Essays on Correlation Modelling

Nielsen, Mads Stenbo

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Essays on Correlation Modelling

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Mads Stenbo Nielsen

The PhD School of Economics and Management

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Mads Stenbo Nielsen

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"The Doctoral School of Economics and Management is an active national and international research environment at CBS for research degree students who deal with economics and management at business, industry and country level in a theoretical and empirical manner".

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Preface

This thesis is the result of my Ph.D. studies at the Department of Finance, Copenhagen Business School. The thesis consists of three essays that cover different aspects of correlation modelling in corporate default risk. Each essay is self-contained and can be read independently.

Structure of the thesis

The common theme across all three essays is the role of correlation in corporate default risk. While the likelihood for a given firm to default depends on a number of firm-specific characteristics such as earnings, debt outstanding, cash holdings, total assets, stock returns etc., there are also cross-sectional comovements in default probabilities that cannot be explained by idiosyncratic factors. E.g. the general state of the economy, sector-wide up- or downswings, and the financial soundness of competitors and business partners may all contribute to a clustering of default risk over time.

Accounting for such correlation is important for both pricing and risk management of portfolios of defaultable assets, and the thesis addresses different ways to capture correlation both in actual default probabilities as well as in prices of defaultable assets and credit derivatives. The common goal of the thesis is to formulate and estimate quantitative models of default risk with specific attention to the importance of default correlation, and use that to gain further understanding of the nature of correlation in default risk.

In the first essay (co-authored with David Lando, Copenhagen Business School), we investigate statistical techniques for testing the adequacy of "conditional independence"–based intensity models of default. Previous literature has used a time-change technique to analyze these models and jointly test the intensity specification *and* the "conditional independence" assumption. Using 24 years of data of both defaulting and non-defaulting

U.S. industrial firms, we show that the time-change technique is, however, mainly a test of the intensity specification. We further demonstrate by a simple example how a violation of the conditional independence assumption may not be captured by the time-change test, and we give the intuition behind this result. We conclude by proposing alternative tests that explicitly account for the impact of previous defaults on the default intensity, but we find little evidence of this type of correlation in our empirical sample.

The second essay (co-authored with Peter Feldhütter, London Business School) addresses the pricing of correlation in CDO tranche spreads, which are essentially call option spreads on default correlation among a portfolio of defaultable entities. We provide an intensity-based model that allows us to split the default risk into a systematic (correlation) and an idiosyncratic component, and we show how to estimate the model without imposing the restrictive parameter constraints appearing in previous literature. We find that the systematic default component is an explosive process with low volatility, whereas the idiosyncratic default risk is more volatile but less explosive. We further find that the model is able to capture both the level and time series dynamics of CDO tranche spreads.

The third and final essay concerns time series variation in corporate bond spreads induced by variation in the state of the economy. The essay documents how the level and slope of empirical credit spread curves vary with the business cycle, and it develops a structural credit risk model with jump risk that allows for explicit dependence on the state of the economy. The model unifies several existing models that focus entirely on *either* jump *or* business cycle risk. Subsequent estimation of the model reveals the importance of accounting for both jump and business cycle risk in order to capture the time-variation in empirical credit spread curves. In addition, the model gives predictions for net benefits to debt and optimal capital structure that are in line with existing literature.

English and Danish summary of each essay is provided below.

Publication details

The first essay is an extended version of a paper published in the *Journal of Financial Intermediation*, volume 19, page 355–372, and the second essay has been accepted for publication in the *Journal of Financial Econometrics*.

Acknowledgements

The essays in this thesis have benefitted greatly from comments and suggestions from a number of people, and they are mentioned with each of the essays. However, a few people deserve a special mention.

First of all, I am deeply indebted to my advisor, professor David Lando, for his constant encouragement, commitment to, and belief in my work – even at times when little progress was made. His guidance has been an invaluable source of inspiration and his critical comments have affected much of the work presented in this thesis. I similarly wish to thank assistant professor Peter Feldhütter for sharing his previous work on CDO pricing with me, for his excellent co-authorship on the second essay, and not least for suggesting me to apply for a Ph.D. scholarship in the first place. Furthermore, I thank current and previous colleagues and fellow Ph.D. students at the Department of Finance, CBS, for many rewarding discussions as well as for many hours of great fun. In particular, I thank Claus Bajlum, Jens Dick-Nielsen, Peter Feldhütter, René Kallestrup, and Morten Nalholm for always taking the time to listen to my ideas and providing me with useful feedback, and a special thanks to Jens Dick-Nielsen for our many stimulating five-minute-seminars.

Moreover, I am indebted to Derek Moore for always supplying instant and accurate IT assistance, and to Peter Raahauge for patiently introducing me to the world of Matlab. In addition, I wish to thank professor Kristian Miltersen and associate professor Christian Riis Flor for participating in my pre-defense and for providing me with a range of critical and useful comments.

Finally, I wish to thank my family and friends for always being there for me and for their unconditional support and belief in me, without which this thesis would not have been completed.

Mads Stenbo Nielsen

Copenhagen, August 2011

Summary

This section contains English and Danish summaries of the three essays that comprise the thesis.

English summary

Essay I: Correlation in corporate defaults: Contagion or conditional independence? (co-authored with David Lando, CBS)

The first essay studies statistical procedures for testing the validity of intensity-based models of *actual* defaults. Such models are often applied under an additional assumption of conditional independence, whereby the default event is assumed to be conditionally independent of the factors appearing in the specification of the default intensity. Das, Duffie, Kapadia, and Saita (2007) (DDKS) propose a statistical procedure to *jointly* test the specification of the default intensity *and* the conditional independence assumption through time-changing observed defaults into independent Poisson-distributed variables. In an empirical application to U.S. default data, DDKS strongly reject the validity of the joint hypothesis of well-specified intensities and conditional independence.

This leaves open the question of whether their rejection is due to incorrectly specified intensities or a violation of the conditional independence assumption. Using an extensive data set covering 24 years and a total of 2,557 U.S. industrial firms, we show that the rejection is likely to be caused by misspecified default intensities. We first confirm the results obtained by DDKS using their intensity specification and subsequently show that by employing an extended specification, we can no longer reject the joint hypothesis of well-specified intensities and the conditional independence assumption. To strengthen our result, we add further Poisson test statistics to those appearing in DDKS, but this does not change our conclusion.

We subsequently nuance our result by showing that the time-change procedure is, in fact, unable to capture certain violations of the conditional independence assumption. We set up a simple example to demonstrate how default contagion that spreads through the variables in the default intensity specification will *not* be captured by the time-change approach. We therefore need additional test procedures to account for the presence of contagion in corporate default data, and we propose to use both regression analysis as well as a Hawkes specification of the intensity. In the latter approach, previous defaults are allowed to directly impact the likelihood of default for firms that are still alive. We apply both types of tests to our empirical data and find only limited evidence of default contagion.

Essay II: Systematic and idiosyncratic default risk in synthetic credit markets (co-authored with Peter Feldhütter, LBS)

The second essay develops a flexible intensity-based model for pricing correlation-dependent credit derivatives. The model features both idiosyncratic and systematic default risk and ensures consistent pricing of single- and multi-name credit derivatives. The key idea behind the model is to infer term structures of risk-neutral default probabilities from single-name Credit Default Swaps (CDSs), and use that to estimate the systematic default component of each firm's default probability from tranche spreads of Collateralized Debt Obligations (CDOs).

The default intensity of each individual firm is assumed to be a sum of an idiosyncratic component and a suitable scaling of a systematic component. We show by a straightforward argument how the scaling of the systematic component may be inferred from a simple linear regression, and we demonstrate in our empirical application that this choice of weighting is consistent with the intuitive scaling of systematic default risk applied in previous literature.

Central to our approach is the fact that we can leave the idiosyncratic default component unmodelled, and thereby avoid the restrictive parameter contraints imposed in existing literature. Thus, we are able to specify a highly flexible model, while retaining a tractable estimation procedure, where only relatively few parameters have to be estimated.

Furthermore, since our model only relies on liquid, synthetic credit derivatives (CDSs and CDOs), we are able to base our estimation of the model on a large data set of daily data. In our implementation we use 90,600 credit spreads covering a total of 120 days.

When estimating the model we find that systematic default risk is explosive and has low volatility, whereas idiosyncratic risk on the other hand is less explosive but has larger volatility. Finally, we find that the model is able to capture both the level and time series dynamics of the CDO tranche spreads in our sample.

Essay III: Credit spreads across the business cycle

The third essay takes a completely different approach to the modelling of default risk correlation than the first two. Instead of using default intensities, the third essay relies on a structural approach in order to describe business cycle variation in corporate credit spreads.

I first demonstrate how the level and slope of empirical credit spreads are negatively, respectively positively correlated with consumption growth, and I show that these patterns are persistent across both investment and speculative grade issuers. In particular, I document that the credit spread curve is generally upward-sloping in times of high economic growth, but becomes flat or even inverted as the economy approaches a trough.

I further show that the variation in the slope of the credit spread curve may result from shifts in the relative distribution between short- and long-term risk. As economic growth declines, not only does the level of default risk increase, but also the relative importance of short-term default risk increases. As a proxy for short-term risk, I consider jumps in equity returns, and I find empirically that both positive and negative jumps covary with the business cycle, with larger jumps in times of low economic growth. I develop a new technique in order to estimate the jumps, and I show that the detected jumps are consistent with the common interpretation of jumps as representing the arrival of new information to the market.

To capture the observed business cycle correlation with both level and slope of corporate credit spread curves, I formulate a structural credit risk model that takes both jump and business cycle risk into account. This model is the first to consider both risk factors in a joint framework, and it unifies several existing models that focus entirely on just one of these two factors.

I estimate the model on a firm-by-firm basis using daily data from 1962 to 2006, and the estimation shows that the model is able to replicate the observed variation in both level and slope of corporate credit spreads. In particular, the model-implied credit spread curves are upward-sloping when economic growth is high, and flat or downward-sloping when economic growth is low. The ability of the model to generate such curves hinges crucially on the *interplay* between jump and business cycle risk, with jump risk increasing during economic downturns. Moreover, the estimated model yields predictions for net benefits to debt and optimal capital structure that are in line with results in the existing literature.

Dansk resumé

Essay I: Korrelation blandt virksomheders fallithændelser: Smitte-effekter eller betinget uafhængighed? (medforfatter David Lando, CBS)

Det første essay undersøger statistiske metoder til at teste brugbarheden af intensitetsbaserede modeller for *observerede* fallithændelser. Sådanne modeller anvendes ofte i sammenhæng med en yderligere antagelse om betinget uafhængighed, hvorved fallithændelsen antages at være betinget uafhængig af de faktorer, der indgår i specifikationen af fallitintensiteten. Das, Duffie, Kapadia, and Saita (2007) (DDKS) foreslår en statistisk metode til på samme tid at teste *både* specifikationen af fallitintensiteten *og* antagelsen om betinget uafhængighed ved at tidstransformere observerede fallithændelser til uafhængige Poisson-fordelte variable. I et empirisk studie af amerikanske fallitdata forkaster DDKS entydigt gyldigheden af den dobbelte hypotese om korrekt specificerede intensiteter og betinget uafhængighed.

Det rejser spørgsmålet om, hvorvidt deres resultat skyldes forkert specificerede intensiteter eller et fravær af betinget uafhængighed. På baggrund af data for i alt 2.557 amerikanske industrivirksomheder over en 24-årig periode viser vi, at resultatet formentlig skyldes fejlagtigt specificerede intensiteter. Vi replikerer først DDKS' resultat ved at bruge deres foreslåede intensitetsspecifikation, og vi viser derefter at ved at bruge en udvidet specifikation, er det ikke længere muligt at forkaste den dobbelte hypotese om korrekt specificerede intensiteter og betinget uafhængighed. For at underbygge vores resultat tilføjer vi yderligere Poisson-teststørrelser til dem, der allerede optræder i DDKS, men det ændrer ikke på vores konklusion.

Vi nuancerer herefter vores konklusion ved at vise, at tidstransformationsmetoden ikke er i stand til at opfange bestemte overtrædelser af antagelsen om betinget uafhængighed. Vi opstiller et simpelt eksempel, der viser hvorledes smitte-effekter, der optræder via variablene i specifikationen af fallitintensiteten, *ikke* fanges af tidstransformationsmetoden. Det er derfor nødvendigt med yderligere tests for at kunne opdage smitte-effekter blandt virksomheders fallithændelser. Vi foreslår i den forbindelse at benytte såvel regressionsmetoder som en Hawkes-specifikation af fallitintensiteten. Sidstnævnte tillader at forudgående fallithændelser kan have en direkte effekt på sandsynligheden for fallit blandt de tilbageværende virksomheder. Vi anvender begge typer af testprocedurer på vores empiriske data, og finder kun begrænsede tegn på eksistens af smitte-effekter.

Essay II: Systematisk og idiosynkratisk fallitrisiko i "syntetiske" kreditrisiko-instrumenter (medforfatter Peter Feldhütter, LBS)

Det andet essay opstiller en fleksibel intensitetsbaseret model til prisfastsættelse af korrelationsafhængige kreditrisiko-instrumenter. Modellen omfatter både idiosynkratisk og systematisk fallitrisiko og sikrer en konsistent prisfastsættelse af afledte instrumenter, der involverer både én enkelt såvel som en hel gruppe af virksomheder. Den grundlæggende idé bag modellen er at udlede kurver af risiko-neutrale fallitsandsynligheder på baggrund af handlede Credit Default Swaps (CDS'er), og bruge disse til at estimere den systematiske del af hver enkelt virksomheds fallitsandsynlighed ved hjælp af Collateralized Debt Obligation (CDO) tranche-spænd.

Hver enkelt virksomheds fallitintensitet antages at være en sum af en idiosynkratisk faktor og en passende skalering af en systematisk faktor. Vi viser med et simpelt argument, hvorledes skaleringen af den systematiske faktor kan udledes fra en almindelig lineær regression, og viser siden hen i den empiriske del af papiret, hvorledes resultatet af denne skalering stemmer overens med den ad hoc skalering af systematisk fallitrisiko, som tidligere studier har anvendt.

Et vigtigt element i vores metode er, at vi ikke behøver modellere den idiosynkratiske del af fallitrisikoen, og at vi derved undgår de strenge parameterrestriktioner, der optræder i den eksisterende litteratur. Vi er således i stand til at specificere en yderst fleksibel model, der samtidig er overkommelig at estimere, idet det samlede antal parametre, der skal estimeres, er forholdsvis begrænset. Eftersom modellen alene bygger på likvide, "syntetiske" kreditrisiko-instrumenter (CDS'er og CDO'er), er det muligt at basere estimation af modellen på en stor mængde af daglige data. I vores implementering af modellen bruger vi 90.600 kreditspænd fordelt over en periode på i alt 120 dage.

Vores estimation af modellen viser, at den systematiske kreditrisiko er "eksplosiv" omend med lav volatilitet, mens den idiosynkratiske risiko er mindre eksplosiv men mere volatil. Endelig viser estimationen, at modellen er i stand til at fange både niveauet og tidsserievariationen i de empiriske CDO tranche-spænd.

Essay III: Konjunkturvariation i virksomheders kreditspænd

Det tredje essay benytter en helt anden tilgang til modellering af korrelation i virksomheders kreditrisiko end de to første essays. I stedet for at basere sig på fallitintensiteter anvendes i stedet en strukturel model til at beskrive konjunkturvariationen i virksomheders kreditspænd.

Indledningsvis viser jeg, hvorledes niveauet og hældningen på empiriske kreditspændskurver er henholdvis negativt og positivt korreleret med væksten i privatforbruget. Mere specifikt så viser jeg, at kreditspændskurver generelt har positiv hældning i perioder med høj økonomisk vækst, mens de flader ud og i visse tilfælde ligefrem inverterer i perioder med lav vækst.

Jeg demonstrerer dernæst, at konjunkturvariationen i kurvernes hældning kan knyttes til ændringer i den relative fordeling mellem kort- og langsigtet kreditrisiko. I takt med at den økonomiske vækst aftager, stiger både det absolutte niveau af kreditrisiko såvel som den relative betydning af kortsigtet kreditrisiko. Som mål for kortsigtet kreditrisiko anvender jeg spring i realiserede aktieafkast, og jeg bruger det til at dokumentere konjunkturfølsomhed i størrelsen af både positive og negative aktiespring, hvor springene generelt er større i perioder med lav økonomisk vækst. Til brug for estimation af aktiespringene udvikler jeg en ny metode, som jeg påviser er i overensstemmelse med den traditionelle fortolkning af aktiespring som udtryk for tilgang af ny information til

aktiemarkedet.

Til beskrivelse af den dokumenterede korrelation mellem samfundsøkonomiske konjunkturer og henholdsvis niveau og hældning på virksomheders kreditspændskurver opstiller jeg herefter en strukturel kreditrisikomodel, der tager højde for både spring- og konjunkturrisici. Dette er den første strukturelle model, som tager hensyn til begge risikofaktorer på samme tid. Som specialtilfælde indeholder den adskillige eksisterende modeller, der alene fokuserer på den ene af de to faktorer.

Jeg estimerer modellen for en række virksomheder på baggrund af daglige data for perioden fra 1962 til 2006, og estimationen viser at modellen er i stand til at replikere den observerede variation i både niveau og hældning på virksomheders kreditspændskurver. Specielt så har modellens kurver positiv hældning, når den økonomiske vækst er høj, mens kurverne er flade eller har negativ hældning, når væksten er lav. Modellens evne til at generere disse kurver er tæt knyttet til *samspillet* mellem spring- og konjunkturrisici, der medfører en forøget springrisiko i perioder med lav økonomisk vækst. Den estimerede model giver desuden anledning til forudsigelser vedrørende nettofordelen ved udstedelse af gæld samt valget af optimal kapitalstruktur. For begge dele gælder, at disse forudsigelser er i overensstemmelse med resultater i den eksisterende litteratur.

Introduction

When a firm borrows money to finance its activities, it pays an interest which, among other things, is influenced by the firm's ability to service its loan. When the lender, say, a bank, has to determine the appropriate interest rate to charge, it therefore has to assess the likelihood that the firm will default on its obligation. If the bank has also granted loans to other firms, it likewise has to assess the likelihood of default for each of these firms. Hence, it is necessary for the bank to have models that it can use to estimate the probability of default for each of its borrowers.

There is strong empirical evidence that defaults cluster over time, simply because in times of low economic growth more firms struggle to repay their existing loans and/or experience increasing difficulties in obtaining new loans. As a result, the bank is likely to suffer excessive losses in such periods, and it is therefore not enough just to estimate the probability of default for each individual borrower. It is equally important to also take into account the *correlation* between defaults in order to capture the clustering of defaults (and hence losses) over time.

If we consider a specific borrower and let τ denote his (stochastic) default time, then the object of interest is the probability distribution of τ. Default risk models are traditionally classified as either *intensity* models or *structural* models, depending on the way they model the distribution of τ. For intensity models, the distribution of τ is described in terms of its default *intensity*

$$\lim_{dt \to 0} \frac{P(t < \tau \leq t + dt \,|\, \tau > t)}{dt} = \lambda_t$$

that determines the probability of instant default at any time t. Intensity models make no a priori assumptions about the behaviour of λ_t and thus provide a highly flexible framework, which is used both for default probability modelling as well as for pricing of credit risky securities. The definition of the default intensity implies that the distribution of τ has the equivalent representation

$$P(\tau > t) = \exp\left(-\int_0^t \lambda_s ds\right)$$

which shows that the mathematical structure of intensity models is closely related to models of default-free interest rates. This has the obvious advantage that many of the techniques used to model fixed income instruments can also be used to model default risk as pointed out e.g. in Lando (1998) and Duffie and Singleton (1999).

Intensity models find their strength in the flexible specification of the default intensity, whereas structural models take a completely different approach. Here, the idea is to set up specific economic *structures* based on underlying factors that are believed to be the drivers of default risk. Hence, structural models are significantly more restrictive in terms of their modelling flexibility, but offer instead important insights into the economic mechanisms behind the distribution of the default time τ. The first papers along these lines were the seminal works of Black and Scholes (1973) and Merton (1973; 1974) for which the latter two authors were awarded the Alfred Nobel Prize in Economic Sciences in 1997.

This thesis contains new results related to both of the classical fields of default risk models. Essay I and II contain empirical applications of intensity-based models to estimation of *actual* default probabilities and pricing of credit derivatives, respectively, and Essay III develops a new structural credit risk model. The common theme across all three essays is the role of correlation between default times for a pool of borrowers, and how to model and estimate this correlation from observed defaults and from prices of traded securities.

The first essay, Essay I (co-authored with David Lando, CBS), studies various specifications of intensity-based models and discuss their ability to match the probability of default in a large sample of U.S. industrial firms. The paper builds on earlier work by Duffie, Saita, and Wang (2007) and Das, Duffie, Kapadia, and Saita (2007) on estimation and test of intensity models under an assumption of conditional independence between default events and default intensities. In this setting default correlation only enters through cross-correlation among the firm-specific and macroeconomic variables appearing in the specification of the default intensities. Das, Duffie, Kapadia, and Saita (2007) suggest a statistical procedure for testing this particular class of models, and in an empirical application the authors find that their proposed test rejects their conditional independence intensity specification.

The first contribution of Essay I is to show that a more careful specification of the default intensities, still working under the conditional independence assumption, changes the conclusion of Das, Duffie, Kapadia, and Saita (2007). Hence, it is no longer possible to reject the validity of intensity models specified using conditional independence. The second contribution of Essay I is then to demonstrate that the proposed test procedure is, in fact, insufficient to test the conditional independence assumption, since the assumption may be violated without the test procedure being able to detect this.

The third and final contribution of Essay I is to propose and apply alternative tests using regression analysis and Hawkes processes (Hawkes 1971a;b). The latter type of process has a long-standing history e.g. in studies of earthquakes (see for example Ogata, Akaike, and Katsura (1982)), but has only recently gained attention in financial applications (see for example Errais, Giesecke, and Goldberg (2010) and Shek (2010)).

In recent work related to Essay I, Duffie, Eckner, Horel, and Saita (2009) find that instead of changing the set of observable variables entering the default intensity, it is also possible to obtain an improved fit to empirical default data by incorporating latent variables. Unfortunately this approach also implies a significant increase in the statistical estimation uncertainty, and it does not provide any economic interpretation of the added latent factors.

Essay II (co-authored with Peter Feldhütter, LBS) also applies an intensity-based model to describe default risk and default correlation among a pool of borrowers. However, in contrast to Essay I default probabilities are not based on observations of actual defaults but instead inferred from prices of credit derivatives. The default correlation structure is again based on a conditional independence assumption and is thus similar to that of Essay I, except that now both idiosyncratic and systematic default risk are modelled as latent factors as opposed to observable factors in Essay I.

Although Essay II concerns the estimation of default risk, the estimation methodology draws heavily on techniques from the literature on default-free term structure modelling. Specifically, to infer term structures of default probabilities from prices of Credit Default Swap (CDS) contracts, we use an approach similar to the derivation of yield curves from observed bond prices suggested by Nelson and Siegel (1987). Similarly, for the parametrization of our systematic default component we use an affine jump-diffusion process in analogy with the extensive literature on affine term structure models (see e.g. Duffie and Kan (1996), Dai and Singleton (2000), Collin-Dufresne, Goldstein, and Jones (2008)).

The first contribution of Essay II is that we exploit the whole term structure of CDS spreads to infer a corresponding term structure of default probabilities for each firm in our sample. This allows us in a novel way to remove the restrictive parameter contraints enforced in earlier work by Duffie and Gârleanu (2001) and Mortensen (2006). Moreover, our approach enables us to split the total amount of default risk into an idiosyncratic and a systematic part. This potentially allows for more detailed analyses of the forces driving market-implied default risk compared e.g. to the papers of Longstaff and Rajan (2008) and Errais, Giesecke, and Goldberg (2010), where only the aggregate default risk is considered.

In the second contribution of the paper, we give a theoretical argument for how to estimate the weight on the systematic default component in each firm's default intensity. We further demonstrate that the resulting empirical estimates are similar to those implied by the ad hoc method applied in previous literature. Our third and final contribution is to estimate our model on a large empirical data set and thereby show that it *is* possible to formulate a default correlation model that can match the level and time series dynamics of both single-name CDS spreads *and* correlation-dependent, multi-name CDO (Collateralized Debt Obligation) spreads at the same time.

The scope of Essay II is to capture the correlation implied by observed market prices of credit risky securities, and not to determine the fundamental economic sources of default correlation. This is instead the focal point of Essay III. Here, I apply the idea that correlation (in actual defaults as well as in prices of credit risky securities) is to some extent caused by common variation in macroeconomic variables. This is already exploited in the intensity-model considered in Essay I, and in Essay III it is used to develop a structural credit risk model with the purpose of explaining business cycle variation in corporate credit spreads.

The basic setting of the model follows the structural framework introduced in Leland (1994b), where a firm's debt and equity are viewed as claims to underlying assets, and the default time τ is the first time asset value falls below some prespecified threshold. The model builds on a large recent literature that has extended Leland's original model in two different directions: *either* to allow for jumps in asset value (Hilberink and Rogers (2002), Cremers, Driessen, and Maenhout (2008), Chen and Kou (2009)) *or* to take business cycle fluctuations in asset value into account (Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a;b), Chen (2010)).

The first contribution of Essay III is to document how both level and slope of observed credit spreads vary with the state of the economy, and to link this to similar fluctuations in empirical jump behaviour. This suggests that both jumps *and* business cycle variation have a role to play in explaining corporate credit spreads, and in the second contribution of the paper I therefore construct a structural credit risk model that incorporates *both* risk factors at the same time. This essentially unifies most of the models mentioned above, and I demonstrate that despite significant additional model complexity, that arises when both jump and business cycle risk are included, it is still possible to obtain closed-form expressions for the market values of debt and equity.

The last two contributions of Essay III regard empirical aspects of the formulated structural model. While there already exists a comprehensive literature on jump parameter estimation using high-frequency data (see e.g. Barndorff-Nielsen and Shephard (2004; 2006), Huang and Tauchen (2005), Andersen, Bollerslev, Diebold, and Vega (2007)), it is not possible to apply these techniques to the estimation of my model, since reliable estimation of the business cycle related parameters requires a sample period of multiple decades over which high-frequency data is not available. Instead, I present an alternative method for estimation of the jump parameters, and I verify that the outcome of this alternative procedure is consistent with the common perception of jumps as representing arrival of new information to the market (see e.g. Maheu and McCurdy (2004), Lee and Mykland (2008)). In the last contribution of the paper, I perform a full firm-by-firm estimation of the model and I show that the resulting model-implied credit spread curves replicate the previously observed business cycle variation in empirical credit spreads.

Briefly summing up, the overall purpose of this thesis is to gain further understanding of the importance of and mechanisms behind corporate default correlation, and the three essays in the thesis describe different aspects of this correlation. Essay I looks at correlation in *actual* default probabilities, Essay II discusses correlation in *market-implied* default probabilities derived from prices of correlation-dependent credit derivatives, and Essay III discusses business cycle variation in *market-implied* default probabilities and default loss rates with particular focus on the underlying economic mechanisms driving these fluctuations.

Essay I

Correlation in corporate defaults: Contagion or conditional independence?*

Co-authored with David Lando, Copenhagen Business School.

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†The thesis version of the paper contains some additional results in section I.4.

Abstract

We revisit a method used by Das, Duffie, Kapadia, and Saita (2007) (DDKS) to test the doubly stochastic assumption in intensity models of default. We show that using a different specification of the default intensity, and using the same test as DDKS, we cannot reject using an almost identical set of default histories recorded by Moody's in the period from 1982 to 2006. We propose additions to the procedure as well as a Hawkes process alternative to test for violations of conditional independence but cannot detect contagion. We then observe that the test proposed by DDKS is mainly a misspecification test in that it will not detect contagion effects as long as individual firms have default intensities and there are no simultaneous jumps to default. Specifically, contagion spread through the explanatory variables ("covariates") that determine the default intensities of individual firms will not be detected. We therefore perform different tests to see if firm-specific variables are affected by occurrences of defaults. Regression tests show that there is no influence from defaults on quick ratios, but some influence on distance-to-default.

I.1 Introduction

Can we think of time variation in the frequency of corporate defaults as controlled by "exogenous" factors with no feedback from actual defaults to these factors? Or can we statistically document "contagion effects" by which one firm's default increases the likelihood of other firms defaulting?

In a recent paper Das, Duffie, Kapadia, and Saita (2007) (DDKS) test whether default events in an intensity-based setting can reasonably be modelled as "doubly stochastic", i.e. as dependent solely on "exogenous" factors. Their approach is to transform the time scale using the sum of the default intensities estimated for individual firms and then test whether defaults on this transformed time scale behave as a standard Poisson process. Based on a time series of U.S. corporate defaults, they strongly reject that defaults can be modelled as doubly stochastic. DDKS view this test as a joint test of the specification of the default intensities of the individual firms *and* the doubly stochastic assumption. A core message of our paper is that the time transformation test should be thought of mainly as a misspecification test. We need – and propose – other tests to look for contagion effects that violate the doubly stochastic assumption.

Our first contribution is to show that a different specification of the intensity will in fact make us unable to reject the tests performed by DDKS. That is, using our specification of the intensity there is no excess default clustering. As DDKS we use the sample of firms listed in Moody's default database. To make sure that the different conclusion is not merely a consequence of deviations in the data, we show that specifying the explanatory variables as in DDKS, we reject the assumption of conditional independence but using our specification, we are not able to reject using a large variety of tests. In essence, our change in specification consists in replacing a measure of the short rate with a measure of steepness of the term structure, adding industrial production (a variable also examined in DDKS) and adding the following three firm-specific variables: quick ratio, short-to-long debt and the book value of assets. We will discuss this choice of the explanatory variables below.

The fact that we are unable to reject the tests performed in DDKS with our covariates could lead us to conclude, that there are no detectable contagion effects in the data. This conclusion is premature, however. Our second contribution is to show that when contagion takes place through firm covariates (as opposed to contagion by "domino effects"), this will not be detected by the test procedure followed in DDKS (and in the first part of our paper). To state this in more economic terms, if default of one firm causes, say, the book value of assets of another firm to fall, and this increases the intensity of default of the other firm, then as long as the book asset value is an explanatory variable in our estimation of default intensities of firms, we will not detect this as a contagion effect using the test based on time transformation. To explain the intuition behind this insight, we set up the simplest structure rich enough to illustrate a contagion effect which occurs through explanatory variables, but which is not detected by the test.

Our final contribution is to analyze contagion effects, both direct and through explanatory variables, and using both likelihood tests based on Hawkes processes and regression analysis. Hawkes processes, or self-exciting processes, are a class of counting processes which allow intensities to depend on the timing of previous events. When we use firm-specific variables in the Cox regressions, the Hawkes specification does not add any explanatory power. If we only condition on macroeconomic variables and look for contagion by checking through a Hawkes specification whether downgrade intensities increase following a default, then we do detect a contagion effect. Since this effect may be due to rating agency behaviour, we also perform regression tests to check for contagion through the firm-specific variables distance-to-default and the quick ratio to be defined below. We find some support for this.

There is ample evidence that corporate defaults are correlated. For example, Lang and Stulz (1992) show that bankruptcy announcements significantly decrease the value of a portfolio of competitor stocks. Several empirical studies document a large time variation in default frequencies and link this variation to, among other variables, business cycle indicators. Examples of this include Nickell, Perraudin, and Varotto (2000), Shumway (2001), Duffie, Saita, and Wang (2007), and many others. Since such indicators simultaneously affect the default probabilities of many firms, their variation induces correlation between default events just as variation of common factors in asset return models induce correlation between returns.

We also have indirect evidence that defaults are correlated from market prices of traded securities. For example, Credit Default Swap premia have significant common movements and prices of tranches of Collateralized Debt Obligations (CDOs) can only be reasonably explained if one assumes a significant amount of default correlation. Of course, market prices of these securities reflect not only the physical probabilities of defaults but also contain an adjustment for risk. Still, it is fair to assume that the price patterns we observe for CDO tranches can at least partially be attributed to correlated default risk.

How to best model the correlation effects is less clear. The most tractable way from an analytical standpoint is to work under a conditional independence assumption, in which a common factor structure induces covariation between the default times of different firms. Conditionally on the evolution of the common factors, defaults are independent. This formulation is also referred to as a doubly stochastic setting. This is a setting in which default dependence is captured by business cycle related variables. The conditional independence structure is analyzed among other places in Jarrow, Lando, and Yu (2005), and it is applied to CDO modelling for example in Duffie and Gârleanu (2001).

A more direct way of inducing dependence between default times is to assume that there is contagion, i.e. that the actual default event of one firm either directly triggers the default of other firms or causes their default probabilities to increase.[1] Some examples of contagion models include Davis and Lo (2001), Jarrow and Yu (2001), Azizpour and Giesecke (2008), and Azizpour, Giesecke, and Kim (2011). This type of contagion is clearly relevant when firms belong to the same corporate family, for example through parent-subsidiary relationships, see for example Emery and Cantor (2005). The question we address here is whether this type of contagion is present even for firms which do not belong to the same corporate family.

Note that our focus in this paper is not on “informational” contagion in prices on equity, corporate bonds or Credit Default Swap premia as studied for example by Collin-Dufresne, Goldstein, and Helwege (2003) and Jorion and Zhang (2009). Rather, we focus on methods for testing for conditional independence in actual defaults. Also, our focus is only on models based on observables. We do not estimate intensity models with frailty as done for example in Azizpour and Giesecke (2008), Duffie, Eckner, Horel, and Saita (2009), and Chava, Stefanescu, and Turnbull (2011).

Before looking at hard evidence, it is interesting to note that when looking through the default histories in Moody’s default database, it is almost impossible to locate any

[1] It is also conceivable that defaults could cause the default probabilities of competing firms to *decrease*, which can also be captured by the model specifications we consider.

examples where the brief description of what caused a firm to default mentions other firms outside the corporate family. The vast majority of cases list reasons such as too much leverage, failing sales in declining markets, and lawsuits – effects that are typically captured through either firm-specific explanatory variables or market-wide conditions. Indeed, looking at the points in time where the defaults seem to cluster more than what can be explained by the aggregate intensity in the DDKS specification, we find that none of the default stories contains any instances of contagion from other firms in the sample. This seems to rule out at least the direct domino effect explanation for clustering of defaults and also raises doubts that earlier defaults in the sample have any effect.

Prior to our study, we inspected all default explanations in Moody's Default Risk Service Database. A typical explanation of a default event (our emphasis added) is as follows:

> ***Heartland Wireless Communications, Inc.****, based in Plano, Texas, develops, owns and operates wireless cable television systems and channel rights in small to mid-size markets in the central United States. Although the company has experienced strong revenue growth since its inception, posting $78.8 million in revenues for 1997 compared to $2.2 million in its first full operating year (1994), substantial start-up capital costs and* **an aggressive expansion strategy** *pursued by management resulted in consecutive operating losses and built up significant amounts of debt. Heartland Wireless incurred a net loss of $134.6 million for 1997, compared to a net loss of $61.1 million a year earlier. The technological limitations of Heartland's major product (MMDS – multichannel multipoint distribution service – has a limited number of channels it can disseminate), an* **inability to achieve sufficient subscriber levels***, and* **intense competition** *from traditional hard-wire cable television firms have applied additional pressure to the company's financial position. Mounting debt service costs and the need for additional capital induced the company to hire Wasserstein Perella & Co., an investment banking firm, to analyze all available options to finance the company's business plan and service its existing debt. In consultation with its financial advisor, Heartland Wireless announced that it would not be making interest payment due April 15, 1998 on its 13% senior notes due 4/15/2003.*

It is clear in this explanation that there is no trace of contagion. What might a contagion story have looked like in the data? The famous Penn Central default – often mentioned as a contagious default event – has the following description:

> *On June 21, 1970, the Penn Central declared bankruptcy and sought bankruptcy protection. As a result,* **the Penn Central was relieved of its obligation to pay fees to various Northeastern railroads – the Lehigh Valley included** *– for the use of their railcars and other operations. Conversely,* **the other railroads' obligations to pay those fees to the Penn Central were not waived**. *This* **imbalance in payments** *would prove fatal to the financially frail Lehigh Valley, and it declared bankruptcy three days after the Penn Central, on June 24, 1970.*

The source of this default history is Wikipedia and if we look in Moody's database, we learn that Penn Central was in fact a majority shareholder in Lehigh Valley, and hence they belonged to the same corporate family by Moody's definition. Since we exclude defaults within the same corporate family which occur less than a month apart, this event would not have been in our data, even if we had extended back to 1970. We *did* find one example of a contagion story in the Moody's data, but here only the company at the receiving end of the contagion channel shows up as part of our final data sample, and hence this specific example of a contagion event will not affect our empirical analysis.

The flow of our paper is as follows. We describe our data and set up a proportional hazard model for default intensities of individual firms. We then estimate the default intensities of the individual firms and show that our specification "survives" the time transformation test used in DDKS. Consistent with DDKS, we find that their specification of the intensity leads to rejection of most tests. We also consider a method for testing for contagion using a Hawkes process alternative. Then we explain why the test in DDKS is really just a misspecification test which will not capture important violations of the doubly stochastic assumption. Our main example involves contagion through explanatory variables. This example motivates our extended testing for conditional independence in which we first look for contagion effects through ratings which are used as a one-dimensional proxy for the firm-specific explanatory variables. We then perform regression tests to see if defaults affect levels of distance-to-default and the quick ratio.

I.2 Data and model specification

Our empirical analysis is based on corporate default data from Moody's Default Risk Service Database (DRSD), which essentially covers the period since 1970. However, the material is sparse until 1982, which we therefore choose as the beginning of our sample period. Other default studies have used the same data supplemented with additional defaults from other sources, see e.g. Li and Zhao (2006), DDKS, Le (2007), and Davydenko (2010).[2] We have chosen to rely only on the data in the Moody's database since these all have explanatory notes associated with each default allowing us to both screen the default histories for traces of contagion and for parent-subsidiary relationships. It also has the advantage of giving us an unambiguous definition of what constitutes a default event.[3]

Thus our estimation will comprise all U.S. industrial firms with a debt issue registered in Moody's DRSD, and for which we are able to obtain accompanying stock market data from CRSP and accounting information from CompuStat. This leaves us for the period January 1982 to December 2005 with a total of 2,557 firms comprising 370 defaults, with an average of 1,142 and a minimum of 1,007 firms in the model at any time throughout the sample period, all of which have at least 6 months of available data.

The time change test involves transforming the time by a cumulative intensity, which is the sum of default intensities estimated for each firm separately. Therefore, we first need to specify a model for each firm's default intensity. Formally, the default of a single debt-issuing firm i is described by the default time τ_i, and we assume that the default

[2]Le (2007) includes defaults registered in the CompuStat database, which he notes in some instances implies that a registered default does not correspond to an actual default, but merely reflects the timing of a stock delisting event. To resolve a similar difficulty, in the case where the actual default date is known but delisting occurs prior to default, Davydenko (2010) applies an extrapolation technique to infer values for the necessary stock market variables at the actual default date, although inspection of the default data in Moody's DRSD reveals that this occasionally leads to extended periods of time, where inference can only be based on imputed data.

[3]We consider as a default any of the following events classified in Moody's DRSD: "Chapter 7", "Chapter 11", "Distressed exchange", "Grace period default", "Missed interest payment", "Missed principal payment", "Missed principal and interest payments", "Prepackaged Chapter 11", and "Suspension of payments". In particular, we do not correct the timing of a "Distressed exchange", which in the DRSD is registered as the time of completion of the exchange, although as suggested by Davydenko (2010), it would probably be more appropriate to instead collect separate information on the announcement date of the exchange.

time can be modelled through its stochastic intensity λ_i. If the firm is alive at time t, then the intensity at time t for firm i satisfies

$$\lambda_i(t) = \lim_{\Delta t \to 0} \frac{P(t < \tau_i \leq t + \Delta t \,|\, \tau_i \geq t, \mathcal{F}_t)}{\Delta t}$$

i.e. the probability of default within a small time period Δt after t is close to $\lambda_i(t)\Delta t$. λ_i depends on information available at time t as represented by $\mathcal{F}_t$. This information contains all intensities of firms and all default histories up to time t (see the appendix for a rigorous formulation). In the intensity setting, modelling the probability of default for firm i thus reduces to modelling its default intensity λ_i.

The critical exercise here is to determine the firm-specific and macro variables which are significant explanatory variables in the Cox regressions used to specify the intensity. In the specification of individual default intensities we employ a selection of four macroeconomic variables collected from CRSP and the U.S. Federal Reserve Board:

- 1-year return on the S&P500 index
- 3-month U.S. Treasury rate
- 1-year percentage change in U.S. industrial production, calculated from monthly data on the gross value of final products and nonindustrial supplies (seasonally adjusted)
- Spread between the 10-year and 1-year Treasury rate

and five firm-specific variables collected from CRSP and CompuStat:

- 1-year equity return
- 1-year distance-to-default
- Quick ratio, calculated as the sum of cash, short-term investments and total receivables divided by current liabilities
- Percentage short-term debt, calculated as debt in current liabilities divided by the sum of debt in current liabilities and long-term debt
- Book asset value (log).

