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An Alternative Approach to Interpret Long-Run Returns**

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Volatility-Adjusted Performance

An Alternative Approach to Interpret Long-Run Returns

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Abstract:

This paper investigates long-run returns by utilizing log-normal distribution properties of cross-sectional buy-and-hold returns. We decompose expected cross-sectional buy-and-hold returns into transformed mean components and volatility components. This decomposition shows that the volatility component contributes positively to the right-skewed buy-and-hold returns due to Jensen's inequality. Given the log-normal distribution properties are fulfilled, the method can be applied to any type of long-horizon event study of security performance. We apply the method to IPO stocks and SEO stocks listed on the Copenhagen Stock Exchange. Using traditional standard techniques, we find that IPO stocks and SEO stocks under perform relative to the market after five years by 27.3 percent and 21.4 percent, respectively. However, the volatility-adjusted performance measure shows that the IPO stocks and SEO stocks under perform relative to the market after five years by 43.7 percent and 38.1 percent, respectively.

Keywords: Wealth relatives; buy-and-hold returns; equity offerings

JEL classification: G14, G32

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I. Introduction

The right-skewed distribution of long-run returns causes an inference problem of the significance of security performance. Fama (1998) addresses this problem as a "bad-model" problem in studies of long-run returns, arguing that the error in the expected return grows faster with time than the volatility does. Lyon, Barber and Tsai (1999) address the problem using a skewness-adjusted t-statistic for testing average long-run returns. However, to the best of our knowledge, none are actually testing the distribution properties of the cross-sectional long-run returns. Therefore, we question the validity of prior tests of long-run abnormal returns.

The contribution of this paper is twofold. First, we formalize the skewness problem utilizing the properties of the log-normal distribution to describe the expected buy-and-hold returns. The methodology hinges on the cross-sectional distribution properties of long-run returns. The log-normal distribution of cross-sectional buy-and-hold returns of wealth relatives (see Ritter (1991) and Loughran and Ritter (1995)) allows for a decomposition of the expected mean buy-and-hold abnormal return into its transformed mean and volatility component. The decomposition provides well-specified and testable maximum likelihood estimators. Second, the method provides an alternative approach to understand and interpret long-run security performance. Applying the method on a sample of unseasoned- and seasoned-equity offerings shows that the volatility-adjusted performance measure provides a better inference of security performance than the usual measure of buy-and-hold abnormal return.

We calculate the cross-sectional averages of the buy-and-hold returns using three different methods. The first method, the most commonly used, is a simple cross-sectional arithmetic average calculation that does not adjust for right skewed long-run returns. The second method is to calculate the average cross-sectional return as the expected value of log-normally distributed buy-and-hold returns. The expected value of log-normally distributed buy-and-hold returns cannot be tested directly due to an upward bias caused by the volatility. The third method decomposes the expected cross-sectional mean buy-and-hold returns into transformed mean components and volatility

components. This decomposition of the average cross-sectional mean buy-and-hold return in levels provides a more distinctive interpretation of the under performance or over performance. This is possible because the decomposition of the average cross-sectional buy-and-hold return accounts for the impact that originates from the transformed mean component and the positive volatility component.

Using a simple arithmetic cross-sectional average of the wealth-relative measure shows that 142 IPO stocks and 413 SEO stocks under perform relative to the market after five years by 27.3 percent and 21.4 percent, respectively. However, looking at the market-to-security wealth-relatives show that the market out performs IPO stocks and SEO stocks by an average of 152.6 percent and 128.4 percent, respectively. The large differences between the two wealth relatives (market-to-security and security-to-market) are due to the volatility component. The cross-sectional average buy-and-hold return of the market-to-security ratio is amplified by the volatility component while the security-to-market wealth relative ratio is dampened by the volatility component. Those cross-sectional averages are contaminated by the right-skewness bias. Adjusting the expected cross-sectional mean buy-and-hold return for the volatility component, IPO stocks and SEO stocks under perform relative to the market by 43.7 percent and 38.1 percent after five years. Equivalently, the market out performs IPO stocks and SEO stocks by 77.5 percent and 61.5 percent, respectively. This under (out) performance is statistically significant at the one-percent critical level of significance.

Applying the volatility-adjusted performance measure on a sample of hot (cold) period equity issues show that hot issues experience significant long-run under performance compared to cold issue periods.¹ The results show that the under performance after one year is similar for hot and cold issues while after five years hot issues under perform significantly by 50.0 percent and cold issues under perform insignificantly by 22.6 percent. This indicates that investing in firms going public in a cold issue period is better in the long run than investing in firms going public in a hot issue period. Catego-

¹ Loughran, Ritter and Rydqvist (1994) document the tendency of high initial returns and volume after a period of high stock market performance. Jakobsen and Sørensen (1999) also document that high initial returns are inversely related to the buy-and-hold returns and with negative long-run returns over a five-year period.

rized IPO stocks by market capitalization provide some evidence that the lowest quartile under perform more than the highest quartile. Volatility-adjusted performance measure confirms that small capitalization issues under perform relative to larger capitalization issues. The out performance after five years is not statistically significant at a five-percent critical level of significance for the largest quartile. This indicates that under performance may be concentrated in small capitalization firms. Categorizing SEO stocks by the proceed-to-market value ratio shows that the market out performs all quartiles after five years and this out performance is statistically significant at a one-percent level. This indicates that the under performance of the proceed-to-market capitalization does not depend on the size of the capitalization.

The structure of the paper is as follows. Section II discusses methodological problems of measuring long-run returns. Section III introduces the methodology that is used for testing and measuring long-run buy-and-hold returns of wealth relatives. Section IV presents prior findings of long-run security performance and the empirical results using the method on IPO and SEO stocks, a hot (cold) issue period, and on a sample partition by market capitalization. Section V provides the concluding remarks.

II. Methodological Problems of Measuring Long-Run Returns

A number of studies find significant over (under) performance by testing long-run buy-and-hold returns. However, conventional test statistics used for testing are not necessarily valid due to distribution properties of the buy-and-hold returns. Recently, Fama (1998) argues that none of the documented long-run performance studies actually tests for whether securities on average are under performing or over performing.² Fama argues that the market efficiency hypothesis is intact because the anomalies disappear with changes in the estimation model of the expected returns, i.e. the estimation meth-

² See Agrawal, Jaffe and Mandelker (1992) for mergers and acquisitions, Ikenberry, Lakonishok and Vermaelen (1995) for share repurchases, Michaelley, Thaler and Womach (1995) for dividend initiations and omissions, Speiss and Affleck-Graves (1995) for seasoned equity offerings, Loughran and Vjih (1997) for corporate acquisitions, and Ritter (1991), Aggarwal and Rivioli (1990), Loughran and Ritter (1995), Jakobsen and Sørensen (1999) for initial public offerings. The results from various kinds of event studies show that over reactions in the marketplace is as common as under reactions, therefore, Fama (1998) argues that the post-event reversals are as frequent as the post-event continuations of pre-event returns.

odology may cause long-run anomalies. Also, inference of long-run returns are problematic due to unavoidable “bad-model” problems caused by incomplete models that are applied to estimate expected returns and sample-specific patterns in average returns which are caused by systematic deviations.

The commonly used event-study methodology (*CAR*) to estimate long-run abnormal returns causes severe problems for testing and interpreting long-run security performance. Conrad and Kaul (1993) document a potential bias (upward or downward) induced by cumulative abnormal returns over the long run, i.e. the long-run cumulative adjusted returns become potentially biased. Barber and Lyon (1997a) and Kothari and Warner (1997) find severe problems related to the fact that abnormal returns are non-normally distributed as they become right skewed after a period of time. Symmetric and independently distributed periodic returns induce right skewness and autocorrelation in accumulated returns. This right skewness arises due to the accumulation method of buy-and-hold returns. Fama (1998) address problems concerning mis-specifications that arise from “re-balancing” bias, “new listing” bias, and right “skewness” biases. These problems affect the conventional test methods and yet many empirical results make inference based on incorrect test statistics.³ Barber and Lyon (1997a) argue that the mis-specification causes the inference based on long-run returns to be incorrect. They argue that it is possible to achieve well-specified test statistics when the usual BHAR is corrected by matching sample firms with control firms of similar size and book-to-market ratios.⁴ Kothari and Warner (1997) suggest a parametric shift in the event window that influences the tests of accumulated returns, e.g. the interference must include the possible increase in variability of abnormal returns. Lyon, Barber and Tsai (1999) suggest a correction of the usual t-test statistic to capture the skewness of the distribution of the buy-and-hold abnormal returns. However, the design of the benchmark is important for the applicability of the skewness-adjusted t-statistics.

³ See Conrad and Kaul (1993), Barber and Lyon (1997), Canina, Michaely, Thaler, and Womack (1997), Kothari and Warner (1997), Cowan and Sergeant (1997), Lyon, Barber and Tsai (1999) for warnings about the buy-and-hold return methodology used to estimate abnormal returns.

⁴ Fama (1998) argues that corrections of expected returns using a matching approach based on size and BE/ME does not limit bad-model problems. Abnormal returns vary whether matching is based on size or size and BE/ME. Fama (1998) argues that this matching approach does not capture the cross-sectional variation in expected returns. Therefore, a matching approach for expected returns is not a panacea for

III. Methodology

The following four sub-sections describe the methodology used in this paper. Sub-section A provides an introduction to the event-study method and its problem in measuring long-run returns. Sub-section A also examines the cross-sectional distribution properties of the long-run returns of the securities and the benchmark. Sub-section B describes, for the long run, the methodology of designing a variable transformation that exhibits log-normally distributed buy-and-hold returns (e.g. wealth relatives). We investigate wealth-relative transformation of the security return series and the benchmark return series. In sub-section C, we decompose the expected average cross-sectional long-run returns into transformed mean and volatility components. Finally, sub-section D specifies the tests of the components.⁵

A. Short-run Returns versus Long-run Returns

Event studies are performed using periodic returns (e.g. daily, weekly or monthly) over a pre-event, event, and post-event window. Define r_{it} as the simple periodic return of security i over period t . The expected return of security i over period t based on the information set \mathbf{W}_t is $E(r_{it}/\mathbf{W}_t)$. \mathbf{W}_t is the relevant information set, e.g. pre-event or post-event windows, of the security performance model. Subtracting the expected return from the observed return is defined as the abnormal return $AR_{it} = r_{it} - E(r_{it}/\mathbf{W}_t)$ over period t . The event window can vary from a short-horizon period (few days) to a long-horizon period (36 to 60 months). To make inference of security performance over an event window with time horizon \mathbf{t} the cumulative abnormal return (CAR) can be used and it is calculated as:

$$CAR_{i,t} = \sum_{t=1}^{\mathbf{t}} AR_{it} \quad [1]$$

bad-model problems. In particular in long-run event studies because the standard error in abnormal return increases with the number of months.

