A component-based analysis of the Danish long-run money demand relation

la Cour, Lisbeth Funding

Institut for Nationaløkonomi

Handelshøjskolen i København

Working paper 6-99

A COMPONENT-BASED ANALYSIS OF THE DANISH LONG-RUN MONEY DEMAND RELATION

Lisbeth Funding la Cour

Department of Economics - Copenhagen Business School
Nansensgade 19, 5. DK - 1366 København K.

A Component-based Analysis of the Danish Long-run Money Demand Relation.

Lisbeth Funding la Cour

Department of Economics,
Copenhagen Business School
Nansensgade 19, 5th
DK1366 Copenhagen K
Denmark
Phone: +45 38152488 (direct line)
Fax: +45 38152576
E-mail: llc.eco@cbs.dk

Abstract:
This paper examines the relation between monetary asset components and some of the variables that traditionally enter into aggregate money demand relations. This is done for Danish data within the natural framework of a multivariate econometric model. The purpose of the study is to investigate issues in relation to the level and weighting of a monetary aggregate. We show that within this model it is possible to identify monetary aggregates at different levels of aggregation and that for the narrow aggregate (M1) equal weighting of the components are permitted. For the broader aggregates, (M2) or (M2-M1), equal weighting is no longer appropriate. These findings are not contradictory to what we would expect from aggregation and index number theory. Finally, the overall identification of the stationary (or long-run) structure points towards a co-existence of a liquid and a less liquid money demand relation which is interpreted as an indication of the possibility of splitting the total money demand relation with respect to different motives for holding money.

I would like to thank K. Juselius, H. Hansen and H.C. Kongsted from the Department of Economics, University of Copenhagen for valuable discussions.

A Component-based Analysis of the Danish Long-run Money Demand Relation

1 Introduction

Money has traditionally been an important variable in the theoretical description of the link between the financial and the real sectors of the economy. Unfortunately, the theoretical concept of money, i.e. monetary services, is difficult to quantify making the implementation of monetary policy based on empirical studies of the link difficult, if not impossible. Any inability to find a stable aggregate demand for money relation might then be due to the measurement problem as well as to an improper formulation of the statistical model of the data, or the even more fundamental failure of the theory model to describe the real world phenomenon of interest - a discussion in the spirit of the famous Haavelmo(1944)-monograph. In the past few decades severe stability problems have been encountered within the traditional short-run demand for narrow money equations in many countries, see Laidler (1985) and Judd & Scadding (1982) for surveys, coinciding with the growing turbulence in the financial environment i.e. financial innovations, capital liberalizations etc., see e.g. Bordo & Jonung (1990). The traditional way to start in these studies is to choose a measure of money (sum- or Divisia-index, broad or narrow) and proceed analysing the correlation between this measure and variables representing the price level and the real side of the economy by applying more or less sophisticated econometric tools, see e.g. Fase (1993) for an attempt of making a synthesis of about 400 empirical money demand studies covering the G7 and the EC (now EU) countries. None of these studies seem to provide a unique solution to the problem - not that improvements have not been made but in the sense that no general agreement on an improved understanding of the link has been reached. This study is based on the idea that it may be useful to look at the problem the other way around: By formulating and analysing a proper multivariate statistical model of a data set containing components of monetary assets together with price level, income etc., it may be possible to learn something about the problem of interest, i.e. if a relation between these variables interpretable as a long-run money demand relation is found it may reveal new insight into the financial versus the real sector linkage. Furthermore, such a relation will provide information concerning the measurement discussion as well. Back in the late sixties attempts of making databased weighting of monetary aggregates can be found in e.g. Timberlake & Fortson (1967) and Laumas (1968), Both studies are based on univariate multiple regression approaches.

Later the single equation estimation of monetary aggregates was followed up by Clements & Nguyen (1980). A more recent study is found in Spanos (1984), but his approach is still basically univariate in nature although the econometric setup is more sophisticated using a state space model and the Kalman filter estimation technique. Hence, the natural extension along this line is to adopt a proper multivariate framework as the basis of the analysis and this is exactly the strategy followed in the present paper. From this point of view the purposes of the present study are clearly of a methodological nature: It discusses what kind of issues it is possible to analyse within a multivariate money demand system, i.e. with respect to index and aggregation theory and with respect to the possibility of identifying long-run money demand relations. Barnett (1990) criticises the whole idea of estimating monetary aggregates. He argues that the theoretical guidelines concerning the construction of monetary aggregates in principle are clear so that other attempts must necessarily be arbitrary in nature. His arguments are of course to some extent valid, yet considering the quite restrictive assumptions needed to derive the microeconomic basis of aggregation, there seems to be plenty of room left to justify alternative approaches e.g. along the lines adopted by the present study.
Indeed, any study that potentially might shed new light on the complicated issues of monetary transmission in general and specifically on the measurement problem connected with liquidity, does seem valuable.

In section 2 both the theory- and the econometric model will be presented and in section 3 the data set is described. Section 4 contains the empirical analysis and section 5 will conclude the paper.

2 Theoretical background

2.1 The theory model

The theory model of the analysis is quite simple. It focuses primarily on the more liquid part of the aggregate money demand relation, hence uses the variables that enter the well-known quantity-equation (see Laidler (1985,1990)):

$$M^D = M^D(P, Y, r_A, r), \qquad (1)$$

where M^D is aggregate money, P is the general price level, Y is income (or the amount of transactions), r_A is an alternative rate of return and r is the own-rate-of-return on money. The econometric model is log-linear (except in the interest rates that enter untransformed) and will be used as the specification of the functional form. As the official monetary aggregates in their broad form include a lot of assets that cannot directly be considered as liquid either quite a narrow aggregate can be used to represent the money-variable of the model, or a weighted (e.g. a Divisia[1]) aggregate can be used. In this study the components will enter disaggregatedly i.e. the M^D-variable of (1) will not be represented by a single time series in the data vector, rather a number (necessarily limited) of component series will take the place of M^D in order to see whether the data can give us any feedback on the proper level and method of aggregation.