Table I.1. Descriptive statistics for covariates

The table reports empirical averages and standard deviations (in parenthesis) for the explanatory variables used in the Cox regressions.

Macro variables:						
1-year S&P500 return	0.110	(0.164)				
3-month Treasury rate	5.469	(2.671)				
Industrial production	0.027	(0.029)				
Treasury term spread	1.371	(0.955)				
Firm-specific variables:						
	Defaulting firms		*Non-def. firms*		*All firms*	
1-year equity return	0.044	(0.497)	0.119	(0.526)	0.109	(0.523)
1-year distance-to-default	0.612	(1.356)	2.063	(2.854)	1.867	(2.746)
Quick ratio	0.507	(6.237)	0.682	(3.091)	0.658	(3.677)
Short-to-long term debt	0.057	(0.154)	0.094	(0.185)	0.089	(0.181)
Book asset value (log)	1.835	(2.882)	3.170	(3.582)	2.990	(3.526)

Table I.1 shows descriptive statistics for the variables to guide the interpretation of the regression coefficients obtained below. We also show average levels of the covariates for defaulting vs. non-defaulting firms.

For all balance sheet variables we substitute, if quarterly data are missing, with the latest yearly observation, and for the calculation of the distance-to-default measure we follow the iterative approach described in Duffie, Saita, and Wang (2007). Moreover, to comply with the mathematical foundations of our model, we require that the value of $\lambda_i(t)$ is known prior to time t, a phenomenon referred to as "predictability" in the technical literature, such that e.g. as a proxy for the book value of assets on, say January 1st, we use the number reported for December of the previous year.[4] Finally, in order to correct for observations of multiple defaults caused by parent-subsidiary relations, we disregard all consecutive default events that occur within a 1-month horizon of any previously registered default ascribed to the same parent company.[5]

Our specification of the individual firm default intensity is

$$\lambda_i(t) = R_{it} e^{\beta_W' W_t + \beta_X' X_{it}}$$

where W_t is a vector containing the covariates that are common to all firms and X_{it} is a vector of firm-specific variables. R_{it} is an indicator which is 1 if firm i is alive and observable at time t and zero otherwise and $N_i(t)$ is the one-jump process which jumps to 1 if firm i defaults at time t. The log (partial) likelihood function takes the form (see Andersen, Ørnulf Borgan, Gill, and Keiding (1992))[6]

$$\log L(\beta) = \sum_{i=1}^{n} \int_0^T \big(\beta_W' W_t + \beta_X' X_{it}\big) dN_i(t) - \sum_{i=1}^{n} \int_0^T R_{it} e^{\beta_W' W_t + \beta_X' X_{it}} 1_{(\tau_i \geq t)} dt$$

where T is the terminal time point of the estimation and n the total number of firms. We can then apply standard maximum likelihood techniques to draw inference about $\beta = (\beta_W, \beta_X)$. Table I.2 reports estimates and asymptotic standard errors from two different intensity specifications: Model I which is the model analyzed in DDKS, and

[4]The issue of delayed public disclosure leads Carling, Jacobson, Lindé, and Roszbach (2007) to argue that it is more appropriate to use lagged values for both macroeconomic and accounting variables, although it is not clear exactly how to choose an appropriate lag length. Similarly, Koopman and Lucas (2005) suggest that macroeconomic variables could be lagged in order to improve causality of the model, arguing that to the extent that default events are consequences of (and thus lagged wrt.) macroeconomic fluctuations, they will appear with a certain time lag which should be corrected for. However, they also demonstrate how estimation results may be highly vulnerable to the choice of lag length.

[5]Davydenko (2010) similarly chooses to disregard all subsequent defaults within a 2-year period, which may be a more appropriate horizon. However, our shorter horizon should make it harder to specify intensities consistent with an assumption of conditional independence.

[6]We work under the usual assumption of independent filtering by assuming that the various filtering mechanisms we employ: left truncation for all firms operating on January 1st 1982 (beginning of the estimation period), (temporary) withdrawal of firms in case of lacking covariates, and right censoring of all firms operating on December 31st 2005 (end of the estimation period) do not alter the likelihood function. For thorough discussions of these issues see Andersen, Ørnulf Borgan, Gill, and Keiding (1992) and Martinussen and Scheike (2006).

Model II which is an extension that incorporates a wider selection of variables. The signs of the various $\beta-$coefficients are largely as expected and consistent with the findings of DDKS (see Duffie, Saita, and Wang (2007) for parameter estimates). The key differences are the following: a measure of the short rate is replaced with a measure of steepness of the term structure, we add growth in industrial production (a variable also examined in DDKS), and the following three firm-specific variables: quick ratio, short-to-long debt and the logarithm of the book value of assets.

Table I.2 also reveals how both model I and II, somewhat surprisingly but consistent with for example Duffie, Saita, and Wang (2007) and Figlewski, Frydman, and Liang (2008), show a positive dependence of default intensities on the yearly return on the S&P500 stock index.[7]

Figure I.1 shows monthly defaults along with the estimated cumulative default intensities for both models. Clearly, the estimated default intensities are different, but the graph also shows that it is difficult from visual inspection to tell which model gives the better fit.

We have examined the influence of additional economy-wide factors besides those appearing in Model I and II through proxies for the U.S. unemployment rate, the wages of U.S. production workers, the U.S. consumer price index, the U.S. gross domestic product in both real and nominal terms, the price of crude oil, and the spread between Moody's Aaa- and Baa-rated corporate bonds, but without finding any significant effects. In a similar fashion, we have looked at a variety of alternative indicators of financial soundness at the firm-specific level including some of the empirical default predictors proposed by Altman (1968) and Zmijewski (1984), but likewise without finding support for further expansion of the set of explanatory variables.

Ideally, we should also take specific account of debt issue characteristics such as the time of issuance, maturity, face value, coupon payments including possible step up-clauses etc. given the empirical evidence presented in Davydenko (2010) who demonstrates the

[7] Duffie, Saita, and Wang (2007) suggest that this may in part reflect business cycle effects as well as be a consequence of correlation with the idiosyncratic stock returns, and perhaps also with other variables.

[8] Calculations are based on the likelihood ratio test statistic and its asymptotic distribution. However, the (asymptotically equivalent) Wald and score test statistics yield similar conclusions thus indicating a limited finite sample bias in the results.

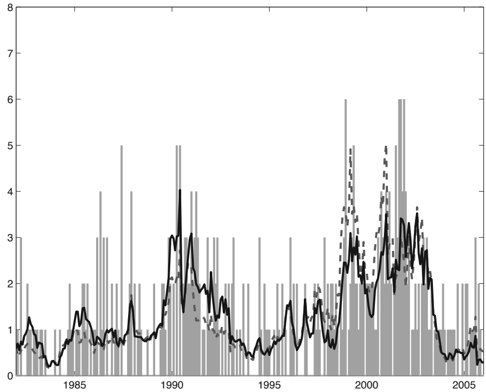
Table I.2. Parameter estimates (doubly stochastic models)

The macro variables entering the models are the 1-year return on the S&P500 index, the level of the 3-month U.S. Treasury yield, the 1-year percentage change in U.S. industrial production, and the spread between the 10-year and 1-year U.S. Treasury yields. The firm-specific variables are the 1-year stock return, the 1-year distance-to-default, the quick ratio, short-term debt as a percentage of total debt, and (log) book value of assets. Asymptotic standard errors are reported in parenthesis and statistical significance is indicated at 5% (*), 1% (**), and 0.1% (***) levels, respectively.[8]

	Model I	Model II
Macro variables:		
Constant	-3.735 ***	-3.480 ***
	(0.179)	(0.299)
1-year S&P500 return	1.566 ***	1.886 ***
	(0.318)	(0.353)
3-month Treasury rate	-0.040	
	(0.024)	
Industrial production		-5.723 **
		(1.956)
Treasury term spread		0.209 ***
		(0.055)
Firm-specific variables:		
1-year equity return	-3.131 ***	-3.151 ***
	(0.202)	(0.213)
1-year distance-to-default	-0.841 ***	-0.794 ***
	(0.039)	(0.043)
Quick ratio		-0.263 ***
		(0.085)
Short-to-long term debt		0.651 ***
		(0.177)
Book asset value (log)		-0.095 **
		(0.031)

influence of this type of information on the probability of default. Similarly, it could also be of importance to allow for specific industry effects given the variation in default rates across industries documented by Li and Zhao (2006). However, the lack of available debt issue information and the limited number of defaults unfortunately prevents us from performing either type of analysis on the current data set. Working with larger data sets and performing out-of-sample tests would naturally lead us to include more variables but

Figure I.1. Aggregate default intensity 1982-2005



Monthly number of U.S. industrial defaults recorded in Moody's DRSD in the period 1982-2005 and estimated default intensities for the simple (Model I, dashed) and the expanded (Model II, solid) model.

as we will see in the next section our specification is rich enough to capture the correlation in the data.

I.3 Testing for conditional independence and contagion

Having estimated the default intensities of each firm, we now follow DDKS and transform the time scale using the cumulative intensity and test whether on the new time scale the default arrivals are a unit rate Poisson process. We also propose and test an extended version of the default intensity which explicitly models the possibility of contagion through a Hawkes process specification.

I.3.1 The time change test

The doubly stochastic assumption is meant to capture a setup in which probabilities of default of individual firms are affected by exogenous "background variables". The variables are exogenous in the sense that they are not affected by actual defaults of firms. A helpful illustration from medical science could be pollution in a city and onsets of asthma attacks among its citizens. When the level of pollution is high, there are more asthma attacks and hence onsets of these attacks are correlated. However, conditioning on the level of pollution the onsets are independent (assuming that asthma is non-contagious). Also, asthma attacks do not affect the level of pollution. For an example with more relevance to default modelling, it is possible that increasing oil prices will cause more firms who use oil as an input in their production to default, but that the defaults will have no effect on oil prices, so conditionally on the level of oil prices defaults are then uncorrelated. In models of stock returns, conditional independence is often assumed in factor models where the residual returns, i.e. the part that is not explained by the factors, are independent across firms.

The test procedure used in DDKS is easy to describe in fairly non-technical terms. First, estimate individual firm intensities using Cox regressions. Then compute the sum of these intensities. Under the assumption of orthogonality, i.e. that there are never exact simultaneous defaults, the sum of the intensities is equal to the aggregate default intensity. Now, transform time using the aggregate intensity and check whether aggregate defaults in the new time scale are a unit rate Poisson process. Testing this uses a range of different properties of the Poisson process, such as moment properties and exponential waiting times between jumps.

To describe the test more rigorously, we first recall that default times are said to be *orthogonal* if $P(\tau_i = \tau_j) = 0$ whenever $i \neq j$. The cumulative number of defaults among n firms is defined as

$$N(t) = \sum_{i=1}^{n} 1_{(\tau_i \leq t)} \qquad t \geq 0$$

and, as noted in the appendix, if the default times are orthogonal the cumulative default process has intensity

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t) 1_{(\tau_i \geq t)} \qquad t \geq 0$$

and the compensator of the cumulative default process is then the integral of the intensity

$$\Lambda(t) = \int_0^t \lambda(s) ds \qquad t \geq 0.$$

Hence, if we time-change the cumulative default process by the compensator, it follows from Meyer (1971) that the cumulative default process becomes a unit rate Poisson process, i.e. the time-scaled process

$$J(t) = N\big(\Lambda^{-1}(t)\big) \qquad t \geq 0$$

is then a unit Poisson process with jump times $V_i = \Lambda(\tau_{(i)})$, where $0 \leq \tau_{(1)} \leq \tau_{(2)} \leq \dots$ denotes the ordered default times. A consequence of this is that $V_1, V_2 - V_1, \dots$ are independent exponentially distributed variables and for any $c > 0$, the binned jump times

$$Z_j = \sum_{i=1}^{n} 1_{]c(j-1),cj]}(V_i)$$

will be independent Poisson(c)−distributed variables. In summary, if default times are orthogonal, we can transform the time scale of the cumulative default process to obtain a unit rate Poisson process and we can then use standard properties of this process for testing. Note that conditional independence or the doubly stochastic assumption is not needed to have orthogonality of the default times. Thus, we really use the time transformation test as a misspecification test. We return to this point in section I.4.

To test whether the default arrivals on the transformed time scale truly follow a unit rate Poisson process, we use various theoretical properties of such a process: that the number of arrivals in a time interval is Poisson distributed with a mean equal to the length of the time interval, that waiting times between jumps are exponentially distributed, that arrivals in disjoint time intervals are independent, and some moment properties.

If we split up the entire time period into intervals in each of which the cumulative intensity increases by an integer c, then the number of arrivals in each of these intervals are independent and Poisson distributed with mean c. We follow DDKS and refer to c as the bin size, since it reflects the expected number of defaults in each time interval. The larger c is, the smaller is the total number k of time intervals (and hence Poisson variables) that we get, thereby weakening the power of our statistical tests. On the other hand, by increasing c we can hope to get a clearer picture of the presence of heavy tails representing excess clustering of defaults. We use the same test statistics as those of DDKS, i.e. the Fisher Dispersion (FD) and the upper tail statistics (UT1, UT2)[9], and supplement with further tests detailed in Karlis and Xekalaki (2000). Since we only have a limited number

[9] We correct for the apparent misprint in DDKS in the description of the upper tail median statistic by comparing the simulated median statistics to the sample *median* (instead of the sample mean). However, this implies that the median statistic by construction only will be efficient for large bin sizes.

of observations and some of the asymptotic distributions of the test statistics require a much larger amount of data (see Karlis and Xekalaki (2000)), we calculate instead for each statistic the $p-$value under the null hypothesis from a history of 100,000 simulated test statistics to improve accuracy.

The tests of the Poisson distribution listed above tend to concentrate on whether the univariate distribution of recorded defaults for a given bin size is Poisson. They therefore ignore the time series aspects. If default contagion takes place with a time lag, it is conceivable that bins with many defaults tend to be followed by bins with many defaults and vice versa. To account for this possibility we use (as an alternative to the regression test in DDKS) the additional test statistics SC1 and SC2.

The Fisher and upper tail tests are outlined in DDKS, so we only describe the remaining statistics, which we define through the following acronyms:

$$\text{BD} = \frac{1}{\overline{Z}\sqrt{2(k-1)}} \sum_{j=1}^{k} \left(Z_j - \overline{Z}\right)^2 - \sqrt{\frac{k-1}{2}}$$

$$\text{CVM} = \frac{1}{k}\sum_{i=0}^{\infty} V_i^2 \qquad \text{with} \qquad V_i = \sum_{s=0}^{i}(|\{j \mid Z_j = s\}| - \text{Expected}_s)$$

$$\text{KK} = \sqrt{k}\frac{\phi_k(t) - \exp\left(\overline{Z}(t-1)\right)}{\exp\left(\overline{Z}(t^2-1)\right) - \exp\left(2\overline{Z}(t-1)\right)\left(1+\overline{Z}(t-1)^2\right)} \text{ with } \phi_k(t) = \frac{1}{k}\sum_{j=1}^{k} t^{Z_j}$$

$$\text{NPA} = \frac{1}{k^3\overline{Z}^{1.45}}\left(\sum_{i,j,l,m=1}^{k} Z_i(Z_i - Z_j - 1)Z_l(Z_l - Z_m - 1)1_{(Z_i+Z_j=Z_l+Z_m)}\right)$$

$$\text{SC1} = \frac{1}{k-1}\sum_{j=1}^{k-1}\left(Z_jZ_{j+1} - c^2\right)^2$$

$$\text{SC2} = \frac{1}{k-1}\sum_{j=1}^{k-1}\left(Z_j - c\right)\left(Z_{j+1} - c\right).$$

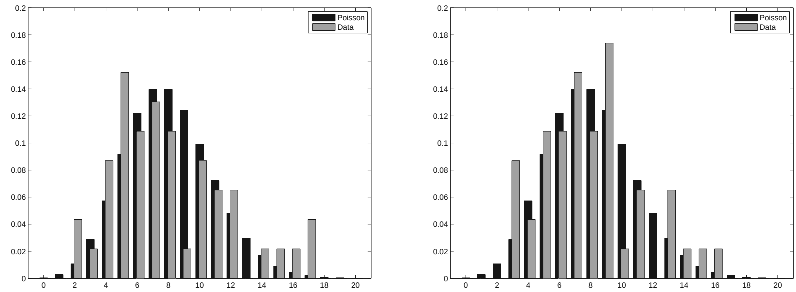
The results of the tests are reported in Table I.3. Model I refers as before to the intensity specification used in DDKS and Model II to our intensity specification. All except one test is rejected at the 5% level using the Model II specification, whereas a number of tests are rejected for the Model I specification – predominantly for the large bin sizes. For bin size 8, for example, Figure I.2 shows that the Model II specification has a less pronounced heaviness in the right tail of the distribution and Figure I.3 shows that it is also better at eliminating serial dependence. Hence we conclude, that using our specification of the firm default intensities, we are not able to reject that the time transformed process is Poisson. Figure I.4 compares visually the two intensity specifications around two very active periods around 1990 and between 1998 and 2002. We show actual defaults, the fitted intensities and the associated time intervals corresponding to an expected number of defaults equal to 8. The visual inspection confirms that our model is a better fit around 1990. In the latter period, the DDKS model has a higher spike but may be overshooting around 2000.

Table I.3. Binned data tests (doubly stochastic models)

The table reports $p-$values for tests of the fit of the transformed default data to the Poisson distribution for bin sizes $c = 1, 2, 4, 6, 8, 10$, based on the intensity specification $\lambda_i(t) = R_{it}e^{\beta'_W W_t + \beta'_X X_{it}}$ estimated in Table I.2. The employed test statistics are: Fisher dispersion (FD), Upper tail mean (UT1), Upper tail median (UT2), Böhning dispersion (BD), Cramer von Mises (CVM), Kocherlakota-Kocherlakota with parameter $t = 0.9$ (KK), Nakamura-Perez-Abreu (NPA), and the serial correlation statistics (SC1 and SC2). $p-$values (rounded to 3rd decimal) are calculated with 2-sided alternatives for the BD, KK, and SC statistics and 1-sided otherwise following Karlis and Xekalaki (2000), and statistical significance is indicated at 5% (*), 1% (**), and 0.1% (***) levels.

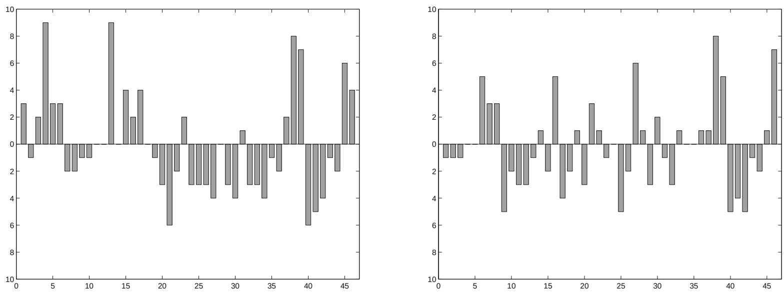
	FD	UT1	UT2	BD	CVM	KK	NPA	SC1	SC2
Model I									
Bin size 1	0.326	0.440	1.000	0.558	0.819	0.548	0.570	0.038 *	0.003 **
Bin size 2	0.110	0.476	1.000	0.156	0.439	0.152	0.090	0.047 *	0.002 **
Bin size 4	0.021 *	0.211	0.972	0.023 *	0.435	0.028 *	0.052	0.011 *	0.001 ***
Bin size 6	0.013 *	0.062	0.136	0.012 *	0.195	0.012 *	0.019 *	0.004 **	0.000 ***
Bin size 8	0.002 **	0.012 *	0.260	0.001 **	0.117	0.002 **	0.001 **	0.003 **	0.000 ***
Bin size 10	0.004 **	0.013 *	0.025 *	0.004 **	0.063	0.006 **	0.000 ***	0.002 **	0.000 ***
Model II									
Bin size 1	0.349	0.302	1.000	0.616	0.847	0.570	0.479	0.225	0.412
Bin size 2	0.324	0.428	0.728	0.582	0.993	0.558	0.981	0.289	0.226
Bin size 4	0.377	0.447	0.972	0.625	0.629	0.768	0.246	0.392	0.209
Bin size 6	0.406	0.501	0.734	0.696	0.853	0.787	0.712	0.722	0.277
Bin size 8	0.113	0.233	0.257	0.154	0.639	0.192	0.191	0.170	0.181
Bin size 10	0.037 *	0.120	0.566	0.051	0.746	0.112	0.111	0.177	0.135

Figure I.2. Distribution of binned data Z_j $(c = 8)$



Empirical distribution for $c = 8$ of the binned data Z_j (gray) for the simple (Model I, left) and the expanded (Model II, right) model against their theoretical counterpart (black).

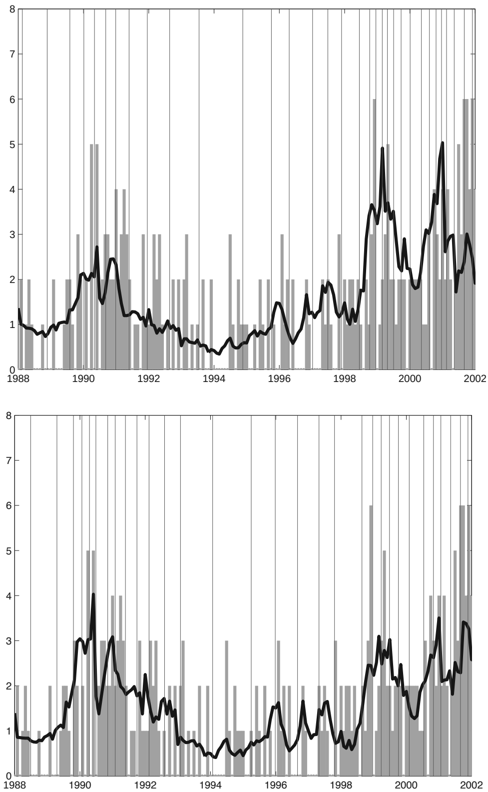
Figure I.3. Sequence of binned data Z_j $(c = 8)$



The sequence of binned, centered data $Z_j - c$ for $c = 8$ for the model used in DDKS (Model I, left) and the our expanded model (Model II, right). The graph shows that model II is better at removing serial dependence.

It is interesting to note that there is a deviation from the Poisson property which is not detected by the test. In Figure I.5 we have plotted the distribution of default events by calendar day and we note that most defaults occur on calendar days 1 and 15. This is consistent with the frequent use of these days for coupon payments on corporate bonds. It is not enough, however, to affect our test results since the defaults are spread out over a 24-year period and thus we do not see any large default clusters on any particular calendar

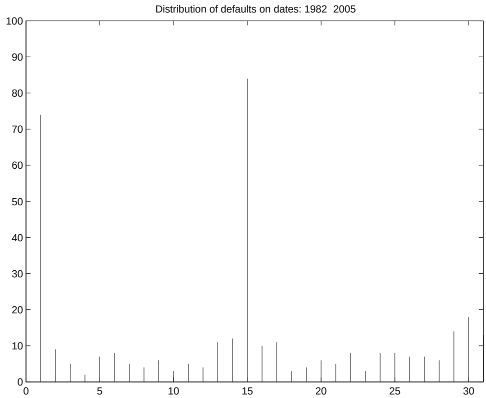
Figure I.4. Time transformation ($c = 8$)



Monthly number of observed defaults (thick gray bars) and estimated aggregate intensities (thick black line) for Model I (above) and Model II (below), 1988–2002. The thin vertical lines indicate time periods in which the expected number of defaults is equal to 8 based on the estimated integrated intensity. Model II (our model) tracks the spike in defaults around 1990 better whereas the period up to 2002 seems to pose problems for both models.

day. Subsequent work by Kramer and Löffler (2010) indicates that an improvement in fit can be obtained by explicitly modelling this effect through a baseline intensity.

Figure I.5. Calender day effects



The distribution of U.S. industrial defaults 1982-2005 on calendar day. Most defaults occur on calendar days 1 and 15. This is consistent with the frequent use of these days for coupon payments on corporate bonds. It is not enough, however, to affect our test results since the defaults are spread out over a 24-year period and thus we do not see any large default clusters on any particular calendar day.

I.3.2 A contagion alternative

All of the tests performed above rely on transforming the time using the estimated intensities. We now perform a different, likelihood-based test which does not rely on the time transformation. We use an extended model which explicitly includes a contagion effect in the intensity specification. To be specific, following Hawkes (1971a;b) we use an intensity of the form[10]

$$\lambda_i^c(t) = R_{it}\left(e^{\beta_W' W_t + \beta_X' X_{it}} + \int_0^t \big(\alpha_0 + \alpha_1 Y_s\big) e^{-\alpha_2(t-s)} dN_s + \delta\right) \qquad t \geq 0 \qquad \text{(I.1)}$$

where Y_s denotes the log book asset value of the firm defaulting at time s. The idea behind the specification is to allow the default of a firm to influence all other intensities. The immediate effect is modelled as an affine function of Y thus allowing for larger firms to have a higher impact on the individual default intensities. The exponential function makes the default impact decay exponentially with time at a rate. The log (partial) likelihood

[10]See Kwiecinski and Szekli (1996) for alternative specifications.

function follows from this expression by standard arguments (Rubin (1972), Ogata and Akaike (1982), Andersen, Ørnulf Borgan, Gill, and Keiding (1992))

$$\begin{aligned}\log L(\alpha,\beta) = \quad & \sum_{i=1}^{n}\int_0^T \log\left(e^{\beta_W' W_t+\beta_X' X_{it}} + \int_0^t \big(\alpha_0+\alpha_1 Y_s\big)e^{-\alpha_2(t-s)}dN_s + \delta\right)dN_{it}\\ & -\sum_{i=1}^{n}\int_0^T R_{it}\left(e^{\beta_W' W_t+\beta_X' X_{it}} + \int_0^t \big(\alpha_0+\alpha_1 Y_s\big)e^{-\alpha_2(t-s)}dN_s + \delta\right)1_{(\tau_i\geq t)}dt\end{aligned}$$

and we can apply maximum likelihood inference as before.[11] Note that α_2 may be taken as a measure of the horizon of influence of a default on the overall default proneness of remaining firms (Hawkes (1971b)).

We use the Hawkes specification to further test for misspecification in the Cox regression used in Model I and II. Since Model I caused a rejection of the Poisson property, it is possible that this is caused by a contagion effect which the Hawkes specification might capture. However, as shown in Table I.4, there is no explanatory power added by this specification. This further supports the hypothesis that the reason for the rejection of Model I is the missing covariates. Even if the Model II specification did not reject the Poisson property of the time-transformed cumulative default process, we use the Hawkes specification as a robustness check. As shown in Table I.5, we find no significance of this addition in the contagion related parameters. However, we do find a significant effect of adding a constant term to the default intensities. Thus, there is a "floor" on all default intensities of 3.5 basis points arising from the constant term δ. This term may be capturing a small misspecification of the proportional hazard regression or of the functional form of the hazard function. The functional form (using the exponential function of a linear function of the covariates) forces intensities to be very small when default covariates are in very "safe territory" far from values held by risky firms. It is possible that even if true intensities are not as small for safe firms as shown in the proportional hazard regression, this deviation is not penalized heavily in the likelihood function and therefore does not affect our time-change test. However, if we allow a constant term in the regression, it does show up as significant, but very small.

[11] Ogata (1978) gives sufficient conditions to ensure consistency and asymptotic normality of the estimators under an additional assumption of stationarity, and Ozaki (1979) presents simulation results that support numerical feasibility of maximum likelihood estimation for self-exciting processes.

Table I.4. Parameter estimates (contagion models)

The explanatory variables in the table are the same as appearing in Table I.2. Asymptotic standard errors are reported in parenthesis and statistical significance is indicated at 5% (*), 1% (**), and 0.1% (***) levels, respectively.[8]

	Model I				
	Without level ($\delta = 0$)		With level ($\delta \neq 0$)		
Macro variables:					
Constant	-3.840 ***	-3.840 ***	-3.845 ***	-3.845 ***	-3.845 ***
	(0.193)	(0.192)	(0.191)	(0.191)	(0.191)
1-year S&P500 return	1.607 ***	1.607 ***	1.604 ***	1.604 ***	1.604 ***
	(0.322)	(0.322)	(0.322)	(0.321)	(0.321)
3-month Treasury rate	-0.038	-0.038	-0.039	-0.039	-0.039
	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
Industrial production					
Treasury term spread					
Firm-specific variables:					
1-year equity return	-3.252 ***	-3.252 ***	-3.270 ***	-3.270 ***	-3.269 ***
	(0.222)	(0.221)	(0.218)	(0.218)	(0.218)
1-year distance-to-default	-0.858 ***	-0.858 ***	-0.856 ***	-0.856 ***	-0.856 ***
	(0.041)	(0.041)	(0.041)	(0.041)	(0.041)
Quick ratio					
Short-to-long term debt					
Book asset value (log)					
Contagion effects:					
Constant (α_0)	$1.2{\cdot}10^{-5}$	$1.3{\cdot}10^{-5}$ *	$2.3{\cdot}10^{-14}$	$2.3{\cdot}10^{-14}$	
	$(8.3{\cdot}10^{-5})$	$(1.7{\cdot}10^{-5})$	$(5.1{\cdot}10^{-5})$	$(1.1{\cdot}10^{-9})$	
Firm size (α_1)	$2.2{\cdot}10^{-14}$		$2.6{\cdot}10^{-14}$		
	$(1.3{\cdot}10^{-5})$		$(4.6{\cdot}10^{-6})$		
Decay rate (α_2)	0.902	0.905	0.982	1.001	
	(1.069)	(1.051)	(—)	(—)	
Level (δ)			$2.1{\cdot}10^{-4}$	$2.1{\cdot}10^{-4}$	$2.1{\cdot}10^{-4}$ **
			$(3.3{\cdot}10^{-4})$	$(1.2{\cdot}10^{-4})$	$(1.2{\cdot}10^{-4})$

Table I.5. Parameter estimates (contagion models)

The explanatory variables in the table are the same as appearing in Table I.2. Asymptotic standard errors are reported in parenthesis and statistical significance is indicated at 5% (*), 1% (**), and 0.1% (***) levels, respectively.[8]

	Model II				
	Without level ($\delta = 0$)		With level ($\delta \neq 0$)		
Macro variables:					
Constant	-3.077 ***	-3.077 ***	-3.086 ***	-3.085 ***	-3.099 ***
	(0.318)	(0.318)	(0.318)	(0.318)	(0.317)
1-year S&P500 return	1.833 ***	1.833 ***	1.832 ***	1.832 ***	1.830 ***
	(0.361)	(0.361)	(0.361)	(0.361)	(0.361)
3-month Treasury rate					
Industrial production	-5.833 **	-5.832 **	-5.834 **	-5.838 **	-5.874 **
	(2.008)	(2.008)	(2.008)	(2.008)	(2.007)
Treasury term spread	0.207 ***	0.207 ***	0.207 ***	0.207 ***	0.208 ***
	(0.056)	(0.056)	(0.056)	(0.056)	(0.056)
Firm-specific variables:					
1-year equity return	-3.236 ***	-3.236 ***	-3.244 ***	-3.244 ***	-3.254 ***
	(0.237)	(0.236)	(0.236)	(0.237)	(0.234)
1-year distance-to-default	-0.799 ***	-0.799 ***	-0.797 ***	-0.798 ***	-0.796 ***
	(0.046)	(0.046)	(0.046)	(0.046)	(0.046)
Quick ratio	-0.765 ***	-0.765 ***	-0.764 ***	-0.765 ***	-0.762 ***
	(0.130)	(0.130)	(0.131)	(0.131)	(0.132)
Short-to-long term debt	0.389 *	0.389 *	0.390 *	0.390 *	0.392 *
	(0.182)	(0.182)	(0.183)	(0.183)	(0.183)
Book asset value (log)	-0.103 **	-0.103 **	-0.102 **	-0.102 **	-0.101 **
	(0.032)	(0.032)	(0.032)	(0.032)	(0.032)
Contagion effects:					
Constant (α_0)	$2.1{\cdot}10^{-5}$	$2.1{\cdot}10^{-5}$ ***	$1.5{\cdot}10^{-6}$	$1.4{\cdot}10^{-5}$	
	($8.7{\cdot}10^{-5}$)	($2.1{\cdot}10^{-5}$)	($1.1{\cdot}10^{-4}$)	($2.5{\cdot}10^{-5}$)	
Firm size (α_1)	$2.4{\cdot}10^{-14}$		$1.7{\cdot}10^{-6}$		
	($1.3{\cdot}10^{-5}$)		($1.4{\cdot}10^{-5}$)		
Decay rate (α_2)	0.868 *	0.869 *	0.737	0.803	
	(0.837)	(0.813)	(1.063)	(0.894)	
Level (δ)			$1.2{\cdot}10^{-4}$	$8.9{\cdot}10^{-5}$	$3.5{\cdot}10^{-4}$ ***
			($3.9{\cdot}10^{-4}$)	($2.9{\cdot}10^{-4}$)	($1.4{\cdot}10^{-4}$)

Azizpour and Giesecke (2008) estimate a model which involves both frailty and Hawkes effects. An important difference to their paper is that we have included both firm-specific and macro variables that are not part of their intensity specification. They use DDKS' specification of macro variables (but do not estimate their influence), whereas we show that both the Treasury term spread and growth in industrial production are significant additional variables to those employed by DDKS. This may explain why frailty is needed in their model whereas we do not need it to pass our misspecification tests. Azizpour and Giesecke (2008) also find significant Hawkes effects in their analysis, but note here that they also choose, for example, to include a specific event involving 24 simultaneous railroad defaults as part of their data sample (which extends further back than ours). However, when looking in Moody's default database, one sees that 22 of these defaults all have Penn Central as their ultimate parent company. That is, these defaults occur within the same corporate family, and contrary to Azizpour and Giesecke (2008) we have chosen in this paper *not* to think of multiple defaults within the same corporate family as contagion. Including multiple defaults from the same parent also makes the data set extremely vulnerable to the exact number of different subsidiaries of a firm that happen to issue bonds. When we look at contagion through covariates in the next section, we will see a further possible reason for the difference between our findings and those of Azizpour and Giesecke (2008).

I.4 Contagion through covariates

If defaults of firms cause intensities of other firms to rise (but never cause an immediate default) then we have orthogonality but not conditional independence. This means that the Poisson property of the transformed process can hold even in cases where there is not conditional independence. Below, we provide the simplest possible example in which there is contagion in the model but the transformation test will not capture this.

We have shown that with a different specification of the explanatory variables in the Cox regressions, we are not able to reject the hypothesis of conditional independence using this specification, but on the same sample we reject using the DDKS specification. It is thus tempting to conclude that contagion effects are eliminated as long as we specify our covariates carefully. There are, however, possibilities of contagion effects which are not captured by the tests performed here and in DDKS. In essence, the time transformation of the intensity may not capture contagion effects which occur through the covariates. That is, if the default of firm A causes, say, the leverage of firm B to rise, and subsequently the increased leverage ratio contributes to the default of firm B, then we will not see this as a contagious default effect since the tests we are performing are conditioning on the evolution of the covariates. The increased leverage will cause the default intensity of firm B to rise, and therefore this will not be seen as a contagion effect violating conditional independence. A full test of contagion should address these "weak" contagion effects as well. In this section we first give a basic illustration of the problem we are addressing using the simplest possible example which is rich enough to capture the effect. We then set up tests for contagion using rating as a proxy for quality of covariates, and looking at covariates directly.

I.4.1 Contagion through covariates – an illustration

It is possible using the language of filtrations to give a rigorous definition of what we are trying to capture, but we believe that the example below is more useful as a reference for the discussion and gives a much clearer illustration of the main point.

Consider a collection of firms whose default risk is entirely determined by their rating which can be either A or B. Firms with rating A have a default intensity of 0.001 and firms in rating class B have a default intensity of 0.01. Assume that there is a "basic" migration intensity of 0.1 from A to B and the same intensity from B to A. In addition to this basic migration, there is a contagion effect in ratings in the following sense: every time a firm defaults from rating class B, it implies that 1% of the A-rated firms are instantaneously downgraded into B. No A or B-rated firm is thrown directly into default because of the default of another firm, but some downgrades from A to B are due to a contagion effect from the defaults of B-rated firms. If we simulate a sample of firms that follow these dynamics, we subsequently estimate the default intensities of all firms as a function of rating, and finally we transform the time of default arrivals by the cumulative intensities of all firms, then we do not see a violation of the conditional independence assumption. Yet it is clear that this setup has contagion through the (only) covariate, namely the rating of the firms.

We performed a simulation study based on 1,000 firms initially rated A and 1,000 firms initially rated B, and we ran the experiment for 24 years. The estimated default intensities from class A and B were very close to the actual intensities (0.01 and 0.001). The plainly estimated transition intensity from A to B was 0.123, i.e. slightly higher than 0.1 due to the number of forced downgrades. However, this "distorted" intensity estimate did not affect our time transformation, since we used instead the true intensity, which is known to us in this designed experiment. We then performed all of the Poisson distribution tests for the same bin sizes that we did for our data set in the previous section and not a single test rejected the Poisson distribution assumption.

The point of this example is to establish that the time transformation test is mostly a misspecification test and not so much a *joint test* of intensity misspecification *and* the doubly stochastic assumption. By construction, one of the hypotheses is satisfied (the model is correctly specified) but the other is not (the model is not doubly stochastic). We have then shown that the test does not reject. This must mean that it is blind in one of the directions. In summary, conditioning on firm-specific covariates and testing for conditional independence using the cumulative intensities may not reveal "contagion through the covariates". We now address a way of testing for such a contagion effect.

I.4.2 Testing for contagion through covariates

As we have just learned from our simulation experiment, it is perfectly possible that there are contagion effects in the data in the sense that observed defaults affect the firm-specific variables X_{it}. As explained above and in the appendix, the Cox regression conditioning on these variables will not detect this source of contagion. We now wish to address this issue of contagion through covariates more closely. It is difficult to test for each covariate whether it is affected by defaults of other firms. We therefore choose to use rating changes as a proxy for changes in firm-specific covariates. For our total sample of 2,557 firms over the period 1982 to 2005, we therefore consider all changes in the rating of their publicly issued debt as recorded in Moody's DRSD. Specifically, we investigate whether defaults cause an increase in the aggregate number of rating downgrades.

To ensure a reasonable comparison of ratings across firms, abstracting from differences caused by special features of the individual debt contracts, we use the Estimated Senior

Rating (ESR) as a measure of the overall default risk of the firm. For firms without an ESR, we complement the ESR data by instead using either an issuer rating if available, or alternatively a corporate family rating, in compliance with the guidelines set up by Moody's for the calculation of ESR, see Hamilton (2005). This procedure reduces the total set of firms in our data set from 2,557 to 2,503 of which the 2,434 have an ESR and the remaining 69 firms a comparable, inferred rating.

We define the aggregate downgrade intensity for the firms as

$$\eta_t = \sum_{i=1}^{n} R_{it} 1_{(\tau_i \geq t)} \left(e^{\tilde{\beta}'_W W_t} + \int_0^t (\tilde{\alpha}_0 + \tilde{\alpha}_1 Y_s) e^{-\tilde{\alpha}_2 (t-s)} dN_s + \tilde{\delta} \right) \qquad t \geq 0$$

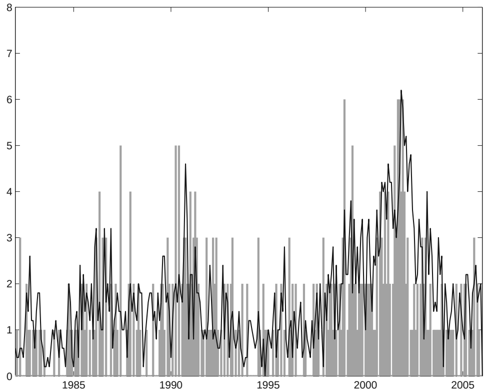
with W_t as before representing various macro variables to account for changes in rating intensities caused by business cycle variations and with R_{it}, Y_t and N_t also as previously defined. We thus allow for the same type of "contagion mechanism" from observed defaults to the intensity of (future) rating transitions as we studied in section I.3.2. Note, however, that we only allow for defaults to affect the future downgrade intensity whereas non-default downgrades do not cause a Hawkes effect. As shown in Table I.6, we find a strongly significant effect in that defaults cause the downgrade intensity to increase. We also find that defaults of larger firms have a larger effect on the downgrade intensity. The decay rate is close to 2 which means that the effect tapers off to roughly $1/8$ after one year. In Figure I.6 we show downgrade occurrences (scaled) and the default events. However, the strong significance of these tests may be difficult to attribute to contagion effects. The problem is that when we measure contagion through the ratings we may really be capturing the reactions of rating agencies to corporate defaults. These reactions could potentially reflect revisions of rating policies or extra scrutiny in light of a recent default. This extra scrutiny could lead to updating of the rating agency's measurement of critical firm characteristics and this in turn cause downgrades. As such, the measurement of contagion would be consistent with contagion taking place through updating of latent variables. However, our main focus is on whether actual, measurable key ratios are affected by economy-wide defaults. We therefore turn to conducting such tests.

