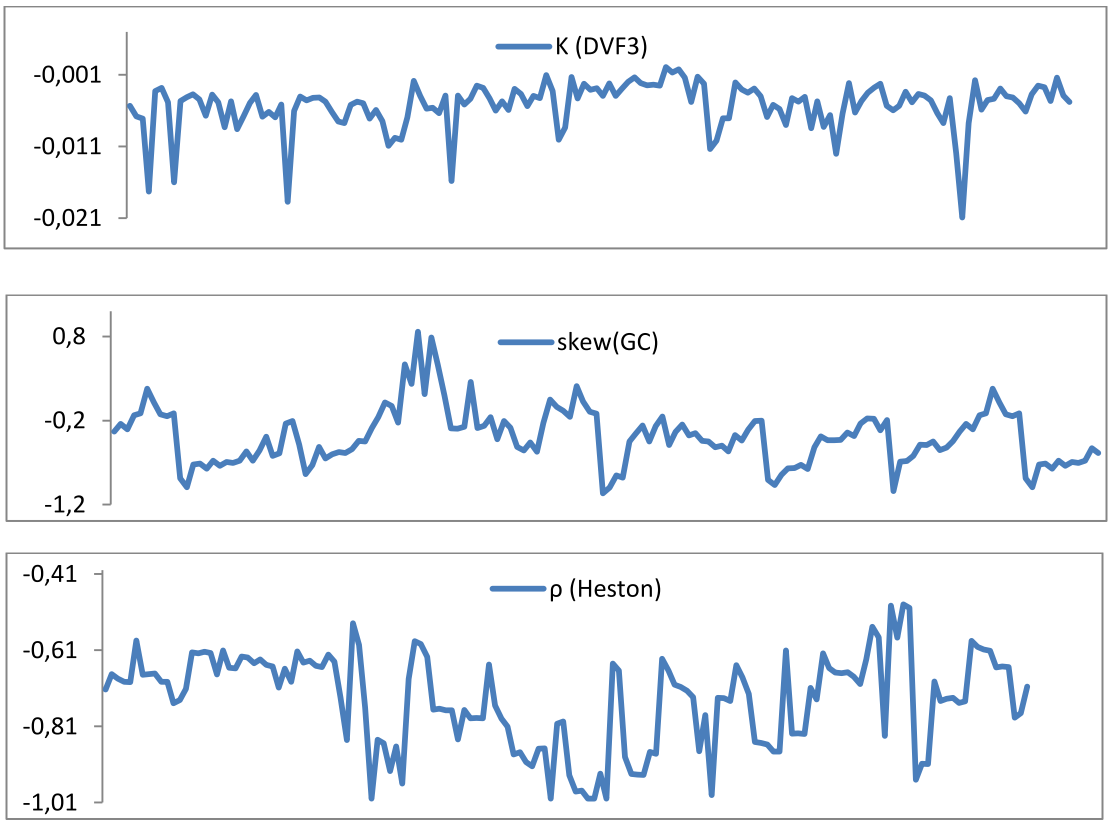
	BS		GC		DVF1		DVF2		DVF3		Heston		GARCH
σ_1	0,2160	*Vol*	0,1389	α_0	3,7672	α_0	2,9785	α_0	2,8942	ρ	-0,6697	α	8,85E-06
	(0,0719)		(0,0389)		(4,5706)		(3,0856)		(2,9951)		(0,2122)		(7,09E-06)
σ_2	0,2788	*Skew*	-0,3504	α_1	-0,0062	α_1	-0,0047	α_1	-0,0045	κ	4,3445	β	0,802723
	(0,0972)		(0,3706)		(0,0082)		(0,00541)		(0,0052)		(2,5268)		(0,1522)
σ_3	0,2604	*Kurt*	0,1543	α_2	2,70E-06	α_2	1,96E-06	α_2	1,9E-06	θ	0,1114	γ	64,068
	(0,0778)		(0,4840)		(3,69E-06)		(2,4E-06)		(2,25E-06)		(0,0943)		(40,428)
						α_3	-0,7790	α_3	-0,8853	σ	0,7880	ω	9,42E-06
							(3,50272)		(3,1635)		(0,3778)		(1,24E-05)
						α_4	7,560E-04	α_4	8,70E-04	ν	0,0683	σ^2_{t+1}	1,69E-04
							(3,11E-03)		(0,0028)		(0,0439)		(1,09E-04)
								α_5	-0,0212				
									(2,2204)				

This table reports the mean and standard deviation (in parenthesis) of the parameter estimates for each model. The parameter estimates are explained in Chapter 3. For BS, σ_1 is fitted volatility using %RMSE, σ_2 is historical volatility N=30, σ_3 is SIXVX ATM implied volatility.

In our case the index moves a lot during our sample and making the model parameters even more volatile. The implied volatility of the year is seen in figure 5 indicating that there are large movements in market sentiment throughout the year. By prohibiting the parameters to adjust can negatively bias the results.

The instability of parameters suggests that out-of-sample pricing can be somewhat mispriced since the parameters are unstable. On the other hand, the table is a summary of all observations and maybe the parameters are in fact more stationary if we decrease the time period. In the following figure the skewness parameters of the different models is graphed, to see how volatile the parameter is. It seems that there are not any distinctive patterns and we conclude that the parameters fluctuate much during the sample period.

Figure 16



The figure illustrates the first 150 estimations for the volatility smirk parameters K, ρ and skewness

Conclusion of the calibration

Calibrating models is essential to obtain good results. In this section we have used Excel VBA to estimate parameters and we are positive that the parameter set obtained from the calibration are reasonable. The Heston model may to some extent be calibrated slightly better without the time constraint, but we believe this is of minor importance. The GARCH would probably also perform better with the "true" estimated parameter γ.

In spite of model theory, the parameters have large standard deviations making them flexible and adaptable to new market conditions. This seems to be a major concern since models that have large standard deviations in parameters seem to perform better (Kim & Kim, 2004). All of the parameters seem to incorporate the volatility smile and leverage effect which gives us confidence in the results.

As a last note, one should not spend infinite time for the perfect calibration when the model by itself is imperfect. It is just as vital to understand the assumptions behind the models and how the different parameters affect the output.

To sum it up, the alternative option pricing models included in this thesis are BS30, BS, GC, PBS with deterministic volatility functions, Heston and GARCH. BS30 volatility parameter is based on the returns from the 30 last trading days to estimate historical volatility, while BS, GC and Heston obtains the volatility parameter by minimizing %RMSE. PBS uses OLS on the implied volatility from the cross-sectional options prices that have passed through the filtering rules.[49]. GARCH on the other hand, arbitrary values are chosen for the skewness parameter γ, while the remaining parameters were updated weekly except for historical value of log returns which is updated daily. Of those weekly estimations, the parameters that gave the best in-sample fit were chosen for the GARCH model.

In the next chapter, we will explain how the pricing errors are calculated for in-sample, out-of-sample and how the regression analysis is commenced and at last, delta hedging errors.

[49] See Chapter 4.

Chapter 6 Methodology

We follow the approach of Bakshi, Cao & Chen (1998) and employ 3 yardsticks to compare empirical performances of the option pricing models: In-sample, out-of-sample pricing errors and hedging. According to the authors, in-sample and out-of-sample errors reflect a model's static performance while hedging errors reflect the model's dynamic performance.

First, in-sample testing shows how consistent the parameters are with the option prices. The authors state that the structural parameters of each model is required to be consistent with implicit in the relevant time-series data. Second, while a model with more parameters will generally give a better fit in-sample, it will not necessarily give a better fit out-of-sample. The model misspecification is measured by the pricing errors in out-of-sample as overfitting may be penalized. Third, hedging errors measures how well a model captures the dynamic properties of option and underlying index prices. In this thesis we will implement delta hedging strategy to determine the forecasting power of the volatility of the underlying index.

To evaulate the pricing errors to compare the performances of the models, we will use the measurements of Kim & Kim (2004) by using

$$\text{MPE} = \frac{1}{n}\sum_{i=1}^{n}(C_i^{model} - C_i^{market})/C_i^{market})$$

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}|(C_i^{model} - C_i^{market})/C_i^{market})|$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|(C_i^{model} - C_i^{market})|$$

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(C_i^{model} - C_i^{market})^2$$

where C_i^{model} is the call price estimated by the model and C_i^{market} is the observed market price of the option. To measure the magnitude of the pricing errors, we use mean absolute errors (MAE) and mean absolute percentage errors (MAPE). Mean percentage errors (MPE) indicate the direction of the pricing errors while mean squared errors (MSE) measures the volatility of errors. This analysis will be based on these 4 measurements, although we will mainly deal with MAPE because the relative comparison is important above all else.

In-sample performance

The in-sample performance of each model is evaluated by comparing market prices with model prices computed by the estimated parameters of the current day. As mentioned earlier, the daily re-estimation of parameters is admittedly potentially inconsistent with constant or slow-changing parameters used to compute option prices. On the other hand, such estimation is useful for indicating market outlook on a daily basis.

Out-of-sample performance

The results of in-sample performance may be a consequence of increasingly larger number of structural parameters and cause overfitting. Including more parameters without improving the structural fit will have the models penalized for out-of-sample pricing. The analysis also evaluates each model's parameter stability over time by analyzing the out-of-sample valuation errors for the next day. To conduct the 1 day ahead out-of-sample analysis, we use the estimated structural parameters from the previous day to price the options today. The pricing errors of the models are then compared to the benchmark BS. If the results show that a model is not able to outperform the benchmark, we will conclude that the model is not appropriate to forecast option prices 1 day ahead.

We follow the same procedure for 3 day ahead out-of-sample pricing where the estimated parameters are used to forecast 3 days ahead. To further check the robustness of the models we can assess if the model parameters are stable through longer time periods and their ability to predict option prices.

Regression analysis

To further analyze the out-of-sample pricing errors, we perform a regression analysis MAPE as the dependent variable and moneyness, maturity and interest rate as the explanatory variables. This approach is adopted by Madan, Carr & Chang (1998) which will let us further infer the degree of errors explained by the well-known biases and will be seen in comparison of the pricing errors produced.

$$\varepsilon_{n.t} = \beta_0 + \beta_1 (S_t/K_n) + \beta_2 (S_t/K_n)^2 + \beta_3 \tau_n + \beta_4 r_t + \eta_n(t)$$

where $\varepsilon_{n.t}$ is denote the 1 or 3 day ahead absolute percentage error on day *t*, S_t/K_n is the moneyness and τ is time to maturity, r_t the risk-free interest rate at time *t.*[50] A negative linear term (β_1) and positive quadratic term (β_2) of moneyness are consistent with the volatility smile and confirms the moneyness bias. Another systematic bias is the maturity bias which is mentioned earlier. In this regression, we expect that if a model has a maturity bias, β_3 should be negative and significant. This means that a model with maturity bias will have a higher MAPE as the days to expiration decreases.

To check for significance of the individual regression coefficients, a *t*-test is conducted. According to Gujarati & Porter (2009, p.115), a test of significance is a procedure by which sample results are used to verify the truth or falsity of a null hypothesis. The null hypothesis is to check if the individual parameters are significant on a certain significance level. The significance level is the probability of rejecting the "true" hypothesis and commonly fixed at the 1, 5 or at the most, 10 percent. An example of a hypothesis can be

$$H_0: \beta_3 = 0$$
$$H_A: \beta_3 \neq 0$$

to check if a model has a maturity bias. The null hypothesis of β_3 being insignificant means the days to maturity do not affect the prediction errors of the model, while the alternative hypothesis states it is significant. The critical *t* value is commonly 1.96 for a two-tailed significance test, which means each of the parameters $|t|$ value will have to exceed the critical value to be statistically significant.[51]

To test the overall significance of the regression coefficients the usual *t*-test cannot be used. According to Fomby, Hill & Johnson (1984, p.37)

…testing a series of single (individual) hypothesis is not equivalent to testing those same hypotheses jointly. The intuitive reason for this is that in a joint test of several hypotheses any single hypothesis is "affected" by the information in the other hypothesis.[52]

[50] Parameter τ is divided by 252 trading days.
[51] The critical value is based on 95% confidence interval.
[52] Thomas B. Fomby, R. Carter Hill & Stanley R. Johnson, *Advanced Econometric Methods*, Springer-Verlag, New York, 1984, p.37.

Since the usual *t*-test cannot be used to test the joint hypothesis that coefficients are zero simultaneously, the *F*-test can be used. The joint hypothesis is

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$
$$H_A: \beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4 \neq 0$$

If the null hypothesis is false, this means the moneyness, maturity and interest rate explains the forecasting errors of a model. The critical *F* value is 2.37 and if exceeded, the null hypothesis is rejected. To compare the regression results relatively, the *F* value can tell us the magnitude of how much the biases explain the absolute percentage pricing errors of a model. In this case, a higher *F* value means a higher degree of the biases affecting the model.

As a penalty of adding regressors to explain the dependent variable, Henry Theil developed the adjusted R^2. On the contrary to R^2, adjusted R^2 only increases if the absolute *t* value of the added variable is greater than 1. In other words, it tells us how much of the absolute percentage errors are explained by the moneyness, maturity and interest rate bias. The derivation of statistics mentioned above has not been included as they are provided by SAS and can be found in *Basic Econometrics* by Gujarati & Porter (2009,p. 493).

Hedging performance

For hedging performance we implement a delta-neutral hedge strategy. The strategy is applied by shorting a call option C_t^i then create a replicating portfolio by investing in $\Delta_s S_t$. To implement this strategy we have to borrow the remaining funds from a risk free asset(r), $\delta = (C_t^i - \Delta_s S_t)$. The next trading day we liquidate this position by buying the same option $C_{t+\Delta}^i$, sell the $\Delta_s S_{t+\Delta t}$ and repay the borrowed money. The loss/gain from this strategy is given by[53]

$$\varepsilon_t = \Delta_S S_{(t+\Delta t)} + \delta e^{r\Delta t} - C_{t+\Delta t}^i$$

[53] The cost of implementing this delta-neutral strategy is the risk-free rate.

For the BS models and PBS, Δ for call options is computed by taking the cumulative density function the normal distribution of d_1 (equation 3.1) which is mentioned in Chapter 3, in mathematical expression, $\Delta Call = N(d_1)$. The difference between the deltas of BS and PBS is that in the PBS the delta is calculated from the fitted volatilities which is different from model to model, while the BS uses the same volatility for every option.