2.2 The econometric model

As mentioned in the introduction the data analysis will be based on a multivariate econometric model: The Johansen VAR-model for analysing integrated time series. A formal definition of the time series concepts used will not be given here but can be found in e.g. Johansen (1992a,b). The basic model is the p-dimensional Vector-AutoRegression (VAR) of order k:

$$Z_t = \Pi_1 Z_{t-1} + \ldots + \Pi_k Z_{t-k} + \mu + \varphi D_t + \varepsilon_t, \qquad t = 1,\ldots,n$$

where Z_t is the p-dimensional data vector at time t, the Π_i's are matrices of coefficients, μ is a vector of constants (allowing for both linear trends and intercepts in the model), D is a set of centred seasonal dummies (or other deterministic variables) with corresponding

---

1 The idea of Divisia-monetary aggregates originates back to Barnett (1980, 1987) and refers to the use of the Törnquist discrete approximation to the continuous Divisia index as the method of aggregation. This formula is known (see Diewert (1976,1981)) to possess some desirable approximation properties in relation to economic index numbers. The index formula is written:

$$\log M_t^D - \log M_{t-1}^D = \sum_{i=1}^{n} s_{it}^* \left(\log m_{it} - \log m_{i(t-1)}\right),$$

$$s_{it}^* = \tfrac{1}{2}(s_{it-1} + s_{it}), \quad s_{it} = \frac{\pi_{it} m_{it}}{\sum_{i=1}^{n} \pi_{it} m_{it}}, \quad \pi_{it} = R_t - r_{it}$$

where t refers to the dating of the variables, s_{it}^*'s are weights and m are the components (of which there are n) to be aggregated. The weights suggested by Barnett can be viewed as a kind of value shares where the price of holding money, the π_i's, basically is an expression of interest differentials between the rate of return of an illiquid asset, R_t, and the own rate of return on the monetary asset components, the r_{it}'s. Hence the index makes %-changes in the aggregate a weighted sum of %-changes in the components.

coefficients φ and ϵ is the residual term assumed to be niid($0,\Omega_\epsilon$). A simple reparametrization of the model is:

$$\Delta Z_t = \Pi Z_{t-1} + \Gamma_1 \Delta Z_{t-1} + \ldots + \Gamma_{k-1} \Delta Z_{t-k+1} + \mu + \varphi D_t + \epsilon_t, \qquad t=1,\ldots,n$$

where Δ is the difference-operator. Cointegration among the series of Z means that the Π matrix has reduced rank: $\Pi = \alpha\beta'$, where α and β are (pxr)-matrices of full column rank, $r<p$ (see Johansen (1995a)). The β-coefficients are interesting from an economic point of view as they describe the long-run (or stationary) relations of the system. Furthermore the β coefficients of the monetary asset components that enter into a long-run money demand relation are interpretable as the weights of the monetary aggregate of that relation. Following the Johansen-procedure the unknown rank of Π can be derived as a result of a statistical test, the so called trace-test (see Johansen & Juselius (1990)). The test is fundamentally a likelihood-ratio test but with a non-standard asymptotic distribution. Because of the multiplicativity of the model originating from the reduced rank of Π it is in fact the space spanned by the β-vectors that is estimated and an economic identification[2] of the individual relations would be needed. For this purpose it is possible to impose linear restrictions on each of the β vectors and to test such restrictions by likelihood ratio tests as suggested by Johansen & Juselius (1992) or Johansen (1995b). These tests are shown to follow χ^2-distributions asymptotically. The number of degrees of freedom (hereafter: df) for these tests is in general found by counting the total number of restrictions on the structure (including the normalizing ones) and then subtracting the number r^2 - the number of restrictions that we can impose for free due to the multiplicativity of the original cointegration restrictions on the Π-matrix. The use of these type of tests as part of the identification procedure is very important in a basically reduced form econometric model.

2 For more on identification see Johansen & Juselius (1994).

3 The data

In order to keep the dimension of the multivariate data vector manageable, only 5 categories of monetary assets are used and M^D from (1) is represented by the following components:

1: Non-interest bearing assets (CURR)
2: Demand deposits at commercial banks and saving banks (DD)
3: Time deposits at commercial banks and saving banks (TD)
4: Special term deposits at commercial banks and saving banks (STD)
5: Treasury bills (TB)

By summation of these components an aggregate at the same level as the official Danish M2 (definition corresponding to the one used at the end of the sample) is obtained. This aggregate is more in line with what is called an M3-aggregate in some countries. (For the sample period studied here no official M3 monetary aggregate for Denmark existed).

In order to construct an aggregate measuring the amount of liquidity in the economy, a natural assumption is that the result of an empirical weighting of the components will to some extent rank the groups in such a way that the highest weights are given to the most liquid groups. This is a feature to be checked as the analysis proceeds.
The own-rates-of-return corresponding to the balance figures are measured as ex post effective average interest rates. Including the macro series of the price level (the deflator of total internal demand), income (total internal demand) and the average bond rate to the data vector consisting of the group balances and interest rate series makes a full data vector of 12 series. This gives rise to a problem due to the rather short sample period (from 2nd quarter 1976 to 4th quarter 1989[3]): There are too few df left for estimation of the system. With the lag-order k=2 the df in each of the 12 equations is 52(number of obs.)-12(lag 1 coeff.'s)-12(lag 2 coeff.)-1(constant)-3 (seasonal dummies)-6(approximately the average number of covariance-parameters of each equation[4])=18.
Therefore the strategy of modelling a 12-dimensional system has been abandoned. Instead a more modest approach in which the own rates of return and the benchmark rate are left out and only an aggregate measure of the opportunity costs of holding money is included

3 It is not possible to make the sample up-to-date because the tables on the monetary component data are not produced any more.

4 Strictly speaking this number ought to be $(12\times(12+1))/2)/12$.

(this opportunity cost measure is called user and is constructed as the average bond rate minus the average deposit rate) has been adopted. The data to be used in the rest of the paper are listed in Appendix A.

