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a welfare-theoretical approach

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Abstract

In the literature on cost-effectiveness analyses, there has recently been considerable debate on the way of measuring the production gain caused by a new treatment as well as its proper evaluation. In this paper, we propose a general framework for the discussion of such questions in the form of a general equilibrium model with separate health variables. It is shown how the standard methods of cost-effectiveness and cost-utility analysis are derived from basic assumptions. Also, the question of whether to choose the human capital or the frictional method in evaluating production gains can be given an answer in this model.

1. Introduction

In a recent paper by Olsen and Richardson (1999), the authors discuss the problems connected with including production gains in a cost-effectiveness analysis. The production gain from a medical treatment is society's indirect benefit to the increased amount of work supplied to society as a result of the patient being cured quicker than previously (this production gain is considered as an indirect benefit in order to distinguish it from the direct benefits of the treatment in form of increased health of the patients). In traditional cost-effectiveness analyses, there has been a certain reluctance to include the production gains, and in addition, there has been a controversy pertaining to the way of measuring the production gain.

In Olsen and Richardson (1999) the production gain is discussed and methods of evaluation are proposed. The approach of the authors, as outlined in their introduction, consists in (1) determining the correct magnitude of the production gain, and (2) deciding what should and what should not enter into a cost-effectiveness analysis, thus dividing their theoretical analysis of the problem into a positive and a normative part. Consequently, the discussion splits into arguments about the *correct* way of measuring production gains followed by a discussion of what *ought to be* taken into consideration when computing a cost-effectiveness ratio.

The purpose of the present note is to argue that the search for a *correct* measurement (in money terms) of the production gain is futile – there is no such thing. The production gain is necessarily relative to an underlying criterion for social welfare; when this criterion changes, the production gain will change its size as well. Therefore, the "positive" and "normative" parts of the investigation cannot really be distinguished, and for both, "correct" should be taken to mean "consistent", that is consistent with the given underlying criterion for social welfare. The fact that this criterion is not known to the investigator and will have to be assessed using other information, does not change the situation.

Unfortunately, this consistency approach to cost-effectiveness analyses has not been given due attendance in the literature, where most authors – including the much cited Washington Panel (see Gold e.a., 1996) – argue about the "right" evaluation methods using intuitive economic arguments but without reference to a common theoretical framework.

In the following section, we give a short presentation of the theoretical basis for cost-effectiveness analysis, and we then use this background to consider some of the questions raised by Olsen and Richardson, among which the controversy about the human capital versus the friction approach to evaluation of the production gain, the asymmetry in evaluation of treatments depending on whether the patients are active in the labor market or retired, and finally the equality-efficiency trade-off in health economic priority setting.

Since the point of the present paper is that health economic evaluations are by nature *relative* (to an underlying criterion for what is good or bad for society) our conclusions cannot have the nature of precise recommendations; what we can hope for is a clarification of which evaluation methods are consistent with the way of measuring other economic effects and which are not. Among the latter is the so-called friction approach according to which the production gain should be evaluated in what economists call shadow prices (the cost for society of having the labor service performed); this method would indeed make sense if everything else was measured in this way – but it is not and could hardly be, and therefore, the isolated measuring of one out of many goods and services in shadow prices is an inconsistency.

The argumentation in Olsen and Richardson (1999) that only part of the production gain represents *relevant* gain to the *rest of society,* an argument which seems to be based on the idea that the patient's gain is already counted in the health dimension and therefore only part of the production gain should be included, is also hard to connect with the theoretical background. If a medical treatment results in a larger labor supply actually used in society, then there is a production gain for society, and it does not matter at all where this labor supply actually came from, whether it was from the patients themselves, their families or for that reason from total strangers.

With respect to the equality-efficiency trade-off, which certainly is an important issue in health economics, what we get from making explicit the theoretical foundations of cost-effectiveness is the insight that an economic evaluation does not necessarily capture *all* relevant aspects of society's preferences with respect to new treatments or drugs; it does not even attempt to do so. What it tries to achieve is to show how society would evaluate a given medical technology *given that it uses the norms and priorities that are revealed by the actual choices of society's citizens.* It goes without saying that the results of cost-effectiveness analyses should be used with corresponding restraint: They show at best the priorities in society evaluated according to the preferences which are used in day-to-day economic decisions.