Table I.6. Parameter estimates for aggregate rating downgrade intensity

The explanatory variables in the table are the same as appearing in Table I.2. Asymptotic standard errors are reported in parenthesis and statistical significance is indicated at 5% (*), 1% (**), and 0.1% (***) levels, respectively.[8]

Macro variables:			
Constant	-3.031 ***	-3.030 ***	-3.030 ***
	(0.307)	(0.444)	(0.130)
1-year S&P500 return	-0.887 *	-0.890 *	-0.887 *
	(0.468)	(0.524)	(0.387)
Industrial production	-4.864 *	-5.429 *	-4.868 *
	(2.760)	(3.941)	(2.169)
Treasury term spread	-0.270 ***	-0.298 ***	-0.270 ***
	(0.135)	(0.206)	(0.087)
Contagion effects:			
Constant ($\tilde{\alpha}_0$)	$2.5{\cdot}10^{-14}$		
	(0.003)		
Firm size ($\tilde{\alpha}_1$)	0.001 ***	0.001 ***	0.001 ***
	($4.4{\cdot}10^{-4}$)	($1.6{\cdot}10^{-4}$)	($1.7{\cdot}10^{-4}$)
Decay rate ($\tilde{\alpha}_2$)	2.007 ***	1.874 ***	2.006 ***
	(0.269)	(0.235)	(0.257)
Level ($\tilde{\delta}$)	$1.2{\cdot}10^{-10}$	$1.2{\cdot}10^{-8}$	
	(0.013)	(0.019)	

Figure I.6. Rating downgrades vs. observed defaults



Monthly number of registered U.S. industrial defaults and (scaled) number of rating downgrades among Moody's rated U.S. industrial firms (solid line) for the period 1982-2005.

I.4.3 Effects through quick ratios and distance-to-default

In this section we carry out simple regression tests to see if the average levels of distance-to-default and quick ratio are affected by corporate defaults. Specifically, we test whether changes in quick ratio and distance-to-default react to the number of defaults occurring in a preceding time window of variable length. At the same time, we control for economy-wide variables that were significant in our Cox regressions. The economic motivation for considering the distance-to-default variable is that the contagion effect through stock prices demonstrated in Lang and Stulz (1992) would have implications for individual firm's distance-to-default since this variable is computed using the equity price. Similarly, if trade credits play an economy-wide role in propagating defaults, as argued for example in Boissay (2006), then since trade credits are short-term assets for the creditor, an increasing number of observed defaults could lead to a lower quick ratio.

As a representative example of our regression tests, we consider the following regression

$$\begin{aligned}\Delta(\text{1-year distance-to-default})_t &= \eta_0 + \eta_1(\text{1-year S\&P500 return})_t + \eta_2(\text{Industrial production})_t \\ &\quad + \eta_3(\text{Treasury term spread})_t + \eta_4(\text{Defaults in } k \text{ mths.})_t\end{aligned}$$

based on monthly observations. Here, Δ(1-year distance-to-default)$_t$ is the difference between time t and $t+1$ in the cross-sectional value of distance-to-default across all firms at risk, and (Defaults in k mths.)$_t$ is the aggregate number of observed defaults within the last k months prior to t. We choose the k-month time window to be of length 1, 3, 6, 12 and 24 months, and consider for both the quick ratio and distance-to-default the median, the 10% quantile and the 90% quantile value. Note that since a low quick ratio and a low distance-to-default both are indicators of high default risk, the 10% quantile represents riskier firms. The median level tests for whether there is an effect of default on the level of the variables overall, whereas the quantiles are meant to capture effects that affect the tails – either the more risky firms or the safer firms. Ideally, we would want to look in specific sectors as well, but our data set is too thin for this purpose.

We are unable to find any effects from the number of defaults to quick ratios. As illustrated in Table I.7–Table I.9 the quick ratio seems unaffected by any information related to the number of defaults, regardless of which quantile we consider and regardless of the width of the default window. However, this is not true for the distance-to-default. As shown in Table I.10–Table I.12, we find that the number of defaults in the prior 6- and 12-month period *do* affect the changes in distance-to-default. There is no effect on the shorter horizons, and only for the 90% quantile do we see an effect from 24-month defaults.

Table I.7. Effect of previous defaults on changes in quick ratio (median)

The table reports estimation results for the time series regression

$$\Delta(\text{Quick ratio})_t = \eta_0 + \eta_1(\text{1-year S\&P500 return})_t + \eta_2(\text{Industrial production})_t + \eta_3(\text{Treasury term spread})_t + \eta_4(\text{Defaults in } k \text{ mths.})_t$$

based on monthly observations. $\Delta(\text{Quick ratio})_t$ is the change from t to $t+1$ in the cross-sectional median across all firms at risk, and $(\text{Defaults in } k \text{ mths.})_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	−0.002 *	−0.002	−0.001	−0.004	−0.004 *
1-year S&P500 return	0.006 *	0.005	0.004	0.007 *	0.005
Industrial production	−0.039 **	−0.039 *	−0.050 **	−0.021	−0.015
Treasury term spread	0.002 ***	0.002 ***	0.002 ***	0.002 ***	0.002 **
Defaults in 1 mth.	$1.7 \cdot 10^{-5}$				
Defaults in 3 mths.		$4.7 \cdot 10^{-5}$			
Defaults in 6 mths.			$-8.1 \cdot 10^{-5}$		
Defaults in 12 mths.				$6.0 \cdot 10^{-5}$	
Defaults in 24 mths.					$5.3 \cdot 10^{-5}$
R^2	0.041	0.038	0.038	0.036	0.033
Obs.	287	285	282	276	264

Table I.8. Effect of previous defaults on changes in quick ratio (10% quantile)

The table reports estimation results for the time series regression

$$\Delta(\text{Quick ratio})_t = \eta_0 + \eta_1(\text{1-year S\&P500 return})_t + \eta_2(\text{Industrial production})_t + \eta_3(\text{Treasury term spread})_t + \eta_4(\text{Defaults in } k \text{ mths.})_t$$

based on monthly observations. $\Delta(\text{Quick ratio})_t$ is the change from t to $t+1$ in the cross-sectional 10% quantile across all firms at risk, and $(\text{Defaults in } k \text{ mths.})_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	−0.001	−0.001	0.000	−0.001	−0.001
1-year S&P500 return	0.001	0.001	−0.001	0.001	−0.001
Industrial production	−0.013	−0.015	−0.028	−0.012	−0.033
Treasury term spread	0.001	0.001	0.001	0.001	0.001
Defaults in 1 mth.	$2.1 \cdot 10^{-4}$				
Defaults in 3 mths.		$4.8 \cdot 10^{-5}$			
Defaults in 6 mths.			$-7.8 \cdot 10^{-5}$		
Defaults in 12 mths.				$2.6 \cdot 10^{-5}$	
Defaults in 24 mths.					$1.6 \cdot 10^{-5}$
R^2	0.008	0.007	0.008	0.007	0.013
Obs.	287	285	282	276	264

Table I.9. Effect of previous defaults on changes in quick ratio (90% quantile)

The table reports estimation results for the time series regression

$$\Delta(\text{Quick ratio})_t = \eta_0 + \eta_1(\text{1-year S\&P500 return})_t + \eta_2(\text{Industrial production})_t + \eta_3(\text{Treasury term spread})_t + \eta_4(\text{Defaults in } k \text{ mths.})_t$$

based on monthly observations. $\Delta(\text{Quick ratio})_t$ is the change from t to $t+1$ in the cross-sectional 90% quantile across all firms at risk, and $(\text{Defaults in } k \text{ mths.})_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	−0.000	−0.005	−0.001	0.001	−0.004
1-year S&P500 return	0.056 ***	0.061 ***	0.060 ***	0.059 ***	0.056 **
Industrial production	−0.228 **	−0.194	−0.221 *	−0.223 *	−0.161
Treasury term spread	0.000	0.001	0.001	0.001	0.001
Defaults in 1 mth.	$1.2 \cdot 10^{-3}$				
Defaults in 3 mths.		$9.4 \cdot 10^{-4}$			
Defaults in 6 mths.			$2.0 \cdot 10^{-5}$		
Defaults in 12 mths.				$-1.7 \cdot 10^{-4}$	
Defaults in 24 mths.					$1.3 \cdot 10^{-5}$
R^2	0.033	0.036	0.035	0.034	0.024
Obs.	287	285	282	276	264

Table I.10. Effect of previous defaults on changes in 1-year distance-to-default (median)

The table reports estimation results for the time series regression
Δ(1-year distance-to-default)$_t = \eta_0 + \eta_1$(1-year S&P500 return)$_t + \eta_2$(Industrial production)$_t + \eta_3$(Treasury term spread)$_t + \eta_4$(Defaults in k mths.)$_t$ based on monthly observations. Δ(1-year distance-to-default)$_t$ is the change from t to $t+1$ in the cross-sectional median across all firms at risk, and (Defaults in k mths.)$_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	−0.006	0.033	0.070 ***	0.075 **	0.045
1-year S&P500 return	−0.036	−0.055	−0.083	−0.090	−0.092
Industrial production	−0.577 **	−0.784 ***	−1.080 ***	−1.074 ***	−0.726 **
Treasury term spread	0.009	0.006	0.005	0.007	0.008
Defaults in 1 mth.	0.004				
Defaults in 3 mths.		−0.002			
Defaults in 6 mths.			−0.004 **		
Defaults in 12 mths.				−0.003 **	
Defaults in 24 mths.					−0.001
R^2	0.036	0.040	0.056	0.049	0.034
Obs.	287	285	282	276	264

Table I.11. Effect of previous defaults on changes in 1-year distance-to-default (10% quantile)

The table reports estimation results for the time series regression
Δ(1-year distance-to-default)$_t = \eta_0 + \eta_1$(1-year S&P500 return)$_t + \eta_2$(Industrial production)$_t + \eta_3$(Treasury term spread)$_t + \eta_4$(Defaults in k mths.)$_t$ based on monthly observations. Δ(1-year distance-to-default)$_t$ is the change from t to $t+1$ in the cross-sectional 10% quantile across all firms at risk, and (Defaults in k mths.)$_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	0.001	0.004	0.034 **	0.035	0.013
1-year S&P500 return	−0.021	−0.025	−0.048	−0.052	−0.052
Industrial production	−0.461 ***	−0.485 ***	−0.720 ***	−0.696 ***	−0.433 **
Treasury term spread	0.013 **	0.012 **	0.010 *	0.011 **	0.012 **
Defaults in 1 mth.	−0.001				
Defaults in 3 mths.		−0.000			
Defaults in 6 mths.			−0.003 **		
Defaults in 12 mths.				−0.002 *	
Defaults in 24 mths.					−0.000
R^2	0.057	0.057	0.075	0.066	0.050
Obs.	287	285	282	276	264

Table I.12. Effect of previous defaults on changes in 1-year distance-to-default (90% quantile)

The table reports estimation results for the time series regression

Δ(1-year distance-to-default)$_t = \eta_0 + \eta_1$(1-year S&P500 return)$_t + \eta_2$(Industrial production)$_t + \eta_3$(Treasury term spread)$_t + \eta_4$(Defaults in k mths.)$_t$ based on monthly observations. Δ(1-year distance-to-default)$_t$ is the change from t to $t+1$ in the cross-sectional 90% quantile across all firms at risk, and (Defaults in k mths.)$_t$ is the aggregate number of observed defaults within the last k months prior to t. Asymptotic standard errors are Newey-West-corrected, and statistical significance is indicated at 10% (*), 5% (**), and 1% (***) levels.

Constant	0.013	0.053 **	0.085 ***	0.104 ***	0.087 **
1-year S&P500 return	−0.034	−0.064	−0.085	−0.104 *	−0.120
Industrial production	−0.709 **	−1.008 ***	−1.264 ***	−1.363 ***	−1.053 ***
Treasury term spread	0.003	−0.001	−0.002	0.000	0.003
Defaults in 1 mth.	0.009				
Defaults in 3 mths.		−0.002			
Defaults in 6 mths.			−0.004 **		
Defaults in 12 mths.				−0.003 **	
Defaults in 24 mths.					−0.001 **
R^2	0.032	0.035	0.042	0.042	0.032
Obs.	287	285	282	276	264

To assess the economic significance of this impact on the distance-to-default variable, we can look at both an aggregate and a marginal effect. By multiplying the average number of defaults within one year, 15.1 in our data set, with the regression coefficient (-0.003) from Table 10, we see that the distance-to-default variable for an average firm is approximately 0.045 lower due to the impact of other firms having defaulted within the last 12 months. This effect approximately transforms into an increase in the average default intensity by a factor of $\exp(-0.8 \cdot (-0.045)) = 1.037$. Similarly, we find that the marginal effect on the distance-to-default variable and hence on the default probability of one extra firm defaulting is an approximate increase in the default intensity of $\exp(-0.8 \cdot (-0.003)) = 1.0024$. Here, it is important to keep in mind that the average estimated default intensity (cross-sectionally and across time) in our model is 0.0061, and hence the effect of contagion through the distance-to-default measure will be small measured in absolute terms.

With better data on the specific financial interactions between firms and sectors, we might be able to explain why the 6-month and 12-month windows turn out to be significant, although the problem of establishing causality is inherently difficult, since one can imagine that the covariates of a given firm may be affected already *prior* to the default of another firm. In this context it is worth noting that in our current implementation, the self-exciting default effect represented by the parameters $\alpha_0, \alpha_1, \alpha_2$ in the Hawkes process (I.1) is not significant once we control for firm-specific variables (see Table I.5). However, unreported results show that it becomes significant at the 1% level, if instead we leave out the firm-specific variables and only use macro variables in our specification of the default intensity. This finding is consistent with the idea that previous defaults *do* affect the probability of other firms defaulting in the future, but that we are able to capture this effect through the influence on the firm-specific explanatory variables. We leave a more detailed analysis of this topic for future research.

I.5 Conclusion

In this paper we re-investigate the time-change method used by DDKS for testing whether company defaults in the U.S. can be viewed as doubly stochastic. While DDKS reject the statistical tests based on the time transformation, we show (on a slightly smaller data set) that if we use a different specification of firms' default intensities we cannot reject the same tests. To show that this is not due to a lack of power, we show that we do reject in most tests with the intensity specification used in DDKS.

The time-change procedure is based on testing Poisson properties of a time-transformed process of aggregate defaults. We observe that the Poisson property may be satisfied even if defaults are not doubly stochastic. Thus, the fact that we cannot reject the Poisson property need not be indicative of conditional independence. The reason for this is that the time transformation, which the test procedure proposed in DDKS relies on, in fact works for a very large class of models, and therefore in particular will not capture contagion through observed covariates. It therefore needs to be adjusted if one wants to rule out such contagion effects. We provide an illustrative example which conveys the intuition.

To specifically test for the possibility of contagion through covariates, we first use rating as a summary statistic for the credit quality of individual firms. The idea is that if a default of one firm significantly affects another firm's credit quality by changing its explanatory variables, then this should also be reflected in the rating. We therefore test whether rating downgrade intensities are affected by previous defaults, and we find a significant contagion effect. Still, this may be an effect caused by the behaviour of rating agencies rather than actual default intensity changes alone, so we also perform regression tests to see if the explanatory variables quick ratio and distance-to-default are affected by the occurrence of defaults after controlling for macroeconomic variables. We find no effects of previous defaults on quick ratio but do find an effect on distance-to-default. This is consistent with the empirical evidence found in Lang and Stulz (1992) in which equity prices are negatively affected by bankruptcy announcements. Our findings suggest that even if we are able to capture the empirical behaviour of default intensities using a Cox regression involving firm-specific covariates, the fact that defaults have an impact on covariates means that the doubly stochastic assumption is not satisfied. Hence, when modelling future exposures of a credit portfolio, it is not enough to simulate the evolution of the covariates and then simulate defaults using a doubly stochastic assumption. One will also have to model the feedback effects of default on the firm-specific variables. This suggests an alternative to frailty modelling in that we are relying exclusively on observable variables. This is a topic for future research.

Appendix

There are no new results about counting processes in our paper. In this appendix we simply restate various theoretical results from standard point process theory that we need for our exposition in the paper. We explain here why the time transformation used in the study of DDKS only needs orthogonality of the counting processes, we take care of a technical problem related to the fact that intensities die out in our model, and finally we address an issue related to bias in the empirical application of the transformation result.

Let τ_i denote the default time of firm i. For the definition of the intensity, it is convenient to work with the single jump counting process N_i which starts at zero and jumps to one at the time of the default, i.e

$$N_i(t) = 1_{(\tau_i \geq t)}.$$

All of our processes are defined on a probability space $(\Omega, \mathcal{F}, P)$ and information on the point processes and on their *intensities* is given by the filtration $(\mathcal{F}_t)_{t\geq 0}$.

Recall that a counting process N_i is said to have intensity λ_i with respect to the filtration $(\mathcal{F}_t)_{t\geq 0}$ if

$$N_i(t) - \int_0^t \lambda_i(s)\, ds$$

is an $\mathcal{F}_t-$martingale. In our analysis we study firms over a time period $[0, T]$, and for a firm that is not under observation at time 0, the intensity is set to 0 until it enters the sample. Similarly, the intensity again falls to 0, when the company exits the sample (because of default or for some other reason, e.g. delisting).

Two point processes N_i and N_j are orthogonal if the probability of having simultaneous jumps is 0, i.e

$$P\big(\Delta N_i(t) \Delta N_j(t)\big) = 0$$

for all t. If we define N as the aggregate counting process $N(t) = \sum_{i=1}^n N_i(t)$, then the following is immediate: if $N_1, N_2, \ldots$ are pairwise orthogonal and have $\mathcal{F}_t-$intensities $\lambda_1, \lambda_2, \ldots$, then N is a counting process with $\mathcal{F}_t-$intensity $\lambda = \sum_{i=1}^n \lambda_i$. This observation follows immediately by noting that orthogonality ensures that N jumps at most by 1 and from the fact that

$$N(t) - \int_0^t \lambda(s) ds = \sum_{i=1}^n \big(N_i(t) - \int_0^t \lambda_i(s)\big)$$

is a sum of martingales and therefore itself a martingale.

A counting process N on the line which has an intensity process can be transformed into a Poisson process under a very simple condition: let N have intensity λ, define $\Lambda(t) = \int_0^t \lambda(s)\,ds$, and assume that $\Lambda(t) \to \infty$ almost surely. Then the process defined as $\tilde{N}(t) = N\big(\Lambda^{-1}(t)\big)$ is a unit rate Poisson process. This result is usually ascribed to Meyer (1971) with a multitude of successive variations and extensions in e.g. Papangelou (1972), Brémaud (Brémaud), Aalen and Hoem (1978), Cocozza and Yor (1980), Brown and Nair (1988) and Kallenberg (1990).[12]

The condition in Meyer's result that the integrated intensity $\Lambda(t)$ converges to infinity almost surely merely ensures that the counting process N "does not run out of jumps" to form the transformed Poisson process $\tilde{N}$. With several thousand firms in our sample and new firms entering the sample continually, this assumption is harmless for our application. Even if new firms do not enter the sample, it is clear that we can alter the process N ever so slightly by adding a Poisson process with rate $\epsilon > 0$ to N. No matter how small ϵ is, this modified process N^ϵ satisfies the requirement of Meyer's theorem, and the transformation of N^ϵ by the cumulated intensity is indistinguishable for ϵ very small from the transformation of N by Λ.

As a final remark on the practical implementation of Meyer's theorem, note that since we do not know the true integrated intensity Λ, we must use the estimated integrated intensity resulting from our maximum likelihood estimation. This, in principle, leads to a minor bias in the time transformation. However, the magnitude of this bias seems to be of minor importance in our tests. For more details on this problem, we refer the reader to Schoenberg (2002).

[12] See Aalen and Hoem (1978) for a brief historical review.

Essay II

Systematic and idiosyncratic default risk in synthetic credit markets*

Co-authored with Peter Feldhütter, London Business School.
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Abstract

We present a new estimation approach that allows us to extract from spreads in synthetic credit markets the contribution of systematic and idiosyncratic default risk to total default risk. Using an extensive data set of 90,600 CDS and CDO tranche spreads on the North American Investment Grade CDX index we conduct an empirical analysis of an intensity-based model for correlated defaults. Our results show that systematic default risk is an explosive process with low volatility, while idiosyncratic default risk is more volatile but less explosive. Also, we find that the model is able to capture both the level and time series dynamics of CDO tranche spreads.

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II.1 Introduction

Campbell and Taksler (2003) show that idiosyncratic firm-level volatility is a major driver of corporate bond yield spreads and that there has been an upward trend over time in idiosyncratic equity volatility in contrast to market-wide volatility. This suggests that in order to understand changing asset prices over time, it is important to separate out and understand the dynamics of both idiosyncratic and systematic volatility. In this paper, we present a new approach to separate out the size and time series behaviour of idiosyncratic and systematic (default intensity) volatility by using information in synthetic credit markets.

Markets for credit derivatives have experienced massive growth in recent years (see Duffie (2008)) and numerous models specifying default and correlation dynamics have been proposed. A good model of multi-name default should ideally have the following properties (see Collin-Dufresne (2009) for a discussion). First, the model should be able to match prices consistently such that for a fixed set of model parameters, prices are matched over a period of time. This is important for pricing non-standard products in a market where prices are available for standard products. Second, the model should have parameters that are economically interpretable, such that parameter values can be discussed and critically evaluated. If a non-standard product needs to be priced and parameters cannot be inferred from existing market prices, economic interpretability provides guidance in choosing parameters. Third, credit spreads and their correlation should be modelled dynamically such that options on multi-name products can be priced. And fourth, since market makers quote spreads at any given time, pricing formulas should not be too time-consuming to evaluate.

In single-name default modelling the stochastic intensity-based framework introduced in Lando (1994) and Duffie and Singleton (1999) has proven very successful and is widely used.[1] Default of a firm in an intensity-based model is determined by the first jump of a pure jump process with a stochastic default intensity. We follow Duffie and Gârleanu (2001) and model the default intensity of a firm as the sum of an idiosyncratic and a common component, where the latter affects the default intensity of all firms in

[1] Examples of empirical applications are Duffie and Singleton (1997), Duffee (1999), and Longstaff, Mithal, and Neis (2005).

the economy. In this setting, credit spreads are matched, parameters are interpretable, and pricing of options is possible. But even though the framework has many attractive properties it has not been used much because estimation poses a challenge.

We present in this paper a new approach to estimate intensity-based models from spreads observed in synthetic credit markets. The main challenge so far has been that the estimation of a model based on an index with 125 names requires simultaneous estimation of a common factor and 125 idiosyncratic factors. The solution has been to impose strong parameter restrictions on the idiosyncratic factors (see among others Mortensen (2006) and Eckner (2007; 2009)). We specify the process for systematic default risk and show how idiosyncratic risk can be left unmodelled. This reduces the problem of estimating 126 factors to estimating one factor. Subsequently, we parameterize and estimate idiosyncratic default factors one at a time. Thus, our approach reduces the problem of estimating 126 factors simultaneously to 126 single-factor estimations. Furthermore, restrictions on idiosyncratic factors are not necessary.

We apply our approach to the North American Investment Grade CDX index, and estimate both systematic and idiosyncratic default risk as affine jump-diffusion processes using CDS and CDO spreads. Papers imposing strong parameter restrictions have found that intensity-based jump-diffusion models can match the levels but not the time series behaviour of CDO tranche spreads (see Eckner (2007) for a discussion). We find that the models can in fact match not only the levels but also the time series behaviour in tranche spreads. That is, once parameter restrictions are not imposed, the model gains the ability to match time series dynamics of systematic and unsystematic default risk. We also find that idiosyncratic default risk is a major driver of total default risk consistent with the findings in Campbell and Taksler (2003). Furthermore, we confirm the finding in Zhang, Zhou, and Zhu (2009) that both diffusion volatility and jumps are important for default risk. More importantly, our analysis allows us to separate idiosyncratic and systematic default risk into a diffusion and a jump part, and this yields new insights: compared to systematic default risk, idiosyncratic default risk has a higher diffusion volatility, a higher contribution from jumps, and is less explosive.

An alternative modelling approach to that of ours is to model aggregate portfolio loss and fit the model to CDO tranche spreads. This is the approach taken in for example Longstaff and Rajan (2008), Errais, Giesecke, and Goldberg (2010), and Giesecke, Goldberg, and Ding (2011). Since the default intensity of individual firms is not modelled, this approach is not useful for examining individual default risk, whether it is systematic or unsystematic.

The paper is organized as follows. Section II.2 formulates the multi-name default model and derives CDO tranche pricing formulas. Section II.3 explains the estimation methodology, and section II.4 describes the data. Section II.5 examines the ability of the model to match CDO tranche spreads and examines the properties of systematic default risk, while idiosyncratic default risks are examined in section II.6 . Section II.7 concludes.

II.2 Intensity-based default risk model

This section explains the model framework that we employ for pricing single- and multi-name credit securities. For single-name Credit Default Swaps (CDSs) we use the intensity-based framework introduced in Lando (1994) and Duffie and Singleton (1999). For multi-name Collateralized Debt Obligation (CDO) valuation we follow Duffie and Gârleanu (2001) and model the default intensity of each underlying issuer as the sum of an idiosyncratic and a common process. Default correlation among issuers thus arises through the joint dependence of individual default intensities on the common factor. Furthermore, we generalize the model in Duffie and Gârleanu (2001) by allowing for a flexible specification of the idiosyncratic processes, while maintaining semi-analytical calculation of the loss distribution as in Mortensen (2006). This extension allows us to avoid the ad hoc parameter restrictions that are common in the existing literature.

II.2.1 Default modelling

We assume that the time of default of a single issuer, τ, is modelled through an intensity $(\lambda_t)_{t\geq 0}$, which implies that the risk-neutral probability at time t of defaulting within a short period of time Δt is approximately

$$Q_t(\tau \leq t + \Delta t | \tau > t) \approx \lambda_t \Delta t.$$

Unconditional default probabilities are given by

$$Q_t(\tau \leq s) = 1 - E_t^Q \left[\exp\left(- \int_t^s \lambda_u du \right) \right] \tag{II.1}$$

which shows that default probabilities in an intensity-based framework can be calculated using techniques from interest rate modelling.

In our model we consider a total of N different issuers. To model correlation between individual issuers we follow Mortensen (2006) and assume that the intensity of each issuer is given as the sum of an idiosyncratic component and a scaled common component

$$\lambda_{i,t} = a_i Y_t + X_{i,t} \tag{II.2}$$

where $a_1, \ldots, a_N$ are non-negative constants and $Y, X_1, X_2, \ldots, X_N$ are independent stochastic processes. The common factor Y creates dependence in default occurrences among the N issuers and may be viewed as reflecting the overall state of the economy, while X_i similarly represents the idiosyncratic default risk for firm i. Thus, a_i indicates the sensitivity of firm i to the performance of the macroeconomy, and we allow this parameter to vary across firms, contrary to Duffie and Gârleanu (2001) that assume $a_i = 1$ for all i and thereby enforce a homogeneous impact of the macroeconomy on all issuers.

We assume that the common factor follows an affine jump diffusion under the risk-neutral measure

$$dY_t = (\kappa_0 + \kappa_1 Y_t)dt + \sigma\sqrt{Y_t}dW_t^Q + dJ_t^Q \tag{II.3}$$

where W^Q is a Brownian motion, jump times (independent of W^Q) are those of a Poisson process with intensity $l \geq 0$, and jump sizes are independent of the jump times and follow an exponential distribution with mean $\mu > 0$. This process is well-defined for $\kappa_0 > 0$. As a special case, if the jump intensity is equal to zero the default intensity then follows a CIR process.

We do *not* impose any distributional assumptions on the evolution of the idiosyncratic factors $X_1, ..., X_N$. In particular, they are not required to be affine jump diffusions. This generalizes the setup in Duffie and Gârleanu (2001), Mortensen (2006), and Eckner (2009), where the idiosyncratic factors are required to be affine jump diffusions with very restrictive assumptions on their parameters.

II.2.2 Risk premium

For the basic affine process in equation (II.3) we assume an essentially affine risk premium for the diffusive risk and constant risk premia for the risk associated with the timing and sizes of jumps. Cheridito, Filipovic, and Kimmel (2007) propose an extended affine risk premium as an alternative to an essentially affine risk premium, which would allow the parameter κ_0 to be adjusted under P in addition to the adjustment of κ_1. However, extended affine models require the Feller condition to hold and since this restriction is likely to be violated, as discussed in Feldhütter (2008), we choose the more parsimonious essentially affine risk premium.[2]

This leads to the following dynamics for the common factor under the historical measure P

$$dY_t = (\kappa_0 + \kappa_1^P Y_t)dt + \sigma\sqrt{Y_t}dW_t^P + dJ_t^P \tag{II.4}$$

where W^P is a Brownian motion, jump times (independent of W^P) are those of a Poisson process with intensity l^P, and jump sizes are independent of the jump times and follow an exponential distribution with mean $\mu^P > 0$.

II.2.3 Aggregate default distribution

Our model allows for semi-analytic calculation of the distribution of the aggregate number of defaults among the N issuers. More specifically, we can at time t calculate in semi-closed form the distribution of the aggregate number of defaults at time $s \geq t$ by conditioning on the common factor. If we let

$$Z_{t,s} = \int_t^s Y_u du$$

denote the integrated common factor, then it follows from (II.1) and (II.2) that conditional on $Z_{t,s}$, defaults are independent and the conditional default probabilities given as

$$p_{i,t}(s|z) := Q_t(\tau_i \leq s|Z_{t,s} = z) = 1 - \exp(-a_i z)E_t^Q\left[\exp\left(-\int_t^s X_{i,u}du\right)\right]. \tag{II.5}$$

The total number of defaults at time s among the N issuers, D_s^N, is then found by the recursive algorithm[3]

$$Q_t(D_s^N = j|z) = Q_t(D_s^{N-1} = j|z)(1 - p_{N,t}(s|z)) + Q_t(D_s^{N-1} = j-1|z)p_{N,t}(s|z)$$

[2] To illustrate why the Feller condition is necessary in extended affine models consider the simple diffusion case, $dY_t = (\kappa_0^Q + \kappa_1^Q Y_t)dt + \sigma\sqrt{Y_t}dW^Q$. The risk premium $\Lambda_t = \frac{\lambda_0}{\sqrt{Y_t}} + \lambda_1\sqrt{Y_t}$ keeps the process affine under P but the risk premium explodes if $Y_t = 0$. To avoid this, the Feller restriction $\kappa_0 > \frac{\sigma^2}{2}$ under both P and Q ensures that Y_t is strictly positive.

[3] The last term disappears if $j = 0$.

due to Andersen, Sidenius, and Basu (2003). The unconditional default distribution is therefore given as

$$Q_t(D_s^N = j) = \int_0^\infty Q_t(D_s^N = j|z) f_{t,s}(z) dz \tag{II.6}$$

where $f_{t,s}$ is the density function for $Z_{t,s}$. Finally, $f_{t,s}$ can be determined by Fourier inversion of the characteristic function $\phi_{Z_{t,s}}$ for $Z_{t,s}$ as

$$f_{t,s}(z) = \frac{1}{2\pi} \int_{-\infty}^\infty \exp(-iuz) \phi_{Z_{t,s}}(u) du \tag{II.7}$$

where we apply the closed-form expression for $\phi_{Z_{t,s}}$ derived in Duffie and Gârleanu (2001).[4]

II.2.4 Synthetic CDO pricing

CDOs began to trade frequently in the mid-nineties and in the last decade issuance of CDOs has experienced massive growth, see BIS (2007). In a CDO the credit risk of a portfolio of debt securities is passed on to investors by issuing CDO tranches written on the portfolio. The tranches have varying risk profiles according to their seniority. A synthetic CDO is written on CDS contracts instead of actual debt securities. To illustrate the cash flows in a synthetic CDO an example that reflects the data used in this paper is useful.

Consider a CDO issuer, called A, who sells credit protection with notional \$0.8 million in 125 5-year CDS contracts for a total notional of \$100 million. Each CDS contract is written on a specific corporate bond, and agent A receives quarterly a CDS premium until the CDS contract expires or the bond defaults. In case of default, agent A receives the defaulted bond in exchange for face value. The loss is therefore the difference between face value and market value of the bond.[5]

Agent A at the same time issues a CDO tranche on the first 3% of losses in his CDS portfolio and agent B "buys" this tranche, which has a principal of \$3 million. No money is exchanged at time 0, when the tranche is sold. If the premium on the tranche is, say, 2,000 basis points, agent A pays a quarterly premium of 500 basis points to agent B on the remaining principal. If a default occurs on any of the underlying CDS contracts, the

[4] Duffie and Gârleanu (2001) derive an explicit solution for $E_t^Q[\exp(q\int_t^s Y_u du)]$ when q is a real number, but as noted by Eckner (2009) the formula works equally well for q complex.

[5] Pricing CDS contracts is explained in Appendix II.A.

loss is covered by agent B and his principal is reduced accordingly. Agent B continues to receive the premium on the remaining principal until either the CDO contract matures or the remaining principal is exhausted. Since the first 3% of portfolio losses are covered by this tranche it is called the $0\% - 3\%$ tranche. Agent A similarly sells $3\% - 7\%$, $7\% - 10\%$, $10\% - 15\%$, $15\% - 30\%$, and $30\% - 100\%$ tranches such that the total principal equals the principal in the CDS contracts. For a tranche covering losses between K_1 and K_2, K_1 is called the attachment point and K_2 the exhaustion point.

Next, we find the fair spread at time t on a specific CDO tranche. Consider a tranche that covers portfolio losses between K_1 and K_2 from time $t_0 = t$ to $t_M = T$, and assume that the tranche has quarterly payments at time $t_1, \ldots, t_M$. The tranche premium is found by equating the value of the protection and premium payments. We denote the total portfolio loss in percent at time s as L_s, i.e. the percentage number of defaults $D_s^N \,/\, N$ times $1 - \delta$, where δ is the recovery rate, which we assume to be constant at 40%. The tranche loss is then given as

$$T_{K_1,K_2}(L_s) = \max\{\min\{L_s, K_2\} - K_1, 0\}$$

and the value of the protection payment in a CDO tranche with maturity T is therefore

$$Prot(t,T) = E_t^Q \left[\int_t^T \exp\left(- \int_t^s r_u du \right) dT_{K_1,K_2}(L_s) \right]$$

while the value of the premium payments is the annual tranche premium $S(t,T)$ times

$$\begin{aligned} & Prem(t,T) \\ = \; & E_t^Q \left[\sum_{j=1}^{M} \exp\left(- \int_t^{t_j} r_u du \right) (t_j - t_{j-1}) \int_{t_{j-1}}^{t_j} \frac{K_2 - K_1 - T_{K_1,K_2}(L_s)}{t_j - t_{j-1}} ds \right] \end{aligned}$$

where r_u is the riskfree interest rate and $\int_{t_{j-1}}^{t_j} \frac{K_2 - K_1 - T_{K_1,K_2}(L_s)}{t_j - t_{j-1}} ds$ is the remaining principal during the period t_{j-1} to t_j. The CDO tranche premium at time t is thus given as $S(t,T) = \frac{Prot(t,T)}{Prem(t,T)}$.

We follow Mortensen (2006) and discretize the integrals appearing in $Prot(t,T)$ and $Prem(t,T)$ at premium payment dates, we assume that the riskfree rate is uncorrelated with portfolio losses, and that defaults occur halfway between premium payments. Under these assumptions the value of the protection payment is

$$Prot(t,T) = \sum_{j=1}^{M} P\left(t, \frac{t_j + t_{j-1}}{2}\right) \left(E_t^Q[T_{K_1,K_2}(L_{t_j})] - E_t^Q[T_{K_1,K_2}(L_{t_{j-1}})] \right)$$

while the expression for the premium payments reduces to

$$\begin{aligned} & Prem(t,T) \\ = & \sum_{j=1}^{M}(t_j - t_{j-1})P(t,t_j)\Big(K_2 - K_1 - \frac{E_t^Q[T_{K_1,K_2}(L_{t_{j-1}})] + E_t^Q[T_{K_1,K_2}(L_{t_j})]}{2}\Big) \end{aligned}$$

where $P(t,s) = E_t^Q[\exp(-\int_t^s r_u du)]$ is the price at time t of a riskless zero coupon bond maturing at time s.

II.3 Estimation

The parameters in our intensity model are estimated in three separate steps. First we imply out firm-specific term structures of risk-neutral survival probabilities from daily observations of CDS spreads, second, we use the inferred survival probabilities to estimate each issuer's sensitivity a_i to the economy-wide common factor Y, and finally we estimate the parameters and the path of the common factor using a Bayesian MCMC approach.[6] An important ingredient in the third step is our explicit use of the calibrated survival probabilities, which implies that we do not need to impose any structure on the idiosyncratic factors X_i.

In other words, we can estimate the model without putting specific structure on the idiosyncratic factors and this has several advantages, which we discuss in section II.3.1. Note also that our estimation approach is consistent with the common view that CDS contracts may be used to read off market views of marginal default probabilities, whereas basket credit derivatives instead reflect the correlation patterns among the underlying entities, see e.g. Mortensen (2006).

In an additional fourth step of the estimation procedure, we take in section II.6 a closer look at the cross-section of the idiosyncratic factors implicitly given by the inferred survival probabilities and the estimated common factor. Here, we impose a dynamic structure on each X_i and then estimate the parameters for each idiosyncratic factor separately, again using MCMC methods.

[6] For a general introduction to MCMC see Robert and Casella (2004) and for a survey of MCMC methods in financial econometrics see Johannes and Polson (2006).

II.3.1 A General Estimation Approach

For each day in our data sample we observe 5 CDO tranche spreads as well as CDS spreads for a range of maturities for each of the 125 firms underlying the CDO tranches. Previous literature on CDO pricing has also studied models of the form (II.2), but only by imposing strong assumptions on the parameters of the idiosyncratic factors X_i, as well as by disregarding the information in the term structure of CDS spreads (Mortensen (2006), Eckner (2007; 2009)). In this paper, we remove both of these shortcomings by allowing the idiosyncratic factors to be of a very general form, while we at the same time use all the available information from each issuer's term structure of CDS spreads.

Theoretically, if we had CDS contracts for any maturity we could extract survival probabilities for any future time-horizon, but in practice CDS contracts are only traded for a limited range of maturities. To circumvent this problem we assume a flexible parametric form for the term structure of risk-neutral survival probabilities, and use that to infer survival probabilities from the observed CDS spreads.[7] That is, on any given day t and for any given firm i we extract from the observed term structure of CDS spreads the term structure of marginal survival probabilities $s \mapsto q_{i,t}(s)$, where

$$q_{i,t}(s) = Q_t(\tau_i > s) = E_t^Q \left[\exp \left(- \int_t^s (a_i Y_u + X_{i,u}) du \right) \right],$$

see appendix II.B for details. Once we condition on the value of the common factor, this directly gives us the idiosyncratic component

$$E_t^Q \left[\exp \left(- \int_t^s X_{i,u} du \right) \right]$$

of the risk-neutral survival probability. Thus, we can use observed CDS spreads to derive values of the function $s \mapsto E_t^Q[\exp(-\int_t^s X_{i,u} du)]$, which is all we need to calculate the aggregate default distribution (and hence compute CDO tranche spreads) using equation (II.5). Therefore, we do not need to explicitly model the stochastic behaviour of each X_i in order to price CDO tranches.

For each firm i in our sample, the parameter a_i measures that firm's sensitivity to the overall state of the economy, and this parameter can be estimated directly from the inferred term structures of survival probabilities $s \mapsto q_{i,t}(s)$. Intuitively, a_i measures

[7] This procedure is essentially similar to the well-known technique for inferring a term structure of interest rates from observed prices of coupon bonds, see Nelson and Siegel (1987).

to what extent the default probability of firm i is correlated with the average default probability (since this average mainly reflects exposure to the systematic risk factor Y), and therefore a consistent estimate of a_i is given by the slope coefficient in the regression of firm i's short-term default probability on the average short-term default probability of all 125 issuers.[8] Appendix II.C provides the technical details.

Once we have inferred marginal default probabilities from CDS spreads and estimated common factor loadings a_i, we can then, given the parameters and current value of the common factor Y, price CDO tranches.

II.3.2 MCMC Methodology

In order to write the CDO pricing model on state space form, the continuous-time specification in equation (II.4) is approximated using an Euler scheme

$$Y_{t+1} - Y_t = (\kappa_0 + \kappa_1^P Y_t)\Delta_t + \sigma\sqrt{\Delta_t Y_t}\epsilon_{t+1}^Y + J_{t+1}Z_{t+1} \tag{II.8}$$

where Δ_t is the time between two observations and

$$\begin{aligned} \epsilon_{t+1}^Y &\sim N(0,1) \\ Z_{t+1} &\sim \exp(\mu^P) \\ P(J_{t+1} = 1) &= l^P \Delta_t. \end{aligned}$$

To simplify notation in the following, we let $\Theta^Q = (\kappa_0, \kappa_1, l, \mu, \sigma)$, $\Theta^P = (\kappa_1^P, l^P, \mu^P)$, and $\Theta = (\Theta^Q, \Theta^P)$.

