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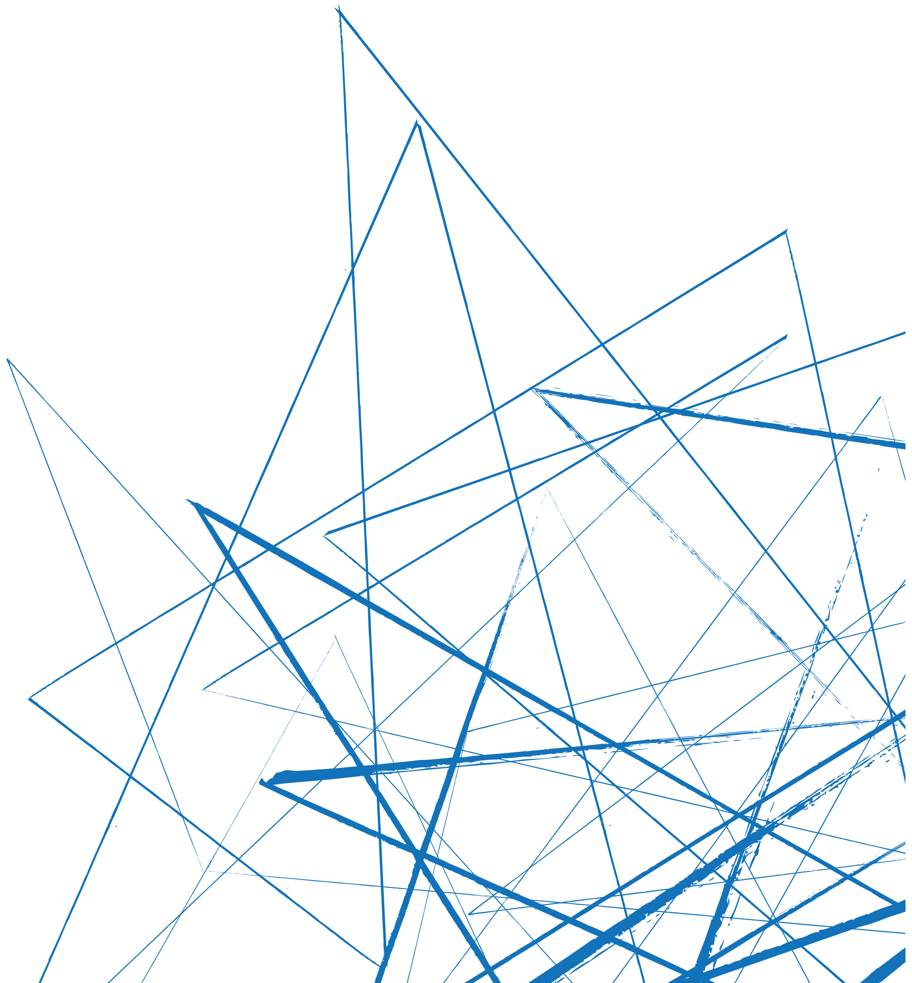

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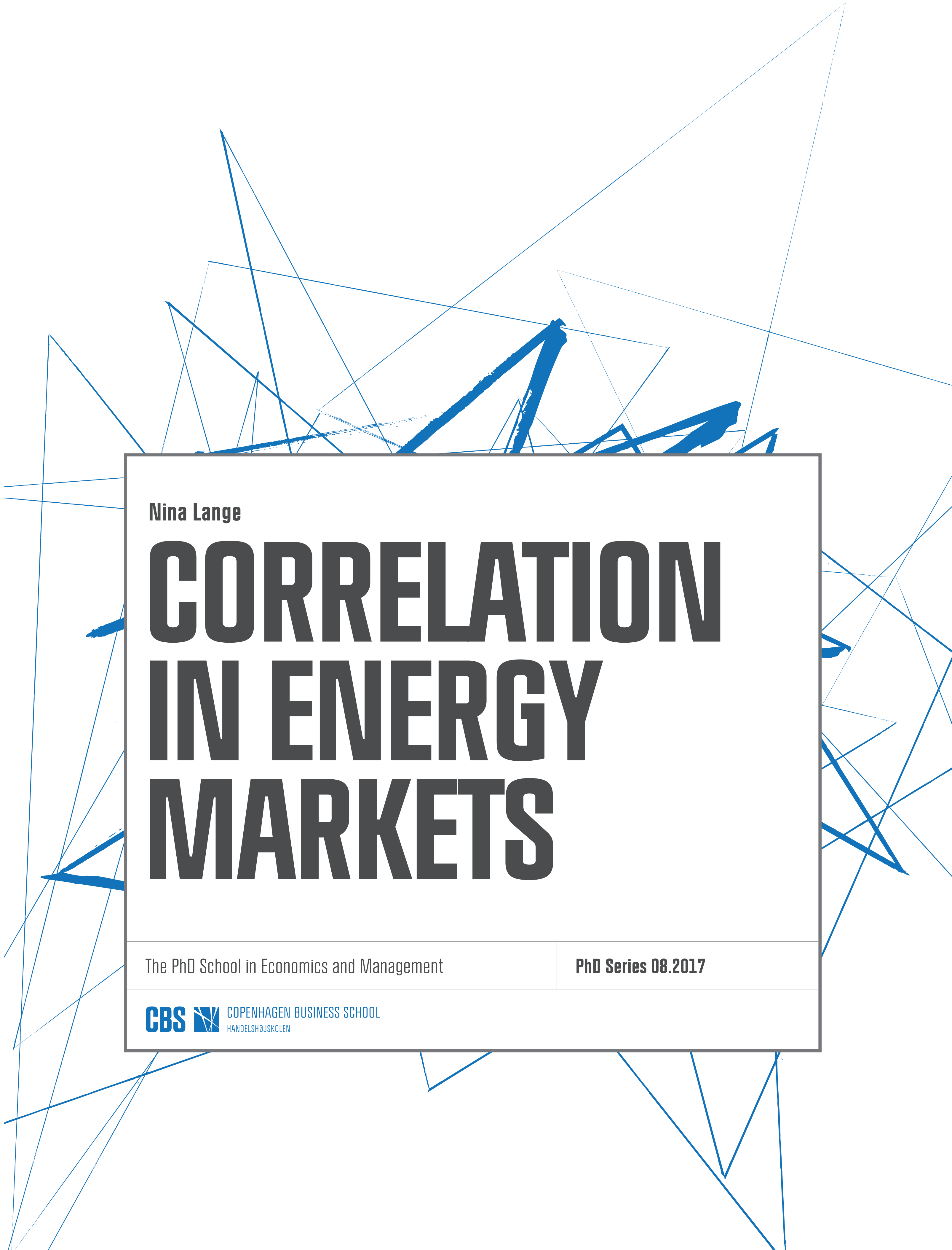

Nina Lange

CORRELATION IN ENERGY MARKETS

The PhD School in Economics and Management

PhD Series 08.2017

CBS COPENHAGEN BUSINESS SCHOOL
HANDELSHØJSKOLEN

CORRELATION IN ENERGY MARKETS

Nina Lange

Submitted: August 31, 2016

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Preface

This dissertation concludes my years as an Industrial PhD student at Department of Finance, Copenhagen Business School and Customers & Markets, DONG Energy A/S. The dissertation consists of four essays within the overall topic of energy markets. Although the last three essays are on the same topic, all four essays are self-contained and can be read independently of each other. Essay I studies the relationship of volatility in oil prices and the EURUSD rate and how it has evolved over time. Essay II-IV are on the so-called energy quanto options – a contract paying the product of two options. Essay II shows why energy quanto options are strong candidates for Over-The-Counter structured hedge strategies. Essay III (co-authored with Fred Espen Benth and Tor Åge Myklebust) studies the pricing of energy quanto option in a log-normal framework and Essay IV presents an approximation formula for the price of an energy quanto option using Greeks, individual option prices and the correlation among assets.

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I am beyond grateful for my collaboration with Fred Espen Benth at the University of Oslo. It has been a privilege to work with such an experienced and structured researcher and I look forward to continue on the projects in our joint pipeline. The time spend lecturing, examining and supervising jointly with Kristian Miltersen within the field of energy markets provided me with a good foundation for my future teaching activities. During my second year, I had the opportunity to visit Andrea Roncoroni at ESSEC Business School in Paris and Singapore. I learned a lot from this visit. During my last year at CBS, I was funded by CBS Maritime. The inclusion in CBS Maritime and cooperation on teaching shipping related topics was a great experience and I hope to pursue this topic also as a research area.

I appreciate the comments from the Committee of my closing seminar on April 29, 2016; Anders Trolle and Bjarne Astrup Jensen, and especially thanks to Bjarne for pointing out literature related to Essay II.

A big thanks to René Kallestrup for the introduction to Bloomberg and to Mads Stenbo Nielsen, David Skovmand, Desi Volker and Michael Coulon for endless discussions about life and life as a researcher.

Last, but not least, Martin; thank you for always being there even when I was impossible.

Nina Lange
Frederiksberg, August 31 2016

Summary

English Summary

Essay I: Volatility Relations in Crude Oil Prices and the EURUSD rate

In this paper, I study the relationship between volatility of crude oil prices and volatility of the EURUSD rate. If there is a common factor in the volatility of crude oil and the volatility of exchange rates, possible explanations could be that the financial crisis caused a volatility spillover between the two markets or that commodity markets, one of the most important being the crude oil market, are strengthening their connection with classic financial markets including exchange rate markets.

I use an extensive data set of crude oil futures and options and EURUSD futures and options spanning a period from 2000 to 2012. This data allows me to analyse the market-perceived volatility rather than investigating volatilities in the form of realised returns.

A model-free analysis supports the presence of a joint factor in the volatilities since mid-2007. As the two markets are asynchronous in futures and options maturity date, a term structure models allow for a description of the observed volatility surfaces by one or more stochastic volatility processes. A term structure model including one joint volatility factor and two market-specific volatility factors is proposed to capture the joint factor in the volatilities from 2007 onwards.

The paper focuses on confirming the existence of a joint factor, but leaves out the explanation of where it comes from. As data only covers until 2012, there is not enough information post-crisis to distinguish whether the joint volatility is a financialization or a crisis effect – or a combination.

Essay II: How Energy Quanto Options can Hedge Volumetric Risk

In this paper, the performance of energy quanto options as a tool for hedging volumetric risk is compared to other proposed hedge strategies. Volumetric risk is defined as the impact of fluctuations in demand on revenue and is not a unique problem for commodity markets. The difference between commodity markets and other markets is however that in commodity markets, market participants has little or no control over the demanded or supplied quantity and corresponding prices. For instance in liberalised energy markets, standard contract structures require energy companies to deliver any amount of energy demanded by the customers at a pre-determined price. The significant positive relationship between demanded quantity and associated market prices makes this contract structure risky for energy companies. When prices are low and they face a positive profit margin on the energy they sell, they sell a relatively small amount. On the other hand, when prices are high and the profit margin is negative, the customer demands a relatively larger quantity, which leads to a loss for the energy company. The lack of control over market prices as well as the sold volume give companies reasons for employing hedge strategies using the financial markets, either in form exchange traded derivatives or OTC-traded derivatives.

Proposed strategies include forward or futures and options written on the underlying energy price. The exact choice of contracts is impacted by the correlation between quantity and prices. A natural extension of these hedging strategies is to include derivatives on weather, as weather and quantity for many cases show significant correlation. Other strategies do not specifically define the derivative type used in the hedge strategy, but derives the hedge as a general function of price. This idea is extended to a hedging strategy depending not only on the price, but also an index, e.g., weather, related to the quantity. The general expression for the hedge is in simple cases obtainable in closed form, but nevertheless it does not have a structure that appeals to an OTC counterpart.

Mathematically, the general hedge strategy written on both price and index can be replicated using, among other, energy quanto options, e.g., options which pays out a product of two standard options. These structures are offered by re-insurance companies and used by energy companies as a way to hedge against an adverse situation, where the energy company risk low revenues. Using a comparative study, the performance of energy quanto options is proved to do almost as well as the optimal hedge. As the optimal hedge is infeasible to obtain in the OTC market, the energy quanto option is a strong candidate for risk management.

The energy quanto strategy also outperforms any other strategy consisting of market traded contracts. Further, the study illustrates that the exact choice of energy quanto options depends on the behaviour of the underlying variables.

Essay III: Pricing and Hedging Energy Quanto Options

In this paper, we study the pricing and hedging energy quanto options. Energy quanto options are tools to manage the joint exposure to weather and price variability. An example is a gas distribution company that operates in an open wholesale market. If, for example, one of the winter months turns out to be warmer than usual, the demand for gas would drop. This decline in demand would probably also affect the market price for gas, leading to a drop in gas price. The firm would make a loss compared with their planned revenue through not only the lower demand also through the indirect effect from the drop in market prices.

Since 2008, the market for standardized contracts has experienced severe retrenchment and a big part of this sharp decline is attributed to the substantial increase in the market for tailor-made contracts. As these tailor-made energy quanto options are often written on the average of price and the average weather index, we convert the pricing problem by using traded futures contracts on energy and a temperature index as underlying assets, as these settle to the average of the spot.

We derive options prices under the assumption that futures prices are log-normally distributed. Using futures contracts on natural gas and the Heating Degree Days (HDDs) temperature index, we estimate a model based on data collected from the New York Mercantile Exchange and the Chicago Mercantile Exchange. We compute prices for various energy quanto options and benchmark these against products of plain-vanilla European options on gas and HDD futures.

Essay IV: A Short Note on Pricing of Energy Quanto Options

In this short note, an approximation for the price of an energy quanto option is proposed. An energy quanto option is an option written on the price of energy and a quantity related index, for instance weather. The option only pays off if both prices and the weather index are in the money and such structures are useful to hedge volumetric risk in energy markets. Under the assumption of a log-normal prices, the energy quanto option price can be approximated

by a pricing formula involving the correlation, current individual asset prices and option prices, volatilities and Greeks. These quantities are known or can be assessed by market participants, thereby yielding a fast pricing method for energy quanto options. The pricing performance is illustrated using a simple numerical example.

Dansk Resumé

Essay I: Sammenhæng mellem olieprisers og valutakursers volatilitet

I denne artikel undersøger jeg sammenhængen mellem volatiliteten af oliepriser og EURUSD kursen. Hvis en fælles faktor driver volatiliteten i både oliepriser og valutakurser, kunne en mulig forklaring være at finanskrisen er skyld i at volatilitet fra det ene market flyder over til det andet market. En anden forklaring kunne være at råvaremarkeder, hvoraf et af de vigtigste er oliemarkedet, har fået styrket deres forbindelse til de klassiske finansielle markeder, herunder valutamarkederne.

Jeg bruger et omfangsrigt datasæt bestående af priser på oliefutures og -optioner samt EURUSD-futures og -optioner fra perioden 2000-2012. Dette datasæt giver mig mulighed for at analysere den markedsimplicitte volatilitet fremfor realiseret volatilitet udledt fra spotpriser.

En modelfri analyse understøtter eksistensen af en fælles faktor i volatiliteterne siden midten af 2007. De to markeders futures og optioner har ikke samme udløbstidspunkter, så jeg opstiller en model som kan sammenkoge hele volatilitetsoverfladen til en eller flere stokastiske volatilitetsprocesser. En model med en fælles volatilitetsproces og to markedsspecifikke volatilitetsprocesser foreslås og estimeres for at fange den fælles faktor i volatiliteter siden 2007.

Artiklen fokuserer på at bekræfte eksistensen af den fælles faktor, men omhandler ikke en forklaring på hvorfra den kommer. Da datasættet kun dækker perioden frem til 2012, er der ikke nok post-krise data til at identificere, om den fælles volatilitet kommer fra finansialisering af råvaremarkederne eller fra finanskrisen – eller en kombination.

Essay II: Double-trigger optioners evne til at hedge volumenrisiko i energimarkeder

I denne artikel sammenlignes det hvordan double-trigger optioner på priser og mængder klarer sig sammenlignet med andre hedgestrategier, når det drejer sig om at at styre volumenrisiko. Volumenrisiko er defineret som indvirkningen på omsætningen som følge af fluktuationer i efterspørgsel og findes ikke kun på råvaremarkeder. Forskellen er imidlertid at markedsdeltagerne i råvaremarkeder har lille eller ingen kontrol over efterspurgt eller produceret mængde og dertilhørende priser. For eksempel, på det liberaliserede elmarked vil en almindelig kontraktstruktur forpligte forsyningsselskabet til at levere en hvilken som helst

efterspurgt mængde af strøm til en på forhånd fastsat pris. Den klart positive relation mellem efterpsurgt mændge og tilhørende markedspriser gør denne kontraktstruktur risikabel for forsyningsselskabet. Når priserne er lave og deres profitmargin er positiv, sælger de relativt lidt. Omvendt, når priserne er høje og profitmarginen er negativ, efterspørger kunden en relativt større mængde, hvilket resulterer i et tab. Manglen på kontrol over priser såvel som mængder giver virksomheder god grund til at anvende hedgestrategier indeholdende derivater, enten ved hjælp af standardiserede kontrakter eller gennem en direkte modpart. Foreslåede strategier inkluderer forwards eller futures og optioner skrevet på energiprisen. Det præcise valg af kontrakter er afhængig af korrelationen mellem pris og mængde. En naturlig udvidelse af dette er at inkludere derivater som afhænger af vejr, da vejr og mængde i mange tilfælde udviser en signifikant korrelation. Andre strategier specificerer ikke den præcise type af derivat, men udleder hedgestrategien som en generel funktion af energiprisen. Denne ide kan udvides til en hedgestrategi, som ikke kun afhænger af pris, men også et index, fx. vejr, som relaterer sig til mængde. I simple tilfælde er kan dette hedge udtrykkes i lukket form, men dets struktur vil på ingen måde appelere til en modpart.

Matematisk kan den generelle hedgestrategi skrevet på både energipris og index replikeres ved hjæpl af blandt andet double-trigger optioner, dvs. optioner der betaler et produkt af to almindelige optioner. Sådanne strukturer tilbydes for eksempel af genforsikringsselskaber og bruges af energiselskaber som en måde at sikre sig mod en situation hvor omsætningen er kritisk lav.

En sammenligning viser at double-trigger optioner skrevet på pris og et vejrindex klarer sig stort set lige så godt som det optimale teoretiske hedge. Da det optimale teoretiske hedge ikke er muligt at opnå i praksis, er double-trigger optioner potentielt en stærk kandidat til en risikostyringsstrategi. Ydermere klarer double-trigger optionerne sig bedre end markedsbaserede stratgier. Endeligt viser eksemplet at det præcise valg af double-trigger optioner er afhængig af fordelingen af de underliggende variable.

Essay III: Prisfastsættelse og hedging af dobbelt-trigger optioner på energi og mængder

I denne artikel studerer vi prisfastsættelse af dobbelt-trigger optioner på energi og mængder. Dobbelt-trigger optioner på energi og mængder er redskaber til at styre den fælles eksponering mod vejr og prisusikkerheder. Et eksempel er et gasdistributionsselskab, som opererer i engrosmarkedet. Hvis, for eksempel, en vintermåned er varmere end normalt vil efterspørgslen efter gas falde. Dette fald i efterspørgsel vil også påvirke markedsprisen for gas i nedadgående retning. Gasdistributionsselskabet vil derfor have et tab sammenlignet med deres budgetterede omsætning og dette tab kommer både fra det lavere salg, men også indirekte fra faldet i priser.

Siden 2008 er markedet for standardiserede vejrkontrakter blevet betydeligt mindre og en stor del af dette fald skyldes den betydelige stigning i markedet for skræddersyede kontrakter på vejr. Eftersom disse skræddersyede kontrakter ofte er skrevet på et gennemsnit af priser og et gennemsnit af et vejrindex og da futureskontrakter ligeledes afregnes mod en gennemsnit af prisen over en periode, omskriver vi prisfastsættelsesproblemet ved at bruge futureskontrakter på energi og vejr som underliggende kontrakter.

Vi udleder optionspriser under en antagelse om at futurespriser er log-normalfordelte. For futureskontrakter på gas og på temperaturindekset Heating Degree Days (HDDs), estimerer vi en model baseret på data fra New York Mercantile Exchange og Chicago Mercantile Exchange. Vi udregner priser for diverse dobbelt-trigger optioner på gas og temperaturindexet og benchmarker disse mod produktet af standard optioner på gas og standard optioner på temperaturindexet.

Essay IV: En kort bemærkning om prisfastsættelse af dobbelt-trigger optioner

I denne korte note præsenteres en approksimation for prisen for en dobbelt-trigger optioner på energi og mængder. En dobbelt-trigger option er et derivat skrevet på prisen af energi og en mængderelateret indeks, for eksempel vejr. Optionen betaler kun, hvis både prisen påenergi og vejrindekset er ”i pengene” og sådanne strukturer er nyttige for at styre volumenrisiko på energimarkederne. Under antagelse af log-normalfordelte priser, kan dobbelt-trigger optionen approximeres af en prisformel, der involverer korrelation, individuelle underliggende priser og optionspriser, volatiliteter og Greeks. Alle disse størrrelser er kendte eller kan blive tilnærmet af markedsdeltagerne og dermed kan dobbelt-trigger optioner nemt og hurtigt prisfastsættes. Nøjagtigheden af formlen er illustreret i et simpelt numerisk eksempel.

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Introduction

In the recent years, the size of commodity derivatives markets have increased. The U.S. Energy Information Administration reports that the number of outstanding WTI crude oil futures on U.S. exchanges has more than quadrupled from 2000 to 2016. The market participant can be both hedgers such as producers and end-users or speculators, who have no interest in the underlying oil as a commodity. Besides the general links of commodity prices and the world economy and the world economy and financial markets, the presence of financial investors in commodity market is a direct link between commodities markets and regular financial markets.

In commodity markets, spot prices are a result of supply and demand in a given location. Surrounding the physical market is a huge commodity derivatives markets, where prices of futures, forwards, swaps and options are related to the spot prices either because physical delivery is possible of because the contract is settled to underlying spot prices. The link between spot prices and the derivatives market differ from market to market. The exact relationship between the spot and derivatives market on commodities suck as oil is explained by the concept of convenience yields, which is a benefit or a cost accruing to the holder of the commodity, but not to the owner of a forward or futures on the commodity. Using this theory, the difference in spot prices today compared to the price of a oil futures is explained by storage costs, funding costs and potential non-monetary benefits of possessing the physical commodity between today and maturity. The convenience yield or equivalently the shape of the futures curve behaves dynamically as a reflection to the spot *and* financial markets. Over the past years, the oil futures curve has been in contango (oil futures with longer maturities are more expensive), in backwardation (oil futures with longer maturities are less expensive) or shown a hump-structure. While it must be expected that spot prices are solely driven by fundamentals, the question of whether the futures prices are driven by fundamentals or by financial investors has attracted much attention over the last decade.

Starting in 2004, commodity markets saw a general increase in prices, in volatility and in co-movement with other asset classes. One string of literature argues that this is due to *financialization of commodity markets* – a term used to describe increased role of financial investors. This theory states that the changes in markets is caused by the inflow of investments into commodity markets from large investors, see e.g., Tang and Xiong (2012) or Carmona (2015). Contradicting this theory is e.g., Kilian (2009) who argues that the change in commodity markets are driven by fundamentals, while others seek a compromise between the two theories as for instance Vansteenkiste (2011), who claims that both investors and fundamentals play a role in price determination with the former domination the majority of the time during the last decade. Similar conclusions are drawn by Basak and Pavlova (2016) who estimate that around 15% of the futures price comes from financialization and the rest from fundamentals.

Essay I in this dissertation is related to the intersection of commodity and financial markets. It considers the oil market and a foreign exchange market. These two markets are often analysed in terms of correlation between spot returns and the results depend on whether the currency in question is from a country with a large export of commodities. In my analysis, I focus specifically on the EURUSD rate. None of the countries in the Euro-zone are considered major commodity exporters, so the EURUSD rate and crude oil price will largely be expected to have a positive correlation when looking a returns or levels, the argument being that as the Dollar depreciates against the Euro (meaning the EURUSD rate increases), the oil-importing Euro-zone will import more and thereby increase oil prices. The relationship is especially strong just before the financial crisis (see Verleger (2008)). How volatilities in these two markets relate is a less studied question. Ding and Vo (2012) find that for realized volatility of spot oil prices and spot EURUSD rates, there are indications of volatility spillovers from one market to another after the financial crisis started. In my analysis, I use options on futures and using first a model-free approach, I find that there is a joint volatility factor for the two markets after 2007. I then propose a term structure model along the lines of Trolle and Schwartz (2009) for future and options and estimate this model. After 2007, adding a joint volatility factor to explain the co-movement in volatilities in the two markets gives a better fit to the observed implied volatility surfaces. The improvement in the volatility fit is highly significant for the oil options and for shorter EURUSD options. Whether the presence of a joint factor is a result of fundamentals, financialization or the financial crisis is left for later studies.

The remaining part of this dissertation it devoted to the study of volumetric risk. Volumetric risk is defined as the impact on revenue from fluctuations in demand. Compared to other markets, market participants in commodity markets has less over the demanded or supplied quantity and corresponding prices. For instance in liberalised energy markets, standard contract structures require energy companies to deliver any amount of energy demanded by the customers at a pre-determined rate. The significant positive relationship between demanded quantity and associated market prices makes this contract structure risky for energy companies. When prices are low and they face a positive profit margin on the energy they sell, they sell a relatively smaller amount. On the other hand, when prices are high and the profit margin is negative, the customer demands a relatively larger quantity resulting in a loss for the energy company. Another example is the owner of a wind park producing power in a competitive electricity market. The wind speed and direction solely determines the quantity produced and the market determines the prices, leaving the owner as both price taker and quantity taker.

First, Essay II shows why energy quanto options[1] arise when hedging the volumetric risk. The name energy quanto option is used for a derivative paying the product of two options. This type of derivative arises from deriving an optimal theoretical hedge strategy based on both the underlying price and a quantity-related and market-traded index, for instance weather. Using a numerical experiment, it is shown that in comparison with the optimal theoretical hedge, energy quanto options are strong candidates for Over-The-Counter structured hedge strategies. By just using one or two quanto options, the hedge performance is 90-98% compared to the preferred theoretical hedge. Further, the energy quanto hedge is superior in the sense, that it has an understandable structure. The optimal theoretical hedge is in best case expressed by conditional profit expectations and density ratios and therefore not a contract structure to request from an OTC counterpart. While the use of an OTC counterpart rather than a market traded hedge might seem as a big step to take for an energy company, it further has the benefit of being able to choose the exact weather index which correlates well with the risk taken by the company rather than having to rely on available weather contracts.

Essay III (co-authored with Fred Espen Benth and Tor Åge Myklebust) studies the pricing of the aforementioned energy quanto options is a log-normal framework. In practice, an energy quanto option is an Asian type option written on the average of a price or an index, and we therefore convert the pricing problem by arguing that writing an option on an average is essentially the same as writing it on a futures contract settling on the average price or index during the same period. With futures contracts being traded assets, we can extract the pricing measure from market data and use this to provide pricing and hedging formulas for energy quanto options. In a log-normal framework, both the price and Greeks are available in closed form. We estimate a model for US NYMEX gas futures and Heating Degree Days futures for New York and Chicago based on the model by Sørensen (2002). We use the estimated model to illustrate the difference of energy quanto option prices and plain vanilla options.

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[1] Other common names are double trigger options or cross-commodity-weather options.

Essay IV provides a pricing approximation formula for energy quanto options. Using a Taylor-expansion around the correlation of prices, it is shown that a first order approximation can be obtained from univariate option prices and Deltas. A second order approximation is obtained by adding Gammas. Many traders will have either information or an educated guess for these quantities, making the pricing formula easy to use. If market prices and Greeks are available, it becomes (approximately) redundant to estimate the model as all information is already incorporated in prices and Greeks. Using a simple example, the second order approximation is shown to be close to the actual price.

Essay I

Volatility Relations in Crude Oil Prices and the EURUSD rate*

Abstract

Studies on the relationship between oil prices and the EURUSD rate is mostly focused on the correlation of returns or levels. However, the volatility of oil price and the volatility of the EURUSD rate is also of importance for e.g., portfolio risk management, margin requirements or derivatives pricing models. In this paper, the relationship between the volatility of crude oil and the volatility of the EURUSD rate is analysed. A model-free analysis shows the presence of a joint factor in the volatilities after mid-2007. A term structure model for futures and options on both oil and EURUSD is proposed and estimated to WTI Crude Oil and EURUSD futures and options traded at the Chicago Mercantile Exchange from 2000-2012. The addition of a joint volatility factor significantly improves the fit to oil options and short term EURUSD options after mid-2007.

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I.1 Introduction

The relationship of oil prices and the EURUSD[1] rate is often investigated in terms of returns or levels. The majority of studies confirms empirically that oil and EURUSD returns or levels are positively correlated. This implies that for a EUR-denominated investor, an increase in oil prices is dampened by the weaker dollar and vice versa. One explanation is that a depreciation of the US Dollar will allow a EUR-denominated investor to buy more oil for the same amount of EUR, thereby putting an upward pressure on the oil price.

In addition to studying the size of the correlation of oil returns and EURUSD returns (or levels), or more generally speaking foreign exchange rates, most research focuses on the causality and forecasting performance. For instance, Chen and Chen (2007) concludes that real oil prices have significant forecasting power when in comes to explaining real exchange returns[2]. Similar conclusions are drawn by Lizardo and Mollick (2010). Research supporting the link from currencies to commodities focuses on the so-called commodity currencies, which are defined as currencies of countries with a large export of commodities, e.g., Canadian Dollars, Australian Dollars or South African Rand. For instance, Chen et al. (2010) find that commodity currencies have large predictive power for commodity prices. Fratzscher et al. (2014) find a bidirectional causality between oil prices and the value of the Dollar.

While there has been considerable attention on the direction of the relationship and the possible explanations, the relationship between volatilities of the two markets have received less attention. Ding and Vo (2012) uses a multivariate stochastic volatility framework to investigate the volatility interaction between the oil market and the foreign exchange markets. Using spot price data, their analysis is using realized volatility and focuses on forecasting. They conclude that volatility spills over from one market to the other in times of turbulence. They attributes the volatility interaction to inefficient information incorporation during the financial crisis. The study includes only the beginning of the financial crisis, and it is therefore not possible to determined if this only occurs during the crisis or if it is a more permanent change.

Christoffersen et al. (2014) investigates the factor structures among different commodities and their relation to the stock market. Using a model-free approach and high-frequency data, they find that the commodity market volatilities have a strong common factor, that is largely driven by stock market volatility. As numerous studies empirically document the relationship between foreign exchange market volatility and stock market volatility, a natural hypothesis is therefore a relationship between the volatility of oil and the volatility of the EURUSD.

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[1] The EURUSD rate is defined as the price of euros measured in US Dollars

[2] They do not consider the EUR, but include Germany in their analysis.

Generally, commodity markets have seen increases in prices, volatilities and correlation among commodities and financial assets during the last decade. One string of literature refer to these changes as *the financialization of commodity markets* and explain them by the entry of institutional investors into commodity markets, see e.g., Tang and Xiong (2012) and Carmona (2015). Other papers, such as Kilian (2009), argue that the increase in volatilities and in prices are solely due to fundamentals, e.g., increased demand from BRIC-countries.

In this paper, I analyse the behaviour of oil price volatility and the EURUSD volatility to find the relationship between those and if this relationship is changing over time. The analysis is based on exchange traded futures and options on futures. The spot price of oil is based on actual physical trades, whereas futures on oil are mainly traded financially[3]. The use of the futures market allows for viewing the volatilities as market-perceived. The use of spot oil prices as in Ding and Vo (2012) includes potential short term impact from the physical market rather than the volatility being seen as a reflection of the financial market.

The analysis of the volatility relation is done in two ways. First, using a model-free approach, at-the-money straddles (written on nearby futures) with maturities up to six months are computed for both oil and for EURUSD. The rolling correlation of short maturity straddles is then analysed for structural breaks. The resulting sub-samples are then analysed separately, to see if the variations in the combined straddle returns are impacted by a common factor. Based on these results, a term structure model for pricing futures and options is proposed and estimated. Both of these analyses show presence of a common joint volatility factor after mid-2007.

Term structure models are well-studied in the literature. Several papers deals with modelling of oil prices. Studies by Cortazar and Naranjo (2006), Trolle and Schwartz (2009), Chiarella et al. (2013) all present a joint model for the term structure of futures prices and option prices. Also FX rates and options has been widely studied, e.g., in Bakshi et al. (2008). The model analysed in this paper is a two-asset variation of Trolle and Schwartz

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[3] Although oil futures on WTI crude oil can be physically settled, the majority of contracts are rolled over before maturity and thus never actually physically delivered, see e.g., the discussion about market volume and open interest in Trolle and Schwartz (2009).

(2009).

The paper continues as follows: Section I.2 provides an overview and a detailed description of the available data and the data cleaning process. Section I.3 presents the model-free analysis. Section I.4 introduces the model, which is essentially a simplified two-asset version of the model proposed by Trolle and Schwartz (2009). Section I.5 presents and discusses the estimation results. Finally, Section I.6 concludes.

I.2 Overview of data

The raw data set for this study consists of daily data from January 1, 1987 to March 26, 2013 for end-of-day settlement prices for all futures and American options on WTI crude oil and daily data from May 19, 1998[4] to March 26, 2013 for end-of-day settlement prices for all futures and American options on EURUSD as well as the spot EURUSD exchange rate during the same period. Futures and spot data are obtained from Bloomberg, while the options data is purchased from the CME Group.

The crude oil futures are listed for each month for the subsequent 5-6 years and semi-annual in the June cycle for further three years. The futures contracts are for physical delivery of 1000 barrels of crude oil and terminates trading around the 21st in the month preceding the delivery month. Among the crude oil futures, a subset of contracts are chosen by investigating liquidity. I disregard all futures contracts with less than ten trading days to maturity and among the remaining I choose the first six monthly contracts (labelled M1-M6), two contracts in the March-cycle following the monthly contracts (labelled Q1-Q2) and four December contracts following the quarterly contracts (Y1-Y4). The choice and labelling is equivalent to the one used by Trolle and Schwartz (2009).

At all times, six EURUSD futures with quarterly expiry in March, June, September and December are listed. One contract equals 125,000 euros. The EURUSD futures expire two business days before the third Wednesday of the expiry month[5]. I include contracts with more than one week to expiry. The EURUSD futures contracts are labelled Q1-Q6.

As the Euro was not introduced until 1999, I start my analysis on January 4, 2000. Figures I.1 and I.2 show the price evolution of the futures contracts included in the estimation. Figures I.18 and I.19 in the Appendix I.A shows the daily returns.

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[4] The first contract listed on this date is for delivery in March 1999, which is after the Euro is introduced.

[5] Where a March oil futures terminates trading in around the 20th of February and delivers during the month of March, a March EURUSD futures trades until mid-March with delivery shortly after.

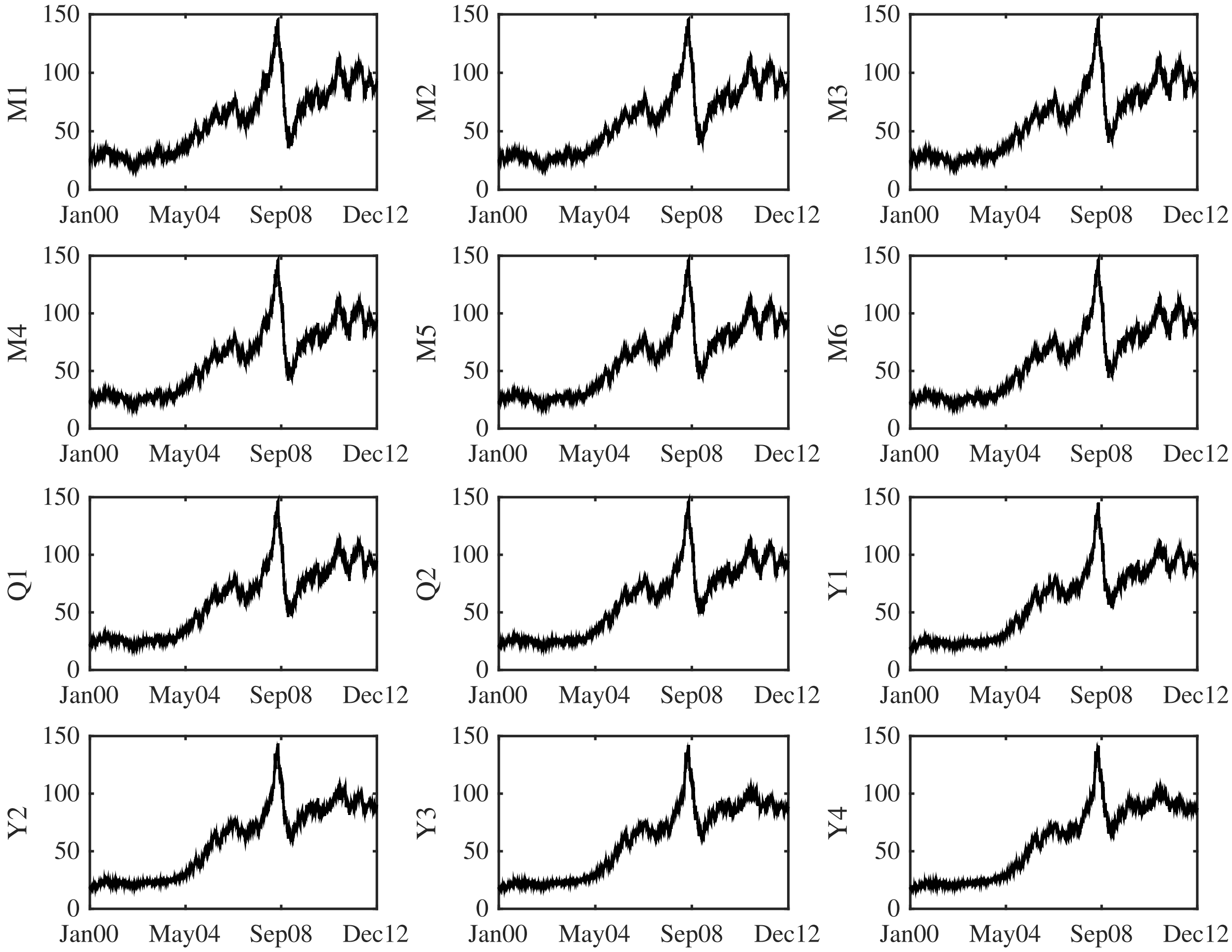


Figure I.1. Evolution of oil futures prices

These figures shows the evolution of the oil futures prices used in the estimation. M1 refers to the shortest futures contract with more than two weeks to maturity. M2-M6 are contracts following the M1 contract. Q1 is the first contract with delivery in March, June, September or December after the M6-contract and Q2 is the following contract with delivery in March, June, September or December after the Q1-contract. Y1-Y4 are the first four December-contracts following the Q2 contract. All prices are in USD.

Crude oil options terminates trading three business days before the futures contract they are written on. Options are listed on all of the futures contracts, but for liquidity reasons and to minimize potential problems when assuming non-stochastic interest rates, I initially restrict the analysis to options on the first eight oil futures contract. This results in a set of option prices than span approximately one year. For each maturity, I choose up to 11 options corresponding to moneyness-intervals $[0.78 - 0.82, \dots, 1.18 - 1.22]$. Only options with a open interest of more than 100 contracts and with a price greater than 0.01

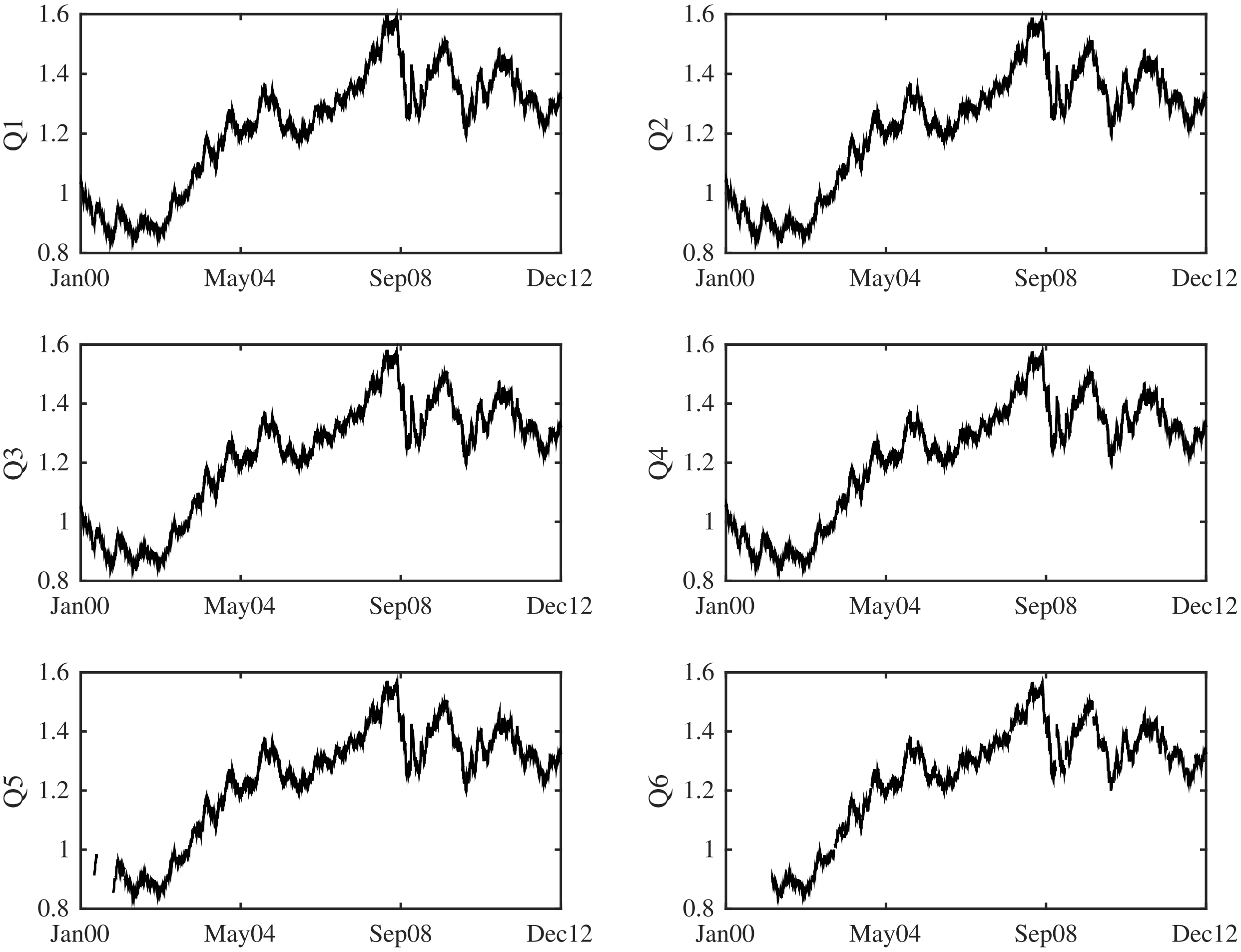


Figure I.2. Evolution of EURUSD futures prices

These figures shows the evolution of the EURUSD futures prices used in the estimation. Q1 refers to the shortest futures contract maturing more than a week later. Q2-Q6 are the contracts delivering 3-15 months after Q1 futures contract. All expiry dates are in the March-cycle. Availability of prices in the long end of the futures is limited for the first couple of years. By definition, the EURUSD rate is the price of EUR measured in USD.

are included. For moneyness-intervals less than one, only put options are chosen and for moneyness-intervals greater than one, only call options are included in the dataset. For the moneyness-interval including 1, the most liquid option is chosen. Within each moneyness-interval, the option closest to the mean of the interval is chosen based the aforementioned criteria. The number of days with observations, implied volatilities, average prices and open interest are summarized in the tables in Appendix I.A and the ATM implied volatilities are plotted in Figure I.3. The Samuelson effect – that futures with short time to expiry are more volatile compared to options with long time to expiry – is clearly seen by comparing across maturities. Further, volatility is evidently stochastic.

EURUSD options are traded on the first four quarterly futures contracts. The options with expiry in the same month as the underlying are denoted Q1-Q4. In addition, two nearby monthly options on the nearby futures are also traded: For instance, in early January, the closest futures contract is the March contract. Besides options expiring in March, the first monthly option, M1, expires in January with the March futures as underlying. The second monthly option, M2, is a February option on the March futures contract. When the January option expires, the February option on the March futures becomes the M1 contract and an April option on the June futures become the M2 contract. See Figure I.4 for an illustration of existence and labelling of EURUSD options. The options expire on the Friday 12 days before the third Wednesday of the option's contract month. Like with the oil options, the sorting results in a set of option prices than span approximately one year. The final set of EURUSD options data is chosen using the same guidelines as with the oil options data. The resulting dataset is summarized in tables I.10-I.16 in Appendix I.A, and the ATM implied volatility is plotted in Figure I.5.

Both the oil option prices and the EURUSD options are for American type options. The impact of early exercise is small in the chosen sample, because only OTM and ATM options were included. Nevertheless, the prices are still converted to European prices by converting quoted American option prices to European option prices using the approach from Barone-Adesi and Whaley (1987). All reported implied volatilities are for corresponding European options.

