Essays on the Design of Contracts and Markets for Power System Flexibility

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"The Doctoral School of Economics and Management is an active national and international research environment at CBS for research degree students who deal with economics and management at business, industry and country level in a theoretical and empirical manner".

Preface

This PhD thesis is the result of my work as PhD Fellow and Research Assistant at the Department of Economics, Copenhagen Business School. I am grateful for the economic support received through the ForskEL-funded research consortium TotalFlex and for other departmental funds that allowed me to complete my studies.

There are several people to whom I wish to show gratitude. First, I would like to mention my main supervisor during the first two years of doctoral training, Peter Møllgaard. I can still recall very clearly the Skype interview I had with him, which ultimately motivated me to move from Venezuela to Denmark - along with my family - to start this project. I wish to thank him for welcoming me to the department he led at the time, but also for giving me and my family a warm welcome to a country we didn't know then. Without the confidence he initially placed in me and the guidance he offered in several respects, none of this would have been possible. Second, but with the same level of gratitude, I wish to mention Peter Bogetoft, my current main supervisor, who took over Peter Møllgaard's role during the last part of my PhD studies. In particular, I wish to thank him for his words of advice and clever approach, but especially for allowing me to pursue my research interests with freedom and showing me the way to the end of this thesis. Another person I would like to say a big and warm thank you is to Anette Boom, who has been my secondary supervisor but has also been a diligent PhD Coordinator during the longest part of my period as PhD student. At different stages of this project, she kindly offered her time to listen but also to counsel and encourage me. Another person I wish to thank is my good friend and co-author Rahmatallah Poudineh. Thanks for accepting to embark with me on two chapters of this PhD thesis, requiring endless hours of discussion. I'm glad that our teamwork, which dates back to the "Surrey days", has endured. I also want to thank the members of my pre-defence committee, Dolores Romero and Per Agrell, for providing useful comments to earlier versions of the chapters that compose this thesis.

I also wish to gratefully acknowledge the Christian og Ottilia Brorsons Rejselegat and the Otto Mønsteds fund for economically supporting my research visit to the Centre of Mathematics for Applications (CMA) at the University of Oslo (UiO) during the period January-April 2014. In particular, I want to thank Fred Espen Benth, who kindly hosted me and spent time discussing interesting research ideas in the field of Energy Finance with me. In addition to my visit to the CMA, I have also received inspiration through my attendance to different courses, seminars, project meetings and conferences in several cities, which I never thought I would get to know. Aalborg, Aarhus, Antalya, Berlin, Copenhagen, Düsseldorf, Jerusalem, Istanbul, Louvain, Nyborg, Oslo, Rome and Zurich were, to some extent, the background scenery for this thesis.

On a personal note, I wish to mention a fundamental person in my life, along whom I have fought the most important battles and dreamed the dearest dreams. I refer to my wife Luisana to whom saying “thank you” only, wouldn’t do justice. This project, which has been *ours* and not only mine from the very beginning, would have been just impossible without your support, patience, encouragement but, above all, your unselfishness. Starting from that conversation we had in that café in Chacaito in 2012, when *we* decided that I would do the PhD, to the present when everything is done, you have always been at my side. Every single line of this thesis is also yours, belongs to you and I dedicate to you. This is only fair, because while I was writing, you were taking care of *everything* else, making our lives easier, making sure everything ran smoothly for the four of us and looking after Lucía and Luisa Amelia who - after all - inspire all our efforts. I love you with all my heart and I hope that I am able to compensate you and our daughters for all the time I have taken away from you. Building stability and a decent future for our family has always been a common goal. Now that this stepping stone is available, let us live the future, which is now! The time belongs to us as a family.

I also wish to thank all my family for their words of encouragement and support but, above all, for having always believed in me. In particular, I want to thank my dad, who passed away so many years ago, but managed to teach me to try and be the best, regardless of what we choose to do in life. I also want to thank my mother who, among many things, taught me the important life lesson of fighting until the very end.

Last, but definitely not least, I want to thank the one who really deserves all the praise: none of the above would have been possible without your protection and perfect grace. Thank you, Jesus.

Summary

This PhD thesis consists of five essays on the economics of Power System Flexibility, a topic that has traditionally been addressed from a technical perspective by engineers, system planners, electricity industry stakeholders and energy policymakers interested in the integration of Variable Renewable Energy. While significant progress has been made in the understanding and characterization of flexibility, its *economic* properties and the required *incentives* to provide it have not been sufficiently analyzed. The present work aims at filling this gap.

In the first chapter, entitled “Business Models for Power System Flexibility: New Actors, New Roles, New Rules”, Rahmatallah Poudineh (from the Oxford Institute for Energy Studies) and I take on the task of identifying and analyzing existing business models that enable power system flexibility, a requirement that is not actually novel but is becoming critical for the successful integration of renewables. We find that technological innovation - with the Smart Grid as catalyst - is essential to enable the flexibility of existing resources in the power system and note that many of these developments are already taking place. We claim that, as a result, an entirely different electricity industry is emerging: one in which new activities are being added to the traditional supply chain, contesting the status quo. Incumbents, who rely on traditional, large scale industrial assets are beginning to compete with entrants who depend on a non-traditional, knowledge-based mode of operation.

In the second chapter, “Power System Flexibility: A Product Design Perspective”, by way of concrete examples from the short-term operation of the Danish and Californinan power systems; I illustrate the need for flexibility when integrating renewables. In addition, I review the existing literature on the topic, which is mostly technically and policy-oriented and organize it according to the specific topics of interest in the current state of the debate. Motivated by the technical characteristics of Power System Flexibility, the paper then presents two simple, yet relevant contributions of normative nature. The first of these consists in three economic postulates that *should* guide the economic modelling of flexibility. Specifically, I claim that flexibility has *multiple attributes*, which are *imperfectly substitutable* and that flexibility is an inherently *heterogeneous* commodity. The second contribution is a set of desirable properties that any product design *should* have to actually enable flexibility, namely *simplicity*, *measurability* and *relevance*. The chapter, which ends with a review of existing product designs for power system flexibility, serves as a transition and establishes general guidelines for the three remaining chapters, which contain the core economic modelling of the thesis.

The third chapter, “Trading Demand-Side Flexibility in Power Markets”, which is joint work with Peter Bogetoft and Peter Møllgaard (from Copenhagen Business School), is a first approximation to the microeconomic modelling of flexibility. With a focus on the particular kind of flexibility that can be harnessed from demand-side resources, as mediated by a technological solution that reduces transaction costs to a negligible level, the paper proposes a baseline model of bilateral trade between a consumer and an aggregator, which highlights their gains from trading. Using the Nash bargaining approach as solution concept, the model and a number of extensions implemented, allows pricing flexibility but also achieves an additional insight into the role of investment costs in a long-term perspective. In particular: relative to the situation in which an aggregator and a consumer symmetrically share investment costs, a consumer is able to obtain a better deal for the flexibility it offers, if it faces a relatively higher cost than the aggregator. By extension, the model shows that economies of scale may be present in the aggregation business. This could induce a network effect in which the gains from trading flexibility increase as the number of consumers offering flexibility to a single aggregator increases. As a result, an open question remains: to what extent can a flexibility marketplace be competitive?

In the fourth chapter, “Flexibility-Enabling Contracts in Electricity Markets”, Rahmatallah Poudineh and I team again to study the procurement problem that a buyer (the principal) faces when acquiring flexibility from a supplier (the agent) in a bi-dimensional adverse selection setting in which both parties have non-separable utility and cost. The assumptions of the model stem directly from the economic characteristics of flexibility (discussed in the second chapter) and relates to specific situations, along the supply chain of flexibility services, in which the competitive procurement of flexibility is infeasible due, for example, to the presence of transaction costs and limited capacity size of the resource provider. Relative to the rent extraction-efficiency trade off conventionally studied under the assumption of separable functions, the model of this paper relates more generally to situations in which the elements that determine the cost of provision and the utility derived from a good or service cannot be separated. We find that, relative to the separable case, this “non-separable externality” among activities leads to further distortion of the inefficient types.

In the fifth and last chapter, “Product-Mix Exchanges, Efficiency and Power System Flexibility”, I develop a number of extensions to the Product-Mix Auction, originally proposed by Paul Klemperer. These are generalized under the name of Product-Mix Exchanges, which are double, multi-unit combinatorial auctions in which participants are restricted to reporting substitutable preferences, and can be on the demand side, the supply side or both sides of the market. The existence of equilibrium for the different variants of the exchange follows immediately from a previous result (coined as "the unimodularity theorem"), but in the paper I propose a simple, linear programing-based approach to checking the existence of equilibrium, given a set of participants' concave valuations. However, the main contributions of the paper are two. First, I show that Product-Mix Exchanges can be supported by Vickrey-Clarke-Groves payments, which ensure that each market participant obtains its marginal product as payoff and that truthful bidding is a weakly dominant strategy. This result is relevant for applications where sufficient market thickness is not guaranteed, a setting in which manipulation of the results is more likely, as participants have the incentive to report their true valuations. Second, I show an application of the exchanges to the design of a marketplace for flexibility, namely the Delta Energy Market.

Dansk Resumé

Denne PhD afhandling består af fem essays som omhandler emnet Power System Flexibility (Strømsystems fleksibilitet). Dette er et emne som traditionelt er blevet behandlet fra et teknisk synspunkt af ingeniør, systemplanlæggere, folk med aktier i energi sektoren og politikere som har udvist interesse i Variable Renewable Energy (variabel vedvarende energi). Påtrods af, at der er sket store fremskridt i forhold til forståelsen og karakteriseringen af fleksibilitet, såer dets *økonomiske* egenskaber endnu ikke blevet analyseret fyldestgørende. Denne afhandling forsøger at udfylde dette hul.

I det første kapitel "Business Models for Power System Flexibility: New Actors, New Roles, New Rule" har Rahmatallah Poudineh (fra the Oxford Institute for Energy Studies) og jeg påtaget os den opgave at identificere og analysere de eksisterende forretningsmodeller, som muliggør power system flexibility, et system der ikke er nyt, men som efterhånden er helt afgørende for en vellykket integration af vedvarende energi. Vi har opdaget, at teknologisk innovation, med Smart Grid som katalysator, er essentiel for et energisystem der skal have et fleksibelt forhold til forskellige energiressourcer. Vi finder ydermere at denne teknologiske udvikling allerede finder sted. Vi påstår, at dette vil lede til en markant anderledes energisektor, hvor nye aktiviteter bliver tillagt den traditionelle forsyningskæde, hvilket udfordrer det nuværende marked. De etablerede operatører, der hidtil har benyttet sig af traditionelle industrielle aktiver, er begyndt at konkurrere med nytilkommende ikke traditionelle og vidensbaseret operatører.

I det andet kapitel Power System Flexibility: A Product Design Perspective, bruger jeg konkrete eksempler fra korttidsbrug pådet danske og californiske strømsystem, for at eftervise behovet for fleksibelt integration af vedvarende energi. Ydemere, laver jeg en systematisk gennemgang af Power System Flexibility litteraturen, hvor jeg kategorisere de specifikke forhold der gør sig gældende i den nuværende debat. Den nuværende litteratur fokuserer primært påde tekniske og politiske aspekter af strømsystemet. Denne afhandling præsenterer to simple, men relevante bidrag af *normativ* natur, som er baseret påden tekniske karakteristisk af Power System Flexibilty. Den første består af tre økonomiske postulater som burde være afgørende for den økonomiske modellering af flexibility. Postulater der i enden betyder, at flexibility har forskellige attributter, som er imperfectly substitutable, og at flexibility i sagens natur er en heterogen vare. Det andet bidrag fremstiller en række af foretrukne egenskaber som et produkt specielt designet til at fremme flexibility burde have. Disse egenskaber er; enkelhed, målbarhed, og relevans. Kapitlet slutter med en gennemgang af de eksisterende produktdesigns af power system flexibility. Denne gennemgang tjener som en overgang der fastlægger de generelle rammer for de tre tilbageværende kapitler, hvori

afhandlingens økonomiske kernemodel bliver opstillet.

Det tredje kapitel "Trading Demand-Side Flexibility in Power Markets" er skrevet i samarbejde med Peter Bogetofte og Peter Møllgaard (fra Copenhagen Business School Handelshøjskolen). I dette kapitel fremstilles det første forsøg påen mikroøkonomiske modellering af flexiblity. Kapitlet fokuserer påden bestemte udgave af flexibility, som kan udnyttes gennem efterspør gselssidens resurser og som bliver formidlet gennem teknologiske løsninger, der reducerer transaktionsomkostningerne til et ubetydeligt niveau. Påbaggrund af dette foreslås der en baseline model for bilateral handel mellem en forbruger og en aggregator, som netop fremhæver parternes profit ved denne form for handel. Ved at gøre brug af Nash bargaining approach, kan man med denne model og et antal af implementerede udvidelser, tillade fleksible priser. Ved brug af denne metode opnås der ydermere indsigt i investeringsudgifternes rolle pålang sigt. Særligt med hensyn til den relative situation, hvor en aggregator og en forbruger deler investeringsudgiften symmetrisk, er det muligt for forbrugerne at opnåbedre fleksibilitet end hvis forbrugerne oplever relativ højere omkostning end aggregatoren. Det ligger op til en diskussion om, i hvilken grad et fleksibelt energimarked kan være konkurrencedygtigt?

Det fjerde kapitel "Flexibility-Enabling Contracts in Electricity Markets", er endnu et samarbejde mellem Rahmatallah Poudineh og jeg. Denne gang undersøger vi, hvorledes der ligger et problem for køberen i forhold til anskaffelse af fleksibilitet fra en udbyder (agent) sat i kontekst af en bi-dimensional adverse selection situation, hvori begge parter har indbyrdes uafhængige nytte- og omkostningsværdier. Modelens antagelser stammer direkte fra de økonomiske karakteristika for flexibility (fremlagt i kapitel to). Antagelserne forholder sig til specifikke situationer i forsyningskæden af servicer, med udgangspunkt i flexibility, hvor competitive indkøb af flexibilty er umulige påbaggrund af fx transaktionsomkostninger og begrænset kapacitet fra udbyderens side. I forhold til de rent extraction efficiente handler som normalt bliver belyst under antagelsen af indbyrdes uafhængige nyttefunktioner, sårelaterer modellen i dette kapitel mere generelt til situationer, hvor i provisionsomkostninger og den nytte man opnår fra forskellige aktiviteter ikke er uafhængige. Vi konkluderer at, set i relation til de sager hvor der er muligt at opnåseparation, at denne ikke uafgæninghedseksternalitet leder til yderligere forvrængninger for den ineffektive type.

I det femte og sidste kapitel, "Product-Mix Exchanges, Efficiency and Power System Flexibility, udvikler jeg nye tillæg til Product-Mix Auction, som oprindeligt blev beskrevet af Paul Klemperer. Disse er grupperet og generaliseret under navnet Product-Mix exchange, som er flerfacetteret kombinatoriske auktioner, hvor deltagere er bundet til at oplyse deres ombyttelige præferencer og som kan tilhøre efterspør gselssiden, udbudssiden eller befinde sig påbegge sider af markedet. Eksistensen af en ligevægt for de forskellige udgaver af udvekslinger er givet af et foregående The Unimodularity Theorem, men i denne afhandling foreslår jeg i stedet en simpel lineær programmeringsbaseret tilgang til at undersøge eksistensen af ligevægt, der er givet ud fra deltagernes konkave værdiansættelser. I kapitlet har to hovedbidrag. 1, at Product Mix Exchange kan understøttes af Vickrey-Clarke-Groves payments, som sikrer, at oprigtige bud er en svagt dominerende strategi for alle aktører. Dette resultat er relevant når det bliver anvendt pårelativt tynde markeder, hvor manipulation af resultaterne ellers ville forekomme i højere grad, da deltagerne nu har fået et incitament til at afrapportere deres sande værdiansættelser. 2, viser jeg en applikation af de transaktioner, som finder sted påen markedsplads, der er designet med henblik påfleksibilitet, navnlig Delta Energy Market

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Introduction

Variable Renewable Energy (VRE) is acquiring increased relevance in the energy mix of many countries throughout the world, as part of their overarching goal to decarbonize their economies and, particularly, their electricity industries. The International Energy Agency (IEA, 2015a)) estimates that by the end of 2012, 13.2% of worldwide total primary energy supply came from renewables. According to the same source, renewables accounted for 22% of global electricity generation in 2013. Furthermore, the IEA also forecasts that this share will increase to 26% by 2020 and their central policy scenario expects that renewable electricity generation will triple between 2012 and 2040 (IEA, 2015b).

However, operating with greater shares of VRE exposes power systems to increased *variability* and *uncertainty*. Not only does the amount of energy available from VRE sources, such as wind and solar, changes stochastically over time but the accuracy of forecasts, on which operators rely, decreases. In contrast to the traditional operation of power systems in which generation has been mostly dispatchable (i.e. able to produce upon request) and demand (load) was the only random variable that operators had to account for, a greater reliance on VRE sources requires operators to focus on *net load*. That is: load minus the output from renewables, which depends on not one but two random variables. Observing the short-term evolution of net load in power systems that rely on VRE sources (see Chapter 2 of this thesis for concrete illustrations) reveals that keeping the system in balance requires that the non-renewable generation base adapts to steeper ramps, which are the consequence of unexpected upward or downward variations in renewable output, and shorter peaks, which result from giving way for VRE generators to produce. In other words, with more VRE sources, power systems must become more *flexible*.

Because of the technical nature of power system flexibility, engineers, system planners, policymakers and industry stakeholders have developed a substantial body of work that has helped to define it, measure it and compare it (e.g. Makarov et al. (2009), Morales et al. (2013), Ulbig and Andersson (2012), IEA (2014), Cochran et al. (2014)). More importantly, they have also proposed a number of solutions to enable it (e.g. Valsomatzis et al. (2014)). Nevertheless, the approach taken by this literature is typically *systemic*, in the sense that it sees flexibility as an exogenously-given trait that the power system either has or hasn't. It also tends to focus on the necessary technicalities of available solutions but overlooks a central *economic* insight: that incentives matter.

In contrast, taking an engineering perspective to economics (Roth, 2002, 2008), this PhD thesis argues for *incentivizing* the provision of flexibility. Consequently, it focuses on the design of contracts and markets

- that is, on *institutional arrangements* - that enable power system flexibility. The thesis, which is composed of five chapters, is applied in nature but builds on and makes contributions to several areas of the economic literature, including contract theory, mechanism design, market design and combinatorial auctions. Because of the topic it discusses, it relates generally to power system economics and policy, a sub-field of the Industrial Organization literature.

The first chapter, "Business Models for Power System Flexibility: New Actors, New Roles, New Rules"(Boscán and Poudineh, 2016a), which is joint work with Rahmatallah Poudineh from the Oxford Institute for Energy Studies (OIES), identifies and analyzes existing business models that enable power system flexibility. Rather than looking at these from an abstract perspective, we investigate a number of recent developments happening in several parts of the world and find that new activities being added to the traditional supply chain are not only fundamental to enable flexibility, but also constitute a technology-driven *layer of innovation*, that is contesting the traditional power industry business model in which utilities enjoy a relatively undisputed position. We claim that, as a result, an entirely different electricity industry is emerging: incumbents, who rely on traditional, large scale industrial assets are beginning to compete with entrants who depend on a non-traditional, knowledge-based mode of operation.

Chapter 2: "Power System Flexibility: A Product Design Perspective", serves as transition and guideline for the remaining chapters, which contain the core economic modelling of the thesis. By way of concrete examples from the short-term operation of two power systems with substantial shares of VRE - namely Denmark and California - I illustrate what flexibility requirements are all about. The chapter also reviews the existing literature on power system flexibility, which is mostly technically and policy-oriented, and organize it according to the main topics of interest in the current state of the debate. I find that defining, measuring and comparing flexibility in different power systems - what I call the systemic perspective to flexibility - preoccupy the contributors of the existing body of work, while very few works refer to the incentives to provide flexibility. Motivated by the technical characteristics of power system flexibility described by the existing literature, the chapter then makes two simple but relevant *normative* contributions. First, I propose three economic postulates to guide the economic modelling of flexibility. Specifically, I claim that flexibility has *multiple attributes*, which are *imperfectly substitutable*, and that flexibility is an inherently *heterogeneous commodity.* Second, I describe a set of desirable properties that any flexibility product design should have to actually enable flexibility, which are *simplicity*, *measurability* and *relevance.* The chapter ends with a review of existing product designs for power system flexibility, including California's Flexible Ramping Product and the Flex-Offer informations technology concept.

The third chapter,"Trading Demand-Side Flexibility in Power Markets", which I wrote together with Peter Bogetoft and Peter Møllgaard from Copenhagen Business School, focuses on the particular kind of flexibility that can be harnessed from demand-side resources. In contrast to price-based demand response, the central contribution of the chapter is a model of incentive-based contracts in the presence of a technology that reduces transaction costs to a negligible level. Using the Nash (1950) bargaining approach as solution concept, the model and a number of extensions implemented, determines the price of demand-side flexibility in a bilateral setting. Relaxing the assumption of single-shot transactions between a consumer - who offers flexibility - and an aggregator - who acts as an intermediary between small-scale suppliers of flexibility and the market - the model describes the role of investment costs associated to the deployment of the technology. Relative to the case in which costs are symmetrically shared, the consumer is able to obtain a better deal for the flexibility it offers if it has a relatively higher cost than the aggregator. By extension, this finding speaks about the possibility of a network effect in the aggregation business, which requires some degree of scale economies in the flexibility-enabling technology, and has implications for the possibility or not of introducing competition in a potential market for flexibility.

In Chapter 4: "Flexibility-Enabling Contracts in Electricity Markets" (Boscán and Poudineh, 2016b), which is another joint work with Rahmatallah Poudineh (from OIES), we study the procurement problem that a buyer (the principal) faces when acquiring flexibility from a supplier (the agent) in a bi-dimensional adverse selection setting. Unlike existing models of multi-dimensional screening with separable functions (e.g.Armstrong and Rochet (1999), Rochet and Stole (2003)), the model proposed in this paper assumes that both the principal and the agent have non-separable utility and cost. Not only is the setup and solution of the model a theoretical contribution in its own right, but the assumptions of the model stem directly from the economic characteristics of flexibility, discussed in Chapter 2 of the thesis. We find that besides the fact that there are multi-dimensional types in the procurement problem for flexibility, the "non-separability effect" leads to further distortion of the less efficient types relative to rent extraction-efficiency tradeoff under separability. This effect is closely related to the work on non-separable externalities discussed by Davis and Whinston (1962) and Marchand and Russell (1973). On the applied side, the model relates to specific situations, along the supply chain of flexibility services, in which the competitive procurement of flexibility is infeasible due, for example, to the presence of transaction costs and limited capacity size of the resource provider. The paper ends with a simulation that elucidates the applicability of the model to *practical* contract design problems in situations like a thermostat-based demand response program.

In the fifth chapter of the thesis, "Product-Mix Exchanges, Efficiency and Power System Flexibility", I introduce Product-Mix Exchanges (PMEs) which are double, multi-unit combinatorial auctions that extend the Product-Mix Auction (PMA) format proposed by Klemperer (2010) as a solution to the liquidity provision problem faced by the Bank of England at the height of the 2007 financial crisis. Besides the substitutable preferences inherent to both PMAs and PMEs, a fundamental characteristic of the latter is that participants can buy and sell without having a fixed role, effectively swapping over the two sides of the market. The chapter makes three contributions. First, applying existing Tropical Geometric techniques - recently introduced to Economics by Baldwin and Klemperer (2012, 2016) - to the analysis of substitutable preferences. To this end, I present several illustrative examples that not only help in the understanding of the concepts but illustrate their advantages in applied work. Second, proposing a linear programming approach to check the conditions under which a set of valuations is guaranteed to have an equilibrium with indivisibility. Third, analyzing the conditions under which Vickrey Clarke Groves (VCG) payments - which ensure that each market participant obtains its marginal product as payoff - support the efficient allocation of a PME. Specifically, I find that truthful bidding is a weakly dominant strategy for all market participants in any variant of the PME with VCG payments. Furthermore, because the kind of susbtitutable preferences imposed on PMEs always satisify the Gross Substitute condition of Kelso and Crawford (1982), VCG payments are *always* in the core of the coalitional game associated to any PME. This is an important finding for applications in which insufficient market thickness is a source of concern, because results are more prone to manipulation. The last contribution of the paper, which relies on the usage of "swap bidding", illustrates how the PME framework can be utilized to design a marketplace for flexibility based on the Flex-Offer information technology concept, which is discussed in Chapter 2. In particular, I propose an implementation of a *Delta Energy* market, which allows to trade two flexibility-enabling products, i.e. "quantity flexibility" and "time shifting".

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Chapter 1

Business Models for Power System Flexibility:

New Actors, New Roles, New Rules

Business models for power system flexibility: New actors, New roles, New rules[*]

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1. Introduction

The significant increase in the share of renewables in the generation mix poses a number of planning and operational challenges to power systems, raising the need for flexibility more than ever. At the same time, the emergence of innovative solutions are catalysing the development of new, flexibility-enabling business models; adding activities to the existing supply chain. New actors, sparking innovation in software, hardware and market design, are defining new roles. For example, aggregators are linking small-scale suppliers of flexibility to electricity markets. Likewise, consumers are not passive anymore, but instead are evolving into active participants: *prosumers*, with an active role in the supply side.

The key element in the emergence of new business models for power system flexibility is, unequivocally, technological change. The context of this evolution is, in most cases, a post-liberalization power system, characterized by unbundling of activities, with transmission and distribution operating as regulated monopolies, and competition being promoted in generation and retail. After several years of experience with reforms throughout the world, market power has been mitigated, efficiency has increased, but many firms still retain a dominant position. On the other hand, market mechanisms are well established now and relied upon. Wholesale and intra-day markets are generally used to allocate and price electric energy. Ancillary services and capacity are also competitively procured.

The chapter by Sioshansi (2016) explored current trends in power systems, including the rapid uptake of distributed generation and renewables, micro-grids, storage, and so on. With the increase in the cost efficiency and the competitiveness of renewable resources, they become a more serious alternative to traditional power plants. However, the operational challenges derived from power system operation with intermittent resources require planners to actively incentivize the adaptability of systems to the challenge posed by stochastic variability.[1] The IEA (2014), for example, claims that integrating a significant share of renewables is dependent on an overall transformation that increases system flexibility, and advocates for

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[1] The technically-oriented reader is referred to Morales et. al (2014), who devote an entire book to the analysis of operational problems associated to the integration of renewables into electricity markets. Chapter five of their book studies flexibility, originating from different sources in the power system, as an alternative to deal with the stochastic nature of renewable sources of generation.

further development of market-based, short-term balancing mechanisms that create reliable price signals for it.

In addition, the rapid progress of information systems, the declining cost of computing, and the swift evolution of software are creating the conditions for smart grid solutions to become feasible. Coupled with progress achieved in areas like electricity storage, home automation and electric vehicle development, synergies among energy sectors, such as transportation and heating, are also becoming viable.

In light of recent developments, this chapter reviews the evolution of operational flexibility issues and its associated business models, with a particular focus on short-term flexibility services and the role of emerging players. Long term issues of market based capacity arrangements have been discussed in the chapter by Woodhouse (2016).

Section 2 discusses the concept of flexibility and reviews the resources that can enable flexible operation of the power system. Section 3 reviews the issue of trading flexibility as a commodity and describes some of the challenges associated with contracting for flexibility services. Section 4 is about the emerging business models for flexibility services and the role of new players followed by the chapter's conclusions.

2. Flexibility in the power system

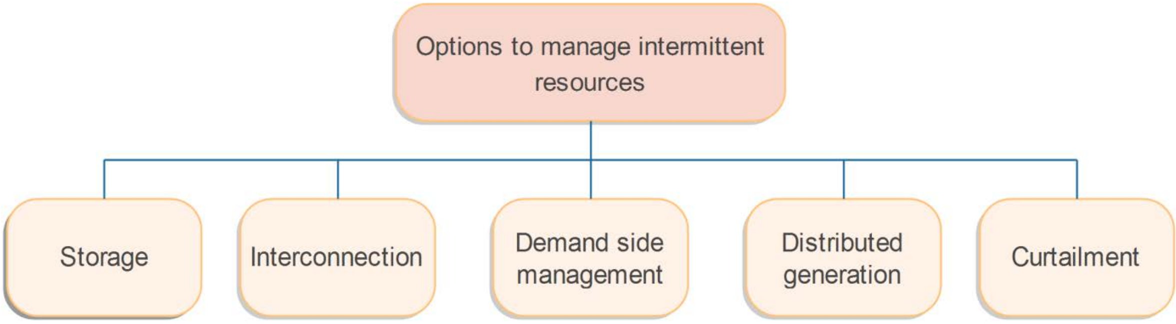
In recent years, the technical literature has coined the term "flexibility" in relation to the requirements of power systems to integrate intermittent resources. However, its definition remains vague and implies different meanings depending on the context. In this chapter, flexibility refers to the ability of power systems to utilise its resources to manage net load variation and generation outage over various time horizons. *Net load* is defined as load minus supply from intermittent resources, such as wind and solar. As a commodity, flexibility has several dimensions including capacity, duration and ramp rate or lead time, for demand-side resources. Boscán and Poudineh (2015) distinguish between short-term flexibility, associated to real-time balancing of the grid, and long-term flexibility, which relates to the adequacy of generation capacity and investment.

It is also helpful to distinguish between *resource flexibility*, which refers to the built-in flexibility of a particular resource, such as demand response; and *system flexibility*, which comprehends transmission, network flexibility, and market design. The transmission network is not an additional source of flexibility *per se*, but the lack of an adequate transmission network severely affects power system flexibility.

2.1 Flexibility-enabling resources

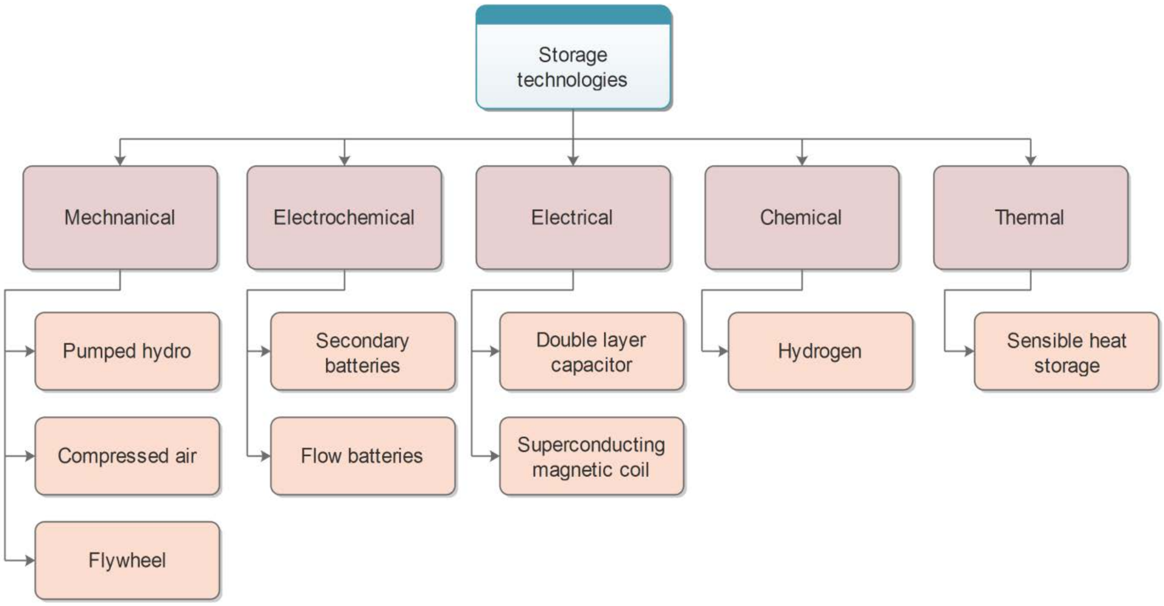
There are various options available to manage the variability of intermittent resources. As shown in Figure 1, these range from storage technologies, interconnections, demand-side management to distributed generation and curtailment.

Figure 1: Options to manage variability of renewables



Electrical energy storage technologies are among the most effective ways of absorbing net load variability, and although there are various options available, not all of them are commercially viable. Figure 2 (IEC, 2011) classifies existing technologies into five main categories: mechanical, electrochemical, chemical, electrical and thermal. Of these, the most widely used form is mechanical: specifically, pumped hydro, which accounts for 99% of global energy storage (127 GW of installed capacity). Given its unparalleled start-up and ramp rate capability, it is a particularly attractive option to address variability from renewables. The second largest electrical energy storage in operation is compressed air but, compared to pumped hydro, it has a negligible global capacity (440 MW). Other means of storage such as batteries, capacitors or heat storage currently have very low penetration levels, but recent improvements in technology and cost of electrochemical batteries (particularly, Lithium-Ion) makes them a promising source of electrical storage with various benefits to the power system, including flexibility. However, the key to the success of storage technologies is the viability of business models that allow the industry to move forward, beyond demonstration cases and towards massive penetration (see Section 4.1.4).

Figure 2: Storage technologies classification



Source: Authors, adapted from IEC (2011)

The interconnectivity of power systems is a determinant factor in the extent to which power systems are flexible. In fact, not only interconnections have the potential to facilitate integration of variable generation, but also can contribute to energy security, decarbonisation and affordability. In Europe, for example, where there is a strong interest to create an integrated, sustainable and competitive energy market, there is a specific target to achieve 10% of interconnection (as a share of the installed production capacity) for each member state. Although the European interconnection capacity has increased considerably during the last decade, there remain member states that have less than the 10% goal and are thus isolated from the internal electricity market (EC, 2015). Figure 3 shows the countries with higher and lower than 10% interconnection. Countries such as the UK, Spain, Italy and Ireland need to invest in their interconnection capacity. In contrast, Denmark, which has a high penetration of wind power, has benefited significantly from the interconnection with countries such as Germany, besides the existing interconnections with NordPool countries. [2] The EU third energy package clearly states the need for cross border interconnections but for this to become a reality; it is required to design an efficient regulatory framework that incentivises investment. The existing legal framework seems to favour a regulated business model for interconnection expansion, but it also allows for private merchant transmission initiatives.

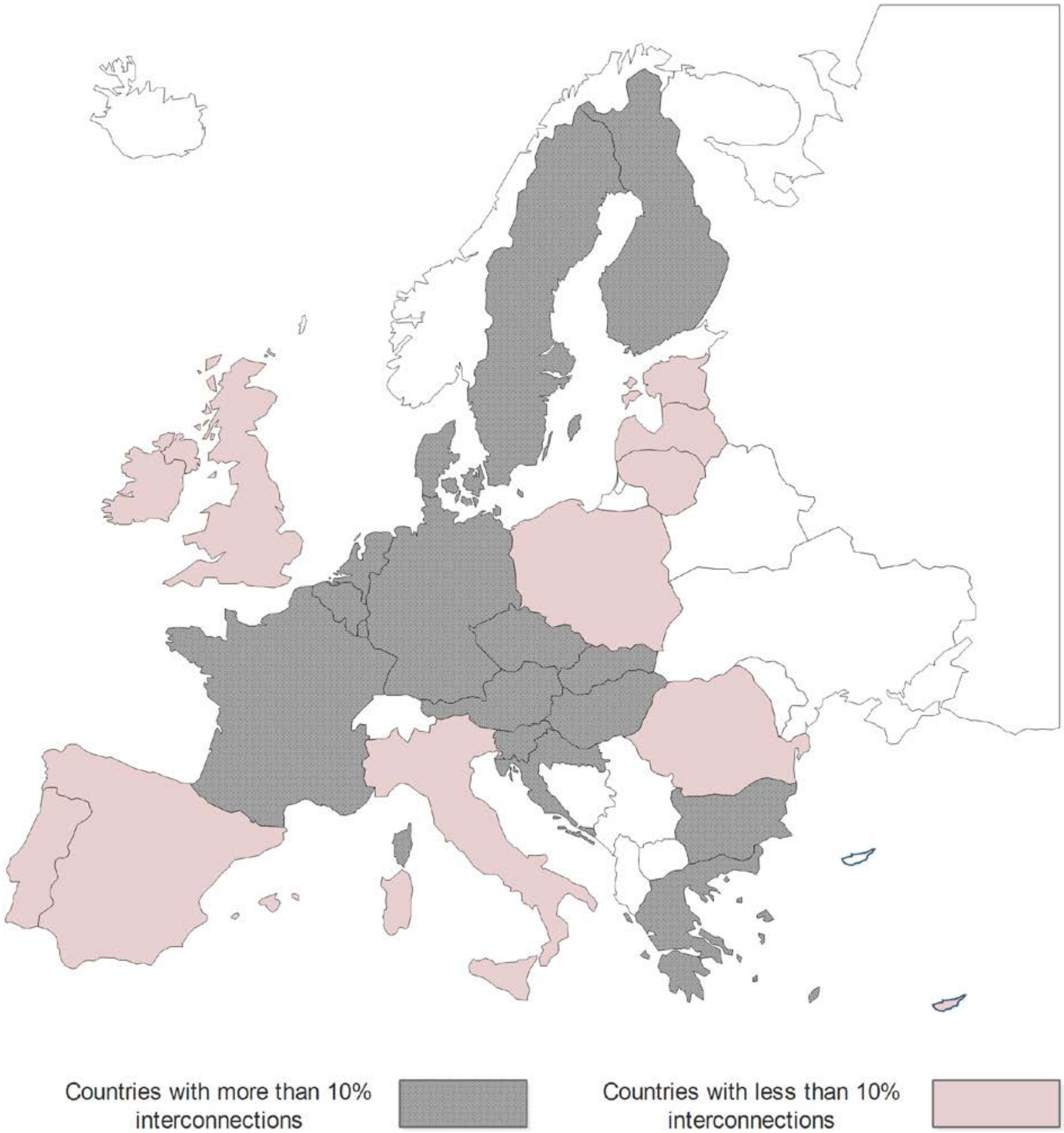
[2] Interestingly and widely cited by various media outlets, on 9 July 2015, Denmark generated 140% of its electricity demand with wind power. However, Denmark managed the excess production by exporting to neighboring Norway, Sweden and Germany. In relation to the relevance of interconnections, Green and Vasilakos (2012) perform an econometric analysis of Denmark's electricity exports and find that exporting on windy days is a cost-effective way to deal with intermittency.

Because of its suitability for relieving network congestion and providing ancillary services, such as fast and long term reserve requirements, distributed generation, e.g. combined heat and power, is well-positioned to increase power system flexibility (IEA, 2005). Traditionally, large conventional power plants served this purpose, and depending on their types, have been an effective source of flexibility. The most important requirements of flexible operation for conventional plants are start-up time, ramp rate and partial load efficiency (Boscán and Poudineh, 2015), but these are not fully available in all types of conventional generation. For instance, cycling capability of most current coal power plants are limited and their ramp rate is generally low[3]. The same applies to nuclear power plants with even more degrees of inflexibility. The most flexible types of thermal generation are gas fired power plants but. However, cycling and ramping increase the wear and tear of plants, as well as their heat rate.

In recent years, the need for an efficient portfolio of flexibility resources has drawn attention to demand-side flexibility. In fact, with the advancement in information and communication technologies (ICT) many of the generation services can also be provided through demand response. In the UK, currently some forms of demand side flexibility are being traded in the balancing market. For example, through National Grid's Frequency Control Demand Management scheme, frequency response is provided through automatic interruption of contracted consumers when the system frequency transgresses the low frequency relay setting on site. Furthermore, National Grid is utilising slower responding demand response for load following services. Similar arrangements exist in other countries as demand side schemes gradually find their way into balancing markets.

[3] Coal power plants, however, can be designed to operate flexibility.

Figure 3: The European electricity interconnection as a share of total installed capacity in 2014



Source: Authors based on the information from EC (2015)

Curtailment, a form of negative dispatch in which the system operator reduces the output of wind and solar generation to maintain stability, happens more frequently in the absence of sufficient flexibility. The issues that trigger curtailment are related to system balancing, system dynamics or grid constraints and, therefore, the level of curtailment can be used as a negative metric for measuring power system flexibility. Although many countries with increasing shares of renewables have attempted to improve the flexibility of their systems, there remain some with high levels of curtailment. For example, China had an average curtailment rate of 18% in 2012 (Li, 2015), whereas this figure was 4% for the US during the same period (NREL, 2014). As more renewables are integrated, these figures will rise, unless more flexibility is enabled. For example, the risk of over-generation in the afternoon (low demand periods) is high in California and this is likely to become even worse when the renewable portfolio requirement increases from 33% by 2020 to 50% by 2030, as currently proposed.

The use of flexibility services is not limited to addressing net load variation. Indeed, flexibility has three different functions in the power system and three final users of flexibility services. An important role of flexibility is to ease the integration of intermittent resources. The transmission system operator (TSO), which is responsible for balancing the grid, is thus one of the main procurers of flexibility services. Another function of flexibility is to manage congestion in the electricity distribution network for which the distribution system operator (DSO) is the buyer of flexibility. The third usage of flexibility is for portfolio optimisation. The market players (e.g., aggregators, suppliers, balancing responsible parties) can obtain flexibility services to fulfil their energy obligations in a cost efficient way by, for example, arbitraging between generation and demand response. Table 1 presents the parties involved in the procurement side of flexibility services in liberalised electricity markets. It is worth mentioning that although TSOs or DSOs procure flexibility services in a competitive manner, these companies recover their costs in a regulated fashion.

Table1: Flexibility service and their final users

Party	Activity	Business model	Commodity	Use	Final objectives
TSO	Balancing the grid	Regulated business	System flexibility service	System-wide	Grid planning and operational efficiency maximisation
DSO	Managing distribution grid	Regulated business	System flexibility service	Local, regional or national	Grid planning and operational efficiency maximisation
Market player	Trading electricity	Price set by market rules	Resource flexibility (Portfolio optimisation)	System-wide	Profit maximisation

Source: Adapted from EDSO (2014)

3. *Trading flexibility services*

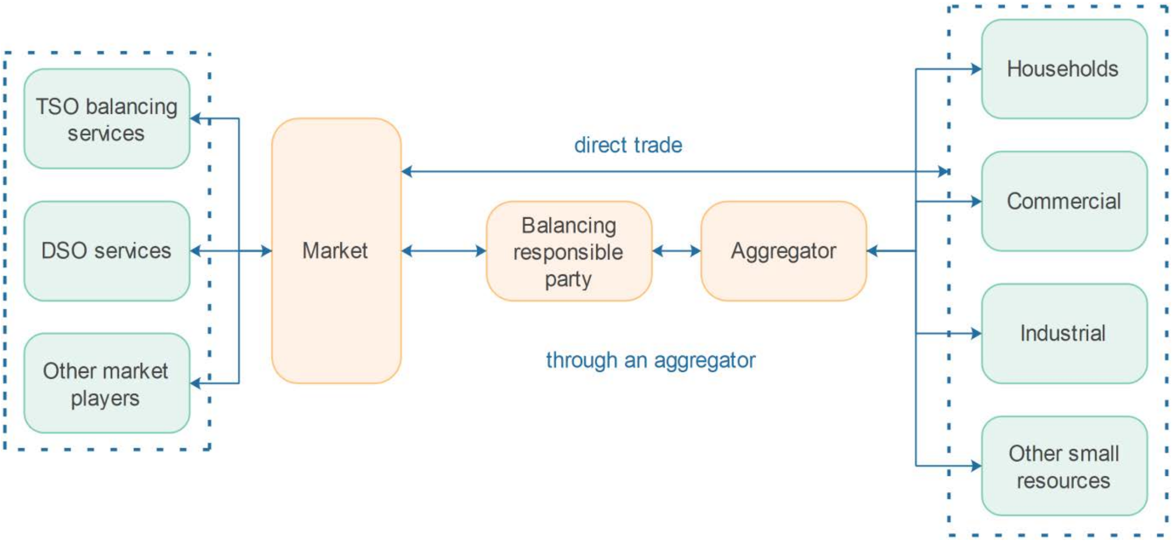
The ability to trade flexibility services is important for the reliable operation of power systems. In the currently liberalised electricity sector, flexibility services are traded in intra-day and day-ahead markets as an energy product or in ancillary service markets as control reserve products (Boscán and Poudineh, 2015). Market design has important implications for procuring flexibility in an efficient and reliable manner: even when there are sufficient resources available for managing variable generation, the market may not have been designed to incentivise efficient use of them. For example, in some US regions where there are no sub-hourly electricity markets, variations in the net load need to be met by regulation services which have a high ramping rate and thus are among the costliest flexibility services. This inefficiency results from the market design because it has been shown that variable generation requires does not require a faster ramping rate than the contingency reserves (Boscán and Poudineh, 2015). [4]

As the current electricity markets in many countries were not originally designed to manage a large share of intermittent resources, further penetration of variable generation might lead to increased market power, reduced competition and reliability degradation (Ela et al., 2014). Additionally, it is not clear whether the current market design can provide a sufficient level of flexibility when the need for it increases in the system. In the US electricity market, several mechanisms are in place to incentivise flexibility, e.g., centralized scheduling and pricing, 5-min settlements, ancillary service markets, make-whole payments, and day-ahead profit guarantees (see Ela et al., 2014). However, a different design might be required to incentivise the right amount of flexibility resources both in the short run and the long run. Non-traditional resources such as demand response, storage and even variable generation itself can contribute to system flexibility when the incentives are provided. Evidence from the GB electricity market shows that, with more uptake of variable generation, the real time price volatility increases much faster than the day-ahead price volatility and flexible resources can take the advantages of this volatility (Pöyry, 2014).

Flexibility enabling contracts can be traded either directly between the final user and the resource provider or through an aggregator. The capacity of supply is an important factor for the way that trade can happen: transaction cost is an impediment for the small capacity resources to participate directly in the market. The small resource providers such as households thus can be aggregated and offered to the market through an intermediary. Figure 4 presents the way that demand side flexibility-enabling contracts can be traded in an electricity market.

[4] On November 1, 2014 California ISO has introduced an energy imbalance market (EIM) mostly to allow grid operators of adjacent areas to share and economically dispatch a broad array of resources for efficient renewable integration.

Figure 4: Trading demand-side flexibility services in the electricity market



3.1 Designing contracts for flexibility services

Flexibility is a multi-dimensional commodity and the marginal cost at each dimension is the private information of the resource provider. Therefore, the procurer should design the contract such that informational rents are minimised and the cost of integrating renewables is efficient. Designing optimal contracts for flexibility services under multidimensional information asymmetry is challenging and becomes even more important when the cost of balancing services increases with an increased uptake of intermittent resources.

In bilateral contracts (between the resource providers and the final users or an aggregator), when the sellers differentiate themselves by concentrating on different dimensions, the procurer can design the contract in a way to extract all informational rents (Li et al., 2015). For example, consider a system operator who aims to control thermostats in two households' premises and for this she offers a contract based on two parameters of lead time and duration of load control[5]. Under the condition that the households are very similar in terms of the disutility they experience at each dimension (lead time and duration of load control), there is no way for the system operator to design a truth-telling contract which extracts all informational rents. In this case, the system operator needs to give up some rents by distorting downward the contract specifications (lead time and duration of load control) from the optimal level for one of the households. However, if the two households differ significantly at each dimension, the system operator can extract all the rents. This happens when, for example, the flexibility procurer knows that one household incurs a high disutility for the short lead time and the other for the long duration of load control. Naturally, the former household prefers a contract with higher lead time but can sacrifice on load control

[5] So the contract is in the form of a payment for specific lead time and duration.

duration, whereas the later value more a contract which has a shorter load control duration. In this case both households select a contract which is optimal for them.

The above results also hold when there are multiple flexibility resource providers. Therefore, more differentiation across the dimensions of flexibility by the resource providers benefits the buyer and vice versa. If the contract is designed (and offered) by flexibility resource provider rather than the system operator, the results are not necessarily symmetric to the previous case. For example, double marginalisation[6] can happen although the supplier can change the specification of contract to avoid this. Additionally, when the resource provider enters into a contract with an aggregator who faces an uncertain demand for flexibility in the market, the optimal mechanism requires reducing the specification of contract at each dimensions, i.e., it is optimal for aggregator to buy less compared to the case of a deterministic demand.

An intermediary (for example, an aggregator) might face a demand for multiple flexibility products with various specifications in terms of capacity, duration, response time, and ramp rate. This is because the impact of intermittent resources on the power system can be considered in four time frames: frequency regulation, load following, scheduling and unit commitment (Boscan and Poudineh, 2015). Frequency regulation requires very speedy response and ramp rates and thus is costly. The requirement for speed of response decreases as the time frame moves towards load following and beyond. Therefore, for each time frame a different flexibility service and consequently flexibility contracts are needed. In this case the intermediary needs to make a decision between supplying all range of flexibility products or only some of them. Theoretically, there is a fundamental trade-off in the intermediary's product selection decision in this case. This trade-off results from the market share of slower responding flexibility resources versus the revenue obtained from more expensive flexibility services (e.g. regulation services).

3.2 Next generation utilities and system flexibility

As the traditional utility model is evolving, next generation utility concepts emerge as a result of rapid advancement in ICT. Demand response, electric vehicles, energy efficiency and intelligent grid management will have an evolved function as described in table 2 (Hansen and Levine, 2008). For example, demand response which traditionally has been used for emergency curtailment to protect grid frequency under an emergency condition (load shedding), gradually enters the electricity markets as a capacity resource as well as balancing service at all time frames. In the current electricity markets the need for system flexibility may not be critical yet, but it is not clear that this will remain the case in the future, as long as renewables gain a greater share in the generation mix. Access to various sources of flexibility services both on the supply and demand side along with appropriate market design provide an opportunity to profit from short term spikes in spot prices, balancing markets and specific contracts with grid operators.

It is likely that next generation utilities will be more reliant on ICT, which are already storming the industry with "smart", programmable, communicable gadgets and "Internet of Things" (see the chapter by Cooper

[6] Double marginalization happens when the two actors across the supply chain apply their own mark-ups over the price which results in higher deadweight losses.

(2016). Although ICT has always been important in the power sector, especially for system protection, with the need for more flexibility and a reliable real time operation, the role of ICT becomes even more critical. Smart grid, smart meters, intelligent home management systems and various forms of advanced technologies will enable utilities to profit from trading flexibility services.

Table 2: Next generation utility concepts

	Traditional approach	Conventional wisdom now	Next generation concepts
Demand response	Emergency curtailment	Peak shaving	Resource for capacity and balancing service
Plug-in electric vehicles	R&D only	Flexible load	Vehicle-to-grid storage resource
Intermittent resources	Marginal fuel saving, no capacity value	Some capacity value with gas fired firming	Resource for capacity and balancing service
Grid automation and intelligence	Unidirectional from source to load	Some intelligence to automate loads	Omnidirectional web of sources &loads
Energy efficiency	Up to the customer	Component-based utility programs	Breakthrough-level system efficiencies

Source: adapted from Hansen and Levine (2008)

4. *New Business Models*

The electricity sector landscape is changing rapidly with the integration of renewables, technological advancement in ICT and the emergence of various new players. Amidst this environment, entrants are coming to participate in electricity markets, but in completely novel ways. Decentralized generation units are beginning to compete with traditional generators. Aggregators, acting as intermediaries, acquire the right to modify energy consumption from end users, and sell it in the form of available capacity. Software and technology developers offer energy management solutions, intelligent devices and storage capability. Ventures among these new players, teaming up to offer new services are becoming more frequent, and the sum of it all depicts a creatively chaotic picture. Yet, these entrants share some common features: relative to incumbents that rely on traditional, large scale industrial assets, entrants have considerably lower fixed costs, and depend on non-traditional, knowledge based assets.[7]

Taken together, they constitute a *layer of innovation* that is being added to the existing structure of power systems and challenges the traditional business model, in which utilities enjoy a relatively undisputed position, and consumers act as the passive end of the supply chain. All of this contests the *status quo*, motivates incumbents to reconsider their roles and, potentially, adopt new ones in accordance with the changing environment. Regulators, in consequence, are being led to consider new, previously unforeseen sources of involvement and potential dispute among entrants and incumbents.

[7] Rodgers (2003) identifies three categories of knowledge-based assets, namely *human assets*: attitudes, perceptions, and abilities of employees; *organizational*: intellectual property such as brands, copyrights, patents, and trademarks; *relational*: knowledge of and acquaintance with communities, competitors, customers, governments, and suppliers in which the company operates.

4.1 A partial taxonomy of new actors, new roles and new business models

The rapidly evolving nature of innovation and frequent function overlap prevents an exhaustive enumeration, and mutually exclusive categorization of agents involved. [8] To contribute in the understanding of business models leading to increased levels of power system flexibility, a simple, yet partial, categorization is proposed as follows:

1. *New actors* are the constituents of the innovation layer, which is composed of entrants sparking innovation through new software, technology, and market design proposals. Firms, researchers and, to a lesser extent, regulators can also be identified here.

2. *New roles* are defined by new actors, and are assumed by existing market participants and new actors alike. Aggregators and prosumers are two good examples of this category.

3. *Business models* are the commercial outcome of innovation brought about by the new actors. In a well-defined business model, the sources of revenue, cost and, therefore, profitability are unambiguously defined. Furthermore, business models are subject to evolution and depend on the overall economic environment: some will appear, consolidate and evolve into new areas of action, while others will disappear, given their lack of viability (for more on this, see the chapter by Nillesen and Pollitt (2016)).

4.1.1 Aggregation for Demand-Side Management

Aggregation for demand-side management is one of the most consolidated existing business models for power system flexibility.[9] The role of aggregator, fulfilled by energy management software developers and other traditional retailers with real-time metering, is to bundle '*negawatts*' (unused capacity)[10] offered by commercial and industrial (C&I), and residential consumers of electricity. In exchange for capacity and energy usage payments or rebates in their electricity bill, consumers adjust consumption at times of peak demand or when required by grid operators. Aggregators sell negawatts in different outlets including capacity, balancing, and ancillary services markets, or as part of demand response programs carried out by utilities.

This business model has grown in several countries, including Europe and Asia, but has shown particular strength in the US. As an example of its relevance, consider the 2014 capacity auction results for PJM, the largest wholesale electricity market in the US: 10.9 GW of demand response capacity were procured, which is equivalent to more than 6% of the total. Nevertheless, in the latest episode of a legal battle

[8] In a recent discussion paper by Ofgem on the topic of non-traditional business models, they find the same difficulty.

[9] The term "demand-side management" is used to encompass both participation of demand as a resource in markets such in capacity markets; and in the conventional demand response sense, which includes interruptible loads, load management, peak shaving and so on.

[10] The term 'negawatts' has been attributed to Amory Lovins, cofounder and Chief Scientist of the Rocky Mountain Institute, by a number of publications, including The Economist's special report on energy and technology (2015), and Maurer and Barroso (2011).

between power companies and aggregators, demand response in the US has recently received a regulatory setback. The federal order that set demand response and generation on equal footing regarding payment received by grid operators was vacated last year on the grounds that demand response is being overcompensated, inducing inefficient prices that discriminate against generators.[11] The final say, though, has not been declared yet: at the time of writing, the US Supreme Court of Justice has decided to re-consider the case.

From a more general perspective, though, the role of aggregation for demand-side management goes beyond conventional demand response. Aggregators typically rely on software solutions and other hardware to realize efficiency gains and, therefore, they are shifting to developing integrated energy management solutions. As a result, some of them are rebranding themselves as software developers while others are emphasizing on the role of hardware as a tool for demand-side management, while retaining their role as aggregators. They are also entering agreements with utilities and grid operators to manage intermittency from renewables with demand-side resources, an element that emphasizes their growing role as a supplier of flexibility. For example, EnerNOC – a US based aggregator known for its demand response operations who is re-focusing its business towards software development – ran a pilot project with the Bonneville Power Administration to show the capabilities of demand response to deliver short-term balancing. Such new services from demand response are especially valuable as the likelihood of over generation increases in places such as California. Also, as the role of distributed assets increases, aggregators will not only manage demand but will make a transition into virtual power plant managers.

4.1.2 Thermostats as a Demand-Side Management Tool

Although thermostats are key for controlling energy consumption in residential, and C&I buildings, they have rarely been a particularly interesting object of attention for retail consumers. With the majority of sales channelled through dealers offering service contracts, well-established products developed by long-standing incumbents have taken the lead.

Nevertheless, the usually undisrupted retail market for thermostats became invigorated once Nest Labs transformed this typically uninteresting device into an appealing gadget for tech-savvy consumers, through the development of user-adaptive technology and a well-designed marketing strategy.

While the argument for significant product differentiation and technological breakthrough by the Nest thermostat is not easily argued for,[12] more significantly, their contribution has been to introduce innovative business models for flexibility, in which smart thermostats are the key element to enable demand-side management.

According to these business models, smart thermostat users are given the choice to surrender control of their load at peak demand hours or when there are seasonal weather variations, and allow the utility to adjust consumption, following user-defined comfort levels. In exchange, utilities compensate consumers

[11] Order 745 issued by FERC (March 15, 2011) on the topic of "Demand Response Compensation in Organized Wholesale Electricity Markets".

[12] Ecobee, a Canadian competitor to Google-owned Nest Labs, introduced the first WiFi connected thermostat at least two years before the Nest thermostat hit the market.

with rebates on their final bill or through direct payments. Not only have other smart thermostat developers followed suit, but utilities have also created *bring-your-own-thermostat* demand-side management programs. Moreover, non-traditional demand response services (increasing consumption as opposed to reduce demand) are also becoming increasingly important. These forms of demand side management can be particularly relevant for places with over-generation, like California, Texas, Denmark and Germany.

Looking forward, there is ample room for these business models to develop further, as the penetration of programmable thermostats still remains very low.[13] The upfront cost of deployment, though, is a barrier for many customers. As a solution, and in a similar vein to business models in the telecommunications industry where service carriers and hardware providers team up, utility companies are subsidizing the deployment of smart thermostats. In summary, thermostat-based demand-side management is setting new standards in the adoption of new technology and in the development of demand-side management models that could easily extend to other devices.

4.1.3 Software Developers

The emerging business models for power system flexibility are closely intertwined to smart grid development. Coupled with hardware, software is pervasive across processes and solutions. Remotely controlling devices, smart metering, and identifying consumption patterns to reduce demand charges, are just some of the many examples that highlight the role of software (for more details, see the chapter by Cooper).

In some models, software is bundled with hardware as part of the complete solution. For example, on-site energy storage vendor Stem describes its system as composed of three elements: software, batteries and a real-time meter. Others focus on software development and work with any kind of hardware. Such is the case of software vendor BuildingIQ, which specializes on demand-side management for heating, ventilation and air conditioning systems in C&I buildings.

More generally, and as part of an emerging trend, many agents currently developing new business models for power system flexibility are expanding their role into *software-as-a-service* suppliers. This licensing and delivery model has consolidated in recent years among software developers, because it allows end users to reduce hardware, upfront and maintenance costs, and has enabled scalable usage and payment. On the other hand, vendors obtain a recurring revenue stream from subscription payments. Firms like EnerNOC, a leading aggregator, are following this trend as a growth strategy and are also creating interactions with other existing business models. In summary, software is already playing a central role in the new business models for power system flexibility and its relevance will only continue to grow.

4.1.4 Storage Providers

[13] Consider, for example, the American market where (according to the US Energy Information), 85% of American homes with central heating own thermostats, but less than half of these are programmable. Similarly, 60% of those with central cooling own them, but approximately a half of these are programmable.

Location within the supply chain largely defines the scale, response time, size and, therefore, suitability of different storage solutions to increase flexibility in the power system. Although not fully consolidated,[14] recent years have witnessed a considerable expansion of electricity storage business models, with greater emphasis on *behind-the-meter* (distributed) than in *front-of-the-meter* (grid-level) solutions. A report by the firm GTM Research (2015), sponsored by the US Energy Storage Association, reveals that distributed storage deployments increased more than threefold between 2013 and 2014 in the US and the non-residential sector accounted for the lion's share of this amount. They expect the distributed storage segment to continue growing in years to come, outpacing grid-level storage, until it reaches 45% of the total market share by 2019.

Distributed storage targeted at C&I, large residential, and institutional consumers is one of these models. In most markets, these clients pay for the energy they consume plus a share of their peak demand within a billing period. Coupled with software analytics and real-time metering to analyze *peak-shaving* opportunities, suppliers offer on-site storage systems to go off-grid when demand is high. Typical agreements between storage suppliers and their customers are based on revenue sharing, but initial investments, operational and price risk are assumed by suppliers.

The economic case for residential energy storage is different and, given current conditions in most retail markets, difficult to make. To begin with, most residential customers have fixed price retail contracts and, therefore, price arbitrage and peak shaving become mostly irrelevant. Furthermore, in markets where residential solar PV systems are becoming widely adopted, it is sensible to acquire storage if customers wish to become entirely independent of the grid. Yet not only are such green-energy-oriented customers a well-off minority, but net metering – an incentive that is particularly relevant in many states of the US – is at odds with it: being completely off grid would imply cutting off a source of revenue that helps to pay the cost of the solar facility investment. Unless the cost of residential storage is competitive enough or there is an economic incentive to install it, this business model is not viable.

However, distributed energy storage is one of the most effective resources to enable power system flexibility, as it can instantaneously balance power supply and demand. The aggregated deployment of storage capability creates virtual power plants that, depending on market design innovations at the distribution level may create sources of revenue for owners of this kind of resources. Grid operators interested in the procurement of capacity, reactive power, and voltage management might well provide the necessary source of revenue to further boost the adoption of distributed storage, including residential applications. In fact, following the recent introduction of Tesla Motor's batteries for C&I and residential applications, at an approximate price of $500/KWh, its partner company Solar City clarified that the 10-year lease agreements, with which they typically operate, for solar and storage systems contemplated revenue sharing of grid service income.

4.1.5 Market Design Innovation

[14] Energy storage, mostly pumped hydro, accounts for 2% of total US generation capacity.

Driven by technology, the new business models are already transforming the way in which power systems operate. However, given the crucial role of incentives, market design and regulation can either hinder or help their consolidation and evolution as a tool to increase flexibility.

Regulators and system operators in areas where renewables are on the way to playing a more relevant role are considering different market design innovations. Many of these, though, still appear to have a piecemeal and tentative approach. Some of them prioritize the role of short-term balancing, whereas others emphasize the role of demand-side management and long-term resource adequacy. While there is no one-size-fits-all solution, restructuring existing electricity market designs to enable flexibility requires a holistic approach. Hogan (2014) argues that adapting existing markets to renewables requires, first, recognizing the value of energy efficiency, including demand-side management; second, upgrading grid operations to increase short-term flexibility; and third, incentivizing long-term flexibility investments, i.e., adopting flexible resources.

An interesting example comes from the California Independent System Operator (CAISO) which is currently developing a *flexible ramping product,* aimed at minimizing short term (5-minute to 5-minute) load variations. In contrast to conventional ancillary services, this product focuses on addressing net load changes between time intervals and not on standby capacity aimed at meeting demand deviations within a time period. In addition, an innovative feature of this proposal is that it is continuously procured and dispatched.

Another interesting experience is Southern California Edison's recent capacity procurement of 2.2 GW of behind-the-meter solar PV generation, storage and demand-side management to alleviate congestion in particular zones of the grid. Besides being a complex process because of the necessary cross-comparisons between technologies, location of assets and the diverse nature of contracts with suppliers, it reveals emerging business models in which generation and distributed energy resources are treated on a par with conventional generation. Of particular interest is the agreement with distributed solar generation company Sun-Power which assumes and enhances the role of aggregator. Upon requirement of the utility, the aggregator commits to achieving savings through solar power, which it procures at specific sites from generation facilities scattered throughout different grid locations –a Virtual Power Plant– without exporting it to the grid.

Also, in the context of a comprehensive review of their power system, the single electricity market for Ireland has decided on a number of measures aimed at adapting it to the 2020 goal of 40% of renewables in Irish electricity demand. On the market design front, relying on a hybrid regulated tariff/auction mechanism to procure contracts with maturities between 1 to 15 years, it has been agreed to increase the number of ancillary services procured from 7 to 14, including specific ramping products with horizons of up to 8 hours.

To sum up, market design innovation is already playing a key role and it will have a substantial impact on the consolidation of emerging business models.

4.2 New Business Models and the Future of Utilities

The absence of large-scale economically viable storage and an entirely passive demand-side have justified the existence of the traditional power system business model, but technological breakthrough has begun to challenge this approach. From this follows a central question for the future, namely: *what is the impact of new business models on existing utilities?*

The immediate consequence is that the business-as-usual operation of utilities is challenged, but the extent of the impact depends on the strategic decisions that both incumbents and entrants make. Incumbents can choose a confrontational approach to deter consolidation of the emerging business models or can accommodate to entry (see the chapter by Burger and Weinmann).

Evidence shows that confrontation is already happening. The extended legal battle between power producers and aggregators in the US over Federal Order 745 mandating equal treatment between demand response and conventional generators in wholesale markets is an example of this. In France, a similar conflict over imbalance mechanisms arose between retailers and aggregator Voltalis.

Nonetheless, the line between confrontation and adaptation is not clearly delineated because several incumbents are extending their activities into new business models. Big players, including large vertically integrated energy holdings are entering the aggregation business, and are acquiring stakes in energy management developments, effectively extending their scope. For example, NRG, which owns 50 GW of fossil-fuel dominated generation assets in the US acquired Energy Curtailment Specialists in 2013, a leading US aggregator with a portfolio of 2 GW. In France, Schneider Electric acquired leading European aggregator Energy Pool in 2010, which controls more than 1.5 GW in demand response assets. Also, Swiss generator Alpiq, which owns a generation portfolio of 6 GW including hydro, fossil and nuclear, acquired British aggregator Flexitricity in 2014.

Entrants, on the other hand, are partnering in their offers, bundling products in markets that show potential first-mover advantages. For example, Tesla and Advanced Microgrid Solutions (AMS) have recently announced a sales deal to install up to 500 MWh in battery capacity as part of a grid-scale storage project. EnerNOC and Tesla have also announced a partnership to bundle batteries with software solutions to enable demand-side management. Google's acquisition of Nest Labs in 2014 for USD 3.2 billion is yet another indication of the rapidly changing face of the new business models.

The future will depend, mostly, on these strategic interactions and while it is impossible to predict the future, one thing is certain: utilities as we know them today will definitely change. Table 3 summarises the emerging players and associated new business models for the flexible power systems of future.

Table 3: Summary of new business models for power system flexibility

Business model	*Characteristics of the agreements*
Aggregation for demand-side-management	- Consumers obtain capacity and/or energy usage payments - Negawatts are sold in organized markets or as part of bilateral agreements with utilities
Thermostats as a demand-side management tool	- Consumers acquire the device with a subsidy from utility - Consumers enter direct-load control agreements, allowing load to be adjusted to pre-defined comfort settings. - Consumers are given rebates or paid for energy not consumed - Utility manages peak load with higher cost efficiency - Hardware sales increase
Storage (C&I clients)	- Consumers pay no upfront cost for software or hardware deployment. Alternatively, supplier delays deployment costs until first revenue streams are realised - Revenue from demand charge reduction is shared between consumer and supplier
Storage (residential)	- Upfront deployment cost is borne by households - Consumers benefit from going off-grid, price arbitrage or grid service payments
Market design innovation	- Utilities procuring services through a number of bilateral contracts with suppliers of flexibility - New ancillary services - New short-term services focused on short –term balancing
Software	- Software-as-a-service - Vendors collect subscription fees - End users reduce hardware, upfront and maintenance costs

5 Conclusions

Efficiently integrating renewables requires increasing flexibility and technological progress is facilitating this process. Over the last few decades, technology has pushed the operational boundary of utilities away from a traditional paradigm, but the changes happening now are paving the way for a next generation of utilities. According to this emerging paradigm, new actors sparking innovation are defining new roles and, as a result, interconnectors, distributed generators, storage providers and suppliers of demand response are competing with incumbents, while relying on the novelty of their business models to provide flexibility services. Meanwhile, the interaction of regulation, technological innovation, and business model evolution are shaping the strategic interaction among players, who have several pieces of private information. Markets, contracts and regulatory frameworks will have to change to become more compatible with the requirements of the new environment.

These trends provide a sense of the forces shaping the emergence of a completely new state of affairs in which existing utilities will have evolved and will coexist with new players in the provision of flexibility. The final shape of power systems will not be unique, as it is path-dependent due to the effects of technological, financial, and institutional legacies. What is certain is that the change is inevitable and utilities as are known today will definitely change. In the new environment, the opportunities for utilising competition among the suppliers of flexibility increase. In conclusion, as flexibility becomes scarce in the system, innovative flexibility-enabling business models initiated by new actors will be highly valuable and critical for the efficient provision of flexibility services.

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Chapter 2

Power System Flexibility:
A Product Design Perspective

Power system flexibility: a product design perspective

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Abstract

Power system flexibility, or the ability to adapt to uncertainty and variability, is a desirable quality of systems operating with substantial shares of variable renewable energy (VRE). Engineers, policymakers and industry stakeholders have so far developed an important body of work -which this paper reviews - describing and quantifying what flexibility is, but its economic properties have not been analyzed thus far. To address this gap, and informed by its technical properties, this paper proposes three postulates that guide the *economic* modelling of flexibility for product, contract and market design purposes. These are: the multi-attribute nature of flexibility, the imperfect substitutability among its elements, and the heterogeneity of the flexibility product space. In addition, this paper proposes three qualitative properties that make a good flexibility-enabling product design, which are simplicity, measurability and relevance. Using these as guiding principles, the paper reviews two existing product designs, namely the FlexOffer based Delta Energy and California's Flexible Ramping Product.

Keywords: Product design, market design, electricity markets, power system flexibility

JEL Classification Numbers: L94, D47

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1 Introduction

Following an upward trend, the International Energy Agency (IEA, 2015) reported that, on a global scale, 130 GW of renewable capacity was added during 2014; the fastest growth rate registered to date. Onshore wind accounted for more than one-third of all renewable capacity additions, while solar photovoltaic (PV) and hydropower represented one-third and one-fifth, respectively. Other renewable technologies grew at a slower pace but, overall, renewable capacity accounted for more than 45% of all net additions in the power sector. With a renewable share of total electricity generation standing at 22% by the end of 2013, the IEA projects it to reach 26% by 2020.

The aforementioned figures confirm the predominant role that variable renewable energy (VRE) has already gained and its expected increase in the years to come, as technological improvements and cost reductions materialize while policy makers prioritize decarbonisation in all economic activities, including electricity generation. However, increasing the share of renewables exposes power systems to greater *uncertainty* and *variability*, particularly in the short term. Not only does the generating source change over time, i.e. neither wind blows always with the same intensity nor the sun shines equally at all times, but any forecast will always include some degree of error, which tends to increase as renewable generation becomes prevalent in power systems. (Katz and Cochran, 2015; Katz et al., 2015b).

Ever since power systems have existed, operators have dealt with the challenge of variability and uncertainty in demand and generation resources, and have developed a number of tools to deal with it. With the advent of market mechanisms in the power industry, many of these are well-established and respond to both physical and economic requirements. But the perspective of relying on larger shares of VRE generation poses a challenge that requires increasing the overall adaptability of the system to uncertain variations, including changes in market design (Ela et al., 2014). In other words, as power systems become more operationally *flexible* by increasing their ability to modify supply or demand within time-frames that can vary between minutes and hours, they become more resilient to fluctuations, and VRE can become more efficiently integrated. As a result, the increased flexibility of a power system becomes evident in the lower risk of renewable curtailment, negative pricing and greater confidence on the revenue streams of power plants. Furthermore, besides its relevance for VRE integration, flexibility matters because it can reduce overall system costs and reduce the environmental impact of power system operations (Cochran et al., 2014).

In the second section of this paper, by way of examples from Denmark and California, I illustrate the impact of VRE on the operation of power systems and how can flexibility facilitate its integration. This serves as motivation for the third section of the paper where I present a literature review on the subject of power system flexibility, which is organized according to the main topics that have attracted the interest of academics, policymakers and stakeholders writing about it.

Despite the fact that, in recent years, the technical literature has shown interest in defining *power system flexibility* and quantifying it at a system-wide level, it can take different forms and imply different meanings. As Ulbig and Andersson (2012) note: "...flexibility is often not properly defined and may refer to very different things, ranging from the quick response times of certain generation units, e.g. gas turbines, to the degree of efficiency of a given power market setup". Section 3 aims, therefore, to demystify the concept and present an overview of the current state of the debate on the topic.

Another related issue is that flexibility has remained so far a chiefly technical concept discussed by engineers, while its economic properties have not been sufficiently studied. Although related to capacity and energy, two typically traded products in existing electricity markets, a fundamental claim of this paper is that *flexibility is a heterogeneous commodity characterized by multiple, imperfectly substitutable attributes.* This view is developed in the fourth part of the paper, where the economic properties of flexibility are analyzed.

But precisely because flexibility can mean many things and the different sources from which it can be obtained may excel in their different attributes, precisions regarding the specific features of any product that is to be labelled with a "flexibility tag" must be made if any fruitful market design developments are to be achieved. That is, rather than a general definition of what flexibility is, traders of flexibility require a *concrete service* that fulfills a specific technical purpose. The most studied market design problems in the economics literature have involved designing marketplaces to address market failures where the traded goods are self-explanatory: "doctors", "kidneys", "licenses" and "electricity", to name just a few. Yet, a product design stage might be required before proceeding to any actual market implementation. Surprisingly, besides *ad hoc* product designs put forward (e.g. **?**) and unlike the existing summaries of lessons learnt so far about the practice of *market* design (Roth, 2002, 2008), the economics literature hasn't addressed so far the question of what constitutes a set of best practices in the *product* design stage of a practical market design problem. In section five of the paper, using flexibility as a guiding example, I describe the ideal characteristics that a product design must have.

In the sixth section of the paper, using the guidelines outlined in section 5, I examine two product designs that enable power system flexibility. First, I study the Flex-Offer concept put forward by engineers and computer scientists to describe the flexibility associated to any resource in the power system and describe how the flexibility contained in a Flex-Offer can be transformed into a tradeable product, namely Delta Energy. Second, I address the Flexible Ramping Product to be traded in the California Independent System Operator (CAISO) real-time market. Section 7 concludes.

2 The need for flexibility in the operation of power systems

With the greater reliance on VRE and its projected increase in years to come, stakeholders, electricity industry analysts and academics have become interested in the question of efficiently integrating these sources of generation into the grid (see, e.g., Morales et al. (2013)). Operating systems with substantial shares of VRE increases the uncertainty and variability of power systems, particularly in the short term, which requires operators, business, and policy makers to incentivize the provision of flexibility, if higher levels of VRE penetration are to be achieved.