4 The modelling procedure: Strategy and results

Using a data vector that consists of the variables just mentioned (curr, dd, td, std, tb, pytr, fytr, user) makes an 8-dimensional system the basis of the data analysis. All series except for the user cost term are logarithmically transformed (graphs of the data are found in Appendix A). Due to certain capital liberalizations in the beginning of 1983 a shift dummy is also included in the model. The dummy is zero until 1982 4th quarter and 1 from 1983 1st quarter and onwards allowing a mean shift in the long-run relations and a change in the trend of the level of the variables.
Within this system the hypotheses of interest relate to the M1 level of aggregation. The (M2-M1) and to some extent the M2-level is also studied and the question of weighting - equal or different weights - is considered at each level: M1, (M2-M1) and M2.

4.1 Model control and rank determination

The first step in the analysis is to examine whether the VAR gives an appropriate description of the data set. The results of these multivariate misspecification tests are reproduced in Table 1. These statistics indicates that the models gives a fairly good description of the data.

Table 1: Multivariate misspecification tests

	Autocorr. LM(1)	Autocorr. LM(4)	Normality
Statistic	73.07	66.72	30.24
p-value	0.20	0.38	0.02

Note: The tests for autocorrelation are Lagrange Multiplier tests and are asymptotically Chi-square distributed with 64 df. The normality test is the Hansen-Doornik test and the statistic is asymptotically Chi-square distributes with 16 df.

Furthermore, there are no signs of ARCH in any of the 8 equations when tested univariately[5].

As the model allows for data that are integrated of either order 1 or order 0 (stationary) the way research usually proceeds is to check the order of integration of the data at quite an early stage of the analysis. In fact this is usually done prior to the multivariate modelling of the series but an alternative and in the present case preferable procedure would be to investigate the time series properties of the data within the multivariate framework. In this case the inclusion of the shift dummy among the deterministic variables of the model is automatically handled properly. The idea of the multivariate stationarity testing procedure is to test whether a unit vector that corresponds to a particular variable in the data set belongs to the cointegration space (the β-space) for each possible value of the cointegration rank, r. The asymptotic distribution of this likelihoodratio test is chi-square with (p-r) df. This procedure is repeated for each of the variables in the dataset. The results of these tests are shown in Table 2:

Table 2: Results of the multivariate stationarity testing

Rank	r=1	r=2	r=3	r=4	r=5	r=6	r=7
Df of test	7	6	5	4	3	2	1
Chi-sq 95%	14.07	12.59	11.07	9.49	7.81	5.99	3.84
Currency	87.53	33.85	23.35	17.85	17.57	10.53	2.37
Dem.dep.	85.40	31.28	24.22	21.59	19.28	9.82	2.41
Time dep.	99.61	45.82	34.37	19.90	17.57	10.29	0.01
Spec. dep.	81.88	28.10	21.62	9.53	7.40	7.38	1.02
Treas.bills	87.66	44.80	33.24	17.95	16.42	6.78	2.48
Prices	100.65	47.01	35.12	22.67	1.70	12.13	2.34
Income	82.41	31.08	22.96	9.21	7.61	2.89	1.53
Opp. costs	76.27	22.80	14.28	3.47	2.91	0.08	0.02

The results in Table 2 indicates that for a rank above 4 the opportunity cost series can be considered stationary (with a shift in mean). Income and Special term deposits are

5 The results of the ARCH tests are available from the author upon request.

borderline cases for r=4 and r=5. Note that the tests are still univariate in the sense that stationarity of just one of the series are tested at a time.

The next step of the analysis is to determine the cointegration rank, r. In table 3 the results of the Johansen Trace test for the cointegration rank are displayed. The critical values are simulated by the programme DISCO developed by Bent Nielsen and Søren Johansen both from Institute of Mathematical Statistics, University of Copenhagen. Hence the asymptotic distributions allow for the shift dummy as well as for the unrestricted constant term.

Table 3: Trace test statistics

Rank: r	Trace stat.	80% quantile	85% quantile	90% quantile	95% quantile
0	264.65	147.67	150.72	154.49	160.36
1	157.23	114.94	117.68	121.13	126.77
2	103.95	86.13	88.62	91.48	96.18
3	63.45	60.81	62.85	65.64	69.70
4	38.44	39.57	41.18	43.29	46.63
5	15.77	21.12	22.39	24.01	26.72
6	2.75	6.01	6.76	7.74	9.57
7	0.03	1.64	2.07	2.71	3.84

The testing procedure starts at the top of the table and according to the preferred level of significance[6] a rank of either 3 (95%) or 4 (85%) is chosen. For the analysis that follows a rank of r=4 is chosen. This choice is made partly for economic reasons as the fourth cointegration vector (c.f. table 4 below) seems to possess many of the properties that is expected of a long-run money demand relation.

Table 4 Unrestricted stationary vectors

6 Remember that for statistical reasons the null-hypothesis is formulated contrary to what is actually desired justifying a more elaborate discussion of the significance level than is usually the case (see also Juselius (1994)).

Variable	β_1	β_2	β_3	β_4
Currency	5.44	11.41	-27.73	-12.23
Dem. deposits	7.86	-23.32	12.69	-3.21
Time deposits	12.02	-4.10	2.34	-1.89
Spec. deposits	0.25	-6.23	5.84	-4.30
Treasury bills	3.55	-0.92	1.60	-0.03
Prices	-28.61	29.86	-3.74	20.83
Income	-42.27	40.26	-22.12	30.17
Opp.costs	-41.77	-48.71	53.81	-102.06

The vectors of Table 3 have not been normalized. From a priori economic considerations the fourth β-vector seems to be of special interest, as it may be interpreted as a long-run demand for liquid money relation. First of all the signs are correct in the sense that the coefficients of the monetary components have signs opposite those of the price and income variables while having the same sign as the opportunity cost measure. Secondly, the sum of the coefficients of the monetary components is of approximately the same magnitude as the price coefficients suggesting that price homogeneity is fulfilled for this relation. Usually the interpretation of a long-run relation also finds support in the α-coefficients but with non-normalized β-vectors this will be rather diffcult due to numeric problems: The coefficients will be too close to zero to be really useful. Hence the information contained in the α-coefficients will be retained for use later in the analysis.

