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BETA-ARBITRAGE

Institutional Implementation: Size, Time, and Short-Selling

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Executive Summary

It is well-documented that the *Capital Asset Pricing Model* does not adequately capture the expected return of securities. Empirically, the *Security Market Line* almost consistently overestimates the performance of high beta securities while it underestimates the performance of low beta securities. The line is too flat, the intercept is too high, and consequently beta may be a valuable tool for arbitrageurs. Using more than 30 million data observations across the US and international markets, in equities and fixed income, we construct 16 *betting against beta* portfolios and confirm the persistence of this apparent disequilibrium. We investigate the effect that time, size, and short-selling frictions may have on the performance of these portfolios. We find that beta-arbitrage performs well on average, but with significant time-variation of returns. Return variability seems positively correlated with implied leverage, as measured by the beta spread. Next, we find that the adverse effects of short-selling costs are negligible and that long-short is mostly preferable to long-only implementation, gross and net of lending fees. Moreover, in the US sample, alpha contribution is roughly equally distributed across size deciles and we find that excluding the smallest equities has little impact on overall performance of the portfolio. Our findings suggest that beta-arbitrage portfolios are implementable and robust when adjusted for size and shorting frictions and present an attractive investment opportunity for sophisticated institutional investors.[1]

Keywords: *Betting against beta, beta-arbitrage, implementation, frictions, leverage constraints, size, short-selling, time-variation of returns*

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Executive Summary *(Danish)*

Det er veldokumenteret, at Capital Asset Pricing Model (CAPM) ikke tilstrækkeligt forklarer det forventede afkast på aktiver. Empirisk overvurderer *Security Market Line* meget konsistent det forventede afkast på høj-beta aktiver og undervurderer det forventede afkast på lav-beta aktiver. Linjen er for flad, interceptet for højt og resultatet er, at beta snarere er blevet et arbitrage værktøj. Med omkring 31,1 millioner observationer i USA og internationale markeder, i aktier og obligationer, konstruerer vi 16 *betting against beta* portføljer og bekræfter denne åbenbare uligevægt . Vi undersøger hvilken effekt tid, størrelse- og kortsalg- friktioner har på porteføljeafkastet. Vi finder at beta-arbitrage er generelt er profitabelt, men med signifikant variation over tid. Tidsvariationen er positivt korreleret med markedets implicitte gældssætning, når denne måles ved spredningen af beta. Vores resultat viser at kortsalgsomkostninger er negligerbare, samt at long-short hovedsageligt er at foretrække over en ensidig lang implementering, før og efter omkostninger. I det amerikanske sample er alpha-bidraget ligeligt fordelt på tværs af markedsværdi percentiler; effekten ved at ekskludere den mindste percentil er minimal. Resultaterne indikerer at beta-arbitrage kan implementeres, er robust med hensyn til størrelse- og kortsalgs- friktioner og kan være en attraktiv investeringsmulighed for institutionelle investorer.

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1 Introduction

According to the Capital Asset Pricing Model, the expected return of any security or portfolio is linearly determined by its market beta. To arrive at this conclusion, the model assumes that all investors hold the market portfolio and engage in lending or borrowing activities to satisfy their preferences. Risk sensitive investors hold a fraction of the market portfolio and invest their remaining capital in riskless money market funds. Conversely, investors that prefer higher returns apply leverage to increase the expected returns of the market portfolio. Deviation from this behaviour is inefficient, as CAPM requires the market portfolio to be the optimal bundle of risky assets, where the return-to-risk relationship is steepest.

However, not all investors have unrestricted access to leverage. Retail-investors exhibit behavioural constraints (debt aversion) and large institutional investors are often restricted by regulatory requirements such as margin requirements and solvency laws. According to Black (1972), the risk-reward relationship might flatten in an equilibrium with restricted borrowing. Rather than explicitly applying leverage, constrained investors may be forced to substitute for implicit leverage by investing in securities that are themselves riskier or where leverage is embedded.[2] High-risk securities – with large market betas – are bid up by risk-capable investors. As a result, low-beta assets are attractive while high-beta assets are unattractive on a risk-adjusted basis. Consequently, it is achievable for unconstrained investors

[2] Investors can invest in equities with high debt levels or certain financial products (out-of-the-money options, leveraged index products, etc.) and get indirect access to debt financing (Frazzini and Pedersen, 2012)

to exploit this asset pricing irregularity by buying securities with low betas while selling securities with high betas. Black, Jensen and Scholes (1972) successfully constructed beta-arbitrage portfolios in US equities with positive returns. Black (1993) later expanded the horizon of the investigation with similar results. Most recently, Frazzini and Pedersen (2014) widened the analysis across markets and asset classes by constructing *betting against beta (BAB)* portfolios with large and statistically significant risk-adjusted returns. *Betting against beta* portfolios are roughly market neutral long-short relative bets, with long positions in leveraged low-beta securities offset by short positions in deleveraged high-beta securities. The results are inconsistent with a pure CAPM-equilibrium, but consistent with an equilibrium with restricted borrowing (Black, 1972).

Regardless of CAPM-beta's inadequacies, perhaps the most influential innovation of the model has been the subsequent employment of linear asset pricing models (Fama and French, 2004). In addition, in the factor-based model, numerous asset pricing inconsistencies have emerged in the literature. Today, this has become a widely recognized and vigorously debated topic in academia and among investment professionals. More recently, the application of factor-models has proliferated into a large industry that is often referred to as factor- or smart beta investing. Rather than loading on the conventional market beta, investors are increasingly demanding exposure to alternative betas, such as size, value and momentum. More recently, large institutional investors such as pension funds, sovereign wealth funds, and others have realized the competitive advantage of their large balance sheets – and are increasingly demanding *betting against beta* products or similar strategies built to exploit leverage asymmetries. Exchange traded products (ETPs) with different low-risk features currently account for 8.6% of alternative beta exposure among European pension funds on a value-weighted basis, this compares with 6.2% in value and only 0.1% in momentum.[3]

The interest in *betting against beta* entails natural questions: *are these strategies implementable and profitable in live trading? Can BAB portfolios remain profitable outside academia, with actual constraints and within a realistic investment universe? Are such strategies attractive to investors sensitive to return variation?*

[3] According to Morningstar Research, June 30th 2014.

What are the costs of frictions? There are substantial limits of arbitrage and paper profits may easily evaporate once implementation frictions have been incurred. To elaborate on these questions we investigate the effect of time, size, and short-selling in a restricted investment universe.

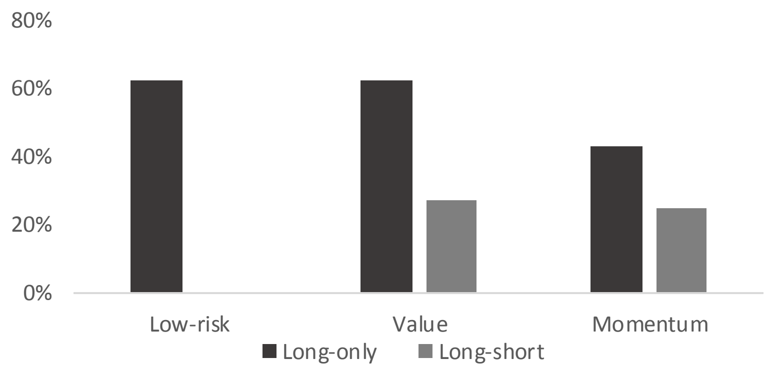
Even sophisticated institutional investors rarely trade *all available securities,* but rather limit their diversified quant-trading portfolios to members of indices like MSCI or the S&P. *Specifically, in equities, to understand how this may impact such investors, we investigate whether conventional BAB portfolios can be constructed only from index constituents in various indices* (**Proposition 1**).

A well-known dilemma in financial markets is that professional money managers rarely invest their own capital, but rather invest on behalf of others. To mitigate information asymmetries, money managers are frequently subject to investor scrutiny and benchmarking. As a result, from the perspective of the institutional investor, large return-variation or frequent drawdowns can severely affect the attractiveness of a strategy. To elaborate, we investigate the performance of BAB portfolios over time. Beta-arbitrage may rest on market-wide leverage constraints and leverage utilization by unconstrained investors. Leveraged investors are exposed to funding liquidity risk and may involuntarily be forced to deleverage when liquidity tightens. *Therefore, we inspect whether BAB returns are time-varying* (**Proposition 2**).

BAB portfolios are long-short, – short securities that empirically have performed poorly on a risk-adjusted basis. Accordingly, the BAB strategy relies on short-selling. Short-selling can be expensive and unattractive even for sophisticated investors; short-positions may be terminated involuntarily at ill times and sellers must pay a lending fee to the lender. A survey of European pension funds show that long-only implementation is often preferred to long-short implementation (Chart 1). Therefore, we inspect the long- and short contribution of BAB portfolios in terms of raw- and risk-adjusted performance. *The empirically documented monotonically declining relationship between betas and alphas indicates that the performance of long-only strategies should remain profitable, albeit less than the long-short implementation* (**Proposition 3**). We further investigate this proposition by examining the trade-off between long-only and long-short implementation net of lending fees.

Chart 1: Summary of European Pension Funds

The chart is based on a survey of European pension funds. It shows the percentage of respondents that have invested in long-short or long-only strategies. There are no statistics of long-short low-risk implementation.



Source: IPE Spring 2015

Large investors are unfortunate to have large pools of capital to allocate and may suffer from a large degree of price impact when trading in illiquid securities. This is particularly likely for investors with capacity to employ substantial leverage[4]. As a result, such investors may be unable to form and properly scale strategies in illiquid or small securities. *We investigate whether beta-arbitrage is size-dependent and examine whether the strategy is attractive even when very large amounts of capital has to be allocated* (**Proposition 4**).

In order to answer these questions we use more than 30 million price observations and roughly 17 million market capitalization values from commonly used and accessible financial databases. The sample periods range from the early 1980s to the beginning of 2015 and spans across several international markets, but mostly developed countries. Additionally, we use a sample of short-selling costs that

[4] Access to substantial leverage requires a balance sheet of a certain size. Large balance sheet investors typically have more capital to allocate and therefore must place larger positions, resulting in market impact risk, especially if securities are very illiquid.

allows us to see the costs of borrowing individual equities at one point in time. We begin by confirming the results of Frazzini and Pedersen (2014) by constructing betting against beta arbitrage portfolios from equity index constituent portfolios and bond markets. Portfolio performances are investigated across multiple markets in equities, equity indices, corporate bonds, and government bonds. We find results consistent with previous literature (Black, Jensen and Scholes 1972; Frazzini and Pedersen 2014), *betting against beta* portfolios have positive and statistically significant excess returns in equities, bonds and three out of four equity indices tests (**proposition 1**). We also find that returns are time-varying with respect to aggregate levels of implicitly embedded leverage in equities (**proposition 2**). Then we separate portfolios by long- and short contribution in order to see how BAB performance depends on short-selling. The results are robust when adjusting for short-selling costs and the performance of BAB portfolios is mostly, but not entirely, driven by the long-leg, indicating that short-constrained investors can engage in beta-arbitrage (**proposition 3**). When sorting US equities into size deciles (size measured by market capitalization, but deciles sliced by rank), we find that excess beta-arbitrage performance is roughly unaffected. The long- and short-contribution is roughly constant across size deciles. When excluding the smallest decile, excess performance is only slightly reduced, but remains positive and statistically significant and the tracking error of the constrained portfolio is minimal (**proposition 4**).

These findings are interesting for several reasons. Beta-arbitrage is one of many factor-based strategies that may suffer from implementation frictions. The relatively naïve models applied for this investigation and conservative performance hair-cuts would indicate that such institutions may advantageously exploit beta-arbitrage strategies. Theoretically, the findings also suggest that size and short-selling frictions cannot explain the flatness of the SML. Rather we regard the persistence as additional evidence for restricted borrowing. However, more importantly, we find relatively few limits of arbitrage that should entirely prevent crowding and eventual price convergence.

The rest of the thesis is structured as follows. Section 2 reviews the relevant theory and empirical evidence of the Capital Asset Pricing Model, beta-arbitrage, and implementation frictions. Section 3 discusses choice of data samples, estimation considerations and how the portfolios are formed. Section 4.1 outlines the results of conventional BAB portfolios and the performance over time, section 4.2 addresses size and short-selling implications and section 4.3 quantifies short selling cost; Section 4.4 provides additional robustness tests. Finally, section 5 concludes.

2 Literature Review

In this section we describe the theoretical foundations of the *Capital Asset Pricing Model* and the all-important *Security Market Line*. We outline the empirical evidence against the pricing-model, the underlying assumptions, and discus the *Arbitrage Pricing Theory* as an alternative. Finally, we discuss how market frictions and implementation costs can impact the pricing of securities.

2.1 The Capital Asset Pricing Model

Modern portfolio theory (MPT) was first conceptualized more than five decades ago when Markowitz (1952, 1959) developed and formalized the efficient frontier, the range of ex ante mean-variance optimal portfolios. Markowitz proposed that investors care only about returns and return variability and argued that a top-down approach to portfolio selection is superior to stock-picking, because the latter fails to account for certain aspects of diversification and correlations between assets. Instead market participants should optimize the expected risk-return relationship of their entire portfolio rather than at the individual stock-level.

Tobin (1958) expanded the framework by introducing a risk free asset. The introduction of risk free lending and borrowing enable investors to leverage and deleverage their portfolios in accordance with their individual preferences towards risk and return. As a result, the efficient frontier was reduced to a single tangency portfolio, the bundle of risky securities that optimizes the return compensation per unit of risk. To obtain a higher expected rate of return, risk-willing investors can simply borrow at the risk-free rate and purchase a larger quantity of the risky bundle. Conversely, investors with lesser preferences for risk can deleverage the tangency portfolio and invest their remaining capital in the same riskless asset. Thus, once the efficient frontier has been computed, investors effectively allocate between two assets. The separation of the risky and riskless portfolio is known as the *Tobin's Separation Theorem*. Since the covariance between the riskless and risky assets is zero by definition, the relationship between return and risk is simplified to a linearity. The linear relationship is known as the Capital Market Line:

$$r^p - r^f = \left(r^{tan} - r^f\right) * \frac{\sigma^p}{\sigma^{tan}}$$

Where superscript tan is the tangency portfolio and superscript p denotes the combined portfolio. The relationship describes how the return of an efficient portfolio depends on the allocation between the risky and riskless assets. It is easy to see that if an investor requires a higher expected return than the tangency portfolio, the allocation into risky assets must be higher than one and borrowing is required $\sigma^p/\sigma^{tan} > 1$ and $(1 - \sigma^p/\sigma^{tan}) < 0$, e.g. a negative allocation in the riskless asset.

Based on the mean-variance framework, Sharpe (1964), Mossin (1966) and Lintner (1965) theorized the *Capital Asset Pricing Model* (CAPM). The most important implication of the model is that the price of any given security can be determined solely based on a simple measure of the covariance between the security and the market portfolio. To derive this property the model makes a range of nontrivial assumptions:

1. *Investors have equal access to information, use the same computational methods and develop homogenous views.*
2. *All investors are single-period mean-variance utility optimizers*
3. *The market is in a competitive equilibrium; all investors are price-takers and have no market impact.*
4. *There are no taxes, transaction costs, or market frictions and all assets are accessible to all investors and are perfectly divisible.*
5. *Investors have unlimited access to lending and borrowing at the risk free rate.*

Most importantly, in addition to the mean-variance framework, the model dictates an informational perfect and efficient capital market with unconstrained access to riskless lending and borrowing. In other words, *every* participant in the market faces the same mean-variance optimization problem, make the same computations, and arrive at the same results. Because everyone goes through the same steps and arrives at the same results, it is unnecessary to compute the optimal bundle of risky assets, because this can already be observed in the market. Specifically, because all investors hold identical weights of every security, the tangency portfolio is also the market portfolio and all investors allocate between the market portfolio and the risk-free asset[5]. In the large diversified market portfolio, investors care only about non-diversifiable risk. The systemic risk component is linearly determined by the market beta coefficient of the portfolio, which in terms is determined by the value-weighted average of every security's beta. The result is that the price (and expected return) of any given security is a simple function of a security's volatility scaled covariance with the market. This leads to the elegant *Security Market Line* (SML), the prediction of a strictly linear relationship between expected returns and betas.

$$\mathbb{E}(r^i) - r^f = \alpha^i + \beta^i(r^m - r^f), \qquad \alpha^i = 0$$

According to the SML, the expected return of any given security depends only on the market beta coefficient, the market risk premium, and the risk-free rate. $(r^m - r^f)$ is the market risk premium (excess compensation for systemic risk) and r^f is the risk-free rate (compensation for time). Since these variables are collectively determined for the entire universe of assets, the most interesting parameter, with respect to the expected return of a single security is β^i, the market beta coefficient. The product of the market premium and the beta coefficient captures the return compensation awarded for incurring systemic risk exposure, due to market return commonality.

An important assumption of the CAPM is that the market is mean-variance efficient. While the precise definition of risk can sometimes be elusive or hard-to-

[5] Identical weights also implicitly restricts short-selling, because if everyone were shorting the same securities the price would drop to zero.

identify, according to the CAPM the only relevant measure of risk is systemic return volatility. The market risk of a security is technically determined by the the covariance between the security and the market portfolio over the variance of the market. As discussed, non-systemic risk, idiosyncratic volatility, is irrelevant as this is diversified to zero in the large portfolio. In the CAPM framework, we can easily see why this relationship must be true. Given that non-market return variance is assumed entirely diversified away, all expected return variability is due to market co-movement. Then, if the equation does not hold, we can construct a simple long-short portfolio with non-zero expected- return and no ex ante variance by buying and selling r^i and $\beta^i r^m$ depending on the sign of the inequality. It would then be possible to construct a portfolio that is ex ante mean-variance superior to the market portfolio.

2.2 Arbitrage Pricing Theory

Since its introduction, the Capital Asset Pricing Model has ranked as one of the most important, cited and controversial equilibrium models in modern financial economics. The model is an elegant attempt to describe a competitive market in equilibrium and determine security prices. Naturally, the model has undergone close scrutiny by academics and the literature extends to a long list of empirical tests. Early empirical studies have suggested that security prices do not behave as the model predicts, that market exposure explains some, but far from all return variability, and that the SML exaggerates the performances of high beta assets while underestimating the performance of low beta assets (Black, Jensen, and Scholes, 1972; Fama and Macbeth, 1973; Miller and Scholes, 1972). In fact, there are many examples of violations of the single-factor model in the literature. The majority of these tests has been performed by estimating the cross-sectional relation between average return on assets and their betas, and rarely is the single market beta coefficient able to explain all or most return variability. In fact in 1992, Nobel Prize Winner Eugene Fama declared the single-factor model dead.[6]

[6] Black (1993)

The Arbitrage Pricing Theory (APT) developed by Ross (1976) relaxes the strict assumptions of the CAPM and suggests a more nuanced approach to asset pricing. The APT merely makes the much weaker no-arbitrage assumption. Security prices are not determined by a single universal market risk factor, but by multivariate and asset-specific factors. Proponents of the Efficient Markets Hypothesis (EMH) have argued that securities with nonzero alphas, when evaluated solely on the single factor, are likely to be exposed to multiple sources of risk that are unidentified by the single factor. Fama and French (1992, 1993) have famously shown how a 3-factor model constructed with size- (Banz, 1981) and value- (Basu, 1977, 1983) factor exposures have significantly greater explanatory power in the cross-section of stock returns. Price momentum (Jegadeesh and Titman, 1993; Carhart, 1997), a portfolio long past winners and short past losers, is commonly used as an extension to the 3-factor model.

The multifactor framework has helped to improve the predictive power of security prices, but at the expense of theoretical inference. Notably, there is no wide agreement as to whether many of these factors constitute rational compensation for risk or market mispricing. Risk-based models propose that securities with certain characteristics are subject to certain systemic risk factors and naturally require a risk premium. In contrast, behavioral models suggest that markets are not always perfectly efficient and security prices may deviate from fundamentals due to investor biases or other market frictions. Specifically, Fama and French (1992, 1993) argue that size – the phenomenon that small firms tend to outperform large firms on average – is a rational compensation for risk because smaller firms are exposed to a non-market, but systemic size related risk factor. Asness, Frazzini, Israel, Moskowitz and Pedersen (2015, working paper) show overrepresentation of low-quality firms in small cap equities, but document that the performance of small high-quality stocks is even stronger. Merton (1987a) has offered a behavioral explanation by arguing that small firms may be systematically underpriced because of incomplete information and investor negligence. The outperformance of value stocks – typically measured by book-to-market or price-to-earnings – is sometimes contributed to risk such as distance-to-default, high operating leverage or simply “cheap for a reason”. High book-to-market may rationally reflect negative prospects (and therefore a low market price) or it can be challenging to utilize large tangible assets in distressed markets. Conversely, it can be argued that the value-effect is an inefficient phenomenon that transpires from behavioral biases; potentially because investors selectively choose to invest in high-reward, exciting business opportunities. There is evidence that investors do not invest rationally (Kumar, 2009). Bali, Cakici, and Whitelaw (2011) find evidence of systematic overpricing of low-probability but high-payoff securities. Moreover, momentum may be exposed to crash risk, but there is some degree of consensus in the literature that trends arise from behavioural biases, such as initial under-reaction and delayed overreactions by investors (Hong and Stein 1999). Frazzini (2006) and Odean (1998) show that investors may be subject to a disposition effect – a tendency to realize profitable investments too early while holding on to losing investments for too long – which may explain trending prices.

A crucial limitation with the liberal application of the multifactor framework, in terms of both the risk-based and behavioural models, is to distinguish between actual phenomena and those discovered by statistical chance. (Merton, 1987b) argues that the danger of data-mining is much larger in financial economics. When a large number of researchers are running thousands of tests, build on other researchers' work, employ similar models, avoid the same pitfalls and apply it to largely the same datasets, tests are hardly independent and conventional criteria of significance are basically flawed (Harvey and Liu, 2014). Out-of-sample asset-pricing observations are scarce and once enough new samples have been obtained the underlying driver may have changed and the phenomenon disappeared (Black, 1993). In an out-of-sample investigation of 600 return factors Levi and Welch (2014, working paper) were able to confirm just half. Adjusting for data-snooping Harvey and Liu (2014) find that out of 315 factors only a handful of factors remain statistically significant. The size-effect discovered in the early 1980s may provide an example of this. Banz (1981) has shown that small-cap outperformed large-cap equities in earlier periods. However, out-of-sample the results have been mixed. The abnormal performance of SMB is concentrated only in the smallest equities (Crain, 2011), mostly in January (Keim, 1983), it has been particularly weak in subsequent periods (Schwert, 2003) and in international markets (Crain, 2011).

The pioneering work of Ross, Fama, and French has led small revolution in empirical asset pricing. Half a century after the introduction of the CAPM, Zhu and Liu (2013) identify hundreds of return drivers documented in the literature. Empirical asset pricing has discovered an exhausting list of factors including size, value, price-, earnings- and customer-momentum, reversals, quality, lottery-assets, low-volatility, liquidity, carry and many others.[7] However, a crucial implication is that the APT does not address what the underlying factors are: *"So, although the Capital Asset Pricing Models are clear about the major item to be measured, it is un-measureable. On the other hand, the Arbitrage Pricing Theory deals with items that might be easily measured but neglects to tell us what they are."* (Sharpe, 1984). The multifactor asset pricing framework relaxes unrealistic CAPM assumptions and provides flexibility, but at the expense of appealing theoretical underpinnings. As a result, *there is no clear consensus about what the relevant factors are, what makes them relevant or whether they will persist in the future.*

[7] References: (Asness et al., 2014; Ball and Brown, 1968; Banz, 1981; Basu, 1977, 1983; Bilson, 1981; Blitz and van Vliet, 2007; Carhart, 1997; Chen and Lu, 2014; Cohen and Frazzini, 2008; Foster, Olsen, and Shevlin, 1984; Jegadeesh and Titman, 1993; Kumar, 2009; Thaler and De Bondt, 1985)

2.3 Restricted Borrowing

Academia has rigorously debated the inadequacy of the CAPM. The multifactor framework offer a nuanced alternative, but at the expense of the appealing theoretical implications of the CAPM equilibrium. Alternatively, academia has discussed the assumptions behind the model. The single factor model rests on strong assumptions about market efficiency, investor behaviour and accessibility. While the assumption about homogeneous investors is strong, Lintner (1969) has shown that violations do not significantly change the predictions of the CAPM. Market efficiency remains a heated topic, however, assumptions about information efficiency, mean-variance investor behavior, and competitive markets are generally considered *reasonable* approximations of reality.[8] If taxes exists, as we know they do, and are differentiated between income and capital gains, as they often are, it could perhaps explain investor preference towards more volatile (non-income) securities, and to some extend higher prices than a frictionless pricing-model would predict (Black and Scholes, 1974; Miller and Scholes, 1982). Moreover, the model assumes that every valuable asset in the economy can be traded without restrictions; in reality this is an unrealistic prerequisite and it is not possible to observe the theoretical market portfolio for empirical purposes. Consequently, beta coefficients are computed erroneously and excess return estimates (alphas) are biased (Roll and Ross, 1994; Roll, 1977).

