Essays on Pensions and Fiscal Sustainability

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The PhD School in Economics and Management is an active national and international research environment at CBS for research degree students who deal with economics and management at business, industry and country level in a theoretical and empirical manner.

Foreword

This body of work comprises the work developed in my doctoral studies at the Department of Economics, Copenhagen Business School (CBS), as a member of the Pension Research Center (PeRCent). I am very grateful to the Danish national institute of statistics, Danmarks Statistik (DST), for the funding and office space provided for my Ph.D. studies, as well as Copenhagen Business School and PeRCent for all the support provided to make my Ph.D. studies successful.

I want to, first, acknowledge my gratitude towards my team of supervisors and co-authors, Svend E. Hougaard Jensen and Jimmy Martinez-Correa for excellent supervision, always keeping me on track, providing insightful comments and suggestions, teaching me how to become a better researcher with great work ethic, though always reminding me the importance of keeping a healthy work-family balance. My doctoral studies have been a process of true professional and personal development, not the least thanks to my team of supervisors. Specifically, I thank my main supervisor, Svend, for always believing in me, pushing me towards pursuing Ph.D. studies, and for the many invitations to speak at conferences and seminars.

I am very grateful to my other two co-authors, Andrew Hughes Hallet, Thorsteinn Sigurdur Sveinsson, and Alvaro Pedraza for developing research together, full of insightful and enriching discussions. Namely, Thorsteinn, who I have known long before my doctoral studies, has been a source of support, motivation, knowledge, and laughs. This balanced mix of elements certainly improved my time spent completing my Ph.D. studies.

I also wish to thank the support of Bo Møller, Jarl Christian Quitzau and Annesofie Thorup Olesen at Danmarks Statistik for their support and guidance in understanding and using the data provided by DST for the development of the research present in this thesis.

A very special thanks goes to my wife, Amalie, and our daughter, Alice, for being a tremendous beacon of light in my life, for the endless source of love and support in guiding and motivating me to achieve more and becoming a better person. Alice, ainda que apenas tenhas um ano de idade, a tua mera existência tem sido a melhor experiência da minha vida. And to you, Amalie, you continue to amaze me with your character, strength, and overall joy for life. Through my PhD studies, you have carried a weight that surely many people would not be able to carry. That was fundamental in the way our joint life has progressed over the past years.

The deepest appreciation goes to my family, that have always given me unconditional support throughout my life, being a central part in my development into the person I am today. You have taught me many things throughout my life, and all that knowledge still finds its way into everything I do to this day. Even though we are currently separated by more 2.700 kilometers, your support and love is still always felt.

To the Danish side of my family, Amalie’s parents and siblings, I say thank you for the continuous support given to Amalie and I in the course of our relationship, which was fundamental in providing me (us) with stability for the, at times, heavy work schedule. You have been an essential pillar in my integration in Denmark and a good example of how to live a fulfilling life.

Last, but not least, I thank my friends Svavar and Nicolaj for all the good times and all the support given throughout these years. My life in Denmark would have not been the same without you. I also thank my Ph.D. colleagues at CBS for the company and for brightening our joint work place.

Filipe António Eslau Bonito Vieira – Copenhagen, Denmark, 2019

Abstract

This thesis is a volume comprised of four independent chapters: The overarching topic of the thesis is pensions and fiscal sustainability. An overriding problematic throughout the different chapters is that of demographic ageing. This ageing process, in the form of higher longevity and lower fertility, will most certainly bring changes to the way our economic lives work, whether the individual or society at large is considered. Each chapter, described below, sheds light on issues related to: fiscal sustainability and optimal public debt levels under demographic change; reform to pension systems as a response to increased longevity; and saving behaviour for retirement - a topic which is more and more relevant, as higher longevity implies a fundamental shift in how individuals ought to save for retirement. More specifically:

Chapter one, titled "*Sustainable Fiscal Strategies under Changing Demographics*", develops an overlapping generations model to evaluate, first, the steady state growth maximizing level of public debt around which an economy needs to stabilize; second, how the optimal level of public debt varies as a function of key population parameters; third, how fiscal rules designed to stabilize the economy around that debt level need to vary with the population parameters; and fourth, how well the model performs as a reasonable and plausible representation of advanced economies that face fiscal strain and deteriorating demographics. The main conclusion is that despite diminished fiscal space and flexibility due to deteriorating population parameters, a relatively benign steady state is feasible and available under mild fiscal restraints. The bigger problem will be how to get there without financial or fiscal breakdowns along the way. We offer some political economy perspectives on how best to manage that risk.

Chapter two, titled "*Pensions, Public Debt and Equilibrium Dynamics under Demographic Change*", shows how demographic change impacts the optimal debt-GDP ratio under the golden rule of public finance and balanced-budgets, and evaluates how the existence of a PAYG pension system impacts that optimality. For the average OECD economy, very small impacts to the optimal debt-GDP ratio occur when fertility and mortality rates decline according to their projected values for the following 85 years (2015-2100). The size of the impact depends on how generous the PAYG pension system is: Countries with stronger PAYG schemes are more vulnerable to demographic ageing in terms of their fiscal balances. The sign of the effect of demographic change on the optimal debt-GDP ratio is also shown to depend on GDP's private capital-intensity. Labour-intensive countries will automatically converge towards the maximal growth steady state by using optimal public investment flows, with no fiscal austerity measures being required. Private capital-intensive countries face the possibility of having to apply austerity measures to control the public debt stock. Overall, fiscal reforms to ensure both sustainability and transition towards maximal economic growth steady states do not imply significant decreases to future public investment rates, even for countries with very generous public pensions.

Chapter three, titled “*Reform and Backlash to Reform: Longevity Adjustment of the Retirement Age*”, uses an overlapping generations model *ad modum* Blanchard-Yaari to analyze the effects of retirement reform on labour supply. It is found that there is not a one-to-one relationship between an increase in the statutory retirement age and the corresponding increase in labour supply. Rather, a backlash effect is observed, where a reform aimed at broadening the labour supply on the extensive margin will have the unintended effect of decreasing labour supply on the intensive margin. A calibration exercise is performed to analyse the robustness of these backlash effects to different designs of the pension scheme. Here, it is found that, under a defined-benefit scheme, the effects on the intensive margin of labour supply are drastically increased compared to the defined-contributions scheme and the fully-funded scheme.

Chapter four, titled “*The Residual Group in Denmark: Who Is at Risk*”, evaluates saving behaviour in Denmark. For the past decades, Denmark has fostered the funded component of its pension system via quasi-mandatory labour-market pension schemes alongside the strengthening of old-age public insurance in the form of means-tested unfunded supplementary benefits. Within this institutional setting, which requires a higher individual responsibility to save for retirement, the group of individuals that are eligible to the full public pension supplement (*folkepensionstillæg*) is analysed - a group termed the “residual group” in Denmark. This pension supplement is means-tested, making this group the collection of individuals with the lowest pension savings in Denmark and which can become a fiscal pressure on the public pillar of the Danish pension system. We investigate the factors associated with higher or lower risk of belonging to this group. We then quantify that risk for cohorts that will reach retirement in the next decade and provide a framework that can be used to measure what level of means-tested pension expenditures the government could expect to pay out in the future.

Resumé (Abstract in Danish)

Denne afhandling består af fire selvstændige kapitler under det overordnede tema ”pensioner og finanspolitisk holdbarhed”. Det gennemgående tema i alle kapitlerne er demografisk aldring. Denne aldringsproces, forårsaget af længere levetid og lavere fertilitet, vil utvivlsomt betyde ændringer i den måde, vores økonomiske liv udfolder sig på, både individuelt og for samfundet generelt.

Hvert kapitel, beskrevet herunder, belyser problemstillinger relateret til: Finanspolitisk holdbarhed og optimale offentlige gældsniveauer ved demografiske ændringer; reform af pensionssystemer som konsekvens af længere levetid; samt opsparingsadfærd til tilbagetrækning – et emne som bliver mere og mere relevant, i takt med at den længere levetid indebærer en fundamental ændring i den måde, vi hver især bør spare op til vores tilbagetrækning på. Mere specifikt:

Kapitel et med titlen “*Sustainable Fiscal Strategies under Changing Demographics*” (“*Bæredygtige finanspolitiske strategier under ændret demografi*”), præsenterer en overlappende generationsmodel til evaluering af 1) Det maksimale niveau af langsigtsvækstraten, omkring hvilket en økonomi bør stabilisere sig; 2) Hvordan det optimale niveau af offentlig gæld varierer som en funktion af væsentlige befolkningsparametre; 3) Hvordan økonomiske indgreb, som har til formål at stabilisere økonomien omkring dette gældsniveau, er nødt til at variere i forhold til befolkningsparametrene; og 4) Hvor godt modellen fungerer som en fornuftig og plausibel repræsentation af de avancerede økonomier, der går en fremtid i møde med svækket demografi og pres på de offentlige finanser. Hovedkonklusionen er: På trods af det formindskede økonomiske råderum og fleksibilitet som konsekvens af svækkede befolkningsparametre, er en relativt gunstig langsigtsligevægt stadig mulig og opnåelig ved mildere økonomiske indgreb. Sværere vil det være at nå dertil uden finansielle eller fiskale sammenbrud undervejs. Vi fremlægger nogle politisk-økonomiske perspektiver på, hvordan man kan håndtere denne risiko bedst muligt.

Kapitel to med titlen “*Pensions, Public Debt and Equilibrium Dynamics under Demographic Change*” (“*Pensioner, offentlig gæld og dynamisk ligevægt under demografiske ændringer*”) viser, hvordan demografiske ændringer påvirker det optimale gælds-BNP-forhold under den gyldne regel om offentlige finanser og balancerede budgetter, og evaluerer hvordan et offentligt, skattefinansieret pensionssystem som den danske folkepension påvirker det optimale gælds-BNP-forhold. For den gennemsnitlige OECD-økonomi ser man en meget lille påvirkning af det optimale gælds-BNP forhold, når fertilitets og dødelighedsraten falder i forhold til deres forventede værdier for de næste 85 år (2015-2100). Hvor stor påvirkningen er, afhænger af hvor generøst det offentlige pensionssystem er. Lande med meget generøse ordninger er mere sårbare over for demografisk aldring i forhold til deres finanspolitiske balance. Tegnene på effekten af de demografiske ændringer på det optimale gælds-BNP-forhold viser sig også at være afhængig af den private kapitalintensitet. Arbejdsintensive lande vil automatisk hælde mod maksimal

langsigtsvækstrate ved at anvende optimale offentlige investerings-flows, der ikke kræver en finanspolitisk sparepolitik. Privatkapital-intensive lande kan være nødt til at implementere en stram sparepolitik for at kontrollere den offentlige gæld. Generelt indebærer økonomiske reformer til at sikre både økonomisk bæredygtighed og en bevægelse mod maksimal økonomisk langsigtsvækstrate ikke nogen betydelige reduceringer i de fremtidige offentlige investeringer, selv for lande med meget generøse offentlige pensionsordninger.

Kapitel tre med titlen "*Reform and Backlash to Reform: Longevity Adjustment of the Retirement Age*" ("*Reform og modreaktion mod reform: levetidsindeksering af tilbagetrækningsalderen*") anvender en overlappende generationsmodel *ad modum* Blanchard-Yaari til at analysere effekterne af en tilbagetrækningsreform på arbejdsudbuddet. Resultaterne viser, at der ikke er et et-til-et forhold mellem en forhøjelse af den lovbestemte tilbagetrækningsalder og tilsvarende forøgelse af arbejdsudbuddet. Tværtimod observeres der en modsatrettet effekt, hvor en reform, der har til hensigt at øge arbejdsudbuddet på den ekstensive margin, vil have den ikke-intenderede effekt, at den mindsker arbejdsudbuddet på den intensive margin. Der gennemføres en kalibreringsøvelse for at analysere betydningen af disse modsatrettede effekter på forskellige pensionsmodeller. Her ses det, at effekterne øges kraftigt på den intensive margin for arbejdsudbuddet ved en ydelsesbaseret ordning sammenlignet med en bidragsbaseret ordning og en opsparingsbaseret ordning.

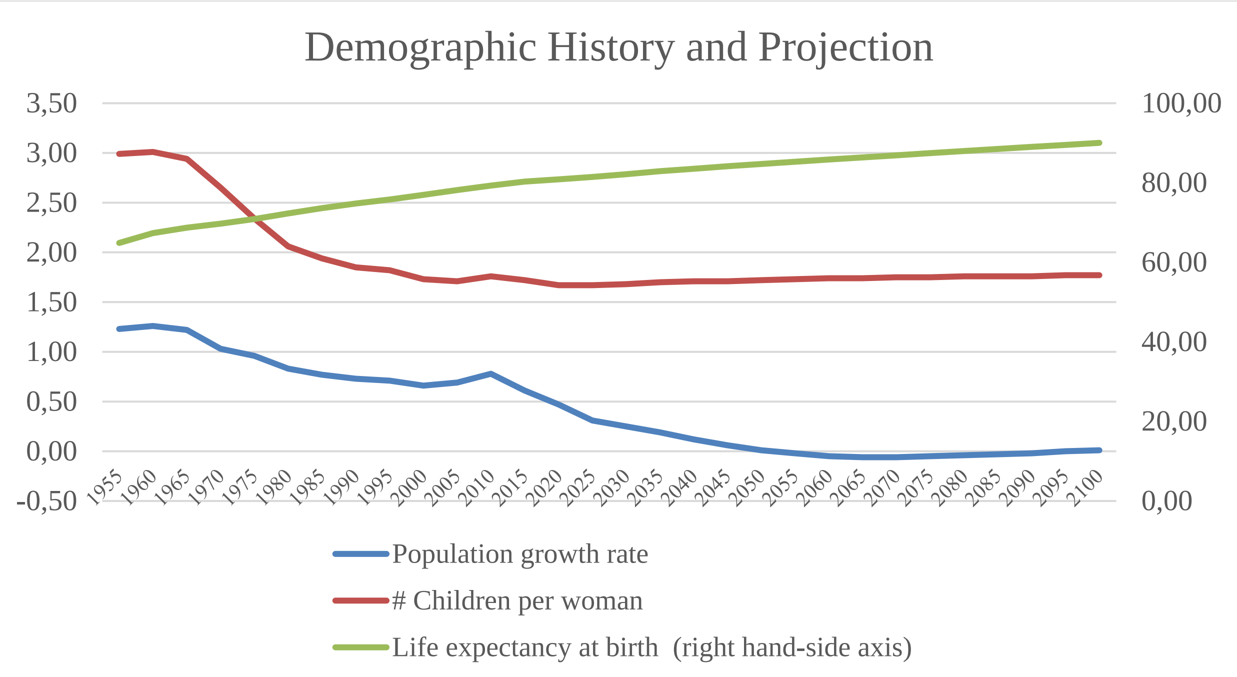
Kapitel fire med titlen "*The Residual Group in Denmark: Who Is at Risk?*" ("*Restgruppen i Danmark: Hvem er i risiko?*") ser på opsparingsadfærd i Danmark. I de sidste årtier har Danmark opbygget de private arbejdsmarkedspensioner som en central del af det samlede pensionssystem. Samtidig er en del af den offentlige folkepension gjort indkomst- og formueafhængig. Inden for denne institutionelle ramme, som kræver et højere individuelt ansvar for at spare op til sin tilbagetrækning fra arbejdsmarkedet, analyseres den gruppe, som er berettiget til det fulde folkepensionstillæg – en gruppe der betegnes som ”restgruppen” i Danmark. Vi undersøger de faktorer, der er forbundet med højere eller lavere risiko for at høre til i denne gruppe. Derefter kvantificerer vi denne risiko for kohorter, der vil nå pensionsalderen i det næste årti, og opstiller en model, der kan bruges til at måle, hvilket niveau af behovsvurderede pensionsudgifter regeringen kan forvente at skulle udbetale i fremtiden.

Contents

Introduction

This thesis is a volume comprised of four independent chapters: The overarching topic of the thesis is pensions and fiscal sustainability. The driving force behind these two topics is the demographic ageing process under way since the middle of the 20th century. All contributions in each article are original in their respective topics.

The figure below shows that since 1950, the number of children per woman has been declining up to nowadays. This number is projected to stabilize from now onwards, but nonetheless, the drop in the fertility rate seen in the 20th century represents a near halving of the fertility rate well below the population replacement rate (children per woman≥2). The figure below also shows the population growth rate, taking into account migration. Since the migration force is a net positive, on average, among high income countries, the population growth rate is not below 0%, but is projected to stabilize around 0% over the course of the 21st century.



Source: UN (2019). Note: Data refers to average within high income countries, as classified by the UN.

Turning to the other side of the demography problematic, in terms of longevity, the figure above shows that life expectancy at birth has increased from 65 years in 1950-1955 being projected to increase up to 90 years by 2100. Such a dual decrease to mortality and fertility rates has economic implications at many levels, some of which are studied in this thesis. In this thesis, topics related to fiscal sustainability issues, in terms of the taxation problematic, the maintenance of public debt stocks, and also in terms of the sustainability of pension systems, are addressed. Issues related to how individuals save for retirement also receives focus.

With the worsening of the old-age dependency ratio, it will arguably become increasingly difficult to finance given levels of public benefits, or other public spending for that matter, if these benefits are both increasing with demographic ageing and if the level of benefits is to be kept constant. Impacts of demographic ageing also affect each of us individually, as we are facing longer and longer lifespans surrounded by less and less children. With longer lives comes a necessity to finance a longer retirement period, if the official pension age offered by governments is not reformed to track expected increases in longevity. With higher longevity, there may come a necessity for stronger saving behavior.

All chapters included in this PhD thesis touch on all these issues raised in the paragraph above. A question that has gone by largely unanswered is how demographic change can affect the optimality of public capital. Chapter 1 evaluates optimal debt policy under demographic change, showing how it depends on different demographic parameters. The idea is that the optimal schedule of public debt issuing might be connected to the demographic structure of an economy, if the government is both responsible to fund productive activities and re-distribute income in the form of benefits that are demography related. Chapter 2 builds on this idea and expands the framework to include a more complete set of demography related public transfers. It also evaluates the implications of demographic ageing to the transition dynamics OECD economies can expect throughout the 21st century.

A common observed reform applied by policymakers worldwide has been to increase the official pension age to relieve financial pressure from the pension system by reducing the number of recipients versus contributors (see for example OECD, 2017). Chapter 3 evaluates precisely these types of reforms to the official pension age as means to face demographic ageing. A problem may arise in the process: inducing individuals to work for more years might induce these individuals to work less *each* year. This is an effect termed the “Backlash effect”, and its impact is evaluated under different pension schemes.

Lastly, the 4th chapter focuses on saving behavior of Danish individuals. We measure the number of Danish individuals that are likely going to become eligible to pension supplements at retirement, supplements that are means-tested. As such, those that become entitled to the full pension supplement are the group of lowest savers in Denmark, a group called the “Residual group”. Characteristics associated with being part of this group are found, as well as an estimate of its size over the population that will retire in the following 10-15 years. This is done by developing a new framework that is risk based. Such a framework is very useful to policymakers, since it enables to quantify risk associated with certain individual types. By knowing the probability distribution of becoming part of the group in Denmark with the lowest savings, more targeted action can be applied to reduce potential under-saving behavior, a topic that is of high relevance nowadays in the Danish public discussion forum. It also informs policymakers what level of supplementary pension benefits they can expect to pay out in the incoming years.

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Chapter 1

Sustainable fiscal strategies under changing demographics

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Sustainable fiscal strategies under changing demographics☆

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ABSTRACT

This paper develops an overlapping generations model to evaluate, first, the steady state growth-maximizing level of public debt around which an economy needs to stabilise; second, how the optimal level of public debt varies as a function of key population parameters; third, how fiscal rules designed to stabilise the economy around that debt level need to vary with the population parameters; and, fourth, how well the model performs as a reasonable and plausible representation of the advanced economies that face fiscal strain and deteriorating demographics. The main conclusion is: despite diminished fiscal space and flexibility due to deteriorating population parameters, a relatively benign steady state is feasible and available under mild fiscal restraints. The bigger problem will be how to get there without financial or fiscal breakdowns along the way. We offer some political economy perspectives on how best to manage that risk.

1. Introduction

General fiscal rules are legislative agreements intended to mitigate the deficit bias usually associated with fiscal policy and typically due to myopia by governments. Recent empirical research suggests that national fiscal rules are helpful in achieving greater budgetary discipline (Debrun et al., 2008; Nerlich and Reuter, 2013; Foremny, 2014). The question of which specific rule is most effective in promoting fiscal discipline has recently attracted a lot of attention. For example, Bergman et al. (2016) find that a combination of an expenditure rule and a balanced budget rule, or a combination of an expenditure rule, a balanced budget rule and debt rules, give significant positive effects on the primary balance for virtually all levels of government efficiency.[1] However that does not guarantee acceptable economic outcomes.

In this paper, we take a different route. Rather than formulating generic rules designed to reduce the deficit bias, we set up specific rules aimed at maximising economic growth.[2] This enables us to condition those rules on population parameters and age-related spending, and to show the impact of population change on fiscal balances and debt. We ask: first, whether, to what extent and when do changing demographics affect the net fiscal position; second, whether it is acceptable to allow larger debt burdens, or whether tax

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[1] For a fuller discussion of what makes a fiscal rule successful, see Eyraud et al. (2018) of the IMF fiscal rules project.

[2] Existing models in the fiscal literature that aim to determine optimal levels of debt include Aschauer (2000), Aiyagari and McGrattan (1998) and Checherita et al. (2014).

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and spending austerity are always necessary when demographic change leads to pressure on public finances.

Then, rather than evaluating alternative forms of fiscal rule, we restrict attention to a rule for public debt. This is in line with previous work where we have argued that debt targets are superior to deficit targets for theoretical and practical reasons (Hughes Hallett and Jensen, 2012).[3] But choosing a debt target is not a trivial task. A key issue is how to account for (the discounted value of) future spending liabilities. If the implicit liabilities created by ageing populations are ignored, the debt criterion will ignore the future revenues required to avoid default despite the obvious need to cover the benefits promised to existing and future beneficiaries.

This is the case for extending debt targeting rules to account for predictable demographic changes. Put differently, forward looking fiscal rules are needed to allow for future liabilities created by adverse demographics. The implication is that governments facing demographic change, or the need for higher social spending, will have to adjust (most likely restrict) their fiscal plans to accommodate those changes. Hence our key contribution is to make debt control forward looking by designing a rule where fiscal policy reacts, not only to changes in existing levels of debt, but also to anticipated changes in future liabilities.

This study provides a comparative static analysis of the problem. Within that framework, the paper makes three new contributions.[4] First, we offer a formal evaluation of the optimal debt level around which the economy needs to stabilise.[5] Second, we study how the optimal debt level varies with the key population parameters. Third, we show how fiscal rules designed to stabilise the economy around that debt level need to react to population age, life expectancy, the birth rate and rising welfare expenditures.[6]

While general fiscal rules primarily address the deficit bias, this paper also looks at specific rules to deal with ageing and lack of growth in the work force. Such policies imply three roles for public policy: i) to improve the incentives to raise children, to maintain the number of taxpayers and a capacity to sustain a certain level of public debt; ii) to improve the volume and effectiveness of public capital in order to boost productivity and growth; and iii) to smooth the distributional consequences of demographic change and offset negative effects on growth. In this paper, we focus on all three roles because they explain *how* we can reach a benign steady state and good fiscal outcomes despite adverse demographics: an explanation that has been missing in the literature so far, but is perhaps the main contribution of the paper.

How and whether the government should incentivize child-rearing is a controversial issue. Since the children of today are the workers of tomorrow, we consider how demographics and sustainable government finances are related. We do that by allowing social spending aimed at alleviating the private cost of child-rearing. We include fiscal balances because a growing population has a feedback that can help maintain a certain level of public spending and debt. Specifically, we model government spending related to child-rearing; and then analyze how those expenditures influence the economy.

We consider public and private capital to be labour productivity-enhancing factors in production. Public capital can be interpreted as education, R&D and the social infrastructure that underpins human capital formation *and* leads to a more skilled and productive workforce. Private capital, however, provides the incentives for innovation and competition. The ratio of public to private capital is therefore a key factor for economic prosperity. The government seeks to improve the volume and effectiveness of public infrastructure, helping to raise productivity and the economy's rate of growth. Having a government with this mandate enables this study to present an optimal (public) debt policy which allows for demographic factors.

The roadmap of the paper is as follows. In Section 2 we set out our analytical framework: an OLG model extended to allow for public debt. Section 3 derives the optimal debt-to-GDP ratio; section 4 outlines policies to manage demographic change and introduces the political economy forces that underlie the main issue: intergenerational equity and transfers. Sections 5 and 6 provide a simulation treatment of the effects of population change on optimal debt. Section 7 then illustrates the political economy trade-offs that underlie policy problems of this type. Section 8 provides our principle conclusions and an agenda for future research.

2. The model

We use a model with overlapping generations, changing demographics, welfare spending, and productive public and private capital accumulation (Yakita, 2008; Bokan et al., 2016). The economy contains homogenous individuals and firms operating in competitive markets. Individuals have two periods in their lives, first as a worker and then as a retiree. To introduce population ageing and entitlement spending, people face a risk of dying in the transition from worker to retirement, so that not every worker will live a full two-period life. However, in each period, working people form the young cohort and make decisions about consumption, savings and child rearing. Surviving retirees then form the old cohort. Public investment decisions are made by a government that issues debt and levies taxes in order to fulfil its objectives.

[3] If debt targeting is preferred, the question is: what debt or debt-to-GDP ratio should be targeted (Auerbach, 2009)? Deriving optimal levels of public debt may involve several complicated trade-offs. For example, how should inter-generational equity be balanced against economic performance and long term fiscal sustainability?

[4] In a follow up paper, Hughes Hallett et al. (2017) focus on the dynamics of how different economies might reach the new steady state. In this paper, by contrast, we focus on establishing (from a political economy perspective) that a feasible steady state does indeed exist before going on to analyse how to reach it. It makes little sense to study the transition before we can show that an acceptable/sustainable steady state exists.

[5] The optimal debt level depends on the marginal productivity of public capital, defined as public spending on investment projects which are (a) productive, (b) have an identifiable rate of return and (c) have a longer horizon than consumption expenditures. Conceptually, this is clear-cut. But in practice there are often problems in separating public investment from public consumption in the data (Checherita et al., 2014).

[6] Previous work has tended to focus on impact of worsening demographics on the financial markets; Auerbach and Herrmann (2002) for example devote 7 out of 8 chapters to that theme, and only one to fiscal issues. On the other hand, Bohn (2002) stresses the importance of managing debt for a successful resolution of the impact of ageing.

2.1. Firms

The economy is composed of a large number of identical firms which produce a homogenous product by utilizing the services of private capital and labour. The production technology for each firm j is characterized by constant returns to scale between the private capital and productivity-adjusted labor inputs that each firm employs:

$$Y_{j,t} = AK_{j,t}^{\alpha}\left(h_t L_{j,t}\right)^{1-\alpha};\ 0 < \alpha < 1 \tag{1}$$

where $Y_{j,t}$, $K_{j,t}$ and $L_{j,t}$ denote, respectively, the output level, the private capital stock, and labour input in firm j for period t. Total factor productivity (TFP) is captured by a standard scale factor, A (which may be time dependent), and labour augmenting productivity (not firm specific) described by h_t, where α is the output elasticity with respect to private capital.

Given the assumption of perfectly competitive markets for production goods and inputs, the equilibrium rental rate of capital, r_t, and wage rate, w_t, can be found in the usual way as the solutions to the firm's profit maximization problem:

$$r_t = A\alpha\left(\frac{K_{j,t}}{L_{j,t}}\right)^{\alpha-1}(h_t)^{1-\alpha}. \tag{2}$$

$$w_t = A(1-\alpha)\left(\frac{K_{j,t}}{L_{j,t}}\right)^{\alpha}(h_t)^{1-\alpha} \tag{3}$$

The labor augmenting productivity, h_t, is determined by both the aggregate public and private capital stocks. From a macroeconomic perspective, it is important to recognize the productivity effects stemming from government productive inputs that combine with the productivity effects of private capital.[7] We do not model these productivity effects as public goods; instead we treat them as rival goods that are universally available to workers.[8] This implies that it is the per capita values of productive activities and private capital that matter, not their absolute level. So, the productivity adjustment process h_t is defined as[9]:

$$h_t = \frac{K_t^{\beta}G_t^{1-\beta}}{L_t};\ 0 < \beta < 1 \tag{4}$$

where G_t is the public capital stock and β is the productivity elasticity of private capital (i.e. private capital's share of any income gains from increased productivity).

Equations (2) and (3) reflect a perfectly competitive environment where the marginal product of an input equals its price. Notice that, since h_t reflects a macro-level measurement, the actions of an individual or firm cannot influence h_t. So, at equilibrium, the competitive pricing of inputs does not take into account marginal changes to h_t: h_t is taken as given by firms and individuals.

The aggregate private capital stock and the aggregate labour input are, respectively, defined as $K_t = \sum_j K_{j,t}$ and $L_t = \sum_j L_{j,t}$. Moreover, given the symmetry of firms, the equilibrium pricing expressions (2) and (3) imply that all firms share the same optimal solution, such that the ratio of input choices of each firm is the same in equilibrium: $K_{j,t}/L_{j,t} = K_t/L_t$. This allows us to rewrite expression (1) in terms of aggregate output at time t:

$$Y_t = AK_t^{\alpha}(h_t L_t)^{1-\alpha} = AK_t^{\alpha+\beta(1-\alpha)}G_t^{(1-\beta)(1-\alpha)} = AK_t^{\omega}G_t^{1-\omega} \tag{5}$$

where $Y_t = \sum_j Y_{j,t}$ and $\omega \equiv \alpha + \beta(1-\alpha)$. Note that Equation (5) does not include the aggregate labor input. This is a simple consequence of our definition of the (endogenous) labour productivity input, h_t.

The firm specific optimality conditions can now be rewritten in aggregate terms as:

$$r_t = A\alpha\left(\frac{K_t}{G_t}\right)^{\omega-1} \tag{6}$$

$$w_t = A(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\omega}\left(\frac{G_t}{L_t}\right) \tag{7}$$

Finally, for the purpose of simplification only, this model assumes that neither private nor public capital depreciates over time. We define the ratio of public-to-private capital, G_t/K_t, as X_t.

[7] We consider aggregate private capital to be as in Arrow's (1962) Learning by Doing model. For a literature review on the importance of both private and public investment in the economic growth process, see Makuyana and Odhiambo (2016).

[8] We take the view of Barro (1990) that actual nonrival government activities are relatively few, so we do not consider them.

[9] See Kalaitzidakis and Kalyvitis (2004).

2.2. Households

The economy is populated by homogeneous individuals. The lives of individuals are divided into working and retirement periods. The working period is of fixed length. Individuals in the working period form the young cohort; those in retirement the old cohort. At the end of the working period, a fraction of the young agents die while the rest move into retirement. The probability of dying by the end of the working period is given by the hazard rate λ; the same for all agents. The probability of being alive at the start of the retirement period is $1-\lambda$. As there is no third period, the size of the retired cohort is $(1-\lambda)$ times the size of the working cohort in the previous period. With N_t being the population of working-age people, the total population of the economy at t is equal to $N_t + (1-\lambda)N_{t-1}$.

