

Empirical studies of price behavior in the Danish stock market

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Ph.D-serie 2.2001

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Empirical Studies of Price Behavior in the Danish Stock Market

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Ph.D.-Dissertation
March 2000

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Introduction

This dissertation includes an introduction, six chapters, a data annex and a Danish summary. The present introduction gives a motivation and a short summary of the studies presented in the following chapters.

Motivation

In recent years, we have witnessed large and persistent increases in stock prices in most of the stock markets in the OECD area including the Danish market. As always when stock prices change substantially, this experience has stimulated interest in stock markets in general and in the behavior of prices in particular. A key issue which has occupied both practitioners and academics has been the question whether the recent surge in stock markets is warranted by the underlying economic determinants of stock prices, that is, the economic 'fundamentals', or whether it represents a 'non-fundamental' phenomenon such as a (rational) bubble, fads or noise trading, see Campbell and Shiller (1998), Cochrane (1997), Kopcke (1997) and Cole *et al.* (1996) for just a few examples. An understanding of the 'fundamental' mechanisms underlying the determination of prices and returns in the stock market is a central prerequisite for settling this issue.

Being able to value stocks and predict future stock returns is obviously of interest to any investor facing the task of allocating his portfolio optimally between stocks and alternative securities such as e.g. bonds. Over the last decade and especially in recent years the state of the stock market has also received increased attention by policy makers, most notably in the US. Based on the Japanese experience of a prolonged recession that was triggered by a collapse of the stock and land markets in the early 1990s and the currency and financial crises in Southeast Asia in 1997-1998, policy makers appear to have recognized that financial markets play a crucial role in determining the outcome of the traditional policy goals of employment and production. In particular, the evidence suggests that the crash of an 'overvalued' stock market may potentially have severe negative impacts on the real economy. The concern that stock markets are at present 'overvalued' and that this may pose a risk for the outlook of the real economy has on several occasions been expressed by the Chairman of the US Federal Reserve System, Alan Greenspan. He has also

"...emphasized the increasing importance of this issue (asset prices) to policy makers and stressed the need for a better understanding of asset price determination..." (Sellon and Buskas, 1999, p. 6).

This dissertation is empirical and studies the economic mechanisms which determine the prices and returns in the Danish stock market. It consists of six independent and self-contained papers (chapters) which address selected issues relating to the formation of stock prices and returns, respectively. In each case, we formulate, estimate and test an *ad hoc* model which is suited for an analysis of the specific question raised. The overall purpose is to provide insight into how stock prices and returns are determined empirically while focusing in particular on the importance and relevance of underlying economic 'fundamentals' variables. For several of the papers, a main inspiration has been the recent debate whether or not stock markets are 'overvalued'.

We confine ourselves to the study of the aggregate market level of stock prices (and returns) rather than the pricing of individual stocks. Thus, we address the issue of the valuation of the market and examine price determinants that are common to all stocks rather than idiosyncratic. This macroeconomic approach is directly devoted to the question whether the market is 'overvalued' or not. Moreover, the pricing of the market appears to be the area within the field of empirical asset pricing where research is most needed. Despite the recent debate about its validity, the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) still seems to provide valuable information on how individual stocks are priced relative to the market, see the survey in Jagannathan and McGrattan (1995)¹. However, attempts at explaining the pricing of the market by models based on first best principles have until now proved less successful, see the survey in Cochrane (1997). The latter observation at the same time justifies a more *ad hoc* type approach to the issue of modeling stock prices at the aggregate market level.

¹ As a possible alternative to the CAPM, the empirically motivated three-factor model of Fama and French (1993) has been suggested to account for the 'anomalies' facing the CAPM.

The informational advantage inherent in studying the national market has been a main motivation for focusing on the Danish stock market. Admittedly, the Danish market is small by international standards. However, providing evidence for the Danish stock market is obviously relevant to any economic agent engaged in the market. Moreover, the literature is dominated by research on the US stock market. We consider it to be a valuable exercise to study stock markets other than the US market because this may give us a more general insight into how stock markets function. By the same token, our research may prove to have a more general applicability.

The individual papers consider the post-World War I-period or in some cases the period since World War II. For dividend payments on stocks and bond interest rates, we have used the recently constructed Danish data set in Nielsen, Olesen and Risager (1999). The official All-Share Stock Price Index by Statistics Denmark is used throughout for the level of stock prices. This index comprises a sample of Danish stocks listed at the Copenhagen Stock Exchange. Since 1983, nearly all listed Danish stocks have been included in the sample. All observations are annual. The data set was originally compiled for the period 1921-1995 and presented in a 1997-working paper version of Nielsen and Risager (1999a). A brief description of the data is provided in the annex of this dissertation².

The dissertation has an empirical orientation and it is within this area that the contributions should be found. The theoretical analysis is limited in scope and originality and only serves the purpose of providing a platform for the formulation of an empirical model. In particular, we keep the analysis simple in order to maintain a model that is tractable empirically. In the papers involved, the basis is the textbook Gordon Growth Model which is a simple partial equilibrium model for the price of a stock in the case of a constant growth in dividends and a constant discount rate, see Gordon (1962) or the outline in Campbell *et al.* (1997). We use a slightly modified version of this model where we have incorporated a time-varying discount rate.

² In Denmark, stocks are listed at the Copenhagen Stock Exchange. For an introduction to the Danish stock market, see the various issues of the Annual Report (in Danish) and the Fact Book by the Copenhagen Stock Exchange (can be downloaded from their homepage <http://www.xcse.dk>). The Fact Book includes a comprehensive set of key statistics on the stock market.

What have We Learned ?

This dissertation has provided a set of empirical insights which in concise terms are:

- Stocks hedge against inflation in the long run in the sense that the real value of stocks is immune to shocks to the general price level over long horizons. This property of stocks seems *apriori* as a reasonable hypothesis because increases in the general price level tend to increase firm profits proportionately in the long run. However, the empirical support to the hypothesis is not a standard result in the international literature (Chapter 1).
- The equity premium, defined as the excess of stock returns over bond returns, is predictable at the medium (5-year) investment horizon but not at the very short and the very long investment horizons (1- and 10-year). When forming a prediction of the future equity premium, it is important to take due account of multiple predictors and the possible non-stationarity of predictor variables (Chapter 2).
- In the long run, stock returns at the 5- and 10-year investment horizons are closely related to the bond yields at the same maturities (Chapter 3).
- Coinciding with the effective opening up of the Danish stock market to foreign investors in the early 1980s, stock returns have shifted from being characterized by a (relatively) low mean return and low volatility to a regime of a (relatively) high mean return and high volatility. Furthermore, in contrast to the early regime where returns were serially uncorrelated, the most recent regime is characterized by a significant degree of mean reversion (Chapter 4).
- We identify economic 'fundamentals' variables that are strongly significant in explaining the variation in the dividend-price ratio (the ratio of current dividends to current stock prices). The key 'fundamentals' variable is a time-varying discount rate which is decomposed into time-varying measures of the growth-adjusted real interest rate and the risk premium on stocks (Chapter 5).
- Firm profits and the nominal bond rate explain the long-run behavior of stock prices. In particular, the movements in the bond rate account for the bearish stock market period before the early 1980s and the subsequent bullish period. The empirical significance of the nominal bond rate could indicate that stock investors

suffer from 'money illusion' because rational asset pricing theories suggest the use of a real discount rate. However, another possibility is that the nominal rate is merely a good proxy for the latent and complex real discount rate used by a rational investor (Chapter 6).

- Our evidence does not suggest that the stock market is substantially 'overvalued'. The outlook for stock returns in the 5-year period 1998-2002 is poor by historical standards but does not suggest a market crash, as predicted by Campbell and Shiller (1998) and Engsted and Tanggaard (1998) (Chapter 2). Moreover, by late 1998 the level of stock prices is roughly in line with 'fundamentals' as the recent surge in the stock market can to a large extent be explained by the contemporaneous decline in the nominal bond rate (Chapter 6).

A Synopsis

The dissertation is organized by the order in which the chapters (papers) were originally written.

In Chapter 1 (the paper '*Stocks Hedge against Inflation in the Long Run: Evidence from a Cointegration Analysis for Denmark*'), we test the classical hypothesis that stocks provide a hedge against inflation. This issue is important to any investor who rationally cares about the *real* value of future wealth. Hence, if stocks hedge against inflation, the real value of a portfolio of stocks will not be subject to the uncertainty arising from uncertainty about future inflation. We focus explicitly on the long-run horizon where the hedge hypothesis *apriori* seems most relevant. In the existing literature, it is standard to test the hedge hypothesis in terms of the estimated effect of inflation on contemporaneous stock returns, see e.g. Fama and Schwert (1977), Gultekin (1983) and Boudoukh and Richardson (1993). We suggest a different approach where we focus on the long-run relationship between the level of stock prices and the general price level, as estimated by cointegration analysis. Using the cointegrated Vector Autoregressive (VAR) method of Johansen (1996) and the single-equation cointegration methods of Engle and Granger (1987) and Phillips and Loretan (1991), we estimate an *ad hoc* cointegrating relation which we interpret as a simple

structural model for stock prices in the long run. We then test the inflation hedge hypothesis in terms of the estimated coefficient to the general price level in this cointegrating relation. Using data for the post-World War II-period and measuring the general price level by the official Consumer Price Index (CPI), we find strong support to the hedge hypothesis in the long run, even in the narrow sense of a 'perfect' hedge which requires stock prices to respond one-to-one to innovations in the general price level. This evidence contrasts with the weak support (if any) found in the literature and also represents stronger and more reliable support than produced by standard methods. We argue that the cointegration approach has the advantage of allowing for a clear distinction between short-run dynamics and long-run properties of stock prices and, in particular, of allowing for a slow adjustment in stock prices to long-run equilibrium. The standard approach of linking stock returns to contemporaneous inflation over a prespecified investment horizon (implicitly) assumes that stock prices respond immediately to innovations in the general price level. Our approach has similarities to Ely and Robinson (1997) who also differ from the standard literature by testing the inflation hedge hypothesis in terms of the relationship between stock prices and the general price level. However, their inference is based on impulse response analysis rather than a parametric test on a cointegrating relation for stock prices. Moreover, their definition of an inflation hedge allows stocks to be an 'imperfect' hedge in the sense that stock prices increase by more than the general price level. If the hedge is 'imperfect', the uncertainty about future inflation leads to uncertainty about the real value of stocks. To our knowledge, the issue whether stocks provide a hedge against inflation has not previously been given a thorough examination for the case of Denmark. Bonnichsen (1983) is an informal study of the relationship between Danish stock returns and inflation in the period 1900-1982. However, he addresses the issue whether the purchasing power of a stock investment increases over long time periods, that is, whether the real return on stocks is positive at long investment horizons (and concludes this to be the case). While this may be interesting in itself, it does not answer the basic question whether stocks *hedge* against inflation. Thus, the latter is concerned with the issue how stock returns (or stock prices) *respond* to *changes* in inflation (or the general price level). In the light of the evidence presented

in Chapter 6, the inflation hedge result is discussed further at the end of this introduction.

Chapter 2 (the paper '*On the Predictability of the Danish Equity Premium*', which is joint work with Ole Risager) is inspired by the study of Campbell and Shiller (1998). Based on the level of stock prices by early 1997, they predict that the US stock market will suffer a *de facto* market crash (however, without specifying the horizon). This study has been replicated for the Danish stock market by Engsted and Tanggaard (1998) and their prediction is almost as gloomy as the forecast by Campbell and Shiller (1998). Hence, they conclude that real stock prices were 'overvalued' by roughly 50% by late 1996 and that we should expect a decline in real stock prices by the same magnitude. Both studies are based on the observation that the dividend-price ratio of stocks is at a level (far) below its historical mean and that this always has been followed by a decline in stock prices in a historical sample. To provide further evidence on the outlook for the stock market, we conduct a thorough study of whether Danish stock returns are predictable. We focus on the prediction of the equity premium, defined as the difference between stock returns and bond yields, at three different investment horizons (1, 5 and 10 years). We test the predictive power of a comprehensive list of financial ratios, interest rates etc., including the dividend-price ratio. In order to exploit all available information, we take a multi-variable approach where we consider all candidate predictors simultaneously and subsequently identify the predictors that are most significant, using a 'general-to-specific' procedure. This leads to a parsimonious predictor model for the equity premium based on multiple predictors. Moreover, we take due account of the (non-) stationarity properties of the predictor variables to ensure that the use of standard regression methods is valid. Using data for the period since World War I, we find that the 5-year equity premium is predictable whereas the premia at the 1- and 10-year horizons are not. This result is consistent with the numerous findings for other stock markets which show that stock returns are predictable at the medium investment horizons, see e.g. Campbell *et al.* (1997, Chapter 7). However, we also conclude that it is important to take account of multiple predictors and the possible non-stationarity of the predictor variables when forming a prediction. Thus, using the 5-year model to

forecast the equity premium in the period 1998-2002, the outlook for the stock market is poor by historical standards but does not suggest a market crash, as predicted by Campbell and Shiller (1998) and Engsted and Tanggaard (1998). The latter studies focus exclusively on the dividend-price ratio which in the Danish case, furthermore, is a non-stationary variable due to a persistent shift towards a lower level of the ratio in the beginning of the 1980s. The latter is related to institutional changes in the Danish economy. Thus, we should not expect the dividend-price ratio to revert to its historical mean.

Chapter 3 ('*On the Relationship between the Danish Stock and Bond Market in the Medium and Long Term*', joint with Ole Risager) is a short paper which studies the empirical relationship between the realized returns in the stock and bond markets at the 5- and 10-year investment horizons, respectively. Using the cointegrated VAR method of Johansen (1996) and data covering the post-World War I-period, we find strong evidence of cointegrating relationships between stock returns and bond yields at both horizons. This result reflects that the stock and bond returns tend to move closely over long periods of time. Hence, an increase in the bond rate tends to be followed by an increase in the stock return at the 5- and 10-year investment horizon, respectively, and *vice versa*. The estimated cointegrating relation can be interpreted as a no-arbitrage relation between the stock and bond markets, saying that the stock return equals the bond yield plus a constant risk premium. This no-arbitrage relation relates to the long run, that is, the relation may not necessarily hold year-by-year but it serves as an 'attractor' for the long-run movements in stock and bond returns. Results suggest that the coefficient to the bond yield is less than one, implying that the stock return only partially reflects changes in the bond yield. The point estimates are well below one and the deviation from a unitary coefficient is statistically significant at the 10-year investment horizon. We conjecture that this result is due to taxation since bond yields are usually taxed at a higher rate than stock returns.

In Chapter 4 ('*Regime-Switching Stock Returns and Mean Reversion*', joint with Steen Nielsen), we estimate a time-series model for the 1-year nominal stock return. The purpose is to test whether the return process has changed over time in terms of its

mean, volatility or degree of serial correlation. Following Hamilton (1990), we estimate a well-specified two-state regime-switching model which allows for an autoregressive specification of order 1 (AR(1)) in each of the regimes. The model identifies two distinct regimes in the post-World War I-period. These are characterized by a low mean return and low variance and a high mean return and high variance, respectively. Except for a few short episodes, the low return-low volatility regime dominated until the early 1980s whereas the subsequent period has been characterized by a high return and high volatility. The shift in the return generating process in the early 1980s coincides with the effective opening up of the Danish stock market to foreign investors. We also develop an alternative test of mean reversion or, more generally, of first order serial correlation in returns. The test explicitly allows for multiple (two) return regimes defined as subperiods with potentially different constant, autoregressive or variance terms in the AR(1) specification for returns. Using this test procedure for the entire sample, we find mean reversion at the 10% but not at the 5% significance level. This represents much weaker evidence of mean reversion than produced by the standard autoregression test introduced by Fama and French (1988). Our finding that the allowance for multiple regimes results in less support for mean reversion is consistent with Kim and Nelson (1998). Furthermore, when analyzing the contributions by the two regimes, we find that the indication of mean reversion is due to the recent high return-high volatility regime. Thus, the regime shift in the early 1980s also implied a shift from a state where returns were serially uncorrelated to a state with a significant negative serial correlation in returns. Our conclusions on the existence of mean reversion parallel the findings in Risager (1998). Using standard variance ratio and autoregression tests, he also finds weak support of mean reversion³. Based on a split of the sample into subsamples, he concludes that mean reversion has been stronger in the most recent part of the sample, that is, since the 1970s. The regime-switching method has previously been applied by Kim and Nelson (1998) and Kim, Nelson and Startz (1998) to take account of regime-shifts in the volatility of US stock returns. Our approach differs by including an

³ For a recent survey of the evidence on mean reversion for the Danish stock market, see Nielsen and Risager (1999b). This survey also includes a comparison with results for the US stock market.

autoregressive term in the return generating process and by allowing for regime-shifts in the mean returns. Both features are shown to be relevant for Danish stock returns.

Chapter 5 ('Modeling the Dividend-Price Ratio: The Role of Fundamentals Using a Regime-Switching Approach'), joint with Steen Nielsen) focuses on the dividend-price ratio of stocks, defined as the ratio of current dividends to current stock prices. This ratio is often used to determine whether stocks are 'fairly' valued by comparing the prevailing level of the ratio to its historical mean and using the latter to indicate the 'fair' level of stock prices. The basic idea in this paper is that we need to take account of the 'fundamentals' variables that influence the dividend-price ratio if we want to extract information about the 'fair' level of stock prices. Thus, a dividend-price ratio which is (say) low by historical standards does not necessarily imply that stocks are 'overvalued' but could, for instance, reflect a low real interest rate. To distinguish between these two possibilities, we need an economic model for the dividend-price ratio. The purpose and contribution of this paper is to identify economic 'fundamentals' variables which are significant in explaining the variations in the dividend-price ratio. We estimate an empirical model using time-varying measures of the growth-adjusted real interest rate and the risk premium on stocks relative to bonds as underlying 'fundamentals'. Together, these two components reflect a time-varying discount rate. The model includes real dividends and the lagged dividend-price ratio as additional explanatory variables. The former allows for the possibility that stock prices respond less than proportionately to innovations in dividends while the latter takes account of a possible slow adjustment in stock prices to long-run equilibrium. Estimating the model over the post-World War I-period, we find that it suffers from structural breaks. We therefore re-estimate the model, using the two-state regime-switching approach of Hamilton (1990) to capture latent and non-modeled structural changes in the economy. Tentatively, these structural changes could be changes in taxation rules or the institutional set-up of the stock market. We find that all 'fundamentals' variables are strongly significant in at least one regime and that the coefficients have the expected signs. Moreover, we obtain a good fit to the dividend-price ratio. The model identifies two very persistent regimes which cover the subsamples 1927-1949 and 1986-1991 and the subsample 1950-1985,

respectively. The regimes are characterized by a 'low' and a 'high' dividend-price ratio, respectively. The high persistence of regimes is consistent with the interpretation that structural changes in the economy account for the regime shifts. We argue that a change in the taxation of pension funds is a possible explanation of the regime shift in 1986. Related literature is Driffill and Sola (1998) who estimate a regime-switching model for US stock prices, assuming a constant discount rate. The existence of multiple regimes for stock prices is motivated by the evidence of multiple regimes in the process for dividends. Our study differs by focusing on the issue of modeling the dividend-price ratio from underlying 'fundamentals' variables, in particular, the time-varying discount rate.

In Chapter 6 ('*A Simple Explanation of Stock Price Behavior in the Long Run: Evidence for Denmark*'), we model the long-run behavior of stock prices, that is, we explain the trends in stock prices over 'long' periods over time. The motivation is two-fold. First, we want to address the question whether or not the recent surge in the stock market can be accounted for by economic 'fundamentals' underlying the prices of stocks? Second, we want to explain the observation that the Danish stock market after World War II can be divided into a *bearish* period (defined as a period with capital gains below average) before the early 1980s and a subsequent *bullish* period (a period with capital gains above average)⁴. We formulate and estimate an empirical long-run model for stock prices which is based on two macroeconomic 'fundamentals' variables, firm profits and the nominal bond rate. Using both multivariate and univariate cointegration methods, we find a stable and strong cointegrating relation between stock prices and these two 'fundamentals' variables. The latter are highly significant in both statistical and economic terms. Moreover, the model gives a fairly good account of the long-run behavior of stock prices. Growth in profits drives the long-run trend in stock prices whereas the bond rate explains the observed large deviations in stock prices from their long-run trend. The bond rate explains the bearish and bullish subperiods of the stock market as it increased in the period before the early 1980s and declined subsequently. Furthermore, the recent

⁴ Our use of the terms 'bearish' and 'bullish' may differ from other definitions in the literature, but they are just short-hand expressions for periods with stock price increases below average and above average, respectively.

surge in the stock market can largely be accounted for by the contemporaneous decline in the bond rate. By late 1998, stock prices are close to their 'fundamental' levels as determined by the prevailing levels of profits and the bond rate and the historical relationship between stock prices and these two variables. We have included the nominal bond rate as the discount rate used by stock investors because it provides a better empirical account of actual stock price behavior than a set of common proxies for a real discount rate. This could suggest that investors suffer from 'money illusion', that is, that they pay more attention to nominal magnitudes than a rational investor would do. The possibility of 'money illusion' has also been noted for the US stock market by Modigliani and Cohn (1979) and Ritter and Warr (1999)⁵. Ritter and Warr (1999) explains the US bull market since 1982 by the observation that inflation has declined and that this has reduced an 'undervaluation' of stocks arising from 'money illusion'. This parallels our explanation of the Danish bull market because the decline in the nominal bond rate since the early 1980s to a large extent reflects a decline in inflation. However, the empirical significance of the nominal bond rate could also just be the result of a high correlation between the nominal rate and the latent and complicated *after-tax and risk-adjusted ex ante real discount rate* used by a rational investor. Results do not differentiate between the 'money illusion' and the 'proxy' explanation. We consider our long-run model for stock prices to be a macroeconomic alternative to the traditional valuation models for stocks which are based on financial and accounting ratios such as e.g. the dividend-price ratio. In spirit, it is closely related to a model presented in a recent market study by a major Danish bank, see Valgreen (1999). He estimates a long-run relation for stock prices using nominal GDP and a nominal bond rate as explanatory variables. This formulation comes close to ours. The main differences are that we take account of variations in the profit share and include a trend as a proxy for the growth in the nominal value of outstanding stocks. Valgreen (1999) estimates the relation for Denmark over the period from 1981 to (early) 1999. However, using the two-step procedure of Engle and Granger (1987), the estimated relation is rejected as a cointegrating relation at any conventional significance level (possibly because of the short sample).

⁵ Recent studies that address the possibility of 'money illusion' in a different context include Shiller (1999), Shafir *et al.* (1997) and Canner *et al.* (1994).

A Remark on Inflation Hedge and Money Illusion

As standard in the literature, the conclusion that stocks hedge against inflation in the long run (Chapter 1) is based on a *partial* definition of an inflation hedge. That is, stock prices are shown to respond one-for-one to a change in the general price level in a situation where all other explanatory variables (in Chapter 1, real production and the real discount rate) are kept constant. In interpreting the inflation hedge result, it is useful to distinguish between two scenarios, viz. a permanent change in the *general price level* and a permanent change in the *rate of inflation*. The result implies that real stock prices are unaffected in the long run in both scenarios *provided that* the remaining explanatory variables are unaffected. The latter seems as a reasonable assumption as long as the remaining variables are real magnitudes such as real production and a real discount rate. Hence, it is standard in most of the macroeconomic literature to assume that real variables are unaffected by nominal variables in the long run. This is basically the property of 'Classical Dichotomy' between the real and the money sectors which is the cornerstone of the Neoclassical-Keynesian Synthesis of traditional macroeconomic theory, see e.g. Grandmont (1988). The evidence that the nominal bond rate is important in explaining actual stock price behavior may be interpreted as an indication of 'money illusion' on behalf of stock investors (Chapter 6). *If one accepts* 'money illusion' as a behavioral assumption, this will in general inflict on standard results in macroeconomic theory including a violation of the 'Classical Dichotomy' property. 'Money illusion' also has implications for the interpretation of the inflation hedge property of stocks. If we consider the scenario where *the rate of inflation* is subject to a permanent (say) increase, we should *a priori* expect an increase in the nominal bond rate in the long run as implied by the Fisher Parity. In the case of 'money illusion', investors will use this higher nominal bond rate as their discount rate when pricing stocks and real stock prices will as a result decline in the long run. Hence, the real value of stocks will *not* remain unaffected by changes in the inflation rate⁶. However, in the scenario of a permanent change in the *general price level*, it seems as a reasonable assumption that the nominal bond rate is unaffected in the long run even in the case of 'money

⁶ In the case of 'money illusion', we might observe an additional long-run impact on real production which also has implications for real stock prices.

illusion'. As a consequence, real stock prices will remain unaffected. Thus, we should not expect 'money illusion' to be detrimental to stocks' purchasing power when considering changes in the general price level. On the other hand, *if we do not accept* the assumption of 'money illusion' but rather interpret the explanatory power of the nominal bond rate as the result of a high correlation with the latent real discount rate used by a rational investor, stocks *will* as before retain a stable real value in both scenarios. This suggests a possible reconciling interpretation of the inflation hedge result. Stocks will in the long run retain a stable purchasing power in the event of inflation as long as we consider variations in the general price level or the rate of inflation within a given inflation regime, defined as a regime where the long-run average inflation rate is fixed. When we consider shifts in the inflation regime, that is, permanent changes in the long-run level of inflation, the response in real stock prices depends on stock investors' discount rate and, in particular, how it adjusts to the regime shift. If investors suffer from 'money illusion', real stock prices will change. If investors are rational, stocks will retain their purchasing power.

A Final Perspective

The Danish stock market has not received much attention in the academic literature in the past. However, in recent years several studies of the Danish stock market have appeared, including Lund and Engsted (1996), Engsted and Lund (1997), Risager (1998), Nielsen (1998), Nielsen and Risager (1999a, 1999b), Engsted and Tanggaard (1999) and the papers collected in this dissertation.

In general, this dissertation contributes by providing original empirical results for the Danish stock market. These results give an empirical insight into the question of what determines stock prices and returns at the aggregate market level in Denmark? The specific contributions are indicated in the Synopsis but discussed more comprehensively in the individual chapters. These also include a more detailed account of the related literature.

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Chapter 1

Stocks Hedge against Inflation in the Long Run:

**Evidence from a Cointegration Analysis for
Denmark**

Stocks Hedge against Inflation in the Long Run: Evidence from a Cointegration Analysis for Denmark *

by

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Abstract

We suggest an alternative approach to testing whether stocks provide a hedge against inflation in the long run. Based on a simple structural model, we test the hedge hypothesis in terms of the long-run linkage between stock prices and the general price level, as estimated by cointegration analysis. Using data for the Danish stock market over the post-World War II-period, results give strong support for the hedge property, defined in the narrow sense of a perfect hedge. This contrasts with the weak support found in the literature and also represents stronger support than produced by standard methods. We argue that our approach has the advantage of allowing for a clear distinction between short- and long-run dynamics of stock prices which adjust slowly to long-run equilibrium.

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1. Introduction

Stocks are said to provide a hedge against inflation if they compensate investors completely (and not by more) for increases in the general price level through corresponding increases in nominal stock returns, thereby leaving real returns unaffected. That is, stocks hedge against inflation if their real value or purchasing power is immune to changes in the general price level.

Whether or not stocks hedge against inflation is relevant to any rational investor who cares about real wealth. The above definition is one of a perfect hedge as it demands a one-for-one compensation for inflation. This contrasts with the weaker notion of an imperfect (or partial) hedge, as often encountered in the literature, which requires the relation between nominal stock returns and inflation (or equivalently, between nominal stock prices and the general price level) to be significant and positive but it may be less or larger than one-for-one. However, with an imperfect hedge, the real value of a portfolio of stocks is subject to uncertainty due to the uncertainty about future inflation. This is not the case when the hedge is perfect. As we interpret an inflation hedge as a device of eliminating the uncertainty deriving from inflation uncertainty, we shall throughout use the term in its most restrictive sense of a perfect hedge.

A priori it can be argued that stocks should provide a hedge against inflation, at least in the long run where firms' profit margins can reasonably be assumed to be fixed. The argument is that stocks are claims on current and future profit opportunities which in the long run (with profit margins being fixed) increase with the general price level in relation one-for-one, that is, in the long run stocks are basically claims on real profit opportunities. As a result, we should expect the real value of stocks to remain unaffected by inflation and, hence, stocks should hedge against inflation in the long run. What happens in the short run is, on the other hand, more ambiguous because slow adjustment in output prices and real production imply that profit margins may be significantly affected by inflation.

Whether stocks also provide a hedge against inflation empirically has been studied extensively in the literature, see e.g. Fama and Schwert (1977), Gultekin (1983), Boudoukh and Richardson (1993), Ely and Robinson (1997) and Barnes *et al.* (1999). With the only exception of Ely and Robinson (1997), cf. below, the literature has based its inference on return regressions where nominal stock returns are regressed on inflation and possibly further explanatory variables such as real production growth and changes in a relevant discount rate measure. The inflation hedge hypothesis is then put to a test by testing whether the coefficient to inflation is significant and equal to 1¹. Results of the literature are fairly mixed, but a general conclusion is that stocks do not hedge against inflation in the short run (investment horizons less than 1-2 years), where inflation usually turns out to have an insignificant effect on stock returns. In fact, at short horizons the estimated relation between nominal stock returns and inflation may even be negative, see e.g. Fama and Schwert (1977) and Gultekin (1983). There is some evidence of a significant positive relationship on longer horizons (more than 2 years) but often with a coefficient different from 1 so that the inflation hedge is not perfect, cf. Boudoukh and Richardson (1993). Hence, the hedge hypothesis comes closer to receiving support at longer horizons but the evidence is still weak. On balance it therefore seems that the empirical evidence tends to reject the hypothesis of stocks providing a (perfect) hedge against inflation.

This paper tests the inflation hedge hypothesis for stocks by taking a different approach to that used in the literature. We test the hypothesis by focusing on the long-run relation between stock prices and the general price level rather than the relation between stock returns and inflation. Most importantly, this shift of focus allows us to take account of slow adjustment in stock prices in the event of inflation. The latter is from the outset precluded in the standard return regressions approach which

¹ Some studies frame the test in terms of real rather than nominal stock returns, testing whether inflation has a significant influence on real stock returns, see for instance Fama (1981) and Kaul (1987). A survey of the literature including a detailed account of the empirical results is provided by Frennberg and Hansson (1993). The latter study at the same time represents an exception in the literature as the authors conclude that Swedish stocks provide a hedge against inflation even at fairly short horizons (down to one month). Another survey of the literature can be found in Sellin (1998). He concludes that "Stocks seem to be a good hedge against both expected and unexpected inflation at longer horizons" (Sellin 1998, p. 25). However, this conclusion is based on an imperfect hedge definition, which allows stock prices (or returns) to respond more than proportionately to shocks to the general price level (or to inflation).

(implicitly) assumes that stock prices adjust completely to inflationary shocks over the prespecified, fixed investment horizon, see section 5 below for a further discussion. We focus explicitly on the long-run horizon where the fixed-profit-margin assumption underlying the hedge hypothesis *a priori* seems most relevant. We proceed as follows. Motivated by a simple theoretical framework, we formulate a structural model for stock prices which includes the general price level, real production and stock investors' discount rate as explanatory variables. We identify the long-run relationships between the variables by cointegration analysis, using the cointegrated VAR-model, see e.g. Johansen (1996). We estimate a cointegrating relation for stock prices and, finally, test the inflation hedge hypothesis by testing whether this relation implies a one-for-one relationship between stock prices and the general price level.

We test the hypothesis for the market portfolio of Danish stocks, using annual data from 1948 to 1996. While the sample may be considered small in terms of the number of observations, the sample period spans many years which is crucial for the analysis of "the long run". In the empirical analysis, we use small sample versions of tests whenever possible. Moreover, we check the robustness of results from the cointegrated VAR model by also using single-equation-cointegration-methods to test the hedge hypothesis.

Our approach has similarities with that of Ely and Robinson (1997) who also differ from the standard literature by focusing on the relation between stock prices and the general price level in testing the inflation hedge hypothesis. Ely and Robinson (1997) test the hypothesis for 16 OECD countries, based on impulse response analysis in a cointegrated VAR model with 4 variables - stock prices, the general price level, real production and money supply. They find for almost all countries that stocks overcompensate for inflation and conclude, using an imperfect hedge definition, that stocks hedge against inflation. However, using the more restrictive definition of a perfect hedge, the evidence in Ely and Robinson (1997) does not give support to the hedge hypothesis.

Our approach differs from Ely and Robinson (1997) in several ways. First of all, we differ in the definition of an inflation hedge. In addition to the use of a perfect rather than an imperfect hedge definition, we define an inflation hedge in terms of the 'partial' sensitivity of stock prices wrt. the general price level within the context of a structural model for the former. Thus, we address the question: What happens to stock prices in the event of shocks to the price level, all other factors (real production and the discount rate) kept constant? Ely and Robinson (1997), on the other hand, examine the response in stock prices within a VAR model which we interpret as a reduced form model for stock prices and the price level where real production and the money stock are the 'driving' (exogenous) variables². Hence, they address the question: What happens to stock prices in the event of shocks to the price level, when other factors (e.g. real production and the discount rate) are allowed to vary? Our *ceteris paribus* definition of an inflation hedge resembles that used in the literature of return regressions.

Second, we test the hedge hypothesis in terms of a cointegrating relation for stock prices and, hence, do not rely on impulse response analysis as in Ely and Robinson (1997). This may be viewed as an advantage, given the critique raised by e.g. Faust and Leeper (1997), who show that results from impulse response analysis depend crucially on the assumptions needed to identify the underlying structural shocks of the VAR model. This may question the robustness of results derived from impulse response analysis. Moreover, by focusing on the cointegrating relation, we can perform an explicit parametric test of the hedge hypothesis instead of the 'qualitative' test criteria used in Ely and Robinson (1997)³.

Finally, we can test whether the underlying framework of our approach - the structural model for stock prices - is reasonable empirically by testing whether it is

² Ely and Robinson (1997) do not provide a theoretical foundation for their VAR model.

³ Based on the impulse response analysis, Ely and Robinson (1997) test the hedge hypothesis at a qualitative level, concluding that "In those cases where the impact on stock prices is significantly positive (negative) and/or where the impact on goods prices is significantly negative (positive), stocks offer (do not offer) a hedge against inflation in the sense that the relative value of stock prices to goods prices rises (falls)" and "Stocks can also be said to offer a hedge in those cases where neither stock price nor goods price innovations are statistically significant", Ely and Robinson (1997, page 151).

validated as a cointegrating relation. This turns out to be the case, implying that we can have (some) confidence in the framework underlying the test of the hedge hypothesis. For instance, the evidence of cointegration suggests that we do not lack an important variable in modeling the long-run linkages between stock prices and the general price level. Such a validity test of the underlying framework is not (directly) possible in the approach of Ely and Robinson (1997).

Compared to the existing literature, the contribution of the paper is three-fold. First of all, we suggest an alternative approach to testing the inflation hedge hypothesis. Second, it turns out that results give strong support to the hypothesis which contrasts with the weak support found in the literature. Third, the paper provides results for Denmark, a case which to our knowledge has not been examined thoroughly before⁴.

The paper is organized as follows. In section 2 an operational empirical model for the long-run is formulated. Section 3 reviews the data and section 4 reports the empirical results. Section 5 concludes the paper with a summary and a comparison of our approach with that used in the literature.

2. An Empirical Model For The Long Run

We formulate an empirical structural model for stock prices based on a simple theoretical framework that links stock prices to the general price level. The framework is *ad hoc* and rests on a set of assumptions which are restrictive but facilitate the formulation of an empirically tractable model. We focus on the long-run horizon with the objective of a model that can act as a good approximation to the long-run movements in stock prices. This provides us with a sound empirical (and a theoretical) foundation for testing the inflation hedge hypothesis in the long run. Whether the model actually is a good approximation, is tested as part of the empirical

⁴ Bonnichsen (1983) is an informal study of the relationship between Danish stock returns and inflation in the period 1900-1982. He examines whether the nominal stock return exceeds inflation at long investment horizons, that is, whether the real return at long horizons is positive, and concludes this to be the case. However, this evidence does not address the basic issue whether stocks hedge against inflation. The latter requires an analysis of how stock returns (or stock prices) respond to changes in the inflation rate (or the general price level). Thus, *a priori* the real return on stocks may still be positive in a situation where the nominal stock return does not respond to changes in the inflation rate, that is, in a situation where stocks do not hedge against inflation.

analysis by testing whether it can be validated as a cointegrating relation for stock prices.

The starting point is the usual 1-period no-arbitrage relation between stocks and bonds under the assumption of perfect capital markets. Excluding risk premia, this relation demands that the expected 1-period holding return on stocks, consisting of a capital gain and a dividend yield, is equal to the 1-period return (yield-to-maturity) on bonds:

$$(1) \quad \frac{Q_{t+1}^e - Q_t}{Q_t} + \frac{D_{t+1}^e}{Q_t} = B_t$$

where Q_t is the (ex dividend) stock price per share at time t , D_{t+1} is the dividend payment per share during period $t+1$ and B_t is the 1-period bond return as of time t . Superscript "e" denotes expectations on unknown future variables. The stock is assumed to be a claim on a representative firm (in our case representative for all firms listed at the Copenhagen Stock Exchange).

We shall assume that investors only form point expectations on future variables, i.e., that 'Certainty Equivalence' applies, and that investors, furthermore, expect bond returns to be constant over time. This, and the exclusion of rational bubbles, gives the forward-looking stock price solution⁵:

$$(2) \quad Q_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+B_t} \right)^{i+1} D_{t+i+1}^e$$

which determines the stock price as the expected discounted value of all future dividend payments.

⁵ Assuming that the forward-looking stock price solution exists, i.e. that dividend payments are expected to grow at a rate less than B_t .

Now make the following assumptions:

- (A1) Constant profit margin π^* , i.e., profits $\Pi_t = \pi^* P_t Y_t$
 (A2) Output price P_t and real production Y_t are expected to grow at constant growth rates g_p and g_y , respectively, i.e.

$$P_T^e = P_t(1+g_p)^{(T-t)} \quad \text{and} \quad Y_T^e = Y_t(1+g_y)^{(T-t)} \quad (T \geq t)$$

- (A3) All profits are paid out as dividends each period, i.e. $D_t = \Pi_t$

P_t is the price of the firm's product, Y_t is the production of the same product and Π_t denotes total 'profits' of the firm, assumed to be a constant fraction π^* (the profit margin) of the value of production. Π_t should be interpreted as the earnings that the firm generates to stock holders and could, to be specific, be defined as the value of production (value-added) less labor costs, accounting for 'pure' profits in a firm without capital, respectively, 'pure' profits plus capital rent in a firm with capital. By assuming a constant profit margin, our basic working hypothesis is that any fluctuations in the profit margin are purely short-run (business-cycle) phenomena which are eliminated in the long run. In particular, we assume that any changes in the relative prices between output and inputs (e.g. real wages and real oil prices) are either reversed or validated by average productivity changes in the long run. In theory, the assumption of a constant profit margin will, for instance, hold for a perfectly competitive firm with a Cobb-Douglas production technology, in which case the capital income share (taken to be the profit margin) is fixed. The assumption may also be justified (for the long term) by empirical observations, as evidence suggests that the aggregate profit share in the Danish economy has been fairly stable over the period since World War II⁶. g_p and g_y , finally, denote the expected inflation rate and the expected real growth rate. We assume that the number of shares in the firm is constant over time and normalize it to 1.

⁶ A possible measure of the aggregate profit share is the ratio of Gross Operating Surplus to GDP at factor cost (both in current prices), as defined by the National Account Statistics. Using annual data for the private sector over the period from 1948 to 1996, this ratio has varied within the narrow interval between its low of 38% in 1980 and its high of 49% in 1951.

From (A3) and the expected dynamics of profits implied by (A1) and (A2), the solution for stock prices becomes:

$$(3) \quad Q_t = \frac{\Pi_t}{R_t} = \frac{\pi^* P_t Y_t}{R_t}$$

where

$$(4) \quad R_t \equiv \frac{1 + B_t}{(1 + g_p)(1 + g_y)} - 1 \approx B_t - g_p - g_y \text{ (for } g_p \text{ and } g_y \text{ "small")}$$

R_t is the (*ex ante*) growth-adjusted real discount rate, defined as the nominal bond return adjusted for expected inflation and expected real growth. (3) is basically just a variant of the standard Gordon-growth-formula for the price of a stock with a constant discount rate and constant dividend growth, cf. e.g. Campbell *et al.* (1997). (3) only differs by allowing for time-variation in the discount rate and by having replaced dividends by profits.

We shall say that stocks provide a hedge against inflation if shocks to the general price level result in proportional changes in stock prices when controlling for other relevant factors. Our simple framework highlights why we should expect stocks to hedge against inflation in the long run. Thus, consider a shock to current prices P_t , reflecting the outcome of past inflation. Such a shock translates *ceteris paribus* into a proportional change in the value of production ($P_t Y_t$) and - due to the constant profit margin - profits (Π_t). Because prices and production are expected to grow over time at fixed (unaffected) rates, expected future profits, likewise, change proportionally. As a result, current stock prices (Q_t) change proportionally, confirming the hedge property, cf. also (3). Note the *ceteris paribus* (or partial) content of the hedge property as real production and the discount rate are held fixed in the argument⁷.

⁷ The literature using the return regressions approach also controls for other relevant factors by regressing stock returns not only on inflation but also on further explanatory variables (e.g. the real growth rate), and focusing on the direct effect from inflation in testing the hedge hypothesis. From an econometric point of view, the inclusion of other relevant factors is important in order to avoid an

Based on (3), we formulate the following empirical model expressed in logarithmic terms (lower case letters denote corresponding log-levels):

$$(5) \quad q_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + \varepsilon_t$$

The β_i 's are coefficients (including a constant term) to be estimated and ε_t is the residual of the equation.

(5) explains the long-run movements in stock prices by the long-run movements in the general price level, real production and the real discount rate. As the variables considered are non-stationary, cf. section 3, we have to use cointegration techniques in estimating (5). If our framework is valid empirically, we should expect (5) to be a cointegrating relation, i.e., a stable, long-run equilibrium relation for stock prices⁸. Whether this is actually the case, is tested as an initial step of the empirical analysis. On a validation of (5), we can then test the inflation hedge hypothesis. Given our definition of the hedge property, a formal test of the hypothesis can be framed in terms of the coefficient to the general price level, β_1 , measuring the direct (or partial) effect of the price level on stock prices. The hedge hypothesis stipulates that there exists a long-run linkage between stock prices and the price level, i.e., that the price level is significant in (5) ($\beta_1 \neq 0$) and, moreover, that the elasticity of stock prices wrt. the price level is exactly one ($\beta_1 = 1$). Hence, the hypothesis is supported if, and only if, the estimated β_1 is significant and, furthermore, not significantly different from one.

3. The Data

All data are annual and cover the period 1948-1996. The source database is Nielsen, Olesen and Risager (1997).

omitted-variables bias in the estimate of the inflation effect. The latter is also true in our approach. We test (indirectly) for having omitted variables important for the long-run modeling of stock prices, by testing whether the model provides a cointegrating relation.

⁸ Formally, we have cointegration if, and only if, the residual term ε_t is stationary. In this case, (5) serves as an 'attractor' for the included variables, see e.g. Engle and Granger (1991) for an interpretation of the concept of cointegration.

Stock prices are measured by the overall stock price index by Statistics Denmark, comprising all firms listed at the Copenhagen Stock Exchange (CSE). For the general price level, we use the official Consumer Price Index (CPI). We consider the question of whether stocks hedge against inflation to be most interesting in terms of CPI inflation because stock investors - ultimately being consumers - care about real wealth in terms of consumption bundles. Moreover, CPI is the price measure encountered in the literature. We choose to proxy real production by a deterministic trend. This may be justified by the fact that we are interested in modeling the movements in production over long horizons and, for this purpose, a trend may be a reasonably good approximation⁹.

In order to estimate (5), we also need a proxy for the unobservable (*ex ante*) growth-adjusted real discount rate r_t . We use a discount rate measure which in a given year is calculated as the difference between the yield-to-maturity of a 10-year government bond and the historical inflation rate over the 5-year period preceding that year. This proxy results - compared to other discount rate proxies that we have examined - in the strongest evidence that (5) is a cointegrating relation. We consider this to be a valid criterion for choosing the proxy because we only want to formulate an empirically valid framework prior to testing the inflation hedge hypothesis, i.e., to formulate a model that captures the important long-run features of stock prices. The latter is evidenced by the presence of cointegration. The stronger cointegration could in fact be interpreted as evidence that this proxy is particularly good at modeling the long-run movements in the 'true' discount rate.

Notice that the use of proxies introduces measurement errors in the explanatory variables in (5). However, as long as the measurement errors are stationary, this does not affect the inference on the cointegrating relation, cf. Hamilton (1994).

⁹ What we need is a proxy for real production which results in (5) being a cointegrating relation. This turns out to be the case when using a deterministic trend. We have tried several explicit production measures (e.g. real GDP for the overall economy, for the private sector and for manufacturing) but without any further success in establishing a cointegrating relation, cf. Appendix A, which reports the results of estimating alternative candidates for a cointegrating relation for stock prices, using alternative measures of both production and the general price level.

Figure 1 shows the data.

< Insert Figure 1 around here >

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Unit root tests are performed using both the Phillips and Perron (1988) Z_t -test (PP) and the test by Kwiatkowski *et al.* (1992) (KPSS), cf. Tables 1.a and 1.b¹⁰. Using conventional significance levels, both tests clearly support that stock prices and the discount rate proxy are integrated of order 1 (I(1)), i.e., are non-stationary with stationary first differences. For CPI, the PP test concludes integration of (at least) order 2 (I(2)), i.e., both levels and first differences are non-stationary, using a strict 5% significance level. However, the PP test consistently supports the I(1) hypothesis at a 10% level. The KPSS test strongly supports the I(1) hypothesis for CPI when allowing for serial correlation in the disturbance term (lag length ≥ 1). Overall, the evidence is therefore in favor of I(1).

To conclude, all series are I(1). The exclusion of the possibility of I(2)-behavior means that the standard Johansen-procedure can be used for estimating (5). Moreover, as (5) is balanced in terms of unit root behavior, single-equation-cointegration techniques can be used for estimation purposes.

4. The Empirical Results

Motivated by (5), we formulate a VAR model using stock prices, the general price level (CPI) and the discount rate proxy as the endogenous variables, and including a deterministic trend. To outline the model, let the endogenous variables be described by the column vector $X_t = (q_t, p_t, r_t)'$. Following the notation of Johansen (1996), the VAR model can be written in its reduced vector error-correction form (VECM) as:

¹⁰ We have used a maximum of 6 lags in both tests because the test statistics become reasonably stable within this lag length. The evidence in Kwiatkowski *et al.* (1992) also suggests that, for our sample size, the KPSS test has a reasonable size and power at a lag length around 4 to 6.

$$(6) \quad \Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \varepsilon_t$$

where k denotes the lag length, Π and Γ_i are matrices of dimensions 3×3 and D_t is a 2×1 vector containing the deterministic terms. We allow for a constant term and a deterministic trend, i.e., $D_t = (1, t)'$. Φ is the 3×2 matrix which contains the coefficients to the deterministic terms. ε_t is the vector of disturbance terms, assumed to be identically distributed "white noise".

The rank of matrix Π , denoted by r , determines the number of cointegrating relations among the three endogenous variables. If Π has zero rank ($r=0$), there is no cointegration in the data and (6) becomes a VAR model in first differences only because the level term disappears. If Π has a non-zero, but reduced rank ($0 < r < 3$), (6) is a cointegrated VAR model with r (linearly independent) cointegrating relations. In this case, Π can be written as the product of two full column rank matrices α and β of dimensions $3 \times r$, i.e., $\Pi = \alpha\beta'$, and (6) can be rewritten as:

$$(7) \quad \Delta X_t = \alpha\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \varepsilon_t$$

Each column vector in the β -matrix corresponds to a cointegrating relation in the sense that the linear combination $\beta_i' X_t$, where β_i (here) denotes the i 'th column vector of β , is stationary. $\beta_i' X_t$ corresponds to the usual error-correction-term in single-equation cointegration analysis. Each vector is called a cointegrating vector and there exists a total of r (linearly independent) cointegrating vectors. The matrix α contains the adjustment coefficients by which each cointegrating relation affects the short-run dynamics of the endogenous variables. For example, element α_{ji} in α captures by how much the short-run dynamics of variable j in X_t (ΔX_{jt}) responds to the equilibrium error in cointegrating relation no. i ($\beta_i' X_t$). Finally, if Π has full rank ($r=3$), we have in principle 3 cointegrating relations, which is only possible if all the variables are stationary.

We restrict the deterministic trend to be in the cointegrating space, precluding the possibility of a quadratic trend in the endogenous variables, cf. Johansen (1996). The latter assumption seems both plausible and, at an informal level, cf. Figure 1, validated by the data. The estimation is therefore based on the VAR specification:

$$(8) \quad \Delta X_t^* = \alpha\beta^* X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \varepsilon_t$$

where $\beta^* \equiv (\beta', \rho_1)'$ and $X_t^* \equiv (X_t', t)'$, i.e., the trend is included as part of the cointegration term. μ_0 is the vector of unrestricted constant terms while the $r \times 1$ vector ρ_1 contains the coefficients to the trend in the cointegrating relations. In the empirical analysis, interest focuses on, first of all whether there exists any cointegrating relations or vectors β^* , and, secondly, on the coefficients of the cointegrating vectors, β^* , in particular, the coefficient to the general price level.

As the initial step in the estimation, the appropriate lag length (k) of the VAR model has to be determined¹¹. Various procedures can be used, including the explicit testing on lag coefficients in a "general-to-specific" procedure and the use of information criteria. Using the "general-to-specific" procedure, we start out with 6 lags which is sufficient to ensure that the white noise requirements on the disturbance term are fulfilled. We then successively remove insignificant lags from the top, performing a Likelihood Ratio test of the hypothesis that all coefficients at the largest lag are zero¹². This procedure results in a lag length of $k=4$, using conventional significance levels. The test for removing all variables at lag 4 leads to a clear rejection (critical significance level of 0.2%), while the hypothesis of reducing the lag length from 5 to 4 is firmly accepted (critical significance level of 58%). A lag length of 4 is supported by the Hannan-Quinn and Akaike information criteria while the Schwarz criterion suggests a shorter lag length of 2.

¹¹ Estimations are performed in PCFIML, cf. Doornik and Hendry (1997).

¹² We use the approximate F-form of the Likelihood Ratio test suggested by Rao, cf. Doornik and Hendry (1997). This F-form which corrects for degrees of freedom is generally considered to have better small sample properties than the uncorrected χ^2 -form.

Table 2 reports both univariate and multivariate specification tests of the VAR model with 4 lags. Diagnostics for each equation in the model, including fitted values for the endogenous variables, are furthermore graphed in Figure 2. The specification tests test whether the residuals from the VAR model fulfill the white noise requirements of being serially uncorrelated, homoskedastic and normally distributed. According to the univariate test, the hypothesis of normally distributed residuals is rejected for the discount rate equation, using conventional significance levels. For the price level equation, the normality hypothesis is close to a rejection. However, the normality assumption is not crucial to the cointegrated VAR model, see Johansen (1996, Part II), who shows that it is a sufficient condition for using this method that the disturbance terms are identically distributed over time. The violation of the normality hypothesis is therefore not a problem for the inference to be drawn. There are no signs of misspecification according to the other, more critical specification tests for serial correlation and heteroskedasticity. Hence, we conclude that the VAR model with 4 lags is well specified and proceed with this specification.

< Table 2 >

< Figure 2 >

The cointegration part of the VAR model (α and β^*) is estimated by Maximum Likelihood, using the Johansen procedure, cf. e.g. Johansen (1996). Table 3 shows the (standardized) estimates of α and β^* together with estimated eigenvalues. Table 3 also reports statistics from trace tests on the rank of Π . Two trace test statistics are shown. The first statistic which is the one used in Johansen (1996) is the outcome of an asymptotic test. The evidence in Reimers (1992) suggests that this test is "over-sized" in small samples, implying that when using this test we tend to accept too many cointegrating relations compared to the significance level which we are actually willing to use. Based on this evidence and the fact that we have to deal with a small sample, we have more faith in the second trace test which adjusts the former test for degrees of freedom in the way discussed by Reimers (1992). This test is reported to have significantly better small sample properties in the sense that the actual

significance levels of the test come close (closer) to the nominal levels in small samples.

< Table 3 >

Both rank tests lead to the conclusion that there is at least one cointegrating relation as both tests firmly reject the hypothesis of no cointegration ($r=0$) at conventional significance levels. The first (asymptotic) trace test also rejects the hypothesis of 1 cointegrating relation in favor of the alternative of more than 1 cointegrating relation. However, this hypothesis cannot be rejected according to the second (degrees-of-freedom-adjusted) trace test. Based on the latter test, we conclude that there is one and only one cointegrating relation between the variables ($r=1$). The second trace test gives a clear rejection of the hypothesis of no cointegration (the critical significance level is 1.9% by linear interpolation). Hence, the evidence of cointegration is strong.

The econometric identification of the cointegrating relations is relatively straightforward with only 1 cointegrating relation because normalizing on one of the variables suffices. Motivated by the modeling framework of section 2 (and the lack of an obvious alternative), we interpret the cointegrating relation as a model for stock prices and normalize on this variable. The resulting estimates of the normalized cointegrating vector and the corresponding adjustment coefficients appear in Table 3 as the first column of β^* (i.e., β_1^*), respectively, the first column of α (i.e., the adjustment coefficients wrt. $\beta_1^* X_t^*$). The assumption that the cointegrating relation is a model for stock prices is actually supported by the estimates of the α -coefficients, because the error-correction in the short-run dynamics is strong in the direction of stock prices, whereas the corrections in the directions of the price level and the discount rate are very small in magnitude and can actually be shown to be insignificant, cf. below. The estimation gives the following long-run model for stock prices (indicative standard errors of the parameter estimates in parenthesis)¹³:

¹³ The constant term in (9) is calculated from the formula $\rho_0 = (\alpha' \alpha)^{-1} \alpha' \mu_0$ where μ_0 is the unrestricted constant term, cf. (8), and ρ_0 is the component of this constant term which enters the cointegrating relation, see Johansen (1996, p. 81). α here denotes the first column of the estimated α -matrix in Table 3.

$$(9) \quad q_t = 0.96 + \underset{(0.13)}{1.04} p_t + \underset{(0.009)}{0.011} t - \underset{(2.4)}{5.42} r_t$$

All coefficients have signs consistent with theory. The trend may appear to be insignificant, using the indicative standard error, but we proceed with (9) because our interest lies with the price level coefficient and we do not want to condition the inference on the coefficients to the remaining variables.

We take the estimated cointegrating relation as evidence in favor of the modeling framework of section 2, hence establishing a firm empirical framework within which to test the inflation hedge hypothesis. The hedge hypothesis is tested by Likelihood Ratio (LR) tests on the coefficient to the price level in the cointegrating relation. These tests compare the likelihood of the unrestricted VAR model (where the price level coefficient can vary freely) with the likelihood of the restricted VAR model (where the price level coefficient is restricted). Testing, first, the null hypothesis that the price level has an insignificant effect on stock prices ($\beta_1=0$), the outcome is a LR test statistic of 11.8 which has to be compared with a $\chi^2(1)$ -distribution. The critical significance level is for all practical purposes zero, leading to a strong rejection of the null. Hence, the price level has a significant effect on stock prices in the long run. Next, testing the null hypothesis that stock prices and the price level move one-for-one ($\beta_1=1$) gives a test statistic of 0.04 which, again, has to be compared to a $\chi^2(1)$ -distribution. The critical significance level is 83% which leads to the unambiguous test result that the null can not be rejected. The conclusion is strong support for the long-run inflation hedge hypothesis.

< Figures 3 and 4 >

To check the robustness of this conclusion, we have examined whether results are stable over time by estimating the cointegrated VAR model recursively. Figures 3 and 4 provide the results, showing the recursive estimates of the three eigenvalues and of the coefficients of the (one) cointegrating vector, respectively. The eigenvalues are

fairly stable over the sample period, so the conclusion of one and only one cointegrating relation in the data is robust over time. Figure 4 shows that the long-run coefficients are reasonably stable, maybe with the exception of a slight instability of the coefficient to the discount rate in the late part of the sample. Most importantly, the coefficient to the price level is very stable. We take these results as evidence that the conclusion in favor of the inflation hedge hypothesis is robust over time.

The cointegrated VAR model approach has the advantages, compared to single-equation-cointegration methods, that it allows for more than one cointegrating relation in the data and, in general, leads to consistent and asymptotically efficient estimates of the long-run parameters (β^*). However, as noted by e.g. Gonzalo and Lee (1998), Johansen (1999) and Juselius (1999), the cointegrated VAR model is sensitive to the number of observations. Thus, evidence based on Monte Carlo simulations suggests that the test of cointegration and the tests of hypotheses on the long-run coefficients may suffer from poor small sample performance (size distortions and low power). Moreover, inference from the model is based on the condition that the VAR specification gives the correct model not only for the variable of interest (stock prices) but also for the remaining variables (the general price level and the discount rate). As a further check on the robustness of conclusions, we have therefore re-estimated (5) by single-equation-cointegration methods. These give valid and efficient inference in our case because we only have one cointegrating relation and because there is only error-correction in the direction of stock prices, implying that the price level and the discount rate are weakly exogenous for the parameters of the cointegrating vector, cf. Johansen (1996, Chp. 8). The latter can be shown by formal testing¹⁴.

¹⁴ We have weak exogeneity if the equilibrium error does not affect the short-run dynamics of the price level and the discount rate, i.e., if the corresponding adjustment coefficients in α (see first column, second and third entry of α in Table 3) are both zero. This hypothesis can be tested formally by a LR test. The LR test statistic is 1.04 which has to be compared with a $\chi^2(2)$ -distribution. The critical significance level is 59%, leading to the conclusion that weak exogeneity can not be rejected.