By inspection of Tables I.9 and I.10, it become apparent that there is still a great deal of asymmetry in the availability of option prices. The model-free analysis in Section I.3 is based on ATM options resulting from the initial sorting. When turning to estimation using multiple moneyness-intervals in Section I.5, the dataset is restricted further to ensure that

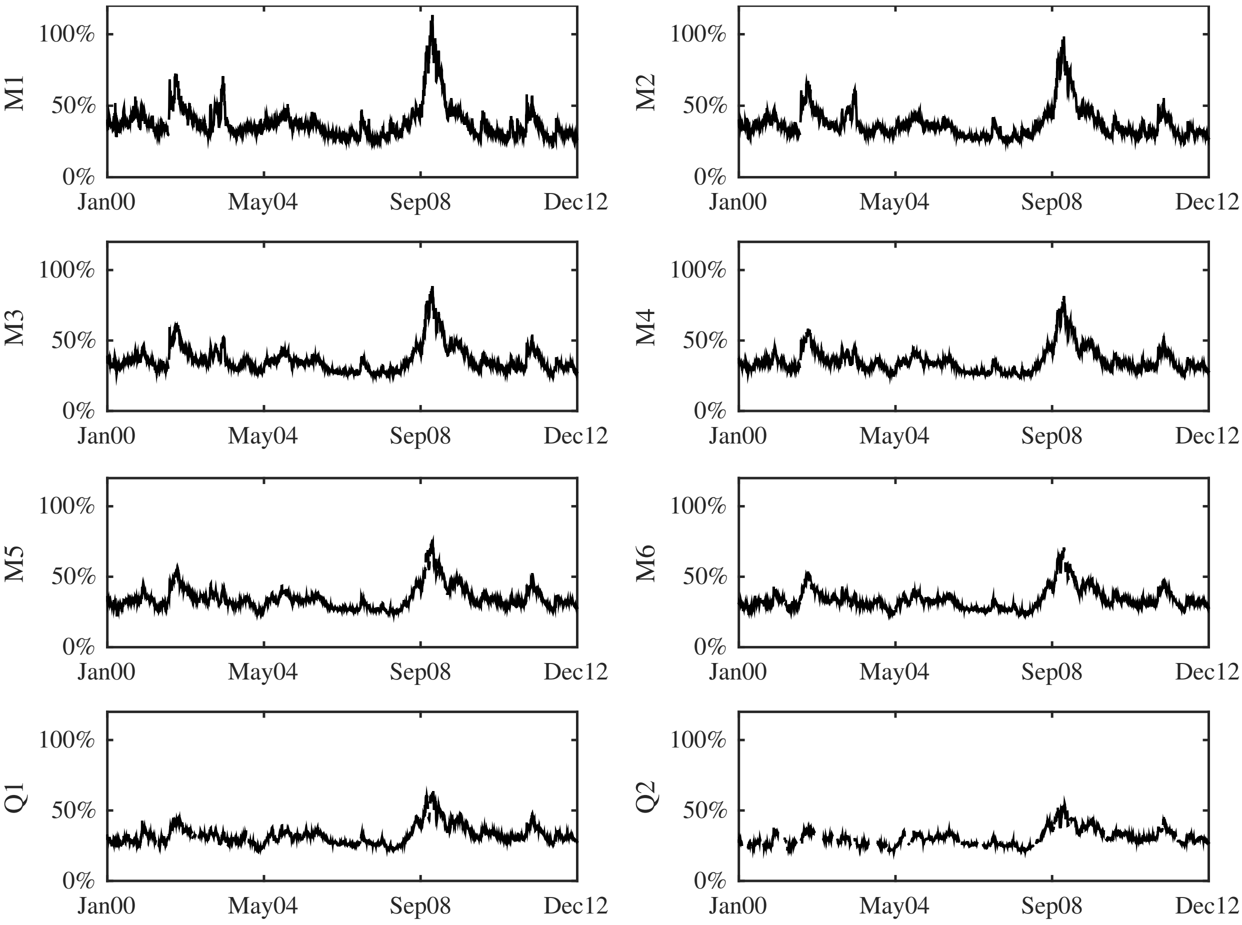
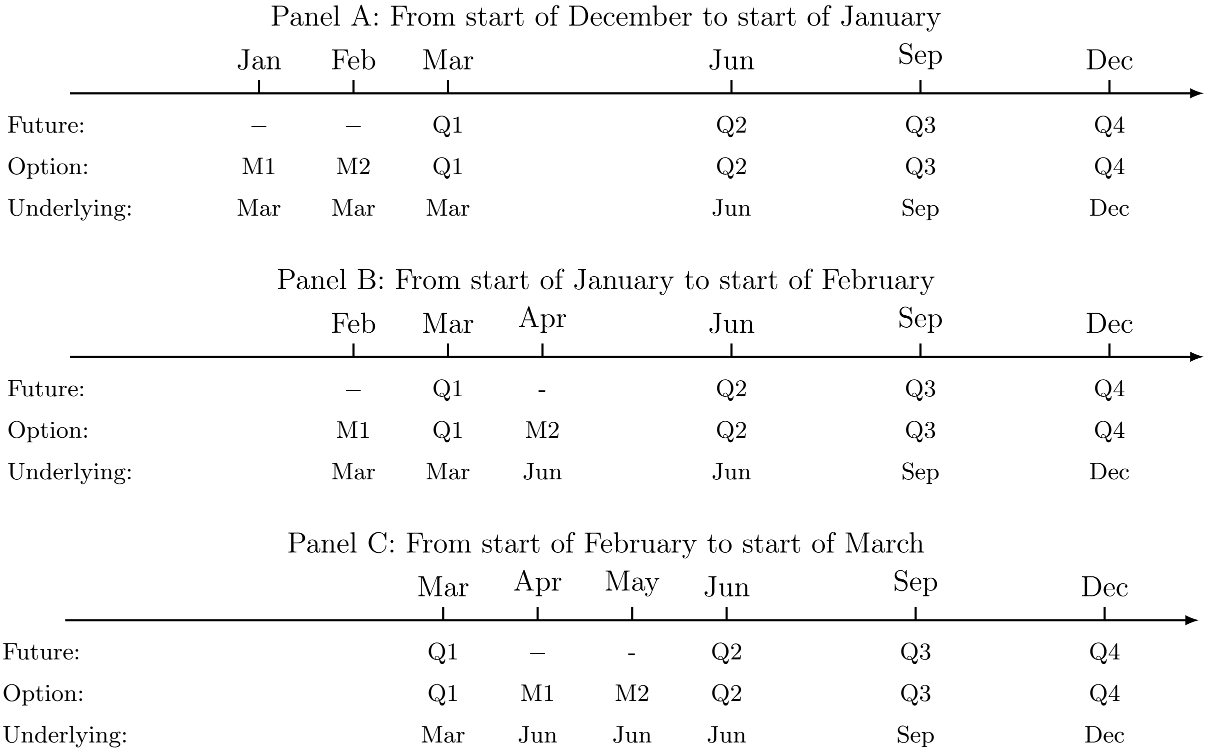


Figure I.3. Oil Options Implied Volatility (BAW)

The figures shows the evolution of the implied volatility of the oil options. The implied volatility is found by first converting the quoted American option prices to European option prices using the method by Barone-Adesi and Whaley (1987). Afterwards the implied volatility is computed using the formulas for options on futures derived in Black (1976). The picture confirms the so-called Samuelson effect; that volatility decreases with time to maturity. Especially for the short dated options, several events can be identified, e.g., the terrorist attack in September 2001, where the short term volatilities jump significantly and the financial crisis starting in the Autumn of 2008, where M1-volatilities increases to more than 100%.



Panel A: From start of December to start of January

	Jan	Feb	Mar	Jun	Sep	Dec
Future:	–	–	Q1	Q2	Q3	Q4
Option:	M1	M2	Q1	Q2	Q3	Q4
Underlying:	Mar	Mar	Mar	Jun	Sep	Dec

Panel B: From start of January to start of February

	Feb	Mar	Apr	Jun	Sep	Dec
Future:	–	Q1	-	Q2	Q3	Q4
Option:	M1	Q1	M2	Q2	Q3	Q4
Underlying:	Mar	Mar	Jun	Jun	Sep	Dec

Panel C: From start of February to start of March

	Mar	Apr	May	Jun	Sep	Dec
Future:	Q1	–	-	Q2	Q3	Q4
Option:	Q1	M1	M2	Q2	Q3	Q4
Underlying:	Mar	Jun	Jun	Jun	Sep	Dec

Figure I.4. EURUSD contracts

This figure illustrates the existence and naming of options on EURUSD futures contracts. In Panel A, the M1- and M2-options are written on the Q1-futures, but expiring 2 resp. 1 month before the underlying. The Q1–Q4-options mature in the same months as their underlying. This happens from start of December to start of January, from start of March to start of April, from start of June to start of July, and from start of September to start of October. Panel B shows the situation where the M1-option is written on the Q1-futures and the M2-option is written on the Q2-futures. This happens from start of January to start of February, from start of April to start of May, from start of July to start of August, and from start of October to start of November. Finally, Panel C shows the situation where the M1- and M2-options are written on the Q2-futures. This happens from start of February to start of March, from start of May to start of June, from start of August to start of September, and from start of November to start of December.

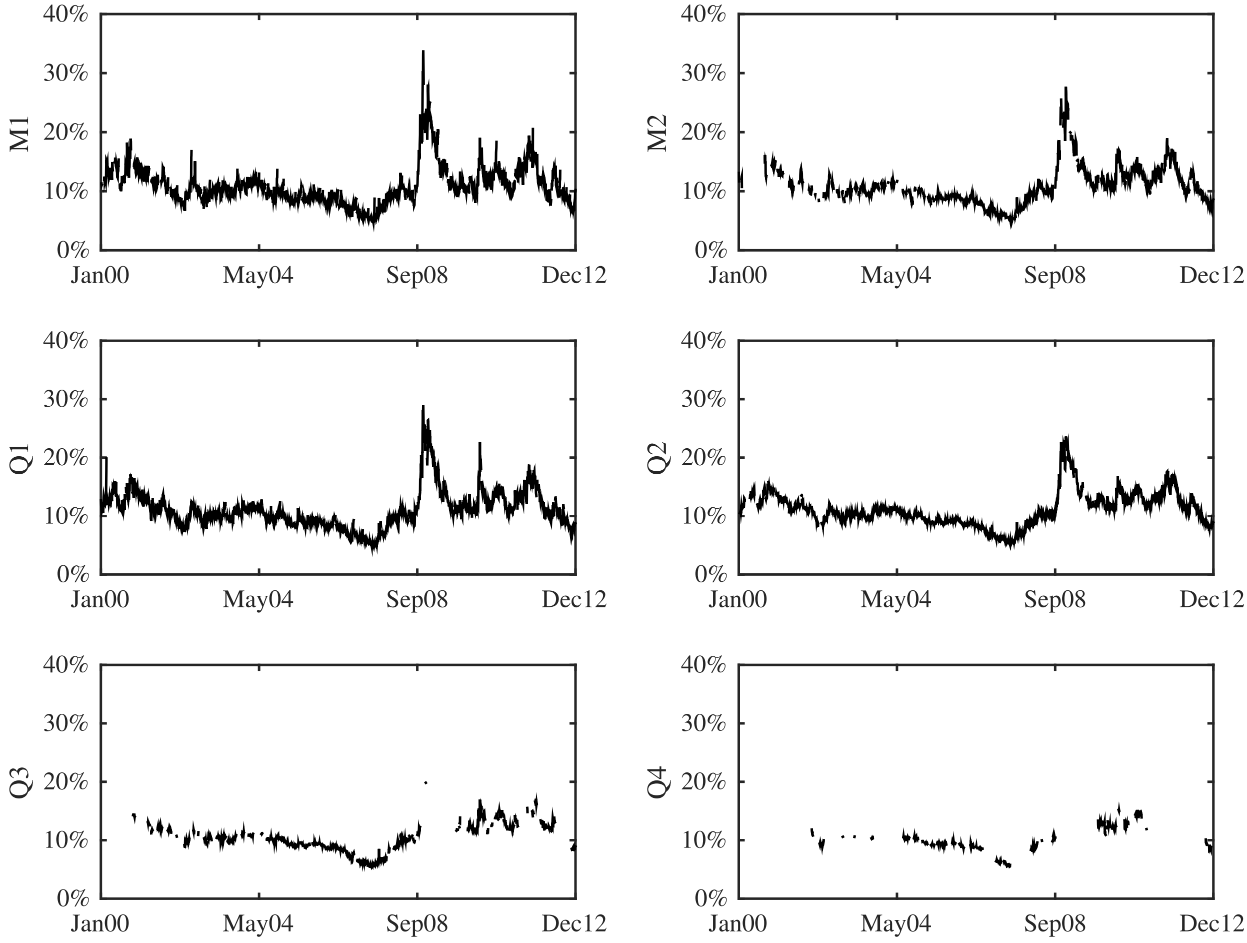


Figure I.5. EURUSD Options Implied Volatility (BAW)

The figures shows the evolution of the implied volatility of the EURUSD options. The implied volatility is found by first converting the quoted American option prices to European option prices using the method by Barone-Adesi and Whaley (1987). Afterwards the implied volatility is computed using the formulas for options on futures derived in Black (1976). Availability of liquid options in the long end is scarce. EURUSD volatility approximately double at the beginning of the financial crisis.

the volatility surfaces span a reasonable time frame and moneyness-intervals, while at the same time not showing too much asymmetry in availability of data.

I.3 Model-free Analysis: Joint Volatility Factors

The investigation of a joint factor in the volatility of crude oil and the volatility of EURUSD starts with a model-free analysis: I construct oil ATM straddle returns by calculating daily prices of ATM straddles, i.e., a put and a call option with strike equal to the value of the underlying futures, with 1-6 months to maturity. The first oil five straddles are on the nearest futures contract expiring approximately one month after the options and the 6M-straddle is on the futures contract in the March cycle that expires 7-9 months out. Similarly, straddle returns for EURUSD are calculated for contracts with 1-5 months to maturity with the nearest following futures contract as underlying. ATM straddles are by construction approximately Delta-neutral, so they are almost unaffected by changes in the underlying, but very sensitive to changes in volatility.

Figure I.6 shows the 3-month rolling correlation of the 1M-straddle returns. Over the full period, the empirical full sample correlation is positive, 0.1718, but a visual inspection of the rolling correlation indicates a development over time. To identify the possible change points in the correlation, I employ the method proposed by Galeano and Wied (2014), which is an extension of the test proposed in Wied et al. (2009). The method provide an algorithm for detecting multiple breaks in correlation structure of random variables. For a sequence of random variables (X_t, Y_t), $t \in [1, \ldots, T]$ with correlation between X_t and Y_t denoted by ρ_t, the hypothesis of all correlations being equal is tested using the test statistic

$$Q_T(X,Y) = \hat{D} \max_{2 \leq j \leq T} \frac{j}{\sqrt{T}} |\hat{\rho}_j - \hat{\rho}_T|,$$

where $\hat{\rho}_j$ is the empirical correlation up to time j and $\hat{D}$ is a normalising constant. The asymptotic distribution of the test statistic is the supremum of the absolute value of a standard Brownian bridge. For a critical level of 5%, the test statistic is compared to a value of 1.358. The algorithm described in Galeano and Wied (2014) is an iterative procedure, where the test statistic is first obtained for the full sample size. If the test statistic is significant, the break point obtained from the test size is determined a break in correlation and the resulting two samples are tested separately. This procedure is continued until no further breaks are obtained[6]. Finally, adjoining sub-samples are tested pairwise to assess if the estimated break point is optimal. If not, the original break point is replaced by the new optimal break point.

This analysis indicates a break in correlation in July 2007 with no additional breaks. The two horizontal lines in Figure I.6 represents the empirical correlation for these two sub-samples. The empirical correlation of 1M-straddle returns is 0.0353 from January 2000 to July 2007 and 0.3236 from July 2007 to December 2012. The structural break is occurring before the start of the financial crisis starting in the Fall of 2008, but several years after financialization is claimed to have started (see e.g., Tang and Xiong (2012)). It is however consistent with remarks from the U.S. Energy Information Administration, who in their presentation on drivers of the crude oil market state correlations between oil futures prices and other financial markets were fleeting prior to 2007.

Analysis of the straddle correlation

Interval	$Q_T(X,Y)$	Change point	Correlation	Date
Step 1				
$[1, 3259]$	4.8956*	1886	0.1718	July 20, 2007
Step 2				
$[1, 1886]$	0.6812	754	0.0353	January 13, 2003
$[1887, 3259]$	1.1542	2580	0.3236	April 23, 2010

This table shows the results of testing for a shift in correlation between 1-month straddles on oil and FX and indicates just a single change point. The initial nominal significance level is $\alpha_0 = 0.05$ and decreases with every iteration to keep the significance level constant.

Table I.1. Results of testing for a shift in correlation.

Next, the two sub-samples are investigated using Principal Components Analysis. Table I.2 shows the common variation in straddle returns. For the oil straddles, one factor explains 81.71% respectively 92.42% of the variation within the two sub-samples. For the FX straddles, one factor explains almost the same percentage of variation, 91.54% respectively 89.59%. Considering all 11 straddle returns series at once, two factors explain 85.89% of the variation in the first sub-sample and 90.90% of the variation in the second sub-sample. The degree of explanation coming from the first common factor increases from 47.40% to

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[6] The critical level for detecting the kth level of break points is lowered to $\alpha_k = 1 - (1 - \alpha_0)^{1/k+1}$ to keep the significance level constant for all tests.

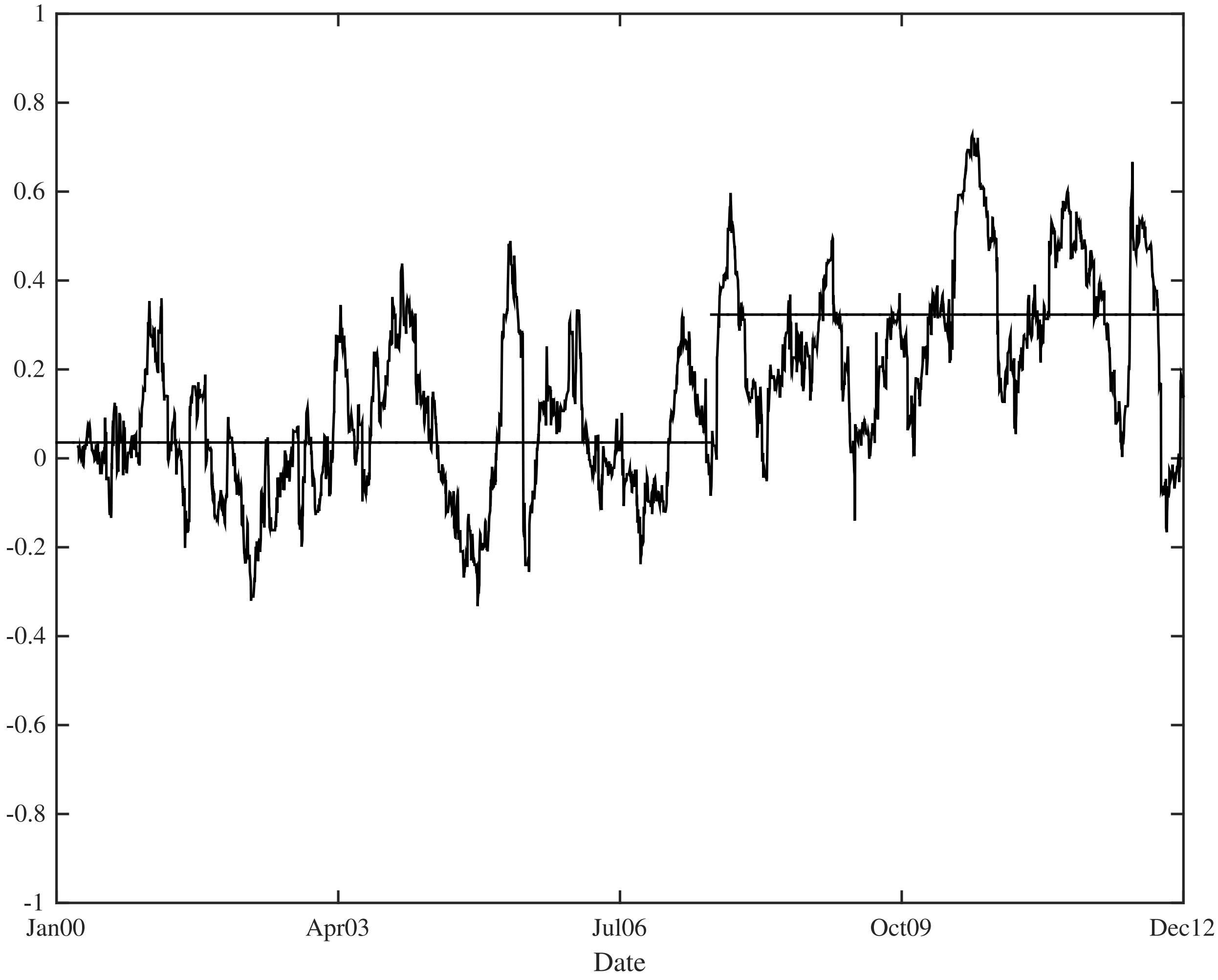

Figure I.6. Rolling 3M correlation of 1M straddle returns for EURUSD and for oil.

The figure shows the rolling three month correlation calculated on daily returns. The horizontal lines shows the average correlation over the periods 2000-2007 and 2007-2012. The exact periods are chosen from test of breaks in correlation and shown in Table I.1.

62.29%.

The above numbers do not guarantee that there is co-variation between straddle returns. To ensure that the two first factors is the combined PCA is not merely one factor for crude oil and one factor for EURUSD, the eigenvectors for the first three joint principal components are presented in Figures I.7 and I.8. For the first sub-sample (Figure I.7), the first joint principal component mainly explains the variations in the oil straddles, as the eigenvector values related to the EURUSD straddles is close to zero. The second joint principal component mainly explains the variations in the EURUSD straddle returns. There is little indication of a common factor driving the two sets of straddle returns and thereby the volatilities of crude oil and EURUSD. For the second sub-sample (Figure I.8) staring in July 2007, the picture is different: The first joint principal component, that explains 62.29% of the total variation in the combined straddles, affect both the oil straddles and the EURUSD straddles. The second principal component also explains variation in both oil and EURUSD straddles, but with opposite signs for oil straddles and EURUSD straddles. The eigenvector is decaying in maturity of the straddles, which is in line with the Samuelson-effect; volatilities (or equivalent straddle returns) are higher for shorter maturities compared to longer maturities.

In conclusion, the model free analysis in this section empirically considers the rolling correlation of short crude oil straddles and short EURUSD straddles and confirms that volatilities (more precisely in the form of straddle returns) exhibit a much higher correlation during the later part of the period analysed compared to the beginning.

Secondly, a Principal Components Analysis shows that around 85-90% of the variation across combined straddle returns can be explained using two factors. For the first sub-sample one factor is attributed to explaining the oil straddles and a second factor is attributed to EURUSD straddles. For the second sub-sample, the two factors both contribute to explaining the variation across the combined set of straddles.

In the next section, a model including a joint volatility factor is proposed. In total, there will be three volatility factors; one joint volatility factor, one volatility factor for oil and one volatility factor for EURUSD.

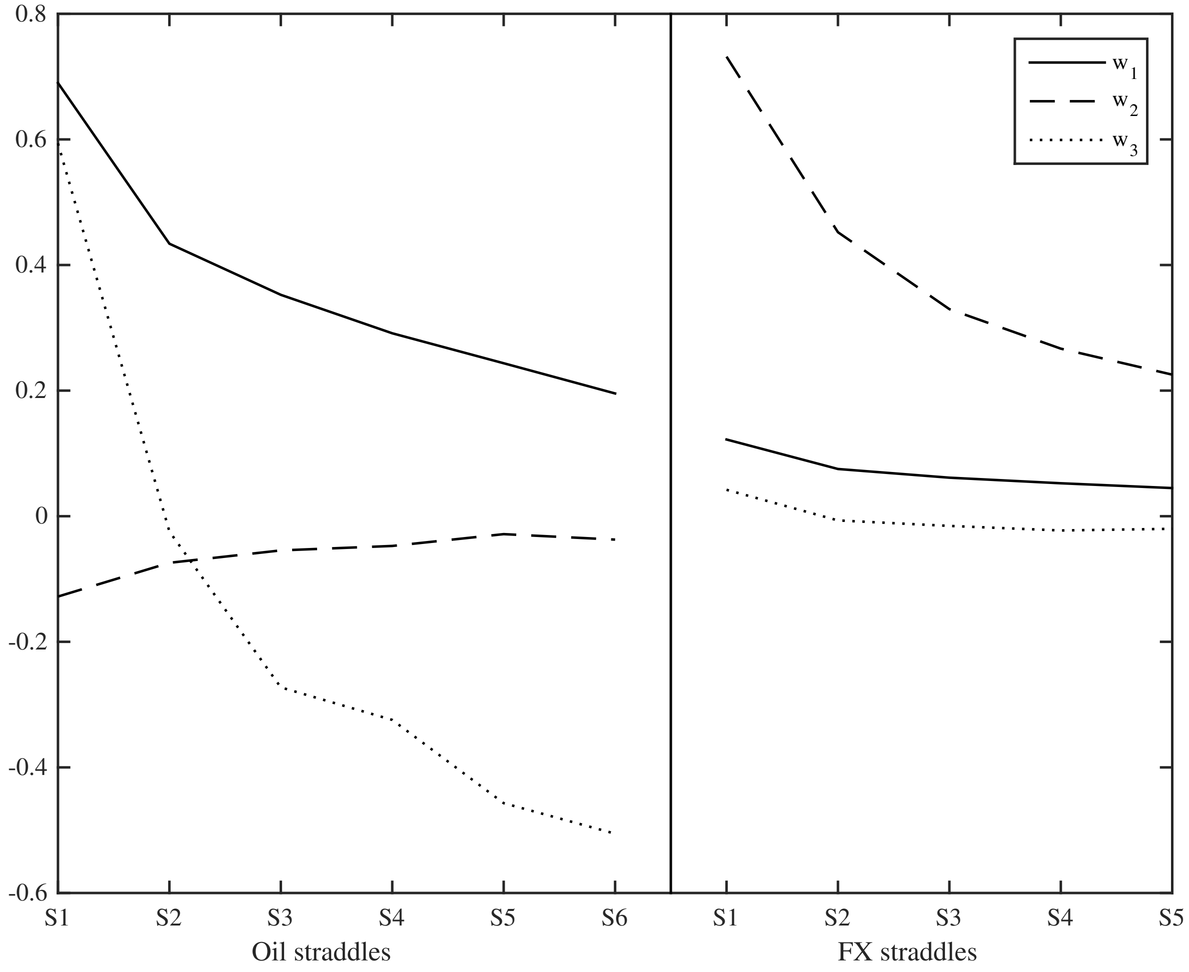

Figure I.7. Eigenvectors for separate and combined oil and EURUSD straddles (2000-2007)

The figure shows the eigenvectors for three first principal components, when looking at the combined straddle returns. The first eigenvector (solid line) for the joint set of straddles is close to zero for the EURUSD straddles and the first principal component is therefore largely explaining the variation in oil straddles. The second eigenvector (dashed line) for the joint set of straddles is close to zero for the oil straddles and the second principal component is therefore largely explaining the variation in EURUSD straddles. The third eigenvector (dotted line) is mainly impacting the oil straddles, but offers very little explanatory power.

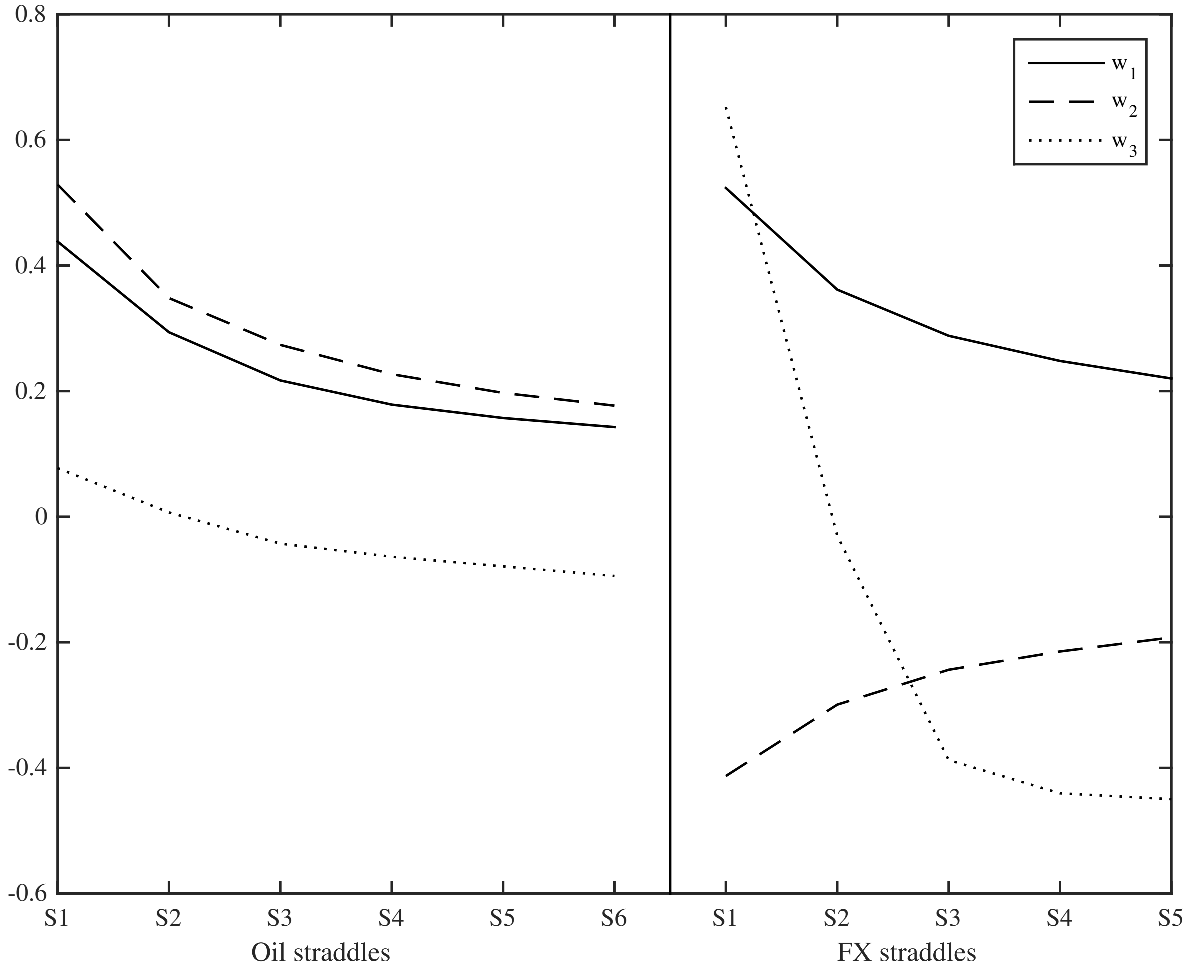

Figure I.8. Eigenvectors for separate and combined oil and EURUSD straddles (2007-2012)

The figure shows the eigenvectors for three first principal components, when looking at the straddle returns together. The first eigenvector (solid line) for the joint set of straddles shows that the first principal component is explaining variation in both oil and EURUSD straddles. The loadings for oil straddles and for EURUSD straddles on the first principal component are of similar size and decaying. The second eigenvector (dashed line) for the joint set of straddles is also explaining variation in both oil and EURUSD straddles. The loadings for oil straddles and for EURUSD straddles on the second principal component are of same magnitude, but with opposite signs. The third eigenvector (dotted line) offers very little explanatory power.

Panel A: PCA for separate straddle returns

		Oil				*FX*		
	# obs	PC_1^{Oil}	PC_2^{Oil}	PC_3^{Oil}	# obs	PC_1^{FX}	PC_2^{FX}	PC_3^{FX}
2000 – 2007	988	81.71	9.51	4.10	988	91.54	6.53	1.52
2007 – 2012	693	92.42	5.27	0.98	693	89.59	7.09	2.40

Panel B: PCA for combined straddle returns

	# obs	PC_1^{Joint}	PC_2^{Joint}	PC_3^{Joint}	PC_4^{Joint}
2000 – 2007	988	47.40%	38.49%	5.49%	2.83%
2007 – 2012	693	62.29%	28.61%	3.88%	2.37%

Panel A shows the percentage of variation explained for the straddle returns. Only days where straddle returns could be computed is are included in the analysis. For both oil straddles and EURUSD straddles, two factors explain a large part of the variation, when the panel data of straddle returns are analysed separately. Panel B shows the percentage of variation explained for the combined set of straddles on oil and on EURUSD. Two common factors explain about the same percentage of the variation as one oil and one EURUSD factor does for the separate staddles.

Table I.2. Common variation in straddle returns.

I.4 A Linked Model for Derivatives on Oil and the EURUSD

In this paper, the model proposed by Trolle and Schwartz (2009) for oil derivatives is used as a starting point[7]. It has proven to fit the market well as documented in the original paper and tested for later years in Cortazar et al. (2016). To have a flexible and still tractable model for the futures and option prices for both oil and FX, two models of the type developed in Trolle and Schwartz (2009) is combined by allowing one of the two asset specific volatility factors, υ, to be the same. As the focus is on the joint volatility factor, the number of factors driving the futures prices are limited to decrease the number of parameters to be estimated. This will favour the fit of options over the fit of the futures curve.

I.4.1 Model

Let E_t denote the time-t spot price of oil with a spot cost-of-carry[8] given by δ_t^E. The spot cost-of-carry is derived from the forward cost-of-carry y^E, i.e., $y_t^E(T)$ is the time-t instantaneous cost-of-carry between t and T and $\delta_t^E = y_t^E(t)$. Under the (domestic) risk neutral measure, the oil price (measured in the domestic currency) is

$$\frac{dE_t}{E_t} = \delta_t^E dt + \sigma_{E1}\sqrt{\upsilon_t^E} dW_t^E + \sigma_{E2}\sqrt{\upsilon_t^J} dW_t^J \tag{I.1}$$

$$dy_t^E(T) = \mu_E(t,T) + \alpha_E e^{-\gamma_E(T-t)}\sqrt{\upsilon_t^E} dB_t^E \tag{I.2}$$

Correspondingly, X_t denotes the time-t EURUSD exchange rate with an spot interest rate differential given by δ_t^X. The interest rate differential is derived from the forward interest rate differential y^X, i.e., $y_t^X(T)$ is the time-t forward rate differential and $\delta_t^X = y_t^X(t)$. Under the risk neutral measure, the EURUSD rate (i.e., the price of EUR measured in the USD) is

$$\frac{dX_t}{X_t} = \delta_t^X dt + \sigma_{X1}\sqrt{\upsilon_t^X} dW_t^X + \sigma_{X2}\sqrt{\upsilon_t^J} dW_t^J \tag{I.3}$$

$$dy_t^X(T) = \mu_X(t,T) + \alpha_X e^{-\gamma_X(T-t)}\sqrt{\upsilon_t^X} dB_t^X \tag{I.4}$$

The three volatility factors evolve according to

$$d\upsilon_t^E = \left(\eta^E - \kappa_E \upsilon_t^E\right) dt + \sigma_{\upsilon^E}\sqrt{\upsilon_t^E} dZ_t^E \tag{I.5}$$

---

[7] Another approach would have been to extend the approach taken in Pilz and Schlögl (2012), who they apply a multi-LIBOR model applied to one interest rate and the oil price. An extension of the would be to include an additional interest rate and thereby modelling the FX rate through the two interest rates.

[8] The cost-of-carry is defined as the interest rate net of the convenience yield

$$d\upsilon_t^X = \left(\eta^X - \kappa_X \upsilon_t^X\right) dt + \sigma_{\upsilon^X}\sqrt{\upsilon_t^X}\, dZ_t^X \tag{I.6}$$

$$d\upsilon_t^J = \left(\eta^J - \kappa_J \upsilon_t^J\right) dt + \sigma_{\upsilon^J}\sqrt{\upsilon_t^J}\, dZ_t^J \tag{I.7}$$

The Brownian motions are correlated in the following way: W^E, B^E, and Z^E are pairwise correlated with ρ_{WB}^E, ρ_{WZ}^E, and ρ_{BZ}^E as are W^X, B^X, and Z^X are pairwise correlated with ρ_{WB}^X, ρ_{WZ}^X, and ρ_{BZ}^X. All other correlations are set to zero.

No-arbitrage conditions will impose a restriction on the drift of the cost-of-carry and the interest rate differential leading to the following prices for futures:

Proposition I.1 (Trolle and Schwartz (2009) Equation 17) The log-futures price is affine in the state variables below and are given by for $j = E, X$:

$$\log F_t^j(T) = \log \frac{F_0^j(T)}{F_0^j(t)} + \log \jmath_t + \underbrace{\tfrac{\alpha_j\left(1-e^{-\gamma_j(T-t)}\right)}{\gamma_j}}_{B_x^j(T-t)} x_t^j + \underbrace{\tfrac{\alpha_j\left(1-e^{-2\gamma_j(T-t)}\right)}{2\gamma_j}}_{B_\phi^j(T-t)} \phi_t^j \tag{I.8}$$

with

$$d\log \jmath_t = \left(y_0^j(t) + \alpha_j\left(x_t^j + \phi_t^j\right) - \tfrac{1}{2}\left(\sigma_{j1}^2\upsilon_t^j + \sigma_{j2}^2\upsilon_t^J\right)\right) dt + \sigma_{j1}\sqrt{\upsilon_t^j}\, dW_t^j + \sigma_{j2}\sqrt{\upsilon_t^J}\, dW_t^J \tag{I.9}$$

$$dx_t^j = \left(-\gamma_j x_t^j - \left(\frac{\alpha_j}{\gamma_j} + \rho_{WB}^j\sigma_{j1}\right)\upsilon_t^j\right) dt + \sqrt{\upsilon_t^j}\, dB_t^j \tag{I.10}$$

$$d\phi_t^j = \left(-2\gamma_j\phi_t^j + \frac{\alpha_j}{\gamma_j}\upsilon_t^j\right) dt \tag{I.11}$$

European options on futures contracts can be priced according to:

Proposition I.2 (Trolle and Schwartz (2009) Propositions 3 and 4) The price of an European put option expiring at time T_0 on a futures contract maturing on time T_1 is for $j = E, X$[9]

$$\begin{aligned}\mathcal{P}^j(t, T_0, T_1, K) &= \mathbb{E}_t^Q\left[e^{-\int_t^{T_0} r_s ds}\left(K - F_{T_0}^j(T_1)\right)1_{F_{T_0}^j(T_1)<K}\right] \\ &\approx P(t, T_0)\left(KG_0^j(\log K) - G_1^j(\log K)\right)\end{aligned} \tag{I.12}$$

where

$$\begin{aligned}G_a^j(y) &= \mathbb{E}_t^Q\left[e^{a\log F_{T_0}^j(T_1)}1_{\log F_{T_0}^j(T_1)<y}\right] \\ &= \frac{\chi^j(a, t, T_0, T_1)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathtt{Im}\left[\chi^j(a + \imath u, t, T_0, T_1)e^{-\imath u y}\right]}{u} du\end{aligned} \tag{I.13}$$

---

[9] Like in Trolle and Schwartz (2009), I disregard the covariance term in the option price. For short-dated options, the covariance term is negligible.

and the transform χ is given as

$$\chi^j(u,t,T_0,T_1) = \mathbb{E}_t^Q\left[e^{u\log F^j_{T_0}(T_1)}\right],$$

with an exponentially affine solution given by

$$\chi^j(u,t,T_0,T_1) = e^{M_j(T_0-t)+N_{1j}(T_0-t)\upsilon_t^j+N_{2j}(T_0-t)\upsilon_t^J+u\log F_t^j(T_1)}. \tag{I.14}$$

where M_j and N_j are solutions to the following ODEs:

$$\begin{aligned}
M_j'(\tau) =& N_{1j}(\tau)\eta^j + N_{2j}(\tau)\eta^J \\
N_{j1}'(\tau) =& \tfrac{1}{2}(u^2-u)\left[\sigma_{j1}^2 + B_x^j(T_1-T_0+\tau)^2 + 2\rho^j_{WB}\sigma_{j1}B_x^j(T_1-T_0+\tau)\right] \\
&+ N_{j1}(\tau)\left[-\kappa^j + u\sigma_{\upsilon^j}\left(\rho^j_{WZ}\sigma_{j1} + \rho^j_{BZ}B_x^j(T_1-T_0+\tau)\right)\right] + \tfrac{1}{2}N_{j1}(\tau)^2\sigma^2_{\upsilon^j} \\
N_{j2}'(\tau) =& \tfrac{1}{2}(u^2-u)\sigma_{j2}^2 - N_{j2}(\tau)\kappa^J + \tfrac{1}{2}N_{j2}(\tau)^2\sigma^2_{\upsilon^J}
\end{aligned}$$

subject to the boundary conditions $M_j(0) = N_{j1}(\tau) = N_{j2}(\tau) = 0$. In practice the integral is evaluated in a set of cleverly chosen points, e.g., using the Gauss-Legendre quadrature.

For the joint model, this results in 34 parameters. In case $\sigma_{E2} = \sigma_{X2} = 0$, each asset is modelled using the SV1-specification in Trolle and Schwartz (2009) and 14 parameters for each model is then needed. The six extra parameters are the loadings of the spot oil price and spot EURUSD rate on the joint volatility factor, the speed (and thereby mean-reversion level) and the volatility of the joint volatility process as well as the market price of risk belonging to the two extra Brownian motions[10].

Using this specification, only the spot price is affected by the joint volatility, whereas the shape of the futures curve is not. For simplicity the three stochastic volatility processes are not connected via their drift. Another possible specification would be to let the separate oil volatility and the separate EURUSD volatility process mean reverts around the joint volatility process. In this situation, the joint volatility process would not need to enter into the spot price specification, but could be obtained from the joint volatility process affecting the future volatility and thereby the option prices.

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[10] It might seem unnecessary restrictive to include the same Brownian motion in the two spot dynamics, but the estimation showed very little difference in the likelihood function when the same Brownian motion was used compared to two different and correlated Brownian motions.

I.4.2 Estimation Approach

The model is estimated using quasi maximum-likelihood applying the extended Kalman filter after formulating the model in state-space form[11]. The observed log futures prices and the option prices are linked to the state vector through the measurement equation

$$y_t = h(\theta_t) + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \Omega) \tag{I.15}$$

where y_t is a vector of observed prices, h is the pricing functions given by (I.8) and (I.12) and ϵ_t are assumed i.i.d. Gaussian measurement errors with covariance matrix Ω. It is assumed that measurement errors are uncorrelated, implying that Ω is a diagonal matrix and that each of the four contract types has the same variance of the measurement error. The states

$$\theta_t = \left(\log E_t, x_t^E, \phi_t^E, \upsilon_t^E, \log X_t, x_t^X, \phi_t^X, \upsilon_t^X, \upsilon_t^J\right)'$$

has the following transition equation

$$\theta_{t+1} = T\theta_t + c + \varepsilon_{t+1} \tag{I.16}$$

with $E_t(\varepsilon_{t+1}) = 0$ and $Cov_t(\varepsilon_{t+1}) = Q_0 + Q^E \upsilon_t^E + Q^X \upsilon_t^X + Q^J \upsilon_t^J$. The state vector needs to be specified under the physical probability measure, which is obtained by specifying the market prices of risk to link the Brownian motions under $\mathbb{P}$ (indicated by a ˜) and $\mathbb{Q}$. The completely affine specification is commonly used in this class of models:

$$d\tilde{W}_t^j = dW_t^j - \lambda_W^j \sqrt{\upsilon_t^j} dt \tag{I.17}$$

$$d\tilde{W}_t^J = dW_t^J - \lambda_W^J \sqrt{\upsilon_t^J} dt \tag{I.18}$$

$$d\tilde{B}_t^j = dB_t^j - \lambda_B^j \sqrt{\upsilon_t^j} dt \tag{I.19}$$

$$d\tilde{Z}_t^j = dZ_t^j - \lambda_Z^j \sqrt{\upsilon_t^j} dt \tag{I.20}$$

The vector c and the matrices and T, Q_0, Q^E, Q^X and Q^J are available in closed form and derived using (I.9)-(I.11) and (I.5)-(I.7) together with the market price of risk specification. The details can be found in Appendix I.B.

At time t, m^E oil futures prices and n^E oil option prices are observed along with the spot FX rate, m^X FX futures prices and n^X FX option prices. To convert option pricing errors to implied volatility errors, the option prices are scaled by their vegas, so the observations z_t are given by

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[11] Details related to the application of Kalman filters can be found in classic references such as Harvey (1989).

$$\begin{aligned} z_t = \Big(& \log F_t^E(T_1), \dots, \log F_t^E(T_{m^E}), \mathcal{O}_t^{E,1}/\mathcal{V}_t^{E,1}, \dots, \mathcal{O}_t^{E,n^E}/\mathcal{V}_t^{E,n^E}, \\ & \log X_t, \log F_t^X(T_1), \dots, \log F_t^X(T_{m^X}), \mathcal{O}_t^{X,1}/\mathcal{V}_t^{X,1}, \dots, \mathcal{O}_t^{X,n^X}/\mathcal{V}_t^{X,n^X} \Big)' \end{aligned}$$

Finally as it is standard in this type of estimation, see e.g., Trolle and Schwartz (2009) and Chiarella et al. (2013), a time-homogeneous version of (I.8) is considered by assuming that the initial cost-of-carry, $y_0^E(T)$, and interest rate differential, $y_0^X(T)$, are flat: $y_0^E(T) = \xi^E$ and $y_0^E(T) = \xi^X$ for all maturities T. To ensure identification, η is set to 1 for all volatility processes .

I.5 Results

The joint likelihood function will favour the fit of oil options more, if the dataset is kept as it is after the initial sorting as there are more oil option observations. To ensure a reasonable surface without too much asymmetry in the number of contracts, the dataset is further limited: The estimation is done on the middle five moneyness intervals, i.e., ranging from 0.9 to 1.1. For oil, options on M1-M6 is included and for EURUSD, options on M1-M2 and Q1-Q2 is used. This gives a maturity span of just over six months. Further, the EURUSD options with less than 10 trading days to maturity is taken out, as the fitting procedure by scaling option prices with Vega is sensitive to short maturities. For the EURUSD options, there is very limited availability of options for the two outermost moneyness intervals during the first sub-sample. The discount rate in the option pricing formula is obtained by interpolation of the 1-month, 3-month, 6-month and 1-year LIBOR-rates and the 2-year swap rate. Interest rate data is downloaded from the Federal Reserve Bank of St. Louis.

I.5.1 Estimation results

Before the full model is estimated presented in Section I.4, the model is estimated using only one volatility factor for the oil futures and options sample and for the EURUSD futures and options sample. The figures in Figure I.9 show the filtered time series of v_t^E and v_t^X, when the models are estimated separately for each of the two sub-samples. During the first sub-sample, the processes seem unrelated. The empirical correlation of changes in the volatility processes is 0.0444. During the second sub-sample, the processes exhibit similarities and their empirical correlation of changes in the volatility processes is 0.3207. The relation between volatilities in the two sub-samples is in line with the findings from Section I.3.

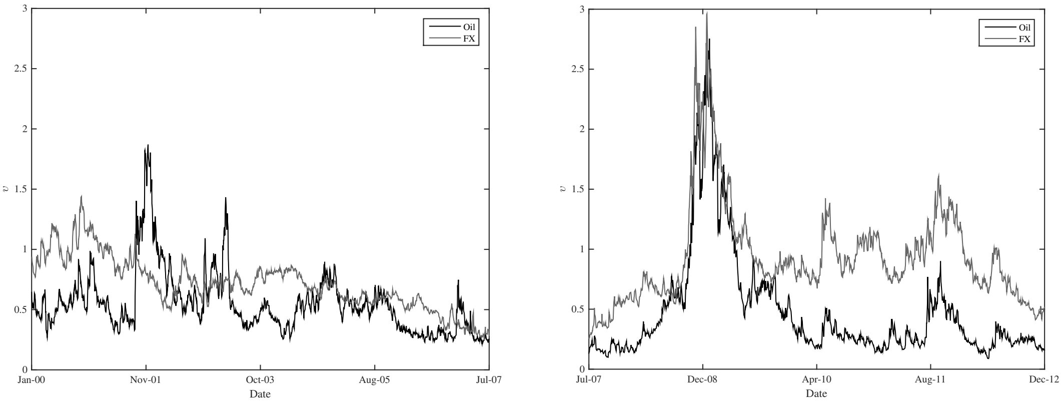

Figure I.9. Filtered υ-processes for the separate estimations

The figures shows the filtered υ^E (the black line) and υ^X (the grey line). The left figure is from January 4, 2000 to July 20, 2007. The empirical correlation of changes in the filtered volatility processes is 0.0444. By visual inspection, there seems to be little co-movement between processes. The volatility process for oil jumps significantly after the 9/11 terrorist attack, while there is little reaction to the EURUSD volatility. The right figure is from July 21, 2007 to December 31, 2012. The empirical correlation of differences changes in the filtered volatility processes is 0.3207. During this period, the two processes exhibit some co-movement, which support the presence of a joint volatility factor during the second period.

Following the estimation of the separate models, the full model is estimated for each of the two sub-samples. The estimation results for oil and for EURUSD are used as a starting point for the estimation of the full model. Table I.3 shows the results of the separate and joint estimation for each of the two sub-samples.