⁵ Jakobsen and Sørensen (1999) propose the decomposition of long-run returns using a geometric Brownian motion.

If there are N securities, i.e. $i=\{1, \dots, N\}$, dividing the sum over i of all the $CAR_{i,t}$ by N provides the cumulative average abnormal return over the time horizon t ($CAAR_t$). CAR only adds-up periodic returns and does not capture the compounding effect of periodic returns. The measure of buy-and-hold returns explicitly captures the compounding of periodic returns. By using the buy-and-hold measure for measuring abnormal returns, we can apply different approaches. The traditional approach to measure abnormal returns of a security i against some benchmark (e.g. the expected return) over the time horizon t is:

$$BHAR_{i,t} = \prod_{t=1}^t (1 + r_{it}) - \prod_{t=1}^t (1 + E(r_{it} | \Omega_{it})) \quad [2]$$

Over the long run, the buy-and-hold abnormal return ($BHAR_{i,t}$) approach reflects a more realistic measure of security i 's performance than the $CAR_{i,t}$ approach. Similar to the $CAAR$ approach, an average of the cross-sectional $BHAR_{i,t}$ can be calculated. Both methods (CAR and $BHAR$) require an appropriate benchmark against which the sample-specific securities can be compared. Usual applied test statistics of both the CAR and the $BHAR$ approach apply the ratio of the sample-specific mean to its estimated standard deviation. However, it is difficult to achieve unbiased test statistics of both the mean and the standard deviation estimates. For the CAR approach, the statistical power in event studies is not the main issue since it approaches one when N increases but the lack of the compounding effect is a problem in the long run.⁶ For the $BHAR$ approach, the compounding effect is intact in the long run but usually applied test statistics are invalid due to the severe right-skewed distribution of the $BHAR$ measure. Therefore it is necessary to address the underlying distribution properties of long-run returns.

⁶ The statistical power increases because the mean CAR increases with N while the standard error of CAR increases with $N^{1/2}$.

B. Buy-and-Hold Returns

Long-run returns are measured by the method of compounding returns, i.e. buy-and-hold returns. The buy-and-hold return is calculated for each security, i , from the announcement day to some future date T . An initial amount $W_{i,0}$ is invested in each security, i , with a stochastic periodic (monthly) return, $r_{i,t}$, that is realized at the end of period t . The invested amount accumulates after T periods to $W_{i,T}$:

$$W_{i,T} = W_{i,0} \cdot \prod_{t=1}^T (1 + r_{i,t}) \quad [3]$$

Without loss of generality, the initial amount $W_{i,0}$ can be set equal to one and the buy-and-hold return is equal to $\prod_{t=1}^T (1 + r_{i,t}) - 1$. Buy-and-hold returns will typically be right-skewed distributed, even if the periodic returns are symmetrically distributed, and that necessitates an investigation of the distribution properties. The wealth relative between two accumulated values is the ratio between the wealth $W_{i,T}$ of investing in a security and the wealth $W_{m,T}$ of investing in the market index after T periods. This accumulated wealth relative follows the spirit of Ritter (1991) and Loughran and Ritter (1995).⁷ The wealth relative can be measured as either the market-to-security ratio, $W_{m-i,T} = (W_{m,T}/W_{i,T})$, or the security-to-market ratio, $W_{i-m,T} = (W_{i,T}/W_{m,T})$. The accumulated wealth relative (security-to-market) after T months is:

$$W_{i-m,T} \equiv W_{i-m,0} \cdot \frac{\prod_{t=1}^T (1 + r_{i,t})}{\prod_{t=1}^T (1 + r_{m,t})} = W_{i-m,0} \cdot \prod_{t=1}^T \frac{(1 + r_{i,t})}{(1 + r_{m,t})} \quad [4]$$

⁷ Ritter (1991) and Loughran and Ritter (1995) use the wealth relatives differently by taking the average in the numerator and the denominator. Our approach is more applicable because the wealth relative is log-normal distributed. An alternative to the wealth relative is the transformed buy-and-hold abnormal return (T -BHAR) where we compounded abnormal returns over time, which can also be accepted log-normal distributed (see appendix A). The T -BHAR states that an investor holds a long position in the market index and a short position in the equity offering in which the proceed are realized monthly and reinvested assuming no transaction costs.

where $i=\{1,\dots,N\}$ and $T=\{1,\dots,60\}$. Taking the logarithm of the wealth relative and assuming that $(W_{i,0}/W_{m,0})=1$ the expression becomes:

$$\text{Log}(W_{i-m,T}) = \sum_{t=1}^T \text{Log}(1+r_{i,t}) - \sum_{t=1}^T \text{Log}(1+r_{m,t}) \quad [5]$$

If the logarithms of the periodic (monthly) gross returns are normally distributed the buy-and-hold returns of the securities, the benchmarks, and the wealth relatives will be log-normally distributed. Even if the securities' or the benchmark's periodic returns are not normally distributed it may still be the case that subtracting the periodic (monthly) returns of the security from the benchmark turns out to be normally distributed. In other words, in [5] the left-hand side may be normally distributed even if the individual terms on the right-hand side are not normally distributed. The transformation of wealth relatives from logarithmic values to level values using exact expressions in discrete time is:

$$W_{i-m,T} = W_{i-m,0} \cdot e^{\sum_{t=1}^T \text{Log}(1+r_{i,t}) - \text{Log}(1+r_{m,t})} \quad [6]$$

If the wealth relative is log-normally distributed an inherent relationship exists between the geometric Brownian motion model and the development in the wealth relatives. The geometric Brownian motion model represents an explicit structure on the wealth relative that describes the development of the buy-and-hold returns over time. The common interpretation of the geometric Brownian motion model calls for constant drift and volatility parameters over time. However, if the purpose is to investigate cross-sectional buy-and-hold returns at different time horizons and allowing the drift parameter and the volatility parameter to vary over time it is not necessary to use the geometric Brownian motion model. It is sufficient to utilize the log-normal distribution properties of long-run returns, i.e. Jensen's inequality. Though we do not need to describe the cross-sectional buy-and-hold returns with the geometric Brownian motion, we show the expression of the wealth relative (security-to-market):

$$W_{i-m,T} = W_{i-m,0} \cdot e^{(\mathbf{m}_T - \frac{1}{2}\mathbf{s}_T^2)T + \mathbf{s}_T \cdot Z_T} \quad [7]$$

where $\mathbf{m}_T = \mathbf{a}_T + \frac{1}{2}\mathbf{s}_T^2$. The parameters \mathbf{a}_T , \mathbf{m}_T , and \mathbf{s}_T are constants and $\{Z_t\}_{t \geq 0}$ is a Wiener process with $dZ_t \sim N(0, dt)$. Expression [7] allows the constants \mathbf{a}_T and \mathbf{s}_T to vary dependent on the time horizon T . This flexibility is based on the assumption that the cross-sectional means at any given time horizon T can be treated as independent observations.

C. Decomposing the Wealth Relative

Given that the wealth relatives (security-to-market) follows a log-normal distribution, the logarithm of the wealth relative is normally distributed with transformed mean $\mathbf{a}_T \cdot T$ and standard deviation $\mathbf{s}_T \sqrt{T}$. The expected mean buy-and-hold return of the wealth relative in levels is log-normally distributed with the expected mean:⁸

$$\begin{aligned} E(W_T) &= e^{\mathbf{m}_T \cdot T} = e^{\mathbf{a}_T \cdot T + \frac{1}{2}\mathbf{s}_T^2 \cdot T} \\ &= \underbrace{e^{\mathbf{a}_T \cdot T}}_{\text{Transformed mean component}} \cdot \underbrace{e^{\frac{1}{2}\mathbf{s}_T^2 \cdot T}}_{\text{Volatility component}} \end{aligned} \quad [8]$$

From expression [8] it is observed that the volatility component implies an upward bias on the average wealth relative. This volatility effect is always positive and identical irrespective of the wealth-relative measure (security-to-market or market-to-security). Therefore, it is necessary to adjust the expected average cross-sectional wealth relative for the volatility component in order to infer coherent long-run security performance. As the volatility component imposes an upward bias on the expected mean buy-and-hold return, we decompose the cross-sectional mean of the wealth-relative ratio, $e^{\mathbf{m}_T \cdot T}$, into its transformed mean component, $e^{\mathbf{a}_T \cdot T}$, and its volatility component, $e^{\frac{1}{2}\mathbf{s}_T^2 \cdot T}$. The

⁸ In general, the expected value in expression [8] does not equal the simple arithmetic average of cross-sectional buy-and-hold returns (method 1, above).

geometric Brownian motion model representation explicitly depicts the transformed mean component and the volatility component. This decomposition also captures the feature that the periodic (monthly) returns may be symmetric and independently distributed while the buy-and-hold returns of the wealth relatives exhibit right skewness.

