

Measuring and Pricing the Risk of Corporate Failures

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MEASURING AND PRICING THE RISK OF CORPORATE FAILURES

Mamdouh Medhat

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Measuring and Pricing the Risk of Corporate Failures

Mamdouh Medhat¹

PhD Dissertation
November 3, 2014

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Preface

These writings constitute my PhD dissertation in financial economics. The dissertation consists of three chapters. Each chapter can be read independently of the others, but all three chapters share the dissertation's overall topic: Measuring and pricing the risk of corporate failures.

The ability to adequately measure and price the risk of corporate failures is vital for creditors, shareholders, and regulators of financial institutions. Whenever a firm uses debt instruments such as loans or bonds to finance its operations, the firm may fail to meet the debt's contractual obligations. Typically, a failure is in the form of a default on a payment of interest or principle, but can also be a violation of a covenant attached to the debt or a bankruptcy filing by the firm or by the creditors on behalf of the firm. If a firm fails, it may be forced to temporarily or permanently halt its operations, which can entail losses: Creditors may realize a loss on their promised payments, while shareholders may see their entire equity stake wiped out. Therefore, creditors and shareholders need to adequately measure the risk of a failure, so that this risk can be reflected in the prices they are willing to pay—and the returns they require—for holding a firm's financial securities. At the same time, regulators must be able to verify that a financial institution is adequately cushioned against this risk, so that losses do not destabilize an important institution or even the financial system itself.

The overall questions addressed in this dissertation are, then, how should we adequately measure the risk of corporate failures, and, is the risk of corporate failures adequately reflected in the market prices of financial securities? As a testimony to the importance of these questions, they have been at the center of much academic and regulation-oriented research since the late 1960s. The chapters of this dissertation thus constitute three contributions to this still highly active field of research within financial economics: Chapters 1 and 2 contribute to the literature on the measurement and prediction of the risk of corporate failures, while Chapter 3 contributes to the literature on the pricing of financial securities exposed to the risk of corporate failures. A review of the historic developments in the literature is given after this preface.

Chapter 1 is titled “Additive Intensity Regression Models in Corporate Default Analysis” and was published in *Journal of Financial Econometrics* in 2013. It presents an additive survival regression framework that allows the estimation and analysis of time-varying effects on default probabilities. The chapter uses data on firm characteristics and default histories for rated public US firms during the 1982-2006 period. The main result is that the ‘distance-to-default’ (an equity-market based measure of volatility-adjusted leverage) and the ratio of short-to-long term debt have significantly time-varying effects on default probabilities. The time-variation in these effects is related to business cycle fluctuations. The chapter also shows that

the inclusion of time-varying effects subsumes the effects of macroeconomics explanatory variables. These findings suggest that time-varying effects can help us understand which risk-factors that are the most important at certain points in a business cycle, and they indicate that at least part of the effects of macroeconomic fluctuations is captured through their influence on firm-specific characteristics.

Chapter 2 is titled “Cyclicality and Firm-size in Private Firm Defaults.” It analyses the validity of an assumption in the Basel II/III and CRD IV accords allowing banks to considerably reduce their risk-weights on loans to small and medium sized enterprises (SMEs)—namely, an assumption that the default probabilities of small firms are less cyclical than the default probabilities of large firms. The chapter uses a large dataset on loan and default histories for private Danish firms during the 2003-2012 period. The main result is that discriminating solely on firm-size, the default probabilities of small firms do exhibit less cyclical than the default probabilities of large firms, but that accounting for firm characteristics other than size, the average small firm’s default probability is equally cyclical or even more cyclical than the default probability of the average large firm. These findings suggest that preferential treatment of capital charges solely based on firm-size is too simplistic and may in fact entail adverse effects on the stability of European banks with a high exposure to the SME-segment.

The third and final chapter is titled “Liquidity Risk and Distressed Equity” and is my job market paper for the 2015 academic job market. It shows theoretically and empirically that firms’ cash holdings can help rationalize the ‘distress puzzle’, i.e. the counterintuitive empirical finding that higher probability of default predicts lower, not higher, future equity returns. The chapter presents a structural model in which a levered firm with financing constraints can default because of either insolvency (insufficient market value of assets relative to total liabilities) or illiquidity (insufficient cash holdings relative to short-term liabilities). The firm chooses its cash holdings so as to maximize equity value. The model implies that as long as the firm is solvent, it should optimally hold a level of cash that allows it to avoid illiquidity. Hence, when the firm follows the optimal cash-policy but is in high risk of insolvency, it will have a large fraction of its assets in cash. This implies that the firm’s equity will have low conditional beta (i.e. a low exposure to priced systematic risk), which helps rationalize the equity’s low expected returns. The chapter uses data on liquidity, solvency, and equity returns for rated public US corporations during the 1970-2013 period. Its main result is that the model’s theoretical predictions are empirically confirmed. In particular: (1) In all solvency levels, the average firm holds enough liquid assets to cover its short-term liabilities; less solvent firms have (2) a higher fraction of their total assets in liquid assets and therefore (3) lower conditional betas and (4) lower returns. These findings suggest that corporate cash holdings are key to rationalizing the distress puzzle.

My intended audience is academics within the field of financial economics, financial engineers working with models that measure or price the risk of corporate failures, and regulators of financial institutions with an exposure to the public or private corporate sector. The intended reader will hold at least a Master’s degree or similar in economics or mathematics-economics, and have some experience with capital structure theory, asset pricing theory, and survival analysis. Some familiarity with survival analysis and stochastic processes in continuous time is preferred for chapter 1; some familiarity with the Basel accords of recommendations

on banking laws and regulations is preferred for chapter 2; and some familiarity with structural models of default and empirical asset pricing anomalies is preferred for chapter 3.

I owe thanks to many people, without whom this dissertation would not be a reality. I first became interested in financial economics during my Bachelor's studies at University of Copenhagen while attending a course on mathematical finance taught by Rolf Poulsen. My first attempt to write about continuous time finance was my Bachelor's thesis, which was supervised by Ernst Hansen. Ernst encouraged me to further pursue the topic and advised me to read David Lando's work. During my Master's studies, I learned about stochastic processes from Martin Jacobsen, about survival analysis from Søren Feodor Nielsen, and about credit risk models from David Lando. David, along with Søren Feodor Nielsen and Rolf Poulsen, supervised work that became my Master's thesis. This would later lay the groundwork for the first chapter of this dissertation. The data that I used for my Master's thesis and the first chapter of this dissertation was provided by Mads Stenbo Nielsen, who also taught me about collecting data from CRSP and CompuStat, and who has been an endless source of support throughout my PhD studies.

During my PhD studies at Copenhagen Business School, I was advised by David Lando and Søren Feodor Nielsen, to whom I am extremely grateful. I learned about corporate finance from Ken Bechmann, about asset pricing theory from Kristian Miltersen, about econometrics from Bent Jesper Christensen, and about survival analysis from Thomas Scheike. I was fortunate to co-author with Thais Lærholm Jensen, David Lando, Søren Feodor Nielsen, and Mads Stenbo Nielsen. I also received much feedback and support from many colleagues at Copenhagen Business School, especially Peter Dalgaard, Søren Hvidkjær, Cathrine Jessen, Michael Reimer Jensen, Mads Vestergaard Jensen, Agatha Murgoci, Jens Dick Nielsen, Lasse Pedersen, Gyuri Venter, Desi Volcker, Christian Wagner, Ramona Westermann, Ida Willumsen, and Davide Tomio.

During my stay at Stanford University, I received much help and advice from Kay Giesecke, and I was extremely fortunate to learn about dynamic asset pricing theory from Darrell Duffie. Darrell has ever since been a great source of inspiration and support.

A special word of thanks goes to Darrell Duffie, Søren Hvidkjær, David Lando, and Lasse Pedersen for their advise and feedback regarding my job market paper (the third chapter of this dissertation).

Lastly, I would like to thank my family, my friends (some long-term and some more recent), and my girlfriend for their unconditional and unwavering support. I dedicate this dissertation to them.

Mamdouh Medhat
Copenhagen, November, 2014

Literature review

The three chapters of this dissertations are contributions to large and still active research areas within financial economics. Although each chapter includes a short review of the most directly related literature, I believe it is instructive with a broader historic review of some of the research milestones that predated the chapters of this dissertation—at least for readers who are not particularly familiar with the literature. Regardless of the reader’s familiarity with the literature, my hope is that the following review will help clarify where the contribution of each chapter fit in the broader literature. In the following, I divide the review into two parts: The first part, which relates to chapters 1 and 2, is dedicated to the literature on measuring the risk of corporate failures, while the second part, which relates to chapter 3, is dedicated to the literature on pricing this risk.

Literature related to chapters 1 and 2:

Measuring the risk of corporate failures

Chapters 1 and 2 provide contributions to the literature on the statistical measurement of the risk of corporate failures. This literature is concerned with risk-factors and statistical models that can help us explain or predict corporate failures. The important risk-factors are usually identified through a data-driven approach using accounting statements and macroeconomics time-series. However, in some cases, the risk-factors are inferred from ‘structural’ models of default, which are economic models of the firm’s fundamentals that directly determine the incentives or ability of the firm to honor its debt. A famous example of such a risk-factor is the ‘distance-to-default’ implied by the models of, for instance, [Black and Scholes \(1973\)](#) and [Merton \(1974\)](#). I give an overview of structural models in the review of the literature related to chapter 3.

The first published application of a statistical method to measure the risk of corporate failures is the paper titled “Financial ratios as predictors of failure” by [Beaver \(1966\)](#). The paper features a pair-matched sample of different accounting ratios for 79 failed and 79 surviving public US firms, and uses simple paired *t*-tests to identify which accounting ratios that can, in a univariate manner, distinguish between failed and surviving firms. As an important extension of this univariate methodology, [Altman \(1968\)](#) applied multivariate discriminant analysis to the accounting ratios of a pair-matched sample of 33 failed and 33 surviving public US firms. The resulting ‘Z-score’ is a linear combination of accounting ratios with estimated coefficients, which, given cutoffs, can be used to discriminate between high and low risk firms. It is shown to have

a 70% success rate in predicting failures two years before the event. Since its proposal, the Z-score and its variants have become a commercial success and are still widely applied by investors and in research.

Modern statistical measures of failure risk are typically based on some form of regression analysis. [Ohlsen \(1980\)](#) was one of the first to apply logistic regression to the accounting ratios of a non-matched sample of 2,058 public US firms, of which 105 are failures. The resulting ‘O-score’ is, like the Z-score, a linear combination of accounting ratios with estimated coefficients, which is transformed into a failure probability using the logistic function, $e^O/(1 + e^O)$. The O-score is about as accurate or slightly more accurate than the Z-score in predicting failures over horizons of one and two years. In a related study, [Zmijewski \(1984\)](#) argues that prior studies suffer from selection bias because they oversampled failures and firms with complete data. He uses probit regression to estimate the probability of failure as a function of accounting ratios for a sample of 40 failed and 800 surviving public US firms.

The models reviewed up to this point are now known as ‘static’ models, because they are estimated using statistical methods where each firm is represented by a single set of accounting ratios measured at a single point in time. [Shumway \(2001\)](#), however, argues that static models overestimate the effects of accounting ratios and produce failure probabilities that are biased and inconsistent. This is mainly because they ignore that a firm’s accounting ratios change from year to year. Instead, he proposes estimating failure probabilities using ‘hazard’ (or ‘duration’) regression models known from survival analysis. A hazard regression model relates the conditional probability of failure at a specific time point, given survival up to that time point, to the values of time-varying explanatory variables. [Shumway \(2001\)](#) argues that hazard regression models are superior to the static ones because they 1) automatically correct for the amount of time that each firm is at risk of failure, 2) take as input the entire observed history of a firm’s accounting ratios and can incorporate macroeconomic variables as predictors, and 3) can utilize much more data than static models.

The theoretical contribution of [Shumway \(2001\)](#) is that a discrete time hazard regression model can be estimated as a ‘dynamic logistic regression model’—that is, a model similar to the one applied by [Ohlsen \(1980\)](#), but estimated using data on each firm in each year as if each firm-year is an independent observation. To account for the dependence between firm-year observations for the same firm, standard errors are clustered at the firm-level. The empirical contributions are that 1) hazard regression models outperform static models in predicting future failures, 2) several of the accounting ratios applied in static models become insignificant predictors of failure in a hazard regression model, and 3) explanatory variables based on market equity data (such as equity market capitalization, past equity return, and equity return volatility) are strong predictors of failure. In a related study, [Chava and Jarrow \(2004\)](#) show that the predictive power of hazard regression models is further improved by using a monthly instead of yearly observation frequency for equity market-based variables and by including industry dummies as explanatory variables.

Hazard regression models are now the state-of-the-art in measuring and predicting the risk of corporate failures, and have been applied to estimate, study, and explain the failure probabilities of both public and private firms. For public firms, [Hillegeist, Keating, Cram, and Lundstedt \(2004\)](#) find that the ‘distance-to-default’ variable implied by the structural models of, for instance, [Black and Scholes \(1973\)](#) and [Merton \(1974\)](#), cannot entirely explain variations in default probabilities across firms. The distance-to-default is a

market-equity based measure of volatility-adjusted leverage and is the basis of the commercial ‘Estimated Default Frequency’ (EDF) measure provided by Moody’s KMV (see [Crosbie and Bohn \(2002\)](#)). [Duffie, Saita, and Wang \(2007\)](#) estimate conditional default probabilities over several future time periods using a hazard regression model augmented with a time-series model for the stochastic development of firm-specific and macroeconomic explanatory variables. They find that the level and shape of the term structure of future default probabilities depend, among other variables, on the distance-to-default, and that variations in the distance-to-default have the greatest effect on the term structure of future default probabilities. Moreover, the model’s out-of-sample predictive performance over different prediction horizons is an improvement over prior models. [Bharath and Shumway \(2008\)](#) show that the performance of distance-to-default in hazard regression models is relatively robust to alternative empirical approximations used to construct the measure.

An important assumption underlying most hazard regression models is that failures are conditionally independent given the paths of the explanatory variables—that is, conditional on the paths of the explanatory variables, failures occur at independent Poisson arrival times with conditionally deterministic intensity paths. [Das, Duffie, Kapadia, and Saita \(2007\)](#) use the data and model of [Duffie et al. \(2007\)](#) to test the joint hypothesis of 1) a well-specified model for conditional default probabilities and 2) conditionally independent defaults given the paths of the explanatory variables. They find no support in the data for the joint hypothesis and conclude that the failures in the data cluster more over time than is predicted by the model of [Duffie et al. \(2007\)](#). Building on these results, [Duffie, Eckner, Horel, and Saita \(2009\)](#) find evidence that firms are exposed to a common dynamic latent risk factor (or ‘frailty’, to borrow a term from survival analysis) that drives failures, even after controlling for observable firm-specific and macroeconomic explanatory variables like distance-to-default and aggregate stock market performance. Using almost the same data as [Das et al. \(2007\)](#), [Lando and Nielsen \(2010\)](#) find that it is possible to specify a model of conditional default probabilities such that the tests conducted by [Das et al. \(2007\)](#) (as well as additional tests) do not lead to a rejection of the hypothesis of conditionally independent failures. Specifically, they make the following changes to the list of explanatory variables used by [Duffie et al. \(2007\)](#): They add the quick ratio (a conservative measure of corporate liquidity), the ratio of short-to-long term debt, the book value of assets, and an indicator of aggregate industrial production, and replace a measure of short-term interest rate with a measure of the steepness of the term structure.

Chapter 1 of this dissertation (that is, [Lando, Medhat, Nielsen, and Nielsen \(2013\)](#)) uses the same data as [Lando and Nielsen \(2010\)](#) and provides evidence of significant time-variation in the effects of distance-to-default and the ratio of short-to-long term debt on conditional default probabilities. The chapter also shows that allowing for time-varying effects for firm-specific explanatory variables subsumes the effects of macroeconomic explanatory variables. Finally, the chapter shows that, after applying model-checking techniques to remove outliers, the quick ratio is a strong predictor of defaults and that there is a significant and economically interpretable interaction-effect between distance-to-default and quick ratio.

The studies mentioned until this point focus on public US firms. However, hazard regression models have also more recently been applied studies of private firms and firms outside the US. One topic that has received much interest in the literature is how the conditional default probabilities of private firms depend

on macroeconomic explanatory variables. One of the reasons is that since we do not have equity-market based explanatory variables like distance-to-default and past equity returns for private firms, macroeconomic explanatory variables should, a priori, have a larger role to play in default prediction for private firms than they do for public firms. [Carling, Jacobson, Lindé, and Roszbach \(2007\)](#) use data on private Swedish firms and find that a model employing both firm-specific and macroeconomic explanatory variables is superior to a model that only includes firm-specific explanatory variables—specifically, the model including both types of explanatory variables can both accurately rank firms with respect to default likelihood as well as capture the aggregate default rate over time. The latter property is important for, for instance, managing the risk of failures for a portfolio of firms. [Bonfim \(2009\)](#) finds similar results using data on private Portuguese firms.

Chapter 2 of this dissertation uses data on private Danish firms to study an assumption in the Basel II/III and CRD IV accords stating that the default probabilities of small private firms are less sensitive to macroeconomic cyclical than are the default probabilities of large private firms. This assumption allows banks a considerable reduction in the capital they are required to hold when lending to small private firms. The main finding is that solely discriminating with respect to firm-size, the conditional default probabilities of small private firms do in fact exhibit less sensitivity to macroeconomic explanatory variables than do the conditional default probabilities of large private firms. However, when correcting for the effects of firm-specific explanatory variables other than size, the conditional default probabilities of the average small private firm is as cyclical or even more cyclical than the conditional default probability of the average large private firm. The results are robust to correcting for time-varying effects, as in Chapter 1, as well as different definitions of ‘small’ and ‘large’ firms.

Literature related to chapter 3:

Pricing the risk of corporate failures

Chapter 3 provides a contribution to each of the theoretical and empirical strands of the literature on pricing the risk of corporate failures. The theoretical strand of this literature deals with pricing models that relate the prices of corporate liabilities (i.e. equity and debt) to risk factors that drive the risk of failures. The empirical strand deals with identifying systematic variations related to the risk of failures in the observed market prices of corporate liabilities, as well as testing the predictions of theoretical pricing models. The following gives a review of both strands of the literature, including the contributions of chapter 3.

The theoretical strand

Theoretical pricing models are typically divided into two main classes: ‘Structural’ models, which explicitly determine the event that triggers a default, and ‘reduced form’ models, which do not. Structural models date back to the seminal option pricing models of [Black and Scholes \(1973\)](#) and [Merton \(1974\)](#). Reduced form models were developed as an alternative, mainly due the complexity of most firms’ capital structure and of the mechanisms behind a failure. The by far most tractable and popular reduced form pricing models

are the ones based on ‘Cox processes’ or ‘doubly stochastic Poisson processes’, which were introduced by Jarrow and Turnbull (1995), Madan and Unal (1998), Jarrow, Lando, and Turnbull (1997), Lando (1998), and Duffie and Singleton (1999). This approach builds on continuous-time survival analysis techniques and assumes that a firm’s default time is the first jump time of a Poisson process whose intensity is conditionally deterministic given the paths of state variables. Because Cox process-based pricing models can be linked to term structure models (which are used to price default-free bonds and bond-derivatives), they are natural in the pricing of corporate and sovereign bonds, and, of course, credit derivatives—see, for instance, Duffie (1999), Duffie (1999), and Duffie, Pedersen, and Singleton (2003). However, since chapter 3 is concerned with the pricing of the risk of failures in equities, and since equity pricing has historically been based on structural models, I focus in the following on reviewing the literature on structural models.

The original Black-Scholes-Merton model considers a single representative firm in a continuous time, perfect market setting. The firm’s assets have a market value which is assumed to follow a geometric Brownian motion (i.e. a log-normal stochastic process in continuous time) with predetermined dynamics. The firm’s liabilities consist of common equity stock and a single zero-coupon bond with predetermined face value and maturity date. Default can only occur at the bond’s maturity date: If, at the maturity date, the market value of the firm’s assets is below the face value of the firm’s debt, the firm defaults. In case of a default, equity holders exercise their limit liability option and abandon the firm’s assets, which are transferred in full to the debt holders as a recovery. Since the payoff to equity and debt holders can be determined at the bond’s maturity date, option pricing techniques can be used to derive the market prices of the firm’s equity and debt at any date prior to maturity as functions of the firm’s fundamentals (i.e. the market value of the assets and the parameters governing the dynamics of the asset value process). Given these theoretical pricing functions, observable market prices can be used to back-out the firm’s fundamentals, which can then be used to calculate the model’s implied probability of default.¹ Conversely, given (estimates of) the firm’s fundamentals, the pricing functions can be used to derive theoretical prices of corporate liabilities. Finally, the theoretical pricing functions can be used to derive predictions regarding how market prices and expected returns are related to firm fundamentals.

There are countless extensions of the Black-Scholes-Merton model that attempt to capture the complexity of most firms’ capital structure and of the mechanisms behind a failure. In the following, I focus on extensions related to optimal capital structure and endogenous default decisions, as these are the ones most closely related to the theoretical contribution of chapter 3. In one of the models studied by Black and Cox (1976), the firm is financed through common equity stock and a perpetual (infinite maturity) bond which pays a constant, predetermined coupon rate continuously in time. If the firm is allowed to sell assets to finance coupon payments, default occurs only when the value of assets falls to zero. If, however, asset sales are not permitted, the firm is assumed to issue additional equity in order to finance coupon payments,

¹In the Black-Scholes-Merton model, the firm’s conditional probability of default is completely determined by the so-called ‘distance-to-default’, which is the number of standard deviations of asset drift by which the current asset level (or expected asset level at a future time) exceeds the face value of the firm’s outstanding zero coupon bond. As mentioned in the review of the literature on statistical measurement of the risk of corporate failures, the empirical approximations of distance-to-default can be viewed as market-equity based measures of volatility-adjusted leverage.

and default thus occurs whenever equity holders no longer find it optimal to continue financing coupon payments—that is, default is endogenously triggered at an asset level that is optimally chosen by equity holders in order to maximize the value of their claim. In the model of [Leland \(1994\)](#) (which extends the ideas of [Fischer, Heinkel, and Zechner \(1989\)](#)), corporate taxes and bankruptcy costs are added to the above mentioned model of [Black and Cox \(1976\)](#), and the coupon rate, which specifies the firm’s capital structure, is also determined optimally by maximizing the total firm value. [Fan and Sundaresan \(2000\)](#) use the model of [Leland \(1994\)](#) to study strategic renegotiation of debt, where equity and debt holders use their relative bargaining power to bargain over the firm value when the firm’s asset value reaches an endogenously determined reorganization boundary. [Duffie and Lando \(2001\)](#) show, in the context of structural models like the one due to [Leland \(1994\)](#), that if the firm’s asset value is only imperfectly observed, the firm’s default time has a stochastic default intensity that depends on the firm’s distance-to-default in addition to other risk-factors that may reveal additional information regarding the firm’s health. The existence of a default arrival intensity produces credit spreads (the difference between the promised yield on a corporate bond and a corresponding treasury security) that are substantially larger in the short end of bond maturities than the ones implied by standard structural models with perfect information.

Parallel to the development of the above mentioned structural models that followed the Black-Scholes-Merton model, another part of the literature developed models of optimal dividend and liquidity management, i.e. models of how a firm should optimally distribute its earnings between dividends to equity holders and an internal cash reserve. The models of [Jeanblanc-Picqué and Shiryaev \(1995\)](#) and [Radner and Shepp \(1996\)](#) feature a firm whose productive assets generate uncertain earnings continuously through time. The firm’s cumulative net earnings are assumed to follow an arithmetic Brownian motion (i.e. a Gaussian stochastic process in continuous time). This implies that the firm’s net earnings may be either positive or negative. If net earnings are positive, they can either be distributed to equity holders as a dividend or retained inside the firm as a liquidity reserve, i.e. ‘cash holdings’. If net earnings are negative, then existing cash holdings may be used to pay out dividends to equity holders. Negative dividends, which correspond to equity issuances by the firm, are not allowed, and so the firm is financially constrained. The firm must have a positive cash reserve to remain operational—that is, default occurs due to ‘illiquidity’ whenever the firm’s cash holdings hit zero. This is in contrast to the above mentioned structural models, including the Black-Scholes-Merton model, where default occurs due to ‘insolvency’ whenever the firm has insufficient asset value. The optimal strategy for paying out dividends and holding cash is the strategy that maximizes the market value of the firm’s equity. The solutions of [Jeanblanc-Picqué and Shiryaev \(1995\)](#) and [Radner and Shepp \(1996\)](#) show that it is optimal for the firm to retain all net earnings as cash whenever cash is below a certain time-constant threshold, and, subsequently, to distribute all net earnings as dividends whenever cash is above the threshold. Therefore, these models predict a role for ‘precautionary’ cash holdings used by the firm as a cushion against liquidity risk that otherwise might drive it into default.

Following the development of the first liquidity management models, several extensions appeared that linked corporate liquidity, or cash holdings, to the prices of corporate liabilities. Here, I focus on the extensions related to the pricing of the risk of corporate failures. [Gryglewicz \(2011\)](#) extends the classical

liquidity management models by considering a firm that faces both solvency and liquidity concerns. Specifically, he randomizes the drift-component of the firm's cumulative earnings-process and explicitly models the firm's debt as a perpetual coupon-paying bond, as in, for instance, [Leland \(1994\)](#). The added uncertainty in the drift of cumulative earnings means that expected earnings are not time-constant and therefore that the value of the firm's productive assets is also not time-constant. This model setup means that the firm can default due to illiquidity, as in the classical liquidity management models, but also due to insolvency, which is endogenously triggered at an asset level that is optimally chosen by equity holders. Given the firm's optimal cash-dividend policy as well as the optimal coupon and the optimal asset level that triggers insolvency, [Gryglewicz \(2011\)](#) studies how solvency and liquidity concerns interact to affect the firm's choice of capital structure and shows that the presence of cash in firms' capital structure reduces the dispersion of credit spreads. [Acharya, Davydenko, and Strebulaev \(2012\)](#) use a pure liquidity model to rationalize why higher cash holdings empirically predicts higher, not lower, credit spreads. Their argument is based on the precautionary motive for holding cash, which causes firms with a higher risk of failure to accumulate higher cash reserves. Also using a pure liquidity model, [Bolton, Chen, and Wang \(2013\)](#) argue, among other things, that the endogenous nature of cash holdings is key to understanding the relation between cash holdings and expected equity returns.

Chapter 3 of this dissertation builds an equity pricing model that relates the expected return on a firm's equity to the firm's solvency and liquidity—that is, the model determines the return that an investor requires for holding a firm's equity as a function of the firm's solvency and liquidity. I argue that this model can help rationalize the so-called 'distress puzzle', i.e. why higher probability of default predicts lower, not higher, average equity returns. The model builds on the setup of [Gryglewicz \(2011\)](#), but deviates by assuming that the firm's cumulative earnings-process follows a geometric Brownian motion instead of an arithmetic Brownian motion with a randomized drift. While a relatively small technical change in the setup, it has the implication that the value of the firm's productive assets also follows a geometric Brownian motion, thereby making the model setup consistent with the classical structural models of [Black and Scholes \(1973\)](#), [Merton \(1974\)](#), [Black and Cox \(1976\)](#), [Leland \(1994\)](#), and others mentioned above. Similar to the classical liquidity management models and the model of [Gryglewicz \(2011\)](#), the optimal policy for holding cash dictates that while the firm is solvent, it will hold a level of cash that allows it to avoid illiquidity. I show that when a firm that follows the optimal cash-policy is in high risk of insolvency, it will have a large fraction of its assets in cash. This, in turn, implies that the firm's equity is relatively insensitive to priced systematic risk (i.e. has a low 'conditional beta') and therefore requires low returns, thereby helping to rationalize the distress puzzle.

The empirical strand

The empirical contribution of chapter 3 concerns the question of whether the risk of corporate failures is correctly reflected in the observed market prices of equities. I therefore focus on the empirical literature related to the pricing of equities.

The empirical literature on the pricing of equities dates back to the first tests of the Capital Asset Pricing

Model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972). The classical CAPM is a single period model of financial markets assuming that 1) investors are risk averse and only care about the expectation and variance of an asset's return, 2) that all investors agree perfectly on the true joint distribution of asset returns, and 3) that there is unrestricted borrowing and lending at a risk-free rate. Under these assumptions, an asset's expected return is, in equilibrium, equal to the risk-free rate plus a risk premium. The risk premium is given by the asset's 'beta' times the excess return on the market above the risk-free rate, where the beta is the slope-coefficient from a linear regression of the asset's return on the return of the market portfolio. The beta is thus the sensitivity of the asset's returns to changes in the excess return of the market portfolio, i.e. to systematic or undiversifiable risk. The model therefore predicts 1) a positive linear relation between expected returns and betas, 2) that the risk premium associated with higher beta is the excess return on the market, and 3) that variations in expected are completely explained by variations in betas. The restricted-borrowing or 'zero-beta' CAPM due to Black (1972) relaxes the assumption of unrestricted borrowing or lending at the risk-free rate and replaces it with an assumption of unrestricted shortselling of assets. The change relative to the classic CAPM is that the risk-free rate is replaced by the expected return on some asset that has zero covariance with the market portfolio.

The early cross-sectional tests of the CAPM find that while there is a positive and roughly linear relation between average equity returns and equity betas, this empirical 'security market line' is too flat (too high intercept and too low slope) compared to what is predicted by the CAPM (see, for instance, Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973)). Similar results are found using time-series regressions of excess returns on excess market returns, where the intercept in such a regression, i.e. Jensen's 'alpha', is significant and positive for low-beta equities and significant and negative for high-beta equities (see, for instance, Black et al. (1972) and Stambaugh (1982)). While these findings are a rejection of the classical CAPM, they are a priori not a rejection of the Black-version of the CAPM, which merely predicts a positive linear relation between expected returns and beta, without specifying the risk premium associated with higher beta. Moreover, the initial cross-sectional tests of Fama and MacBeth (1973) find that beta seems to suffice in explaining variations in average returns, which further strengthened the belief in the Black-version of the CAPM.

However, following the initial tests, several findings indicate that firm characteristics other than equity beta can help explain and predict variations in average equity returns, thereby also rejecting the Black-version of the CAPM. Basu (1977) finds that equities with high earnings-to-price ratio have higher average future returns than is predicted by the CAPM, i.e. too high average future returns relative to their equity beta. Banz (1981) finds that equities with a small (big) market value have higher (lower) future average returns than is predicted by the CAPM. Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) report similar findings for equities with a high ratio of book equity to market equity, while Bhandari (1988) finds the same for equities with a high debt-to-equity (i.e. leverage) ratio. These findings are now commonly referred to as the original 'asset pricing anomalies', where 'anomaly' is to be understood relative to the CAPM. The seminal work of Fama and French (1992) combines and reinforces the mentioned evidence on the existence of asset pricing anomalies. They show, using cross-sectional tests, that while the univariate relation between

equity beta and average equity returns is statistically insignificant, the univariate relations between average equity returns and the firm characteristics mentioned above are statistically significant. Furthermore, in multivariate tests, they find that size (i.e. the market value of equity) and the ratio of book-to-market equity combine to subsume the effects of beta, earnings-to-price, leverage, and other variables.

The evidence pointing towards the existence of asset pricing anomalies lead to the three-factor asset pricing model of Fama and French (1993, 1996). In this model, the expected excess return on a firm's equity is given by a linear combination of three risk-factors with estimated coefficients. The first risk-factor is the excess return on the market portfolio or 'market risk premium', as implied by the CAPM. The second risk-factor, called the 'size-factor' or 'size-premium', is the return on a diversified equity portfolio which is long small firms (i.e. firms with low market equity value) and short big firms (i.e. firms with high market equity value). The third and final risk-factor, called the 'value-factor' or 'value-premium', is the return on a diversified equity portfolio which is long firms with high book-to-market equity and short firms with low book-to-market equity. Fama and French (1993, 1996) show, using time-series regressions, that the three-factor model can capture much of the variation in average returns for equity portfolios formed on firm size, book-to-market equity, as well as other firm characteristics. They argue that the size- and value-factors are consistent with the intertemporal (continuous time) CAPM due to Merton (1973) and are merely proxies for state variables that imply undiversifiable risks which are not captured by the market risk premium.

Several strands of the literature were concerned with explanations of asset pricing anomalies like the size and the value premia. Chan and Chen (1991) were among the first to suggest that the size premium is due to investors requiring higher returns as compensation for holding the equities of firms with a higher risk of failure or 'distress'. They argue that small firms (i.e. firms with low market equity value) require high equity returns because (1) they are likely to have high leverage and low earnings, which makes them less likely to survive aggregate downturns, and (2) their equity returns tend to be highly correlated, which makes their risks difficult to diversify away. Similarly, Fama and French (1995, 1996) suggest that the value premium is due to the fact that firms with a high risk of distress also have high book-to-market equity.

Dichev (1998) was among the first to refute the idea that the size and value premia are compensation for distress risk. Using the failure prediction models of Altman (1968) and Ohlson (1980), he shows that higher risk of failure predicts lower, not higher, average future equity returns. Furthermore, he finds that (1) while higher risk of failure is generally associated with higher book-to-market equity, the firms with the highest risk of failure actually have relatively low book-to-market equity, and (2) that there is no statistically significant relation between the risk of failure and firm size (market value of equity). These findings are now what is commonly known as the 'distress puzzle'. In a related study, Griffen and Lemmon (2002) use the failure prediction model of Ohlson (1980) to show that firms with a high risk of failure and low book-to-market equity have particularly low average future equity returns. Vassalou and Xing (2004) use the distance-to-default measure as a proxy for failure risk and find some evidence that lower distance-to-default predicts higher average future equity returns, but their results are entirely driven by small and high book-to-market firms. Campbell, Hilscher, and Szilagyi (2008) estimate a dynamic logistic regression model to predict failures and use it to confirm and reinforce previous evidence that higher probability of failure persistently and

robustly predicts lower average future returns. [Avramov, Chordia, Jostova, and Philipov \(2009\)](#) find similar results using credit ratings, which can be viewed as the rating agency's subjective measure of failure risk.

Chapter 3 of this dissertation fits into a recent strand of the literature which attempts to rationalize the distress puzzle using arguments based on the capital structure of firms with a high risk of failure. In general, this strand of the literature proposes structural models of default which predict that the equity of a firm in distress has a low exposure to systematic risk and therefore requires low expected returns. The models differ, however, in the mechanisms behind this low exposure to systematic risk. [Garlappi, Shu, and Yan \(2008\)](#) use the Estimated Default Frequency (EDF) measure by Moody's to show that the empirical relation between average future returns and default risk is not monotonically decreasing but rather hump-shaped. They rationalize this finding using the structural model of [Fan and Sundaresan \(2000\)](#), in which shareholders can use their bargaining power to recover part of the firm's value upon resolution of distress. Extending this analysis, [Garlappi and Yan \(2011\)](#) show theoretically that shareholder recovery upon resolution of distress implies a hump-shaped relation between expected returns and probability of default, and they find empirical support for the model's predictions using conditional (time-varying) equity betas estimated at the firm-level. The structural model of [Opp \(2013\)](#) rationalizes the empirical relation between expected returns and probability of default by means of increased learning about firm-solvency in aggregate downturns, which, he argues, implies low exposure to systematic risk and therefore low expected returns. [McQuade \(2013\)](#) proposes a structural model in which the equity of a firm with a high risk of failure is negatively exposed to persistent volatility risk, and therefore commands lower expected returns.

Chapter 3 of this dissertation argues that corporate cash holdings can help rationalize the distress puzzle. The chapter contributes to the above mentioned literature by proposing a model which separates the liquidity and solvency components of distress. The model considers a levered firm with financing constraints that can default because of either insolvency or illiquidity, but the firm is allowed to choose its cash holdings optimally. I show that an equity-maximizing firm will, as long as it is solvent, optimally hold a level of cash that allows it avoid illiquidity. When the firm follows the optimal cash-policy and is in high risk insolvency, it will have a large fraction of its assets in cash. I show that this implies that the firm's equity will have low exposure to systematic risk and therefore command low expected returns. The model's central predictions are that, for firms that hold enough cash to avoid illiquidity, higher probability of insolvency is generally associated with lower expected returns, and the relation between expected returns and probability of insolvency is, in fact, hump-shaped. Using data on solvency, liquidity, and equity returns for rated US firms, I find evidence consistent with these central prediction, and I show that my empirical results hold using either conditional (time-varying) firm-specific equity betas; cross-sectional regressions of firm-specific returns on measures of solvency; or the realized returns and alphas of value-weighted portfolios formed on measures of solvency. Other predictions derived from the model, for instance regarding the relation between expected returns and liquidity, are also confirmed in the data.

Chapter summaries in English and Danish

The following are summaries of the dissertation's chapters in both English and Danish.

Chapter 1

English summary. We consider additive intensity (Aalen) models as an alternative to the multiplicative intensity (Cox) models for analyzing the default risk of a sample of rated, non-financial U.S. firms. The setting allows for estimating and testing the significance of time-varying effects. We use a variety of model checking techniques to identify misspecifications. In our final model we find evidence of time-variation in the effects of distance-to-default and short-to-long term debt, we identify interactions between distance-to-default and other covariates, and the quick ratio covariate is significant. None of our macroeconomic covariates are significant.

Dansk resumé. Vi betragter additive (Aalen) intensitetsmodeller som et alternativ til multiplikative (Cox) intensitetsmodeller i analysen af fallitrisikoen af ikke-finansielle amerikanske virksomheder. Vores setup tillader estimation og test af tidsvarierende effekter. Vi bruger en række modeltjek til at identificere misspecifikationer. I vores endelige model finder vi evidens for tidsvariation i effekten af distance-to-default og kort-til-langfristet gæld, vi identificerer vekselvirkninger mellem distance-to-default og andre kovariater, og quick ratio-kovariaten har en signifikant effekt. Ingen af vores makroøkonomiske kovariater er signifikante.

Chapter 2

English summary. The Basel II/III and CRD IV Accords treat the default probabilities of small firms as less sensitive to macroeconomic cyclicity than the default probabilities of large firms. We investigate whether this is an appropriate assumption in a default intensity regression framework using a large sample of loans to private firms. We find that discriminating solely on firm-size, the default probabilities of small firms do exhibit less cyclicity than the default probabilities of large firms. However, accounting for firm characteristics other than size, we find that the average small firm's default probability is equally cyclical or even more cyclical than the default probability of the average large firm. Our results hold using both a multiplicative Cox model and an additive Aalen model.

Dansk resumé. Basel II/III og CRD IV antager, at små virksomheders fallitsandsynligheder er mindre konjunkturfølsomme end store virksomheders fallitsandsynligheder. Vi undersøger validiteten af denne antagelse ved hjælp af intensitetsregressioner og et stort datasæt af lån til private virksomheder. Vi finder, at skelner man udelukkende på størrelse, udviser små virksomheders fallitsandsynligheder mindre konjunkturfølsomhed end store virksomheders fallitsandsynligheder. Vi finder dog, at tager man højde for andre virksomhedskarakteristika end størrelse, er fallitsandsynligheden for den gennemsnitlige lille virksomhed lige så konjunkturfølsom eller mere konjunkturfølsom end fallitsandsynligheden for den gennemsnitlige store virksomhed. Vores resultater holder både i en multiplikativ Cox-model og en additiv Aalen-model.

Chapter 3

English summary. I show theoretically and empirically that cash holdings can help rationalize the low returns to distressed equity. In my model, levered firms with financing constraints optimally choose their cash holdings to manage liquidity risk and optimally default when insolvent. Using data on solvency, liquidity, and returns for US firms, I find evidence consistent with the model's predictions: (1) In all solvency levels, the average firm holds enough liquid assets to cover its short-term liabilities; less solvent firms have (2) a higher fraction of their total assets in liquid assets and therefore (3) lower conditional betas and (4) lower returns; (5) the profits of long-short solvency strategies are highest among firms with low liquidity; and (6) the profits of long-short liquidity strategies are highest among firms with low solvency.

Dansk resumé. Jeg viser teoretisk og empirisk at likvide beholdninger kan hjælpe med at rationalisere de lave afkast til nødstedte aktier. Jeg modellerer gearede virksomheder uden adgang til ekstern finansiering. Virksomhederne vælger deres likvide beholdninger for at styre likviditetsrisiko og vælger optimalt at gå fallit når de er insolvent. Ved brug af data om solvens, likviditet og aktieafkast for amerikanske virksomheder, finder jeg evidens i overensstemmelse med modellens resultater: (1) I alle solvensniveauer har den gennemsnitlige virksomhed nok likvide aktiver til at dække sine kortfristede forpligtelser; mindre solvente virksomheder har (2) en større andel af deres samlede aktiver i likvide aktiver, og derfor (3) lavere betaer og (4) lavere afkast; (5) afkastet fra lang/kort-handelstrategier baseret på solvens er højest blandt virksomheder med lav likviditet; og (6) afkastet fra lang/kort-handelstrategier baseret på likviditet er højest blandt virksomheder med lav solvens.

Bibliography

- Acharya, V. V., S. A. Davydenko, and A. Strebulaev (2012). Cash Holdings and Credit Risk. *Review of Financial Studies* 25(12), 3572–3609.
- Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance* 23, 589–609.
- Avramova, D., T. Chordiab, G. Jostovac, and A. Philipovd (2009). Credit ratings and the cross-section of stock returns. *Journal of Financial Markets* 12(3), 469–499.
- Banz, R. W. (1981, 18). The Relationship Between Return and Market Value of Common Stocks. *Journal of Financial Economics* 9(1), 3.
- Basu, S. (1977). Investment Performance of Common Stocks in Relation to Their Price- Earnings Ratios: A Test of the Efficient Market Hypothesis. *Journal of Finance* 12(3), 129–156.
- Beaver, W. (1966). Financial ratios and the prediction of failure. *Journal of Accounting Research. Supplement : Empirical research in accounting : Selected studies 1966* 4, 77–111.
- Bhandari, L. C. (1988). Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence. *Journal of Finance* 43(2), 507–528.
- Bharath, S. T. and T. Shumway (2008). Forecasting Default with the Merton Distance to Default Model. *Review of Financial Studies* 21(3), 1339–1369.
- Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. *Journal of Business* 45(3), 444–454.
- Black, F. and J. Cox (1976, May). Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance* 31(2), 351–367.
- Black, F., M. C. Jensen, and M. Scholes (1972). The Capital Asset Pricing Model: Some Empirical Tests. In M. C. Jensen (Ed.), *Studies in the Theory of Capital Markets*, pp. 79–121. Praeger.
- Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–654.
- Bolton, P., H. Chen, and N. Wang (2013). Market timing, investment, and risk management. *Journal of Financial Economics* 109, 40–62.
- Bonfim, D. (2009). Credit risk drivers: Evaluating the contribution of firm level information and of macroeconomic dynamics. *The Journal of Banking and Finance* 28, 281–299.
- Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In Search of Distress Risk. *The Journal of Finance* 63(6), 2899–2939.

- Carling, K., T. Jacobson, J. Lindé, and K. Roszbach (2007). Corporate credit risk modeling and the macroeconomy. *Journal of Banking and Finance* 31(3), 845–868.
- Chan, K. C. and N.-F. Chen (1991). Structural and Return Characteristics of Small and Large Firms. *Journal of Finance* 46(4).
- Chava, S. and R. Jarrow (2004). Default risk and diversification : Theory and empirical implications. *Review of Finance* 8(4), 537–569.
- Crosbie, P. J. and J. R. Bohn (2002). Modeling default risk. Technical report, Moody's KMV, LLC.
- Das, S., D. Duffie, N. Kapadia, and L. Saita (2007). Common failings: How corporate defaults are correlated. *Journal of Finance* 62, 93–117.
- Dichev, I. A. (1998). Is the risk of bankruptcy a systematic risk? *Journal of Finance* 53(3), 1131–1147.
- Duffee, G. (1999). Estimating the price of default risk. *Review of Financial Studies* 12, 197–226.
- Duffie, D. (1999). Credit swap valuation. *Financial Analysts Journal* 55(1), 73–87.
- Duffie, D., A. Eckner, G. Horel, and L. Saita (2009, October). Frailty correlated default. *The Journal of Finance* LXIV(5), 2089–2123.
- Duffie, D. and D. Lando (2001). Term structures of credit spreads with incomplete accounting information. *Econometrica* 58, 635–665.
- Duffie, D., L. H. Pedersen, and K. J. Singleton (2003). Modeling sovereign yield spreads: A case study of Russian debt. *Journal of Finance* 58(1), 119–159.
- Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics* 83, 635–665.
- Duffie, D. and K. Singleton (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies* 12(2), 687–720.
- Fama, E. F. and K. French (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance* 42(2), 427–465.
- Fama, E. F. and K. R. French (1993). Common risk factors in the returns of stocks and bonds. *Journal of Financial Economics* 33, 3–56.
- Fama, E. F. and K. R. French (1995). Size and Book-to-Market Factors in Earnings and Returns. *Journal of Finance* 50(1), 131–155.
- Fama, E. F. and K. R. French (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance* 51(1), 55–84.
- Fama, E. F. and J. D. MacBeth (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81(3), 607–636.
- Fan, H. and S. Sundaresan (2000). Debt Valuation, Renegotiation, and Optimal Dividend Policy. *Review of Financial Studies* 13(4), 1057–1099.
- Fischer, E. O., R. Heinkel, and J. Zechner (1989). Dynamic Capital Structure Choice: Theory and Tests. *Journal of Finance* 44(1), 19–40.
- Garlappi, L., T. Shu, and H. Yan (2008). Default Risk, Shareholder Advantage, and Stock Returns. *Review of Financial Studies* 21(6), 2743–2778.

- Garlappi, L. and H. Yan (2011). Financial Distress and the Cross-section of Equity Returns. *Journal of Finance* 66(3), 789–822.
- Griffen, J. M. and M. L. Lemmon (2002). Book-to-market equity, distress risk and stock returns. *The Journal of Finance* 57(5), 2317–2336.
- Gryglewicz, S. (2011). A theory of corporate financial decisions with liquidity and solvency concerns. *Journal of Financial Economics* 99, 365–384.
- Hillegeist, S. A., E. K. Keating, D. P. Cram, and K. G. Lundstedt (2004). Assessing the probability of bankruptcy. *Review of Accounting Studies* 9, 5–34.
- Jarrow, R., D. Lando, and S. Turnbull (1997). A Markov model for the term structure of credit risk spreads. *Review of Financial Studies* 10(2), 481–523.
- Jarrow, R. and S. Turnbull (1995). Pricing options on financial securities subject to credit risk. *Journal of Finance* 50, 53–85.
- Jeanblanc-Picqué, M. and A. N. Shiryaev (1995). Optimization of the flow of dividends. *Russian Mathematical Surveys* 50, 257–277.
- Lando, D. (1998). On cox processes and credit risky securities. *Review of Derivatives Research* 2, 99–120.
- Lando, D., M. Medhat, M. S. Nielsen, and S. F. Nielsen (2013). Additive Intensity Regression Models in Corporate Default Analysis. *Journal of Financial Econometrics* 11(3), 443–485.
- Lando, D. and M. S. Nielsen (2010). Correlation in Corporate Defaults: Contagion or Conditional Independence? *Journal of Financial Intermediation* 19(3), 335–372. <http://ssrn.com/paper=1338381>.
- Leland, H. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* 49(4), 1213–1252.
- Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics* 47(1), 13–37.
- Madan, D. and H. Unal (1998). Pricing the risks of default. *Review of Derivatives Research* 2, 121–160.
- McQuade, T. J. (2013). Stochastic volatility and asset pricing puzzles. *Working Paper, Stanford GSB*.
- Merton, R. C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica* 41(5), 867–887.
- Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29, 449–470.
- Ohlsen, J. S. (1980). Financial ratios and the probabilistic prediction of bankruptcy. *Journal of Account. Res.* 19, 109–31.
- Opp, C. C. (2013). Learning, Active Investors, and the Returns of Financially Distressed Firms. *Working paper, University of Pennsylvania - The Wharton School*. Available at SSRN: <http://ssrn.com/abstract=2181441>.
- Radner, R. and L. Shepp (1996). Risk vs. profit potential: A model for corporate strategy. *Journal of Economic Dynamics and Control* 20, 1373–1393.
- Rosenberg, B., K. Reid, and R. Lanstein (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management Spring*(11), 9–17.

- Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance* 19(3), 425–442.
- Shumway, T. (2001). Forecasting bankruptcy more efficiently : A simple hazard model. *Journal of Bus.* 74, 101–124.
- Stambaugh, R. F. (1982). On The Exclusion of Assets from Tests of the Two-Parameter Model: A Sensitivity Analysis. *Journal of Financial Economics* 10(3), 237–268.
- Stattman, D. (1980). Book Values and Stock Returns. *The Chicago MBA: A Journal of Selected Papers* 4, 25–45.
- Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *Journal of Finance* 59, 831–868.
- Zmijewski, M. E. (1984). Methodological issues related to the estimation of financial distress prediction models. *Journal of Account. Res.* 22, 59–82.

Chapter 1

Additive Intensity Regression Models in Corporate Default Analysis

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1.1 Introduction

Intensity regression models provide flexible and powerful tools for studying one of the most basic questions of credit risk modeling: Which observable variables influence the default risk of corporations? Consequently, the models are useful for risk management of loan portfolios by providing a strong statistical basis for credit scoring. They also play an important role in academic studies investigating risk premia on corporate bonds or credit default swaps. There, the purpose of the hazard regressions is to provide estimates of the “physical” or “real-world” default probabilities. Combining these with “implied” or “risk-neutral” default probabilities obtained from prices of financial instruments, we can measure the risk premium required by investors for assuming default risk.

The statistical analysis of default data dates back at least to [Beaver \(1966\)](#) and [Altman \(1968\)](#), but the use of survival analysis techniques (parametric and semiparametric Cox regressions and non-parametric methods) is more recent. Examples of this literature includes [Shumway \(2001\)](#), [Lando and Skødeberg \(2002\)](#), [Fledelius, Lando, and Nielsen \(2004\)](#), [Couderc and Renault \(2004\)](#), [Das et al. \(2007\)](#), [Duffie et al. \(2007\)](#), [Chava, Stefanescu, and Turnbull \(2011\)](#), [Duffie et al. \(2009\)](#), [Lando and Nielsen \(2010\)](#), [Figlewski, Frydman, and Liang \(2012\)](#).

The studies that employ regression models typically look at Cox models. Some of these studies use a non-parametric baseline intensity – others used a constant baseline. But all other parameters remain fixed over time. Furthermore, for reasons discussed later in the paper, there is often very little model checking after the insignificant explanatory variables have been eliminated.

In this paper, we use additive Aalen models as an alternative to the Cox model. Using both non-parametric and semi-parametric versions we are able to study not only whether explanatory variables are significant, but also whether their effects vary with time. Both graphical techniques and formal tests are employed. To allow for a comparison with a Cox regression study by [Lando and Nielsen \(2010\)](#), we use the exact same data set.