Under the risk neutral measure, the separate volatility components υ^E and υ^X mean reverts quickly as the κ's are estimated to lie between 4.7471 and 8.6996, while the the joint volatility component is much slower with κ_J less than 1. In the joint model, the υ^J expresses the more persistent shocks to volatility with υ^E and υ^X capturing the transitory shocks to volatility.

For oil, the correlation between shocks to the spot price and shocks to the cost-of-carry curve is close to -1. The same feature is reported in other papers like Schwartz (1997) and Trolle and Schwartz (2009).

Both volatility processes affect the volatility of the spot prices, but the impact is scaled by σ_{j1} and σ_{j2}. For oil, σ_{E1} is three-four times higher than σ_{E2}, but combined with the level of the υ's, about 58% of the instantaneous variance is explained by υ_E during the first sub-sample and about 51% during the second sub-sample. For EURUSD, σ_{X1} is around seven times higher than σ_{X2} during the first sub-sample and υ_X explains 96% of the variance. During the second sub-sample, the ratio of σ_X's decreases to four and υ_X now explains only 86% of the variance.

The separate models are per construction not connected. Estimating the two models jointly will give the value of the log-likelihood function as the sum of the two separate log-likelihood functions. This model would be nested in the full model and it would therefore be possible to test the parameter restriction in the separate models using likelihood ratio tests. Given the large number of observations, the separate models would be rejected at high levels of statistical significance. Instead the pricing performance is compared both at an aggregated level, where time series of root mean squared errors are time series of errors are investigated, and later in Section I.5.2 for each combination of moneyness interval and maturity.

Based on the filtered values of the latent processes, fitted futures prices and log-normal implied volatilities are computed and compared to actual values. The figures showing the time series of RMSEs for oil futures in Figure I.10 and for EURUSD futures in Figure I.11 are very similar. This is not surprising, as the additional volatility factor only affects the futures prices through the spot price.

Adding the extra volatility factor instead benefits the fit of the option prices. Table I.4 presents the results on an aggregated level: The average[12] RMSE of the actual and fitted log-normal implied volatilities decrease from 1.558% to 1.072% for the oil options in the first period. For the same period, average RMSE of the actual and fitted log-normal implied volatilities for EURUSD remain almost unchanged in the two specifications. For the second sub-sample, there is both a decrease in oil option RMSE and EURUSD option RMSE. The oil option RMSE decrease from 2.068% to 1.572% which is about 25 percent in the relative terms. The EURUSD option RMSE from 0.809% to 0.766% which is about 5 percent in relative terms. Together with the summary of RMSEs, Table I.4 also presents the results of the test from by Diebold and Mariano (1995), where two time siries of RMSEs are compared: The pairs of time series of RMSEs obtained by the separate and the full models are compared by computing the mean of the differences between the separate and joint model and the associated t-statistics. If the mean is significantly positive, the joint model provides a better fit. The tests show that for the first sub-sample, oil options are significantly better fitted by

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[12] The daily RMSE are the root of the mean of the squared errors from all options fitted on that day. The average RMSE is the average of the daily RMSEs.

	2000-2007				2007-2012			
	Separate		Joint		Separate		Joint	
	Oil	FX	Oil	FX	Oil	FX	Oil	FX
κ_j	2.0908	7.6684	4.7948	8.6699	2.5370	5.5802	6.1828	8.4629
	(0.0380)	(0.0978)	(0.0002)	(0.1204)	(0.0064)	(0.0762)	(0.0082)	(0.0937)
κ_J	–	–	0.8985		–	–	0.6140	
	–	–	(0.0238)		–	–	(0.0139)	
σ_{υ_j}	0.8458	0.3982	2.0994	0.6345	0.8858	0.8866	3.2983	1.0904
	(0.0155)	(0.0118)	(0.0023)	(0.0166)	(0.0011)	(0.0242)	(0.0128)	(0.0293)
σ_{υ_J}	–	–	2.3840		–	–	2.6616	
	–	–	(0.0778)		–	–	(0.0384)	
σ_{j1}	0.5385	0.2059	0.7064	0.2103	0.6439	0.1963	0.8114	0.2401
	(0.0028)	(0.0029)	(0.0015)	(0.0036)	(0.0022)	(0.0034)	(0.0016)	(0.0027)
σ_{j2}	–	–	0.1791	0.0289	–	–	0.1052	0.0259
	–	–	(0.0030)	(0.0008)	–	–	(0.0017)	(0.0005)
α_j	0.4317	0.0215	0.9305	0.0194	0.3449	0.0157	0.4709	0.0204
	(0.0079)	(0.0011)	(0.0019)	(0.0013)	(0.0032)	(0.0008)	(0.0062)	(0.0009)
γ_j	1.0131	0.4534	1.1174	0.4310	0.4374	0.0836	0.4318	0.2592
	(0.0052)	(0.0080)	(0.0029)	(0.0080)	(0.0048)	(0.0243)	(0.0000)	(0.0031)
ρ^j_{WB}	−0.9098	0.6201	−0.9651	0.4544	−0.8050	0.2337	−0.9283	−0.2911
	(0.0050)	(0.0266)	(0.0027)	(0.0450)	(0.0082)	(0.0423)	(0.0027)	(0.0050)
ρ^j_{WZ}	−0.3503	0.2466	−0.1881	0.1757	−0.5842	−0.4437	−0.2958	−0.5142
	(0.0085)	(0.0203)	(0.0032)	(0.0157)	(0.0063)	(0.0124)	(0.0037)	(0.0132)
ρ^j_{BZ}	0.3095	0.1888	−0.0278	0.1271	0.3598	−0.1004	0.0032	0.0382
	(0.0184)	(0.0425)	(0.0130)	(0.0644)	(0.0215)	(0.0422)	(0.0024)	(0.0214)
λ^j_Z	−0.1987	14.8593	1.2216	3.5157	2.8523	6.6109	4.3867	5.2140
	(0.4863)	(0.6807)	(0.0013)	(0.6160)	(0.0061)	(0.4058)	(0.0399)	(0.1416)
λ^J_Z	–	–	−0.0028		–	–	−0.0396	
	–	–	(0.2563)		–	–	(0.0075)	
λ^j_W	1.2568	0.4929	0.7767	0.3167	0.5043	−2.8914	0.7739	0.5947
	(0.4861)	(0.4350)	(0.8858)	(0.4950)	(0.1032)	(0.1718)	(0.0083)	(0.1611)
λ^J_W	–	–	−0.7173		–	–	−0.8766	
	–	–	(0.3051)		–	–	(0.0006)	
λ^j_B	−0.5958	0.1020	−2.5758	−3.9569	−2.8832	−1.6761	−4.6689	−6.5189
	(0.4862)	(0.4358)	(0.8184)	(0.7293)	(0.4075)	(0.1068)	(0.0069)	(0.5644)
ξ_j	−0.0167	0.0119	−0.0112	0.0123	0.0080	0.0091	0.0057	0.0012
	(0.0002)	(0.0003)	(0.0001)	(0.0003)	(0.0013)	(0.0042)	(0.0006)	(0.0004)
σ_{fut}	0.0135	0.0009	0.0131	0.0009	0.0093	0.0010	0.0102	0.0010
	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
σ_{opt}	0.0164	0.0053	0.0089	0.0052	0.0204	0.0084	0.0121	0.0079
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)	(0.0001)
$\log L$	206275	96927	333883		147250	86418	252035	

Table I.3. Parameter estimates with standard errors

the joint model, while the EURUSD options experience a slightly worse fit. For the second sub-sample, the oil options are again fitted significantly better, while the EURUSD options are estimated better, although not significantly.

The time series of RMSEs tested are plotted in Figures I.12 and I.13 and in Figures I.14 and I.15 the difference between the daily RMSE for the separate model and the daily RMSE for the joint model is shown. Inspection of the figures confirms the results in Table I.4: The first sub-sample shows a improvement in oil option RMSE time series, while the fit is approximately the same for the EURUSD options. The second sub-sample again shows an improvement in the fit of oil options, but also an improvement for the EURUSD options.

The significant improvement of the oil options fit and the improvement (although non-significant) of the EURUSD options fit during the second period supports the presence of a joint factor in volatilities. During the second sub-sample, oil options are mainly improved during the financial crisis, making it relevant to investigate further if the joint volatility factor is a crisis or a financialization phenomenon – or a combination.

	2000-2007		2007-2012	
	Oil	FX	Oil	FX
		Futures contracts		
Separate	1.879	0.474	1.811	0.577
Joint	1.851	0.475	1.900	0.548
Separate - Joint	0.028	−0.001	−0.090	0.030
	(0.63)	(−0.58)	(−3.43)	(4.82)***
		Options on futures contracts		
Separate	1.558	0.486	2.068	0.809
Joint	1.072	0.492	1.572	0.766
Separate - Joint	0.486	−0.006	0.496	0.043
	(3.82)***	(−0.67)	(2.99)***	(1.32)

The table shows the average RMSE for the fit of the separate and the joint model. The top panel shows the average RMSE for futures. The futures fit is almost unaffected and in some cases even deteriorated, when the extra volatility factor is added. The bottom panel shows the average RMSE for implied volatility differences. The numbers in parenthesis are test-statistics from the Diebold-Mariano test. There is a significant improvement of oil options, when an extra volatility factor is added. For the first period, the fit of the EURUSD options is slightly worse, while it for the second period is improved non-significantly. $*$, $**$, $***$ identifies significance at the 10%, 5% and 1% levels.

Table I.4. Summary of Root Mean Squared Errors

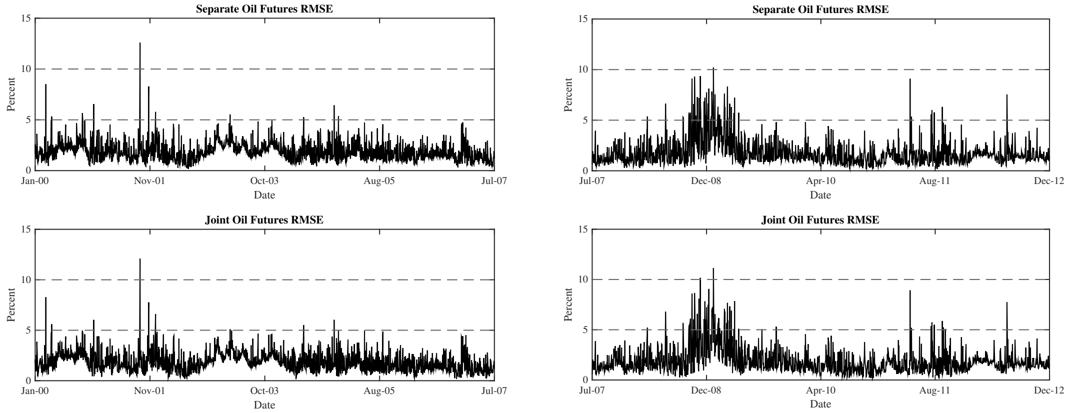

Figure I.10. The time series of RMSE across all oil futures.

The upper figures show the oil futures root mean sqaured errors (RMSE) across all oil futures, when the separate models are estimated. The lower figures show the oil futures RMSE, when the joint model is estimated. The left figures show the first sub-sample and the right figures show the second sub-sample. The two specifications seems to perform equally well, which is not surprising as the same number of factors are driving the futures prices. During the financial crisis, RMSEs were much higher compared to the general level.

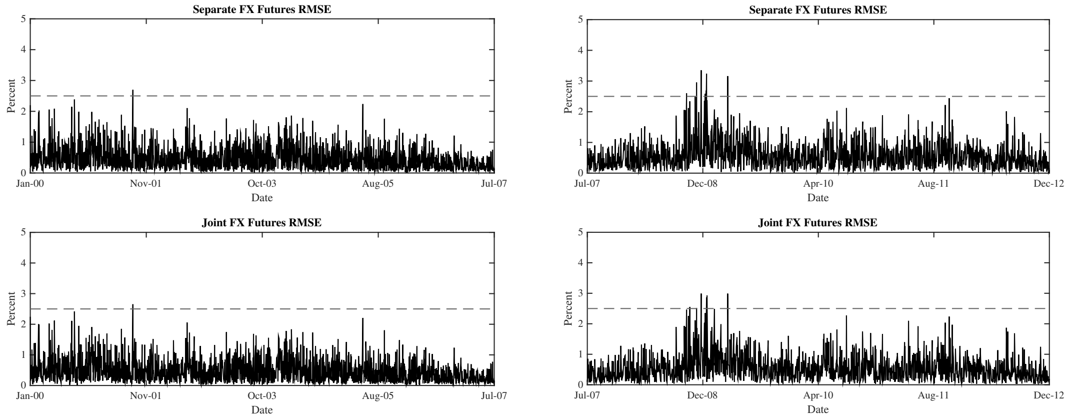

Figure I.11. The time series of RMSE across all EURUSD futures.

The upper panels show the EURUSD spot and futures RMSE, when the SV1 specification is estimated. The lower panels show the EURUSD spot and futures RMSE when the joint model is estimated.

I.5.2 Volatility surface fit

Tables I.5 and I.7 show the Mean Absolute Errors (MAEs) for each combination of moneyness and maturity for the two model specifications. In the first sub-sample, oil options MAEs ranges from 0.73% to 2.17% in the separate model and from 0.55% to 1.48% in the joint model. Except for a single case, there is an overall improvement in MAEs when the joint volatility factor is added. For the EURUSD options, the MAE ranges from 0.29% to

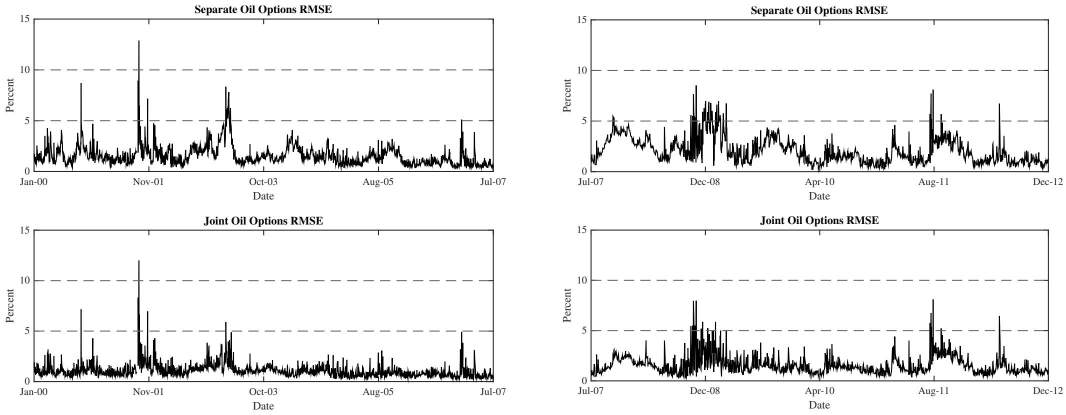

Figure I.12. The time series of RMSEs of the differences between fitted and actual log-normal implied option volatilities across all 30 oil options. The upper figures show time series of RMSEs of the differences between fitted and actual log-normal implied option volatilities, when the separate models are estimated. The lower figures show the oil options RMSE when the joint model is estimated. The left figures show the first sub-sample and the right figures show the second sub-sample. There is a clear improvement to the overall fit of option prices, when an additional volatility factors is included.

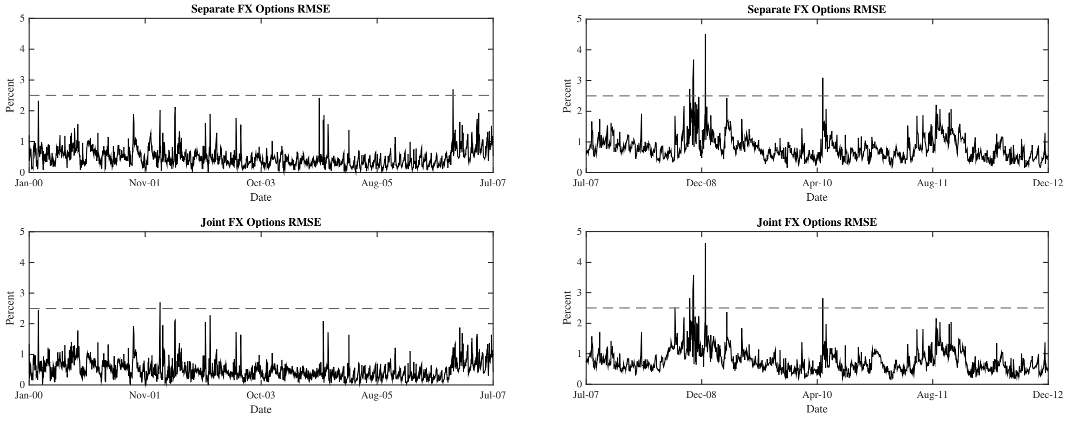

Figure I.13. The time series of RMSEs of the differences between fitted and actual log-normal implied option volatilities across all 20 EURUSD options. The upper figures show the EURUSD options RMSEs when the separate models are estimated. The lower panels show the EURUSD options RMSEs when the joint model is estimated. The improvement is evident in the period just before the financial crisis.

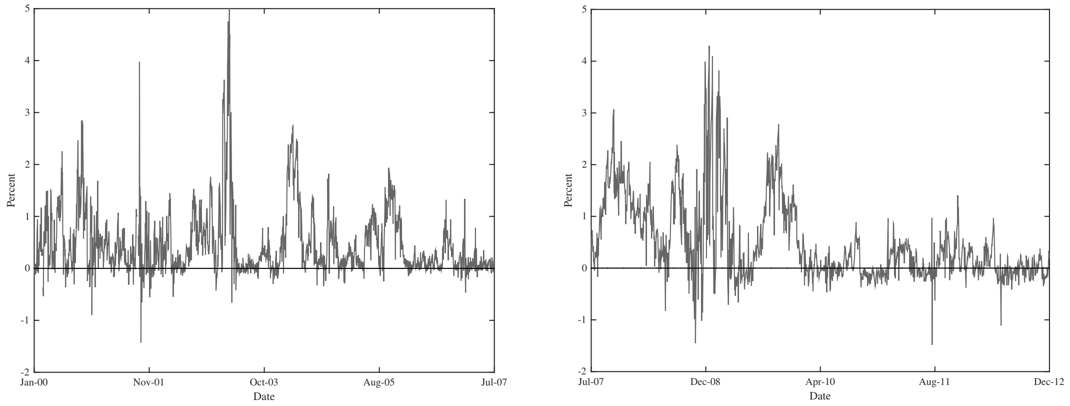

Figure I.14. The time series of differences in oil option RMSEs for the two specifications

The left figure shows the time series of differences in RMSEs for the two model specifications for the first sub-sample. As stated in table I.4, the average difference is 0.485%. Most of the days in the first sub-sample sees an improvement in daily RMSEs. The right figure shows the time series of differences in RMSEs for the two model specifications in the second sub-sample. Table I.4 gives the the average difference as 0.496%. From mid 2007 to end 2009, the average improvement is very high and relatively smaller from 2010 to 2012.

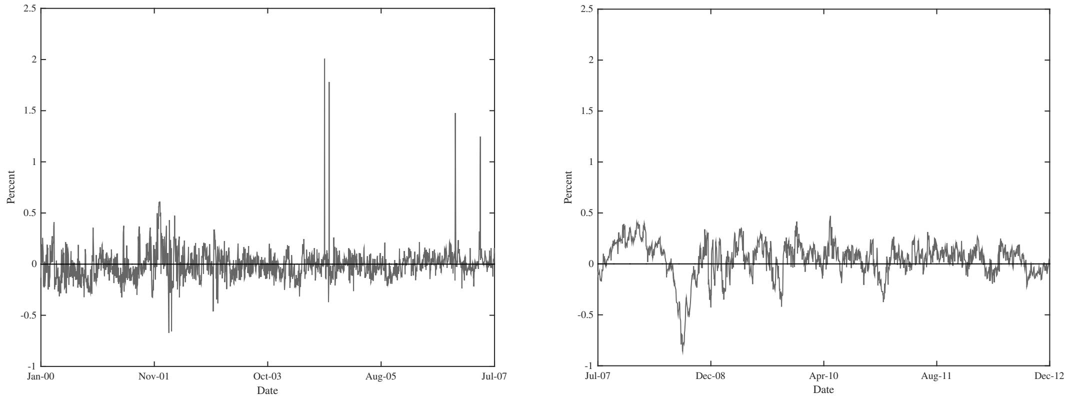

Figure I.15.

The time series of difference in EURUSD option RMSEs for the two specifications

The left figure shows the time series of differences in RMSEs for the two model specifications for the first sub-sample. As stated in table I.4, the fits are almost the same and the daily differences fluctuate around 0. The right figure shows the time series of differences in RMSEs for the two model specifications in the second sub-sample. Table I.4 gives the the average difference as 0.043%. There is a general picture of an improvement in fit, although there is deterioration of fit just as the financial crisis starts.

0.89% in the separate model and from 0.25% to 0.83% in the joint model. While there is a general improvement of fit in the M1, M2 and Q1 options, the fit to Q2-options deteriorates significantly. In the second sub-sample, pricing errors are generally higher. For oil options they range from 1.01% to 3.48% in the separate model and from 0.82% to 2.51% in the joint model and for EURUSD options, the MAEs ranges from 0.54% to 1.17% in the separate model and 0.47% to 1.13% in the joint model. Again, there is generally an overall improvement in oil options MAEs when the joint volatility factor is added, as well an improvement in the EURUSD options MAE except for the longer dated options.

Tables I.6 and I.8 compare the two models in terms of their ability to price options for each combination of moneyness and maturity. The tables report the mean differences in absolute pricing errors and associated t-statistics for the Diebold and Mariano (1995) test. For the first sub-sample, there is a significant improvement in the fit of oil options, when the extra volatility factor is added. The improvement is substantial for the shortest oil option and for the longer dated oil options. Table I.4 reported a very small, insignificant deterioration in overall EURUSD pricing performance, when adding the joint volatility factor. If EURUSD options are considered across maturities, there is in fact a significant improvement in the shorter OTM EURUSD options and the M2 ATM EURUSD option, whereas the options on the Q2 EURUSD futures are fitted significantly worse. For the second sub-sample, a similar picture shows for the oil options; a significant improvement for the shortest options and for longer dated options. For the EURUSD options, Table I.4 reported an overall positive, but not significant difference in the EURUSD pricing performance, when adding the joint volatility factor. Considered across maturities, there is a significant improvement in the shorter options and a much smaller difference in fit of the Q2 options.

I.5.3 Reconstruction of correlation behaviour

Finally, I consider if the estimated model is able to replicate the empirical behaviour of correlations during the second sub-sample. The imposed volatility structure will result in a correlation between the spot oil price and spot EURUSD rate that depends on the joint volatility factor. Figure I.17 shows the theoretical correlation given by (I.9) using the latent volatility backed out from the estimation along with the rolling correlation of front month oil futures and the spot exchange rate. The rolling correlation based on the nearby futures contract and further, a rolling correlation is a different measure than the spot correlation,

		Oil						FX			
Moneyness	Model	M1	M2	M3	M4	M5	M6	M1	M2	Q1	Q2
0.90-0.94	Separate	2.10	1.21	0.99	1.14	1.31	1.56	NaN	NaN	NaN	0.70
	Joint	1.48	1.10	1.03	0.91	0.79	0.82	NaN	NaN	NaN	0.83
0.94-0.98	Separate	1.98	1.09	0.83	1.01	1.20	1.44	0.89	0.39	0.33	0.30
	Joint	1.27	0.91	0.80	0.71	0.58	0.61	0.72	0.25	0.27	0.36
0.98-1.02	Separate	2.05	1.13	0.79	0.95	1.21	1.41	0.61	0.46	0.51	0.36
	Joint	1.31	0.89	0.73	0.60	0.55	0.63	0.62	0.35	0.45	0.40
1.02-1.06	Separate	2.02	1.11	0.73	0.94	1.22	1.42	0.63	0.41	0.36	0.29
	Joint	1.31	0.93	0.70	0.61	0.58	0.71	0.48	0.30	0.26	0.35
1.06-1.10	Separate	2.17	1.21	0.81	1.01	1.32	1.50	NaN	NaN	NaN	0.46
	Joint	1.41	1.01	0.78	0.70	0.70	0.84	NaN	NaN	NaN	0.57

The table presents the Mean Absolute Pricing Errors as defined by the difference of fitted and actual log-normal implied volatilities. Numbers are reported for each of the model specifications for the period January 4, 2000 to July 20, 2007. Missing values are due to lack of options data in that moneyness-maturity combination.

Table I.5. Mean Absolute Errors across moneyness and maturities (2000-2007)

	Oil						FX			
Moneyness	M1	M2	M3	M4	M5	M6	M1	M2	Q1	Q2
0.90-0.94	0.62^{***}	0.11	-0.04	0.23^{***}	0.52^{***}	0.74^{***}	NaN	NaN	NaN	-0.12
	3.01	1.29	-2.27	4.22	5.10	5.18	NaN	NaN	NaN	-40.43
0.94-0.98	0.71^{***}	0.19^{**}	0.03^{**}	0.30^{***}	0.62^{***}	0.83^{***}	0.17^{***}	0.14^{***}	0.06^{***}	-0.06
	4.17	2.10	2.08	3.84	4.73	4.64	26.40	14.93	6.97	-3.14
0.98-1.02	0.74^{***}	0.24^{***}	0.06^{***}	0.35^{***}	0.65^{***}	0.78^{***}	-0.01	0.11^{***}	0.06	-0.04
	4.99	3.10	3.56	3.19	3.45	3.48	-3.55	6.33	1.05	-2.58
1.02-1.06	0.71^{***}	0.18^{***}	0.03	0.33^{***}	0.64^{***}	0.71^{***}	0.15^{***}	0.11^{***}	0.09^{***}	-0.05
	4.16	2.59	1.53	2.99	3.07	2.81	17.54	9.52	8.29	-4.14
1.06-1.10	0.76^{***}	0.20^{**}	0.03	0.31^{***}	0.62^{***}	0.66^{**}	NaN	NaN	NaN	-0.11
	3.44	2.25	0.92	4.07	3.08	2.57	NaN	NaN	NaN	-11.65

The table compares the two model's ability to price options within each combination of moneyness and maturity. The differences in MAEs are given and the it reports the mean differences in absolute pricing errors between the two specifications, when they are estimated on the entire data set. Underneath the difference in MAE are the t-statistics for the Diebold-Mariano test of difference in pricing errors. Each statistic is computed on the basis of a maximum of 1499 daily observations from January 4, 2000 to July 20, 2007. $*$, $**$, $***$ identifies significance at the 10%, 5% and 1% levels.

Table I.6. Test for improvement in errors (2000-2007)

		Oil						FX			
Moneyness	Model	M1	M2	M3	M4	M5	M6	M1	M2	Q1	Q2
0.90-0.94	Separate	2.57	1.31	1.24	1.61	1.91	2.08	0.96	1.05	0.96	0.70
	Joint	2.04	1.60	1.45	1.42	1.35	1.30	1.06	1.01	0.90	0.73
0.94-0.98	Separate	2.50	1.27	1.11	1.43	1.74	1.95	0.90	0.55	0.72	0.68
	Joint	1.62	1.21	1.08	1.05	1.01	0.98	0.83	0.49	0.63	0.77
0.98-1.02	Separate	2.79	1.43	1.01	1.25	1.56	1.77	0.91	0.57	0.75	0.58
	Joint	1.61	1.16	0.91	0.82	0.81	0.82	0.87	0.47	0.63	0.65
1.02-1.06	Separate	3.12	1.67	1.03	1.13	1.41	1.69	1.06	0.85	0.80	0.54
	Joint	2.06	1.53	1.15	0.91	0.84	0.87	0.95	0.70	0.68	0.52
1.06-1.10	Separate	3.48	2.02	1.28	1.17	1.37	1.59	1.17	1.07	1.10	0.87
	Joint	2.51	1.99	1.55	1.25	1.12	1.08	0.91	0.99	1.13	0.82

The table presents the Mean Absolute Pricing Errors as defined by the difference of fitted and actual log-normal implied volatilities. Numbers are reported for each of the model specifications for the period July 21, 2007 to December 31, 2012.

Table I.7. Mean Absolute Errors across moneyness and maturities (2007-2012)

	Oil						FX			
Moneyness	M1	M2	M3	M4	M5	M6	M1	M2	Q1	Q2
0.90-0.94	0.53^{*}	−0.30	−0.20	0.20^{***}	0.56^{***}	0.79^{***}	−0.09	0.04^{***}	0.06^{***}	−0.03
	1.89	−2.11	−2.90	2.90	3.46	3.22	0.46	4.56	9.21	−2.40
0.94-0.98	0.88^{***}	0.06	0.04	0.39^{***}	0.73^{***}	0.97^{***}	0.07^{***}	0.06^{***}	0.09^{***}	−0.09
	2.86	0.23	0.68	3.36	3.49	3.34	7.21	3.15	4.27	−2.60
0.98-1.02	1.18^{***}	0.27^{**}	0.10^{**}	0.44^{***}	0.75^{***}	0.95^{***}	0.05^{*}	0.10^{***}	0.12^{***}	−0.07
	3.79	2.37	2.45	3.27	3.48	3.29	1.82	4.82	2.95	−1.97
1.02-1.06	1.07^{***}	0.15^{**}	−0.12	0.23^{**}	0.57^{***}	0.82^{***}	0.11^{***}	0.15^{***}	0.12^{***}	0.02
	3.91	1.98	−1.29	2.23	3.16	3.20	5.29	4.94	5.01	0.45
1.06-1.10	0.98^{***}	0.03	−0.26	−0.08	0.25^{**}	0.51^{***}	0.25^{***}	0.08^{***}	−0.03	0.05
	3.35	1.06	−2.30	0.31	1.98	2.58	11.98	14.99	−0.68	1.48

The table compares the two model's ability to price options within each combination of moneyness and maturity. The differences in MAEs are given and the it reports the mean differences in absolute pricing errors between the two specifications, when they are estimated on the entire data set. Underneath the difference in MAE are the t-statistics for the Diebold-Mariano test of difference in pricing errors. Each statistic is computed on the basis of a maximum of 1373 daily observations from July 21, 2007 to December 31, 2012. $*$, $**$, $***$ identifies significance at the 10%, 5% and 1% levels.

Table I.8. Test for improvement in errors (2007-2012)

but the picture confirms that the joint model with correlation driven by the joint volatility factor yields a returns correlation level of magnitude similar to the empirical correlation.

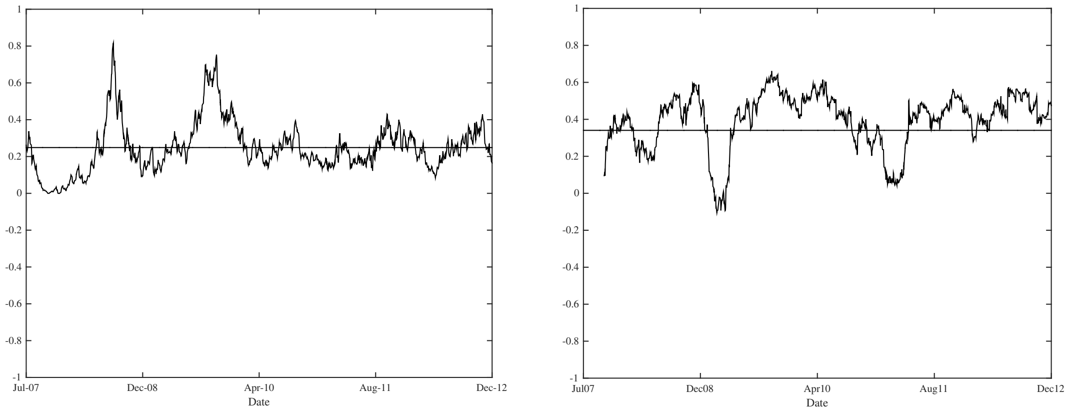

Figure I.16. Model backed-out correlation vs. empirical correlation of short contracts

The left figure shows the theoretical spot correlation in the joint model. The right figure shows the 3 month rolling correlation of spot EURUSD exchange rate and front month futures (with more than ten days to maturity) returns. The models average level of correlation is 0.25, whereas the average empirical correlation of spot EURUSD rate and front month futures of 0.32.

In Figure I.17, the rolling correlation of the model implied volatilities are show alongside the rolling correlation of straddle returns for the second sub-sample. The model implied correlation of volatility matches the level and largely matches the dynamics over time.

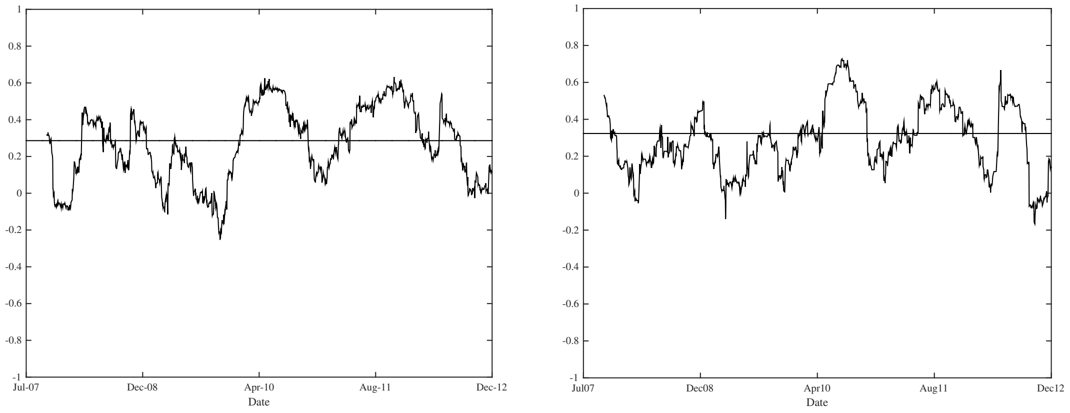

Figure I.17. Model correlation vs. empirical correlation of volatility

The left figure shows the model implied correlation of volatility changes in the joint model. The right figure shows the 3 month rolling correlation of 1 month straddle returns (equivalent to Figure I.6). The average level and the general pattern is the same in the two figures.

I.6 Conclusion

Several studies argue that a change in the relationship between commodity markets and financial markets has occurred after 2004 and other studies document a stronger relationship during and after the financial crisis. In this paper, the correlation between the market implied volatility in crude oil and the market implied volatility of the EURUSD rate has been studied. Using first a model-free approach, straddle returns are computed for crude oil and for EURUSD. The rolling correlation of short straddles increased in level around 2007. A Principal Components Analysis showed that a the variation across straddles was driven by joint factors in the period 2007-2012, but not in the period 2000-2007.

Using a term structure model for futures and options, a model including a joint volatility factor is estimated. In line with the conclusions from the model free analysis, there is improvement in options price fit for both oil options and EURUSD options for the period 2007-2012. When the same model was fitted to the period 2000-2007, the extra volatility factor mainly served to improve the fit of oil options while leaving the fit of EURUSD options largely unchanged.

Although the improvement in EURUSD options was not significant using a Diebold-Mariano test, the fit was improved around 5% on average compared to the separate model. Across moneyness and maturities, the fit was improved for EURUSD options on shorter futures and the results could potentially be strengthened by using option surfaces of equal size, such that an improvement in the fit of oil options is not over-emphasized compared to an improvement in the fit of EURUSD options. Further the liquidity of the Q2-futures is very low compared to the Q1-futures.

In this paper, there is no attempt to explain whether the relation between oil volatility and EURUSD volatility is a crisis phenomenon or the result of financialization. My analysis ends in 2012, which is not sufficiently long after the crisis to determine, if the volatility relation is persistent. I leave the explanation of the existence of the joint volatility factor to future research.

Appendix

I.A Data Description

Figures I.18 and I.19 show the returns of the futures data. The magnitude of daily EURUSD futures returns is much smaller than for oil futures. During the financial crisis, returns data show more volatility. There is a large downwards spike in oil futures contracts at the 9/11 terrorist attack, but the event does not stand out in the EURUSD futures contracts.

Tables I.9 and I.10 shows the number of options after the initial sorting.During the 3260 trading days, oil options with an open interest of more than 100 contracts exists across all moneyness-intervals and maturities, except for short out-of the money options and some of the longer dated contracts. For EURUSD options on the other hand, there is limited availability. For the M1, Q1 and Q2 contracts, almost all days have data for the ATM options. For slightly OTM options, the three shortest maturities have prices up to 30% of the time, whereas the Q2 contract have around 80% of the time. The choice of five moneyness-intervals and six respectively four maturities was made in order to include a reasonable option surface to represent market volatility across time and moneyness, while at the same time ensuring that the number of options were not too uneven.

Tables I.11 and I.12 show the average implied volatility after the initial sorting. The average value is between $0.30 - 0.55$ for oil options and $0.10 - 0.30$ for the EURUSD options. As indicated from the returns plot, the implied volatility of oil is much higher for shorter dated options. Further, a for each maturity a smirk is observed, i.e., the volatility for OTM puts are higher than for OTM calls. The degree of the smirk is diminishing with increasing time to maturity. The EURUSD implied ATM volatility is almost flat for all maturities, but a bigger smile is seen for shorter dated options, although values for far OTM options are based on very few observations and thereby not very good representation of the entire period.

Tables I.13 and I.14 show the average option prices. As expected, OTM options are less expensive as are shorter dated options.

Finally, I.15 and I.16 shows the average open interest. The depth of the oil options market is larger than that of the EURUSD market[13]. For oil options, the general picture is that there are more open interest in OTM options than for ATM option. This is most likely because options closer to a moneyness level of 1 could have been in the money (ITM) and thereby being exercised prematurity. For FX options the picture is somewhat the same for the middle moneyness intervals, but dips to low levels for the far OTM options. As very few observations were available, the average open interest is not representative for OTM options.

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[13] It would be more correct to compare notional values, but as a EURUSD futures is for 125,000 EUR and an oil futures is for 1,000 barrels of oil, the difference in value of contracts will not result in a similar depth market if measured in value rather than number of contracts.

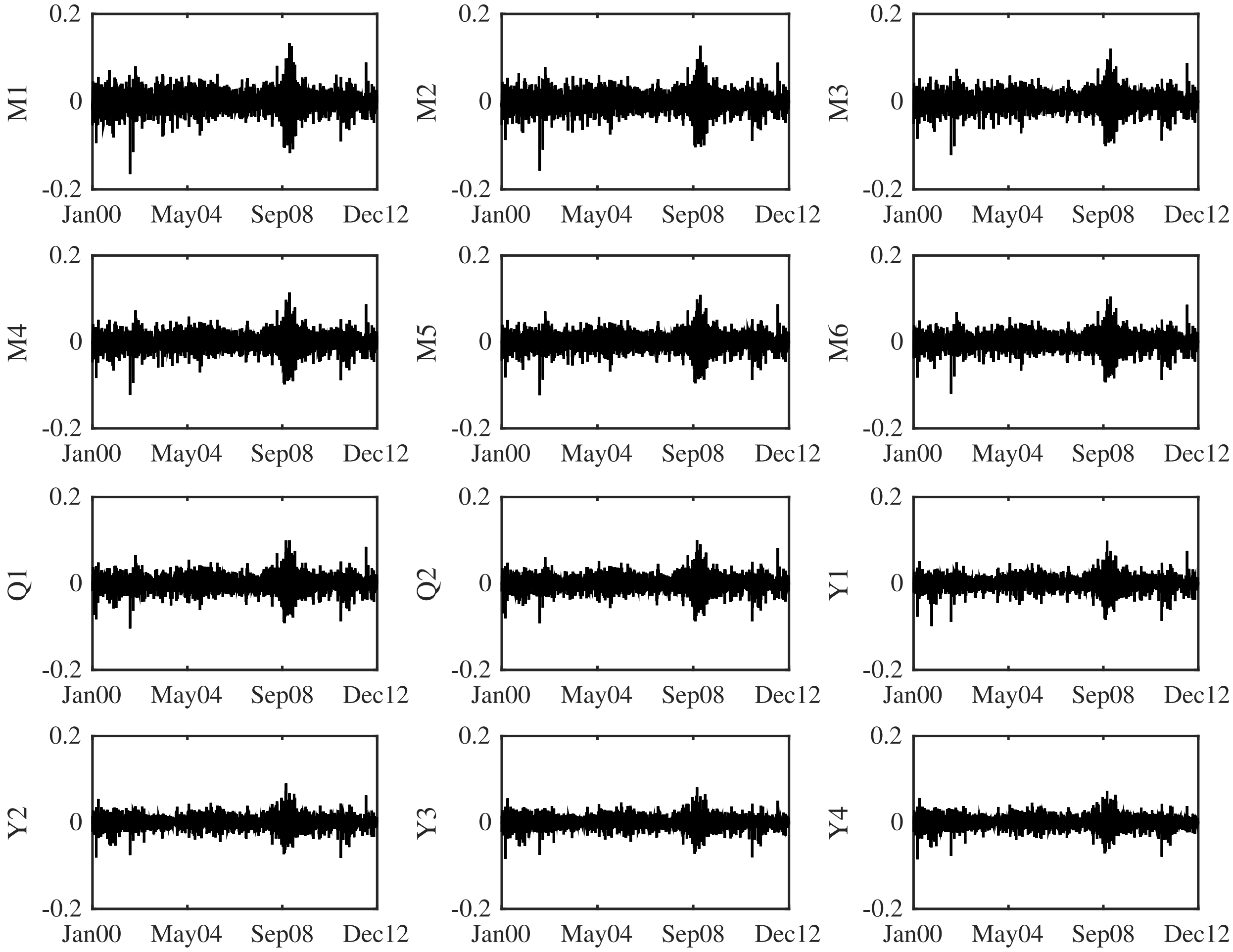

Figure I.18. Oil futures returns

Daily log returns of the chosen oil futures contracts. The returns exhibit more variation during the financial crisis. The large negative spike in the beginning is the 9/11 terrorist attack, which affected the oil markets quite significantly. Also, the Samuelson effect shows as the shorter contracts are much more fluctuating than the longer contracts.

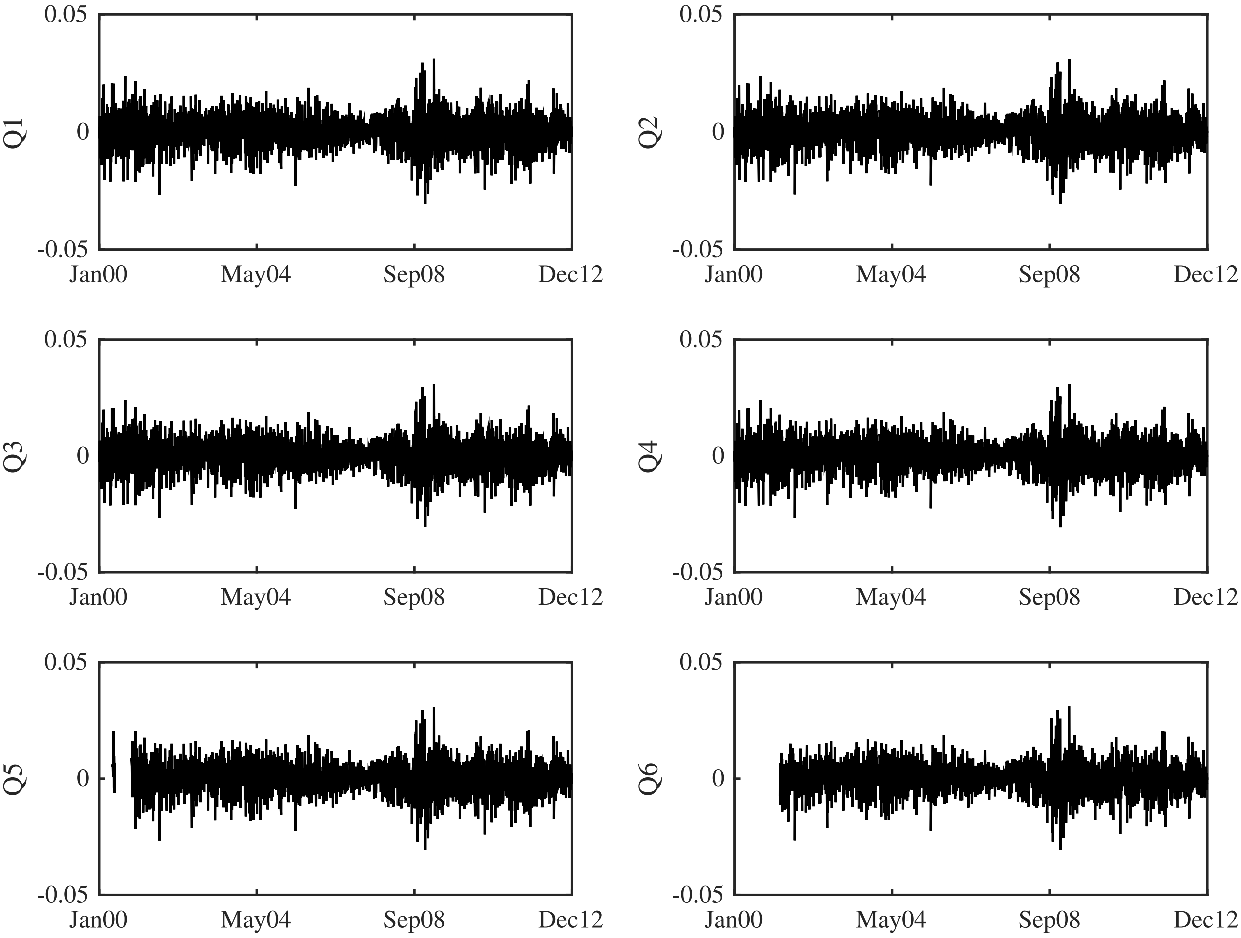

Figure I.19. EURUSD futures returns

Daily log returns of the chosen EURUSD futures contracts. The returns exhibit many spikes and show more variation during the financial crisis. The magnitude of returns is fairly equal across maturities.

	Moneyness	M1	M2	M3	M4	M5	M6	Q1	Q2	All
	0.78-0.82	1344	3057	3182	3120	3040	2864	2828	2180	21615
	0.82-0.86	2264	3229	3225	3166	3130	2944	2888	2282	23128
Puts	0.86-0.90	2989	3246	3244	3208	3125	2965	2992	2318	24087
	0.90-0.94	3234	3251	3240	3221	3147	3091	3032	2375	24591
	0.94-0.98	3235	3248	3237	3203	3155	3082	3059	2362	24581
	0.98-1.02	3239	3252	3253	3235	3219	3164	3118	2487	24967
	1.02-1.06	3235	3250	3231	3181	3077	2858	2941	2230	24003
	1.06-1.10	3235	3249	3216	3108	2926	2821	2809	2020	23384
Calls	1.10-1.14	3021	3240	3221	3043	2779	2537	2661	1885	22387
	1.14-1.18	2456	3188	3097	2833	2591	2303	2503	1758	20729
	1.18-1.22	1769	3037	2997	2662	2390	2051	2277	1643	18826
	All	30021	35247	35143	33980	32579	30680	31108	23540	252298

Table I.9. Number of oil option observations from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	Q1	Q2	Q3	Q4	All
	0.78-0.82	NaN	NaN	NaN	1	1	NaN	2
	0.82-0.86	NaN	NaN	3	9	14	7	33
Puts	0.86-0.90	NaN	3	24	233	306	39	605
	0.90-0.94	20	65	129	881	662	150	1907
	0.94-0.98	318	800	957	2647	1510	305	6537
	0.98-1.02	2822	2158	3103	3047	1623	673	13426
	1.02-1.06	264	678	974	2682	1261	488	6347
	1.06-1.10	18	49	92	721	509	132	1521
Calls	1.10-1.14	NaN	3	22	141	60	22	248
	1.14-1.18	NaN	NaN	4	66	11	NaN	81
	1.18-1.22	NaN	NaN	2	18	7	NaN	27
	All	3442	3756	5310	10446	5964	1816	30734

Table I.10.