D. Tests of the Transformed Mean and Volatility Components

The wealth relative that is described in expression [7] is log-normally distributed. Thus, when the logarithm is taken it becomes normally distributed:

$$\log(W_{i-m,T}) \sim (\mathbf{a}_T \cdot T, \mathbf{s}_T^2 \cdot T) \quad [9].$$

Given that the logarithm of the wealth relative is normally distributed, we can estimate the transformed mean parameters and variance parameters:

$$\hat{\mathbf{a}}_T = \frac{1}{T \cdot N} \sum_{i=1}^N \log(W_{i-m,T}) \quad [10]$$

$$\hat{\mathbf{s}}_T^2 = \frac{1}{T \cdot (N-1)} \sum_{i=1}^N (\log(W_{i-m,T}) - \hat{\mathbf{a}}_T \cdot T)^2 \quad [11]$$

The parameter estimates: $\hat{\mathbf{a}}_T$ and $\hat{\mathbf{s}}_T^2$ are marginal parameter estimates at any point in time. The marginal parameter estimates are only based on the expected cross-sectional wealth relative at time T . Both the marginal parameter estimates at time T are treated independently of marginal parameter estimates of other time periods. Therefore, serial correlation or other time-series dependencies do not influence the marginal parameter estimates. At time T , the transformed mean and standard deviation of $\log(W_{i-m,T})$ are $\hat{\mathbf{a}}_T \cdot T$ and $\hat{\mathbf{s}}_T \cdot \sqrt{T}$, respectively. The marginal estimates $\hat{\mathbf{a}}_T \cdot T$ and $\hat{\mathbf{s}}_T \cdot \sqrt{T}$ can be tested against alternatives using the cross-sectional wealth relatives at time T . For example, two independent hypotheses could be the following:

$$H_0: \mathbf{a}_T \mathcal{A} = 0, \quad \text{versus} \quad H_1: \mathbf{a}_T \mathcal{A} \neq 0$$

or

$$H_0: \mathbf{s}_T \cdot \sqrt{T} = \mathbf{I}, \quad \text{versus} \quad H_1: \mathbf{s}_T \cdot \sqrt{T} \neq \mathbf{I}$$

where \mathbf{I} can be any positive value. The marginal confidence interval of the marginal estimate $\mathbf{a}_T \sqrt{T}$ is t -distributed at time T and the marginal confidence interval of the marginal estimate $\mathbf{s}_T \cdot \sqrt{T}$ is χ^2 -distributed at time T both with $N-1$ degrees of freedom. The marginal estimates can be transformed back to levels through the exponential function and the resulting expected mean buy-and-hold returns can be compared to the wealth relative. This ensures that the transformed mean components, $e^{\hat{\mathbf{a}}_T \cdot T}$, and the volatility components, $e^{\frac{1}{2} \mathbf{s}_T^2 \cdot T}$ can be compared to the expected mean buy-and-hold returns, $e^{m_T \cdot T}$.

To summarize, the methodology, used to investigate cross-sectional buy-and-hold returns of wealth relatives, identifies log-normally distributed long-run returns and decomposes the expected average cross-sectional buy-and-hold returns into transformed mean components and volatility components. Adjusting for the volatility component, the transformed mean component provides a testable and correct estimate of long-run security performance.⁹

IV. Results

In the following section, we apply the method that implements the volatility-adjusted performance measures to equity offerings stocks. We partition the unseasoned and seasoned equity offerings in different groups and test the expected volatility-adjusted performance. We apply two approaches, the market model and the market-adjusted model, to infer the performance of unseasoned and seasoned equity offerings stocks.

⁹ Appendix B describes the relationship between the geometric Brownian motion and the log-normal distribution.

A. Long-Run Performance of IPO and SEO Stocks

Empirically, unseasoned (IPO) and seasoned (SEO) equity offerings have been reported to under perform in the long run, and this under performance continues several years after the equity issues. In the United States, the long-run market adjusted returns are reported to be negative during the three- and five-year period after the issue (see Ritter (1991), Eckbo and Masulis (1995), Loughran and Ritter (1995), and Spiess and Affleck-Graves (1995)). Loughran, Ritter, and Rydqvist (1994) compare unseasoned equity offerings for 24 countries that demonstrate similar patterns, although some find positive long-run returns (see Loderer and Zimmermann (1988), Bøhren, Eckbo and Michalsen (1997), Bigelli (1997), and Dubois and Jeanneret (1998)).¹⁰ Lewis (1993) reports a three-year under performance in IPO stocks from the United Kingdom and the IPO firms experience an average 30 percent loss of value compared to the market. Lee, Taylor and Walter's (1996) result also shows that the market-adjusted cumulative average returns from Australian IPO stocks under perform over a three-year period.¹¹

Ritter (1991) and Spiess and Affleck-Graves (1995) state that the persistency of long-run returns after equity offerings is puzzling (this paper, however, suggest that the volatility is important in explaining the observed persistence of long-run returns). Ritter (1991) examines 1,526 IPO stocks and finds a negative 15.08 percent average cumulative matching firm-adjusted return after 36 months. Comparing returns with firms of similar size and industry, the average IPO stock's cumulative abnormal returns are negative 26 percent. Ritter (1991) argues that the result is consistent with investors being overoptimistic about potential growth firms. Loughran and Ritter (1995) examine the five-year long-run return for 4,753 IPO stocks and 3,702 SEO stocks in the United States during 1970-1990. The average long run IPO return adjusted for size and industry under performed with 20 percent after three years. While the SEO long-run returns under perform with 22 percent. Both studies calculate buy-and-hold returns and argue that the results show a deviation from the long-run efficient market theory. Spi-

¹⁰ For comprehensive surveys see Smith (1986), Eckbo and Masulis (1995), Ibbotson and Ritter (1995) for IPO stocks, and Eckbo and Masulis (1995) for SEO stocks. Other findings from non-US studies showing positive returns are Hietala and Löytyniemi (1991) for Finland, Dhatt, Kim and Mukherji (1996) for Korea, Tsangarakis (1996) for Greece, and Kang and Stulz (1996) for Japan.

¹¹ Corresponding results are found in Aggarwal et al. (1993) for Brazil and Chile, Keloharjo (1993) for Finland, Kunz and Aggarwal (1994) for Switzerland, and Page and Reyneke (1997) for South Africa.

ess and Affleck-Graves (1995) examine 1,246 seasoned equity offerings during the period 1975-1989 and find a negative abnormal return of 22 percent when compared to matching firms over a five-year post-event period. The results are persistent when adjusting for trading system, offer size, and the issuing firm's age and book-to-market value.

There are several similarities between IPO stocks and SEO stocks, e.g. transfer of ownership rights from existing to new shareholders. Presently, there exists no convincing theory that explains long-run under performance in unseasoned and seasoned equity offerings. The most promising explanation concerns an argument about investors' overreaction and over optimistic behavior to the information revealed through equity offerings (See Loughran and Ritter (1995) and Jakobsen and Sørensen (1999)). Spiess and Affleck-Graves (1995) examine the abilities of managers to take advantages and exploit overvaluation. They argue for an overvaluation exploitation opportunity for managers in SEO stocks if it is possible in IPO stocks, i.e. their results confirm that managers issue equity when the stock is overvalued because they are able to take advantage of firm-specific information. Comparing the under performance of IPO stocks and SEO stocks shows support for Spiess and Affleck-Graves (1995) result that managers, also, can exploit overvaluation opportunities in SEO stocks, i.e. it is not only the vast information asymmetry in IPO stocks that allow managers to exploit overvaluation.¹² Empirically, the findings in this paper complement Spiess and Affleck-Graves's (1995) result and the methodology supports the exploitation explanation. However, the task of future research is still to develop a consistent theory.

B. Data Material

We construct a sample by gathering information of unseasoned and seasoned equity offerings during the period 1983-1998. The data source is Account Data that contains information on equity offerings by firms that are listed on the Copenhagen Stock Ex-

¹² Two alternative and promising explanations are the theory of investor sentiment. Barberis, Shleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) explain long-run performance with behavioral model of investor sentiment. The behavior models provide an alternative explanation of investors' behavior but not a better explanation of the long-run anomalies.

change, and it identifies the type of equity offering and its characteristics.¹³ For the equity offering to be included in the sample it must meet the following three criteria: 1) the issue must be a primary offering, i.e. offerings that include preferred and secondary shares are excluded; 2) joint offerings are excluded; and 3) the offering firm must be listed on the Account Data's security price information at the time of the issue. In addition, we evaluate the pricing information to identify and exclude any equity offerings in which missing observations exist during the six months following the equity issue. Further, if the offering firm is de-listed prior to five years following the equity offering, the buy-and-hold returns of the firm and the matching index are truncated at the same time. For equity offerings after 1993, de-listing and buy-and-hold returns are less than five years because the last information gathered in Account Data ends in December 1998. Applying these criteria provides a sample of 142 unseasoned and 413 seasoned equity offerings. Table I provides the distribution of the total sample of IPO stocks and SEO stocks partition by years.

[INSERT TABLE I]

Table I shows an aggregate issue amount for IPO stocks of 6,604 million *DKK* and 17,908 million *DKK* for SEO stocks that yields proceeds of 23,849 million *DKK* and 46,717 million *DKK*, respectively. We find a large number of equity offerings in the early- and mid-eighties. The samples used in Ritter (1991) and Spiess and Affleck-Graves (1995) show similar characteristics. Unseasoned equity offerings experienced a large number of issues particularly in 1984, 1985, and 1986. Table I also shows that the unseasoned offerings are, on average, about as many as seasoned equity offerings in those years, while subsequently there are four to five times as many seasoned as unseasoned equity offerings. The number of SEO stocks is more evenly distributed than IPO stocks over the years 1983 to 1998 varying between 43 offerings in 1986 and 15 offerings in 1987. Of the 413 SEO stocks represented in our sample 242 firms made equity offerings more than once during the sample period.¹⁴ A total of 224 firms of the

¹³ Account Data is a database that contains information about all firms that are listed on the Copenhagen Stock Exchange. It contains annual reports and market information about each firm/security.

¹⁴ 107 firms made two, 61 made three issues; and 34 made four issues, 19 made five issues, 11 made six issues, 5 made seven issues, 3 made eight issues, 2 made nine issues.

sample did not make a second equity issue within a five-year post-offering period. Loughran and Ritter (1995) argue that the under performance in periods with few unseasoned equity offerings is modest, therefore, we partition the sample in sub periods of hot-issue periods and cold-issue periods to confirm whether or not this is evident in our sample. Likewise, we partition the sample by capitalization in order to identify whether under performance is concentrated around small capitalization offers.