For the Gram-Charlier model, Δ is calculated by first calculating the BS delta, and then adds the effect of kurtosis and skewness to the delta. For a full proof of this approach we refer to Chapter 3. As for Heston and GARCH model, authors of the articles argue that the Δ of their models is the probability P_1.[54]

[54] Chapter 3 explains how to obtain the probability.

Chapter 7 Empirical Findings

In-sample Pricing Performance

DVF 3 shows best performance with respect to MAPE, MAE and MSE for the whole sample period. As the table below shows, it is within our expectations that DVF 3, one of the models with the most parameters, has the best fit in-sample and BS30 has the highest errors throughout the sample. All of the models fit the prices better than BS30 and BS. In-sample results suggest that the BS30 cannot fit the option prices satisfactory as BS30 has nearly 5 times higher MAPE than BS, which indicate the model is not correctly specified. BS does not have the best performance, but makes a good fit considering it uses a single parameter compared to the more advanced models. According to Bates (1996a, 1996c), the parameters should be consistent with in-sample data. Consequently, we find the parameter of BS30 inappropriate and the results are therefore not reported for the rest of the in-sample and out-of-sample pricing performance.[55] BS is then left as the only benchmark.

Table 0.1
In-sample pricing performance

	BS30	BS	GC	DVF 1	DVF 2	DVF 3	Heston	GARCH
MPE	57,72 %	-6,01 %	-4,75 %	1,26 %	0,21 %	-0,12 %	-1,09 %	-4,04 %
MAPE	63,54 %	13,10 %	10,28 %	8,01 %	3,91 %	2,40 %	5,34 %	10,09 %
MAE	6,61	2,90	2,67	1,08	0,48	0,24	1,16	2,13
MSE	74,99	15,83	17,90	3,22	0,79	0,22	3,30	8,81

The table reports in-sample pricing errors. Due to weekly estimation model parameters, GARCH estimated 1045 of 4846 option prices compared to the other models. Denoting $\varepsilon_n = C_n^* - C_n$, where C_n^* is the model price and C_n is the market price. MPE, MAPE, MAE and MSE are calculated using the equations in Chapter 6.

Table 7.2 sorts the pricing errors according to days to maturity using intervals of less than 20 days (short maturity), between 20 and 39 days (medium maturity) and above 40 days (long maturity). GARCH results are only included in the table above due to the limitations we had for estimating GARCH parameters. When sorting errors for maturity (and moneyness later) the lack of observations may be distorted for certain maturity or moneyness categories and hence GARCH is excluded to give a fair interpretation of the results.

[55] All the pricing performance tests were still conducted for BS30. As the model has the highest pricing errors for in-sample and out-of-sample pricing, the results are not reported.

As MAPE stays relatively the same, DVF 3 is still the best model to fit the option prices in all of the maturity categories. It is shown among the DVF functions, having *T*, T^2 and *KT* improves the fitting of the models because DVF 2 and 3 are able to model the short maturity options better than DVF 1, while the difference is less prominent for medium and long maturity. Both Heston and GC perform better than DVF 1 for short maturity options suggesting the parameters of DVF 1 cannot fully describe the prices when the volatility smile is the most prominent. BS has larger pricing errors for short maturities compared to medium and long term maturities in terms of MAPE. This confirms the maturity bias which has been documented empirically where the volatility smile is less prominent for longer expiration dates.

Table 0.2
In-sample pricing performance sorted by option maturity

3 ≤ T < 20

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	-10,56 %	-6,01 %	1,22 %	-0,18 %	-0,25 %	-1,41 %
MAPE	14,52 %	7,74 %	9,48 %	3,34 %	2,42 %	4,68 %
MAE	2,47	1,66	0,67	0,20	0,15	0,83
MSE	11,26	7,80	0,97	0,09	0,05	1,89

20 ≤ T < 40

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	-4,72 %	-2,71 %	1,80 %	0,56 %	-0,29 %	-1,14 %
MAPE	12,18 %	11,29 %	6,69 %	4,38 %	2,29 %	5,80 %
MAE	3,03	2,92	1,01	0,59	0,26	1,32
MSE	17,45	19,55	2,63	0,94	0,20	3,67

40 ≤ T

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	-1,66 %	-5,78 %	0,60 %	0,27 %	0,29 %	-0,60 %
MAPE	12,43 %	12,33 %	7,79 %	4,03 %	2,52 %	5,63 %
MAE	3,29	3,70	1,73	0,72	0,35	1,41
MSE	19,77	29,20	6,99	1,52	0,47	4,69

Table reports in-sample pricing errors sorted by days to maturity. GARCH is not included as the number of observations is significantly lower compared to the other models. T represents the remaining trading days to expiration of the option.

To see the degree of moneyness biased errors for in-sample pricing, table 7.3 sorts the pricing errors for both maturity and moneyness and figure 17 to 19 illustrates MAPE across moneyness for each maturity category. The intervals of moneyness categories expands from less than 0.94 (very deep OTM), 0.94-

0.97 (deep OTM), 0.97-1.00 (OTM) and 1.00-1.03 (ITM), 1.03-1.06 (deep ITM) and above 1.06 (very deep ITM).

Considering short maturity options, all models show moneyness-based errors and exhibit the worst fit for OTM options except for GC where the highest errors are centered ATM options. Figure 17 illustrates MAPE for short term maturity options. MPE show that Heston, GC and BS seem to undervalue across moneyness while the DVF 2 and 3 undervalue very deep OTM and ITM options and overvalue near ATM options. As we move from DOTM to DITM options the errors measured by MAPE steadily decrease for Heston and DVF models. DVF 1 and BS pricing errors do however peak for OTM options. Among the DVF models, there is a significant difference by adding parameter *T* as seen in figure 17. DVF 1 has more than twice as high MAPE compared to DVF for all options below moneyness of 1.03. For very deep OTM options, GC is able to model fit the prices nearly on par with DVF 3. Furthermore, BS is outperformed by all of the models except for GC for very deep ITM options.

Figure 17

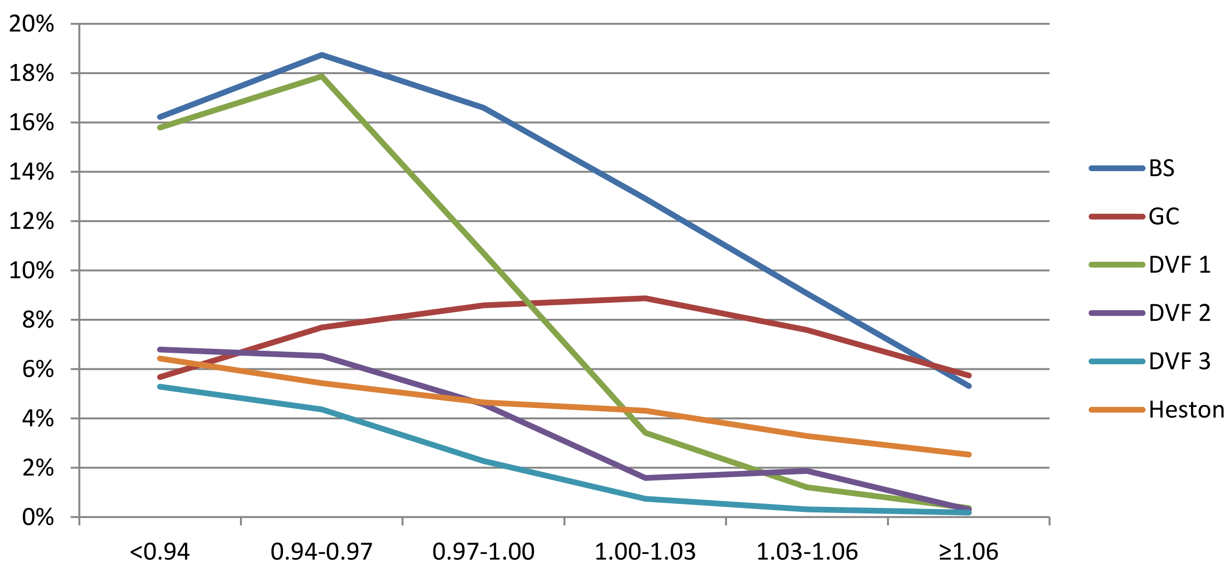


Figure 17 shows in-sample MAPE of the models for short maturity options across moneyness.

Table 0.3
In-sample pricing performance sorted by maturity and moneyness

		3 ≤ T < 20						20 ≤ T <40						40 ≤ T					
	S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**
MPE	*BS*	1,96 %	-14,79 %	-14,93 %	-12,06 %	-9,07 %	-5,31 %	8,74 %	-6,68 %	-10,63 %	-9,97 %	-10,14 %	-8,42 %	5,68 %	-2,24 %	-7,07 %	-6,50 %	-8,38 %	-8,36 %
	GC	-2,94 %	-5,74 %	-6,61 %	-6,96 %	-7,32 %	-5,59 %	13,64 %	-1,38 %	-9,67 %	-12,53 %	-12,33 %	-11,87 %	5,94 %	-5,97 %	-13,64 %	-16,19 %	-17,60 %	-17,29 %
	DVF 1	0,52 %	2,09 %	3,01 %	0,19 %	-0,51 %	-0,08 %	5,56 %	0,66 %	0,02 %	1,32 %	0,97 %	-0,12 %	-0,31 %	0,75 %	0,64 %	2,10 %	1,72 %	2,12 %
	DVF 2	-0,82 %	-0,48 %	-0,04 %	0,24 %	0,38 %	0,03 %	0,06 %	0,86 %	0,39 %	0,30 %	0,22 %	0,11 %	1,36 %	0,49 %	-0,60 %	-0,12 %	0,04 %	0,39 %
	DVF 3	-3,29 %	-0,34 %	1,14 %	0,13 %	-0,18 %	-0,08 %	-0,03 %	-0,89 %	-0,72 %	0,38 %	0,44 %	0,26 %	1,84 %	-0,06 %	-1,15 %	-0,20 %	0,15 %	0,41 %
	Heston	-0,41 %	-0,41 %	-1,23 %	-2,07 %	-2,95 %	-2,31 %	2,24 %	-1,50 %	-2,40 %	-2,20 %	-3,21 %	-3,98 %	0,84 %	-1,01 %	-1,40 %	-0,94 %	-3,32 %	-2,27 %
MAPE	*BS*	16,21 %	18,73 %	16,60 %	12,90 %	9,07 %	5,31 %	14,00 %	9,89 %	13,17 %	12,95 %	11,09 %	8,42 %	12,23 %	12,57 %	13,25 %	11,91 %	10,74 %	9,16 %
	GC	5,68 %	7,68 %	8,58 %	8,86 %	7,58 %	5,74 %	15,62 %	6,33 %	10,51 %	12,90 %	12,33 %	11,87 %	10,18 %	10,32 %	14,23 %	16,49 %	17,60 %	17,29 %
	DVF 1	15,79 %	17,86 %	10,69 %	3,41 %	1,21 %	0,36 %	9,95 %	7,40 %	5,85 %	5,16 %	2,79 %	0,73 %	7,60 %	9,12 %	7,71 %	6,77 %	5,27 %	4,10 %
	DVF 2	6,79 %	6,53 %	4,57 %	1,58 %	1,87 %	0,30 %	5,80 %	5,59 %	2,59 %	2,74 %	0,65 %	0,36 %	5,29 %	3,96 %	3,66 %	0,51 %	1,20 %	0,69 %
	DVF 3	5,29 %	4,37 %	2,28 %	0,73 %	0,31 %	0,19 %	4,44 %	2,69 %	1,24 %	1,13 %	0,78 %	0,40 %	3,55 %	2,81 %	1,84 %	1,23 %	0,75 %	0,59 %
	Heston	6,43 %	5,42 %	4,64 %	4,31 %	3,29 %	2,53 %	6,58 %	5,81 %	5,57 %	5,92 %	4,59 %	3,98 %	5,62 %	5,70 %	6,02 %	5,12 %	5,01 %	3,82 %
MAE	*BS*	0,36	0,89	2,28	3,87	4,59	4,05	0,82	1,41	3,46	5,38	6,89	7,17	1,53	2,30	4,48	5,83	7,62	8,78
	GC	0,14	0,28	1,08	2,56	3,82	4,47	0,87	0,83	2,80	5,37	7,60	10,28	1,08	1,96	4,91	8,00	12,17	16,31
	DVF 1	0,34	0,60	0,90	0,84	0,58	0,26	0,44	0,79	1,24	1,74	1,51	0,62	0,89	1,52	2,21	2,80	3,17	3,63
	DVF 2	0,24	0,17	0,67	0,41	0,85	0,23	0,26	0,47	0,44	0,95	0,37	0,32	0,41	0,38	0,92	0,20	0,77	0,66
	DVF 3	0,09	0,12	0,19	0,18	0,15	0,14	0,16	0,22	0,24	0,38	0,43	0,34	0,26	0,30	0,43	0,50	0,45	0,53
	Heston	0,15	0,25	0,61	1,26	1,68	1,97	0,41	0,73	1,37	2,28	2,81	3,46	0,68	1,01	1,93	2,27	3,43	3,60
MSE	*BS*	0,26	1,62	8,02	19,26	26,45	22,38	1,40	3,80	16,61	35,78	54,10	60,32	5,24	8,39	26,24	44,98	73,09	98,86
	GC	0,04	0,17	2,37	10,79	23,25	29,95	1,43	1,38	11,71	37,77	69,51	121,57	1,95	6,50	32,29	75,23	164,08	282,33
	DVF 1	0,26	0,82	1,36	1,54	0,93	0,15	0,40	1,15	2,59	6,44	7,25	0,86	1,89	4,44	9,00	16,43	18,26	27,24
	DVF 2	0,18	0,05	1,09	0,60	2,08	0,22	0,19	0,56	0,58	2,40	0,68	0,33	0,39	0,44	2,16	0,11	1,95	0,78
	DVF 3	0,01	0,02	0,06	0,06	0,05	0,04	0,05	0,10	0,13	0,42	0,57	0,39	0,18	0,31	0,89	0,73	0,60	0,51
	Heston	0,05	0,17	0,84	2,87	4,90	6,36	0,40	0,99	3,05	7,30	10,89	15,39	1,18	2,01	7,85	8,67	16,65	15,19

For medium maturity options, figure 18 graphically illustrates MAPE for all models. BS is still outperformed except by GC for very deep OTM options and moneyness above 1.03. However, all of the DVF models fit the prices better than Heston for moneyness above 1.00.