Number of EURUSD option observations from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	M3	M4	M5	M6	Q1	Q2	All
	0.78-0.82	0.53	0.43	0.41	0.39	0.38	0.37	0.36	0.34	0.39
	0.82-0.86	0.47	0.42	0.40	0.38	0.37	0.36	0.35	0.34	0.38
Puts	0.86-0.90	0.43	0.40	0.39	0.38	0.36	0.35	0.34	0.33	0.38
	0.90-0.94	0.41	0.39	0.38	0.37	0.36	0.35	0.34	0.32	0.37
	0.94-0.98	0.39	0.38	0.37	0.36	0.35	0.34	0.33	0.31	0.36
	0.98-1.02	0.38	0.37	0.36	0.35	0.35	0.34	0.32	0.31	0.35
	1.02-1.06	0.38	0.37	0.36	0.35	0.34	0.33	0.32	0.31	0.35
	1.06-1.10	0.38	0.37	0.36	0.35	0.34	0.33	0.32	0.31	0.35
Calls	1.10-1.14	0.39	0.37	0.36	0.35	0.34	0.33	0.32	0.31	0.35
	1.14-1.18	0.41	0.37	0.36	0.35	0.34	0.33	0.32	0.30	0.35
	1.18-1.22	0.44	0.38	0.37	0.36	0.35	0.34	0.32	0.31	0.36
	All	0.41	0.39	0.37	0.36	0.35	0.34	0.33	0.32	0.36

Table I.11.

Average Implied Volatility of Oil Options from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	Q1	Q2	Q3	Q4	All
	0.78-0.82	NaN	NaN	NaN	0.28	0.20	NaN	0.24
	0.82-0.86	NaN	NaN	0.30	0.23	0.20	0.18	0.21
Puts	0.86-0.90	NaN	0.29	0.25	0.19	0.16	0.15	0.18
	0.90-0.94	0.22	0.22	0.20	0.16	0.14	0.11	0.15
	0.94-0.98	0.16	0.14	0.14	0.12	0.11	0.10	0.12
	0.98-1.02	0.11	0.11	0.11	0.11	0.10	0.10	0.11
	1.02-1.06	0.16	0.13	0.13	0.11	0.10	0.10	0.12
	1.06-1.10	0.22	0.22	0.20	0.14	0.12	0.11	0.14
Calls	1.10-1.14	NaN	0.23	0.24	0.19	0.14	0.13	0.18
	1.14-1.18	NaN	NaN	0.28	0.21	0.18	NaN	0.21
	1.18-1.22	NaN	NaN	0.30	0.23	0.20	NaN	0.23
	All	0.12	0.12	0.12	0.12	0.11	0.10	0.12

Table I.12.

Average Implied Volatility of EURUSD Options from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	M3	M4	M5	M6	Q1	Q2	All
	0.78-0.82	0.24	0.37	0.64	0.92	1.20	1.45	1.95	2.69	1.18
	0.82-0.86	0.27	0.56	0.95	1.30	1.64	1.93	2.50	3.42	1.53
Puts	0.86-0.90	0.37	0.87	1.37	1.80	2.18	2.54	3.17	4.17	1.98
	0.90-0.94	0.60	1.33	1.95	2.47	2.91	3.31	3.96	4.99	2.60
	0.94-0.98	1.08	2.01	2.72	3.30	3.82	4.21	4.89	5.97	3.40
	0.98-1.02	2.84	3.89	4.62	5.17	5.60	5.95	6.77	7.80	5.25
	1.02-1.06	1.69	2.73	3.46	4.01	4.48	4.84	5.70	6.81	4.08
	1.06-1.10	0.97	1.87	2.54	3.06	3.53	3.85	4.66	5.77	3.11
Calls	1.10-1.14	0.60	1.27	1.86	2.35	2.77	3.12	3.81	4.96	2.42
	1.14-1.18	0.43	0.88	1.36	1.81	2.18	2.54	3.18	4.14	1.93
	1.18-1.22	0.36	0.63	1.02	1.44	1.74	2.09	2.69	3.55	1.59
	All	0.96	1.51	2.06	2.54	2.96	3.33	4.00	5.01	2.70

Table I.13. Average Oil Option Price from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	Q1	Q2	Q3	Q4	All
	0.78-0.82	NaN	NaN	NaN	0.0104	0.0106	NaN	0.0105
	0.82-0.86	NaN	NaN	0.0103	0.0139	0.0133	0.0144	0.0134
Puts	0.86-0.90	NaN	0.0113	0.0106	0.0131	0.0150	0.0159	0.0141
	0.90-0.94	0.0112	0.0122	0.0112	0.0134	0.0171	0.0160	0.0147
	0.94-0.98	0.0112	0.0127	0.0116	0.0150	0.0206	0.0247	0.0158
	0.98-1.02	0.0151	0.0210	0.0198	0.0355	0.0382	0.0444	0.0260
	1.02-1.06	0.0145	0.0148	0.0157	0.0192	0.0221	0.0262	0.0191
	1.06-1.10	0.0125	0.0155	0.0158	0.0154	0.0144	0.0164	0.0152
Calls	1.10-1.14	NaN	0.0114	0.0143	0.0163	0.0127	0.0117	0.0148
	1.14-1.18	NaN	NaN	0.0131	0.0143	0.0127	NaN	0.0140
	1.18-1.22	NaN	NaN	0.0112	0.0136	0.0121	NaN	0.0130
	All	0.0146	0.0179	0.0172	0.0219	0.0244	0.0307	0.0208

Table I.14. EURUSD Option Average Price from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	M3	M4	M5	M6	Q1	Q2	All
	0.78-0.82	4267	4133	3485	2947	2449	2199	3124	2436	3078
	0.82-0.86	4681	3970	3133	2784	2388	2223	2746	2390	3015
Puts	0.86-0.90	5075	3811	3207	2720	2372	1893	2647	2040	3003
	0.90-0.94	5118	3665	2947	2353	2152	1637	2251	1690	2776
	0.94-0.98	4606	3101	2448	2136	1703	1348	1874	1520	2384
	0.98-1.02	3452	2436	1887	1541	1329	1041	1524	1339	1838
	1.02-1.06	3834	2805	2141	1863	1406	1348	1739	1535	2128
	1.06-1.10	3764	2811	2277	2031	1615	1358	1988	1793	2254
Calls	1.10-1.14	3680	2612	2337	2132	1900	1560	2194	1961	2339
	1.14-1.18	3273	2854	2258	2221	2007	1774	2586	2008	2398
	1.18-1.22	3346	2888	2377	2488	2214	1959	2451	2250	2498
	All	4123	3186	2592	2289	1952	1653	2269	1886	2514

Table I.15. Average Oil Option Open Interest from January 1 2000 to December 31 2012.

	Moneyness	M1	M2	Q1	Q2	Q3	Q4	All
	0.78-0.82	NaN	NaN	NaN	118	301	NaN	210
	0.82-0.86	NaN	NaN	330	159	179	221	196
Puts	0.86-0.90	NaN	267	658	1344	282	233	703
	0.90-0.94	336	246	710	759	251	221	515
	0.94-0.98	923	417	1086	473	276	219	520
	0.98-1.02	635	307	820	333	242	256	490
	1.02-1.06	471	359	730	436	249	232	422
	1.06-1.10	161	288	487	427	329	372	385
Calls	1.10-1.14	NaN	126	399	382	353	232	360
	1.14-1.18	NaN	NaN	245	314	266	NaN	304
	1.18-1.22	NaN	NaN	118	194	334	NaN	224
	All	645	338	839	460	264	248	481

Table I.16.

Average EURUSD Option Open Interest from January 1 2000 to December 31 2012.

I.B Kalman Filter

To calculate the likelihood function, the Extended Kalman Filter is applied. To ease the notation, we assume that $\Delta t = 1$. Let $\hat{\theta}_t = E_t(\theta_t)$ and $\hat{\theta}_{t|t-1} = E_{t-1}(\theta_t)$ denote the predictors of the states (with and without the observations z_t) and P_t and $P_{t|t-1}$ denotes the estimation error covariance matrices. The measurement equation (I.15) is linearised around the one-period ahead predicted state, $\hat{\theta}_{t|t-1}$ and the measurement errors are approximated by a Gaussian distribution:

$$z_t = h(\hat{\theta}_{t|t-1}) - H_t'\hat{\theta}_{t|t-1} + H_t'\theta_t + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \Omega) \tag{I.21}$$

where

$$H_t' = \left.\frac{\partial h'(\theta_t)}{\partial \theta_t'}\right|_{\theta_t = \hat{\theta}_{t|t-1}}. \tag{I.22}$$

In this setting the predictors and the error estimation covariance matrices are

$$\hat{\theta}_{t|t-1} = c + T_1\hat{\theta}_{t-1} \tag{I.23}$$

$$P_{t|t-1} = T_1 P_{t-1} T_1' + Q_t \tag{I.24}$$

$$\hat{\theta}_t = \hat{\theta}_{t|t-1} + P_{t|t-1} H_t' F_t^{-1} \nu_t \tag{I.25}$$

$$P_t = P_{t|t-1} - P_{t|t-1} H_t' F_t^{-1} H_t P_{t|t-1} \tag{I.26}$$

The log-likelihood function is given by

$$2 \log L = -\log 2\pi \sum_{i=1}^{T} N_t - \sum_{i=1}^{T} \log |F_t| - \sum_{i=1}^{T} \nu_t' F_t^{-1} \nu_t, \tag{I.27}$$

where N_t is the number of observations at time t, T is the number of observation dates and

$$\nu_t = y_t - h(\hat{\theta}_{t|t-1}), \tag{I.28}$$

$$F_t = H_t' P_{t|t-1} H_t' + \Omega. \tag{I.29}$$

I.B.1 Details of state equation

By looking at the structure of $\mathcal{K}$, it follows that υ_t^E, υ_t^X and υ_t^J has dynamics and conditional mean:

$$\begin{aligned} d\upsilon_t^j &= \left(\eta^j - \kappa_j \upsilon_t^j\right) dt + \sigma_{\upsilon^j}\sqrt{\upsilon_t^j}\, dZ_t^j \\ E_t(\upsilon_s) &= e^{-\kappa_j(s-t)}\upsilon_t^j + \int_t^s e^{-\kappa_j(s-u)}\eta^j du = e^{-\kappa_j(s-t)}\upsilon_t^j + \frac{\eta^j}{\kappa_j}\left(1 - e^{-\kappa_j(s-t)}\right) \end{aligned} \tag{I.30}$$

Using (I.9), (I.10), (I.11), (I.5), and (I.6), the state equations' terms are collected:

$$d\theta_t = (\Phi - \mathcal{K}\theta_t)\,dt + \sqrt{\upsilon_t^E} R_E \begin{pmatrix} dW_t^E \\ dB_t^E \\ dZ_t^E \end{pmatrix} + \sqrt{\upsilon_t^X} R_X \begin{pmatrix} dW_t^X \\ dB_t^X \\ dZ_t^X \end{pmatrix} + \sqrt{\upsilon_t^J} R_J \begin{pmatrix} dW_t^J \\ dZ_t^J \end{pmatrix}, \tag{I.31}$$

where the R-terms are matrices consisting of 0's, 1's and σ's.

The conditional mean of $\theta_{t+\Delta t}$ given θ_t is:

$$E_t(\theta_{t+\Delta t}) = \underbrace{e^{-\mathcal{K}\Delta t}}_{T_{\Delta t}}\theta_t + \underbrace{e^{-\mathcal{K}\Delta t}\int_0^{\Delta t} e^{\mathcal{K}u}du \cdot \Phi}_{c_{\Delta t}} \tag{I.32}$$

Both $T_{\Delta t}$ and $c_{\Delta t}$ are easily computed. The conditional covariance of θ_t has the following form

$$Cov_t(\theta_{t+\Delta t}) = Q_0 + Q^E\upsilon_t^E + Q^X\upsilon_t^X + Q^J\upsilon_t^J$$

and is given by:

$$\begin{aligned}
Cov_t(\theta_{t+\Delta t}) = &\left[\int_t^{t+\Delta t} E_t\left(\upsilon_u^E\right) e^{-\mathcal{K}(t+\Delta t-u)} R_E\Sigma_E R_E' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \left[\int_t^{t+\Delta t} E_t\left(\upsilon_u^X\right) e^{-\mathcal{K}(t+\Delta t-u)} R_X\Sigma_X R_X' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \left[\int_t^{t+\Delta t} e^{-\mathcal{K}(t+\Delta t-u)} R_J\Sigma_J R_J' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
=&\upsilon_t^E \left[\int_t^{t+\Delta t} e^{-\kappa_E(u-t)} e^{-\mathcal{K}(t+\Delta t-u)} R_E\Sigma_E R_E' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \upsilon_t^X \left[\int_t^{t+\Delta t} e^{-\kappa_X(u-t)} e^{-\mathcal{K}(t+\Delta t-u)} R_X\Sigma_X R_X' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \upsilon_t^J \left[\int_t^{t+\Delta t} e^{-\kappa_J(u-t)} e^{-\mathcal{K}(t+\Delta t-u)} R_J\Sigma_J R_J' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \left[\int_t^{t+\Delta t} \frac{\eta^E}{\kappa_E}\left(1-e^{-\kappa_E(u-t)}\right) e^{-\mathcal{K}(t+\Delta t-u)} R_E\Sigma_E R_E' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \left[\int_t^{t+\Delta t} \frac{\eta^X}{\kappa_X}\left(1-e^{-\kappa_X(u-t)}\right) e^{-\mathcal{K}(t+\Delta t-u)} R_X\Sigma_X R_X' e^{-\mathcal{K}'(t+\Delta t-u)}du\right] \\
&+ \left[\int_t^{t+\Delta t} \frac{\eta^J}{\kappa_J}\left(1-e^{-\kappa_J(u-t)}\right) e^{-\mathcal{K}(t+\Delta t-u)} R_J\Sigma_J R_J' e^{-\mathcal{K}'(t+\Delta t-u)}du\right]
\end{aligned}$$

where Σ_j is the covariance matrices for the Brownian motions.

I.B.2 Details of the measurement equation

For the futures contracts the corresponding rows of the H_t matrix is straightforward (and corresponds to a standard Kalman filter). For the options, the price is expressed in terms of an integral, so both the price and the derivative must be approximated by a sum. To ease the notation, subscripts indicating asset are removed:

$$\begin{aligned}
\mathcal{P}(t,T_0,T_1,K)P(t,T_0)^{-1} \approx\ & KG_{0,1}(\log K) - G_{1,1}(\log K) \\
=& K\left\{\frac{\chi(0,t,T_0,T_1)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\texttt{Im}\left[\chi(\imath u,t,T_0,T_1)e^{-\imath u\log K}\right]}{u}du\right\} \\
&-\left\{\frac{\chi(1,t,T_0,T_1)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\texttt{Im}\left[\chi(1+\imath u,t,T_0,T_1)e^{-\imath u\log K}\right]}{u}du\right\} \\
\approx& K\left\{\frac{\chi(0,t,T_0,T_1)}{2} - \frac{1}{\pi}\sum_{i=1}^{M}\texttt{Im}\left[\chi(\imath u_i,t,T_0,T_1)e^{-\imath u_i\log K}\right]\frac{w_i}{u_i}\right\} \\
&-\left\{\frac{\chi(1,t,T_0,T_1)}{2} - \frac{1}{\pi}\sum_{i=1}^{M}\texttt{Im}\left[\chi(1+\imath u_i,t,T_0,T_1)e^{-\imath u_i\log K}\right]\frac{w_i}{u_i}\right\} \\
=& K\left\{\frac{1}{2} - \frac{1}{\pi}\sum_{i=1}^{M}\texttt{Im}\left[e^{A_i+B_i\imath}\right]\frac{w_i}{u_i}\right\} - \left\{\frac{F_t(T_1)}{2} - \frac{1}{\pi}\sum_{i=1}^{M}\texttt{Im}\left[e^{C_i+D_i\imath}\right]\frac{w_i}{u_i}\right\} \\
=& K\left\{\frac{1}{2} - \frac{1}{\pi}\sum_{i=1}^{M}e^{A_i}\sin(B_i)\frac{w_i}{u_i}\right\} - \left\{\frac{F_t(T_1)}{2} - \frac{1}{\pi}\sum_{i=1}^{M}e^{C_i}\cos(D_i)\frac{w_i}{u_i}\right\}
\end{aligned} \tag{I.33}$$

where u_i denotes the chosen integration points, w_i the corresponding weights and

$$\begin{aligned}
A_i &= M^R_{u=\imath u_i}(T_0-t) + N^{1,R}_{u=\imath u_i}(T_0-t)\upsilon^1_t + N^{2,R}_{u=\imath u_i}(T_0-t)\upsilon^2_t, \\
B_i &= M^I_{u=\imath u_i}(T_0-t) + N^{1,I}_{u=\imath u_i}(T_0-t)\upsilon^1_t + N^{2,I}_{u=\imath u_i}(T_0-t)\upsilon^2_t + u_i\log(\tfrac{F_t(T_1)}{K}), \\
C_i &= M^R_{u=1+\imath u_i}(T_0-t) + N^{1,R}_{u=1+\imath u_i}(T_0-t)\upsilon^1_t + N^{2,R}_{u=1+\imath u_i}(T_0-t)\upsilon^2_t + \log F_t(T_1), \\
D_i &= M^I_{u=1+\imath u_i}(T_0-t) + N^{1,I}_{u=1+\imath u_i}(T_0-t)\upsilon^1_t + N^{2,I}_{u=1+\imath u_i}(T_0-t)\upsilon^2_t + u_i\log(\tfrac{F_t(T_1)}{K}).
\end{aligned}$$

Essay II

How Energy Quanto Options can Hedge Volumetric Risk[*]

Abstract

Many corporations do not have full control over the quantities they sell or purchase and at which price. Their operation therefore includes volumetric risk and in most cases the volumetric risk is not tradable, and sufficient management of risk can thus be difficult to obtain. But when the quantity is closely related to another tradable asset, risk management strategies based on both prices and the correlated asset shows to be superior compared to hedges based on only the price. Although it can be derived for very simple frameworks, closed form solutions for the optimal hedge are not available for the general case. But when a closed form solution is available, the theoretical hedge is not a straightforward function of price and the other traded asset, so finding a counterpart can be difficult. This paper shows that energy quanto options arises naturally in any optimal trading strategy. Through a comparative study, it is shown that a hedge using just one or two energy quanto options do remarkably well compared to the theoretical optimal hedge. The comparative study further illustrates that the choice of energy quanto options depends on the behaviour of the underlying variables.

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II.1 Introduction

Producers and consumers of energy are facing several types of risks. Two key uncertainties are the quantity sold or purchased and the market price. Some businesses might have the option to adjust the quantity in response to the price, but a farmer, a wind park owner or a provider of energy must in many situations just accept both the market price and the quantity produced or demanded. For instance, in the retail and wholesale markets for gas and electricity, typical contracts allows the costumer to consume as much gas or electricity as needed at a pre-determined fixed rate. E.g., for electricity the fixed rate could be set on a monthly, quarterly or annual basis. For the consumer, the only uncertainty faced is therefore the quantity she uses. If the paid rate is not sufficient to cover the purchase or production of her electricity, she is not financially punished nor is she rewarded, if prices are lower than the agreed rate. The provider of electricity – often denoted as the Load Serving Entity (LSE) – faces all the price risk and this price risk is potentially amplified by the uncertainty in demanded quantity. The positive relationship between prices and consumption leads the following stylized problem: When demand is high, the price of electricity is also high, leading to a negative profit margin multiplied with a high quantity, i.e., a loss for the LSE. When quantity and thereby prices are low, the profit margin might be positive, but when selling a low quantity, the revenue for the LSE is low.

The reasons for engaging in risk managements are many and the topic has been addressed frequently in the literature. Without addressing the particular reasons a company might have for hedging certain risks, the focus in this paper is *how* to hedge the risk. The same approach was used in Brown and Toft (2002), who take as given a non-financial firm that hedges using derivatives contracts and focus on the design of an appropriate hedge strategy based on the future price without specifying the functional form of the hedge. Contrary to many other papers, they refrain from assuming that the hedge is made up of e.g., forwards or futures and options, but derive the ”perfect exotic hedge”. They study this in a simple set-up, where a company produces an uncertain quantity of a single good, which is sold at an uncertain price. The company is value-maximizing and faces a deadweight cost, which gives them incentives to hedge. Their derived perfect exotic hedge depends on the correlation between the price and quantity as well as the uncertainty in these together with the risk-aversion as represented in the deadweight costs. This enables the company to hedge some of the otherwise unhedgeable quantity risk by exploiting the correlation of price and quantity.

This approach has been studied in the context of energy markets and the related literature is discussed in more detailed in Section II.2. The challenge when hedging quantity risk is that contrary to the market price risk, there are no derivatives contracts available to target the quantity risk[1] resulting in an incomplete market as long as the correlation is imperfect. But as the link between quantity and weather is well-established for commodity markets (although the nature of this relationship differs across markets and locations depending on the type of commodity considered), a natural extension is to let the perfect hedge depend on not only the price, but also a weather-related index on which financial contracts exist. Id Brik and Roncoroni (2016a) generalise the idea of a perfect exotic hedge to two dimensions and let the hedge be a general function of underlying price and an index that correlates with quantity. The major problem is however, that the hedge strategy has a functional form that depends on density ratios and the conditional mean of the unhedged profit. For some distributional assumptions, the functional form of the hedge is available in closed form, but even for simple settings like the (log-)normal distribution, the functional form is not a straightforward expression. In practice, it would be difficult to approach a counterpart and ask for a such structured deal.

The issue of quantity risk is widely present in agricultural markets. A farmer's unhedged revenue is the quantity at harvest time times the price earned for the quality of corn he produces. Setting aside basis risk, the farmer is exposed to the product of two negatively correlated variables. If total crop production is high, there is generally an over-supply in the market which affects prices negatively and vice versa[2]. The farmer is concerned about low revenue, i.e., that the quantity times price is low[3]. In stylized terms, the risk management problem for the LSE is similar to the risk management problem of farmer making the solutions proposed in the agricultural finance literature relevant to relate to energy markets.

The focus of this paper is to show that energy quanto options – also sometimes referred to as double structure options, double trigger options or weather cross-commodity options – are strong alternatives to the perfect exotic hedge, when engaging in a hedge with an over-the-counter counterpart. Counterparts in such a deal could for instance be re-insurance companies[4]. For a general hedge strategy expressed as a function of two underlyings, the replication result in Carr and Madan (2001) is extended to two dimensions and energy quanto options arise as a natural part of the perfect exotic hedge. Following this, a comparative simulation study is done to show that hedges using just one or two energy quanto option perform remarkably well compared to the perfect exotic hedge as proposed by Id Brik and Roncoroni (2016a).

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[1] Futures contracts related to quantities in the agricultural markets were traded on CBOT during the 90s, but was shortly after introduction delisted again because of low interest.

[2] In real life, basis risk plays a major role as the individual farmer's production might not correlate with total crop production and he might face locational spreads in the price.

[3] The risk management problem of the farmer is basically equivalent to that of the LSE. Using the notation introduced in Section II.3, let $S_t = R - P_t$. The LSE is then worried about low S_tQ_t, where S_t and Q_t are negatively correlated, as P_t and Q_t were positively correlated.

The paper continues as follows: In Section II.2, literature related to dealing with quantity risk is discussed. In Section II.3, the general risk management problem is presented and then simplified to a one-period setting. It is shown that energy quanto options arises in the perfect exotic hedge. Section II.4 contains a simulation study that compares hedges using energy quanto hedge strategies with other both customized and market available strategies. Section II.5 rounds off the paper including a discussion of further analyses.

II.2 Background and related literature

The application of the approach proposed by Brown and Toft (2002) to energy markets was first done by Oum and Oren (2006). Where Brown and Toft (2002) use an assumption of normally distributed price and quantity and employ maximization of profits net of dead weight costs[5], Oum and Oren (2006) solve the hedging problem for a general utility function and separate probability functions for the physical measure and the risk-neutral pricing measure. For a CARA utility function and for a mean-variance approach, the customized hedge can be expressed in terms of the two densities and functions hereof. For the mean-variance approach, the hedge from Oum and Oren (2006) hedge is later presented as Strategy II.3. They illustrate the results of their model for a log-normal price and both normal and log-normal quantity. They discuss the replication using forwards and options and in the later paper, Oum and Oren (2010), the discretization error of the replicating strategy as well as when to optimally enter the hedge is analysed. Also, Näsäkkälä and Keppo (2005) analyse the timing issue, when only using forward contracts.

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[4] As an example, Munich Re mentions this type of contracts in their Topics Risk Insurance in 2014 (see Munich Re (2014)). On http://www.swissre.com/corporate_solutions/weather_risk_solutions_double_trigger.html, Swiss Re also mentions these type of products. Eydeland and Wolyniec (2003) describes this type of options as well under the title Synthetic Peaker.

[5] Korn (2009) later relaxes the assumption of normality and shows that the optimal payoff function takes different forms arising from the assumed dependence structure rather than being a second order polynomial in price

Several studies demonstrate the link between quantity and weather, e.g., Engle et al. (1992), Timmer and Lamb (2007) and Swindle (2014), making the next natural step to include weather contracts in the above-mentioned strategies. While not presenting the details, Eydeland and Wolyniec (2003) study the efficiency of weather hedges by looking at the residual cash flow. For electricity markets, they argue that weather hedges do not add much value after application of a price hedge, whereas for gas markets it can be worthwhile to consider basing the hedge on weather contracts.

Lee and Oren (2009) introduce an equilibrium economy, where market participants can invest in a customized hedge based on the energy price and a weather contract whose price is determined by supply and demand in the economy rather than being priced under an exogenously given pricing measure. Id Brik and Roncoroni (2016b) and Id Brik and Roncoroni (2016a) extend the customized hedge to be a general function of price and a linked index. The former paper employs a customized hedge which is the sum of a general function of price and a general function of the linked index and derive closed form solutions under the assumption that price and index is independent, while price-quantity and quantity-index are correlated. The latter paper views the customized hedge as a general function of price and index. The exact functional form of the hedge depends on the underlying joint distribution of price, quantity and index. An assumption of log-normality will for instance result in a functional form that partly depends on the product of price and index raised to various powers.

In the agricultural finance literature, a similar development in hedge strategies are seen. Papers analysing hedges based on prices only, e.g., Moschini and Lapan (1995). They conclude that options are needed to hedge the joint production and price risk. When standardized yield futures was introduced in 1995 on the Chicago Board of Trade, a tool was provided for managing the quantity risk directly rather than through the correlation of price and quantity. Aase (2004) develops a pricing model for combined price futures and yield futures, which can price contracts depending on both underlying and further points out that (a synthetic) revenue futures can be obtained via dynamic trading in underlying futures under the assumption of no basis risk and no transactions costs. The yield futures contracts were de-listed shortly after introduction due to low trading interest.

Lien and Hennessy (2004) compare revenue futures to price futures and highlight the trade-off in contract design: To ensure market depth, a revenue futures contract must be based on a broad geographical area. But at the same time this will lead to high idiosyncratic risk making the hedge inefficient compared to a pure price hedge. As also pointed out by Poitras (2013), the source of uncertainty that a farmer needs to insure is the income, rather than the quantity and price components of the income. There seems to be little need for construction of revenue futures or yield futures in the North America, as crop insurance products are administered and state subsidized. Since 1996, specific revenue insurance contract have been available. Cornaggia (2013) points out that the use of insurance can further be attributed to pure risk management, whereas the use of derivatives could also be a sign of speculation. In conclusion, although a farmer generally faces the same type of risk management problems as an energy company, there has been a strong development towards a state-subsidized solution in terms of crop insurance programs[6].

In most of the papers referred to above, the focus is on a one-period static hedging problem is studied. While this is fairly realistic when risk managing crop production, where harvest is an annual or bi-annual occurrence, the risk management problem becomes more complex for energy markets as the owner of a wind farm or a LSE are selling or buying different quantities of power at different hourly rates. The profit or revenue will therefore be a sum of products of hourly quantities and prices. The available market traded contracts are on the other hand energy forwards, i.e., depending on the average price over a period, for instance a month, a quarter or all peak-hours in a given week, energy options written on the forwards and index futures on an accumulated index over a calender month. Kleindorfer and Li (2005) analyse a multi-period framework by imposing a regularity condition on the distribution of cash flows and illustrate results for the PJM market. In general, it is not possible to derive an expression for an optimal hedge in a multi-period framework. A second issue that an LSE should be consider is the portfolio effect. Most LSE also holds generation power and the risks from generation of power should not be isolated from the risk arising from providing power to end-customers.

Another issue which is seem not to have been addressed is the peculiar relationship between price and quantity. As pointed out in e.g., Eydeland and Wolyniec (2003), the price and demanded quantity have a strong relationship for low and medium prices, while it breaks down for high prices. When closed form hedge strategies are solved, the assumptions made regarding the price-quantity correlation in most studies does not incorporate this feature. If hedge strategies are analysed with real data rather than in a stylized market, this should be addressed using the exact characteristics of the market in question. In this paper, the use of the OTC market for structuring tailor made hedges is promoted. However, the classic trade-off of credit risk vs. basis risk when deciding between OTC structured derivatives and market traded contracts should be taken into consideration. In many cases, the availability of market traded contracts is so limited that an OTC contract provide a strong alternative to market based hedges due to the basis risk and the liquidity premium in the latter. Golden et al. (2007) analyses this trade-off in a one period model without price risk.

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[6] The fact that so many farmers use insurance could indicate that farmers are risk averse as a low revenue in one season can have severe consequences for the future of their business.

On a final note, alternatives to risk management using derivatives should mentioned. The LSE could pass on their price risk to the customer by charging the actual spot price (plus a profit margin). This could be based one real-time data, but requireds the technology to read off the consumption in real time as well as provide price information to the consumers to which they can adjust to in an either automated or manual way. For instance, Pacific Northwest GridWise Demonstration Project saw a decrease in peak loads (in which case the price and quantity have more a complex relationship) as well as a decrease in the average electricity bill[7]. This way of handling the profit risk is arguably an initiative that has more merits in the context of grid reliability, and potential risk management benefits should more be seen as the bi-product. A simpler approach to transfer of the price risk is to charge customers the actual average costs per unit consumed[8]. For instance, in 2015 the default contract for a customer in the Danish energy company DONG Energy changed from quarterly fixed rates to monthly rates based on the actual costs (including balancing and trading costs) for purchase of power on Nord Pool, while at the same time raising the monthly subscription.

Although these spot based contract structures, whether they are hourly or average based, are becoming increasingly available in some markets, the fixed rate contract is still a widely used contract. For the rest of this paper, we continue the string of literature considering the hedging problem an LSE faces when offering fixed rate contracts.

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[7] https://www.smartgrid.gov/files/GridWise_Demonstration_Project_Fast_Facts_200708.pdf

[8] Or weighted according to a pre-determined template.

II.3 Risk management problem for an energy retailer

In this section, the risk management problem is presented in a general form and subsequently simplified to a one-period static hedging problem. As discussed in the previous section, many solutions to the simple hedging problem have been proposed in the literature and the resulting strategies are presented in Section II.4.1. Using the solution in Id Brik and Roncoroni (2016a) combined with the representation result proposed by Carr and Madan (2001), energy quanto contracts arise naturally as a part of the customized hedge.

II.3.1 The general profit expression

Consider an energy retailer who charges a fixed price of R over the time horizon $[0, T]$. Before hedging, the profit and loss for an energy retailer over this period is

$$V_{[0,T]} = \int_0^T e^{-r_t t}(R - P_t)Q_t dt \tag{II.1}$$

where Q_t is the amount of energy delivered to customers at time t and P_t the corresponding price. In practice, the price P_t and the demanded quantity Q_t are piece-wise linear functions. I.e., if the retailer is facing n prices per year, the profit and loss will be

$$V_{[0,T]} = \sum_{j=1}^{nT} e^{-r_{t_j} t_j}(R - P_{t_j})Q_{t_j} \tag{II.2}$$

where $t_j = \frac{j}{n}$. If T is measured in years, setting $n = 365$ corresponds to daily prices and $n = 24 \times 365$ corresponds to hourly prices. r_{t_j}, P_{t_j} and Q_{t_j} are the interest, price and quantity faced over the period $[t_{j-1}, t_j]$.

II.3.2 Structuring of hedge contract

In the most simple case $n = T = 1$. This is the case of delivery of an uncertain amount of energy Q at an unknown price P. Further, it is assumed that there exists an index I on which contracts can be written and that the volume Q is correlated with this index. The main question is then; if it is possible to structure a payoff depending on price P and this index I, what is then the optimal structure? The value after entering such a hedge would be: $V_h = Q(R - P) + x(P, I)$[9]. The optimal hedge choice can mathematically be expressed as the solution to:

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[9] The condition in II.3 ensures that the cost of the hedge in included in the structure of x.

$$\max_{x(p,i)} \mathbb{E}(\mathcal{U}(V_h)) \qquad \text{s.t.} \qquad \mathbb{E}^Q(x(P,I)) = 0 \tag{II.3}$$

where $\mathcal{U}$ is a utility function. The solution to this problem $x^*(p,i)$ is a function of p and i and depends on the choice of utility function as well as the distribution of P, Q and I under the physical probability measure and the distribution of P and I under a risk-neutral measure. This is the main result in Id Brik and Roncoroni (2016a), who also gives the functional form for a generalised mean-variance utility function. In Proposition II.2, the general functional form of the hedge is replicated using various contracts. It is a straightforward extension of the one-dimensional version from by Carr and Madan (2001), which is stated below:

LEMMA II.1 (CARR AND MADAN (2001)) Any twice differentiable function $x : \mathbb{R} \to \mathbb{R}$ can be represented as

$$x(p) = x(F_P) + x'(F_P)(p - F_P) + \int_0^{F_P} x''(K)(K-p)^+ dK + \int_{F_P}^{\infty} x''(K)(p-K)^+ dK$$

If the customized hedge x is only a function of the price, the formula shows that customized hedge can be replicated in terms of a combination of a bank account, a forward contract, a continuum of put options with strike smaller than the forward price and a continuum of call options with strike larger than the forward price[10]. In reality, the customized hedge can only be approximated by a finite number of options as not all strikes are available.

PROPOSITION II.2 Any bivariate function $x : \mathbb{R}^2 \to \mathbb{R}$ that is twice differentiable in both arguments can be represented as

$$\begin{aligned} x(p,i) = {}& x(F_P, F_I) && \text{(II.4)}\\ & + x_i(F_P,F_I)(i - F_I) + x_p(F_P,F_I)(p - F_P) && \text{(II.5)}\\ & + x_{pi}(F_P,F_I)\left((i - F_I)(p - F_I)\right) && \text{(II.6)}\\ & + \int_0^{F_I} x_{ii}(F_P,M)(M-i)^+ dM + \int_{F_I}^{\infty} x_{ii}(F_P,M)(i-M)^+ dM && \text{(II.7)}\\ & + \int_0^{F_P} x_{pp}(K,F_I)(K-p)^+ dK + \int_{F}^{\infty} x_{pp}(K,F_I)(p-K)^+ dK && \text{(II.8)}\\ & + (p - F_P)\int_0^{F_I} x_{pii}(F_P,M)(M-i)^+ dM && \text{(II.9)} \end{aligned}$$

---

[10] The expansion does not have to be around the current forward price F_P, but this choice makes the most economic sense because the second term then equals the payoff of $x'(F_P)$ forward contracts.

$$+ (p - F_P) \int_{F_I}^{\infty} x_{pii}(F_P, M)(i - M)^+ dM \tag{II.10}$$

$$+ (i - F_I) \int_{0}^{F_P} x_{ppi}(K, F_I)(K - p)^+ dK \tag{II.11}$$

$$+ (i - F_I) \int_{F_P}^{\infty} x_{ppi}(K, F_I)(p - K)^+ dK \tag{II.12}$$

$$+ \int_{0}^{F_P} \int_{F_I}^{\infty} x_{ppii}(K, M)(i - M)^+ (K - p)^+ dM dK \tag{II.13}$$

$$+ \int_{0}^{F_P} \int_{0}^{F_I} x_{ppii}(K, M)(M - i)^+ (K - p)^+ dM dK \tag{II.14}$$

$$+ \int_{F_P}^{\infty} \int_{0}^{F_I} x_{ppii}(K, M)(M - i)^+ (p - K)^+ dM dK \tag{II.15}$$

$$+ \int_{F_P}^{\infty} \int_{F_I}^{\infty} x_{ppii}(K, M)(i - M)^+ (p - K)^+ dM dK \tag{II.16}$$

where subscripts denote differentiation with respect to the function arguments. This result states that the optimal hedge consists of several different contract types:

- (II.4): bank account
- (II.5): forward/future on energy and the index
- (II.6): energy quanto swap[11]
- (II.7): continuum of options on the index
- (II.8): continuum of options on energy
- (II.9)-(II.12): the product of a forward and a continumm of options[12]
- (II.13)-(II.16): continuum of energy quanto options □

The proposition states that the customized hedge can be replicated using the contract types listed above, one of which is the energy quanto option. The importance of a specific type of contract is given by the value of various derivatives of the customized hedge. For the simple model in Id Brik and Roncoroni (2016a), it would be possible to derive and analyse the shape of these quantities and for instance analyse the discretisation error as done by Oum and Oren

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[11] Energy quanto swaps are also studied in Cucu et al. (2016)

[12] These terms can also be expressed as a continuum of regular quanto options net of a continuum of univariate options.

(2010). But as the counterpart for an energy quanto option is found in the OTC market, buying (or selling if the derivatives of x^* is negative) multiple energy quanto options would be an expensive strategy in practice. Instead, Proposition II.2 is used in a more pragmatic sense: The contracts making up the replication of the preferred hedge are standard forwards and options on the price and on the index and a whole range of contracts related to the product of price and index, so a *realistic* choice of strategy is therefore a combination of market available contracts and a number of energy quantos. The exact choice of energy quanto swaps vs. energy quanto options, strikes and quantity must in practice be chosen by case or simulation studies.

Closely connected to the choice of contract is of course the pricing, as the costs of hedging must be included in the decision. For the purpose of this paper, it is assumed that one unique pricing measure exists and that all market participants agree on the measure. Further, there are no restrictions to financing, no transactions costs and no bid-ask spreads and no credit risk. Surely this is highly unrealistic, but it allows for illustration of how effective energy quanto options are in comparison with other studies using the same assumptions. The next section develops a comparative simulation study.

II.4 Comparative study

In this section, a numerical experiment is conducted and hedge strategies involving energy quantos are compared to other strategies. A hedge strategy solves (II.3) for different choices of x assuming a generalized mean-variance utility function. The mean-variance utility function allows for a trade-off between risk and return by choice of a and η, where $\eta = 0, 1$[13]:

$$\mathcal{U}[V_h] = \eta \mathbb{E}[V_h] - \frac{a}{2}\mathrm{Var}[V_h] \tag{II.17}$$

In the following, V denotes the unhedged profits $Q(R - P)$. When expressing expectations and densities they are denoted without superscript for the physical measure and with superscript ^{Q} for the risk-neutral probability measure.

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[13] In the case of $\eta = 0$, the objective is solely to minimize variance, which is a questionable objective for most businesses, see for instance the discussion in Poitras (2013).

II.4.1 Hedge strategies

The best possible hedge is the strategy set forward by Id Brik and Roncoroni (2016a) and this will be denoted the *preferred hedge*. The preferred hedge solves (II.3) using no assumption about the functional form of x using a generalized mean-variance utility:

Strategy 1 (Id Brik and Roncoroni (2016a) Theorem 1) The general solution to (II.3) is given by

$$x^*(p,i) = \left(\mathbb{E}^Q\left[\mathbb{E}\left[V|P=p,I=i\right]\right] - \frac{\eta}{a}\right)\frac{f^Q(p,i)/f(p,i)}{\mathbb{E}^Q\left[\frac{f^Q(P,I)}{f(P,I)}\right]} - \left(\mathbb{E}\left[V|P=p,I=i\right] + \frac{\eta}{a}\right) \tag{II.18}$$

The preferred hedge is superior to any other hedge strategy written on price and/or index and will therefore serve as a benchmark for how well energy quanto hedge strategies perform. The preferred hedge can be replicated by the strategy outlined in Proposition II.2, so in theory a hedge strategy using energy quanto options can get arbitrarily close to the preferred hedge. However, as discussed in the introduction, this will not be realistic. In practice, structuring an energy quanto hedge strategy would consist of choosing an appropriate combination of (double) forward- and option-like structures.

Entering into an energy quanto hedge only makes sense if it is superior to hedges consisting of market traded contracts. The energy quanto hedges will therefore also be compared to other hedge strategies proposed and discussed in the literature. First, the *additive hedge*:

Strategy 2 (Id Brik and Roncoroni (2016b) Theorem 3.2) Under the restriction that x is additive, $x(p,i) = h(p) + g(i)$, and that the price and index are independent, the solution[14] to (II.3) is given by

$$\begin{pmatrix} h^*(p) & g^*(i) \end{pmatrix}' = \mathbf{b}(p,i) - A(p,i)\left\{\mathbb{E}^Q\left[A(P,I)\right]\right\}^{-1}\mathbb{E}^Q\left[\mathbf{b}(P,I)\right] \tag{II.19}$$

where

$$A(p,i) = \begin{pmatrix} \frac{f_P^Q(p)}{f_P(p)} & \frac{f_P^Q(p)}{f_P(p)} - 1 \\ \frac{f_I^Q(i)}{f_I(i)} - 1 & \frac{f_I^Q(i)}{f_I(i)} \end{pmatrix}$$

$$\mathbf{b}(p,i) = \begin{pmatrix} \frac{\eta}{a} - \mathbb{E}\left[V|P=p\right] + \left(\mathbb{E}\left[V\right] - \frac{\eta}{a}\right)\frac{f_P^Q(p)}{f_P(p)} \\ \frac{\eta}{a} - \mathbb{E}\left[V|I=i\right] + \left(\mathbb{E}\left[V\right] - \frac{\eta}{a}\right)\frac{f_I^Q(i)}{f_I(i)} \end{pmatrix}$$

---

[14] Id Brik and Roncoroni (2016b) does not include η, but this is easily included.

Using the representation in Proposition 1 on each of the functions h and g, the additive hedge can be interpreted as the best possible hedge obtainable if both forwards and options for all strikes were available for both the energy and the index. For markets such as gas and crops, it is not unreasonable to have a liquid forward/futures market and a range of listed options, while it is mainly possible to trade forward/futures for electricity and weather[15], so the additive hedge will always perform better than any realistically market traded hedge.

A realistic market traded hedge will be one of the following three. The first is denoted the *pure price hedge*, where the functional form of x is restricted to be a function of price:

STRATEGY 3 (OUM AND OREN (2006) SECTION 3.1.4) If the hedge strategy consists of only a general function of the price the solution to (II.3) is

$$h^*(p) = \frac{\eta}{a} - \mathbb{E}\left[V|P=p\right] + \frac{\frac{f_P^Q(p)}{f_P(p)}}{\mathbb{E}^Q\left[\frac{f_P^Q(p)}{f_P(p)}\right]}\left(\mathbb{E}^Q\left[\mathbb{E}\left[V|P=p\right]\right] - \frac{\eta}{a}\right) \tag{II.20}$$

This hedge can be seen as the limit of a hedge using contracts on the price only and obtainable (to a degree given by the discretization errors) in case of a liquid options market. If there are no options market, the two forward hedges; the *forward hedge* and the *price forward hedge* are possible market traded hedges:

STRATEGY 4 (ID BRIK AND RONCORONI (2016A) PROPOSITION 1) If the hedge strategy consists only of forward contracts on the energy and on the index, the number of long forward contracts held to solve (II.3) are

$$\theta_P^* = \frac{\mathrm{var}(I)\left[\eta(\mathbb{E}(P)-F_P) - a\mathrm{cov}(V,P)\right] - \mathrm{cov}(P,I)\left[\eta(\mathbb{E}(I)-F_I) - a\mathrm{cov}(V,I)\right]}{a(\mathrm{var}(P)\mathrm{var}(I) - \mathrm{cov}^2(P,I))} \tag{II.21}$$

$$\theta_I^* = \frac{\mathrm{var}(P)\left[\eta(\mathbb{E}(I)-F_I) - a\mathrm{cov}(V,I)\right] - \mathrm{cov}(P,I)\left[\eta(\mathbb{E}(P)-F_P) - a\mathrm{cov}(V,P)\right]}{a(\mathrm{var}(P)\mathrm{var}(I) - \mathrm{cov}^2(P,I))} \tag{II.22}$$

STRATEGY 5 (ID BRIK AND RONCORONI (2016A) PROPOSITION 1) If the hedge strategy consists only of a forward contract on the energy, the number of long forward contract held to solve (II.3) is

$$\theta_P^* = \frac{\eta(\mathbb{E}(P)-F_P) - a\mathrm{cov}(V,P)}{a\mathrm{var}(P)} \tag{II.23}$$

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[15] Nord Pool electricity futures are traded for several maturities. For electricity options on Nord Pool, only one option was traded (with a volume of 10 contracts) during the first three weeks of August 2016. On EEX, the European Energy Exchange, options on futures are restricted to one or two strike values. Weather contracts listed on Chicago Mercantile Exchange show low trading in futures and almost no trading in options.

These five strategies together with no hedge (the *naked hedge*) are compared to two different quanto strategies. As previously mentioned, the structuring of energy quanto hedge strategies requires different decisions regarding contract type, swap rates/strikes and contract size. Using various combinations of contract structures from Proposition II.2, different contract choices are analysed after inspecting the *preferred hedge*. Given the contract structure, the choice of strikes and swap rates as well at the optimal number of each contract are found using numerical optimization. The analysis is restricted to two simple strategies, where the first uses a call-call energy quanto options (II.16) and the second uses a call-call together with a put-put energy quanto option:

STRATEGY 6 (*Simple quanto hedge*) The hedge consists of forwards on the asset and the index and one energy quanto call-call option with strikes K_P and K_I. The price of the quanto option is denoted p_Q:

$$x(p,i) = \theta_P(p - F_P) + \theta_I(i - F_I) + \theta_Q \left[(p - K_P)^+(i - K_I)^+ - p_Q\right] \tag{II.24}$$

STRATEGY 7 (*Diagonal quanto hedge*) The hedge consists of forwards on the asset and the index, an energy quanto call-call option and an energy quanto put-put option with strikes K_P and K_I. The prices of the quanto options are denoted p_Q^{CC} and p_Q^{PP}:

$$\begin{aligned} x(p,i) = {} & \theta_P(p - F_P) + \theta_I(i - F_I) + \\ & + \theta_Q^{CC}\left[(p - K_P)^+(i - K_I)^+ - p_Q^{CC}\right] + \theta_Q^{PP}\left[(K_P - p)^+(K_I - i)^+ - p_Q^{PP}\right] \end{aligned} \tag{II.25}$$

II.4.2 Market design

The stylized gas market analysed by Id Brik and Roncoroni (2016a) is applied in this comparative study and is described in Table II.1. All of the above strategies are easily implementable for normally distributed and log-normally distributed variables. In Id Brik and Roncoroni (2016a), price, quantity and index are all log-normally distributed[16]. In e.g, Oum and Oren (2006) the analysis is also done under the assumption of quantity being normally distributed. The comparative study is done for both set of distributional assumptions. Parameters in (II.26) and (II.27) are chosen such that the mean and standard deviations corresponds to those of Table II.1.