To measure the long-run return of an equity offering stock, we calculate the buy-and-hold return from the first day of trading over a period of 60 months. Two benchmark returns of the post-offering period are considered: market-adjusted returns and market-model returns. To estimate the market-model returns, we use the post-offering period as the estimation window.¹⁵ The reason is that the seasoned equity offerings typically experience large pre-issue increases in stock prices and stable prices in the post-issue period (Lee et al., 1996). For the initial-public offering stocks, the lack of pre-event returns necessitates a post-offering estimation window of the market-model returns.

C. Long-Run Security Performance

Table II shows the average buy-and-hold returns of unseasoned (panel A) and seasoned (panel B) equity offerings using cross-sectional averages, wealth relatives, and transformed buy-and-hold returns.¹⁶ The average cross-sectional buy-and-hold returns show that the unseasoned and seasoned equity offerings constantly under perform the market when the benchmark is the market-adjusted return. Similar results are documented in Spiess and Affleck-Graves (1995) using average cumulative returns that are adjusted for size, industry and size, and book-to-market and size.

[INSERT TABLE II]

¹⁵ Estimating parameters prior to SEO stocks, as an alternative, provides results (not reported) that are similar to those of the using post parameters.

¹⁶ Ritter (1991) argues that the use of equally weighted monthly returns imply an increasing investment in poorly performing firms is avoided using independent monthly rebalancing. However, the long-run returns using this technique may be downwards biased. In addition, Lee et al. (1996) argue that this rebalancing assumption is in conflict with the problem concerning calendar time intervals, i.e. it is not possible to create a feasible investment strategy.

The results in table II need to be interpreted with caution because the distribution properties of the buy-and-hold returns are unknown. Therefore, it is important to investigate the distribution properties of buy-and-hold returns. The cross-sectional standard deviations are not sufficient statistics if the buy-and-hold returns are not symmetric distributed around the averages. Panel A shows that the average cross-sectional buy-and-hold return after 60 months is 2.5 percent for IPO stocks and 15.8 percent for the market index. After the first 12 months the IPO stocks outperform the market by 7.75 percent $(\frac{1.057}{0.981} - 1)$ while after 60 months the IPO firms under perform the market by 11.5 percent $(\frac{1.025}{1.158} - 1)$. This under performance is of the same magnitude as the findings in Spiess and Affleck-Graves (1995) and Loughran and Ritter (1995) using data from the United States. The 60 months buy-and-hold return from wealth relatives defined as the IPO stocks relative to the market return is negative 27.3 percent. However, for the revised wealth relative (the wealth relative of the market against the IPO stocks) the average buy-and-hold return is 152.6 percent. For the market model, the average transformed buy-and-hold abnormal return (*T-BHAR*) is negative 21.4 percent and negative 21.3 percent using abnormal returns $r_{it} - (\mathbf{a} + \mathbf{b}\mathbf{x}_{mt})$ and $(\mathbf{a} + \mathbf{b}\mathbf{x}_{mt}) - r_{it}$, respectively. Panel B shows that the average buy-and-hold return after 60 months is 6.1 percent for SEO stocks and 40.4 percent for the market index. The SEO stocks constantly under perform the market, e.g. after the first 12 months the SEO firms under perform the market by 5.4 percent and after 60 months the under performance is 22.3 percent. The 60 months buy-and-hold return from the wealth relative defined as the SEO stocks against the market return is negative 21.4 percent. However, for the revised wealth relative (the wealth relative of the market against the SEO stocks) the average buy-and-hold return is 128.4 percent. The average transformed buy-and-hold abnormal return (*T-BHAR*) is negative 18.9 percent and negative 17.6 percent using abnormal returns $r_{it} - (\mathbf{a} + \mathbf{b}\mathbf{x}_{mt})$ and $(\mathbf{a} + \mathbf{b}\mathbf{x}_{mt}) - r_{it}$, respectively. The results in table II shows that the expected mean buy-and-hold return depends on the method used to calculate the long-run returns. Overall, the results show that the buy-and-hold returns of the market vary less than the buy-and-hold returns of the IPO and SEO stocks. The buy-and-hold returns that are reported in table II are not applicable for testing the long-run performance of IPO and SEO stocks.

D. Results with Log-Normal Transformation

The differences in the average buy-and-hold return for the market-to-equity wealth relative and the equity-to-market wealth relative are due to the volatility component. Figure 1, panel A and panel C show the development in the expected mean buy-and-hold return of wealth relatives shown as the normalized measure: $e^{m_{y,T}^T} - 1$. (Table III summaries results for selected years.) Figure 1, panel B and panel D show the decomposition of the expected mean buy-and-hold return into the normalized transformed mean component, $e^{a_{j,T}^T} - 1$ and the normalized volatility component, $e^{\frac{1}{2}s_j^2 T} - 1$, where $J = \{i-m, m-i\}$, $i = \{IPO\ stocks, SEO\ stocks\}$ and $T = \{1, \dots, 60\}$.

[INSERT FIGURE 1]

Figure 1 illustrates that it is not possible to draw inference only based on expected mean buy-and-hold returns (panel A and panel C). A total of 10 percent of the IPO stocks' cross-sectional buy-and-hold returns are rejected as being log-normally distributed at a 5 percent level of significance, while 19 percent of the SEO stocks' cross-sectional buy-and-hold returns are rejected as being log-normally distributed at a 5 percent level of significance.¹⁷ Figure 1A provides the expected cross-sectional buy-and-hold returns and the decomposition of the wealth relatives related to the IPO stocks. Panel A and panel B in figure 1A shows the results of the IPO-to-market wealth relative. Panel A shows that the IPO stocks under perform relative to the market by 22.9 percent after five years. Using the volatility-adjusted performance measure (panel B), the IPO stocks under perform relative to the market by 43.7 percent after five years. Using the market-to-IPO wealth relative (panel C), the market out performs the IPO stocks by 142.9 percent. The difference between the result in table II and figure 1 arises from the transformation of the buy-and-hold returns into log-normality. The ratio of the two wealth relatives (142.9 to 152.6) expresses the accuracy of the

¹⁷ The test statistics are based on Doornik and Hansen (1994) that adjust for sample size. The problem is severe for the market model's estimates of buy-and-hold returns while the buy-and-hold returns of wealth relatives are acceptable log-normally distributed. Figures summarizing the probability level of normality and corresponding chi-squared statistics for wealth relatives in the spirit of Loughran and Ritter (1995) are shown in appendix C. Test statistics for distribution properties from the transformed buy-and-hold abnormal returns are available from the authors.

geometric Brownian motion ability to form an approximation of the average buy-and-hold returns. The volatility-adjusted performance of the market-to-IPO wealth relative shows that the market out performs the IPO stocks with 77.5 percent after five years (panel D). Comparing panel B and panel D confirms an identical volatility component for the IPO-to-market wealth relative and the market-to-IPO wealth relative.

Figure 1B provides the expected cross-sectional buy-and-hold returns and the decomposition of the wealth relatives related to the SEO stocks. Panel A and panel B in figure 1B shows the results of the SEO-to-market wealth relative. Panel A in figure 1B shows that the SEO stocks under perform relative to the market by 17.8 percent after five years. Panel B shows volatility-adjusted performance measure, the results show that the SEO stocks under perform by 38.1 percent after five years. Panel C in figure 1B shows the market-to-SEO wealth relative. It shows that the market index out performs SEO stocks by 114.4 percent after five years. Applying the volatility-adjusted performance measure, the market out performs the SEO stocks by 61.5 percent after five years.

A correct inference of the mean buy-and-hold returns is to account for the volatility component in the reported returns. Accounting for the volatility component shows that the mean buy-and-hold returns tend to under-estimate the under performance of equity offerings. For example, for IPO stocks the volatility-adjusted under performance is 43.7 percent compared to the average under performance, as reported in table II, of 27.3 percent. Similarly, for SEO stocks the volatility-adjusted under performance is 38.1 percent compared to an average under performance of 21.4 percent. Overall, figure 1 shows that the transformed mean component and the volatility component either amplify or weaken each other. Therefore, the volatility-adjusted performance depends on whether the transformed mean component is positive or negative. Next, we test the transformed mean component and the volatility component to evaluate whether the under performance or the over performance is significant.

E. Test of Expected Mean and Volatility Component

The test statistics are displayed graphically rather than numerically and are shown in figure 2. Figure 2: panel A and C show the transformed mean components, and panel B and D show the volatility components. Tests of the market relative to IPO stocks and SEO stocks show that the transformed mean component of the expected cross-sectional buy-and-hold returns is significantly different from zero at a 5% level of significance after 15 months and after 5 months, respectively. Figure 2 shows the marginal estimates for transformed mean components and volatility components and their respective 95-percent marginal confidence intervals. Any number outside the confidence intervals are significant against the marginal estimates.

[INSERT FIGURE 2]

The volatility component always contributes positively to the expected cross-sectional buy-and-hold returns. If the transformed mean component is zero and the volatility is positive, and constant, then the level average will increase with the time horizon. The reason is the positive gains accumulate more than the losses even if the transformed mean component is zero. This is the observed right skewness in the development of the cross-sectional buy-and-hold returns.

F. Categorized into Hot Issue Period versus Cold Issue Period

The unseasoned and seasoned equity offerings are partitioned by a hot (cold) issue period. For the IPO stocks the hot issue period 1983-1986 is investigated. High initial returns and high volumes characterize hot issue periods, i.e. an IPO stock rises above the offering price gaining a higher than average premium in the aftermarket (see Ibbotson and Jaffe (1975) and Loughran, Ritter and Rydqvist (1994)). IPO stocks on the Copenhagen Stock Exchange show a strong relationship between high initial returns and long-run performance in which high initial returns experience negative long-run returns after 5 years. Table III presents the results of hot issues for the period 1983-1986 and the cold issues for the period 1987-1993. A similar hot issue period from 1983-1986 is reported in Uhlir (1989) for the German IPO market.