2. The welfare economic foundations of cost-benefit analysis

In this section, we give a short overview of the economic theory of project evaluation, which lies behind economic appraisals of treatments or medicine. The basic idea behind the economic appraisals is quite simple and can be outlined in a few lines: Assume that a function $f : \mathbf{R}^k \to \mathbf{R}$ is maximized at x^0 subject to a constraint $x \in A \subset \mathbf{R}^k$. If the solution x^0 is given a small displacement dx, made possible by a change in the feasible set A, we can evaluate the effect on the objective function by computing

$$\sum_{h=1}^{k} f'_h dx_h,$$

where f'_h is the partial derivative of f w.r.t. x_h, evaluated at x^0. This method of evaluation is useful even if we do not know the exact functional form of f, as long as we have access to information about the partial derivatives from some other source, and in the context of economic evaluations, we have this kind of information in the form of the market prices.

Now we turn to elaborating this basic idea in the context of allocation in a society. In this and the following sections, we assume that (1) social welfare depends only on the utility levels of the individuals, and (2) utility of any individual depends only on the consumption of the available commodities (in other words, health carries no value per se and is valuable only by making it possible to obtain a higher level of consumption). Following Sen, we use the term *consumerism* for this setup. While (1) is rather standard in economic welfare theory, also in its relation to health economics, assumption (2) may seem more doubtful, but even so it is common enough in the literature (as e.g., in Grossman (1971)).

The theoretical approach to cost-benefit analysis, as developed by e.g. Lessourne (1975), can now be described as follows: Assume that society's consumption and production is given by $(x_1, \dots, x_m)$ and $(y_1, \dots, y_n)$, where for $i = 1, \dots, m$, x_i is a consumption bundle (of the l available commodities) of consumer i, and for $j = 1, \dots, n$, y_j is a vector of net production of each of the l commodities in firm j. By (1), society's welfare can be measured as

$$S(u_1(x_1), \dots, u_m(x_m)),$$

where $S : \mathbf{R}^m \to \mathbf{R}$ is a social welfare function, and $u_i : \mathbf{R}^l_+ \to \mathbf{R}$, for $i = 1, \dots, m$, are the individual utility functions (which by (2) depend only on the achieved levels of consumption x_i. We assume that both S and each of the individual utilities u_i are differentiable.

If the allocation in society is changed by some small amount $dx_i = (dx_{ih})_{h=1}^{l}$, $i = 1, \ldots, m$, for each consumer, and dy_j, $j = 1, \ldots, n$ for the producers, then society's welfare changes by

$$dS = \sum_{i=1}^{m} S_i' \sum_{h=1}^{l} u_{ih}'(x_i) dx_{ih}. \tag{1}$$

Here, S_i' is the society's marginal welfare with respect to individual i, and u_{ih}' is the marginal utility of individual i for commodity h. The latter quantity is generally unknown to the analyst, so in order to get along, one has to make the following assumption.

Assumption 1. *In the initial allocation, consumers obtain their bundles by trading in the market.*

The assumption says that there is some price vector $p \in \mathbf{R}_+^l$ such that for each i, the consumption bundle x_i maximizes u_i on all x_i' satisfying the budget constraint

$$p \cdot x_i' \leq p \cdot x_i = w_i,$$

where we have introduced the notation w_i for the total budget or income of consumer i, $i = 1, \ldots, m$. Letting $\xi_i(p, w_i)$ be the demand function of consumer i (which will be assumed differentiable), we introduce the indirect utility function v_i of consumer i by $v_i(p, w_i) = u_i(\xi_i(p, w_i))$.

We now have:

Lemma 1. *Under Assumption 1, the change in social welfare may be written as*

$$dS = \sum_{i=1}^{m} S_i' \sum_{h=1}^{l} \lambda_i u_{ih}' dx_{ih}, \tag{2}$$

where

$$\lambda_i = \frac{\partial v_i(p, w_i)}{\partial w_i}$$

is the marginal utility of income for consumer i.