Black (1972) has argued that perhaps the most unrealistic assumption is the unlimited access to risk free lending and borrowing. By relaxing the assumption of unrestricted lending and borrowing Black (1972) shows that the slope of the SML may flatten and the intercept increase above the riskless rate. A general proposition of the CAPM is that investors always hold the tangency portfolio and leverage or deleverage to adjust their portfolios to their individual preferences. Investors that require higher-than-market returns to optimize their utility functions will simply purchase more of the market portfolio paid for with debt. However, as

[8] This is further discussed by Black (1972) in his presentation of CAPM with restricted borrowing.

very few investors have unconstrained access to debt financing, investors with insufficient capacity for leverage are forced to bid up high-beta asset prices and make low-beta assets attractive on a risk-adjusted basis. Thus, by introducing borrowing restrictions, Black's model suggests that low-beta securities will perform relatively well while high-beta securities will perform relatively poorly when evaluated on the standard CAPM.

The proposition finds empirical support. In fact, the flatness of the SML is arguable the most persistent anomaly in empirical asset pricing (Black, Jensen and Scholes 1972; Blume and Friend 1973; Lakonishok and Shapiro 1986). The line consistently overestimates the performance of high beta securities while underestimating the performance of low beta securities. Put differently, the sign of alpha coefficients tend to be positive for low-beta securities and negative for high beta securities. Black, Jensen and Scholes (1972) constructed a beta-factor portfolio: a portfolio of stocks, long low-beta stocks and short lesser amounts of high-beta stocks. The authors reported evidence of abnormal risk-adjusted returns in US equities in the period from 1931 to 1965, and thereby provided evidence for the theory of CAPM with restricted borrowing. Black (1993) later expanded the analysis to 1991. More recently, Frazzini and Pedersen (2014) constructed *betting against beta* portfolios, long low-beta and short high-beta leveraged to an ex ante beta of one and found similar evidence in the US and international equity markets, in bond-, credit-, futures-, and commodity- markets.

The notion and impact of restricted borrowing in financial markets finds evidence beyond beta-arbitrage. When risk is measured by return- or earnings volatility, riskier assets tend to underperform safer assets. This is true when comparing bonds and equities (Asness, Frazzini and Pedersen, 2012), within equities sorted by return volatility, idiosyncratic- and earnings- volatility (Ang, Hodrick, Xing, and Zhang, 2006; Baker, Bradley, and Wurgler, 2011; Blitz and van Vliet, 2007). Hence, the ramifications of restricted borrowing are evident outside the CAPM-framework.

It can be argued that leverage constraints have not been alleviated since the early literature (Black, Jensen, and Scholes, 1972; Black, 1972). In fact borrowing restrictions may have increased; investor protection remains or has strengthened.

Since the crisis of 2007-2008 new solvency laws require large institutional investors to maintain larger equity buffers than before. Counterparty risk is getting more attention, margin requirements remains in effect and asymmetric debt-financing has vastly increased in the low interest rate environment. From a behavioral perspective, there is no reason to believe that the fundamental behavior of investors has changed to become less debt averse. Therefore, the borrowing restrictions and the effects introduced by Black (1972) remains valid; the SML is apparently still flat and abnormal beta-arbitrage performance seems to persist (Frazzini and Pedersen 2014), making it perhaps one of the most interesting and robust asset-pricing puzzles in contemporary finance.

There are, however, competing explanations. (Bali, Brown, Murray, and Tang (2014) suggest that investor demand for lottery-like stocks, stocks with extreme tail-return distributions, produces a flatter SML and explains the excess performance of BAB portfolios. Cohen, Polk, and Vuolteenaho (2005) finds evidence that the SML is flatter when expected inflation is high, indicative of the money illusion (Modigliani and Cohn, 1979) whereby investors irrationally discount high premium (beta) securities relatively less compared with safer assets.

2.4 Beta-arbitrage with Frictions

Beta-arbitrage BAB portfolios have historically performed very well on average. One explanation is an equilibrium with restricted borrowing. It is plausible that constrained investors bid up the price of riskier assets, but less clear why arbitrageurs have not forced prices to converge. Baker, Bradley and Wurgler (2011) suggest that while behavioral or regulatory biases may force prices out of equilibrium, significant limits to arbitrage that discourages arbitrageurs from forcing prices to converge are essential to maintain disequilibrium.

Arbitrage barriers can largely be divided into implementation difficulty and investor constraints. It is interesting to consider whether these limits of arbitrage apply to beta-arbitrage; *may time-varying performance hold back arbitrageurs subject to benchmarks? Is illiquidity? Short-selling?* Next we consider investor constraints with regard to time-varying performance and implementation difficulty with regard to liquidity, size, and short-selling.

2.4.1 Time

Institutional investors may be faced with considerable barriers when engaging in arbitrage activity (Shleifer and Vishny, 1997). Textbook arbitrage is risk free and requires no capital upfront because investment outflows are exactly offset by proceeds from short-selling. Reality is rarely – if ever – completely risk free. The structure of the financial industry and the separation of agents and investors create additional barriers to arbitrage. Indeed, the well-known agent-principal asymmetry is an important obstacle to effectuating arbitrage opportunities, because agents are capital constraint and must compete for capital. Moreover, principal sentiments and capital allocation may be based on incomplete information. Consequently, if investments suffer temporary drawdowns, capital is scarce and reallocated quickly, capital outflow may occur before prices converge and force agents to liquidate otherwise profitable positions prematurely. Ironically, constraints and adverse sentiments may be highest when arbitrage is most attractive. Mitchell, Pedersen and Pulvino (2007) have shown this in merger- and convertible bond- arbitrage trades. As a result, arbitrage may be riskier in trades with frequent drawdowns, because agents are constrained by principals, compete for capital and face short-term performance pressure (Shleifer and Vishny, 1997).

Brennan and Li (2008) have argued that benchmarking may cause managers to care only about excess return and variance relative to the benchmark.[9] Baker, Bradley, and Wurgler (2011) have argued that benchmark mandates (given by principals) may impact the investment decision by discouraging investment professionals from exploiting low- volatility or beta arbitrage trades. Money managers are frequently mandated to maximize the return of their portfolios relative to a benchmark. Without leverage, low-risk strategies may then be unattractive for agents, despite being superior on a risk-adjusted basis. Thus, capital constrained investors may be unwilling (or unable) to pursue profitable arbitrage strategies with significant return volatility or prolonged periods of underperformance relative to a benchmark.

BAB portfolios are leveraged in order to exploit market-wide investor leverage constraints. Frazzini and Pedersen (2014), Huang, Lou, and Polk (2014, working paper) document time-variation of BAB returns. Frazzini and Pedersen (2014) suggest that the beta-arbitrage could be partially driven by funding liquidity risk; when funding constraints tighten, highly leveraged investors become exposed to margin calls and may be forced to descale their positions. In a new paper Chen and Lu (2014, working paper) show that the performance of hedge funds (read: leveraged investors (Ang, Gorovyy, and Inwegen, 2011))) are on average adversely affected 2% p.a. by a one standard deviation funding liquidity shock, evidence that leveraged BAB investors may be sensitive to funding liquidity shocks. Consequently, arbitrageurs with tight capital constraints or rigorous principal monitoring may favor strategies with high benchmark-specific information ratios or strategies where even transitory drawdowns are uncommon. To rephrase, if BAB portfolios

[9] Essentially, this is equivalent to the information ratio, excess return over the residual variation when evaluated on the benchmark: $IR^p = \frac{R^p - R^{BM}}{\sigma(R^p - R^{BM})}$

are subject to drawdowns when investor liquidity is already scarce arbitrageurs may not have the necessary flexibility to converge prices.

2.4.2 Size

In addition to the abovementioned asymmetric information obstacles, there are substantial implementation barriers. Foremost, investors have to pay large setup costs, file legal documentation, and hire expensive personal. Moreover, many quant-like strategies require frequent turnover and investors must pay broker commissions, transaction costs, and post collateral when buying on margin. The latter is especially strict when trading illiquid securities.

The empirical investigations of beta-arbitrage highlight the profit potential for leverage-unconstrained investors. However, in order to exploit the cross-sectional mispricing of securities, the BAB portfolios are constructed as leveraged market neutral long-short strategies, with frequent rebalancing and with no concern of size- or trade- difficulty. Yet, paper profits may substantially exaggerate live trading results. In one study Keim and Madhavan (1997) investigate the effect of transaction costs and market impact on different investment styles. The authors find that costs are economically significant and that transaction costs can substantially lower paper profits; the adverse effect decrease with size and increase with demand for trade immediacy. The effect is particularly strong in the lowest size-segments and for large trades, indicating that micro-cap strategies or strategies that rely on quick rebalancing might not be net profitable because trade difficulty is too high. Brennan (1993b) finds that large cap stocks are favored by institutional managers.

Trade difficulty tends to be higher when liquidity is low and liquidity is typically lower for smaller firms (Pastor and Stambaugh, 2003). Implicit costs increase because trading activity tends to be lower and prices more sensitive. Explicit transaction costs increase, because market-makers demand higher margin requirements and additional collateral for providing liquidity. From a purely risk-based perspective we would expect prices of the smallest securities, where liquidity is lowest, to be relatively lower because investors must be compensated for the additional risk. Acharya and Pedersen (2005) have developed a liquidity-based CAPM where the expected return of a security depends both on the market risk- and liquidity risk premia. Gârleanu and Pedersen (2011) derive a margin-based CAPM where the expected return of a security increases with margin requirements. Brunnermeier and Pedersen (2008) show in a model that market- and funding- liquidity can be mutually enforcing. Pedersen (2009) discusses shocks and liquidity spirals. In this view liquidity affect prices, not because arbitrageurs fail to force prices to converge, but because liquidity-risk is rationally priced.

According to Chen and Lu (2014, working paper) the margin for equities with a market capitalization less than \$250m is 100% with at least one large US broker. If frictions are negatively related with size, we can infer that prices can deviate more from fundamentals in the smallest securities, because there are larger obstacles to price convergence and ferocious margin constraints prevent arbitrageurs from offsetting this effect. According to Baker, Bradley and Wurgler (2011) low-risk performance is mostly persistent in the smallest equities. Malkhozov, Vedolin, Mueller, and Venter (2014) show that BAB portfolios perform better in the most illiquid securities, suggesting that – with size as a proxy for liquidity – beta-arbitrage may suffer from large trade difficulty. If beta-arbitrage is concentrated in the smallest securities, then large arbitrageurs may be inhibited from engaging in arbitrage, because price impact may be too great for managers with substantial assets under management. Leveraged investors may not have capacity to absorb the risk. Liu and Longstaff (2004) propose that underinvesting is often the optimal strategy because of margin constraints.

The size effect can be particularly damaging when positions have to be closed quickly (Keim and Madhavan, 1997). Adverse funding conditions may force capital constrained BAB investors to suddenly unwind large positions. If market and funding liquidity is scarcer in illiquid securities, then beta-arbitrage – a leveraged strategy – may not be attractive if it is too size-dependent, especially for large investors, and especially if unexpected funding shocks may lead to high trade immediacy.

2.4.3 Short-selling

The financial literature identifies a range of situations where short-selling is both feasible and vital in order to exploit situations where one asset seems overvalued relative to another. However, the market for short-selling is not frictionless. In fact, short-selling is far from frictionless and may in some circumstances be very costly.

In order to sell short a stock, a short-seller have to locate lendable shares, negotiate a lending fee, post collateral and equity margin, resell the share and – at the request of the lender – be able to return the share within three days.[10] Collateral is usually mark-to-market with daily settlements, if the margin account runs low the position will be (prematurely) closed. Moreover, the lender is sanctioned to recall the share at any time, at which time the borrower must repurchase the share in the marketplace. If the borrower fails to deliver, the lender can use the collateral to rebuy the owed shares (D'Avolio, 2002). If lendable shares are scarce and the investment has yet to materialize, the arbitrageur risk foregoing profits when the relative values eventually converges. Thus, the borrower is margin constrained, exposed to price- and alpha decay- risk. If lenders consider small equities riskier, it is likely that that short-selling costs and size is positively correlated, because investors demand a higher premium for owning shares to lend. In support, D'Avolio (2002) finds that size and short-selling fees are negatively associated and that the costs of lending are particularly high while availability is particularly low for the smallest equities.

Short-selling constraints may tilt the equilibrium price in favor of the most optimistic market participants. Miller (1977) theorizes that when short constraints are nontrivial and investors disagree, prices will be upward biased because short-sellers are cost restricted while buyers are not. Duffie, Gârleanu, and Pedersen (2002) theoretically show that when lendable shares are scarce, prices may be lifted temporarily, but gradually decline. Absence of shares to lend can occur when non-lending owners hold a large quantity of the share float (low supply) or when short interest is very high (high demand). Merger-arbitrage provides an example. Essentially the strategy involves buying the acquisition target while (frequently)

[10] According to US Regulation T (D'Avolio, 2002)

shorting the acquirer. Consequently demand for shares to sell can be very high and Geczy, Musto, and Reed (2002) finds that the performance of such strategies drop substantially net of costs. Similarly the authors find that IPO stocks are significantly more expensive to lend, albeit strategies remain profitable. Short constraints span across other return premia, (D'Avolio, 2002) show that low-momentum and glamour stocks – the short legs of momentum and value strategies – are more expensive to sell on average. However, the literature indicates that lending fees are trivial on average.

There is some evidence that short-selling high beta equities is costlier in general. Empirically and theoretically, the cost of short-selling is correlated with demand (D'Avolio 2002; Geczy, Musto and Reed 2002; Duffie, Gârleanu and Pedersen 2002). Brent, Morse and Stice (1990) find that beta and short interest is positively correlated. Similarly, MacDonald and Baron (1973) show that stocks with higher idiosyncratic risk have higher short interest. Others have reported that short interest is negatively correlated with future excess returns (Figlewski, 1981; Seneca, 1967). Similarly, Jones and Lamont (2002) report results that indicate securities that are expensive to short have low subsequent returns. Moreover, costs may be higher when arbitrage is most profitable. Huang, Lou, and Polk (2014, working paper) document that BAB returns are higher when beta spreads are larger and arbitrage activity is higher. BAB portfolios are short high beta portfolios and demand for lendable shares should be higher as more investors crowd to beta-arbitrage. If short-selling is too costly it might not be profitable to sell high beta equities short, and prices may stay high as a result.

3 Data & Methodology

In this section we describe the choice of samples used to construct and analyze BAB portfolios in a context of size, time and short-selling. Then we explain and outline the computation of variables and formation of portfolios and the factors used for evaluating the performance subsequently.

3.1 Data Samples

Data was collected from multiple sources. US equities data was collected from The Center for Research in Security Prices (CRSP) through the Wharton Research Data Services, while European equities data was gathered from Thomson Reuters' Datastream. Equity indices, corporate bonds, and government bond data was collected through Datastream and Bloomberg Services. All prices are dividend-adjusted total returns. Daily prices are used for individual equities and monthly prices for equity indices and bonds, although with some exceptions for the latter. Missing observations are filled with the last known traded value. Relatively rare events such as sudden delisting or trade suspensions have been removed entirely[11]. Finally, short-selling costs were obtained from Danske Bank A/S.[12] An overview is given in table 1 below. All returns are in excess of the risk free rate. For US denominated prices the one-month US Treasury bill rates were used. Treasury data was obtained from Ken French's website.[13] Gilts (UK government bonds) are in excess of the one-month UK Treasury rate. EUR returns are in excess of the one-month EURIBOR, and finally DKK returns are in excess of the one-month CIBOR.

[11] CRSP marks these events with "-99". We realize that this may create a survivorship bias, but we assessed that treating all such events as 100% losses would be less reflective of reality.

[12] Courtesy of Risk Advisory Division in Danske Bank A/S

[13] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Table 1: Overview of data sets and sources

Asset Class	Asset	Period (primo-ultimo)	Frequency	Currency	Source
Indices	MSCI AC	1995 – 2014	Monthly	USD	Datastream
	MSCI World	1995 – 2014	Monthly	USD	Datastream
	MSCI EM	1995 – 2014	Monthly	USD	Datastream
	S&P Industries	1989 – 2014	Monthly	USD	Datastream
	European Corporate	2000 – 2014	Daily	EUR	Bloomberg
	US Credit Indices	2004 – 2014	Daily	USD	Bloomberg
Govt. Bonds	US Treasuries	1988 – 2014	Monthly	USD	Datastream
	German Bunds	2000 – 2014	Daily	EUR	Bloomberg
	UK Gilts	1988 – 2014	Monthly	GBP	Datastream
	French OATs	1985 – 2014	Monthly	EUR	Datastream
	Spanish Public Treasuries	1998 – 2011	Daily	EUR	Datastream
	Danish Govt. Bonds	1998 – 2014	Daily	DKK	Datastream
Equities	S&P 1500	1995 – 2014	Daily	USD	CRSP
	Euro Stoxx 600	2000 – 2014	Daily	EUR	Datastream
	FTSE all shares	1994 – 2014	Daily	GBP	Datastream
	Nordic Equities	2000 – 2014	Daily	DKK	Bloomberg
Other	Risk factors	*Full period*	Daily	USD*	AQR**
	Short-selling costs	*January-March 2015*	NA	NA	

**Originally in USD but converted to local currencies for return calculated from local prices.*

*** https://www.aqr.com/library/data-sets*

3.1.1 Equities

To investigate the beta-anomaly in more investable markets, the regional equity market portfolios are constructed based on index representatives. Indices were chosen to capture the majority of the floating regional equity markets. For example, the US portfolio is constructed based on all members of the S&P 1500 Composite Index (SPX) in the period from January 1995 to December 2014. The index covers about 90% of total US traded market capitalization across all recognized sectors from small-, mid- and large cap segments. The full dataset with a relatively high constituent turnover totals of 3,294 unique stocks.

In Europe equity members from Euro Stoxx 600 (STOXX), FTSE All-Share (FTSE) and each of the most liquid equity market indices in the Nordic countries were used (NRDS). STOXX is a continental index that covers 18 European countries while FTSE covers UK listings, both representing stocks across all size segments[14]. Constructed to cover 90% and 99% of the freely floating market capitalization, STOXX and FTSE are very reasonably representatives of each of these markets, at least in equities. The data on FTSE begins in January 1994 until December 2014, while STOXX – a much newer index - begins in January 2000 and ends ultimo 2014. During the full sample period there are approximately 3,038 unique stocks in both indices combined.

The Nordic indices investigated hardly cover the entire market portfolio, but in order to calculate and compare results gross and net of short-selling costs – which was available only for the largest securities – the analysis was limited to these securities. The indices include about 100 securities from Denmark, Finland, Norway and Sweden in the period from January 2000 to December 2014. In the full periods

[14] We realize that there is some degree of constituent overlap in both indices and it should be no surprise to find similar results. An obvious implication is that we should be careful not to overemphasize the results as two completely independent studies confirming each other. We are not too worried about this, since our primary focus is not to verify the low-beta anomaly but investigate the details around it.

there are 208 unique constituents. To compute the international Nordic portfolio calendar days were standardized to Denmark, and blanks arising from non-synchronous trading were substituted for the last traded price.[15]

An implication of the index approach is constituent turnover. The constituent lists are updated annually and any firm not a member at the beginning of the year will not be included until the subsequent year. Likewise, firms leaving the index during the year will be included until year-end. Finally, firms that entered and left intra-year are excluded entirely. We are not too worried about biases as indices are usually only updated on a semi-annual basis.

Daily price data was collected for all equities. Prices are in local currencies, except for European and the Nordic equities, which are in EUR and DKK, respectively, because these are cross-country portfolios. Returns are dividend-adjusted and in excess of the appropriate risk free rate as reported in section 3.1. Additionally, daily market capitalization values were collected for US equities. Market values are in USD and are later used to rank stock portfolios on size.

3.1.2 Equity Indices

In order to test the low-beta anomaly in more liquid securities additional data was obtained on four overlapping equity market indices. For convenience equity indices were extracted directly rather than combining the sectors based on industry tickers ourselves. In addition, this approach would more accurately reflect investors trying to pursue the low-beta anomaly in liquid futures or derivatives securities only.

Monthly dividend-adjusted total returns were collected from the Datastream database in the period from January 1994 to December 2014. Returns are in USD and in excess of the US 1 month Treasury bill. The data includes three different MSCI indices and the S&P index. Each of these indices cover the majority of the tradable equity markets in both different regions and globally. We further divide

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[15] This effect happens when markets are open and closed on different days across countries. In section 3.2.1 (beta estimation) we will discuss how we handle this with regard to beta coefficients.

them into subsets of 24 industry groups based on the *Global Industry Classification Standards* (GICS). This allows us to construct BAB portfolios based on entire industries rather than individual securities. In other words this approach explicitly bets on low- versus high- beta industries as opposed to individual stocks (sector neutral or not).

We use the *MSCI All Countries World Index (MSCI ACWI)* which covers 2,469 constituents across 46 developed and developing countries and approximately 85% of the investable equity in the respective markets. The index is value-weighted and naturally dominated by the US (about 50%). Sectors are distributed relatively evenly, although with a slight tilt towards financials.

The *MSCI World (MSCI World) and MSCI Emerging Market (MSCI EM)* indices are subsets of the aforementioned ACWI index, each consisting of 23 developed and developing countries, with 1,633 and 836 constituents, respectively. Similarly, these indices are value-weighted and the former is clearly dominated by the US while EM is more evenly distributed, although with some tilt towards China and South Korea.

Finally, the *S&P 500* index (SP Industries) is considered. The index consists of 500 members, it is designed to capture the bulk US traded equities, or equivalent to 80% of outstanding market capitalization. Unlike the MSCI indices, the S&P 500 is a large-cap index. For better comparison we would have preferred to use the MSCI USA Index, but unfortunately the timespan available to us was much shorter, and a longer time-series albeit with less comparability was preferred.

It must be emphasized that these securities share significant constituent commonality and we realize that the results cannot be treated as entirely independent samples. As Black (1993) rightly points out we are not surprised to confirm an anomaly in an aggregated format already registered in a subset. However, if we think about it as a top-down approach it will allow us to see if the low-beta anomaly is consistent globally and then especially whether it holds in both developed and developing markets.

3.1.3 Bonds

Bond data was collected from Datastream and Bloomberg in the period from January 1988 to March 2015, although not all return series for the full period. The data includes six different government bond indices, a US Credit index, and a European corporate bond index. All returns are in local currencies and in excess of the appropriate risk free rate as discussed above. To follow the approach of Frazzini and Pedersen (2014) bonds are sorted by maturity with maturities ranging from 1-3 years up to +15 years. Maturity, as a proxy for duration, proxies for beta as a measure of sensitivity with the underlying market.

Non-government bonds were sorted by maturity as well. Frazzini and Pedersen (2014) show that speculative debt is associated with low risk adjusted returns, e.g. investment grade outperforms low grade bonds. In the sample used for this investigation we only consider investment grade bonds (ratings from AAA to BBB). Thus, while default risk is not removed, the analysis differs from the literature by only investigating the low-beta anomaly in interest rate- rather than across credit-risk arbitrage.

3.1.4 Short Sample

Short-selling costs were obtained from a Danske Bank A/S. The sample includes lending fees – the cost of borrowing equity securities to sell short – the number of shares available to lend as a proportion of total lendable shares and total short interest. Short interest is the number of shares sold short out of the total equity float. Short utilization is the proportion of available shares to short actually shorted. This information is used for a comparison of excess returns gross- and net of lending costs and a brief discussion of the propensity of early forced termination of the short positions. The sample is limited in several ways. By availability, it is restricted to the Nordic region and only includes members of the largest indices, reducing the sample size to 100 individual equities at any one time. The Nordic large cap market is a tiny fraction of financial markets and may as a result be a poor representative. Large-cap securities tend to be more liquid and cheaper to sell short. The sample does not contain data on firms that were index constituents in prior periods nor does it contain information about the cost at different times, this is problematic given that lending fees are not static (Jones and Lamont, 2002). The cost variables are computed as the daily average cost of selling short each of the 100 securities over the past 30 and 90 days. The data was obtained in early March 2015 and thus describe the average cost back to approximately the beginning of December 2014.