4.3 Identification

In order to increase the understanding of the long-run structure of the system an attempt to identify each of the long-run relations economically as well as statistically is done. The hypothesis is that in order for a monetary aggregate at the M1-level to exist one of the long-run relations must be a relation between the currency , the demand deposit component, prices and income. Due to the long-run nature of the cointegrating relations imposition of price homogeneity on this vector will also be considered appropriate. Usually the user cost series would be expected to enter this relation as well but recalling the result of the individual stationarity testing the strong indications of stationarity on this series conditionally on the dummy variable implies that a separation of the user cost series from the M1 money demand relation will be appropriate in the long-run space[7]. Accordingly the next part of the identifying hypothesis is that the user cost series will be stationary by itself. The third economic identification concerns the less liquid parts of the monetary components. In other words we try to split the traditional kind of a money demand relation into a liquid and a less liquid part the latter being a relation between time- and special term deposits, treasury bills, prices and income. The user cost series is for the same reasons as above not directly in this relation. The fourth relation is only statistically identified and three just-identifying zero-restrictions are imposed. In the first three relations more than three restrictions are imposed so that the whole structure will be over-identified and a likelihood-ratio test as suggested in Johansen (1995b) can be used to test these over-identifying restrictions. The results of the identification is given below in Table 5:

Table 5 Identified long-run structure.

Variable	β_1	β_2	β_3	β_4
Currency	0.47*	0.00	0.00	1.00*
Dem. deposits	0.53*	0.00	0.00	-0.78*
Time deposits	0.00	0.75*	0.00	0.34*
Spec. deposits	0.00	0.02	0.00	-0.29*
Treasury bills	0.00	0.23*	0.00	0.10*
Prices	-1.00*	-1.00*	0.00	0.00
Income	-0.66*	-2.14*	0.00	0.00
Opp.costs	0.00	0.00	1.00	0.00

A * means significance at the 5% level based on the Wald-like standard-errors in Johansen (1995b). A * on a normalising variable has been obtained from a different normalisation (this was not possible to do for the third β-vector that only contains one variable (this does of course not mean that the coefficient on this variable is insignificant).

Notice that the weights of the monetary components within the M1 money demand relation are very close to each other. The income elasticity of that relation is somewhat below unity. In the M2-M1 money demand relation a weight of 0.75 is given to time deposits and a weight of 0.23 is given to treasury bills. The weight of the special term

7 Notice that this separation does not mean that changes in the different monetary components will not be influenced by the user cost series. Using the α-coefficients it is in principle possible to construct a weighted disequilibrium term containing the elements of both the M1 long-run relation and the user costs.

deposits is insignificant. Notice that the income elasticity is around 2. An elasticity of this magnitude is quite normal for broad money measures, see Fase (1993).

The structure of table 5 contains 23 restrictions (remember that the coefficients of 1 to the price series in the first two relations are both normalisations and "true" restrictions) of which $4^2 = 16$ are normalizing ones. The LR-test of the over-identifying restrictions has a statistic of 9.39 and compared to a Chi-square distribution with (23-16=7) degrees of freedom produces a p-value of 23%. The attempt to identify the long-run structure proves successful to the extent that at least 3 out of the 4 relations have an economic meaning while the 4th relation must only be considered statistically identified probably due to the choice of information set.

In table 6 below the α-coefficients of the restricted and normalized structure are found. The information contained in these coefficients in general support the economic interpretation of the first three relations: Assuming that a change in a monetary aggregate is, considering the index theoretical background, associated with an income effect[8] which, under "normal" circumstances, will ensure that the changes in the individual components are all in the same direction. Expected significant error correction effects in the equations of components of the corresponding aggregate are: Excess demand for the aggregate in question should lead to a downward adjustment in the components of the aggregate. For the user cost relation the expected direct effect of a deviation from the mean should be an adjustment back towards the mean.

---

8 Substitutional effects, on the other hand, open up the possibility of different signs of the α's of the asset components. In the current context substitutional effects must be associated with a demand system setup rather than with a macro money demand relation, but a priori such effects cannot be excluded, as the data vector contains groups of asset components. These effects, however, will not be taken into account in the attempt to simultaneously identify a long-run demand-for-money relation and a macro monetary aggregate.

Table 6 The adjustment coefficients of the identified long-run structure

Variable	α_1	α_2	α_3	α_4
Currency	-0.42(-4.24)	0.10(1.13)	0.29(0.56)	-0.36(-3.31)
Dem. deposits	-0.11(-1.24)	-0.09(-1.01)	-0.55(-1.16)	0.23(2.29)
Time deposits	-0.25(-1.94)	-0.30(-2.52)	-1.52(-2.29)	0.30(2.19)
Spec. deposits	0.06(0.28)	0.47(2.24)	1.12(0.96)	-0.22(-0.88)
Treasury bills	-0.52(-1.72)	-0.67(-2.36)	1.47(0.92)	-0.27(-0.83)
Prices	0.02(1.28)	-0.03(-1.63)	0.14(0.92)	0.01(0.65)
Income	0.11(2.18)	0.23(4.78)	0.71(2.69)	-0.24(-4.35)
Opp.costs	0.06(3.27)	0.08(4.79)	-0.33(-3.49)	-0.05(-2.55)

The numbers in parentheses are t-values.

Based on these estimates it is seen that there is additional support for the economic interpretation of the relations. And even though the α-coefficient of the special term deposits in the (M2-M1) demand relation is significant and with a sign opposite the expected one this may be no major problem as the STD series was insignificant in this long-run relation. With 8 variables in the data vector some significant dynamic effects which are not directly explicable is expected to exist and is in fact present but this issue will not be pursued here.