Figure 18 2

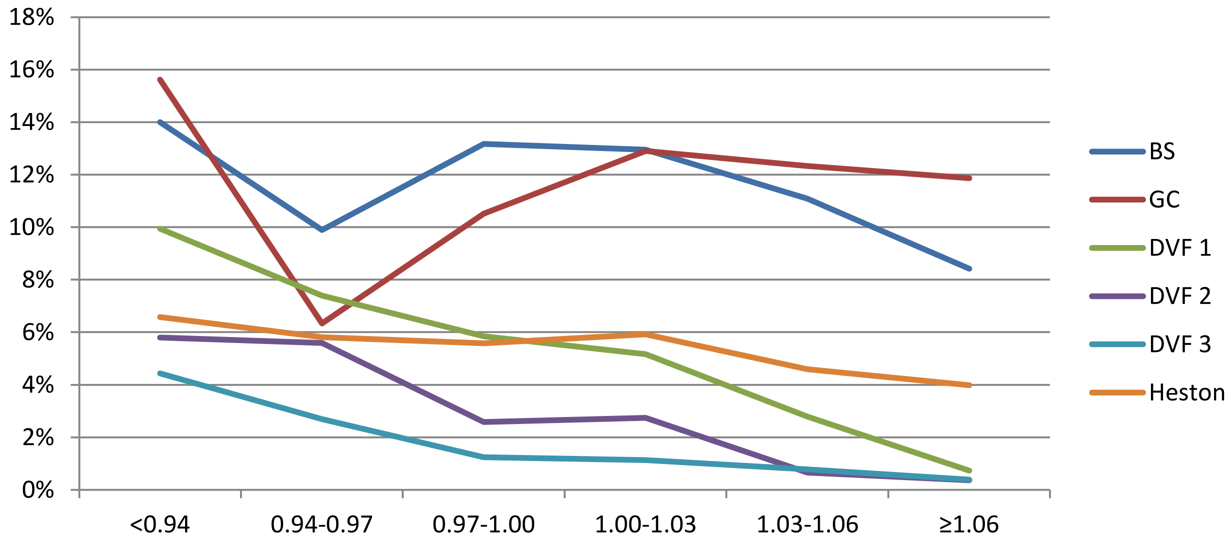


Figure 18 shows in-sample 1 day ahead MAPE of the models for medium maturity options across moneyness.

For long term maturity, the pattern of decreasing MAPE across moneyness remains the same for Heston and DVF models and the ranking remains the same. The graphs illustrates that the pricing errors diminish as expiration increases.

Figure 19

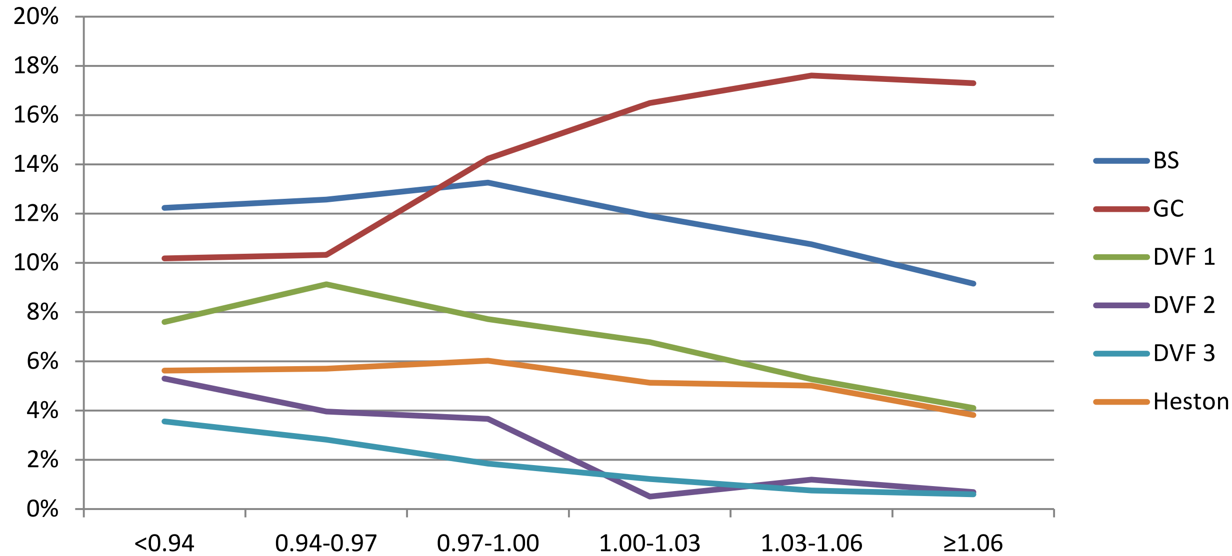


Figure 19 shows in-sample 1 day ahead MAPE of the models for long maturity options across moneyness.

To sum it up, DVF 3 shows the best in-sample performance and BS30 has the worst performance. We find that by adding more parameters to describe the volatility smile decreases the pricing errors significantly as all of the models outperform the benchmarks on average. Among the DVF models, including parameters *T*, T^2 and *KT* significantly increases the models ability to fit the option prices. Meanwhile for GC, the percentage loss function is not able to improve pricing errors for OTM and DOTM options compared to BS, and it is more prominent for medium and long maturity options. For in-sample pricing, all models are unable to fit the prices perfectly where the worst fit for OTM options, which is to be expected. Nonetheless, the models significantly improve the in-sample pricing errors and are able to model the volatility smile compared to BS. The results therefore suggest that on average, the models (except for BS 30) included in the thesis are consistent with option prices in our sample period.

Out-of-sample Pricing Performance

1 day ahead results

For all of the models, the pricing errors worsen when shifting from in-sample pricing to 1 day ahead out-of-sample pricing. Looking at both table 7.1 and 7.4, MAPE increases for all models as expected. The last row of table 7.4 shows how many times MAPE multiplies from in-sample to 1 day ahead out-of-sample. The multiples indicate the degree of penalization of including more parameters for a better in-sample fit compared to out-of-sample. The multiplier shows the in-sample fit does not necessarily mean better prediction of option prices the following day.

Table 0.4
1 day ahead out-of-sample pricing errors

	BS	GC	DVF 1	DVF 2	DVF 3	Heston	GARCH
MPE	-4,41 %	-3,82 %	2,25 %	1,24 %	1,41 %	0,47 %	-2,97 %
MAPE	17,13 %	17,23 %	12,16 %	9,83 %	9,61 %	13,40 %	18,46 %
MAE	3,07	3,18	1,61	1,29	1,19	2,00	2,95
MSE	17,31	21,70	5,56	3,58	2,76	8,31	16,72
Multiplier	1,31	1,68	1,52	2,52	4,01	2,51	1,83

The table reports 1 day ahead out-of-sample pricing errors. Multiplier is calculated by taking each of models 1 day ahead MAPE divided by in-sample MAPE to indicate the degree of overfitting. Due to weekly estimation model parameters, GARCH estimated 2059 of 4600 option prices compared to the other models. Denoting $\varepsilon_n = C_n^* - C_n$, where C_n^* is the model price and C_n is the market price. MPE, MAPE, MAE and MSE are calculated using the equations in Chapter 6. For MPE and MAPE, ε_n is divided by the option market price today.

For the whole sample period, the DVF models perform the best followed by Heston, BS, GC and GARCH. Comparing the DVF models, they all overprice the options on average and it seems that including the *T* parameter increases the forecasting ability of the model, although DVF 3 only marginally performs better than DVF 2 with the quadratic parameter of *T*. As mentioned in Chapter 3, Dumas et al. (1998) found that prediction errors grow larger with DVF functions were less parsimonious. Our results are on the contrary of the authors results on S&P 500 as T, T^2 and *KT* decrease pricing errors for 1 day ahead.

Between Heston and PBS, the ranking is changed compared to the in-sample ranking. Heston which has a better in-sample fit compared to DVF 1, now falls behind all of the DVF models in forecasting the prices 1 day ahead. In contrast of the in-sample results, both Heston and DVF 3 now overvalue the option prices as shown by MPE.

Note that GARCH has the highest MAPE of all of the models, although with merely half of the observations. Considering BS is more accurate in predicting the option prices the next day, it is surprising GC and GARCH perform worse than the benchmark model for the whole sample period. This indicates both GC and GARCH are not appropriate for forecasting 1 day ahead as they are outperformed by the benchmark. Since DVF models and Heston produce the lower pricing errors in terms of MAPE, MAE and MSE, we find the models superior to BS in forecasting 1 day ahead.

Table 0.5
1 day ahead pricing errors sorted by maturity

3 ≤ T < 20

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	-9,03 %	-6,19 %	2,59 %	1,29 %	1,62 %	0,39 %
MAPE	19,34 %	18,21 %	14,32 %	11,57 %	11,92 %	15,26 %
MAE	2,62	2,23	1,26	1,12	1,12	1,55
MSE	12,64	10,92	2,99	2,50	2,45	4,66

20 ≤ T < 40

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	-2,94 %	-0,93 %	2,71 %	1,32 %	1,18 %	0,13 %
MAPE	16,23 %	17,04 %	10,88 %	9,12 %	8,31 %	12,78 %
MAE	3,23	3,40	1,60	1,32	1,18	2,11
MSE	18,75	22,93	5,13	3,36	2,62	8,23

40 ≤ T

	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	0,29 %	-4,27 %	1,11 %	1,08 %	1,41 %	1,04 %
MAPE	15,13 %	16,09 %	10,74 %	8,26 %	8,03 %	11,56 %
MAE	3,50	4,24	2,15	1,49	1,29	2,52
MSE	22,10	35,59	9,86	5,43	3,41	13,66

Table reports 1 day ahead out-of-sample pricing errors sorted by days to maturity. GARCH is not included as the number of observations is significantly lower compared to the other models. T represents the remaining trading days to expiration of the option.

The table above sorts the errors according to maturity for 1 day ahead pricing errors. For short maturity, all of the models actually outperform BS. Surprisingly, DVF 2 has lower MAPE than DVF 3 indicating that T^2 does not improve forecasting ability for short term maturity options. MPE tells us DVF 3 on average, overprice the short maturity options more than DVF 2. This is unexpected as T^2 is added to better fit the volatility smile across moneyness and time to expiration. However, for both medium and long term categories, DVF 3 outperforms all of the models while GC has higher MAPE, MAE and MSE than BS. Since the number of option prices are higher for medium and long maturity combined than for short maturity, in overall the results confirms the findings of GC inferior to BS.

Figure 20

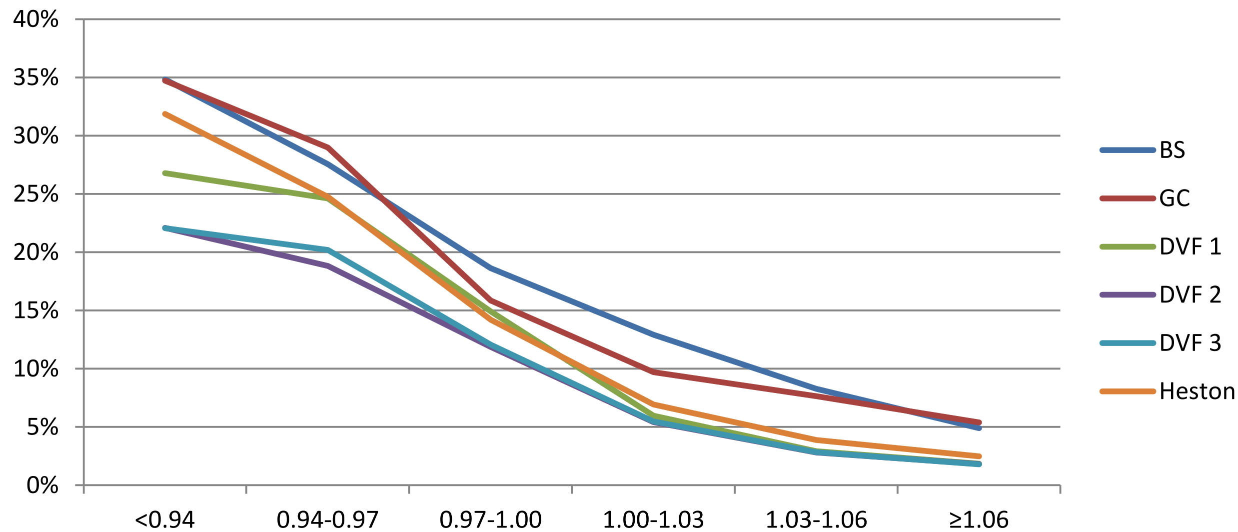


Figure 20 illustrates 1 day ahead out-of-sample MAPE of the models for short maturity options across moneyness.

Table 7.6 reports the pricing errors sorted by maturity and moneyness. Figure 20 illustrates MAPE of the models for short maturity options and shows DVF 2 has lower MAPE in 4 of 6 moneyness categories than DVF 3. As mentioned earlier, this suggests parameter T^2 does not improve the structural fit and penalize the model with higher errors compared to DVF 2 for DOTM and DITM for options with less than 20 days to maturity. The pattern of highest MAPE for very deep OTM options and steadily decreases across moneyness is common for all of the models. This is in contrast of the in-sample results for BS, GC and DVF 1 for short maturity options. The pattern of the pricing errors indicates the moneyness bias, where the largest errors are concentrated at DOTM options. However, this can also be due to the fact that DOTM option prices relatively are lower than DITM options.[56]

[56] See table 4.3 for option prices sorted by maturity and moneyness.

