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*Document Version*  
Final published version

*Publication date:*  
2007

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*Citation for published version (APA):*  
Keuschnigg, C., & Nielsen, S. B. (2007). *Self-Selection and Advice in Venture Capital Finance*. Centre for Economic and Business Research, Copenhagen Business School.

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Centre for  
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Discussion Paper

2007-07

Self-Selection and Advice in  
Venture Capital Finance

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# Self-Selection and Advice in Venture Capital Finance\*

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January 12, 2007

## Abstract

In financing start-up firms, venture capitalists carefully select among alternative projects, design incentive compatible financial contracts and support portfolio companies with value enhancing managerial advice. This paper considers how venture capitalists can induce self-selection among entrepreneurial firms with different qualities by designing appropriate contracts and offering commercial support. We study the efficiency of the competitive market equilibrium with respect to the level and quality of entrepreneurship and the level of effort by entrepreneurs and venture capitalists. We also provide comparative statics results with respect to basic preference and technology parameters.

**JEL-Classification:** D82, G24, M13.

**Keywords:** Venture capital, entrepreneurship, self-selection, moral hazard.

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\*We greatly appreciate the hospitality and stimulating environment at the ifo-institute in Munich where we started this paper during a research visit. We thank seminar participants at the University of St. Gallen as well as participants at the 5th journée Gérard-Varet in Marseille, PET06 in Hanoi, CEPR conference on entrepreneurship 2006 in Copenhagen and the Swiss Society of Economics and Statistics 2006 in Lugano for constructive comments.

# 1 Introduction

Venture capital backed firms are more innovative, grow larger and create more value than other, bank-financed firms. Kortum and Lerner (2000) find that a disproportionately large share of industrial innovation originates in firms financed with venture capital (VC).<sup>1</sup> Based on a sample of start-up firms in Silicon valley, Hellmann and Puri (2000, 2002) estimate the value added of venture capitalists (VCs) to company development. The strategic advice and the monitoring activities of VCs promote the commercialization of portfolio companies and help them to exploit their growth potential.<sup>2</sup> It has often been argued, however, that much of the superior performance of VC-backed compared to bank-financed companies might not be due to the value added activities of VCs. It might rather result from the fact that VCs are simply more successful in selecting the more promising firms in the pool of all start-ups. Kaplan and Stromberg (2001, 2004) indeed point to the importance of both the screening and advising activities of VCs. Quite consistent with this, the empirical study of Sorensen (2005) finds that about 50 percent of the extra performance of VC backed firms is due to the advisory support of VC firms, while the rest results from the fact that the more promising entrepreneurs tend to match with experienced VCs. This paper argues that the VCs' use of convertible contracts may be very instrumental in attracting a better selection of firms.

A good descriptive model of the VC industry should therefore pay due attention to the implications of both selection and advice for the quality of VC financing. We build on our own previous research in modeling the productive contribution of VCs to their portfolio companies in terms of advice and managerial support (Kanniainen and Keuschnigg, 2003, and Keuschnigg and Nielsen, 2003, 2004a,b). However, we now extend this research by allowing for quality differences among projects as emphasized in the literature on adverse selection.<sup>3</sup> There are three main differences with this literature: (i) we allow for only

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<sup>1</sup>See Da Rin, Nicodano and Sembenelli (2006) on innovation financing with VC and the impact of public policy in Europe.

<sup>2</sup>See Gompers and Lerner (1999) for a standard reference on empirical work on VC financing and Kanniainen and Keuschnigg (2005) for a collection of policy oriented contributions.

<sup>3</sup>See e.g. DeMeza and Webb (1987) and the survey of the subsequent literature by De Meza (2002). Boadway and Keen (2004) synthesize different models which mostly consider pooling equilibria. For an

two qualities instead of a continuum to simplify the model; (ii) we add to this the self-selection model of Hall (2005) to endogenize entry into entrepreneurship; and (iii) we include a double moral hazard problem after a contract is signed. The moral hazard relates to the entrepreneur's managerial effort in building the company and the VC's advisory support to magnify company growth. The result is a quite tractable model that rationalizes the use of convertible debt in VC financing and allows to consider the characteristics and efficiency of the market equilibrium.<sup>4</sup> We show how certain structural parameters of the VC industry lead VCs to adjust their convertible debt contracts to attract a better selection of firms and to ensure optimal incentives for managerial effort and advisory support. At the same time, we are able to characterize the quantity and quality of VC-financed entrepreneurship.

When start-ups invent new and untested products, the technological risk in making the product ready for production and the market potential of the innovation may be bigger or smaller. It is thus assumed that ideas of entrepreneurs have either high or low quality, leading to large or small market potential in case of success. Entrepreneurs do not have enough own capital and therefore need outside finance to start a firm. Initially, neither the entrepreneur nor the VC knows the true quality of the project. Entrepreneurs, however, receive an informative signal on the potential of their project that allows them to revise their prior expectations. Depending on the financing contracts on offer, agents self-select into entrepreneurship, if they receive a sufficiently good signal indicating that their project is likely to be a high quality one. Once a firm is started and the collaboration between the VC and entrepreneur begins, the project quality becomes known. The probability of success of either type may still be advanced by managerial effort and VC advice, either to increase expected profits or to cut losses. On average, bad quality firms result in a loss and high quality firms yield profits.

To shed light on policy implications, we clarify the welfare properties of the market analysis of separating equilibria see Innes (1990) and Fuest and Tillessen (2005), among others.

<sup>4</sup>See Casamatta (2003), Schmidt (2003) and Repullo and Suarez (2004) for theoretical analysis on the role of convertible instruments. Kaplan and Stromberg (2003) and Cumming (2005) document the empirical importance of convertible instruments in VC financing. Cumming (2006) empirically shows that different securities do indeed attract different types of firms in VC financing.

equilibrium with respect to self-selected entry and the managerial and advisory effort levels. Entry determines the average quality of start-ups, while effort levels determine the success probability of each type of firm. We find that the double moral hazard between entrepreneurs and VCs leads to an inefficiently low effort level. The reasons for this underinvestment are well known from our own previous research, or from the analysis of Schmidt (2003) and Inderst and Müller (2004). Since an entrepreneur has insufficient own resources to start a project, she needs outside finance and must share the returns with the financier. Consequently, during the start-up phase, the entrepreneur and VC must each bear the full cost of their own effort, but must share the returns to effort among them.

On the other hand, entrepreneurial entry is excessive. VCs incur a loss on bad projects and need to cross-subsidize them with profits from good projects. Limited liability prevents entrepreneurs from paying for these losses. As a result of cross-subsidization, entrepreneurs who are endowed with a project which with high probability is bad get a too favorable deal. They are thus too eager to start a firm which should not get started. This result is in line with the adverse selection models of DeMeza and Webb (1987, 2002).

The policy implications are immediate. The model calls for higher quality, but a smaller number of start-up firms. To improve the efficiency of the market equilibrium, one needs to look for policies that are able to stimulate effort, but at the same time reduce entry into VC-backed entrepreneurship.

The next section sets up the model and analyzes the constrained optimal allocation. Section 3 derives market equilibrium and investigates its efficiency properties by comparing it to the optimal allocation. Section 4 discusses the results of comparative statics analysis of the industry equilibrium with respect to key taste and technology parameters. Finally, section 5 concludes and points out possible avenues for future research.

## 2 The Model

### 2.1 Overview

Potential entrepreneurs have ideas for new products, either good or bad, but are endowed with limited own wealth,  $k$ . To start a firm, they need outside finance, since the necessary capital investment  $I > k$  exceeds their own capital. Entrepreneurs lack not only sufficient capital, but also managerial experience. They would thus benefit from professional support of seasoned VCs who have industry knowledge, can give advice and add value by sharing their commercial know-how.