Based on the identified long-run structure of Table 5 a demand for liquid money, M1, relation is found suggesting that M1 would be an acceptable level for monetary aggregation. Also from β_2 the level M2-M1 must be considered appropriate for aggregation. Next, the method of aggregation is a most relevant issue and therefore a set of additional restrictions are imposed on the structure:
The log-linear form of the econometric model implies that the derived monetary aggregate is of the geometric kind and alternative aggregation methods are not directly testable within the currant framework. It is, however, possible to test for equal[9] as opposed to different weights of the components within the geometric index. Such tests will be conducted for both the M1 and (M2-M1) aggregates below. First, the weights (coeffi-

9 As an example: Summation (implying equal weights to the components of the aggregate) is valid as a method of aggregation only if the components of the aggregate are perfect substitutes.

cients) of the monetary components in the M1-relation are restricted to be equal. At the same time the income elasticity is restricted to 1. In the second relation only the insignificant coefficient on the special term deposits is restricted to zero. In total this means three additional restrictions and the LR-test statistic of this hypothesis becomes: 10.62. Compared to the Chi-square distribution with 10 degrees of freedom the p-value of the test is 39%. Hence equal weighting of the components within the liquid M1-aggregate would be appropriate. Acceptance of equal weights of a narrow monetary aggregate like M1 is not alarming from an index- and aggregation theoretical point of view: The components of a narrow aggregate will a priori be expected to be much closer substitutes than those of a broader aggregate. In table 7 the estimates of the long-run structure with equal weighting of the M1-components are shown:

Table 7 Long-run structure with additional restrictions.

Variable	β_1	β_2	β_3	β_4
Currency	0.50	0.00	0.00	1.00*
Dem. deposits	0.50	0.00	0.00	-0.90*
Time deposits	0.00	0.76*	0.00	0.46*
Spec. deposits	0.00	0.00	0.00	-0.32*
Treasury bills	0.00	0.24*	0.00	0.14*
Prices	-1.00	-1.00*	0.00	0.00
Income	-1.00	-1.84*	0.00	0.00
Opp.costs	0.00	0.00	1.00	0.00

A * again means significance at the 5% level, see table 5.

The α-coefficients do not change much and the new set of estimates are left out to save space. They are available from the author upon request.

Next, imposing the additional restriction that the weights of the components within the (M2-M1) aggregate should be equal is strongly rejected: The test statistic is 40.03 and compared to the asymptotic Chi-square distribution with 11 degrees of freedom the p-value becomes 0.00.

Finally, an analysis of a monetary aggregate at the M2-level is done. Note that due to the stationarity of the M1-money demand and the stationarity of the (M2-M1) money demand any linear combination of these relations (e.g. based on the α-coefficients) will be stationary suggesting that a money demand relation at the M2-level exists. The identification of this relation is not straight forward, however. One way to proceed would be to look for significant adjustment coefficients in the error correction equations of the system and base an identification on these. There will, however, be no guarantee of consistency for the different equations. An alternative procedure - and the one used in the present case - is to consider a liquid M2-money demand which would allow the imposition of an additional restriction on the long-run space: In the M2-money demand relation the coefficients of the two most liquid components should still be equal. The coefficients of the significant components from (M2-M1) are different and price homogeneity is imposed. Finally, income homogeneity is imposed as the necessary identifying restriction that allows the interpretation of the M2 aggregate to be a liquid one. Long-run relations 2-4 are restricted as in the previous case. The results for a monetary aggregate at the M2-level are found in table 8 below. The structure is accepted as stationary with a test statistic: Q=6.68 and a p-value of 0.57 in the asymptotic Chi-square distribution with 8 degrees of freedom.

Table 8 Long-run structure with M2 money demand.

Variable	β_1	β_2	β_3	β_4
Currency	0.42*	0.00	0.00	1.00*
Dem. deposits	0.42*	0.00	0.00	-0.82*
Time deposits	0.14*	0.73*	0.00	0.37*
Spec. deposits	0.00	0.00	0.00	-0.30*
Treasury bills	0.01	0.27*	0.00	0.13*
Prices	-1.00*	-1.00*	0.00	0.00
Income	-1.00*	-1.99*	0.00	0.00
Opp.costs	0.00	0.00	1.00	0.00

A * again means significance at the 5% level, see table 5.

The liquid M2 aggregate derived from relation 1 has the largest weight to the most liquid components and an insignificantly small weight to treasury bills.

4.3 The data-weighted monetary aggregates.

The analysis in section 4.1 and 4.2 shows that it is possible to construct data-weighted or estimated monetary aggregates for Denmark at different aggregation levels: M1, (M2-M1) and M2. Below a discussion of the properties of these aggregates is found. Due to the logarithmic transformation of the monetary components the aggregates to be extracted from the cointegration analysis are, as mentioned above, of the geometrically weighted kind:

$$Q_t^C = \prod_{i=1}^{n} m_{it}^{\beta_i}$$

where Q means quantity index, C is "calculated", the large Π means product over the n components chosen to be in the aggregate, the m's represent the monetary components and the β's are the estimated weights. In Figure 1 and Figure 2 the calculated aggregates at the M1 level is compared to their simple sum counterparts in levels and in percentage changes respectively. In both cases the indices are normalised to begin at 100 in 1976 2nd quarter.