Given the evidence of cointegration, OLS estimation of (5) produces consistent estimates of the coefficients¹⁵. However, testing coefficient hypotheses based on these estimates is in general difficult due to a (possible) correlation between the error term in the cointegrating relation and the innovations in the regressors, cf. Hamilton (1994). In particular, usual t-test statistics calculated from the OLS coefficients and the OLS standard errors do not have standard (known) distributions. Therefore, we have to refine the estimation of the cointegrating relation. Several approaches have been suggested for this purpose, cf. e.g. Phillips and Loretan (1991), Stock and Watson (1993) and Phillips and Hansen (1990). Hamilton (1994) and Mills (1993) provide surveys. We employ two of these procedures, both suggested by Phillips and Loretan (1991); the Phillips-Loretan OLS procedure (PLOLS) and the Phillips-Loretan Non-linear least squares procedure (PLNLS).

In both approaches, the static regression of (5) is augmented by stationary terms which capture the short-run dynamics of the explanatory variables. The PLOLS procedure augments (5) with current, lagged and leaded first differences of the explanatory variables (the price level and the discount rate), leading to the dynamic regression:

$$(10) \quad q_t = \beta_0 + \beta_1 p_t + \beta_2 t + \beta_3 r_t + \sum_{i=-N_1}^{N_1} \gamma_{1i} \Delta p_{t-i} + \sum_{i=-N_2}^{N_2} \gamma_{2i} \Delta r_{t-i} + u_t$$

t denotes as before the deterministic time trend (replacing y_t in (5)) and u_t is the new residual term. N_1 and N_2 which determine the number of lags and leads in the regression have to be specified prior to estimation. We use different specifications, cf. below, in order to check the sensitivity of coefficient estimates. (10) is estimated by OLS.

¹⁵ Using the two-step procedure of Engle and Granger (1987), we can, at the 10% significance level, confirm (5) as a cointegrating relation, see Appendix A (the alternative based on CPI and a trend).

The PLNLS procedure augments (5) further by adding lagged levels of the error correction term, i.e., the difference between stock prices and their long-run equilibrium level as determined by (5), $[q_t - (\beta_0 + \beta_1 p_t + \beta_2 t + \beta_3 r_t)]$:

$$(11) \quad q_t = \beta_0 + \beta_1 p_t + \beta_2 t + \beta_3 r_t + \sum_{i=-N_1}^{N_1} \gamma_{1i} \Delta p_{t-i} + \sum_{i=-N_2}^{N_2} \gamma_{2i} \Delta r_{t-i} + \sum_{i=1}^{N_3} \phi_i (q_{t-i} - \beta_0 - \beta_1 p_{t-i} - \beta_2(t-i) - \beta_3 r_{t-i}) + v_t$$

The error correction terms are included in order to eliminate serial correlation in the disturbance term (v_t) and increase the efficiency of the coefficient estimates, cf. Hamilton (1994). Because the coefficients of the cointegrating relation enter the lagged error correction terms, (11) is estimated by Non-linear least squares (NLS).

< Table 4 >

Results including t-tests on the price level coefficient are reported in Table 4. In the first entry, results from estimating (5) by OLS (no augmentation) are shown together with OLS standard errors which are indicative only. The PLOLS procedure is used in three regression specifications which differ according to the included first differences of the price level and the discount rate (entries 2 through 4). In the first application (entry 2), current first differences and first differences at lead 1 and lag 1, respectively, are included. The disturbance term shows serial correlation up to lag 5 so standard errors and t-statistics have to be corrected. We use the adjustment method suggested by Hamilton (1994), based on an AR(5)-model fitted to the residuals of the PLOLS regression¹⁶. In the second application (entry 3), first differences of up to 2 leads and 2 lags are included. This further augmentation only has a minor effect on the estimated price level coefficient. It turns out that the disturbance term shows no sign of misspecification in this formulation (no serial correlation) so usual OLS standard errors can be used. Finally, in the third application (entry 4), we use a "specific-to-general" procedure and augment (5) with current, lagged and leaded first

¹⁶ The adjusted t-statistics reported in Table 4 (entry 1) are calculated as the ordinary OLS t-statistics multiplied by the ratio (s/λ) , where s is the ordinary standard error of the residual in (10) while λ is calculated from an AR(5)-model fitted to the residual, see Hamilton (1994, p. 610). λ can, heuristically, be interpreted as an estimate of the residual standard error in 'long-run equilibrium' of the AR(5)-model. The reported standard errors of the coefficient estimates are adjusted accordingly.

differences until the disturbance term fulfills the white noise requirements. The resulting regression is just a reduced version of the second PLOLS regression (entry 3) where insignificant first difference terms have been omitted. Again, the estimated price level coefficient is only mildly affected. PLOLS regressions have also been carried out with more leads and lags but the coefficients and, in particular, the price level coefficient are stable wrt. this further augmentation.

The PLNLS regression in entry 5 has the augmenting terms shown in the first column of the table, including one lag of the error correction term. The augmenting terms are chosen in a "specific-to-general" manner in order to ensure a white noise disturbance. The reported standard errors are NLS calculated standard errors.

The results show that while the coefficient estimates for the trend and especially the discount rate are sensitive to the estimation procedure used, the estimate of the price level coefficient is fairly robust (and also comes close to the estimate obtained from the cointegrated VAR model). Turning to the inflation hedge hypothesis, the t-tests show that the price level coefficient is significant in all four cases. Moreover, in none of the cases we can reject the hypothesis that the price level coefficient is 1. The evidence in terms of critical significance levels is very strong. Hence, we conclude that single-equation-cointegration methods confirm the strong evidence in favor of the hedge hypothesis.

5. Conclusion and Discussion

We have examined whether Danish stocks provide a hedge against inflation, focusing explicitly on the long-run horizon. We have tested the hypothesis based on the long-run relation between stock prices and the general price level, estimated by cointegration analysis. Using the Consumer Price Index as the relevant price measure, results give strong support to the hedge hypothesis. The evidence supports the hedge property in its most restrictive sense of a perfect hedge. The conclusion is confirmed by both multivariate and univariate cointegration methods and is robust over time.

The inflation hedge hypothesis is tested within a firm modeling framework which is validated by the data as a cointegrating relation for stock prices.

The inflation hedge property of stocks (defined as a perfect hedge) only receives weak support, if any, in the literature. The strong support in this paper is therefore not a standard result. We do not believe that the Danish stock market has unique characteristics compared to other stock markets but rather attribute the difference to the literature to other factors. First of all, the use of different investment horizons is one possible explanation. We test the inflation hedge hypothesis in a long-run framework whereas others, e.g. Fama and Schwert (1977) and Gultekin (1983), examine relatively short investment horizons (less than 6 months). A plausible and reconciling interpretation of this evidence is that stocks hedge against inflation in the long run, but not in the short run.

Second, the use of different sample periods may be important. In this paper, we include observations until 1996, while other studies, e.g. Fama (1981) and Gultekin (1983), use samples that only cover the period until the end of the 1970s. As well-known, the 1970s were in almost all OECD countries a period of very high and increasing inflation due to the 1973 and 1979 oil price shocks. The use of a sample ending shortly after the oil price shocks ignores the subsequent and major adjustment in stock prices and may have triggered the (false) conclusion that stocks do not hedge against inflation. In this context, it may in particular be important that real oil prices, while increasing substantially during the oil crises with a deteriorating effect on profit margins, have by the beginning of the 1990s returned to the pre-oil crises level, hence allowing for a restoration of "normal" profit margins. Our study differs from the older literature by including the important adjustment period after the 1970s.

Finally, we have taken a different approach compared to the literature where it has been standard to test the inflation hedge hypothesis based on return regressions. We use cointegration methods to disentangle the short-run dynamics of and the long-run linkages between stock prices and the general price level, explicitly allowing for slow adjustment in stock prices to long-run equilibrium in the event of shocks to (not least)

the general price level. This approach has the advantage of allowing for a clear identification of long-run stock price behavior. The return regressions approach, on the other hand, does not distinguish between short-run dynamics and long-run linkages and the identification of long-run stock price behavior is conducted merely by investigating a sufficiently 'long' investment horizon. However, this muddles short-run dynamics and long-run linkages. Moreover, by linking stock returns to contemporaneous inflation, return regressions from the outset preclude slow adjustment in stock prices. In principle, this could trigger a false conclusion that stocks do not hedge against inflation in the long run. That is, stocks may be a perfect hedge against inflation with a lagged response in stock prices, but return regressions may fail to establish this because they do not explicitly take account of the lagged adjustment. As an illustration, assume that stocks hedge against inflation after a lagged adjustment over (say) 3 years, i.e., stock prices adjust completely to current inflation after 3 years. A return regression for even a long investment horizon of e.g. 5 years may not be able to detect this because stock returns do not reflect (completely) inflation in the last 3 years of each horizon, while at the same time, stock returns in the first 3 years are a result of adjustment to inflation in the preceding years¹⁷.

To highlight the difference between the standard return regressions approach and our approach (the cointegration approach) more formally, consider the cointegrated VAR model of (8) with a lag length of (for simplicity) $k=1$ and let us assume that this is the 'true' reduced form model. The structural form of the cointegrated VAR model is formally derived by premultiplying this reduced form by a non-singular matrix, cf. Johansen (1996). The resulting dynamic equation for stock prices can be written as (ignoring the disturbance term):

$$(12) \quad \Delta q_t = \alpha_0 + \alpha_1 \Delta p_t + \alpha_2 \Delta r_t + \alpha_3 \beta^* X_{t-1}^*$$

¹⁷ The possibility of a slow adjustment in stock prices or rather stock returns to a change in the inflation rate has also been noted by Barnes *et al.* (1999). They test the inflation hedge hypothesis on a large sample of countries using the standard return regressions approach. To take account of the possible slow adjustment, they include both the contemporaneous and the lagged inflation rate in the return regressions. However, this does not alter the evidence significantly. The general result in Barnes *et al.* (1999) is a rejection of the inflation hedge hypothesis for stocks.

where the α_i 's are structural coefficients. The term $\beta^* X_{t-1}^*$ denotes as before the error-correction term from the long-run stock price relation (as of period $t-1$). Now notice, that the first differences part of (12) resembles a return regression for an investment horizon of 1 year, by regressing the first differences of log-to-stock prices (Δq_t), which is a proxy for the 1-year stock return, on 1-year inflation (Δp_t) and the 1-year change in the discount rate (Δr_t). The 1-year return regression, therefore, can be viewed as a special case of (12) where the level term (the cointegration term) has been excluded. In terms of (12), what distinguishes the cointegration approach from the standard approach is that the former is concerned with the long-run coefficients to stock prices and the general price level, i.e., the cointegrating vector β^* . The return regressions approach, on the other hand, is concerned with the dynamic coefficient to the price level, i.e., the coefficient α_1 which captures the short-run or contemporaneous response in stock prices to inflation. This difference reflects our explicit focus on the long-run horizon whereas the existing literature has mainly examined the inflation hedge hypothesis over relatively short horizons.

(12) also suggests a possible shortcoming of the standard approach. In standard return regressions, the level term of (12) is excluded which (implicitly) assumes either that stock prices adjust immediately to their long-run equilibrium level as determined by the cointegrating relation (i.e., the equilibrium error $\beta^* X_{t-1}^*$ is always zero), or that there is no cointegration between the level variables (i.e., the cointegrating rank is zero and no cointegrating vectors β^* exist). In our case, both possibilities are rejected by the data. Therefore, the 1-year return regression must be misspecified because it omits a significant regressor (the level term of (12)), which reflects the slow adjustment in stock prices. In general, the result is inconsistent coefficient estimates, which affects the inference on the coefficient to inflation and, hence, the inflation hedge hypothesis. In the cointegration approach, we explicitly allow for slow adjustment¹⁸.

¹⁸ The cointegrated VAR model is more general than implied by (12) because it allows for more short-run dynamic terms, i.e., lagged first differences, when the lag length k is larger than 1. Moreover, in the case of a cointegrated VAR model with an explicit measure for real production, we would also have included the 1-year real growth rate (Δy_t) as a first-differences regressor in (12). Note that (12) focuses on the 1-year investment horizon. For longer horizons, the implied model for returns will be

For the purpose of comparison, we have also tested the inflation hedge hypothesis for Danish stocks using standard return regressions over the same sample period as considered above. We have run a return regression where nominal stock returns are regressed (OLS) on contemporaneous inflation and a constant term for each of the five investment horizons of 1 to 5 years duration¹⁹. The point estimates of the coefficient to inflation range from 0.27 (1-year horizon) to 0.94 (5-year) so the estimates for the longest horizons come close to one, the value consistent with the inflation hedge hypothesis. We can test whether the response in stock returns to inflation is statistically significant. Using the Newey and West (1987) coefficient standard error, which is consistent to heteroskedasticity and serial correlation in the regression residual (up to lag 5), we get a t-test statistic of 1.5 for the null hypothesis that the coefficient to inflation is insignificant (zero) at the 5-year horizon (where the inflation coefficient is most significant). The conclusion is that, even though the point estimate is high and close to one, the estimation uncertainty is substantial and the inflation effect is, in statistical terms, only weakly significant. Using conventional significance levels, we would accept the null that inflation has no effect on stock returns. At best, these results only provide weak support to the inflation hedge hypothesis. Moreover, it turns out that the high point estimates of the inflation coefficient hinge primarily on two events, that is, the exceptionally large stock returns encountered in the years 1972 and 1983 (see Figure 1 for the large capital gains on stocks in these two years). If we eliminate the influence of the returns realized in these two years by including dummies in the regressions of stock returns on contemporaneous inflation, the resulting point estimates of the inflation coefficient

more complicated but the fundamental insight remains that return regressions omit significant level terms.

¹⁹ The stock return is annualized, discretely compounded and includes capital gain and dividend yield, cf. Nielsen, Olesen and Risager (1997). Inflation is measured by the continuously compounded annual growth rate in the Consumer Price Index (CPI). The return regressions use overlapping observations for each of the horizons of 2 to 5 years. Appendix B provides further evidence from using the return regressions approach to test the hedge hypothesis, including results for the 10-year horizon and results from an extended return regression with additional explanatory variables. At first sight, the results in Appendix B give more support to the hedge hypothesis than the regression results presented here. In particular, the results for the 10-year horizon seem to confirm the hedge hypothesis. However, a closer examination raises serious doubts about the reliability of this conclusion, in particular, because the return regressions suffer from highly unstable coefficient estimates over the sample, including an unstable estimate of the coefficient to inflation.

are substantially lower and now range from -0.11 (1-year horizon) to 0.21 (2-year)²⁰. The coefficient at the 5-year horizon is estimated to be 0.14. None of these coefficients are significantly different from zero.

Hence, using the cointegration approach gives stronger and more reliable support for the inflation hedge hypothesis than produced by standard methods. This evidence suggests that the differences in approach could be important in understanding why we find much stronger support for the hedge hypothesis than in the literature. On this background, and recalling the possible shortcomings of the return regressions approach, cf. above, an obvious topic for future research would be to use the cointegration approach (and a more recent sample) to re-examine the evidence on the inflation hedge hypothesis for other stock markets.

In testing the inflation hedge hypothesis, we have focused exclusively on stock prices, thereby ignoring dividends as part of overall stock returns. In principle, a hedge against inflation demands that the total stock portfolio consisting both of stocks bought at the time of initial investment and stocks bought subsequently by the reinvestment of dividends retains a stable purchasing power in the event of inflation. Our stock price approach takes a shortcut by focusing on the value of initial stocks only. One reason is that we want to test the hedge hypothesis within a firm modeling framework which in our case is a model for stock prices. Moreover, it can be argued that for all practical purposes the exclusion of dividends is not important because dividend yields are fairly modest (in particular in recent history). Thus, whether the neglect of dividend payments is of importance is reflected by how much future reinvestments amount to as a fraction of total future portfolio value. This, again, is determined by the dividend yield (in absolute terms). Because the dividend yields for

²⁰ The regressions include two impulse dummies that eliminate *all* the effects of the stock returns in 1972 and 1983. That is, for the 2-year regression where we use overlapping observations, we include one dummy which has the value of one for 1972 and 1973 and is zero otherwise, and another dummy which has the value of one for 1983 and 1984 and is zero otherwise. Similarly, for the 3-year regression each dummy takes on the value of one three years in a row, and so forth. We want to exclude the returns of these two years because they represent clear outliers (annual returns of 95% and 118%, respectively) which can, furthermore, be explained by one-off exceptional changes in the Danish economy such as the Danish referendum in 1972 leading to membership of the EEC and the introduction of a new, separate pension fund tax on bond returns in 1983.

Danish stocks over the sample period considered are small (between 1% and 7%), the stock price approach should give a fairly good approximation to the 'overall' inflation hedge question in terms of total portfolio value.

We have, as standard in the literature, ignored costs of stock transactions and investor taxes on stock returns, i.e., dividend and capital gains taxes. While the neglect of transaction costs may not be so important because the hedge property of stocks relate to a passively held portfolio with limited active trading, it is a more open question whether the neglect of taxes matters. Dividend taxes should not matter because the behavior of stock prices is the crucial issue for the inflation hedge property, cf. above. In the case of a capital gains tax it could, tentatively, be argued that the inflation hedge property of stocks is retained on an after-tax basis as long as the tax is fixed and proportional because the 'after-tax' stock price (or portfolio value) would be proportional to the 'before-tax' stock price, thereby preserving a one-for-one relation with the general price level. However, capital gains taxes are not proportional and have not been fixed over time, so this aspect needs closer investigation. This is an interesting but in the Danish case also highly challenging question to address because taxation rules are complex and differ markedly between different types of investors.

The result that (Danish) stocks in the long run hedge against inflation should be of interest to any rational investor who cares about the real value of his investments and who has a long-run investment horizon, e.g. a pension saver. Hence, the hedge property has *ceteris paribus* implications for optimal portfolio choice because not all assets are immune to inflation uncertainty. For instance, nominal bonds can at most compensate for expected inflation, leading to real uncertainty of a bond investment. However, one should be careful with the proper interpretation of the inflation hedge result.

First of all, the result relates to the market portfolio of stocks, i.e. the highly-diversified portfolio consisting of all stocks listed at the CSE. A high degree of diversification must be expected to be necessary in order to sustain the hedge property

against the overall price level because relative prices of goods and services and, hence, relative firm profits change over time.

Secondly, the inflation hedge is a long-run phenomenon, so that stocks should be expected to compensate for inflation in the 'long run' and the 'long run' only. Our analysis does not answer the question how long the 'long run' is, but the lag length of the VAR model and the estimated adjustment parameter (together with return regressions, see above and Appendix B) loosely indicate that a time span of 5-10 years is the appropriate horizon. It should be emphasized that the interpretation is not that stocks compensate for contemporaneous inflation over a fixed (say) 5-year horizon, but rather that stock prices have adjusted completely to current inflation after an adjustment period of 5 years. Thus, over a fixed investment horizon of 5 years an investor may only get compensated for the first year of inflation but not (fully) for the remaining 4 years of inflation. This interpretation distinguishes our approach from the return regressions approach which focuses on a fixed horizon, and is related to the different assumptions on the adjustment-speed of stock prices.

Finally, the hedge result is framed within a structural model for stock prices where real production and a discount rate also enter as explanatory variables. The result that stock prices move one-for-one with the general price level therefore applies to what could be called a controlled or *ceteris paribus* 'experiment' where the price level is changed while real production and the discount rate are kept fixed. This means that in the event of inflation, stock prices may not always end up by increasing proportionately, i.e., with the same relative change as the general price level, because (and only because) real production and the discount rate may have changed simultaneously. This does not contradict the hedge hypothesis but rather reflects the fact that stock prices do not depend only on the price level. Thus, real stock prices may change due to innovations in real fundamentals. The standard return regression literature also focuses on a *ceteris paribus* 'experiment' when drawing inference on the inflation hedge hypothesis.

In the field of monetary economics, it is standard to distinguish between the 'neutrality' and 'superneutrality' of money, see e.g. Grandmont (1988). 'Neutrality' means that a change in the *level* of the money stock has no effects on real economic variables (production, employment, and so forth) whereas 'superneutrality' implies that a change in the *growth rate* of the money stock has no real consequences. Using a corresponding terminology, the evidence provided on the long-run relationship between stock prices and the general price level can be given the interpretation that inflation is both neutral and superneutral to stocks in the long run. That is, a permanent change in the general price *level* will, according to (9), eventually lead to a proportionate change in stock prices, leaving real stock prices unaffected. Similarly, from (9), a permanent change in the *rate of inflation*, i.e., the growth rate in the general price level, will in the long run result in an equivalent change in the growth rate of stock prices and there will be no impact on real stock prices. Notice that it is a prerequisite for both neutrality and superneutrality that real production and the real discount rate are unaffected. This seems as the reasonable assumption when considering the long-run responses to one-off changes in the price *level*. As to the case of superneutrality, changes in the inflation rate will have no impact on real production and the discount rate and, hence, real stock prices *if* the economy is characterized by the property of Classical Dichotomy between the real and the money sectors in the long run²¹. The latter assumption is often used both in macroeconomic theory (it forms, in particular, the corner stone of the traditional Neoclassical-Keynesian Synthesis) and applied business-cycle research.

²¹ Following the definition by Grandmont (1988, p.2), Classical Dichotomy applies if real magnitudes are determined exclusively by the real sector while absolute prices are determined by the equilibrium condition for money.

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Appendix A: Alternative Candidates for a Cointegrating Relation

This appendix reports the results from estimating alternative candidates for a cointegrating relation for stock prices, cf. (5), while using alternative measures of the general price level and real production, respectively. For the general price level, we have examined three different measures, the official Consumer Price Index (CPI) (denoted by p_t in the following), the implicit price deflator for total GDP in factor prices (pyf_t) and, finally, the implicit price deflator for GDP in factor prices in the sector of manufacturing ($pyfi_t$). For real production, we use data on total GDP (denoted by yf_t), respectively, GDP in manufacturing (yfi_t), both in fixed 1980-factor prices. The price deflators and production measures are taken from National Accounts and all series are in log-levels. For the growth-adjusted real discount rate (r_t in (5)), we, throughout, use the same proxy as in the main text, cf. section 3. The estimations and tests are performed by a single-equation cointegration method, that is, the Engle and Granger (1987) two-step procedure (EG2).

To begin with, we have to test for the stationarity properties of the data series (stock prices, all price and production measures and the discount rate proxy). Unit root tests have been performed following the same approach as in section 3 and the conclusion is that all series are integrated of order 1, i.e., non-stationary in levels but stationary in first differences (tests not reported). Hence, the regression in (5) is balanced which is a prerequisite for using the EG2 procedure for estimation purposes.

< Table A.1 >

Table A.1 reports the alternative estimates of (5) and the corresponding tests for cointegration. The measures used for the general price level and real production are indicated in the first column. For example, the regression of the first entry uses the price deflator for total GDP and, correspondingly, total GDP in fixed prices as the relevant measures. The second column shows the OLS estimates of the coefficients of (5) (stated as a cointegrating vector which is normalized on stock prices), together with *indicative* OLS standard errors. For instance, the price level coefficient in the

regression of the first entry (the estimated β_1) is 1.38 with an indicative OLS standard error of 0.14. A residual-based test for cointegration is performed by testing the null hypothesis of a unit root in the process for the OLS residuals. If the null is rejected, the residuals are stationary and the regression (5) is concluded to be a cointegrating relation. The test results and conclusions on cointegration are reported in the remaining columns of the table, using the cointegrating regression Dickey-Fuller test (CRDF), cf. Engle and Granger (1987) or Hamilton (1994). The number of augmenting lags (of the first differences of the residuals) in the CRDF test is chosen according to a "specific-to-general" procedure, taking the simple Dickey-Fuller regression without augmentation as the starting point and - in case this regression shows signs of being misspecified (serial correlation in the disturbance term) - including lags until diagnostic tests are passed. A maximum of 1 augmenting lag suffices in the tests reported in Table A.1²².

The choice of production measure is important for whether or not (5) is a cointegrating relation. The first two regressions in Table A.1 which both use an explicit measure of production show no cointegration at the 10% significance level. Because the OLS estimates *indicate* that the production measures are insignificant, a regression is run (third entry) where prices and production are combined in nominal production (using total GDP in current prices as the relevant measure, denoted by y_t^* in the table) to check whether this enhances the presence of cointegration. This is not the case²³.

In the last three regressions in Table A.1, we have replaced the explicit production measure by a deterministic trend (denoted by t), which can be interpreted as a proxy for the trend growth in production. Results show that the inclusion of a deterministic trend leads to cointegration at the 10% significance level when measuring the general

²² The level of augmentation used in Table A.1 is the same as one would get from a "general-to-specific" procedure, starting out with a Dickey-Fuller regression with 5 augmenting lags and then, successively, removing insignificant lags from the highest order.

²³ We have examined alternative measures of real production including GDP for the private sector and GDP for the private sector excluding farming and housing, but without any further success. The lack of cointegration and the apparent insignificance of the production measures, basically, suggests that we have not been able to find a good proxy for the production of goods and services by the representative firm on the Copenhagen Stock Exchange.

price level by CPI (entry 4) or the factor price deflator for manufacturing (entry 6). Cointegration is most evident in the latter case with cointegration being accepted also at the 5% significance level. Cointegration is just rejected when using the factor price deflator for total GDP as the price measure (entry 5).

Despite the fact that the evidence of cointegration is strongest when using the price deflator for manufacturing as the price measure, we prefer to test the hedge hypothesis in terms of CPI inflation for the reasons stated in section 3²⁴.

²⁴ While CPI seems most relevant to stock investors, a deflator for GDP in factor prices (or a net price index) may actually be more relevant to firm profits and, hence, more adequate for the theoretical framework of section 2. Thus, CPI includes indirect taxes paid by the consumers. Furthermore, CPI measures the prices of (domestically consumed) consumer goods and services and, thereby, ignores (say) the prices of investment goods. A factor price deflator captures the prices of all goods and services produced. However, whatever price measure used, it is just a proxy for what we really want to measure, and that is the prices of goods and services produced by the representative firm at the Copenhagen Stock Exchange. In particular, what we need is a good proxy for the long-run movements in the 'true' prices and, in this respect, CPI may do as well as e.g. a factor price deflator. It should also be recalled that the use of a proxy for the price level does not undermine the asymptotic consistency of the coefficient estimates in a cointegrating relation, cf. Hamilton (1994), provided cointegration is preserved.

Appendix B: Evidence from Return Regressions

In the literature, the inflation hedge hypothesis is tested by examining the link between stock returns and contemporaneous inflation. This appendix provides comparable results for Denmark, focusing on the three investment horizons of 1, 5 and 10 years.

The Empirical Model

Based on the theoretical framework of section 2, we use the following empirical model for stock returns over the k -year investment horizon²⁵:

$$(B1) \quad Sk_t = \beta_0 + \beta_1 Ik_t + \beta_2 GYk_t + \beta_3 GRk_t + \varepsilon_t, \quad k = 1, 5 \text{ and } 10 \text{ years}$$

Sk_t denotes the annualized total stock return over the k -year investment horizon, including both capital gains and dividend yield. $Ik_t \equiv (\Delta_k \ln P_t) / k$ and $GYk_t \equiv (\Delta_k \ln Y_t) / k$ are, respectively, the (continuously compounded) annual inflation rate and the (continuously compounded) annual growth rate in real production over the same k year horizon. $GRk_t \equiv (\Delta_k \ln R_t) / k$ is the per annum relative change in the discount rate over the investment horizon while ε_t , finally, denotes the usual disturbance term. According to (B1), stock returns should be regressed on a constant term and contemporaneous values of the inflation rate, the real growth rate and the relative change in the discount rate. Whether or not stocks provide a hedge against inflation is captured by the coefficient to inflation, β_1 , measuring the direct or partial effect from inflation to stock returns. A formal test of the hedge hypothesis is performed in two steps, by testing (i) whether inflation has a significant effect on stock returns ($\beta_1 \neq 0$), and, if the inflation effect is significant, (ii) whether the relationship between (changes in) stock returns and inflation, furthermore, is one-to-one ($\beta_1 = 1$).

²⁵ The theoretical counterpart of (B1) is obtained by taking k -year differences in the logarithmic analog to (3) and dividing through by k to obtain *per annum* continuous growth rates. We substitute total stock returns (including dividend yields) for capital gains as the endogenous variable to allow for a comparison with the literature. It can be shown that this does not affect the empirical results significantly. The latter reflects the fact that the variation in dividend yields have played only a minor role for the variation in Danish stock returns over the sample period.

The Data

Data for total stock returns are from the database by Nielsen, Olesen and Risager (1997) and relate to the market portfolio of all Danish stocks listed at the Copenhagen Stock Exchange. Inflation is measured by the (annualized and continuously compounded) growth in the official Consumer Price Index (CPI) by Statistics Denmark, while we as data for real growth use the (annualized and continuously compounded) growth in total GDP in fixed 1980 factor prices, taken from National Accounts. For the unobservable discount rate, we use the same proxy as in the main text, cf. section 3, and GRk_t is calculated as the (annualized) change in this proxy over a k -year period. All data are annual. In order to cover the same sample period as in the main text, we consider the period 1949-1996 for the 1-year investment horizon, 1953-1996 for the 5-year horizon and 1958-1996 for the 10-year horizon. At the 5- and 10-year horizons, we use overlapping observations. Figure B1 shows the stock return and the inflation at the three horizons.

< Figure B1 >

< Tables B1.a and B1.b >

Tables B1.a and B1.b report the outcome of tests for unit roots, using the Phillips and Perron (1988) Z_t -test (PP) and the test by Kwiatkowski *et al.* (1992) (KPSS). For the 1-year horizon, we conclude that all variables are stationary. The two tests give conflicting results for the real growth rate but the evidence seems most robust across lag lengths for the PP-test which strongly points to stationarity. For the inflation rate, the PP-test consistently concludes stationarity at the 10% significance level, while the KPSS test at the same time gives firm evidence in favor of stationarity when allowing for serial correlation in the disturbance term (lag length ≥ 1). At the 5-year horizon, stock returns and the change in the discount rate ($GR5_t$) are stationary. Results for inflation are ambiguous as the PP-test points to (at least one) unit root whereas the KPSS test supports stationarity. Real growth is non-stationary according to both tests. Finally, for the 10-year horizon, results for both stock returns and inflation are ambiguous. $GR10_t$ is stationary while real growth is non-stationary. To conclude, the static regression of (B1) which requires the data series to be stationary is valid for the

1-year horizon whereas conclusions are less clear for the 5- and 10-year horizons. At the latter horizons, the real growth should be excluded to allow for a valid regression and the regression results should, in general, be interpreted with caution due to the possible non-stationary behavior of the data series.

The Results

< Table B2 >

Results are shown in Table B2. Regressions of the type (B1) are performed for each of the three investment horizons. Furthermore, for each investment horizon three distinct regressions are examined, cf. below. For all regressions, OLS is used for estimating the parameters, producing consistent estimates. For the 5- and 10-year horizons standard errors of the parameter estimates are estimated by the Newey and West (1987) method to take account of heteroskedasticity and serial correlation up to lag 5 in the disturbance term. The non-standard behavior of the disturbance term can be motivated by the use of overlapping observations. The truncation at lag 5 seems appropriate as the Newey and West (1987) standard error of the inflation coefficient becomes stable at this lag length. For the 1-year horizon, we include impulse dummies for 1972 and 1983 in order to exclude the exorbitant and exceptionally high stock returns these years (returns of 95% and 118%, respectively). These outliers can be explained by exceptional changes in the Danish economy including, in particular, the Danish favorable EEC referendum in 1972, the major shift towards a new economic policy regime in late 1982 and the introduction of a new pension fund tax on bonds in 1983. Having included these dummies, the regression residual fulfills the white noise requirements of being serially uncorrelated and homoskedastic and, hence, standard errors of the coefficients can be estimated by OLS at the 1-year horizon.

The first regression for each horizon (first entry) shows the results for "the simple model", which is the specification that has been used most extensively in the literature. This formulation is a special case of (B1) where any effects from real

production and the discount rate are ignored ($\beta_2 = \beta_3 = 0$) so that stock returns are explained by inflation only. The problem with this formulation is that it, according to (B1), ignores potentially relevant explanatory variables. As well known, the omission of relevant regressors leads to biased estimates for the remaining regressors to the extent that the omitted and included regressors are correlated. Thus, in the simple model, the estimated coefficient to inflation could potentially be biased, as it may also capture relevant effects from real growth and a changing discount rate. Results for the simple model should, therefore, in general be interpreted with caution.

The second regression for each horizon (second entry) shows results for (B1) including both real growth and the change in the discount rate ("the extended model"). The latter enters significantly and with the correct (minus) sign for all three horizons whereas real growth has the wrong sign (minus) in each case. However, the effect from real growth is also insignificant at the 5% significance level. For this reason, we exclude it from the regression (which is also preferable from unit root considerations, cf. above) and arrive at the "reduced extended model" (third entry for each horizon) which can be interpreted as the parsimonious model formulation. Comparing the reduced extended model with the simple one, we find that the inclusion of the discount rate matters at both the 5- and 10-year horizons as it increases the point estimate of the inflation coefficient.

In testing the inflation hedge hypothesis, we focus on the "reduced extended model" which provides the best specification in terms of included regressors. The impact of inflation on the stock return is clearly insignificant at the 1-year horizon, where the estimated inflation coefficient for all practical purposes is zero. At the 5-year horizon, the coefficient of 1.01 is very close to one, but the coefficient standard error is large (0.55) so that the inflation effect is only at the boarder of being significant, using conventional significance levels (the critical significance level of a two-sided t-test for significance is 6.6%). If the inflation effect is accepted to be significant, the hypothesis that stock returns and inflation move one-for-one is clearly accepted. At the 10-year horizon, the inflation coefficient of 1.01 is strongly significant with a t-

statistic of almost 5. Moreover, the hypothesis that the inflation effect is one ($\beta_1=1$) receives strong support.

To judge the robustness of the latter evidence, a recursive estimation of the reduced extended model at the 10-year horizon is performed, cf. Figure B2.

< Figure B2 >

The support of the inflation hedge hypothesis at the 10-year horizon is certainly not stable over time. It is only with the inclusion of the observations in the mid-1980s that the hypothesis can be supported. Using a sample from 1958 to the early 1980s, the inflation effect is largely insignificant. This seriously questions the robustness of the inflation hedge result at the 10-year horizon. A similar picture can be shown for the 5-year horizon.

Conclusion

Using the standard return regressions approach, we find that stocks are certainly no hedge against inflation at the short 1-year horizon. Apparently, the hedge hypothesis receives mild support at the medium 5-year horizon and strong support at the long 10-year horizon. However, a closer examination raises serious doubts about the validity of this conclusion because estimates of the parameters in the return regressions and, in particular, the estimated coefficients to inflation, are highly unstable over time. Moreover, the regressions at the 5- and 10-year horizons suffer from a potential problem with non-stationary data series²⁶. Hence, we conclude that the return regressions approach does not produce reliable support to the hedge hypothesis.

²⁶ Other econometric problems include a small number of non-overlapping observations and a possible measurement error bias in coefficient estimates due to the use of a proxy for the discount rate regressor.

Figure 1. The Data

Levels and first differences of (by row and from the top) stock prices, the general price level (CPI) and the real discount rate (proxy). Variables in logs. Sample: 1948-1996.

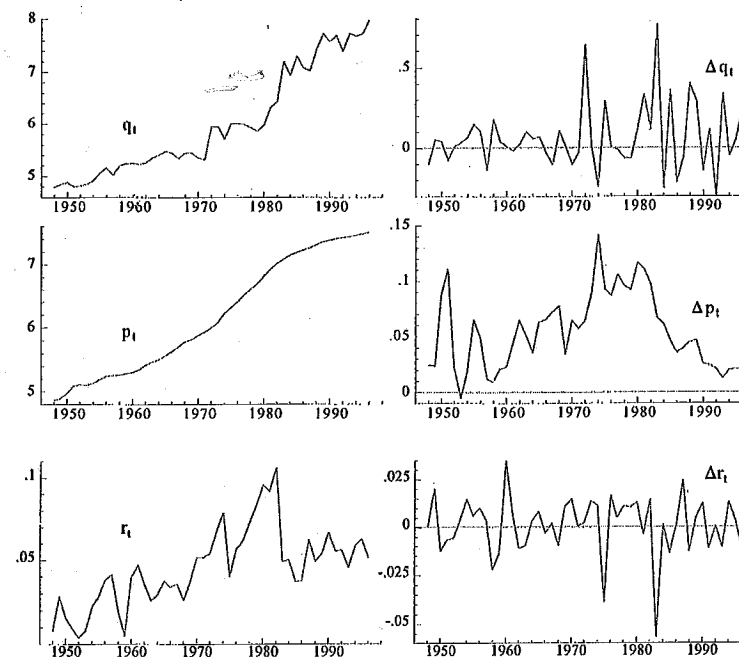
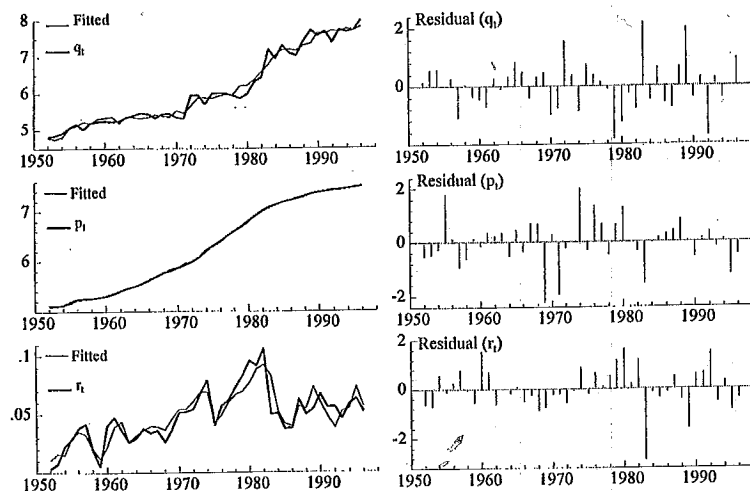
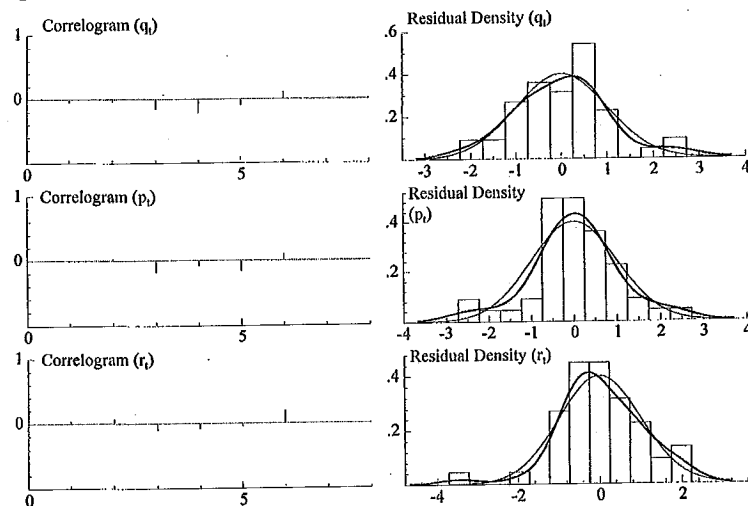
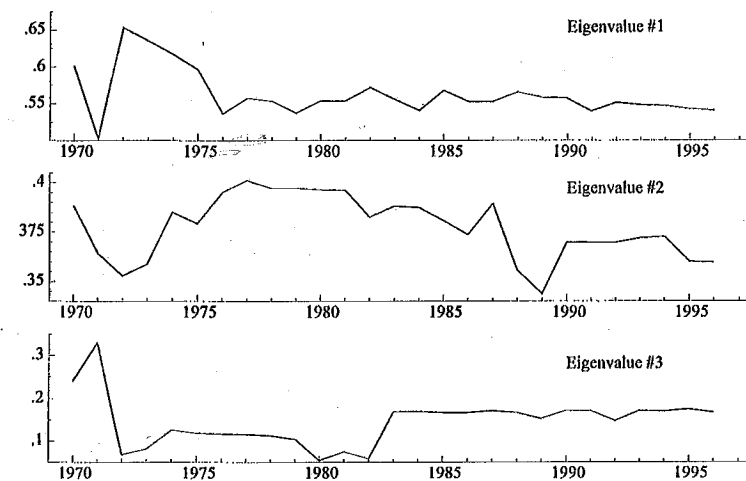


Figure 2. Diagnostic Graphics for the VAR Model

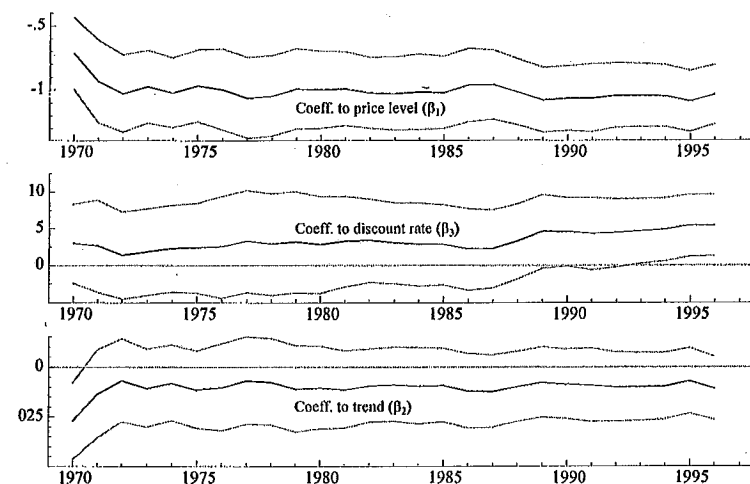
Actual and fitted values (in levels), residuals, residual correlogram and residual density for the equation for (by row and from the top) stock prices, the general price level and the real discount rate.

**Figure 2, continued.**

Note: Density plots include standard normal density for comparison (thin curve).

Figure 3. Recursive Estimates of the Eigenvalues of the Unrestricted VAR Model (Full sample: 1952-1996)**Figure 4. Recursive Estimates of the Cointegrating Vector**

Recursive point estimates (solid line) and 95% confidence bands for the coefficient to (from the top) the general price level, the real discount rate and the deterministic trend. Cointegrating rank restricted to 1 and cointegrating vector normalized on stock prices. Full sample: 1952-1996.



Note (both figures): The method of recursive estimation keeps the short-run dynamics fixed at the one estimated for the entire sample, cf. Hansen and Johansen (1996) and Doornik and Hendry (1997).

Table 1.a. Phillips and Perron (1988) Z_t -Test for Unit Root
1948-1996

Series:	Lag length (l)						
	0	1	2	3	4	5	6
<u>Levels:</u>							
q_t (T)	-2.42	-2.24	-2.23	-2.24	-2.22	-2.23	-2.25
p_t (T)	-0.84	-1.08	-1.22	-1.34	-1.44	-1.53	-1.61
r_t	-2.68*	-2.64*	-2.61*	-2.59	-2.54	-2.53	-2.51
<u>First differences:</u>							
Δq_t	-8.95***	-8.97***	-9.04***	-9.13***	-9.29***	-9.43***	-9.52***
Δp_t	-2.79*	-2.94**	-2.80*	-2.70*	-2.73*	-2.78*	-2.86*
Δr_t	-8.14***	-8.15***	-8.25***	-8.41***	-8.69***	-9.00***	-9.45***
<u>Critical test values:</u>							
		10 %	5 %	2.5 %	1 %		
Without trend		-2.60	-2.93	-3.22	-3.58		
With trend		-3.18	-3.50	-3.80	-4.15		

Note: The Phillips and Perron (1988) unit root test is based on the first order autoregression $x_t = \alpha + \rho x_{t-1} + u_t$ (without trend), respectively, $x_t = \alpha + \rho x_{t-1} + \delta t + u_t$ (with trend) where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed (heteroskedastic) and serially correlated up to lag l , see also Hamilton (1994). The Z_t test statistic is as a modified t-statistic for the null hypothesis of a unit root ($\rho=1$), corrected for the possible non-standard properties of u_t . The null is rejected in favor of the stationary alternative ($\rho<1$) if Z_t is negative and sufficiently large in absolute value. Critical values are from Hamilton (1994, Table B.6) for a sample size of 50. *, ** and *** denote rejection of a unit root at the 10%, 5% and 1% significance level, respectively. All regressions include a constant term. (T) indicates that the regression includes a deterministic trend.

Table 1.b. Kwiatkowski *et al.* (1992) Test for Unit Root
1948-1996

Series:	Lag length (l)						
	0	1	2	3	4	5	6
<u>Levels:</u>							
q_t (T)	0.84***	0.48***	0.35***	0.28***	0.24***	0.21**	0.19**
p_t (T)	0.70***	0.36***	0.25***	0.19**	0.16**	0.14*	0.13*
r_t	2.58***	1.47***	1.07***	0.86***	0.73**	0.65**	0.58**
<u>First differences:</u>							
Δq_t	0.12	0.17	0.19	0.21	0.24	0.26	0.27
Δp_t	0.61**	0.36*	0.28	0.23	0.20	0.17	0.16
Δr_t	0.05	0.06	0.07	0.09	0.11	0.12	0.15
<u>Critical test values:</u>							
		10 %	5 %	1 %			
Without trend		0.35	0.46	0.74			
With trend		0.12	0.15	0.22			

Note: The Kwiatkowski *et al.* (1992) test for a unit root is a Lagrange Multiplier test of the null hypothesis that the series can be described by a stationary process (possibly around a deterministic trend), against the alternative that the process also includes a random walk component. The null of stationarity is rejected in favor of the unit root alternative if the test statistic is sufficiently large. Critical values are from Kwiatkowski *et al.* (1992). *, ** and *** denote rejection of the null (i.e., a unit root is accepted) at the 10%, 5% and 1% significance level, respectively. Lag length l is the number of lags allowed for in the stationary component of the process. (T) after a series indicates that the test allows for a deterministic trend, i.e., the null hypothesis is trend-stationarity. Otherwise, the null is mean-stationarity.

Table 2. Specification Tests of the VAR Model
Estimation sample 1952-1996

<u>Multivariate tests:</u>			
Vector Autocorrelation order 2	F(18,65) =	1.05	[0.42]
Vector Autocorrelation order 4	F(36,50) =	0.92	[0.60]
Vector Autocorrelation order 6	F(54,33) =	0.98	[0.54]
Vector Heteroskedasticity (squares)	F(156,2) =	0.02	[1.00]
Normality	$\chi^2(6)$ =	7.60	[0.27]
<u>Univariate tests:</u>			
	Δq_t	Δp_t	Δr_t
Autocorrelation order 2, F(2,29):	0.50 [0.61]	0.06 [0.95]	1.80 [0.18]
Autocorrelation order 4, F(4,27):	1.34 [0.28]	0.71 [0.59]	1.15 [0.35]
Autocorrelation order 6, F(6,25):	1.14 [0.37]	0.56 [0.76]	0.94 [0.49]
ARCH (1), F(1,29):	0.15 [0.71]	1.36 [0.25]	0.02 [0.89]
Heteroskedast. (squares), F(26,4):	0.23 [0.99]	0.07 [1.00]	0.54 [0.85]
Normality, $\chi^2(2)$:	2.58 [0.27]	5.43 [0.07]	9.18 [0.01] *
<u>Goodness-of-fit:</u>			
ρ	0.98	1.00	0.81
σ_e	0.184	0.018	0.012

Note: The VAR model has a lag length of 4 ($k=4$). The F-tests are small sample approximations to Lagrange Multiplier tests, being adjusted for degrees of freedom. Normality test of Doornik and Hansen (1994). For a description of the tests, see Doornik and Hendry (1997). Numbers in brackets are critical significance levels. * and ** indicate misspecification at the 5% and 1% significance level, respectively. ρ is the correlation between actual and fitted values for each equation (variables in levels). σ_e is the standard deviation of the residual term.

Table 3. Cointegration Analysis in the VAR Model
Estimation sample 1952-1996

<u>Cointegrating rank:</u>			
Rank(II) ($r =$)	0	1	2
Eigenvalue	0.54	0.36	0.17
Trace test ¹⁾	63.3 ***	28.3 **	8.2
Trace test (adj. for df.) ²⁾	46.4 **	20.7	6.0
95 % critical test value	42.2	25.5	12.4
97.5 % critical test value	45.0	27.9	14.1
99 % critical test value	48.6	30.7	16.4
<u>Standardized eigenvectors β^*:</u>			
	β_1^*	β_2^*	β_3^*
q_t	1.000	0.066	0.031
p_t	-1.037	1.000	0.008
r_t	5.423	-15.338	1.000
t	-0.011	-0.053	-0.004
<u>Standardized loadings α_i ³⁾</u>			
	$\beta_1^* X_t^*$	$\beta_2^* X_t^*$	$\beta_3^* X_t^*$
Δq_t	-0.877	-0.136	8.049
Δp_t	-0.017	-0.051	-0.565
Δr_t	-0.017	0.010	-0.593

Note: Maximum Likelihood Estimation by the Johansen-method, cf. Johansen (1996). The trace tests test for each value of r the null hypothesis $H_0: \text{rank}(\Pi) \leq r$ against the alternative $H_A: \text{rank}(\Pi) > r$. The null is rejected iff the trace statistic is larger than the critical test value. Critical values from Table 15.4 in Johansen (1996). *, ** and *** indicate rejection of the null at the 5%, 2.5% and 1% significance level, respectively. The standardized eigenvectors are normalized on the diagonal wrt. the endogenous variables. Corresponding α -loadings.

¹⁾ The asymptotic trace test of the Johansen-method.

²⁾ Small sample approximation to the asymptotic trace test, obtained by adjusting for degrees of freedom, cf. Reimers (1992).

³⁾ $X_t^* = (q_t, p_t, r_t, t)'$

Table 4. Estimation and Testing of the Cointegrating Relation: Single-Equation-Analysis

The regression is: (*)

$$q_t = \beta_0 + \beta_1 p_t + \beta_2 f_t + \beta_3 \Delta r_{t-1} + \sum_{i=1}^N \gamma_i (\Delta p_{t-i} + \sum_{j=1}^N \gamma_j \Delta r_{t-j}) + v_t, \text{ where } \text{ecm}_t = q_t - \beta_0 - \beta_1 p_t - \beta_2 f_t - \beta_3 \Delta r_{t-1}$$

(*) is the static cointegrating regression augmented by current, leaded and lagged first differences of p_t and r_t and lagged error correction terms ecm_t . The augmenting terms in each regression are indicated in the first column.

Regression	Sample (no. obs.) No. of regressors	Coefficient Estimates (standard errors)			t-test on price level coeff. (critical sign. level) ¹⁾
		β_0	β_1	β_2	$H_0: \beta_1=0$ $H_0: \beta_1=1$
1. No augmentation	1948-1996 (49) 4	0.372 (0.99)	0.898 (0.22)	0.024 (0.014)	-11.32 (1.80)
2. 1 lead, 1 lag and current first differences of p_t and r_t ($N_1=N_2=1$, $N_3=0$ in (*))	1950-1995 (46) 10	0.283 (0.60)	0.948 (0.13)	0.015 (0.009)	7.18 * (0.000)
3. 2 leads, 2 lags and current first differences of p_t and r_t ($N_1=N_2=2$, $N_3=0$ in (*))	1951-1994 (44) 14	0.001 (1.18)	1.024 (0.26)	0.012 (0.016)	3.94 * (0.000)
4. Δp_{t-1} , Δr_{t-1} and Δr_{t-2}	1950-1994 (45) 7	0.464 (0.83)	0.913 (0.18)	0.024 (0.012)	5.02 * (0.000)
5. Δp_{t-1} , Δr_{t-2} and ecm_{t-1} (NLS)	1950-1994 (45) 7	-0.003 (1.31)	1.007 (0.29)	0.016 (0.019)	3.46 * (0.001)

Note: Entry 1 shows the results from estimating (OLS) the static cointegrating regression (no augmentation), including indicative OLS standard errors. Entries 2 through 4 give the results from the Phillips and Loretan (1991) OLS procedure using different augmentations (as tabulated). In entry 2 the standard errors of the coefficient estimates are adjusted to take account of AR(5) serial correlation in the disturbance term (v_t), using the method suggested by Hamilton (1994, p. 6088). Standard errors in entries 3 and 4 are OLS standard errors as the disturbance term fulfills the white noise requirements. Entry 5 uses the Phillips and Loretan (1991) NLS procedure. NLS standard errors, calculated from numerical derivatives of the sum of squared residuals.

¹⁾ Critical significance level for two-sided t-test, calculated from standard normal distribution (asymptotic test). A ** indicates that the null is rejected at the 5% significance level.

Table A.1. Cointegration Analysis: Estimates and Tests

Single-equation cointegration analysis following the Engle and Granger (1987) two-step procedure. Residuals-based tests for cointegration.

In the cointegrating regression, stock prices (q_t) are regressed on a constant term (const), measures for the general price level and real production, and the discount rate proxy (r_t), cf. (5). Sample 1948-1996

Model	Candidate cointegrating vector [OLS standard errors] ¹⁾	CRDF (no. of lags 2)	Critical test value at significance level ³⁾	Test conclusion: Cointegration at 10% significance level ?
$q_t, \text{const}, \text{pyf}_t, v_t, r_t$	(1; -1.67; -1.38; 0.16; 11.17) [0; 0.65; 0.14; 0.28; 2.10]	-2.834 (0)	10% 5% -3.990 -4.338 (N=4, No Trend, T=48)	No
$q_t, \text{const}, \text{pyf}_t, \text{fl}_t, v_t, r_t$	(1; 0.02; -1.41; -0.13; 8.78) [0; 0.29; 0.08; 0.13; 1.76]	-3.717 (0)	-3.990 -4.338 (N=4, No Trend, T=48)	No
$q_t, \text{const}, v_t, r_t$	(1; -3.38; -0.87; 12.98) [0; 0.11; 0.04; 2.52]	-3.162 (1)	-3.586 -3.927 (N=3, No Trend, T=47)	No
$q_t, \text{const}, p_t, r_t$	(1; -0.37; -0.90; -0.02; 11.32) [0; 0.99; 0.21; 0.01; 1.80]	-4.163 (1)	-4.032 -4.381 (N=3, With Trend, T=47)	Yes
$q_t, \text{const}, \text{pyf}_t, r_t$	(1; -2.56; -0.79; -0.03; 11.74) [0; 0.57; 0.23; 0.01; 1.90]	-3.920 (1)	-4.032 -4.381 (N=3, With Trend, T=47)	No
$q_t, \text{const}, \text{pyf}_t, \text{fl}_t, r_t$	(1; -1.29; -1.10; -0.02; 9.18) [0; 0.52; 0.18; 0.01; 1.59]	-4.460 (1)	-4.032 -4.381 (N=3, With Trend, T=47)	Yes

Note: For definition of variables entering the model, see text.

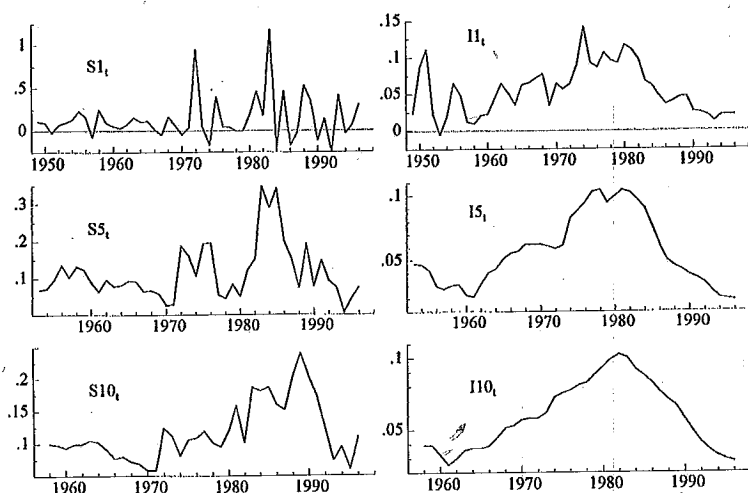
1) The candidate cointegrating vector is normalized on stock prices. In terms of (5), the vector is $(1; -\beta_0; -\beta_1; -\beta_2; \beta_3)$. OLS standard errors are indicative only.

2) CRDF is the Dickey and Fuller (1979) t-test statistic of the null of a unit root in the OLS residuals from the cointegrating regression. The null is rejected in favor of the stationary alternative (and cointegration is accepted) if the test statistic is negative and larger in absolute value than the critical test value. Number of augmenting lags of the first differences of the OLS residuals used in the unit root regression shown in parenthesis.

3) Small sample critical values from MacKinnon (1991). N=number of (1) variables in the model; 'No Trend' and 'With Trend' indicate whether a deterministic trend is included in the cointegrating regression; T=number of observations in the unit root regression, cf. Table 1 in MacKinnon (1991).

Figure B1. Stock Return and Inflation

Annual stock return and inflation for the (by row) 1-, 5- and 10-year horizon.
Sample periods 1949-1996, 1953-1996 and 1958-1996, respectively.

**Table B1.a. Phillips and Perron (1988) Z_t -Test for Unit Root**

Series:	Lag length (l)						
	0	1	2	3	4	5	6
1-Year Horizon, 1949-1996:							
$S1_t$	-8.52***	-8.52***	-8.53***	-8.55***	-8.61***	-8.67***	-8.72***
$I1_t$	-2.79*	-2.94**	-2.80*	-2.70*	-2.73*	-2.78*	-2.86*
$GR1_t$	-8.14***	-8.15***	-8.25***	-8.41***	-8.69***	-9.00***	-9.45***
$GY1_t$	-5.27***	-5.24***	-5.24***	-5.32***	-5.41***	-5.50***	-5.55***
5-Year Horizon, 1953-1996:							
$S5_t$	-3.16**	-3.11**	-3.23**	-3.22**	-3.28**	-3.15**	-3.08**
$I5_t$	-0.31	-0.63	-0.78	-0.91	-1.01	-1.08	-1.13
$GR5_t$	-3.25**	-3.35**	-3.41**	-3.36**	-3.31**	-3.11**	-2.96**
$GY5_t$	-1.28	-1.41	-1.43	-1.49	-1.56	-1.54	-1.49
10-Year Horizon, 1958-1996:							
$S10_t$	-2.19	-2.15	-2.18	-2.19	-2.18	-2.20	-2.20
$I10_t$	-0.25	-0.57	-0.74	-0.88	-0.99	-1.08	-1.16
$GR10_t$	-2.80*	-2.81*	-2.80*	-2.78*	-2.79*	-2.82*	-2.82*
$GY10_t$	-0.28	-0.40	-0.41	-0.49	-0.58	-0.64	-0.68
Critical test values:							
		10 %	5 %	2.5 %	1 %		
Without trend		-2.60	-2.93	-3.22	-3.58		

Note: See note to Table 1.a. All regressions include a constant term, while no trend is allowed for.
*, ** and *** denote rejection of the null of a unit root at the 10%, 5% and 1% significance level, respectively.