We find that both the additive structure and the use of time varying coefficients change the conclusions of [Lando and Nielsen \(2010\)](#) somewhat. Model checking leads us to identify outliers from the data and helps us resolve problems with model misspecification. We find evidence of time-variation in the effects of distance-to-default and short-to-long

term debt, and we identify the effect of interactions between distance-to-default and two other covariates: the quick ratio and (log) pledgable assets. Before removing outlier, the quick ratio covariate is insignificant – this, however, changes after the removal of outliers. None of our macroeconomic covariates are significant which may indicate that their effects are captured through their influence on firm-specific covariates.

The flow of the paper is as follows: We first recall the specification of non-parametric and semi-parametric Aalen models. We then summarize the estimation and testing procedures used, and after a data review we set up a model using a time-varying baseline intensity and firm-specific variables only. This leads us to conclude that the effects of certain firm-specific covariates are time-varying and that apparent model misspecifications may be resolved by including two interaction terms. We then replace the time varying baseline intensity by a constant baseline and global covariates (including a trailing monthly default rate), and replace regression functions by constant parameters for those firm-specific covariates which did not have significantly time-varying effects. After describing and implementing our model checking procedure, we discover that we need to remove several outliers from the data. We finish up by testing the revised model and looking at the model check once more, before we conclude.

1.2 General model setup

Intensity models of default focus on describing the default time(s) of a debt-issuing firm through a stochastic intensity process. Fix a probability space (Ω, \mathcal{F}, P) and a finite time horizon $[0, T]$. For a cohort of n firms, the default-history of firm i is summarized by a piecewise-constant, right-continuous counting process $(N_{it})_{t \in [0, T]}$ with jumps of size 1 at the firm's default times. The counting processes are assumed adapted to a common filtration $(\mathcal{F}_t)_{t \in [0, T]}$, corresponding to the flow of information. In the absolute continuous case, the default intensity of firm i is the non-negative, integrable, (\mathcal{F}_t) -predictable process $(\lambda_{it})_{t \in [0, T]}$ such that

$$M_{it} = N_{it} - \int_0^t \lambda_{is} ds$$

is an (\mathcal{F}_t) -local martingale. Intuitively,

$$\lambda_{it} = \lim_{h \rightarrow 0} \frac{1}{h} E(N_{i(t+h)} - N_{it} \mid \mathcal{F}_t) = \lim_{h \rightarrow 0} \frac{1}{h} P(N_{i(t+h)} - N_{it} = 1 \mid \mathcal{F}_t), \quad (1.1)$$

so λ_{it} is the \mathcal{F}_t -conditional mean arrival rate of default: Given \mathcal{F}_t and survival up to time t , firm i 's probability of default within $[t, t+h)$ is $\lambda_{it}h + o(h)$.

Usually, $(N_{it})_{t \in [0, T]}$ is a one-jump process, corresponding to a single default. In this case, $N_{it} = \mathbf{1}_{(\tau_i \leq t)}$, where τ_i is the single stochastic default-time for firm i . However, ambiguous definitions of real-world defaults and the possibility of restructuring will cause some firms to have several registered defaults over time. We therefore allow the counting processes in this paper to take any nonnegative integer value.

In the following, we will write the intensity as

$$\lambda_{it} = Y_{it}\alpha_i(t), \quad (1.2)$$

where $(Y_{it})_{t \in [0, T]}$ is a left-continuous, (\mathcal{F}_t) -predictable ‘‘at-risk indicator process,’’ taking the value 1 if firm i is at risk of defaulting just before time t , and 0 otherwise. Here, $\alpha_i(t)$ is the (\mathcal{F}_t) -predictable ‘‘pre-default’’ intensity that may depend on covariates and past events. With a slight abuse of language we will also refer to $\alpha_i(t)$ as the intensity of default of firm i .

In regression models, the variation in the intensities across firms is solely due to covariates. This means that $\alpha_i(t) = \alpha(t | \mathbf{x}_{it})$ for a common function α , specifying the functional form of dependency on a p -dimensional, locally bounded vector $\mathbf{x}_{it} = (x_{i1,t}, \dots, x_{ip,t})^T$ of covariate values at time t for firm i . The covariates might be constant (industry classification, for example), but will in this paper always be time-varying. Some covariates will be specific to firm i , and some will be macroeconomic variables shared by all firms. The predictability condition on the intensity then boils down to predictability of covariates. In practice, this means that covariate values entering the models at time t are required to be known just before time t .

The focus of this paper is the specification of $\alpha(t | \mathbf{x}_{it})$, i.e. determining which firm-specific and macroeconomic variables are significant explanatory variables, and how well α describes the data.

1.2.1 Relative and excess survival regression: The Cox and Aalen models

The Cox model was introduced by [Cox \(1972\)](#) in a survival data setting and extended to the general counting process framework by [Andersen and Gill \(1982\)](#). In this model, the intensity for firm i as

$$\alpha(t | \mathbf{x}_{it}) = \alpha_0(t) \exp(\boldsymbol{\beta}^T \mathbf{x}_{it}),$$

where $\alpha_0(t)$ is a locally integrable baseline intensity, which is left unspecified, while the vector $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^T$ of regression coefficients gives the time-constant effects of the covariates. The baseline intensity $\alpha_0(t)$ corresponds to the default intensity at time t when all covariates are identically equal to zero. The model thus assumes that all firm-specific intensities are proportional to the same baseline intensity.

Consider two time- t covariate vectors \mathbf{x}_{1t} and \mathbf{x}_{2t} , and assume that these are identical except for the j th coordinate, where $x_{2j,t} = x_{1j,t} + 1$. Forming the ratio of the intensities then gives

$$\frac{\alpha(t | \mathbf{x}_{2t})}{\alpha(t | \mathbf{x}_{1t})} = \exp(\boldsymbol{\beta}^T (\mathbf{x}_{2t} - \mathbf{x}_{1t})) = e^{\beta_j},$$

so the effect at time t of a one-unit increase in the j th covariate, when all other covariates are kept fixed, is to multiply the intensity by the ‘‘relative risk’’ e^{β_j} . Note that e^{β_j} is constant over time – the Cox model thus assumes that covariate effects are time-invariant and proportional to a baseline intensity.

In the models due to [Aalen \(1980, 1989\)](#), covariate effects act in an additive way on a baseline intensity. In the nonparametric case, the additive model specifies the intensity for firm i as

$$\alpha(t | \mathbf{x}_{it}) = \beta_0(t) + \boldsymbol{\beta}(t)^T \mathbf{x}_{it}, \tag{1.3}$$

where $\beta_0(t)$ is a locally integrable baseline intensity, left unspecified, and $\boldsymbol{\beta}(t) = (\beta_1(t), \dots, \beta_p(t))^T$ is a vector of locally integrable regression coefficient functions, also left unspecified. The vector $\boldsymbol{\beta}(t)$ gives the time-varying effects of the covariates. The baseline $\beta_0(t)$ again corresponds to the intensity at time t when all covariates are identically equal to zero. We will also consider semiparametric versions of the additive model, where some covariate effects are time-constant parameters.

As for the Cox model, consider two time- t covariate vectors \mathbf{x}_{1t} and \mathbf{x}_{2t} that are identical except for the j th coordinate, where $x_{2j,t} = x_{1j,t} + 1$. Subtracting the intensities then gives

$$\alpha(t | \mathbf{x}_{2t}) - \alpha(t | \mathbf{x}_{1t}) = \boldsymbol{\beta}(t)^T (\mathbf{x}_{2t} - \mathbf{x}_{1t}) = \beta_j(t),$$

so the effect at time t of a one-unit increase in the j th covariate, when all other covariate are kept fixed, is to add the

“excess-” or “absolute risk” $\beta_j(t)$ to the intensity.

In classical survival applications, the time-scale is usually duration-time, where t is measured as age or time from entry to exit for each subject at risk. In historical default studies, however, the natural time-scale is calendar-time. The fact that this does not vary across firms makes it impossible to simultaneously identify a time-dependent baseline intensity and the effects of global time-dependent covariates. In the Cox model, this is handled by using a constant baseline, and similarly in the additive models, one sets $\beta_0(t) = \theta_0$ for all t and some real parameter θ_0 , which leads to a semiparametric additive model.

1.2.2 Contrasting the Cox and the Aalen models

Using additive models for default intensities is unconventional, since one would a priori prefer models where intensities are forced to stay positive. In fact, an advantage of the Cox model is that intensities are born strictly positive. In additive models, we always run the risk of negative intensities, either as a result of estimation, or when extrapolating to more extreme covariate values.

Why, then, do we propose applying additive models in default studies? A main reason is the flexibility gained by relaxing the assumption of time-constant covariate effects. The additive models allow for simple estimation of time-varying effects using least-squares methods known from ordinary linear regression, and the resulting estimators are on a closed form that is easy to interpret and study. No smoothing is needed during estimation. There are extensions of the Cox model that incorporate time-varying effects, they require an iterative estimation procedure with smoothing in each iteration, which does not produce closed-form estimators and may blur the time-variation of effects due to repeated smoothing (see [Zucker and Karr \(1990\)](#) or [Martinussen and Scheike \(2006\)](#) and references therein).

The additive structure of the Aalen model is more robust towards model misspecification than the multiplicative structure of the Cox model. When we talk about model misspecification in this paper, it is in the sense of misspecification of the intensity. This may be a question of omitting relevant covariates or including them in an incorrect functional form (e.g. specifying a linear effect of a covariate when the true effect is nonlinear).

If a covariate is omitted, this corresponds to conditioning on a smaller filtration in the conditional expectation in (1.1). With an additive structure on this conditional mean and time varying parameters, the intensity will still be additive, though the remaining covariates may need to be transformed. In a Cox model, the proportionality is ruined when covariates are omitted, and the relative risk estimates will be biased, as shown by [Struthers and Kalbfleisch \(1986\)](#).

Including covariates in the wrong functional form in an additive model is a true misspecification, but the parameter estimates will still be interpretable as e.g. linear effects (“how much does the intensity change per change in the covariate overall?”). In a Cox model, the relative risk estimates are not interpretable as relative risks when the model is not correctly specified.

In the case of a wrong functional form for covariates, $(M_{it})_{t \in [0, T]}$ is no longer a martingale. This means that traditional variance estimates, which are based on martingale theory, will be biased. We will therefore use alternative variance estimators that are robust towards this type of misspecification. Having robust variance estimates and martingale based variance estimates allows us to detect model misspecification by comparing the two. Any large differences may suggest that the martingale based variance estimator is biased and this will be due to model misspecification. This idea is quite similar to the “Information Matrix Test” of [White \(1982\)](#).

The additive structure also permits methods from ordinary linear regression to carry over to the additive models. We will, for instance, in our data analysis in Section 1.4, include linear interaction terms as a supplement to the marginal effects of covariates and use a method known from ordinary linear regression to identify potential outliers

that may be a source of model misspecification.

Studying excess- or absolute risks may give a more nuanced picture of risk-factor importance than solely relying on relative risks. Relative risks may be misleading and overstate the actual importance of risk-factors, especially if the event is relatively rare, as has historically been the case with defaults. If a one unit increase in a covariate raises annual default probabilities from, say, 1 bps to 2 bps, the covariate’s relative risk increase is 100% per year (a relative risk of 2), while its absolute risk increase is only 0.01% per year (an excess risk of 1bps). In this example, the relative risk may indicate an economically important default-predictor, while the excess risk may indicate that the same covariate is only moderately important or perhaps even economically insignificant.

As a final note, it is straightforward to obtain a non-negative estimate of the (past) intensity of a specific firm: Estimate the integrated intensity based on the parameter estimates obtained from the Aalen model and modify this estimate to be non-decreasing (by pooling adjacent violators – see [Robertson, Wright, and Dykstra \(1988\)](#)). Smoothing this modified integrated estimate will produce a non-negative estimate of the intensity.

1.2.3 Frailty and dynamic effects

Unobserved or latent effects are receiving increased attention in empirical default studies. Such effects may be due to omitted covariates or covariates subject to measurement error, but they may also correspond to effects which are actually unobservable.

There are two dominating approaches for correcting for unobserved effects in survival models. First, one may include “frailty” effects, where latent risk factors proxy unobserved effects – frailty is thus the survival analog of random effects and is often used to model dependence between event times. For instance, [Duffie, Eckner, Horel, and Saita \(2009\)](#) found evidence of a frailty process influencing historic U.S. corporate default probabilities by including a latent Ornstein-Uhlenbeck process alongside observable risk factors in a Cox model. A key purpose of this paper is to conduct a thorough check of time-varying effects, functional form, and interactions, hoping to detect possible sources of misspecification that might otherwise show up as frailty effects. We therefore do not include frailty effects in our paper but focus on means of teasing out more information on the effects of observable variables.

Second, one may use the internal history of the observed counting processes as a correction for missing effects. This is done by including time-dependent covariates directly linked to the observed history of the counting processes in the regression models. We follow [Aalen, Fekjær, Borgan, and Husebye \(2004\)](#), who included such covariates in an additive setting similar to ours, and call such covariates “dynamic.” In our data analysis in Section 1.4, we will include a global dynamic covariate in the form of the trailing monthly default rate for the previous month as a correction for unobserved effects in an additive regression. This dynamic covariate may at a given time be viewed as reflecting the instantaneous default risk in the cohort. A significant effect may thus suggest that the additive model at hand is missing global effects that drive default intensities upwards.

1.3 Additive regression models

In this section, we describe how the nonparametric and semiparametric additive models are estimated, and how we test the relevant hypotheses of significance and possible time-variation of regression coefficients.

The focus is on two main model structures. First, the nonparametric additive model (1.3) with all covariate effects as unspecified functions of time, restated here for convenience

$$\alpha(t | \mathbf{x}_{it}) = \beta_0(t) + \beta_1(t)x_{i1,t} + \cdots + \beta_p(t)x_{ip,t}. \quad (1.4)$$

Second, the semiparametric sub-model first introduced by [MacKeague and Sasieni \(1994\)](#), where more structure is put on some of the regression coefficients. This is of interest when some effects are believed to be time-invariant, but also when including global time-dependent covariates of both the macroeconomic or dynamic type. Covariates with time-varying effects are collected in the p -dimensional covariate vector \mathbf{x}_{it} , whereas the q -dimensional vector $\mathbf{z}_{it} = (z_{i1,t}, \dots, z_{iq,t})^T$ captures the time-invariant effects. Both are assumed to be predictable and locally bounded. The semiparametric additive model then specifies the intensity as

$$\alpha(t | \mathbf{x}_{it}, \mathbf{z}_{it}) = \beta_0(t) + \beta_1(t)x_{i1,t} + \dots + \beta_p(t)x_{ip,t} + \theta_1 z_{i1,t} + \dots + \theta_q z_{iq,t}, \quad (1.5)$$

where $\beta_0(t), \dots, \beta_p(t)$ are as before while $\theta_1, \dots, \theta_q$ are real-valued parameters giving the time-invariant effects of a one-unit increase in each component of \mathbf{z}_{it} . Note that, as discussed in Section 1.2.1, $\beta_0(t) = \theta_0$ when we include global time-dependent covariates in the regressions, in order to make all parameters identifiable. [Martinussen and Scheike \(2006\)](#) propose a resampling-based inference procedure that allows the time-invariance of effects to be tested, so that an initial nonparametric additive model may be reduced to a semiparametric. This is implemented in the `aalen`-function as a part of their `timereg` package in R ([R Development Core Team \(2011\)](#)) which will be used in the data analyses of Sections 1.4 and 1.6.

1.3.1 The nonparametric additive regression model

In the general model (1.4), the regression functions $\beta_j(t)$ for $j = 0, 1, \dots, p$ are unrestricted and consequently difficult to estimate nonparametrically. However, similar to estimating a cumulative distribution function rather than a density or a cumulative hazard instead of the hazard itself, it turns out that the cumulative regression functions,

$$B_j(t) = \int_0^t \beta_j(s) ds, \quad j = 1, \dots, p,$$

are easier to estimate than the regression functions themselves. As in the case with the density or hazard, estimators of the regression functions may be obtained by smoothing the cumulative estimates.

Estimation

The basic idea is to estimate the cumulative regression coefficients by step functions. We can write the increment of $(N_{it})_{t \in [0, T]}$ over the small time interval $[t, t + dt)$ as

$$dN_{it} = Y_{it} dB_0(t) + \sum_{j=1}^p Y_{it} x_{ij,t} dB_j(t) + dM_{it}, \quad (1.6)$$

where

$$M_{it} = N_{it} - \int_0^t \left(Y_{is} \beta_0(s) + \sum_{j=1}^p Y_{is} x_{ij,s} \beta_j(s) \right) ds$$

defines a local martingale $(M_{it})_{t \in [0, T]}$ due to the assumed local boundedness of $x_{ij,t}$ and local integrability of $\beta_j(t)$ for all i and j . At each time t , the model (1.6) has the form of an ordinary linear regression with dN_{it} as the response, $Y_{it} x_{ij,t}$ as the predictors, $dB_j(t) = \beta_j(t) dt$ as the parameters of interest, and dM_{it} as the noise. For the cohort of n firms,

the full model may thus be written as

$$d\mathbf{N}_t = \mathbf{X}_t d\mathbf{B}(t) + d\mathbf{M}_t, \quad (1.7)$$

where $\mathbf{N}_t = (N_{1t}, \dots, N_{nt})^T$, $\mathbf{B}(t) = (B_0(t), \dots, B_p(t))^T$, and $\mathbf{M}_t = (M_{1t}, \dots, M_{nt})^T$, while \mathbf{X}_t is the $n \times (1 + p)$ -dimensional, locally bounded matrix with i th row $(Y_{it}, Y_{it}x_{i1,t}, \dots, Y_{it}x_{ip,t})$. When \mathbf{X}_t has full rank, the ordinary least squares estimator of the increment of $\mathbf{B}(t)$ is given by

$$\widehat{d\mathbf{B}}(t) = \mathbf{X}_t^- d\mathbf{N}_t,$$

where $\mathbf{X}_t^- = (\mathbf{X}_t^T \mathbf{X}_t)^{-1} \mathbf{X}_t^T$ is the usual least squares generalized inverse of \mathbf{X}_t . When \mathbf{X}_t has less than full rank, $d\mathbf{B}(t)$ is not identifiable from the data, and we put $\widehat{d\mathbf{B}}(t) = 0$. Also, note that $\widehat{d\mathbf{B}}(t) = 0$ when $d\mathbf{N}_t = 0$, so that all the increments are at default times.

To obtain an estimator for the vector $\mathbf{B}(t)$ of cumulative regression functions, we let $J(t)$ be the indicator of $\mathbf{X}(t)$ having full rank and aggregate $\widehat{d\mathbf{B}}(t)$ over the ordered default-times $\tau_1 < \tau_2 < \dots$ to obtain

$$\widehat{\mathbf{B}}(t) = \int_0^t J(s) \mathbf{X}_s^- d\mathbf{N}_s = \sum_{\tau_k \leq t} J(\tau_k) \mathbf{X}_{\tau_k}^- \Delta \mathbf{N}_{\tau_k}, \quad (1.8)$$

where $\Delta \mathbf{N}_{\tau_k}$ is a vector of zeros except for a one at the component corresponding to the firm with a default at τ_k . Note that when there are no covariates in the model (i.e. $p = 0$), the estimator (1.8) is just the usual Nelson-Aalen estimator of the cumulative hazard – in this sense, the nonparametric additive model is the natural generalization of nonparametric hazard estimation to the situation with covariates.

Using (1.7), we obtain

$$\begin{aligned} \widehat{\mathbf{B}}(t) - \mathbf{B}(t) &= \int_0^t J(s) \mathbf{X}_s^- d\mathbf{M}_s + \int_0^t (J(s) - 1) d\mathbf{B}(s) \\ &= \int_0^t J(s) \mathbf{X}_s^- d\mathbf{M}_s + o_P\left(\frac{1}{\sqrt{n}}\right), \end{aligned}$$

where the last equality holds under reasonable regularity assumptions (Martinussen and Scheike, 2006). Hence, the deviation $\widehat{\mathbf{B}}(t) - \mathbf{B}(t)$ is a vector-valued local martingale except for a negligible remainder term. The asymptotic properties of the estimator (1.8) may be obtained by the martingale central limit theorem. In case of model misspecifications, the process $(\mathbf{M}_t)_{t \in [0, T]}$ is no longer a local martingale, and care must be taken when deriving the asymptotic properties of the estimator $\widehat{\mathbf{B}}(t)$. Both cases will be treated below.

When the model is well-specified, we can estimate the covariance function of $\widehat{\mathbf{B}}(t)$ by the optional variation process of the martingale part of the deviation $\widehat{\mathbf{B}}(t) - \mathbf{B}(t)$:

$$\widehat{\Sigma}_{\text{mar}}(t) = \sum_{\tau_k \leq t} J(\tau_k) \mathbf{X}_{\tau_k}^- \text{diag}(\Delta \mathbf{N}_{\tau_k}) \mathbf{X}_{\tau_k}^{-T}. \quad (1.9)$$

If the model is misspecified, $\widehat{\Sigma}_{\text{mar}}(t)$ will be biased. In this case Martinussen and Scheike (2006) show that

$$\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t)) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \mathbf{Q}_i(t) + o_P(1), \quad (1.10)$$

with

$$\mathbf{Q}_i(t) = \int_0^t J(s) (n^{-1} \mathbf{X}_s^T \mathbf{X}_s)^{-1} X_{is}^T (dN_{is} - X_{is} d\mathbf{B}(s)),$$

and where X_{it} is the i th row of \mathbf{X}_t . When $n \rightarrow \infty$, $J(t) (n^{-1} \mathbf{X}_t^T \mathbf{X}_t)^{-1}$ converges in probability, and (1.10) is a normalized sum of independent and identically distributed processes. The covariance function of $\widehat{\mathbf{B}}(t)$ is consistently estimated by

$$\widehat{\boldsymbol{\Sigma}}_{\text{rob}}(t) = \frac{1}{n} \sum_{i=1}^n \widehat{\mathbf{Q}}_i(t) \widehat{\mathbf{Q}}_i(t)^T, \quad (1.11)$$

where $\widehat{\mathbf{Q}}_i(t)$ is obtained by replacing $\mathbf{B}(t)$ with its estimator in $\mathbf{Q}_i(t)$. Since $\widehat{\boldsymbol{\Sigma}}_{\text{rob}}(t)$ is derived without use of the local martingale property of $(\mathbf{M}_t)_{t \in [0, T]}$, it is robust to model misspecifications of the intensity, and is therefore a robust covariance function estimator.

Large sample properties

When the model is correctly specified and regularity conditions are fulfilled, it follows from the martingale central limit theorem that the normalized deviation $\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t))$ converges (for $n \rightarrow \infty$ and fixed T) in distribution to a mean-zero multivariate Gaussian martingale with a covariance function that may be estimated consistently by the martingale-based estimator $\widehat{\boldsymbol{\Sigma}}_{\text{mar}}(t)$. In case of misspecifications, when $(M_{it})_{t \in [0, T]}$ is not a martingale, the main asymptotic result is instead that $\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t))$, using (1.10), converges in distribution to a mean-zero Gaussian process (but not a martingale) with a covariance function which may be estimated consistently by the robust estimator $\widehat{\boldsymbol{\Sigma}}_{\text{rob}}(t)$. [Martinussen and Scheike \(2006\)](#) give the details and proofs.

The asymptotic results imply that an approximate $100(1 - \alpha)\%$ martingale-based pointwise confidence band for the j th cumulative regression coefficient is given by

$$\widehat{B}_j(t) \pm z_{1-\alpha/2} \sqrt{\widehat{\sigma}_{\text{mar},jj}^2(t)},$$

where $z_{1-\alpha/2}$ is the $(1 - \alpha/2)$ -quantile of the standard normal distribution and $\widehat{\sigma}_{\text{mar},jj}^2(t)$ is the j th diagonal element of $\widehat{\boldsymbol{\Sigma}}_{\text{mar}}(t)$. Alternatively, an approximate $100(1 - \alpha)\%$ robust confidence band for the j th cumulative regression coefficient may be obtained as

$$\widehat{B}_j(t) \pm z_{1-\alpha/2} \sqrt{\widehat{\sigma}_{\text{rob},jj}^2(t)},$$

where $\widehat{\sigma}_{\text{rob},jj}^2(t)$ is the j th diagonal element of $\widehat{\boldsymbol{\Sigma}}_{\text{rob}}(t)$.

In large samples, if the two types of variance estimates (i.e. the martingale-based and the robust) are markedly different, it is an indication of model misspecification.

Kernel smoothing

$\widehat{B}_j(t)$ estimates the cumulated regression function $\int_0^t \beta_j(s) ds$. However, we are really interested in its slope – i.e. the regression function $\beta_j(t)$ itself. As assessing the slope of a step function graphically may be difficult, we smooth the estimators of the cumulative regression functions to obtain estimators of the regression functions themselves.

We will use kernel function smoothing, as first proposed in a survival model setting by [Ramlau-Hansen \(1983\)](#).

Then $\boldsymbol{\beta}(t) = (\beta_0(t), \dots, \beta_p(t))$ is estimated at time t as a weighted sum of the increments of $\widehat{\mathbf{B}}(t)$ over the interval $[t - b, t + b]$,

$$\widehat{\boldsymbol{\beta}}(t) = \frac{1}{b} \sum_{\tau_k} K\left(\frac{t - \tau_k}{b}\right) \Delta \widehat{\mathbf{B}}(\tau_k), \quad (1.12)$$

where $b > 0$ is a bandwidth, determining the size of the interval $[t - b, t + b]$, while $K(x)$ is a bounded kernel-function vanishing outside $[-1, 1]$ and integrating to 1, determining the weights. A typical choice is the Epanechnikov kernel, where $K(x) = \frac{3}{4}(1 - x^2)$ for $|x| \leq 1$, and zero otherwise, but [Aalen, Borgan, and Gjessing \(2008\)](#) discuss other kernels. Like other kernel estimators, (1.12) suffers from boundary effects: For small values of t (i.e. when $t - b < 0$) the estimator is severely biased towards zero. We handle this problem by using a boundary kernel, as also discussed by [Aalen et al. \(2008\)](#).

When the model is correctly specified and $(\mathbf{M}_t)_{t \in [0, T]}$ is a vector-valued local martingale, an estimator of the covariance function of $\widehat{\boldsymbol{\beta}}(t)$ is simply obtained as

$$\widehat{\text{Cov}} \widehat{\boldsymbol{\beta}}(t) = \frac{1}{b^2} \sum_{\tau_k} K\left(\frac{t - \tau_k}{b}\right)^2 \Delta \widehat{\boldsymbol{\Sigma}}_{\text{mar}}(\tau_k),$$

where $\Delta \widehat{\boldsymbol{\Sigma}}_{\text{mar}}(\tau_k) = J(\tau_k) \mathbf{X}_{\tau_k}^- \text{diag}(\Delta \mathbf{N}_{\tau_k}) \mathbf{X}_{\tau_k}^{-T}$ is the increment of the martingale-based covariance function estimator (1.9). If the model is misspecified, the covariance function of the function $\widehat{\boldsymbol{\beta}}(t)$ has the same form as above, but with $\Delta \widehat{\boldsymbol{\Sigma}}_{\text{mar}}(\tau_k)$ replaced by $\Delta \widehat{\boldsymbol{\Sigma}}_{\text{rob}}(\tau_k)$, i.e, the increment of the robust covariance function estimator (1.11). These covariance function estimators may be combined with the large sample properties of $\widehat{\mathbf{B}}(t)$ to construct approximate confidence bands for $\widehat{\boldsymbol{\beta}}(t)$ in the usual way.

There exist techniques (such as cross-validation) for objectively choosing the bandwidth with an optimal (in some sense) trade-off between bias and variances, but these will not be considered in this paper. We simply rely on a subjective assessment of what is a reasonable degree of smoothing.

Resampling-based inference

The tests we use will be based on a resampling procedure presented and implemented by [Martinussen and Scheike \(2006\)](#) – however, [Aalen et al. \(2008\)](#) give a more traditional martingale-based approach.

We are primarily interested in testing two hypotheses: The hypothesis of no effect of the j th covariate,

$$H_0^{\text{effect}} : \beta_j(t) = 0 \quad \text{for all } t \in [0, T].$$

and the hypothesis of a time-invariant effect of the j th covariate,

$$H_0^{\text{time}} : \beta_j(t) = \theta_j \quad \text{for all } t \in [0, T].$$

The null of H_0^{time} is the semi-parametric additive model with a parametric coefficient for the effect of the j th covariate. Both hypotheses may be of interest over a shorter time-interval than the entire study time, but this is usually not the case, and will not be considered here. Since the estimated cumulative regression coefficients have nicer distributional properties than their smoothed counterparts, the hypotheses are usually formulated in the equivalent forms

$$H_0^{\text{effect}} : B_j(t) = 0 \quad \text{and} \quad H_0^{\text{time}} : B_j(t) = \theta_j t,$$

both for all $t \in [0, T]$.

Martinussen and Scheike (2006) propose testing the hypothesis of no influence, H_0^{effect} , by the supremum test statistic

$$T_{\text{sup}} = \sup_{t \in [0, T]} \left| \frac{\sqrt{n} \widehat{B}_j(t)}{\sqrt{\widehat{\sigma}_{\text{rob},jj}^2(t)}} \right|, \quad (1.13)$$

where, again, $\widehat{\sigma}_{\text{rob},jj}^2(t)$ is the j th diagonal element of $\widehat{\Sigma}_{\text{rob}}(t)$ given in (1.11). This evaluates the maximal deviation of the estimated cumulative regression coefficient $\widehat{B}_j(t)$ from the zero function, relative to its variation.

With regards to testing the hypothesis of time-invariance, H_0^{time} , Martinussen and Scheike (2006) propose the process

$$\sqrt{n} \left(\widehat{B}_j(t) - \frac{\widehat{B}_j(T)}{T} t \right) \text{ for } t \in [0, T] \quad (1.14)$$

as a basic starting point for evaluating the time-invariance of the j th regression coefficient – the idea is that $\widehat{B}_j(T)/T$ is an estimator of the time-constant coefficient θ_j under the null. This process may then be turned into the Kolmogorov-Smirnov test statistic

$$T_{\text{KS}} = \sup_{t \in [0, T]} \sqrt{n} \left| \widehat{B}_j(t) - \frac{\widehat{B}_j(T)}{T} t \right|, \quad (1.15)$$

or the Cramer-von Mises test statistic

$$T_{\text{CvM}} = n \int_0^T \left(\widehat{B}_j(t) - \frac{\widehat{B}_j(T)}{T} t \right)^2 dt. \quad (1.16)$$

The former is a maximal deviations test statistic, sensitive to single large deviations from the null, while the latter is a sum of squared deviations type statistic, sensitive to small but persistent deviations from the null.

To test the hypotheses, Martinussen and Scheike (2006) propose evaluating the variability of test statistics through a resampling-scheme that approximates the distribution of the estimated vector of cumulative regression coefficients $\widehat{\mathbf{B}}(t)$. Based on the iid. representation (1.10), their main result is that, conditional on the data $(N_{it}, Y_{it}, \mathbf{x}_{it})$ for $i = 1, \dots, n$, the normalized deviation $\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t))$ has the same limiting distribution as

$$\mathbf{R}(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^n u_i \widehat{\mathbf{Q}}_i(t),$$

where u_1, \dots, u_n are iid. standard normally distributed stochastic variables, and $\widehat{\mathbf{Q}}_i(t)$ is as in (1.11). The result utilizes the “conditional multiplier central limit theorem” (van der Vaart and Wellner, 1996). Note that $\mathbf{R}(t)$ a weighted sum of the observable $\widehat{\mathbf{Q}}_i(t)/\sqrt{n}$ with standard normally distributed weights. In fact, any normalized distribution could have been used for the weights, but the choice of a normal distribution fits well with the limiting distribution.

Obtaining P -values for the tests may be done through replication of $\mathbf{R}(t)$. The idea is to hold the observed data fixed whilst repeatedly generating series of iid. standard normal variables $u_1^{(r)}, \dots, u_n^{(r)}$, and approximating the distribution

of T_{sup} by the empirical distribution of the processes

$$\sup_{t \in [0, T]} \left| \frac{\sqrt{n} R_j^{(r)}(t)}{\sqrt{\widehat{\sigma}_{\text{rob}, jj}^2(t)}} \right|; \quad r = 1, 2, \dots,$$

where $R_j^{(r)}(t)$ denotes the r th resample of the j th element of $\mathbf{R}(t)$. Similarly, approximations of the distributions of T_{KS} and T_{CVM} are obtained by approximating the distribution of the process (1.14) by the empirical distribution of the processes

$$\sqrt{n} \left(R_j^{(r)}(t) - \frac{R_j^{(r)}(T)}{T} t \right); \quad r = 1, 2, \dots$$

Finally, the deviation of the estimated regression coefficient from the null of time-invariance may be assessed by plotting the observed process (1.14) as a function of study time along with a number of the resampled processes under the null. A similar behavioral pattern in the observed and resampled processes would suggest consistency with the null of time-invariance. The advantage of this graphical method is that it pinpoints where in time deviations from the null might occur.

1.3.2 The semiparametric additive regression model

The methods used to estimate and conduct inference in the semiparametric Aalen model (1.5) are to a large extent similar to the ones presented in the previous sections for the nonparametric model. We will therefore only briefly outline the procedure of finding estimators and the resampling-based method of inference. [Martinussen and Scheike \(2006\)](#) give a detailed account.

Estimation

For the semiparametric Aalen model, the increments of the counting processes are given as

$$d\mathbf{N}_t = \mathbf{X}_t d\mathbf{B}(t) + \mathbf{Z}_t \boldsymbol{\theta} dt + d\mathbf{M}_t, \quad (1.17)$$

where \mathbf{N}_t , \mathbf{X}_t , $\mathbf{B}(t)$, and \mathbf{M}_t are as in (1.7), while $\boldsymbol{\theta} = (\theta_1, \dots, \theta_q)^T$ and \mathbf{Z}_t is the $n \times q$ -dimensional, locally bounded matrix with i th row $(Y_{it} z_{i1,t}, \dots, Y_{it} z_{iq,t})$.

We estimate the unknown regression functions and parameters by minimizing the integrated sum of squares,

$$\int_0^T J(t) (d\mathbf{N}_t - \mathbf{X}_t d\mathbf{B}(t) - \mathbf{Z}_t \boldsymbol{\theta} dt)^T (d\mathbf{N}_t - \mathbf{X}_t d\mathbf{B}(t) - \mathbf{Z}_t \boldsymbol{\theta} dt), \quad (1.18)$$

where, again, $J(t)$ is the indicator of \mathbf{X}_t having full rank. Re-writing $d\mathbf{N}_t - \mathbf{X}_t d\mathbf{B}(t) - \mathbf{Z}_t \boldsymbol{\theta} dt$ as the sum of its projections on $\text{span } \mathbf{X}_t$ and its orthogonal complement, we obtain, when $J(t) = 1$,

$$\mathbf{X}_t (\mathbf{X}_t^- d\mathbf{N}_t - d\mathbf{B}(t) - \mathbf{X}_t^- \mathbf{Z}_t \boldsymbol{\theta} dt) + (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) (d\mathbf{N}_t - \mathbf{Z}_t \boldsymbol{\theta} dt),$$

where \mathbf{I} is the $n \times n$ identity matrix, while, as usual, \mathbf{X}_t^- is the least squares generalized inverse of \mathbf{X}_t . Hence, (1.18)

splits into a sum of two terms,

$$\int_0^T J(t) (\mathbf{X}_t^- d\mathbf{N}_t - d\mathbf{B}(t) - \mathbf{X}_t^- \mathbf{Z}_t \boldsymbol{\theta} dt)^T \mathbf{X}_t^T \mathbf{X}_t (\mathbf{X}_t^- d\mathbf{N}_t - d\mathbf{B}(t) - \mathbf{X}_t^- \mathbf{Z}_t \boldsymbol{\theta} dt) \\ + \int_0^T J(t) (d\mathbf{N}_t - \mathbf{Z}_t \boldsymbol{\theta} dt)^T (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) (d\mathbf{N}_t - \mathbf{Z}_t \boldsymbol{\theta} dt),$$

as $(\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-)^T (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) = (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-)$ and $(\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-)^T \mathbf{X}_t = \mathbf{0}$. Minimizing the latter term first and the former second yields the estimators

$$\widehat{\boldsymbol{\theta}} = \left(\int_0^T J(t) \mathbf{Z}_t^T (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Z}_t dt \right)^{-1} \int_0^T J(t) \mathbf{Z}_t^T (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) d\mathbf{N}_t, \quad (1.19)$$

$$\widehat{\mathbf{B}}(t) = \int_0^t J(s) \mathbf{X}_s^- (d\mathbf{N}_s - \mathbf{Z}_s \widehat{\boldsymbol{\theta}} ds). \quad (1.20)$$

In the non-parametric case, the minimizer of the integrated sum of squares and the pointwise sums of squares is the same. Thus, the present estimator (1.20) generalizes the one given for the nonparametric case in (1.8).

Combining (1.19) and (1.20) with (1.17) gives the normalized deviations

$$\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = \mathbf{U}^{-1} \mathbf{M}_T^{(1)}, \quad (1.21)$$

$$\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t)) = \mathbf{M}_t^{(2)} - \mathbf{V}(t) \mathbf{U}^{-1} \mathbf{M}_T^{(1)} + o_P(1), \quad (1.22)$$

where

$$\mathbf{U} = \frac{1}{n} \int_0^T J(t) \mathbf{Z}_t^T (\mathbf{I} - \mathbf{X}_t \mathbf{X}_t^-) \mathbf{Z}_t dt, \quad \mathbf{V}(t) = \frac{1}{n} \int_0^t J(s) \mathbf{X}_s^- \mathbf{Z}_s ds,$$

$$\mathbf{M}_t^{(1)} = \frac{1}{\sqrt{n}} \int_0^t J(s) \mathbf{Z}_s (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) d\mathbf{M}_s, \quad \mathbf{M}_t^{(2)} = \frac{1}{\sqrt{n}} \int_0^t J(s) \mathbf{X}_s^- d\mathbf{M}_s.$$

When the model is correctly specified, $(\mathbf{M}_t^{(1)})_{t \in [0, T]}$ and $(\mathbf{M}_t^{(2)})_{t \in [0, T]}$ are vector-valued, local martingales (due to the assumed local boundedness of \mathbf{X}_t and \mathbf{Z}_t), thus showing that $\widehat{\boldsymbol{\theta}}$ is an unbiased estimator of $\boldsymbol{\theta}$, while $\widehat{\mathbf{B}}(t)$ is an approximately unbiased estimator of $\mathbf{B}(t)$. In case of a misspecified model, the process $(\mathbf{M}_t)_{t \in [0, T]}$ is no longer a local martingale. In the following, we handle the two cases separately when deriving variance-estimates and asymptotic properties for the estimators (1.19) and (1.20).

When the model fit is reasonable, the deviations (1.21) and (1.22) imply that a martingale-based estimator of the covariance matrix of $\widehat{\boldsymbol{\theta}}$ is given by

$$\widehat{\boldsymbol{\Psi}}_{\text{mar}} = \mathbf{U}^{-1} [\mathbf{M}^{(1)}](T) \mathbf{U}^{-1},$$

while a martingale-based estimator of the covariance function of $\widehat{\mathbf{B}}(t)$ is given by

$$\widehat{\boldsymbol{\Upsilon}}_{\text{mar}}(t) = [\mathbf{M}^{(2)}](t) + \mathbf{V}(t) \widehat{\boldsymbol{\Psi}}_{\text{mar}} \mathbf{V}(t)^T \\ - [\mathbf{M}^{(1)}, \mathbf{M}^{(2)}](t) \mathbf{U}^{-1} \mathbf{V}(t)^T - \mathbf{V}(t) \mathbf{U}^{-1} [\mathbf{M}^{(1)}, \mathbf{M}^{(2)}](t),$$

where the optional variation and covariation processes of $(\mathbf{M}_t^{(1)})_{t \in [0, T]}$ and $(\mathbf{M}_t^{(2)})_{t \in [0, T]}$ are

$$\begin{aligned} [\mathbf{M}^{(1)}](t) &= \frac{1}{n} \int_0^t J(s) \mathbf{Z}_s^T (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) \text{diag}(d\mathbf{N}_s) (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) \mathbf{Z}_s, \\ [\mathbf{M}^{(2)}](t) &= n \int_0^t J(s) \mathbf{X}_s^- \text{diag}(d\mathbf{N}_s) \mathbf{X}_s^-, \\ [\mathbf{M}^{(1)}, \mathbf{M}^{(2)}](t) &= \int_0^t J(s) \mathbf{Z}_s^T (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) \text{diag}(d\mathbf{N}_s) \mathbf{X}_s^{-T}. \end{aligned}$$

If the semiparametric additive model is misspecified, $\widehat{\Psi}_{\text{mar}}$ and $\widehat{\Upsilon}_{\text{mar}}(t)$ will be biased. To address this, [Martinussen and Scheike \(2006\)](#) show that

$$\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = \frac{1}{\sqrt{n}} \mathbf{U}^{-1} \sum_{i=1}^n \mathbf{K}_i,$$

where

$$\mathbf{K}_i = \int_0^T J(t) \left(\mathbf{Z}_{it}^T - \mathbf{Z}_t^T \mathbf{X}_t (\mathbf{X}_t^T \mathbf{X}_t)^{-1} \mathbf{X}_{it}^T \right) (dN_{it} - X_{it} d\mathbf{B}(t) - Z_{it} \boldsymbol{\theta} dt),$$

with X_{it} as the i th row of \mathbf{X}_t and Z_{it} as the i th row of \mathbf{Z}_t . When n is large, this represents the deviation $\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})$ as a normalized sum of iid. terms, showing that the covariance function of $\widehat{\boldsymbol{\theta}}$ may be estimated by the sandwich-type estimator

$$\widehat{\Psi}_{\text{rob}} = \mathbf{U}^{-1} \left(\frac{1}{n} \sum_{i=1}^n \widehat{\mathbf{K}}_i \widehat{\mathbf{K}}_i^T \right) \mathbf{U}^{-1},$$

where $\widehat{\mathbf{K}}_i$ is obtained by replacing $\boldsymbol{\theta}$ and $\mathbf{B}(t)$ with their estimators in \mathbf{K}_i . As $\widehat{\Psi}_{\text{rob}}$ was derived without relying on the martingale property, it is robust to model misspecifications. Similarly,

$$\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t)) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \mathbf{O}_i(t) + o_p(1),$$

where

$$\mathbf{O}_i(t) = \int_0^t J(s) (n^{-1} \mathbf{X}_s^T \mathbf{X}_s)^{-1} X_{is}^T (dN_{is} - X_{is} d\mathbf{B}(s) - Z_{is} \boldsymbol{\theta} ds) - \mathbf{V}(t) \mathbf{U}^{-1} \mathbf{K}_i.$$

Hence, copying the above argument, this suggests that the covariance function of $\widehat{\mathbf{B}}(t)$ may be estimated by the robust estimator

$$\widehat{\Upsilon}_{\text{rob}}(t) = \frac{1}{n} \sum_{i=1}^n \widehat{\mathbf{O}}_i(t) \widehat{\mathbf{O}}_i(t)^T. \tag{1.23}$$

Large sample properties

The asymptotic properties of $\widehat{\boldsymbol{\theta}}$ are in case of a well-specified model obtained through a standard application of the martingale central limit theorem, which under suitable regularity assumptions implies that $\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})$ converges (for

$n \rightarrow \infty$ and fixed T) in distribution to a mean-zero, multivariate normal variable with a covariance which is consistently estimated by $\widehat{\Psi}_{\text{mar}}$. If the model is not well specified, the same convergence result holds, but $\widehat{\Psi}_{\text{mar}}$ has to be replaced by $\widehat{\Psi}_{\text{rob}}$, which may also be shown to be a consistent estimator of the asymptotic covariance. The details are given by [Martinussen and Scheike \(2006\)](#).

The situation is slightly more complicated for $\widehat{\mathbf{B}}(t)$ due to the more involved nature of the corresponding deviation (1.22), which depends on $\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})$. When the model is well-specified and the necessary regularity conditions are fulfilled, [Martinussen and Scheike \(2006\)](#) show that combining the martingale central limit theorem and the continuous mapping theorem gives that $\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t))$ converges in distribution to a mean-zero, multivariate Gaussian process (i.e. not a martingale). Its asymptotic covariance function is estimated consistently by $\widehat{\mathbf{T}}_{\text{mar}}(t)$. If the model is misspecified, the same result holds, but $\widehat{\mathbf{T}}_{\text{mar}}(t)$ has to be replaced by $\widehat{\mathbf{T}}_{\text{rob}}(t)$. Note that the asymptotic distribution of $\sqrt{n}(\widehat{\mathbf{B}}(t) - \mathbf{B}(t))$ is not a martingale in either case. These results may in large samples be used to construct approximate pointwise confidence bands for $\widehat{\mathbf{B}}(t)$ in the usual way.

Resampling-based inference

Three hypotheses arise naturally when conducting inference in the semiparametric additive model. The first two are the null hypotheses of no influence and time-invariance of the time-varying regression coefficients that we also considered for the nonparametric model. The third is the simple null hypothesis $\theta_l = \tilde{\theta}$, where the l th time-constant regression coefficient is tested against the null of some known, real-valued parameter $\tilde{\theta}$. Usually, $\tilde{\theta} = 0$, in which case we are testing the influence of the l th time-constant covariate effect. This hypothesis is tested in the usual manner using e.g. the Wald test statistic, which in case of a well-specified model is given by

$$T_{\text{Wald}} = \frac{\widehat{\theta}_l - \tilde{\theta}}{\sqrt{\psi_{\text{mar},ll}^2}},$$

where $\psi_{\text{mar},ll}^2$ is the l th diagonal element of $\widehat{\Psi}_{\text{mar}}$. Alternatively, in case of a misspecified model, we instead use $\psi_{\text{rob},ll}^2$, the l th diagonal element of $\widehat{\Psi}_{\text{rob}}$. In either case, the asymptotic properties of $\widehat{\boldsymbol{\theta}}$ imply that T_{Wald} will under the null be approximately standard normally distributed in large samples.

With regards to the hypotheses of no influence and time-invariance for the a priori time-varying regression coefficients, [Martinussen and Scheike \(2006\)](#) propose a resampling-based method analogous to what was done for the nonparametric additive model. As for the nonparametric additive model, we can assess the deviation of an estimated, time-varying regression coefficient from the null of time-invariance by plotting the observed process (1.14), as a function of study time, along with a number of the resampled processes under the null.

1.4 Data and model specification

Our empirical results are based on an analysis of U.S. industrial corporate defaults occurring between 1982 and 2006. In this period, the U.S. economy suffered three major economic recessions between '81-'82, '91-'92, and '01-'02. We use the exact same data set as [Lando and Nielsen \(2010\)](#), but, for convenience, we repeat the description of the data below. Other default studies have used the same data supplemented with additional defaults from other sources – e.g. [Li and Zhao \(2006\)](#), [Das et al. \(2007\)](#), [Davydenko \(2007\)](#), and [Le \(2007\)](#). Our main purpose here is to investigate the role of time-varying coefficients and the importance of model checking in an additive intensity regression setting.

1.4.1 Data

The sample includes all U.S. industrial firms with a debt issue registered in Moody's Default Risk Service Database, or DRSD (Moody's Investor's Service©, 2010), which essentially covers the period since 1970, including universal identifiers facilitating the combination with other data sources. However, due to sparseness of data, the study time was chosen as the period between January 1st 1982 and January 1st 2006, giving a total of 289 observation months. The sample was restricted to firms for which accompanying stock market data from CRSP (Center for Research in Security Prices©, 2010) and accounting data from CompuStat (Standard & Poor's©, 2010) could be obtained, and for which there were at least 6 months of available data. All consecutive default events occurring within a 1-month horizon of any previously registered default ascribed to the same parent company were excluded from the data in order to correct for observations of multiple defaults caused by parent-subsidiary relations within the same corporate family. The final result is a study cohort of 2,557 firms comprising a total of 370 defaults.

The database classifies any of the following 9 events as constituting a default: *Chapter 7, chapter 11, distressed exchange, grace period default, missed interest payment, missed principle payment, missed principal and interest payment, prepackaged chapter 11, and suspension of payments*. In particular, we do not correct the timing of a "Distressed exchange," which in the DRSD is registered as the time of completion of the exchange, although as suggested by [Davydenko \(2007\)](#), it would probably be more appropriate to instead collect separate information on the announcement date of the exchange.

Of the 370 realized defaults in the cohort, 25 defaults happened to firms which had already defaulted earlier. To be precise, 22 firms experienced 2 defaults and a single firm defaulted 4 times.

Covariate specification

The idiosyncratic covariate specification to be employed in the regressions consists of the following 5 balance sheet variables obtained from CRSP and CompuStat:

- **Quick ratio** for the previous month, calculated as the book value of cash and short-term investments added the book value of total receivables, all relative to the book value of total current liabilities. This measures the firm's ability to use its near cash assets to immediately extinguish current liabilities.
- **Pledgeable assets** for the previous month, calculated as the book value of total current assets plus the book value of net property, plant, and equipment. This measures the firm's ability to convert liquid assets to collateral that may be transferred to a lender to secure debt.
- **Trailing 1-year equity return** for the previous month. This measures the firm's efficiency at generating profits from shareholders' equity (or assets less liabilities) and is thus a reflection of how well the firm uses investment funds to generate growth on earnings.
- **Trailing 1-year distance-to-default** for the previous month and estimated over a one-year rolling window. Our distance measure is the option-implied measure as used for example in [Duffie et al. \(2007\)](#) (see page 660 of that paper). This measures roughly how far - measured in standard deviations of log returns - assets are from hitting a default triggering boundary. The default triggering level is the sum of the notional of short-term debt and half of the notional of long-term debt.
- **Percentage short-term debt** for the previous month calculated as the book value of debt in current liabilities divided by the sum of the book value of debt in current liabilities and the book value of total long-term debt. This is a measure of the firm's vulnerability to a sudden funding shock.

All the above are time-dependent covariates which are clearly predictable due to the 1-month lagging. In case of missing monthly values, the latest quarterly observation was substituted as a proxy, and if also this was missing, the latest yearly observation was used. In addition, we examined the influence of the firm's book asset value, its equity value, and the value of its fixed assets. The book assets and the pledgeable assets were highly correlated, and including both in the analysis would give problems with collinearity. A preliminary analysis showed that our estimates are almost identical regardless of which one we choose. The equity return was preferred over the equity value since the latter seemed to cause instabilities during estimation and unintuitive results due to excessively large values and high correlation with other covariates. The fixed assets also caused instabilities during estimation, and were dropped altogether from the covariate specification.