Figure 1: M1-level Aggregates

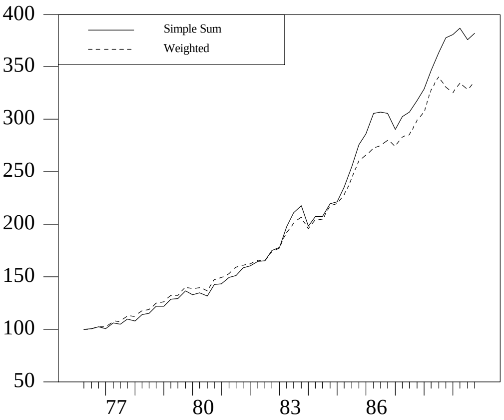
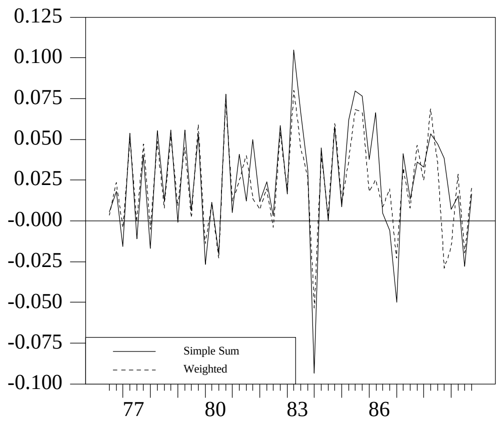


Figure 2: M1 Aggregates: % changes



From the graphs it is seen that during the first part of the sample period the two aggregates move very closely together. During the second part of the sample period the simple sum aggregate seems to be slightly more volatile than the weighted one and in the levels graph the curve of the simple sum aggregates lies above that of the weighted one. Hence at least for some sub periods there are important differences in the behaviour of the two aggregates: The method of aggregation is important! In Figures 3 and 4 below the same kind of comparisons are done for an aggregate at the M2-M1 level.

Figure 3: (M2-M1)-level Aggregates

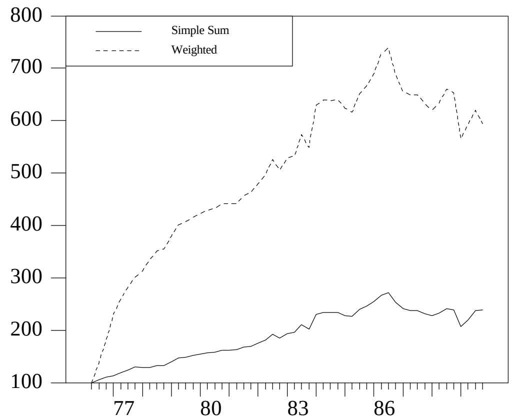
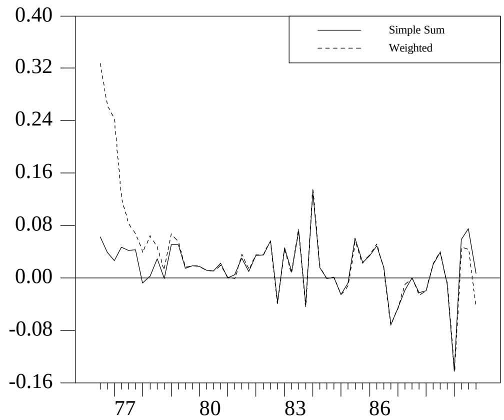


Figure 4: (M2-M1) Aggregates: % changes



From these graphs it becomes apparent that something dramatic happens at the beginning of the sample period. From Figure 4 it is clear that only during the first few years of the sample period any significant differences in the behaviour of the two aggregates exist. These differences, however, being in the first part of the sample period induce the large discrepancies between the two aggregates in levels, cf. Figure 3. There is, however, a simple explanation for this: Financial innovation! - Just in the beginning of 1976 the treasury bills that constitutes a part of the M2-M1 aggregate are introduced in Denmark and in the introductory phase there is a rapid growth in the amount of treasury bills in circulation. From the graphs above and especially those of Figure 3 and 4 it is apparent that an estimated monetary aggregate will be vulnerable to the introduction of new financial assets /monetary components. With constant weights over the sample period the newly introduced component will during the introductory phase obtain a weight that is too large based on economic considerations as it is an average weight calculated based on data for the whole sample period. Hence there must be a general warning against the use of the method in such situations though this should not be a general warning against the use of estimated indices as such. An index type which is capable of coping with the above mentioned problem would be a weighted index that allows for different weights over the sample period such as e.g. a Divisia index. Hence the estimated or data based monetary index must be considered to lie somewhere in between the most restrictive index: The simple sum one and the more flexible index: The Divisia one (or any other index belonging to the Diewert-class (see e.g. Diewert (1976) or Diewert(1981))of superlative index numbers - i.e. index numbers with different and time varying weights that at the same time possess nice approximating properties vis-a-vis index numbers derived from optimizing behaviour in economic theory).

5. Evaluations and conclusions

The purpose of this paper has been to simultaneously investigate the question of monetary aggregation and the possibilities of identifying a stationary relation between variables that traditionally enter into macro money demand relations. For this purpose a multivariate modelling approach is of inestimable importance. The example that illustrates the ideas comes from the Danish economy and the multivariate framework within which the analysis has been carried out is the Johansen-procedure for analysing integrated time series within a vector autoregressive model. Some of the ideas originally suggested turned out to be too ambitious to be carried out, e.g. an analysis of a 12-dimensional system containing both component balance- and interest data, prices and income was impossible due to the rather small number of observations in the sample. Therefore, a partial system containing the balance data, the price level, the income measure, a general user cost measure and a shift dummy was formulated, and within this latter system hypotheses-testing has been performed. The hypothesis of interest relates to the level as well as to the method of monetary aggregation and a summary of the results are: It is possible to identify long-run money demand relations with monetary components belonging to the M1, the (M2-M1) and the M2 level of aggregation. Hence any of these levels can be considered appropriate for the constructions of monetary aggregates. At the M1-level equal weighting of the components within the aggregate is data consistent and with the income elasticity restricted to one, the M1-stationary relation is interpreted as a demand-for-liquid-money relation. Equal weighting of monetary components at the M1 level is not directly contradictory to index and economic aggregation theory, because the components within M1 can be considered quite homogeneous. At the (M2-M1) level it is again possible to identify a stationary demand-for-money relation. Equally weighting of the components of this group is, however, not supported by the data. This conclusion is also expected a priori as the components at the (M2-M1) level must be considered less homogeneous than those of M1. As the (M2-M1)- money demand relation describes a demand for less liquid money, the income elasticity is not restricted to one. The estimated value of the income elasticity is around 2, and a value of this size is quite common in international studies of the demand for broad money. Next the M2-level of monetary aggregation is studied. A stationary money demand relation is not surprisingly (as M1 and (M2-M1) relations are already identified) found. In order to interpret and identify the M2-relation as a demand-for-liquid-money relation an income elasticity of one is imposed (and accepted), and with both liquid and less liquid monetary assets in M2 the conclusion that different weighting of the less liquid components within the aggregate is apprioriate is drawn. This result is expected, intuitively appealing and consistent with our earlier findings. Finally, an overall economic interpretation of the results is that we find evidence that a traditional money demand relation can be separated into partial demand-for-money relations that relate to the different motives for holding money. The fact that the user cost series pops up as a stationary series (with a mean shift) does not imply that monetary adjustment is not affected by this variable. The only outstanding issue is the last stationary vector that seems to relate to a demand system approach describing substitutional effects, as opposed to the money demand relations that emphasize the income-effects among the monetary asset components. Apparently, the data set does not contain sufficient information for a firm interpretation of this relation.