VC financing of new firms involves the following sequence of events: (i) VCs offer outside financing, covering at least  $I - k$ , and announce the contract. (ii) Agents have an idea for a project which may be good or bad,  $j \in \{G, B\}$ . The true share of good projects is  $\varepsilon$  but agents do not know in advance whether their project is good. They do receive a signal  $y$  indicating that the project is good with probability  $q$ . A higher signal value means a higher probability of the project being good. If  $y$  is sufficiently large, agents opt for entrepreneurship and apply for outside finance. If not, they turn to alternative employment in industry, earning a fixed wage  $w$  and deriving end of period wealth  $w + k$ .<sup>5</sup> (iii) After the business is launched and investment  $I$  is sunk, the true quality of the project is revealed to both VC and entrepreneur as a result of their early collaboration. (iv) Knowing quality, agents spend effort and advice to boost the success probability. Even if the project turns out to have low potential, effort is spent in order to limit expected losses. (v) Good and bad projects yield outcome  $v_G$  or  $v_B$ , if successful, and the payments according to the terms of contract are executed.

### 2.2 Venture Capital Financing

In the following we explain in detail the determinants of project success; the parameters of the financial contract; expected marginal and average quality of start-up firms; their surpluses; and GDP and welfare.

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<sup>5</sup>Interest on assets is normalized to zero.

### 2.2.1 Project Success

If a project succeeds, it generates a value  $v_G > v_B$  on the output market. When it fails, revenue is zero, leaving expected revenue  $p_j v_j$ . The success probability  $p_j$  is specific to the type which is revealed after the project starts. Depending on its type, the success probability of the firm is enhanced by the entrepreneur's effort and the degree of VC support. To simplify, we assume that managerial effort is discrete and can be either high or low,  $l_j \in \{0, 1\}$ . The entrepreneur's effort is thus critical for the success chances of the firm. The company will always fail if the entrepreneur shirks and puts in low effort. VC advice  $a_j$  is a more gradual matter and is treated as a continuous variable. Both efforts determine the success probability,

$$p_j = p(l_j, a_j) = l_j \cdot (a_j)^\alpha, \quad 0 < \alpha < 1. \quad (1)$$

The key condition is that  $p_j$  is increasing in each effort level and strictly concave in VC advice. Efforts are complements.

### 2.2.2 Contract Parameters and Project Rents

When the VC offers a contract, she does not yet know the type of the company, but she anticipates learning the project type after commencing collaboration. The contract thus includes a convertible option that can be exercised when this information becomes available after the investment is sunk, but before the outcome is realized.

Specifically, the contract consists of (i) a credit  $I - k$  and a possible supplementary payment  $b_B$ ; (ii) a VC share  $s_B$  of profits if the project turns out bad;<sup>6</sup> and (iii) an option to increase the equity stake to  $s_G > s_B$  at a conversion price  $b \equiv b_G - b_B$ , if the project turns out good. The option can be exercised after the project is started and its type is revealed to both parties. Whatever the type, both agents receive a zero repayment if the project fails and yields no revenue. The VC thus gets paid back only in case of success.

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<sup>6</sup>The upfront payment  $b_B$  will in fact be optimally set to zero in the most realistic case. The amount  $s_B v_B$  can thus be interpreted as a debt repayment on the initial credit  $I - k$ , leaving  $v_B - s_B v_B$  to the entrepreneur in case of success.



After learning the type of the project through early collaboration, and after possibly adjusting the profit share, efforts  $a_j$  and  $l_j$  can be tailored to the particular project type, giving a success rate  $p_j$  as in (1). Agents choose their own effort, taking the input by the other party as given, and thereby maximize the rent

$$\begin{aligned}
R_j^E &= \max_{l_j} p(l_j, a_j) \cdot (1 - s_j) \cdot v_j - \beta l_j, \\
R_j^F &= \max_{a_j} p(l_j, a_j) \cdot s_j \cdot v_j - \gamma a_j, \\
R_j &= R_j^E + R_j^F = p(l_j, a_j) \cdot v_j - \beta l_j - \gamma a_j.
\end{aligned} \tag{2}$$

The superscripts  $E$  and  $F$  refer to entrepreneurs and VC financiers, while parameters  $\beta$  and  $\gamma$  measure the cost per unit of effort exerted by the entrepreneur and the VC, respectively. Since the success probability is concave in advice, VCs can expect a rent which compensates them for their effort cost and earlier pecuniary expenses.

Of the uniform capital investment  $I$  a part  $k$  is financed out of the entrepreneur's wealth and  $I - k$  by the VC. Apart from this credit, the contract thus specifies profit shares  $s_j$  and fixed payments  $b_j$ . As will be clear later, optimality could necessitate a negative price  $b_B < 0$  or a payment from the entrepreneur to the VC, but this is prevented by limited liability. Hence we require  $b_B \geq 0$ . Taking account of the payment  $b_j$ , the entrepreneur expects a total value  $R_j^E + b_j$  while the VC claims  $R_j^F - b_j$ .

### 2.2.3 Occupational Choice and Signals

Agents are endowed with good or bad ideas. Nobody knows in advance whether an idea is good or bad, not even the entrepreneur herself. The true proportion of good ideas in the population is  $\varepsilon$ , whence the fraction  $1 - \varepsilon$  of agents are endowed with a bad idea. The share  $\varepsilon$  is the prior probability for a high quality project for all potential entrepreneurs.

However, agents receive a signal  $y$  that is positively correlated with their project's quality. Those with a better signal are more likely to have a good idea,  $q' > q$  for  $y' > y$ . The individual agent updates her perceived probability of a good idea to  $q(y)$ , giving a probability  $1 - q(y)$  of being stuck with a bad idea. Even though agents are identical in other respects, they differ continuously in the signal received and therefore in the expected

project quality  $q$ . For an agent with a relatively high signal,  $q(y) > \varepsilon$ ; conversely  $q(y) < \varepsilon$  for an agent with a relatively low signal. Given a marginal probability  $q$  of an agent with signal  $y$ , the average probability  $Q$  of having a good project among all agents with signals  $y' > y$  fulfils  $Q > q$ . Also, the average quality increases with the marginal quality,  $dQ/dq > 0$ . Appendix A explains the details.<sup>7</sup> From now on, we mean by the quality of a project the perceived probability that it is of a good type.

#### 2.2.4 Surpluses in Start-up Firms

When an agent contemplates entrepreneurship, she reckons with two possible events: (i) with probability  $q$ , her project is good and has value  $R_G^E + b_G$ ; (ii) with probability  $1 - q$ , it is bad but is financed nevertheless, yielding  $R_B^E + b_B$ , where  $b_B \geq 0$ . In both cases, the entrepreneur gives up  $k$  to pay part of  $I$ . The signal must be sufficiently good to warrant entry,  $q(R_G^E + b_G) + (1 - q)(R_B^E + b_B) \geq w + k$ . The expected surpluses of the entrepreneur and VC, and their joint surplus are then

$$\begin{aligned}\pi^E &= \sum_j q_j \cdot (R_j^E + b_j) - k - w, \\ \pi^F &= \sum_j q_j \cdot (R_j^F - b_j) - (I - k), \\ \pi &= \pi^F + \pi^E = \sum_j q_j \cdot R_j - w - I.\end{aligned}\tag{3}$$

To simplify notation we use  $q_j$  meaning  $q_G = q$  and  $q_B = 1 - q$ . Using the same notation  $Q_j$  for the average likelihood of good and bad projects, respectively, we obtain the average surpluses over all projects financed. Appendix B gives details:

$$\begin{aligned}\Pi^E &= \sum_j Q_j \cdot (R_j^E + b_j) - k - w, \\ \Pi^F &= \sum_j Q_j \cdot (R_j^F - b_j) - (I - k), \\ \Pi &= \Pi^E + \Pi^F = \sum_j Q_j \cdot R_j - w - I.\end{aligned}\tag{4}$$

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<sup>7</sup>By assuming a specific functional form of the distribution of signals in the population as in (A.8), one can conveniently derive a closed form expression for average quality as in (A.10):  $Q = \frac{q}{q + (1-q)\theta} > q$ , where  $\theta < 1$  parameterizes the information content of the signal. If  $\theta = 1$ , the signal is not informative and agents cannot update their prior probability  $\varepsilon$ . In this case, all would expect a good project with the same probability  $q = \varepsilon$ , implying  $Q = \varepsilon$  as well. If the signal is informative,  $\theta < 1$ , then the perceived probability of a good project  $q(y)$  increases with the signal received, and so does the average probability  $Q$  taken over all agents with even better signals.