Proof: From standard textbook results, see e.g., Green, MasColell and Whinston (1993), we have that the necessary conditions for maximum of $u_i(x_i')$ under the budget constraints $p \cdot x_i' \leq w_i$ (assuming an interior solution) are

$$u_{ih}' = \lambda_i p_h, \ h = 1, \ldots, l,$$

where λ_i is the Lagrangian multiplier of the constrained maximization problem. The last statement of the lemma is found by direct computation,

$$\frac{\partial v_i(p, w_i)}{\partial w_i} = \sum_{h=1}^{l} u_{ih}' \frac{\partial \xi_{ih}}{\partial w_i} = \sum_{h=1}^{l} \lambda_i p_h \frac{\partial \xi_{ih}}{\partial w_i},$$

where

$$\sum_{h=1}^{l} \lambda_i p_h \frac{\partial \xi_{ih}}{\partial w_i} = 1$$

as can be seen by differentiating the budget equation $\sum_{h=1}^{l} p_h \xi_{ih}(p, w_i) = w_i$ with respect to w_i. □

With the expression (2) we got rid of the unknown terms u'_{ih} but we have got the new terms λ_i, the interpretation of which is given by Lemma 1, but which still are not easily assessed from the data of the problem. To circumvent this problem, we need another assumption:

Assumption 2. *The distribution of incomes sustaining the consumptions at the initial allocation is optimal measured by the social welfare function.*

What Assumption 2 states is that the incomes w_i, $i = 1, \ldots, m$, which entered into the individual consumer's utility maximization problems above, cannot be redistributed in such a way that society's welfare increases, or, otherwise put, $(w_1, \ldots, w_m)$ maximizes $S(v_1(p, w'_1), \ldots, v_m(p, w'_m))$ on all $(w'_1, \ldots, w'_m)$ with

$$w'_1 + \cdots + w'_m = w_1 + \cdots + w_m.$$

Necessary conditions for maximum gives

$$S'_i \frac{\partial v_i}{\partial w_i} = \mu, \; i = 1, \ldots, m$$

where μ is a (positive) Lagrange multiplier, and we may now rewrite (2) as

$$dS = \mu \sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih}.$$

We summarize our findings in the following:

Lemma 2. *Under Assumptions 1 and 2, the change in social welfare has the form*

$$dS = \mu \sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih}. \tag{3}$$

This expression may be used for evaluation purposes: If the value at the prices p of the aggregate changes in consumption $\sum_{i=1}^{m} dx_i$ is positive, then by (3) society's welfare increases, and if $p \cdot \sum_{i=1}^{m} dx_i$ is negative, then society's welfare will decrease. Thus, we have a criterion for assessing the gain or losses of society of any displacement of the initial allocation.

However, for practical purposes it is usually inconvenient to check the consumption changes of each and every individual in society; it is much easier to check the changes in production. Fortunately, since only aggregate displacements matter, it almost amounts to the same thing. Formally, however, we need a final assumption (of a closed economy):

Assumption 3. *The change in aggregate consumption equals the change in aggregate production.*

Formally, this means that $\sum_{i=1}^{m} dx_i = \sum_{j=1}^{n} dy_j$. Inserting in (3), we get the expression

$$dS = \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh},$$

which can be used to check society's gain or loss from a change of allocation.

We state the result of our considerations in the following proposition:

Proposition 1. *Assume that the economy is initially at the allocation* $(x_1, \ldots, x_m, y_1, \ldots, y_n)$ *and that this allocation is subjected to a displacement* $(dx_1, \ldots, dx_m, dy_1, \ldots, dy_n)$. *Under Assumptions 1 – 3, the displacement is an improvement for society according to the social welfare function* S *if and only if*

$$dS = \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh} > 0. \tag{4}$$

The derivation above is a rather standard application of the general economic welfare theory and it can be found in the literature (e.g. Lessourne (1975)). It gives the theoretical background for project evaluation or cost-benefit analysis; it is however surprisingly little used in the health economic literature, where it can shed light on some of the controversies connected with the treatment of indirect costs in health economic appraisals.

3. Production gains: the human-capital and friction methods

In the previous section, we have assumed that *all* available commodities enter into the consumption of any consumer – or, more specifically, the consumption of each and every commodity enters into the utility function of each consumer. In practice, some commodities are *intermediate,* which means that they are made available for producers who use them as input while producing other commodities, and only the end products are used directly by the consumers.

Clearly, the arguments of the present section carry over without problems; the level of consumption of each intermediate good stays at 0 throughout the investigation, so it is immaterial whether or not the consumer would be able to buy such goods in the market (which, according to our Assumption 1, should be the case). We may consequently consider our Assumption 1 to pertain only to such commodities which do enter into the utility functions of the consumers. This means, among other things, that the evaluation based on the displacement of the consumption carried out by the individuals can be carried through *also* in the case where the initial allocation is not efficient (so that there is no system of efficiency prices on all commodities) but where the consumption side of the economy is still sustained by prices in the sense of Assumption 1.