Figure 21

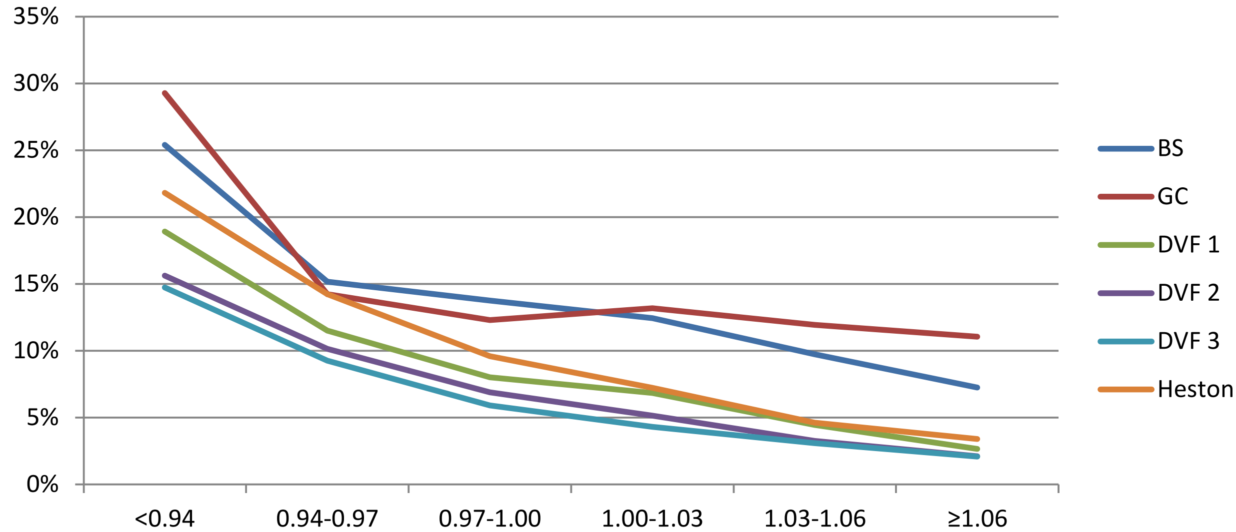


Figure 21 illustrates 1 day ahead out-of-sample MAPE of the models for medium maturity options across moneyness.

For medium maturity options, DVF 3 has the lowest MAPE, MAE and MSE for across moneyness followed by DVF 2, DVF 1 and Heston. Those models still outperforms BS in every moneyness category. The difference between DVF 1 and Heston is now larger compared to options with shorter expiration date. Considering options with long expiration date, the difference diminishes between the two of them as figure 22 displays. Note that Heston has the least MAPE, MAE and MSE for very DITM options while DVF 2 outperforms DVF 3 for very DOTM options.

Table 0.6
1 day ahead out-of-sample pricing errors sorted by maturity and moneyness

		3 ≤ T < 20						20 ≤ T < 40						40 ≤ T					
	S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**
MPE	*BS*	4,12 %	-11,94 %	-13,29 %	-11,63 %	-8,28 %	-4,88 %	12,01 %	-5,51 %	-9,32 %	-9,04 %	-8,53 %	-7,26 %	7,96 %	0,70 %	-5,75 %	-6,25 %	-7,80 %	-7,28 %
	GC	-8,27 %	-6,56 %	-4,65 %	-6,33 %	-6,55 %	-5,11 %	18,35 %	-0,52 %	-8,89 %	-12,13 %	-11,76 %	-11,02 %	8,33 %	-3,75 %	-13,35 %	-16,10 %	-18,41 %	-15,21 %
	DVF 1	1,83 %	4,82 %	5,07 %	0,65 %	-0,40 %	-0,13 %	6,86 %	1,58 %	0,99 %	1,81 %	1,50 %	0,04 %	-0,71 %	2,68 %	1,46 %	1,98 %	1,43 %	-1,37 %
	DVF 2	-2,96 %	2,70 %	4,13 %	0,58 %	0,07 %	-0,11 %	3,18 %	0,64 %	0,61 %	0,81 %	1,10 %	0,47 %	1,47 %	1,92 %	-0,13 %	0,57 %	0,11 %	4,16 %
	DVF 3	-2,84 %	3,78 %	4,50 %	0,60 %	0,05 %	-0,15 %	3,13 %	0,34 %	0,40 %	0,74 %	1,16 %	0,58 %	2,76 %	1,81 %	-0,22 %	0,17 %	0,54 %	2,35 %
	Heston	1,33 %	3,44 %	1,02 %	-1,71 %	-2,19 %	-1,84 %	4,88 %	-0,93 %	-1,75 %	-1,43 %	-1,59 %	-2,41 %	2,79 %	1,30 %	-0,65 %	0,42 %	-2,47 %	-1,56 %
MAPE	*BS*	34,83 %	27,55 %	18,63 %	12,92 %	8,28 %	4,88 %	25,40 %	15,17 %	13,76 %	12,45 %	9,73 %	7,26 %	18,33 %	15,22 %	13,65 %	11,36 %	10,47 %	8,25 %
	GC	34,73 %	28,99 %	15,84 %	9,70 %	7,63 %	5,38 %	29,27 %	14,24 %	12,29 %	13,17 %	11,93 %	11,04 %	17,99 %	14,10 %	15,14 %	16,46 %	18,41 %	15,21 %
	DVF 1	26,79 %	24,60 %	14,91 %	5,96 %	2,90 %	1,83 %	18,93 %	11,50 %	8,01 %	6,84 %	4,45 %	2,65 %	12,29 %	12,17 %	9,39 %	7,61 %	6,67 %	3,92 %
	DVF 2	22,07 %	18,83 %	11,86 %	5,42 %	2,81 %	1,81 %	15,62 %	10,15 %	6,89 %	5,14 %	3,25 %	2,11 %	11,56 %	8,60 %	5,68 %	5,16 %	4,47 %	4,99 %
	DVF 3	22,06 %	20,19 %	12,06 %	5,47 %	2,83 %	1,79 %	14,73 %	9,26 %	5,92 %	4,32 %	3,09 %	2,08 %	12,23 %	8,56 %	4,91 %	3,58 %	3,28 %	3,63 %
	Heston	31,86 %	24,75 %	14,20 %	6,93 %	3,89 %	2,47 %	21,82 %	14,23 %	9,60 %	7,23 %	4,63 %	3,41 %	16,11 %	11,60 %	8,54 %	7,40 %	6,63 %	3,36 %
MAE	*BS*	0,87	1,32	2,52	3,88	4,19	3,73	1,48	2,11	3,60	5,10	6,06	6,20	2,12	2,62	4,46	5,63	7,29	7,91
	GC	0,92	1,30	1,91	2,81	3,86	4,21	1,66	1,87	3,24	5,45	7,38	9,51	1,96	2,66	5,27	8,00	12,83	14,24
	DVF 1	0,65	0,96	1,45	1,59	1,42	1,37	0,88	1,26	1,77	2,42	2,54	2,24	1,27	1,84	2,63	3,31	4,19	3,60
	DVF 2	0,51	0,78	1,25	1,45	1,38	1,36	0,78	1,13	1,50	1,82	1,90	1,79	1,06	1,20	1,66	2,24	2,82	4,67
	DVF 3	0,52	0,80	1,25	1,46	1,39	1,34	0,73	0,98	1,29	1,55	1,82	1,78	1,03	1,16	1,44	1,57	2,06	3,37
	Heston	0,83	1,14	1,63	1,96	1,97	1,92	1,33	1,87	2,37	2,75	2,81	2,98	1,96	2,15	2,80	3,56	4,50	3,20
MSE	*BS*	1,55	3,66	11,05	21,57	24,37	19,32	5,09	8,19	19,95	34,97	46,17	46,28	10,33	12,03	27,98	42,64	71,13	78,91
	GC	1,62	3,33	6,83	14,19	26,72	29,25	6,20	6,79	16,97	40,53	71,70	108,44	8,18	11,85	40,45	81,63	190,97	222,49
	DVF 1	0,80	1,82	3,59	4,61	3,56	2,82	1,48	2,73	5,26	10,09	12,82	7,55	3,53	6,28	11,69	21,83	31,95	21,63
	DVF 2	0,50	1,34	2,93	3,92	3,23	2,82	1,13	2,20	3,92	5,96	6,44	4,60	2,21	3,09	5,46	10,94	15,04	60,17
	DVF 3	0,49	1,32	2,82	3,86	3,26	2,74	1,01	1,69	2,92	4,37	5,37	4,24	1,80	2,62	3,98	4,71	8,02	26,92
	Heston	1,44	2,74	4,90	6,57	6,87	6,31	4,06	6,45	9,35	12,03	13,54	13,43	8,84	8,56	15,04	26,82	36,76	16,74

Figure 22

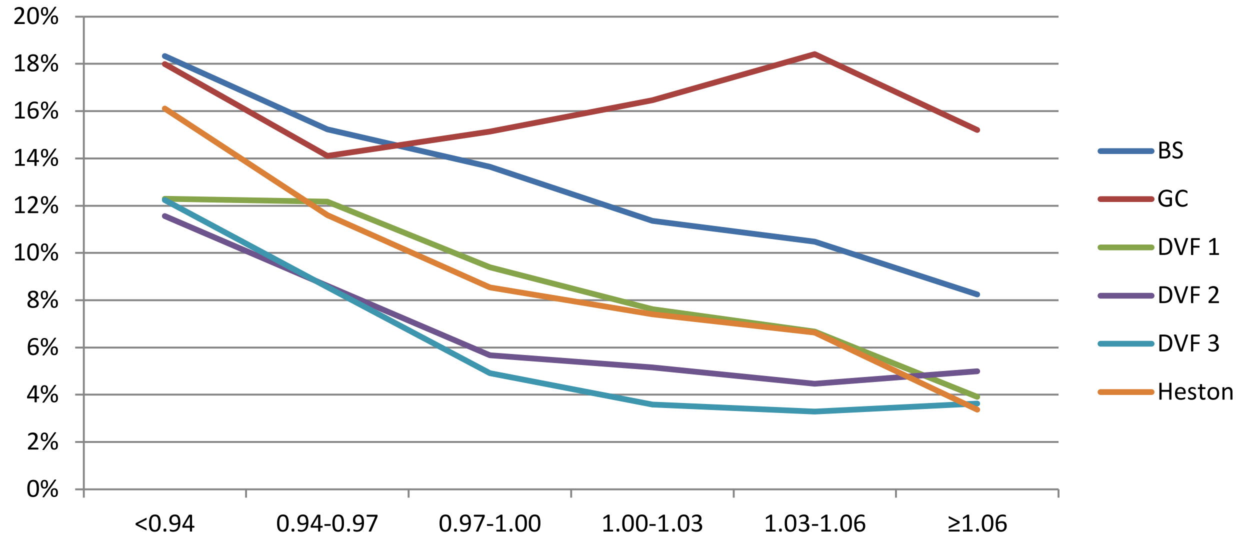


Figure 22 illustrates 1 day ahead out-of-sample MAPE of the models for long maturity options across moneyness.

Across all of the maturity categories, the pricing errors decreases for all of the models as the maturity increases, except for BS30. This indicates the models suffer from a maturity bias and the fact that MAPE decreases across moneyness confirms a moneyness bias. These biases will be further examined in the regression analysis.

3 day ahead results

The table below reports the 3 day ahead pricing errors. For all of the models, MAPE, MAE and MSE increase the longer period of time ahead each model tries to forecast. PBS ranks 1st again followed by Heston, however the difference between the DVF models is diminishing compared to the results from 1 day ahead. GC and GARCH still have higher errors than BS and we can therefore suggest when it comes to predict option prices 1 and 3 day ahead, the models are inferior to the classic BS. We find that DVF models and Heston overvalues the options on average while BS and GC undervalues.

Table 0.7
3 day ahead out-of-sample pricing errors

	BS	GC	DVF 1	DVF 2	DVF 3	Heston	GARCH
MPE	-3,32 %	-3,35 %	2,79 %	1,81 %	1,89 %	1,20 %	-1,90 %
MAPE	22,86 %	23,89 %	16,97 %	15,19 %	15,15 %	21,07 %	24,10 %
MAE	3,50	3,77	2,13	1,88	1,81	2,94	3,73
MSE	23,13	29,23	9,66	7,49	7,01	18,45	26,74
Multiplier	1,75	2,32	2,12	3,89	6,31	3,94	2,39

The table reports 3 day ahead out-of-sample pricing errors. Multiplier is calculated by taking each of models 3 day ahead MAPE divided by in-sample MAPE to indicate the degree of overfitting. Due to weekly estimation model parameters, GARCH estimated 1688 of 4074 option prices compared to the other models. Denoting $\varepsilon_n = C_n^* - C_n$, where C_n^* is the model price and C_n is the market price. MPE, MAPE, MAE and MSE are calculated using the equations on p.60. For MPE and MAPE, ε_n is divided by the option market price today.

When sorting for maturity, the ranking remains the same for the DVF models as the ranking for 1 day ahead results. On average, both DVF models and Heston overvalues the option price for all maturities (except Heston for medium maturities) where the overvaluing is prominent for short maturity options. This suggests that the parameters that accounts for the volatility smile, overestimate the smile or smirk when forecasting a longer time period ahead. For short maturity options, DVF 2 is the superior model followed by DVF 3, DVF 1 and Heston. Furthermore, considering medium and long maturity, DVF 3 is now back on top followed by DVF 2, DVF 1 and Heston in terms of MAPE, MAE and MSE. The ranking is consistent with the 1 day ahead results and confirms that T^2 is excessive for short term options for both 1 and 3 day ahead pricing performance. On the other hand, it improves the forecasting accuracy for medium and long maturity options on average.

Table 0.8

3 ≤ T < 20

	BS	GC	DVF 1	DVF 2	DVF 3	Heston
MPE	-6,86 %	-5,92 %	3,78 %	2,36 %	2,85 %	2,63 %
MAPE	26,12 %	26,69 %	19,91 %	18,02 %	19,09 %	24,21 %
MAE	2,91	2,81	1,69	1,62	1,64	2,24
MSE	16,06	16,12	5,54	5,51	5,66	10,05

20 ≤ T < 40

	BS	GC	DVF 1	DVF 2	DVF 3	Heston
MPE	-2,57 %	-0,72 %	2,51 %	1,30 %	0,99 %	-0,22 %
MAPE	21,54 %	22,77 %	15,50 %	14,15 %	13,14 %	20,04 %
MAE	3,77	4,08	2,25	2,02	1,89	3,26
MSE	26,74	32,70	10,55	8,31	7,42	20,46

40 ≤ T						
	BS	**GC**	**DVF 1**	**DVF 2**	**DVF 3**	**Heston**
MPE	1,53 %	-2,90 %	1,54 %	1,62 %	1,59 %	0,89 %
MAPE	19,35 %	20,85 %	14,21 %	11,97 %	11,49 %	17,32 %
MAE	4,08	4,92	2,69	2,10	1,97	3,65
MSE	29,71	46,13	15,27	9,61	8,65	29,60

Table reports 3 day ahead out-of-sample pricing errors sorted by days to maturity. GARCH is not included as the number of observations is significantly lower compared to the other models. T represents the remaining trading days to expiration of the option.