[INSERT TABLE III]

Table III besides showing sub-sample periods also displays the different methods of calculating long-run returns. All three methods show that the market outperforms equity offerings after five years for the hot issue period. Method 1, which does not adjust for right-skewness, shows that the market outperforms IPO stocks by 172.9 percent, while the volatility-adjusted buy-and-hold return shows an over performance of 99.9 percent (method 3). For the cold issue period, using method 1, the market outperforms IPO stocks with 64.6 percent after five years while the volatility-adjusted out performance is 29.3 percent after five years. These reported out performance percentages are numbers that arise when the market is measured relative to the equity issues stocks. However, for the reverse wealth-relative measure (IPO-to-market) using method 1 the under performance in the hot issue period is only 36.4 percent after five years and for the cold issues period the under performance is only 9.0 percent. This may seem odd but it is caused by the fact that the volatility component is identical for the two wealth-relative measures: IPO-to-market and market-to-IPO. Adjusting for the volatility, i.e. using method 3, shows that hot (cold) issue period under perform by 50.0 percent (22.0 percent). The measures of volatility-adjusted performance are compatible irrespectively of whether out performance or under performance is considered.

Loughran and Ritter (1995) argue that the under performance in periods with few unseasoned equity offerings are modest which is confirmed in our cold issue period. Hot issue periods experience a more severe long-run under-performance than cold issue periods. Loughran, Ritter and Rydqvist (1994) also document the tendency of high initial returns and volumes after a period of high stock market performance. We investigate whether or not this inverse relationship exists between the one-year return and the five-years return. The results show that the under performance after one year for hot (cold) issue periods is 4.1 percent (5.9 percent) while the under performance after five years for hot (cold) issue periods is 50.0 percent (22.6 percent) using method 3. Thus, the results may indicate that investors that invest in initial public offerings in cold issue periods are better off in the long run than investors that invest in initial public offerings in a hot issue period.

All three methods in table III show under performance of the 142 IPO stocks and 413 SEO stocks for the total sample period. Method 3 shows that the IPO stocks and the SEO stocks significantly under perform the market by 43.7 percent and 38.1 percent using the volatility-adjusted performance measure. The results confirm the similarities between IPO stocks and SEO stocks long-run performance and that the IPO stocks under perform more relative to the market than SEO stocks.

G. Categorized by Market Capitalization

We partition the sample by the size of the equity offerings to investigate whether under performance is concentrated around small capitalization offers. This partition may reveal differences in observed long-run returns. Table IV shows the unseasoned equity offerings categorized by market capitalization and seasoned equity offerings categorized by the proceed-to-market value ratio.

[INSERT TABLE IV]

Panel A provides the results of the post-performance of IPO stocks using all three methods. Comparing the results of method 1 and method 2 provides some evidence that the lowest quartile under performs more than the highest quartile. Method 3 confirms that small capitalization issues under perform relative to large capitalization issues. The out performance by the market after five years for the highest quartile is not statistically significant at a 5 percent level. This indicates that under performance seems to be concentrated in small capitalization firms. Whereas, only little evidence indicates that the poor long-run performance of IPO stocks is concentrated in small capitalization stocks. In addition, the under performance after one year for three of the quartiles are not statistically significant at any level.

Panel B provides the results of the performance of SEO stocks group by the proceed-to-market value ratio. This ratio is chosen to capture the performance of equity offerings relative to existing market value. The result shows that the market outperforms SEO stocks after five years for all quartiles and the out performance is statistically sig-

nificant at a 1-percent critical level of significance. The sample provides no evidence of different under performance for the market capitalization quartiles. Method 3 shows that the volatility-adjusted buy-and-hold returns after five years vary between 42.3 percent (highest quartile) and 95.0 percent (third quartile). Spiess and Affleck-Graves (1995) find some evidence that smaller firm's experience more severe under performance when partitioning their sample by book-to-market value. We find no such evidence looking at the relative size of the Danish equity offering.

V. Conclusion

This paper shows that the volatility-adjusted performance measure is a better measure of long-run returns than just considering a simple arithmetic average. First, we identify transformations of data that exhibit log-normally distributed cross-sectional buy-and-hold returns. Secondly, we decompose the expected cross-sectional buy-and-hold returns into transformed mean components and volatility components. For the wealth relative measure, we document that it is necessary to correct the expected cross-sectional mean buy-and-hold return for the volatility component. We report that whether accumulated returns are calculated using the market-to-security or the security-to-market wealth relative, the volatility component introduces an identical upward bias. The expected buy-and-hold returns must be adjusted for the volatility component to show coherent security performance.

We investigate the long-run security performance made by 142 IPO stocks and 413 SEO stocks at the Copenhagen Stock Exchange during the period 1983-1998. The results document volatility-adjusted buy-and-hold returns that show that unseasoned and seasoned equity offerings under perform over a five-year period by 43.7 percent and 38.1 percent, respectively. The under performance is more evident for equity offerings in a hot issue period compared to a cold issue period. Moreover, there is little evidence that small capitalization issues under perform relative to large capitalization issues. We document that IPO stocks with large capitalization issues do not under perform in the

long run. Also, SEO stocks grouped by the proceed-to-market value ratio show no evidence of different under performance for various market capitalization quartiles.

Appendix A: Transformed Buy-and-Hold Returns

An alternative measure of long-run abnormal returns denoted by the transformed buy-and-hold return (T - $BHAR$) is:

$$T-BHAR_{i-m,T} = \prod_{t=1}^T (1 + ar_t) \quad (\text{A.1})$$

where the abnormal return is defined as $ar_t = r_{i,t} - r_{m,t}$. The usual measurement of cross-sectional buy-and-hold return is defined $BHAR_{i-m,T} = \left(\prod_{t=1}^T (1 + r_{i,t}) - \prod_{t=1}^T (1 + r_{m,t}) \right)$. The average cross-sectional T - $BHAR$ is:

$$\overline{T-BHAR}_{i-m,T} = \frac{1}{N} \sum_{i=1}^N \left(\prod_{t=1}^T (1 + ar_t^{i-m}) \right) = \frac{1}{N} \sum_{i=1}^N \left(e^{\sum_{t=1}^T \log(1 + ar_t^{i-m})} \right) \quad (\text{A.2})$$

Applying a geometric Brownian motion in continuous times:

$$d \log T-BHAR = \mathbf{m}_a dt + \mathbf{s}_a dz_t \quad (\text{A.3})$$

where

$$T-BHAR_{i-m,T} = e^{\mathbf{m}_a T + \mathbf{s}_a z_T} \quad (\text{A.4})$$

where $z_T \sim N(0, T)$. A serious problem arises when applying (A.4) if the abnormal return ar_t at any time is less than minus 1 because the logarithm cannot be taken on $(1 + ar_t)$. An explicit structure of the geometric Brownian motion can be enforced on the $BHAR$. But, $BHAR$ consists of a difference between two exponential expressions. Therefore, it does not assist in separating the mean component from the volatility component. Empirically, the various combinations $T-BHAR_{i-m,T}$ and $T-BHAR_{m-i,T}$ are equivalent to the wealth relatives with the only difference being the sizes of the numbers.

Appendix B: Decomposing Expected Mean Buy-and-Hold Returns

Define a wealth relative at time T by the variable W_T . If the logarithm of the wealth relative at time T is normally distributed with transformed mean $\mathbf{a}_T \cdot T$ and variance $\mathbf{s}_T^2 \cdot T$:

$$\log(W_T) \sim N(\mathbf{a}_T \cdot T ; \mathbf{s}_T^2 \cdot \sqrt{T}) \quad (\text{B.1})$$

then the wealth relative, W_T , is log-normally distributed: $W_T = e^{\log(W_T)}$. As W_T is log-normally distributed the expected mean of the wealth relative is given by:

$$\begin{aligned} E(W_T) &= e^{\mathbf{m}_T \cdot T} = e^{\mathbf{a}_T \cdot T + \frac{1}{2} \mathbf{s}_T^2 \cdot T} \\ &= \underbrace{e^{\mathbf{a}_T \cdot T}}_{\text{Transformed mean component}} \cdot \underbrace{e^{\frac{1}{2} \mathbf{s}_T^2 \cdot T}}_{\text{Volatility component}} \end{aligned} \quad (\text{B.2})$$

where the definition $\mathbf{m}_T \equiv \mathbf{a}_T + \frac{1}{2} \mathbf{s}_T^2$ is used. The variance of W_T is given by:

$$\begin{aligned} \text{Var}(W_T) &= e^{(2\mathbf{m}_T + \mathbf{s}_T^2) \cdot T} \cdot (e^{\mathbf{s}_T^2 \cdot T} - 1) \\ &= e^{(2(\mathbf{a}_T + \frac{1}{2} \mathbf{s}_T^2) + \mathbf{s}_T^2) \cdot T} \cdot (e^{\mathbf{s}_T^2 \cdot T} - 1) \end{aligned} \quad (\text{B.3})$$

Using the structure of a geometric Brownian motion with the dynamics

$dW_t = \mathbf{m}_t W_t dt + \mathbf{s}_t W_t dZ_t$ and applying Ito's lemma on $\log(W_t)$:

$$\begin{aligned} d \log(W_t) &= \frac{\partial \log(W_t)}{\partial t} dt + \frac{\partial \log(W_t)}{\partial W_t} dW_t + \frac{1}{2} \cdot \frac{\partial^2 \log(W_t)}{\partial W_t^2} (dW_t)^2 dt \\ &= \mathbf{m}_t dt + \mathbf{s}_t Z_t - \frac{1}{2} \mathbf{s}_t^2 dt \\ &= \left(\mathbf{m}_t - \frac{1}{2} \mathbf{s}_t^2 \right) dt + \mathbf{s}_t Z_t \end{aligned} \quad (\text{B.4})$$

where $\frac{\partial \log(W_T)}{\partial t} = 0$, $\frac{\partial \log(W_T)}{\partial W_T} = \frac{1}{W_T}$, $\frac{\partial^2 \log(W_T)}{\partial W_T^2} = -\frac{1}{W_T^2}$