On each day $t = 1, ..., T$, 5 CDO tranche spreads are recorded and stacked in the 5×1 vector S_t, and we let S denote the $5 \times T$ matrix with S_t in the t'th column. The logarithm of the observed CDO spreads are assumed to be observed with measurement error, so the observation equation is

$$\log(S_t) = \log(f(\Theta^Q, Y_t)) + \epsilon_t, \qquad \epsilon_t \sim N(0, \Sigma_\epsilon) \tag{II.9}$$

where f is the CDO pricing formula. Appendix II.D gives details on how to calculate f in the estimation of the common factor Y. For the estimation of each of the idiosyncratic factors X_i in section II.6, Y is replaced by X_i and f is instead the model-implied

[8] The average of the a_i's are without loss of generality normalized to 1.

idiosyncratic part of the survival probability, i.e. $E_t^Q[\exp(-\int_t^{t+s} X_{i,u}du)]$, calculated for each of the time horizons $s = 0.5$, 1, 2, 3, 4, and 5 years.

The interest lies in samples from the target distribution $p(\Theta, \Sigma_\epsilon, Y, J, Z|S)$. The Hammersley-Clifford Theorem (Hammersley and Clifford (1970), Besag (1974)) implies that samples are obtained from the target distribution by sampling from a number of conditional distributions. Effectively, MCMC solves the problem of simulating from a complicated target distribution by simulating from simpler conditional distributions. If one samples directly from a full conditional the resulting algorithm is the Gibbs sampler (Geman and Geman (1984)). If it is not possible to sample directly from the full conditional distribution one can sample by using the Metropolis-Hastings algorithm (Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953)). We use a hybrid MCMC algorithm that combines the two since not all conditional distributions are known. Specifically, the MCMC algorithm is given by (where $\Theta_{\setminus\theta_i}$ is defined as the parameter vector Θ without parameter θ_i)[9]

$$\begin{aligned}
p(\theta_i|\Theta^Q_{\setminus\theta_i}, \Theta^P, \Sigma_\epsilon, Y, J, Z, S) &\sim \text{Metropolis-Hastings} \\
p(\kappa_1^P|\Theta^Q, \Theta^P_{\setminus\kappa_1^P}, \Sigma_\epsilon, Y, J, Z, S) &\sim \text{Normal} \\
p(l^P|\Theta^Q, \Theta^P_{\setminus l^P}, \Sigma_\epsilon, Y, J, Z, S) &\sim \text{Beta} \\
p(\mu^P|\Theta^Q, \Theta^P_{\setminus\mu^P}, \Sigma_\epsilon, Y, J, Z, S) &\sim \text{Inverse Gamma} \\
p(\Sigma_\epsilon|\Theta, Y, J, Z, S) &\sim \text{Inverse Wishart} \\
p(Y|\Theta, \Sigma_\epsilon, J, Z, S) &\sim \text{Metropolis-Hastings} \\
p(J|\Theta, \Sigma_\epsilon, Y, Z, S) &\sim \text{Bernoulli} \\
p(Z|\Theta, \Sigma_\epsilon, Y, J, S) &\sim \text{Exponential or Restricted Normal}
\end{aligned}$$

Details of the derivations of the conditional and proposal distributions in the Metropolis-Hastings steps are given in Appendix II.E. Both the parameters and the latent processes are subject to constraints and if a draw is violating a constraint it can simply be discarded (Gelfand, Smith, and Lee (1992)).

[9] All random numbers in the estimation are draws from Matlab 7.0's generator which is based on Marsaglia and Zaman (1991)'s algorithm. The generator has a period of almost 2^{1430} and therefore the number of random draws in the estimation is not anywhere near the period of the random number generator.

II.4 Data

In our estimation we use daily CDS and CDO quotes from MarkIt Group Limited. MarkIt receives data from more than 50 global banks and each contributor provides pricing data from its books of record and from feeds to automated trading systems. These data are aggregated into composite numbers after filtering out outliers and stale data and a price is published only if a minimum of three contributors provide data.

We focus in this paper on CDS and CDO prices (i.e. spreads) for defaultable entities in the Dow Jones CDX North America Investment Grade (NA IG) index. The index contains 125 North American investment grade entities and is updated semi-annually. For our sample period March 21, 2006 to September 20, 2006, the latest version of the index is CDX NA IG Series 6. We specifically select the most liquid CDO tranches, the 5-year tranches, with CDX NA IG 6 as the underlying pool of reference CDSs. These tranches mature on June 20, 2011. Daily spreads of the five CDO tranches we consider: $0\% - 3\%$, $3\% - 7\%$, $7\% - 10\%$, $10\% - 15\%$, and $15\% - 30\%$, are not available for the first 7 days of the period, so the data we use in the estimation covers the period from March 30, 2006 to September 20, 2006. There are holidays on April 14, April 21, June 3, July 4, and September 4, thus leaving a total of 120 days with spreads available.

The quoting convention for the equity tranche (i.e. the $0\% - 3\%$ tranche) differs from that of the other tranches. Instead of quoting a running premium, the equity tranche is quoted in terms of an upfront fee. Specifically, an upfront fee of 30% means that the investor receives 30% of the tranche notional at time 0 plus a fixed running premium of 500 basis points per year, paid quarterly.[10]

In addition to the CDO tranche spreads, we also use 0.5-, 1-, 2-, 3-, 4-, and 5-year

[10] Upfront payments may be converted to running spreads using so-called "risky duration", see e.g. Amato and Gyntelberg (2005). This calculation requires a fully parametric model, and hence is not possible within our modelling framework. Instead we use the original upfront payment quotes available from MarkIt for the equity tranche.

CDS spreads for each of the 125 index constituents.[11] The total number of observations in the estimation of the multi-name default model is therefore 90,600: 125×6 CDS spreads and 5 CDO tranche spreads observed on 120 days. Table II.1 shows summary statistics of the CDS and CDO data.

As a proxy for riskless rates we use LIBOR and swap rates since Feldhütter and Lando (2008) show that swap rates are a more accurate proxy for riskless rates than Treasury yields. Thus, prices of riskless zero coupon bonds with maturities up to 1 year are calculated from 1-12 month LIBOR rates (taking into account money market quoting conventions), and for longer maturities are bootstrapped from 1-, 2-, 3-, 4-, and 5-year swap rates (using cubic spline to infer swap rates for semi-annual maturities). This gives a total of 20 zero coupon bond prices on any given day (maturities of 1-12 months, 1.5, 2, 2.5, ..., 5 years) from which zero coupon bond prices at any maturity up to 5 years can be found by interpolation (again using cubic spline).

II.5 Results

II.5.1 Marginal Default Probabilities

As the first step in the estimation of the multi-name default model, we calibrate for each firm daily term structures of risk-neutral default probabilities using all the available information from CDS contracts with maturities up to 5 years. With 125 firms and a sample period of 120 days, we calibrate a total of 125×120=15,000 term structures of default probabilities, with each term structure based on 6 CDS contracts.

Figure II.1 plots for each day in the sample the average term structure of default probabilities across the 125 firms. By definition, the term structures are upward sloping since the probability of defaulting increases as maturity increases. Also, the graph shows

[11] The 5-year CDS contracts for the period March 21, 2006 to June 19, 2006 mature on June 20, 2011, consistent with the maturity of the 5-year CDO tranches, but for the period June 20 to September 19, 2006, the maturity of the 5-year CDS contracts is September 20, 2011 (and the maturity of the other CDS contracts are similarly shifted forward by 3 months from June 20 and onwards). However, this maturity mismatch between the CDS and CDO contracts in the latter part of our sample period is automatically corrected for, when we imply out the term structures of firm-specific survival probabilities from observed CDS spreads (see appendix II.B), and hence poses no problem to the estimation of the model.

Table II.1. Summary statistics

Panel A reports summary statistics for CDS spreads of the 125 constituents of the CDX NA IG 6 index over the period March 30, 2006 to September 20, 2006. Panel B reports summary statistics for the five CDO tranches: 0% – 3%, 3% – 7%, 7% – 10%, 10% – 15%, and 15% – 30% of the CDX NA IG 6 index over the same period.

Panel A: CDS spreads for CDX NA IG 6 constituents

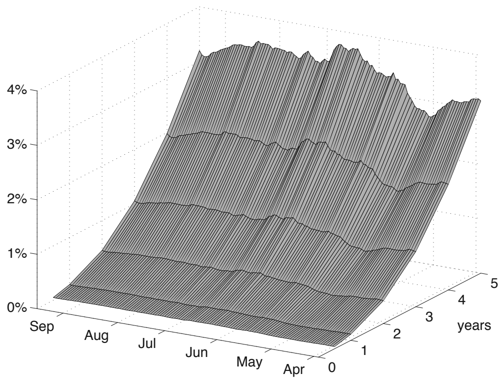
Maturity	0.5 yr (in bps)	1 yr (in bps)	2 yrs (in bps)	3 yrs (in bps)	4 yrs (in bps)	5 yrs (in bps)
Mean	6.78	8.75	14.58	21.52	30.52	39.14
Std.	5.77	6.62	11.25	16.74	23.44	29.82
Median	4.86	6.56	10.72	16.07	22.51	29.03
Min.	0.41	1.73	2.65	2.94	3.99	5.45
Max.	56.46	59.82	103.73	140.48	181.60	222.19
Observations	15,000	15,000	15,000	15,000	15,000	15,000

Panel B: CDO tranche spreads for CDX NA IG 6 tranches

Tranche	0% – 3% (in %)	3% – 7% (in bps)	7% – 10% (in bps)	10% – 15% (in bps)	15% – 30% (in bps)
Mean	29.95	91.83	20.43	9.33	5.13
Std.	2.92	15.37	4.27	1.58	0.74
Median	30.29	92.48	20.31	9.06	5.17
Min.	21.97	65.52	13.96	6.40	3.54
Max.	35.75	125.02	28.97	13.02	6.84
Observations	120	120	120	120	120

that on average the first derivative with respect to maturity is increasing.[12] Thus, forward default probabilities $\frac{\partial Q_t(\tau \leq s)}{\partial s}$, which measure the probability of defaulting at time s given that the firm has not yet defaulted, are upward-sloping. Hence, the market expects the marginal probability of default to increase over time for the average firm. This is likely caused by the fact that the CDX NA IG index consists of solid investment grade firms with low short-term default probabilities, and it is therefore more probable that credit conditions worsen for a given firm than improve.

Figure II.1. Default probabilities



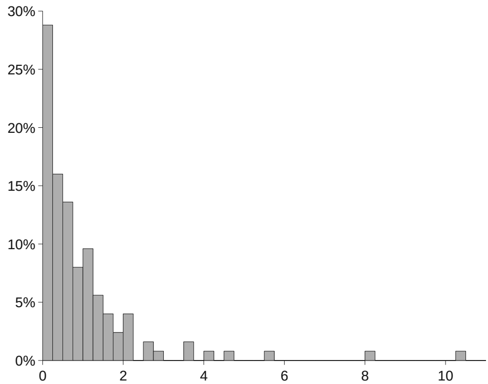
The figure shows the average calibrated term structure of risk-neutral default probabilities for 0 to 5 years over the period March 30, 2006 to September 20, 2006, averaging across all 125 constituents of the CDX NA IG 6 index. Default probabilities are calibrated on a firm-by-firm basis following the procedure outlined in appendix II.B.

The sensitivity of each firm's default probability to the economy-wide factor Y is captured in the parameter a_i, which is estimated model-independently through the covariance between firm-specific instantaneous default probabilities and market-wide instantaneous default probabilities. Figure II.2 shows the distribution of a_i's across firms (remember that the a_i's are normalized such that the average across firms is 1). There is a significant amount of variation in the a_i's, and for a large fraction of the firms the default probabilities are quite insensitive to market-wide fluctuations in credit risk. This

[12]This observation is apparent from a visual inspection of the graph, and quantitative estimates are available upon request.

suggests that the assumption in Duffie and Gârleanu (2001) to let all firms have the same sensitivity through identical a_i's is not supported by the data.

Figure II.2. Common factor sensitivities



The figure shows the distribution of the estimated common factor sensitivities a_i for the 125 constituents of the CDX NA IG 6 index. The sensitivities are estimated following the procedure outlined in appendix II.C.

To examine whether the subset of firms with large a_i's have common characteristics, we split the index into its five subindices (fraction of total index in parenthesis): Energy (11%), Financials (19%), Basic Industrials (23%), Telecommunications, Media and Technology (18%), Consumer Products and Retail (29%), and we find that firms with large a_i's are fairly evenly distributed across these five sectors.[13]

The correlation between a_i's and the average 5-year CDS spread for firm i (averaging across the 120 days) is 0.78 across the 125 firms. This strong positive correlation indicates that the ad hoc assumption in Mortensen (2006) and Eckner (2007; 2009), where a_i is exogenously set based on the firm-specific 5-year CDS spread, is reasonable.[14]

[13] The distribution on sectors of the firms with the 20% largest market sensitivities a_i is: Energy (8%), Financials (8%), Basic Industrials (16%), Telecommunications, Media and Technology (28%), Consumer Products and Retail (40%).

[14] Mortensen (2006) fixes a_i implicitly through a parameter restriction but notes that it effectively corresponds to setting a_i equal to the fraction of firm-specific to average (across all firms) 5-year CDS spread.

II.5.2 CDO Parameter Estimates and Pricing Results

The multi-name default model is estimated on the basis of a panel data set of daily CDS and CDO tranche spreads as described in section II.3, and we assume that the measurement error matrix Σ_ϵ in (II.9) is diagonal and use diffuse priors. We run the MCMC estimation routine using a burn-in period of 20,000 simulations and a subsequent estimation period of another 10,000 simulations, where we use every 10th simulation to calculate parameter estimates.

The parameter estimates are given in Table II.2, and the first thing we note is that the volatility of the common factor is $\sigma = 0.0166$, which is low compared to estimates in the previous literature: Duffee (1999) fits CIR processes to firm default intensities using corporate bond data and finds an average σ of 0.074, and Eckner (2009) uses a panel data set of CDS and CDO spreads similar to the data set used here and estimates σ to be 0.103. An important factor in explaining this difference in the estimated size of σ is the extent to which systematic and idiosyncratic default risk is separated. Duffee (1999) is not concerned with such a subdivision of the default risk and therefore estimates a factor that includes both systematic and unsystematic risk. Eckner (2009) has a model that is similar to ours, but when estimating the model he imposes strong restrictions on the parameters of the systematic and idiosyncratic factors. For example, he requires σ^2 of the common factor to be equal to the average σ_i^2 of the idiosyncratic factors.

Our results suggest that separating default risk into an idiosyncratic and a common component, and letting these factors be fully flexible during the estimation, reveals that the common factor is "slow-moving" in the sense that the volatility is low. In addition, we estimate the total contribution of jumps $l \times \mu$ to be $6 \cdot 10^{-5}$ which is lower than the estimate of $3 \cdot 10^{-3}$ in Eckner (2009), further underlining that the total volatility of the common factor is low when properly estimated.[15] Finally, we note that although the common factor is not very volatile, it is explosive with a mean reversion coefficient of 0.94 under the risk-neutral measure. Under the actual measure, the factor is estimated to be mean-reverting, although the mean-reversion coefficient is hard to pin down with any precision due to the relatively short time span of our data sample.

[15] In a previous version of this paper, we imposed parameter restrictions similar to Eckner (2009), which resulted in parameter estimates consistent with those that he reports.

Table II.2. Parameter estimates (common factor)

The table reports point estimates and 95% confidence intervals (in parenthesis) for the parameters of the multi-name default model outlined in section II.2.

$\kappa_0\ (\times 10^5)$	κ_1	$\sigma\ (\times 10^2)$
2.32 (2.15, 2.58)	0.94 (0.90, 0.99)	1.66 (1.48, 1.81)
$l\ (\times 10^3)$	$\mu\ (\times 10^2)$	
3.74 (2.54, 4.59)	1.59 (1.11, 2.12)	
κ_1^P	$l^P\ (\times 10^2)$	$\mu^P\ (\times 10^{10})$
-3.45 (-15.09, 5.08)	2.54 ($3.40 \cdot 10^{-13}$, $2.18 \cdot 10^4$)	8.34 (8.19, $1.57 \cdot 10^8$)
$\sqrt{\Sigma_{11}}$	$\sqrt{\Sigma_{22}}$	$\sqrt{\Sigma_{33}}$
0.11 (0.10, 0.33)	0.19 (0.15, 0.43)	0.16 (0.11, 0.51)
$\sqrt{\Sigma_{44}}$	$\sqrt{\Sigma_{55}}$	
0.35 (0.30, 0.64)	0.38 (0.28, 0.67)	

We now examine the pricing ability of our model by considering the average pricing errors and RMSEs (Root-Mean-Squared-Errors) given in Table II.3. We see that on average the model underestimates spreads for the $3\% - 7\%$ tranche by 7 basis points and overestimates the $10\% - 15\%$ tranche by 4 basis points. For comparison, Mortensen (2006) reports average bid-ask spreads for the $3\% - 7\%$ tranche to be 10.9 basis points and for the $10\% - 15\%$ to be 5 basis points. In both cases, average pricing errors are smaller than the bid-ask spread. The RMSEs of the model are larger than the average pricing errors, so the model errors are not consistently within the bid-ask spread, but RMSEs and pricing errors do suggest a good overall fit.

Figure II.3 shows the observed and fitted CDO tranche spreads over time, and the graphs confirm a reasonable fit to all tranches apart from a slight underestimation of the $3\% - 7\%$ tranche and overestimation of the $15\% - 30\%$ tranche. It is particularly noteworthy that the time series variation in the most senior tranches – especially the

Table II.3. CDO pricing errors

The table reports mean and standard deviation of the daily pricing errors for each of the five CDO tranches: $0\%-3\%$, $3\%-7\%$, $7\%-10\%$, $10\%-15\%$, and $15\%-30\%$ of the CDX NA IG 6 index over the period March 30, 2006 to September 20, 2006. The pricing errors are calculated as model-implied minus observed tranche spreads, and the model spreads are based on the parameter point estimates in Table II.2.

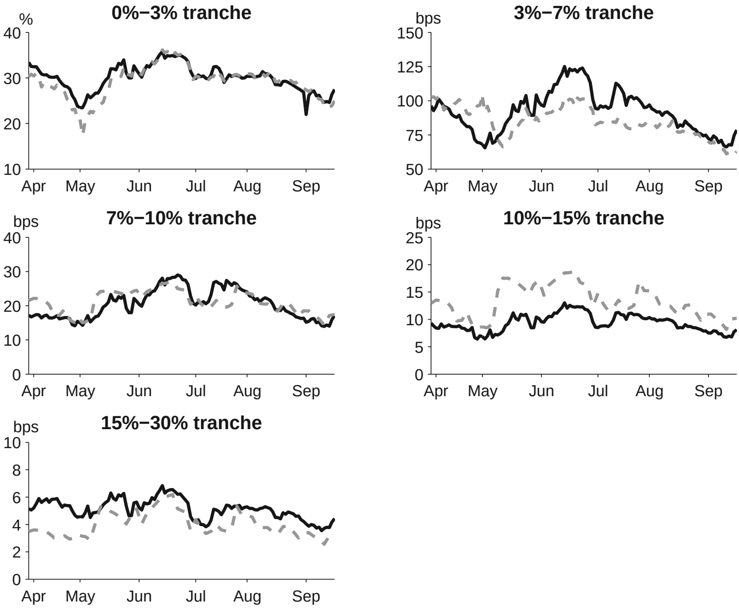
Tranche	$0\% - 3\%$ (in %)	$3\% - 7\%$ (in bps)	$7\% - 10\%$ (in bps)	$10\% - 15\%$ (in bps)	$15\% - 30\%$ (in bps)
Mean	– 0.7888	– 6.93	0.55	3.96	– 1.12
RMSE	1.7907	14.31	2.94	4.42	1.31

$15\% - 30\%$ tranche – is well matched. This is surprising because both the level and the time series variation of the $15\% - 30\%$ tranche have been difficult to capture by models in the previous literature. Mortensen (2006) finds that jumps in the common factor are necessary to generate sufficiently high senior tranche spreads, but even with jumps it has been difficult to reproduce the observed time series variation in senior tranche spreads, as argued by Eckner (2009) and in a previous version of this paper.[16] What enables our model to fit the time series variation of senior tranche spreads well is that we have not imposed the usual set of strong assumptions on the parameters of the common and idiosyncratic factors as done in Mortensen (2006), Eckner (2009), and in a previous version of this paper. Thus, a careful implementation of the multi-name default model frees up the model's ability to fit tranche spreads in important dimensions.

To examine the contribution of systematic default risk to the total default risk across different maturities, we calculate the following: for each maturity, date, and firm we use the estimated sensitivities a_i and the path and parameters of the common factor Y to calculate the systematic part of the risk-neutral default probability according to equation (II.1) and (II.2). We then find an average term-structure of systematic default risk by averaging across firms and dates and plot the result in Figure II.4 together with the average total default risk inferred from observed CDS spreads. The figure shows that

[16] The previous version of the paper entitled "An empirical investigation of an intensity-based model for pricing CDO tranches" is available upon request.

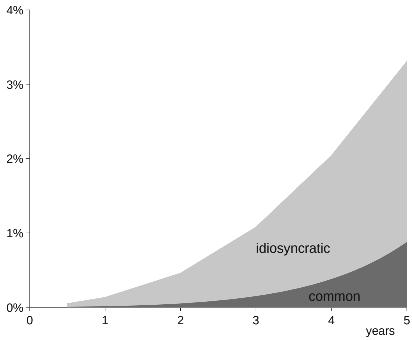
Figure II.3. CDO tranche spreads



The graphs show the observed (solid black) and model-implied (dashed gray) CDO tranche spreads for the five CDX NA IG 6 tranches: 0% − 3%, 3% − 7%, 7% − 10%, 10% − 15%, and 15% − 30% over the period March 30, 2006 to September 20, 2006. The model-implied spreads are based on the parameter estimates reported in Table II.2.

the systematic contribution to the overall default risk is small for short maturities but increases with maturity. As shown in Table II.4 the average exposure to systematic default risk on a 6-month horizon is merely 0.003% and constitutes only 6% of the overall default risk, but increases to 0.874% and a fraction of 26% of the total default risk for a 5-year horizon. Hence, out of the total average 5-year default probability of 3.309%, 0.874% is systematic and non-diversifiable.

Figure II.4. Average default probabilities



The figure shows the average term structure of risk-neutral default probabilities, averaging across all 125 constituents of the CDX NA IG 6 index and across all trading days in the period March 30, 2006 to September 20, 2006. The default probabilities are decomposed into their common (dark gray) and idiosyncratic (light gray) parts. The total default probabilities (dark and light gray) are calibrated from CDS spreads (see appendix II.B), and the common part is calculated using the parameter estimates reported in Table II.2.

Table II.4. Average default probabilities

The table reports average risk-neutral default probabilities (DP), averaging across all 125 constituents of the CDX NA IG 6 index and across all trading days in the period March 30, 2006 to September, 2006. "Total DP" reports the total default probability and corresponds to the total gray area (light and dark) in Figure II.4. "Common part of total DP" similarly expresses the common factor part of the total default probability corresponding to the dark gray area in Figure II.4.

Maturity	0.5 yr	1 yr	2 yrs	3 yrs	4 yrs	5 yrs
Total DP	0,051%	0,134%	0,460%	1,080%	2,042%	3,309%
Common part of total DP	0,003%	0,010%	0,048%	0,147%	0,376%	0,874%
Common part in % of total DP	5,88%	7,64%	10,40%	13,59%	18,42%	26,41%

II.6 Idiosyncratic default risk

So far in the estimation we have put structure on the systematic part of default risk through the specification of the common factor, while total default risk has been estimated model-independently. Combining the two elements gives us for each firm and each date a term structure of idiosyncratic default risk calculated as the "difference" between total default risk and its systematic component.[17] Thus, for each firm we have a data set consisting of the idiosyncratic part of the survival probability $E_t^Q[\exp(-\int_t^{t+s} X_{i,u}du)]$ for maturities of $s = 0.5, 1, 2, 3, 4, 5$ years and for each of the 120 days in the sample. Given this panel data set we can now put structure on the idiosyncratic default risk and estimate the parameters of this structural form.

We can allow idiosyncratic default risk to be the sum of several factors and the factors can be of any distributional form subject only to the requirements of non-negativity and that we can calculate the expectation

$$E_t^Q\left[\exp\left(-\int_t^s X_{i,u}du\right)\right].$$

We choose to let the idiosyncratic factors have the same functional form as the common factor, namely be a one-factor affine jump-diffusion

$$dX_{i,t} = (\kappa_{i,0} + \kappa_{i,1}X_{i,t})dt + \sigma_i\sqrt{X_{i,t}}dW_{i,t}^Q + dJ_{i,t}^Q$$

with an essentially affine risk premium for diffusive risk and constant risk premium for the jump risk. This allows us to compare the results of our general estimation approach with those in previous literature, where a number of restrictions are placed jointly on the common and idiosyncratic factors. Thus, for each of the 125 firms in the sample, we estimate by MCMC the parameters of the idiosyncratic factor in the same way as the parameters of the common factor, but in the estimation we now use a panel data set of the idiosyncratic part of default probabilities instead of CDO prices. Note that structural assumptions on the idiosyncratic risk were not necessary in order to price CDOs in the

[17] More specifically, the relation

$$Q_t(\tau_i > s) = E_t^Q\left[\left(-a_i\int_t^s Y_u du\right)\right] \cdot E_t^Q\left[\left(-\int_t^s X_{i,u}du\right)\right]$$

allows us to infer the idiosyncratic survival probabilities directly from the estimated common and total survival probabilities $E_t^Q\left[\left(-a_i\int_t^s Y_u du\right)\right]$ and $Q_t(\tau_i > s)$, respectively.

previous section, but adding structure here enables us to gain further understanding of the nature of the idiosyncratic default risk.

The results from the estimation of the idiosyncratic default factors are given in Table II.5. We see that the average volatility across all firms is $\sigma = 0.14$, almost 10 times higher than the volatility estimate of 0.017 for the common factor. Combined with the parameter estimates discussed in the previous section, this shows that the idiosyncratic factors are more volatile than the systematic factor. The fact that the volatility of our systematic factor is lower than that reported in previous papers reflects that our estimation procedure allows us to fully separate the dynamics of the systematic factor from the dynamics of the idiosyncratic factors. This leads to a low-volatility systematic factor and high-volatility idiosyncratic factors, while previous research finds something in-between. In addition, we see that the average total (risk-neutral) contribution from jumps is $l \times \mu = 4 \cdot 10^{-2}$, which is higher than the total jump contribution in the systematic factor of $6 \cdot 10^{-5}$, reinforcing the conclusion that volatilities of the idiosyncratic factors are higher than that of the systematic factor.

We see that κ_1 is positive on average, so the idiosyncratic factors are on average explosive under the risk-neutral measure. However, they are less explosive than the systematic factor, implying that when pricing securities sensitive to default risk, the relative importance of systematic risk increases as maturity increases in accordance with our observations in Figure II.4.

II.7 Conclusion

We present a new approach to estimate the relative contributions of systematic and idiosyncratic default risks in an intensity-based model. Based on a large data set of CDS and CDO tranche spreads on the North American Investment Grade CDX index, we find that our model is able to capture both the level and time series dynamics of CDO tranche spreads. We then go on and split the total default risk of a given entity into its idiosyncratic and systematic part. We find that the systematic default risk is explosive but has low volatility and that the relative contribution of systematic default risk is small for short maturities, but of growing importance as maturity increases. Our subsequent parametric estimation of the idiosyncratic default risks shows that idiosyncratic risk is

Table II.5. Parameter estimates (idiosyncratic factors)

The table reports mean, median and standard deviation (in parenthesis) of the 125 parameter point estimates resulting from the idiosyncratic factor estimations in the multi-name default model outlined in section II.2.

$\kappa_0\ (\times 10^6)$	κ_1	σ
9.08	0.80	0.14
0.41	0.87	0.16
(36.61)	(0.24)	(0.08)
$l\ (\times 10^3)$	μ	
4.48	8.93	
2.68	0.30	
(6.05)	(59.12)	
κ_1^P	$l^P\ (\times 10^{-2})$	$\mu^P\ (\times 10^9)$
0.14	1.31	1.66
-0.60	1.30	1.66
(5.79)	(0.34)	(0.07)
$\sqrt{\Sigma_{11}}\ (\times 10^4)$	$\sqrt{\Sigma_{22}}\ (\times 10^4)$	$\sqrt{\Sigma_{33}}\ (\times 10^4)$
1.10	1.19	1.37
1.01	1.04	1.18
(0.26)	(0.42)	(0.52)
$\sqrt{\Sigma_{44}}\ (\times 10^4)$	$\sqrt{\Sigma_{55}}\ (\times 10^4)$	
1.40	1.66	
1.23	1.09	
(0.69)	(2.55)	

more volatile and less explosive than systematic risk.

Appendix

II.A CDS pricing

This section briefly explains how to price Credit Default Swaps (CDSs). More thorough introductions are given in Duffie (1999) and O'Kane (2008).

A CDS contract is an insurance agreement between two counterparties written on the default event of a specific underlying reference obligation. The protection buyer pays fixed premium payments periodically until a default occurs or the contract expires, whichever happens first. If default occurs, the protection buyer delivers the reference obligation to the protection seller in exchange for face value.

For a CDS contract covering default risk between time $t_0 = t$ and $t_M = T$ and with premium payment dates $t_1, \ldots, t_M$, the value of the protection payment is given as

$$Prot(t,T) = E_t^Q \left[(1-\delta) \exp\left(-\int_t^\tau r_u du \right) 1_{(\tau \le T)} \right]$$

where δ is the recovery rate, while the value of the premium payment stream is $S \cdot Prem(t,T)$, where S is the annual CDS premium and

$$Prem(t,T) = E_t^Q \left[\sum_{j=1}^M \exp\left(-\int_t^{\min\{t_j,\tau\}} r_u du \right) \int_{t_{j-1}}^{t_j} 1_{(\tau > s)} ds \right].$$

The CDS premium at time t is settled such that it equates the two payment streams, i.e. $S(t,T) = \frac{Prot(t,T)}{Prem(t,T)}$.

In order to calculate the CDS premium $S(t,T)$ we make the simplifying assumptions that the recovery rate δ is constant at 40%, that the riskfree interest rate is independent of the default time τ, and finally that default, if it occurs, will occur halfway between two premium payment dates. With these assumptions we can rewrite the two expressions above as

$$Prot(t,T) = (1-\delta) \sum_{j=1}^M P\left(t, \tfrac{t_{j-1}+t_j}{2}\right) \cdot \left(Q_t(\tau > t_{j-1}) - Q_t(\tau > t_j) \right) \tag{II.10}$$

$$\begin{aligned} Prem(t,T) = & \sum_{j=1}^M P\left(t, \tfrac{t_{j-1}+t_j}{2}\right) \cdot \frac{t_j - t_{j-1}}{2} \cdot \left(Q_t(\tau > t_{j-1}) - Q_t(\tau > t_j) \right) \\ & + \sum_{j=1}^M P(t,t_j) \cdot \left(t_j - t_{j-1} \right) \cdot Q_t(\tau > t_j). \end{aligned} \tag{II.11}$$

II.B Calibration of survival probabilities

For the calibration of firm-specific survival probabilities from observed CDS spreads we assume that risk-neutral probabilities take the flexible form

$$Q_t(\tau > s) = \frac{1}{1+\alpha_2+\alpha_4}\left(e^{-\alpha_1(s-t)} + \alpha_2 e^{-\alpha_3(s-t)^2} + \alpha_4 e^{-\alpha_5(s-t)^3}\right) \qquad s \geq t \qquad \text{(II.12)}$$

with all $\alpha_j \geq 0$. The calibrated survival probabilities $s \mapsto Q_t(\tau > s)$ for a given firm at time t are then calculated by minimizing relative pricing errors using (II.10)–(II.12)

$$\sum_T \left(\frac{Prot(t,T)/Prem(t,T) \; - \; S_{obs}(t,T)}{S_{obs}(t,T)}\right)^2$$

where $S_{obs}(t,T)$ is the empirically observed CDS spread at time t on a contract with maturity T. The calibration is based on observed CDS spreads for maturities of $T = 0.5, 1, 2, 3, 4, 5$ years and is carried out separately for each firm, at each time t, and results in a very accurate fit to the observed CDS term structure.[18]

II.C Estimation of common factor sensitivities

The common factor sensitivities a_i appearing in the specification (II.2) of individual default intensities can be estimated by ordinary linear regression, and without exploiting specific assumptions on the dynamic evolution of the processes $Y, X_1, \ldots, X_N$ except for a mild stationarity condition. As we argue in the following, this model-independent technique only relies on the availability of term structures of risk-neutral survival probabilities for each of the N issuers in the portfolio.

The simple idea that we build upon is the fact that (II.1) and (II.2) imply

$$-\lim_{s\searrow 0} \frac{\partial}{\partial s} Q_t(\tau_i > t+s) = \lambda_{i,t} = a_i Y_t + X_{i,t}$$

and that we can calculate this quantity simply by inserting the calibrated survival probabilities on the left-hand-side of this expression.

If we now for fixed i consider the regression

$$W_{i,t} = \beta_{0,i} + \beta_{1,i}(V_t - \bar{V}) + \varepsilon_t \qquad t = 1, \ldots, T$$

[18] This calibration approach is close to the industry benchmark of fitting the observed CDS term structure perfectly using piecewise constant intensities, see O'Kane (2008).

where

$$\begin{aligned} W_{i,t} &= a_i Y_t + X_{i,t} \\ \bar{W}_i &= \frac{1}{T}\sum_{t=1}^{T} W_{i,t} \\ V_t &= \frac{1}{N}\sum_{j=1}^{N} W_{j,t} \\ \bar{V} &= \frac{1}{T}\sum_{t=1}^{T} V_t \end{aligned}$$

and ε_t is a Gaussian noise term, then it follows by standard estimation theory that

$$\hat{\beta}_{1,i} = \frac{\sum_t \big(W_{i,t} - \bar{W}_i\big)\big(V_t - \bar{V}\big)}{\sum_t \big(V_t - \bar{V}\big)^2}.$$

Under the assumption of stationarity of each of the processes $X_1, \ldots, X_N, Y$ (and hence also of W_i and V), we can rewrite the estimated regression coefficient as

$$\hat{\beta}_{1,i} = \frac{\widehat{Cov(W_i, V)}}{\widehat{Var(V)}}. \tag{II.13}$$

Since $X_1, \ldots, X_N, Y$ are mutually independent then for sufficiently large N

$$Cov(W_i, V) = \frac{1}{N} Var(X_i) + a_i Var(Y) \approx a_i Var(Y) \tag{II.14}$$

and similarly

$$Var(V) = \frac{1}{N^2}\sum_{j=1}^{N} Var(X_j) + Var(Y) \approx Var(Y) \tag{II.15}$$

where we have applied the normalization $\frac{1}{N}\sum_i a_i = 1$. By combining (II.13), (II.14) and (II.15) it is now straightforward to see that $\hat{\beta}_{1,i}$ is an approximate estimator of the unknown sensitivity a_i.

To increase numerical robustness of the calculations, we make a small approximation and replace everywhere the derivative

$$-\lim_{s \searrow 0} \frac{\partial}{\partial s} Q_t(\tau_i > t + s)$$

with the one-year default probability

$$1 - Q_t(\tau_i > t + 1) = -\frac{Q_t(\tau_i > t + 1) - Q_t(\tau_i > t)}{1 - 0} \approx -\lim_{s \searrow 0} \frac{\partial}{\partial s} Q_t(\tau_i > t + s)$$

since our calibration of the term structure of survival probabilities uses CDS contracts with maturities from 0.5 to 5 years, which results in minor numerical instabilities (across calendar time) in the very short end of the term structure.

II.D Estimation of common factor

Once we have inferred marginal risk-neutral survival probabilities $s \mapsto q_{i,t}(s)$ from CDS spreads and estimated the common factor sensitivities a_i, we are ready to estimate the parameters and the path of the common factor process Y. Throughout the estimation of the common factor process, all $q_{i,t}(s)$ and all a_i are taken as given (and thus held fixed).

Given an initial path of Y and initial values of the common factor parameters, the estimation procedure runs as follows:

(i) Calculate the common factor component of survival probabilities

$$E_t^Q\left[\left(-a_i \int_t^s Y_u du\right)\right]$$

for all firms i, all dates t and all maturities s.

(ii) Use the common factor components $E_t^Q[(-a_i \int_t^s Y_u du)]$ from (i) and the calibrated term structures of survival probabilities $q_{i,t}(s)$ to determine the idiosyncratic component of survival probabilities

$$E_t^Q\left[\left(-\int_t^s X_{i,u} du\right)\right]$$

for all firms i, all dates t and all maturities s using the relation

$$q_{i,t}(s) = E_t^Q\left[\left(-a_i \int_t^s Y_u du\right)\right] \cdot E_t^Q\left[\left(-\int_t^s X_{i,u} du\right)\right]$$

(iii) Use the idiosyncratic components $E_t^Q[(-\int_t^s X_{i,u} du)]$ from (ii) as input to equation (II.5) and calculate spreads for the 5 CDO tranches for all dates t (this is what is referred to as the “pricing formula” f in section II.3.2).

(iv) Use the MCMC estimation routine to update the parameters and the path of the common factor Y, and repeat steps (i)-(iv) until convergence.

II.E Conditional posteriors in MCMC estimation

In this section the conditional posteriors stated in the main text and used in the MCMC estimation are derived. Bayes' rule

$$p(X|Y) \propto p(Y|X)p(X)$$

is used repeatedly in the calculations.

II.E.1 Conditionals of S, Y, J, and Z

The conditional posteriors of S, Y, J, and Z are used in most of the conditional posteriors for the parameters and are therefore derived in this subsection.

$p(Y|\Theta, \Sigma_\epsilon, J, Z)$ **and** $p(S|\Theta, \Sigma_\epsilon, Y, J, Z)$

With the discretization in (II.8) we have that

$$\begin{aligned} p(Y|\Theta, \Sigma_\epsilon, J, Z) &= \Big(\prod_{t=1}^{T} p(Y_t|Y_{t-1}, \Theta, \Sigma_\epsilon, J, Z)\Big)p(Y_0) \\ &= p(Y_0)\prod_{t=1}^{T}\frac{1}{\sigma\sqrt{\Delta_t Y_{t-1}}}\exp\Big(-\frac{1}{2}\frac{[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1}+J_tZ_t)]^2}{\sigma^2\Delta_t Y_{t-1}}\Big) \\ &\propto p(Y_0)\sigma^{-T}Y_x^{-\frac{1}{2}}\exp\Big(-\frac{1}{2}\sum_{t=1}^{T}\frac{[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1}+J_tZ_t)]^2}{\sigma^2\Delta_t Y_{t-1}}\Big) \qquad \text{(II.16)} \end{aligned}$$

where $Y_x = \prod_{t=1}^{T} Y_{t-1}$. Note that the posterior $p(Y|\Theta, \Sigma_\epsilon, J, Z)$ differs from $p(Y|\Theta, \Sigma_\epsilon, J, Z, S)$.

The conditional posterior of S is found as

$$\begin{aligned} p(S|\Theta, \Sigma_\epsilon, Y, J, Z) &= \prod_{t=1}^{T}|\Sigma_\epsilon|^{-\frac{1}{2}}\exp\Big(-\frac{1}{2}[S_t - f(\Theta^Q, Y_t)]'\Sigma_\epsilon^{-1}[S_t - f(\Theta^Q, Y_t)]\Big) \\ &= |\Sigma_\epsilon|^{-\frac{T}{2}}\exp\Big(-\frac{1}{2}\sum_{t=1}^{T}\hat{e}_t'\Sigma_\epsilon^{-1}\hat{e}_t\Big), \qquad \text{(II.17)} \end{aligned}$$

where $\hat{e}_t = S_t - f(\Theta^Q, Y_t)$. If Σ_ϵ is diagonal this simplifies to

$$p(S|\Theta, \Sigma_\epsilon, Y, J, Z) \propto \prod_{i=1}^{N}\Sigma_{\epsilon,ii}^{-\frac{T}{2}}\exp\Big(-\frac{1}{2\Sigma_{\epsilon,ii}}\sum_{t=1}^{T}\hat{e}_{t,i}^2\Big).$$

This posterior does not depend on J, Z, κ_0^P, and κ_1^P.

$p(Z|\Theta, \Sigma_\epsilon, Y, J, S)$ **and** $p(J|\Theta, \Sigma_\epsilon, Y, Z, S)$

Since Z_t is exponentially distributed we have that

$$\begin{aligned} p(Z|\Theta, \Sigma_\epsilon, Y, J, S) &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(Z|\Theta, \Sigma_\epsilon, Y, J) \qquad \text{(II.18)} \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)p(Z|\Theta, \Sigma_\epsilon, J) \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)\prod_{t=1}^{T}\frac{1}{\mu^P}\exp(-\frac{Z_t}{\mu^P}) \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)(\mu^P)^{-T}\exp(-\frac{Z_\bullet}{\mu^P}) \qquad \text{(II.19)} \end{aligned}$$

where $Z_\bullet = \sum_{t=1}^{T} Z_t$.