However, for the replacement of consumption by production data, the prices on the intermediate goods matter. The equality of the values of aggregate consumption and production displacements will hold true only if the production prices are meaningful. We summarize these considerations in the following proposition.

Proposition 2. *Assume that the economy is initially at the allocation* $(x_1, \ldots, x_m, y_1, \ldots, y_n)$ *and that this allocation is subjected to a displacement* $(dx_1, \ldots, dx_m, dy_1, \ldots, dy_n)$. *Assume that the individual utility functions* u_i *depend only on consumption of the first* $k < l$ *commodities, that Assumption 1 is fulfilled with respect to the commodities* $h = 1, \ldots, k$*, and that Assumptions 2 and 3 are fulfilled.*

Then the social desirability of the displacement may be evaluated by the criterion

$$dS = \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}$$

if and only if the production prices reflect the value of the consumption displacement, that is iff $\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh} = \sum_{h=1}^{k} p_h \sum_{i=1}^{m} dx_{ih}$.

The proposition, which is a straightforward consequence of the definitions, is stated in detail since it sheds light on a famous controversy in the debate about cost-effectiveness analyses, that of evaluating the so-called *production gain.* This production gain arises as the result of a treatment, if the patient may return to productive work sooner than without the treatment and thus produces a benefit for society (often referred to as an indirect benefit to distinguish from the direct benefits in terms of improved health or quality of life). If the effect of a new treatment is evaluated from the production side, which indeed is the usual or rather the only feasible way of evaluating displacements in allocation, then (assuming that work has no utility per se) this additional work is an intermediate good and as such it is not covered by Assumption 1, at least in its relaxed version.

Since there are usually no obvious counterparts of (more or less) free markets for consumption goods pertaining to labor markets, the question arises of how to evaluate the above mentioned production gain. In the literature, two quite different methods are proposed: (1) the *human capital* approach amounts to using the actual wages paid in the labor market for the type of work in question, and (2) the *friction method,* according to which the production gain should be evaluated as replacement cost, the cost for society of obtaining the appropriate replacement for the person who has gone out of work due to illness. The differences in indirect benefits calculated according to the two methods are quite significant, so it does matter which one is used.

The proposition above gives us an answer to this: The evaluation of the production gain should be in prices which reflect the value of the additional consumption goods produced (in the market prices for consumption goods). In the friction method, labor services are measured in shadow prices (which are low if there are idle resources of labor), and if all the remaining prices are also measured in such true shadow prices (so that the price of each and any commodity reflects what it would cost society to provide one more unit), then all is well and the friction method is the right one. Usually, the prices on ordinary goods which can be observed are *not* scarcity or shadow prices, and then the friction method runs into an inconsistency. What happens is essentially the following: If the production gain should be measured in shadow prices of labor, then all the other displacements should similarly be measured in these prices according to their labor content (since "true" labor costs are much smaller than nominal costs), meaning that also the direct costs (of medicin, treatment etc.) should be reduced, unfortunately in a way which is largely unknown since the true scarcity costs of the commodities are not revealed by the market.

The fact that the friction method is not theoretically founded does not mean that the human capital method is correct. The crucial point is whether or not the prices on intermediate products, and among these the price of labor services, are such that the change in aggregate production reflects the change in aggregate consumption. The actual wages paid may not have this property; however, in one particular case, namely if all prices are competitive, meaning that the initial allocation is a market equilibrium at the prices $p_1, \dots, p_l$, the human capital method is $exactly$ the right one. Otherwise put, if one is willing to accept the initial allocation together with the price and wage structure observed and used in a cost-effectiveness analysis as a good approximation to an equilibrium of the market mechanism, then the human capital method does the right thing.

4. The foundation of cost-effectiveness analysis

Assume now that social welfare depends not only on achieved consumption levels, but also on certain health parameters of the individuals, so that the social welfare function has the form

$$S(h_1, \dots, h_m, u_1(x_1), \dots, u_m(x_m)),$$

where h_i is a real number, an indicator of the health level of individual i. In this context, the analysis of section 1 must be slightly modified: A project is now described not only by its consequences on consumption, $dx_1, \dots, dx_m$, but also by the changes $dh_1, \dots, dh_m$ in the health status of each individual which it gives rise to. With this modification, the change in social welfare is

$$dS = \sum_{i=1}^{m} \frac{\partial S}{\partial h_i} dh_i + \sum_{i=1}^{m} \frac{\partial S}{\partial u_i} \sum_{h=1}^{l} u'_{ih} dx_{ih}.$$

Proceeding as before (under Assumptions 1 – 3), we get the expression

$$dS = \sum_{i=1}^{m} \frac{\partial S}{\partial h_i} dh_i + \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}; \tag{5}$$

thus, the effects of the changes in allocation of ordinary commodities may (as previously) be evaluated by the change in aggregate value of (net) production. Unfortunately, (5) still contains the unobservable quantities $\partial S/\partial h_i$, for $i = 1, \dots, m$.