Figure 23

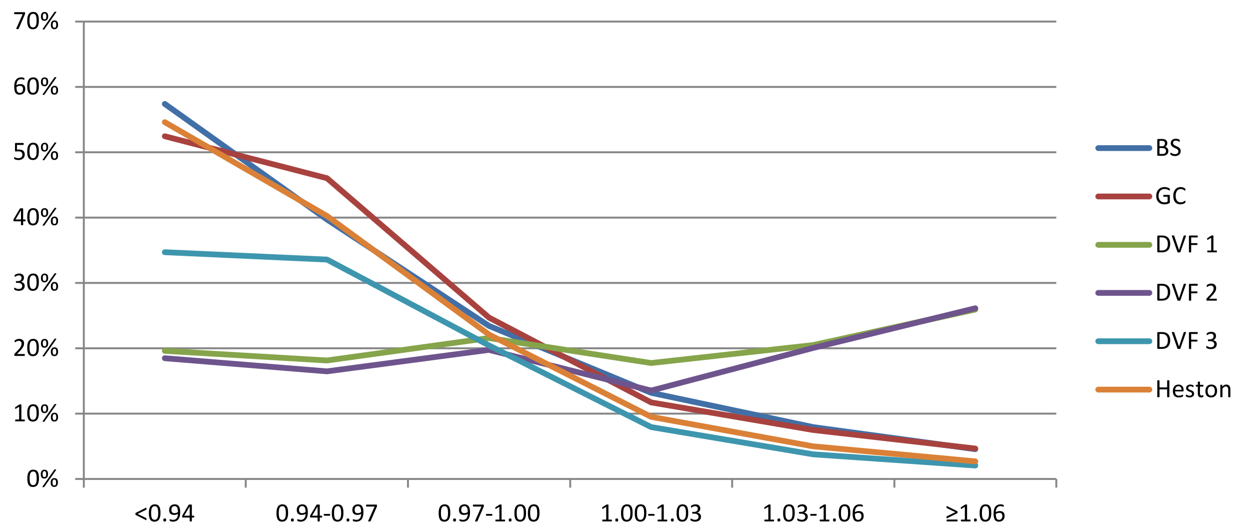


Figure 23 illustrates out-of-sample 3 day ahead MAPE of the models for short maturity options across moneyness.

The figure above displays the 3 day ahead MAPE while table 7.9 reports the errors sorted for moneyness and maturity. As the figure shows, the pattern is different from the 1 day ahead errors. Both DVF 1 and DVF 2 have a steady MAPE near the 20% mark, while BS, GC, DVF 3 and Heston have the familiar declining MAPE across moneyness. DVF 2 and DVF 1 outperform DVF 3 in terms of MAPE for options with moneyness up to 1.00. This means T^2 is excessive for all options that are OTM, but improves the pricing errors for options above 1.00. Heston is on par with BS for DOTM options up to 1.00.

Table 0.9
3 day ahead out-of-sample pricing performance sorted by maturity and moneyness

		3 ≤ T < 20						20 ≤ T <40						40 ≤ T					
	S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**
MPE	*BS*	10,01 %	-8,05 %	-11,22 %	-10,84 %	-7,68 %	-4,52 %	11,37 %	-6,20 %	-8,92 %	-8,35 %	-7,81 %	-4,55 %	10,36 %	0,73 %	-4,60 %	-5,80 %	-8,13 %	-5,93 %
	GC	-16,83 %	-3,32 %	-3,16 %	-4,93 %	-4,95 %	-3,95 %	17,64 %	-1,42 %	-8,76 %	-11,77 %	-11,99 %	-10,69 %	11,16 %	-3,80 %	-12,41 %	-16,12 %	-18,89 %	-13,49 %
	DVF 1	0,04 %	-0,53 %	15,12 %	8,38 %	-9,54 %	-5,85 %	3,45 %	3,77 %	3,77 %	2,72 %	-8,51 %	-1,20 %	0,92 %	2,37 %	6,84 %	-1,63 %	-18,15 %	-8,14 %
	DVF 2	-2,67 %	0,27 %	10,93 %	5,39 %	-3,48 %	-10,92 %	1,63 %	3,08 %	2,44 %	0,51 %	-6,52 %	-2,56 %	2,14 %	3,45 %	4,77 %	-4,59 %	-11,71 %	-11,18 %
	DVF 3	-5,23 %	7,99 %	7,00 %	1,07 %	-0,45 %	0,02 %	2,79 %	0,33 %	-0,08 %	0,71 %	1,18 %	0,79 %	4,58 %	0,22 %	0,31 %	-0,54 %	-1,84 %	3,28 %
	Heston	4,41 %	8,78 %	3,56 %	-1,04 %	-1,89 %	-1,57 %	3,80 %	-2,07 %	-1,94 %	-1,57 %	-1,66 %	-1,05 %	2,61 %	0,67 %	0,70 %	-0,89 %	-3,08 %	-0,85 %
MAPE	*BS*	57,39 %	39,72 %	23,42 %	13,19 %	7,93 %	4,57 %	37,24 %	21,38 %	16,35 %	13,18 %	9,20 %	5,14 %	26,28 %	19,88 %	15,59 %	11,17 %	9,44 %	6,97 %
	GC	52,43 %	46,04 %	24,70 %	11,73 %	7,52 %	4,65 %	41,59 %	20,98 %	15,41 %	13,71 %	12,42 %	10,87 %	27,08 %	18,91 %	16,94 %	16,69 %	18,91 %	13,49 %
	DVF 1	19,60 %	18,12 %	21,57 %	17,76 %	20,49 %	25,92 %	16,00 %	10,11 %	11,07 %	26,23 %	20,66 %	19,48 %	10,99 %	8,44 %	19,04 %	24,70 %	21,93 %	10,59 %
	DVF 2	18,50 %	16,47 %	19,77 %	13,54 %	20,07 %	26,13 %	15,05 %	9,19 %	9,11 %	23,84 %	21,18 %	17,18 %	8,71 %	7,54 %	16,52 %	20,33 %	18,22 %	11,87 %
	DVF 3	34,72 %	33,58 %	20,45 %	7,96 %	3,78 %	2,05 %	23,16 %	14,11 %	9,93 %	6,65 %	3,51 %	3,40 %	16,97 %	11,23 %	7,94 %	5,73 %	5,64 %	3,87 %
	Heston	54,62 %	40,22 %	22,10 %	9,51 %	5,03 %	2,69 %	34,93 %	21,53 %	14,77 %	10,51 %	6,46 %	4,73 %	24,79 %	17,47 %	13,20 %	9,06 %	9,34 %	3,27 %
MAE	*BS*	1,39	1,91	3,03	3,93	3,97	3,47	2,26	3,06	4,28	5,50	5,78	4,37	2,93	3,70	4,90	5,59	6,76	6,47
	GC	1,44	2,10	3,01	3,44	3,74	3,58	2,50	3,00	4,10	5,75	7,63	9,31	3,00	3,77	5,67	8,07	13,07	12,31
	DVF 1	1,95	2,18	1,60	1,18	1,78	1,49	2,79	2,09	1,84	2,54	2,01	1,00	2,38	2,26	3,28	3,39	3,00	1,56
	DVF 2	1,82	2,09	1,58	1,01	1,71	1,69	2,59	1,90	1,57	2,16	1,99	0,88	1,81	1,99	2,54	2,51	1,87	1,16
	DVF 3	0,81	1,24	1,95	2,13	1,83	1,52	1,25	1,65	2,23	2,50	2,09	2,80	1,52	1,77	2,27	2,49	3,53	3,50
	Heston	1,37	1,87	2,54	2,72	2,48	2,03	2,98	2,99	3,68	4,23	4,02	4,03	2,92	3,43	4,11	4,47	6,80	3,02
MSE	*BS*	3,76	7,79	17,40	25,27	23,95	17,10	11,18	19,01	30,76	47,21	50,21	24,56	17,71	24,31	37,62	47,75	62,04	58,70
	GC	3,99	9,38	17,77	20,91	26,53	22,89	12,82	18,79	29,78	53,84	87,21	99,10	17,54	25,44	52,13	92,50	221,90	184,67
	DVF 1	8,47	7,40	4,29	2,32	7,16	5,64	16,46	7,34	6,74	14,22	8,18	1,92	12,25	8,70	21,40	24,14	25,61	5,96
	DVF 2	7,67	7,61	4,59	1,87	6,43	7,79	13,50	6,28	4,89	9,16	8,88	1,57	7,41	7,32	13,68	14,54	6,86	2,49
	DVF 3	1,30	3,27	7,62	8,80	6,29	3,48	3,25	5,35	9,66	11,98	8,11	16,30	4,44	6,09	10,28	15,01	31,27	22,10
	Heston	3,69	7,54	13,14	13,39	11,19	6,63	10,82	18,27	24,13	30,09	27,93	21,52	20,49	21,96	33,83	43,11	102,93	10,37

Figure 24

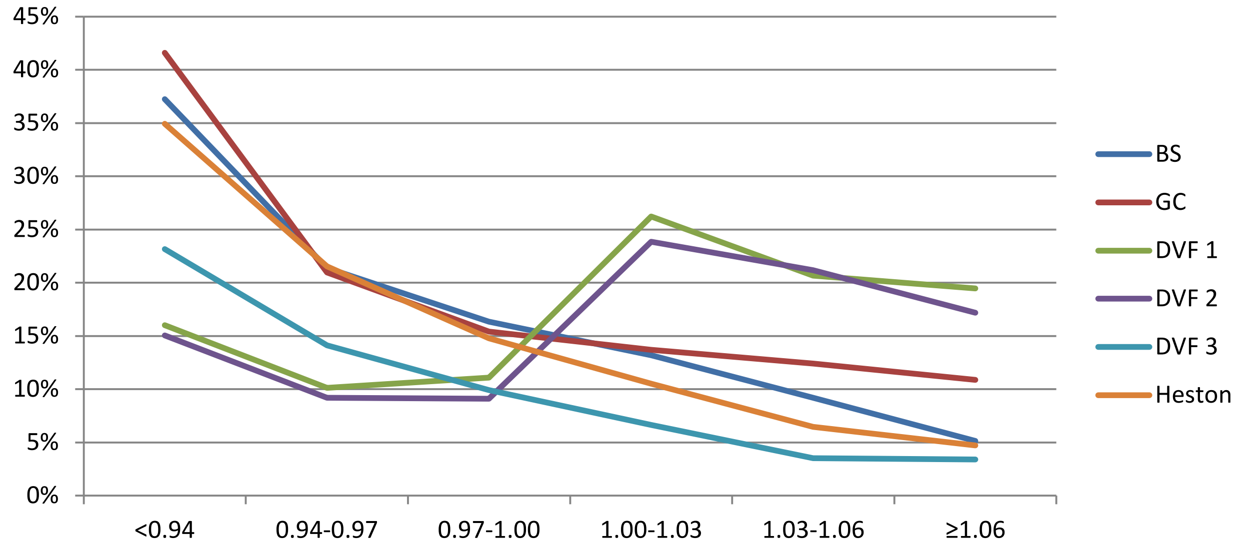


Figure 24 illustrates out-of-sample 3 day ahead MAPE of the models for medium maturity options across moneyness.

For medium and long maturity options, DVF 1 and DVF 2 still have MAPE pattern that stands out from the other models. Both models outperform DVF 3 for options up to 0.97 moneyness. This implies that when forecasting option prices a longer time period ahead, T^2 increases pricing errors for options up to 0.97 in moneyness and reduces pricing errors for options above 0.97. The findings are consistent across maturity for 3 day ahead forecasting. We also see that Heston outperforms all models for options above 1.06 moneyness and long maturity which is the same for 1 day ahead.

Figure 25

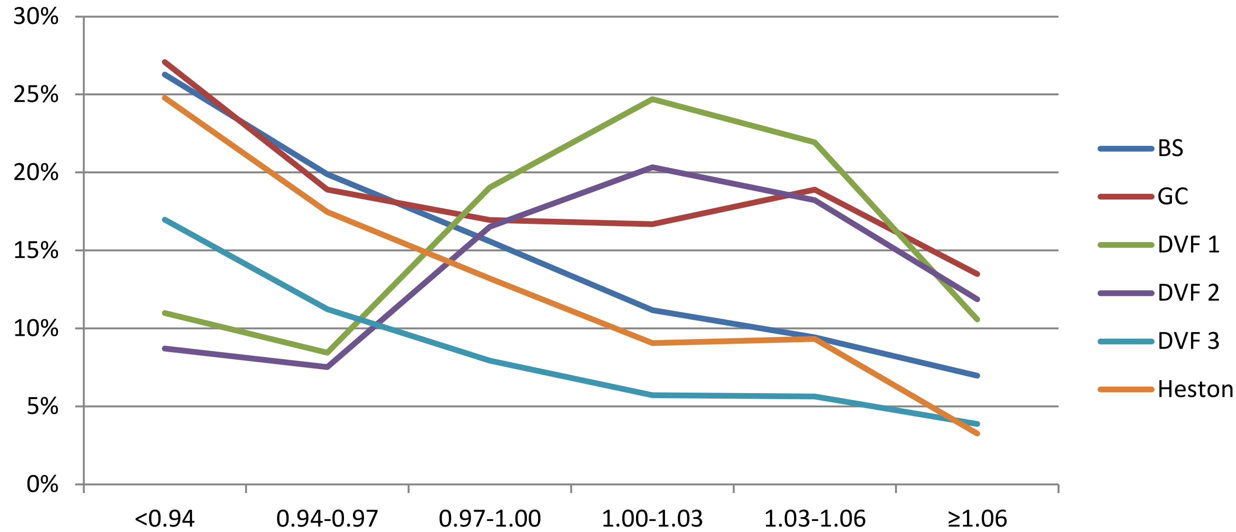


Figure 25 illustrates 3 day out-of-sample ahead MAPE of the models for long maturity options across moneyness.