Table 2: Overview of short-selling sample

The table reports the value- and equally-weighted averages across 100 Nordic equities from the largest indices. Fees are reported as the annualized averages of the past 30 and 90 days, respectively. Short interest and utilization reports the current statistics as of March 2015.

Index	No. of constituents	30 days fee p.a.	90 days fee p.a.	Short Interest	Short Utilization	Total Market Capitalization DKKb
Market capitalization weighted averages						
OMX	30	0.25%	0.18%	1.59%	9.43%	4,122
KFX	20	0.38%	0.27%	1.50%	8.22%	1,587
OBX	25	0.50%	0.45%	2.58%	24.71%	831
HEX	25	0.45%	0.3%	3.81%	24.33%	104
Total	**100**	**0.32%**	**0.24%**	**1.73%**	**11.29%**	**6,644**
Equally-weighted averages						
OMX	30	0.32%	0.24%	2.23%	13.51%	4,122
KFX	20	0.4%	0.3%	2.54%	13.85%	1,587
OBX	25	1.12%	0.99%	5.43%	47.15%	831
HEX	25	0.82%	0.55%	4.34%	29.57%	104
Total	**100**	**0.68%**	**0.52%**	**3.6%**	**25.89%**	**6,644**

In aggregate the numbers are similar with D'Avolio (2002) who finds that the annual cost of lending stocks in the US was 25bps between April 2000 and September 2001. In NRDS, within the 90 days interval 16 out of 100 shares fall into the *specials* category[16]. The highest fee in that period was slightly above 400bps. Within 30 days twenty shares classify as specials with the highest fee at nearly 436bps. Several of the costliest stocks to lend operate in the energy sector, a challenged industry given low energy prices at this time. The value-weighted average of specials was 1.87% in the last 90 days and 1.84% in the last 30 days. Only a single stock has zero short interest (Nordea), the quoted fee was not particularly high or low. When specials are excluded the value-weighted average fee drops to roughly 20bps for both 30 and 90 days, roughly equivalent to the medians of the entire sample. This compares – and is quite similar – to the US

[16] Defined by D'Avolio (2002) as stocks with lending fees above 1% p.a.

sample from April 2000 to September 2001 investigated by D'Avolio (2002), who finds a value-weighted average of specials at 4.3% and a small 1% of non-sellable securities. Here the value-weighted lending fee of non-specials was at 17 bps.

The smallest fees in the sample were 6 and 8 bps p.a. and the value-weighted average of the bottom quintile (by rank, not market capitalization) was only 13 (16) bps over the last 90 (30) days. The quintile composes nearly 50% of total market capitalization and a small majority of stocks are classified as low-beta stocks. Appendix 1 provides an overview of the securities in both ends of the spectrum.

The descriptive summary hints that short-selling constraints are relatively small, at least in the Nordic sample. This is unsurprising given that the sample only includes the largest and most traded equities in the Nordics. Size is negatively correlated with short-selling constraints D'Avolio (2002). Within the space of short-sellable securities the sample seems consistent with prior expectations and might not be too far off in terms of representing the broader equity markets, at least within large-cap securities.

3.2 Methodology

For the purpose of constructing and testing the performance of BAB portfolios we apply a similar methodology as Frazzini and Pedersen (2014). First, we show how ex ante betas are estimated. Then we go through the portfolio formations and how the performance of the final portfolios will be evaluated. All portfolios are constructed using only information that was *then* available.

3.2.1 Estimating Betas

In order to rank and assign securities into portfolios ex ante betas are estimated. Betas are estimated with rolling univariate least square regressions. The approach is no different than conventional financial literature, formally we write:

$$R_t^i = \alpha_t^i + \beta_t^i R_t^M + e_t^i, \qquad where\ \beta_t^i = \frac{COV\left(R_{t-1,t-1-l}^i, R_{t-1,t-1-l}^M\right)}{VAR\left(R_{t-1,t-1-l}^M\right)}$$

Where capital R denotes excess returns and l denotes the look-back period.

Betas are computed using excess returns regressed on the appropriate underlying market portfolio. Excess returns are calculated as the net change in the dividend-adjusted price, less the appropriate risk free rate. The underlying market portfolios are asset- and regionally- specific. Bond market portfolios are constructed in every country as an equally-weighted average of all bond maturities in that country. Equity market portfolios are the value-weighted average of the underlying indices. These were obtained directly from Datastream. The NRDS is the equally-weighted average of the value-weighted KFX, OBX, HEX and OMX indices. In line with Frazzini and Pedersen (2014), we use segmented market proxies in the respective markets to reduce noise and easier correction for nonsynchronous trading.

Technically the results will be biased, because regressing BAB-returns on a false market portfolio may lead to erroneous beta and alpha coefficients. However, the authors find that the results are robust using either approach, segmented or an aggregated world market portfolio.

High data frequency is preferred. When available we use daily observations, but employ monthly observations when necessary. At least 250 trading days are required for our beta estimations using daily observations and at least 36 observations when only monthly data is available. Betas are computed as rolling one-factor coefficients over the past 250 (36) observations. To ensure that the portfolio approach is implementable, all computations are based only data available at formation.

Daily frequencies offer superior precision, but also introduce potential market friction issues such as nonsynchronous trading and delayed price reactions. When securities trade on different calendar days – for example due to country-specific calendars – beta coefficients of individual securities will be erroneous because price movements will be asynchronous. Similarly, market frictions such as incomplete information and limited market participation can lead to delayed price discovery (Hou and Moskowitz, 2005; R. Merton, 1987). As a result the true covariance between a security and the market is not completely revealed in the simultaneous price movements alone. To avoid these biases betas are adjusted by employing four periods of lagged market returns in addition to the ordinary regression so that any delays should be captured in the adjusted beta. Hou and Moskowitz (2005) argue that most assets respond to information within a month and the procedure is avoided entirely when using monthly returns.

Securities with high beta coefficients are placed into one portfolio while low beta securities are placed into another. We cannot observe the true betas and the coefficients are estimated with some measurement error. This bias will tend to be positive for high-beta values and a negative for low-beta values, even if the measurement error is symmetric (Vasicek, 1973). Consequently, the alpha values of high beta securities will be underestimated while the alpha values of low beta securities will be overestimated. To account for this and deal with extreme outliers beta coefficients are adjusted towards the cross-sectional mean:

$$E(\beta_{t-1}^{i}) = \delta_{t-1}^{i}\beta_{t-1}^{i,TS} + (1-\delta_{t-1}^{i})\beta_{t-1}^{XS}$$

Where β^{XS} is the mean of the beta coefficients and δ^{i} is a shrinkage factor. Frazzini and Pedersen (2014) employ a fixed shrinkage factors $\delta = 0.6$ and

$\beta^{XS} = 1$. Fixed shrinkage parameters are common in finance, for example Bloomberg's adjusted betas are calculate with $\delta = 2/3$ and $\beta^{XS} = 1$. The disadvantage of applying a fixed shrinkage factor is that more accurate estimates will be over-adjusted while less accurate estimates will be under-adjusted. Vasicek (1973) has showed that the estimation accuracy of beta coefficients with regard to security prices can be improved and propose a dynamic Bayesian shrinkage factor computed as the variation in the beta coefficient of a security with respect to the cross-sectional variation of betas.[17] Thus, security betas with large relative variation are *penalized* harder and adjusted closer towards the best prior estimate of the center (median applied here). Accurate ex ante beta estimation is important in order to construct beta-neutral portfolios, but it has no effect on the long or short assignment of securities.

The universe of securities under investigation differ significantly from (Frazzini and Pedersen, 2014) and the abovementioned shrinkage factors might not be optimal for this analysis. Following Vasicek (1973) we compute rolling shrinkage factors using 250 daily beta observations. The estimated factors are relatively constant over time, but vary significantly across securities and sample sets. In fact, when tested on a few samples we found that simply applying fixed factors carried very similar results. Frazzini and Pedersen (2014) attempted several beta estimation techniques with similar results as well. However, we found that beta coefficients were more accurate when applying dynamic shrinkage factors so we chose this approach. For simplicity, shrinkage factors were not applied to bonds or equity indices. Bond portfolios are formed with only five to six maturity intervals

The BAB strategies are evaluated using realized, ex post, betas not subject to estimation bias and these are not adjusted.

[17] Vasicek (1973) calculates the shrinkage factor as $\delta_{t-1}^{i} = 1 - \frac{VAR(\beta_{t-1}^{i})}{VAR(\beta_{t-1}^{i}) + VAR(\beta_{t-1}^{XS})}$.

3.2.2 Betting Against Beta Portfolios

Following the approach of Frazzini and Pedersen (2014) the BAB factor is constructed from two portfolios; a long portfolio consisting of low-beta assets and a short portfolio consisting of high-beta assets. Low and high betas are defined with respect to the median of the sample under investigation. In the equity portfolios, the assets are weighted by beta rankings, where the lowest beta assets carry the highest weight in the long low-beta portfolio and highest beta assets carry the highest weight in the short high-beta portfolio. Only data available at formation is used. The beta-weight relationship is constructed to be linear:

$$w_t^{i,L} = \left(Rank(\beta_{t-1}^i) - Rank(\beta_{t-1}^{XS})\right) X_L^{-1}, \qquad iff.\ \beta_{t-1}^i \leq \beta_{t-1}^{XS}$$

$$w_t^{i,H} = \left(Rank(\beta_{t-1}^i) - Rank(\beta_{t-1}^{XS})\right) X_H^{-1}, \qquad iff.\ \beta_{t-1}^i > \beta_{t-1}^{XS}$$

Where ranks are descending with beta, ranked β^{XS} is the median rank and X_k are the two constants that sets the combined weights of each leg to $w^L = w^H = 1.$

The procedure is advantageous to beta-weighting because it is less sensitive to estimation error. When weighting securities by rank only relative coefficients matter. For convenience portfolios created from only relatively few securities (government bonds and equity indices) are equally-weighted. However, the results were almost identical when tested on government bonds, albeit with a slight tilt towards the shortest and highest maturities. Moreover, the beta factors constructed by and later extended by Black (1993) was created with equally weighted portfolios. So while the monotonic relationship between alpha and beta documented explicitly by Frazzini and Pedersen (2014) suggests that ranked weights will perform better, we expect to find positive alpha nonetheless.

After assigning and weighting securities into a long and short portfolio both legs are leveraged to an ex ante beta of one. The long leg is leveraged while the short leg is deleveraged. The purpose is twofold. First, we want to eliminate the market exposure of the portfolio. Second, while low-beta securities tend to yield higher risk-adjusted returns the nominal returns of high-beta securities tend to be higher.

Therefore, the returns of an unlevered portfolio are likely to be negative. Portfolio rebalancing and leveraging occurs monthly. The final time t return is written as:

$$R_t^{BAB} = \left(\frac{R_t^L}{\beta_{t-1}^L}\right) - \left(\frac{R_t^H}{\beta_{t-1}^H}\right)$$

Where capital letters denote returns in excess of the risk free rate.

3.2.3 Performance evaluation

To investigate the performance of the equity portfolios, the BAB returns are evaluated against common asset pricing factors. Risk factors are constructed as self-financing dollar market neutral portfolios with specific tilts that empirically have carried abnormal risk-adjusted returns when evaluated in the CAPM. The long-short portfolio approach enables an investigation of specific return drivers separately and reduces colinearity between factors, reducing estimation variance. More specifically the excess performance, alpha, is computed as the intercept of a multivariate regression between BAB returns and five factors:

$$R_t^{BAB} = \alpha_t^{BAB} + \beta^M MKT_t + \beta^{SMB} SMB_t + \beta^{HML} HML_t + \beta^{UMD} UMD_t + \beta^{QMJ} QMJ_t$$

Where MKT is the underlying market used for estimating ex ante beta coefficients. The remaining factors were obtained through AQR's website on a country-specific basis. The data was retrieved from AQR's website rather than from Ken French in order to obtain factors for local markets. When necessary return series are adjusted to local currencies with spot prices from Bloomberg, but the difference was trivial. Following the approach of Fama and French (1992, 1993) SMB and HML are conventional size and value factors sorted on market capitalization and book-to-price. Value is long high book-to-price and short low book-to-price. Size is long small firms and short large firms. Portfolios are equally weighted. UMD is price momentum (Carhart, 1997; Jegadeesh and Titman, 1993; Moskowitz, Ooi, and Pedersen, 2011) a portfolio long winners and short losers of the past 12 months excluding the last month. Finally, QMJ is a long-short portfolio long high quality and short low quality firms (Asness, Frazzini and Pedersen, 2014, working paper). While size, value and momentum are almost invariably used for investigating securities' returns in the cross-section, the quality-factor is added because it can be argued that low return variability is a measure of quality. We expect at least some loading, since low-beta is one of many components used to compute the quality score in article. We do not control for a liquidity premium (Pastor and Stambaugh 2005) for practical reasons.

As discussed, there is no broad consensus as to the underlying explanations of these factors. Regardless, whether these return premia reflect risk or market inefficiency, evaluating the performance of BAB returns provides valuable information. The factors have been well-documented, can easily be replicated or purchased by relatively sophisticated investors. Thus multivariate evaluation highlights the novelty of beta-arbitrage and the value in terms of excess performance for institutional *smart-beta* investors.

4 Results & Analysis

In this section we review the performance of conventional *betting against beta* portfolios based on Frazzini and Pedersen (2014). Then we compute and discuss factor-loadings and alternative sources of risk. Israel and Moskowitz (2012) investigate the effects of size, short-selling and time on classic value and momentum strategies. Similarly, we expand our investigation by considering the performance of BAB returns over time, across size and with respect to short-selling.

4.1 Betting Against Beta

The first step is to confirm the low-beta anomaly, then we discuss equity factor exposure and return variation. We construct and review 16 *betting against beta* portfolios across markets and securities. The portfolios are constructed as Frazzini and Pedersen (2014) with the notable exception that equity portfolios are formed only with constituents of certain equity indices.

4.1.1 Bonds

Out-of-sample evidence of the low-beta anomaly has been amply documented in equity markets, but so far the empirical investigation of fixed income securities has been limited to the US. Below the investigation is expanded across multiple international bond markets. Bond portfolios are sorted by maturity, long low-maturity and short high-maturity. All portfolios are constructed as self-financing long-short strategies. The results confirm those of Frazzini and Pedersen (2014) in both US Treasuries and US credit indices, and provide additional evidence of the low-beta anomaly across countries not previously investigated in the bond markets.

Table 3: Summary of BAB performance in Bonds

The table summarizes bond portfolios sorted on maturities. Alphas are annualized and in excess of the risk free rate. Betting against beta portfolios are market neutral strategies long low maturity and short high maturity bonds leveraged to an ex ante beta of one. The portfolio legs are equally weighted and rebalanced monthly. Betas are estimated using 250 daily (36 monthly) excess returns regressed on the returns of an equally-weighted average portfolio. Test statistics are shown below in brackets. Maturity portfolios are reported unlevered.

Asset	1-3 years	3-5 years	5-7 years	7-10 years	+10 years (10-15 years)	+15 years	BAB
US Treasury	0.6%** (3.51)	0.45% (1.75)	0.21% (1.09)	-0.39%** (-3.06)	-0.86% (-1.43)	NA NA	1.50%* (2.58)
German Bunds	0.82%** (4.76)	0.6%* (2.35)	0.34% (1.42)	-0.04% (-0.21)	-1.72% (-2.46)*	NA NA	1.60%* (2.36)
UK Gilts	1.34%** (4.84)	1.20%** (3.35)	0.59% (1.83)	-0.18% (-0.74)	-0.72%** (-2.66)	-2.23%* (-2.51)	2.72%** (3.87)
Spanish Public Treasury	2.06%** (3.93)	0.11% (0.16)	-0.12% (-0.21)	-0.74% (-1.27)	-1.31% (-1.29)	NA NA	3.20%** (2.54)
French OATs	0.52%** (2.84)	0.38% (1.86)	0.13% (0.93)	-0.09% (-0.74)	-0.94% (-2.02)	NA NA	1.20%* (2.24)
Danish Govt. Bonds	1.64%** (4.59)	0.33% (0.96)	-0.01% (-0.02)	-0.19% (-0.62)	-1.77%** (-2.36)	NA NA	3.34%** (3.69)
European Corporate	1.2%** (6.83)	0.7%** (3.52)	0.3% (1.54)	-0.3%* (-2.23)	-1.8%** (-3.4)	NA NA	2.7%** (4.93)
US Credit Indices	1.49%* (3.22)	1.07%* (2.53)	0.94%* (2.06)	-0.66% (-1.62)	-0.55% (-0.81)	-2.28%* (-2.22)	3.40%** (4.26)

** Indicates strong significance (99%), * Indicates significance (95%).

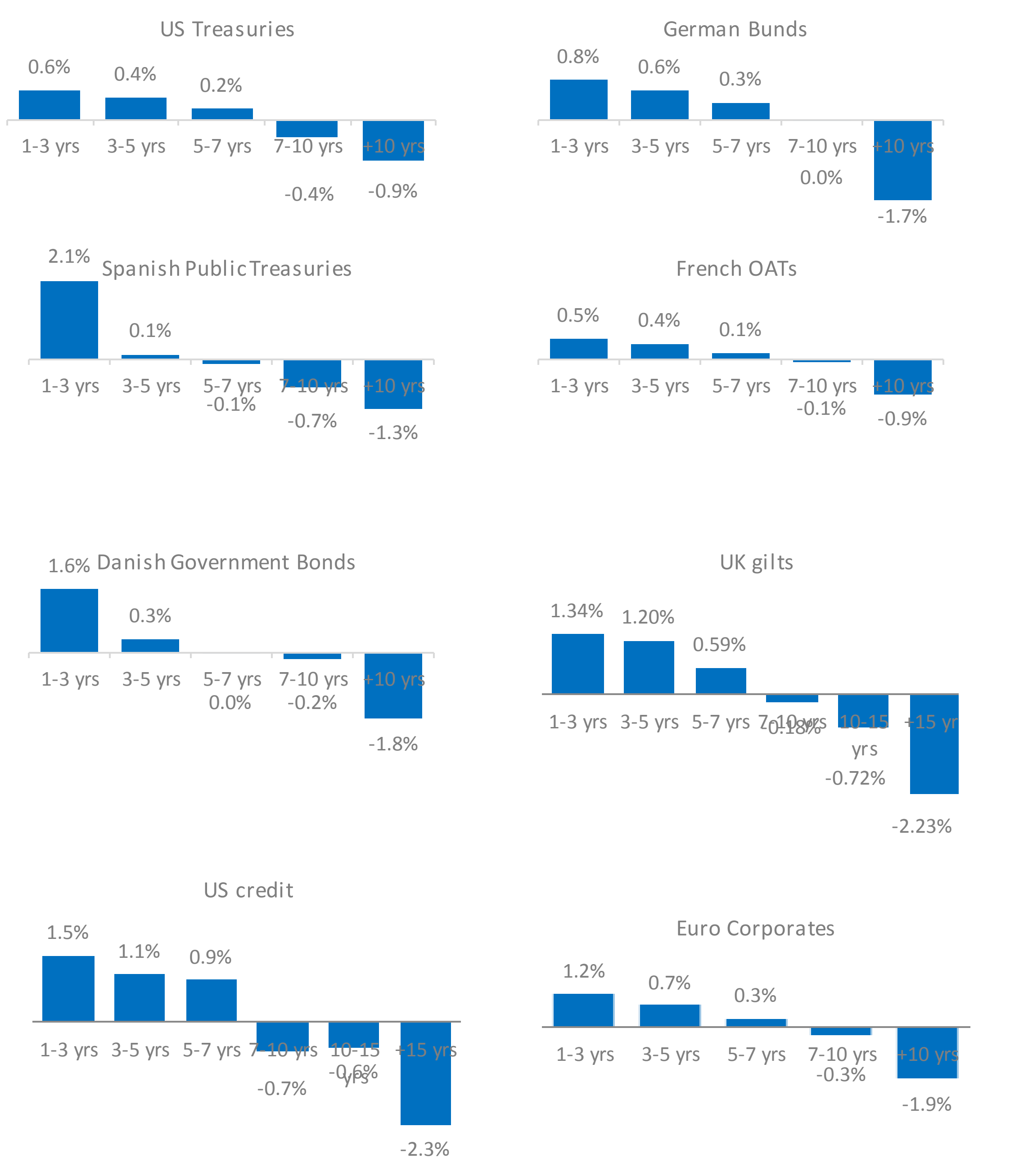
The alphas and test statistics of eight *betting against beta* bond portfolios are summarized in table 3. Below, Chart 2 illustrates the relationships. Each of the eight BAB portfolios is statistically significant. Alphas decline monotonically with betas (maturity) in every instance without exception. The consistency in the relationship between alphas and betas is evidence for the flatter security market line proposed by Black (1972) and the severe impact that borrowing restrictions might have. Further performance statistics are presented in appendix 2.

While the equity sample cover periods of expansion and recession in equity markets, this is not the case for bonds. The bond market has been in a long-trending bull market for several decades with ever-falling interest rates, currently a significant number of which are near zero. This has two broad implications: *First,* in this analysis we are unable to cover the performance of bond BAB portfolios when interest rates are increasing. Given that this environment is unlikely to continue indefinitely, it is unrepresentative across time and we might expect a different behavior in the bond market in the future. *Second,* in a market of exceptionally low interest rates investors have been crowding towards anything with higher yields, across asset classes (as is evident from the recent equity bull market) and – we suspect – within asset classes. Fixed income investors looking for higher yields are forced to buy more duration, and given that duration is empirically, and intuitively, equivalent to beta this might have created a market where high-beta bonds performed better than it would have in different interest-rate cycles.

Regardless, low maturity delivers a superior return-risk trade-off versus high maturity, e.g. alphas and Sharpe ratios decline as maturity increases. Several of these tests range back only a decade, a period when interest rates have been particularly low and even moderate yields increasingly scarce. While this could tempt some investors into higher maturity bonds, from a mean-variance perspective unrestricted investors should strictly prefer lower maturities levered to the desired level of expected returns. However, as we have only considered maturity risk, we cannot discard other sources of risk as explanatory factors for the apparently abnormal performance.

Chart 2: Summary of maturity and risk-adjusted performance

The charts show annualized alphas of bond indices in eight different markets. Alphas are calculated in excess of the relevant risk free return and evaluated on an equally-weighted bond portfolio of the underlying market. Alphas are unlevered. Maturity increase from left to right with decreasing alphas. This is evidence that the CAPM-SML is consistently flatter than predicted.



4.1.2 Equity

Table 4A below reports the alphas and t-statistics of eight betting against beta equity portfolios. With one exception the results confirm those of Black, Jensen, and Scholes (1972), Black (1993), Frazzini and Pedersen (2014). Returns, standard deviations, and Sharpe Ratios are summarized in Appendix 3.

Table 4A: Summary of BAB performance in equities

The table reports 1-factor alphas and t-statistics in brackets. Alphas are annualized and in excess of the risk free rate. Portfolios are sorted into two portfolios; betas below the median are "low" while betas above are "high". Betting against beta portfolios are market neutral long-short strategies, long low- and short high- portfolios leveraged to an ex ante beta of one. The left section summarizes the results of BAB portfolios constructed on 25 industry groups. Industry groups are equally-weighted. The right section summarizes the results of BAB portfolios constructed on individual equity securities. Equities are weighted according to their betas, giving higher weights to low betas in the long portfolio and lower weights to low betas in the short portfolio. Betas are estimated using 250 daily excess returns regressed on the respective value-weighted market index. The reported "Low" and "High" portfolios are unlevered.