Traditionally, system operators have focused on forecasting system demand (load), which varies randomly. In contrast, with large shares of VRE, it is sensible for operators to focus on the *net load* - electricity demand *minus* VRE generation - which represents the demand that must be met by *non-renewable* sources.

The impact of uncertainty can be readily perceived in the net load, given that it depends on not one but two random variables (i.e. load and generation), reducing thus the accuracy of forecasts. The effect of variability, on the other hand, becomes evident in the *shorter peaks*, *steeper ramps* and *lower turn-downs* required from the non-renewable sources of generation (Cochran et al., 2015). In consequence, operators require flexibility-enabling assets (resources) that can adapt to these patterns.

But because "sometimes examples of *inflexibility* are easier to document than flexibility" (Cochran et al., 2014), it is interesting to describe what happens when flexibility is unavailable. On the *technical* side, the following impacts may be perceived:

1. *Difficulty balancing demand and supply* which result in frequency excursions and dropped load.[1]

2. *Significant VRE curtailments*, as a result of excess supply or transmission constraints

3. *Area balance violations* which reflects that a system cannot meet its balancing responsibility

In relation to *markets*, the following may happen:

[1] "The two fundamental characteristics of power delivered to a customer are frequency and voltage. As long as these remain correct the customer will have access to the needed power, and it will have the required characteristics" (Stoft, 2002). Integrating VRE challenges the stability of both frequency and voltage, which are the responsibility - respectively - of transmission system operators and distribution system operators. Flexibility is currently used to keep frequency within acceptable ranges. In the future, DSOs may have an increasing role as end users of flexibility for voltage control purposes (EDSO, 2014)

1. *Negative market prices* which might reflect that conventional generators are unable to reduce output, demand that cannot absorb excess supply, surplus of renewable energy or limited transmission capacity

2. *Price volatility* which can reflect insufficient transmission capacity, limited ramping availability, insufficiently fast response or limited ability to reduce demand

Illustrating the need for power system flexibility can be understood better by way of examples. The first of these comes from the Danish power system, which follows.

In the early hours of the 10th of July, 2015, wind blew in Denmark so strongly that 140% of electricity demand was met by the output of windfarms, exporting the excess power generation through the interconnections with Sweden, Norway and Germany.[2]

Figure 1 illustrates the evolution of the Danish load, wind power output, net load (calculated as load minus wind power output) and the net exchange through all interconnections with neighboring Germany, Sweden and Norway for both Danish bidding areas (DK1,i.e. western Denmark and DK2, i.e. eastern Denmark), over the course of four days (08/07/2015 – 11/07/2015). The typical short-term features of systems operating with large shares of renewables become evident in the figure.

Ideally, to make good use of VRE, generation and demand would have a positive correlation, that is: demand is high when renewables are available or, similarly, demand is low when renewables become scarce. However, a consequence of this situation is that non-renewable, dispatchable generators face shorter peaks and, consequently, are compensated for less operating hours, which adversely affects their cost recovery. As an example of this situation, EURELECTRIC (2011) cites the Spanish case - where renewables are dispatched first and at zero variable cost - which has led thermal units to have plummeting utilisation rates. Specifically, in a comparison of this indicator for the 2005-2007 with the 2009-2010 period,

[2]Many media outlets rejoiced with the record figure, as it proved that relying entirely on VRE was indeed feasible. For example, see the article in The Guardian: `http://www.theguardian.com/environment/2015/jul/10/denmark-wind-windfarm-power-exceed-electricity-demand`

the publication reports a 47% decrease for coal-fired power plants and a 30% decrease for combined cycle gas turbines (CCGT) plants.

Back to the Danish example, as can be observed in figure 1, during the daytime hours of Wednesday 8th, Thursday 9th and Friday 10th of July, 2015, the net load showed (relative to load) shorter peaks which coincided with high levels of demand and generation of wind energy. For example, between 10 and 11 on the 9th, net load peaked when 84% of total demand was being satisfied by wind generation. On the same day, net load peaked again between 17 and 18 when 90% of load was being met by wind.

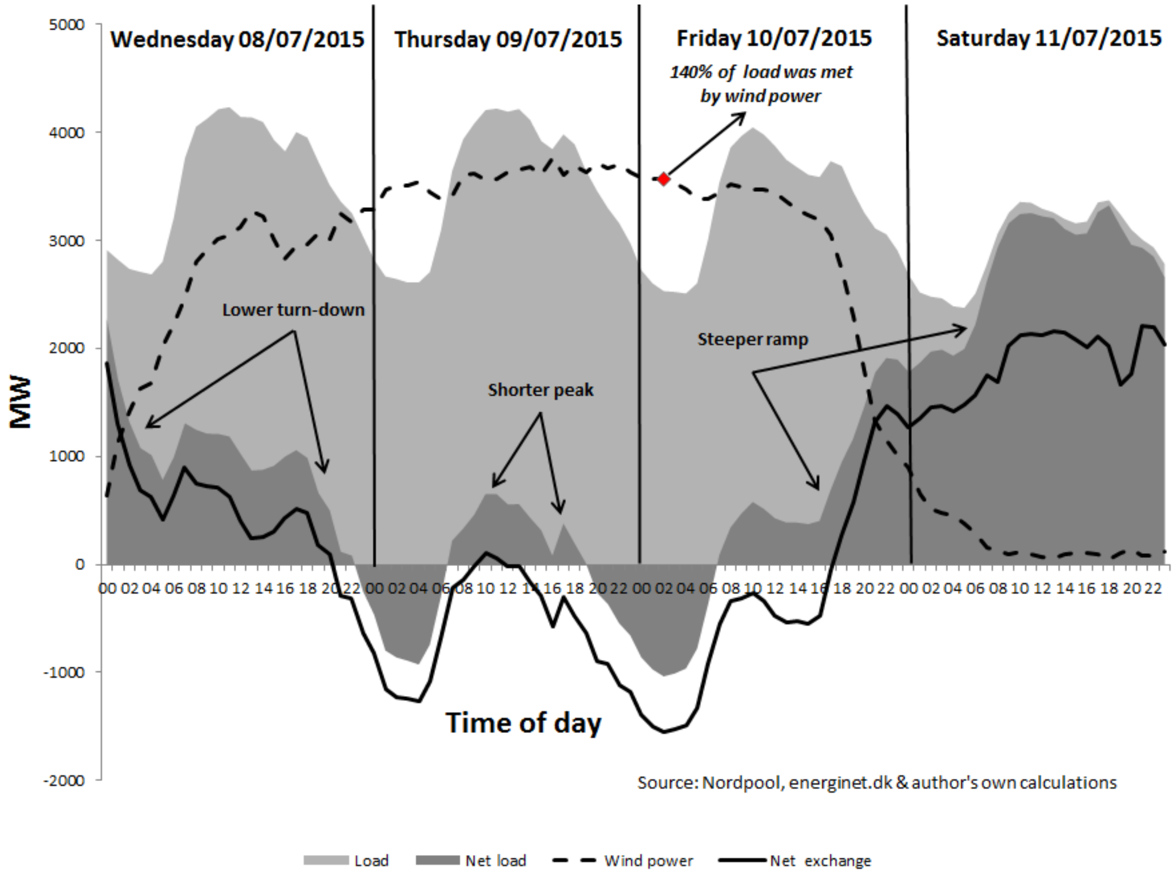

Figure 1: Net load (load minus wind power generation) and net power exchange in the Danish power system 08/07/2015 – 11/07/2015

However, VRE supply and load are not always positively correlated. In an empirical study about the Nordic countries, Holttinen (2005) finds that wind power output and load were slightly positively (31%) correlated in the period 2000-2002, but when the sample is restricted to the winter months, correlation is very close to zero.

In contrast to positive correlation between generation and demand, whenever these two variables are negatively - and thus unfavorably - correlated, two different ramping effects on the remaining generation base are induced.

If renewable supply decreases together with increases in demand, system operators must dispatch generation that is able to ramp up quickly. In figure 1, this can be readily observed from the afternoon hours of Friday 10th of July onwards, when wind generation began to decrease at the same time that load increased. Specifically, between 17 and 22 on that date, net load exhibited a steep ramp rate, whereas load declined.

In contrast, if renewable supply is high when demand is low, dispatchable generators face deeper turn downs as they must give way for renewables to satisfy demand. In figure 1, this becomes evident in the early hours of Wednesday 8th of July when net load declined considerably more steeply than load, as wind power generation increased. In fact, the achievement of the 140% record happened as a coincidence of low demand (during early morning hours), high wind power output and the existence of interconnections with neighboring countries, which explains the negative values for the net load. Instead of curtailing generation from wind turbines - a sign of inflexibility - power was exported. Specifically, in relation to Denmark and its approach to manage the intermittency of wind power, Green and Vasilakos (2011) find that on windy days, Denmark uses exports as a kind of electricity storage. In coincidence with their finding "... that short-term fluctuations in wind output are highly correlated with short-term fluctuations in net exports of electricity, which is exactly the efficient pattern of operation dictated by (their) model", net load and net exchanges are almost perfectly correlated (98%) in figure 1. Beyond the lesson learnt about Danish system operation with high shares of wind power their finding illustrates, more generally, the role of interconnections as an asset to enable power system flexibility.

A second figure (see figure 2) completes the illustration of the Danish power system during the days of the record wind power output of July 2015. In it, the net loads of each Danish bidding area (which differs from the aggregated Danish net load shown in figure 1) are shown together with the day-ahead (Elspot) system price and Danish price (note that prices coincided in both Danish bidding areas). Observe (in red) the prices that cleared the day-ahead market when the record output was achieved. It is easy to see that western (DK1) net load exhibits greater variability than its eastern (DK2) counterpart. Steeper ramps and lower turn-downs (including negative values, associated to exports) in the west than in the east can be explained by the greater concentration of windfarms in this area of the country and to differences in weather conditions. Regarding prices it is interesting to note that, relative to the Nordpool system price, prices in the two DK bidding areas were always slightly below in the period 08/07/2015-10/07/2015, coinciding with the build-up of wind power output. Sufficient capacity in the interconnections allowed exporting without curtailing when there was excess supply. However, as soon as wind power output decreased and net load (particularly in DK1) exhibited a steep ramp, the DK price increased. On average, 92% of load was covered by wind power on the 10th of July, whereas only 8% was covered on the 11th. The lower power output had to be compensated with imports on the 11th: on average, 62% of the load was covered by trade with neighboring areas on that day. However, the remainder had to be covered with ramping capability. The fact that the price average between the 10th and the 11th spiked so markedly[3] shows insufficient flexibility. Although it is difficult to make precisions with the data considered in this example, it is likely that relatively inefficient dispatchable generators had to ramp up. Had there been other flexibility options, such as demand-response or storage or other kind of market mechanisms, the flexibility procured by the operator would have been less costly.

Another example comes from power system operation in California, the most populous state in the US[4], which has one of the most ambitious energy and environmental goals in

[3]The average DK price quadrupled, from 5.41 EUR/MWh on 10th of July to 21.83 EUR/MWh on 11th of July

[4]39.1 million inhabitants as of 2015, according to Wikipedia `https://en.wikipedia.org/wiki/List_of_U.S._states_and_territories_by_population`

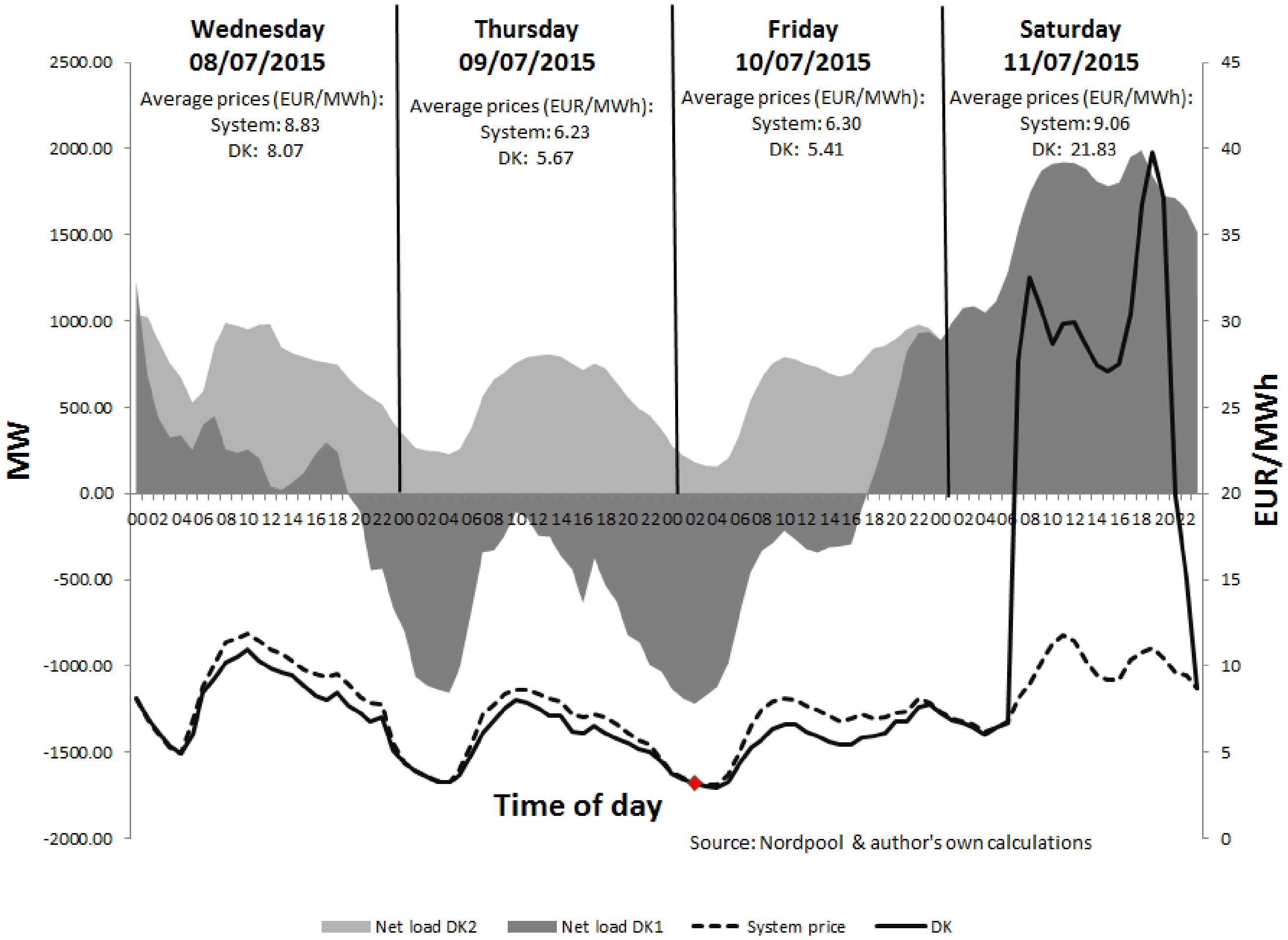

Figure 2: Net load (load minus wind power generation) in the DK1 (west), DK2 (east) bidding areas together with system and DK Elspot (day-ahead) prices 08/07/2015 – 11/07/2015

the whole country, including a 50% of retail electricity from renewable sources by 2030.[5] Consequently -in an analysis that has involved every single day of operation between 2012 and 2020 - the California Independent System Operator (CAISO, 2016b) has identified a number of prospective operational challenges, including :

1. *Short, steep ramps* in both upward and downward directions, which requires either dispatching or shutting down generation resources quickly and for short periods of

[5]According to the Energy Information Administration (EIA): "In 2014, California ranked fourth in the nation in conventional hydroelectric generation, second in net electricity generation from other renewable energy resources, and first as a producer of electricity from both solar energy and geothermal energy" `http://www.eia.gov/state/?sid=CA`

time.

2. *Risk of oversupply* when excess generation, including renewables, exceeds real-time demand

3. *Reduced frequency response* when less operating resources are available to automatically adjust electricity output for grid reliability purposes

The first two elements of the previous list can be observed in figures 3 and 4, known in energy circles as "duck curves". In the first of these, actual and projected net load for the 11th of January (i.e. a winter day) is shown. Note three ramps: the first (known as the duck's tail, estimated by the CAISO in an amount of 8000 MW) happens during the early morning hours, builds up from around 4 AM and lasts until 7 AM, sunrise time. The second (known as the duck's belly) is downwards and reflects the increasing renewable supply (particularly solar) displacing conventional generation. At around 4 PM, sunset time, an upward ramp (known as the duck's neck, estimated at around 11.000 MW) appears again as demand increases again towards the night hours.

On spring days, as shown in figure 4, the risk of oversupply accentuates as the sun rises earlier and sets later, inducing a more pronounced "duck belly", where the afternoon ramp is estimated in 13000 MW in approximately three hours.

Consequently, (CAISO, 2016b) requires flexible resources that are able to:

1. Sustain an upward/downward ramp,

2. Respond for a defined period of time,

3. Change ramp directions quickly,

4. Store energy or modify use,

5. React quickly and meet expected operating levels,

6. Start with short notice from a zero or low-electricity operating level,

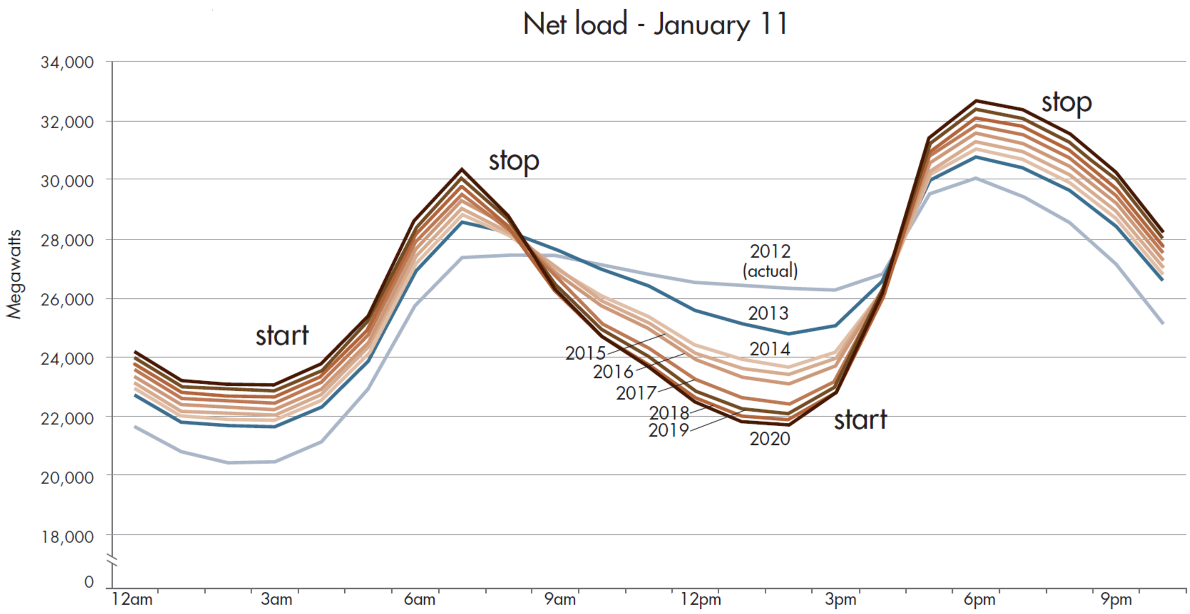

Figure 3: California's "Duck curve" on a winter day (net load on January 11th 2012-2020). Source: (CAISO, 2016b)

7. Start and stop multiple times per day, and

8. Accurately forecast operating capability

To summarize: with high shares of VRE, system operators require *flexibility-enabling assets* (resources) that increase the flexibility in the short-term operation of power systems. These must be able to adapt to the shorter peaks, steeper ramps and lower turn-downs evident in the system's net load.

3 Power system flexibility: a literature review

Flexibility has traditionally been associated to rapidly dispatchable generation: power plants that, given their technological characteristics, can respond quickly at the request of operators (IEA, 2014). However, a common feature of the existing literature is that it recognizes that power system flexibility can be obtained from a wider array of resources besides generation (see figure 5):

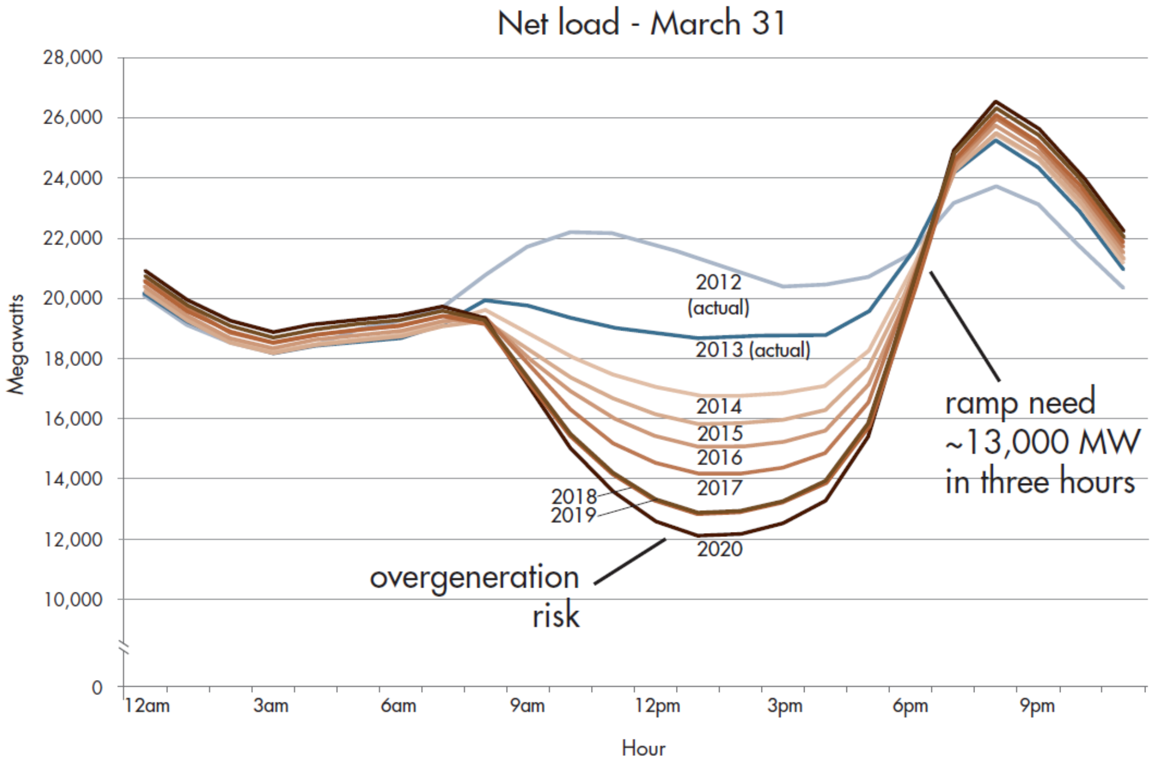

Figure 4: California's "Duck curve" on a spring day (net load on March 31st 2012-2020). Source: (CAISO, 2016b)

> "Sources of flexibility exist - and can be enhanced- across all of the *physical* and *institutional* elements of the power system, including system operations and markets, demand-side resources and storage; generation; and transmission networks" Cochran et al. (2015).

Not only has this been recognized since the publication of two early institutional reports on VRE integration (namely IEA (2005) and EWEA (2005))[6] but emerging business models are clearly showing that flexibility may originate from different sources in the system (Boscán and Poudineh, 2016a).

EURELECTRIC (2011), for example, claims that demand-side participation and storage, interconnections, and market tools are all options to manage variability and ensure security

[6]These studies are considered by EURELECTRIC (2011) to be "the first comprehensive reports describing the flexibility challenge and possible options . . . ". In particular, see Annex I in their report for a critical overview of the existing publications about flexibility requirements.

Cochran et al. (2015)	IEA (2014)	EURELECTRIC (2011)
❑ System operations and markets	❑ Dispatchable power plants	❑ Dispatchable flexible and back-up generation
❑ Demand-side resources and storage	❑ Storage	❑ Demand-side participation and storage
❑ Generation	❑ Demand-side management/response	❑ Interconnections
❑ Transmission networks	❑ Grid infrastructure	❑ Market tools

Figure 5: Sources of power system flexibility

of supply. The IEA (2014) admits that "while dispatchable power plants are of great importance, other resources that may potentially be used for balancing are storage, demand-side management or response, and grid infrastructure".

Clearly, not all technologies perform in the same way and have intrinsic features that distinguish them from one another in several dimensions, i.e. a multi-faceted and differing *resource flexibility*, also known as *technical flexibility*. As Ela et al. (2014) describe it: "different types of resource excel at different forms of flexibility, and they have different cost impacts when providing flexibility". Such costs, like in the provision of energy, can be fixed (associated to capital expenses) or variable (associated to operational expenses).

According to the IEA (2014), a power plant is more flexible if it: 1) can start at short notice (i.e. has a short lead time), 2) can operate at a wide range of generation levels, and 3) can move quickly between different levels of generation (i.e. can ramp up or down quickly). Similarly, in a comparison of the existing European generation fleet, EURELECTRIC (2011) considered several relevant characteristics to assess the technical flexibility of nuclear, hard coal-fired, lignite-fired, combined cycle gas-fired and pumped storage power plants. Specifically, they considered the following characteristics relevant to the provision of flexibility: 1) Start-up time in "cold" conditions, 2) Start-up time in "warm" conditions, 3) Rapidity of load change (ramp rate), 4) Minimum load level, and 5) Minimum shutdown time.

EURELECTRIC (2011) finds that pumped storage is the most efficient technology, given its short lead time (i.e. time to dispatch) and fast ramp rate. By comparison, the second fastest ramping technology are nuclear plants, which can increase or decrease output at a rate of 5% per minute, but are considerably less efficient in their start-up time from "cold" or "warm" conditions.[7]

Along the same lines as the IEA and EURELECTRIC, Ulbig and Andersson (2012) - building on the contribution of Makarov et al. (2009) - propose a *flexibility trinity* to measure the " technical capabilities, and related constraints, of individual power system units to modulate *power* and *energy* in-feed into the grid, respectively out-feed out of the grid". Specifically, they suggest: 1) Power capability P for up/down regulation (measured in MW), 2) Energy storage capability E (measured in MWh), and 3) Power ramping capability R (measured in MW/min). The three metrics are related via integration and differentiation over the time domain, as figure 6 shows. A fourth, related metric is ramping duration D, which is defined as the ratio of power to the ramp rate, $D = P/R$.

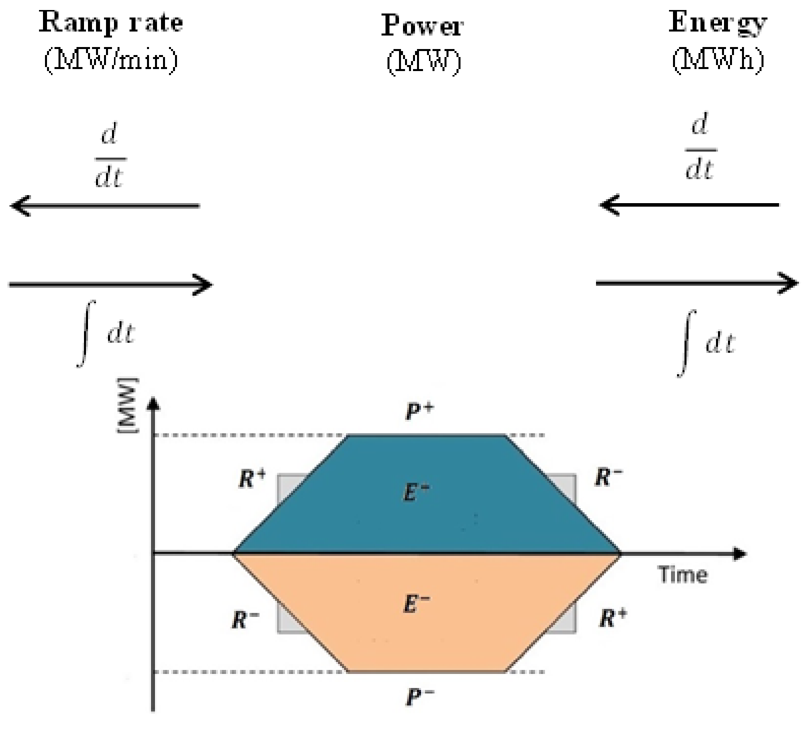

Figure 6: The flexibility trinity. Source: Ulbig and Andersson (2012)

[7]An interesting finding of the study by EURELECTRIC (2011) is that "contrary to common belief, nuclear power plants may perform in a rather flexible mode if the appropriate (technical) design has been implemented (as witnessed in countries like Germany and France)".

An interesting feature of the flexibility trinity proposed by Ulbig and Andersson (2012) is that it represents a sufficiently general framework to characterize the flexibility coming from resources other than generation, as it illustrates the multi-dimensionality and heterogeneity of flexibility, together with the imperfect substitutability of the elements that compose it.[8]

However, there are important differences between supply and demand resources. On the technical side, it is not accurate to include the ramp rate as an element of demand-side flexibility as this characteristic depends on purely technical constraints, whereas in the case of demand the change is instantaneous. instead, a more sensible characteristic for demand-side flexibility is the *lead time*, i.e. the time elapsed between scheduling and delivery, a key element in the pricing problem of flexible electricity contracts discussed by Bjorgan et al. (2000) and in existing demand-response programs, such as Nest thermostat's "Rush Hour Rewards".

Furthermore, unlike supply-side resources, another key characteristic of demand-side flexibility is that a significant *human behavior* component determines its provision, particularly when it comes to residential and small and medium sized enterprise consumers, as recognized by a number of experimental and quasi-experimental studies (e.g. Allcott (2011), Allcott and Mullainathan (2010)). Consequently, any framework that adequately analyses residential demand-side flexibility must consider the incentives to *voluntarily* modify consumption behavior, as suggested by He et al. (2013).

Relative to industrial consumers, small scale consumers have higher barriers to trade as a result of higher transaction costs, technological differences, lower trading skills and different objectives, which motivates the emergence of business models like *aggregation*: "a commercial function of pooling de-centralized generation and/or consumption to provide energy within the system"(EURELECTRIC, 2014).

Given that "flexibility is system specific" (Cochran et al., 2014), the majority of publications emphasize on a *system perspective* when defining flexibility (see figure 7) (e.g. Chan-

[8]Which are the economic properties of flexibility, discussed in section 4

dler (2008), Silva (2010), Ma (2012), Lannoye et al. (2012), Bertsch et al. (2012), Katz and Cochran (2015), Zhao et al. (2016)). In essence, they coincide in characterizing flexibility as the inherent ability of an entity - the power system - to adapt to the challenge of facing exogenously given, uncertain variations.

For instance, consider the following definition by the IEA:

> "In its widest sense power system flexibility describes the extent to which a power system can adapt to the patterns of electricity generation and consumption in order to maintain the balance between supply and demand in a cost-effective manner" (IEA, 2014)

The system perspective to defining flexibility stems from the interest on measurement, which supports the assessment of a given system's flexibility, the comparison among systems, and informing policy decisions. Relevant questions like: "where does system A obtain its flexibility from?", "how does flexibility in system A compare to system B?" and "what can be done to increase the flexibility of system A?" are underlying in this strand of the literature:

> "The concept of flexibility often arises when policymakers ask system planners how much wind and solar generation can be *reliably* added to the power system. The question can lead to debate about how flexible a power system is and the corresponding impacts of adding renewables" (Cochran et al., 2014).

Implicit in these definitions is the (technical) hierarchical assumption that "flexibility is one element to reliability and ... a subset of frequency stability; other stability impacts such as voltage stability can arise when integrating wind and solar into power grids" (Cochran et al., 2014).

A somewhat different - and influential [9] - perspective to defining flexibility is, for example, that of EURELECTRIC (2014) (see figure 7) because it emphasizes on the *individual* behavior of flexibility-enabling assets in response to an external signal, such as price or activation.

[9] Influential because it has been adopted by a number of power system stakeholders, e.g. EDSO (2014), OFGEM (2015)

Author	Definition
EURELECTRIC (2014)	*"On an individual level flexibility is the modification of generation injection and/or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service within the energy system. The parameters used to characterise flexibility include the amount of power modulation, the duration, the rate of change, the location etc."*
Lannoye et al. (2012)	*"Flexibility is defined here as the ability of a system to deploy its resources to respond to changes in net load, where net load is defined as the remaining system load not served by variable generation. Hence, an isolated power system containing mostly generation units with long start up times and low ramp rates will find it more difficult to successfully integrate variable generation than a well interconnected power system, containing many generation units which can start up and ramp quickly"*
Katz and Cochran (2015)	*"The ability of a power system to respond to changes in electricity demand and supply"*
Silva (2010)	*"In general terms, the flexibility of the system represents its ability to accommodate increasing levels of uncertainty while maintaining satisfactory levels of performance at minimal additional cost for any timescale . Flexibility can be viewed from either from a system planning or system operation perspective."*
IEA (2008)	*"A power system is flexible if it can - within economic boundaries - respond rapidly to large fluctuations in demand and supply, both scheduled and unforeseen variations and events, ramping down production when demand decreases, and upwards when it increases"*
Zhao (2016)	*"Flexibility is the ability of a system to respond to a range of uncertain future states by taking an alternative course of action within acceptable cost threshold and time window"*
Kiviluoma (2013)	*"Flexibility is influenced by the types and numbers of power plants in the system, the availability of reservoir hydro power or pumped storage, transmission lines to other power systems, transmission constraints within the system, and availability of demand response including electricity use in transport and heat generation"*
Ela et al. (2014)	"The ability of a resource, whether any component or collection of components of the power system, to respond to the known and unknown changes of power system conditions at various operational timescales"
Ma (2012)	*"Flexibility describes the system ability to cope with events that may cause imbalance between electricity supply and demand while maintaining system reliability in a cost-effective manner"*
IEA (2014)	*"In its widest sense, power system flexibility describes the extent to which a power system can adapt to the patterns of electricity generation and consumption in order to maintain the balance between supply and demand in a cost-effective manner. In a narrower sense, the flexibility of a power system refers to the extent to which generation or demand can be increased or reduced over a timescale ranging from minutes to hours in response to variability, expected or otherwise"*
Bertsch et al. (2012)	*"Flexibility is the capability to balance rapid changes in renewable generation and forecast errors within power systems"*

Figure 7: Definitions of flexibility

This perspective is more fruitful *economically* as it relates to the question of *incentivizing* the provision of flexibility, rather than assuming that flexibility is a given characteristic of power systems. Aside exceptions, such as the work by Ela et al. (2012, 2014) and Ela et al. (2014), the power systems literature has left the question of creating the incentives to provide flexibility largely unaddressed. This is in spite the fact that it recognizes that innovative market designs are able to enable flexibility.

Measuring power system flexibility is yet another topic of interest in the literature. With increasing levels of complexity, Cochran et al. (2014) review three different frameworks to measure flexibility. The first of these are the visually oriented assessments of flexibility, which are amenable to non-technical audiences willing to make quick flexibility comparisons among countries and systems but are limited in scope and precision. Within this kind of assessment, Yasuda et al. (2012) propose a flexibility chart - a polygonal radar - to measure the existing installed capacities (GW) of resource flexibility in a given system, including dispatchable power plants (hydropower, combined cycle gas turbine (CCGT) and combined heat and power (CHP)), pumped-hydro storage and interconnection.

A second example of a visually-oriented flexibility assessment is the IEA (2014)'s Grid Integration of Variable Renewables (GIVAR) project, which employed a visual comparison tool of fundamental system properties that enable power system flexibility. Specifically, this approach accounts for:

1. *Power area size* (total installed capacity of the system under consideration): the greater the area and the more diverse the power mix is, the greater the likelihood for keeping the system in balance and the lower the impact of variability and uncertainty is felt.

2. *Internal grid strength* (installed transmission capacity to transport power within an area): if there is enough capacity to transport power from one place to the other within an area, the lower the possibilities that congestion is experienced and thus resources in the system are less likely to address the challenges of variability.

3. *Interconnection* (available capacity to transport power to adjacent areas): if interconnections are available, systems are able to use the flexible resources from its neighbors

4. *The number of power markets:* there is a trade-off between the number of markets and the volume of trade. When several, un-coordinated markets exist, there is less volume of trade and less possibilities of enabling flexibility. On the contrary, if markets are integrated and coordinated, greater volumes are traded and higher possibilities of enabling flexibility exist.

5. *The geographical spread of VRE generation:* the greater the dispersion of VRE generation facilities, the smoother output will be, as a result of divergent weather conditions in specific areas within a system.

6. *The flexibility of the dispatchable generation portfolio:* if the generation fleet is flexible enough, this impacts favorably the overall flexibility of the system

7. *The existence of investment opportunities:* power systems are *stable* if its prospects for demand growth or renewal of infrastructure are low or inexistent, whereas they are *dynamic* if investments are required regardless of VRE integration. Integrating renewables into dynamic systems is easier because opportunities to create the necessary infrastructure to efficiently integrate VRE exist.

A second approach to measure power system flexibility described by Cochran et al. (2014) is the IEA (2014)'s Flexibility Assessment Tool (FAST2) which accounts for the fact that "power system flexibility is a time-specific quality" (Cochran et al., 2014). Using time series data of power demand and time-synchronized wind and solar generation, and given a fixed time horizon and operating state (i.e. the operation level of power plants), this approach measures the maximum upward or downward change in the supply/demand balance that a power system is capable of meeting.

The third approach to measure flexibility described by Cochran et al. (2014) are flexibility assessments in the context of detailed power system planning, which requires detailed data on:

1. *Physical characteristics of the system* including transmission constraints, balancing area size, VRE source characteristics and generator
2. *Institutional characteristics*, such as system operation practices and economic and market contexts
3. *Integration with other energy systems*, e.g. transportation, heat.

Nevertheless,

> "the data requirements to calculate flexibility in this manner are significant ... and may be more appropriate for analyzing systems that might be flexibility challenged ... (which) could arise with higher penetration levels of renewable energy, when flexibility is likely insufficient, or with low levels of renewable energy being integrated into a portfolio of inflexible conventional generation (e.g., Japan, Alberta)" (Cochran et al., 2014)

Another important topic in the discussion about power system flexibility refers to the timescale in which it is provided. With a long-term (planning) perspective, the relevant question is if the incentives to invest in resources with flexible attributes are in place. This is important because "if flexible resources are never built, it does not matter what incentives are introduced in the short-term market - the operator would clearly not have access to resources that are not built" (Ela et al., 2014). However, according to the study by Bertsch et al. (2012), no particular emphasis on flexibility must be placed because "any market design that incentivizes investment in least (total system) cost generation investment does not need additional incentives for flexibility ... (because) flexibility (is an) inevitable complement" of a least cost capacity mix. Bertsch et al. (2012)'s modelling approach is, nevertheless, problematic as it assumes that a benevolent social planner is able to minimize discounted total system costs subject to a number of technical constraints, hardly accounting for actual incentive issues.

With a short-term (operational) perspective, the question is if owners of the existing flexibility-enabling assets in the system have the incentives to provide the necessary flexibility to cope with the challenge of increased variability and uncertainty due to a higher reliance on renewables. That is: if existing services, contracts and markets are enough to enable flexibility. In relation to this issue, Ela et al. (2014) consider that "it is unclear whether or not the current market designs have the right incentives to provide this (operational) flexibility when the system flexibility need is increased with variable generation", leaving open the possibility of introducing market design innovations that incentivize the provision of flexibility in short-term operations.

With regards to the users of flexibility, it is worth noting that:

> "*Today*, TSOs procure flexibility in the form of ancillary services, mostly from large power producers. *In the future*, DSOs could additionally procure ancillary services from distributed generation and other distributed energy resources (including demand response and decentralized storage) as one of the tools for maintaining the quality of service and the security of supply in their networks" (EURELECTRIC, 2014). [10]

In addition to the previously mentioned, commercial parties (e.g. suppliers, aggregators, balancing responsible parties) may also trade with flexibility for portfolio optimization purposes (EDSO, 2014).

Furthermore, the following describes how flexibility is traded today:

[10] The statement applies equally to Transmission System Operators (TSOs) and Independent System Operators (ISOs). The former *own* and *operate* the high-voltage transmission network, whereas the latter only operate it. Furthermore, the acronym DSO stands for Distribution System Operator, who operates the medium and low voltage networks.

> "In liberalized power systems, operational flexibility is traded in the form of *energy products* via power markets, i.e. day-ahead and intra-day spot markets, as well as *control reserve products*, i.e. primary/secondary/tertiary frequency control reserves, from Ancillary Services (AS) markets" Ulbig and Andersson (2015)

Concerning the procurement method for flexibility services, Katz et al. (2015a) review existing administrative and market-based mechanisms with an emphasis on the US experience. The former includes "contracts, requests for proposals and internal acquisitions to procure a variety of grid services to support system flexibility", while the latter comprehends "market designs - with clear definitions of performance requirements[11] - To incentivize the provision of power system flexibility". In addition, EURELECTRIC (2014) foresees that "the DSO can procure ancillary services from local providers if there is a market. If there is no market DSO must have other tools to maintain the stability of the grid". Nonetheless, a dedicated, economics-based study of the problem of designing products, contracts and markets for power system flexibility has, so far, not been addressed by the literature. The papers by Bogetoft et al. (2016) which analyses demand-side flexibility; Boscán and Poudineh (2016a) which focuses on emerging business models that enable power system flexibility; Boscán and Poudineh (2016b) which studies bilateral contracts to enable power system flexibility; and Boscán (2016), which proposes a number of extensions to the Product-Mix Auction and applies it to the design of a flexibility marketplace, constitute an attempt to fill the previously mentioned gap in the literature.

4 The economic properties of power system flexibility

As section 3 has shown, the existing literature has emphasized on the *technical* properties of power system flexibility and has been advanced mostly by engineers. Significant progress

[11] That is, product designs.

has been made but, so far, its *economic* properties have not been studied. Understanding them is relevant because the adequate design of products, markets and contracts to incentivize the provision of power system flexibility depends on the existence of well-reasoned economic principles. Typically, these are based on simple economics but lead to fruitful results.

Informed by its technical features, but based particularly on the work by Makarov et al. (2009) and Ulbig and Andersson (2012, 2015), in this section I take on an axiomatic perspective and propose a number of postulates that guide the economic modelling of power system flexibility. These are as follow:

P1: Flexibility F is a bundle of $f_1, \dots f_n$ attributes that can be represented by an n-dimensional vector $F = [f_1, \dots f_n]$

P1 allows applying standard consumer and production theory as well as their duality results. A well understood property of these branches of microeconomic theory is that utility maximization and profit maximization are mathematically equivalent problems, which is recognized in textbook treatments such as Mas-Colell et al. (1995), Jehle and Reny (2011), Simon et al. (1994) [12].

The modeler may assume that $U(F)$ is the utility that a user of flexibility derives from bundle F. Alternatively, if $Y(F)$ is a production function, such that $y = Y(F)$, y is the output that results from combining the attributes of flexibility, which may be used in an internal production process or might be sold in a marketplace. Likewise, a cost function $C(F)$ associated to, say, technology Y can also be specified. This simple postulate allows modelling the two sides of a transaction where flexibility is traded, namely buyers and sellers. Further assumptions regarding the presence or absence of competition can be left to specific applications.

[12] For example, Jehle and Reny (2011) write in their opening comments to Chapter 3 (Theory of the Firm) that "You will see we can now move rather quickly through much of this material because there are many formal similarities between producer theory and the consumer theory we just completed"

For example, in the baseline model proposed by Bogetoft et al. (2016), flexibility has two attributes, namely quantity (load) - which is fixed to unity- and time, which can take on a discrete set of values. Because in their model two parties bargain over the price of a flexible load, it is equivalent to assuming that there is a bilateral monopoly. Similarly, in the bilateral contracting model with bidimensional types of Boscán and Poudineh (2016b), flexibility also has two continuous attributes, q and t, which can stand for ramp rate, duration, lead time, or other flexibility attributes. In this model, which focuses on situations in which competition among sellers of flexibility is infeasible, a principal -characterized by a multi-attribute gross utility function - procures flexibility from an agent with a cost function that depends on the elements of flexibility and a pair of unit cost parameters, which the principal cannot observe. Also, the multi-unit nature of the Product-Mix Exchanges (PMEs) discussed by Boscán (2016) -applied to the Delta Energy market design [13] - implies that participants bid for *bundles* of goods rather than on individual goods.

P2: The attributes that compose flexibility are imperfectly substitutable

An alternative way of stating *P2* is that consumers and producers of flexibility have, respectively, convex indifference curves and convex isoquants. This postulate is based on the *technical* fact that it is impossible to produce (or derive utility from) flexibility with only one of its attributes, neither is it possible to perfectly substitute among them, nor are these to be generally combined in production (or consumption) in fixed proportions. In other words, the marginal rate of technical substitution (or simply the marginal rate of substitution, in the case of indifference curves) of isoquants is not zero nor is it a constant number but a function of the attributes. To illustrate the point more clearly, observe figure 8 and assume that flexibility has only two attributes, namely f_1 and f_2. *P2* states that isoquants (or indifference curves) are similar to I_1 with a curvature that can be somewhere between that of I_2 or I_3 but, in general, rules out I_2 (perfect complements) and I_3 (perfect substitutes).

[13] Discussed in section 6

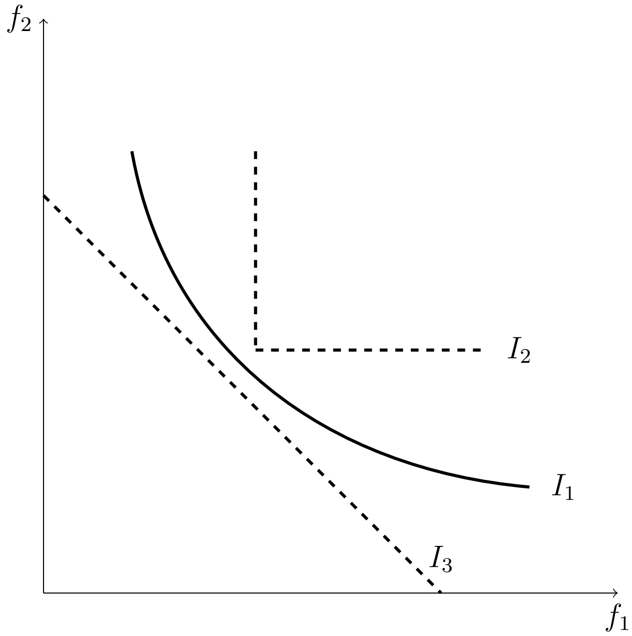

Figure 8: Isoquants/Indifference curves

So, for example, if a system operator procures flexibility in the form of, say, ramping duration (as described by Ulbig and Andersson (2012)[14]) two strictly positive quantities of power and ramping rate are required, but there is imperfect substitutability between them. This is in accordance with Makarov et al. (2009) who propose "a *concurrent* consideration of the regulation and load following capacity, ramping and ramp duration requirements" to account for the fact that "the regulation capacity and ramping requirements are inherently related. Insufficient ramping capability could cause additional capacity requirements".

Specific choices of functional forms could be the Constant Elasticity (CES) production and utility function or the Cobb-Douglas, which is a special case of a CES. An example of the application of *P2* to model flexibility trading in a competitive setting is in the work by Boscán (2016), where the participants bid for bundles whose components they perceive as imperfect substitutes.

P3: Flexibility is a *heterogeneous* (i.e. differentiated) product

[14]See also section 3

A key assumption of the competitive market model is that firms sell *homogeneous* (i.e. identical) products across its attributes. Consequently, consumers perceive all goods as the same and firms lack the incentive to raise their price above the market clearing price because a customer can easily buy exactly the same good elsewhere at a lower price. In other words, the demand faced by a single firm in a competitive market is perfectly elastic (i.e. a horizontal line).

However, the homogeneity assumption about goods is unrealistic as "goods are almost always differentiated by some characteristic" (Tirole, 1988). Accordingly, *P3* states that flexibility is heterogeneous. Nonetheless, in the case of flexibility, *P3* is not suggested for the sake of greater realism alone -which usually comes at a cost in the tractability of economic models- but because there are technical reasons as to why flexibility should be assumed to be heterogeneous. As noted in section 3, several authors coincide in stating that there are different forms of flexibility with differing levels of efficiency, that it can come from different sources and can imply different meanings. Therefore, any economic model should account for the fact that there isn't such a thing as *the* flexibility but *several kinds* of it.

It is necessary to make a further precision in relation to the kind of flexibility product space differentiation that *P3* refers to. "In a *vertically differentiated* product space, all consumers agree over the most preferred mix of characteristics and, more generally, over the preference ordering" (Tirole, 1988). A good example comes from petroleum markets, where lighter varieties of crude oil are valued at higher prices than heavier ones because, inter alia, the former have lower refining costs. That is, there is an objective attribute[15] in the commodity that determines if one given variety should be considered better than another.

Alternatively, when there is *horizontal differentiation*, "for some characteristics, the optimal choice (at equal prices) depends on the particular consumer. Tastes vary in the population" (Tirole, 1988). The analogy with flexibility is straightforward as not all users of flexibility have the same purpose for it and will, therefore, have different requirements as to

[15]Namely, the API gravity index which measures the density of a petroleum liquid relative to water

what sort of flexibility serves best their purpose. For example, a TSO might require ramping capability at some hours of the day, whereas a DSO might require services for voltage control. The former can come from conventional generators whereas the latter can be sourced from distributed generation or demand response.

In summary, *P3* states that flexibility is a horizontally differentiated product but does not rule out the possibility that in some well-developed markets for flexibility, final users agree that one variety is preferred over a different one because of some objective attributes that make it more desirable than others.

5 Best practices in product design: lessons from power system flexibility

In the context of a product design proposal for the Colombian electricity market, Cramton (2007) argues that:

> "Product design is the critical first step in the design of any market. It defines *what is being traded*. Good product design can play an important role in reducing complexity and increasing liquidity in the market"

Despite the clarity of the claim but unlike existing reviews of the lessons learnt about *practical* and *theoretical* market design that exist (e.g. Roth (2002, 2008)), a specific treatment of what are the best practices in the product design stage of a market design problem does not exist in the economics literature. The reason for this is, perhaps, that the best documented market design problems require very little clarification when it comes to the traded product. Although the problems are theoretically involved, matching *doctors* to workplaces, *students* to schools, *kidneys* to transplant recipients or allocating *licenses* to broadcasting corporations are all understandable problems, if explained in layman's terms. Similarly, in existing electricity markets, the two best known traded products are *energy* (measured in

MWh) and *capacity* (measured in MW). Yet in many situations, specifications about the traded product are required. For example, kidney transplants require three medical tests before a donor is considered apt. But when it comes to flexibility -currently traded in some markets as ancillary services in the form of energy products-further precision is required precisely because flexibility can mean many things, as follows from the literature review of section 3.

In this section, I take a *normative* approach and attempt to answer the question of *what makes a good product design*? Without making a claim of generality, I propose three qualitative properties that make a good product design for flexibility services. These are:

1. *Simplicity:* A product design is simple if its description is precise, sets clearly defined requirements on the parties involved and does not require additional knowledge, information or skills to be understood and traded.

2. *Measurability:* A product design is measurable if there is an unambiguous unit of measurement for each attribute that composes a flexibility bundle.

3. *Relevance:* A product design is relevant if it satisfies a specific technical requirement that attracts the interest of a sufficiently large number of buyers and sellers. That is: for the market to be thick enough.

In the following section, I use these properties to characterize existing product designs that enable power system flexibility.

6 A review of product designs in power system flexibility

6.1 FlexOffers and Delta Energy

FlexOffers are an information systems concept initially developed by a group of engineers and computer scientists in the European Union (EU)-funded research project MIRABEL[16], which emphasized on the development of Smart Grid infrastructure to enable power system flexibility. Boehm et al. (2012) define FlexOffers as "an energy planning object" composed of a profile, an earliest start time and a latest start time, associated to an energy-consuming or producing appliance in the power system. According to the MIRABEL approach, all forms of flexible *demand* (e.g. heat pumps, dishwashers, washing machines, freezers), *supply* (e.g. from private solar panels) or supply and demand together (e.g. an electric vehicle that can be charged and discharged within a period of time) can be *expressed* by a FlexOffer. Indeed, a useful way to think about FlexOffers is as general tools to express the flexibility (i.e. in the time and amount dimensions) which an issuer offers to give away in exchange for an economic compensation.

In a successor research project, named TotalFlex[17], the FlexOffer concept has been developed further and applied to a number of relevant questions, including flexibility aggregation. Another key question - which I briefly document in this subsection - has been trading and demonstration, i.e. *how to create a market-based approach to trade with the flexibility contained in a FlexOffer?* More specifically, *how to translate the flexibility contained in a FlexOffer into a tradeable product?*

[16]The MIRABEL (Micro-Request-Based Aggregation, Forecasting and Scheduling of Energy Demand, Supply and Distribution) was supported by the European Commissions Seventh Framework Program (FP7) and had as its main goal developing "an approach on a conceptual and an infrastructural level that allows energy distribution companies to balance the available supply of renewable energy sources and the current demand in ad-hoc fashion". Further information can be found in `http://www.mirabel-project.eu/`

[17]The TotalFlex research consortium has been funded by energinet.dk's (Denmark's Transmission System Operator) ForskEL research and development fund. It envisions "a cost-effective, market-based system that utilizes total flexibility in energy demand and production, taking balance and grid constraints into account". Further information can be found in `http://www.totalflex.dk/`

To answer the previously mentioned questions, consider the following formal definition of a FlexOffer, due to Valsomatzis et al. (2014):

> "A flex-offer f is a tuple $f = (T(f), profile(f))$ where $T(f)$ is the start time flexibility interval and $profile(f)$ is the data profile[18]. Here, $T(f) = [t_{es}, t_{ls}]$ where t_{es} and t_{ls} are the earliest start time and latest start time, respectively.
>
> The data profile $profile(f) = s^{(1)}, \ldots, s^{(m)}$ where a slice $s^{(i)}$ is a tuple $([t_s, t_e], [a_{min}, a_{max}])$ where $[a_{min}, a_{max}]$ is a continuous range of the amount and $[t_s, t_e]$ is a time interval defining the extent of $s^{(i)}$ in the time dimension."

There are three types of FlexOffers: *positive*, which correspond to a consumption profile, with all slices positive; *negative*, which describe a supply resource and have all slices negative; and *mixed* with both negative and positive slices, which contain both demand and supply offers of flexibility. Note, as well, that a FlexOffer can also contain a minimum energy requirement, i.e. a constraint that establishes that the sum of slices adds up to a certain number to reflect, for example, a minimum charging requirement of an electric vehicle. In addition, it follows from the definition that FlexOffers with time flexibility only or amount flexibility only also exist.

Figure 9, also due to Valsomatzis et al. (2014), illustrates a mixed FlexOffer $f = ([1, 6], s^1, s^2, s^3, s^4)$. The *time flexibility* in this FlexOffer is five periods, i.e. $6 - 1 = 5$, because the earliest period at which the profile can start is one, while the latest is six. The *amount flexibility* of the FlexOffer is the sum of the amount flexibilities (i.e. $a_{max} - a_{min}$) over all slices in the profile. In the example, this is: $(3-(-1))+(4-2)+(2-(-4))+(-1-(-3)) = 14$. Total flexibility, which is the product of the time and amount flexibility, is therefore 70.

It is worth noting that FlexOffers have infinitely many possible assignments, but figure 9 shows a specific one. The profile, which is composed of four slices, is drawn with *actual* start period 2 and ending period 6, while timewise there could have been 5 other starting times

[18] "Data profile" is the term used in this informations system context but it actually refers to a load (or supply) profile associated to the operation of a flexibility-enabling resource.

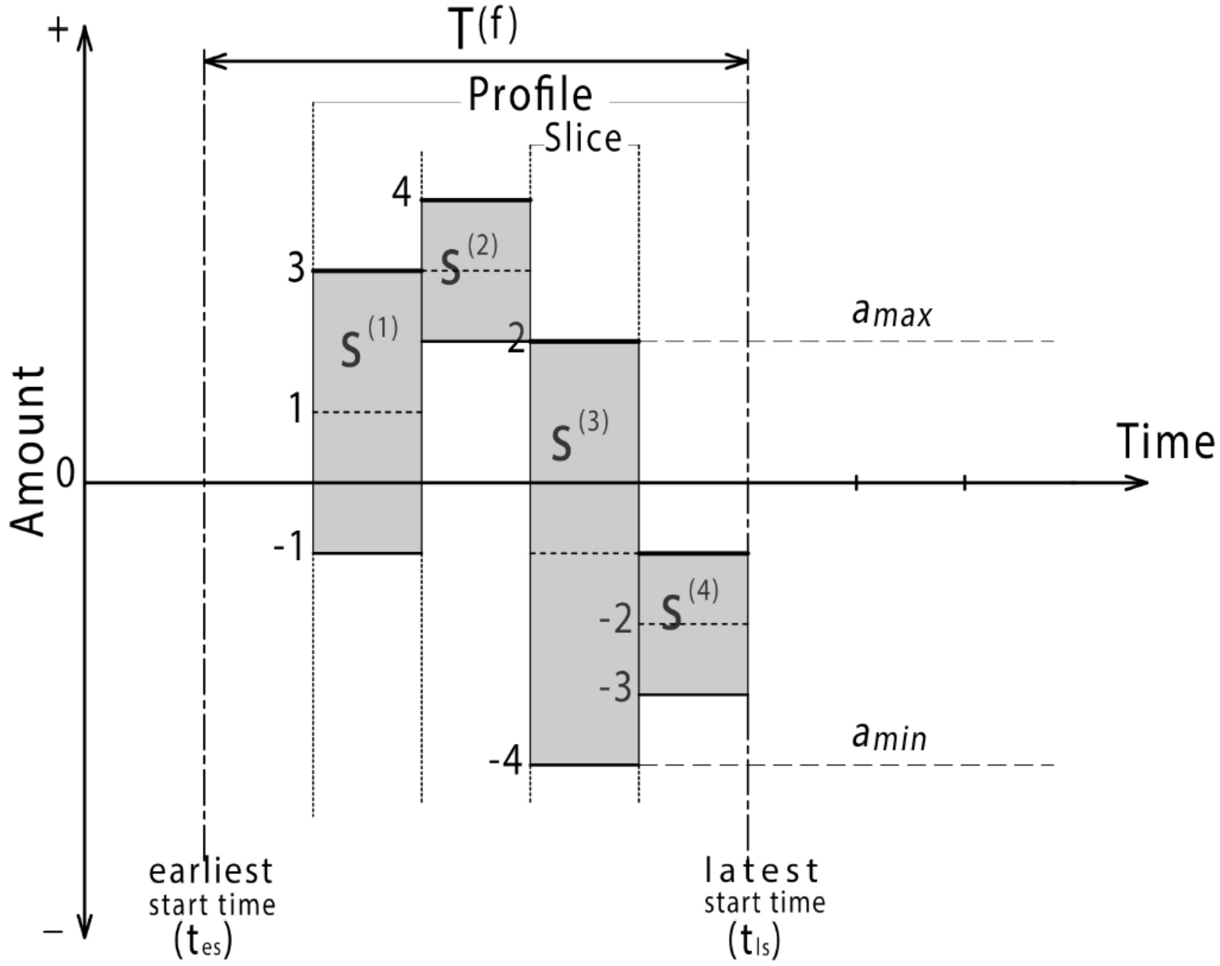

Figure 9: A mixed FlexOffer. Source: Valsomatzis et al. (2014)

(i.e. periods 1, 3, 4, 5 and 6). Assuming that the dashed line in every slice is the actual energy delivered, note that slice 1 is set at an amount of 1, slice 2 is set at an amount of 3, slice 3 is set at an amount of -1 and slice 4 is set at -2. Furthermore, given that a FlexOffer reflects an offer to supply flexibility, it is the user's prerogative to decide on the actual usage of the flexibility contained in it, that is: on the assignment that suits the final user best. Which takes the discussion back to the questions posed before: how can a flexibility-enabling product be designed if there are infinitely many feasible assignments to a FlexOffer?

The answer is the *Delta Energy*, which is the available *energy* contained in a Flex-Offer, relative to a baseline assignment, determined by the issuer. In other words, in a transaction involving flexibility described by FlexOffers, the issuer offers *the deviation in energy consumption (or output) relative to a reference plan.* This simplification allows specifying flexibility-enabling products.

To illustrate the point, consider figure 10, which depicts a four-sliced positive FlexOffer with time flexibility only and assume that the profile shown in the picture is the baseline assignment.[19] Establishing a baseline assignment for a FlexOffer like this results in a *time-shifting* product. Given a FlexOffer like this, the final user is able to "place" the profile (i.e. schedule and dispatch) anywhere between the earliest start time and the latest start time. In Delta Energy terms, this is equivalent to a reduction (or increase) of energy consumption at each of the time slices composing the profile, which is matched by an amount-equivalent increase (or reduction) between the earliest and latest start times.

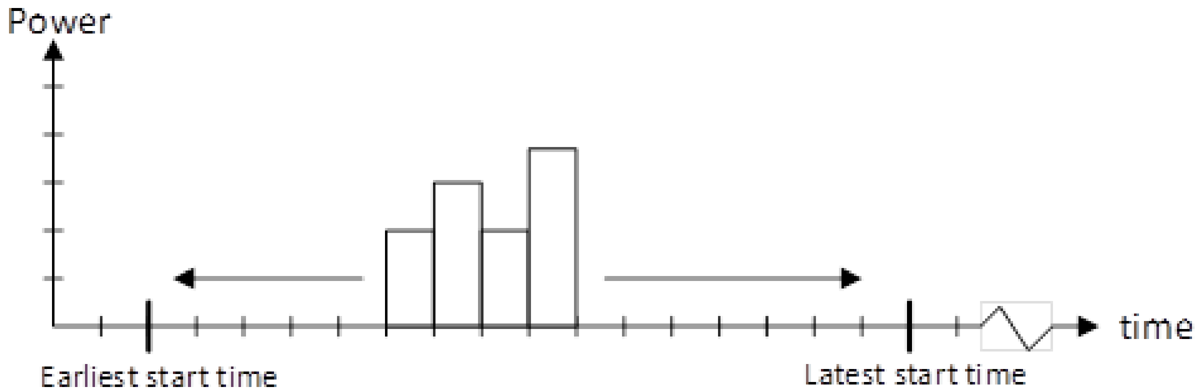

Figure 10: Flex-Offer with time flexibility. Source: TotalFlex internal documents

Another flexibility-enabling product that can be derived from FlexOffers is *quantity flexibility*, as in figure 11, which shows a positive FlexOffer with quantity flexibility only.[20] Note that there can be negative or downward flexibility (the blue portion) relative to a baseline assignment (the yellow line), which is a reduction in consumption. Alternatively, there can also be positive or upward flexibility (the red portion), which is an increase in consumption.

[19] The baseline model of demand-side flexibility trading of Bogetoft et al. (2016) analyses a similar setting to the one described by a time-flexible FlexOffer, but focuses on a one-sliced FlexOffer.

[20] I have illustrated FlexOffers with time flexibility only and with quantity flexibility alone, but other FlexOffers with both kinds of flexibilities can also supply Delta Energy. To keep the discussion simple, I do not discuss these cases.

In Delta Energy terms, the issuer determines the baseload and offers the upward and downward flexibility to the user of flexibility. The latter has the right to schedule and dispatch an instance that lies within either the blue or red portion.

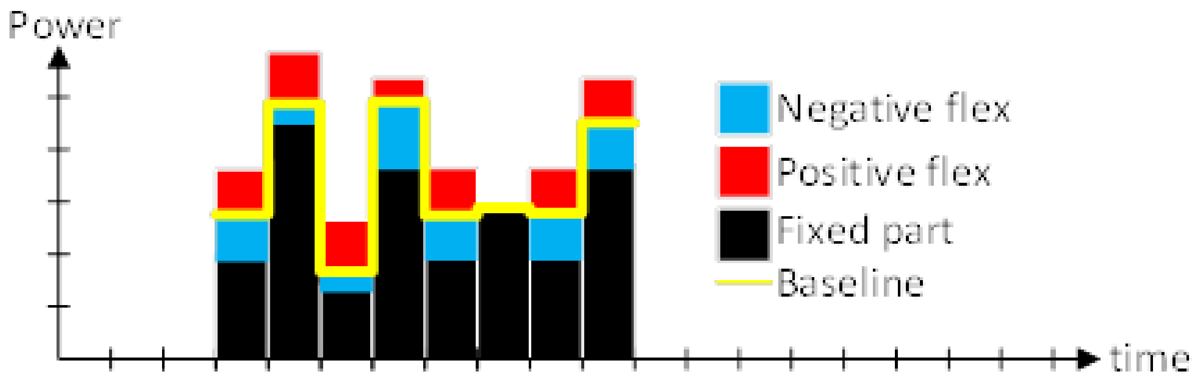

Figure 11: FlexOffer with quantity flexibility. Source: TotalFlex internal documents

Clearly, differing comfort levels associated to the flexibility expressed in a FlexOffer may exist. Assume that figure 12 shows the FlexOffer associated to a heat pump or thermostat user. According to comfort level 1 (the solid upward and downward areas), the user offers less flexibility, whereas comfort level 2 (the hatched areas) indicates greater flexibility. The issuer of this FlexOffer may wish to accept the second comfort level in exchange for a higher economic compensation than what would have been required by comfort level 1 [21].

According to the TotalFlex view, FlexOffers can be utilized in a flexibility marketplace where the *Delta Energy is the traded product* as in figure 13. [22]

Buyers of flexibility, in the form of Delta Energy, are DSOs (but also TSOs) who are final users, and Balancing Responsible Parties (BRPs) who trade for portfolio optimization purposes (see figure 13). On the supply side, large-scale suppliers may access the market directly, together with aggregators who act as intermediaries between small-scale suppliers

[21] Similarly, each FlexOffer may represent the degree of flexibility of a specific supplier. The more flexible it is, the lower the disutility it experiences and vice versa. However, the buyer of flexibility need not know the disutility a supplier experiences, which creates an *adverse selection* problem. This relevant economic issue in the provision of flexibility is studied by Boscán and Poudineh (2016b).

[22] The paper by Boscán (2016) describes how a Delta Energy market can be cleared with Product-Mix Exchanges (PMEs).

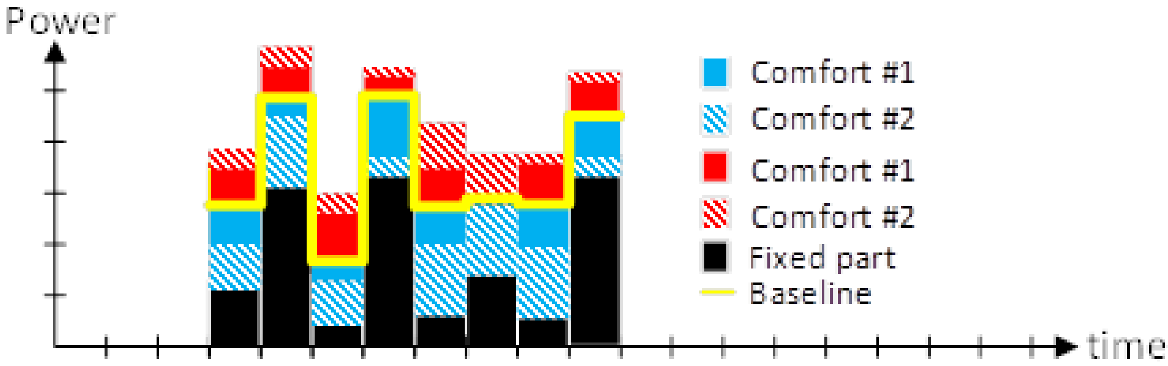

Figure 12: FlexOffer with quantity flexibility and differing comfort levels. Source: TotalFlex internal documents

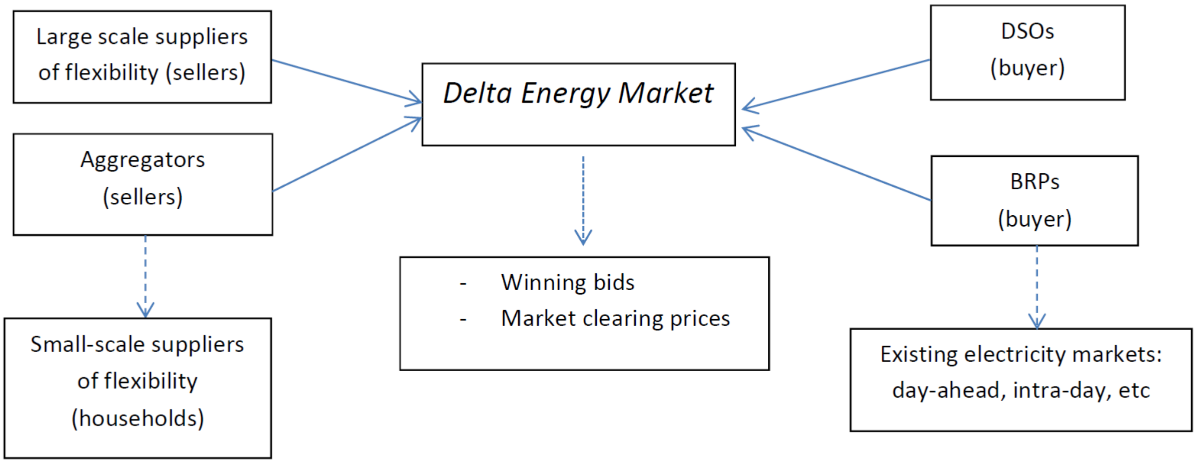

Figure 13: A Delta Energy marketplace. Source: TotalFlex internal documents

and the market.

To conclude, I evaluate the Delta Energy product design according to the categories proposed in section 5:

1. *Simplicity*: Delta Energy is sufficiently simple to be understood by any market participant of an electricity market. It translates flexibility into energy, which is convenient.

2. *Measurability*: As any energy product, it is measured in kWh or MWh.

3. *Relevance*: While FlexOffers provide a sufficiently rich and general way of expressing the flexibility with which an asset can operate, so far it is unclear how do Delta Energy

products fit into the existing market architecture of existing power systems, such as the Nordic one. Open questions such as the timescale with which Delta Energy products will be supplied, the geographical area that will be covered and the specific technical requirements that will be satisfied are crucial to determine its relevance.

6.2 The Californian flexible ramping product

The California Independent System Operator (CAISO) runs a competitive wholesale electricity market, comprised of both day-ahead and real-time processes in which energy, ancillary services and congestion revenue rights are traded. Locational marginal prices are calculated for all nodes in the network.

The day-ahead market opens seven days prior to the trading day and closes one day before gate closure. In addition to energy, capacity is also traded in the form of ancillary services and Residual Unit Commitment (RUC) to meet the gap between the operator's forecasted demand and supply-side economic bids from the day-ahead market. Committed RUC resources are required to submit a bid for energy in the real-time market, which opens as soon as the day-ahead market results are published, and closes seventy-five minutes before the trading hour.

The real-time market, whose main purpose is to procure *balancing energy* to meet instantaneous demand, is composed of the Fifteen-Minute Market (FMM) and the Real-Time Economic Dispatch (RTED) which, respectively, clear every fifteen and every five minutes.[23] Supplemental energy bids, to increase or reduce supply, and committed RUC resources from the day-ahead market are the main inputs to the real-time market. For every run of the FMM and the RTED, the operator optimizes over multiple intervals and dispatches resources over the whole horizon. However, only the initial interval is financially *binding*, while the remaining are *advisory*. As conditions change, the real-time market often dispatches resources differently from the previous runs.

[23]The FFM was implemented by the CAISO in response to Federal Energy Regulatory Commission (FERC) Order No. 764, which required the operator to offer intra-hour transmission scheduling.

Since December 2011, the CAISO has implemented an upward flexible ramping constraint in the FMM to ensure that there is sufficient *upward* ramping capability in the RTED. Although the arrangement has increased the available ramping capability, a limitation is that it assumes that actions that should be taken in the RTED actually aren't. For example, in the pre-dispatch calculations it is assumed that generation will decrease in the current interval to make more ramping capability available in future intervals but this actually doesn't happen. In response to this, it has been decided to replace the existing mechanism (the flexible ramping constraint) with the enhanced Flexible Ramping Product (FRP) to account for *both* upward and downward ramping uncertainty of net load forecasts in the CAISO's multi-interval optimization. (Bushnell et al., 2016). The FRP is defined as:

> "The 5-minute ramping capability, which will be dispatched to meet the 5-minute to 5-minute *net system demand* changes or net system movement in real time dispatch" (CAISO, 2015)

Note in the definition that the target magnitude is net system demand (i.e. net load), defined as "load plus export minus all resources' schedules that are not 5-minute dispatchable, which may include renewable resources, imports and self-schedules" (CAISO, 2015). The FRP comes in two versions: a) procurement of ramping capacity for the forecasted net load ramp, between a financially binding interval and the subsequent advisory interval, and b) procurement of ramping uncertainty. According to the CAISO's management final decision to implement the FRP, the market will award, price, and settle the flexible ramping product in both the FMM and the RTED, and resources that provide the ramping capability will receive a separate payment for this capability. (CAISO, 2016a). To understand the two varieties of the FRP, consider the following two illustrations.

Figure 14 shows the evolution of net system demand and its forecast, together with upper and lower limits, during a binding interval $[t, t+5]$ and an advisory interval $[t+5, t+10]$. Suppose that the time currently is a point *earlier than* time t and that the RTED is being run. From this standpoint, the next interval is binding and the net system demand is considered certain, that is: a random variable that almost surely will follow the shown trajectory (green dashed line) during the interval. Consequently, resources to meet net load are dispatched. In contrast, for the advisory interval, net system demand is a random variable that can take any value between the lower and upper limit. The FRP rewards the *real* ramping need: the potential net demand change between the two intervals. More precisely:

> "If load or supply resources increase the forecast ramp, the market will *charge* the load or supply resource for the flexible ramping product. If load or supply resources decrease the forecasted ramp, the market will *compensate* the load or supply resource" (CAISO, 2016a)

The desired outcome is that load serving entities are incentivized to have a portfolio of dispatchable and non-dispatchable resources able to follow their load profiles.