To get further beyond this point, we need further assumptions; an obvious first one is the counterpart of Assumption 2 with respect to the health status variables:

Assumption 4. *Health is optimally distributed initially, that is,*

$$\frac{\partial S}{\partial h_1} = \cdots = \frac{\partial S}{\partial h_m} = H > 0.$$

We remark that Assumption 4 might be more problematic than Assumption 2 since we have not specified a mechanism for redistributing health in society.

Using Assumption 4, we get that welfare gains for society can be expressed as

$$dS = H \sum_{i=1}^{m} dh_i + \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}. \tag{6}$$

Thus, welfare gains of society may be assessed as a weighted sum of the health gains and the changes in value of net production.

The two members of the right hand side correspond to the outcome and the cost side of a traditional cost-effectiveness analysis. However, the apparent cornerstone of cost-effectiveness analysis, the cost-effectiveness ratio, has been replaced by a weighted sum. And what is even worse, the weights, even the relative weights, are generally unknown, being society's marginal rate of substitution between health and consumption-related utility.

Thus, so far we have obtained that project evaluation may be reduced to evaluation of pairs (dh, dy) with

$$dh = \sum_{i=1}^{m} dh_i,\ dy = \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}. \tag{7}$$

According to (6), pairs (dh, dy) are ordered according to a linear criterion with unknown weights.

As long as the weights remain unknown, there is no simple criterion (comparable to (4) above) for deciding whether society gains or not from the project under consideration. However, a comparison between different projects may still be possible, at least under the additional assumption of scale invariance.

Proposition 3. *If two projects are given by displacements in health parameters* $dh_1^i, \ldots, dh_m^i$ *together with consumption and production displacements* $dx_1^i, \ldots, dx_m^i$, $dy_1^i, \ldots, dy_n^i$, *for* $i = 1, 2$, *from a given allocation* $(x_1, \ldots, x_m, y_1, \ldots, y_n)$, *then under Assumptions 1 – 4 project 1 is at least as good for society as project 2 in the sense that*

$$S(h_1 + t dh_1^1, \ldots, h_m + t dh_m^1, u_1(x_1 + t dx_1^1), \ldots, u_m(x_m + t dx_m^1)) \\ \geq S(h_1 + dh_1^2, \ldots, h_m + dh_m^2, u_1(x_1 + dx_1^2), \ldots, u_m(x_m + dx_m^2))$$

for some $t > 0$ *iff*

$$\frac{\sum_{i=1}^{m} dh_i^1}{\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}^1} \geq \frac{\sum_{i=1}^{m} dh_i^2}{\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}^2} \tag{8}$$

(that is project 1 yields at least as much health per dollar as project 2).

Proof: The comparison of project 1 and 2 amounts to comparing the two pairs (dh^1, dy^1), (dh^2, dy^2) written according to (7); for small enough displacements (technically, for (dh^i, dy^i) going to 0, we get from (6) that $dS^1 \geq dS^2$ if and only if (8) holds. $\square$

Proposition 3 gives the theoretical foundation for the celebrated cost-effectiveness ratio used widely in the literature. It should be stressed that the comparison according to cost-effectiveness ratios is less simple than what is usually assumed; projects are not compared directly, rather some (scaled) version of one project is compared to the other one. As a result, computing cost-effectiveness ratios does not directly answer the question whether one treatment is socially preferable to another (even given our assumptions); it does so only if the projects may be scaled arbitrarily. Unfortunately, this is very rarely the case; treatments can usually not be limited to a smaller class of patients than those suffering from the relevant disease, and even more obviously, they cannot be scaled upwards to include more patients than are actually available. This is a price which must be paid in order to compare two-dimensional vectors (given by effects in terms of aggregate health and by cost) without knowledge of society's weights upon each of the variables.