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[16] Although temperature can be negative, the index can for instance be thought of as the HDD index, which is by definition always non-negative, so the assumption of log-normality is not inconsistent with representing temperature.

	Average value	Standard deviation
Price P (USD/mmBtu)	4	1
Quantity Q (mmBtu)	10,000,000	5,000,000
Index I ($^\circ F$)	60	30

Notes: The average gas consumption is 10,000,000 mmBtu (million British thermal units) and has a standard deviation of 5,000,000. The gas price has an average value of 4 USD/mmBtu with a standard deviation of 1. Temperature has a average value of 60 ° Fahrenheit and a standard deviation of 30.

Table II.1. Stylized gas market

Log-normal model:

$$\begin{pmatrix} \log P \\ \log Q \\ \log I \end{pmatrix} \overset{\mathbf{P}}{\sim} \mathcal{N}\left(\begin{bmatrix} m_{P1} \\ m_Q \\ m_{I1} \end{bmatrix}, \left(\begin{bmatrix} \sigma_P^2 & \sigma_P\sigma_Q\rho_{PQ} & \sigma_P\sigma_Q\rho_{PQ} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_Q^2 & \sigma_I\sigma_Q\rho_{QI} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_I\sigma_Q\rho_{QI} & \sigma_I^2 \end{bmatrix} \right) \right) \tag{II.26}$$

Normal/log-normal model:

$$\begin{pmatrix} \log P \\ Q \\ \log I \end{pmatrix} \overset{\mathbf{P}}{\sim} \mathcal{N}\left(\begin{bmatrix} m_{P1} \\ m_Q \\ m_{I1} \end{bmatrix}, \left(\begin{bmatrix} \sigma_P^2 & \sigma_P\sigma_Q\rho_{PQ} & \sigma_P\sigma_Q\rho_{PQ} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_Q^2 & \sigma_I\sigma_Q\rho_{QI} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_I\sigma_Q\rho_{QI} & \sigma_I^2 \end{bmatrix} \right) \right) \tag{II.27}$$

Under the risk neutral measure, the mean value of $\log P$ shifts to m_{P2} (parametrised by market price of risk λ_P, such that $m_{P1} + \lambda_P\sigma_P$) and the mean value of $\log I$ shifts to m_{I2} (parametrized by λ_I, such that $m_{I1} + \lambda_I\sigma_I$). The log price and (log) quantity is moderately correlated with a correlation coefficient of 0.5. Log price and log index are mildly correlated with a correlation coefficient of 0.15. Log index and (log) quantity are strongly correlated with a correlation coefficient of 0.8. The level of risk aversion a is set to 0.1 and the analysis is restricted to mean-variance utility by choosing η to be 1. This is chosen to reflect that a LSE also caring about expected profits. Following Id Brik and Roncoroni (2016a), the market price of risk is set to 0.01 for the energy asset and slightly higher at 0.05 for the index. Studies such as e.g, Bellini (2005) and Benth and Benth (2013) report a significant risk premium for weather contracts, which is included by setting λ_I slightly higher than λ_P.

Using the parameters from Tables II.2 and II.3, 10,000,000 simulations are done both for the assumption of log-normally distributed quantity and for normally distributed quantity.

	Log-normal model			Mixed normal log-normal model		
Variable	m_1	m_2	σ	m_1	m_2	σ
Price P	1.356	1.358	0.2462	1.356	1.358	0.2462
Quantity Q	16.007	–	0.4724	10,000,000	–	5,000,000
Index I	3.983	4.006	0.4724	3.983	4.006	0.4724

Notes: Under the given model, the parameters in this table match the stylized market in Table II.1.

Table II.2. Mean and standard deviations for log price, (log) quantity and log index values.

ρ_{PQ}	ρ_{PI}	ρ_{QI}	a	η	R	λ_P	λ_Q
0.50	0.15	0.8	0.1	1	6	0.01	0.05

Table II.3. Additional model parameters.

II.4.3 Analysis

For each of the strategies described previously, the hedge is computed and the resulting payoff after hedge, V_h is analysed. The choice of quanto strategies in Section II.4.1 is supported by inspection of the shape of the preferred hedge and secondly to be as simple as possible.

As in Brown and Toft (2002) and Id Brik and Roncoroni (2016a), performance indices are computed for the normalised mean-variance and the standard deviation. For a strategy i, they are defined by

$$PI_{\text{NMV}}(i) = \frac{\text{NMV(naked position)} - \text{NMV(strategy } i)}{\text{NMV(naked position)} - \text{NMV(preferred hedge)}}$$

$$PI_{\text{SD}}(i) = \frac{\text{SD(naked position)} - \text{SD(strategy } i)}{\text{SD(naked position)} - \text{SD(preferred hedge)}}$$

and expresses how close strategy i is to the *preferred hedge*. Finally to compare the improvement or deterioration when moving from one strategy to another, ID_{NMV} measures the improvement in Normalized Mean Variance relative to the Normalized Mean Variance of the naked strategy:

$$ID_{\text{NMV}}(i, j) = 100 \times \frac{\text{NMV(strategy } j) - \text{NMV(strategy } i)}{|\text{NMV(naked position)}|}.$$

II.4.4 Results under assumption of log-normally distributed quantity

The surface plots in Figure II.1 shows the *preferred hedge* and the *forward hedge* as a function of price p and index i and illustrates what the LSE is concerned about: Looking at the dimensions one by one, it is seen that the higher the price, the more payoff the LSE would like from their hedge. This is also obtained in the *forward hedge*. But for the index, the LSE would like less for higher values of the index if the price is low, while they would like more for higher values of the index if the price is high. With the forward hedge, they can only obtain the same slope regardless of price. For this example, it results in shorting forwards on the index.

A payoff in the high price-high index situation is exactly obtained by buying a call-call quanto option together with a number of forwards. This was denoted to as the *simple quanto hedge*. Looking more closely at the *preferred hedge*'s surface in II.1, there is also a slight increase in the function value for low price and low index. It might therefore add extra value to add a put-put quanto option to the simple quanto strategy. This was referred to as the *diagonal quanto hedge*.

In Figure II.3 selected profit distributions are compared: The left panel shows the *preferred hedge* together with the *naked strategy* as well as the *simple quanto hedge*. The right panel shows the *preferred hedge*, the *simple quanto hedge* and the *additive hedge*. The *simple quanto hedge* seems to do better than the *additive hedge* and as this will outperform all other described strategies, the *simple quanto hedge* is a strong candidate for a hedge strategy with a feasible design.

Table II.4 shows performance statistics for the hedge strategies described in section II.4.1. The columns present the expected utility for a mean-variance hedge, the expected profit, the standard deviation, the normalised mean-variance (defined as the expected profit net of the standard deviation), the probability of a loss, the 5% Value-at-Risk and the 5% Expected Shortfall[17]. Regardless of performance measure, when quantity is log-normally distributed, the *simple quanto hedge* and the *diagonal quanto hedge* is doing almost equally well.

Table II.5 shows the performance both for the normalised mean-variance and for a pure variance measure of performance. The discrepancy between any of the two *quanto hedges* and the *preferred hedge* is merely two efficiency points. The pure variance measure of performance shows similar values compared to the mean-variance performance criteria for all strategies.

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[17] For VaR and ES, a negative number indicates a gain. The average margin is 2$ and a loss is therefore not very likely in this setup. The unrealistically large margin is kept to allow for a direct comparison with Id Brik and Roncoroni (2016a).

Table II.6 provide a comparison of all eight strategies. Hedge strategies involving price and index in a multiplicative way clearly outperform a hedge which only relies on the price and index separately. Comparing to market available strategies, there is a great advantage of adding just a single quanto option to a market available strategy. A market traded available strategy would as noted under the description of strategies perform somewhere between the *forward hedge* and the *additive hedge* depending on the options market for the energy in question.

II.4.5 Results under assumption of normally distributed quantity

The same analysis is now made under the assumption of normally distributed quantity. The surface plots in Figure II.4 shows the *preferred hedge* and the *forward hedge* as a function of price p and index i. In this case the *diagonal quanto hedge* is expected to perform better than the *single quanto hedge* because of the shape of the preferred hedge in the low price-low index scenario. Figure II.5 shows the payoff of the *diagonal quanto hedge* after strikes and number of contracts are estimated.

In Figure II.6 selected profit distributions are compared: The left figure shows the *preferred hedge* together with the *naked strategy* as well as the *diagonal quanto hedge*. The right figure shows the *preferred hedge*, the *simple quanto hedge* and the *diagonal quanto hedge*. For the profit distribution, *diagonal quanto hedge* and the *simple quanto hedge* do not display the same performance and they also do not match the profit distribution of the *preferred hedge* as well as in the case of log-normally distributed quantity. The left tail of the *simple quanto hedge* is fatter than the *diagonal quanto hedge*. Contrary to before, two quanto options are needed to obtain a hedge comparable to the *preferred hedge*. II.8 and II.9 confirms these observations: The quanto hedges have a relative performance of around 90%. The loss in relative performance when shifting from the *diagonal quanto hedge* to the *simple quanto hedge* is much more severe than before. They however still perform slightly better than the (infeasible) *additive hedge* and much better the (realistic) *forward hedge*.

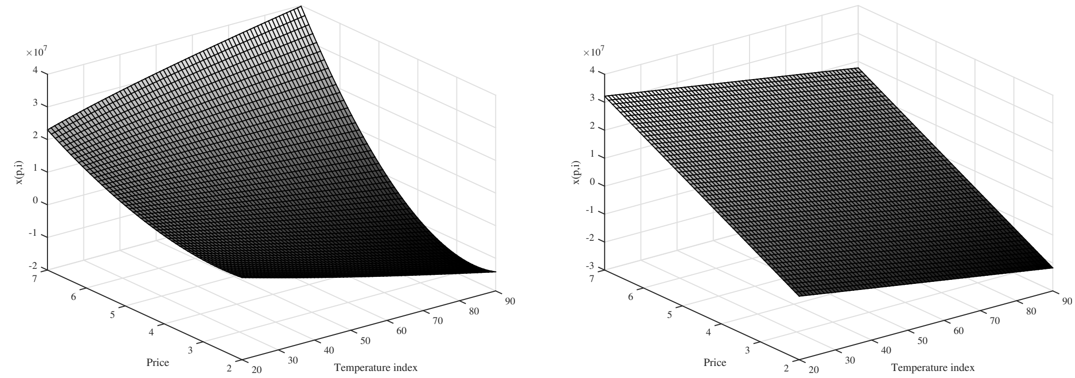

Figure II.1.

3D plot of preferred hedge and forward hedge for log-normally distributed quantity. The left panel of this figure shows the *preferred hedge* as a function of energy price and temperature index and the right panel shows the *forward hedge*. The main qualitative difference is that the high price-high index scenario has a too low payoff in the *forward hedge* compared to the *preferred hedge*.

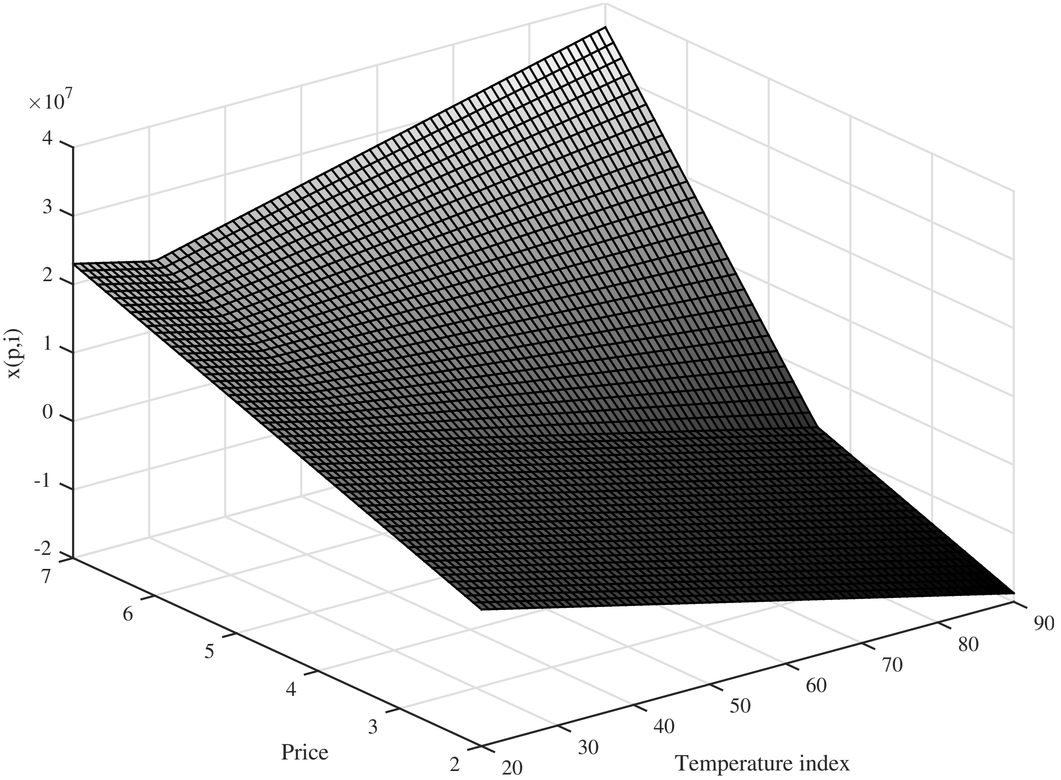

Figure II.2. 3D surface plot of the simple quanto hedge.

This figure shows the payoff for *simple quanto hedge*, which combines forwards on the energy asset and on the temperature index with a call-call energy quanto option. The exact strikes and the optimal number of contracts are chosen via numerical optimisation of the utility function.

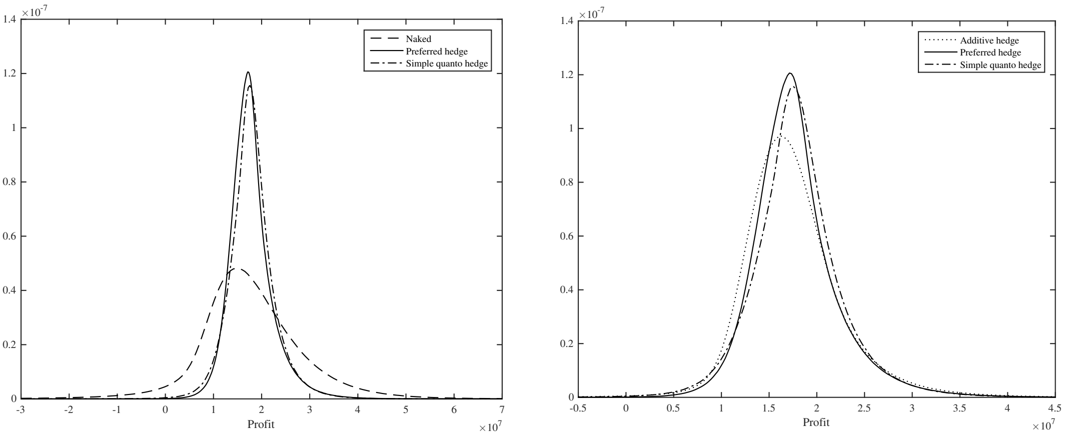

Figure II.3. Profit distributions for log-normally distributed quantity.

The left panel shows the profit distribution for the *preferred hedge*, the naked strategy and *simple quanto hedge*; a hedge strategy involving forward on the energy asset and the temperature index as well as a double-call quanto option. The right panel shows the the *preferred hedge*, *simple quanto hedge* and the *additive hedge*. By inspection of the profit distribution, the *simple quanto hedge* is seen to almost mimic the *preferred hedge*.

Strategy	Utility (10^9s)	$E(V_h)$ (1,000s)	SD (1,000s)	NMV (1,000s)	P(loss) %	VaR (1,000s)	ES (1,000s)
Preferred hedge	-1,060	17,799	4,605	13,194	0.04	-11,505	-9,465
Diagonal quanto	-1,174	17,928	4,845	13,083	0.19	-10,723	-7,716
Simple quanto	-1,192	17,943	4,882	13,061	0.21	-10,730	-7,622
Additive hedge	-1,554	17,456	5,575	11,881	0.41	-10,545	-6,745
Forward hedge	-2,226	17,824	6,673	11,151	0.93	-8,096	-2,006
Pure price	-3,400	17,495	8,246	9,249	0.09	-6,729	-4,662
Price forward	-4,113	17,531	9,070	8,461	1.22	-5,077	-518
Naked	-6,579	17,597	11,471	6,126	3.85	-2,122	8,614

Quantity is log-normally distributed. Strategies are ranked according to expected utility, but exhibit largely the same ranking for other performance measures.

Table II.4. Performance statistics for the different hedge strategies (log-normally quantity)

Strategy	Performance (NMV)	Performance (SD)
Preferred hedge	1.00	1.00
Diagonal quanto	0.98	0.97
Simple quanto	0.98	0.96
Additive hedge	0.81	0.86
Forward hedge	0.71	0.70
Pure price	0.44	0.47
Price forward	0.33	0.35
Naked	0.00	0.00

Notes: The table dispays the performance of hedge strategies compared to the *preferred hedge* and the naked profits, when quantity is log-normally distributed. If the performance index is equal to 1, the strategy is doing as well as the *preferred hedge*. The quanto strategies both have a 0.98 performance when measured in relation to the normalised mean-variance and a performance of 0.96-0.97 in terms of decreasing the standard deviation of profits.

Table II.5. Performance of strategies

	Preferred hedge	Diagonal quanto	Simple quanto	Additive hedge	Forward hedge	Pure price	Price forward	Naked
Preferred hedge	0.00	-1.82	-2.16	-21.42	-33.34	-64.39	-77.25	-115.36
Diagonal quanto	1.82	0.00	-0.35	-19.61	-31.52	-62.58	-75.44	-113.54
Simple quanto	2.16	0.35	0.00	-19.26	-31.18	-62.23	-75.09	-113.20
Additive hedge	21.42	19.61	19.26	0.00	-11.92	-42.97	-55.83	-93.93
Forward hedge	33.34	31.52	31.18	11.92	0.00	-31.05	-43.91	-82.02
Pure price	64.39	62.58	62.23	42.97	31.05	0.00	-12.86	-50.96
Price forward	77.25	75.44	75.09	55.83	43.91	12.86	0.00	-38.10
Naked	115.36	113.54	113.20	93.93	82.02	50.96	38.10	0.00

Notes: The tables show the relative improvement or deterioration, when going from one hedge to another in the case of log-normally distributed quantity. The preferred hedge and the two quanto hedges are very close to each other and there is almost no difference between the two quanto hedges.

Table II.6. Relative improvement or deterioration for hedge strategies

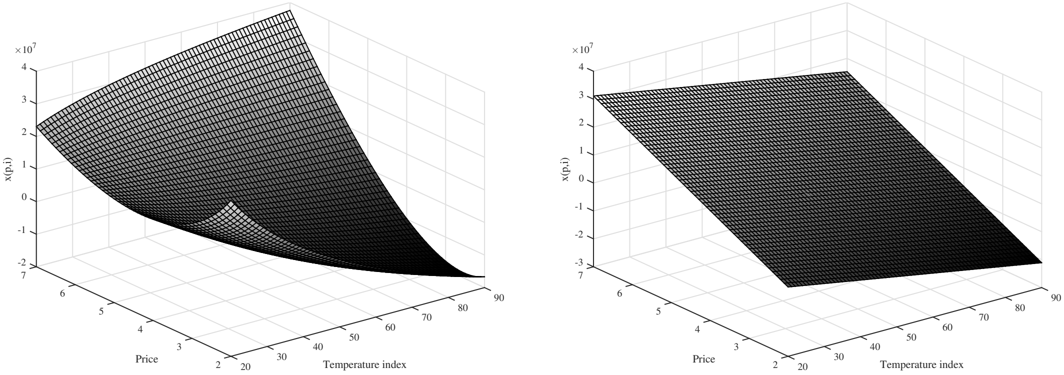

Figure II.4. 3D plot of preferred hedge and forward hedge for normally distributed quantity.

The left panel of this figure shows the *preferred hedge* as a function of energy price and temperature index and the right panel shows the *forward hedge*. The main qualitative difference is that the high price-high index scenario and the low price-low index scenario has a too low payoff in the *forward hedge* relative to the *preferred hedge*. Compared to the log-normally distributed quantity, the LSE would like a much higher payoff in the low price-low index scenario.

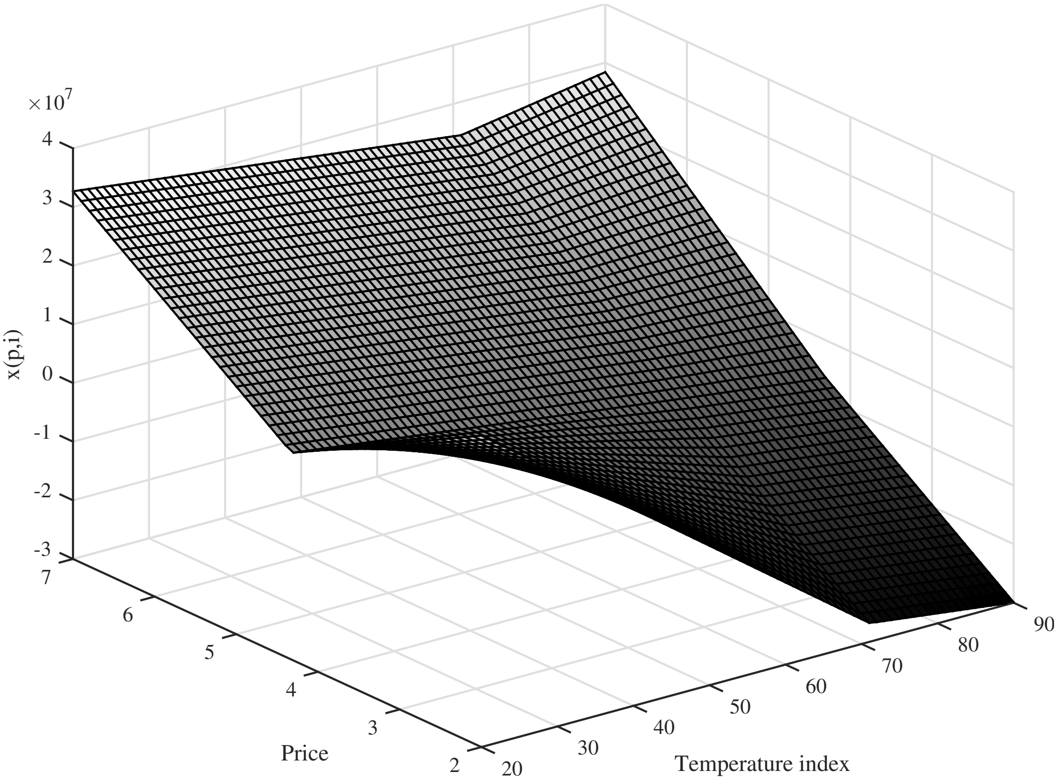

Figure II.5. 3D surface plot of the diagonal quanto hedge.

This figure shows the payoff for *diagonal quanto hedge*, which combines forwards on the energy asset and on the temperature index with a call-call and a put-put energy quanto option. The exact strikes and the optimal number of contracts are chosen via numerical optimisation of the utility function.

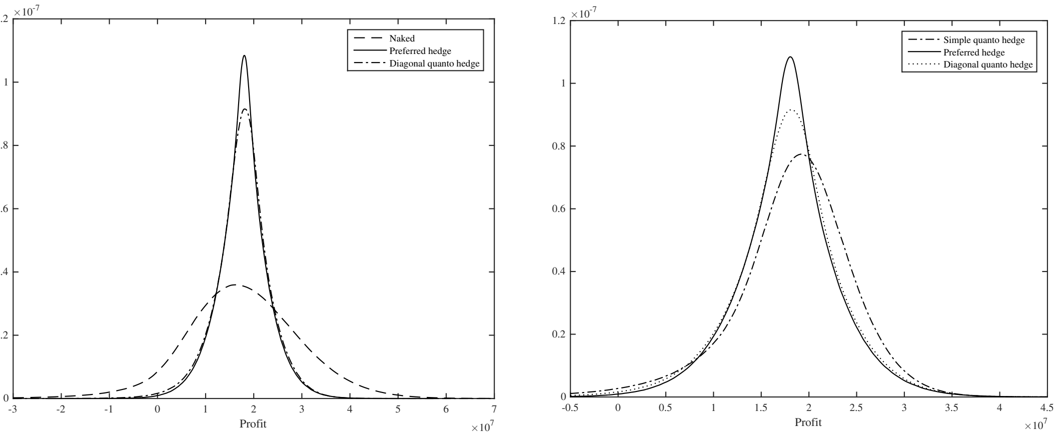

Figure II.6. Profit distributions for normally distributed quantity.

The left panel shows the profit distribution for the *preferred hedge*, the naked strategy and *diagonal quanto hedge*; a hedge strategy involving forward on the energy asset and the temperature index as well as a call-call and a put-put quanto option. The right panel shows the the *preferred hedge*, *simple quanto hedge* and the *diagonal quanto hedge.* By inspection of the profit distribution, the *diagonal quanto hedge* seems to be required to get close to the *preferred hedge*.

Strategy	Utility (10^9s)	$E(V_h)$ (1,000s)	SD (1,000s)	NMV (1,000s)	P(loss) %	VaR (1,000s)	ES (1,000s)
Preferred hedge	-1,366	17,794	5,227	12,568	0.25	-8,999	-5,870
Diagonal quanto	-1,616	17,673	5,685	11,989	0.67	-8,074	-4,036
Simple quanto	-2,172	18,261	6,591	11,670	1.54	-6,631	-906
Additive hedge	-2,141	17,915	6,544	11,372	0.62	-7,601	-3,911
Forward hedge	-3,364	17,793	8,203	9,590	3.53	-2,644	5,266
Pure price	-4,687	17,481	9,682	7,799	4.10	-1,239	4,587
Price forward	-5,640	17,476	10,621	6,855	5.58	890	8,728
Naked	-7,633	17,537	12,355	5,182	6.12	1,500	10,643

Notes: Quantity is normally distributed. Strategies are ranked according to expected utility, but exhibit largely the same ranking for other performance measures.

Table II.7. Performance statistics for the different hedge strategies (normal quantity)

Strategy	Performance (NMV)	Performance (SD)
Preferred hedge	1.00	1.00
Diagonal quanto	0.92	0.94
Simple quanto	0.88	0.81
Additive hedge	0.84	0.82
Forward hedge	0.60	0.58
Pure price	0.35	0.38
Price forward	0.23	0.24
Naked	0.00	0.00

Notes: The table dispays the performance of hedge strategies compared to the *preferred hedge* and the naked profits, when quantity is normally distributed. If the performance index is equal to 1, the strategy is doing as well as the *preferred hedge*. The quanto strategies have a performance of 0.88-0.92 when measured in relation to the normalised mean-variance and slightly lower in terms of decreasing the standard deviation of profits.

Table II.8. Performance for the different strategies

	Preferred hedge	Diagonal quanto	Simple quanto	Additive hedge	Forward hedge	Pure price	Price forward	Naked
Preferred hedge	0.00	-11.17	-17.32	-23.08	-57.46	-92.02	-110.23	-142.52
Diagonal quanto	11.17	0.00	-6.14	-11.91	-46.29	-80.85	-99.06	-131.35
Simple quanto	17.32	6.14	0.00	-5.77	-40.15	-74.70	-92.92	-125.20
Additive hedge	23.08	11.91	5.77	0.00	-34.38	-68.94	-87.15	-119.44
Forward hedge	57.46	46.29	40.15	34.38	0.00	-34.56	-52.77	-85.05
Pure price	92.02	80.85	74.70	68.94	34.56	0.00	-18.21	-50.50
Price forward	110.23	99.06	92.92	87.15	52.77	18.21	0.00	-32.28
Naked	142.52	131.35	125.20	119.44	85.05	50.50	32.28	0.00

Notes: The tables show the relative improvement or deterioration, when going from one hedge to another in the case of normally distributed quantity. The relative deterioration of the diagonal quanto hedge compared to the preferred hedge is about 11% and the simple quanto hedge is more comparable to the additive hedge than under the assumption of log-normally distributed quantity.

Table II.9. Relative improvement or deterioration for hedge strategies

II.5 Concluding remarks

The representation of a hedge strategy based on both price and an index correlated to quantity contains energy quanto options and swaps. Using a standard one-period example, it was shown that realistic strategies including one or two quanto options perform almost as well as the unrealistic ideal hedge strategy.

The randomness in price, quantity and index was modelled using a multivariate normal distribution. The quantity had in both analyses the same mean and variance, but followed either a log-normal or a normal distribution. The difference impacted the shape of the ideal hedge and thereby also which quanto strategy was needed. It is reasonable to assume that there is no universal energy quanto structure, when it comes to hedging volumetric and price risk jointly. Rather, the exact choice must be based on studies of the marginal and joint distribution of price, quantity and index for the exact market in question.

As discussed in Section II.2, a one period model is not realistic for many energy applications. In a multi-period framework, simulation studies must be conducted including necessary analyses of correlation structures across variables as well as choosing an appropriate pricing approach. If the counterpart is a re-insurance company, pricing under the physical measure and adding a premium to ensure an acceptable risk level could be reasonable. It is reasonable to assume that strategies involving energy quanto contracts can improve risk management also in the multi-period setting, as it offers a multiplicative contract structure to take care of the embedded multiplicative risk.

The results of this simple analysis provide an argument for why a Load Serving Entity should also consider OTC options rather than only market traded instruments, when they are designing their hedge.

Appendix

II.A Details of benchmark strategies

II.A.1 Preferred hedge

The preferred hedge using a mean-variance approach is a function of price value p and index value i. It is denoted x^* and is given as

$$x^*(p,i) = \left(\mathbb{E}^Q\left[\mathbb{E}\left[V|P=p,I=i\right]\right] - \frac{\eta}{a}\right) \frac{f^Q(p,i)/f(p,i)}{\mathbb{E}^Q\left[\frac{f^Q_{PI}(P,I)}{f_{PI}(P,I)}\right]} - \left(\mathbb{E}\left[V|P=p,I=i\right] + \frac{\eta}{a}\right). \tag{II.28}$$

The above expression is derived under that assumption that both price, quantity and index are log-normally distributed:

$$\begin{pmatrix} \log P \\ \log Q \\ \log I \end{pmatrix} \overset{\mathbf{P}}{\sim} \mathcal{N}\left(\begin{bmatrix} m_{P1} \\ m_Q \\ m_{I1} \end{bmatrix}, \left(\begin{bmatrix} \sigma_P^2 & \sigma_P\sigma_Q\rho_{PQ} & \sigma_P\sigma_Q\rho_{PQ} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_Q^2 & \sigma_I\sigma_Q\rho_{QI} \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_I\sigma_Q\rho_{QI} & \sigma_I^2 \end{bmatrix}\right)\right). \tag{II.29}$$

Under the risk neutral measure, the mean value shifts to $\begin{bmatrix} m_{P2} & m_Q & m_{I2} \end{bmatrix}'$. In the end of this subsection, (II.28) is also derived for normally distributed quantity and index. In the following the notation from Id Brik and Roncoroni (2016a) is adopted:

$$\mathbf{u} = \begin{pmatrix} \log p \\ \log i \end{pmatrix}, \ \mathbf{U} = \begin{pmatrix} \log P \\ \log I \end{pmatrix}, \ m_{u1} = \mathbb{E}^P(\mathbf{U}), \ m_{u2} = \mathbb{E}^Q(\mathbf{U}),$$

$$\Sigma = \mathrm{Var}(\mathbf{U}), \ \mathbf{b} = \begin{pmatrix} \sigma_P\sigma_Q\rho_{PQ} \\ \sigma_I\sigma_Q\rho_{QI} \end{pmatrix}$$

The individual terms of (II.28) are now considered. First the density ratio of the marginal distributions:

$$\begin{aligned} \frac{f^Q_{PI}(p,i)}{f_{PI}(p,i)} &= \exp\left\{\tfrac{1}{2}\left(\mathbf{u}-m_{u1}\right)'\Sigma^{-1}\left(\mathbf{u}-m_{u1}\right) - \tfrac{1}{2}\left(\mathbf{u}-m_{u2}\right)'\Sigma^{-1}\left(\mathbf{u}-m_{u2}\right)\right\} \\ &= \exp\left\{\left(m_{u2}-m_{u1}\right)'\Sigma^{-1}\mathbf{u} - \tfrac{1}{2}\left(m_{u2}-m_{u1}\right)'\Sigma^{-1}\left(m_{u1}+m_{u2}\right)\right\} \end{aligned} \tag{II.30}$$

The expected value of this under $\mathbb{Q}$ is then assessed. Everything inside the $\exp\{\}$ is normally distributed with mean value $(m_{u2}-m_{u1})'\,\Sigma^{-1}m_{u2}-\frac{1}{2}\,(m_{u2}-m_{u2})'\,\Sigma^{-1}\,(m_{u1}+m_{u2})$ and variance $(m_{u2}-m_{u1})'\,\Sigma^{-1}\,(m_{u2}-m_{u1})$ This results in

$$\mathbb{E}^{Q}\left[\frac{f_{PI}^{Q}(P,I)}{f_{PI}(P,I)}\right]=\exp\left\{(m_{u2}-m_{u1})'\,\Sigma^{-1}\,(m_{u2}-m_{u1})\right\} \tag{II.31}$$

Next, the expected profit equals

$$\mathbb{E}\left[V|P=p,I=i\right]=(R-p)\mathbb{E}\left[Q|P=p,I=i\right]=(R-p)\mathbb{E}\left[e^{\log Q}|P=p,I=i\right]$$

As $\log Q$ given $P=p$ and $I=i$ is conditionally normal with mean and variance

$$m_Q+\mathbf{b}'\Sigma^{-1}(\mathbf{u}-m_{u1})$$

$$\sigma_Q^2-\mathbf{b}'\Sigma^{-1}\mathbf{b}$$

the conditional expected value of the unhedged profit is given as

$$\mathbb{E}\left[V|P=p,I=i\right]=(R-p)\exp\left\{m_Q+\tfrac{1}{2}\sigma_Q^2+\mathbf{b}'\Sigma^{-1}(\mathbf{u}-m_{u1}-\mathbf{b}/2)\right\}. \tag{II.32}$$

The expectation under Q value of this is

$$\begin{aligned}\mathbb{E}^{Q}\left[\mathbb{E}\left[V|P,I\right]\right]&=\mathbb{E}^{Q}\left[(R-P)\exp\left\{m_Q+\tfrac{1}{2}\sigma_Q^2+\mathbf{b}'\Sigma^{-1}(\mathbf{U}-m_{u1}-\mathbf{b}/2)\right\}\right]\\&=R\exp\left\{m_Q+\tfrac{1}{2}\sigma_Q^2\right\}\mathbb{E}^{Q}\left[\exp\left\{\mathbf{b}'\Sigma^{-1}(\mathbf{U}-m_{u1}-\mathbf{b}/2)\right\}\right]\\&\quad-\exp\left\{m_Q+\tfrac{1}{2}\sigma_Q^2\right\}\mathbb{E}^{Q}\left[\exp\left\{\begin{bmatrix}1&0\end{bmatrix}\mathbf{U}+\mathbf{b}'\Sigma^{-1}(\mathbf{U}-m_{u1}-\mathbf{b}/2)\right\}\right]\\&=\exp\left\{m_Q+\tfrac{1}{2}\sigma_Q^2+\mathbf{b}'\Sigma^{-1}(m_{u2}-m_{u1})\right\}\\&\quad\times\left(R-\exp\left\{m_{P2}+\tfrac{1}{2}\sigma_P^2+\sigma_P\sigma_Q\rho_{PQ}\right\}\right)\end{aligned} \tag{II.33}$$

Note that (II.30) and (II.31) would also hold if the price and/or index was normally distributed instead of assumption (II.29). It would only require a change of the definition of $\mathbf{u}$. The two other expressions (II.32) and (II.33) are also available in closed form for any of the eight normal/log-normal combination of (II.29). The derivation is restricted to the case where price is log-normally distributed and the quantity is normally distributed. The index may follow either distribution. When Q is normally distributed, then

$$\mathbb{E}\left[V|P=p,I=i\right]=(R-p)\left(m_Q+\mathbf{b}'\Sigma^{-1}(\mathbf{u}-m_{u1})\right) \tag{II.34}$$

with $\mathbf{u}$ defined appropriately. The expectation under Q is:

$$\begin{aligned}
\mathbb{E}^{Q}\left[\mathbb{E}\left[V|P,I\right]\right] &= \mathbb{E}^{Q}\left[(R-P)\left(m_{Q}+\mathbf{b}'\Sigma^{-1}(\mathbf{U}-m_{u1})\right)\right] \\
&= R\left(m_{Q}+\mathbf{b}'\Sigma^{-1}(m_{u2}-m_{u1})\right)-m_{Q}e^{m_{P2}+\frac{\sigma_P^2}{2}} \\
&\quad -\mathbf{b}'\Sigma^{-1}\mathbb{E}^{Q}\begin{bmatrix} PU_1 \\ PU_2 \end{bmatrix}+\mathbf{b}'\Sigma^{-1}m_{u1}e^{m_{P2}+\frac{\sigma_P^2}{2}}
\end{aligned}$$

For log-normally distributed price (i.e., $U_1 = \log P$), then

$$\mathbb{E}^{Q}\left[P\log P\right] = e^{m_{P2}+\frac{\sigma_P^2}{2}}\left(\sigma_P^2+m_{P2}\right)$$

and

$$\mathbb{E}^{Q}\left[PU_2\right] = e^{m_{P2}+\frac{\sigma_P^2}{2}}\left(\sigma_P\sigma_I\rho_{PI}+m_{I2}\right),$$

resulting in

$$\mathbb{E}^{Q}\left[\mathbb{E}\left[V|P,I\right]\right] = \left(R-e^{m_{P2}+\frac{\sigma_P^2}{2}}\right)\left(m_{Q}+\mathbf{b}'\Sigma^{-1}(m_{u2}-m_{u1})\right)-e^{m_{P2}+\frac{\sigma_P^2}{2}}\mathbf{b}'\Sigma^{-1}\begin{bmatrix} \sigma_P^2 \\ \sigma_P\sigma_I\rho_{PI} \end{bmatrix} \tag{II.35}$$

II.A.2 Additive hedge

The additive hedge using a mean-variance approach consists of two functions, one of price value p and one on index value i. They are denoted $h^*(p)$ and $g^*(i)$ and are given as

$$\begin{pmatrix} h^*(p) & g^*(i) \end{pmatrix}' = \mathbf{b}(p,i)-A(p,i)\left\{\mathbb{E}^{Q}\left[A(P,I)\right]\right\}^{-1}\mathbb{E}^{Q}\left[\mathbf{b}(P,I)\right] \tag{II.36}$$

where

$$\begin{aligned}
A(p,i) &= \begin{pmatrix} \frac{f_P^Q(p)}{f_P(p)} & \frac{f_P^Q(p)}{f_P(p)}-1 \\ \frac{f_I^Q(i)}{f_I(i)}-1 & \frac{f_I^Q(i)}{f_I(i)} \end{pmatrix} \\
\mathbf{b}(p,i) &= \begin{pmatrix} \frac{1}{a}-\mathbb{E}\left[V|P=p\right]+\left(\mathbb{E}\left[V\right]-\frac{1}{a}\right)\frac{f_P^Q(p)}{f_P(p)} \\ \frac{1}{a}-\mathbb{E}\left[V|I=i\right]+\left(\mathbb{E}\left[V\right]-\frac{1}{a}\right)\frac{f_I^Q(i)}{f_I(i)} \end{pmatrix}
\end{aligned}$$

The above expression is derived under that assumption that price, quantity and index are log-normally distributed (again, the assumption regarding quantity is later replaced by a normal distribution). Further, it is assumed that price and index is uncorrelated:

$$\begin{pmatrix} \log P \\ \log Q \\ \log I \end{pmatrix} \overset{\mathbf{P}}{\sim} \mathcal{N}\left(\begin{bmatrix} m_{P1} \\ m_{Q} \\ m_{I1} \end{bmatrix}, \left(\begin{bmatrix} \sigma_P^2 & \sigma_P\sigma_Q\rho_{PQ} & 0 \\ \sigma_P\sigma_Q\rho_{PQ} & \sigma_Q^2 & \sigma_I\sigma_Q\rho_{QI} \\ 0 & \sigma_I\sigma_Q\rho_{QI} & \sigma_I^2 \end{bmatrix}\right)\right).$$

As before; under the risk neutral measure, the mean value shifts to $\begin{bmatrix} m_{P2} & m_Q & m_{I2} \end{bmatrix}'$. The individual expressions of (II.36) are now considered. First the ratio of the marginal densities:

$$\begin{aligned} \frac{f_P^Q(p)}{f_P(p)} &= \exp\left\{\tfrac{1}{2\sigma_P^2}\left(\log p - m_{P1}\right)^2 - \tfrac{1}{2\sigma_P^2}\left(\log p - m_{P2}\right)^2\right\} \\ &= \exp\left\{\tfrac{1}{\sigma_P^2}\left(m_{P2} - m_{P1}\right)\log p - \tfrac{1}{2\sigma_P^2}\left(m_{P2} - m_{P1}\right)\left(m_{P1} + m_{P2}\right)\right\} \end{aligned} \tag{II.37}$$

The expected value of this under $\mathbb{Q}$ is then calculated. Everything inside the exp{} is normally distributed with mean value $\frac{1}{\sigma_P^2}\left(m_{P2} - m_{P1}\right)m_{P2} - \frac{1}{2\sigma_P^2}\left(m_{P2} - m_{P1}\right)\left(m_{P1} + m_{P2}\right)$ and variance $\frac{1}{\sigma_P^2}\left(m_{P2} - m_{P1}\right)^2$ This results in

$$\mathbb{E}^Q\left[\frac{f_P^Q(P)}{f_P(P)}\right] = \exp\left\{\tfrac{1}{\sigma_P^2}\left(m_{P2} - m_{P1}\right)^2\right\}. \tag{II.38}$$

The same expressions holds for the index. Next, the expected profit expressions:

$$\mathbb{E}\left[V|P = p\right] = (R - p)\mathbb{E}\left[Q|P = p\right] = (R - p)\mathbb{E}\left[e^{\log Q}|P = p\right]$$

As $\log Q$ given $P = p$ is conditionally normal with mean and variance

$$m_Q + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(\log p - m_{P1}) \text{ and } \sigma_Q^2(1 - \rho_{PQ}^2)$$

the conditional expected value of the unhedged profit is given as

$$\mathbb{E}\left[V|P = p\right] = (R - p)\exp\left\{m_Q + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(\log p - m_{P1}) + \tfrac{1}{2}\sigma_Q^2(1 - \rho_{PQ}^2)\right\}. \tag{II.39}$$

The expectation under Q value of this is

$$\begin{aligned} \mathbb{E}^Q\left[\mathbb{E}\left[V|P\right]\right] &= \mathbb{E}^Q\left[(R - P)\exp\left\{m_Q + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(\log p - m_{P1}) + \tfrac{1}{2}\sigma_Q^2(1 - \rho_{PQ}^2)\right\}\right] \\ &= R\exp\left\{m_Q + \tfrac{1}{2}\sigma_Q^2(1 - \rho_{PQ}^2)\right\}\mathbb{E}^Q\left[\exp\left\{\rho_{PQ}\frac{\sigma_Q}{\sigma_P}(\log p - m_{P1})\right\}\right] \\ &\quad - \exp\left\{m_Q + \tfrac{1}{2}\sigma_Q^2(1 - \rho_{PQ}^2) - \rho_{PQ}\frac{\sigma_Q}{\sigma_P}m_{P1}\right\}\mathbb{E}^Q\left[\exp\left\{\left(1 + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}\right)\log p\right\}\right] \\ &= \exp\left\{m_Q + \tfrac{1}{2}\sigma_Q^2 + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(m - m_{P1})\right\}\left(R - \exp\left\{m_{p2} + \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_Q\sigma_P\right\}\right) \end{aligned} \tag{II.40}$$

Expected unhedged profits given $I = i$:

$$\mathbb{E}\left[V|I = i\right] = \mathbb{E}\left[(R - P)Q|I = i\right] = R\mathbb{E}\left[e^{\log Q}|I = i\right] - \mathbb{E}\left[e^{\log P + \log Q}|I = i\right]$$

As $(\log P, \log Q)$ given $I = i$ is bivariate normal with mean and variance

$$\begin{pmatrix} m_{P1} \\ m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(\log i - m_{I1}) \end{pmatrix} \text{ and } \begin{pmatrix} \sigma_P^2 & \rho_{PQ}\sigma_P\sigma_Q \\ \rho_{PQ}\sigma_P\sigma_Q & \sigma_Q^2(1-\rho_{QI}^2) \end{pmatrix},$$

the expected value of the unhedged profit conditional on $I = i$ is given as

$$\begin{aligned} \mathbb{E}\left[V|I=i\right] = \left(R - \exp\left\{m_{P1} + \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_P\sigma_Q\right\}\right) \times \\ \exp\left\{m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(\log i - m_{I1}) + \tfrac{1}{2}\sigma_Q^2(1-\rho_{QI}^2)\right\}. \end{aligned} \tag{II.41}$$

Taking Q expectations of this yields

$$\mathbb{E}^Q\left[\mathbb{E}\left[V|I\right]\right] = \left(R - \exp\left\{m_{P1} + \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_P\sigma_Q\right\}\right) \exp\left\{m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(m_{I2} - m_{I1}) + \tfrac{1}{2}\sigma_Q^2\right\}. \tag{II.42}$$

The unconditional P-expectation of the unhedged profit becomes

$$\mathbb{E}\left[V\right] = \mathbb{E}\left[\mathbb{E}\left[V|I\right]\right] = \left(R - \exp\left\{m_{P1} + \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_P\sigma_Q\right\}\right) \exp\left\{m_Q + \tfrac{1}{2}\sigma_Q^2\right\}. \tag{II.43}$$

If Q instead of $\log Q$ given $P = p$ is conditionally normal, then the conditional expected value of the unhedged profit is given as

$$\mathbb{E}\left[V|P=p\right] = (R-p)\left\{m_Q + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(\log p - m_{P1})\right\} \tag{II.44}$$

and the expectation under Q is

$$\mathbb{E}^Q\left[\mathbb{E}\left[V|P\right]\right] = \left(R - e^{m_{P2}+\frac{\sigma_P^2}{2}}\right)\left\{m_Q + \rho_{PQ}\frac{\sigma_Q}{\sigma_P}(m_{p2} - m_{P1})\right\} - e^{m_{P2}+\frac{\sigma_P^2}{2}}\rho_{PQ}\sigma_Q\sigma_P. \tag{II.45}$$

The expected unhedged profit given $I = i$ becomes:

$$\mathbb{E}\left[V|I=i\right] = \mathbb{E}\left[(R-P)Q|I=i\right] = R\mathbb{E}\left[Q|I=i\right] - \mathbb{E}\left[Qe^{\log P}|I=i\right].$$

As $(\log P, Q)$ given $I = i$ is bivariate normal with mean and variance

$$\begin{pmatrix} m_{P1} \\ m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(\log i - m_{I1}) \end{pmatrix} \text{ and } \begin{pmatrix} \sigma_P^2 & \rho_{PQ}\sigma_P\sigma_Q \\ \rho_{PQ}\sigma_P\sigma_Q & \sigma_Q^2(1-\rho_{QI}^2) \end{pmatrix},$$

the conditional on i expected value of the unhedged profit is given as

$$\mathbb{E}\left[V|I=i\right] = (R - e^{m_{P1}+\frac{1}{2}\sigma_P^2})\left(m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(\log i - m_{I1})\right) - e^{m_{P1}+\frac{1}{2}\sigma_P^2}\sigma_P\sigma_Q\rho_{PQ} \tag{II.46}$$

Taking Q expectations of this yields

$$\mathbb{E}^Q\left[\mathbb{E}\left[V|I\right]\right] = (R - e^{m_{P1}+\frac{1}{2}\sigma_P^2})\left(m_Q + \rho_{QI}\frac{\sigma_Q}{\sigma_I}(m_{I2} - m_{I1})\right) - e^{m_{P1}+\frac{1}{2}\sigma_P^2}\sigma_P\sigma_Q\rho_{PQ} \quad \text{(II.47)}$$

with the unconditional P-expectation of the unhedged profit:

$$\mathbb{E}\left[V\right] = \mathbb{E}\left[\mathbb{E}\left[V|I\right]\right] = (R - e^{m_{P1}+\frac{1}{2}\sigma_P^2})m_Q - e^{m_{P1}+\frac{1}{2}\sigma_P^2}\sigma_P\sigma_Q\rho_{PQ} \quad \text{(II.48)}$$

Essay III

Pricing and Hedging Energy Quanto Options*

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Abstract

In energy markets, the use of quanto options has increased significantly in recent years. The payoff from such options are typically written on an underlying energy index and a measure of temperature. They are suited to managing the joint price and volume risk in energy markets. Using a Heath-Jarrow-Morton approach, we derive a closed-form option pricing formula for energy quanto options under the assumption that the underlying assets are lognormally distributed. Our approach encompasses several interesting cases, such as geometric Brownian motions and multifactor spot models. We also derive Delta and Gamma expressions for hedging. Further, we illustrate the use of our model by an empirical pricing exercise using New York Mercantile Exchange-traded natural gas futures and Chicago Mercantile Exchange-traded heating degree days futures for New York.