The conclusion concerning the methodological aspects of the study is: The multivariate approach has indeed made it possible to study a lot of relevant issues with respect to monetary aggregation and existence of stationary long-run money demand relations. And even though some caution need to be taken concerning the actual construction of a monetary aggregate based on the estimated weights the method is probably as far as we can come in this direction and is indeed an interesting supplement to traditional approaches to the construction of monetary aggregates.

References:

Barnett, W.A. (1980), ‹Economic Monetary Aggregates - An Application of Index Number and Aggregation Theory'. Journal of Econometrics, 14 : 11-48.

Barnett, W.A. (1987), ‹The Microeconomic Theory of Monetary Aggregation', in: Barnett and Singleton (eds.) New Approaches to Monetary Economics, Cambridge University Press.

Barnett, W.A. (1990), ‹Developments in Monetary Aggregation Theory’. Journal of Policy Modelling, 12 no 2, 205-258.
Bordo, M.D., Jonung, L. (1990), ‹The Long-Run Behaviour of Velocity: The Institutional Approach Revisited’, Journal of Policy Modelling, 12 no 2, 165-198.
Clements, K.W., Nguyen, P. (1980), ‹Economic Monetary Aggregates - Comment’, Journal of Econometrics, 14, 49-53.
Diewert, W.E. (1976), ‹Exact and Superlative Index Numbers’, Journal of Econometrics, 4, 115-145.
Diewert, W.E. (1981), ‹The economic theory of index numbers: a survey’, in: Deaton (ed) Essays in the Theory and Measurement of Consumer behaviour, Cambridge University Press : 163-208.
Fase, M.M.G. (1993), ‹The Stability of the Demand for Money in the G7 and the EC Countries: A Survey’, Research Memorandum WO&E nr 9321, De Nederlandsche Bank NV.
Haavelmo, T. (1944), ‹The Probability Approach in Econometrics’, Supplement to Econometrica, 12.
Johansen, S. (1992a), ‹A Representation of Vector Autoregressive Processes Integrated of Order 2', Econometric Theory, 8, 188-202.
Johansen, S. (1992b), ‹Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models’, Econometrica, 59, 1551-1580.
Johansen, S. (1995a), ‹A Statistical Analysis of Cointegration for I(2) Variables’, Econometric Theory, 11, 25-59.
Johansen, S. (1995b): ‘Likelihood-based Inference in Cointegrated Vector Autoregressive Models’, Oxford University Press.
Johansen, S., Juselius, K. (1990), ‹The Full Information Maximum Likelihood Procedure for In-ference on Cointegration - With Applications’, Oxford Bulletin of Economics and Statistics, 52, no 2.
Johansen, S. Juselius, K. (1992), ‹Some Structural Hypotheses in a Multivariate Cointegration Analysis of the Purchasing Power Parity and the Uncovered Interest Parity for UK’, Journal of Econometrics, 53, 211-244.
Johansen, S. Juselius, K. (1994), ‹Identification of the Long-run and the Short-run Structure. An Application to the IS-LM Model’, Journal of Econometrics, 63, 7-36.
Judd, J.P., Scadding, J.L., (1982), ‹The Search for a Stable Money Demand Function: A Survey of the Post-1973 Literature’, Journal of Economic Literature, XX, 993-1023.

Juselius, K. (1994), ‘On the Duality between Long-run Relations and Common Trends in the I(1) versus I(2) Model. An application to Aggregate Money Holdings’, Econometric Reviews, 13, 151-178.

Laidler, D.E.W. (1985), ‘The Demand for Money. Theories and Evidence’, Harper & Row.

Laidler, D.E.W. (1990), ‘Taking Money Seriously, Philip Allan.

Laumas, G.S. (1968), ‘The Degree of Moneyness of Savings Deposits’, American Economic Review, 501-503.

Spanos, A. (1984), ‘Liquidity as a Latent Variable - An Application of the MIMIC Model’, Oxford Bulletin of Economics and Statistics, 46, 125-143.

Timberlake, R.H., Fortson. J. (1967), ‘Time Deposits in the Definition of Money’, American Economic Review, 57, 190-194.

Statistical information have been collected from relevant numbers of:

"Statistical News", Danmarks Statistik

"Statistics Service", Danmarks Statistik

"Report and Accounts", Danmarks Nationalbank

Appendix A The data.