Young individuals work, consume, save and raise children. The retirees do not have children but they consume based on the return on savings from the previous period. The representative individual's decisions in both periods are based on the maximization of life-time utility which is given by the following function in period t:

$$\ln\ c_t + (1-\lambda)\rho\ \ln\ d_{t+1} + \varepsilon \ln\ n_t \tag{8}$$

where c_t (d_{t+1}) is consumption in the working (retirement) period, n_t is the number of children the individual has, ε is the priority/weight of having children in a person's life-time utility, and ρ denotes the usual time discount factor, being a value between 0 and 1.

The time endowment for a young individual is normalized to 1. The model does not include leisure explicitly, but rather includes the child rearing time during which the young individual receives no wages.[10] Labour income is then allocated between current consumption, savings and tax payments. However, the individual receives a subsidy for child rearing which is also subject to taxation. This subsidy, denoted by ρ_w, is a fixed ratio of the wage rate, w_t. The tax rate is the same for wages and the child-rearing subsidy.[11]

The budget constraint of the representative young agent at time t therefore reads as:

$$(1-\theta_t)[w_t(1-zn_t) + \rho_w w_t z n_t] = c_t + s_t \tag{9}$$

where $z > 0$ is the rearing time per child, and θ_t denotes the flat tax rate. Savings are given by s_t and are exclusively invested in purchasing annuity assets. We assume that not all of each individual's time endowment can go towards child rearing: $(1-zn_t) > 0$.[12]

As in Blanchard (1985), we assume the existence of an actuarially fair insurance company operating in a perfectly competitive market for insurance. This insurance company collects savings from agents and invests them in private capital and/or government bonds. We assume that government bond purchases can crowd out private capital. The insurance company pays a return on savings to the agents that survive into retirement. The retirees therefore receive a rate of return of $\frac{r_{t+1}}{(1-\lambda)}$ on their savings. The returns (not the principal) on savings are taxed at a rate equal to that on labour income. The second period budget constraint therefore reads as:

$$\frac{1+(1-\theta_{t+1})r_{t+1}}{(1-\lambda)} s_t = d_{t+1} \tag{10}$$

The maximization problem of young individuals consists of choosing the optimal amount of savings and number of children to maximize life-time utility subject to the budget constraints for the working and retirement periods. The amount of savings directly influences the amount consumed in both periods. Meanwhile labour income not saved in the working period goes to current consumption, c_t; while that saved goes towards consumption in retirement, d_{t+1}. The overall maximization problem now is:

$$\max_{c_t,\ d_{t+1},\ n_t} \ln\ c_t + (1-\lambda)\rho\ \ln\ d_{t+1} + \varepsilon \ln\ n_t$$

$$s.t.\ \ (i)\ c_t + s_t = (1-\theta_t)[w_t(1-zn_t) + \rho_w w_t z n_t]$$

$$(ii)\ d_{t+1} = \frac{1+(1-\theta_{t+1})r_{t+1}}{(1-\lambda)} s_t$$

By rearranging the first order conditions for this maximization, the following relationships are obtained:

$$\frac{d_{t+1}}{c_t} = \rho[1+(1-\theta_{t+1})r_{t+1}] \tag{11}$$

[10] Hence the desire to have children crowds out the time spent working. A more general approach might allow extra saving, to leave bequests or invest in education. But to do that, workers have to have the children and pay carers to rear the children while agents are working extra hours to make the extra savings. The result is little or no ***net*** effect on incomes, savings or the fiscal position. So we treat these cases in a simpler way in Section 3 below.

[11] We can think of the child-rearing subsidy as a cost that is covered by the government. As such, the subsidy accounts for the amount of income that would be spent by the individual on child-rearing. Hence the subsidy is a labour income augmenting subsidy. Because that is income that could end up not being spent on child-rearing (hiring a nanny for example), this subsidy is taxed at the same rate as labour income. This interpretation forces us to see the subsidy as something that diminishes out-of-pocket costs of child-rearing.

[12] $(1-zn_t) > 0$ is guaranteed if the interval between the lower bound on ε in (15), and the upper bound on ε when $(1-zn_t) = 0$, is nonempty. That interval is always nonempty if $z < 1$, a restriction that has to hold since not all agents can spend all of their allotted life time rearing children and still survive. The parameter restrictions in this model therefore guarantee $(1-zn_t) > 0$.

$$\frac{n_t}{c_t} = \frac{\varepsilon}{(1-\theta_t)(1-\rho_w)w_t z} \tag{12}$$

From these two conditions we find optimal solutions for savings s_t, and number of children per individual, n_t:

$$s_t = \frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}(1-\theta_t)w_t \tag{13}$$

$$n_t = \frac{\varepsilon}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]} \equiv n > 0 \tag{14}$$

Two insights follow. First, we see the fraction in equation (13) is solely composed of demographic related factors, meaning that savings are a fixed share of after-tax income *only* if population parameters do not change. Second, the same is true for the fraction in equation (14), except that it includes ρ_w which is set by the government. Moreover, n is increasing in ρ_w. This should not be a surprise since increasing ρ_w lowers the financial burden of raising children, which gives individuals an incentive to have more children. The economy will then continue to survive so long as $n_t \geq 1$, which holds true as long as:

$$\varepsilon \geq \frac{z(1-\rho_w)[1+(1-\lambda)\rho]}{1-z(1-\rho_w)} \tag{15}$$

In this analysis we focus on the case of non shrinking populations ($n \geq 1$). If $n < 1$, the economy would disappear asymptotically. For the sustainability of the steady state, we need to assume that the inequality in (15) holds. In practice, to ensure such a steady state exists, we do not restrict n to be greater than the replacement rate of the population, but instead impose a restraint on the preferences of agents for children such that it does. The reality is that, with immigration, population growth is positive (if small in some places) almost everywhere – as data from the *World Development Indicators* show. Revealed preference therefore justifies (15) as the appropriate restriction. Population growth may also be improved by changes in mortality (λ) as well as by fertility decisions (n, ε).

2.3. Government

The government collects taxes from the wage income of the working population, from the child-rearing subsidy, and from the returns on savings of the retired population. The tax rate is denoted by θ_t and is fixed regardless of income type. The government also issues public debt, b_t, and invests the proceeds in public capital, G_t. In addition, the government pays the child-rearing subsidy as specified above. The government budget constraint is therefore:

$$b_{t+1} = (1+r_t)b_t + (G_{t+1} - G_t) + \rho_w w_t z n_t N_t - \theta_t(w_t L_t + \rho_w w_t z n_t N_t + r_t s_{t-1} N_{t-1}) \tag{16}$$

We assume debt interest payments and public consumption are financed by taxes on wages, subsidies and returns on savings. That leads to the period-by-period budget constraint:

$$r_t b_t + \rho_w w_t z n_t N_t = \theta_t(w_t L_t + \rho_w w_t z n_t N_t + r_t s_{t-1} N_{t-1}) \tag{17}$$

Public debt is then issued as needed to finance public capital formation, that is:

$$b_t = G_t \tag{18}$$

The model is thus stated in terms of the "golden rule" for public finance: the government only borrows to invest, not to finance its consumption or transfer payments.[13] The question is then, what is the optimal level of public debt and how is it determined?

3. The optimal level of public debt

3.1. Growth maximising public debt

The representative insurance company, and hence the young generation, can invest in both private and public capital. Equilibrium in capital markets requires that

$$s_t N_t = K_{t+1} + G_{t+1} \tag{19}$$

We can use conditions (17), (18) and (19), and combine them with capital rents and wages, (6) and (7), and with the fact that $L_t = (1-zn)N_t$ (labour supply equals the time endowment less child rearing time), to derive an expression for the income tax rate, θ_t, needed to satisfy current public spending:

[13] The rationale for this golden rule is given in Blanchard and Givazzi (2002) and Fatas et al. (2003). It is, or has been, practiced in Germany, the UK and several other advanced economies.

$$\theta_t = 1 - \frac{1}{\alpha X + \rho_w(1-\alpha)zn/(1-zn) + 1} \tag{20}$$

Using the equilibrium condition for capital markets (19), and the solution for s_t in (13), we obtain:

$$\frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}(1-\theta_t)w_t N_t = K_{t+1} + G_{t+1} \tag{21}$$

This equation determines the dynamic relationship between wages, working population, and public and private capital. We define balanced growth to be a situation in which public and private capital grow at a constant rate. Specifically, let the aggregate steady state growth rate γ^A be defined as:

$$\frac{G_{t+1}}{G_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \gamma n \equiv \gamma^A \tag{22}$$

where n is the constant growth rate of the population and γ is the aggregate growth rate per capita. This now implies that the public-to-private capital ratio is constant in steady state. It also implies that the tax rate, θ_t, and the interest rate, r_t, are constant in steady state.

Next we use equation (21), together with (7) and (20), to obtain a relationship from which we can derive a closed form expression for the economy's growth rate (see Appendix A for the full solution and definition of $\tilde{C}$):

$$\gamma^A = \frac{A\tilde{C}[(1-\alpha)/(1-zn)]}{X^{\omega-1}(X+1)(\alpha X + C + 1)} \tag{23}$$

In order to derive the public-to-private capital ratio which maximizes the aggregate growth rate along the balanced growth path, we take the first derivative of equation (23) with respect to the public-to-private capital ratio, X, set it equal to zero, and solve for X. The result is a general solution of the form (Appendix A):

$$X_{1,2} = \frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right) \pm \sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2 - 4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)} \tag{24}$$

With our parameter restrictions, it is easy to show that the positive solution to this equation is also positive (see Appendix B)[14]:

$$X^* = \frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right) + \sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2 - 4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)} \tag{25}$$

Given equation (5), the optimal debt to GDP ratio in this model is therefore:

$$d^* = \frac{1}{A}X^{*\omega} \tag{26}$$

3.2. Optimal debt policy and demographic factors

It is now apparent that the optimal debt policy depends on the child-rearing subsidy rate. This subsidy rate is also an important component in the balanced-budget tax rate. As a result, the subsidy plays a central role in the savings-investment balance. It also affects fertility choices and hence population growth. The subsidy rate is thus inescapably part of the optimal debt policy. Furthermore, by maximizing aggregate growth per capita, $\gamma = \frac{\gamma^A}{n}$, we can reach the same results as for maximal aggregate growth, γ^A: equations (24)–(26). This is expected since the optimal level of government debt does not affect childbearing decisions of agents. But the optimal debt level is influenced by childbearing through the subsidy ρ_w. Hence demographics affect the optimal debt ratio; but the debt ratio does not affect population growth.

As we might expect, in the special case without subsides ($\rho_w \to 0$) the optimal debt ratio becomes independent of the population parameters; it becomes solely dependent on the elasticities of private capital to public capital (ω) and labour (α). It is important to note that the choice of ρ_w is not treated as a policy tool in this analysis, in the sense that we do not attempt to derive an optimal subsidy policy. An optimal subsidy rate would depend on efficiency and welfare concerns in relation to endogenous population growth, which would turn this analysis into a normative one with respect to fertility choices. Instead, the choice of ρ_w is a parameter used to explore different calibrations, to show the economic effects of manipulating the child-rearing subsidy. The ultimate focus is on providing the growth-maximizing optimal debt-to-GDP ratio with balanced budgets for any *given* child-rearing subsidy.

Moreover, the structure of the labour productivity-enhancing factors in production is key to the optimality of debt policy. Notice that if public capital becomes irrelevant relative to private capital in the productivity process h_t in equation (4), we will have (other things equal) $\beta \to 1$ and consequently $(1-\omega) \to 0$. In this case, both the optimal public-private capital ratio, X^* in equation (25), and the

[14] We are only interested in a positive solution since capital stock ratios must be nonnegative.

optimal level of debt, d^* in equation (26), tend to zero. This is in line with Greiner (2010) and Checherita et al. (2014).

Finally, the subsidies described above may be interpreted in different ways to cover the different types of age-related spending or social support available. The subsidies may be designed to support raising children. For example, the UK provides child benefit payments which were taxed at standard rates until the 1980s and again from 2012. In fact, *any* grant which is either taxed or means tested can be written as equation (9) under uniform grant or means test rates. Most tax codes, including those in the US, offer a tax free element per-child equivalent to the net income supplement of $\rho_w(1-\theta_t)wzn$, which is then taxed at the standard rate (where ρ_w is set to make this expression equal to the tax saving).

But the subsidies could also be in the form of support to higher education. The US, for example, taxes certain training grants and fee waivers. Subsidised loans or means tested tax reductions on fees operate like child benefits, except that z now refers to time spent in education (child rearing by society, rather than by parents). Subsidies could also support education in general, where state spending per pupil is related to the average wage and taxes (levied at standard rates on steady state earnings) that fund that spending. In this case, z is the proportion of the young population in state funded schools.

In addition, subsidies could be directed at health care costs, where caregivers are paid via a state subsidy; or where those costs contain hidden subsidies. Another possibility is sick pay. For example, most EU countries pay a fraction of the wage for time off sick, but then tax it as income. Similarly, caregivers may be paid with a state subsidy, but taxed at standard rates.

4. Policies to manage demographic change

This section examines how two major demographic trends, reduced mortality and reduced fertility, affect fiscal policy and optimal levels of public debt. These two trends lead to ageing populations. In addition, we study the effects on optimal debt of an increase in preferences for current consumption over future consumption, as has been observed for the baby-boomer generation throughout the OECD area. Finally, we also study optimal debt responses to changes in public child-rearing support.

4.1. Increased longevity

Within the framework developed above, we can evaluate how increased longevity impacts the debt-to-GDP ratio. Increased longevity is simulated by lowering the probability of death in transition between periods, λ. By differentiating the optimal debt-to-GDP ratio in equation (26) with respect to λ, we get:

$$d^*_\lambda = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\ \underbrace{X^*_n}_{>0}\ \underbrace{n_\lambda}_{>0} \geq 0 \tag{27}$$

where X^*_n denotes the first derivative of X^* with respect to n; and n_λ is the first derivative of n with respect to λ. Note that equation (14) implies $n_\lambda > 0$. The sign of the partial derivative X^*_n is determined in Appendix C.

Expression (27) implies that rising life expectancy, in the form of a lower probability of death in the transition between periods, leads to a fall in the optimal debt ratio for the whole economy.[15] As the hazard rate decreases, consumption during retirement becomes a more pressing source of utility. Since consumption is financed by labor activities, people need to spend more time at work. As a result they have fewer children. Individuals thus shift from utility derived from having children, to utility derived from consumption.

With lower population growth and higher longevity (lower λ), it is no longer optimal for the government to sustain the same public debt level as it did before. The fact that agents choose to have fewer children has a direct impact on the government's budget constraint through the child-rearing subsidy. Since the government spends less on the subsidy, the tax rate needed to balance the current account budget becomes lower, as shown in equation (20). The servicing costs of public debt will also be less since the optimal debt level falls. This further lowers debt servicing cost through a drop in the interest rate payable, this can be seen by equation (6). In short, a fall in λ leads to a rise in savings by the working population ($\partial s_t / \partial \lambda < 0$) that now realizes it needs to save more for retirement than before, at the cost of fewer children.

4.2. A fall in the preference for children

We next consider the case of a fall in the preference towards children, defined as a drop in ε which leads to a fall in the birth rate. In this case we can write, upon differentiating the optimal debt-to-GDP ratio (26) with respect to ε,

$$d^*_\varepsilon = \frac{1}{A}\omega X^{*\omega-1}X^*_n n_\varepsilon \tag{28}$$

where d^*_ε denotes the derivative of d with respect to ε; and where

[15] A variation on this result is that a rise in the retirement age would be reflected, all else equal, in a rise in λ since fewer agents would survive into the retirement period. This is the opposite to the previous example: higher debt is now required to finance the extra output agents would need to produce during their working lives.

$$n_\varepsilon = \frac{1+(1-\lambda)\rho}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]^2} > 0 \tag{29}$$

which confirms that a fall in the preference for children leads to a fall in the birth rate. We may therefore work with change in either ε or n. It follows that

$$d^*_\varepsilon = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\ \underbrace{X^*_n}_{>0}\ \underbrace{n_\varepsilon}_{>0} > 0 \tag{30}$$

So, for positive subsidies, a fall in the birth rate would lead to lower optimal debt levels. Since the sign of n_ε is the same as n_λ, the story here is similar to the one of greater longevity. The explanation: a lower preference for children increases labour force participation in period t, due to less time spent rearing children. That leads to increased rates of consumption, now and in the future. Additionally, the drop in fertility eases the tax burden of the child-rearing subsidy, leading to lower tax rates and greater savings and consumption, yielding an optimal fiscal policy with lower public debt.

4.3. *Higher discount factors*

The third example is the effect of an increase in the discount factor, taking the form of a fall in ρ. This is often referred to as the baby boomer problem. It is exactly what happened with the baby-boomer generation in Europe, albeit less dramatically than in the US or UK, where the preference for current consumption over future consumption increased and savings rates fell.[16] Those are exactly the changes implied by (8), which then implies, *ceteris paribus*, $\partial s_t/\partial\rho > 0$ by (13).

To examine the debt consequences of this, we need to determine the sign of

$$d^*_\rho = \frac{1}{A}\omega X^{*\omega-1}X^*_n n_\rho \tag{31}$$

where, by eqn. (14),

$$n_\rho = \frac{-\varepsilon(1-\lambda)}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]^2} < 0. \tag{32}$$

Hence we have:

$$d^*_\rho = \underbrace{\omega X^{*\omega-1}}_{>0}\ \underbrace{X^*_n}_{>0}\ \underbrace{n_\rho}_{<0} < 0 \tag{33}$$

In words, a lower concern for the future (the revealed preference of the baby boomers) leads to higher levels of public debt. This matches exactly what has happened in nearly all European and OECD countries over the past two decades, although baby-boomers may not have been the only cause. However, higher future discounting is not the only effect in play. Mortality trends and fertility choices coexist in the observed data associated with the baby boomer generation. The end result depends on the *net* combined effects of a drop in ρ, λ and ε. The model would then match observed data; lower savings combined with fewer children.

The intuition is that baby boomers have lower preferences for saving and higher preferences for immediate gratification which, in this model, is present consumption and child rearing at the cost of future consumption. Consequently the birth rate increases, but so does the child-rearing subsidy bill. Thus, even though individuals save less now, implying lower economic growth, the optimal level of debt rises (a higher public-to-private capital ratio, X) for the same reason as in the hazard rate and preference for children examples.

This poses a policy challenge: not only do current levels of debt need to be reduced to normal levels, but they need to fall further than that as the baby boomers retire. This might be taken as a justification for imposing a "granny tax" on retirees in cases where social support and excess debt are allowed to continue unchecked. However, a proper analysis of that case would need a model with separate tax rates for young and old, since those rates may affect working and saving behaviour.

4.4. *An increase in support for child-rearing*

Finally, we consider the effects of ρ_w on growth-maximizing debt. This highlights how different government policies related to fertility can impact optimal fiscal policy. The subsidy rate reduces the cost of raising children, so higher subsidies naturally lead to higher fertility n by equation (14). This would be useful if fertility falls below the replacement rate.

Using the previous framework, we now need to determine the sign of $d^*_{\rho_w}$. The relevant derivative of d^* with respect to ρ_w can be written as:

[16] This example assumes social support, ρ_w, is unchanged. If not, savings may increase – as in Japan or China.

$$d^*_{\rho_w} = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\left(\underbrace{X^*_{\rho_w}}_{>0} + \underbrace{X^*_n n_{\rho_w}}_{>0}\right) > 0 \tag{34}$$

where $X^*_{\rho_w}$ and n_{ρ_w} denote first derivatives with respect to ρ_w. The signs of n_{ρ_w} and $X^*_{\rho_w}$ are determined in Appendix C.

It follows from equation (34) that the optimal debt burden in equation (26), d^*, will increase with subsidies ρ_w. The reason: the subsidy reduces the private cost of raising children, so higher subsidies lead to higher fertility n: (14). It also increases the tax rate θ_t and time spent on child rearing (zn). Anything not related to public debt maintenance (rb_t) that increases the tax rate and budget cover, increases the optimal level of public debt and public-private capital ratio, directly and indirectly, as can be seen from (34) where subsidy rates and the fertility rate are increased at the same time.

5. Calibration

To test the performance of our model as a reasonable and plausible representation of the type of advanced economy that we might be interested in, we calibrate it to match the typical OECD economy for the period 1969–2013.[17] We then alter the underlying demographic structure to match the following 45 year periods. This allows us to assess fiscal sustainability under social or demographic changes of different sorts; how that compares to the long-run fiscal position and growth rates observed in the OECD economies in the past; and whether these economies would find it easy or feasible to transition to the corresponding steady state.[18]

To gain some insight into the potential problems of OECD economies facing demographic change, we first perform a baseline calibration of those economies using the model in section 2. The calibration is based on a period of 45 years in length. This assumes that agents in the model start working at age 20, work for 45 years and retire at age 65. We choose the remaining parameters to produce a realistic demographic and production structure for the average OECD economy. Table 1 sets out the parameter values; the motivation for each value follows below.

5.1. Production parameters

We focus first on production. The value of the output elasticity of private capital, α, is 0.37. Given the production function is Cobb-Douglas, this value then matches the average private wage income share of GDP in the OECD from 1969 to 2013, as measured by the AMECO database: "Adjusted wage share, total economy, as percentage of GDP at current factor cost".

Parameter β captures the private sector share in labor productivity gains: that is, the share of income gains that come from productivity increases due to private as opposed to public capital. This is not a figure that is reported in national accounts. But Checherita et al. (2014) estimate ω to be equal to 0.716.[19] Having α equal to 0.37, the definition of ω implies that β equals 0.55. In section 7 we analyse the sensitivity of our results to this assumption.

The TFP scale effect A is subject to interpretation. In this model, productivity contributes to growth in two ways: one endogenous, the other exogenous. Labor-augmenting productivity h_t represents the endogenous component, and the TFP parameter A is the exogenous component (Fehr et al., 2013; Hviding and Merette, 1998). We use the expression for production in equation (5) to calibrate A. This expression links GDP, the private capital stock and public capital stock, all available figures. We obtain these figures from IMF's Investment and Capital Stock Dataset and MEI database (2017). We then use the value for ω specified above. Solving equation (5) with these values then yields $A = 34$.

5.2. Preference and demographic parameters

Parameter ρ is designed to capture the value an agent puts on future consumption relative to present consumption; that is, the discount to be applied at the start of the working period to decisions made for retirement 45 years later. Given the long time period relating the preference for consumption in the present vs. that in the future, we apply an annual discount rate of 1% hyperbolically. For a period of 45 years, this yields ρ equal to 0.69.

Parameter z captures time taken off work due to child-rearing activities. We use data from the US Census Bureau's "Annual Social and Economic Supplement to the Current Population Survey" for "Persons who did not work or look for work by age, gender, and reason for

[17] Given the time period at hand and the focus group (the OECD countries), some of the data required for the calibration exercise is missing. We use mean imputation to fill those gaps.

[18] In future work, we could go on to look at the steady states implied in particular economies, including: (1) the "Anglo-Saxon economies" (US, Canada, UK); (2) countries where the process of population ageing is particularly strong (Japan, Italy, Germany); (3) the Nordic welfare states with flexible markets and strong social provision (Denmark, Sweden); (4) economies with slow population growth and limited social support (Russia, China); or (5) those with young and fast growing populations (India, Mexico, the Philippines).

[19] This paper uses a slightly more general production function than we do, the difference being that Checherita et al. do not include per-employee productivity gains to labour inputs to production. We have the following production function $Y_t = AK_t^{\omega}G_t^{1-\omega}$ whereas they have $Y_t = A(K/L)_t^{\varphi}K_t^{\omega}L_t^{1-\omega}$ (in our notation, where φ is the output elasticity of the labour-capital ratio to GDP). For our specification to match theirs, φ must be zero. In fact, their estimate of the OECD average for φ is very close to zero at 0.037 (Table 2, Model 2, Checherita et al., 2014).

Table 1
Parameter values of the baseline simulation, OECD area.

α	Private capital's share in national income	0.37
β	Private capital elasticity in productivity gains	0.55
ω	Private capital elasticity in production	0.716
A	TFP scale factor	34
ρ	Discounting	0.69
z	Rearing time per child	0.281
ρ_w	Child rearing subsidy rate	0.193
λ	Hazard rate	0.7
ε	Importance of children in utility	0.539

Table 2
OECD population growth, old-age dependency ratios and life expectancy (45 year averages).

	1970–2015	2015–2060	2060–2100
45-year population growth rate	36.1%	6.3%	−5.2%
Old-age dependency ratio	21.9%	44.0%	60.1%
Life expectancy at birth	75.2	83.9	89.2

Source: United Nations.

not working in 2014". The proportion of working-age people not working in order to undertake home responsibilities is 28.1%. Interpreting "home responsibilities" as a measure of people not working for child-rearing reasons, z simply becomes 0.281.

Next we calibrate the child-rearing subsidy rate. We use the OECD's Family Database, and the "Total public social expenditure on families as a percentage of GDP" data in particular. Social spending by families in the OECD from 2001 to 2013 averaged 2.36% of GDP. This number does not include primary to tertiary schooling. For this, we use the OECD "Education at a glance" reports. The OECD average public expenditure on primary to tertiary education, for 2001–2013, is 5.15% of GDP. Total public expenditures on children measured as a percent-age of GDP then becomes 7.51%. In order to match those expenditures as a proportion of GDP with the model's expenditures measured as a proportion of wages, we match total expenditures in the real world to total public expenditures in the model with children, $Nwzn\rho_w$ (where Nw is total spending in the economy). Knowing that the population growth factor in the OECD for 1970–2015 is $n = 1.361$ (see Table 2), $\alpha = 0.37$ and $z = 0.281$, we get $\rho_w = 0.193$.

Next, λ and ε are the key demographic parameters. Both influence the age structure and the growth of the population. Starting with λ, the hazard rate, it needs to be set so that the model's old-age dependency ratio matches the observed data. The model's old-age dependency ratio is equal to $(1-\lambda)/n$. And for the old-age dependency ratio in the data, as seen in Table 2, is 21.9% for the period 1970–2015. Table 2 also reveals that $n = 1.361$ for the same period. Solving for lambda, we obtain $\lambda = 0.7$.

Finally, the children preference parameter, ε, needs to be set so that the model's fertility rate per individual in the model matches OECD data. In the model, the 45-year population growth rate is simply equal to the fertility rate per individual, n. Again, Table 2 indicates that the 45-year population growth rate for the period 1970–2015 is 36.1% or 0.69% annually. Solving (14), the expression that defines fertility choices, for ε yields $\varepsilon = 0.539$.

Notice that $\varepsilon = 0.539$ ensures that a non-trivial steady state will exist; the lowest permissible value, by (15), being $\varepsilon \approx 0.35$. We make an important distinction between reality and the model here. The model does not recognise that not every woman is able or willing to have children. The rule of thumb is that 10% of adults do not have children which means that, to maintain a constant population, each woman needs to have 2.2 children. These figures therefore ensure we have a non-trivial steady state, but also a realistic population growth in steady state.

6. Steady state characterisation

6.1. A first look at the results

Table 3 summarizes the output of the baseline simulation. We see that the demographic parameters, the number of children per agent and the age dependency ratio, are in line with what was experienced in a typical OECD economy as represented in Table 2 for the period 1970–2015. Life expectancy at birth, as calculated by $20 + 45 + 45(1-\lambda)$, stands at 78.5 years, which is not far from the value observed in the data for the OECD (75.3 years, source: United Nations Population Division).

The optimal debt to GDP ratio, d^*, is then 49.67% which indicates a value that is somewhat lower than the historical value for most OECD economies. This does not come as a surprise, given the fact that most countries are far from respecting the golden rule of public finance. The optimal ratio of public-to-private capital is roughly 25.5%, while the optimized growth rate of output is 90% over the 45-year period. This *per period* growth implies *annual* average growth rates of 1.44% in steady state (0.75% per capita). These values are a bit lower than the OECD economies actually experienced in the 1969–2013 period (an average of 2.8%, or 1.91% per capita[20]). The

[20] The OECD Economic Outlook dataset for growth rates; World Bank WDI report for per capita growth rates.

Table 3
Results of steady state baseline simulation.

d^*	Optimal debt to gdp ratio in steady state (no TFP)	49.67%
X	Public private capital ratio	0.2546
γ^A	Annual economic growth rate	1.44%
γ	Annual economic growth rate per capita	0.75%
θ	Tax rate	14.50%
$\bar{C}$	Saving rate (of disposable income)	11.86%
r	Annualized interest rate	5.14%
n	Number of children per person	1.36
	Old-age dependency ratio	22%
	Life expectancy at birth	78.5

reason for this is that we only include endogenous growth components, and not exogenous growth effects (a growing TFP parameter A).[21]

The resulting tax rate is 14.5%, which is a reasonable rate for taxes that finance (service) only public debt and child-related spending. Furthermore, the savings rate from disposible income is 11.86%, in line with OECD data in the AMECO database (11.3%). Also, the *annualized* steady-state interest rate sits at 5.14%, somewhat higher than the observed average OECD rate interest rate of 4.37% in the period 1969–2013.

What are the lessons from these simulation results? First: that a steady state solution and a sensible and sustainable set of fiscal policies is both feasible and available. Second: based on reasonable parameter values for the average OECD economy, the steady state solution demonstrates realistic properties in view of the OECD experience in 1969–2013.

They also suggest that public debt at just below 50% of GDP is a reasonable target level to aim at. This accords well with the Maastricht Treaty/Fiscal Compact upper limit of 60% to create a debt target with a 10% safe zone to accommodate shocks. Moreover, it accords well with the pre-crisis experience of non-Eurozone economies (the US or UK) where debt was held in the 35%–45% range. It also suggests a mixed economy with a public sector share of around 25.5%: that is lower than in Scandinavia or the Eurozone (Sweden, Denmark, France, Italy, Germany, the Netherlands), but similar to that in the US, Canada, Australia or the UK in the pre-crisis era. Finally, it suggests population growth very close to that projected for the OECD (Table 2).[22]

A first impression of these results is that the demographic situation is perhaps not as bad as many commentators fear. But it will not be easy for most OECD countries to get themselves down to this optimal steady state level of debt (that is to remove an average of 65% points from current debt ratios for those OECD economies with excessive debt-GDP ratios in 2013[23]), and to maintain the steady state rate of growth in productivity at the same time, currently a problem of major concern to OECD policymakers.

Our results are also consistent with standard growth theory models that show GDP growth ultimately comes from either growth in productivity or growth in the workforce. Since we have little of the latter (the annual population growth rate of 0.69%, is only a little above the constant population replacement rate), GDP growth has to come from productivity growth. Demographics do not affect that: the population growth is simply too slow to make up the difference. However, the transition to steady state growth could become more difficult for economies that currently find themselves a long way from that steady state.