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III.1 Introduction

Many industries are exposed to the variability of the weather. Take, as an example, a gas distribution company that operates in an open wholesale market. Their planned sales volumes per day and the market price are the two main factors to which they are exposed. If, for example, one of the winter months turns out to be warmer than usual, the demand for gas would drop. This decline in demand would probably also affect the market price for gas, leading to a drop in gas price. The firm would make a loss compared with their planned revenue, which is equal to the shortfall in demand multiplied by the difference between the retail price at which they would have sold had their customers bought the gas, and the market price at which they must now sell their excess gas. So, they face not only a direct weather effect, eg, the lower demand, but also an indirect effect through the drop in market prices. The above example clearly illustrates that the adverse movements in market price and demand due to higher temperatures represent a kind of correlation risk, which is difficult to properly hedge against, as it leads to a heavier tailed profit-and-loss distribution. Using standard weather derivatives as offered by the Chicago Mercantile Exchange (CME) would most likely represent an imperfect and rather expensive hedging strategy, as it accounts only for the direct earnings effect from the change in demand, and not the indirect earnings effect from price changes. If standardized weather products are insufficient as hedging tools, the companies must turn to over-the-counter (OTC) markets for weather derivatives. Davis (2010) and Pérez-González and Yun (2013) refer to surveys conducted by the Weather Risk Management Association (WRMA) and reports from the CME about market sizes and expected development:[1] the market for standardized weather derivatives peaked in 2007 with a total volume of trades close to 930,000 and a corresponding notional value of US$17.9 billion. Since 2008, the market for standardized contracts has experienced severe retrenchment. In 2009, the total volume of trades dipped below 500,000, amounting to a notional value of around US$5.3 billion. A big part of this sharp decline is attributed to the substantial increase, eg, 30% from 2010 to 2011, in the market for tailor-made contracts, especially the quantity-adjusting weather contracts (”quanto”). Contracts of this type worth US$100 million have been reported. Market participants indicate that the demand for quanto contracts is international, with transactions being executed in the United States, Europe, Australia and South America. In 2010, the WRMA believed that the developing market in India alone had a potential value of US$2.35 billion.

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[1] Although the reported numbers are small compared with other markets, weather-exposed utilities can use weather derivatives to reduce extreme losses from weather incidents and increase the valuation of the company (see Pérez-González and Yun (2013) for an extensive study of the effect of weather derivatives on firm value, investments and leverage).

The label "quanto options" has traditionally been assigned to a class of derivatives in financial markets where the investor wishes to be exposed to price movements in the foreign asset without the corresponding exchange rate risk. The pricing of currency quanto options has been extensively researched and dates back to the original work of Garman and Kohlhagen (1983). Although the same term is used for the specific type of energy options that we study in this paper, these two types of derivatives contracts are different: a typical currency quanto option has a regular call-put payoff structure, whereas the energy quanto options we study have a payoff structure similar to a product of call-put options, and energy quanto options are therefore mainly used to hedge exposure to the joint price and volume risk.[2] In comparison with studies of currency quantos, research related to the pricing of quanto options in energy markets is scarce. One exception is Caporin et al. (2012), who propose a bivariate time-series model to capture the joint dynamics of energy prices and temperature. In particular, they model the energy price and the average temperature using a sophisticated parameter-intensive econometric model. Since they aim to capture features such as seasonality in means and variances, long memory, autoregressive patterns and dynamic correlations, the complexity of their model leaves no other option than simulation-based procedures to calculate prices. Moreover, they leave the issue of how we should hedge such options unanswered.

In order for quanto contracts to provide a superior risk management tool compared with standardized futures contracts, it is crucial that there is a significant correlation between the two underlying assets. In energy markets, the payoff of a quanto option is determined by the level of both the energy price and an index related to weather. This correlation has been studied by, for example, Engle et al. (1992), who documents that temperature is important in forecasting electricity prices, and Timmer and Lamb (2007), who document a strong relationship between natural gas prices and heating degree days (HDD).

In this paper, we also study the pricing of energy quanto options. However, unlike Caporin et al. (2012), we derive analytical solutions to the option pricing problem. Such closed-form solutions are easy to implement, fast to calculate and, most importantly, they give a clear answer as to how the energy quanto option should be properly hedged. We convert the pricing problem by using traded futures contracts on energy and a temperature index as underlying assets, rather than energy spot prices and temperature. We are able to do this because the typical energy quanto options have a payoff that can be represented as an "Asian" structure on the energy spot price and the temperature index. The markets for energy and weather organize futures with delivery periods, which will coincide with the aggregate or average spot price and temperature index at the end of the delivery period. Hence, any "Asian payoff" on the spot and temperature for a quanto option can be viewed as a "European payoff" on the corresponding futures contracts. This insight is the key to our solution and the main contribution of this paper. The analytical solution also gives the desirable feature that we can hedge the quanto option in terms of traded instruments, namely the underlying futures contracts that – unlike temperature and spot power/gas – can be easily bought and held.

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[2] This double-call structure was also studied by Jørgensen (2007) for the case of interest rates and stock prices.

Using a Heath-Jarrow-Morton (HJM) approach, we derive options prices under the assumption that futures dynamics are lognormally distributed with a possibly time-varying volatility. Furthermore, we explicitly derive Delta-hedging and cross-Gamma hedging parameters. Our approach encompasses several models for the underlying futures prices, such as the standard bivariate geometric Brownian motion (GBM) and the two-factor model proposed in papers such as Schwartz and Smith (2000), Sørensen (2002) and Lucia and Schwartz (2002). The latter class of models allows for time-varying volatility, which is a stylized fact for many commodities. We include an extensive empirical example to illustrate our findings. Using futures contracts on natural gas and the HDD temperature index, we estimate relevant parameters in the seasonal two-factor model of Sørensen (2002) based on data collected from the New York Mercantile Exchange (NYMEX) and the CME. We compute prices for various energy quanto options and benchmark these against products of plain-vanilla European options on gas and HDD futures. The latter can be priced by the classical Black (1976) option pricing formula and corresponds to the case of the energy quanto option for independent gas and temperature futures. In Section III.2, we discuss the structure of energy quanto options and introduce the pricing problem. In Section III.3, we derive the pricing and hedging formulas and show how the model of futures price dynamics in Sørensen (2002) is a special case of our general framework. Section III.4 presents the empirical case study, and Section III.5 concludes.

III.2 The contract structure and pricing of energy quanto options

In this section, we first discuss typical examples of energy quanto options. We then argue that the pricing problem can be simplified using standardized futures contracts as underlying assets.

III.2.1 Contract structure

Most energy quanto contracts have payoffs that are triggered by two underlying "assets", temperature and energy price. Since these contracts are tailor made, rather than standardized, the contract design varies. In its simplest form, a quanto contract has a payoff function S:

$$S = (T_{\text{var}} - T_{\text{fix}}) \times (E_{\text{var}} - E_{\text{fix}}) \tag{III.1}$$

Payoff is determined by the difference between some variable temperature measure (T_{var}) and some fixed temperature measure (T_{fix}), multiplied by the difference between variable and fixed energy price (E_{var} and E_{fix}). Note that the payoff might be negative, indicating that the buyer of the contract pays the required amount to the seller.

Entering into a quanto contract of this type might be risky, since the downside may potentially become large. For hedging purposes, it seems more reasonable to buy a quanto structure with optionality, thereby eliminating all downside risk. In Table III.1, we show a typical example of how a quanto option might be structured (see also Caporin et al. (2012) for a discussion of the design of the energy quanto option). The example contract has a payoff that is triggered by an average gas price denoted E (defined as the average of daily prices for the last month). It also offers an exposure to temperature through the accumulated number of HDD in the corresponding month. The HDD index is commonly used as the underlying variable for temperature derivatives and is defined as how much the average temperature over a day has deviated below a pre-set level. We denote the accumulated number of HDD over interval $[\tau_1, \tau_2]$ by $I_{[\tau_1,\tau_2]}$:

$$I_{[\tau_1,\tau_2]} = \sum_{t=\tau_1}^{\tau_2} \text{HDD}_t = \sum_{t=\tau_1}^{\tau_2} \max(c - T_t, 0) \tag{III.2}$$

where c is some pre-specified temperature threshold ($65°F$ or $18°C$) and T_t is the mean temperature on day t. If the number of HDD I and the average gas price E are above the high strikes ($\overline{K}_I$ and $\overline{K}_E$ respectively), the owner of the option would receive a payment

	Nov	Dec	Jan	Feb	Mar
(a) High Strike (HDDs)	$\overline{K}_I^{11}$	$\overline{K}_I^{12}$	$\overline{K}_I^{1}$	$\overline{K}_I^{2}$	$\overline{K}_I^{3}$
(b) Low Strike (HDDs)	$\underline{K}_I^{11}$	$\underline{K}_I^{12}$	$\underline{K}_I^{1}$	$\underline{K}_I^{2}$	$\underline{K}_I^{3}$
(a) High Strike (Price/mmBtu)	$\overline{K}_E^{11}$	$\overline{K}_E^{12}$	$\overline{K}_E^{1}$	$\overline{K}_E^{2}$	$\overline{K}_E^{3}$
(b) Low Strike (Price/mmBtu)	$\underline{K}_E^{11}$	$\underline{K}_E^{12}$	$\underline{K}_E^{1}$	$\underline{K}_E^{2}$	$\underline{K}_E^{3}$
Volume (mmBtu)	200	300	500	400	250

The underlying process triggering payouts to the option holder is accumulated number of heating-degree days I and monthly index gas price E. As an example the payoff for November will be: (a) In cold periods - $\max(I - \overline{K}_I, 0) \times \max(E - \overline{K}_E, 0) \times$ Volume. (b) In warm periods - $\max(\underline{K}_I - I, 0) \times \max(\underline{K}_E - E, 0) \times$ Volume. We see that the option pays out if both the underlying temperature and price variables exceed (dip below) the high strikes (low strikes).

Table III.1. A specification of a typical energy quanto option.

equal to the pre-specified volume multiplied by the actual number of HDD minus the strike $\overline{K}_I$, multiplied by the difference between the average energy price and the strike price $\overline{K}_E$. On the other hand, if it is warmer than usual and the number of HDD dips below the lower strike of $\underline{K}_I$ and the energy price at the same time is lower than $\underline{K}_E$, the owner receives a payout equal to the volume multiplied by $\underline{K}_I$ less the actual number of HDDs multiplied by the difference between the strike price $\underline{K}_E$ and the average energy price. Note that the volume adjustment is varying between months, reflecting the fact that "unusual" temperature changes might have a stronger impact on the option holder's revenue in the coldest months like December and January. Also note that the price strikes may vary between months.

This example illustrates why quanto options might be a good alternative to more standardized derivatives. The structure of the contracts takes into account the fact that extreme temperature variations might affect both demand and prices, and compensates the owner of the option by making payouts contingent on both prices and temperatures. The great possibility of tailoring these contracts provides potential customers with a powerful and efficient hedging instrument.

III.2.2 Pricing Using Terminal Value of Futures

As described above energy quanto options have a payoff which is a function of two underlying assets, temperature and price. We focus on a class of energy quanto options which has a payoff function $f(E, I)$, where E is an index of the energy price and I an index of temperature. To be more specific, we assume that the energy index E is given as the average spot price over some measurement period $[\tau_1, \tau_2]^3$, $\tau_1 < \tau_2$,

$$E = \frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{\tau_2} S_u,$$

where S_u denotes the energy spot price. Further, we assume that the temperature index is defined as

$$I = \sum_{u=\tau_1}^{\tau_2} g(T_u)$$

for T_u being the temperature at time u and g some function. For example, if we want to consider a quanto option involving the HDD index, we choose $g(x) = \max(18 - x, 0)$. The quanto option is exercised at time τ_2, and its arbitrage-free price C_t at time $t \leq \tau_2$ is defined as by the following expression:

$$C_t = e^{-r(\tau_2 - t)} \mathbb{E}_t^{\mathbb{Q}} \left[f\left(\frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{\tau_2} S_u, \sum_{u=\tau_1}^{\tau_2} g(T_u) \right) \right]. \tag{III.3}$$

Here, $r > 0$ denotes the risk-free interest rate, which we for simplicity assumes constant. The pricing measure is denoted $\mathbb{Q}$, and $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ is the expectation operator with respect to $\mathbb{Q}$, conditioned on the market information at time t given by the filtration $\mathcal{F}_t$.

We now argue how to relate the price of the quanto option to futures contracts on the energy and temperature indexes E and I. Observe that the price at time $t \leq \tau_2$ of a futures contract written on some energy price, (eg, natural gas) with delivery period $[\tau_1, \tau_2]$ is given by

$$F_t^E(\tau_1, \tau_2) = \mathbb{E}_t^{\mathbb{Q}} \left[\frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{\tau_2} S_u \right].$$

At time $t = \tau_2$, we find from the conditional expectation that

$$F_{\tau_2}^E(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{\tau_2} S_u,$$

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[3] Technically we should write $\tau_2 - \tau_1 + 1$, when using the discretely computed average. To ease the notation, we keep $\tau_2 - \tau_1$ to determine the average over the period.

ie, the futures price is exactly equal to what is being delivered. Applying the same argument to a futures written on the temperature index, with price dynamics denoted $F_t^I(\tau_1, \tau_2)$, we immediately see that the following must be true for the quanto option price:

$$\begin{aligned} C_t &= e^{-r(\tau_2 - t)} \mathbb{E}_t^{\mathbb{Q}} \left[f \left(\frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{\tau_2} S_u, \sum_{u=\tau_1}^{\tau_2} g(T_u) \right) \right] \\ &= e^{-r(\tau_2 - t)} \mathbb{E}_t^{\mathbb{Q}} \left[f \left(F_{\tau_2}^E(\tau_1, \tau_2), F_{\tau_2}^I(\tau_1, \tau_2) \right) \right]. \end{aligned} \tag{III.4}$$

Equation (III.4) shows that the price of a quanto option with payoff being a function of the energy index E and temperature index I must be the same as if the payoff was a function of the terminal values of two futures contracts written on the energy and temperature indexes, and with the delivery period being equal to the contract period specified by the quanto option. Hence, we view the quanto option as an option written on the two futures contracts, rather than on the two indexes. This is advantageous from the point of view that the futures are traded financial assets. We note in passing that we may extend the above argument to quanto options where the measurement periods of the energy and the temperature indexes are not the same.

To compute the price in (III.4) we must have a model for the futures price dynamics $F_t^E(\tau_1, \tau_2)$ and $F_t^I(\tau_1, \tau_2)$. The dynamics must account for the dependency between the two futures, as well as their marginal behaviour. The pricing of the energy quanto option has thus been transferred from modelling the joint spot energy and temperature dynamics, followed by computing the $\mathbb{Q}$-expectation of an index of these, to modelling the joint futures dynamics and pricing a European-type option on these. The former approach is similar to pricing an Asian option, which for most relevant models and cases is a highly difficult task. Remark also that by modelling and estimating the futures dynamics to market data, we can easily obtain the market-implied pricing measure $\mathbb{Q}$. We will see this in practice in Section III.4, where we analyse the case of gas and HDD futures. If we choose to model the underlying energy spot prices and temperature dynamics, one obtains the dynamics under the market probability $\mathbb{P}$, rather than under the pricing measure $\mathbb{Q}$. Additional hypotheses must be made in the model to obtain this. Moreover, for most interesting cases, the quanto option must be priced by Monte Carlo or some other computationally demanding method (see Caporin et al. (2012)). Finally, but no less importantly, with the representation in (III.4) at hand we can discuss the issue of hedging energy quanto options in terms of the underlying futures contracts.

In many energy markets, the futures contracts are not traded within their delivery period. That means that we can only use the market for futures up to time τ_1. This has a clear consequence on the possibility to hedge these contracts, as a hedging strategy inevitably will be a continuously rebalanced portfolio of the futures up to the exercise time τ_2. As this is possible to perform only up to time τ_1 in many markets, we face an incomplete market situation where the quanto option cannot be hedged perfectly. Moreover, it is to be expected that the dynamics of the futures price have different characteristics within the delivery period than prior to start of delivery, if it can be traded for times $t \in (\tau_1, \tau_2]$. The reason being that we have less uncertainty as the remaining delivery period of the futures become shorter. In this paper we will restrict our attention to the pricing of quanto options at times $t \leq \tau_1$. The entry time of such a contract is most naturally taking place prior to delivery period. However, for marking-to-market purposes, one is interested in the price C_t also for $t \in (\tau_1, \tau_2]$. The issuer of the quanto option may be interested in hedging the exposure, and therefore also be concerned of the behaviour of prices within the delivery period.

Before we start looking into the details of pricing quanto options, we investigate the option contract of the type described in section III.2.1 in more detail. This contract covers a period of five months, from November through to March. Since this contract is essentially a sum of one-period contracts, we focus our attention on an option covering only one month of delivery period $[\tau_1, \tau_2]$. Recall that the payoff in the contract is a function of some average energy price and accumulated number of HDD. From the discussion in the previous section we know that rather than using spot price and HDD as underlying assets, we can instead use the terminal value of futures contracts written on price and HDD, respectively. The payoff function $p(F^E_{\tau_2}(\tau_1, \tau_2), F^I_{\tau_2}(\tau_1, \tau_2), \overline{K}_E, \overline{K}_I, \underline{K}_E, \underline{K}_I) = p$ of this quanto contract is defined as

$$\begin{aligned} p = \gamma \big[&\max\big(F^E_{\tau_2}(\tau_1, \tau_2) - \overline{K}_E, 0\big) \max\big(F^I_{\tau_2}(\tau_1, \tau_2) - \overline{K}_I, 0\big) \\ &+ \max\big(\underline{K}_E - F^E_{\tau_2}(\tau_1, \tau_2), 0\big) \max\big(\underline{K}_I - F^I_{\tau_2}(\tau_1, \tau_2), 0\big)\big], \end{aligned} \tag{III.5}$$

where γ is the contractual volume adjustment factor. Note that the payoff function in this contract consists of two parts, the first taking care of the situation in which temperatures are colder (and prices higher) than usual, and the second taking care of the situation in which temperatures are warmer than usual (and prices lower than usual). The first part is a product of two call options, whereas the second part is a product of two put options. To illustrate our pricing approach in the simplest possible way it suffices to look at the product call structure with the volume adjuster γ normalized to 1, ie, we want to price an option with the following payoff function:

$$\hat{p}\left(F^E_{\tau_2}(\tau_1,\tau_2), F^I_{\tau_2}(\tau_1,\tau_2), \overline{K}_E, \overline{K}_I\right) = \max\left(F^E_{\tau_2}(\tau_1,\tau_2) - \overline{K}_E, 0\right)\left(F^I_{\tau_2}(\tau_1,\tau_2) - \overline{K}_I, 0\right). \tag{III.6}$$

In the remainder of this paper, we will focus on this particular choice of a payoff function for the energy quanto option. It corresponds to choosing the function f as $f(E,I) = \max(E - \overline{K}_E, 0)\max(I - \overline{K}_I, 0)$ in (III.4). Other combinations of put-call mixes, as well as different delivery periods for the energy and temperature futures can easily be studied by a simple modification of what follows.

III.3 Asset Price Dynamics and Option Prices

Suppose that the two futures price dynamics under the pricing measure $\mathbb{Q}$ can be expressed as

$$F^E_T(\tau_1,\tau_2) = F^E_t(\tau_1,\tau_2)\exp(\mu_E + X) \tag{III.7}$$

$$F^I_T(\tau_1,\tau_2) = F^I_t(\tau_1,\tau_2)\exp(\mu_I + Y) \tag{III.8}$$

where $t \leq T \leq \tau_1$, and X, Y are two random variables independent of $\mathcal{F}_t$, but depending on t, T, τ_1 and τ_2. We suppose that (X,Y) is a bivariate normally distributed random variable with mean zero. We define

$$\sigma_X^2 = \mathrm{var}(X), \quad \sigma_Y^2 = \mathrm{var}(Y) \quad \text{and} \quad \rho_{X,Y} = \mathrm{cov}(X,Y)$$

Obviously, σ_X, σ_Y and $\rho_{X,Y}$ are depending on t, T, τ_1 and τ_2. Moreover, as the futures price naturally is a martingale under the pricing measure $\mathbb{Q}$, we have $\mu_E = -\sigma_X^2/2$ and $\mu_I = -\sigma_Y^2/2$.

Our general representation of the futures price dynamics III.7 and III.8 encompasses many interesting models. For example, a bivariate geometric Brownian motion looks like

$$F^E_T(\tau_1,\tau_2) = F^E_t(\tau_1,\tau_2)\exp\left(-\frac{1}{2}\sigma_E^2(T-t) + \sigma_E(W_T - W_t)\right)$$

$$F^I_T(\tau_1,\tau_2) = F^I_t(\tau_1,\tau_2)\exp\left(-\frac{1}{2}\sigma_I^2(T-t) + \sigma_I(B_T - B_t)\right)$$

with two Brownian motions W and B being correlated. We can easily associate this GBM to the general set-up above by setting $\mu_E = -\sigma_E^2(T-t)/2$, $\mu_I = -\sigma_I^2(T-t)/2$, $\sigma_X = \sigma_E\sqrt{T-t}$,

$\sigma_Y = \sigma_I\sqrt{T-t}$, and $\rho_{X,Y}$ being the correlation between the two Brownian motions. In section III.3.2, we show that also the two-factor model by Schwartz and Smith (2000) and the extension by Sørensen (2002) fits this framework.

III.3.1 A General Solution for the Quanto Option Price and Hedge

The price of the quanto option at time t is

$$C_t = e^{-r(\tau_2-t)}\mathbb{E}_t^{\mathbb{Q}}\left[\hat{p}\left(F^E_{\tau_2}(\tau_1,\tau_2), F^I_{\tau_2}(\tau_1,\tau_2), \overline{K}_E, \overline{K}_I\right)\right] \tag{III.9}$$

where the notation $\mathbb{E}^{\mathbb{Q}}$ states that the expectation is taken under the pricing measure $\mathbb{Q}$. Given these assumptions, Proposition 1 below states the closed-form solution of the energy quanto option.

Proposition III.1 For two assets following the dynamics given by (III.7) and (III.8), the time t market price of an European energy quanto option with exercise at time τ_2 and payoff described by (III.6) is given by

$$\begin{aligned} C_t = e^{-r(\tau_2-t)}\Big(& F^E_t(\tau_1,\tau_2)F^I_t(\tau_1,\tau_2)e^{\rho_{X,Y}\sigma_X\sigma_Y}M(y_1^{***},y_2^{***};\rho_{X,Y}) \\ & - F^E_t(\tau_1,\tau_2)\overline{K}_I M\left(y_1^{**},y_2^{**};\rho_{X,Y}\right) \\ & - F^I_t(\tau_1,\tau_2)\overline{K}_E M\left(y_1^{*},y_2^{*};\rho_{X,Y}\right) + \overline{K}_E\overline{K}_I M\left(y_1,y_2;\rho_{X,Y}\right)\Big) \end{aligned} \tag{III.10}$$

where

$$\begin{aligned} y_1 &= \frac{\log(F^E_t(\tau_1,\tau_2)) - \log(\overline{K}_E) - \frac{1}{2}\sigma_X^2}{\sigma_X} & y_2 &= \frac{\log(F^I_t(\tau_1,\tau_2)) - \log(\overline{K}_I) - \frac{1}{2}\sigma_Y^2}{\sigma_Y} \\ y_1^* &= y_1 + \rho_{X,Y}\sigma_Y & y_2^* &= y_2 + \sigma_Y \\ y_1^{**} &= y_1 + \sigma_X & y_2^{**} &= y_2 + \rho_{X,Y}\sigma_X \\ y_1^{***} &= y_1 + \rho_{X,Y}\sigma_Y + \sigma_X & y_2^{***} &= y_2 + \rho_{X,Y}\sigma_X + \sigma_Y \end{aligned}$$

Here $M(x,y;\rho)$ denotes the standard bivariate normal cumulative distribution function with correlation ρ. □

Proof: Observe that the payoff function in (III.6) can be rewritten in the following way:

$$\begin{aligned} \hat{p}(F^E,F^I,\overline{K}_E,\overline{K}_I) =& \max(F^E-\overline{K}_E,0)\max(F^I-\overline{K}_I,0) \\ =& \left(F^E-\overline{K}_E\right)\left(F^I-\overline{K}_I\right)1_{\{F^E>\overline{K}_E\}}1_{\{F^I>\overline{K}_I\}} \\ =& F^EF^I1_{\{F^E>\overline{K}_E\}}1_{\{F^I>\overline{K}_I\}} - F^E\overline{K}_I1_{\{F^E>\overline{K}_E\}}1_{\{F^I>\overline{K}_I\}} \\ & - F^I\overline{K}_E1_{\{F^E>\overline{K}_E\}}1_{\{F^I>\overline{K}_I\}} + \overline{K}_E\overline{K}_I1_{\{F^E>\overline{K}_E\}}1_{\{F^I>\overline{K}_I\}} \end{aligned}$$

The problem of finding the market price of the European quanto option is thus equivalent to the problem of calculating the expectations under the pricing measure $\mathbb{Q}$ of the four terms above. The four expectations are derived in Appendix III.A in details. $\blacksquare$

Based on III.10 we derive the Delta and cross-Gamma hedging parameters, which can be straightforwardly calculated by partial differentiation of the price C_t with respect to the futures prices. All hedging parameters are given by the current futures price of the two underlying contracts and are therefore simple to implement in practice. The delta hedge with respect to the energy futures is given by

$$\begin{aligned}\frac{\partial C_t}{\partial F_t^E(\tau_1,\tau_2)} &= F_t^I(\tau_1,\tau_2)e^{-r(\tau_2-t)+\rho_{X,Y}\sigma_X\sigma_Y}\left(M\left(y_1^{***},y_2^{***};\rho_{X,Y}\right)+B(y_1^{***})N(y_2^{***}-\rho_{X,Y})\frac{1}{\sigma_X}\right)\\ &-\overline{K}_I e^{-r(\tau_2-t)}\left(M\left(y_1^{**},y_2^{**};\rho_{X,Y}\right)+B(y_1^{**})N(y_2^{**}-\rho_{X,Y})\frac{1}{\sigma_X}\right)\\ &-\frac{F_t^I(\tau_1,\tau_2)\overline{K}_E}{F_t^E(\tau_1,\tau_2)\sigma_X}e^{-r(\tau_2-t)}B(y_1^{*})N(y_2^{*}-\rho_{X,Y})\\ &+\frac{\overline{K}_E\overline{K}_I}{F_t^E(\tau_1,\tau_2)\sigma_X}e^{-r(\tau_2-t)}B(y_1)N(y_2-\rho_{X,Y}) \qquad \text{(III.11)}\end{aligned}$$

where $N(\cdot)$ denotes the standard normal cumulative distribution function, and

$$B(x)=\frac{e^{\left(x^2-\rho_{X,Y}^2\right)}}{4\pi^2\left(1-\rho_{X,Y}^2\right)}$$

The Delta hedge with respect to the temperature index futures is of course analogous to the energy Delta hedge, only with the substitutions $F_t^E(\tau_1,\tau_2)=F_t^I(\tau_1,\tau_2)$, $y_1^{***}=y_2^{***}$, $y_1^{**}=y_2^{**}$, $y_1^{*}=y_2^{*}$, $y_1=y_2$, $\sigma_Y=\sigma_X$ and $\sigma_X=\sigma_Y$. The cross-Gamma hedge is given by

$$\begin{aligned}&\frac{\partial C_t^2}{\partial F_t^E(\tau_1,\tau_2)\partial F_t^I(\tau_1,\tau_2)}\\ &\quad= e^{-r(\tau_2-t)+\rho_{X,Y}\sigma_X\sigma_Y}\left(M\left(y_1^{***},y_2^{***};\rho_{X,Y}\right)+B(y_2^{***})N(y_1^{***}-\rho_{X,Y})\frac{1}{\sigma_Y}\right)\\ &\quad+e^{-r(\tau_2-t)+\rho_{X,Y}\sigma_X\sigma_Y}B(y_1^{***})\left(N(y_2^{***}-\rho_{X,Y})\frac{1}{\sigma_X}+n(y_2^{***}-\rho_{X,Y})\frac{1}{\sigma_Y}\right)\\ &\quad-\frac{\overline{K}_I}{F_t^I(\tau_1,\tau_2)\sigma_Y}e^{-r(\tau_2-t)}\left(B(y_2^{**})N(y_1^{**}-\rho_{X,Y})+B(y_1^{**})n(y_2^{**}-\rho_{X,Y})\frac{1}{\sigma_X}\right)\\ &\quad-\frac{\overline{K}_E}{F_t^E(\tau_1,\tau_2)\sigma_X}e^{-r(\tau_2-t)}B(y_1^{*})\left(N(y_2^{*}-\rho_{X,Y})+n(y_2^{*}-\rho_{X,Y})\frac{1}{\sigma_Y}\right)\\ &\quad+\frac{\overline{K}_E\overline{K}_I}{F_t^E(\tau_1,\tau_2)F_t^I(\tau_1,\tau_2)(\sigma_X+\sigma_Y)}e^{-r(\tau_2-t)}B(y_1)n(y_2-\rho_{X,Y}) \qquad \text{(III.12)}\end{aligned}$$

where $n(\cdot)$ denotes the standard normal probability density function (pdf). In our model it is possible to hedge the quanto option perfectly, with positions described above by the three Delta and Gamma parameters. In practice, however, this would be difficult due to low liquidity in, for example, the temperature market. Furthermore, as discussed in Section III.2.2, we cannot in all markets trade futures within the delivery period, which puts additional restrictions on the suitability of the hedge. In such cases, the parameters above will guide in a partial hedging of the option.

III.3.2 Two-dimensional Schwartz-Smith Model with Seasonality

The popular commodity price model proposed by Schwartz and Smith (2000) is a natural starting point for deriving dynamics of energy futures. In this model, the log-spot price is the sum of two processes, one representing the long-term dynamics of the commodity prices in form of an arithmetic Brownian motion and one representing the short term deviations from the long run dynamics in the form of an Ornstein-Uhlenbeck process with a mean reversion level of zero. Other papers such as Lucia and Schwartz (2002) and Sørensen (2002) uses the same two driving factors and extends the model to include seasonality. We choose the seasonality parametrization of the latter and further extend to a two-asset framework by linking the driving Brownian motions. The dynamics under $\mathbb{P}$ is given by

$$\begin{aligned}\log S_t &= \Lambda(t) + X_t + Z_t \\ dX_t &= \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma d\widetilde{W}_t \\ dZ_t &= -\kappa Z_t dt + \nu d\widetilde{B}_t\end{aligned}$$

Here $\widetilde{B}$ and $\widetilde{W}$ are correlated Brownian motions and μ, σ, κ and η are constants. The deterministic function $\Lambda(t)$ describes the seasonality of the log-spot prices. In order to price a futures contract written on an underlying asset with the above dynamics, a measure change from $\mathbb{P}$ to an equivalent probability $\mathbb{Q}$ is made:

$$\begin{aligned}dX_t &= \left(\alpha - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t \\ dZ_t &= -\left(\lambda_Z + \kappa Z_t\right) dt + \nu dB_t\end{aligned}$$

Here, $\alpha = \mu - \lambda_X$, and λ_X and λ_Z are constant market prices of risk associated with X_t and Z_t respectively. This corresponds to a Girsanov transform of $\widetilde{B}$ and $\widetilde{W}$ by a constant drift so that B and W become two correlated $\mathbb{Q}$-Brownian motions. As is well-known for the Girsanov transform, the correlation between B and W is the same under $\mathbb{Q}$ as the one for $\widetilde{B}$ and $\widetilde{W}$ under $\mathbb{P}$ (see Karatzas and Shreve (2000)). Following Sørensen (2002), the futures price $F_t(\tau)$ at time $t \geq 0$ of a contract with delivery at time $\tau \geq t$ has the following form on log scale (note that it is the Schwartz-Smith futures prices scaled by a seasonality function):

$$\log F_t(\tau) = \Lambda(\tau) + A(\tau - t) + X_t + Z_t e^{-\kappa(\tau-t)} \tag{III.13}$$

where

$$A(\tau) = \alpha\tau - \frac{\lambda_Z - \rho\sigma\nu}{\kappa}\left(1 - e^{-\kappa\tau}\right) + \frac{\nu^2}{4\kappa}\left(1 - e^{-2\kappa\tau}\right).$$

The log futures prices are affine in the two factors X and Z driving the spot price and scaled by functions of time *to* delivery $\tau - t$ and by functions of time *of* delivery, τ. Sørensen (2002) chooses to parametrize the seasonality function Λ by a linear combination of cosine and sine functions:

$$\Lambda(t) = \sum_{k=1}^{K} \left(\gamma_k \cos(2\pi k t) + \gamma_k^* \sin(2\pi k t)\right) \tag{III.14}$$

In this paper, we have highlighted the fact that the payoff of energy quanto options can be expressed in terms of the futures prices of energy and temperature index. We can use the above procedure to derive futures price dynamics from a model of the spot. However, we can also state directly the futures price dynamics in the fashion of Heath-Jarrow-Morton (HJM) using the above model as inspiration for the specification of the model. The HJM approach was proposed to model energy futures by Clewlow and Strickland (2000), and later investigated in detail by Benth and Koekebakker (2008) (see also Benth, Benth, and Koekebakker (2008) and Miltersen and Schwartz (1998)). We follow this approach here, proposing a joint model for the energy and temperature index futures price based on the seasonal Schwartz-Smith model.

In stating such a model, we must account for the fact that the futures in question are delivering over a period $[\tau_1, \tau_2]$, and not at a fixed delivery time τ. An attractive alternative to the additive approach by Lucia and Schwartz (2002), is to let $F_t(\tau_1, \tau_2)$ itself follow a dynamics of the form (III.13) with some appropriately chosen dependency on τ_1 and τ_2. For example, we may choose $\tau = \tau_1$ in (III.13), or $\tau = (\tau_1 + \tau_2)/2$, or any other time within the delivery period $[\tau_1, \tau_2]$. In this way, we will account for the delivery time-effect in the futures price dynamics, sometimes referred to as the Samuelson effect. We remark that it is well-known that, for futures delivering over a period, the volatility will not converge to that of the underlying spot as time to delivery goes to zero (see Benth et al. (2008)). By the above choices, we obtain such an effect.

In order to jointly model the energy and temperature futures price, two futures dynamics of the type in (III.13) are connected by allowing the Brownian motions to be correlated across assets. We will have four Brownian motions W^E, B^E, W^I and B^I in our two-asset, two-factor model. These are assumed correlated as follows: $\rho_E = corr(W_1^E, B_1^E)$, $\rho_I = corr(W_1^I, B_1^I)$, $\rho_W = corr(W_1^E, W_1^I)$ and $\rho_B = corr(B_1^E, B_1^I)$. Moreover, we have cross-correlations given by

$$\begin{aligned}\rho_{I,E}^{W,B} &= corr(W_1^I, B_1^E)\\ \rho_{E,I}^{W,B} &= corr(W_1^E, B_1^I)\end{aligned}$$

In a HJM-style, we assume that the joint dynamics of the futures price processes $F_t^E(\tau_1, \tau_2)$ and $F_t^I(\tau_1, \tau_2)$ under $\mathbb{Q}$ is given by

$$\frac{dF_t^i(\tau_1, \tau_2)}{F_t^i(\tau_1, \tau_2)} = \sigma_i dW_t^i + \eta_i(t) dB_t^i \tag{III.15}$$

for $i = E, I$ and with

$$\eta_i(t) = \nu_i e^{-\kappa^i(\tau_2 - t)} \tag{III.16}$$

Note that we suppose the futures price is a martingale with respect to the pricing measure $\mathbb{Q}$, which is natural from the point of view that we want an arbitrage-free model. Moreover, we have made the explicit choice here that $\tau = \tau_2$ in (III.13) when modelling the delivery time effect. Note that

$$d\log F_t^i(\tau_1, \tau_2) = -\frac{1}{2}\left(\sigma_i^2 + \eta_i(t)^2 + 2\rho_i\sigma_i\eta_i(t)\right)dt + \sigma_i d\tilde{W}_t^i + \eta_i(t) d\tilde{B}_t^i$$

for $i = E, I$. Hence, we can make the representation $F_T^E(\tau_1, \tau_2) = F_t^E(\tau_1, \tau_2)\exp\left(-\mu_E + X\right)$ by choosing

$$X \sim \mathcal{N}\Bigg(0, \underbrace{\int_t^T \left(\sigma_E^2 + \eta_E(s)^2 + 2\rho_E\sigma_E\eta_E(s)\right) ds}_{\sigma_X^2}\Bigg), \qquad \mu_E = -\frac{1}{2}\sigma_X^2$$

and similarly for $F_T^I(\tau_1, \tau_2)$. These integrals can be computed analytically in the above model, where $\eta_i(t) = \nu_i e^{-\kappa^i(\tau_2 - t)}$. We can also compute the correlation $\rho_{X,Y}$ analytically, since $\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X\sigma_Y}$ and

$$cov(X,Y) = \rho_W\int_t^T \sigma_E\sigma_I ds + \rho_{E,I}^{W,B}\int_t^T \sigma_E\eta_I(s) ds + \rho_{I,E}^{W,B}\int_t^T \eta_E(s)\sigma_I ds + \rho_B\int_t^T \eta_E(s)\eta_I(s) ds$$

A closed-form expression of this covariance can be computed. In the special case of zero cross-correlations this simplifies to

$$cov(X, Y) = \rho_W \int_t^T \sigma_E \sigma_I ds + \rho_B \int_t^T \eta_E(s)\eta_I(s)ds$$

The exact expressions for σ_X, σ_Y and $\mathrm{cov}(X, Y)$ in the two-dimensional Schwartz-Smith model with seasonality are presented in Appendix III.B.

This bivariate futures price model has a form that can be immediately used for pricing energy quanto options by inferring the result in Proposition 1. We shall come back to this model in the empirical case study in Section III.4. The general setup in section III.3 above includes in fact the implied forward dynamics from general multi-factor spot models, with stationary and non-stationary terms. Hence, this is a very general pricing mechanism, where the basic essential problem is to identify the overall volatilities σ_X and σ_Y, and the cross-correlation $\rho_{X,Y}$. As a final remark, we note that our pricing approach only looks at futures dynamics up to the start of the delivery period τ_1. As briefly discussed in Section III.3.2 it is reasonable to expect that the dynamics of a futures contract should be different within the delivery period $[\tau_1, \tau_2]$. For times t within $[\tau_1, \tau_2]$ we will, in the case of the energy futures, have

$$F_t(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \sum_{u=\tau_1}^{t} S_u + \mathbb{E}_t^{\mathbb{Q}} \left[\frac{1}{\tau_2 - \tau_1} \sum_{u=t+1}^{\tau_2} S_u \right].$$

Thus, the futures price must consist of two parts, the first simply the observed energy spot prices up to time t, and next the second the current futures price of a contract with delivery period $[t, \tau_2]$. This latter part will have a volatility that must go to zero as t tends towards τ_2.

III.4 Empirical Analysis

In this section, we present an empirical study of energy quanto options written on NYMEX natural gas futures and the Heating Degree Days temperature index. We present the futures price data which constitute the basis of our analysis, and estimate the parameters in the joint futures price model (III.15). We then discuss the impact of correlation on the valuation of the option to be priced.

III.4.1 Data

Futures contracts for delivery of gas are traded on NYMEX for each month ten years out. The underlying is delivery of gas throughout a month and the price is per unit. The contract trades until a couple of days before the delivery month. Many contracts are closed prior to the last trading day, and we choose the first 12 contracts for delivery at least one month later. For example, for January 2 2007, we use March 2007 to February 2008 contracts. We denote the time-t futures price for a contract delivering one month ($= \Delta$) up till τ_2 by $F_t^E([\tau_2 - \Delta, \tau_2])$ and let the price follow a process of the type (III.13) discussed in the section III.3.2. When investigating data, there is a seasonality pattern over the year, where prices, in general, are lowest in late spring and early fall, slightly higher in between these periods and highest in the winter. These two "peaks" during the year are modelled by setting $K = 2$ in equation (III.14) similar to the seasonal pattern of the commodities studied in Sørensen (2002). The choice is statistically supported by the significance of parameter estimates and standard errors for the γ's. The evolution of the futures gas curves is shown in Figure III.1.

Futures contracts on accumulated HDD are traded on CME for several cities for the months October, November, December, January, February, March, and April, a couple of years out. The contract value is \$20 for the number HDD accumulated over the month for a specific location, ie, a day with temperature is $60°F$ adds 5 to the index and thereby \$100 to the final settlement, whereas a day with temperature is $70°F$ does not add to or subtract from the index. The contract trades until the beginning of the concurrent month. The futures price is denoted by $F_t^I(\tau_2 - \Delta, \tau_2)$ and settled on the accumulated index, $\sum HDD_u$. Liquidity is basically non-existent after the first year, so for every day we choose the first seven contracts, where the index period has not yet started, ie, for January 2, 2007, we use the February 2007, March 2007, April 2007, October 2007, November 2007, December 2007 and January 2008 contracts.

Again, we let the futures price follow a price process of the type (III.13). The stationary part represents the short term random fluctuations in the underlying temperature deviation. Over a long time, we might argue that temperature and thereby a month of accumulated HDD has a long term drift, but during the time period our data covers, the effect of long-term environmental changes are negligible. The short time period covered speaks justifies leaving out the nonstationary part, X. However, estimation of the full model led to significant parameter estimates for σ_I (see Section III.4.2), so we choose to keep the

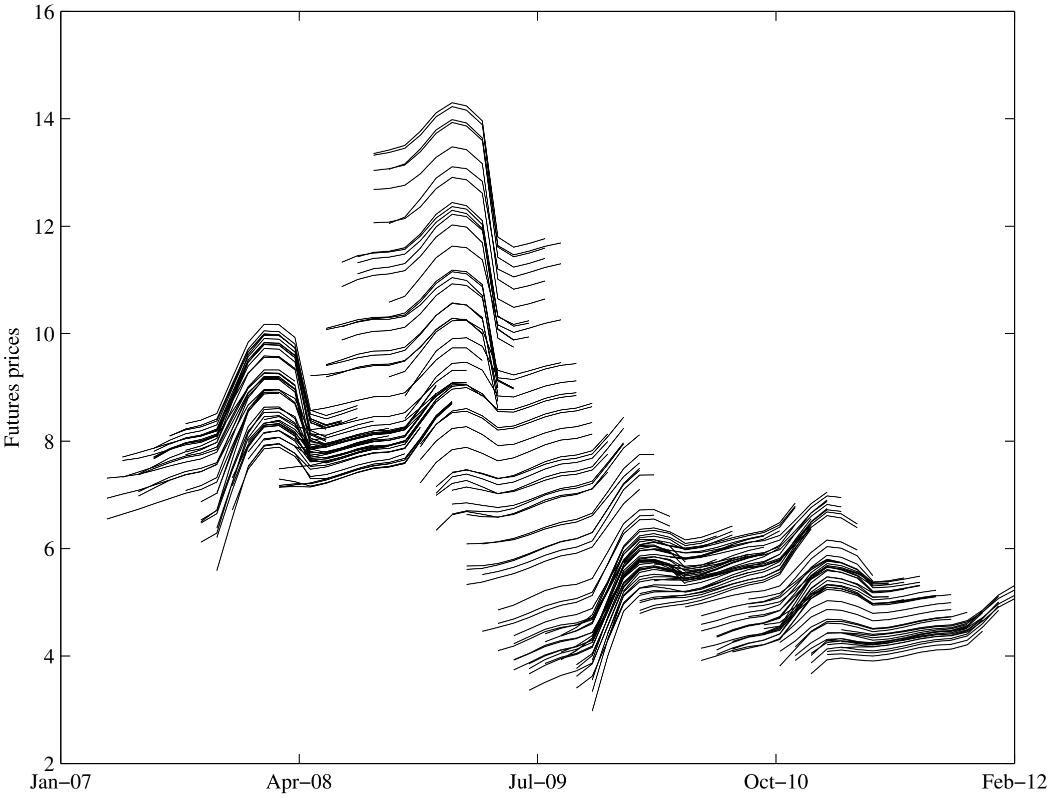


Figure III.1. Evolution of the gas futures curve as a function of maturity τ_2.

For each day t, the observed futures curve $F_t(\tau_2 - \Delta, \tau_2)$ with $\Delta = 1$ month is plotted as a function of τ_2. We observe up to 12 maturities at each observation point t. From t = 1-Jan-2007 to 31-Dec-2010 one observed futures curve per week is plotted in the figure.

long-term component in for the temperature index as well.

Inspection of data makes it clear that there is a deterministic level for each month, which does not change much until we get close to index period and the weather reports thereby starts to add information and affect prices. An obvious choice for modelling this deterministic seasonal is along the lines of Lucia and Schwartz (2002), where the seasonality is modelled by dummy for each month. With seven observed contracts, this would give us four additional parameters to estimate. Due to this and for keeping the two models symmetrical, we choose to keep the same structure as for the gas, but with $K = 1$ in equation (III.14). The chosen locations are New York (and Chicago in Appendix III.E), due to their being areas with fairly large gas consumption. The development in the term structure of HDD futures prices are shown in Figures III.2, where the daily observed futures curves are plotted as a function of τ_2.