For the upward ramping requirement, the calculation is:

$$\text{Upward} = \max\{\text{Upper limit}\,[t+5, t+10] - \text{Net system demand in}\,[t, t+5]\,, 0\}$$

and for the downward ramping requirement:

$$\text{Downward} = \max\{\text{Net system demand in}\,[t, t+5] - \text{Lower limit}\,[t+5, t+10]\,, 0\}$$

The spread between the upper and lower limits in the advisory interval reflects the operator's intended coverage but the actual realization of net demand might differ when the advisory interval becomes binding (CAISO, 2015).

As with any forecast, it is uncertain and the CAISO plans to procure a flexible ramping requirement to account for this fact. Figure 15 depicts the forecasted net demand, the realization of the net demand and the lower limit of the forecast. The maximum downward forecast error is the difference between the forecast and the lower limit. If the realized net demand is greater than the lower limit, as in the upper picture of figure 15, the operator procures the difference between the maximum downward net forecast error and the lower limit of the forecast, which corresponds to the region between the green dashed line and the orange dashed line. Otherwise, if the maximum downward forecast error coincides with the difference between the forecast and the lower limit (see the lower picture of figure 15)

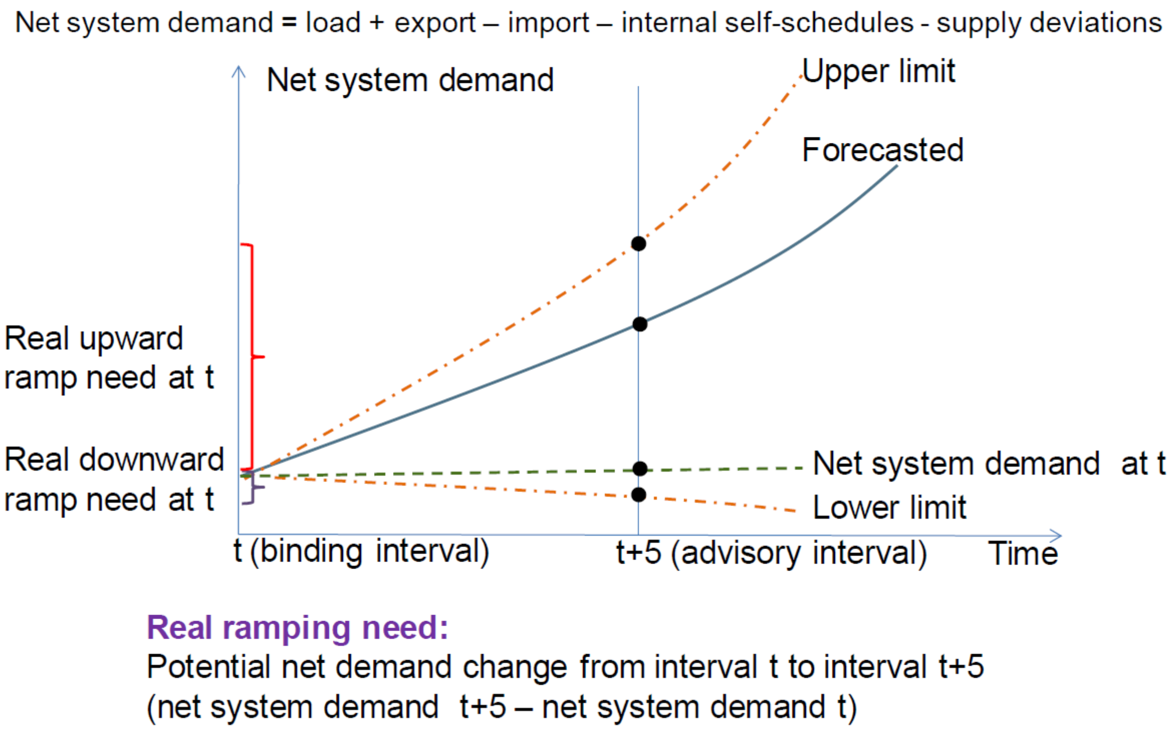

Figure 14: Procurement of real ramping need between an advisory and a binding interval

such that the realized net demand lies below the lower limit, there is no need to procure additional ramping capability.

Regarding settlement of the FRP due to uncertainty, and unlike the FRP between intervals, there cannot be a direct settlement between those requiring ramping capacity and those supplying it. Because the market may not use this capability, it is not possible to attribute it to a specific resource. Instead, it will determine a resource's contribution over month. (CAISO, 2016a)

Two significant differences exist between the FRP and existing capacity products traded in the CAISO markets. First, unlike ancillary services which consists of standby capacity that is dispatched to meet demand deviations *within* an interval, FRP targets system demand changes between intervals. Second, FRP is continuously dispatched through the RTED. (CAISO, 2015)

Regarding the qualitative categories to evaluate the FRP design proposed in section 5, I

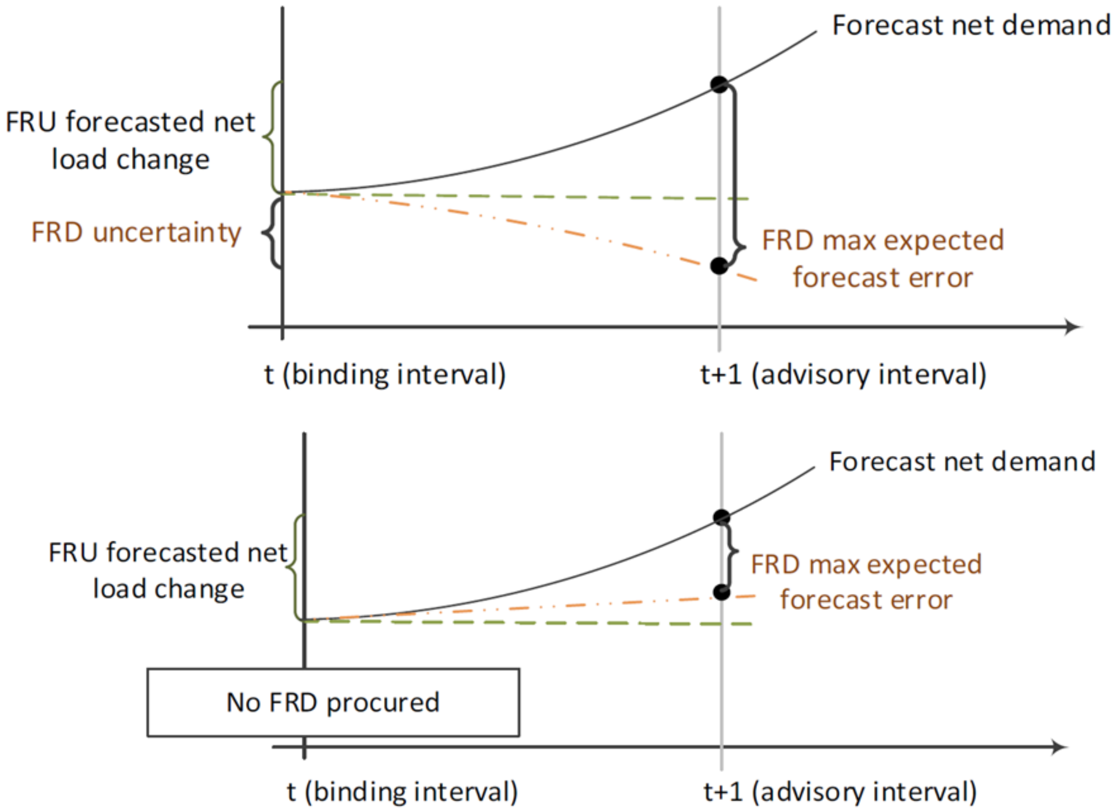

Figure 15: Procurement of FRP due to uncertainty

consider the following:

1. *Simplicity*: The FRP definition is indeed simple because it does set precise requirements for the involved parties and does not require any additional knowledge or skills to be implemented or traded. However, the CAISO market architecture is already complex, as it includes a considerable number of simultaneous processes, which involve optimization problems of large scale and asks bidders to submit a large number of inputs. Although this is technically plausible and in line with the goal of increasing VREs, an open question is if participants are acting strategically or not. That is, it is difficult to establish the kind of incentive issues operating behind CAISO's market mechanisms. As Bushnell (2013) eloquently puts it:

> "What the Morgan story may say something about is the trend in US markets for increasingly complex price-setting processes. For example, the California market today is way more *complex* than the one Enron made infamous 12 years ago. Back then, the California power exchange took bids for price and quantity, but not much else. Today, an offer to sell power from a generation unit can contain all sorts of parameters describing what the plant can and cannot do, from the "ramp rates" to a minimum running time. All these parameters are fed into optimization routines such as mixed integer programs in which the system operator theoretically finds the "best" solution for everyone, taking all these operating constraints as given.
>
> The problem is, the "best" solution when everyone is telling the truth about their costs and capabilities can be very different than the solution when firms are *strategically* bidding those parameters."

2. *Measurability*: The FRP measures the ramp rate in MW/min as is the standard, but because the FRP is traded and priced in an energy market, settlements are made in dollars per MWh.

3. *Relevance*: The very introduction of the ramping constraints and its later enhancement in the form of a FRP clearly shows that the CAISO deems the operational challenges associated to the "duck curve" relevant. Furthermore, the fact that the FRP is traded in CAISO's real-time energy market, a mature market, ensures a sufficiently large number of transactions. However, an open question that requires further (empirical) analysis is if potential suppliers of ramping capability -a form of flexibility - have greater incentives to participate as a result of the introduction of the product. In other words: it is not clear if suppliers of this ramping capability find the opportunity relevant per se.

7 Conclusions

Power system flexibility is an increasingly relevant quality for power systems operating with greater shares of VRE, as these require becoming resilient to the challenge of uncertainty and variability. Engineers have successfully developed a rich literature filled with models that describe what flexibility is and how to measure it but have, so far, left the question of how to *incentivize* its provision largely unaddressed.

However, the question of creating incentives is a topic beyond the realm of the technically-oriented. Instead, economists - armed with an engineer's perspective - are better suited for the task of designing products, contracts and markets -in a word: *institutions*- that enable flexibility in a power system. Informed by the technical features of flexibility, this paper has taken a product design perspective and has contributed to the analysis of flexibility from an *economic* perspective to inform the design of concrete services that can be traded and valued by markets. According to this view, flexibility is a heterogeneous product, which has multiple attributes among which buyers and sellers cannot perfectly substitute.

Another contribution of this paper has been *normative*, namely three simple characteristics that an ideal flexibility-enabling product should have. These are: simplicity, measurability and relevance. Using these as guidelines, I have explored two specific product designs put forward to enable flexibility.

First, I have described FlexOffers, an information systems concept that can express the flexibility of any energy-consuming or supplying appliance in the power system with a great degree of generality. In connection with FlexOffers, I describe a simple way to translate the flexibility contained in any FlexOffer, that is: the Delta Energy or the deviation in energy consumption relative to a baseline. With this, it is straightforward to define flexibility-enabling products such as time shifting and quantity flexibility. I deem the Delta Energy approach as simple and measurable but question its relevance on the grounds of insufficient details regarding the integration with existing market architecture, such as the Nordic.

Second, I review the Flexible Ramping Product put forward by the California Independent System Operator (CAISO), as an enhancement to the existing flexible ramping constraint. This proposal, recently approved by the operator's board of governors and soon to be implemented, seeks to procure and reward ramping capability between intervals in the (5-minute) real-time market as well as ramping capacity to address the uncertainty caused by forecast errors. I consider that the flexible ramping product is an example of good product design in all three categories, but raise questions regarding the overall simplicity of the Californian market design in general.

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Chapter 3

Trading Demand-Side Flexibility in Power Markets

Trading Demand-Side Flexibility in Power Markets

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Abstract

This paper focuses on the particular kind of flexibility that can be harnessed from demand-side resources, as mediated by a technological solution, i.e. Smart Grid, which reduces transaction costs to a negligible level. In contrast to price-based demand response, the bilateral baseline model of flexibility trading of this paper models incentive-based contracts in which consumers are remunerated for their willingness to modify demand. With a Nash bargaining approach, the baseline model of this paper shows that it is possible for an aggregator and a consumer to gain from trading flexibility in single-shot transactions. Taking a long-term perspective, the model shows that agents must be able to trade for a sufficiently long period to cover investment costs. Furthermore, the way in which these are shared determines how favorable the conditions are for consumers. Relative to the case in which costs are symmetrically shared, when the consumer faces a relatively higher cost than the aggregator, the consumer is able to obtain a better deal for its flexibility. Such a finding relates to the possibility of a network effect in the aggregation business, which requires some degree of scale economies in the flexibility-enabling technology. This result speaks about the possibility of introducing competition among aggregators in a potential market for flexibility.

Keywords: Flexibility, Smart Grids, Demand Response

JEL Classification Numbers: L94, C78, L14

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1 Introduction

The need to account for the time and space-varying valuation and cost of electric energy in its pricing and trading has been acknowledged for a long time. With the liberalization of the electricity industry throughout the world and the adoption of market-based arrangements, electricity has come to be understood as a commodity, and price signals tend to reflect its relative scarcity (Joskow, 2008; Joskow and Schmalensee, 1988; Newbery, 1997). While this clearly is a sign of progress that has led to increased economic efficiency, a critical element of any well-designed marketplace, namely "a demand side with varying demands which can adapt to price changes"(Schweppe et al., 1988) has not been successfully implemented so far. A number of technical and economic reasons explain this fact.

From the early days of the electricity industry, supply has traditionally responded to demand requirements but not vice versa. Power plants were usually located close to centers of demand and output - mainly from coal - was swiftly modified in accordance with consumers' requirements. Seen as a fundamental sign of economic progress, end users would rarely reflect on the environmental impact of generating electricity or the fact that its cost could vary. In the modern-day industry, even after the inception of wholesale electricity markets, the majority of residential and commercial end users are charged per kilowatt-hour retail prices that do not reflect wholesale price variations and have their consumption measured through conventional meters, which register aggregate measurements, and pay on the basis of periodical readings, i.e. monthly, bi-monthly, quarterly, and so on (Joskow, 2012). In contrast, because of the impact that energy consumption has on their cost structure, large scale consumers have a greater potential to adapt to the fluctuations in wholesale prices. Indeed, in many places, these customers are metered in real-time and pay prices that more closely reflect the dynamically-varying marginal cost of electricity.

However, the emergence of Smart Grid[1] solutions is becoming a reality and considerable

[1]Despite diverse definitions, Agrell et al. (2013) find consensus in relation to six areas in which Smart Grids operate, namely: (i) optimization of grid operation and utilization, (ii) optimization of grid infrastructure, (iii) integration of decentralized energy resources, (iv) enhanced information and communications

emphasis is being placed by policymakers to put it in place. In the UK, for example, at least 53 million smart meters will be replaced between 2015 and 2020 in homes and small businesses, at a cost of £10.9 billion and expected net benefits of £6.2 billion (DECC, 2014).[2] British customers will thus have the possibility to obtain real time information about their electricity consumption and its cost, but whether they will choose to modify demand accordingly is another matter.

Besides the technical barriers to have an active demand-side in electricity markets, there are economic reasons that prevent small-scale consumers from modifying their behavior, even if there are potential economic gains from reacting to dynamically varying prices, and taking real-time metered electricity consumption for granted. *Transaction costs* associated to monitoring the evolution of price and adjusting energy-consuming appliances might hinder these gains from being fully realized, as Joskow and Tirole (2006) note. Therefore, consumers on real-time meters and dynamically-varying prices can be assumed to be reactive as long as they rationally trade off transaction costs and savings in their electricity bill (Joskow and Tirole, 2007).

But economic efficiency is not the only motive to promote an active demand-side in power markets. The electricity industry throughout the world is undergoing a process of technological change, as decarbonisation has climbed up on the list of priorities of policymakers. As a result, variable renewable energy, such as the generated from wind turbines and solar panels, is becoming more relevant to attain this goal. However, integrating large shares of energy from these sources into the power grid poses challenges that call for designing and adopting innovative *business models* that enable *flexibility* (Boscán and Poudineh, 2016a). That is, the modification of generation injection or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service within the energy

technology, (v) active distribution grid, (vi) new markets and end-user services. This paper adheres to this multi-faceted view of the Smart Grid.

[2]In contrast, using French data, Léautier (2014) estimates the yearly surplus of high-elasticity residential consumers at less than 1€, and of high elasticity households at 4€. This view challenges the benefits of mandating the roll-out of smart meters.

system (EURELECTRIC, 2014). As power systems increase their flexibility, they become more resilient to the uncertainty posed by the inherent variability of renewables and are able to integrate them more efficiently.[3]

In light of the greater need for flexibility to integrate renewables into power systems, this paper focuses on the particular kind of flexibility that can be harnessed from demand-side resources, particularly from small and medium scale consumers of electric energy. In addition, the paper studies the aggregation business model, which consists on pooling loads from consumers or output from de-centralized generation assets to provide energy and services to actors within the system (EURELECTRIC, 2014).

Also, because of its relevance for the feasibility and consolidation of the aggregation business model, the paper studies the role of the Smart Grid as a catalyst for demand-side flexibility. Through its progress in applications like lightning, temperature control, and smart appliance development, the home automation industry - a key element of the Smart Grid - is on its way to reducing the transaction costs associated to modifying consumption behavior. Consumers now have the possibility of revealing preferences for electricity-consuming services and activities, while relying on technology to adjust demand. Furthermore, if two-way communication capabilities exist, dishwashing, laundry-making, heating, among other activities, can be remotely controlled, according to the requirements of a third party, e.g. an aggregator.

More specifically, the paper tries to answer the following two questions: *what is gained from trading demand-side flexibility? what is required from an aggregator and a consumer to gain from trading it?*.

In contrast to price-based demand response, the bilateral baseline model of flexibility trading of this paper models incentive-based contracts in which consumers are remunerated for their willingness to modify demand (Cooke, 2011; DOE, 2006). The proposed framework encompasses specific instances of these agreements, such as direct load control programs,

[3]For a review of the current debate on power system flexibility and its economic properties, the reader is referred to Boscán (2016).

interruptible supply contracts and dynamic load capping contracts, as described by He et al. (2013). The model also applies to the Delta Energy-based agreements for time shifting and quantity flexibility described by Boscán (2016).

The model that this paper proposes relates to the literatures on power system flexibility, renewable energy integration, demand response and the Smart Grid, which are mostly engineering-oriented, e.g. the work by Morales et al. (2013),Kiviluoma (2013) Silva (2010), Nicolosi (2012). Many of these studies evaluate policies under alternative scenarios, using optimization models from a centralized energy planner's perspective and, therefore, take on a *systemic* view.

In contrast, this paper takes a *microeconomic* perspective to analyze the incentives to trade flexibility. Because of the bilateral Nash (1950) bargaining approach utilized, it relates generally to the vast body of theoretical and applied work that has followed from Nash's seminal contribution, e.g. Osborne and Rubinstein (1990), Muthoo (1999). For example, the model by Horn and Wolinsky (1988) relates to ours because, like them, we focus on "relations between suppliers and buyers that are often characterized by elements of bilateral monopoly" which may arise when "the supplier's product is an intermediate good that is specially tailored to the needs of the buyer". These elements are key in the vision of demand-side flexibility trading that this paper develops. First, the main inputs for an aggregator to produce flexibility that can be valued by a final user are disaggregated loads. Second, competition among suppliers is typically infeasible due to transaction costs and scale issues, i.e. a single household may be too small to participate in a marketplace. Because of their focus on contractual agreements from a bargaining perspective, the papers by Lippman et al. (2013) and Yu et al. (2012) are also examples of related work. By assuming a partial equilibrium approach as the model of this paper does, light is shed into the market structure of the aggregation business and, in this way, a general link with the industrial organization literature (Tirole, 1988) is established.

The paper is structured as follows. Section 2 describes the assumptions of the baseline model of flexibility trading, explains the aggregator's delivery policy and presents two simple but fundamental results, namely the price of flexibility in single-shot transactions, and a proof that it is possible for an aggregator and a consumer to gain from trading flexibility. Given that the set of assumptions on which these conclusions depend are reasonable but restrictive, section 3 modifies them to validate the generality of the claims. The key message of the section is that under less restrictive assumptions, such as a non-unitary load and the aggregator's role being fulfilled by a retailer, single-shot transactions of flexibility lead to gains from trade. In contrast to the previous sections, section 4 takes a long-term perspective and analyzes the role of investment costs. The claim that flexibility trading is welfare enhancing is based on the assumption that a transaction cost reducing technology, i.e. a Smart Grid solution, is in place. But the question of who pays for this technology remains unaddressed up to this point. Therefore, the results from section 4 present a more nuanced vision of flexibility trading. That is: flexibility trading is welfare enhancing as long as the investment cost is sufficiently low and consumer and aggregator are able to trade for a sufficiently long period of time. Moreover, a consumer can obtain a better deal if it trades flexibility with an aggregator of a certain scale, such that a kind of *positive network effect* operates. [4] Section 5 presents concluding remarks.

[4] Weiller and Pollitt (2014) suggest that "the electricity supply can be conceived of as a platform-mediated, two-sided market" which may stimulate competition and innovation. The network effect view of the aggregation business we suggest relates to theirs.

2 Baseline model: time-shifting of an indivisible unit load

2.1 Assumptions

The baseline model of flexibility trading satisfies the following assumptions:

BL1: There is one consumer (C).

Small, medium enterprises and households are naturally thought of as consumers of electric energy, who can take the role of itsuppliers of demand-side flexibility if incentivized to shift demand in time or to modify the amount of energy they consume.[5]

BL2: Part of C's demand for electric energy is inherently inflexible, while another part of it is potentially itflexible. With regard to the latter, C is indifferent concerning the time n of delivery as well as the exact amount of energy that it will consume at each moment as long as a desired final outcome (e.g. laundry is done, vehicle is charged) is achieved within a itflexibility time frame $\mathcal{F} = [1, \ldots, N]$, i.e., insofar as $n \in \mathcal{F}$, $N > 1$. Note that if $N = 1$, C does not offer any flexibility, i.e. $\mathcal{F} = [1]$.

Inflexible demand is associated with appliances whose usage cannot be modified, unless C experiences disutility as a result of interrupting the service associated with it. In contrast, flexible demand is related to appliances for which C is capable of modifying its usage without experiencing loss of utility. Matching our definitions of flexible and inflexible demand to the categorization of loads proposed by He et al. (2013), we find that inflexible loads are *non-shiftable* (e.g. computers, TV sets, lightning). In contrast, flexible demand is either *storable* or *shiftable*. The former is characterized by a decoupling of power consumption and end

[5]Provided that the adequate regulatory framework and technology are in place, consumers can also take the role of producers if they own generation facilities and sell energy to the system, in which case they would be adequately called itprosumers, a portmanteau word that subsumes the roles of consumer and producer into a single economic agent. However, this paper focuses on the particular form of flexibility that arises from modifying consumption behavior.

use, typically mediated by battery storage or thermal inertia (e.g. electric vehicles, heating), while the latter is associated with non-interruptible processes that can be shifted in time (e.g. laundry-making, dish-washing).

More fundamentally, BL2 states that the consumer is risk-neutral regarding the time and exact amount of energy consumed. Furthermore, the model assumes that the demand for energy is *derived*: C does not obtain utility from the energy it uses but from the goods and services associated with its delivery, such as transportation, heating or entertainment (see Hausman (1979) for more on this assumption).

BL3: There is one aggregator (A) to whom C gives control of the delivery of the flexible load. That is, both parties can enter a contract in which C lets A decide on the time of delivery n, provided that this is chosen from the flexibility time frame $\mathcal{F}$ that C has specified. If A gains control over the flexible load, C pays A a price p per kWh of energy it consumes.

Note that BL3 refers to a contract for a one-time delivery of the load. Extensions to the baseline model that relax this assumption and consider several deliveries over longer periods of time are analyzed in section 4.

Moreover: despite the fact that retailers, not aggregators, are responsible for delivering electricity to consumers, BL3 states otherwise for analytical simplicity and to highlight the gains from trading flexibility experienced by the consumer through savings in its electricity bill. This is equivalent to assuming that p is the price of electric energy that C consumes minus the flexibility remuneration that it receives from A. Figure 1 illustrates the transactions in the single-shot contract for flexibility between A and C (solid lines reflect a flow of indivisible commodities, while the dashed line reflects a flow of money).

BL4: C has a contract for delivery of electricity with a retailer (R) which establishes a known, fixed price $\overline{p}$ per kWh of inflexible demand.

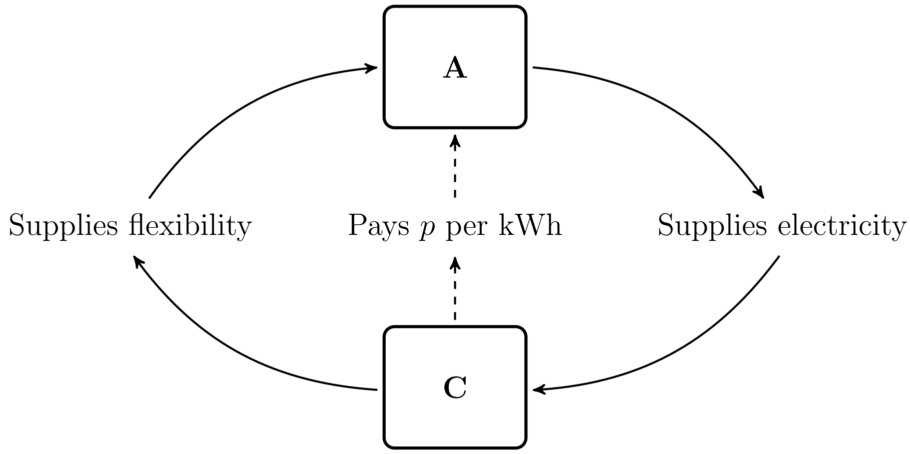

Figure 1: Transactions in the single-shot contract between A and C

The price $\overline{p}$ that C pays for electricity illustrates C's outside option, that is, the price that it would pay for energy if it kept flexibility to itself. Under the retail contract, C unilaterally decides *when* to consume energy and *how much* of it. In summary, C faces a choice between remaining as a conventional, inflexible consumer and becoming a supplier of flexibility. In line with BL3, and for comparison purposes, BL4 refers to a one-time delivery of the inflexible load. Figure 2 illustrates the transactions in the contract between retailer and consumer (as in figure 1, a solid line indicates a commodity flow, while the dashed line indicates a flow of money).

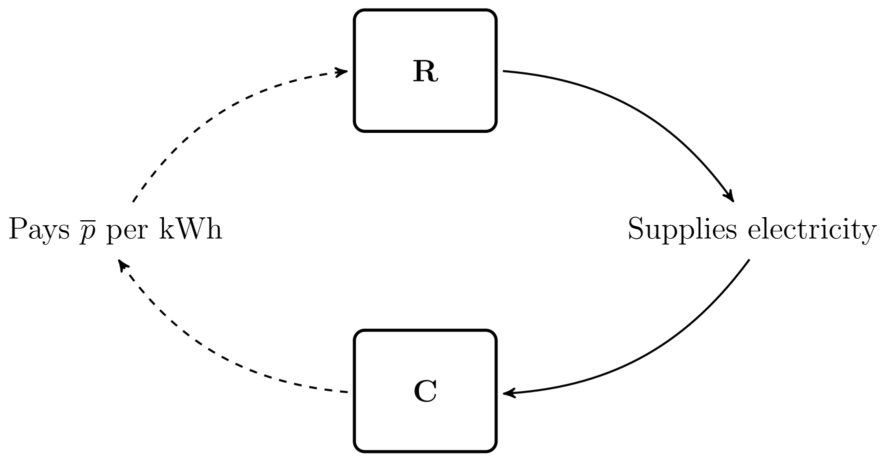

Figure 2: Transactions in the single-shot contract between R and C

BL5: C's load, flexible and flexible, is indivisible and equals one kWh.

BL6: Both parties bargain à la Nash (1950) over the value of p and have symmetric bargaining power.

Besides its simplicity, the bargaining approach emphasizes the incentives of both parties to cooperate: by agreeing to trade flexibility, consumer and aggregator can be better off.

BL7: A's activity is independent of R's.

Implicit in this assumption is that the flexible load and the inflexible load are two distinct products which give rise to two separate economic activities. In the flexible load business, the aggregator buys flexibility and makes a profit from managing risk. In the inflexible load business - which is exogenous to the model - the consumer is the passive end of the traditional electricity supply chain. Note, however, that nothing prevents a conventional retailer from entering into a different but related line of business: that of flexible loads; a development that is actually happening in the electricity industry [6] and is analyzed in subsection 3.2, as an extension to the baseline model.

BL8: The wholesale price w_n per kWh of electricity, at which A and R buy the load, is a sequence of independently and identically distributed (iid) random variables, which follow a uniform distribution between zero and one. That is to say, $w_n \sim U(0, 1), n \in \mathcal{F}$.

BL9: There is a Smart Grid solution in place that reduces the transaction costs associated to adjusting consumption and delivering the load to a negligible level. Neither A nor C are liable to any costs associated to the deployment of this technology.

In the absence of such a technological solution, to be flexible, the consumer would have to cover transaction costs associated to monitoring the evolution of price and adjusting demand, and the aggregator would be unable to automatically control delivery.

[6] Boscán and Poudineh (2016a) cite as an example the fact that NRG, a US utility which owns 50 GW in fossil-fuel dominated generation assets, acquired in 2013 an aggregator with a 2 GW portfolio.

Before ending this subsection, as a reality check, consider the following view of the aggregation business by EURELECTRIC (2014) :

> "Aggregation is a commercial function of pooling de-centralized generation and/or consumption to provide *energy* and services to actors within the system. Aggregators can be *retailers* or *third parties*. They may act as an intermediary between customers who provide flexibility (both demand and generation) and procurers of this flexibility. They would identify and gather customer flexibilities and intermediate their joint participation. This could be done via flexibility products or simply by *selling and buying aggregated energy* (kilowatt-hours) *at optimal points in time*"

2.2 Aggregator's Delivery Policy

Suppose that C and A have bargained over the value of p, given a flexibility time frame $\mathcal{F}$ specified by C, and have thus agreed on the terms of the contract for delivery of the flexible load, which equals one kWh. Accordingly, C has given A the option to decide on the time n of delivery. The aggregator, who is obliged to deliver within the time frame, bears all the price risk resulting from variations in w_n and has the incentive to deliver it when conditions are most favorable. A's problem is to choose, over a discrete set of periods, the best moment to deliver a load with a stochastically varying price. Technically, this is an optimal stopping problem for which dynamic programming gives a simple structure and solution. See, e.g., Lindley (1961) for a theoretical discussion on the topic and an introduction to the "marriage problem" of which the following results are instances.

Let $ED_n(N)$ be the expected cost of delivery at the beginning of period n in a flexibility time frame $\mathcal{F}$ of length N.

At $n = N$, the last period in the time frame, the aggregator cannot postpone delivery unless it breaches the agreement. Therefore, his expected cost of delivery is:

$$ED_N(N) = E(w_N)$$

At time $n = N - 1$, the aggregator has two opportunities to deliver: now (at $N - 1$) or later (at N). Consequently, A observes w_{N-1} and determines if it is favorable to postpone or not. If $w_{N-1} < ED_N(N)$, A prefers to deliver now. If $w_{N-1} = ED_N(N)$, it is indifferent between the two periods, but if $w_{N-1} > ED_N(N)$, it will postpone.

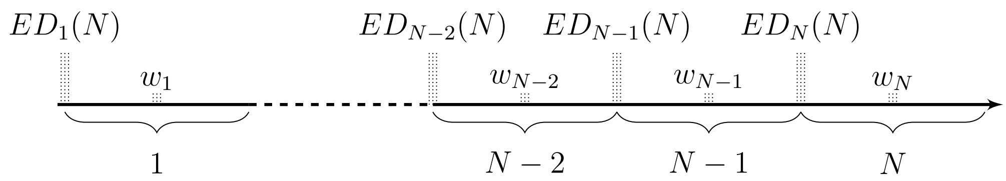

Figure 3: Aggregator's delivery policy with a flexibility time frame $\mathcal{F}$ of length N

The expected cost of delivery at $n = N - 1$ is:

$$\begin{aligned} ED_{N-1}(N) = {} & E(w_{N-1} \mid w_{N-1} \leq ED_N(N))Pr(w_{N-1} \leq ED_N(N)) \\ & + E(w_N \mid w_{N-1} > ED_N(N))Pr(w_{N-1} > ED_N(N)) \end{aligned}$$

where $E(w_{N-1} \mid w_{N-1} \leq ED_N(N))$ is the expected value of w_{N-1}, conditional on w_{N-1} being less or equal than $ED_N(N)$, in which case it is best to deliver before N.

If, on the contrary, w_{N-1} is greater than $ED_N(N)$, then A expects to pay for the wholesale price w_N, conditioned on the event that w_{N-1} is greater than $ED_N(N)$, i.e., $E(w_N \mid w_{N-1} > ED_N(N))$.

In the previous expression, $Pr(w_{N-1} \leq ED_N(N))$ is the probability that w_{N-1} is less than or equal to $ED_N(N)$ and $Pr(w_{N-1} > ED_N(N))$ is its complement.

From the expression for $n = N - 1$, applying recursion, it is straightforward to derive the expression for $n = N - 2$:

$$\begin{aligned} ED_{N-2}(N) = {} & E(w_{N-2} \mid w_{N-2} \leq ED_{N-1}(N))Pr(w_{N-2} \leq ED_{N-1}(N)) \\ & + E(w_{N-1} \mid w_{N-2} > ED_{N-1}(N))Pr(w_{N-2} > ED_{N-1}) \end{aligned}$$

Similarly for $N - 3$, $N - 4$, and so on. Note the recursive structure of the decision in which the expected cost of delivery at n is conditional on the expected cost of delivery at $n + 1$.

The following formalizes the argument:

Proposition 1 *Given the flexibility time frame $\mathcal{F} = [1, \ldots, N]$, $N > 1$, specified by C, the optimal delivery policy for A is:*

- *At $n = N$, deliver and face expected cost of delivery $ED_N(N) = E(w_N)$.*
- *At $n \in \mathcal{F}$ such that $n \in [1, \ldots, N-1]$:*

$$\text{If } w_n \leq ED_{n+1}(N) \implies \text{deliver}$$
$$\text{If } w_n > ED_{n+1}(N) \implies \text{postpone}$$

The expected cost of delivery at n is:

$$\begin{aligned} ED_n(N) &= \int_0^1 \min\{w_n, ED_{n+1}(N)\}\, \mathrm{d}w_n \\ &= \frac{2ED_{n+1}(N) - (ED_{n+1}(N))^2}{2} \\ &= ED_{n+1}(N) - \frac{(ED_{n+1}(N))^2}{2} \end{aligned} \tag{1}$$

A consequence of this delivery policy is that as flexibility increases, the expected cost of delivery decreases. More precisely, fix $n = 1$ to define an instance of $ED_n(N)$, i.e. $ED_1(N)$, which is the expected *expected cost of delivery at period 1 in any time frame $\mathcal{F}$ when its length is N*. In the sequel, we shall refer to this magnitude as the expected unitary cost of delivery with N periods of flexibility.

Thus, without flexibility, i.e. when $\mathcal{F} = [1]$, $N = 1$:

$$ED_1(1) = E(w_1) = \frac{1}{2}$$

When there are two periods of flexibility, $\mathcal{F} = [1, 2]$, $N = 2$:

$$ED_2(2) = E(w_2) = \frac{1}{2}$$
$$ED_1(2) = ED_2(2) - \frac{(ED_2(2))^2}{2} = \frac{3}{8}, \quad \text{from equation (1)}$$

With three periods of flexibility, $\mathcal{F} = [1, 2, 3]$:

$$\begin{aligned}
ED_3(3) &= E(w_3) = \frac{1}{2} \\
ED_2(3) &= ED_3(3) - \frac{(ED_3(3))^2}{2} = \frac{3}{8} \\
ED_1(3) &= ED_2(3) - \frac{(ED_2(3))^2}{2} = \frac{39}{128}
\end{aligned}$$

where $ED_2(3)$ and $ED_1(3)$ follow from equation (1)

Note, more generally, that as the length N of $\mathcal{F}$ increases, $ED_1(N)$ decreases:

$$\begin{aligned}
ED_1(1) &= E(w_N) = \frac{1}{2} \\
ED_1(2) &= ED_1(1) - \frac{(ED_1(1))^2}{2} = \frac{3}{8} \\
ED_1(3) &= ED_1(2) - \frac{(ED_1(2))^2}{2} = \frac{39}{128} \\
\vdots \qquad & \qquad \vdots \\
ED_1(N) &= ED_1(N-1) - \frac{(ED_1(N-1))^2}{2}
\end{aligned}$$

which defines a sequence $\{ED_1(N)\}_{N \geq 1}$.

Proposition 2 *The sequence* $\{ED_1(N)\}_{N \geq 1}$ *is monotonically decreasing and convergent to zero.*

Figure 4 summarizes the intuition behind the aggregator's delivery policy: as the flexibility offered by the consumer increases, the expected cost of delivery faced by the aggregator decreases.

An interesting feature of this delivery policy is that deciding on myopic information is almost as good as having perfect information. To see this, suppose that A knew all the prices $w_1, \ldots, w_N$ that will actually happen during a flexibility time frame $\mathcal{F}$ of length N and, thus, it could rank them from lowest to highest, assigning each of them the number associated with its rank, i.e. 1 corresponds to the lowest, 2 corresponds to the second lowest, and so on. The N prices divide the unit interval $[0, 1]$ in sub-intervals of average length $\frac{1}{N+1}$, i.e.

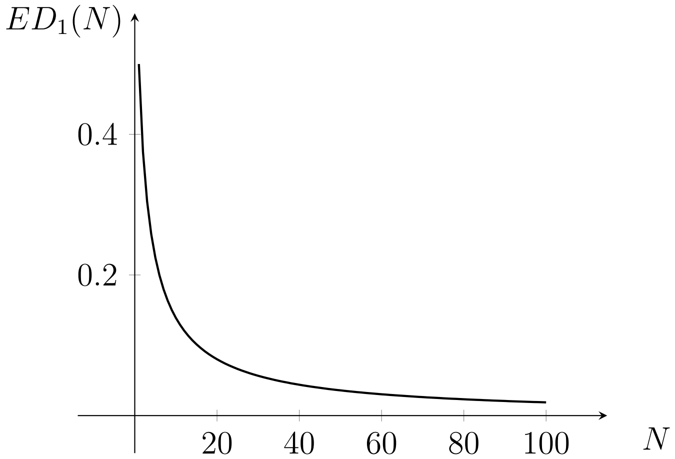

Figure 4: Expected cost of delivery at $n = 1$ with N periods of flexibility $(ED_1(N))$

each price has an expected value of $\frac{1}{N+1}$. In particular, the second lowest price has expected value $\frac{2}{N+1}$. Simple numerical examples show that as N increases:

$$ED_1(N) \approx \frac{2}{N+1}$$

As an illustration, note that if $N = 70$, $ED_1(70) = 0.026$ and $\frac{2}{71} = 0.028$. In summary, the delivery policy is almost expost efficient.

2.3 Single-shot Contracts

With the setup established so far, this subsection describes the terms of the single-shot contract between A and C and its welfare implications.

On the demand side, C can buy the load from R and choose to remain as a conventional customer who doesn't act flexibly. In this case, his payoff is $\overline{U}_C$. Otherwise, if C decides to be flexible, his payoff is $\widetilde{U}_C$:

$$U_C = \begin{cases} v - p \equiv \widetilde{U}_C & \text{if } C \text{ acts flexibly} \\ v - \overline{p} \equiv \overline{U}_C & \text{if } C \text{ does not act flexibly} \end{cases} \tag{2}$$

where v is the value that C obtains from consuming energy. For our analysis, we assume that $v > p, \overline{p}$.

On the supply side, A must procure the load it will deliver to C at the wholesale price w_n. If it acquires the right to control delivery of the load, it faces $ED_1(N)$: the unitary cost of delivery with N periods of flexibility $ED_1(N)$. If it doesn't, A remains out of business and makes a profit of zero. Hence, A's payoff is:

$$U_A = \begin{cases} p - ED_1(N) \equiv \widetilde{U}_A & \text{if } A \text{ gains control} \\ 0 \equiv \overline{U}_A & \text{if } A \text{ does not gain control} \end{cases} \tag{3}$$

Note that the problem is of interest if $v > ED_1(N)$. The price of the contract is the solution to a Nash bargaining problem:

Proposition 3 *Under assumptions $BL1 - BL9$, the single-shot contract between C and A has a price:*

$$p = \frac{\overline{p} + ED_1(N)}{2} \tag{4}$$

per kWh

The result of Proposition 3 is appealing because p stands at the midpoint between the retail price $\overline{p}$ and A's expected unitary cost of delivery $ED_1(N)$ with N periods of flexibility. Hence, C and A are able to gain from trading flexibility.

In addition, the more flexible C is, as measured by the length N of $\mathcal{F}$, the lower the price p is and the higher total welfare is. In simple terms, as Proposition 4 shows, the "size of the pie" can be made bigger if the two parties (C and A) cooperate.

Proposition 4 *Under assumptions $BL1-BL9$, C and A enter a single-shot contract leading to total welfare:*

$$TW_{flex} = v - ED_1(N)$$

which is higher than total welfare under the alternative of not entering the contract:

$$TW = v - ED_1(1)$$

3 Extensions to the baseline model

The baseline model presented in section 2 allows formalizing the claim that A and C can be better off by trading flexibility which, in consequence, increases total welfare. However, the statement relies on a set of reasonable but restrictive assumptions. First, the baseline model assumes that A and C have symmetric bargaining power. Second, A and R are assumed to be two independent entities. Third, it was assumed that the load is indivisible and equal to one. This section investigates the generality of the results of section 2 by relaxing some of the assumptions on which they are based.

3.1 Asymmetric bargaining

According to $BL6$, A and C have symmetric bargaining power. Instead, assume now that either A or C are in a position to make an ultimatum offer, which would be the case if there were one aggregator and several identical consumers or, vice versa, many aggregators and one consumer. The agreed price differs from the result of Proposition 3 and total welfare is lower:

Proposition 5 *Let ϵ be an arbitrarily small positive number:*

1. *If A makes an ultimatum offer to C, it proposes:*

$$p = \overline{p} - \epsilon \tag{5}$$

such that $U_C = \epsilon$ and $U_A = \overline{p} - \epsilon - ED_1(N)$

2. Similarly, if C makes a take-it-or-leave-it offer to A, it suggests:

$$p = ED_1(N) + \epsilon \tag{6}$$

such that $U_A = \epsilon$ and $U_C = \overline{p} - \epsilon - ED_1(N)$.

In both cases, the contract between A and C leads to total welfare:

$$TW_{flex} = \overline{p} - ED_1(N)$$

which is lower than $v - ED_1(N)$ because $v > p, \overline{p}$.

3.2 Role of A is fulfilled by R

According to assumption $BL7$, A's role is independent of R's. Alternatively, in this subsection let us assume that A's role is fulfilled by R with whom C has a contract. In consequence, A's outside option is to deliver the load as a retailer. Thus, instead of equation (7), A's payoff is:

$$U_A = \begin{cases} p - ED_1(N) \equiv \widetilde{U}_A & \text{if } A \text{ gains control} \\ \overline{p} - ED_1(1) \equiv \overline{U}_A & \text{if } A \text{ does not gain control} \end{cases} \tag{7}$$

Thus, if A acquires control of the flexible load, it gets paid the lower p, and faces the lower $ED_1(N)$. In contrast, as a retailer, A receives the higher $\overline{p}$ but faces the higher $ED_1(1)$.

Proposition 6 *If A's role is fulfilled by R, the price of the contract between A and C is:*

$$p = \overline{p} + \frac{ED_1(N) - ED_1(1)}{2} \tag{8}$$

or equivalently

$$p = \frac{\overline{p} + ED_1(N)}{2} + \frac{\overline{p} - ED_1(1)}{2} \tag{9}$$

Re-arranging terms, we obtain the following expression:

$$\overline{p} - p = \frac{ED_1(1) - ED_1(N)}{2}$$

From Proposition 2, which established that $ED_1(1) > ED_1(N)$, it follows that $p < \overline{p}$ and a profitable deal to both parties can be reached.

Akin to the result with an independent aggregator, as the number of flexibility periods (N) offered by the consumer increases, the price becomes lower. Unlike the asymmetric bargaining assumption considered in section 3.1, total welfare remains unchanged when the role of aggregator is fulfilled by the retailer.

Corollary 1 *If A's role is fulfilled by R, Proposition 4 holds.*

Nonetheless, because A can charge now a higher p for the flexible load, the division of surplus changes to his advantage. Note that the first term on the right-hand side of equation (9) is equation (4). In consequence, it is clear that:

$$p = \frac{\overline{p} + ED_1(N)}{2} + \frac{\overline{p} - ED_1(1)}{2} > \frac{\overline{p} + ED_N(N)}{2}$$

This result relates to the presence of economies of scope: as R participates in a distinct but related economic activity, its bargaining power increases, which works to the disadvantage of C.

3.3 Size of flexible load depends on p

According to BL5, the baseline model has assumed that the flexible load that C consumes is indivisible and equals one kWh. Furthermore, no consideration about the price elasticity of flexible demand has been made. Instead, it is now assumed that C has a downward-sloping demand curve for the flexible load:

$$q = max\left[0, 1 - p\right] \tag{10}$$

and

$$\overline{q} = max\,[0, 1 - \overline{p}] \tag{11}$$

for the inflexible load. This allows delving into the implications of a load different from unity, which may be divisible, and a price-elastic demand.

In addition, we resort to consumer surplus $CS(\cdot)$ and producer surplus $PS(\cdot)$ as adequate measures of welfare.

For C:

$$\begin{aligned} CS(p) &= \frac{(1-p)^2}{2}, \quad \text{if } C \text{ pays } p \text{ for the flexible load } q \\ CS(\overline{p}) &= \frac{(1-\overline{p})^2}{2}, \quad \text{if } C \text{ pays } \overline{p} \text{ for the inflexible load } \overline{q} \end{aligned}$$

For A:

$$PS(p) = (p - ED_1(N))(1-p), \quad \text{if } A \text{ is paid } p \text{ for the flexible load } q$$

Therefore, C's payoff is:

$$U_C = \begin{cases} \frac{(1-p)^2}{2} \equiv \widetilde{U}_C & \text{if } C \text{ is flexible} \\ \frac{(1-\overline{p})^2}{2} \equiv \overline{U}_C & \text{if } C \text{ is not flexible} \end{cases} \tag{12}$$

Similarly, the aggregator's payoff is:

$$U_A = \begin{cases} (p - ED_1(N))(1-p) \equiv \widetilde{U}_A & \text{if } A \text{ gains control over the flexible load } q \\ 0 \equiv \overline{U}_A & \text{if } A \text{ does not gain control over } q \end{cases} \tag{13}$$

Unlike for the baseline model and extensions already studied, a closed-form solution for p was not obtained. Instead, from the following expression for p as a function of $\overline{p}$ and the expected unitary cost of delivery $ED_1(N)$:

$$(1-p)^2 = (1-\overline{p})^2 \left[\frac{1-2p+ED_1(N)}{1-4p+3ED_1(N)}\right] \tag{14}$$

it is possible to show that the same results of section 2 hold qualitatively [7].

Specifically, bargaining between A and C leads to a price p that is always below the price $\overline{p}$ that R would charge as a pure monopolist. Moreover, p decreases as the length N of the flexibility time frame $\mathcal{F}$ increases. These results are summarized in the following:

Proposition 7 *Under the assumption of a downward-sloping demand curve for C given by $q = max\,[0, 1-p]$, the price p agreed by A and C:*

1. *Is such that $CS(\overline{p}) < CS(p)$ for $p < \frac{1+ED_1(N)}{2}$*

2. *Is a decreasing function of $ED_1(N)$*

Given that a closed form solution for p was not obtained, the condition that $p < \frac{1+ED_1(N)}{2} = p_m$ in Proposition 7 sets a benchmark price to illustrate the point that trading flexibility always leads to higher consumer welfare, relative to C's outside option to remain as conventional, inflexible consumer of electricity. Note that p_m is the price that A would set as a pure monopolist (i.e. without bargaining). In addition, if $N = 1$ and R were a pure monopolist[8], then $p_m = \overline{p}$ because the retailer delivers the load without any flexibility.

To illustrate the results obtained in this subsection, we estimated the roots of the cubic polynomial from the first order condition of the Nash bargaining problem (see equation 22 in the Appendix). Specifically, table 1 reports numerically obtained values of p for given pairs of expected unitary cost of delivery with $N = 2, \ldots, 5$ periods of flexibility, i.e. for $ED_1(2), \ldots, ED_1(5)$, and retail prices $\overline{p} = \{0.75, 0.70, 0.65\}$. Note that $\overline{p} = \frac{1+ED_1(1)}{1} = 0.75$ corresponds to the monopoly price that R would charge: the highest possible price. For the sake of completeness, we also use values of $\overline{p}$ below the monopoly level in our estimations.

[7]For details of the derivation of equation 14, see the Appendix

[8]An admittedly strong assumption which, nonetheless, serves well for illustrative purposes.

$\overline{p}$	$ED_1(2) = 0.38$	$ED_1(3) = 0.30$	$ED_1(4) = 0.26$	$ED_1(5) = 0.22$
0.75	0.51	0.46	0.42	0.40
0.70	0.50	0.45	0.42	0.39
0.65	0.48	0.44	0.40	0.38

Table 1: Numerical values of p for fixed $\overline{p}$ and $ED_1(2), \ldots, ED_1(5)$. Independent A.

As stated in Proposition 7, $p < \overline{p}$ and, as more flexibility is offered by C, the agreed price p becomes lower. For example: given a retail price $\overline{p} = 0.75$, and two periods of flexibility (such that $ED_1(2) = 0.38$), the agreed price for the flexible load is $p = 0.51$. For the same retail price as before, and five periods of flexibility (such that $ED_1(5) = 0.22$), the agreed price is $p = 0.4$. Likewise, if the retail price is lower (say, $\overline{p} = 0.65$), and C offers 4 periods of flexibility (such that $ED_1(4) = 0.26$) the price of the flexible load is $p = 0.4$.

As a consequence, relative to the inflexible case, both A and C gain from trading flexibility and total welfare is greater:

Proposition 8 *Under the assumption that C has a downward-sloping demand curve of the form $q = max\,[0, 1 - p]$, the contract between A and C leads to total welfare:*

$$TW_{flex} = \frac{(1-p)^2}{2} + (p - ED_1(N))(1-p)$$

which is greater than the alternative of not entering a contract?

$$TW = \frac{(1-\overline{p})^2}{2} + (\overline{p} - ED_1(1))\,(1-\overline{p})$$

3.4 Role of A is fulfilled by R, size of flexible load depends on p

One further extension of the model results from modifying assumptions $BL5$ and $BL7$ simultaneously. In addition to assuming, as in subsection 3.3, that the flexible load is different from unity, now we add the assumption that A's role is fulfilled by R. By implication, equation (13) is now re-written as:

$$U_A = \begin{cases} (p - ED_1(N))(1-p) \equiv \widetilde{U}_A & \text{if } A \text{ gains control over the load } q \\ (\overline{p} - ED_1(1))\,(1-\overline{p}) \equiv \overline{U}_A & \text{if } A \text{ does not gain control over the load } q \end{cases} \tag{15}$$

Note that (15) is similar to (7) in subsection 3.2 but differs because the load is not unitary anymore.

If A and C do not agree on a contract, now its outside option is to operate as a retailer. As in subsection 3.3, a closed-form solution for p was not obtained, but from an expression of p as a function of $\overline{p}$, $ED_1(1)$ and $ED_1(N)$:

$$(1-p)^2 = (1-\overline{p})^2 \left[\frac{(1 - 2p + ED_1(N))(1-p)}{(1 - 4p + 3ED_1(N))(1-p) + 2(\overline{p} - ED_1(1))(1-\overline{p})} \right] \tag{16}$$

the conclusions outlined previously (in the baseline model, and in Propositions 7 and 8) carry over to the modified set of assumptions just outlined (see Appendix for further details). Consequently, the following:

Corollary 2 *Propositions 7 and 8 hold under the assumptions that:*

1. *C has a downward-sloping demand curve $q = max\,[0, 1-p]$*

2. *A's role is fulfilled by R.*

The fundamental difference is that the price agreed in bargaining between A and C is always higher than if A's role remained independent of R. This coincides with the result of section 3.2, given that A's outside option is now better and, therefore, its bargaining power has increased. In table 2 below, we report analogous[9] results to the ones shown in table 1.

With two periods of flexibility offered by C, the expected cost of delivery of the flexible load faced by A is $ED_1(2) = 0.38$. But A's outside option is to deliver the (inflexible) load

[9] Table 2 shows the roots of the third degree polynomial associated to the first order condition of the Nash bargaining problem (equation 23 in the Appendix) under the assumptions outlined above. Reported values are numerical calculations of p given $\overline{p}$, R's expected unitary cost of delivery $ED_1(1) = 0.50$, and pairwise combinations of $ED_1(2) \ldots ED_1(5)$ and $\overline{p} = \{0.75, 0.70, 0.65\}$.

$\overline{p}$	$ED_1(2) = 0.38$	$ED_1(3) = 0.30$	$ED_1(4) = 0.26$	$ED_1(5) = 0.22$
0.75	0.59	0.53	0.49	0.46
0.70	0.57	0.51	0.48	0.45
0.65	0.55	0.49	0.46	0.43

Table 2: Numerical values of p for fixed $\overline{p}, ED_1(N)$ and $ED_1(1) = 0.5$. Role of A is fulfilled by R.

as a monopolist retailer for which it would charge $\overline{p} = 0.75$ while facing an expected cost of $ED_1(1) = 0.5$ per kWh. Symmetric bargaining between C and A leads to a price $p = 0.59$ for delivery of the flexible load, which is higher than the price agreed if A's role remained independent of R (i.e. $p = 0.51$, as shown in table 1). As a result, while total welfare is always greater with flexibility trading than without it, if A is independent of R, welfare is higher than if A's role is fulfilled by R.[10] The following example clarifies this point.

Example 3 *Consider a consumer who has a contract with a retailer for delivery of the inflexible load for a price* $\overline{p} = 0.75$ *per kWh. The payoff to* C *is* $U_C = CS(0.75) = \frac{(1-0.75)^2}{2} = 0.03$ *while that of* R*, who faces unitary cost of delivery* $ED_1(1) = 0.5$ *with no flexibility, is* $U_R = PS(0.75) = (0.75 - 0.5)(1 - 0.75) = 0.06$. *Hence,* $TW = 0.09$. *Figure 4 illustrates this initial situation as the sum of areas* **A** *and* **B**.

Suppose C *offers* A *a flexibility time frame of length* $N = 10$, *i.e.* $\mathcal{F} = [1, \dots, 10]$. *Thus,* A*'s expected unitary cost of delivery is* $ED_1(10) = 0.14$.

If C *agrees on a contract with an independent* A *(as per assumption Bl7), then* $p = 0.34$ *and the payoff to* C *is* $U_C = CS(0.34) = \frac{(1-0.34)^2}{2} = 0.22$ *while the payoff to* A *is* $U_A = PS(0.34) = (0.34 - 0.14)(1 - 0.34) = 0.13$. *Therefore,* $TW_{flex} = 0.35$. *In figure 6 this corresponds to* $A' + B'$*, including both (yellow and green) hatched areas.*

In contrast, if C *enters a contract with an* A *whose role is fulfilled by* R*, the price is* $p = 0.39$. *The payoff to* C *is* $U_C = CS(0.39) = \frac{(1-0.39)^2}{2} = 0.19$ *and* A*'s payoff is* $U_A = PS(0.39) = (0.39 - 0.14)(1 - 0.39) = 0.15$. *This leads to* $TW_{flex} = 0.34$. *In figure 5*

[10] The reason behind this result is that A operates where demand is inelastic and, hence, its revenue is increasing in price.

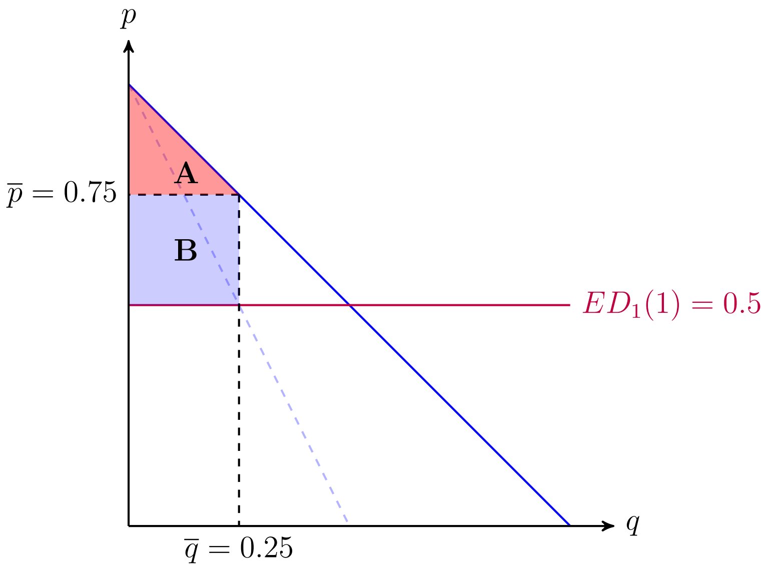

Figure 5: Total welfare without flexibility. Downward-sloping demand curve for C

this is indicated as area $\mathbf{A}' + \mathbf{B}'$*, minus the yellow hatched area. The green hatched area is transferred from* C *to* A*.*

4 Long-term contracts

Up to this point, this paper has focused on single-shot bargaining situations between A and C, under a number of variations. So far, as per assumption BL3, the contracts between A and C are for single-shot transactions. In contrast, this section analyzes long-term contracts between both agents. In addition, according to assumption BL9, there is a technology in place that reduces transaction costs to a negligible level, allowing both parties to gain from trading flexibility. However, the analysis has overlooked the investment cost (I) required to deploy such technology *before* both parties are able to realize an increase in their payoffs. To account for this, this section assumes instead that $I = I_A + I_C$, where I_A and I_C are, respectively, A's and C's investment costs required to establish a bilateral trade of flexibility. Clearly, for any A to be in business, it should be able to control a sufficiently large number of loads but this is exogenous to the model. Thus, I_A refers to the specific bilateral trade

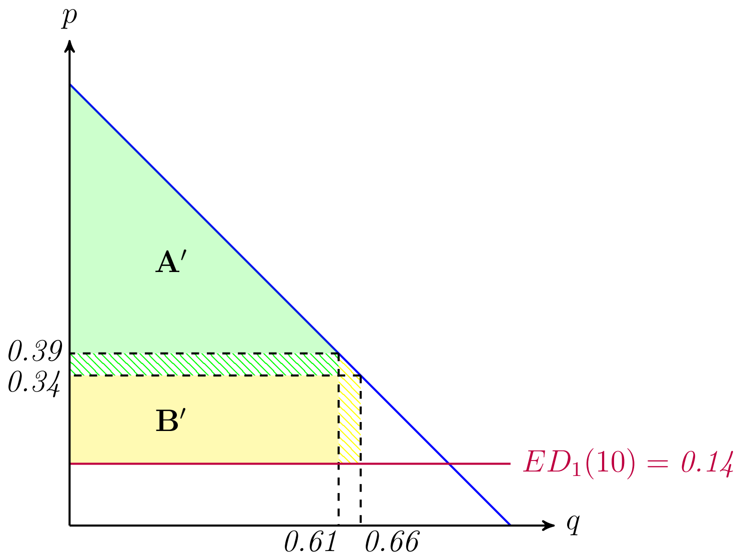

Figure 6: Total welfare with flexibility. Numerical example with length of flexibility time frame $\mathcal{F} = [1, \ldots, 10]$. Downward-sloping demand curve for C.

with C.

In addition, let $T > N$ be the *duration of the contract* and suppose that flexibility trading happens at times $t \in [1, \ldots T]$ such that there is a flexibility time frame $\mathcal{F}_1, \ldots, \mathcal{F}_T$ associated to each t. Note that while both T and N refer to time, it is safe to assume - without loss of generality - that they are measured in two different units, e.g. days and hours, respectively. For example, a flexible C can offer $N = 5$ hours of flexibility to A over the course of $T = 60$ days, that is, 60 time frames of length N, i.e. $\mathcal{F}_1, \ldots, \mathcal{F}_{60}$. Instead of bargaining at each time t, i.e. repeatedly, A and C can agree on p for the whole duration of the contract. Indeed, as simplifying assumptions, the analysis of this section assumes that the flexibility time frames are of equal length and that A and C bargain over the terms of the contract once: before flexibility trading takes place in the period $[1, \ldots T]$.

4.1 Unit load

Assuming that C commits to supplying N periods of flexibility in each transaction at time t, its payoff over T periods is:

$$\sum_{t=1}^{T}(v_t - p_t) - I_C, \quad \text{if it enters a contract with } A$$

If it doesn't, then C's payoff is:

$$\sum_{t=1}^{T}(v_t - \overline{p}_t)$$

But because it is assumed that v_t, $\overline{p}$ remain unchanged throughout the duration of the contract, and p is the outcome of bargaining, it is simpler to write:

$$\begin{array}{l} T(v-p) - I_C, \quad \text{if } C \text{ enters a contract with } A \\ T(v-\overline{p}), \quad \text{if } C \text{ doesn't enter a contract with } A \end{array}$$

Therefore, C's payoff is:

$$\mathbf{U_C} = \begin{cases} T(v-p) - I_C \equiv \widetilde{\mathbf{U}}_\mathbf{C} & \text{if } C \text{ is flexible} \\ T(v-\overline{p}) \equiv \overline{\mathbf{U}}_\mathbf{C} & \text{if } C \text{ is not flexible} \end{cases} \tag{17}$$

Similarly, if A gains control over delivery of the flexible load, it obtains a payoff of:

$$\sum_{t=1}^{T}(p_t - ED_1(N)_t) - I_A$$

but if it does not, then A does not enter into the load aggregation business. For the same reasons as before, and the fact that the expected unitary cost of delivery $ED_1(N)$ is determined before the parties trade with flexibility, it is correct and simpler to write:

$$T(p - ED_1(N)) - I_A$$

Thus, the payoff to A is:

$$\mathbf{U_A} = \begin{cases} T(p - ED_1(N)) - I_A \equiv \widetilde{\mathbf{U}}_\mathbf{A} & \text{if } A \text{ gains control} \\ 0 \equiv \overline{\mathbf{U}}_\mathbf{A} & \text{if } A \text{ does not gain control} \end{cases} \tag{18}$$

The conditions of the long-term contract are established before any flexibility trading takes place. But, specifically, the price p remains fixed for the duration T of the contract and it's determined by Nash bargaining between A and C:

Proposition 9 *Over a period $T > N$ and with investment cost $I = I_A + I_C$, the price of the contract between A and C is:*

$$p = \frac{ED_1(N) + \overline{p}}{2} + \frac{I_A - I_C}{2T}$$

The price p for the long-term contract has two terms. The first coincides with equation (4): the midpoint between the expected unitary cost of delivery $ED_1(N)$ with N periods of flexibility and the retail price $\overline{p}$. The second is a fixed term that depends on I_A, I_C, and T, with which p is inversely related: the longer the duration of the contract, the more periods that are available to distribute the impact of the investment cost, $I_A - I_C$, on the long-term price of flexibility.

An interesting feature of the agreed price is that three possible cases emerge:

1. If $I_A = I_C$, two cost-symmetric agents trade and the price agreed over T periods of trading equals the price agreed in a single transaction, which is shown in equation 4.
2. If $I_A > I_C$, A faces a greater cost than C and the agreed price is greater than equation 4
3. If $I_A < I_C$, C faces a greater cost than A and the price is less than equation 4

Although the bargaining model of this paper has not made any assumption about the specific technology with which A and C trade with flexibility, the size of I_A and I_C says something about it. If the technology is such that there are scale economies, as the number of loads that A controls increases, both I_A and I_C should decrease too. That is, a network externality effect operates: if more consumers agree to let an aggregator control its load, it is possible for A to distribute its total investment cost among more consumers and I_A should decrease. By the same argument, the more consumers trade with a given aggregator, the lower the investment cost I_C that C faces should be. In light of this, the third case gives the consumer a better deal: C is able to obtain a lower p over the lifetime of a contract if it trades flexibility with an "established" A. On the other hand, the presence of scale economies may imply some degree of market power for A.

In summary, relative to the single-shot contracts analyzed earlier in the paper, a more nuanced vision emerges when long-term contracts are considered. One in which flexibility trading increases welfare relative to the welfare without flexibility trading, but depends on the size of the required investment:

Proposition 10 *If C, who faces investment cost I_C, and A, who faces investment cost I_A, enter a long-term contract of duration T to trade with flexibility, total welfare is:*

$$TW_{flex} = T(v - ED_1(N)) - I$$

where $I = I_A + I_C$

In contrast, if C and A do not enter a contract, total welfare is:

$$TW = T(v - ED_1(1))$$

$TW_{flex} > TW$ *holds if* $I < T(v - ED_1(N)) - T(v - ED_1(1))$

4.2 Downward-sloping demand

Following the approach of section 3, in which baseline model assumptions are modified in turn, this subsection investigates the validity of the results of the previous subsection (4.1) under the assumption of downward-sloping demand, as in equations 10 and 11. In the same way described in subsections 3.3 and 3.4, consumer surplus and producer surplus (as functions of price) are used to measure the welfare of C and A, respectively.

If C agrees to offer flexibility, then his payoff is the sum of consumer surpluses over the duration T of the contract:

$$\sum_{t=1}^{T} CS(p_t) - I_C = \sum_{t=1}^{T} \frac{(1-p_t)}{2} - I_C$$

Alternatively, if C does not offer any flexibility and remains as a conventional consumer, his payoff is:

$$\sum_{t=1}^{T} CS(\overline{p}_t) = \sum_{t=1}^{T} \frac{(1-\overline{p}_t)}{2}$$

But because the valuation v and the price $\overline{p}$ remain fixed while the contract is valid and p is determined via bargaining, the payoff to C is written as:

$$\mathbf{U_C} = \begin{cases} T\frac{(1-p)^2}{2} - I_C \equiv \widetilde{\mathbf{U}}_\mathbf{C} & \text{if } C \text{ is flexible} \\ T\frac{(1-\overline{p}^2)}{2} \equiv \overline{\mathbf{U}}_\mathbf{C} & \text{if } C \text{ is not flexible} \end{cases} \tag{19}$$

On the other hand, over T periods of flexibility trading, A obtains a payoff of:

$$\sum_{t=1}^{T} PS(p) - I_A = \sum_{t=1}^{T} (p_t - ED_1(N)_t)(1-p_t) - I_A$$

Because the expected unitary cost of delivery is determined before trading and p is determined in bargaining for the contract duration, A's payoff is written as:

$$\mathbf{U_A} = \begin{cases} T[(p - ED_1(N))(1-p_t)] - I_A \equiv \widetilde{\mathbf{U}}_\mathbf{A} & \text{if } A \text{ gains control} \\ 0 \equiv \overline{\mathbf{U}}_\mathbf{A} & \text{if } A \text{ does not gain control} \end{cases} \tag{20}$$

As in subsections 3.3 and 3.4, closed form solutions for the flexibility price were not obtained. However, the following expression, which follows from the first order condition for the Nash bargaining problem (see the Appendix), allows establishing connections with results proven earlier in the paper.

$$\left[\frac{(1-p)^2}{2}(1-4p+3ED_1(N))\right]+\frac{1}{T}\left[I_A(1-p)-I_C(1-2p+ED_1(N))\right] \\ =\left[\frac{(1-\overline{p})^2}{2}(1-2p+ED_1(N))\right] \tag{21}$$

Note in the equation that p, as in (14), is a function of the expected unitary cost of delivery $ED_1(N)$ and the retail price $\overline{p}$ but also of I_A, I_C and T.

Corollary 4 *If A and C enter a long-term contract of duration T to trade with flexibility, Proposition 7 holds*

However, in accordance with Proposition 10, total welfare with flexibility trading is greater than without it, only if the investment cost I is sufficiently small:

Proposition 11 *If C, who has downward-sloping demand $q = max\,[0, 1-p]$ and has investment cost I_C enters a long-term contract for flexibility trading of duration T with A, who has investment cost I_A, total welfare is:*

$$TW_{flex} = T\left[\frac{(1-p)^2}{2}+(\overline{p}-ED_1(N))(1-p)\right]-I$$

where $I = I_A + I_C$.

In contrast, if A and C do not enter a long-term contract

$$TW = T\left[\frac{(1-\overline{p})^2}{2}+(p-ED_1(1))(1-\overline{p})\right]$$

$TW_{flex} > TW$ holds if

$$I < T\left[\frac{(1-p)^2}{2}+(p-ED_1(N))(1-p)\right]-T\left[\frac{(1-\overline{p})^2}{2}+(p-ED_1(1))(1-\overline{p})\right]$$

To illustrate the results of this subsection, the following three tables (namely, tables 3, 4 and 5) present agreed long-term prices of flexibility under different situations. In all cases, assume that $T = 90$ days is the duration of a contract for flexibility between A and C and that $I = 10$. Furthermore, the prices reported in the following tables are estimated from equation 24 in the appendix. As in the results of tables 1 and 2, the reported results are calculated given the expected unitary cost of delivery for flexibility time frames of length 2 to 5, i.e. $ED_1(2), \ldots, ED_1(5)$ and retail prices $\overline{p} = [0.75, 0.70, 0.65]$.

Table 3 assumes that $I_A = I_C = 5$, a situation in which A and C face equally-sized investment costs to trade with flexibility over T periods.

$\overline{p}$	$ED_1(2) = 0.38$	$ED_1(3) = 0.30$	$ED_1(4) = 0.26$	$ED_1(5) = 0.22$
0.75	0.53	0.48	0.44	0.42
0.70	0.51	0.46	0.43	0.41
0.65	0.50	0.45	0.42	0.40

Table 3: Agreed prices of long-term contracts for flexibility between A and C. Symmetric investment costs.

$\overline{p}$	$ED_1(2) = 0.38$	$ED_1(3) = 0.30$	$ED_1(4) = 0.26$	$ED_1(5) = 0.22$
0.75	0.59	0.52	0.48	0.46
0.70	0.57	0.51	0.47	0.45
0.65	0.56	0.50	0.46	0.43

Table 4: Agreed prices of long-term contracts for flexibility between A and C. Investment costs are asymmetric towards A.

Table 4 assumes that $I_A = 7, I_C = 3$, a situation in which A faces a greater investment cost than C to trade with flexibility. Note that relative to table 3, prices are consistently higher in table 4.

Table 5 assumes that $I_A = 3, I_C = 7$, a situation in which A's investment cost is lower than C's.

In accordance with the discussion of subsection 4.1, the numerical results shown in the previous tables give a more nuanced vision of flexibility trading in a long-term setting. First, the total investment cost must be sufficiently small and the trading period sufficiently long to be able to realize a profitable trade. Second, the distribution of investment costs between

$\overline{p}$	$ED_1(2) = 0.38$	$ED_1(3) = 0.30$	$ED_1(4) = 0.26$	$ED_1(5) = 0.22$
0.75	0.48	0.43	0.40	0.38
0.70	0.47	0.42	0.39	0.37
0.65	0.45	0.41	0.38	0.36

Table 5: Agreed prices of long-term contracts for flexibility between A and C. Investment costs are asymmetric towards C.

the two agents has a direct impact on the agreed prices. Relative to the situation in which both parties bear the same cost (table 3), the situation in which A faces a greater cost than C is (table 4) is the least advantageous for C. In contrast, whenever $I_A < I_C$, the agreed prices are the lowest. Although not directly modelled in this paper, this feature says something about the presence of scale economies and network externalities in the aggregation business. The more loads A is able to aggregate, the lower the investment cost required in *each* long-term bilateral contract. Similarly, the more consumers offer their flexibility to an aggregator, the lower the required I_C. The key economic message is that some degree of market power may be required to cover the investment cost associated to the establishment of the necessary technology to trade flexibility.

5 Conclusions

This paper has dealt with power system flexibility from demand-side resources, focusing on the contractual relationship between an aggregator and a consumer. Under a number of reasonable but restrictive assumptions, *the key message of the paper is that flexibility trading is welfare enhancing to both parties*. Such is the consequence of the aggregator's optimal delivery policy, which gives him the possibility of managing price risk, together with the consumer's possibility to reduce its electricity bill.

Under a set of relaxed assumptions, discussed in section 3, the same conclusion - with caveats - stands. For example, if a retailer enters the related but distinct business of delivering flexible loads, essentially expanding the *scope* of its economic activity, the gains from trading flexibility can still be increased but the agreed price of flexibility is higher, which operates to the disadvantage of the consumer. Likewise, if one of the parties is able to make an ultimatum offer, welfare can still be increased relative to the situation in which no flexibility trading exists, but one of the parties appropriates the gains from trading.

Taking a long-term perspective, in which single-shot transactions are not the focus of the contracts, and accounting for the required investment costs to put in place a technology that reduces transaction costs, i.e. a Smart Grid solution, gives a more nuanced vision of the key result of the paper. First, aggregator and consumer must trade for a sufficiently long period of time to cover the investment cost and be able to realize the single-transaction gains from trading flexibility. Second, the agreed price depends on the way that costs are shared. Relative to the case in which they are symmetrically shared, when the consumer faces a relatively higher cost than the aggregator, the consumer is able to obtain a better deal for its flexibility. Such a finding relates to the possibility of a network effect, which requires some degree of scale economies in the flexibility-enabling technology. Clearly, a trade-off may exist between the presence of scale economies and the possibility to introduce competition among aggregators in a potential market for flexibility.

The analysis presented in this paper, naturally, has its limitations. First, all of the results are under symmetric information and risk neutrality.[11] It might be useful to modify some of these assumptions in further extensions. Second, apart from the impact of wholesale prices on the aggregator's revenue, the full impact of demand uncertainty in the aggregator's profit function has not been explicitly modelled. That is: the aggregator is an intermediary and, consequently, faces double uncertainty. On the cost side, it must optimize its acquisition of loads. On the revenue side, the flexibility it "assembles" may or may not be desirable by the final users. A third limitation relates to the insufficiently detailed analysis of the investment cost sharing arrangements, i.e. well-designed contracts for flexibility should not avoid this crucial topic.

Last but not least, from a methodological perspective, what we have presented is just a theoretical economic model which, after all, is not much more than what Rubinstein (2012) calls an "economic fable", a tale that seeks to impart a lesson about an economic situation by "clarifying concepts, evaluating assumptions, verifying conclusions and acquiring insights that will serve us when we return from the model to real life". But, as with any fable, its message should be applied judiciously.