The macroeconomic covariate specification consisted of the following 7 variables obtained from CRSP and the U.S. Federal Reserve Board:

- **Trailing 1-year return on the S&P 500-index** for the previous month. The change on this index is considered one of the major predictors for the future state of the U.S. economy.
- **Spread between yields on Moody's Baa- and AAA-rated corporate bonds** for the previous month. This is a measure of the credit risk that the market is factoring on lower grade bonds. A widening usually suggests that the market is forecasting greater credit risk due to a slowing economy.
- **Trailing 1-year percentage change in U.S. Consumer Price Index (CPI)** for the previous month. The index measures the average price of consumer goods and services purchased by households and its percentage change thus measures the level of inflation.
- **Trailing 1-year percentage change in average weekly earnings** for the previous month. This is an indicator of short-term earnings growth.
- **Trailing 1-year percentage change in U.S. domestic crude oil First Purchase Price (FPP)** for the previous month. High oil prices might have a negative impact on U.S. economic growth.
- **Spread between the 10- and 1-year U.S. Treasury yields** for the previous month. This measures the extra cost of holding long-term debt compared to the cost of holding short-term debt.
- **Trailing 1-year percentage change in U.S. unemployment rate (UR)** as percentage of civilian labor force (seasonally adjusted) for the previous month. The rate itself is a major macroeconomic indicator, and its change measures the difference between labour relationships newly broken and labour relationships newly initiated.

As for the idiosyncratic covariates, predictability is ensured through the 1-month lagging. Alternative versions of the above macroeconomic variables were also considered along with the U.S. industrial production and several versions of the U.S. gross domestic product (GDP), but only the above showed some signs of reasonable significance or did not cause instability during estimation.

Lastly, the following global dynamic covariate will be used:

- **Trailing monthly default rate** for the previous month. The trailing monthly default rate is calculated at each month as the realized number of defaults during that month relative to the number of firms at risk at the beginning of the month.

Including the lagged version, and not the monthly default rate itself, ensures predictability. This dynamic covariate may at a given time be viewed as a reflection of the instantaneous default risk in the cohort – a significant effect may thus suggest missing global effects driving default intensities upwards.

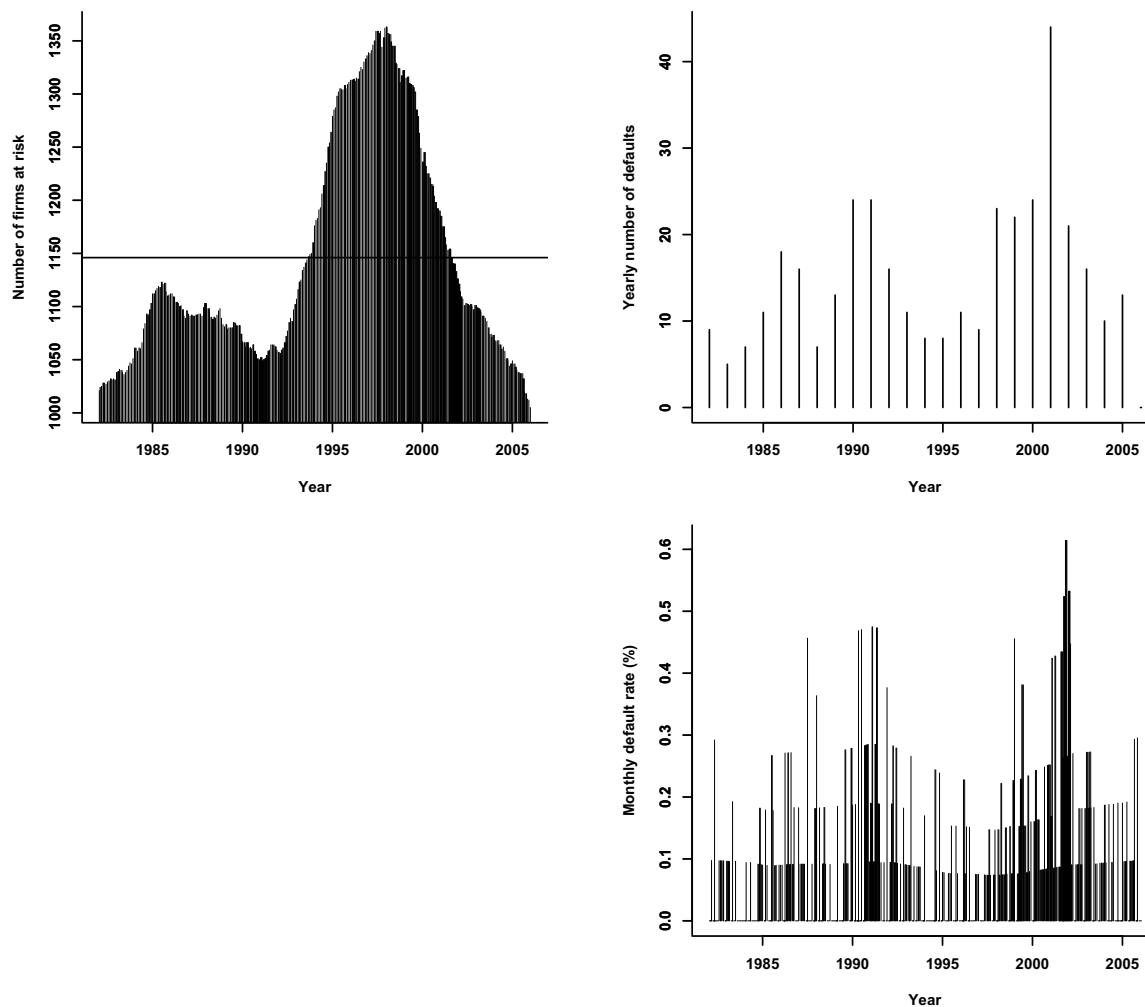


Figure 1.1. *At-risk and empirical default patterns of the study cohort.* Upper left panel: Number of firms at-risk at each time point in the study time, with the time-average of 1,142 firms at-risk indicated by the horizontal line. Upper Right panel: Yearly number of defaults. Lower right panel: Monthly default rate in % at each time point in the study time.

The at-risk and empirical default patterns of the study cohort are illustrated in Figure 1.1. The upper left panel shows that the final dataset contains a minimum of 1,005, an average of 1,142, and a maximum of 1,363 firms at-risk at any given point in the study time. The upper and lower right panels show the effect on the cohort of the recessions in the U.S. economy. The monthly default rate reaches its maximum of 0.611% during the ‘01-‘02 recession. Note that the lower right panel also depicts the path of the trailing monthly default rate used in the analysis in a 1-month lagged version as a global dynamic covariate.

Figure 1.2 shows nonparametric Nelson-Aalen estimates of the default intensity in the cohort due to the passing of time itself – i.e. without correcting for covariate effects. The left panel shows the cumulative estimate, From this graph, the slope seems fairly constant except for during the recessions of ‘91-‘92 and ‘01-‘02. The smoothed plot in the right panel, obtained through kernel smoothing of the increments of the Nelson-Aalen estimator, is better at showing the time-variation. It captures the same tendencies as the empirical default patterns in Figure 1.1 and clearly shows that the default intensity is much larger during the recessions compared to periods of economic upturns. During the ‘01-‘02 recession the smoothed monthly default intensity peaks at just over 3%.

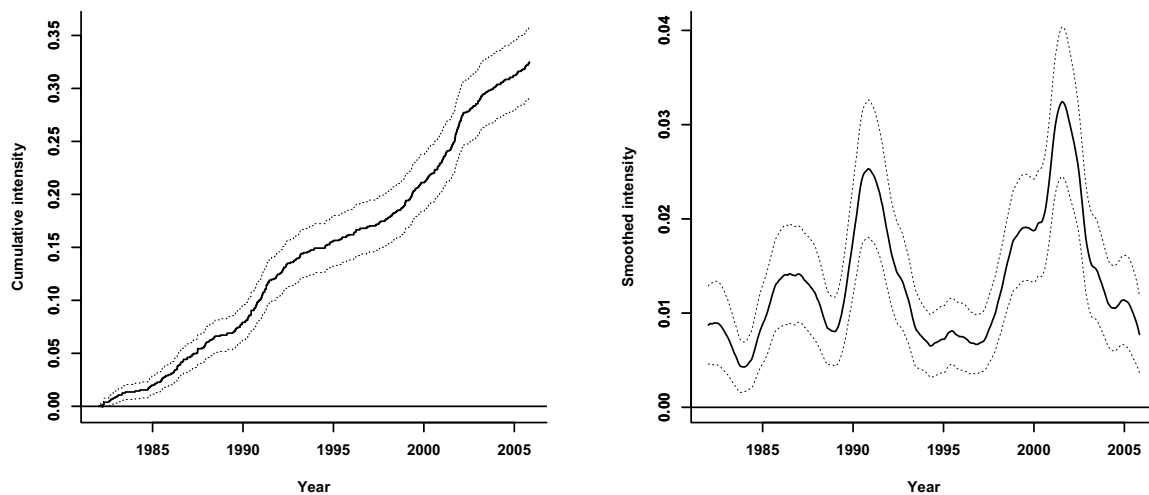


Figure 1.2. *Nelson-Aalen estimates of the default intensity in the cohort.* Left panel: *Estimated cumulative default intensity due to the passing of time itself with approximate 95% martingale-based pointwise confidence bands.* Right panel: *Smoothed default intensity due to the passing of time itself with approximate 95% martingale-based pointwise confidence bands. Smoothing done using the Epanechnikov kernel with bandwidth 1.1 years and boundary correction at the left endpoint of the study time.*

Table 1.1 shows summary statistics for the covariates included in our analysis and we indicate the expected sign of the effects of each covariate on the default intensities of firms. The firm-specific covariates are grouped with respect to defaults and non-defaults to give a rough idea of whether the values of covariates of defaulting firms differ from those of non-defaulted firms in a way consistent with the expected sign of the effect of the covariate. This seems to be the case, although the ratio of short-to-long term debt is seen to be of almost the same magnitude (or maybe even slightly greater) for non-defaulting firms compared to defaulting firms across all statistics.

Note that the table shows extreme maximum values for both the quick ratio, the pledgeable assets, and the 1-year equity return covariates when compared to their 75% quantiles. The influence of these potential outliers will be examined during model check in Section 1.6.

Table 1.1. Covariate summary statistics.

Summary statistics for the five idiosyncratic, the seven macroeconomic, and the global dynamic covariate included in the analysis. The idiosyncratic covariates are grouped according to defaults and non-defaults, while the macroeconomic covariates and the global dynamic covariate are for the full cohort. Expected default intensity increasing effects are indicated with “(+)” while expected default intensity decreasing effects are indicated with “(-).” Effects where the sign is not clear, even when based on univariate economic reasoning, are indicated with “(?)”.

	Min	25% quant.	Median	Mean	75% quant.	Max	SE
Idiosyncratic covariates							
(-) Quick ratio							
<i>Defaults</i>	0.005	0.540	0.898	1.677	1.392	900.900	11.750
<i>Non-defaults</i>	0.003	0.666	0.980	1.460	1.484	828.500	4.611
(-) Pledgeable assets (\$mio.)							
<i>Defaults</i>	1.244	138.900	304.800	1,168.000	911.800	42,120.000	3,109.598
<i>Non-defaults</i>	0.050	213.800	667.300	2,953.000	2,070.000	228,400.000	8,590.747
(-) 1-year equity return (\$mio.)							
<i>Defaults</i>	-0.996	-0.408	-0.092	0.089	0.310	9.948	0.875
<i>Non-defaults</i>	-0.999	-0.161	0.092	0.228	0.410	9.980	0.748
(-) 1-year distance-to-default							
<i>Defaults</i>	-3.659	0.580	1.595	1.862	2.896	11.580	1.787
<i>Non-defaults</i>	-5.119	2.440	4.009	4.337	5.910	19.430	2.695
(+) Short-to-long term debt							
<i>Defaults</i>	0.000	0.026	0.086	0.185	0.254	1.000	0.233
<i>Non-defaults</i>	0.000	0.031	0.109	0.199	0.290	1.000	0.228
Macroeconomic covariates							
(-) 1-year S&P 500 return (\$mio.)	-0.275	-0.027	0.106	0.090	0.218	0.534	0.171
(+) Baa-AAA yield spread	0.550	0.740	0.900	1.010	1.210	2.690	0.385
(-) 1-year CPI change (%)	1.100	2.300	3.000	3.052	3.700	6.400	1.090
(?) 1-year earnings change (%)	0.700	2.400	3.000	3.041	3.500	6.500	0.884
(?) 1-year oil price change (%)	-0.615	-0.152	-0.008	0.079	0.324	1.963	0.397
(+) Treasury yield spread	-0.370	0.490	1.260	1.337	2.140	3.290	0.976
(+) 1-year UR change (%)	-28.700	-8.900	-4.300	0.296	7.400	46.200	13.898
Global dynamic covariate							
(+) Monthly default rate (%)	0.000	0.000	0.093	0.138	0.189	0.611	0.138

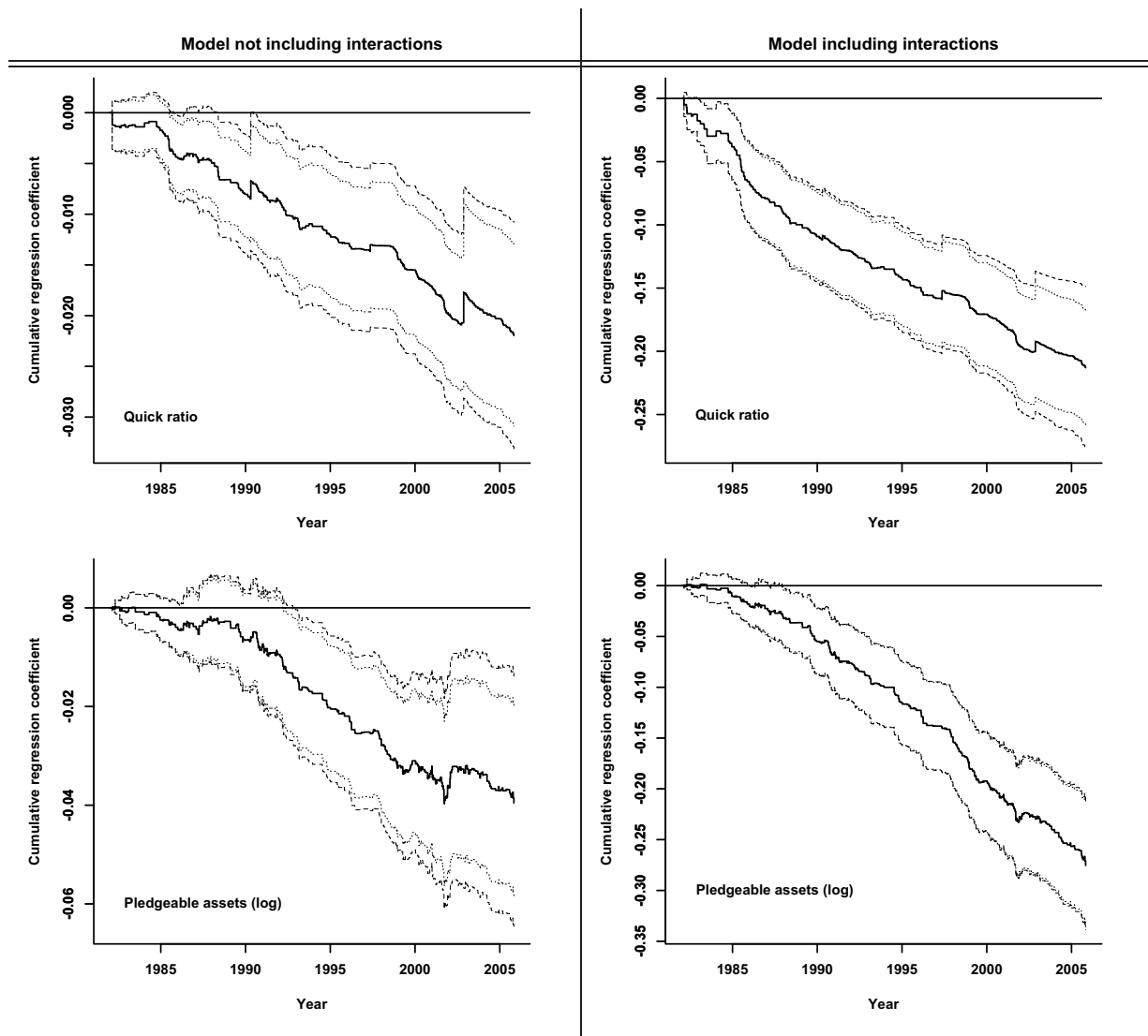


Figure 1.3. Missing interactions. Estimated cumulative regression coefficients for the quick ratio and (log) pledgeable assets covariates with martingale-based 95% confidence bands (dotted lines) and robust 95% confidence bands (dashed lines). Left panels: Estimates from the model without interactions. Right panels: Estimates from extended model including interactions. Note that the value axis' are not the same for the two sets of plots.

1.4.2 Nonparametric Aalen analysis

Initially, a nonparametric additive model including time-varying effects for all five idiosyncratic covariates was fitted using the aalen function from the `timereg` package in R (R Development Core Team (2011)) with robust standard errors. Apart from the additive specification replacing the Cox specification used by e.g. Lando and Nielsen (2010), our main departure in this initial fit is the time-varying coefficients on the idiosyncratic covariates and a general time-varying baseline intensity.

As mentioned in Section 1.2.1, it is impossible to simultaneously identify a time-varying baseline and the time-constant effects of global (both macroeconomic and dynamic) covariates when the time-scale is calendar-time. We therefore initially fit the model with a time-varying baseline, which can be viewed as a time-varying proxy for all global tendencies. Afterwards, in Section 1.4.3, we consider the semiparametric submodel where the time-varying

baseline is replaced by a constant term and where the 7 macroeconomic covariates and the global dynamic covariate all have time-constant effects.

Figure 1.3 illustrates the first interesting observation of our study: The 1-year distance-to-default is, in the terminology of Kalbfleisch and Prentice (2002, p. 199), “responsive,” in the sense that it alone weakened the effect and increased the misspecification of both the quick ratio and the pledgeable assets. Such behavior is a sign of missing interactions in the model.

Estimated cumulative regression coefficients $\widehat{B}_j(t) = \int_0^t \widehat{\beta}_j(s) ds$ for the quick ratio and the pledgeable assets from the reference model without interactions are shown in Figure 1.3 with confidence bands along with the corresponding estimates from an extended model including interactions between the two and the 1-year distance-to-default. The estimates from the model not including interactions are considerably smaller than the estimates from the extended model and have robust confidence bands which are much wider than the bands based on martingale theory. Including the interaction terms corrects this behavior. In fact, the estimates from the extended model are almost identical to estimates from a model not including the 1-year distance-to-default (not shown). Note that the interactions are economically plausible, as it is reasonable to expect that the effect of a firm’s distance-to-default on default risk depends on asset level and short-term liquidity. However, even in the model including interactions, the robust confidence bands for the quick ratio coefficient are still somewhat larger near the end of the study time than the ones based on martingale-theory, suggesting that there is still some misspecification with regards to this covariate. Inspired by this finding, we tried all other possible combinations of two-way interactions, but none seemed to improve the overall fit.

Estimated cumulative regression coefficients from the extended model including the two interactions are shown with confidence bands in Figure 1.4 for all but the quick ratio and the pledgeable assets, which were given in the right panels of Figure 1.3. The effect of each of the firm specific covariates is as expected. The percentage short-term debt is seen to increase default intensities, but the effect seems to wear off near the end of the study time, from around 2004 and onwards. On the other hand, default intensities fall as we increase the 1-year equity return, the 1-year distance-to-default, the quick ratio, or the book asset value. The pledgeable assets are unimportant until about 1987, or for nearly the first 5 years of the study time. The cumulative estimates for the interactions are both increasing, but have much lower magnitude than the effects of the 1-year distance-to-default, the quick ratio, and the (log) pledgeable assets, so that they only dampen, or correct, but do not completely remove the marginal effects of the covariates. The baseline is increasing suggesting that, given the covariate specification used here, it proxies for default increasing effects.

Judging from the confidence bands of the cumulative coefficients, all effects are significant over most of the study time. The robust bands for all but the quick ratio and its interaction with the 1-year distance-to-default are almost identical to the martingale-based bands, suggesting limited model misspecification with respect to the rest of the covariates.

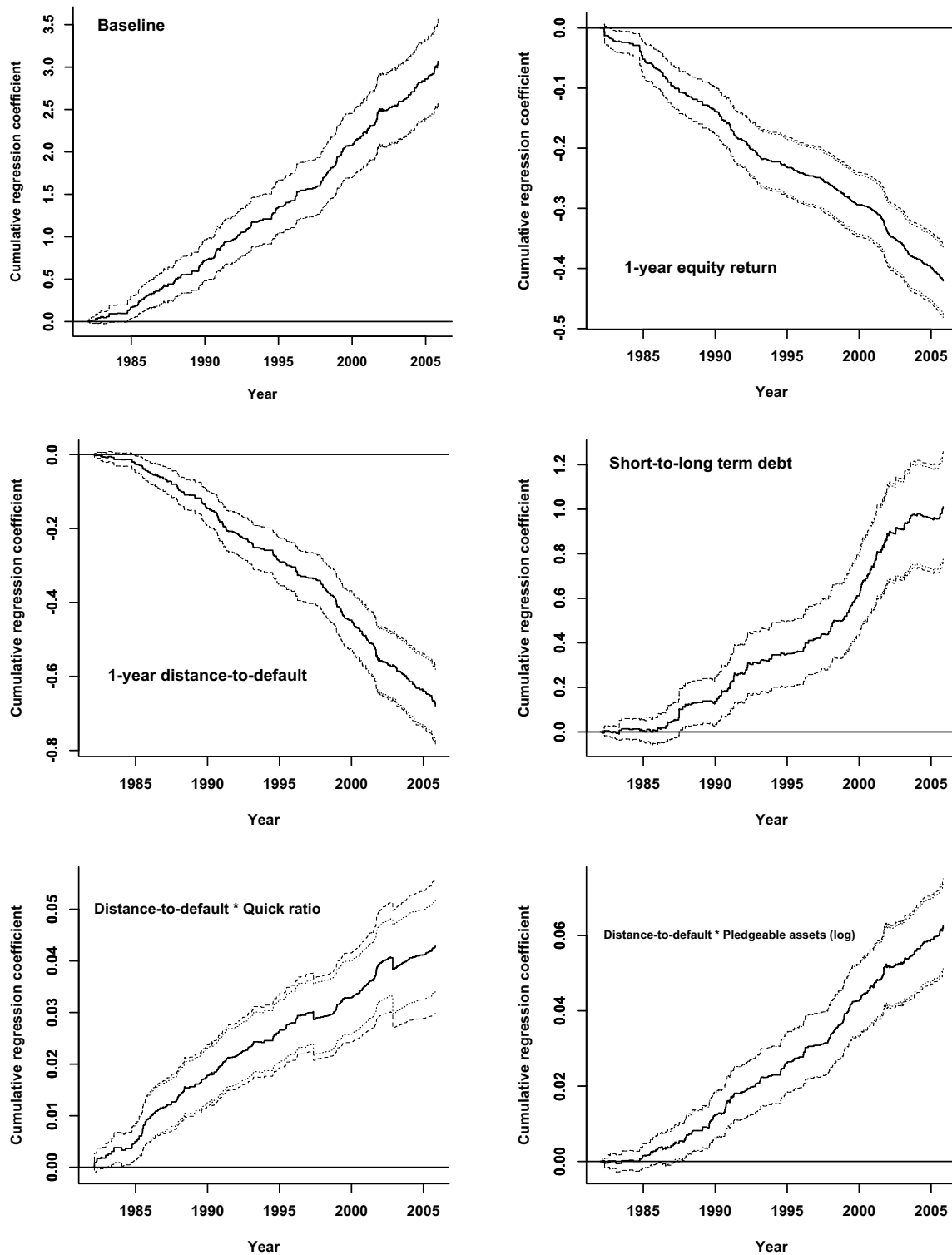


Figure 1.4. Cumulative coefficients from initial nonparametric Aalen analysis. Estimated cumulative regression coefficients for the baseline, the 1-year equity return, the 1-year distance-to-default, the short-to-long term debt ratio, and the interactions of the extended nonparametric additive model including interactions with martingale-based 95% confidence bands (dotted lines) and robust 95% confidence bands (dashed lines). The estimates from this model for the quick ratio and the (log) pledgeable assets are shown in the right panels of Figure 1.3.

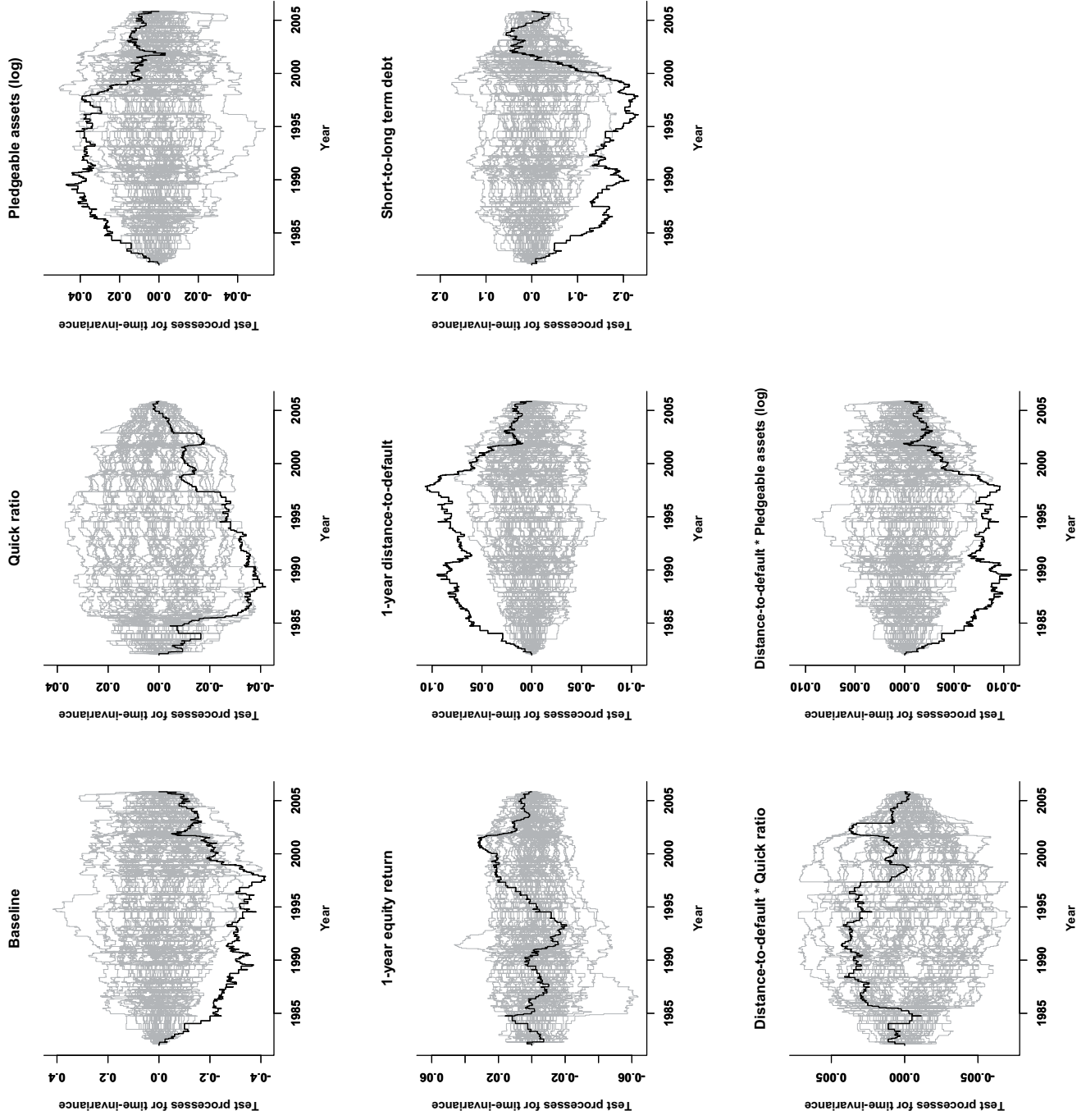


Figure 1.5. Testing for time-invariance. Observed test processes (1.14) for the cumulative regression coefficients from the extended nonparametric additive model including interactions (thick black lines), along with 50 resampled test processes under the null of time-invariance (thin grey lines).

Table 1.2. Initial nonparametric Aalen analysis.

Supremum test of significance (1.13), Kolmogorov-Smirnov test of time-invariance (1.15) and Cramer-von Mises test of time-invariance (1.16) for the cumulative coefficients from the extended nonparametric Aalen model including interactions. Associated P -values are based on 1,000 resampled test processes.

Effect	Supremum statistic	P -value	Kolmogorov-Smirnov statistic	P -value	Cramer-von Mises statistic	P -value
Baseline	12.10	0.00	0.42	0.01	1.69	0.00
Quick ratio	7.50	0.00	0.04	0.03	0.01	0.05
Pledgeable Assets (log)	8.54	0.00	0.05	0.01	0.02	0.01
1-year equity return	13.50	0.00	0.03	0.14	0.00	0.27
1-year distance-to-default	12.70	0.00	0.11	0.00	0.11	0.00
Short-to-long term debt	8.00	0.00	0.23	0.00	0.52	0.00
Distance-to-default \times Quick	7.74	0.00	0.00	0.39	0.00	0.32
Distance-to-default \times Assets	9.95	0.00	0.01	0.00	0.00	0.00

To determine if the changes in the slopes of the cumulative estimates are significant, the observed test processes (1.14) for the extended model including interactions are plotted in Figure 1.5 along with 50 resampled test processes under the null of time-invariance. Only the observed test processes corresponding to the short-to-long term debt ratio, the 1-year distance-to-default, and the interaction between the latter and the pledgeable assets truly exhibit extreme behavior over longer periods of time compared to the resampled test processes. The rest are at best borderline extreme, and only over limited periods of time. Hence, we only expect the three mentioned effects to be significantly time-varying.

The graphical considerations are supported by test statistics and associated P -values based on 1,000 resamples of each cumulative coefficient in Table 1.2. All the estimated cumulative coefficients are highly significant. The Kolmogorov-Smirnov and Cramer-von Mises tests of time-invariance agree fairly well for all cumulative coefficients, and the hypothesis of a time-constant effect is only clearly rejected for the effects of the short-to-long term debt ratio, the 1-year distance-to-default, and the interaction between the latter and the (log) pledgeable assets. The test results correspond well with the impressions from the plots.

Note that while the overall conclusions are comparable to findings by e.g. Lando and Nielsen (2010), we obtain a description of how the effects of covariates vary with time and how this may cause model misspecification.

1.4.3 Semiparametric Aalen analysis

The next step is to introduce the macroeconomic covariates and the global dynamic covariate instead of the general time-varying baseline intensity in a semiparametric Aalen model. As mentioned above, to identify the parameters, this means that we have to use a constant baseline intensity. Based on the results from the nonparametric Aalen model, the semiparametric model is fitted allowing for time-varying effects of the short-to-long term debt ratio, the 1-year distance-to-default, and the interaction between the 1-year distance-to-default and the (log) pledgeable assets.

Table 1.3 shows the time-constant parameter estimates from this model, while estimated regression coefficients $\widehat{\beta}_j(t)$ for the short-to-long term debt ratio, the 1-year-distance-to-default and the interaction between the latter and the (log) pledgeable assets are shown as smoothed functions with confidence bands in Figure 1.6.

Consider first the time-constant effects. The most pronounced time-constant default decreasing effect is the 1-year equity return, which when increased by 1% lowers intensities by 1.52 percentage points. Note, however, that using robust standard errors, the effects of the quick ratio and the interaction between the quick ratio and the 1-year distance-to-default are insignificant. The robust standard errors for these two effects are also seen to be somewhat

Table 1.3. Initial semiparametric Aalen analysis.

Parameter estimates, regular (martingale-based) standard errors, robust standard errors, Wald test statistics, and associated P -values for the time-constant effects from the semiparametric Aalen model. Test statistics and P -values are based on robust standard errors. Effects are grouped according to covariate type: Idiosyncratic, macroeconomic, and dynamic.

Effect	Estimate	Standard error	Robust SE	Wald statistic	P -value
Quick ratio	-0.000590	0.000095	0.000468	-1.26	0.21
Pledgeable Assets (log)	-0.009850	0.001250	0.001330	-7.41	1.30×10^{-13}
1-year equity return	-0.015200	0.000916	0.001000	-15.20	0.00
Distance-to-default \times Quick	0.000235	0.000024	0.000114	2.02	0.04
Baseline	0.115000	0.018200	0.018500	6.21	5.09×10^{-10}
1-year S&P 500 return	-0.021200	0.019200	0.019100	-1.10	0.27
Baa-AAA yield spread	-0.009660	0.006360	0.006220	-1.55	0.12
1- year CPI change	-0.003790	0.002280	0.002220	-1.71	0.09
1-year earnings change	0.001540	0.003050	0.003080	0.5	0.61
1-year oil price change	-0.004990	0.005910	0.005830	-0.86	0.39
Treasury yield spread	0.002290	0.003250	0.003170	0.72	0.47
1- year UR change	0.000343	0.000272	0.000274	1.25	0.21
Monthly default rate	0.069700	0.023800	0.023600	2.95	3.31×10^{-3}

larger than the martingale-based standard errors. This shows that the misspecifications first observed in Figures 1.3 and 1.4 are also present for the corresponding time-constant effects. In fact, had the Wald test statistics for these two effects been calculated using the martingale-based standard errors, we would have mistakenly judged them as being clearly significant. The robust standard errors for the rest of the time-constant coefficients, both idiosyncratic, macroeconomic, and dynamic, are, however, almost identical to the standard errors based on martingale theory.

The table also shows that the global dynamic effect is of large magnitude and highly significant, suggesting a high level of unobserved variation in the cohort: A 1% increase in the monthly default rate implies an excess risk of 6.97 percentage points, which is a large shift, but plausible given that the observed monthly default rate in the data has a maximum of 0.611%. On the other hand, none of the macroeconomic effects are significant at conventional levels. A slight indication of a default decreasing effect is found for the 1-year CPI change, but only at the 10% level. In contrast to what is expected from univariate economic reasoning, the Baa-AAA yield spread decreases intensities, but the effect is very small, and its sign is therefore not a big concern. Finally, note that the 1-year return on the S&P500 index has, as expected, a default decreasing effect in the additive model, although not significant at conventional levels. This particular covariate has been reported in other studies to have the “wrong” sign in Cox models, see for example Duffie et al. (2009) and Lando and Nielsen (2010).

Turning to the time-varying coefficients $\widehat{\beta}_j(t)$ in Figure 1.6, the variational pattern over time is in all three cases closely connected to the recessions in the U.S. economy, with all three effects being most pronounced during the recessions as compared to in-between periods. The magnitude of the short-to-long term debt ratio effect varies greatly depending on the recession at hand: A 1% increase corresponds to an excess risk of around 7.00 percentage points during the ‘91-‘92 recession, while the same increase implies an excess risk of almost 14.00 percentage points, or twice as much, during the ‘01-‘02 recession. The effect of the 1-year distance-to-default is, on the other hand, fairly much the same during both mentioned recessions, where an increase of one standard deviation lowers intensities by about 3 percentage points. Comparing this with the time-constant estimates in Table 1.3, the 1-year distance-to-default has the most influential default decreasing effect of all included covariates. The effect of the interaction is a time-varying correction to the marginal effects of the 1-year distance-to-default and the pledgeable assets.

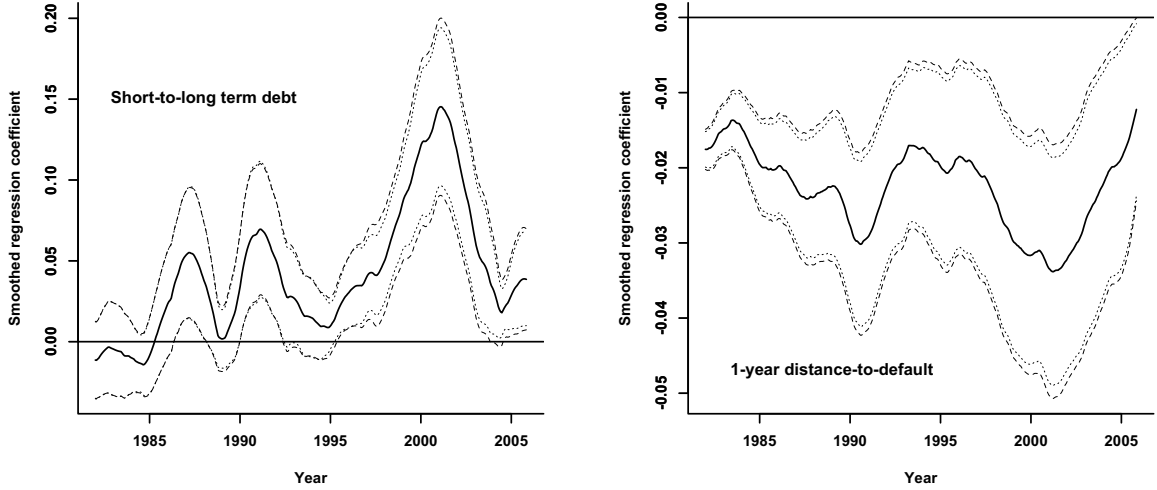


Figure 1.6. Smoothed regression coefficients. Smoothed estimates (1.12) of regression coefficients for the short-to-long term debt ratio, the 1-year-distance-to-default, and the interaction between the latter and the (log) pledgeable assets from the semiparametric model with martingale-based 95% confidence bands (dotted lines) and robust 95% confidence bands (dashed lines). Smoothing done using the Epanechnikov kernel with bandwidth 1.3 years and boundary correction at the left endpoint of the study time.

1.5 Model check

Despite the existence of several goodness-of-fit methods for survival models, actual model checking is still somewhat overlooked in practice. The problem is that classical model check procedures looking at residuals all originate from similar methods used in ordinary statistics, but the way in which they are to be interpreted in the survival data setup with censoring is unclear. For instance, the usual martingale residuals first considered by Barlow and Prentice (1988) are not normally distributed – hence, the interpretation of such residuals is not as straightforward as in classical regression analysis, and it may therefore be difficult to judge the quality of a model-fit from a plot of the residuals.

Nevertheless, model checking may actually lead us to alter the model specification. In our case, it leads us to conclude that potential problems with the fit of the additive model is due to over-influence of firms with extreme covariate values. Before presenting these findings, we describe graphical model checking methods for the nonparametric Aalen model.

An advantage of the nonparametric Aalen model is that its additive structure fits well with martingale theory, producing residual processes which are exact local martingales when the model is correct – this is neither the case for the semiparametric additive model nor for the Cox model, for which the residual processes are only approximate (asymptotic) local martingales. As the semiparametric Aalen model is a submodel of its nonparametric counterpart, we will focus on model-check for the nonparametric model.

Aalen et al. (2008) discuss a graphical method based on so-called “martingale residual processes” – a method that has also been applied to the Cox model. Using the nonparametric additive model form (1.7), define the martingale residual process $(\mathbf{M}_{\text{res},t})_{t \in [0, T]}$ as the accumulated difference between the vector of counting processes and the vector of estimated cumulative intensity processes at the time points where the model is estimable,

$$\mathbf{M}_{\text{res},t} = \int_0^t J(s) d\mathbf{N}_s - \int_0^t J(s) \mathbf{X}_s d\widehat{\mathbf{B}}(s). \quad (1.24)$$

Inserting the expression (1.8) for the estimator $\widehat{\mathbf{B}}(t)$, applying the model form (1.7), and using the definition of the

least squares generalized inverse \mathbf{X}_t^- , we get

$$\begin{aligned}\mathbf{M}_{\text{res},t} &= \int_0^t J(s) (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) d\mathbf{N}_s = \int_0^t J(s) (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) (\mathbf{X}_s d\mathbf{B}(s) + d\mathbf{M}_s) \\ &= \int_0^t J(s) (\mathbf{I} - \mathbf{X}_s \mathbf{X}_s^-) d\mathbf{M}_s.\end{aligned}$$

Under the assumption that \mathbf{X}_t is locally bounded, this proves that the process $(\mathbf{M}_{\text{res},t})_{t \in [0, T]}$ is a vector-valued local martingale when the nonparametric additive model is true. As mentioned, this exact result does not carry over to semi-parametric models – whether of the Aalen or the Cox type.

1.5.1 Covariate misspecifications: Grouped martingale residual processes

The firm specific martingale residual processes are typically not of much use on their own due to the relatively few number of recurrent default events. It is more useful to aggregate over groups of firms with respect to covariate values and conduct graphical model-check based on the grouped residual processes. Specifically, assume that a grouping of the firms is given, and let $G(t)$ denote the set of all firms belonging to group G at time t . The grouping is allowed to depend on time, such that firms may move from one group to another during the study time, as would naturally be the case when grouping is based on time-dependent covariates. It is, however, essential that the grouping is predictable, such that information needed to group firms at time t is available just before time t . The martingale residual process of group G at time t then takes the form

$$M_{\text{res},t}^{(G)} = \int_0^t \sum_{i \in G(s)} dM_{\text{res},it}, \quad (1.25)$$

where $M_{\text{res},it}$ is the i th element of the time- t vector $\mathbf{M}_{\text{res},t}$ in (1.24). These grouped residual processes may then be plotted as functions of study time in order to assess the model fit with respect to covariates. If the model fits well, the resulting plots will fluctuate around zero and show no overdispersion or particular trends.

1.5.2 The model fit as a whole: Model-based covariance of residual processes

The martingale residual processes may also be of use in a different manner, aimed at judging the model fit as a whole. The method is explained by [Aalen et al. \(2008\)](#), and involves estimating a model-based covariance function for the martingale residual processes. The idea is that if we standardize martingale residual processes by dividing them with their estimated standard deviation at each time t , then the time-varying mean and standard deviation of the standardized processes should stay close to 0 and 1, respectively, if the model fits well.

To elaborate, assume that the nonparametric additive model is true, so that the process $(\mathbf{M}_{\text{res},t})_{t \in [0, T]}$ is a vector-valued local martingale. Its covariance function may then be estimated by its optional variation process

$$\boldsymbol{\Sigma}_{\text{res}}(t) = \sum_{\tau_k \leq t} J(\tau_k) (\mathbf{I} - \mathbf{X}_{\tau_k} \mathbf{X}_{\tau_k}^-) \text{diag}(\mathbf{X}_{\tau_k} \mathbf{X}_{\tau_k}^- \Delta \mathbf{N}_{\tau_k}) (\mathbf{I} - \mathbf{X}_{\tau_k} \mathbf{X}_{\tau_k}^-)^T. \quad (1.26)$$

Standardized residual processes are then obtained at each time t by calculating

$$\frac{M_{\text{res},it}}{\sqrt{\sigma_{\text{res},ii}^2(t)}}; \quad i = 1, \dots, n,$$

where $\sigma_{\text{res},ii}^2(t)$ is the i th diagonal element of $\Sigma_{\text{res}}(t)$. A plot of the mean and standard deviation of the standardized residual processes as a function of time will then give a graphical assessment of the model fit as a whole: If the model gives a reasonable fit, the mean will stay close to 0, and the standard deviation to 1.

1.5.3 Outliers and over-influence: Diagonal cumulative hat processes

The leverages are a useful diagnostic for finding observations that may be overly influential in linear regression models. The leverages are the diagonal elements of the so-called hat matrix, which is given by the product of the design matrix and its least squares generalized inverse. For the additive model, we define a cumulative hat matrix as

$$\mathbf{H}_{\text{cum}}(t) = \sum_{\tau_k \leq t} \mathbf{X}_{\tau_k} \mathbf{X}_{\tau_k}^{-1}. \quad (1.27)$$

The idea is to look for observed firm processes with particularly high influence by plotting the diagonal elements $h_{\text{cum},ii}(t)$ of $\mathbf{H}_{\text{cum}}(t)$, called the diagonal cumulative hat processes, as functions of the successive default times in the cohort.

In ordinary linear regression, the average value of the diagonal elements of the hat matrix is r/n , where r is the rank of the hat matrix, which is the number of independent parameters of the model, and n is the number observations. Diagonal elements well above r/n are said to have high leverage, and [Aalen et al. \(2008\)](#), amongst others, recommend investigation of observations with diagonal hat matrix element above $2r/n$.

With respect to the nonparametric additive model, this theory may be applied at each default time in the cohort. Hence, the outlying process criterion is

$$h_{\text{cum},ii}(t) > 2r \sum_{\tau_k \leq t} \frac{1}{Y_{\cdot, \tau_k}}, \quad (1.28)$$

where Y_{\cdot, τ_k} is the number of firms at risk at default time τ_k .

1.6 Revising the model

The initial analyses of Section 1.4 using the Aalen models gave intuitive results about the influence of the included risk factors that are largely consistent with results from Cox models found by e.g. [Duffie et al. \(2009\)](#) and [Lando and Nielsen \(2010\)](#). There were, however, also signs of model misspecifications. First, robust standard errors were considerably larger than regular standard errors for several covariates, even after correcting for interactions. Second, several significantly time-varying effects were identified, indicating that the time-invariant proportionality assumption of the Cox model applied elsewhere is a misspecification for these covariates. Finally, the significance of the global dynamic effect suggests that the covariate specification applied here is missing essential elements. Hence, there is good reason for a deeper analysis of the misspecifications and the data. All results presented in this section are based on own implementations in R ([R Development Core Team \(2011\)](#)).

1.6.1 Martingale residual processes

Graphical assessment of martingale residual processes, suitably grouped with respect to the values of covariates, may give a good indication of functional form misspecifications and outliers due to extreme covariate values. We study the

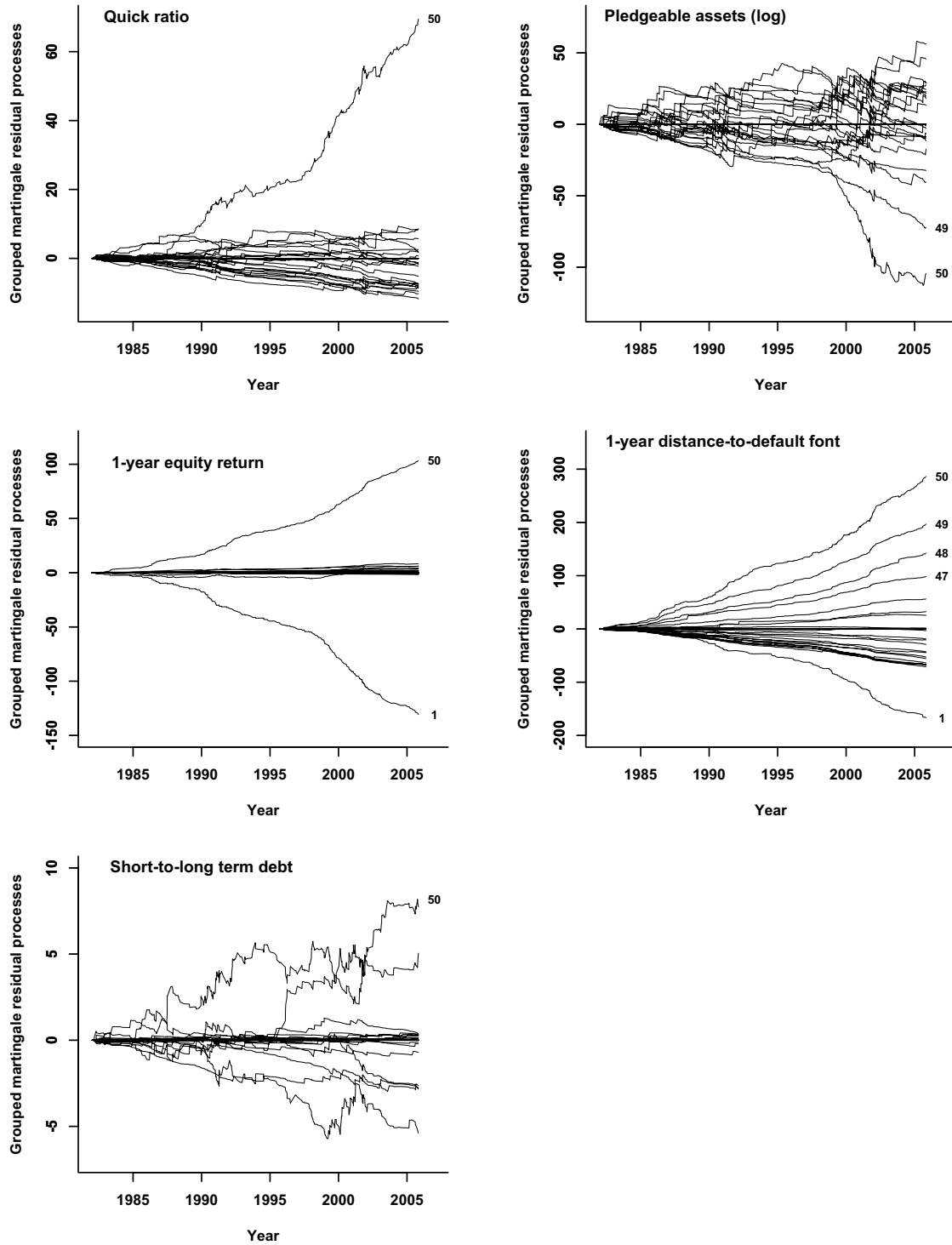


Figure 1.7. Covariate misspecifications. Grouped martingale residual processes (1.25) for the nonparametric Aalen model including all five idiosyncratic covariates and the two interactions. Grouping done at each default time according to the quantiles of each idiosyncratic covariate, with 50 equidistant groups in each case: Group 1 corresponds to values between the 0% and 2% quantiles, group 2 corresponds to values between the 2% and 4% quantiles, and so on. Groups with outlying residual processes are marked by group number.

martingale residual processes of the nonparametric additive model including all five idiosyncratic covariates and the two interactions.

Figure 1.7 shows grouped martingale residual processes (1.25) for each of the five idiosyncratic covariates. Grouping is done at each default time according to the quantiles of each covariate, with 50 equidistant groups in each case: Group 1 corresponds to values between the 0% and 2% quantiles, group 2 corresponds to values between the 2% and 4% quantiles, and so on. This allowed for the isolation of the very extreme covariate values while maintaining a reasonable amount of firms in each group at each default time.

If the model fits well, all residual processes would stay close to zero and not show signs of over-dispersion or trends. The impression from the figure is therefore that deviations from the model mostly occur in the extreme (both low and high) quantiles of the covariates, with especially the high quantiles causing misspecifications. This is expected, but the plots allow us to give a detailed account of the misspecifications. More defaults than expected occur in the highest quantile groups of the quick ratio, the 1-year equity return, and the 1-year distance-to-default. There are slightly more defaults than expected in the highest quantile group of the short-to-long term debt ratio, but the deviation is limited to around the year 2001, which had a historically high default rate. On the other hand, fewer defaults than expected occur in the lowest quantile groups of the 1-year-equity return and the 1-year distance-to-default, while it is in the highest quantile groups of the (log) pledgeable assets that fewer defaults than expected occur. In the latter case, the deviation is again limited to around the year 2001. Most of the non-extreme quantile groups show no truly alarming signs.