PERIOD	CURR	DD	TD	STD	TB	Price	Income	User costs
76: 2	10512.83	41895.74	32932.59	13794.37	50.50	0.68846	381.46390	0.07511871
76: 3	10511.17	42188.81	34938.36	14264.63	163.83	0.69926	374.78900	0.08057870
76: 4	10859.83	42809.21	36052.10	14593.00	444.00	0.72336	378.31080	0.05883250
77: 1	11031.17	41801.45	36296.23	14506.29	1192.50	0.73318	374.90210	0.06958140
77: 2	11555.50	44186.75	37516.20	15080.88	1776.50	0.74935	381.15430	0.07502131
77: 3	11756.67	43375.10	38715.10	15211.80	2269.67	0.76771	385.59580	0.07424661
77: 4	12479.33	44916.82	40096.44	15680.11	2687.17	0.79616	378.69240	0.07588960
78: 1	12679.17	43758.06	39022.60	15219.54	3445.33	0.80854	377.52700	0.07273710
78: 2	13151.50	46487.98	37515.88	15328.31	5100.83	0.82078	378.79690	0.08770751
78: 3	13155.00	47218.88	37737.26	15520.63	6127.67	0.83071	390.04600	0.09019331
78: 4	13774.33	50054.78	36938.12	15843.93	6903.50	0.85050	390.66280	0.10500410
79: 1	14156.33	49630.42	37253.12	15895.46	8882.50	0.87094	389.90580	0.09504340
79: 2	14535.67	52920.13	38172.17	16614.33	10354.83	0.88609	398.85270	0.09192930
79: 3	14450.50	53439.98	37789.99	16668.86	11433.50	0.91624	396.44290	0.09227500
79: 4	15497.17	56099.31	38660.59	17273.76	11503.33	0.94288	392.99350	0.07944630
80: 1	15637.00	54090.19	38670.10	16965.07	12402.17	0.96600	397.15560	0.08206670
80: 2	15791.83	54725.93	40013.07	17209.00	11676.67	0.99025	380.89480	0.07570000
80: 3	15359.67	53747.29	40716.38	17213.53	11555.67	1.01290	366.53580	0.07130000
80: 4	16444.83	58244.39	42745.50	18486.95	10738.50	1.03409	366.35220	0.07596670
81: 1	16847.17	58223.03	42554.19	18573.54	10920.50	1.06578	364.40650	0.07953330
81: 2	16757.50	61454.86	43860.31	19283.09	9892.17	1.10880	363.47890	0.08386670
81: 3	18349.17	60809.07	44102.93	19246.17	11284.00	1.13341	358.59230	0.09223330
81: 4	17443.00	65749.59	43603.41	21232.50	12332.33	1.16212	362.57420	0.08333330
82: 1	17416.17	66777.58	44244.52	20038.17	13660.00	1.19224	366.73020	0.09626670
82: 2	17634.67	68583.90	45000.20	21650.50	15008.17	1.21827	374.58170	0.09976670
82: 3	17361.17	69127.74	47311.76	20268.50	16188.83	1.24605	382.80910	0.09786670
82: 4	18208.67	73472.85	44531.15	23846.33	16683.17	1.27753	375.19090	0.08706670
83: 1	18612.50	74607.65	45718.65	25205.83	18382.17	1.29641	371.40590	0.05510000
83: 2	19374.83	84135.72	45613.88	28369.83	19087.33	1.30947	380.28640	0.05130000
83: 3	19600.17	90990.09	50082.31	25697.50	19368.17	1.32882	376.34570	0.05870001
83: 4	20025.83	94041.23	47194.57	37648.00	19455.50	1.35027	392.45000	0.04880001
84: 1	20240.33	83646.20	56114.40	44795.73	19920.50	1.37267	392.34230	0.04910001
84: 2	20959.33	87695.90	57083.40	50869.27	20114.17	1.38916	408.98970	0.05870001
84: 3	21138.83	87505.50	57289.80	53332.50	19838.50	1.40484	403.62690	0.05980001
84: 4	22515.33	92581.40	58023.10	67123.47	19147.67	1.42195	393.07720	0.04955276
85: 1	22863.17	93238.10	57651.70	65402.67	17584.83	1.44012	408.36020	0.04143541
85: 2	22757.83	100778.30	59292.20	64168.93	15325.83	1.45522	409.56010	0.03196482
85: 3	23945.33	109838.80	64423.00	61235.43	14865.83	1.45953	425.34720	0.02695742
85: 4	25218.50	119177.80	66299.90	65581.70	14894.33	1.46923	440.40060	0.02811645
86: 1	24925.50	125006.40	68598.00	66114.67	15462.17	1.47411	442.81350	0.02532086
86: 2	24071.50	136126.90	71224.00	63787.33	17059.00	1.49579	446.69050	0.03094166
86: 3	24403.50	136580.80	72239.50	58840.27	17411.33	1.50507	454.85550	0.04355921
86: 4	25826.83	134244.40	67550.80	74101.60	15941.83	1.52306	441.52440	0.03887130
87: 1	26287.33	126010.00	64819.00	78229.17	14949.50	1.54042	433.13290	0.04268004
87: 2	26783.83	131938.00	59942.00	84850.17	18387.83	1.57723	434.70150	0.04107463
87: 3	26779.33	134053.00	58490.00	83186.83	19856.17	1.58664	423.87470	0.04380381
87: 4	28474.67	138312.00	55103.00	89608.67	21467.67	1.60448	440.35910	0.04440365
88: 1	28851.67	143523.00	54378.00	75888.17	20684.83	1.63245	423.03100	0.03691507
88: 2	31653.00	150169.00	56125.00	69894.83	20512.67	1.65100	428.63320	0.03795920
88: 3	32258.17	158338.00	59903.00	63132.33	19760.50	1.66418	419.36990	0.03481795
88: 4	28408.67	169694.00	58823.00	62003.00	20056.17	1.68767	422.36570	0.03131718
89: 1	27042.00	172477.00	50042.00	68807.50	18406.50	1.71024	432.83700	0.03106851
89: 2	28331.33	174289.00	49626.00	69813.83	22966.00	1.73726	427.49490	0.03298389
89: 3	28115.50	168943.00	48429.00	68618.50	29807.83	1.74606	424.35490	0.03298389
89: 4	28888.17	171468.00	43465.00	84292.83	35323.50	1.77060	414.69330	0.03298389

Graphs of the data