This provides the mean component and the volatility component:

$$W_T = W_0 + \int_0^T \mathbf{m}_t W_t dt + \int_0^T \mathbf{s}_t W_t dZ_t$$

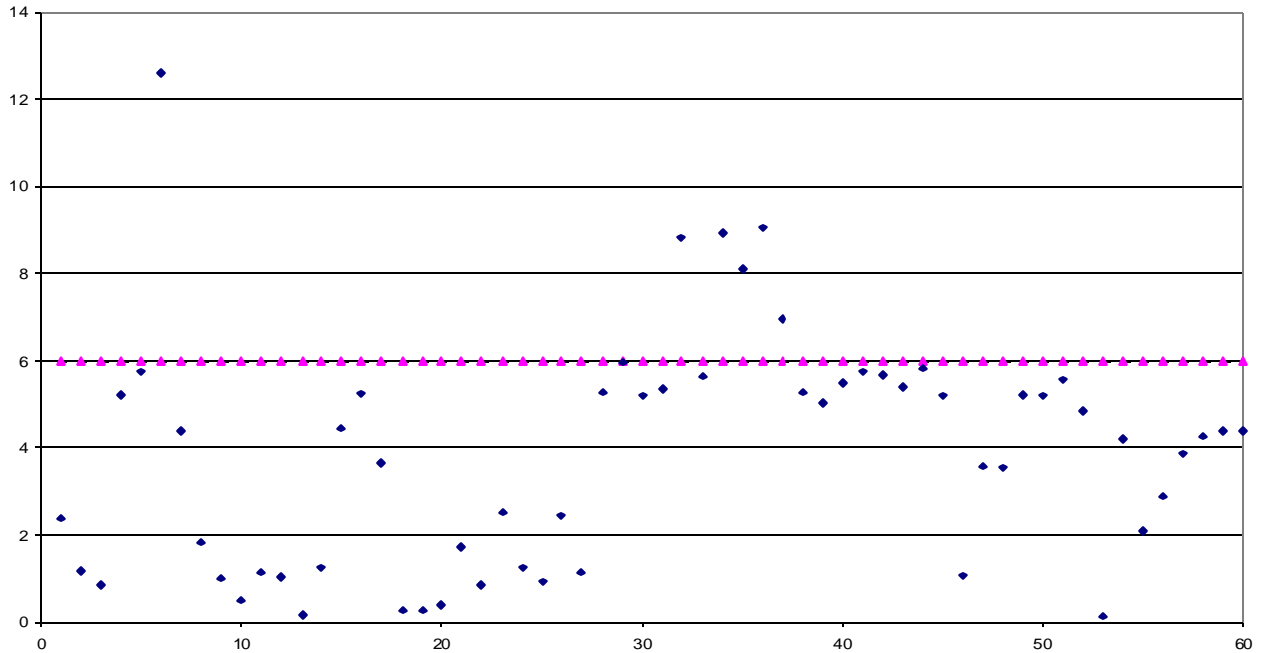
$$W_T = e^{(\mathbf{m}_T - \frac{1}{2} \mathbf{s}_T^2)T + \mathbf{s}_T \cdot Z_T} \quad (\text{B5})$$

where $\mathbf{m}_T \equiv \mathbf{a}_T + \frac{1}{2} \mathbf{s}_T^2$ is the drift and \mathbf{a}_T and \mathbf{s}_T^2 are the marginal parameter estimates at any point in time. At time T the transformed mean component and the volatility component of $\log(W_T)$ is $\mathbf{a}_T \cdot T$ and $\mathbf{s}_T^2 \cdot \sqrt{T}$.

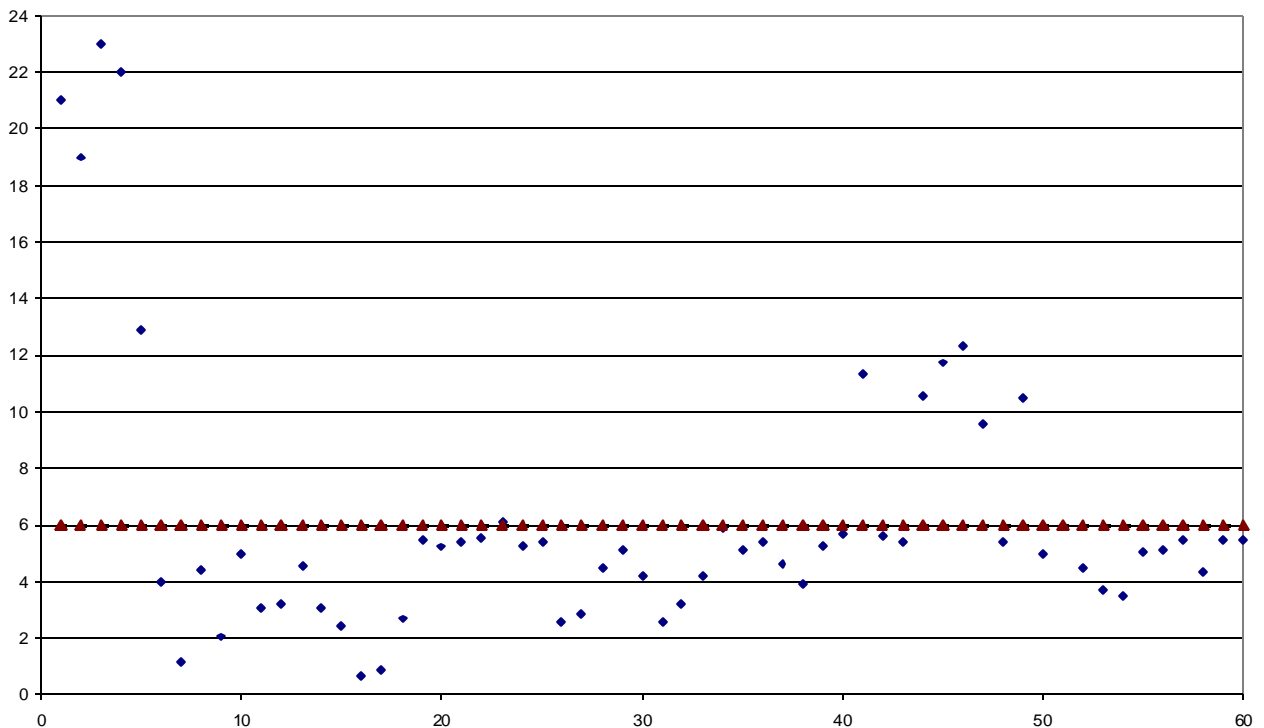
Appendix C Test Statistics for Normality

The test statistics that adjusts for sample size are based on Doornik & Hansen (1994). Panel A and B summarize the probability level of normality and corresponding chi-squared statistics for wealth relatives in the spirit of Ritter (1991) and Loughran & Ritter (1995). The horizontal line shows the 5 percent level of significance for being log-normally distributed.

Panel A Chi-squared of normality for wealth relatives of IPO's



Panel B Chi-squared of normality for wealth relatives of SEO's



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Table I Distribution of Equity Offerings on the Copenhagen Stock Exchange

We construct the sample by gathering information of unseasoned and seasoned equity offerings during the period 1983-1998. The data source is Account Data for identifying security offerings and offerings characteristics. The total sample consists of all equity offerings on the Copenhagen Stock Exchange. For an equity offering to be included in the sample it must meet the following three criteria: 1) the issue must be a primary offering, i.e. offerings that include secondary shares are excluded; 2) joint offerings are excluded; and 3) the offering firm must be listed on the Account Data's database of security price information at the time of the issue. In addition, we evaluate the pricing information to identify and exclude any equity offerings in which missing observations exists during the six months following the offering. Offering is the nominal equity issue and proceed is the market capitalization of the issue.

Year	Unseasoned Equity Offerings			Seasoned Equity Offerings		
	Number	Offering ^a	Proceed ^a	Number	Offering ^a	Proceed ^a
1983	7	63	220	24	533	779
1984	23	298	786	27	893	1,731
1985	12	547	3,882	25	955	1,603
1986	23	381	845	43	1,389	3,453
1987	4	21	59	15	619	1,353
1988	3	252	325	24	364	1,283
1989	9	1,160	1,276	32	1,280	3,727
1990	12	731	1,582	24	1,123	3,579
1991	8	682	1,646	31	2,674	10,258
1992	4	257	292	27	2,639	4,569
1993	4	503	1,732	16	159	287
1994	6	1,152	3,007	26	2,517	6,112
1995	10	207	2,344	27	613	2,164
1996	6	273	4,778	23	1,122	2,167
1997	4	17	234	33	753	2,808
1998	7	59	840	16	276	846
Total	142	6,604	23,849	413	17,908	46,717

^a Amount issued and proceeds in million DKK.

Table II Long-Run Returns for Unseasoned and Seasoned Equity Offerings

The time horizon is shown in the first column. The buy-and-hold returns are calculated assuming that an investor invests in an equally weighted portfolio of firms at the day of the equity offering. The wealth relatives are used to calculate buy-and-hold returns, $W_{i,t} = W_{0,t} \cdot \prod_{t=1}^T (1 + r_{i,t})$. Applied wealth relatives are market-to-equity offering ($W_{m,T}/W_{i,T}$) and equity offering-to-market ($W_{i,T}/W_{m,T}$), respectively. Two versions of the market model are applied, the standard method where the abnormal return (1) is $MAR_{it} = r_{it} - (\mathbf{a} + \mathbf{b}\mathbf{x}_{mt})$, but also by subtracting the firm-specific return from the market return (2) $MAR_{it} = (\mathbf{a} + \mathbf{b}\mathbf{x}_{mt}) - r_{it}$. The MAR_{it} is used to calculate the transformed buy-and-hold abnormal return (T -BHAR). Standard deviations are shown in parentheses.