5. Cost-utility analysis and the problem of interpersonal comparisons

In the previous section, we assumed throughout that the health variable of the individual was a one-dimensional quantity. This, unfortunately, does not fit very well with real life, where health and displacement of health have several aspects which should be taken into consideration in an analysis of a new medical technology.

If individual health is measured as a vector $h_i = (h_{i1}, \dots, h_{ir})$ of different attributes of health, then the formalism in the previous section changes slightly; the change in social welfare $S(h_1, \dots, h_m, u_1(x_1), \dots, u_m(x_m))$ caused by displacements in health $dh_1, \dots, dh_m$ and in consumption $dx_1, \dots, dx_m$ should now be evaluated as

$$dS = \sum_{i=1}^{m} \sum_{k=1}^{r} \frac{\partial S}{\partial h_{ik}} dh_{ik} + \sum_{i=1}^{m} \frac{\partial S}{\partial u_i} \sum_{h=1}^{l} u'_{ih} dx_{ih}.$$

In order to proceed, we need Assumption 1 – 3, so that the last member on the right hand side may be replaced by $\mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}$, and assumptions pertaining to health variables allowing us to perform the same kind of reduction. Here we have a problem, however; it might be acceptable to assume that each of the different aspects of health is distributed optimally at the outset, so that

$$\frac{\partial S}{\partial h_{1k}} = \cdots = \frac{\partial S}{\partial h_{mk}} = H_k$$

for $k \in \{1, \dots, r\}$, but this still leaves us with r different variables, one for each of the relevant aspects of health.

In the literature, this problem has been taken care of by using indices for health which performs a consolidation of individual health variables into an index of overall health (or rather, quality of life), that is a rule I which transforms individual health status $(h_{i1}, \ldots, h_{ir})$ into a single variable $I_i = I(h_{i1}, \ldots, h_{ir})$. Assuming the existence of such a rule – and that social welfare depends on individual health only through the index values – we now get that the change in social welfare has the form

$$dS = \sum_{i=1}^{m} \frac{\partial S}{\partial I_i} dI_i + \mu \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh},$$

where $\partial S/\partial I_i$ denotes the marginal social welfare of an increase in the health index value of individual i. This expression still does not give us a cost-utility ratio criterion; we need the usual assumption of socially optimal distribution of health as measured by the health index:

Assumption 5. *Health as expressed by the health index I is optimally distributed initially, that is,*

$$\frac{\partial S}{\partial I_1} = \cdots = \frac{\partial S}{\partial I_m} = H > 0.$$

With Assumption 5 added to the our assumptions, we have the obvious counterpart of Proposition 3:

Proposition 4. *If two projects are given by displacements in the value of the health index $dI_1^i, \ldots, dI_m^i$ together with consumption and production displacements $dx_1^i, \ldots, dx_m^i, dy_1^i, \ldots, dy_n^i$, for $i = 1, 2$ from a given allocation $(x_1, \ldots, x_m, y_1, \ldots, y_n)$, then under Assumptions 1 – 3 and 5 project 1 is at least as good for society as project 2 in the sense that*

$$\begin{aligned} S(h_1 + tdh_1^1, \ldots, h_m + tdh_m^1, u_1(x_1 + tdx_1^1), \ldots, u_m(x_m + tdx_m^1)) \\ \geq S(h_1 + dh_1^2, \ldots, h_m + dh_m^2, u_1(x_1 + dx_1^2), \ldots, u_m(x_m + dx_m^2)) \end{aligned}$$

for some $t > 0$ iff

$$\frac{\sum_{i=1}^{m} dI_i^1}{\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}^1} \geq \frac{\sum_{i=1}^{m} dI_i^2}{\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}^2}. \tag{9}$$

While the result of Proposition 4 formally supports the use of cost-utility analyses in the literature, its assumptions are stronger than those made in the previous sections. The assumption about equality of marginal social welfare with respect to individual health index values is easy to formulate but not quite as easy to interpret. If individual i obtains a change in health characteristics $dh_i = (dh_{i1}, \ldots, dh_{ir})$, then the index changes by

$$dI = \sum_{k=1}^{r} I_k' dh_{ik},$$

where I_k' is the partial derivative of the index w.r.t. the kth health variable. By Assumption 5, all the displacements dh_i such that the weighted sum $\sum_{k=1}^{r} I_k' dh_{ik}$ is the same will give rise to exactly the same change in social welfare, meaning that locally the index is obtained as a weighted sum (with weights I_k') of the displacements in each health variable. Since the weights depend only on the health variables and not on the individuals, we have implicitly assumed that *all individuals evaluate the different aspects of health in the same way.*