The jump time J_t can only take on two values so the conditional posterior for J_t is Bernoulli. The Bernoulli probabilities are given as

$$\begin{aligned} p(J|\Theta, \Sigma_\epsilon, Y, Z, S) &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(J|\Theta, \Sigma_\epsilon, Y, Z) \qquad \text{(II.20)} \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)p(J|\Theta, \Sigma_\epsilon, Z) \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)p(J|\Theta) \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)\prod_{t=1}^{T}\Big((l^P\Delta_t)^{J_t}(1-l^P\Delta_t)^{1-J_t}\Big) \\ &\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)(l^P\Delta_t)^{J_\bullet}(1-l^P\Delta_t)^{T-J_\bullet} \qquad \text{(II.21)} \end{aligned}$$

with $J_\bullet = \sum_{t=1}^{T} J_t$

II.E.2 Conditional Posteriors

The conditional posteriors are derived and the choice of priors for the posteriors are discussed in this subsection.

(i) The conditional posterior of the error matrix Σ_ϵ is given as

$$\begin{aligned} p(\Sigma_\epsilon|\Theta, Y, J, Z, S) &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(\Sigma_\epsilon|\Theta, Y, J, Z) \\ &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(\Sigma_\epsilon|\Theta) \\ &\propto |\Sigma_\epsilon|^{-\frac{T}{2}}\exp\Big(-\frac{1}{2}\sum_{t=1}^{T}\hat{e}_t'\Sigma_\epsilon^{-1}\hat{e}_t\Big)p(\Sigma_\epsilon|\Theta) \\ &= |\Sigma_\epsilon|^{-\frac{T}{2}}\exp\Big(-\frac{1}{2}tr(\Sigma_\epsilon^{-1}\sum_{t=1}^{T}\hat{e}_t\hat{e}_t')\Big)p(\Sigma_\epsilon|\Theta). \end{aligned}$$

The last line follows because $-\frac{1}{2}\sum_{t=1}^{T}\hat{e}_t'\Sigma_\epsilon^{-1}\hat{e}_t = -\frac{1}{2}\sum_{t=1}^{T}tr(\hat{e}_t'\Sigma_\epsilon^{-1}\hat{e}_t) = -\frac{1}{2}\sum_{t=1}^{T}tr(\Sigma_\epsilon^{-1}\hat{e}_t\hat{e}_t') = -\frac{1}{2}tr(\sum_{t=1}^{T}\Sigma_\epsilon^{-1}\hat{e}_t\hat{e}_t') = -\frac{1}{2}tr(\Sigma_\epsilon^{-1}\sum_{t=1}^{T}\hat{e}_t\hat{e}_t')$. If the prior on Σ_ϵ is independent of the other parameters and has an inverse Wishart distribution with parameters V and m then $p(\Sigma_\epsilon|...)$ is inverse Wishart distributed with parameters $V+\sum_{t=1}^{T}\hat{e}_t\hat{e}_t'$ and $T+m$. The special case of V equal to the zero matrix and $m=0$ corresponds to a flat prior.

(ii) The conditional posterior of κ_1^P is found as

$$\begin{aligned}
p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon,Y,J,Z,S) &\propto p(S|\Theta,\Sigma_\epsilon,Y,J,Z)p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon,Y,J,Z)\\
&\propto p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon,Y,J,Z)\\
&\propto p(Y|\Theta,\Sigma_\epsilon,J,Z)p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon).
\end{aligned}$$

According to equation (II.16) we have

$$p(\kappa_1^P|...) \propto \exp\Big(-\frac{1}{2}\sum_{t=1}^{T}\frac{[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1}+J_tZ_t)]^2}{\sigma^2\Delta_tY_{t-1}}\Big)p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon)$$

so

$$p(\kappa_1^P|...) \propto \exp\Big(-\frac{1}{2}\sum_{t=1}^{T}\frac{[a_t\kappa_1^P-b_t]^2}{\sigma^2\Delta_tY_{t-1}}\Big)p(\kappa_1^P|\Theta_{\backslash\kappa_1^P},\Sigma_\epsilon)$$

where

$$\begin{aligned}
a_t &= -\Delta_tY_{t-1}\\
b_t &= \kappa_0\Delta_t+Y_{t-1}+J_tZ_t-Y_t.
\end{aligned}$$

Using the result in Frühwirth-Schnatter and Geyer (1998, p.10) and assuming flat priors we have that $\kappa_1^P \sim N(Qm,Q)$ where

$$\begin{aligned}
m &= \sum_{t=1}^{T}\frac{a_tb_t}{\sigma^2\Delta_tY_{t-1}}\\
Q^{-1} &= \sum_{t=1}^{T}\frac{a_t^2}{\sigma^2\Delta_tY_{t-1}}.
\end{aligned}$$

(iii) For the jump size parameter μ^P the conditional posterior is found as

$$p(\mu^P|\Theta_{\backslash\mu^P},\Sigma_\epsilon,Y,J,Z,S) \propto p(S|\Theta,\Sigma_\epsilon,Y,J,Z)p(\mu^P|\Theta_{\backslash\mu^P},\Sigma_\epsilon,Y,J,Z)$$

$$
\begin{aligned}
&\propto p(Y|\Theta, \Sigma_\epsilon, J, Z)p(\mu^P|\Theta_{\setminus\mu^P}, \Sigma_\epsilon, J, Z) \\
&\propto p(Z|\Theta, \Sigma_\epsilon, J)p(\mu^P|\Theta_{\setminus\mu^P}, \Sigma_\epsilon, J) \\
&\propto p(Z|\Theta)p(\mu^P|\Theta_{\setminus\mu^P}, \Sigma_\epsilon) \\
&\propto (\mu^P)^{-T} \exp(-\frac{Z_\bullet}{\mu^P})p(\mu^P|\Theta_{\setminus\mu^P}, \Sigma_\epsilon).
\end{aligned}
$$

If the prior on μ^P is flat then the conditional posterior is inverse gamma distributed with parameters $Z_\bullet$ and $T-1$.

(iv) The same calculations as for the jump-size parameter μ^P yields the conditional posterior of the jump-time parameter l^P as

$$
\begin{aligned}
p(l^P|\Theta_{\setminus l^P}, \Sigma_\epsilon, Y, J, Z, S) &\propto p(J|\Theta)p(l^P|\Theta_{\setminus l^P}, \Sigma_\epsilon) \\
&\propto \Big((l^P\Delta_t)^{J_\bullet}(1-l^P\Delta_t)^{T-J_\bullet}\Big)p(l^P|\Theta_{\setminus l^P}, \Sigma_\epsilon).
\end{aligned}
$$

Assuming a flat prior on l^P the conditional posterior of $l^P\Delta_t$ is beta distributed, $l^P\Delta_t \sim B(J_\bullet + 1, T - J_\bullet + 1)$.

(v) The parameters σ and κ_0 are sampled by Metropolis-Hastings since the conditional distributions are not known. Denoting any of the two parameters θ_i, the conditional distribution is found as

$$
\begin{aligned}
p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, Y, J, Z, S) &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, Y, J, Z) \\
&\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(Y|\Theta, \Sigma_\epsilon, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, J, Z) \\
&\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(Y|\Theta, \Sigma_\epsilon, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon).
\end{aligned}
$$

Flat priors on both parameters are assumed.

(vi) The parameters κ_1^Q, l^Q, and μ^Q are sampled by Metropolis-Hastings. The only difference in the derivation of their conditional distributions compared to derivation of the distributions of σ and κ_0 is that the distribution of Y does not depend on these three parameters. Letting θ_i represent any of the three parameters, the conditional distribution is found as

$$
\begin{aligned}
p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, Y, J, Z, S) &\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, Y, J, Z) \\
&\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(Y|\Theta, \Sigma_\epsilon, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon, J, Z) \\
&\propto p(S|\Theta, \Sigma_\epsilon, Y, J, Z)p(\theta_i|\Theta_{\setminus\theta_i}, \Sigma_\epsilon).
\end{aligned}
$$

Flat priors on all three parameters are assumed.

(vii) The latent jump indicators J_t's are sampled individually from Bernoulli distributions. To see this, note that equation (II.21) implies that

$$p(J|\Theta,\Sigma_\epsilon,Y,Z,S) \propto \prod_{t=1}^{T}\exp\Big(-\frac{1}{2}\frac{[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1}+J_tZ_t)]^2}{\sigma^2\Delta_tY_{t-1}}\Big)\Big(\frac{l^P\Delta_t}{1-l^P\Delta_t}\Big)^{J_t}.$$

In the actual implementation we use

$$p(J|\Theta,\Sigma_\epsilon,Y,Z,S) \propto \prod_{t=1}^{T}\exp\Big(-\frac{1}{2}\frac{(-2[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1})]+J_tZ_t)J_tZ_t}{\sigma^2\Delta_tY_{t-1}}\Big)\Big(\frac{l^P\Delta_t}{1-l^P\Delta_t}\Big)^{J_t}$$

since this is numerically more robust.

(viii) For the latent jump sizes Z_t we have according to equation (II.19) that

$$p(Z|\Theta,\Sigma_\epsilon,Y,J,S) \;\propto\; \prod_{t=1}^{T}\exp\Big(-\frac{1}{2}\frac{[Y_t-(\kappa_0\Delta_t+(\kappa_1^P\Delta_t+1)Y_{t-1}+J_tZ_t)]^2}{\sigma^2\Delta_tY_{t-1}}-\frac{Z_t}{\mu^P}\Big)$$

so the Z_ts are conditionally independent and are sampled individually. If $J_t=0$ then Z_t is sampled from an exponential distribution with mean μ^P. If $J_t=1$ tedious calculations show that

$$p(Z_t|\Theta,\Sigma_\epsilon,Y,J,Z_{\setminus Z_t},S) \propto \frac{[((\kappa_1^P+\mu^P\sigma^2)\Delta_t+1)Y_{t-1}-(Y_t-\kappa_0\Delta_t)+Z_t]^2}{\sigma^2\Delta_tY_{t-1}}\Big),$$

where $Z_t\geq 0$. Therefore, Z_t is drawn from a $N((Y_t-\kappa_0\Delta_t)-((\kappa_1^P+\mu^P\sigma^2)\Delta_t+1)Y_{t-1},\sigma^2\Delta_tY_{t-1})$ distribution and the draw is rejected if $Z_t<0$. In practice the number of rejections are small.[19]

(ix) The latent Y_ts are sampled individually by Metropolis-Hastings and for $t=1,...,T-1$ the conditional posterior is

$$\begin{aligned}
p(Y_t|\Theta,\Sigma_\epsilon,Y_{\setminus Y_t},J,Z,S) &\propto p(S|\Theta,\Sigma_\epsilon,Y,J,Z,S)p(Y_t|\Theta,\Sigma_\epsilon,Y_{\setminus Y_t},J,Z)\\
&\propto p(S_t|\Theta,\Sigma_\epsilon,Y_t,J,Z,S)p(Y_t|\Theta,\Sigma_\epsilon,Y_{t-1},Y_{t+1},J,Z)\\
&\propto p(S_t|\Theta,\Sigma_\epsilon,Y_t,J,Z,S)\\
&\quad\times p(Y_t|\Theta,\Sigma_\epsilon,Y_{t-1},J,Z)p(Y_{t+1}|\Theta,\Sigma_\epsilon,Y_t,J,Z)
\end{aligned}$$

[19] If the draws were frequently rejected the method in Gelfand, Smith, and Lee (1992) could be used.

For Y_T the conditional posterior is

$$\begin{aligned} p(Y_T|\Theta, \Sigma_\epsilon, Y_{\backslash Y_T}, J, Z, S) &\propto p(Y_T|\Theta, \Sigma_\epsilon, Y_{T-1}, J, Z, S) \\ &\propto p(S_T|\Theta, \Sigma_\epsilon, Y_T, J, Z, S)p(Y_T|\Theta, \Sigma_\epsilon, Y_{T-1}, J, Z) \end{aligned}$$

while for Y_0 it is

$$\begin{aligned} p(Y_0|\Theta, \Sigma_\epsilon, Y_{\backslash Y_0}, J, Z, S) &\propto p(Y_0|\Theta, \Sigma_\epsilon, Y_1, J, Z) \\ &\propto p(Y_1|\Theta, \Sigma_\epsilon, Y_0, J, Z)p(Y_0). \end{aligned}$$

II.E.3 Implementation Details

In the RW-MH steps of the MCMC sample, the proposal density is chosen to be Gaussian, and the efficiency of the RW-MH algorithm depends crucially on the variance of the proposal normal distribution. If the variance is too low, the Markov chain will accept nearly every draw and converge very slowly while it will reject a too high portion of the draws if the variance is too high. We therefore do an algorithm calibration and adjust the variance in the first half of the burn-in period in the MCMC algorithm. Roberts, Gelman, and Gilks (1997) recommend acceptance rates close to $\frac{1}{4}$ and therefore the standard deviation during the algorithm calibration is chosen as follows: every 100'th draw the acceptance ratio of each parameter is evaluated. If it is less than 10 % the standard deviation is doubled while if it is more than 50 % it is cut in half. This step is prior to the second half of the burn-in period since the convergence results of RW-MH only applies if the variance is constant (otherwise the Markov property of the chain is lost).

The Fourier inversion in equation (II.7) is calculated by using Fast Fourier Transform and the number of points used in FFT is 2^{18}. We use Simpson's rule in the Fast Fourier Transform routine as suggested by Carr and Madan (1999), and our results show that this gives a significant improvement in overall accuracy. The characteristic function is not evaluated in every Fourier transform point. Instead, since the characteristic function is exponentially affine with affine coefficient functions A and B, the functions A and B are splined from a lower number of points. The spline uses a total number of 60 points. Also, the integration in (II.6) is done using Gauss-Legendre integration and the number of integration points is 60.

Essay III

Credit spreads across the business cycle*

Abstract

This paper studies how corporate bond spreads vary with the business cycle. I show that both level and slope of empirical credit spread curves are correlated with the state of the economy and I link this to idiosyncratic jump risk. I develop a structural credit risk model that accounts for both business cycle and jump risk, and show by estimation that the model captures the counter-cyclical level and pro-cyclical slope of empirical credit spread curves. In addition, I provide a new procedure for estimation of idiosyncratic jump risk, which is consistent with observed shocks to firm fundamentals.

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III.1 Introduction

The yield on a corporate bond exceeds the risk-free rate by a spread, which is commonly linked to the credit riskiness and liquidity of the bond (Duffee (1999), Longstaff, Mithal, and Neis (2005), Chen, Lesmond, and Wei (2007)). Since the liquidity premium is moderate for most bonds (Dick-Nielsen, Feldhütter, and Lando (2011)), spreads move counter-cyclically with the state of the economy to reflect that default risk is larger when economic growth is low (Fama and French (1989), Chen (1991), Amato and Luisi (2006)).[1]

In this paper, I demonstrate that *short-term* spreads move relatively more than *long-term* spreads as the distribution between short- and long-term risk is shifted to put more weight on imminent default risk during economic downturns. Specifically, I find that credit spreads are low and the credit spread curve upward-sloping when economic growth is high, and conversely that spreads are high and the spread curve flat or downward-sloping when growth is low. These movements are persistent across both investment and speculative grade bonds, and I further find that the variation in level and slope is related to changes in idiosyncratic jump risk, as shocks to firm fundamentals are larger during periods of economic slowdown. Based on this link I formulate a structural credit risk model that allows for interaction between business cycle and jump risk in order to capture the observed variation in empirical credit spreads. The model extends previous literature that has focused solely on *either* business cycle *or* jump risk, with no attention to the intrinsic relation between the two risk factors.

The structural model in this paper is founded on a relation between business cycle and jump risk, and this aligns well with the common interpretation of jumps as the market reaction to arrival of new information. When the economy is near a trough, firms are believably more vulnerable and their market values therefore react more strongly to new information. Following this line of reasoning Maheu and McCurdy (2004) interpret jumps in equity returns as a consequence of the arrival of unexpected information, and Lee and Mykland (2008) show that the vast majority of jumps in equity returns can be linked to company-specific news about earnings, sales, strategic decisions, etc. Jiang and Yao (2009)

[1] Collin-Dufresne, Goldstein, and Martin (2001) show that leverage is a significant determinant of credit spread changes but find little explanatory power in macroeconomic variables. This may be explained by the fact that leverage itself has strong cyclical patterns as pointed out by Korajczyk and Levy (2003).

similarly show that jumps relate to news and that the frequency of jumps is related to firm characteristics. Furthermore, Andersen, Bollerslev, and Diebold (2007) and Bollerslev, Law, and Tauchen (2008) find evidence of jumps associated with macroeconomic news announcements, and Andersen, Bollerslev, and Diebold (2007) document considerable time variation in the intensity and size of jumps.

While the structural credit risk model in this paper centers around the importance of jump risk, the inclusion of jumps in models of debt and equity returns is not new, but dates back at least to Press (1967) and Merton (1976). Since then many papers have documented the relevance of jumps in equity returns e.g. for capturing the distributional properties of returns (Ball and Torous (1983), Jorion (1988), Eraker, Johannes, and Polson (2003)), pricing equity options (Bakshi, Cao, and Chen (1997), Andersen, Benzoni, and Lund (2002), Eraker (2004)), and forecasting equity volatility (Maheu and McCurdy (2004), Andersen, Bollerslev, and Diebold (2007)). Similarly, a related line of papers has focused on the impact of jump risk in explaining credit spreads (Collin-Dufresne, Goldstein, and Martin (2001), Cremers, Driessen, and Maenhout (2008)), and more recently on how jumps in equity returns can help predict credit spreads (Zhang, Zhou, and Zhu (2009), Tauchen and Zhou (2010)). Hence, a structural credit risk model that includes jump risk unifies several strands of literature by creating an explicit link between debt and equity returns through the value of the firm's assets, while at the same time also taking jump risk into account.[2]

The structural framework was initiated with the seminal work of Black and Scholes (1973) and Merton (1974), and since then a vast literature has extended the original model in multiple directions (see Leland (2009) for a survey). Recently, particular attention has been paid to the inclusion of jumps in asset value (Merton (1990), Zhou (2001), Hilberink and Rogers (2002), Cremers, Driessen, and Maenhout (2008), Chen and Kou (2009), Gorbenko and Strebulaev (2010)), and to the integration of business cycle risk (Hackbarth, Miao, and Morellec (2006), David (2008), Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010a;b), Chen (2010)). The former series of papers

[2] Several empirical papers have similarly tried to search for common factors driving both debt and equity returns, but with no particular attention to jump risk. These include Campbell and Ammer (1993), Fama and French (1993), Elton, Gruber, Agrawal, and Mann (2001), Campbell and Taksler (2003), Vassalou and Xing (2004), and Chordia, Sarkar, and Subrahmanyam (2005) among others.

is motivated by the inability of previous models to generate empirically plausible credit spreads (Jones, Mason, and Rosenfeld (1984), Huang and Huang (2003)), and the latter by how the state of the economy influences firms' operating conditions and thus also their likelihood of default and loss rate given default occurs (Chen (2010), Tang and Yan (2010), Doshi (2011)). The structural model in this paper differs from previous literature by incorporating both jumps *and* macroeconomic variation, consistent with the empirical evidence that short-term risk (as modelled by jumps) varies with the business cycle. The model is most closely related to the papers of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010a;b) in terms of the modelling of business cycle risk, but can at the same time also be viewed as extending the jump models of Cremers, Driessen, and Maenhout (2008) and Chen and Kou (2009).

I demonstrate that despite the additional complexity that results from combining two inherently different model extensions (jump and business cycle risk), it is still possible to derive closed-form expressions for the value of debt and equity while allowing for an arbitrary (finite) number of future states of the economy. The closed-form solution of the model, which relies on a technique developed in Jiang and Pistorius (2008), facilitates a detailed firm-by-firm estimation of the model with particular attention to the identification of the jump parameters. To this end, I develop a new procedure for estimating jump parameters from daily equity returns, and I show empirically that the resulting estimates are consistent with the interpretation of jumps as the market reaction to new and mainly firm-specific information. This paper thus provides evidence that not only are jumps connected to information dissemination, but jump sizes are also related to general market conditions.

The estimated model delivers a series of promising results. First and foremost, the model captures well the observed variation in empirical credit spreads with low levels and upward-sloping curves in good times, and high levels and flat to downward-sloping curves during recessions. Second, utilizing a simple Markov structure the model is able to accurately describe historical business cycle variation and of particular importance, to fit the time the economy spends in recession (as defined by NBER). Third, the model provides estimates of both realized and optimal net benefits to debt that are comparable to those found in existing literature, and it reveals an interesting implication for optimal capital structure. Although business cycle variation is essential to accurately capture credit spreads, the optimal capital structure turns out to be largely a-cyclical. This is a consequence of the fact that the model takes the expected future business cycle variation into account, and incorporates this into the choice of capital structure. As the economy moves through periods of both high and low growth on a regular basis, this implies that optimal capital structure decisions display low sensitivity to the current state of the economy.

In summary, the contributions of the paper are fourfold: it documents the business cycle variation in level and slope of empirical credit spreads and links that to jump risk; it develops a structural credit risk model that takes both business cycle and jump risk into account; it demonstrates how to estimate the model including consistent estimation of jump parameters; and finally, it shows that the estimated model captures the observed variation in credit spreads well and gives predictions for capital structure that align with the existing literature.

The remainder of the paper is organized as follows. Section III.2 contains the empirical analysis of historical credit spreads. Section III.3 formulates the structural credit risk model. Section III.4 describes the estimation methodology including the procedure for identifying jump parameters. Section III.5 reports the results of the estimation, and section III.6 concludes. Appendices III.A-III.C contain details on data, estimation, and the model expressions for debt and equity.

III.2 Empirical evidence

In this section I document two stylized facts about business cycle variation in corporate credit spreads: as economic growth declines, the level of credit spreads increases, and at the same time the credit spread curve shifts from upward-sloping to flat or downward-sloping. Moreover, both effects reverse when growth starts to increase again. Thus, not only do credit spreads increase during economic downturns, but equally important short-term spreads increase significantly more than their long-term counterparts. These characteristics are persistent across both investment and speculative grade bonds, and I show that time variation in the growth of firm debt is too small to be the only source of explanation. I further present empirical evidence suggesting that business cycle variation in the occurrence of shocks to firm fundamentals is an important factor in explaining changes in both level and slope of the credit spread curve.

III.2.1 Level and slope of credit spreads

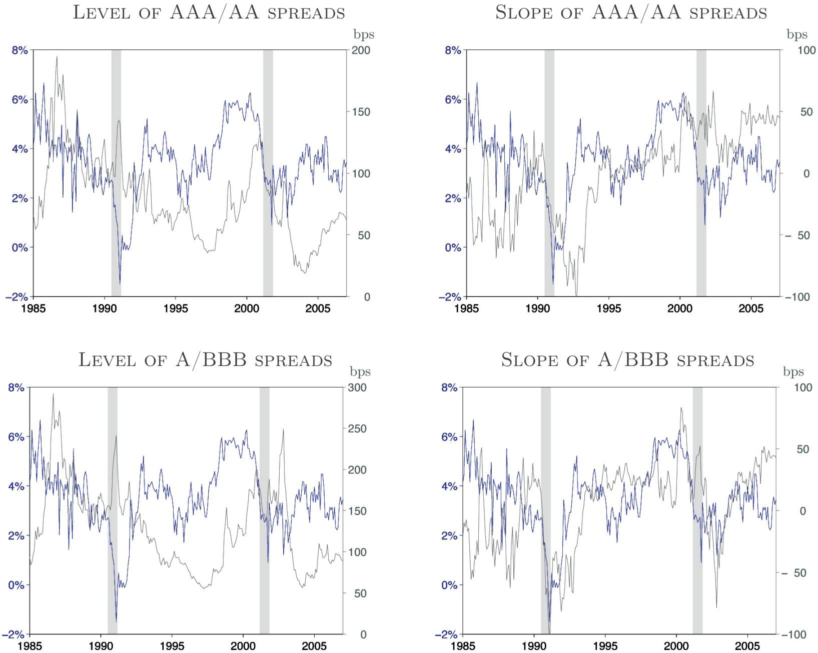
To explore business cycle variation in corporate credit spreads I use time series of yield to maturity on Merrill Lynch U.S. Corporate Investment Grade and High Yield bond indices and subtract corresponding U.S. Treasury rates to obtain historical spreads. As a proxy for economic growth I use monthly data on the U.S. real personal consumption growth rate. From the monthly time series I construct a trailing 1-year growth rate covering the period from January 1962 to December 2006, and I interpret variation in this rate as "business cycle variation" in agreement with existing literature.[3]

Figure III.1 shows how consumption growth exhibits a negative covariation with the level of both AAA/AA and A/BBB credit spreads, while the slopes of the AAA/AA and A/BBB credit spread curves at the same time display a distinct positive relation with consumption growth. These patterns are even more pronounced for the speculative grade yields in Figure III.2, so both investment and speculative grade credit spreads tend to be high and decreasing with maturity, when consumption growth is low, and low and increasing with maturity, when consumption growth is high. This finding may help explain the mixed results in previous literature that speculative grade yield curves can be both up- and downward-sloping (Sarig and Warga (1989), Helwege and Turner (1999), Lando and Mortensen (2005)). In particular, Helwege and Turner (1999) find most curves to be upward-sloping and argue that earlier findings of downward-sloping curves suffer from sample selection bias as relatively safer firms tend to issue longer maturity bonds. While this may be the case, it is less clear that such maturity bias should be changing over time, and hence this cannot explain the finding in this paper of both up- and downward-sloping curves for both speculative and investment grade issuers. Moreover, previous studies do not explicitly account for business cycle effects, but as Figure III.2 shows such effects are particularly important for spreads on low credit quality bonds. This is further confirmed by the fact that both the negative correlation between consumption growth and credit spread *level*, and the positive correlation between consumption growth and the credit

[3] In related work, Chen (2010) relies on consumption and dividend growth to determine the dynamics of the business cycle, while Lettau, Ludvigson, and Wachter (2008) in a different setting use volatility of consumption growth to proxy for macroeconomic risk.

spread *slope*, are almost monotone with respect to bond rating.[4]

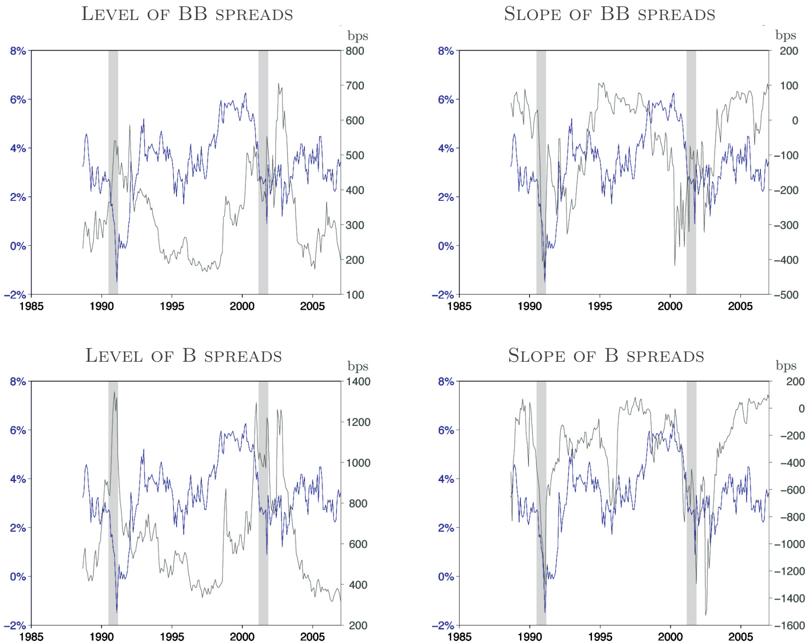
Figure III.1.
Investment grade credit spreads and consumption growth



Trailing 1-year U.S. real personal consumption growth rate (blue) versus level and slope of AAA/AA and A/BBB credit spreads (gray). The upper (lower) left graph displays the credit spread on 3–5 year maturity bonds in the Merrill Lynch U.S. Corporate AAA/AA (A/BBB) index. The upper (lower) right graph similarly displays the difference between credit spreads on 10–15 year and 1–3 year maturity bonds in the same index. The vertical bars (light gray) indicate the official NBER recession periods.

[4] The time series correlations between consumption growth and credit spread levels range from –6.8% for AAA/AA credit spreads to –28.8% for B spreads, and the similar correlations for credit spread slopes from 12.1% for AAA/AA spreads to 41.3% for B spreads.

Figure III.2.
Speculative grade credit spreads and consumption growth



Trailing 1-year U.S. real personal consumption growth rate (blue) versus level and slope of BB and B credit spreads (gray). The upper (lower) left graph displays the credit spread on 3–5 year maturity bonds in the Merrill Lynch U.S. Corporate BB (B) index. The upper (lower) right graph similarly displays the difference between credit spreads on 10–15 year and 1–3 year maturity bonds in the same index. The vertical bars (light gray) indicate the official NBER recession periods.

III.2.2 Leverage

In the extensive theoretical literature on structural credit risk modelling (see e.g. Merton (1974), Leland (1994b), Collin-Dufresne and Goldstein (2001)) firm leverage is a main determinant of credit spreads, and this link is confirmed in several empirical studies (Campbell and Taksler (2003), Ericsson, Jacobs, and Oviedo (2009), Tang and Yan (2010)). To investigate the causes of business cycle variation in credit spreads it is therefore natural to look at variation in firm leverage using a standard definition of leverage as the ratio of book value of debt to the sum of book value of debt and market value of equity (see e.g. Welch (2004), Leary and Roberts (2005), Bhamra, Kuehn, and Strebulaev (2010b)).

To study the time series behaviour of firm leverage I collect market values of equity and book values of debt for firms that were in the S&P 500 Industrials stock index as of January 1962.[5] See appendix III.A for a complete description of the data. The data sample consists of 170 firms for which the necessary data is available, and for these firms there is a time series correlation between consumption growth and cross-sectional average firm leverage of –54.1%. When paired with the similarly strong correlation between consumption growth and credit spreads observed in the previous section, this lends further empirical support to a strong link between credit spreads and leverage.[6] Moreover, the finding that firm leverage is strongly influenced by the business cycle is consistent with existing evidence that firms adjust their leverage towards time-varying targets (Korajczyk and Levy (2003), Leary and Roberts (2005)).[7]

Fluctuations in leverage are by definition related to changes in either debt or equity growth. Figure III.3 shows the time variation for both growth rates and reveals two notable facts. First, debt growth displays signs of both positive and negative comovement with consumption growth throughout the sample period, and therefore cannot be the main factor driving the cyclical leverage ratio. Second, equity growth is strongly pro-cyclical and therefore, when combined with the slower-moving debt growth, is what effectively leads to the observed counter-cyclical leverage ratio (see also Welch (2004)).[8]

III.2.3 Jumps in equity returns

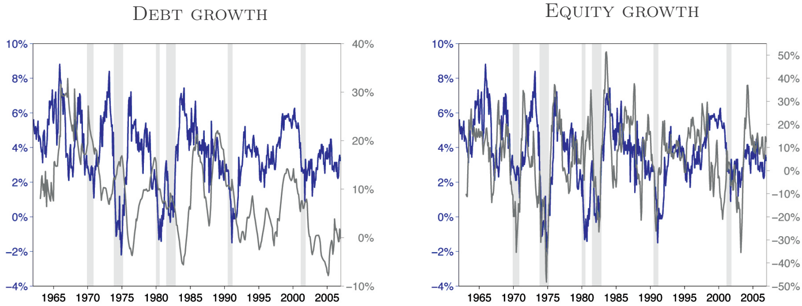
[5] 1962 is the first year with information available in Compustat about the constituents of the major S&P indices.

[6] In the absence of reliable credit spread data at the firm-level dating back as far as 1962, establishing a link between credit spreads and leverage via their common business cycle variation provides an alternative approach.

[7] In a recent study, Lemmon, Roberts, and Zender (2008) find that leverage is largely time-invariant, but this conclusion is based on a static "event time" sorting technique that effectively prevents the authors from drawing conclusions about calendar time variation in leverage ratios.

[8] Consumption growth has a time series correlation with debt growth of 4.7%, and with equity growth of 36.3%.

Figure III.3. Consumption vs. debt and equity growth



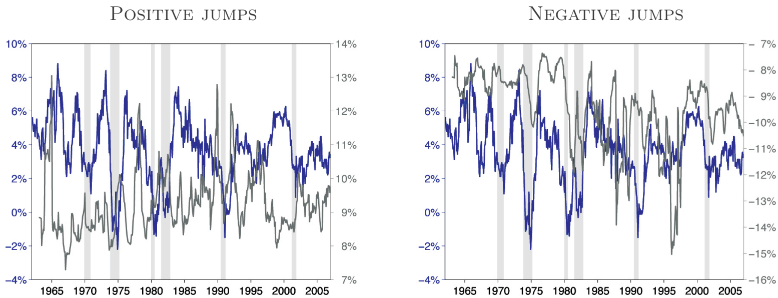
Trailing 1-year U.S. real personal consumption growth rate (blue) versus debt and equity growth rate (gray). Debt and equity rates are cross-sectional average 1-year trailing rates based on the 170 firms in the data sample. The vertical bars (light gray) indicate the official NBER recession periods.

The empirical evidence above suggests that both credit spreads and equity returns are strongly correlated with the business cycle, indicating that these fluctuations may be driven by business cycle variation in their common underlying factor: firm fundamentals. In particular, in times of low economic growth the levels of credit spreads increase, the slopes of credit spread curves decrease, and equity growth drops dramatically. The former may simply be a consequence of an increase in volatility, but a higher volatility cannot explain the relative shift towards more short-term firm risk that changes the credit spread curve from upward- to downward-sloping, nor can it explain the downward spikes in equity growth. Instead, both of these effects could be related to large and sudden shocks to firm fundamentals. To investigate this hypothesis I show in Figure III.4 the average size of large daily changes – i.e. jumps – in equity value over the sample period 1962–2006. The size of both positive and negative shocks display a distinct covariation with the business cycle, with larger jumps mainly occuring in times of low economic growth, thereby confirming the findings of Tauchen and Zhou (2010) that jumps in equity returns are time-varying both with respect to frequency and size.[9] To the extent that jumps in equity value proxy for large sudden changes in the value of firm fundamentals, the larger negative shocks in

[9]Consumption growth has a time series correlation with positive jumps of –23.0%, and with negative jumps of 24.4%.

economic downturns provide an explanation for both the spikes in equity growth as well as the declining slope of the credit spread curve.

Figure III.4. Consumption vs. size of equity jumps



Trailing 1-year U.S. real personal consumption growth rate (blue) versus size of daily jumps in equity returns (gray). Jump sizes are cross-sectional average 1-year trailing sizes based on the 170 firms in the data sample. The vertical bars (light gray) indicate the official NBER recession periods.

The jumps in equity returns reported in Figure III.4 are calculated using a novel procedure developed in section III.4.3. To investigate whether the detected jumps relate to company specific events or instead are results of general market turmoil, trading patterns, or other non-firm factors, I list for one of the 170 firms in the sample, Eastman Kodak Company, all daily jumps in equity returns that exceed 10% in absolute value. Table III.1 shows that by searching in Bloomberg for corporate news related to Eastman Kodak Company, all identified jumps since 1990 can be directly linked to the dissemination of company-specific news. This relation between jumps in equity and arrival of corporate news is consistent with similar findings in Lee and Mykland (2008), and provides evidence that jumps are a natural component of any firm value model, and with a straightforward interpretation as the market reaction to the arrival of new information. While jumps in equity value do not necessarily correspond to jumps in total firm value, as a jump in equity value could, in principle, be offset by a simultaneous, opposite jump in debt value, the results in Table III.1 show that this is rarely the case, as almost all jumps can be linked to genuine information about firm fundamentals.

Table III.1. Large equity returns for Eastman Kodak Co.

The table displays the dates between January 2 1962 and December 29 2006 where the daily equity return for Eastman Kodak Co. exceeds 10% in absolute value. This is the set of dates for which the largest jumps in the daily equity return for Eastman Kodak Co. are detected (see section III.4.3 for details). If possible, any corporate news or other events related to the abnormal returns are listed (with EPS: Earnings Per Share, DPS: Dividends Per Share).

Year	*Date*	*Equity return*	*Corporate news / Event*
1974	October 16	–12.9%	
1987	October 19	–36.0%	1987 stock market crash
1987	October 20	21.6%	1987 stock market crash
1987	October 21	10.1%	1987 stock market crash
1988	January 8	–10.5%	
1990	August 1	10.6%	Increased EPS ($1.19 in 1990Q2 vs. $0.24 in 1989Q2)
1993	April 28	–10.3%	High-profiled CFO leaves the company after less than 3 months
1993	December 15	–12.3%	Earnings forecast considerably below market estimates
1994	January 4	–19.9%	Spin-off of chemical business unit creating 10th largest U.S. chemical company
1997	March 21	–11.1%	Lower-than-expected sales for Jan-Feb 1997 (zero growth compared with Jan-Feb 1996)
1997	July 16	–11.7%	Decreased EPS ($1.12 in 1997Q2 vs. $1.30 in 1996Q2)
1998	July 15	11.2%	Increased EPS ($1.38 in 1998Q2 vs. $1.12 in 1997Q2)
1998	October 13	–16.0%	Decreased revenue ($3.39 bil. in 1998Q3 vs. $ 3.77 bil. in 1997Q3)
1999	January 14	–11.0%	Lower-than-expected EPS ($1.05 vs. expected $1.15 in 1998Q4) and forecasts of future EPS below market expectations
2000	September 26	–28.5%	Lowered EPS forecast (from $1.56-$1.66 to $1.31-$1.46 in 2000Q3)
2001	September 20	–14.4%	Lowered EPS forecast (from $0.90-1.20 to not above $0.65 in 2001Q3)
2001	October 24	–10.7%	Lowered EPS forecast (not above $0.15 vs. expected $0.45 in 2001Q4) and plan for 9% reduction in workforce
2002	July 11	10.5%	Increased EPS forecast (from $0.60-$0.70 to $0.97 in 2002Q2)
2003	January 22	–12.5%	Lower-than-expected EPS ($0.39 vs. expected $0.68 in 2002Q4)
2003	June 18	–10.6%	Lowered EPS forecast (from $0.60-$0.80 to $0.25-$0.35 in 2003Q2)
2003	September 25	–19.8%	Reduced DPS (from $1.80 to $0.50) and announcement of new strategy
2004	January 22	12.0%	Plan for 20% reduction in workforce
2006	August 1	–14.7%	Lower-than-expected EPS ($–0.98 vs. expected $0.22) and lowered 2006 revenue forecast

Thus, jumps in equity value generally reflect substantial changes to the value of firm fundamentals, and the occurence of these jumps are closely linked to variation in the overall growth of the economy. Similar patterns are observed in the level and slope of firms' credit spreads, which leads to the conjecture that jump risk may be an important driver of firms' credit spread curves. Any reasonable model of equity and debt value should take this into account, and the next section shows how to do that in a structural credit risk framework.

III.3 Model

The empirical results in section III.2 show that accounting for the time-varying nature of jumps has the potential to explain business cycle variation in the level and slope of the credit spread curve. Moreover, it is well-documented that credit spreads also depend on volatility (Campbell and Taksler (2003), Cremers, Driessen, Maenhout, and Weinbaum (2008), Zhang, Zhou, and Zhu (2009)), and that volatility displays significant cyclical behaviour (Christie (1982), Schwert (1989), Campbell, Lettau, Malkiel, and Xu (2001)). In this section I formulate a theoretical model that accommodates all of these features by developing a structural credit risk model that allows for business cycle variation in both expected growth rate, volatility and jump behaviour of firm fundamentals.

III.3.1 Model specification

The setting of the model follows the classical framework of Leland (1994a;b) and concerns a firm with debt and equity, both modelled as claims to the firm's underlying assets. The firm has an incentive to issue debt to secure a tax benefit from its coupon payments, and balances this benefit against the potential deadweight costs it incurs in case of bankruptcy. The market value V_t of the firm's assets is assumed to evolve according to

$$d(\log V_t) = \theta^{\mathbb{P}}(Z_t)dt + \sigma(Z_t)dW_t^{\mathbb{P}} + dJ_t^{\mathbb{P}} \tag{III.1}$$

under the physical measure $\mathbb{P}$, with all $\theta_i^{\mathbb{P}}$ non-zero and all $\sigma_i > 0$.[10] Here, $(Z_t)_{t\geq 0}$ is an $n-$state Markov chain with intensity matrix Ξ that describes the state of the economy, $(W_t^{\mathbb{P}})_{t\geq 0}$ is a standard Brownian motion, and $(J_t^{\mathbb{P}})_{t\geq 0}$ is a regime-switching jump process

[10] I use interchangeably the notation $x(i)$ and x_i for the ith element of a vector $(x_1, \ldots, x_n)$.

$J_t^{\mathbb{P}} = \sum_{i=1}^n 1_{(Z_t=i)} J_{i,t}^{\mathbb{P}}$. The processes $(Z_t)_{t\geq 0}, (W_t^{\mathbb{P}})_{t\geq 0}, (J_{1,t}^{\mathbb{P}})_{t\geq 0}, \ldots, (J_{n,t}^{\mathbb{P}})_{t\geq 0}$ are mutually independent, and each $(J_{i,t}^{\mathbb{P}})_{t\geq 0}$ is a compound Poisson process with jump intensity $\lambda_i^{\mathbb{P}}$ and jump size density

$$\alpha_i^{\mathbb{P}} \kappa_i^{+,\mathbb{P}} e^{-\kappa_i^{+,\mathbb{P}} y} 1_{(y>0)} + (1-\alpha_i^{\mathbb{P}}) \kappa_i^{-,\mathbb{P}} e^{\kappa_i^{-,\mathbb{P}} y} 1_{(y<0)}$$

with $\kappa_i^{+,\mathbb{P}} > 1, \kappa_i^{-,\mathbb{P}} > 0$ and $0 \leq \alpha_i^{\mathbb{P}} \leq 1$. For ease of interpretation I will assume that the n macroeconomic states are ordered according to the growth of the economy, with state 1 being the highest and state n the lowest state of growth.