Industry sorted equities (indices)				Regular BAB portfolios			
	Low	High	BAB		Low	High	BAB
MSCI ACWI	2.11%* (2.09)	-1.88%* (-2.18)	4.73%* (2.25)	SPX	2.1%* (2.28)	-4.4%** (-3.61)	7.04%** (2.76)
MSCI World	2.02%* (2.52)	-1.89%* (-2.45)	3.96%* (2.19)	FTSE	2.8%** (2.88)	-1% (-1.24)	11.2%** (4.45)
MSCI EM	4.17%** (4.33)	-3.7%** (4.46)	7.51%** (4.39)	STOXX	5.6%** (3.17)	-5.5%** (-4.79)	11%** (4.63)
S&P Industries	-0.8% (-0.68)	0.1% (0.14)	-1.9% (-0.79)	NRDS	3.7%** (2.04)	-2.1% (-0.67)	14.1%** (3.84)

** Indicates strong significance (99%), * Indicates significance (95%)

As proposed, BAB portfolios carry positive excess returns and superior risk-adjusted returns with little or no market exposure. This holds when BAB portfolios are constructed from index constituents, rather than the entire investable universe. More precisely, the right panel reports regular BAB portfolios long low-beta and short-high beta equities. Sophisticated investors – defined as those with capabilities to buy and sell short large numbers of equities – with liberal access to debt financing could have traded this opportunity and received abnormal risk-adjusted returns. 1-factor alpha coefficients are positive and significant in all four BAB portfolios; low beta yield positive returns while high beta yield negative risk-adjusted returns.

The left panel comprise of four portfolios long low-beta industries and short high-beta industries. As suspected these portfolios are on average long sectors such as health, utilities, and tobacco while being short automobiles, construction, and technology. The leftmost BAB portfolios explicitly bet on defensive versus aggressive industries. The results are generally consistent with Asness, Frazzini, and Pedersen (2014, working paper), who finds that the beta factor delivers abnormal risk-adjusted returns across and within industries. The beta anomaly is persistent in three of four tests, but the results of the industry-sorted S&P portfolio are inconsistent with our expectations and BAB returns are negative. Plotting the cumulative return spread, in our sample US high-beta outperform low-beta industries for nearly a decade beginning after the DOTCOM crisis and while low-beta industries perform well both before and after the BAB portfolio never recovers in the sample.

Semi-constrained investors – unable to construct equity-quant portfolios in a large universe of securities or with too much market impact to trade individual securities, but have access to leverage – can potentially implement BAB strategies across entire sectors. However, the evidence reported is much weaker compared to a regular implementation in individual securities.

Chart 3 visualizes the performance of the four equity BAB portfolios. The periods are not time-matched and the visual presentation cannot be interpreted as such. The underlying markets are deleveraged to match the ex post volatility of the BAB portfolios for better comparison. BAB portfolios outperform the respective markets, but performance is variable and all four portfolios experience drawdown periods. Visually, large movements are affected by compounding effects because the vertical axes are not log-scaled.

Chart 3: Cumulative return of BAB portfolios compared with the underlying market

The four charts plot the cumulative returns of the four equity BAB portfolios compared to the underlying market using daily returns. Returns are in excess of the risk free rate and scaled to the same volatility for better visual comparison. The charts plot the full periods of respective samples, excluding the initial years of data required to create the BAB portfolios.

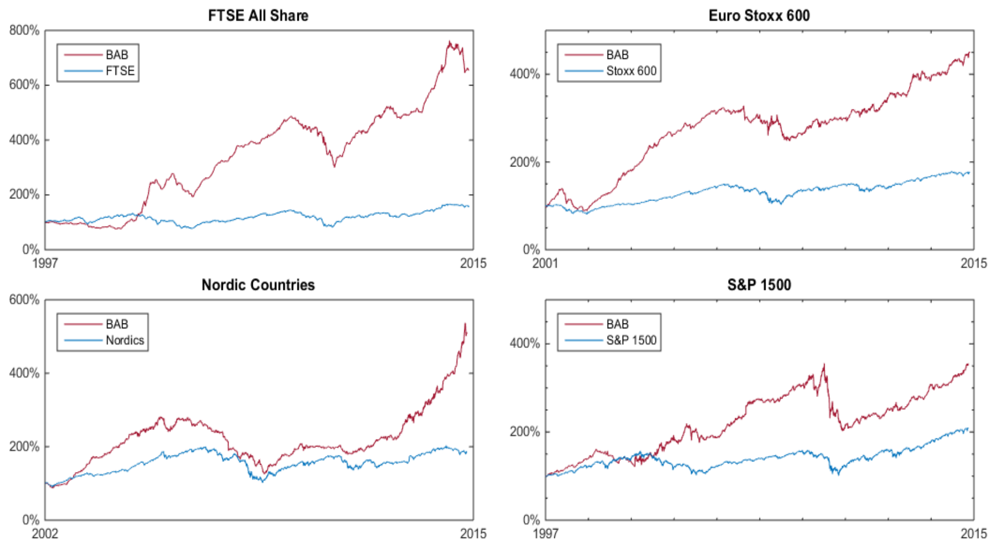


Table 4 below presents the factor-loadings and excess annualized alphas of 1-, 3- and 5-factor models. From a theoretical perspective this is important in order to distinguish the novelty of the BAB factor, and from a practitioner's perspective it is interesting because beta exposures can mostly be traded at very little cost. We use the standard 3-factor Fama-French model (1992, 1993) with size (SMB) and value (HML) factor loadings. The 5-factor model further includes price momentum (UMD) and quality (QMJ) factor loadings.

Table 4B: BAB portfolio alphas and factor-loadings

The left panel reports BAB alphas in excess of 1-, 3- and 5-factor model and corresponding t-statistics in brackets. Alphas are annualized. The right panel reports factor loadings regressed as one combined multivariate regression with all 5-factors. The factors include the underlying market index (MKT), the size-effect (SMB), value (HML), price momentum (UMD) and quality (QMJ). Betas are computed based over the entire back-test period. T-statistics are shown below in brackets.

	Excess Alphas			5-Factor Betas				
Asset	1-factor	3-factor	5-factor	MKT	SMB	HML	UMD	QMJ
SPX	7.04%** (2.76)	7.65%** (3.07)	4.3% (1.81)	0.10** (11.45)	-0.23** (12.13)	-0.00 (-0.05)	0.24** (21.42)	0.11** (5.03)
FTSE	11.2%** (4.45)	9.7%** (4.0)	7.1%** (2.98)	0.08** (3.47)	0.24** (9.11)	0.32** (9.45)	0.03 (1.2)	0.31** (6.92)
STOXX	11%** (4.63)	11.2%** (4.74)	10.2%** (4.35)	0.06** (7.63)	-0.09** (-5.27)	-0.02 (-0.54)	0.02 (1.02)	0.08* (2.57)
NRDS	14.2%** (3.99)	14%** (3.82)	14.1%** (3.83)	0.01 (1.4)	-0.04 (-1.64)	0.00 (0.04)	0.00 (0.12)	-0.03 (-1.08)
MSCI ACWI	4.73%* (2.25)	4.4%* (2.16)	0.6% (0.27)	0.26** (4.38)	-0.17 (-1.21)	0.03 (0.22)	0.18** (3.55)	0.27 (1.62)
MSCI World	3.96%* (2.19)	3.6%* (2.01)	-0.1% (-0.43)	0.29** (5.63)	0.02 (0.19)	0.1 (1.63)	0.14** (4.12)	0.33** (3.03)
MSCI EM	7.51%** (4.39	6.9%** (4.0)	4.4%* (2.18)	0.02 (0.44)	0.03 (0.27)	0.29** (2.58)	0.1* (2.3)	0.2 (1.52)
SP industries	-1.9% (-0.79)	-1.5% (-0.63)	-2.2% (-0.9)	0.09 (1.88)	-0.13 (-1.47)	-0.11 (-1.45)	0.04 (0.83)	0.05 (0.58)

** Indicates strong significance (99%), * Indicates significance (95%).

Table 4B summary generally shows that 3- and 5-factor alphas are lower than single factor alphas, because factor-loadings and returns are mostly positive. Nonetheless, excess 5-factor alphas remain positive and statistically significant in 4 out 8 tests (3 individual equity portfolios included). The lower test statistics of the industry-wide portfolios (MSCI ACWI, MSCI World, MSCI EM, and SP Industries) are most likely underestimated by the lower number of observations (using monthly data).

The BAB portfolios generally load positive on quality (QMJ), indicating that the beta anomaly to some extent is captured by the quality factor constructed by Asness et al. (2014, working paper). QMJ is partially constructed with low-risk signals, therein low-beta, and the portfolios are therefore naturally correlated. In addition, *quality* is characterized by low- leverage, credit, and profitability risk. Low-beta, or noncyclical stocks, might plausible share these characteristics. Or, conversely, high-beta stocks that vary substantially with aggregate market sentiment, might well offer the opposite characteristics.

Asness, Frazzini and Pedersen (2014, working paper) find that BAB portfolios load positive on value, but that the value exposure is mostly driven by industry tilts. A value tilt could possibly be explained by exposure to income stocks. However, our results do not display definite value exposure.

Consistent with the literature, the BAB portfolios load positively on UMD. The sample periods spans across two historically large equity crashes, and as Daniel and Moskowitz (2013) note, subsequent to a market collapse a momentum portfolio will be long low betas and short high betas and we might expect the portfolios to behave very similarly.

More interestingly, there is no clear pattern as to the loading on small stocks, loadings are generally insignificant or slightly negative (except for FTSE), suggesting that BAB portfolios are neither explicitly betting on small or large firms; and that sorting portfolios by size-deciles (as we do in section 4.2.2) will not have a significant impact on performance.

The one-factor market betas are clearly biased. Most extreme are the MSCI ACWI and MSCI World portfolios, where the MKT coefficients increase from roughly 0.1 to 0.3 (but the pattern seems consistent across most tests). Both portfolios load positively on QMJ and the correlations between MKT and QMJ are highly negative in the sample. Consequently, the ex post market coefficients are downward biased when omitting the quality factor. The omitting variables bias means that our one-factor alphas are exaggerated.

While the BAB portfolios are not entirely independent of existing return premia, excess alphas are generally positive and statistically significant after controlling for 1-, 3- and 5-factors, suggesting that either the beta anomaly is in fact a riskless arbitrage or that it is driven by yet-unidentified risk factors. There is slight evidence that the portfolios have fatter and negatively skewed tail-distribution relative to the market, however, an investigation of the time-series' returns indicates that drawdowns are fewer than the underlying markets.[18]

4.1.3 Time-variation

While alphas are positive and large on average we find significant time-variation in raw and the excess performance of the BAB portfolios. Appendix 4A-B reports the average return by year for the BAB portfolios over the full sample periods. The BAB portfolio formed with French OATs, the longest sample, has historically been profitable 17 out of 26 years (63%). Equivalently, the US portfolio has been profitable in 13 out of 24 years (54%). The most consistently profitable equity portfolio has been the SPX with 15 out of 18 years (83%), in equity indices MSCI EM has been profitable in 10 out of 11 years (91%). On the other hand FTSE was only profitable in 11 out of 19 years (58%) and SP Industries 13 out of 22 years (59%). In any given year the probability to incurring losses has been nontrivial. Investment professionals subject to benchmarking or peer group risk, may refrain from engaging in beta-arbitrage of any significant scale unless they are confident competing have already done so.[19]

To expand, 250-days rolling one-factor alphas are reported for equities and four bond portfolios and plotted in chart 4 below.

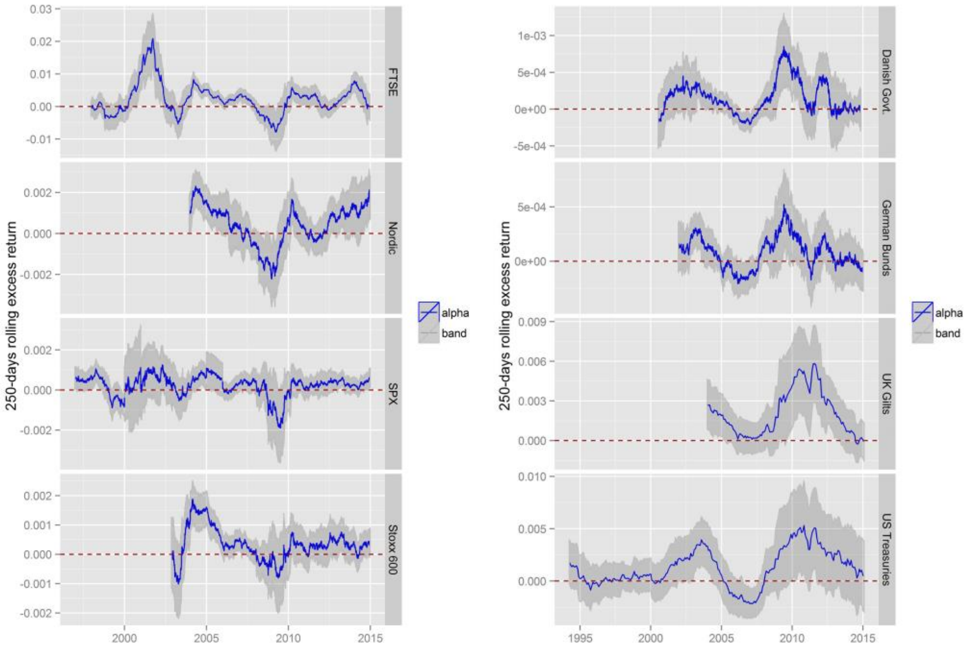
[18] Drawdowns measured as the number of months below high water mark.

[19] The argument is that peer group- or career- risk will induce money managers to construct portfolios that resemble rivals', so that any drawdowns can more easily be excused.

Chart 4: Alpha variation over time

Daily alphas are computed as the 250-days one-factor excess performance of BAB returns when evaluating on the corresponding underlying market. The highlighted grey areas surrounding the alpha-line illustrate the 95% confidence intervals.

Equities	Bonds



While alphas are positive on average there are recurring periods of negative performance.[20] Frazzini and Pedersen (2014) propose that BAB-arbitrage is exposed to funding liquidity risk. As funding conditions tighten, leveraged investors have to descale their positions and profits decline. Using the TED spread[21], the authors report that a tightening in funding constraints relates to lower lagged and contemporaneous BAB returns, indicating that BAB performance is sensitive to perceptions of credit risk and worsening funding constraints.

[20] We realize that the sample periods in our investigation are relatively short. To understand the cyclicality over longer periods we also obtained BAB returns from AQR's website from 1931 to present, or roughly 80 years, and computed one year rolling one-factor alphas and found similar results.

[21] The spread between 3-month Treasury Bill rate and LIBOR.

Huang, Polk and Lou (2014, working paper) provide a novel explanation. Beta-arbitrage experiences periods of booms and busts as a function of arbitrage activity. Specifically, the authors document that when activity is low beta-arbitrage profits materializes much slower and conversely when activity is high profits short-run more than triple, but then subsequently normalizes.

Beta is positively correlated with corporate leverage and return-volatility, a debt-like characteristic (Black 1993), indicating that beta-arbitrage may be relatively more attractive when the high and low beta spread is wider. As more arbitrageurs crowd to buy low-beta stocks and sell high-beta stocks the cross-sectional beta spread may widen (Huang, Polk and Lou 2014, working paper). To further investigate the time-variation of returns the aggregate beta spread is calculated as the difference between the portfolio betas of the unlevered high and low legs divided by the product:

$$SPRD_t = \frac{\beta_t^H - \beta_t^L}{\beta_t^H \beta_t^L}$$

A wider spread is indicative of larger BAB return. The regression coefficient between the time-series BAB returns and the beta spreads is positive in all four equity portfolios with large significant test statistics in three out of four tests. We also regressed one-period lagged spreads on BAB return series with similar, albeit less significant, results. This indicates that ex ante spreads can be employed as a trading signal, possibly providing investors with a valuable timing tool.[22] Table 6 below reports the arithmetic average return of BAB portfolios when spreads a *high* and *low*, where the ex post spread median is the discriminant.

The results in table 6 clearly states performance is higher when spreads are wide.

[22] Table 5 reports the lagged and simultaneous regression result

Table 5: Beta Spread Regression Results

The table reports summary statistics of SPRD as a predictor of BAB return. SPRD is calculated as the difference between β_H and β_L divided with the product: $(\beta^H - \beta^L)/(\beta^H \beta^L)$. Statistics are reported for long-short BAB portfolios. T-statistics are reported in brackets below.

	STOX	NRDS	FTSE	SPX
$BAB_t, SPRD_t$	0.00954**	0.00108*	0.006345**	0.000327
(t-stat)	(4.23)	(2.47)	(9.01)	(0.97)
$BAB_t, SPRD_{t-1}$	0.00093**	0.001791**	.006158**	0.000106
(t-stat)	(4.12)	(4.10)	(8.71)	(0.31)

**Indicates strong significance (99%), *indicates significance (95%)

Table 6: Beta Spreads and Performance

The table shows the arithmetic average annualized performance and Sharpe ratios of BAB portfolios when spreads are high and low. The high and low states are discriminated using the ex post median of the full sample. The beta spread is calculate as the difference between the high and low portfolio betas divided by the product.

	STOXX	NRDS	FTSE	SPX
High	19.2%	27.1%	18.5%	11.7%
Low	3.5%	1.3%	3.4%	16%
Difference	15.7%**	25.9%**	15.1%**	-4.29%
(t-stat)	(3.32)	(3.54)	(3.02)	(-0.32)

**Indicates strong significance (99%), *indicates significance (95%)

The results in table 5 and 6 provide clear evidence that beta spreads and beta-arbitrage performance is interdependent. If spreads measures the leverage embedded in equity securities, we can interpret the results as evidence that constrained investors tilt towards and overweight riskier assets to compensate as proposed by Frazzini and Pedersen (2014). Alternatively, we can interpret the spread as a secondary response to arbitrage activity, which consequently improves beta-arbitrage performance. It also indicates that the strategy can be timed. By scaling the BAB exposure on the $t-1$ spread.

In summary, BAB portfolios have positive returns on average in both bond and equity markets. Beta-arbitrage works with index constituents. There is some degree of return-variation over time, but our investigation hints that arbitrageurs can exploit this through tactical timing, as signalled by the beta spread. Moreover, time-variation is significant, but most years BAB portfolios perform with positive returns. Portfolios tend to load positively on QMJ and UMD, which is unsurprising, but alphas remain positive and mostly significant.

4.2 The Effect of Size and Short-Selling

The BAB portfolios investigated yield abnormal risk-adjusted returns across markets and asset classes. This clearly presents a challenge to modern asset pricing in general and the Capital Asset Pricing Model in particular. It is difficult to fully determine whether the beta factor is a rationally priced unknown risk exposure or whether it is in fact a behavioral or institutional market inefficiency. However, it is far less challenging to determine the implementability of the portfolios. Onward the thesis focuses less on the existence of the beta-anomaly and more on potential implications of implementation. Regardless of the underlying explanations for the beta-factor, if market frictions and implementation costs exceed the benefits even for very sophisticated investors, then we would not expect the phenomenon to be arbitraged away. This section less is theoretical and we instead focus more on the practical implications of implementing beta-arbitrage. More specifically, we focus on short-selling and size effects.

The BAB portfolios are constructed as long-short strategies in order to exploit relative mispricing, consequently making strong assumptions about short-selling. However, there are limits of arbitrage (Schleifer and Vishny, 1997). Ignoring the previously discussed agency barriers, successful short-selling cannot be guaranteed. Moreover, proceeds are not freely available to the seller, but must be posted as collateral to the lender. Contrary to theoretical arbitrage, short-selling ties up capital and in fact introduces additional sources of risk. Short-selling can be expensive to implement and the borrower is exposed to recall risk and possible alpha decay as the short position is reestablished[23] In other cases less sophisticated investors may be entirely blocked from short-selling, and more regulated investors can be prohibited.[24]

[23] D'Avolio (2002) finds that the average time to re-establish a short-position after a recall is 23 trading days and that on average 2% of shorts are recalled every calendar month.

[24] In 2000 Almazan, Brown, Carlson, and Chapman (2004) find in a sample of 1,838 mutual funds, that 66.1% are restricted from short selling.

Keim and Madhavan (1997) show that implementation costs can be particularly severe in smaller equities. The BAB portfolios are constructed across all sizes in the selection investment space. Smaller segments are less liquid and tend to be more costly to trade and perhaps even untradeable for large arbitrageurs with market impact. If the constraints of implementation are nontrivial it can potentially evaporate the excess profits to an extent where it is no longer worthwhile for arbitrageurs. It is therefore interesting to consider the effect of size and short-selling on the performance of various BAB portfolios.

4.2.1 Short-Sale Frictions

Negative momentum and glamour (low HML) stocks are more likely to be expensive to sell short (D'Avolio, 2002). Brent et al. (1990) find that high beta assets are more likely to be expensive to sell short as well.[25] Consistent with that, based on the average lending fees over the past 90 and 30 days, 87% and 80% of the *special firms* are also ranked as high beta in our BAB portfolio as of March 2015. Exactly the stocks that supposedly carry negative alphas. If high beta stocks are more short sale constrained this perhaps partially explains the flatness of the SML and why the beta-factor anomaly at the high end of the spectrum seems to persist.[26]

[25] In fact the authors find a positive correlation between short interest and beta. However, short interest is positively correlated with costs.

[26] Various studies have found that large short-sale constraints in the cross-section of equities lead to subsequently lower returns (Figlewski 1981; Jones and Lamont 2002)

Table 7: Beta and short-selling constraints in Nordic equities

Special are defined as stocks with annualized lending fees above 1%. Panel A is based on the last 90 days average while panel B is based on the last 30 days average. Beta coefficients are computed based on the last 250 trading days. The total number of observations is only 96 given that betas could not be calculated for the final 4. Lower case "s" denotes the *special* criteria and β_H denotes high beta.

Panel A		High	Low	$\mathbb{P}(\boldsymbol{\beta_H}\,|\,\boldsymbol{s})$	Panel B		High	Low	$\mathbb{P}(\boldsymbol{\beta_H}\,|\,\boldsymbol{s})$
Special	Yes	13	2	**87%**	Special	Yes	15	4	**79%**
	No	35	46	**43%**		No	33	44	**43%**
	$\mathbb{P}(\boldsymbol{s_y}|\boldsymbol{\beta})$	**27%**	**4%**			$\mathbb{P}(\boldsymbol{s_{Yes}}|\,\boldsymbol{\beta})$	**31%**	**8%**	

Table 7 is based on a small sample, but the pattern is clear: The probability of being special is much larger for high beta securities and similarly the probability of being high beta given specialness is much higher than low beta. To consider implementation frictions Israel and Moskowitz (2012) investigate the role of shorting on classic investment strategies and find that – in terms of risk-adjusted returns - the contribution of the short leg of both value and momentum is positive and non-trivial, but not entirely sufficient to explain the alphas observed. Following the same methodology, below in Table 8 we have plotted Sharpe ratios, betas, and return- and alpha- contributions of both legs of the BAB portfolios. Both legs are leveraged to ex ante betas of one for better comparison.

Panels A and B reports the performance statistics of leveraged portfolios in equities and equity indices. With the exception of the SP Industries the excess returns of the leveraged low-beta portfolios are larger than both the deleveraged high-beta and BAB portfolios. The lower returns of the deleveraged portfolios are consistent with the overpricing of high beta securities and the flatness of the SML. In equities, the risk-adjusted returns are generally highest in the BAB portfolios, but the low-beta portfolios clearly outperforms the underlying markets and alphas are positive and significant in 7 out of 8 tests. Conflicting with the basic premise of return to risk, the deleveraged high-beta portfolios have positive, but lower average returns. The returns are lower than the low-beta leg and the market portfolios, but generally higher than the BAB portfolios.

Table 8: Long- and short- decomposition and summary of all the BAB portfolios

The table examines the contributions of the short and long legs of the BAB portfolios. Alphas and returns are annualized and in excess of the risk free rate. Both legs are ex ante leveraged to a beta of one for better comparison.. Panel A reports the results of the equity indices, whereas Panel B reports summary statistics of individual equity portfolios. Finally, panel C reports summary statistics for bonds. Cross-sectional performances should not be over-interpreted as the length of the portfolio formation periods vary considerably.