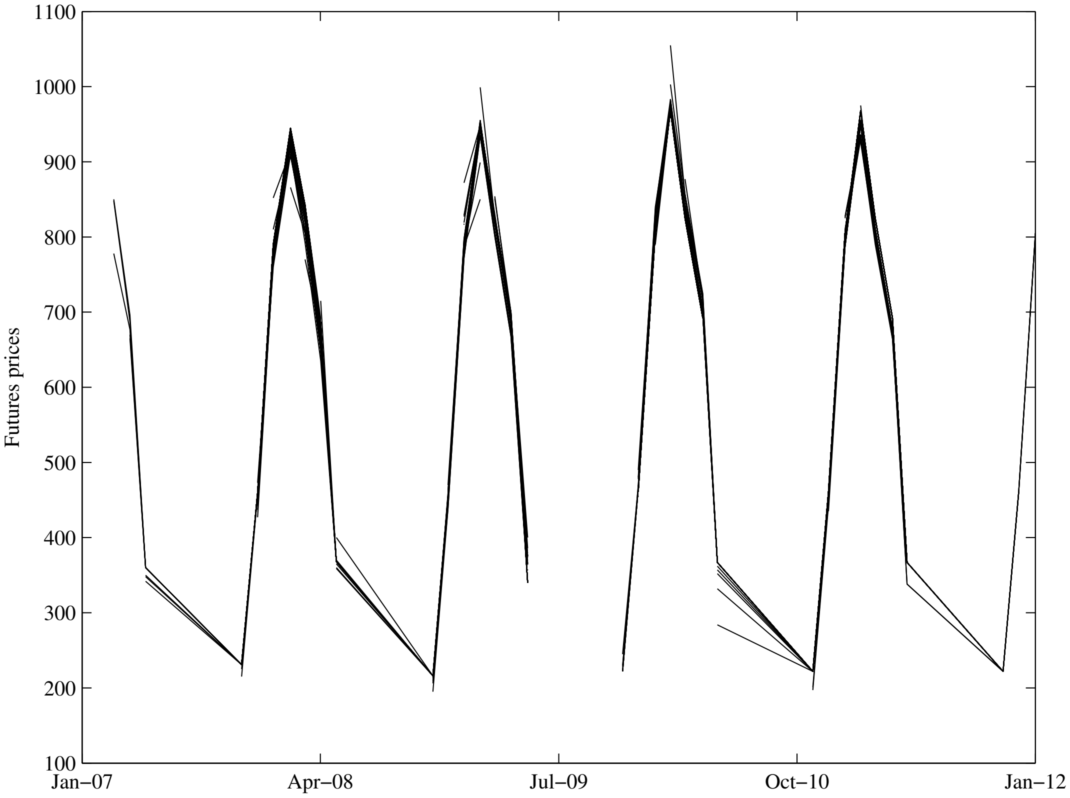


Figure III.2. Evolution of the New York HDD futures curve as a function of maturity τ_2.

For each day t, the observed futures curve $F_t(\tau_2 - \Delta, \tau_2)$ with $\Delta = 1$ month is plotted as a function of τ_2. We observe up to seven maturities at each observation point t. From t = 1-Jan-2007 to 31-Dec-2010 one observed futures curve per week is plotted in the figure. There is the same number of curves as in Figure 1, but because of low liquidity, HDD futures prices do not fluctuate much from day to day except for the first contracts. Therefore, many of the curves are lying on top of each other.

	HDD (NY)		Gas
μ	0.0063 (0.0247)		−0.0850 (0.0989)
κ	16.5654 (1.1023)		0.6116 (0.0320)
σ	0.0494 (0.0059)		0.2342 (0.0200)
ν	3.6517 (0.6197)		0.6531 (0.0332)
ρ	−0.6066 (0.0801)		−0.6803 (0.0656)
α	0.0027 (0.0049)		−0.3366 (0.0246)
λ	−5.9581 (2.3059)		−0.9191 (0.1968)
σ_ϵ	0.0655 (0.0006)		0.0199 (0.0001)
γ_1	0.9044 (0.0023)		0.0500 (0.0003)
γ_1^*	0.8104 (0.0018)		0.0406 (0.0003)
γ_2	N/A		0.0128 (0.0003)
γ_2^*	N/A		0.0270 (0.0003)
ρ^W		−0.2843 (0.0904)	
ρ^B		0.1817 (0.0678)	
ℓ		36198	

Table III.2. Parameter estimates for the two-dimensional two-factor model with seasonality, when New York HDD futures and NYMEX gas futures are modeled jointly.

III.4.2 Estimation Results

We estimate the parameters using Maximum Likelihood Estimation via the Kalman filter technique (see Appendix III.D), as in Sørensen (2002). The resulting parameter estimates for gas and New York HDD are reported in Table III.2 with standard errors based on the Hessian of the log-likelihood function given in parentheses. Estimates obtained by using HDD for Chicago are reported in Appendix III.E. Both under the physical and the risk neutral measure, the drift of the long term component for gas is negative. This matches the decrease in gas prices over time. The volatility parameters corresponds to a term structure of volatility that for gas starts around 50%. For HDD futures, the annualized volatility starts at a very high level of more than 100% for the closest contract and then quickly drops. For both types of contracts, we see a negative correlation between the long- and the short-term factors. For gas, this is obvious, because it creates a mean reversion effect that is characteristic of commodities. The positive short-term correlation reflect the connection between temperature and prices. If there is a short term shock in temperature, this is reflected in the closest HDD futures contract. At the same time, there is an increase in demand for gas leading to a short term increase in gas prices. The standard deviation of the estimation errors for the log prices is, on average, 2% for the gas contracts and a bit higher (around 6%) for the HDD contracts. Figure III.3 show the the model fit along with observed data and Figures III.4-III.5 in show plots of the squared pricing errors.

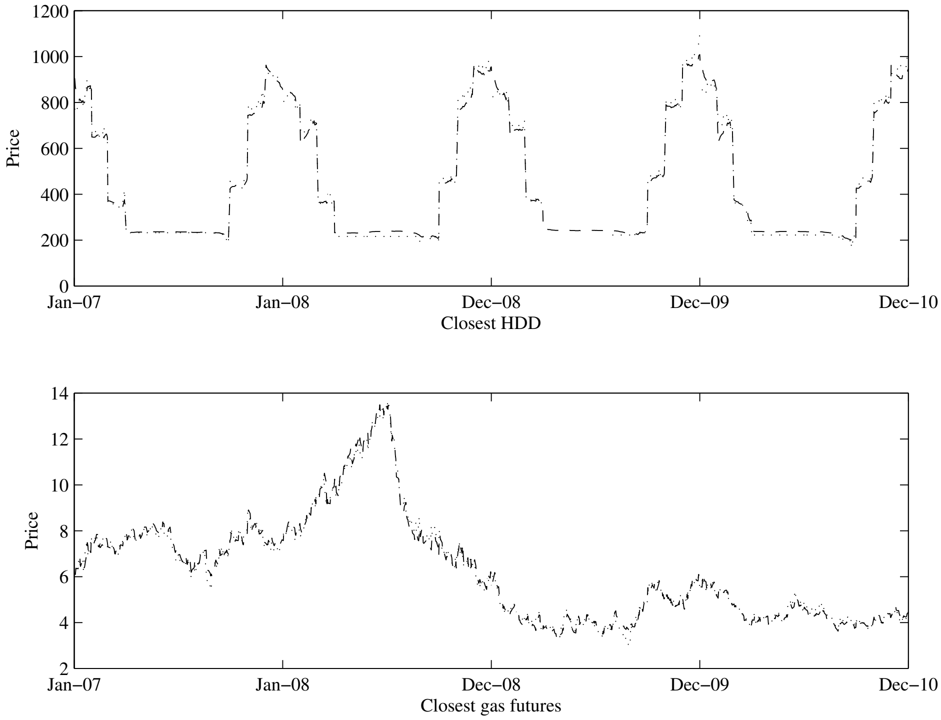


Figure III.3. Model prices and observed prices for New York HDD and NYMEX gas

The figures shows model prices (dashed line) and observed prices (dotted line) for the closest maturity, when prices of natural gas futures (bottom panel) and New York HDD futures (top panel) are modelled jointly. The errors between model and observed prices has a standard deviation of around 2% resp. 6.5%. Especially for the HDD futures contracts, the roll time of the futures contract is identifiable by the jump in prices. For the period April to September, the closest HDD future is the October contract, which is seen in the figure as a the longer, flatter lines.

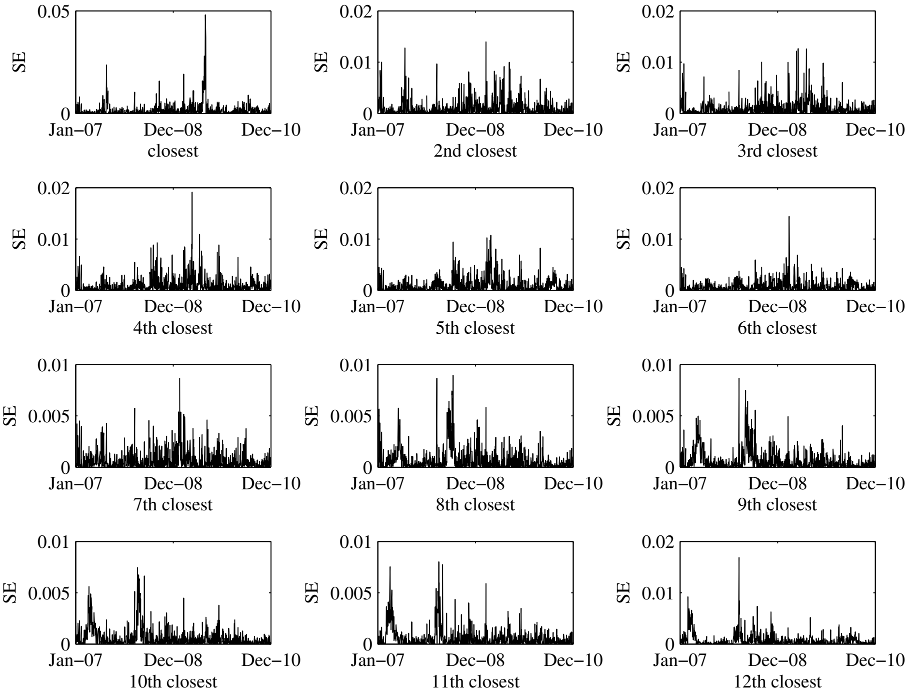


Figure III.4. Time series of squared percentage pricing errors for gas.

The figure shows the time series of squared pricing errors of the percentage difference between fitted and actual NYMEX natural gas futures prices when modelled jointly with New York HDD futures. The pricing errors are largest around the 2008 boom/bust in energy prices.

III.4.3 A case study

To consider the impact of the connection between gas prices and temperature (and thus gas and HDD futures), we compare the quanto option prices with prices obtained under the assumption of independence, and thus priced using the model in Black (1976) (see Appendix III.C). If the two futures were independent, we would get (C_t^0 denotes the price under the zero correlations assumption)

$$C_t^0 = e^{-r(\tau_2-t)}\mathbb{E}_t^Q\left[\max\left(F_{\tau_2}^E(\tau_1,\tau_2)-\overline{K}_E,0\right)\right]\times\mathbb{E}_t^Q\left[\max\left(F_{\tau_2}^I(\tau_1,\tau_2)-\overline{K}_I,0\right)\right], \quad \text{(III.17)}$$

which can be viewed as the product of the prices of two plain-vanilla call options on the gas and HDD futures respectively. In fact, we have the price C_t^0 given in this case as the product of two Black-76 formulas using the interest rate $r/2$ in the two respective prices. From the Figures III.6 and III.7, it is clear that the correlation between the gas and HDD futures significantly impacts the quanto option price. The left graphs on figures III.6 and III.7 shows the quanto option price on December 31, 2010 for two different settlement

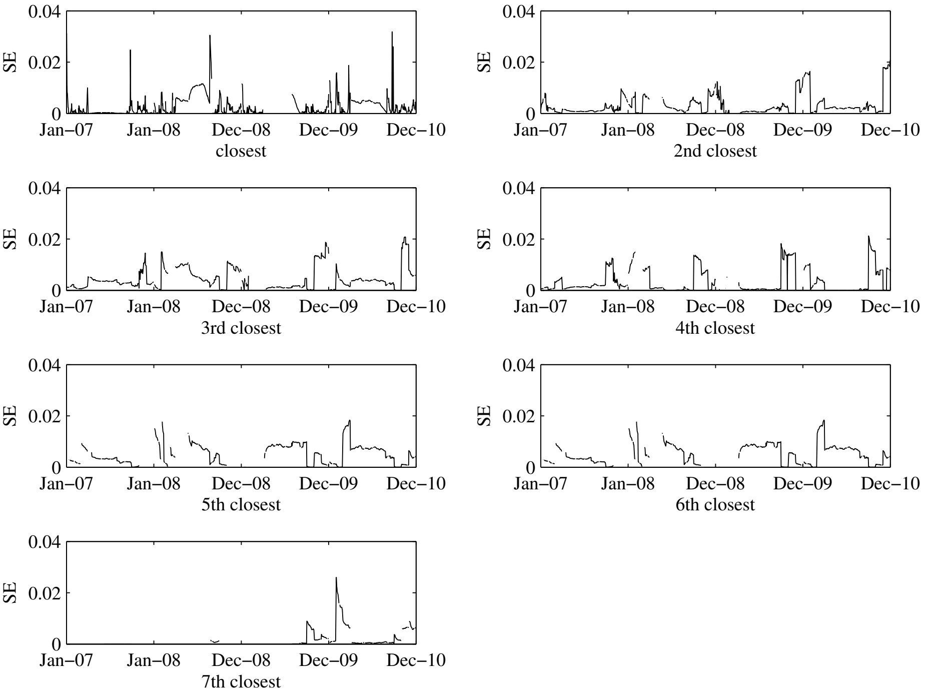


Figure III.5. Time series of squared percentage pricing errors for New York HDD.
The figure shows the time series of squared pricing errors of the percentage difference between fitted and actual New York HDD futures prices when modelled jointly with NYMEX natural gas futures. The pricing errors jump when the contract roll.

months; December 2011 and February 2011 respectively. The right graphs on figures III.6 and III.7 shows the relative pricing error between the quanto option price with and without correlation across assets. The ratio of the change in quanto option price to the product of the marginal options, ie, $(C_t - C_t^0)/C_t$ is plotted. For a short time to maturity, we see a relative pricing error of more than 75% for the high strikes. The fact that the observed correlation increases the quanto option price compared with the product of the two marginal options indicates that more probability mass lies in the quanto's exercise region. For short time to maturity especially, ignoring correlation can lead to significant underpricing of the quanto option.

III.5 Concluding remarks

In this paper, we presented a closed-form pricing formula for an energy quanto option under the assumption that the underlying assets were lognormal. Taking advantage of the fact that energy and temperature futures are designed with a delivery period, we showed how

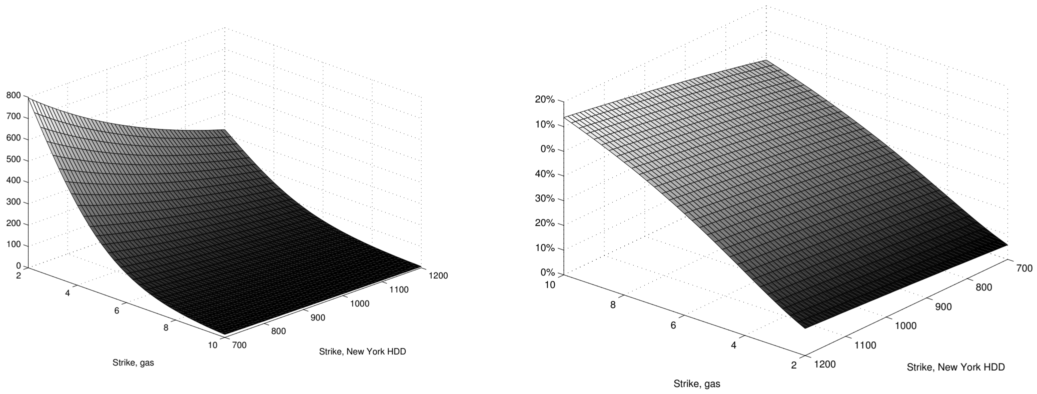


Figure III.6. Quanto option prices and relative pricing differences for a one-year option
The left figure shows the price of a quanto option as a function of the two strike values. The contract is priced on December 31, 2010 for settlement in December 2011. The right figure shows the relative pricing error between the quanto option price calculated with and without correlation across assets. The interest rate is set to 2%. Current futures prices are 5.0920 and 805 respectively. Depending on the strikes, the relative price error is up to 40%.

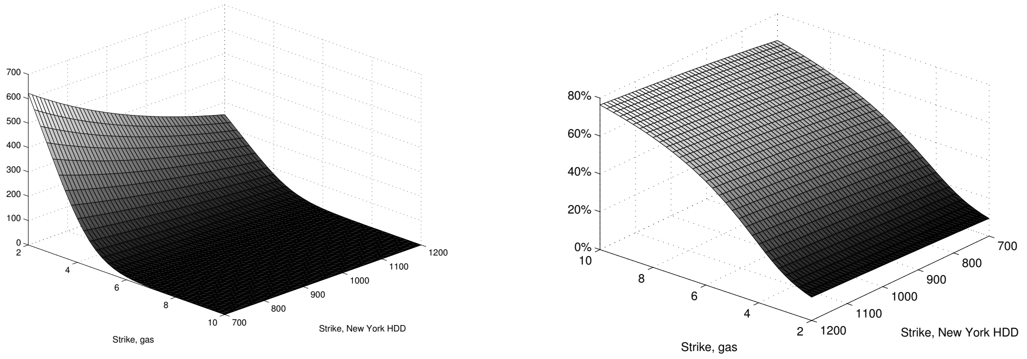


Figure III.7. Quanto option prices and relative pricing differences for a one-month option
The left figure shows the price of a quanto option as a function of the two strike values. The contract is priced on December 31, 2010 for settlement in February 2011. The right figure shows the relative pricing error between the quanto option price calculated with and without correlation across assets. The interest rate is set to 2%. Current futures prices are 4.405 and 797 respectively. Depending on the strikes, the relative price error can be more than 75%.

quanto options can be priced using futures contracts as underlying assets. Correspondingly, we adopt an HJM approach, and modelled the dynamics of the futures contracts directly. We showed that our approach encompasses relevant cases, such as GBMs and multifactor spot models. Importantly, our approach enabled us to derive hedging strategies and perform hedges with traded assets. We illustrate the use of our pricing model by estimating a two-dimensional two-factor model with seasonality using NYMEX data on natural gas and CME data on temperature HDD futures. We calculated quanto energy option prices and show how correlation between the two asset classes significantly impacts the prices.

Appendix

III.A Proof of pricing formula

In Section III.3.1 we showed that the payoff function in (III.6) could be rewritten in the following way:

$$\begin{aligned}
\hat{p}(F_T^E, F_T^I, \overline{K}_I, \overline{K}_E) &= \max(F_T^I - \overline{K}_I, 0) \cdot \max(F_T^E - \overline{K}_E, 0) \\
&= \left(F_T^E - \overline{K}_E\right) \cdot \left(F_T^I - \overline{K}_I\right) \cdot 1_{\{F_T^E > \overline{K}_E\}} \cdot 1_{\{F_T^I > \overline{K}_I\}} \\
&= F_T^E F_T^I \cdot 1_{\{F_T^E > \overline{K}_E\}} \cdot 1_{\{F_T^I > \overline{K}_I\}} - F_T^E \overline{K}_I \cdot 1_{\{F_T^E > \overline{K}_E\}} \cdot 1_{\{F_T^I > \overline{K}_I\}} \\
&\quad - F_T^I \overline{K}_E \cdot 1_{\{F_T^E > \overline{K}_E\}} \cdot 1_{\{F_T^I > \overline{K}_I\}} + \overline{K}_E \overline{K}_I \cdot 1_{\{F_T^E > \overline{K}_E\}} \cdot 1_{\{F_T^I > \overline{K}_I\}}.
\end{aligned}$$

Now, let us calculate the expectation under $\mathbb{Q}$ of the payoff function, ie, $\mathbb{E}_t^{\mathbb{Q}}\left[\hat{p}(F_T^E, F_T^I, \overline{K}_I, \overline{K}_E)\right]$. We have

$$\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}\left[\hat{p}(F_T^E, F_T^I, \overline{K}_I, \overline{K}_E)\right] &= \mathbb{E}_t^{\mathbb{Q}}\left[\max(F_T^I - \overline{K}_I, 0) \cdot \max(F_T^E - \overline{K}_E, 0)\right] \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[F_T^E F_T^I 1_{\{F_T^E > \overline{K}_E\}} 1_{\{F_T^I > \overline{K}_I\}}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[F_T^E \overline{K}_I 1_{\{F_T^E > \overline{K}_E\}} 1_{\{F_T^I > \overline{K}_I\}}\right] \\
&\quad - \mathbb{E}_t^{\mathbb{Q}}\left[F_T^I \overline{K}_E 1_{\{F_T^E > \overline{K}_E\}} 1_{\{F_T^I > \overline{K}_I\}}\right] + \mathbb{E}_t^{\mathbb{Q}}\left[\overline{K}_E \overline{K}_I 1_{\{F_T^E > \overline{K}_E\}} 1_{\{> \overline{K}_I\}}\right].
\end{aligned} \tag{III.18}$$

In order to calculate the four different expectation terms we will use the same trick as Zhang (1995), namely to rewrite the pdf of the bivariate normal distribution in terms of the marginal pdf of the first variable times the conditional pdf of the second variable given the first variable. Remember that we assume F_T^E and F_T^I to be lognormally distributed under $\mathbb{Q}$ (ie, (X, Y) are bivariate normal):

$$F_T^E = F_t^E e^{\mu_E + X} \tag{III.19}$$

$$F_T^I = F_t^I e^{\mu_I + Y} \tag{III.20}$$

where σ_X^2 denotes variance of X, σ_Y^2 denotes variance of Y, and they are correlated by $\rho_{X,Y}$. Consider the fourth expectation term first:

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[\overline{K}_E\overline{K}_I 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right]\\
&\qquad= \overline{K}_E\overline{K}_I\mathbb{E}_t^{\mathbb{Q}}\left[1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right]\\
&\qquad= \overline{K}_E\overline{K}_I\mathbb{Q}_t\left(\left\{F_T^E>\overline{K}_E\right\}\cap\left\{\ F_T^I>\overline{K}_I\right\}\right)\\
&\qquad= \overline{K}_E\overline{K}_I\mathbb{Q}_t\left(\left\{F_t^E e^{\mu_E+X}>\overline{K}_E\right\}\cap\left\{F_t^I e^{\mu_I+Y}>\overline{K}_I\right\}\right)\\
&\qquad= \overline{K}_E\overline{K}_I\mathbb{Q}_t\left(\left\{X>\log\left(\frac{\overline{K}_E}{F_t^E}\right)-\mu_E\right\}\cap\left\{Y>\log\left(\frac{\overline{K}_I}{F_t^I}\right)-\mu_I\right\}\right)\\
&\qquad= \overline{K}_E\overline{K}_I\mathbb{Q}_t\left(\left\{-X<\log\left(\frac{F_t^E}{\overline{K}_E}\right)+\mu_E\right\}\cap\left\{-Y<\log\left(\frac{F_t^I}{\overline{K}_I}\right)+\mu_I\right\}\right)\\
&\qquad= \overline{K}_E\overline{K}_I\cdot M\left(y_1,y_2;\rho_{X,Y}\right)
\end{aligned}$$

where (ϵ_1,ϵ_2) are standard bivariate normal with correlation $\rho_{X,Y}$ and

$$y_1=\frac{\log\left(\frac{F_t^E}{\overline{K}_E}\right)+\mu_E}{\sigma_X}\quad\text{and}\quad y_2=\frac{\log\left(\frac{F_t^I}{\overline{K}_I}\right)+\mu_I}{\sigma_Y}.$$

Next, consider the third expectation term,

$$\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}\left[F_T^I\overline{K}_E 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right] &= F_t^I\overline{K}_E e^{\mu_I}\mathbb{E}\left[e^Y 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right]\\
&= F_t^I\overline{K}_E e^{\mu_I}\mathbb{E}\left[e^{\sigma_Y\epsilon_2}1_{\{\epsilon_1<y_1\}}1_{\{\epsilon_2<y_2\}}\right]\\
&= F_t^I\overline{K}_E e^{\mu_I}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}e^{\sigma_Y\epsilon_2}f\left(\epsilon_1,\epsilon_2\right)d\epsilon_1 d\epsilon_2\\
&= F_t^I\overline{K}_E e^{\mu_I}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}e^{\sigma_Y\epsilon_2}f\left(\epsilon_2\right)f\left(\epsilon_1|\epsilon_2\right)d\epsilon_1 d\epsilon_2\\
&= F_t^I\overline{K}_E e^{\mu_I}\int_{-\infty}^{y_2}\int_{-\infty}^{y_1}e^{\sigma_Y\epsilon_2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\epsilon_2^2\right)\times\\
&\qquad\frac{1}{\sqrt{2\pi}\sqrt{1-\rho_{X,Y}^2}}\exp\left[\frac{-1}{2(1-\rho_{X,Y}^2)}\left(\epsilon_1-\rho_{X,Y}\epsilon_2\right)^2\right]d\epsilon_1 d\epsilon_2.
\end{aligned}\tag{III.21}$$

Using the substitution $w=-\epsilon_1+\rho_{\epsilon_1,\epsilon_2}\sigma_Y$ and $z=-\epsilon_2+\sigma_Y$, the exponent in the above expression becomes:

$$\begin{aligned}
&\sigma_Y\epsilon_2-\frac{1}{2}\epsilon_2^2-\frac{1}{2(1-\rho_{X,Y}^2)}\left(\epsilon_1^2+\rho_{X,Y}^2\epsilon_2^2-2\rho_{X,Y}\epsilon_1\epsilon_2\right)\\
&=-\frac{1}{2(1-\rho_{X,Y}^2)}\left(-2\sigma_Y(1-\rho_{X,Y}^2)\epsilon_2+(1-\rho_{X,Y}^2)\epsilon_2^2+\epsilon_1^2+\rho_{X,Y}^2\epsilon_2^2-2\rho_{X,Y}\epsilon_1\epsilon_2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2(1-\rho_{X,Y}^2)}\left(\epsilon_1^2 - 2\sigma_Y(1-\rho_{X,Y}^2)\epsilon_2 + \epsilon_2^2 - 2\rho_{X,Y}\epsilon_1\epsilon_2\right)\\
&= -\frac{1}{2(1-\rho_{X,Y}^2)}\left(w^2 + z^2 - 2\rho_{X,Y}zw - (1-\rho_{X,Y}^2)\sigma_Y^2\right)\\
&= -\frac{1}{2(1-\rho_{X,Y}^2)}\left(w^2 + z^2 - 2\rho_{X,Y}zw\right) + \frac{\sigma_Y^2}{2},
\end{aligned}
$$

which enable us to rewrite (III.21) as

$$
\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[F_T^I\overline{K}_E 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right] =\\
&\quad F_t^I\overline{K}_E e^{\mu_I+\frac{\sigma_Y^2}{2}}\int_{-\infty}^{y_2^*}\int_{-\infty}^{y_1^*}\frac{1}{2\pi\sqrt{1-\rho_{X,Y}^2}}\exp\left[-\frac{1}{2(1-\rho_{X,Y}^2)}\left(w^2+z^2-2\rho_{X,Y}zw\right)\right]dwdz\\
&\quad = F_t^I\overline{K}_E e^{\mu_I+\frac{\sigma_Y^2}{2}} M\left(y_1^*, y_2^*; \rho_{X,Y}\right)
\end{aligned}
$$

where

$$
y_1^* = y_1 + \rho_{X,Y}\sigma_Y \quad \text{and} \quad y_2^* = y_2 + \sigma_Y.
$$

The second expectation term can be calculated in the same way as we calculated the third term. The only difference is that we now use the substitution $\bar{w} = -\epsilon_1 + \sigma_X$ and $\bar{z} = -\epsilon_2 + \rho_{X,Y}\sigma_X$, so we can write

$$
\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[F_T^E\overline{K}_I 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right] =\\
&\quad F_t^E\overline{K}_I e^{\mu_E+\frac{\sigma_X^2}{2}}\int_{-\infty}^{y_2^{**}}\int_{-\infty}^{y_1^{**}}\frac{1}{2\pi\sqrt{1-\rho_{X,Y}^2}}\exp\left[-\frac{1}{2(1-\rho_{X,Y}^2)}\left(w^2+z^2-2\rho_{X,Y}zw\right)\right]dwdz\\
&\quad = F_t^E\overline{K}_I e^{\mu_E+\frac{\sigma_X^2}{2}} M\left(y_1^{**}, y_2^{**}; \rho_{X,Y}\right)
\end{aligned}
$$

where

$$
y_1^{**} = y_1 + \sigma_X \quad \text{and} \quad y_2^{**} = y_2 + \rho_{X,Y}\sigma_X.
$$

Finally, consider the first expectation term in (III.18),

$$
\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}\left[F_T^E F_T^I 1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right] &= F_t^E F_t^I e^{\mu_E+\mu_I}\mathbb{E}_t^{\mathbb{Q}}\left[e^{X+Y}1_{\{F_T^E>\overline{K}_E\}}1_{\{F_T^I>\overline{K}_I\}}\right]\\
&= F_t^E F_t^I e^{\mu_E+\mu_I}\mathbb{E}_t^{\mathbb{Q}}\left[e^{\sigma_X\epsilon_1+\sigma_Y\epsilon_2}1_{\{\epsilon_1<y_1\}}1_{\{\epsilon_2<y_2\}}\right]\\
&= F_t^E F_t^I e^{\mu_E+\mu_I}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} e^{\sigma_X\epsilon_1+\sigma_Y\epsilon_2} f(\epsilon_1,\epsilon_2)d\epsilon_2 d\epsilon_1 \qquad \text{(III.22)}
\end{aligned}
$$

Using the same trick as before with the substitution $u = -\epsilon_1 + \rho_{X,Y}\sigma_Y + \sigma_X$ and $v = -\epsilon_2 + \rho_{X,Y}\sigma_X + \sigma_Y$, expression (III.22) can be written

$$\mathbb{E}_t^{\mathbb{Q}}\left[F_T^E F_T^I 1_{\{F_T^E>\overline{K}_E\}} 1_{\{F_T^I>\overline{K}_I\}}\right] = F_t^E F_t^I e^{\mu_E+\mu_I+\frac{1}{2}(\sigma_X^2+\sigma_Y^2+2\rho_{X,Y}\sigma_X\sigma_Y)} M(y_1^{***}, y_2^{***};\rho_{X,Y}) \tag{III.23}$$

where

$$y_1^{***} = y_1 + \rho_{X,Y}\sigma_Y + \sigma_X \quad \text{and} \quad y_2^{***} = y_2 + \rho_{X,Y}\sigma_X + \sigma_Y.$$

Thus, the expectation of the payoff function is

$$\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}\left[\hat{p}(F_T^E, F_T^I, \overline{K}_I, \overline{K}_E)\right] = {} & F_t^E F_t^I e^{\mu_E+\mu_I+\frac{1}{2}(\sigma_X^2+\sigma_Y^2+2\rho_{X,Y}\sigma_X\sigma_Y)} M(y_1^{***}, y_2^{***};\rho_{X,Y}) \\
& - F_t^E \overline{K}_I e^{\mu_E+\frac{\sigma_X^2}{2}} M\left(y_1^{**}, y_2^{**};\rho_{X,Y}\right) \\
& - F_t^I \overline{K}_E e^{\mu_I+\frac{\sigma_Y^2}{2}} M\left(y_1^{*}, y_2^{*};\rho_{X,Y}\right) \\
& + \overline{K}_E \overline{K}_I \cdot M\left(y_1, y_2;\rho_{X,Y}\right).
\end{aligned}$$

Discounting the expected payoff gives us the price of the option.

III.B Closed-form solution for σ and ρ in the two-dimensional Schwartz-Smith model with seasonality

$$\begin{aligned}
\sigma_X^2 &= \int_t^T \left(\sigma_E^2 + \left(\nu_E e^{-\kappa^E(\tau-s)}\right)^2 + 2\rho_E\sigma_E\left(\nu_E e^{-\kappa^E(\tau-s)}\right)\right) ds \\
&= \sigma_E^2(T-t) + \nu_E \int_t^T e^{-2\kappa^E(\tau-s)} ds + 2\rho_E\sigma_E\nu_E \int_t^T e^{-\kappa^E(\tau-s)} ds \\
&= \sigma_E^2(T-t) + \frac{\nu_E}{2\kappa^E} e^{-2\kappa^E\tau}\left(e^{2\kappa^E T} - e^{2\kappa^E t}\right) + 2\frac{\rho_E\sigma_E\nu_E}{\kappa^E} e^{-\kappa^E\tau}\left(e^{\kappa^E T} - e^{\kappa^E t}\right); \\
cov(X,Y) &= \rho_W \int_t^T \sigma_E\sigma_I ds + \rho_B \int_t^T \left(\nu_E e^{-\kappa^E(\tau-s)}\right)\left(\nu_I e^{-\kappa^I(\tau-s)}\right) ds \\
&= \rho_W\sigma_E\sigma_I(T-t) + \rho_B\nu_E\nu_I e^{-(\kappa^E+\kappa_I)\tau} \int_t^T e^{(\kappa^E+\kappa_I)s} ds \\
&= \rho_W\sigma_E\sigma_I(T-t) + \frac{\rho_B\nu_E\nu_I}{\kappa^E+\kappa^I} e^{-(\kappa^E+\kappa_I)\tau}\left(e^{(\kappa^E+\kappa^I)T} - e^{(\kappa^E+\kappa^I)t}\right); \\
\rho_{X,Y} &= \frac{cov(X,Y)}{\sigma_X\sigma_Y}.
\end{aligned}$$

When $T=\tau$, this simplifies to

$$\begin{aligned}
\sigma_X &= \sigma_E^2(\tau-t) + \frac{\nu_E}{2\kappa^E}\left(1 - e^{-2\kappa^E(\tau-t)}\right) + 2\frac{\rho_E\sigma_E\nu_E}{\kappa^E}\left(1 - e^{\kappa^E(\tau-t)}\right), \\
\rho_{X,Y} &= \frac{\rho_W\sigma_E\sigma_I(\tau-t) + \frac{\rho_B\nu_E\nu_I}{\kappa^E+\kappa^I}\left(1 - e^{-(\kappa^E+\kappa_I)(\tau-t)}\right)}{\sigma_X\sigma_Y}.
\end{aligned}$$

III.C One-dimensional option prices

In this section, option prices on one underlying is presented. As for the joint case, assume that the dynamics of a gas futures contract is given by:

$$F_T^E(\tau_1, \tau_2) = F_t^E(\tau_1, \tau_2) \exp(\mu_E + X).$$

Consider now a call option written on gas futures only. The price c_t of this option is then given by the Black (1976) formula, i.e.

$$c_t = e^{-r(T-t)} \left[F N(d_1) - K N(d_2) \right],$$

where

$$d_1 = \frac{\ln \frac{F_t^E}{K_E} - \mu_E}{\sigma_X}, \qquad d_2 = \frac{\ln \frac{F_t^E}{K_E} + \mu_E}{\sigma_X}.$$

The same formula of course applies to an option written only on temperature futures.

III.D Estimation using Kalman filter techniques

Given a set of observed futures prices, it is possible to estimate the parameters using Kalman filter techniques. Let

$$Y_n = \left(f_{t_n}^I\left(T_n^1\right), \ldots, f_{t_n}^I\left(T_n^{M_n^I}\right), f_{t_n}^E\left(T_n^1\right), \ldots, f_{t_n}^E\left(T_n^{M_n^E}\right) \right)'$$

denote the set of log futures prices observed at time t_n with maturities $T_n^1, \ldots, T_n^{M_n^I}$ for the temperature contracts and maturities $T_n^1, \ldots, T_n^{M_n^E}$ for the gas contracts. The measurement equation relates the observations to the unobserved state vector $U_n = (X_{t_n}, Z_{t_n})'$ by

$$Y_n = d_n + C_n U_n + \epsilon_n$$

where the ϵ's are measurement errors assumed iid normal with zero mean and covariance matrix H_n. In the present framework we have

$$d_n = \begin{pmatrix} \Lambda^I\left(T_n^1\right) + A^I\left(T_n^1 - t_n\right) \\ \vdots \\ \Lambda^I\left(T_n^{M_n^I}\right) + A^I\left(T_n^{M_n^I} - t_n\right) \\ \Lambda^E\left(T_n^1\right) + A^E\left(T_n^1 - t_n\right) \\ \vdots \\ \Lambda^E\left(T_n^{M_n^E}\right) + A^E\left(T_n^{M_n^E} - t_n\right) \end{pmatrix}, C_n = \begin{pmatrix} 1 & e^{-\kappa^I\left(T_n^1 - t_n\right)} \\ \vdots & \vdots \\ 1 & e^{-\kappa^I\left(T_n^{M_n^I} - t_n\right)} \\ 1 & e^{-\kappa^E\left(T_n^1 - t_n\right)} \\ \vdots & \vdots \\ 1 & e^{-\kappa^E\left(T_n^{M_n^E} - t_n\right)} \end{pmatrix}$$

and $H_n = \begin{pmatrix} \sigma^2_{\epsilon,I} I_{M_n^I} & 0 \\ 0 & \sigma^2_{\epsilon,E} I_{M_n^E} \end{pmatrix}$
The state-vector evolves according to

$$U_n = c + TU_n + \eta_n$$

where η_n are i.i.d. normal with zero-mean vector and covariance matrix Q and where

$$c = \begin{pmatrix} \mu^I - \frac{1}{2}\left(\sigma^I\right)^2 \\ 0 \\ \mu^E - \frac{1}{2}\left(\sigma^E\right)^2 \\ 0 \end{pmatrix}, \Delta_{n+1}, \qquad T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\kappa^I \Delta_{n+1}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-\kappa^E \Delta_{n+1}} \end{pmatrix},$$

$$Q = \begin{pmatrix} \left(\sigma^I\right)^2 \Delta_{n+1} & 0 & \rho^S \sigma^I \sigma^E \Delta_{n+1} & 0 \\ 0 & \frac{\left(\nu^I\right)^2}{2\kappa^I}\left(1 - e^{-2\kappa^I \Delta_{n+1}}\right) & 0 & \frac{\rho^L \nu^I \nu^E}{\left(\kappa^I + \kappa^E\right)}\left(1 - e^{-\left(\kappa^I + \kappa^E\right)\Delta_{n+1}}\right) \\ \rho^S \sigma^I \sigma^E \Delta_{n+1} & 0 & \left(\sigma^E\right)^2 \Delta_{n+1} & 0 \\ 0 & \frac{\rho^L \nu^I \nu^E}{\left(\kappa^I + \kappa^E\right)}\left(1 - e^{-\left(\kappa^I + \kappa^E\right)\Delta_{n+1}}\right) & 0 & \frac{\left(\nu^E\right)^2}{2\kappa^E}\left(1 - e^{-2\kappa^E \Delta_{n+1}}\right) \end{pmatrix}.$$

III.E Result of analysis using data from Chicago

This section contains the results of the estimation on Chicago HDD futures modelled jointly with NYMEX gas futures. The results are very similar to those presented in Section III.4.

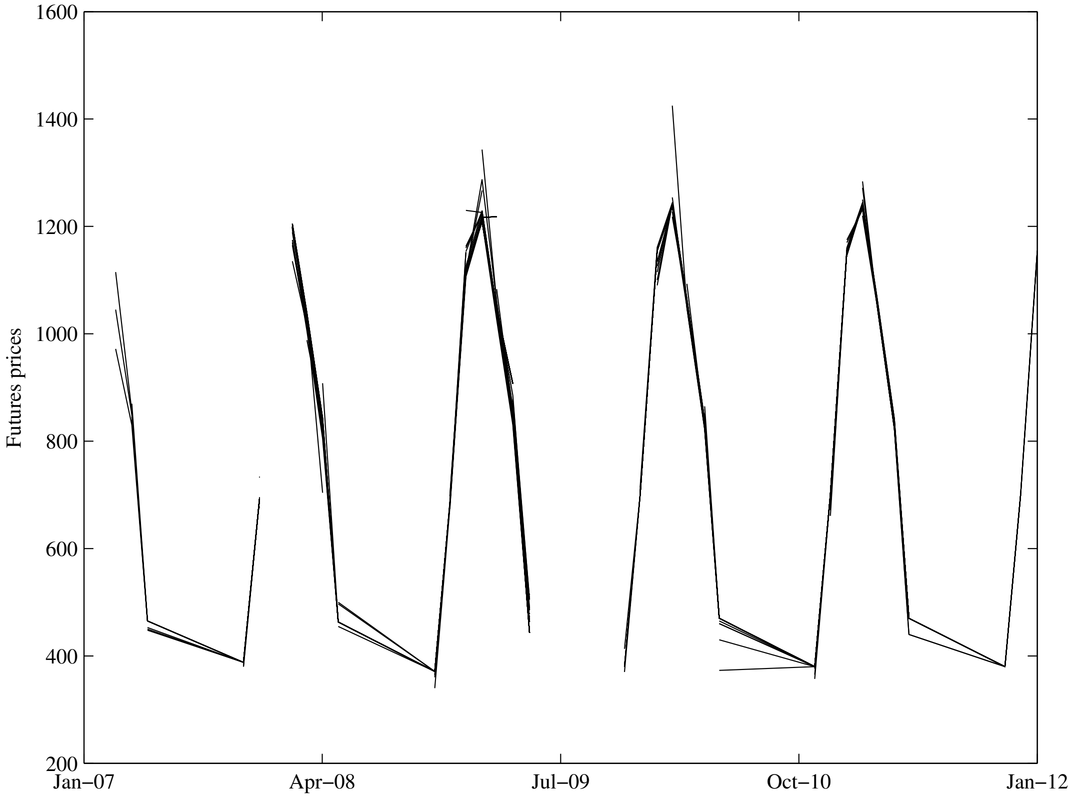


Figure III.8. Evolution of the Chicago HDD futures curve as a function of maturity τ_2. For each day t, the observed futures curve $F_t(\tau_2 - \Delta, \tau_2)$ with $\Delta = 1$ month is plotted as a function of τ_2. We observe up to seven maturities at each observation point t. From t = 1-Jan-2007 to 31-Dec-2010 one observed futures curve per week is plotted in the figure. There is the same number of curves as in Figure 1, but because of low liquidity, HDD futures prices do not fluctuate much from day to day except for the first contracts. Therefore, many of the curves are lying on top of each other.

The parameters are again estimated using Maximum Likelihood Estimation and the Kalman filter. The resulting parameter estimates for gas and Chicago HDD are reported in Table III.3 with standard errors based on the Hessian of the log-likelihood function given in parentheses. Figure III.9 show the the model fit along with observed data and Figures III.10-III.11 in show plots of the squared pricing errors.

	HDD (Chicago)	Gas
μ	0.0126 (0.0191)	-0.0817 (0.0998)
κ	18.8812 (1.3977)	0.6034 (0.0317)
σ	0.0379 (0.0051)	0.2402 (0.0209)
ν	4.3980 (0.8908)	0.6647 (0.0335)
ρ	-0.5509 (0.0948)	-0.7038 (0.0611)
α	0.0107 (0.0040)	-0.3403 (0.0249)
λ	-5.8799 (2.9083)	-0.9438 (0.1988)
σ_ϵ	0.0554 (0.0005)	0.0199 (0.0001)
γ_1	0.8705 (0.0019)	0.0499 (0.0003)
γ_1^*	0.6391 (0.0015)	0.0406 (0.0003)
γ_2	N/A	0.0128 (0.0003)
γ_2^*	N/A	0.0270 (0.0003)
ρ^W	-0.2707 (0.0909)	
ρ^B	0.1982 (0.0643)	
ℓ	37023	

Table III.3. Parameter estimates for the two-dimensional two-factor model with seasonality, when Chicago HDD futures and NYMEX gas futures are modeled jointly.

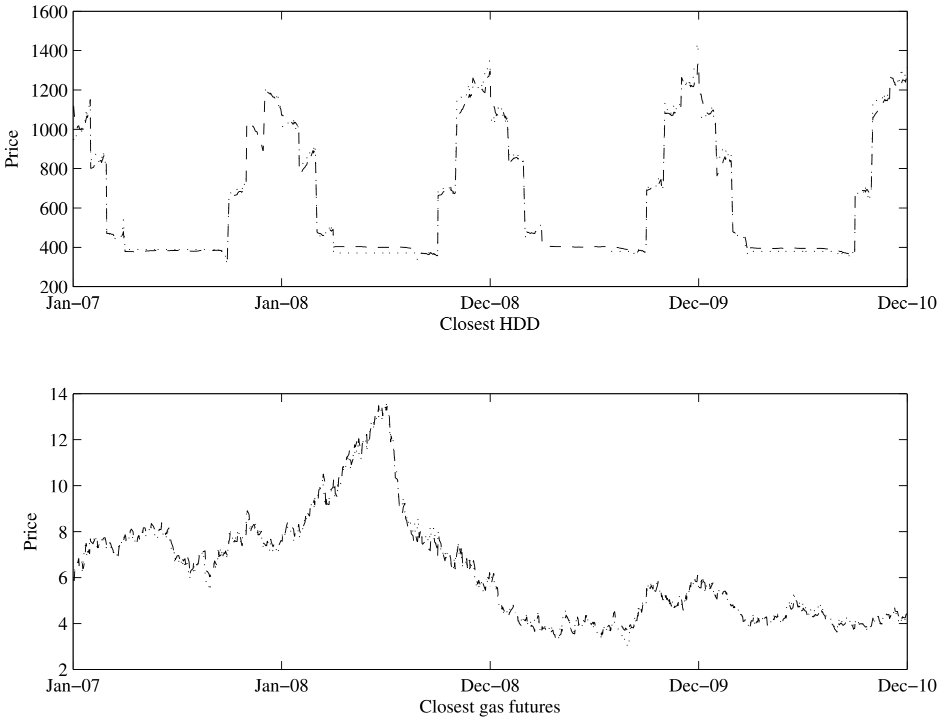
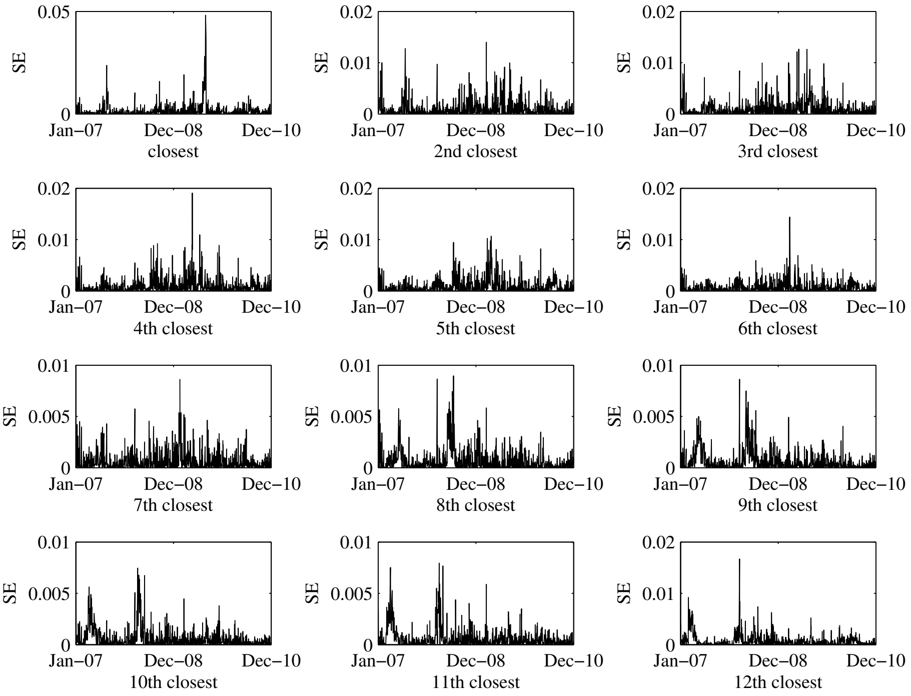


Figure III.9. Model prices and observed prices for Chicago HDD and NYMEX gas

The figures shows model prices (dashed line) and observed prices (dotted line) for the closest maturity, when prices of natural gas futures (bottom panel) and Chicago HDD futures (top panel) are modeled jointly. The errors between model and observed prices has a standard deviation of around 2% resp. 6.5%. Especially for the HDD futures contracts, the roll time of the futures contract is identifiable by the jump in prices. For the period April to September, the closest HDD future is the October contract, which is seen in the figure as a the longer, flatter lines.



closest, 2nd closest, 3rd closest, 4th closest, 5th closest, 6th closest, 7th closest, 8th closest, 9th closest, 10th closest, 11th closest, 12th closest

Figure III.10. Time series of squared percentage pricing errors for gas.