### 2.2.5 GDP and Welfare

We close the model and thereby obtain GDP and welfare in the entrepreneurial economy. To this end, substitute (2) into (4) to get the average surpluses per project,

$$\begin{aligned}
\Pi^E &= \sum_j Q_j \cdot [(1 - s_j) p_j v_j - \beta l_j + b_j] - k - w, \\
\Pi^F &= \sum_j Q_j \cdot [s_j p_j v_j - \gamma a_j - b_j] - (I - k), \\
\Pi &= \sum_j Q_j \cdot [p_j v_j - \beta l_j - \gamma a_j] - w - I.
\end{aligned} \tag{5}$$

With population normalized to unity, the number of entrepreneurs obtaining VC financing is equal to the number of applications,  $E < 1$ . A share  $Q$  of these is endowed with a good project and a share  $1 - Q$  with a bad one. GDP, or end of period income, is  $k$  plus the output of workers, each producing  $w$ , and output of start-up firms, minus various costs. The welfare measure must also take account of all non-pecuniary effort costs. Welfare is  $W = w + k + \Pi E$  where  $\Pi$  is the average joint surplus of a start-up firm. Upon substitution of  $\Pi E$  and using the identity  $L + E = 1$ ,

$$W = (w + k) L + \left[ \sum_j Q_j \cdot (p_j v_j - \beta l_j - \gamma a_j) - (I - k) \right] \cdot E. \tag{6}$$

GDP consists of wealth plus output of workers,  $(w + k) L$ , plus output of (good and bad) firms, net of start-up costs  $(I - k) E$ . Subtracting effort costs yields welfare  $W$ .

## 2.3 Efficient Allocation

The constrained optimal allocation  $q^*$ ,  $l_j^*$ ,  $a_j^*$  maximizes welfare  $W = w + k + \Pi E$  by directly allocating resources subject to the restriction that the government does not know more than private parties. Since  $w$  and  $k$  are exogenous, we need to examine only  $\Pi E$ . With some details given in Appendix B, we derive the following efficiency conditions,

$$\begin{aligned}
l_j^* &: \frac{dW}{dl_j} = [p_l(a_j, l_j) \cdot v_j - \beta] \cdot Q_j E \geq 0 \quad \Rightarrow \quad l_j^* = 1, \\
a_j^* &: \frac{dW}{da_j} = [p_a(a_j, l_j) \cdot v_j - \gamma] \cdot Q_j E = 0, \\
q^* &: \frac{dW}{dq} = - \left[ \sum_j q_j \cdot (p_j v_j - \gamma a_j - \beta l_j) - I - w \right] \cdot e\phi'(q) = 0.
\end{aligned} \tag{7}$$

The first two conditions in (7) determine efficient effort levels. If the VC advises more intensively, the success probability increases which strengthens the entrepreneur's incentives for high effort. Given the functional form in (1), the project never succeeds and yields no revenue if the entrepreneur shirks ( $l_j = 0$ ). That cannot be optimal and must be ruled out. Optimal VC advice follows by the second condition above. Advice is positive only if entrepreneurial effort is high. Hence,  $p_l(a_j, l_j) \cdot v_j \geq \beta$  must hold by the first condition. Substituting optimal advice, this condition gives a restriction on parameters to ensure an interior solution,

$$a_j^* = (v_j \cdot \alpha / \gamma)^{1/(1-\alpha)}, \quad p_l(a_j^*, 1)v_j \geq \beta \quad \Rightarrow \quad v_j \geq (\gamma/\alpha)^\alpha \beta^{1-\alpha}. \quad (8)$$

Advice is higher with the good project. Hence, if the condition for high entrepreneurial effort is fulfilled for the bad project, it is a fortiori fulfilled for the good project as well.

One should note that the first two conditions result from maximizing the joint rent in (2). Rents strictly increase in project value  $v_j$  on account of the envelope theorem with respect to advice,  $R'(v_j) = p_j > 0$ . We denote by  $R_j^*$  the joint rent with the first-best levels of effort. The condition for optimal entry thus emerges as

$$q^* : \quad \frac{dW}{dq} = - \left[ \sum_j q_j \cdot R_j^* - I - w \right] \cdot e\phi'(q) = 0. \quad (9)$$

Recalling  $q_G = q$  and  $q_B = 1 - q$ , socially optimal entry is given by the marginal quality

$$q^* = \frac{I + w - R_B^*}{R_G^* - R_B^*}. \quad (10)$$

This condition reveals that bad projects must make a loss for there to be a well defined interior solution of the constrained optimal allocation. We henceforth assume

$$R_B^* - I - w < 0. \quad (11)$$

### 3 Competitive Market Equilibrium

Having looked at the constrained optimum, we proceed to the market equilibrium. Deriving the equilibrium by means of backward induction implies the following steps: (i)

we solve for effort and advice and find the resulting rents to effort; (ii) we characterize the overall surplus and derive the optimal VC contract given by profit shares  $s_j$  and the conversion price  $b$ ; (iii) we determine entrepreneurial entry as it results from self-selection based on signals and the offered contracts.

### 3.1 Effort, Advice and Rents

Given profit shares, entrepreneurs and VCs strive to maximize the rent from their inputs to the company. When maximizing (2), each party takes the action of the other as given. Incentive compatibility is assured by the first order condition with respect to advice and the inequality reflecting the discrete comparison of rents for low and high managerial effort. Quite obviously, the VC leaves a high enough profit share  $1 - s_j$  to the entrepreneur to ensure her critical effort. Otherwise there would be no revenue at all:

$$p_a(l_j, a_j) s_j v_j = \gamma, \quad p_l(l_j, a_j) (1 - s_j) v_j - \beta \geq 0. \quad (12)$$

Profit maximization also leads the VC to raise  $s_j$  as much as possible until the entrepreneur's incentive compatibility condition in (12) binds with equality. The equilibrium profit share and level of VC support are thus given by the two constraints in (12) holding with equality. Using the functional form in (1), we compute

$$a_j = (s_j v_j \alpha / \gamma)^{1/(1-\alpha)} < a_j^*, \quad (13)$$

i.e. smaller than the first best level noted in (8). From a social perspective, the VC should be full residual claimant on her input. However, the need to provide incentives to the entrepreneur limits her share in the market equilibrium, leading to underinvestment in advice. Knowing the VC's stake  $s_j$  and her level of support  $a_j$ , we can infer her rent.

**Proposition 1** (a) *Given the form in (1), the VC's profit shares fulfil  $1 > s_G > s_B > \alpha$ . (b) The VC's share  $s_j$  and the level of advice  $a_j$  increase in project value  $v_j$  but decline in marginal effort costs  $\beta$  of entrepreneurs and  $\gamma$  of VCs. (c) The entrepreneur's rent  $R_j^E$  is zero, her profit share exactly compensating for her effort cost. The VC gets the entire joint rent  $R_j$  which increases with project value but falls with marginal effort costs.*

**Proof.** See Appendix C. ■

Noting  $a_j \cdot p_a(a_j) = \alpha \cdot p_j$  as implied by (1) and using the optimality condition (12) to substitute for  $\gamma$  yields a convenient closed form for the VC's rent

$$R_j^F = (1 - \alpha) s_j p_j v_j, \quad R_j^E = (1 - s_j) p_j v_j - \beta = 0. \quad (14)$$

The entrepreneur must appropriate a surplus to make her willing to forgo other career opportunities. Her expected surplus derives from selling the share  $s_j$  to the VC at a price that will exceed the VC's investment costs by an amount  $b_j$  to be determined below.