In conclusion, the results of 3 day ahead out-of-sample have shown us that only DVF models and Heston are able to outperform the benchmark. Among the DVF models, T^2 is found to excessive for options with moneyness up to ATM options, but improves the pricing errors for all of the short maturity ITM options. For medium and long maturity options, DVF 2 has the lowest MAPE up to 0.97 moneyness before T^2 starts improving the pricing errors of DVF 3. On the other hand, Heston is the best performer for very deep ITM options with long maturity.

Regression results

As the Heston and the DVF models are the only models to outperform the benchmark, the regression results for the other models are not reported (see appendix). The results of regression analysis are presented in the table below. For all of the models we observe a high degree of predictability in pricing errors with a high adjusted R^2 which tells us moneyness, maturity and interest rate biases are systematically related to MAPE. The errors are also negatively related to the interest rates. The adjusted R^2 shows us how much of MAPE is explained by the explanatory variables. The F statistic reported shows that we must reject the hypothesis that the biases do not explain the pricing errors for all models.

The Heston model appears to be the most affected by the biases as the each of the coefficients are more significant than the other models. Note that DVF 1 does not have a significant quadratic moneyness variable on a 5% level, but still have a higher MAPE than DVF 2 and DVF 3 (table 7.10).

Table 0.10
Regression coefficients for 1 day ahead pricing errors

Coefficients	DVF 1	DVF 2	DVF 3	Heston
β_0	3.81***	5.16***	5.83***	7.53***
β_1	-5.09***	-8.22***	-9.45***	-12.69***
β_2	1.86*	3.56***	4.16***	5.60***
β_3	-0.52***	-0.45***	-0.51***	-0.53***
β_4	-17.70***	-15.43***	-17.00***	-11.88***
Adj. R^2	0.1633	0.1687	0.1769	0.2058
F	225.39	234.35	248.05	298.96

, **, *, Indicates significance at 10%, 5% and 1%, respectively. The regression results is based on the equation $\varepsilon_{n.t} = \beta_0 + \beta_1 (S_t/K_n) + \beta_2 (S_t/K_n)^2 + \beta_3 \tau_n + \beta_4 r_t + \eta_n(t)$ using OLS to analyze the 1 day ahead out-of-sample pricing errors.*

DVF 1 shows the best performance in the regression analysis because of the lowest F values and adjusted R^2. In other words, 16.87% of absolute percentage errors are explained by moneyness, maturity and interest rate biases. Based on these results, we can infer that even though a model is less affected these biases, it does not necessarily mean that the model's ability to forecast 1 day ahead improves. Despite the fact that DVF 3 has the highest adjusted R^2 and F value among the DVF models, we found earlier that DVF 3 in general is the most suited model to forecast 1 day ahead due to the lowest pricing errors.

These findings imply that if a model is less affected by moneyness, maturity and interest rate biases do not necessarily improve the 1 day ahead forecasting ability of the model as there are more errors that may stem from parameter misspecification.

Table 0.11
Regression coefficients for 3 day ahead pricing errors

Coefficients	DVF 1	DVF 2	DVF 3	Heston
β_0	6.76***	6.20***	6.41***	12.23***
β_1	-10.05***	-9.13***	-9.39***	-21.07***
β_2	4.00***	3.62**	3.73**	9.22***
β_3	-0.87***	-0.86***	-0.96***	-1.00***
β_4	-20.09***	-20.21***	-22.29***	-5.62*
Adjusted R^2	0.1716	0.1518	0.1393	0.1580
F	211.90	183.26	165.71	191.98

, **, *, Indicates significance at 10%, 5% and 1%, respectively. The regression results is based on the equation $\varepsilon_{n.t} = \beta_0 + \beta_1 (S_t/K_n) + \beta_2 (S_t/K_n)^2 + \beta_3 \tau_n + \beta_4 r_t + \eta_n(t)$ using OLS to analyze the 3 day ahead out-of-sample pricing errors.*

For the 3 day ahead regression results as reported in table 7.11, DVF 3 is now the best performer in terms of lowest adjusted R^2 and F value. All of the models are less affected by the biases as the predictability of the errors is dropped compared to 1 day ahead. This infers that the longer the time frame, less pricing errors is attributable to the biases. DVF 2 and 3 which include time variable T and T^2 are less affected by the biases mentioned earlier than DVF 1 model which is on the contrary of 1 day ahead regression results.

For 3 day ahead, it can be inferred the importance of reducing the errors stemming from the biases as it improves the models ability to forecast option prices. However, although Heston and DVF models are superiors to BS in forecasting 3 day ahead, the pricing errors are higher compared to 1 day ahead. This suggests the parameters of the models are less stable for a longer forecasting period.

Hedging Performance

Table 7.12 presents 1 day ahead delta hedging results. The delta hedge strategy is implemented by shorting a call option, long Δ shares, borrow at risk-free rate and then the position is liquidated the day after.

Table 0.12
1 day ahead delta-hedging errors

	MPE	MAPE	MAE	MSE
BS30	-7,18 %	24,69 %	1,63	5,69
BS	-4,33 %	16,24 %	1,39	4,92
GC	-7,54 %	24,24 %	1,80	6,88
DVF 1	-3,74 %	15,80 %	1,30	3,76
DVF 2	-3,61 %	15,49 %	1,29	3,75
DVF 3	-3,54 %	15,38 %	1,29	3,74
Heston	-6,47 %	20,71 %	1,72	6,52
GARCH	-4,99 %	16,58 %	1,43	4,64

Denoting $\varepsilon_n = C_n^* - C_n$, where C_n^* is the model price and C_n is the market price. MPE, MAPE, MAE and MSE are calculated using the equations on p.60. For MPE and MAPE, ε_n is divided by the option market price at the time of liquidation.

For the whole sample period, all models show negative MPE which means the delta hedged portfolio has losses on average. This is on the contrary of the findings of negative risk premium associated with the volatility risk (Bakshi & Kapadia, 2003) found in Kim & Kim (2004). The authors find that all models have positive mean hedging errors on average across moneyness.

DVF 3 has the best hedging performance, but the difference is small among the DVF models. It seems that an additional parameter barely improves the performance in hedging of the DVF models. Surprisingly, BS performs better than Heston regardless of only using a single parameter while the difference between DVF models and BS is not large. In delta hedging, GARCH estimate the same number of observations as the other models. However, due to weekly estimations the parameters do not update every day which can either improve or worsen the hedging results of the model. GARCH now performs similar to BS, with a barely higher MAPE, but a lower MSE. We recognize that GARCH shows a weak point in pricing 1 and 3 day ahead, but a strong point in forecasting volatility. The hedging results for GARCH are consistent with the findings of Kim & Kim (2004).

In terms of MAPE, GC, Heston and GARCH underperform the benchmark. BS30 is still ranked last but shows more similar results with the other models contrary to in-sample and out-of-sample pricing. The results suggests that even though a model is theoretical advanced, it does not imply a better delta hedging performance than BS. It seems that parameters that incorporates the volatility smile and maturity for options prices, has less effect in hedging.

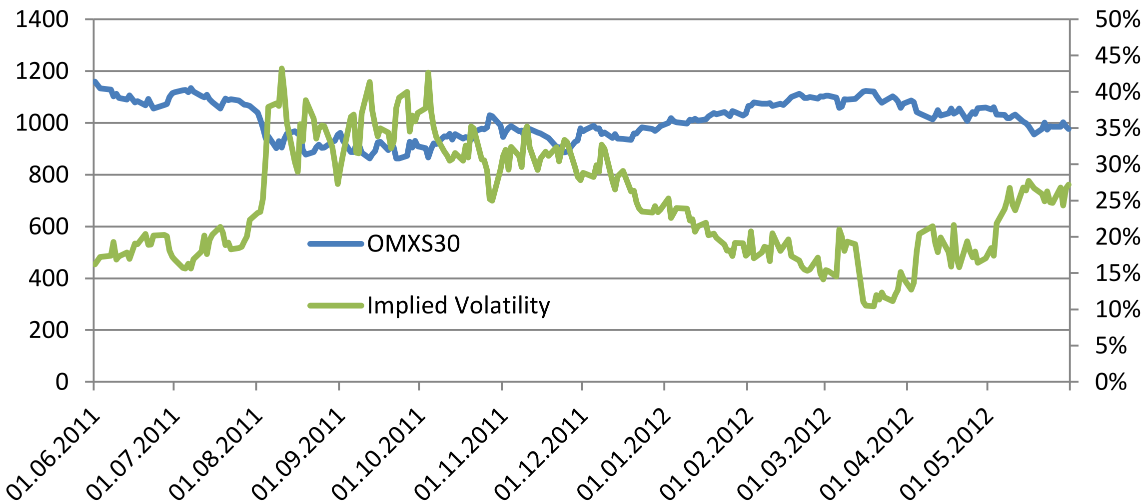
We can see in table 7.13 that the losses on average are most profound for DOTM options where actually BS has the least errors when sorting for moneyness. The hedging errors are highest for OTM and decreases as we move to ITM options. However, all models show positive gains for very DOTM options while only BS30, DVF models and GARCH have positive gains for very DITM options. In overall, DVF models perform the best except in terms of MAPE for DOTM and OTM options.

Table 0.13
Delta hedging errors sorted by moneyness

	S/K	**<0,94**	**0,94-0,97**	**0,97-1,00**	**1,00-1,03**	**1,03-1,06**	**>1,06**
MPE	*BS30*	6,86 %	-18,37 %	-10,33 %	-3,90 %	-1,71 %	0,14 %
	BS	3,68 %	-8,53 %	-6,48 %	-4,67 %	-2,96 %	-0,23 %
	GC	5,66 %	-16,48 %	-11,21 %	-5,74 %	-2,76 %	-0,05 %
	DVF 1	5,16 %	-8,19 %	-6,63 %	-3,95 %	-1,30 %	0,16 %
	DVF 2	5,10 %	-7,83 %	-6,46 %	-3,99 %	-1,29 %	0,15 %
	DVF 3	5,07 %	-7,56 %	-6,45 %	-3,99 %	-1,28 %	0,15 %
	Heston	4,09 %	-12,72 %	-10,01 %	-5,51 %	-2,83 %	-0,11 %
	GARCH	2,63 %	-8,89 %	-7,65 %	-4,95 %	-2,84 %	0,02 %
MAPE	*BS30*	32,68 %	37,38 %	21,29 %	9,70 %	4,95 %	1,87 %
	BS	18,41 %	20,82 %	16,88 %	10,37 %	6,16 %	2,23 %
	GC	30,84 %	33,92 %	23,06 %	11,63 %	5,97 %	2,07 %
	DVF 1	15,28 %	22,04 %	17,26 %	9,59 %	4,63 %	1,85 %
	DVF 2	14,82 %	21,44 %	17,04 %	9,62 %	4,61 %	1,85 %
	DVF 3	14,72 %	21,12 %	17,04 %	9,62 %	4,62 %	1,85 %
	Heston	23,84 %	28,01 %	21,40 %	11,44 %	6,04 %	2,11 %
	GARCH	16,65 %	22,10 %	17,84 %	10,87 %	6,21 %	2,36 %
MAE	*BS30*	1,30	1,58	1,72	1,89	1,89	1,39
	BS	0,78	1,09	1,54	2,00	2,29	1,60
	GC	1,37	1,68	1,91	2,21	2,24	1,47
	DVF 1	0,70	1,10	1,48	1,83	1,77	1,32
	DVF 2	0,70	1,09	1,47	1,83	1,78	1,32
	DVF 3	0,69	1,09	1,47	1,83	1,78	1,32
	Heston	1,09	1,52	1,93	2,26	2,27	1,51
	GARCH	0,81	1,15	1,56	2,07	2,33	1,68
MSE	*BS30*	3,45	5,25	6,35	7,48	7,71	2,97
	BS	1,91	2,97	5,37	8,61	11,06	4,37
	GC	3,90	5,73	7,58	10,01	10,63	3,78
	DVF 1	0,93	2,43	4,49	7,09	6,53	2,99
	DVF 2	0,92	2,41	4,45	7,13	6,51	3,01
	DVF 3	0,91	2,38	4,43	7,15	6,50	3,00
	Heston	3,06	5,07	7,49	10,17	10,40	3,92
	GARCH	1,19	2,53	5,02	8,97	11,46	4,58

Since the results are quite even among DVF models and BS, it is interesting to see if the ranking of their performances change for the different periods in our sample. Figure 26 illustrates the index and the average Black-Scholes implied volatility of all options traded on the same day.[57] We can see that the index movement and the implied volatility of the options are highly negatively correlated and is very similar to the movements of SIXVX for the same period.

Figure 26



The figure illustrates the movements of OMXS30 and the average implied volatility of the options during our sample period. The index values are displayed on the left axis while implied volatility values are displayed on the right axis.

Based on the pattern of the implied volatility, the sample can be divided into three periods where the market conditions are different from each other.[58] Table 7.14 shows the statistics for each period and the market condition is based on the characteristics of the index movement and implied volatility.

A noticeable characteristic in period 1 is that the market is somewhat stable until the sudden jump of the index falling 137 points at the start of August 2011.

[57] The average is calculated for the filtered options based on the filtering rules in Chapter 5.
[58] The sample period were divided for in-sample and out-of-sample pricing. The ranking did not change for different periods and are therefore not reported.

Table 0.14
Period statistics

	Start	End	Return	Volatility	Impl.Vol	Market Condition
Period 1	1.6.2011	20.7.2011	-0,18 %	0,2386	0,1827	Low volatility
Period 2	21.07.2011	7.12.2011	-0,07 %	0,3884	0,3194	High volatility
Period 3	8.12.2011	31.05.2012	0,01 %	0,2318	0,2004	Bull, then bear
All	1.6.2011	31.05.2012	-0,05 %	0,3052	0,2460	

The table reports statistics of each period. The statistics show the average return, historical and implied volatility for that period. The historical volatility of the returns is annualized by taking the standard deviation of the return and multiplied with the root of 252 trading days. Implied volatility is calculated by taking the daily BS implied volatility of the filtered options and then averaged over the specific period.