		Panel A				Panel B				Panel C							
		MSCI ACWI	MSCI World	MSCI EM	SP	STOXX	FTSE	NRDS	SPX	Euro Corp.	US Credit	US T-Bills	Bunds	OATs	UK Gilts	Spain	DK
Low	Return	13.1%	9.8%	16.8%	7.2%	17.8%	14.5%	24.2%	18.2%	6.3%	7.4%	4.6%	5.6%	4.6%	6%	7.3%	7.3%
	Alpha	3.1%*	2.5%*	4.2%**	-1.6%	6.9%**	9.8%**	13.4%**	3.7%*	2%**	2.4%**	1.1%**	4.5%**	0.8%*	1.9%**	2.7%**	3%*
	(t-stat)	(2.22)	(2.1)	(4.38)	(-1.1)	(3.69)	(3.92)	(3.52)	(2.11)	(5.21)	(4.38)	(2.62)	(2.6)	(2.26)	(3.86)	(2.85)	(4.1)
	Sharp	0.7	0.55	0.79	0.42	0.8	0.75	0.89	0.76	1.73	1.76	0.91	1.23	1.01	1.27	1.01	1.49
	e	1.06	1.07	0.98	1.04	1.06	0.5	1.00	1.03	0.98	0.99	0.94	1.01	0.98	0.96	0.95	1.03
	Beta																
High	Return	7.5%	5.2%	9.9%	8.7%	6.4%	3.5%	10%	11%	3.7%	4.1%	3.4%	4.1%	3.5%	3.5%	4.5%	3.9%
	Alpha	-1.7%*	-1.5%*	-3.3%**	0.2%	-4%**	-1.3%	-0.7%	-3.3%**	-0.7%	-1%**	-0.4%*	-0.3%	-	-0.8%**	-0.5%	-0.3%
	(t-stat)	(-2.29)	(-2.3)	(-4.4)	(0.21)	(-4.18)	(-1.76)	(-0.23)	(-3.54)	(-4.16)	(-3.89)	(-2.37)	(-1.42)	0.3%*	(-3.87)	(-1.07)	(-1.46)
	Sharp	0.44	0.32	0.44	0.56	0.31	0.21	0.39	0.48	1.06	1.02	0.67	0.94	(-2.16)	0.76	0.63	0.98
	e	0.98	0.99	1.02	1.00	1.01	0.99	0.99	1.01	1.01	1.01	1.02	1.02	0.81	1.02	1.04	1.04
	Beta													1.01			
BAB	Return	5.6%	4.5%	6.9%	-1.5%	11.4%	10.8%	14.2%	11.%	2.6%	3.3%	1.2%	1.5%	1.1%	2.46%	2.8%	3.3%
	Alpha	4.7%*	4%*	7.5%**	-1.9%	10.9%**	11.2%**	14.1%**	7.04%**	2.7%**	3.4%**	1.5%*	1.6%*	1.2%*	2.7%**	3.2%*	3.3%*
	(t-stat)	(2.25)	(2.19)	(4.39)	(-0.8)	(4.63)	(4.45)	(3.84)	(2.76)	(4.93)	(4.26)	(2.58)	(2.36)	(2.24)	(3.87)	(2.54)	*
	Sharp	0.77	0.6	1.2	-0.14	1.3	1.03	1.1	0.64	1.25	1.47	0.41	0.59	0.39	0.93	0.64	(3.69)
	e	0.09	0.08	-0.04	0.05	0.05	-0.04	0.01	0.01	-0.02	-0.02	-0.09	-0.02	-0.03	-0.06	-0.08	0.91
	Beta																-0.01

**Indicates strong significance (99%), *indicates significance (95%)

Next, considering the bond portfolios in panel C we find similar results. The low maturity leg strictly outperforms the high maturity leg. Because deleveraged high portfolios yield positive returns, as we would expect, the absolute returns of the long-only strategies are higher than the long-short BAB portfolios. All long-only alphas are positive and significantly different from zero, albeit somewhat lower than the BAB portfolios. Completely unconstrained investors would still strictly prefer the long-short implementation. Nonetheless, the bulk of abnormal performance lies with the low beta leg, as with equities.

The relatively high Sharpe ratios in table 8 deserve a comment. At first glance, the trade-offs between return and risk are exceptionally high, especially in the bond portfolios. Several of the low portfolios have Sharpe ratios above one. The long-term trend of interest rates has been consistently downward-sloping, pushing up prices and returns to bondholders. Equity markets have experienced similar bullish trends. More surprising is the relatively low Sharpe ratios of the bond BAB portfolios. The return per unit of risk is more modest, partially because of the much lower nominal excess returns and partially explained by the non-trivial residual deviations. Nominal returns are very low because even highly deleveraged high-maturity portfolios have yielded substantial rewards, likely explained by the interest rate environment. While idiosyncratic risk is captured in the risk-reward relationship, in a portfolio context such variation is somewhat trivial and systematic risk exposure should be much more alarming. The BAB portfolios are (successfully) constructed as market neutral strategies, and consequently the marginal contribution from adopting these would be much larger.

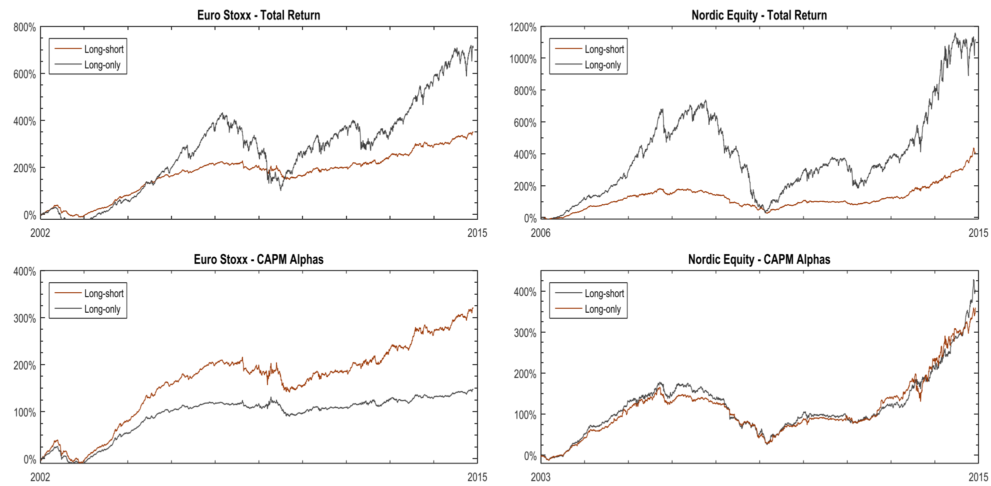
The Sharpe ratios of the leveraged and unleveraged portfolios of both legs are different (see Appendix 2 and 3 for unlevered results). In fact (de)leverage generally has a beneficial effect on the risk-return relationship. The explanation is that the portfolios are leveraged dynamically. Dynamic scaling rather than simply "levering up" the ex post results serves two purposes. It has the advantage of presenting the behavior of both legs more realistically as standalone strategies and – more importantly – puts the portfolios into a context where the beta anomalies are exploited. This happens because the portfolios are leveraged so that more (less) leverage is applied, as the weighted average beta of the long (short) leg is lower (higher). Therefore, the portfolios are more (less) exposed when betas are lower (higher), consequently exploiting that risk-adjusted returns are superior for lower beta coefficients.

Chart 5: Cumulative return of long-only and long-short portfolios

The charts plot the cumulative performance of long-only and short-short beta-arbitrage BAB portfolios. Charts to the left plot total returns while excess returns are illustrated to the right. Excess returns are estimated as one-factor alphas, where total returns are evaluated on the CAPM. Panel A contains equities and panel B contains bonds. The remaining portfolios are illustrated in appendix 5.

Panel A: Equities

The long-short portfolios are conventional *betting against beta* portfolios. The long-only portfolios are long low beta equities leveraged to an ex ante beta of one.



Panel B: Bonds

The long-short portfolios are conventional betting against beta portfolios, long low maturity and short high maturity. Long-only portfolios are leveraged to an ex ante beta of one.

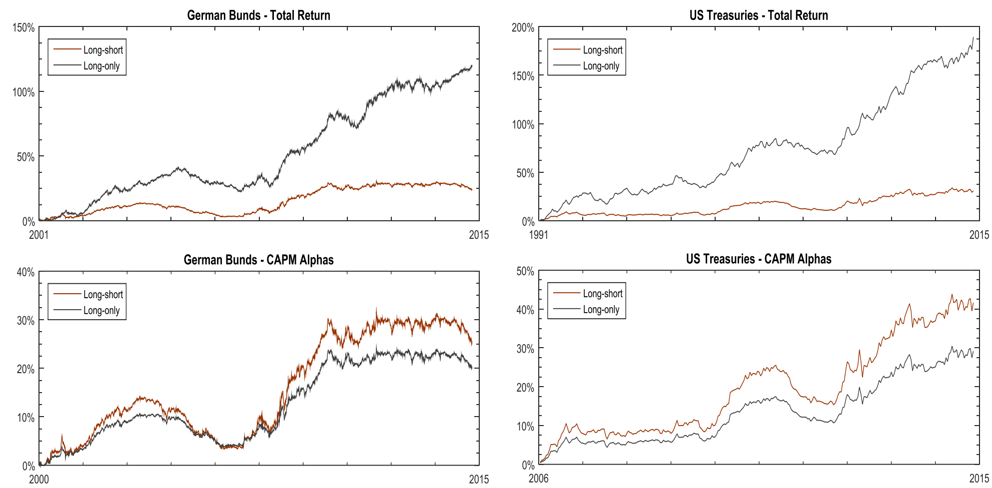


Chart 5 plots the leveraged (but not volatility-scaled) long-only and long-short performance in terms of raw and excess returns. Excess returns are estimated as the intercept of a single factor regression between BAB returns and the underlying market returns. Appendix 5 plots similar charts for the remaining portfolios. The pattern is remarkably persistent. The total (raw) return of leveraged low-beta portfolios is clearly higher than BAB portfolios, but market risk drive a substantial fraction. Rather, on a risk-adjusted basis (excess performance) the BAB portfolios outperform the long low-beta because high-betas have negative alphas. However, the excess performance of the long-only portfolios is positive and statistically significant, just slightly lower.

In short, to summarize, long-only or otherwise short-constrained investors can still benefit from the beta anomaly. High-beta portfolios have statistically significant negative intercepts, but in nominal terms the long leg drives the bulk of both alphas and raw returns. The returns supersede the underlying index portfolios in all instances. Rather than employing long-short strategies, an investor can simply buy into the low-beta anomaly and refrain from selling the somehow less important high-beta anomaly. Nonetheless, unlike the BAB portfolios the long-only strategies have significant market exposure (indeed, ex ante betas were deliberately scaled to one). As an overlay investment strategy on an already market exposed portfolio Sharpe ratios alone are not necessarily the best measure of performance and the market neutral BAB portfolios might provide better diversification. However, in portfolio context the net difference is small. Investors can simply choose to invest slightly less in the underlying market portfolio to get the same net exposure.

While short-selling constraints cannot explain the performance of low-beta securities, the findings do not dispute explanations of other alternative investor constraints. In fact, the lower risk-adjusted – but higher absolute returns of unlevered high beta portfolios – relative to low beta portfolios confirm the theory of leverage aversion. The portfolio returns are obtained with leverage (sometimes up to 200-300%) at the risk free rate, while most investors must pay more. Investors who are risk-averse or unable to apply debt financing at very low and risk free levels cannot replicate these results. Leverage aside, for investors with preference towards returns are – above a certain point – forced to construct portfolios from riskier assets such as long maturity bonds or high beta equities.

Consequently flattening the risk-reward relationship if constrained investors are well represented in the market.

4.2.2 Size Implications

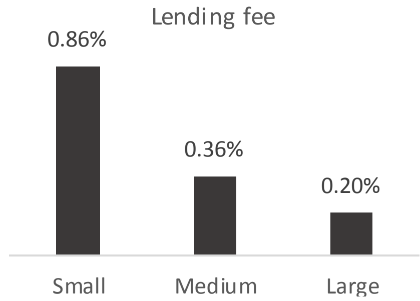
While the cost of short-selling is more pronounced in the high-beta equity space, the driver of abnormal profits primarily rests with the long leg. Investors unwilling or unable to sell short can implement variations of the long-only low beta portfolios. Having examined the long- and short- performance contribution of BAB portfolios we now expand the investigation and consider the effect of size. For simplicity the tests are only performed in the US equity sample (SPX), it is the sample with the largest number of individual stocks – ensuring that the smaller portfolios will have enough stocks to be relatively diversified.

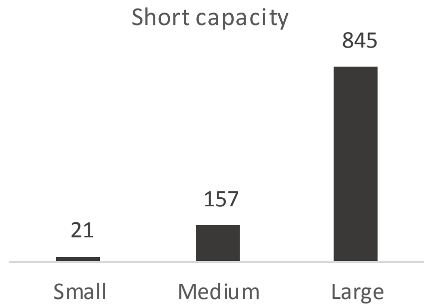
Keim and Madhavan (1997) show that market capitalization and transaction size are important determinants of transaction costs. Costs tend to be lower and capacity higher when investing in large cap equities. The marginal trade-off between cost and size is especially steep in equities smaller than $500m. Israel and Moskowitz (2013) find that the momentum premium is distributed evenly across size, whereas the value premium is largely concentrated among small stocks and insignificant among the largest quintiles of stocks. This represents an implementation challenge, as small stocks are less liquid and harder to sell short. The relationship between size and liquidity is well-documented (Pastor and Stambaugh, 2003). Moreover, from a short-seller perspective lending can be much costlier or frequently impossible in the smallest equities (D'Avolio, 2002). To illustrate, table 9 below summarizes the lending costs, short utilization, and short capacity of the Nordic sample sorted by market capitalization.

Table 9: Short-selling costs and size

The table is based on 100 Nordic equities. Lending fee, short utilization, and short capacity is calculated as value-weighted average based on market capitalization. Short capacity is derived from short utilization, short interest and market capitalization. The lending fees reported are the annualized daily average fees over the past 90 days. The data was collected around March 2015. Size groups are determined by market cap-rank, e.g. each groups contains one third of the total sample.

Market Capitalization	30 days	90 days	Short utilization	Short capacity DKKb	Average Size DKKb
1 (Largest)	0.27%	0.2%	1.34%	845	171
2	0.45%	0.36%	3.25%	157	32
3 (Smallest)	1.09%	0.86%	5.26%	21	17



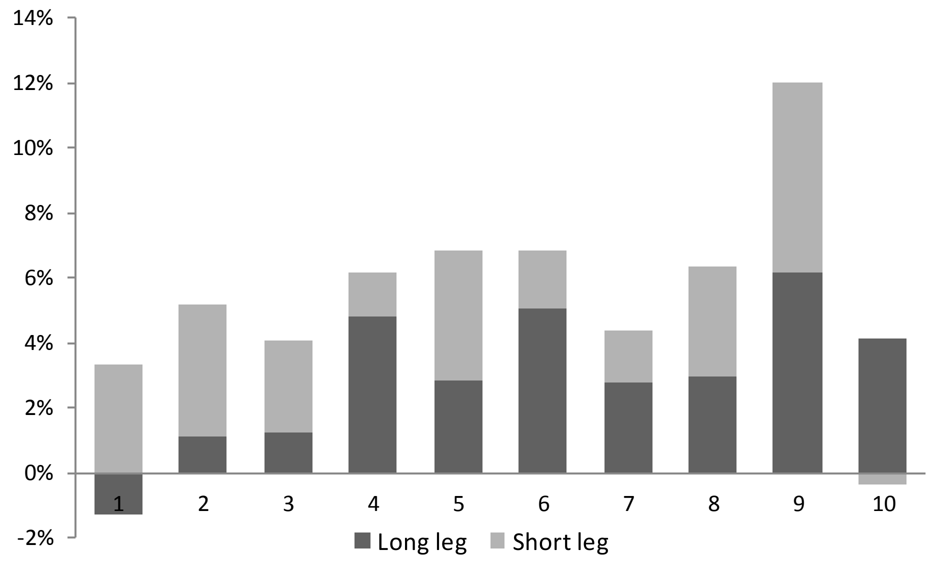


To a large extent short capacity reflects size because it is derived from market capitalization. Fees and short utilization decrease with size, while capacity increases. Large cap equities are easier and less expensive to sell short. Fees are explicitly costs of lending and implicitly recall and buy-in risk. Appendix 12 reports the correlation matrix of size, beta, fee and short utilization. Beta is positively correlated with fees, but we suspect that size is a more important determinant of short constraints. An investigation of the beta-factor with respect to size is therefore interesting for two reasons. First, the adverse effects of short-sale constraints are higher in smaller equities. Second, for liquidity reasons costs and capacity is preferable in larger securities. Moreover capacity reduces the costs of price impact and permits larger employment of capital and higher profits.

To investigate the interdependence between size and performance, annualized statistics of size-sorted BAB portfolios are presented in table 10. Portfolios are sorted by market capitalization into deciles, where size ascends from left to right. In order to construct the portfolios we considered two methodologies, one approach is to use a size-specific beta-discriminant coefficient. This would create more balanced portfolios in terms of securities count (each portfolio would always hold 75 stocks long and short). It would also be equivalent size-neutral BAB portfolios. The alternative approach is to use a beta discriminant irrespective of size; that is the median of the entire sample. The latter approach is not necessarily balanced on count (there are still exactly 150 equities in each portfolio but not necessarily 75 in each leg). However, we found it more suitable for dissecting the relationship between size and beta-arbitrage when the purpose is to understand whether alpha-drivers are conditional on size. Basically, the methodology tells what the average performance of high and low beta stocks is in each size group. In addition, when counting the number of stocks on each leg on every size decile the, the portfolios are relatively balanced, indicating that there is no significant relationship between size and beta. We would have liked to construct size-neutral BAB as an additional analysis to measure the performance of size-neutral BAB portfolios, but for practical reasons we decided not to. Size-sorted portfolios are equally-weighted, but everything else follow the same methodology discussed earlier.

Chart 6: Long and short alpha contribution sorted by size

Every month equities are sorted into size deciles based on past market capitalization rank. Low and high portfolios are constructed in each size group based on beta coefficients. The discriminant value is the median of all index constituents. Both legs (low and high) are equally-weighted. The chart plots annualized one-factor alphas of each leg.



The risk-adjusted performance of the combined long-short portfolio is relatively constant across size deciles, with no clear trend. This is somewhat surprising, Frazzini and Pedersen (2011, working paper) document a negative relationship between size and BAB performance. Malkhozov et al. (2014, working paper) document that BAB performs better in more illiquid securities. Low-beta alpha increases with size, while high-beta contribution is somewhat larger in the smallest equities. Nonetheless high beta equities have negative alphas in every decile except for the highest decile. Meanwhile, low beta equities have positive alphas in every decile except for the smallest decile; indicating that the alpha-beta relationship is not specifically size-dependent. Israel and Moskowitz (2013) find that while momentum is robust across size deciles, the value effect is more pronounced in the smallest securities.

Table 10 reports alphas, returns and Sharpe ratios of high and low beta portfolios sorted by size. The four rightmost columns report similar statistics for equally- and beta-weighted constrained and unconstrained BAB portfolios. According to Chen and Lu (2014, working paper) stocks with market capitalization below $250m can be much more challenging to buy on margin. Keim and Madhaven (1997) document that trade difficulty is particularly pronounced in equities smaller than $500m. The average market value of the smallest decile is only $208m, indicating that beta-arbitrage can be particular difficult in this segment. Therefore it is interesting to investigate the performance of a BAB portfolio where the smallest percentile of stocks have been excluded. Following the same approach, but each month excluding the smallest stock as measured by market capitalization we report the performance of the constrained BAB portfolio. The last four columns from the right separately contain the performance statistics of the unconstrained and the (slightly) constrained SPX *betting against beta* portfolios using beta-weighting (as before) and equal-weighting. The unconstrained beta-weighted portfolio is essentially the same as reported earlier. The constrained variants are constructed from all shares in the underlying market except for the bottom percentile. As we would expect, returns, Sharpe ratios and alphas (test statistics) are all modestly inferior when the portfolio is subject to constraints. Regressing the results of the time-series returns of the constrained and unconstrained conventionally beta-weighted portfolios show that while the constrained underperforms by 0.95% p.a. ($t = -2.35$) the tracking error is minimal ($R^2 = 0.99$).

Size may be a bigger obstacle than short-selling for sophisticated institutions. While investment professionals with billions of assets under management may have access to cheap financing, less restrictive margin requirements and low lending fees, the market impact of such actors is likely much higher. Ironically, investors with little market impact are much less likely to have sufficient debt capacity or access to competitive lending rates. The results indicate that investors with high price impact or unable to leverage equities smaller than $250m with their brokers can advantageously construct the BAB factor formed without the smallest 10%.

Table 10: Long- and short- decomposition of S&P 1500 BAB portfolios sorted on size

The table reports statistics on BAB portfolios of 1500 equities belonging to the S&P 1500 Composite Index using data from January 1995 to December 2014. Portfolios are sorted on size. Long and short positions are separated. Statistics are annualized and in excess of the risk free rate. Size is determined by ranking on past market capitalization every month. For every percentile, the table reports the performance of long-short BAB portfolios within that size-rank. Each decile contains 150 equities at any one time. The four rightmost columns report the performance of portfolios either equally-weighted (EW) or beta-weighted (BW) containing every security, when 1-10 and excluding the smallest decile when 2-10. The High and Low rows report the statistics of unlevered independently sorted BAB portfolios across sizes. The bottom rows reports the long-short BAB portfolios, where the long and short portfolios has been (de)levered to a beta of one. The reported average market capitalization is the time- and cross-sectional equally-weighted average across all securities in the full sampling period.

		(smallest) 1	2	3	4	5	6	7	8	9	(largest) 10	EW 1-10	EW 2-10	BW 1-10	BW 2-10
	Average size	208m	450m	700m	1.0bn	1.5bn	2.2bn	3.bn	5.7bn	11.1bn	18.5bn				
Low	Returns	8.1%	8.5%	9.3%	11.8%	10.2%	11.3%	10.3%	9.8%	12%	10.6%	11.6%	10.5%	10.3%	10.1%
	Alpha	-0.5%	0.1%	0.6%	3.1%	1.6%	2.8%	2%	1.1%	3.5%	2.2%	3 %	2%	2.1%	1.9%
	(t-stat)	(-0.15)	(0.03)	(0.16)	(0.88)	(0.44)	(0.79)	(0.58)	(0.31)	(1.01)	(0.59)	(0.92)	(0.60)	(0.68)	(0.59)
	Sharpe	0.53	0.57	0.6	0.77	0.66	0.74	0.70	0.64	0.79	0.66	0.80	0.73	0.76	0.74
	Beta	0.63	0.61	0.63	0.63	0.63	0.63	0.61	0.63	0.62	0.62	0.62	0.62	0.58	0.59
	#Stocks	46%	49%	50%	50%	50%	49%	49%	52%	53%	53%	50%	50%	50%	50%
High	Returns	13.3%	11.3%	14.3%	15.6%	11.6%	15 %	15.6%	12.7%	9.9%	17.9%	15%	13.8%	13.9%	14%
	Alpha	-3.5%	-5.6%	-2.9%	-1.6%	-5.5%	-2.5%	-2%	-4.6%	-7.6%	0.6%	-2.1%	-3.3%	-4.4%	-4.3%
	(t-stat)	(-0.54)	(-	(-	(-	(-	(-	(-	(-0.70)	(-1.12)	(0.09)	(-0.32)	(-	(-0.66)	(-
	Sharpe	0.47	0.86)	0.44)	0.24)	0.83)	0.37)	0.29)	0.43	0.34	0.60	0.54	0.52)	0.47	0.64)
	Beta	1.22	0.4	0.5	0.54	0.40	0.51	0.53	1.26	1.28	1.26	1.25	0.49	1.29	0.48
	#Stocks	54%	1.23	1.25	1.26	1.25	1.28	1.28	48%	47%	47%	50%	1.25	50%	1.29
			51%	50%	50%	50%	51%	51%					50%		50%
BAB	Returns	2.4%	5.3%	4.3%	6.7%	7.5%	7. 4%	4.5%	7.0 %	12.2%	3.5%	6.8%	6.5%	7.2%	6.3%
	Alpha	2.1%	5.2%	4.1%	6.1%	6.9%*	6.8%	4.4%	6.3%	12%**	3.8%	6.8%**	6.1%*	7.0%**	6.1%*
	(t-stat)	(0.68)	(1.58)	(1.27)	(1.83)	(2.0)	(1.93)	(1.37)	(1.91)	(3.55)	(0.91)	(3.05)	(2.54)	(2.76)	(2.49)
	Sharpe	0.18	0.37	0.31	0.45	0.5	0.48	0.33	0.48	0.82	0.19	0.70	0.62	0.64	0.59
	Beta	0.02	0.01	0.02	0.04	0.04	0.04	0.01	0.04	0.01	-0.02	0.00	0.03	0.01	0.01

**Indicates strong significance (99%), *indicates significance (95%)

4.3 Quantifying Short-Selling Costs

So far we have investigated important relationships between beta, size, and short-selling constraints. More concretely, we considered the effect of size and short-selling on the performance of BAB portfolios and found that while the short-leg contributes positively, the low-beta driver is more important. The effect of size is more ambiguous, but the tracking error of a size-constrained portfolio remains very small.