### 3.2 Competition Among VCs

The profit shares are chosen to maximize the VC's rent which coincides with the joint rent. On the other hand, VCs must compete for financing start-up firms by offering an overall attractive package to entrepreneurs. They can do so by offering a high price  $b_j + I - k$  for their share  $s_j$  which must cover at least the unfinanced part of start-up cost, but can also include a lump-sum, success independent component  $b_j$ . In offering a price, VCs aim to attract good projects and to deter bad ones, especially if the bad ones result in a loss. Since VCs can convert their profit shares  $s_j$  after the firm is started and quality is revealed, they can also differentiate the prices  $b_j$ , so that  $b \equiv b_G - b_B$ , the conversion price to be paid if the option to increase the share from  $s_B$  to  $s_G$  is exercised, is positive.

The VC's surplus from a project of type  $j$  is  $\pi_j^F = R_j^F - b_j - (I - k)$ . VCs try to attract entrepreneurs with good signals who are likely to have a good project, by offering a high conversion price resulting in a high value of  $b_G$  for the same share  $s_G$ , possibly until  $\pi_G^E = 0$ . Similarly, VCs will offer only a low price  $b_B$  to deter entrepreneurs who are likely to be endowed with a bad project that might result in a loss. If even the bad project is profitable, the VC competes for this project by offering a low but positive price  $b_B$ . If it is unprofitable, she would ask for a negative price, i.e. a payment from entrepreneur to VC, to avoid losses that would have to be covered by cross-subsidization. However, limited liability  $b_j \geq 0$  prevents this, since entrepreneurs have already invested their entire wealth  $k$  in the project and have no funds left. The VC cannot cut her losses on a bad project

any further than paying the lowest possible price  $b_B = 0$  after  $I - k$  is sunk and project quality is revealed. In this case, the VC will have to make strictly positive profits on good projects to cover her losses by means of cross subsidization and break even at least on average. To sum up, we must distinguish two cases:

$$\begin{aligned}
(a) \quad & b_j > 0; \quad R_j^F - (I - k) = b_j, \\
(b) \quad & b_G > 0; \quad R_G^F - (I - k) > b_G, \\
& b_B = 0; \quad R_B^F - (I - k) < 0.
\end{aligned} \tag{15}$$

In case (a), both projects are profitable and the VC competes with prices  $b_j$  that allow her to break even on each project separately and, hence, on average as well. In case (b), only the good project is profitable, and the bad one makes a loss. The VC thus sets the price for a bad project at the lowest possible value  $b_B = 0$  and offers a conversion price such that she makes a strictly positive profit on good projects. In this case, she cannot avoid cross-subsidizing from good to bad projects.

Entry of the marginal entrepreneur with quality  $q$  determines the average probability  $Q(q)$  as explained in Appendix A. Since the VC might in fact face any entrant, she must consider the average probability. Given  $Q$ , competition among VCs forces them to raise prices for high quality ventures and cut them for bad ones until they just break even. The zero profit condition is  $\Pi^F = \sum_j Q_j \pi_j^F = 0$ , or

$$Q \cdot (R_G^F - b_G) + (1 - Q) \cdot (R_B^F - b_B) = I - k. \tag{16}$$

Breaking even on average is possible in two distinct ways. In case (a), the VC competes for each project separately and breaks even on each of them individually. There is no cross-subsidization. In case (b) of (15),  $b_B = 0$  due to limited liability. Consequently, the VC makes losses on bad projects which must be covered with profits from good ones.

The type specific profit shares together with prices  $b_j$  can be understood as a very simple representation of convertible debt. The interpretation rests on the fact that debt and equity are really the same in our framework, if we consider only a single project. The VC provides a total amount of funds equal to  $b_j + I - k$  to pay for capital expenses and to compensate for the foregone outside option that is not covered by the entrepreneur sharing

in the profit.<sup>8</sup> The VC gets the return either as a profit share or as a debt repayment.

Define  $s_B v_B \equiv D_B > b_B + I - k$  as the debt repayment of a bad project that goes to the VC if the project succeeds. The debt repayment  $D_B$  exceeds the initial credit by an implicit premium which must cover the credit losses from failed projects as well as any effort costs by the VC. Repayment of debt leaves all residual profits  $(1 - s_B) v_B = v_B - D_B$  to the entrepreneur. This way we can understand the project specific profit share as debt for a bad project, allowing the VC to convert to a higher equity share  $s_G > s_B$  if the project turns out good and if she is willing to pay the conversion price  $b_G - b_B$ . The conversion is done after the project is started and quality is revealed. Converting to a higher share reinforces the incentives to advise the good project more intensively.

**Proposition 2** (a) *When bad projects are profitable, competition among VCs raises prices  $b_j + I - k$  for the profit shares  $s_j$ . The entire surplus goes to entrepreneurs. VCs break even separately on each project type without cross-subsidization.* (b) *When bad projects are unprofitable and limited liability binds, high quality entrepreneurs obtain less and low quality entrepreneurs more than the joint surplus. VCs make positive profits on high quality ventures that subsidize losses on low quality projects. They break even on average.*

**Proof.** Discussion of equations (15) and (16). ■

### 3.3 Self-Selection of Entrepreneurs

Agents who receive sufficiently good signals, start a firm and apply for VC financing. The marginal entrant, whose project is good with probability  $q$  and bad with probability  $1 - q$ , is indifferent between entrepreneurship and work,  $\pi^E = 0$ . Free entry establishes  $q(R_G^E + b_G) + (1 - q)(R_B^E + b_B) = w + k$  and identifies the marginal quality

$$q = \frac{w + k - R_B^E - b_B}{R_G^E + b_G - R_B^E - b_B}. \quad (17)$$

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<sup>8</sup>In fact, the conversion price must cover the entire outside option  $w + k$  since the profit share just suffices to compensate for managerial effort cost and leaves a zero rent to the entrepreneur.



Although an entrepreneur obtains no rent beyond the compensation for effort,  $R_j^E = 0$ , we keep these terms for better interpretation. The more realistic case of loss-making bad projects implies  $b_B = 0$  and  $b_G = b$ . The critical quality would be  $q = (w + k) / b$ .

Use  $R_j^E = R_j - R_j^F$  and write the denominator in (17) as  $R_G^E + b_G - R_B^E - b_B = R_G - R_B - \nabla$  with  $\nabla$  defined below. Taking the derivative then yields

$$\begin{aligned}\frac{dq}{db_G} &= -\frac{q}{R_G - R_B - \nabla} < 0, \\ \frac{dq}{db_B} &= -\frac{1 - q}{R_G - R_B - \nabla} < 0, \\ \nabla &\equiv (R_G^F - b_G) - (R_B^F - b_B).\end{aligned}\tag{18}$$

Raising the conversion price  $b_G$  lowers the critical probability  $q$  and thereby encourages entry. The denominator stands for the income difference that the entrepreneur would realize, if she could exchange a bad for a good project, and  $\nabla$  gives the corresponding income difference of the VC. Quite intuitively, the entrepreneur's gain  $R_G - R_B - \nabla$  is the total gain minus the VC's share in the income gain.