While there can be little doubt that an assumption of this type is necessary in order to reduce a multidimensional problem to one with a single or at most two variables, it still remains quite strong. When dealing with the economic variables (consumption or production of the many different commodities) we, have, at least in principle, a mechanism which works in the direction of equalizing the weights by which these quantities are aggregated, namely the market. There is no such counterpart for the health variables (for an investigation along these lines, see Hougaard and Keiding, 1999) , so Assumption 4 remains a postulate without theoretical foundation or even a plausible intuitive foundation. Therefore, the result of Proposition 4 is of much less value than that of the previous section.

6. Production gains in cost-effectiveness analysis with explicit health variables

Having now discussed the foundations of cost-effectiveness and cost-utility analyses, we may return to the main theme, which was the evaluation of production gains. In the first approach, where health variables were assumed away, we saw that production entered only by replacement – the production gain was a simple way of approaching the gain in utility of consumption obtained by additional production.

The inclusion of health variables does not change this story, but it adds a further dimension to the problem; more specifically, Assumption 3 may not be quite reasonable since the production efforts in society could be directed towards health rather than commodities for consumption, so that aggregate consumption no longer corresponds to what is available in endowment or through production. The question arising in this case is whether in an evaluation which uses data from the production side of the economy we may run a risk of double-counting by including *both* the health gain *and* the production gain.

The advantage of having a formal model is that such questions may be answered unambiguously without appeal to plausible ad-hoc arguments. What matters in our model is the change in social welfare, which again is triggered by changes in individual health variables and in (utility of) consumption. Indeed, from the previous sections we know that what we look for can be expressed as

$$dS = \sum_{i=1}^{m} \sum_{k=1}^{r} \frac{\partial S}{\partial h_{ik}} dh_{ik} + \mu \sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih},$$

and if Assumption 3 – which states that changes in the aggregate consumption of any of the marketed goods must correspond to a change in aggregate net production of this good – we may still substitute $\sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh}$ for $\sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih}$. Nothing has changed by adding special health variables; as previously, the "production gain" is only a proxy for the increased amount of consumption goods which are made available for the population as a whole. In particular, there is no need to distinguish between patients who are net contributors and those who are net recipients and then evaluate the net production gain for each type (as proposed in Olsen and Richardson, 1999). Cost-effectiveness analysis and the concepts used in it all relate to society as a whole, and any assignment of gain and losses to single individuals is arbitrary.

Since the addition of non-economic "goods" or variables to the analysis does not change the approach, it might be tempting to reduce this case to the first one, treating the health variables as ordinary economic commodities for which there happens to be no markets (the particular features of health being individual and not exchangable have been abstracted from anyway by e.g. Assumption 5). In order to formulate this as a final result, we need to introduce some further structure on the problem. We assume that there is some connection between the health enjoyed by the individuals and the consumption and production carried out in the economy, described by the equation

$$\Phi(I(h_1), \ldots, I(h_m), x_1, \ldots, x_m, y_1, \ldots, y_n) = 0,$$

where Φ is a differentiable function, which gives the trade-off between economic activity and health. We have assumed that this trade-off depends only on the value of the health index discussed in the previous section; this assumption is, as we noted, not realistic, but since the additional assumptions are much worse in this respect, we have chosen it for simplicity of exposition.

Now we state a final assumption, which formally allows us to put a money value on a unit of the health index:

Assumption 6. *The initial allocation and the achieved levels of health $(I_1, \ldots, I_m)$ are socially optimal in the sense that there is no alternative way of producing and consuming in society so that overall social welfare, when health is taken into consideration, is greater.*

Proposition 5. *Assume that the economy is initially at the allocation $(x_1, \ldots, x_m, y_1, \ldots, y_n)$ and that its individuals enjoy levels $I_1, \ldots, I_m$ of the health index. If this allocation is subjected to displacements $(dx_1, \ldots, dx_m, dy_1, \ldots, dy_n)$ and $(dh_1, \ldots, dh_m)$, then under Assumptions 1, 2 and 4 – 6, there is a number $Q > 0$ such that these displacements represent an improvement for society according to the social welfare function S if and only if*

$$Q \sum_{i=1}^{m} dI_i + \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh} \geq 0. \tag{10}$$