6.2. Changes in the population parameters

To provide insight into the effect of demographic change on the steady state, we evaluate the partial derivatives in Section 4 around the baseline in Table 3. The results are in Table 4.

A general finding is that the changes in outcomes are all rather small with respect to both X^* and d^*. Table 5 goes further; it reports the effects on X^* and d^* when we calibrate the model to match variations in the demographics that might be expected in the future (Table 2). We see that the effect in 2015–2060 is a decrease of 0.3 percentage points in both X^* and d^*. That is extremely small. For 2060–2100, the decrease is even smaller: 0.1 percentage points. These negligible impacts illustrate the economy's *structural* robustness to age-related transfers. But ε and λ influence optimal debt through n and the child-rearing subsidy. So other performance indicators may be more sensitive to variations in age-related government spending.[24]

Thus, we conclude that not only is the optimal steady state under the new demographics relatively benign, but likely future demographic and social *changes* will make little difference to that steady state once the economy gets there. In short, there is not much long-term sensitivity to demographic or economic shocks. Instead, for most OECD economies, the difficulty will be how best to manage the transition from where they are now to their steady state.

[21] The model is not well equipped to have a growing TFP parameter A, since doing so may imply ever growing steady state interest rates (see equation (6)).

[22] Our calibration also yields an old-age dependency ratio of 22%, see Table 2. This is rather low and suggests a new a risk: if migrants arrive but then adopt the mores of the domestic population, as assumed by our model, then age-dependency ratios will rise without a corresponding increase in long term population growth rates.

[23] These countries are all OECD economies, except Chile, Estonia, Luxembourg, Norway, and Turkey.

[24] Some generalisations of the model might be useful here: removing the assumption of no capital depreciation, or equal taxes on wages and subsidies, or subsidies as a fixed ratio to wages.

Table 4
Sensitivities to demographic parameters, OECD economies.

X_n^*	$\partial X^*/\partial n$	0.0067
n_λ	$\partial n/\partial \lambda$	0.538
n_ε	$\partial n/\partial \varepsilon$	1.746
n_ρ	$\partial n/\partial \rho$	−0.234
d_λ^*	$\partial d^*/\partial \lambda$	0.005
d_ε^*	$\partial d^*/\partial \varepsilon$	0.016
d_ρ^*	$\partial d^*/\partial \rho$	−0.002

Table 5
Optimal public debt/investment policies with balanced budgets.

	1970–2015	2015–2060	2060–2100
λ	0.7	0.532	0.43
ε	0.539	0.42	0.382
Life expectancy at birth	78.5	86.1	90.65
X^*	25.5%	25.3%	25.2%
d^*	49.7%	49.4%	49.3%

6.3. Changes in the child-rearing subsidy

We now analyze how changes to the child-rearing subsidy rate ρ_w impact the steady state and fiscal policy. As an example, we consider a case where the government uses the child-rearing subsidy to reverse adverse demographic shocks. Specifically, suppose a preference shift takes place such that agents have a lower propensity to raise children, directing their income towards more consumption instead. This could be problematic for society if fertility rates drop below the replacement rate (as is expected to happen sometime in the period 2060–2100, see Table 2). Faced with this problem, the government might wish to raise the child-rearing subsidy. From Section 4, we know that (a) a drop in the preference for children leads to a lower optimal debt-GDP ratio; and (b) a higher subsidy rate leads to increases in the optimal debt-GDP ratio. These two effects conflict, so the end result on the debt ratio is ambiguous. It all depends on which effect on d^* is larger.

To answer that question, we introduce a calibration exercise where we look at the steady state effects of a composite shock where there is first a drop in the preference for children, sufficient to bring the fertility rate to the population replacement rate (1.1 per person), and then a rise in the child-rearing subsidy to bring the fertility rate back to its baseline value (1.36 per person).

The results are reported in Table 6. All parameters are as in the baseline calibration (Table 1) except for the explicit changes made to ε and ρ_w. For clarity, the tax rate has been decomposed into the taxes necessary to service public debt and taxes needed to cover the subsidies. From this calibration, we see that, ceteris paribus, a drop in the preference for children decreases the total tax rate since, with fewer children, there is less strain on the government's budget. There is also some extra saving as agents divert their time allocation away from child-rearing to employment. Taxes to cover debt service therefore increase when the preference for children goes down because more public investment will be needed to generate the extra output and jobs. There are two opposing forces here. On one hand, optimal public debt decreases, equation (30), as do interest rates (though by a negligible amount, as Table 6 reveals). On the other, tax revenues decrease because subsidy payments (a *partial* income replacement) are subject to tax, so fewer children enter the work force. This may create fiscal pressures in order to service public debt. Ultimately, if the tax revenue effect dominates the public debt effect, a higher tax rate will be necessary for debt servicing. However, growth per capita may increase even if aggregate economic output falls because fewer children are available for work in the future.

When we allow increasing subsidies to recover the baseline fertility rate, a different picture emerges. The additional subsidy spending is covered by taxation, increasing the total tax rate to well above baseline. This is due to the higher subsidy rate; the tax rate

Table 6
Subsidies used as a policy tool.

	Baseline	Decreased preference for children	Increased subsidy
Preference for children (ε)	0.539	0.401	0.401
Child-rearing subsidy (ρ_w)	0.193	0.193	0.348
Fertility rate	1.36	1.10	1.36
Total tax rate …	14.5%	12.9%	18.8%
…of which debt servicing	8.1%	8.2%	7.8%
…of which subsidy coverage	6.4%	4.7%	11.0%
Saving rate	11.86%	12.87%	12.87%
Interest rate (per year in steady state)	5.14%	5.14%	5.15%
Public-private capital ratio	25.5%	25.3%	25.9%
Government debt-GDP ratio	49.7%	49.5%	50.3%
GDP growth (per year in steady state)	1.44%	1.41%	1.51%
GDP growth per capita (per year)	0.75%	1.2%	0.82%

associated with debt servicing actually decreases as a result of the extra tax revenues stemming from the increased subsidy rates. The saving rate from post-tax income remains the same. The public-private capital ratio and optimal public debt are now both above baseline. Interestingly, per capita GDP growth and GDP growth are also both above baseline, revealing that, in the face of a decreasing preference for children, an economy can still benefit from increased public support for child-rearing at the aggregate level. When the increase to the child-rearing subsidy is considered in isolation to the drop in the preference towards children, we see that aggregate economic growth is increased, whereas its per capita measure decreases. This reveals a trade-off in fertility-boosting policies: changes in the subsidies may have opposing effects on economic growth and growth per capita.

In sum, when a government attempts to preserve the fertility rate at current levels by increased public support for child-rearing, the optimal level of public debt increases along with higher tax rates and lower economic growth per capita. These results illustrate the trade-off between population growth and economic growth when policies are used to influence population size. Our analysis does not address the issue of an optimal child subsidy of course; it merely reveals alternative steady state outcomes.

7. Sensitivity analysis and political economy trade-offs

7.1. Sensitivity analysis

The optimal debt level depends, to a large extent, on the composition of labour productivity, in which β plays a key role. Fig. 1 therefore demonstrates the sensitivity of our results to different choices for β.

As β increases, we expect to see private capital become increasingly important to output and labor productivity (h_t in equation (4)), while public capital becomes less important (decreasing returns to scale). The public-private capital ratio should therefore decrease, along with the optimal level of debt (26). That much we see, with rising growth rates, in Fig. 1. Conversely, X^* and d^* *rise* with falling β; but growth rates fall (if decreasingly so).

In other words, the economy becomes increasingly dependent on public capital and lower growth when the private sector's income share from productivity gains falls. This is hardly a surprise. The question is, why would that income share fall; and why are the impacts on X^*, d^* so large? It is worth stressing though that the impacts on growth are relatively small if we restrict ourselves to *plausible* changes in β, say $0.4 \leq \beta \leq 0.7$. The share of private capital income in GDP is not likely to change beyond those bounds – in which case annual growth rates would vary between roughly 1.25% and 1.75%, a fairly narrow range. However, X^* and d^* then vary from 18% to 35%, and 30%–70% respectively, a larger range but still consistent with a feasible and sustainable steady state solution (and less debt than many currently have).

To some extent, the sensitivity of economic growth to changes in β is due to the impact of taxation. Fig. 1 depicts a negative relationship between economic growth and taxation. A lower value of β implies an economy where public capital plays a larger role in determining labor productivity. However taxes have to rise to cover the cost of servicing the debt used to create that public capital. The negative effects of taxation manifests itself directly through a reduction in savings at any given generation; this partly explains why the

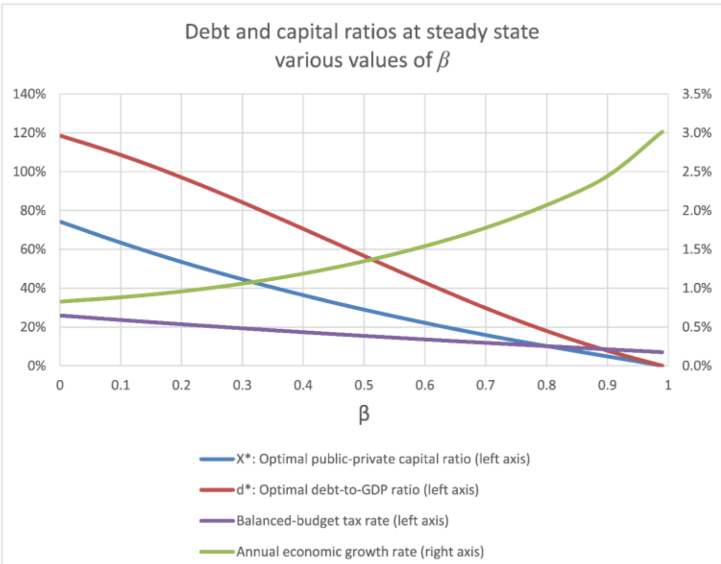

Fig. 1. Debt and capital ratios in steady state, various beta values.

economic growth curve is upward sloping.

Having to issue more government bonds crowds out the quantity of private capital that can be generated from a finite amount of aggregate savings; this further increases the sensitivity of X^* with respect to the issuing of public capital. Coupling this crowding-out with the fact that ω falls when β falls, equation (26) reveals why the debt-GDP ratio is the more sensitive variable. This suggests that fiscal discipline (rules) should focus on the debt burden, rather than on the deficit. At the same time, real wages are likely to fall, and returns to investment rise, if private capital's income share were to fall (see (6) and (7)) when taxes and interest rates are low (as they are now). This gives further insight into current concern for inequality.

7.2. Sensitivity to different subsidy and hazard rates

Next we explore sensitivities due to different child-rearing subsidy rates ρ_w and hazard rates λ. Fig. 2 depicts the change in the optimal debt-to-GDP ratio, d^*, and optimal public-private capital ratio X^* as a result of doubling the child-rearing subsidy rate; Fig. 3 depicts the effects of increased longevity (lower λ values) on the same variables. The main observation is that the impacts on X^* and d^* are rather small for plausible values of β, say $0.4 \leq \beta \leq 0.7$. So the effects on growth rates will be considerably smaller.

Since β is a key determinant of the positive effects public capital on growth, the d^* and X^* curves are downward sloping in Fig. 2. Thus, a government that attempts to incentivize higher fertility choices in an economy that is heavily dependent on public capital will tend to have higher optimal debt-GDP ratios (increases of more than 2 percentage points for $\beta < 0.5$). In the opposite case, an economy where labor productivity is little dependent on public capital, doubling of the subsidy rate produces low to negligible effects on both d^* and X^*. Similar effects can be seen from increased longevity, in Fig. 3: the effect on both d^* and X^* is now negative and negligible across the board when the hazard rate is halved. In sum, economies that depend on public capital to boost productivity (and thus, wage rates) are more sensitive to demographic and public finance shocks; though these shocks never translate into meaningful changes to optimal public debt schedule for the whole range of β values.

In general, the value of 0.55 for β in the baseline calibration seems to be a good middle ground for plausible β values: $0.4 \leq \beta \leq 0.7$. Extremes on both sides are eliminated as we neither observe, or are likely to observe, the extremes associated with low β values relative to the small variations associated with high β values. A key takeway is that the value of β does not change the sign of the impacts of demographic change, but only their size.

7.3. Political economy considerations

Fig. 4 provides isoquant lines for different values of growth and debt. The isoquants are determined by considering different mixes of α and β values. In this sense, who benefits from productivity gains can be discussed in the wider context of economic performance and debt sustainability.

The political economy implications here depend on specific interpretations of α and β. As the value of β increases, private capital's share of the income generated by labour augmenting productivity gains rises. By contrast, a fall in β implies greater importance for public capital. At the same time, a rise in α increases the share of private capital relative to the share of labor income in national income.

Fig. 4 can be interpreted as follows: If we go northeast in the diagram, private capital becomes the main driving force of economic growth, both by having a major influence on labor productivity and by being the chief input in the production function. This area represents economies marked by higher private capital income shares (relative to both public capital and labor). Here we get lower debt,

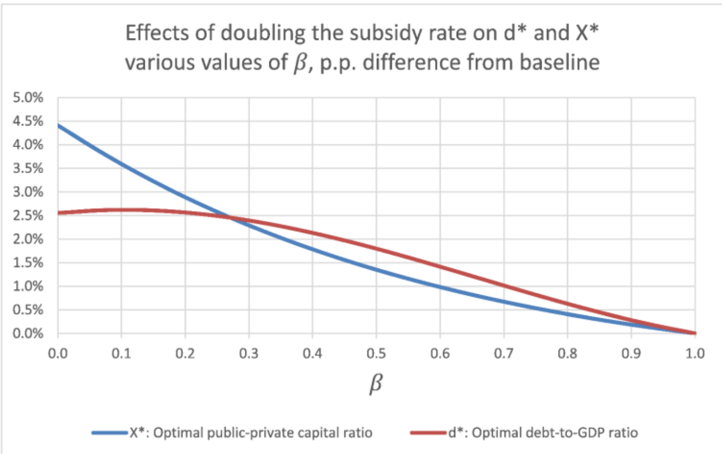

Fig. 2. Changing the subsidy rate, impacts on d* and X*.

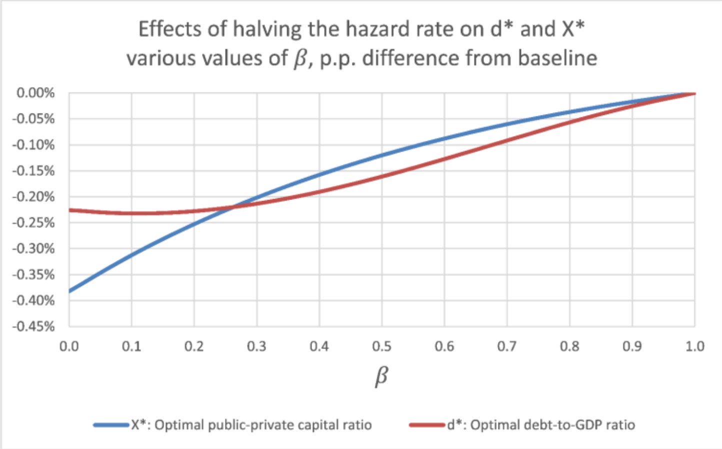

Fig. 3. Changes in life expectancy, the effects on d* and X*.

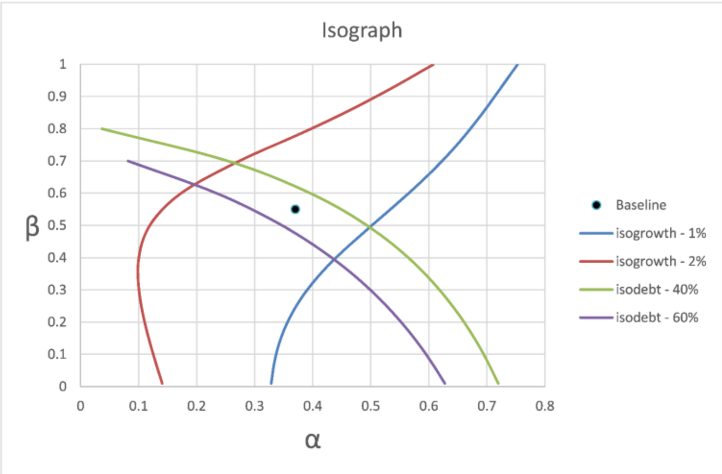

Fig. 4. Steady state growth-public debt possibility frontiers under different income shares for private and public capital.

but little (if any) additional growth, and none at all if the movement is mostly east. Conversely, movement to the south-west would result in an economy largely dependent on public capital, resulting in higher debt levels (and higher growth if the movement is sufficiently to the west).

By contrast, moving northwest up the diagram, an area where there is a larger share of labor income at the same time that private capital is still highly relevant for the productivity of labor, brings higher growth more rapidly. Debt reductions will become increasingly elusive (and may not materialize at all if there is too much movement west). The income gains from higher productivity will increasingly accrue to workers. Conversely, movement to the south-east could lead to low shares of income going to workers while there might be little change in the debt level. This is because, as α decreases, the marginal productivity of labor decreases.

The implication of these results is that, in steady state, public sector productivity is crucial to economic performance. The political economy trade-offs, in terms of performance and fiscal imbalances, are laid out in uncompromising reality once we account for changing demographics and how the gains from productivity are distributed. It would appear that productivity plays a key role, perhaps even more than we thought before.

Furthermore, even if demographic change has small impacts on d^* and X^*, it is important to highlight that changes to tax rates due to demographic change may not be trivial. As Fig. 5 shows, the child-rearing subsidy is heavily correlated with the tax rate, so that, for countries with stronger subsidies, a balanced budget rule obviously requires stronger shifts in the tax rate to balance the fiscal current

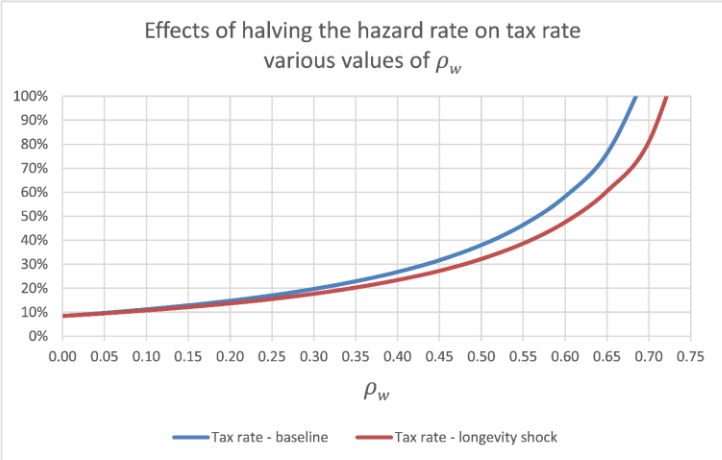

Fig. 5. The effect of changes in the subsidy rate on the general tax rate.

account. Thus, if there is any impact of demographic change on optimal fiscal policy, it won't be on public debt, but on tax rates and fiscal policy.

The process of demographic change has economic growth effects that are quite small, but may cumulate into something larger over time. We also saw that economic growth depends on population growth as well as productivity growth. This just confirms the standard models of growth theory. However, once fertility choices are endogenised, the problem of maximizing economic growth is not the same as maximising growth per capita.

In fact, if the government turns fertility into a policy objective by introducing child-rearing subsidies (and nearly all advanced economies do), the maximization of economic growth may imply reductions in growth per capita. The reasoning is as follows: an increase in the subsidy rate at time t has three effects. One, it stimulates a higher *incoming* rate of workers (higher N_{t+1}, N_{t+2}, etc.), since more children are born at t. Two, labor supply permanently decreases [lower $(1 - zn)$] because more time has to be spent on child rearing. The effect of increased subsidies on labour supply is therefore ambiguous. Three, tax rates increase as a consequence of a higher subsidy (and the rise in zn). But the increase in the number of future workers does not automatically lead to higher savings in the future since the extra workers face a decrease in disposable wages. In addition, any increases in labor supply will lead to a permanent decrease in real wages, as seen in equation (7). Those effects, if they prevail over increased post-tax earnings and employment, will decrease the flow of funds available for investment. That is not guaranteed however. The overall effect of increased subsidies on aggregate savings is therefore also ambiguous.

Be that as it may, even if higher subsidies lead to stronger wealth accumulation and stronger growth, the impact on growth per capita γ^A/n, via the increase to the denominator of this expression, may outweigh the positive effects of any decrease in labor supply $1/(1 - zn)$. Thus, increasing the subsidy rate may lead to reduced economic growth per capita.

8. Concluding remarks

1) Our model and simulations show that steady state outcomes for an advanced economy facing today's demographic and social trends are always feasible and available. They are relatively benign and resemble those of the OECD economies a few decades back.
2) Plausible changes to population parameters and social conditions in the future make rather little difference to the steady state outcomes for the optimal public-private capital ratio and public debt once we get there.
3) Instead the steady state is driven by economic fundamentals: productivity, the production structure, and income generation. The real problem is therefore to get to the chosen steady state without being derailed by undue strains or fiscal collapse along the way. That is the next step in our research (Hughes Hallett et al., 2017).
4) Fiscal sustainability is a matter of changes in tax or spending in transition, not so much long term debt.
5) Sensitivity tests show we can get a variety of outcomes for the public ownership mix and debt level in the economy, but without much change in the performance levels. In that sense our results are robust to parameter or assumption variations.

Hence demographic change is not necessarily a problem, if properly handled, compared to the fiscal laxity seen over the past 20–30 years. This is contrary to a conventional reading of the IMF's earlier advice. But it is consistent with the IMF view if we reinterpret that to mean "ageing and social change are not a problem, in steady state, so long as reliable fiscal rules or credible fiscal restraints are set up in advance to manage fiscal policy and the transition". We have shown that the golden rule of public finance (the least intrusive form of fiscal restraint) is sufficient to do the job.

But to imply that ageing and changing demographics are not a problem in steady state does *not* say that all OECD economies will find it easy to get used to living in an era of slower growth and lower debt than they enjoyed for three or four decades past. In fact, the real problem is likely to be how to create and then safeguard the transition to that steady state. In short, the challenge is to find and maintain durable dynamic adjustment paths.

Appendix A. The solution to the optimal level of debt problem

We start from (21), with (20) and (7), to obtain an expression from which we can derive a closed form expression for the economy's growth rate γ^A:

$$\tilde{C}\left(\frac{1}{\alpha X+\rho_w(1-\alpha)\frac{zn}{1-zn}+1}\right)A(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\omega}\frac{G_t}{L_t}N_t=K_{t+1}+G_{t+1} \tag{A1}$$

where $\tilde{C}=\frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}$. Next we set $C=\rho_w(1-\alpha)\frac{zn}{1-zn}$ so that (A1) becomes

$$A\tilde{C}\left(\frac{1}{\alpha X+C+1}\right)\frac{1-\alpha}{1-zn}\frac{1}{X^{\omega}}=\frac{K_{t+1}}{G_{t+1}}\frac{G_{t+1}}{G_t}+\frac{G_{t+1}}{G_t} \tag{A2}$$

Using (22) we can rewrite (A2) as

$$A\tilde{C}\left(\frac{1}{\alpha X+C+1}\right)\frac{1-\alpha}{1-zn}\frac{1}{X^{\omega}}=\gamma^A\left(1+\frac{1}{X}\right)$$

This allows us to solve for the economy's aggregate growth rate, γ^A, to get

$$\gamma^A=\frac{A\tilde{C}\frac{1-\alpha}{1-zn}}{X^{\omega-1}(X+1)(\alpha X+C+1)} \tag{A3}$$

which is the growth rate we need to optimize with respect to the public-private capital ratio.

To perform this optimisation, define $C_3=A\tilde{C}\frac{1-\alpha}{1-zn}$. Take the first order derivative of γ^A (denoted by $\hat{\gamma}$) with respect to X:

$$\hat{\gamma}(x)=-C_3\frac{(\omega-1)X^{\omega-2}[\alpha X^2+(\alpha+C+1)X+C+1]+X^{\omega-1}(2\alpha X+\alpha+C+1)}{[X^{\omega-1}(X+1)(\alpha X+C+1)]^2}$$

Setting this expression equal to zero, we can write

$$\alpha(\omega-1)X^{\omega}+(\omega-1)(\alpha+C+1)X^{\omega-1}+(\omega-1)(C+1)X^{\omega-2}+2\alpha X^{\omega}+(\alpha+C+1)X^{\omega-1}=0$$

which results in a quadratic equation in the optimal public-private capital ratio X:

$$\alpha(\omega+1)X^2+\omega(\alpha+C+1)X+(\omega-1)(C+1)=0$$

The optimal solution for X is therefore:

$$X_{1,2}=\frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)\pm\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)}$$

which is (24) in the main text.

Appendix B. The positive root

We now demonstrate that our solution for X^* is postive. We start from (25) in the text. Since $\omega>0,\ 0<z<1,\ n\geq 1,$ *and* $0<\alpha<1$, it follows that $\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)\rangle 0$ and $2\alpha(1+\omega)>0$. Therefore to get a positive solution, $X^*>0$, we need the numerator of (25) to be positive. It will be positive if and only if

$$-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)+\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}$$

is positive. We therefore need to check if this condition is in fact satisfied for the parameter constraints that we have imposed. We require $\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}>\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)$, or

$$\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)\rangle\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2;$$

that is, $-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)\rangle 0$

which is true since $-4\alpha(1+\omega)(\omega-1)=-4\alpha(\omega^2-1)>0$ and $\rho_w\frac{(1-\alpha)zn}{1-zn}+1>0$ both necessarily hold. Hence X^* in (25) is positive.

Appendix C. Signs of partial derivatives of X^* and d^*

In order to determine the sign of the partial derivative X_n^*, we introduce a shorthand notation to write the solution for X^*. Let $f(n)=\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha$, so $f(n)-\alpha=\rho_w\frac{(1-\alpha)zn}{1-zn}+1$. We define $c=-\frac{\omega}{2\alpha(1+\omega)}$. We now rewrite X^* in equation (25) as

$$X^*=cf(n)-\frac{c}{\omega}\sqrt{\omega^2f^2(n)-4\alpha(\omega^2-1)[f(n)-\alpha]}$$

Now let $\alpha F(n)=f(n)$, so that

$$X^*=c\alpha F(n)-\frac{c}{\omega}2\alpha\omega\sqrt{\frac{1}{4}F^2(n)-[F(n)-1]+\frac{1}{\omega^2}[F(n)-1]}$$

After taking the first order derivative with respect to n, we get:

$$X_n^*=c\alpha F_n(n)-\alpha c\frac{1}{\sqrt{\frac{1}{4}F^2(n)-[F(n)-1]+\frac{1}{\omega^2}[F(n)-1]}}\left[\frac{1}{2}F(n)F_n(n)+\left(\frac{1}{\omega^2}-1\right)F_n(n)\right]$$

Now let $G(n)=\frac{1}{2}F(n)+\left(\frac{1}{\omega^2}-1\right)$. Rewrite the denominator above as,

$$\sqrt{\frac{1}{4}F^2(n)+[F(n)-1]\left(\frac{1}{\omega^2}-1\right)}=\sqrt{\left[\frac{1}{2}F(n)+\left(\frac{1}{\omega^2}-1\right)\right]^2-\left(\frac{1}{\omega^2}-1\right)^2-\left(\frac{1}{\omega^2}-1\right)}$$

so that

$$X_n^*=c\alpha F_n(n)\left[1-\frac{G(n)}{\sqrt{G^2(n)-\left(\frac{1}{\omega^2}-1\right)^2-\left(\frac{1}{\omega^2}-1\right)}}\right]$$

To see the sign of X_n^*, we need to determine the signs of c, F_n and the expression in brackets. Since $c=-\frac{\omega}{2\alpha(1+\omega)}<0$, and $F_n(n)=\rho_w\frac{(1-\alpha)z}{\alpha(1-zn)^2}\geq 0$, the expression in square brackets is negative because the numerator is larger than denominator.

We now determine the sign of $d_{\rho_w}^*$. Note that, through the chain rule, the differentiation of d^* with respect to ρ_w can be written as:

$$d_{\rho_w}^*=\frac{1}{A}\omega X^{*\omega-1}\left(X_{\rho_w}^*+X_n^*n_{\rho_w}\right)$$

where $X_{\rho_w}^*$ and n_{ρ_w} denote first derivatives with respect to ρ_w. We already know the sign of X_n^*. We are left to determine the sign of n_{ρ_w} and $X_{\rho_w}^*$. For the first of the two, we have that:

$$n_{\rho_w}=\frac{\varepsilon}{z(1+(1-\lambda)\rho+\varepsilon)(1-\rho_w)^2}>0$$

For the second, in order to determine the sign of $X_{\rho_w}^*$ we can perform an identical analysis to that done for X_n^* in sub-section 4.1, except that now we define instead $f(\rho_w)=\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha$, $\alpha F(\rho_w)=f(\rho_w)$, and $G(\rho_w)=\frac{1}{2}F(\rho_w)+\left(\frac{1}{\omega^2}-1\right)$. With these definitions, we write:

$$X_{\rho_w}^*=c\alpha F_{\rho_w}(\rho_w)\left[1-\frac{G(\rho_w)}{\sqrt{G^2(\rho_w)-\left(\frac{1}{\omega^2}-1\right)^2-\left(\frac{1}{\omega^2}-1\right)}}\right]$$

As before, to see the sign of $X_{\rho_w}^*$ we need to determine the signs of c, F_{ρ_w} and the expression in brackets. We still have that $c=-\frac{\omega}{2\alpha(1+\omega)}<0$ and that the expression in square brackets is negative because the numerator is larger than denominator. But now we have a different expression to evaluate: $F_{\rho_w}(\rho_w)=\frac{(1-\alpha)zn}{\alpha(1-zn)}$. The numerator of this expression is always positive. As for the denominator, since we have assumed that not all of the individual's time endowment goes towards child rearing, i.e. $(1 - zn_t) > 0$, we have that $F_{\rho_w}(\rho_w) > 0$. Thus $X^*_{\rho_w}$ is positive, and $d^*_{\rho_w}$ is positive.

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Chapter 2

Pensions, Public Debt and Equilibrium Dynamics under Demographic Change

Pensions, Public Debt and Equilibrium Dynamics under Demographic Change*

By

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Abstract: This paper shows how demographic change impacts the optimal debt-GDP ratio under the golden rule of public finance and balanced-budgets, and evaluates how the existence of a PAYG pension system impacts that optimality. For the average OECD economy, very small impacts to the optimal debt-GDP ratio are found when fertility and mortality rates decline according to their projected values for the following 85 years (2015-2100). The size of the impact depends on how generous the PAYG pension system is. Countries with stronger PAYG schemes are more vulnerable to demographic ageing in terms of their fiscal balances. The sign of the effect of demographic change on the optimal debt-GDP ratio is also shown to depend on GDP's private capital intensity. Labor-intensive countries will automatically converge towards the maximal growth steady state by using optimal public investment flows, with no fiscal austerity measures being required. Private capital-intensive countries face the possibility of having to apply austerity measures to control the public debt stock. Overall, fiscal reforms to ensure both sustainability and transition towards maximal economic growth steady states do not imply significant decreases to future public investment rates, even for countries with very generous public pensions.