The deviations in the high quantile groups of the quick ratio, the 1-year equity return, and the 1-year distance-to-default are either due to outliers, or due to certain firms having excessively high values of these covariates, yet still defaulting. Analogously, the deviations in the lowest quantile groups of the 1-year equity return and the 1-year distance-to-default are due to certain firms having negative values for these covariates, yet still not defaulting. The deviation in the highest quantile groups of the (log) pledgeable assets is somewhat unintuitive, but may be due to a considerable amount of defaults amongst firms with relatively high values of pledgeable assets around the year 2001.

The deviations seen in the figure suggest that simple transformations or inclusion of, say, quadratic terms, would not correct the misspecifications, since they are limited to the extreme quantile groups. Transformations yielding reasonable residual processes would have to “squeeze” together the distribution of the covariates, so that outlying values are neutralized, and such would tend to blur effects. Transformations of the form $x \mapsto -e^{-ax}$ for $a > 0$ were used in a preliminary version of this paper. Hence, a detailed analysis of influential observations, particularly ones due to extreme covariate values, is necessary.

1.6.2 Model-based covariance of residual processes

The grouped martingale residual processes revealed a considerable amount of model misspecification at the extreme quantiles of the covariates. However, looking back at the results from Section 1.4.2, robust standard errors were quite close to martingale-based standard errors for most covariates, suggesting that the misspecifications might not be of crucial overall importance for the final results.

To get an idea if the model fits as a whole, we standardize the martingale residual processes using the model-based covariance function (1.26) and compare in Figure 1.8 their time-varying mean and standard deviation with what would be the true values if the model fit well: The mean should stay close to 0 while the standard deviation should stay close to 1 if the model gives an adequate description of the data. The figure thus shows a misspecification with respect to the standard deviation of the standardized residual processes, which increases with time to a maximum value of just over 2. Hence, the standard deviation of the calculated residual processes are greater than is expected under the model.

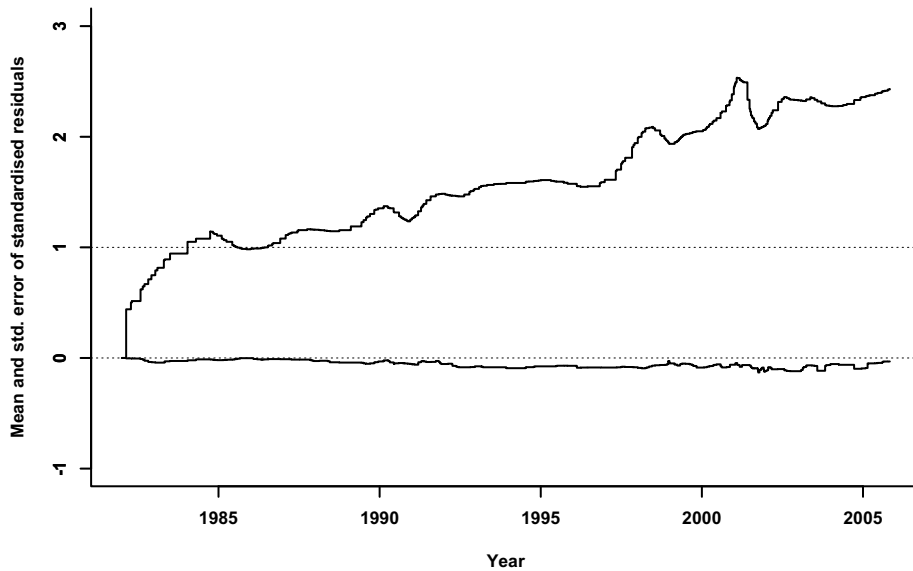


Figure 1.8. *The model fit as a whole.* Mean (lower curve) and standard deviation (upper curve) of standardized martingale residual processes as functions of study time from the nonparametric Aalen model including all five idiosyncratic covariates and the two interactions. The true values under the model, 0 and 1, respectively, are indicated by the dotted lines.

The plot therefore shows that while the additive model on average captures the patterns present in the data throughout the study time, the magnitude of variation in the data which the model fails to explain is considerable. It thus seems that while the model is not entirely wrong in its description of the defaults in the data, it apparently lacks essential explanatory risk factors.

1.6.3 Diagonal cumulative hat processes

To check whether the deviations from martingale-behavior seen in Figures 1.7 and 1.8 are due to over-influence of some firms in the cohort, we calculated the cumulative hat matrix (1.27) for the nonparametric additive model as a function of the successive default times in the cohort.

The left panel of Figure 1.9 shows the 2,557 diagonal cumulative hat processes corresponding to each firm in the cohort along with the expected hat matrix and the outlying process criterion (1.28). Several processes exceed the outlying process criterion, meaning that these firms have the highest influence on the results of the analysis. In total, 177 processes were identified as exceeding the outlying process criterion at some default time, and these are emphasized in the right panel of the figure. These firms were the ones with the most extreme (both high and low) values of the idiosyncratic covariates. Among those firms there were 19 defaults distributed across 16 firms, with 3 firms defaulting twice.

When examining the 177 firms more closely, it was clear that they all have some covariate values that are outliers which, for example, were many magnitudes higher than the maximum in the rest of the cohort. This identification of over-influential firms confirms the observations from the study of the grouped martingale residual processes, where it was noted that deviations from the additive model were primarily limited to the extreme quantile groups.

1.6.4 Analysis based on clean data

Based on the results from the model check, we conduct an analysis of the data, omitting the 177 firms from the cohort who were identified as over-influential. Omitting these firms resulted in a “clean” data set consisting of 2,380 firms, comprising a total of 351 defaults. The 19 omitted defaults were fairly evenly distributed across study time, and an

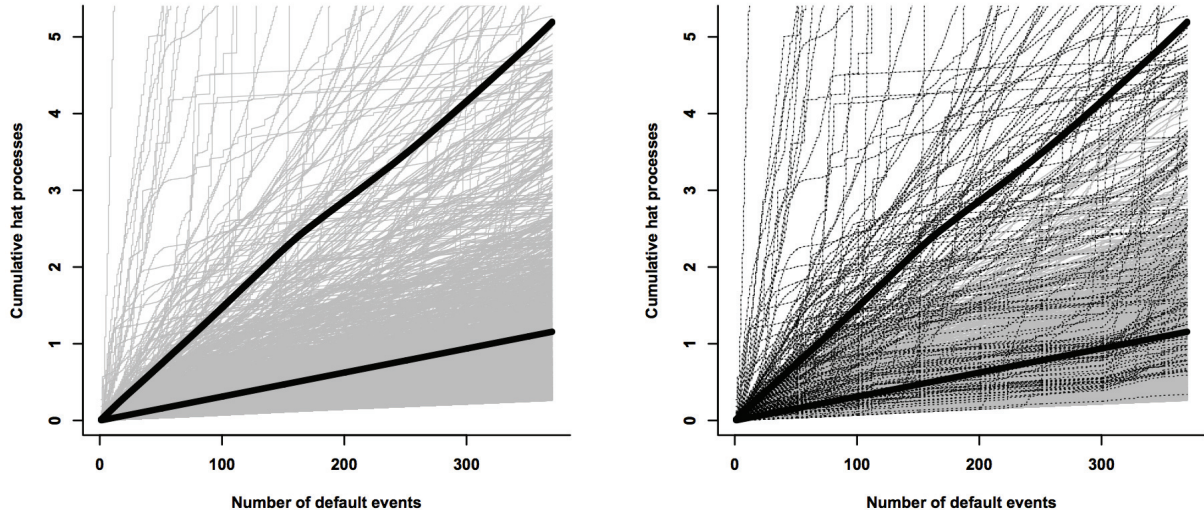


Figure 1.9. Outliers and over-influence. Analysis of over-influence through the cumulative hat matrix (1.27) of the nonparametric additive model including all five idiosyncratic covariates and the two interactions. The plots show the 2,557 firm-specific hat processes (thin grey lines) as functions of the default times in the cohort, along with the expected hat matrix (lower thick solid line) and the outlying process criterion (upper thick solid line). In the right plot, the 177 processes which at some default time exceed the outlying process criterion are emphasised (thin dotted lines).

analysis of the at-risk and default patterns in the clean data set showed no particular changes compared to the full data set.

Model check for the cleaned data

Grouped martingale residual processes for a nonparametric additive model fitted to the clean data, including the two interactions, showed much less dispersion in the extreme quantile groups and generally a better fit across all covariates. As shown in Table 1.4, and detailed below, martingale-based and robust standard errors are also in close correspondence. The standard deviation of the standardized residual processes did not change much compared to the analysis on the full data set. There may therefore be room for additional covariates which we have not considered. Overall, however, the model fit was considerably improved by omitting outliers in the data. The trade-off is, of course, that the model is then not fitted to the full data set. On the other hand, we avoid ad-hoc transformations of the functional form that are sensitive to a few extreme outliers.

Model fits for the cleaned data

A nonparametric Aalen model was fitted to the clean data including all five idiosyncratic covariates and the two interactions. Only the effects of the 1-year distance-to-default and the short-to-long term debt ratio had significantly time-varying effects – the hypothesis of time-invariance was clearly not rejected for the rest of the effects. Hence, cleaning the data removed the time-variation of the effect of the interaction between the 1-year distance-to-default and the (log) pledgeable assets compared to the analysis on the full data set.

Based on these findings, a semiparametric additive model was fitted to the clean data with time-varying effects for the 1-year distance-to-default and the short-to-long term debt ratio, while the rest of the idiosyncratic covariates had time-constant effects. The time-varying baseline was replaced by a constant term, the macroeconomic covariates, and the global dynamic covariate. Table 1.4 gives the estimates for the time-constant effects. The smoothed coefficients for

Table 1.4. Semiparametric Aalen analysis of cleaned data.

Parameter estimates, regular (martingale-based) standard errors, robust standard errors, Wald test statistics, and associated P -values for the time-constant effects from a semiparametric Aalen model fitted to the clean data. Test statistics are based on robust standard errors. Effects are grouped with respect to covariate type: Idiosyncratic, macroeconomic, and dynamic.

Effect	Estimate	Standard error	Robust SE	Wald statistic	P -value
Quick ratio	-0.021800	0.002150	0.002580	-8.45	0.00
Pledgeable Assets (log)	-0.012500	0.001810	0.001840	-6.79	1.10×10^{-11}
1-year equity return	-0.015600	0.000993	0.001060	-14.72	0.00
Distance-to-default \times Quick	0.004860	0.000421	0.000520	9.34	0.00
Distance-to-default \times Assets	0.003200	0.000371	0.000379	8.44	0.00
Baseline	0.157000	0.023200	0.023200	6.77	1.31×10^{-11}
1-year S&P 500 return	-0.020700	0.022000	0.021900	-0.95	0.34
Baa-AAA yield spread	-0.009920	0.007620	0.007430	-1.34	0.18
1-year CPI change	-0.003120	0.002650	0.002570	-1.21	0.22
1-year earnings change	0.001610	0.003580	0.003610	0.45	0.65
1-year oil price change	-0.006590	0.006360	0.006260	-1.05	0.29
Treasury yield spread	0.003450	0.003670	0.003590	0.96	0.34
1-year UR change	0.000353	0.000304	0.000305	1.16	0.24
Monthly default rate	0.077200	0.026700	0.026600	2.90	3.70×10^{-3}

the time-varying effects of the 1-year distance-to-default and the short-to-long term debt ratio did not change compared to what is given in Figure 1.6, and are not repeated here.

Consider first the time-constant idiosyncratic effects in Table 1.4. Comparing with the results in Table 1.3 for the semiparametric model fitted to the full data set, the effects of the quick ratio and its interaction with the 1-year distance-to-default are now of considerably larger magnitude, and their robust standard errors are in good correspondence with the martingale-based standard errors. Both effects are therefore significant regardless of which standard errors we use to construct test statistics. Thus, cleaning the data has corrected the misspecification first observed for these effects. In fact, the quick ratio now has the most pronounced default decreasing time-constant effect, lowering default intensities by 2.18 percentage points per unit increase.

Finally, with respect to the global effects, the borderline significant effects of the 1-year CPI change found in Table 1.3 is now clearly insignificant at conventional levels. On the other hand, the global dynamic effect is still highly significant, and its magnitude has not changed much compared to the model fitted to the full data set. In conclusion, the significance of the global dynamic effect was not an artifact of over-influence of certain firms. It is conceivable, that the dynamic covariate captures an effect related to the business cycle which our macro variables do not capture and which is not transmitted through the firm specific covariates. Note that this fits well with the analysis of the standardized martingale residual processes which indicated that the model is missing essential risk factors, even after cleaning the data. This is a topic of future research.

1.7 Conclusion

This paper has studied additive models for stochastic default intensities. We model the mean arrival rate of default events using nonparametric and semiparametric versions of Aalen's additive regression model. Using both firm-specific and global covariates, we fit the models and conduct model check on a sample of rated, non-financial U.S. corporates, covering the period 1982 to 2005.

In additive models, covariates act in an additive way on a baseline intensity. The nonparametric model has time-varying effects for all covariates, while the semiparametric model allows for some parametric, time-constant effects.

This makes the additive models more flexible than the often applied Cox model in describing covariate effects and how they may change over time.

An advantage of the Cox models is that their log-linear structure ensures that estimated intensities are positive. The way we estimate the Aalen model does not rule out negative values of estimated intensities. Negative intensities are inevitable for extrapolations using extreme values of covariates, but even for data with firms that have very small default intensities, estimated intensities for observed covariates may be negative. The possibility of negative intensities and the non-parametric nature of the intensity function imply that we should not think of Aalen models as a tool for prediction. Its purpose is to analyze how macro-economic and firm-specific covariates have affected the default intensity in the past. The strength of the Aalen model is its ability to model time-varying effects of covariates in a flexible, non-parametric manner.

We present the theory behind estimation and inference for both nonparametric and semiparametric additive models, including how to test whether a covariate effect is time-varying. We use model checking techniques based on visual inspection of martingale residual processes that help us identify model misspecifications, as well as a method for identifying outliers that is very similar to what is known from ordinary linear regression. Our model checking strongly influences the final model specification.

In our final model we find evidence of time-variation in the effects of distance-to-default and short-to-long term debt, and we identify the effect of interactions between distance-to-default and two other covariates: the quick ratio and (log) pledgable assets. In our final specification, which excludes outliers, the effect of the interaction terms is not time-dependent. Furthermore, the quick ratio covariate is significant. None of our macroeconomic covariates are significant which may indicate that their effects are captured through their influence on firm-specific covariates.

Bibliography

- O. O. Aalen. A model for non-parametric regression analysis of life times. In J. Rosinski W. Klonecki, A. Kozek, editor, *Mathematical Statistics and Probability Theory*, volume 2 of *Lecture Notes in Statistics*, pages 1–25, New York, USA, 1980. Springer-Verlag.
- O. O. Aalen. A linear regression model for the analysis of life times. *Statistics in medicine*, 8:907–925, 1989.
- O. O. Aalen, H. W. Fekjær, Ø. Borgan, and E. Husebye. Dynamic analysis of multivariate failure time data. *Biometrics*, 60(3):764–773, September 2004.
- O. O. Aalen, Ø. Borgan, and H. K. Gjessing. *Survival and Event History Analysis: A Process Point Of View*. Springer-Verlag, New York, USA, 1st edition, 2008.
- E. I. Altman. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance*, 23:589–609, 1968.
- P. K. Andersen and R. D. Gill. Cox’s regression model for counting processes: A large sample study. *Annals of Statistics*, 10:1100–1120, 1982.
- W. E. Barlow and R. L. Prentice. Residuals for relative risk regression. *Biometrika*, No. 75:65–74, 1988.
- W. Beaver. Financial ratios and the prediction of failure. *Journal of Accounting Research. Supplement : Empirical research in accounting : Selected studies 1966*, 4:77–111, 1966.
- S. Chava, C. Stefanescu, and S. Turnbull. Modeling the loss distribution. *Management Science*, 57(7):1267–1287, 2011.
- F. Couderc and O. Renault. Time-to-default : Life-cycle, global and industry cycle impacts. Unpublished manuscript, University of Geneva and Warwick Business School, 2004.
- D. R. Cox. Regression models and life-tables (with discussion). *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 34:187–220, 1972.
- S. Das, D. Duffie, N. Kapadia, and L. Saita. Common failings: How corporate defaults are correlated. *Journal of Finance*, 62:93–117, 2007.
- S. A. Davydenko. When do firms default? A study of the default boundary. Unpublished manuscript, Rotman School of Management, University of Toronto, 2007.
- D. Duffie, L. Saita, and K. Wang. Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics*, 83:635–665, 2007.
- D. Duffie, A. Eckner, G. Horel, and L. Saita. Frailty correlated default. *The Journal of Finance*, LXIV(5):2089–2123, October 2009.

- S. Figlewski, H. Frydman, and W. Liang. Modeling the effect of macroeconomic factors on corporate default and credit rating transitions. *International Review of Economics & Finance*, 21(1):87–105, 2012.
- P. Fledelius, D. Lando, and J. Perch Nielsen. Non-parametric analysis of rating transition and default data. *Journal of Investment Management*, 2(2):71–85, 2004.
- J. D. Kalbfleisch and R. L. Prentice. *The Statistical Analysis of Failure Time Data*. Wiley, Hoboken, N.J., 2nd edition, 2002.
- D. Lando and M. S. Nielsen. Correlation in Corporate Defaults: Contagion or Conditional Independence? *Journal of Financial Intermediation*, 19(3):335–372, 2010. <http://ssrn.com/paper=1338381>.
- D. Lando and T. Skødeberg. Analyzing Rating Transitions and rating Drift with Continuous Observations. *The Journal of Banking and Finance*, 26:423–444, 2002.
- A. Le. Separating the components of default risk : A derivative-based approach. Unpublished manuscript, Kenan-Flagler Business School, University of North Carolina, 2007.
- X. Li and X. Zhao. Macroeconomic effect in corporate default. Unpublished manuscript, York University, 2006.
- I. W. MacKeague and P. D. Sasieni. A partly parametric additive risk model. *Biometrika*, No. 81:501–514, 1994.
- T. Martinussen and T. H. Scheike. *Dynamic Regression Models for Survival Data*. Springer-Verlag, New York, USA, 1. edition, 2006.
- R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2011. URL <http://www.R-project.org/>. ISBN 3-900051-07-0.
- H. Ramlau-Hansen. Smoothing counting processes by means of kernel functions. *Annals of Statistics*, 11:453–466, 1983.
- T. Robertson, F.T. Wright, and R. Dykstra. *Order restricted statistical inference*. Wiley series in probability and mathematical statistics: Probability and mathematical statistics. Wiley, 1988.
- T. Shumway. Forecasting bankruptcy more efficiently : A simple hazard model. *Journal of Bus.*, 74:101–124, 2001.
- C. A. Struthers and J. D. Kalbfleisch. Misspecified proportional hazard models. *Biometrika*, 73:363–369, 1986.
- A. W. van der Vaart and J. A. Wellner. *Weak Convergence and Empirical Processes: With Applications to Statistics*. Springer-Verlag New York, 1996.
- H. White. Maximum Likelihood estimation of Misspecified Models. *Econometrica*, 50(1-25), 1982.
- D. M. Zucker and A. F. Karr. Nonparametric survival analysis of with time-dependent covariate effects: A penalized partial likelihood approach. *Annals of Statistics*, 18:329–353, 1990.

Chapter 2

Cyclicality and Firm-size in Private Firm Defaults

With David Lando and Thais Lærkholm Jensen.

Introduction

Small and medium-sized enterprises (SMEs) typically depend more heavily on funding from banks than do larger firms. It is therefore conceivable that SMEs are hit harder during a financial crisis in which banks' capital constraints are binding. Perhaps as a recognition of this dependence, the Basel II Accord awards preferential treatment of bank loans to SMEs, effectively and significantly lowering capital charges for lending to the SME-segment. Technically, the reduction follows by prescribing lower asset correlation with a common risk-driver when the calculating capital charges. To the extent that asset correlation arises because of common dependence on macroeconomic shocks, the reduction corresponds to assuming that the default probabilities of SMEs are less sensitive to macroeconomic cyclicality relative to larger firms. These deductions in capital charges were recently reaffirmed and extended in the Basel III Accord and in the fourth Capital Requirements Directive (CRD IV).

This paper investigates whether there is empirical support for the assumption that the default probabilities of smaller firms are less sensitive to macroeconomic cyclicality. Using a default intensity regression framework, our results indicate that solely discriminating with respect to firm-size, the default probabilities of small firms do in fact exhibit less sensitivity to macroeconomic cyclicality compared to the default probabilities of large firms, in the sense that the effects of macroeconomic variables are of smaller magnitude for smaller firms. However, when we account for differences in firm-characteristics other than size, our results indicate that the default probability of the average small firm is as cyclical or even more cyclical than the default probability of the average large firm. The results are robust to different regression models and different divisions of our sample into small and large firms. Our findings suggest that preferential treatment of capital charges solely based on firm-size is too simplistic and may in fact entail adverse effects on the stability of European banks with a high exposure to the SME-segment.

Our data covers the period 2003-2012 and consists of obligor, loan, and default histories for a large sample of private Danish firms. We use both the popular multiplicative Cox regression model and the more flexible additive Aalen regression model to estimate the effects of firm-specific and macroeconomic variables on default intensities. Our main goal is analyze whether the effects of macroeconomic variables are of smaller magnitude for small firms compared to large firms. The crux of our analysis is that our regression models also include accounting ratios that control for

firm characteristics other than size. In this way, we rule out the possibility that our division into small and large firms is merely capturing other characteristics that differentiate the two types of firms. Our regression models confirm previous findings that accounting ratios and macroeconomic variables play distinct roles in default prediction for private firms: (1) Accounting ratios are necessary for accurately ranking private firms according to default likelihood, but cannot by themselves capture the cyclical nature of aggregate default rates, while (2) macroeconomic variables are indispensable for capturing the cyclical nature of aggregate default rates, but do not aid in the ranking of firms with respect to default likelihood. These findings confirm that our method of focusing on the effects of macroeconomic variables is adequate for our intended analysis, as it is these variables, and not firm-specific characteristics, that add the cyclical component to our regression models.

Using the Cox model, we find that solely discriminating with respect to firm size, and keeping all other firm characteristics equal, the default intensities for smaller firms do in fact exhibit less sensitivity to macroeconomic cyclical nature. However, when we account for the Cox model's non-linear form and use averaging techniques adapted from other non-linear regression models, our results indicate that the default intensity of the average small firm may be as cyclical or even more cyclical than the default intensity of the average large firm. Because of this ambiguity in the results from the Cox model, we also conduct the investigation using an additive Aalen model which, due to the linearity of its effects, allows us to directly compare the effects of macroeconomic variables for small and large firms. Using the additive model, we find no evidence that there is different sensitivity to macroeconomic variables for small firms compared to large firms.

We finally conduct robustness and model checks for our results based on the Cox model. First, we show that the signs, magnitudes, and significance levels of the effects of our variables (both firm-specific and macroeconomic) are robust to the use of alternative size-criteria when splitting the full sample into the subsamples of small and large firms. Second, we show that the effects of our variables are robust to the exclusion of certain sample periods. Third, we show that the effects of our variables are robust to alternative choices of lag lengths. Finally, we show that the residual processes from our preferred Cox model do not exhibit systematic trends when grouped according to sectors and size quarterlies.

The flow of the rest of the paper is as follows. Following this introduction is a brief review of the most related literature. Section 1 details our data, variable selection, and estimation methodology. Section 2 presents our regression results, where we confirm previous finds that accounting ratios and macroeconomic variables play distinct roles in default prediction for private firms. Section 3 presents our results regarding the sensitivity to macroeconomic cyclical nature for small and large firms. Section 4 shows results related to robustness and model check. Section 5 concludes.

Related literature

Statistical models using accounting ratios to estimate default probabilities date back to at least [Beaver \(1966\)](#) and [Altman \(1968\)](#), followed by [Ohlson \(1980\)](#) and [Zmijewski \(1984\)](#)—we use many of the same accounting ratios as in these studies. [Shumway \(2001\)](#) was among the first to demonstrate the advantages of intensity models with time-varying covariates compared to traditional discriminant analysis, and was also among the first to include equity return as a market-based predictor of default probabilities—we use a similar estimation setup, although we do not have market-based variables for our private firms. [Chava and Jarrow \(2004\)](#) improved the setup of [Shumway \(2001\)](#) using covariates measured at the monthly level and showed the importance of industry effects—our data frequency is also at the monthly level and we correct for industry effects in all our regressions.

Structural models of credit risk, like the models of [Black and Scholes \(1973\)](#), [Merton \(1974\)](#), and [Leland \(1994\)](#), usually assume that a firm defaults when its assets drop to a sufficiently low level relative to its liabilities. The

connection between structural models and intensity models was formally established by [Duffie and Lando \(2001\)](#), who showed that when the firm’s asset value process is not perfectly observable, a firm’s default time has a default intensity that depends on the firm’s observable characteristics as well as other covariates. Studies demonstrating the importance of covariates implied from structural models, like distance-to-default or asset volatility, include [Duffie et al. \(2007\)](#), [Bharath and Shumway \(2008\)](#), [Lando and Nielsen \(2010\)](#), and [Chava et al. \(2011\)](#) among many others.

Default studies using data on public firms and employing macroeconomic variables include [McDonald and de Gucht \(1999\)](#), [Peseran, Schuermann, Treutler, and Weiner \(2006\)](#), [Duffie et al. \(2007\)](#), [Lando and Nielsen \(2010\)](#), [Figlewski, Frydman, and Liang \(2012\)](#), among many others. Recent default studies of private firms that also employ macroeconomic variables include [Carling et al. \(2007\)](#), who use Swedish data, and [Bonfim \(2009\)](#), who uses Portuguese data. We employ many of the same macroeconomic variables as in these studies.

The provisions in the Basel II Accord permitting “[banks] to separately distinguish exposures to SME borrowers (defined as corporate exposures where the reported sales for the consolidated group of which the firm is a part is less than 50 million euro) from those to large firms” and specifying the reduction for SMEs through the asset correlation formula can be found in Articles 273 and 274 of [The Basel Committee on Banking Supervision’s \(2006\)](#) report. [Lopez \(2004\)](#) examines the relationship between Basel II’s assumed asset correlation and firm size, and finds an increasing relationship between the two using a sample of US, Japanese, and European firms. Using data on French and German firms, [Dietsch and Petey \(2004\)](#) find that SMEs are riskier than larger firms, and that the asset correlations for SMEs are weak and decrease with firm size. [Chionsini, Marcucci, and Quagliariello \(2010\)](#) find evidence in support of the size-sensitive treatment in the Basel II Accord for Italian SMEs, though not during severe financial crises like that of 2008-09. The corresponding provisions in the Basel III Accord and the recently adopted CRD IV can be found in Articles 153.4 and 501.1 of [The European Parliament and the Council of the European Union’s \(2013\)](#) report. Discussions of the treatment of SMEs in the Basel III Accord and CRD IV prior to the adoption can, for instance, be found in reports by [The Association of Chartered Certified Accountants \(ACCA\) \(2011a,b\)](#) and [The European Banking Authority \(EBA\) \(2012\)](#)—see also the references therein.

In our analysis of the cyclicity of the default probabilities of small and large firms, we apply averaging techniques to the Cox regression model resembling those commonly applied to generalized linear models—see, for instance, [Wooldridge \(2009, p. 582-83\)](#) for an overview. The additive regression model due to [Aalen \(1980, 1989\)](#), which we apply in our analysis, was first applied in a default prediction setting by [Lando et al. \(2013\)](#)—they study the time-variation of firm-specific effects on the default probabilities of public US firms.

2.1 Data and methodology

This section presents our data, the variables that we employ as firm-specific and macroeconomic drivers of default probabilities, and the methodology that we use to estimate the effects of the variables on default probabilities.

Our raw data comprises 28,395 firms and 114,409 firm-year observations of obligor and loan histories, accounting statements, and default indicators over the period 2003 to 2012. The data is obtained from a large Danish financial institution. A firm is included in this dataset if, in at least one of the years underlying the period of analysis, it has an engagement over DKK 2 million, which is the largest segmentation category used by the financial institution for its corporate clients. An engagement is defined in terms of loans or granted credit lines. After removing sole proprietorships, government institutions, holding companies without consolidated financial statements, firms that do not have Denmark as their residency, and firms that do not fulfill balance sheet checks, we are left with 10,671 firms

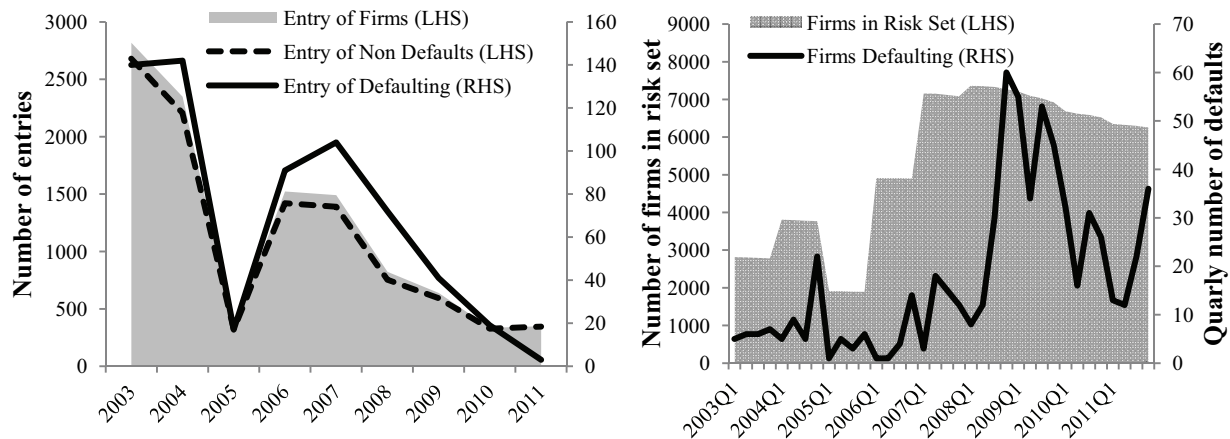


Figure 2.1. Entry and at-risk pattern in the sample. The left panel shows the yearly number of firms entering the sample (grey mass) along with the yearly number of entries that do not default (black, solid line) or eventually do default (black, dashed line). The right panel shows the quarterly number of firms at risk of defaulting (i.e. in the “risk set”; grey mass) along with the actual number of defaulting firms in each quarter (black, solid line).

and 48,703 firm-year observations. In the cleaned dataset, a total of 633 firms experienced a default event, defined by the Basel II Accord as more than 60 days delinquency. Moreover, 54 of the 633 defaulting firms experience a second default, in the sense that they became delinquent a second time during the sample period. Other default studies have treated a firm that re-emerges from default as a new firm, as merited by bankruptcy protection laws. However, due to the Basel II Accord’s definition of a default event as a period of delinquency, we choose to disregard multiple default events, and our results should hence be interpreted as specific to a firm’s initial default.

Figure 2.1 shows the patterns with which firms enter and potentially leave our final sample. The right panel shows the number of firms that enter the sample at each year along with an indication of the number of entries eventually corresponding to defaults and non-defaults. Despite discussions with the financial institution providing the data, the low number of firms entering the sample in 2005 remains a conundrum. It appears, however, that the firms that eventually default do not seem to differ systematically from the non-defaulting firms based on when they enter the sample. The right panel shows the number of firms at risk of defaulting, i.e. firms in the “risk set,” at each quarter, along with the quarterly number of defaults. The risk set is seen to contain at least 2,000 firms at each quarter, and the 2008–09 financial crisis is readily visible from the sharp rise in the number of defaults.

In order to incorporate quarterly macroeconomic variables, we re-code the accounting variables for each firm from annual to quarterly observations. This will naturally induce persistence in the accounting variables from quarter to quarter, which we correct for by basing all inference on standard errors clustered at the firm-level. The final dataset thus consists of a total of 192,196 firm-quarter observations.

Figure 2.2 compares the observed default rate in our sample to the number of registered bankruptcies in Denmark. The comparison is feasible because the total number of firms at risk of default in Denmark is relatively stable over time. We see that, due to the relatively few incidences, the default rate in the sample fluctuates while still co-moving with the aggregate level in Denmark. This indicates that our results are not necessarily specific to the financial institution that provided us with data, but may be applied to Danish firms in general.

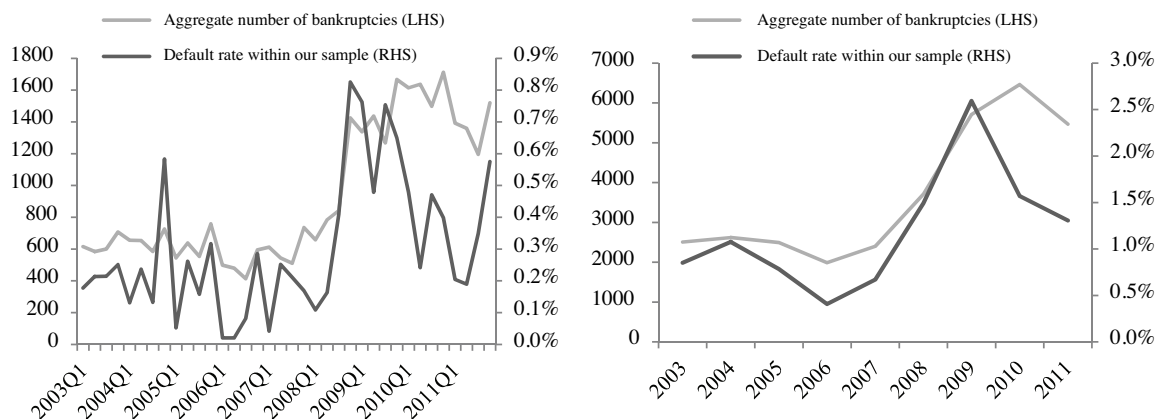


Figure 2.2. Default rates in the sample and the general Danish economy. The left panel shows the quarterly default rate from the sample along with the corresponding aggregate number of quarterly bankruptcies in the general Danish economy. The right panel shows the yearly default rate from the sample along with the aggregate quarterly number of default in the general Danish economy.

2.1.1 Firm-specific explanatory variables

Table 2.1 provides an overview of the firm-specific explanatory variables which we employ in our regression analysis. Our firm-specific explanatory variables measure size, age, leverage, profitability, asset liquidity, collateralization, and equity-value. The table also gives the accounting ratios which we use to proxy for the firm-specific explanatory variables along with the expected sign of each accounting-based variable’s effect on default probabilities. All our accounting-based variables have been applied in previous default studies—see, for instance, Ohlson (1980), Shumway (2001), Duffie et al. (2007), and Lando and Nielsen (2010). We also correct for industry effects as in Chava and Jarrow (2004). The main difference between our list of firm-specific variables and the ones used in default studies of public firms is the lack of market-based measures like stock return and distance-to-default.

In order to avoid discriminating against smaller companies that report financial statements in less detail, we use high-level, aggregated data to construct the accounting-based variables. We also control for industry effects since certain industry characteristics may prescribe a certain leverage structure, particularly linked to the volatility of cash flows. We use the sector affiliation by Statistics Denmark to identify a firm’s primary industry as either “Construction,” “Manufacturing,” or “Wholesale and Retail,” as these have above average default rates, but are at the same time coarse enough to ensure a sufficient number of firms in each sector.

An analysis of the firm-specific variables revealed a few miscodings and extreme values. Due to the anonymized nature of the data, we were not able to check the validity of these data points manually, and we therefore choose to winsorize all the firm-specific variables at the 1st and 99th percentile—a practice also used by Chava and Jarrow (2004), Shumway (2001), and Bonfim (2009), among others. The winsorized summary statistics are presented in Table 2.2. The average firm has DKK 275 million assets, a ratio of 68% between total debt to total assets, and interest payments corresponding to 3% of total assets. Further, the average firm had a relationship with the bank for 23 years and remains in the sample for 7 out of the 9 years.

Due to Danish reporting standards, firms below a certain size may refrain from reporting revenue and employee count, and hence these variables are zero for a large proportion of firms in the sample. We therefore choose not to use these two variables in our further analysis in order not to discriminate against smaller firms. In the table, firm age is taken to be time since the bank recorded the first interaction with the client; entry year specifies the year at which the

Table 2.1. Firm-specific explanatory variables and corresponding, observable accounting-based variables. The left column shows our list of firm-specific explanatory variables, the center column shows the observable accounting-based variables which we use as proxies, and the right column shows the expected effect of each accounting-based variables on default probabilities. Industry Effects are included in the list for completeness, although we only use this variable as a control (see details in the text).

Explanatory Variable	Investigated Measurements	Expected effect on probability of default
Size	Log of book value of assets	Negative
Age	Years active in the bank	Negative
Leverage	Short term debt to total assets	Positive
	Total debt to total assets	Positive
	Interest bearing debt to total assets	Positive
	Interest payments to total assets	Positive
Profitability	Net income to total assets	Negative
	EBIT to total assets	Negative
	EBITDA to total assets	Negative
Liquidity	Current ratio	Negative
	Quick ratio	Negative
Collateralization	Fixed assets to total assets	Negative
	PPE to total assets	Negative
Negative equity	Dummy for negative equity	Positive
Industry Effects	Sector affiliation	Control variable

firm enters the sample; while duration is the number of years a firm is observed in the sample since its entry year.

2.1.2 Macroeconomic explanatory variables

Table 2.3 provides an overview of the macroeconomic explanatory variables which we employ in our regression analysis. Our macroeconomic explanatory variables cover the stock market, interest rates, GDP, credit supply, inflation, industrial production, as well as demand of consumer goods. The table also gives the observable time-series which we use to proxy for the macroeconomic explanatory variables along with the expected sign of each time-aerie’s effect on default probabilities. The macroeconomic time-series are primarily obtained from Ecwin, with additional data from Statistics Denmark, OECD, and Stoxx.

The inclusion of lagged macroeconomic variables allows entering these variables as growth rates, differences, or levels. We select the appropriate form by 1) computing the correlation between each form of the macroeconomic variable and the observed default rate, and 2) visually inspecting the relationship of each form with the observed default rate. Note, however, that some pairs of the macroeconomic variables exhibit collinearity—for example, the Danish GDP growth and the European GDP growth rate, as well as the return on the OMX index and the Stoxx index, have pair-wise correlations of 0,92 and 0,77, respectively. The high degree of collinearity should be kept in mind when interpreting the estimated regression coefficients in the following sections.

2.1.3 Estimation of firm-specific default intensities

We now present the methodology which we apply to estimate the effects of our explanatory variables on firm-specific default probabilities.

Suppose we have a sample of n levered firms observed over a time-horizon $[0, T]$, where firm i may default at a

Table 2.2. Descriptive statistics for the firm-specific variables. The table shows descriptive statistics for the firm-specific variables of the cleaned sample, winsorized at the 1st and 99th percentile. The total number of observations is 192,196 firm-quarters. Age is time since the bank recorded the first interaction with the client. Entry is the year where the firm entered the sample. Duration is the number of years the firm remains in the sample. All other variable have standard interpretations.

Variable	Mean	Std	1%	5%	25%	50%	75%	95%	99%
Total assets (tDKK)	275.074	1.060.373	811	2.784	9.553	28.222	100.723	1.025.875	8.656.000
Revenue (tDKK)	242.031	885.431	0	0	0	0	70.925	1.097.486	6.733.409
Employees	113	348	0	0	1	18	64	484	2.666
Age (years)	23	20	1	3	9	18	30	71	97,25
Log(total assets) (tDKK)	10,46	1,81	6,70	7,93	9,16	10,25	11,52	13,84	15,97
Short term debt to total assets	0,51	0,28	0,01	0,09	0,30	0,49	0,69	0,96	1,58
Total debt to total assets	0,68	0,28	0,02	0,18	0,53	0,70	0,84	1,04	1,80
Interestbearing debt to total assets	0,39	0,28	0,00	0,00	0,17	0,37	0,56	0,87	1,38
Interest payments to total assets	0,03	0,03	0,00	0,00	0,01	0,02	0,03	0,07	0,17
Current ratio	1,66	2,42	0,03	0,28	0,88	1,17	1,59	3,80	20,21
Quick ratio	1,28	2,37	0,02	0,14	0,49	0,81	1,19	3,15	19,67
Fixed assets to total assets	0,40	0,29	0,00	0,01	0,14	0,36	0,63	0,93	0,99
Tangible Assets to total assets	0,30	0,28	0,00	0,00	0,06	0,23	0,49	0,87	0,97
Net Income to total assets	0,03	0,14	-0,68	-0,18	0,00	0,03	0,09	0,24	0,43
EBIT to total assets	0,06	0,15	-0,59	-0,16	0,00	0,05	0,12	0,29	0,51
EBITDA to total assets	0,10	0,15	-0,51	-0,12	0,02	0,09	0,17	0,34	0,54
Entry Year	2005	1,94	2003	2003	2003	2004	2006	2009	2010
Duration (Years)	7	2	1	2	5	7	9	9	9

stochastic time τ_i . At each time t , the firm's financial state is determined by a vector \mathbf{X}_t of firm-specific variables, with values specific to the firm, as well as a vector \mathbf{Z}_t of macroeconomic variables, with values common to all firms in the sample. Default at time t occurs with intensity $\lambda_{it} = \lambda(\mathbf{X}_{it}, \mathbf{Z}_t)$, meaning that λ_{it} is the conditional mean arrival rate of default for firm i , measured in events per time unit. Intuitively, this means that, given survival and the observed covariate histories up to time t , firm i defaults in the short time-interval $[t, t + dt)$ with probability $\lambda_{it} dt$.¹ We assume τ_i is doubly-stochastic driven by the combined history of the internal and external covariates (see for instance Duffie et al., 2007).

In our analysis of which accounting ratios and macroeconomic variables that significantly predict defaults below, we specify the firm-specific default intensities using the “proportional hazards” regression model of Cox (1972). The intensity of firm i at time t is thus modeled as

$$\lambda(\mathbf{X}_{it}, \mathbf{Z}_t) = Y_{it} \exp(\boldsymbol{\beta}^\top \mathbf{X}_{it} + \boldsymbol{\gamma}^\top \mathbf{Z}_t),$$

where Y_{it} is an at-risk-indicator for i , taking the value 1 if firm i has not defaulted “just before” time t and 0 otherwise, while $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of regression coefficients. The effect of a one-unit increase in the j th internal covariate at time t is to multiply the intensity by the “relative risk” e^{β_j} . The same interpretation applies to the external covariates. We let the first component of the vector \mathbf{Z}_t be a constant 1, so that the first component of the vector $\boldsymbol{\gamma}$ is a baseline intensity, corresponding to the (artificial) default intensity of firm i when all observable covariates are identically equal

¹Precisely, a martingale is defined by $1_{(\tau_i \leq t)} - \int_0^t 1_{(\tau_i > s)} \lambda_{is} ds$ with respect to the filtration generated by the event $(\tau_i > t)$ and the combined history of the internal and external covariates up to time t .

Table 2.3. Macroeconomic explanatory variables and corresponding observable time-series. The left column shows our list of macroeconomic explanatory variables, the center column shows the macroeconomic time-series which we use to measure each macroeconomic explanatory variable, and the right column shows the expected effect of each macroeconomic time-series on default probabilities.

Explanatory Variable	Investigated Measurements	Expected effect on probability of default
Stock return	Return of OMX index	Negative
Stock volatility	Volatility of OMX index	Unknown
Interest rates	Slope of yield curve	Negative
GDP	Real growth in Danish GDP	Negative
Loan growth	Loan growth to non-financial firms	Positive
Credit availability	Funding costs	Positive
Aggregate defaults	Danish bankruptcies	Positive
Inflation	Headline CPI	Unknown
Demand side effects	Consumer confidence	Negative
	House prices	Negative
Supply side effects	Business indicator, manufacturing	Negative
	Capacity utilization	Negative
International exposure	Exports to Danish GDP	Unknown
	Return of Stoxx50 index	Negative
	EU 27 GDP growth	Negative

to zero.²

Following, for instance, Andersen et. al (1993), and under the standard assumptions that late-entry, temporal withdrawal, right-censoring, and covariate distributions are uninformative on regression coefficients, the (partial) log-likelihood for estimation of the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ based on a sample of n firms becomes

$$l(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{i=1}^n \int_0^{\tau_i} (\boldsymbol{\beta}^\top \mathbf{X}_{it} + \boldsymbol{\gamma}^\top \mathbf{Z}_t) dN_{it} - \int_0^T \sum_{i=1}^n Y_{it} \exp(\boldsymbol{\beta}^\top \mathbf{X}_{it} + \boldsymbol{\gamma}^\top \mathbf{Z}_t) dt,$$

where $N_{it} = 1_{(\tau_i \leq t)}$ is the one-jump default counting process for firm i . We investigate the assumption of independent censoring and entry-pattern in Section 2.4, and find that our parameter estimates are robust to the exclusion of firm-years that could potentially induce bias.

Estimation, inference, and model selection for the Cox model may then be based on maximum likelihood techniques. Given maximum likelihood estimators (MLEs) of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, we can judge the influence of covariates on default intensities by judging the significance of the corresponding regression coefficients, and we can predict firm-specific and aggregate default intensities by plugging the MLEs back into intensity specification of the Cox model. Model check may be based on the so-called “martingale residual processes,”

$$N_{it} - \int_0^t Y_{is} \exp(\widehat{\boldsymbol{\beta}}^\top \mathbf{X}_{is} + \widehat{\boldsymbol{\gamma}}^\top \mathbf{Z}_s) ds, \quad i = 1, \dots, n, \quad t \in [0, T], \quad (2.1)$$

which, when the model fit is adequate and in large samples, are asymptotic mean-zero martingales. Hence, when aggregated over covariate-quantiles or sectors, the grouped residuals processes should not exhibit any systematic

²Note that while the usual Cox model includes an (unspecified) time-varying baseline-intensity, thereby making it a semi-parametric survival regression model, we cannot simultaneously identify the vector $\boldsymbol{\gamma}$ of macroeconomic regression coefficients as well as a time-varying baseline-intensity – we therefore restrict to a fully parametric model with a constant baseline intensity.

trends when plotted as functions of time.

In addition to the Cox model, we will in our analysis of the cyclicity of default probabilities for small and large firms in Section 2.3 also employ the additive regression model of Aalen (1980, 1989). This specifies the default intensity of firm i as a linear function of the covariates, allowing an easy comparison of regression coefficients across firm size-subsamples.

2.2 Default prediction for private firms

In this section, we provide and discuss the results of our empirical analysis of which accounting ratios and macroeconomic variables that significantly predict defaults in our sample. First, we show the result from a model using only firm-specific variables. We will see that this model cannot adequately predict the cyclical variation in the aggregate default rate. We then add macroeconomic variables to the model and show that this addition allows the model to much more accurately predict the aggregate default rate over time. However, when judging the different models' ability to correctly rank firms with respect to default likelihood, we will see that macroeconomic variables only marginally improve the ranking based on accounting ratios alone. In summary, to capture cyclicity of default rates, it is sufficient to focus on macroeconomic variables—however, the accounting variables are necessary controls for variations in firm-specific default risk not related to size.

2.2.1 Using accounting ratios alone

Initially, we fit a model of firm-by-firm default intensities using only firm-specific variables. We will use this model to examine to what extent macroeconomic variables add additional explanatory power to default prediction of non-listed firms.

Estimation results for the intensity models using only firm-specific variables are provided in Table 2.4. Due to the high degree of correlation among the measurements within the same categories, we perform a stepwise elimination of variables in a given category, removing the least significant variables in each step. The outcome is that interest bearing debt to total assets, net income to total assets, quick ratio, and tangible assets (PPE) to total assets remain in the model, along with age of banking relationship, log of total assets, and a negative equity dummy.

Interpreting the preferred model (Model 5 in Table 2.4) the effect of age is negative, implying that the longer a firm has had a relationship with the bank, the less likely it is that the firm will default. The effect of size, as measured by book assets, appears insignificant in the specification. This might potentially be explained by the sample pertaining to only the largest corporate clients, where size is less relevant as an explanation of default. The leverage ratio of interest bearing debt to total assets is, as expected, positively related to default probability. Likewise, past profitability is negatively related to default probability. The quick ratio enters with a significant negative sign confirming the hypothesis that the more liquidity a firm has, the higher its ability to service unexpected cash shortfalls which would otherwise have resulted in a default. Tangible assets, or Plant Property and Equipments (PPE), to total assets does not appear to have a significant effect, confirming the findings of Bonfim (2009) that tangible assets to total assets remain insignificant in explaining corporate defaults. The negative equity dummy enters with a positive sign in all specifications, confirming that a firm with negative equity is in fact a sign of a firm in trouble and at increased risk of default. The sign of the sectoral dummies are all positive and significant, confirming that these sectors have above average default rates.

Using the results of Table 2.4, we calculate a predicted quarterly default intensity for each firm in the sample, and then aggregate these to get a predicted aggregate intensity for each quarter. Figure 2.3 shows the observed number of

Table 2.4. Estimation results for Cox models including only accounting ratios. The table shows parameter estimates, standard errors, and model summary statistics for Cox models of the quarterly default intensity of firms in the sample. All variables are lagged one year to allow for one-year prediction. The full list of firm-specific variables are included in model (1). Models (2) through (4) show the stepwise elimination, keeping only the most significant measure within the groups of (1) leverage, (2) profitability, (3) liquidity, and (4) collateralization. Model (5) (shaded grey) is the preferred specification when only firm-specific variables are used as covariates. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm-level.