Table B1.b. Kwiatkowski *et al.* (1992) Test for Unit Root

Series:	Lag length (l)						
	0	1	2	3	4	5	6
1-Year Horizon, 1949-1996:							
SI_t	0.12	0.16	0.16	0.17	0.19	0.20	0.21
II_t	0.61**	0.36*	0.28	0.23	0.20	0.17	0.16
$GR1_t$	0.05	0.06	0.07	0.09	0.11	0.12	0.15
$GY1_t$	1.05***	0.84***	0.74***	0.64**	0.56**	0.51**	0.48**
5-Year Horizon, 1953-1996:							
SS_t	0.42*	0.26	0.20	0.18	0.16	0.16	0.16
IS_t	1.01***	0.52**	0.36*	0.28	0.23	0.20	0.18
$GR5_t$	0.32	0.20	0.17	0.15	0.16	0.17	0.19
$GY5_t$	2.43***	1.28***	0.90***	0.70**	0.59**	0.52**	0.47**
10-Year Horizon, 1958-1996:							
SI_{10t}	1.31***	0.74***	0.54**	0.44*	0.38*	0.34*	0.31*
II_{10t}	1.19***	0.61**	0.42*	0.33	0.27	0.24	0.21
$GR10_t$	0.73**	0.44*	0.34	0.29	0.26	0.24	0.22
$GY10_t$	3.04***	1.57***	1.07***	0.83***	0.68**	0.58**	0.52**
Critical test values:							
		10 %	5 %	1 %			
Without trend		0.35	0.46	0.74			

Note: See note to Table 1.b. No trend is allowed for in the tests, i.e., the null hypothesis is mean-stationarity. ** and *** denote rejection of the null (i.e., a unit root is present) at the 10%, 5% and 1% significance level, respectively.

Table B2. Return Regressions for the 1-, 5- and 10-Year Investment Horizon.

The model is:

$$S_{kt} = \beta_0 + \beta_1 \pi_{kt} + \beta_2 GR_{kt} + \beta_3 GY_{kt} + \varepsilon_t, \quad k = 1, 5, 10 \text{ years}$$

The 'simple' model in the table only includes inflation as an explanatory variable ($\beta_2 = \beta_3 = 0$), while the 'extended' model includes all variables in (*). In the 'reduced extended' model, we have removed the insignificant variables from the extended model.

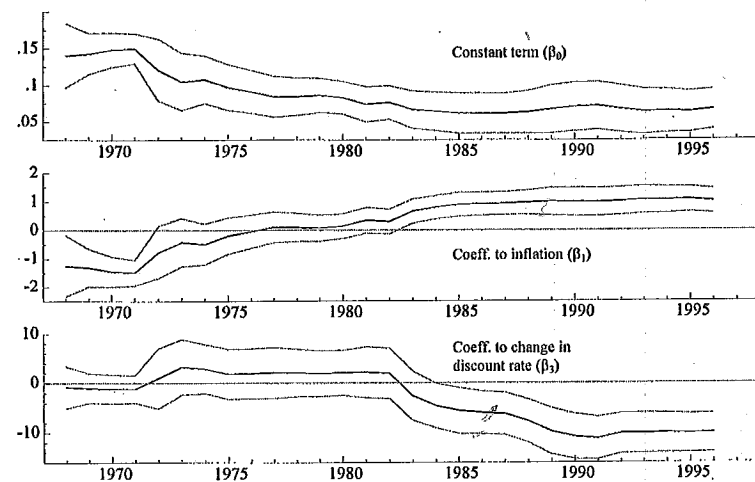
Horizon (years)	Sample (sample size)	Model	Coefficient estimates (standard errors)				Goodness-of-fit 2)	t-test on inflation coeff. (critical sign. level) 3)	
			β_0	β_1	β_2	β_3		$H_0: \beta_1 = 0$	$H_0: \beta_1 = 1$
1	1949-1996 (48)	Simple 1)	0.102 (0.029)	-0.106 (0.61)	-	-	0.57	0.18	-0.17 (0.865)
		Extended 1)	0.152 (0.045)	-0.205 (0.68)	-1.02 (0.88)	-5.63 (1.8)	0.67	0.16	-0.30 (0.764)
		Reduced extended 1)	0.111 (0.032)	-0.0368 (0.73)	-	-6.01 (1.7)	0.66	0.16	-0.05 (0.959)
5	1953-1996 (44)	Simple	0.0617 (0.025)	0.936 (0.61)	-	-	0.11	0.073	1.53 (0.126)
		Extended	0.0908 (0.041)	0.936 (0.57)	-0.702 (0.69)	-7.27 (2.8)	0.34	0.064	1.64 (0.101)
		Reduced extended	0.0643 (0.025)	1.01 (0.55)	-	-7.35 (2.8)	0.33	0.064	1.84 (0.066)
10	1958-1996 (39)	Simple	0.0665 (0.014)	0.858 (0.28)	-	-	0.21	0.041	3.06* (0.002)
		Extended	0.109 (0.031)	0.808 (0.28)	-0.944 (0.55)	-8.85 (2.4)	0.59	0.030	2.89* (0.004)
		Reduced extended	0.0668 (0.014)	1.01 (0.21)	-	-9.99 (2.6)	0.54	0.031	4.81* (0.000)

Note: All coefficients are estimated by OLS. Standard errors of the coefficient estimates are OLS errors for the 1-year horizon, respectively Newey and West (1987) errors for the 5- and 10- year horizons. The reported Newey and West (1987) errors allow for heteroskedasticity and serial correlation in the disturbance term (ε_t) up to lag 5. Impulse dummies included for 1972 and 1983.

1) Not corrected for serial correlation or heteroskedasticity in the disturbance term.

2) Critical significance level for two-sided t-test, calculated from standard normal distribution (asymptotic test). Based on Newey and West (1987) standard errors for the 5- and 10-year horizons. A ** indicates that the null is rejected at the 5% significance level.

Figure B2. Recursive Estimation of the 10-Year Return Regression
Recursive point estimates (solid line) and 95% confidence bands for the coefficients of the 10-year return regression (reduced extended model). Recursive least squares. Full sample: 1958-1996.



Chapter 2

On the Predictability of the Danish Equity Premium

On the Predictability of the Danish Equity Premium *

by

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Abstract

This paper analyzes whether, and to what extent, the Danish 1, 5 and 10-year equity premia are predictable. We examine the predictive power of a comprehensive list of financial ratios, interest rates and so forth. The results show that the 5-year premium is predictable in the sense that the model explains a non-trivial proportion of the variability of the equity premium. Moreover, the model is good at predicting turning points in the premium. We also analyze the portfolio implications of the model and find that the model is useful in predicting the optimal return maximizing portfolio choice. Finally, the paper presents forecasts for the 5-year equity premium.

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1. Introduction

The relationship between the stock market and the bond market and between the return on the two assets has been an active research area for many years in economics. In the academic literature on the topic, two approaches can be identified. The first approach attempts to explain the fundamental nature of the relationship between the two asset returns using general equilibrium theory. The second approach investigates the empirical relationship between the two asset returns and other variables that may be of importance within a partial equilibrium framework. This literature has in particular focused on whether and to what extent it is possible to predict the movement of the stock market relative to the bond market, which has bearings for the efficient market hypothesis.

In recent years, a large proportion of the general equilibrium research on stock and bond returns has been influenced by the Consumption-CAPM. According to this theory, the high return on stocks relative to bonds reflects the different covariances the two assets have with consumption. Because stock returns tend to covary more with consumption than bond returns, stocks are a poorer hedge against consumption fluctuations, and due to that stocks require a premium for investors to be willing to hold them. Kocherlakota (1996) surveys this literature and arrives at the conclusion that the magnitude of the equity premium remains a puzzle for the US, see also the pioneering paper by Mehra and Prescott (1985).

The other strand of the literature has searched for and actually found variables that have predictive power against the equity premium. This literature has shown that several financial ratios like the dividend price ratio, the price earnings ratio but also short and long term interest rates may have predictive power against the equity premium, see e.g. Lamont (1998) and Blanchard (1993). To the extent that it is possible to predict the return on stocks relative to the known or predetermined bond yield, this may be interpreted as a signal of market inefficiency. A related literature has solely been concerned with the predictability of stock returns and found that the aforementioned financial statistics also have predictive power against stock returns in particular in the medium and long term, see e.g., Campbell and Shiller (1998) and the survey in Campbell *et al.* (1997).

The purpose of this paper is to analyze the Danish equity premium and in particular to study whether the premium is predictable. While results for the Danish market should be of interest

in themselves, the paper may also be of interest in a broader perspective as the predictability literature has mainly focused on the US, whereas less is known for other markets. Moreover, the paper examines the predictive power of a fairly comprehensive list of potential predictor variables both in a single and multi-variable setting, that is, we investigate whether a candidate variable (e.g. the dividend-price ratio) is a useful predictor variable both when used in isolation and when other predictor variables are allowed for. We examine the 1-year, the 5-year and the 10-year equity premium within the period 1922-97. Our results show that the 5-year premium is predictable in the sense that there are predictor variables that explain a non-trivial proportion of the variability in the premium. In contrast to several of the earlier studies, however, we do not stop at this stage but proceed to investigate whether the statistical model is actually useful for forecasting purposes. To this end we check the stability of the model within the sample, and we also calculate the risk adjusted return we would have obtained had we followed the predictions of the model from 1971 and onwards in choosing between investments in stocks and bonds. We compare this risk adjusted return to a pure bond and a pure stock strategy, and find that the model outperforms these strategies. An important explanation of the success of the model is its ability to predict turning points and significant movements in the equity premium and hence to predict when it pays to choose either a diversified stock portfolio or a bond portfolio.

The paper also presents the prediction of the model for the 5-year period 1998-2002. This is of importance also from a practical view point because several analysts have predicted that stock markets will display large declines in the near future. Thus, Campbell and Shiller (1998) have argued that the stock market outlook in the US is extraordinarily bearish. Their prediction, frequently cited in the Financial Press, is entirely based on the current low dividend-price ratio, which they argue is likely to increase to its historical mean via essentially large declines in stock prices. Engsted and Tanggaard (1998) have replicated this analysis on Danish data, and their prediction is almost as gloomy as the forecast by Campbell and Shiller (1998). An important contribution of this paper is to demonstrate that the outlook for Denmark is not nearly as pessimistic when proper account is taken of other predictor variables. Thus, by allowing for not only the dividend-price ratio but also for other variables, the stock market forecast is certainly not a crash. Moreover, in the Danish case there is no reason to believe that the dividend-price ratio should return to its mean simply because this variable is not stationary. This has to do with institutional changes in the Danish economy that took place in the

beginning of the 1980s where the dividend-price ratio declined sharply, see Nielsen and Olesen (1999).

Section 2 of the paper presents the historical magnitude and movement of the equity premium at the three horizons and, furthermore, sketches the framework for the predictability analysis. Section 3 discusses a list of variables that might have predictive power and we also briefly comment on their statistical properties, which is of importance for the way we can formulate the regression equations. Section 4 presents the regression results for the 1-, 5- and 10-year horizon. Section 5 evaluates the statistical models in terms of parameter stability in-sample while section 6 evaluates the 5-year model in a portfolio performance setting. Section 7 reports and discusses the forecast for the 5-year period 1998-2002, and section 8 summarizes the paper.

2. Stock Returns, Bond Yields and Equity Premia

The 1-year, 5-year and 10-year nominal stock return along with the 1-year, 5-year and 10-year nominal yield to maturity on government bonds are illustrated in Figures 1, 2 and 3¹. The stock returns consist of the dividend yield and the capital gain (the equity price change).

Figure 1 shows that the 1-year stock return is highly volatile as compared to the short interest rate. There is very little correlation between the stock return and the interest rate in the short term; the simple correlation coefficient equals 0.19. Because stocks tend to yield higher return than bonds the 1-year equity premium is positive in the majority of the years. The average annual equity premium over the period 1924-1996², defined as the difference between the 1-year stock and bond return equals 2.3 per cent, and is fairly low by international comparison.

< Insert Figures 1, 2 and 3 around here >

¹ All returns are log returns (defined as the log to one plus the return) and they are all annualized. Moreover, they are forward looking. Our data are from the Nielsen, Olesen and Risager (1998) Database, Copenhagen Business School.

² Note that the notation adopted implies that the last recorded return is ultimo 1996 and is for the calendar year 1997.

Figure 2 depicts the 5-year stock return (5-year geometric average of the annual returns) and the 5-year interest rate. Because economic theory predicts a close relationship between the return on equity and the interest rate, cf. below, it is encouraging to notice the existence of a high degree of correlation in the medium term as witnessed also by the correlation coefficient which equals 0.62. Figure 3 displays the corresponding series for the 10-year horizon. At this horizon, the bond and stock return are also closely correlated. The correlation coefficient equals 0.78.

The close relationship between the two asset returns is familiar from the theory of equity pricing, see e.g. Campbell *et al.* (1997)³. In this context stock prices are determined as the expected discounted value of future dividends. This forward looking pricing equation is related to or follows from a no-arbitrage equation between the stock and the bond markets, which states that the expected k -period stock return Sk_t^e (where $k=1, 5$ and 10 years), consisting of both a dividend yield and a capital gain component, is equal to the k -period interest rate Bk_t properly adjusted for a risk premium γk_t ,

$$(1) \quad Sk_t^e = Bk_t + \gamma k_t$$

If the realized stock return Sk_t exceeds this expected or equilibrium return, stocks earn excessive returns. Likewise, bonds yield excessive returns if the realized stock return turns out to be lower than its expected level in (1)⁴. According to the efficient market hypothesis, it is impossible to earn excess returns in a systematic way, that is, over long horizons excess returns should on average be zero. A necessary condition for this to be the case is that realized excess returns are unforecastable on the basis of available information (i.e. white noise), meaning that the best estimate (the point estimate) of excess returns is always zero. Thus, predictability is a sign of market inefficiency. What we need in order to make the efficiency hypothesis testable is an operational model for the equilibrium returns in (1).

³ The relationship between stock and bond returns is also known from the neoclassical investment model, which produces a first order condition for optimality that says that the total return on a unit of capital, measured as the dividend yield plus the capital gain, should equal the opportunity cost of capital appropriately adjusted for depreciation and risk, see e.g. Blanchard and Fisher (1989).

⁴ By use of (1), excess returns can be determined as $EXCK_t = Sk_t - Bk_t - \gamma k_t$.

The often used hypothesis is that the risk premium is constant over time. Under this assumption, the efficient market condition is equivalent to the statement that the equity premium PRk_t ($\equiv Sk_t - Bk_t$) is unforecastable except for the constant term capturing the constant risk premium. The efficient market hypothesis is tested by examining whether the premium PRk_t can be forecasted on the basis of available information. This test is conducted by regressing the realized premium PRk_t on a set of potential predictor variables, cf. below, and if these variables explain a non-trivial proportion of the variability of the premium we may conclude that there is departure from efficiency under the null that the risk premium is constant.

If the risk premium is time variant, predictability may but need not signal market inefficiency, see also Campbell *et al.* (1997). In the light of the inherent difficulty in testing the efficiency hypothesis, we interpret predictability as weak evidence against efficiency, because the potential time variability of the risk premium endows proponents of the efficient market hypothesis with an escape clause.

3. Potential Predictor Variables and Their Statistical Properties

The close medium and long term relationship between stock returns and interest rates is naturally an important focus for a paper that investigates whether, and to what extent, it is possible to predict stock returns. However, since the paper also deals with the efficiency issue, which relates to the predictability of excess returns rather than stock returns, we have chosen the equity premium as the dependent variable. Another motivation for this choice is that 5- and 10-year stock returns and interest rates are non-stationary, whereas the return difference is stationary, see Appendix 1 for unit root tests. Hence, we can analyze the equity premium by conventional statistical methods. In the statistical analysis that follows, the 1-year premium is defined as the (natural) log of one plus the 1-year stock return minus the log of one plus the 1-year interest rate, denoted $PR1$. The corresponding 5- and 10-year premia are denoted $PR5$ and $PR10$, respectively. Below, we discuss potential predictor variables. The variables that all relate to fundamentals are selected because they often enter in both academic research and practitioners' applications.

The dividend price ratio. Several studies for the US have shown that the ratio between current dividends ${}_tD_t$ and the end of period stock price P_t has predictive power in the medium and long term in particular, see e.g. Campbell *et al.* (1997). According to these empirical

studies a low ratio signals falling future stock prices (and not increasing dividends). Hence, a low dividend price ratio is a warning of low future stock returns. Because the Danish dividend price ratio is non-stationary, according to standard unit root tests, we do not use the ratio as it is but subtract an equally weighted moving average of the dividend-price ratio (the current and past five years observations) from the current dividend-price ratio, resulting in a stationary variable. This stochastically detrended dividend price ratio is denoted $D7P^5$.

The dividend yield. Another measure of fundamentals is the dividend yield Yld , defined as $_{t-1}D_t/P_{t-1}$, that is, current dividends divided by beginning-of-period stock prices. The dividend yield can be viewed as an alternative to the dividend price ratio with the difference being the timing of stock prices. This variable is also non-stationary, and we shall therefore also work with the dividend yield subtracted by an equally weighted average of the current and past five observations. This modified dividend yield is labeled Yld .

Interest rates. For each horizon we use the appropriate interest rate subtracted by a moving average of the current and past five observations. A motivation for introducing interest rates as predictors for the equity premia is that the empirical relation between stock returns and bond returns may not be a one-to-one relationship as implicitly assumed when using the premium as the dependent variable. This may be captured by including interest rates. The modified interest rate variables are henceforth labeled $B1$, $B5$, and $B10$, respectively.

Term structure variables. An upward sloping yield curve may signal higher economic activity in the future, which in turn may be positively correlated with earnings and stock returns. To the extent that this potentially valuable information is not incorporated (correctly) in current stock prices, the term structure may have predictive power for the equity premium. The term structure variable for the 10- versus 1-year horizon is defined as the log of one plus the 10-year interest rate minus the log of one plus the 1-year interest rate and denoted $TE10-1$. The other term structure variables are defined analogously and denoted $TE5-1$, and $TE10-5$, respectively.

⁵Any of the detrended variables that we are using can be written as: $\bar{X}_t = X_t - (X_t + \dots + X_{t-5})/6 = (5/6)(X_t - (X_{t-1} + \dots + X_{t-5})/5)$. Hence, the impact of X_t on the dependent variable in the predictor model is $(5/6)$ times the coefficient to the detrended variable.

Mean reversion. In the Danish case there is evidence of mean reversion in the sense that good years in the stock market are followed by bad years and *vice versa*, see Risager (1998). To capture this we include lagged 1-year equity premia, denoted PRI_{t-1} , PRI_{t-2} and PRI_{t-3} , respectively. Further lags have proved to be insignificant as predictors ⁶. It is important to emphasize that these are the lagged equity premia and that there is no overlap between these variables and the dependent variable.

It is customary to distinguish between three kinds of market (or informational) efficiency, depending on the specification of the information set that can be used for predicting excess returns: weak-form efficiency (information set consisting of past realizations of returns and prices), semi-strong-form efficiency (all publicly available information) and strong-form efficiency (all information, including privately held information). Evidence of mean-reversion can be interpreted as evidence against market efficiency in its weakest form, while forecastability on the basis of the other predictor variables is evidence against market efficiency in its semi-strong form, albeit not definitive evidence as noted earlier.

4. Regression Results

In the analysis to be reported below we first perform a single variable analysis where each candidate predictor variable is entered separately. The well-known weakness of this approach is that the parameter estimates will be biased if some of the other (omitted) variables have explanatory power and if there is correlation between the included and the omitted variables. We therefore also run regressions with all variables entering at the same time, and from the general specification we derive the parsimonious representation in which all variables are significant⁷. The latter model is the preferred one for both econometric and economic reasons because the model includes all relevant information. All coefficient estimates are obtained by OLS. Due to the use of overlapping observations, there are potential serial correlation and heteroskedasticity problems at the 5 and 10 year horizons. The standard errors of the coeffi-

⁶Note that in the 1-year premium equation this implies that the lagged dependent variable appears as an regressor. This leads to biased OLS estimates, but the OLS estimates are still consistent as there is no serial correlation in the disturbance term.

⁷The least significant variable is first omitted. After the model has been reestimated, the next insignificant variable is deleted, and so forth. We use the standard 5 per cent significance level in the modeling reduction process.

cients are therefore estimated using the Newey-West method, cf. Newey and West (1987), which gives consistent estimates. For the 1 year horizon, White's heteroskedasticity consistent method is used for estimating the standard errors. For the parsimonious representations at each horizon diagnostic graphics including a plot of the fit of the model are shown in Appendix 2.

4.1. The 1-Year Equity Premium

The results for the 1-year horizon are reported in Table 1. Rows 1 to 7 give the single variable regressions while row 8 shows the full model with all predictor variables included simultaneously. Row 9 is the parsimonious equation, which is the preferred model.

The results show that the term structure (10 minus 1 year) is significant at the 1 per cent level in the parsimonious regression. A rise in the 10-year interest rate relative to the 1-year rate signals a higher equity premium. The parsimonious regression also includes the lagged 1-year equity premium. High past equity premia are associated with declining future premia, that is, there is evidence of mean reversion. It is interesting to note that the dividend price ratio, emphasized by Campbell *et al.* (1997) and Campbell and Shiller (1998), is insignificant (even at the 10 percent level) when entered separately and is removed in the modeling reduction process leading to the parsimonious model. Altogether, the two significant variables only explain 16 per cent of the variability in the dependent variable. The conclusion is therefore that there is a lot of noise in the 1-year premium and due to that predictor variables do a poor job in forecasting the premium in the short term.

< Table 1 >

4.2. The 5-Year Equity Premium

The results for the 5-year premium are statistically stronger than for the 1-year premium. In the parsimonious equation reported in Table 2, both the dividend yield, the interest rate and the past 1-year equity premia are all highly significant. Furthermore, the parsimonious model explains 44 per cent of the variability of the 5-year premium, which is a satisfactory result for a pure predictor model.

< Table 2 >

The parsimonious model can be rewritten in a form where only the 5-year stock return appears on the left hand side⁸,

$$(2) \quad S5_t = 0.026 + 1.07B5_t^* + 2.72(Yld_t - Yld_t^*) - 0.16PR1_{t-1} - 0.06PR1_{t-3} + u_t$$

where a '*' denotes moving averages over the past 5 years (excluding the current value) and u_t is the residual. This equation has several interesting characteristics. First, a one percentage point increase in the dividend yield is associated with a 2.72 per cent increase in the 5-year stock return (and equity premium). This estimate may seem high, but the high degree of statistical significance underscores the point that the dividend yield is an important predictor variable. Second, the interest rate affects stock returns through the average of the past 5 years' interest rates. The effect is roughly a one-to-one effect, meaning that a one percentage point increase in the average interest rate over the past 5 years approximately predicts a one percentage point increase in the 5-year stock return. It is important to note that a change in the contemporaneous interest rate has a negligible effect if this change does not persist into the future. Third, past equity premia also play an important role at the 5-year horizon. High returns in the past signal low stock returns in the future. According to the coefficient estimate as much as 80 per cent of any 1-year premium will *ceteris paribus* be reversed within the coming 5-year period.

4.3. The 10-Year Equity Premium

The parsimonious equation for the 10-year premium is given in line 9 in Table 3. The dividend yield is again significant at the one per cent level. The lagged annual premium is also significant at the one per cent level. According to the coefficient estimate, the 40 per cent excess return on stocks in 1997 is associated with a 1.96 per cent lower premium in the forthcoming 10-year period. Thus, there is also at this horizon a strong tendency to mean reversion. The modified 10-year interest rate is not significant suggesting that the current long interest rate has a one to one effect on the 10-year stock return, without any effect from lagged

⁸ We have ignored the term $-0.066B5_t$, because this is negligible, and the coefficient is not significantly different from zero.

interest rates. The R^2 equals 0.26 and hence is considerably lower than for the 5-year horizon.

< Table 3 >

4.4. Summing up

The results obtained so far show that the forecasting variables are most useful in a medium term perspective, which is a conclusion that will be further strengthened when we examine the parameter stability of the models. At the 1-year horizon, there is a substantial amount of fluctuations in the equity premium that cannot be explained by the movements of the broad spectrum of forecasting variables (fundamentals) that we have looked at, and it is therefore likely that the short term is dominated by non-fundamental factors, noise trading etc. The fundamentals also explain relatively little of the variability of the 10-year premium, which may simply reflect that contemporaneous financial statistics have very little to say about returns over such a long time span.

5. Model Evaluation: Parameter Stability

A necessary condition for equity premium predictability is that the predictor variables in question should be able to explain a non-trivial proportion of the variability of the premium over the sample. In order for the model to be useful for forecasting purposes it is, however, also important that the relationship between the predictor variables and the premium is stable over time. The only way to judge stability is to examine the historical relationship between the premium and the predictor variables. If the historical parameter estimates are stable, we may have some confidence also in future parameter stability and hence in out-of-sample forecastability. However, there is of course always a risk that a forecasting rule which has been successful in the past may become obsolete in the future due to learning behavior in the market (or some other structural breaks). It is not possible to hedge against this risk.

In order to analyze the within sample stability of the regressions, we have for each horizon estimated the parameters recursively. It turns out that the parameters in the parsimonious equations for the 1-year and 10-year horizon are unstable in particular towards the end of the sample period, see Appendix 3. In the model for the 5-year horizon, the parameters are reasonably stable after the beginning of the 1970s, see Figures 4-8. Similarly, by estimating the three models on data only for the post World War II period it turns out that it is only the

model for the 5-year horizon that has (reasonably) stable parameters across the full sample and sub-sample period, see the regression results in Appendix 4⁹.

< Figures 4-8 >

Another way to test the forecasting ability of a statistical model is to estimate the model over a given sub-sample, construct forecasts for the remainder of the sample and then compare the forecasts with the realizations of the dependent variable. For the 5-year horizon we therefore estimate the model on the sub-sample 1927 to 1970 and subsequently make forecasts for the period 1971 to 1992¹⁰. Figure 9 compares the actual 5-year premium with the predicted premium.

< Figure 9 >

The diagram shows that the model performs well in a qualitative sense, that is, in predicting the significant movements of the premium and in particular the important turning points. Thus, when the premium has risen (fallen) by significant amounts the model correctly predicts this change in almost all cases. However, it is also clear that the model's quantitative performance is less impressive; often the realized equity premia are close to the boundaries of the OLS forecasting interval. Moreover, there is a tendency to either over- or underpredict the premium. This phenomenon, however, is almost inevitable when forecasts are made over overlapping horizons. To understand the nature of this phenomenon, suppose we are at the New Years Eve in 1991 and that we attempt to forecast the five year premium for 1992-96. Let us further suppose that 1994 turns out to yield an extremely high return in the stock market for some unforeseeable reasons. Given that this is something we cannot know in 1991, the predicted 5-year premium is likely to underestimate the actual premium. Moreover, the model will for the same reason also underpredict in 1992, and so forth. The implication is that we may observe persistent over- or underprediction *ex post* but we cannot correct for it *ex*

⁹ For the 1- and 10-year model we even observe that the relevant predictors change.

¹⁰ We have conducted similar forecasting exercises for the 1 and 10 year horizons, leading to the conclusion that the actual forecasting ability of the premium models are poor, see Appendix 5. This confirms the conclusion from the recursive estimation that the model coefficients are unstable.

*ante*¹¹. In spite of the shortcomings of the model it is of interest to note that the forecast for 1993-97 is almost exactly equal to the actual premium, which is very high (10.8 per cent per annum) due to the exceptionally good stock market years 1996 and 1997. The most important explanation underlying this prediction is the mean reversion component (the lagged equity premia) which predicts a large equity premium for this period due to a very poor stock market performance in the years 1989 to 1991. Notice also that the model prediction in 1991 is very close to the realized premium (for 1992-96).

6. Model Evaluation: Portfolio Strategies (5 Year Model)

In order to shed further light on the usefulness of the 5-year predictor model, it is informative to analyze the consequences of making investment decisions on the basis of this model. To arrive at the most clear-cut insights, assume that the investor picks a pure stock or bond portfolio depending on what the model is recommending. Moreover, assume that the investor demands a (constant) premium in order to be willing to invest in stocks. Let this premium be equal to the unconditional equity premium. As the model for this purpose is estimated over the period 1927-1970, where the unconditional (logarithmic) mean equity premium equals 1.91 per cent, we assume that this is the investor's risk premium. Thus, if the model in late 1971 predicts a premium that exceeds 1.91 per cent, the potential investor goes into stocks. If the predicted premium is below the critical 1.91 per cent, the investor goes into bonds. The performance of this strategy over the period 1971-92 is then compared to the risk adjusted return on the two benchmark strategies, namely, a pure stock strategy and a pure bond portfolio. We ignore transaction costs, but they are not likely to influence our results in a crucial way. We also ignore investor taxes, so the case is mostly relevant for investors who are taxed symmetrically, e.g. banks.

Figure 10 plots the risk adjusted returns on the pure stock and bond benchmark strategies as well as the outcome of following the model recommendation. The returns are risk adjusted in

¹¹ The point is that we use 5 year ahead forecasts. That is, the forecast for the 5 year premium as of 1997 is based on observations on the 5 year premium up to 1992. As the 5 year periods beginning in 1992 and 1997, respectively, are non-overlapping no serial correlation should be expected. If we on the other hand were to make 1 year ahead forecasts (e.g. forecasting the premium as of 1993) we would have to take account of serial correlation. This could for instance be done by explicitly allowing for serial correlation in the disturbance term when setting up and estimating the premium model.

the sense that the risk premium 1.91 per cent is subtracted from the pure stock return and from the return associated with the model recommendation whenever the model recommendation has resulted in a stock investment.

< Figure 10 >

The diagram shows that by following the model, the investor makes the maximizing return decision almost every year; there are only three years where the return associated with the model based choice is not the highest attainable. The average (arithmetic) annualized risk adjusted return from following the model recommendation is 14.0 per cent. The pure bond strategy yields 12.1 per cent, whereas the pure stock strategy gives 10.7 per cent after the risk adjustment. Hence, the yield difference to a pure bond investment is around 2 per cent per year, whereas the yield difference compared to a pure stock strategy is around 3 per cent. A simple mean t-test suggests that the differences in returns between the model strategy and the benchmark stock strategy is highly significant, that is, the return differences between the model strategy and the pure stock strategy has a mean that is significantly larger than zero, using a one per cent significance level.¹² The corresponding comparison between the model and the bond strategy yields, unfortunately, less clear-cut results. The t-test is significant almost at the 5 per cent level, but the result hinges primarily on the three observations in the period 1978-1980.¹³

7. The 5-Year Premium for 1998-2002: Model Prediction and Discussion

Due to the 5-year model's forecasting ability and in particular the model's track record in the recent past, it is of interest to discuss the prediction for 1998-2002¹⁴.

¹² The t-ratio is 4.2 with 22 degrees of freedom. Note that the return difference can be shown to be normally distributed using the Doornik and Hansen (1994) small sample test.

¹³ Moreover, the return difference is not normally distributed.

¹⁴ It should be noted that the precise premium forecasts that one arrives at depends on which of the predictor variables one includes in the model. Thus, using for instance the modified dividend-price ratio as the sole predictor will lead to different forecasts. We think that by using a multi-variable setting and a general-to-specific model reduction process we have identified the predictor variables that are most relevant of the candidate predictors at hand. We therefore rely more on the forecasts of the parsimonious model than the ones one would obtain from the different specifications in Table 2.

By plugging the values of the modified dividend yield, the modified 5-year interest rate and the lagged 1-year premium variables ($PR1_{t-1}$ and $PR1_{t-3}$) by late 1997 into the 5-year model we arrive at the forecast for the period 1998-2002. The point forecast for the equity premium is roughly zero per cent which is low compared to its historical average. The contributions of each of the predictor variables are given in Table 4. The constant (risk premium) contributes with 2.5%, the dividend yield adds 0.5% to this, whereas the interest rate variable further adds 2%. Because of the high premium in the past, we shall, however, subtract 5.3%. Hence, altogether the premium is expected to be close to zero in the 5-year period 1998-2002.

Recalling that the model has been successful at predicting turning points and significant movements in the premium historically, it is interesting to note that the model predicts a turning point in the 5 year premium with a significant reduction of the premium compared to the last observation in 1992. If we follow the portfolio decision strategy in section 6, this suggests that investors should have gone into bonds in late 1997.

< Table 4 >

Given that the 5-year interest rate is 5 per cent in late 1997, the forecast for the premium implies that the (annualized) 5-year stock return should be 5 per cent. Assuming that the dividend yield is 1.5 per cent throughout the 5 year forecast period - which corresponds to the level in recent years - the stock market price index is predicted to rise by 3.5 per cent annually in the period 1998-2002. The model therefore predicts a much less optimistic outlook than experienced in the recent past. The considerable fall in interest rates that the Danish economy has experienced in recent years is a key explanation of the less optimistic future stock return scenario. Another explanation is the very high premium in the recent past and due to the highly significant tendency to mean reversion this also produces a less optimistic outlook. The current dividend yield is broadly in line with its level in the past 5 years, thus having a rather small effect on the stock return prediction.

As is evident from the forecasting exercise in section 5, the point forecasts for the 5 year premium are sometimes imprecise, implying that more emphasis should be put on interval forecasts when deriving specific numbers for the future premium. As usual when forecasting stock market returns, the uncertainty attached to the forecast is considerable. Given that the

residuals are normally distributed, the 95% confidence bands can be estimated from twice the standard error of the equation, that is, as $\pm 7.6\%$ relative to the point forecast, see Table 2. Hence, in terms of the annual return on the stock market the confidence band is $(-2.6\%, +12.6\%)$. Assuming a dividend yield equal to 1.5% per year, the confidence band for the annual growth in the share price index is $(-4.1\%, +11.1\%)$. Because many analysts are very pessimistic at the moment, see e.g. Cole *et al.* (1996) and Campbell and Shiller (1998), let us briefly focus attention on the bearish side of these confidence intervals. If the share price index stands at 100 to begin with and if the index declines by 4.1% annually, the share price index may fall to roughly 80 after 5 years. Hence, a 20% fall in the index is the rough lower bound of the confidence interval¹⁵.

The forecast reported above is the forecast that historical experience and the use of standard predictors can provide us with. The point forecast is the central estimate of the model. Due to the (considerable) uncertainty inherent in the model the actual outcome may deviate from the point forecast but the deviations are equally likely in both directions, as set out by the forecast interval. Thus, the model does not attach greater probability to negative deviations than to positive ones, and *vice versa*. Due to that it may be valuable to add judgmental factors in order to find out which part of the confidence band - the lower or the upper one - we will attach most probability to. In the Danish case, we think that there are some factors that may suggest a negative outcome. First, there is a risk that the economy moves into recession after four years with high economic activity, which will dampen earnings growth. Second, there is a risk of an American stock market crash, which may spread to the rest of the World. In this context it is, however, important to note that stocks in the US have increased much faster than in Denmark. On the other hand, there are also more bright sides. First, the Danish economy is in a transition phase to an economy with much more emphasis on stock investment. Thus, institutional investors have increased the share of stocks in their portfolios quite considerably in recent years, but they are far from the long run equilibrium level. The stock market has also received much more attention in recent years from ordinary citizens. Hence, there appears to have been a structural shift in the demand curve, which makes it easier to support a fairly high price level provided liquidity plays a role. Second, there is nowadays much more emphasis on share holder value and the notion that firms should make money. Due to that it is easier for

¹⁵ By the same line of reasoning, the upper bound of the confidence interval predicts an increase in the stock market index by 55 per cent over the 5 year period.

firms to make rational business decisions in order to maintain profitability. Third, the Danish equity premium has been fairly low by international comparison over a long historical period. With capital being highly mobile it is possible that the Danish premium will approach the higher Anglo-Saxon level, notwithstanding that the US equity premium might fall but from a very high level compared to the Danish premium, see Blanchard (1993).

As a final piece of information in judging the forecast, Figure 11 plots the model's consecutive 5-year stock return forecasts until 1997 along with the realized returns until 1992. The trend in the (forecasts of the) 5-year stock return is declining such that the gap between the expected stock return and the bond yield gradually disappears. Thus, as noted earlier the forecast in late 1997 is that the stock market over the period 1998-2002 will give a return that is equal to the bond return. The figure also shows that the anticipated premium in the period 1993-97 to a large extent compensates for the negative premium in the period 1987-91. Hence, the very high 5-year stock return recorded in 1992 along with the predicted returns can be interpreted as a compensation for poor stock returns in the preceding period. In this context it should be noted that for the whole period 1987-97, the premium is only 0.9 per cent per year compared to a historical average that equals 1.4 per cent.

< Figure 11 >

8. Summary

This paper has examined whether, and to what extent, the return on Danish stocks relative to bonds can be predicted by financial ratios and other financial statistics. We have examined both the 1-year, 5-year and 10-year equity premium. We have investigated the predictor ability of the dividend-price ratio, the dividend yield, various short and long term interest rates, and we have also allowed for past equity premia to have an effect on the current equity premium, reflecting the possibility of mean reversion. The issue of forecastability has not only been examined by testing the significance of the aforementioned predictor variables, but we have also investigated whether parameters are stable and whether the model is helpful in predicting when stocks outperform bonds and *vice versa*.

The main result that comes out of our analysis is that the 5-year premium is predictable. Thus,

the preferred model is good at predicting significant movements and turning points in the 5-year premium. Due to that the model is also a useful tool for portfolio decisions, that is, to predict when it pays to be more exposed to stocks than to bonds, and *vice versa*. Thus, the results show that if investors had followed the model in deciding between stock and bond investments, they would have made systematic excess returns compared to a pure stock strategy. The results also show that it is only a subset of the variables that have predictive power. More specifically, the dividend yield is of some value, but it is really interest rates and the past equity premium that are the key predictor variables. Finally, the ability to predict the equity premium is evidence against market efficiency in its semi-strong form if there is a constant risk premium in the market. In any case, the predictability result is evidence against the simultaneous hypothesis of efficient markets and a constant risk premium. Due to parameter instability and low explanatory power, the 1- and 10-year equity premia can not be said to be predictable.

The preferred 5 year premium model can be used for forecasting the equity premium and the stock return over the years 1998-2002. It is mainly due to a historically low 5-year interest rate and very high returns in the recent past, that the model predicts a low 5-year stock return that is roughly equal to the contemporaneous 5-year interest rate, implying a zero equity premium. The expected outcome is not impressive, but not a disaster either. Thus, the outlook for the Danish market is not extraordinarily bearish as argued by Engsted and Tanggaard (1998), using the single variable dividend-price approach due to Campbell and Shiller (1998). Whether or not our conclusion, based on a multi-variable approach, carries over to the US is another matter that we have not addressed.

It is important to emphasize that the reported forecasts are based on historical relationships between stock returns and financial ratios. Any forecast that has to be used in real-life situations will of course also depend on other judgmental factors and broad perspectives on the outlook for the economy in general. The paper has discussed a few factors that should be taken into account when making such a 'normative' forecast.

Postscript

It is of interest to compare the model's forecast with the actual performance of the market. According to the model the market should go up by 3.5% per year as from late 1997. In the two years 1998 and 1999 that have passed since the first model forecast was made, stock prices have altogether increased by 14.6%. Thus, the model is on track.

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Figure 1a: 1-Year Stock and Bond Return, 1924-96

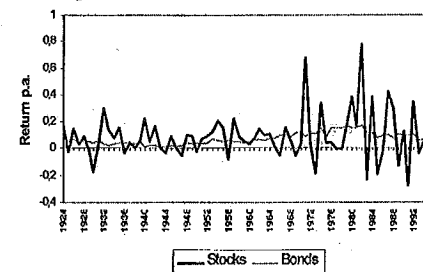


Figure 1b: 1-Year Equity Premium, 1924-96

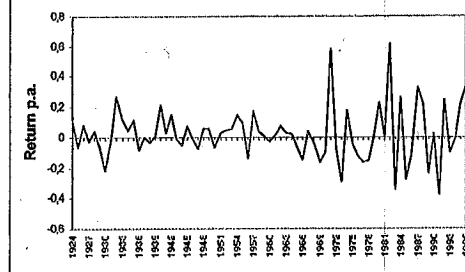


Figure 2a: 5-Year Stock and Bond Return, 1921-92

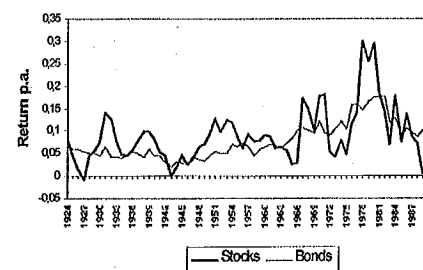


Figure 2b: 5-Year Equity Premium, 1924-92

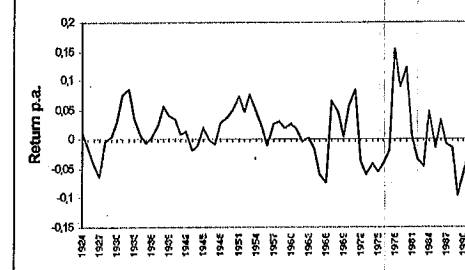


Figure 3a: 10-Year Stock and Bond Return, 1924-87

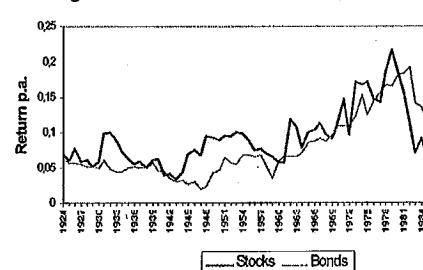
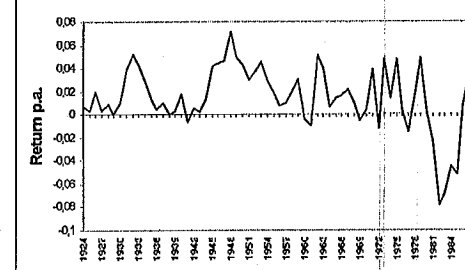
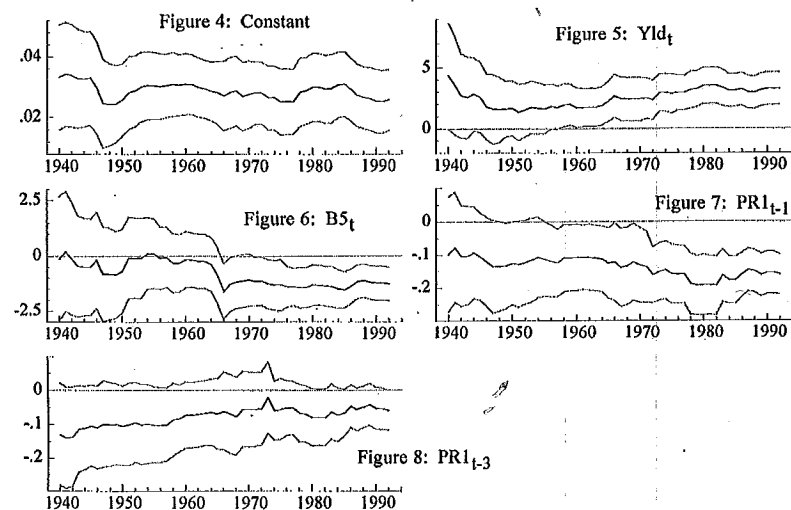


Figure 3b: 10-Year Equity Premium, 1924-87

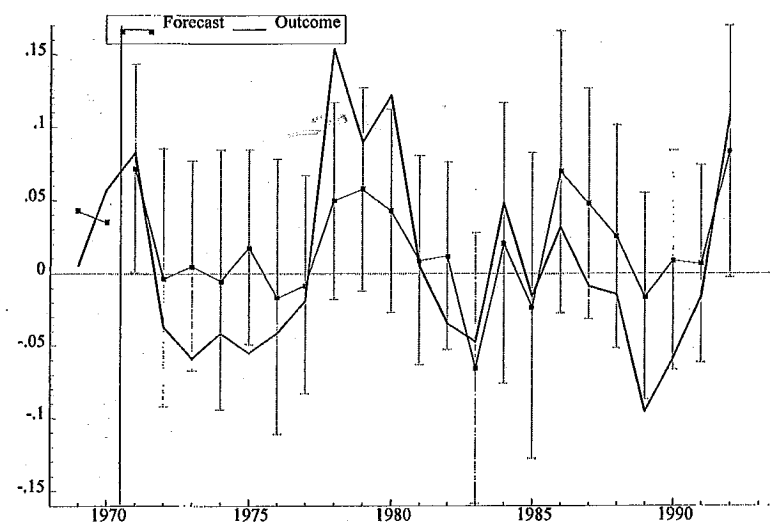


Figures 4-8 Recursive Parameter Estimates for 5-Year Equity Premium Model 1940-1992 (starting year 1927)



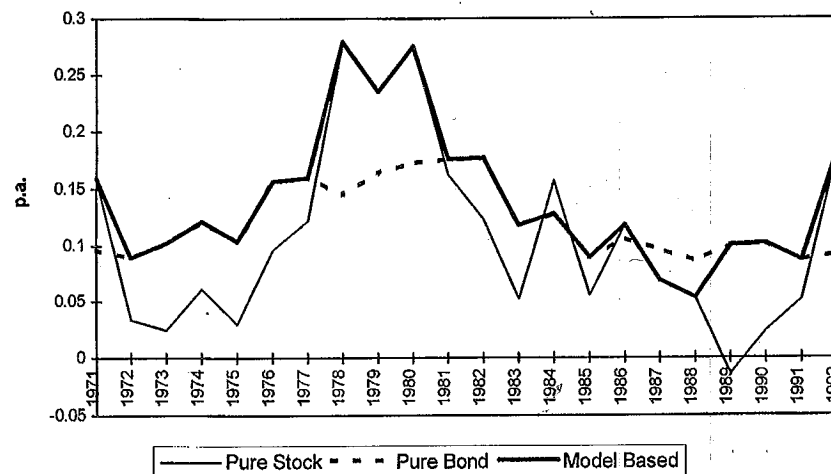
Note: Confidence bands only indicative (based on OLS coefficient standard errors).

Figure 9 Forecasting the 5-Year Equity Premium $PR5_t$, 1971-1992



Note: OLS confidence bands only indicative.

Figure 10 5-Year Risk-Adjusted Return to Portfolio Strategies 1971-1992



**Figure 11 5-Year Stock and Bond Return, 1927-97
Consecutive Stock Return Forecasts for 1993 to 1997**

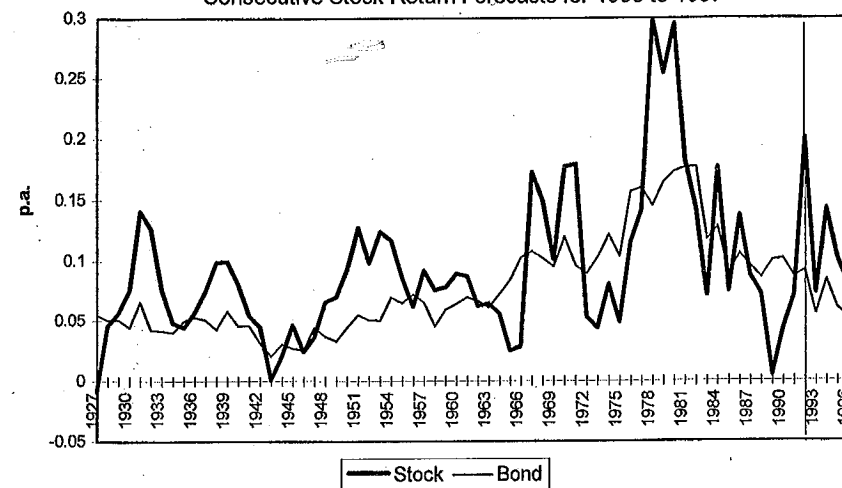


Table 1: The 1-Year Equity Premium $PR1_t$, 1929-96^{1,2,3}

	CONS	$D\tilde{T}P_t$	$\tilde{Y}Id_t$	$\tilde{B}1_t$	TE10-1 _t	TE10-5 _t	TE5-1 _t	$PR1_{t-1}$	$PR1_{t-2}$	$PR1_{t-3}$	Lag	σ	R^2
1.	0.011 (0.018)	2.885 (2.298)									-	0.153	0.05
2.	0.010 (0.017)		2.523 (2.934)								-	0.154	0.05
3.	0.005 (0.018)			-1.015 (1.375)							-	0.154	0.05
4.	-0.016 (0.019)				2.782* (1.490)						-	0.150	0.05
5.	0.003 (0.017)					0.982 (1.954)					-	0.155	0.05
6.	-0.011 (0.022)						3.287* (1.762)				-	0.150	0.05
7.	0.009 (0.018)							-0.184 ⁴⁾ (0.118)			-	0.152	0.05
8.	0.004 (0.021)	-10.02 (6.110)	13.54*** (5.237)	-1.808 (1.578)	3.056* (1.640)	-0.729 (2.512)		-0.664*** (0.240)	-0.023 (0.168)	0.005 (0.136)	-	0.141	0.05
9.	-0.018 (0.018)				3.883*** (1.268)			-0.280** (0.124)			-	0.143	0.05

Table 2: The 5-Year Equity Premium $PR5_t$, 1927-92^{1,2}

	CONS	$D\tilde{T}P_t$	$\tilde{Y}Id_t$	$\tilde{B}5_t$	TE10-1 _t	TE10-5 _t	TE5-1 _t	$PR1_{t-1}$	$PR1_{t-2}$	$PR1_{t-3}$	Lag	σ	R^2
1.	0.019*** (0.006)	2.469*** (0.622)									5	0.043	0.24
2.	0.018*** (0.006)		2.071** (0.834)								5	0.045	0.14
3.	0.014** (0.006)			0.349 (0.328)							5	0.049	0.01
4.	0.013* (0.007)				0.152 (0.355)						5	0.049	0.00
5.	0.015** (0.006)					-0.243 (0.674)					5	0.049	0.00
6.	0.012 (0.008)						0.434 (0.400)				5	0.049	0.01
7.	0.021*** (0.007)							-0.133*** (0.018)	-0.126*** (0.029)	-0.108*** (0.024)	5	0.039	0.38
8.	0.021*** (0.006)	-0.702 (0.892)	2.95*** (1.10)	-0.963** (0.382)	0.770*** (0.284)	-0.328 (0.646)	---	-0.195*** (0.037)	-0.063 (0.038)	-0.080** (0.032)	5	0.037	0.49
9.	0.026*** (0.006)		3.258*** (0.696)	-1.279*** (0.272)				-0.160*** (0.026)		-0.059*** (0.019)	5	0.038	0.44

Notes: 1) OLS and White's heteroskedasticity consistent estimator of coefficient standard errors. σ denotes the standard error of the residual term. σ and R^2 are calculated from the OLS formula, excluding the two years 1971 and 1982 for which dummies are introduced.

2) Two impulse dummies for 1971 and 1982, respectively, control for the abnormal high return during the years 1972 and 1983.

3) *** = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

4) Lags 2 and 3 are insignificant.

Notes: 1) OLS and Newey-West estimation of coefficient standard errors (number of lags used in Newey-West shown in "lag"-column).

2) *** = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

Table 3: The 10-Year Equity Premium PR_{10} , 1927-87^{1,2}

	CONS	$D\tilde{I}P_t$	$\tilde{Y}d_t$	$\tilde{B}_{10,t}$	TE10-1 _t	TE10-5 _t	TE5-1 _t	PR1 _{t-1}	PR1 _{t-2}	PR1 _{t-3}	Lag	σ	R^2
1.	0.017*** (0.005)	1.405** (0.576)									5	0.025	0.23
2.	0.017*** (0.005)		1.377*** (0.449)								5	0.026	0.19
3.	0.014** (0.006)			0.376 (0.237)							8	0.028	0.08
4.	0.019*** (0.005)				-0.503 (0.534)						5	0.028	0.08
5.	0.016*** (0.005)					-0.606 (0.606)					5	0.028	0.08
6.	0.015*** (0.006)						-0.138 (0.282)				5	0.029	0.08
7.	0.018*** (0.005)							-0.057*** (0.019)	-0.060* (0.034)	-0.060*** (0.020)	5	0.026	0.23
8.	0.021*** (0.005)	-0.317 (1.398)	1.758 (1.278)	-0.307 (0.288)	-0.187 (0.367)	-0.037 (0.372)		-0.065 (0.044)	-0.014 (0.034)	-0.023 (0.018)	5	0.026	0.18
9.	0.018*** (0.005)		1.433*** (0.456)					-0.049*** (0.018)			5	0.025	0.23

Notes: 1) OLS and Newey-West estimation of coefficient standard errors (number of lags used in Newey-West shown in "lag"-column).
 2) *** = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

Table 4: Forecast of 5-Year Premium on Stocks PR_5 , 1998-2002

Forecast	Contributions			
	CONS	$\tilde{Y}d_t$	\tilde{B}_5	$PRI_{t-1 \text{ \& } t-3}$
-0.0021	0.0256	0.0044	0.0209	-0.0531

Appendix 1:

Statistical Properties of the Variables

Table A1.1: Univariate Summary Statistics

Variable	Sample	Sample Mean (pct.)	Sample std.dev. (pct.)	Min. (pct.)	Max. (pct.)	Skewness	Excess Kurtosis	Doornik-Hansen statistic	AR(1)-coefficient
PR1	1929-96	2.24	17.9	-37.4	61.0	0.70	1.65	8.01**	-0.22 (0.12)
PR5	1927-92	1.43	4.8	-9.6	15.3	0.27	0.20	1.46	0.46 (0.11)
PR10	1927-87	1.46	2.9	-7.9	7.1	-0.96	1.57	8.87**	0.63 (0.10)
\tilde{r}^P	1927-96	-0.19	0.9	-3.2	1.5	-0.64	0.26	5.49*	0.52 (0.10)
$\tilde{Y}ld$	1927-96	-0.19	0.9	-3.4	1.3	-0.83	1.49	7.98**	0.66 (0.10)
$\tilde{B}1$	1929-96	-0.02	1.8	-4.9	4.0	-0.39	-0.09	1.99	0.48 (0.11)
$\tilde{B}5$	1927-92	0.12	1.6	-5.4	4.5	-0.44	1.67	9.52***	0.62 (0.10)
$\tilde{B}10$	1927-87	0.22	1.6	-4.7	3.7	-0.99	1.46	9.53***	0.76 (0.09)
TE10-1	1927-96	0.77	1.4	-2.0	3.8	0.02	-0.42	0.09	0.33 (0.12)
TE5-1	1927-96	0.49	1.1	-2.9	3.5	-0.44	0.87	4.52	0.26 (0.12)
TE10-5	1927-96	0.28	1.1	-2.3	3.2	0.02	0.35	1.78	0.30 (0.12)

Note: Sample mean, standard deviation (based on T), skewness and excess kurtosis relate to the first four moments of a given distribution. For the standard normal distribution the numbers would be 0, 1, 0 and 0, respectively. A positive (negative) skewness indicates that the distribution is skewed to the right (left), i.e. has its weight to the left (right) and a long tail to the right (left). The skewness is zero for any symmetric distribution. A distribution has positive (negative) excess kurtosis if it is more peaked (more flat topped and fat tailed) than the normal distribution. The Doornik-Hansen χ^2 -test statistic indicates whether the four moments are from a normal distribution, see Doornik and Hansen (1994). A large value of the test statistic leads to rejection of the null of normality. *, ** and *** denote rejection of the null at 10%, 5% and 1% significance level, respectively. The Doornik-Hansen test has better size properties in small samples than the usual (asymptotic) Jarque-Bera test. The reported AR(1)-coefficient is based on a regression of each variable on itself lagged one-period. OLS coefficient standard errors in parentheses.

Table A1.2: Phillips-Perron Z_t -test for Unit Root

Variables	Lag length (l)						
	0	1	2	3	4	5	6
<u>1927-96 (70 obs)</u>							
Yld	-1.617	-1.678	-1.629	-1.672	-1.633	-1.598	-1.573
<i>Yld</i>	-3.745***	-3.949***	-3.959***	-3.977***	-3.856***	-3.682***	-3.536**
D/P	-2.078	-1.972	-1.906	-1.921	-1.904	-1.904	-1.860
<i>D/P</i>	-4.687***	-4.790***	-4.829***	-4.829***	-4.738***	-4.597***	-4.447***
TE10-1	-5.633***	-5.537***	-5.668***	-5.817***	-5.837***	-5.883***	-5.907***
TE10-5	-5.955***	-5.918***	-5.972***	-6.065***	-6.067***	-6.061***	-6.043***
TE5-1	-6.231***	-6.180***	-6.292***	-6.330***	-6.402***	-6.528***	-6.579***
<u>1929-96 (68 obs)</u>							
B1	-1.917	-1.468	-1.533	-1.599	-1.645	-1.709	-1.727
PR1	-9.817***	-9.844***	-9.940***	-10.07***	-10.30***	-10.64***	-11.00***
S1	-9.792***	-9.813***	-9.860***	-9.917***	-10.02***	-10.14***	-10.22***
<i>B1</i>	-4.624***	-4.376***	-4.611***	-4.771***	-4.879***	-4.922***	-4.922***
<u>1927-92 (66 obs)</u>							
B5	-1.621	-1.443	-1.440	-1.485	-1.507	-1.526	-1.555
PR5	-4.738***	-4.843***	-4.889***	-4.757***	-4.663***	-4.436***	-4.292***
S5	-3.714***	-3.710***	-3.839***	-3.820***	-3.858***	-3.701***	-3.640***
<i>B5</i>	-3.877***	-3.766***	-3.851***	-3.951***	-3.992***	-3.964***	-3.956***
<u>1927-87 (61 obs)</u>							
B10	-1.096	-1.077	-1.111	-1.141	-1.173	-1.194	-1.204
PR10	-3.626***	-3.641***	-3.680***	-3.664***	-3.653***	-3.647***	-3.625***
S10	-2.265	-2.265	-2.250	-2.244	-2.200	-2.186	-2.173
<i>B10</i>	-2.690*	-2.666*	-2.754*	-2.801*	-2.831*	-2.816*	-2.793*

Note: The Phillips-Perron unit root test is based on the first order autoregression $x_t = \alpha + \rho x_{t-1} + u_t$ (without trend) where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed and serially correlated, see Hamilton (1994, Table 17.2). The Z_t test statistic is a modified t-statistic for the null hypothesis of a unit root ($\rho=1$), correcting for the possible non-standard properties of u_t . The null of a unit root is rejected in favour of the stationary alternative ($\rho < 1$) if Z_t is negative and sufficiently large in numerical value. *, ** and *** denote rejection of a unit root at the 10%, 5% and 1% significance level, respectively. Critical values are from Hamilton (1994, Table B.6). All regressions include a constant term w . No deterministic trend is allowed for. Serial correlation is allowed for up to the selected lag length of l .