Keywords: Optimal public debt, PAYG pensions, demographic change

JEL Codes: E62, H54, H55, H63, J11

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1. Introduction

Given expected developments of fertility and mortality rates for advanced economies, the future sustainability of fiscal policy is being questioned when the current level of public spending is considered. For a given structure of public transfers, lower mortality rates imply future increases in public expenditure. Adding lower fertility rates to the socio-economic conjuncture, balanced-budget policies will require higher contributions per taxpayer, if the current level of benefits and pension age is to be maintained.

Remarkably, demographic ageing impacts to fiscal policy are not so obvious when public debt is considered. One reason is that the benefits stemming from public debt creation are much less measurable when compared to pecuniary public transfers. But the benefits do exist, with two main examples being that public debt enables governments to smooth the taxation schedule along business cycles and it enables governments to finance public capital investments. Focusing on the latter point, since governments are usually responsible for the provision of a significant proportion of an economy's public capital stock (an average of 26,2% for the OECD for the last 45 years[1]), the question becomes whether a government should shift the size of productive public capital in response to increasing old-age dependency ratios.

In this paper, I address the question of whether and by how much demographic change can impact the optimal public debt schedule and its sustainability in the long-run in the context of a government that follows the golden rule of public finance and a PAYG pension scheme. By utilizing adjustments to public investment as the policy tool, I focus on two policy objectives: one is just achieving fiscal sustainability, the other is achieving maximum economic growth under demographic change. I do not consider changes to the level of public benefits (see for example Ueshina, 2018, Haan and Prowse, 2014, or Maebayashi, 2013, for such analyses). The sole focus is on determining the existence of appropriate steady states after demographic change takes place, and if feasible transition paths are available. This is done without considering the benefits system as a policy tool, though I evaluate the steady state and transition paths under different intensities of unfunded public pension benefits.

The approach used here is one of maximizing intergenerational welfare by maximizing long-run economic growth[2]. This approach is in line with arguments made by Barr (2009), who states that economic growth is the important metric for pension systems, and not just how to fund these systems. This is because pensions, one way or another, are claims made in present time on future output. Maximizing long-run growth is then a proper goal to ensure increasing welfare of future generations when benefit accrual rules remain static. Under such a goal, the optimal public debt-GDP ratio is evaluated as well as the implications to this ratio of providing more or less generous unfunded public transfers to retirees.

Since public investment requires obtaining the resources through taxation and/or attracting private

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1 Derived from IMF Investment and Capital Stock Dataset, 2017. See IMF (2015) for more details.

2 Barro (1990) shows that maximizing economic growth is equivalent to maximizing social welfare in that model. The model in the current paper has the same feature in what (total) consumption is concerned (ignoring utility derived from having children).

investment (ignoring cases of monetary seigniorage), these public funding activities may compete for resources with other types of public spending. For example, if a PAYG pension system is in place and demographic ageing takes place, that PAYG system will now be collecting more resources from the common pool of private production to non-productive objectives[3], making it harder to finance the creation and maintenance of a given stock of productive public capital. This poses a question of optimality in terms of the level of productive public capital when the government is also mandated to respect the transfer of public benefits such as old-age pensions and child-care related benefits. As such, models that do not consider public investment may be missing on relevant transmission mechanisms when evaluating the impact of demographic change on the optimal schedule of public investment and public debt (if a golden rule of public finance is in place).

In short, the taxation process will change along with a demographic shift if there are pecuniary public benefits that require more funds when that demographic shift takes place. The availability of private funds for public investment will also change along with a demographic shift, if the existence of certain public transfers introduces incentives to save more or less. These two transmission channels, the tax and the saving ones, imply that increased non-productive spending due to a hike in the old-age dependency ratio makes it relatively harder to finance a given stock of public capital/debt. Thus, the optimality of public debt, in view of its use to finance public investment, may well depend on the demographic ageing process.

When considering the impacts of demographic change on the production side of the economy, I focus on two mechanisms: First, on the saving side of the economy, where the existence of, for example, a PAYG pension scheme might disincentivize private savings, affecting the growth capacity of the economy; Second, the taxation side, where a given level of public debt might be harder to sustain in an economy with higher old-age dependency ratios, when this ageing process induces higher old-age public transfers competing more strongly for resources from the "common pool" of production.

This model is rooted in the work of Barro (1990) and Futagami et al. (1993). A model of endogenous growth is built with a productive public capital stock. The model is enriched with finite lives, endogenous fertility decisions, fertility-dependent labor supply, variable life-expectancy, and public transfers, like in Hughes Hallett et al. (2019). But where Hughes Hallett et al. (2019) consider only private funded saving, this paper develops the framework further by also considering unfunded pension benefits. Labor supply is tied to child-rearing commitments, as it introduces a realistic channel by which demographic change impacts GDP growth and public fiscal balances. For example, Cruz and Ahmed (2018) find that a reduction to the child-dependency ratio is the main driver of growth per capita increases within projected demographic changes).

Within the literature of models incorporating productive public capital and public debt, Checherita-

3 Non-productive here means activities that do not yield produced goods to be marketed. It is of course simple to conceptualize how the provision of old-age pensions can be socially "productive", or in other words, useful.

Westphal et al. (2014) provide the first optimal debt-GDP ratios rooted under a theoretical approach based on Aschauer (2000). This approach includes the assumption that governments follow the golden rule of financing, by which public debt is only used for the purpose of public investment. Under this fiscal rule, they find an average optimal debt-GDP ratio of 55-65% for 22 OECD countries, an optimal target if the maximization of economic growth is the objective. Bokan et al. (2016) take a further theoretical step and consider a theoretical approach that combines productive public capital, public debt, and a fully-fledged demographic structure, where, again under the golden rule of financing, the optimality of public debt is shown to depend on demographic parameters. Hughes Hallett et al. (2019) builds on the same model as Bokan et al. (2016), it shows how demographic change impacts the optimal schedule of public debt and provides a numerical exercise. They find an optimal debt-GDP ratio of 49.7%, again, with the objective of maximizing economic growth, as well as negligible impacts to the optimal schedule of public debt with expected incoming demographic shifts.

Under the Hughes Hallett et al. (2019) framework, optimality of public debt is directly related to fertility. With lower fertility comes a higher old-age dependency ratio. This results in a decreased tax base from which public transfer costs are financed from, pushing the optimal debt-GDP ratio downwards. A missing component there was unfunded public pensions, which is a public transfer that is expected to face considerable pressure from demographic ageing. In this paper, and its original contribution, is to introduce public pensions and consider how it is linked to the saving and taxation mechanisms described above in determining the optimality of public debt.

This paper finds that, for the average OECD economy, there are both reasonable steady states and transition paths available under a golden rule of public finance for the expected demographic ageing processes throughout the 21st century. Under this golden rule, the optimal public debt-GDP ratio suffers very small impacts stemming from demographic ageing. The model indicates an initial optimal public debt-GDP ratio of 40.6%, suffering negligible changes throughout the expected demographic structure for the rest of the 21st century. As the old-age dependency ratio increases, public debt sustainability does not require significant reductions in public investment.

It is also concluded that countries with more generous unfunded pension systems are at a higher risk of seeing their public debt stocks turning unsustainable upon a reduction in fertility and mortality rates. Demographic ageing processes imply strong reductions to the optimal public debt-GDP ratio, though ensuring its sustainability requires relatively small reductions to public investment. Hence, the outlook for public debt seems benign. The problem with strong unfunded pension benefits is the dramatic increase in funding requirements in order to finance increased unfunded pension outlays upon demographic ageing. Focus should be posited on reforming the pension system and its benefits level, and not in managing the public debt stock that finances public investment for a country that follows the golden rule of public finance.

Moreover, this paper finds that the transition mechanics depend on the labor intensity of the economy.

For labor intense countries, demographic ageing processes do not de-stabilize the public stock of debt. Convergence is automatically ensured upon demographic ageing. The situation is reversed for capital intensive countries, where demographic ageing turns a sustainable public debt stock into an unsustainable one. Though the numerical results show that sustainability-inducing policies require negligible interventions to the public investment schedule, this result reveals a potential for trouble: With the advent of robotics, IT technologies, artificial intelligence and automation, economies that are becoming more capital intensive may need to re-evaluate their unfunded pension systems, or risk having to cut on public investment in the future, yielding lower economic growth prospects for future generations.

A brief structure of the rest of the paper follows: Section 2 lays the foundation of the model, and section 3 derives its equilibria and transitional dynamics. Section 4 introduces the baseline calibration and the initial steady state. Section 5 reveals the main results. Section 6 concludes.

2. The model

This model features overlapping generations, changing demographics, welfare spending, and productive public and private capital accumulation as in Yakita (2008), Bokan et al., (2016) and Hughes Hallett et al., (2019). Each generation of individuals is homogenous in their characteristics. Firms are also homogenous and operate in perfectly competitive markets. There are two overlapping generations, one comprising working individuals and the other retirees. I introduce a demographic structure featuring both endogenous and exogenous factors by having individuals make fertility decisions during their working period and face an exogenous risk of death upon the transition from the working to the retired period of life. Public investment decisions are made by a government that issues debt and levies taxes to fulfil its objectives, namely servicing debt and financing social security. These public investments are productive in nature and introduce a component of endogenous growth. This model is thus specified in terms of growth and not levels[4].

2.1. Firms

The production technology of each homogeneous firm j is characterized by constant returns to scale between the private capital firm j employs and its productivity-augmented labor force:

$$Y_{j,t} = AK_{j,t}^{\alpha}\left(h_t L_{j,t}\right)^{1-\alpha};\ 0 < \alpha < 1 \tag{1}$$

$Y_{j,t}$, $K_{j,t}$ and $L_{j,t}$ respectively symbolize the output level, the private capital stock, and labor input in firm j for period t. α is the income share of private capital for each firm j. There are two productivity structures: One, total factor productivity (TFP), which is not time dependent, denoted by A; Two, labor-augmenting productivity, which is not firm specific, described by h_t.

4 Endogenous growth differentiates the type of analysis from the classical setting of, for example, Diamond (1965), where long-run growth is exogenous and equal to population growth. Since our objective is to analyze optimal public debt in the context of demographic change and public unfunded pensions, I focus on endogenous growth and not on dynamic efficiency considerations.

With homogenous firms operating in perfectly competitive markets, firm j's profit maximization problem yields the following equilibrium rental rate of capital, r_t, and wage rate, w_t:

$$r_t = A\alpha\left(k_{j,t}\right)^{\alpha-1}(h_t)^{1-\alpha} \tag{2}$$

$$w_t = A(1-\alpha)\left(k_{j,t}\right)^{\alpha}(h_t)^{1-\alpha} \tag{3}$$

Lower case letters refer to variables expressed in *per unit of labor* form, such as $k_{j,t} \equiv K_{j,t}/L_{j,t}$. The labor augmenting productivity, h_t, seen in equations (2) and (3) is modelled as being determined by the level of aggregate public and private capital, where h_t is then not firm specific.

As Hughes Hallett et al. (2019) argues, the inclusion of public capital in the production process is taken as an important feature to study optimal public debt in the context of demographic change, since one important objective of public debt is to raise high amounts of funds for public capital[5]. A measure of aggregate capital is included in the labor productivity factor in spirit of Arrow's (1962) Learning by Doing model. These productivity effects are not interpreted as non-rivalrous public goods; here, productive externalities from capital are taken as rival goods that are universally available to workers[6]. The rival use of these externalities imply that they are effective to the extent of the value of capital per capita, and not its absolute level.

The productivity adjustment process h_t is defined as:

$$h_t = \frac{K_t^{\beta} G_t^{1-\beta}}{L_t} = X_t^{-\beta} g_t \,;\; 0 < \beta < 1 \tag{4}$$

The public capital stock per unit of labor is denoted by $g_t \equiv G_t/L_t$, β is the labor productivity share of private capital, and X_t is the ratio of public-to-private capital, G_t/K_t. The aggregate private capital stock and the aggregate labor input are, respectively, defined as $K_t = \sum_j K_{j,t}$ and $L_t = \sum_j L_{j,t}$.

In order to derive the aggregate production and factor prices, note that the labor-augmenting productivity process, h_t, is not firm specific; it instead reflects a macro-level measurement. Since all firms are assumed to be symmetric and each firm's size is assumed to be small compared to the size of the economy, the decision making of the individual firm does not consider its own impact on h_t. As such, this firm environment implies that all the market participants share the same optimal solution. We thus have that $k_{j,t} = k_t$. Furthermore, defining $Y_t = \sum_j Y_{j,t}$ and $\omega \equiv \alpha + \beta(1-\alpha)$ allows us to rewrite expression (1) in terms of aggregate output at time t:

$$Y_t = AK_t^{\alpha}(h_t L_t)^{1-\alpha} = AK_t^{\alpha+\beta(1-\alpha)} G_t^{(1-\beta)(1-\alpha)} = AX_t^{-\omega} G_t \tag{5}$$

Remarkably, expression (5) does not include aggregate labor input, L_t. This is a simple consequence of the definition of the (endogenous) labor productivity input, h_t, where it is not how many workers that

5 There is relative wide (though not unanimous) support for the idea that public capital plays a key role in production (see Makuyana and Odhiambo (2016) for a literature review). Moreover, in a model of endogenous growth, it is important to consider motivations for the existence of public debt beyond dynamic efficiency-type arguments.

6 Barro (1990) argues that actual nonrival government activities are relatively few.

matters for total production, but it is how much capital per capita there is for that labor to produce and how this capital is balanced between public and private sources.

As for the aggregate factor prices, they become:

$$r_t = A\alpha(X_t)^{1-\omega} \tag{6}$$

$$w_t = A(1-\alpha)(X_t)^{-\omega} g_t \tag{7}$$

Expression (7) reveals how growth in wages depends on public capital per unit of labor as well as the ratio of public to private capital in X. As Dalenberg and Partridge (1997) argue, government productive activities, measured in G, can be seen as acting both as an unpaid factor to firms, boosting productivity, and as an amenity to individuals, boosting employment levels. Such an amenity channel tends to attract workers, consequently reducing wages. Here, equation 7 reveals that both channels, factor/productivity and amenity/employment, are present. For the first channel, higher public capital contributes to increase wage rates up to a certain point of the optimal mix of public and private capital (as measured in X). For the second channel, public capital benefits equilibrium wage rates only to the extent of how much capital there is per unit of labor input.

2.2. Population

The population is composed of homogeneous individuals who plan for two distinct periods of their lives: a working and a retirement period. Each period is of fixed length, but survival into retirement is not ensured. At the end of the working period, a fraction of the working agents dies while the rest move into retirement. The probability of dying at the beginning of retirement period t is given by the hazard rate λ_t, which is the same for all agents. This implies that the current size of the retiree group is proportional to $(1-\lambda_t)$ of the working population in the previous period. Using N_t as the number of currently working individuals, current population size at time t is equal to $N_t + (1-\lambda_t)N_{t-1}$. With changing fertility and mortality profiles, the period-to-period population growth factor is:

$$\gamma_{t+1}^N \equiv \frac{N_{t+1} + (1-\lambda_{t+1})N_t}{N_t + (1-\lambda_t)N_{t-1}} = \frac{n_t + (1-\lambda_{t+1})}{n_{t-1} + (1-\lambda_t)} n_{t-1}$$

This expression considers both the effects of death at old-age and the number of children at different time points. It shows that when the population is at a growth steady state, with $\lambda_t = \lambda_{t+1}$ and $n_{t-1} = n_t = n$, we obtain the usual population growth factor $\gamma^N = n$ (note that γ always denotes the growth *factor* of variables). Moreover, the old-age dependency ratio, denoted μ_t, becomes:

$$\mu_t \equiv \frac{(1-\lambda_t)N_{t-1}}{N_t} = \frac{(1-\lambda_t)}{n_{t-1}} \tag{8}$$

So, lower fertility and/or higher longevity in this model both produce a higher old-age dependency ratio.

As individuals enter the economy, they become workers and make fertility and saving-consumption choices. They receive labor income and a child-rearing subsidy. Upon reaching retirement, conditional on survival, no more decisions are made, on fertility or otherwise, where individuals simply consume out of their savings and PAYG pension benefits. Within this structure, individuals enter the economy and make choices at the beginning of time t to maximize lifetime utility as expressed by:

$$\ln c_t + (1-\lambda_{t+1})\rho \ln d_{t+1} + \varepsilon_t \ln n_t \tag{9}$$

In the expression above, c_t (d_{t+1}) is consumption in the working (retirement) period, n_t is the number of children the individual has, ε_t measures the importance of having children in a person's life-time utility, and ρ denotes the usual time discount factor, being a value between 0 and 1. Following Eckstein and Wolpin (1985), fertility choices are modelled in a weak sense, where parents derive utility only from the number of children they have[7].

The time endowment for a working individual is normalized to 1. Having children comes with a child-rearing time-penalty, reducing the time an individual can spend working. Parameter z measures rearing time per child. It is assumed that agents never stop working due to child-rearing commitments: $(1-zn_t) > 0$. After-tax labor income is then allocated between current consumption, c_t, and savings, s_t. The individual receives a child-rearing subsidy, which is taxable (thus modelled as a tax deduction/allowance[8] for simplicity). The child-rearing subsidy, denoted by ρ_w, is a fixed proportion of the lost wage income to child-rearing, $w_t z n_t$. The tax rate, denoted by θ_t, is the same for wages and the child-rearing subsidy. The budget constraint of the working agent at time t becomes:

$$[(1-zn_t) + \rho_w z n_t](1-\theta_t)w_t = c_t + s_t \tag{10}$$

Following the methodology in Blanchard (1985), the existence of homogenous actuarially fair insurance companies operating in a perfectly competitive market is assumed. Individuals exclusively save by means of this insurance market, and the insurance companies invest the savings proceeds in private capital and public debt. Survival into retirement plays a key role in the return to the insurance product, as the assets of those who die before retirement get re-distributed to surviving individuals by means of a higher return to savings conditional on survival. Assuming full depreciation of capital over a period, we have that the real interest rate is the rental rate of capital subtracted of depreciation. Given a zero-profit condition being assumed on the insurance market, the surviving retirees therefore receive a rate of return of $\frac{r_{t+1}-1}{(1-\lambda_{t+1})}$ on their savings.

The PAYG public pension system is set up in a defined-benefit fashion. Pension benefits are calculated as a fraction ρ_p of total income earned in the working period of life and are tax-free[9]. This income measurement considers income lost due to child-rearing and the compensatory subsidy, so that labor supply effects impact accrued public pension benefits. The accrued pension benefits are indexed according to a

7 Parents do not incorporate their children's lifetime utility as their own, nor do they consider any measure of the "quality" of children or explicit benefits (transfers/help at old-age, for example). See Zhang and Zhang (1998) for an analysis over alternative motives for having children and how it relates to social security.

8 An alternative to this approach would be to model child-rearing subsidies like a tax credit. I do not follow this path for simplicity reasons, as introducing tax credits unnecessarily complicates the model, thus diluting the focus of the analytical results, which is public pensions.

9 Most OECD economies do not levy social contribution taxes on public pension transfers (OECD, 2017). I follow this setup, except that the taxation of pension income is removed altogether for simplicity. Introducing other non-social contributory taxes on public pensions unnecessarily introduces distortions into the model that dilute the mechanism being evaluated in this paper. It will not change the theoretical results of the paper, but it may have a limited impact on the numerical results.

given criteria via the variable i_{t+1}, which, in this setting, is wage indexation[10]. In what follows, I define[11]:

$$i_{t+1} = \frac{w_{t+1}}{w_t} \tag{11}$$

The second period budget constraint, relating consumption at retirement with savings returns and PAYG pension benefits, reads as follows:

$$d_{t+1} = \frac{r_{t+1}}{(1-\lambda_{t+1})} s_t + [(1 - zn_t) + \rho_w zn_t]\rho_p w_t i_{t+1} \tag{12}$$

The maximization problem that individuals face is described as follows:

$$\max_{c_t, d_{t+1}, n_t} \ln c_t + (1 - \lambda_{t+1})\rho \ln d_{t+1} + \varepsilon_t \ln n_t$$

$$s.t.\ (i)\ c_t + s_t = [(1 - zn_t) + \rho_w zn_t](1 - \theta_t)w_t$$

$$(ii)\ d_{t+1} = \frac{r_{t+1}}{(1 - \lambda_{t+1})} s_t + [(1 - zn_t) + \rho_w zn_t]\rho_p w_t i_{t+1}$$

Solving the maximization problem of individuals, indexing the pension benefits via expression (11), and defining the "pseudo" saving rate[12] $\tilde{C}_t \equiv \frac{(1-\lambda_{t+1})\rho}{1+(1-\lambda_{t+1})\rho+\varepsilon_t}$, we obtain the Euler equation and the optimal saving, s_t, and fertility, n_t:

$$\frac{d_{t+1}}{c_t} = \rho r_{t+1} \tag{13}$$

$$s_t = \tilde{C}_t(1 - \theta_t)w_t - \tilde{C}_t \frac{\rho_p}{\rho} \frac{w_{t+1}}{r_{t+1}} \tag{14}$$

$$n_t = \frac{\varepsilon_t}{[1+(1-\lambda_{t+1})\rho+\varepsilon_t](1-\rho_w)z} \tag{15}$$

Expression (14) reveals that how much is saved is a function of demography related factors and impatience, hidden in $\tilde{C}_t$, as well as PAYG pension benefits. Unsurprisingly, the existence of a PAYG pension system decreases the amount saved by individuals, since these pension benefits are at least an imperfect substitution to own savings. This decrease is dictated by the replacement rate of the PAYG pension weighed by impatience factor and the ratio of future wages to the future interest rate. Higher pension replacement rates naturally disincentivize private savings further, whereas a higher subjective discount rate makes an individual care less for consumption during retirement, which attenuates the negative effect PAYG pension benefits have on private saving. As for factor prices: On one hand, the higher future wages are, the higher pension benefits an individual expects to get, making private saving less appealing; On the other hand, the higher future interest rates are, the more one earns from private savings, incentivizing private saving.

10 This indexation regime is the most common one in indexing the *valorization* of pension contributions of earnings-related PAYG pensions. During the pay-out phase, price indexation is most commonly seen (Piggott and Sane, 2009).

11 We do not consider other indexation schemes. Price (nominal) indexation is not possible in the setting of the model. One could consider GDP growth indexation. In fact, wage indexation is equivalent to GDP per capita growth. If pensions are indexed to GDP growth, then pension benefits get proportionally increased by growth in the labor supply, relative to wage indexation.

12 This variable is the saving rate out of disposable income if the public pension system did not exist. This can be confirmed by looking at expression (14) and setting ρ_p to 0.

Expression (15), governing fertility choices, is composed of demographic factors, the subjective discount rate, the child-rearing subsidy rate ρ_w and how much time is spent child-rearing, z. First, n_t is increasing in ρ_w. This should not come as a surprise, since the existence of the subsidy ρ_w lowers the financial burden of raising children. Second, higher longevity (lower λ) or lesser future discounting behavior (higher ρ) both decrease fertility choices, since the individual then decides to derive more utility from consumption during retirement at the cost of deriving less utility by having children. Third, it is important to note that fertility choices are not impacted directly by the equilibrium market wage rate or GDP levels, but fertility choices are directly related to how much income a household obtains through labor activities. Higher fertility choices imply lower personal income due to more time being spent on non-labor activities (child-rearing)[13].

2.3. Government

The government provides public capital, child-rearing subsidies and public pensions, financing these activities via taxation and public debt. Issued public debt, denoted B_t, can only finance investment in public capital, G_t. This implies that deficit spending is impossible. Via the usual arbitrage argument, the interest cost of public debt is equal to the real interest rate $(r_t - 1)$. The government budget constraint becomes:

$$B_{t+1} = B_t + (r_t - 1)B_t + G_{t+1} + \rho_w w_t z n_t N_t + [(1 - zn_{t-1}) + \rho_w z n_{t-1}] w_{t-1} i_t \rho_p (1 - \lambda_t) N_{t-1} - \theta_t (w_t L_t + \rho_w w_t z n_t N_t) \tag{16}$$

In line with the no debt deficit-financing assumption, interest payments and public transfers are solely tax-financed. Public debt's existence is defined by the need to finance public capital formation. In other words, the model is stated in terms of the "golden rule" of public finance[14]:

$$b_t = g_t \tag{17}$$

Given expressions (5) and (17), the debt to GDP ratio in this model is now:

$$d_t = \frac{X_t^{\omega}}{A} \tag{18}$$

The government budget can be further simplified using the pension indexing rule (11), the dependency ratio, μ_t, defined in (8) and some algebra. The budget constraint, in per unit of labor form, becomes:

$$r_t b_t + \rho_w w_t \frac{zn_t}{(1 - zn_t)} + [(1 - zn_{t-1}) + \rho_w z n_{t-1}] \frac{\mu_t}{(1 - zn_t)} w_t \rho_p = \theta_t \left(w_t + \rho_w w_t \frac{zn_t}{(1 - zn_t)} \right) \tag{19}$$

We can use conditions (17) and (19), combine them with capital rents and wages, expressions (6) and (7), as well as $L_t = (1 - zn_t)N_t$ (labour supply equals the time endowment less child-rearing time), to derive an expression for the income tax rate, θ_t, needed to satisfy current public spending:

13 In fact, the model also displays the observed link between higher fertility rates and lower GDP per capita. Whenever a fertility-related parameter gets changed, such as z, ε or ρ_w, producing a higher fertilty rate, this change will yield a decreased GDP per capita as expected (see for example Doepke and Tertilt, 2016).

14 This rule is discussed by Musgrave (1939). The rationale for this golden rule is given in Blanchard and Giavazzi (2004) and Fatas et al. (2003).

$$\theta_t = 1 - \frac{1 - \frac{\alpha}{(1-\alpha)} X_t - C_{p,t}}{1 + C_{c,t}} \tag{20}$$

where I define expressions:

$$C_{c,t} \equiv \rho_w \frac{zn_t}{(1-zn_t)} \tag{21}$$

$$C_{p,t} \equiv [(1 - zn_{t-1}) + \rho_w zn_{t-1}] \frac{\mu_t}{(1-zn_t)} \rho_p \tag{22}$$

Expression (21) captures costs related to the child-rearing subsidy, and expression (22) captures costs related to the PAYG pension system. Note that the balanced-budget tax rate in expression (20) only depends on variables at time t, with exception for the number of children at time $t-1$, n_{t-1}, in expression (22). The equilibrium tax rate in expression (20) thus depends on two generations' fertility choices.

The relevant dimension of analysis in this paper is demographic ageing in the form of slowing population growth and longer-living individuals. These two dimensions have key consequences for the balanced-budget tax rate. Considering slowing population growth, say we start with higher past fertility, at time $t-1$. This implies more time spent outside of labor activities for this past generation (from the perspective at time t), meaning that generation $t-1$'s public pension benefits will equally be reduced. Similarly, higher fertility at time $t-1$ implies a lower old-age dependency ratio, creating more future workers versus retirees, *ceteris paribus*. In other words, higher fertility in the past creates more taxpayers today, which relieves pressure that the pension system exerts on *each* member of the working population. Both these effects reduce the taxation effort needed to balance the PAYG pension system.

If population growth then slows at time t with a lower fertility rate, it impacts both the costs associated with the child rearing subsidy and public pensions. For the former, lower current fertility implies both a decrease in the benefit level an individual is entitled to and an increase in labor activities of that same individual. Naturally, the combined effect here is a drop in expenses with the child-rearing subsidy. For the latter, in terms of public pensions, a lower fertility at time t increases labor market participation of the workers in the economy, increasing labor income balances and decreasing the fiscal burden the public pension system exerts on *each* of these taxpayers.

Turning to increased longevity, reflected as a drop in λ, it is a part of the balanced-budget tax rate expression (20), through the old-age dependency ratio in the public pension balance ,$C_{p,t}$, and the fertility rate n_t present in $C_{c,t}$ and $C_{p,t}$. The total effect of current and future longevity on pension payouts per unit of income is:

$$\frac{dC_{p,t}}{d\lambda_t} = \frac{\partial C_{p,t}}{\partial \mu_t}\frac{\partial \mu_t}{\partial \lambda_t} + \frac{\partial C_{p,t}}{\partial n_{t-1}}\frac{\partial n_{t-1}}{\partial \lambda_t} = -\frac{[1 + 2(1-\lambda_t)\rho]\rho_p(1-\rho_w)z}{\varepsilon_{t-1}(1-zn_t)} < 0$$

$$\frac{dC_{p,t}}{d\lambda_{t+1}} = \frac{\partial C_{p,t}}{\partial n_t}\frac{\partial n_t}{\partial \lambda_{t+1}} = \frac{(1-\lambda_t)(1-\rho_w)[1 - n_{t-1}(1-\rho_w)z]\varepsilon_t\rho_p\rho}{n_{t-1}[1 + (1-\lambda_{t+1})(1-\rho_w)\rho - (1+\varepsilon_t)\rho_w]^2} > 0$$

When current longevity increases (lower λ_t), current pension costs increase since more individuals survive into retirement. When *future* longevity increases (lower λ_{t+1}), we see the opposite result where *current* pension costs decrease. The reason for it is that current fertility choices, n_t, drop when future longevity increases, since individuals require more labor income to sustain consumption in the now longer lifetime. This implies a higher labor supply and lower future pension benefits, consequently lowering public pension contributions per working individual. For the effect of longevity on the costs with the child-rearing subsidy, current longevity λ_t does not affect $C_{c,t}$, whereas future longevity does:

$$\frac{dC_{c,t}}{d\lambda_{t+1}} = \frac{\partial C_{c,t}}{\partial n_t}\frac{\partial n_t}{\partial \lambda_{t+1}} = \frac{(1-\rho_w)\rho_w\rho\varepsilon_t}{[1+(1-\lambda_{t+1})(1-\rho_w)\rho-(1+\varepsilon_t)\rho_w]^2} > 0$$

As with the effects described above, higher future longevity (lower λ_{t+1}) reduces the fertility rate, which reduces child-rearing benefits to be transferred.

Overall, these results reveal two key details: one is the importance of acknowledging the different cohorts in society when accounting for demography-specific social spending; the other is the importance of analyzing longevity effects alongside fertility choices. These insights make it so that an increase in longevity together with a decrease in fertility provokes both an inflationary and a reductionist impact on pension costs. The net effect remains ambiguous and requires further numerical evaluation.