Panel A: Unseasoned Equity Offerings

Months	N	IPO's	Market	Wealth Relatives		T -BHAR	
		$W_{i,t}-1$	$W_{m,t}-1$	$(W_{m,T}/W_{i,T})-1$	$(W_{i,T}/W_{m,T})-1$	$\prod_{t=1}^T (MAR_{m-i,t})^{-1}$	$\prod_{t=1}^T (MAR_{i-m,t})^{-1}$
12	142	0.057 (0.44)	-0.019 (0.18)	0.121 (0.45)	0.002 (0.36)	0.063 (0.31)	-0.062 (0.28)
24	142	0.089 (0.69)	0.013 (0.27)	0.335 (0.83)	-0.024 (0.52)	0.075 (0.46)	-0.097 (0.33)
36	142	0.025 (0.88)	0.034 (0.32)	1.008 (1.99)	-0.157 (0.58)	-0.051 (0.41)	-0.106 (0.39)
48	142	0.013 (0.78)	0.129 (0.32)	1.133 (2.57)	-0.201 (0.56)	-0.113 (0.34)	-0.169 (0.24)
60	142	0.025 (0.81)	0.158 (0.36)	1.526 (2.89)	-0.273 (0.53)	-0.213 (0.14)	-0.214 (0.15)

Panel B: Seasoned Equity Offerings

Months	N	SEO's	Market	Wealth Relatives		T -BHAR	
		$W_{i,t}-1$	$W_{m,t}-1$	$(W_{m,T}/W_{i,T})-1$	$(W_{i,T}/W_{m,T})-1$	$\prod_{t=1}^T (MAR_{m-i,t})^{-1}$	$\prod_{t=1}^T (MAR_{i-m,t})^{-1}$
12	413	0.019 (0.35)	0.077 (0.21)	0.161 (0.49)	-0.038 (0.32)	0.039 (0.28)	-0.052 (0.27)
24	413	0.051 (0.51)	0.163 (0.29)	0.404 (1.07)	-0.069 (0.46)	0.032 (0.36)	-0.075 (0.34)
36	413	0.065 (0.59)	0.253 (0.34)	0.621 (1.31)	-0.121 (0.50)	-0.013 (0.36)	-0.109 (0.33)
48	413	0.060 (0.60)	0.253 (0.35)	0.781 (1.64)	-0.157 (0.49)	-0.066 (0.30)	-0.175 (0.24)
60	413	0.061 (0.69)	0.404 (0.38)	1.284 (3.16)	-0.214 (0.55)	-0.176 (0.15)	-0.189 (0.15)

Table III Test of Long-Run Returns for Unseasoned Equity Hot and Cold Issues and Seasoned Equity Offerings

The unseasoned equity offerings are partitioned into a hot issue group from 1983-1986 and a cold issue group from 1987-1993. The period 1983-1998 long run returns for all IPO's and SEO's are reported. Method 1 is a simple arithmetic average calculation after T years, which does not adjust for right skewness. Applied wealth relatives are market-to-equity offering ($W_{m,T}/W_{i,T}$) and equity offering-to-market ($W_{i,T}/W_{m,T}$), respectively. Method 2 describes the buy-and-hold returns with expected mean of $e^{(a_T + \frac{1}{2}s^2_T)T}$. The method includes the problem of testing the cross-sectional average buy-and-hold returns in levels. Method 3 transforms the wealth relatives to logarithms and tests the transformed mean component and the volatility component. The reported figure is $e^{a_T T} - 1$ while the volatility component, $e^{\frac{1}{2}s^2_T T} - 1$, is filtered out. The volatility is shown in {x}. Standard deviations are provided in (x) for method 1 and 2. The time horizon (T) is 1 year, 3 years, and 5 years after the first day of trading.

Test	Wealth Relative	Unseasoned Equity Offerings									Seasoned Equity Offerings		
		Hot Issue 1983 - 1986			Cold Issue 1987 - 1993			Period 1983 - 1998			Period 1983 - 1998		
		1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Method 1	$(W_{m,T}/W_{i,T})$	10.68 (49.59)	76.44 (123.18)	172.88 (293.64)	11.13 (43.28)	105.83 (274.10)	64.56 (177.41)	12.14 (44.97)	100.79 (198.83)	152.56 (289.11)	16.06 (49.26)	62.09 (131.08)	128.35 (315.89)
	$(W_{i,T}/W_{m,T})$	1.01 (37.57)	-15.85 (63.97)	-36.43 (49.27)	-1.76 (37.63)	-12.16 (53.60)	-9.03 (56.46)	-0.21 (36.21)	-15.76 (57.50)	-27.29 (53.50)	-3.85 (31.98)	-12.09 (49.57)	-21.36 (54.51)
	$(W_{m,T}/W_{i,T})$	11.86 (43.56)	79.88 (131.74)	167.14 (236.80)	13.82 (43.60)	87.21 (175.20)	63.81 (127.50)	12.74 (42.33)	95.09 (165.58)	142.88 (226.99)	15.71 (39.82)	59.09 (103.91)	114.44 (187.24)
Method 2	$(W_{i,T}/W_{m,T})$	2.95 (40.09)	-14.59 (62.55)	-33.15 (59.26)	0.75 (38.60)	0.19 (93.76)	-1.97 (76.29)	1.19 (37.98)	-11.82 (74.84)	-22.86 (72.09)	-3.34 (39.81)	-10.32 (103.913)	-17.81 (187.24)
	$(W_{m,T}/W_{i,T})$	4.24 {0.19}	45.12 ^a {23.95}	99.90 ^a {33.63}	6.29 {7.09}	36.69 ^b {36.96}	29.27 ^b {26.72}	5.55 ^b {6.81}	48.74 ^a {31.16}	77.45 ^a {36.87}	9.41 ^a {5.76} ^a	33.18 ^a {19.44}	61.53 ^a {32.75}
	$(W_{i,T}/W_{m,T})$	-4.07 {0.19}	-31.09 {23.95}	-49.98 ^a {33.63} ^a	-5.92 {7.09}	-26.84 ^b {36.96}	-22.64 ^b {26.72}	-5.25 ^a {6.81}	-32.77 ^a {31.16}	-43.65 ^a {36.87}	-8.60 ^a {5.76}	-24.91 ^a {19.44}	-38.09 ^a {32.75}

^{a,b} Statistical significance at 1 % and 5 % level, respectively.

Table IV IPO's and SEO's Partition Market Capitalization Quartiles

The market capitalization for IPO's is used to generate four quartiles. The ratio of market capitalization of proceeds to the market value is used to generate four quartiles for SEO's. Method 1 is a simple arithmetic average calculation after T years, which does not adjust for right skewness. The applied wealth relative is market-to-equity offering ($W_{m,T}/W_{i,T}$). Method 2 describes the buy-and-hold returns with the expected value $e^{(a_T + \frac{1}{2}s^2T)}$. Method 3 transforms the wealth relatives to logarithms and tests the transformed mean component and the volatility component. The reported figure is $e^{a_T T} - 1$ while the volatility component, $e^{\frac{1}{2}s^2 T} - 1$, is filtered out. Standard deviations are provided in (x) for method 1 and 2. The probability (p-values) of values being equal to zero are provided in {x}. The time horizon (T) is 1 year, 3 years, and 5 years after the first day of trading.

Panel A: Unseasoned Equity Offerings

Quartile	Method 1			Method 2			Method 3		
	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Lowest	4.14 (43.92)	91.61 (178.84)	165.38 (364.08)	7.21 (40.85)	94.23 (174.10)	151.21 (246.63)	0.18 {0.49}	44.63 {0.01} ^a	79.26 {0.00} ^a
2	8.91 (52.45)	72.24 (125.56)	162.70 (237.25)	10.98 (42.57)	75.37 (124.96)	161.52 (180.45)	3.63 {0.29}	42.83 {0.00} ^a	115.25 {0.00} ^a
3	15.56 (42.73)	155.74 (313.83)	51.70 (98.99)	21.55 (45.16)	149.59 (255.47)	65.05 (98.13)	13.94 {0.03} ^b	74.42 {0.00} ^a	41.87 {0.00} ^a
Highest	4.25 (40.01)	42.96 (102.59)	126.93 (257.29)	10.18 (41.76)	61.09 (115.51)	146.74 (326.94)	3.03 {0.33}	30.91 {0.03} ^b	48.63 {0.06}

Panel B: Seasoned Equity Offerings

Quartile	Method 1			Method 2			Method 3		
	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Lowest	16.01 (43.92)	47.18 (130.56)	101.58 (175.87)	16.63 (37.93)	45.48 (95.97)	102.29 (168.06)	10.91 {0.00} ^a	21.44 {0.00} ^a	55.60 {0.00} ^a
2	10.83 (36.54)	51.61 (137.05)	88.60 (165.41)	11.95 (33.03)	51.40 (93.09)	93.84 (145.73)	7.38 {0.01} ^a	28.98 {0.00} ^a	54.94 {0.00} ^a
3	19.96 (71.17)	78.64 (141.82)	199.87 (517.17)	19.84 (51.66)	78.38 (123.55)	166.84 (249.15)	10.05 {0.01} ^b	46.64 {0.00} ^a	95.03 {0.00} ^a
Highest	14.02 (38.76)	59.26 (112.19)	98.56 (219.81)	14.83 (36.82)	60.12 (102.23)	94.39 (180.92)	9.34 {0.01} ^a	34.95 {0.00} ^a	42.29 {0.00}

^{a,b} Statistical significance at 1 % and 5 % level, respectively.

Figure 1A. The Expected Mean and Volatility in Unseasoned Equity Offerings

The development in expected mean and volatility are shown for the average buy-and-hold return. The development in average buy-and-hold returns is shown in Panel A and C for wealth relative market-to-equity offering ($W_{m,T}/W_{i,T}$) and equity offering-to-market ($W_{i,T}/W_{m,T}$), respectively. In panels B and D, the cross-sectional average buy-and-hold return are decomposed in a transformed mean component, $e^{a_T T} - 1$, and in a volatility component, $e^{\frac{1}{2}s_T^2 T} - 1$. The volatility component is independent of whether wealth relative ($W_{m,T}/W_{i,T}$) and ($W_{i,T}/W_{m,T}$) are used, and the volatility component has a positive influence on the average long-horizon wealth relative performance. The transformed mean component of the buy-and-hold return depends on the used transformation of wealth relatives.