The firm asset dynamics (III.1) allow for both positive and negative jumps to occur as well as for the macroeconomy to impact asset value in multiple ways: through the expected growth rate, the asset volatility, and via both the intensity and magnitude of jumps. The model thereby extends the work of Chen and Kou (2009) to include macroeconomic variation, and that of David (2008), Bhamra, Kuehn, and Strebulaev (2010a;b), and Chen (2010) to allow for jumps in asset value. The latter string of papers all give detailed accounts of how such models can be motivated by fundamental assumptions about a utility-maximizing representative agent or a stochastic discount factor linked to the dynamic evolution of prices and aggregate output. While these are important considerations for understanding the theoretical background of the structural modelling framework, I focus in this paper on the empirical implications for the valuation of equity and debt without specifying a similar set of underlying economic assumptions, but merely note that this can be done (see also Kou (2002)).

Apart from the extended generality in the specification (III.1) of the asset dynamics, the model aligns with several of the above-mentioned papers in the sense that the shareholder-owned firm is assumed to continuously issue bonds to enjoy a tax shield to operating income caused by the bond coupon payments. The tax advantage to debt is balanced against the bankruptcy costs lost in case of default, and shareholders initially guarantee the coupon payments to bond holders (if necessary by issuing additional shares). However, due to their limited liability they will stop disbursements, if the total market value of assets falls below some threshold $b(Z_s)$ that depends on the state of the economy. Firm default thus occurs at time

$$\tau = \inf\{s \geq t \,|\, V_s < b(Z_s)\}$$

at which point bond holders take over the firm after paying liquidation and reorganization costs amounting to a fraction $l(Z_\tau)$ of the remaining asset value. I specifically require the default boundaries to be counter-cyclical, i.e. $b_1 < \ldots < b_n$, consistent with the interpretation of the n macroeconomic states as representing high to low growth (going from state 1 to state n). As argued in Chen (2010), the economic intuition behind this is that the more favourable a state the economy is in, the more willing are shareholders to accept a low current asset value and still keep the firm as a going concern.[11]

To have a tractable modelling of the maturity of issued debt I impose the "roll-over" debt structure suggested by Leland (1994a) and further detailed in Hilberink and Rogers (2002), which involves a constant retirement of old debt and simultaneous reissuance of new. Thus, the firm is assumed to constantly issue debt with a face value of p and corresponding coupon rate c, and the redemption of each issuance is determined by the maturity profile $\phi(\cdot) \geq 0$ satisfying $\int_0^\infty \phi(s)ds = 1$, i.e. $p\phi(s)$ is the amount of face value issued at time t which will be retired s periods later. Consequently, at time t the total face value of previously issued debt to be redeemed at time $s \geq t$ is

$$p_t(s) = \int_{-\infty}^{t} p\phi(s-u)du$$

and hence in particular the amount maturing at time t is

$$p_t(t) = \int_{-\infty}^{t} p\phi(t-u)du = p$$

which equals the face value of the simultaneously issued new debt. This implies that the total face value of outstanding debt

$$P = \int_{t}^{\infty} p_t(s)ds = p\int_{0}^{\infty}\int_{s}^{\infty} \phi(u)duds$$

is constant through time and therefore results in a constant total coupon payment of $C = cP$. Recent models by Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010b) allow the firm to pursue a dynamic refinancing policy by taking into account the possibility of issuing further debt in the future. However, the numbers reported in Bhamra, Kuehn, and Strebulaev (2010b) show that allowing for future debt restructuring has little impact

[11] Bhamra, Kuehn, and Strebulaev (2010b) similarly impose a counter-cyclical default boundary by placing restrictions on the first and second moments of the growth rate of firm earnings and aggregate consumption.

on the model-implied credit spreads, and Chen (2010) similarly finds that the average firm restructures only once every 20 years. Based on these results I prefer to keep a parsimonious modelling of the capital structure and not model the possibility to relever.

I assume that operating assets generate a continuous payout to bond- and shareholders at a state-dependent rate $\beta(Z_s)$, which reflects that payouts may vary over time in response to variation in firm growth. Thus, at any point in time $s \geq t$ the firm's net debt service payment equals the sum of coupon payments (C) and principal retirement (p) less the tax benefits to debt, modelled as a constant inflow of ζC, asset payouts (at rate $\beta(Z_s)$), and the market value of newly issued debt.[12]

To ease notation, I henceforth let $t = 0$ and take $V_0 = v, Z_0 = i$. The market value of firm debt is now given as

$$\begin{aligned}\textsc{Debt}(v,i) = \quad & \mathrm{E}^{\mathbb{Q}}_{v,i}\left[\int_0^\infty \left(\int_0^{\tau\wedge s} e^{-ru} c\, p_0(s) du\right) ds\right] \\ & + \mathrm{E}^{\mathbb{Q}}_{v,i}\left[\int_0^\tau e^{-rs} p_0(s) ds + e^{-r\tau}\big(1 - l(Z_\tau)\big) V_\tau \frac{\int_\tau^\infty p_0(s) ds}{P} 1_{(\tau<\infty)}\right]\end{aligned}$$

where the two terms cover the value of coupon payments and the value of repaid principal, respectively. Here, $r > 0$ is the riskless rate, which I assume to be constant for parsimony, and $\mathbb{Q}$ is a risk-neutral pricing measure specified below. The trade-off between tax benefits and bankruptcy costs determines total firm value as

$$\textsc{Firm}(v,i) = v + \mathrm{E}^{\mathbb{Q}}_{v,i}\left[\int_0^\tau e^{-rs}\zeta C\, ds\right] - \mathrm{E}^{\mathbb{Q}}_{v,i}\left[e^{-r\tau} l(Z_\tau) V_\tau 1_{(\tau<\infty)}\right]$$

and the market value of equity is therefore given as the residual claim

$$\textsc{Equity}(v,i) = \textsc{Firm}(v,i) - \textsc{Debt}(v,i). \tag{III.2}$$

To facilitate explicit calculations I consider the specific debt maturity profile $\phi(s) = me^{-ms}$, where $1/m$ is the average maturity of outstanding debt, and the above expressions then reduce to

$$\textsc{Debt}(v,i) = \big(C + p\big)\, \mathrm{E}^{\mathbb{Q}}_{v,i}\left[\frac{1 - e^{-(m+r)\tau}}{m + r}\right] + \mathrm{E}^{\mathbb{Q}}_{v,i}\left[e^{-(m+r)\tau}\big(1 - l(Z_\tau)\big) V_\tau 1_{(\tau<\infty)}\right] \tag{III.3}$$

$$\textsc{Firm}(v,i) = v + \zeta C\, \mathrm{E}^{\mathbb{Q}}_{v,i}\left[\frac{1 - e^{-r\tau}}{r}\right] - \mathrm{E}^{\mathbb{Q}}_{v,i}\left[e^{-r\tau} l(Z_\tau) V_\tau 1_{(\tau<\infty)}\right]. \tag{III.4}$$

[12] Here, I follow Leland (1994a) in assuming that if debt is issued below par, new shares are simultaneously issued to cover the difference from par value, and conversely, that firm payouts in excess of the net debt service payment are paid out to shareholders as dividends.

The default triggering exit levels $(b_i)_{i=1,\dots,n}$ are set in order to maximize total firm value subject to shareholders' limited liability. This implies that exit levels are set according to the n smooth pasting conditions[13]

$$\lim_{v \to b_i+} \frac{\partial \text{EQUITY}}{\partial v}\big(v, i\big) = 0 \qquad i = 1, \dots, n.$$

III.3.2 Risk premia

The structural model is incomplete due to the presence of jumps and regime-switching behaviour and consequently has no uniquely defined risk premia. I therefore fix a specific risk-neutral pricing measure $\mathbb{Q}$, and for reasons of tractability I choose $\mathbb{Q}$ such that it leads to the same type of dynamics for V_t under $\mathbb{Q}$ as in (III.1). Cremers, Driessen, and Maenhout (2008) consider a model with both idiosyncratic and systematic jumps in asset value, and only attach a jump risk premium to the latter. In my model there is no distinction between the two types of jumps, and I therefore allow all jumps to carry a risk premium although some of the jump risk may, in fact, be diversifiable. Chen (2010) does not consider asset jump risk but focuses instead on the importance of business fluctuations and thus attaches risk premia to the macroeconomic regime shifts, which, in principle, is also possible within my model. However, as outlined in section III.4, the way I estimate the model is to take the historical path of the business cycle process $(Z_t)_{t\geq 0}$ as given, and then estimate the model on a firm-by-firm basis. This approach makes it difficult to estimate *aggregate* macroeconomic jump risk premia, since the estimation procedure would dictate them to vary from firm to firm, and I therefore choose to only allow for risk premia on the diffusion and jump risk factors.

I link the pricing measure $\mathbb{Q}$ to the physical probability measure $\mathbb{P}$ through the nominal stochastic discount factor $(M_t)_{t\geq 0}$

$$d(\log M_t) = \theta_M(Z_t)dt + \sigma_M(Z_t)dW_t^{\mathbb{P}} + \gamma_M(Z_t)dJ_t^{\mathbb{P}}.$$

This specific choice of $(M_t)_{t\geq 0}$ can be motivated by equilibrium considerations based on the existence of a utility-maximizing representative agent (see Kou (2002)), and furthermore

[13] The optimality of these conditions is verified in Chen and Kou (2009) for the special case with only one regime. However, for more general types of asset dynamics than those considered in this paper, Boyarchenko and Levendorskii (2002) and Kyprianou and Surya (2007) show that smooth pasting is not necessarily the appropriate criterion.

has the benefit of preserving the dynamic structure of the asset value process $(V_t)_{t\geq 0}$ under $\mathbb{Q}$. Hence, under the pricing measure $(V_t)_{t\geq 0}$ is still a state-dependent jump-diffusion process with double-exponential log jump sizes

$$d(\log V_t) = \theta^{\mathbb{Q}}(Z_t)dt + \sigma(Z_t)dW_t^{\mathbb{Q}} + dJ_t^{\mathbb{Q}} \tag{III.5}$$

with parameters (state-dependent subscript i is suppressed for notational convenience)

$$\begin{aligned}
\theta^{\mathbb{Q}} &= r - \beta - \lambda^{\mathbb{Q}}\left(\delta^{\mathbb{Q}} - 1\right) - \frac{\sigma^2}{2} \\
\lambda^{\mathbb{Q}} &= \lambda^{\mathbb{P}} \cdot \delta^{\mathbb{P}} \\
\alpha^{\mathbb{Q}} &= \frac{\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}}}{\left(\kappa^{+,\mathbb{P}} - \gamma_M\right)\delta^{\mathbb{P}}} \\
\kappa^{+,\mathbb{Q}} &= \kappa^{+,\mathbb{P}} - \gamma_M \\
\kappa^{-,\mathbb{Q}} &= \kappa^{-,\mathbb{P}} + \gamma_M
\end{aligned}$$

where

$$\begin{aligned}
\delta^{\mathbb{P}} &= \frac{\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}}}{\kappa^{+,\mathbb{P}} - \gamma_M} + \frac{(1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}}}{\kappa^{-,\mathbb{P}} + \gamma_M} \\
\delta^{\mathbb{Q}} &= \frac{\alpha^{\mathbb{Q}}\kappa^{+,\mathbb{Q}}}{\kappa^{+,\mathbb{Q}} - 1} + \frac{(1-\alpha^{\mathbb{Q}})\kappa^{-,\mathbb{Q}}}{\kappa^{-,\mathbb{Q}} + 1}
\end{aligned}$$

and subject to the parameter restrictions $\kappa^{+,\mathbb{Q}} > 1, \kappa^{-,\mathbb{Q}} > 0, \gamma_M < 0$.[14] Absence of arbitrage determines the parameters θ_M, σ_M of the stochastic discount factor through the conditions

$$\begin{aligned}
\theta_M + \frac{\sigma_M^2}{2} &= -r - \lambda^{\mathbb{P}} \cdot \left(\delta^{\mathbb{P}} - 1\right) \\
\sigma \cdot \sigma_M &= r - \beta - \theta^{\mathbb{P}} - \lambda^{\mathbb{Q}} \cdot \left(\delta^{\mathbb{Q}} - 1\right) - \frac{\sigma^2}{2}
\end{aligned}$$

and thus there is effectively only one parameter γ_M to control asset risk premia in each of the n macroeconomic states. Following Cremers, Driessen, and Maenhout (2008) I define the total asset risk premium η as the difference in drift rates under the physical and risk-neutral measure

$$\eta = \eta_W + \eta_J = \left(\theta^{\mathbb{P}} - \theta^{\mathbb{Q}}\right) + \left(\lambda^{\mathbb{P}}\zeta^{\mathbb{P}} - \lambda^{\mathbb{Q}}\zeta^{\mathbb{Q}}\right)$$

[14] $\gamma_M < 0$ comes out as a natural condition in case the stochastic discount factor $(M_t)_{t\geq 0}$ is motivated by the existence of a representative power-utility agent (see Kou (2002)).

and I split it into separate risk premia related to diffusion (η_W) and to jump (η_J) risk. Here

$$\zeta^{\mathbb{P}} = \frac{\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}}}{\kappa^{+,\mathbb{P}}-1} + \frac{(1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}}}{\kappa^{-,\mathbb{P}}+1} - 1, \qquad \zeta^{\mathbb{Q}} = \frac{\alpha^{\mathbb{Q}}\kappa^{+,\mathbb{Q}}}{\kappa^{+,\mathbb{Q}}-1} + \frac{(1-\alpha^{\mathbb{Q}})\kappa^{-,\mathbb{Q}}}{\kappa^{-,\mathbb{Q}}+1} - 1$$

are the expected jump sizes under the physical and risk-neutral measure, and the parameter restriction $\gamma_M < 0$ implies that $\eta_J \geq 0$ always, see Appendix III.B.4 for details.

III.3.3 Calculation of equity and debt

The market values of equity and debt in equations (III.2)–(III.4) are calculated by evaluating expectations of the form

$$\mathrm{E}^{\mathbb{Q}}_{v,i}\left[e^{-\int_0^{\tau} r(Z_s)ds}\,(V_\tau)^a\,f(Z_\tau)1_{(\tau<\infty)}\right] \qquad a \geq 0 \tag{III.6}$$

where

$$\tau = \inf\left\{s \geq 0 \mid V_s < b(Z_s)\right\}$$

with $b_1 < \ldots < b_n$, and log asset value has the dynamics

$$d(\log V_t) = \theta^{\mathbb{Q}}(Z_t)dt + \sigma(Z_t)dW_t^{\mathbb{Q}} + dJ_t^{\mathbb{Q}}$$

detailed in the previous section.[15] Compared to existing credit risk models with business cycle effects, the calculation of (III.6) is complicated by the fact that the model allows for jumps in asset value, which implies non-continuous sample paths of the asset value process. While this is an important feature to fit observed credit spreads, as the analysis in section III.5 will show, it also turns the solution of the model into a non-trivial mathematical problem. Previous literature has shown how to handle models without jumps in asset value (Jobert and Rogers (2006), Chen (2010), Bhamra, Kuehn, and Strebulaev (2010a;b)), and the basic idea behind the solution of the current model is to use a state space expansion to circumvent the discontinuity problem, and thereby be able to exploit the same approach as in models without jumps. Jiang and Pistorius (2008) develop these ideas and show in a general framework that the expectation in (III.6) has a representation in terms of

[15]For reasons of generality the risk-free rate r appearing in (III.6) is allowed to depend on the state of the economy, although the model specified in section III.3.1 does not exploit this features but instead assumes the risk-free rate to be constant.

solutions to a series of matrix equations, but they do not consider how to solve the equations. For the model in this paper I solve the relevant equations in closed form using an eigenvalue approach similar to Jobert and Rogers (2006). Appendix III.B.1 gives the solution to the matrix equations, and appendix III.B.2 derives the resulting closed-form expression for the expectation in (III.6). Despite the substantial complexity of the current model compared to existing models, the results in the appendices show that it is possible to obtain closed-form expressions for the values of equity and debt with an arbitrary number of macroeconomic states n even in the presence of jumps in asset value. The expressions in appendix III.B.2 deviate slightly from those found in Jiang and Pistorius (2008) as I correct for an error appearing in one of their main theorems. In the interest of completeness, I show in appendix III.B.3 how to correct this error in the full generality of their framework.

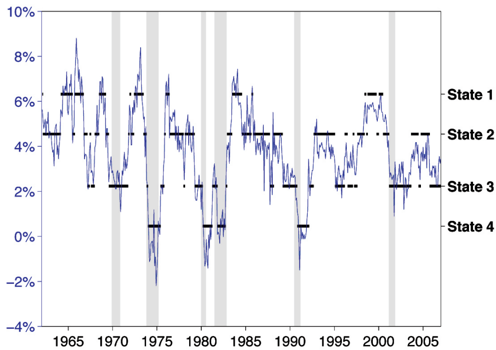
III.4 Estimation methodology

Estimation of the structural credit risk model from section III.3 requires separate identification of its two different sources of risk: jump and diffusion. The estimation procedure must take into account that both sources are allowed to vary with the state of the economy, and that the asset value process $(V_t)_{t\geq 0}$ is unobservable. These issues are solved in a series of steps. First, I determine the parameters of the Markov chain $(Z_t)_{t\geq 0}$ in a way that is consistent with the interpretation of $(Z_t)_{t\geq 0}$ as describing the state of the business cycle. Next, I calibrate all directly observable model parameters before I turn to estimation of the asset process parameters. Here, I first develop a new technique for identifying the parameters of the asset jump process $(J_t^{\mathbb{P}})_{t\geq 0}$, and then I combine two well-known estimation procedures to estimate all remaining parameters.

III.4.1 Business cycle

The estimation of business cycle variation is based on the trailing 1-year U.S. real personal consumption growth rate discussed in section III.2.1. The time series covers the period January 1962 to December 2006, and following the approach of Tauchen and Hussey (1991) I split the range of observed 1-year growth rates into four different regions, spanning from a state 1 of high growth (above 5.43%) over states 2 and 3 of high-to-medium and medium-to-low growth (3.39%–5.43% and 1.34%–3.39%) to a state 4 of low growth (below 1.34%).[16] Figure III.5 shows the variation over time in the 1-year consumption growth rate together with the four calibrated states of the economy, and we see how all of the official NBER recession periods correspond to periods of medium-to-low or low consumption growth (state 3 or 4) in the calibration.

Figure III.5. Consumption growth and the state of the economy



Trailing 1-year U.S. real personal consumption growth rate (blue) and the resulting four regions (black horizontal lines) that describe the state of the business cycle. The vertical bars (light gray) indicate the official NBER recession periods.

The time series of calibrated states gives the evolution of the Markov chain $(Z_t)_{t\geq 0}$ over the sample period, and based on this it is straightforward to estimate the intensity matrix Ξ. Details on the calibration of the four states and estimation of the intensity matrix are given in appendix III.C.1. Table III.2 contains the estimated transition intensities and reveals that the average duration of a period of high growth is $1/1.66 = 0.60$ years, whereas the average duration of a period of low growth is $1/0.87 = 1.15$ years, almost two-times as long. In particular, the latter is consistent with the average time span of 0.90 years of the 6 NBER recession periods occurring throughout the sample period.

[16] The calibration technique of Tauchen and Hussey (1991) is used to determine four levels of annual consumption growth: 0.46%, 2.23%, 4.55%, 6.32%, from which the boundaries of the four regions of consumption growth are found by calculating the midpoints.

Table III.2. Shifts between macroeconomic states

Estimated intensity matrix for changes in the state of the U.S. economy. The estimation is based on the calibrated path of the macroeconomic state process $(Z_t)_{t\geq 0}$ in Figure III.5. Asymptotic standard errors are calculated by outer product and reported in parenthesis.

	State 1	State 2	State 3	State 4
State 1 (high growth)		1.66		
		(0.48)		
State 2	0.63		0.80	
	(0.19)		(0.21)	
State 3		0.99		0.30
		(0.27)		(0.15)
State 4 (low growth)			0.87	
			(0.44)	

To further evaluate the fit of the calibrated Markov chain $(Z_t)_{t\geq 0}$ to the observed time series of consumption growth rates, I compare in Table III.3 the estimated stationary distribution for $(Z_t)_{t\geq 0}$ with its empirical counterpart. Looking at both the distribution across states as well as at the first four standardized moments supports the impression that the Markov chain gives a satisfactory description of historical business cycle behaviour. Furthermore, the agreement between the Markov chain and the official NBER recession periods is once again confirmed by noting that the total part of the estimation period January 1962 to December 2006 spent in any of the NBER recession periods amounts to 12.0%, which is close to the estimated probability of 11.4% of being in the low growth state (state 4).

The above results show that a model with four macroeconomic states captures the observed business cycle variation well. David (2008) similarly considers a four-state model based on inflation and earnings growth, whereas Chen (2010) uses nine states to obtain even richer dynamics by calibrating to the consumption and dividend growth model of Bansal and Yaron (2004). A model with just two states is, in fact, sufficient to demonstrate the *qualitative* implications of business cycle variation for credit risk in a single-firm setting, and it may also be enough to capture the effects at the aggregate level (Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a;b)). However, as seen

Table III.3. Distribution of macroeconomic states

Panel A displays the average time spent in each of the four macroeconomic states. The occupation time is calculated from the empirically calibrated path for the Markov chain $(Z_t)_{t\geq 0}$ (the black horizontal lines in Figure III.5) and from the stationary distribution for $(Z_t)_{t\geq 0}$ (corresponding to the estimated transition intensities in Table III.2). Panel B displays the first four standardized moments of the observed consumption growth rate (the blue line in Figure III.5) and the stationary distribution for $(Z_t)_{t\geq 0}$. The stationary distribution is distributed on the following consumption growth rate levels (the y-axis in Figure III.5): 0.46% (State 4), 2.23% (State 3), 4.55% (State 2), 6.32% (State 1).

Panel A: Occupation time

Macroeconomic state	*Empirical*	*Stationary*
State 1 (high)	17.2%	15.3%
State 2	41.3%	40.5%
State 3	30.9%	32.7%
State 4 (low)	10.6%	11.4%

Panel B: Moments

	Empirical	*Stationary*
Mean ($\times 10^2$)	3.69	3.18
Variance ($\times 10^4$)	3.65	3.24
Skewness	–0.35	0.15
Kurtosis	3.21	2.00

for example in Figure III.5, capturing empirical business cycle variation using only two states provides a very coarse approximation in a quantitative, firm-level analysis.

To ensure consistent estimation of the credit risk model on a firm-by-firm basis, I take the evolution of the business cycle process $(Z_t)_{t\geq 0}$ as given by fixing it to its historical path shown in Figure III.5. This implies that exactly the same time periods are taken as states of high/medium/low growth for all firms, which is necessary to facilitate a meaningful comparison of estimated model parameters across firms. In addition, this also simplifies the estimation procedure, which would otherwise have to involve the state of the business cycle as an additional, latent variable (David (2008)).

III.4.2 Observable parameters

The credit risk model employs several parameters that are directly observable: the average maturity of outstanding debt $1/m$, the asset payout rate β, the coupon rate c, the corporate tax rate ζ, the risk-free rate r, and the corporate bond loss rate l. For each firm the average debt maturity $1/m$ is fixed at its time series average, and the state-dependent payout rate β is calibrated to a time series of observed payout rates by OLS to give a fixed payout rate within each of the four macroeconomic states. Calculations of the time series of debt maturity and payout rate are detailed in appendix III.A.1. Although the bond coupon rate c could be observed directly from coupon rates on outstanding bonds, I prefer instead to set it by requiring initial debt value to equal the debt principal P. This avoids subtle considerations about exactly which bond to use for fixing the coupon rate, and also ensures that the coupon rate is set in accordance with empirical evidence showing that most bond issues are offered at or close to par.[17]

For the corporate tax rate I follow existing literature and set $\zeta = 35\%$ (Leland and Toft (1996), Graham (2000), Chen (2010)), and the risk-free rate is fixed at $r = 6.18\%$, the mean of the 1-year U.S. Treasury rate over the period January 1962 to December 2006. Throughout the sample the 1-year U.S. Treasury rate does show some signs of counter-cyclicality wrt. consumption growth, but the correlation is heavily fluctuating over time.[18] In addition, there is little consensus in existing literature on the importance of non-constant interest rates for structural credit risk models (Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995)), so since the focus in this paper is on modelling credit *spreads*, keeping the interest rate constant seems like a reasonable

[17] In an empirical study of corporate bond issues, Fung and Rudd (1986) find that the offer yield generally lies very close to the coupon rate. Similar evidence is found by studying all corporate bond issues registered in Moody's Default Risk database. Among all issues with an offer price available, 95.1% of the observations lie within ±2% of par value.

[18] The 5-year rolling window correlation between consumption growth and the 1-year Treasury rate ranges from –74.3% to 85.1% during the sample period.

approximation.

There is on the other hand substantial empirical evidence that the corporate bond recovery rate $1-l$ is varying with the business cycle (Duffie and Singleton (1999), Altman, Brady, Resti, and Sironi (2005), Acharya, Bharath, and Srinivasan (2007)), although as noted by Bhamra, Kuehn, and Strebulaev (2010b) numerical estimates are subject to considerable uncertainty. In the theoretical model, $1-l$ specifies the recovery rate of firm asset value in case of default, which is difficult to observe empirically, and I therefore follow existing literature (Bhamra, Kuehn, and Strebulaev (2010b), Chen (2010)) and estimate $1-l$ by looking at recovery of par values instead. The recovery rate data are taken from Moody's Default Risk Service database, where I compute a trailing 1-year recovery rate compounded from recovery rates on the most frequent debt classes and debt seniorities and weigh by the amount outstanding at default. Details are in appendix III.A.2. I subsequently use OLS to convert the time series of recovery rates into estimates of the state-dependent recovery rates $1-l$, subject to the condition that recovery should be decreasing with the state of the economy as indicated by empirical evidence. This leads the recovery rates for state 1 and 2, and similarly the rates for state 3 and 4, to collapse and results in recovery levels of $1-l(1) = 1-l(2) = 36.5\%$ and $1-l(3) = 1-l(4) = 23.7\%$.

III.4.3 Jumps

Since the asset value process $(V_t)_{t\geq 0}$ is unobservable, it is necessary to use time series of either debt or equity to identify the asset process parameters, but the presence of jumps in the asset value dynamics (III.1) puts certain requirements on these time series. In section III.3.1 both equity and debt are modelled as continuous functions of V_t, and this implies a one-to-one relation between jumps in asset value and jumps in debt and equity value. A historical time series of debt or equity returns should therefore be sufficient to infer distributional characteristics about jumps in asset value. However, to be able to discriminate between diffusion $(\theta^{\mathbb{P}}, \sigma)$ and jump parameters $(\lambda^{\mathbb{P}}, \alpha^{\mathbb{P}}, \kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}})$, it is crucial that the time series is sampled at a sufficiently high frequency[19], which effectively means at least at a daily frequency (Aït-Sahalia (2004), Johannes, Polson, and Stroud

[19] Jorion (1988) studies jumps and diffusive heteroscedasticity in equity and exchange rate returns and finds that a weekly and a monthly sampling frequency give considerably different results.

(2009)). Furthermore, because a sudden, abnormally large positive or negative return is a rare event, it requires a long estimation period to accurately estimate the jump distribution parameters (Maheu and McCurdy (2004)). Altogether, this creates a “curse of frequency” problem: the combination of high-frequency (daily) observations with low-frequency shifts in the state of the economy implies that estimation of the model has to involve a substantial amount of data to ensure parameter identification. I therefore only consider firms with at least 20 years of data available, which results in a total sample of 170 firms from the S&P 500 Industrials index.[20] From the state occupation times reported in Panel A of Table III.3 this implies that there are at least two years of daily observations available for the estimation within each of the four macroeconomic states.

The absence of long time series of daily market prices of debt on a firm-by-firm basis means that jump identification in practice has to be based on daily equity returns. Several recent studies of jumps in equity returns even use intra-daily data together with so-called “realized variation” estimation techniques (see e.g. Huang and Tauchen (2005), Andersen, Bollerslev, and Diebold (2007), Bollerslev, Law, and Tauchen (2008), Lee and Mykland (2008)). While it is in general sufficient for jump detection to use daily data, it is crucial for the “realized variation” methodology that it is applied to intra-daily data, and such data have only recently become available. Instead, I estimate the parameters of the jump distribution from daily equity returns by developing a simple explorative technique that relies on the same basic intuition as the “realized variation” statistics, but does not require the availability of intra-daily data.

Since a jump represents a large and instant change in equity value, I classify a daily equity return as containing at least one jump, if the observed return is “sufficiently far” away from the return I would expect to see in the absence of jumps. In addition, I follow existing literature and assume that due to the infrequency of jumps there can be at most one jump on any given day, and that a jump always dominates any other shock to the stock price on that day (see e.g. Eraker, Johannes, and Polson (2003), Tauchen and Zhou (2010)). I can therefore use the entire daily return as a measure of the size of the jump,

[20] Lemmon, Roberts, and Zender (2008) similarly impose a lower bound of 20 years of available data in their study of firm leverage, Maheu and McCurdy (2004) use 17 to 38 years of daily data to study jumps in equity returns, and Bollerslev and Todorov (2011) use 19 years of *intra*-daily data to study the importance of jumps for equity and variance risk premia.

and thus estimate the jump intensity as the average number of jumps per year, and the jump size as the average return on days with jumps.

More specifically, for a given firm and in a given macroeconomic state, I identify daily return observations containing a jump by first determining the expected range of the non-jump returns. Using only return observations in the 5% to 95% percentile range of the distribution of all daily returns within that state, I calculate the empirical mean $\mu_{5/95}$ and standard deviation $\sigma_{5/95}$ and take these as measures of the mean and standard deviation of non-jump returns. Jump-returns are then identified as those observations in the sample of all daily returns that lie more than 5 standard deviations $\sigma_{5/95}$ away from the expectation $\mu_{5/95}$.[21]

This simple procedure, that only uses the center part of the return distribution to characterize non-jump returns, effectively mitigates possibly deceptive effects from a jump detection procedure that uses the entire return distribution, where a few abnormally large return observations could potentially distort the inference.[22] Figure III.6 exemplifies the procedure for one of the 170 firms in the data sample, Eastman Kodak Company, by showing the distribution of all daily equity returns split across the four macroeconomic states. The dashed vertical lines indicate the 5% and 95% percentile cutoff points used to calculate the mean $\mu_{5/95}$ and standard deviation $\sigma_{5/95}$ within each state, and the solid line marks the average return in the 5%–95%-percentile truncated distribution.

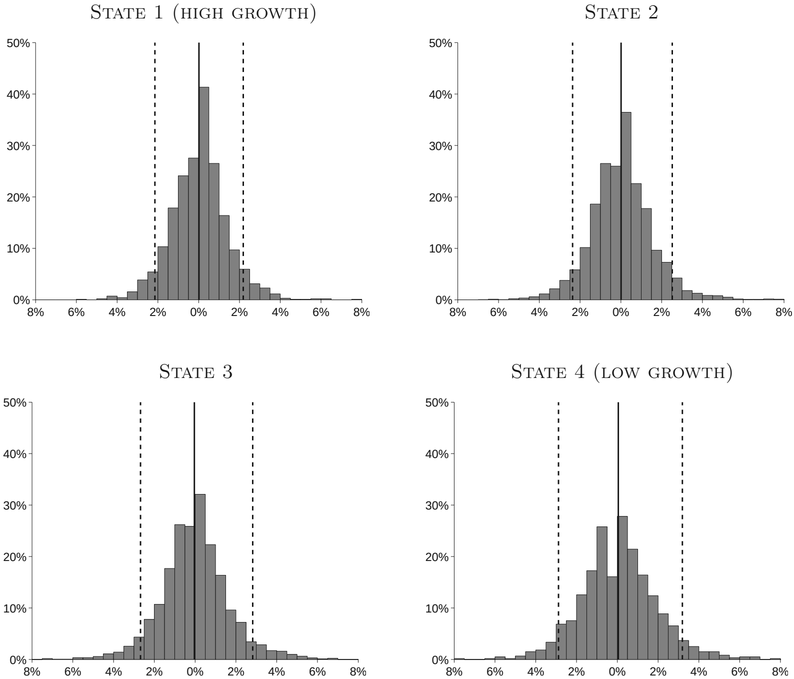
III.4.4 Asset value parameters

To complete the estimation of the model, it remains to estimate the parameters of the asset value process $(V_t)_{t\geq 0}$. All parameters could, in principle, be determined in a joint maximum likelihood estimation, but it is well-known that jump distribution parameters are difficult to estimate accurately regardless of the type of estimation procedure (Lee and Mykland (2008)). To reduce this fundamental estimation inaccuracy I employ a two-stage

[21] Aït-Sahalia (2004) shows that a distance of more than 4 standard deviations from the mean is required to reliably disentangle the jump component from its non-jump part.

[22] I choose a cutoff level of 5% to single out parts of the return distribution that are very unlikely to be affected by jumps. Applying the procedure with either 1% or 0.1% changes both the estimated frequency and size of jumps, but leaves the relative jump pattern across the four macroeconomic states essentially unaltered.

Figure III.6. Equity returns, Eastman Kodak Co.



Distribution of daily equity returns for Eastman Kodak Co. in the four macroeconomic states over the period January 2, 1962 to December 29, 2006. Returns are continuously compounded and in percentages. The dashed vertical lines indicate 5% and 95% percentiles in the state-dependent return distributions, and the solid lines indicate the mean return in the 5%–95% range of the distributions.

estimation procedure, where I first determine the jump intensity $\lambda^{\mathbb{P}}$ and the proportion of positive jumps $\alpha^{\mathbb{P}}$ using the jump detection procedure in section III.4.3, and then subsequently estimate the remaining parameters by maximum likelihood.[23] An additional outcome of the jump detection procedure is that positive and negative jumps appear to be

[23] Ericsson and Reneby (2005) show that maximum likelihood is suitable for estimating a variety of structural credit risk models.

of approximately the same size, and I therefore set $\kappa^{+,\mathbb{P}} = \kappa^{-,\mathbb{P}}$ in the estimation below.[24]

While the jump identification procedure could be used to estimate both the jump intensity $\lambda^{\mathbb{P}}$ and all the jump size parameters $(\alpha^{\mathbb{P}}, \kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}})$, I only use it to determine $\lambda^{\mathbb{P}}$ and $\alpha^{\mathbb{P}}$ for two reasons. First, using estimates of $\kappa^{+,\mathbb{P}}$ and $\kappa^{-,\mathbb{P}}$ obtained from the procedure in section III.4.3 means that the estimation will be based on the implausible assumption that jumps in asset and equity value are always of the same magnitude. Secondly, separate estimation of all jump parameters $(\lambda^{\mathbb{P}}, \alpha^{\mathbb{P}}, \kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}})$ without taking the diffusive parameters $(\theta^{\mathbb{P}}, \sigma^{\mathbb{P}})$ into account, severely weakens the possibilities of the model to fit the data. Note, namely, that estimating the diffusion growth rate $\theta^{\mathbb{P}}$ while keeping all jump parameters fixed would set strong bounds on $\theta^{\mathbb{P}}$, since it would then have to be set to match both the empirical growth rates of equity and debt as well as the (fixed) jump growth rate. Unreported results show that for some firms this becomes a severe restriction, whereas allowing $\kappa^{+,\mathbb{P}}$ and $\kappa^{-,\mathbb{P}}$ to be estimated jointly with the diffusion parameters significantly increases the fit of the model.

Having estimated $(\lambda^{\mathbb{P}}, \alpha^{\mathbb{P}})$ the remaining parameters $(\kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}}, \theta^{\mathbb{P}}, \sigma, \gamma_M)$ are found by combining ordinary maximum likelihood estimation (Duan (1994; 2000)) based on time series of debt and equity with an extension of the iterative approach suggested by Vassalou and Xing (2004). The data requirement of at least 20 years of daily data implies that estimation for a single firm involves between 5,000 and 11,000 daily observations which, in combination with the numerical complexity of the expressions (III.2)-(III.4) for equity and debt, renders standard maximum likelihood estimation infeasible.[25] I circumvent this problem by splitting the likelihood estimation into a two-step iterative procedure: first, estimation of parameters $(\kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}}, \theta^{\mathbb{P}}, \sigma, \gamma_M)$ *conditional* on time series of implied asset value V, debt coupon C, and default boundaries b, and next a recalculation of implied asset values, debt coupon and default boundaries *conditional* on the updated parameter estimates. The two steps are then repeated until parameter estimates converge. Splitting the estimation into a parameter estimation and a time series calibration part makes a

[24] For most firms the estimated value of $\alpha^{\mathbb{P}}$ lies around 0.5–0.6. In those cases setting $\kappa^{+,\mathbb{P}} = \kappa^{-,\mathbb{P}}$ essentially corresponds to requiring the expected contribution from jumps to asset returns to be close to zero, which is consistent with similar results for equity returns in Maheu and McCurdy (2004).

[25] Standard maximum likelihood estimation requires inversion of the expression for the market price of equity *as part of* the parameter optimization, and this becomes numerically intractable for large data sets.

huge difference from a computational perspective, and as noted in Lando (2004) such an approach is closely related to ordinary maximum likelihood estimation and has rapid numerical convergence. Details on the implementation are given in appendix III.C.2.

III.5 Results

I consider in this section the results of an empirical estimation of the structural credit risk model following the procedure outlined in section III.4. I compare estimated model parameters to the existing literature and analyze implications of the estimated model for credit spreads, net benefits to debt, and optimal leverage.

III.5.1 Model parameters

Estimation of the credit risk model is computationally challenging since it is based on daily data spanning multiple decades as discussed in section III.4. I therefore focus the firm-by-firm estimation on the 15 largest firms in the sample (as measured by market capitalization on January 2nd, 1962), and Table III.4 reports the cross-sectional average parameter estimates.

Table III.4. Model parameters

Cross-sectional average parameter values from estimation of the structural credit risk model on the 15 largest firms in the S&P 500 Industrials index as of January 1962. Panel A lists cross-sectional parameter estimates, and Panel B lists asset growth rates and risk premia.

Panel A: Parameter estimates

	State 1	State 2	State 3	State 4
$\lambda^{\mathbb{P}}$	2.25	2.04	2.48	1.41
$\kappa^{+,\mathbb{P}}$	31.85	24.79	22.53	24.18
$\kappa^{-,\mathbb{P}}$	31.85	24.79	22.53	24.18
$\alpha^{\mathbb{P}}$	0.63	0.53	0.51	0.64
$\theta^{\mathbb{P}}$	0.04	0.07	0.03	0.01
σ	0.14	0.13	0.14	0.17
γ_M	–6.79	–1.83	–1.57	–4.21
β	0.04	0.05	0.05	0.06

Panel B: Asset growth rates and risk premia

	State 1	State 2	State 3	State 4
Asset growth rate ($\theta^{\mathbb{P}} + \lambda^{\mathbb{P}}\zeta^{\mathbb{P}}$)	6.84%	7.98%	4.08%	4.32%
Diffusion ($\theta^{\mathbb{P}}$)	3.58%	6.61%	3.43%	1.06%
Jump ($\lambda^{\mathbb{P}}\zeta^{\mathbb{P}}$)	3.26%	1.36%	0.65%	3.26%
Asset risk premium (η)	7.89%	10.31%	7.34%	8.08%
Diffusion (η_W)	1.26%	3.11%	–0.84%	0.68%
Jump (η_J)	6.63%	7.20%	8.18%	7.40%

The table shows that if we for a moment disregard the jumps, then the asset growth rate ($\theta^{\mathbb{P}}$) is pro-cyclical and the asset volatility (σ) counter-cyclical. Both findings are consistent with similar results about business cycle patterns in the expected value and volatility of stock returns in e.g. Fama and French (1989) and Campbell, Lettau, Malkiel, and Xu (2001). If we now consider the jumps, then it is surprising to note that the jump parameters do not display particular signs of business cycle variation, despite the evidence in Andersen, Bollerslev, and Diebold (2007) and Bollerslev, Law, and Tauchen (2008) that large jumps may be related to macroeconomic news. An average of 0.9–1.4 positive jumps ($\alpha^{\mathbb{P}}\lambda^{\mathbb{P}}$) and 0.5–1.2 negative jumps ($(1-\alpha^{\mathbb{P}})\lambda^{\mathbb{P}}$) occur each year, and both the proportion of positive jumps ($\alpha^{\mathbb{P}}$) and the jump sizes ($\kappa^{\mathbb{P}}$) are remarkably stable across the four macroeconomic states. The estimated number of jumps are comparable to those reported in Ball and Torous (1983), Honoré (1998), and Eraker (2004), but in general smaller than those found in Maheu and McCurdy (2004) and Lee and Mykland (2008). Table III.4 also shows that although the parameter restriction $\kappa^{+,\mathbb{P}} = \kappa^{-,\mathbb{P}}$ was motivated by empirical evidence in section III.4.4, it appears to be a constraint in the actual estimation and most likely the main reason for the relatively small and almost a-cyclical asset jump sizes of 3%-4% ($1/\kappa^{\mathbb{P}}$), which contrast the observed equity jump sizes of 7%-15% in Figure III.4.