3. The Aggregate Economy

3.1. Equilibria and transition

In this section, the transition function and the steady state curve are characterized, as well as the public-private capital ratio, X, that maximizes economic growth. The representative insurance company, and hence the working generation, invests in both private and public capital. Equilibrium in capital markets requires that:

$$s_t N_t = K_{t+1} + G_{t+1} \Leftrightarrow \frac{s_t}{(1-zn_{t+1})n_t} = k_{t+1} + g_{t+1} \tag{23}$$

Using the equilibrium condition for capital markets (23) and the solution for s_t in (14), we obtain:

$$\frac{\tilde{C}_t}{(1-zn_{t+1})n_t}(1-\theta_t)w_t - \frac{\tilde{C}_t}{(1-zn_{t+1})n_t}\frac{\rho_p}{\rho}\frac{w_{t+1}}{r_{t+1}} = k_{t+1} + g_{t+1} \tag{24}$$

To determine transitional dynamics, expressions (6), (7) are used, along with the definition $\gamma_{t+1}^g \equiv \frac{g_{t+1}}{g_t}$ to solve expression (24) for next period's public-private capital ratio, X_{t+1}:

$$X_{t+1}\left(X_t, \gamma_{t+1}^g\right) = \frac{\tilde{C}_t\frac{(1-\alpha)}{\alpha}\frac{\rho_p}{\rho}+(1-zn_{t+1})n_t}{\frac{(1-\theta_t)}{\gamma_{t+1}^g}\tilde{C}_t A X_t^{-\omega}-(1-zn_{t+1})n_t} \tag{25}$$

This solution is dependent on the policy variable γ_{t+1}^g, which is the growth *factor* of the public capital stock per unit of labor. I assume that the government always preserves a balanced-budget tax rate. Even though the denominator of (25) reveals that increases to the growth of public capital, γ_{t+1}^g, or discretionary increases to the tax rate, θ_t, both increase X_{t+1}, it is assumed that the government only uses public investment, $\Delta\gamma_{t+1}^g$, as a policy tool to perform the desired fiscal adjustments to ensure convergence or maintenance of a steady state.

Expression (25) shows that the existence of a public pension system creates an inflationary pressure on the future public-private capital ratio. The reduction in private savings due to the public pension system, seen in the numerator of (25), is proportional to the ratio of future wages to the interest rate. Since this ratio decreases when future public-private capital ratio increases, X_{t+1}, higher public pension replacement rates apply positive pressure on the future public-private capital ratio, *ceteris paribus*. The implication is that sustaining a *given* level of public capital becomes more cumbersome the higher public pension replacement rates are.

Through expression (5), we can define the economic growth factor per unit of labor:

$$\gamma_{t+1}^{y} \equiv \frac{y_{t+1}}{y_t} = \left(\frac{X_t}{X_{t+1}}\right)^{\omega} \gamma_{t+1}^{g} \tag{26}$$

Expression (26) tells us that the balanced growth path is a situation where public and private capital grow at a constant rate equal to economic growth, since the ratio of public to private capital should remain constant at steady state. We thus have at steady state (no time subscript implies time stationarity): $\gamma^{g} = \gamma^{k} = \gamma^{y} = \gamma^{Y}/n$. It implies that the tax rate, θ_t, and the interest rate, r_t, are constant at steady state, whereas wages grow at the same rate as GDP per capita. But most importantly, a steady state implies that the growth in the public capital stock per unit of labor, γ^{g}, must be equal to the economic growth counterpart, γ^{y}.

Using expression (24) and definition (26), we can obtain the general endogenous expression for the economic growth factor per unit of labor from period t to $+1$, γ_{t+1}^{y}:

$$\gamma_{t+1}^{y}(X_t, X_{t+1}) = \frac{A\tilde{C}_t(1-\alpha)X_t^{-\omega}}{\tilde{C}_t\frac{\rho p(1-\alpha)}{\rho}\frac{1}{\alpha}\frac{1}{X_{t+1}}+\left(\frac{1}{X_{t+1}}+1\right)(1-zn_{t+1})n_t}(1-\theta_t) \tag{27}$$

Next, I use expression (27) and substitute all variables by their steady state counterparts, most notably introducing a constant fertility rate, n, and public-private capital ratio, X, to obtain an expression for the economic growth factor per labor unit at steady state, the steady state curve (the conditions for short- and long-run equilibria can be found in Appendix A):

$$\gamma^{y}(X) = \frac{A\tilde{C}(1-\alpha)}{\tilde{C}\frac{\rho p(1-\alpha)}{\rho}\frac{}{\alpha}X^{\omega-1}+(X^{\omega-1}+X^{\omega})(1-zn)n}(1-\theta_t) \tag{28}$$

Expression (28) is proportional to $\frac{X^{1-\omega}}{(1+X)}$. Given that $0<\omega<1$, we have that expression (28) initially grows while the numerator dominates, until the denominator becomes dominant and expression (28) starts to permanently decrease. Hence, expression (28) admits one maximum.

3.2. Growth-Maximizing Public Debt

The steady state curve expressed in (28) can be used to determine the steady state public-private capital ratio that maximizes economic growth. There are essentially three layers to this optimality problem. One is determining the optimal X that would maximize growth if the financing of public capital was free and no public transfers existed. This can be thought of as the "pure" X ratio that maximizes production. If one removes public transfers and taxation from expression (28), this "pure" optimal X would be determined by:

$$\frac{(1-\omega)}{\omega}$$

This is simply the equalization of the marginal product of public capital with the marginal product of private capital, as determined by aggregate production in expression (5). Since public capital must be financed by public debt, it requires taxation to fund the interest rate payments on that debt. Expression (28) shows that taxation hinders growth proportionally to it. So, adding the second layer to the growth optimization problem, which is the tax rate (but still ignoring public transfers), the optimal public-private capital ratio becomes:

$$\frac{-2\alpha(1-\omega)-\omega+\sqrt{4\alpha(1-\omega)+\omega^2}}{2\alpha(1-\omega)}$$

The introduction of the taxation system lowers optimal X due to the fiscal effort exerted on each individual's income as measured by the relative composition of capital vs. labor intensity: $\alpha/(1-\alpha)$ (see the numerator of the tax rate in expression 20). Notably, no demographic parameters are part of this solution. This tells us that the impact of demographic change on the optimality of public debt (or public capital) is linked to the public transfers in place.

The third layer to the growth optimization problem is related to the existence of a PAYG pension system (and child-rearing subsidies). Expression (28) shows that the existence of this pension system affects economic growth[15] in two ways:

1) Through negative impacts to the amount of saving each individual does, as seen in the denominator of (28). The impact of the PAYG pension is weighed by the ratio of the pension replacement rate and the subjective discount factor, ρ_p/ρ and the relative composition of labor vs. capital intensity, $(1-\alpha)/\alpha$. Here:
    a. The more impatient individuals are, or the smaller the pension replacement ratios are, the less impact PAYG pensions have on saving behavior and, consequently, growth.
    b. High labor intensity economies are more sensitive to the existence of a PAYG system in terms of economic growth. The higher the labor intensity, the lower economic growth will be for a given pension replacement, ρ_p.
2) Through the taxation system, seen as $(1-\theta_t)$ in expression (28). The negative effects of the PAYG system are then measured by the funding requirements specified in expression (22). The analysis of that expression showed that the effects of demographic ageing on the tax rate θ_t is

15 Note that PAYG pension systems can have socially useful objectives not considered in this paper. As such, I refrain from making normative statements on the existence of PAYG pensions. They are considered in the model as a reflection of reality, only being present due to the hypothesized link between age-related expenditures and the provision of public capital described in the introduction of this paper.

ambiguous, such that the net effect of PAYG pension on economic growth can only be verified in the numerical section.

The optimal public-private capital ratio that maximizes economic growth, as seen in expression (28), is:

$$
\begin{aligned}
X^* &= \frac{1}{2\alpha n\rho(1-\omega)(1-zn)}\Bigg\{n^2\left[\omega\left(1-C_p\right)+\alpha\left[2-\left(2-C_p\right)\omega\right]\rho z\right] \\
&- \left[\omega\left(1-C_p\right)+\alpha\left[2-\left(2-C_p\right)\omega\right]\right]\rho n-(1-\alpha)\tilde{C}(2-\omega)\rho_p \\
&+ \sqrt{\begin{matrix} 4(1-\alpha)\left(1-C_p\right)n(1-\omega)^2\rho(1-zn)\left[\tilde{C}\rho_p(1-\alpha)+\alpha n\rho(1-zn)\right]+ \\ \begin{bmatrix} -2\tilde{C}\rho_p(2-\omega)-\left(1-C_p\right)n\omega\rho(1-zn)+ \\ \alpha\left[-n\left(2-\left(2-C_p\right)\omega\right)\rho+\tilde{C}(2-\omega)\rho_p+n^2\left(2-\left(2-C_p\right)\omega\right)\rho z\right]\end{bmatrix}^2\end{matrix}}\Bigg\}
\end{aligned}
\tag{29}
$$

Beyond the usual production parameters α and ω found in the formulas above, X^* now becomes a function of both the child-rearing subsidy as well as the PAYG pension replacement rate (along with many other parameters, such as impatience, child-rearing time). The formula is too complex to make simpler analysis on it. The results on the impact of demographic ageing to the optimal public-private capital ratio is then left to the numerical section. It should be noted that Appendix C shows that this capital ratio is indeed a maximizing solution, and not a minimizing one. We thus have that $\gamma^y(X^*)$ is the unique maximum growth point in the steady state curve.

3.3. Transitional dynamics and policy tool

So far, the transition equation, the steady state curve, and the public-private capital ratio that maximizes economic growth were defined. A remaining piece of the puzzle is the transitional dynamics along the steady state curve, and especially, the dynamics around the maximum growth steady state.

For the steady state curve (28), there always exists a range of public-private capital ratios which are locally stable, including a non-hyperbolic steady state (see Appendix B for the proof). The non-hyperbolic steady state is in fact the maximum growth steady state with special stability properties (see Appendix C for proof). The transition curve found in expression (25) has the shape found in Figure 1 below:

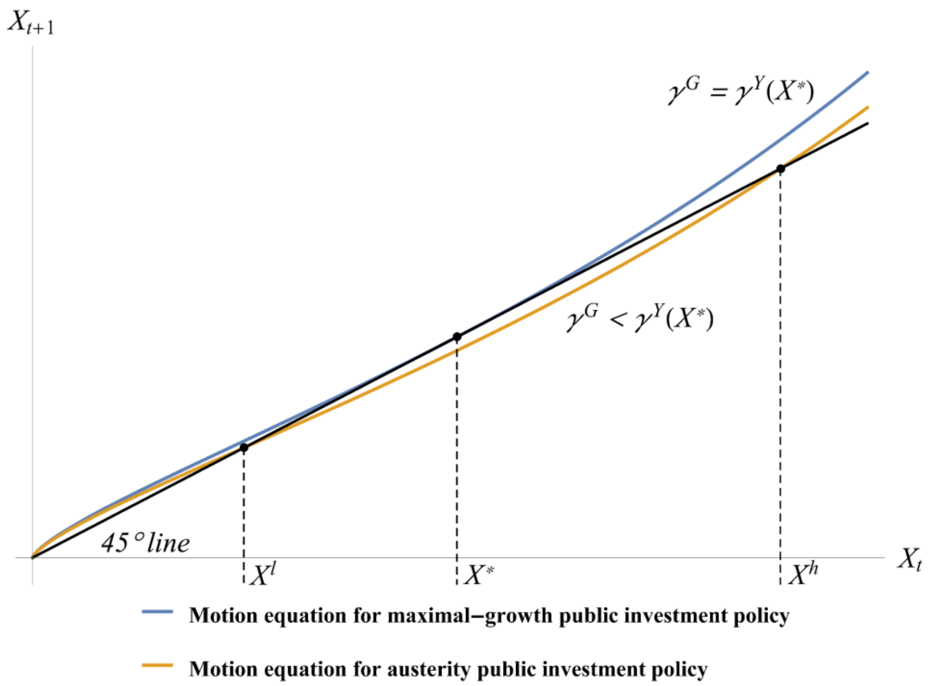

Figure 1 – Plot of motion equation for X_{t+1}.

Looking at the blue line in Figure 1, graphing the motion equation for the maximal growth public investment policy, it has one steady state associated with it at X^*, the maximal growth steady state. It is stable from the left, meaning that if an economy is currently at an X level lower than X^* and the government sets the growth rate of public capital to the maximal growth rate of the economy, the economy will converge towards the maximal growth steady state and it will stay there. The maximal growth steady state is unstable from the right, yielding the exact opposite result: an economy currently at an X level higher than X^* must restrict its borrowing for public investment, relative to the maximal growth policy, to prevent the economy from collapsing.

Most importantly, there will always be two steady states when the growth rate of the public capital stock is set to any value between 0 and the maximal growth rate. The orange line in Figure 1 is an example of an austerity motion equation. One of the two steady states will have a low public-private capital ratio, termed X^l, and the other a high ratio, termed X^h, where the maximal growth steady state will necessarily be between them (see Figure 1). Both X^l and X^h steady states will have the same economic growth rate (by definition), though the X^l steady state is locally stable, whereas its X^h counterpart is unstable (see Appendix C for proof). This implies that X^h is a critical value, beyond which an economy's public debt stock explodes. (under a golden rule of public finance).

The main take-away here is that there exists an instrument with which the government can stabilize an economy's fiscal situation by temporarily setting low enough public investment (lowering the issue of public debt) to contain an otherwise exploding X. The containment region is determined by the position of the high ratio steady state X^h, and this ratio itself can be increased by further decreasing the growth of the public capital stock, γ_{t+1}^{G}. Denoting X^c as the current public-private capital ratio, the minimum level of austerity to contain X^c is the one that reduces public capital growth just enough to make $X^h > X^c$. If enough austerity is applied, the economy's current public-private capital ratio X^c will converge towards the low ratio steady state, X^l, over time. Recalling that X^l will always be a lower value than the growth-maximizing optimal ratio X^*, once the current ratio X^c goes below this optimal ratio the government can go back to applying the growth maximizing public investment rate, and the economy will converge to and stabilize at X^*.

4. Baseline Calibration

Now the benchmark calibration that will be used for the numerical results is presented. The benchmark calibration is set to match the 45-year period 1969-2013 for the average OECD economy[16] with respect to key data points of relevance to the study. This will be the starting point to analyze how demographic change and its interaction with the provision of public pensions and child-rearing subsidies impact the optimal debt-GDP ratio over the rest of the 21st century.

4.1. Production parameters

Starting with production-side parameters, the value of α is set to 0.36. Having a Cobb-Douglas production function, this value was obtained from AMECO database's "Adjusted wage share, total economy, as percentage of GDP at current factor cost" measurement.

Next, measuring the influence of private capital (as opposed to public capital) in the efficiency of labor, is parameter β, which is not readily available in the literature. Checherita-Westphal et al. (2014) estimates ω to be equal to 0.716.[17] Having α equal to 0.36, the definition of ω implies that β equals 0.56.

Finally, the TFP parameter A is calibrated through the production function in expression (5), by making use of the calibrated value for ω and data for the public and private capital stock from IMF's Investment and Capital Stock Dataset, 2017. This operation yields $A = 34$.

4.2. Preference and demographic parameters

Parameter ρ represents the subjective discount factor individuals apply to consumption in their retired period of life. Given that this discount factor represents a 45-year leap, it is arguably appropriate to acknowledge declining discount rates for longer periods of inter-temporal planning (see for example the

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16 Certain data only exists up to 2013, hence the choice of 2013 as the end year. Given the time period at hand and the focus group (the OECD countries), some of the data required for the calibration exercise is missing. Whenever there are data points consistently throughout the entire 45-year period, I use averages over the period at hand. Whenever data is not available for a portion of the period, linear inter-/ extrapolation is used. For demographic projections, the only data points available are 1970-2015-2060-2100. This is slightly mismatching the rest of the data, but this detail should not significantly bias the numerical results presented here.

17 First, the referred paper uses data for 22 of the biggest OECD countries for the period 1960-2010, which closely matches the current setting. Second, it uses a slightly more general production function than I do, the difference being that they do not include per-employee productivity gains to labor inputs to production. I have the following production function $Y_t = AK_t^{\omega}G_t^{1-\omega}$ whereas they have $Y_t = A(K/L)_t^{\varphi}K_t^{\omega}L_t^{1-\omega}$ (in my notation, where φ is the elasticity of the labor-capital to GDP). For my specification to match theirs, φ must be estimated to be zero. In fact, their estimate of the OECD average for φ is very close to zero at 0.037 (see Table 2, for Model 2, Checherita-Westphal et al., 2014).

treatise on discounting by Frederick et al. 2002, and a substantiation of hyperbolic discounting for saving behavior by Cao and Werning, 2018). In this light, the discount rate is calibrated hyperbolically at an annual rate of 1%. We thus have $\rho = 1/(1 + 45 \times 0{,}01) = 0.69$[18].

Parameter z measures the loss of work time to child-rearing activities. Given the difficulty in obtaining such a parameter, a proxy measurement is utilized of "Persons who did not work or look for work by age, gender, and reason for not working in 2014" for "people not working in order to undertake home responsibilities", taken from the US Census Bureau's "Annual Social and Economic Supplement to the Current Population Survey". Interpreting "home responsibilities" as a proxy for not working due to child-rearing, averaging across the entire US population in 2014 yields $z = 0.28$.

The child-rearing subsidy rate is calibrated it by making use of government spending data from OECD's "Family Database" under "Total public social expenditure on families as a percentage of GDP", and OECD's "Education at a glance" reports. For the period 1969-2013, social spending, including primary to tertiary schooling, amounted to 7% of GDP. I then match the calibration's child-rearing costs as a percentage of GDP to the value found in data. Knowing that the population growth factor in the OECD from 1970-2015 implies $n = 1.361$ (see Table 1), $\alpha = 0.36$ and $z = 0.28$, one can solve for ρ_w: $(Nwzn\rho_w)/Y = 0{,}07 \Leftrightarrow [(1-\alpha)zn\rho_w]/(1-zn) = 0.07 \Leftrightarrow \rho_w \approx 0.18$[19].

	1970-2015	**2015-2060**	**2060-2100**
45-year population growth rate	36.1%	6.3%	-5.2%
Old-age dependency ratio	21.9%	44.0%	60.1%
Life expectancy at birth	75.2	83.9	89.2

Table 1: Demographic structure, historical values and projections. Source: United Nations

Next, fertility and mortality parameters are calibrated. Overall, three demographic structures will be calibrated for the periods 1970-2015, 2015-2060 and 2060-2100. I start by focusing on the first one. The parametrization of the latter two demographic structures can be found in Table 4. The mortality rate, λ, is found by matching the model's old-age dependency ratio to that observed in the data. The model's old-age dependency ratio is equal to $(1-\lambda)/n$. The old-age dependency ratio in the data (see Table 1), is 21.9% for the period 1970-2015. With $n = 1.361$ for the same period, λ becomes 0.7. Following a similar procedure for the preference for having children parameter ε, I match the model's fertility rate, n, to the one observed in the data. I then use expression (15), which does not include any endogenous variables, and solve it for ε. This procedure yields $\varepsilon = 0.55$.

Finally, the pension replacement rate is calibrated. Public pension expenditures as a proportion of GDP can be written as:

18 Note that Frederick et al. 2002 reports a survey of longer-term discounting results (average measurement > 1 year), where the average discount factor found is consistently around $0{,}8$ for all time horizons considered. The parametrization here of $0{,}69$ is then perhaps overly discounting the future, and not otherwise.

19 Note that this procedure does not make use of endogenous variables. The calibration procedure overall is strictly based on other (non-optimized) parameters.

$$\frac{[w_{t-1}(1-zn_{t-1})+\rho_w w_{t-1}zn_{t-1}]i_t\rho_{p,t}(1-\lambda_t)N_{t-1}}{Y_t}=$$

$$=\frac{[(1-zn)+\rho_w zn]w_t(1-\lambda)\rho_p N_{t-1}}{AX_t^{-\omega}g_t L_t}=$$

$$=[(1-zn)+\rho_w zn]\frac{(1-\lambda)}{(1-zn)n}(1-\alpha)\rho_p \tag{30}$$

The pension replacement rate is found by matching public pension expenditures as a proportion of GDP to real world values, utilizing the calibrated parameters for z, ε, ρ_w, λ, and α. Using OECD's Pensions at a Glance database, I estimate pension expenditures as a proportion of GDP to be 6% for the period 1969-2013. By equating this proportion to the last part of expression (30) and solving for ρ_p, we obtain $\rho_p=0.21$.

Table 2 summarizes the parameter values for the baseline calibration:

α	Private capital share in national income	0.36
β	Private capital share in labor productivity	0.56
ω	Private capital share in aggregate production	0.72
A	TFP scale factor	34
ρ	Discounting	0.69
z	Rearing time per child	0.28
ρ_w	Child-rearing subsidy rate	0.18
λ	Hazard rate	0.70
ε	Importance of children in utility	0.55
ρ_p	Public pension replacement rate	0.21

Table 2: Summary of baseline parametrization

4.3. Initial Steady state

Table 3 summarizes the modeling results for the baseline steady state in 1970-2015. The baseline calibration results in an optimal debt-GDP ratio of 40.6%, tied together with an optimal public-private capital ratio of approximately 0.19. The optimal debt-GDP ratio is quite smaller than what OECD countries like Japan, Greece, Italy, Portugal, and the US (among others) have nowadays. The average OECD country has a 65% debt-GDP ratio for the period 1995-2013, though some countries keep lower-than-optimal debt-GDP ratios, such as Estonia, Chile, and Luxembourg.

d^*	Optimal debt-GDP ratio at steady state (annualized)	40.6%
X^*	Optimal public-private capital ratio	0.193
γ^Y	Economic growth rate (annualized)	0.99%

γ^y	Economic growth rate *per capita* (annualized)	0.3%
θ	Tax rate	24.4%
$r-1$	Interest rate (annualized)	4.64%
n	Number of children per person	1.361
	Saving rate out of disposable income	13.2%
	Old-age dependency ratio	22%
	Life expectancy at birth	78.5
	Taxation revenue as a proportion of GDP	10.7%
	Public pension expenditure as a proportion of GDP	6.1%
	Return on private savings (annualized)	7.5%
	Implied return to public pensions (annualized)	3.7%

Table 3: Summary of benchmark calibration results

Note that these countries can have more public debt than public capital, since they do not follow the golden rule of public finance. Considering these cases, where the golden rule of public finance is not in place, it is important to compare the public-private capital ratio, instead of public debt, to the optimal value indicated by the model. For this, IMF's Investment and Capital Stock Dataset, 2017, indicates an average public-private capital ratio X of 0.383 for the OECD group for the period 1969-2013. This value is in fact higher than what the model indicates, a public-private capital ratio of 0.193. The baseline thus suggests that, on average, OECD economies are either not employing enough private capital or are employing to much public capital.

Countries with public-private capital ratios higher than the optimal value suggested by this model are not necessarily performing excessive public investment. The ratio X can be too high if public capital is too high or if private capital is too low. The stock of private capital can in fact be "artificially" low if there exists excessive discretionary[20] public debt crowding it out. This means that policies to reduce the public-private capital ratio, X, do not have to include reductions in public capital investment; in a closed-economy perspective, such an objective can be fulfilled simply by reducing "discretionary" public debt. In this case, private capital crowd-out is reduced, and a lower public-private capital ratio follows while maintaining the same level of public capital.

The optimized yearly economic growth rate for the economy stands at roughly 1.0%, with *per capita* growth at 0.3%. These values are lower than what the OECD economies experienced in the 1969-2013 period (an average of 2.8%, 1.91% per capita[21]). The detail to consider here is that exogenous TFP growth is not included in the modelling; only labor-related endogenous productivity growth is included. The saving

20 "Discretionary" in the sense that it is debt that is not raised for the purpose of public investment.

21 The OECD Economic Outlook dataset for growth rates; World Bank WDI report for per capita growth rates.

rate stands at 13.2%[22] and it is very close to the average gross saving rate out of disposable income reported in the AMECO database (13.9% for the period 1969-2013), but larger than the average net saving rate of 8.3% reported in OECD's Economic Outlook database[23].

As for the interest rate, the annualized value stands at 4.6%, a reasonable value, close to the observed average OECD real interest rate of 4.4% in the period 1969-2013[24]. The balanced-budget tax rate is 24.4%. To gain a better perspective into what this number means, the tax revenue as a proportion of GDP is also reported, which stands at 10.7%. Data from OECD's Revenue Statistics indicates that the OECD average is 9.4% for income and profits earned by individuals. The resulting balanced-budget tax rate represents a reasonable value, though a bit on the high-side given that many other public expenditure items exist. The excess can also stem from the fact that only labor income and the child-rearing subsidy are taxed. In reality, many other incomes/profits are taxed.

Values are reported for the resulting fertility rate, old-age dependency ratio, and public pension expenditures as a proportion of GDP. By parametrization design, these values should be as close as possible to the observable data. The fertility rate (per individual) stands at 1.361 (or an annual population growth rate of 0.7%), the old-age dependency ratio is 22%, and public expenditures amount to 6.1% of GDP, all virtually identical to their observed data counterparts. As an additional check, I report life expectancy at birth, as calculated by $20 + 45 + 45(1 - \lambda)$. It stands at 78.5 years, a value higher than the one observed in the data for the OECD, 75.3 years[25], which is expected since a 45-year working career starting at the age of 20 is assumed. The model value for life expectancy is still quite close to the real counterpart, such that this should not represent an issue for the analysis. Finally, I report the return to retirement savings and implied return to public pensions measured as the ratio of pension payouts to pension pay-ins. This is calculated as $\left[(1 + C_c)/C_p\right]\rho_p\gamma^y|_{X=X^*}$. An annualized return of 7.5% for private savings is obtained, versus 3.7% for public pensions. Private savings represent a better deal for households, though subjective discounting of future consumption makes households less willing to save. This discounting effect does not impact consumption at retirement provided by public pension income, since the benefit accrual rate is not chosen by households.

5. Results

So far, steady state equilibria, transitional behavior, and the existence of a policy tool with which to preserve public debt sustainability have been shown. This theoretical setting is now applied numerically

22 This saving rate is derived by dividing savings, as it stands in expression (14), and dividing it by disposable income, $(1-\theta)w\left(1 - zn(1-\rho_w)\right)$.

23 It is unlikely that the difference between gross and net saving rates explains the wedge between the AMECO and OECD databases. This is revealing a more symptomatic reason, where methodologies for the derivation of saving rates likely yield markedly different results.

24 Data from the World Development Indicators database. Only countries with more than 15 data points were considered 18 out of the 35 OECD countries were included. These are Australia, Belgium, Canada, Chile, Czech Republic, Finland, France, Germany, Greece, Hungary, Iceland, Italy, Japan, Mexico, Spain, Switzerland, the United Kingdom, the United States of America.

25 Source: United Nations Population Division.

to analyze demographic shocks and their impact on the economy. Relevant dimensions, such as the fiscal stance of the government and the optimal public debt level, are discussed, as well as the impact of demographic ageing on the stability of the initial benchmark steady state. The new maximal growth steady states are shown for each future expected demographic structure, along with their convergence properties relative to the previous steady state. An analysis into how key parameters in the model affect the baseline results then follows.

5.1. Benchmark scenario

Demographic parameters will now be modified to match the projections in Table 1 in terms of the old-age dependency ratio and the fertility rate by re-calibrating λ and ε. Table 4 summarizes all three steady states for each of the demographic structures in Table 1, as well as the reductions in public investment necessary to ensure that the public debt stock does not spiral out of control:

Period:	**1970-2015**	**2015-2060**	**2060-2100**
λ	0.7	0.53	0.43
ε	0.55	0.427	0.388
Life expectancy at birth	78.5	86.15	90.65
X^*	0.193	0.192	0.191
d* (a)	40.6%	40.5%	40.3%
γ^Y (a)	0.99%	1.55%	1.7%
γ^y (a)	0.3%	1.42%	1.82%
θ	24.4%	26.4%	28.8%
Saving rate out of disp. income	13.2%	15.3%	15.4%
Interest rate (a)	4.64%	4.64%	4.63%
Tax revenue / GDP	10.7%	12.8%	14.4%
Public pension expenditure / GDP	6.1%	7.2%	7.74%
Return on private savings (a)	7.5%	6.4%	5.9%
Implied return to public pensions (a)	3.7%	3.3%	3.0%
Public investment - Sustainability (a)	**---**	**$6\text{x}10^{-6}$ pp.**	**$2\text{x}10^{-5}$ pp.**
Public investment – Max. growth (a)	**---**	**0.007 pp.**	**0.013 pp.**

Table 4: Projected demographic shifts and future steady state results; (a) indicates annualized values.

Starting with the most important result, we see that with decreasing fertility and mortality comes a lower optimal public-private capital ratio, X^*[26]. This reduction in X^* can have serious implications in terms of stability, since, upon a demographic shock, the steady state for the period 1970-2015 lies to the right of the 2015-2060 steady state. As the stability analysis in sub-section 3.3 concluded, this implies that the government must apply austerity to public investment relative to the maximal growth policy. Even though there is the need for a reduction in public investment, and by extension, a need for reducing public debt issuing, table 4 shows that the impact of the demographic ageing on X^* is very small.

Two different public investment policies are described in table 4 to face demographic ageing in what the

26 Though not included in the text, fertility and mortality decreases have each a negative impact on X^*.

sustainability of public debt is concerned. The first is termed "Public investment – Sustainability" and the other "Public investment – Max. growth". The former policy just ensures the sustainability of the public debt stock at its level in the previous steady state after the demographic shift takes place. This will be a *permanent* state of lower economic growth compared to its current potential. Since an effort is not made to transition into the new maximal growth steady state, the economy will be stabilized at a lower growth state, which by steady state definition also requires a permanently reduced public investment policy. Turning to the "maximum growth" policy, it ensures convergence towards the new maximal growth steady state in one 45-year period, which requires a more aggressive reduction in public investment, though temporary. After the transition period, the economy stabilizes at maximal growth forever if no further shocks take place.

Table 4 shows that, for both demographic shocks at 2015-2060 and 2060-2100, public capital growth must be reduced by a negligible amount to just ensure its sustainability. For example, transitioning from a 1975-2015 to 2015-2060 demographic structure requires a sustainability-inducing austerity policy of reducing public capital growth by an annual $6\text{x}10^{-6}$ percentage points for eternity (with no further demographic change). Transitioning to the 2060-2100 period, a minimum sustainability-inducing policy of $2\text{x}10^{-5}$ is observed, which remains negligible. As for the 1-period convergence public investment policy, though it requires stronger adjustments now, it still implies negligible reductions to public investment of an annual 0.007 percentage points per year for 45 years when transitioning from the 1970-2015 period to the 2015-2060 one. This number becomes 0.013pp. to transition to the 2060-2100 maximal growth steady state in one period. We thus have that austerity policies over the optimal public capital policy are virtually unnecessary in what demographic change is concerned. Even if a government attempts to induce convergence in only one-period, the amount of austerity remains negligible[27].

The public debt-GDP ratio tracks X^* proportionally by definition (see expression 17), and it also sees very small changes to it. Interest rates also barely change negatively, meaning that interest servicing costs per unit of public debt will virtually remain the same. Effectively, the predicted demographic shifts, as large as they are (especially the old-age dependency ratio), virtually do not require accommodation in terms of the optimal public debt policy. We expectedly see a rather large change to the balanced-budget tax rate, and reforms to accommodate demographic change should instead focus on balancing taxation revenue and public expenditure on demography related items.[28]

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27 It is always worthwhile forcing the fastest convergence possible towards the maximal growth steady state in what concerns total inter-generational welfare. This will never be a Pareto improvement, since inducing (even marginally) an economy to converge to a new higher growth steady state after a demographic shock always implies at least one period of lower wage growth. This means that those living today will have lower life-time consumption and all future generations will be as well off as the economy allows them to. Applying the minimum austerity policy does not penalize the first generation of workers relative to all future workers, but all these future workers will not be able to be as well off as the economy actually allows them too because the government is forced to permanently restrict public investment to allow for a higher than optimal debt-GDP ratio (or equivalently, the optimal public-private capital ratio).