Table A1.3: Kwiatkowski *et al.* (1992) (KPSS) test for Unit Root

Variables	Lag length (l)						
	0	1	2	3	4	5	6
<u>1927-96 (70 obs)</u>							
Yld	2.553***	1.375***	0.9762***	0.776***	0.6595**	0.5829**	0.5296**
<i>Yld</i>	0.2589	0.1564	0.1252	0.1128	0.1103	0.1128	0.1175
D/P	2.865***	1.585***	1.138***	0.9093***	0.7741***	0.6849**	0.6225**
<i>D/P</i>	0.1845	0.1212	0.1010	0.0948	0.0969	0.1040	0.1138
TE10-1	0.1967	0.1500	0.1220	0.1061	0.0993	0.0945	0.09161
TE10-5	0.4895**	0.3798*	0.3243	0.2900	0.2783	0.2740	0.2738
TE5-1	0.5000**	0.3996*	0.3319	0.3001	0.2727	0.2474	0.2337
<u>1929-96 (68 obs)</u>							
B1	3.915***	2.085***	1.423***	1.089***	0.8890***	0.7561***	0.6621**
PR1	0.02835	0.03572	0.0405	0.0458	0.0530	0.0634	0.0723
S1	0.2289	0.2860	0.3106	0.3346	0.3679*	0.4047*	0.4241*
<i>B1</i>	0.5955**	0.4098*	0.3104	0.2613	0.2355	0.2222	0.2156
<u>1927-92 (66 obs)</u>							
B5	4.542***	2.366***	1.614***	1.235***	1.010***	0.8598***	0.7534***
PR5	0.1929	0.1343	0.1175	0.1180	0.1262	0.1453	0.1666
S5	1.201***	0.7473***	0.5776**	0.5031**	0.4640**	0.4489*	0.4370*
<i>B5</i>	0.3484*	0.2157	0.1636	0.1380	0.1250	0.1190	0.1167
<u>1927-87 (61 obs)</u>							
B10	4.518***	2.314***	1.570***	1.199***	0.9775***	0.8314***	0.7286**
PR10	0.9330***	0.5757**	0.4485*	0.3878*	0.3535*	0.3327	0.3207
S10	3.000***	1.657***	1.188***	0.9478***	0.8028***	0.7034**	0.6308**
<i>B10</i>	0.4069*	0.2355	0.1766	0.1493	0.1351	0.1281	0.1249

Note: The KPSS test for a unit root is a Lagrange Multiplier test of the null hypothesis that the variable in question can be described by a stationary process possibly around a deterministic trend, against the alternative that the process also includes a random walk component, that is, the null is one of stationarity, see Kwiatkowski *et al.* (1992). The null is rejected in favour of the unit root alternative if the test statistic is sufficiently large. *, ** and *** denote rejection of the null (i.e., a unit root is present) at the 10%, 5% and 1% significance level, respectively. Critical values are from Kwiatkowski *et al.* (1992). The lag length l determines how many lags are allowed for in the stationary component of the process. No trend is allowed for in the test, that is, the null is one of mean-stationarity.

Appendix 2: Diagnostic Graphics

Figure A2.1 Diagnostic Graphics For Parsimonious 1-Year Equity Premium Model

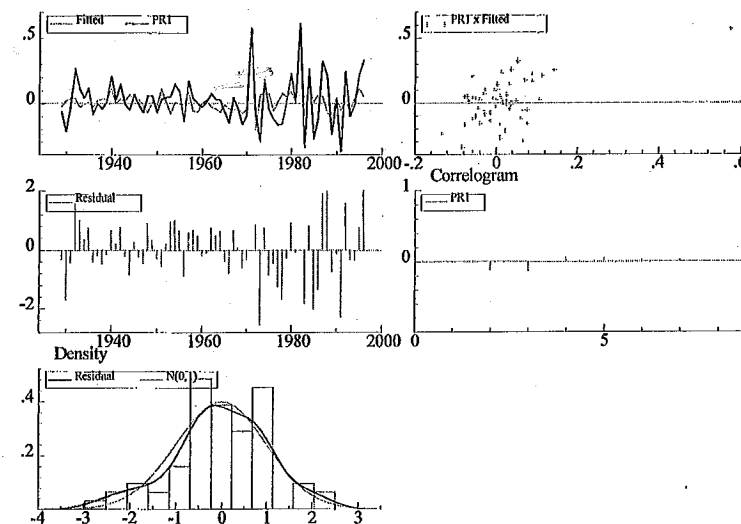


Figure A2.2 Diagnostic Graphics For Parsimonious 5-Year Equity Premium Model

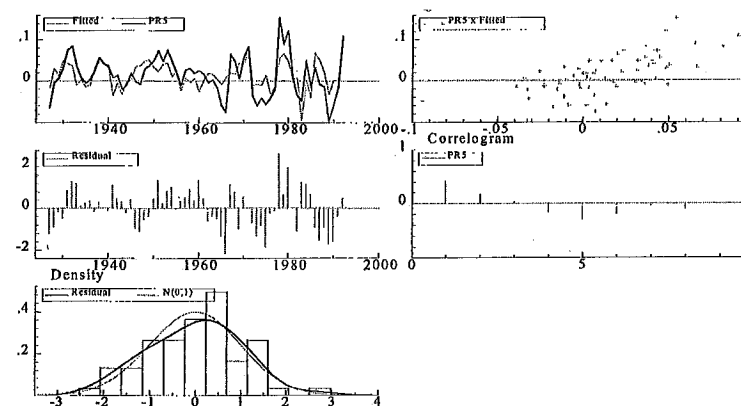
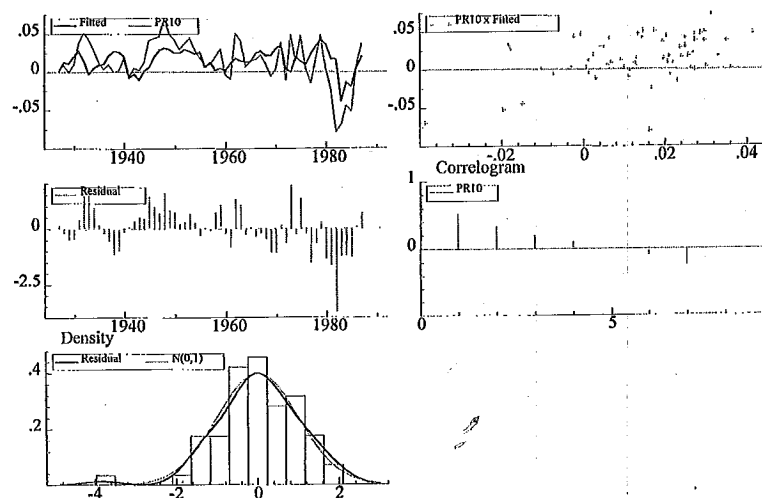


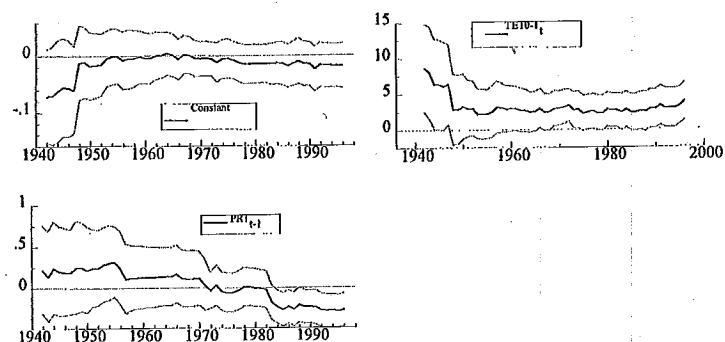
Figure A2.3 Diagnostic Graphics For Parsimonious 10-Year Equity Premium Model



Appendix 3:

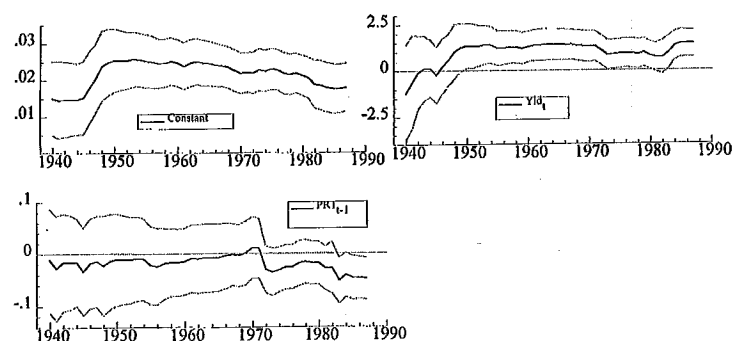
Recursive Parameter Estimates for the Parsimonious 1 and 10 Year Equity Premium Model

Figure A3.1 Recursive Parameter Estimates for 1-Year Equity Premium Model, 1940-1996 (starting year 1929)



Note: Coefficients to dummies not shown. Indicative OLS confidence bands.

Figure A3.2 Recursive Parameter Estimates for 10-Year Equity Premium Model, 1940-1987 (starting year 1927)



Note: Indicative OLS confidence bands.

Appendix 4:

Parameter Estimates Over Samples Beginning in 1952

Table A4.1: The 1-Year Equity Premium $PR1_t$, 1952-96^{1,2,3,4}

	CONS	$D\tilde{T}P_t$	$\tilde{Y}d_t$	$\tilde{B}1_t$	TE10-1 _t	TE10-5 _t	TE5-1 _t	PR1 _{t-1}	PR1 _{t-2}	PR1 _{t-3}	Lag	σ	R^2
1	0.012 (0.024)	6.775** (3.309)									-	0.16 8	0.12
2	0.005 (0.023)		3.978 (4.003)								-	0.17 5	0.05
3	-0.003 (0.027)			-0.636 (1.554)							-	0.17 9	0.01
4	-0.018 (0.024)				2.552 (1.936)						-	0.17 4	0.05
5	-0.007 (0.023)					0.914 (2.234)					-	0.17 9	0.00
6	-0.012 (0.027)						3.251 (2.313)				-	0.17 5	0.05
7	0.000 (0.025)							-0.225* ³⁾ (0.126)			-	0.17 2	0.07
8	0.022 (0.025)	1.841 (8.323)	9.967 (6.080)	-3.500** (1.713)	1.492 (1.980)	0.438 (3.016)	----	-0.384 (0.282)	0.180 (0.206)	0.067 (0.146)	-	0.16 2	0.33
9	0.029 (0.023)		9.205 *** (3.058)	-3.835 *** (1.324)				-0.444 *** (0.112)			-	0.15 5	0.29

Notes: 1) OLS and White's heteroskedasticity consistent estimator of coefficient standard errors. σ denotes the standard error of the residual term. σ and R^2 are calculated from the OLS formula, excluding the two years 1971 and 1982 for which dummies are introduced.

2) Two impulse dummies for 1971 and 1982, respectively, control for the abnormal high return during the years 1972 and 1983.

3) * = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

4) Lags 2 and 3 are insignificant.

Table A4.2: The 5-Year Equity Premium $PR5_t$, 1952-92^{1,2}

	CONS	$D\tilde{T}P_t$	$\tilde{Y}d_t$	$\tilde{B}5_t$	TE10-1 _t	TE10-5 _t	TE5-1 _t	PR1 _{t-1}	PR1 _{t-2}	PR1 _{t-3}	Lag	σ	R^2
1.	0.017** (0.008)	3.037*** (1.045)									5	0.049	0.25
2.	0.015* (0.008)		2.009* (1.119)								5	0.053	0.12
3.	0.009 (0.008)			0.285 (0.321)							5	0.056	0.01
4.	0.010 (0.009)				0.011 (0.481)						5	0.057	0.00
5.	0.012 (0.009)					-0.414 (0.738)					5	0.056	0.01
6.	0.009 (0.009)						0.477 (0.460)				5	0.056	0.01
7.	0.015* (0.009)							-0.134*** (0.023)	-0.132*** (0.038)	-0.110*** (0.031)	5	0.045	0.39
8.	0.022** (0.009)	-0.176 (1.83)	3.126*** (1.14)	-1.212** (0.493)	0.732* (0.381)	-0.597 (0.717)	----	-0.185*** (0.042)	-0.048 (0.053)	-0.068 (0.044)	5	0.044	0.50
9.	0.025*** (0.009)		3.783*** (0.932)	-1.427*** (0.291)				-0.175*** (0.033)		-0.052*** (0.018)	5	0.043	0.47

Notes: 1) OLS and Newey-West estimation of coefficient standard errors (number of lags used in Newey-West shown in "lag"-column).

2) *** = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

Table A4.3: The 10-Year Equity Premium PR_{10} , 1952-87^{1,2,3}

	CONS	$D\tilde{T}P_t$	$\tilde{Y}d_t$	$\tilde{B}_{10,t}$	$TE_{10-1,t}$	$TE_{10-5,t}$	$TE_{5-1,t}$	$PR_{1,t-1}$	$PR_{1,t-2}$	$PR_{1,t-3}$	Lag	σ	R^2
1.	0.012** (0.005)	1.896*** (0.561)									5	0.027	0.33
2.	0.011* (0.006)		1.364*** (0.451)								5	0.029	0.19
3.	0.007 (0.007)			0.473** (0.194)							8	0.031	0.08
4.	0.013*** (0.005)				-0.603 (0.709)						5	0.031	0.08
5.	0.011* (0.005)					-0.479 (0.665)					5	0.032	0.04
6.	0.010 (0.007)						-0.362 (0.455)				5	0.032	0.02
7.	0.010 (0.007)							-0.044*** (0.016)	3)	-0.040*** (0.010)	5	0.031	0.14
8.	0.015*** (0.006)	3.292** (1.373)	-0.994 (1.349)	-0.238 (0.248)	-0.335 (0.510)	0.480 (0.375)	----	0.047 (0.048)	-0.005 (0.035)	-0.050* (0.026)	5	0.028	0.40
9.	0.013** (0.006)	1.819*** (0.477)								-0.030*** (0.009)	5	0.026	0.36

Notes: 1) OLS and Newey-West estimation of coefficient standard errors (number of lags used in Newey-West shown in "lag"-column).

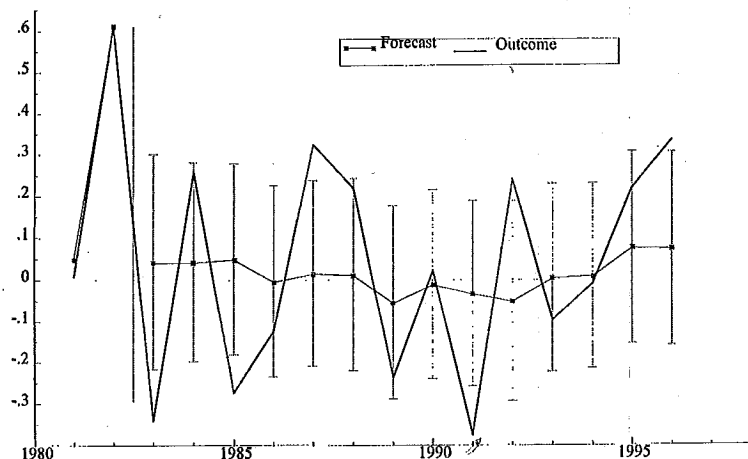
2) *** = significant at 1%; ** = significant at 5%; * = significant at 10%. Asymptotic normal distribution, two-sided test.

3) Lag 2 insignificant.

Appendix 5:

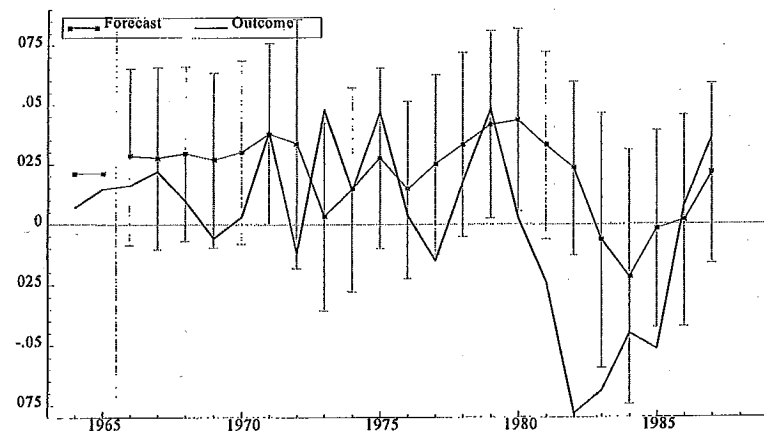
Forecasting 'Out-of-sample' for the Parsimonious 1 and 10 Year Equity Premium Model

Figure A5.1 Forecasting the 1-Year Equity Premium $PR1_t$, 1983-1996



Note: Forecasting exercise begins in 1983 which is the first year after the latest dummy, i.e. the dummy for 1982. OLS confidence bands only indicative.

Figure A5.2 Forecasting the 10-Year Equity Premium $PR10_t$, 1966-1987



Note: OLS confidence bands only indicative.

Chapter 3

On the Relationship between the Danish Stock and Bond Market in the Medium and Long Term

On the Relationship between the Danish Stock and Bond Market in the Medium and Long Term

by

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Abstract

This short paper studies the empirical relationship between realized stock returns and bond yields at the 5- and 10-year investment horizons, respectively. Using annual Danish data since 1927, we find that stock returns and bond yields are closely linked in the medium and long term, as we estimate strong cointegrating relations at both horizons. Hence, at the 5- and 10-year investment horizons a high bond yield tends to go hand in hand with a high stock return, and vice versa. Results show that stock returns tend to respond less than one-to-one to changes in the bond yield.

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1. Introduction

This short paper studies the relationship between Danish stock returns and government bond yields in the period 1927 to 1997. The paper is concerned with the relationship between the two markets in a medium term perspective, defined as an investment horizon of 5 years, and in a long term perspective, defined as a 10-year horizon. The specific question we want to address is whether the stock returns and bond yields at the 5- and 10-year horizon, respectively, form cointegrating relationships²?

If the stock and the bond market are interdependent, an expected abnormal high return in one of the markets is likely to attract funds from the other market, which in turn will result in an equilibrating price increase in the first market and in a declining price in the latter market. In that case, expected returns in the first market will decline whereas returns in the other market will go up. As a result of this arbitrage process, the return gap will decline³. The question we address is whether the returns in the two markets are closely linked in the long run? Because of the interdependence between the two markets, it is appropriate to apply a cointegration technique that allows for interdependence or simultaneity in the jargon of econometrics. We use the VAR method of Johansen (1996). Another advantage of the Johansen method is that it uses both the short and long-run information in the data to extract the long-run relationship between the two asset markets.

The paper is in 7 sections. The next section briefly describes the historical movements of the stock returns and bond yields. The third section outlines the simple theoretical arbitrage type framework that helps us organize our thoughts about the relationship between stock and bond returns. The fourth section sketches the cointegration method. In section 5 and 6, we present the empirical results. Section 7 concludes.

² For an introduction to the concept of cointegration, see Engle and Granger (1991, Chapter 1).

³ The gap between the expected returns on stocks and bonds is related but not exactly identical to the notion of a risk premium. It is not exactly a risk premium in the conventional sense (a premium relative to a riskfree asset) because bonds are also risky due to inflation and default risk and (because we use the yield-to-maturity on bonds) the reinvestment risk attached to the coupons paid before maturity.

2. A Look at the Data

Figure 1 shows the 5-year nominal stock return and the 5-year yield-to-maturity on government bonds in the period 1927-92, using overlapping annual observations⁴. All returns in this paper are annualized logarithmic returns, that is, they are defined as the logarithm to one plus the annualized return. Moreover, they are forward-looking and relate to investments by the end of the year, which means that a return recorded in say year t measures the realized return in the periods $t+1$ through $t+5$. Hence, the 5-year observation for 1992 covers the years 1993 to 1997. The data source is Nielsen, Olesen and Risager (1998).

< Insert Figure 1 around here >

The 5-year interest rate is relatively constant from the 1920s to the beginning of the 1960s, where it starts climbing up. It reaches a peak in 1982. In 1983 the interest rate displays the largest fall observed in the entire sample period. The considerable fall is usually attributed to the shift in economic policy regime that took place in late 1982, see e.g. Andersen and Risager (1987). Following the dramatic fall in 1983, the interest rate continues to decline until it stabilizes towards the end of the sample.

The 5-year stock return is obviously much more volatile than the interest rate, but follows also a pattern that resembles the interest rate. Thus, stock returns oscillate around a fairly constant mean until the beginning of the 1960s. Thereafter, stock returns tend to increase. There is a drastic fall in stock returns in the beginning of the 1980s. The decline sets in at a time with very high oil prices and large wage increases and where it is widely recognized that the overall macroeconomic policy stance is unsustainable. In the remainder of the 1980s, stock returns tend to decline. Moreover, both the 1970s and the 1980s are decades where bond yields often exceed stock returns.

⁴ The yield-to-maturity concept assumes essentially a flat yield curve or that coupons can be reinvested at the (constant) yield-to-maturity rate. This is, of course, a weakness of this measure and the alternative return on a zero-coupon bond would, in principle, have been a superior measure of the return on bond investments. However, data for zero-coupon rates are not available over the historical sample. Moreover, what we focus on in this study are the long-run, non-stationary movements in the level of bond rates, and for this purpose, the yield-to-maturity measure should suffice.

Figure 2 illustrates the stock return and yield-to-maturity at the 10-year investment horizon in the period 1927-1987 (the last observation covers the 10-year period from 1988 to 1997). The correlation between the two series is now much stronger. This shows that stock returns and bond yields are very closely connected in the long term. It is also interesting to observe that in this time perspective, stocks do not appear to be more volatile. In a 10-year perspective, the ups and downs in the stock market tend to cancel out and this explains why there is not much difference between the two series in terms of volatility, see Nielsen and Risager (1999).

< Figure 2 >

A formal test of the stationarity properties of the above series, see the Appendix, shows that they are integrated of order 1 ($I(1)$) within the sample period, i.e., the series are non-stationary in levels but stationary in first differences. We treat the series as such, notwithstanding the fact that the $I(1)$ assumption may appear counter-intuitive on more economic grounds. The point is that in our small (or rather, finite) sample the returns behave *as if* they are $I(1)$ series and treating them as such should improve the small sample properties of the statistical methods. Because of the non-stationarity of returns, we use cointegration methods.

3. Arbitrage between Stocks and Bonds

Economic theory suggests that there is a simple no-arbitrage relation between stocks and bonds,

$$(1) \quad E(S_t | I_t) = E(B_t | I_t) + PR_t$$

which says that the expected forward-looking return on stocks $E(S_t | I_t)$ equals the expected forward-looking bond return $E(B_t | I_t)$ plus an additive risk premium PR_t on stocks relative to bonds. The returns relate to the same (5- or 10-year) investment horizon and the expectations are conditioned on the available information set I_t . Over the business cycle, the risk premium is likely to vary, whereas it seems plausible that the premium is constant in the long term. Because this paper focuses entirely on the long run relationship between the two markets, we assume that the risk premium is constant⁵. For an empirical analysis of the variation in the

⁵ The risk premium could, in principle, be related to past or (for the predetermined yield-to-maturity on bonds) current levels of returns. One possibility is that the risk premium

risk premium in the short run, see Olesen and Risager (1999).

Equation (1) is an ex-ante equilibrium relationship that is consistent with the idea that the marginal investor will move money in or out of the markets until the expected stock return equals the expected bond yield plus a risk premium. For a rational marginal investor, the above equation should incorporate investor taxation insofar as income from the two assets is taxed at different rates⁶. In theory, the marginal investor is a well defined agent, whereas the marginal investor is harder to identify in practice and in particular over a long historical time period. Candidates to the title are numerous. In modern times, it can be large Danish institutional investors like pension funds. Over the recent decades, it may also be foreign investors, reflecting the capital account liberalizations concerning stock investments which took place in the 1970s. It could also be wealthy private citizens, foundations and so forth even though it seems plausible that these investors played a bigger role in the past. As Danish banks traditionally have held both stocks and bonds in large quantities, the banking sector may also at times have been the marginal investor.

declines if the stock market has done well in the past because investors' memory may be short. In this case, investors may become less risk averse because they tend to forget how risky the market can be. Similarly, the risk premium could also be negatively related to the level of the bond rate. Thus, in a situation where bonds may be perceived as being more risky, e.g. due to increased default or inflation risk, we should at the same time expect a higher bond rate and (for given risk attitude towards stocks) a lower risk premium on stocks relative to bonds. However, as high bond rates may be associated with a high rate of business failure due to firms facing a higher interest burden, stocks could also be expected to be more risky and the net effect on the relative risk premium on stocks is unclear. These concerns are, however, most likely to be of greatest relevance over the business cycle. Due to our focus on the long run, we take a simple approach to the risk premium.

⁶ With taxation of investment income, equation (1) is replaced by an after-tax no-arbitrage relation, saying that the expected after-tax return on stocks is equal to the expected after-tax return on bonds plus a risk premium. By rearranging this relation, we get the following equation for the before-tax expected stock return (ignoring the risk premium),

$$(1') \quad E(S_t | I_t) = \frac{1 - \tau_B}{1 - \tau_S} E(B_t | I_t)$$

where τ_S (τ_B) is the representative marginal investor's rate of tax on stock (bond) returns. If the tax rate on bond returns is higher than on stock returns, i.e., $\tau_B > \tau_S$, the coefficient to the expected bond return $E(B_t | I_t)$ in (1') is less than one, and *vice versa*.

The bottom line is that the marginal investor is not well defined in practice. Due to these complexities, we have not attempted to include taxation in the analysis. However, for now, it is important to be aware that the general picture is that stock returns often have been taxed at a lower rate than bond yields. Thus, pension funds were from 1984 to the end of the sample not taxed on stocks but taxed on their interest income at a varying rate, related to the rate of inflation in the economy. Private households have in the same period been taxed lighter on their income from stocks. To the extent that banks acquire stocks and bonds for their own deposits, they are taxed uniformly at the rate applicable to taxable earnings in the banking sector. As regards the arbitrage process, one could at an informal level argue that it is the agent with the highest tax rate on bond investments, i.e., the agent who is taxed in the most asymmetric way, that will dominate the scene and hence become the marginal investor. This agent will have the highest reservation price for stocks. Assuming that short selling in bonds is allowed (or that there are no restrictions on borrowing in banks), this agent will tend to buy out the other agents in the stock market and bid up the market price to his reservation price⁷. In practice, there are limitations to the marginal investor's willingness to take extreme positions, including liquidity constraints, constraints stemming from risk aversion and possibly legal quantitative constraints on the allocation of portfolios. However, this very simple line of reasoning suggests that there is an inherent tendency for the agent suffering the highest tax rates on bond investments to become the marginal investor. As a result, we might expect that the coefficient to the bond yield is below one in the estimated relation between stock returns and bond yields.

Equation (1) assumes rational expectations. In general, the realized return on an asset equals the expected return plus a component reflecting forecast errors, and under rational expectations the mean of this component will be zero over a long time period. Hence, realized returns will also be related to each other in a linear fashion. The error term will reflect forecast errors in both markets but have a mean that is zero in the long run. However, because we use

⁷ If we allow for short selling in stocks, we could, actually, turn the argument around and conclude that it might be the agent with the lowest tax rate on bond investments that will become the marginal investor in the stock market, not as a holder of stocks but as a supplier of stocks. Thus, this agent will have a high reservation price for *bonds* and will have an incentive to buy bonds financed by a sale of stocks. If both short selling in stocks and bonds is allowed, no definite equilibrium exists. However, in practice, short selling in stocks has not been possible (or customary) in the Danish stock market over the historical period.

overlapping observations, the error term may be serially correlated. This is purely a statistical artifact.

Given this, we can now turn to the cointegration analysis with the purpose of detecting whether there exists a linear long run relationship between realized stock and bond returns.

4. Cointegration Analysis

As mentioned earlier, the VAR method of Johansen is a simultaneous equation method that allows for interdependence between the stock and the bond market. It is also a full information maximum likelihood (FIML) estimation method which uses both the short and long run information in the data. In error-correction form, the dynamic Vector Autoregressive (VAR) system to be estimated is

$$(2) \quad \Delta Z_t = \tau_1 \Delta Z_{t-1} + \dots + \tau_{k-1} \Delta Z_{t-k+1} + \pi Z_{t-1} + \mu + d83_t + \varepsilon_t$$

where $Z_t = (S_t, B_t)'$ is the (2×1) vector of the endogenous stock and bond returns, μ is a (2×1) vector of constants, $d83_t$ is a (2×1) vector of impulse dummies for the year 1983⁸, and ε_t is a (2×1) vector of white noise errors. The lag length k is chosen such that the residuals satisfy the white noise assumptions of being serially uncorrelated and homoskedastic. In the 5-year horizon model, we choose $k=5$ and in the 10-year model, we set $k=3$, cf. below. These lag lengths can, furthermore, be validated by formal testing of the significance of individual lag lengths in a general-to-specific-procedure. The impulse dummy for 1983 has, likewise, been included to ensure that the 5- and 10-year models are well specified. Thus, without this dummy the residuals show serial correlation in both models. Because of the shift in the economic policy regime and the resulting dramatic fall in interest rates in 1983, the dummy is also warranted on economic grounds. In the estimation, we restrict the constant terms in μ to lie within the cointegrating space. This precludes a deterministic time trend in the endogenous variables, cf. Johansen (1996), which appears to be consistent with the data, cf. Figures 1 and 2.

The parameter matrix that this paper is concerned with is π . This matrix can be decomposed

⁸ Each impulse dummy has a value of one for 1983 and is zero, otherwise.

in a γ and a β matrix according to

$$(3) \quad \pi = \gamma\beta'$$

where γ and β are $2 \times r$ matrices, where r is the rank of π . Notice that if π is of full rank ($r=2$), the long run solution for Z_t is unique and equal to a vector of constants. However, since Z_t is $I(1)$ (and not stationary), this is false, and π cannot be of full rank. As explained by e.g. Johansen (1996), the rank of π determines the number of cointegrating vectors. Below, we find that there for both investment horizons exists one cointegrating vector β ($r=1$). The elements of this β -vector are the long run coefficients which we focus on in this paper. The elements of the estimated γ vector measure the average speed of adjustment towards long run equilibrium. These parameters also have an interpretation related to the concept of weak exogeneity, which we return to.

5. Results for the 5-Year Horizon

We first estimate the dynamic system (2) on the basis of overlapping 5-year returns for Danish stocks and government bonds. As it is important for the inference that the error term in (2) fulfills the white noise assumptions, we perform a number of specification tests, see Table 1. Table 1 reports the outcome of single-equation specification tests for serial correlation, heteroskedasticity (ARCH) and normality. Both equations pass the tests at the conventional 5% significance level. We have also performed specification tests using a system approach (multivariate tests for serial correlation, normality and heteroskedasticity, not reported for expositional reasons); the outcome is, again, that there is no sign of misspecification at the 5% level. We therefore conclude that the dynamic model is well specified.

< Table 1 >

Tests for the rank of π can then be performed, see Table 2. We report both the standard asymptotic trace test, cf. Johansen (1996), and the small-sample-adjusted trace test, as suggested by Reimers (1992). The critical values for the trace tests are simulated using the simulation program DisCo, cf. Johansen and Nielsen (1993), to take account of the inclusion

of the impulse dummy for 1983⁹. The conclusion is very clear; there is one and only one cointegrating vector, i.e., $r=1$. Furthermore, the evidence of cointegration is strong; based on the simulated test values, the critical significance level for the null that there is not cointegration is virtually zero.

< Tables 2 and 3 >

Table 3 reports the estimates of the β and γ vector under the restriction that there is one cointegrating vector. The π matrix is also reported. The estimated β vector leads to the following long-run equilibrium relation between stock and bond returns at the 5-year horizon (indicative standard errors of coefficient estimates in parentheses):

$$(4) \quad S_t = 0.86 B_t + 0.026 \quad (5 \text{ year})$$

(0.09) (0.007)

The coefficient to the bond yield, estimated to be 0.86,¹⁰ is clearly significant. We furthermore note that this point estimate is below one. Equation (1) suggests a one-to-one relationship between stock and bond returns, and it is therefore of relevance to test the hypothesis that the ('true') coefficient is one¹⁰. The Likelihood Ratio (LR) test statistic for this hypothesis is 2.77; the asymptotic test distribution is $\chi^2(1)$ and the critical test value equals 3.84 at the 5% significance level. Hence, the hypothesis cannot be rejected at the conventional 5% level¹¹. However, there appears to be a tendency for the bond yield coefficient to be less than one. Based on the normal distribution and the indicative standard error of the coefficient estimate, the (indicative) 95% confidence interval can be shown to be (0.69, 1.03). The fact that it is likely that the ('true') bond yield coefficient is below one may reflect the tendency for stocks to be taxed at a lower rate than bonds, cf. section 3.

⁹ For comparison, the standard 95% critical test values are 20.0 (for $r=0$) and 9.1 ($r=1$), cf. Johansen (1996, Table 15.2).

¹⁰ Formally, we test the validity of the restricted cointegrating vector ($\beta_{11}=1, \beta_{21}=-1, \beta_{31}$), where we have normalized on stock returns (this augmented 3×1 vector includes the constant term (β_{31}), restricted to be part of the cointegrating relation, cf. section 4).

¹¹ The critical significance level of the test is 9.6%, so at a strict 10% significance level we would reject the null.

Over the sample 1927-1992, the average difference between the 5-year return on stocks and the 5-year return on bonds is 1.4% per year. This long run return difference between the two assets is reflected in the constant term of (4), estimated to be 2.6% per year (slightly higher because the estimated bond yield coefficient is below one). The premium on stocks may be considered fairly low, in particular, by international standards. In judging the difference between stock and bond yields, it should be noted, though, that we are dealing with 5-year horizons, and that the 5-year bond rate on average is higher than a short Treasury Bill rate, which is a common estimator of the riskfree rate in the equity premium literature, see Kocherlakota (1996) for a survey of this literature.

Having estimated the long-run equilibrium relation between the stock and bond markets, we can test whether deviations from this relationship trigger adjustments in the bond yield and stock returns, respectively. These tests are concerned with the estimated adjustment coefficients in the γ vector and amount to testing for weak exogeneity of bond and stock returns, cf. Johansen (1996). To begin with, we test the null hypothesis that the adjustment coefficient in the direction of bond yields (estimated to be -0.13, cf. Table 3) is zero¹². The LR test of this hypothesis has a critical significance level of 3.4%, so we reject the null at conventional significance levels. In other words, the bond yield is *not* exogenous as deviations from the long run stock and bond yield relationship trigger adjustments in the bond yield. Likewise, we can test whether stock returns are exogenous by testing the null that the adjustment coefficient in the direction of stock returns (estimated to be -1.45) is zero. This hypothesis is clearly rejected as the critical significance level of the corresponding LR test is effectively zero (the LR test statistic is 43 which should be compared to the $\chi^2(1)$ distribution). Thus, stock returns take on a significant burden in the adjustment to long-run equilibrium between the stock and bond markets¹³.

¹² Formally, we test the validity of the restricted adjustment vector ($\gamma_{11}, \gamma_{21}=0$).

¹³ In interpreting the results on exogeneity, it is important to recall that the yield-to-maturity on bonds, in contrast to stock returns, is a predetermined variable as it is determined and known at the beginning of any 5-year investment period. The apparent endogeneity of the bond yield, therefore, in principle, implies that future stock return *realizations* are significant in explaining the *current* bond yield. At the 10-year horizon we also find that the bond yield is endogenous. This result could tentatively be explained by expectational effects and a slow arbitrage process, cf. section 6. However, at the 5-year horizon, this interpretation is not so obvious. The estimated adjustment coefficients suggest that if stock returns are excessively

6. Results for the 10-Year Horizon

The results for the 10-year horizon are reported in Tables 4, 5 and 6. We have performed the same specification tests as before and find that there are no signs of serial correlation or heteroskedasticity (ARCH) in the residuals of the dynamic model, using the conventional 5% significance level. We reject the hypothesis of normally distributed residuals in the equation for stock returns, a standard assumption underlying the statistical results of the Johansen method. However, as shown in Johansen (1996, Part II), this assumption is not crucial as the asymptotic inference of the Johansen method is also valid in the less restrictive case where residuals are identically (and not necessarily normally) distributed over time. We therefore conclude that the dynamic model (2) is acceptable from a statistical point of view.

< Tables 4, 5 and 6 >

We find one and only one cointegrating relation in the data and, again, the evidence of cointegration is strong. The estimated long-run equilibrium relationship between stock returns and the bond yield at the 10-year horizon is:

$$(5) \quad S_t = 0.71 B_t + 0.035 \quad (10 \text{ year})$$

(0.05) (0.004)

The point estimate of the coefficient to the 10-year bond yield is highly significant and equals 0.71. Thus, the coefficient is also below one at the 10-year horizon. By testing whether the coefficient is one, we get a clear rejection at any significance level (the LR test statistic is 22 with a critical significance level that is virtually zero). The indicative 95% confidence interval for the coefficient is given as (0.61, 0.81). Hence, at this horizon there is a clear deviation from a unitary coefficient to the bond yield. The estimate of the constant equals 3.5% per year, slightly higher than at the 5-year horizon¹⁴.

high relative to bond yields, stock returns tend to decline *by more* than is necessary to restore equilibrium (as the adjustment coefficient in the direction of stock returns is negative and above one in absolute value) while at the same time the bond yield also tends to *decline* (as the adjustment coefficient is negative). This suggests a rather complex (and puzzling) dynamic adjustment to long-run equilibrium.

¹⁴ For comparison, the average difference between the 10-year stock return and the 10-year bond yield is 1.5% over the sample 1927-1987.

Our results on the exogeneity status of the variables parallel the findings at the 5-year horizon, as both the stock return and the bond yield are endogenous¹⁵. Hence, deviations from the long run cointegrating relation trigger adjustments in both the stock return and the bond yield. The estimated adjustment coefficients show that if stock returns exceed the long run level implied by the cointegrating relation, stock returns tend to fall (negative adjustment coefficient) whereas the bond yield tends to increase (positive adjustment coefficient). Thus, the returns in both markets contribute to restore the long-run equilibrium relation.

Tentatively, the endogeneity of the predetermined bond yield may reflect expectational effects and a slow arbitrage process between the two markets. Thus, consider a situation where the representative investor expects that the stock return over the next 10-year investment period will be higher than the equilibrium level implied by the cointegrating relation. In an effective market, this expected abnormal return will lead to an immediate transfer of funds from the bond to the stock market which eliminates the arbitrage opportunity. However, if for some reason the arbitrage process is slow¹⁶, the expected stock return will remain high. Moreover, we will see a gradual adjustment in the expected stock return (which will decline) and the bond yield (which will increase) as funds are transferred across markets over time. During this adjustment phase the expected excess return on stocks leads the adjustment in the bond yield. Because the realized stock return signals the expected stock return under rational expectations, we may as the outcome observe that an abnormally high *realized* stock return over the next 10-year period leads changes in the *current* bond yield. That is, the bond yield may appear to be endogenous even though it is predetermined.

7. Concluding Remarks

This short paper has shown that the realized stock returns and bond yields at the 5-year horizon and at the 10-year horizon form strong cointegrating relationships. Thus, a high bond yield tends to go hand in hand with a high stock return in the medium and long term, and *vice versa*. The stable relationships between the two markets are likely to reflect an arbitrage

¹⁵ In both cases, the critical significance level of the relevant LR test is, for all practical purposes, zero.

¹⁶ Possible reasons for a slow arbitrage process could be the existence of transaction costs, a slow transmission of information in the market or legal constraints on the representative investor's portfolio allocation.

process that works in the medium and long term.

Our results also show that stock returns tend to respond less than one-to-one to changes in the interest rate. This result is strongest at the 10-year horizon where the deviation is statistically significant. In a world where investors are taxed symmetrically, arbitrage considerations suggest that there should be a one-to-one relationship between stock returns and bond yields. Hence, we think that the result reflects asymmetric taxation because bond yields often have been taxed at a higher rate than stock returns; that is the picture that applies to the majority of the investors.

To get a deeper understanding of the effects of taxation is, however, a large and complex project in itself. It requires first of all a careful study of the tax laws (that differ across investors) in the entire sample period. It also requires that one is able to identify the marginal investor at different points in time. This is an interesting project with many potential externalities. We therefore consider this issue to be an obvious topic for future research.

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Appendix: Unit Root Tests

We have examined the stationarity properties of each of the return series, using the unit root tests of Phillips and Perron (1988) (PP) (see Table A.1) and Kwiatkowski *et al.* (1992) (KPSS) (see Table A.2). The tests are introduced in the notes to the tables.

< Tables A.1 and A.2 >

The outcome for the 10-year stock return and the 5- and 10-year bond yields is clear; using conventional significance levels, all three series are concluded to be integrated of order 1 ($I(1)$), i.e., to be non-stationary in levels, but stationary in first differences. The outcome for the 5-year stock return is mixed, as the PP test indicates $I(0)$ (i.e., that the series is stationary in levels) while the KPSS test suggests $I(1)$. To provide further evidence, we have performed an augmented Dickey-Fuller test of the null that the process for the 5-year stock return contains a unit root, cf. Dickey and Fuller (1979). Based on a well-specified augmented Dickey-Fuller regression where we include the fifth lag of the first differences of the stock return (this lag structure is chosen by a general-to-specific procedure, eliminating the insignificant lags in a general regression), we get a test statistic of -2.5. Hence, at conventional significance levels, we can not reject the null of a unit root, i.e., the level of returns is non-stationary. Overall, we therefore conclude that also the 5-year stock return is $I(1)$.

Finally, it should be noted that in order to use the (standard) Johansen method, all we need is that each return series is at most $I(1)$. This is clearly accepted by the unit root tests.

Figure 1. 5-Year Stock and Bond Return, 1927-1992.

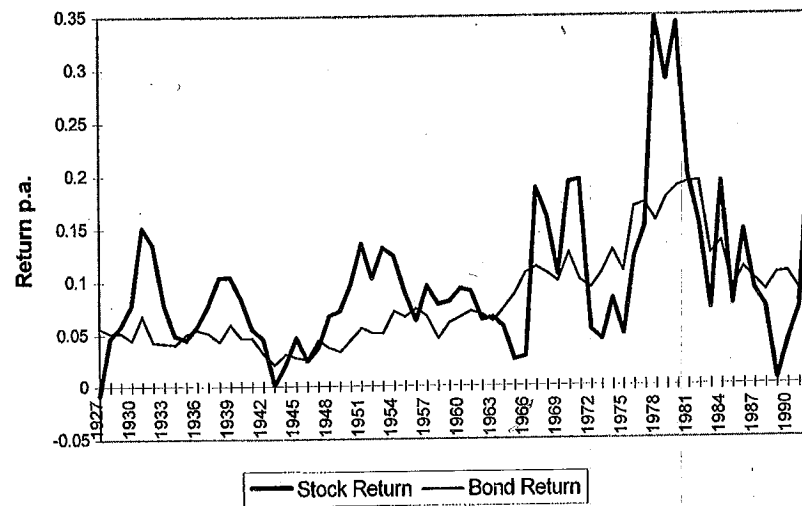


Figure 2. 10-Year Stock and Bond Return, 1927-1987.

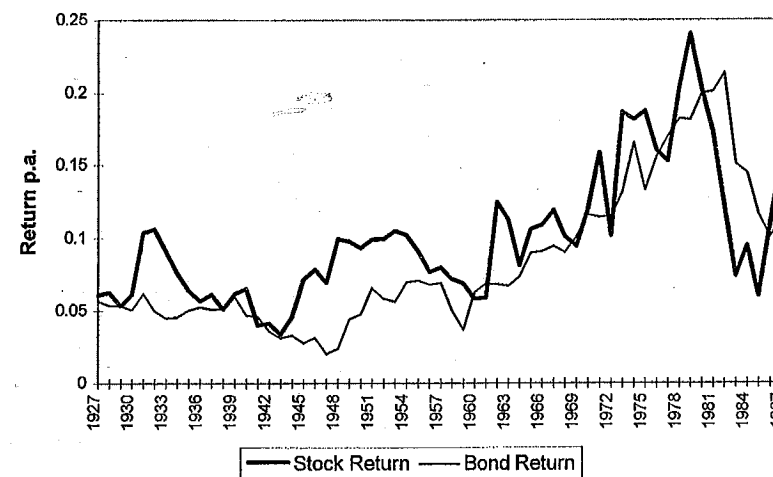


Table 1. Specification Tests of the VAR Model ¹⁾. 5-Year Horizon. 1927-92.

	Correl. Actual & Fitted ²⁾	S.E. ³⁾	Normality $\chi^2(2)$	ARCH, F-test				Autocorrelation, F-test			
				Lags				Lags			
				1 to 1	1 to 2	1 to 4	1 to 6	1 to 1	1 to 2	1 to 4	1 to 6
Eq. ΔS_t	0.845	0.037	0.52 [0.77]	0.40 [0.53]	2.50 [0.09]	1.30 [0.29]	1.12 [0.37]	2.90 [0.09]	1.67 [0.20]	1.01 [0.41]	0.71 [0.64]
Eq. ΔB_t	0.963	0.012	1.37 [0.50]	0.69 [0.41]	2.09 [0.13]	1.15 [0.35]	1.06 [0.40]	0.01 [0.93]	0.70 [0.50]	0.80 [0.53]	0.57 [0.75]

Note: The VAR model has a lag length of 5 and includes a constant term (restricted to the cointegrating space) and an impulse dummy for 1983 (enters unrestricted). F-tests are small sample approximations to Lagrange Multiplier tests (F-form). Normality test of Doornik and Hansen (1994), cf. also Doornik and Hendry (1997). Critical significance levels in brackets.

1) Single-equation specification tests for Normality, ARCH and serial correlation in residuals. We have also undertaken vector specification tests (based on the whole system) for Normality, heteroskedasticity (squares) and serial correlation (not reported). None of these signal misspecification problems at the 5% significance level when we apply the small sample F-test.

2) Coefficient of correlation between actual and fitted value for variable in levels.

3) Standard error of residuals.

Table 2. Tests for Cointegrating Rank. 5-Year Horizon. 1927-92.

Rank	Eigenvalue	Trace Statistic ¹⁾	Trace Statistic (small sample) ²⁾	Trace 95% Quantil ³⁾
r=0	0.49	45.08	38.25	16.5
r=1	0.01	0.93	0.79	5.9

Note: Maximum Likelihood estimation by method of Johansen (1996). 1) Asymptotic trace test statistic, cf. Johansen (1996).

2) Trace test statistic adjusted for small sample, cf. Reimers (1992) and Doornik and Hendry (1997). 3) 95% critical test value for trace test, simulated by the simulation program DisCo, cf. Johansen and Nielsen (1993). Allows for the impulse dummy for 1983.

Table 3. Restricted II-Matrix, γ - and β -Vector (r=1). 5-Year Horizon. 1927-92.

γ	β'	II			
-1.445 (0.20)	1	-0.864 (0.087)	-0.026 (0.007)	-1.445 (0.20)	1.249 (0.17)
-0.130 (0.07)				-0.130 (0.07)	0.113 (0.06)
					0.037 (0.005)
					0.003 (0.002)

Note: Maximum Likelihood estimation by method of Johansen (1996). Rank r restricted to 1. The β -vector is normalized on stock returns and includes the constant term (restricted to the cointegrating space). Indicative standard errors of parameter estimates in parentheses.

Table 4. Specification Tests of the VAR Model ¹⁾. 10-Year Horizon. 1927-87.

	Correl. Actual & Fitted ²⁾	S.E. ³⁾	Normality $\chi^2(2)$	ARCH, F-test				Autocorrelation, F-test			
				Lags				Lags			
				1 to 1	1 to 3	1 to 5	1 to 7	1 to 1	1 to 3	1 to 5	1 to 7
Eq. ΔS_t	0.866	0.022	7.80 [0.02]	1.84 [0.18]	0.72 [0.54]	0.76 [0.59]	0.51 [0.82]	0.38 [0.54]	1.21 [0.32]	0.73 [0.61]	1.40 [0.23]
Eq. ΔB_t	0.981	0.009	0.06 [0.97]	2.91 [0.09]	1.72 [0.18]	1.36 [0.26]	1.20 [0.32]	0.96 [0.33]	1.64 [0.19]	0.99 [0.43]	0.69 [0.68]

Note: The VAR model has a lag length of 3 and includes a constant term (restricted to the cointegrating space) and an impulse dummy for 1983 (enters unrestricted). F-tests are small sample approximations to Lagrange Multiplier tests (F-form). Normality test of Doornik and Hansen (1994), cf. also Doornik and Hendry (1997). Critical significance levels in brackets.

1) Single-equation specification tests for Normality, ARCH and serial correlation in residuals. We have also undertaken vector specification tests (based on the whole system) for Normality, heteroskedasticity (squares) and serial correlation (not reported). None of these signal misspecification problems at the 5% significance level when we apply the small sample F-test.

2) Coefficient of correlation between actual and fitted value for variable in levels.

3) Standard error of residuals.

Table 5. Tests for Cointegrating Rank. 10-Year Horizon. 1927-87.

Rank	Eigenvalue	Trace Statistic ¹⁾	Trace Statistic (small sample) ²⁾	Trace 95% Quantil ³⁾
r=0	0.53	47.22	42.57	16.1
r=1	0.03	1.62	1.46	6.0

Note: Maximum Likelihood estimation by method of Johansen (1996). 1) Asymptotic trace test statistic, cf. Johansen (1996).

2) Trace test statistic adjusted for small sample, cf. Reimers (1992) and Doornik and Hendry (1997). 3) 95% critical test value for trace test, simulated by the simulation program DisCo, cf. Johansen and Nielsen (1993). Allows for the impulse dummy for 1983.

Table 6. Restricted II-Matrix, γ - and β -Vector (r=1). 10-Year Horizon. 1927-87.

γ	β'	II			
-0.571 (0.17)	1	-0.709 (0.050)	-0.035 (0.004)	-0.571 (0.17)	0.405 (0.12)
0.384 (0.07)				0.384 (0.07)	-0.272 (0.05)
					0.020 (0.006)
					-0.014 (0.003)

Note: Maximum Likelihood estimation by method of Johansen (1996). Rank r restricted to 1. The β -vector is normalized on stock returns and includes the constant term (restricted to the cointegrating space). Indicative standard errors of parameter estimates in parentheses.

Table A.1. Phillips and Perron (1988) Z_t -Test for Unit Root

Series:	Lag length (<i>l</i>)						
	0	1	2	3	4	5	6
<i>5 Year Horizon (1927-1992):</i>							
S_t	<u>-3.71</u>	<u>-3.71</u>	<u>-3.84</u>	<u>-3.82</u>	<u>-3.86</u>	<u>-3.70</u>	<u>-3.64</u>
B_t	-1.62	-1.44	-1.44	-1.49	-1.51	-1.53	-1.56
ΔS_t	<u>-9.14</u>	<u>-9.13</u>	<u>-9.12</u>	<u>-9.15</u>	<u>-9.17</u>	<u>-9.37</u>	<u>-9.61</u>
ΔB_t	<u>-10.51</u>	<u>-10.51</u>	<u>-10.48</u>	<u>-10.41</u>	<u>-10.38</u>	<u>-10.36</u>	<u>-10.36</u>
<i>10 Year Horizon (1927-1987):</i>							
S_t	-2.27	-2.27	-2.25	-2.24	-2.20	-2.19	-2.17
B_t	-1.10	-1.08	-1.11	-1.14	-1.17	-1.19	-1.20
ΔS_t	<u>-8.11</u>	<u>-8.11</u>	<u>-8.12</u>	<u>-8.14</u>	<u>-8.21</u>	<u>-8.28</u>	<u>-8.38</u>
ΔB_t	<u>-8.00</u>	<u>-8.00</u>	<u>-8.00</u>	<u>-8.00</u>	<u>-8.01</u>	<u>-8.01</u>	<u>-8.01</u>
<i>Critical test values:</i>							
			10%	5%	1%		
Without trend			-2.59	-2.91	-3.54		

Note: The Phillips and Perron (1988) unit root test is based on the first order autoregression $x_t = \alpha + \rho x_{t-1} + u_t$ (without trend) where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed and serially correlated up to lag l , see also Hamilton (1994). The Z_t test statistic is a modified t -statistic for the null hypothesis of a unit root ($\rho=1$), correcting for the possible non-standard properties of u_t . The null is rejected in favor of the stationary alternative ($\rho<1$) if Z_t is negative and sufficiently large in absolute value. Critical test values are small-sample values calculated from MacKinnon (1991). Underlining indicates rejection of a unit root at the 5% significance level.

Table A.2. Kwiatkowski *et al.* (1992) Test for Unit Root

Series:	Lag length (<i>l</i>)						
	0	1	2	3	4	5	6
<i>5 Year Horizon (1927-1992):</i>							
S_t	<u>1.20</u>	<u>0.75</u>	<u>0.58</u>	<u>0.50</u>	<u>0.46</u>	0.45	0.44
B_t	<u>4.54</u>	<u>2.37</u>	<u>1.61</u>	<u>1.24</u>	<u>1.01</u>	<u>0.86</u>	<u>0.75</u>
ΔS_t	0.02	0.03	0.03	0.03	0.03	0.05	0.05
ΔB_t	0.07	0.09	0.10	0.10	0.10	0.10	0.10
<i>10 Year Horizon (1927-1987):</i>							
S_t	<u>3.00</u>	<u>1.66</u>	<u>1.19</u>	<u>0.95</u>	<u>0.80</u>	<u>0.70</u>	<u>0.63</u>
B_t	<u>4.52</u>	<u>2.31</u>	<u>1.57</u>	<u>1.20</u>	<u>0.98</u>	<u>0.83</u>	<u>0.73</u>
ΔS_t	0.04	0.04	0.04	0.05	0.05	0.06	0.07
ΔB_t	0.12	0.12	0.12	0.11	0.11	0.11	0.11
<i>Critical test values:</i>							
			10%	5%	1%		
Mean-stationarity			0.35	0.46	0.74		

Note: The Kwiatkowski *et al.* (1992) test for a unit root is a Lagrange Multiplier test of the null hypothesis that the series can be described by a stationary process around a constant mean, against the alternative that the process also includes a non-stationary random walk component. The null of stationarity is rejected in favor of the unit root alternative if the test statistic is sufficiently large. Critical values are from Kwiatkowski *et al.* (1992). Underlining indicates rejection of the null (i.e., a unit root is present) at the 5% significance level. The lag length l is the number of lags allowed for in the stationary component of the process.

Chapter 4

Regime-Switching Stock Returns and Mean Reversion

Regime-Switching Stock Returns and Mean Reversion *

by

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Abstract

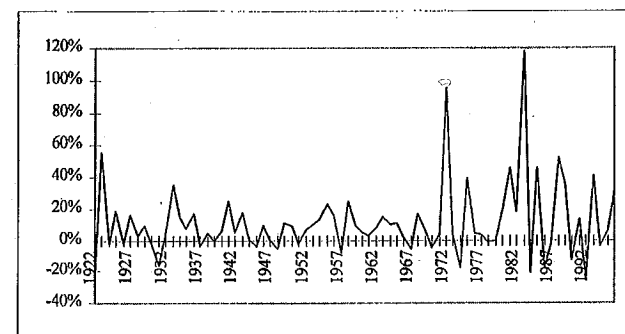
We estimate a well-specified two-state regime-switching model for Danish stock returns. The model identifies two regimes which have low return-low volatility and high return-high volatility, respectively. The low return-low volatility regime dominated, except in a few, short episodes, until the beginning of the 70s whereas the 80s and 90s have been characterized by high return and high volatility. We propose an alternative test of mean reversion which allows for multiple regimes with potentially different constant and autoregressive terms and different volatility. Using this test procedure we find mean reversion at 10% but not at 5% significance level which is weaker evidence than produced by estimating a standard autoregressive model for returns. Furthermore, when analyzing contributions of the two regimes we find that the indication of mean reversion is due to the recent high return-high volatility regime only.

* We thank participants in the workshop "Stock Market Economics" at Copenhagen Business School, May 1999, for useful comments. The activities of the Economic Policy Research Unit (EPRU) are financed by a grant from the Danish National Research Foundation.

1. Introduction

A plot of Danish stock returns over time suggests that returns were low and relatively stable from the 1920s until the beginning of the 1970s whereas the period since then has been characterized by higher average return and more volatility:

Figure 1. Annual Nominal Stock Returns in Denmark 1922-96



Note: Market portfolio of stocks listed at the Copenhagen Stock Exchange. Data are from the Nielsen, Olesen and Risager (1999) database.