When bad projects are unprofitable to the VC, condition (15.b) applies and limited liability binds,  $b_B = 0$ . In this case, the conversion price  $b = b_G$  is fixed by the VC's average break even condition in (16),

$$Q \cdot \nabla + R_B^F - b_B - (I - k) = 0.\tag{19}$$

Recall that when bad projects are unprofitable, (15.b) implies  $\nabla > 0$ .

## 3.4 Efficiency

### 3.4.1 The Entry Margin

We first turn to the identity of the marginal entrepreneur and the implied efficiency of market entry. In principle, one could directly compare the constrained optimal allocation  $q^*$  in (10) with the market allocation in (17). Since  $q^*$  follows from  $W_q = 0$ , we evaluate instead the welfare derivative  $W_q$  at the market allocation. If this derivative is zero,

market entry is optimal. Rewrite the break even condition of the marginal entrepreneur, using  $R_j^E = R_j - R_j^F$  and the definition of  $\nabla$  in (18), to get

$$\pi^E = q \cdot [R_G - R_B - \nabla] + [R_B - (R_B^F - b_B)] - k - w = 0. \quad (20)$$

Using this to substitute for  $w$  in (9), and noting  $R_j^* = R_j$  if evaluated in the market equilibrium, yields

$$W_q = - [q \cdot \nabla + R_B^F - b_B - (I - k)] \cdot e\phi'(q). \quad (21)$$

Finally, replacing  $I - k$  by the VC's break even condition (19) gives

$$W_q = (Q - q) \cdot \nabla \cdot e\phi'(q). \quad (22)$$

The correlation of the signal with project quality implies  $\phi'(q) > 0$ , see (A.6), where  $y = \phi(q)$  is the inverted relationship of (A.2). Since  $Q > q$ , the welfare effect of entry thus depends on the sign of  $\nabla$ . If, as in case (a), limited liability is not binding, the VC breaks even on each project separately without cross-subsidization which gives  $\nabla = 0$ . Entry is efficient in this case. If case (b) applies and the limited liability condition is binding, then  $\nabla > 0$ . Preventing entry of the marginal entrepreneur and thereby raising the value of the marginal probability  $q$  will boost welfare,  $W_q > 0$  by (22). Low quality entrepreneurs thus get a too favorable deal which results in excess entry.

Further intuition about the reasons for excess entry is obtained by writing  $\pi^E$  as in (20) and using the last equation in (3) to replace  $w$ , giving  $\pi^E = \pi - q\nabla - [R_B^F - b_B - (I - k)]$ . Substituting the VC's zero profit condition in (19) for the square bracket results in  $\pi^E = \pi + (Q - q) \cdot \nabla$ . If there is no cross-subsidization,  $\nabla = 0$  and the entrepreneur gets the entire social surplus,  $\pi^E = \pi$ . If instead the limited liability constraint necessitates cross-subsidization,  $\nabla > 0$ , entrepreneurs who are very likely to be endowed with a high quality project ( $q > Q$ ) get less than the social surplus. In consequence, low quality entrepreneurs with  $q < Q$  acquire more than the joint surplus which induces excess entry.

### 3.4.2 Effort Decisions

The other potential distortion in this model has to do with the level of VC advice. Managerial effort cannot be distorted since it is assumed to be discrete, and the VC will always

ensure the socially optimal high effort  $l = 1$ . Nevertheless, it is instructive to treat managerial effort as continuous for a moment. According to (7), the overall welfare impact is  $dW = W_q \cdot dq + \sum_j (W_{a_j} \cdot da_j + W_{l_j} \cdot dl_j)$  in general. Substituting the private condition in (12) for  $\gamma$  in  $W_a$  in (7), and substituting similarly for  $\beta$  in  $W_l$ , yields

$$dW = W_q \cdot dq + \sum_j [(1 - s_j) p_a(a_j, l_j) v_j \cdot da + s_j p_l(a_j, l_j) v_j \cdot dl] Q_j E. \quad (23)$$

We have already discussed the first term. The other two terms indicate inefficiently low effort by the VC and, if managerial effort  $l_j$  were continuous, by the entrepreneur.

Welfare would increase if VC advice could be raised. In putting in more advice, the VC creates extra value  $p_a v_j$ , but can appropriate only a share  $s_j p_a v_j$  since she must cede a share  $1 - s_j$  to the entrepreneur to secure her cooperation. The difference between the social and private returns to VC advice is a spill-over to entrepreneurs that the VC does not take account of when she decides on her own input to the company. Advice is thus inefficiently low. A symmetric argument would in general apply to the entrepreneur's managerial effort. The social return to effort is  $p_l v_j$  but the entrepreneur gets only a share  $(1 - s_j) p_l v_j$ . The rest accrues to the VC. If the entrepreneur's effort were continuous, she would underinvest in effort as well. Given our discrete formulation, entrepreneurial effort is efficient in the market equilibrium.

**Proposition 3** (a) *VC advice is inefficiently low in equilibrium.* (b) *When limited liability binds for unprofitable projects, entry is excessive on the low quality margin.*

**Proof.** See equations (22-23). ■

It is instructive to deliberate under what conditions the distortions in VC financing could be avoided. If we had a budget breaking third party as in Holmstrom (1982) which can also be replicated as a tax transfer mechanism supplied by the government as in Keuschnigg and Nielsen (2003), advice could be made efficient. Basically, the mechanism subsidizes the VC's revenues ex post until she is full residual claimant on the returns to advisory effort, and finances the subsidy by a tax ex ante. If, in addition, the entrepreneur could be made a residual claimant on the project type (to be distinguished from residual claimant on the returns to effort at the moral hazard stage), entry would be efficient as

well. The entrepreneur would be residual claimant on the project type if she appropriated the entire income difference between good and bad projects. In competitive market equilibrium, this is possible only if low quality projects are profitable, and is otherwise prevented by the limited liability constraint.

## 4 Comparative Statics

**(a) The unrestricted case.** Equilibrium in the unrestricted case is fully recursive. The VC breaks even separately on each venture, yielding  $b_j = R_j^F - I + k > 0$  even for the bad project. The VC gets the entire joint rent,  $R_j^E = 0$  and  $R_j^F = R_j$ , but the entrepreneur appropriates the total surplus  $\pi_j^E = \pi_j$  because competitive VCs bid up prices  $b_j$  until their own surplus from financing the project is exhausted,  $\pi_j^F = 0$ . Substituting these prices into (17) reveals the quality of the marginal entrant,  $q = (w + I - R_B) / (R_G - R_B)$ . This is the same formula as for socially optimal quality  $q^*$  in (10). Marginal quality, however, will not be the same since VC advice is inefficiently low, implying  $R_j < R_j^*$ . Nevertheless, according to (22), welfare cannot be improved by encouraging entry since  $\nabla = 0$  when limited liability is not binding. The complete optimum is obtained only by encouraging VC advice until  $a = a^*$  which yields  $R_j = R_j^*$  and therefore  $q = q^*$  as well. This completes the solution of the unconstrained case since all other variables are implied.

**(b) The restricted case.** More realistic is the case where bad projects result in a loss, implying  $b_B = 0$  on account of limited liability and  $b = b_G$ . One is left with two unknowns,  $q$  and  $b$ , and two restrictions: free entry of entrepreneurs in (17) and the average break-even condition of VCs in (19). Using  $\nabla = R_G - R_B - b > 0$  yields

$$VC^D : q = (w + k) / b, \quad VC^S : Q(q, \theta) \cdot (R_G - R_B - b) = I - k - R_B. \quad (24)$$

Entry of entrepreneurs creates demand for VC finance. The identity of the marginal entrepreneur, as given by marginal quality  $q$ , is a downward sloping function of the conversion price  $b$ . Figure 1 illustrates. The break-even condition of competitive VCs in (23) stands for the supply of VC. VCs take entry and average quality as given and respond

by offering a conversion price. The supply function hence gives the break even price  $b$  for any given quality of the pool of applicants. Since average and marginal qualities are positively related, as captured by  $Q(q, \theta)$ , it is an upward sloping function of  $q$ . When the quality of projects increases, in the sense that a given venture is more likely to be of the profitable type, then the expected surplus of financiers rises. They compete by offering an even higher conversion price, until they hit the break even condition.