[11] The work by Boscán and Poudineh (2016b) addresses the asymmetric information issue. Their model studies the procurement problem under adverse selection problem. Their results can be applied to, *inter alia*, the aggregator's problem when procuring loads from consumers.

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Appendix

Proof of Proposition 1.

We prove the claim by backward induction. Let $\mathcal{F} = [1, \ldots, N]$ be a flexibility time frame of length $N, N > 1$.

At $n = N$, A must deliver the load and cannot postpone any longer, unless it breaches the agreement. Therefore, it faces the expected cost of delivery:

$$ED_N(N) = E\left(w_N\right)$$

At $n \in [1, \ldots, N-1]$, A compares w_n with $ED_{n+1}(N)$. If $w_n \leq ED_{n+1}(N)$, A delivers. If $w_n > ED_{n+1}(N)$, A postpones. This results in the following recursive expression for $ED_n(N)$:

$$\begin{aligned}
ED_n(N) &= \int_0^1 \min\left\{w_n, ED_{n+1}(N)\right\} \, \mathrm{d}w_n \\
&= \int_0^{ED_{n+1}(N)} w_n \, \mathrm{d}w_n + \int_{ED_{n+1}(N)}^1 ED_{n+1}(N)\mathrm{d}w_n \\
&= \frac{(ED_{n+1}(N))^2}{2} + (ED_{n+1}(N))(1 - ED_{n+1}(N)) \\
&= \frac{(ED_{n+1}(N))^2 + 2ED_{n+1}(N) - 2(ED_{n+1}(N))^2}{2} \\
&= \frac{2ED_{n+1}(N) - (ED_{n+1}(N))^2}{2} \\
&= ED_{n+1}(N) - \frac{(ED_{n+1}(N))^2}{2}
\end{aligned}$$

$\blacksquare$

Proof of Proposition 2. We will use the monotone convergence theorem[12], which states that if a sequence is monotonic and bounded then it converges, to prove the claim that $\left\{ED_1(N)\right\}_{N\geq 1}$ converges to zero.

1. By induction, we show that the sequence is monotonically decreasing in N:

[12]For a proof and discussion of this theorem see, for example, de la Fuente (2000) or Hoy et al. (2001).

For $N = 1$ and $N = 2$ we can easily verify that:

$$\begin{aligned} ED_1(N) = ED_1(1) &= \frac{1}{2} \\ > ED_1(2) &= \frac{2ED_1(N-1) - (ED_1(N-1))^2}{2} = \frac{3}{8} \end{aligned}$$

Assume that $ED_1(N) > ED_1(N + 1)$ holds for N sufficiently large.

From:

$$ED_1(N) = \frac{2ED_1(N-1) - (ED_1(N-1))^2}{2}, \quad \text{it follows that:}$$

$$\begin{aligned} \frac{2ED_1(N) - (ED_1(N))^2}{2} &= ED_1(N+1) \\ &> ED_1(N+2) = \frac{2ED_1(N+1) - (ED_1(N+1))^2}{2} \end{aligned}$$

2. By assumption $BL8$,$w_n \sim U\,[0, 1]$, it is clear that the sequence is bounded below by 0

We have proved that the sequence converges and now we evaluate its limit:

$$\begin{aligned} \lim_{N \to \infty} ED_1(N) &= \lim_{N\to\infty} \frac{2ED_1(N-1) - (ED_1(N-1))^2}{2} \\ &= \frac{2}{2} \lim_{N\to\infty} ED_1(N-1) - (ED_1(N-1))^2 \\ &= \lim_{N\to\infty} ED_1(N-1) - (ED_1(N-1))^2 \end{aligned}$$

Letting $\lim_{N\to\infty} ED_1(N) = L$ we can write:

$$\begin{aligned} L &= L - L^2 \\ L &= 0 \end{aligned}$$

■

Proof of Proposition 3. Under assumptions BL1-BL9, the agreed price between C and A is the solution to the following Nash (1950) bargaining problem, where $(U_C U_A)$ is the well-known "Nash product":

$$p = \arg\max_p \,(U_C U_A)$$

Note that C's net payoff equals the difference between acting flexibly and remaining as a conventional, non-flexible electricity consumer (C's "outside option"):

$$\begin{aligned} U_C &= \widetilde{U}_C - \overline{U}_C \\ &= v - p - v + \overline{p} \\ &= -p + \overline{p} \end{aligned}$$

Similarly, A's net payoff is the difference between its two options, which are acquiring control of the load and remaining out of business:

$$\begin{aligned} U_A &= \widetilde{U}_A - \overline{U}_A \\ &= p - ED_1(N) - 0 \end{aligned}$$

The first-order condition is:

$$\begin{aligned} U_C' U_A + U_C U_A' &= 0 \\ -p + ED_1(N) + \overline{p} - p &= 0 \end{aligned}$$

Thus, the agreed price is

$$p = \frac{\overline{p} + ED_1(N)}{2}$$

■

Proof of Proposition 4.

If C and A enter a single-shot contract over the control of the flexible load, $U_C = \widetilde{U}_C = v - p$ and $U_A = \widetilde{U}_A = p - ED_1(N)$, because the load is delivered to C by A. Therefore,

$$\begin{aligned} TW_{flex} &= U_C + U_A \\ &= \widetilde{U}_C + \widetilde{U}_A \\ &= v - p + p - ED_1(N) \\ &= v - ED_1(N) \end{aligned}$$

In contrast, if C and A and do not reach an agreement, then $U_C = \overline{U}_C = v - \overline{p}$ but $U_A = \overline{U}_A = 0$ because it is the retailer who delivers the load, whose payoff is $U_R = \overline{p} - ED_1(1)$. Hence,

$$
\begin{aligned}
TW &= U_C + U_R \\
&= \overline{U}_C + U_R \\
&= v - \overline{p} + \overline{p} - ED_1(1) \\
&= v - ED_1(1)
\end{aligned}
$$

Clearly, $TW_{flex} > TW$. This is because, $ED_1(1) > ED_1(N)$ whenever $N > 1$, which follows from Proposition 2.

■

Proof of Proposition 5.

1. If A has all the bargaining power, it will propose a price p such that the consumer's net payoff is:

$$U_C = v - p - v + \overline{p} = \epsilon$$

where ϵ is an arbitrarily small, positive number. Therefore,

$$p = \overline{p} - \epsilon$$

A's payoff is then:

$$
\begin{aligned}
U_A &= p - ED_1(N) \\
U_A &= (\overline{p} - \epsilon) - ED_1(N)
\end{aligned}
$$

2. Similarly, if C has all the bargaining power, he will set a price p such that A's net payoff is:

$$U_A = p - ED_1(N) = \epsilon$$

Therefore,

$$p = ED_1(N) + \epsilon$$

C's payoff is then:

$$U_C = v - p - v + \overline{p}$$
$$U_C = \overline{p} - ED_1(N) - \epsilon$$

■

Proof of Proposition 6.

Relaxing assumption BL7, i.e. assuming that R takes the role of A, but with equal bargaining power between the two parties, the price p is the solution to the following Nash bargaining problem:

$$p = \arg\max_{p} (U_C U_A)$$

where

$$\begin{aligned} U_C &= \widetilde{U}_C - \overline{U}_C \\ &= v - p - v + \overline{p} \\ &= -p + \overline{p} \end{aligned}$$

and

$$\begin{aligned} U_A &= \widetilde{U}_A - \overline{U}_A \\ &= p - ED_1(N) - \overline{p} + ED_1(1) \end{aligned}$$

The first order condition to the Nash bargaining problem is:

$$\begin{aligned} U_C' U_A + U_C U_A' &= 0 \\ -p + ED_1(N) + \overline{p} - ED_1(1) + \overline{p} - p &= 0 \end{aligned}$$

from which the following price is derived:

$$p = \overline{p} + \frac{ED_1(N) - ED_1(1)}{2}$$

■

Calculations associated with subsection 3.3

Derivation of equation (14) on the main text:

With C's downward-sloping demand curve $q = max[0, 1-p]$, the Nash bargaining problem is:

$$p = \arg\max_{p} (U_C U_A)$$

where C's net payoff is $U_C = \frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}$ and A's is $U_A = (p - ED_1(N))(1-p)$. Therefore:

$$p = \arg\max_{p} \left[\frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}\right] [(p - ED_1(N))(1-p)]$$

which is equivalent to

$$p = \arg\max_{p} (1-p)^3(p - ED_1(N)) - (1-\overline{p})^2(1-p)(p - ED_1(N))$$

This has the first order condition:

$$-3(1-p)^2(p - ED_1(N)) + (1-p)^3 + (1-\overline{p})^2(p - ED_1(N)) - (1-\overline{p})^2(1-p) = 0$$

or

$$(1-p)^2(1 - 4p + 3ED_1(N)) = (1-\overline{p})^2(1 - 2p + ED_1(N)) \tag{22}$$

which is the equation used to estimate numerical values of p in table 1 on the main text. Re-arranging this expression gives equation (14) in the main text:

$$(1-p)^2 = (1-\overline{p})^2 \left[\frac{1 - 2p + ED_1(N)}{1 - 4p + 3ED_1(N)}\right]$$

On the condition that $p < \frac{1+ED_1(N)}{2}$:

If A were a pure monopolist, then it would solve:

$$\max_{p_m} (p_m - ED_1(N))(1 - p_m)$$

which leads to the first order condition $1 - p_m - p_m + ED_1(N) = 0$ or

$$1 - 2p_m + ED_1(N) = 0$$

Thus,

$$p_m = \frac{1 + ED_1(N)}{2}$$

Of course, if $N = 1$, then $ED_1(N) = ED_1(1)$ and R were a pure monopolist, then $\overline{p} = p_m$. For $N > 1$, $ED_1(1) > ED_1(N)$ as shown in Proposition 2. It follows that $\overline{p} \geq p_m > p$.

Proof of Proposition 7.

1. To see the first part of the claim, divide both sides of equation (22) (in the Appendix) by 2 and re-arrange terms to obtain:

$$\begin{aligned} \frac{(1-p)^2}{2}(1 - 4p + 3ED_1(N)) &= \frac{(1-\overline{p})^2}{2}(1 - 2p + ED_1(N)) \\ CS(p)\left[\frac{1 - 4p + 3ED_1(N)}{1 - 2p + ED_1(N)}\right] &= CS(\overline{p}) \end{aligned}$$

Note that:

$$\frac{1 - 4p + 3ED_1(N)}{1 - 2p + ED_1(N)} = \frac{1 - 2p + ED_1(N) - 2p + 2ED_1(N)}{1 - 2p + ED_1(N)} = 1 - 2\frac{p - ED_1(N)}{1 - 2p + ED_1(N)} < 1$$

for $p < \frac{1+ED_1(N)}{2} = p_m$

2. The second part of the claim follows from:

$$\begin{aligned}
\frac{d}{dED_1(N)}\left(\frac{1-4p+3ED_1(N)}{1-2p+ED_1(N)}\right) &= \\
\frac{d}{dED_1(N)}\left(1-2\frac{p-ED_1(N)}{1-2p+ED_1(N)}\right) &= \\
-\frac{d}{dED_1(N)}\left(2(p-ED_1(N))(1-2p+ED_1(N))^{-1}\right) &= \\
&= 2\left(1-2p+ED_1(N)\right)^{-1}+2\left(p-ED_1(N)\right)(\\
&= \frac{2-2p}{\left(1-2p+ED_1(N)\right)^{-2}} \\
&= 2\frac{1-p}{\left(1-2p+ED_1(N)\right)^{2}}
\end{aligned}$$

Note that this expression is strictly positive for p. Hence, as $ED_1(N)$ increases $CS(p)$ and $CS(\overline{p})$ tend to be equal. Put differently: the lower $ED_1(N)$, the bigger the increase in $CS(p)$ relative to $CS(\overline{p})$.

■

Proof of Proposition 8.

If C has a downward-sloping demand curve given by $q = max\,[0, 1-p]$ and enters a contract for delivery of the flexible load q with A, then $U_C = \frac{(1-p)^2}{2} = \widetilde{U}_C$ and $U_A = (p-ED_1(N))(1-p) = \widetilde{U}_A$, because the load is delivered to C by A. Therefore,

$$\begin{aligned}
TW_{flex} &= U_C + U_A \\
&= \widetilde{U}_C + \widetilde{U}_A \\
&= \frac{(1-p)^2}{2} + (p-ED_1(N))(1-p)
\end{aligned}$$

In contrast, if C and A do not agree on a contract to deliver the flexible load, then $U_C = \frac{(1-\overline{p})^2}{2} = \overline{U}_C$ but $U_A = \overline{U}_A = 0$ because it is R who delivers the load and, consequently, its profit is $U_R = (\overline{p} - ED_1(1))(1-\overline{p})$. Hence,

$$\begin{aligned}
TW &= U_C + U_R \\
&= \overline{U}_C + U_R \\
&= \frac{(1-\overline{p})^2}{2} + (\overline{p}-ED_1(1))(1-\overline{p})
\end{aligned}$$

Now, we show that $TW_{flex} > TW$. Term-to-term comparison of the two expressions shows that:

1. Consumer surplus is greater when flexibility is traded:

$$\frac{(1-p)^2}{2} > \frac{(1-\overline{p})^2}{2}$$

   This is because $CS(\cdot)$ is decreasing in p and we have proved that $p < \overline{p}$.

2. A's profit is greater than R's profit:

$$(p - ED_1(N))(1-p) > (\overline{p} - ED_1(1))(1-\overline{p})$$

   Subtracting $(1-p)$ from $(1-\overline{p})$ and $(p - ED_1(N))$ from $(\overline{p} - ED_1(1))$, we obtain the following expression:

$$\begin{aligned} 0 &> \left[(\overline{p} - ED_1(1)) - (p - ED_1(N))\right]\left[(1-\overline{p}) - (1-p)\right] \\ &> \left[(\overline{p} - p) - (ED_1(1) - ED_1(N))\right]\left[(p - \overline{p})\right] \end{aligned}$$

   From Proposition 7 and 2, $(p-\overline{p}) < 0$, $(\overline{p}-p) > 0$ and $(ED_1(1) - ED_1(N)) > 0$ must hold. Furthermore, if flexibility is traded, then $(\overline{p} - p) > (ED_1(N) - ED_1(N))$ must hold as well. This proves that the inequality above holds.

■

Calculations associated with Section 3.4

Derivation of equation (16):

If the role of aggregator is assumed by a retailer and the consumer has downward-sloping demand curve $q = max\,[0, 1-p]$, the Nash bargaining problem is:

$$p = \arg\max_{p} \left(U_C U_A\right)$$

where C's net payoff is $U_C = \frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}$ and A's is $U_A = (p - ED_1(N))\,(1-p) - (\overline{p} - ED_1(1))\,(1-\overline{p})$. Therefore:

$$p = \arg\max_p \left[\frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}\right]\left[(p - ED_1(N))\,(1-p) - (\overline{p} - ED_1(1))\,(1-\overline{p})\right]$$

After some algebraic manipulation:

$$p = \arg\max_p\,(p - ED_1(N))(1-p)^3 - (\overline{p} - ED_1(1))(1-\overline{p})(1-p)^2 - (p - ED_1(N))(1-p)(1-\overline{p})^2 + (\overline{p} - ED_1(1))(1-\overline{p})^3$$

The first order condition of this problem is:

$$-3(p-ED_1(N))(1-p)^2+(1-p)^3+2(1-p)(\overline{p}-ED_1(1))(1-\overline{p})-(1-p)(1-\overline{p})^2+(p-ED_1(N))(1-\overline{p})^2 = 0$$

or

$$(1-p)^2\left[-3(p - ED_1(N)) + (1-p) + 2\frac{(\overline{p} - ED_1(1))\,(1-\overline{p})}{(1-p)}\right] = (1-\overline{p})^2\left[1 - 2p + ED_1(N)\right]$$

We obtain:

$$(1-p)^2\left[\frac{(1-4p+3ED_1(N))(1-p) + 2(\overline{p} - ED_1(1))(1-\overline{p})}{(1-p)}\right] = (1-\overline{p})^2\left[1 - 2p + ED_1(N)\right] \tag{23}$$

Re-arranging terms leads to equation (16) in the main text:

$$(1-p)^2 = (1-\overline{p})^2\left[\frac{(1-2p+ED_1(N))(1-p)}{(1-4p+3ED_1(N))(1-p) + 2(\overline{p} - ED_1(1))(1-\overline{p})}\right]$$

Corollary 2:

1. Dividing both sides of equation 23 by 2 and manipulating the expression further, we get:

$$\frac{(1-p)^2}{2}\left[\frac{(1-2p+ED_1(N)-2p+2ED_1(N))(1-p)}{(1-2p+ED_N(N))(1-p)}+2\frac{(\overline{p}-ED_1(1))(1-\overline{p})}{(1-2p+ED_1(N))(1-p)}\right]=\frac{(1-\overline{p})^2}{2}$$

or

$$\frac{(1-p)^2}{2}\left[1-2\frac{(p-ED_1(N))}{(1-2p+ED_1(N))}+2\frac{(\overline{p}-ED_1(1))(1-\overline{p})}{(1-2p+ED_1(N))(1-p)}\right]=\frac{(1-\overline{p})^2}{2}$$

which is equivalent to

$$CS(p)\left[1-\frac{2}{1-2p+ED_1(N)}\frac{(p-ED_1(N))(1-p)+(\overline{p}-ED_1(1))(1-\overline{p})}{1-p}\right]=CS(\overline{p})$$

Clearly,

$$\left[1-\frac{2}{1-2p+ED_1(N)}\frac{(p-ED_1(N))(1-p)+(\overline{p}-ED_1(N))(1-\overline{p})}{1-p}\right]<1$$

for $p<\frac{1+ED_1(N)}{2}$, which proves that $CS(\overline{p})<CS(p)$ and, therefore, $p<\overline{p}$. Note that this is equivalent to the first claim in Proposition 7.

In addition, note that

$$\begin{aligned}
&\frac{d}{dED_1(N)}\left(1-\frac{2}{1-2p+ED_1(N)}\frac{(p-ED_1(N))(1-p)+(\overline{p}-ED_1(1))(1-\overline{p})}{1-p}\right)\\
&=2\frac{(p-ED_1(N))(1-p)+(\overline{p}-ED_1(1))(1-\overline{p})}{(1-p)(1-2p+ED_1(N))^2}\\
&\quad-2\left(\frac{(p-1)}{(1-p)(1-2p+ED_1(N))}\right)\\
&=\left(\frac{2}{(1-2p+ED_1(N))}\right)\left(\frac{(p-ED_1(N))(1-p)+(\overline{p}-ED_1(1))(1-\overline{p})}{(1-p)(1-2p+ED_1(N))}+1\right)\\
&>0
\end{aligned}$$

which is equivalent to the second claim in Proposition 7.

2. If A and C enter a contract for delivery of the flexible load q and C has a downward-sloping demand curve given by $q = max\,[0, 1-p]$, then $U_C = CS(p) = \frac{(1-p)^2}{2} = \widetilde{U}_C$ and $U_A = (p - D_N)(1-p) = \widetilde{U}_A$. Thus,

$$\begin{aligned} TW_{flex} &= U_C + U_A \\ &= \widetilde{U}_C + \widetilde{U}_A \\ &= \frac{(1-p)^2}{2} + (p - ED_1(N))(1-p) \end{aligned}$$

In contrast, if C and A do not agree on a contract to deliver the flexible load, then $U_C = CS(\overline{p}) = \frac{(1-\overline{p})^2}{2} = \overline{U}_C$ and $U_A = (\overline{p} - ED_1(1))(1-\overline{p})$ because A delivers the load in its role of retailer. Hence,

$$\begin{aligned} TW &= U_C + U_A \\ &= \overline{U}_C + \overline{U}_A \\ &= \frac{(1-\overline{p})^2}{2} + (\overline{p} - D_1)(1-\overline{p}) \end{aligned}$$

Furthermore, $TW_{flex} > TW$ as shown in Proposition 8.

Proof of Proposition 9

Relaxing Proposition $BL9$ and considering a period T, the agreed price between C and A is the solution to the following Nash bargaining problem:

$$p = \arg\max_p \left(\mathbf{U_C U_A}\right)$$

where

$$\begin{aligned} \mathbf{U_C} &= \widetilde{\mathbf{U}}_\mathbf{C} - \overline{\mathbf{U}}_\mathbf{C} \\ &= T(v-p) - I_P - T(v-\overline{p}) \\ &= T(\overline{p} - p) - I_C \end{aligned}$$

and

$$\begin{aligned} \mathbf{U_A} &= \widetilde{\mathbf{U}}_\mathbf{A} - \overline{\mathbf{U}}_\mathbf{A} \\ &= T(p - ED_1(N)) - I_A \end{aligned}$$

The first order condition to the Nash bargaining problem is:

$$\begin{aligned} \mathbf{U'_C U_A + U'_A U_C} &= 0 \\ -T(T(p - ED_1(N)) - I_A) + T(T(\overline{p} - p) - I_C) &= 0 \end{aligned}$$

from which the following price is derived:

$$p = \frac{ED_1(N) + \overline{p}}{2} + \frac{I_A - I_C}{2T}$$

■

Proof of Proposition 10.

If C and A enter a long-term contract of duration T over the control of the flexible load, $U_C = \widetilde{U}_C = T(v-p) - I_C$ and $U_A = \widetilde{U}_A = T(p - ED_1(N)) - I_A$, because the load is delivered to C by A. Therefore,

$$\begin{aligned} TW_{flex} &= U_C + U_A \\ &= \widetilde{U}_C + \widetilde{U}_A \\ &= T(v - p) - I_C + T(p - ED_1(N)) - I_A \\ &= T(v - ED_1(N)) - I \end{aligned}$$

where $I = I_C + I_A$.

In contrast, if C and A and do not reach an agreement, then $U_C = \overline{U}_C = T(v - \overline{p})$ but $U_A = \overline{U}_A = 0$ because it is the retailer who delivers the load, whose payoff is $U_R = T(\overline{p} - ED_1(1))$. Hence,

$$\begin{aligned} TW &= U_C + U_R \\ &= \overline{U}_C + U_R \\ &= T(v - \overline{p}) + T(\overline{p} - ED_1(1)) \\ &= T(v - ED_1(1)) \end{aligned}$$

Clearly, $T(v - ED_1(N)) > T(v - ED_1(1))$, which follows from Proposition 2. But $TW_{flex} > TW$ only if $I < T(v - ED_1(N)) - T(v - ED_1(1))$

■

Calculations associated with Section 4.2

Derivation of equation (21):

With duration of a long-term contract T, and assuming that C has downward-sloping demand curve $q = max\,[0, 1-p]$, the Nash bargaining problem between A and C is:

$$p = \arg\max_p (\mathbf{U_C U_A})$$

where C's net payoff is $\mathbf{U_C} = T\left[\frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}\right] - I_C$ and A's is $\mathbf{U_A} = T\left[(p - ED_1(N))(1-p)\right] - I_A$.

Thus:

$$p = \arg\max_p \left\{T\left[\frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}\right] - I_C\right\}\left\{T\left[(p - ED_1(N))(1-p)\right] - I_A\right\}$$

Expanding the expression above:

$$p = \arg\max_p \; T^2\left[\frac{(p - ED_1(N))(1-p)^3}{2} - \frac{(p - ED_1(N))(1-p)(1-\overline{p})^2}{2}\right] - TI_A\left[\frac{(1-p)^2}{2} - \frac{(1-\overline{p})^2}{2}\right] - TI_C\left[(p - ED_1(N))(1-p)\right] + I_C I_A$$

This has the first order condition:

$$\frac{T^2}{2}\left[-3(1-p)^2(p - ED_1(N)) + (1-p)^3 + (1-\overline{p})^2(p - ED_1(N)) - (1-\overline{p})^2(1-p)\right] + TI_A(1-p) - TI_C(1 - 2p + ED_1(N)) = 0$$

Simplifying and re-organizing the equality:

$$\frac{T^2}{2}\left[(1-p)^2(1 - 4p + 3ED_1(N))\right] + T\left[I_A(1-p) - I_C(1 - 2p + ED_1(N))\right] = \frac{T^2}{2}\left[(1-\overline{p})^2(1 - 2p + ED_1(N))\right]$$

Multiplying the whole expression by $\frac{1}{T^2}$:

$$\left[\frac{(1-p)^2}{2}(1-4p+3ED_1(N))\right] + \frac{1}{T}\left[I_A(1-p) - I_C(1-2p+ED_1(N))\right] = \left[\frac{(1-\overline{p})^2}{2}(1-2p+ED_1(N))\right] \quad (24)$$

to obtain equation 21 on the main text.

Corollary 4:

1. There is a price $p < \overline{p}$ such that $CS(p) > CS(\overline{p})$:

   Equation 21 is equivalent to:

$$CS(p)(1-4p+3ED_1(N)) + \frac{1}{T}\left[I_A(1-p) - I_C(1-2p+ED_1(N))\right] = CS(\overline{p})(1-2p+ED_1(N))$$

   Dividing the whole expression by $(1-2p+ED_1(N))$:

$$CS(p)\left[\frac{(1-4p+3ED_1(N))}{(1-2p+ED_1(N))}\right] + \frac{1}{T}\left[I_A\frac{(1-p)}{(1-2p+ED_1(N))} - I_C\frac{(1-2p+ED_1(N))}{(1-2p+ED_1(N))}\right] = CS(\overline{p})\left[\frac{(1-2p+ED_1(N))}{(1-2p+ED_1(N))}\right]$$

   which is equivalent to:

$$CS(p)\left[\frac{(1-4p+3ED_1(N))}{(1-2p+ED_1(N))}\right] + \frac{1}{T}\left[I_A\frac{(1-p)}{(1-2p+ED_1(N))} - I_C\right] = CS(\overline{p})$$

   From the proof of Proposition 7, we know that:

$$\frac{1-4p+3ED_1(N)}{1-2p+ED_1(N)} = 1 - 2\frac{p-ED_1(N)}{1-2p+ED_1(N)} < 1$$

for $p < \frac{1+ED_1(N)}{2} = p_m$

Furthermore,

$$\frac{(1-p)}{1-2p+ED_1(N)} < 1$$

for $p < \frac{ED_1(N)}{2}$

2. The price p is a decreasing function of $ED_1(N)$

   It follows from the proof of proposition 7 and the fact that

$$\frac{d}{dED_1(N)}\left(\frac{(1-p)}{(1-2p+ED_1(N))}\right) = -\frac{(1-p)}{(1-2p+ED_1(N))}$$

Proof of Proposition 11.

If C and A enter a long-term contract of duration T over the control of the flexible load, $U_C = \widetilde{U}_C = T\left[\frac{(1-p)^2}{2}\right] - I_C$ and $U_A = \widetilde{U}_A = T\left[(p - ED_1(N))(1-p)\right] - I_A$, because the load is delivered to C by A. Therefore,

$$\begin{aligned} TW_{flex} &= U_C + U_A \\ &= \widetilde{U}_C + \widetilde{U}_A \\ &= T\left[\frac{(1-p)^2}{2}\right] - I_C + T\left[(p-ED_1(N))(1-p)\right] - I_A \\ &= T\left[\frac{(1-p)^2}{2} + (p-ED_1(N))(1-p)\right] - I \end{aligned}$$

where $I = I_C + I_A$.

In contrast, if C and A and do not reach an agreement, then $U_C = \overline{U}_C = T\left[\frac{(1-\overline{p})^2}{2}\right]$ but $U_A = \overline{U}_A = 0$ because it is the retailer who delivers the load, whose payoff is $U_R = T\left[(\overline{p} - ED_1(1))(1-\overline{p})\right]$. Hence,

$$\begin{aligned}
TW &= U_C + U_R \\
&= \overline{U}_C + U_R \\
&= T\left[\frac{(1-\overline{p})^2}{2}\right] + T\left[(\overline{p} - ED_1(1))(1-\overline{p})\right] \\
&= T\left[\frac{(1-\overline{p})^2}{2} + (\overline{p} - ED_1(1))(1-\overline{p})\right]
\end{aligned}$$

From Proposition 8,

$$T\left[\frac{(1-p)^2}{2} + (p - ED_1(N))(1-p)\right] > T\left[\frac{(1-\overline{p})^2}{2} + (\overline{p} - ED_1(1))(1-\overline{p})\right]$$

but $TW_{flex} > TW$ holds only if

$$I < T\left[\frac{(1-p)^2}{2} + (p - ED_1(N))(1-p)\right] - T\left[\frac{(1-\overline{p})^2}{2} + (\overline{p} - ED_1(1))(1-\overline{p})\right]$$

■

Chapter 4

Flexibility Enabling Contracts in Electricity Markets

Flexibility-Enabling Contracts in Electricity Markets

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Abstract

As the share of intermittent renewable energy increases in the generation mix, power systems are exposed to greater levels of uncertainty and risk, which requires planners, policy and business decision makers to incentivise flexibility, that is: their adaptability to unforeseen variations in generation and demand. As a commodity, flexibility has multiple attributes such as capacity, ramp rate, duration and lead time, among which there are complementarities. This paper asks the fundamental question of *how should the provision of flexibility, as a multi-dimensional commodity, be incentivised?* To answer it, this paper proposes a model of bilateral trade in an environment characterized by multidimensional adverse selection. Through a simulation analysis, the paper also elucidates the applicability of the proposed model and demonstrates the way in which it can be utilised in, for example, a thermostat-based demand response programme.

Keywords: Flexibility, multi-attribute, bilateral contracts, electricity markets, mechanism design

JEL classification: L94, D82, D86

1. Introduction

Integrating renewables into electricity systems poses a number of operational challenges (on this topic see, e.g. Morales et al. (2014)). Transmission system operators, which ensure system balancing at all times, must face higher short-term uncertainty as renewable production is stochastic. In the absence of widely available electricity storage, there is a growing need for power systems to adapt to the fluctuation imposed by renewables, that is: to increase their flexibility. While a universal definition for *flexibility* doesn't exist in the technical literature on power systems, it is the encompassing word used to describe the ability of power systems to respond to demand and supply variations over various time horizons.[1] Moreover, while the term is relatively new, the concept is not, as the challenges related to variable demand and generation outage have existed since the dawn of the power industry.

Given the increased deployment of renewables, the technical and policy literatures on power systems have shown greater interest in the topic of flexibility. For example, Lannoye et al. (2012) and Ulbig and Adersson (2015) have focused on the important question of measuring power system flexibility, while associations like EURELECTRIC (2015) have focused on regulatory recommendations to increase the role of demand side flexibility in European electricity markets.

However, very few studies have recognised the need to incentivise the provision of power system flexibility or that it has multiple attributes or dimensions. One notable exception is the report by Ela et al. (2014) who focus on the incentives to enable flexibility in short-term power system operation and claim that "*different types* of resources excel at *different forms of flexibility*, and they also have *different cost impacts* when providing flexibility".

In line with Ela et al. (2014), a key claim of this paper is that - unlike commodities traded in existing electricity markets, such as energy or capacity - power system flexibility is a commodity with multiple attributes. These include, for example, capacity, ramp rate, duration and lead time for demand-side resources. Therefore, buyers have different preferences over the elements that compose flexibility, but sellers are also constrained by the technology they possess, creating thus heterogeneity in the commodity space of flexibility. From the buyer perspective, the value of different attributes of flexibility depends on the specific purpose and conditions (e.g., sometimes for the user, ramp rate is more important than other features of flexibility and some other times duration of response). On the supply side of the market, sellers of flexibility also have different degrees of efficiency across flexibility components.

The inherent multi-attribute nature of flexibility, its heterogeneity and the imperfect complementarity among the consumption and production of its composing elements create a set of unique characteristics that have not been analysed thus far. Its economic implications are much more than a theoretical curiosity and have implications of practical relevance for energy policy makers in general and system operators in particular, because they must incentivise the efficient provision of flexibility if an increasing reliance on renewables is to be achieved.

Furthermore, flexibility is topically relevant as recent technological innovations have sparked new business models that are attracting new actors to trade with different forms of flexibility. Among the new players are distribution system operators, who are expected to have an increasing role as buyers of flexibility to manage congestion. Other market players include retailers, aggregators and balancing-responsible parties who trade for portfolio optimisation purposes (Boscán and Poudineh, 2016).

[1] The reader is referred to Boscán (2016b) for a literature review on power system flexibility and its product design perspective.

The emergence of new technologies along with the greater use of ICT in the power infrastructure have enabled the provision of flexibility from many small resource providers (such as households). This is, in fact, a feature of the future decentralised power systems in which the role of small players will become more valuable to the system especially at an aggregated level. Against this background, the central question of this paper is of an *economic* nature and can be summarised in a sentence, namely: *how should the provision of flexibility be incentivised from these small resources?* Specifically, how does a utility company, system operator or an aggregator compensate owners of flexibility-enabling assets in a power system to supply flexibility? The incentive can be provided through bilateral contracts or auctions (see Boscán, 2016a) specifically designed to account for the economic properties of flexibility. However, due to presence of high transaction cost relative to the size of resource, the small resources providers cannot participate directly in an organised market and compete against each other. Therefore, this situation creates a trading environment in which "efficient" bilateral contracts are the natural method of procurement.

This paper fills an existing gap and contributes to the literature in three different ways. In section two, we discuss the concept of power system flexibility by explaining its relevance, the different sources from which it can be obtained, how it is traded in existing markets, and its main economic properties.

The second and the main contribution of this paper is in power system economics, as section three proposes and solves a static bilateral contracting model with bi-dimensional adverse selection, which appeals to the topic of flexibility but, more generally, to the procurement problem. [2] A buyer of flexibility - the principal - procures the two composing elements of flexibility from a seller - the agent - who has private information about the unit cost of supplying each component. While the model is presented in a bilateral setting, it can be extended to instances where competition among suppliers is feasible, but this paper focuses on trading environments where competition is not realistically viable. For example, contracts between an aggregator and a household or contracts between a DSO and a household for obtaining flexibility through the installation of smart energy management systems.[3] In such cases, transaction costs associated to market access or scale of the offer from a specific agent prevent competition to exist.

The model is presented in a sufficiently general form, which accounts for a wide range of specific functional forms, and the solution assumes non-separability[4] in both the principal's gross utility and the agent's cost function.[5] In this way, instances where separability holds are special cases. To gain tractability, we solve a relaxed version of the fully constrained optimisation program which gives rise to the most economically relevant situations. Within this program, we analyse five different cases, which stem from the covariance of types – which determines if an agent's efficiency in *one* dimension of flexibility can be used to predict or not its efficiency in the *second* dimension of flexibility – namely: perfect correlation, positive correlation,

[2] Although flexibility is *multi*-dimensional in nature, our model is *bi*-dimensional. Such a simplification was introduced for the sake of tractability.

[3] In separate work, Boscán and Poudineh (in preparation) analyse bilateral contracts in a decentralised competitive market and multi-attribute auction models for procuring flexibility services.

[4] Non-separability means two (or more) elements of commodity (here flexibility) must be produced or consumed together. In other words, a provider of flexibility either produces all attributes (e.g., capacity, ramp rate, duration) at each time or none of them. Similarly, a user of flexibility either consumes all of the elements together or none of them as they cannot be separated. Later in the paper we show that this non-separability feature has a great impact on the specifications of efficient contracts for flexibility services.

[5] Throughout the paper we refer to the principal's "gross utility" but a mathematically equivalent interpretation is that of a non-separable, multiple input production function where the inputs are the attributes that compose flexibility and the output is used in an internal production process. For example, a DSO that procures "capacity" and "duration" utilises both elements to produce flexibility that alleviates congestion in the network, although this output does not necessarily have a market value but a value that is relevant for the DSO's overall profitability. In the DSO's case, an alternative to procuring the composing elements of flexibility and using it in the internal production process would be expanding the network, which may not be the most efficient alternative.

weak correlation and negative correlation with asymmetry towards each one of the two existing middle types.

The third contribution appears in section four of the paper, where specific functional forms to characterize the optimal contracts in simulated bilateral environments are presented. Specifically, we analyse existing thermostat-based response programs. The last part of the paper concludes.

Related economic literature

On the theoretical side, there is a considerable body of economic related research. First, it has been acknowledged by Che (1993), Parkes and Kalagnanam (2005), and Asker and Cantillon (2010) that procurement is rarely concerned exclusively with one attribute and its price. Buyers of products and services in different industries usually take quality, materials, managerial performance among other considerations into account when offering a contract.

Unlike Che (1993), Asker and Cantillon (2010) and, more recently, Li et al. (2015) depart from the uni-dimensional mechanism design paradigm and analyse procurement in environments with multi-dimensional private information. While our work coincides with theirs in the multi-dimensional procurement approach, it differs with Asker and Cantillon's (2010) in the competitiveness of the environment and with Li et al.'s (2015) in the number of agent types considered.

The second area of related literature is concerned with the multi-dimensional screening approach surveyed by Rochet and Stole (2003). Within this area of research, the paper by Armstrong and Rochet (1999) has been highly influential in our work as it presents a complete and tractable analysis of optimal contracts with four types of agents. Our model, however, generalizes their approach in a significant way as we assume non-separability, whereas they assume additively separable utility and cost functions. This is an important distinction as it introduces a relevant economic insight into the analysis, which we have termed as the "non-separability effect", closely related to the topic of non-separable externalities discussed by Davis and Whinston (1962) and Marchand and Russell (1973). The model by Dana (1993) is also related, as it coincides with ours in the number of agents considered.

From the principal's perspective, the composing elements of flexibility are never perfectly substitutable and create an externality in the sense that the marginal utility of *one* of the components always depends on the *other* component (in a bi-dimensional setting). Symmetrically, from the agent's perspective, the marginal cost to produce one of the composing elements of flexibility depends on the output level selected for the second component. Therefore, whenever the principal is more interested in a specific element (for example, duration of response), he will have to compensate the agent with a price that not only depends on the marginal cost of that element but also on the level of output of the second element (for example, ramp rate or capacity), regardless of his valuation for it. Most other models of procurement under multi-dimensional screening, e.g., Asker and Cantillon (2010), Li et al. (2015), Laffont and Martimort (2002), have avoided such complications.

2. Power system flexibility

A distinctive feature of power systems is that they require instantaneous equilibrium between supply and demand. Traditionally, utilities have operated with fairly predictable and mature technologies. To deal with the challenges of *uncertainty* and *variability*, which aren't new, a stock of balancing services and reserves have been available to system operators to ensure that the system remains in balance second by second.

However, as decarbonisation climbs up in the policy agenda and renewable generation becomes more relevant in power systems throughout the world, increased uncertainty and variability represent greater challenges for system operation. With substantial shares of renewables, the system operator's problem is to predict fluctuations in the *net load*, which is the difference between total demand (load) and variable generation, that is: demand that must be met by *other sources* if all renewable generation is utilised.[6] This magnitude is harder to predict accurately – i.e. contains greater uncertainty– as it depends on two random variables, namely demand and renewable generation.

Variability of the net load has technical and economic impact on the overall generation base of power systems. In ideal circumstances, demand and renewable generation would be positively correlated: demand is high when renewables are available or, conversely, demand is low when renewables become scarce. If this is the case, generators face *shorter peaks*, implying fewer operating hours and lower economic compensation for existing power plants. Which, of course, raises the related question of resource adequacy: How can system reliability be ensured? How can investments in baseload power plants be incentivised if, as a consequence of the greater reliance on renewables, these receive lower compensation? This is, however, a fundamentally distinct question from that of renewable integration: *How to tackle the operational challenges implied by the greater variability imposed by renewables?* The short answer to this question is "flexibility".

When demand and renewable supply are negatively – and therefore unfavourably – correlated, the remaining generation base experiences *steeper ramp ups* and *deeper turn downs* (Katz and Cochran, 2015).[7] If renewable supply decreases together with increases in demand, system operators must dispatch generation that is able to ramp up quickly. On the contrary, if renewable supply is high when demand is low, the generation base faces deeper turn downs as they must give way for renewables to satisfy demand. In other words, operators require resources – *flexibility-enabling assets* – that modify demand or output in order to follow net load fluctuations. But the question remains: what should the owners of these assets modify in order to help the operator meet net load variations? Supply and demand must indeed be modified, but for how long, for how much and at what cost? More precisely, what are the *exact* requirements of the operator? Is it capacity? Is it duration? Is it the ramp rate? Is it the lead time? Or is it a combination of these elements?

From a *technical* perspective, Ulbig and Andersson (2012), extending the work of Marakov et al. (2009), elucidate these questions. Focusing on "individual power system units" (a synonym of flexibility-enabling assets), they propose the following *flexibility trinity* to measure flexibility:

a) Power capability P for up/down regulation (measured in MW),
b) Energy storage capability E (measured in MWh), and
c) Power ramping capability R (measured in MW/min),

[6] The term "sources" is employed here in its widest possible sense: it could refer to generation, conventional or not, but it could also involve any change in demand that helps to keep the system in balance.
[7] Morales et al. (2013) note that this is typically the case in places like Northern Europe and Texas: renewable supply and demand are negatively correlated.

The three magnitudes are related via integration and differentiation over the time domain, as figure 1 shows. A fourth, related metric is ramping duration D, which is defined as the ratio of power to the ramp rate, $D = \frac{P}{R}$.

While Ulbig and Andersson (2012)'s trinity could be considered incomplete by some to measure flexibility,[8] their approach highlights its fundamental *economic* characteristics:

1. *Flexibility has multiple attributes*: unlike other commodities traded in existing electricity markets, such as energy or capacity, it is not possible to measure flexibility with a single metric. This feature is economically relevant because ranking the flexibility coming from enabling assets or the agents' cost of supplying flexibility is not straightforward unless precisions regarding the multiple attributes that compose flexibility are made. Statements such as "Flexibility-enabling asset A is more flexible than flexibility-enabling asset B" are not valid, unless a clarification of what dimension of flexibility the statement refers to. Instead of a single good, it is convenient to think of flexibility as a bundle of goods.[9]

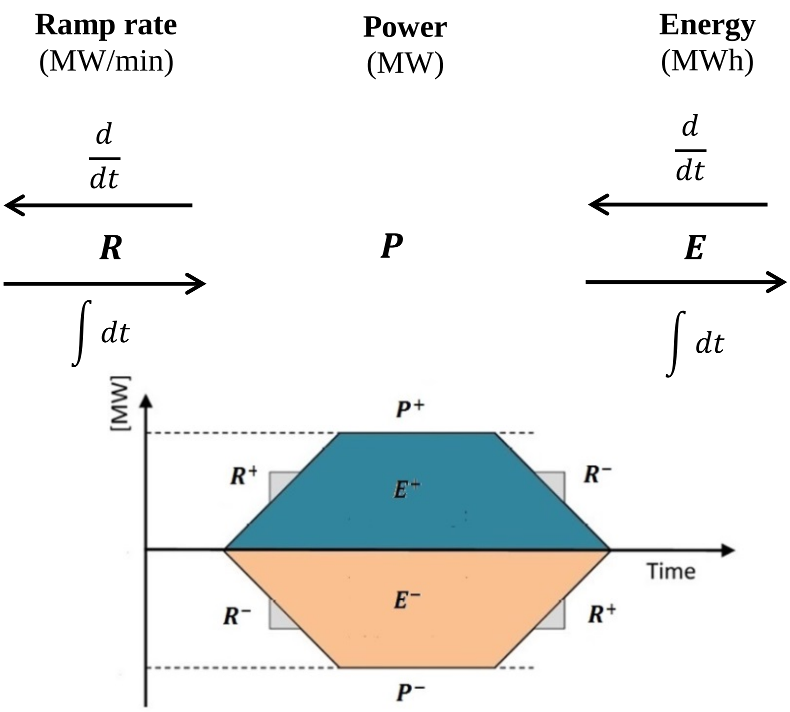

Figure 1:
Flexibility in Power Systems Operation: Ramp rate (R), Power (P), Energy (E)
Source: Ulbig and Andersson (2012)

[8] For example, EURELECTRIC (2014) considers location, while Zhao et al. (2015) include uncertainty and cost as composing elements of flexibility.
[9] It could be argued that *any* commodity has multiple attributes as well. All pencils, for example, have height and colour. But in existing markets (e.g. the day-ahead market), electricity is measured in MWh, whereas flexibility has several dimensions.

2. *Flexibility is a heterogeneous commodity*: in contrast to homogenous commodities, which have the *same* characteristics across its attributes, flexibility naturally has *different* characteristics across them. A flexibility-enabling asset or agent can be efficient in one dimension but inefficient in another, creating heterogeneity. Consider, for example, a comparison between pumped-storage and nuclear power: the first has a very short lead time (can be started up from "cold" quickly) and has a steep ramp rate, whereas the second has a long lead time (cannot be started from "cold") and a much flatter ramp rate (EURELECTRIC, 2011).

3. *The elements that compose flexibility are imperfect complements*: in addition to being technically interrelated as explained before, the components of flexibility are imperfect complements because it is not possible to create any value (e.g. to the system operator) without having positive quantities of at least two flexibility components. Therefore, it is convenient to assume that the final user of flexibility has convex indifference curves (or isoquants) over the bundle of flexibility components.[10] Interestingly, the imperfect complementarity nature of flexibility can be traced back to the work of Marakov et al. (2009) who propose a "*concurrent* consideration of … capacity, ramping and … duration".

The techno-economic vision of flexibility taken in this paper is summarised in figure 2: a multi-dimensional (not necessarily three-dimensional as in figure 2), heterogeneous commodity whose components are imperfect complements. Supply-side resources (e.g. generation) are assumed to be composed – at least – by capacity, duration and ramp rate. Likewise, demand-side resources are assumed to be composed by – at least – capacity, duration and lead time (the time elapsed between agreement and delivery). From a different perspective, if each axis in the figure 2 represents the cost or disutility (when the flexibility provider is for example a household) of providing that dimension of flexibility, then an efficient flexibility contract can be interpreted as a mechanism that minimises the size of the cube shown in the figure.

Moreover, it is worth noting that the economic properties of flexibility refer both to the flexibility-enabling assets and to the agents who own the assets. However, when it comes to incentivising the provision of flexibility, there are significant distinctions between both. When considering supply-side resources like generation, owners of power plants have technical constraints to supply flexibility but will typically behave in an economically rational way, i.e. as profit maximisers. Owners of flexibility-enabling assets participating in, say, a demand response program, can be assumed to be relatively uniform regarding the assets they possess (e.g air conditioners), but their cost of provision (disutilities) should not be straightforwardly assumed to be uniform. The latter are consumers and may react to behavioural elements beyond utility maximisation.

[10] Read footnote 5.

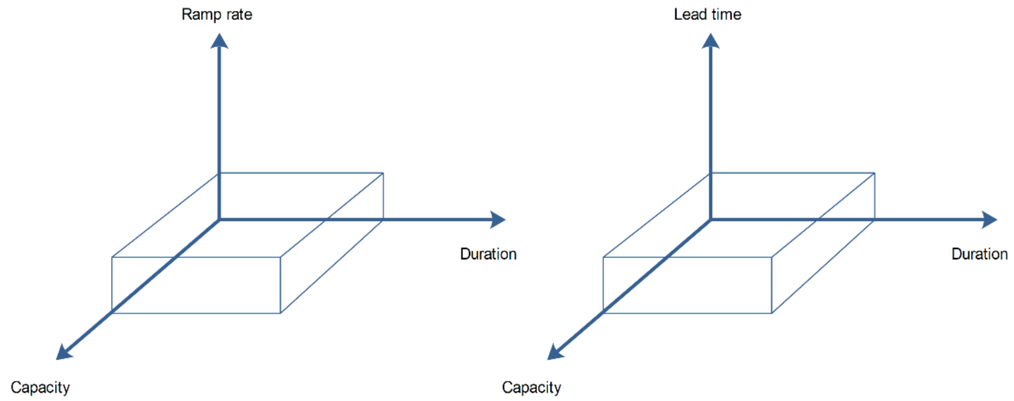

Figure 2:
Dimensions of flexibility for a) a supply side resource (left) b) a demand side resource (right)
Source: Authors

There are a number of resources that enable operational flexibility in power systems, including both physical (i.e. flexibility-enabling assets) and other elements, including institutional design. The former includes generation facilities – which depending on its attributes – are positioned to supply flexibility,[11] storage (pumped hydro, thermal storage and batteries), interconnections with neighbouring networks and demand-side management. The latter comprehends elements like the power grid, which does not actually provide additional flexibility but severely limits the provision of flexibility if sufficient transmission capacity is unavailable. Market and contract design are another relevant element that can enable flexibility: in the absence of adequate trading mechanisms, market participants are unable to trade flexibility even if sufficient physical resources exist. Figure 3 summarises these elements.

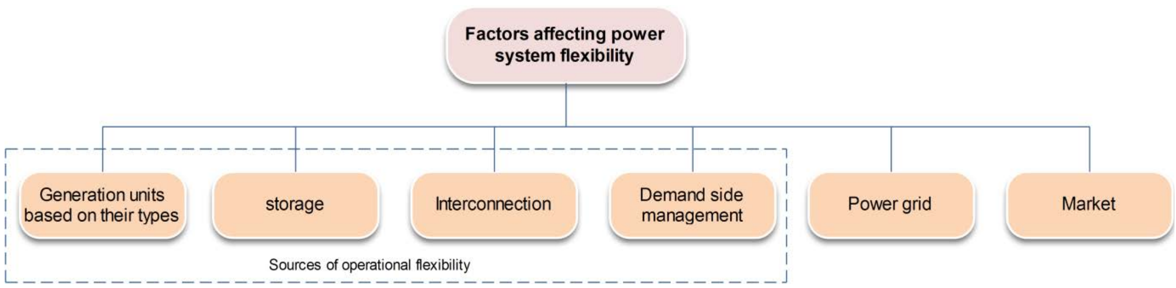

Figure 3: Factors affecting flexibility
Source: Authors

It is worth noting that despite the fact that flexibility has always been needed and indeed used in the operation of power systems, it has never been traded as a distinct commodity. Usually, conventional generators have been able to provide the flexibility that the system requires and have been compensated on the basis of its energy and/or capacity component. However, as the requirement for flexibility increases in

[11] In a comparative study of the European generation fleet in terms of its technical flexibility, EURELECTRIC (2011) considered that warm and cold start-up times, the rapidity of load change (i.e. the ramp rate), the minimum load level and the shutdown time were the relevant attributes to describe the *technical* flexibility of power plants, also referred to in this paper, as *resource* flexibility.

power systems, there is a need for designing a product that can be bought and sold, along with other components of electricity services such energy and capacity (on this topic, see Boscán (2016b)).

Mainly, final users of flexibility are transmission system operators (TSOs), Independent System Operators (ISOs) – who are responsible for balancing the high-voltage grid – distribution system operators (DSOs) – who are responsible for the reliable operation of distribution networks and quality of service – together with other market players who may use flexibility to meet their energy and balancing obligations. Access to flexible resources to manage network constraints leads to an effective integration of distributed energy resources, allows network companies to optimise on their network reinforcement capital investments, and improve the reliability and quality of service. It also enables other market players to optimise their energy portfolio in order to meet their energy market and balancing obligations at minimum cost by, for example, arbitrating between generation and demand response (Boscán and Poudineh, 2016).

Overall, electricity markets can be centralised or decentralised: they can be based on a tightly controlled pool, centralised exchange or bilateral contracts. Trade can include physical and/or financial obligations through forward and spot contracts and the market may allow for financial hedging. The main official market can be mandatory or optional and allow or disallow secondary markets.

Depending on the specific situation, flexibility can be procured in a competitive setting, where a single buyer incentivises suppliers to compete. Alternatively, when competition isn't feasible – as assumed in this paper – a buyer of flexibility can offer contracts to sellers who do not compete against each other. For example, a contract in which a DSO or an aggregator offers a household to provide demand side flexibility services.

Bilateral procurement contracts without competition among sellers exist across the supply chain of flexibility services for various reasons. For example, direct participation of the flexibility resource provider in an organised market may not be feasible always because of insufficient capacity size. This is important as there are diverse classes of consumers, ranging from households to large industrial units who can provide flexibility. Due to high transaction costs, small resources may face barriers to directly access the market as opposed to large industrial or commercial consumers. Aggregation provides an opportunity for small generation and demand resources to offer their flexibility in the market (Eurelectric, 2014). In this case, the aggregator can be a retailer or a third party who acts as intermediary between providers and buyers of flexibility. Additionally, in many European countries intermittent renewables are being treated as conventional generation in the sense they have the same obligations for their imbalance position and entitlement to participate in balancing market (for example, in Denmark, Finland, Estonia, Netherlands, Spain and Sweden renewable resources have full balancing responsibility). This encourages not only improved forecasting but also entry of competitive aggregators whose role is to minimise balancing risks and offer ancillary services from renewable resources. Furthermore, this provides incentives for renewables to be firmed up by, for example, entering into separate contracts with owners of flexible resources such as residential demand response and storage facilities.

The Nest Learning Thermostat is a good real-world example of how bilateral contracts for flexibility services can be utilised in the integration of renewable resources. Nest, as the manufacturer of smart thermostat technology, partners with utility companies to provide a residential demand response program. Under the so-called "Rush hour scheme", the contracted consumer's consumption (air conditioner temperature for instance) is adjusted automatically by a utility company to manage fluctuation of demand and supply. The consumers are offered a menu of contracts with different lead times (from on-demand to 24-hour notice in advance, for example), duration of adjustments in consumption (30 minutes to 4 hours for instance) and payments. The contract design problem arises because different consumers experience different disutilities for the various dimensions of flexibility they provide, and such disutilities are a privately held piece of information held by the resource provider. For example, one household may incur a high disutility for the short lead time and another household for long duration of load control. Logically, the former household prefers a contract with higher lead time but can sacrifice on load control duration, whereas the latter values more a contract with shorter load control duration. Therefore, the contracts *should* be (and currently are not necessarily being) designed in a way that each participating agent truthfully self-selects its own contract, given the presence of multidimensional information asymmetry between the buyer and sellers. The bilateral contract model of this paper focuses on this category of contractual settings in electricity markets.

3. The model

Let q and t be any two composing elements of flexibility that a buyer – the principal – procures, in exchange for a transfer T, from a seller – the agent – who has asymmetric information parameters θ_q, θ_t. Note that in contrast to the claim that flexibility is a multi-attribute commodity stated in section 2, and for the sake of tractability, the model reduces the *multi*-dimensionality of flexibility to a *bi*-dimensional setting.

In the context of flexibility, q and t can be the lead time, duration, capacity or the ramp rate of a generator, for example. More generally, beyond the context of flexibility, q and t can be any two activities or characteristics of a product delegated on an agent by a principal, who is uncertain about the unit cost of production θ_q, θ_t. These can also be thought of as relevant parameters of any production process with multiple attributes. For example, materials, design, quality, product features (Li et al., 2015; Asker and Cantillon, 2010).

Net payoffs to the principal and agent are, respectively:

$$W = v(q, t) - T$$

and

$$U = T - c(q, t, \theta_q, \theta_t)$$

where $v(q, t)$ and $c(q, t, \theta_q, \theta_t)$ are the utility and cost function of the principal (buyer) and the agent (seller) respectively, which results from consuming and producing flexibility.[12] Besides linearity of T in W and U, which ensures the risk neutrality of the principal and the agent, few additional assumptions are made:

Assumption 1: the gross utility $v(q, t)$ of the buyer and the seller's cost $c\big(q, t, \theta_q, \theta_t\big)$ are twice differentiable functions.

Assumption 2 :$v(q, t)$ satisfies:

2a: Monotonicity, i.e. $\frac{\partial v}{\partial q}, \frac{\partial v}{\partial t} > 0$ and

[12] A mathematically equivalent interpretation of $v(q, t)$ is that of a production function with inputs q and t which combined produce an output (flexibility) employed in an internal production process.

2b: Concavity, i.e. $\frac{\partial^2 v}{\partial q^2}$, $\frac{\partial^2 v}{\partial t^2} < 0$

Assumption 3: $c(q, t, \theta_q, \theta_t)$ satisfies:

3a: Monotonicity in all its parameters, i.e., $\frac{\partial c}{\partial q}, \frac{\partial c}{\partial t}, \frac{\partial c}{\partial \theta_q}, \frac{\partial c}{\partial \theta_t} > 0$,

3b: Concavity in θ_q, θ_t, i.e. $\frac{\partial^2 c}{\partial {\theta_q}^2}$, $\frac{\partial^2 c}{\partial {\theta_t}^2} < 0$,

However, no specific curvature assumption of $c(q, t, \theta_q, \theta_t)$with respect to q and t is made. That is, $c(q, t, \theta_q, \theta_t)$ can be convex (if $\frac{\partial^2 c}{\partial q^2}, \frac{\partial^2 c}{\partial t^2} > 0$), concave (if $\frac{\partial^2 c}{\partial q^2}, \frac{\partial^2 c}{\partial t^2} < 0$) or both (i.e., linear).

Assumption 4: the Spence-Mirrlees (constant sign) conditions hold for U with respect to q and t:

$$\frac{\partial}{\partial \theta_q}\left(\frac{\partial U/\partial q}{\partial U/\partial T}\right), \frac{\partial}{\partial \theta_t}\left(\frac{\partial U/\partial t}{\partial U/\partial T}\right) < 0$$

These assumptions are sufficiently general to account for a wide range of specific functional forms. For example, $v(q, t)$ and $c(q, t, \theta_q, \theta_t)$ can be non-separable, weakly separable or additively separable. The solution to the model, though, assumes non-separability such that instances where separability holds are special cases.

As is standard in static bilateral contracting, nature determines the agent type which, in this case, is a pair of parameters(θ_q, θ_t) in the agent's cost function with two possible realizations. Each parameter can be either "High" (H) or "Low"(L), i.e., $\theta_q \in \{\theta_q^H, \theta_q^L\}$ and $\theta_t \in \{\theta_t^H, \theta_t^L\}$. Nature reveals the type to the agent but not to the principal, resulting thus in an adverse selection problem.

However, the distribution of types is common knowledge to both players. Namely, there are four types of agent:

1. Agent type LL characterised by (θ_q^L, θ_t^L) with probability p_{LL}
2. Agent type LH characterised by (θ_q^L, θ_t^H) with probability p_{LH}
3. Agent type HL characterised by (θ_q^H, θ_t^L) with probability p_{HL}
4. Agent type HH characterised by (θ_q^H, θ_t^H) with probability p_{HH}.

We refer to the LL type as the "efficient" because it is capable of offering the lowest cost product in both dimensions of flexibility. The HH type is called the "inefficient" because it has the highest cost in both activities. Likewise, LH and HL types are the "middle types", because they are efficient in one of the activities but inefficient in the other one. We refer to the LH, HL and HH together as the "less efficient" types.

3.1. Dependence of events and covariance of types

The following table summarizes the joint probability distribution of parameters θ_q and θ_t:

		θ_q		
		θ_q^L	θ_q^H	
θ_t	θ_t^L	p_{LL}	p_{HL}	$p_t^L = p_{LL} + p_{HL}$
	θ_t^H	p_{LH}	p_{HH}	$p_t^H = p_{LH} + p_{HH}$
		$p_q^L = p_{LL} + p_{LH}$	$p_q^H = p_{HL} + p_{HH}$	

Table 1: Joint and marginal probability distribution of types

Here, p_q^L, p_q^H and p_t^L, p_t^H are the marginal (unconditional) probabilities that events θ_q and θ_t happen to be, respectively, "High" or "Low". Clearly, it must always be the case that $p_q^L + p_q^H = p_t^L + p_t^H = 1$.

A crucial question is how to formalise a general relationship among the probability distribution parameters to reflect the possibility of dependence or independence of events. For example, if the result of a random experiment in which the two asymmetric information parameters are drawn is the event $\theta_q = \theta_q^H$, what should the principal expect to happen: the event $\theta_t = \theta_t^H$ or $\theta_t = \theta_t^L$? Or are these two events independent of one another?

To analyze this, we use the fact that the probabilistic covariance of two events[13] $\theta_q = \theta_q^i,\ \theta_t = \theta_t^j,\ i = L, H;\ j = L, H$ is defined as:

$$cov\left(\theta_q = \theta_q^i, \theta_t = \theta_t^j\right) = p\left(\theta_q = \theta_q^i \cap \theta_t = \theta_t^j\right) - p\left(\theta_q = \theta_q^i\right)p\left(\theta_t = \theta_t^j\right)$$

which leads to the following concise expression for the *covariance of types*:[14]

$$cov = p_{LL}p_{HH} - p_{HL}p_{LH}$$

It is without loss of generality to define the following three possible cases:

i. Positive correlation if and only if $p_{LL}p_{HH} > p_{HL}p_{LH}$
ii. Negative correlation if and only if $p_{LL}p_{HH} < p_{HL}p_{LH}$
iii. Absence of correlation if and only if $p_{LL}p_{HH} = p_{HL}p_{LH}$

[13] The formula for covariance presented in the text is based on a probabilistic view of θ_q and θ_t as events, not as random variables. There is a probability associated to the occurrence of each event. Formally: if A, B are events then $cov(A, B) = P(A \cap B) - P(A)P(B)$.
[14] The discrete set of agents considered and the expression for covariance of types coincides with the setup of Armstrong and Rochet (1999) and Dana (1993).

3.2. Relaxing the fully constrained program

In this bi-dimensional setting, $S(q_{ij}, t_{ij}) = W_{ij} + U_{ij}$ is the total surplus derived from trading goods q_{ij} and t_{ij}. W_{ij} and U_{ij} stand for, respectively, the net payoffs to the principal and agent associated to the ij-type.

Without informational asymmetry, the principal is able to observe cost and obtains first-best levels q_{ij}^{fb} and t_{ij}^{fb} where $q_{ij}^{fb}, t_{ij}^{fb} = \arg\max_{q_{ij}, t_{ij}} S(q_{ij}, t_{ij})$, such that:

$$\frac{\partial v(q_{ij}^{fb}, t_{ij})}{\partial q_{ij}} = \frac{\partial c(q_{ij}^{fb}, t_{ij}, \theta_q{}^i, \theta_t{}^j)}{\partial q_{ij}} \text{ for all } q_{ij}$$

$$\frac{\partial v(q_{ij}, t_{ij}^{fb})}{\partial t_{ij}} = \frac{\partial c(q_{ij}, t_{ij}^{fb}, \theta_q{}^i, \theta_t{}^j)}{\partial t_{ij}} \text{ for all } t_{ij}$$

for all $i = L, H\,; j = L, H$

With adverse selection, the principal's problem is to offer a menu of contracts $\{q_{ij}, t_{ij}, T_{ij}\}$ that maximizes expected surplus,

$$E(\pi) = \sum_{ij} p_{ij} S(q_{ij}, t_{ij}) - \sum_{ij} p_{ij} U_{ij} \quad (1)$$

subject to individual rationality (IR):

$$T_{ij} - c(q_{ij}, t_{ij}, \theta_q^i, \theta_t^j) = U_{ij} \geq 0 \text{ for all } i = L, H\,; j = L, H$$

and incentive compatibility (IC) constraints:

$$U_{ij} \geq U_{i'j'} + c\left(q_{i'j'}, t_{i'j'}, \theta_q^{i'}, \theta_t^{j'}\right) - c(q_{i'j'}, t_{i'j'}, \theta_q^i, \theta_t^j) \text{ for all pairs } ij \text{ and } i'j'$$

At this point, it is convenient to introduce the following notational convention:

$$\Delta_{ij}^{i'j'}(q_{i'j'}, t_{i'j'}) = c(q_{i'j'}, t_{i'j'}, \theta_q^{i'}, \theta_t^{j'}) - c(q_{i'j'}, t_{i'j'}, \theta_q^i, \theta_t^j)$$

where $\Delta_{ij}^{i'j'}(q_{i'j'}, t_{i'j'})$ stands for the *total cost difference* between the $i'j'$ and the ij type, evaluated at output levels $(q_{i'j'}, t_{i'j'})$. Note that $\Delta_{ij}^{i'j'}(q_{i'j'}, t_{i'j'})$ can be positive, negative or zero.

Using this convention, it is also possible to write:

$$\frac{\partial \Delta_{ij}^{i'j'}(q_{i'j'}, t_{i'j'})}{\partial z_{i'j'}} = \frac{\partial c(q_{i'j'}, t_{i'j'}, \theta_q^{i'}, \theta_t^{j'})}{\partial z_{i'j'}} - \frac{\partial c(q_{i'j'}, t_{i'j'}, \theta_q^i, \theta_t^j)}{\partial z_{i'j'}} \text{ for } z \in \{q, t\}$$

to denote the *difference in marginal cost* between the $i'j'$ and the ij type when producing either $q_{i'j'}$ or $t_{i'j'}$.

With this notation, IC constraints can be simplified to:

$$U_{ij} \geq U_{i'j'} + \Delta_{ij}^{i'j'}(q_{i'j'}, t_{i'j'}) \text{ for all pairs } ij \text{ and } i'j'$$

The fully constrained program consists of four IR constraints and twelve IC constraints from which the following monotonicity conditions for production levels follow (for details, see sections A.1 and A2.1 in the appendix):

$$q_{LL} \geq q_{HL}\,,\; q_{LH} \geq q_{HH}\,, t_{LL} \geq t_{LH}\,, t_{HL} \geq t_{HH} \quad (2)$$

Adding local incentive constraints two by two reveals that it is optimal for the principal to incentivize the efficient types to produce more than the inefficient ones in both dimensions. However, this procedure does not clarify how the outputs of the middle types are ordered relative to each other. It can be shown that assuming a given order in the output levels of one of the middle types in one dimension, does not lead to a definite conclusion about the order in the corresponding output levels of the other dimension. For example, assuming that $q_{LH} \geq q_{HL}$ holds, t_{HL} can be equal, greater or lower than t_{LH} and still satisfy the corresponding cost inequality that results from adding IC constraints. Conversely, assuming a given order for t_{HL} and t_{LH} does not lead to a clear-cut conclusion about the order of q_{LH} and q_{HL}.

To gain tractability, it is convenient to relax the fully constrained program. To this end, economic reasoning helps identifying a number of constraints that can be ignored to construct a relaxed program, which is relevant to the extent that its solution satisfies the general, fully constrained program. First note that it is in the interest of the more efficient types to mimic the relatively less efficient agents because by doing so, the former choose to produce sub-optimal output levels while obtaining positive informational rents. In contrast, the less efficient types lack the incentives to mimic the relatively more efficient ones because this would imply incurring in an unnecessarily high cost. Therefore, it is reasonable to focus on the *upward* IC constraints only. But in the presence of a set of incompletely ordered agents, who mimics whom?

The efficient (LL) can mimic any of the three remaining agents but it is not immediately obvious which would give the LL-type the highest possible informational rent. Consequently, the efficient's optimal choice could involve the possibility of mimicking one, two or three of the less efficient types. Likewise, the middle types can choose to mimic the inefficient (HH) but they could also have incentives to mimic each other (see sections A2.2 and A2.3 in the appendix, for more details).

In consequence, there are seven relevant IC constraints that must be considered in *any* relaxed program that attempts to solve the fully constrained program:

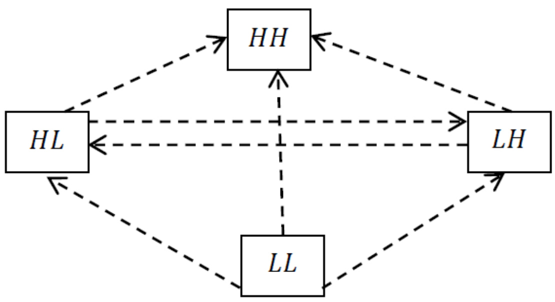

Figure 4:
Relevant IC constraints in *any* relaxed program

$$U_{LL} \geq \max\{U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH}), U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})\}$$

$U_{LH} \ \geq \max\{U_{HL} + \Delta_{LH}^{HL}(q_{HL}, t_{HL}), U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})\}$

$U_{HL} \ \geq \max\{U_{LH} + \Delta_{HL}^{LH}(q_{LH}, t_{LH}), U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})\}$

This situation is reflected in figure 4, where the dashed arrows represent the *potentially* binding IC constraints of a general program relaxation. However, analysing them simultaneously introduces considerable complexity. Instead, this paper focuses on a baseline relaxed program (one of the two possible relaxed programs) which accounts for the most economically relevant cases.[15]

3.3. A baseline relaxed program

The widest range of practically relevant cases can be covered if the IC constraints of the middle type agents to mimic each other are ignored. Specifically, by assuming that:

$$U_{HL} + \Delta_{LH}^{HL}(q_{HL}, t_{HL}) < U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})$$

and

$$U_{LH} + \Delta_{HL}^{LH}(q_{LH}, t_{LH}) < U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})$$

hold, it is always optimal for the middle types to mimic the inefficient type (HH) while ignoring their counterpart's contract. That is, the baseline relaxed program assumes that the "horizontal" constraints will never bind at the optimum.

The baseline relaxed program thus reduces to maximizing the principal's expected profit (equation (1)) subject to the following five IC constraints:

$U_{LL} \ \geq \max\{U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH}), U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})\}$

$U_{LH} \geq U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})$

$U_{HL} \geq \ U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})$

and the IR constraint for the inefficient: $U_{HH} \geq 0$

Figure 5 depicts the IC constraints of the baseline relaxed program. Note that the solid lines indicate optimally binding constraints, whereas the dashed lines indicate potentially binding constraints where *at least one* of them binds at the optimum. That is, the middle types' IC constraints with respect to the inefficient always bind while at least one of the inefficient's IC constraints with respect to the less efficient bind.

[15] The other relaxed program is the "alternative" relaxed program which accounts for situations in which it may be profitable for the middle types to mimic each other.

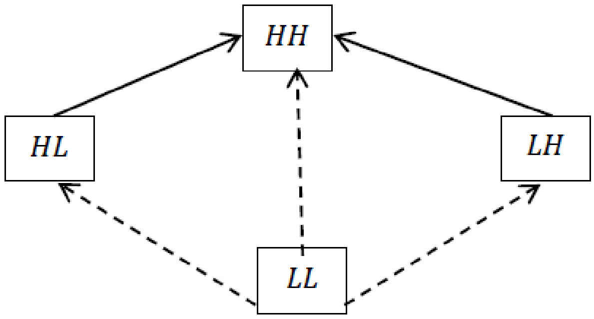

Figure 5:
Relevant IC constraints in the baseline relaxed program

At the optimum, the IR constraint for the inefficient binds ($U_{HH} = 0$), but the efficient's and middle types' IC constraints also bind. Then, substituting for the latter into the efficient's IC constraints simplifies the set of constraints to:

$$U_{LL} = \max\{\Delta_{LH}^{HH}(q_{HH}, t_{HH}) + \Delta_{LL}^{LH}(q_{LH}, t_{LH}), \Delta_{HL}^{HH}(q_{HH}, t_{HH}) + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), \Delta_{LL}^{HH}(q_{HH}, t_{HH})\} \quad (3)$$

$$U_{LH} = \Delta_{LH}^{HH}(q_{HH}, t_{HH})$$

$$U_{HL} = \Delta_{HL}^{HH}(q_{HH}, t_{HH})$$

The setup of the baseline relaxed program gives rise to four mutually exclusive cases, which depend on the optimally binding constraints for the efficient type. As will become clear, the emergence of these cases is closely related to the covariance of types and the difference in marginal cost between the least efficient type (HH) and the three other agent types: LL, HL and LH.

3.3.1. Solution to the baseline relaxed program[16]

The output levels produced under adverse selection are always second-best and are written as:

$$q_{ij}^{sb} = q(\cdot), \ t_{ij}^{sb} = t(\cdot) \qquad (4)$$

Here, $q(\cdot), t(\cdot)$ denote that the second-best output levels are, respectively, outputs q and t that deviate from the first-best solution by the expression in the argument. *So, the greater the argument, the greater the deviation from the first best solution is*. In contrast, if the argument is zero, then first-best and second-best output levels coincide[17]. That is, if $q_{ij}^{sb} = q(0) \Rightarrow q_{ij}^{sb} = q_{ij}^{fb}$; $t_{ij}^{sb} = t(0) \Rightarrow t_{ij}^{sb} = t_{ij}^{fb}$.

In the baseline relaxed program, the efficient's output levels always coincide with the first-best:

$$q_{LL}^{sb} = q_{LL}^{fb}, t_{LL}^{sb} = t_{LL}^{fb} \qquad (5)$$

but the outputs of the remaining types (HL, LH, HH) are distorted. The size of the distortion, however, will depend on each case.

[16] Technical details are outlined in Appendix, subsection A.3
[17] This happens when marginal cost equals marginal benefit and thus at the optimum point their difference is zero such that the agent produces with no distortion.

Case 1: Positive correlation

In the first case, equation (3) has no single maximum, implying that the efficient type does not have an incentive to mimic one specific agent but all the three less efficient agents. Note that perfect correlation is a subcase of case 1. The LL type does not ignore the global IC constraint with the inefficient (HH), as it would give him an equally rewarding informational rent as mimicking any of the two adjacent types, LH or HL.

Graphically, this situation is represented in figure 6, which indicates that all the IC constraints of the baseline relaxed program bind at the optimum. In particular, the LL type's IC constraints bind with respect to the three less efficient types.

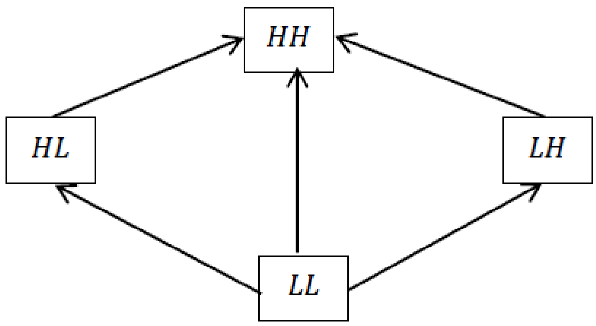

Figure 6:
Binding IC constraints under case 1 (Positive correlation)

Intuitively, this situation arises whenever the types are positively correlated such that the efficient and inefficient types are more likely than the middle types in a given distribution. This case holds whenever the covariance of types is:

$$cov\left(\theta_q^i, \theta_t^j\right) > (1 - p_{HH})\left(p_{LH} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) - p_{LH}\, p_{HL} \text{ for } z \in \{q, t\}$$

In this case, the principal finds it optimal to incentivize the efficient to produce at first-best levels, giving this agent type a rent that equals a convex combination of the informational rents that he would obtain by mimicking the less efficient agents *simultaneously*.