This observation was also made on an informal basis by Nielsen and Risager (1999)¹. In this paper, we fit a time series model to the nominal return data which allows for the presence of more than one regime. This provides for a formal analysis of whether there have been several regimes and when changes of regime occurred. Furthermore, this approach enables us to test the hypotheses that mean return and volatility are higher in one regime than in the other. Identification of multiple regimes is important for understanding the time series properties of stock returns and may, in particular, be valuable for forecasting purposes.

The plot also indicates that annual stock returns display negative serial correlation (most obviously in the latter part of the sample), i.e., that stock prices mean-revert. This question was first raised by Fama and French (1988) and Poterba and Summers (1988) and has been

¹ However, they view the return in 1972 as an outlier and conclude that the change of regime takes place in 1983.

examined for Denmark by Risager (1998). These papers all report weak evidence of mean reversion². The present paper provides an alternative test of this issue within the framework of the regime-switching model. Thus, our approach leads to a mean reversion test which allows for multiple regimes in the return process.

Our procedure takes into account the specific pattern of heteroskedasticity, i.e., regime shifts in volatility level, identified by the regime-switching model. There are two related papers by Kim and Nelson (1998) and Kim, Nelson and Startz (1998) in which a similar model for returns is estimated. They standardize returns by estimated volatility and calculate variance ratio and autoregression tests for standardized returns. Our approach, on the other hand, is a parametric test of negative serial correlation which directly utilizes estimates obtained for the regime-switching model.

Furthermore, the paper provides new evidence about the extent to which serial correlation differs across regimes, i.e., whether the visual impression, that negative serial correlation is stronger in the latter part of the sample, is correct. In order to apply the tests we calculate analytical expressions for unconditional and state-specific means, variances and serial correlations for the regime-switching model with an autoregressive term.

The following section fits a regime-switching model to our return data. Section 3 derives analytical means and variances of the model and tests hypotheses. Similarly, serial correlation and implications for mean reversion is considered in section 4. Finally, section 5 concludes.

2. Estimating a Regime-Switching Model for Returns

Given the apparent change in behavior of Danish stock returns we are led to estimate a model which accounts for stochastic changes in regime. We employ a two-state version of the model developed by Hamilton (1990). According to this model there is an unobserved state variable, s_t , which takes on the values 0 or 1. The state variable is assumed to follow a Markov chain, i.e., the transition probabilities satisfy $p_{00} = P(s_t=0 | s_{t-1}=0) = P(s_t=0 | s_0=i_0, \dots, s_{t-2}=i_{t-2}, s_{t-1}=0)$ and

² The former paper analyzes real return, the second real and excess return, and the latter real and nominal return. In the present paper, we examine nominal returns.

$p_{11} = P(s_t=1 | s_{t-1}=1) = P(s_t=1 | s_0=i_0, \dots, s_{t-2}=i_{t-2}, s_{t-1}=1)$ for any sequence i_0, \dots, i_{t-2} and any t . The observed stock return depends on the state variable:

$$(1) \quad R_t = \mu_0 + (\mu_1 - \mu_0)s_t + \phi_0 R_{t-1} + (\phi_1 - \phi_0)s_t R_{t-1} + \sigma_0 \epsilon_t + (\sigma_1 - \sigma_0)s_t \epsilon_t,$$

where $\epsilon_t \sim \text{i.i.d. } (0,1)$.

Thus,

$$(2) \quad R_t | s_t=0 = \mu_0 + \phi_0 R_{t-1} + \sigma_0 \epsilon_t$$

and

$$(3) \quad R_t | s_t=1 = \mu_1 + \phi_1 R_{t-1} + \sigma_1 \epsilon_t.$$

Note, that this version of the model allows for distinct μ 's and σ 's, and that an autoregressive term is included in each state.

The parameter vector is estimated by numerically maximizing the log likelihood function. The likelihood function and the maximizing procedure are standard for regime-switching models and described in Hamilton (1994), section 22.4. The algorithm used to evaluate the log likelihood has two other interesting byproducts. First, it is possible to evaluate the probability that a given observation was generated by, say, state 0 conditional on information available at that time (filtered probabilities), i.e., current and past stock returns. This provides insight about timing of regime changes. Second, the algorithm generates one-period-ahead probabilities which can be used to construct return forecasts.

Estimating the model described above does not immediately give satisfactory results. The main problem is that the estimate of one of the transition probabilities is at a corner, $\hat{p}_{00}=0$, and that the estimate of the autoregressive term in state 0 is above 1, $\hat{\phi}_0=1.59$. Both of these estimates thus violate the assumptions under which specification tests proposed in Hamilton

(1996) are derived. Hence, the distribution of test statistics is unknown. However, informal diagnostic tests of standardized residuals of the three-state model suggests that the three-state model suffers from autocorrelation in the error term, cf. Appendix A. In this formulation, the filtered probabilities conditional on information available at time t only assign three observations to state 0, namely 1923, 1972 and 1983 which all represent years with extraordinary returns (cf. figure 1). Thus, state 0 may be viewed as a state which picks up outliers whereas state 1 is the ordinary state.

To pursue the question of whether there exist two states in addition to the outlier state we estimate a three-state version of the model. This results in an outlier state for 1972 and 1983 and two ordinary states for the remaining observations. The ordinary regimes have low return-low volatility and high return-high volatility, respectively, and the timing of regimes is in line with what we anticipated from looking at data. However, transition probabilities and the autoregressive term of the outlier state cause the same problem as above.

To be able to perform the Hamilton (1996) specification tests of the model and given the indication of misspecification revealed by residual-based tests we therefore choose to introduce dummies for 1972 and 1983 in the two-state model. The two dummy variables have zeroes every year except in 1972 and 1983, respectively, where the value is 1. They are added to equation (1) as two additional variables with potentially distinct coefficients in the two states to allow maximum flexibility. Thus, the resulting model is:

$$(1') \quad R_t = \mu_0 + (\mu_1 - \mu_0)s_t + \mu_0^{72}d72_t + (\mu_1^{72} - \mu_0^{72})s_t d72_t + \mu_0^{83}d83_t + (\mu_1^{83} - \mu_0^{83})s_t d83_t \\ + \phi_0 R_{t-1} + (\phi_1 - \phi_0)s_t R_{t-1} + \sigma_0 \epsilon_t + (\sigma_1 - \sigma_0)s_t \epsilon_t,$$

where $s_t \in \{0,1\}$ and $\epsilon_t \sim \text{n.i.d. } (0,1)$. μ_0^{72} and μ_0^{83} are the coefficients to the dummy variables in

state 0, and likewise for state 1.³

The fundamental difference between the three-state and the dummy model is the assumption of the latter that 1972 and 1983 are abnormal and non-recurring events which can be ignored while fitting a model for the remaining observations. On the other hand, the three-state model views 1972 and 1983 as belonging to a separate, extreme state which there is a (small) positive probability of returning to.

Our choice of the two-state dummy model is motivated by the fact that there are solid economic reasons for treating these years as special. In 1972 Denmark decided to join the EEC and agreed to allow foreign ownership of Danish stocks. In 1983 nominal interest rates were dramatically reduced as a result of the adoption of a fixed exchange rate policy and further capital market liberalizations, and a new pension fund tax was introduced on bond yields only. These events are potential explanations of the outstanding stock returns of these particular years.

The following estimates are obtained for the two-state model with dummies⁴:

³ The likelihood function is identical to the one presented in Hamilton (1994), p. 692, where the elements in η_t are (using the notation of this paper)

$$\frac{1}{\sqrt{2\pi}\sigma_t} \exp\left\{-\frac{(R_t - \mu_t - \mu_t^{72}d72_t - \mu_t^{83}d83_t - \phi_t R_{t-1})^2}{2\sigma_t^2}\right\}, \quad t=0,1$$

⁴ Parameter estimates of the two-state dummy model are similar to estimates of the three-state model, cf. Appendix A.

Table 1. Two-state model with dummies for 1972 and 1983, sample 1923-96

μ_0	0.0601 (0.0244)	P_{00}	0.8497 (0.1430)
μ_1	0.1802 (0.0461)	P_{11}	0.8304 (0.1400)
ϕ_0	-0.0446 (0.0955)	μ_0^{72}	0.8925 (0.0825)
ϕ_1	-0.3297 (0.1256)	μ_1^{72}	0.7819 (0.5881)
σ_0^2	0.0056 (0.0030)	μ_0^{83}	1.1265 (0.2028)
σ_1^2	0.0385 (0.0126)	μ_1^{83}	1.0582 (0.2150)

Note: Standard errors in parentheses estimated by second derivatives of log likelihood.

Point estimates of μ and σ are smaller in state 0 than in state 1, and as we are going to see in section 3 a non-trivial implication of table 1 is that state 0 is the low return-low volatility state whereas state 1 is characterized by high return and high volatility. ϕ_0 is insignificant but we choose to keep it for use in the next section. Finally, to determine whether the regimes are statistically different we may for example test a hypothesis that the μ 's are equal across states. A Wald test rejects this hypothesis (the critical significance level is 0.0244) which confirms that there are 2 distinct regimes.

Note also, that the problem of corner solutions is avoided and that both AR-terms are numerically less than 1. Hence, specification tests suggested by Hamilton (1996) may be applied. These are reported in table 2.

Table 2. Specification tests.

White tests, $\chi^2(4)$		
Autocorrelation	0.7832	(0.9403)
ARCH	6.4781	(0.1677)
Markov property	0.3786	(0.9835)
Lagrange multiplier tests, $\chi^2(1)$		
Autocorrelation in regime 0	0.2250	(0.6394)
Autocorrelation in regime 1	0.0116	(0.9151)
Autocorrelation across regimes	0.5266	(0.4717)
ARCH in regime 0	1.3324	(0.2547)
ARCH in regime 1	0.1414	(0.7079)
ARCH across regimes	0.9079	(0.3454)

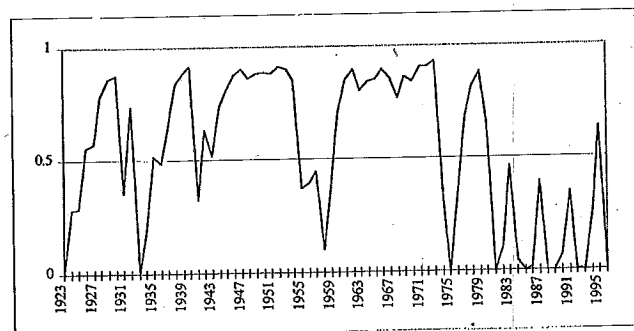
Note: Critical significance levels in parentheses. Large sample tests of Hamilton (1996).

The tests show that the residuals of (1') fulfil the white noise requirements, i.e., they are serially uncorrelated and homoskedastic (no ARCH), both within and across regimes. Furthermore, the Markov property of the transition probabilities cannot be rejected, i.e., the probabilities of the future state outcome are determined exclusively by the most recent state realization.

The model clearly passes all specification tests at the conventional significance level using large sample distributions. Using the small sample corrections suggested by Hamilton (1996) leads to even clearer acceptance of the model. Furthermore, informal diagnostic tests confirm that standardized residuals is white noise, cf. Appendix B.

We are now ready to analyze the timing of regimes. Figure 2 shows the filtered probabilities, i.e., the probability that observation t belongs to state 0 given the information on current and past stock returns available at time t .

Figure 2. Probability that observation t is in state 0 given information available.



This confirms that after a long period of state 0 dominance state 1 has recently become more frequent. Except for a few, short episodes, returns were in the low return-low volatility state with probability greater than one half until 1973. The exceptions are in the beginning of the 20s which was a period of financial distress in Danish financial and industrial companies, the beginning of the 30s which covers both the decline and recovery in the wake of the Wall Street crash, and the latter half of the 50s which marks the beginning of a long business cycle boom. All the episodes occur in periods of volatile stock returns, cf. figure 1. Since 1973, and especially during the 80s and 90s, the high return-high volatility regime has dominated. One possible explanation is that liberalization has made the Danish stock market more vulnerable to foreign volatility.⁵ A similar argument is made by Sellin (1996) in relation to a recent Swedish liberalization.

Figure 3 shows the return forecast of the model⁶ for time t given information available at $t-1$.

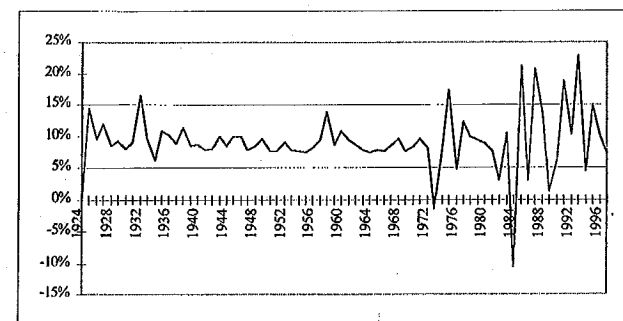
⁵ Although the Danish stock market is formally opened to foreigners around 1972, foreign holding of Danish stocks does not accelerate until the beginning of the 1980s, cf. Eskesen et al. (1984). This explanation is consistent with the observation that a persistent regime-shift seems to take place in the beginning of the 1980s.

⁶ We have excluded the dummy terms in forming forecasts which is natural since the necessity of dummies could not have been anticipated. The forecast is calculated by (note 6 is continued):

$$E(R_t | \Omega_{t-1}) = P(s_t=0 | \Omega_{t-1})(\mu_0 + \phi_0 R_{t-1}) + P(s_t=1 | \Omega_{t-1})(\mu_1 + \phi_1 R_{t-1})$$

Assuming that market participants know the return process, we may interpret the model forecast as a measure of market expectations at time $t-1$ about time t return. We see that the market almost always expected returns within the 5 - 15 per cent per year range in the long period from 1924 to 1972. Since then, and in particular since 1981, market expectations have been extremely volatile and, in fact, more often outside the 5 - 15% range than inside. This reflects that returns have been more volatile in the latter part of the sample and that current returns affect forecasted returns significantly in the state which dominates towards the end.

Figure 3. Model's return forecast at $t-1$



3. Means and Variances of the Two States

In this section, we calculate means and variances of the return process estimated in the previous section. Both unconditional and conditional means and variances are calculated. We consider an 'ordinary' year, i.e., the dummy terms are ignored.

The calculations in sections 3 and 4 are complicated by the presence of the AR-term and have to our knowledge not been presented elsewhere. It is important to include the AR-term for two reasons. First, table 1 shows that the AR-term is statistically significant. Hence, a model without this component would be misspecified and mean and variance calculations would be

where the probabilities are one-period ahead probabilities, cf. Hamilton (1994), section 22.4.

invalid. Second, in section 4, we suggest an alternative test for mean reversion in returns which, basically, tests the significance of the AR-term.

3.1. Means

The unconditional mean of model (1) is:

$$(4) \quad E(R_t) = P(s_t=0)E(R_t|s_t=0) + P(s_t=1)E(R_t|s_t=1) \\ = \pi_0(\mu_0 + \phi_0 E(R_{t-1}|s_t=0)) + \pi_1(\mu_1 + \phi_1 E(R_{t-1}|s_t=1)),$$

where $\pi_0 = P(s_t=0) = (1-p_{11})/(2-p_{00}-p_{11})$ and $\pi_1 = P(s_t=1) = 1-\pi_0$ are unconditional (ergodic) probabilities of being in the particular state, cf. Hamilton (1994). Note, that the mean depends on expected return in the previous period conditional on the current state⁷:

$$(5) \quad E(R_{t-1}|s_t=1) = P(s_{t-1}=0|s_t=1)E(R_{t-1}|s_{t-1}=0) + P(s_{t-1}=1|s_t=1)E(R_{t-1}|s_{t-1}=1) \\ = pE(R_{t-1}|s_{t-1}=0) + (1-p)E(R_{t-1}|s_{t-1}=1),$$

where $p = P(s_{t-1}=0|s_t=1)$ is the probability that the state variable in the previous period was in state 0 given it currently is 1 which can be interpreted as an 'inverse' transition probability.

Using Bayes' rule it can be shown that:

$$(6) \quad p = \frac{\pi_0 p_{01}}{\pi_0 p_{01} + \pi_1 p_{11}} = p_{10}$$

Thus, the inverse transition probability equals the ordinary transition probability.

Assuming covariance stationarity, ie., that means and autocovariances are constant over time, the dating on the right hand side of (5) may be changed:

$$(7) \quad E(R_{t-1}|s_t=1) = pE(R_t|s_t=0) + (1-p)E(R_t|s_t=1).$$

⁷ This is derived in Appendix C.

Similarly,

$$(8) \quad E(R_{t-1}|s_t=0) = qE(R_t|s_t=0) + (1-q)E(R_t|s_t=1),$$

where $q = P(s_{t-1}=0|s_t=0)$ is another inverse transition probability. Using Bayes' rule, it can be shown that:

$$(9) \quad q = \frac{\pi_0 p_{00}}{\pi_0 p_{00} + \pi_1 p_{10}} = p_{00}$$

(7) and (8) can be inserted in:

$$(10) \quad E(R_t|s_t=0) = \mu_0 + \phi_0 E(R_{t-1}|s_t=0)$$

$$(11) \quad E(R_t|s_t=1) = \mu_1 + \phi_1 E(R_{t-1}|s_t=1)$$

(derived from (1)) to get two equations in two unknowns. The solutions are⁸:

$$(12) \quad E(R_t|s_t=1) = \frac{A}{B} \\ E(R_t|s_t=0) = \frac{\phi_0 (1-q) A + \mu_0 B}{(1-\phi_0 q) B},$$

where $A = \mu_1 - \phi_0 q \mu_1 + \phi_1 p \mu_0$ and $B = 1 - \phi_0 q - \phi_1 (1-p) - \phi_0 \phi_1 (p-q)$. Finally, insert (12) in (4) to get the unconditional mean.

$E(R_t|s_t=i)$ is the expected return in state i . It depends not only on the parameters of state i but also on the parameters of the alternative state. This is due to the AR-terms in returns which force us to consider expected return, and hence the value of the unobserved state variable, in

⁸ Assuming $B \neq 0$.

the previous period to form expectations about returns in this period. For example, if $\phi_1 > 0$ and $p > 0$, state 1 expected return increases in μ_0 since there is some probability, p , that the state variable was 0 in the previous period in which case μ_0 affects expected return last period which, in turn, affects expected return in the present period via the positive AR-term in state 1 (ϕ_1).

Given the analytical means in (4) and (12) we are able to estimate:

Table 3. Unconditional and conditional means

$E(R_t)$	0.0955 (0.0177)
$E(R_t s_t=0)$	0.0570 (0.0211)
$E(R_t s_t=1)$	0.1390 (0.0330)
Wald test, $H_0: E(R_t s_t=0) = E(R_t s_t=1)$	3.9806 [0.0460]

Note: Each of the means are calculated as a function, $f(\theta)$, (cf. (4) and (12)) of the estimated parameter vector, $\theta = [\hat{\mu}_0, \hat{\mu}_1, \hat{\mu}_0^2, \hat{\mu}_1^2, \hat{\mu}_0^3, \hat{\mu}_1^3, \hat{\phi}_0, \hat{\phi}_1, \hat{\sigma}_0^2, \hat{\sigma}_1^2, \hat{p}_{00}, \hat{p}_{11}]'$. Standard errors in parentheses are calculated as: $\text{Std}(f(\theta)) = [J' \text{Var}(\theta) J]^{1/2}$, where $J = [\partial f / \partial \theta]$. The restriction being tested has been reformulated as $g(\theta) = 0$, and the test statistic is calculated as: $W = g(\theta)' [J' \text{Var}(\theta) J]^{-1} g(\theta)$, where $J = [\partial g / \partial \theta]$. W is asymptotically χ^2 with degrees of freedom equal to number of restrictions (i.e., 1). Critical significance levels in square brackets.

The estimated unconditional expected return is 9.5% per year which is close to the simple average⁹ of 9.1%. State 0 expected return is estimated to 5.7% per year whereas state 1 has an expected return of 13.9%. The Wald test just rejects (at 5% significance) the hypothesis that

⁹ From 1923 to 1996 excluding 1972 and 1983.

means are equal in favor of the alternative that means are different.¹⁰ Thus, we are justified in saying that regime 0 has lower expected return than regime 1.

3.2. Variances

Unconditional variance is:

$$(13) \quad \text{Var}(R_t) = E(R_t^2) - E(R_t)^2$$

Consider,

$$(14) \quad \begin{aligned} E(R_t^2) &= P(s_t=0)E(R_t^2 | s_t=0) + P(s_t=1)E(R_t^2 | s_t=1) \\ &= \pi_0(\mu_0^2 + 2\mu_0\phi_0 E(R_{t-1} | s_t=0) + \phi_0^2 E(R_{t-1}^2 | s_t=0) + \sigma_0^2) + \\ &\quad \pi_1(\mu_1^2 + 2\mu_1\phi_1 E(R_{t-1} | s_t=1) + \phi_1^2 E(R_{t-1}^2 | s_t=1) + \sigma_1^2) \end{aligned}$$

using the model, that is, (2) and (3).

In this expression, we have that

$$(15) \quad \begin{aligned} E(R_{t-1}^2 | s_t=0) &= qE(R_{t-1}^2 | s_{t-1}=0) + (1-q)E(R_{t-1}^2 | s_{t-1}=1) \\ E(R_{t-1}^2 | s_t=1) &= pE(R_{t-1}^2 | s_{t-1}=0) + (1-p)E(R_{t-1}^2 | s_{t-1}=1) \end{aligned}$$

Assuming covariance-stationarity, we need to solve¹¹

$$(16) \quad \begin{aligned} E(R_t^2 | s_t=0) &= E[(\mu_0 + \phi_0 R_{t-1} + \sigma_0 \epsilon_0)(\mu_0 + \phi_0 R_{t-1} + \sigma_0 \epsilon_0) | s_t=0] \\ &= \mu_0^2 + 2\mu_0\phi_0 E(R_{t-1} | s_t=0) + \phi_0^2 (qE(R_{t-1}^2 | s_t=0) + (1-q)E(R_{t-1}^2 | s_t=1)) + \sigma_0^2 \end{aligned}$$

and a similar expression for $E(R_t^2 | s_t=1)$ to obtain $E(R_t^2)$. The solutions for $E(R_t^2 | s_t=i)$

¹⁰ In addition to the Wald test, we have performed a Likelihood Ratio test of the same hypothesis which has a critical significance level of 0.0643 leading to acceptance of the hypothesis at 5%. We have more confidence in the Wald test, however, since filtered probabilities change completely under the restriction which in our opinion makes the test hard to interpret. Possibly, the existence of multiple local maxima of the unrestricted likelihood function reduce the power of Likelihood Ratio tests.

¹¹ $E(R_{t-1} | s_t=0)$ is known from section 3.1.

are in appendix D. Subtracting the squared means derived earlier gives expressions for unconditional and conditional variances.

Unconditional and conditional variances can now be estimated:

Table 4. Unconditional and conditional variances

Var(R_t)	0.0246 (0.0074)
Var($R_t s_t=0$)	0.0056 (0.0030)
Var($R_t s_t=1$)	0.0425 (0.0143)
Wald test, $H_0: \text{Var}(R_t s_t=0) = \text{Var}(R_t s_t=1)$	7.7977 [0.0052]

Note: See note to table 3 where 'f' now relates to the variance formulae derived above. Standard errors in parentheses and critical significance levels in square brackets.

The estimated unconditional standard error of annual returns is 15.7% which should be compared to the sample standard error of 16.4%.¹² State 0 standard deviation is 7.5% whereas state 1 standard deviation is 20.6%. The hypothesis that conditional variances are equal is strongly rejected with a critical significance level of less than 1 per cent.¹³ Hence, volatility is lower in state 0 than in state 1.

Finally, a Wald test rejects the joint hypothesis that both means and variances are equal across states (the critical significance level is 0.0013).

¹² From 1923 to 1996 excluding 1972 and 1983.

¹³ A Likelihood Ratio test of the hypothesis has a critical significance level of 0.1193 leading to acceptance of H_0 . However, the test is not easily interpretable, cf. footnote 9.

4. Serial Correlation: Test for Mean Reversion

The question of whether stock prices are mean-reverting has received a lot of attention since the papers by Fama and French (1988) and Poterba and Summers (1988). A number of studies have produced evidence of mean reversion using variance-ratio and autoregression tests, see Risager (1998) for an analysis of the Danish return data. In this section, we provide evidence based on an alternative test procedure which has the important feature that it explicitly allows for regime-shifts in the return process.

We choose to focus attention on first order serial correlation. Specification tests in table 2 and Appendix B show no sign of autocorrelation in the error term, so any higher order serial correlation is due to first order serial correlation. We calculate the analytical first order serial correlations of the two-state Markov switching model, see appendix E. Then we obtain point estimates and standard errors:

Table 5. Unconditional and conditional first order serial correlation

Corr(R_t, R_{t-1})	-0.1993 (0.1104)
Corr($R_t, R_{t-1} s_t=0$)	0.0297 (0.2482)
Corr($R_t, R_{t-1} s_t=1$)	-0.3340 (0.1214)
Wald test, $H_0: \text{Corr}(R_t, R_{t-1})=0$	3.2567 [0.0711]
Wald test, $H_0: \text{Corr}(R_t, R_{t-1} s_t=0)=0$	0.0143 [0.9048]
Wald test, $H_0: \text{Corr}(R_t, R_{t-1} s_t=1)=0$	7.5669 [0.0059]

Note: See note to table 3 where 'f' relates to the serial correlation formulae displayed in Appendix E.

Our estimate of first order serial correlation across regimes is -0.2 which is significantly less than zero at 10% significance level but cannot be rejected to be zero at the 5% level. Hence, there is weak evidence of mean reversion in nominal stock returns which is consistent with

findings of others.¹⁴

Interestingly, the same hypothesis has a critical significance level of 0.0042 in a standard one-regime AR 1-specification with dummies for 1972 and 1983 and using OLS standard errors which leads to clear acceptance of mean reversion.¹⁵ Hence, allowing for multiple regimes results in much less support for mean reversion than the standard AR-regression introduced by Fama and French (1988). This finding is consistent with the results of Kim and Nelson (1998) who also conclude that accounting for the observed pattern of heteroskedasticity stemming from regime-switching volatility of returns weakens the evidence of mean reversion according to autoregression tests.¹⁶

Thus, it is important to take account of heteroskedasticity when making inference about mean reversion, in particular, since the critical significance levels are close to the conventional significance level even small changes may have large qualitative importance for conclusions. Although OLS gives consistent estimates of coefficients, a procedure which allows for heteroskedasticity (of the correct form) leads to more efficient inference. Moreover, usual OLS estimates of variances including coefficient standard errors are biased. Heteroskedasticity consistent standard errors (such as White) improve inference asymptotically, but may have problems in small samples. For example, in our case, using White standard errors only increases the critical significance level to 0.0064, whereas we found a critical significance level of around 7%.

Our regime-switching model includes the standard one-regime model as a special case, and, hence, is more general. Therefore, we have more confidence in results of the regime-switching model. We interpret the conflicting inference as evidence of weaknesses of the standard

¹⁴ Risager (1998) finds slightly more support for mean reversion in real than in nominal returns which indicates that the critical significance level would be slightly less than 7.11% if our analysis were applied to real returns.

¹⁵ The estimated coefficient to lagged returns is -0.235 with a t-statistic of -2.957. These results are similar to the findings in Nielsen and Risager (1999) and Risager (1998).

¹⁶ A similar conclusion has been found for the variance ratio test by Kim, Nelson and Startz (1998).

approach.

Our analysis highlights two important points. First, in the presence of multiple persistent regimes which have the feature that some but not all regimes exhibit mean reversion, it is important to have observations from each regime in order to draw correct inference. In the case of nominal Danish stock returns it is particularly important to have enough observations after the beginning of the 80s to be able to detect two regimes. This parallels the so-called peso problem encountered in the exchange rate literature, see e.g. Evans (1996), ie., in order to identify a process with rare, discrete events, a large sample is needed.¹⁷ Second, there are two sources to negative serial correlation if the true return generating process is regime-switching. First of all, a negative autoregressive term creates mean reversion as in the usual one-state AR case. But, even if the autoregressive term is zero in both states serial correlation may be different from zero just because the process shifts between states (assuming these have different means).

Within our framework, we are able to distinguish serial correlation of the two states. As table 5 shows, our estimate of serial correlation is only negative in state 1. In fact, only in state 1 is serial correlation significantly different from zero. Hence, we conclude that the weak evidence of mean reversion presented in table 5 is (mainly) a result of serial correlation in the high return-high volatility state which has dominated the most recent decades. This is in contrast to results for the US which indicate that mean reversion was stronger before World War II than after, see Kim, Nelson and Startz (1991) and Kim and Nelson (1998).

The evidence of mean reversion parallels the findings in Risager (1998). Using standard autoregressive and variance ratio tests, he finds weak support of the mean reversion hypothesis. Furthermore, the paper suggests splitting the sample into subsamples. This analysis indicates that mean reversion has been stronger in the most recent part of the sample, that is, since the 1970s. This conclusion is consistent with the results of the regime-switching model in the present paper.

¹⁷ We conjecture that since our model is constructed to identify regime-shifts, it will stand a better chance of solving peso problems and lead to more reliable inference on mean reversion in small samples.

Given the strong presence of mean reversion in recent years, what should we expect for the future? This basically depends on whether one believes that the current regime is absorbing or not. From a purely statistical point of view, there is a probability of returning to the no-mean-reversion state which implies that unconditional serial correlation is the right measure, thus suggesting only weak evidence for mean reversion. From an economic point of view, however, it is essential to focus on the underlying factors which cause regime changes and, in particular, to analyze whether all the variables causing the most recent regime-shift are reversible. It is perhaps not likely that the liberalizations, which we argue led to the latest transition to high volatility, will be reversed within a foreseeable future. However, other factors, such as a decrease of US stock market volatility, may be able to cause a return to low volatility. In other words, we use capital market liberalizations as one (of several) component to explain the latest transition to high volatility but do not view deliberalization as necessary for a return to the low volatility regime. Hence, economic considerations have ambiguous implications for the question of mean reversion.

5. Conclusion

We have estimated a well-specified two-state regime-switching model for Danish stock returns. The model identifies two regimes which have low return-low volatility and high return-high volatility, respectively. The low return-low volatility regime dominated, except in a few, short episodes, until the beginning of the 70s whereas the 80s and 90s have been characterized by high return and high volatility.

We propose an alternative test of mean reversion which allows for multiple regimes with potentially different constant and autoregressive terms and different volatility. Using this test procedure we find mean reversion at 10% but not at 5% significance level. This is weaker evidence than produced by the standard method of testing for significance of the AR-term in a one-regime autoregressive model. Furthermore, when analyzing contributions of the two regimes we find that the indication of mean reversion is due to the recent high return-high volatility regime only.

The regime-switching model has also been applied by Kim and Nelson (1998) and Kim,

Nelson and Startz (1998) on stock returns using US data. Our approach differs by allowing for an autoregressive term and by incorporating regime-shifts in the mean. Both features are shown to be relevant for Danish data.

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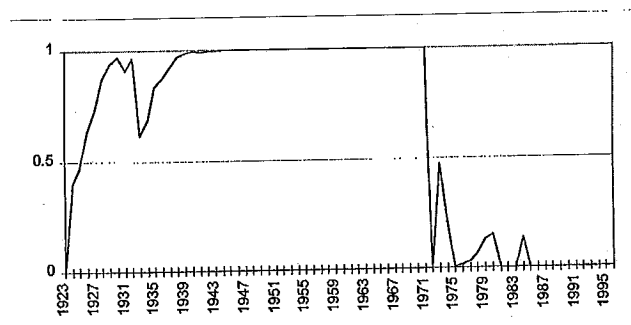
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Appendix A: Three-State Model**Parameter estimates**

μ_0	0.0781 (0.0167)	P_{00}	0.9703 (0.0228)
μ_1	0.1614 (0.0493)	P_{01}	0.0000
μ_2	0.8923 (0.0021)	P_{02}	0.0297
ϕ_0	-0.0888 (0.1163)	P_{10}	0.0000
ϕ_1	-0.2616 (0.1234)	P_{11}	0.9328 (0.0495)
ϕ_2	1.5922 (0.0127)	P_{12}	0.0672
σ_0^2	0.0091 (0.0019)	P_{20}	0.2741
σ_1^2	0.0440 (0.0130)	P_{21}	0.7259
σ_2^2	0.0000 (0.0000)	P_{22}	0.0000

Note: Standard errors in parentheses estimated by second derivatives of log likelihood. Omitted standard errors cannot be calculated due to corner solutions.

Filtered probabilities for state 0

The outlier state has filtered probabilities close to 1 in 1923, 1972 and 1983 and zero otherwise.

All point estimates of the three-state model are within one standard deviation of the two-state dummy model estimates. The main difference is that the regimes are estimated to be more persistent in the three-state model. This has the implication that inference about the state and the timing of regime shifts is much clearer than in the two-state model. Another difference between the models is that the three-state model assigns some probability to the event that s_t returns to the outlier state.

Diagnostic tests of standardized residuals:¹⁸

	Test statistic	Critical significance level
AR(1)	0.0000	0.9967
AR(2)	3.2539	0.0445 *
AR(3)	2.1393	0.1030
AR(4)	1.853	0.1286
AR(5)	1.5575	0.1839
AR(6)	1.5812	0.1663
AR(7)	1.3459	0.2433
AR(8)	1.2564	0.2818
ARCH(1)	0.0661	0.7978
Normality	0.9729	0.6148

¹⁸ Standardized residuals are calculated as in Appendix B except for the extra state.

Appendix B: Analysis of Standardized Residuals of Two-State Dummy Model

Standardized residuals are calculated as the difference between actual and fitted return divided by conditional standard deviation, ie., the square root of (derived in Nielsen and Olesen, 1999):

$$Var(R_t | \Omega_{t-1}) = P(s_t=0 | \Omega_t) \sigma_0^2 + P(s_t=1 | \Omega_t) \sigma_1^2 + \\ P(s_t=0 | \Omega_t) P(s_t=1 | \Omega_t) (E(R_t | \Omega_{t-1}, s_t=0) - E(R_t | \Omega_{t-1}, s_t=1))^2,$$

where Ω_t contains information about current and past stock returns. Fitted returns are:

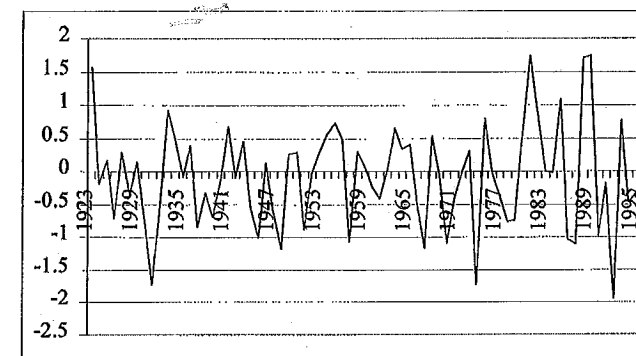
$$\hat{R}_t = P(s_t=0 | \Omega_t) (\mu_0 + \mu_0^{72} d72_t + \mu_0^{83} d83_t + \phi_0 R_{t-1}) + \\ P(s_t=1 | \Omega_t) (\mu_1 + \mu_1^{72} d72_t + \mu_1^{83} d83_t + \phi_1 R_{t-1})$$

which is conditioned on information on past stock returns and uses filtered probabilities for each state (that is, probabilities conditioned on Ω_t which includes all available stock returns of the sample). The standardized residuals are estimates of ϵ_t in (1').

The standardized residuals have been tested for autocorrelation from lag 1 to 8, ARCH and normality:

	Test statistic	Critical significance level
AR(1)	0.0390	0.8440
AR(2)	1.5828	0.2126
AR(3)	1.2332	0.3042
AR(4)	0.9278	0.4530
AR(5)	0.8111	0.5458
AR(6)	0.7359	0.6225
AR(7)	0.6219	0.7360
AR(8)	0.6334	0.7468
ARCH(1)	1.6207	0.2071
Normality	0.4076	0.8156

The following plot confirms that the standardized residuals are well-behaved:

Standardized Residuals

Appendix C: Derivation of (5)

$$\begin{aligned}
E(R_{t-1}|s_t=1) &= \int R_{t-1} f(R_{t-1}|s_t=1) dR_{t-1} \\
&= \int R_{t-1} \sum_{j=0}^1 f(R_{t-1}, s_{t-1}=j|s_t=1) dR_{t-1} \\
&= \int R_{t-1} \sum_{j=0}^1 f(R_{t-1}|s_{t-1}=j, s_t=1) P(s_{t-1}=j|s_t=1) dR_{t-1} \\
&= \int R_{t-1} \sum_{j=0}^1 f(R_{t-1}|s_{t-1}=j) P(s_{t-1}=j|s_t=1) dR_{t-1} \\
&= \sum_{j=0}^1 \int R_{t-1} f(R_{t-1}|s_{t-1}=j) P(s_{t-1}=j|s_t=1) dR_{t-1} \\
&= \sum_{j=0}^1 P(s_{t-1}=j|s_t=1) \int R_{t-1} f(R_{t-1}|s_{t-1}=j) dR_{t-1} \\
&= \sum_{j=0}^1 P(s_{t-1}=j|s_t=1) E(R_{t-1}|s_{t-1}=j) \\
&= P(s_{t-1}=0|s_t=1) E(R_{t-1}|s_{t-1}=0) + P(s_{t-1}=1|s_t=1) E(R_{t-1}|s_{t-1}=1)
\end{aligned}$$

Appendix D: Solutions for $E(R_t^2|s_t=i)$

$$\begin{aligned}
E(R_t^2|s_t=0) &= \frac{CD + \phi_0^2(1-q)[(1-\phi_0^2q)E + \phi_1^2pD]}{(1-\phi_0^2q)C} \\
E(R_t^2|s_t=1) &= \frac{(1-\phi_0^2q)E + \phi_1^2pD}{C}
\end{aligned}$$

where

$$\begin{aligned}
C &= 1 - \phi_0^2q - \phi_0^2\phi_1^2p(1-q) - \phi_1^2(1-p) + \phi_0^2\phi_1^2(1-p)q \\
D &= \mu_0^2 + 2\mu_0\phi_0 E(R_{t-1}|s_t=0) + \sigma_0^2 \\
E &= \mu_1^2 + 2\mu_1\phi_1 E(R_{t-1}|s_t=1) + \sigma_1^2
\end{aligned}$$

Appendix E: Serial Correlation

Unconditional first order serial correlation is defined as (assuming covariance stationarity):

$$\text{Corr}(R_t, R_{t-1}) = \frac{\text{Covar}(R_t, R_{t-1})}{\text{Var}(R_t)}$$

Thus, we need:

$$\begin{aligned} E(R_t R_{t-1}) = & \pi_0(q[\mu_0^2 + \mu_0\phi_0 E(R_{t-1}|s_t=0) + \mu_0\phi_0 E(R_t|s_t=0)] + \\ & (1-q)[\mu_0\mu_1 + \mu_0\phi_1 E(R_{t-1}|s_t=1) + \mu_1\phi_0 E(R_t|s_t=1)]) + \\ & \pi_1(p[\mu_0\mu_1 + \mu_1\phi_0 E(R_{t-1}|s_t=0) + \mu_0\phi_1 E(R_t|s_t=0)] + \\ & (1-p)[\mu_1^2 + \mu_1\phi_1 E(R_{t-1}|s_t=1) + \mu_1\phi_1 E(R_t|s_t=1)]) + \\ & (\pi_0 q \phi_0 + \pi_1 p \phi_1)(\phi_0 E(R_t R_{t-1}|s_t=0) + \sigma_0^2) + \\ & (\pi_0(1-q)\phi_0 + \pi_1(1-p)\phi_1)(\phi_1 E(R_t R_{t-1}|s_t=1) + \sigma_1^2) \end{aligned}$$

Hence, we must solve

$$\begin{aligned} E(R_t R_{t-1}|s_t=0) = & q(\mu_0^2 + \mu_0\phi_0 E(R_{t-1}|s_t=0) + \mu_0\phi_0 E(R_t|s_t=0) + \phi_0^2 E(R_t R_{t-1}|s_t=0) + \phi_0\sigma_0^2) + \\ & (1-q)(\mu_0\mu_1 + \mu_0\phi_1 E(R_{t-1}|s_t=1) + \mu_1\phi_0 E(R_t|s_t=1) + \phi_0\phi_1 E(R_t R_{t-1}|s_t=1) + \phi_0\sigma_1^2) \end{aligned}$$

and a similar expression for $E(R_t R_{t-1}|s_t=1)$. The solutions are:

$$\begin{aligned} E(R_t R_{t-1}|s_t=0) = & \frac{CF + (1-q)\phi_0\phi_1[(1-\phi_0^2)G + p\phi_0\phi_1F]}{(1-\phi_0^2)C} \\ E(R_t R_{t-1}|s_t=1) = & \frac{(1-\phi_0^2)G + p\phi_0\phi_1F}{C}, \end{aligned}$$

where

$$\begin{aligned} F = & q\mu_0^2 + (1-q)\mu_0\mu_1 + q\mu_0\phi_0 E(R_{t-1}|s_t=0) + (1-q)\mu_0\phi_1 E(R_{t-1}|s_t=1) + q\mu_0\phi_0 E(R_t|s_t=0) + \\ & (1-q)\mu_1\phi_0 E(R_t|s_t=1) + q\phi_0\sigma_0^2 + (1-q)\phi_0\sigma_1^2 \\ G = & p\mu_0\mu_1 + (1-p)\mu_1^2 + p\mu_1\phi_0 E(R_{t-1}|s_t=0) + (1-p)\mu_1\phi_1 E(R_{t-1}|s_t=1) + p\mu_0\phi_1 E(R_t|s_t=0) + \\ & (1-p)\mu_1\phi_1 E(R_t|s_t=1) + p\phi_1\sigma_0^2 + (1-p)\phi_1\sigma_1^2 \end{aligned}$$

Inserting these solutions and the results from the previous sections above gives $E(R_t R_{t-1})$.

Subtract $E(R_t)^2$ to obtain $\text{Covar}(R_t, R_{t-1})$. Similarly for state dependent covariances.

Chapter 5

Modeling the Dividend-Price Ratio:

The Role of Fundamentals Using a Regime-Switching Approach

Modeling the Dividend-Price Ratio:

The Role of Fundamentals Using a Regime-Switching Approach^{*}

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Abstract

Using annual data over the post-World War I-period, we estimate a fundamentals-based empirical model for the dividend-price ratio of Danish stocks. The key fundamentals-variable is a time-varying discount rate, decomposed into time-varying measures for the growth-adjusted real interest rate and the risk premium on stocks. In addition, the model includes real dividends and the lagged dividend-price ratio as explanatory variables. Results show that the model suffers from structural breaks over the sample. Using a two-state regime-switching approach to capture non-modeled shifts in the economic environment, we find that all fundamentals are highly significant in at least one regime and, moreover, obtain a good fit. The model identifies two very persistent regimes characterized by a 'low', respectively, 'high' dividend-price ratio.

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1. Introduction

In empirical finance the dividend-price ratio, defined as the ratio between a given periods dividend payments per share and the end-of-period stock price per share, is often - explicitly or implicitly - used as an indicator of whether stock prices are (too) high or (too) low. For instance, Campbell and Shiller (1998) report a very gloomy prediction for the US stock market based on the fact that the dividend-price ratio has fallen far below its historical mean, suggesting an overvalued stock market. Fama and French (1988) and Hodrick (1992) are other examples of the numerous studies that use dividend-price to forecast future stock returns, see also the survey in Campbell *et al.* (1997, Chp. 7).

However, according to standard finance theory one should expect time variation in the dividend-price ratio as a result of changes in the underlying economic fundamentals, in particular changes in the (*ex ante*) real interest rate and the risk premium on stocks relative to bonds. Hence, it is crucial to consider the economic fundamentals when using the dividend-price ratio to judge whether stocks are fairly valued or not. For this purpose we need an economic model for dividend-price. This is the topic of the present paper. Motivated by a Gordon growth type model which is modified to incorporate a time-varying discount rate, we formulate an empirical model using a real interest rate proxy, a proxy for the risk premium and the level of real dividend payments as explanatory variables. The real interest rate and risk premium proxies together capture the effects from the time-varying discount rate while the inclusion of real dividends allows for the possibility that innovations in dividends are reflected less than proportionately in stock prices. We also include lagged dividend-price in the model to allow for slow adjustment in the dividend-price ratio to long-run equilibrium.

The economic model is estimated for the aggregate Danish stock market, using annual observations for the period 1927-1996. All variables turn out to be significant with the right signs and a reasonably good fit is obtained. However, the model suffers from structural breaks as the coefficients to the explanatory variables are highly unstable. This suggests that we have omitted an important (or several important) fundamental variable(s). In the Danish case, a possible explanation for a structural break is a change in investor taxation as of 1983,

i.e., the introduction of a separate pension fund tax on bond investments, affecting the relative profitability of stock investments (cf. below). Modifying the economic framework in order to take account of the omitted variable is obviously the ideal solution in such a situation. However, in practice this may not always be realistic or even possible because the omitted variable may be difficult (or impossible) to identify and, subsequently, quantify. When modeling the effects of investor taxes in a heterogeneous tax system as the Danish where tax rates differ significantly across investor groups, it is essential to correctly identify, at every single point of time, the 'marginal investor', defined as the stock investor who has a reservation price or willingness-to-pay for stocks which at the margin is equal to the prevailing stock price. However, the 'marginal investor' is unobservable and, hence, the inclusion of investor taxes in the model is a difficult task. In the case of the new pension fund tax, matters are, moreover, complicated by the gradual implementation of the tax.

In this paper, we take a short-cut by estimating the economic model using the two-state regime-switching approach of Hamilton (1990). We consider this approach to be a practical tool of incorporating and indirectly modeling the omitted factor(s) which give rise to the structural breaks that we encounter in the one-regime specification, without having to explicitly model those factors. The regime-switching approach is based on the assumption that the economic model differs across (a finite number of) distinct regimes, whose timing is governed by an exogenous, discrete and latent state-variable. This means that the type of omitted factors which we can capture by this approach are the more persistent factors that relate to the 'economic environment' of the model and that result in the outcome of distinct regimes over time with distinct economic models. Such factors often relate to the institutional or policy framework of the economy, leading to distinct policy or institutional regimes over time, and are typically also the factors that are difficult to model. We find in our case that the regimes identified by the regime-switching approach are highly persistent which is consistent with the interpretation that the omitted factor(s) represents changes in the economic environment rather than being an additional temporary explanatory variable. In particular, we conjecture that the identified regimes may be given the interpretation of different tax policy regimes.

Beyond providing a practical modeling tool, we also consider the analysis based on the regime-switching approach to be a useful step in identifying the possible omitted factor(s) because the results provide valuable insight regarding the timing of regime-shifts, without being conditioned on *apriori* information. Hence, the regime-switching model lets the data determine if and when regime shifts occur. This information can consequently be used to identify candidates for omitted factors by examining relevant institutional or policy changes around these dates of regime-shifts.

The regime-switching approach of Hamilton (1990) has previously been used in the empirical literature to model asset pricing in situations where the pricing process changes over time, e.g. due to shifts in the process governing economic fundamentals (for instance as a result of policy regime shifts), shifts in the predominance of different investor types over time or changes in the institutional set up or taxation rules of relevance for the stock market. The importance of regime-shifts in the pricing process has recently been emphasized for the US stock market by Driffill and Sola (1998) who motivate shifts in the pricing process with regime-shifts in the underlying process for dividends, cf. the discussion at the end of this paper. The possible influence of different investor types with different investment rules has been examined for the currency market by Vigfusson (1996), who assumes that the market on a high-frequency (daily) basis shifts between being driven by chartists and fundamentalists. In the context of the stock market, a potential motivation for time differences in investment and, hence, pricing rules could be that the market misprices stocks in high-inflation regimes by using nominal rather than real interest rates, whereas investors may price stocks more correctly in low-inflation regimes, cf. Modigliani and Cohn (1979), who argue that US stocks were mispriced (undervalued) in the high-inflation regime of the 1970s. In such a setting we should *apriori* expect the regimes identified by the regime-switching approach to be identical to different inflationary regimes. In this paper, we do not attempt at formally explaining the regime shifts but the working hypothesis motivating the use of the regime-switching approach is that the regime shifts are related to (persistent) changes in the 'economic environment', leading to (persistent) shifts in the economic model linking dividend-price to the economic fundamentals. We conjecture that changes in investor taxation is a prime candidate but institutional changes or changes in the processes for the economic

fundamentals leading to changes in expectations formation and hence the economic model may do as well¹. In any case, a closer examination of the causes underlying the regime shifts would be interesting but this is left for future work.

Results from estimating the regime-switching model show that all the fundamentals variables including the real interest rate and the risk premium are highly significant in at least one regime. Hence, we have succeeded in modeling a time-varying discount rate, here decomposed into a time-varying real interest rate and a time-varying risk premium, which is empirically significant in explaining the dividend-price ratio. This is an innovation compared to the existing empirical literature where the discount rate is either assumed to be fixed or not quantified directly (no closed-form measure) when modeling the behavior of the dividend-price ratio or, more generally, stock prices, cf. e.g. Driffill and Sola (1998), Froot and Obstfeld (1991) and Campbell *et al.* (1997, Chp. 7). Our model is not perfect in terms of misspecification tests but passes at a 5% significance level, is stable over time and provides a rather good fit to dividend-price. Moreover, results show that two regimes are both necessary and sufficient to remove the structural breaks from the underlying economic model. The model clearly identifies 3 distinct subperiods over which the regimes reign (1927-1949, 1950-1985 and 1986-1991), thereby providing valuable insight which can be used in inferring the possible causes of the two regimes.

The outline of the paper is as follows. In section 2, we formulate an operational empirical model, based on a simple and *ad hoc* theoretical framework which is derived from the standard Gordon growth model by allowing for a time-varying discount rate. The data is reviewed in section 3. In section 4 we, first, estimate the economic model under the assumption that only one regime applies, i.e., assuming that the model is stable over the entire sample. In section 5, we estimate the regime-switching model allowing for two distinct regimes. Section 6 finally concludes the paper.

¹ To illustrate, consider a change in the process for the real interest rate leading to increased short-run volatility. This may imply that investors put less emphasis on the current level of the real interest rate when forming expectations about the future long-run, average real interest rate, which is the relevant measure for the pricing of stocks. The implication is a change in the economic model with a smaller coefficient to the current real interest rate.

2. The Empirical Model

In formulating the empirical model, we take as a starting point the textbook Gordon growth model for the price of a stock with a constant discount rate and constant expected dividend-growth, see Gordon (1962) or Campbell *et al.* (1997). We modify Gordon's model in a rather simple way to allow for time variation in the discount rate, reflecting time variation in both the real interest rate and the risk premium on stocks. The resulting theoretical framework is *ad hoc* but allows us to formulate an operational empirical model with specific candidates for economic variables that may explain dividend-price. The theoretical model can, basically, be interpreted as an assumption that market participants at each point in time price stocks according to the constant-discount-rate-and-constant-dividend-growth Gordon model, i.e., as if the discount rate and dividend-growth were in fact constant, while using the prevailing levels for nominal bond returns, expected nominal dividend growth and the risk premium on stocks as determinants².

Thus, let the equilibrium in stock and bond markets at each point in time t be described by a no-arbitrage relation stating that the expected (nominal) return on stocks $E_t[S_{t+1}]$ from time t to $t+1$ should be equal to the corresponding (nominal) return on bonds B_t augmented by a risk premium γ_t on stocks relative to bonds:

$$(1) \quad E_t[S_{t+1}] = B_t + \gamma_t$$

We take B_t to be the yield-to-maturity on a (one-period) bond so that it is predetermined and known as of time t .

The return on stocks is given as the sum of capital gains and dividend yield:

² Campbell and Shiller (1988) have generalized Gordon's growth model to take account of a stochastic, time-varying discount rate, the so-called "dynamic Gordon growth model". However, their model is (at least in its general version) not as operational as the one we formulate. In particular, the Campbell and Shiller (1988) model does not entail a closed-form expression for the time-varying discount rate. Our assumptions on expectations formation, basically, imply that stocks can be priced within the original Gordon model despite the fact that the discount rate (and dividend growth) vary over time.

$$(2) \quad E_t[S_{t+1}] = \frac{E_t[P_{t+1}] - P_t}{P_t} + \frac{E_t[D_{t+1}]}{P_t}$$

where P_t is the *ex dividend* price per share as of time t (i.e., at the beginning of period $t+1$) and D_{t+1} is the dividend payment per share paid during period $t+1$.

Even though B_t and γ_t are allowed to vary stochastically over time, we shall assume that market participants only form *point* expectations wrt. future bond returns and risk premia, i.e., "Certainty Equivalence" is assumed to apply. Moreover, we assume that market participants expect bond returns and risk premia to be constant over time, so that any innovations in the two variables are expected to be permanent. These assumptions - while clearly restrictive in a theoretical setting - allow us to set up an empirically tractable model. Thus, under the additional Gordon assumption of constant expected dividend growth, (1) can be solved by forward recursion to give the following no-bubble solution for the dividend-price ratio (assuming that $R_t + \gamma_t > 0$):

$$(3) \quad \frac{D_t}{P_t} = \frac{R_t + \gamma_t}{1 + G_t} \approx R_t + \gamma_t, \quad \text{where } R_t \equiv B_t - G_t$$

G_t is the expected *nominal* growth in dividends per share as of time t . G_t is also allowed to vary over time. According to (3), the dividend-price ratio is in equilibrium equal to the sum of the (*ex ante*) growth-adjusted real interest rate $R_t \equiv B_t - G_t$ and the risk premium on stocks γ_t . (3) resembles the solution of the standard Gordon growth model with the main difference being the allowed variation in the real interest rate and the risk premium and, hence, the discount rate (the sum of the two).

Based on (3), we set up the empirical model:

$$(4) \quad \frac{D_t}{P_t} = \beta_0 + \beta_1 R_t + \beta_2 \gamma_t + \beta_3 D R_t + \beta_4 \frac{D_{t-1}}{P_{t-1}} + \varepsilon_t$$

where ε_t is the residual of the equation. We have augmented the model by including the lagged dividend-price ratio $(D/P)_{t-1}$ and the log-level of real dividends per share DR_t as additional candidate explanatory variables. The introduction of the former allows for slow or partial adjustment in the dividend-price ratio so that (3) (or rather the long-run solution to (4)) is basically thought of as a model for the long run, providing us with an equilibrium relation to which dividend-price adjusts in the long run. The introduction of DR_t allows for the possibility that real stock prices may react more or less than proportional to innovations in real dividend payments. According to (3), the relation between real stock prices and real dividends should be proportional as the dividend-price ratio is unaffected by innovations in dividends. The reason is that market participants expect any innovation in current dividends to be permanent under the Gordon constant-dividend-growth setting. However, this may not be the case empirically. Froot and Obstfeld (1991) and Driffill and Sola (1998) also include real dividends in their models for the price-dividend ratio (the inverse of the dividend-price) in order to capture the possibility of "intrinsic bubbles" in stock prices, i.e., rational bubbles that depend on fundamental variables. As standard in empirical analysis, we allow for a constant term in (4), even though not strictly implied by the theoretical model. Hence, we intend to explain the variations in rather than the actual levels of the dividend-price ratio³.

The challenge facing (4) is the fact that the real interest rate R_t and the risk premium γ_t are unobservable. We, therefore, have to use suitable proxies for these two variables.

3. The Data

The data are depicted in Figures 1-4. The source database is Nielsen, Olesen and Risager (1999) which comprises data for the Danish stock and bond markets. Stock market data relate to the aggregate market level of all Danish firms listed at the Copenhagen Stock Exchange. The market index by Statistics Denmark is used for stock prices while dividend payments are estimated from a large sample of firms, cf. Nielsen, Olesen and Risager (1999) for further details. Bond data relate to the markets for government bonds. All observations are annual.

³ Note that according to the constant-discount-rate Gordon model, all regressors in (4) should be insignificant, implying that the dividend-price ratio would, basically, follow a white noise process around a constant mean. However, the latter assumption is clearly violated by the data on dividend-price, cf. below, as we observe systematic deviations in the ratio from its mean. It is these systematic deviations that we intend to explain.

The empirical analysis in the following sections uses the sample period 1927-1991 which is the longest available sample for all variables.

< Insert Figures 1-4 around here >

Figure 1 shows the dividend-price ratio over the period 1927-1996. The plot suggests a cyclical component in the ratio with large and often persistent deviations from its sample mean, in particular, in the first half of the period. For instance, stock prices seem to have been persistently low compared to dividends in the first half of the 1950s while stock prices were high during World War II. In relative terms, the ratio is often subject to large year-by-year changes where in particular the drop in the ratio from 5.2% in 1982 to 1.8% in 1983 (a decrease of 65% in relative terms) attracts attention. This fall which is a result of capital gains on stocks of 114% that year coincides with at least two important events in the Danish economy. First of all, there was a major shift in economic policy as a new conservative-liberal government came into office in September 1982, emphasizing tight economic policies including a fixed exchange rate policy. Second, a new tax was introduced on the returns on pensions funds' bond holdings while the returns on stocks were exempted from taxation⁴. This *ceteris paribus* gave pension funds an incentive to invest more in stocks and less in bonds. We can also observe that the dividend-price ratio has been at a historically low level since 1983. The post-1983 average is 1.7% which compares to an average of 5.1% over the years before 1983. This persistent low level is a key issue in understanding the mechanisms which determine the dividend-price ratio and it is, in particular, of interest to know whether it can be explained by economic fundamentals or whether it marks a new regime compared to the pre-1983 history.

The proxy used for the latent growth-adjusted real interest rate R_t is plotted in Figure 2. The real interest rate as of time t is constructed as the 5-year yield-to-maturity on government bonds at time t minus the realized growth in *nominal* dividends over the corresponding 5-year period following time t . The proxy is an *ex post* (or *perfect foresight*) growth-adjusted

⁴ This tax was passed by the Parliament in 1983 and took effect as of Jan 1 1984. Because pension savings before 1984 were exempted from taxation, the tax was phased in gradually.

real interest rate where the use of growth in nominal dividends to take account of inflation and real growth is consistent with the theoretical framework of section 2. Because of the forward-looking nature of the proxy, we lose 5 observations towards the end of the period so that the sample effectively ends in 1991⁵.