Dependent variable: Default (0/1)	(1)	(2)	(3)	(4)	(5)
Variables (all lagged 1 year)	Coef	Coef	Coef	Coef	Coef
Intercept	-6,788 ***	-6,494 ***	-6,668 ***	-6,684 ***	-6,729 ***
Years active in the bank	-0,011 ***	-0,011 ***	-0,011 ***	-0,011 ***	-0,011 ***
Log of total assets	0,043 *	0,014	0,020	0,019	0,016
(1) Short term debt to total assets	-0,572 **				
Total debt to total assets	0,644 **				
Interest bearing debt to total assets	0,661 ***	1,226 ***	1,262 ***	1,261 ***	1,255 ***
Interest payments to total assets	7,843 ***				
(2) Net income to total assets	-0,961 **	-1,790 ***	-2,012 ***	-2,025 ***	-2,015 ***
EBIT to total assets	1,403 *	1,963 **			
EBITDA to total assets	-2,615 ***	-2,514 ***			
(3) Quick Ratio	-0,078	-0,045	-0,057	-0,212 **	-0,202 **
Current ratio	-0,192	-0,179	-0,161		
(4) Fixed assets to total assets	-0,455	-0,288	-0,308	-0,280	
PPE to total assets	0,529 **	0,571 **	0,473 **	0,466 **	0,255
Negative equity, dummy	0,467 **	0,547 ***	0,555 ***	0,563 ***	0,570 ***
Construction, dummy	0,926 ***	0,885 ***	0,928 ***	0,921 ***	0,951 ***
Wholesale and retail trade, dummy	0,214	0,216 *	0,267 **	0,250 *	0,275 **
Manufacturing, dummy	0,404 ***	0,399 ***	0,420 ***	0,406 ***	0,417 ***
Number of observations	192.196	192.196	192.196	192.196	192.196
Number of firms	10.671	10.671	10.671	10.671	10.671
Number of events	633	633	633	633	633
Sector effects	YES	YES	YES	YES	YES
QIC	7.677,2	7.716,4	7.724,8	7.722,8	7.721,9
QICu	7.669,2	7.710,5	7.719,2	7.717,8	7.717,5

quarterly defaults in the sample along with the predicted number of defaults based on the preferred Cox model only including accounting ratios. As evident in panel (a), the model based on accounting ratios alone is unable to explain the cyclical nature of the observed defaults. However, acknowledging that the firm-specific data can only change yearly through annual financial statements, it may be more appropriate to aggregate the predicted and observed number of defaults on a yearly basis. This is shown in panel (b), and the conclusion is the same: The model based solely on firm-specific variables is not capable of capturing the cyclical variation in defaults.

2.2.2 Including macroeconomic variables

Given that firm-specific variables are unable to explain the cyclical nature of defaults in our sample, this section attempts to incorporate macroeconomic effects. In order to assess if macroeconomic variables add explanatory power beyond what is implied by the firm-specific variables, the preferred model of the firm-specific variables is used as the basis of the covariate specification.

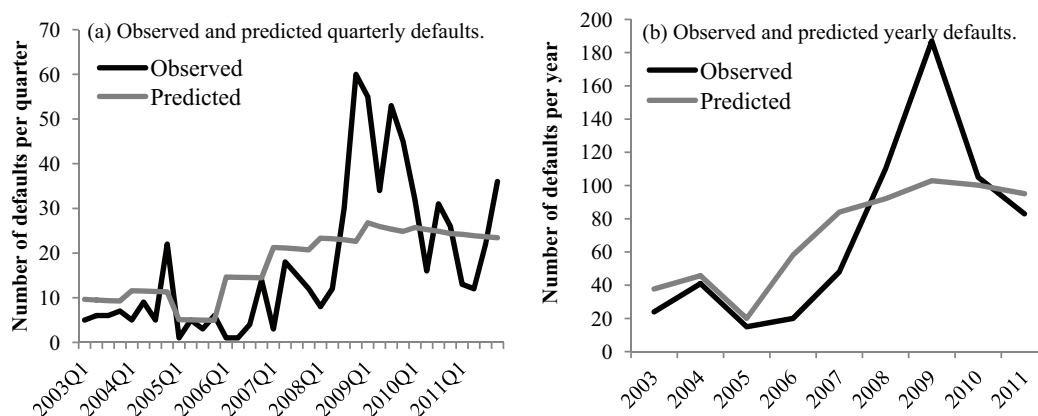


Figure 2.3. Default prediction based on the preferred Cox model only including accounting ratios. Panel (a) shows the observed number of quarterly defaults in the sample along with the predicted number of defaults based on the preferred Cox model only including accounting ratios (Model (5) in Table 2.4). Panel (b) is similar, except that the aggregation is done on a yearly basis.

Table 2.5 presents estimation results for the models incorporating macroeconomic variables. The selection procedure has been to perform a stepwise elimination of insignificant variables until only significant macroeconomic variables remain in the model. Model (7) is the preferred model including both firm-specific and microeconomic variables, while Model (8) is this preferred model excluding the firm-specific variables.

The effects of the firm-specific variables remain robust to the inclusion of the macroeconomic variables. In the preferred model (Model (7) of Table 2.5), the significant macroeconomic variables are as follows: The return of the OMX stock market index, the volatility of OMX index, the difference between CIBOR and the policy rate, slope of the yield curve, change in consumer confidence, change in the capacity utilization, the return of the Stoxx 50 index, and, finally, the European GDP growth rate. On the other hand, the aggregate number of defaults, the Danish real GDP growth, exports as a fraction of GDP, inflation, changes in the cyclical indicator for construction, as well as the loan growth to non-financials are all insignificant in predicting default events.

When interpreting the coefficients of the macroeconomic variables in multivariate intensity regression models, one should bear in mind that it would be unrealistic to obtain a complete *ceteris paribus* effect of one macroeconomic variable, as this variable cannot be viewed in isolation from other macroeconomic variables. While not done here, an appropriate interpretation would involve testing the model from the perspective of internally consistent scenarios of macroeconomic variables. For instance, a further analysis shows that the volatility of the stock market, the slope of the yield curve, the return of the Stoxx 50 index, and the European GDP growth rate appear with an opposite sign in the preferred model compared to a model where they enter separately.

Nonetheless, a positive return of the OMX stock market would, controlling for other macroeconomic effects, imply a lower number of default occurrences one year after. An increased spread between CIBOR and the policy rate would be associated with an increased number of default occurrences, thereby supporting the notion that the higher funding costs of the banks would generally be passed through to clients. Both growth in house prices and changes in the consumer confidence index tend to be negatively linked to defaults, illustrating the importance of demand side effects. Capacity utilization is also negatively associated with default occurrences, meriting the interpretation that higher level of idle capacity could result in price competition that would ultimately lead a number of firms to default.

Table 2.5. Estimation results for Cox models with both accounting- and macroeconomic variables. The table shows parameter estimates, standard errors, and model summary statistics for Cox models of the quarterly default intensity of firms in the sample. All variables are lagged one year to allow for one-year prediction. The full list of firm-specific and macroeconomic variables are included in model (6), and model (7) is the preferred specification after stepwise elimination of variables. Model (8) is the preferred specification in Model (7) excluding the firm-specific variables. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm-level.

Dependent variable: Default (0/1)	(6)	(7)	(8)
Variables (all lagged 1 year)	Coef	Coef	Coef
Intercept	-13,341 ***	-8,051 ***	-7,206 ***
Years active in the bank	-0,011 ***	-0,011 ***	
Log of total assets	0,007	0,007	
Interest bearing debt to total assets	1,232 ***	1,231 ***	
Net income to total assets	-1,877 ***	-1,877 ***	
Quick Ratio	-0,200 **	-0,200 **	
PPE to total assets	0,282 *	0,281 *	
Dummy for negative equity	0,592 ***	0,591 ***	
Aggregate quarterly number of Danish bankruptcies	0,005		
Danish Real GDP growth	-0,027		
Export / GDP	9,633 *		
Inflation, pct point	-0,124		
OMX stock market return	-0,052 ***	-0,045 ***	-0,038 ***
OMX stock market volatility	-0,119 ***	-0,101 ***	-0,099 ***
Difference CIBOR - policy rate, pct. Point	1,837 ***	2,006 ***	2,117 ***
Yield curve slope 10y - 3m, pct. Point	0,986 ***	0,588 ***	0,634 ***
Growth in house prices	-0,114 ***	-0,104 ***	-0,115 ***
Change in consumer confidence Indicator	-0,110 ***	-0,099 ***	-0,110 ***
Change in cyclical indicator, construction	0,002		
Change in capacity utilization in the industrial sector	-0,338 ***	-0,300 ***	-0,292 ***
Loan growth to non-financial institutions	0,023 **		
Stoxx50 stock market return	0,031 ***	0,034 ***	0,029 ***
EU27 Real GDP growth	0,250 ***	0,227 ***	0,213 ***
Number of observations	192.196	192.196	192.196
Number of firms	10.671	10.671	10.671
Number of events	633	633	633
Sector effects	YES	YES	YES
QIC	7.513	7.513,6	8.265,0
QICu	7.518	7.508,6	8.264,7

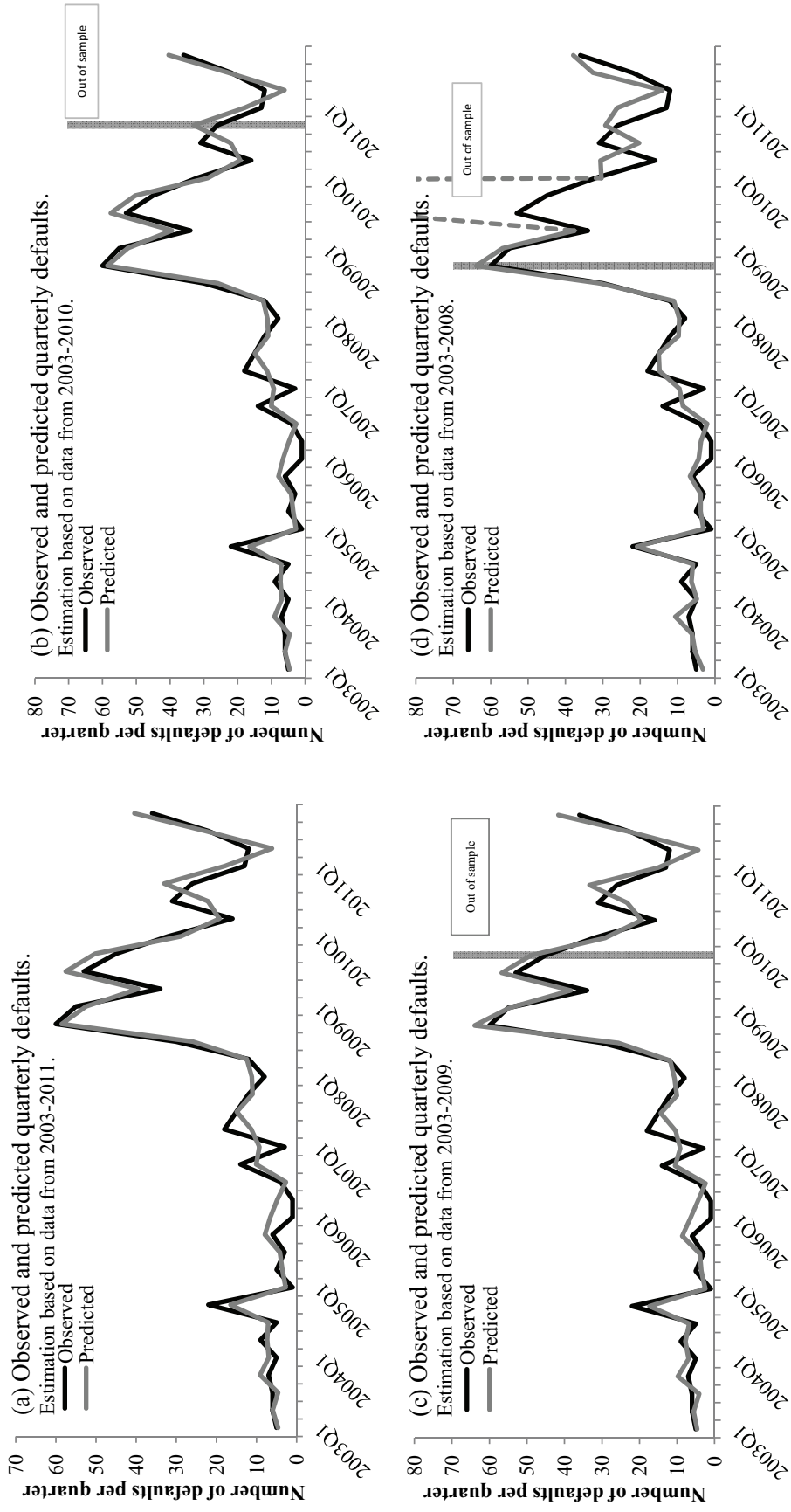


Figure 2.4. Default prediction based on the preferred Cox model with both accounting ratios and macroeconomic variables. All panels show the observed quarterly number of defaults in the sample along with a predicted number of defaults based on the preferred Cox model with both firm-specific and macroeconomic covariates (Model (7) in Table 2.5). In Panel (a), the model is estimated using the full sample from 2003 to 2011. In Panels (b), (c), and (d), the model is estimated on shorter subsamples, allowing in each case out-of-sample prediction on the remaining part of the full sample.

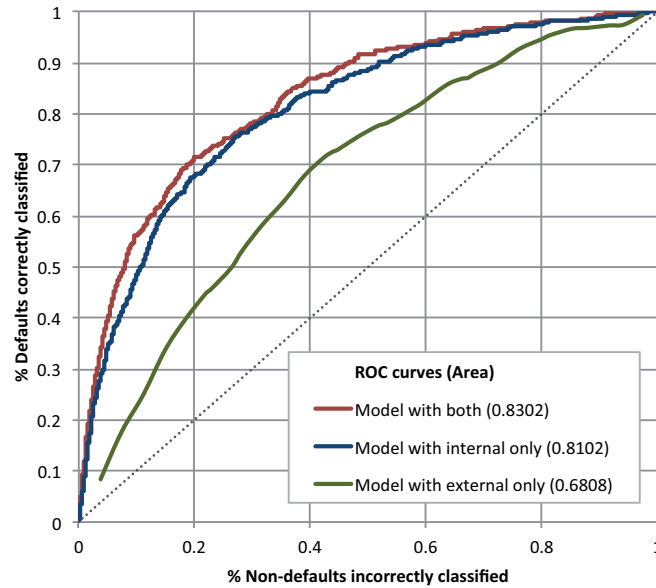


Figure 2.5. Comparison of firm-ranking accuracy for different covariate-specifications. The figure shows receiver operating characteristic (ROC) curves for Cox models with different covariate-specifications fitted to the sample. Each curve illustrates the model’s ability to correctly discriminate between defaults and non-defaults, and plots the percentage correctly classified defaults (true positives) against the percentage incorrectly classified non-defaults (false positives) at all possible cut-off points of default intensity. The area under each curve serves as a goodness-of-fit measure, where a value of 1 means a model with perfect discriminatory ability, while a value of 0.5 means a model that discriminates based on a random guess.

Figure 2.4 illustrates the relationship between the observed and predicted number of defaults taking into account both the firm-specific variables and the macroeconomic variables in Model (7) of Table 2.5. Adding macroeconomic variables as explanatory factors improves the model’s ability to predict the cyclical variation in quarterly default occurrences. While there is a potential that the good fit of the preferred model may be a result of the numerical optimization techniques deployed for estimating the parameters, the out of sample prediction obtained from estimating the same model on only part of the data puts comfort in the chosen model. Panel (b) and (c) of the figure estimates the model on the sample excluding observations from 2010 and both 2010 and 2011 respectively. The obtained coefficients from the models estimated on the reduced samples are then used to estimate the aggregate intensities for all 36 quarters, thereby generating out of sample predictions. Hence, Panel (b) of the figure shows one and Panel (c) shows two years of out of sample prediction. The out of sample prediction based on the reduced sample estimation adequately captures both the level and cyclical variation in default rates.

Panel (d) of Figure 2.4 shows prediction based on excluding the years 2009, 2010 and 2011 from the estimation. For the out of sample prediction in Panel (d), large deviations occur in 2009 (which pertains to 2008 covariates observations because of the one year lag). However, it should be emphasized that the latter model has been fitted to a period of economic expansion, and therefore it is of little surprise that the model cannot be used to predict future defaults in a period of economic contraction. This finding also highlights the importance of estimating default predicting occurrences on a full business cycle.

2.2.3 Ranking firms with respect to default likelihood

The out of sample estimation results presented in Figure 2.4 showed that the preferred model including both firm-specific and macroeconomic variables adequately captures the level and cyclicity of defaults. However, given the

stochastic nature of a default event, it will never be possible to completely predict which firms will default in a given quarter. Nonetheless, by specifying a particular cut-off point for the intensities, the model's ability to correctly discriminate between defaults and non-defaults can be evaluated in terms of how many outcomes that are correctly predicted and how many outcomes that are incorrectly predicted. This, however, necessitates arbitrarily specifying the cut-off point.

A more general approach is to plot the Receiver Operating Curve (ROC), as shown in Figure 2.5. The curve illustrates the percentage of defaults that are correctly classified as defaults on the vertical axis against the percentage of non-defaults that are mistakenly classified as defaults on the horizontal axis for all possible cutoff points. The area under the curve (AUC) is then used as measure of the model goodness of fit where a value of 1.0 implies a model with perfect discriminatory ability and a value of 0.5 is a completely random model.

In terms of discriminatory power, the addition of macroeconomic variables does not improve the model's ability to effectively determine which firms eventually default beyond what is implied by the accounting ratios. From the ROC curves, it can be seen that the model with both external and firm-specific variables is only marginally better in correctly determining defaults compared to the model with just firm-specific variables. This is consistent with the notion that it is the firm-specific characteristics that provide the ordinal ranking of firms, and therefore also ultimately determine *which* firms that actually default. Including the macroeconomic factors only improves the model's ability to capture the cyclical nature in the aggregate default rate, which is related to *when* defaults occurs.

2.3 The macroeconomy's impact on small and large firms' default risk

The results of the previous section show that our final model specification including both firm-specific and macroeconomic variables is able to both accurately rank firms and predict the aggregate default rate over time. We now use this model to investigate whether there is empirical support in our data for the provisions in the Basel II Accord that implicitly assume that the default probabilities of smaller firms are less impacted by the macroeconomy compared to larger firms (The Basel Committee on Banking Supervision, 2006, Articles 273-74). The same provisions are carried forward and extended in the Basel III Accord and the recently adopted CRD VI (The European Parliament and the Council of the European Union, 2013, Articles 153.4 and 501.1). Specifically, Basel II and III allow banks to estimate capital requirements for small and medium-sized corporations (SMEs) using a risk weight formula that includes a lower asset correlation with macroeconomic risk-drivers compared to larger corporations. This allows banks to have effectively smaller capital reserve for this particular segment than would have been the case if they were treated as standard large corporations.

Precisely, the Basel II and III Accords specify the correlation between obligor i 's assets and a common (macroeconomic) risk-driver as

$$\rho_i = 0.12 \times \frac{1 - e^{-50 \times PD_i}}{1 - e^{-50}} + 0.24 \times \left(1 - \frac{1 - e^{-50 \times PD_i}}{1 - e^{-50}}\right) - 0.04 \times \left(1 - \frac{S_i - 5}{45}\right),$$

where PD_i is the one-year probability of default while $S_i = \min\{50, \max\{5, S_i^*\}\}$ where S_i^* is total revenue in millions of Euros. The last term is the deduction in asset correlation specific to SMEs, and equals zero for firms with annual revenue above 50 million euros. To gauge the economic significance of the reduction, note that with an annual default probability of 1%, an SME with an annual revenue of 25 million Euro achieves a deduction in asset correlation of around 11.5% relative to an equally risky non-SME.

Before moving forward, let us underline that the specific asset correlation formula from the Basel II and III

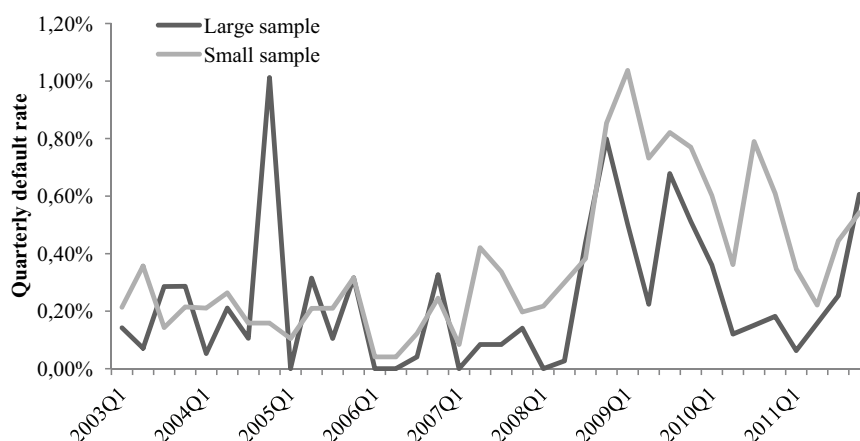


Figure 2.6. Aggregate quarterly default rate in each of the subsamples of small and large firms. This figure shows the aggregate quarterly default rate in each of the subsamples of small and large firms. Based on the year a firm enters the sample, it is classified as “large” if its first-year asset level is above the median asset level that year. Similarly, a firm is classified as “small” if its asset value at the time of entry is below the median asset level that year.

Accords is *not* in itself the focus of our analysis, as the capital reduction for SMEs can be achieved in other ways—for instance through the use of capital multipliers, as in CRD IV.³ Instead, we focus on testing the implicit assumption that the default probabilities of smaller firms are less cyclical compared to the default probabilities of larger firms—that is, whether the capital reduction for SMEs *can* be merited by the fact that their default probabilities are less cyclical, and therefore should have lower correlation with the common risk-driver compared to larger firms.

Our tests examine the extent to which the default intensities of small and large firms are impacted differently by macroeconomic variables. First, we split the sample into two subsamples corresponding to “small” and “large” firms. Based on the year a firm enters the sample, it is classified as “large” if its first-year asset level is above the median asset level that year. Similarly, a firm is classified as “small” if its asset value at the time of entry is below the median asset level that year. This particular division is done so as to ensure approximately equal sample sizes with a sufficient amount of default events in each category and to allow for the classification of firms in a predictable manner. However, we will later show (in the section on robustness and model checks) that our results are robust to a division into four subsamples based on the quartiles of the asset value at the year of entry.

Figure 2.6 shows the aggregate quarterly default rate in each of the subsamples of small and large firms. The two default rates are seen to exhibit a high degree of comovement throughout our entire sample period. In the beginning of our sample period, the two default rates are seen to be virtually indistinguishable except for the spike in the default of large firms around 2005. However, near the end of our sample period, the default rate of small firms tends to be systematically higher than the default rate of large firms, indicating that small firms were hit the hardest by the financial crises of 2008-10.

³Comment (44) on p. 6 of [The European Parliament and the Council of the European Union’s \(2013\) report](#) states the following: “Small and medium sized enterprises (SMEs) are one of the pillars of the European economy given their fundamental role in creating economic growth and providing employment. [...] The limited amount of alternative sources of funding has made EU SMEs even more sensitive to the impact of the banking crisis. It is therefore important to fill the existing funding gap for SMEs and ensure an appropriate flow of bank credit to SMEs in the current context. Capital charges for exposures to SMEs should be reduced through the application of a supporting factor equal to 0.7619 to allow credit institutions increase lending to SMEs.”

2.3.1 Macro-sensitivity analysis based on the Cox model

Table 2.6 shows estimation results for the final model specification including both firm-specific and macroeconomic variables fitted to each of the two subsamples. With the exception of OMX stock market volatility, the magnitude of all macroeconomic effects are larger for large firms compared to small firms. The difference between the coefficients of macroeconomic factors for the two subsamples is significant for the slope of the yield curve, growth in house prices, and European GDP growth, and all are larger in magnitude in the subsample of large firms.⁴ Hence, if we suppose there exists a large and a small firm whose only difference is their size (which is reasonable since all accounting ratios in our models are relative to total assets), the apparent interpretation of these results might be that the small firm's default intensity is less exposed to macroeconomic fluctuations.

On the other hand, the estimation results for the two subsamples also show substantial differences with regards to the coefficients of the firm-specific variables: Firm size appears with a significant positive coefficient for small firms, but an insignificant (yet negative) coefficient for large firms; neither quick ratio nor the ratio of tangible assets to total assets have significant effects for small firms, whereas they have significant effects for large firms; and, finally, negative equity has a significant effect for small firms, but not for large.

While the results for the macroeconomic variables may corroborate the lower asset correlation adopted in Basel II for SMEs, the direct comparison of coefficients in the two subsamples is arguably naïve, because it ignores that a covariate's marginal effect in a non-linear model, like Cox regression, actually depends on the values of all the other covariates. This implies that comparing the coefficients for the macroeconomic variables in the two subsamples is potentially problematic, because such a comparison fails to take into account that the firm-specific characteristic for the small and large firms are generally different and have different effects on default intensity.

⁴In an unreported analysis, we find that the importance of the European GDP growth for the large firms is merited by the tendency of large firms in the sample to engage more actively in exports.

Table 2.6. Estimation results for Cox models fitted to the subsamples of small and large firms. The table shows parameter estimates, standard errors, model summary statistics, and comparison criteria for Cox models of the quarterly default intensity for small and large firms in the sample. The covariate list corresponds to the preferred model including both firm-specific and macroeconomic variables (Model (7) in Table 2.5). All variables are lagged one year to allow for one-year prediction. In each year, an entering firm is deemed “small” if its book value of assets is under the median assets level for that particular year—otherwise, it is deemed “large.” Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm level. “Same sign” indicates whether or not the estimated coefficients are of the same sign for small and large firms; “Magnitude” indicates which type of firm has the largest estimated coefficient; and “Sig. Diff.” indicates whether or not the coefficients for small and large firms are significantly different at the 5% level using a Welch t-test. Finally, “PEA” indicates whether the partial effect at the average is largest for the large firms or the small firms, while “APE” indicates whether the average partial effect is largest for the large firms or the small firms.

Variables (all lagged 1 year)	Small firms			Large firms			Comparison				
	Coef	Se		Coef	Se		Same sign	Magnitude	Sig. diff.	PEA	APE
Intercept	-8.290	***	0.676	-9.057	***	0.765	Yes	Large	No	Small	Small
Years active in the bank	-0.008	**	0.004	-0.012	***	0.004	Yes	Large	No	Small	Small
Log of total assets	0.122	**	0.056	-0.006	0.043		No	NA	No	NA	NA
Interest bearing debt to total assets	1.048	***	0.224	1.628	***	0.351	Yes	Large	No	Small	Small
Net income to total assets	-1.788	***	0.302	-2.432	***	0.422	Yes	Large	No	Small	Small
Quick ratio	-0.112		0.074	-0.462	***	0.137	Yes	Large	Yes	Large	Large
PPE to total assets	0.040		0.197	0.508	**	0.255	Yes	Large	No	Large	Large
Dummy for negative equity	0.782	***	0.199	0.269	0.291		Yes	Small	No	Small	Small
OMX stock market return	-0.045	***	0.015	-0.045	**	0.018	Yes	Large	No	Small	Small
OMX stock market volatility	-0.102	***	0.038	-0.100	**	0.049	Yes	Small	No	Small	Small
Difference CIBOR - policy rate, pct. point	1.655	***	0.447	2.463	***	0.553	Yes	Large	No	Small	Small
Yield curve slope 10y - 3m, pct. point	0.372	***	0.130	0.869	***	0.154	Yes	Large	Yes	Small	Large
Growth in house prices	-0.040		0.032	-0.200	***	0.047	Yes	Large	Yes	Large	Large
Change in consumer confidence indicator	-0.083	***	0.026	-0.122	***	0.036	Yes	Large	No	Small	Small
Change in capacity utilization	-0.194	**	0.086	-0.420	***	0.106	Yes	Large	No	Small	Large
Stoxx50 stock market return	0.027	***	0.009	0.043	***	0.013	Yes	Large	No	Small	Small
EU27 Real GDP growth	0.101	*	0.055	0.410	***	0.070	Yes	Large	Yes	Large	Large
PEA scaling factor	0.0023			0.0009							
APE scaling factor	0.0041			0.0025							
Number of observations	91,182			101,014							
Number of firms	5,333			5,338							
Number of defaults	376			257							
Sector effects	Yes			Yes							

To elaborate, note that the marginal effect in a Cox regression of a change in the j th macroeconomic variable on the default intensity of firm i is given by

$$\frac{\partial \lambda(\mathbf{X}_{it}, \mathbf{Z}_t)}{\partial Z_{jt}} = \gamma_j Y_{it} \exp(\boldsymbol{\beta}^\top \mathbf{X}_{it} + \boldsymbol{\gamma}^\top \mathbf{Z}_t) = \gamma_j \lambda(\mathbf{X}_{it}, \mathbf{Z}_t),$$

which depends on all the characteristics of firm i through \mathbf{X}_{it} , as well as all other macroeconomic variables through the dependence on \mathbf{Z}_t .

A somewhat crude way to facilitate comparison between subsamples in non-linear models, like Cox regression, is to compute the marginal effect of a covariate at “average levels” in each of the subsamples. This gives rise to the *partial effect at the average* (PEA) and the *average partial effect* (APE)—see, for instance, Wooldridge (2009, p. 582-83). In the setting of an intensity model, the PEA plugs a subsample’s average covariate values into the subsample’s estimated intensity, while the APE takes the average across the estimated intensity values for each subsample. Due to the non-linearity of the intensity, Jensen’s inequality implies that the two ways of averaging will generally produce different results.

In our analysis, the PEA is a measure of a covariate’s marginal effect for the “average firm” and at “average macroeconomic levels” in each of the two subsamples of small and large firms. We thus compute the PEA for the j th macroeconomic variable in subsample k as

$$\text{PEA}_{kj} = \gamma_{kj} \underbrace{\exp(\widehat{\boldsymbol{\beta}}_k \bar{\mathbf{X}}_k + \widehat{\boldsymbol{\gamma}}_k^\top \bar{\mathbf{Z}}_k)}_{=s_k^{\text{PEA}}},$$

where $k \in \{\text{small, large}\}$, $\bar{\mathbf{X}}_k$ is the average firm-specific covariate vector for firms in subsample k , $\bar{\mathbf{Z}}_k$ is the average macroeconomic covariate vector in subsample k , $\widehat{\boldsymbol{\beta}}_k$ and $\widehat{\boldsymbol{\gamma}}_k$ are the estimated regression coefficients in subsample k , while s_k^{PEA} is a subsample-specific scaling factor for each PEA. On the other hand, the APE is a measure of a covariate’s marginal effect at the “average intensity level” across firms and time in each of the two subsamples. The APE for the j th macroeconomic variable in subsample k is thus computed as

$$\text{APE}_{kj} = \gamma_{kj} \underbrace{\frac{1}{T} \int_0^T \frac{1}{|k(t)|} \sum_{i \in k(t)} \exp(\widehat{\boldsymbol{\beta}}_k^\top \mathbf{X}_{it} + \widehat{\boldsymbol{\gamma}}_k^\top \mathbf{Z}_t) dt}_{=s_k^{\text{APE}}},$$

where $k(t)$ denotes the firms belonging to subsample $k \in \{\text{small, large}\}$ at time t , and s_k^{APE} is again a subsample-specific scaling factor for each APE.

The two right-most columns of Table 2.6 show the PEAs and APEs for each covariate in each of the two subsamples. Focusing on the effects of the macroeconomic variables, the PEA seems to suggest that most macro effects are, on average, strongest in the sample of small firms. On the other hand, the APEs for the macroeconomic variables suggest that it is entirely dependent on the macro variable at hand whether its average effect is strongest for the smaller or the larger firms.

In sum, while the naïve comparison of regression coefficients seems to indicate that there is merit to the assumption that smaller firms are less cyclical than larger firms, the more refined analysis based on the PEA and APE, which takes the non-linearity of the Cox model into account, indicates that smaller firms may “on average” be as cyclical, or perhaps even more cyclical, than larger firms. We are thus reluctant to draw any conclusions from a comparison of the subsamples based on the Cox regression model. We, therefore, in the following section, perform an additional

macro-sensitivity analysis based on a model that allows for an easier comparison of coefficients across subsamples.

2.3.2 Macro-sensitivity analysis based on the additive Aalen model

A direct comparison of coefficients in subsamples is possible using the additive intensity regression model. This model was first proposed by Aalen (1980, 1989) and has recently been applied in a default study of public US corporations by Lando et al. (2013). The Aalen model specifies the default intensity for firm i as

$$\lambda(\mathbf{X}_{it}, \mathbf{Z}_t) = \boldsymbol{\beta}(t)^\top \mathbf{X}_{it} + \boldsymbol{\gamma}^\top \mathbf{Z}_t,$$

where $\boldsymbol{\beta}(t)$ is a vector of unspecified regression *functions*, giving the linear effects of the firm-specific covariates at time t , while $\boldsymbol{\gamma}$ is a (constant) vector of regression coefficients for the macroeconomic variables. In contrast to the multiplicative effects in the Cox model, covariate effects in the additive model are easy to interpret and compare across subsamples. We will therefore use the additive model to check the assumption that the default probabilities of smaller firms are less sensitive to macroeconomic cyclicalities. Note, however, that the Cox model still has the advantage that it automatically produces nonnegative intensities and its constant regression coefficients allow out of sample prediction.

The linearity of the additive model allows for estimation of both time-varying and constant parameters using ordinary least squares-methods. For the time-varying coefficients, the focus is on the *cumulative regression coefficients*, $B_j(t) = \int_0^t \beta_0(s) ds$, which are easy to estimate non-parametrically. Further, formal tests of the significance and time-variation of regression functions is possible through resampling schemes. We refer to Aalen et al. (2008), Martinussen and Scheike (2006), and Lando et al. (2013) for a detailed presentation of estimation and inference procedures for the additive model.

Initially, we fit an additive model for our entire sample of firms, including the same covariate specification as our final Cox model (Model (7) of Table 2.5), and with time-varying coefficients for the firm-specific covariate. The hypothesis of a time-constant effect could not be rejected for all firm-specific covariates but the following four: The (log of) total assets, interest bearing debt to total assets, quick ratio, and the construction sector indicator. The time-varying effects of these four firm-specific covariates are shown as cumulative regression coefficients with 95% pointwise confidence bands in Figure 2.7. When interpreting these effects, one should focus on the *slope* of the cumulative coefficients, which estimates the regression coefficients themselves. We see, for instance, that interest bearing debt to total assets has a negligible effect up to around the year 2009, before the effect becomes quite strong and associated with higher default intensity for the rest of the sample period. All four covariate effects have the expected signs in periods where they have a non-negligible influence on default intensities.

The estimation results for the time-constant regression coefficients from the additive model fitted to the entire sample are given in Table 2.7. We see that all coefficients corresponding to firm-specific variables have the same sign as in our final Cox model (Model (7) of Table 2.5) and roughly the same significance level. The macroeconomic variables, however, appear to have lost much of their importance compared to the analysis based on the Cox models. In the additive setting, only OMX stock market return, change in consumer confidence indicator, and Stoxx50 stock market return have significant effects. The latter is of the reversed sign compared to intuition, but is nonetheless consistent with results for public-firms found by Duffie et al. (2007, 2009), Lando and Nielsen (2010), and Figlewski et al. (2012), amongst others.⁵ The general lack of importance of macroeconomic effects is a consequence of the

⁵Recent work by Giesecke, Lando, and Medhat (2013) shows that univariately significant but multivariately insignificant or even reversed effects may be observed for macroeconomic variables if these have indirect effects mediated through other covariates included in the models. This is in particular the case for stock market returns.

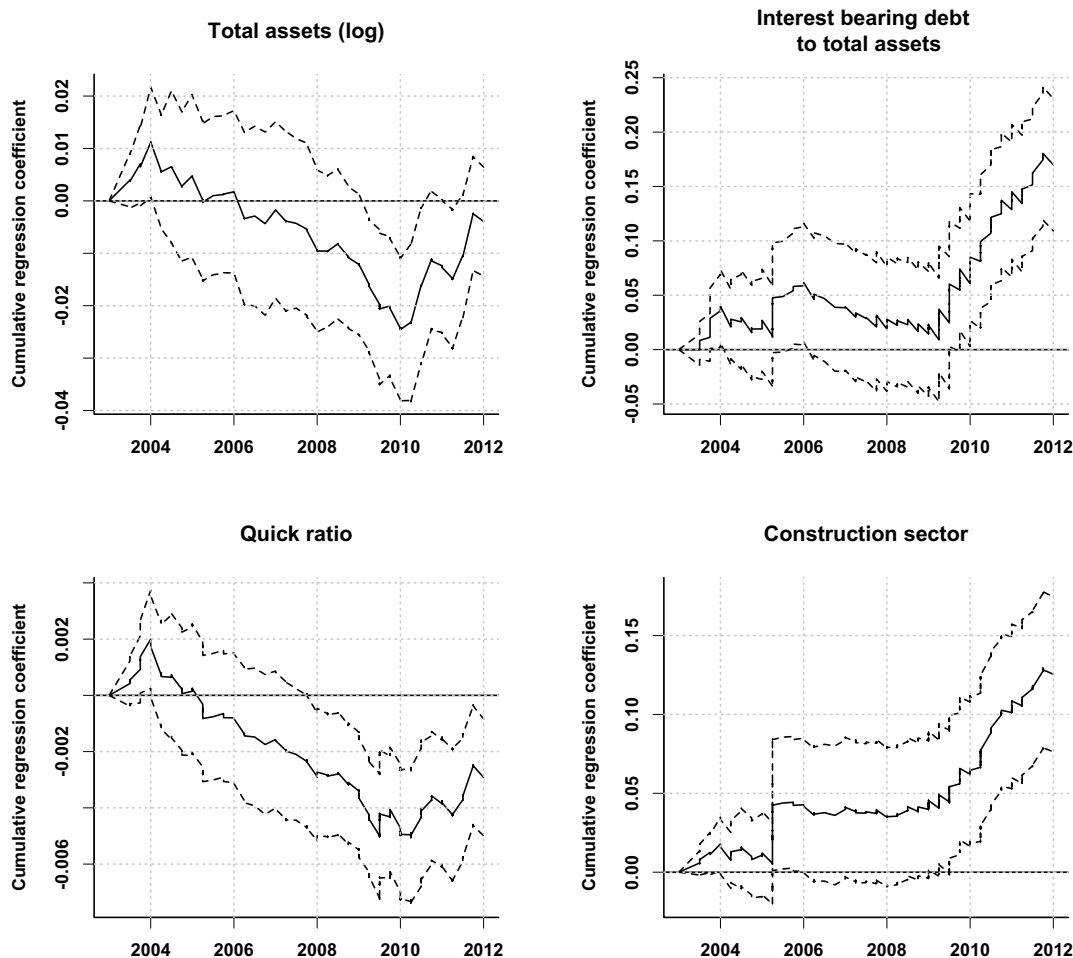


Figure 2.7. Cumulative regression coefficients from Aalen analysis of full sample. The panels show (cumulative) estimation results for the significantly time-varying firm-specific covariates from an analysis based on the additive Aalen model including the same covariate specification as our final Cox model (Model (7) of Table 2.5). All variables are lagged one year to allow for one-year prediction. The dotted lines are asymptotic 95% pointwise confidence bands.

significantly time-varying effects for four of the firm-specific variables: Allowing firm-specific effects to be time-varying implies that the effects will to some extent vary with the macroeconomy, and this reduces the added explanatory power of macroeconomic variables.

We now fit the additive model including both the preferred firm-specific and macroeconomic variables to each of the two subsamples containing small and large firms. Even though the macroeconomic variables did not have much explanatory power in the additive model fitted to the whole sample, an additive model, due to its linearity in the regression coefficients, still allows us to directly compare effects for macroeconomic variables in each subsample. One could perhaps imagine that some macroeconomic variables had significant additive effects in the subsample of large firms, but not in the subsample of small firms—this would be evidence that macro-dependence differs for small and large firms, as is the working assumption in the Basel II/ III and CRD IV Accords.

Estimating the additive model in each of the two subsamples did not change the conclusions regarding which firm-specific covariates have significant time-varying effects compared to the full sample. The estimation results for the time-constant regression coefficients in each of the two subsamples is shown in Table 2.8. We see no difference in

Table 2.7. Constant regression coefficients from Aalen analysis of full sample. The table shows the estimation results for the time-constant regression coefficients for the firm-specific and macroeconomic variables from an analysis based on the additive Aalen model including the same covariate specification as our final Cox model (Model (7) of Table 2.5). All variables are lagged one year to allow for one-year prediction. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on robust standard errors.

Variables (all lagged 1 year)	Coef		Se
Intercept	0.0025		0.0407
Years active in the bank	-2.05×10^{-5}	***	5.34×10^{-6}
Net income to total assets	-0.0140	***	0.0017
PPE to total assets	-0.0014	**	0.0006
Dummy for negative equity	0.0078	***	0.0012
Wholesale and retail trade, dummy	0.0005		0.0004
Manufacturing, dummy	0.0012	**	0.0004
OMX stock market return	-0.0036	**	0.0016
OMX stock market volatility	-0.0064		0.0080
Difference CIBOR - policy rate, pct. point	0.0059		0.0510
Yield curve slope 10y - 3m, pct. point	0.0110		0.0111
Growth in house prices	-0.0006		0.0024
Change in consumer confidence indicator	-0.0105	**	0.0047
Change in capacity utilization	-0.0077		0.0085
Stoxx50 stock market return	0.0042	***	0.0012
EU27 Real GDP growth	0.0050		0.0077

sign, and virtually no difference in significance or magnitude for small firms compared to large firms. Hence, from the analysis based on the additive model, we find no evidence that the default intensities of small firms are less sensitive to macroeconomic cyclicality.

2.4 Robustness and model check

In this section, we perform robustness checks and examine the model fit of our preferred Cox model. First, we show that the effects of our variables within the subsamples of small and large firms remain largely unaltered when we use alternative size-criteria to split the full sample. Second, we show that the effects of our variables are robust to the exclusion of certain sample periods. Third, we show that the effects of our variables are robust to alternative choices of lag lengths. Finally, we verify that our preferred Cox model gives an adequate fit to the data by showing that the model's residual processes do not exhibit systematic trends when grouped according to sectors and size quarterlies.

2.4.1 Alternative criteria for defining small and large firms

In our analysis of small and large firms in Table 2.6, the criterion defining the two subsamples of small and large firms was the median asset level of the year where a particular firm enters the sample. We now investigate alternative criteria for defining the subsamples of small and large firms and how they affect the effects of our variables within the two subsamples.

Table 2.9 shows estimation results for Cox models within subsamples of small and large firms, where the subsamples are using defined different, time-varying size-criteria. All the models use the covariate list corresponding to our preferred Cox model including both firm-specific and macroeconomic variables, i.e. Model (7) in Table 2.5.

Models (A-1) and (A-2) show the estimation results when the full sample is divided into subsamples of small and a large firms according to the following criterion: A particular firm in a particular year is classified as small (large) if the

Table 2.8. Constant regression coefficients from Aalen analysis of small and large firms. The table shows the estimation results for the time-constant regression coefficients for the firm-specific and macroeconomic variables from an analysis based on the additive Aalen model for the two subsamples of small and large firms. All variables are lagged one year to allow for one-year prediction. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on robust standard errors.

Variables (all lagged 1 year)	Small firms			Large firms		
	Coef		Se	Coef		Se
Intercept	0.0597		0.1883	0.0600		0.1918
Years active in the bank	-1.83×10^{-5}	*	1.14×10^{-5}	-1.70×10^{-5}		1.18×10^{-5}
Net income to total assets	-0.0160	***	0.0025	-0.0168	***	0.0025
PPE to total assets	-0.0021	**	0.0010	-0.0021	**	0.0010
Dummy for negative equity	0.0082	***	0.0016	0.0085	***	0.0016
Wholesale and retail trade, dummy	0.0003		0.0006	0.0003		0.0006
Manufacturing, dummy	0.0011		0.0007	0.0011		0.0007
OMX stock market return	-0.0084		0.0066	-0.0135	*	0.0069
OMX stock market volatility	0.0192		0.0324	0.0089		0.0317
Difference CIBOR - policy rate, pct. point	-0.3409		0.2317	-0.3048		0.2416
Yield curve slope 10y - 3m, pct. point	0.0003		0.0524	0.0134		0.0560
Growth in house prices	0.0136		0.0105	0.0098		0.0115
Change in consumer confidence indicator	-0.0324	*	0.0191	-0.0283	*	0.0196
Change in capacity utilization	-0.0625		0.0402	-0.0713	*	0.0404
Stoxx50 stock market return	0.0111	**	0.0050	0.0146	**	0.0054
EU27 Real GDP growth	0.0049		0.0351	0.0131		0.0370

firm's book asset value is below (above) the 25th percentile of the full sample's book asset values. This classification is time-varying in the sense that the same firm can be classified as small in some years but large in other years. Similarly, models (A-3) and (A-4) show the estimation results when a particular firm in a particular year is classified as small (large) if the firm's book asset value is below (above) the 50th percentile of the full sample's book asset values. Lastly, models (A-5) and (A-6) show the estimation results when a particular firm in a particular year is classified as small (large) if the firm's book asset value is below (above) the 75th percentile of the full sample's book asset values.

A comparison of the estimation results within the three subsamples of small firms—i.e. models (A-1), (A-3), and (A-5)—shows that the signs, magnitudes, and significance levels of the effects of both the firm-specific and the macroeconomic variables remain largely unaltered as the size-criterion defining the subsample of small firms increases. The only notable changes for the firm-specific variables are for the size variable itself, which loses magnitude and significance as the size-criterion defining a small firm increases, and for the manufacturing sector dummy, which gains magnitude and significance. The only notable changes for the macroeconomic variables are for the slope of the yield curve and the change in capacity utilization, which gain magnitude and significance as the size-criterion defining a small firm increases. Furthermore, a comparison of models (A-1), (A-3), and (A-5) with the model for small firms in our main analysis in Table 2.6 shows that using a time-varying classification of small firms entails no material changes to the signs, magnitudes, or significance levels of the effects of our variables within the subsamples of small firms.

Similarly, a comparison of the estimation results within Table 2.9's three subsamples of large firms—i.e. models (A-2), (A-4), and (A-6)—shows that the signs, magnitudes, and significance levels of the effects of both the firm-specific and the macroeconomic variables remain largely unaltered as the size-criterion defining the subsample of small firms increases. Finally, a comparison of models (A-2), (A-4), and (A-6) with the model for large firms in our main analysis in Table 2.6 shows that using a time-varying classification of large firms entails no material changes to the signs, magnitudes, or significance levels of the effects of our variables within the subsamples of large firms.

Table 2.9. Estimation results for Cox models fitted to subsamples of small and large firms using different, time-varying size-criteria. The table shows parameter estimates, standard errors, and model summary statistics for Cox models of the quarterly default intensity for small and large firms in the sample. The covariate list corresponds to the preferred model including both firm-specific and macroeconomic variables (Model (7) in Table 2.5). All variables are lagged one year to allow for one-year prediction. The firm-specific variables are kept in all models to focus on the added explanatory effect of macroeconomic variables. In models (A-1) and (A-2), a particular firm in a particular year is classified as small (large) if the firm's book asset value is below (above) the 25th percentile of the full sample's book asset values (which is DKK 9,553,000). In models (A-3) and (A-4), the cut-off is instead the 50th percentile of the full sample's book asset values (which is DKK 28,222,000). Finally, in models (A-5) and (A-6), the cutoff is the 75th percentile of the full sample's book asset values (which is DKK 100,723,000). This classification is time-varying in the sense that the same firm can be classified as small in some years but large in other years. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm level.

Dependent variable: Default (0/1)	Cutoff: 25th percentile			50th percentile			75th percentile		
	(A.1)	(A.2)	Se	(A.3)	(A.4)	Se	(A.5)	(A.6)	Se
Variables (all lagged 1 year)	(A.1)	(A.2)	Se	(A.3)	(A.4)	Se	(A.5)	(A.6)	Se
Intercept	-9,00 ***	1,08	0,01	-8,32 ***	0,71	-8,58 ***	0,81	-8,14 ***	0,56
Years active in the bank	-0,01	-0,01	0,00	-0,01 **	0,00	0,00	0,00	-0,01 ***	0,00
Log of total assets	0,28 **	0,11	0,04	0,12 **	0,06	0,05	0,06	0,06	0,04
Interest bearing debt to total assets	0,87 ***	0,26	0,27	0,89 ***	0,22	0,36	1,09 ***	0,21	0,21
Net income to total assets	-1,65 ***	0,34	0,34	-2,02 ***	0,28	0,51	-1,96 ***	0,26	0,26
Quick Ratio	-0,09	0,07	0,15	-0,12 *	0,07	0,14	-0,16 *	0,09	0,09
PPE to total assets	-0,01	0,26	0,20	0,23	0,19	0,25	0,31 *	0,17	0,17
Dummy for negative equity	0,93 ***	0,23	0,22	0,79 ***	0,19	0,31	0,67 ***	0,17	0,24
Construction	0,41 **	0,20	0,19	0,65 ***	0,17	0,24	0,79 ***	0,15	0,15
Wholesale and retail trade	0,01	0,19	0,17	0,19	0,15	0,21	0,24 *	0,14	0,14
Manufacturing	0,17	0,23	0,17	0,29 *	0,18	0,21	0,42 ***	0,15	0,15
OMX stock market return	-0,05 ***	0,02	0,02	-0,04 ***	0,01	0,02	-0,05 ***	0,01	-0,04
OMX stock market volatility	-0,12 **	0,05	0,04	-0,10 ***	0,04	0,05	-0,11 ***	0,03	-0,09
Difference CIBOR - policy rate, pct. point	1,45 ***	0,54	0,44	1,79 ***	0,44	0,55	1,94 ***	0,38	2,10 ***
Yield curve slope 10y - 3m, pct. point	0,21	0,16	0,13	0,36 ***	0,13	0,16	0,47 ***	0,11	1,08 ***
Growth in House prices	-0,03	0,04	0,04	-0,03	0,03	0,05	-0,06 **	0,03	-0,33 ***
Change in Consumer Confidence Indicator	-0,08 **	0,03	0,03	-0,09 ***	0,03	0,04	-0,10 ***	0,02	-0,10 *
Change in Capacity utilization in the industrial sector	-0,14	0,11	0,09	-0,16 *	0,08	0,11	-0,24 ***	0,07	-0,48 ***
Stoxx50 stock market return	0,04 ***	0,01	0,01	0,03 ***	0,01	0,01	0,03 ***	0,01	0,04 *
EU27 Real GDP growth	0,07	0,07	0,06	0,10 *	0,05	0,07	0,16 ***	0,05	0,56 ***
Number of observations	48.049	144.147		96.083	96.113		144.150		48.046
Number of firms	3.817	8.291		6.420	5.530		8.728		2.762
Number of events	242	391		396	237		523		110
Sector effects	YES	YES		YES	YES		YES		YES
QIC	2.739,2	4.733,6		4.586	2.881,4		6.119,9		1.365,5
QICu	2.736,3	4.729,9		4.582	2.880,1		6.115,3		1.363,6

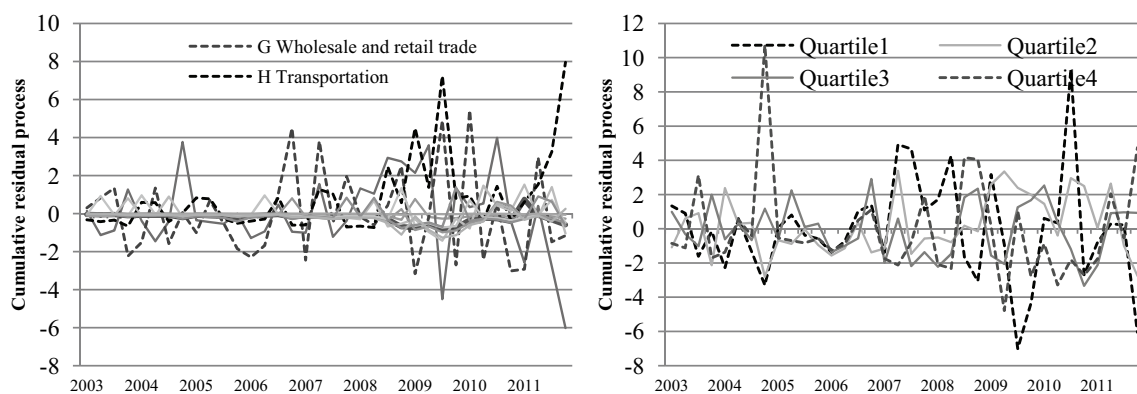


Figure 2.8. Model check based on grouped martingale residual processes. The figure shows cumulative martingale residual processes for the preferred Cox model including both firm-specific and macroeconomic variables (Model (7) in Table 2.5). In the left panel, grouping corresponds to each firm’s sector, which is stationary across the sample period, while in the right panel, the grouping is time-varying and done According to asset quartiles.