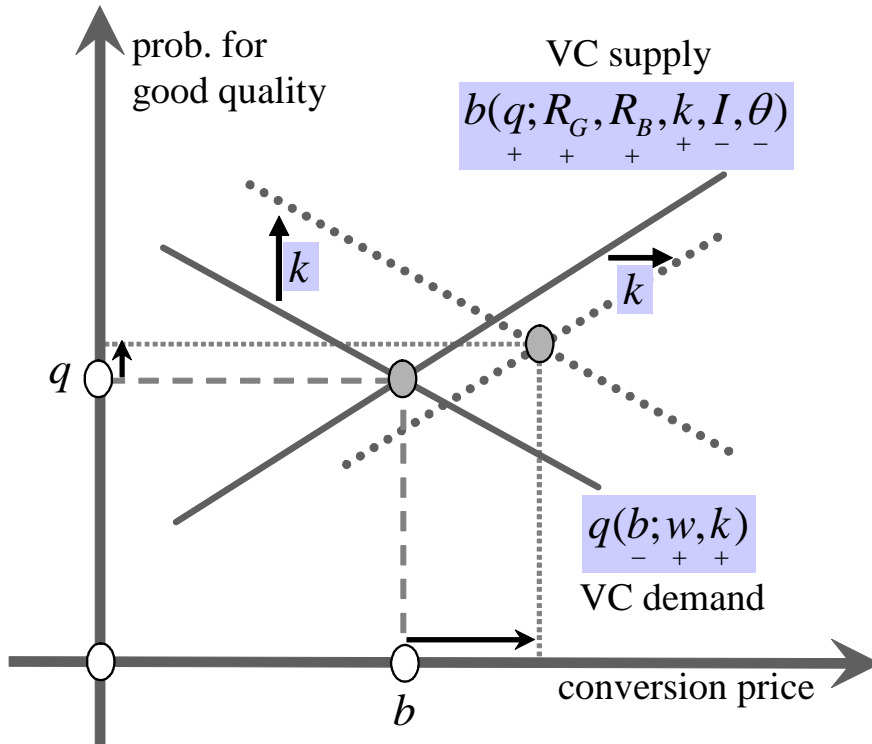


Fig. 1: Demand and Supply of Venture Capital

The comparative statics in case (b) can be easily understood with Figure 1. The signs noted below the exogenous variables in the supply and demand schedules indicate in which direction the curves are shifted when the variable is increased. Consider, for example, an increase in the market values  $v_j$  of ventures. By proposition 1, rents  $R_j$  from managing and advising a company increase which boosts the surplus  $\pi_j^F$  of the VC. Given the average quality in the pool of applicants, VCs bid up the conversion price to attract more business, until they break even. The supply schedule shifts to the right while the demand schedule remains unchanged (not drawn). A new equilibrium results with a higher conversion price and more entry at the lower quality margin. Because the

VC is willing to pay a higher price when converting, the deal becomes more attractive to entrepreneurs and attracts additional entry. Average quality declines.

As another example, consider a change in the informativeness of signals. When market and technological uncertainty increases, the potential of a new idea is more difficult to assess and self-selection into entrepreneurship a more shaky process. The signals received by entrepreneurs become less informative which is captured by the parameter  $\theta < 1$  increasing towards one, see Appendix A. People will revise their expectations of having a good project by less. So the average and marginal probabilities  $Q > q$  of being endowed with a good business idea will move closer to the prior probability  $\varepsilon$ . In consequence, the wedge between average and marginal quality shrinks as (A.11) in the Appendix shows. Hence, for any given  $q$ , a lower information content reduces the average quality in the pool of applicants and thereby forces VCs to offer less favorable deals by reducing the conversion price. The supply schedule shifts to the left. A less informative signal thus reduces entrepreneurship. The effect on average quality as stated in (A.11) seems ambiguous at first sight. On the one hand, a lower information content directly reduces average quality in the pool, but the fact that some low quality marginal entrepreneurs stay out works in the opposite direction. According to (D.5) in the appendix, the direct effect dominates. The net effect is a reduction in the average quality of VC backed start-up firms.

The last experiment we consider in detail is an increase in the entrepreneur's assets. When more own capital is at stake, the opportunity cost of starting a firm rises and thereby shifts up the demand curve for VC. The fear of losing own capital in case of business failure raises the required marginal quality  $q$  and diminishes demand for VC. On the other hand, more own capital reduces the need for external financing. Given that the VC finances a smaller amount  $I - k$ , she will compete with a higher conversion price  $b$  to acquire the same profit shares  $s_j$ . This in itself shifts the supply schedule to the right and will attract more rather than fewer entrepreneurs. At first sight, the net effect of more assets on entry seems ambiguous. The Appendix shows that in fact it is negative, see (D.3-4). When entrepreneurs can invest more own wealth, VCs bear less downside risk which relaxes the limited liability constraint and reduces the need to cross-subsidize bad projects. Marginal entrepreneurs thus face a worse deal, and some retire from the

market, raising the marginal quality of entrepreneurs.

The effects of remaining shocks follow unambiguously from Figure 1 and are not discussed in detail. The results are summarized for the case when limited liability binds:

**Proposition 4** *(a) Higher market values  $v_j$  or lower marginal effort costs  $\beta$  and  $\gamma$  lead to a higher conversion price, more entry and lower average quality. (b) More own capital  $k$  and a larger outside wage  $w$  result in a higher price, less entry and higher average quality. (c) A lower information content of signals (reflecting increased technological and market uncertainty) reduces the conversion price, entry and average quality. A larger capital investment  $I$  has the same effects, except that average quality increases.*

**Proof.** Proposition 1, Figure 1 and Appendix D, suitably combined. ■

## 5 Conclusions

Despite the fact that project selection is presumably as important as the value added role of VCs for the performance of VC-backed compared to other firms, the literature has largely focussed on the value-added role. This paper proposed a descriptive model of VC financing where entrepreneurs possess projects with high or low market potential. Our model of the VC industry features (i) an advisory role of VCs that is conditional on the quality of the venture which is revealed once the firm is started and close cooperation begins; (ii) a financial contract that is interpreted as a simple form of convertible debt; and (iii) self-selection into entrepreneurship of those persons who perceive themselves as more likely to have a high potential business idea.

The proposed framework replicates a number of important stylized facts in VC financing. In our model, VCs acquire a larger stake in high quality ventures and advise them more intensively. Since self-selection of entrepreneurs according to quality is imperfect, some are inevitably stuck with low potential firms. Should a venture turn out to be of low quality, the firm will receive less intensive, but still positive VC support which is mainly motivated to cut losses from that firm.

Our model rationalizes a simple form of convertible debt, one of the most widely used financial instruments in VC financing. The contract posts a low profit share that is suitable for low quality firms at a price that covers no more than the capital expenses that cannot be financed out of the entrepreneur's own pocket. This part of the contract can also be interpreted as debt. In addition, the contract includes an option to convert, at a prespecified conversion price, to a higher equity stake, should the firm turn out profitable.

The contract induces self-selection in the right direction, because the conversion price strengthens the entrepreneurs' reward when the project is revealed to have high potential. Competition among VCs for good projects bids up the conversion price to the largest possible extent, until VCs hit their break even condition. Entrepreneurs thereby appropriate most of the surplus from good projects, while they obtain no surplus from a bad project. The contract is thus particularly attractive for entrepreneurs who have received a good signal and thus perceive themselves as likely to have a high quality project, compared to others who have received a less inviting signal.