It is evident from Figure 2 that the real interest rate proxy is highly volatile. The fluctuations are mainly driven by the variation in the dividend growth part of the proxy, as the 5-year nominal bond return is much more stable on a year-by-year basis. For instance, the low levels of the real interest rate in the 1940s is due to high future dividend growth which is not accompanied by high nominal bond returns. Due to a non-credible stance of economic policy (amongst other things), the Danish economy experienced very high nominal and real interest rates towards the end of the 1970s and in the beginning of the 1980s. Nominal interest rates declined following the new policy regime as of late 1982, but nominal dividend growth declined likewise, sustaining the high real interest rate level until the end of the 1980s.

To construct a proxy for the risk premium on stocks γ_t , we draw on Olesen and Risager (1999) who test whether the Danish premium on stocks, defined as the excess of stock returns over bond returns, can be predicted from a set of possible predictor variables such as the dividend price ratio, dividend yield, bond returns, lagged equity premia etc. They conclude that the 5-year premium on stocks is predictable from the dividend yield, the 5-year bond return and past 1-year equity premia, see Olesen and Risager (1999) for details. This predicted or fitted 5 year premium can in an efficient markets framework be interpreted as an estimate of the risk premium on stocks relative to bonds. However, Olesen and Risager (1999) use the dividend yield as a predictor, and the dividend yield comes close to the dividend-price ratio variable. In terms of (4), one could therefore possibly argue that when using the fitted premium in Olesen and Risager (1999) as the risk premium proxy γ_t , we

⁵ Data for bond yields are available for the 1-, 5- and 10-year maturities, cf. Nielsen, Olesen and Risager (1999). In choosing between these horizons, we excluded the latter because using a 10-year *ex post* real interest rate would imply a loss of more observations towards the end of the sample period. We excluded the 1-year horizon because the resulting 1-year real interest rate turned out to be a very 'noisy' measure with large year-to-year variability and no explanatory power wrt. the dividend-price ratio. Intuitively, the 1-year maturity also appears to be too short in order to be of relevance for the pricing of stocks in practice.

would basically explain the dividend-price ratio by a variable that comes close the ratio itself, the dividend yield.

In order to be immune to this critique, we have therefore (re-)estimated a predictor model excluding the dividend yield and the dividend-price ratio as potential predictor variables. Following the approach of Olesen and Risager (1999), the resulting model is (standard errors of coefficient estimates in parentheses)⁶:

$$(5) \quad \hat{PR5}_t = 2.804 - \frac{0.113}{(0.727)} PR1_{t-1} - \frac{0.106}{(0.037)} PR1_{t-2} - \frac{0.093}{(0.030)} PR1_{t-3}$$

$PR1_t$ and $PR5_t$ are the equity premia, calculated as the simple difference between stock and bond returns, over the 1-year, respectively, 5-year holding period starting at time t . $\hat{PR5}_t$ is the 5-year premium predicted or fitted from the model. According to (5), the premium on stocks over the next 5-year period can be predicted from the preceding three years' (known) realizations of 1-year equity premia. The coefficients are negative which suggests a significant mean-reverting component in stock prices. We use (5) as the proxy for the risk premium, i.e., $\gamma_t \equiv \hat{PR5}_t$ ⁷. Notice that from (1), the risk premium γ_t should be equal to the predicted premium on stocks so our proxy is consistent with the theoretical framework.

γ_t is plotted in Figure 3. The risk premium proxy is also highly volatile, in particular, towards the end of the sample. The large drop in the risk premium in the beginning of the 1980s partially coincides with the shift in the economic policy regime, cf. above. The large negative risk premia in the years 1983-1985 may possibly (to some extent) be explained by the

⁶ (5) is estimated according to a 'general-to-specific' procedure by, first, estimating a full model where the 5-year premium is regressed on all candidate predictor variables (bond returns, term structure components, past 1-year equity premia) and, subsequently, removing all insignificant predictors successively, using a 5% significance level. (5) is the resulting parsimonious model. All parameters are estimated by OLS while Newey-West standard errors which are consistent to heteroskedasticity and serial correlation in the disturbance term (up to lag 5) are used for standard errors of the coefficient estimates. The sample is the available period 1927-1992, using overlapping observations. (5) explains 36% ($=R^2$) of the variation in the actual 5-year equity premium. The residual has a standard deviation of 4.5%. Notice that we differ from Olesen and Risager (1999) by using absolute rather than logarithmic returns.

⁷ The predictions of (5) come close to those reported in Olesen and Risager (1999), in particular, as regards the significant movements and turning points in the 5-year equity premium.

introduction of the new pension fund tax on bond returns as we *ceteris paribus* should expect pension funds to demand a smaller before-tax 'risk' premium on stocks relative to bonds under the new tax regime. To see this, note that with the pension fund tax the no-arbitrage relation between stock and bond returns is modified from (1) to:

$$(1') \quad E_t[S_{t+1}] = (1-\tau)B_t + \gamma_t^*$$

where we have assumed that a pension fund is the (representative) marginal investor. τ is the pension fund tax on bond returns, $(1-\tau)B_t$ is the after-tax bond return and γ_t^* denotes the ('pure') risk premium on stocks. From (1'), the before-tax premium on stocks γ_t , i.e., the expected before-tax excess return on stocks $\gamma_t \equiv E_t(S_{t+1}) - B_t$, is related to the after-tax premium γ_t^* by $\gamma_t = -\tau B_t + \gamma_t^*$. Thus, the introduction of the pension fund tax *ceteris paribus* lowers the before-tax premium. For sufficiently high bond returns B_t - and bond returns were still high in the years 1983-1985 - we may even expect a negative before-tax premium.

Notice that we could, in principle, have constructed a proxy for the 'pure' risk premium γ_t^* if we had known the relevant tax rate τ for each year in the sample. However, constructing a data series for τ is impeded both by interim arrangements for the pension fund tax and by the fact that we need to know the relevant but latent 'marginal investor' (which we do not). These complications motivate our use of the 'before-tax' proxy γ_t .^{8 9}

⁸ We proceed by calling γ_t a 'risk premium' even though this is not entirely adequate in the presence of the pension fund tax. Thus, with the tax, γ_t captures both the 'true' risk premium γ_t^* and the distortionary tax effect $-\tau B_t$.

⁹ We can also introduce taxes in the theoretical framework of section 2. Using (1') instead of (1), the with-tax solution for the dividend-price ratio is:

$$(3') \quad \frac{D_t}{P_t} \approx (1-\tau)B_t - G_t + \gamma_t^* = B_t - G_t + \gamma_t$$

where the final equation follows from the relationship between γ_t and γ_t^* . (3') is actually identical to the without-tax solution in (3). Thus, in terms of the theoretical framework, the introduction of the pension fund tax does not matter, i.e., it does not change the solution (the structural equation) for dividend-price. The reason is that we include the before-tax 'risk premium' γ_t which fully incorporates the stock price effects of the new tax. Note, however, that the pension fund tax *ceteris paribus* lowers the level of dividend-price by lowering γ_t . Moreover, it is crucial for the result that the real interest rate and the risk premium have the same quantitative effects on dividend-price. Thus, allowing for taxes in the empirical model (4) (replacing R_t and γ_t with the after-tax real interest rate $(1-\tau)B_t - G_t$ and the 'true' risk premium γ_t^* , respectively, and rewriting) we get:

Both the real interest rate proxy and the risk premium proxy are negative in some of the years and the sum of the two proxies also turns out to be negative occasionally. The latter is obviously not consistent with the theoretical framework of section 2 which requires the discount rate $R_t + \gamma_t$ to be strictly positive in order to result in a well-defined (finite) forward-looking stock price solution. The estimation results to follow, however, suggest that market participants - in contrast with theory - expect a significant degree of mean reversion in the real interest rate and the risk premium so that negative values for the *current* real interest rate and the *current* risk premium may *a priori* be perfectly valid because it is expected to be a temporary phenomenon. Moreover, in terms of the empirical model (4), what matters are the variations in the real interest rate and the risk premium proxies over time (in which we may have more confidence) rather than the actual levels.

Finally, Figure 4 illustrates the log-level of real dividend payments. Dividends show some turbulence in the beginning and towards the end of the sample but have otherwise shown a steady declining trend.

4. Results Using a One-Regime Approach

Column 2 in Table 1 reports the results from estimating (4) over the entire sample 1927-1991, assuming that only one regime prevails. The estimation is performed by the Maximum Likelihood (ML) method under the assumption that the disturbance term of (4) is normal and independent distributed over time with homoskedastic variance ($\varepsilon_t \sim \text{Nid}(0, \sigma^2)$). The ML coefficient estimates are identical to those obtained by Ordinary Least Squares (OLS).

$$(4') \quad \frac{D_t}{P_t} = \beta_0 + \beta_1 R_t + \beta_2 \gamma_t + (\beta_2 - \beta_1)\tau B_t + \beta_3 DR_t + \beta_4 \frac{D_{t-1}}{P_{t-1}} + \varepsilon_t$$

The pension fund tax leaves the structural model unchanged iff the coefficients β_1 and β_2 are identical. If the coefficients differ, an additional explanatory variable τB_t , capturing the tax distortion, is introduced into the model. As the results will show, cf. below, the latter is the relevant case empirically and we should *a priori* expect a regime-shift in the empirical model at the time of the introduction of the new tax (because the additional variable is not included). Therefore, in a more general setting than (3), we can not be sure that the structural model for dividend-price will be unaffected by the pension fund tax and, hence, the question of whether or not the model survives becomes an empirical issue. The empirical results suggest that the inclusion of the pension fund tax in the risk premium construction γ_t does not sufficiently account for the effects of this tax on dividend-price, as we estimate a structural break in the model (a regime-shift) in the mid-1980s, cf. section 5 below.

< Table 1 >

Using the ML standard errors, all coefficients are highly significant. The real interest rate and the risk premium have the expected positive effects on the dividend-price ratio. The magnitudes are, however, less than predicted by theory. This applies both to the 'short run effects' (coefficients of 0.0345 and 0.1444, respectively, cf. Table 1) and the 'long run effects' (0.062 and 0.259, respectively)¹⁰, where the latter take account of the evident slow adjustment in the dividend-price process, cf. below. According to the theoretical framework of section 2, we should expect a coefficient of one for both variables but this value falls far above the point estimates and the deviations are much larger than what the uncertainty of the coefficient estimates allows for. When inspecting Figures 1 through 3, the result is not surprising as the variation intervals for the real interest rate and the risk premium are much larger than for the dividend-price ratio. This result suggests, tentatively, that market participants do not expect innovations in the two variables to be permanent, as assumed in the theoretical framework, but that they expect some significant degree of mean reversion in the real interest rate and the risk premium so that current realizations of the variables receive (relatively) less weight¹¹. The mean reversion feature seems perfectly reasonable from the time series behavior of the two variables, cf. Figures 2 and 3.

Real dividends also have a significant effect on dividend-price. The effect is positive, implying that an increase in real dividends gives rise to a less than proportional increase in real stock prices. This is, again, a deviation from theory and suggests that market participants do not consider innovations in dividends to be permanent but expect some degree of mean reversion.

¹⁰ By dividing through by one minus the autoregressive coefficient 0.4433 in the model of Table 1, the long-run equilibrium model for dividend-price becomes (ignoring the residual term):

$$\frac{D_t}{P_t} = -4.883 + 0.062 * R_t + 0.259 * \gamma_t + 2.308 * DR_t$$

¹¹ Of course, the result may also suggest that the proxies used for the real interest rate and the risk premium are too volatile. However, the high significance of the proxies validates their use.

Finally, the significance of lagged dividend-price indicates slow or partial adjustment in the dividend-price process to its long-run equilibrium.

Figure 5 illustrates the fit of the model. The one-regime model seems to work reasonably well and is, in particular, able to track the significant fall in dividend-price in 1983. There are, however, also episodes of systematic under- or overvaluation of dividend-price, see for instance the periods 1946-1956 and 1985-1991.

< Figure 5 >

The model passes the White and Lagrange Multiplier (LM) specification tests for serial correlation (at lag 1) and heteroskedasticity (ARCH) in the residual term, using conventional significance levels, see the bottom half of Table 1¹². There is, however, strong evidence of serial correlation at higher lags (AR(3) and AR(5)) leading to a rejection of the model. Note that the documented serial correlation implies that the coefficient estimates are inconsistent, given the presence of the lagged dependent dividend-price as a regressor. The coefficients should therefore be interpreted with caution.

Another severe problem with the model is that it is highly unstable over time. Figures 6-10 show recursive estimates of the model coefficients including 95% confidence bands, obtained by recursive least squares. With the exception of the risk premium, the coefficients are very unstable and there is a strong indication of a structural break in the model both in the beginning and towards the end of the sample.

< Figures 6-10 >

The apparent instability of the model can be further documented by formal testing. The Andrews test, see Table 1, allows one to perform a test for structural break without having to

¹² The tests are documented in Hamilton (1996). We use the suggested small-sample versions of the tests whereby the asymptotic test is transformed to a small-sample test based on the F-distribution. The tests for autocorrelation are tests for an AR(1) process in the residual term.

pre-specify a candidate time for a breakpoint, see Andrews (1993) and Hamilton (1996) for details. The Andrews test procedure basically performs a LM test for a shift in the mean at each point in the sample, except for the first 15% and the last 15% of the observations. One then chooses the observation with the highest LM test value and compares with critical test values, as tabulated in Andrews (1993). The evidence for the one-regime model is a clear indication of a (at least one) structural break in the sample. Hence, the Andrews (1993) test statistic is 23 which should be compared to critical values of 8.85 (5% significance level) and 12.35 (1%), that is, a clear rejection of the null hypothesis of no structural breaks. The test statistic is obtained for the year 1947¹³.

To conclude, the estimation results suggest that we have identified economic fundamentals variables which have power in explaining the variations in the dividend-price ratio, including the large fall in 1983. There are, however, specification problems with the model and there is, in particular, strong evidence that the one-regime model is unstable over time, suggesting that more than one regime applies over the sample period.

5. Results Using a Regime-Switching Approach

Motivated by the evident instability of the economic model in (4), we estimate a version of the model which allows for more than one regime. A regime is here defined as a subperiod (or several subperiods) over which (4) is stable, i.e., over which the coefficients of the economic fundamentals (including the constant term) and the explanatory power of the model (as measured by the residual variance) are constant. A regime shift takes place whenever the underlying structural framework for dividend-price changes either because of a change in the impacts of the various fundamentals or because of a change in the non-explained part of the volatility in dividend-price. In other words, a regime shift can be interpreted as a structural break in the underlying economic model. We incorporate the possibility of multiple regimes by using the Markovian regime-switching model developed by Hamilton (1990). This approach has the advantage of letting the data - as opposed to *a priori* information - determine whether there is more than one regime and, if affirmative, when the regime shifts take place.

¹³ The individual LM test values over the sample are reported in the Appendix.

In order to keep the model as simple as possible, we only allow for two regimes from the outset and, subsequently, test whether two regimes are sufficient to eliminate the structural breaks in (4).

In the regime-switching approach, the economy can at each point of time be in one of two possible states, as indexed by an unobservable state-variable s_t which takes on the values 1 or 2¹⁴. Each regime is described by a distinct model for the dividend-price ratio:

(6)

$$\frac{D_t}{P_t} = \beta_0(s_t) + \beta_1(s_t) * R_t + \beta_2(s_t) * \gamma_t + \beta_3(s_t) * DR_t + \beta_4(s_t) * \frac{D_{t-1}}{P_{t-1}} + \sigma(s_t) * \varepsilon_t, \quad s_t = 1, 2$$

where the parameters depend on the prevailing state s_t . (6) is identical to (4) except for the state-dependence so that the underlying economic framework is fundamentally unchanged. The crucial difference in (6) is that we here operate with (possibly) two distinct models which differ wrt. parameters, i.e., the coefficients $\beta_i(s_t)$ and the residual variance $\sigma(s_t)^2$.

Which model applies at a given point of time is governed by the state-variable s_t . s_t is stochastic and is assumed to follow a two-state Markov Chain with constant transition probabilities p_{ij} , where the latter is defined as the probability that the state (or regime) is j in period t conditional on the state i in period $t-1$, i.e., $p_{ij} = \Pr\{s_t = j | s_{t-1} = i\}$ ($i, j = 1, 2$). s_t is by assumption independent of the residual term ε_t across all time periods, so that the state process is exogenous to the dynamics of dividend-price.

Under the assumption that ε_t is independent standard normal ($\varepsilon_t \sim \text{Nid}(0,1)$), we can estimate (6) by the method of Maximum Likelihood (ML), see Hamilton (1994, Section 22). The results are reported in columns 3 and 4 of Table 1¹⁵.

¹⁴ For a detailed outline of the regime-switching model including the statistical foundations, we refer to Hamilton (1990), Hamilton (1996) or the textbook Hamilton (1994, Section 22). Numerous applications of the model can be found, including those in Driffill and Sola (1998), Engel and Hamilton (1990) and Hamilton and Lin (1996).

¹⁵ We have performed the estimation under the assumption that the state probabilities of the initial observation are given by the ergodic probabilities. Including an estimation of the initial probabilities does not change the

First of all, we note that all coefficients have the expected signs. In regime #2, all coefficients can be shown to be significant at the 1% significance level, whereas the real interest rate and lagged dividend-price turn out to be insignificant in regime #1¹⁶. The two remaining variables (the risk premium and real dividends) are highly significant in regime #1. In fact, for the risk premium and real dividends, the coefficient estimates do not differ much across regimes. The results suggest that we have two regime-dependent, underlying models for dividend-price, one in which there is partial adjustment in dividend-price and where both the real interest rate, the risk premium and real dividends matter (regime #2), and one in which there is an immediate adjustment and where only the risk premium and real dividends are important (regime #1). Because the estimated residual variance is markedly higher in regime #2 than in regime #1, the uncertainty attached to the model's fit is larger in the former regime (despite the model having more significant explanatory variables in this regime)¹⁷.

The autoregressive term in the dividend-price model, reflecting partial adjustment, implies that the impact of the fundamentals variables is (slightly) stronger in the long term than in the short term. This difference between the two horizons is most pronounced for regime #2 where the autoregressive term has the largest coefficient and the adjustment to long run equilibrium, hence, is the slowest. The long run equilibrium relation pertaining to each

results significantly. The computations are performed with the BFGS algorithm in GAUSS using a variety of different starting values. We identify more than one local maximum (a total of 5) depending on the starting values and, moreover, encounter a singularity problem of the likelihood function, that is, for certain starting values the likelihood becomes 'large' without convergence as one of the regime-dependent variances goes to zero. The results of Table 1 apply to the local maximum with the highest likelihood. This choice is consistent with Kiefer (1978) who shows for the mixed-distribution model - where a global maximum does not exist - that there is a bounded local maximum of the likelihood function (with variances being positive) which exhibits the usual maximum likelihood properties of being consistent and asymptotically efficient.

¹⁶ A Likelihood Ratio test of the joint hypothesis that the real interest rate and lagged dividend-price are insignificant in regime #1 gives a test statistic of 3.4 with two degrees of freedom, corresponding to a critical significance value of 18.4%. Hence, the hypothesis is accepted at conventional significance levels. We keep the two variables in the model because the resulting parsimonious model fails the specification tests.

¹⁷ From a probabilistic inference, cf. below, we can estimate regime #2 to have reigned over the period 1950-1985. Using the model's overall fit (see equation (8) below), the coefficient of determination (R^2) over this subperiod is 81%. This is considerably lower than the R^2 of 96% in the remaining periods 1927-1949 and 1986-1991 (regime #1), indicating a lower explanatory power for the model in regime #2. Over the whole sample, the R^2 is 91%. Notice that the dating of regimes is not certain so the differences should be interpreted with caution.

regime can be calculated from Table 1 by dividing through by one minus the autoregressive coefficient. This leads to (ignoring the error term)¹⁸:

$$(7) \quad \begin{aligned} \frac{D_t}{P_t} &= -6.562 + 0.019 * R_t + 0.158 * \gamma_t + 2.676 * DR_t & (\text{regime \#1}) \\ \frac{D_t}{P_t} &= -6.319 + 0.094 * R_t + 0.199 * \gamma_t + 2.787 * DR_t & (\text{regime \#2}) \end{aligned}$$

The main difference between the two regimes is the real interest rate impact which is insignificant in regime #1. The impact of the risk premium is also somewhat larger in regime #2, whereas the coefficients to real dividends (and the constant terms) are almost equal across regimes.

As was also the case for the one-regime model, both the real interest rate and the risk premium have smaller effects than expected *a priori*, that is, the coefficients are less than one. In regime #2, a permanent increase in the real interest rate by 1 percentage point will *ceteris paribus* lead to an increase in the (expected) dividend-price ratio by 0.09 percentage point in the long run. Thus, only 9% (rather than the 100% implied by the theoretical framework) of the change in the real interest rate shows up in the dividend-price ratio. The impact of the risk premium is somewhat higher, as an increase in the premium by 1 percentage point raises the expected long-run dividend-price ratio by 0.16 percentage point in regime #1 (an impact of 16%) and by roughly the same magnitude in regime #2 (20%). A possible explanation is, like before, that market participants expect a significant part of the shocks to the two variables to be transitory. For the real interest rate, this may in particular be true in regime #1 (covering the subperiods 1927-1949 and 1986-1991, cf. below) where the real interest rate is subject to very large fluctuations, implying that a relatively large portion of the variation in the current real interest rate is transitory (see Figure 2). This could potentially explain the low and insignificant impact of the real interest rate in regime #1.

¹⁸ As shown in Nielsen and Olesen (1999), the computation of the regime-dependent mean $E[(D/P)_t | s_t]$ is complicated when allowing for an autoregressive dependent term. (7) should therefore correctly be interpreted as the expected dividend-price ratio conditional on being in regime #1, respectively regime #2 *both in the current and previous period*, i.e., as $E[(D/P)_t | s_t = s_{t-1} = i]$ ($i=1,2$). However, this mean will come close to that of $E[(D/P)_t | s_t]$ whenever the regimes are persistent, which turns out to be the case for our model.

The level of real dividends has a significant positive impact on dividend-price so that stock prices appear to underreact to shocks to dividends, as compared to theory. A prime candidate for explaining this feature is, again, that shocks to dividends are expected, to some extent, to be transitory. Because we measure dividends in log-levels, the coefficients can be interpreted as semielasticities. From (7), a permanent 1% relative increase in real dividends will in the long run lead to an increase in the (expected) dividend-price ratio by approximately 0.03 percentage point in both regimes.

< Figures 11 and 12 >

The fit of the regime-switching model is illustrated in Figure 11, while Figure 12 shows the standardized residuals. Both the fit and the residuals are calculated using the filtered probabilities for the latent state variable s_t . The fitted (or expected) dividend-price ratio is calculated across regimes as¹⁹:

$$(8) \quad E\left[\frac{D_t}{P_t}\right] = E\left[\frac{D_t}{P_t} | s_t = 1\right] * p_t^1 + E\left[\frac{D_t}{P_t} | s_t = 2\right] * p_t^2$$

$p_t^i \equiv \Pr(s_t = i | I_T)$ ($i=1,2$) is the filtered probability of state i at time t , conditional on the information set I_T which contains all available information on observables (including dividend-price) in the sample, cf. Hamilton (1994, Section 22). The state-conditioned means $E[\cdot | s_t]$ follow immediately from (6), using the fact that the residual term has a zero mean. The variance of dividend-price around its fitted value, $\text{VAR}(D_t/P_t) \equiv E[D_t/P_t - E(D_t/P_t)]^2$, can be derived by using a formula similar to (8) for the second moment $E(D_t/P_t)^2$ and exploiting the fact that $E((D_t/P_t)^2 | s_t) \equiv \sigma(s_t)^2 + (E(D_t/P_t | s_t))^2$ (by the definition of variances). Subtracting the term $(E(D_t/P_t))^2$ (follows from (8)), then gives the variance. The result is:

¹⁹ All moments in (8) and (9) and the following derivations are conditioned on the information set containing the past and current levels of the explanatory variables (including lagged dividend-price) as of period t (omitted for notational convenience).

$$(9) \quad \text{VAR}\left(\frac{D_t}{P_t}\right) = p_t^1 \sigma(1)^2 + p_t^2 \sigma(2)^2 + p_t^1 p_t^2 \left(E\left[\frac{D_t}{P_t} | s_t = 1\right] - E\left[\frac{D_t}{P_t} | s_t = 2\right]\right)^2$$

The uncertainty of dividend-price is a result of both the unknown error term (captured by the first two terms in (9)) and the uncertainty arising from the fact that the state is unknown and the state-dependent means differ (the last term in (9)). The standardized residual which is a point estimate of the error term ε_t in (6) can, finally, be calculated as the difference between actual and fitted dividend-price, divided by the standard error of dividend-price (the square root of (9)).

Figures 11 and 12 show that the model captures the significant movements in dividend-price over most of the sample and, in particular, performs well in the beginning and towards the end of the sample. Like the one-regime model, the regime-switching model tracks the significant fall in dividend-price in 1983. Less appealing features are that the 1974 observation seems to be an outlier and that there are two subperiods (1947-1955 and 1958-1968) over which the model systematically underestimates, respectively, overestimates actual dividend-price.

The specification tests of Table 1 (bottom half) test whether the residual term of (6) is serially uncorrelated, homoskedastic and normal distributed. The tests reveal no misspecification at the standard 5% significance level, except for the LM test for ARCH in regime #2. However, using an alternative small sample correction to that used in Table 1, the test for ARCH in regime #2 is (just) passed²⁰. Notice that the tests for serial correlation, including the tests for serial correlation within the two regimes, are passed so that the tendency to a systematic under- and overestimation as noted above is not statistically significant. The Andrews (1993) test for structural break gives a test statistic close to its critical value at the 5% significance

²⁰ Hamilton (1996) suggests two possible small sample corrections to the asymptotic Likelihood Ratio (LR) test; either to transform the test to a small sample version based on the F-distribution (which is the one used in Table 1) or to use a 1% significance level for the LR test. According to Monte Carlo simulations, both help in correcting for an 'over-size' of the specification tests (that is, the tests tend to indicate misspecification too often) in small samples. For the test for ARCH in regime #2, the LR test statistic which is asymptotically χ^2 -distributed with 1 degree of freedom, is 6.17. The critical significance level is 1.3%. Hence, the null of no ARCH in regime #2 is (just) accepted at the 1% significance level.

level. Hamilton (1996) suggests that a 1% significance level is used for this test in small samples to correct for a possible 'over-size', i.e., a tendency to indicate structural breaks too often in small samples. Using a 1% significance level, the test is passed with a comfortable margin, that is, we accept the null that the model has no structural breaks²¹.

We conclude that the regime-switching model is, overall, well specified. In particular, the model performs well in regime #1. The Andrews test suggests that two regimes are sufficient to remove the structural breaks in the one-regime model.

According to the estimated transition probabilities, cf. Table 1, both regimes are highly persistent with the probability of continuing in a given regime being 96-97%. The state variable s_t is unobservable, but it is possible from the estimated transition probabilities and the estimated regime-dependent models to draw a probabilistic inference about the state for each year in the sample. This inference is expressed by the filtered state probability, that is, the probability of the economy being in (say) regime #1 in year t , conditional on all available sample information on observables (dividend-price and the economic fundamentals), cf. also above. The estimated filtered probabilities, expressed as the probability of regime #1, are shown in Figure 13. This plot confirms that the regimes are highly persistent. Furthermore, it gives a very clear inference about the state variable, suggesting that we can divide the sample into three distinct subperiods (using the 50% probability value as the dividing line between subperiods); 1927-1949, where the model of regime #1 governed the dividend-price process; 1950-1985 (regime #2), and 1986-1991 (regime #1). The identification of regimes corresponds quite well with the recursive plots of Figures 6-10 which, tentatively, indicate that there are two regime-shifts in the sample, one in the beginning and one towards the end.

< Figure 13 >

A further understanding of the two regimes can be facilitated by inspecting Figure 14 which shows the fit of each of the two regime-dependent models together with the actual dividend-

²¹ The individual LM statistics used in the Andrews test are reported in the Appendix. The test value is obtained for the year 1969.

price ratio. It is evident that the model of regime #1 systematically predicts a lower dividend-price ratio than that of regime #2 (over the sample, the average difference is 1.1 percentage point). This suggests that regime #1 (#2) is one with a low (high) dividend-price ratio and - correspondingly - a high (low) level of stock prices, taking due account of the underlying economic fundamentals. The recent period from 1986 where regime #1 has reigned could therefore be interpreted as a period with relatively high stock prices (even when taking account of fundamentals). The identification of regimes suggests that this period resembles the subperiod 1927-1949 in the beginning of the sample. The long intervening period from 1950 to 1985 has, on the other hand, been characterized by relatively low stock prices.

< Figure 14 >

The results from estimating the regime-switching model leads to the conclusion that the underlying economic model was subject to a structural break in both 1950 and 1986. The evidence that a regime-shift towards a lower dividend-price ratio occurred in the 1980s seems plausible given the large changes in the Danish economy in that period, cf. section 3. The timing of the regime-shift (1986) may be slightly surprising because the large adjustment in dividend-price as well as the structural changes took place in 1983. Thus, the economic model is able to explain the significant fall in dividend-price and the underlying increase in stock prices in 1983 without referring to a regime-shift. Using the estimated coefficients for the prevailing regime #2, the prime factor in explaining this event is the huge fall in the risk premium by about 11 percentage point that year which in itself explains a decline in dividend-price by 1.7 percentage point²². Moreover, a fall in the real interest rate (by nearly 12 percentage points) and real dividends (by 30% in relative terms) contribute an estimated 0.9 and 0.6 percentage point, respectively, to the decline in dividend-price. The regime-shift instead occurs in 1986. This shift is needed in order to explain why the dividend-price ratio remains low despite a reversal in the real interest rate, the risk premium and real dividends

²² Recall that this large fall in the premium is partially motivated by the introduction of the new pension fund tax, cf. section 3, so that this particular variable incorporates one of the big structural changes in 1983. Also notice that within the regime-switching model, the pension fund tax may have induced a decline in the dividend-price ratio via two channels; by lowering the before-tax (risk) premium γ_t and, potentially, by triggering the shift to the 'low' dividend-price regime after 1986.

towards the levels prevailing before 1983. Tentatively, this lagged regime-shift might be consistent with the gradual phasing in of the pension fund tax. As a more general insight, the timing of the regime-shift in 1986 rather than 1983 highlights the importance of taking due account of underlying economic fundamentals when estimating whether or not a regime-shift has taken place.

The result that a regime-shift occurred in 1950 and that the pre-1950 regime should resemble that of the post-1986 period is harder to explain and a closer examination is needed.

It is evident from Table 1 that the allowance for two regimes significantly alters the estimated coefficients. The one-regime model does not come close to any of the regime-dependent models and we, in particular, encounter differences for the coefficients of real dividends and lagged dividend-price. The regime-switching model is better than the one-regime model in terms of fit (as measured by the likelihood or the estimated residual variances) which is no surprise as the regime-switching model contains more parameters. However, even after correcting for the number of parameters the regime-switching model seems superior. Table 1 shows the values of three information criteria often used as the basis for model selection; the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ) and the Schwarz criterion (SC). According to the first two, the regime-switching model is the preferable one, while the SC does not give a clear answer.

There are two more evident reasons for choosing the regime-switching model. First of all, the allowance for two regimes solves a clear problem with structural breaks in the one-regime model. Second, within the context of the regime-switching model, a one-regime model is valid if and only if the two regime-dependent models do not differ in any statistically significant way. This hypothesis can be put to a formal test by, for instance, testing whether all coefficients (including the constant term) are identical across the two regimes²³. The

²³ As noted by Hamilton (1990), it is not possible to perform a Likelihood Ratio (LR) test of the more adequate hypothesis that all parameters including the variances are identical across regimes. The reason is that the asymptotic information matrix becomes singular under the null because the transition probabilities can not be identified in the case of two identical regime-dependent models. This is a violation of one of the standard regularity conditions underlying the LR test.

Likelihood Ratio test of this null hypothesis gives a test statistic of 29.7 with 5 degrees of freedom, which corresponds to a critical significance level of 0.00%. Hence, the null is clearly rejected. We can conclude that there are two distinct regimes in the data and the evidence in favor of the regime-switching model is strong.

6. Conclusion

We have estimated a fundamentals-based economic model for the dividend-price ratio. Results show that our proxies for the growth-adjusted real interest rate and the risk premium on stocks are significant in explaining dividend-price empirically. This identification of a time-varying discount rate which is useful for empirical modeling is the main contribution of this paper. The existing empirical literature on modeling stock price behavior often ignores the time variation in the discount rate by assuming it to be constant. The estimated coefficients of the real interest rate and the risk premium are significantly less than one, the value predicted by a Gordon-type theoretical model where all innovations in the two variables are expected by market participants to be permanent. This result suggests that the innovations are considered to be partially transitory. Lagged dividend-price and the level of real dividends are also important explanatory variables. The former captures slow adjustment in the dividend-price process while the significance of real dividends (with a positive coefficient) shows that stock prices tend to respond less than proportionately to dividend shocks. The latter may, again, reflect that market participants expect some of the shocks to dividends to be transitory.

We estimate the economic model using both a one-regime and a regime-switching approach. The latter is used to account for non-modeled changes in the exogenous 'economic environment', leading to structural changes in the economic model. Results show that it is important to allow for more than one regime over the sample in order to avoid structural breaks. Two regimes seem to suffice. The regimes correspond to two distinct versions of the economic model which differ wrt. the relative importance of the fundamentals variables. A main difference is that the real interest rate (proxy) is only significant in one of the regimes.

The two regimes also differ wrt. the level of the dividend-price ratio as one of the regimes has a systematically higher level for dividend-price than the other over the sample period considered. One way to interpret the two regimes is therefore to distinguish them as a 'low-dividend-price' regime (corresponding to a high level of stock prices) and a 'high-dividend-price' regime (low stock prices). The terms 'high' and 'low' are to be used within the context of the economic model which explicitly takes account of the underlying economic fundamentals. Results clearly identify three distinct subperiods (1927-1949, 1950-1985 and 1986-1991) in which the regimes (submodels) apply. The high persistence of the regimes gives plausibility to the working hypothesis that structural changes in the economy account for the shifts in the underlying economic model. The latest regime-shift in 1986 may possibly be explained by the gradual phasing in of a new separate tax on the returns on pension funds' bond holdings, initiated in 1983.

Related literature is Driffill and Sola (1998) who estimate a two-state regime-switching model for the US stock market over the period 1900-1987, using the price-dividend ratio as the endogenous variable. Within the context of the standard Gordon model, they motivate the existence of two distinct states for price-dividend by the existence of two states of the underlying dividend process - a 'low-growth-and-high-variance' state and a 'high-growth-and-low-variance' state, respectively. These states result in two different fundamental solutions for the price-dividend ratio. Driffill and Sola (1998) find evidence of two states being present in the processes for dividends and price-dividend. Furthermore, they find that the allowance for two regimes leads to a significant improvement on the one-regime model, in particular, in terms of fit. Driffill and Sola (1998) also test for intrinsic bubbles in stock prices, as originally proposed by Froot and Obstfeld (1991), by allowing for the level of real dividends to explain price-dividend. Even though they cannot formally reject the existence of intrinsic bubbles, they conclude, based on the explanatory power of the models, that the inclusion of intrinsic bubbles is not important when first having allowed for two different regimes.

Our analysis differs from Driffill and Sola (1998) by using economic fundamentals, in particular, a time-varying real interest rate and a time-varying risk premium, in explaining

dividend-price. Driffill and Sola (1998) focus exclusively on the regime-switching element, assuming a constant discount rate²⁴. Our motivation for using the regime-switching approach is *ad hoc* in the sense that we do not provide a specific account of the causes of the regime-shifts. Driffill and Sola (1998), on the other hand, have a more firm theoretical foundation for the existence of distinct states in the pricing process, which is based on the existence of distinct states in the (exogenous) underlying dividend process.

The significance of real dividends in our analysis could - as in Driffill and Sola (1998) and Froot and Obstfeld (1991) - be suggestive of intrinsic bubbles in stock prices. However, this conclusion is only valid if certain restrictions on the parameters of the dividends and price processes are fulfilled, cf. Driffill and Sola (1998) and Froot and Obstfeld (1991). These have not been tested in the present paper.

The model we estimate provides a good fit to dividend-price, is overall well specified and does in particular work well in regime #1 (the periods 1927-1949 and 1986-1991). The model is not entirely satisfactory over subperiods in the middle of the sample (concentrated in regime #2) where we encounter a tendency to under-, respectively, overestimate dividend-price. The latter is a point where the model may be improved upon. Even though two regimes suffice according to formal testing, one possibility would be to allow for three regimes as the 'problematic' subperiods may be suggestive of an additional regime. However, allowance for a third regime increases the number of parameters to be estimated significantly (by 10).

The regime-switching model identifies regime-shifts in 1950 and 1986. An obvious but also challenging issue for future research is to identify the causes of the regime-shifts and, if possible, formally incorporate these as additional explanatory variables in the model. We have conjectured that the introduction of a new pension fund tax is a possible explanation of the regime-shift in 1986. By incorporating taxation in the economic model, the validity of this conjecture can be tested. Moreover, it would allow us to test whether changes in taxation also can account for the regime-shift in 1950. If so (and taxation is the sole explanation of the

²⁴ We should add that the approach of Driffill and Sola (1998) is not applicable for Denmark as there is no evidence of distinct states in the process for dividends.

regime-shifts), the inclusion of taxation would remove the structural breaks from the underlying economic model.

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Figure 1. Dividend-Price Ratio 1927-1996

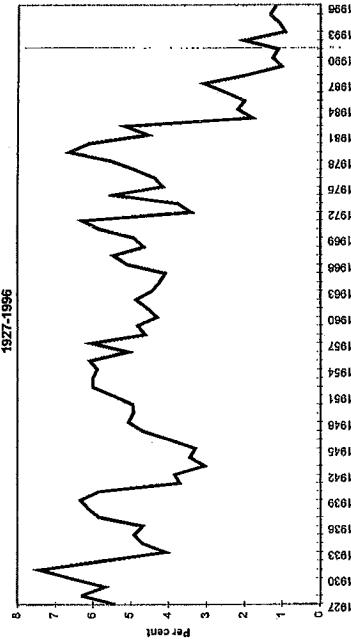


Figure 2. Growth-Adjusted Real Interest Rate 1927-1991

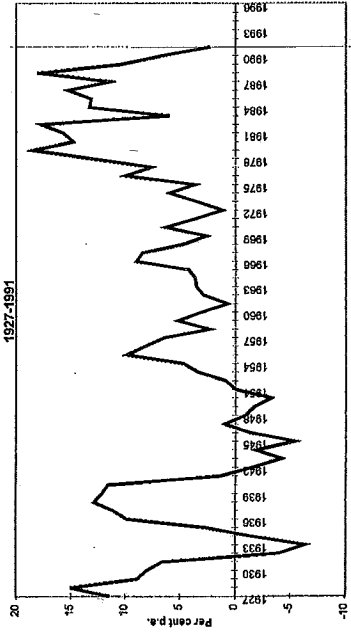


Figure 3. Risk Premium on Stocks 1927-1992

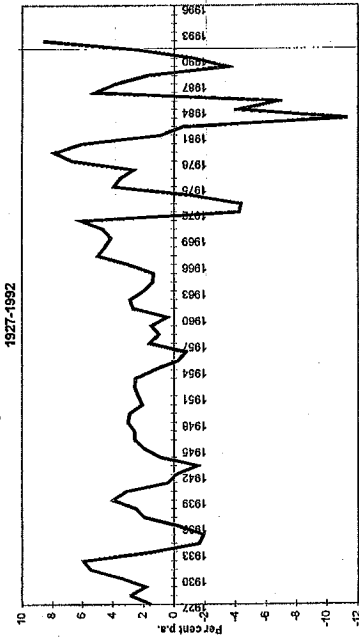


Figure 4. (Log-) Real Dividend Payments Per Share 1927-1996

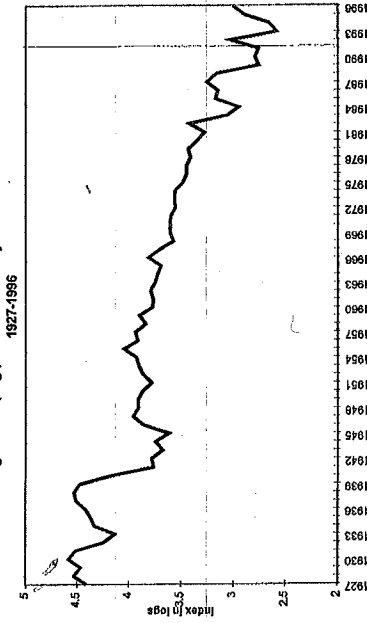
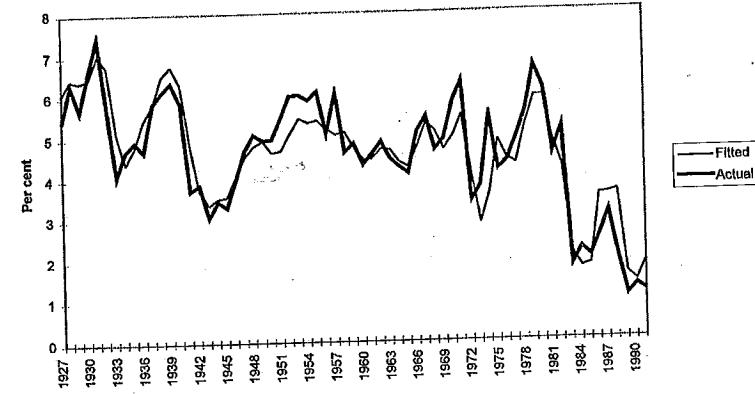


Figure 5. Dividend-Price Ratio: Actual and Fitted One-Regime Model



Note: Vertical line marks end of estimation sample (1991).

Figures 6-10. Recursive Parameter Estimates for One-Regime Model

Recursive least squares point estimates (bold line) and 95% confidence band limits, 1942-1991. Sample start 1927.

Figure 6. Constant term

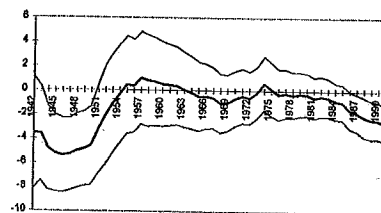


Figure 7. Real Interest Rate

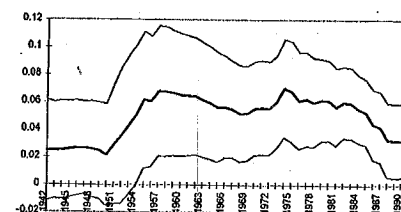


Figure 8. Risk Premium

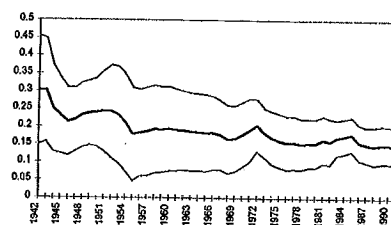


Figure 9. Real Dividends

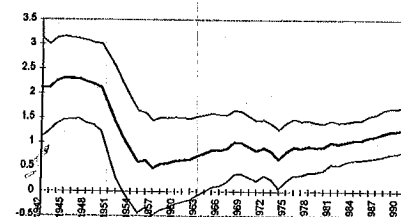
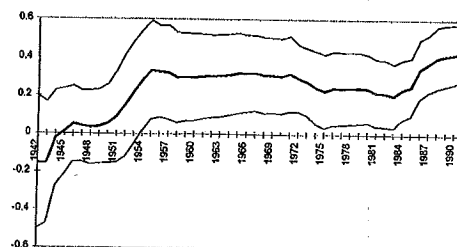
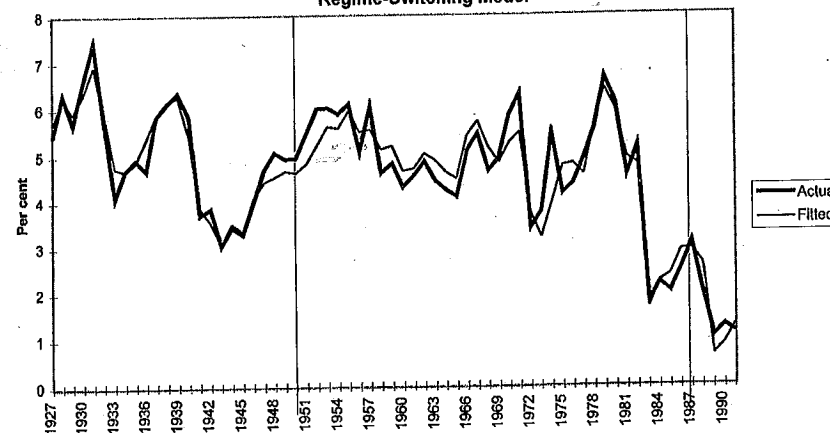


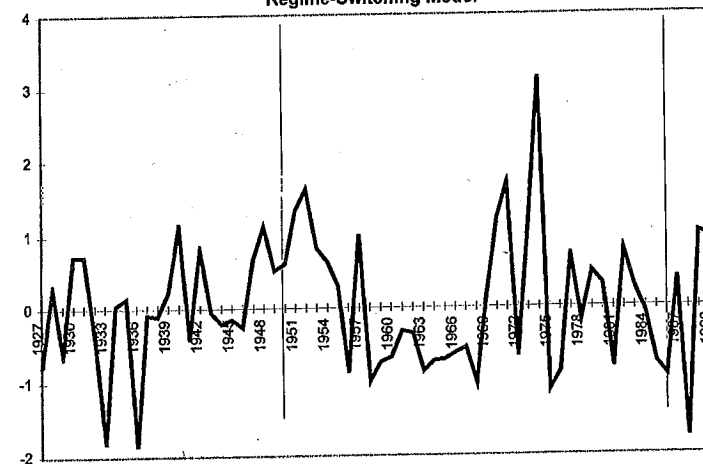
Figure 10. Lagged Dividend-Price Ratio



**Figure 11. Dividend-Price Ratio: Actual and Fitted
Regime-Switching Model**

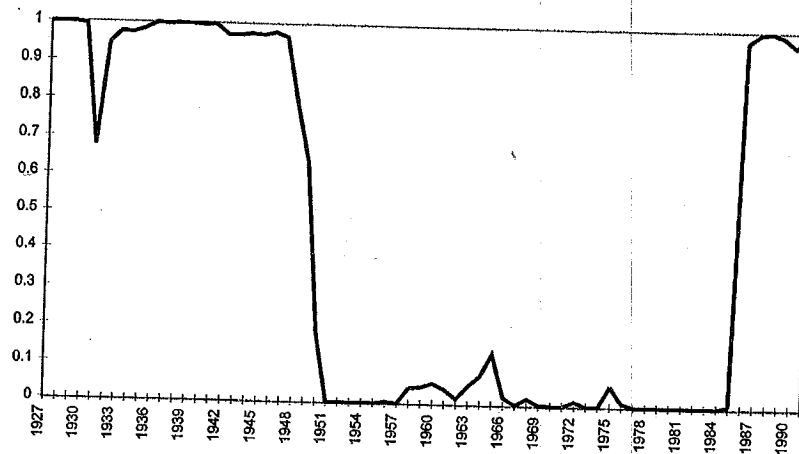
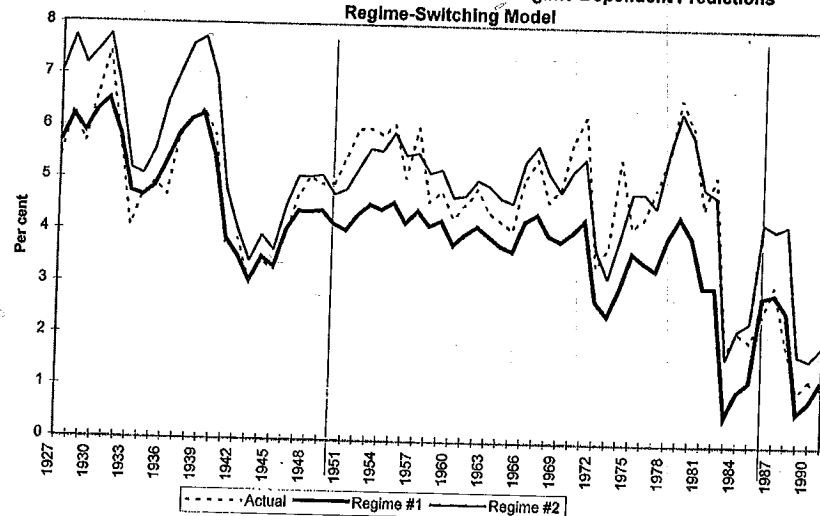


**Figure 12. Standardized Residuals
Regime-Switching Model**



Note: Vertical lines indicate datings of regime #1 (1927-1949 and 1986-1991) and regime #2 (1950-1985).

Figure 13. Filtered Probabilities for Regime #1

Figure 14. Dividend-Price Ratio: Actual and Regime-Dependent Predictions
Regime-Switching Model

Note: Vertical lines indicate datings of regime #1 (1927-1949 and 1986-1991) and regime #2 (1950-1985).

Table 1. Maximum Likelihood Estimates and Specification Testing:
Models with and without Regime-Switching

		One-Regime Model	Regime-Switching Model	
<i>Parameter estimates</i>			Regime #1	Regime #2
Constant term	β_0	-2.7186 ** (0.6976)	-5.8118 ** (0.6552)	-4.8685 ** (1.615)
Real interest rate	β_1	0.0345 ** (0.0127)	0.0172 (0.0112)	0.0725 ** (0.0195)
Risk premium	β_2	0.1444 ** (0.0250)	0.1399 ** (0.0418)	0.1535 ** (0.0242)
Real dividends	β_3	1.2848 ** (0.2202)	2.3701 ** (0.2414)	2.1473 ** (0.4386)
Lagged D/P	β_4	0.4433 ** (0.0728)	0.1143 (0.0876)	0.2456 ** (0.0913)
Variance	σ^2	0.3719 (0.0652)	0.1255 (0.0366)	0.2296 (0.0525)
Transition probability	p_{ii}	—	0.9740 (0.0291)	0.9626 (0.0295)
Ergodic probability		—	0.5901	0.4099
<i>Log-likelihood</i>		-60.0815	-43.7682	
AIC		132.2	115.5	
HQ		137.3	127.6	
SC		145.2	146.0	
<i>White specification test</i> ¹⁾				
Autocorrelation	F(4,51)	2.737 (0.103)	2.333 (0.068)	
ARCH	F(4,51)	1.970 (0.166)	1.834 (0.137)	
Markov specification	F(4,51)	—	2.199 (0.082)	
<i>LM specification test</i> ¹⁾				
Autocorr. regime #1	F(1,51)	—	0.529 (0.470)	
Autocorr. regime #2	F(1,51)	—	2.659 (0.109)	
Autocorrelation	F(1,51)	2.732 (0.104)	1.249 (0.269)	
ARCH regime #1	F(1,51)	—	2.493 (0.121)	
ARCH regime #2	F(1,51)	—	4.844 (0.032) *	
ARCH	F(1,51)	1.913 (0.172)	0.287 (0.595)	
<i>Standardized residuals</i> ^{1) 2)}				
AR(1)	F(1,63)	3.106 (0.083)	0.863 (0.356)	
AR(3)	F(3,61)	5.519 (0.002) **	2.402 (0.076)	
AR(5)	F(5,59)	4.543 (0.002) **	1.403 (0.237)	
Normality	$\chi^2(2)$	2.810 (0.245)	3.775 (0.151)	
<i>Andrews test for structural break</i> ³⁾				
		23.009 **	8.964 *	

Note: Asymptotic standard errors of parameter estimates shown in parentheses, based on second derivatives of log likelihood function. A ** shows significance at the 5% level, *** at 1% level. The Akaike, Schwarz and Hannan-Quinn model selection criteria are calculated as: $AIC = -2l + 2k$, $HQ = -2l + 2k \ln(T)/k$, and $SC = -2l + k \ln(T)$, where l is the log-likelihood value, k is the number of freely estimated parameters and T is the number of observations.

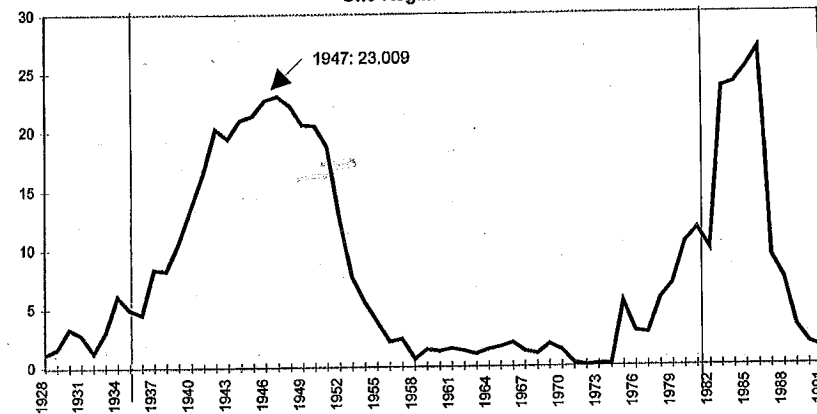
1) Test distributions apply to regime-switching model. For one-regime model, White and Lagrange Multiplier (LM) tests are distributed $F(1,59)$. Tests are small-sample approximations based on the F-distribution, as suggested by Hamilton (1996). Critical significance levels in parentheses. The White and LM tests are described in Hamilton (1996).

2) For regime-switching model, the serial correlation (AR) tests are standard LM specification tests applied to a regression of the standardized residuals on a constant term. For one-regime model, standard LM tests on the regression equation. Normality test by Doornik and Hansen (1994).

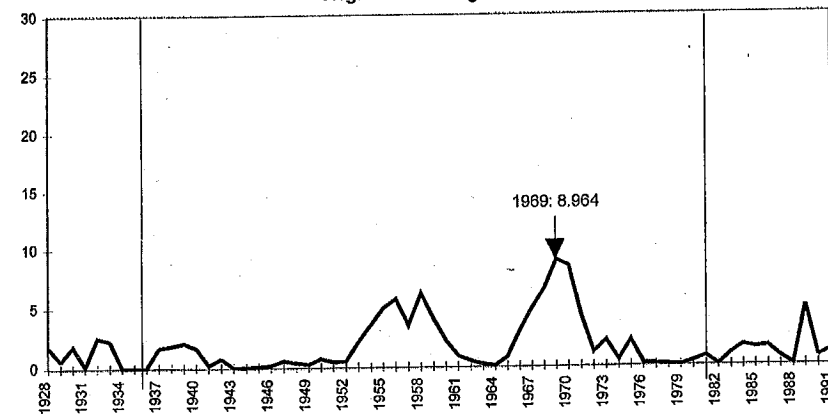
3) Asymptotic critical test values are 8.85 (5% significance level) and 12.35 (1%), see Andrews (1993).

Appendix:
Andrews (1993) Tests for Structural Break

**Figure A1. Individual LM Statistics Used in Andrews Test
One-Regime Model**



**Figure A2. Individual LM Statistics Used in Andrews Test
Regime-Switching Model**



Note: Only observations between 1936 and 1981 (both years included) are used in the Andrews test. Period indicated by vertical lines.

Chapter 6

A Simple Explanation of Stock Price Behavior in the Long Run:

Evidence for Denmark

A Simple Explanation of Stock Price Behavior in the Long Run:

Evidence for Denmark *

by

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Abstract

Using Danish data for the post-World War II-period, we estimate a simple model for the long-run behavior of stock prices. We find a stable and strong cointegrating relation between stock prices and two macroeconomic "fundamentals" variables, firm profits and the nominal bond rate. Both "fundamentals" are highly significant. Growth in profits drives the long-run trend in stock prices while the bond rate explains the observed large deviations from trend growth. The behavior of the bond rate accounts for the evident split of the Danish stock market into a bearish period before the early 1980s and a subsequent bullish period. Likewise, a decline in the bond rate explains a major part of the large capital gains realized in recent years.

* We appreciate comments from Lisbeth Funding la Cour, Steen Nielsen and Ole Risager, Copenhagen Business School.

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1. Introduction

Like most stock markets in the OECD area, the Danish market experienced large capital gains in 1996 and 1997. In total, the increase in stock prices amounted to 84%. These bullish stock markets have stimulated interest in the behavior of stock prices both among academics and practitioners and the interest has, in particular, focused on the question whether stock markets are currently overvalued, see Campbell and Shiller (1998), Cochrane (1997), Kopcke (1997) and Cole *et. al* (1996) for just a few examples.

When examining the recent experience in a historical perspective, it is evident that the stock market developments in 1996 and 1997 are remarkable. However, they are not without precedents, see Figure 1 which shows the Danish All-Share Stock Price Index in the post-World War II-period. The Danish stock market has in three previous episodes experienced capital gains of a similar or even bigger magnitude, that is, in 1972 (89%), 1980-1983 (276%, with a capital gain of 114% in 1983 alone) and 1988-1989 (99%)¹. It is also interesting to note that the increases in 1996 and 1997 follow a period of several years where stock prices have effectively moved in a horizontal direction.

< Insert Figure 1 around here >

Moreover, 1996 and 1997 may be viewed as merely the most recent part of a long-lasting bull market commencing in the early 1980s. Over the subperiod from 1980 to 1998, stock prices have increased at a trend growth rate of 10.3% per year. This is much faster than over the preceding years 1948-1980 where the trend in stock prices showed a growth rate of 4.0% per year, see Figure 1 where the straight lines indicate the trend growth rates. This evidence suggests that we can divide the post-World War II -period into a long-lasting bear market period before (around) 1980 where capital

¹ The stock market increases in 1972 and 1983 coincided with major changes in the Danish economy. In 1972, the Danish population decided by a referendum to join the EEC as of Jan 1 1973. In late 1982, a new conservative-liberal government came into office, marking the change to a new economic policy regime of tight fiscal policy, wage moderation and a fixed exchange rate policy. In 1983, a new tax on pension funds' bond returns was passed by the Parliament, favoring pension fund investments in stocks. The tax took effect as of Jan 1 1984.

gains were below average and a subsequent long-lasting bull market period with capital gains above average².