2.4.2 Independent censoring and entry-pattern

Given that data is only available from 2003 onwards, the existing stock of firms entering the sample in 2003 may potentially be of better average quality than the firms entering at a later point in time. This bias would violate the assumption of independent censoring. To address this issue, estimation was done on a reduced sample that excludes firms entering the sample in 2003 (where a considerable part of these entries ties to the existing stock of the bank clients). The results (not presented here, but available upon request) are that all estimated coefficients remain significant and of the same sign as the final model (Model (7) in Table 2.5). To address the concern that the very low number of entries in 2005 might have an impact on the results, the final model specification was re-estimated using two samples: One that exclude entries from 2005, and another that excludes all entries up until 2006. The estimates from these model fits (not presented here, but available upon request) are still all significant and of the same sign as the model estimated on the full sample.

2.4.3 Lag length

We have throughout chosen to focus on a lag length of one year for the covariates employed in our intensity models. One may, however, believe that for macroeconomic variables, this is not the appropriate lag, as aggregate changes may take longer to impact firms, since these operate with a capital buffer that allow them to operate though an extended period of time before a default is observed. To address this concern, we estimated the preferred model with all macroeconomic variables lagged eight quarters instead of four. The results (not presented here, but available upon request) showed that the macroeconomic variables are generally less able to explain defaults when lagged eight quarters, as indicated by the loss in significance for most of the coefficients.

Still, this analysis does not consider the possibility that different macroeconomic variables are operating though different lag periods. Considering all possible combinations of lag periods would result in an extensive number of permutations of the model for us to check. Instead, we construct a correlation matrix for the observed default rate and each of the macroeconomic time series lagged from zero to eight quarters. It generally shows that, while the lag length of four quarters does not provide the highest correlation with the default rate for all macroeconomic variables, it appears that a unified lag period of four quarters is at least a very appropriate choice.

2.4.4 Grouped martingale residual processes

We check the fit of our final model using the martingales residual processes (2.1). Specifically, we consider to what extent the model is systematically over or under estimating the default frequency in different sectors and size-groups.

By definition, the martingale residual processes are the difference between the observed default frequency and the default frequency predicted by the model. Since a single firm can at most have one default event in our estimation setup, the firm-specific processes contain too little information. However, when grouped in sufficiently large clusters, the increments of the grouped processes should not be systematically positive or negative if the model fit is adequate. An increasing grouped residual process would imply that the model is under-predicting the number of defaults this particular group, whereas a decreasing grouped residual process would imply that the model is predicting too many defaults for this group.

The left panel of Figure 2.8 shows grouped residual processes by sector as a function of time. We see that the residual processes fluctuate around zero with both positive and negative increments for all sectors. This is support for the model performing equally well for all sectors. However, noting that the sectors “wholesale and retail trade” and “transportation” have the largest deviances, we re-estimated our final model excluding firms in these two sectors – the results (not presented here, but available upon request) do not change.

The right panel of the figure shows grouped residual processes by asset quartiles as a function of time. We again observe no truly systematic deviations. We note, however, that the largest quarterly deviances occur in the largest and smallest quartiles, further motivating the point that default prediction models should take firm size into account.

2.5 Concluding remarks

The Basel II and III Accords award preferential treatment to bank loans to SMEs on the basis that the default probabilities of smaller firms are less sensitive to macroeconomic cyclicalities compared to the default probabilities of larger firms. This effectively and significantly lowers capital charges for lending to the SME-segment.

This paper investigates whether there is empirical support for the assumption that the default probabilities of smaller firms are less sensitive to macroeconomic cyclicalities. Using a default intensity regression framework, our results indicate that solely discriminating with respect to firm-size, the default probabilities of small firms do in fact exhibit less sensitivity to macroeconomic cyclicalities compared to the default probabilities of large firms, in the sense that the effects of macroeconomic variables are of smaller magnitude for smaller firms. However, when we account for differences in firm-characteristics other than size, our results indicate that the default probability of the average small firm is as cyclical or even more cyclical than the default probability of the average large firm. The results are robust to different regression models and different divisions of our sample into small and large firms.

Our findings suggest that preferential treatment of capital charges solely based on firm-size is too simplistic and may in fact entail adverse effects on the stability of European banks with a high exposure to the SME-segment.

Bibliography

- Aalen, O. O. (1980). A model for non-parametric regression analysis of life times. In J. R. W. Klonecki, A. Kozek (Ed.), *Mathematical Statistics and Probability Theory*, Volume 2 of *Lecture Notes in Statistics*, New York, USA, pp. 1–25. Springer-Verlag.
- Aalen, O. O. (1989). A linear regression model for the analysis of life times. *Statistics in medicine* 8, 907–925.
- Aalen, O. O., Ø. Borgan, and H. K. Gjessing (2008). *Survival and Event History Analysis: A Process Point Of View* (1st ed.). New York, USA: Springer-Verlag.
- Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance* 23, 589–609.
- Beaver, W. (1966). Financial ratios and the prediction of failure. *Journal of Accounting Research. Supplement : Empirical research in accounting : Selected studies 1966* 4, 77–111.
- Bharath, S. T. and T. Shumway (2008). Forecasting Default with the Merton Distance to Default Model. *Review of Financial Studies* 21(3), 1339–1369.
- Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–654.
- Bonfim, D. (2009). Credit risk drivers: Evaluating the contribution of firm level information and of macroeconomic dynamics. *The Journal of Banking and Finance* 28, 281–299.
- Carling, K., T. Jacobson, J. Lindé, and K. Roszbach (2007). Corporate credit risk modeling and the macroeconomy. *Journal of Banking and Finance* 31(3), 845–868.
- Chava, S. and R. Jarrow (2004). Default risk and diversification : Theory and empirical implications. *Review of Finance* 8(4), 537–569.
- Chava, S., C. Stefanescu, and S. Turnbull (2011). Modeling the loss distribution. *Management Science* 57(7), 1267–1287.
- Chionsini, G., J. Marcucci, and M. Quagliariello (2010, March). The Treatment of Small and Medium Enterprises in Basel 2: So Right, So Wrong? Technical report, Center for Economic Policy Research (CEPR), <http://dev3.cepr.org/meets/wkcn/1/1737/papers/Marcucci.pdf>.
- Cox, D. R. (1972). Regression models and life-tables (with discussion). *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 34, 187–220.
- Dietsch, M. and J. Petey (2004). Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. *Journal of Banking and Finance* 28(4), 773–788.

- Duffie, D., A. Eckner, G. Horel, and L. Saita (2009, October). Frailty correlated default. *The Journal of Finance* *LXIV*(5), 2089–2123.
- Duffie, D. and D. Lando (2001). Term structures of credit spreads with incomplete accounting information. *Econometrica* *58*, 635–665.
- Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics* *83*, 635–665.
- Figlewski, S., H. Frydman, and W. Liang (2012). Modeling the effect of macroeconomic factors on corporate default and credit rating transitions. *International Review of Economics & Finance* *21*(1), 87–105.
- Giesecke, K., D. Lando, and M. Medhat (2013, September). The Macroeconomy and Credit Risk: Direct and Indirect effects. *Working Paper, Copenhagen Business School and Stanford University*.
- Lando, D., M. Medhat, M. S. Nielsen, and S. F. Nielsen (2013). Additive Intensity Regression Models in Corporate Default Analysis. *Journal of Financial Econometrics* *11*(3), 443–485.
- Lando, D. and M. S. Nielsen (2010). Correlation in Corporate Defaults: Contagion or Conditional Independence? *Journal of Financial Intermediation* *19*(3), 335–372. <http://ssrn.com/paper=1338381>.
- Leland, H. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* *49*, 1213–1252.
- Lopez, J. A. (2004). The empirical relationship between average asset correlation, firm probability of default, and asset size. *Journal of Financial Intermediation* *13*(2), 265–283.
- Martinussen, T. and T. H. Scheike (2006). *Dynamic Regression Models for Survival Data* (1. ed.). New York, USA: Springer-Verlag.
- McDonald, C. G. and L. M. V. de Gucht (1999). High-Yield Bond Default and Call Risks. *The Review of Economics and Statistics* *81*(3), 409–419.
- Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* *29*, 449–470.
- Ohlsen, J. S. (1980). Financial ratios and the probabilistic prediction of bankruptcy. *Journal of Account. Res.* *19*, 109–31.
- Peseran, M. H., T. Schuermann, B.-J. Treutler, and S. M. Weiner (2006). Macroeconomic Dynamics and Credit Risk: A Global Perspective. *Journal of Money, Credit, and Banking* *38*, 1211–1262.
- Shumway, T. (2001). Forecasting bankruptcy more efficiently : A simple hazard model. *Journal of Bus.* *74*, 101–124.
- The Association of Chartered Certified Accountants (ACCA) (2011a, December). CRD IV and small businesses: revisiting the evidence in Europe. Technical report, Association of Chartered Certified Accountants (ACCA), <http://www.accaglobal.com/content/dam/acca/global/PDF-technical/small-business/pol-af-crdiv.pdf>.
- The Association of Chartered Certified Accountants (ACCA) (2011b, July). Framing the Debate: Basel III and SMEs. Technical report, Association of Chartered Certified Accountants (ACCA), <http://www.accaglobal.com/content/dam/acca/global/PDF-technical/small-business/pol-af-ftd.pdf>.
- The Basel Committee on Banking Supervision (2006, June). International Convergence of Capital Measurement and Capital Standards – A Revised Framework (Comprehensive Version). Technical report, Bank for International settlements (BIS).

The European Banking Authority (EBA) (2012, September). Assessment of SME proposals for CRD IV/CRR. Technical report, European Banking Authority (EBA), <http://www.eba.europa.eu/documents/10180/16148/EBA-SME-Report.pdf>.

The European Parliament and the Council of the European Union (2013, June). Regulation (eu) no 575/2013 of the european parliament and of the council of 26 june 2013 on prudential requirements for credit institutions and investment firms and amending regulation (eu) no 648/2012. Technical report, The European Parliament and the Council of the European Union, <http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2013:176:0001:0337:EN:PDF>.

Wooldridge, J. M. (2009). *Introductory Economics—A Modern Approach* (4th ed.). South-Western.

Zmijewski, M. E. (1984). Methodological issues related to the estimation of financial distress prediction models. *Journal of Account. Res.* 22, 59–82.

Chapter 3

Liquidity Risk and Distressed Equity

Job Market Paper for the 2015 Academic Job Market.

Introduction

“Cash is a bad investment over time. But you always want to have enough so that nobody else can determine your future, essentially.” —Warren Buffet (CNBC, November 12, 2009)

Intuition suggests that rational investors should require higher returns for holding the equity of firms with higher distress risk (Chan and Chen, 1991; Fama and French, 1995, 1996). Yet, empirically, higher probability of default robustly and persistently predicts lower future returns (Dichev, 1998; Griffen and Lemmon, 2002; Campbell et al., 2008). Recently, capital structure theory has been invoked to rationalize the low returns to distressed equity. Garlappi and Yan (2011) argue that potential shareholder recovery upon resolution of distress increases the value of the default option and lowers expected returns near the default boundary. Opp (2013) argues that faster learning about firm solvency in aggregate downturns has the same effect, while McQuade (2013) argues that distressed equity hedges against persistent volatility risk and therefore commands lower expected returns.

In this paper, I argue that firms’ cash holdings can help rationalize the low returns to distressed equity. In contrast to extant models of distressed equity, my model features levered firms with financing constraints that can default because of illiquidity or insolvency and seek to choose their cash holdings optimally. I show that an equity-maximizing firm will hold a level of cash that allows it to avoid illiquidity. When such a firm is in high risk of insolvency, it has a large fraction of its assets in cash and therefore a low conditional beta, which helps rationalize its low expected returns. Using data on solvency, liquidity, and equity returns for rated US firms over the period 1970-2013, I find empirical evidence consistent with the model’s predictions.

The model setup is similar to Gryglewicz (2011) and Davydenko (2013). I consider a levered firm whose assets consist of productive assets-in-place and unproductive liquid assets; namely, cash holdings. The productive assets-in-place generate uncertain earnings. Earnings can be either positive (a profit) or negative (a loss). The firm’s liabilities consist of common equity and coupon-bearing debt. The coupon level is predetermined and stays fixed during the firm’s lifetime. Because of capital market frictions, the firm has no access to additional external financing and can, in particular, not issue additional equity to cover coupon payments. Positive net earnings can be distributed as dividends or retained as cash holdings. Cash holdings can be used to cover coupon payments and dividend payouts in case

of negative or insufficient earnings.¹ If the firm has no means to cover a coupon payment, it becomes illiquid (i.e. financially distressed) and defaults. If the value of the firm's equity reaches zero, the firm becomes insolvent (i.e. economically distressed) and, acting in the interest of equity holders, defaults voluntarily. The firm chooses its policies for holding cash, paying dividends, and triggering insolvency so as to maximize equity value.

The model solution consists of the firm's optimal policies for holding cash, paying dividends, and triggering insolvency. At the center of the model solution is a *target cash level* that allows the firm to avoid illiquidity ([Proposition 1](#)). The target cash level increases with the firm's earnings because higher earnings mean a more solvent firm with a higher continuation value, and such a firm is willing to cover higher earnings-shortfalls with cash holdings. I show that before default, it is optimal (i.e. equity maximizing) for the firm to retain all positive earnings if cash is below the target level, and, once cash has reached the target level, to pay dividends that maintain cash at the target level ([Proposition 2](#)). For a firm with cash at the target level, I solve for the optimal insolvency trigger and show that such a firm reaches insolvency with a large fraction of its assets in cash ([Proposition 3](#)). Consistent with the model solution, I find for my sample of rated US firms that solvency and (balance sheet) liquidity are positively correlated, but that the average firm in all solvency levels holds enough liquid assets to cover short-term liabilities. Furthermore, I find that less solvent have a higher fraction of their total assets in liquid assets.

The model implies a conditional CAPM (capital asset pricing model) in which conditional equity beta depends on the firm's asset composition. For a firm with cash at the target level, I show that equity beta is higher than asset beta when the firm is sufficiently solvent, but, conversely, that equity beta is lower than asset beta when the firm is sufficiently close to insolvency (Part 1 of [Proposition 4](#)). Hence, the model implies a generally positive relation between equity beta and solvency. The reason is that when the firm is sufficiently solvent, its asset value is largely comprised of the value of the productive assets-in-place. Because equity is a levered claim on the firm's assets, the equity of a sufficiently solvent firm will be riskier than the productive assets-in-place, implying that equity beta is higher than asset beta. Conversely, when the firm is sufficiently close to insolvency, its asset value is to a larger degree comprised of the value of cash. This implies that the equity of a firm that is sufficiently close to insolvency, despite being levered, is less risky than the productive assets-in-place, and therefore that equity beta is less than asset beta. The model thus provides a novel theoretical rationale for the low returns to distressed equity: Because it is optimal for the firm to hold a level of cash that allows it to avoid illiquidity, the firm will have a large fraction of assets in cash when it is close to insolvency, which leads to a low equity beta and therefore low expected returns. In addition, the model complements the insights of [Garlappi and Yan \(2011\)](#), [Opp \(2013\)](#), and [McQuade \(2013\)](#) by separating the liquidity and solvency components of distress.

The model also has implications for the slope of the relation between equity beta and solvency, i.e. how changes in solvency are related to changes in equity beta. For a firm with cash at the target level, I show that the slope is negative when the firm is sufficiently solvent, but that the slope is positive when the firm is sufficiently close to insolvency (Part 2 of [Proposition 4](#)). Hence, the model implies that while the relation between equity beta and solvency is generally positive, it is in fact non-monotonic and "hump-shaped." To see this, note that an increase in solvency means lower leverage, which decreases equity beta, but also a lower fraction of assets in cash, which increases equity beta. When the firm is sufficiently solvent, the effect of lower leverage dominates, so that an increase in solvency leads to a decrease in equity beta. However, when the firm is sufficiently close to insolvency, the effect of a lower fraction of assets in cash dominates, so that an increase in solvency leads to an increase in equity beta.

¹There is ample empirical evidence suggesting that the precautionary motive is the most important determinant of corporate cash holdings and their secular growth since the 1980s—see, for instance, [Opler, Pinkowitz, Stulz, and Williamson \(1999\)](#), [Bates, Kahle, and Stultz \(2009\)](#), [Acharya et al. \(2012\)](#), and references therein.

I derive four asset pricing predictions from the model's conditional CAPM. I first consider the cross-sectional relation between expected returns and probability of insolvency. For firms with cash at the target level, the model predicts high expected returns for those with low probability of insolvency, but low expected returns for those with high probability of insolvency. Furthermore, the model predicts that the slope of the relation between expected returns and probability of insolvency is positive when the probability of insolvency is low, but negative when the probability of insolvency is high (Corollary 4.1). Consistent with these predictions, I find for my sample of rated US firm's that more solvent firms generally have higher average returns, but that the relation between average returns and solvency is hump-shaped. These results hold using conditional (time-varying) firm-specific equity betas; cross-sectional regressions of firm-specific returns on measures of solvency; and the realized returns and alphas of value-weighted portfolios formed on measures of solvency.

Second, I consider the cross-sectional relation between expected returns and optimal liquidity, defined here as the optimal (target) cash level divided by the value of coupon liabilities. Note that optimal liquidity is bounded from below by 1. The model predicts high expected returns for firms with high optimal liquidity, but low expected returns for firms with low optimal liquidity (Corollary 4.2). The reason is that firms with high (low) optimal liquidity also have a high (low) value for their productive assets-in-place, and therefore high (low) equity betas. Furthermore, the model predicts that the slope of the relation between expected returns and optimal liquidity is positive when optimal liquidity is high, but negative when optimal liquidity is low. To see this, note that an increase in optimal liquidity means, on the one hand, a higher fraction of assets in cash, which decreases equity beta. On the other hand, an increase in optimal liquidity is an endogenous response to an increase in the value of the productive assets, which will increase equity beta. When optimal liquidity is sufficiently high, the effect of a larger fraction of assets in cash dominates, leading to a decrease in equity beta, while when optimal liquidity is sufficiently low, the effects of a higher value for the productive assets-in-place dominates, leading to a higher equity beta. Consistent with these predictions, I find for my sample that firms with higher liquidity generally have higher average returns, but that the relation between liquidity and average returns is hump-shaped. These results hold using either conditional betas, cross-sectional regressions, or value-weighted portfolios.

The above results can be exploited in long-short portfolio strategies that produce positive expected returns. To this end, I derive predictions regarding the performance of two long-short portfolio strategies formed on liquidity and solvency. First, I consider the expected returns on a higher-liquidity minus lower-liquidity (HLmLL) portfolio as a function of the probability of insolvency. HLmLL is a portfolio strategy that goes long firms with higher optimal liquidity and shorts firms with lower optimal liquidity. The model predicts that HLmLL has negative expected returns for firms with a low probability of insolvency, but positive expected returns for firms with a high probability of insolvency (Corollary 4.3). In particular, the expected return on HLmLL is a generally increasing function of the probability of insolvency. The reason is that within the set of firms with low (high) probability of insolvency, the ones with higher (lower) liquidity are the ones with the lower (higher) expected returns. I test this prediction using HLmLL portfolios constructed using conditional sorts—first on solvency and then on liquidity. Consistent with the model's prediction, I find that the realized returns and alphas of HLmLL are highest among firms with low solvency.

Finally, I consider the expected returns on a higher-solvency minus lower-solvency (HSmLS) portfolio as a function of optimal liquidity. HSmLS is a portfolio strategy that goes long firms with higher solvency and shorts firms with lower solvency. The model predicts that HSmLS has negative expected returns for firms with high optimal liquidity, but positive expected returns for firms with low optimal liquidity (Corollary 4.4). In particular, the expected return on HSmLS is a generally decreasing function of optimal liquidity. The reason is that within the set of firms with high (low) optimal liquidity, the ones with higher (lower) solvency are the ones with the lower (higher) expected returns. I test this prediction using HSmLS portfolios constructed using conditional sorts—first on liquidity and then on sol-

gency. Consistent with the model's prediction, I find that the realized returns and alphas of HSMLS are highest among firms with low liquidity.

My results shed new light on the distress puzzle, and, more generally, on the relation between firms' cash holdings and expected equity returns. The theoretical results suggest that separating the solvency and liquidity components of distress risk is central in understanding the returns that investors require for holding distressed equities. The empirical results confirm that firms' optimal cash holdings are closely related to their solvency and, consequently, to their realized equity returns.

Related literature

Chan and Chen (1991) were among the first to suggest that asset pricing anomalies (relative to the CAPM) like the size premium are due to investors requiring higher returns as compensation for holding the equities of firms with a higher risk of default or 'distress'. Similarly, Fama and French (1995, 1996) suggest that the value premium is due to the fact that firms with a high risk of distress also have high book-to-market equity.

Dichev (1998) was among the first to refute the idea that the size and value premia are compensation for distress risk by showing that higher default probability predicts lower, not higher, average future returns. This finding is now commonly known as the 'distress puzzle'. In a related study, Griffen and Lemmon (2002) show that firms with a high risk of failure and low book-to-market equity have particularly low average future equity returns. Vassalou and Xing (2004) find some evidence that lower Merton's distance-to-default predicts higher average future equity returns, but their results are entirely driven by small and high book-to-market firms. Campbell et al. (2008) estimate a dynamic logistic regression model to predict defaults and use it to confirm and reinforce previous evidence that higher probability of default persistently and robustly predicts lower average future returns. Avramov et al. (2009) find similar results using credit ratings.

Garlappi et al. (2008) show that the empirical relation between average returns and probability of default is not monotonically decreasing but rather hump-shaped, and they rationalize this using simulations from a model in which shareholders can use their bargaining power to recover part of the firm's value upon resolution of distress. Extending these results, Garlappi and Yan (2011) show theoretically that shareholder recovery implies a hump-shaped relation between conditional betas and probability of default, and they find empirical support using estimated time-varying betas. The models of Opp (2013) and McQuade (2013) provide further theoretical rationals for the hump-shaped relation in models featuring shareholder learning and persistent volatility risk. I complement these theories by separating the liquidity and solvency components of distress, and I show that in a model where firms endogenously eliminate their liquidity risk, there is a hump-shaped relation between expected returns and probability of insolvency. Furthermore, I provide empirical evidence in support of these and other model predictions.

My model builds on the classical time-homogenous capital structure framework of e.g. Leland (1994) and Goldstein, Ju, and Leland (2001), but augments this framework with a liquidity risk component similar to Gryglewicz (2011). The latter paper extends the the classical pure-liquidity models of Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996) and studies the optimal capital structure of a firm facing liquidity and solvency concerns. Davydenko (2013) uses a related framework to study whether corporate defaults are driven by insolvency or illiquidity. My model deviates from Gryglewicz (2011) by specifying the firm's cumulated earnings as a geometric Brownian motion instead of an arithmetic Brownian motion with a randomized drift. This simplifies the model's information structure, which in turn simplifies the analysis of expected equity returns, while still maintaining an optimal insolvency decision for equity holders. Furthermore, specification implies that the value of the firm's productive assets-in-place (the expected discounted value of future earnings) is again a geometric Brownian motion, which makes my model

consistent with the classical asset-value based models of e.g. Black and Scholes (1973), Merton (1974), Black and Cox (1976), Leland (1994), Fan and Sundaresan (2000), Goldstein et al. (2001), Duffie and Lando (2001), etc.

Lastly, this paper is also related to the growing literature on the determinants and implications of corporate cash holdings. Relevant empirical studies include Opler et al. (1999), Bates et al. (2009), Acharya et al. (2012), and Davydenko (2013), while relevant theoretical studies include Décamps, Mariotti, Rochet, and Villeneuve (2011), and Bolton, Chen, and Wang (2011); Bolton et al. (2013).

3.1 Model

This section develops an equity valuation model for a levered firm with financing constraints. The setup is related to Gryglewicz (2011) and Davydenko (2013): The firm can default because of illiquidity or insolvency and chooses its policies for holding cash, paying dividends, and triggering insolvency by maximizing equity value. In Section 3.2, derive the optimal policies for holding cash and paying dividends, and then, for a firm following these policies, I derive the optimal insolvency trigger and the expected equity return.

The financially constrained firm

I consider a levered firm in a continuous-time economy with infinite time-horizon, $[0, \infty)$. The firm's assets consist of productive assets-in-place that generate uncertain earnings (revenue net of expenses) and unproductive liquid assets, namely cash holdings, that earn interest. Earnings can be either positive (a profit) or negative (a loss). The firm's liabilities consist of common equity stock and consol (infinite maturity) bonds with total coupon rate $k > 0$ per time unit. The total coupon rate is assumed to be predetermined and remain fixed throughout the firm's life-time. Because taxes are not essential for the paper's results, I abstract away from them. The economy's instantaneous risk-free interest rate, $r > 0$, is assumed to be a constant.

The model's main state variable is the firm's earnings. For my purpose of modeling cash holdings, it is convenient to specify the *stock* rather than the *flow* of earnings. Let therefore X_t be the firm's *cumulated earnings* up to time t . I assume that the process $(X_t)_{t \geq 0}$, under a physical probability measure, \mathbb{P} , is a geometric Brownian motion with dynamics

$$dX_t = \mu^{\mathbb{P}} X_t dt + \sigma X_t dB_t^{\mathbb{P}}. \quad (3.1)$$

Here, $(B_t^{\mathbb{P}})_{t \geq 0}$ is a standard \mathbb{P} -Brownian motion driving the firm's total (idiosyncratic and systematic) earnings risk. *Instantaneous earnings* (per dt) are thus given by the increment dX_t , which can be either positive or negative, depending on the realization of the *total earnings shock*, $dB_t^{\mathbb{P}}$.² The drift parameter, $\mu^{\mathbb{P}}$, is the \mathbb{P} -*expected growth rate* of earnings while the diffusion parameter, σ , is the *volatility rate* of earnings.³

²Modeling instantaneous earnings as the increment (rather than the level) of a stochastic process is common in models of liquidity management and related topics. See, for instance, Jeanblanc-Picqué and Shiryaev (1995), Radner and Shepp (1996), Demarzo and Sannikov (2006), Décamps et al. (2011), Gryglewicz (2011), Acharya et al. (2012), and Bolton et al. (2011, 2013).

³In contrast to Gryglewicz (2011), I specify instantaneous earnings in (3.1) as the increments of a geometric Brownian motion rather than the increments of an arithmetic Brownian motion with a drift that is randomized over a known two-point distribution. This still implies a state-dependent value for the firm's productive assets, which ensures the existence of an insolvency trigger (see footnote 7), but simplifies the model's information structure and the analysis of expected equity returns. Moreover, I show below that in my model, the value of the productive assets is again a geometric Brownian motion, which makes my model consistent with the classical capital structure models (see footnote 8).

After coupons, the firm's instantaneous net earnings are given by $dX_t - kdt$. Positive net earnings can be distributed to equity holders as dividends or retained inside the firm to increase cash holdings. Cash holdings can be used to cover coupons and dividends in case of negative or insufficient earnings. If the firm cannot or chooses not to pay a coupon, it defaults and is immediately liquidated. Bond holders have absolute priority in default and recover the market value of the firm's productive assets (derived below) at the time of default. Because bankruptcy or liquidation costs are not essential for the paper's results, I abstract away from them.

I assume that due to capital market frictions, the firm cannot raise additional external capital.⁴ Coupons must therefore be financed through internal liquidity, namely positive earnings or cash holdings. If the firm has insufficient liquidity to cover a coupon, it is *illiquid* and defaults. Such a default is driven by *short-term* or *financial distress*. If the firm's equity value becomes negative, the firm is *insolvent* and acts in the interest of equity holders by defaulting voluntarily. Such a default is driven by *long-term* or *economic distress*.

Without financing constraints, illiquidity never occurs because the firm finances any earnings-shortfalls by issuing additional equity. Such an issuance is typically modeled as a negative dividend, following [Black and Cox \(1976\)](#) and [Leland \(1994\)](#). Since negative dividends are possible as long as equity value is positive, only insolvency occurs when there are no financing constraints. In contrast, when external financing is unavailable, the firm has a *precautionary motive* to retain some earnings as cash that serves as a cushion against liquidity risk.

I assume that the managers of the financially constrained firm act in the best interest of equity holders. The managers thus determine the policies for holding cash, paying dividends, and triggering insolvency so as to maximize equity value. To simplify the analysis, I assume that the firm's cash holdings can be distributed to equity holders at any time before default. This assumption is, however, not essential for the paper's qualitative results for two reasons. First, bond covenants that limit distributions just before default are difficult to enforce because equity holders would try to preempt them. Second, if cash cannot be distributed just before default, equity holders would never trigger insolvency while the firm still has positive cash holdings. However, once the firm has used up its cash to make coupons, the optimal policy for triggering insolvency becomes identical to the one derived below.

Cash holdings, dividends, and default

Let C_t be the firm's cash holdings at time t and let D_t be its cumulated dividends up to time t . Because the firm cannot raise additional external capital, $(C_t)_{t \geq 0}$ is a nonnegative process and $(D_t)_{t \geq 0}$ is a non-decreasing process.⁵ For simplicity, I assume that cash holdings earn the risk-free rate, r , per time unit (for instance through investment in short-term marketable securities), and that equity holders opt for dividends whenever they weakly prefer so.⁶

At each time t before default, the firm's managers choose how to distribute net earnings between cash holdings and dividends. Therefore, the instantaneous dividend at time t , dD_t , equals the difference between net earnings and the change in cash holdings, $(dX_t - k dt) - dC_t$, added the interest earned on current cash holdings, $rC_t dt$:

$$dD_t = (dX_t - k dt) - dC_t + rC_t dt. \quad (3.2)$$

⁴The assumption of no additional external financing can be justified by the debt-overhang problem of [Myers \(1977\)](#) or the information asymmetry problems of [Leland and Pyle \(1977\)](#) and [Myers and Majluf \(1984\)](#). The assumption can be replaced by the milder assumption of sufficiently high issuance costs without affecting the results qualitatively as long as (i) liquidation of the firm is costly, and (ii) only a fraction of future earnings can be pledged as collateral. See [Acharya et al. \(2012\)](#) for a further discussion.

⁵Usually, negative negative cash holdings is interpreted as the firm drawing on a credit line, while a decrease in cumulated dividends (i.e. a negative instantaneous dividend) is interpreted as a capital injection by equity holders. Since I assume that the firm cannot raise additional external financing, the firm has access to neither a credit line nor equity injections.

⁶Other interest rates on cash would alter the amount of cash held by the firm but not qualitatively alter the theoretical results.

Illiquidity occurs when the firm runs out of cash, i.e. at the stopping time $\tau_C = \inf_{t>0}\{C_t \leq 0\}$. Insolvency is triggered when earnings hit a sufficiently low level, i.e. at the stopping time $\tau^* = \inf_{t>0}\{X_t \leq \underline{X}^*(C_t)\}$, which is chosen to maximize equity value.⁷ Here, $\underline{X}^*(C) \geq 0$ is an *insolvency trigger*, which in general depends on the level of cash holdings. Nonetheless, I show below that the insolvency trigger is independent of the level of cash when the firm's cash holdings are sufficiently high.

Pricing setup and managers' optimization problem

The managers choose the optimal policies for holding cash, paying dividends, and triggering insolvency by maximizing the market value of equity. Market values are calculated through the following pricing setup.

I assume that the economy's stochastic discount factor, $(\Lambda_t)_{t \geq 0}$, has \mathbb{P} -dynamics

$$d\Lambda_t = -r\Lambda_t dt - \lambda\Lambda_t dZ_t^{\mathbb{P}}. \quad (3.3)$$

Here, $(Z_t^{\mathbb{P}})_{t \geq 0}$ is a standard \mathbb{P} -Brownian motion that drives systematic earnings risk and has instantaneous correlation ρ with the firm's total earnings risk, $(B_t^{\mathbb{P}})_{t \geq 0}$, as given in (3.1). The constant λ is then the market price of systematic earnings risk. In the appendix, I detail how (3.3) defines a risk-neutral pricing measure, \mathbb{Q} , under which the cumulated earnings process, $(X_t)_{t \geq 0}$, is a geometric Brownian motion with dynamics

$$dX_t = \mu^{\mathbb{Q}}X_t dt + \sigma X_t dB_t^{\mathbb{Q}}, \quad (3.4)$$

where $\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \rho\sigma\lambda$ is the risk-neutral growth rate of earnings and where $B_t^{\mathbb{Q}} = B_t^{\mathbb{P}} + \rho\lambda t$ defines a standard \mathbb{Q} -Brownian motion. Under the parameter restriction $\mu^{\mathbb{Q}} < r - \frac{1}{2}\sigma^2$, the time- t market value of the firm's productive assets (i.e. the value of the unlevered firm) becomes

$$U(X_t) \equiv \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(u-t)} dX_u \right] = \frac{\mu^{\mathbb{Q}} X_t}{r - \mu^{\mathbb{Q}}}, \quad (3.5)$$

which is again a geometric Brownian motion.⁸ This is a Gordon growth-type valuation, where expected earnings, $\mu^{\mathbb{Q}}X$, are discounted by the risk-neutral expected rate of return net of the growth rate of earnings, $r - \mu^{\mathbb{Q}}$. In the following, I also impose the parameter restriction $\mu^{\mathbb{Q}} > \frac{1}{2}r$ to ensure that $U(X) > X$. Otherwise, it would be optimal for equity holders to immediately liquidate the firm (see the discussion of "asset shifting" due to too low expected earnings in Acharya et al. (2012)).

Given this pricing setup, the market value of the firm's equity is the \mathbb{Q} -expected, discounted value of future dividends until default plus a liquidation payout of any remaining cash. The liquidation payment of cash follows from the assumption that cash can be distributed to equity holders at any time before default. Given an insolvency policy,

⁷The existence of an insolvency trigger follows from the assumption that $(X_t)_{t \geq 0}$ is a geometric Brownian motion. If $(X_t)_{t \geq 0}$ were an arithmetic Brownian motion with constant drift, say μ , no insolvency trigger exists, as the firm would either never be insolvent or always be insolvent, depending on the sign of the difference $\mu - k$.

⁸By Itô's Formula and (3.4), $U(X_t)$ has \mathbb{Q} -dynamics $dU(X_t) = \mu^{\mathbb{Q}}U(X_t)dt + \sigma U(X_t)dB_t^{\mathbb{Q}}$. This makes the specification of cumulated earnings in (3.4) (and, equivalently, in (3.1)) consistent with the classical capital structure models assuming that the value of the unlevered firm is a geometric Brownian motion—e.g. Black and Scholes (1973), Merton (1974), Black and Cox (1976), Leland (1994), Fan and Sundaresan (2000), Goldstein et al. (2001), and Duffie and Lando (2001). The parameter restriction $\mu^{\mathbb{Q}} < r - \frac{1}{2}\sigma^2$ ensures that $U(X_t)$ is well-defined and given by (3.5) through an application of the Itô Isometry.

τ^* , the time- t market value of the firm's equity is thus defined by

$$E(X_t, C_t, \tau^*) \equiv \sup_{(D_u)_{u \geq t}} \mathbb{E}_t^Q \left[\int_t^{\tilde{\tau}} e^{-r(u-t)} dD_u + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}} \right], \quad (3.6)$$

where $\tilde{\tau} = \tau_C \wedge \tau^*$ is the firm's default time due to either illiquidity or insolvency, whichever first occurs. The supremum in (3.6) is taken over all dividend processes, $(D_t)_{t \geq 0}$, which are non-decreasing, satisfy the relation in (3.2), and are adapted to the filtration generated by the cumulated earnings-process in (3.4).

3.2 Theoretical results

In this section, I solve the model by first deriving the optimal policies for holding cash and paying dividends. I then solve for the optimal policy for triggering insolvency for a firm that follows the optimal cash-dividend policy. Finally, I derive the expected equity return when the firm follows the optimal policies. The main goal is to show that an equity-maximizing firm will hold the minimal level of cash that allows it to avoid illiquidity and, consequently, that such a firm will have a low equity beta when it is close to insolvency. These theoretical results lead to the predictions that I test in the paper's empirical part.

3.2.1 Optimal policies

I first solve for the firm's optimal policy for holding cash and paying dividends. Then, for a firm following this policy, I derive the market value of equity and use it to solve for the optimal insolvency trigger.

Target cash level

In the model, the firm uses cash to manage its liquidity risk. In general, higher cash holdings mean lower liquidity risk. However, too high cash holdings are inefficient from equity holders' point-of-view. It is therefore of special interest to derive, for a given level of earnings, *the minimal level of cash that allows the firm to eliminate liquidity risk*—that is, for given level of earnings, the minimal level of cash that ensures that the firm avoid illiquidity and only defaults due to insolvency. I call this the *target cash level* because I later show that an equity-maximizing firm will aim for this level of cash. The following proposition gives an expression for the target cash level. The proof relies on the assumptions that cash holdings and instantaneous dividends satisfy (3.2) and have to remain nonnegative before default. (All proofs are in the appendix.)

Proposition 1 (The target cash level).

- (i) For each level of cumulated earnings, X , there exists a minimal level of cash, $\underline{C}(X)$, such that the firm eliminates liquidity risk (i.e. avoids illiquidity) if its cash is at or above $\underline{C}(X)$.
- (ii) When cash, C , is at or above $\underline{C}(X)$, the insolvency trigger is independent of C : $\underline{X}^*(C) = \underline{X}^* \geq 0$.
- (iii) Given the insolvency trigger, \underline{X}^* , the cash level $\underline{C}(X)$ is given by

$$\underline{C}(X) = X + \left[\frac{k}{r} - \underline{X}^* \right]. \quad (3.7)$$

The cash level $\underline{C}(X)$ increases in X , decreases in \underline{X}^* , and is bounded from below by the present value of future coupons: $\underline{C}(X) \geq \frac{k}{r}$ for all $X \geq \underline{X}^*$.

The target cash level $\underline{C}(X)$ has several intuitive properties. The quantity $X - \underline{X}^*$ ensures that dividends remain nonnegative even if an earnings-shock brings X down to \underline{X}^* . The quantity $\frac{k}{r}$ ensures that the interest on the target cash level is always at least as large as a coupon payment.⁹ Further, the target cash level increases in X and decreases in \underline{X}^* because a more solvent firm (higher X or lower \underline{X}^*) firm has a higher continuation value and is willing to cover higher earnings-shortfalls with cash holdings.

Optimal policy for holding cash and paying dividends

Next, I show that the optimal policy for holding cash and paying dividends is to aim for the target cash level. More formally, I first conjecture a dividend process, $(D_t^*)_{t \geq 0}$, which aims at maintaining cash at the target level, and then show that this dividend process solves the equity maximization problem in (3.6).

Suppose that the firm is not in default and that it aims for the target cash level. Then, if cash is below the target level, the firm retains all earnings and pays no dividends. If cash is at the target level, the firm pays out dividends that maintain cash at the target level. Finally, if cash is above the target level, the firm distributes excess cash. This suggests that before default, i.e. when $t < \tilde{\tau}$, the dividend process $(D_t^*)_{t \geq 0}$ is given by

$$dD_t^* = \begin{cases} 0 & \text{if } C_t < \underline{C}(X_t) \\ [r\underline{C}(X_t) - k] dt & \text{if } C_t = \underline{C}(X_t) \\ C_t - \underline{C}(X_t) & \text{if } C_t > \underline{C}(X_t). \end{cases} \quad (3.8)$$

In default, equity holders receive any remaining cash. For convenience, I model this as a liquidation payment rather than a part of the dividend process. The form of dD_t^* when $C_t = \underline{C}(X_t)$ and $t < \tilde{\tau}$ follows by applying Itô's Formula to the target cash level in (3.7) and using the identity in (3.2). This case may also be written as

$$dD_t^* = r(X_t - \underline{X}^*) dt \quad \text{if } C_t = \underline{C}(X_t) \quad \text{and} \quad t < \tilde{\tau}. \quad (3.9)$$

Given $(D_t^*)_{t \geq 0}$, I further conjecture, through Itô's Formula, that the corresponding time- t equity value function, $W(X_t, C_t)$, satisfies the differential equations

$$\left. \begin{aligned} rW(X, C) &= \mathcal{A}_{(X, C)} W(X, C) && \text{if } 0 < C < \underline{C}(X) \quad \text{and} \quad X > \underline{X}^*(C) \\ rW(X, C) &= \mathcal{A}_X W(X, C) + r(X - \underline{X}^*) && \text{if } C \geq \underline{C}(X) \quad \text{and} \quad X > \underline{X}^* \end{aligned} \right\} \quad (3.10)$$

along with the conditions

$$\left. \begin{aligned} W(X, C) &= C && \text{if } C \geq 0 \quad \text{and} \quad X \leq \underline{X}^*(C) \\ W_C(X, C) &\geq 1 && \text{if } C > 0 \quad \text{and} \quad X > \underline{X}^*(C) \end{aligned} \right\} \quad (3.11)$$

⁹An inspection of the proof of Proposition 1 reveals that the expression in (3.7) is derived independently of the assumption that the cumulated earnings process, $(X_t)_{t \geq 0}$, is a geometric Brownian motion (see (3.1) or (3.4)). In fact, the expression in (3.7) holds for a general $(X_t)_{t \geq 0}$ that is consistent with the existence of an insolvency trigger. As argued in footnote 7, this is in particular the case when $(X_t)_{t \geq 0}$ is a geometric Brownian motion, but not if it were an arithmetic Brownian motion with constant drift.

Moreover, an instructive way to derive (3.7) without appealing to the specific dynamics of $(X_t)_{t \geq 0}$ is as follows. First, if $\underline{C}(X_t)$ is to eliminate liquidity risk, it is reasonable that the interest on $\underline{C}(X_t)$ can cover a coupon: $r\underline{C}(X_t) dt \geq k dt$. Given this, (3.2) implies that dD_t is nonnegative if and only if $d\underline{C}(X_t) \leq dX_t$. Combined, these two conditions imply that

$$\underline{C}(X_t) \geq \underline{C}(\underline{X}^*) + [X_t - \underline{X}^*] \geq \frac{k}{r} + [X_t - \underline{X}^*].$$

Choosing the minimal $\underline{C}(X_t)$ that satisfies the last inequality gives (3.7).

In (3.10), $\mathcal{A}_{(X,C)}$ is the infinitesimal generator of the two-dimensional process $(X_t, C_t)_{t \geq 0}$ with dynamics in (3.4) and (3.2), while \mathcal{A}_X is the infinitesimal generator of the process $(X_t)_{t \geq 0}$ with dynamics in (3.4). The conditions in (3.11) follow from the assumption that cash holdings can be paid out at any time before default. The first states that in default equity holders receive any remaining cash. The second states that an extra dollar in cash is at least worth its face value to equity holders. This is because, under an optimal dividend policy, the value of equity with C in cash must be at least equal to the value of equity with $C - \epsilon$ in cash plus a dividend of ϵ : $W(X, C) \geq W(X, C - \epsilon) + \epsilon$. Rearranging and letting $\epsilon \rightarrow 0$ gives $W_C(X, C) \geq 1$.

The following proposition proves that the function $W(X, C)$ in (3.10)–(3.11) corresponds to the dividend policy $(D_t^*)_{t \geq 0}$ in (3.8), and that no other dividend policy provides a higher equity value.

Proposition 2 (The optimal policy for holding cash and paying dividends). *The dividend process $(D_t^*)_{t \geq 0}$ in (3.8) solves the firm's equity-maximization problem in (3.6): The function $W(X, C)$ given in (3.10)–(3.11) satisfies*

- (i) $W(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\bar{\tau}} e^{-r(u-t)} dD_u^* + e^{-r(\bar{\tau}-t)} C_{\bar{\tau}} \right]$, and
- (ii) $W(X_t, C_t) \geq \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\bar{\tau}} e^{-r(u-t)} dD_u + e^{-r(\bar{\tau}-t)} C_{\bar{\tau}} \right]$ where $(D_t)_{t \geq 0}$ is any adapted, non-decreasing dividend process satisfying the relation in (3.2).

Intuitively, the cash-dividend policy in (3.8) maximizes equity value because it optimally exploits that $W_C(X, C) \geq 1$. By (3.8), earnings are retained whenever an additional dollar in cash decreases liquidity risk, i.e. whenever $W_C(X, C) > 1$, while earnings are paid out whenever an additional dollar in cash no longer decreases liquidity risk, i.e. whenever $W_C(X, C) = 1$.

Optimal insolvency trigger

I complete the model solution by deriving the optimal insolvency trigger in the case where cash is at the target level, $C = \underline{C}(X)$, because Proposition 2 implies that any other cash level is suboptimal. Indeed, if $C \neq \underline{C}(X)$, the dividend process in (3.8) dictates that all variations in C are due to the adjustment towards $\underline{C}(X)$. Moreover, once cash reaches the target level, the associated dividend flow in (3.9) gives that cash remains at the target level until insolvency. Hence, I focus on the case of $C = \underline{C}(X)$, which implies that the firm avoids illiquidity and allows me to derive closed-form expressions.

Let $E(X) = W(X, \underline{C}(X))$ be the market value of equity when $C = \underline{C}(X)$ for all X . By Proposition 2 and (3.10)–(3.11), $E(X)$ solves the differential equation

$$rE(X) = \frac{1}{2}\sigma^2 X^2 E_{XX}(X) + \mu^{\mathbb{Q}} X E_X(X) + r(X - \underline{X}^*) \quad \text{for } X > \underline{X}^*, \quad (3.12)$$

along with the conditions

$$E(X) \rightarrow \underline{C}(X) + U(X) - \frac{k}{r} \quad \text{for } X \rightarrow \infty, \quad (3.13)$$

$$E(X) = \underline{C}(X) \quad \text{for } X \leq \underline{X}^*, \quad (3.14)$$

$$E_X(\underline{X}^*) = \underline{C}_X(\underline{X}^*). \quad (3.15)$$

The first is a *value matching* condition stating that as the firm becomes infinitely solvent, equity approaches its risk-free value: Total assets (cash plus productive assets) minus risk-free debt. The second is a *limited liability* condition

stating that at the insolvency trigger, equity consists solely of cash, which is consistent with the assumption that cash can be distributed at any time before default. The third is a *smooth-pasting* condition which follows from the limited liability condition and states that insolvency is triggered when an extra dollar in earnings increases equity value by no more than the increase in cash.

Combining the differential equation in (3.12) with the boundary conditions in (3.13)–(3.15) gives the following proposition.

Proposition 3 (The optimal insolvency trigger). *When the firm's cash holdings are at the target level, the following holds:*

(i) *The market value of equity is given by*

$$E(X) = \underline{C}(X) + U(X) - \frac{k}{r} + \left[\frac{k}{r} - U(\underline{X}^*) \right] \pi^{\mathbb{Q}}(X), \quad (3.16)$$

$$\text{where } \pi^{\mathbb{Q}}(X) = \left(\frac{X}{\underline{X}^*} \right)^{\phi^-} \text{ and where } \phi^- = \frac{\sigma^2 - 2\mu^{\mathbb{Q}} - \sqrt{(\sigma^2 - 2\mu^{\mathbb{Q}})^2 + 8r\sigma^2}}{2\sigma^2} < 0.$$

(ii) *The optimal insolvency trigger is given by*

$$\underline{X}^* = \frac{k}{r} \frac{r - \mu^{\mathbb{Q}}}{\mu^{\mathbb{Q}}} \frac{\phi^-}{\phi^- - 1}. \quad (3.17)$$

(iii) *When the firm is sufficiently solvent, its productive assets have higher value than its cash holdings, while when the firm is sufficiently close to insolvency, its productive assets have lower value than its cash holdings. Precisely: There exists $X' > \underline{X}^*$ such that $U(X) > \underline{C}(X)$ for all $X > X'$, while $U(X) < \underline{C}(X)$ for all $X < X'$.*

The first part of Proposition 3 shows that for a firm with cash at the target level, equity is the value of total assets (cash plus productive assets) minus future coupon liabilities plus the insolvency option. The scaling factor on the option-value term, $\pi^{\mathbb{Q}}(X)$, goes to 0 as X goes to infinity and goes to 1 as X goes to \underline{X}^* . It can thus, similar to Garlappi and Yan (2011), be interpreted as a long-term \mathbb{Q} -probability of insolvency.

The third part of Proposition 3 shows that when a firm with cash at the target level is sufficiently solvent, its productive assets have higher value than its cash. Conversely, when such a firm is sufficiently close to insolvency, its productive assets have lower value than its cash. In particular, for a firm with cash at the target level and a high risk of insolvency, cash comprises a relatively large fraction of total assets. This, as I detail in the next subsection's analysis of expected returns, implies that such a firm's conditional equity will be lower than its asset beta.

3.2.2 Expected returns under the optimal policies

This subsection derives the expected equity return for a firm that follows the model's optimal policies for holding cash, paying dividends, and triggering insolvency. The goal is to show that such a firm has a low conditional equity beta when its probability of insolvency is high. This provides a theoretical rationale of low equity returns for firms that choose their cash to avoid illiquidity but have a high risk of insolvency. To verify this rationale empirically, I derive several cross-sectional implications, which I will test in the paper's empirical part.

Expected returns, conditional beta, and their relation to solvency

Consider a firm whose cash at time t is at the target level, $C_t = \underline{C}(X_t)$. Then the firm's time- t equity value, $E(X_t)$, is given in Proposition 3. Applying Itô's Formula and using the differential equation in (3.12), the excess return on the

firm's equity, under \mathbb{Q} , is given by

$$\frac{dE(X_t) + dD_t^*}{E(X_t)} - rdt = X_t \frac{E_X(X_t)}{E(X_t)} \sigma dB_t^{\mathbb{Q}}.$$

Using the translation $B_t^{\mathbb{Q}} = B_t^{\mathbb{P}} + \rho\lambda t$, it follows that, at time t , the conditional \mathbb{P} -expected instantaneous excess return on the firm's equity may be expressed as

$$\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^E] - r = \Omega^E(X_t) \rho\sigma\lambda, \quad (3.18)$$

where the key quantity $\Omega^E(X_t) = X_t \frac{E_X(X_t)}{E(X_t)} = \frac{\partial \log E(X_t)}{\partial \log X_t}$ is the *earnings-sensitivity* (or *earnings-elasticity*) of equity. It measures the percentage change in equity value for a one percent change in earnings. A value of $\Omega^E(X_t)$ above (below) 1 means that equity value changes more (less) than proportionally to changes in earnings, i.e. that a change in earnings has a relatively large (small) effect on equity value. The relation in (3.18) says that given the correlated earnings volatility, $\rho\sigma$, and the market price of systematic earnings risk, λ , higher earnings-sensitivity leads to higher expected returns.