Essentially, the principal distinguishes between the efficient, which is able to offer q and t at the lowest cost, and the less efficient types, who compose a rather indistinguishable group. There is a resemblance of this case with that of the unidimensional adverse selection problem, in which a single parameter determines the difference between the efficient and the inefficient. By implication, the outputs of the inefficient are distorted in a way that leads to a bunching solution, i.e., $q_{LH}^{sb} = q_{HL}^{sb} = q_{HH}^{sb}$; $t_{LH}^{sb} = t_{HL}^{sb} = t_{HH}^{sb}$.

Specifically, the deviations of output relative to the first-best levels are given by:

- For the LH, HL and HH in the q attribute:

$$q_{LH}^{sb} = q_{HL}^{sb} = q_{HH}^{sb} = q\left(\frac{p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}}{p_{HH} + \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}(p_{HL} + p_{LH}) - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}}\right) \quad (6a)$$

- For the LH, HL and HH in the t attribute:

$$t_{LH}^{sb} = t_{HL}^{sb} = t_{HH}^{sb} = t\left(\frac{p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}}{p_{HH} + \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}(p_{HL} + p_{LH}) - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}}\right) \quad (6b)$$

Case 2: Weak correlation

In the second case, the global IC constraint between the efficient (LL) and inefficient (HH) becomes irrelevant. This is because the LL type can realize a higher rent by mimicking any of the middle, adjacent types (LH or HL) than by mimicking the inefficient type. In contrast to case two, this case emerges whenever the middle types become more likely in the distribution, such that the covariance of types is still positive or negative but closer to zero. The situation is depicted in figure 7, where the binding IC constraints are shown.

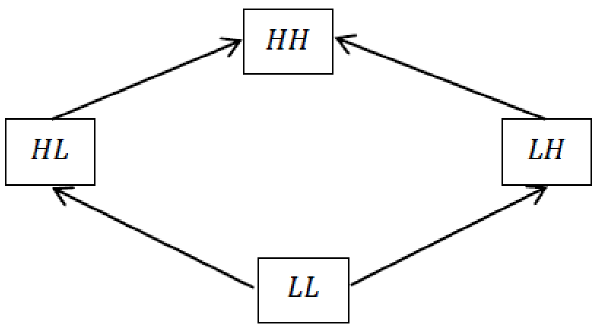

Figure 7:
Binding IC constraints under case 2 (Weak correlation)

It is rational for the efficient to always mimic the least efficient type because by doing so he obtains a higher informational rent. To avoid the rent becoming too costly, it is in the principal's interest to make the efficient type indifferent between any of the two middle types. This means that the following equality should hold:

$$\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) + \Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right) = \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) + \Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right) \qquad (7)$$

Equation (7) depends on a real number λ, and the principal's problem reduces to finding a $0 < \lambda < 1$ for which the informational rent that the LL type would realize from mimicking any of the two middle types is equal, as in (7).

To this end, the principal distorts output levels which, in turn, depend on the degree of marginal cost symmetry between them. Depending on the assumption about the marginal cost symmetry of the middle types three possibilities emerge.

Output levels, which depend on λ, are:

- For the LH type:

$$q_{LH}^{sb} = q\left(\lambda \frac{p_{LL}}{p_{LH}}\right), t_{LH}^{sb} = t\left(\lambda \frac{p_{LL}}{p_{LH}}\right) \quad \text{(8a)}$$

- For the HL type:

$$q_{HL}^{sb} = q\left((1-\lambda) \frac{p_{LL}}{p_{HL}}\right), t_{HL}^{sb} = t\left((1-\lambda) \frac{p_{LL}}{p_{HL}}\right) \quad \text{(8b)}$$

- For the HH type:

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[(\lambda\, p_{LL} + p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}} + \left((1-\lambda)\, p_{LL} + p_{HL}\right) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}}\right]\right)$$

(8c)

$$t_{HH}^{sb} = t\left(\frac{1}{p_{HH}}\left[(\lambda\, p_{LL} + p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}} + \left((1-\lambda)\, p_{LL} + p_{HL}\right) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}}\right]\right)$$

The first case is that *middle types are equally marginally efficient*, i.e., assume that:

$$\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} = \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \text{ for } z \in \{q, t\}$$

holds, meaning that both middle types would realize the same informational rent by mimicking the HH type because they are equally efficient at the margin.

It is, however, too restrictive to assume cost symmetry between the middle types and even in the cases in which there is marginal cost asymmetry we show that there is $0 < \lambda < 1$ that satisfies equation (7). One possibility is that *LH type is more inefficient than the HL type:*

$$\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} > \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \text{ for } z \in \{q, t\}$$

The other possibility is that *HL type is more inefficient than the LH type:*

$$\frac{\partial \Delta_{\mathrm{LH}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} < \frac{\partial \Delta_{\mathrm{HL}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} \text{ for z} \in \{\mathrm{q, t}\}$$

The appendix (section A.3) contains a proof for the existence of a λ that satisfies the indifference condition in (7) in all previous three possibilities.

The sufficient condition for the existence of λ and thus a solution for case three is that the following inequality holds.

$$\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) - \Delta_{LL}^{HL}(q(0), t(0)) < \Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) - \Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$$
$$< \Delta_{LL}^{LH}(q(0), t(0)) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$$

This implies that weak correlation of types case happens when there is no large asymmetry between the middle types.

Case 3: Negative correlation with asymmetry towards the LH type

This case arises whenever the distribution of types is negatively correlated because the middle types are more likely than the LL and HH type together, and contains a higher probability of observing an LH type than any other type. In this situation, the efficient will have an incentive to mimic the LH type but will, essentially, ignore the HL type. The situation is graphically illustrated in figure 8.

It is optimal for the principal to distort the LH type but not the HL type, whose output will coincide with the first-best. The middle types will continue to mimic the inefficient but because of the asymmetry, HH's output will have a higher distortion relative to the LH than to the HL.

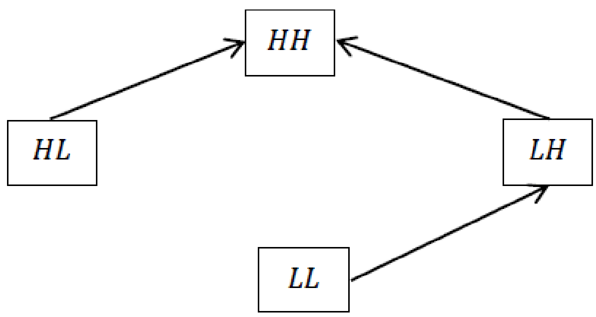

Figure 8:
Binding IC constraints under case 3 (Negative correlation with asymmetry towards the LH type)

The output levels in this case are:

- For the LH type:

$$q_{LH}^{sb} = q\left(\frac{p_{LL}}{p_{LH}}\right), t_{LH}^{sb} = t\left(\frac{p_{LL}}{p_{LH}}\right) \quad (9a)$$

- For the HL type:

$$q_{HL}^{sb} = q(0) = q_{HL}^{fb},\ t_{HL}^{sb} = t(0) = t_{HL}^{fb} \quad (9b)$$

- For the HH type:

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[(p_{LL} + p_{LH})\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]\right) \tag{9c}$$

$$t_{HH}^{sb} = t\left(\frac{1}{p_{HH}}\left[(p_{LL} + p_{LH})\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]\right) \tag{9d}$$

Case 4: Negative correlation with asymmetry towards the HL type

This case is symmetric to case 3 and happens when the probability of observing the HL type is higher than any other type in the distribution. Consequently, the efficient will have an incentive to mimic the HL while ignoring the LH type. In this situation, shown in figure 9, the principal distorts HL type's output but not LH's, who produces first-best levels.

The middle types will continue to mimic the inefficient but because of the asymmetry, HH's output will have a higher distortion relative to the HL than to the LH.

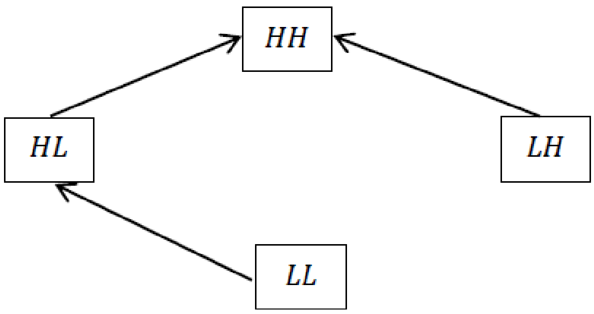

Figure 9:
Binding constraints under case 5 (Negative correlation with asymmetry towards the HL type)

The output levels in this case are:

- For the LH type:

$$q_{LH}^{sb} = q(0) = q_{LH}^{fb}, t_{LH}^{sb} = t(0) = q_{LH}^{fb} \tag{10a}$$

- For the HL type:

$$q_{HL}^{sb} = q\left(\frac{p_{LL}}{p_{HL}}\right), t_{HL}^{sb} = t\left(\frac{p_{LL}}{p_{HL}}\right) \tag{10b}$$

- For the HH type:

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + (p_{LL} + p_{HL})\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]\right) \tag{10c}$$

$$t_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + (\,p_{LL} + \,p_{HL})\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]\right)$$

4. Simulating bilateral flexibility-enabling contracts

In this section, we emphasise on the applications of the model and delve into its concrete implications for policy and business decision makers. We find inspiration in existing thermostat-based demand response programs in which utilities incentivise their customers to modify their consumption during peak hours or when the system reliability is at stake. Relying on automation, customers allow utilities to automatically reduce their air conditioning (during summer) or electric heating (during winter) consumption in exchange for payments. Some companies pay customers for each season in which the customer enrols or give a rebate on the device. Others give a flat credit on the customer's electricity bill, while others pay per peak hour. If smart metering technology is available, companies will compare actual vs. typical consumption and reward them accordingly.

While an important feature of this approach is that it reduces the customers' transaction cost to act flexibly – a relevant barrier to successfully achieving price responsiveness – a demand response programs cannot be based on a "representative agent" approach in which customers do not differ from one another. We claim that designing "efficient" contracts based on this kind of approach is not possible given that suppliers naturally differ in their cost (or disutility) of provision across the different dimensions of flexibility. This is true even when, for example, consumers have identical flexibility enabling assets (e.g., similar air conditioners).

In contrast, we take a *normative* approach to illustrate how the multi-dimensional adverse selection model discussed in this paper can be employed to design bilateral flexibility-enabling contracts that ensure economic efficiency. The main ingredient required to apply the proposed contract design approach is information regarding the distribution of types and the hidden unit cost parameters in the suppliers' cost functions. A key question that follows is thus how to elicit the relevant information from flexibility suppliers.

For illustration purposes, consider the case of a utility company – the principal – that seeks to procure flexibility through a demand response program from its customer base – the agents – in an area where electric heating or air conditioning is widely used (in practice the programme can include all types of flexible loads such as washing machine, electricity vehicles, among others). The company aims at entering into bilateral contracts with customers who aren't competing against each other and thus designs a menu of contracts $\{q_{ij}, t_{ij}, T_{ij}\}$into which suppliers self-select.. In the menu of contract q_{ij} is the capacity of response (e.g., measured in kW but it is closely correlated with the temperature of air conditioner or electric heater), t_{ij} is duration that the control of the flexibility-enabling asset is surrounded to the utility company (e.g., measured in hours) and T_{ij} is the payment to the consumer in exchange for proving flexibility (e.g., measured in dollar or any other unit of money). Subscript i and j refer to the type of resource provider in terms of its cost efficiency at each dimensions (capacity and duration) which can be either low cost (L) or high cost (H). This creates four types of consumers LL, HL, LH, HH which their descriptions are presented in Table 2.

Type of consumer	Description
LL	Experiences a low disutility with regard to change in both the air conditioner temperature and duration of surrounding the control of air conditioner to the utility company.
HL	Experiences a high disutility with respect to change in temperature but a low disutility with respect to duration of control.
LH	Experiences a low disutility with respect to change in temperature but a high disutility with respect to duration of control.
HH	Experiences a high disutility with respect to both change in temperature and duration of control.

Table 2: type of customers and their descriptions

In the absence of a Smart Grid infrastructure, which would allow the customer base anonymously revealing their type, an alternative approach would involve conducting experiments among the utility's customer base in which the emphasis is placed on eliciting consumer's preferences (not the technical flexibility of the asset).

Regarding the values of hidden cost parameters (i.e., marginal disutility that customer experience with respect to change in temperature and duration of surrounding the control to the utility company) in the agents' cost functions, these could be estimated through dedicated empirical studies, laboratory experiments or a combination of both. A relatively simple, yet suitable approach would involve conducting an experiment with randomly chosen subjects from the customer base who have identified themselves as LL, HL, LH, HH in the survey.

In what follows, however, we limit ourselves to presenting results that reflect the *simulated* application of the previously described approach. To this end, we take the following steps:

1. *Generate simulation data for survey results to determine the distribution of types in the customer base:* Assuming a sufficiently large sample and an equally likely outcome coming from a uniform distribution for the binary outcome (i.e., High or Low) that represents the disutility which consumers experience with respect to temperature variation and duration of change, we follow a Monte Carlo approach to generate a discrete, joint probability distribution of types (p_{LL}, p_{LH} p_{HL}, and p_{HH} which shows probability of observing each type of agent). Using the Microsoft Excel add-ins developed by Myerson (2005), it is a straightforward process to generate this data in a spreadsheet. In our model, the information about distribution of types is concentrated in one parameter named covariance of types. Covariance of type (cov) show how different dimensions of flexibility are correlated across customer base.

2. *Determine the optimal contracts under the four different cases that emerge from the baseline relaxed program (section 3.3) assuming, respectively* $v(q,t) = A{q_{ij}}^{\alpha}{t_{ij}}^{1-\alpha}$ and $c\big(q,t,\theta_q,\theta_t\big) =$

$\left(\theta_q{}^i\right)^\beta \left(\theta_t{}^j\right)^{1-\beta} (qt)^\gamma$ as the principal's gross utility and the agent's cost function. With this specification, net payoffs to the principal and agent are $W = Aq_{ij}{}^\alpha t_{ij}{}^{1-\alpha} - T$

$U = T - \left(\theta_q{}^i\right)^\beta \left(\theta_t{}^j\right)^{1-\beta} \left(q_{ij}t_{ij}\right)^\gamma$. Note that $v(q,t)$ and $c\left(q,t,\theta_q,\theta_t\right)$ comply with the assumptions of the model introduced in section 3. First, both are twice differentiable functions. Second, $v(q,t)$ is always monotone and concave, given that $0 < \alpha < 1$. Third, $c\left(q,t,\theta_q,\theta_t\right)$ is monotone in all its parameters and concave in θ_q and θ_t, which is guaranteed because $0 < \beta < 1$. Furthermore, the value assigned to γ determines the convexity, concavity or linearity of $c\left(q,t,\theta_q,\theta_t\right)$ with respect to q and t. If $\gamma < 1$, the cost function is concave and exhibits *economies of scale*. In contrast, if $\gamma > 1$, the cost function is convex and has *diseconomies of scale*. But if $\gamma = 1$, the cost function is linear. Section A4 in the appendix contains complete technical details of the equations and numerical procedures to compute the simulation results. Section A6 in the appendix contains MATLAB script files used in the computation.

4.1 Results

Before proceeding into setting the value of cost and utility function parameters a caveat worth noting is that in the absence of actual data to feed into the model, the parameters chosen for simulation are arbitrary. Since the simulation is just for illustration of the way our model works therefore, we do not try to give a technical interpretation to the numerical results. However, the results are of sufficient information to establish the point we are trying to make about designing efficient contracts for flexibility services. If actual data becomes available, a more realistic set of results can be produced.

In all simulations, we set the following parameter values: $A = 5$, $\alpha = 0.5$ in $v(q,t)$ and $\beta = 0.5$, $\gamma = 0.6$ in $c\left(q,t,\theta_q,\theta_t\right)$. Further, we assume that $\theta_q \in \left\{\theta_q{}^L = 2, \theta_q{}^H = 4\right\}$ and that $\theta_t \in \left\{\theta_t{}^L = 4, \theta_t{}^H = 8\right\}$, giving rise to four agent types, as summarised in the following table:

	θ_q	θ_t
LL	2	4
LH	2	8
HL	4	4
HH	4	8

Table 3: Cost parameters for the four agent types

That is: the efficient (LL) type experiences the lowest discomfort in both the q and the t dimensions, the inefficient (HH) experiences the highest discomfort in both dimensions, while the middle types (LH and HL) are efficient in only one of the dimensions.

The principal procures one dimension of flexibility, q, which in our case stands for capacity (correlated with change in temperature), for a fixed quantity of second dimension (t) , which stands for duration. The desired capacity is obtained through the change in temperature of air conditioner or electric heating at consumers' premises. In all cases that follow, and for simplicity alone, we assume three different possible values for duration of control, $t_{ij} = 2, 4, 6$ hours. That is: the principal designs an optimal menu of contracts to procure capacity q, given three different options for duration.

The company announces that it will reward provision of capacity, i.e. modifying of household temperature, during 2, 4 or 6 hours, in exchange for a payment T_{ij} and the customer who enters into contact with company must provide the specified amount. Each agent type is expected to (truthfully) self-select into one of the contracts. Under the baseline relaxed program, the incentive compatibility (IC) constraints between the middle types (LH and HL) and the inefficient (HH) always bind at the optimum (see Figure 5), but the correlation of types determines which of the efficient's IC constraints bind at the optimum, as depicted in figures 6 to 9.

The details of the results are presented in cases 1 to 4 below but they can be broadly summarized as follows. The optimal menu of contract for flexibility services have the following properties: (i) the most efficient type (LL) always receives the first best contract (i.e., undistorted) and the least efficient type' contract (HH) is always the second best (i.e., distorted) (ii) the middle type contracts are the second best except when there is negative correlation with asymmetry in which case one of the middle type receives the first best contract (iii) under most distributions assumed (except when there is strong positive correlation) the least efficient type is shut down in the sense that in practice it provides no flexibility and receives no compensation (in theory the production and compensation for this type are not zero but extremely low). This is likely due to cost parameters assumed in Table 3 in which the marginal cost (disutility) of production (θ_q, θ_t), for the least efficient type is assumed to be twice of the most efficient one. This makes provision of flexibility by the least efficient type very costly to principal.

Case 1: Positive correlation

In a simple language positive correlation means that having information about one dimension of flexibility of a customer we can deduce about its efficiency in other dimension. That is if we randomly select a customer and observe that it is efficient in q dimension it is highly probable that it is also efficient in t dimension (LL). Similarly, if it is inefficient in q dimension it is highly probable that is also inefficient in t dimension (HH). Mathematically, positive correlation happens when the customers are distributed in a way that the product of probabilities of being efficient in both dimensions (LL) and inefficient in both dimensions (HH) is strictly higher than the product of probabilities of being efficient in one dimension and inefficient in the other dimension (HL or LH). This means $p_{LL}p_{HH} > p_{HL}p_{LH}$ and therefore covariance of types is positive ($cov = p_{LL}p_{HH} - p_{HL}p_{LH} > 0$). To illustrate this case, consider the following distributions, with high probability for LL and HH types:

Distribution 1A: where $p_{LL} = 0.5, p_{LH} = 0, p_{HL} = 0, p_{HH} = 0.5, cov = 0.25$

Table 5 contains the menu of contracts for this distribution of types. In table 5, each row contains the menu of contracts for duration of $t_{ij} = 2{,}4{,}6$ hours respectively, and each column contains the optimal menu of contracts for each of the four agent types. The fifth column contains the sum of capacity procured, given each duration. For example, when the duration is 2, it is optimal for the principal to procure 24.07 units of q from the efficient type in exchange for a transfer $T_{LL} = 28.95$. The inefficient type (HH), in contrast, produces an output of 4.8E-04 units, which is virtually zero, in exchange for a transfer $T_{HH} = 0.08$.[18] The rows given a duration of 4 and 6 hours can be equivalently interpreted: note that as duration increases, the capacity procured decreases. This is the consequence of the specified $v(q,t)$ in which both inputs are imperfect complements.

In the next two examples for this case the probability of observing middle types (LH and HL) increases as that of the inefficient (HH) and efficient (LL) decrease. The results of for these distributions have been presented in Tables 6 and 7.

In all aforementioned distributions, the three incentive compatibility constraints of the efficient type bind at the optimum, as in figure 6 in Section 3-i.e., for the three different duration considered, the efficient is simultaneously indifferent between his contract, the inefficient's (HH), and the two middle types (HL and LH). The details of calculations of incentive compatibility constraints are presented in appendix A5.

Two points needs to be noted in the case of positive correlation. First, as seen in the theoretical model in Section 3, under positive correlation there is a bunching for the second best contacts offered to the less efficient types, while the efficient type produces at first best levels. This means that although there are four possible types of agents the principal bunch middle type contract with that of inefficient type and thus offer them the same contract. This can be readily confirmed from the simulation results presented in Tables 5,6 and 7. Such a result is true under any alternative distributions that exhibit positive correlation.

Note, in the results for distributions 1B and 1C, that as the middle types become more likely in the distribution of types, the rent that the principal gives to the efficient becomes costlier. In consequence, the expected profit decreases. Furthermore, there is a relevant qualitative characteristic of the positive correlation case that sets it apart from the remaining cases: first, the principal determines the output level of the inefficient (HH) type and, accordingly, determines the middle types' (LH and HL) output levels, such that the efficient stays indifferent among the three less efficient types' contracts. This explains that even within the same type distribution, the optimal contract differs from one another: given a fixed t_{HH}, the principal determines q_{HH} which, in turn, determines the (q_{LH}, t_{LH}) and (q_{HL}, t_{HL}) output pairs.

Additionally, when there is strong positive correlation, the optimal menu of contract, in practice, leads to shutdown of less efficient types. This can be seen from Table 5 where LH, HL and HH virtually produce nothing and receive no compensation. The sum of q_{ij} for each duration is almost equal to production level of the most efficient type. However, as seen from Tables 6 and 7 when probability of middle type increases the optimal contract involves a non-zero level of production for less efficient types.

[18] The inefficient produces an arbitrarily low amount, which is very close to zero. Actually, it is correct to assume that the inefficient does not produce any amount.

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of*** q_{ij}
{24.07, 2 ,28.95}	{0.00, 2 ,0.08}	{0.00, 2 ,0.08}	{4.8E-04, 2 ,0.08}	24.07
{12.03, 4 , 28.95 }	{0.00, 4 ,0.08}	{0.00, 4 ,0.08}	{2.4E-04, 4 ,0.08}	12.04
{8.02, 6 , 28.95 }	{0.00, 6 ,0.08}	{0.00, 6 ,0.08}	{1.35E-04, 6 ,0.08}	8.03

Table 5: Menu of contracts under case 1 and type distribution 1A

Distribution 1B: where $p_{LL} = 0.49$, $p_{LH} = 0.01$, $p_{HL} = 0.01$, $p_{HH} = 0.49$, $cov = 0.24$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of*** q_{ij}
{24.07, 2, 28.99}	{0.0014, 2 ,0.16}	{0.0014, 2 ,0.16}	{0.0014, 2 ,0.16}	24.07
{12.03, 4 , 29.04 }	{0.0015, 4 ,0.26}	{0.0015, 4 ,0.26}	{0.0015, 4 ,0.26}	12.04
{8.02, 6 , 29.10 }	{0.002, 6 ,0.38}	{0.002, 6 ,0.38}	{0.002, 6 ,0.38}	8.03

Table 6: Menu of contracts under case 1 and type distribution 1B

Distribution 1C: where $p_{LL} = 0.45$, $p_{LH} = 0.05$, $p_{HL} = 0.05$, $p_{HH} = 0.45$, $cov = 0.20$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of*** q_{ij}
{24.07, 2, 29.30}	{0.02, 2 ,0.79}	{0.02, 2 ,0.79}	{0.02, 2 ,0.79}	24.12
{12.03, 4 , 29.83}	{0.04,4,1.84}	{0.04,4,1.84}	{0.04,4,1.84}	12.15
{8.02, 6 , 30.46 }	{0.06, 6 , 3.10 }	{0.06, 6 ,3.10 }	{0.06, 6 , 3.10 }	8.21

Table 7: Menu of contracts under case 1 and type distribution 1C

Case 2: Weak correlation

When there is weak correlation, the middle types (LH and HL) become more likely in the distribution of types, while the efficient (LL) and inefficient (HH) become less likely. Therefore, types are weakly correlated, i.e. the covariance of types is closer to zero but still positive, and the incentive compatibility (IC) constraint between the efficient (LL) and inefficient (HH) does not bind. Instead, the efficient finds it more profitable to mimic any of the middle types. Therefore, it is in the principal's best interest to make the LL type indifferent between mimicking any of the two middle types (LH or HL). This is ensured by finding a real number $0 < \lambda < 1$ that leads to output levels for which equation (7) in Section 3 holds, which we obtain numerically. Likewise, IC constraints behave as in figure 7 (see Section 3).

The following two type distributions illustrate the relevant features of weak correlation:

Distribution 2A where $p_{LL} = 0.3$, $p_{LH} = 0.25$, $p_{HL} = 0.19$, $p_{HH} = 0.26$, $cov = 0.03$. For $\lambda = 0.57$, the the output levels that satisfy equation (7) have been presented in Table 8.

Distribution 2B where where $p_{LL} = 0.28$, $p_{LH} = 0.22$, $p_{HL} = 0.22$, $p_{HH} = 0.28$, $cov = 0.03$. For $\lambda = 0.50$, the output levels that satisfy equation (7) have presented in Table 9.

As seen from Table 8 and 9, the middle types produce more flexibility compare to the previous case but the least efficient type (HH) almost does not produce. The share of middle type output increases when its probability of being observed increases. Unlike the positive correlation case, in which the principal determines the middle types' output levels in accordance with the inefficient type's (HH) output, under weak correlation it is optimal for the principal to determine output levels that will satisfy equality (7), which ignores the global IC constraint between the HH and LL. If a number λ satisfying this equality exists, then the same number will satisfy it for different levels of the fixed t_{ij}. This explains that the *same* contracts are optimal for different duration and that the efficient's rent and expected profit remain unchanged.

For both 2A and 2B distributions, the efficient type is indifferent between his contract and that of any of the two middle types. However, as predicted in theory presented in Section 3 the efficient's incentive compatibility constraint with respect to the inefficient does not bind. The details of calculations for incentive compatibility check can be found in Appendix A5.

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	**Sum of q_{ij}**
{24.07, 2, 29.42}	{0.12, 2 ,1.72}	{0.12, 2 ,1.72}	{5.47E-05, 2 ,0.02}	24.31
{12.03, 4 , 29.42 }	{0.06,4,1.72}	{0.06,4,1.72}	{2.74E-05,4,0.02}	12.15
{8.02, 6 , 29.42}	{0.05, 6 ,1.72 }	{0.05, 6 ,1.72 }	{1.82E-05, 6 ,0.02}	8.10

Table 8: Menu of contracts under type distribution 2 A

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	**Sum of q_{ij}**
{24.07, 2, 29.45}	{0.14, 2 ,1.84}	{0.14, 2 ,1.84}	{8.57E-05, 2, 0.03}	24.34
{12.03, 4 , 29.45 }	{0.07,4,1.84}	{0.07,4,1.84}	{4.28E-05,4, 0.03}	12.17
{8.02, 6 , 29.45 }	{0.05, 6 ,1.84 }	{0.05, 6 ,1.84 }	{2.86E-05, 6, 0.03}	8.11

Table 9: Menu of contracts under type distribution 2B

Case 3: Negative correlation with asymmetry towards the LH type

If there is negative correlation, because both of the middle types (LH and HL) are more likely than the efficient (LL) and inefficient (HH) taken together, and the LH type has a greater probability of being observed in the distribution than any other type, then the general model is under case 3. In consequence, at the optimum, the efficient's only binding constraint is the one with respect to the LH type (see figure 8 in Section 3). To avoid the efficient's rent becoming too costly, the principal distorts the LH type's and the HH's output level, while letting the HL type produce at first-best levels.

The following two distributions illustrate the features of case 3:

Distribution 3A where $p_{LL} = 0.125$, $p_{LH} = 0.5$, $p_{HL} = 0.25$, $p_{HH} = 0.125$, $cov = -0.11$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of*** q_{ij}
{24.07, 2, 29.45}	{0.37, 2 ,1.84}	{0.75, 2 ,1.84}	{3.37E-07, 2, 0.00}	25.19
{12.03, 4 , 29.30 }	{0.19,4,3.34}	{0.38,4,5.11}	{1.68E-07,4, 0.00}	12.59
{8.02, 6 , 29.89 }	{0.12, 6 ,3.34 }	{0.25, 6 ,5.11 }	{1.12E-07, 6, 0.00}	8.40

Table 10: Menu of contracts under type distribution 3A

Distribution 3B where where $p_{LL} = 0.08$, $p_{LH} = 0.74$, $p_{HL} = 0.1$, $p_{HH} = 0.08$, $cov = -0.11$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of*** q_{ij}
{24.07, 2, 30.15}	{0.55, 2, 4.24}	{0.75, 2 , 5.11}	{9.2E-09, 2 ,0.00}	25.37
{12.03, 4 , 30.15 }	{0.28,4, 4.24}	{0.38,4,5.11}	{4.6E-09,4,0.00}	12.68
{8.02, 6 , 30.15}	{0.18, 6 , 4.24 }	{0.25, 6 ,5.11 }	{3E-09, 6 ,0.00}	8.46

Table 11: Menu of contracts under type distribution 3B

Unlike all previous cases in which only the most efficient type receives the first best contract and the rest are offered the second best contract, in the case of negative correlation with asymmetry, one of the middle type receives the first best contract-i.e., the middle type that is least probable to be observed (here HL type). This situation creates a sharp distinction between bilateral contracts under unidimensional and multidimensional information asymmetry in the sense that when there is more than one dimension it is possible to have first best contract for a less efficient type even under information asymmetry. In fact the low probability of HL type averts the need to distort its contract.

In both distribution 3A and 3B, the efficient type is indifferent between his contract and that of the LH type. With the closed form solutions of section A4 in the appendix, it is straightforward to compute output levels for case 3 by letting $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 0$.

Case 4: Negative correlation with asymmetry towards the HL type

Symmetrically to case 3, if both middle types (LH and HL) are more likely than the efficient (LL) and inefficient (HH) together, and the HL type has a greater probability of being observed in the distribution than any other type, then the general model is under case 4. At the optimum, the efficient's only binding constraint is the one with respect to the HL type (see figure 9 in Section 3), and the principal optimises by distorting both the HL type's and the HH's output level, while letting the LH type produce at first-best levels.

Consider the following two type distributions, which satisfy case 4:

Distribution 4A, where $p_{LL} = 0.125$, $p_{LH} = 0.25$, $p_{HL} = 0.5$, $p_{HH} = 0.125$, $cov = -0.11$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of q_{ij}***
{24.07, 2, 29.89}	{0.75, 2, 5.11}	{0.37, 2 , 3.34}	{3.3E-07, 2 ,0.00}	25.19
{12.03, 4 , 29.89 }	{0.38,4, 5.11}	{0.19,4,3.34}	{1.6E-07,4,0.00}	12.59
{8.02, 6 , 29.89}	{0.25, 6 , 5.11 }	{0.12, 6 ,3.34 }	{1.12E-07, 6 ,0.00}	8.40

Table 12: Menu of contracts under type distribution 4A

Distribution 4B, where $p_{LL} = 0.04$, $p_{LH} = 0.05$, $p_{HL} = 0.87$, $p_{HH} = 0.04$, $cov = -0.04$

$\{q_{LL}, t_{LL}, T_{LL}\}$	$\{q_{LH}, t_{LH}, T_{LH}\}$	$\{q_{HL}, t_{HL}, T_{HL}\}$	$\{q_{HH}, t_{HH}, T_{HH}\}$	***Sum of q_{ij}***
{24.07, 2, 30.29}	{0.75, 2, 5.11}	{0.66, 2 , 4.72}	{2.1E-11, 2 ,0.00}	25.48
{12.03, 4 , 30.29 }	{0.38,4, 5.11}	{0.33,4, 4.72}	{1.05E-11,4,0.00}	12.74
{8.02, 6 , 30.29}	{0.25, 6 , 5.11 }	{0.22, 6 , 4.72}	{7E-12, 6 ,0.00}	8.49

Table 13: Menu of contracts under type distribution 4B

The results in this case is symmetric of case 3. The most efficient type and one of the middle types (here LH) receives the first best contract and the rest receive the second best. As under most distributions assumed previously the least efficient type (HH) contract in practice leads to no production and compensation.

In terms of incentive compatibility, the efficient type is indifferent between his contract and that of the HL type under both distributions 4A and 4B. The results for incentive compatibility check have been presented in Appendix A5. Using the formulas derived in section A4 in the appendix, it is straightforward to compute output levels for case 5 by letting $\lambda_2 = 1$, $\lambda_1 = \lambda_3 = 0$.

5. Conclusions

Given the increased reliance on renewables taking place in many power systems throughout the world, incentivising the provision of flexibility is becoming a priority for system operators, policy and business decision makers alike. The technological improvement in digital communication has resulted in emergence of new players in the electricity market with flexible loads (e.g., households with electric vehicles, electric heater, air conditioner, solar PV with storage) which are valuable sources of flexibility for the system at an aggregated level.

On the other hand, flexibility is inherently *multi-dimensional* as both the cost of producing it and the utility (or output if one considers the equivalent mathematical interpretation of a production function) that is derived from utilizing it depends on more than one factor. Capacity, ramp rate, duration and/or lead time are among the many elements that describe flexibility. Because of this, different flexibility-enabling resources possess differing levels of efficiency, implying that flexibility is not a homogenous commodity. Additionally, an equally relevant economic property of power system flexibility is that its composing elements (i.e., capacity, ramp rate, duration) are best understood as *imperfect complements*: the cost of production and the utility derived from it are always *non-separable*. Not only do buyers and sellers of flexibility have utility and cost functions that depend on more than one factor but these factors enter into these functions multiplicatively.

Therefore, as flexibility differs significantly from commodities traditionally traded in existing electricity markets, such as energy or capacity, correctly accounting for its economic properties is essential to create the incentives to enable it in electricity systems. More specifically, designing a market for flexibility services needs to be compatible with properties of the traded commodity, an important point that has so far remained unaddressed in the existing literature of power system economics. For example, where competition among flexibility providers are possible, a multi-attribute auction is needed to procure flexibility in an efficient manner. The multi-attribute auction is an allocation mechanism in which more than one feature of the commodity is valued (e.g., MW, MW/min and emission performance). Therefore, it allows the principal to incentivise, for example, capacity, flexibility and emission performance simultaneously in a single auction. In situations where competition is not feasible, multi-dimensional bilateral contracts are the alternative method of procurement.

The bilateral contracts are specifically relevant in the case of emerging small resources as, due to high transaction cost relative to the size of resource, these resources cannot directly participate in an organised market and compete. Taking the aforementioned considerations as guiding principles rather than mere theoretical considerations, this paper has taken a contract design perspective to incentivise the bilateral exchange of flexibility from small resources, already happening in some electricity markets in the form of demand response programs and ancillary services. More transactions of the kind are expected to play a greater role in the not so distant future as renewables grow further as an alternative to fossil fuels, and emerging business models that enable power system flexibility – led by technological innovation – consolidate (Boscán and Poudineh, 2016).

The paper introduces an adverse selection model of procurement with multi-dimensional types. The model innovates by presenting solutions to the non-separable case in a baseline relaxed program that accounts for the vast majority of economically relevant cases. The results of this model provide important insights on designing efficient contract for flexibility services which can be utilised in a, for example, demand response programme.

First, the results show that optimal contract for flexibility crucially depends on the way that flexibility providers are distributed-e.g., if we assume flexibility with two dimensions (capacity and duration) which is procured from a group of flexibility providers then distribution of type is a set of probabilities that show what percentage of group is efficient (low cost) in both dimensions (LL) or inefficient (high cost) in both dimensions (HH) or efficient in one dimension and inefficient in the other dimension (LH or HL). In the model presented in the paper, the information about distribution of types is concentrated in one parameter- i.e., covariance of type. The information about distribution of types can be obtained through well-designed surveys or through indirectly observing the consumers' behavior in a smart grid environment.

Depending on the covariance of types – whether efficiency in one dimension is independent or not of the other dimension – four mutually exclusive cases arises:

If there is *positive correlation*, meaning most flexibility providers are either low cost (LL) or high costs (HH) in both dimension (middle types are a small percentage of group when there is positive correlation), the efficient type does have incentive to mimic the three less efficient types (HH, H and HL). The optimal contract in this case has two properties: (a) it incentivises the efficient type (LL) to produce at first-best levels (no distortion as if there is no information asymmetry), giving this agent type a rent that equals a convex combination of the informational rents that he would obtain by mimicking the less efficient agents *simultaneously* (b) the other three types are offered the same contract and produce at the second best level (distorted because of information asymmetry). The bunching of the contracts for three less efficient types happens because probability of observing middle types (LH or HL) is low in this case and thus there is no need to offer them a separate contract.

If there is *weak correlation*, meaning that most flexibility providers are middle type agents (LH or HL), the efficient type (LL) can realize a higher rent by mimicking any of the middle, adjacent types (LH or HL) than by mimicking the inefficient type (HH). In this case the optimal contact involves offering the efficient type the first best and the other three types each separately the second best contract. In contrast to the case of positive correlation, there is no bunching of contracts in this case and middle types produce at level which is higher than the least efficient type (HH) although all three are distorted to maintain incentive compatibility.

If there is *negative correlation with asymmetry towards the LH type*, meaning that middle type agents constitute a higher proportion in the group of flexibility providers and probability of observing an LH type is higher than the other middle type, the efficient type will have an incentive to mimic the LH type but will, essentially, ignore the HL type. The optimal contract in this case include the first best for the efficient type (LL) and (HL) types but the rest will receive the second best contracts. An important difference of this case with previous cases is that in additional to efficient type, one of the middle types also receives the first best contract. This is one of the important differences between unidimensional and multi-dimensional adverse selection procurement in the sense that when the number of dimesons increases, sometimes a first best contract can be given to a not fully efficient agent (here HL) even when there is information asymmetry.

Symmetrically, if there is n*egative correlation with asymmetry towards the HL type,* the probability of observing the HL is higher than the other middle type in a distribution in which middle type agents dominate. Consequently, the efficient will have an incentive to mimic the HL while ignoring the LH type. In this situation, the principal gives the first best contract to LL and LH types but distorts HL and HH's types output.

The second important results of the model in this paper is that designing the optimal contract for flexibility services is complicated not only because of multidimensional information asymmetry but also because of the fact that the composing elements of flexibility are non-separable (capacity, ramp rate, duration, cannot be produced or consumed separately). This "non-separable externality" leads to further distortion of inefficient types beyond the fundamental rent efficiency trade off prevailing under information asymmetry. The non-separability distortion is unique to non-conventional commodities such as flexibility and it does not exist when cost and utility function of agent and principal is separable (a condition that has been assumed almost always in the contract theory literature).

Besides the conceptual discussion about flexibility and the theoretical contribution and that constitutes the model itself, by way of a real-life example, the last section of the paper shows that the model is *applicable* by decision makers who wish to incentivise the provision of flexibility or, for that matter, any multi-dimensional commodity with imperfectly substitutable components. Relying on surveys, empirical analyses and suitably designed experiments that elicit the cost of acting flexibly together with the distribution of types in a given area, it is *feasible* to actually design optimal contracts for flexibility services. Existing contracts fail to take any of this information into account and are, therefore, ill-positioned to deliver economic efficiency when for example applied to a demand response programme.

The analysis of the paper has, of course, its limitations. First, we claim that flexibility is multi-dimensional but our model is bi-dimensional only: this is, of course, to gain tractability. The economic intuition, of course, carries over to a multi-dimensional framework. Second, unlike most treatments of the principal-agent model in the literature, we deal with a discrete set of types. However, in line with Vohra (2011) "we know of no modeling reason to prefer a continuous type space to a discrete one". Third, we are aware that the simulation results could cover a wider range of applications and considerations. For example, it would be interesting to obtain real empirical data to apply the contract modelling framework with real information. It would also be interesting to extend the model to consider the situation in which the principal is an intermediary (e.g. an aggregator) who faces *downstream* uncertainty in relation to the distribution of types and its hidden cost parameters but also faces *upstream* uncertainty coming from market fluctuations. Furthermore, analysing the impact of risk attitudes and competitive environments are relevant but beyond the scope of this paper.

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Appendix

A1. Fully constrained program

This section presents the fully constrained program.

The principal maximises expected profit:

$$\max_{\{q_{ij},t_{ij},U_{ij}\}} E(\pi) = \sum_{ij} p_{ij} S(q_{ij}, t_{ij}) - \sum_{ij} p_{ij} U_{ij}$$

subject to:

<u>Individual Rationality (IR) constraints:</u>

$T_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) = U_{LL} \geq 0$ (A1)

$T_{LH} - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) = U_{LH} \geq 0$ (A2)

$T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) = U_{HL} \geq 0$ (A3)

$T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) = U_{HH} \geq 0$ (A4)

<u>Incentive Compatibility (IC) constraints:</u>

For the efficient (LL) type:

$T_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) \geq T_{LH} - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^L)$ which is equivalent to

$U_{LL} \geq U_{LH} + c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^L) = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$ (A5)

$T_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) \geq T_{HL} - c(q_{HL}, t_{HL}, \theta_q^L, \theta_t^L)$ which is equivalent to

$U_{LL} \geq U_{HL} + c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) - c(q_{HL}, t_{HL}, \theta_q^L, \theta_t^L) = U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL})$ (A6)

$T_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) \geq T_{HH} - c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^L)$ which is equivalent to

$U_{LL} \geq U_{HH} + c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^L) = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$ (A7)

For the middle (LH) type:

$T_{LH} - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) \geq T_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^H)$ or

$U_{LH} \geq U_{LL} - c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^H) + c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) = U_{LL} + \Delta_{LH}^{LL}(q_{LL}, t_{LL})$ (A8)

$T_{LH} - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) \geq T_{HL} - c(q_{HL}, t_{HL}, \theta_q^L, \theta_t^H)$ or

$U_{LH} \geq U_{HL} + c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) - c(q_{HL}, t_{HL}, \theta_q^L, \theta_t^H) = U_{HL} + \Delta_{LH}^{HL}(q_{HL}, t_{HL})$ (A9)

$T_{LH} - c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) \geq T_{HH} - c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^H)$ or

$U_{LH} \geq U_{HH} + c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^H) = U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})$ (A10)

For the middle (HL) type:

$T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) \geq T_{LL} - c(q_{LL}, t_{LL}, \theta_q^H, \theta_t^L)$ or

$U_{HL} \geq U_{LL} - c(q_{LL}, t_{LL}, \theta_q^H, \theta_t^L) + c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) = U_{LL} + \Delta_{HL}^{LL}(q_{LL}, t_{LL})$ (A11)

$T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) \geq T_{LH} - c(q_{LH}, t_{LH}, \theta_q^H, \theta_t^L)$ or

$U_{HL} \geq U_{LH} + c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) - c(q_{LH}, t_{LH}, \theta_q^H, \theta_t^L) = U_{LH} + \Delta_{HL}^{LH}(q_{LH}, t_{LH})$ (A12)

$T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) \geq T_{HH} - c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^L)$ or

$U_{HL} \geq U_{HH} + c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^L) = U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})$ (A13)

For the inefficient (HH) type:

$T_{HH} - c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) \geq T_{LL} - c(q_{LL}, t_{LL}, \theta_q^H, \theta_t^H)$ which is equivalent to

$U_{HH} \geq U_{LL} - c(q_{LL}, t_{LL}, \theta_q^H, \theta_t^H) + c(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L) = U_{LL} + \Delta_{HH}^{LL}(q_{LL}, t_{LL})$ (A14)

$T_{HH} - c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) \geq T_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^H)$ or

$U_{HH} \geq U_{HL} - c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^H) + c(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L) = U_{HL} + \Delta_{HH}^{HL}(q_{HL}, t_{HL})$ (A15)

$T_{HH} - c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) \geq T_{LH} - c(q_{LH}, t_{LH}, \theta_q^H, \theta_t^H)$ or

$U_{HH} \geq U_{LH} - c(q_{LH}, t_{LH}, \theta_q^H, \theta_t^H) + c(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H) = U_{LH} + \Delta_{HH}^{LH}(q_{LH}, t_{LH})$ (A16)

A2. Analysis of the incentive compatibility constraints

A2.1 Monotonicity of output levels

The monotonicity of output levels follows from the addition of local incentive constraints two by two.

For q:

- Adding (A6) and (A11) establishes $q_{LL} \geq q_{HL}$:

$$c\left(q_{LL}, t_{LL}, \theta_q^H, \theta_t^L\right) - c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L\right) \geq c\left(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L\right) - c\left(q_{HL}, t_{HL}, \theta_q^L, \theta_t^L\right)$$

- And adding (A10) with (A16) establishes $q_{LH} \geq q_{HH}$:

$$c\left(q_{LH}, t_{LH}, \theta_q^H, \theta_t^H\right) - c\left(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H\right) \geq c\left(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H\right) - c\left(q_{HH}, t_{HH}, \theta_q^L, \theta_t^H\right)$$

Similarly, for t:

- Adding (A5) and (A8) establishes $t_{LL} \geq t_{LH}$:

$$c\left(q_{LL}, t_{LL}, \theta_q^L, \theta_t^H\right) - c\left(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H\right) \geq c\left(q_{LL}, t_{LL}, \theta_q^L, \theta_t^L\right) - c\left(q_{LH}, t_{LH}, \theta_q^L, \theta_t^L\right)$$

- Adding (A13) and (A15) establishes $t_{HL} \geq t_{HH}$:

$$c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^H\right) - c\left(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H\right) \geq c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L\right) - c\left(q_{HH}, t_{HH}, \theta_q^H, \theta_t^L\right)$$

However, adding (A9) with (A12) results in:

$$c\left(q_{LH}, t_{LH}, \theta_q^H, \theta_t^L\right) - c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L\right) \geq c\left(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H\right) - c\left(q_{HL}, t_{HL}, \theta_q^L, \theta_t^H\right)$$

which is satisfied if $q_{LH} \geq q_{HL}$ and $t_{HL} \geq t_{LH}$, but also if $q_{LH} \geq q_{HL}$ and $t_{HL} < t_{LH}$.

Re-organizing the inequality,

$$c\left(q_{HL}, t_{HL}, \theta_q^L, \theta_t^H\right) - c\left(q_{LH}, t_{LH}, \theta_q^L, \theta_t^H\right) \geq c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L\right) - c\left(q_{LH}, t_{LH}, \theta_q^H, \theta_t^L\right)$$

it is easy to verify that it holds if $t_{HL} \geq t_{LH}$ and $q_{LH} \geq q_{HL}$, but also if $t_{HL} \geq t_{LH}$ and $q_{LH} < q_{HL}$. Therefore, monotonicity of the middle types' output levels cannot be established.

A2.2 Binding constraints for the LL type

Inequalities (A5), (A6) and (A7) are the three IC constraints for the efficient type, and all of them are upward.

By mimicking the LH type, the efficient obtains an informational rent of

$$U_{LL} \geq U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$$

If the efficient mimics the HL type, he would obtain an informational rent of

$$U_{LL} \geq U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL})$$

And if the LL mimics the HH, the informational rent is

$$U_{LL} \geq U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$$

Because there is no *a priori* ordering of the unit cost difference between the efficient and the inefficient in both dimensions, i.e. $(\theta_t^H - \theta_t^L)$ can be equal, greater or lower than $(\theta_q^H - \theta_q^L)$, and because monotonicity for the middle-types' output levels cannot be established, it is not possible to determine which of the incentive constraints will bind at the optimum or if only one of them will bind. Therefore:

$$U_{LL} \geq \max\{U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH}), U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})\}$$

A.2.3 Binding constraints for the middle types LH and HL

The LH type agent can mimic the HL type in which case he would realize a rent of:

$$U_{HL} + \Delta_{LH}^{HL}(q_{HL}, t_{HL})$$

Agent LH can also mimic the HH, obtaining a rent of:

$$U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})$$

Similarly, the HL can mimic the LH realizing:

$$U_{LH} + \Delta_{HL}^{LH}(q_{LH}, t_{LH})$$

Or it can mimic HH obtaining:

$$U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})$$

For the same reasons mentioned previously, that is: an absence of an *a priori* ordering of the unit cost difference between the efficient and the inefficient in both dimensions, i.e. $(\theta_t^H - \theta_t^L)$ and $(\theta_q^H - \theta_q^L)$, and an incomplete ordering of the middle types' output levels, it is not possible to determine which of the incentive constraints will bind at the optimum or if only one of them will bind. Therefore, U_{LH} and U_{HL} are respectively,

$$U_{LH} \geq \max\{U_{HL} + \Delta_{LH}^{HL}(q_{HL}, t_{HL}), U_{HH} + \Delta_{LH}^{HH}(q_{HH}, t_{HH})\}$$

$$U_{HL} \geq \max\{U_{LH} + \Delta_{HL}^{LH}(q_{LH}, t_{LH}), U_{HH} + \Delta_{HL}^{HH}(q_{HH}, t_{HH})\}$$

as in the program relaxation described in section 2.2.

A3. Solution to the baseline relaxed program (technical details)

To solve the problem, it is convenient to re-write equation (3) in the main text as a convex combination of the rents that result from mimicking the (adjacent) middle types and the (non-adjacent) inefficient type:

$$U_{LL} = \lambda_1 \left(\Delta_{LH}^{HH}(q_{HH}, t_{HH}) + \Delta_{LL}^{LH}(q_{LH}, t_{LH})\right) + \lambda_2 \left(\Delta_{HL}^{HH}(q_{HH}, t_{HH}) + \Delta_{LL}^{HL}(q_{HL}, t_{HL})\right) + \lambda_3(\Delta_{LL}^{HH}(q_{HH}, t_{HH}))$$

where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$, $i = 1,2,3$ are real numbers. This approach, used by Armstrong and Rochet (1999), provides a simple way to characterize all the economically relevant cases emerging from the baseline relaxed program.

Substituting for the binding IC constraints into the principal's expected payoff (equation (1) on the main text) leads to:

$$p_{LL}\Big[S(q_{LL},t_{LL})-\lambda_1\left(\Delta_{LH}^{HH}(q_{HH},t_{HH})+\Delta_{LL}^{LH}(q_{LH},t_{LH})\right)-\lambda_2\left(\Delta_{HL}^{HH}(q_{HH},t_{HH})+\Delta_{LL}^{HL}(q_{HL},t_{HL})\right)$$
$$-\lambda_3(\Delta_{LL}^{HH}(q_{HH},t_{HH}))\Big]+p_{LH}[S(q_{LH},t_{LH})-\Delta_{LH}^{HH}(q_{HH},t_{HH})]$$
$$+p_{HL}[S(q_{HL},t_{HL})-\Delta_{HL}^{HH}(q_{HH},t_{HH})]+p_{HH}[S(q_{HH},t_{HH})]$$

The first-order conditions for the LL type are:

- With respect to q_{LL}:

$$p_{LL}\frac{\partial s(q_{LL},t_{LL})}{\partial q_{LL}}=\ p_{LL}[\frac{\partial v(q_{LL},t_{LL})}{\partial q_{LL}}-\frac{\partial c\left(q_{LL},t_{LL},{\theta_q}^L,{\theta_t}^L\right)}{\partial q_{LL}}]=0 \quad \text{(A17)}$$

- With respect to t_{LL}:

$$p_{LL}\frac{\partial s(q_{LL},t_{LL})}{\partial t_{LL}}=\ p_{LL}[\frac{\partial v(q_{LL},t_{LL})}{\partial t_{LL}}-\frac{\partial c\left(q_{LL},t_{LL},{\theta_q}^L,{\theta_t}^L\right)}{\partial t_{LL}}]=0 \quad \text{(A18)}$$

So, q_{LL}^{sb} and t_{LL}^{sb} solve the equations above, and it follows that $q_{LL}^{sb}=q_{LL}^{fb}$ and $t_{LL}^{sb}=t_{LL}^{fb}$. This proves equations numbered with (4) in the main text.

In contrast, the less efficient types are deviated from their first-best output levels.

Consider the first-order conditions for the LH type:

- With respect to q_{LH}:

$$-\lambda_1\,p_{LL}\frac{\partial\Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial q_{LH}}+\ p_{LH}\frac{\partial s(q_{LH},t_{LH})}{\partial q_{LH}}=0$$

  Expanding $\partial s(q_{LH},t_{LH})/\partial q_{LH}$ and re-arranging:

$$\frac{\partial v(q_{LH},t_{LH})}{\partial q_{LH}}=\frac{\partial c\left(q_{LH},t_{LH},\theta_q^L,\theta_t^H\right)}{\partial q_{LH}}+\lambda_1\frac{p_{LL}}{p_{LH}}\frac{\partial\Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial q_{LH}} \quad \text{(A19)}$$

  Further manipulation leads to:

$$\frac{\frac{\partial v(q_{LH},t_{LH})}{\partial q_{LH}}-\frac{\partial c\left(q_{LH},t_{LH},\theta_q^L,\theta_t^H\right)}{\partial q_{LH}}}{\frac{\partial\Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial q_{LH}}}=\lambda_1\frac{p_{LL}}{p_{LH}} \qquad \text{(A20)}$$

- With respect to t_{LH}:

$$-\lambda_1\,p_{LL}\frac{\partial\Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial t_{LH}}+\ p_{LH}\frac{\partial S(q_{LH},t_{LH})}{\partial t_{LH}}=0$$

Expanding $\partial S(q_{LH}, t_{LH})/\partial t_{LH}$ and re-arranging:

$$\frac{\partial v(q_{LH},t_{LH})}{\partial t_{LH}} = \frac{\partial c\left(q_{LH},t_{LH},\theta_q^L,\theta_t^H\right)}{\partial t_{LH}} + \lambda_1 \frac{p_{LL}}{p_{LH}} \frac{\partial \Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial t_{LH}} \quad \text{(A21)}$$

Re-arranging to obtain:

$$\frac{\frac{\partial v(q_{LH},t_{LH})}{\partial t_{LH}} - \frac{\partial c\left(q_{LH},t_{LH},\theta_q^L,\theta_t^H\right)}{\partial t_{LH}}}{\frac{\partial \Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial t_{LH}}} = \lambda_1 \frac{p_{LL}}{p_{LH}} \quad \text{(A22)}$$

The second best levels of output q_{LH}^{sb}, t_{LH}^{sb} satisfy equations (A19) and (A21), respectively, and both q_{LH}^{sb} and t_{LH}^{sb} deviate from first-best levels q_{LH}^{fb}, t_{LH}^{fb} by an amount equal to the second term of the corresponding equations. Equations (A20) and (A22) show that $q_{LH}^{sb} = q\left(\lambda_1 \frac{p_{LL}}{p_{LH}}\right), t_{LH}^{sb} = t\left(\lambda_1 \frac{p_{LL}}{p_{LH}}\right)$. Note also that $\frac{\partial \Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial q_{LH}}, \frac{\partial \Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial t_{LH}} \neq 0$ for non-separable cost functions.

The first-order conditions for the HL type are:

- With respect to q_{HL}:

$$-\lambda_2\, p_{LL} \frac{\partial \Delta_{LL}^{HL}(q_{LH}, t_{LH})}{\partial q_{HL}} + p_{HL} \frac{\partial s(q_{HL}, t_{HL})}{\partial q_{HL}} = 0$$

Expanding $\partial s(q_{HL}, t_{HL})/\partial q_{HL}$ and re-arranging:

$$\frac{\partial v(q_{HL},t_{HL})}{\partial q_{HL}} = \frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^L\right)}{\partial q_{HL}} + \lambda_2 \frac{p_{LL}}{p_{HL}} \frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial q_{HL}} \quad \text{(A23)}$$

Re-arranging further to obtain:

$$\frac{\frac{\partial v(q_{HL},t_{HL})}{\partial q_{HL}} - \frac{\partial c\left(q_{HL},q_{HL},\theta_q^H,\theta_t^L\right)}{\partial q_{HL}}}{\frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial q_{HL}}} = \lambda_2 \frac{p_{LL}}{p_{HL}} \quad \text{(A24)}$$

- With respect to t_{HL}:

$$-\lambda_2\, p_{LL} \frac{\partial \Delta_{LL}^{HL}(q_{HL}, t_{HL})}{\partial t_{HL}} + p_{HL} \frac{\partial s(q_{HL}, t_{HL})}{\partial t_{HL}} = 0$$

Expanding $\partial s(q_{HL}, t_{HL})/\partial t_{HL}$ and re-arranging:

$$\frac{\partial v(q_{HL},t_{HL})}{\partial t_{HL}} = \frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^L\right)}{\partial t_{HL}} + \lambda_2 \frac{p_{LL}}{p_{HL}} \frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial t_{HL}} \quad \text{(A25)}$$

Re-arranging again shows:

$$\frac{\frac{\partial v(q_{HL},t_{HL})}{\partial t_{HL}}-\frac{\partial c\left(q_{HL},q_{HL},\theta_q^H,\theta_t^L\right)}{\partial t_{HL}}}{\frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial t_{HL}}}=\lambda_2\frac{p_{LL}}{p_{HL}} \qquad \text{(A26)}$$

The second best levels of output q_{HL}^{sb}, t_{HL}^{sb} are, respectively, the solutions to (A23) and (A25). In both cases, q_{HL}^{sb} and t_{HL}^{sb} deviate from first-best levels q_{HL}^{fb}, t_{HL}^{fb} by an amount equal to the second term of equations the corresponding equations. Equations (A24) and (A26) show that $q_{HL}^{sb}=q\left(\lambda_2\frac{p_{LL}}{p_{HL}}\right), t_{HL}^{sb}=t\left(\lambda_2\frac{p_{LL}}{p_{HL}}\right)$. Note that $\frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial q_{HL}}, \frac{\partial \Delta_{LL}^{HL}(q_{HL},t_{HL})}{\partial t_{HL}} \neq 0$ for non-separable cost functions.

Consider the first-order conditions for the HH type:

- With respect to q_{HH}:

$$-p_{LL}\left[\lambda_1\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+\lambda_2\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+\lambda_3\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]-p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} - p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+p_{HH}\frac{\partial S(q_{HH},t_{HH})}{\partial q_{HH}}=0$$

  Expanding $\partial s(q_{HH},t_{HH})/\partial q_{HH}$ and re-arranging:

$$\frac{\partial v(q_{HH},t_{HH})}{\partial q_{HH}}=\frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^H\right)}{\partial q_{HH}}+\frac{(\lambda_1\,p_{LL}+p_{LH})}{p_{HH}}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+\frac{(\lambda_2\,p_{LL}+p_{HL})}{p_{HH}}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+\frac{\lambda_3\,p_{LL}}{p_{HH}}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}$$

  It follows that the deviation from the first-best depends on the difference in marginal cost between the HH type relative to the LH, the HL and the LL:

$$\frac{\partial v(q_{HH},t_{HH})}{\partial q_{HH}}=\frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^H\right)}{\partial q_{HH}}+\frac{1}{p_{HH}}\left[(\lambda_1\,p_{LL}+\,p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+(\lambda_2\,p_{LL}+p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}+\lambda_3\,p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]$$

- With respect to t_{HH}:

$$-p_{LL}\left[\lambda_1\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}+\lambda_2\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}+\lambda_3\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]-p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} - p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}+p_{HH}\frac{\partial S(q_{HH},t_{HH})}{\partial t_{HH}}=0$$

  Expanding $\partial s(q_{HH},t_{HH})/\partial t_{HH}$ and re-arranging:

$$\frac{\partial v(q_{HH},t_{HH})}{\partial t_{HH}}=\frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^H\right)}{\partial t_{HH}}+\frac{(\lambda_1\,p_{LL}+p_{LH})}{p_{HH}}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}+\frac{(\lambda_2\,p_{LL}+p_{HL})}{p_{HH}}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}+\frac{\lambda_3\,p_{LL}}{p_{HH}}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}$$

Thus the deviation from the first-best depends on the difference in marginal cost between the HH type relative to the LH, the HL and the LL:

$$\frac{\partial v(q_{HH},t_{HH})}{\partial t_{HH}} - \frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^H\right)}{\partial t_{HH}} = \frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} +\ p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + (\lambda_2\, p_{LL} + p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + \lambda_3\, p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]$$

So,

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} +\ p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + (\lambda_2\, p_{LL} +\ p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + \lambda_3\, p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]\right)$$

$$t_{HH}^{sb} = t\left(\frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} +\ p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + (\lambda_2\, p_{LL} +\ p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + \lambda_3\, p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]\right)$$

To summarize, the *general* solutions to the baseline relaxed program are:

- For the LL type:

$$q_{LL}^{sb} = q_{LL}^{fb}\,,\ t_{LL}^{sb} = t_{LL}^{fb} \quad \text{(A27)}$$

- For the LH type:

$$q_{LH}^{sb} = q\left(\lambda_1 \frac{p_{LL}}{p_{LH}}\right), t_{LH}^{sb} = t\left(\lambda_1 \frac{p_{LL}}{p_{LH}}\right) \text{(A28)}$$

- For the HL type:

$$q_{HL}^{sb} = q\left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right), t_{HL}^{sb} = t\left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) \text{(A29)}$$

- For the HH type:

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} +\ p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + (\lambda_2\, p_{LL} +\ p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + \lambda_3\, p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}\right]\right)$$

$$t_{HH}^{sb} = t\left(\frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} +\ p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + (\lambda_2\, p_{LL} +\ p_{HL})\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + \lambda_3\, p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}\right]\right)$$

(A30)

Case 1: Positive correlation

The second case arises when $\lambda_1, \lambda_2, \lambda_3 > 0$ such that:

$$U_{LL} = \begin{cases} \left(\Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right)\right) \\ \left(\Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right)\right) \\ \left(\Delta_{LL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})\right) \end{cases}$$

That is, in this case it is optimal for the principal to give the efficient type a rent that will make him indifferent between his own contract and any of the three remaining agent's.

- Setting $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) + \Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right) = \Delta_{LL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$ and expanding:

$$c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^H\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^H\right) + c\left(q_{LH}^{sb}, t_{LH}^{sb}, \theta_q{}^L, \theta_t{}^H\right) - c\left(q_{LH}^{sb}, t_{LH}^{sb}, \theta_q{}^L, \theta_t{}^L\right) \\ = c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^H\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^L\right)$$

Simplifying and re-arranging:

$$c\left(q_{LH}^{sb}, t_{LH}^{sb}, \theta_q{}^L, \theta_t{}^H\right) - c\left(q_{LH}^{sb}, t_{LH}^{sb}, \theta_q{}^L, \theta_t{}^L\right) = c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^H\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^L\right)$$

$$\Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right) = \Delta_{LL}^{LH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) \Leftrightarrow q_{LH}^{sb} = q_{HH}^{sb}; t_{LH}^{sb} = t_{HH}^{sb}$$

- Likewise, $\Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) + \Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right) = \Delta_{LL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$ and expanding:

$$c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^H\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^L\right) + c\left(q_{HL}^{sb}, t_{HL}^{sb}, \theta_q{}^H, \theta_t{}^L\right) - c\left(q_{HL}^{sb}, t_{HL}^{sb}, \theta_q{}^L, \theta_t{}^L\right) \\ = c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^H\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^L\right)$$

Simplifying and re-arranging:

$$c\left(q_{HL}^{sb}, t_{HL}^{sb}, \theta_q{}^H, \theta_t{}^L\right) - c\left(q_{HL}^{sb}, t_{HL}^{sb}, \theta_q{}^L, \theta_t{}^L\right) = c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^H, \theta_t{}^L\right) - c\left(q_{HH}^{sb}, t_{HH}^{sb}, \theta_q{}^L, \theta_t{}^L\right)$$

$$\Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right) = \Delta_{LL}^{HL}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) \Leftrightarrow q_{HL}^{sb} = q_{HH}^{sb}; t_{HL}^{sb} = t_{HH}^{sb}$$

Thus, output levels are equal for the less efficient types: $q_{LH}^{sb} = q_{HL}^{sb} = q_{HH}^{sb}; t_{LH}^{sb} = t_{HL}^{sb} = t_{HH}^{sb}$.

In consequence, from (A28), (A29) and (A30), the deviation from the first best for $q_{LH}^{sb}, q_{HL}^{sb}, q_{HH}^{sb}$ and $t_{LH}^{sb}, t_{HL}^{sb}, t_{HH}^{sb}$ is, respectively:

For element q:

$$\lambda_1 \frac{p_{LL}}{p_{LH}} = \lambda_2 \frac{p_{LL}}{p_{HL}} = \frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} + p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}} + (\lambda_2\, p_{LL} + p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}} + \lambda_3\, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}}\right]$$

For element t:

$$\lambda_1 \frac{p_{LL}}{p_{LH}} = \lambda_2 \frac{p_{LL}}{p_{HL}} = \frac{1}{p_{HH}}\left[(\lambda_1\, p_{LL} + p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}} + (\lambda_2\, p_{LL} + p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}} + \lambda_3\, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}}\right]$$

that is, two different systems of three equations in three unknowns. Because of symmetry between the systems, it is more compact to write and solve:

$$\lambda_1 \frac{p_{LL}}{p_{LH}} = \lambda_2 \frac{p_{LL}}{p_{HL}} = \frac{1}{p_{HH}} \left[(\lambda_1 \, p_{LL} + \, p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (\lambda_2 \, p_{LL} + \, p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \lambda_3 \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right]$$

where $z \in \{q, t\}$.

Finding the second-best output levels for elements q, t thus amounts to solving the following:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \text{ (A31)}$$

$$\lambda_1 \frac{p_{LL}}{p_{LH}} = \frac{1}{p_{HH}} \left[(\lambda_1 \, p_{LL} + \, p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (\lambda_2 \, p_{LL} + \, p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \lambda_3 \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right]$$
(A32)

$$\lambda_2 \frac{p_{LL}}{p_{HL}} = \frac{1}{p_{HH}} \left[(\lambda_1 \, p_{LL} + \, p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (\lambda_2 \, p_{LL} + \, p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \lambda_3 \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right]$$
(A33)

Substituting for $\lambda_3 = 1 - \lambda_1 - \lambda_2$ into (A32) and (A33) simplifies the system of equations to two equations in two unknowns:

$$\lambda_1 \frac{p_{LL}}{p_{LH}} = \frac{1}{p_{HH}} \left[(\lambda_1 \, p_{LL} + \, p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (\lambda_2 \, p_{LL} + \, p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (1 - \lambda_1 - \lambda_2) \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right] \text{ (A34)}$$

$$\lambda_2 \frac{p_{LL}}{p_{HL}} = \frac{1}{p_{HH}} \left[(\lambda_1 \, p_{LL} + \, p_{LH}) \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (\lambda_2 \, p_{LL} + \, p_{HL}) \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + (1 - \lambda_1 - \lambda_2) \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right] \text{ (A35)}$$

Re-organizing the equations:

$$\lambda_1 \left(\frac{p_{LL} \, p_{HH}}{p_{LH}} - \, p_{LL} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) + \lambda_2 \left(- \, p_{LL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) = p_{LH} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \, p_{HL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}$$

$$\lambda_1 \left(\, p_{LL} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) + \lambda_2 \left(\, p_{LL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \frac{p_{LL} \, p_{HH}}{p_{HL}} \right) = - \, p_{LH} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - p_{HL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \, p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}$$

After simplification, the expressions for $\lambda_1, \lambda_2, \lambda_3$ are:

$$\lambda_1 = \frac{\left(p_{LH} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right)}{\frac{p_{LL}}{p_{LH}} \left(p_{HH} - p_{HL} \left(\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) - p_{LH} \left(\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) \right)} \text{ (A36)}$$

$$\lambda_2 = \frac{\left(p_{LH} \frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HL} \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{LL} \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right)}{\frac{p_{LL}}{p_{HL}} \left(p_{HH} - p_{HL} \left(\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) - p_{LH} \left(\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \right) \right)} \text{ (A37)}$$

$$\lambda_3 = 1 - \frac{\left(p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} \right)}{p_{LL}\left(p_{HH} - p_{HL}\left(\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right) - p_{LH}\left(\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)\right)} (p_{LH} + p_{HL})$$

(A38)

Finally, it remains to establish that $\lambda_1, \lambda_2, \lambda_3 > 0$. To this end, we prove the following:

Lemma

1. $\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}, \frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}, \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} > 0$ for $z \in \{q, t\}$
2. $\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} < 0$ and $\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} < 0$, for $z \in \{q, t\}$

Proof:

From the agent's payoff function $U = T - c(q, t, \theta_q, \theta_t)$, we know that $\frac{\partial U}{\partial \theta_z} = -\frac{\partial c}{\partial \theta_z}$, $z \in \{q, t\}$ the payoff is decreasing in the agent's type. Alternatively, we write $\frac{\partial c}{\partial \theta_z} > 0$, so the cost increases with the agent's type.

Therefore, if the HH type produces outputs (q_{HH}, t_{HH}), its cost is always strictly greater than if it were produced by any other type. So we can write:

For $ij \in \{LH, HL, LL\}$:

$$c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) > c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j)$$

$$c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j) > 0$$

$$\frac{\partial c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H)}{\partial z_{HH}} - \frac{\partial c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j)}{\partial z_{HH}} > 0$$

$$\frac{\partial\Delta_{ij}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} > 0$$

which proves statement 1.

Also note that, for $ij \in \{LH, HL\}$:

$$c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j) > c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^L)$$

$$-c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j) < -c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^L)$$

Adding $c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H)$ at both sides of the inequality:

$$c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^i, \theta_t^j) < c(q_{HH}, t_{HH}, \theta_q^H, \theta_t^H) - c(q_{HH}, t_{HH}, \theta_q^L, \theta_t^L)$$

$$\Delta_{ij}^{HH}(q_{HH}, t_{HH}) < \Delta_{LL}^{HH}(q_{HH}, t_{HH})$$

Deriving with respect to $z \in \{q, t\}$:

$$\frac{\partial \Delta_{ij}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} < \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}$$

$$\frac{\partial \Delta_{ij}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} < 0$$

which proves statement 2.

It follows that λ_1, λ_2 are strictly positive. It remains to prove that $\lambda_3 > 0$:

$$\lambda_3 = 1 - \frac{\left(p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} \right)}{p_{LL}\left(p_{HH} - p_{HL}\left(\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right) - p_{LH}\left(\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)\right)}(p_{LH} + p_{HL}) > 0$$

$$\frac{\left(p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} \right)}{p_{LL}\left(p_{HH} - p_{HL}\left(\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right) - p_{LH}\left(\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)\right)}(p_{LH} + p_{HL}) < 1$$

$$\left(p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} \right)(p_{LH} + p_{HL}) < p_{LL}\left(p_{HH} - p_{HL}\left(\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right) - p_{LH}\left(\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)\right)$$

$$\begin{aligned} &{p_{LH}}^2 \frac{\partial \Delta_{LH}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} + p_{LH}\,p_{HL}\frac{\partial \Delta_{LH}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} + p_{HL}\,p_{LH}\frac{\partial \Delta_{HL}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} + {p_{HL}}^2\frac{\partial \Delta_{HL}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} \\ &< p_{LL}\,p_{HH} - p_{LL}\,p_{HL}\frac{\partial \Delta_{HL}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} - p_{LL}\,p_{LH}\frac{\partial \Delta_{LH}^{HH}\left(q_{HH},t_{HH}\right)}{\partial z_{HH}} \end{aligned}$$

which simplifies to the following condition:

$cov\left(\theta_q^i, \theta_t^j\right) > (1 - p_{HH})\left(p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right) - p_{LH}\,p_{HL}$ for $z \in \{q,t\}$ (A39)

Substituting for $\lambda_1, \lambda_2, \lambda_3$ in (A28), (A29), (A30) shows:

$$\begin{aligned} &q_{LH}^{sb} = q_{HL}^{sb} = q_{HH}^{sb} \\ &= q\left(\frac{p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} + p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}}{p_{HH} + \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}(p_{HL} + p_{LH}) - p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}} - p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial q_{HH}}}\right) \end{aligned}$$

$$\begin{aligned} &t_{LH}^{sb} = t_{HL}^{sb} = q_{HH}^{sb} \\ &= t\left(\frac{p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} + p_{LL}\frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}}{p_{HH} + \frac{\partial \Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}(p_{HL} + p_{LH}) - p_{HL}\frac{\partial \Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}} - p_{LH}\frac{\partial \Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial t_{HH}}}\right) \end{aligned}$$

which are equations (6a-6c) in the main text.

Case 2: Weak correlation

The third case arises when $\lambda_3 = 0$ and $\lambda_1 + \lambda_2 = 1$. For simplicity, let $\lambda_1 = \lambda$ so $\lambda_2 = 1 - \lambda$

$$U_{LL} = \begin{cases} \left(\Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb})\right) \\ \left(\Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb})\right) \end{cases}$$

That is, in this case it is optimal for the principal to give the efficient type a rent that will make him indifferent between his own contract and any of the two adjacent, middle types:

$$\Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb}) = \Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) + \Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb}) \quad \text{(A40)}$$

which is the same as equation (7) in the main text.

The "deltas" in (A40) are increasing in their arguments. From the lemma proved above (see the subsection on Case 2), we know that $\Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$, $\Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$ are increasing in $(q_{HH}^{sb}, t_{HH}^{sb})$.