To make better inferences about the adverse effects of short-selling, we now attempt to compute the actual lending costs of a long-short portfolio. The lending data is available only for the Nordic equities and the remainder of this section is mainly a study of the Nordic sample. However, given the asymmetric alpha contributions of especially the Nordic BAB portfolios we extrapolate lending costs and apply them to the remaining equity samples in order to include samples where the short-leg contribution is more important.

4.3.1 Estimating Lending Fees

To compute lending costs we calculate a one-shot lending fee across all securities sold short had portfolio formation been around the end of 2014. The fee sample is the average fee over the past 30 and 90 days as of March 2015. Had the portfolio been formed at the beginning of 2015 the fees paid by a short-seller would be very close to the sample. The fee is simply the weighted average fee of every short position where weights are determined by beta-ranking as discussed in sections 3.2 and 3.3.[27] At inception the total lending fees would have annualized to 0.94% (30 days) and 0.71% (90 days) before leverage. Assuming beta coefficients have been relatively constant over the period, the 90 days fee should come relatively close to

[27] $fee_0 = \sum_{j=1}^{i} w_0^{H,i} fee_0^{H,i}$

the actual fee paid by a beta-arbitrageur. Excluding hard-to-borrow stocks and the lending fees drop substantially. Table 11 summarizes fees under different conditions. Frazzini et al. (2012) find the short-selling costs of value and momentum strategies can be significantly mitigated without considerable tracking error (alpha decay). Geczy et al. (2002) finds similar results that portfolios without expensive-to-short stocks produce nearly the same results. We would have liked to compute similar short-optimized portfolios where specials are not allowed in order to investigate the trade-off between tracking error and cost-savings, but the data does not allow us to identify the hard-to-borrow stocks across time and simply excluding stocks that are currently expensive would not provide meaningful information.

Table 11: Variations of lending fees

The table reports annualized lending fees across different weighting schemes sorted by beta coefficients. Betas are calculated using 250 trading days regressed on an equally-weighted average of the OMX, KFX, OBX and HEX indices. Low and High is determined by the median coefficient. Beta-weighted is the BAB weighted-average fee at formation, the last trading day of 2014. "No specials" exclude all securities with lending fees above 1% p.a.

		High beta	Low beta	All
Equally-weighted	30 days	0.91%	0.44%	0.68%
	90 days	0.7%	0.34%	0.52%
Value-weighted	30 days	0.35%	0.3%	0.32%
	90 days	0.31%	0.21%	0.24%
Beta-weighted*	30 days	0.94%	0.51%	0.83%
	90 days	0.71%	0.39%	0.63%
*(no specials)***	30 days	0.4%	0.3%	0.3%
	90 days	0.2%	0.2%	0.%

*Beta-weighted symbolizes how the securities would have weighted in the BAB portfolios
**With beta-weights

The 90 days fee more accurately predicts the true fee (because of the lag from the beginning of 2015 to March), but to be conservative, we apply the highest fee (30 days). The total fee paid is variable with respect to the amount of lending activity on the short-leg. When the high-beta portfolio is being deleveraged relatively more, less equity is shorted and the total portfolio fee declines. Conversely, when the combined beta coefficients are relatively low less deleveraging occurs and the aggregated fee is higher. This is exactly equivalent to lending and borrowing at the risk free rate.[28]

$$R_t^{BAB} = \frac{R_t^L}{\beta_{t-1}^L} - \frac{R_t^H + fee_0\theta_t}{\beta_{t-1}^H}, \qquad fee_0 = \frac{0.94\%}{250}$$

Where θ_t is an index variable that determines the size of the fee relative to the 0.94% level. We determine thetas in three ways, as a function of market-volatility of the underlying market, as a function of the aggregate beta-spread and as a constant equal to one in all periods so that the fee is simply fixed. Variable fees are higher when spreads (volatility) are wider (larger) and lower when spreads are narrower (lower). The model is linear so that if volatility increases by 10% the fee does as well.

Beta-spreads are calculated as described in section 4.1.3 on time-variation of BAB performance. Intuitively it makes sense to link beta-spreads and short-selling costs. Frazzini and Pedersen (2014) hypothesize that alpha systematically declines with beta. Empirically the authors document a consistent and almost linear decline. Our examination of maturity sorted bond portfolios shows a similar relationship in eight different countries. Intuitively high-beta stocks offer investors implicit access to leverage. Frazzini and Pedersen (2012, working paper) show that securities with high embedded leverage such as deep out-of-the-money options or leveraged-index products are overpriced. The regression coefficient between the beta-spread and the return of the high-beta portfolio is negative. When the leverage embedded in high beta securities increases (and similarly decreases in low beta securities) the beta spread widens, BAB profits strengthen and beta-arbitrage activity intensifies (Huang, Luo and Polk 2014, working paper). We can use this intuition as the basis for lending fees because, all else equal, fees should increase with short-selling

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[28] While the borrower *receives* the rebate, given that all returns are excess returns, this has no effect and we can simply add the fee to the short-leg $R_t^{BAB} = \frac{R_t^L}{\beta_{t-1}} - \frac{\left(r_t^H - (r_f - fee_0\theta_t)\right)}{\beta_{t-1}^H}$

demand. We do not lag the spread because the purpose is to estimate how fees would have been, not to construct a buy or sell signal.

The rationale for positively linking volatility with short-selling costs is that, all else equal, we expect steeper lending fees when differences in opinions are high. While short-sale constraints are small on average, they are systematically larger when investor dispersion is high (D'Avolio, 2002). The author identified several proxies for disagreement among investors, such as high turnover and high dispersion in analyst forecasts. In relative terms, when non-lenders are optimistic while short-sellers are pessimistic the demand for lendable shares is high while the supply is reduced, thus driving up returns. Similarly, if lenders are less optimistic than non-lenders, the former will demand a higher lending fee for holding the share and provide supply to short-sellers.

4.2.2 Short-Constrained Portfolios

Table 12 below summarizes the results of BAB portfolios under different assumptions of short-selling constraints. The results compare with the unconstrained BAB portfolio and a *long-only-but-hedged* low-beta portfolio (highlighted) with portfolio statistics net of lending costs. The long-only-but-hedged low-beta portfolio is long the in long-leg of the BAB-portfolio and short a market future (assumed costless).

Table 12: Short-selling costs and BAB performance

The table tables annualized statistics of four BAB portfolios under different conditions. The leftmost column is the conventional portfolio discussed and described in section 4.1. The long-only-hedged portfolio (highlighted) is long Nordic low-beta equity and short the underlying market index. The remaining results are long-short BAB portfolios less lending fees. Fixed fee is long-short reduced by lending costs equivalent to $0.94/250$ bps before leverage. The rightmost two columns are depressed by the same lending fee but adjusted by an index variable θ_t. Theta is determined by beta spread between high and low or the volatility of the underlying market.

	Unconstrained BAB	Long-only (hedged)	Fixed fee (30 days)	SPRD fee	VOL fee
Return	14.2%	13.4%	13.3%	13.1%	11.8%
Alpha	14.1%**	13.4%**	13.2%**	13%**	11.7%**
(t-stat)	(3.84)	(3.52)	(3.61)	(3.55)	(3.04)
SD	12.8%	13.3%	12.8%	12.8%	12.9%
Sharpe	1.11	1.01	1.04	1.03	0.91
Beta	0.01	0.00	0.01	0.01	0.01

**Indicates strong significance (99%), *Indicates significance (95%).

Several points should be emphasized. The long-only portfolio performs nearly as well as the unconstrained BAB portfolio with a slightly lower intercept (-0.7% p.a. and t-statistics $= -1.93$). From a short-selling perspective the long-only-hedged portfolio is theoretically interesting because – other potential market frictions aside – while investors cannot construct the short-leg without incurring short-selling costs, simply selling in the futures market is nearly costless and margin requirements can often be satisfied with other long positions in the portfolio.[29] Consequently, investors should strictly prefer the long-only-hedged portfolio at the estimated level of lending fee. And even short-constrained investors should be able to seek the arbitrage opportunity.

Given that the long-leg of the Nordic equity portfolio carries a very large proportion of alpha in the sample the results cannot necessarily be extrapolated to the remaining BAB portfolios where long and short contribution was much more balanced. Accordingly, the optimal preference between the long-only-hedged and

[29] However, when investors use risky assets as collateral the lender usually require a slight haircut.

a regular BAB portfolio may be different in other samples. For example, if a significant proportion, say a third, of alpha contribution was generated from shorting high-beta securities then the performance of the long-only-hedged portfolio would be inferior to all the short-depressed long-short BAB portfolios. Appendix 6 illustrates this. Put differently, at current fee levels an unconstrained beta-arbitrageur would strictly prefer short-selling, if we disregard other frictions that might be related to trading individual securities (commissions and price impact will almost certainly be higher than trading a futures contract or a total return swap in the underlying market).

To illustrate we set portfolio alphas equal and derive the break-even level of lending costs that will make investors equally well-off between short-selling individual shares and the underlying market only. In Nordic equities the fees would have to be roughly 80%, 60% and 20% of current levels for the long-short cost-depressed portfolios to be equivalent to the long-only-hedged portfolio (of course investors could not have known this at inception). In comparison, contrary to the Nordic BAB portfolios, the results in the remaining portfolios indicate that the long-only-but-hedged portfolios are inferior to actually short-selling individual securities because alpha contributions of short-selling high betas supersede costs. The level of the fixed fee has to increase four-, five, and almost eightfold in the remaining equity samples to make an investor equally well of in terms of alpha from a long-only hedged or a traditional long-short BAB with short selling cost. Break-even levels are reported in table 13. Coincidentally while below one in the Nordic sample all levels are above one in the remaining three samples, indicating that the long-only-hedge approach is inferior.

For the sake of thoroughness, the cost depressed performance statistics of SPX, FTSE and STOXX are reported in appendix 7.

Table 13: Break-even levels

The table reports lending fee break-even levels. Break-even is when the ex post performance of long-only-hedged equals the conventional BAB portfolio, net of short-selling costs. Levels below one are indicative that fees at current level depress portfolio performance more than alpha contribution from the short leg, conversely, break-even levels above one indicates that the performance of regular BAB portfolios net of costs supersede the performance of the long-only-hedged portfolios. Fees are calculated as of March 2015, but only for the Nordic sample, as the weighted average fees over the past 90 days. Weights are determined in accordance with the weights of the BAB portfolio at formation March 2015. Fees are set to be fixed, linearly indexed to the aggregated beta spread and the volatility of the underlying market. Volatility is calculated as a 250-days rolling average. Beta spreads are calculated as the difference between the betas of the high and low beta portfolios divided by its product.

Break-even levels	Fixed fee	SPRD fee (linear)	VOL fee (linear)
Nordics	0.80	0.64	0.63
Stoxx 600	5.61	4.10	4.24
FTSE	7.90	4.62	8.84
S&P	4.68	3.22	3.01

It does not make sense to derive the zero-profit fee level of the portfolios, because investors can always implement the long-only or long-only-hedged variation.

Finally, it should be noted that while lending fees are low, another concern of short-selling is the availability of lendable stocks and therein the buy-in risk and alpha decay during repositioning.[30] Thus although fees are low investors might still prefer the long-only variant. Appendix 8 reports aggregated short-utilization under different assumptions. At formation the unlevered utilization of the short-leg would be nearly 35% (but less than 19% excluding specials).

[30] The time after a recall to establish the short position again.

4.4 Robustness

Financial asset pricing is subject to vigorous analysis by financial economists. Over the last few decades the number of return drivers identified by academics (and practitioners) has vastly proliferated. John Cochrane has referred to the current collection of factors as a zoo (Cochrane, 2011). For observers or researchers the difficulty lies in discriminating between viable and unviable return anomalies. Many of these factors may simply have been discovered by chance or statistical error. Levi and Welch (2014) examined 600 factors (from academia and investment professionals) and found that only 51% were statistical significant out-of-sample. McLean and Pontiff (2013) could in a re-examination of 82 factors, all published in respected academic journals, only replicate 72, suggesting that nearly an eight of the attempted tests could be the result of reporting mistakes in database or statistical errors. Adjusted for data-snooping Harvey and Liu (2014) reports that only a handful of 315 factors remain statistical significant. Specifically, they confirm the existence of value, low volatility, and momentum. Hsu and Kalesnik (2014) test many of the reported factors in non-American datasets and also find value, low volatility, and momentum to be robust, but must reject other factors as mere artifacts of American data.

We tested the beta-anomaly and found statistically significant results, but there is one important caveat. Conventional test statistics are deemed significant at the 0.05 threshold. The null hypothesis is rejected when the probability of “chance” of a single and independent test is sufficiently low. Rarely, however, is only a single test performed. Admittedly, we have not reported *everything* we tried and we have no count of the total number of tests that we ran. For example we have tried different shrinkage factors, but the changes only seemed to affect ex ante beta accuracy and the market neutrality of the portfolios. Ex post low-beta alphas remained positive while high-beta alphas remained negative. Moreover, our procedure is based on the models and inferences of Black, Jensen, and Scholes (1972), Black (1972, 1993) and Frazzini and Pedersen (2014) and to some extent we apply the same data. Clearly the assumption of independence is violated as well.

To correct for multiple testing, we follow the advice of Harvey and Liu (2014) and apply the BHY haircut.[31] All tests are sorted on the basis of their statistical significance in descending order. The evaluation of the tests starts from the lowest k, where p-values sequentially are compared to their threshold. Once a test's p-value is lower than the corresponding threshold all ranks results above are accepted and ranks below are rejected. Consequently, the tests with the weakest significance are put to another test.

The results are mixed. Only eight out of sixteen one-factor alphas remain statistically significant; but the tests below the threshold have been tested using monthly observations or very short sample periods (where we would expect weaker results). MSCI ACWI and MSCI World indices are no longer significant when correcting for multiple testing. The MSCI EM BAB portfolio is the only remaining significant equity index. The strongest evidence is found in European Corporate Bonds; but STOXX, FTSE, SPX, NRDS hold as well. Contrarily, US Treasuries, OATs, Bunds and Spanish Public Treasuries are not significant at the tougher BHY hurdle. Only three out of eight five-factor alphas remain significant, but the pattern is similar: except for the SPX BAB portfolio the insignificant results are based on monthly observations. The results are summarized in appendix 9.

To summarize the above, when t-statistics are adjusted the number of statistically significant BAB portfolios are reduced to roughly half. Especially portfolios with few sample observations suffer. Nonetheless we will argue that beta-arbitrage is reliable. Hsu and Kalesnik (2014) argues that in order to be reliable, an investment strategy (as the portfolio method applied here) needs to have been tested out-of-sample, it has to be robust with regard to different estimation techniques, have large test statistics and have a theoretical explanation. Many of the prerequisites for reliability have been rigorously tested before us. The beta factor was formalized by Black, Jensen, and Scholes (1972) years ago and the results have proved robust out-of-sample many years later (Black, 1993), in different markets and asset classes, from different databases (Frazzini and Pedersen, 2014), within and across industries (Asness, Frazzini, and Pedersen, 2014, draft), thus the evidence for the

[31] (Benjamini & Hochberg, 1995): The methodology is more carefully explained in the appendix

beta factor is very strong. The phenomenon is persistent with respect to various beta estimation techniques, shrinkage factors, market proxies, sampling periods, and riskless rates (Frazzini and Pedersen, 2014). Our investigation (mostly) confirms the results. There are slight methodological differences, but comparison of results reveals the same relationships, confirming the robustness of long low-beta and short high-beta portfolios. We have not tested for different financing rates, but when we do a simple scaling of the risk free rates, the results (mostly) remain significant. We also tested for different shrinkage factors, and while affecting the beta estimation accuracy, ex post performance largely remained the same. In a few instances we constructed equally-weighted (opposed to beta-weighted) portfolios and the results were robust.

To elaborate we provide additional tests of robustness in the appendices. First, appendix 10 reports bond portfolios sorted by maturity and BAB portfolios split into subgroups. While test statistics are naturally lower (given the smaller sample size), the pattern is consistent in both periods. Low beta bonds have positive alphas, while high beta bonds have negative alphas. Considering the maturity sorted bond portfolios, alphas decline monotonically. Appendix 11 reports equities and equity indices in a similar fashion, and the results are mostly consistent. As discussed earlier – in accordance with Black, Jensen and Scholes (1972) – the results show that the BAB-factor is not stationary over time and there is significant time-variation of excess returns (alpha) and the statistical significance is naturally lower given the relatively short sample periods, but alphas (mostly) remain positive in both sub-periods. A word of caution, slicing one large sample into two smaller samples is not a true out-of-sample test. Rather, we merely consider it evidence that BAB portfolios have not worked solely based on a single event or a particularly set of short-term circumstances.

In section 4.2 we also investigated the robustness of the BAB performance across size, for constrained and unconstrained investors. It was evident that beta-arbitrage is mostly unaffected and remains statistically significant when depressed with lending fees. The short-selling sample was limited to the Nordic Region and we only had access to a single point-in-time average fee, but applying the more conservative estimate had very little impact on BAB performance. To eliminate the abnormal returns, performance fees would have to increase so dramatically, which is why we are doubtful it can be a sampling fluke, and even then low-beta securities would remain attractive on a risk-adjusted basis.

In section 4.3 we tested the effect of size. When sorted low- and high- beta equities by size we found that low-beta firms in especially the smallest size decile held very large risk-adjusted returns, but low-beta equities remained attractive while high-beta equities remained unattractive in most size groups. We tested only the performance of equally-weighted legs, to expand it would be interesting to construct conventional size-neutral BAB portfolios. Similarly, it would be interesting to investigate the performance of idiosyncratic- or total volatility sorted BAB portfolios.

Finally, but of equal importance we have a theoretical explanation, when facing leverage constraints investors are forced to overweight securities with higher expected returns, thus risk-willing investors are forced to bid up high-beta asset prices and make low-beta assets more attractive on a risk-adjusted basis.

4.5 Further Issues

In the examination of beta-arbitrage portfolios we have confirmed the abnormal risk-adjusted performance of BAB portfolios. The results are mostly consistent with what we expected, although we were surprised to find poor performance in S&P sorted industries. The investigation was based on a relatively wide but short sample. Previous literature has reported similar, or stronger, results over longer periods, but it would be interesting to conduct similar analyses in the future to see if the anomaly persists. It would also be interesting to conduct similar examinations more rigorously in emerging- and frontier- markets such as Africa and the Middle East.

We believe it would add valuable information to further investigate or quantify the underlying explanations for why beta-arbitrage exists and why arbitrageurs have historically been unable to converge prices. Our results show that BAB portfolios are positively correlated with beta spreads, a measure of implicit leverage or leverage constraints, but we have not properly investigated whether strategic timing may improve the beta-arbitrage portfolios, as we would expect given our results. We did not thoroughly investigate how funding shocks may affect arbitrageurs in particular or whether these can be alleviated. Nor have we shed further light on whether funding liquidity risk may be a sufficient risk factor to explain the abnormal performance of BAB portfolios. Finally, we have not directly tested for a market liquidity risk premium.

Measured by size, our examination found no evidence that trade-difficulty is a particular challenge to beta-arbitrage. Beta mispricing is roughly constant across all size groups. While beta and size does not seem correlated, it would be interesting to form size-neutral portfolios to understand the relative performance. Another limitation is that we only considered the size effect in the US market, future research should expand this to international markets and possibly over a longer period.

We also considered short-selling. Beta-arbitrage is more pronounced in the long low-beta leg, but long-short implementation is mostly preferable. Unfortunately, given our sample limitations, we were unable to properly examine the exact costs of short-selling securities. Moreover, the sample relies on a single source, the representativeness of which is unknown. It would be interesting if investigators with better data access were to construct a more accurate examination. We would also like to investigate the performance of short-cost optimized portfolios, for example, formed without expensive-to-short stocks.

While size and short-selling are interesting topics of implementation limitations, costs such as commissions, bid-ask spreads and a better estimate of price-impact (as opposed to size) could very well be investigated in the future.

Our results suggest that for relatively sophisticated and unconstrained investors BAB portfolios offer an attractive method of implementation to exploit the well-documented and apparently very profitable beta anomaly, both in equities and bonds, the US and international markets.

5 Conclusion

The Capital Asset Pricing Model dictates that all investors hold the market portfolio, where the return-to-risk relationship is steepest. Investors then leverage or deleverage the risky portfolio in order to adjust the expected return and risk profiles of their combined portfolios. But not all investors have unconstrained access to debt financing. Investors are subject to regulatory requirements and behavioural biases. When met with leverage constraints, investors are forced to buy riskier assets in order to form portfolios with the desired level of expected returns. As a result, constrained investors bid up prices of high-beta securities, returns decline and become unattractive on a risk-adjusted basis. Unconstrained investors can exploit this asymmetry by constructing *betting against beta* portfolios that are long leveraged low-risk (beta) securities and short deleveraged high-risk (beta) securities.

Following the framework of Black, Jensen and Scholes (1972), Black (1993) and Frazzini and Pedersen (2014), using historical data from the US and international markets, in equities and bonds, we constructed 16 BAB portfolios and tested the excess performance with index constituents (**proposition 1**). The analysis extends the literature by testing several international bond markets that have not previously been investigated. In order to replicate the strategy as we suspect actual investment professionals would, we form portfolios with index constituents rather than the entire universe of investable equities. Our results are consistent with Black's theory of restricted borrowing. The SML line is too flat and long-short beta-arbitrage portfolios have positive abnormal risk-adjusted returns when evaluated on 1-, 3-, and 5-factor models with size, value, momentum and quality. We construct BAB portfolios using the same methodological approach as Frazzini and Pedersen (2014) and find almost identical results, albeit the reported test statistics here are substantially lower, potentially due to the much shorter sampling period.

The results are mostly robust to different shrinkage factors, time-periods and weighting schemes.

While the excess returns are positive on average we find significant return time variation. According to Huang, Polk and Lou (2014, working paper) beta-arbitrage profits goes through cycles of booms and busts. High beta securities offer implicit leverage to constrained investors. When the aggregate beta spread widens the implicit leverage increases and beta-arbitrage activity intensifies. Arbitrageurs buy and short low and high betas, respectively, and short-term beta-arbitrage profits increase. Subsequently, when arbitrage activity takes hold or is reduced the subsequent performance declines thus explaining the time variation. Our investigation shows that when beta spreads are wide BAB profits are large and vice versa. This suggests that timing can crucially affect the short-term profits that institutional investors will be able to reap in the application of beta-arbitrage strategies (proposition 2).

The performance of paper strategies can be severely overstated. Short-selling can be difficult or expensive and many investment professionals prefer long-only strategies. Leverage capable investors have to allocate large quantities of capital and are frequently unable to trade microcap stocks. To investigate potential implications, we investigated the effect of size and short-selling on the performance of BAB-arbitrage (proposition 3, 4). We found that while alphas on high beta securities are negative on average, the performance of long-only BAB portfolios is positive and statistically significant because both raw and excess returns are mostly driven by the long low beta leg. Put differently, short constrained investors may advantageously construct long-only portfolios to exploit the mispricing of low beta securities without engaging in short selling activities. The investigation is similar to Israel and Moskowitz' (2012) investigation of value, size and momentum portfolios.

To evaluate the trade-off between long-only and long-short more realistically we attempted to measure the costs of lending equity securities to sell short (proposition 3). To do so we use an samples of the largest Nordic equity indices, KFX, OMX, HEX and OBX. Lending fees are generally low, but slightly higher for high-beta securities. We found that when lending fees were considered – because alphas were predominantly driven by the long leg – investors would strictly prefer long-only implementation hedged with market futures in NRDS. The results were different in the remaining equity markets where short-selling individual securities remained preferable because the contribution of high beta alphas were substantially. We note that these results are somewhat biased, because fees were estimated using only a sample of the largest and most liquidly trading Nordic equities and we cannot be sure of their representativeness in global markets. We did not consider the impact of commissions or transaction costs, which might further reduce the excess performance of long-short portfolios. We would have liked to construct short-optimized portfolios by removing specials, but the sample set did not allow us to identify these securities across time.