28 All these results come with one caveat: this analysis is performed on the average OECD country. The weak effect results might be much stronger for certain OECD economies. We provide a robustness check to gain insight into this possibility.

As for economic growth (per capita), it is seen that it increases with the demographic shift. The driver of stronger economic growth stems mainly from an increase in the saving rate. There are two other items that are dragging economic growth down: the increased tax rate, and the decreased fertility rate. The tax rate suffers an increase, from 24.4% in 1970-2015 to 26.4% in 2015-2060, and then to 28.8% in 2060-2100. In a different perspective, tax revenues as a proportion of GDP goes from 10.7% to 12.8% to 14.4%. We are thus in a situation where demographic ageing implies increased financing requirements. Adding to this is the projected decrease in fertility, from 1.361 children per person on average in 1970-2015 to less than 1 child per person for 2060-2100. This hinders economic growth in two ways: first, by having less workers come into the economy; second, by increasing the negative effect public pensions have on growth, since there are now fewer working age individuals to pay into the public pension system.

We observe that the increase in saving behavior, due to a higher life expectancy, dominates all these negative effects, thus increasing the economic growth rate. This reveals a potential problem that economies may face: if the saving rate of households does not increase as expected, in line with increases in longevity, or if the retirement age is not adjusted accordingly to roughly maintain the same relative length of retirement, we will not see the positive growth effects from increased saving, but we will have all the negative effects from higher taxation and lower fertility.

Finally, the demographic shift implies an increase with public pension costs, from an initial 6.1% to 7.2% in the following 45-year period, to 7.7% in the 2 period after the initial one. More people surviving into retirement and less people contributing into the pension system justifies this result. The implied public pension return, defined as how much one individual receives in public pension compared to how much that individual contributes into it, drops after the demographic shift takes place. Public pension payouts become more attractive as economic growth per capita increases due to wage indexing, but pension contributions into the public system also become higher when longevity increases. The latter effect dominates the former effect, thus worsening the implied return to the public pension system.

5.2. Weakening/Strengthening PAYG pensions

The analysis so far focused on a benchmark case of the average OECD country. Further investigation is thus warranted to understand how the model's results change with key parameters. This is important to understand in which dimensions the model's results are generalizable, and which should be given further thought at a country-specific level. To begin, different economies are considered, differing only in the public pension replacement rate, ρ_p, offered by the government. The experiment is done for the range 0 to 100%.

Figure 2 plots the key results from the model over the pension replacement rate on the x-axis. Starting with non-demographic observations: The optimal public-private capital and debt-GDP ratios are remarkably stable across all replacement rates *within* period. This comes in contrast with what is observed from tax rates in the first period, 1970-2015, which increase from 20%, when no public pensions are offered, to 40% when full replacement of lifetime income is offered by the government at retirement.

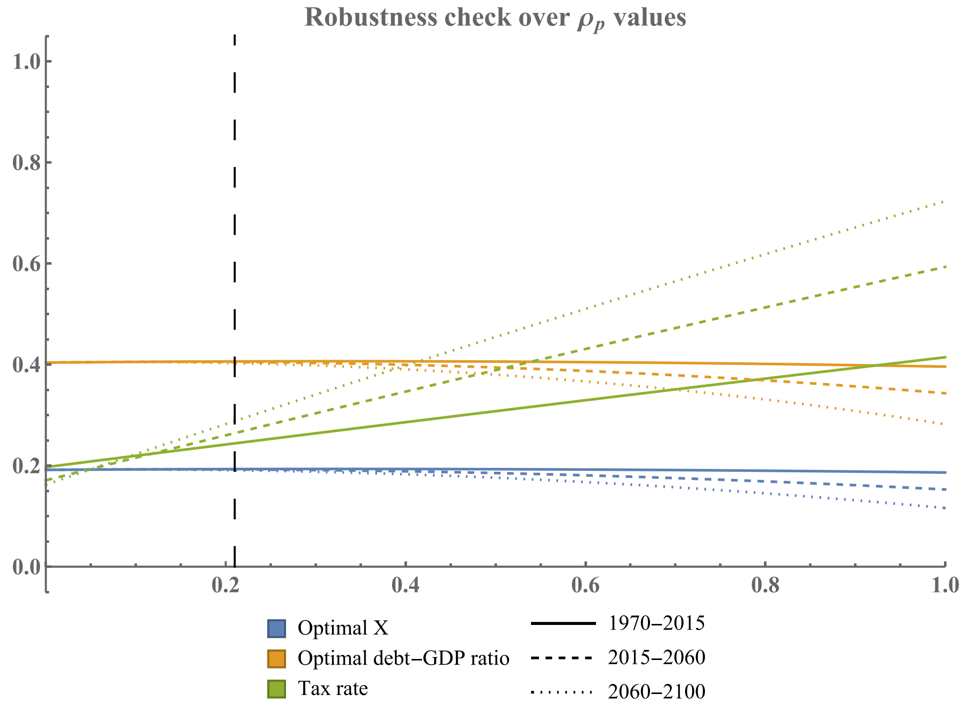

Figure 2 - Results for various ρ_p values. Vertical dashing line represents benchmark value.

Now considering the results *across* periods, the lower end of replacement rates sees negligible impacts on all three variables, optimal public debt/capital and the tax rate, stemming from demographic shifts. The existence of public pensions is the main driver of the results seen in the benchmark calibration. The child-rearing subsidy alone has little impact to the optimal public debt policy, a result already confirmed in the work of Hughes Hallett et al. (2019). Only when public pension replacement rates rise do optimal public-private capital and debt-GDP ratios begin to curve downwards after demographic change takes place.

Two effects take place to the optimal public debt-GDP ratio when the replacement rate increases:

- One, the PAYG pension benefits disincentivize more and more private saving, which is damaging to economic growth. Increasing the public-private capital ratio X softens the impact the PAYG system does to the saving mechanism (see denominator of expression 28).
- Two, tax collection to preserve the fiscal balance of the PAYG pension system progressively increases as the replacement rate is higher. For a given old-age dependency ratio, higher benefits become relatively harder to finance, which in turn make a given stock of public debt harder to finance out of the common pool of resources (see the numerator of tax rate expression 19). This implies a lower optimal public debt-GDP ratio for higher pension replacement rates.

The two effects described above impact the optimal public debt-GDP ratio in different directions. The numerical simulation above shows that the second mechanism dominates the first, such that with higher PAYG pension replacement rates comes a lower optimal public debt-GDP ratio.

Table 5 summarizes the results for no PAYG pensions vs. 100% PAYG pension replacement rate. First, for the key result on transitional impacts, an economy that does not offer PAYG pension benefits sees virtually no impact to its optimal public debt policy. With no PAYG pension system, the reductions in fertility translate into a lower tax rate when the demographic structure evolves from the 1970-2015 period onward, as expenditures with the child-rearing subsidy drop. As public pensions are introduced, its tax-increasing effects quickly overshadow the tax decreasing effects from child-related transfers.

For full pension benefits, at a 100% replacement rate, we now observe that the effects of demographic change become very pronounced for the optimal public-private capital and debt-GDP ratios. Initially, between 1970-2015, it is optimal to keep a debt-GDP ratio of 39.6%. Two periods ahead, between 2060-2100, that number decreases to 28.2%, which is a significant drop. Figure 2 also shows that the drop in the optimal debt-GDP ratio starts to get significant in size from a replacement rate of ~60% and above. Demographic shifts are a double-edged sword for countries offering very high replacement rates: not only the optimal debt-GDP ratio becomes markedly lower (by roughly 5 pp. every 45 years), setting a more demanding target for public debt reduction, it also raises tax rates significantly (by roughly 15 pp. every 45 years), making it harder to create the budget surpluses required to reduce the public debt stock.

Pension replacement rate:	$\boldsymbol{\rho_{\mathrm{p}} = 0}$			$\boldsymbol{\rho_{\mathrm{p}} = 1}$		
Period:	**1970-2015**	**2015-2060**	**2060-2100**	**1970-2015**	**2015-2060**	**2060-2100**
X^{*}	0.1916	0.1916	0.1916	0.1864	0.1528	0.1161
d^{*} (a)	40.4%	40.4%	40.4%	39.6%	34.3%	28.2%
θ	19.7%	17.1%	16.2%	41.4%	59.3%	72.3%
Tax revenue / GDP	8.7%	8.3%	8.1%	18.2%	28.7%	36.2%
Public investment - Sustainability (a)	**-**	**≈0pp.**	**≈0pp.**	**-**	**0.015pp.**	**0.03pp.**
Public investment – Max. growth (a)	**-**	**≈0pp.**	**≈0pp.**	**-**	**0.31pp.**	**0.44pp.**

Table 5: Model's results when ρ_p, the public pension replacement rate, is 100%; (a) indicates annualized values

Table 5 reports the sustainability-preserving and maximal growth-inducing adjustment to the government's policy function of public investment. First, the steady state adjustment to public investment required to stabilize the economy around the previous period's public-private capital ratio is 0.015 pp. for the transition from 1970-2015 to 2015-2060. The next jump to 2060-2100 requires a minimum adjustment of 0.03 pp. Though these values are much higher than in the benchmark scenario, they are still relatively negligible. If we turn our attention to the austerity policy of enforcing a 45-year convergence towards the new maximal growth steady state, we now see that this type of austerity policy requires lowering the growth of the public capital stock by around 0.31-0.44 percentage points every year, for 45 years. The results are not as negligible as they were, though a value just below half a percentage point is arguably no cause for serious concern.

It is thus concluded that paths to ensure stability or convergence towards a new steady state under a new demographic structure exist for the entire range of possible public pension replacement rates. Even for governments that offer very high public pension replacement rates, ensuring stability or convergence towards the new steady states requires only negligible reductions in public investment. Even if transition to the new steady state is to be made in one period (45 years), a reasonable path still exists, only requiring a drop in public investment in the order of 0.31-0.44 percentage points a year. The point to note in terms of fiscal sustainability is then the implied balanced-budget tax rates seen for countries with very strong PAYG pension schemes. Keeping these systems as they currently are imply massive increases to tax collection in order to keep these PAYG schemes balanced. Action is required on reforming these systems, and not so much on reforming the way public investment is taking place today vs. the long-term future.

5.3. Production function parameters – α and β

Now, production function parameters, key elements in defining the optimal public-private capital ratio X^*, are considered in terms of α and β. Figure 3 plots the model's results for the entire range of possible values for α. Starting with tax rate effects of increasing α, we see that tax rates display a certain concavity over α values for *each* period. As α increases, the costs with servicing public debt increase, but the optimal public debt level decreases at the same time. These two elements counteractively remain balanced, which explains the low variance of the tax rate across different values of α. Looking at how demographic change impacts the equilibrium tax rate *over* periods, it uniformly impacts the tax rate across the range by increasing it.

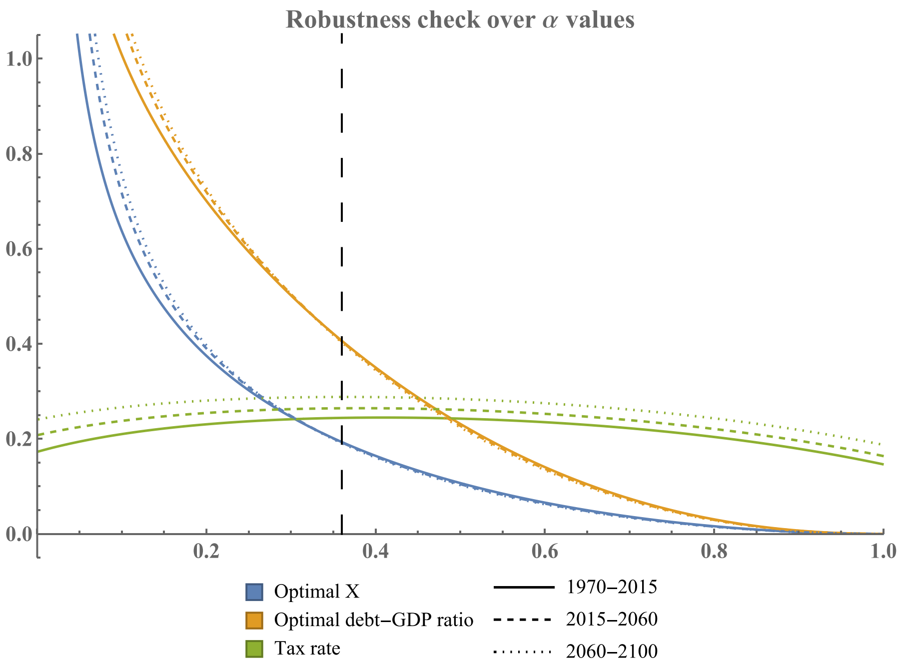

Figure 3 – Results for various α values. Vertical dashing line represents benchmark value.

Focusing now on the optimal debt-GDP ratio (or equivalently the optimal public-private capital ratio), we see that increasing α provokes a monotonic decrease in the optimal debt-GDP ratio for a given period. This is not surprising, given that increasing the proportion of production that stems from private capital, as opposed to labor activities, decreases the importance of public capital for a given level of production. The interesting detail to underline is that demographic change impacts the optimal debt-GDP ratio in different directions depending on the value of α. The reasoning follows:

- When α is low, two effects take place:
  - One, the costs with financing a given stock of public debt are lower (see tax rate expression 20).
  - Two, the negative impact of PAYG pensions on the saving mechanism becomes stronger. Thus, stimulating wage rates further up by increasing X becomes optimal as a government policy (see denominator of economic growth expression 28).

These two effects together yield a higher X^* when demographic ageing takes place. When α is high, these two effects reverse, and demographic ageing implies new steady states with a lower maximal growth public-private capital ratio X^*. The benchmark case is performed at $\alpha = 0.36$. In the data, countries with low α values, as of 2013, are: Belgium, France, the Netherlands, Slovenia, the UK, Iceland, Switzerland and South Korea. All other OECD countries face a negative impact on optimal public investment due to demographic change, however small that impact may be. This result also reveals future concerns over technological change: if private capital is increasing in importance over time, relative to labor, demographic change may have an increasing negative impact on economies as time moves forward.

Turning to the parameter β (see Figure 4), it determines the contribution of aggregate private capital, as opposed to public capital, in the effectiveness of labor in the production process. Within period, the optimal public-private capital and public debt ratios change considerably over β, and the curves are virtually independent of demographic change. For a reasonable value interval of $[0.3 - 0.8]$, X^* (d^*) ranges from 0.33 (71%) to 0.09 (16%). Like with parameter α, the debt-GDP ratio is much more sensitive than the public-private capital ratio because the expression for the debt-GDP ratio is scaled by $\omega \equiv \alpha + \beta(1 - \alpha)$ (see expression 17).

The tax rate is downwards sloping, since a lower β is associated with a higher need for public capital, which requires more taxation to fund the public debt servicing costs. Demographic change increases tax rates across all β values equally, but it virtually has no impact to the optimal debt-GDP ratio. A changing β does not affect the taxation structure of the economy, as it does not enter the balanced-budget tax rate expression. It only affects the production side of the economy. Since public capital is costly to run due to the taxation requirements (as opposed to private capital), a lower β makes the economy more dependent on public capital, which in turn makes it more sensitive to demographic change. Nonetheless, the impact is negligible, even for small values of β.

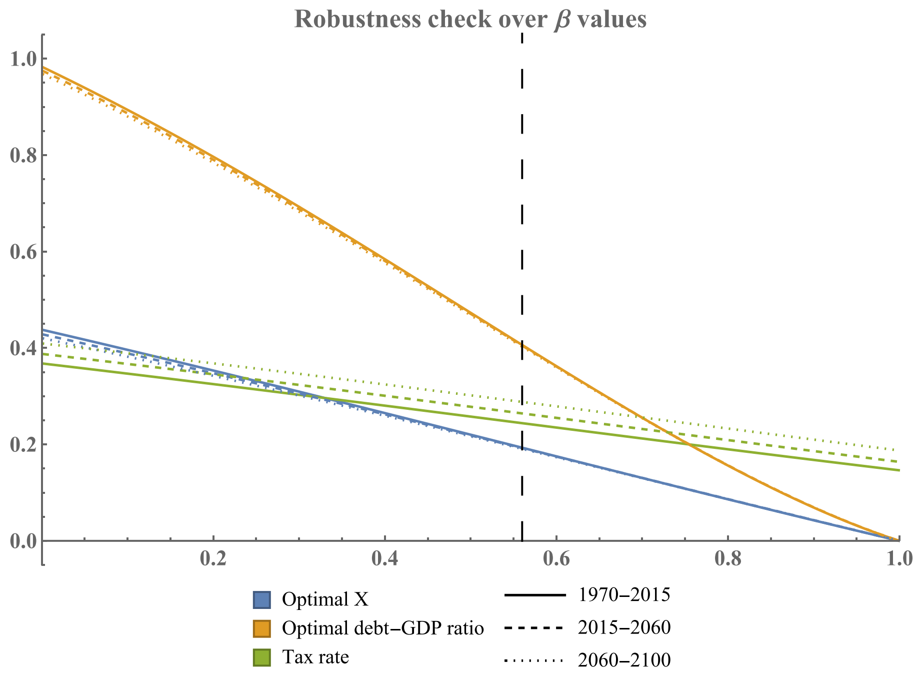

Figure 4 - Results for various β values. Vertical dashing line represents benchmark value.

Like in the α situation, it is confirmed that it is the taxation system that suffers a heavy impact with the expected demography shift, and not the optimal public debt schedule, for a reasonable range of production parameters.

6. Conclusion

Under a regime of old-age and child-related public transfers, I show how future expected demographic shifts, represented by a lowering of both fertility and mortality rates, represent a potential for future disruption of current balanced-budget fiscal policies. For the average OECD economy, there are both reasonable steady states and transition paths available under a golden rule of public finance for the expected demographic ageing processes throughout the 21st century. Under this golden rule, the optimal public debt-GDP ratio suffers very small impacts stemming from demographic ageing. The model indicates an initial optimal public debt-GDP ratio of 40.6%, suffering negligible changes throughout the expected demographic structure for the rest of the 21st century. As the old-age dependency ratio increases, public debt sustainability does not require significant reductions in public investment.

Countries with more generous unfunded pension systems are at a higher risk of seeing their public debt stocks turning unsustainable upon a reduction in fertility and mortality rates. When higher benefits are distributed under PAYG pension schemes, demographic ageing implies stronger funding requirements, making it harder to finance a given stock of public debt, as well as stronger interference in the saving decisions of households. The combined effect of these two transmission channels is a strong reduction to the future optimal public debt-GDP ratio, though ensuring its sustainability requires relatively small reductions to public investment. Hence, the outlook for public debt seems benign. The problem with strong unfunded pension benefits is the dramatic increase in funding requirements in order to finance increased unfunded pension outlays upon demographic ageing. Focus should be posited on reforming the pension system and its benefits level, and not in managing the public debt stock that finances public investment for a country that follows the golden rule of public finance.

The transition mechanics depend on the labor intensity of the economy. Labor intensive economies have stronger wage rates, which makes it easier to finance a given stock of public debt. Stronger wage rates also augment the negative impact that PAYG pensions have in disincentivizing private saving. These two mechanisms make it so that demographic ageing increases the optimal public debt-GDP ratio. The conclusion here is that demographic ageing processes do not de-stabilize the public stock of debt for labor intensive countries. Convergence is automatically ensured upon demographic ageing. The situation is reversed for capital intensive countries, where demographic ageing turns a sustainable public debt stock into an unsustainable one. Though the numerical results show that sustainability inducing policies require negligible interventions to the public investment schedule, this result reveals a potential for trouble: With the advent of robotics, IT technologies, artificial intelligence and automation, economies that are becoming more capital intensive may need to re-evaluate their unfunded pension systems, or risk having to cut on public investment in the future, yielding lower economic growth prospects for future generations.

For future work, allowing for budget surpluses/deficits would enrich the analysis by providing a more comprehensive approach to fiscal policy reform in terms of achieving fiscal sustainability by either reductions (increases) to public investment or by creating budget surpluses (deficits). Furthermore, different taxation structures, including variations to the tax treatment of subsidies and savings, exploring lump-sum transfers, or the introduction of taxation on other income items could be explicitly explored for the enrichment of the results shown in this paper. Considering public debt not associated with public capital would also be interesting follow-up work, as most advanced economies do not follow the golden rule of public finance.

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Appendix A – Short- and long-run conditions for equilibria

A short-run equilibrium can be described as a point in time where: consumption of working and retired individuals and next period's capital stock is positive, while public capital does not completely crowd-out private capital and the government budget is respected:

- Positive consumption of workers: starting from expression (10), and substituting optimal savings described in expression (14):

$$c_t > 0 \Leftrightarrow (1-\theta_t)[w_t(1-zn_t)+\rho_w w_t z n_t] - s_t > 0 \Leftrightarrow (1-\theta_t)w_t + (1-\lambda_{t+1})\rho_{p,}\frac{w_{t+1}}{r_{t+1}} > 0$$

$$\Leftrightarrow (1-\theta_t) > -(1-\lambda_{t+1})\frac{\rho_{p,}}{r_{t+1}}\frac{w_{t+1}}{w_t}$$

This in turn implies: $\begin{cases} X_t > 0 \\ X_{t+1} > 0 \\ \theta_t < 1 + (1-\lambda_{t+1})\frac{\rho_{p,}}{r_{t+1}}\frac{w_{t+1}}{w_t} \Leftrightarrow \theta_t < 1 \end{cases}$

- Positive consumption of retirees: starting from expression (12), and substituting in expression (11) and (14) measured at times t and $t+1$, respectively:

$$d_t > 0 \Leftrightarrow \frac{r_t}{(1-\lambda_t)}s_{t-1} + [(1-zn_{t-1})+\rho_w z n_{t-1}]\rho_{p,}w_t > 0 \Leftrightarrow r_t(1-\theta_{t-1})w_{t-1} + (1-\lambda_t)\rho_{p,}w_t$$

$$> 0 \Leftrightarrow (1-\theta_{t-1}) > -(1-\lambda_t)\frac{\rho_{p,}}{r_t}\frac{w_t}{w_{t-1}}$$

This in turn implies: $\begin{cases} X_{t-1} > 0 \\ X_t > 0 \\ \theta_{t-1} < 1 + (1-\lambda_t)\frac{\rho_{p,}}{r_t}\frac{w_t}{w_{t-1}} \end{cases}$

- Positive capital stock/no private capital full crowd-out: starting from expression (23) and substituting in expression (14), and using the definition of the debt-GDP ratio (18):

$$s_t N_t - G_{t+1} > 0 \Leftrightarrow \left[\tilde{C}_t(1-\theta_t)w_t - \tilde{C}_t\frac{\rho_{p,}}{\rho}\frac{w_{t+1}}{r_{t+1}}\right]N_t - G_{t+1} > 0$$

$$\Leftrightarrow \tilde{C}_t(1-\theta_t)A(1-\alpha)X_t^{-\omega}\frac{1}{\gamma_{t+1}^g}\frac{1}{(1-zn_t)} - \tilde{C}_t\frac{\rho_{p,}}{\rho}\frac{(1-\alpha)}{\alpha}\frac{1}{X_{t+1}}\frac{1}{(1-zn_{t+1})n_t} - 1$$

$$> 0$$

This in turn implies: $\theta_t < 1 - \left[\frac{(1-zn_t)}{(1-\alpha)\tilde{C}_t} + \frac{\rho_{p,}}{\rho}\frac{1}{\alpha X_{t+1}}\frac{(1-zn_t)}{(1-zn_{t+1})n_t}\right]d_t\gamma_{t+1}^g$

Summing up the short-run conditions for equilibrium, positive consumption of both workers and retirees yield the same conditions but measured at different time periods. The first conditions simply require a positive public-private capital ratio, which is trivial. The non-trivial condition requires the tax-rate to be lower than a number bigger than 1. Since it is not possible to tax more than gross income in this model, this restriction collapses to the tax-rate not being higher than 100%. The positive capital stock/no private capital crowd-out restriction presents itself as an expression that is dependent on the public pension replacement rate, the debt-GDP ratio, and the growth factor of the public capital stock. If any of these elements become higher, the more restrictive the condition becomes for how high the tax-rate can be if a short-run equilibrium is to exist.

A long-run equilibrium is, under initial conditions X_0 and G_0 and the path $\left(\rho_w, \rho_p\right)_{t\geq}$, characterized by the sequences $(X_t, G_t)_{t>0}$ and $(\theta_t)_{t\geq 0}$ such that these two conditions are verified:

$$\begin{cases} s_t N_t = K_{t+1} + G_{t+1} \\ B_{t+1} = B_t + (r_t - 1)B_t + (G_{t+1} - G_t) + \rho_w w_t z n_t N_t + \\ +[(1 - z n_{t-1}) + \rho_w z n_{t-1}] w_{t-1} i_t \rho_p (1 - \lambda_t) N_{t-1} - \theta_t (w_t L_t + \rho_w w_t z n_t N_t) \end{cases}$$

I assume a golden rule of public finance, so that:

$$B_t = G_t$$

Simplifying further, this makes it so that the long-run equilibrium conditions can be re-stated as:

$$\begin{cases} X_{t+1} = \dfrac{1 + \dfrac{(1-\alpha)}{\alpha}\dfrac{\rho_p}{\rho}\dfrac{\tilde{C}_t}{(1 - z n_{t+1}) n_t}}{\dfrac{\tilde{C}_t}{(1 - z n_{t+1}) n_t} A(1-\alpha)\dfrac{(1-\theta_t)}{X_t^{\omega} \gamma_{t+1}^{g}} - 1} \\ B_t = G_t \\ \theta_t = 1 - \dfrac{1 - \dfrac{\alpha}{(1-\alpha)} X_t - C_{p,t}}{1 + C_{c,t}} \end{cases}$$

These conditions are expressions (25), (17) and (20), respectively.

Appendix B – The Jacobian and Hessian of transition equation

The Jacobian of the transition equation in expression (25) is now derived. Taking the derivative of this expression and evaluating it at steady state with $\gamma_{t+1}^{g} = \gamma^{y}$ yields:

$$\frac{\partial}{\partial X_t} X_{t+1}\left(X_t, \gamma_{t+1}^{G}\right)\Big|_{\gamma_{t+1}^{G}=\gamma^{y}(X)} = \frac{\left[(1-\alpha)\left(1-C_p\right)\omega + \alpha(1-\omega)X\right]\left[(1-\alpha)\tilde{C}\rho_p + (1-zn)(1+X)\alpha n\rho\right]}{\left[(1-\alpha)\left(1-C_p\right) - \alpha X\right]\left[(1-\alpha)\tilde{C}\rho_p + (1-zn)\alpha n\rho\right]}$$

The sign of the Jacobian is completely determined by the element $\left[(1-\alpha)\left(1-C_p\right) - \alpha X\right]$ in the denominator. Thus, the sign of the Jacobian is:

$$\begin{cases} > 0 \; if \; X < \dfrac{(1-\alpha)}{\alpha}\left(1-C_p\right) \\ < 0 \; if \; X > \dfrac{(1-\alpha)}{\alpha}\left(1-C_p\right) \\ undefined \; if \; X = \dfrac{(1-\alpha)}{\alpha}\left(1-C_p\right) \end{cases}$$

This implies that the Jacobian starts at a positive value and converges to $+\infty$ as X approaches $\frac{(1-\alpha)}{\alpha}\left(1-C_p\right)$. When $X \to 0$, the Jacobian value converges to ω, which is below 1 by definition. This proves that there exists a section of the steady state curve that is locally stable, where the Jacobian value is between ω and 1.

I now derive the Hessian of the transition curve in expression (25), twice differentiating it with respect to X_t, and again calculate it evaluated at steady state with $\gamma_{t+1}^{g} = \gamma^{y}$:

$$\frac{\partial^2}{\partial X_t^2} X_{t+1}\left(X_t, \gamma_{t+1}^{g}\right)\Big|_{\gamma_{t+1}^{g}=\gamma^{y}(X)}$$

$$= \Bigg[\alpha^3\rho^3\left(n(1-zn) + \frac{(1-\alpha)\tilde{C}\rho_p}{\alpha\rho}\right)\left[-n(1+X)(1-zn) - \frac{(1-\alpha)\tilde{C}\rho_p}{\alpha\rho}\right]^2 \Bigg\{2\left[(1-\alpha)\left(1-C_p\right)\omega + \alpha(1-\omega)X\right]^2$$

$$+ \frac{\omega\left[\alpha X - (1-\alpha)\left(1-C_p\right)\right]\left[(1-\alpha)\left(1-C_p\right)(1+\omega) + \alpha(1-\omega)X\right]\left[(1-\alpha)\tilde{C}\rho_p + (1-zn)\alpha n\rho\right]}{(1-\alpha)\tilde{C}\rho_p + (1-zn)(1+X)\alpha n\rho}\Bigg\}\Bigg]$$

$$\times \frac{1}{X\left[\alpha\left(1-C_p+X\right) - \left(1-C_p\right)\right]^2\left[(1-\alpha)\tilde{C}\rho_p + (1-zn)\alpha n\rho\right]^3}$$

The sign of the Hessian is dependent on the value of X. The sign of the Hessian depends on the expression $\left[\alpha X - (1-\alpha)\left(1-C_p\right)\right]$ in the numerator of the fraction inside the curly brackets. When $X \to 0$, the expression inside the curly brackets converges to $-(\alpha-1)^2\left(C_p-1\right)^2(1-\omega)\omega$, and the denominator of the last fraction converges to infinity. As such, the Hessian converges to $-\infty$ when $X \to 0$. In turn, X can become large enough such that $\left[\alpha X-(1-\alpha)\left(1-C_p\right)\right]$ becomes positive. Hence, the Hessian necessarily becomes positive for X high enough. The conclusion is that the transition curve expressed in (25) starts out concave and necessarily becomes convex for X high enough.

Finally, I show the derivative of the transition equation with respect to growth of the public capital stock, γ^g, evaluated at steady state with $\gamma_{t+1}^g=\gamma^y$:

$$\frac{\partial}{\partial\gamma_{t+1}^g}X_{t+1}\left(X_t,\gamma_{t+1}^g\right)\bigg|_{\gamma_{t+1}^g=\gamma^y(X)}=\frac{(1+C_c)X^{\omega}\left[(1-\alpha)\tilde{C}\rho_p+(1-zn)(1+X)\alpha n\rho\right]^2}{A\alpha\tilde{C}\rho\left[(1-\alpha)\left(1-C_p\right)-\alpha X\right]\left[(1-\alpha)\tilde{C}\rho_p+(1-zn)\alpha n\rho\right]}$$

This function is proportional to the Jacobian of the transition curve. As such, it has the same sign as the Jacobian. Another very important consequence here is that changes to the policy function γ_{t+1}^g affect the transition curve monotonically, meaning that increasing or decreasing public investment respectively tilt the transition curve upwards or downwards from the origin $(0,0)$.