Proof: By Assumption 6, the social welfare is maximized at the given allocation (including the health variables) under the additional constraint

$$\Phi(I(h_1), \ldots, I(h_m), x_1, \ldots, x_m, y_1, \ldots, y_n) = 0,$$

and from the first order conditions for constrained maximum we get that

$$\frac{\partial S}{\partial I_i} = \nu \frac{\partial \Phi}{\partial I_i}, \text{ all } i,$$
$$\frac{\partial S}{\partial u_i}\frac{\partial u_i}{\partial x_{ih}} = \nu \frac{\partial \Phi}{\partial x_{ih}} + \tau_h, \text{ all } i \text{ and } h$$
$$\tau_h = \nu \frac{\partial \Phi}{\partial y_{jh}}, \text{ all } j \text{ and } h$$

from which we get (using Assumptions 1, 2, and 5) that

$$\frac{H}{\mu p_h} = \left(\frac{\partial S}{\partial u_i}\right)^{-1} \frac{\partial S}{\partial I_i} = \left(\frac{\partial \Phi}{\partial x_{ih}} + \frac{\partial \Phi}{\partial y_{jh}}\right)^{-1} \frac{\partial \Phi}{\partial I_i}$$

for each commodity $h \in \{1, \dots, l\}$, showing that the marginal rate of substitution for society between aggregate health and consumption of commodity h has the magnitude $H/\mu p_h$.

Let $Q = H/\mu$, so that

$$\frac{Q}{p_h} = \left(\frac{\partial S}{\partial u_i}\right)^{-1} \frac{\partial S}{\partial I_i}.$$

Then the change in social utility associated with the displacements $(dx_1, \dots, dx_m, dy_1, \dots, dy_n)$ in allocation and $(dh_1, \dots, dh_m)$ in health may be evaluated as

$$dS = \sum_{i=1}^{m} \frac{\partial S}{\partial I_i} dI_i + \mu \sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih} = \mu \left[Q \sum_{i=1}^{m} dI_i + \sum_{h=1}^{l} p_h \sum_{i=1}^{m} dx_{ih} \right],$$

and using Assumption 3, we finally get that

$$dS = \mu \left[Q \sum_{i=1}^{m} dI_i + \sum_{h=1}^{l} p_h \sum_{j=1}^{n} dy_{jh} \right],$$

showing that the sign of dS is positive if and only if (10) holds. □

The result in Proposition 5 may be seen as a theoretical basis for cost-benefit analyses in problems involving health variables. Unfortunately, it has as yet little or no practical applicability; it states that there exists a correct "dollar per QALY" value which may be used in economic appraisals, but it gives no clue to the problem of finding this value. This should come as no surprise; it is just another way of stating that Assumption 6 may or may not be satisfied, but that we cannot point to an institutional framework inside which the assumption would be fulfilled as a result of the economic behavior of individuals or groups of individuals.

It might be noticed that Assumption 6 together with Assumption 1 tells us that we have an equilibrium where the prices paid by the consumers in the market reflect the true cost of society, even though we in the formulation of the health production relationship have opened up for "indirect" production of health via the individual consumptions. This shows once more that the result uses too restrictive assumptions to be of much practical use.

7. Concluding comments

In the present paper, we have considered the welfare theoretical basis for economic appraisals of drugs and treatments, with a special regard to the role of the so-called indirect costs, and in particular to the production gain. We found that this production gain enters because the change in allocation is easier to trace on the production side of the economy than by checking all individual consumption bundles. What matters is the change in health *and* consumption, the change in production is only of secondary importance. This insight helps us to understand why the so-called frictional method of evaluating the production does not work: The prices used should be the prices at which the individuals actually trade, so that they reflect the marginal rate of substitutions of the consumers. Shadow prices in the labor market do not reflect market choices; it might be argued that neither does the nominal wages, and this is correct, but the difference is much smaller.

The basic idea of our analysis has been to make precise the formal model in which we argue, so that practical rules of evaluation may be deduced from a few basic principles rather than argued on general considerations of intuition or consensus among experts. In order for such an approach to be succesfull, the model should be sufficiently well developed to capture all essential aspects of the situation, in our case economic appraisals of medicine and of medical treatments. In its present version, our model still has some limitations, among which the absence of externalities, which certainly are an important feature of health economics, in particular when society's evaluation of alternative technologies are involved. A model incorporating this and yielding conclusions as to the proper way of handling such externalities will remain a topic of future research.

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