By a similar argument, we show that $\Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb})$ is increasing in $(q_{LH}^{sb}, t_{LH}^{sb})$ and that $\Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb})$ is increasing in $(q_{HL}^{sb}, t_{HL}^{sb})$:

For $ij \in \{LH, HL\}$:

$$c(q_{ij}, t_{ij}, \theta_q^i, \theta_t^j) > c(q_{ij}, t_{ij}, \theta_q^L, \theta_t^L)$$

$$c(q_{ij}, t_{ij}, \theta_q^i, \theta_t^j) - c(q_{ij}, t_{ij}, \theta_q^L, \theta_t^L) > 0$$

$$\frac{\partial c(q_{ij}, t_{ij}, \theta_q^i, \theta_t^j)}{\partial z_{ij}} - \frac{\partial c(q_{ij}, t_{ij}, \theta_q^L, \theta_t^L)}{\partial z_{ij}} > 0$$

$$\frac{\partial \Delta_{ij}^{HH}(q_{ij}, t_{ij})}{\partial z_{ij}} > 0 \text{ for } z \in \{q, t\}$$

However, substituting for $\lambda_1 = \lambda$, $\lambda_2 = 1 - \lambda$, $\lambda_3 = 0$ in (A28), (A29), (A30):

$$q_{LH}^{sb} = q\left(\lambda \frac{p_{LL}}{p_{LH}}\right), t_{LH}^{sb} = t\left(\lambda \frac{p_{LL}}{p_{LH}}\right) \quad \text{(A41)}$$

$$q_{HL}^{sb} = q\left((1-\lambda)\frac{p_{LL}}{p_{HL}}\right), t_{HL}^{sb} = t\left((1-\lambda)\frac{p_{LL}}{p_{HL}}\right) \quad \text{(A42)}$$

$$q_{HH}^{sb} = q\left(\frac{1}{p_{HH}}\left[(\lambda\, p_{LL} + p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}} + \left((1-\lambda)\, p_{LL} + p_{HL}\right)\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}}\right]\right) \quad \text{(A43)}$$

$$t_{HH}^{sb} = t\left(\frac{1}{p_{HH}}\left[(\lambda\, p_{LL} + p_{LH})\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}} + \left((1-\lambda)\, p_{LL} + p_{HL}\right)\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}}\right]\right) \quad \text{(A44)}$$

Which are equations (8a-8c) in the main text.

reveals that the deviation of $(q_{LH}^{sb}, t_{LH}^{sb})$ relative to the first best $(q_{LH}^{fb}, t_{LH}^{fb})$ is increasing in λ, whereas the deviation of $(q_{HL}^{sb}, t_{HL}^{sb})$ relative to $(q_{HL}^{fb}, t_{HL}^{fb})$ is decreasing in this parameter.

In contrast, the effect of λ on $(q_{HH}^{sb}, t_{HH}^{sb})$ is ambiguous and depends on the marginal cost difference between the HH and the LH types and the marginal cost difference between the HH and the HL types, $\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}, \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}$ for $z \in \{q, t\}$.

So, under weak correlation, the principal's problem of defining the optimal output reduces to proving the existence of a λ for which (A40) holds. There are three cases to consider:

i. *Marginal cost symmetry between the middle types: s*uppose that

$$\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} = \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \text{ for } z \in \{q, t\} \qquad \text{(A45)}$$

holds.

Substituting (A45) into equations (A43) and (A44) leads to the following expressions for q_{HH}^{sb}, t_{HH}^{sb}, which are independent of λ:

$$q_{HH}^{sb} = q\left(\frac{1 - p_{HH}}{p_{HH}}\left[\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial q_{HH}}\right]\right)$$

$$t_{HH}^{sb} = t\left(\frac{1 - p_{HH}}{p_{HH}}\left[\frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial t_{HH}}\right]\right)$$

Therefore, to prove the existence of a λ that satisfies (A40), the arguments of $\Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb})$ and $\Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb})$ are the only relevant terms. The former is decreasing whereas the latter is increasing in λ.

Re-arranging (A40), we obtain:

$$\Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb}) - \Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb}) = \Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) - \Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$$

From (A45) we know that the difference $\Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) - \Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb})$ is a constant k^*, that is:

$$\Delta_{HL}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) - \Delta_{LH}^{HH}(q_{HH}^{sb}, t_{HH}^{sb}) = k^* \text{ (A46)}$$

Substituting k^* into (A46) and rearranging yields a function $f(\lambda), 0 \leq \lambda \leq 1$:

$$f(\lambda) = \Delta_{LL}^{LH}(q_{LH}^{sb}, t_{LH}^{sb}) - \Delta_{LL}^{HL}(q_{HL}^{sb}, t_{HL}^{sb}) - k^* = 0 \qquad \text{(A47)}$$

The problem now reduces to showing that (A47) has a root and to show this we use a classical result, namely the:

Lemma *(Intermediate Value Theorem, as stated in de La Fuente, 2000)):* Let f be a continuous real-valued function on the closed bounded interval $[a, b]$. Then for each number γ (strictly) between $f(a)$ and $f(b)$, there exists a point $c \in (a, b)$ such that $f(c) = \gamma$.

To be able to apply the result, $f(\lambda)$ must be continuous over λ, which can be easily verified by simple inspection of (A41) and (A42), which are well-defined over the admissible range of λ. Therefore, the sufficient condition for the existence of a root for (A47) over the domain of $f(\lambda)$ is that $f(0)f(1) < 0$.

If $\lambda = 0$:

$$f(0) = \Delta_{LL}^{LH}(q(0), t(0)) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right) - k^*$$

If $\lambda = 1$:

$$f(1) = \Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) - \Delta_{LL}^{HL}\big(q(0), t(0)\big) - k^*$$

We now show that $\Delta_{LL}^{LH}(q(0), t(0)) > \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$ and that $\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) < \Delta_{LL}^{HL}(q(0), t(0))$ hold.

<u>Proof:</u>

Let us assume that $\Delta_{LL}^{LH}(q(0), t(0)) > \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$ does not hold. In other words, $\Delta_{LL}^{LH}(q(0), t(0)) < \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$ or $\Delta_{LL}^{LH}(q(0), t(0)) = \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$ hold. We know from monotonicity of the cost difference that $\Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right) < \Delta_{LL}^{HL}\big(q(0), t(0)\big)$ always holds and thus $\Delta_{LL}^{LH}(q(0), t(0)) < \Delta_{LL}^{HL}\big(q(0), t(0)\big)$. In this case, the LL type will have a strict preference to mimic the HL type, which contradicts our assumption in (A40). The same logic applies when $\Delta_{LL}^{LH}(q(0), t(0)) = \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$ holds. A similar argument proves the claim that $\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) < \Delta_{LL}^{HL}\big(q(0), t(0)\big)$ holds.

Therefore, if there is no large difference between $\Delta_{HL}^{HH}\big(q_{HH}^{sb}, t_{HH}^{sb}\big)$ and $\Delta_{LH}^{HH}\big(q_{HH}^{sb}, t_{HH}^{sb}\big)$, that is, if the value of k^* is within the following interval:

$$\Delta_{LL}^{LH}\left(q(\frac{p_{LL}}{p_{LH}}), t(\frac{p_{LL}}{p_{LH}})\right) - \Delta_{LL}^{HL}(q(0), t(0)) < k^* < \Delta_{LL}^{LH}(q(0), t(0)) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$$

then we always have $f(0) > 0$ and $f(1) < 0$ implying that $f(0)f(1) < 0$ and, by the Intermediate Value Theorem, there is certainly a $0 < \lambda^* < 1$ satisfying (A46). QED.

ii. *The LH type is more marginally inefficient than the HL type:* to analyse this case suppose that

$$\frac{\partial \Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} > \frac{\partial \Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} \text{ for } z \in \{q, t\}$$

which implies $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) - \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) > k^*$, where k^* is as in (A46), defined in the case (i), so $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) - \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right)$ belongs to the interval (k^*, ∞). This means that there is a constant $k \in (k^*, \infty)$ for which $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) - \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) = k$ holds.

Then, in a similar way to (A47) we can write the following equation:

$$f(\lambda) = \Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right) - \Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right) - k = 0 \qquad \text{(A48)}$$

(A48) is the same as (A47), except that the constant is now k rather than k^* where $k \in (k^*, \infty)$. Thus, the same approach used to prove that $f(0) > 0$ and $f(1) < 0$ for (A47) can also be used for (A48). We avoid repeating this step and only present the acceptable range of k that guarantees the existence of $0 < \lambda^* < 1$ that satisfies (A48):

$$\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) - \Delta_{LL}^{HL}\left(q(0), t(0)\right) < k^* < k$$
$$< \Delta_{LL}^{LH}\left(q(0), t(0)\right) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$$

iii. *The HL type is more marginally inefficient than the LH type:* to analyse this case suppose that

$$\frac{\partial \Delta_{\mathrm{LH}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} < \frac{\partial \Delta_{\mathrm{HL}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} \text{ for } \mathrm{z} \in \{\mathrm{q}, \mathrm{t}\}$$

This means $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) - \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) < k^*$ where k^* is as in (A46) same constant as in case (i). In a similar manner to the previous case there is a $k' \in (-\infty, k^*)$ for which $\Delta_{LH}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) - \Delta_{HL}^{HH}\left(q_{HH}^{sb}, t_{HH}^{sb}\right) = k'$.

This means we need prove that there is a $0 < \lambda^* < 1$ that satifies equation below.

$$f(\lambda) = \Delta_{LL}^{LH}\left(q_{LH}^{sb}, t_{LH}^{sb}\right) - \Delta_{LL}^{HL}\left(q_{HL}^{sb}, t_{HL}^{sb}\right) - k' = 0$$

Repeating the previous proof procedure we show that the acceptable range of k' that guarantees the existence of $0 < \lambda^* < 1$ as a root to above equation is:

$$\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) - \Delta_{LL}^{HL}\left(q(0), t(0)\right) < k' < k^* < \Delta_{LL}^{LH}(q(0), t(0)) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$$

The above results show that irrespective of how marginal cost differences are (i.e., $\frac{\partial \Delta_{\mathrm{LH}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} = \frac{\partial \Delta_{\mathrm{HL}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}}$ or $\frac{\partial \Delta_{\mathrm{LH}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} > \frac{\partial \Delta_{\mathrm{HL}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}}$ or $\frac{\partial \Delta_{\mathrm{LH}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}} < \frac{\partial \Delta_{\mathrm{HL}}^{\mathrm{HH}}(\mathrm{q_{HH}}, \mathrm{t_{HH}})}{\partial \mathrm{z_{HH}}}$), there is always a λ that satisfies (A40). The condition for the existence of λ across three aforementioned subcases can be compactly presented as follows:

$$\Delta_{LL}^{LH}\left(q\left(\frac{p_{LL}}{p_{LH}}\right), t\left(\frac{p_{LL}}{p_{LH}}\right)\right) - \Delta_{LL}^{HL}\big(q(0), t(0)\big) < k' < k^* < k < \Delta_{LL}^{LH}(q(0), t(0)) - \Delta_{LL}^{HL}\left(q\left(\frac{p_{LL}}{p_{HL}}\right), t\left(\frac{p_{LL}}{p_{HL}}\right)\right)$$

Case 3:

Let $\lambda_1 = 1, \lambda_2 = 0, \lambda_3 = 0$ in (A28), (A29), (A30) to obtain equations (9a-9c) in the main text.

Case 4:

Let $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 0$ in (A28), (A29), (A30) to obtain equations (10a-10c) in the main text.

A4. Equations for the simulation in Section 4

Consider the following Cobb-Douglas specification

$$v(q,t) = A{q_{ij}}^{\alpha}{t_{ij}}^{1-\alpha}$$

$$c(q,t,\theta_q,\theta_t) = \left({\theta_q}^i\right)^{\beta}\left({\theta_t}^j\right)^{1-\beta}\left(q_{ij}t_{ij}\right)^{\gamma}$$

A4.1 Output levels

1. *For the inefficient (HH) type:*

From:

$$\frac{\partial v(q_{HH},t_{HH})}{\partial z_{HH}} = \frac{\partial c\left(q_{HL},t_{HL},\theta_q^H,\theta_t^H\right)}{\partial z_{HH}} + \frac{(\lambda_1\, p_{LL} + p_{LH})}{p_{HH}}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + \frac{(\lambda_2\, p_{LL} + p_{HL})}{p_{HH}}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + \frac{\lambda_3\, p_{LL}}{p_{HH}}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}$$

for $z \in \{q,t\}$

- q_{HH}:

With respect to q_{HH}:

$$\begin{aligned} A\alpha{q_{HH}}^{\alpha-1}{t_{HH}}^{1-\alpha} = {} & (q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta} \\ & + \frac{(\lambda_1\, p_{LL} + p_{LH})}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)\right) \\ & + \frac{(\lambda_2\, p_{LL} + p_{HL})}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\right) \\ & + \frac{\lambda_3\, p_{LL}}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\right) \end{aligned}$$

Let:

$$K_1 = \gamma\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}$$

$$K_2 = \gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)$$

$$K_3 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}\right)$$

$$K_4 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right)$$

The expression is now:

$$A\alpha {q_{HH}}^{\alpha-1}{t_{HH}}^{1-\alpha} = (q_{HH}t_{HH})^{\gamma-1}t_{HH}\,K_1 + \frac{(\lambda_1\, p_{LL} +\, p_{LH})}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}t_{HH}\,K_2 + \frac{(\lambda_2\, p_{LL} +\, p_{HL})}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}t_{HH}\,K_3 + \frac{\lambda_3\, p_{LL}}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}t_{HH}\,K_4$$

From which a closed form solution for q_{HH} can be derived:

$$q_{HH} = \left[\frac{1}{A\alpha}\left(K_1 + \frac{(\lambda_1\, p_{LL}+ p_{LH})}{p_{HH}}K_2 + \frac{(\lambda_2\, p_{LL}+ p_{HL})}{p_{HH}}\,K_3 + \frac{\lambda_3\, p_{LL}}{p_{HH}}K_4\right)\right]^{\frac{1}{\alpha-\gamma}} {t_{HH}}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}} \text{ (A48)}$$

- t_{HH}:

Likewise, with respect to t_{HH}:

$$\begin{aligned} A(1-\alpha){q_{HH}}^{\alpha}{t_{HH}}^{-\alpha} &= (q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} \\ &+ \frac{(\lambda_1\, p_{LL} +\, p_{LH})}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^H)^{1-\beta}\right)\right) \\ &+ \frac{(\lambda_2\, p_{LL} +\, p_{HL})}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}\right)\right) \\ &+ \frac{\lambda_3\, p_{LL}}{p_{HH}}\left((q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right)\right) \end{aligned}$$

Recall that:

$$K_1 = \gamma\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta}$$

$$K_2 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^H)^{1-\beta}\right)$$

$$K_3 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}\right)$$

$$K_4 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right)$$

The expression is now:

$$A(1-\alpha){q_{HH}}^{\alpha}{t_{HH}}^{-\alpha} = (q_{HH}t_{HH})^{\gamma-1}q_{HH}\,K_1 + \frac{(\lambda_1\, p_{LL}+ p_{LH})}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}q_{HH}\,K_2 + \frac{(\lambda_2\, p_{LL}+ p_{HL})}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}q_{HH}K_3 + \frac{\lambda_3\, p_{LL}}{p_{HH}}(q_{HH}t_{HH})^{\gamma-1}q_{HH}\,K_4$$

From which a closed form solution for t_{HH} can be derived:

$$t_{HH} = \left[\frac{1}{A(1-\alpha)}\left(K_1 + \frac{(\lambda_1\, p_{LL}+ p_{LH})}{p_{HH}}K_2 + \frac{(\lambda_2\, p_{LL}+ p_{HL})}{p_{HH}}\,K_3 + \frac{\lambda_3\, p_{LL}}{p_{HH}}K_4\right)\right]^{\frac{1}{1-\gamma-\alpha}} {q_{HH}}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}} \text{ (A49)}$$

2. *For the middle (LH) type:*

From:

$$\frac{\partial v(q_{LH},t_{LH})}{\partial z_{LH}}=\frac{\partial c\left(q_{LH},t_{LH},\theta_q^L,\theta_t^H\right)}{\partial z_{LH}}+\lambda_1\frac{p_{LL}}{p_{LH}}\frac{\partial \Delta_{LL}^{LH}(q_{LH},t_{LH})}{\partial z_{LH}}$$

for $z\in\{q,t\}$

- q_{LH}:

With respect to q_{LH}:

$A\alpha {q_{LH}}^{\alpha-1}{t_{LH}}^{1-\alpha}=(q_{LH}t_{LH})^{\gamma-1}t_{LH}\,\gamma\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}+\left(\lambda_1\frac{p_{LL}}{p_{LH}}\right)(q_{LH}t_{LH})^{\gamma-1}t_{LH}\,\gamma\left[\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right]$

Let:

$Q_1=\gamma\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}$

$Q_2=\gamma\left[\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right]$

and we have:

$$A\alpha {q_{LH}}^{\alpha-1}{t_{LH}}^{1-\alpha}=(q_{LH}t_{LH})^{\gamma-1}t_{LH}\,Q_1+\left(\lambda_1\frac{p_{LL}}{p_{LH}}\right)(q_{LH}t_{LH})^{\gamma-1}t_{LH}\,Q_2$$

A closed form solution for q_{LH} is:

$$q_{LH}=\left[\frac{1}{A\alpha}\left(Q_1+\left(\lambda_1\frac{p_{LL}}{p_{LH}}\right)Q_2\right)\right]^{\frac{1}{\alpha-\gamma}}{t_{LH}}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}\ \text{(A50)}$$

- t_{LH}:

With respect to t_{LH}:

$A(1-\alpha){q_{LH}}^{\alpha}{t_{LH}}^{-\alpha}=(q_{LH}t_{LH})^{\gamma-1}q_{LH}\,\gamma\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}+\left(\lambda_1\frac{p_{LL}}{p_{LH}}\right)(q_{LH}t_{LH})^{\gamma-1}q_{LH}\,\gamma\left[\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right]$

Recall that:

$Q_1=\gamma\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}$

$Q_2=\gamma\left[\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right]$

to obtain:

$$A(1-\alpha){q_{LH}}^{\alpha}{t_{LH}}^{-\alpha}=(q_{LH}t_{LH})^{\gamma-1}q_{LH}\,Q_1+\left(\lambda_1\frac{p_{LL}}{p_{LH}}\right)(q_{LH}t_{LH})^{\gamma-1}q_{LH}\,Q_2$$

A closed form solution for t_{LH} is:

$$t_{LH} = \left[\frac{1}{A(1-\alpha)}\left(Q_1 + \left(\lambda_1 \frac{p_{LL}}{p_{LH}}\right) Q_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{LH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}} \text{ (A51)}$$

3. For the middle (HL) type:

From:

$$\frac{\partial v(q_{HL}, t_{HL})}{\partial z_{HL}} = \frac{\partial c\left(q_{HL}, t_{HL}, \theta_q^H, \theta_t^L\right)}{\partial z_{HL}} + \lambda_2 \frac{p_{LL}}{p_{HL}} \frac{\partial \Delta_{LL}^{HL}(q_{HL}, t_{HL})}{\partial z_{HL}}$$

for $z \in \{q, t\}$

- q_{HL}:

$$\begin{aligned} A\alpha q_{HL}{}^{\alpha-1} t_{HL}{}^{1-\alpha} \\ &= (q_{HL} t_{HL})^{\gamma-1} t_{HL}\, \gamma\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} \\ &+ \left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) (q_{HL} t_{HL})^{\gamma-1} t_{HL}\, \gamma\left[\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right] \end{aligned}$$

Let:

$R_1 = \gamma\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}$

$R_2 = \gamma\left[\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right]$

A closed form solution for q_{HL} is:

$$q_{HL} = \left[\frac{1}{A\alpha}\left(R_1 + \left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) R_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HL}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}} \text{ (A52)}$$

- *t_{HL}:*

$$\begin{aligned} A(1-\alpha) q_{HL}{}^{\alpha} t_{LH}{}^{-\alpha} \\ &= (q_{HL} t_{HL})^{\gamma-1} q_{HL}\, \gamma\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} \\ &+ \left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) (q_{HL} t_{HL})^{\gamma-1} q_{HL}\, \gamma\left[\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right] \end{aligned}$$

Recall that:

$R_1 = \gamma\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}$

$R_2 = \gamma\left[\left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right]$

$A(1-\alpha) q_{HL}{}^{\alpha} t_{LH}{}^{-\alpha} = (q_{HL} t_{HL})^{\gamma-1} q_{HL}\, R_1 + \left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) (q_{HL} t_{HL})^{\gamma-1} q_{HL}\, R_2$

A closed form solution for t_{HL} is:

$$t_{HL} = \left[\frac{1}{A(1-\alpha)}\left(R_1 + \left(\lambda_2 \frac{p_{LL}}{p_{HL}}\right) R_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HL}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}} \text{ (A53)}$$

4. For the efficient (LL) type:

From :

$$p_{LL}\left[\frac{\partial v(q_{LL},t_{LL})}{\partial z_{LL}}-\frac{\partial c\left(q_{LL},t_{LL},{\theta_q}^{L},{\theta_t}^{L}\right)}{\partial z_{LL}}\right]=0$$

for $z\in\{q,t\}$

- q_{LL}:

$$A\alpha {q_{LL}}^{\alpha-1}{t_{LL}}^{1-\alpha}=(q_{LL}t_{LL})^{\gamma-1}t_{LL}\,\gamma\left(\theta_q^L\right)^{\beta}\,(\theta_t^L)^{1-\beta}$$

Let:

$$S_1=\gamma\left(\theta_q^H\right)^{\beta}\,(\theta_t^L)^{1-\beta}$$

to obtain

$$q_{LL}=\left[\frac{1}{A\alpha}S_1\right]^{\frac{1}{\alpha-\gamma}}{t_{LL}}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}\quad\text{(A54)}$$

- t_{LL}:

$$A(1-\alpha){q_{LL}}^{\alpha}{t_{LL}}^{-\alpha}=(q_{LL}t_{LL})^{\gamma-1}q_{LL}\,\gamma\left(\theta_q^L\right)^{\beta}\,(\theta_t^L)^{1-\beta}$$

Recall that:

$$S_1=\gamma\left(\theta_q^H\right)^{\beta}\,(\theta_t^L)^{1-\beta}$$

to obtain:

$$t_{LL}=\left[\frac{1}{A(1-\alpha)}S_1\right]^{\frac{1}{1-\gamma-\alpha}}{q_{LL}}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}\quad\text{(A55)}$$

A4.2 Cases

Case 1: Positive correlation

Focus on the λs:

The solution to the system of equations is:

$$\lambda_1=\frac{p_{LH}\left(p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}+p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}+p_{HH}p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)}{p_{LL}\left(p_{HH}-p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}-p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}+\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}(p_{HH}p_{HL})+\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}(p_{HH}p_{LH})\right)}$$

More compactly:

$$\lambda_1 = \frac{p_{LH}\left(p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HH}p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\right)}{p_{LL}\left(p_{HH}\left(1 + \frac{\partial\Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}(p_{HL} + p_{LH})\right) - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\right)}$$

$$\lambda_2 = \frac{p_{HL}\left(p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + p_{HH}p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)}{p_{LL}\left(p_{HH} - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}(p_{HH}p_{HL}) + \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}(p_{HH}p_{LH})\right)}$$

More compactly:

$$\lambda_2 = \frac{p_{HL}\left(p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} + p_{HH}p_{LL}\frac{\partial\Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\right)}{p_{LL}\left(p_{HH}\left(1 + \frac{\partial\Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}(p_{HL} + p_{LH})\right) - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\right)}$$

$$\lambda_3 = \frac{\left(p_{HH}\,p_{LL} - (p_{LH}p_{LL})\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - (p_{HL}\,p_{LH})\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - (p_{LH}{}^2)\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - (p_{HL}\,p_{LL})\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - (p_{HL}\,p_{LH})\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - (p_{HL}{}^2)\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\right)}{p_{LL}\left(p_{HH} - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}} + \frac{\partial\Delta_{LL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}(p_{HH}{}^2 p_{HL}\,p_{LH})\right)}$$

More compactly:

$$\lambda_3 = \frac{\left(p_{HH}\,p_{LL} - \frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\,p_{LH}(p_{LL} + p_{LH} + p_{HL}) - \frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\,p_{HL}(p_{LL} + p_{LH} + p_{HL})\right)}{p_{LL}\left(p_{HH}\left(1 + \frac{\partial\Delta_{LL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}(p_{HL} + p_{LH})\right) - p_{LH}\frac{\partial\Delta_{LH}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}} - p_{HL}\frac{\partial\Delta_{HL}^{HH}(q_{HH}, t_{HH})}{\partial z_{HH}}\right)}$$

Now we derive the Cobb-Douglas instances of $\lambda_1, \lambda_2, \lambda_3$.

1. Consider the **denominator** in the expressions for $\lambda_1, \lambda_2, \lambda_3$:

The full expressions are:

- With respect to q_{HH}:

$$\begin{aligned} p_{LL}\Bigg(p_{HH}\bigg(1 + \Big((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^L)^{1-\beta}\right)\Big)(p_{HL} + p_{LH})\bigg) \\ - p_{LH}\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^\beta (\theta_t^H)^{1-\beta}\right)\right) \\ - p_{HL}\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}\right)\right)\Bigg) \end{aligned}$$

Let:

$x = \left(q_{HH} t_{HH}\right)^{\gamma-1} t_{HH}$

Recall that:

$K_2 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta}\right)$

$K_3 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta}\right)$

$K_4 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta}\right)$

The denominator for $\lambda_1, \lambda_2, \lambda_3$ with respect to q_{HH}:

$p_{LL}\left(p_{HH}\big(1 + (xK_4)(p_{HL} + p_{LH})\big) - p_{LH}(x\, K_2) - p_{HL}(xK_3)\right)$

Or more compactly:

$p_{LL}(p_{HH}(1 + K_4(p_{HL} + p_{LH})x) - (p_{LH}K_2 + p_{HL}K_3)x)$

- With respect to t_{HH}:

$$\begin{aligned} p_{LL}\Bigg(p_{HH}\Bigg(1 + \Big((q_{HH}t_{HH})^{\gamma-1} q_{HH}\, \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta}\right)\Big)(p_{HL} + p_{LH})\Bigg) \\ - p_{LH}\left((q_{HH}t_{HH})^{\gamma-1} q_{HH}\, \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta}\right)\right) \\ - p_{HL}\left((q_{HH}t_{HH})^{\gamma-1} q_{HH}\, \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta}\right)\right)\Bigg) \end{aligned}$$

Let:

$y = \left(q_{HH} t_{HH}\right)^{\gamma-1} q_{HH}$

Recall that:

$K_2 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta}\right)$

$K_3 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta}\right)$

$K_4 = \gamma\left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta}\right)$

The denominator for $\lambda_1, \lambda_2, \lambda_3$ with respect to t_{HH}:

$p_{LL}\left(p_{HH}\big(1 + (yK_4)(p_{HL} + p_{LH})\big) - p_{LH}(y\, K_2) - p_{HL}(yK_3)\right)$

Or more compactly:

$p_{LL}(p_{HH}(1 + K_4(p_{HL} + p_{LH})y) - (p_{LH}K_2 + p_{HL}K_3)y)$

2. Consider the **numerator** in the expressions for λ_1, λ_2:

With respect to q_{HH}:

- The numerator of λ_1 is:

$$p_{LH} x T_1$$

- The numerator of λ_2 is:

$$p_{HL} x T_1$$

where $x = \left(q_{HH} t_{HH}\right)^{\gamma-1} t_{HH}$

With respect to t_{HH}:

- The numerator of λ_1 is:

$$p_{LH}(q_{HH} t_{HH})^{\gamma-1} q_{HH} T_1$$

- The numerator of λ_2 is:

$$p_{HL}(q_{HH} t_{HH})^{\gamma-1} q_{HH} T_1$$

where $y = \left(q_{HH} t_{HH}\right)^{\gamma-1} q_{HH}$

$$T_1 = \gamma \Big[p_{LH} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta} \right) + p_{HL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta} \right) + p_{HH} p_{LL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta} \right) \Big]$$

Recall that the full expressions are:

With respect to q_{HH}:

- The numerator of λ_1 is:

$$p_{LH}(q_{HH} t_{HH})^{\gamma-1} t_{HH}\, \gamma \Big[p_{LH} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta} \right) + p_{HL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta} \right) + p_{HH} p_{LL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta} \right) \Big]$$

- The numerator of λ_2 is:

$$p_{HL}(q_{HH} t_{HH})^{\gamma-1} t_{HH}\, \gamma \Big[p_{LH} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^H)^{1-\beta} \right) + p_{HL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^{\beta} (\theta_t^L)^{1-\beta} \right) + p_{HH} p_{LL} \left(\left(\theta_q^H\right)^{\beta} (\theta_t^H)^{1-\beta} - \left(\theta_q^L\right)^{\beta} (\theta_t^L)^{1-\beta} \right) \Big]$$

With respect to t_{HH}:

- The numerator of λ_1 is:

$$p_{LH}(q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\Big[p_{LH}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)+\ p_{HL}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)$$
$$+\ p_{HH}p_{LL}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\Big]$$

- The numerator of λ_2 is:

$$p_{HL}(q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\Big[p_{LH}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)+\ p_{HL}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)$$
$$+\ p_{HH}p_{LL}\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\Big]$$

3. Consider the **numerator** in the expression for λ_3:

$$\left(p_{HH}\,p_{LL}-\frac{\partial\Delta_{LH}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\,p_{LH}(\,p_{LL}+\ p_{LH}+\ p_{HL})-\frac{\partial\Delta_{HL}^{HH}(q_{HH},t_{HH})}{\partial z_{HH}}\,p_{HL}(1-\ p_{HH})\right)$$

The full expression is:

- With respect to q_{HH}:

$$\Bigg(p_{HH}\,p_{LL}-\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)\right)p_{LH}(1-\ p_{HH})$$
$$-\left((q_{HH}t_{HH})^{\gamma-1}t_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\right)p_{HL}(1-\ p_{HH})\Bigg)$$

Recall that:

$$x=\left(q_{HH}t_{HH}\right)^{\gamma-1}t_{HH}$$

$$K_2=\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)$$

$$K_3=\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)$$

And substitute to obtain:

$$\left(\,p_{HH}\,p_{LL}-(xK_2)\,p_{LH}(\,p_{LL}+\ p_{LH}+\ p_{HL})-(xK3)\,p_{HL}(1-\ p_{HH})\right)$$

- With respect to t_{HH}:

$$\Bigg(p_{HH}\,p_{LL}-\left((q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)\right)p_{LH}(1-\ p_{HH})$$
$$-\left((q_{HH}t_{HH})^{\gamma-1}q_{HH}\,\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^H\right)^{\beta}(\theta_t^L)^{1-\beta}\right)\right)p_{HL}(1-\ p_{HH})\Bigg)$$

Recall that:

$$y=\left(q_{HH}t_{HH}\right)^{\gamma-1}q_{HH}$$

$$K_2=\gamma\left(\left(\theta_q^H\right)^{\beta}(\theta_t^H)^{1-\beta}-\left(\theta_q^L\right)^{\beta}(\theta_t^H)^{1-\beta}\right)$$

$K_3 = \gamma\left(\left(\theta_q^H\right)^\beta (\theta_t^H)^{1-\beta} - \left(\theta_q^H\right)^\beta (\theta_t^L)^{1-\beta}\right)$

$\left(p_{HH}\, p_{LL} - (yK_2)\, p_{LH}(p_{LL} + p_{LH} + p_{HL}) - (yK_3)\, p_{HL}(1 - p_{HH})\right)$

Finally, the λs are:

- With respect to q_{HH}:

$$\lambda_1 = \frac{p_{LH} x T_1}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)x\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)x\right)}$$

$$\lambda_2 = \frac{p_{HL} x T_1}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)x\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)x\right)}$$

$$\lambda_3 = \frac{p_{HH}\, p_{LL} - (xK_2)\, p_{LH}(1 - p_{HH}) - (xK_3)\, p_{HL}(1 - p_{HH})}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)x\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)x\right)}$$

- With respect to t_{HH}:

$$\lambda_1 = \frac{p_{LH} y T_1}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)y\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)y\right)}$$

$$\lambda_2 = \frac{p_{HL} y T_1}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)y\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)y\right)}$$

$$\lambda_3 = \frac{p_{HH}\, p_{LL} - (yK_2)\, p_{LH}(1 - p_{HH}) - (yK3)\, p_{HL}(1 - p_{HH})}{p_{LL}\left(p_{HH}\left(1 + K_4\left(p_{HL} + p_{LH}\right)y\right) - \left(p_{LH}K_2 + p_{HL}K_3\right)y\right)}$$

<u>Focus on the output levels:</u>

The HH type:

- q_{HH}:

To obtain q_{HH} numerically, plug the λs and substitute for x into:

$$A\alpha q_{HH}{}^{\alpha-1} t_{HH}{}^{1-\alpha} = x\, K_1 + \frac{\left(\frac{p_{LH} x T_1}{p_{LL}(p_{HH}(1 + K_4(p_{HL} + p_{LH})x) - (p_{LH}K_2 + p_{HL}K_3)x)}\, p_{LL} + p_{LH}\right)}{p_{HH}} x\, K_2$$
$$+ \frac{\left(\frac{p_{HL} x T_1}{p_{LL}(p_{HH}(1 + K_4(p_{HL} + p_{LH})x) - (p_{LH}K_2 + p_{HL}K_3)x)}\, p_{LL} + p_{HL}\right)}{p_{HH}} x\, K_3$$
$$+ \frac{\frac{p_{HH}\, p_{LL} - (xK_2)\, p_{LH}(1 - p_{HH}) - (xK_3)\, p_{HL}(1 - p_{HH})}{p_{LL}(p_{HH}(1 + K_4(p_{HL} + p_{LH})x) - (p_{LH}K_2 + p_{HL}K_3)x)}\, p_{LL}}{p_{HH}} x\, K_4$$

- t_{HH}:

To obtain t_{HH} numerically, plug the λs and substitute for y into:

$$A(1-\alpha)q_{HH}{}^{\alpha}t_{HH}{}^{-\alpha} = y\,K_1 + \frac{\left(\frac{p_{LH}yT_1}{p_{LL}(\,p_{HH}(1+K_4(\,p_{HL}+\,p_{LH})y)-(\,p_{LH}K_2+\,p_{HL}K_3)y)}\,p_{LL}+\,p_{LH}\right)}{p_{HH}}y\,K_2 + \frac{\left(\frac{p_{HL}yT_1}{p_{LL}(\,p_{HH}(1+K_4(\,p_{HL}+\,p_{LH})y)-(\,p_{LH}K_2+\,p_{HL}K_3)y)}\,p_{LL}+\,p_{HL}\right)}{p_{HH}}yK_3 + \frac{\frac{p_{HH}\,p_{LL}-(yK_2)\,p_{LH}(1-\,p_{HH})-(yK3)\,p_{HL}(1-\,p_{HH})}{p_{LL}(\,p_{HH}(1+K_4(\,p_{HL}+\,p_{LH})y)-(\,p_{LH}K_2+\,p_{HL}K_3)y)}\,p_{LL}}{p_{HH}}y\,K_4$$

The LH type:

- q_{LH}:

Use the closed form solution derived above and plug λ_1 (with respect to q_{HH}) to obtain:

$$q_{LH} = \left[\frac{1}{A\alpha}\left(Q_1+\left(\frac{p_{LH}xT_1}{p_{LL}\big(\,p_{HH}\big(1+K_4\big(\,p_{HL}+\,p_{LH}\big)x\big)-\big(\,p_{LH}K_2+\,p_{HL}K_3\big)x\big)}\frac{p_{LL}}{p_{LH}}\right)Q_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{LH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{LH}:

Use the closed form solution derived above and plug λ_1 (with respect to t_{HH}) to obtain:

$$t_{LH} = \left[\frac{1}{A(1-\alpha)}\left(Q_1+\left(\frac{p_{LH}yT_1}{p_{LL}(\,p_{HH}(1+K_4(\,p_{HL}+\,p_{LH})y)-(\,p_{LH}K_2+\,p_{HL}K_3)y)}\frac{p_{LL}}{p_{LH}}\right)Q_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{LH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

The HL type:

- q_{HL}:

Use the closed form solution derived above and plug λ_2 (with respect to q_{HH}) to obtain:

$$q_{HL} = \left[\frac{1}{A\alpha}\left(R_1+\left(\frac{p_{HL}xT_1}{p_{LL}\big(\,p_{HH}\big(1+K_4\big(\,p_{HL}+\,p_{LH}\big)x\big)-\big(\,p_{LH}K_2+\,p_{HL}K_3\big)x\big)}\frac{p_{LL}}{p_{HL}}\right)R_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HL}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HL}:

Use the closed form solution derived above and plug λ_2 (with respect to q_{HH}) to obtain:

$$t_{HL} = \left[\frac{1}{A(1-\alpha)}\left(R_1+\left(\frac{p_{HL}yT_1}{p_{LL}(\,p_{HH}(1+K_4(\,p_{HL}+\,p_{LH})y)-(\,p_{LH}K_2+\,p_{HL}K_3)y)}\frac{p_{LL}}{p_{HL}}\right)R_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HL}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

Case 2: Weak correlation

In case 3, simply let $\lambda_1 = \lambda$, $\lambda_2 = 1 - \lambda$ and $\lambda_3 = 0$ and plug these values into the closed form solutions derived above.

For computational purposes, it is convenient to use the following expressions:

For the LH type:

- q_{LH}:

$$q_{LH} = \left[\frac{1}{A\alpha}\left(Q_1 + \left(\lambda \frac{p_{LL}}{p_{LH}}\right) Q_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{LH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{LH}:

$$t_{LH} = \left[\frac{1}{A(1-\alpha)}\left(Q_1 + \left(\lambda \frac{p_{LL}}{p_{LH}}\right) Q_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{LH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HL type:

- q_{HL}:

$$q_{HL} = \left[\frac{1}{A\alpha}\left(R_1 + \left((1-\lambda) \frac{p_{LL}}{p_{HL}}\right)\right) R_2\right]^{\frac{1}{\alpha-\gamma}} t_{HL}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HL}:

$$t_{HL} = \left[\frac{1}{A(1-\alpha)}\left(R_1 + \left((1-\lambda) \frac{p_{LL}}{p_{HL}}\right) R_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HL}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HH type:

- q_{HH}:

$$q_{HH} = \left[\frac{1}{A\alpha}\left(K_1 + \frac{\left(\lambda\, p_{LL} + p_{LH}\right)}{p_{HH}} K_2 + \frac{\left((1-\lambda)\, p_{LL} + p_{HL}\right)}{p_{HH}} K_3\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HH}:

$$t_{HH} = \left[\frac{1}{A(1-\alpha)}\left(K_1 + \frac{(\lambda\, p_{LL} + p_{LH})}{p_{HH}} K_2 + \frac{\left((1-\lambda)\, p_{LL}\, p_{LL} + p_{HL}\right)}{p_{HH}}\; K_3\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

Case 3: Negative correlation with asymmetry towards the LH type

It holds when $\lambda_1 = 1$ and $\lambda_2 = \lambda_3 = 0$:

For the LH type:

- q_{LH}:

$$q_{LH} = \left[\frac{1}{A\alpha}\left(Q_1 + \left(\frac{p_{LL}}{p_{LH}}\right) Q_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{LH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{LH}:

$$t_{LH} = \left[\frac{1}{A(1-\alpha)}\left(Q_1 + \left(\frac{p_{LL}}{p_{LH}}\right) Q_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{LH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HL type:

- q_{HL}:

$$q_{HL} = \left[\frac{1}{A\alpha} R_1\right]^{\frac{1}{\alpha-\gamma}} t_{HL}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HL}:

$$t_{HL} = \left[\frac{1}{A(1-\alpha)} (R_1)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HL}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HH type:

- q_{HH}:

$$q_{HH} = \left[\frac{1}{A\alpha}\left(K_1 + \frac{(p_{LL} + p_{LH})}{p_{HH}} K_2 + \frac{p_{HL}}{p_{HH}}\; K_3\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HH}:

$$t_{HH} = \left[\frac{1}{A(1-\alpha)}\left(K_1 + \frac{(p_{LL} + p_{LH})}{p_{HH}} K_2 + \frac{p_{HL}}{p_{HH}} K_3\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

Case 4: Negative correlation with asymmetry towards the HL type

It holds when $\lambda_2 = 1$ and $\lambda_1 = \lambda_3 = 0$:

For the LH type:

- q_{LH}:

$$q_{LH} = \left[\frac{1}{A\alpha} Q_1\right]^{\frac{1}{\alpha-\gamma}} t_{LH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{LH}:

$$t_{LH} = \left[\frac{1}{A(1-\alpha)} Q_1\right]^{\frac{1}{1-\gamma-\alpha}} q_{LH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HL type:

- q_{HL}:

$$q_{HL} = \left[\frac{1}{A\alpha}\left(R_1 + \left(\frac{p_{LL}}{p_{HL}}\right) R_2\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HL}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HL}:

$$t_{HL} = \left[\frac{1}{A(1-\alpha)}\left(R_1 + \left(\frac{p_{LL}}{p_{HL}}\right) R_2\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HL}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

For the HH type:

- q_{HH}:

$$q_{HH} = \left[\frac{1}{A\alpha}\left(K_1 + \frac{p_{LH}}{p_{HH}} K_2 + \frac{(p_{LL} + p_{HL})}{p_{HH}} K_3\right)\right]^{\frac{1}{\alpha-\gamma}} t_{HH}{}^{\frac{\gamma+\alpha-1}{\alpha-\gamma}}$$

- t_{HH}:

$$t_{HH} = \left[\frac{1}{A(1-\alpha)}\left(K_1 + \frac{p_{LH}}{p_{HH}}K_2 + \frac{\left(p_{LL} + p_{HL}\right)}{p_{HH}}\,K_3\right)\right]^{\frac{1}{1-\gamma-\alpha}} q_{HH}{}^{\frac{\gamma-\alpha}{1-\gamma-\alpha}}$$

A5. Incentive compatibility constraints for simulation results

Case 1 (positive correlation):

Distribution 1A: where $p_{LL} = 0.5$, $p_{LH} = 0$, $p_{HL} = 0$, $p_{HH} = 0.5$, $cov = 0.25$

- The efficient is *simultaneously* indifferent between his contract, the inefficient's (HH), and the two middle types' (HL and LH):

  $U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.04$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.04$

  $U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.04$, $U_{LH} = 0.02$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.02$

  $U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.04$, $U_{HL} = 0.02$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.02$

  Expected profit is $E(\pi) = 2.90$

Distribution 1B: where $p_{LL} = 0.49$, $p_{LH} = 0.01$, $p_{HL} = 0.01$, $p_{HH} = 0.49$, $cov = 0.24$

When the principal fixes $t_{ij} = 2$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.08$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.08$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.08$, $U_{LH} = 0.05$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.03$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.08$, $U_{HL} = 0.05$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.03$

Expected profit is $E(\pi) = 2.84$

When the principal fixes $t_{ij} = 4$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.13$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.13$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.13$, $U_{LH} = 0.08$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.05$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.13$, $U_{HL} = 0.08$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.05$

Expected profit is $E(\pi) = 2.83$

When the principal fixes $t_{ij} = 6$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.19$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.19$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.19$, $U_{LH} = 0.11$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.08$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.19$, $U_{HL} = 0.11$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.08$

Expected profit is $E(\pi) = 2.82$

Distribution 1C: where $p_{LL} = 0.45$, $p_{LH} = 0.05$, $p_{HL} = 0.05$, $p_{HH} = 0.45$, $cov = 0.20$

When the principal fixes $t_{ij} = 2$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.40$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.40$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.40$, $U_{LH} = 0.23$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.17$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.40$, $U_{HL} = 0.23$ $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.17$

Expected profit is $E(\pi) = 2.62$

When the principal fixes $t_{ij} = 4$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.92$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.92$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.92$, $U_{LH} = 0.54$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.38$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 0.92$, $U_{HL} = 0.54$ $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.38$

Expected profit is $E(\pi) = 2.47$

When the principal fixes $t_{ij} = 6$:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 1.55$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 1.55$

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 1.55$, $U_{LH} = 0.91$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.64$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{LH}, t_{LH})$, because $U_{LL} = 1.55$, $U_{HL} = 0.91$ $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.64$

Expected profit is $E(\pi) = 2.23$

Case 2: Weak correlation

Distribution 2A where where $p_{LL} = 0.3$, $p_{LH} = 0.25$, $p_{HL} = 0.19$, $p_{HH} = 0.26$, $cov = 0.03$. For $\lambda = 0.57$, the following output levels hold and satisfy equation (7):

The efficient is indifferent between his contract and that of any of the two middle types:

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.51$, $U_{LH} = 0.01$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.50$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL})$, because $U_{LL} = 0.51$, $U_{HL} = 0.01$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.50$

However, the efficient's incentive compatibility constraint with respect to the inefficient does not bind:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.55$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.01$

Expected profit is $E(\pi) = 1.96$

Distribution 2B where where $p_{LL} = 0.28$, $p_{LH} = 0.22$, $p_{HL} = 0.22$, $p_{HH} = 0.28$, $cov = 0.03$. For $\lambda = 0.50$, the following output levels hold and satisfy equation (7).

The efficient is indifferent between his contract and that of any of the two middle types:

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.55$, $U_{LH} = 0.01$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.54$

$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL})$, because $U_{LL} = 0.55$, $U_{HL} = 0.01$, $\Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.54$

However, the incentive compatibility constraint with respect to the inefficient does not bind:

$U_{LL} = U_{HH} + \Delta_{LL}^{HH}(q_{HH}, t_{HH})$, because $U_{LL} = 0.55$, $U_{HH} = 0$, $\Delta_{LL}^{HH}(q_{HH}, t_{HH}) = 0.02$

Expected profit is $E(\pi) = 1.84$

Case 3: Negative correlation with asymmetry towards the *LH* type

Distribution 3A where $p_{LL} = 0.125$, $p_{LH} = 0.5$, $p_{HL} = 0.25$, $p_{HH} = 0.125$, $cov = -0.11$

The efficient is indifferent between his contract and that of the LH type:

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 0.98$, $U_{LH} = 0$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 0.98$

Expected profit is $E(\pi) = 1.34$

Distribution 3B where where $p_{LL} = 0.08$, $p_{LH} = 0.74$, $p_{HL} = 0.1$, $p_{HH} = 0.08$, $cov = -0.11$

The efficient is indifferent between his contract and that of the LH type:

$U_{LL} = U_{LH} + \Delta_{LL}^{LH}(q_{LH}, t_{LH})$, because $U_{LL} = 1.24$, $U_{LH} = 0$, $\Delta_{LL}^{LH}(q_{LH}, t_{LH}) = 1.24$

Expected profit is $E(\pi) = 1.2$

Case 4: Negative correlation with asymmetry towards the HL type

Distribution 4A, where $p_{LL} = 0.125$, $p_{LH} = 0.25$, $p_{HL} = 0.5$, $p_{HH} = 0.125$, $cov = -0.11$

The efficient is indifferent between his contract and that of the HL type:

$$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), \text{ because } U_{LL} = 0.98,\ U_{LH} = 0,\ \Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 0.98$$

Distribution 4B, where $p_{LL} = 0.04$, $p_{LH} = 0.05$, $p_{HL} = 0.87$, $p_{HH} = 0.04$, $cov = -0.04$

The efficient is indifferent between his contract and that of the LH type:

$$U_{LL} = U_{HL} + \Delta_{LL}^{HL}(q_{HL}, t_{HL}), \text{ because } U_{LL} = 1.38,\ U_{HL} = 0,\ \Delta_{LL}^{HL}(q_{HL}, t_{HL}) = 1.38$$

A6. MATLAB script files to compute simulation results

For Case 1 (Positive Correlation):

```
%***************************************************%
%       Parameters of the cost function             %
%***************************************************%
A = 5;
alpha = 0.5;
beta = 0.5;
gama = 0.6;
%***************************************************%
%       Distribution of types                       %
%****************************************************
pll = 0.45;
plh = 0.05;
phl = 0.05;
phh = 0.45;
%***************************************************%
%      Asymmetric information parameters            %
%***************************************************%
theta_qL = 2;
theta_qH = 4;
theta_tL= 4;
theta_tH= 8;
%***************************************************%
%        Deltas                                     %
%****************************************************
delta_HH_LH = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tH)^(1-beta));
delta_HH_HL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qH)^beta)*((theta_tL)^(1-beta));
```

```
delta_HH_LL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_LH_LL = ((theta_qL)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_HL_LL = ((theta_qH)^beta)*((theta_tL)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
%**************************************************%

K1 = gama*((theta_qH)^beta)*((theta_tH)^(1-beta));
K2 = gama*delta_HH_LH;
K3 = gama*delta_HH_HL;
K4 = gama*delta_HH_LL;


Q1 = gama*((theta_qL)^beta)*((theta_tH)^(1-beta));
Q2 = gama*delta_LH_LL;

R1 = gama*((theta_qH)^beta)*((theta_tL)^(1-beta));
R2 = gama*delta_HL_LL;

S1 = gama*((theta_qL)^beta)*((theta_tL)^(1-beta));

T1 = gama*(plh*delta_HH_LH + phl*delta_HH_HL + phh*pll*delta_HH_LL);

%**************************************************%
%               Lambdas wrt q                      %
%**************************************************%

syms qhh

thh = 6;


lambda1 = (plh * (plh * (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH) + ...
                 phl * (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)+ ...
                 phh*pll*(gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)))/...
                 ((phh - (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)*phl ...
                 + (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*phl - ...
                 (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH)*plh +...
                 (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*plh)* pll);

lambda2 = (plh * (phl * (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH) + ...
                 phl * (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)+ ...
                 phh*pll*(gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)))/...
                 ((phh - (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)*phl ...
                 + (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*phl - ...
                 (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH)*plh +...
                 (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*plh)* pll);

 lambda3 = (phh*pll - (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)*phl*(1-
phh) - ...
           (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH)*plh*(1-phh))/...
           ((phh - (gama*((qhh *thh)^(gama-1))*thh*delta_HH_HL)*phl ...
                 + (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*phl - ...
```

```
                 (gama*((qhh*thh)^(gama-1))*thh*delta_HH_LH)*plh +...
                 (gama*((qhh *thh)^(gama-1))*thh*delta_HH_LL)*phh*plh)* pll);

%*************************************************%
%        Numerical computation of qhh             %
%*************************************************%

x= (((qhh*thh)^(gama-1))*thh);

qhh = vpasolve(A*alpha*(qhh^(alpha - 1))*(thh^(1 - alpha)) == x*K1 ...
     + ((lambda1*pll + plh)/phh)*(x*K2)...
     + ((lambda2*pll + phl)/phh)*(x*K3)...
     + ((lambda3*pll)/phh)*(x*K4),qhh, [0,0.1]);

 qlh = qhh;
 qhl = qhh;

%*************************************************%
%       Computing qlh, given tlh and lambda       %
%*************************************************%

tlh = 6;


qlh = (((1/(A*alpha))*(Q1 + lambda1*(pll/plh)*Q2))^(1/(alpha - gama)))*...
    tlh^((gama + alpha - 1 )/(alpha - gama));

%*************************************************%
%    Computing qhl, iven tlh and lambda           %
%*************************************************%

thl = 6;

qhl = (((1/(A*alpha))*(R1 + lambda2*(pll/phl)*R2))^(1/(alpha - gama)))*...
    thl^((gama + alpha - 1 )/(alpha - gama));

% %*************************************************%
% %                Computing qll                    %
% %*************************************************%

tll = 6;

qll = (((1/(A*alpha))*S1)^(1/(alpha - gama)))*...
    tll^((gama + alpha - 1 )/(alpha - gama));
```

For Case 2 (Weak Correlation):

```
%*************************************************%
A = 5;
alpha = 0.5;
beta = 0.5;
gama = 0.6;
```

```
thh = 6;
tlh = 6;
thl = 6;
tll = 6;
%***************************************************%
pll = 0.3;
plh = 0.25;
phl = 0.19;
phh = 0.26;
%***************************************************%
theta_qL = 2;
theta_qH = 4;
theta_tL= 4;
theta_tH= 8;
%***************************************************%
delta_HH_LH = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tH)^(1-beta));
delta_HH_HL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qH)^beta)*((theta_tL)^(1-beta));
delta_HH_LL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_LH_LL = ((theta_qL)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_HL_LL = ((theta_qH)^beta)*((theta_tL)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));

%***************************************************%


K1 = gama*((theta_qH)^beta)*((theta_tH)^(1-beta));
K2 = gama*delta_HH_LH;
K3 = gama*delta_HH_HL;
K4 = gama*delta_HH_LL;



Q1 = gama*((theta_qL)^beta)*((theta_tH)^(1-beta));
Q2 = gama*delta_LH_LL;

R1 = gama*((theta_qH)^beta)*((theta_tL)^(1-beta));
R2 = gama*delta_HL_LL;

S1 = gama*((theta_qL)^beta)*((theta_tL)^(1-beta));

T1 = gama*(plh*delta_HH_LH + phl*delta_HH_HL + phh*pll*delta_HH_LL);
%***************************************************%
%  Finding the lambda that satisfies eq. A40       %
%***************************************************%
%
 init_guess=[0 1];
% %
 syms lambda
% %
 vpasolve( ((( ((((1/(A*alpha))*(K1 + ((lambda*pll + plh)/phh)* K2 + ...
       (((1 - lambda)*pll + phl)/phh)*K3))...
```

```
        ^(1/(alpha - gama)))*thh^((gama - 1 + alpha)/(alpha - gama))) *
thh)^(gama))*delta_HH_LH) + ((( ((((1/(A*alpha))*(Q1 + (lambda *
pll/plh)*Q2))...
        ^(1/(alpha - gama)))*tlh^((gama - 1 + alpha)/(alpha - gama)))
*tlh)^(gama))*delta_LH_LL)   == ((( ((((1/(A*alpha))*(K1 + ((lambda*pll +
plh)/phh)* K2 + ...
         (((1 - lambda)*pll + phl)/phh)*K3))...
         ^(1/(alpha - gama)))*thh^((gama - 1 + alpha)/(alpha - gama)))
*thh)^(gama))*delta_HH_HL) + ((( ((((1/(A*alpha))*(R1 + ((1-lambda) *
pll/phl)*R2))...
         ^(1/(alpha - gama)))*thl^((gama - 1 + alpha)/(alpha - gama)))
*thl)^(gama))*delta_HL_LL), lambda, init_guess)

%**************************************************%
%         Computing output levels                  %
%**************************************************%

lambda = 0.56818181818181817247476191987596;

%**************************************************%
%  Output levels for the inefficient (HH) type     %
%**************************************************%
%
  qhh = ((((1/(A*alpha))*(K1 + ((lambda*pll + plh)/phh)* K2 + ...
                (((1 - lambda)*pll + phl)/phh)*K3))...
                ^(1/(alpha - gama)))*thh^((gama - 1 + alpha)/(alpha -
gama)));

% %**************************************************%
% %  Output levels for middle (LH) type              %
% %**************************************************%
%
   qlh = ((((1/(A*alpha))*(Q1 + (lambda * pll/plh)*Q2))...
             ^(1/(alpha - gama)))*tlh^((gama - 1 + alpha)/(alpha - gama)));
% %**************************************************%
% %  Output levels for middle (HL) type              %
% %**************************************************%
%
     qhl = ((((1/(A*alpha))*(R1 + ((1-lambda) * pll/phl)*R2))...
            ^(1/(alpha - gama)))*thl^((gama - 1 + alpha)/(alpha - gama)));
% %**************************************************%
% %  Output levels for efficient (LL) type              %
% %**************************************************%

qll = (((1/(A*alpha))*S1)^(1/(alpha - gama)))*tll^((gama - 1 + alpha)/(alpha
- gama));
```

For Case 3 (Asymmetry towards the *LH* type):

```
%**************************************************%
A = 5;
alpha = 0.5;
beta = 0.5;
gama = 0.6;
%**************************************************%
```

```
pll = 0.08;
plh = 0.74;
phl = 0.1;
phh = 0.08;
%**************************************************%
theta_qL = 2;
theta_qH = 4;
theta_tL= 4;
theta_tH= 8;
%**************************************************%
delta_HH_LH = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tH)^(1-beta));
delta_HH_HL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qH)^beta)*((theta_tL)^(1-beta));
delta_HH_LL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_LH_LL = ((theta_qL)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_HL_LL = ((theta_qH)^beta)*((theta_tL)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
%**************************************************%

K1 = gama*((theta_qH)^beta)*((theta_tH)^(1-beta));
K2 = gama*delta_HH_LH;
K3 = gama*delta_HH_HL;
K4 = gama*delta_HH_LL;


Q1 = gama*((theta_qL)^beta)*((theta_tH)^(1-beta));
Q2 = gama*delta_LH_LL;

R1 = gama*((theta_qH)^beta)*((theta_tL)^(1-beta));
R2 = gama*delta_HL_LL;

S1 = gama*((theta_qL)^beta)*((theta_tL)^(1-beta));

T1 = gama*(plh*delta_HH_LH + phl*delta_HH_HL + phh*pll*delta_HH_LL);


%**************************************************%
%                Computing qhh                     %
%**************************************************%

thh = 6;

qhh = (((1/(A*alpha))*(K1 + ((pll + plh)/phh)*K2 + ...
    (phl/phh)*K3))^(1/(alpha - gama)))* thh^((gama - 1 + alpha)/(alpha -
gama));

%**************************************************%
%                Computing qlh                     %
%**************************************************%
tlh = 6;
```

```
qlh = (((1/(A*alpha))*(Q1 + (pll/plh)*Q2))...
    ^(1/(alpha - gama)))*tlh^((gama - 1 + alpha)/(alpha - gama));

%**************************************************%
%                Computing qhl                     %
%**************************************************%

thl = 6;

qhl = (((1/(A*alpha))*(R1))^(1/(alpha - gama)))...
    *thl^((gama - 1 + alpha)/(alpha - gama));

%**************************************************%
%                Computing qll                     %
%**************************************************%

tll = 6;
%
 qll = (((1/(A*alpha))*S1)^(1/(alpha - gama)))*tll^((gama - 1 + alpha)/(alpha
- gama));
```

For Case 4 (Asymmetry towards the *HL* type):

```
%**************************************************%
A = 5;
alpha = 0.5;
beta = 0.5;
gama = 0.6;
%**************************************************%
pll = 0.125;
plh = 0.25;
phl = 0.5;
phh = 0.125;
%**************************************************%
theta_qL = 2;
theta_qH = 4;
theta_tL= 4;
theta_tH= 8;
%**************************************************%
delta_HH_LH = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tH)^(1-beta));
delta_HH_HL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qH)^beta)*((theta_tL)^(1-beta));
delta_HH_LL = ((theta_qH)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_LH_LL = ((theta_qL)^beta)*((theta_tH)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
delta_HL_LL = ((theta_qH)^beta)*((theta_tL)^(1-beta))...
    -((theta_qL)^beta)*((theta_tL)^(1-beta));
%**************************************************%

K1 = gama*((theta_qH)^beta)*((theta_tH)^(1-beta));
```

```
K2 = gama*delta_HH_LH;
K3 = gama*delta_HH_HL;
K4 = gama*delta_HH_LL;


Q1 = gama*((theta_qL)^beta)*((theta_tH)^(1-beta));
Q2 = gama*delta_LH_LL;

R1 = gama*((theta_qH)^beta)*((theta_tL)^(1-beta));
R2 = gama*delta_HL_LL;

S1 = gama*((theta_qL)^beta)*((theta_tL)^(1-beta));

T1 = gama*(plh*delta_HH_LH + phl*delta_HH_HL + phh*pll*delta_HH_LL);



%**************************************************%
%                Computing qhh                     %
%**************************************************%

thh = 6;

qhh = (((1/(A*alpha))*(K1 + (plh/phh)*K2 + ...
    ((pll+phl)/phh)*K3))^(1/(alpha - gama)))* thh^((gama - 1 + alpha)/(alpha
- gama));

%**************************************************%
%                Computing qlh                     %
%**************************************************%
tlh = 6;


qlh = (((1/(A*alpha))*(Q1))...
    ^(1/(alpha - gama)))*tlh^((gama - 1 + alpha)/(alpha - gama));

%**************************************************%
%                Computing qhl                     %
%**************************************************%

thl = 6;

qhl = (((1/(A*alpha))*(R1+(pll/phl)*R2))^(1/(alpha - gama)))...
    *thl^((gama - 1 + alpha)/(alpha - gama));

%**************************************************%
%                Computing qll                     %
%**************************************************%

tll = 6;
%
 qll = (((1/(A*alpha))*S1)^(1/(alpha - gama)))*tll^((gama - 1 + alpha)/(alpha
- gama));
```

Chapter 5

Product-Mix Exchanges, Efficiency and Power System Flexibility

Product-Mix Exchanges, Efficiency and Power System Flexibility

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Abstract

This paper develops extensions to the Product-Mix Auction (PMA), which are generalized under the name of Product-Mix Exchanges (PMEs): double, multi-unit combinatorial auctions in which buyers and sellers report substitutable preferences over bundles of goods. A key feature of PMEs is that participants can buy and sell without having a fixed role as buyers or sellers, effectively swapping over the two sides of the market. Furthermore, PMEs are applicable when goods are divisible or indivisible and strong substitute or ordinary substitute preferences are imposed on market participants. The main contributions of the paper are, first, applying existing tropical geometric techniques to the analysis of substitutable preferences. Second, proposing a linear programming approach to identify if a set of valuations have equilibrium with indivisibility. Third, analyzing the conditions under which Vickrey Clarke Groves (VCG) payments can support the efficient allocation of a PME. Finally, I apply the PME framework to the design of a marketplace for power system flexibility, namely the Delta Energy Market in which quantity flexibility and time shifting products are traded.

Keywords: Multi-Unit Auctions, Combinatorial Auctions, Vickrey-Clarke Groves Mechanism

JEL Classification Numbers: D44, D47, D82, C61, C65

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1 Introduction

At the height of the financial crisis, central banks throughout the world were in the difficult position of supplying liquidity to troubled banks while accounting for the different qualities of collateral they possessed. In particular, the Bank of England (BoE) asked for the advice of Paul Klemperer, who came up with an auction format - the Product-Mix Auction (PMA) - in which buyers and sellers perceive the traded goods as imperfect substitutes. In his proposal, Klemperer (2010) suggests that the more traditional multiple round format is "impractical" because financial markets are fast-paced, bidders may change their minds in the auction rounds and financial market themselves may be influenced by the results. Klemperer's suggested format is closely related to a number of existing mechanisms in the Economics literature and has a number of "easy extensions" which he left as avenues for future research. The central goal of this paper is to develop these extensions.

To this end, I introduce Product-Mix Exchanges (PMEs): double, multi-unit combinatorial auctions in which buyers and sellers report substitutable preferences over bundles of goods, and participants buy and sell without having a fixed role, effectively swapping over the two sides of the market. Moreover, the paper analyzes some of the incentive properties of PMEs, with an emphasis on *efficiency*, and studies a relevant application, namely the *Delta Energy* market design in which power system flexibility originating from Flex-Offers is traded.[1]

PMEs extend and generalize PMAs, which are direct mechanisms that allow participants expressing preferences over sets of goods, perceived by bidders as imperfect substitutes. In the PMA original formulation, a seller asks buyers to submit bids containing mutually exclusive offers for different varieties of a good on sale, together with a single, maximum quantity to acquire. Bidders are allowed to submit one or more bids and, within each bid, to decide if they want to make offers for one, some or all of the varieties. Designed in

[1]Flex-Offers are, essentially, load profiles that describe the supply or demand of any flexibility-enabling asset in the grid, including the time and amount flexibility. Further details can be found in the paper by Boscán (2016).

this way, one-to-one substitutable preferences are imposed such that bidders approximate *multiple-variety demand correspondences* which, upon aggregation, cross with a seller-defined *multiple-variety supply correspondence*. For each variety, the mechanism simultaneously determines highest-loser, uniform prices that clear the market, such that bids below are rejected and bids above are accepted. In general, the mechanism allocates at most one of the varieties to each bidder and selects the one that gives the bidder the highest payoff, given the market clearing prices.

The best -if not the only- documented application of PMAs is in the BoE's Indexed Long-Term Repo operations, introduced in June 2010. In this implementation, the BoE is the seller, while banks, building societies and broker dealers are the buyers. The central bank privately determines its funding supply preferences across loan types, which are backed by different qualities of collateral. Naturally, because of risk considerations, loans that have lower quality collateral require the borrower to pay higher interest rates than the ones that have a superior kind.[2]

However, the applicability of PMAs is considerably wide and exceeds the realm of financial markets. Harbord et al. (2011), for example, have considered it as one of the alternatives to auction natural gas contracts in Colombia. Similarly, although not explicitly referred to as a PMA, the "multiple bid auction" discussed by SEM (2014) as one of the options to procure system services in the Irish Electricity Market corresponds to one. Furthermore, according to Klemperer (2010), the initial PMA design can be easily extended in the following ways:

1. There can be multiple buyers and multiple sellers

2. Swap bids, which allow market participants to exchange one variety for another at the market clearing price difference, can be implemented

[2] After a number of extensions to the framework applied by the BoE, implemented since February 2014, the number of collateral qualities was extended from two to three and two automatic supply responses were introduced. First, if there is greater demand for loans backed by a certain collateral quality, the fund allocation for this variety will increase. Second, total supply is not exogenously fixed by the seller but is dependent on the overall demand. For further details on the BoE's implementation, see Frost et al. (2015)

3. In a given bid, bidders can be allowed to ask for *different* amounts of the available varieties, as opposed to the basic implementation where bidders express preferences over the *same* amount of any of the available varieties

4. Variable total quantity offered by the seller can be made dependent on prices, as in the BoE's current implementation

As mentioned before, Klemperer has not hitherto developed any of the extensions. Instead, Baldwin and Klemperer (2012, 2016) have made a more fundamental contribution. They have developed geometric techniques and results that underpin the original PMA and any possible extension to it. More generally, their work facilitates the analysis of agents' preferences in matching and market design and establishes a pioneering link between Tropical Geometry and Economics. Applying the results emerging from this nascent connection and establishing a link between PMAs and combinatorial auctions, I take on the task of implementing the first three extensions to PMAs suggested by Klemperer (2010) in the previous enumeration.

This paper is structured as follows. In section 2, I describe PMAs and its connection to other existing mechanisms described in the economics literature. To fix ideas and to illustrate the difference between PMAs and PMEs, I present two simple examples: one for each of them. The section ends with a detailed description of the basic notation, assumptions and bid expression used throughout the paper.

Section 3 discusses the notions of substitutable preferences relevant for PMEs, its connection with the well-known Gross Substitutes (GS) condition of Kelso and Crawford (1982), and introduces the essential concepts in tropical geometry, which are relevant for the understanding of the following sections. There are no new results in this section because the focus is on applying existing concepts. Thus, the interested reader is referred to the original sources for proofs and further details. The section contains several examples that show how substitutable preferences can be represented with the aid of tropical geometric techniques.

The remaining sections contain the central contributions of the paper. In section 4, I deal with the winner determination problem of PMEs and the existence of equilibrium with both indivisible and divisible goods. Notably, the linear programs presented there are of *practical* relevance, given that these allow estimating the equilibrium allocations and associated supporting prices in all variations of PMEs. Furthermore, proposition 4.8, which builds on the linear programming proof to Baldwin and Klemperer (2012, 2016)'s unimodularity theorem done by Tran and Yu (2015), is a straightforward way to verify if a set of valuations can be expected to have equilibrium with indivisibility.

Section 5 deals with incentive issues in PMEs, a topic that has not been dealt with before but for which a number of existing results characterizing the incentive properties of combinatorial auctions can be readily applied. The central message of this section is that because the GS condition holds in all variations of the PME, the Vickrey outcome is always in the core. Furthermore, if non-linear pricing is allowed for, it is always possible to run PMEs in which Vickrey payments are implemented. Once again, this is a result of practical relevance: when markets are thin and thus prone to manipulation, resorting to a mechanism in which bidding truthfully is a weakly dominant strategy safeguards efficiency.[3]

Finally, section 6 illustrates how PMEs can solve a relevant application. Delta Energy markets, where the energy associated to Flex-Offers is traded, can be cleared with PMEs. In this section, I illustrate how this market design proposal for flexibility can accommodate relevant flexibility-enabling products such as quantity flexibility and time shifting. Section 7 presents concluding remarks.

[3]Subsection A.3 in the Appendix contains a number of PME market-clearing examples in which VCG payments are implemented as supporting prices for the PME.

2 From Product-Mix Auctions to Product-Mix Exchanges

The extensions to PMAs proposed by Klemperer (2010) have already been addressed by Milgrom (2009) in parallel work. He introduces assignment messages and exchanges, which extend the classic assignment model by Shapley and Shubik (1971). Assignment messages are linear programs that express agents' substitutable preferences to reduce the length of report problem in direct mechanisms, while the exchanges are Walrasian mechanisms where participants are restricted to reporting assignment messages. In this framework, PMAs are special cases of Milgrom (2009)'s model who defines these as an "... assignment auction design (which) ... is simply an exchange with one seller and many buyers or one buyer and many sellers".

Yet, there is a fundamental difference between Milgrom's approach and PMAs: the latter are inextricably linked to their geometric representation, whereas the former aren't. The focus on geometry works in favor of the increased applicability of PMAs, as it facilitates the bid expression of agents participating in the mechanism and provides an alternative way to determine the conditions under which equilibrium with indivisibilities is guaranteed to exist.

Clearly, there is a connection between PMEs and PMAs because of their common reliance on tropical geometry. Also, PMEs and assignment messages and exchanges have similarities because they both extend the original PMA.[4] However, there is an important contrast with both approaches. In contrast to Klemperer's original PMA formulation and to Milgrom's assignment exchanges, who focus on bids over goods, but in a similar fashion to Tran and Yu (2015) and Lee et al. (2015), I emphasize on the combinatorial nature of PMAs to implement PMEs. Therefore, the following definition:

Definition 2.1 *A Product-Mix Exchange is a multi-unit combinatorial auction with multiple buyers and multiple sellers who are restricted to reporting substitutable preferences. Market*

[4] In fact, the term "exchange" in the Product Mix-Exchange is used in Milgrom (2009)'s sense to highlight the double auction nature of PMEs and the equivalence of his model with mine.

participants in a Product-Mix Exchange can be on the supply side or the demand side of the market, or both.

In combinatorial auctions, bidders place bids on combinations of items or "packages" rather than on individual items (Cramton et al., 2006). Consequently, the winner determination problem must account for the fact that participants are allocated indivisible bundles of goods, potentially imposing computational complexity.

Focusing on packages reflects the large number of possible applications of PMEs with indivisible goods as well as the emphasis placed by existing studies on the PMA on the existence of equilibrium with indivisible goods (e.g. Tran and Yu (2015)). On the other hand, the best known application of PMAs is for a divisible good, i.e., money. But even if goods are divisible, focusing on bundles of goods facilitates the desired extensions of PMAs, as a simple linear programming relaxation can straightforwardly determine the winning bids.

As in PMAs, the key characteristic of PMEs is that participants interpret good bundles in a bid as imperfect substitutes. It is well established, though, that the notion of substitutability is not unique in economics (Milgrom and Strulovici, 2009). To allow for a sufficiently rich bid expressiveness by market participants while ensuring desirable properties, such as the existence of equilibrium and supporting prices, I restrict bidders to having "strong substitutes" or "ordinary substitutes" demand types (Baldwin and Klemperer, 2012, 2016).

The divisibility of the traded goods together with the substitutability concept considered gives rise to four variants of the PME:

	Strong Substitutes (SS)	*Ordinary substitutes* (OS)
Indivisible goods	Variant 1	Variant 3
Divisible goods	Variant 2	Variant 4

Table 1: Variants of a PME

The strong substitutes demand type refers to the situation in which goods of the *same type* are regarded as distinct Milgrom and Strulovici (2009). In this case, goods can potentially have their own prices, and substitutability *may* fail. This is precisely the assumption imposed on the original BoE PMA implementation, where participants were limited to expressing one-to-one substitution rates: banks bid to obtain loans backed by two different kinds of securities, the "strong" and the "weak". Some banks (buyers) would be willing to report interest rates (prices) at which they would substitute one kind of loan (good) for the other, but always for the same amount of money. Each participating bank is allocated to one or the other loan, but not both. Other participating banks, in contrast, would want to express interest for only one kind of loan, actually indicating that the goods are not substitutes for them.

The ordinary (also known as weak in Milgrom and Strulovici (2009)'s terminology) substitutes demand type allows participants to express trade-offs among goods that are considered distinct and satisfies the price-theoretic notion of substitutes in a multi-unit context. That is: whenever the price of a certain commodity in a class of goods increases, demand for an alternative commodity will weakly increase. As a conventional textbook example, consider the case of a consumer of fruits in a world of apples and oranges only. Whenever the price of an apple increases, the consumer will buy fewer apples and possibly more oranges, adjusting his consumption bundle. Section 3 discusses the implications of each substitutability assumption in a geometric context, together with its relevance for the PME design.

2.1 Simple examples

To illustrate the topic, in this subsection I present a concrete example of a PMA, as implemented by the BoE. A second example - of a PME - is also presented to emphasize the fact that PMAs are special cases of PMEs

Example 2.2 *(Product-Mix Auction) Consider a PMA in which there are two indivisible goods on sale. For simplicity, we refer to each good as "commodity 1" and "commodity 2", respectively. In this example, there are 20 bids (shown in figure 1) and each one is represented in price space with coordinates denoting the price at which the buyer wishes to* *acquire commodity 1 (abscissa) or commodity 2 (ordinate) and a mutually exclusive quantity demanded. So, for example, the "bubble" marked with 29 shows that the buyer wishes to acquire either 29 units of commodity 1 or commodity 2. If the buyer acquires commodity 1, he pays at most 37 units of money but if he acquires commodity 2, he is willing to pay at most 25. Similarly, a buyer willing to buy only commodity 1 may place a bid as the one with a 28 on the abscissa which indicates that he would pay at most 45 for commodity 1 but would pay nothing for commodity 2.*

Note that total demand adds up to 600 units but the seller fixes supply at 200 for commodity 1 and 250 for commodity 2, therefore rejecting 150 units of any of the two goods on sale. This choice determines the seller-defined two-commodity supply correspondence, drawn in figure 2 as three line segments that determine which bids are allocated to what commodity. The seller's problem is to optimally determine which bids to reject and the prices to charge for each commodity. Figure 2 shows the optimal solution to the seller's problem, which consists on rationing the bids marked with 35, 22 and 33 while rejecting all bids below price coordinates of (28,30).