To determine the market price of systematic earnings risk, λ , I assume there exists a traded, diversified portfolio with (cum-dividend) value process $(M_t)_{t \geq 0}$, where M_t is instantaneously perfectly negatively correlated with the stochastic discount factor, Λ_t , in (3.3). Its return, under \mathbb{P} , is then given by

$$\frac{dM_t}{M_t} = r^M dt + \sigma^M dZ_t^{\mathbb{P}}. \quad (3.19)$$

Since $Z_t^{\mathbb{Q}} = Z_t^{\mathbb{P}} + \lambda t$ and since the \mathbb{Q} -expected, instantaneous return on the portfolio has to be the risk-free rate, r , it follows that $\lambda = \frac{r^M - r}{\sigma^M}$, i.e. the expected excess return on the portfolio relative to its volatility, or its Sharpe ratio. Combining this with (3.18) gives the familiar relation

$$\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^E] - r = \underbrace{\Omega^E(X_t) \beta^U}_{\beta^E(X_t)} (r^M - r), \quad (3.20)$$

where $\beta^U = \frac{\rho\sigma}{\sigma^M}$ is the correlated volatility of earnings relative to the volatility of the diversified portfolio, i.e. the firm's asset (or unlevered) beta. The relation in (3.20) is a conditional capital asset pricing model (CAPM), in which the firm's time- t conditional equity beta, $\beta^E(X_t) = \Omega^E(X_t) \beta^U$, equals the product of the earnings-sensitivity and the asset beta. Clearly, higher equity beta leads to higher expected equity returns. Furthermore, expected equity returns are high (low) relative to expected asset returns when equity beta is higher (lower) than asset beta, i.e. when $\Omega^E(X_t)$ is above (below) 1.

The following proposition provides the model's main implications for the expected returns of a firm with cash at the target level.

Proposition 4 (Equity beta and firm-solvency). *Suppose that the firm's cash holdings are at the target level. Then the following holds:*

- (i) *When the firm is sufficiently solvent, conditional equity beta is higher than asset beta, while, when the firm is sufficiently close to insolvency, conditional equity beta is lower than asset beta. Precisely: There exists $X'' > \underline{X}^*$ such that $\Omega^E(X_t) > 1$ for all $X_t > X''$, while $\Omega^E(X_t) < 1$ for all $X_t < X''$.*
- (ii) *When the firm is sufficiently solvent, there is a negative relation between conditional equity beta and solvency, while when the firm is sufficiently close to insolvency, there is a positive relation between conditional equity*

beta and solvency. Precisely: There exists $X''' > \underline{X}^*$ such that $\frac{\partial \Omega^E}{\partial X_t} < 0$ for all $X_t > X'''$, while $\frac{\partial \Omega^E}{\partial X_t} > 0$ for all $X_t < X'''$.

The first part of [Proposition 4](#) shows that when a firm with cash at the target level is sufficiently solvent (i.e. when X_t is above the threshold X'') equity beta is higher than asset beta. The reason is that when the firm is sufficiently solvent, the value of its total assets is to a large degree comprised of the value of the productive assets-in-place (cf. the third part of [Proposition 3](#)). Because equity is a levered claim on the firm's total assets, the equity of a sufficiently solvent firm will be riskier (i.e. have higher earnings-sensitivity) than the firm's productive assets-in-place, implying that equity beta is higher than asset beta. Conversely, the first part of [Proposition 4](#) also shows that when a firm with cash at the target level is sufficiently close to insolvency (i.e. when X_t is below the threshold X''), equity beta is lower than asset beta. This is because when the firm is sufficiently close to insolvency, the value of its total assets will to a large degree be comprised of the cash that it holds to avoid illiquidity. This implies that equity, despite being a levered claim on total assets, will be less risky than the productive assets-in-place, and therefore that equity beta is less than asset beta. In sum, the first part of [Proposition 4](#) implies a generally positive relation between equity beta and solvency. It therefore provides a theoretical rationale for the low returns to distressed equity: Because it is optimal for the firm to hold a level of cash that allows it to avoid liquidity distress, the firm will have a high fraction of its assets in cash when it is in solvency distress, which leads to a low equity beta. In addition, this result complements the insights of [Garlappi and Yan \(2011\)](#), [Opp \(2013\)](#), and [McQuade \(2013\)](#) by separating the illiquidity and insolvency components of distress.

The second part of [Proposition 4](#) shows that when the firm is sufficiently solvent (i.e. when X_t is above the threshold X'''), there is a negative relation between equity beta and solvency, while when the firm is sufficiently close to insolvency (i.e. when X_t is below the threshold X'''), there is a positive relation between equity beta and solvency. To see this, note an increase in solvency means lower leverage, which decreases equity beta, but also a lower fraction of assets in cash, which increases equity beta. When the firm is sufficiently solvent, the effect of lower leverage dominates, so that an increase in solvency leads to a decrease in equity beta. However, when the firm is sufficiently close to insolvency, the effect of a lower fraction of assets in cash dominates, so that an increase in solvency leads to an increase in equity beta. In sum, the second part of [Proposition 4](#) implies that while the relation between equity beta and solvency is generally positive, it is in fact non-monotonic and "hump-shaped."

In the following subsection, I use [Proposition 4](#) to derive testable predictions for the behavior of expected equity returns in the cross-section.

Testable predictions for expected returns

In this subsection, I derive testable predictions from [Proposition 4](#). First, I determine the relation between expected returns and probability of insolvency. Second, I determine the relation between expected returns and optimal liquidity.

I first determine the relation between expected returns and probability of insolvency. Recall, from the expression for the equity value in [Proposition 3](#), that the time- t , long-term \mathbb{Q} -probability of insolvency can be proxied by $\pi^{\mathbb{Q}}(X_t) = \left(\frac{X_t}{\underline{X}^*}\right)^{\phi^-}$, where $\phi^- < 0$. Hence, $\pi^{\mathbb{Q}}(X_t)$ is monotonically decreasing in X_t . Therefore, the results of [Proposition 4](#) can be directly translated into results concerning the relation between expected returns and probability of insolvency.

Corollary 4.1 (Expected returns and probability of insolvency). *Under the assumptions of Proposition 4, the following holds:*

- (i) *When the probability of insolvency is sufficiently close to 0, expected returns are relatively high, i.e. $\Omega^E(X_t) > 1$, while when the probability of insolvency is sufficiently higher than 0, expected returns are relatively low, i.e. $\Omega^E(X_t) < 1$.*
- (ii) *When the probability of insolvency is sufficiently close to 0, there is a positive relation between expected earnings and probability of insolvency, i.e. $\frac{\partial \Omega^E}{\partial \pi^Q} > 0$, while when the probability of insolvency is sufficiently higher than 0, there is a negative relation between expected returns and probability of insolvency, $\frac{\partial \Omega^E}{\partial \pi^Q} < 0$.*

The first part of Corollary 4.1 predicts that, in general, firms with low probability of insolvency have high expected returns (and high conditional betas) relative to firms with high probability of insolvency. The second part of Corollary 4.1 predicts that the relation between expected returns and probability of insolvency (as well as the relation between equity beta and probability of insolvency) is non-monotonic and “hump-shaped.” This follows directly from the second part of Proposition 4 because the chain-rule gives that $\frac{\partial \Omega^E}{\partial \pi^Q} = \frac{\partial \Omega^E}{\partial X_t} / \frac{\partial \pi^Q}{\partial X_t}$ while $\frac{\partial \pi^Q}{\partial X_t} < 0$. In the paper’s empirical part, I test these predictions using observable firm-specific solvency measures as proxies for the probability of insolvency.

Next, I determine the relation between expected returns and optimal liquidity. In the following, I measure the firm’s optimal time- t liquidity, which I denote $\underline{L}(X_t)$, as the ratio of the firm’s target cash level and the value of future coupon payments, i.e.

$$\underline{L}(X_t) = \frac{C(X_t)}{k/r}.$$

Recall, from Proposition 1, that the time- t target cash level is given by $C(X_t) = X_t - \underline{X}^* + \frac{k}{r}$. It is therefore monotonically increasing in X_t and bounded from below by the value of coupon liabilities. Hence, optimal liquidity is monotonically increasing in X_t and bounded from below by 1. The following corollary translates the results of Proposition 4 into results concerning the relation between expected returns and optimal liquidity.

Corollary 4.2 (Expected returns and optimal liquidity). *Under the assumptions of Proposition 4, the following holds:*

- (i) *When optimal liquidity is sufficiently higher than 1, expected returns are relatively high, i.e. $\Omega^E(X_t) > 1$, while when optimal liquidity is sufficiently close to 1, expected returns are relatively low, i.e. $\Omega^E(X_t) < 1$.*
- (ii) *When optimal liquidity is sufficiently higher than 1, there is a negative relation between expected returns and optimal liquidity, i.e. $\frac{\partial \Omega^E}{\partial \underline{L}} < 0$, while when optimal liquidity is sufficiently close to 1, there is a positive relation between expected returns and optimal liquidity, $\frac{\partial \Omega^E}{\partial \underline{L}} > 0$.*

The first part of Corollary 4.2 predicts that, in general, firms with high optimal liquidity have high expected returns (and high conditional betas) relative to firms with low optimal liquidity. The second part of Corollary 4.2 predicts that the relation between expected returns and optimal liquidity (as well as the relation between equity beta and optimal liquidity) is non-monotonic and “hump-shaped.” Note that the second part captures the common notion that firms with higher reserves of liquid assets should have lower equity betas and therefore lower expected returns, but makes the notion more precise, as the result says that the negative relation between expected returns and liquid reserves only holds for firm’s with sufficiently high levels of liquid reserves. In the paper’s empirical part, I test these predictions using observable firm-specific liquidity measures as proxies for optimal liquidity.

Testable predictions for long-short portfolio strategies

The results of [Corollaries 4.1](#) and [4.2](#) can be exploited in long-short portfolio strategies that produce positive expected returns. Therefore, in this subsection, I consider the performance of two long-short portfolios formed on optimal liquidity and probability of insolvency.

I first consider the expected return on a *higher-liquidity minus lower-liquidity* (HLmLL) portfolio as a function of the probability of insolvency. To this end, consider two identical firms with time- t optimal liquidity $\underline{L}(X_t)$. Suppose that the first firm experiences a positive earnings-shock $\Delta_X > 0$. Then the first firm's optimal liquidity will increase to $\underline{L}(X_t + \Delta_X)$. Let r_{t+dt}^{HL} be the return of the higher-liquidity firm and let r_{t+dt}^{LL} be the return of the lower-liquidity firm. Then the expected return-difference between the two firms can be expressed as a function of the probability of insolvency by using the relation in [\(3.20\)](#) to write

$$\begin{aligned}\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^{HL} - r_{t+dt}^{LL}] &= (\Omega^E(X_t + \Delta_X) - \Omega^E(X_t))\beta^U(r^M - r) \\ &\approx \frac{\partial\Omega^E}{\partial\pi^Q} \frac{\partial\pi^Q}{\partial X_t} \Delta_X \beta^U(r^M - r).\end{aligned}$$

I now define, for a given earnings-shock, Δ_X , the expected return on the HLmLL portfolio as the right-hand side of this last expression, i.e. $\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^{HLmLL}] = \frac{\partial\Omega^E}{\partial\pi^Q} \frac{\partial\pi^Q}{\partial X_t} \Delta_X \beta^U(r^M - r)$. The following corollary follow from the second part of [Corollary 4.1](#).

Corollary 4.3 (The HLmLL portfolio). *Under the assumptions of [Proposition 4](#), the following holds for the expected return on the HLmLL portfolio:*

- (i) *When the probability of insolvency is sufficiently close to 0, the expected return on the HLmLL portfolio is negative.*
- (ii) *When the probability of insolvency is sufficiently higher than 0, the expected return on the HLmLL portfolio is positive.*

[Corollary 4.3](#) predicts that the expected return of the HLmLL portfolio is a generally increasing function of the probability of insolvency and that the positive expected returns on HLmLL are concentrated among firms with high probabilities of insolvency. In the paper's empirical part, I test these predictions using HLmLL portfolios that are constructed through conditional sorts—first on solvency, and then on liquidity.

Finally, I consider the expected return on a *higher-solvency minus lower-solvency* (HSmLS) portfolio as a function of optimal liquidity. Consider two identical firms with time- t insolvency probability $\pi^Q(X_t)$. Suppose that the first firm experiences a positive earnings-shock $\Delta_X > 0$. Then the first firm's probability of insolvency will decrease to $\pi^Q(X_t + \Delta_X)$, i.e. it will have higher solvency. Let r_{t+dt}^{HS} be the return of the higher-solvency firm and let r_{t+dt}^{LS} be the return of the lower-solvency firm. The expected return-difference between the two firms can be expressed as function of optimal liquidity by writing

$$\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^{HL} - r_{t+dt}^{LL}] \approx \frac{\partial\Omega^E}{\partial L} \frac{\partial L}{\partial X_t} \Delta_X \beta^U(r^M - r),$$

I now define the expected return on the HSmLS portfolio as $\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^{HSmLS}] = \frac{\partial\Omega^E}{\partial L} \frac{\partial L}{\partial X_t} \Delta_X \beta^U(r^M - r)$. The following corollary follow from the second part of [Corollary 4.2](#).

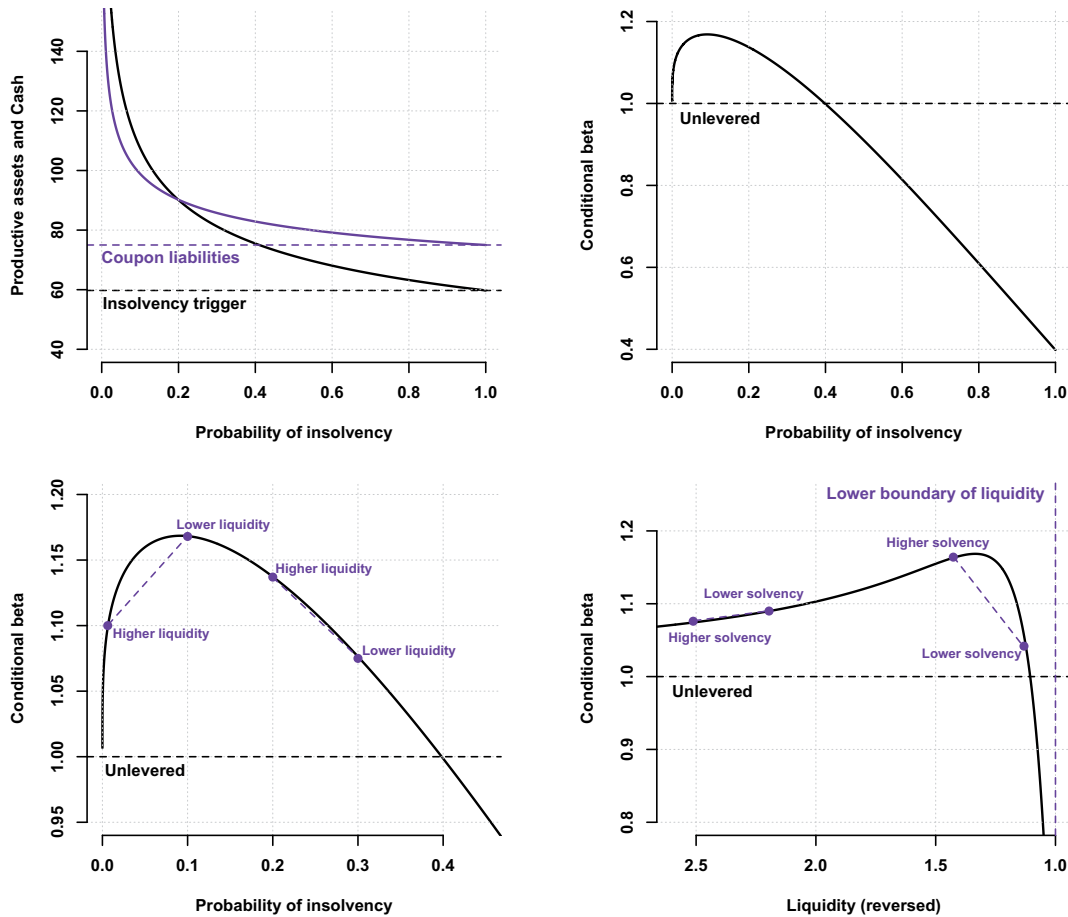


Figure 3.1. Asset-composition and conditional beta. This figure shows a numerical illustration of Corollary 4 and its cross-sectional implications. *Top left:* Earnings-generating asset value (black curve) and target cash holdings (purple curve) as a function of probability of insolvency with an indication of the insolvency trigger (black dashed line) and the lower boundary of cash holdings (dashed purple line). *Top right:* Conditional beta as a function of the probability of insolvency with an indication of the unlevered (asset) beta (dashed line). *Bottom left:* Conditional beta as a function of the probability of insolvency (from 0 to 0.4) with an indication of the higher-liquidity minus lower-liquidity strategy (dashed purple line segments). *Bottom right:* Conditional beta as a function of optimal liquidity (i.e. target cash holdings divided by the value of future coupon payments) with an indication of the higher-solvency minus lower-solvency strategy (dashed purple line segments). The parameters are $\mu^Q = 0.04$, $\sigma = 0.15$, $k = \$4.5$, $r = 0.06$, $\rho = 0.33$, and $\sigma^M = 0.05$.

Corollary 4.4 (The HSmLS portfolio). *Under the assumptions of Proposition 4, the following holds for the expected return on the HSmLS portfolio:*

- (i) *When optimal liquidity is sufficiently higher than 1, the expected return on the HSmLS portfolio is negative.*
- (ii) *When optimal liquidity is sufficiently close to 1, the expected return on the HSmLS portfolio is positive.*

Corollary 4.4 predicts that the expected return of the HSmLS portfolio is a generally decreasing function of optimal liquidity and that the positive expected returns on HSmLS are concentrated among firms with low optimal liquidity. In the paper’s empirical part, I test these predictions using HSmLS portfolios that are constructed through conditional sorts—first on liquidity, and then on solvency.

Numerical illustration

Figure 3.1 illustrates the results of Proposition 4 and its cross-sectional implications in Corollaries 4.1–4.4. I consider a representative firm and economy given by the following parameters:

$$\begin{aligned}\mu^Q &= 0.04, \sigma = 0.15, k = \$4.5, \\ r &= 0.06, \rho = 0.33, \sigma^M = 0.05.\end{aligned}$$

This choice of parameters implies that insolvency is triggered when cumulated earnings fall to $\underline{X}^* = \$29.87$ (corresponding to $U(\underline{X}^*) = \$59.75$ for the value of productive assets) and that the firm's unlevered asset beta is normalized to $\beta^U = \frac{\rho\sigma}{\sigma^M} = 1$. For instance, if the firm's time- t level of cumulated earnings is $X_t = \$50.00$, the value of productive assets is $U(X_t) = \$100$, equity value is $E(X_t) = \$122.15$, the target cash level is $\underline{C}(X_t) = \$95.13$, the proxy for the long-term \mathbb{Q} -probability of insolvency is $\pi^Q(X_t) = 0.13$, and, finally, conditional beta is $\beta_t^E = \Omega^E(X_t)\beta^U = 1.16$.

The figure's top panels show the value of productive assets, target cash holdings, and conditional beta as functions of the probability of insolvency. For low levels of probability of insolvency, the target cash level is high, but the value of productive assets is higher. This is associated with a conditional beta that is above the asset beta and is upwards-sloping in the probability of insolvency. However, as the probability of insolvency increases, the target cash level decreases, but the value of productive assets decreases by more and ultimately fall below the value of target cash. At the same time, conditional beta falls below the asset beta and becomes downwards-sloping in the probability of insolvency.

The lower left panel zooms in on the conditional beta as a function of probability of insolvency and indicates the long and short legs of the HLmLL portfolio. Because conditional beta initially increases in the probability of insolvency, the short leg of HLmLL has higher expected returns than the long leg when the probability of insolvency is high, so HLmLL has negative expected returns. This is, however, reversed when the probability of insolvency is low, where the long leg of HLmLL has higher expected returns than the short leg, leading to positive expected returns for the HLmLL portfolio.

Finally, the lower right panel shows conditional beta as a function of decreasing optimal liquidity and also indicates the long and short legs of the HSmLS portfolio. Because optimal liquidity moves opposite the probability of insolvency, conditional beta is above 1 and downward-sloping when optimal liquidity is high, but below 1 and upwards-sloping when optimal liquidity is close to its lower boundary of 1. Therefore, the short leg of HSmLS has higher expected returns than the long leg for high levels of optimal liquidity. The situation is, however, reversed for low levels of optimal liquidity.

3.3 Data and variables

This section presents the data and the variables that I use to test the model's predictions in [Section 3.4](#). I use firm-level data on stock prices, accounting numbers, and credit ratings for US firms during the period 1970-2013. In the following, I first detail my sample construction and then define the measures of solvency and liquidity which I employ to in the analysis of equity returns.

3.3.1 Data

I search for data in the intersection of US industrial firms with stock prices in the CRSP database, accounting fundamentals in the Compustat North American database, and credit ratings or default records in the Moody's DRS (Default Risk Service) database.

For every US debt issuer in DRS' industrial category with an available third party identifier, I search for the corresponding security-level PERMNO-identifiers in the daily CRSP file and in the quarterly and yearly Compustat files, taking name changes, mergers, accusations, and parent-subsidiary relations into account, and excluding issuers which I cannot reliably match. I only include common stocks (CRSP's SHRCD 10-11) and I exclude utilities and financial firms (CRSP's SIC codes 4900-4999 and 6000-6999). The final sample has 3,947 unique firms, spanning 720,371 firm-months, and covers January 1970 to December 2013.

Because distress may ultimately result in a default or bankruptcy, I track these events for the firms in the sample. I identify a default- or bankruptcy event if it is recorded in DRS, or in CRSP (DLSTCD 400-490 or 574, or SECSTAT 'Q'), or in Compustat (DLRSN 2-3 or STALTQ 'TL'), and I count multiple events for the same firm occurring within a month as a single event. This results in a total of 874 events incurred by 683 firms, of which 529 events were identified solely through DRS, 134 solely through CRSP, and 137 solely through Compustat—the remaining 87 events were identified simultaneously by two or more sources.

I use the daily stock data to calculate market equity values, ME (the product of CRSP's PRC and SHROUT, both adjusted by their cumulative adjustment factors), and I accumulate daily log-returns (\ln of 1 plus CRSP's simple return, RET) over a 21 trading day rolling window to obtain monthly returns at a daily frequency. I require at least 10 trading days to calculate a monthly return and I use delisting returns (CRSP's DLRET) whenever possible.

I use the quarterly accounting data to calculate balance-sheet based measures of firm solvency and liquidity as well as regression controls (to be detailed below). When possible, I substitute yearly accounting numbers for missing quarterly accounting numbers. I align the quarterly accounting data and the daily stock data as follows: On a given trading day, the corresponding accounting numbers are the latest ones available prior to that day. Except for returns, all variables are winsorized at the 1st and 99th percentile to remove the influence of divisions by near-zero denominators, recording errors, and statistical outliers.

3.3.2 Measuring solvency and liquidity

The model argues that because it is optimal for a firm to hold a level of cash that allows it to avoid liquidity distress, such a firm will have a high fraction of its assets in cash when it is in high risk of solvency distress, and this leads to a low conditional beta and therefore low expected returns. I now present the variables which I employ to measure solvency and liquidity. These will constitute the 'sorting' or 'explanatory' variables in the analysis of conditional betas and expected returns in [Section 3.4](#).

Solvency measures

Solvency is a firm's ability to honor its long-term debt obligations. To measure solvency, I primarily use Moody's senior unsecured long-term credit ratings. However, for robustness, and because numeric variables are better suited for regressions, I also use two balance-sheet based variables: Leverage and interest coverage.

The long-term credit rating is a categorical measure of solvency, giving Moody's relative assessment of a firm's ability to honor its financial obligations with an original maturity of one year or more (Moody's Investors Service, 2014). Credit ratings incorporate not only firms' economic and financial characteristics, but also industry conditions and 'soft' information like experts' outlook. Moody's assigns 9 long-term credit ratings: Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C. Firms rated above Baa are considered 'investment grade,' while firms rated Baa and below are considered 'speculative grade.' Within speculative-grade firms, a rating of Caa or below corresponds to 'distressed,' with C-rated firms typically being in default. In some cases, I augment Moody's original rating categories with a D-category, corresponding to an identified default or bankruptcy.

I measure leverage using the (quasi) market leverage ratio, $LT/(LT + ME)$ (Compustat's total liabilities, LTQ, divided by the sum of total liabilities and market equity), but other leverage ratios produce similar results. This is a 'stock' variable measuring the firm's total liabilities as a fraction of its total market value. The closer the ratio is to 1, the higher the risk of 'accounting insolvency.' Although this does not automatically trigger default, a firm in solvency distress will likely be forced by its debt holders to take actions in response to its deteriorated solvency—this could be a restructuring of its operations, a renegotiation of its debt obligations, or even an involuntary bankruptcy filing.

As a final measure of solvency, I use the interest coverage ratio, OI/IX (Compustat's operating income or EBITDA variable, OIBDPQ, divided by interest expense, XINTQ). This is a 'flow' variable measuring the firm's ability to generate earnings in excess of its interest expense on a quarter-by-quarter basis. If interest coverage is below 1, the firm is currently not generating enough earnings to cover its interest expense, and a cash-flow based assessment of its asset value may thus indicate solvency distress. Because firms with negative operating income do not have meaningful interest coverage ratios, I set interest coverage to zero whenever operating income is negative.

Liquidity measures

Liquidity is, in this paper, a firm's ability to honor its short-term debt obligations. To measure liquidity, I use three balance-sheet based variables that compare a firm's stock of liquid assets (including cash and marketable securities) to its short-term liabilities (due within one year): The current ratio, the quick ratio, and the working capital ratio.

The current ratio, CA/CL (Compustat's current assets, ACTQ, divided by current liabilities, LCTQ), measures a firm's total stock of liquid assets relative to its short-term liabilities. A current ratio below 1 means that the firm's total stock of liquid assets is insufficient to meet its short-term liabilities. This is an indication of liquidity distress, since the firm will default in the short-term if it cannot generate sufficient earnings or obtain external financing to cover its liquidity short-fall.

The quick ratio, QA/CL (Compustat's current assets, ACTQ, minus inventories, INVTQ, the difference divided by current liabilities, LCTQ), measures a firm's assets that can 'quickly' be converted into cash at near book value in order to pay off short-term liabilities. The current- and quick ratios are similar, but the quick ratio does not regard inventories as 'quick' assets, and is thus more conservative. Like the current ratio, a quick ratio below 1 is an indication of liquidity distress.

Finally, the working capital ratio (Compustat's current assets, ACTQ, minus current liabilities, LCTQ, the difference divided by total assets, ATQ) measures a firm's net liquid assets as a percentage of its total book assets. It is thus

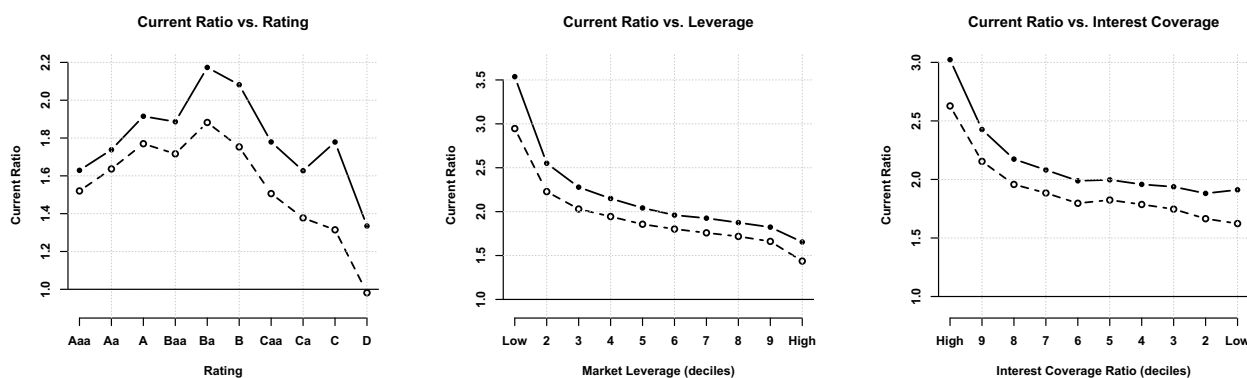


Figure 3.2. Even the least solvent firms hold liquid assets that can cover their short-term liabilities. This figure shows the current ratio, CA/CL , plotted against the three solvency measures (Moody's long-term credit rating, deciles of market leverage, $LT/(LT + ME)$, and deciles of interest coverage, OI/IX). Solid lines indicate means within groups while dashed lines indicate medians. In all panels, a horizontal move to the right corresponds to lower solvency, while a vertical move downwards corresponds to lower liquidity.

the dollar-amount of total liquid assets in excess of current liabilities relative to total book assets. Therefore, a negative working capital ratio is an indication of liquidity distress.

Liquidity vs. solvency

Figure 3.2 shows how liquidity varies with solvency in the sample. For brevity, I focus on the current ratio as the measure of liquidity, although the results are very similar for the quick ratio and the working capital ratio. The leftmost column shows the current ratio across credit ratings (the 'total' measure of solvency); the middle column shows the current ratio against market leverage (the 'stock' measure of solvency); and the rightmost column shows the current ratio against interest coverage (the 'flow' measure of solvency). Two observations are important.

First, liquidity generally decreases as solvency decreases. More precisely, average and median liquidity levels are hump-shaped in decreasing credit ratings (increasing up to Ba but decreasing thereafter) and almost monotonically decreasing in increasing leverage and decreasing interest coverage. Note, however, that investment grade firms (rated Aaa-A) generally have higher average and median liquidity levels than do distressed and defaulted firms (rated Caa-C and D). The relatively modest reserves of liquid assets for investment grade firms is probably due to their high ratings, which means that they need to worry less about financing constraints.

Second, even the least solvent firms hold levels of liquid assets that can cover their short-term liabilities. The average and median distressed firm (rated Caa-C and D) is still liquid and the same is true for the average and median firm in the highest leverage decile and the lowest interest coverage decile.

In sum, Figure 3.2 shows that the distribution of liquidity across solvency within the sample closely resembles the model's optimal policy for holding cash: Less solvent firms are less willing to cover earnings-shortfalls with cash holdings and thus hold less cash, but even the least solvent firms hold cash levels that allow them to service short-term liabilities (Proposition 1–2).

Liquid assets as a fraction of total assets vs. solvency

Figure 3.3 shows liquid assets, as a fraction of the (quasi) market value of total assets, plotted against solvency. In the figure, I measure liquid assets as the firm's current assets, CA , and the (quasi) market value of total assets is the book value of total liabilities added the market value of equity, $LT + ME$. The figure is, however, very similar if liquid

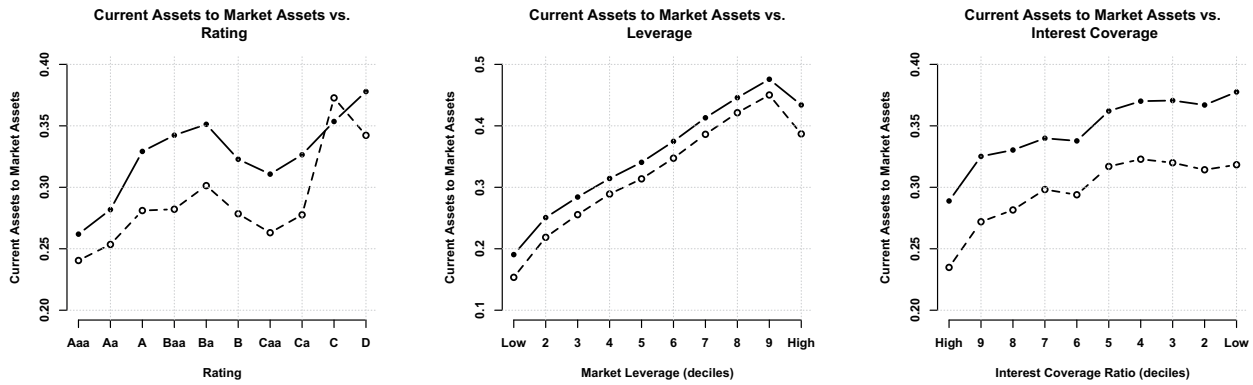


Figure 3.3. Less solvent firms have a higher fraction of their total assets in liquid assets. This figure shows current assets, CA , as a fraction of the (quasi) market value of total assets, $ME + LT$, plotted against the three solvency measures (Moody’s long-term credit rating, deciles of market leverage, $LT/(LT + ME)$, and deciles of interest coverage, OI/IX). Solid lines indicate means within groups while dashed lines indicate medians.

assets are measured as quick assets, QA (i.e. current assets less inventories), or if the book value of total assets is used in the denominator.

The figure shows that less solvent have a higher fraction of their total assets in liquid assets. For instance, while the average Aaa-rated firm has around 25% of its total assets in liquid assets, the fraction is over 35% for the average distressed firm (rated Caa-C or D). Similar relations hold when solvency is measured as leverage or interest coverage. These results are consistent with the model’s prediction that firms with a sufficiently high probability of insolvency have a large fraction of their assets in cash (part (iii) of Proposition 3).

3.4 Empirical results

This section tests the model’s predictions for expected returns. The model’s predictions are derived for a firm that follows the optimal policy for holdings cash and thus avoid illiquidity. Hence, one could argue that the model’s predictions should be tested in the subsample of firm-observations where liquid assets meet or exceed current liabilities. However, since Figure 3.2 suggests that even the least solvent firms in the sample are on average still liquid, I test the model’s predictions using the entire sample. This biases my analysis against finding support for the model’s rationale of the distress puzzle. At the same time, it increases the available number of observations and alleviates concerns regarding data-mining.

First, the model predicts that, in general, firms with high solvency have high expected returns (and high conditional betas) relative to firms with low solvency. Also, the relation between expected returns and solvency (as well as between conditional betas and solvency) is non-monotonic and “hump-shaped” (Corollary 4.1). Second, the model predicts that, in general, firms with high liquidity have high expected returns (and high conditional betas) relative to firms with low liquidity. Also, the relation between expected returns and liquidity (as well as between conditional betas and liquidity) is non-monotonic and “hump-shaped” (Corollary 4.2). In the model, there is a one-to-one correspondence between conditional betas and expected returns (cf. relation (3.20)). However, in practice, the relation may not be as clearcut: Conditional betas are subject to estimation noise and there is mixed evidence regarding the performance of conditional

betas in asset pricing tests.¹⁰ Therefore, I test these predictions separately for conditional betas and expected returns: First for conditional betas in the cross-section of solvency and liquidity-levels, and then for expected returns using 1) cross-sectional regressions of firm-level returns on the solvency and liquidity measures, and 2) returns and alphas for portfolios formed on solvency and liquidity.

Third, the model predicts that a higher-liquidity minus lower-liquidity (HLmLL) portfolio has negative expected returns for firms with high solvency, but positive expected returns for firms with low solvency (Corollary 4.3). I test this predictions using the returns and alphas of HLmLL portfolios in the cross-section of solvency levels.

Finally, the model predicts that a higher-solvency minus lower-solvency (HSmLS) portfolio has negative expected returns for firms with high liquidity, but positive expected returns for firms with low liquidity (Corollary 4.3). I test this predictions using the returns and alphas of HSmLS portfolios in the cross-section of liquidity levels.

3.4.1 Conditional betas

To test the predictions of Corollary 4.1 and Corollary 4.2 for conditional betas, Figure 3.4 plots estimated conditional betas across the solvency and liquidity measures.

The conditional betas are estimated at the firm-level at a monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted “market” index (available in the CRSP file or on prof. Ken French’s website as the MKT-factor). The risk-free rate is proxied by the 1-month US T-bill rate. I account for nonsynchronous trading data as in Dimson (1979) using an estimation window of 3 days. Specifically, I estimate frazfirm i ’s monthly beta at month t as $\beta_{it} = \sum_{k=-1}^1 \beta_{it,d+k}$ from the time-series regression

$$r_{it,d}^E - r_{t,d}^f = \alpha_{it} + \sum_{k=-1}^1 \beta_{it,d+k} (r_{t,d+k}^M - r_{t,d+k}^f) + \epsilon_{it,d},$$

where $r_{it,d}^E$ is firm i ’s equity return, $r_{t,d}^f$ is the risk-free rate, and $r_{t,d}^M$ is the market return on day d of month t , while $\epsilon_{it,d}$ is a mean-zero error term. I require a minimum of 5 trading days to estimate a monthly beta and, to reduce the influence of outliers, I exclude each firm’s lowest and highest monthly beta.

Figure 3.4 shows strong evidence in favor of the model’s predictions that conditional betas are generally positively related to solvency and liquidity. The most solvent firms (high credit rating, low leverage, or high interest coverage) have average monthly betas between 0.9 and 1.2, while the least solvent firms have average monthly betas between 0.6 and 0.8. Similarly, the firms with highest liquidity also have monthly betas close to 1, while the firms with the lowest liquidity have monthly betas around 0.8. If higher conditional beta is associated with higher expected returns—at least on average—then this is also in favor of the model’s predictions that higher-solvency firms outperform lower-solvency firm and that higher-liquidity firms outperform lower-liquidity firms.

On the other hand, the figure shows mixed evidence for the model’s prediction that conditional betas are hump-shaped in solvency and liquidity, i.e. that conditional betas are initially upwards-sloping but eventually downwards-sloping as both solvency and liquidity decrease. For the solvency measures, the monthly betas are hump-shaped as credit ratings deteriorate, but are almost monotonically decreasing as leverage increases and as interest coverage decreases. For the liquidity measures, the monthly betas are almost monotonically decreasing as all three liquidity measures decrease.

¹⁰For instance, Lewellen and Nagel (2006) refute the early results of Jagannathan and Wang (1996) and others that conditional betas can explain asset-pricing anomalies. On the other hand, recent studies by Adrian and Franzoni (2009) and Bali, Engle, and Tang (2014) find that conditional betas do have explanatory power in asset-pricing tests using more refined estimation procedures and tests.

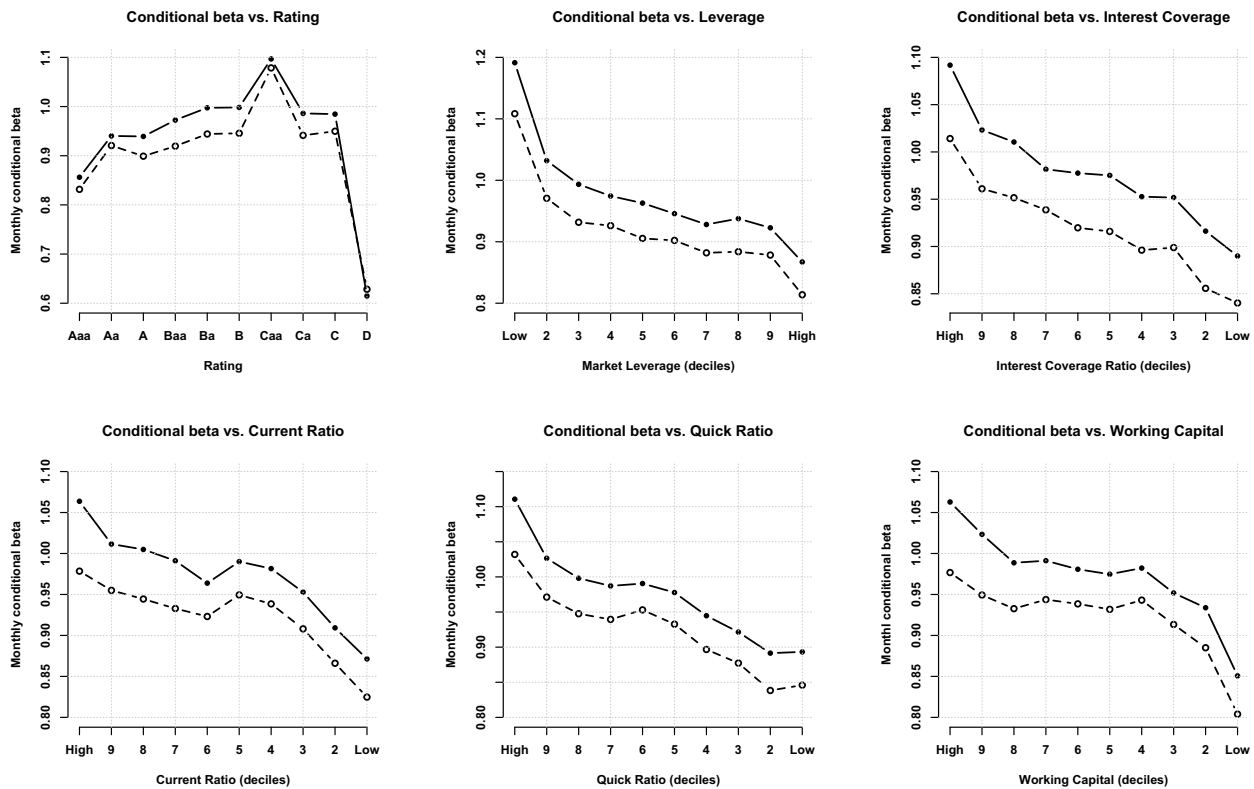


Figure 3.4. Conditional betas are positively related to solvency and liquidity. This figure shows conditional (time-varying) betas plotted against the three solvency measures (top panels: Moody’s credit rating, deciles of market leverage, $LT/(LT + ME)$, and deciles of interest coverage, OI/IX) and the three liquidity measures (bottom panels: Deciles of current ratio, CA/CL , quick ratio, QA/CL , and working capital, WC/AT). Conditional betas are estimated at the firm-level at a monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted “market” index. The risk-free rate is proxied by daily values for the 1-month US T-bill rate. Beta estimates are adjusted for nonsynchronous trading data as in Dimson (1979) using an estimation window of 3 days. Each monthly beta is estimated using a minimum of 15 trading days, and, to remove the influence of outliers, each firm’s beta estimates are winsorized at a 5% level. Solid lines indicate means while dashed lines indicate medians. In the top (bottom) panels, a horizontal move to the right corresponds to lower solvency (liquidity).

To summarize, Figure 3.4 strongly confirms the model’s predictions that conditional betas are positively related to solvency and liquidity. This extends the findings of Garlappi and Yan (2011), in that the decrease in conditional betas is prevalent in both the insolvency and illiquidity dimensions of default risk. On the other hand, the figure shows mixed evidence for the predicted “hump-shaped” relations. In the following subsection, I use cross-sectional regressions of firm-level returns to further investigate these predictions.

3.4.2 Fama-MacBeth regressions

I now test the predictions of Corollary 4.1 and Corollary 4.2 for expected returns using predictive cross-sectional regressions of firm-level returns on solvency and liquidity. In the following, I consider both linear and linear-quadratic regression specifications. A linear specification allows me to test whether the correlation between expected returns and a given solvency or liquidity measure is as predicted by the model. A linear-quadratic specification allows me to conduct a simple but direct test of a “hump-shaped” relation by testing whether the coefficient on the quadratic term is negative.

Specifically, I consider linear and linear-quadratic regression specifications of the forms

$$\begin{aligned} \text{Linear:} \quad r_{it}^E &= \gamma_0 + \gamma_1 V_{i,t-1} + \text{controls} + \epsilon_{it}, \\ \text{Linear-Quadratic:} \quad r_{it}^E &= \tilde{\gamma}_0 + \tilde{\gamma}_1 V_{i,t-1} + \tilde{\gamma}_2 V_{i,t-1}^2 + \text{controls} + \tilde{\epsilon}_{it}. \end{aligned}$$

Here, r_{it}^E is firm i 's 1-month return at month t , $V_{i,t-1}$ is a given measure of the firm's solvency or liquidity at month $t-1$, while ϵ_{it} and $\tilde{\epsilon}_{it}$ are mean-zero error terms. In the linear specification, the parameter γ_1 estimates the marginal effect of V on expected (next-month) returns. In the linear-quadratic specification, the parameter $\tilde{\gamma}_1$ estimates the marginal effect of V when V is close to zero, while the parameter $\tilde{\gamma}_2$ estimates the curvature of the relation between expected returns and V . Hence, a hump-shaped relation between expected returns and V can be tested by testing whether $\tilde{\gamma}_2 < 0$.

Table 3.1 shows results from monthly Fama and MacBeth (1973) regressions of firm-level 1-month returns on lagged values for the two numeric solvency measures and the three liquidity measures. I log-transform nonnegative variables that do not have a naturally bounded distribution. All specifications include 1-month lagged controls for firm size ($\log(ME)$) and book-to-market equity ($\log(BE/ME)$), as well as the firm's return from 2 months ago until a month ago ($r_{t-2,t-1}$, a control for short-term-reversal) and the firm's return from 12 months ago until 2 months ago ($r_{t-12,t-2}$, a control for momentum).¹¹ The table reports average monthly slope-coefficients with t -statistics based on standard errors that are adjusted for heteroskedasticity and autocorrelation as in Newey and West (1987) using a lag length of 12 months.

Specification (1) shows that higher market leverage (i.e. lower solvency measured in 'stock' terms) is associated with significantly lower expected returns. Specification (2) shows that the curvature of the relation between expected returns and market leverage is significantly negative—that is, if the relation between expected returns and market leverage can be reasonably approximated by a linear-quadratic function, then this function has a significantly negative second derivative. That market leverage has a positive but insignificant linear-effect in the linear-quadratic specification means that when market leverage is close to zero, an increase in market leverage is associated with a positive but insignificant increase in expected returns.

Specification (3) shows that higher interest coverage (i.e. higher solvency measured in 'flow' terms) is associated with significantly higher expected returns. Interestingly, the t -statistic on the effect of interest coverage is 12 standard deviations away from zero and over twice as large as the t -statistic on the effect of book-to-market equity from the same specification. Specification (4) shows that the curvature of the relation between expected returns and interest coverage is significantly negative, and that, when interest coverage is close to zero, an increase in interest coverage is associated with significantly higher expected returns.

¹¹Book-to-market equity is book equity, BE , divided by market equity, ME . Similar to Fama and French (1993), I calculate quarterly book equity as stockholders' equity (Compustat's SEQQ) plus deferred taxes and investment tax credit (Compustat's TXDITCQ) minus preferred stock (Compustat's PSTKQ). I exclude negative BE -values and, following Novy-Marx (2013), I use 6-month lagged ME to avoid taking unintentional positions in momentum. The return from 2 months ago until a month ago, $r_{t-2,t-1}$, is calculated by compounding daily log-returns, and similarly for the return from 12 months ago until 2 months ago, $r_{t-12,t-2}$.

Table 3.1. Fama-MacBeth regressions of firm-level returns on solvency and liquidity measures. This table shows Fama-MacBeth slope-coefficients ($\times 10^2$) from monthly cross-sectional regressions of firm-level returns on solvency and liquidity measures. At each calendar month in the sample, firm-level 1-month returns (the dependent variable) are regressed on lagged values of the explanatory variables (the independent variables). Market leverage is $LT/(LT + ME)$, interest coverage is OI/IX , current ratio is CA/CL , quick ratio is QA/CL where QA is current assets less inventories, and working capital is WC/AT where WC is current assets less current liabilities. All regressions include controls for firm size ($\log(ME)$), book-to-market ($\log(B/M)$), the return from 2 months ago until 1 month ago ($r_{t-2,t-1}$), and the return from 12 months ago until 2 months ago ($r_{t-12,t-2}$). Parentheses in subscript give the t -statistics for a null value of zero, based on standard errors that are adjusted for heteroskedasticity and autocorrelation as in Newey and West (1987) using a lag length of 12 months. Statistical significance at the 5% level is indicated in bold.

Independent variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Market leverage	-0.89 _(-3.04)	1.39 _(1.70)								
[Market leverage] ²		-2.50 _(-2.94)								
log(Interest coverage)			0.48 _(12.12)	1.38 _(12.44)						
[log(Interest coverage)] ²				-0.17 _(-10.48)						
log(Current ratio)					0.23 _(3.18)	0.73 _(6.63)	0.27 _(4.05)	0.29 _(4.43)		
[log(Current ratio)] ²					-0.38 _(-5.60)		-0.12 _(-3.36)			
log(Quick ratio)									1.13 _(4.31)	3.01 _(6.55)
[log(Quick ratio)] ²									-3.85 _(-5.16)	
Working capital										
[Working capital] ²										
log(BE/ME)	0.47 _(6.02)	0.45 _(5.76)	0.42 _(5.52)	0.37 _(4.89)	0.35 _(4.56)	0.33 _(4.39)	0.36 _(4.89)	0.35 _(4.87)	0.37 _(4.88)	0.34 _(4.67)
log(ME)	0.05 _(1.11)	0.03 _(0.82)	-0.08 _(-1.83)	-0.13 _(-3.16)	0.07 _(1.60)	0.06 _(1.38)	0.07 _(1.64)	0.07 _(1.52)	0.09 _(1.89)	0.07 _(1.58)
$r_{t-2,t-1}$	-2.00 _(-3.42)	-2.08 _(-3.60)	-2.42 _(-4.15)	-2.58 _(-4.46)	-1.83 _(-3.02)	-1.86 _(-3.07)	-1.86 _(-3.10)	-1.89 _(-3.16)	-1.87 _(-3.10)	-1.90 _(-3.14)
$r_{t-12,t-2}$	1.42 _(6.05)	1.38 _(5.95)	0.90 _(3.79)	0.75 _(3.16)	1.46 _(5.96)	1.44 _(5.87)	1.45 _(5.99)	1.45 _(6.00)	1.43 _(5.83)	1.41 _(5.74)

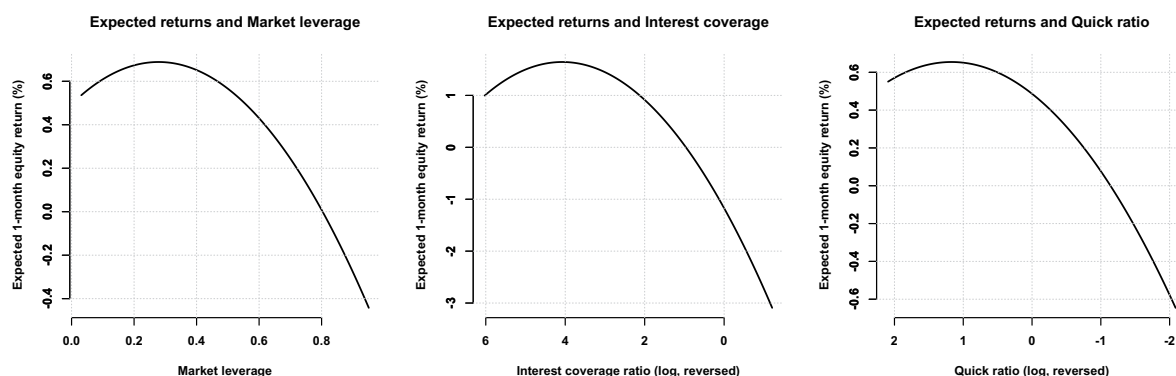


Figure 3.5. Hump-shaped relations between expected returns and the measures of solvency and liquidity. This figure shows the estimated relations from the linear-quadratic regressions of firm-level returns on the solvency and liquidity measures in Table 3.1. *Left panel:* Estimated relation between 1-month expected returns and lagged market leverage from specification (2). *Middle panel:* Estimated relation between 1-month expected returns and lagged (log) interest coverage from specification (4). *Right panel:* Estimated relation between 1-month expected returns and lagged (log) quick ratio from specification (8). In all panels, the control variables from the regression specifications are fixed at their sample means and the range of the horizontal axis is the observed range in the sample.