The firm payout rate is slightly counter-cyclical and hence in line with numbers reported in Lettau and Ludvigson (2005) showing that consumption and dividend growth display minor negative correlation, and the total asset growth rate is strongly pro-cyclical, which by looking at the data appears to be mainly due to large increases in debt financing in the high growth states.

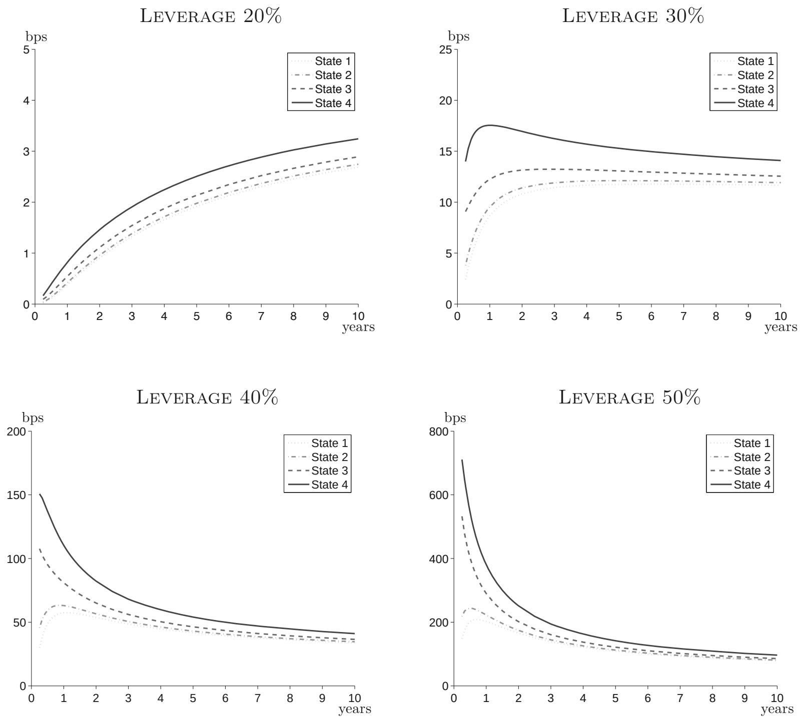
The results regarding risk premia are less encouraging albeit largely consistent with those found in Cremers, Driessen, and Maenhout (2008). Structural credit risk models that include jump risk tend to favour jump over diffusive risk, and jumps therefore easily become the main driver of risk premia. In one of the states this even leads to a negative diffusion risk premium similar to the result in Cremers, Driessen, and Maenhout (2008). However, although jump risk is the main determinant of the total asset risk premium, I do not find that it completely drives out the diffusive risk premium as in Cremers, Driessen, and Maenhout (2008). The estimated risk premia display a surprising lack of business cycle variation, and there may be at least two reasons that can help explain this. First of all, the risk premium specification I employ in this model corresponds to assuming a representative agent with constant relative risk aversion within each of the four macroeconomic states (Kou (2002)), and this is likely to be too simple to match observed risk premia (Cremers, Driessen, and Maenhout (2008)). Secondly, the estimation in Table III.4 is only based on a selection of the largest U.S. industrial firms, but as shown in Perez-Quiros and Timmermann (2000) business cycle variation in firm risk is in general more pronounced for small firms e.g. due to tighter credit market conditions.

III.5.2 Credit spreads

The introduction of time-varying jumps in asset value was motivated in section III.2 by the substantial business cycle variation in level and slope of empirical credit spread curves. To examine if the theoretical model is able to capture these stylized facts, I graph in Figure III.7 model-implied credit spread curves for different levels of leverage based on the parameter estimates in Panel A of Table III.4.

It is evident that both level and slope of the credit spread curve are strongly dependent on firm leverage and the state of the economy. The model generates credit spreads close to zero for firms with leverage below 20%, and credit spreads increase to several hundred basis points as the amount of debt financing increases. The importance of leverage for the *level* of credit spreads is not surprising given the empirical evidence in e.g. Campbell and Taksler (2003) and Ericsson, Jacobs, and Oviedo (2009), but Figure III.7 shows that leverage is also an important determinant of the *slope* of the credit spread curve, with the curve being mainly upward-sloping for firms with low to moderate levels of leverage and downward-sloping for more levered firms. For the latter group, keeping leverage fixed while changing the state of the economy also leads to substantial changes in the short end of the curve, with spreads almost tripling from the best to the worst state of the economy. However, state-dependence only plays a role for short-term spreads since they are mainly determined by the current state of the economy, as opposed to spreads on longer term debt issues that are essentially weighted averages across all states of the economy. The latter is a consequence of the estimated state transition intensities from Table III.2, since they imply that the probability of remaining within any given state until maturity vanishes as debt maturity increases. This contrasts results in Chen (2010), where the 10-year credit spread varies substantially across different states of the economy,

Figure III.7. Credit spread



Model-implied credit spread as function of average debt maturity for leverage ratios between 20% and 50%.

which is due to considerably different state transition dynamics that cause the economy to spend most of its time in a limited part of the state space. Therefore, in his model, long-term spreads cannot be viewed as weighted averages across all possible states of the economy, and this introduces state-dependence also in long-term credit spreads.

It may appear from Figure III.7 as if credit spreads are mostly determined by leverage and only to a minor extent by the state of the economy, but empirical leverage ratios are highly state-dependent as mentioned in section III.2.2, and this has to be taken into consideration when accounting for the aggregate effect of state-dependence in credit spreads.[26] The increase in level and decrease in slope of empirical credit spread curves when economic conditions deteriorate, as observed in Figure III.1 and III.2, are therefore consequences of two effects. There is a direct effect through a change in the expected growth, volatility, and jump behaviour of the firm's assets, and there is an indirect effect through an increase in the firm's leverage. Figure III.7 only focuses on the former but nevertheless indicates that the estimated model captures the changes in both level and slope well for medium and highly levered firms. For low-leverage firms it is the magnitude of the indirect effect via leverage that for any given firm determines, whether the model is able to give an appropriate description of short- and long-term credit spreads.

III.5.3 Implied asset values, default barriers, and coupon rates

Although the model does not per se allow debt coupon C and default barriers b to be time-varying, time series of both variables appear as natural biproducts of the estimation. They are both set on a daily basis to ensure that debt always trades at par (see section III.4.2) and the smooth pasting conditions are satisfied, and Table III.5 give their time series averages relative to model-implied asset value V. Despite the fact that the debt coupon C is implicitly determined from the par value of debt and not inferred from observed bond coupon rates, the model is still able to generate economically plausible values. The model-implied interest expense C/P increases from 7.23% in the high growth state to 8.39% in the low growth state, and these numbers lie close to the sample averages of 6.44% and 9.60%, respectively.

While the default barriers b do not have obvious empirical counterparts, their range of 13% to 23% of asset value aligns well with similar numbers reported for the median default boundary in Chen (2010). Note also that within any given state, all default boundaries $b_1, \ldots, b_4$ lie close together. This indicates that the jump risk mechanism employed in Bhamra, Kuehn, and Strebulaev (2010a;b) and Chen (2010), which is the instant change of default barrier resulting from a sudden shift in the prevailing economic regime, is of minor importance once the model is extended to allow for jumps in asset value.

The trade-off between tax benefits and bankruptcy costs generates counter-cyclical

[26] For the full data sample of 170 firms, the 1st (3rd) quartile in the cross-sectional distribution of leverage is 11.7% (31.4%) in state 1 and increases to 22.4% (48.8%) in state 4.

Table III.5. Asset value, debt coupon, and exit levels

Cross-sectional average statistics based on time series of unlevered asset value V, total coupon C, and default barriers $b_1, \ldots, b_4$. All time series are implied from the estimation of the structural credit risk model on the 15 largest firms in the S&P 500 Industrials index as of January 1962.

Macroeconomic state	$\frac{C}{V}$	$\frac{C}{P}$	$\frac{b_1}{V}$	$\frac{b_4}{V}$	$\frac{\text{Firm} - V}{V}$
State 1 (high)	1.41%	7.23%	12.95%	13.58%	2.60%
State 2	2.12%	7.85%	18.35%	19.14%	3.16%
State 3	2.38%	8.16%	19.58%	20.58%	3.30%
State 4 (low)	2.69%	8.39%	21.97%	22.88%	3.36%
All states	2.14%	7.90%	18.19%	19.02%	3.13%

net tax benefits to debt of 2.60% to 3.36% of unlevered asset value, comparable to the 3.8%–4.3% in Korteweg (2010) and the 1.1% reported in van Binsbergen, Graham, and Yang (2010). Moreover, van Binsbergen, Graham, and Yang (2010) decompose their 1.1% net gain into a 9.0% gross benefit and a 7.9% cost, which are slightly bigger than the numbers found in the current estimation, where the model-implied gross tax benefit ranges from 4.95% (state 1) to 8.30% (state 4) and costs from 2.35% (state 1) to 4.94% (state 4). Counter-cyclicality of the estimated tax benefits may at first seem surprising, given that bankruptcy costs should be higher when the economy is in a downturn, and in fact they are: bankruptcy costs constitute 2.35% of asset value in state 1 and 4.94% in state 4. However, coupon payments relative to assets are also considerably higher in the low growth state and that is what causes net tax benefits to be higher, when growth is lower. Note that this conclusion may be reversed if the model is extended to take a possible loss of tax shield into account (Leland and Toft (1996), since this will presumably reduce gross tax benefits more in times of low economic growth.

III.5.4 Optimal capital structure

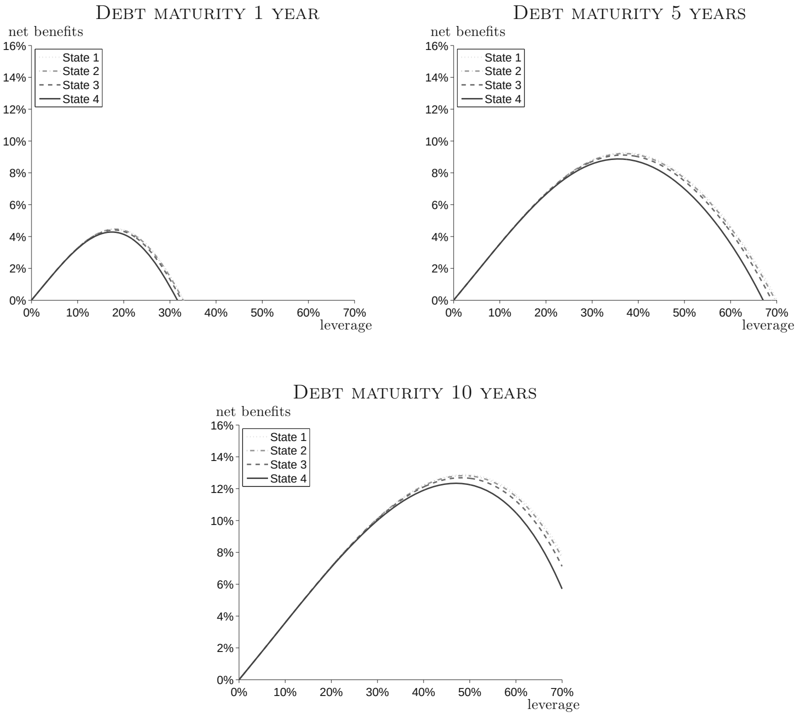
The tax benefits to issuing debt and the disadvantage in terms of default risk can be weighted against each other to obtain an optimal trade-off and hence an optimal choice of leverage. While the previous section addressed net benefits at *actual* leverage, I consider in this section how much larger benefits would be if leverage was *optimally* chosen. Here, I follow existing literature and neglect any costs to issuing debt, and thus the maximal net benefit to debt may be viewed as an upper bound on the maximum attainable gain once issuance and refinancing costs are taken into account.

Based on the parameters in Panel A of Table III.4 I plot in Figure III.8 net tax benefits against leverage for debt maturities of 1, 5, and 10 years, and the figure shows how longer maturities make debt less risky and therefore leads to larger benefits (through a higher coupon C and lower exit levels b). The estimated maximal net benefits of 4.28%–4.48% for the 1 year maturity and 12.34%–12.84% for the 10 year maturity debt are compatible with the 1%-10% found by Korteweg (2010) and the 0%–14% in van Binsbergen, Graham, and Yang (2010). Interestingly, the model implies little state-dependence in the optimal level of net benefits, which is due to the same mechanism that led to state-invariance of long-term credit spreads in section III.5.2. The fact that the economy passes through all four states on a regular basis implies that the optimal leverage ratio is based on a firm's operating conditions in all four states of the business cycle and only pays minor attention to the current state of the economy. Figure III.8 further shows that the net benefit curves are almost flat around the maximal benefit, which implies that the maximal debt issuance gain has a limited sensitivity to leverage. Hence, in the absence of debt restructuring costs there is a whole range of close-to-optimal capital structures, which is consistent with the empirical finding in Leary and Roberts (2005) that leverage is slow-moving because adjustment costs reduce the incentive to restructure.

A comparison of results for each of the 15 firms in the estimation shows that the cross-sectional average *maximal* net benefits to debt range from 7.2% (state 1) to 9.0% (state 4), which should be compared to the *realized* net benefits of 2.60% (state 1) to 3.36% (state 4) reported in Table III.5. In terms of leverage this corresponds to observed average leverage ratios of 15.5% (state 1) to 25.5% (state 4) and estimated optimal leverage ratios between 34.1% and 35.7%. These numbers are in line with those reported in Korteweg (2010) for firms with interest-bearing debt, where the average observed leverage is 25.9% and the optimal leverage 32.8%.

While the numbers seem to suggest that the 15 firms in general are underlevered according to the model, the conclusion is not that clear-cut. A comparison of actual

Figure III.8. Optimal leverage



Model-implied net benefits to debt as function of leverage for average debt maturities of 1, 5, and 10 years.

and model-implied optimal leverage ratios shows that while most of the firms (87%) are underlevered in the high growth state (state 1), this reduces to just over half of the firms (60%) in the low growth state (state 4). In other words, although structural credit risk models traditionally have been known to project unrealistically high optimal leverage ratios, this does not seem to be the case once business cycle and jump risk is taken into account.

III.6 Conclusion

I demonstrate how empirical credit spreads on both investment and speculative grade corporate bonds vary with the state of the economy as changes in economic growth induce shifts in both the level of risk and the distribution between short- and long-term risk. I provide evidence that these movements in short- and long-term risk are linked to idiosyncratic jump patterns and develop a structural credit risk model that encompasses both business cycle variation and jump risk.

Corporate credit spreads are low (high) and the credit spread curve upward-sloping (flat or downward-sloping), when economic growth is high (low), and this implies a counter-cyclical level and pro-cyclical slope of the credit spread curve. Using an extensive data set of daily data spanning 45 years I estimate the structural model on a firm-by-firm basis, and show that it replicates the observed variation in both level and slope of the credit spread curve. As part of the estimation I provide a new procedure for estimation of idiosyncratic jump risk and show that this approach is consistent with observed shocks to firm fundamentals.

The estimated model implies that long-term credit spreads, optimal leverage, and net benefits to debt all have low sensitivity to the business cycle. This is a natural consequence of the fact that the model explicitly incorporates expectations about future changes in the economy, and the current state of the economy therefore has minimal impact on long-term decisions such as the choice of optimal capital structure.

The results in the paper indicate that the inclusion of *either* business cycle *or* jump risk in existing models is insufficient for an adequate description of corporate credit spreads, since it is the *interaction* between the two factors that is crucial for capturing business cycle variation in short- and medium-term credit spreads. Given that the current model emphasizes the importance of business cycle variation in jump risk but has some difficulties in explaining the related risk premia, it is an interesting topic for future research to reach a more detailed description of how jump risk premia move with the business cycle.

Appendix

III.A Data description

The analysis of historical corporate bond spreads in section III.2 and the model estimation in section III.4 are based on the firm and macroeconomic variables listed below.

III.A.1 Firm variables

The sample consists of all firms in the S&P500 500 Industrials stock index as of January 1962 for which more than 20 years of data is available. For each firm information on the following four variables is collected: market value of equity, book value of debt, firm payout, and average debt maturity. Market capitalization is based on daily time series from CRSP of the number of shares outstanding and the price per share, and the other variables are computed from quarterly or annual book values from Compustat. Annual values are used to supplement whenever quarterly values are missing, and the quarterly observations are converted into a daily time series by linear interpolation as in Ericsson, Jacobs, and Oviedo (2009). Book value of debt is constructed as the sum of short- (STD) and long-term debt (LTD), where the former is calculated as the maximum of "debt in current liabilities" and "debt due in 1 year", and the latter as the maximum of "total long-term debt" and "debt maturing in 2–5 years" (DD2+DD3+DD4+DD5). The continuously compounded, annualized firm payout rate is determined as the sum of (annualized) "interest expense" and "common dividends" divided by the sum of market value of equity and book value of debt. Finally, the average maturity of outstanding debt is calculated as

$$\frac{0.5 \cdot \text{STD} + 1.5 \cdot \text{DD2} + 2.5 \cdot \text{DD3} + 3.5 \cdot \text{DD4} + 4.5 \cdot \text{DD5} + 8.5 \cdot (LTD - \sum_{i=2}^{5} DD_i)}{STD + LTD}$$

where an average maturity of 8.5 years for debt maturing in more than 5 years is based on the empirical evidence in Larsen (2006).

III.A.2 Macroeconomic variables

Interest rate

Daily data on the continuously compounded 1-year U.S. Treasury rate is from the Federal Reserve Board (Gürkaynak, Sack, and Wright (2007)).

Consumption growth

Monthly data on the real personal consumption growth rate is from the U.S. Department of Commerce, Bureau of Economic Analysis, Table 2.8.1.

Corporate bond yield

Monthly yields to maturity are from the AAA/AA and A/BBB subindices of the Merrill Lynch U.S. Corporate Investment Grade bond index, and the BB and B subindices of the Merrill Lynch U.S. Corporate High Yield, Cash Pay bond index. All yields are collected from Thomson Datastream.

Recovery rate

Recovery rate data from Moody's Default Risk Service database are used to construct a 1-year trailing recovery rate. The time series covers the period 1985 to 2006 and comprises 1,754 corporate bond recovery observations. The recovery rate is measured as the market value one month after default, and the sample consists of bonds from two debt classes: "Conv./Exch. Bond/Debenture" (11%), "Regular Bond/Debenture" (89%), and four debt seniority categories: "Senior Secured" (6%), "Senior Subordinated" (17%), "Senior Unsecured" (49%), "Subordinated" (28%).

A restriction to only consider recovery rates on regular senior unsecured bonds would be more in line with the theoretical model in section III.3, but the lack of available data requires a less restrictive approach to reduce noise and obtain a robust time series of recovery estimates. Limiting the data set to only contain recovery rates on regular senior unsecured bonds would reduce the amount of observations from 1,754 to 846, and in particular cut the number of observations in the period 1984 to 1996 from 700 to just 200. Moreover, visual inspection of the time series of recovery rates split on either seniority or debt class suggests that recovery rates are not particularly sensitive to either characteristic over the sample period.

III.B Model calculations

All calculations in appendix III.B.1 and III.B.2 are based on the dynamic evolution of the asset value process V_t under the risk-neutral measure, and the superscript $\mathbb{Q}$ is therefore skipped throughout these sections for notational convenience.

III.B.1 Matrix equations and their solutions

In order to state and solve the matrix equations appearing in Jiang and Pistorius (2008), it is necessary to first introduce some notation, so let Θ and Σ be $3n \times 3n$ diagonal matrices with diagonal elements

$$\Theta_{jj} = \begin{cases} 1 & j = 1, \dots, n \\ \theta_{j-n} & j = n+1, \dots, 2n \\ -1 & j = 2n+1, \dots, 3n \end{cases}$$

and

$$\Sigma_{jj} = \begin{cases} 0 & j = 1, \dots, n \\ \sigma_{j-n} & j = n+1, \dots, 2n \\ 0 & j = 2n+1, \dots, 3n \end{cases}$$

and let Π be the $3n \times 3n$ matrix

$$\Pi = \begin{pmatrix} \mathrm{diag}(-\kappa_i^+) & \mathrm{diag}(\kappa_i^+) & \mathrm{O}_n \\ \mathrm{diag}(\alpha_i \lambda_i) & \Xi - \mathrm{diag}(r_i + \lambda_i) & \mathrm{diag}((1-\alpha_i)\lambda_i) \\ \mathrm{O}_n & \mathrm{diag}(\kappa_i^-) & \mathrm{diag}(-\kappa_i^-) \end{pmatrix}$$

where $\mathrm{diag}(y_i)$ denotes the $n \times n$ diagonal matrix with diagonal elements $(y_1, \dots, y_n)$, $\mathrm{O}_{n' \times n''}$ is a $n' \times n''$ zero matrix, and $\mathrm{O}_{n'} = \mathrm{O}_{n' \times n'}$. Finally, let $\mathrm{I}_{n'}$ denote the $n' \times n'$ identity matrix.

Lemma III.B.1 *Assume that the $4n \times 4n$ matrix*

$$\begin{pmatrix} \left((\Theta^{-1}\Pi)_{ij}\right)_{\substack{i=1,\dots,n\\ j=1,\dots,3n}} & \mathrm{O}_n \\ \mathrm{O}_{n\times 3n} & \mathrm{diag}\left(\frac{2\theta_i}{\sigma_i^2}\right) \\ \left((\Theta^{-1}\Pi)_{ij}\right)_{\substack{i=2n+1,\dots,3n\\ j=1,\dots,3n}} & \mathrm{O}_n \\ \left(-(\Theta^{-1}\Pi)_{ij}\right)_{\substack{i=n+1,\dots,2n\\ j=1,\dots,3n}} & \mathrm{diag}\left(\frac{2\theta_i}{\sigma_i^2}\right) \end{pmatrix}$$

has exactly $2n$ eigenvalues e_k^- with strictly negative real part and $2n$ eigenvalues e_k^+ with strictly positive real part. Then

(i) a solution $(A, B) \in \mathbb{R}^{n\times 2n} \times \mathbb{R}^{2n\times 2n}$ to the matrix equation

$$\mathrm{O}_{3n\times 2n} = \frac{1}{2}\Sigma^2 \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} B^2 - \Theta \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} B + \Pi \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \tag{III.7}$$

is given by

$$A = \left(\mathrm{w}_1^- \;\cdots\; \mathrm{w}_{2n}^-\right)\left(\mathrm{v}_1^- \;\cdots\; \mathrm{v}_{2n}^-\right)^{-1}$$
$$B = \left(e_1^-\mathrm{v}_1^- \;\cdots\; e_{2n}^-\mathrm{v}_{2n}^-\right)\left(\mathrm{v}_1^- \;\cdots\; \mathrm{v}_{2n}^-\right)^{-1}$$

where v_k^- are the first $2n$ elements and w_k^- the next n elements of an eigenvector corresponding to e_k^-, $k = 1, \dots, 2n$.

(ii) a solution $(A, B) \in \mathbb{R}^{n\times 2n} \times \mathbb{R}^{2n\times 2n}$ to the matrix equation

$$\mathrm{O}_{3n\times 2n} = \frac{1}{2}\Sigma^2 \begin{pmatrix} A \\ \mathrm{I}_{2n} \end{pmatrix} B^2 + \Theta \begin{pmatrix} A \\ \mathrm{I}_{2n} \end{pmatrix} B + \Pi \begin{pmatrix} A \\ \mathrm{I}_{2n} \end{pmatrix} \tag{III.8}$$

is given by

$$A = \left(\mathrm{w}_1^+ \;\cdots\; \mathrm{w}_{2n}^+\right)\left(\mathrm{v}_1^+ \;\cdots\; \mathrm{v}_{2n}^+\right)^{-1}$$
$$B = -\left(e_1^+\mathrm{v}_1^+ \;\cdots\; e_{2n}^+\mathrm{v}_{2n}^+\right)\left(\mathrm{v}_1^+ \;\cdots\; \mathrm{v}_{2n}^+\right)^{-1}$$

where w_k^+ are the first n elements and v_k^+ the next $2n$ elements of an eigenvector corresponding to e_k^+, $k = 1, \dots, 2n$.

PROOF. Fix k and let $\tilde{\mathrm{v}}_k = (\tilde{\mathrm{v}}_{ik})_{i=1,\dots,4n}$ denote an eigenvector corresponding to the eigenvalue e_k^-. Then

$$\begin{cases} e_k^-(\tilde{\mathrm{v}}_{ik})_{i=n+1,\dots,2n} &= \left(\frac{2\theta_i}{\sigma_i^2}\tilde{\mathrm{v}}_{3n+i,k}\right)_{i=1,\dots,n} \\ \mathrm{O}_{3n\times 1} &= \Theta^{-1}\Pi\,(\tilde{\mathrm{v}}_{ik})_{i=1,\dots,3n} - e_k^- \begin{pmatrix} (\tilde{\mathrm{v}}_{ik})_{i=1,\dots,n} \\ -(\tilde{\mathrm{v}}_{ik})_{i=3n+1,\dots,4n} \\ (\tilde{\mathrm{v}}_{ik})_{i=2n+1,\dots,3n} \end{pmatrix} - \begin{pmatrix} \mathrm{O}_{n\times 1} \\ \left(\frac{2\theta_i}{\sigma_i^2}\tilde{\mathrm{v}}_{3n+i,k}\right)_{i=1,\dots,n} \\ \mathrm{O}_{n\times 1} \end{pmatrix} \end{cases}$$

which by definition of v_k^- and w_k^- reduces to

$$\mathrm{O}_{3n\times 1} = \Pi \begin{pmatrix} \mathrm{v}_k^- \\ \mathrm{w}_k^- \end{pmatrix} - e_k^-\Theta \begin{pmatrix} \mathrm{v}_k^- \\ \mathrm{w}_k^- \end{pmatrix} + \frac{1}{2}(e_k^-)^2\Sigma^2 \begin{pmatrix} \mathrm{v}_k^- \\ \mathrm{w}_k^- \end{pmatrix}.$$

Since $A(\mathrm{v}_1^- \ \cdots \ \mathrm{v}_{2n}^-) = (\mathrm{w}_1^- \ \cdots \ \mathrm{w}_{2n}^-)$ and thus

$$\begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \mathrm{v}_k^- = \begin{pmatrix} \mathrm{v}_k^- \\ \mathrm{w}_k^- \end{pmatrix}$$

we can rewrite the equation as

$$\mathrm{O}_{3n\times 1} = \Pi \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \mathrm{v}_k^- - e_k^-\Theta \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \mathrm{v}_k^- + \frac{1}{2}(e_k^-)^2\Sigma^2 \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \mathrm{v}_k^-.$$

The definition of B implies that e_k^- is also an eigenvalue for B with corresponding eigenvector v_k^-, so

$$\mathrm{O}_{3n\times 1} = \Pi \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} \mathrm{v}_k^- - \Theta \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} B\mathrm{v}_k^- + \frac{1}{2}\Sigma^2 \begin{pmatrix} \mathrm{I}_{2n} \\ A \end{pmatrix} B^2\mathrm{v}_k^-$$

and hence part (i) follows from the regularity of $(\mathrm{v}_1^- \ \cdots \ \mathrm{v}_{2n}^-)$. Part (ii) follows by similar arguments. □

The proof of lemma III.B.1 does not per se exploit the partitioning of the eigenvalues $e_k^\pm$ according to the sign of their real part. However, for the matrices solving (III.7) and (III.8), henceforth denoted A^-, B^- respectively A^+, B^+, to be valid in the sequel, it is necessary to make the additional requirements that each row in $A^\pm$ is a subprobability vector and that $B^{\pm}$ is a subintensity matrix. It is for these characteristics to hold that it is important to distinguish between the two types of eigenvalues (see Jiang and Pistorius (2008) theorem 4.2). Although I do not have a proof verifying that the expressions for $A^{\pm}$ and $B^{\pm}$ in lemma III.B.1 automatically satisfy these extra conditions, I have checked in all of the numerical calculations that this is the case.

Moreover, following Jiang and Pistorius (2008) the solutions to (III.7) and (III.8) will be unique under suitable parameter restrictions and subject to the additional requirements of $A^{\pm}$ having subprobability rows and $B^{\pm}$ being subintensity. A comparison with results for similar models (Barlow, Rogers, and Williams (1980), Jacobsen (2005)) indicates that in this case it may even be possible to show that the $4n \times 4n$ matrix will always have exactly half of its eigenvalues with strictly negative real part and the other half with strictly positive real part.

Finally, lemma III.B.1 implicitly assumes that the matrices $(\mathrm{v}_1^{\pm} \ \cdots \mathrm{v}_{2n}^{\pm})$ of (partial) eigenvectors are regular. While I have no theoretical justification for this, related results in Jacobsen (2005) suggest that singularity of these matrices is not likely to be of any concern in numerical implementations, and indeed I have encountered no singularities in the computations in this paper.

III.B.2 Calculating the expectation

To calculate the expectation in (III.6) I follow Jiang and Pistorius (2008) with a minor adjustment, see subsection III.B.3 for details. The calculation is based on multiple applications of lemma III.B.1, so continuing the notation of the previous section, I now introduce the $3n' \times 3n'$ submatrices

$$\Theta_{n'} = \begin{pmatrix} (\Theta_{ij})_{\substack{i=1,\dots,n' \\ j=1,\dots,n'}} & (\Theta_{ij})_{\substack{i=1,\dots,n' \\ j=n+1,\dots,n+n'}} & (\Theta_{ij})_{\substack{i=1,\dots,n' \\ j=2n+1,\dots,2n+n'}} \\ (\Theta_{ij})_{\substack{i=n+1,\dots,n+n' \\ j=1,\dots,n'}} & (\Theta_{ij})_{\substack{i=n+1,\dots,n+n' \\ j=n+1,\dots,n+n'}} & (\Theta_{ij})_{\substack{i=n+1,\dots,n+n' \\ j=2n+1,\dots,2n+n'}} \\ (\Theta_{ij})_{\substack{i=2n+1,\dots,2n+n' \\ j=1,\dots,n'}} & (\Theta_{ij})_{\substack{i=2n+1,\dots,2n+n' \\ j=n+1,\dots,n+n'}} & (\Theta_{ij})_{\substack{i=2n+1,\dots,2n+n' \\ j=2n+1,\dots,2n+n'}} \end{pmatrix}$$

$$\Sigma_{n'} = \begin{pmatrix} (\Sigma_{ij})_{\substack{i=1,\ldots,n' \\ j=1,\ldots,n'}} & (\Sigma_{ij})_{\substack{i=1,\ldots,n' \\ j=n+1,\ldots,n+n'}} & (\Sigma_{ij})_{\substack{i=1,\ldots,n' \\ j=2n+1,\ldots,2n+n'}} \\ (\Sigma_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=1,\ldots,n'}} & (\Sigma_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=n+1,\ldots,n+n'}} & (\Sigma_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=2n+1,\ldots,2n+n'}} \\ (\Sigma_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=1,\ldots,n'}} & (\Sigma_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=n+1,\ldots,n+n'}} & (\Sigma_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=2n+1,\ldots,2n+n'}} \end{pmatrix}$$

$$\Pi_{n'} = \begin{pmatrix} (\Pi_{ij})_{\substack{i=1,\ldots,n' \\ j=1,\ldots,n'}} & (\Pi_{ij})_{\substack{i=1,\ldots,n' \\ j=n+1,\ldots,n+n'}} & (\Pi_{ij})_{\substack{i=1,\ldots,n' \\ j=2n+1,\ldots,2n+n'}} \\ (\Pi_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=1,\ldots,n'}} & (\Pi_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=n+1,\ldots,n+n'}} & (\Pi_{ij})_{\substack{i=n+1,\ldots,n+n' \\ j=2n+1,\ldots,2n+n'}} \\ (\Pi_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=1,\ldots,n'}} & (\Pi_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=n+1,\ldots,n+n'}} & (\Pi_{ij})_{\substack{i=2n+1,\ldots,2n+n' \\ j=2n+1,\ldots,2n+n'}} \end{pmatrix}$$

for $n' = 1, \ldots, n$, and I henceforth use $A^{\pm}_{n'}$ and $B^{\pm}_{n'}$ to denote the matrices resulting from applying lemma III.B.1 to the triple $\left(\Theta_{n'}, \Sigma_{n'}, \Pi_{n'}\right)$. Next, I specify for $n' = 1, \ldots, n-1$ the auxiliary matrices

$$\begin{aligned}
C^{-}_{n'} &= \begin{pmatrix} \mathrm{O}_{n'} \quad \mathrm{I}_{n'} \\ A^{-}_{n'} \end{pmatrix} \exp\left(B^{-}_{n'} \log\left(\frac{b_{n'+1}}{b_{n'}}\right)\right) \\
C^{+}_{n'} &= \begin{pmatrix} A^{+}_{n'} \\ \mathrm{I}_{n'} \quad \mathrm{O}_{n'} \end{pmatrix} \exp\left(B^{+}_{n'} \log\left(\frac{b_{n'+1}}{b_{n'}}\right)\right) \\
D^{-}_{n'} &= \begin{pmatrix} \mathrm{I}_{2n'} \\ A^{-}_{n'} \end{pmatrix} \\
D^{+}_{n'} &= \begin{pmatrix} A^{+}_{n'} \\ \mathrm{I}_{2n'} \end{pmatrix} \\
E^{-}_{n'} &= \begin{pmatrix} \mathrm{I}_{2n'} & \mathrm{O}_{2n' \times n'} \end{pmatrix} \\
E^{+}_{n'} &= \begin{pmatrix} \mathrm{O}_{2n' \times n'} & \mathrm{I}_{2n'} \end{pmatrix}
\end{aligned}$$

and the $3n' \times 3n'$ diagonal matrix $F_{n'}$ with diagonal elements

$$(F_{n'})_{jj} = \begin{cases} 0 & j = 1, \ldots, n' \\ \dfrac{\displaystyle\sum_{k=n'+1}^{n} (\Pi_n)_{n+j-n',n+k} f_k}{\displaystyle\sum_{k=1}^{3n'} (\Pi_{n'})_{jk}} & j = n'+1, \ldots, 2n' \\ 0 & j = 2n'+1, \ldots, 3n' \end{cases}$$

and use these to define the matrix-valued function

$$\mathrm{G}(v) = D_n^+ \exp\left(B_n^+ \log\left(\frac{v}{b_n}\right)\right)$$

and for $n' = 1, \ldots, n-1$ the functions

$$\begin{aligned} \mathrm{H}_1(v, n') = & \left[D_{n'}^- \exp\left(B_{n'}^- \log\left(\frac{b_{n'+1}}{v}\right)\right) - D_{n'}^+ \exp\left(B_{n'}^+ \log\left(\frac{v}{b_{n'}}\right)\right) C_{n'}^-\right] \\ & \cdot \left[\mathrm{I}_{2n'} - C_{n'}^+ C_{n'}^-\right]^{-1} \\ \mathrm{H}_2(v, n') = & \left[D_{n'}^+ \exp\left(B_{n'}^+ \log\left(\frac{v}{b_{n'}}\right)\right) - D_{n'}^- \exp\left(B_{n'}^- \log\left(\frac{b_{n'+1}}{v}\right)\right) C_{n'}^+\right] \\ & \cdot \left[\mathrm{I}_{2n'} - C_{n'}^- C_{n'}^+\right]^{-1} \\ \mathrm{H}_3(v, n', a) = & -\left[v^a \mathrm{I}_{3n'} - {b_{n'+1}}^a \mathrm{H}_1(v, n') E_{n'}^- - {b_{n'}}^a \mathrm{H}_2(v, n') E_{n'}^+\right] \\ & \cdot \left[\frac{a^2}{2} \Sigma_{n'}^2 + a \Theta_{n'} + \Pi_{n'}\right]^{-1} F_{n'} \Pi_{n'} 1_{n'} \end{aligned}$$

with $1_{n'}$ a $3n' \times 1$ column vector of ones. I can then introduce the real-valued function

$$
\mathcal{E}(v,i,a) = \begin{cases}
d_{n,i}^{\top} \mathrm{G}(v)\bar{c}_n, & \textit{when} \quad v > b_n, \\
& i = 1,\dots,3n \\[2ex]
d_{n',i}^{\top}\Big[\mathrm{H}_1(v,n')\underline{c}_{n'} + \mathrm{H}_2(v,n')\bar{c}_{n'} + \mathrm{H}_3(v,n',a)\Big], & \\
& \textit{when} \quad b_{n'} < v \le b_{n'+1}, \\
& i = 1,\dots,n', \\
& n' = 1,\dots,n-1 \\[2ex]
d_{n',i-(n-n')}^{\top}\Big[\mathrm{H}_1(v,n')\underline{c}_{n'} + \mathrm{H}_2(v,n')\bar{c}_{n'} + \mathrm{H}_3(v,n',a)\Big], & \\
& \textit{when} \quad b_{n'} < v \le b_{n'+1}, \\
& i = n+1,\dots,n+n', \\
& n' = 1,\dots,n-1 \\[2ex]
d_{n',i-2(n-n')}^{\top}\Big[\mathrm{H}_1(v,n')\underline{c}_{n'} + \mathrm{H}_2(v,n')\bar{c}_{n'} + \mathrm{H}_3(v,n',a)\Big], & \\
& \textit{when} \quad b_{n'} < v \le b_{n'+1}, \\
& i = 2n+1,\dots,2n+n' \\
& n' = 1,\dots,n-1 \\[2ex]
v^a f(i), & \textit{otherwise}
\end{cases}
$$

where $d_{n',i} = (d_{n',i,j})_{j=1,\dots,3n'}$ are column vectors with

$$
d_{n',i,j} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}
$$

and

$$
\underline{c}_{n'} = \begin{pmatrix} (c_{n'+1,j})_{j=1,\dots,n'} \\ (c_{n'+1,j})_{j=n+1,\dots,n+n'} \end{pmatrix} \qquad \bar{c}_{n'} = \begin{pmatrix} (c_{n',j})_{j=n+1,\dots,n+n'} \\ (c_{n',j})_{j=2n+1,\dots,2n+n'} \end{pmatrix}
$$

are column vectors of constants $c_{n',j}$ determined by the linear system

$$\begin{cases} c_{n',n+n'} = {b_{n'}}^a f_{n'} & \textit{for} \quad n'=1,\ldots,n \\ c_{n',2n+n'} = {b_{n'}}^a f_{n'} \dfrac{\kappa_{n'}^-}{a+\kappa_{n'}^-} & \textit{for} \quad n'=1,\ldots,n \\ \lim\limits_{v\to b_{n'}+} \dfrac{\partial}{\partial v}\mathcal{E}(v,i,a) = \lim\limits_{v\to b_{n'}-} \dfrac{\partial}{\partial v}\mathcal{E}(v,i,a) & \textit{for} \quad i=n+1,\ldots,n+n'-1, \\ & \qquad n'=2,\ldots,n \\ \lim\limits_{v\to b_{n'}+} \mathcal{E}(v,i,a) = \lim\limits_{v\to b_{n'}-} \mathcal{E}(v,i,a) & \textit{for} \quad i=1,\ldots,n'-1, \\ & \qquad\quad 2n+1,\ldots,2n+n'-1, \\ & \qquad n'=2,\ldots,n \end{cases}$$

The expectation in (III.6) is now given as

$$\mathrm{E}_{v,i}\left[e^{-\int_0^\tau r(Z_s)ds}\,(V_\tau)^a\, f(Z_\tau)1_{(\tau<\infty)}\right] = \mathcal{E}(v,n+i,a) \qquad\qquad i=1,\ldots,n.$$

III.B.3 A note on Jiang and Pistorius (2008)