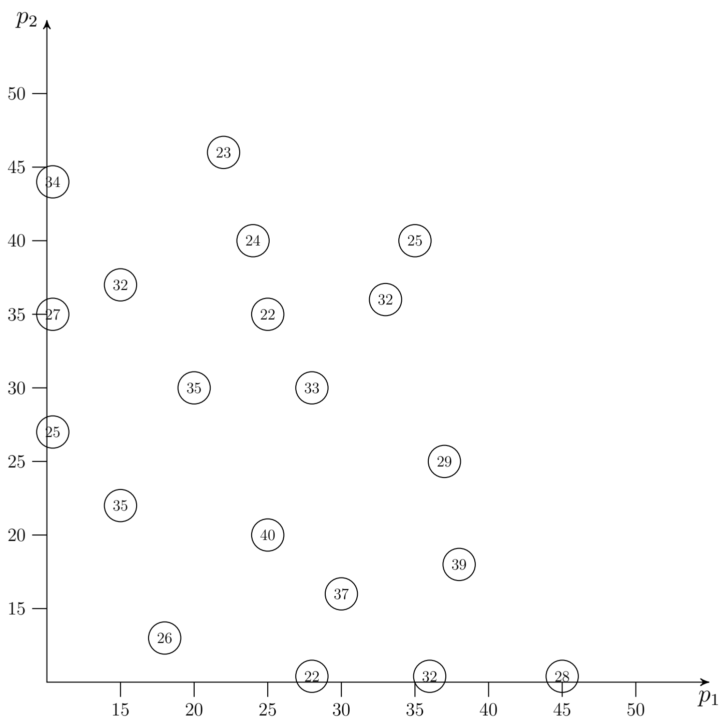

Figure 1: Example 1 (PMA): bids in price space

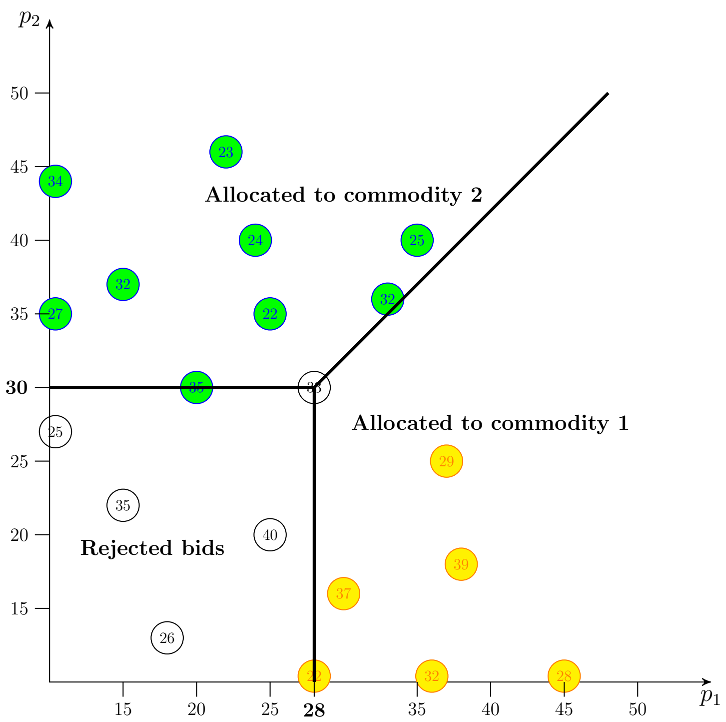

Figure 2: Example 1 (PMA): market clearing

Example 2.3 *(Product-Mix Exchange) Now consider a PME with two indivisible goods on sale, as before. In this specific example, there are 20 selling bids and 20 buying bids, but buyers and sellers can be on either side of the market: the same agent can place buying bids or selling bids with no restriction. Figure 3 shows demand-side bids in solid gray and supply-side bids with red circles, in price space. As before, every demand "bubble" represents a mutually exclusive quantity to acquire (at most) at the prices denoted by the corresponding coordinates. Supply side bubbles represent a maximum quantity to sell (at least) at the prices indicated by the coordinates.*

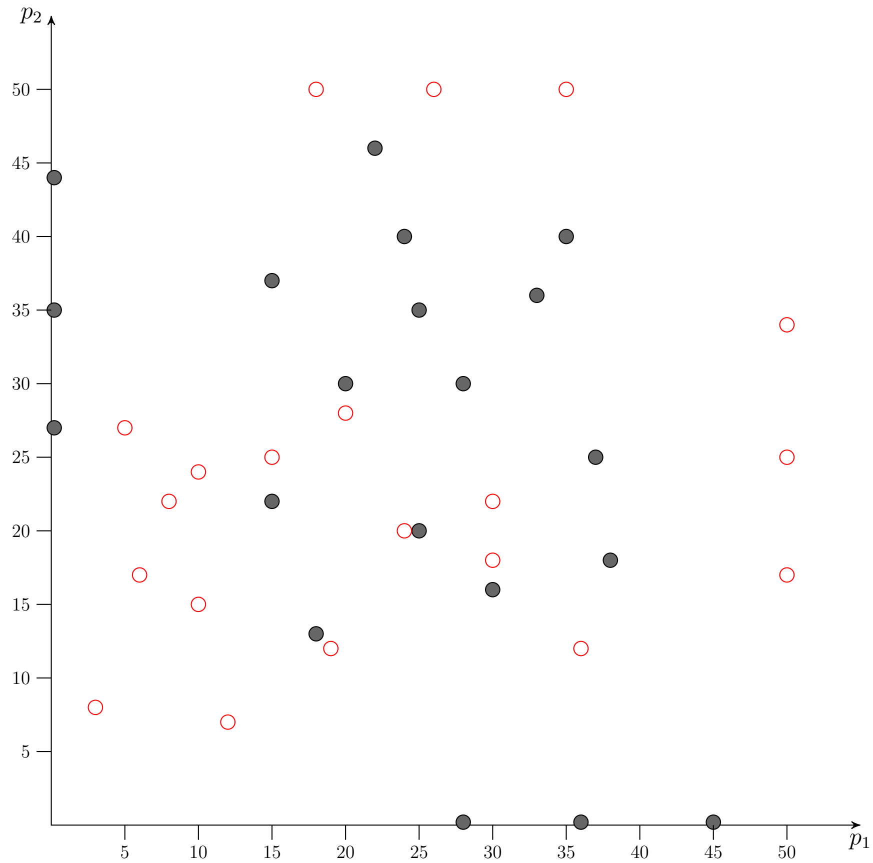

Figure 3: PME: bids in price space

In this example, total demand adds up to 600 units and supply adds up to 450 units. However, roughly 220 units of commodity 1 and 183 units of commodity for a total of 404 units can be matched at prices (28,33). In this case, there is no demand for the two highest *supply-side bids. Figure 4 shows accepted and rejected demand-side bids and the supply correspondence, which unlike the PMA, in this case is defined by the supply- side of the market.*

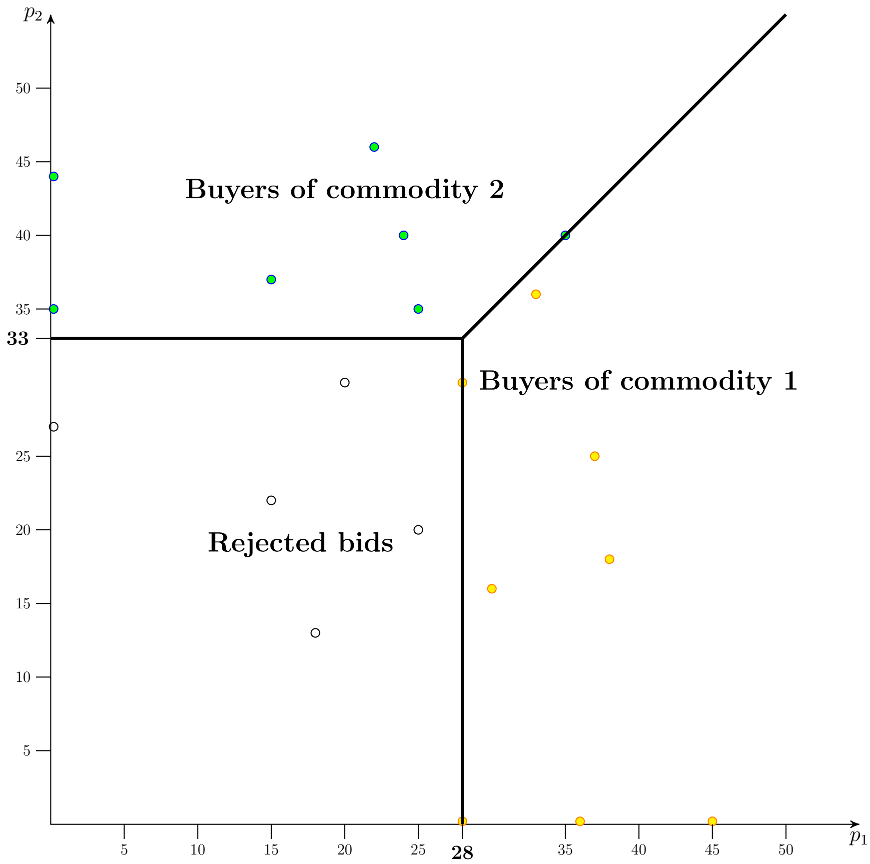

Figure 4: PME: market clearing (Demand-side bids)

Figure 5, on the other hand, shows the supply side of the market indicating the rejected and the accepted bids. In this case, there is a demand correspondence defined by the aggregate demand of all buyers in the economy.

An alternative view of market clearing is shown in figures 6 and 7, where all the profitable trades for each commodity in the exchange are shown. In figure 6 note that buyers of commodity 1 pay at least 28, while sellers are paid 28 or less. Similarly, in figure 7, buyers of commodity 2 pay at least 33 and sellers are paid 33 or less.

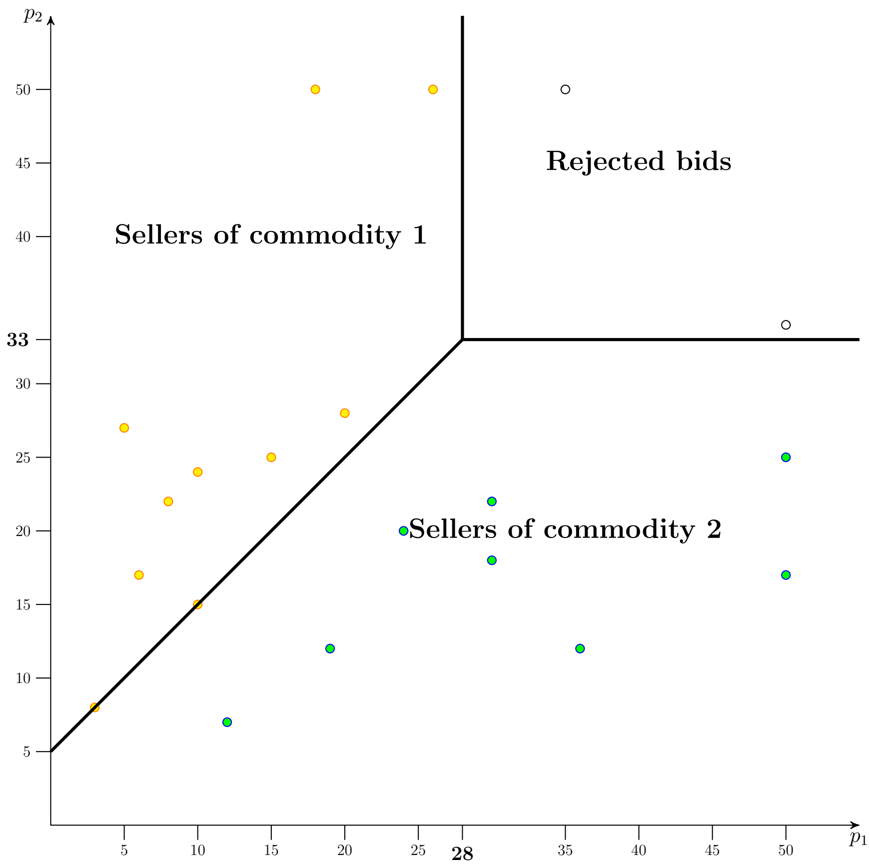

Figure 5: PME: market clearing (Selling-side bids)

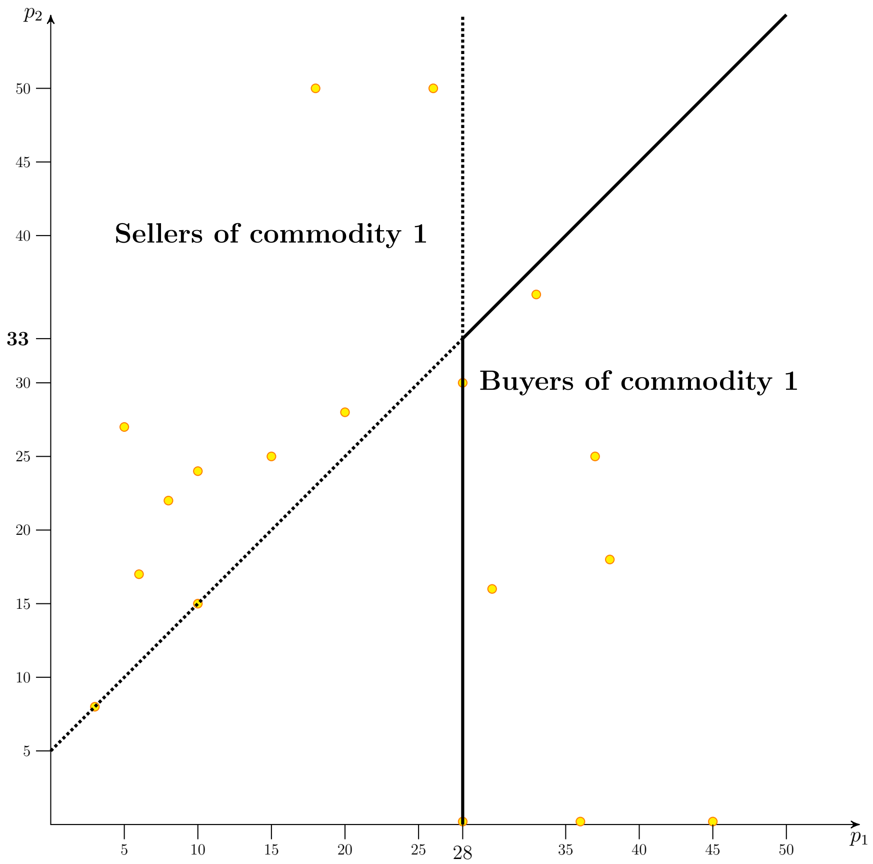

Figure 6: PME:profitable trades of commodity 1

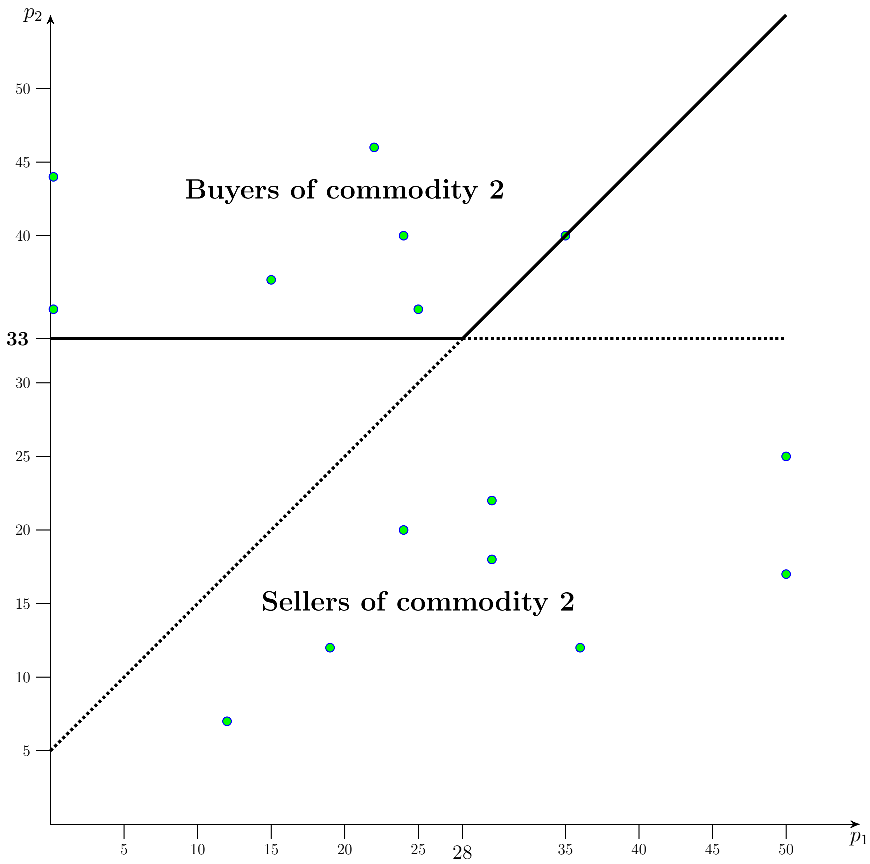

Figure 7: PME: profitable trades of commodity 2

2.2 Basic notation, assumptions and bid expression

In a PME, there is a set of commodities $\mathcal{N} = \{1, \dots, N\}$, indexed by n, and a set of market participants $\mathcal{J} = \mathcal{B} \cup \mathcal{S}$. The set $\mathcal{B} = \{1, \dots, B\}$ contains buyers, indexed by b, and the set $\mathcal{S} = \{1, \dots, S\}$ is composed of sellers, indexed by s. Any participant, buyer or seller, is indexed by j.

A1: All participants have concave valuation functions $u^j : A^j \to \mathbb{R}$ over a finite domain $A^j \subset \mathbb{R}^N_+$.

Note that while I refer to u^j as a participant's valuation throughout the paper, when referring to sellers, it is more accurate to think of u^s to think in terms of cost. Moreover, a buyer's valuation u^b is usually positive, whereas a seller's is usually negative. This is because of the distinction between buyers and sellers I have introduced in the model.

The organizer of the exchange - a clearinghouse - calls participants to report their bids. Note that, without loss of generality, the described setting encompasses situations in which there is one seller and many buyers (a PMA), one buyer and many sellers (a procurement PMA) or many buyers and many sellers (a PME). In all cases, the clearinghouse has a valuation of zero for all commodities and its unique goal is to ensure an efficient allocation.

Bids are composed of pairs $(\mathbf{q}, u^j(\mathbf{q}))$ where $\mathbf{q} = (q_1, \dots, q_N)$ is a bundle in the participant's domain and u^j is her valuation for the bundle $\mathbf{q}$. That is, bids are sets $\{(\mathbf{q}, u^j(\mathbf{q})) \mid \forall \mathbf{q} \in A^j\}$. In the sequel, I refer to q_n as the quantity of commodity n in the bundle $\mathbf{q}$. But in all cases:

A2: All participants have the zero bundle in their domain: $\mathbf{q} = \mathbf{0} \in A^j, \ \forall j \in \mathcal{J}$

Depending on the kind of substitute preferences allowed for in the PME, the clearinghouse restricts the participants' domains, but A^j can be generally written as a cartesian product:[5]

[5]The representation of market participants' domains as cartesian products is similar to Milgrom and Strulovici (2009).

$$A^j = \prod_{n \in \mathcal{N}} \mathbf{x}_n$$

where $\mathbf{x}_n$ is a set containing all possible quantities of each commodity.

When the clearinghouse imposes *strong substitute* preferences, each participant j is restricted to having a *binary domain*, where each bundle in the domain A^j either has zero or K units of each good n. Therefore:

$$\mathbf{x}_n = \{0, K\}$$

Example 2.4 *(Binary domain)*

The set of commodities is $\mathcal{N} = \{A, B\}$ *and thus* $N = 2$*. Bidder* j *has defined* $K = 3$ *and thus* $\mathbf{x}_A = \{0, 3\}$*,* $\mathbf{x}_B = \{0, 3\}$*. In consequence:*

$$\begin{aligned} A^j = \prod_{n \in \mathcal{N}} \mathbf{x}_n &= \{(q_A, q_B) \mid q_A \in \mathbf{x}_A, q_B \in \mathbf{x}_B\} \\ &= \{(0,0), (3,0), (0,3), (3,3)\} \end{aligned}$$

If the exchange establishes *ordinary substitute* preferences, each participant j has a *multi-unit* domain, where each bundle in the domain A^j contains up to K_n units of each good n. Thus the sets containing all possible quantities of the n goods are:

$$\mathbf{x}_n = \{0, 1, \ldots, K_n\}$$

Example 2.5 *(Multi-unit domain)*

As in the previous example, there is a set of commodities $\mathcal{N} = \{A, B\}$ *in the exchange and participant* j *defines* $K_A = 2, K_B = 1$*. So,* $\mathbf{x}_A = \{0, 1, 2\}$ *and* $\mathbf{x}_B = \{0, 1\}$

$$\begin{aligned} A^j = \prod_{n \in \mathcal{N}} \mathbf{x}_n &= \{(q_A, q_B) \mid q_A \in \mathbf{x}_A, q_B \in \mathbf{x}_B\} \\ &= \{(0,0), (0,1), (1,0), (1,1), (2,0), (2,1)\} \end{aligned}$$

The role of the clearinghouse is to allocate bundles $\mathbf{q} \in A^j$ and to define a price $P : \mathcal{J} \to \mathbb{R}^N$, that is, the prices that each agent in the exchange pays for (or gets paid for) a bundle of goods.[6] As Bikhchandani and Ostroy (2002, 2006), in this paper I consider two kinds of prices, namely:[7]

- A *non-anonymous*, (possibly) *non-linear* and, consequently, *discriminatory* price $P^j = \mathbf{p}^j$, where the amount paid for an allocated bundle $\mathbf{q}^j$ depends on the identity of agent j and may not satisfy linearity. That is, for any price vector $\mathbf{p}$ and bundles $\mathbf{q}, \mathbf{q}'$ it may be that $\mathbf{p} \cdot (\mathbf{q} + \mathbf{q}') \neq \mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{q}'$.
- An *anonymous* and *linear* price where $P^j = \mathbf{p}, \forall j \in \mathcal{J}$

A3: All participants have quasi-linear utility $\upsilon^j(\mathbf{q}, P^j)$, which depends on the bundle of goods $\mathbf{q} \in A^j$ allocated to the participant and the price P^j.

Thus, for buyers:

$$\upsilon^b(\mathbf{q}, P^b) = \left\{u^b(\mathbf{q}) - P^b \cdot \mathbf{q}\right\}, \quad \forall \mathbf{q} \in A^b$$

And for sellers:

$$\upsilon^s(\mathbf{q}, P^s) = \left\{P^s \cdot \mathbf{q} + u^s(\mathbf{q})\right\}, \quad \forall \mathbf{q} \in A^s$$

The demand correspondence for the b-th buyer, $D_{u^b}(P^b)$, and the supply correspondence for the s-th seller, $S_{u^s}(P^s)$ are respectively:

$$D_{u^b}(P^b) = \underset{\mathbf{q} \in A^b}{\arg\max} \left\{u^b(\mathbf{q}) - P^b \cdot \mathbf{q}\right\} \tag{1}$$

$$S_{u^s}(P^s) = \underset{\mathbf{q} \in A^s}{\arg\max} \left\{P^s \cdot \mathbf{q} + u^s(\mathbf{q})\right\} \tag{2}$$

The value function (indirect utility or dual profit function) for buyers is:

$$\pi^b \max_{\mathbf{q} \in A^b} \left\{u^b(\mathbf{q}) - P^b \cdot \mathbf{q}\right\} \tag{3}$$

while that of sellers is:

$$\pi^s = \max_{\mathbf{q} \in A^s} \left\{P^s \cdot \mathbf{q} + u^s(\mathbf{q})\right\} \tag{4}$$

[6] Alternatively, I write $\mathbf{q}^j$ to refer to a bundle that belongs to the j's participant domain

[7] This is an important distinction because a PME with VCG payments, a topic discussed in section 5 cannot be implemented unless prices are non-anonymous.

The assumptions introduced so far are equivalent to the extensions to the PMA discussed by Klemperer (2010) and the model by Milgrom (2009), where agents can buy and sell without having a fixed role. The setting I describe is, however, more general than what is described by Tran and Yu (2015), because any PME with one seller and many buyers is a PMA, and any PME with one buyer and many sellers is a procurement PMA. It is convenient, therefore, to emphasize that:

A4: Participants of a PME can bid as buyers or as sellers, or both

Implicit in this formulation is the assumption that the cardinality of $\mathcal{J}$ is equal to the number of bids, i.e.,"one bidder, one bid". This, however, does not restrict the number of bids that a bidder can place. From an auctioneer's perspective, bids are treated independently, as if each bid were placed by a different bidder, even if they aren't. This is in contrast to assignment exchanges (Milgrom, 2008), where the number of bids placed by a bidder is known by the auctioneer.

Let A be the set of all possible aggregate bundles in the economy, defined as the Minkowski difference of the aggregated individual valuation domains in the demand side of the economy $\sum_{b\in\mathcal{B}}$ *minus* those in the supply side $\sum_{s\in\mathcal{S}} A^s$:

$$A = \sum_{b\in\mathcal{B}} A^b - \sum_{s\in\mathcal{S}} A^s$$

.

Note that A, which I also refer to as the *aggregate valuation domain*, contains all the possible aggregate outcomes of the exchange, including the ones in which total demand is not equal to supply. Naturally, any element in A can be disaggregated into individual bundles.

The aggregate valuation function $U : A \to \mathbb{R}$ is the total surplus derived from trading any bundle in A. Given buyers' and sellers' reported valuations over bundles in their respective domains, U is constructed as the maximum valuation taken over all possible decompositions of an aggregate outcome in A. Thus, for an arbitrary bundle $\mathbf{y} \in A$, U is:

$$U(\mathbf{y}) = \max \left\{ \sum_{b \in \mathcal{B}} u^b(\mathbf{q}^b) - \sum_{s \in \mathcal{S}} u^s(\mathbf{q}^s) \mid \mathbf{q}^b \in A^b, \mathbf{q}^s \in A^s, \mathbf{y} = \sum_{b \in \mathcal{B}} \mathbf{q}^b - \sum_{s \in \mathcal{S}} \mathbf{q}^s \right\}$$

Given U, the clearinghouse's problem is to find an efficient outcome $\mathbf{q}_* \in A$ such that:

$$\mathbf{q}_* = \arg\max_{\mathbf{q} \in A} \left\{ U(\mathbf{q}) \mid \mathbf{q} = \sum_{b \in \mathcal{B}} \mathbf{q}^b - \sum_{s \in \mathcal{S}} \mathbf{q}^s = \mathbf{0} \right\} \tag{5}$$

In other words: of all the possible decompositions that add up to $\mathbf{q} = \mathbf{0} \in A$, $\mathbf{q}_*$ is the *efficient aggregate bundle* that solves the clearinghouse's problem of maximizing the gains from trade U. Because $\mathbf{q}_* = \mathbf{0}$, it can be decomposed as:

$$\mathbf{q}_* = \sum_{b \in \mathcal{B}} \mathbf{q}_*^b - \sum_{s \in \mathcal{S}} \mathbf{q}_*^s = \mathbf{0} \tag{6}$$

where $\mathbf{q}_*^b, \mathbf{q}_*^s$ are the *efficient allocations* to buyers and sellers.

As De Vries and Vohra (2003) note, the problem in (5) amounts to solving a problem without knowing the objective function, given that participants need not report their true valuations.

A dual problem is that of finding a price P_* that supports the allocation $\mathbf{q}_*$, i.e., finding P_* such that:

$$\mathbf{q}_* \in \left\{ \sum_{b \in \mathcal{B}} D_{u^b}(P_*^b) - \sum_{s \in \mathcal{S}} S_{u^s}(P_*^s) = \mathbf{0} \right\}$$

To summarize, any PME works as follows:

1. There is a set $\mathcal{N}$ goods and the clearinghouse calls all the participants to report their sealed bids

2. The j-th bid is a set $\{(\mathbf{q}, u^j(\mathbf{q})) \mid \forall \mathbf{q} \in A^j\}$ and each bid is *always* allocated at most one bundle. Therefore, it is assumed that $\mathbf{q} = \mathbf{0} \in A^j$, such that losing bids are also part of the optimal aggregate allocation

3. If the clearinghouse imposes strong substitute preferences on the bids reported by participants, every participant has a binary valuation domain. Otherwise, if ordinary substitute preferences are allowed for, participants have multi-unit valuation domains

4. The optimal aggregate allocation $\mathbf{q}^*$ is the one that maximizes the aggregate valuation function U. Associated to $\mathbf{q}^*$, there is a market clearing price vector, such that $\mathbf{q}^* \in D_U(P^*)$. Depending on the specific application, prices can be linear and anonynous or non-linear and non-anonymous

3 The tropical geometry of substitutable preferences

In a two-commodity economy, the price-theoretic notion of substitute goods for demand states that increasing the price of one good does not decrease the demand for the other (Milgrom and Strulovici, 2009).

For example, if the set of commodities is $\mathcal{N} = \{A, B\}$ as in examples 2.4 and 2.5, a buyer that observes an increase in the price of A will not reduce its demand for B. On the contrary, its demand for B will *weakly* increase. Similarly, a seller that observes an increase in the price of A will not increase its supply for B, but will *weakly* decrease it instead. More precisely:

Definition 3.1 *(Substitutable preferences)*[8]

Let $P^b, \tilde{P}^b$ be prices perceived by a buyer b such that $P^b \leq \tilde{P}^b$:

- *If there is a bundle $\tilde{\mathbf{q}}$ such that $\mathbf{q} \leq \tilde{\mathbf{q}}$, where $\mathbf{q} \in D_{u^b}(P^b)$ and $\tilde{\mathbf{q}} \in D_{u^b}(\tilde{P}^b)$, then buyer b has substitutable preferences.*

Likewise, consider a seller s who faces prices $P^s, \tilde{P}^s$, such that $P^s \leq \tilde{P}^s$:

- *If there is a bundle $\tilde{\mathbf{q}}$ such that $\mathbf{q} \geq \tilde{\mathbf{q}}$, where $\mathbf{q} \in S_{u^s}(P^s)$ and $\tilde{\mathbf{q}} \in S_{u^s}(\tilde{P}^s)$, then seller s has substitutable preferences.*

[8] For the sake of precision note that if $\mathbf{q} \leq \tilde{\mathbf{q}}$ then $q_n \leq \tilde{q}_n$ for all $n \in \mathcal{N}$. Similarly, if $\mathbf{q} \geq \tilde{\mathbf{q}}$ then $q_n \geq \tilde{q}_n$ for all $n \in \mathcal{N}$.

Note, however, that the Gross Substitutes (GS) condition of Kelso and Crawford (1982)[9] is more restrictive. Roughly stated, the GS condition is satisfied if an increase in the price of substitute goods never leads a participant to modify its demand or supply for goods whose prices have not risen. Formally:

Definition 3.2 *(Gross Substitutes)*

Let $P^b, \tilde{P}^b$ *be prices perceived by a buyer* b *such that:* $P^b \leq \tilde{P}^b$ *and* $\mathbf{q} \in D_{u^b}(P^b)$, $\tilde{\mathbf{q}} \in D_{u^b}(\tilde{P}^b)$ *hold. Likewise, let* $P^s, \tilde{P}^s$ *be prices perceived by a seller* s *such that:* $P^s \leq \tilde{P}^s$ *and* $\mathbf{q} \in D_{u^s}(P^s)$, $\tilde{\mathbf{q}} \in D_{u^s}(\tilde{P}^s)$ *hold.*

The GS condition holds if the bundle $\tilde{\mathbf{q}}$ *satisfies:*

- $q_n = \tilde{q}_n$ *if and only if* $p^b_n = \tilde{p}^b_n$ *for some* $n \in \mathcal{N}$*, in the case of buyers*
- $q_n = \tilde{q}_n$ *if and only if* $p^s_n = \tilde{p}^s_n$ *for some* $n \in \mathcal{N}$*, in the case of sellers*

As noted in section 2, the different variants of a PME depend on the kind of substitute preferences imposed by the clearinghouse on market participants and the indivisibility of the goods. Both the Ordinary Substitute (OS) and the Strong Substitutes (SS) concepts are special cases of definition 3.1 and satisfy the GS condition. The distinction between them, however, can only become entirely clear after some *essential* tropical geometric concepts, as recently applied in Economics by Baldwin and Klemperer (2012, 2016), are introduced.

In the discussion that follows I focus on agents that have substitutable preferences and introduce some basic concepts required to analyze these with tropical geometry.[10] Hopefully, the reader will be convinced that the application of geometric concepts pays off as they provide considerable visual intuition into an *individual* agent's preferences but also describe very well the *joint* preferences of a group of agents, regardless of the fact that each agent expresses substitutable, complementary or other kind of preferences. Besides the visual

[9] Which plays a central role in theoretical economics because the existence of equilibrium with indivisibility depends on it

[10] The interested reader is referred to detailed expositions of the subject, e.g. the book by Maclagan and Sturmfels (2015). Note, in addition, that the appendix (section A.1.1) of this paper contains a brief exposition of polyhedral geometry concepts that can facilitate the understanding of the concepts that follow.

intuition, the mathematical structure underlying such representation of agents' preferences has been shown by to determine the existence of equilibrium in environments like the PME, a topic that is discussed in section 4.

As examples 2.2 and 2.3 show, PMEs rely on the representation of bids in price space. Therefore, to draw the supply or demand correspondence of an *individual* market participant as well as the market's *aggregate supply* correspondence or *aggregate demand* correspondence, it is important to determine how supply and demand vary in accordance with price variations.

Specifically, for a particular buyer, the *demand correspondence in price space* is the *locus* where demand varies as a result of changes in price. Formally:

$$\left\{P^b \mid \#D_{u^b}(P^b) > 1\right\} \tag{7}$$

where # stands for "number". That is, for buyer b, who has valuation u^b, it is the set of prices P^b at which the demand correspondence in equation (1) varies. The depiction is of economic interest as long as demand changes and, therefore, the requirement that the number of demand correspondences is greater than one.

Similarly, for an individual seller, a *supply correspondence in price space* is defined as:

$$\{P^s \mid \#S_{u^s}(P^s) > 1\} \tag{8}$$

Namely: for seller s and valuation u^s, it is the set of prices P^s at which the supply correspondence in equation (2) varies.

Note that the geometric structures just defined are anything but new to economics. For example, in their exposition of the kind of preferences that imply the GS condition, Kelso and Crawford (1982) (page 1501, figures 1 and 2) show demand correspondences in price space when, respectively, preferences are subadditive and superadditive and there are only two goods in the economy.[11] Similarly, in an example shown in chapter 11 of Murota (2003) (discussed later by Baldwin and Klemperer (2012, 2016)) similar structures are drawn to analyze the conditions when a pricing equilibrium with indivisibility fails.

[11]Kelso and Crawford (1982) explain that, in a two-good economy, whenever preferences are subadditive, the GS condition holds but it never does when preferences are superadditive. They also emphasize that if there are more than two goods in the economy, subadditivity does not generally imply the GS condition.

What is new to the economics literature is the realization that such structures are much more than several lines partitioning the price space according to valuations that result in optimal demand or supply bundles. Indeed, both demand and supply correspondences are *tropical hypersurfaces* (THs) associated with market participants' valuations:

Definition 3.3 *(Adapted from Baldwin and Klemperer (2016)) The tropical hypersurface $\mathcal{T}_u$ associated with any valuation u is a weighted rational polyhedral complex such that:*

1. *Its underlying set is $\left\{P^b \mid \#D_{u^b}(P^b) > 1\right\}$ in the case of buyers and $\{P^s \mid \#S_{u^s}(P^s) > 1\}$ in the case of sellers.*

2. *The weight w_F of the facet F is the integer number defined by $w_F \cdot \mathbf{v} = \mathbf{q}' - \mathbf{q}$ in which bundles $\mathbf{q}, \mathbf{q}'$ are uniquely demanded at each side of the facet and form Unique Bundle Regions (UBRs), and $\mathbf{v}$ is the primitive integer normal vector (i.e. a vector whose greater common divisor is one) pointing from $\mathbf{q}'$ to $\mathbf{q}$.*

The aforementioned definition contains terminology that can be best understood by way of examples:

Example 3.4 *(Demand-side correspondence for a buyer with binary domain) Let $\mathcal{N} = \{A, B\}$ be the set of commodities in the PME and consider a buyer b with the binary domain of example 2.4, i.e. $A^b = \{(0,0),(3,0),(0,3),(3,3)\}$. Suppose that this buyer's valuation function is: $u^b(0,0) = u^b(3,3) = 0; u^b(0,3) = 9; u^b(3,0) = 12$.*

Figure 8 shows the demand correspondence in price space for this buyer, which has the structure of a TH associated to the buyer's valuation. Specifically, note that there are three facets (solid red lines) dividing the price space and at each side of the facet there are three distinct UBRs: one corresponding to the $(0,0)$ bundle, another for the $(3,0)$ bundle and another one for the $(0,3)$ bundle.[12]

[12]Strictly speaking, figure 8 is a demand correspondence restricted to prices in which the participant is limited to buying. Indeed, with the specified valuation function, and given price vector $P^b = (-3,-4)$, the participant would be indifferent about *selling* bundles $(3,0),(0,3)$ and $(3,3)$. In practical applications, there is nothing that prevents the buyer to exclude the $(3,3)$ bundle from its domain, if he does not wish to show any valuation for it.

Observe, for example, that whenever the difference between the price for commodity A and commodity B equals one, the buyer is indifferent between bundles $(3, 0)$ *and* $(0, 3)$*. Also, whenever* $P^b = (p^b_A > 4, p^b_B > 3)$*, the buyer will optimally demand bundle* $(0, 0)$*.*

Standing at each of the UBRs, a simple calculation shows that the weight of all facets is $w_F = 3$*. For example, using the same notation of definition 3.3, let* $\mathbf{q}' = (0, 3), \mathbf{q} = (3, 0)$ *and it follows that* $\mathbf{q}' - \mathbf{q} = (-3, 3)$*. Clearly, the integer* w_F *satisfying* $w_F \cdot \mathbf{v} = \mathbf{q}' - \mathbf{q}$ *is* $w_F = 3$ *and* $\mathbf{v} = (-1, 1)$*. Similar calculations show that the set of all primitive integer vectors is* $(-1, 1), (1, -1), (1, 0), (0, 1), (-1, 0), (0, -1)$*.*

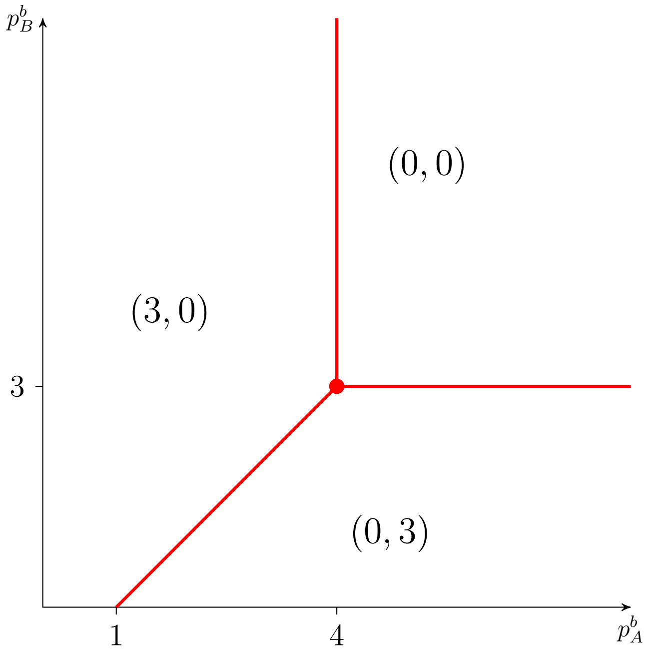

Figure 8: Demand-side correspondence for a buyer with binary domain, example 3.4

Example 3.5 *(Supply-side correspondence for a seller with binary domain)*

As before, let $\mathcal{N} = \{A, B\}$ *be the set of commodities in the PME and suppose that the seller* s *also has the binary domain of example 2.4. Specifically, suppose that* $A^s = \{(0, 0), (3, 0), (0, 3), (3, 3)\}$ *and that the seller's valuation function is:* $u^s(0, 0) = 0; u^s(0, 3) = -6; u^s(3, 0) = -3; u^s(3, 3) = -60.$

Figure 9 shows the supply correspondence in price space for this seller. Its TH has

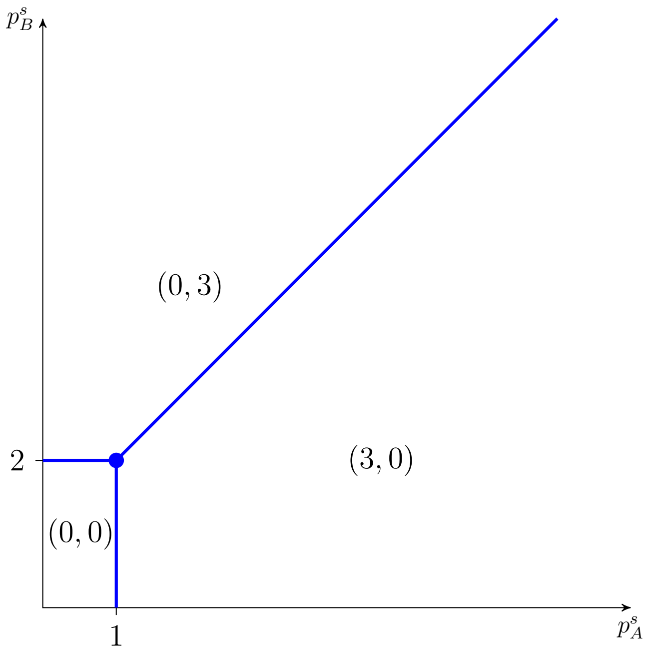

Figure 9: Supply-side correspondence for a seller with binary domain, example 3.5

three facets (solid blue lines) dividing the price space and at each side of a facet there are three different UBRs: one for the $(0,3)$ *bundle, another for the* $(3,0)$ *and one for the* $(0,0)$ *bundle.*[13]

Note that, for instance, if the price vector $P^s = (p_A^s < 1, p_B^s < 2)$ *the buyer will optimally supply the bundle* $(0,0)$ *and whenever* $p_A^s - p_B^s < 1$*, the seller is indifferent between offering bundle* $(0,3)$ *and bundle* $(3,0)$*.*

A simple calculation shows that the weight of all facets in this example is $w_F = 3$*, as in example 3.4. For example, using the same notation of definition 3.3, let* $\mathbf{q}' = (3,0), \mathbf{q} = (0,0)$ *and thus* $\mathbf{q}' - \mathbf{q} = (3,0)$*. Clearly, the integer* w_F *satisfying* $w_F \cdot \mathbf{v} = \mathbf{q}' - \mathbf{q}$ *is* $w_F = 3$ *and* $\mathbf{v} = (1,0)$*. Similar calculations show that the set of all primitive integer vectors for this example is* $(-1,1),(1,-1),(1,0),(0,1),(-1,0),(0,-1)$*.*

Besides the possibility to represent an individual participant's demand and supply, an

[13] A comment similar to the one in footnote 12 applies: figure 9 is restricted to prices in which the supplier has a trade-off between selling $(0,3),(3,0)$ and $(0,0)$ but at price vector $P^s = (18,19)$ the seller would be indifferent between offering bundles $(0,3),(3,0)$ and $(3,3)$. Likewise, there is nothing that prevents the seller to exclude the $(3,3)$ bundle from its domain, if he does not wish to show any valuation for it.

interesting feature of THs is that they "encode" all that is required to determine the kind of preferences of a given market participant. That is, any TH is enough to describe the *demand type* of a market participant's valuation:

Definition 3.6 *(Baldwin and Klemperer (2016)) A valuation is of demand type $\mathcal{D}$ if all the primitive integer normal vectors to the facets of its associated TH lie in a set $\mathcal{D}$ of primitive integer vectors in $\mathbb{Z}^N$, such that if $\mathbf{v} \in \mathcal{D}$ then $-\mathbf{v} \in \mathcal{D}$.*

As an illustration, consider the demand type $\mathcal{D}$ of examples 3.4 and 3.5. It follows immediately from definition 3.6 that the demand type of these is the set $\mathcal{D} = \{\pm(1,0), \pm(0,1), \pm(1,-1)\}$.

Armed with the concepts described so far, I state that the substitute concept relevant for PMEs is what Baldwin and Klemperer (2016) have defined as Ordinary Substitutes (OS):

Definition 3.7 *(Adapted from Baldwin and Klemperer (2016)) A valuation u^j is Ordinary Substitutes (OS) if:*

- ***In the case of buyers:*** *For any prices $P^b \leq \tilde{P}^b$ in a UBR, $D_{u^b}(P^b) = \{\mathbf{q}\}$ and $D_{u^b}(\tilde{P}^b) = \{\tilde{\mathbf{q}}\}$ we have $q_n \leq \tilde{q}_n$ for all n such that $\tilde{p}^b_n = p^b_n$*
- ***In the case of sellers:*** *For any prices $P^s \leq \tilde{P}^s$ in a UBR, $S_{u^s}(P^s) = \{\mathbf{q}\}$ and $S_{u^s}(\tilde{P}^s) = \{\tilde{\mathbf{q}}\}$ we have $q_n \geq \tilde{q}_n$ for all n such that $\tilde{p}^s_n = p^s_n$*

Note that, unlike definition 3.1, definition 3.7 focuses on demand and supply correspondences composed of singletons, such that each one corresponds to a UBR. Furthermore, it also ensures that the GS condition holds. In terms of demand types, the OS condition can be easily described as follows:

Definition 3.8 *(Baldwin and Klemperer (2012))*[14] *A demand type $\mathcal{D}$ is Ordinary Substitutes (OS) if it consists of vectors in $\mathbb{Z}^N$ with at most one positive and at most one negative entry, and all others are zero.*

[14]Proposition 3.3 in Baldwin and Klemperer (2016) also gives a proof of this fact.

Furthermore, note that the SS valuation type is a special case of the OS valuation type:

Definition 3.9 *(Baldwin and Klemperer (2012)) A demand type $\mathcal{D}$ is Strong Substitutes (SS) if it consists of vectors in $\mathbb{Z}^N$ with at most one $+1$ and at most one -1 entry, and all others are zero.*

From definition 3.9, it follows that the valuations of examples 3.4 and 3.4 are of the SS demand type.

The following two propositions, which complete the minimum set of results required to analyze substitutable preferences, allow extending the results presented so far to an aggregate set of valuations. First, the sum of individual demands over the set of all buyers, and the set of individual supplies over the set of sellers, results in aggregate demand and aggregate supply:

Proposition 3.10 *Given buyers' aggregate valuation $U^{\mathcal{B}}$, aggregate demand $D_{U^{\mathcal{B}}}(P)$ is the sum of individual demands over the set of buyers $\mathcal{B}$, $D_{U^{\mathcal{B}}}(P) = \sum_{b \in \mathcal{B}} D_{u^b}(P^b)$*

Likewise, given sellers' aggregate valuation $U^{\mathcal{S}}$, aggregate supply $S_{U^{\mathcal{S}}}(P)$ is the sum of individual demands over the set of sellers $\mathcal{S}$, $S_{U^{\mathcal{S}}}(P) = \sum_{s \in \mathcal{S}} S_{u^s}(P^s)$

Proof See the appendix, section A.1.2. ■

While there is nothing surprising about Proposition 3.10, it is relevant because a simple superimposition of individual participants' THs results in an aggregate TH. That is, an aggregate demand or supply correspondence in price space also has the structure of a TH. In consequence, demand types are preserved under aggregation:

Proposition 3.11 *(Adapted from Baldwin and Klemperer (2016)) Valuations u^j are of demand type $\mathcal{D}$ for all $j \in \mathcal{J}$ if and only if the aggregate TH ($\mathcal{T}_U$) is of demand type $\mathcal{D}$.*

As hinted in the introductory discussion about PMAs and PMEs in section 2 of the paper, the SS demand type is equivalent to what Milgrom and Strulovici (2009) have defined as strong substitutes valuation because it satisfies their "binary substitutes" property. Similarly, the OS demand type corresponds to their weak substitute valuation, as it satisfies their "multi-unit substitutes property" [15].

The following example illustrates how can tropical geometry be employed to analyze substitutable preferences in an aggregate setting:

Example 3.12 [16] *Consider, as in the earlier examples, a set of commodities $\mathcal{N} = \{A, B\}$. Let the set of buyers be $\mathcal{B} = \{b1, b2\}$ and the set of sellers be $\mathcal{S} = \{s1, s2\}$ and suppose all participants have multi-unit valuation domains and are restricted to expressing OS preferences.*

Focusing on the demand side of the market:

- *Buyer $b1$'s domain is $A^{b1} = \{(0,0), (1,0), (0,2)\}$ and has valuation function: $u^{b1}(0,0) = 0; u^{b1}(1,0) = 6; u^{b1}(0,2) = 8$. His TH is depicted in figure 10 and has demand type $\mathcal{D}_{b1} = \{\pm(1,-2), \pm(1,0), \pm(0,1)\}$, and so is of the OS demand type. All the facet weights are $w_F = 1$, except for the facet between $(0,0)$ and $(0,2)$ which has a weight of $w_F = 2$.*

- *Buyer $b2$'s domain is $A^{b2} = \{(0,0), (3,0), (0,2)\}$ and has valuation function: $u^{b2}(0,0) = 0; u^{b1}(3,0) = 9; u^{b1}(0,2) = 4$. Figure 11 shows his TH which has demand type $\mathcal{D}_{b2} = \{\pm(1,0), \pm(0,1), \pm(3,-2)\}$, which is of the OS type. The weight of the facets are: $w_F = 3$ for the one between $(3,0)$ and $(0,0)$, $w_F = 2$ for the facets between $(0,0)$ and $(0,2)$, and $w_F = 1$ for the facets between $(3,0)$ and $(0,2)$.*

- *Now consider the aggregate TH (see figure 12) for the set of buyers $\mathcal{B}$ in the example. It contains six UBRs, one for each aggregate bundle that can be uniquely decomposed as a sum of demands. The demand type of the aggregate TH is: $\mathcal{D}_{\mathcal{B}} = \{\pm(1,-2), \pm(3,-2), \pm(1,0), \pm(0,1)\}$, clearly an OS demand type.*

[15] See definitions 3 and 4 (page 216) in Milgrom and Strulovici (2009)

[16] Further details about this example can be found in the Appendix, subsection A.3.3.

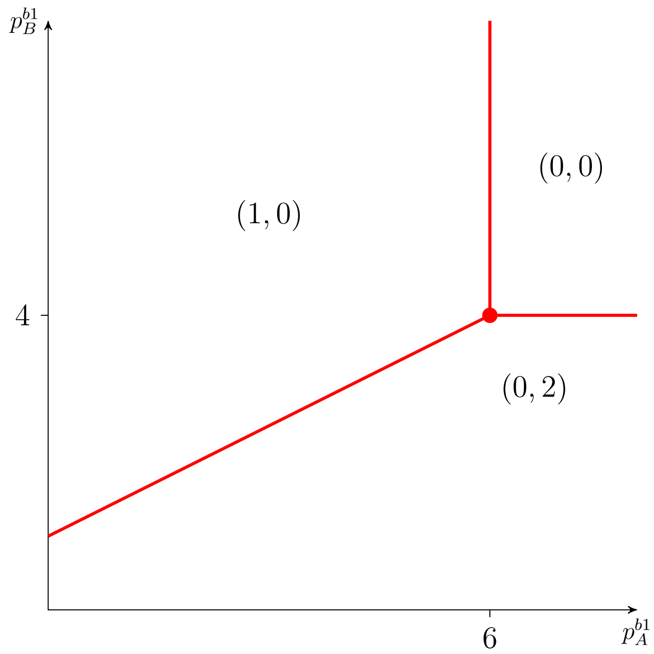

Figure 10: Demand-side correspondence for buyer $b1$, example 3.12

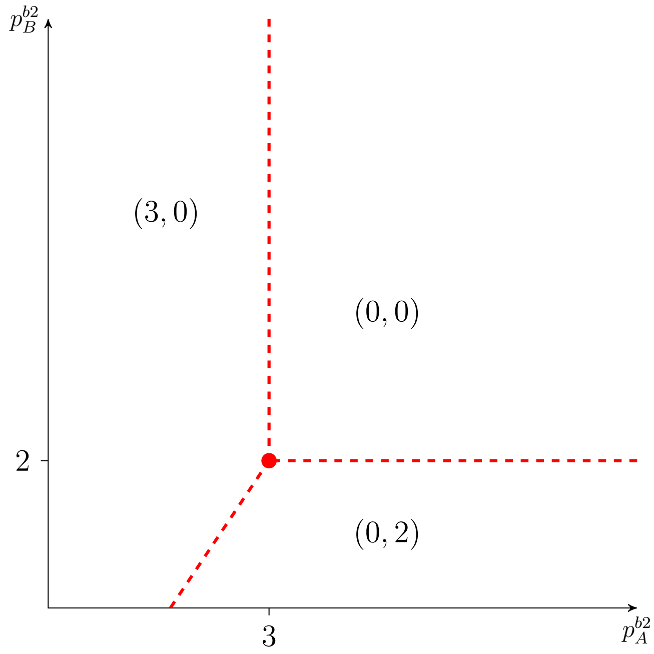

Figure 11: Demand-side correspondence for buyer $b2$, example 3.12

Focusing on the supply side of the market:

- *Seller s1's domain is* $A^{s1} = \{(0,0),(4,0),(0,1)\}$ *and has valuation function::*

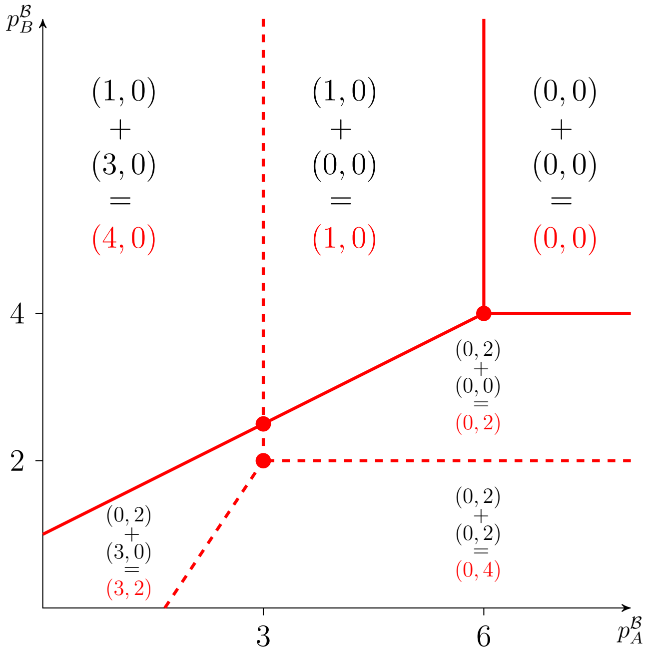

Figure 12: Aggregate demand-side correspondence for the set of buyers $\mathcal{B}$, example 3.12

$u^{s1}(0,0) = 0; u^{b1}(4,0) = -8; u^{b1}(0,1) = -3$. *His TH is depicted in figure 13 and has demand type* $\mathcal{D}_{s1} = \{\pm(4,-1), \pm(1,0), \pm(0,1)\}$, *and so is of the OS demand type. All the facet weights are* $w_F = 1$, *except for the facet between* $(0,0)$ *and* $(4,0)$ *which has a weight of* $w_F = 4$.

- *Seller* $s2$*'s domain is* $A^{s2} = \{(0,0), (3,0), (0,6)\}$ *and has valuation function:* $u^{s2}(0,0) = 0; u^{b1}(3,0) = -12; u^{b1}(0,6) = -18$, *His TH, shown in figure 14, has demand type* $\mathcal{D}_{s2} = \{\pm(-1,2), \pm(1,0), \pm(0,1)\}$, *and so is of the OS demand type. The facet weights between bundles* $(0,0)$, $(3,0)$, *and between bundles* $(0,6)$, $(3,0)$ *are* $w_F = 3$, *whereas the facet weight between bundles* $(0,0)$, $(0,6)$ *is* $w_F = 6$.

- *Now consider the aggregate TH for the set of sellers* $\mathcal{S}$ *in figure 15. Because of the way in which individual THs intersect, there are five and not six UBRs in the aggregate TH, one for each aggregate bundle. The demand type for the sellers' TH is* $\mathcal{D}_{\mathcal{S}} = \{\pm(-1,2), \pm(4,-1), \pm(1,0), \pm(0,1)\}$

Finally, visual inspection can help to visually determine in which UBR can this PME

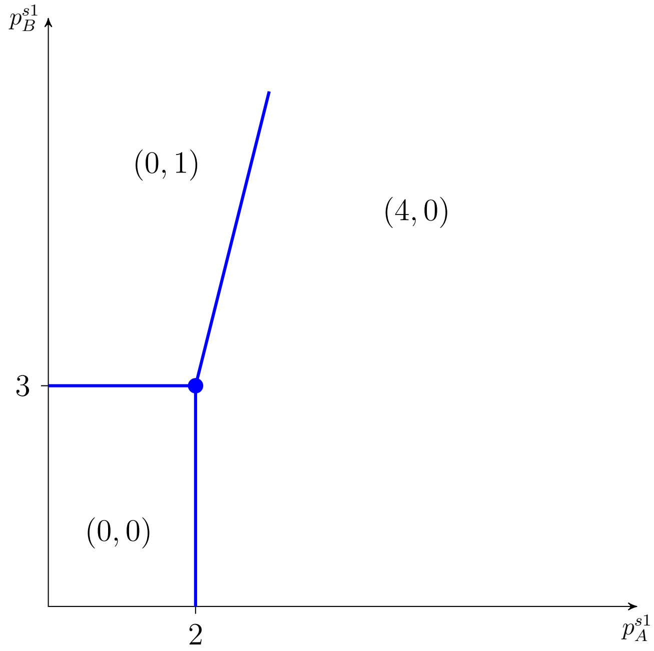

Figure 13: Suply-side correspondence for seller $s1$, example 3.12

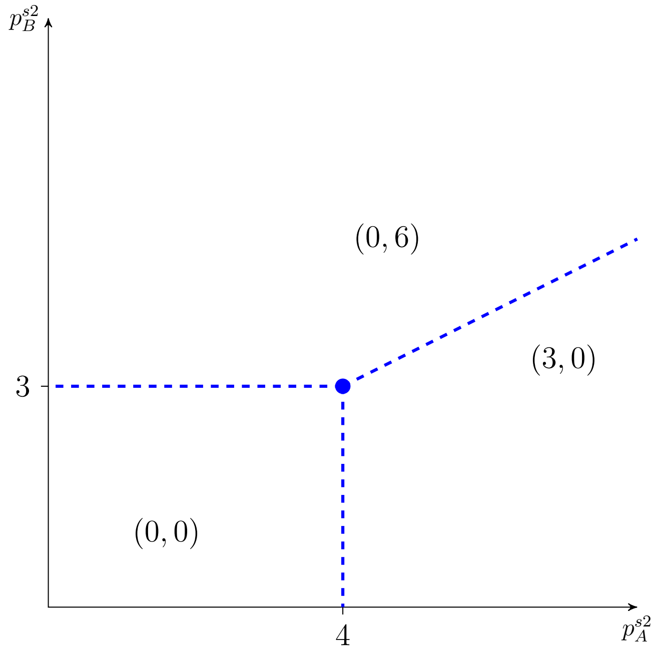

Figure 14: Suply-side correspondence for seller $s2$, example 3.12

clear. Note in figure 16, in which non-essential facets are erased, that this PME clears if buyer $b1$ *obtains bundle* $(1, 0)$ *and buyer* $b2$ *buys* $(3, 0)$*, while seller* $s1$ *sells* $(4, 0)$ *and seller*

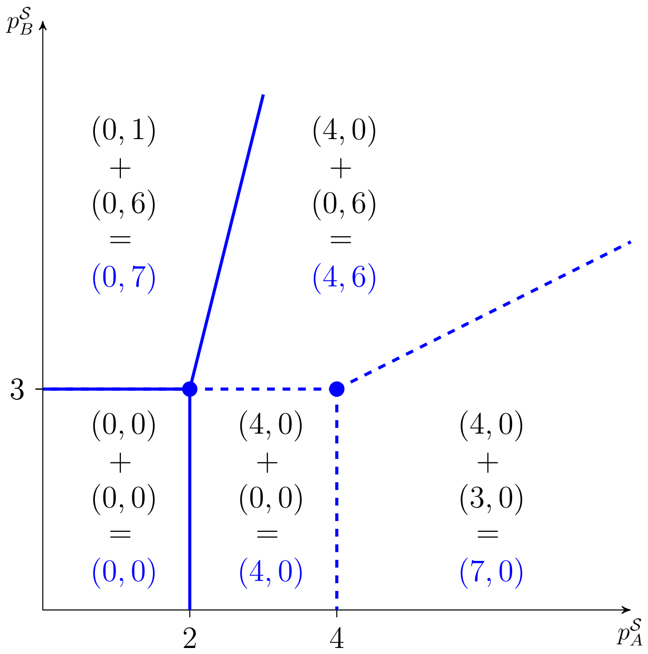

Figure 15: Aggregate supply-side correspondence for the set of sellers $\mathcal{S}$, example 3.12

s2 offers $(0, 0)$*, i.e. nothing. DLP6 in section 4 shows that this allocation is possible if the price vector is* $P = (2.34, 2.54)$.

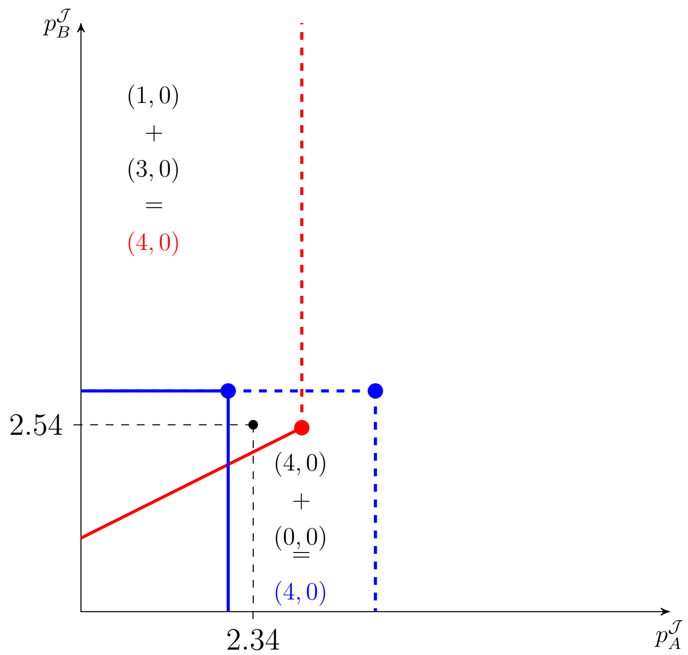

Figure 16: Market clearing in example 3.12

4 Winner determination problem and pricing equilibria in Product-Mix Exchanges

One of the advantages of formulating PMEs as combinatorial auctions is that a number of existing results in the vast literature on this topic can be readily applied or simply modified for the problem at hand. Tran and Yu (2015) proposed a weighted set packing formulation for the winner determination problem in PMAs which I modify to account for multiple buyers and multiple sellers.

The following integer program, in decision variables $y(\mathbf{q}, b), y(\mathbf{q}, s)$, solves the clearinghouse's problem stated in equation (5), which consists on ensuring an efficient allocation:

$$
\begin{aligned}
\max \quad & \sum_{b\in\mathcal{B}} \sum_{\mathbf{q}\in A^b} u^b(\mathbf{q}) y(\mathbf{q}, b) + \sum_{s\in\mathcal{S}} \sum_{\mathbf{q}\in A^s} u^s(\mathbf{q}) y(\mathbf{q}, s) \\
\text{s.t.} \quad & \sum_{\mathbf{q}\in A^b} y(\mathbf{q}, b) = 1 \quad \forall b \in \mathcal{B} \\
& \sum_{\mathbf{q}\in A^s} y(\mathbf{q}, s) = 1 \quad \forall s \in \mathcal{S} \\
& \sum_{b\in\mathcal{B}} \sum_{\mathbf{q}\in A^b} y(\mathbf{q}, b)\mathbf{q} - \sum_{s\in\mathcal{S}} \sum_{\mathbf{q}\in A^s} y(\mathbf{q}, s)\mathbf{q} = 0 \quad \forall n \in \mathcal{N} \\
& y(\mathbf{q}, b), y(\mathbf{q}, s) \in \{0, 1\}
\end{aligned} \tag{9}
$$

If the decision variable $y(\mathbf{q}, b) = 1$, the clearinghouse *sells bundle* $\mathbf{q} \in A^b$ *to* buyer b, and refer to the bundle in question as $\mathbf{q}^b_*$. Likewise, if $y(\mathbf{q}, s) = 1$ the clearinghouse *buys bundle* $\mathbf{q} \in A^s$ *from* seller s, and write $\mathbf{q}^s_*$. As noted in section 2.2, in these cases we refer generally to $\mathbf{q}^b_*, \mathbf{q}^s_*$ as the efficient allocations to buyers and sellers. The summation of allocations to buyers over the set $\mathcal{B}$ minus the summation of allocations to sellers over the set $\mathcal{S}$ equals the efficient aggregate bundle $\mathbf{q}_*$, as equation (6) shows. Else, if the bundles are not part of the efficient allocation, $y(\mathbf{q}, b) = 0, y(\mathbf{q}, s) = 0$.

The first constraint in (9) restricts each buyer to be allocated exactly one bundle of goods, while the second constraint ensures the same condition for each seller. The third constraint imposes that for each commodity traded in the exchange, total supply must equal total demand and, therefore, its right-hand side equals zero. This formulation is sufficiently general to account for all versions of PMEs.

Also, the model in (9) is closely related to the models in Bikhchandani and Ostroy (2002, 2006) and can be considered as an extension of their work into the multi-unit combinatorial auction framework, which is described, for example, by De Vries and Vohra (2003) and Leyton-Brown et al. (2000). A relevant innovation in the work of Bikhchandani and Ostroy is that, following the theory of personalized trading developed by Makowski (1979), they introduce the notion of non-linear and non-anonymous pricing in a competitive setting. Accordingly, I adopt their definition of pricing equilibrium:

Definition 4.1 *(Pricing equilibrium, adapted from Bikhchandani and Ostroy (2006)) A pricing equilibrium is* $(\mathbf{q}_*, P_*)$ *if:*

- *Each buyer* $b \in \mathcal{B}$ *maximizes utility and obtains* $\pi^b = \max_{\mathbf{q} \in A^b} \left\{ u^b(\mathbf{q}) - P^b \cdot \mathbf{q} \right\}$
- *Each seller* $s \in \mathcal{S}$ *maximizes profit and obtains* $\pi^s = \max_{\mathbf{q} \in A^s} \left\{ P^s \cdot \mathbf{q} + u^s(\mathbf{q}) \right\}$
- *The market clears:* $\mathbf{q}_* \in \left\{ \sum_{b \in \mathcal{B}} D_{u^b}(P_*^b) - \sum_{s \in \mathcal{S}} S_{u^s}(P_*^s) = \mathbf{0} \right\}$

Note that this definition is more general than Walrasian equilibrium, which holds when the pricing rule is linear and anonymous:

Definition 4.2 *(Walrasian equilibrium, adapted from Bikhchandani and Ostroy (2006)) A pricing equilibrium* $(\mathbf{q}_*, P_*)$ *is Walrasian if* $P_* = \mathbf{p}_* \; \forall j \in \mathcal{J}$

Unlike Bikhchandani and Ostroy, recent work on PMAs emphasizes the geometric nature of equilibrium with indivisibilities, based on the duality between quantity space and price space explained in section 3. Therefore, the following definition - which does not collide with pricing equilibria - is also relevant for the study of PMEs when the traded goods are indivisible:

Definition 4.3 *(Equilibrium with indivisibility, adapted from Tran and Yu (2015)) A set of agents $\mathcal{J}$ with valuations $\{u^j\}$ has competitive equilibrium at aggregate bundle $\mathbf{q}_* \in \mathrm{conv}(A) \cap \mathbb{Z}^N$ (read $\mathrm{conv}(A)$ as the convex hull of aggregate domain A) if there exists a price $P_* \in \mathbb{R}^N$ such that $\mathbf{q}_* \in \left\{ \sum_{b \in \mathcal{B}} D_{u^b}(P_*^b) - \sum_{s \in \mathcal{S}} S_{u^s}(P_*^s) = \mathbf{0} \right\}$*

There are two interesting aspects of this definition. The first one refers to the fact that the aggregate bundle at which equilibrium exists is in the convex hull of A. More than just another geometric condition, this is in accordance with the notion of *pseudo-equilibrium* introduced by Milgrom and Strulovici (2009) which is relevant in auctions, e.g. ascending, where sellers are willing to adjust their supply to obtain prices that support the efficient allocation. The second interesting feature is that, although in their work, Tran and Yu (2015) assume a linear pricing rule to obtain the prices that support the outcome $\mathbf{q}_*$, a discriminatory pricing rule also serves the purpose, as I show in section 5.

As in the original PMA, PMEs are also applicable when the goods are divisible. The corresponding definition of equilibrium is less restrictive than its counterpart for the indivisible case.

Definition 4.4 *(Equilibrium with divisibility) A set of agents $\mathcal{J}$ with valuations $\{u^j\}$ has competitive equilibrium at aggregate bundle $\mathbf{q}_* \in A$ if there exists a price $P_* \in \mathbb{R}^N$ such that $\mathbf{q}_* \in \{D_U(P_*) - S_U(P_*) = \mathbf{0}\}$*

The conditions under which linear programming relaxations to integer programs yield integer solutions has received considerable attention over the course of decades. Specifically, with respect to (9), Tran and Yu (2015) proved that a competitive equilibrium with indivisibility exists at the (aggregate) seller-supplied bundle if and only if the optimum of the linear programming relaxation and that of the integer program coincide. Note that in the case of the PME formulation, the aggregate allocation corresponds to the zero bundle in the aggregate domain. For simplicity, in the remainder of the paper, I refer to the Linear

Programming (LP) relaxation of integer program (9) as LP6, which is the same model as (9) replaced with the conditions that $y(\mathbf{q}, b), y(\mathbf{q}, s) \geq 0$).

Knowing that it is possible to drop the integrality constraint in (9) and use LP6 instead is important for applications. First, there are situations in which the computational cost can become a burden, and this becomes more manageable if it is possible to solve LP6 and still be able to find an allocation with indivisibilities. On the other hand, being able to compute the dual of the relaxed program allows for the application of linear programming duality theory which, inter alia, permits estimating the supporting prices of an allocation when goods are indivisible or not. Consider thus the dual to the relaxation of (9), to which I refer to as DLP6 in the remainder of the paper:

$$
\begin{aligned}
\min \quad & \sum_{b \in \mathcal{B}} \pi^b + \sum_{s \in \mathcal{S}} \pi^s \\
\text{s.t.} \quad & \pi^b + P^b \cdot \mathbf{q} \geq u^b(\mathbf{q}) \ \forall b \in \mathcal{B}, \mathbf{q} \\
& \pi^s - P^s \cdot \mathbf{q} \geq u^s(\mathbf{q}) \ \forall s \in \mathcal{S}, \mathbf{q} \\
& \pi^b, \pi^s, P^b, P^s \text{ unrestricted}
\end{aligned}
\tag{10}
$$

The decision variables in DLP6, i.e. in (10), are the dual profit function of each buyer π^b, each seller π^s and the prices paid by buyers P^b and to sellers P^s for each for each valuation $u^b(\mathbf{q}), u^s(\mathbf{q})$ reported to the exchange. The solutions to this linear program yield the pricing equilibria of definition (4.1).

4.1 The indivisible goods case

Rather than asking if it is possible to obtain an efficient allocation with indivisible goods with the model in (9) or with its linear programming relaxation, a question of greater economic interest relates to the conditions under which participants' valuations result in an equilibrium with indivisibility. The following theorem gives an answer:

Theorem 4.5 *(Danilov et al. (2001), Baldwin and Klemperer (2016),Tran and Yu (2015))*

The set $\mathcal{D}$ is unimodular if and only if every collection of concave utility functions $\{u_j \mid \forall j \in \mathcal{J}\}$ of demand type $\mathcal{D}$ has a competitive equilibrium (with indivisibility).

Tran and Yu (2015) give an alternative proof of this theorem and illustrate its geometric implications. Although not new, their result is relevant because they were the first to establish a connection between the theorem and linear programming, which makes it more amenable to applications. In this subsection, I restate their model, give an economic interpretation to it and prove an ancillary result: a linear programming approach to test the unimodularity of a set of valuations.[17]

In their paper, Baldwin and Klemperer (2016) refer to the longest-standing definition of unimodularity: a set of linearly independent vectors with which a nonsingular, integral matrix with determinant ± 1 can be constructed. To check if a set of valuations has the property, they suggest constructing square matrices with the primitive integer vectors in a demand type $\mathcal{D}$ and then calculate the determinants, something that can be readily done in most cases. However, in practical applications where the number of dimensions and the number of participants increases or the geometric intuition is not immediately evident, the process can easily become cumbersome.

Moreover, the concept of unimodularity can be generalized to not necessarily nonsingular matrices, as the following definitions in increasing order of generality show:

Definition 4.6 *(Unimodular matrices, Schrijver (1998)) A matrix $\mathbf{V}$ is unimodular if:*

1. *It is integral, square and has determinant ± 1*
2. *It is integral, has dimension $M \times N$, full row rank and each of its nonsingular submatrices of order M has determinant ± 1*
3. *It is integral, has rank R, and for each submatrix of rank R the greatest common divisor of its subdeterminants is 1.*

[17] The work by Tran and Yu (2015) is aimed at mathematicians. In their abstract, they claim that "We introduce auction theory for a mathematical audience ...". My contribution is to present the same result to an economics audience.

By definition, a TH is a polyhedral complex: a collection of polyhedra called *cells* satisfying certain conditions.[18] Therefore, assuming that participants in an exchange have revealed their valuations together with their preferred bundles of goods, it is possible for the clearinghouse (or a single seller) to precisely define the prices at which participants prefer one bundle over another. This can be represented with a simple inequality description, i.e. finitely many half-spaces that upon intersection form a polyhedron containing the prices at which a given allocation is feasible. Given a fixed bundle of goods, it is then straightforward to write it as a linear function of the prices and maximize it over the polyhedron. This, clearly, amounts to maximizing the revenue of the clearinghouse (or seller), a *distinct* objective to the one in (5) or (9).