Specification (5) shows that higher current ratio (i.e. higher liquidity) is associated with significantly higher expected returns, while specification (6) shows that the curvature of the relation between expected returns and current ratio is significantly negative. The same is true for the quick ratio in specifications (7) and (8), and for the working capital ratio in specification (9) and (10).

To summarize, the the linear specifications provide strong support for the model’s predictions that firms with higher solvency (i.e. lower leverage or higher interest coverage) and firms with higher cash (i.e. higher values for the liquidity measures) have higher expected returns. Furthermore, the linear-quadratic specifications show support for a hump-shaped relation between expected returns and the measures of solvency and liquidity. As an illustration of the latter point, Figure 3.5 shows the estimated relations from the linear-quadratic regressions of returns on market leverage in specification (2); returns on interest coverage in specification (4); and returns on quick ratio in specification (8) [the corresponding plots for the current ratio and the working capital ratio are very similar to the one for the quick ratio, and are omitted here for brevity]. The estimated relations are seen to be the empirical counterparts of the model’s implied relations in the lower panels of Figure 3.1.

3.4.3 Portfolio returns and alphas

The tests of the two previous subsections were based on estimated conditional betas and Fama-MacBeth regression coefficients. Since both are potentially sensitive to estimation noise, misspecified parametric forms, and an over-weighting of small firms, I now test [Corollary 4.1](#) and [Corollary 4.2](#) non-parametrically using the realized returns and alphas of value-weighted portfolios.

[Table 3.2](#) shows average excess returns and 1-4 factor alphas (i.e. ‘risk-adjusted’ returns) as well as Sharpe and Information ratios for portfolios formed on the three solvency measures, while [Table 3.3](#) shows the same for portfolios formed on the three liquidity measures.

At the beginning of each calendar month, I sort firms into portfolios according to their solvency or liquidity levels for the previous month. The portfolios are value-weighted, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. Following [Asness, Moskowitz, and Pedersen \(2013\)](#) and [Frazzini and Pedersen \(2014\)](#), I construct the portfolio weights using ranks—in this case ranks of the previous month’s market equity values. Using ranks instead of raw market equity values mitigates the extremely skewed distribution of market equity values in the cross-section (which over-weights large firms) and implies a much higher degree of diversification, since more firms are given a nonzero weight and the weights are less extreme. Specifically, the weight of firm i at month t in portfolio $J(t)$ is given by

$$w_{it}^J = \frac{\text{rank}(ME_{it}) \cdot 1_{(i \in J(t))}}{\sum_{i \in J(t)} \text{rank}(ME_{it})}.$$

The tables report time-series averages of monthly portfolio returns in excess of the 1-month US T-bill rate. To reduce the influence of outliers, I exclude each portfolio’s highest and lowest realized return. The corresponding alphas are the intercepts from monthly time-series regressions of excess returns on the “market” factor (MKT); the size (SMB) and value (HML) factors of [Fama and French \(1993\)](#); and the momentum factor (UMD) of [Carhart \(1997\)](#). Finally, each reported Sharpe (Information) ratio is calculated as the annualized average excess return (annualized 4-factor alpha) divided by the annualized volatility of excess returns (annualized residual standard error from the 4-factor time-series regression).

Panel A of [Table 3.2](#) shows the results for the portfolios formed on credit ratings. Consistent with the model’s prediction that high-solvency firms outperform low-solvency firms, it is seen that investment grade firms (ratings above Baa) have positive and significant excess returns and alphas, while speculative grade and distressed firms have insignificant and even significantly negative excess returns and alphas. The Sharpe and Information ratios are also clearly higher for investment grade firms compared to speculative grade and distressed firms. Furthermore, and consistent with the model’s prediction that expected returns are hump-shaped in solvency, it is seen that excess returns, alphas, and Sharpe/Information ratios slightly increase as ratings deteriorate from Aaa to Aa, and then decreasing monotonically as ratings deteriorate from Aa to C.

Panels B and C of [Table 3.2](#) show the results for the portfolios formed on deciles of market leverage and interest coverage. Once again, high-solvency firms (low leverage deciles or high interest coverage deciles) outperform low-solvency firms. For the portfolios formed on market leverage, the outperformance is strongest in excess returns and Sharpe ratios because the alphas are mostly insignificant, but the magnitudes of the alphas and Information ratios follow the same pattern as the magnitudes of the excess returns and Sharpe ratios. For the portfolios formed on interest coverage, the outperformance is, however, prevalent in excess returns, alphas, and Sharpe/Information ratios. Finally, it is seen that the performance-measures are hump-shaped in leverage and interest coverage—the only exception is the Information ratio for the portfolios formed on interest coverage, which is monotonically decreasing as interest

coverage decreases.

Table 3.3 shows the results for the portfolios formed on the deciles of the three liquidity measures. Consistent with the model's prediction that higher-liquidity firms outperform lower-liquidity firms, it is seen that firms in the highest deciles outperform firms in the lowest deciles for all three liquidity measures. The outperformance is strongest in excess returns and Sharpe ratios because the alphas are mostly insignificant, but the magnitudes of the alphas and Information ratios follow the same pattern as the magnitudes of the excess returns and Sharpe ratios. Furthermore, and consistent with the model's prediction that expected returns are hump-shaped in liquidity, it is seen that excess returns, alphas, and Sharpe/Information ratios increase as liquidity decreases from high to mid-levels, but decrease as liquidity decreases from mid to low-levels.

To summarize, the tests based on portfolio returns and alphas confirm the previous tests based on conditional betas and cross-sectional regressions and show strong support for the predictions of Corollary 4.1 and Corollary 4.2.

A note on unconditional betas

For completeness, Tables 3.2 and 3.3 also report each portfolio's realized CAPM-beta with a *t*-statistic for a null value of 1.

For the portfolios formed on the solvency measures, the unconditional betas are either U-shaped or monotonically increasing as solvency levels decrease. Similarly, for the portfolios formed on the liquidity measures, the unconditional betas are decreasing or slightly U-shaped as liquidity levels decrease. This is in stark contrast to the generally decreasing betas shown in Figure 3.4. Because the conditional betas more precisely reflect firms' capital structure, this discrepancy between conditional and unconditional betas suggests that the unconditional betas are only capturing the part of the exposure to systematic risk that is due to the higher levels of leverage associated with lower solvency and lower liquidity (cf. the middle column of Figure 3.2). Importantly, the unconditional betas seem to ignore other important aspects of firms' capital structure—in particular, the presence of cash held to avoid illiquidity.

3.4.4 Long-short portfolios

In this subsection, I test the model's predictions for the higher-liquidity minus lower-liquidity (HLmLL) portfolio and the higher-solvency minus lower-solvency (HSmLS) portfolio.

Higher-liquidity minus lower-liquidity as a function of solvency

The model predicts that a higher-liquidity minus lower-liquidity (HLmLL) portfolio has negative expected returns for firms with high solvency, but positive expected returns for firms with low solvency (Corollary 4.3). In particular, the expected return on HLmLL increases as firms become less solvent and the positive expected returns on HLmLL are concentrated among firms with low solvency. I now test this predictions using the returns and alphas of HLmLL portfolios in the cross-section of solvency levels.

Table 3.4 shows the performance of HLmLL portfolios across credit ratings. To ensure a sufficient number of firms in each portfolio, I re-code the original 9 credit ratings into the following three groups: Aaa-A, Baa-B, and Caa-C. Similar to the construction of long-short portfolios in e.g. Fama and French (1993), I form the HLmLL portfolios using conditional sorts: First into the three credit rating portfolios, and then into three liquidity portfolios, where the liquidity breakpoints are the 30th and the 70th percentile. I then calculate the return-spread for each credit rating group as the difference between the value-weighted returns of the top 30% "higher-liquidity" firms and the bottom 30% "lower-liquidity" firms. As in Subsection 3.4.3, I use the ranks of the market equity values to construct the portfolio weights, and, to reduce the influence of outliers, I exclude the highest and lowest realized return for each portfolio. The results for HLmLL portfolios formed using market leverage or interest coverage as the measure of solvency produce very similar results and are omitted here for brevity.

Consistent with the model's prediction, the HLmLL portfolios have monotonically increasing average returns, 1-4 factor alphas, and Sharpe/Information ratios as credit ratings deteriorate. For firms rated Aaa-A, the HLmLL portfolios have insignificant average returns between -0.09% and -0.07% on a monthly basis, insignificant alphas in the same range, and low annualized Sharpe/Information ratios between -0.13 and 0.06 . As the credit ratings deteriorate into Baa-B, the returns and alphas increase somewhat but remain insignificant. Finally, for the riskiest firms rated Caa-C, the HLmLL portfolios have large and significantly positive average returns between 1.84% and 2.81% on a monthly basis, significant alphas in the same range, and fairly large annualized Sharpe/Information ratios between 0.58 and 0.86 .

Table 3.4. Higher-liquidity minus lower-liquidity portfolios within credit rating groups. This table shows returns, alphas, and related quantities for higher-liquidity minus lower-liquidity (HLMLL) portfolios within credit rating groups. At the beginning of each calendar month, I assign firms into 3 portfolios according to their credit rating (Aaa-A, Baa-B, and Caa-C) and then into three portfolios according to the 30th and 70th percentile of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C). The portfolios are value-weighted using the ranks of the previous calendar month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns on “higher liquidity” firms minus “lower liquidity” firms. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP “market” index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give *t*-statistics for a null value of zero. Statistical significance at the 5% level is indicated in bold.

Panel A: Current Ratio

	Aaa-A	Baa-B	Caa-C
	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.
Return (avg. mon. %)	-0.07 _(-0.56)	0.15 _(1.20)	2.81 _(5.02)
CAPM alpha (mon. %)	-0.14 _(-1.08)	0.07 _(0.57)	2.81 _(4.99)
Three-factor alpha (mon. %)	-0.05 _(-0.38)	0.19 _(1.79)	2.85 _(5.01)
Four-factor alpha (mon. %)	0.05 _(0.41)	0.23 _(2.13)	2.74 _(4.72)
Sharpe Ratio (ann.)	-0.09	0.18	0.86
Information Ratio (ann.)	0.06	0.34	0.84
CAPM beta (unconditional)	0.15 _(5.47)	0.18 _(6.97)	-0.01 _(-0.07)
Months	504	505	405

Panel B: Quick Ratio

	Aaa-A	Baa-B	Caa-C
	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.
Return (avg. mon. %)	-0.09 _(-0.82)	0.09 _(0.76)	1.84 _(3.39)
CAPM alpha (mon. %)	-0.13 _(-1.28)	0.01 _(0.10)	1.73 _(3.18)
Three-factor alpha (mon. %)	-0.02 _(-0.17)	0.20 _(1.98)	2.07 _(3.86)
Four-factor alpha (mon. %)	-0.01 _(-0.13)	0.18 _(1.71)	2.19 _(4.02)
Sharpe Ratio (ann.)	-0.13	0.12	0.58
Information Ratio (ann.)	-0.02	0.27	0.72
CAPM beta (unconditional)	0.11 _(5.07)	0.19 _(7.54)	0.19 _(1.58)
Months	503	504	404

Panel C: Working Capital Ratio

	Aaa-A	Baa-B	Caa-C
	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.	Higher-liq. minus lower-liq.
Return (avg. mon. %)	-0.07 _(-0.56)	0.15 _(1.20)	2.81 _(5.02)
CAPM alpha (mon. %)	-0.14 _(-1.08)	0.07 _(0.57)	2.81 _(4.99)
Three-factor alpha (mon. %)	-0.05 _(-0.38)	0.19 _(1.79)	2.85 _(5.01)
Four-factor alpha (mon. %)	0.05 _(0.41)	0.23 _(2.13)	2.74 _(4.72)
Sharpe Ratio (ann.)	-0.09	0.18	0.86
Information Ratio (ann.)	0.06	0.34	0.84
CAPM beta (unconditional)	0.15 _(5.47)	0.18 _(6.97)	-0.01 _(-0.07)
Months	504	505	405

High-solvency minus low-solvency as a function of liquidity

Finally, the model predicts that a higher-solvency minus lower-solvency (HSmLS) portfolio has negative expected returns for firms with high liquidity, but positive expected returns for firms with low liquidity (Corollary 4.4). In particular, the expected returns on HSmLS increase as firms' liquidity decreases and the positive expected returns on HSmLS are concentrated among firms with low liquidity. I now test this prediction using the returns and alphas of HSmLS portfolios in the cross-section of liquidity levels.

Table 3.5 shows the performance of HSmLS portfolios implemented using credit ratings across the liquidity measures. The credit ratings are again re-coded into the three (Aaa-A, Baa-B, and Caa-C), and the HSmLS portfolios are again formed using conditional sorts: First into three liquidity portfolios, where the liquidity breakpoints are the 30th and the 70th percentile, and then into the three credit rating portfolios. I then calculate the return-spread for each liquidity group as the difference between the value-weighted returns on the Aaa-A rated "higher-solvency" firms and the Caa-C rated "lower-solvency" firms. I again use the ranks of the market equity values to construct the portfolio weights and, to reduce the influence of outliers, I exclude the highest and lowest realized return for each portfolio. The results for HSmLS portfolios formed using market leverage or interest coverage as the measure of solvency produce very similar results and are omitted here for brevity.

Consistent with the model's prediction, the HSmLS portfolios have generally increasing average returns, 1-4 factor alphas, and Sharpe/Information ratios as the liquidity measures decrease. For firms with the highest liquidity levels, the HSmLS portfolios have high and significant average returns between 1.26% and 1.99% on a monthly basis, even higher alphas, and annualized Sharpe/Information ratios between 0.50 and 0.93. While this performance is relatively strong, it is in fact dwarfed by the performance of the HSmLS portfolio for firms with the lowest liquidity levels, where average returns are between 3.56% and 3.63% on a monthly basis, alphas are even higher, and annualized Sharpe/Information ratios are between 1.09 and 1.62. Furthermore, the increase in returns, alphas, and Sharpe/Information ratios is monotonic for the quick ratio and the working capital ratio.

Table 3.5. Higher-solvency minus lower-solvency across liquidity measures. This table shows returns, alphas, and related quantities for higher-solvency minus lower-solvency (HSmLS) portfolios that are long firms rated Aaa-A and short firms rated Caa-C within liquidity groups. At the beginning of each calendar month, I assign firms into 3 portfolios according to the 30th and 70th percentile of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C), and then into three groups according to credit ratings (Aaa-A, Baa-B, and Caa-C). The portfolios are value-weighted using the ranks of the previous calendar month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns of Aaa-A rated firms and Caa-C rated firms. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP "market" index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give *t*-statistics for a null value of zero. Statistical significance at the 5% level is indicated in bold.

Panel A: Current Ratio

	High liquidity Aaa-A minus Caa-C	Medium liquidity Aaa-A minus Caa-C	Low liquidity Aaa-A minus Caa-C
Return (avg. mon. %)	1.99 _(4.18)	0.96 _(2.16)	3.56 _(6.37)
CAPM alpha (mon. %)	2.31 _(4.98)	1.37 _(3.27)	4.03 _(7.49)
Three-factor alpha (mon. %)	2.46 _(5.68)	1.52 _(3.84)	4.28 _(8.87)
Four-factor alpha (mon. %)	2.23 _(5.09)	1.11 _(2.84)	3.79 _(7.89)
Sharpe Ratio (ann.)	0.74	0.37	1.09
Information Ratio (ann.)	0.93	0.51	1.41
CAPM beta (unconditional)	-0.54 _(-5.31)	-0.69 _(-7.48)	-0.77 _(-6.41)
Months	388	401	413

Panel B: Quick Ratio

	High liquidity Aaa-A minus Caa-C	Medium liquidity Aaa-A minus Caa-C	Low liquidity Aaa-A minus Caa-C
Return (avg. mon. %)	1.69 _(3.65)	1.88 _(3.83)	3.62 _(6.80)
CAPM alpha (mon. %)	2.15 _(4.89)	2.28 _(4.82)	4.04 _(7.85)
Three-factor alpha (mon. %)	2.29 _(5.54)	2.27 _(5.14)	4.38 _(9.83)
Four-factor alpha (mon. %)	2.01 _(4.83)	1.81 _(4.13)	4.07 _(9.09)
Sharpe Ratio (ann.)	0.64	0.68	1.16
Information Ratio (ann.)	0.87	0.77	1.62
CAPM beta (unconditional)	-0.70 _(-7.23)	-0.65 _(-6.36)	-0.70 _(-5.98)
Months	397	379	412

Panel C: Working Capital Ratio

	High liquidity Aaa-A minus Caa-C	Medium liquidity Aaa-A minus Caa-C	Low liquidity Aaa-A minus Caa-C
Return (avg. mon. %)	1.26 _(2.86)	1.49 _(3.22)	3.63 _(6.31)
CAPM alpha (mon. %)	1.57 _(3.68)	2.01 _(4.65)	4.06 _(7.26)
Three-factor alpha (mon. %)	1.75 _(4.40)	2.15 _(5.30)	4.24 _(8.47)
Four-factor alpha (mon. %)	1.59 _(3.93)	1.82 _(4.49)	3.78 _(7.56)
Sharpe Ratio (ann.)	0.50	0.56	1.10
Information Ratio (ann.)	0.71	0.81	1.37
CAPM beta (unconditional)	-0.53 _(-5.58)	-0.81 _(-8.50)	-0.71 _(-5.66)
Months	399	400	398

3.5 Concluding remarks

To conclude, this paper has shown, theoretically and empirically, that firms' cash holdings can help rationalize low returns to distressed equity.

I presented a model in which levered firms with financing constraints can default because of illiquidity or insolvency and seek to choose their cash holdings optimally. I show that an equity-maximizing firm optimally holds a level of cash that allows it to avoid illiquidity. When such a firm is in high risk of insolvency, it has a high fraction of its assets in cash and therefore a low conditional beta, which helps rationalize its low expected returns.

Using data on solvency, liquidity, and equity returns for rated US firms over the period 1970-2013, I found empirical evidence consistent with the model's predictions. First, solvency and liquidity are positively related, but, in all solvency levels, the average firm holds enough liquid assets to cover short-term liabilities. Second, firms with lower solvency have a higher fraction of their total assets in liquid assets. Third, firms with sufficiently high solvency have high expected returns relative to firms with sufficiently low solvency, but the relation between expected returns and solvency is hump-shaped. Fourth, firms with sufficiently high liquidity have high expected returns relative to firms with sufficiently low liquidity, but the relation between expected returns and liquidity is hump-shaped. Fifth, the profitability of long-short liquidity strategies is concentrated among firms with low solvency. And, finally, the profitability of long-short solvency strategies is concentrated among firms with low liquidity.

My results shed new light on the distress puzzle, and, more generally, on the relation between firms' cash holdings and expected equity returns. The theoretical results suggest that separating the solvency and liquidity components of distress risk is central in understanding the returns that investors require for holding distressed equities. The empirical results confirm that firms' cash holdings are closely related to their solvency and, consequently, to their realized equity returns.

Appendices

A Additional details and proofs

This appendix gives additional details of the model's development as well as the proofs of the paper's propositions.

A.1 Change of measure

The firm's cumulated earnings, $(X_t)_{t \geq 0}$ and the economy's stochastic discount factor, $(\Lambda_t)_{t \geq 0}$, are defined by the \mathbb{P} -dynamics in (3.1) and (3.3), i.e.

$$dX_t = \mu^{\mathbb{P}} X_t dt + \sigma X_t dB_t^{\mathbb{P}}$$

and

$$d\Lambda_t = -r\Lambda_t dt - \lambda\Lambda_t dZ_t^{\mathbb{P}}.$$

Here, $(B_t^{\mathbb{P}})_{t \geq 0}$ and $(Z_t^{\mathbb{P}})_{t \geq 0}$ are standard \mathbb{P} -Brownian motions with instantaneous correlation ρ . Therefore, there exists a standard \mathbb{P} -Brownian motion, $(\widetilde{Z}_t^{\mathbb{P}})_{t \geq 0}$, independent of $(Z_t^{\mathbb{P}})_{t \geq 0}$, such that

$$B_t^{\mathbb{P}} = \rho Z_t^{\mathbb{P}} + \sqrt{1 - \rho^2} \widetilde{Z}_t^{\mathbb{P}}.$$

Now, suppose $\Lambda_0 = 1$, let $L_t = e^{rt}\Lambda_t$ for each t , and note that L_t may be written as

$$L_t = \exp\left(-\frac{1}{2}\lambda' \lambda t - \lambda' Z_t^{\mathbb{P}}\right),$$

where $\lambda = (\lambda, 0)'$ and $Z_t^{\mathbb{P}} = (Z_t^{\mathbb{P}}, \widetilde{Z}_t^{\mathbb{P}})'$. Fix T and define a risk-neutral pricing measure \mathbb{Q} by $\frac{d\mathbb{Q}}{d\mathbb{P}} = L_T$. Girsanov's Theorem then gives that the two-dimensional process $(Z_t^{\mathbb{Q}})_{t \geq 0}$ defined by

$$Z_t^{\mathbb{Q}} = (Z_t^{\mathbb{Q}}, \widetilde{Z}_t^{\mathbb{Q}}) = Z_t^{\mathbb{P}} + \lambda t$$

is a standard \mathbb{Q} -Brownian motion.

Finally, let $B_t^{\mathbb{Q}} = \rho Z_t^{\mathbb{Q}} + \sqrt{1 - \rho^2} \widetilde{Z}_t^{\mathbb{Q}}$ for each t , and note that $(B_t^{\mathbb{Q}})_{t \geq 0}$ is standard \mathbb{Q} -Brownian motion which may be written as

$$B_t^{\mathbb{Q}} = \rho(Z_t^{\mathbb{P}} + \lambda t) + \sqrt{1 - \rho^2} \widetilde{Z}_t^{\mathbb{P}} = B_t^{\mathbb{P}} + \rho\lambda t.$$

It thus follows that the \mathbb{Q} -dynamics of the firm's cumulated earnings process may be written as

$$dX_t = \mu^{\mathbb{Q}} X_t dt + \sigma X_t dB_t^{\mathbb{Q}},$$

where $\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \rho\sigma\lambda$, exactly as in (3.4).

A.2 Proof of Proposition 1

Proof of Part (i)

The task is to show that for each X , there exists a minimal level of cash, $\underline{C}(X)$, such that the firm avoids illiquidity if its cash holdings are at or above $\underline{C}(X)$. Suppose first that $\underline{C}(X)$ is twice continuously differentiable in X . It then follows

by Itô's Formula along with the accounting identity in (3.2) and the \mathbb{Q} -dynamics of $(X_t)_{t \geq 0}$ in (3.4) that

$$\begin{aligned} dD_t &= \left[r\underline{C}(X_t) + (\mu^{\mathbb{Q}}X_t - k) - \underline{C}_X(X_t)\mu^{\mathbb{Q}}X_t - \frac{1}{2}\underline{C}_{XX}(X_t)\sigma^2X_t^2 \right] dt \\ &+ \left[\sigma X_t - \underline{C}_X(X_t)\sigma X_t \right] dB_t^{\mathbb{Q}}, \end{aligned} \quad (21)$$

where subscripts denote partial derivatives. Since the firm has no access to external financing, it can, in particular, not issue additional equity. This implies that the cumulated dividend process, $(D_t)_{t \geq 0}$, has to be non-decreasing for all t . This is satisfied if and only if i) the drift-term in (21) is nonnegative and ii) the volatility-term in (21) is zero.

The requirement that the volatility-term in (21) is zero implies the differential equation $\underline{C}_X(X) = 1$, which has the general solution

$$\underline{C}(X) = X + L \quad (22)$$

for some constant L . Plugging this solution into the drift-term in (21), and imposing the requirement that the drift is nonnegative, it follows that the solution in (22) has to satisfy

$$\underline{C}(X) \geq \frac{k}{r} > 0 \quad (23)$$

for all X . Hence, if for a given X the firm's cash holdings are at or above the level $\underline{C}(X)$, cash holdings are strictly positive and the interest on the firm's cash holdings is enough to cover a coupon, implying that the firm avoids illiquidity.

The remaining part of the proof of part (i) is to assert that the cash level $\underline{C}(X)$ must be twice continuously differentiable in X . If X' is a discontinuity point for $\underline{C}(X)$, then $\underline{C}(X)$ must jump downwards at X' , because an upwards jump would mean an injection of external capital. However, even a downwards jump at X' would imply that $\underline{C}(X)$ is not the minimal cash level that allows the firm to avoid illiquidity an instance before X' . Hence, $\underline{C}(X)$ must be continuous in X .

On the other hand, if $\underline{C}(X)$ is continuous but non-differentiable at some X'' , then $\underline{C}(X)$ would still satisfy the differential equation in (21) except at X'' . However, this differential equation and the associated inequalities imply a solution that is twice continuously differentiable in X .

Proof of Part (ii)

If $C = C(X)$, then X fully determines the level of cash, implying that $\underline{X}^*(C) = \underline{X}^*(C(X))$ is only a function of X . If $C > \underline{C}(X)$, then, because cash holdings by assumption can be paid out to equity holders at any time before default, the positive difference $C - C(X)$ can be distributed immediately to equity holders. This would bring C back down to $C(X)$, implying again that the insolvency decision is only a function of X .

Proof of Part (iii)

From the proof of part (i), the cash level $\underline{C}(X)$ takes the general form in (22) subject to the lower bound in (23). To solve for the constant L , note that since $\underline{C}_X(X) > 0$, the target cash level is increasing in X . As \underline{X}^* is a lower bound for X , it follows that $\underline{C}(X) \geq \underline{C}(\underline{X}^*) \geq \frac{k}{r}$ for all X . Hence, the constant L must satisfy

$$L \geq -\underline{X}^* + \frac{k}{r} = \left[-\underline{X}^* + \frac{k}{r} \right].$$

By choosing L as low as possible (i.e. the value where the inequality is binding), the expression for $\underline{C}(X)$ given in Proposition 1 follows. \square

A.3 Proof of Proposition 2

Proof of Part (i)

The task is to show that the function $W(X, C)$ conjectured in (3.10)–(3.11) satisfies

$$W(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\tilde{\tau}} e^{-r(u-t)} dD_u^* + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}} \right],$$

where $(D_t^*)_{t \geq 0}$ is the dividend policy conjectured in (3.8).

For future reference, note that for any twice continuously differentiable function $f(X, C)$, it follows by Itô's Formula that the infinitesimal generator of the two-dimensional process $(X_t, C_t)_{t \geq 0}$ with dynamics in (3.4) and (3.2) is given by

$$\begin{aligned} \mathcal{A}_{(X,C)} f(X, C) &= \mu^{\mathbb{Q}} X f_X(X, C) + \frac{1}{2} \sigma^2 X^2 f_{XX}(X, C) \\ &\quad + \left[rC + (\mu^{\mathbb{Q}} X - k) \right] f_C(X, C) + \frac{1}{2} \sigma^2 X^2 f_{CC}(X, C) \\ &\quad + \sigma^2 X^2 f_{XC}(X, C). \end{aligned}$$

Similarly, for any twice continuously differentiable $g(X)$, the infinitesimal generator of the process $(X_t)_{t \geq 0}$ with dynamics in (3.4) is given by

$$\mathcal{A}_X g(X) = \mu^{\mathbb{Q}} X g_X(X) + \frac{1}{2} \sigma^2 X^2 g_{XX}(X).$$

Define the process $(G_t)_{t \geq 0}$ by

$$G_t = e^{-rt} W(X_t, C_t) + \int_0^t e^{-rs} dD_s^*,$$

where the function $W(X, C)$ satisfies (3.10)–(3.11) and $(D_t^*)_{t \geq 0}$ is given in (3.8). Note that while $(D_t^*)_{t \geq 0}$ is non-continuous, it may for all $t < \tilde{\tau}$ be decomposed into the sum of its purely continuous part and its jumps: $D_t^* = D_t^{*c} + \sum_{s \leq t} (D_s^* - D_{s-}^*)$. By (3.2), the jumps of the cash process are given by $C_t - C_{t-} = -(D_t^* - D_{t-}^*)$ for all $t < \tilde{\tau}$. Using this along with the generalized Itô's Formula and the dynamics of $(X_t, C_t)_{t \geq 0}$ in (3.2) and (3.4), it follows that

$$\begin{aligned} dG_t &= e^{-rt} [-rW(X_t, C_{t-}) + \mathcal{A}_{(X,C)} W(X_t, C_{t-})] dt + e^{-rt} [1 - W_C(X_t, C_{t-})] dD_t^* \\ &\quad + e^{-rt} \sigma X_t [W_X(X_t, C_{t-}) + W_C(X_t, C_{t-})] dB_t^{\mathbb{Q}} \\ &\quad + e^{-rt} [W(X_t, C_t) - W(X_t, C_{t-}) - W_C(X_t, C_{t-})(C_t - C_{t-})]. \end{aligned} \tag{24}$$

Suppose first that $0 < C < \underline{C}(X)$ and $X > \underline{X}^*(C)$. Then $dD_t^* = 0$ by (3.8), which defines a continuous process, and $-rW(X, C) + \mathcal{A}_{(X,C)} W(X, C) = 0$ by (3.10). Hence, (24) becomes

$$d(e^{-rt} W(X_t, C_t)) = e^{-rt} \sigma X_t [W_X(X_t, C_t) + W_C(X_t, C_t)] dB_t^{\mathbb{Q}}.$$

Assuming that the derivatives of $W(X, C)$ are sufficiently integrable, it follows by integrating this expression between

t and $v \wedge \tilde{\tau}$ for any $v \geq t$ and taking expectations that

$$e^{-rt} W(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-r(v \wedge \tilde{\tau})} W(X_{v \wedge \tilde{\tau}}, C_{v \wedge \tilde{\tau}}) \right].$$

Since v was arbitrary, it follows by letting $v \rightarrow \infty$ and applying the first condition in (3.11) that

$$W(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}} \right],$$

which proves part (i) in this case.

Next, suppose that $C = \underline{C}(X)$ and $X \geq \underline{X}^*$. Then $dD_t^* = r(X_t - \underline{X}^*) dt$ by (3.8), which defines a continuous process, and, by (3.10),

$$-rW(X, C) + \mathcal{A}_{(X,C)} W(X, C) = -rW(X, \underline{C}(X)) + \mathcal{A}_X W(X, \underline{C}(X)) = -r(X - \underline{X}^*).$$

Furthermore, Proposition 3 and its proof (Section A.4 of this appendix) give that $W_C(X, \underline{C}(X)) = 1$. Hence, (24) implies for any $v \geq t$ that

$$W(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} dD_u^* + e^{-r((v \wedge \tilde{\tau})-t)} W(X_{v \wedge \tilde{\tau}}, C_{v \wedge \tilde{\tau}}) \right].$$

Letting $v \rightarrow \infty$ and applying the first condition in (3.11) thus proves part (i) for this case.

Finally, suppose that $C > \underline{C}(X)$ and $X \geq \underline{X}^*$. Then $dD_t^* = C_t - \underline{C}(X_t)$ by (3.8), which defines a pure jump-process. However, this case occurs if and only if $C_{t-} = \underline{C}(X_t)$, which implies $W(X_t, C_{t-}) = W(X_t, \underline{C}(X_t))$. Therefore, (3.10) gives

$$-rW(X_t, C_{t-}) + \mathcal{A}_{(X,C)} E(X_t, C_{t-}) = -rW(X_t, \underline{C}(X_t)) + \mathcal{A}_X W(X_t, \underline{C}(X_t)) = -r(X_t - \underline{X}^*).$$

Furthermore, Proposition 3 and its proof give that $W_C(X, \underline{C}(X)) = 1$, which implies both

$$W(X_t, C_t) - W(X_t, C_{t-}) = C_t - \underline{C}(X_t)$$

and

$$W_C(X_t, C_{t-}) = W_C(X_t, \underline{C}(X_t)) = 1.$$

These two results imply that the jump-term in (24) becomes

$$W(X_t, C_t) - W(X_t, C_{t-}) - W_C(X_t, C_{t-})(C_t - C_{t-}) = C_t - \underline{C}(X_t) - 1 \times (C_t - \underline{C}(X_t)) = 0.$$

Using the implied form of (24) and repeating the steps of the proof in the second case thus proves part (i) in this final case.

Proof of Part (ii)

The task is to show that

$$W(X_t, C_t) \geq \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\tilde{\tau}} e^{-r(u-t)} dD_u + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}} \right],$$

where $(D_t)_{t \geq 0}$ is any adapted, non-decreasing dividend process satisfying the relation in (3.2).

Let $(D_t)_{t \geq 0}$ be such a dividend process and define the process $(H_t)_{t \geq 0}$ by

$$H_t = e^{-rt} W(X_t, C_t) + \int_0^t e^{-rs} dD_s,$$

where the function $W(X, C)$ satisfies (3.10)–(3.11). By the generalized Itô's formula,

$$\begin{aligned} dH_t &= e^{-rt} [-rW(X_t, C_{t-}) + \mathcal{A}_{(X,C)}W(X_t, C_{t-})] dt + e^{-rt} [1 - W_C(X_t, C_{t-})] dD_t \\ &\quad + e^{-rt} \sigma X_t [W_X(X_t, C_{t-}) + W_C(X_t, C_{t-})] dB_t^{\mathbb{Q}} \\ &\quad + e^{-rt} [W(X_t, C_t) - W(X_t, C_{t-}) - W_C(X_t, C_{t-})(C_t - C_{t-})], \end{aligned} \quad (25)$$

where $C_t - C_{t-} = -(D_t - D_{t-})$ for all $t < \tilde{\tau}$, using the decomposition $D_t = D_t^c + \sum_{s \leq t} (D_s - D_{s-})$ and (3.2). Hence, rearranging (25), it follows for all $v \geq t$ that

$$\begin{aligned} W(X_t, C_t) &= e^{-r(v \wedge \tilde{\tau} - t)} W(X_{v \wedge \tilde{\tau}}, C_{v \wedge \tilde{\tau}}) - \int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} [-rW(X_u, C_{u-}) + \mathcal{A}_{(X,C)}W(X_u, C_{u-})] du \\ &\quad + \int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} [1 - W_C(X_u, C_{u-})] dD_u^c \\ &\quad + \int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} \sigma X_u [W_X(X_u, C_{u-}) + W_C(X_u, C_{u-})] dB_u^{\mathbb{Q}} \\ &\quad - \sum_{t \leq u \leq v \wedge \tilde{\tau}} e^{-r(u-t)} [W(X_u, C_u) - W(X_u, C_{u-})]. \end{aligned} \quad (26)$$

By (3.10), it holds that $-rW(X, C) + \mathcal{A}_{(X,C)}W(X, C) = 0$ for $0 < C < \underline{C}(X)$ and $X > \underline{X}^*(C)$, while $-rW(X, \underline{C}(X)) + \mathcal{A}_X W(X, \underline{C}(X)) = -r(X - \underline{X}^*) < 0$ for $C \geq \underline{C}(X)$ and $X > \underline{X}^*$. Combined, these relations imply the following lower bound:

$$- \int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} [-rW(X_u, C_{u-}) + \mathcal{A}_{(X,C)}W(X_u, C_{u-})] du \geq 0.$$

Furthermore, by (3.11), it holds that $W_C(X, C) \geq 1$ for $C > 0$ and $X > \underline{X}^*(C)$, while $C_t - C_{t-} = -(D_t - D_{t-}) \leq 0$. Combined, these relations imply the following lower bound:

$$- \sum_{t \leq u \leq v \wedge \tilde{\tau}} e^{-r(u-t)} [W(X_u, C_u) - W(X_u, C_{u-})] \geq \sum_{t \leq u \leq v \wedge \tilde{\tau}} e^{-r(u-t)} (D_u - D_{u-}).$$

Assuming that the derivatives of $W(X, C)$ are sufficiently integrable, it follows by using these lower bounds with (26) and taking expectations that

$$\begin{aligned} W(X_t, C_t) &\geq \mathbb{E}_t^{\mathbb{Q}} \left[e^{-r((v \wedge \tilde{\tau}) - t)} W(X_{v \wedge \tilde{\tau}}, C_{v \wedge \tilde{\tau}}) \right] + \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} dD_u^c \right] + \mathbb{E}_t^{\mathbb{Q}} \left[\sum_{t \leq u \leq v \wedge \tilde{\tau}} e^{-r(u-t)} (D_u - D_{u-}) \right] \\ &= \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{v \wedge \tilde{\tau}} e^{-r(u-t)} dD_u \right] + \mathbb{E}_t^{\mathbb{Q}} \left[e^{-r((v \wedge \tilde{\tau}) - t)} W(X_{v \wedge \tilde{\tau}}, C_{v \wedge \tilde{\tau}}) \right], \end{aligned}$$

where the last equality uses the decomposition $D_t = D_t^c + \sum_{s \leq t} (D_s - D_{s-})$. Letting $v \rightarrow \infty$ and applying the second condition in (3.11) thus proves part (ii), which completes the proof. \square

A.4 Proof of Proposition 3

Proof of Part (i)

The ODE (3.12) has the general solution

$$\begin{aligned} E(X) &= r \left[\frac{X}{r-\mu^Q} - \frac{X^*}{r} \right] + M_1 X^{\phi^+} + M_2 X^{\phi^-} \\ &= \underline{C}(X) + U(X) - \frac{k}{r} + M_1 X^{\phi^+} + M_2 X^{\phi^-}, \end{aligned}$$

where the second equality follows from the form of the target cash level, $\underline{C}(X)$, in (3.7), and the value of productive assets, $U(X)$, in (3.5). Here, M_1 and M_2 are real-valued constants to be determined by boundary conditions, while the exponents $\phi^+ > 1$ and $\phi^- < 0$ are given by

$$\phi^\pm = \frac{\sigma^2 - 2\mu \pm \sqrt{(\sigma^2 - 2\mu)^2 + 8r\sigma^2}}{2\sigma^2}.$$

Since $\phi^+ > 1$, the value matching condition (3.13) implies $M_1 = 0$. Given this, the limited liability condition (3.14) implies

$$M_2 = (\underline{X}^*)^{-\phi^-} \left[\frac{k}{r} - U(\underline{X}^*) \right].$$

By plugging the constants into the general solution, and defining $\pi^Q(X) = (X/\underline{X}^*)^{\phi^-}$, the expression for $E(X)$ in the proposition follows.

Proof of Parts (ii) and (iii)

The expression for \underline{X}^* given in (3.17) easily follows by solving the smoothing pasting condition in (3.15). To proof part (iii), suppose first that $X_t \rightarrow \infty$. Then the expressions for $\underline{C}(X_t)$ in (3.7) and $U(X_t)$ in (3.5) give that

$$\lim_{X_t \rightarrow \infty} \underline{C}(X_t) = \lim_{X_t \rightarrow \infty} U(X_t) = \infty,$$

while

$$\lim_{X_t \rightarrow \infty} \frac{\frac{\partial \underline{C}(X_t)}{\partial X_t}}{\frac{\partial U(X_t)}{\partial X_t}} = \frac{r - \mu^Q}{\mu^Q} < 1,$$

since $r > \mu^Q > \frac{1}{2}r$ by assumption. Therefore, by l'Hôpital's rule,

$$\lim_{X_t \rightarrow \infty} \frac{\underline{C}(X_t)}{U(X_t)} < 1.$$

Conversely, suppose that $X_t \rightarrow \underline{X}^*$. Then

$$\lim_{X_t \rightarrow \underline{X}^*} \frac{\underline{C}(X_t)}{U(X_t)} = \frac{\frac{k}{r}}{\frac{\mu^Q \underline{X}^*}{r - \mu^Q}} = \frac{\phi^- - 1}{\phi^-} > 1,$$

by the expression for \underline{X}^* in (3.17). Combining these two cases, it follows that there exists $X' > \underline{X}^*$ such that $U(X) > \underline{C}(X)$ for all $X > X'$, while $U(X) < \underline{C}(X)$ for all $X < X'$. This completes the proof. \square

A.5 Proof of Proposition 4

The proofs of parts (i) and (ii) will make use of the following results about the earnings-sensitivity, $\Omega^E(X_t)$. First, using the expression for $\underline{C}(X)$ in (3.7), the sensitivity can be compactly written as

$$\Omega^E(X_t) = 1 + \frac{\underline{X}^* - (1 - \phi^-) M \pi^Q(X_t)}{E(X_t)}, \quad (27)$$

where $M = \left[\frac{k}{r} - U(\underline{X}^*) \right]$. Second, by differentiating the form of $\Omega^E(X_t)$ in (27) with respect to X_t , it follows that

$$\begin{aligned} \frac{\partial \Omega^E(X_t)}{\partial X_t} &= - \frac{(1 - \phi^-) M \frac{\phi^- \pi^Q(X_t)}{X_t}}{E(X_t)} - \frac{E_X(X_t) \left[\underline{X}^* - (1 - \phi^-) M \pi^Q(X_t) \right]}{E^2(X_t)} \\ &= - \frac{(1 - \phi^-) M \frac{\phi^- \pi^Q(X_t)}{X_t}}{E(X_t)} - \frac{\Omega^E(X_t) (\Omega^E(X_t) - 1)}{X_t}, \end{aligned} \quad (28)$$

where the last equality uses the definition $\Omega^E(X_t) = X_t \frac{E_X(X_t)}{E(X_t)}$ as well as (27).

Proof of Part (i)

Suppose first that $X_t \rightarrow \infty$. Since $\phi^- < 0$, Proposition 3 gives that $\pi^Q(X)$ approaches zero faster than $E(X)$ approaches infinity. Combined with the form of $\Omega^E(X_t)$ in (27), it thus follows that

$$\lim_{X_t \rightarrow \infty} \Omega^E(X_t) > 1. \quad (29)$$

Conversely, suppose that $X_t \rightarrow \underline{X}^*$. By the limited liability condition (3.14) and the expression for $\underline{C}(X)$ in (3.7), it follows that

$$\begin{aligned} \lim_{X_t \rightarrow \underline{X}^*} \Omega^E(X_t) &= 1 + \frac{\underline{X}^* - (1 - \phi^-) M}{\underline{C}(\underline{X}^*)} \\ &= \frac{r - \mu^Q}{\mu^Q} \frac{\phi^-}{\phi^- - 1} < 1, \end{aligned} \quad (30)$$

because the assumption $\mu^Q > \frac{1}{2}r$ implies $\frac{r - \mu^Q}{\mu^Q} < 1$ and $\phi^- < 0$ implies $\frac{\phi^-}{\phi^- - 1} \in (0, 1)$. Note that $\mu^Q > \frac{1}{2}r$ is a sufficient but not necessary condition for (30) to hold. Combining these two cases, it follows that there exists $X'' > \underline{X}^*$ such that $\Omega^E(X_t) > 1$ for all $X_t > X''$, while $\Omega^E(X_t) < 1$ for all $X_t < X''$

Proof of Part (ii)

Consider the expression in (28) and suppose first that $X_t \rightarrow \infty$. Because $E(X_t) \rightarrow \infty$, it follows that the first term vanishes. By (29), the numerator of the second term is asymptotically positive. The second term is therefore as a whole asymptotically negative, implying in total that

$$\lim_{X_t \rightarrow \infty} \frac{d\Omega^E(X_t)}{dX_t} < 0.$$

Conversely, suppose that $X_t \rightarrow \underline{X}^*$. Because $E(X_t) \rightarrow \underline{C}(\underline{X}^*)$ by the limited liability condition in (3.14), and since $\phi^- < 0$, the first term as a whole is asymptotically positive. By (30), the numerator of the second term will be

asymptotically negative. The second term is therefore as a whole asymptotically positive, implying in total that

$$\lim_{X_t \rightarrow \underline{X}^*} \frac{d\Omega^E(X_t)}{dX_t} > 0.$$

Combining these two cases, it follows that there exists $X''' > \underline{X}^*$ such that $\frac{\partial \Omega^E}{\partial X_t} < 0$ for all $X_t > X'''$, while $\frac{\partial \Omega^E}{\partial X_t} > 0$ for all $X_t < X'''$. This completes the proof. \square

Bibliography

- Acharya, V. V., S. A. Davydenko, and A. Strebulaev (2012). Cash Holdings and Credit Risk. *Review of Financial Studies* 25(12), 3572–3609.
- Adrian, T. and F. Franzoni (2009). Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. *Journal of Empirical Finance* 16(4), 537–556.
- Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and Momentum Everywhere. *Journal of Finance* 68(3), 929–986.
- Avramov, D., T. Chordia, G. Jostova, and A. Philipov (2009). Credit ratings and the cross-section of stock returns. *Journal of Financial Markets* 12(3), 469–499.
- Bali, T. G., R. F. Engle, and Y. Tang (2014). Dynamic Conditional Beta is Alive and Well in the Cross-Section of Daily Stock Returns. *Working Paper*. Available at <http://ssrn.com/abstract=2089636>.
- Bates, T. W., K. M. Kahle, and R. M. Stultz (2009). Why do u.s. firms hold so much more cash than they used to? *The Journal of Finance* 54(5), 1985–2021.
- Black, F. and J. Cox (1976, May). Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance* 31(2), 351–367.
- Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–654.
- Bolton, P., H. Chen, and N. Wang (2011, October). A Unified Theory of Tobin’s q, Corporate Investment, Financing, and Risk Management. *Journal of Finance* 66(5), 1545.
- Bolton, P., H. Chen, and N. Wang (2013). Market timing, investment, and risk management. *Journal of Financial Economics* 109, 40–62.
- Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In Search of Distress Risk. *The Journal of Finance* 63(6), 2899–2939.
- Carhart, M. M. (1997, March). On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1), 57–82.
- Chan, K. C. and N.-F. Chen (1991). Structural and Return Characteristics of Small and Large Firms. *Journal of Finance* 46(4).
- Davydenko, S. A. (2013, February). Insolvency, illiquidity, and the risk of default. *Working Paper, Joseph L. Rotman School of Management, University of Toronto*.
- Décamps, J. P., T. Mariotti, J. C. Rochet, and S. Villeneuve (2011, October). Free Cash Flow, Issuance Costs, and Stock Prices. *Journal of Finance* 66(5), 1501–1544.

- Demarzo, P. and Y. Sannikov (2006). Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance* 61(6), 2681–2724.
- Dichev, I. A. (1998). Is the risk of bankruptcy a systematic risk? *Journal of Finance* 53(3), 1131–1147.
- Dimson, E. (1979, June). Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics* 7(2), 197–226.
- Duffie, D. and D. Lando (2001). Term structures of credit spreads with incomplete accounting information. *Econometrica* 58, 635–665.
- Fama, E. F. and K. R. French (1993). Common risk factors in the returns of stocks and bonds. *Journal of Financial Economics* 33, 3–56.
- Fama, E. F. and K. R. French (1995). Size and Book-to-Market Factors in Earnings and Returns. *Journal of Finance* 50(1), 131–155.
- Fama, E. F. and K. R. French (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance* 51(1), 55–84.
- Fama, E. F. and J. D. MacBeth (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81(3), 607–636.
- Fan, H. and S. Sundaresan (2000). Debt Valuation, Renegotiation, and Optimal Dividend Policy. *Review of Financial Studies* 13(4), 1057–1099.
- Frazzini, A. and L. H. Pedersen (2014). Beta against beta. *Journal of Financial Economics* 111, 1–25.
- Garlappi, L., T. Shu, and H. Yan (2008). Default Risk, Shareholder Advantage, and Stock Returns. *Review of Financial Studies* 21(6), 2743–2778.
- Garlappi, L. and H. Yan (2011). Financial Distress and the Cross-section of Equity Returns. *Journal of Finance* 66(3), 789–822.
- Goldstein, R., N. Ju, and H. Leland (2001). An EBIT-based Model of Dynamic Capital Structure. *Journal of Business* 74(4), 483–512.
- Griffen, J. M. and M. L. Lemmon (2002). Book-to-market equity, distress risk and stock returns. *The Journal of Finance* 57(5), 2317–2336.
- Gryglewicz, S. (2011). A theory of corporate financial decisions with liquidity and solvency concerns. *Journal of Financial Economics* 99, 365–384.
- Jagannathan, R. and W. Wang (1996). The conditional CAPM and the cross-section of stock returns. *Journal of Finance* 51, 3–53.
- Jeanblanc-Picqué, M. and A. N. Shiryaev (1995). Optimization of the flow of dividends. *Russian Mathematical Surveys* 50, 257–277.
- Leland, H. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* 49(4), 1213–1252.
- Leland, H. and D. Pyle (1977, May). Information Asymmetries, Financial Structure, and Financial Intermediation. *Journal of Finance* 32(2), 371–387.
- Lewellen, J. and S. Nagel (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics* 82(2), 289–314.

- McQuade, T. J. (2013). Stochastic volatility and asset pricing puzzles. *Working Paper, Stanford GSB*.
- Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29, 449–470.
- Moody's Investors Service (2014, April). *Rating Symbols and Definitions*. Moody's Investors Service.
- Myers, S. C. (1977). Determinants of Corporate Borrowing. *Journal of Financial Economics* 5, 145–175.
- Myers, S. C. and N. S. Majluf (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics* 13(2), 187–221.
- Newey, W. K. and K. D. West (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55, 703–708.
- Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics* 108, 1–28.
- Opler, T., L. Pinkowitz, R. Stulz, and R. Williamson (1999). The determinants and implications of corporate cash holdings. *Journal of Financial Economics* 52, 3–46.
- Opp, C. C. (2013). Learning, Active Investors, and the Returns of Financially Distressed Firms. *Working paper, University of Pennsylvania - The Wharton School*. Available at SSRN: <http://ssrn.com/abstract=2181441>.
- Radner, R. and L. Shepp (1996). Risk vs. profit potential: A model for corporate strategy. *Journal of Economic Dynamics and Control* 20, 1373–1393.
- Vasicek, O. A. (1973, December). A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas. *Journal of Financial Economics* 28(5), 1233–1239.
- Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *Journal of Finance* 59, 831–868.

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