When sorting beta-portfolios into size deciles (based on market capitalization) we found no significant relationship between size and alphas (**proposition 4**). Surprisingly, the risk-adjusted performance across size revealed no size-dependency. The long leg produced positive alphas across all deciles except for the smallest, and the short leg produced negative alphas across all deciles, except for the largest. We tested the performance of the US BAB portfolio excluding the smallest decile to investigate the impact. When excluding the smallest securities (measured by market capitalization, in terms of number of firms) the performance of the BAB portfolio remains positive and statistically significant. The performance is reduced, but with very modest tracking error. We only investigated the effect of size in US equities, it would be interesting to perform similar analyses in international markets.

The investigation adds to a long range of empirical studies of the beta-anomaly. The results are interesting for several reasons. Institutional investors are increasingly demanding exposure to alternative risk factors that BAB and similar strategies offer. Our results indicate that such investors, with liberal access to debt financing, can implement strategies in multiple markets and asset classes. Short hesitant investors can construct long-only portfolios with very similar results. Low-beta alphas are largest in the smallest equities, but we find that size frictions cannot alone explain the superior performance of low-beta equities. Moreover, long-short investors can engage in beta-arbitrage with little adverse effects from lending costs. Although we observe that high-beta securities tend to be more expensive to sell short, we find no evidence that lending fees are sufficiently large to explain the inferior performance of high-beta equities.

The results are challenging to the standard theory of market efficiency. The implementation frictions discussed do not seem adequate to explain deviations from equilibrium prices. However, we do not further attempt to determine whether beta-arbitrage is truly *alpha* or exposed to yet unclarified sources of risk. There is return variation, but returns are positive on average. Further investigations may emphasize the underlying explanations of BAB performance over time.

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7 Appendices

Appendix 1: Summary of firms with high and low lending fees

The table shows the top and bottom 15% of the Nordic equity sample in terms of lending fees over the past 90 days.

#	Name	Fee 30	Fee 90	Short Interest	Utilization	MV	Beta
	Highest 15% (descending)						
1	Petroleum Geo-Services	4.4%	4.0%	13.6%	75%	8.84	0.63
2	Outotec	4.2%	3.8%	12.3%	94%	0.89	1.11
3	Norwegian Air Shuttle	4.3%	3.7%	7.8%	89%	7.08	0.96
4	TGS Nopec Geophysical Co	3.0%	3.0%	13.3%	71%	18.82	1.33
5	YIT	4.2%	2.6%	9.9%	65%	0.70	0.88
6	SSAB	2.8%	1.9%	8.5%	77%	13.64	1.33
7	Golden Ocean Group	1.8%	1.8%	14.0%	98%	1.33	1.42
8	Det Norske Oljeselskap	3.1%	1.8%	5.6%	71%	3.91	1.67
9	Cargotec	1.8%	1.6%	3.8%	37%	1.40	1.19
10	Royal Caribbean Cruises	1.7%	1.6%	0.3%	51%	39.88	1.29
11	FLSmidth & Co	1.7%	1.4%	14.7%	63%	15.01	1.38
12	REC Silicon ASA	1.9%	1.4%	9.5%	62%	4.31	NA
13	Outokumpu OYJ	1.5%	1.2%	7.0%	60%	2.37	0.99
14	Opera Software ASA	1.5%	1.1%	11.4%	85%	7.94	2.02
15	Askastor	1.3%	1.1%	5.9%	89%	2.58	1.26
	Lowest 15% (ascending)						
100	Atlas Copco AB	0.1%	0.1%	0.5%	3%	94.51	1.26
99	Investor AB	0.1%	0.1%	0.6%	3%	148.03	0.75
98	Telefonaktiebolaget LM Ericsson	0.1%	0.1%	0.7%	3%	324.42	0.70
97	Investment AB Kinnevik	0.2%	0.1%	0.6%	2%	63.42	0.55
96	Nordea Bank AB	0.1%	0.1%	1.3%	9%	448.33	0.83
95	Svenska Cellulosa AB SCA	0.2%	0.1%	0.9%	4%	126.72	0.55
94	Electrolux AB	0.2%	0.1%	2.0%	10%	79.18	0.80
93	Assa Abloy AB	0.2%	0.1%	0.8%	4%	173.63	0.90
92	Swedbank AB	0.2%	0.1%	1.0%	7%	246.66	0.79
91	Swedish Match AB	0.2%	0.1%	1.9%	8%	52.77	0.26
90	Pandora A/S	0.2%	0.1%	0.7%	3%	76.43	0.94
89	Orkla ASA	0.2%	0.1%	1.1%	13%	48.99	1.19
88	Skanska AB	0.2%	0.2%	0.7%	4%	79.56	0.81
87	Boliden AB	0.2%	0.2%	0.9%	3%	45.07	1.21
86	Hennes & Mauritz AB	0.2%	0.2%	1.0%	8%	497.65	0.54
85	Svenska Handelsbanken AB	0.2%	0.2%	1.7%	10%	254.44	0.70

Appendix 2: Bond performance statistics

The table summarizes bond portfolios sorted on maturities. Alphas are annualized and in excess of the risk free rate. Betting against beta portfolios are market neutral strategies long low maturity and short high maturity leveraged to an ex ante beta of one. The portfolio legs are equally weighted and rebalanced monthly. Betas are estimated using 250 daily (36 monthly) excess returns regressed on the returns of an equally-weighted average portfolio. T-statistics are shown below in brackets.

Panel A: US Treasuries 1988 – 2015

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	1.5%	2.9%	3.7%	4.3%	6.2%	1.2%
Alpha	0.6%**	0.4%	0.2%	-0.4%**	-0.9%	1.5%**
(t-stat)	(3.51)	(1.75)	(1.09)	(-3.06)	(-1.43)	(2.58)
Sigma	1.5%	3.5%	4.7%	6.2%	9.8%	2.9%
Sharpe	1.03	0.84	0.79	0.69	0.63	0.41
Beta	0.24	0.66	0.94	1.26	1.89	-0.09

Panel B: German Bunds 2001 – 2015

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	1.8%	3.1%	4.2%	5%	7.2%	1.5%
Alpha	0.8%**	0.6%*	0.3%	-0.0%	-1.7%*	1.6%*
(t-stat)	(4.76)	(2.34)	(1.42)	(-0.21)	(-2.46)	(2.36)
Sigma	1.1%	2.6%	3.8%	5%	9%	2.6%
Sharpe	1.57	1.19	1.1	1.02	0.8	0.59
Beta	0.22	0.59	0.9	1.19	2.1	-0.02

Panel C: Spanish Public Treasuries 1985 – 2011

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	2.1%	4.2%	5%	5.6%	7.1%	2.8%
Alpha	2.1%**	0.1%	-0.1%	-0.7%	-1.3%	3.2%*
(t-stat)	(3.93)	(0.16)	(-0.21)	(-1.27)	(-1.22)	(2.54)
Sigma	1.8%	6.2%	7.4%	9%	12.4%	4.3%
Sharpe	1.18	0.68	0.67	0.61	0.56	0.64
Beta	0.01	0.86	1.06	1.13	1.76	-0.08

Panel D: French OATs 1987 – 2015

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	1.7%	2.9%	3.8%	4.6%	6.2%	1.1%
Alpha	0.5%**	0.4%	0.1%	-0.1%	-0.9%*	1.2%*
(t-stat)	(2.84)	(1.87)	(0.93)	(-0.74)	(-2.02)	(2.24)
Sigma	1.6%	3%	4%	5.2%	8.2%	2.8%
Sharpe	1.06	0.98	0.93	0.89	0.76	0.39
Beta	0.3	0.66	0.95	1.22	1.87	-0.03

Panel E: UK Gilts 2001 – 2014

	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	+15 years	BAB
Return	2.35%	3.37%	4.00%	4.65%	5.18%	5.86%	2.46%
Alpha	1.34%	1.20%**	0.59%	-0.18%	-0.72%**	-2.23%*	2.72%**
(t-stat)	(4.84)	(3.35)	(1.83)	(-0.74)	(-2.66)	(-2.51)	(3.87)
Sigma	1.49%	2.68%	3.85%	5.25%	6.40%	9.26%	2.63%
Sharpe	1.57	1.26	1.04	0.89	0.81	0.63	0.93
Beta	0.24	0.51	0.81	1.14	1.39	1.91	-0.06

Panel F: Danish Government Bonds 1999 – 2014

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	1.8%	2.9%	3.8%	4.9%	7.3%	3.3%
Alpha	1.6%**	0.3%	-0.0%	-0.2%	-1.8%*	3.3%**
(t-stat)	(4.59)	(0.96)	(-0.02)	(-0.62)	(2.36)	(3.69)
Sigma	1.4%	2.7%	3.7%	4.8%	8.9%	3.6%
Sharpe	1.26	1.06	1.03	1.01	0.83	0.91
Beta	0.04	0.62	0.92	1.22	2.19	-0.01

Panel G: US Credit Indices 2006 – 2014

	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	+15 years	BAB
Return	2.84%	4.12%	5.33%	5.14%	5.73%	6.73%	3.32%
Alpha	1.49%*	1.07%*	0.94%*	-0.66%	-0.55%	-2.28%*	3.40%**
(t-stat)	(3.22)	(2.53)	(2.06)	(-1.62)	(-0.81)	(-2.22)	(4.26)
Sigma	1.7%	2.7%	3.7%	4.7%	5.3%	7.7%	2.3%
Sharpe	1.68	1.54	1.45	1.09	1.08	0.88	1.47
Beta	0.27	0.61	0.88	1.16	1.26	1.81	-0.02

Panel H: European Corporate Bonds

	1-3 years	3-5 years	5-7 years	7-10 years	+10 years	BAB
Return	2.40%	3.59%	4.45%	5.03%	6.44%	2.59%
Alpha	1.16%**	0.75%**	0.30%	-0.33%*	-1.88**	2.69%**
(t-stat)	(6.93)	(3.52)	(1.54)	(-2.24)	(-3.42)	(4.93)
Sigma	1.15%	2.34%	3.29%	4.18%	6.76%	2.08%
Sharpe	2.08	1.53	1.35	1.20	0.95	1.25
Beta	0.28	0.65	0.95	1.22	1.90	-0.02

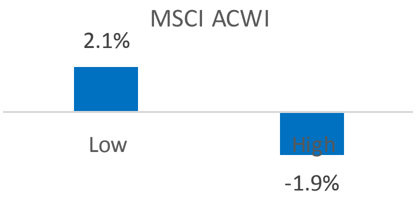
Appendix 3A: Equity indices performance statistics

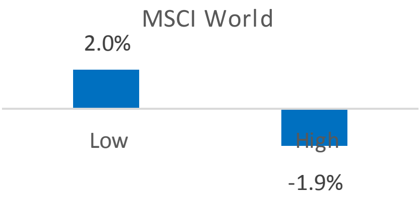
The table shows low-, high- and BAB portfolios of four equity indices. Low and high portfolios are unlevered. Statistics are annualized. Below alphas are visualized.

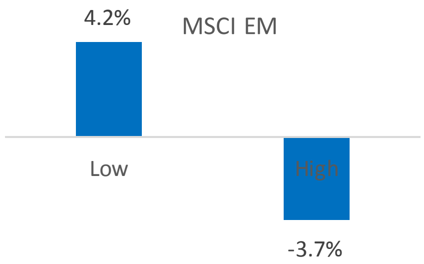
	MSCI ACWI			MSCI World		
	Low	High	BAB	Low	High	BAB
Return	9.7%	9.2%	5.6%	7.3%	6.2%	4.5%
Alpha	2.11%*	-1.88%*	4.73%*	3.96%*	-1.89%*	3.96%*
(t-stat)	(2.09)	(-2.18)	(2.25)	(2.19)	(-2.45)	(2.19)
Std.	14%	20%	7.2%	12.9%	19.3%	7.6%
Sharpe ratio	0.69	0.45	0.78	0.57	0.32	0.6
Beta	0.8	1.17	0.09	0.78	1.21	0.08

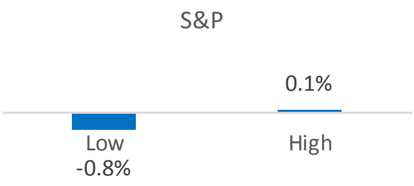
	S&P Industries			MSCI EM		
	Low	High	BAB	Low	High	BAB
Return	6.2%	10.3%	-1.4%	16.8%	11.1%	6.9%
Alpha	-0.8%	0.1%	-1.9%	4.17%**	-3.7%**	7.5%**
(t-stat)	(-0.68)	(0.14)	(-0.8)	(4.34)	(-4.46)	(4.39)
Std.	13.3%	18.4%	11.1%	21.3%	24.8%	5.8%
Sharpe ratio	0.47	0.56	0.05	0.79	0.45	1.2
Beta	0.82	1.19	-0.13	0.98	1.14	-0.04

** Indicates strong significance, *Indicates significance







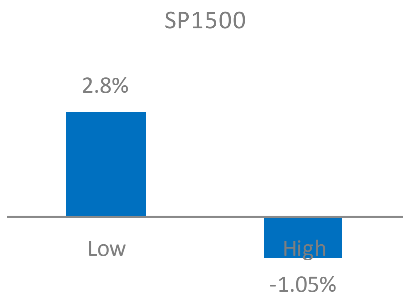


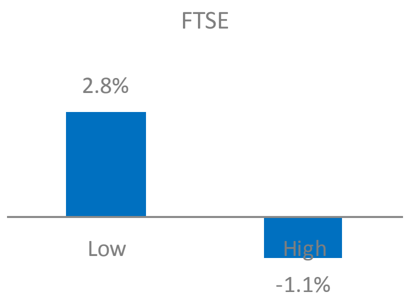
Appendix 3B: Equities performance statistics

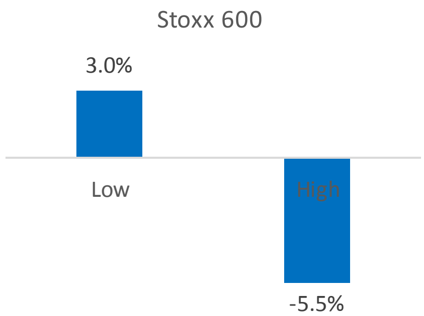
The table shows low-, high- and BAB portfolios of four equity samples. Low and high portfolios are unlevered. Statistics are annualized. Below alphas are visualized.

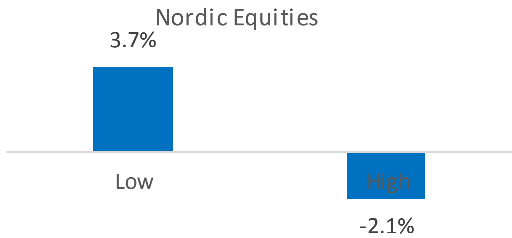
	S&P 1500 Composite			FTSE All-Share		
	Low	High	BAB	Low	High	BAB
Return	10.3%	13.9%	7.04%	5.09%	4.56%	10.97%
Alpha	2.1%*	-4.4%**	7%**	2.83%**	-1.05%	11.02%**
(t-stat)	(2.28)	(-3.61)	(2.76)	(2.88)	(-1.24)	(4.45)
Std.	13.5%	29.2%	11.1%	8.71%	19.36%	10.66%
Sharpe	0.76	0.47	0.64	0.58	0.24	1.03
Beta	0.58	1.29	0.01	0.46	1.13	-0.04

	Euro Stoxx 600			Nordic Equities		
	Low	High	BAB	Low	High	BAB
Return	9.7%	8.3%	11.4%	9.9%	10.8%	14.2%
Alpha	3%**	-5.5%**	10.9%**	3.7%*	-2.1%	14.1%**
(t-stat)	(3.18)	(-4.79)	(4.63)	(2.04)	(-0.66)	(3.84)
Std.	13.3%	26.8%	8.8%	14.9%	30.6%	12.8%
Sharpe ratio	0.73	0.31	1.3	0.66	0.31	1.11
Beta	0.65	1.34	0.05	0.56	1.2	0.01









** Indicates strong significance, *Indicates significance

Appendix 4A: Average annual returns of equity BAB portfolios

The table reports annualized averaged daily returns for given years. For every portfolio the start year may not cover an entire year.

	Equities				Equity indices			
Year	SPX	FTSE	NRDS	STOXX	MSCI World	MSCI EM	MSCI ACWI	SP Industries
1993	-	-	-	-	-	-	-	0%
1994	-	-	-	-	-	-	-	-2%
1995	-	-	-	-	-	-	-	11%
1996	-	-20%	-	-	-	-	-	0%
1997	18%	-3%	-	-	-	-	-	7%
1998	1%	-14%	-	-	11%	-	-	6%
1999	-17%	-7%	-	-	-1%	-	-	-12%
2000	31%	55%	-	-	5%	-	-	-16%
2001	13%	57%	-	55%	-11%	-	-	-31%
2002	7%	-6%	12%	-4%	-7%	-	-	-26%
2003	6%	23%	30%	30%	18%	-	-	-3%
2004	16%	27%	39%	46%	15%	-1%	12%	2%
2005	13%	11%	22%	20%	8%	9%	12%	1%
2006	8%	16%	9%	13%	11%	7%	8%	8%
2007	11%	-2%	-11%	3%	1%	4%	-10%	8%
2008	-27%	-23%	-46%	-9%	-7%	6%	-9%	-13%
2009	-4%	15%	21%	-3%	-5%	7%	-7%	1%
2010	3%	19%	9%	6%	2%	10%	2%	-6%
2011	9%	1%	-1%	10%	14%	5%	12%	13%
2012	6%	4%	14%	7%	2%	5%	2%	6%
2013	12%	33%	41%	13%	13%	10%	22%	1%
2014	14%	-6%	44%	11%	9%	9%	16%	13%
Profitable years	84.2%	57.9%	76.9%	78.6%	70.6%	90.9%	72.7%	59.1%

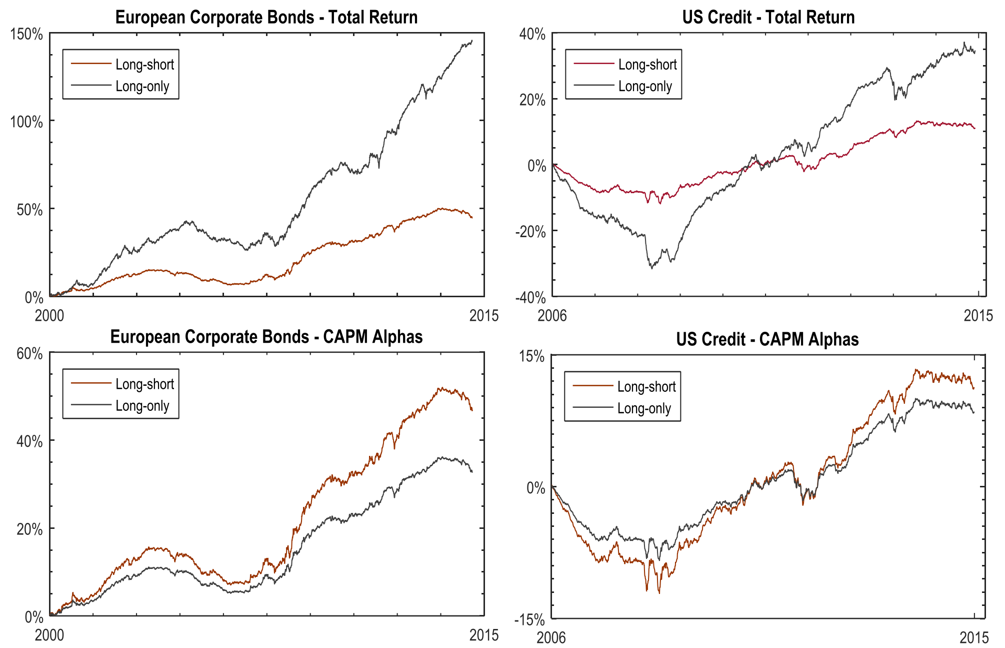
Appendix 4B: Average annual returns of bond BAB portfolios

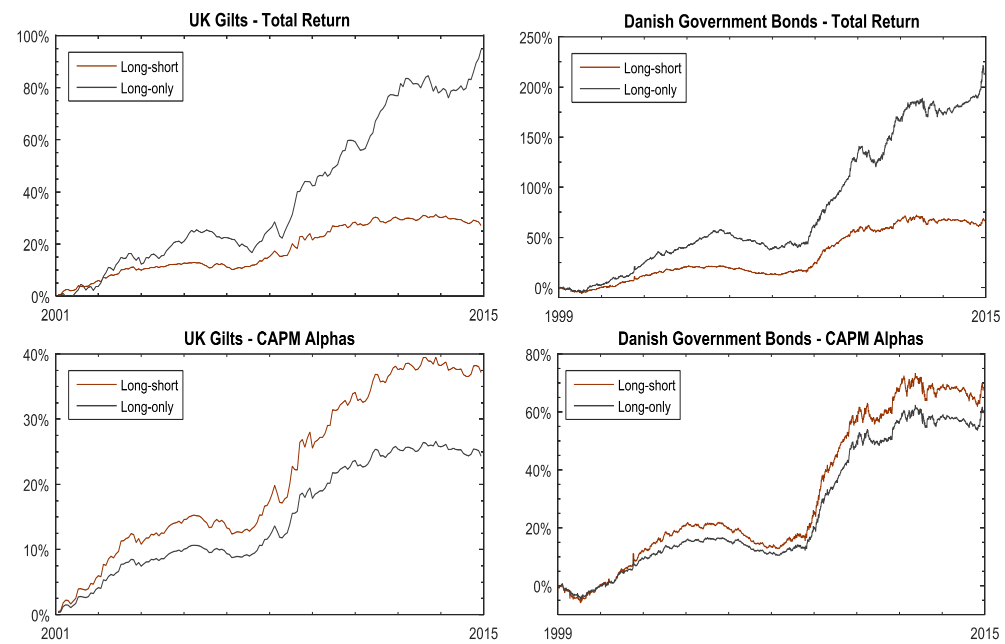
The table reports annualized averaged daily returns for given years. For every portfolio the start year may not cover an entire year.