The expression for the expectation in (III.6) derived in section III.B.2 exploits the novel probabilistic results developed in Jiang and Pistorius (2008) (JP). However, in order to apply these result, I have to correct for an error appearing in one of their main theorems, theorem 6.1. For clarity of exposition, I deviate in this section from the notation used in the remainder of the paper and switch instead to the notation employed by JP. Thus, let

$$\rho_j = \inf\{t \geq 0 \mid Y_t \in \{1,\ldots,j-1\}\} \qquad j=2,\ldots,N.$$

and consider fixed $j \in \{2,\ldots,N\}$, $i \in \widetilde{E}_j$ and $k_j < x < k_{j-1}$. As in JP it follows that

$$\begin{aligned} v_{b,k}(x,i) = {} & \mathrm{E}_{x,i}\left[e^{bA_{\tilde{T}(k)}} h(Y_{\tilde{T}(k)}) 1_{(\tilde{T}(k)<\zeta)}\right] \\ = {} & \mathrm{E}_{x,i}\left[v_{b,k}(k_{j-1}, Y_{\tau_{k_j,k_{j-1}}}) 1_{(\tau_{k_j,k_{j-1}}<\rho_j\wedge\zeta,\ A_{\tau_{k_j,k_{j-1}}}=k_{j-1})}\right] \\ & +\mathrm{E}_{x,i}\left[v_{b,k}(k_j, Y_{\tau_{k_j,k_{j-1}}}) 1_{(\tau_{k_j,k_{j-1}}<\rho_j\wedge\zeta,\ A_{\tau_{k_j,k_{j-1}}}=k_j)}\right] \\ & +\mathrm{E}_{x,i}\left[e^{bA_{\rho_j}} h(Y_{\rho_j}) 1_{(\rho_j<\zeta,\ \rho_j<\tau_{k_j,k_{j-1}})}\right] \end{aligned}$$

and JP give explicit expressions for all three right-hand-side terms. However, the expression for the last term is incorrect, and to see this I apply their proposition 5.1

to get

$$
\begin{aligned}
&\mathrm{E}_{x,i}\left[e^{bA_{\rho_j}}h(Y_{\rho_j})1_{(\rho_j<\zeta,\,\rho_j<\tau_{k_j,k_{j-1}})}\right] \\
&=\mathrm{E}_{x,i}\left[e^{bA_{\rho_j}}\mathrm{E}\big[h(Y_{\rho_j})1_{(\rho_j<\zeta,\,\rho_j\wedge\zeta<\tau_{k_j,k_{j-1}})}\;\big|\;\mathcal{F}_{(\rho_j\wedge\zeta)-}\big]\right] \\
&=\mathrm{E}_{x,i}\left[e^{bA_{\rho_j}}1_{(Y_{(\rho_j\wedge\zeta)-}\in\{j,\dots,N\})}\sum_{s=1}^{j-1}\frac{(Q_a)_{Y_{(\rho_j\wedge\zeta)-},s}}{-(Q_a^{(j)}1)_{Y_{(\rho_j\wedge\zeta)-}}}h(s)1_{(\rho_j\wedge\zeta<\tau_{k_j,k_{j-1}})}\right].
\end{aligned}
$$

Here, I have exploited the fact that a jump to one of the absorbing states $\{1,\dots,j-1\}\cup\partial$ can only occur when $Y_t\in\{j,\dots,N\}$ (i.e. $(Q_a^{(j)}1)_\ell=0$ for $\ell\in\widetilde{E}_j\backslash\{j,\dots,N\}$). Thus if I introduce the $|\widetilde{E}_j|\times|\widetilde{E}_j|$ diagonal matrix R_j with diagonal elements

$$
(R_j)_{\ell,\ell}=\begin{cases}\frac{\sum_{s=1}^{j-1}(Q_a)_{\ell,s}h(s)}{(Q_a^{(j)}1)_\ell} & \ell\in\{j,\dots,N\}\\ 0 & \ell\in\widetilde{E}_j\backslash\{j,\dots,N\}\end{cases}
$$

then I can reapply their proposition 5.1 to express the third term as

$$
\mathrm{E}_{x,i}\left[e^{bA_{\rho_j}}h(Y_{\rho_j})1_{(\rho_j<\zeta,\,\rho_j<\tau_{k_j,k_{j-1}})}\right]=e_i^\top\Psi_j^\circ(b,x)R_jQ_a^{(j)}1
$$

and the correct expression for $v_{b,k}(x,i)$ in their theorem 6.1 therefore reads

$$
v_{b,k}(x,i)=\begin{cases}e_i^\top\Phi_{k_1}^-(x)h_1^- & x>k_1,i\in E\\ e_i^\top\left[\Psi_j^+(x)h_j^++\Psi_j^-(x)h_j^-+\Psi_j^\circ(b,x)R_jQ_a^{(j)}1\right] & j\geq 2,k_j<x\leq k_{j-1},i\in\widetilde{E}_j\end{cases}.
$$

III.B.4 Non-negative jump risk premium

The jump risk premium

$$
\eta_J=\lambda^{\mathbb{P}}\zeta^{\mathbb{P}}-\lambda^{\mathbb{Q}}\zeta^{\mathbb{Q}}=\lambda^{\mathbb{P}}\left(\zeta^{\mathbb{P}}-\delta^{\mathbb{P}}\zeta^{\mathbb{Q}}\right)
$$

is non-negative if $\zeta^{\mathbb{P}}-\delta^{\mathbb{P}}\zeta^{\mathbb{Q}}>0$, and this condition is satisfied exactly when $\gamma_M<0$ as the following argument shows. Note first that

$$
\delta^{\mathbb{P}}\alpha^{\mathbb{Q}}=\frac{\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}}}{\kappa^{+,\mathbb{Q}}},\qquad\delta^{\mathbb{P}}(1-\alpha^{\mathbb{Q}})=\frac{(1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}}}{\kappa^{-,\mathbb{Q}}}
$$

which implies

$$
\delta^{\mathbb{P}}\zeta^{\mathbb{Q}}=\delta^{\mathbb{P}}\left(\frac{\alpha^{\mathbb{Q}}\kappa^{+,\mathbb{Q}}}{\kappa^{+,\mathbb{Q}}-1}+\frac{(1-\alpha^{\mathbb{Q}})\kappa^{-,\mathbb{Q}}}{\kappa^{-,\mathbb{Q}}+1}-1\right)
$$

$$
\begin{aligned}
&= \frac{\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}}}{\kappa^{+,\mathbb{P}} - \gamma_M - 1} + \frac{(1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}}}{\kappa^{-,\mathbb{P}} + \gamma_M + 1} - \delta^{\mathbb{P}} \\
&= \alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}} \left(\frac{1}{\kappa^{+,\mathbb{P}} - \gamma_M - 1} - \frac{1}{\kappa^{+,\mathbb{P}} - \gamma_M} \right) \\
&\quad + (1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}} \left(\frac{1}{\kappa^{-,\mathbb{P}} + \gamma_M + 1} - \frac{1}{\kappa^{-,\mathbb{P}} + \gamma_M} \right)
\end{aligned}
$$

and hence

$$
\begin{aligned}
\frac{\partial}{\partial \gamma_M}\left(\zeta^{\mathbb{P}} - \delta^{\mathbb{P}}\zeta^{\mathbb{Q}}\right) = -\alpha^{\mathbb{P}}\kappa^{+,\mathbb{P}} & \underbrace{\left(\frac{1}{(\kappa^{+,\mathbb{P}} - \gamma_M - 1)^2} - \frac{1}{(\kappa^{+,\mathbb{P}} - \gamma_M)^2} \right)}_{>0} \\
+ (1-\alpha^{\mathbb{P}})\kappa^{-,\mathbb{P}} & \underbrace{\left(\frac{1}{(\kappa^{-,\mathbb{P}} + \gamma_M + 1)^2} - \frac{1}{(\kappa^{-,\mathbb{P}} + \gamma_M)^2} \right)}_{<0} < 0.
\end{aligned}
$$

Since $\delta^{\mathbb{P}}\zeta^{\mathbb{Q}} = \zeta^{\mathbb{P}}$ if $\gamma_M = 0$ it follows that $\zeta^{\mathbb{P}} - \delta^{\mathbb{P}}\zeta^{\mathbb{Q}} > 0$ when $\gamma_M < 0$.

III.C Estimation details

III.C.1 Macroeconomic states

To estimate the intensity matrix Ξ for the macroeconomic state process $(Z_t)_{t\geq 0}$, I first calibrate an observed path $(z_{t_k})_k$ for $(Z_t)_{t\geq 0}$ and subsequently apply maximum likelihood estimation based on $(z_{t_k})_k$.

To infer the n possible states for $(Z_t)_{t\geq 0}$ I follow Tauchen and Hussey (1991) and apply $n-$point Gauss-Legendre quadrature to the monthly time series of consumption growth rates. This yields quadrature points $\ell_n < \ldots < \ell_1$ that I use to define the end points of the n intervals

$$
\left]-\infty, \tfrac{\ell_n + \ell_{n-1}}{2}\right], \quad \ldots, \quad \left]\tfrac{\ell_2 + \ell_1}{2}, \infty\right[
$$

numbered $n, n-1, \ldots, 1$. The monthly time series of the state of economy (z_{t_k}) now follows from the time series of observed growth rates by assigning for each month the number of the interval that contains the corresponding growth rate observation.

Applying the methodology with $n = 4$ gives the quadrature points $\ell_4 = 0.46\%, \ell_3 = 2.23\%, \ell_2 = 4.55\%, \ell_1 = 6.32\%$, and Figure III.5 shows the observed monthly growth rates together with the resulting business cycle path $(z_{t_k})_k$. To limit the influence from potential outliers, I apply the quadrature rule only to the range of growth rates between the 5%– and 95%–percentiles of the empirical growth rate distribution. To further correct for numerical anomalies, I adjust the calibrated path $(z_{t_k})_k$ on the few occasions where $(z_{t_k})_k$ is set to spend only a single month in a given state and then return to its previous state. Such short-lived jumps back and forth are artifacts of the calibration procedure and not representative of fundamental macroeconomic changes, and I therefore ignore these ephemeral jumps.

The state process $(Z_t)_{t\geq 0}$ is assumed to be an $n-$state Markov process with intensity matrix $\Xi = (\xi_{ij})_{i,j=1,\ldots,n}$, which I estimate from $(z_{t_k})_k$ by maximizing the log likelihood function

$$\sum_k \log P(Z_{t_{k+1}} = z_{t_{k+1}} \,|\, Z_{t_k} = z_{t_k}).$$

where each element in the sum is calculated from the approximation

$$P(Z_{t_{k+1}} = j \,|\, Z_{t_k} = i) \approx \begin{cases} \exp\left(-(t_{k+1}-t_k)\sum_{l\neq i}\xi_{il}\right) & j = i \\ \left(1-\exp\left(-(t_{k+1}-t_k)\sum_{l\neq i}\xi_{il}\right)\right)\dfrac{\xi_{ij}}{\sum_{l\neq i}\xi_{il}} & j \neq i \end{cases}.$$

This corresponds to assuming that $(Z_t)_{t\geq 0}$ can jump at most once in each interval $]t_k, t_{k+1}]$, and since the estimation is based on monthly growth rate observations (i.e. $t_{k+1} - t_k = \frac{1}{12}$), I consider this to be a reasonable approximation. Table III.2 gives the estimated intensity matrix Ξ, and Table III.3 contains the resulting stationary distribution $(\pi_i)_{i=1,\ldots,n}$ and its first four standardized moments.[27] These moments are calculated from the $n-$point probability distribution that assigns probability π_i to observing the quadrature point ℓ_i for $i = 1, \ldots, n$.

III.C.2 Asset value parameters

The asset process parameters $(\kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}}, \theta^{\mathbb{P}}, \sigma, \gamma_M)$ are estimated using an extension of the iterative procedure presented in Vassalou and Xing (2004):

[27] Since the estimated intensity matrix $\Xi = (\xi_{ij})$ is irreducible, the stationary distribution $(\pi_i)_{i=1,\ldots,n}$ for $(Z_t)_{t\geq 0}$ is given as the unique solution to the linear system

$$\sum_{j=1,j\neq i}^{n} \pi_i \xi_{ij} = \sum_{j=1,j\neq i}^{n} \pi_j \xi_{ji} \qquad i = 1, \ldots, n$$

subject to $\sum_{i=1}^{n} \pi_i = 1$ and $\pi_i \geq 0$, $i = 1, \ldots, n$ (see e.g. Norris (1997)).

(i) Fix initial daily time series of asset value V, debt coupon C, and default boundaries b.

(ii) Calculate maximum likelihood estimates of $(\kappa^{+,\mathbb{P}}, \kappa^{-,\mathbb{P}}, \theta^{\mathbb{P}}, \sigma, \gamma_M)$ conditional on V, C, b based on the likelihood function outlined below.

(iii) Conditional on parameter estimates from (ii) determine for each daily data point updated values of: asset value V, debt coupon C, and default boundaries b such that model-implied market value of equity matches observed market capitalization, model-implied debt equals observed book value of debt, and the n smooth pasting conditions are satisfied.

(iv) Repeat steps (ii)–(iii) until parameter estimates converge.

The likelihood function $\mathcal{L}$ in step (ii) is constructed using a transformation approach (Duan (1994; 2000)) that exploits the tractability of log asset value $X = \log V$[28]

$$\log \mathcal{L} = \sum_j \log P\big(X_{s_{j+1}} = \log v_{s_{j+1}} \,\big|\, (z_{t_k})_k, \mathcal{F}_{s_j}\big) - \sum_j \log \frac{\partial \text{E}\textsc{quity}}{\partial v}(v_{s_{j+1}}, z_{s_{j+1}}) - \sum_j \log v_{s_{j+1}}$$

Here, v_{s_j} is the market value of assets, z_{s_j} the state of the economy, and $\mathcal{F}_{s_j}$ the total information set at time s_j. By conditioning on the number of jumps in asset value on a given day it follows that (Aït-Sahalia (2004))

$$\begin{aligned} &P\big(X_{s_{j+1}} = X_{s_j} + x \,\big|\, (z_{t_k})_k, \mathcal{F}_{s_j}\big) \\ = &\sum_{m=0}^{\infty} P\Big(\theta_i^{\mathbb{P}} \Delta_j + \sigma_i (W^{\mathbb{P}}_{s_{j+1}} - W^{\mathbb{P}}_{s_j}) + \sum_{l=1}^{m} Y_{i,l} = x\Big) \frac{\big(\lambda_i \Delta_j\big)^m}{m!} \exp\left(-\lambda_i \Delta_j\right) \end{aligned}$$

where $\Delta_j = s_{j+1} - s_j$, and $Y_{i,1}, Y_{i,2}, \ldots$ are the jump sizes corresponding to jumps in $(J^{\mathbb{P}}_{i,t})_{t \geq 0}$. Consistent with the jump detection procedure in section III.4.3 I limit the

[28] Maximum likelihood estimation of a jump-diffusion process is complicated by the fact that the likelihood function may be unbounded at a discrete set of points (Honoré (1998), Craine, Lochstoer, and Syrtveit (2000)). However, several empirical studies have found that this is mostly a theoretical concern (see e.g. Jorion (1988), Hamilton (1994, chp. 22), Aït-Sahalia (2004), or the extensive simulation-based evidence in Tauchen and Zhou (2010)).

occurrence of jumps to at most one each day, which implies that

$$\begin{aligned}
&P\big(X_{s_{j+1}} = X_{s_j} + x \,\big|\, (z_{t_k})_k, \mathcal{F}_{s_j}\big) \\
= &\frac{1}{\sqrt{2\pi\sigma_i^2\Delta_j}} \exp\left(-\frac{(x-\theta_i^{\mathbb{P}}\Delta_j)^2}{2\sigma_i^2\Delta_j} - \lambda_i^{\mathbb{P}}\Delta_j\right) \\
&+ \int_0^{\infty} \frac{\alpha_i^{\mathbb{P}}\kappa_i^{+,\mathbb{P}}\lambda_i^{\mathbb{P}}\Delta_j}{\sqrt{2\pi\sigma_i^2\Delta_j}} \exp\left(-\frac{(x-y-\theta_i^{\mathbb{P}}\Delta_j)^2}{2\sigma_i^2\Delta_j} - \kappa_i^{+,\mathbb{P}}y - \lambda_i^{\mathbb{P}}\Delta_j\right) dy \\
&+ \int_{-\infty}^{0} \frac{(1-\alpha_i^{\mathbb{P}})\kappa_i^{-,\mathbb{P}}\lambda_i^{\mathbb{P}}\Delta_j}{\sqrt{2\pi\sigma_i^2\Delta_j}} \exp\left(-\frac{(x-y-\theta_i^{\mathbb{P}}\Delta_j)^2}{2\sigma_i^2\Delta_j} + \kappa_i^{-,\mathbb{P}}y - \lambda_i^{\mathbb{P}}\Delta_j\right) dy
\end{aligned}$$

The two integrals can be rewritten in terms of incomplete Gamma functions, and this gives a closed-form expression for the loglikelihood function. While the structural credit risk model, in principle, requires firm asset value to be above the default boundary at all times, I consider it a minor approximation to only check that this condition is satisfied for each of the daily observations during the estimation.

Estimation of structural credit risk models that allow for jump risk are known, for some firms, to disproportionally favour jump risk at the expense of the diffusion component (Cremers, Driessen, and Maenhout (2008)). I therefore set the following limits on the parameters during the estimation based on a subjective assessment of what seems to be a reasonable degree of jump risk: $0.25 \leq \alpha^{\mathbb{P}} \leq 0.75$, $1/\kappa^{\mathbb{P}} \leq 15\%$, $-15\% \leq \eta_J \leq 15\%$, $-15\% \leq \eta_W \leq 15\%$, $-15\% \leq \eta_J + \eta_W \leq 15\%$.

Conclusion

This thesis contains three essays about correlation in corporate default risk. The essays document the important role of default correlation in explaining time series behaviour of both actual default events and prices of credit risky securities. The analyses and conclusions in each essay are founded on extensive use of mathematical models followed by detailed empirical studies.

The first essay demonstrates the inadequacy of previous literature in detecting default contagion in intensity-based models of actual defaults. It further gives empirical evidence in favour of both misspecification and default contagion in these models, and it proposes a new way to test for these effects. The second essay formulates a detailed model for joint pricing of Credit Default Swap (CDS) and Collateralized Debt Obligation (CDO) contracts. The model features both idiosyncratic and systematic default risk and circumvents the restrictive parameter contraints enforced in earlier literature. Estimation of the model reveals a good fit to observed CDS and CDO spreads. The third and final essay documents substantial business cycle variation in the level and slope of corporate credit spreads and links this to idiosyncratic jump risk. It develops a structural credit risk model that incorporates both jump and business cycle risk, and shows by estimation that the structural model is able to replicate the observed variation in empirical credit spreads.

The thesis analyses several different aspects of default risk correlation, and highlights a series of theoretical challenges related to proper modelling of default correlation. It documents that some of these issues can be handled by more careful application of existing models and techniques, but it also shows that additional work – both theoretical and empirical – is needed to further increase our understanding of the economic forces that drive default risk correlation.

Bibliography

Aalen, O. O. and J. M. Hoem (1978). Random time changes for multivariate counting processes. *Scandinavian Actuarial Journal*, 81–101.

Acharya, V. V., S. T. Bharath, and A. Srinivasan (2007). Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries. *Journal of Financial Economics 85*, 787–821.

Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance 23*, 589–609.

Altman, E. I., B. Brady, A. Resti, and A. Sironi (2005). The link between default and recovery rates: Theory, empirical evidence, and implications. *Journal of Business 78*, 2203–2228.

Amato, J. D. and J. Gyntelberg (2005). CDS index tranches and the pricing of credit risk correlations. *BIS Quarterly Review, March 2005*, 73–87.

Amato, J. D. and M. Luisi (2006). Macro factors in the term structure of credit spreads. *BIS Working paper no. 203*.

Andersen, L., J. Sidenius, and S. Basu (2003, November). All your hedges in one basket. *Risk*, 67–72.

Andersen, P. K., Ørnulf Borgan, R. D. Gill, and N. Keiding (1992). *Statistical models based on counting processes*. Springer-Verlag.

Andersen, T. G., L. Benzoni, and J. Lund (2002). An empirical investigation of continuous-time equity return models. *Journal of Finance 57*, 1239–1284.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *Review of Economics and Statistics 89*, 701–720.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and C. Vega (2007). Real-time price discovery in global stock, bond and foreign exchange markets. *Journal of International Economics 73*, 251–277.

Aït-Sahalia, Y. (2004). Disentangling diffusion from jumps. *Journal of Financial Economics 74*, 487–528.

Azizpour, S. and K. Giesecke (2008). Self-exciting corporate defaults: Contagion vs. frailty. *Working paper, Stanford University*.

Azizpour, S., K. Giesecke, and B. Kim (2011). Premia for correlated default risk. *Journal of Economic Dynamics and Control 35*, 1340–1357.

Bakshi, G., C. Cao, and Z. Chen (1997). Empirical performance of alternative option pricing models. *Journal of Finance 52*, 2003–2049.

Ball, C. A. and W. N. Torous (1983). A simplified jump process for common stock returns. *Journal of Financial and Quantitative Analysis 18*, 53–65.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance 59*, 1481–1509.

Barlow, M., L. Rogers, and D. Williams (1980). Wiener-Hopf factorization for matrices. *Séminaire de Probabilités XIV, Lecture Notes in Math. 784*, 324–331.

Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics 2*, 1–37.

Barndorff-Nielsen, O. E. and N. Shephard (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics 4*, 1–30.

Besag, J. (1974). Spatial interaction and the statistical analysis of lattice systems. *Journal of the Royal Statistical Association Series B 36*, 192–236.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010a). The aggregate dynamics of capital structure and macroeconomic risk. *Review of Financial Studies 23*, 4187–4241.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010b). The levered equity risk premium and credit spreads: A unified framework. *Review of Financial Studies 23*, 645–703.

BIS (2007). Triennial central bank survey. *Bank for International Settlements*.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy 81*, 637–654.

Boissay, F. (2006). Credit chains and the propagation of financial distress. *ECB Working paper no. 573*.

Bollerslev, T., T. H. Law, and G. Tauchen (2008). Risk, jumps, and diversification. *Journal of Econometrics 144*, 234–256.

Bollerslev, T. and V. Todorov (2011). Tails, fears and risk premia. *Journal of Finance (forthcoming)*.

Boyarchenko, S. I. and S. Z. Levendorskii (2002). *Non-Gaussian Merton-Black-Scholes theory*. World Scientific.

Brémaud, P. A martingale approach to point processes. *Memorandum ERL-M345, University of California, Berkeley*.

Brown, T. C. and M. G. Nair (1988). A simple proof of the multivariate random time change theorem for point processes. *Journal of Applied Probability 25*, 210–214.

Campbell, J. Y. and J. Ammer (1993). What moves the stock and bond markets? A variance decomposition for long-term asset returns. *Journal of Finance 48*, 3–37.

Campbell, J. Y., M. Lettau, B. G. Malkiel, and Y. Xu (2001). Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance 56*, 1–43.

Campbell, J. Y. and G. B. Taksler (2003). Equity volatility and corporate bond yields. *Journal of Finance 58*, 2321–2349.

Carling, K., T. Jacobson, J. Lindé, and K. Roszbach (2007). Corporate credit risk modeling and the macroeconomy. *Journal of Banking and Finance 31*, 845–868.

Carr, P. and D. B. Madan (1999). Option valuation using the Fast Fourier Transform. *Journal of Computational Finance*, 61–73.

Chava, S., C. Stefanescu, and S. Turnbull (2011). Modeling the loss distribution. *Management Science 57*, 1267–1287.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*, 2171–2212.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2009). On the relation between the credit spread puzzle and the equity premium puzzle. *Review of Financial Studies 22*, 3367–3409.

Chen, L., D. A. Lesmond, and J. Wei (2007). Corporate yield spreads and bond liquidity. *Journal of Finance 62*, 119–149.

Chen, N. and S. G. Kou (2009). Credit spreads, optimal capital structure, and implied volatility with endogenous default and jump risk. *Mathematical Finance 19*, 343–378.

Chen, N.-F. (1991). Financial investment opportunities and the macroeconomy. *Journal of Finance 46*, 529–554.

Cheridito, P., D. Filipovic, and R. L. Kimmel (2007). Market price of risk specifications for affine models: Theory and evidence. *Journal of Financial Economics 83*, 123–170.

Chordia, T., A. Sarkar, and A. Subrahmanyam (2005). An empirical analysis of stock and bond market liquidity. *Review of Financial Studies 18*, 85–129.

Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics 10*, 407–432.

Cocozza, C. and M. Yor (1980). Démonstration d'un théorème de F. Knight à l'aide de martingales exponentielles. *Séminaire de Probabilités de Strasbourg 14*, 496–499.

Collin-Dufresne, P. (2009). A short introduction to correlation markets. *Journal of Financial Econometrics 7*, 12–29.

Collin-Dufresne, P., R. Goldstein, and J. Helwege (2003). Is credit event risk priced? Modeling contagion via the updating of beliefs. *Working paper, Carnegie Mellon University*.

Collin-Dufresne, P. and R. S. Goldstein (2001). Do credit spreads reflect stationary leverage ratios? *Journal of Finance 56*, 1929–1957.

Collin-Dufresne, P., R. S. Goldstein, and C. S. Jones (2008). Identification of maximal affine term structure models. *Journal of Finance 63*, 743–795.

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin (2001). The determinants of credit spread changes. *Journal of Finance 56*, 2177–2207.

Craine, R., L. A. Lochstoer, and K. Syrtveit (2000). Estimation of a stochastic volatility jump-diffusion model. *Revista de Analisis Economico 15*, 61–87.

Cremers, K. J. M., J. Driessen, and P. Maenhout (2008). Explaining the level of credit spreads: Option-implied jump risk premia in a firm value model. *Review of Financial Studies 21*, 2209–2242.

Cremers, M., J. Driessen, P. Maenhout, and D. Weinbaum (2008). Individual stock-option prices and credit spreads. *Journal of Banking and Finance 32*, 2706–2715.

Dai, Q. and K. J. Singleton (2000). Specification analysis of affine term structure models. *Journal of Finance 55*, 1943–1978.

Das, S., D. Duffie, N. Kapadia, and L. Saita (2007). Common failings: How corporate defaults are correlated. *Journal of Finance 62*, 93–117.

David, A. (2008). Inflation uncertainty, asset valuations, and the credit spreads puzzle. *Review of Financial Studies 21*, 2487–2534.

Davis, M. and V. Lo (2001). Infectious defaults. *Quantitative Finance 1*, 382–387.

Davydenko, S. A. (2010). What triggers default? A study of the default boundary. *Working paper, University of Toronto*.

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2011). Corporate bond liquidity before and after the onset of the subprime crisis. *Journal of Financial Economics (forthcoming)*.

Doshi, H. (2011). The term structure of recovery rates. *Working paper, McGill University*.

Duan, J.-C. (1994). Maximum likelihood estimation using price data of the derivative contract. *Mathematical Finance 4*, 155–167.

Duan, J.-C. (2000). Correction: Maximum likelihood estimation using price data of the derivative contract. *Mathematical Finance 10*, 461–462.

Duffee, G. R. (1999). Estimating the price of default risk. *Review of Financial Studies 12*, 197–226.

Duffie, D. (1999). Credit swap valuation. *Financial Analysts Journal 55*(1), 73–87.

Duffie, D. (2008). Innovations in credit risk transfer: Implications for financial stability. *BIS Working paper no. 255*.

Duffie, D., A. Eckner, G. Horel, and L. Saita (2009). Frailty correlated default. *Journal of Finance 64*, 2089–2123.

Duffie, D. and N. Gârleanu (2001). Risk and valuation of Collateralized Debt Obligations. *Financial Analysts Journal 57*(1), 41–59.

Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance 6*, 379–406.

Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics 83*, 635–665.

Duffie, D. and K. Singleton (1997). An econometric model of the term structure of interest rate swap yields. *Journal of Finance 52*, 1287–1321.

Duffie, D. and K. J. Singleton (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies 12*, 687–720.

Eckner, A. (2007). Risk premia in structured credit derivatives. *Working paper, Stanford University*.

Eckner, A. (2009). Computational techniques for basic affine models of portfolio credit risk. *Journal of Computational Finance 13*, 63–97.

Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann (2001). Explaining the rate spread on corporate bonds. *Journal of Finance 56*, 247–277.

Emery, K. M. and R. Cantor (2005). Default correlation among non-financial corporate affiliates. *Journal of Fixed Income 15*(2), 87–96.

Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *Journal of Finance 59*, 1367–1403.

Eraker, B., M. Johannes, and N. Polson (2003). The impact of jumps in volatility and returns. *Journal of Finance 58*, 1269–1300.

Ericsson, J., K. Jacobs, and R. Oviedo (2009). The determinants of Credit Default Swap premia. *Journal of Financial and Quantitative Analysis 44*, 109–132.

Ericsson, J. and J. Reneby (2005). Estimating structural bond pricing models. *Journal of Business 78*, 707–735.

Errais, E., K. Giesecke, and L. R. Goldberg (2010). Affine point processes and portfolio credit risk. *SIAM Journal of Financial Mathematics 1*, 642–665.

Fama, E. F. and K. R. French (1989). Business conditions and expected return on stocks and bonds. *Journal of Financial Economics 25*, 23–49.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*, 3–56.

Feldhütter, P. (2008). Can affine models match the moments in bond yields? *Working paper, Copenhagen Business School*.

Feldhütter, P. and D. Lando (2008). Decomposing swap spreads. *Journal of Financial Economics 88*, 375–405.

Figlewski, S., H. Frydman, and W. Liang (2008). Modeling the effect of macroeconomic factors on corporate default and credit rating transitions. *Working paper, NYU Stern School of Business*.

Frühwirth-Schnatter, S. and A. L. Geyer (1998). Bayesian estimation of econometric multifactor Cox-Ingersoll-Ross models of the term structure of interest rates via MCMC methods. *Working paper, Vienna University of Economics and Business Administration*.

Fung, W. K. H. and A. Rudd (1986). Pricing new corporate bond issues: An analysis of issue cost and seasoning effects. *Journal of Finance 41*, 633–643.

Gelfand, A. E., A. F. Smith, and T.-M. Lee (1992). Bayesian analysis of constrained parameter and truncated data problems using Gibbs sampling. *Journal of the American Statistical Association 87*, 523–532.

Geman, S. and D. Geman (1984). Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence 6*, 721–741.

Giesecke, K., L. R. Goldberg, and X. Ding (2011). A top-down approach to multiname credit. *Operations Research 59*, 283–300.

Gorbenko, A. S. and I. A. Strebulaev (2010). Temporary versus permanent shocks: Explaining corporate financial policies. *Review of Financial Studies 23*, 2591–2647.

Graham, J. R. (2000). How big are the tax benefits of debt? *Journal of Finance 55*, 1901–1941.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007). The U.S. Treasury yield curve: 1961 to the present. *Journal of Monetary Economics 54*, 2291–2304.

Hackbarth, D., J. Miao, and E. Morellec (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics 82*, 519–550.

Hamilton, D. T. (2005). Moody's Senior Ratings Algorithm and Estimated Senior Ratings. *Moody's Investors Service*.

Hamilton, J. D. (1994). *Time series analysis*. Princeton University Press.

Hammersley, J. and P. Clifford (1970). Markov fields on finite graphs and lattices. *Unpublished Manuscript*.

Hawkes, A. G. (1971a). Point spectra of some mutually exciting point processes. *Journal of the Royal Statistical Society. Series B 33*, 438–443.

Hawkes, A. G. (1971b). Spectra of some self-exciting and mutually exciting point processes. *Biometrika 58*, 83–90.

Helwege, J. and C. M. Turner (1999). The slope of the credit yield curve for speculative-grade issuers. *Journal of Finance 54*, 1869–1884.

Hilberink, B. and L. Rogers (2002). Optimal capital structure and endogenous default. *Finance and Stochastics 6*, 237–263.

Honoré, P. (1998). Pitfalls in estimating jump-diffusion models. *Working paper, Aarhus School of Business*.

Huang, J. and M. Huang (2003). How much of the corporate-Treasury yield spread is due to credit risk? *Working paper, Stanford University*.

Huang, X. and G. Tauchen (2005). The relative contribution of jumps to total price variance. *Journal of Financial Econometrics 3*, 456–499.

Jacobsen, M. (2005). The time to ruin for a class of Markov additive risk process with two-sided jumps. *Advances in Applied Probability 37*, 963–992.

Jarrow, R., D. Lando, and F. Yu (2005). Default risk and diversification: Theory and empirical implications. *Mathematical Finance 15*, 1–26.

Jarrow, R. A. and F. Yu (2001). Counterparty risk and the pricing of defaultable securities. *Journal of Finance 56*, 1765–1799.

Jiang, G. J. and T. Yao (2009). Stock price jumps and cross-sectional return predictability. *Working paper, University of Arizona*.

Jiang, Z. and M. Pistorius (2008). On American perpetual option valuation and first passage in a regime-switching model with jumps. *Finance and Stochastics 12*, 331–355.

Jobert, A. and L. Rogers (2006). Option pricing with Markov-modulated dynamics. *SIAM Journal on Control and Optimization 44*, 2063–2078.

Johannes, M. and N. Polson (2006). MCMC methods for financial econometrics. *Handbook of Financial Econometrics* (chapter).

Johannes, M. S., N. G. Polson, and J. R. Stroud (2009). Optimal filtering of jump diffusions: Extracting latent states from asset prices. *Review of Financial Studies 22*, 2759–2799.

Jones, E. P., S. P. Mason, and E. Rosenfeld (1984). Contingent claims analysis of corporate capital structures: An empirical investigation. *Journal of Finance 39*, 611–627.

Jorion, P. (1988). On jump processes in the foreign exchange and stock markets. *Review of Financial Studies 1*, 427–445.

Jorion, P. and G. Zhang (2009). Credit contagion from counterparty risk. *Journal of Finance 64*, 2053–2087.

Kallenberg, O. (1990). Random time change and an integral representation for marked stopping times. *Probability Theory and Related Fields 86*, 167–202.

Karlis, D. and E. Xekalaki (2000). A simulation comparison of several procedures for testing the Poisson assumption. *Journal of the Royal Statistical Society, Series D 49*, 355–382.

Kim, J., K. Ramaswamy, and S. Sundaresan (1993). Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management 22*(3), 117–131.

Koopman, S. J. and A. Lucas (2005). Business and default cycles for credit risk. *Journal of Applied Econometrics 20*, 311–323.

Korajczyk, R. A. and A. Levy (2003). Capital structure choice: Macroeconomic conditions and financial constraints. *Journal of Financial Economics 68*, 75–109.

Korteweg, A. (2010). The net benefits to leverage. *Journal of Finance 65*, 2137–2170.

Kou, S. G. (2002). A jump-diffusion model for option pricing. *Management Science 48*, 1086–1101.

Kramer, F. and G. Löffler (2010). Corporate bond defaults are consistent with conditional independence. *Journal of Credit Risk 6*(2).

Kwiecinski, A. and R. Szekli (1996). Some monotonicity and dependence properties of self-exciting point processes. *Annals of Applied Probability 6*, 1211–1231.

Kyprianou, A. E. and B. A. Surya (2007). Principles of smooth and continuous fit in the determination of endogenous bankruptcy levels. *Finance and Stochastics 11*, 131–152.

Lando, D. (1994). *Three essays on contingent claims pricing*. Ph. D. thesis, Cornell University.

Lando, D. (1998). On Cox processes and credit risky securities. *Review of Derivatives Research 2*, 99–120.

Lando, D. (2004). *Credit risk modeling*. Princeton University Press.

Lando, D. and A. Mortensen (2005). Revisiting the slope of the credit spread curve. *Journal of Investment Management 3*(4), 1–27.

Lang, L. H. P. and R. M. Stulz (1992). Contagion and competitive intra-industry effects of bankruptcy announcements. An empirical analysis. *Journal of Financial Economics 32*, 45–60.

Larsen, P. T. (2006). Default risk, debt maturity and levered equity's risk shifting incentives. *Working paper, University of Aarhus*.

Le, A. (2007). Separating the components of default risk: A derivative-based approach. *Working paper, University of North Carolina*.

Leary, M. T. and M. R. Roberts (2005). Do firms rebalance their capital structures? *Journal of Finance 60*, 2575–2619.

Lee, S. S. and P. A. Mykland (2008). Jumps in financial markets: A new nonparametric test and jump dynamics. *Review of Financial Studies 21*, 2535–2563.

Leland, H. E. (1994a). Bond prices, yield spreads, and optimal capital structure with default risk. *IBER Working paper no. 240*.

Leland, H. E. (1994b). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*, 1213–1252.

Leland, H. E. (2009). Structural models of the firm: An underview. *Working paper, University of California Berkeley*.

Leland, H. E. and K. B. Toft (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance 51*, 987–1019.

Lemmon, M. L., M. R. Roberts, and J. F. Zender (2008). Back to the beginning: Persistence and the cross-section of corporate capital structure. *Journal of Finance 63*, 1575–1608.

Lettau, M. and S. C. Ludvigson (2005). Expected returns and expected dividend growth. *Journal of Financial Economics 76*, 583–626.

Lettau, M., S. C. Ludvigson, and J. A. Wachter (2008). The declining equity premium: What role does macroeconomic risk play? *Review of Financial Studies 21*, 1653–1687.

Li, X. and X. Zhao (2006). Macroeconomic effect in corporate default. *Unpublished manuscript, York University*.

Longstaff, F. A., S. Mithal, and E. Neis (2005). Corporate yield spreads: Default risk or liquidity? New evidence from the Credit Default Swap market. *Journal of Finance 60*, 2213–2253.

Longstaff, F. A. and A. Rajan (2008). An empirical analysis of the pricing of Collateralized Debt Obligations. *Journal of Finance 63*, 529–563.

Longstaff, F. A. and E. S. Schwartz (1995). A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance 50*, 789–819.

Maheu, J. M. and T. H. McCurdy (2004). News arrival, jump dynamics, and volatility components for individual stock returns. *Journal of Finance 59*, 755–793.

Marsaglia, G. and A. Zaman (1991). A new class of random number generators. *Annals of Applied Probability 3*, 462–480.

Martinussen, T. and T. H. Scheike (2006). *Dynamic regression models for survival data.* Springer-Verlag.

Merton, R. C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science 4*, 141–183.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance 29*, 449–470.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics 3*, 125–144.

Merton, R. C. (1990). *Continuous-time finance.* Blackwell Publishing.

Metropolis, N., A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller (1953). Equations of state calculations by fast computing machines. *Journal of Chemical Physics 21*, 1087–1091.

Meyer, P.-A. (1971). Démonstration simplifiée d'un théorème de Knight. *Séminaire de probabilités de Strasbourg 5*, 191–195.

Mortensen, A. (2006). Semi-analytical valuation of basket credit derivatives in intensity-based models. *Journal of Derivatives 13*(4), 8–26.

Nelson, C. R. and A. F. Siegel (1987). Parsimonious modeling of yield curves. *Journal of Business 60*, 473–489.

Nickell, P., W. Perraudin, and S. Varotto (2000). Stability of rating transitions. *Journal of Banking and Finance 24*, 203–227.

Norris, J. R. (1997). *Markov Chains*. Cambridge University Press.

Ogata, Y. (1978). The asymptotic behaviour of maximum likelihood estimators for stationary point processes. *Annals of the Institute of Statistical Mathematics 30*, 243–261.

Ogata, Y. and H. Akaike (1982). On linear intensity models for mixed doubly stochastic Poisson and self-exciting point processes. *Journal of the Royal Statistical Society, Series B 44*, 102–107.

Ogata, Y., H. Akaike, and K. Katsura (1982). The application of linear intensity models to the investigation of causal relations between a point process and another stochastic process. *Annals of the Institute of Statistical Mathematics 34* (Part B), 373–387.

O'Kane, D. (2008). *Modelling single-name and multi-name credit derivatives*. Wiley.

Ozaki, T. (1979). Maximum likelihood estimation of Hawkes' self-exciting point processes. *Annals of the Institute of Statistical Mathematics 31*, 145–155.

Papangelou, F. (1972). Integrability of expected increments of point processes and a related random change of scale. *Transactions of the American Mathematical Society 165*, 483–506.

Perez-Quiros, G. and A. Timmermann (2000). Firm size and cyclical variations in stock returns. *Journal of Finance 55*, 1229–1262.

Press, S. J. (1967). A compound events model for security prices. *Journal of Business 40*, 317–335.

Robert, C. P. and G. Casella (2004). *Monte Carlo statistical methods* (2 ed.). Springer-Verlag.

Roberts, G., A. Gelman, and W. Gilks (1997). Weak convergence and optimal scaling of Random Walk Metropolis algorithms. *Annals of Applied Probability 7*, 110–120.

Rubin, I. (1972). Regular point processes and their detection. *IEEE Transactions on Information Theory 18*, 547–557.

Sarig, O. and A. Warga (1989). Some empirical estimates of the risk structure of interest rates. *Journal of Finance 44*, 1351–1360.

Schoenberg, F. P. (2002). On rescaled Poisson processes and the Brownian bridge. *Annals of the Institute of Statistical Mathematics 54*, 445–457.

Schwert, G. W. (1989). Why does stock market volatility change over time? *Journal of Finance 44*, 1115–1153.

Shek, H. (2010). Modeling high frequency market order dynamics using self-excited point process. *Working paper, Stanford University*.

Shumway, T. (2001). Forecasting bankruptcy more efficiently: A simple hazard model. *Journal of Business 74*, 101–124.

Tang, D. Y. and H. Yan (2010). Market conditions, default risk and credit spreads. *Journal of Banking and Finance 34*, 743–753.

Tauchen, G. and R. Hussey (1991). Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica 59*, 371–396.

Tauchen, G. and H. Zhou (2010). Realized jumps on financial markets and predicting credit spreads. *Journal of Econometrics 160*, 102–118.

van Binsbergen, J. H., J. R. Graham, and J. Yang (2010). The cost of debt. *Journal of Finance 65*, 2089–2136.

Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *Journal of Finance 59*, 831–868.

Welch, I. (2004). Capital structure and stock returns. *Journal of Political Economy 112*, 106–132.

Zhang, B. Y., H. Zhou, and H. Zhu (2009). Explaining Credit Default Swap spreads with the equity volatility and jump risks of individual firms. *Review of Financial Studies 22*, 5099–5131.

Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking and Finance 25*, 2015–2040.

Zmijewski, M. E. (1984). Methodological issues related to the estimation of financial distress prediction models. *Journal of Accounting Research 22*, 59–82.

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