Appendix C – Stability analysis of non-hyperbolic steady state

This appendix provides details on the stability features over the steady state curve, which includes one non-hyperbolic point where the Jacobian of the transition curve (25) is equal to 1. Drawing from Croix and Michel (2002), this appendix starts by showing that the bifurcation point is in fact the maximal growth steady state. With a bifurcation point, a further look beyond the system's Jacobian is warranted for the maximal growth steady state, in order to understand it. I then characterize its stability and show the stability of all other steady states relative to the maximal growth.

Expression (25) defines the transition equation for the optimal public-private capital ratio, X, at time $t+1$. It is dependent, among other parameters, on the previous' period capital ratio X_t and the government's policy function, γ_{t+1}^{g}. Recall that, by definition, we have that the policy function is steady state compatible only when $\gamma_{t+1}^{g} = \gamma_{t+1}^{y}(X_t)$. I define $\varphi(X,\gamma^g) = X(X_t,\gamma^g) - X$. Differentiating $\varphi(X,\gamma^g) = 0$ with respect to X and evaluating it at steady state yields:

$$\varphi'_X[X,\gamma^y(X)] + \varphi'_{\gamma^g}[X,\gamma^y(X)]\gamma^{y\prime}(X) = 0 \quad \text{(C1)}$$

Appendix B shows that the Jacobian of the system at the steady state curve is initially positive and equal to ω and increasing towards $+\infty$, meaning that it attains the value of 1 at some point. Denoting $\bar{X}$ as the public-private capital ratio that is associated with a Jacobian value of 1, we have that $\varphi'_X[\bar{X},\gamma^y(\bar{X})] = 0$, and expression (C1) then implies that when $\varphi'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})] \neq 0$ we necessarily must have that $\gamma^{y\prime}(\bar{X}) = 0$. In this case, there are three possibilities: one, the function $\gamma^y(X)$ attains a maximum at point $\bar{X}$; two, it attains a minimum at that same point; and three, there is an inflection point at $\bar{X}$. A look at the Hessian of function $\gamma^y(X)$ is thus useful. For this, I twice differentiate $\varphi[X,\gamma^g] = 0$ with respect to X and evaluate it at steady state:

$$\varphi''_{XX}[\bar{X},\gamma^y(\bar{X})] + \varphi''_{X\gamma^g}[\bar{X},\gamma^y(\bar{X})]\gamma^{y\prime}(\bar{X}) + \varphi''_{\gamma^g X}[\bar{X},\gamma^y(\bar{X})]\gamma^{y\prime}(\bar{X}) + \varphi''_{\gamma^g\gamma^g}[\bar{X},\gamma^y(\bar{X})]\left(\gamma^{y\prime}(\bar{X})\right)^2 + \varphi'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})]\gamma^{y\prime\prime}(\bar{X}) = 0$$

With $\gamma^{y\prime}(\bar{X}) = 0$, this expression reduces to:

$$\varphi''_{XX}[\bar{X},\gamma^y(\bar{X})] + \varphi'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})]\gamma^{y\prime\prime}(\bar{X}) = 0$$

Noting that $\varphi''[\bar{X},\gamma^y(\bar{X})] = X''(X_t,\gamma^g)$, solving the expression just above for $\gamma^{y\prime\prime}(\bar{X})$ yields:

$$\gamma^{y\prime\prime}(\bar{X}) = -\frac{\varphi''_{XX}[\bar{X},\gamma^y(\bar{X})]}{\varphi'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})]} = -\frac{X''_{XX}[\bar{X},\gamma^y(\bar{X})]}{X'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})]}$$

I can now establish in which of the three scenarios the function $\gamma^y(X)$ is on. We need to find the signals $X''_{XX}[\bar{X},\gamma^y(\bar{X})]$ and $X'_{\gamma^g}[\bar{X},\gamma^y(\bar{X})]$. Starting with the former, given that Appendix B showed that there always exists a section of the steady state curve that is stable, the hessian of the system in that interval is necessarily negative. That appendix also showed that the Hessian starts with a concave shape, then turning into a convex shape. But since the maximal growth steady state is a bifurcation point, it must be that an inversion point exists in the Hessian in the interval of X $[0, \bar{X})$. This implies that the Hessian is convex at the maximal growth steady state, $\bar{X}$. Thus, $X''_{XX}[\bar{X}, \gamma^y(\bar{X})]$ >0. Appendix B also shows that $X'_{\gamma^g}[\bar{X}, \gamma^y(\bar{X})]$ has the same sign as the system's Jacobian. Given that the Jacobian at the maximal growth steady state is positive, we then necessarily have that $X'_{\gamma^g}[\bar{X}, \gamma^y(\bar{X})]$ is also positive there. Both these findings imply that $\gamma^{y''}(\bar{X})$ above is negative. I thus conclude that $\gamma^y(X)$ attains a maximum at $\bar{X}$. Since this is the economic growth function, $\gamma^y(\bar{X})$ is the maximal growth steady state, where $\bar{X}$ is labeled as X^* in the main text.

I now focus on the stability properties of the maximal growth steady state and characterize the stability of the rest of the steady state curve relative to it. Having a non-hyperbolic steady state with the characteristics displayed so far, where $X'_X[\bar{X}, \gamma^y(\bar{X})] = 1$, $X'_{\gamma^g}[\bar{X}, \gamma^y(\bar{X})] > 0$, $X''_{XX}[\bar{X}, \gamma^y(\bar{X})] > 0$ and $\gamma^{y''}(\bar{X}) < 0$, comes with a very specific property: When $\gamma^y(X)$ attains a maximum at $X = \bar{X}$, picking any value for the policy function γ^g that is below $\gamma^y(\bar{X})$ creates two hyperbolic steady states. The steady state with lower X, denoted X^l in the main text, is stable. The other steady state with higher X, denoted X^h is unstable. On the other hand, picking any value for the policy function γ^g that is above $\gamma^y(\bar{X})$ implies the non-existence of non-trivial steady states. The reason is simple: since the Jacobian of $X(X_t, \gamma^g)$ is locally monotonic, and the Hessian evaluated at the maximal growth steady state is positive, it means that the Jacobian value for any steady state located at $X < \bar{X}$ ($X < \bar{X}$) is lower (higher) than 1. Stability conclusions immediately follow for these two hyperbolic steady states. Given this result, if we consider a neighborhood around the maximal growth steady state, and let that neighborhood get infinitesimally small, one can conclude that the stability of that infinitesimal neighborhood implies that, when $\gamma^g = \gamma^y(\bar{X})$, X will converge towards $\bar{X}$ when $X < \bar{X}$ and X will converge towards infinity when $X > \bar{X}$. Figure 1 in the main text summarizes these results.

Chapter 3

Reform and Backlash to Reform: Longevity Adjustment of the Retirement Age

Reform and Backlash to Reform: Longevity Adjustment of the Retirement Age*

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Abstract: Using an overlapping generations model, *ad modum* Blanchard-Yaari, we analysed the effects of retirement reform on labour supply, and found there is not a one-to-one relationship between an increase in the statutory retirement age and the corresponding increase in labour supply: Rather, we observed a backlash effect, where a reform aimed at broadening the labour supply on the extensive margin, has the unintended effect of decreasing labour supply on the intensive margin. Following a calibration exercise to analyse the robustness of these backlash effects on different pension scheme designs, we found that, under a defined-benefit scheme, the effects on the intensive margin of labour supply are drastically increased, compared to the defined-contribution scheme and the fully-funded scheme.

Keywords: Demographic change, pension reform, retirement.
JEL codes: E71, J26, H55

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1. Introduction

The phenomenon of demographic ageing driven by increased longevity and lower fertility has led to a serious policy and academic debate about the sustainability of pension systems. Many industrialized countries rely on pay-as-you-go (PAYG) pension schemes, where payments to retirees are covered by current contributions by the working population. The demographic structure is a key determinant of the sustainability of such schemes and how generous they can be. In general, there are three ways of adjusting PAYG schemes faced by ageing populations: The contribution rate can be increased; the benefit rate can be decreased; or the retirement age can be increased.

Preserving the living standards of both the retired and the working-age population in an era of demographic ageing presents a very complex challenge. For example, while lower pension benefits would improve the sustainability of the PAYG pension system, they would also redistribute income from retirees to workers. The opposite distributional effect holds for an increase in contributions to the pension system. The point is that any distinct shift in either the benefit or the contribution rate will necessarily imply some degree of redistribution of intergenerational welfare.[1]

An alternative solution to keeping the pension system on a sustainable path in an era of ageing populations is to raise the retirement age. This would broaden the labour supply by having individuals work for a longer period in their lives (the extensive margin), while simultaneously decreasing the number of retirees by the same amount. Such a policy would counter the adverse effects of population ageing on PAYG pension systems, by exogenously decreasing the old-age dependency ratio and thus increasing the number of contributors relative to beneficiaries.[2]

Retirement decisions of individuals are complex, and affected by financial and social factors. While financial incentives undoubtedly matter, the empirical evidence on retirement behaviour shows a large spike in actual retirement at the statutory (or official) retirement age. Furthermore, there is evidence that indicates large and sharp hikes in the actual retirement age in response to changes in the statutory retirement age.

The statutory retirement age can serve as a reference point to which agents anchor their retirement plans. For example, Lumsdaine et al. (1996) finds that 48% of men working at age 64 retire at age 65 in Fortune 500 companies. Rather than being explained by financial or demographic factors, the study suggests that the spike is simply caused by an “age-65, rule of thumb” based on customs or accepted practice. This observation is consistent with the findings by Cribb et al. (2016) who find that changes in actual retirement

1 The issue of intergenerational equity/fairness is indeed complicated and contentious, depending, among other things, on dynamic efficiency concerns and the definition of fairness. See, for example, Howse (2007) for a discussion.

2 A deeper discussion of fairness is warranted here. For example, Queisser and Whitehouse (2006) elaborates on pension design in the context of actuarial fairness and neutrality. We do not pursue this discussion in this paper.

age, when early retirement provisions are changed, are not driven by financial incentives but rather a signal about the appropriate time to retire.

A number of studies related to retirement reform further demonstrate the role of the statutory retirement age as a reference point. Based on Austrian data on reforms of the public pension system between 2000 and 2004, Staubli and Zweimuller (2013) analyse the effects of an increase in official early retirement age on labour market participation. They find that an increase in the early retirement age has significant effects on delaying pension claims.

The labour market effects of the Austrian reforms are also evaluated by Manoli and Weber (2016). They find that the reforms both increased pension-claiming ages and job market exits. They further state that the official early retirement age is potentially an important reference point for retirement decisions. The effects of an increase in the Swiss official retirement age is found to have a strong effect on the actual retirement age by Lalive and Staubli (2015). This effect is also observed in the US, where Mastrobouni (2009) finds a strong reaction to increases in the actual retirement age as a response to a reform passed in 1983 to increase the default retirement age.

This effect of official retirement age on actual retirement age holds even when accounting for the financial incentives agents are subject to when leaving the labour market. This is investigated by Manoli and Weber (2014), who find that financial incentives alone cannot explain retirement decisions; implying that non-financial factors also play an instrumental role in retirement planning. Brown and Laschever (2012) observe a peer effect on retirement timing, implying that individuals tend to retire at the same time as their peers.

The peer effect on retirement decisions is also identified by Chalmers et al. (2008) who found that individuals are more likely to retire when one or more of their co-workers retire: This further highlights that retirement decisions are not solely driven by financial incentives. These anchoring effects, coupled with specific schemes to incentivize older workers to retire, tie the official retirement age tightly to the actual retirement age.

The focus of this paper is on how such exogenous changes in the statutory retirement age interact with the endogenous labour supply decisions of households. Changes in the intensive margin of labour supply have already attracted some attention, with a focus on the tax-like disincentives of pension contributions; see, for example, Fisher and Keuschnigg (2007) and Keuschnigg and Fisher (2011). Weil (2006) has also argued that the distortionary effect of PAYG systems is the most important channel through which ageing will affect output.

Backlash effects, where labour supply decreases on the intensive margin as a response to increases on the extensive margin, have been found in previous theoretical studies: Jensen and Jorgensen (2010) find the numerical existence of these backlash effects under a public DB scheme when the retirement age increases in response to decreased fertility rates. They find that a more-than-proportional increase to the retirement age is necessary when fertility rates decrease. Börsch-Supan and Ludwig (2010, 2013) numerically find similar backlash effects for public DB and private FF schemes. In line with our results, they also find that backlash effects are stronger in DB schemes as opposed to only private savings. We add to this line of research by, first, showing the analytical existence of these backlash effects regardless of calibration, and second, numerically show the size of these backlash effects under three different pension schemes.

To our best knowledge, the empirical literature on backlash effect is very scarce. This could be (partly) due to the rigidity of the labour market on the intensive margin. Typically, workers face a choice between working full time or not at all. There is, however, significant literature on the effects of taxation on the labour supply, which relates indirectly to the themes addressed in this paper. For example, Meghir and Phillips (2008), review the empirical literature and perform their own empirical analysis to demonstrate the effects of taxation on labour supply on the intensive and extensive margin. Their main conclusion is that incentives matter and that different groups in society react to taxation differently. Women, and especially women with young children, respond substantially to tax and benefits incentives on the intensive margin. However, hours worked for males (intensive margin) is irresponsive to incentives, while participation (extensive margin) is highly responsive.

When designing retirement reforms, our basic insight is that a higher statutory retirement age can discourage workers from working, leading to a drop in hours worked as an endogenous mechanism. Thus, retirement reform, implemented through increases in labour supply on the extensive margin, might come hand-in-hand with a backlash effect on the intensive margin. Identifying the existence of these backlash effects, and their potential magnitude, is the key objective of this paper.

Our analytical framework for demonstrating these effects is a simple lifecycle model over consumption-leisure choices at the steady state level.[3] In addition, we check to what extent the sensitivity of these backlash effects depends on the design of the pension system. Specifically, we distinguish three alternative schemes: a public PAYG scheme, either of the defined-benefit (DB) or the defined-contributions (DC) version, and a private, fully funded (FF) scheme. It turns out that the backlash effect critically depends on the pension system in operation. The intended positive effect on the labour supply is most successfully obtained with a public DC scheme in place, followed by a private FF scheme, whereas the public DB scheme displays the strongest backlash effects.

The roadmap of the paper is as follows: In section 2 we explain the basic assumptions and build the model; Section 3 elaborates on the implications of demographic change and shows the existence of the backlash effects following pension reform; Section 4 features a calibration exercise to quantify the

3 Our focus on steady state impacts should be regarded as a starting point. In later work we plan to include transitional dynamics which, admittedly, may be even more important.

implications of longevity shocks and retirement reform under different pension regimes; Section 5 provides an additional robustness analysis, providing further perspective on the numerical results for different calibrations; Section 6 concludes.

2. The model

2.1. Basic assumptions

In order to analyse the effects of an increase in the retirement age on individual labour supply, we base the analysis on the overlapping generations model in line with Yaari (1965) and Blanchard (1985). We consider a small open economy and incorporate endogenous labour supply decisions. Since the analysis is performed only at steady state, the small open economy is assumed to start at some point in the past to the point where it is at steady state at time $t = 0$. Hence, the analysis is carried out via the age of a generation which enters the labour market at $t = 0$.

The economy is populated by agents who face a stochastic risk of death. This *hazard rate*, or the instantaneous probability of death, is age-independent. We assume a fixed population size, resulting in the number of agents "born" into the labour market being equal to how many die at each moment in time. The hazard rate can be manipulated to simulate longevity, since a lower hazard rate implies that agents live longer.[4] Agents save through actuarial notes, which include a component of life insurance that pays a constant amount until the agent's death, thus mitigating the loss of utility sustained from the uncertainty of death.

Working agents can choose to spend a fraction of their time endowment working, earning a wage at rate w. An individuals' productivity is uniform and the wage is therefore assumed to be constant both between individuals and throughout the lifecycle. However, the labour supply decision of working agents varies with age. A mandatory retirement age is also implemented, labelled g, where agents are forced to leave the labour market.

We abstract from welfare impacts that this mandatory retirement age may have, since our analysis is not dependent on social welfare considerations. We simply focus on the labour supply effects of shifting that retirement age under different pension regimes. The aim of the analysis is to shed light on the labour market response on the intensive margin from changes on the extensive margin. The analysis therefore considers policy reform in a narrower perspective, evaluating the actual performance of the reform in question versus its original intent, a key consideration. Introduction of a welfare analysis of different pension schemes and their effect on the intensive (and extensive) margin labour supply is interesting, but in order to demonstrate

4 Having a fixed population size is compatible with our research objective, since what matters is that we simulate a worker/retiree composition shift. A change in the hazard rate with fixed population size provides us exactly that, a decrease in fertility and an equal decrease in mortality. We acknowledge that it would be interesting to see individual results for both reductions in fertility and mortality, but we leave that choice for future work and use the simpler setting here.

clearly the labour supply effects of changes to the statutory retirement age, these considerations are left to future work.

2.2. Pension system

The introduction of a pension system in the style of Nielsen (1994) allows us to extend the analysis to include judgements about the extent to which different types of pension systems mitigate or amplify the adverse effects on individual labour supply caused by pension reforms.

Since the mortality rate, β, is age-independent, the fraction of the population that is retired can be expressed as:

$$\int_g^\infty \beta e^{-\beta\tau} d\tau = e^{-\beta g} \tag{1}$$

Hence, the fraction of working agents is $(1 - e^{-\beta g})$. The pension system introduced consists of contributions denoted by k and paid by workers. The collected contributions are transferred as pension benefits, denoted by b, to the retired population. These contributions act as lump-sum “taxes” on those who have not reached the retirement age. The pension system is a balanced budget period-by-period, meaning that it must always hold that:

$$\left(1 - e^{-\beta g}\right)k = e^{-\beta g} b \tag{2}$$

This allows us to analyse three different pension schemes: First, a DC scheme where b is a function of k. Second, a DB scheme, where k is a function of b. Third, an FF scheme, where $b = k = 0$. Most pension schemes are either DC or FF, here we also analyse DB. In this setting (DB and DC) the agents are forced to contribute the same amount to the pension scheme regardless of how much they work.

For an agent born at t=0 with age a, the discounted stream of future transfers from the pension system is denoted by $p(a)$ and for a working agent is defined as:

$$p(a) = \int_a^g -ke^{(r+\beta)(a-\tau)} d\tau + \int_g^\infty be^{(r+\beta)(a-\tau)} d\tau =$$
$$= \begin{cases} \dfrac{\left(-1 + e^{\beta g + (r+\beta)(a-g)}\right)}{r+\beta} k \; for \; DC \\ \dfrac{\left(-1 + e^{\beta g + (r+\beta)(a-g)}\right)}{(-1 + e^{\beta g})(r+\beta)} b \; for \; DB \\ 0 \; for \; FF \end{cases} \tag{3}$$

The first integral represents the pension system transfer for working agents until retirement. The latter integral represents pension benefits that agents receive after retirement. In the case of a retired agent, the first integral equals zero and the pension system transfers are found using only the second integral.

For both the DC and DB cases, note that the value of the pension system at labour market entry (age 0 in the model) is always negative. This is due to the static productivity and population size, which makes the implicit return on unfunded pension systems zero. Combined with positive returns on investment for the private savings market, the pension contributions are worth less than an equal amount of savings.[5]

With the existence of a retirement age, there is a moment in an individual's life where the present value of pension transfers becomes positive, a moment determined when $a = rg/(\beta + r)$. This implies that, with a positive chance of death before retirement, the present value of pension transfers necessarily becomes positive during the working period. How early in the working period of life the present value of the pension system becomes positive is determined by the relative size of the hazard rate to the interest rate.

With a lower chance of dying at any moment (lower β), the present value of pension benefits to an individual, increases relative to the contributions made to the pension system, since the agent's expectation of survival until and throughout retirement is now higher. This puts pressure on the pension system as a whole, which now needs to either increase contributions or decrease benefits (for DB and DC schemes, respectively) to maintain balance. Therefore, at the individual level, a lower β makes $p(a)$ become positive for an agent later on.

We aggregate values for pension transfers at steady state, denoted by P. That is, the present value of all future pension transfers of all agents currently alive in the economy, if individual valuations of the pension system, $p(a)$, remain stable over time. This yields:

$$P = \beta \int_0^\infty e^{-\beta a} p(a) da = \begin{cases} \dfrac{\beta(1 - e^{-rg})}{r(r + \beta)} k \; for \; DC \\ \dfrac{\beta(e^{-rg} - 1)}{r(1 - e^{\beta g})(r + \beta)} b \; for \; DB \end{cases} \tag{4}$$

We see that the present value of the aggregated stream of future pension transfers of those agents currently alive is dependent on the pension system type, the size of the contributions (benefits) for a DC (DB) scheme, the demographic structure (through the hazard rate, β) and the interest rate, r. Note that this function is not negative, even though the pension stream for agents at labour-market entry is negative. This is a result of the fact that the present value of pension transfers is never negative during retirement, meaning that $P > 0$ when $\beta > 0$ and $P = 0$ when $\beta \to 0$.

5 Our results are thus within the context of a dynamically efficient economy, where long-run growth, as dictated by population growth, is smaller than the interest rate. Abel et al. (1989) and, more recently, Luo et al. (2018) find dynamic efficiency to be the case for the US and most other developed nations.

2.3. Production and wages

The production side of the economy is now specified. The aggregate production function is given by $Y = \omega L$ where ω is exogenous and potentially time varying and represents the economy's technological level. L represents aggregate labour supply. Perfect markets are assumed, which implies that wages are also exogenous and given by $w = \omega$. This is due to the marginal productivity of labour being equal to the producer's cost of employing labour. The labour supply and technology level determine the economy's output.

With wages set exogenously, we can determine the potential income over the life cycle of the agent. The present value of *earnings potential*, or the potential future wages of an individual aged a working in all his/her available time, where the time allowance is normalized to one, is defined as:

$$h(a) = \int_a^g we^{(r+\beta)(a-\tau)} d\tau = \frac{(1 - e^{(r+\beta)(a-g)})}{(r+\beta)} w \tag{5}$$

If the agent has already retired, the earnings potential is equal to zero. The actual income of the agent is dependent on the labour-leisure choice of the agent, while the potential income is not. Therefore, the potential earnings separate the exogenous part of the agent's income from the endogenous part. Aggregate earnings potential at steady state is then:

$$H = \beta \int_0^g e^{-\beta a} h(a) da = w \frac{\beta}{r+\beta} \left(\frac{1 - e^{-\beta g}}{\beta} - \frac{e^{-\beta g}(1 - e^{-rg})}{r} \right) \tag{6}$$

Aggregate earning potential depends positively on the retirement age. In terms of longevity, its effect on aggregate earnings potential is ambiguous. There are two forces in effect: on one hand, higher longevity increases the chances that workers live for longer, thus increasing total labour supply on the extensive margin; on the other hand, higher longevity decreases the proportion of workers in the population, which has a negative effect on total labour supply. There is no closed-form solution to the total effect, and the result is reverted to the numerical exercise section.

2.4. Consumption and labour choices

The agent derives utility from instantaneous consumption, denoted by c, and instantaneous leisure, denoted by $1 - l$. The instantaneous utility function and discounted stream of lifetime utility at age a are respectively denoted as:

$$\mathrm{u} = \ln \left(c^{\varepsilon}[1 - l]^{1-\varepsilon}\right) \tag{7}$$

$$u(a) = \int_a^{\infty} \varepsilon \ln c(\tau)\, e^{(\rho+\beta)(a-\tau)} + (1-\varepsilon) \ln\big(1 - l(\tau)\big)\, e^{(\rho+\beta)(a-\tau)} d\tau \tag{8}$$

Here, $\rho > 0$ is the discount rate and $0 \leq \varepsilon \leq 1$ dictates the relative weight the agent puts on consumption and leisure respectively. Agents can work until they reach the age g, after which they are forced to retire. Note that as the agent reaches retirement age, $l(\tau)$ is set to zero and the second term reduces to zero.

When faced with a positive risk of death, an agent would be less inclined to save for the future. We thus introduce actuarial notes issued at a *fair rate* (Yaari, 1965) that include a component of life insurance. The purchaser of an actuarial note gets a constant stream of payments until his/her death, and the assets of those who die are redistributed to those currently living. These assets therefore act as savings for individuals with an extra component of life insurance to the purchaser, indicated by $i(a)$. The rate of return on the actuarial note is then $r^A = r + \beta$. We have adopted the small open economy case, so the rate r is exogenous and equals the world interest rate.

Agents in the economy can freely lend and borrow at the international interest rate. Parameter values distinguish whether a nation is a debtor nation, with relatively impatient inhabitants, a creditor nation, with relatively patient inhabitants, or the razor edge case of a perfectly balanced economy. In our setting, there is no intrinsic characteristic that distinguishes foreign and domestic assets, so that, in the context of a small open economy, it can be assumed that all an agent's wealth is held in an unspecified mix of national and foreign assets. Note that the only traded assets for saving purposes are the actuarial notes described above.

The working agent is faced with the budget constraint:

$$\int_a^\infty c(\tau)e^{(r+\beta)(a-\tau)}\,d\tau = i(a) + \int_a^g wl(\tau)e^{(r+\beta)(a-\tau)}d\tau + p(a) \tag{9}$$

$i(a)$ denotes total assets, or stock of actuarial notes, the agent holds at age a. For a retired agent, the integral on the right-hand side reduces to zero. Solving the utility maximization yields a relationship between labour supply and consumption for a working agent:

$$wl(a) = w - \frac{1-\varepsilon}{\varepsilon}c(a) \tag{10}$$

Differentiating the first-order conditions, and acknowledging that an agent is forced to retire at age g, yields the Euler equations:

$$\frac{\dot{c}(a)}{c(a)} = r - \rho \tag{11}$$

$$\begin{cases} \dfrac{-\dot{l}(a)}{[1-l(a)]} = r - \rho \; for \; a < g \\ 0 \; for \; a \geq g \end{cases} \tag{12}$$

This implies that consumption throughout the lifecycle follows the path $c(a+\tau) = c(a)e^{(r-\rho)\tau}$. A similar relationship can be found for the time path of leisure (valid only for when the agent is a worker).

Applying the first-order condition for consumption with the consumption/labour supply relationship, equation (10), and the Euler equation, equation (11), on the budget constraint and integrating, we arrive at a relation for the consumption at each age for the agent:

$$c(a) = \mu[i(a) + p(a) + h(a)] \tag{13}$$

Here, we define $\mu \equiv (\rho + \beta)\left\{\frac{\varepsilon}{1-(1-\varepsilon)e^{(\rho+\beta)(a-g)}}\right\}$ for the working agent, which is a corrective term for consumption and leisure choices that account for the forced retirement age. For a retired agent, the term reduces to $\mu \equiv (\rho + \beta)\left\{\frac{\varepsilon}{1+\varepsilon}\right\}$. For the working agent, between the ages $0 < a < g$, we arrive at the following relation for leisure:

$$1 - l(a) = \frac{(1-\varepsilon)}{\varepsilon w}\mu[i(a) + p(a) + h(a)] \tag{14}$$

Consumption is thus directly related to the time until retirement (also captured in μ), the risk of death and the discount rate. For a working agent, the μ expression always starts at a value lower than 1. Up towards retirement, this expression converges to $(\rho + \beta)$ and stays there throughout retirement, leading to the standard Blanchard-Yaari style relation for consumption.

Instantaneous consumption, $c(a)$, is related to the anticipated stream from the pension system, $p(a)$, and the discounted maximum potential for future wage income, $h(a)$, and the level of current assets, $i(a)$. Since the pension system is actuarially unfair at labour market entry, it exerts negative pressure on consumption until age $a = rg/(\beta + r)$. If an agent survives until then, the expectation of future pension benefits exerts a positive effect on consumption even before the agent retires, when $\beta > 0$. The same conclusion is drawn for leisure choices.

We can aggregate consumption over all households by applying the consumption path based on the Euler equation. At steady state, aggregate consumption, C, and aggregate labour supply, L, become:

$$C = \beta\int_0^\infty c(a)e^{-\beta a}da = \beta c(0)\int_0^\infty e^{(r-\rho-\beta)a}da = \frac{\beta c(0)}{\rho + \beta - r} \tag{15}$$

$$L = \beta\int_0^g l(a)e^{-\beta a}da = \frac{\beta(1-l(0))\left(1 - e^{(r-\beta-\rho)g}\right)}{r - \beta - \rho} + (1 - e^{-\beta g}) \tag{16}$$

2.5. Assets

From the budget constraint and realization about the aggregate pension transfers and earning potential we establish the steady state aggregate stock of actuarial notes, I:

$$I=\beta\int_0^{\infty}e^{-\beta a}\int_a^{\infty}c(\tau)e^{(r+\beta)(a-\tau)}d\tau\,da-\beta\int_0^{\infty}e^{-\beta a}\,p(a)da-\beta\int_0^{g}e^{-\beta a}\int_a^{g}wl(\tau)e^{(r+\beta)(a-\tau)}d\tau\,da$$

By applying the relationship between labour supply and consumption and by plugging in aggregate earnings potential and aggregate future stream of pension income we arrive at:

$$I=\beta\int_0^{\infty}e^{-\beta a}\frac{c(a)}{\rho+\beta}da-P-H+\frac{1-\varepsilon}{\varepsilon}\beta\int_0^{g}e^{-\beta a}\frac{c(a)}{\rho+\beta}\left(e^{(r+\beta)(a-g)}-1\right)da$$

$$\Leftrightarrow I=\frac{C}{\rho+\beta}-P-H+\frac{1-\varepsilon}{\varepsilon}\beta\int_0^{g}e^{-\beta a}\frac{c(a)}{\rho+\beta}\left(e^{(r+\beta)(a-g)}-1\right)da$$

Solving the integral, the steady state aggregate stock of assets can be expressed as:

$$I=\frac{\gamma}{\rho+\beta}C-P-H \tag{17}$$

Here, for convenience, we define $\gamma=1+\frac{1-\varepsilon}{\varepsilon}\left(1-\frac{\rho+\beta}{r}e^{-g(\rho+\beta-r)}+\frac{\rho+\beta-r}{r}e^{-g(\rho+\beta)}\right)>1$

Note, that when we abolish the retirement age by making g approach infinity, the γ expression reduces to $1/\varepsilon$. Adding to this, by abolishing the pension scheme, we arrive at the same expression as in the standard Blanchard-Yaari model, i.e., $C=\varepsilon(\rho+\beta)(I+H)$.

3. Impact of population ageing and pension reform

3.1. Effects of demographic ageing

The effects of ageing in the form of a decrease in mortality and fertility rates are now analysed for an agent just entering the labour market at the beginning of their economic life ($a=0$)[6]. This analysis is performed, first, in a steady state perspective only, and second, for the razor-edge case, when $r=\rho$, though this assumption is relaxed in the numerical analysis (below). Under this assumption, agents have perfectly flat consumption (and labour supply) smoothing over their life cycle. If we relax this assumption and allow $r>\rho$, agents plan their life cycle consumption and labour supply paths with constant exponential growth when they enter the labour market; The existence of backlash effects is not dependent on this assumption.