This picture is confirmed when relating the increase in stock prices to general inflation or the growth in economy-wide nominal income, cf. Figure 2. Real stock prices denote the level of stock prices deflated by the general price level (as measured by the factor price deflator for GDP in the private sector) whereas the growth-adjusted stock prices show the level of stock prices relative to nominal income (GDP in factor prices for the private sector). Stock prices have not been able to keep track with either the general price level or nominal income in the period before (around) 1980, the bear market period, but they have outpaced both the price level and nominal income in the subsequent period, the bull market period. The observation of a long-lasting bull market since the beginning of the 1980s has previously been made by Ritter and Warr (1999) for the US stock market³.

< Figure 2 >

The purpose of this paper is to explain the long-run behavior of the aggregate Danish stock market based on a few macroeconomic "fundamentals" variables. In particular, we want to address the question what can explain the apparent division into bearish and bullish subperiods of the stock market. We also provide some evidence on the question whether the large capital gains in 1996 and 1997 can be motivated by developments in fundamentals variables or whether they represent a non-fundamental innovation (a bubble, a fad or noise trading). We confine ourselves to the analysis of stock price behavior in the long run because this is the horizon where we expect fundamentals variables to have the most predominant effects. We focus on stock prices at the aggregate market level, that is, we formulate a model for the overall

² Our use of the terms 'bear market' and 'bull market' may differ from other definitions in the literature, but in this paper they are just short-hand expressions for periods with stock price increases below average and above average, respectively.

³ The observation that the Danish stock market can be divided into bearish and bullish subperiods is consistent with the findings in Nielsen and Olesen (1999a). They estimate a regime-switching model for the 1-year nominal stock return and find a persistent regime shift in the return process in the early 1980s. This regime shift includes an increase in the mean return.

market price index of Danish stocks. By focusing on the pricing of the market rather than individual stocks, we reduce the importance of idiosyncratic shocks.

The main finding is that we are able to explain the long run behavior of stock prices by only two "fundamentals" variables, firm profits and the nominal bond rate. We establish a strong and stable cointegrating relation between stock prices and these fundamentals variables. The latter are highly significant in both statistical and economic terms. We find that the increase in the bond rate until 1982 is a main explanation of the poor stock market performance in the bear market period and, likewise, that the decline in the bond rate following 1982 can account for the subsequent bull market. Furthermore, we conclude that a major part of the capital gains since 1995 can be motivated by the contemporaneous decline in the bond rate. However, the analysis also suggests a substantial degree of over-shooting in stock prices by late 1997. This overpricing of stocks was rectified during 1998.

Besides contributing to an understanding of the empirical issues raised above, this paper provides an empirical and macroeconomic alternative to more traditional valuation models for stocks, that is, we provide an alternative way to determine whether stocks are mispriced or fairly valued. Traditional models typically rely on financial or accounting ratios such as e.g. the price-earnings ratio, the dividend-price ratio or the book-to-market ratio. We take a different approach where we formulate and estimate an explicit empirical model for stock prices. We establish a well-determined and stable historical link between the long-run behavior of stock prices and a set of macroeconomic fundamentals variables. The level of stock prices predicted by this model (the fit) is interpreted as the fundamental level of stock prices. Thus, any deviations between actual stock prices and this prediction is taken as evidence of a mispricing of stocks in the market. Our approach is directed towards identifying an empirically valid structural model for stock prices which makes it possible to *explain* the behavior of stock prices either by contemporaneous fundamentals variables or by referring to a mispricing error. This distinguishes our approach from traditional models. These typically derive from predictability studies which have established that a certain (say) financial ratio has predictive power.

The outline of the paper is as follows. In section 2, we formulate an empirical long-run model for stock prices and take a look at the data. In section 3, we present the results from estimating the model. We use the VAR method of Johansen (1996) and include a check of the robustness of conclusions by using the two-step procedure of Engle and Granger (1987). Section 4 provides an interpretation and application of the results and section 5, finally, concludes with a summary and a discussion.

2. The Empirical Framework

The empirical long-run model for stock prices is based on a very simple theoretical framework. Let us assume that the fundamental value of stocks Q_t^* , defined as the expected discounted value of future profits, is determined as current profits per share divided by the discount rate used by investors to discount future profits:

$$(1) \quad Q_t^* = \frac{\Pi_t / N_t}{\gamma_t}$$

Π_t denotes the firm's total profits, N_t is the nominal value or number of all outstanding stocks in the firm and γ_t is the discount rate. (1) is basically an assumption about how investors value the uncertain stream of profits. As a special case, (1) comprises the textbook Gordon Growth Model for the price of stock with constant dividend growth and a constant discount rate if we assume that all profits are paid out as dividends, see Gordon (1962) or the outline in Campbell *et al.* (1997). In the Gordon case, the relevant discount rate is identical to the growth-adjusted real interest rate, defined as the excess of the nominal bond rate over inflation and real growth (in dividends). (1) can also be motivated as the relevant stock pricing formulae in the case where investors price stocks from a consideration of the price-earnings ratio of the firm (the ratio of stock price to earnings) including a comparison with a relevant discount rate. This type of valuation is often used in practice.

In order to estimate the model, we need to identify the unobservable discount rate γ_t . In this paper, we use the nominal bond rate as the discount rate. Theoretically, this is a controversial assumption because all rational asset pricing theories suggest that a real

discount rate is the proper measure. This is for instance reflected in the use of a growth-adjusted real interest rate in the Gordon model. The main reason for this choice is that the nominal bond rate provides a good account of actual stock price behavior as the following results will show. On the other hand, the use of common proxies for a real discount rate fail to establish a cointegrating relation for stock prices, see section 4. The use of a nominal bond rate might also be motivated *a priori*. There are basically two possibilities. First of all, variations in the nominal bond rate may simply serve as a rough proxy for the variations in the 'true' but latent real discount rate used by a rational investor. To the extent that changes in the nominal bond rate are mainly driven by innovations in the underlying real interest rate, this proxy may do well.

The other possibility is that stock investors suffer from 'money illusion'. In general, 'money illusion' is an irrational behavior where economic agents confuse nominal and real magnitudes or simply care more about the former. Hence, in the case of 'money illusion' stock investors may actually use the nominal bond rate as their discount rate even though a rational investor would do otherwise. This possibility has originally been advocated by Modigliani and Cohn (1979) for the US stock market. They argue that stocks were heavily undervalued by the late 1970s because investors irrationally used the high nominal bond rate in pricing stocks, thereby ignoring the fact that the high bond rate reflected a high level of inflation more than a high level of the real interest rate. In a rational world, an inflation-induced increase in the nominal bond rate will have no effect on stock prices because the increase in inflation eventually leads to a higher growth in future profits which compensates for the higher nominal bond rate. The result of 'money illusion' is that stock prices include an 'irrational' price component which reflects the gap between the nominal and the real interest rates, that is, the level of inflation. According to Modigliani and Cohn (1979), this lead to an irrational underpricing of stocks by the late 1970s. This argument has recently been resumed by Ritter and Warr (1999). They argue that the US bull market since 1982 can in part be explained by the decline in inflation which has reduced the irrational underpricing of stocks originating in money illusion (or 'inflation illusion',

using their terminology)⁴. Moreover, they find that the level of inflation is a predictor of the undervaluation of stocks and interpret this as evidence that investors suffer from 'money illusion'.

The possibility of 'money illusion' has also received attention in other parts of the academic literature. Recent studies that argue in favor of or incorporate the existence of 'money illusion' include Shiller (1999), Shafir *et al.* (1997), Shiller (1997) and Canner *et al.* (1997). Shafir *et al.* (1997) provide evidence from experimental studies suggesting that 'money illusion' is to be taken serious as a real world phenomenon⁵.

Based on (1), we formulate the empirical model for *actual* stock prices:

$$(2) \quad q_t = \beta_0 + \beta_1 \pi_t + \beta_2 b_t + \beta_3 n_t + u_t$$

(2) is obtained by taking the logarithm of (1) and using small letters to denote corresponding log-values. q_t is the log-value of *actual* stock prices (which we denote by Q_t) while b_t is the log-value of the nominal bond rate. The β_i 's are parameters to be estimated. We allow for a constant term because data for stock prices consists of an indexed series and not observations of stock prices *per se*. Thus, (2) is intended to model the behavior of stock prices over time rather than the actual levels. In the empirical specification, the fundamental value of stock prices is determined by the first part of (2), i.e., $\ln(Q_t^*) = \beta_0 + \beta_1 \pi_t + \beta_2 b_t + \beta_3 n_t$. The coefficients ($\beta_1, \beta_2, \beta_3$) denote partial elasticities of (fundamental) stock prices wrt. profits, the bond rate and the nominal value of outstanding stocks, respectively. According to (1), these should

⁴ Notice that Ritter and Warr (1999)'s argument is not that stock investors have become more rational (less inflicted by 'money illusion') but that the source and hence impact of 'money illusion' (inflation) has been reduced.

⁵ Anecdotal support to the existence of 'money illusion' can be found in recent market studies by two of the largest Danish banks, see DDB (1999) and Valgreen (1999). DDB (1999) evaluates the prices of stocks in a sample of OECD markets by focusing on the so-called 'earnings-yield-ratio' which is a standard practical tool for pricing stocks. The latter is defined as the ratio of the earnings-yield (earnings per share as a percentage of the stock price) to the *nominal* bond rate. The use of a nominal bond rate represents 'money illusion' because it ignores the future growth (inflation and real growth) in earnings. Thus, the Gordon model suggests the use of a growth-adjusted real interest rate. Similarly, Valgreen (1999) uses a nominal bond rate rather than a real discount rate in modeling stock prices. This is also an example of 'money illusion', see section 5 for details about this study.

have a unit value in absolute terms but the empirical model allows for elasticities different from unity. u_t is the residual term of the equation and can be interpreted as the degree of mispricing of stocks. Hence, u_t is determined as the log-difference

between actual and the fundamental level of stock prices, i.e., $u_t = \ln\left(\frac{Q_t}{Q_t^*}\right)$.

< Figure 3 >

The data for stock prices, profits and the bond rate are shown in Figure 3. All observations are annual and cover the period 1948-1998. For stock prices, we use the official All-Share Stock Price Index by Statistics Denmark, comprising (today) all Danish stocks listed at the Copenhagen Stock Exchange (CSE). This market index captures the aggregate level of stock prices over a variety of firms in different branches. For profits, we rely on National Accounts. We use the measure of gross operating income, defined as value added at factor prices minus the labor costs or equivalently, as sales minus the indirect taxes, costs of material inputs (including raw materials) and labor costs. This is not an exact measure of the profits or earnings of the firm because it ignores direct taxes, depreciation and interest payments on debt. However, it serves as a rough guide and captures what we believe to be the most important sources of variations in profits, namely, variations in sales and variable costs. For adequacy, we consider the gross operating income of the private sector⁶. For the bond rate, we use the yield-to-maturity on a 10-year government bond, as constructed by Nielsen, Olesen and Risager (1999).

In modeling stock prices, we have to take account of the fact that the nominal value or number of stocks listed at the CSE has increased substantially over time. Due to a lack of data for the entire sample period, we take a short-cut and use a deterministic trend to proxy the growth in the nominal value of outstanding stocks, that is, n_t in equation

⁶ New National Accounts figures, based on ENS95, are only available as of 1988. The data for profits before 1988 are therefore based on the old National Accounts figures (ENS79). A consistent series for profits is constructed by a multiplicative chaining backwards in time as of 1988. Note, that data for the years 1996-1998 are provisional. The data source is the official ADAM database by Statistics Denmark (the April 1997- and August 1999-versions).

(2) is replaced by a simple time trend. The estimated coefficient, β_3 , can then be interpreted as an estimate of the actual growth in the nominal value of stocks. Parum (1997) reports data for the total nominal value of Danish stocks listed at the CSE for selected years in our sample⁷. These suggest that the assumption of a trendwise growth is reasonable for a period of a couple of years (which is the relevant horizon as (2) provides a model for the long run, cf. below). Moreover, they suggest an estimate of β_3 in the neighborhood of 8.0% per year which is the average (continuous) growth rate in the nominal value of stocks in the period from 1952 to 1996.

It is evident from Figure 3 that both stock prices and profits exhibit a strong time trend. Furthermore, unit root tests show that all three data series including the bond rate contain a unit root (and only one), that is, the series are integrated of order 1, see Appendix A. This implies that (2) has to be estimated by cointegration methods. If (2) is a valid model empirically, it forms a cointegrating regression defined by the statistical criterion that the residual term is stationary. The economic interpretation is that (2) forms a long-run equilibrium model for stock prices by determining an equilibrium level - the fundamental value - to which stock prices will be 'attracted' in the long run. That is, in the short run stock prices may deviate from their fundamental value ($u_t \neq 0$) but in the long run any mispricing disappears because stock prices approach their fundamental value (u_t reverts to its unconditional mean of zero), see Engle and Granger (1991) for a discussion of the concept of cointegration.

3. Results⁸

We estimate (2) by the cointegrated Vector Autoregressive (VAR) method of Johansen (1996). This multivariate method has the advantages of allowing for more than one cointegrating relation in the data and of leading to consistent and asymptotically efficient estimates. To briefly sketch the procedure, we assume that stock prices, profits and the bond rate which all are modeled endogenously can be

⁷ The reported figures in mill DKK nominal value are (the year in parenthesis): 1420 (1952), 2439 (1962), 5869 (1972), 14123 (1982), 43360 (1992), 48764 (1996), cf. Parum (1997, Appendix Tables B1.5-B1.10).

⁸ The estimations are performed in PCFIML, see Doornik and Hendry (1997).

described by a VAR model of lag length k . Written in error-correction-form, this model is:

$$(3) \quad \Delta X_t = \Pi X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \varepsilon_t$$

X_t denotes the (3×1) vector having the three endogenous series as elements and ΔX_t contains the corresponding first differences. We allow for an unrestricted constant term in each of the dynamic equations (captured by the vector of constants μ_0) and a deterministic time trend which is restricted to the cointegrating space. The latter restriction is enforced by including the trend in the cointegrating part of the VAR model, that is, as part of the augmented (4×1) vector $X_t^* \equiv (q_t, \pi_t, b_t, t)'$. The trend proxies the nominal value of listed stocks in the cointegrating regression, cf. section 2. By restricting the trend to the cointegrating space, we preclude the possibility of a quadratic trend in the endogenous variables, cf. Johansen (1996). This seems warranted by the data, see Figure 3. ε_t is the vector of disturbance terms in the dynamic equations, assumed to fulfill the standard assumptions of being normally and independently distributed with mean zero and constant variance. The parameters to be estimated are $(\Pi, \Gamma_1, \dots, \Gamma_{k-1}, \mu_0)$.

Our interest in (3) is attached to the first term on the right hand side which captures the cointegration features of the data. The number of cointegrating relations is determined by the rank of the (3×4) matrix of parameters, Π . If Π has zero rank, there will exist no cointegrating relation between the endogenous series (including the trend). If Π has a non-zero but reduced rank r ($0 < r < 3$), we will have r cointegrating relations⁹. In this case, the Π -matrix can be written as the product of two full column rank matrices α and β^* with dimensions $3 \times r$ and $4 \times r$, respectively; $\Pi = \alpha(\beta^*)'$. Each column vector of the β^* -matrix is a cointegrating vector and contains the coefficients to stock prices, profits, the bond rate and the time trend in a cointegrating relation.

⁹ In the case where Π has a full rank of 3, there will be three cointegrating relations. This is only possible if all the endogenous series are stationary. However, this is ruled out by the prior unit root testing, cf. above.

The α -matrix contains the loadings by which the equilibrium errors from the cointegrating relations, i.e., the deviations from long-run equilibrium $(\beta^*)' X_{t-1}^*$, affect the short-run dynamics of the endogenous variables. Assuming that only one cointegrating relation exists ($r=1$), the β^* -vector can in terms of (2) be formulated as the vector $(1, -\beta_1, -\beta_2, -\beta_3)$ where we have normalized on stock prices (the constant term β_0 is estimated as part of the unrestricted constant terms in the μ_0 -vector). Both α and β^* are estimated in the Johansen (1996) method.

The VAR model specification we use has a lag length of 3 ($k=3$) and includes an impulse dummy for the year 1960¹⁰. This specification is chosen by a general-to-specific procedure where we initially estimate a 'general' VAR model with a large number of lags (5 lags). In this 'general' version, specification tests indicate a significant heteroskedasticity (ARCH) in the residual term of the equation for the bond rate. This arises from an abrupt pattern in the bond rate in the period from 1957 to 1960 which ends with a sharp increase in 1960, cf. Figure 3. To eliminate ARCH, we include the dummy for 1960. The proper lag length is, subsequently, chosen by formal testing where we successively test for the significance of all terms at the highest lag order and eliminate any insignificant lags. Using conventional significance levels, we end up with the 'specific' VAR model with 3 lags¹¹.

< Table 1 >

< Figure 4 >

Specification tests and diagnostic graphics for the VAR model are shown in Table 1 and Figure 4. The specification tests test whether the residuals of the VAR model

¹⁰ The impulse dummy has a value of one in 1960 and is zero otherwise. In terms of (3), the dummy gives rise to an additional (unrestricted) term $+\mu_{60}d60_t$, where $d60_t$ is the scalar dummy and μ_{60} denotes the (3×1) vector of coefficients to the dummy. The term enters in the same manner as the μ_0 -vector of constants.

¹¹ The simultaneous F-test of the null that all terms at lag 3 can be excluded from the VAR model with 3 lags leads to a clear rejection of the null at conventional significance levels (the critical significance level is 1.5%). Similarly, the null that the 'general' VAR model can be reduced from 4 to 3 lags is accepted as the critical significance level is 13.6%. The use of information criteria in choosing the lag length gives ambiguous results: the Schwarz criterion supports the use of 1 lag, the Hannan-Quinn criterion 2 lags and the Akaike criterion 4 lags.

fulfill the requirements of being homoskedastic, serially uncorrelated and normally distributed. The model passes all tests for serial correlation and heteroskedasticity (including ARCH) at the standard 5% significance level and with the exception of the test for first order serial correlation in the equation for profits also at the 10% level. The univariate test for normally distributed residuals in the equation for stock prices leads to a clear rejection¹². However, for inference purposes the normality assumption can be relaxed in favor of the weaker assumption that the residuals are identically distributed over time, cf. Johansen (1996, Part II). Hence, the rejection is not critical. We conclude that the VAR model is well specified.

< Table 2 >

Table 2 reports the results from the cointegration analysis. We report two versions of the trace test on the cointegrating rank, the asymptotic version used in Johansen (1996) and the small-sample approximation suggested by Reimers (1992). Because our sample is relatively small, we rely on the latter version. We also report two sets of 95% critical test values, the standard values tabulated in Johansen (1996) and a set of critical values which have been simulated using the simulation program DisCo, cf. Johansen and Nielsen (1993), to take account of the inclusion of the impulse dummy for 1960. Using the small-sample trace test and the simulated critical values, we find strong evidence of cointegration. Hence, the null of no cointegration ($r=0$) is strongly rejected in favor of the alternative hypothesis that the rank is at least one. The critical significance level of the test is close to 1%. Testing the null that there is only one cointegrating relation, we accept the null by a clear margin using standard significance levels (the critical significance level is approximately 15%). The conclusion therefore is that there is one and only one cointegrating relation in the data.

< Table 3 >

¹² The rejection of normality is a result of right skewness which is driven mainly by the large capital gains in 1972 and 1983.

The estimates of the long-run parameters α and β^* are shown in entry (I) of Table 3. Because we interpret the relation as an equilibrium relation for stock prices, we have normalized on this variable. The table also reports indicative standard errors of the parameter estimates (in parentheses) and for each parameter, the critical significance level of a Likelihood Ratio (LR) test of the null that the given parameter is zero (squared brackets). For instance, the LR test of the null that profits are weakly exogenous for the long-run parameters ($\alpha_2=0$) has a critical significance level of 64.9%.

All β^* -coefficients are highly significant. Among the α -loadings, only the loading in the direction of the bond rate (α_3) is significant when using conventional significance levels. In particular, the loading in the direction of stock prices (α_1) appears to be insignificant which suggests that a deviation from long-run equilibrium has no effect on stock prices. These results may actually be interpreted as an indication that the cointegrating relation identifies an equilibrium for the bond rate rather than stock prices. However, this would imply that the bond rate could be explained by the levels of stock prices, profits and a deterministic time trend which seems to lack any economic-theoretical or -intuitive foundation¹³. This is in particular true in the Danish case where the stock market has been small compared to the bond market historically and where the bond rate, furthermore, has been determined mainly by the foreign bond rate since the capital account liberalization in 1982. Moreover, it turns out that the lack of a significant error-correction in the direction of stock prices may be attributed to the small sample size. Thus, in the general version of the VAR model in (3) we have included several dynamic terms (lagged terms of ΔX_t) which turn out to be insignificant empirically. Excluding these terms, we end up with a more parsimonious VAR model where α_1 is significant at standard significance levels¹⁴. We

¹³ The implied relation for the bond rate would be (ignoring the constant term):

$$b_t = 2.15\pi_t - 1.43q_t - 0.076t$$

¹⁴ Estimating the VAR model by Full Information Maximum Likelihood and using consecutive LR tests to exclude individual terms in the stock price equation, we end up with a well specified and (more) parsimonious VAR model where the stock price equation only includes a constant, the second lag of $\Delta\pi$, and the equilibrium-error from the cointegrating relation. Performing a LR test of the null that α_1 is zero, we get a test statistic of 4.9, corresponding to a critical significance level of 3%. This leads to a rejection of the null at standard significance levels. The result can be confirmed by single-equation estimation of the dynamic stock price equation. Furthermore, using the two-step procedure of

therefore adhere to our interpretation that the cointegrating relation determines an equilibrium for stock prices.

Entries (II) and (III) show the estimates of the (restricted) long-run parameters under two hypotheses on the α -loadings. In (II), we have assumed that profits are weakly exogenous for the long-run parameters, that is, that an equilibrium error has no effect on profits ($\alpha_2=0$). This hypothesis is validated by the data as the relevant LR test on α_2 in the unrestricted specification (I) has a critical significance level of 64.9%. In (III), we have assumed that both profits and the bond rate are weakly exogenous ($\alpha_2=\alpha_3=0$). This hypothesis is rejected by the data as a LR test of the null that $\alpha_2=\alpha_3=0$ in (I) leads to a test statistic of 8.6 and a corresponding critical significance level of 1.4%.

By comparing entries (I) through (III), we find the ensuring result that the estimates of the coefficients in the cointegrating relation (the β^* -vector) are rather robust to the assumptions on the α -loadings. We shall use the parsimonious specification in (II)¹⁵, leading to the following long-run equilibrium relation for stock prices (indicative standard errors in parentheses)¹⁶:

$$(4) \quad q_t = -15.1 + \underset{(0.17)}{1.55} \pi_t - \underset{(0.058)}{0.72} b_t - \underset{(0.015)}{0.057} t$$

All coefficients are strongly significant and have the expected signs. The point estimates of the coefficients of profits and the bond rate are significantly different from unity (which is the value expected from the theoretical framework of section 2)¹⁷ but the differences are not substantial in economic terms. According to the point estimates, a permanent increase in the level of profits by 1% (continuous rate) will

Engle and Granger (1987), we find that the (corresponding) error-correction term has a very significant effect on the short-run dynamics of stock prices, cf. below.

¹⁵ (II) is preferred to (I) because it is (more) parsimonious and hence leads to a more efficient inference. Moreover, recursive estimation suggests that the β^* -vector of (II) is more stable over the sample period than that of (I). Recursive estimates for (II) are shown below.

¹⁶ The constant term is calculated to ensure that the deviations between actual stock prices and the model's fitted stock prices have a sample mean of zero.

¹⁷ This can be shown by formal LR testing (not reported). Indicative 95% confidence intervals for the coefficients, constructed on the basis of the indicative standard errors, suggest that estimation uncertainty can sustain a coefficient to profits of 1.22 and a coefficient to the bond rate of -0.83 as the respective lower limits of the confidence intervals.

ceteris paribus lead to a 1.55% increase in stock prices in the long-run, that is, when prices have adjusted to their new equilibrium or fundamental value. This somewhat stronger response than in the unit-elasticity case could be interpreted as an 'overreaction' in stock prices even in the long run, but could also merely reflect that the 'true' measure of profits of the firms considered is more volatile than the profit proxy we have used.

A permanent 1% *relative* increase in the bond rate (e.g. from 10.0% to 10.1%) will in the long run lead to a decline in the level of stock prices by 0.72%. This 'underreaction' compared to the unit-elasticity-case is consistent with other findings for the Danish stock market, see Nielsen and Olesen (1999b) and Olesen and Risager (1999). Nielsen and Olesen (1999b) find that the dividend-price ratio of stocks (the ratio of dividends to stock prices) responds less to innovations in the bond rate than one should expect in the unit-elasticity (Gordon Growth Model) case. Olesen and Risager (1999) find that stock returns tend to respond less than one-to-one to changes in bond yields at the 5- and 10-year investment horizons. A one-to-one relationship would be the outcome under a simple no-arbitrage relation, saying that the expected return on stocks equals the expected return on bonds plus a risk premium. Actually, our estimate of the bond rate elasticity comes close to the estimated long-run relationship between the 10-year stock return and the 10-year bond yield in Olesen and Risager (1999)¹⁸. Possible explanations of this 'underreaction' in stock prices are that investors may expect innovations in the bond rate to be partially transitory, cf. Nielsen and Olesen (1999b), or that bond returns are taxed at a higher rate than stock returns, cf. Olesen and Risager (1999).

The time trend is highly significant. This shows the importance of taking account of the growth in the nominal value of outstanding stocks. The estimated coefficient suggests that aggregate stock prices on average and in the long run tend to decline by a (continuous) rate of 5.7% per year due to new issuances that increase the number of stocks over which profits have to be distributed. The point estimate of 5.7% comes

¹⁸ Olesen and Risager (1999) estimate a bond yield coefficient of 0.71 in a cointegrating relation for the 10-year stock return.

close to the average growth rate of 8.0% in the actual number of outstanding stocks on the CSE, see section 2. Performing a statistical test, we find that estimation uncertainty can account for the difference¹⁹.

Finally, we note from Table 3 that the sign and magnitude of the estimated loading in the direction of stock prices ($\alpha_1 = -0.287$) is consistent with the interpretation that the cointegrating relation determines a long-run equilibrium (fundamental) level of stock prices. Thus, if stocks are currently (say) overpriced, that is, if stock prices currently are above their fundamental value ($Q_t > Q_t^*$), prices will tend to decline in the next period by a fraction of the amount of mispricing. The estimate of α_3 suggests a simultaneous adjustment in the bond rate. Hence, the bond rate tends to decline whenever stocks are overpriced, and *vice versa*. This reinforces the adjustment to long-run equilibrium because a decline in the bond rate leads to an increase in the fundamental value of stocks, and *vice versa*.

< Figure 5 >

< Figure 6 >

We have performed two types of tests of the robustness of the estimated long-run model. First of all, we have tested whether results are stable over time by performing a recursive estimation of the key parameters. Figure 5 shows the recursively estimated eigenvalues of the (unrestricted) Π -matrix. All three eigenvalues seem fairly stable. This suggests that the conclusion that there is one and only one cointegrating relation in the data is robust over time. Figure 6 shows the recursive estimates of the β^* -vector, having normalized on stock prices and assumed that profits are exogenous ($\alpha_2 = 0$). The result is that the coefficient estimates are reasonably stable over time. Finally, it can be shown by recursive testing (not reported) that restricting profits to be exogenous is a valid hypothesis over the entire sample. We therefore conclude that results are stable over time.

¹⁹ A LR test of the null that the (true) coefficient of the trend is 8.0% ($\beta_4^* = 0.08$), gives a test statistic of 1.8. The critical significance level is 18.3%.

Secondly, we have checked (indirectly) whether results are affected by the relatively small sample size of our study. Evidence suggests that the cointegrated VAR method of Johansen (1996) or more precisely, the embedded tests for cointegration and tests of hypotheses on the long-run coefficients, may suffer from a poor small sample performance (size distortions and low power of tests), see e.g. Gonzalo and Lee (1998), Juselius (1999) and Johansen (1999). To get an indication of whether results have been distorted by the small sample size, we have re-estimated the long-run model for stock prices using an alternative estimation method, that is, the single-equation two-step procedure of Engle and Granger (1987) (EG2) combined with methods suggested by Phillips and Loretan (1991) for testing hypotheses on the cointegrating vector. According to the Monte Carlo simulations in Gonzalo and Lee (1998), the EG2 procedure appears more robust than the Johansen (1996) method when testing for cointegration in small samples.

< Table 4 >

Table 4 reports results of the alternative estimation. In entry a, we have listed the candidate cointegrating vector estimated in the first step of the EG2 procedure. This is obtained by regressing (OLS) stock prices on a constant term, profits, the bond rate and a time trend. Notice that the estimated coefficients come close to those estimated by the Johansen (1996) method. In the second step of the EG2 procedure, we test whether this regression represents a cointegrating relation by testing whether the residuals are stationary. Performing an augmented Dickey-Fuller test (we have included two significant lags of the first differences of the error-correction term which results in a well specified augmented Dickey-Fuller regression) of the null that the residuals are non-stationary, that is, that we do not have cointegration, we get a test statistic of -5.7. The critical small-sample test value at the 1% significance level is -5.1, cf. MacKinnon (1991). Hence, we clearly reject the null and conclude that there is strong evidence in favor of cointegration. Estimating an error-correction model, we

can furthermore show that there is a highly significant error-correction in the direction of stock prices²⁰.

Having established cointegration, we can perform tests on the cointegrating vector. Because of a slow adjustment in stock prices or the regressors (profits and the bond rate) being endogenous, current, past or future innovations in the regressors may be correlated with the residual of the cointegrating relation. This leads to non-standard test distributions, cf. Hamilton (1994, Section 19.3). As a consequence, hypothesis tests on the cointegrating vector have to be conducted within an augmented regression where the tests have well-known distributions. In Table 4, we report the outcomes of estimating two types of augmented regressions, the Phillips and Loretan (1991) OLS (POLS) regression where the static cointegrating regression is augmented by current, lagged and leaded first differences of the regressors (results shown in entries b through d for three alternative lag- and lead-lengths) and the Phillips and Loretan (1991) Non-Linear Least Squares (PLNLS) regression which also includes lagged error-correction terms (entries e through g for three alternative lag- and lead-lengths)²¹. In the latter case, the appropriate number of lags of the error-correction term is determined by a specific-to-general procedure, whereby lags are included until the residuals of the augmented regression fulfill the white noise requirements.

The results in Table 4 show that the augmented point estimates of the cointegrating vector are sensitive to the specification chosen. However, in most cases the differences are of a magnitude consistent with estimation uncertainty (which occasionally is large). Comparing with the estimates obtained by the Johansen (1996) method, we find that except for the large bond rate elasticities obtained in two cases (for $N_1=N_2=1$ for both the POLS and the PLNLS regression) the augmented coefficient estimates come reasonably close. In particular, the deviations usually (with the two exceptions mentioned) fall within one standard error of the augmented

²⁰ The t-test statistic on the coefficient to the lagged error-correction term in a well specified parsimonious error-correction model for stock prices is -4.2, implying that the error-correction term is significant at any conventional significance levels. Results are very similar to those reported in Appendix B where we estimate an error-correction model for stock prices, based on the cointegrating relation obtained by the Johansen (1996) method.

²¹ See Phillips and Loretan (1991) or Hamilton (1994, Section 19.3) for an outline of the two methods.

coefficient estimators. We conclude that the evidence provided by the single-equation analysis seems to support the earlier results.

4. Interpretation and Applications

The fit of the model in (4) is shown in Figure 7. The fitted value is interpreted as the estimated fundamental value of stock prices, defined as the equilibrium level which stock prices will approach in the long-run. The deviations between actual stock prices and their long-run levels show the degree of mispricing. These long-run residuals or mispricing errors are plotted in Figure 8 together with a correlogram and a density plot.

< Figure 7 >

< Figure 8 >

We see that the long-run model tracks actual stock prices remarkably well over the entire sample. This is reflected in the fact that any mispricing error disappears, that is, is eliminated or replaced by an error of the opposite sign within a few years. The observed maximum duration of a mispricing is seven years (corresponding to the underpricing of stocks over the period 1974-1980) but on average an episode of mispricing lasts only 3 (2.7) years. This low persistence of the mispricing error which is also confirmed by its low degree of serial correlation reflects the strong cointegration in (4).

Even though a period of mispricing is usually short, the magnitude of a mispricing can be substantial. The largest overpricing error in the sample is 0.40 for the year 1983, corresponding to an overpricing of stocks by 49% relative to their fundamental value. The largest magnitude of an underpricing occurred in 1979 with a 27% undervaluation of stocks relative to fundamental value (a mispricing error of -0.32). As is evident from the density plot in Figure 8, the sample distribution of the mispricing error comes close to the normal distribution. In fact, according to the test of Doornik and

Hansen (1994) we can not reject the hypothesis that the error distribution is normal²². The sample standard deviation of the mispricing error is 0.160 which implies that 95% of the errors in the unconditional distribution fall in the range between -0.31 and +0.31, i.e., in the range between an underpricing of stocks by 27% and an overpricing of stocks by 37% in relative terms.

< Figure 9 >

The contributions by the individual determinants of stock prices are illustrated in Figure 9. This figure shows actual stock prices, the fundamental level of stock prices implied by (4) and the contributions by the bond rate (the bond rate term in (4)) and 'profits per share' (the sum of the profits and the trend terms in (4)), respectively. The latter component is constructed as the sum of the contributions by total profits and the trend (the nominal value of outstanding stocks) and is, tentatively, interpreted as a proxy for the profits per share. In order to use a common scale, all series are normalized as deviations from their respective sample means.

As we might have expected, the strong trend in stock prices is (primarily) the result of the trend in profits per share. From 1951 to 1998 total profits have on average increased by 7.7% per year (continuous growth rate). Adjusting for the growth in the nominal value of outstanding stocks (5.7% per year), the trend in profits per share has according to (4) contributed by an annual growth in stock prices of 6.2% per year. For comparison, actual stock prices have increased by 7.4% per year over the sample.

Profits (per share) dominates the bond rate in modeling the long-run trend in stock prices. However, the bond rate is important in explaining the observed variations in stock prices around their long-run trend. In particular, the bond rate is the crucial variable in explaining the evident split of the Danish stock market into bearish and bullish subperiods. Thus, reflecting an almost continuous increase, the bond rate rose from 6.6% in 1951 to 21.4% in 1982. This represents more than a trebling or a relative

²² The Doornik and Hansen (1994) test of the null that the mispricing error is normally distributed gives a test statistic of 1.1 which has to be compared to the $\chi^2(2)$ -distribution. The critical significance level is 57%, leading to an acceptance of the null at any conventional significance levels.

increase by 3.8% per year (continuous growth rate). The rise in the bond rate increased the discount rate by which future profits were discounted and, hence, dampened the growth in stock prices. According to (4), the bond rate can account for a decline in the fundamental value of stocks by 2.7% per year over the period from 1951 to 1982. The latter roughly mirrors the difference between the average capital gain over the bearish period (4.0% per year) and the average capital gain over the entire sample (7.4% per year). Likewise, the trend in the bond rate following 1982 explains the subsequent bullish stock market. From its high in 1982, the bond rate declined to 4.2% by 1998, corresponding to only one fifth of the 1982-level or a relative decrease by 10.1% per year. According to (4), this enhanced the growth in stock prices by 7.3% per year. The latter is significantly larger than the difference between the average capital gain over the bullish period (10.1% per year) and the average capital gain over the entire sample (7.4% per year) which reflects the fact that growth in profits was below average in the bullish period, cf. Figure 9. In explaining the bear and bull market characteristics of the stock market, it is interesting to note that the behavior of profits actually works in the opposite direction. Hence, the fastest (above average) growth in profits occurred in the period from the late 1950s to the early 1980s (the bearish period) whereas growth in profits was relatively slow in the following period (the bullish period). The changes in the bond rate more than compensates for this behavior of profits²³.

The explanatory power of the bond rate is evident when examining the behavior of the growth-adjusted stock prices (stock prices relative to nominal income). These do not exhibit the same strong time trend as the level of stock prices. By re-formulating (4), we can immediately derive a model for growth-adjusted stock prices. Thus, subtracting (log-to) nominal income on both sides of (4), using the fact that profits can be written as the product of nominal income and the profit share (by the identity

²³ Even though an exact dating is difficult, the turn in the stock market seems to occur already in 1980, that is, two years before the decline in the bond rate. This earlier turn-around could, in principle, be explained by more temporary features such as the fact that stock prices according to (4) were significantly below their fundamental value in 1980. This underpricing disappeared from 1980 to 1982. Moreover, profits increased significantly in 1981 and 1982, leading to an increase in fundamental stock prices.

defining the profit share) and re-arranging terms, we get the following (implied) long-run model:

$$(5) \quad qy_t = -15.1 + 1.55\pi y_t - 0.72b_t + 0.55y_t - 0.057t$$

qy_t denotes growth-adjusted stock prices, y_t is the measure of nominal income and πy_t is the fraction of profits out of total income, all in logs. According to (5), the growth-adjusted stock prices are driven by four components. In economic terms, the most interesting are the first two, the profit share and the bond rate. Growth-adjusted stock prices will *ceteris paribus* increase, that is, stock prices will outpace growth in nominal income, whenever we see an increase in the profit share. This is because an increase in the profit share means that a larger fraction of total income in the economy accrues to stock holders. Because of the usual discount rate effect, an increase in the bond rate leads to a decline in the growth-adjusted stock prices, implying that stock prices will grow by less than nominal income. The trend term captures as before the growth in the nominal value of outstanding stocks. Finally, (5) includes an income term which reflects the result that innovations in profits and, hence, broad income y_t have a more than proportional effect on the level of stock prices. An increase in nominal income will therefore lead to an increase in the growth-adjusted stock prices²⁴.

< Figure 10 >

Panel a in Figure 10 shows the fit implied by (5). In calculating the growth-adjusted stock prices, we have used private sector GDP in current factor prices as the income measure. This corresponds to our measure of profits. Accordingly, the profit share is calculated as the ratio of gross operating income to GDP in factor prices, both for the private sector. This figure is basically a 'mirror image' of Figure 7 as the residuals are identical across the two figures. Panel b illustrates the contributions by the individual

²⁴ Re-formulating (2), the implied model for growth-adjusted stock prices is in general (ignoring the error term):

$$(5') \quad qy_t = \beta_0 + \beta_1 \pi y_t + \beta_2 b_t + \beta_3 n_t + (\beta_1 - 1)y_t$$

In the unit-elasticity case $\beta_1 = 1$, the income term disappears.

components of (5), showing the trend and the income terms together as a single 'residual component'. In Panel c, we illustrate the explanatory power of the bond rate, having omitted the other components for expositional reasons. We observe that the by far most important component in explaining the long-run behavior of growth-adjusted stock prices is the bond rate. In particular, the bond rate almost by itself explains the continuous decline in growth-adjusted stock prices until the beginning of the 1980s (the bear market) and the subsequent continuous increase (the bull market), confirming the earlier insight obtained from the stock price model. The profit share has some explanatory power. Our measure of the profit share shows an almost steady declining trend from 1951 where the profit share was 49% until (around) 1980 where it reached a low of 38%. According to (5), this shift in the profit share can explain a decline in (growth-adjusted) stock prices by 1.4% per year (continuous rate) over the same period. Following a period of a steady increasing trend, the profit share was almost restored by 1998 (45%). This reversal increased the (growth-adjusted) stock prices by an equivalent 1.4% per year in the period from 1980 to 1998. Hence, the behavior of the profit share is consistent with the bear- and bull-market pattern of the stock market. However, the variations in the profit share have been rather modest and the explanatory power is therefore much lower than for the bond rate. Finally, the 'residual component' has some significance towards the beginning and towards the end of the sample, reflecting the fact that income growth deviated from its mean in these two periods. However, in most of the sample this component is fairly constant and can, hence, be ignored in explaining the behavior of growth-adjusted stock prices.

Our model also contributes to an understanding of the surge in the stock market in recent years. From late 1995 to late 1998, average capital gains amount to 18.5% per year (continuous rate), mainly reflecting the large capital gains realized in 1996 and 1997. The contemporaneous decline in the bond rate can to a large extent account for these stock price increases. Thus, by late 1995 stock prices were roughly in line with their fundamental value as implied by (4) (the mispricing error for 1995 is -0.037, indicating a slight underpricing of stocks by 4% in relative terms). The bond rate has fallen gradually from a level of 8.3% by late 1995 to 4.2% by late 1998 which according to (4) has increased fundamental stock prices by 16.0% per year over the

same period, see also Figure 9. At the same time, profits per share have roughly been constant. Consequently, actual stock prices have only increased by slightly more than fundamental stock prices from 1995 to 1998. This suggests that the recent surge in the stock market was to a large extent warranted by fundamental factors, that is, by the decline in the bond rate. We can also conclude that stock prices were close to their fundamental level by late 1998. According to (4), stocks were overpriced by 7.1% (the mispricing error for 1998 is 0.069). Given the usual magnitudes of a mispricing, we interpret this as evidence that stocks were roughly fairly valued. However, it is also interesting to note that the adjustment in stock prices differs significantly from that in their fundamental level, cf. Figure 7. Prices show a substantial overreaction in 1996 and 1997 with stocks being overpriced by 37% relative to fundamentals by late 1997. This mispricing was eliminated during 1998 as a result of a capital loss on stocks and the continued decline in the bond rate which led to a further increase in the fundamental level of stock prices.

< Figure 11 >

We have modeled the discount rate of investors by the nominal bond rate rather than a real discount rate measure. The reason is that the bond rate leads to a strong cointegrating relation for stock prices. As possible alternatives, we have experimented with a comprehensive set of common proxies for a real discount rate. However, none of these resulted in a cointegrating relation. To highlight this, we provide results for two possible real discount rate proxies and these are the proxies that came closest to sustaining cointegration. Both proxy the expected growth-adjusted real interest rate used in the Gordon Growth Model, see section 2. We take account of real growth and inflation by adjusting for the growth in nominal profits. Figure 11.a illustrates the proxies. The *ex post* real rate is a perfect foresight proxy for the growth-adjusted real interest rate. That is, the *ex post* real rate for e.g. year 1972 is constructed as the difference between the 10-year nominal bond rate in 1972 and the realized rate of growth in profits in the 10-year period from 1972 to 1982. This assumes that investors on average forecast growth in profits correctly. Because of the forward-looking nature of the proxy, data is only available until 1988. The *ex ante* real rate is backward-

looking as it uses the historical growth in profits as a proxy for the investors' expectations about future profit growth. Thus, the *ex ante* real rate in (say) year 1972 is constructed as the difference between the 10-year nominal bond rate in 1972 and the average growth rate in profits in the historical 5-year period from 1967 to 1972. This proxy is available from 1953. In terms of long-run behavior, it is evident from Figure 11.a that the *ex ante* and the *ex post* real rates show a resemblance to the nominal bond rate. In particular, the hike in interest rates in the late 1970s and the early 1980s is common to all three series. However, there are also differences. The most important difference is that the real discount rate proxies fail to track the increase in the nominal bond rate during the first two decades of the sample. From 1951 to the early 1970s the bond rate is roughly doubled which is a main contribution in explaining the large fall in the growth-adjusted stock prices over this period, see Figure 10.c. In comparison, the levels of the real rate proxies are stable or declining in the same period. We can also note that the real rates and, in particular, the *ex ante* real rate decline very rapidly from the high levels in the early 1980s.

Using the two-step procedure of Engle and Granger (1987), we have estimated a cointegrating relation for stock prices based on these alternative discount rate measures. The residuals obtained from regressing (OLS) stock prices on a constant term, profits, a deterministic trend and each proxy in turn (the first step of the procedure) are shown in Figures 11.b and 11.c²⁵. These also include the residuals from the cointegrating relation where the nominal bond rate is used as the discount rate measure. This is the relation outlined in the single-equation analysis of section 3, see Table 4 (entry a). For both real rates, we find that the residuals are very persistent, that is, there is a tendency for the residuals to be systematically negative or systematically positive several years in a row. In particular, the residuals are more persistent than when using the nominal bond rate. It is mainly in the bearish period until the early 1980s that the real rate relations suffer. Thus, from the mid-1960s to the early 1980s,

²⁵ Because the real rate proxies are negative in some of the years, the regressions include the proxies in levels rather than log-levels. Notice that in the Gordon model the real discount rate is not allowed to be negative because this would imply that the investor's willingness to pay for stocks is infinitely large. However, in the empirical model we include a constant term, cf. (2), so what matters are the variations in the discount rate and not the levels.

actual stock prices are systematically overestimated when using the real rates. The bearish period includes the first two decades of the sample where the behavior of the real rates differs most evidently from that of the nominal bond rate. The residuals from the real rate relations also tend to be larger than the residuals from the bond rate relation. The residual has a standard error of 19.8% and 27.2%, respectively, for the *ex post* and *ex ante* real rate relations. The standard error is 16.2% for the relation based on the nominal bond rate.

We can test whether the regressions using the real rates form cointegrating relations by testing whether the residuals are stationary (the second step of the Engle and Granger (1987) procedure)²⁶. For the *ex ante* real rate, we get a Dickey-Fuller t-test statistic of -2.6 for the null hypothesis that the residuals are non-stationary. The critical test value at the 10% significance level is -4.0, cf. MacKinnon (1991). Hence, we accept the null by a clear margin and conclude that cointegration is strongly rejected. Likewise, for the *ex post* real rate, the Dickey-Fuller t-test statistic is -2.8 which should be compared to a critical test value of -4.1 at the 10% level. Again, we strongly reject that the regression forms a cointegrating relation. Finally, recall from section 3, that we find strong evidence of cointegration in the Engle and Granger (1987) analysis when using the nominal bond rate as the discount rate measure. From an empirical point of view, it therefore appears that the nominal bond rate performs significantly better than common proxies for a real discount rate.

²⁶ Unit root tests show that the *ex ante* and the *ex post* real rates are integrated of order 1 (tests not reported). For the *ex ante* real rate (denoted by γ_t^A), the cointegrating regression is (indicative OLS standard errors in parentheses):

$$q_t = \frac{0.41}{(5.2)} + \frac{0.27}{(0.40)} \pi_t - \frac{0.048}{(0.013)} \gamma_t^A + \frac{0.056}{(0.04)} t + \hat{u}_t \quad (1953-1998)$$

For the *ex post* real rate (γ_t^P), the regression is:

$$q_t = \frac{-27.7}{(5.3)} + \frac{2.41}{(0.40)} \pi_t - \frac{0.065}{(0.013)} \gamma_t^P - \frac{0.135}{(0.03)} t + \hat{u}_t \quad (1951-1988)$$

\hat{u}_t is the residual term. The test for cointegration in the second step of the Engle and Granger (1987) procedure is in both cases conducted within a simple Dickey-Fuller regression without augmenting terms. This lag structure is determined by a 'general-to-specific'-procedure where we initially estimate an augmented Dickey-Fuller regression with 5 lags and then successively remove insignificant lags at the 5% significance level. The simple Dickey-Fuller regression is well specified, that is, there is no evidence of serial correlation or heteroskedasticity in the disturbance term.

We have focused exclusively on modeling the long-run behavior of stock prices. However, the model may also be used in explaining and possibly predicting short-run (1-year) movements in stock prices by formulating an error-correction model (ECM) which includes the adjustment to long-run equilibrium as part of the short-run dynamics. In Appendix B, we have estimated such an ECM, using a single-equation specification. The general insights are that the ECM has power in explaining short-run capital gains and that the ECM term, capturing the adjustment to the long-run fundamental level of stock prices, is an important explanatory variable. The most significant explanatory variable is (the change in) the contemporaneous bond rate.

5. Conclusion

We have formulated and estimated an empirical model which performs well in explaining the long-run behavior of stock prices. The model is simple as it only includes two economic explanatory variables, profits and the nominal bond rate. Both are highly significant in statistical and economic terms. Profits is the crucial variable when modeling the strong long-run trend in stock prices whereas the bond rate is important in explaining the large variations in stock prices around this trend, as reflected in the behavior of growth-adjusted stock prices. Despite its simplicity, the model gives rise to a strong cointegration as any mispricing (deviation between actual and fitted stock prices) disappears rather fast. All model coefficients are highly significant, have the right signs and plausible magnitudes and are stable over time. Furthermore, results appear to be robust to the use of a small sample.

We interpret the model as a model for the fundamental level of stock prices, defined as the equilibrium level which stock prices will revert to in the long run, based on the current levels of profits and the bond rate. Our definition of fundamental stock prices is empirically motivated as the point of reference is the way in which stocks have been priced over a historical sample. Hence, when determining whether stocks are fairly valued or not we examine whether the current formation of stock prices differs significantly from the way in which stocks have been priced historically. The fundamental stock price may therefore also be given the interpretation that it

determines what the level of stock prices 'should be' when judged from the historical experience about the relationship between stock prices and the prevailing levels of the two fundamentals variables. We take results as evidence that we have established a good model for this fundamental level of stock prices.

As one application, the evident split of the Danish stock market into a bear market period before 1980 and a subsequent bull market period can be explained by the behavior of the bond rate. The decline in the bond rate from 1995 to 1998 also contributes to an understanding of the recent surge in the stock market. By late 1998, stock prices were roughly fairly valued, that is, in line with their fundamental levels. However, this only occurred after a substantial 'overshooting' in 1997. It is important to notice that a fair valuation of stocks by 1998 does not exclude the possibility that stock prices will decline in the future. The proper interpretation is that we do not find evidence of a significant mispricing error in the current level of stock prices which *in itself* will trigger a decline in stock prices as they adjust to their long-run equilibrium level. However, as a hypothesis we might observe (say) an increase in the bond rate. By reducing the fundamental level of stock prices, this may lead to a decline in actual stock prices.

In spirit, our long-run model is closely related to a model presented in a recent market study by a major Danish bank, see Valgreen (1999). He estimates a long-run relation for stock prices using nominal GDP (GDP at market prices for the total economy) and a nominal bond rate (5-year maturity) as explanatory variables. This formulation comes close to ours. The main differences are that we take account of variations in the profit share and include a trend as a proxy for the growth in the nominal value of outstanding stocks. Valgreen (1999) estimates the relation for Denmark over the period from 1981 to (early) 1999 while imposing a coefficient of one to GDP. However, using the two-step procedure of Engle and Granger (1987), the estimated relation is rejected as a cointegrating relation at any conventional significance level. The lack of cointegration might possibly be explained by the short sample.

In modeling stock prices, we use the nominal bond rate rather than a real interest rate measure as the relevant discount rate of investors. This is motivated by the empirical finding that the bond rate performs well in explaining actual stock price behavior and, in particular, performs better than the set of common real interest rate proxies we have examined. The explanatory power of the nominal bond rate could be interpreted as evidence of 'money illusion' on behalf of stock investors, a possibility that has also been addressed for the US stock market by Modigliani and Cohn (1979) and Ritter and Warr (1999). The result that the increase in the bond rate can account for the bearish stock market period before the early 1980s is consistent with the findings of Modigliani and Cohn (1979). They refer to the increase in the nominal bond rate when explaining the poor performance of the US stock market during the 1970s. Ritter and Warr (1999) explain the US bull market over the last two decades by the decline in inflation. This parallels our conclusion on the Danish bull market because the decline in the nominal bond rate since the early 1980s mainly reflects the contemporaneous decline in inflation. Following Ritter and Warr (1999), the Danish bull market could therefore be interpreted as the outcome of a reduced impact of 'money illusion' on stock prices.

In terms of the definition of fundamental stock prices, there is a difference to Modigliani and Cohn (1979) and Ritter and Warr (1999). In the latter studies, fundamental stock prices have a theoretical foundation as they are calculated on the basis of a stylized Gordon Growth type of model. Their definition of fundamental stock prices is intended to capture the valuation of stocks by a rational investor (within their specific theoretical framework). On the other hand, we have used an empirically motivated definition of fundamental stock prices. This implies that any 'irrational' components in stock prices, e.g. those arising from 'money illusion', are actually part of our measure of fundamental stock prices as long as they show up as empirically significant and regular components in actual stock price behavior. In particular, it means that the nominal bond rate and not a real interest rate measure is used as a fundamentals variable. This conceptual difference, for instance, explains why Ritter and Warr (1999) find that stocks are severely undervalued in the early 1980s whereas we find that stocks are close to their fundamental levels in (say)

1982²⁷. An advantage of using an empirical rather than a theoretical approach is that the valuation of stocks is not theory-dependent. It depends on a specific empirical model but a model that has proved useful in explaining actual stock price behavior over a historical sample.

It should be emphasized that even though results may suggest so, we have not provided strict evidence in favor of 'money illusion'. Thus, we have not explicitly addressed the validity of this hypothesis. Moreover, the reason why the nominal bond rate performs so well in explaining actual stock price behavior may simply be that it provides a good proxy for the latent real discount rate used by a rational investor and, in particular, a better proxy than the real interest rate proxies we have examined. To quantify the discount rate of a rational investor is inherently difficult because we have to take account of his *rational expectations* about future inflation and real growth. In the case of a risk averse investor, we also have to include a risk premium on stocks, all of which are unobservable. A further complication may arise from taxation. Hence, if investment income is taxed, a rational investor will use an after-tax discount rate. The bottom line is that the discount rate of a rational investor can be a highly complex after-tax and risk-adjusted *ex ante* real discount rate and it may simply be the case that the nominal bond rate shows a higher correlation with this 'true' discount rate than the real interest rate proxies we have examined. Therefore, the empirical significance of the nominal bond rate may not necessarily be inconsistent with rational asset pricing. Our study does not differentiate between the 'proxy' and the 'money illusion' explanations. In particular, results do not depend on which explanation is the relevant one. However, determining whether or not stock investors suffer from 'money illusion' is obviously an interesting and important issue for future research.

²⁷ The (possible) presence of 'irrational' components in our measure of fundamental stock prices, of course, raises the subtle question whether a rational investor can beat the market, i.e., gain an abnormal return even in a situation where the market is deemed fairly valued. The answer is not obvious as indicated by the results of De Long *et al.* (1990). They show that 'noise traders' which by definition are irrational can actually survive in a market where rational, but risk averse investors take positions against them. Thus, in the end stock prices will persistently contain an irrational component ('noise') and, moreover, rational investors will not gain an abnormal return. An analysis of this question is outside the scope of the present paper.

Olesen (1998) tests the hypothesis that stocks hedge against inflation in the long run, that is, that the real value of stocks will remain unaffected by inflation in the long run. Using data for Denmark in the post-World War II-period, the result is a strong support to the hypothesis. As standard in the literature, the conclusion that stocks hedge against inflation is based on a *partial* definition of an inflation hedge. That is, stock prices are shown to respond one-for-one to a change in the general price level in a situation where all other explanatory variables (in Olesen (1998), real production and the real discount rate) are kept constant. In interpreting the inflation hedge result, it is useful to distinguish between two scenarios, *viz.* a permanent change in the *general price level* and a permanent change in the *rate of inflation*. The evidence in Olesen (1998) shows that real stock prices are unaffected in the long run in both scenarios *provided that* the remaining explanatory variables are unaffected. The latter seems as a reasonable assumption as long as the remaining variables are real magnitudes such as real production and a real discount rate. Hence, it is standard in most of the macroeconomic literature to assume that real variables are unaffected by nominal variables in the long run. This is basically the property of 'Classical Dichotomy' between the real and the money sectors, which is the cornerstone of the Neoclassical-Keynesian Synthesis of traditional macroeconomic theory, see e.g. Grandmont (1988).

As argued above, the evidence that the nominal bond rate is important in explaining actual stock price behavior may be interpreted as an indication of 'money illusion' on behalf of stock investors. *If one accepts* 'money illusion' as a behavioral assumption, this will in general inflict on standard results in macroeconomic theory including a violation of the 'Classical Dichotomy' property. 'Money illusion' also has implications for the interpretation of the inflation hedge property of stocks. If we consider the scenario where *the rate of inflation* is subject to a permanent (say) increase, we should *a priori* expect an increase in the nominal bond rate in the long run, as implied by the Fisher Parity. In the case of 'money illusion', investors will use this higher nominal bond rate as their discount rate when pricing stocks and real stock prices will as a result decline in the long run. Hence, the real value of stocks will *not*

remain unaffected by changes in the inflation rate²⁸. However, in the scenario of a permanent change in the *general price level*, it seems as a reasonable assumption that the nominal bond rate is unaffected in the long run even with 'money illusion'. As a consequence, real stock prices will eventually remain unaffected. Thus, we should not expect 'money illusion' to be detrimental to stocks' purchasing power when considering changes in the general price level.

If we do not accept the assumption of 'money illusion' but rather interpret the explanatory power of the nominal bond rate as the result of a high correlation with the latent real discount rate used by a rational investor, stocks will as before retain a stable real value in both scenarios. This suggests a possible reconciling interpretation of the inflation hedge result in Olesen (1998). Stocks will in the long run retain a stable purchasing power in the event of inflation as long as we consider variations in the general price level or the rate of inflation within a given inflation regime, defined as a regime where the long-run average inflation rate is fixed. When we consider shifts in the inflation regime, that is, permanent changes in the long-run level of inflation, the response in real stock prices depends on stock investors' discount rate and, in particular, how it adjusts to the regime shift. If investors suffer from 'money illusion', real stock prices will change. If investors are rational, stocks will retain their purchasing power.

We have set up a model which has proved helpful in explaining certain historical features of the Danish stock market. A related question is to what extent the model is also useful for prediction purposes. Being able to predict future stock returns is, obviously, of interest to stock investors. It may also be important to policy makers if they are concerned that an overvalued stock market might collapse and that this will have a severe negative impact on the real economy and the financial sector. The policy relevance of valuing and predicting the stock market has become increasingly recognized in recent years, in particular, in the US and has, for instance, been advocated on several occasions by the Chairman of the US Federal Reserve System,

²⁸ In the case of 'money illusion', we might observe a long-run impact on real production which also has implications for real stock prices.

see e.g. Sellon and Buskas (1999). This paper provides a simple empirical model for the valuation of the Danish stock market. The mere essence of cointegration implies some degree of predictability because we can predict that any mispricing error by definition will be eliminated over time. However, the issue of predictability needs a closer examination and needs, in particular, to be quantified.