Figure 2A – Panel A
Expected BHR of Wealth Relative
IPO versus Market

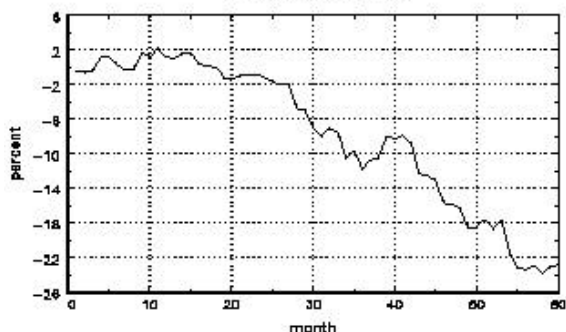


Figure 2A – Panel B
Mean and volatility components
IPO versus Market

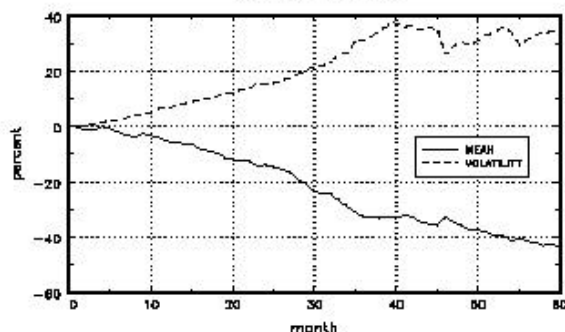


Figure 2A – Panel C
Expected BHR of Wealth Relative
Market versus IPO

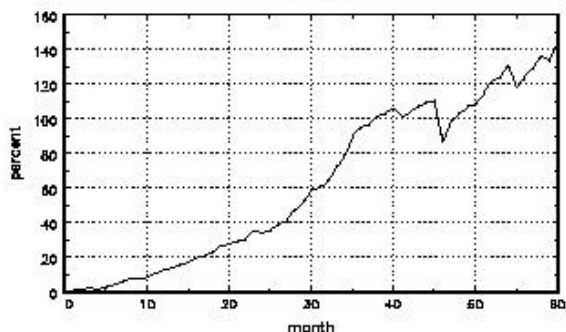


Figure 2A – Panel D
Mean and volatility components
Market versus IPO

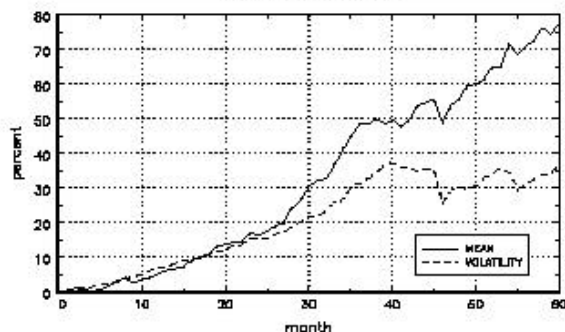


Figure 1B. The Expected Mean and Volatility in Seasoned Equity Offerings.

The development in expected mean and volatility are shown for the average buy-and-hold return. The development in average buy-and-hold returns is shown in Panel A and C for wealth relative market-to-equity offering ($W_{m,T}/W_{i,T}$) and equity offering-to-market ($W_{i,T}/W_{m,T}$), respectively. In panels B and D, the cross-sectional average buy-and-hold return are decomposed in a transformed mean component, $e^{a_T T} - 1$, and in a volatility component, $e^{\frac{1}{2}s_T^2 T} - 1$. The volatility component is independent of whether wealth relative ($W_{m,T}/W_{i,T}$) and ($W_{i,T}/W_{m,T}$) are used, and the volatility component has a positive influence on the average long-horizon wealth relative performance. The transformed mean component of the buy-and-hold return depends on the used transformation of wealth relatives.

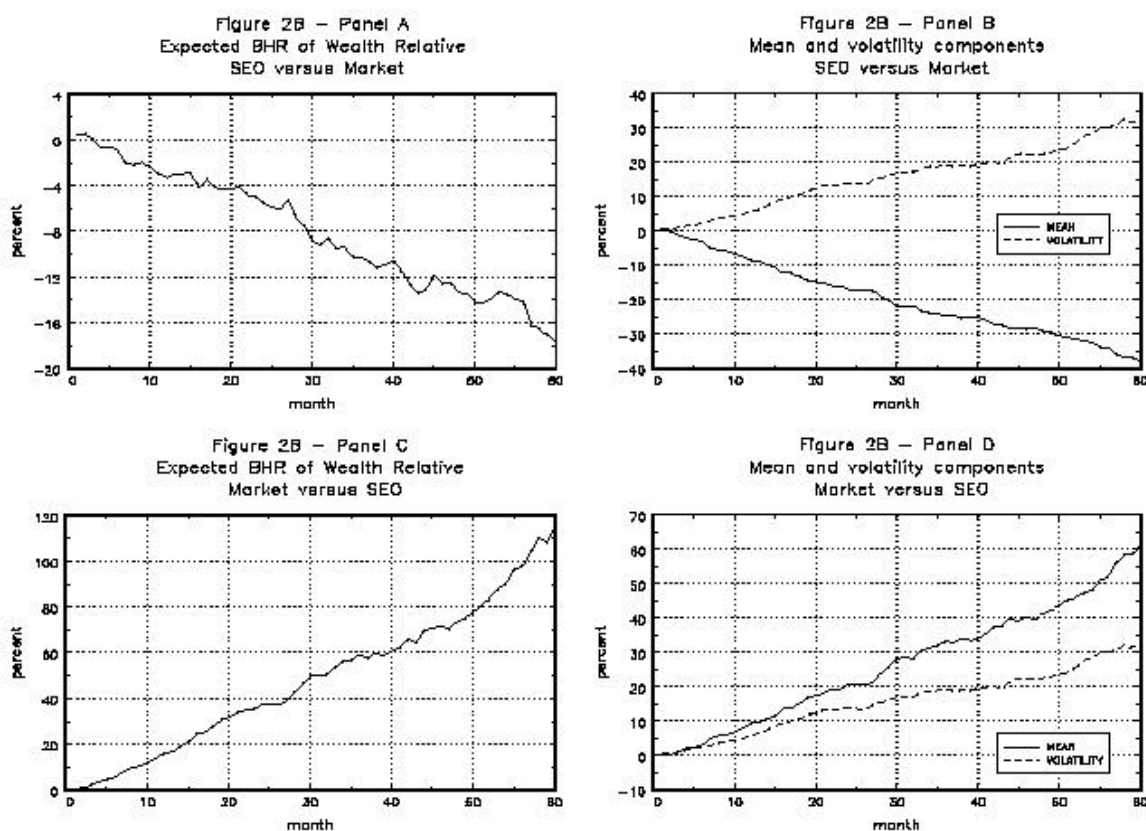


Figure 2 Test of the Volatility-Adjusted Security Performance

We test the maximum likelihood estimate $\mathbf{a}_T T$. The test is $H_0: \mathbf{a}_T T = 0$, $H_1: \mathbf{a}_T T \neq 0$ where $T = \{1, \dots, 60\}$. The maximum likelihood estimate $\mathbf{a}_T T$ is shown as 95 percent marginal confidence intervals. The transformed mean component $e^{\mathbf{a}_T T} - 1$ for the wealth relative $(W_{i,T}/W_{m,T})$ and its confidence intervals are shown in panel A and C, respectively. The volatility component $e^{\frac{1}{2}S_T^2 T} - 1$ and its confidence intervals are shown in panel B. The volatility component is the same whether the wealth relative $(W_{m,T}/W_{i,T})$ or $(W_{i,T}/W_{m,T})$ is used and therefore it can be considered with both panel A and C.

Figure 3 – Panel A: Mean component
IPO versus Market

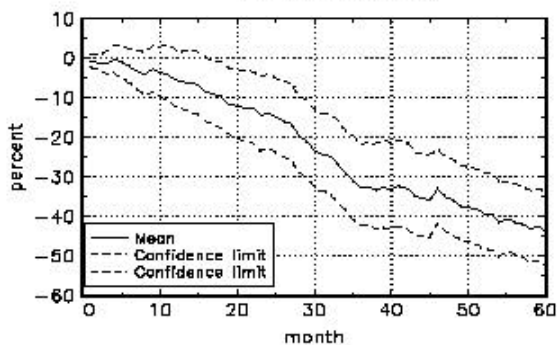


Figure 3 – Panel B: Volatility component
IPO versus Market

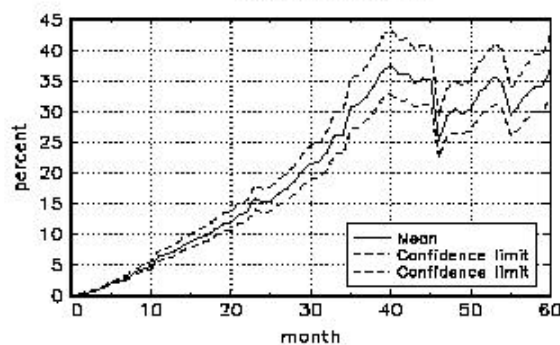


Figure 3 – Panel C: Mean component
SEO versus Market

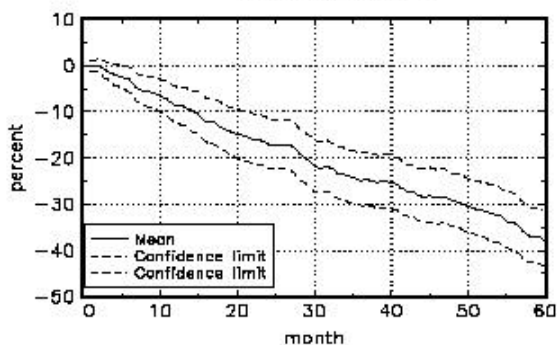


Figure 3 – Panel D: Volatility component
SEO versus Market