When considering the efficiency of the market equilibrium, we have identified two conditions for an efficient allocation of effort and entry. First, optimal effort requires that an agent is full residual claimant on the returns to her own effort input, as the literature on double moral hazard has emphasized. This is not possible since VCs must share profits with entrepreneurs. Advice is therefore inefficiently low in our model. Second, efficient entry requires that the entrepreneur is full residual claimant on the project type, meaning that she gets all the return differential between a good and a bad project. Again this is not possible when bad projects are unprofitable to the VC and limited liability binds, so that the entrepreneur cannot be asked to pay for the loss. In this case, the VC must cross-subsidize from good to bad projects which implies that low quality entrepreneurs get a too favorable deal and entry is excessive at the low quality margin.

Given these results on efficiency, the policy implications are in principle clear. Policy should find ways to stimulate VC effort by favorably treating ex post returns on projects. On the other hand, policy should attempt to restrict entry of marginal entrepreneurs who are rather likely to be endowed with low quality projects. What specific policy instruments could achieve these goals is left for future research.



With our quite tractable framework it should also be possible to examine the coexistence of bank- and VC-financed firms in market equilibrium, for example, by extending Ueda (2004) analysis of the entrepreneurs' choice between bank and VC financing. It would be particularly interesting to see whether an extended framework can explain the differential performance of bank versus VC financed firms, and to what extent it is due to selection effects rather than the value added role of VCs. Recent empirical work by Sorensen (2005) has disentangled the reasons for such differential performance. Applied theoretical work could try to identify the structural parameters governing the relative importance of selection and advice.

## Appendix

### A Signals and Probabilities

Projects are either good or bad,  $j \in \{G, B\}$ . The true proportion of good ideas is  $\varepsilon$ . In the beginning, agents do not know the type but receive an informative signal  $y$  that is positively correlated with project quality. A good signal is thus received by the good type with higher probability than by the bad type. The distribution of signals in the population is

$$E_j(y) = \int_y^\infty e_j(y') dy', \quad E_j'(y) = -e_j(y). \quad (\text{A.1})$$

The *marginal probability* of having a good project is

$$q = \Pr(G|y) = \frac{\varepsilon e_G(y)}{e(y)}, \quad e(y) \equiv \varepsilon e_G(y) + (1 - \varepsilon) e_B(y). \quad (\text{A.2})$$

The average probability of observing a specific signal  $y$  by any of the two types is  $e$ . Good types are much less likely than bad types to receive a low signal, implying  $e_G(y) < e_B(y)$  for  $y$  small. In contrast, high signal values are more frequently received by good types, implying  $e_G(y) > e_B(y)$  for  $y$  large. Consequently, the marginal probability increases in the signal value.

The *average probability* for a good idea among all agents with signals  $y' > y$  is

$$Q = \Pr(G|y' \geq y) = \frac{\varepsilon E_G(y)}{E(y)}, \quad E(y) \equiv \varepsilon E_G(y) + (1 - \varepsilon) E_B(y). \quad (\text{A.3})$$

Agents pursue entrepreneurship only if they perceive a sufficiently high probability  $q$  of having a good project. They will thus need to receive a sufficiently good signal  $y$ . Inverting (A.2) establishes a correspondence  $y = \phi(q)$ . By (A.3), the average probability  $Q$  of a good project among all agents with signals better than  $y = \phi(q)$  is

$$Q = \Pr(G|y' \geq \phi(q)). \quad (\text{A.4})$$

Entry decision thus establishes a cut-off value or marginal probability  $q$  and an average probability  $Q > q$  of a good project in the entire pool of applicants.

We now determine the impact of the marginal entrepreneur with probability  $q$  on the average probability  $Q$ . Using (A.2) and (A.3), we first calculate

$$\frac{dQ}{dy} = (Q - q) \cdot \frac{e}{E} > 0. \quad (\text{A.5})$$

The effect is positive with increasing signals and probabilities, yielding  $Q > q$ . It will also be useful to get, from equation (A.2),

$$\frac{dq}{dy} = q \cdot (1 - q) \cdot \left[ \frac{e'_G}{e_G} - \frac{e'_B}{e_B} \right] > 0. \quad (\text{A.6})$$

The sign of the square bracket must be positive for the signal to be informative of the true quality as will be shown below. Divide (A.5) by (A.6) to obtain the desired effect,

$$\frac{dQ}{dq} = \frac{(Q - q) \cdot e/E}{q \cdot (1 - q) \cdot [e'_G/e_G - e'_B/e_B]} > 0. \quad (\text{A.7})$$

We now use a special functional form for the density of signals. The parameter  $\theta$  conveniently measures the informativeness of signals,

$$e_G(y) = \theta \exp(-\theta y) = \theta E_G(y), \quad e_B(y) = \exp(-y) = E_B(y), \quad \theta < 1. \quad (\text{A.8})$$

If  $\theta = 1$ , then  $e_G = e_B$  and  $E_G = E_B$ . In this case,  $q = Q = \varepsilon$  by (A.2-3), and the signal is not informative. If  $\theta < 1$ , the good type receives a low signal with smaller density than the bad type,  $e_G(0) = \theta < e_B(0) = 1$ . The density of higher signals falls more rapidly with the bad type so that  $e_G(y) > e_B(y)$  for  $y$  large enough. The two density functions cross exactly once. The relative slopes are governed by  $\theta$ , implying

$$e'_G/e_G - e'_B/e_B = 1 - \theta. \quad (\text{A.9})$$

The functional form in (A.8) allows for a convenient closed form solution of the relation between average and marginal quality which also shows how this relation depends on the informativeness parameter. Substitute  $e_G = \theta E_G$  and  $e_B = E_B$  from (A.8) into (A.2), divide the numerator and denominator by  $E$  and use the definition of  $Q$  in (A.3),

$$q = \frac{\theta Q}{\theta Q + 1 - Q} \quad \Rightarrow \quad Q = \frac{q}{q + (1 - q)\theta} > q. \quad (\text{A.10})$$

The informativeness assumption  $\theta < 1$  implies  $Q > q$ . If the signal were not informative, then average and marginal quality would be equal,  $Q = q$ , as argued before. The elasticity of  $Q$  with respect to  $q$  follows from the log-linearization where the hat notation indicates a relative change,  $\hat{Q} \equiv d \ln Q = dQ/Q$ . Rewrite (A.10) as  $(1 - Q)q = (1 - q)\theta Q$  and get the log-linear form

$$\hat{Q} = \mu \cdot \hat{q} - (1 - Q) \cdot \hat{\theta}, \quad \mu \equiv (1 - Q) / (1 - q). \quad (\text{A.11})$$

As a consistency check, we use (A.9) and write the coefficient of (A.7),  $\hat{Q} = \mu \hat{q}$ , as  $\mu = \frac{(Q - q)e/E}{(1 - q)(1 - \theta)Q}$ . Using again (A.8) to rewrite  $e$  yields  $e/E = \theta Q + (1 - Q)$ . Rearranging (A.10) as  $(1 - Q)q = (1 - q)\theta Q$ , one obtains  $\theta Q = (1 - Q)q / (1 - q)$ , and thereby  $(1 - \theta)Q = (Q - q) / (1 - q)$ . Using these expressions to replace  $\theta Q$  in the numerator and  $(1 - \theta)Q$  in the denominator yields  $\mu = \frac{(Q - q)[\theta Q + (1 - Q)]}{(1 - q)(1 - \theta)Q} = \frac{1 - Q}{1 - q}$  as in (A.11).