The index stays fairly under 1000 for period 2 and the implied volatility is nearly twice as much as in period 1, and therefore considered as a high volatility period. It can also be seen that due to the movements of the index in the period, realized volatility is substantially higher. Period 3 can be seen as a trending bullish market followed by a trending bearish market. The implied volatility pattern follows the index inversely while the index is trending upwards and then downwards. As the returns do not have any big jumps as compared to period 1 and 2, the realized volatility is the lowest of all periods. One could ask why period 3 is not split into a bull and bear market conditions, the ranking of all of the hedging performance errors employed in the thesis does not change if the split is made and therefore considered as one.

Table 0.15
Delta hedging results for Period 1

	Period 1			
	MPE	**MAPE**	**MAE**	**MSE**
BS30	-9,43 %	20,01 %	0,89	1,41
BS	-6,22 %	12,90 %	0,71	0,99
GC	-12,92 %	26,60 %	1,20	2,37
DVF 1	-5,87 %	14,72 %	0,69	0,92
DVF 2	-5,17 %	13,35 %	0,68	0,90
DVF 3	-5,12 %	13,31 %	0,68	0,89
Heston	-10,23 %	19,99 %	1,12	2,07
GARCH	-7,95 %	16,28 %	0,88	1,45

The table reports delta hedging errors from 1.6.2011 to 20.7.2011.

In period 1, MPE are more negative on average than for the whole period which can be interpreted as higher losses on the delta neutral portfolio. This suggests that in a declining market an option holder are willing to pay a larger premium for hedging. Bakshi & Kapadia (2003) propose the impact of the volatility risk premium is more prominent during times of greater stock market uncertainty and emphasize that the effect may be related to demand for options as hedging instruments. As figure 26 illustrates the index movement starts with a declining trend then followed by two spikes. The tree declining stages in period 1 may explain the highly negative MPE. Table 7.15 shows that BS has the least and GC has the highest hedging errors for period 1 in terms of MAPE. It is also unexpected that Heston barely performs better than BS30 and worse than BS.

For period 2 which we consider as a high volatility period, BS still have the best performance in terms of MAPE while Heston and GC now ranks in front of BS30. However, DVF models do have a slightly better MAE and MSE. MPE decrease in period 2 for all of the models and MAPE is slightly improved for GC, DVF 1, Heston and GARCH.

Table 0.16
Delta hedging results for Period 2

	Period 2			
	MPE	**MAPE**	**MAE**	**MSE**
BS30	-4,09 %	20,45 %	1,77	5,37
BS	-1,90 %	13,38 %	1,51	4,29
GC	-4,45 %	19,74 %	2,01	6,82
DVF 1	-1,85 %	14,10 %	1,50	4,06
DVF 2	-1,72 %	13,97 %	1,50	4,08
DVF 3	-1,68 %	13,93 %	1,50	4,07
Heston	-4,65 %	18,28 %	1,98	7,03
GARCH	-2,52 %	14,56 %	1,64	4,89

The table reports delta hedging errors from 21.7.2011 to 7.12.2011.

Furthermore, BS falls behind the DVF models for period 3 and the rest of models ranking remain unchanged. In overall, the DVF models have fewer errors on average for the whole sample period because period 3 has a longer timeframe than period 1 and 2. In period 3 where the index has a growing and then a declining phase produce similar MPE as period 1.

Table 0.17
Delta hedging results for Period 3

	Period 3			
	MPE	**MAPE**	**MAE**	**MSE**
BS30	-10,08 %	30,79 %	1,68	7,22
BS	-6,56 %	20,42 %	1,44	6,71
GC	-9,57 %	28,72 %	1,74	8,20
DVF 1	-5,29 %	18,02 %	1,23	4,20
DVF 2	-5,33 %	17,80 %	1,22	4,16
DVF 3	-5,21 %	17,59 %	1,21	4,14
Heston	-7,51 %	23,67 %	1,59	7,16
GARCH	-6,99 %	18,97 %	1,35	5,24

The table reports delta hedging errors from 8.12.2011 to 31.5.2012.

DVF models are now the best performing models followed by GARCH and BS. As one can see from figure 26, period 3 seems to have less volatile and mean-reverting properties which may be in favor for the GARCH model as it has fewer errors than BS.

In sum, BS is only outperformed by the DVF models in terms of delta hedging performance. Considering that DVF 3 uses 5 parameters, BS does a good job as the MAPE difference is less than a percentage point for the whole sample. This shows BS is still competent in delta hedging with the advantage of simplicity, compared to in- and out-of-sample pricing. Based on implied volatility traits of our data set, our results suggests that BS is the recommended model for delta hedging when market has a high or low volatility periods while PBS is recommended when market is trending upwards or downwards, although PBS is the best performer on average. The hedging results of the models suggests that including parameters to incorporate the volatility smile have less effect on hedging compared to in- and out-of-sample pricing.

The ranking of our models differs with the findings of Kim & Kim's (2003) article on the Korean KOSPI 200 index. On the contrary of our results of Heston ranking 6th place, they find that Heston improves hedging errors by 1.16% compared to BS, making Heston the best performer.[59] Their results show PBS is the worst performer, although they only use a version of DVF 1. On the contrary, our results indicate that by adding parameter T, T^2 and KT marginally increase the delta hedge performance.

[59] See table 8 of Kim & Kim (2003), p. 137. Ranking is based on MAPE.

Chapter 8 Conclusion

For our thesis, we have studied pricing and hedging performances of alternative option pricing models: Black Scholes (1973), Practitioner Black-Scholes (Dumas et al. (1998), Christoffersen & Jacobs (2004)) that fits the implied volatility surface, Heston's (1993) continuous-time stochastic volatility model, Gram-Charlier (2004) which incorporates skewness and kurtosis and Heston and Nandi's (2000) GARCH type discrete model. A daily cross-section of the OMXS30 index option prices has been used to estimate each model.

Our sample shows that the option prices produce the well-known implied volatility smile, where the smile is prominent for short-term maturity options. The sample statistics show the implied volatility varies across moneyness and maturity. Therefore, any acceptable alternative model must show ability to incorporate the volatility smile and maturity effects better than the benchmark model BS. Our results are as follows.

We find that none of the models can fully approximate the market, but they can however improve the pricing errors significantly compared to BS in terms of in-sample and out-of-sample pricing. PBS outperforms the other models in terms of in-sample pricing, out-of-sample pricing for both 1 day and 3 day ahead. As expected with additional parameters to fit the option prices in-sample, we find that PBS performs better in general with parameters such as T, T^2 and KT. However, it is surprising that the Heston model does not fit the prices any better than DVF 1.

For out-of-sample pricing, none of models are able to avoid pricing errors stemming from the biases. For 1 day ahead, PBS has the least errors followed by Heston, BS, GC, GARCH and BS30. Only PBS and Heston outperform the benchmark models. Table 10.3 in the appendix lists the models categorically best for option pricing and the results are consistent with the findings of Singh (2013) where PBS and Heston are recommended. The regression analysis infers that models which are less affected by the biases mentioned earlier, do not necessarily improve the accuracy in forecasting 1 day ahead.

As for 3 day ahead, DVF 3 has the lowest MAPE, F value and adjusted R^2. Among the DVF models, for options with less than 20 days to maturity, DVF 2 performs better than DVF 3. The same results was found for 1 day ahead as well, which suggests T^2 is excessive when it comes to forecast options with less than 20 days to maturity. On the contrary of 1 day ahead results, 3 day ahead regression analysis imply that models which are less affected by the less affected a model is by moneyness, maturity and interest rate biases, the better is the model to forecast option prices, but only given the model has lower MAPE than BS.

In delta hedging, DVF 3 again has the least errors. Even though DVF 3 uses 5 parameters, BS barely has a percentage point higher MAPE than DVF 3. To see if there are any differences between PBS and BS in certain market conditions, we divided our sample period based on implied volatility patterns. We find that BS outperforms all models for a high and low implied volatility market, while PBS outperforms all when the market is trending upwards or downwards. Table 10.4 in the appendix sorts the best models in delta hedging categorically across moneyness.

Both PBS from deterministic volatility family and Heston from the stochastic volatility family have outperformed the benchmark BS in option pricing. As our thesis shows evidence that even if a model is more theoretical advanced, the application is just as an important factor. By using Excel VBA, the implementation of PBS is rather less time-consuming and can be effortlessly applicable by others. On the other hand, we suspect Heston model may improve its performance by using a more powerful statistical program.

In conclusion, to efficiently price call options there are alternative models which incorporate the volatility smile to the simple BS. We find that PBS and Heston are the better suited models and should be used in forecasting the option prices.[60] In respect of our problem statement, these models which are able to incorporate the volatility smile and mitigate the maturity bias perform better than BS in terms of fitting and forecasting the volatility smile. On the contrary of in-sample and out-of-sample results, we found that BS is still useful and may be used as a delta hedging model. However, by adding parameters of PBS, the improvement of hedging errors is marginal. We conclude that incorporating volatility smile in hedging is less important and can actually perform worse than the simple BS.

[60] A diffusion process it can be used to price path dependent options. This is not possible with the PBS and one has to use the simple BS model to capture the dynamics of path dependent options. According to our studies the Heston outperformed the BS model with a margin, and should be favored

Future research

We propose two models that we believe could be interesting for further research study

Heston with a Jump Component

It would be interesting to investigate the Heston model with a jump component. The problem for a model like Heston is its ability to price “extreme” OTM options. Since the Heston model follows a diffusion process, it is hard to incorporate ”very unlikely” deep OTM options which can arise for short-dated options. In an economical perspective a large jump can occur, but a model that follows diffusion process without jumps will have problems incorporating extreme smiles for short-dated options.[61] Bakshi et al. (1997) also discuss the fact that diffusion stochastic models are not flexible enough to capture this jump component. Gatheral (2006) suggests a way to cope with this problem in the Heston framework is to add a stochastic jump. He states that introducing jumps can create extreme short-dated skews in the data that quickly dies out. According to his studies the Heston with jumps performs substantially better than the original Heston model, especially when there are many short-dated options. Therefore, in further research this could be an interesting extension of the Heston model. Since the Swedish OMXS30 is a much smaller index than S&P 500 and FT 100, jumps may be more likely than in bigger indices.

The Variance Gamma model

The Variance Gamma model (VG) by Madan et al. (1998) uses a three parameter stochastic process that generalizes Brownian motion as model for the dynamics of the logarithm of stock price. These additional parameters provide control over the skewness and kurtosis of the return distribution. The process is obtained by evaluating Brownian motion (with constant drift and volatility) at a random time change given by a gamma process. In their article, the authors conduct orthogonality tests on the pricing errors and show that the model is relatively free of moneyness and maturity biases and consequently superior to BS. Kim & Kim (2003) find that VG ranks 2nd in 1 day ahead pricing and 1st one week ahead and confirms that VG is the least affected by these biases. Therefore VG is recommended in future research to see if the model will perform likewise on OMXS30.[62]

[61] DOTM options can have a significant positive price even though the probability(statistical speaking) of this happening ~ 0

[62] See table 5 and 6 of Kim & Kim (2003), p. 130-132. Ranking is based on MAPE.

Non-Synchronous Bias

As we have mentioned earlier in the thesis, our data is not time-stamped and may therefore be subject to non-synchronous bias. Although we have taken precautions to mitigate the bias, it can be interesting if our results hold for a data set that is free of the non-synchronous bias. Kim & Kim (2003), have all used last reported transaction price in a specific time window to minimize problems stemming from intra-day variation in volatility, Bakshi et al. (1997) used index levels recorded at the same time as the corresponding bid-ask quote.[63] Also, including put option prices is recommended to see if the models perform the same way as for call options.

[63] All of the articles avoid the non-synchronous price issue, except the index itself may contain stale component stock prices at each point of time.

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Appendix

Appendix A

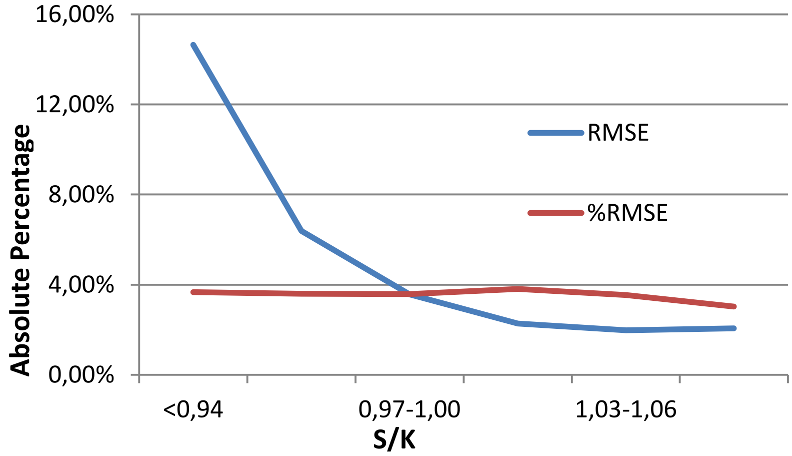
The figure below illustrates the effect of changing the loss function for the Heston model. The top fis

S/K	Loss functon	<0,94	0,94-0,97	0,97-1,00	1,00-1,03	1,03-1,06	≥1,06
MPE	RMSE	31,85 %	21,70 %	5,69 %	1,63 %	0,98 %	-1,52 %
	%RMSE	-1,83 %	-0,68 %	-1,93 %	-3,25 %	-2,48 %	-3,96 %
MAPE	RMSE	32,28 %	25,66 %	12,68 %	5,36 %	3,78 %	1,65 %
	%RMSE	8,15 %	9,11 %	8,14 %	7,47 %	5,77 %	4,15 %
MAE	RMSE	0,54	0,59	1,13	1,53	1,84	1,46
	%RMSE	0,27	0,54	1,31	2,41	2,95	3,65
MSE	RMSE	0,49	0,57	1,98	3,38	4,87	2,92
	%RMSE	0,19	0,65	3,19	8,70	11,81	16,14

S/K	Loss functon	<0,94	0,94-0,97	0,97-1,00	1,00-1,03	1,03-1,06	≥1,06
MPE	RMSE	7,71 %	4,80 %	0,33 %	-0,48 %	-1,84 %	-1,52 %
	%RMSE	0,83 %	0,28 %	-2,20 %	-2,48 %	-3,42 %	-2,61 %
MAPE	RMSE	14,65 %	6,38 %	3,57 %	2,28 %	1,97 %	2,06 %
	%RMSE	3,66 %	3,59 %	3,59 %	3,81 %	3,54 %	3,02 %
MAE	RMSE	0,48	0,57	0,41	0,77	1,24	1,87
	%RMSE	0,14	0,40	0,73	1,44	2,29	2,66
MSE	RMSE	0,46	0,60	0,29	1,06	3,09	4,52
	%RMSE	0,05	0,41	0,95	3,02	9,66	8,50

S/K

The figure below illustrates the effect of changing the loss function for the Heston model. The RMSE indicates the loss function which focuses on the dollar difference between the model and the market price, while the %RMSE focuses on the percentage difference. The sample is the months of July and October.