The figure shows the time series of squared pricing errors of the percentage difference between fitted and actual NYMEX natural gas futures prices when modeled jointly with Chicago HDD futures. The pricing errors are largest around the 2008 boom/bust in energy prices.

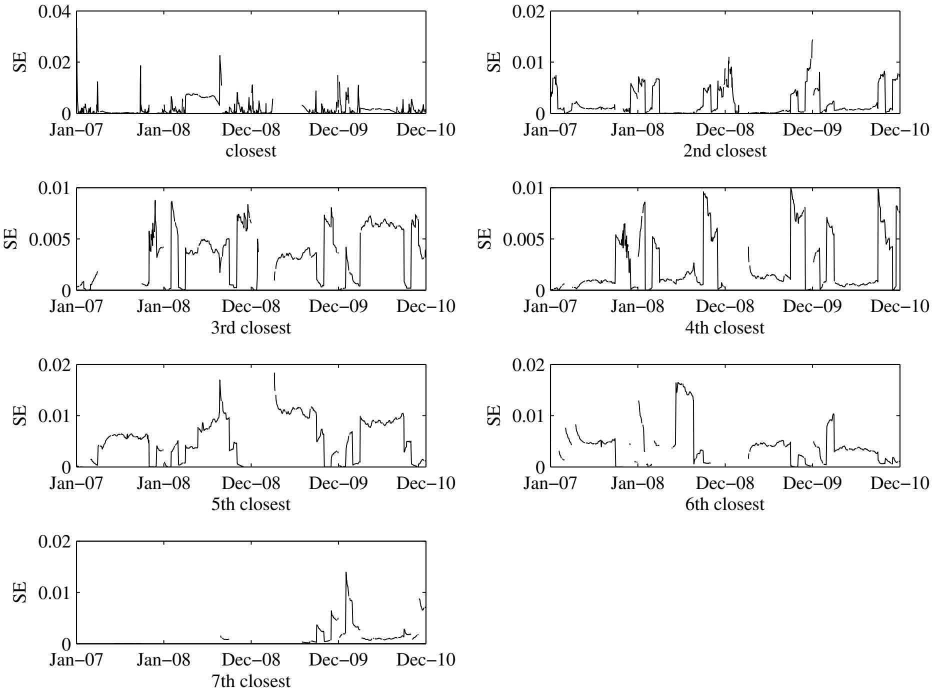


Figure III.11. Time series of squared percentage pricing errors for Chicago HDD.

The figure shows the time series of squared pricing errors of the percentage difference between fitted and actual Chicago HDD futures prices when modeled jointly with natural gas futures. The pricing errors jump when the contract roll.

Essay IV

A Short Note on Pricing of Energy Quanto Options

Abstract

In energy markets, market prices and demand are often positively correlated leading to an amplified risk for e.g., retail energy companies. Because of standardized derivatives' inability to hedge this risk, market participants have turned to using contracts that pay off dependent on both energy prices and a load-correlated variable such as a weather index. An example of such a contract is the so-called "energy quanto" option that only pays off, if both prices and the weather index are high (or if both are low). Under the assumption of a log-normal framework, the energy quanto option price can be approximated by a pricing formula involving the correlation, current individual asset prices and option prices, volatilities and Greeks. These quantities are known or can be assessed by market participants, thereby yielding a fast pricing method for energy quanto options.

IV.1 Introduction

A major risk in commodity markets is the quantity risk arising from the participants not having full control over production or consumption. A classic example from the world of energy is the Load Serving Entity, who needs to provide their electricity retail customers with the demanded quantity at a pre-determined rate. As price and quantity is positively correlated, this could lead to an undesired situation, when price and quantity are high as this leads to a loss for the LSE, or when price and quantity are low as this leads to only a small revenue. Also in agriculture, a farmer faces quantity risk as he is selling his crops at market prices, while not being able to control the exact production.

In Essay II, it was argued why double structure options or energy quanto options as discussed in Caporin et al. (2012) and Benth et al. (2015) is a natural contract structure when hedging quantity risks. The applicability of these is further supported by statements from re-insurance companies as Swiss Re[1] and Munich Re[2]. The pricing question, also addressed in e.g., Jørgensen (2007), Caporin et al. (2012) and Benth et al. (2015), is in practice complicated, as the underlying contracts are often not traded liquidly in the market and pricing methods would either rely on assumptions about risk-neutral distribution or an actuarial approach, where the price is set according to an acceptable loss.

In this note, a price approximation formula is derived. Assuming a log-normal framework for the two underlying assets, a price approximation is stated in terms of Deltas, Gammas, single options, (integrated) volatility and correlation of underlying (log-)prices. Some of the values are given in case of liquidly traded markets and otherwise a value that a market participant has an opinion about. The log-normal framework encompasses models like Schwartz and Smith (2000), Sørensen (2002) or the multi-factor model first introduced by Clewlow and Strickland (1999) and further analysed in Benth and Koekebakker (2008).

The approximation formula is presented in the next section and a numerical example illustrates the pricing error and behaviour. All proofs are given in Appendix IV.A and IV.B.

IV.2 The Quick and Dirty Formulas

Assume that the energy quanto option is written on two underlying assets that follows a joint log-normal distributed. More specifically, the future energy price can be written as

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[1] http://www.swissre.com/corporate_solutions/weather_risk_solutions_double_trigger.html

[2] Munich Re: Topics Risk solutions, Issue 4, 2014

$E_T = E_0 e^{\mu_E + \sigma_E X}$ and the future index contract's price can be written as $I_T = I_0 e^{\mu_I + \sigma_I Y}$, where (X, Y) is standard bivariate normal with correlation ρ. The price of an energy quanto option with strikes K_E resp. K_I is then given as

$$C(E_0, I_0; \rho) = e^{-rT} \mathbb{E}^Q \left[\max\left(E_0 e^{mu_E + \sigma_E X} - K_E, 0 \right) \times \max\left(I_0 e^{\mu_I + \sigma_I Y} - K_I, 0 \right) \right] \tag{IV.1}$$

To approximate the price of the energy quanto option, a first order Taylor approximation of (IV.1) around $\rho = 0$ gives the following approximation for the energy quanto prices[3]:

Proposition IV.1 (First order approximation for energy quanto option)
Using the above introduced notation; when the underlying asset are log-normally distributed under the pricing measure, the energy quanto option price can be approximated using individual option prices, current asset values, Deltas, volatilities and correlation of log-prices:

$$\begin{aligned} C(E_0, I_0; \rho) \approx & C(E_0, I_0; 0) + \rho \frac{\partial C}{\partial \rho}(E_0, I_0; 0) \\ = & e^{rT} \left[C_E(E_0) C_I(I_0) + \rho E_0 I_0 \sigma_E \sigma_I \Delta_E \Delta_I \right] \end{aligned} \tag{IV.2}$$

Here, $C_E(E_0)$ denote the price and Δ_E the option Δ for a corresponding call option on energy with strike K_E and similar for the index[4]. □

Proof: This results is derived from a first order Taylor approximation of (IV.1) around $\rho = 0$. See the details in appendix IV.A. ▮

Proposition IV.2 (Second order approximation for energy quanto option)
When the underlying asset are log-normally distributed under the pricing measure, the energy quanto option price can be approximated using individual option prices, current asset prices, Deltas and Gammas, volatilities and correlation of log-prices:

$$\begin{aligned} C(E_0, I_0; \rho) \approx & C(E_0, I_0; 0) + \rho \frac{\partial C}{\partial \rho}(E_0, I_0; 0) + \tfrac{1}{2} \frac{\partial^2 C}{\partial \rho^2}(E_0, I_0; 0) \\ = & e^{rT} \big[C_E(E_0) C_I(I_0) + \rho E_0 I_0 \sigma_E \sigma_I \Delta_E \Delta_I \\ & + \tfrac{1}{2} \rho^2 E_0 I_0 \sigma_E^2 \sigma_I^2 \left[\Gamma_E \Delta_I E_0 + \Delta_E \Delta_I + \Gamma_E \Gamma_I E_0 I_0 + \Gamma_I \Delta_E I_0 \right] \big] \end{aligned} \tag{IV.3}$$

---

[3] Similar price approximations can be derived for a product of swaps (the so-called energy quanto swaps), but not for spread options as the derivative with respect to ρ is not multiplicative.

[4] σ_i denotes the square root of the integrated variance of the contract from today to maturity. In the most simple case such as Black-Scholes, this equals $\sigma\sqrt{T}$

Proof: This results is derived from a second order Taylor approximation of (IV.1) around $\rho = 0$. See the details in appendix IV.A. ∎

As an energy quanto option price has to be non-negative, the price approximations should further be capped at zero.

IV.3 Numerical Example

The proposed formula enables us to write the price of an energy quanto option in terms of individual option prices, deltas and volatilities. Obviously, the approximation will be best around $\rho = 0$. Figures IV.1-IV.2 show the performance of IV.1 and IV.2 for the values given in Table IV.1.

E_0	K_E	σ_E	I_0	K_I	σ_I	r	T
4	4	0.4	1000	1000	0.5	0	1

Table IV.1. Parameters for numerical example.

Figure IV.1 shows the exact energy quanto option price for the parameters given in Table IV.1 as well as the two price approximations. The energy quanto option price is convex in correlation, so with the price approximation in Proposition IV.1 being linear, the approximation will always underestimate the true price. Further, it will at not be a relevant approximation once the correlation becomes sufficiently negative after which the approximation returns a negative value. The approximation in Proposition IV.2 underestimates the price as correlation increases and overestimates the price when correlation decreases.

Figure IV.2 show the percentage price error:

$$\text{Error} = \frac{\text{Price} - \text{Approximation}}{\text{Price}}$$

The first order price approximation quickly becomes large, when the correlation deviates from zero. The second order price approximations is more accurate. For the parameters given in Table IV.1, the absolute pricing error is less than 1%, when correlation is between -0.4 and 0.4. The price approximations is less than 4% for positive correlations, but grows to (minus) infinity as the true price converges to zero. The large pricing error for strong

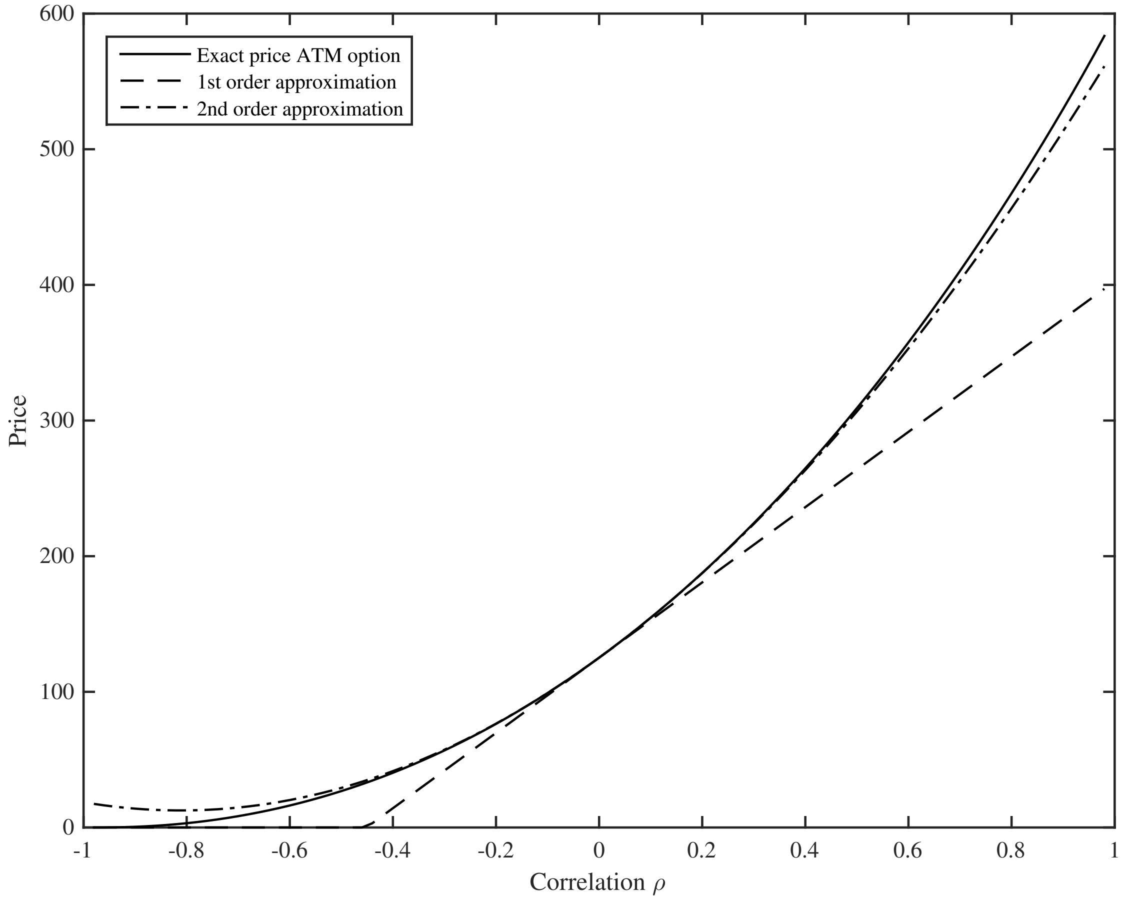

Figure IV.1. Exact price and price approximations

The figure shows the exact price energy quanto price for the parameters given in Table IV.1. The first order approximation is bounded at zero when the correlation is low. The first order approximation consistently underestimates the price due to the convexity of the energy quanto price as a function of correlation. The second order approximation underestimates the price for high correlations and overestimates the price for negative correlations. As there is little reason for entering a double call-structure for strong negative correlations, the pricing error is of less importance for strong negative correlations.

negative correlation is not concerning, as there is little reason for entering into a double call-structure, if the assets move in opposite directions.

IV.4 Concluding remarks

This short note showed how the price of an energy quanto option can be approximated using Greeks and other quantities. The numerical example showed how well the approximation worked within model. Of course it is a different question how accurate the approximations are, if data does not match the distributional assumptions.

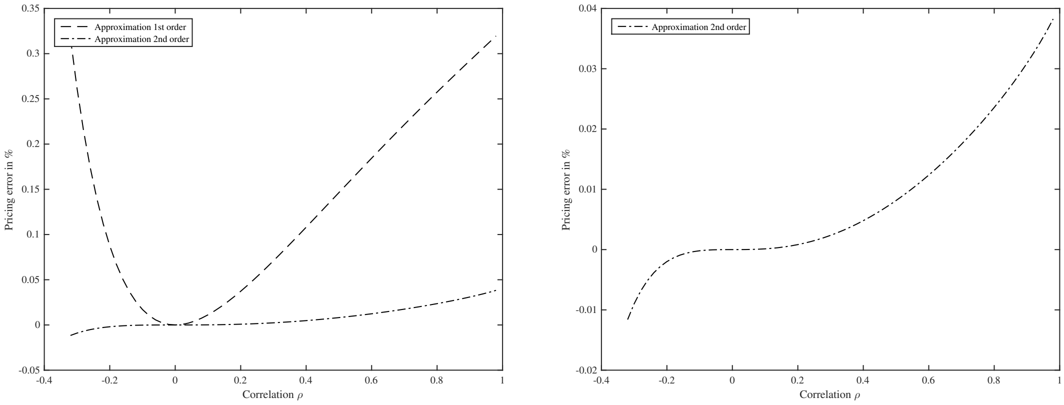

Figure IV.2. Approximation errors for first and second order price approximations. The figures shows the percentage price error for the parameters given in Table IV.1. The left figure shows the percentage price error for both the first and the second order price approximation. The first order price approximation quickly becomes large, when the correlation deviates from zero. The second order price approximations is much more accurate in comparison. The right figure shows a zoom of the second order price approximations. This percentage price error is for these numbers less than 4% for positive correlations, but grows to (minus) infinity as the true price converges to zero. For a correlation between -0.4 and 0, the pricing error is less than 1%.

Some of the inputs in Propositions IV.1 and IV.2 are easily obtainable; namely the strikes and the current values of the underlying contracts. The volatility input is not current volatility of the underlying asset, but rather the (time scaled) implied volatility of a corresponding univariate option. The Deltas and Gammas are likewise for corresponding univariate options. A trader or a structurer familiar with options traded in the market might have a good estimates for both the Greeks and the implied volatility.

On a final note, the approximation formulas can be adjusted to a put-put, a put-call or a call-put structure. The former will also experience a higher price error for negative correlation, but again less important. For the two latter, the conclusion is the opposite – high errors for positive correlations, but then that would not be important.

Appendix

IV.A Proof of propositions IV.1-IV.2

Let $Y = \rho X + \sqrt{1-\rho^2}Z$, where $Z \sim \mathcal{N}(0,1)$ and independent of X. Then $(X, \rho X + \sqrt{1-\rho^2}Z)$ has the same distribution as (X,Y). The derivative of the quanto option price with respect to ρ and later evaluated at $\rho = 0$ is given by:

$$\begin{aligned}
\frac{\partial C}{\partial \rho}(E_0, I_0; \rho) &= \mathbb{E}^Q\left[e^{-rT}(E_0 e^{\mu_E + \sigma_E X} - K_E)^+ \times \frac{\partial}{\partial \rho}\left(I_0 e^{\mu_I + \sigma_I(\rho X + \sqrt{1-\rho^2}Z)} - K_I\right)^+\right] \\
&= e^{-rT}\mathbb{E}^Q\left[\left(E_0 e^{\mu_E + \sigma_E X} - K_E\right)^+ \times 1\left\{I_0 e^{\mu_I + \sigma_I(\rho X + \sqrt{1-\rho^2}Z)} > K_I\right\}\right. \\
&\qquad \left.\times\, I_0 e^{\mu_I + \sigma_I(\rho X + \sqrt{1-\rho^2}Z)}\sigma_I\left(X - \frac{\rho}{\sqrt{1-\rho^2}}Z\right)\right] \qquad \text{(IV.4)} \\
\frac{\partial C}{\partial \rho}(E_0, I_0; 0) &= e^{-rT}\mathbb{E}^Q\left[X\left(E_0 e^{\mu_E + \sigma_E X} - K_E\right)^+ \times 1\left\{I_0 e^{\mu_I + \sigma_I Z} > K_I\right\} \times I_0 e^{\mu_I + \sigma_I Z}\sigma_I\right] \\
&= e^{-rT}\mathbb{E}^Q\left[X(E_0 e^{\mu_E + \sigma_E x} - K_E)^+\right]\mathbb{E}^Q\left[1\left\{I_0 e^{\mu_I + \sigma_I Z} > K_I\right\}e^{\mu_I + \sigma_I Z}\right] I_0 \sigma_I \\
&= e^{rT}\Delta_E \Delta_I E_0 I_0 \sigma_E \sigma_I
\end{aligned}$$

The latter equality follows by Lemmas IV.3 and IV.4.

To find the second derivative with respect to ρ, the first derivative as given in equation (IV.4) is first expressed in terms of integrals and then differentiated. To ease the notation, let $d_E = \frac{\log(K_E/E_0) - \mu_E}{\sigma_E}$ and $d_I(x,\rho) = \frac{\log(K_I/I_0) - \mu_I}{\sigma_I\sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}}x$:

$$\begin{aligned}
e^{rT}\frac{\partial C}{\partial \rho}(E_0, I_0; \rho) &= \int_{d_E}^{\infty}\left(E_0 e^{\mu_E + \sigma_E x} - K_E\right)\phi(x) I_0 \sigma_I \\
&\qquad \left[\int_{d_I(x,\rho)}^{\infty} e^{\mu_I + \sigma_I\left(\rho x + \sqrt{1-\rho^2}z\right)}\left(x - \frac{\rho}{\sqrt{1-\rho^2}}z\right)\phi(z)dz\right]dx \\
e^{rT}\frac{\partial^2 C}{\partial \rho^2}(E_0, I_0; \rho) &= \int_{d_E}^{\infty}\left(E_0 e^{\mu_E + \sigma_E x} - K_E\right)\phi(x) I_0 \sigma_I \qquad \text{(IV.5)} \\
&\qquad \frac{\partial}{\partial \rho}\left[\int_{d_I(x,\rho)}^{\infty} e^{\mu_I + \sigma_I\left(\rho x + \sqrt{1-\rho^2}z\right)}\left(x - \frac{\rho}{\sqrt{1-\rho^2}}z\right)\phi(z)dz\right]dx
\end{aligned}$$

Set $f(z,\rho) = e^{\mu_I+\sigma_I\left(\rho x+\sqrt{1-\rho^2}z\right)}\left(x-\frac{\rho}{\sqrt{1-\rho^2}}z\right)\phi(z)$. Then $\frac{\partial}{\partial\rho}\left[\cdots\right]$ in the equation above is given by

$$\begin{aligned}
\frac{\partial}{\partial\rho}\left[\int_{d_I(x,\rho)}^{\infty} f(z,\rho)dz\right] &= \int_{d_I(x,\rho)}^{\infty}\frac{\partial}{\partial\rho}f(z,\rho)dz - f(d_I(x,\rho),\rho)\frac{\partial}{\partial\rho}d_I(x,\rho)\\
&= -\int_{d_I(x,\rho)}^{\infty} e^{\mu_I+\sigma_I\left(\rho x+\sqrt{1-\rho^2}z\right)}\frac{z}{(1-\rho^2)^{3/2}}\phi(z)dz\\
&\quad+\int_{d_I(x,\rho)}^{\infty}\sigma_I e^{\mu_I+\sigma_I\left(\rho x+\sqrt{1-\rho^2}z\right)}\left(x-\frac{\rho}{\sqrt{1-\rho^2}}z\right)^2\phi(z)dz\\
&\quad-\frac{K_I}{I_0}\left(x-\frac{\rho}{1-\rho^2}\left(\frac{\log(K_I/I_0)-\mu_I}{\sigma_I}-\rho x\right)\right)\phi(d_I(x,\rho))\\
&\quad\times\left(\frac{\log(K_I/I_0)-\mu_I}{\sigma_I}\rho - x\right)\frac{1}{(1-\rho^2)^{3/2}}
\end{aligned}$$

Evaluating this in $\rho = 0$ yields (with $d_I(x,0) = d_I$):

$$\int_{d_I}^{\infty}\sigma_I e^{\mu_I+\sigma_I z}x^2\phi(z)dz - \int_{d_I}^{\infty} e^{\mu_I+\sigma_I z}z\phi(z)dz + \frac{K_I}{I_0}x^2\phi\left(d_I\right)$$

Inserting in (IV.5):

$$\begin{aligned}
e^{rT}\frac{\partial^2 C}{\partial\rho^2}(E_0,I_0;0) &= I_0\sigma_I\int_{d_E}^{\infty}x^2\left(E_0e^{\mu_E+\sigma_E x}-K_E\right)\phi(x)dx\int_{d_I}^{\infty}\sigma_I e^{\mu_I+\sigma_I z}\phi(z)dz\\
&\quad+I_0\sigma_I\int_{d_E}^{\infty}x^2\left(E_0e^{\mu_E+\sigma_E x}-K_E\right)\phi(x)dx\frac{K_I}{I_0}\phi\left(d_I\right)\\
&\quad-I_0\sigma_I\int_{d_E}^{\infty}\left(E_0e^{\mu_E+\sigma_E x}-K_E\right)\phi(x)dx\int_{d_I}^{\infty}e^{\mu_I+\sigma_I z}z\phi(z)dz\\
&= I_0\sigma_I^2\mathbb{E}^Q\left[\max(E_0e^{\mu_E+\sigma_E X}-K_E,0)X^2\right]\mathbb{E}^Q\left[1\left\{I_0e^{\mu_I+\sigma_I Z}>K_I\right\}e^{\mu_I+\sigma_I Z}\right]\\
&\quad+\sigma_I\mathbb{E}^Q\left[\max(E_0e^{\mu_E+\sigma_E X}-K_E,0)X^2\right]K_I\phi\left(d_I\right)\\
&\quad-I_0\sigma_I\mathbb{E}^Q\left[\max(E_0e^{\mu_E+\sigma_E X}-K_E,0)\right]\mathbb{E}^Q\left[1\left\{I_0e^{\mu_I+\sigma_I Z}>K_I\right\}e^{\mu_I+\sigma_I Z}Z\right]\\
&= e^{2rT}I_0E_0\sigma_I^2\sigma_E^2(E_0\Gamma_E\Delta_I+\Delta_E\Delta_I+I_0E_0\Gamma_I\Gamma_E+I_0\Gamma_I\Delta_E)
\end{aligned}$$

which with the help of Lemmas IV.4-IV.7 gives Proposition IV.2.

IV.B Delta and Gamma expressions

Assume that the asset price is log-normally distributed under the pricing measure $\mathbb{Q}$; $E_T = E_0e^{\mu+\sigma X}$. If it is a futures contract, then $\mu = -\frac{1}{2}\sigma^2$. If it is a stock $\mu = rT - \frac{1}{2}\sigma^2$. In the following interest rates are held constant. Without loss of generality, it is assumed that today is time 0.

Lemma IV.3 (Delta expression 1) Assume $E_T = E_0 e^{\mu+\sigma X}$. Then

$$\Delta = \frac{\partial}{\partial E_0}\mathbb{E}^Q\left[e^{-rT}(E_T-K)^+\right] = \frac{e^{-rT}}{E_0\sigma}\mathbb{E}^Q\left[X\left(E_0e^{\mu+\sigma X}-K\right)^+\right]$$

Proof: The expression is shown by first doing integration by substitution and then changing the order of differentiation and integration:

$$\begin{aligned}
\Delta &= \frac{\partial}{\partial E_0}\mathbb{E}^Q\left[e^{-rT}(E_T-K)^+\right] \\
&= \frac{e^{-rT}}{\sqrt{2\pi}}\frac{\partial}{\partial E_0}\int_{\mathbb{R}}\left(E_0e^{\mu+\sigma x}-K\right)^+e^{-\frac{1}{2}x^2}dx \\
&= \frac{e^{-rT}}{\sqrt{2\pi}\sigma}\frac{\partial}{\partial E_0}\int_{\mathbb{R}}\left(e^{\mu+t}-K\right)^+e^{-\frac{1}{2\sigma^2}(t-\log E_0)^2}dt \\
&= \frac{e^{-rT}}{\sqrt{2\pi}\sigma}\int_{\mathbb{R}}\left(e^{\mu+t}-K\right)^+e^{-\frac{1}{2\sigma^2}(t-\log E_0)^2}\frac{1}{E_0\sigma^2}(t-\log E_0)dt \\
&= \frac{e^{-rT}}{\sqrt{2\pi}}\int_{\mathbb{R}}\left(E_0e^{\mu+\sigma x}-K\right)^+e^{-\frac{1}{2}x^2}\frac{1}{E_0\sigma}xdx \\
&= \frac{e^{-rT}}{E_0\sigma}\mathbb{E}^Q\left[X\left(E_0e^{\mu+\sigma X}-K\right)^+\right]
\end{aligned}$$

∎

Lemma IV.4 (Delta expression 2) Assume $E_T = E_0 e^{\mu+\sigma X}$. Then

$$\Delta = \frac{\partial}{\partial E_0}\mathbb{E}^Q\left[e^{-rT}(E_T-K)^+\right] = e^{-rT}\mathbb{E}^Q\left[1\left\{E_0e^{\mu+\sigma X}>K\right\}e^{\mu+\sigma X}\right]$$

Proof: The expression is shown by changing the order of differentiation and integration

$$\begin{aligned}
\Delta &= \frac{\partial}{\partial E_0}\mathbb{E}^Q\left[e^{-rT}(E_T-K)^+\right] \\
&= \frac{\partial}{\partial E_0}\int_{-\infty}^{\infty}e^{-rT}(E_0e^{\mu+\sigma x}-K)^+\phi(x)dx \\
&= e^{-rT}\int_{-\infty}^{\infty}\frac{\partial}{\partial E_0}(E_0e^{\mu+\sigma x}-K)^+\phi(x)dx \\
&= e^{-rT}\int_{-\infty}^{\infty}\left[1\left\{E_0e^{\mu+\sigma x}>K\right\}e^{\mu+\sigma x}\right]\phi(x)dx \\
&= e^{-rT}\mathbb{E}^Q\left[1\left\{E_0e^{\mu+\sigma X}>K\right\}e^{\mu+\sigma X}\right]
\end{aligned}$$

Note, that the normality assumption was not needed for this Delta-expression. ∎

LEMMA IV.5 (GAMMA EXPRESSION 1) Assume $E_T = E_0 e^{\mu+\sigma X}$. Then

$$\Gamma = \frac{\partial \Delta}{\partial E_0} = \frac{e^{-rT}}{\sigma E_0} \mathbb{E}^Q \left[1 \left\{ E_0 e^{\mu+\sigma X} > K \right\} e^{\mu+\sigma X} X \right] - \frac{\Delta}{E_0}$$

Proof: This expression is derived by differentiating the Δ-expression in Lemma IV.3:

$$\begin{aligned}
\Gamma &= \frac{\partial}{\partial E_0} \frac{e^{-rT}}{E_0 \sigma} \mathbb{E}^Q \left[X \left(E_0 e^{\mu+\sigma X} - K \right)^+ \right] \\
&= \frac{e^{-rT}}{E_0 \sigma} \frac{\partial}{\partial E_0} \left(\mathbb{E}^Q \left[X \left(E_0 e^{\mu+\sigma X} - K \right)^+ \right] \right) + \frac{\partial}{\partial E_0} \left(\frac{e^{-rT}}{E_0 \sigma} \right) \mathbb{E}^Q \left[X \left(E_0 e^{\mu+\sigma X} - K \right)^+ \right] \\
&= \frac{e^{-rT}}{E_0 \sigma} \frac{\partial}{\partial E_0} \int_{\frac{\log(K/E_0)-\mu}{\sigma}}^{\infty} x \left(E_0 e^{\mu+\sigma x} - K \right) \phi(x) dx - \frac{\Delta}{E_0} \\
&= \frac{e^{-rT}}{E_0 \sigma} \mathbb{E}^Q \left[X e^{\mu+\sigma X} 1 \left\{ E_0 e^{\mu+\sigma X} > K \right\} \right] - \frac{\Delta}{E_0}
\end{aligned}$$

∎

LEMMA IV.6 (GAMMA EXPRESSION 2) Assume $E_T = E_0 e^{\mu+\sigma X}$. Then

$$\Gamma = \frac{\partial \Delta}{\partial E_0} = \frac{e^{-rT}}{\sigma E_0} \frac{K}{E_0} \phi \left(\frac{\log(E_0/K) + \mu}{\sigma} \right)$$

Proof: This expression is derived by differentiating the Δ-expression in Lemma IV.4:

$$\begin{aligned}
\Gamma &= \frac{\partial}{\partial E_0} e^{-rT} \mathbb{E}^Q \left[1 \left\{ E_0 e^{\mu+\sigma X} > K \right\} e^{\mu+\sigma X} \right] \\
&= e^{-rT} \frac{\partial}{\partial E_0} \int_{\frac{\log(K/E_0)-\mu}{\sigma}}^{\infty} e^{\mu+\sigma x} \phi(x) dx \\
&= e^{-rT} \frac{\partial}{\partial E_0} \frac{\log(K/E_0) - \mu}{\sigma} e^{\mu+\sigma \frac{\log(K/E_0)-\mu}{\sigma}} \phi \left(\frac{\log(K/E_0)-\mu}{\sigma} \right) \\
&= \frac{e^{-rT}}{\sigma E_0} \frac{K}{E_0} \phi \left(\frac{\log(K/E_0)-\mu}{\sigma} \right)
\end{aligned}$$

∎

LEMMA IV.7 (GAMMA EXPRESSION 3) Assume $E_T = E_0 e^{\mu+\sigma X}$. Then

$$\Gamma = \frac{\partial \Delta}{\partial E_0} = \frac{e^{-rT}}{\sigma^2 E_0^2} \mathbb{E}^Q \left[\max(E_0 e^{\mu+\sigma X} - K, 0) X^2 \right] - \frac{C}{\sigma^2 E_0^2} - \frac{\Delta}{E_0}$$

Proof: This expression is derived by differentiating the Δ-expression in Lemma IV.3 and doing integration by substition:

$$\begin{aligned}
\Gamma &= \frac{e^{-rT}}{E_0\sigma}\frac{\partial}{\partial E_0}\left(\mathbb{E}^Q\left[X\left(E_0e^{\mu+\sigma X}-K\right)^+\right]\right)-\frac{\Delta}{E_0}\\
&= \frac{e^{-rT}}{E_0\sigma\sqrt{2\pi}}\frac{\partial}{\partial E_0}\int_{\mathbb{R}} x\left(E_0e^{\mu+\sigma x}-K\right)^+ e^{-\frac{1}{2}x^2}dx-\frac{\Delta}{E_0}\\
&= \frac{e^{-rT}}{E_0\sigma^2\sqrt{2\pi}}\frac{\partial}{\partial E_0}\int_{\mathbb{R}}\frac{t-\log E_0}{\sigma}\left(e^{\mu+t}-K\right)^+ e^{-\frac{1}{2\sigma^2}(t-\log E_0)^2}dt-\frac{\Delta}{E_0}\\
&= \frac{e^{-rT}}{E_0\sigma^2\sqrt{2\pi}}\int_{\mathbb{R}}\frac{t-\log E_0}{\sigma}\left(e^{\mu+t}-K\right)^+ \frac{\partial}{\partial E_0}e^{-\frac{1}{2\sigma^2}(t-\log E_0)^2}dt\\
&\quad+\frac{e^{-rT}}{E_0\sigma^2\sqrt{2\pi}}\int_{\mathbb{R}}\frac{\partial}{\partial E_0}\frac{t-\log E_0}{\sigma}\left(e^{\mu+t}-K\right)^+ e^{-\frac{1}{2\sigma^2}(t-\log E_0)^2}dt-\frac{\Delta}{E_0}\\
&= \frac{e^{-rT}}{E_0^2\sigma^2\sqrt{2\pi}}\int_{\mathbb{R}} x^2\left(E_0e^{\mu+\sigma x}-K\right)^+ e^{-\frac{1}{2}x^2}dx\\
&\quad+\frac{e^{-rT}}{E_0^2\sigma^2\sqrt{2\pi}}\int_{\mathbb{R}}\left(E_0e^{\mu+\sigma x}-K\right)^+ e^{-\frac{1}{2}x^2}dx-\frac{\Delta}{E_0}\\
&= \frac{e^{-rT}}{\sigma^2E_0^2}\mathbb{E}^Q\left[\max(E_0e^{\mu+\sigma X}-K,0)X^2\right]-\frac{C}{\sigma^2E_0^2}-\frac{\Delta}{E_0}
\end{aligned}$$

∎

Conclusion

This dissertation presented four essays related to correlation in energy markets. How prices relate is generally important for risk management and derivatives pricing purposes and the understanding of these relationships and the risk arising from them is therefore interesting.

The first essay investigated if and how volatility of two different markets, the WTI crude oil market and the EURUSD market, changed over time. The analysis is done using market-perceived volatilities in the form of options and futures and shows a joint factor after mid-2007. The conclusion was made based on both a model-free approach and the estimation of a term structure model for futures and options on oil and futures and options on EURUSD. The cause of this volatility-relation is a question left unanswered. A natural extension of this essay would be to investigate the effect of fundamentals, financialization and the latest financial crisis.

The second essay compares energy quanto options as a risk management tool to other approaches proposed in the literature and shows that that it performs well in a simple one-period model. An energy quanto option has an easy and understandable structure and as they are already offered in the market, their potential for risk management using real data is a relevant next area to study, if it is possible to obtain load data from a Load Serving Entity.

The third essay and fourth essay concentrates on pricing energy quanto options. If the energy quanto option is written as an average over a period of time, it is possible to convert the rewrite the problem in terms of options on futures covering the same period of time. In a log-normal framework, closed form solutions are available both for prices and Greeks and prices can further be approximated using the prices and Greeks of the univariate options along with the correlation.

Bibliography

Aase, K. K. (2004). A Pricing Model for Quantity Contracts. *The Journal of Risk and Insurance 71*(4), 617–642.

Alexander, C. (1999). Correlation and Cointegration in Energy Markets. In *Managing Energy Price Risk*, pp. 297–304. RISK Publications.

Amano, R. A. and S. van Norden (1998). Exchange Rates and Oil Prices. *Review of International Economics 6(4)*, 683–694.

Bakshi, G., P. Carr, and L. Wu (2008). Stochastic Risk Premiums, Stochastic Skewness in Currency Options, and Stochastic Discount Factors in International Economies. *Journal of Financial Economics 87*(1), 132–156.

Barone-Adesi, G. and R. E. Whaley (1987). Efficient Analytical Approximation of American Option Values. *The Journal of Finance 42*(2), 301–320.

Basak, S. and A. Pavlova (2016). A Model of Financialization of Commodities. *Journal of Finance 71*(4), 1511–1556.

Bellini, F. (2005). *The weather derivatives market: Modeling and pricing temperature.* Ph. D. thesis, University of Lugano.

Benth, F. E. and J. Š. Benth (2013). *Modeling and Pricing in Financial Markets for Weather Derivatives.* World Scientific.

Benth, F. E., J. Š. Benth, and S. Koekebakker (2008). *Stochastic Modelling of Electricity and Related Markets.* World Scientific.

Benth, F. E. and S. Koekebakker (2008). Stochastic Modeling of Financial Electricity Contracts. *Energy Economics 30*(3), 1116–1157.

Benth, F. E., N. Lange, and T. Å. Myklebust (2015). Pricing and Hedging Quanto Options in Energy Markets. *Journal of Energy Markets 8*(1), 1–36.

Black, F. (1976). The pricing of commodity contracts. *Journal of Financial Economics 3*(1-2), 167–179.

Brown, G. W. and K. B. Toft (2002). How Firms Should Hedge. *Review of Financial Studies 15*(4), 1283–1324.

Caporin, M., J. Preś, and H. Torro (2012). Model based Monte Carlo pricing of energy and temperature Quanto options. *Energy Economics 34*(5), 1700–1712.

Carmona, R. (2015). Financialization of the commodities markets: A non-technical introduction. *Fields Institute Communications 74*, 3–37.

Carr, P. and D. Madan (2001). Optimal positioning in derivative securities. *Quantitative Finance 1*(1), 19–37.

Chen, S.-S. and H.-C. Chen (2007). Oil prices and real exchange rates. *Energy Economics 29*(3), 390–404.

Chen, Y.-C., K. Rogoff, and B. Rossi (2010). Can exchange rates forecast commodity prices? *The Quarterly Journal of Economics*, 1145–1194.

Chiarella, C., B. Kang, C. S. Nikitopoulos, and T.-D. Tô (2013). Humps in the volatility structure of the crude oil futures market: New evidence. *Energy Economics 40*, 989–1000.

Christoffersen, P., A. Lunde, and K. V. Olesen (2014). Factor Structure in Commodity Futures Return and Volatility Factor Structure in Commodity Futures Return and Volatility. *CREATES research paper, Aarhus University 31*.

Clewlow, L. and C. Strickland (1999). A Multi-Factor Model for Energy Derivatives. *QFRC Research Paper Series, University of Technology, Sydney*.

Clewlow, L. and C. Strickland (2000). *Energy Derivatives: Pricing and Risk Management.* Lacima Publications.

Considine, G. (2000). Introduction to Weather Derivatives. *Technical report, CME Group*.

Cornaggia, J. (2013). Does risk management matter? Evidence from the U.S. agricultural industry. *Journal of Financial Economics 109*(2), 419–440.

Cortazar, G., S. Gutierrez, and H. Ortega (2016). Empirical Performance of Commodity Pricing Models: When is it Worthwhile to Use a Stochastic Volatility Specification? *Journal of Futures Markets 36*(5), 457–487.

Cortazar, G. and L. Naranjo (2006). An N-factor Gaussian model of oil futures prices. *Journal of Futures Markets 26*(3), 209–313.

Cucu, L., R. Döttling, P. Heider, and S. Maina (2016). Managing temperature-driven volume risks. *Journal of Energy Markets 9*(2), 95–110.

Davis, A. (2010). A new direction for weather derivatives. *http://www.risk.net/energy-risk/feature/1652654/a-direction-weather-derivatives*.

Diebold, F. and R. Mariano (1995). Comparing Predictive Accuracy. *Journal of Business and Economic Statistics 13*, 252–263.

Ding, L. and M. Vo (2012). Exchange rates and oil prices: A multivariate stochastic volatility analysis. *Quarterly Review of Economics and Finance 52*(1), 15–37.

Engle, R. F., C. Mustafa, and J. Rice (1992). Modelling peak electricity demand. *Journal of Forecasting 11*(3), 241–251.

Eydeland, A. and K. Wolyniec (2003). *Energy and Power Risk Management.* John Wiley & Sons.

Fratzscher, M., D. Schneider, and I. V. Robays (2014). Oil Prices, Exchange Rates and Asset Prices. *ECB Working Paper Series* (1689), 1–43.

Galeano, P. and D. Wied (2014). Multiple break detection in the correlation structure of random variables. *Computational Statistics and Data Analysis 76*, 262–282.

Garman, M. B. and S. W. Kohlhagen (1983). Foreign currency option values. *Journal of International Money and Finance 2*(3), 231–237.

Geman, H. (2005). *Commodities and commodity derivatives.* John Wiley & Sons.

Geman, H. (2015). *Agricultural Finance.* John Wiley & Sons.

Golden, L. L., M. Wang, and C. Yang (2007). Handling weather related risks through the financial markets: Considerations of credit risk, basis risk, and hedging. *Journal of Risk and Insurance 74*(2), 319–346.

Harvey, A. C. (1989). *State space models and the Kalman filter.* Cambridge University Press.

Id Brik, R. and A. Roncoroni (2016a). Hedging Size Risk: Theory and Application to the US Gas Market. *http://papers.ssrn.com/sol3/papers.cfm?abstract_ id=2790350*.

Id Brik, R. and A. Roncoroni (2016b). Static Mitigation of Volumetric Risk. *Journal of Energy Markets 9*(2), 111–150.

Jørgensen, P. L. (2007). Traffic light options. *Journal of Banking & Finance 31*(12), 3698–3719.

Karatzas, I. and S. E. Shreve (2000). *Brownian Motion and Stochastic Calculus*. Springer.

Kilian, L. (2009). Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. *The American Economic Review 99*(3), 1053–1069.

Kleindorfer, P. R. P. R. and L. Li (2005). Multi-period VaR-constrained portfolio optimization with applications to the electric power sector. *The Energy Journal 26*(1), 1–26.

Korn, O. (2009). How firms should hedge: An Extension. *Journal of Futures Markets 30*(9), 834–845.

Lee, Y. and S. S. Oren (2009). An equilibrium pricing model for weather derivatives in a multi-commodity setting. *Energy Economics 31*(5), 702–713.

Lien, D. and D. A. Hennessy (2004). A note on price futures versus revenue futures contracts. *Journal of Futures Markets 24*(2), 503–512.

Lizardo, R. A. and A. V. Mollick (2010). Oil price fluctuations and U.S. dollar exchange rates. *Energy Economics 32*(2), 399–408.

Lucia, J. J. and E. S. Schwartz (2002). Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research 5*, 5–50.

Miltersen, K. R. and E. S. Schwartz (1998). Pricing of Options on Commodity Futures with Stochastic Term Structure of Convenience Yields and Interest Rates. *Journal of Financial and Quantitative Analysis 33(1)*, 33–59.

Moschini, G. and H. Lapan (1995). The hedging role of options and futures under joint price, basis and production risk. *International Economic Review 36*(4), 1025–1049.

Munich Re (2014). Topics Risk solutions, Insurance solutions for industry, Issue 4/2014. *https://www.munichre.com/*.

Myers, R. (2008). What every CFO needs to know about weather risk management. *Technical report, CME Group*.

Näsäkkälä, E. and J. Keppo (2005). Electricity load pattern hedging with static forward strategies. *Managerial Finance 31*(6), 116–137.

Oum, Y. and S. Oren (2006). Hedging quantity risks with standard power options in a competitive wholesale electricity market. *Naval Research Logistics 53*(7), 1–33.

Oum, Y. and S. Oren (2010). Optimal Static Hedging of Volumetric Risk in a Competitive Wholesale Electricity Market. *Decision Analysis 7*(1), 107–122.

Pérez-González, F. and H. Yun (2013). Risk management and firm value: Evidence from weather derivatives. *The Journal of Finance 68*(5), 2143–2176.

Pilz, K. F. and E. Schlögl (2012). A Hybrid Commodity and Interest Rate Market Model. *Quantitative Finance 13*(1), 1–18.

Poitras, G. (2013). *Commodity Risk Management.* Routledge.

Schwartz, E. S. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance 52*(3), 923–973.

Schwartz, E. S. and J. E. Smith (2000). Short-term variations and long-term dynamics in commodity prices: Implications for valuation and hedging. *Management Science 46*, 893–911.

Sørensen, C. (2002). Seasonality in Agricultural Commodity Futures. *The Journal of Futures Markets 22*(5), 393–426.

Swindle, G. (2014). *Valuation and Risk Management in Energy Markets.* Cambridge University Press.

Tang, K. and W. Xiong (2012). Index investment and financialization of commodities. *Financial Analysts Journal 68*(6), 54–74.

Timmer, R. P. and P. J. Lamb (2007). Relations between temperature and residential natural gas consumption in the Central and Eastern United States. *Journal of Applied Meteorology and Climatology 46*(11), 1993–2013.

Trolle, A. B. and E. S. Schwartz (2009). Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives. *Review of Financial Studies 22*(11), 4423–4461.

Vansteenkiste, I. (2011). What is driving oil futures prices? Fundamentals versus speculation. *European Central Bank Working Paper*.

Verleger, P. K. (2008). The Oil-Dollar Link. *The International Economy*, 46–50.

Wied, D., W. Krämer, and H. Dehling (2009). Testing for a Change in Correlation At an Unknown Point in Time Using an Extended Functional Delta Method. *Econometric Theory 28*(3), 570–589.

Zhang, P. G. (1995). Correlation Digital Options. *Journal of Financial Engineering 4*(1), 75–96.

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