## B Marginal and Average Surplus

We first relate marginal and average surplus in (3-4). To get the average surplus over all projects financed, substitute  $q_j = \varepsilon_j e_j / e$  from (A.2) into (3) and multiply the result by  $e$ , yielding  $e(y)\pi(y) = \sum_j \varepsilon_j e_j(y) R_j - (w + I)e(y)$ . Next, integrate over  $y' \geq y$ , note  $E_j(y)$  as well as  $E(y) = \int_y^\infty e(y') dy'$  by (A.1-3), and get  $\int_y^\infty \pi(y') e(y') dy' = \sum_j \varepsilon_j E_j(y) R_j - (w + I)E(y)$ . By definition, the average surplus *per applicant* is

$$\Pi \equiv \int_y^\infty \pi(y') e(y') dy' / E(y). \quad (\text{B.1})$$

Divide the preceding equation by  $E(y)$  and use (B.1) as well as the definition of  $Q_j(y) = \varepsilon_j E_j(y) / E(y)$  in (A.3). This yields the average joint surplus  $\Pi = \sum_j Q_j R_j - (w + I)$  which proves (4).

Next, we prove the derivative of  $W = w + k + \Pi E$  in (7). Since  $w$  and  $k$  are constants, we need to consider only  $\Pi E$ . Substitute the definitions  $Q_j E = \varepsilon_j E_j$  and  $E = \sum_j \varepsilon_j E_j$  into (5),  $\Pi E = \sum_j \varepsilon_j E_j(y) \cdot [p(l_j, a_j) v_j - \beta l_j - \gamma a_j] - (w + I) E(y)$ . Since the inverse of (A.2) establishes  $y = \phi(q)$ , we can take the derivative with respect to  $q$ . Using  $E'(y) = -e(y)$  and  $E'_j(y) = -e_j(y)$  and substituting  $q_j = \varepsilon_j e_j / e$  yields the last condition in (7).

## C Proof of Proposition 1

**Part (a):** Section 3.1 showed that  $a_j$  and  $s_j$  satisfy the two conditions in (12) with equality. Using (1) and substituting (13) into the condition on  $l_j$  in (12) yields

$$z_j \equiv (1 - s_j)^{1-\alpha} (s_j)^\alpha = \beta^{1-\alpha} (\gamma/\alpha)^\alpha / v_j, \quad \frac{dz_j}{ds_j} = \frac{\alpha - s_j}{(1 - s_j)^\alpha (s_j)^{1-\alpha}}. \quad (\text{C.1})$$

This equation implicitly determines the profit share. The  $z_j$ -function returns a zero for values  $s_j = 0$  and  $s_j = 1$ , and is positive and concave in between. Its slope turns from positive to negative as  $s_j$  starts from zero and moves beyond  $\alpha$ . With an interior solution, there are two values for  $s_j$  of which the larger is the relevant profit maximizing one by the arguments in the paragraph following (12). Hence, the slope of the  $z_j$ -function must be negative at the optimal value of  $s_j$ , implying  $\alpha < s_j$ . The inequality  $s_G > s_B$  follows from the fact that a higher value  $v_G > v_B$  reduces the r.h.s. of (C.1) and shifts down the horizontal line which intersects the  $z$ -function.

**Part (b):** We show this by linearizing the system in (12). The notation  $\hat{a} \equiv da/a$  indicates a percentage change where  $da$  is the absolute deviation from an initial value of  $a$ . The functional form (1) yields together with the equilibrium value  $l_j = 1$ ,

$$p_j = l_j \cdot (a_j)^\alpha \quad \Rightarrow \quad a_j \cdot p_a^j = \alpha \cdot p_j, \quad \hat{p}_j = \alpha \hat{a}_j, \quad \hat{p}_a^j = - (1 - \alpha) \hat{a}_j. \quad (\text{C.2})$$

The comparative static effects of shocks to exogenous parameters can be uncovered by log-linearization of (12). Using (C.1) yields

$$(1 - \alpha) \hat{a}_j = \hat{s}_j + \hat{v}_j - \hat{\gamma}, \quad \frac{s_j}{1 - s_j} \hat{s}_j = \alpha \hat{a}_j + \hat{v}_j - \hat{\beta}. \quad (\text{C.3})$$

The first equation shows how a VC increases advice upon receiving a larger profit share, the second relates to the entrepreneur's incentives. If she receives more advice, her own incentives for effort can be ensured with a lower share  $1 - s_j$ , or a higher share  $s_j$  for the VC. Solving (C.3) for the two unknowns and noting  $s_j > \alpha$  by part (a) yields

$$\hat{a}_j = \frac{1}{s_j - \alpha} \left[ \hat{v}_j - s_j \hat{\gamma} - (1 - s_j) \hat{\beta} \right], \quad \hat{s}_j = \frac{1 - s_j}{s_j - \alpha} \left[ \hat{v}_j - \alpha \hat{\gamma} - (1 - \alpha) \hat{\beta} \right]. \quad (\text{C.4})$$

**Part (c):** With  $R_j^E = 0$  in (14), the financier's rent coincides with the joint rent,  $R_j^F = R_j$ . Log-linearizing (14) and substituting (C.2) and (C.4) yields

$$\hat{R}_j = \hat{R}_j^F = \hat{s}_j + \hat{p}_j + \hat{v}_j = \frac{1}{s_j - \alpha} \left[ \hat{v}_j - (1 - s_j) \hat{\beta} - \alpha \hat{\gamma} \right]. \quad (\text{C.5})$$

## D Comparative Statics

Log-linearize (24) to obtain comparative statics results. Define the share  $\delta \equiv k / (w + k)$  of own capital in the total opportunity cost of entrepreneurs. The share of foregone wages is  $1 - \delta = w / (w + k)$ . Use this to obtain the log-linearized form of the demand schedule. The relative change in average quality was already shown in (A.11). Use this together with  $\nabla = R_G - R_B - b > 0$  (or  $\nabla \hat{\nabla} = R_G \hat{R}_G - R_B \hat{R}_B - b \hat{b}$ ) in the log-linearized form of the supply schedule in (24),  $Q \nabla (\hat{Q} + \hat{\nabla}) = I \hat{I} - k \hat{k} - R_B \hat{R}_B$ , which gives

$$\begin{aligned} \hat{q} &= -\hat{b} + (1 - \delta) \hat{w} + \delta \hat{k}, \\ Qb \cdot \hat{b} &= Q \nabla \mu \hat{q} - I \hat{I} + k \hat{k} - (1 - Q) Q \nabla \hat{\theta} + Q R_G \hat{R}_G + (1 - Q) R_B \hat{R}_B. \end{aligned} \quad (\text{D.1})$$

Substituting the demand function  $\hat{q}$  into the supply function yields

$$\hat{b} = \frac{(1 - \delta) Q \nabla \mu \hat{w} + (\delta Q \nabla \mu + k) \hat{k} - (1 - Q) Q \nabla \hat{\theta} - I \hat{I} + \sum_j Q_j R_j \hat{R}_j}{(b + \nabla \mu) Q}. \quad (\text{D.2})$$

Substituting back into the demand schedule  $\hat{q}$  yields the marginal entrant,

$$\hat{q} = \frac{(1 - \delta) Qb \hat{w} + (bQ - w - k) \delta \hat{k} + (1 - Q) Q \nabla \hat{\theta} + I \hat{I} - \sum_j Q_j R_j \hat{R}_j}{(b + \nabla \mu) Q}. \quad (\text{D.3})$$

All results are unique except for the impact of  $k$ . Substituting the demand schedule in (24) for  $w + k$  yields

$$Qb - w - k = (Q - q) b > 0. \quad (\text{D.4})$$

The effect of  $\theta$  on average quality seems ambiguous at first sight. Substituting the effect in (D.3) into (A.11) and finally replacing  $\mu$  yields, after some manipulations,

$$\hat{q} = \frac{1-Q}{b + \nabla\mu} \nabla\hat{\theta}, \quad \hat{Q} = -\frac{(1-Q)b}{b + \nabla\mu} \cdot \hat{\theta}. \quad (\text{D.5})$$

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