Specifically, for each buyer $b \in \mathcal{B}$ and bundles $\mathbf{q}, \mathbf{q}' \in A^b$ such that each bundle constitutes a unique demand region, write:

$$u^b(\mathbf{q}) - P^b\mathbf{q} \geq u^b(\mathbf{q}') - P^b\mathbf{q}'$$

So, at prices P^b the buyer *weakly* prefers bundle $\mathbf{q}$ over bundle $\mathbf{q}'$ and the inequality can be re-written as:

$$P^b \cdot (\mathbf{q} - \mathbf{q}') \leq u^b(\mathbf{q}) - u^b(\mathbf{q}') \tag{11}$$

Likewise, for sellers $s \in \mathcal{S}$ participating in the exchange and bundles $\mathbf{q}, \mathbf{q}' \in A^s$ at two sides of a facet, write:

$$P^s \cdot \mathbf{q} + u^s(\mathbf{q}) \geq P^s \cdot \mathbf{q}' + u^s(\mathbf{q}')$$

Equivalently:

[18] The reader is referred to A.1.1 for a definition of a polyhedral complex but, more generally, I suggest reading Maclagan and Sturmfels (2015) for an introduction to the topic of Tropical Geometry.

$$P^s \cdot (\mathbf{q}' - \mathbf{q}) \leq u^s(\mathbf{q}) - u^s(\mathbf{q}') \tag{12}$$

Note that $(\mathbf{q} - \mathbf{q}')$ in the left hand side of inequality (11) and $(\mathbf{q}' - \mathbf{q})$ in inequality (12) can be written as the product of the weight of a facet times a primitive integer vector. For example, $(\mathbf{q} - \mathbf{q}') = w_F \cdot \mathbf{v}$, where w_F is the weight of a facet and $\mathbf{v}$ is a primitive integer vector. Not only is this the essential information required to determine the prices at which a participant wishes to cross a facet but, when several inequalities like the ones just described are included, they encode the demand type $\mathcal{D}$ of a valuation.

By the same principle, given that the union of individual THs results in an aggregate TH, it is also possible to construct a cell describing the prices required for an aggregate bundle to be feasible. That is, for a set of buyers, and assuming a linear price $\mathbf{p}$, aggregate valuation U and bundles $\mathbf{q} = \sum_{b \in \mathcal{B}} D_{u^b}(\mathbf{p})$ and $\mathbf{q}' = \sum_{b \in \mathcal{B}} D_{u^b}(\mathbf{p})$ in the aggregate domain A, it is possible to write:

$$\mathbf{p} \cdot (\mathbf{q} - \mathbf{q}') \leq U(\mathbf{q}) - U(\mathbf{q}') \tag{13}$$

Similarly, for a set of sellers and bundles $\mathbf{q} = \sum_{s \in \mathcal{S}} S_{u^s}(\mathbf{p})$ and $\mathbf{q}' = \sum_{s \in \mathcal{S}} S_{u^s}(\mathbf{p})$ in A:

$$\mathbf{p} \cdot (\mathbf{q}' - \mathbf{q}) \leq U(\mathbf{q}) - U(\mathbf{q}') \tag{14}$$

More generally, suppose that there are M such inequalities - one for each facet - and let $\mathbf{V}$ be a $M \times N$ constraint matrix, where each row is composed of $(\mathbf{q} - \mathbf{q}')$ in the case of buyers and $(\mathbf{q}' - \mathbf{q})$ in the case of sellers. Define a M-dimensional column vector $\mathbf{u}$ containing the right hand sides of the inequalities just described, e.g. $U(\mathbf{q}') - U(\mathbf{q})$. Fix a bundle, say $\mathbf{q}_*$, and write the following linear program in decision variable P_*:

$$\begin{aligned} \max \quad & P^* \mathbf{q}_* \\ \text{s.t.} \quad & \mathbf{V} P_* \leq \mathbf{u} \end{aligned} \tag{15}$$

Which has the following dual:

$$\begin{aligned} \min \quad & \mathbf{u}^\intercal x \\ \text{s.t.} \quad & \mathbf{V}^\intercal x = \mathbf{q}_*, \; x \geq 0 \end{aligned} \tag{16}$$

Clearly, when goods are indivisible, $\mathbf{V}$ is an integral matrix and even if they are not, bundles are usually formed by integers. Therefore, the following lemma can be applied:

Lemma 4.7 *(Schrijver, 1998) Let $\mathbf{A}$ be an integral matrix. Then $\mathbf{A}^\intercal$ is unimodular if and only if both sides of the linear programming duality equation*

$$\max\{cx|\mathbf{A}x \leq b\} = \min\{yb|y \geq 0; y\mathbf{A} = c\}$$

are attained by integral vectors x and y, for all integral vectors b,c (where y, c are row vectors and x, b are column vectors.

As defined, $\mathbf{V}$ corresponds to facets of a TH in price space and $\mathbf{V}^\intercal$ corresponds to vertices of an Subdivided Newton Polytope cell defined in quantity space.[19] By linear programming duality, it is also possible to define the vertices of an SNP cell as in $\mathbf{V}^\intercal$ and obtain the facets of a TH. Furthermore, by focusing on concave valuations, it can be assured that no bundle is "hidden" between facets in the TH or in a SNP cell. Or, in geometric terms, by focusing on concave valuations, it is possible to ensure that any given bundle is a marked point of the SNP cell. All of this in accordance with the theory developed by Baldwin and Klemperer (2016). [20]

The result is summarized in the following:

Proposition 4.8 *A set of concave valuations $\{u^j\}$ that can be described in price space by a matrix $\mathbf{V}$ is unimodular if and only if the linear programming duality equation holds with integral vectors $P^*, x, \mathbf{u}, \mathbf{q}_*$:*

[19]An SNP is also a polyhedral complex, dual to a TH.

[20]The example in subsection A.4 of the Appendix shows duality between the TH and an SNP.

$$\max\left\{\mathbf{P}^{*}q_{*}|\mathbf{V}P_{*}\leq\mathbf{u}\right\}=\min\left\{\mathbf{u}^{\intercal}x|\mathbf{V}^{\intercal}x=\mathbf{q}_{*}\geq 0;x\geq 0\right\}$$

Proof A simple application of lemma 4.7 ■

The linear programs in (15) and (16) have a clear economic interpretation. For a given bundle of goods, the primal solves revenue maximization for a clearinghouse (or seller) and the dual solves the optimal allocation of bundles among participants. The rows of $\mathbf{V}$ can be interpreted as the "effectiveness" coefficient described by Milgrom (2009): a substitution rate among commodities in an exchange. In theorem 4, he claims that if the constraints that describe the substitution rate among commodities fail to have a "tree"structure, substitutability among goods fails. The unimodularity condition is clearly simpler to verify.

Furthermore, linear programming model (15) relates to existing results on dominant strategy incentive compatibility (Vohra, 2011). Although not interpreted here in this way - as I have not introduced a set of types - the model responds the important question of how to compensate different types of agents participating in a direct revelation mechanism, such that it is a weakly dominant strategy for each agent to reveal their type. The dual in (16) is precisely the problem of finding an all-pairs shortest path, the key to proving that a mechanism is incentive compatible as in Rochet (1987).[21]

4.2 The divisible good case

When goods are divisible, the requirements for the existence of equilibrium are considerably less restrictive than the indivisible goods case. A simple computation of LP6 will ensure an efficient allocation and supporting prices can be computed with DLP6. Furthermore, Milgrom and Strulovici (2009) have proven that whenever goods are weak substitutes (a condition equivalent to the OS condition), Walrasian equilibria exist, the Vickrey outcome is in the core and submodularity of dual profit functions is ensured.

[21] All of these are avenues for future research.

5 Efficiency, the core and Vickrey Clarke Groves payments in Product-Mix Exchanges

In the previous section I discussed the winner determination problem and the conditions under which pricing equilibria are guaranteed to exist in PMEs. As noted earlier, the goal of the clearinghouse is ensuring an efficient allocation: that buyers who value bundles the most trade with sellers who offer them at the lowest cost. Thus, gains from trade are maximized for all participants.

When goods are indivisible, by theorem (4.5), LP6 is guaranteed to have an integral solution. If goods are divisible, an integer solution is not required, and LP6 always determines the efficient aggregate bundle $\mathbf{q}_*$. Furthermore, regardless of the substitutability concept imposed on participants' preferences - SS or OS - LP6 determines the efficient aggregate bundle. In all variants of a PME, DLP6 allows calculating supporting prices for the allocation. The following proves that every pricing equilibrium in a PME is efficient:

Proposition 5.1 *All pricing equilibria* $(\mathbf{q}_*, P_*)$*, where* $\mathbf{q}_*$ *is obtained from LP6 and prices* P_* *are obtained from DLP6, are efficient for all variants of a PME.*

Note that Proposition 5.1 applies to non-anonymous, non-linear prices and to Walrasian prices, which are anonymous and linear.

To deepen the discussion, I associate a coalitional game with transferable payoff to the winner determination problem of PMEs and to this end I introduce additional notation. The set of players is the set $\mathcal{J}$ of market participants and the characteristic function of the game is V, which coincides with the gains from trading in a PME. In particular, let $V(\mathcal{J})$ be equal to the optimal value of the objective function of LP6 (see the model in (9)) when all market participants are included which, by LP duality, equals to the optimal objective function value of DLP6. That is, $V(\mathcal{J})$ are the maximum possible gains from trade in a PME:

$$
\begin{aligned}
V(\mathcal{J}) &= \max \sum_{b \in \mathcal{B}} \sum_{\mathbf{q} \in A^b} u^b(\mathbf{q}) y(\mathbf{q}, b) + \sum_{s \in \mathcal{S}} \sum_{\mathbf{q} \in A^s} u^s(\mathbf{q}) y(\mathbf{q}, s) \\
&= \min \sum_{b \in \mathcal{B}} \pi^b + \sum_{s \in \mathcal{S}} \pi^s
\end{aligned}
$$

Furthermore, let $\mathcal{C}$ be a coalition of market participants, such that $\mathcal{C} \subseteq \mathcal{J}$. Define $\mathcal{C}^B = \{\mathcal{B} \cap \mathcal{C}\}$ and $\mathcal{C}^S = \{\mathcal{S} \cap \mathcal{C}\}$ as, respectively, the set of buyers and sellers in the coalition $\mathcal{C}$.

It follows that $V(\mathcal{C})$ are the gains from trade in a PME, when only the members of a coalition trade among themselves:

$$
\begin{aligned}
V(\mathcal{C}) &= \max \sum_{b \in \mathcal{C}^B} \sum_{\mathbf{q} \in A^b} u^b(\mathbf{q}) y(\mathbf{q}, b) + \sum_{s \in \mathcal{C}^S} \sum_{\mathbf{q} \in A^s} u^s(\mathbf{q}) y(\mathbf{q}, s) \\
&= \min \sum_{b \in \mathcal{C}^B} \pi^b + \sum_{s \in \mathcal{C}^S} \pi^s
\end{aligned}
$$

The following are the assumptions of the coalitional game:

1. If no buyers or no sellers participate, i.e. if $\mathcal{C}^B = \emptyset$ or $\mathcal{C}^S = \emptyset$, the coalition produces no value, i.e. $V(\mathcal{C}) = 0$

2. If $\mathcal{C}_1, \mathcal{C}_2$ are any two coalitions such that $\mathcal{C}_1 \subset \mathcal{C}_2$, then $V(\mathcal{C}_1) \leq V(\mathcal{C}_2)$

3. V is superadditive, i.e., if $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$ then $V(\mathcal{C}_1) + V(\mathcal{C}_2) \leq V(\mathcal{C}_1 \cup \mathcal{C}_2)$

Let π^b, π^s be the payoffs (i.e. imputations) to buyers and sellers, respectively. A set of payoffs $\left\{\pi^b, \pi^s\right\}$ is in the core of the game, denoted as $core(\mathcal{J}, V)$, if:

$$
\sum_{b \in \mathcal{C}^B} \pi^b + \sum_{s \in \mathcal{C}^S} \pi^s \geq V(\mathcal{C}), \quad \forall \mathcal{C} \subseteq \mathcal{J} \tag{17a}
$$

$$
\sum_{b \in \mathcal{B}} \pi^b + \sum_{s \in \mathcal{S}} \pi^s = V(\mathcal{J}) \tag{17b}
$$

The first requirement states that every coalition of market participants should obtain at least as much as $V(\mathcal{C})$. That is, a set of payoffs $\left\{\pi^b, \pi^s\right\}$ is in the core if no coalition of participants in a PME can deviate to obtain a better outcome for all its members.

Proposition 5.2 *All pricing equilibria* $(\mathbf{q}_*, P_*)$ *of PMEs, where* $\mathbf{q}_*$ *is obtained from LP6 and prices* P_* *are obtained from DLP6, lead to payoffs that are in the core, i.e.* $\left\{\pi^b, \pi^s\right\} \in core(\mathcal{J}, V)$.

It is convenient at this stage to introduce the *marginal product* of each market participant $j \in \mathcal{J}$, which is its contribution to $V(\mathcal{J})$. To this end, note that the dual variables corresponding to the first (buyers') and second (sellers') constraint in LP7, namely π^b, π^s have, respectively, $V(\mathcal{J}) - V(\mathcal{J}\backslash b)$ and $V(\mathcal{J}) - V(\mathcal{J}\backslash s)$ as upper bounds. Thus, the following:

Proposition 5.3 *Let the pair* $\left\{\pi^b_*, \pi^s_*\right\}$ *be part of the optimal solution to DLP6. Then the buyers' payoffs are* $\pi^b_* \leq V(\mathcal{J}) - V(\mathcal{J}\backslash b)$ *while the sellers' are* $\pi^s_* \leq V(\mathcal{J}) - V(\mathcal{J}\backslash s)$

It is also possible to derive the expressions for the participants' marginal product from the definition of the core, (17a - 17b). For buyers, consider a coalition that excludes a particular buyer, say b', such that $\mathcal{C} = \{\mathcal{J}\backslash b'\}$:

$$\sum_{b\,\in(\mathcal{C}^B\backslash b')} \pi^b + \sum_{s\in\mathcal{C}^S} \pi^s \geq V(\mathcal{J}\backslash b')$$
$$\sum_{b\in\mathcal{B}} \pi^b + \sum_{s\in\mathcal{S}} \pi^s = V(\mathcal{J})$$

Negating the first expression and adding it to the second yields:

$$\left(\sum_{b\in\mathcal{B}} \pi^b - \sum_{b\in(\mathcal{C}^B\backslash b')} \pi^b\right) + \left(\sum_{s\in\mathcal{S}} \pi^s - \sum_{s\in\mathcal{C}^S} \pi^s\right) \leq V(\mathcal{J}) - V(\mathcal{J}\backslash b')$$

Similarly, for sellers define a coalition that excludes a seller s': $\mathcal{C} = \{\mathcal{J}\backslash s'\}$, such that:

$$\sum_{b\,\in\mathcal{C}^B} \pi^b + \sum_{s\,\in(\mathcal{C}^S\backslash s')} \pi^s \geq V(\mathcal{J}\backslash s')$$
$$\sum_{b\in\mathcal{B}} \pi^b + \sum_{s\in\mathcal{S}} \pi^s = V(\mathcal{J})$$

As before, negating the first expression and adding it to the second results in:

$$\left(\sum_{b\in\mathcal{B}} \pi^b - \sum_{b\in\mathcal{C}^B} \pi^b\right) + \left(\sum_{s\in\mathcal{S}} \pi^s - \sum_{s\in(\mathcal{C}^S\backslash s')} \pi^s\right) \leq V(\mathcal{J}) - V(\mathcal{J}\backslash s')$$

In summary:

Definition 5.4 *The marginal product for buyer b is:*

$$V(\mathcal{J}) - V(\mathcal{J}\backslash b)$$

The marginal product for seller s is:

$$V(\mathcal{J}) - V(\mathcal{J}\backslash s)$$

Note that $V(\mathcal{J}\backslash b), V(\mathcal{J}\backslash s)$ requires the solution of linear programs that, respectively, exclude a buyer b and a seller s. Therefore, to obtain the marginal values of all participants $\mathcal{J}$ of a PME, the clearinghouse must solve $(B+S)+1$ optimization problems, where $B+S$ is the total number of participants in a PME (Bikhchandani et al., 2002).

However, a key question relates to the conditions under which all market participants *simultaneously* obtain payoffs equal to their marginal product. Specifically, I am interested in defining the conditions under which participants of a PME are able to trade with Vickrey Clarke Groves (VCG) payments as supporting prices of the efficient allocation (Vickrey (1961), Clarke (1971), Groves (1973)).

The VCG payment P^b_{VCG} that a buyer b pays for an efficient allocation $\mathbf{q}^b_*$ is the one that ensures that the buyer obtains its marginal product:

$$\pi^b_* = u^b(\mathbf{q}^b_*) - P^b_{VCG} = V(\mathcal{J}) - V(\mathcal{J}\backslash b)$$

where $P^b_*\mathbf{q}^b_*$ is substituted for P^b_{VCG} in the corresponding (buyers') constraint of DLP6. Noting that $V(\mathcal{J}) = u^b(\mathbf{q}^b_*) + \sum_{j\in(\mathcal{J}\backslash b)} u^j(\mathbf{q}^j_*)$ leads to the following expression:

$$P^b_{VCG} = V(\mathcal{J}\backslash b) - \sum_{j\in(\mathcal{J}\backslash b)} u^j(\mathbf{q}^j_*) \tag{18}$$

The VCG payment P^s_{VCG} that is payed to a seller s for an efficient allocation $\mathbf{q}^b_*$ is derived in a symmetric way:

$$\pi^s_* = P^s_{VCG} + u^s(\mathbf{q}^s_*) = V(\mathcal{J}) - V(\mathcal{J}\backslash s)$$

where $P_*^s \mathbf{q}_*^s$ is substituted for P_{VCG}^s in the corresponding (i.e. sellers') constraint of DLP6. Because $V(\mathcal{J}) = u^s(\mathbf{q}_*^s) + \sum_{j \in (\mathcal{J} \backslash s)} u^j(\mathbf{q}_*^j)$, it follows that:

$$P_{VCG}^s = \sum_{j \in (\mathcal{J} \backslash s)} u^j(\mathbf{q}_*^j) - V(\mathcal{J} \backslash s) \tag{19}$$

Note that VCG payments are always non-anonymous and non-linear, which amounts to introducing price discrimination among participants of a PME. Furthermore, the payments in (18) and (19) do not depend on the seller's or buyer's own valuation, implying that these cannot be manipulated by a single participant. It is well known that the VCG mechanism produces an outcome that is individually rational, as every participant obtains its marginal product as payoff, and truthful bidding is a weakly dominant strategy. The same holds for PMEs with VCG payments:

Proposition 5.5 *Truthful bidding is a weakly dominant strategy in PMEs with VCG payments.*

A well-known condition for the existence of VCG payments in the core of the coalitional game associated to the auction dates back to Shapley (1962) and has been further investigated byBikhchandani and Ostroy (2002, 2006), Bikhchandani et al. (2002) and Vohra (2011), among others. Usually termed as the "Agents are Substitutes Condition" or "Bidders are Substitutes Condition", it has typically been stated for single seller environments. The condition conveys the intuition that a coalition of agents can benefit more by cooperating than by acting individually. For example, a union of workers can obtain better terms via collective bargaining than by negotiating individually with a firm. In accordance with this condition, I define the Participants Are Substitutes condition (PSC), which applies to the multiple-buyer and multiple-seller environment of PMEs:

Definition 5.6 *The PSC holds if for any coalition of participants* $\mathcal{C} \subseteq \mathcal{J}$,

$$V(\mathcal{J}) - V(\mathcal{J} \backslash \mathcal{C}) \geq \sum_{j \in \mathcal{C}} [V(\mathcal{J}) - V(\mathcal{J} \backslash j)]$$

That is, the marginal product of the coalition $\mathcal{C}$ is at least as large as the sum of the marginal products of all of its members. Note that this applies equally to buyers and sellers.

Proposition 5.7 *In a PME with VCG payments: if the PSC holds, then there is a point in the core of the associated coalitional game such that all participants simultaneously obtain their marginal products. That is, a pair of payoffs* $\left\{\pi_*^b, \pi_*^s\right\} \in core(\mathcal{J}, V)$ *such that* $\pi_*^b = V(\mathcal{J}) - V(\mathcal{J}\backslash b)$ *for all* $b \in \mathcal{B}$ *and* $\pi_*^s = V(\mathcal{J}) - V(\mathcal{J}\backslash s)$ *for all* $s \in \mathcal{S}$.

The result is relevant because the PSC is implied by the GS condition, and a number of proofs exist in the literature:

Lemma 5.8 *(Ausubel and Milgrom (2002), Bikhchandani et al. (2002), Vohra (2011)) If the value function of each participant satisfies the GS condition, then PSC holds*

This implies that it is always possible to implement PMEs with VCG payments. However possible, this comes at a possibly non-negligible computational cost.

6 An application of Product-Mix Exchanges: Delta Energy Markets

This section contains an example application of PMEs. Specifically, it shows one of the possible ways in which the capabilities of PMEs can be utilized to create a *potential* marketplace for flexibility services based on the FlexOffer systems information concept.[22]

The following is a simplified description in “Q&A” (Question & Answer) format of a Delta Energy marketplace (see figure 17):

1. *What is the traded product?* The traded product is Delta Energy, which is the available *energy* contained in a Flex-Offer, relative to a baseline assignment, determined by its

[22] The problem of designing power system flexibility-enabling products based on this concept is discussed by Boscán (2016).

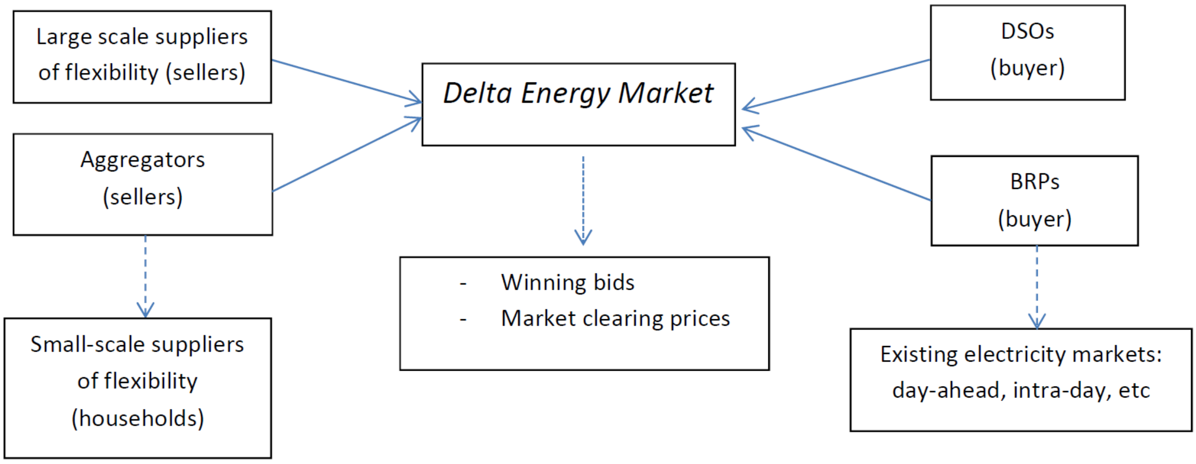

Figure 17: A Delta Energy marketplace. Source: TotalFlex internal documents

issuer. In the simplest possible terms, a Flex-Offer is a load profile[23] associated to the operation of a device in the power system (e.g. electric vehicles, heat pumps), which describes the time and quantity flexibility with which it can operate. Figure 18 shows an instance of a Flex-Offer, which contains quantity flexibility only but no flexibility in the time dimension. The yellow line indicates a baseline assignment for this Flex-Offer and the red and blue portions of the drawing, which respectively correspond to positive and negative flexibility, are the Delta Energy contained in the Flex-Offer. More generally, and for the purpose of this section, it suffices to say that the traded product is *the deviation in energy consumption relative to a reference plan* of operation of a device in the power system. To achieve a sufficiently large scale, the Delta Energy contained in several Flex-Offers are aggregated and are then offered in the marketplace. Furthermore, because the product is measured in units of energy (say kWh), it clearly is indivisible in nature and I assume this is the case.[24]

2. *Who buys the product?* Buyers of flexibility in a marketplace can be Distribution System Operators (DSOs), Transmission System Operators (TSOs) who are final users of

[23] A load profile is a graph that depicts load in the abscissa (measured in kW, for example) versus time in the ordinate

[24] For technical reasons, it may be necessary to restrict this assumption.

flexibility or Balancing Responsible Parties (BRPs), who trade for portfolio optimization purposes [25]

3. *Who sells the product?* Large-scale suppliers of flexibility (such as industrial consumers) may access the market directly, together with aggregators who act as intermediaries between small-scale suppliers and the marketplace.

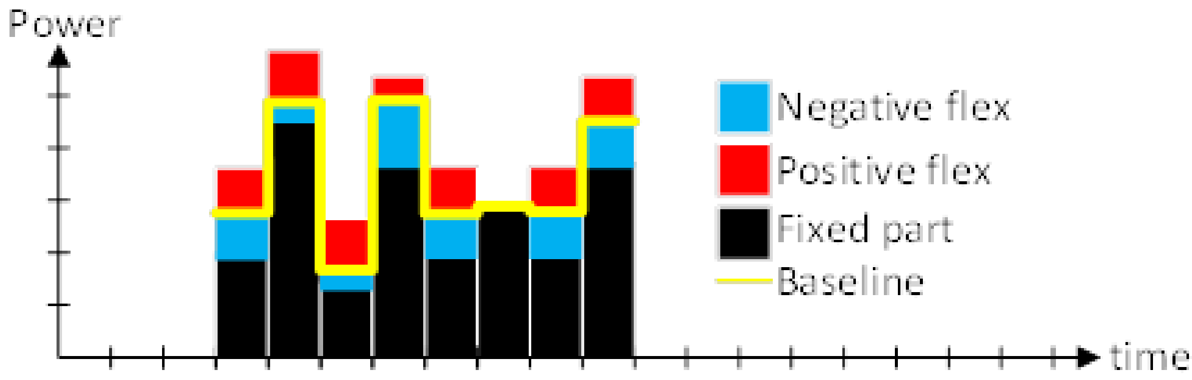

Figure 18: A Flex-Offer with quantity flexibility only. Source: TotalFlex internal documents

In the exposition, I focus on two specific products, namely *quantity flexibility* and *time shifting*. To this end, let the set of commodities in the PME be $\mathcal{N} = \{\text{“Now”}, \text{“Later”}\}$, i.e. there are two periods and these are the commodities of the PME. That is, the set $\mathcal{J}$ of market participants buy and sell units of Delta Energy and place bids for “Now” and for “Later”[26]. Note that this simplified, two-period framework can be easily extended to several periods. Furthermore, in what follows, I assume that the set $\mathcal{S}$ has $S = 10$ sellers and that the set $\mathcal{B}$ has $B = 10$ buyers.

In addition, suppose that each market participant has a domain composed of only three bundles, namely $\mathbf{0}, \mathbf{q}$ and $\mathbf{q}'$:

$$A^j = \{\mathbf{0}, \mathbf{q}, \mathbf{q}'\}, \quad \text{for all } j \in \mathcal{J}$$

[25]For further details on who buys, who sells power system flexibility and their motives, the reader is referred to Boscán (2016) Boscán and Poudineh (2016).

[26]Using the Flex-Offer terminology, I assume here that Flex-Offers like the one shown in figure 18, which contain quantity flexibility only and no time flexibility, are mapped into the bids of this example.

where $\mathbf{0} = (0, 0)$, $\mathbf{q} = (Now, 0)$, $\mathbf{q}' = (0, Later)$ and that the domain is binary, meaning that there is a quantity of energy K such that $Now = Later = K$. In other words, the clearinghouse imposes SS preferences and bidders are restricted to buying or selling an amount of energy K. Thus, the clearinghouse will optimally decide if a participant obtains a bid containing "Now" or containing "Later". This follows from the fact that in PMEs at most one bundle can be allocated to each market participant.

6.1 Quantity flexibility

How can the described framework accommodate offers to increase consumption and offers to decrease consumption relative to a specified baseline assignment? In other words: how can the described framework accommodate quantity flexibility, given that the set of commodities in the PME refers to the time dimension?

One simple and logical approach, which I follow here, is to map the "positive flex" of figure 18, which is an *increase* in consumption relative to the baseline, to an offer to *buy* Delta Energy. Symmetrically, the "negative flex" of 18, which is a *decrease* in consumption relative to the baseline, can be mapped to an offer to *sell* energy.[27]

Because each participant can restrict its bid to only one of the available commodities, i.e. to one of the periods, buyers and sellers of Delta Energy can offer quantity flexibility for one period only. For buyers, this is achieved by specifying a valuation equal to zero for the bundle that it does not. So, for example, a buyer willing to increase consumption relative to the baseline "Now" will submit a positive valuation for bundle $\mathbf{q}$, i.e. $u^b(\mathbf{q}) \neq 0$ and a valuation of zero for bundle $\mathbf{q}'$, i.e. $u^b(\mathbf{q}') = 0$.

For sellers, a converse approach can be applied. Suppose that a seller wishes to offer Delta Energy in only one of the periods: this can be done by specifying an arbitrarily high valuation for the bundle it does not want. For example, if the seller wishes to offer Delta

[27]Increasing energy consumption relative to a baseline is equivalent to increasing the quantity demanded, thus it amounts to *buying* Delta Energy. Similarly, decreasing consumption relative to a baseline is equivalent to reducing the quantity demanded or increasing production. Therefore, it amounts to *selling* Delta Energy.

Energy "Now", it will submit a negative valuation for bundle $\mathbf{q}$, i.e. $u^b(\mathbf{q}) \leq 0$ and an arbitrarily low valuation for bundle $\mathbf{q}'$, e.g. $u^b(\mathbf{q}') = -1000$.[28]

Table 2 contains all demand-side bids. The first column indexes the bid by b, the second is the valuation for bundle $\mathbf{q}$, which contains an amount K of Delta Energy "Now" and 0 Delta Energy "Later"; the third column contains bids for bundle $\mathbf{q}'$, containing an amount of 0 Delta Energy "Now"and K units of Delta Energy "Later"; the fourth column contains the quantity K, which is to be placed either "Now" or "Later". In this example, there are 624 units of Delta Energy being sold.

b	$u^b(\mathbf{q})$	$u^b(\mathbf{q}')$	K
1	36	0	56
2	37	29	78
3	0	38	49
4	36	31	60
5	19	32	34
6	32	0	85
7	33	28	85
8	0	36	53
9	0	34	80
10	28	32	44
		Total	*624*

Table 2: Demand-side bids in the Delta Energy Market example (quantity flexibility)

For example, the first bid ($b = 1$) shows no interest for "Later" and is an offer to buy 56 units of Delta Energy "Now". Symmetrically, the eight bid ($b = 8$) shows no interest for "Now" and is an offer to buy 53 units of energy "Later". In contrast, the fourth bid ($b = 4$) shows interest for either 60 units of Delta Energy "Now" or "Later".

In a similar way to table 2, table 3 contains all supply-side bids. Note that bid $s = 2$ contains an arbitrarily high cost (i.e. low valuation) for Delta Energy "Now", meaning that the seller has interest for Delta Energy "Later" only. Bid $s = 6$ has an arbitrarily high cost for Delta Energy "Later", effectively showing interest for "Now" only. In contrast to both

[28]Recall from subsection 2.2 that sellers submit *negative* valuations, i.e., they report their cost to the clearinghouse. A *very low* valuation is equivalent to a *very high* cost in this model.

$s = 2$ and $s = 6$, $s = 8$ presents an actual trade-off to sell between both periods.

s	$u^s(\mathbf{q})$	$u^s(\mathbf{q}')$	K
1	-19	-1900	40
2	-2700	-27	87
3	-28	-24	85
4	-23	-29	70
5	-2400	-24	35
6	-29	-2900	46
7	-30	-20	65
8	-26	-22	75
9	-22	-2200	86
10	-25	-32	35
		Total	*624*

Table 3: Supply-side bids in the Delta Energy Market example (quantity flexibility)

The solution to the clearinghouse's allocation problem is shown in tables 4 and 5, which were calculated with LP6. Market-clearing prices, calculated with DLP6, are $P_* = (29, 32)$.

b	$\mathbf{q}_*^b$	$\mathbf{q'}_*^b$	K
1	1	0	56
2	1	0	78
3	0	1	49
4	1	0	60
5	0	1	34
6	1	0	85
7	1	0	85
8	0	1	53
9	0	1	80
10	0	1	44
		Total	*624*

Table 4: Efficient allocations (demand-side) in the Delta Energy Market example (quantity flexibility)

s	$\mathbf{q}_*^s$	$\mathbf{q'}_*^s$	K
1	1	0	40
2	0	1	87
3	0.55	0.45	85
4	1	0	70
5	0	1	35
6	1	0	46
7	0	1	65
8	0.53	0.47	75
9	1	0	86
10	1	0	35
		Total	*624*

Table 5: Efficient allocations (supply-side) in the Delta Energy Market example (quantity flexibility)

6.2 Time shifting

Under the same assumptions as before, in which two commodities ("Now" and "Later") are traded in a PME, it is also possible to specify another flexibility-enabling product, which originates from Flex-Offers, namely time-shifting.

With respect to the quantity flexibility example just discussed in subsection 6.1, the main difference is that a participant buying or selling time shifting needs to secure that he will obtain an *opposite position* in the other commodity, i.e. in the other period. This requires "swap bidding", one of the extensions mentioned by Klemperer (2010) in his initial description of PMAs, which is now achieved under PMEs.

So, for example, a buyer of time shifting willing to buy energy "Now" will want to make sure he is able to sell "Later". Similarly, a seller willing to sell energy "Later" needs to secure that he is buying "Now". Furthermore, the only rational thing to do for a buyer or seller is to buy at low prices and sell at high prices. In consequence, bidders require a *price difference* and not a *specific price* for the time shifting product. Unlike before, this is a case of *complementarity* between commodities (i.e. complementarity between time periods), which can be expressed via substitutable preferences, a feature discussed by Milgrom (2009)

[29].

How to use the bidding language built into PMEs to deal with these kind of bids? The numerical example that follows attempts to give an answer:

1. There are four demand-side bids in the market. Two of these ($b = 1$ and $b = 2$) are to buy Delta Energy (see table 6). On the supply side (see table 7), there are also four bids, where bids $s = 1$ and $s = 2$ are to sell Delta Energy. These bids for Delta Energy are in the exact same way as described in subsection 6.1.

2. Consider the case of a market participant willing to time shift by buying "Now" and selling "Later" for a price difference of 5.

3. Bid $b = 3$ is the *demand side* of a time shifting bid: an offer to buy 70 units of Delta Energy at an arbitrarily high price, such that the price of "Later" is 5 units higher than what it is "Now". This bid will make sure that he buys "Now" if the price difference is of at least 5. He will buy "Later" otherwise.

4. Note that this time shifting bid would be incomplete without the *supply side* of the time shifting: the bidder that placed $b = 3$ needs to secure that he is able to sell "Later". Therefore, the clearinghouse must assume that he has already won (i.e. that he is endowed) the right to sell 70 units "Later". This corresponds to bid $s = 3$ on the supply side of the market (see table 7). Note that this bid contains a valuation of zero for bundle $\mathbf{q}'$, i.e. the bundle that contains "Later", and an arbitrarily high cost (i.e. low valuation) for bundle $\mathbf{q}$, i.e. the bundle that contains "Now". In this way, "Later" is always a winning bid and "Now" is always a losing bid.

5. By placing these bids, the time shifter makes sure that he ends up with nothing if the

[29] Milgrom (2009) states that "The ability to report swap bids makes the integer assignment exchange applicable to some resource allocation problems involving *complementary* goods for which package exchange mechanisms might have been thought necessary. This is, perhaps, surprising given that assignment messages can directly only express substitutable preferences." Recall from the discussion in section 2 that Milgrom's assignment exchanges are equivalent to PMEs.

price difference is less than 5 and with the desired position (buy “Now”, sell “Later”) if the price difference is of at least 5.

6. To complete the example, there is another market participant willing to shift by selling “Later” and buying “Now”. He places a positive bid on the supply side of the market (bid $s = 4$, table 7), i.e. instead of getting paid, the seller offers to pay. To secure the opposite position he places an arbitrarily high bid on the demand side of the market (bid $b = 4$, table 6), which ensures his right to buy “Now”.

7. Note that the bidder requires a price difference of at least 4 to time shift. And if he doesn't secure this price difference, his position cancels out.

8. According to DLP6, the PME clears at $P_* = (23, 29)$.

9. As a result, both time shifters in this example are able to secure their preferred positions as can be verified in tables 7 and 8, which contain. Note that if the time shifters' preferred price difference had not been as they wished, their positions would have cancelled out completely.

b	$u^b(\mathbf{q})$	$u^b(\mathbf{q}')$	K
1	26	34	80
2	22	29	60
3 (Time shifting)	120	125	70
4 (Endowment for bid $s = 4$)	1000	0	70
		Total	*280*

Table 6: Demand-side bids in the Delta Energy Market example (time shifting)

s	$u^s(\mathbf{q})$	$u^s(\mathbf{q}')$	K
1	-19	-25	80
2	-17	-24	60
3 (Endowment for bid $b = 3$)	-1000	0	70
4 (Time shifting)	8	4	60
		Total	*280*

Table 7: Demand-side bids in the Delta Energy Market example (time shifting)

b	$\mathbf{q}^b_*$	$\mathbf{q'}^b_*$	K
1	0	1	80
2	0	1	60
3	1	0	70
4	1	0	70
		Total	*280*

Table 8: Efficient allocations (demand-side) in the Delta Energy Market example (time shifting)

s	$\mathbf{q}^s_*$	$\mathbf{q'}^s_*$	K
1	1	0	80
2	1	0	60
3	0	1	70
4	0	1	70
		Total	*280*

Table 9: Efficient allocations (supply-side) in the Delta Energy Market example (time shifting)

7 Conclusions

This paper has developed extensions to PMAs, which were initially proposed by Klemperer (2010) in the context of the financial crisis as a solution to the BoE's problem of supplying liquidity to troubled banks in the UK. The extensions are generalized under the encompassing term of "Product-Mix Exchanges" (PMEs) which share similarities with Milgrom (2009)'s model of assignment messages and exchanges, and the original formulation proposed by Klemperer. Specifically, Klemperer (2010)'s formulation is a special case of the PME studied in this paper.

However, Product-Mix Exchanges have an important difference relative to both PMAs and Milgrom (2009)'s Assignment Messages and Exchanges. Specifically, PMEs are *multi-unit combinatorial auctions in which market participants report substitutable preferences over bundles of goods, without having a fixed role.* By formulating PMEs in this way, a wealth of existing results in the combinatorial auction literature can be readily applied.

For example, the winner determination problem of PMEs, discussed in section 4, has a remarkably simple structure, which allows estimating the efficient allocations under all different variants of a PME, which result from the interaction of the substitutability concept imposed on PMEs and the divisibility of the traded goods. When goods are divisible or indivisible, strong substitutes or ordinary substitutes preferences are imposed on market participant, a single linear program and its dual are enough to determine the efficient allocation and the supporting prices. The main contribution of section 4, however, is not the linear programming itself or the analysis of the conditions under which equilibrium with indivisibility does not fail. Instead, the main contribution of that section is a simple, linear programming approach that verifies if a set of concave valuations has or not an equilibrium with indivisibility.

The incentive issues of PMEs, with a particular emphasis on efficiency, are the topic of section 5. The key result of that section is that, due to the fact that PMEs rely on substitutable preferences, the Vickrey Clarke Groves(VCG) outcome is always in the core of the associated coalitional game with transferrable utility considered in the section. Thus, VCG payments - which ensure that each market participant obtains its marginal product as payoff - can be straightforwardly computed with the aid of the linear programs presented in section 4. Of course, the computation of VCG payments to support the allocation comes at a computational cost, which may be non-negligible: $(B+S)+1$ linear programs must be computed (where $B+S$ is the number of market participants in the PME).

However, being able to support the PME allocation with VCG payments is an important result that facilitates the applicability of PMEs in contexts where efficiency is the main goal of the clearinghouse. Not only is truthful bidding a weakly dominant strategy if VCG payments are applied, but the possibility of avoiding manipulation of results when there are market thickness concerns is important.

Finally, a concrete application of PMEs is shown in section 6, where the Delta Energy market design is described. In this section, I show how relevant flexibility-enabling products, such as quantity flexibility and time shifting can be accommodated into this framework. One limitation of the approach presented in that section is, however, that the role of VCG payments was not explored.

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A Appendix

A.1 Further details about results of section 3

A.1.1 Some notions of polyhedral geometry

The following concepts appear in Maclagan and Sturmfels (2015):

1. A *polyhedron* $\mathcal{P} \subset \mathbb{R}^N$ is the intersection of finitely many closed half spaces:

$$\mathcal{P} = \left\{ \mathbf{x} \in \mathbb{R}^N | A\mathbf{x} \leq \mathbf{b} \right\}$$

2. A *polyhedral complex* is a collection Σ of polyhedra satisfying two conditions:

   i. If the polyhedron $\mathcal{P}$ is in Σ, then so is any face of $\mathcal{P}$. A "face" is what Baldwin and Klemperer (2012, 2016) refer to as facets in the main text.

   ii. If polyhedra $\mathcal{P}$ and $\mathcal{Q}$ lie in Σ, then their intersection $\mathcal{P} \cap \mathcal{Q}$ is either empty or a *face* of both $\mathcal{P}$ and $\mathcal{Q}$

3. The polyhedra in polyhedral complex Σ are called the *cells* of Σ.

4. The *support* of a polyhedral complex is the set $\left\{ \mathbf{x} \in \mathbb{R}^N | \mathbf{x} \in \mathcal{P}, \mathcal{P} \in \Sigma \right\}$

A.1.2 Proofs

Proof of Proposition 3.10

1. To see the first claim, define the buyer's aggregate utility at bundle $\mathbf{x} \in \sum\limits_{b \in \mathcal{B}} A^b$ as:

$$U^{\mathcal{B}}(\mathbf{x}) = \max \left\{ \sum_{b \in \mathcal{B}} u^b(\mathbf{q}^b) | \mathbf{q}^b \in A^b, \mathbf{x} = \sum_{b \in \mathcal{B}} \mathbf{q}^b \right\}$$

   The sum of individual demands is:

$$\sum_{b\in\mathcal{B}} D_{u^b}(P^b) = \sum_{b\in\mathcal{B}} \arg\max_{\mathbf{q}\in A^b} \left\{u^b(\mathbf{q}^b) - P^b\mathbf{q}\right\}$$
$$= \arg\max_{\mathbf{q}\in A^b} \left\{\sum_{b\in\mathcal{B}} u^b(\mathbf{q}^b) - \sum_{b\in\mathcal{B}} P^b\mathbf{q}^b\right\}$$

The buyers'aggregate demand is:

$$D_{U^{\mathcal{B}}}(P) = \arg\max_{\mathbf{x}\in\sum_{b\in\mathcal{B}} A^b} \left\{U^{\mathcal{B}}(\mathbf{x}) - P\cdot\mathbf{x}\right\}$$
$$= \arg\max_{\mathbf{x}\in\sum_{b\in\mathcal{B}} A^b} \left\{\max\left\{\sum_{b\in\mathcal{B}} u^b(\mathbf{q}^b)|\mathbf{q}^b\in A^b, \mathbf{x} = \sum_{b\in\mathcal{B}}\mathbf{q}^b\right\} - P\cdot\mathbf{x}\right\}$$
$$= \arg\max_{\mathbf{q}^b\in A^b} \left\{\sum_{b\in\mathcal{B}} u^b(\mathbf{q}^b) - \sum_{b\in\mathcal{B}} P^b\mathbf{q}^b\right\}$$

2. Likewise, to see the second claim define seller's aggregate utility at bundle $\mathbf{x}\in\sum_{s\in\mathcal{S}} A^s$ as:

$$U^{\mathcal{S}}(\mathbf{x}) = \max\left\{\sum_{s\in\mathcal{S}} u^s(\mathbf{q}^s)|\mathbf{q}^s\in A^s, \mathbf{x} = \sum_{s\in\mathcal{S}}\mathbf{q}^s\right\}$$

The sum of individual supplies is:

$$\sum_{s\in\mathcal{S}} S_{u^s}(P^s) = \sum_{s\in\mathcal{S}} \arg\max_{\mathbf{q}\in A^s} \left\{P^s\mathbf{q} - u^b(\mathbf{q}^b)\right\}$$
$$= \arg\max_{\mathbf{q}\in A^s} \left\{\sum_{s\in\mathcal{S}} P^s\mathbf{q}^s - \sum_{s\in\mathcal{S}} u^s(\mathbf{q}^s)\right\}$$

The sellers' aggregate supply is:

$$S_{U^{\mathcal{S}}}(P) = \arg\max_{\mathbf{x}\in\sum_{s\in\mathcal{S}} A^s} \left\{P\cdot\mathbf{x} - U^{\mathcal{S}}(\mathbf{x})\right\}$$
$$= \arg\max_{\mathbf{x}\in\sum_{s\in\mathcal{S}} A^s} \left\{P\cdot\mathbf{x} - \max\left\{\sum_{b\in\mathcal{B}} u^b(\mathbf{q}^b)|\mathbf{q}^s\in A^s, \mathbf{x} = \sum_{s\in\mathcal{S}}\mathbf{q}^s\right\}\right\}$$
$$= \arg\max_{\mathbf{q}^s\in A^s} \left\{\sum_{s\in\mathcal{S}} P^s\mathbf{q}^s - \sum_{s\in\mathcal{S}} u^s(\mathbf{q}^s)\right\}$$

■

A.2 Proofs of section 5

Proof of Proposition 5.1

First, consider the case when the commodities are indivisible. By theorem (4.5), if the demand type $\mathcal{D}$ of a concave set of valuations is unimodular, LP6 is guaranteed to have an integral solution and DLP6 can be obtained to determine supporting prices. Thus, whenever each market participant has SS valuations, there will always be an integral allocation. But if valuations are OS, unimodularity *may* fail (see Theorem 3.5.11 in Baldwin and Klemperer (2012)) but if does not, an integral solution is guaranteed to exist.

Let $\{y(\mathbf{q}, b), y(\mathbf{q}, s) | \forall b \in \mathcal{B}, \forall s \in \mathcal{S}\}$ be an optimal solution to LP6, i.e. a set of zeroes and ones that determine the efficient allocations to buyers and sellers, $\mathbf{q}_*^b, \mathbf{q}_*^s$. By LP duality, there are π_*^b, π_*^s and prices P_* such that the objective function of DLP6 is minimized.

Because of complementary slackness, the following must hold:

$$(u^b(\mathbf{q}) - \pi^b - P_*^b \mathbf{q}) y(\mathbf{q}, b) = 0, \quad \text{for buyers}$$

$$(u^s(\mathbf{q}) - \pi^s + P_*^s \mathbf{q}) y(\mathbf{q}, s) = 0, \quad \text{for sellers}$$

If $y(\mathbf{q}, b) = 0, y(\mathbf{q}, s) = 0$,the corresponding bundles are not efficient allocations and the slack in the dual is always negative.

In contrast, if $y(\mathbf{q}, b) = 1$ for $b \in \mathcal{B}$, the clearinghouse sells the bundle to the buyer b and it is part of the efficient allocation. Then:

$$\pi^b = (u^b(\mathbf{q}_*^b) - P_*^b \mathbf{q}_*^b)$$

Similarly, if $y(\mathbf{q}, s) = 1$ for $s \in \mathcal{S}$, the clearinghouse buys the bundle from the seller s and it is part of the efficient allocation. Thus:

$$\pi^s = (P_*^s \mathbf{q}_*^s + u^s(\mathbf{q}_*^s))$$

But note that because of the formulation of LP6, in which only one bundle will be allocated to each participant:

$\pi^b = \max\limits_{\mathbf{q} \in A^b} \left\{ (u^b(\mathbf{q}_*^b) - P_*^b \mathbf{q}_*^b) \right\}$, which corresponds to the buyer's value (indirect utility) function

$\pi^s = \max\limits_{\mathbf{q}\in A^s}\{(u^s(\mathbf{q}_*^s) + P_*^s\mathbf{q}_*^s)\}$, which corresponds to the seller's value (indirect profit) function

Thus, suppose that the clearinghouse was able to ask each participant to mention their preferred bundle at prices $\{P_*^b, P_*^s\}$. The bundles would clearly be the ones indicated by LP6.

The same line of reasoning applies to the divisible goods case. The only difference is that the optimal solution to LP6 need not be integral. ■

Proof of Proposition 5.2 To prove the claim, in the same way as in the proof of Proposition 5.1, consider first the indivisible goods case. Thus, let $\{y(\mathbf{q}, b), y(\mathbf{q}, s)|\forall b \in \mathcal{B}, \forall s \in \mathcal{S}\}$ be an optimal solution to LP6, i.e., a set of zeroes and ones that determine the efficient allocations to buyers and sellers, $\mathbf{q}_*^b, \mathbf{q}_*^s$. By duality, there are π_*^b, π_*^s and prices P_* such that the objective function of DLP6 is minimized.

In addition, from the definition of the core ((17a) in the main text) write:

$$\sum_{b\,\in\mathcal{C}^B}\pi^b + \sum_{s\,\in\mathcal{C}^S}\pi^s = \sum_{b\in\mathcal{C}^B}\left\{u^b(\mathbf{q}_*^b) - P_*^b\mathbf{q}_*^b\right\} + \sum_{s\in\mathcal{C}^S}\{P_*^s\mathbf{q}_*^s + u^s(\mathbf{q}_*^s)\} \geq V(\mathcal{C})$$

where the inequality follows from the fact that $(P_*^b, P_*^s, \mathbf{q}_*^b, \mathbf{q}_*^s)$ are feasible solutions of DLP6 when restricted to coalitions $\mathcal{C} \subseteq \mathcal{J}$.

Likewise, from (17b):

$$\sum_{b\,\in\mathcal{B}}\pi^b + \sum_{s\,\in\mathcal{S}}\pi^s = \sum_{b\in\mathcal{B}}\left\{u^b(\mathbf{q}_*^b) - P_*^b\mathbf{q}_*^b\right\} + \sum_{s\in\mathcal{S}}\{P_*^s\mathbf{q}_*^s + u^s(\mathbf{q}_*^s)\} = V(\mathcal{J})$$

where the equality follows from the fact that $(P_*^b, P_*^s, \mathbf{q}_*^b, \mathbf{q}_*^s)$ are optimal solutions to DLP6.

The divisible goods case is proved in the same way, where the optimal solution $\{y(\mathbf{q}, b), y(\mathbf{q}, s)|\forall b \in \mathcal{B}, \forall s \in \mathcal{S}\}$ is not necessarily integral. ■

Proof of Proposition 5.3 To prove the claim note that in LP6 that the first and second constraints restrict each buyer and each seller to be assigned exactly one bundle, i.e. $\sum\limits_{\mathbf{q}\in A^b} y(\mathbf{q}, b) = 1$ and $\sum\limits_{\mathbf{q}\in A^s} y(\mathbf{q}, s) = 1$. If the right hand sides of each of these constraints are reduced to zero, then the corresponding dual variables, π^b, π^s disappear from DLP6. In consequence, if $\{\pi^b, \pi^s\}$ are part of the optimal solution to DLP6, then $\pi_*^b \leq V(\mathcal{J}) - V(\mathcal{J}\backslash b)$ for each $b \in \mathcal{B}$, and $\pi_*^s \leq V(\mathcal{J}) - V(\mathcal{J}\backslash s)$ for each $s \in \mathcal{S}$. ■

Proof of Proposition 5.5 Let $u^j(\mathbf{q})$ be participant j's true valuation for the bundle $\mathbf{q}$ and $\tilde{u}^j(\mathbf{q})$ its reported valuation to the clearinghouse and suppose that $u^j(\mathbf{q}) \neq \tilde{u}^j(\mathbf{q})$. Suppose that the participant reports $\tilde{u}^j(\mathbf{q})$ to the clearinghouse, which (by LP6) determines that the optimal aggregate bundle is $\tilde{\mathbf{q}}_*$ and $V(\mathcal{J}) = \tilde{u}^j(\tilde{\mathbf{q}}_*) + \sum_{j \in (\mathcal{J}\backslash j)} u^j(\tilde{\mathbf{q}}_*)$.

Therefore, if participant j misreports its true valuation, its payoff in a PME with VCG payments is:

$$\left[\tilde{u}^j(\tilde{\mathbf{q}}_*) + \sum_{j \in (\mathcal{J}\backslash j)} u^j(\tilde{\mathbf{q}}_*)\right] - V(\mathcal{J}\backslash j) \leq V(\mathcal{J}) - V(\mathcal{J}\backslash j)$$

where the inequality follows from Proposition 5.3.

■

Proof of Proposition 5.7

Consider a PME with VCG payments and let $\sum_{b \in \mathcal{B}} \pi^b + \sum_{s \in \mathcal{S}} \pi^s = V(\mathcal{J})$ hold when all participants trade.

Now consider a non-empty coalition $\mathcal{C} \subseteq \mathcal{J}$ conformed of at least one buyer b and one seller s. Then for the coalition to be in the core:

$$\begin{aligned}
\sum_{b \in \mathcal{C}^B} \pi^b + \sum_{s \in \mathcal{C}^S} \pi^s &= V(\mathcal{J}) - \left(\sum_{b \in \mathcal{C}^B} [V(\mathcal{J}) - V(\mathcal{J}\backslash b)] + \sum_{s \in \mathcal{C}^S} [V(\mathcal{J}) - V(\mathcal{J}\backslash s)]\right) \\
&= V(\mathcal{J}) - \left(\sum_{j \in \mathcal{C}} V(\mathcal{J}) - \sum_{j \in \mathcal{C}} V(\mathcal{J}\backslash j)\right) \\
&\geq V(\mathcal{J}) - V(\mathcal{J}\backslash C)
\end{aligned}$$

where the inequality follows from the fact that the PSC holds.

■

A.3 Some detailed market clearing examples of PMEs

In the three examples that follow, the set of buyers is $\mathcal{B} = \{b1, b2\}$ is composed of $B = 2$ buyers and the set of sellers is $\mathcal{S} = \{s1, s2\}$ composed of $S = 2$ sellers.

To simplify notation, let $A^j = \{\mathbf{0}, \mathbf{q}, \mathbf{q}'\}$ for all $j \in \mathcal{J}$ where $\mathbf{0} = (0, 0)$

A.3.1 Example 1: An example with SS preferences and all facet weights are $w_F = 1$

In this example, all market participants have: $\mathbf{q} = (1, 0), \mathbf{q}' = (0, 1)$

- Buyer $b1$ has the following valuation function:

$$u^{b1}(\mathbf{0}) = 0; u^{b1}(\mathbf{q}) = 5; u^{b1}(\mathbf{q}') = 10$$

- Buyer $b2$ has the following valuation function:

$$u^{b2}(\mathbf{0}) = 0; u^{b2}(\mathbf{q}) = 8; u^{b1}(\mathbf{q}') = 4$$

- Seller $s1$ has the following valuation function:

$$u^{s1}(\mathbf{0}) = 0; u^{s1}(\mathbf{q}) = -2; u^{s1}(\mathbf{q}') = -5$$

- Seller $s2$ has the following valuation function:

$$u^{s2}(\mathbf{0}) = 0; u^{s2}(\mathbf{q}) = -6; u^{s2}(\mathbf{q}') = -3$$

Participant j	$\mathbf{0}$	$\mathbf{q}^j_*$	$\mathbf{q}'^j_*$
$b1$	0	0	1
$b2$	0	1	0
$s1$	0	1	0
$s2$	0	0	1

Table 10: Efficient allocations in Example 1 (subsection A.3.1)

According to table 10, buyer $b1$ buys $\mathbf{q}' = (0, 1)$, buyer $b2$ buys $\mathbf{q} = (1, 0)$. Seller $s1$ sells $\mathbf{q} = (1, 0)$, seller $s2$ sells $\mathbf{q}' = (0, 1)$. The efficient aggregate bundle is $\mathbf{q}_* = (1, 1)$. Walrasian prices that support the allocation are $\mathbf{p}_* = (5.54, 6.03)$ and if these are implemented, $V(\mathcal{J}) = 13$.

Otherwise, if VCG payments are implemented, then $V(\mathcal{J}) = 26$. The VCG payments supporting this allocation are:

- $b1$ pays $P^{b1}_{VCG} = 3$
- $b2$ pays $P^{b2}_{VCG} = 2$
- $s1$ gets paid $P^{s1}_{VCG} = 8$
- $s2$ gets paid $P^{s2}_{VCG} = 10$

A.3.2 Example 2: An example with SS preferences but facet weights are $w_F \neq 1$

In this example, each market participant defines its own valuation domain but this is restricted to being binary.

- Buyer $b1$ has $\mathbf{q} = (5, 0), \mathbf{q}' = (0, 5)$ and the following valuation function:

$$u^{b1}(\mathbf{0}) = 0; u^{b1}(\mathbf{q}) = 15; u^{b1}(\mathbf{q}') = 35$$

- Buyer $b2$ has $\mathbf{q} = (3, 0), \mathbf{q}' = (0, 3)$ and the following valuation function:

$$u^{b2}(\mathbf{0}) = 0; u^{b2}(\mathbf{q}) = 12; u^{b1}(\mathbf{q}') = 9$$

- Seller $s1$ has $\mathbf{q} = (3, 0), \mathbf{q}' = (0, 3)$ and the following valuation function:

$$u^{s1}(\mathbf{0}) = 0; u^{s1}(\mathbf{q}) = -3; u^{s1}(\mathbf{q}') = -6$$

- Seller $s2$ has $\mathbf{q} = (5, 0), \mathbf{q}' = (0, 5)$ and the following valuation function:

$$u^{s2}(\mathbf{0}) = 0; u^{s2}(\mathbf{q}) = -30; u^{s2}(\mathbf{q}') = -15$$

Participant j	$\mathbf{0}$	$\mathbf{q}_*^j$	$\mathbf{q}'^j_*$
$b1$	0	0	1
$b2$	0	1	0
$s1$	0	1	0
$s2$	0	0	1

Table 11: Efficient allocations in Example 1 (subsection A.3.2)

According to table 11, buyer $b1$ buys $\mathbf{q}' = (0, 5)$, buyer $b2$ buys $\mathbf{q} = (3, 0)$. Seller $s1$ sells $\mathbf{q} = (3, 0)$, seller $s2$ sells $\mathbf{q}' = (0, 5)$. The efficient aggregate bundle is $\mathbf{q}_* = (3, 5)$. Walrasian prices that support the allocation are $\mathbf{p}_* = (3.40, 3.94)$ and if these are implemented, $V(\mathcal{J}) = 29$.

Otherwise, if VCG payments are implemented, then $V(\mathcal{J}) = 55$. The VCG payments supporting this allocation are:

- $b1$ pays $P_{VCG}^{b1} = 15$
- $b2$ pays $P_{VCG}^{b2} = 6$
- $s1$ gets paid $P_{VCG}^{s1} = 12$
- $s2$ gets paid $P_{VCG}^{s2} = 35$

A.3.3 Example 3: An example with OS preferences

[30]

In this example, each market participant defines its own valuation domain and this can be of the multi-unit type.

[30] This example presents further details about example 3.12.

- Buyer $b1$ has $\mathbf{q} = (1, 0), \mathbf{q}' = (0, 2)$ and the following valuation function:

$$u^{b1}(\mathbf{0}) = 0; u^{b1}(\mathbf{q}) = 6; u^{b1}(\mathbf{q}') = 8$$

- Buyer $b2$ has $\mathbf{q} = (3, 0), \mathbf{q}' = (0, 2)$ and the following valuation function:

$$u^{b2}(\mathbf{0}) = 0; u^{b2}(\mathbf{q}) = 9; u^{b1}(\mathbf{q}') = 4$$

- Seller $s1$ has $\mathbf{q} = (4, 0), \mathbf{q}' = (0, 1)$ and the following valuation function:

$$u^{s1}(\mathbf{0}) = 0; u^{s1}(\mathbf{q}) = -8; u^{s1}(\mathbf{q}') = -3$$

- Seller $s2$ has $\mathbf{q} = (3, 0), \mathbf{q}' = (0, 6)$ and the following valuation function:

$$u^{s2}(\mathbf{0}) = 0; u^{s2}(\mathbf{q}) = -12; u^{s2}(\mathbf{q}') = -18$$

Participant j	$\mathbf{0}$	$\mathbf{q}_*^j$	$\mathbf{q}'^j_*$
$b1$	0	1	0
$b2$	0	1	0
$s1$	0	1	0
$s2$	1	0	0

Table 12: Efficient allocations in Example 1 (subsection A.3.3)

According to table 12, buyer $b1$ buys $\mathbf{q} = (1, 0)$, buyer $b2$ buys $\mathbf{q} = (3, 0)$. Seller $s1$ sells $\mathbf{q} = (4, 0)$, seller $s2$ sells $\mathbf{0} = (0, 0)$, i.e. sells nothing. The efficient aggregate bundle is $\mathbf{q}_* = (4, 0)$. Walrasian prices that support the allocation are $\mathbf{p}_* = (2.34, 2.54)$ and if these are implemented, $V(\mathcal{J}) = 7$.

Otherwise, if VCG payments are implemented, then $V(\mathcal{J}) = 14$. The VCG payments supporting this allocation are:

- $b1$ pays $P^{b1}_{VCG} = 2$
- $b2$ pays $P^{b2}_{VCG} = 6$
- $s1$ gets paid $P^{s1}_{VCG} = 13$
- $s2$ gets paid $P^{s2}_{VCG} = 0$

A.4 An example that focuses on duality between a TH and a SNP

There are $N = 2$ indivisible, imperfectly substitutable goods in the economy, and $B = 2$ buyers, each with the following valuation functions $u^b : A^b \to \mathbb{R}$:

Table 13: Valuation of buyer 1

$q_1 = 1$	$q_1 = 0$	u^1
10	0	$q_2 = 0$
12	8	$q_2 = 1$

Table 14: Valuation of buyer 2

$q_1 = 1$	$q_1 = 0$	u^2
8	0	$q_2 = 0$
14	12	$q_2 = 1$

The regular subdivision or Subdivided Newton Polytope (SNP)[31] for the first buyer is in 19:

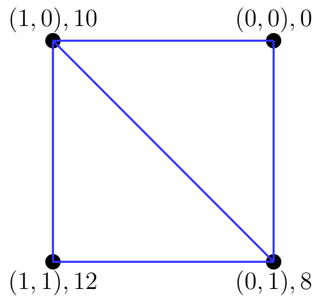

Figure 19: Regular subdivision (SNP) for agent 1

[31] According to terminology by Baldwin and Klemperer (2016), I call it SNP. Following their convention (page 9), I draw the SNP with increasing values to the left and down

The partition of price space (tropical hypersurface-TH) according to buyer 1's valuation is in figure 20:

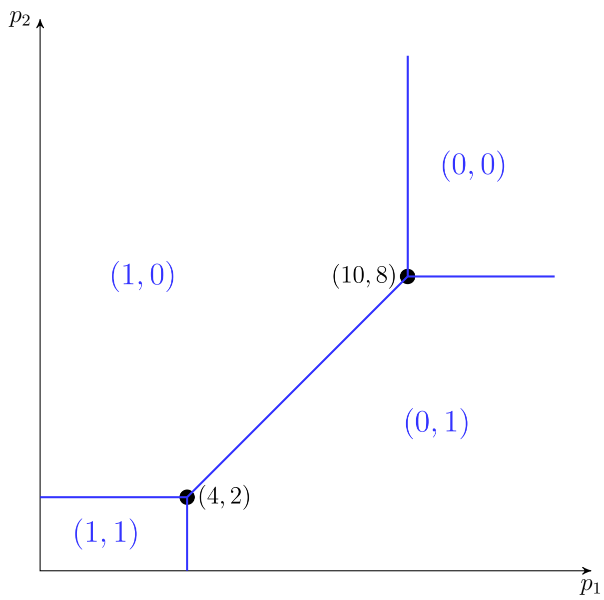

Figure 20: Agent 1's partition of price space induced by u^1 (Buyer 1's TH)

And the SNP corresponding to buyer 2 is in figure 21:

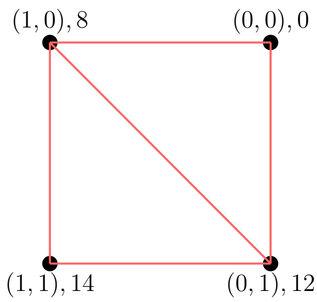

Figure 21: Regular subdivision (SNP) for buyer 2

Accordingly, buyer 2's TH is in figure 22:

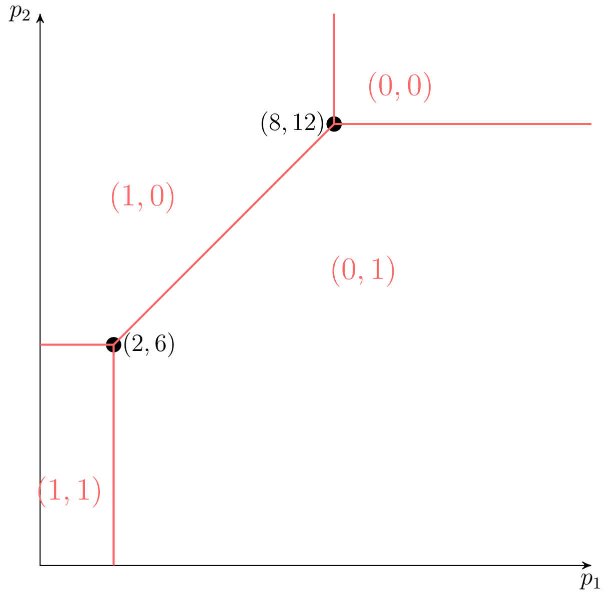

Figure 22: Agent 2's partition of price space induced by u^2 (Buyer 2's TH)

The Minkowski sum of individual domains A^b gives the set of all bundles in the economy: $A = \{(0,0),(1,0),(0,1),(1,1)(2,0),(0,2),(2,1),(2,2)\}$.The aggregate utility function $U : A \to \mathbb{R}$ is:

Table 15: Aggregate valuation

$q_1 = 2$	$q_1 = 1$	$q_1 = 0$	U
18	10	0	$q_2 = 0$
24	22	12	$q_2 = 1$
26	24	20	$q_2 = 2$

And the regular mixed subdivision (aggregate SNP) is in figure 23:

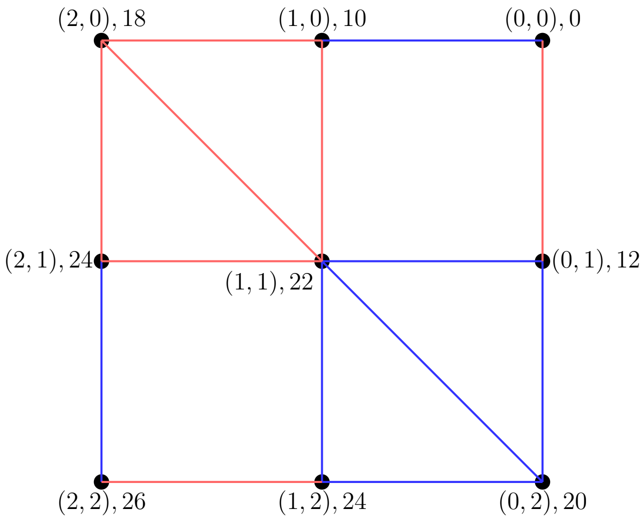

Figure 23: Mixed regular subdivision (aggregate SNP)

The TH dual to the mixed regular subdivision is for the aggregate valuation is in figure 24:

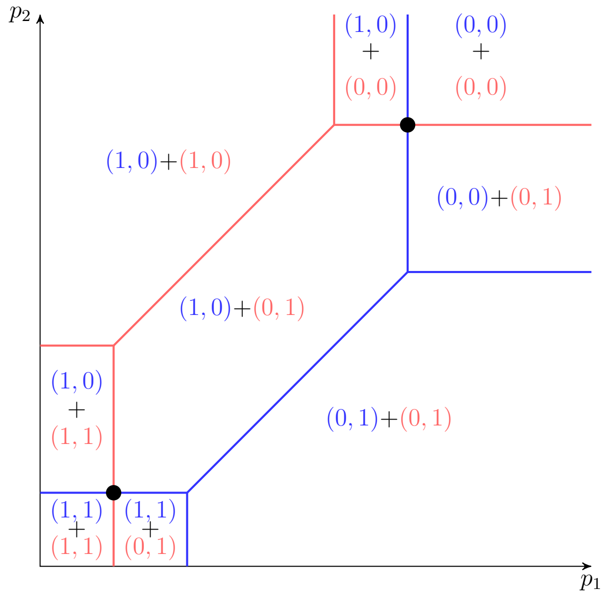

Figure 24: Aggregate TH

Conclusion

This PhD thesis has contributed to the economics of power system flexibility, a topic that is *technical* in nature but can benefit from the fundamental *economic* insight that accounting for incentives is crucial. The five chapters that compose the thesis gradually introduce the reader into a variety of topics and models that lie at the forefront of a timely question: *how to integrate renewables, while ensuring that power systems operate reliably?* As is the case with any problem approached from a (contract and market) designer's perspective, specific details cannot be ignored but once these are sufficiently understood, the responsibility of the economic modeller is to try and derive conclusions of greater generality. In this effort, new economic models are developed and connections with other problems of relevance can be established with greater ease. I have humbly striven to achieve this goal in my PhD thesis.

In the first chapter, rather than going into the abstract world of economic models all at once, my co-author (Rahmatallah Poudineh) and I looked at the real world to find out what kind of business innovation that enables flexibility is actually happening. In an interesting exercise of documental research, we found that technologically-led innovation is well and alive and is beginning to play a fundamental role in the emergence of an entirely different electricity industry. One in which flexibility can be efficiently supplied, as long as business models consolidate, while new activities are added to the traditional supply chain and the role of incumbents is redefined. In summary, a creatively chaotic picture can be depicted. From this chapter, two questions remain open. First, to further refine the partial taxonomy of new actors, new roles and new business models presented. Second, to empirically assess the implications of the emergence of a *layer of innovation* (entrants with innovative business models) on the structure of the electricity industry. Specifically, are incumbents deterring the change or are they adapting to it? If they are adapting, does it imply an expansion into related but different areas of economic activity within the electricity supply chain, leading to economies of scope?

The second chapter of the thesis analyzed the existing body of work on power system flexibility, and concluded that while an important set of questions have been answered by engineers, industry stakeholders and policymakers preoccupied with the integration of VRE, another important set of questions have so far remained unanswered. In particular, I found that the question of *incentivizing* the provision of flexibility, rather than assuming that it is an exogenously given trait of a power system, has largely remained unaddressed. Informed by its technical characteristics and by the fact that its economic properties have not been studied so far, I proposed a set of economic postulates that should guide the modelling of flexibility as well as the desirable properties that a product design to enable flexibility should have. Rather than an abstract exercise, these normative suggestions are meant to guide concrete solutions. However, one limitation of this chapter is that the desirable properties of a flexibility product design are not easily tested unless these are associated to measurable features of the market. Thus, an open task is to conduct robust empirical analyses of the impact of such product designs on concrete market features. For example, to test the relevance of the CAISO flexible ramping product, one could investigate if more or less regulation services have been activated.

In Chapter 3, my co-authors (Peter Bogetoft and Peter Møllgaard) and I made a first approximation to modelling power system flexibility from a microeconomic perspective with a focus on demand-side resources. Based on a set of mildly restrictive but reasonable assumptions, we set up a simple model of bilateral bargaining between a consumer and an aggregator of flexible loads. Relaxing the most relevant assumptions in turn, we obtained a set of clear economic intuitions. First, provided that a transaction cost-reducing technology is in place, single-shot trading of flexibility increases the welfare of the parties involved. This is the consequence of two effects: consumers who derive utility not from energy consumption itself but from the comfort it provides, and aggregators who have increased chances of managing the risk they face when flexibility is provided. Second, in a longer term perspective and accounting for the role of investment costs associated to the transaction cost-reducing technology, consumer and aggregator must trade for a sufficiently long period to be able to gain from trading flexibility. Third, the model shows that whenever the aggregator is able to control more loads, it is able to offer the consumer a better deal for the flexibility it sells. By extension, this could indicate that economies of scale are present. While the model presented in chapter 3 is plausible, it remains to test its applicability for concrete policy analysis. For example, a debate that has recently attracted the attention of Nordic energy regulators is what aggregation model adapts best to the existing regulatory framework of countries like Sweden, Finland, Norway and Denmark. Given that all these countries require anyone acting on behalf of final consumers to assume balancing responsibility, it has been preliminarily suggested that retailers are best suited for this task. However, this conclusion has overlooked the fact that retailers would be endowed with an incumbent position that would deter the entry of independent aggregators. The model of Chapter 3 can be easily adapted to answer a question like this. However, it is a pending task.

In the fourth chapter of the thesis, my co-author (Rahmatallah Poudineh) and I account for the economic properties of flexibility to design efficient procurement contracts in environments characterized by bi-dimensional asymmetric information. Such a modelling effort reflects the fact that flexibility services may be traded bilaterally when competition among suppliers is not feasible. In addition, the model accounts for the imperfect substitutability of the elements that compose flexibility, which justifies the assumption of non-separability in the gross utility and cost of both principal and agent. The thermostat-based demand-response programs implemented by utilities in conjunction with technology providers are good real-world examples of the situations that this paper models. Consequently, we show via simulations how the model can be implemented to design efficient contracts for this purpose. Nonetheless, a question that this paper has not addressed is how the model developed can be adapted to situations in which competition is feasible. Furthermore, it also remains to show how the conclusions derived for flexibility trading apply to more general procurement situations.

The fifth and last chapter of the thesis was motivated by the need to find a solution to the practical problem of designing a marketplace for flexibility based on the Flex-Offer information technology concept. As a result, I extended the Product-Mix Auction format which had many desirable properties but was not completely adequate for the task at hand. The extensions are summarized under the encompassing term of Product-Mix Exchanges, which are double, multi-unit combinatorial auctions in which market participants are able to simultaneously place bids on the two sides of the market. When developing the extensions, I relied upon a number of existing results and Tropical Geometric techniques, which allow expressing and analyzing the kind of substitutable preferences imposed on PME participants. Furthermore, I studied the conditions under which Vickrey-Clarke-Groves (VCG) payments could support the efficient allocation and found that because the well-known Gross Substitutes condition always holds for participants of a PME, the VCG outcome, i.e. the one in which all market participants obtain their marginal product as payoff, is in the core. Finally, this paper illustrated the specific application for which it was originally meant, i.e. a *Delta Energy* market in which two specific flexibility-enabling products are traded. The work presented in this chapter could be extended in two directions. On the theoretical front, a more careful analysis of the impact of payment rules on bidding behavior should be developed. In particular, rather than proposing VCG payments as a solution against strategic manipulation, it is important to understand bidding behavior in the absence of it. It would also be relevant to develop results that help understanding the role of PMEs when optimality, and not efficiency, is the goal of the organizer of the exchange. With an applied perspective in mind, it would also be interesting to analyze whether the framework could be applied in settings where capacity and not energy is the traded good.

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