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Appendix A: Unit Root Tests

The data series have been tested for unit roots, using both the Z_t -test by Phillips and Perron (1988) (PP), cf. Table A.1, and the unit root test by Kwiatkowski *et al.* (1992) (KPSS), cf. Table A.2. The former test tests the null hypothesis of a unit root against the stationary alternative while the latter tests the stationary specification as the null, treating the unit root specification as the alternative hypothesis. Both tests are performed for a prior specified lag length, see the notes to the two tables. Given the sample size, we report test statistics for up to six lags.

< Table A.1 >

< Table A.2 >

Using conventional significance levels, the PP test gives firm evidence that all series - stock prices, profits and the bond rate - are integrated of order 1, i.e., that the series are non-stationary in levels but stationary in first differences. The KPSS test confirms this result for stock prices while the evidence for profits depends on the lag length, as the KPSS test is indecisive between a unit root process and a stationary process with long lags. However, the KPSS test may suffer from a very low power for long lag lengths, cf. the simulations in Kwiatkowski *et al.* (1992), so we decide on the unit root alternative. For the bond rate, the KPSS test indicates that this series is integrated of order 2, i.e., that the series is also non-stationary in first differences, when not allowing for a deterministic trend. However, this test result is entirely driven by the apparent negative trend in Δb_t over the sample (cf. Figure 3) as the KPSS test leads to the unambiguous result that Δb_t is stationary when allowing for a trend in this series.

We, therefore, conclude the evidence that all series are integrated of order 1. This conclusion can be confirmed by the augmented tests of Dickey and Fuller (1979) (results available on request).

Appendix B: An ECM for Stock Prices

Based on the cointegrating relation identified by the Johansen (1996) analysis, we can estimate a (structural) Error-Correction-Model (ECM) for the short-run (1-year) changes in (log-to-)stock prices, i.e., for the short-run capital gains, using as explanatory variables the lagged and contemporaneous changes in the 'fundamentals' variables (profits and the bond rate) and an ECM-term which captures the adjustment in stock prices to long-run equilibrium. In this Appendix, we provide preliminary results, where we, for simplicity, have used a single-equation approach to estimate an ECM (rather than a more appropriate simultaneous approach, such as e.g. FIML estimation of a structural VAR model). We recognize that this approach may lead to inconsistent estimates of the dynamic coefficients because contemporaneous values of stock prices and the fundamentals variables may be determined simultaneously and, in this case, the regressors will be endogenous (correlated with the residual term in the stock price equation).

We use a 'general-to-specific' approach where we start out with a fairly general ECM for stock prices, having included up to five lags of the first differences of stock prices, profits and the bond rate. We estimate a structural ECM as we include the contemporaneous changes in the fundamentals variables. We also include two impulse dummies, one for 1972 and one for 1983, to exclude the extraordinary high capital gains observed in these two years. These extreme observations can be attributed to exceptional changes in the Danish economy, cf. Note 1 in the text, and we do not intend to model these changes. Having estimated the general model, we test for significance of the individual terms (t-test) and, consecutively, remove the insignificant terms. Using a 5% significance level, we end up with the following parsimonious ECM for stock prices, estimated by OLS over the period 1952 to 1998 (standard errors in parentheses):

$$(B1) \quad \Delta q_t = \underset{(0.046)}{0.097} - \underset{(0.12)}{0.668} \Delta b_t + \underset{(0.11)}{0.202} \Delta b_{t-1} + \underset{(0.44)}{1.04} \Delta \pi_t - \underset{(0.38)}{1.56} \Delta \pi_{t-2} \\ + \underset{(0.14)}{0.298} d72_t + \underset{(0.13)}{0.483} d83_t - \underset{(0.15)}{0.511} ecm_{t-1} + \varepsilon_t$$

ecm_{t-1} is the (lagged) error-correction-term, defined as the residual (in the previous period) from the cointegrating relation for stock prices in (4). $d72_t$ and $d83_t$ are the impulse dummies and ε_t is the residual of this dynamic equation.

< Figure B.1 >

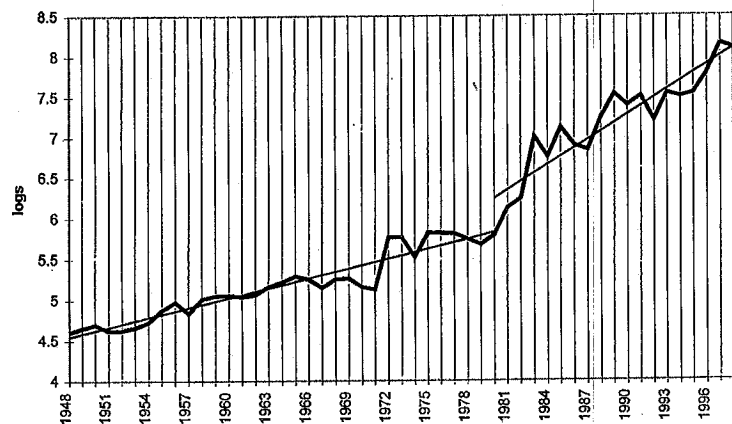
< Table B.1 >

The fit of the ECM is depicted in Figure B.1 and the outcome of specification tests are reported in Table B.1. The specification tests show that the model is well specified as there is no evidence of serial correlation or heteroskedasticity in the residual term. Moreover, the model gives a good fit to the short-run fluctuations in stock prices. Excluding the two dummy years 1972 and 1983, the model explains 56% (= the coefficient of determination) of the sample variation in capital gains (when including 1972 and 1983, the coefficient of determination is 73%). This appears to be a satisfactory result, when recalling that the model explains the fluctuations in stock prices over a short (1-year) horizon.

With the exception of Δb_{t-1} (which is retained in the model to keep it well specified), all coefficients are significant at the 5% significance level. The most significant explanatory variable is the contemporaneous change in the bond rate, Δb_t (t-test statistic of 5.4). The bond rate exerts a strong negative influence on stock prices, with a 1% *relative* increase in the bond rate *ceteris paribus* leading to a 0.67% decline (continuous rate) in stock prices over the same 1-year period (and *vice versa*). The contemporaneous change in profits, $\Delta \pi_t$, has a significant positive effect on stock prices in the short run, with a change in profits inducing a roughly proportional change in stock prices over the same period (coefficient of 1.04). We find that the short-run effects from the fundamentals variables are of the same sign as the long-run effects, cf. (4), but according to the point estimates stock prices are (slightly) less sensitive to changes in the fundamentals variables in the short-run (in the contemporaneous period) than in the long-run, suggesting an element of (immediate) 'underreaction'.

We observe that the error-correction-term is highly significant with a t-test statistic of 4.0. This shows the importance of incorporating the long-run equilibrium relation for stock prices when modeling the short-run capital gains. The estimated adjustment coefficient of -0.511 is consistent with a slow and smooth adjustment to the equilibrium or fundamental level of stock prices in the long-run. The estimate implies that roughly half of any mispricing error, relative to the cointegrating relation, is eliminated, *ceteris paribus*, within one year which, at an informal level, seems consistent with the evidence on the duration of a mispricing provided in section 4.

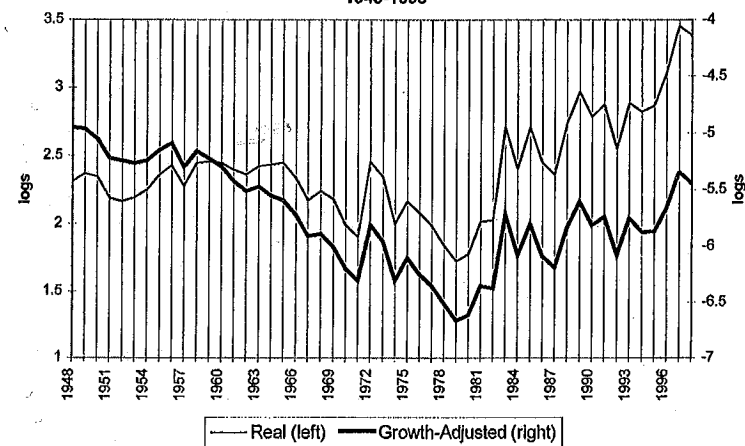
**Figure 1. Stock Prices
1948-1998**



Note: Straight lines indicate trend in stock prices over subsamples 1948-1980 and 1980-1998, respectively. The trend is estimated by a regression (OLS) of stock prices on a constant term and a time trend.

Source: All-Share Stock Price Index by Statistics Denmark.

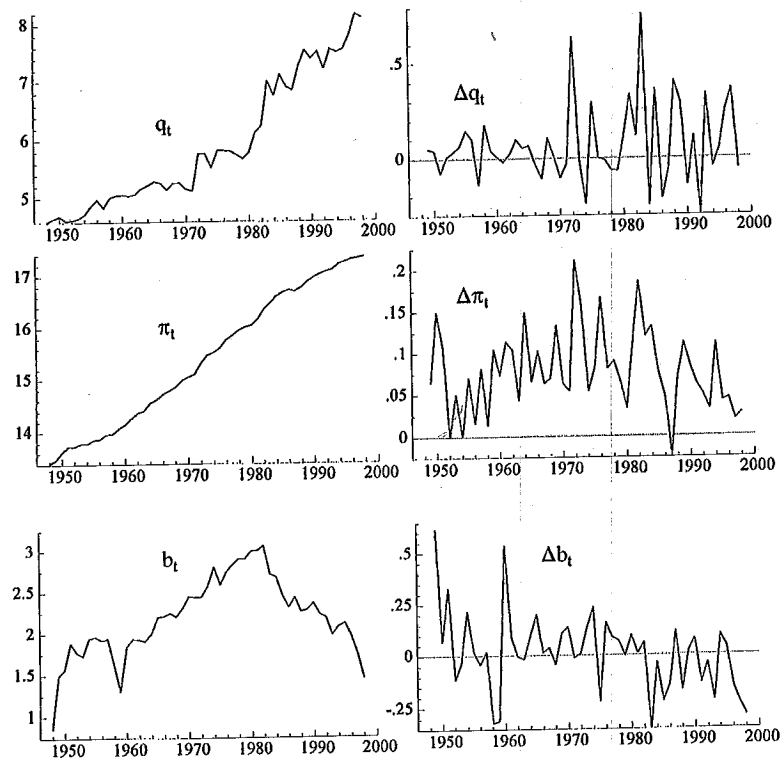
**Figure 2. Real and Growth-Adjusted Stock Prices
1948-1998**



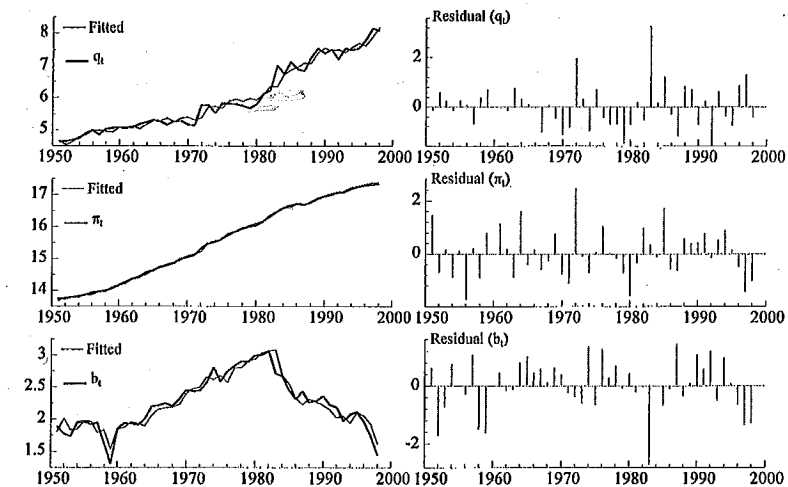
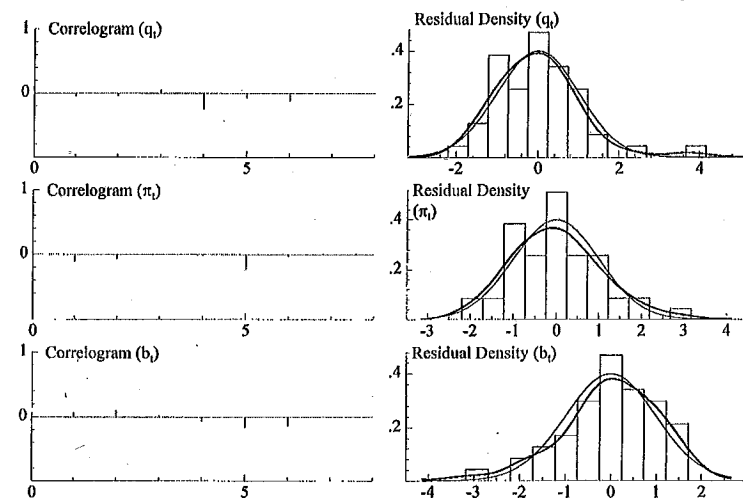
Note: Real and Growth-Adjusted stock prices show the level of stock prices deflated by the general price level and nominal income, respectively.

Figure 3. The Data

Levels and first differences of (by row and from the top) stock prices, profits and the bond rate (all in logs), 1948-1998.

**Figure 4. Diagnostic Graphics for the VAR Model**

Actual and fitted values (in levels), residuals, residual correlogram and residual density for the equation for (by row and from the top) stock prices, profits and the bond rate.

**Figure 4, continued.**

Note: Density plots include standard normal density for comparison (thin curve).

Figure 5. Recursive Estimates of the Eigenvalues
Eigenvalues labelled by numbers 1 to 3, 1970-1998.

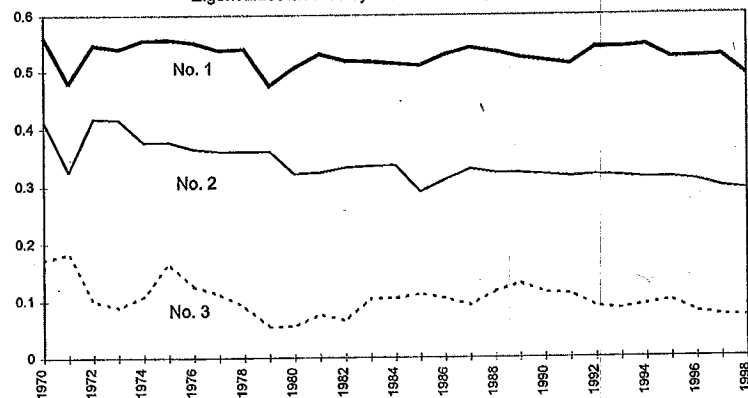
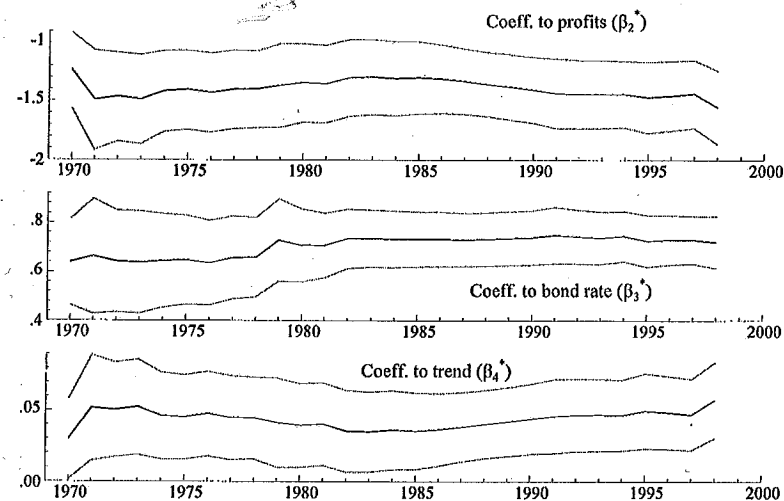


Figure 6. Recursive Estimates of the Cointegrating Vector

Recursive point estimates (solid line) and 95% confidence bands for the coefficient to, respectively, profits, the bond rate and the time trend, 1970-1998.

Cointegrating rank 1, cointegrating vector normalized on stock prices, and profits restricted to be exogenous for long-run parameters ($\alpha_2=0$).



**Figure 7. Stock Prices: Actual and Long-Run Fit
1951-1998**

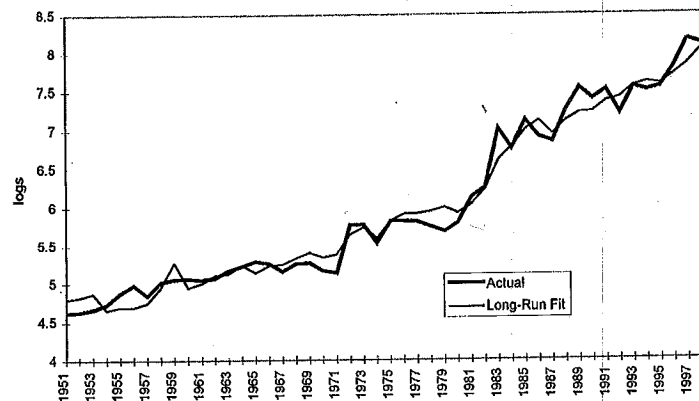
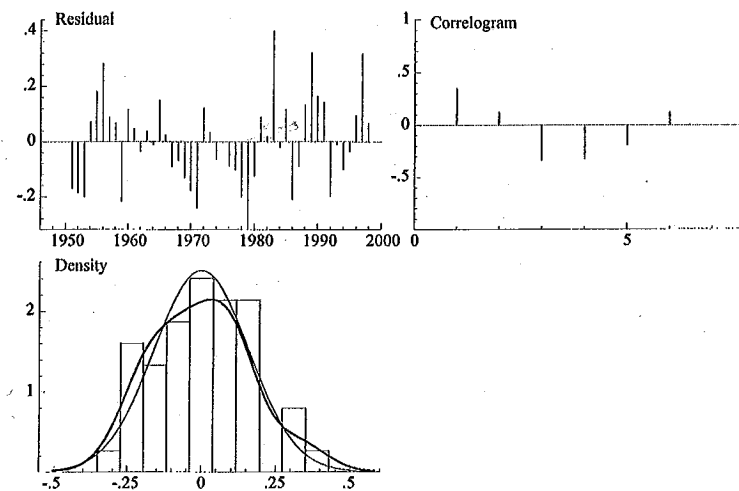


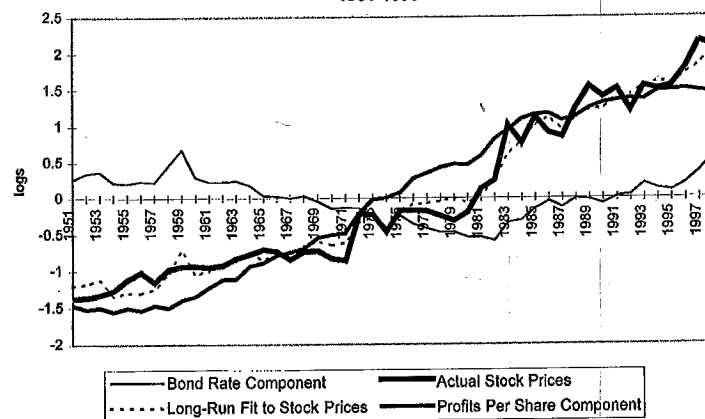
Figure 8. Mispricing Errors

Values, correlogram and density of residuals from cointegrating relation, 1951-1998.



Note: Density plot includes standard normal density for comparison (thin curve). The sample standard deviation of the residuals (using T-1 as degrees of freedom) is 0.160.

**Figure 9. Stock Prices and Contributions by Components
1951-1998**



Note: Series shown as deviations from respective sample means.

**Figure 10.a. Growth-Adjusted Stock Prices:
Actual and Long-Run Fit**

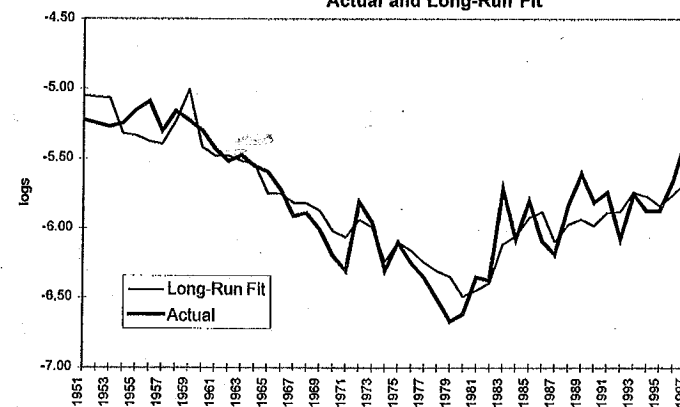


Figure 10.b. Contributions by Components

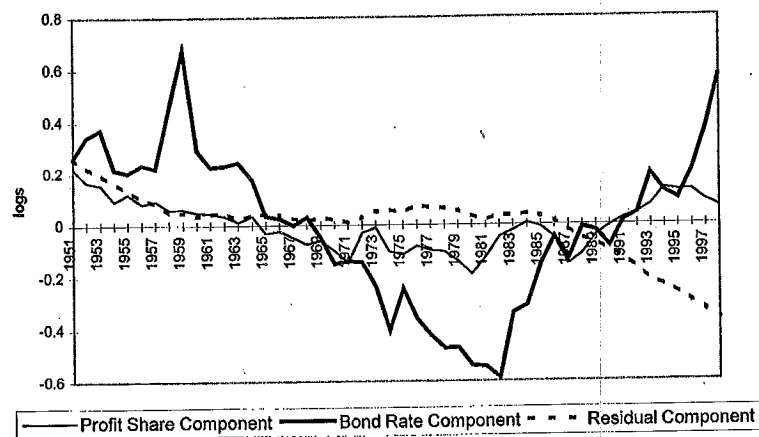
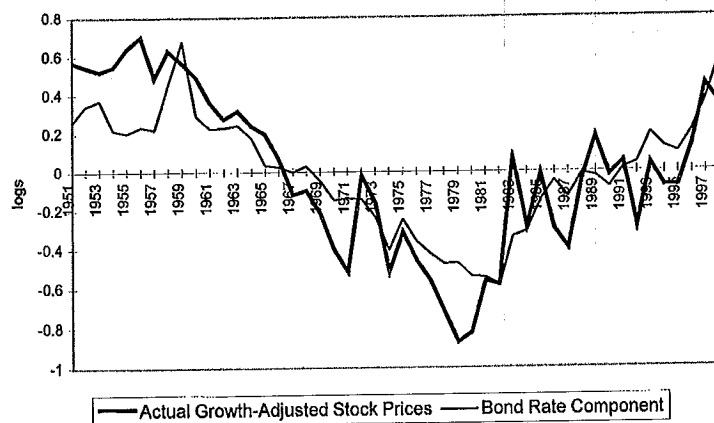


Figure 10.c. Growth-Adjusted Stock Prices and the Contribution by the Bond Rate



Note: In Panels b and c, the series are shown as deviations from respective sample means.

Figure 11.a. Alternative Discount Rate Measures 1951-1998

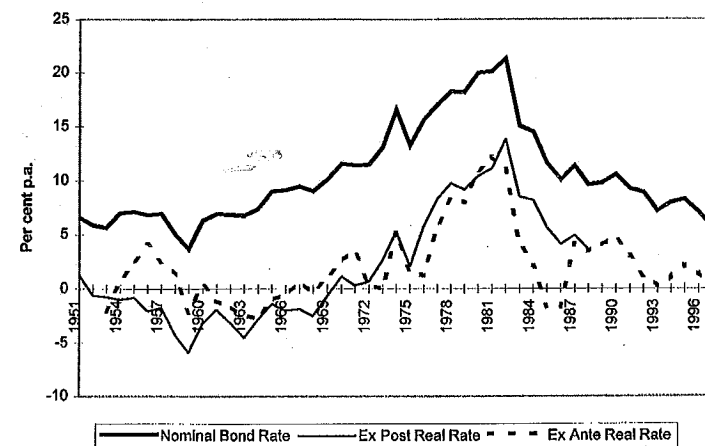


Figure 11.b. Cointegration Residuals:
Ex Ante Real Rate vs. Nominal Bond Rate

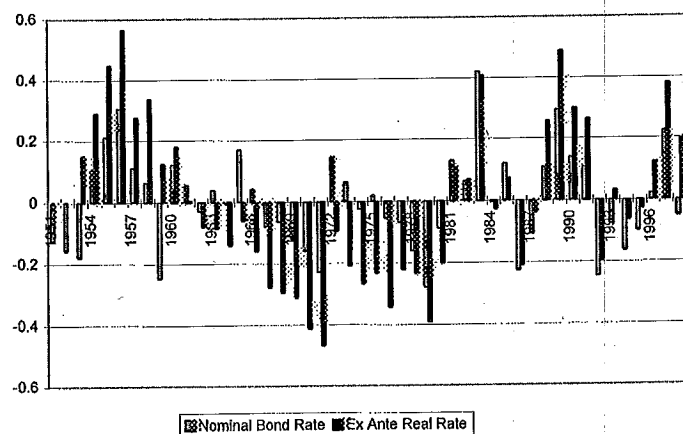
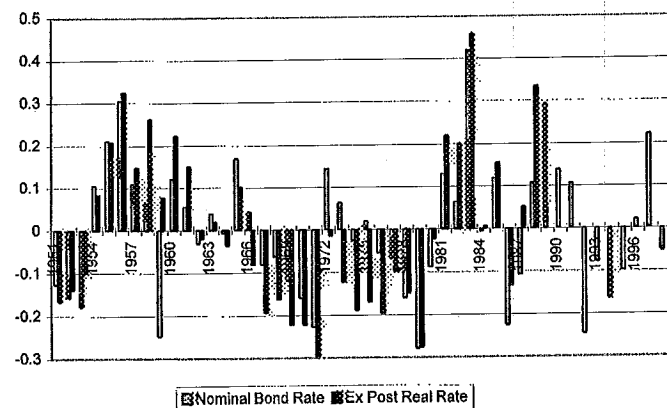


Figure 11.c. Cointegration Residuals:
Ex Post Real Rate vs. Nominal Bond Rate



Note: Ex ante real rate = difference between the 10-year nominal bond rate and the growth rate in profits in the preceding 5-year period. Ex post real rate = difference between the bond rate and the growth rate in profits for the same 10-year period.

Table 1. Specification Tests of the VAR Model
Estimation sample 1951-1998

Multivariate tests:

Vector Autocorrelation order 1	$F(9,75) =$	0.81	[0.61]
Vector Autocorrelation order 2	$F(18,79) =$	1.20	[0.28]
Vector Autocorrelation order 4	$F(36,65) =$	1.42	[0.11]
Vector Autocorrelation order 6	$F(54,48) =$	1.32	[0.16]
Vector Heteroskedasticity (squares)	$F(120,65) =$	0.45	[1.00]
Normality	$\chi^2(6) =$	8.65	[0.19]

Univariate tests:

	Δq_t	$\Delta \pi_t$	Δb_t
Autocorrelation order 1, $F(1,35):$	0.65 [0.43]	2.97 [0.09] *	0.98 [0.33]
Autocorrelation order 2, $F(2,34):$	0.37 [0.70]	1.56 [0.22]	0.69 [0.51]
Autocorrelation order 4, $F(4,32):$	1.24 [0.31]	0.74 [0.57]	0.37 [0.83]
Autocorrelation order 6, $F(6,30):$	1.33 [0.27]	0.78 [0.59]	0.43 [0.85]
ARCH (1), $F(1,34):$	0.31 [0.58]	0.48 [0.49]	0.33 [0.57]
Heteroskedast. (squares), $F(20,15):$	0.58 [0.87]	0.49 [0.93]	0.43 [0.96]
Normality, $\chi^2(2):$	9.58 [0.01] ***	1.65 [0.44]	4.52 [0.10]

Goodness-of-fit:

ρ	0.99	1.00	0.96
σ_e	0.203	0.040	0.138

Note: The VAR model has a lag length of 3 and includes an impulse dummy for 1960. F-tests are small sample approximations to Lagrange Multiplier tests, being adjusted for degrees of freedom. Normality test by Doornik and Hansen (1994). For a description of the tests, see Doornik and Hendry (1997). Critical significance levels in brackets. *, ** and *** indicates misspecification at the 10%, 5% and 1% significance level, respectively. ρ denotes the correlation between actual and fitted values for each equation. σ_e is the estimated standard deviation of the residuals.

Table 2. Cointegration Analysis in the VAR Model
Estimation sample 1951-1998

Cointegrating rank:			
Rank(II) ($r =$)	0	1	2
Eigenvalue	0.49	0.29	0.07
Trace test ¹⁾	52.5 ***	20.1 **	3.5
Trace test (adj. for df.) ²⁾	42.7 **	16.3	2.9
95% critical value (standard)	42.2	25.5	12.4
95% critical value (simulated)	36.5	19.9	NA
Standardized eigenvectors β^* :			
	β_1^*	β_2^*	β_3^*
q_t	1.000	-0.050	1.212
π_t	-1.504	1.000	1.006
b_t	0.699	-0.435	1.000
t	0.053	-0.079	-0.133
Standardized loadings $\alpha:$ ³⁾			
	$\beta_1^* X_t^*$	$\beta_2^* X_t^*$	$\beta_3^* X_t^*$
Δq_t	-0.361	-0.438	0.062
$\Delta \pi_t$	-0.035	-0.164	-0.004
Δb_t	-0.557	0.094	-0.044

Note: Maximum Likelihood Estimation by the procedure of Johansen (1996). The trace tests test for each value of r the null hypothesis $H_0: \text{rank}(\Pi) \leq r$ against the alternative $H_A: \text{rank}(\Pi) > r$. The null is rejected iff the trace statistic is larger than the critical value. The first (standard) critical values are from Johansen (1996, Table 15.4). The second (simulated) set of values are obtained using the simulation program DisCo, taking account of the dummy for 1960. *, ** and *** denote rejection of the null at the 10%, 5% and 1% significance level, respectively, using the simulated critical values.

The standardized eigenvectors are normalized on the diagonal wrt. the endogenous variables. Corresponding α -loadings.

NA: Not available in DisCo.

¹⁾ The asymptotic trace test of Johansen (1996).

²⁾ Small sample approximation to the asymptotic trace test, obtained by adjusting for degrees of freedom, cf. Reimers (1992).

³⁾ $X_t^* = (q_t, \pi_t, b_t, t)'$

Table 3. Hypothesis Testing

Parameter restriction	α -loadings			Cointegrating vector			
	α_1	α_2	α_3	β_1^*	β_2^*	β_3^*	β_4^*
(I) Unrestricted	-0.361 (0.299) [0.241]	-0.035 (0.065) [0.649]	-0.557 (0.196) [0.004]	1 - [0.000]	-1.504 (0.172) [0.000]	0.699 (0.059) [0.000]	0.053 (0.015) [0.010]
(II) $\alpha_2=0$	-0.287 (0.272) [0.270]	0 - -	-0.558 (0.192) [0.004]	1 - [0.000]	-1.554 (0.171) [0.000]	0.720 (0.058) [0.000]	0.057 (0.015) [0.000]
(III) $\alpha_2=\alpha_3=0$	-0.867 (0.160) -	0 - -	0 - -	1 - [0.000]	-1.556 (0.201) [0.000]	0.687 (0.068) [0.000]	0.056 (0.017) [0.004]

Note: All estimations are conducted with cointegrating rank restricted to 1. In (II), profits are assumed to be exogenous for the long-run parameters. In (III), both profits and the bond rate are restricted to be exogenous. Numbers in parentheses are indicative standard errors of the parameter estimates. Numbers in squared brackets give the critical significance level of a Likelihood Ratio test for significance, i.e., for testing the null that the specific parameter is zero. Cointegrating vector is normalized on stock prices. When testing for significance of stock prices, the cointegrating vector is normalized on the bond rate.

Table 4. Estimates of the Cointegrating Relation: Single-Equation-Analysis

The regression is:

$$q_t = \beta_0 + \beta_1 \pi_t + \beta_2 b_t + \beta_3 f_t + \sum_{i=1}^{N_1} \gamma_{1i} \Delta \pi_{t-i} + \sum_{i=1}^{N_2} \gamma_{2i} \Delta b_{t-i} + \sum_{i=1}^{N_3} \phi_i ecm_{t-i} + v_t, \text{ where } ecm_t = q_t - \beta_0 - \beta_1 \pi_t - \beta_2 b_t - \beta_3 f_t$$

The static cointegrating regression is augmented by current, leaded and lagged first differences of π_t and b_t , and lagged equilibrium-correction-terms ecm_t . The augmenting terms, as indicated by N_1 , N_2 and N_3 , are shown in the first column.

Regression	Estimation sample	Coefficient Estimates (standard errors)			
		β_0	β_1	β_2	β_3
<i>a. Static</i>	1951-1998	-14.2 (3.1)	1.50 (0.24)	-0.786 (0.070)	-0.0502 (0.021)
<u>PLOLS:</u>					
<i>b. $N_1=N_2=0$</i>	1951-1998	-13.9 (2.5)	1.47 (0.19)	-0.793 (0.063)	-0.0478 (0.017)
<i>c. $N_1=N_2=1$</i>	1951-1997	-16.6 (2.9)	1.69 (0.23)	-0.936 (0.079)	-0.0661 (0.019)
<i>d. $N_1=N_2=2$</i>	1951-1996	-15.9 (2.1)	1.64 (0.16)	-0.803 (0.057)	-0.0659 (0.014)
<u>PLNLS:</u>					
<i>e. $N_1=N_2=0, N_3=3$</i>	1951-1998	-12.9 (3.2)	1.39 (0.25)	-0.752 (0.081)	-0.0410 (0.022)
<i>f. $N_1=N_2=1, N_3=3$</i>	1951-1997	-18.0 (5.2)	1.80 (0.41)	-0.926 (0.14)	-0.0727 (0.034)
<i>g. $N_1=N_2=2, N_3=4$</i>	1952-1996	-18.0 (6.4)	1.80 (0.50)	-0.830 (0.14)	-0.0770 (0.043)

Note: Entry a shows results from estimating (OLS) the static cointegrating regression, corresponding to the first step of the Engle and Granger (1987) two-step procedure (no augmentation). Standard errors of coefficient estimates are computed by OLS and are indicative only. Entries b through d give results from the Phillips and Loretan (1991) OLS procedure (PLOLS). $N_3=0$ in all three cases. The regressions are estimated by OLS. Standard errors take account of serial correlation in the disturbance term (v_t) by the method suggested by Hamilton (1994, p. 608f). An AR(3) (AR(4)) specification is used in cases b and c (case d). Entries e through g report results from the Phillips and Loretan (1991) NLS procedure (PLNLS). The regressions are estimated by the Non-Linear Least Squares method. Standard errors are calculated from numerical derivatives of the sum of squared residuals.

Table A.1. Phillips and Perron (1988) Z_t -Test for Unit Root 1948-1998

Series:	Lag length (l)						
	0	1	2	3	4	5	6
<u>Levels:</u>							
q_t (T)	-2.35	-2.18	-2.16	-2.15	-2.13	-2.12	-2.13
π_t (T)	-1.02	-1.21	-1.28	-1.34	-1.38	-1.42	-1.46
b_t	-2.55	-2.56	-2.57	-2.58	-2.60*	-2.62*	-2.64*
<u>First differences:</u>							
Δq_t	-8.96***	-8.97***	-9.04***	-9.12***	-9.24***	-9.39***	-9.50***
$\Delta \pi_t$	-5.62***	-5.62***	-5.62***	-5.62***	-5.63***	-5.64***	-5.65***
Δb_t	-7.22***	-7.23***	-7.27***	-7.26***	-7.25***	-7.22***	-7.20***
<u>Critical test values:</u>							
		10 %	5 %	2.5 %	1 %		
Without trend		-2.60	-2.93	-3.22	-3.58		
With trend		-3.18	-3.50	-3.80	-4.15		

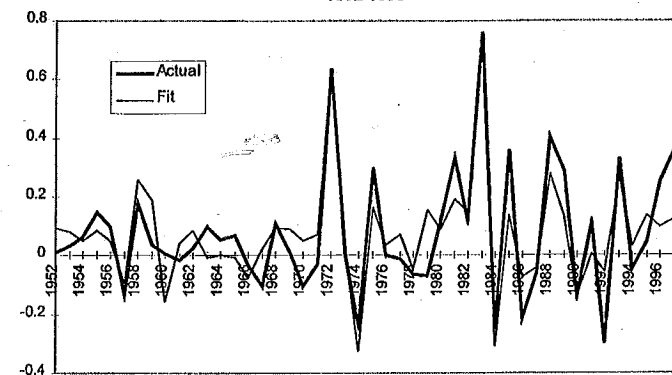
Note: The Phillips and Perron (1988) Z_t -test statistic is a modified t-statistic for the null hypothesis of a unit root ($\rho=1$) in the first order autoregression $x_t = \alpha + \rho x_{t-1} + u_t$ (without trend), respectively $x_t = \alpha + \rho x_{t-1} + \delta t + u_t$ (with trend), where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed and serially correlated up to lag l , see also Hamilton (1994). The null is rejected in favor of the stationary alternative ($\rho < 1$) if Z_t is negative and sufficiently large in numerical value. Critical values are from Hamilton (1994, Table B.6) for a sample size of 50. *, ** and *** denote rejection of a unit root at 10%, 5% and 1% significance level, respectively. All regressions include a constant term. (T) indicates that a deterministic trend is included.

Table A.2. Kwiatkowski *et al.* (1992) Test for Unit Root
1948-1998

Series:	Lag length (<i>l</i>)						
	0	1	2	3	4	5	6
Levels:							
q_t (T)	0.90***	0.51***	0.37***	0.30***	0.25***	0.22***	0.20**
π_t (T)	0.54***	0.29***	0.21**	0.16**	0.14*	0.12*	0.11
b_t	1.89***	1.04***	0.74***	0.58**	0.48**	0.42*	0.37*
b_t (T)	0.93***	0.52***	0.37***	0.30***	0.25***	0.22**	0.20**
First differences:							
Δq_t	0.14	0.19	0.22	0.24	0.26	0.30	0.31
$\Delta \pi_t$	0.25	0.21	0.20	0.19	0.18	0.18	0.17
Δb_t	0.72**	0.69**	0.67**	0.65**	0.63**	0.59**	0.56**
Δb_t (T)	0.06	0.06	0.07	0.07	0.07	0.07	0.07
Critical test values:		10 %	5 %	1 %			
Without trend		0.35	0.46	0.74			
With trend		0.12	0.15	0.22			

Note: The Kwiatkowski *et al.* (1992) test for unit root is a Lagrange Multiplier test of the null hypothesis that the series can be described by a stationary process (possibly around a deterministic trend) of lag length l against the alternative that the process also includes a random walk component. The null of stationarity is rejected in favor of the unit root alternative if the test statistic is sufficiently large. Critical values are from Kwiatkowski *et al.* (1992). *, ** and *** denote rejection of the null (unit root is present) at 10%, 5% and 1% significance level, respectively. (T) indicates that the null hypothesis is trend-stationarity. Otherwise, the null is mean-stationarity.

Figure B.1. Annual Capital Gain on Stocks: Actual and Fit
1952-1998



Note: Capital gain is calculated as the (backward looking) 1-year change in log-to-stock prices.

Table B.1. Specification Tests of the ECM
Estimation sample 1952-1998

Autocorrelation order 1	F(1,38)	=	0.07	[0.80]
Autocorrelation order 2	F(2,37)	=	0.79	[0.46]
Autocorrelation order 3	F(3,36)	=	0.54	[0.66]
Autocorrelation order 5	F(5,34)	=	1.07	[0.39]
ARCH (1)	F(1,37)	=	0.02	[0.89]
Heteroskedasticity (squares)	F(12,26)	=	0.99	[0.48]
Heteroskedasticity (cross-products)	F(22,16)	=	0.57	[0.89]
Normality	$\chi^2(2)$	=	0.04	[0.98]

Memo:

$$\sigma_e = 0.119$$

Note: F-tests are small sample approximations to Lagrange Multiplier tests, being adjusted for degrees of freedom. Normality test by Doornik and Hansen (1994). Critical significance levels in brackets. σ_e is the estimated standard deviation of the residuals.

Annex

A Description of the Stock and Bond Market Data

Annex: A Description of the Stock and Bond Market Data¹

This annex documents briefly the key data series used on the stock and bond markets. The data source is the "Danish Stock Market and Macroeconomic Database" (the 1997, 1998 and 1999 versions) by Steen Nielsen, Jan O. Olesen and Ole Risager, Department of Economics, Copenhagen Business School. This database was originally constructed by Steen Nielsen and Ole Risager for the period 1921-1995 and contains original data for dividend payments and bond returns. The included series for stock prices is compiled by Statistics Denmark. All the data series described in the following are annual.

1. Stock Market Data

1.1. Stock Prices

The series for stock prices is identical to the official All-Share Stock Price Index ("Totalindekset") calculated by the Copenhagen Stock Exchange. This index is published by Statistics Denmark in the Statistical Yearbook ("Statistisk Årbog"), Statistical News ("Statistiske Efterretninger") and the Monthly Review of Statistics ("Statistisk Månedsoversigt"). It is (now) calculated on a daily basis. The annual series in the database uses the prices quoted by the end of December each year (last trading day). Data are available from 1921.

The stock price index describes the overall price development of Danish stocks listed at the Copenhagen Stock Exchange. By 1998, the index comprises all listed stocks excluding stocks in foreign companies, unit trusts ("Investeringsforeninger") and a few holding companies (holding companies with the only purpose of holding stocks in affiliated companies). Nearly 250 companies were included in the index by late 1998. These cover the sectors of Banking, Insurance, Commerce and Services, Shipping, Industry and Investment Trusts. The sample of companies included in the compilation of the index has gradually expanded from around 50 companies in 1921 to nearly all Danish listed companies from 1983 and onwards.

The index is value-weighted, that is, stocks enter the index with their official price weighted in proportion to the share of the overall market capitalization. Weights are changed at emissions and withdrawals from the exchange.

In the entire period, prices have been corrected to remove the effect of the timing of dividend payments. However, there has been a change in the correction method. Until 1983 a standard rate (6%) of equity value was used for expected dividends. This was changed to share-specific rates based on previous years' dividends. From 1983 a correction has also been made in case of emissions with price discounts to previous stockholders to make return calculations reflect

¹ This annex draws extensively on a note written together with Steen Nielsen and Ole Risager, Copenhagen Business School, as a documentation of the database. The raw data set has not been published but is available on request for the purpose of reproducing and checking the results of this dissertation.

actual return for the pre-emission investor.

Further documentation of the All-Share Stock Price Index can be found in "Retningslinier for beregning af aktieindeks - Totalindeks og branche-indeks", January 1998, Copenhagen Stock Exchange (in Danish).

1.2. Dividend Yield

To calculate the dividend yield, a sample of companies was chosen. The average dividend yield in this sample is then viewed as an estimate of the dividend yield on the overall market portfolio of stocks.

The sample of companies is listed at the end of this annex, cf. section 3. In any year, our sample covers between 50% and 80% of the total market capitalization at the Copenhagen Stock Exchange ("Hovedbørsen" and later "Børs I").

For each company, the dividend yield is defined as dividend paid during the calendar year divided by the stock price quoted by the end of the previous year. Thus, we assume, that dividend payments do not earn interest until the following December. A deduction is made in the beginning-of-period price whenever a discounted emission with dividend rights in the current year has taken place. This takes account of the fact that in this case the stockholder is receiving dividend payments on a larger portfolio.

Individual companies' dividend yields are finally aggregated by using their share of total market capitalization in the sample as weights.

Data are available from 1921.

1.3. Total Stock Return

The total stock return equals the sum of the capital gain (calculated as the relative change in the All-Share Stock Price Index, cf. section 1.1) and the dividend yield. That is, the one-year return expresses the sum of the capital gain over the period and the dividends paid during the year as a percentage of last year's price. The return relates to an investment by the end of December in a given year.

The total return may be underestimated due to the assumption that dividends are not reinvested within the year. This bias may be important in particular until the beginning of the 1980s, where the dividend yield plays an important role for the total stock return. To illustrate the bias, consider the following simple example:

Over the period 1922-96, the average (arithmetic) dividend yield is 4.7% under the assumption that dividends are not reinvested within the year. The average capital gain equals 6.6% and the average total return, hence, equals 11.3%.

Case 1. Suppose dividends are paid out after 6 months. Suppose share prices increase "linearly" such that the semiannual increase equals 3.25%. In case dividends are reinvested when paid out, the yield associated with dividend payments and reinvestment of these funds

equals 4.85%. On top of this we have the pure capital gain equal to 6.6%. Total return is therefore 11.45% or 0.15% more than the estimate without reinvestment of dividends.

Case 2. Suppose dividends are paid out after 3 months. In this case, the quarterly growth rate in stocks equals 1.61%. The resulting bias is 0.20%.

This bias makes our total return series a conservative estimator of the market return. However, there may be a bias that works in the other direction, namely, the bankruptcy bias.

Business bankruptcies were widespread in the beginning of the 1920s. The most famous case is the default of Landmandsbanken (the largest Bank) in 1922. Statistics Denmark therefore constructed two share price indexes; one without and one with Landmandsbanken, where the latter takes into account the losses associated with the bankruptcy of Landmandsbanken. We use the latter index in the calculation of the capital gain component. On the basis of our own data we have also checked Statistics Denmark's calculation of the fall in the share price from December 1921 to December 1922 (equals 29.1%). By calculating the value weighted fall in share prices using the 26 shares we have for this year and using all available information including the bankruptcy of Landmandsbanken and (partial) bankruptcy of other firms (e.g. Superfos), we arrive at exactly the same estimate as Statistics Denmark.

Statistics Denmark does not, however, report how it has dealt with this problem subsequently. Hence, it is possible that the share price index is upward biased (in case there has not been proper adjustments for business failures). Casual evidence suggests that the magnitude of this bias roughly equals the bias associated with the treatment of dividends, when we disregard abnormal years like 1922, cf. Parum (1998)², who estimates the bankruptcy bias to be between 0.35% and 0.45% p.a. over the period 1990-1996.

2. Bond Market Data: 1, 5 and 10-Year Bond Returns

Bond returns are available for the 1, 5 and 10 year investment horizons. The bond return is constructed as the annualized yield-to-maturity on government bonds with approximately 1, 5 and 10 years to maturity, respectively. The returns relate to investments by the end of December each year.

In cases where no bonds with the desired maturity exist, the Government bond that comes closest in maturity is chosen. Consequently, the maturity of the 5-year horizon series typically varies from 4 to 6 years. The lowest maturity is 1 year and 7 months and the highest is 10 years and 8 months, both occurring in the thirties where the outstanding stock of Government bonds were exceptionally low. The typical maturity of the 10-year series is 9 to 11 years, the lowest being 6 years and 9 months (for the year 1925) and the highest 14 years and 5 months (1933). For the 1-year return series, the typical maturity is between 9 and 12 months. The shortest maturity is 2½ months (1941) and the longest maturity is close to 3 years (1973).

² Parum, C. (1998), "Historiske afkast af aktier og obligationer i Danmark", Working Paper 98-9, Department of Finance, Copenhagen Business School (in Danish).

The yield-to-maturity is calculated on the basis of the price of the bond on the last trading day in December, the nominal interest rate and the dates of coupon payments. There is taken account of the fact that in trading Danish bonds sellers are paid for accrued interest at the day where trading takes place (the concept of "Vedhængende rente"). Over a long period of time, it was customary to issue bonds with some redemption each term. In these cases, expected payment streams are used.

From 1960 and onwards, observations in the 10-year series are from OECD.

Data are available from 1921 for the 5- and 10-year bond returns and from 1924 for the 1-year bond return.

The 5- and 10-year bond return series were constructed as part of the original database. The 1-year bond return series has been added later on.

3. Companies in the Database

The sample of companies used for calculating the dividend series includes (all companies included in the list):

- Banks:*
- Aktivbank
 - Amagerbanken
 - Amtssparekassen Fyn
 - Andelsbanken
 - C & G Banken
 - Den Danske Bank
 - Fyens Disconto Kasse
 - Handelsbanken
 - Privatbanken
 - Provinsbanken
 - UniDanmark
 - Aarhus Privatbank
- Insurance:*
- Alm Brand B
 - Alm Brandass A
 - Alm Brandass B
 - Baltica
 - Codan
 - Københavnske Reassurance A
 - Københavnske Reassurance B
 - Københavnske Reassurance C
- Service:*
- Andersen & Martini
 - Sophus Berendsen A
 - Sophus Berendsen B
 - F L Bie
 - Brdr Dahl
 - D G Holding B
 - Dalhoff Larsen & Horneman
 - Det danske Kulkompani
 - Danske Luftfartsselskab
 - Ford Motor Co
 - Peder P Hedegaard
 - ISS A
 - ISS B
 - Brdr A & O Johansen
 - Jydsk Telefon
 - Korn- og Foderstofkompagniet
 - Københavns Telefon
 - Nesa
 - C O Olesen Holding B
 - Tivoli A

Tivoli B
 Wessel og Vett C
 Th Wessel og Vett præference
 Østasiatisk Kompagni
 Østasiatisk Kompagni Holding

Shipping:

DFDS
 D/S 1912 A
 D/S 1912 B
 D/S Bornholm
 D/S Dannebrog
 D/S Myren
 D/S Norden
 D/S Orient
 D/S Torm
 J Lauritzen
 D/S Svendborg A
 D/S Svendborg B

Industry:

Albani A
 Albani B
 Ove Arkil
 Atlas
 Bang & Olufsen
 Bing & Grøndahl
 Burmeister & Wain Stamaktier
 Calkas A
 Calkas B
 Cheminova Holding B
 Chemitalic B
 Christiani & Nielsen B
 Coloplast B
 CUBIC Modulsystem B
 Dancall Radio A
 Dancall Radio B
 Danisco
 Dansk Data Elektronik
 Danske Spritfabrikker
 Danske Sukkerfabrikker A
 Danske Vin- og Konserverfabrikker
 Forenede Bryggerier A
 Forenede Bryggerier B
 Forenede Bryggerier C
 Forenede Papirfabrikker
 Brdr Hartmann
 Incentive

Kastrup Glasværk
 Københavns Brødfabrikker
 Nordisk Fjerfabrik A
 Nordisk Fjerfabrik B
 Nordisk Kabel- og Trådfabrikker
 Novo Industri
 C W Obel B
 Royal Copenhagen A
 Royal Copenhagen B
 Schouw & Co A
 Schouw & Co B
 F L Smidt A
 F L Smidt B
 Superfos
 Superfos præference
 Thrige-Titan A
 Thrige-Titan B
 Aarhus Oliefabrik A
 Aarhus Oliefabrik B

Dansk resumé (Danish Summary)

Denne Ph.d.-afhandling omfatter en introduktion, seks kapitler, et databilag samt dette kortfattede danske resumé. Afhandlingen er empirisk og undersøger de økonomiske mekanismer, som ligger til grund for fastlæggelsen af kurser og afkast på det danske aktiemarked. Det analytiske bidrag består i seks selvstændige og uafhængige papirer (kapitler), som hver især søger at besvare et konkret empirisk spørgsmål med det overordnede formål at opnå en bedre forståelse af de faktorer, som er af betydning for aktiekurser og -afkast i praksis. I hvert papir opstiller, estimerer og tester vi en *ad hoc* empirisk model, som er designet til at belyse det konkrete spørgsmål. Analyserne fokuserer generelt på økonomiske forhold med grundlæggende eller 'fundamental' betydning for aktiekurserne. Vi har begrænset os til en analyse af danske aktier på det aggregerede markedsniveau. Kapitlerne betragter perioderne efter 1., henholdsvis 2. Verdenskrig.

I Kapitel 1 (papiret '*Stocks Hedge against Inflation in the Long Run: Evidence from a Cointegration Analysis for Denmark*') undersøges det empirisk, hvorvidt danske aktier yder et værn (hedge) mod inflation på langt sigt i den forstand, at købekraften af aktier er upåvirket af ændringer i det generelle prisniveau over 'lange' perioder. Tilgangen til problemstillingen afviger fra standarden i den internationale litteratur, idet vi tester inflationshypotesen på grundlag af den estimerede langsigtsrelation mellem niveauet for aktiekurserne og det generelle prisniveau. Ud fra årlige observationer efter 2. Verdenskrig samt forbrugerprisindekset som mål for det generelle prisniveau, finder vi stærkt belæg for hypotesen, dvs. at danske aktier yder et værn mod inflation på langt sigt. Denne konklusion adskiller sig fra standardkonklusionen i litteraturen.

I Kapitel 2 (papiret '*On the Predictability of the Danish Equity Premium*', som er fælles arbejde med Ole Risager) tester vi empirisk, hvorvidt det er muligt at forudsige merafkastet på aktier i forhold til obligationer. Undersøgelsen betragter danske aktier og obligationer samt 3 forskellige investeringshorisonter (1, 5 og 10 år). Ved

systematisk at undersøge informationsindholdet i en række forskellige nøgletal (dividende-kurs-forholdet, obligationsrenter, historiske merafkast på aktier, etc.) bliver konklusionen, at det er muligt at forudsige merafkastet på 5 års sigt, men ikke på 1 og 10 års sigt. Vi bruger den estimerede model for 5 års investeringshorisonten til at forudsige merafkastet på aktier over den 5 års periode, der startede ved udgangen af 1997. Modellen forudsiger et afkast, der er lavt efter en historisk målestok, men forudsigelsen er ikke nær så pessimistisk som i andre undersøgelser.

Kapitel 3 ('*On the Relationship between the Danish Stock and Bond Market in the Medium and Long Term*', skrevet sammen med Ole Risager) indeholder et kort papir, hvor vi estimerer den empiriske sammenhæng mellem de realiserede afkast på aktie- og obligationsmarkederne for en investeringshorisont på henholdsvis 5 og 10 år. Resultatet er, at der på begge horisonter er en stærk kointegrerende sammenhæng mellem aktieafkastet og den effektive rente på obligationer. Dette resultat afspejler en stærk tendens til, at aktieafkast og obligationsrente bevæger sig parallelt over 'lange' tidsperioder. Den kointegrerende sammenhæng kan tolkes som en langsigtet ligevægtsrelation, som udelukker muligheden for arbitrage mellem aktie- og obligationsmarkederne. Estimerne viser, at en ændring i obligationsrenten kun delvist slår igennem på aktieafkastet.

I Kapitel 4 ('*Regime-Switching Stock Returns and Mean Reversion*', sammen med Steen Nielsen) estimerer vi en tidsrækkemodel for det 1-årige nominelle aktieafkast. Formålet er at teste, hvorvidt afkastprocessen har ændret sig over tid for så vidt angår dens middelværdi, volatilitet eller autokorrelation. På grundlag af observationer for perioden efter 1. Verdenskrig estimerer vi en velspecificeret 'regime-switching' model med 2 tilstande, idet vi tillader for en 1. ordens autoregressiv specifikation i hver tilstand. Modellen identificerer to forskellige regimer, som er kendetegnet ved lav middelværdi og lav volatilitet henholdsvis høj middelværdi og høj volatilitet. Vi opstiller endvidere et alternativt test for 'mean reversion' (defineret som negativ autokorrelation i afkastet), som inkorporerer muligheden for to forskellige regimer i afkastprocessen. Testresultatet er et svagt belæg for 'mean reversion' i aktieafkastet.

set over hele perioden. Indikationen på 'mean reversion' hidrører fra det seneste høj-middelværdi-og-høj-volatilitet regime, hvor der er en signifikant 'mean reversion'.

Formålet med **Kapitel 5 ('Modeling the Dividend-Price Ratio: The Role of Fundamentals Using a Regime-Switching Approach')**, skrevet sammen med Steen Nielsen) er at give en empirisk forklaring på udviklingen i det såkaldte dividende-kurs-forhold, defineret som forholdet mellem aktuelle dividendebetalinger og aktuelle aktiekurser. Vi estimerer en økonomisk model for dividende-kurs-forholdet for danske aktier over perioden fra 1. Verdenskrig. I modellen indgår som underliggende forklarende variabel en 'proxy' for en tidsvarierende diskonteringsrate, som vi har dekomponeret i en tidsvarierende realrente og en tidsvarierende risikopræmie på aktier. Endvidere indgår niveauet for de reale dividender samt sidste periodes dividende-kurs-forhold. For indirekte at tage højde for ikke-modellerede eller ikke-observerede ændringer i skatteforhold, institutionelle rammer el.lign., estimeres en 'regime-switching' model med 2 tilstande af den økonomiske model. Resultaterne viser, at alle forklarende variable er stærkt signifikante i mindst ét regime, og at modellen giver en god forklaring af dividende-kurs-forholdet. Modellen identificerer to forskellige, men meget vedvarende regimer med et henholdsvis 'højt' og 'lavt' niveau for dividende-kurs-forholdet.

Kapitel 6 ('A Simple Explanation of Stock Price Behavior in the Long Run: Evidence for Denmark') giver en empirisk forklaring på de langsigtede bevægelser i aktiekurserne. Vi formulerer og estimerer en empirisk økonomisk model for aktiekurserne på langt sigt. Som forklarende variable anvendes to grundlæggende ('fundamentale') makroøkonomiske variable, nemlig virksomhedsprofitten og den nominelle obligationsrente. På grundlag af observationer for danske aktier efter 2. Verdenskrig, finder vi en stærk og stabil kointegrerende sammenhæng mellem aktiekurserne og de to 'fundamentale' variable. Sidstnævnte er stærkt signifikante både statistisk og økonomisk. Modellen giver endvidere en god beskrivelse af de langsigtede tendenser i aktiekurserne. Specielt er modellen i stand til at forklare, hvorfor det danske aktiemarked har været kendetegnet ved relativt beskedne kapitalgevinster i hele perioden før (omkring) 1980 og relativt store kapitalgevinster

efterfølgende. Endelig viser modellen, at aktiekurserne ved udgangen af 1998 var tæt på deres 'fundamentale' værdier, idet den forudgående markante opgang i aktiemarkedet kan forklares ved et fald i obligationsrenten.

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