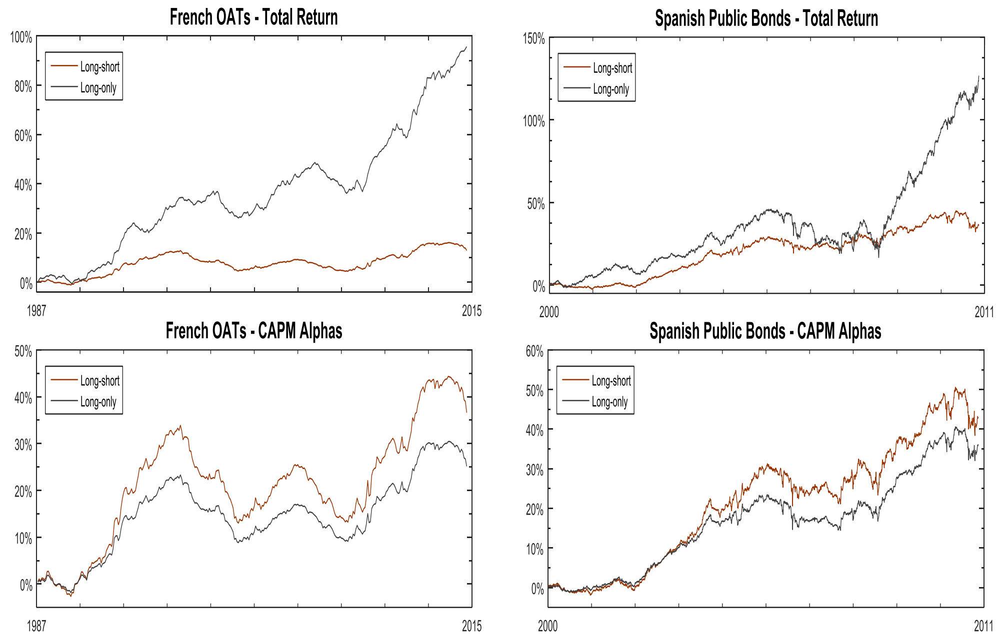
	Government						Corporate	
Year	US Treasury	German Bunds	UK Gilts	Spanis h	French AOTs	Danis h	Credi t	Euro Corp
1988	-	-	-	-	-1%	-	-	-
1989	-	-	-	-	-2%	-	-	-
1990	-	-	-	-	4%	-	-	-
1991	6%	-	-	-	2%	-	-	-
1992	3%	-	-	-	9%	-	-	-
1993	-1%	-	-	-	4%	-	-	-
1994	-1%	-	-	-	5%	-	-	-
1995	-1%	-	-	-	4%	-	-	-
1996	1%	-	-	-	2%	-	-	-
1997	-1%	-	-	-	-7%	-	-	-
1998	1%	-	-	-	-2%	-	-	-
1999	2%	-	-	-	-2%	-6%	-	-
2000	-1%	-	-	-1%	-4%	2%	-	12%
2001	7%	3%	4%	1%	2%	6%	-	3%
2002	3%	5%	6%	6%	3%	8%	-	5%
2003	1%	3%	3%	8%	2%	4%	-	4%
2004	-2%	-1%	2%	4%	-1%	1%	-	0%
2005	-5%	-4%	0%	5%	-5%	-1%	-	-4%
2006	-1%	-3%	-1%	-2%	-2%	-5%	3%	-2%
2007	5%	3%	4%	3%	1%	2%	2%	2%
2008	0%	6%	5%	3%	3%	14%	0%	5%
2009	7%	8%	6%	7%	8%	12%	9%	11%
2010	4%	4%	3%	-3%	2%	4%	4%	3%
2011	-1%	1%	3%	0%	5%	9%	-1%	3%
2012	0%	0%	1%	-	3%	-2%	8%	5%
2013	4%	0%	0%	-	0%	-1%	5%	4%
2014	-2%	-3%	-1%	-	-4%	1%	-1%	-3%
Profitabl e years	54%	57%	80%	67%	63%	69%	77%	80%

Appendix 5: Performance overview of long-only and long-short portfolios

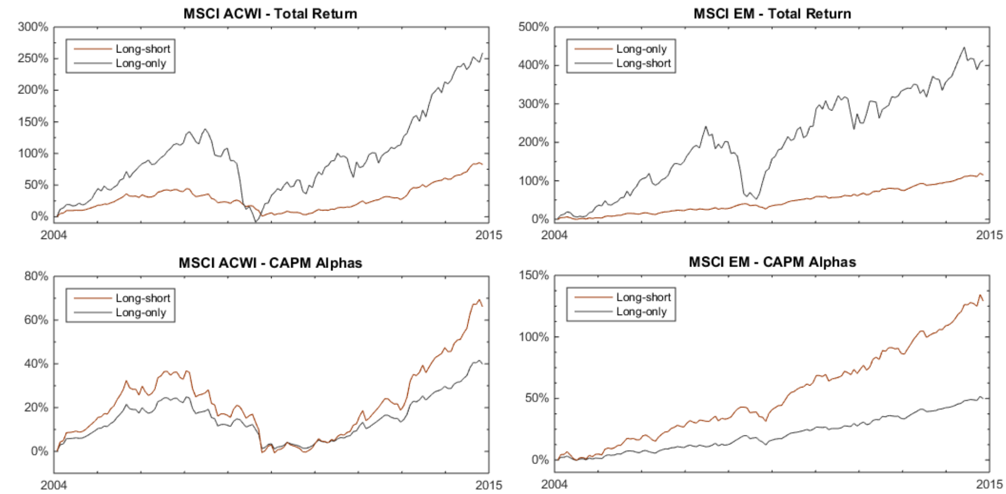
The charts plot the cumulative performance of long-only and short-short beta-arbitrage BAB portfolios. Charts to the left illustrate total excess returns, while charts to the right illustrate excess risk-adjusted returns (one-factor alphas evaluated on CAPM). Long-short portfolios are conventional *betting against beta* portfolios, long low-beta and short high-beta (maturity). Bond and equity index portfolios are equally-weighted, equity portfolios are beta-weighted, giving higher weights to low betas in the long leg and higher weights to high betas in the short leg. Both legs are leveraged to ex ante beta of one. The long-only portfolios are leverage low-beta (maturity) portfolios as well. Panel A contains government and corporate bond portfolios, panel B contains equity index portfolios and panel C contains regular equity BAB portfolios.

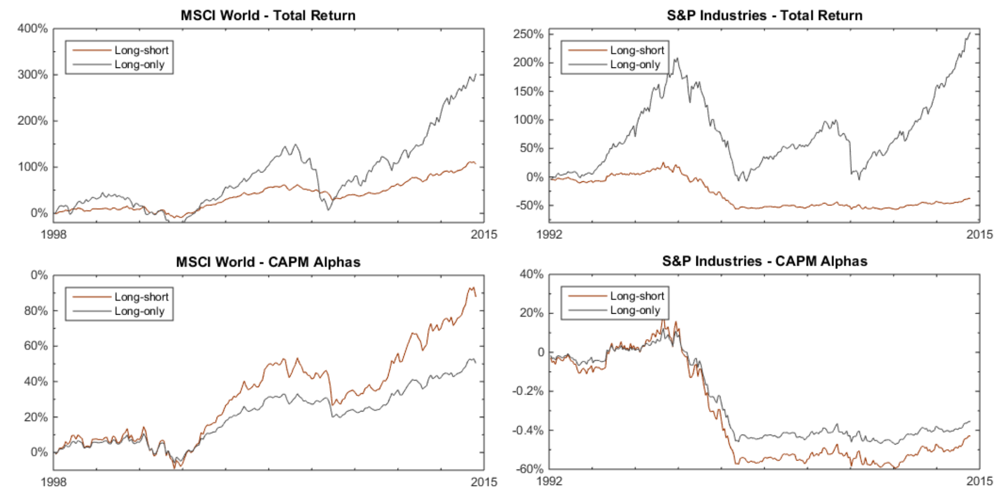




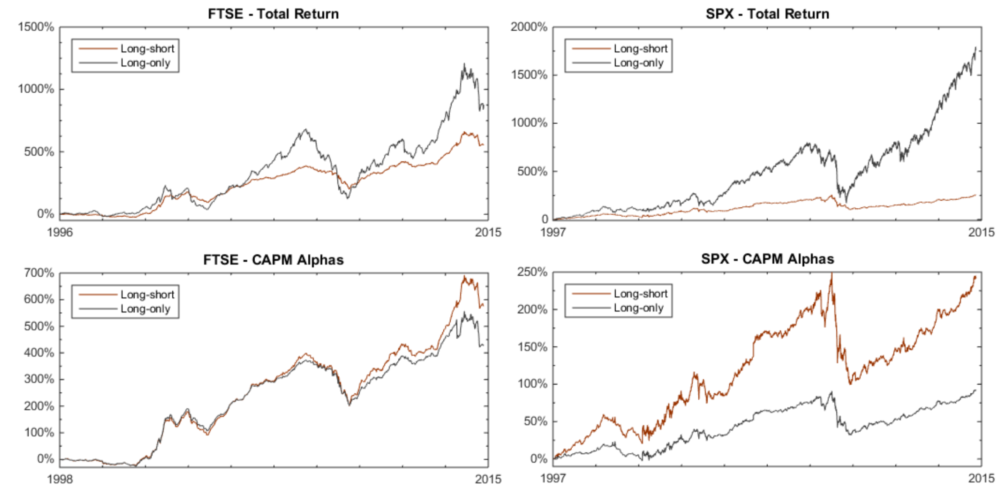

French OATs - Total Return
Long-short
Long-only
100%
80%
60%
40%
20%
0%
1987
2015
Spanish Public Bonds - Total Return
Long-short
Long-only
150%
100%
50%
0%
2000
2011
French OATs - CAPM Alphas
Long-short
Long-only
50%
40%
30%
20%
10%
0%
1987
2015
Spanish Public Bonds - CAPM Alphas
Long-short
Long-only
60%
50%
40%
30%
20%
10%
0%
2000
2011

Panel B


MSCI ACWI - Total Return
Long-short
Long-only
300%
250%
200%
150%
100%
50%
0%
2004
2015
MSCI EM - Total Return
Long-only
Long-short
500%
400%
300%
200%
100%
0%
2004
2015
MSCI ACWI - CAPM Alphas
Long-short
Long-only
80%
60%
40%
20%
0%
2004
2015
MSCI EM - CAPM Alphas
Long-short
Long-only
150%
100%
50%
0%
2004
2015

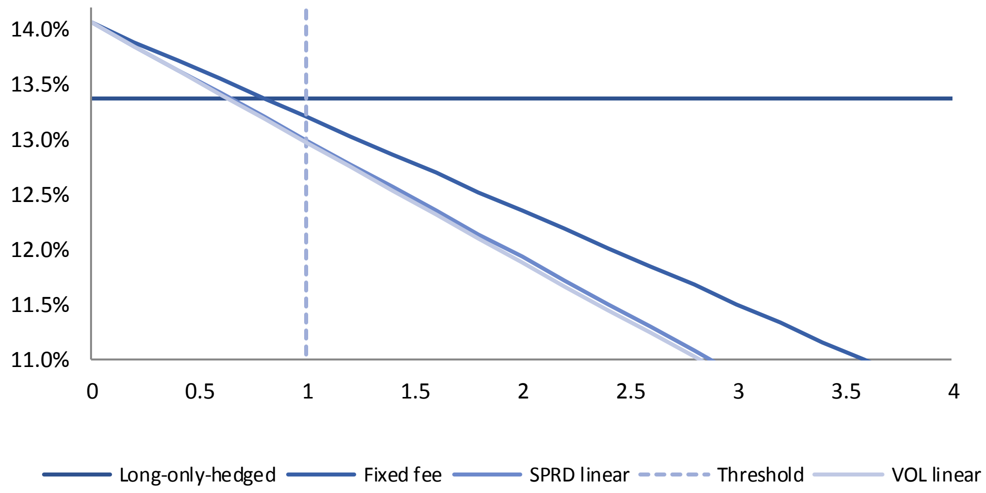

MSCI World - Total Return
400%
300%
200%
100%
0%
1998
2015
Long-short
Long-only
S&P Industries - Total Return
250%
200%
150%
100%
50%
0%
-50%
1992
2015
Long-short
Long-only
MSCI World - CAPM Alphas
0%
80%
60%
40%
20%
0%
1998
2015
Long-short
Long-only
S&P Industries - CAPM Alphas
40%
20%
0
-0.2%
-0.4%
-60%
1992
2015
Long-short
Long-only

Panel C


FTSE - Total Return
1500%
1000%
500%
0%
1996
2015
Long-short
Long-only
SPX - Total Return
2000%
1500%
1000%
500%
0
1997
2015
Long-short
Long-only
FTSE - CAPM Alphas
700%
600%
500%
400%
300%
200%
100%
0%
1998
2015
Long-short
Long-only
SPX - CAPM Alphas
250%
200%
150%
100%
50%
0%
1997
2015
Long-short
Long-only

Appendix 6: Break-even level of lending fees

The vertical axis shows annualized alpha coefficients. The horizontal axis shows percentage changes in lending fees, with 1 equal to the current fees. The intercept (14.1%) is the annualized alpha of an unconstrained BAB portfolio. The long-only hedged portfolio alpha is fixed because it is insensitive to changes in lending costs. The lines of the short-cost portfolios are downward-sloping because alphas are depressed as aggregate fees increase.



Appendix 7: Constrained and unconstrained BAB portfolios

The table reports the performance of seven BAB portfolios under different assumptions. The rightmost is the previously presented unconstrained BAB, it compared six portfolios of different constraints. The *long-only-hedged* portfolio is long low-beta and short the market index (equally-weighted). The fixed fee is calculated at March 2015 and added to the short-leg at every period. Variable fees are computed as a linear functions of an index-level of SPRD and historical volatility. SPRD is calculated as the difference between beta coefficients of the long and short legs divided by the product of betas. Statistics are annualized and in excess of the risk free rate.

		Unconstrained BAB	Long-only (hedged)	Fixed fee (30 days)	SPRD fee (linear)	VOL fee (linear)
Stoxx 600	Alpha	10.9%**	6.9%**	10.2%**	9.9%**	9.9%**
	(t-stat)	(4.63)	(3.69)	(4.32)	(4.21)	(4.22)
FTSE	Alpha	11.16%	9.78%	10.98%	10.86%	11.00%
	(t-stat)	(4.45)**	(3.92)**	(4.38)**	(4.34)**	(4.39)**
S&P1500	Alpha	7.04%**	3.67%*	6.31%*	5.99%*	5.91%*
	(t-stat)	(2.76)	(2.11)	(2.48)	(2.35)	(2.32)
Nordic	Alpha	14.1%**	13.4%**	13.2%**	13.0%**	11.7%**
	(t-stat)	(3.84)	(3.52)	(3.61)	(3.55)	(3.04)

Appendix 8: Variations of short utilization

The table reports short utilization across different weighting schemes sorted by beta coefficients. Betas are calculated using 250 trading days regressed on and equally-weighted average of the OMX, KFX, OBX and HEX indices. Low and High is determined by the median coefficient. Beta-weighted is the BAB weighted-average fee at March 2015 formation. "No specials" exclude all securities with lending fees above 1% p.a.

	High beta	Low beta	All
Equally-weighted	33.4%	18%	25.9%
Value-weighted	16.5%	8.9%	11.3%
Beta-weighted	34.8%	18.3%	30.7%
(no specials)	18.8%	15.2%	17.6%

Appendix 9: Statistical results under multiple testing

The table ranks the alphas according to their p-values and compare them to the hurdle rate obtained from BHY method. Panel A tests the 1-factor CAPM alphas and panel B the five-factor alphas.

Panel A: one-factor alphas

Rank	Name	p-value	Hurdle	Pass	# test to insignificance
1	Euro corp	0.000001	0.000924	TRUE	5,782
2	Euro Stoxx	0.000004	0.001849	TRUE	3,014
3	FTSE	0.000009	0.002773	TRUE	2,021
4	EM	0.000030	0.003697	TRUE	909
5	Nordic	0.000126	0.004622	TRUE	313
6	Danish Gov	0.000233	0.005546	TRUE	216
7	UK gilts	0.001969	0.00647	TRUE	41
8	SP1500	0.005825	0.007395	TRUE	20
9	Spanish	0.011142	0.008319	FALSE	-
10	US treasury	0.012397	0.009244	FALSE	-
11	German bunds	0.018596	0.010168	FALSE	-
12	ACWI	0.027892	0.011092	FALSE	-
13	World	0.031003	0.012017	FALSE	-
14	French OAT	0.031068	0.012941	FALSE	-
15	US credit	0.091098	0.013865	FALSE	-
16	SP industries	0.432838	0.01479	FALSE	-

Panel B: five-factor alphas

Rank	Name	p-value	Hurdle	Pass	# test to insignificance
1	Euro stoxx	0.000014	0.006250	TRUE	526
2	Nordic	0.000128	0.008333	TRUE	141
3	FTSE	0.002992	0.010227	TRUE	15
4	EM	0.031179	0.012000	FALSE	-
5	SP1500	0.070108	0.013686	FALSE	-
6	SP industries	0.370507	0.015306	FALSE	-
7	World	0.664104	0.016873	FALSE	-
8	ACWI	0.790589	0.018397	FALSE	-

Appendix 10: Summary of BAB performance in Bonds across sample periods

The table summarizes bond portfolios sorted on maturities. Alphas are annualized and in excess of the risk free rate. Betting against beta portfolios are market neutral strategies long low maturity and short high maturity leveraged to an ex ante beta of one. The portfolio legs are equally weighted and rebalanced monthly. Betas are estimated using 250 daily (36 monthly) excess returns regressed on the returns of an equally-weighted average portfolio. T-statistics are shown below in brackets. Non-government portfolios have not been interest rate hedged.

Asset	1-3 years	3-5 years	5-7 years	7-10 years	+10 years (10-15 years)	+15 years	BAB
US Treasury	0.6%**	0.45%	0.21%	-0.39%**	-0.86%	NA	1.50%*
	(3.51)	(1.75)	(1.09)	(-3.06)	(-1.43)	NA	(2.58)
1988 – 2001	0.63**	0.28%	0.09%	-0.47%	-0.53%	NA	1.14%
	(3.22)	(1.14)	(0.46)	(-2.85)	(0.87)	NA	(1.89)
2001 – 2015	0.38%	0.47%	0.25%	-0.28%	-0.82%	NA	1.62%
	(1.46)	(1.04)	(0.67)	(-1.22)	(-0.76)	NA	(1.44)
German	0.82%**	0.6%*	0.34%	-0.04%	-1.72%	NA	1.60%*
Bunds	(4.76)	(2.35)	(1.42)	(-0.21)	(-2.46)*	NA	(2.36)
	0.76%**	0.39%	0.18%	-0.19%	-1.13%	NA	0.65%
2000-2007	(3.75)	(1.34)	(0.63)	(-1.04)	(-1.41)	NA	(0.85)
	0.94%**	0.93%*	0.55%	0.00%	-2.41%*	NA	2.41%*
2007-2015	(3.50)	(2.26)	1.39%	(0.00)	(2.13)	NA	(2.07)
UK Gilts	1.11%**	0.8%**	0.07%	-0.73%**	-1.25%**	-2.38%*	2.30%**
	(4.6)	(2.9)	(0.39)	(-3.27)	(2.93)	(-1.86)	(3.85)
1996 – 2006	1.36%**	0.62%*	0.04%	-0.71%**	1.31%**	-2.47%	1.89%*
	(5.43)	(2.59)	(0.26)	(-4.28)	(-2.90)	(-1.69)	(2.67)
2006 – 2015	0.68%**	0.78%*	-0.07	-0.40%	-1.00%	-1.13%	1.18%
	(3.45)	(2.36)	(-0.29)	(-1.59)	(-1.79)	(0.53)	(1.46)
Spanish Public	2.06%**	0.11%	-0.12%	-0.74%	-1.31%	NA	3.20%**
Treasury	(3.93)	(0.16)	(-0.21)	(-1.27)	(-1.29)	NA	(2.54)
1998 – 2005	2.70%**	0.38%	-0.16%	-0.90%	-2.02%	NA	3.99%**
	(4.71)	(0.56)	(-0.31)	(-1.45)	-1.71	NA	(3.40)
2005 – 2011	3.99%**	-0.43%	-0.33%	-0.60%	-0.56%	NA	2.69%
	(3.40)	(-0.35)	(-0.29)	(-0.57)	(-0.30)	NA	(1.14)
French OATs	0.52%**	0.38%	0.13%	-0.09%	-0.94%	NA	1.20%*
	(2.84)	(1.86)	(0.93)	(-0.74)	(-2.02)	NA	(2.24)
1985 – 1999	0.75%**	0.45%	0.12%	-0.32%	-0.32%	NA	1.38%
	(2.73)	(1.52)	(0.66)	(-1.82)	(-1.82)	NA	(1.80)
2000 – 2015	0.38	0.39%	0.21%	0.12%	-1.11%	NA	1.27%
	(1.47)	(1.27)	(0.87)	(0.63)	(1.43)	NA	(1.53)
Danish Govt.	1.64%**	0.33%	-0.01%	-0.19%	-1.77%**	NA	3.34%**
Bonds	(4.59)	(0.96)	(-0.02)	(-0.62)	(-2.36)	NA	(3.69)
1998-2006	1.60%**	-0.06%	-0.22%	-0.42%	-0.90%	NA	1.92%
	(3.03)	(-0.13)	(-0.65)	(-1.20)	(-1.00)	NA	(1.75)
2006-2015	1.99%**	0.62%	0.11%	0.01%	-2.73%*	NA	5.28%**
	(3.69)	(1.21)	(0.21)	(0.02)	(2.34)	NA	(3.35)

European	1.2%**	0.7%**	0.3%	-0.3%*	-1.8%**	NA	2.7%**
Corporate	(6.83)	(3.52)	(1.54)	(-2.23)	(-3.40)	NA	(4.93)
2000-2007	0.80%**	0.41%	0.12%	-0.14%	-1.19%	NA	1.11%
	(4.08)	(1.61)	(0.54)	(-0.98)	(-1.93)	NA	(1.70)
2007-2015	1.64%**	1.32%**	0.63%*	-0.56%*	-3.03%**	NA	4.62%**
	(5.96)	(3.88)	(1.99)	(-2.08)	(-3.39)	NA	(5.22)
US Credit	-0.91%	-0.24	0.58%	-0.12%	0.21%	0.58%	1.34%
Indices	(-1.95)	(-0.57)	(1.06)	(-0.29)	(0.31)	(0.56)	(1.70)
	-2.20%**	-0.97%	0.13%	-0.09%	0.90%	2.22%	-0.72%
2007-2010	(-2.62)	(-1.53)	(0.19)	(-0.12)	(0.76)	(1.51)	(-0.53)
	1.35%**	1.15%**	1.26%*	-0.22%	-0.99%	-2.54%*	2.28%*
2010-2014	(5.42)	(2.50)	(2.39)	(-0.68)	(-1.60)	(2.14)	(2.00)

** Indicates strong significance, * Indicates significance.

Appendix 11: Performance statistics of equity portfolios across sample periods

The table reports 1-, 3- and 5-factor alphas and t-statistics for BAB-portfolios. Alphas are annualized and in excess of the risk free rate. Samples have been divided into two sub-periods of equal size.

Industry sorted equities (indices)					Regular BAB portfolios				
	Perio	1-factor	3-factor	5-factor		Period	1-factor	3-factor	5-factor
MSCI ACW	2003-2015	4.73%* (2.25)	4.4%* (2.16)	0.6% (0.27)	SPX	1995-2014	7.04%** (2.76)	7.65%** (3.07)	4.3% (1.81)
	2003-2009	0.08% (0.25)	-0.01% (-0.26)	-0.01% (-0.14)		1995-2005	10.60%* (2.63)	9.68%* (2.42)	6.56% (1.69)
	2009-2015	8.90%** (3.45)	8.62%** (3.69)	2.1% (0.90)		2005-2014	3.18% (1.02)	1.79% (0.66)	3.96% (1.15)
MSCI World	1998-2015	3.96%* (2.19)	3.6%* (2.01)	-0.1% (-0.43)	FTSE	1994-2015	11.2%** (4.45)	9.7%** (4.0)	7.1%** (2.98)
	1998-2008	3.40% (1.44)	1.30% (0.59)	-1.16% (-0.47)		1994-2004	15.81%** (3.98)	12.31%** (3.25)	9.33%* (2.55)
	2008-2015	6.63%* (2.45)	6.47%** (2.72)	0.73% (0.29)		2004-2015	6.49%* (2.12)	6.64%* (2.26)	3.65% (1.27)
MSCI EM	2003-2015	7.51%** (4.39	6.9%** (4.0)	4.4%* (2.18)	STOXX	2001-2014	11%** (4.63)	11.2%** (4.74)	10.2%** (4.35)
	2003-2009	5.99%* (2.28)	4.44% (1.68)	2.56% (0.77)		2001-2008	15.07%** (4.60)	16.45%** (4.80)	16.49%** (4.80)
	2009-2015	8.9%** (3.90)	8.90%** (3.94)	6.11%* (2.40)		2008-2014	6.20% (1.88)	6.20% (1.88)	5.03% (1.52)
S&P Industries	1992-2015	-1.9% (-0.79)	-1.5% (-0.63)	-2.2% (-0.9)	NRDS	2002-2014	14.2%** (3.99)	14%** (3.82)	14.1%** (3.83)
	1992-2003	-7.48%* (-2.13)	-6.92% (1.94)	-7.09% (-1.89)		2002-2008	7.25% (1.44)	7.86% (1.55)	7.64% (1.50)
	2004-2015	3.70% (1.18)	3.57% (1.14)	2.40% (0.75)		2009-2014	20.70%** (3.91)	20.58%** (3.89)	20.82%** (3.92)

** Indicates strong significance (99%), * Indicates significance (95%)

Appendix 12: short selling cost from Nordic sample

The table presents linear coefficient variables as of March 2015. Beta coefficients are calculated with excess returns regressed on a value-weighted portfolio of the Nordics in the period from January 2000 to March 2015. Four stocks with less than 250 observations were removed. Fees are expressed as the annualized daily average cost of short-selling a security.

	Fee 30	Fee 90	Interest	Utilization	Size	Beta
Fee 30 days	*1*					
Fee 90 days	*0.91*	*1*				
Interest	*0.59*	*0.63*	*1*			
Utilization	*0.69*	*0.73*	*0.89*	*1*		
Size	*-0.24*	*-0.23*	*-0.32*	*-0.36*	*1*	
Beta	*0.24*	*0.31*	*0.32*	*0.36*	*-0.09*	*1*