This figure graph of the difference between the %RMSE and $RMSE for Heston for a random day in July.

Appendix B

To illustrate the difference between Excel finding an “optimal” solution without time restrictions compared to stopping the calibration after 2000 seconds or 15 iterations for the Heston model. The minimum required number of Iterations was set to 10 no matter the time constraint.[64] Two random dates from July and October is chosen.

July

July - No time constraint or iteration constraint

Målcelle (Minimum)

Celle	Navn	Opprinnelig verdi	Sluttverdi
C26	lfh_13	0,2230	0,0460

Variabelceller

Celle	Navn	Opprinnelig verdi	Sluttverdi	Integer
C8	rho	-0,50	-0,73	Forts
C9	kapp	2	2,378253201	Forts
C10	theta	0,0625	0,084566784	Forts
C11	lambda	0	0	Forts
C12	sigmav	0,5	0,915858141	Forts
C13	V	0,0256	0,043762672	Forts

Resultat: Har kommet til gjeldende løsning. Alle begrensninger er oppfylt.
Problemløsermotor
Motor: Ikke-lineær GRG
Løsningstid: 3659,768 Sekunder.
Gjentakelser: 23 Delproblemer: 0

Alternativer for Problemløser
Maksimal tid 20000 sek, Gjentakelser 60, Precision 0,000001
Sammenfall 0,0001, Populasjonsstørrelse 100, Tilfeldig seeding 0, Differensialkvotient for Fremover, Krev grenser
Maksimalt antall delproblemer Ubegrenset, Maksimalt antall heltallsløsninger Ubegrenset, Heltallstoleranse 5%, Løs

July -Time and iteration constrained

Celle	Navn	Opprinnelig verdi	Sluttverdi
C26	lfh_13	0,2230	0,0465

Celle	Navn	Opprinnelig verdi	Sluttverdi	Integer
C8	rho	-0,50	-0,73	Forts
C9	kapp	2	1,908899222	Forts
C10	theta	0,0625	0,089223937	Forts
C11	lambda	0	0	Forts
C12	sigmav	0,5	0,818711296	Forts
C13	V	0,0256	0,042494198	Forts

Maximum time of 2000 seconds was reached

October

October- No time constraint or iteration constraint

Microsoft Excel 14.0 Svarrapport
Regneark: [Chapter5Heston_oct.xlsm]11
Rapport opprettet: 10.08.2013 13:02:20
Resultat: Har kommet til gjeldende løsning. Alle begrensninger er oppfylt.
Problemløsermotor

Motor: Ikke-lineær GRG
Løsningstid: 5795,796 Sekunder.
Gjentakelser: 22 Delproblemer: 0

Alternativer for Problemløser

Maksimal tid 20000 sek, Gjentakelser 60, Precision 0,000001
Sammenfall 0,0001, Populasjonsstørrelse 100, Tilfeldig seeding 0,
Differensialkvotient for Fremover, Krev grenser
Maksimalt antall delproblemer Ubegrenset, Maksimalt antall heltallsløsninger Ubegrenset,
Heltallstoleranse 5%, Løs uten heltallsbegrensninger

Målcelle (Minimum)

Celle	Navn	Opprinnelig verdi	Sluttverdi
C26	lfh_11	0,3487	0,0558

Variabelceller

Celle	Navn	Opprinnelig verdi	Sluttverdi	Integer
C8	rho	-0,50	-1,00	Forts
C9	kapp	2	2,068309277	Forts
C10	theta	0,09	0,484525236	Forts
C11	lambda	0	0	Forts
C12	sigmav	0,9	0,886070675	Forts
C13	V	0,0625	0,07767207	Forts

October Time and Iteration constrained

Regneark: [Chapter5Heston_oct.xlsm]11
Rapport opprettet: 10.08.2013 14:50:34
Resultat: Problemløseren ble stoppet av brukeren.
Problemløsermotor

Motor: Ikke-lineær GRG
Løsningstid: 2096,591 Sekunder.

Alternativer for Problemløser Gjentakelser: 8 Delproblemer: 0

Maksimal tid 2000 sek, Gjentakelser 15, Precision 0,000001
Sammenfall 0,0001, Populasjonsstørrelse 100, Tilfeldig seeding 0,
Differensialkvotient for Fremover,
Krev grenser
Maksimalt antall delproblemer Ubegrenset, Maksimalt antall heltallsløsninger Ubegrenset,
Heltallstoleranse 5%, Løs uten heltallsbegrensninger

Målcelle (Minimum)

Celle	Navn	Opprinnelig verdi	Sluttverdi
C26	lfh_11	0,3487	0,0674

Variabelceller

Celle	Navn	Opprinnelig verdi	Sluttverdi	Integer
C8	rho	-0,50	-1,00	Forts
C9	kapp	2	2,007699384	Forts
C10	theta	0,09	0,441157117	Forts
C11	lambda	0	0	Forts
C12	sigmav	0,9	0,886813916	Forts
			0,08006739	Forts

Appendix C - The formulas

Gram-Charlier

Wu etc breaks the integral into four different part where the first is the familiar BS expression while the bracket adjust for excess kurtosis and skewness

They use the Gram-Charlier expansion to calculate the probability function for the standardized T period return $w_t = \left(\frac{R_{t+1}^T - \mu_T}{\sigma_T}\right)$

$$f(w_t) = \phi(w_t)\left[1 - \frac{\gamma_{1T}}{3!}\phi^{(3)}(w_T) + \frac{\gamma_{2T}}{4!}\phi^{(4)}(W_t)\right]$$

The first is the density of the standard normal distribution, while the second equation is the k derivative of this density function, which is used to obtain skewness and kurtosis.

$$\phi(x) = \exp\left(\frac{\frac{-x^2}{2}}{\sqrt{2\pi}}\right)$$

$$\phi^{(k)} = Kth\ derivative\ of\ \phi(x)$$

$$C_{GC} = e^{-rT}E[\max(S_{t+t} - K), 0] = e^{-rT}\int_{\log(\frac{K}{S_t})}^{\infty}(S_t e^x - K)f(x)dx$$

$$C_{GC} = \int_{w^*}^{\infty}(S_t e^{\mu_T + \sigma_T w_t} - K)f(w_T)dw_T$$

$$C_{GC} = \int_{w^*}^{\infty}(S_t e^{\mu_T + \sigma_T w_t} - K)\left(\phi(w_T)\left[1 - \frac{\gamma_{1T}}{3!}\phi^{(3)}(w_T) + \frac{\gamma_{2T}}{4!}\phi^{(4)}(w_T)\right]\right)dw_T$$

$$w^* = (\log\left(\frac{K}{S_T}\right) - \mu_T)/\sigma_T$$

The Heston formulas

The full Heston formula is stated below. The PDE and the derivation of Itos lemma is not included and we refer to the original article written by Heston. The closed form approach is solved by using the characteristic functions to obtain probabilities P_j

$$f_j = exp(C_j + D_j v + i\phi x)$$

If we have knowledge about the characteristic functions we can find the probability density function of the stochastic variable using an Inverse Fourier transform. This is often easier than working directly with the density functions. This approach can be used since the characteristic functions depend on the same state variables as the probabilities P. In P_j for j(0,1), The *i* is the complex number $\sqrt{-1}$. In the integral it is only the real part of this function and is solved by using a numerical integration called trapezoidal integration.

$$\mathrm{P_j} = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{\mathrm{e}^{-\mathrm{i}\phi \ln(\mathrm{k})}\mathrm{f_j}}{\mathrm{i}\phi}\right] \mathrm{d}\phi$$

$$\mathrm{f_j} = \exp(\mathrm{C_j} + \mathrm{D_j v} + \mathrm{i}\phi \mathrm{x})$$

$$\mathrm{C_j} = \mathrm{r}\phi\mathrm{i}\tau + \frac{\kappa\theta}{\sigma^2}\left\{(\mathrm{b_j} - \rho\sigma\phi\mathrm{i} + \mathrm{d_j})\tau - 2\ln\left[\frac{1-\mathrm{g_j e^{d_j \tau}}}{1-\mathrm{g_j}}\right]\right\}$$

$$\mathrm{D_j} = \frac{\mathrm{b_j} - \rho\sigma\phi\mathrm{i} + \mathrm{d_j}}{\sigma^2}\left[\frac{1-\mathrm{e^{d_j\tau}}}{1-\mathrm{g_j e^{d_j\tau}}}\right]$$

$$\mathrm{g}_{\mathrm{j}=}\frac{\mathrm{b_j} - \rho\sigma\phi\mathrm{i} + \mathrm{d_j}}{\mathrm{b_j} - \sigma\phi\mathrm{i} - \mathrm{b_j}}$$

$$\mathrm{d_j} = \sqrt{(\rho\sigma\phi\mathrm{i} - \mathrm{b_j})^2 - \sigma^2(2\mathrm{u_j}\phi\mathrm{i} - \phi^2)}$$

$$\mathrm{b_1} = \kappa + \lambda - \rho\sigma \text{ and } \mathrm{b_2} = \kappa + \lambda$$

$$\mathrm{u_1} = \frac{1}{2} \text{and } \mathrm{u_2} = -1/2$$

Heston Nandi Garch (1,1)

$$C = e^{-rT}E_t^*[(S_{t+T} - K, 0)] = S_t P_1 - Ke^{-rT}P_2$$

$$P_1 = \frac{1}{2} + \frac{e^{-rT}}{\pi S_t}\int_0^{\infty} Re\left[\frac{K^{-i\phi}f^*(i\phi+1)}{i\phi}\right] d\phi$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{K^{-i\phi}f^*(i\phi)}{i\phi}\right] d\phi$$

Hestn and Nandi have proved in their article that the generating function takes a log-linear form for Garch (1,1)

$$f(\phi) = S_t^{\phi} \exp(A_t + B_t \sigma_{t+1}^2)$$

$$A_t = A_{t+1} + \phi r + B_{t+1}\omega + \frac{1}{2}\log(1 - 2\alpha B_{t+1})$$

$$B_t = \phi(\lambda + \gamma) - \frac{1}{2}\gamma^2 + \beta B_{t+1} + \frac{\frac{1}{2}(\phi - \gamma)^2}{1 - 2\alpha B_{t+1}}$$

It is important to keep in mind that the values $A_t$and B_t are found by working recursively from the time to maturity t+T. The starting values are $A_{t+T} = B_{t+T} = 0$.

Appendix D

Table 0.1
Regression coefficients for 1 day ahead

Coefficients	BS30	BS	GC	DVF 1	DVF 2	DVF 3	Heston	GARCH
β_0	116.54***	7.41***	11.02***	3.81***	5.16***	5.83***	7.53***	1.68
β_1	-197.02***	-12.41***	-19.89***	-5.09***	-8.22***	-9.45***	-12.69***	-0.65
β_2	89.18***	5.53***	9.34***	1.86*	3.56***	4.16***	5.60***	-0.38
β_3	-1.65**	-0.53***	-0.42***	-0.52***	-0.45***	-0.51***	-0.53***	-0.60***
β_4	-313.75***	-13.86***	-12.22***	-17.70***	-15.43***	-17.00***	-11.88***	-19.32***
Adj. R^2	0.1159	0.1697	0.1401	0.1633	0.1687	0.1769	0.2058	0.1692
F	151.68	235.98	188.34	225.39	234.35	248.05	298.96	105.88

Table 0.2
Regression coefficients for 3 day ahead

Coefficients	BS30	BS	GC	DVF 1	DVF 2	DVF 3	Heston	GARCH
β_0	133.90***	14.68***	14.29***	6.76***	6.20***	6.41***	12.23***	6.54***
β_1	-230.75***	-25.86***	-25.32***	-10.05***	-9.13***	-9.39***	-21.07***	-10.19**
β_2	105.93***	11.68***	11.41***	4.00***	3.62**	3.73**	9.22***	4.13*
β_3	-1.42**	-0.99***	-0.98***	-0.87***	-0.86***	-0.96***	-1.00***	-0.85***
β_4	-330.99***	-10.12***	-4.91	-20.09***	-20.21***	-22.29***	-5.62*	-11.26***
Adjusted R^2	0.1242	0.2004	0.1394	0.1716	0.1518	0.1393	0.1580	0.1687
F	145.35	256.19	165.85	211.90	183.26	165.71	191.98	86.59

Appendix E

Table 0.3

Categorically segregation of option pricing models

1 day ahead						
S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**
Short maturity (T < 20)	*DVF 3*	*DVF 2*	*DVF 2*	*DVF 2*	*DVF 2*	*DVF 3*
Medium maturity (20 ≤ T < 40)	*DVF 3*	*DVF 3*	*DVF 3*	*DVF 3*	*DVF 3*	*DVF 3*
Long maturity (40 ≤ T)	*DVF 2*	*DVF 3*	*DVF 3*	*DVF 3*	*DVF 3*	*Heston*

The table sorts the best model for 1 day ahead across moneyness. The ranking is based on MAPE.

Table 0.4

3 day ahead						
S/K	**<0.94**	**0.94-0.97**	**0.97-1.00**	**1.00-1.03**	**1.03-1.06**	**≥1.06**
Short maturity (T < 20)	*DVF 2*	*DVF 2*	*DVF 2*	*DVF 3*	*DVF 3*	*DVF 3*
Medium maturity (20 ≤ T < 40)	*DVF 2*	*DVF 2*	*DVF 2*	*DVF 3*	*DVF 3*	*DVF 3*
Long maturity (40 ≤ T)	*DVF 2*	*DVF 2*	*DVF 3*	*DVF 3*	*DVF 3*	*Heston*

The table sorts the best model for 3 day ahead across moneyness. The ranking is based on MAPE.