Overall, a longevity shock can be broken down into three parts: First, an extensive margin effect on labour supply, where an increase in longevity increases the demographic old-age dependency ratio, if the official retirement age remains the same. This implies that the extension of life spent working is now relatively smaller when compared to the new expected lifetime; Second, an intensive margin effect on labour supply due to an income effect where agents now feel wealthier due to a higher potential to attain work income; Third, another intensive margin effect on labour supply, but now due to changes in the perceived value of

6 Given homogenous agents and zero population growth, the case in point in this section can be generalized to any other generation or point in time (if at steady state).

an unfunded pension system (if in place). When longevity increases, the expected lifetime wealth of the agent will vary according to changes in the value of such an unfunded pension scheme.

An agent entering the labour market chooses his consumption and leisure according to:

$$c(0) = \mu[p(0) + h(0)] \tag{18}$$

$$1 - l(0) = \frac{(1 - \varepsilon)}{\varepsilon w} \mu[p(0) + h(0)] \tag{19}$$

We see that any effects on the mandatory retirement age adjustment factor, μ, the pension value, $p(0)$, and earnings potential, $h(0)$, stemming from longevity shifts in β will yield a level impact to consumption and leisure choices. Hence, these types of shocks work only through income channels, and not through substitution channels between consumption and leisure (see Euler equations 11 and 12 and the absence of longevity factors there).

Starting with the first component of the longevity shock related to labour supply on the extensive margin for new incoming generations with higher longevity, the effect of a drop in the hazard rate on the mandatory retirement age adjustment factor μ is negative. The reason is that a decreased hazard rate makes it more likely that an individual survives into a (now longer) retirement period. With longer life expectancy and a fixed official retirement age, an agent now works for a smaller portion of their lifetime. The downwards adjustment to consumption-leisure during the working period must therefore be larger, compared to a generation of individuals with lower longevity.

Turning to the second component of the longevity shock, in terms of labour supply effects on the intensive margin, the effect of a change in the hazard rate on the present value of earnings potential, $h(a)$, is independent of the pension system type, and it reacts positively to decreases to the hazard rate, β. This makes intuitive sense, since a lower β implies that an agent is more likely to survive throughout their working life. This makes the agent feel wealthier, resulting in a desire for higher consumption and leisure.

The third component of the longevity shock is related to labour supply intensity and the existence of unfunded pension schemes: The individual values of these pension schemes are described by $p(a)$, and they react negatively to a decreasing β for both DC and DB pension systems. This is easily seen by recalling that the value of an unfunded pension system always starts negatively for an agent just entering the labour market, turning positive when the agent reaches the age $a = rg/(\beta + r)$. When longevity increases, the hazard rate, β, decreases. That implies that the age at which the value of the unfunded pension scheme turns positive for the agent is now higher, which makes the value of the pension system even more negative at labour market entry. This has a negative impact on consumption and leisure choices.

Given the different directions of the effect on μ, h and p from a decrease in β, the effect of a mortality/fertility reduction on leisure choices (or conversely, labour supply choices) can only be asserted with a numerical exercise for all pension types (performed below).

3.2. Increase in the retirement age

Since a decrease in the hazard rate exerts negative pressure on the pension system, a natural pension reform would be to simply increase the retirement age so that the portion of working life gets rebalanced. This is the reform type in focus, since it is this type of reform in which backlash effects happen. As before, such a retirement reform is analysed in a steady state context and considers the same three components as described above.

First, increasing the official retirement age is analogous to increasing labour supply at the extensive margin. The impact of such a reform is reflected in the mandatory (or official) retirement age adjustment factor μ, and the impact is negative; Second, at the intensive margin, the present value of earnings potential increases in the retirement age g, as that will increase the amount of time an agent can work; Third, at the intensive margin. But where unfunded pension systems are concerned, the impact of retirement age increase now depends on the type of unfunded pension system in place. In short, pensions of the DC type will see their value decreased and pensions of the DB type will see the opposite.

Basically, when the pension system is of the DC type, an exogenous increase to the retirement age will decrease the present value at birth of the pension system, $p(0)$, since the contributions to the pension system are fixed. Additionally, a longer working life implies a longer schedule of contributions relative to the expected benefits upon retirement. In the DB case, an increase in g has a positive effect on $p(0)$, since the amount of contributions necessary to finance the pension system is now lower.

The overall effect on the intensity of labour supply is determined by the joint effects on both $h(a)$ and $p(a)$. In the DC case, the overall effect is ambiguous since the effect on the value of the pension system is opposite to the effect on earnings potential. In the DB case, the increased value of the pension system, when the retirement age is increased, goes in tandem with the effects on earnings potential, and always dominates the negative extensive margin effects measured in μ. In the FF scheme, we have $p(0) = 0$ and the sole effect of the retirement reform is captured by $h(0)$ and μ. Again, the positive effects on $h(0)$ always dominate μ.

The conclusion is that, in the DB and the FF case, consumption and leisure (labour supply) choices will increase (reduce) when the retirement age is raised for new incoming generations. Therefore, in economies operating under DB and FF pension systems, we get:

$$\frac{\partial c(a)}{\partial g} > 0 \tag{20}$$

$$\frac{\partial\, l(a)}{\partial g} < 0 \tag{21}$$

A retirement reform aimed at increasing the labour supply on the extensive margin will have the unintended effect of lowering the labour supply on the intensive margin at steady state. Thus, with DB and FF pension systems, there will always be backlash effect to reform, where retirement age increases need to be more-than-proportionally relative to increases in longevity, in order for aggregate labour supply to return to its previous level. In these cases, agents who are faced with working for more *years*, choose to work less each *year*. In the DC case, the effect is ambiguous, and the result is reverted to the numerical exercise.

Finally, note that the retirement age does not enter the aggregation of consumption at steady state, C, in any other way than through $c(0)$. We can therefore conclude, that under a DB or an FF pension system, aggregate consumption will necessarily increase as the retirement age increases. But the retirement age makes an extra appearance on aggregate labour supply outside of $l(0)$, as seen in the two exponentials in the equation (16). These extra factors react positively to retirement age increases and ensure that aggregate labour supply will always increase with the raising of the retirement age. The effects of ageing, or a pension reform of increasing the retirement age under DC has ambiguous effects on both aggregate consumption and leisure. Again, we come back to this in the numerical exercise.

4. A numerical exercise

4.1. Calibration baseline

So far, we have laid out the theoretical building blocks of our model. We now turn to a numerical exercise that will allow us to explore the differences and dimensions of ageing and reform of the pension system at steady state in the form of a retirement age increase. The razor-edge assumption of $r = \rho$ will now be relaxed. This numerical exercise will be performed for different pension schemes, whether it is FF, unfunded with DC or unfunded with DB plans. This analysis quantifies the backlash effects of increasing the extensive margin of labour supply. It will also tell us, first, the direction of impact from a change in the hazard rate on leisure choices for all pension types, and second, the direction of the impact on leisure of changing the retirement age in the DC case.

We begin by establishing a baseline calibration: Since 47% of 15-24 year-olds participated in the labour market in 2015 (OECD, 2019), we assume agents will enter the labour market at the age of 20. The average effective retirement age in the OECD was 64 years in 2015 (OECD, 2017), making $g = 64 - 20 = 44$. In terms of the hazard rate, β, we calibrate it to obtain a realistic old-age dependency ratio. The observed dependency ratios in OECD countries in 2015 is 29% (United Nations, 2017), which implies $\beta = 0.034$.[7]

7 Since life expectancy is at every time 1/β for all members of society (aged 20+), an alternative approach would be to calibrate β to match average years left of life of all currently living. This turns out to be roughly 33 years for the European members of the OECD in 2015 (Eurostat, 2018a, 2018b). The baseline calibration is based

The interest rate, r, is assumed to be 1.8%. This is in line with recent calibration of the Blanchard-Yaari model in Benhabib, Bisin and Zhu (2014). The subjective discount rate, ρ, is set at 1.5% which implies that the condition, $r > \rho$ holds. It is thus a case where the inhabitants of the economy are relatively patient compared to the outside world, implying growing consumption and leisure choices over time. Following Heijdra (2003), the relative weight of consumption in leisure is set to $\varepsilon = 0.25$. Furthermore, we normalize the wage rate to one. The average net replacement rate of unfunded PAYG pension schemes in the OECD in 2013[8] was 53.3% of life-time labour income (DICE Database, 2015). This results in the steady state lump-sum benefit rate of 0.15 and a lump-sum contribution rate of 0.044.

The baseline calibration is summarized in the table below:

β	hazard rate	0.034
ρ	discount rate	0.015
r	interest rate	0.018
g	retirement age	44
w	wage rate	1
b	pension benefit rate	0.15
k	pension contribution rate	0.044
ε	weight of consumption in utility	0.25

Given these parameter values we can calculate the relevant aggregate values for the baseline case. We can compare the two cases: with a mandatory DB PAYG pension scheme, and without a mandatory PAYG pension scheme. We do not include the DC case in the table, since the results are identical to the DB ones. These results are summarized in the table below for each pension system:

		FF pension scheme	**PAYG pension scheme**
C	Aggregate consumption	0.254	0.247
L	Aggregate labour supply	0.208	0.223
P	Aggregate pension transfers	0	0.869
H	Aggregate earnings potential	10.47	10.47
I	Aggregate assets	2.564	1.347

The introduction of the pension system implies that aggregate consumption and the aggregate stock of actuarial notes, or assets, drop significantly. This is due to the foregone interest income the agents would have earned if their pension transfers were invested instead. Consumption in the FF case is also higher,

on a realistic dependency ratio, and it implies an average life expectancy of 29.5 years, which resonates with the observed data.

8 We use year 2013 here since data for year 2015 is missing.

since the existence of the unfunded pension system makes agents poorer. The same reasoning justifies a higher labour supply in the DB and DC cases.

4.2. Fully-funded (FF) pension scheme

We now look at the effect of increased longevity on the lifecycle path of leisure. We simulate the demographic shock by decreasing the value of β to 0.025, which represents an increase in the average lifespan of 10 years.[9] The effects of the longevity shock over the lifecycle of an agent is shown in Figure 1 below.

The baseline labour supply is the dashed blue line and the path of labour supply after the shock is the yellow line. Agents who are faced with higher longevity substitute current consumption and leisure for future consumption. This is a combined result of a higher chance of surviving and an expectation of being able to work for longer, and individuals discounting the future less heavily because they are more likely to be alive to enjoy their savings.

From Figure 1 we can see that the effect of increased longevity on leisure. Leisure is subject to a negative shock from increased longevity, such that the negative effects in the retirement age adjustment, factor μ outweigh the increased expectation of earnings potential $h(0)$. But as the expected agent lifespan has increased dramatically for *all age groups*, aggregate consumption has risen (see table below). We see an increase in aggregate consumption of 1.3%. In this case, there is a hike in the labour supply each time of around 4.7% while the agent is working and aggregate labour supply drops by 9.3%. This drop is caused by the higher proportion of individuals in retirement, where labour supply is 0.

We now analyse the effects of countering the life expectancy increase with an increase in the retirement age.[10] We assume that the government attempts to preserve the initial demographic structure as it was before the longevity shock took place, in that the old-age dependency ratio goes back to its original value. In this case, we choose a retirement age increase of 15 years, making g equal to 59. The implication with this experiment is that, because the new retirement age makes the new old-age dependency ratio equal to the original one before the longevity increase, the proportion of life spent as part of the labour force relative to time spent in retirement remains the same. In this case, changes in aggregates stem solely from individual consumption and labour supply choices, and not aggregate demographic composition effects.

When faced with a hike in the retirement age, the new higher longevity agents anticipate being able to work longer during their lives, making them less likely to survive into retirement. This has the effect of

9 Since we are performing an analysis for fully matured retirement age increase reforms (at steady state), we consider an increase in life expectancy that corresponds to the life expectancy after everybody born in 2015 is, on average, dead. As such, we choose a 10-year increase in life expectancy, since life expectancy will be 90 years from 2015 (United Nations, 2017). The implicit assumption here is that the economy will have converged to the new steady state after longevity shock and reform in one lifetime.

10 Again, the results displayed here are for when the retirement age increase has fully matured, meaning that the economy is at steady state under the new longevity level and under the new retirement age, where everybody in the economy is also under the new retirement age for the entirety of their lives.

lowering labour supply on the intensive margin and increasing consumption at each time, as there is less of a need to save for retirement. The effects are summarized in figure 2.

Figure 1 Life Cycle Path of Labour Supply in a Fully-Funded System: Baseline and Longevity Shock

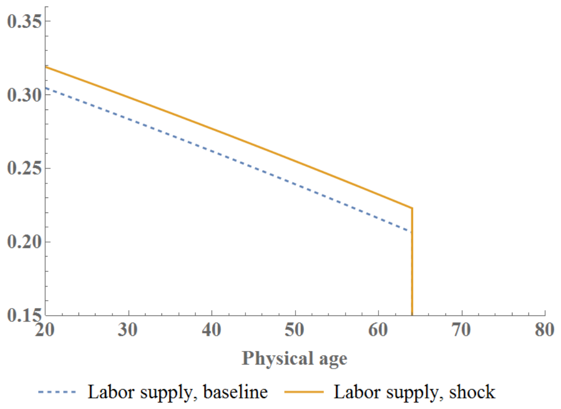
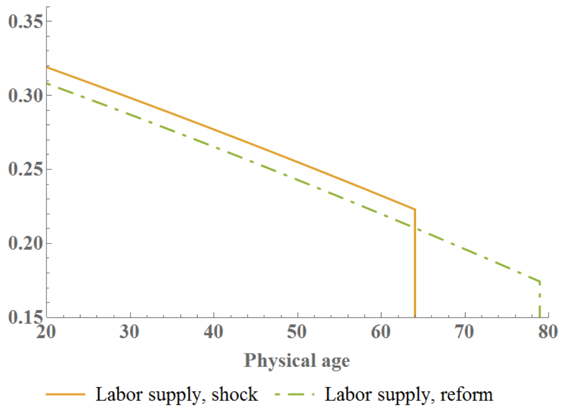


Figure 2 Life Cycle Path of Labour Supply in a Fully-Funded system: Longevity Shock and Retirement Reform



Even though aggregate labour supply increases by 6.5%, it does not return to the original value before the longevity shock. This is a result of decreased labour supply at the intensive margin due to the existence of the backlash effect, which is reflected in a reduction on individual labour supply at each point in time of around 3.4%, a result seen in Figure 2 where the yellow line shifts to the dashed green line when the reform is implemented. The following table summarizes the aggregates:

Variable	Initial scenario	Longevity shock	Retirement age increase
C	0.254	0.257	0.2262
L	0.208	0.189	0.201
P	0	0	0
H	10.47	9.678	13.12
I	2.564	3.822	3.376
Backlash	-	$\mathbf{4.7\%}$	$\mathbf{-3.4\%}$

Again, we see that the effect of a 15-year hike in the retirement rate on aggregate labour supply has a smaller effect than the longevity shock. Showing this result on a smaller scale, in order to arrive at the same labour supply level on the intensive margin, an increase in longevity of 1 year should be followed by an approximately 1.6-year increase in the retirement age, under the FF pension regime.

4.3. Defined-benefits (DB) pension scheme

We now perform the same experiments for an economy with a DB pension scheme: This case has an important consideration: When the size of the retired population changes relative to the working population, the lump-sum contributions of the currently working automatically change. An agent just entering the labour market will certainly experience this change in contributions, but will not necessarily experience the pension benefits, since they are not guaranteed to survive until retirement.

As before, we start by looking at the case of a longevity increase of 15 years. This has the effect of increasing the survival probability of all agents. Those entering the labour market are now more likely to reach retirement, and those retired are more likely to survive longer after retirement. These two factors weigh in the same direction causing the lump-sum pension contributions to increase.

As seen in the FF case, agents are expecting to enjoy a longer retirement, incentivising them to increase their labour supply and decrease their current consumption in order to finance consumption during retirement. The existence of the DB pension scheme introduces a new channel of effects, where the increase in the mandatory lump-sum pension contribution due to demographic ageing has the effect of making the agents poorer, which further drives them to decrease consumption and leisure.

Combining both channels, we see that the negative pension effect dominates the positive earnings potential effect when the hazard rate goes down. Hence, if a longevity shock is allowed to fully mature in the economy, the intensive margin of labour supply would see an increase of 8.7% with consumption dropping by 4.1% at each point in time. Aggregate consumption would drop by 0.9%, which is a change in the opposite direction relative to the effects seen with the FF pension scheme. This is due to the reduced present value of the unfunded public pension system at birth, which is not present in the FF case. The lifecycle paths of consumption and leisure for the baseline case and after the longevity shock are displayed in figure 3.

We now perform the same pension reform of increasing the retirement age by 15 years, such that the old-age dependency ratio remains the same as before the longevity shock took place. The effects of an increase in labour supply on the extensive margin has the effect of increasing the relative size of the working population and decreasing the retired population by the same amount. In addition to the effects associated with a shorter retirement, as traced above in the context of figure 2, the retirement reform has the effect of decreasing pension contributions, since the number of retirees receiving pension benefits has shrunk and the number of workers has increased. This causes pension contributions per person to decrease. These two effects work in the same direction of lowering lump-sum contributions.

As a result, the working agents have more disposable income, leading to an increase in both consumption and leisure. The backlash effect here, measured again as a lowering in labour supply at each point in time, is 6.2%. These effects can be seen in figure 4, below. This effect works in the same direction as the effect of a longer working life, but as seen when comparing figures 2 and 4, the observed backlash effects under an unfunded pension scheme are much more pronounced compared to the case without them.

The implication is that the setup of the DB pension system affects the size of the backlash effect. When the retirement age is increased, it rebalances the pension scheme such that its value for individuals is improved. This in turn further stimulates the backlash effect, reducing labour supply on the intensive margin. The following table summarizes the aggregates:

Variable	Initial scenario	Longevity shock	Retirement age increase
C	0.247	0.245	0.254
L	0.223	0.212	0.219
P	0.869	1.310	0.930
H	10.47	9.678	13.12
I	1.347	1.873	1.935
Backlash	–	**8.7%**	**−6.2%**

Figure 3 Life Cycle Path of Labour Supply in a Defined-Benefits Scheme: Baseline and Longevity Shock

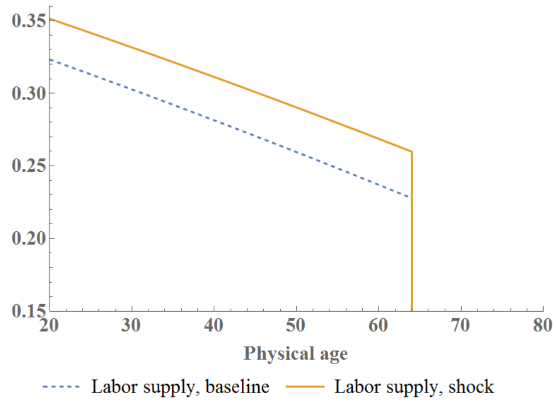
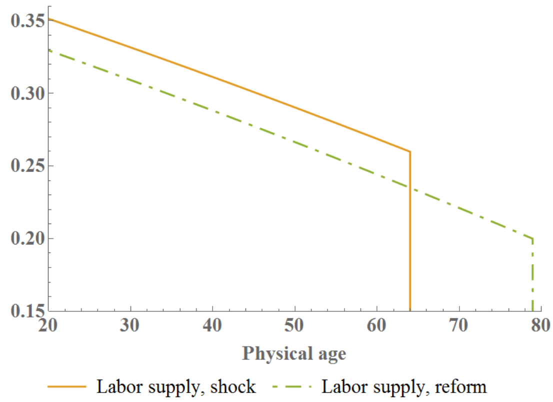


Figure 4 Life Cycle Path of Labour Supply in a Defined-Benefits Scheme: Longevity shock and Retirement Reform



4.4. Defined-contributions (DC) pension scheme

Finally, we perform our experiment in the setting of a DC pension scheme. The effects on the labour supply and consumption decisions of agents when faced with demographic ageing or pension reform have a different scale than in the DB scheme. As longevity increases the proportional size of the retired population, pension benefits drop as a result. This has the impact of decreasing the present value of future pension transfers at labour market entry. The agent is more likely to reach retirement, but when they do, pension benefits are drastically reduced. Once again, the negative effect of the reduced value of life-time pension transfers and the negative μ effect both dominate the positive effect of increased earnings potential when the hazard rate drops. This causes agents to increase their labour supply by approximately the same amount as in the FF case, as well as saving more through private saving schemes. This results in the ability to have a higher lifetime consumption. This is demonstrated in the table and figure 5 below.

Figure 5 Life Cycle Path of Labour Supply in a Defined-Contributions Scheme: Baseline and Longevity Shock

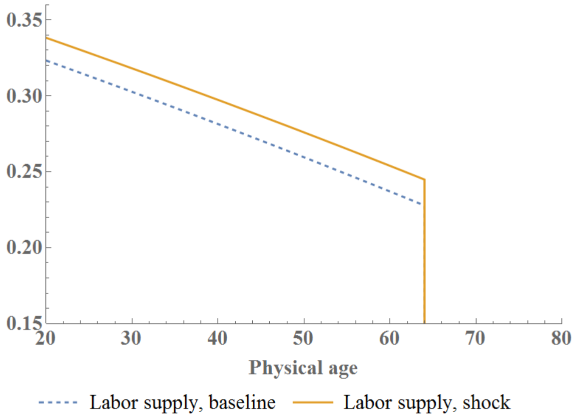
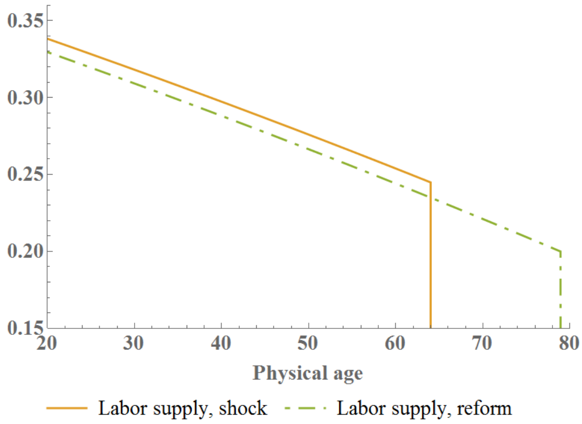


Figure 6 Life Cycle Path of Labour Supply in a Defined-Contributions Scheme: Longevity Shock and Retirement Reform



When faced with an increase in the retirement age of 15 years, we see a backlash effect of reduced labour supply at each point in time of 2.6%. This effect is far smaller than in the DB case, and even smaller than in the FF case. The reason is that the increase to the retirement age now reduces the value of the pension system to the individual, which softens the backlash effect.[40] Once again, this demonstrates how important the existence and type of unfunded pension schemes are for the behaviour of the intensive margin of labour supply. Whether retirement age increases have negative or positive effects on the value of the pension system to an individual just entering the labour market matters in explaining the sizes of backlash effects. This explains why the aggregates of longevity shock and retirement age reform are the same in the DB and DC cases, but the observed backlash effects are nonetheless different. The following table summarizes the aggregates for the DC case:

Variable	**Initial scenario**	**Longevity shock**	**Retirement age increase**
C	0.247	0.250	0.254
L	0.223	0.202	0.219
P	0.869	0.778	0.930
H	10.47	9.678	13.12
I	1.347	2.665	1.935
Backlash	$-$	$\mathbf{4.6\%}$	$\mathbf{-2.6\%}$

5. Robustness analysis and further discussion

We now perform a robustness check on the sensitivity of the observed baseline backlash effects to certain key parameters. We continue with a discussion on how the lack of certain modelling devices can influence our results. These modelling devices are endogenous retirement decisions, realistic mortality and productivity profiles, and generalising the CRRA utility function to assume an

40 This effect is dependent on the size of the contributions, but here we begin with the same baseline for the DC as in the DB case.

intertemporal elasticity of substitution different than 1. The numerical experiments we perform in the following robustness analysis, and the associated parameter changes, are summarized in the following table:

<table>
<tr><th colspan="2">Experiment</th><th>Old parameter(s)</th><th>New parameter(s)</th></tr>
<tr><td colspan="2">Increased importance of consumption in utility</td><td>$\varepsilon = 0.25$</td><td>$\varepsilon = 0.5$</td></tr>
<tr><td rowspan="2">Debtor nation scenario</td><td>Stronger discounting</td><td>$\rho = 0.015$</td><td>$\rho = 0.02$</td></tr>
<tr><td>Lower interest rates</td><td>$r = 0.018$</td><td>$r = 0.012$</td></tr>
<tr><td colspan="2">Higher interest rate</td><td>$r = 0.018$</td><td>$r = 0.024$</td></tr>
<tr><td colspan="2">Higher pension replacement ratio</td><td>53.3%</td><td>100%</td></tr>
</table>

5.1. Decreased importance of leisure in utility

We simulate an economy with a much lower preference towards leisure as opposed to consumption. This makes individuals less sensitive to leisure choices, serving as a check into how important this sensitivity is to the size of the baseline backlash results. We do so by doubling the baseline consumption preference parameter ε to 0.5. The backlash effects, measured by the decrease in labour supply at the intensive margin as a response to an increased retirement age, are now summarized for each pension system type and put in context of the baseline results:

Backlash effect:
Baseline and lower preference for leisure

Pension system	Baseline case	Robustness case
FF	-3.4%	-3.0%
DB	-6.2%	-4.8%
DC	-2.6%	-2.4%

The robustness checks confirm that the baseline results are not heavily dependent on the consumption preference parameter ε for any of the pension types, especially considering the extreme experiment performed here of doubling the parameter. When the preference towards consumption is increased, the backlash effects are diminished, since the individual now values consumption more compared to the baseline case, making the individual want to work relatively more when the retirement age is increased to finance a higher desire for consumption. The differences between pension systems are still preserved as in the baseline case for the same reasons. The case with the strongest reduction is still the DB case, where the backlash weakened by 1.4 pp. Both the FF and DC pension types saw decreases of less than half a percentage point, with the DC system seeing the smallest backlash effect of them all.

5.2. Debtor nation scenario

We now consider a debtor nation scenario under two different experiments. For the first debtor nation scenario, we perform an increase to the subjective discount rate of one third, making $\rho = 0.02$. This makes the discount rate relatively bigger than the interest rate, making individuals choose decreasing paths for consumption and leisure (see equations 11 and 12). The economy, in turn, becomes one composed of impatient individuals, making it a debtor nation. The results are now summarized:

Backlash effect:
Baseline and higher time preference

Pension system	Baseline case	Robustness case
FF	-3.4%	-7.5%
DB	-6.2%	-10.6%
DC	-2.6%	-5.9%

The backlash effects get quite exacerbated by the increase in ρ, making the backlash effects higher in the order of 3-4 pp. By increasing how much individuals discount the future, consumption-leisure choices become stronger during the working period since μ increases with ρ. Note that the present value of earnings potential and pensions are unaffected by changes in ρ. The result is an increase in the sensitivity of labour-leisure choices to changes in the retirement age.

The second debtor nation scenario is performed by reducing the interest rate, r by one third, making it equal to 0.012. Again, the nation becomes an external debtor, with relatively fewer patient individuals. The backlash results are summarized in the following table:

Backlash effect:
Baseline and lower interest rate

Pension system	Baseline case	Robustness case
FF	-3.4%	-10.2%
DB	-6.2%	-12.4%
DC	-2.6%	-8.5%

We see that backlash effects are more sensitive to interest rate decreases than to increased subjective discount rates. Decreasing the interest rate by one third, doubles or even triples the observed backlash effects. The mechanism of action is different compared to changes to the subjective discount rate. In this case, when the interest rate is reduced, the value of μ remains the same, but the present value of earnings potential and pensions increase. Now, when the retirement age is raised, it produces stronger value changes for earnings potential and pension wealth compared to when the interest rate was higher.

Individuals feel even more wealthy when the retirement age is increased, exacerbating the backlash effect when compared to the baseline case.

5.3. Higher interest rate

We next simulate an economy that has an interest rate one third higher than in the baseline case, making the economy a stronger creditor nation, with individuals saving more aggressively in the beginning of their lives, choosing steeper consumption-leisure paths when compared to the baseline case. The backlash results are:

Backlash effect:
Baseline and higher interest rate

Pension system	Baseline case	Robustness case
FF	-3.4%	-0.2%
DB	-6.2%	-2.8%
DC	-2.6%	0.2%

Overall, the more relatively patient individuals are, the weaker backlash effects become. The reason is opposite to the case studied just above, where the value of earnings potential and pension wealth drops with a higher interest rate, which attenuate the backlash effects associated with a higher retirement age.

Interestingly, the backlash effect vanishes in the DC case, where the retirement age increase raises individual labour supply on the intensive margin by 0.2%. Backlash effects are nonetheless still present in the FF and DB cases. The takeaway is that, if the differential between interest rates and subjective discount rates are large enough, backlash effects tend to be mitigated or disappear altogether. Though it is important to underline that large positive differentials between interest rates and subjective discount rates are unlikely to be observed in reality, so that our results remain robust for the average OECD economy.

5.4. Stronger unfunded pensions

We also consider a case where the economy has a 100% unfunded pension replacement ratio, as opposed to the baseline scenario of 53.3%. Since the FF situation represents the case of 0% replacement rate, it is interesting to reveal the backlash effects in the opposite situation as well:

Backlash effect:
Baseline and stronger unfunded pensions

Pension system	Baseline case	Robustness case
DB	-6.2%	-8.5%
DC	-2.6%	-1.8%

Here, the increased volume of inter-generational transfers introduces stronger impacts of the individual value of unfunded pensions $p(a)$ in how wealthy an individual feels. As asserted in section 2, when a DB system is in place, a later retirement age increases the value of the pension system to the individual. When the replacement ratio is 100%, this effect is amplified, so the backlash effects are now stronger compared to the baseline case.

If a DC pension system is in place, a later retirement age reduces the value the pension transfers at birth to the agent. That means that, with a higher pension replacement ratio, the agent feels even poorer when compared to the baseline scenario when the retirement age is increased. This results in diminished backlash effects, as seen in the table above. Note that changes to the pension replacement ratio result in opposite impacts to the backlash effects. Countries with full public pension replacement of the DB type are more vulnerable to backlash effects than countries with DC systems.

5.5. Generalized CRRA utility function

When considering changing the intertemporal elasticity of substitution from 1 (the log case) to a more inelastic setting, it is important to consider that the model as it is, has no substitution effect where longevity and the retirement age are concerned, meaning that the relative “price” of consumption and leisure remains static when an individual now lives for longer or has a longer period to work in. As such, changing the intertemporal elasticity should have no relevant impact on the income effects we measure.

We take the position that the lack of substitution effects, with respect to longevity and the retirement age is not a completely unrealistic setting. After all, when thinking about one’s lifetime, longevity and retirement issues are more closely related to financially planning the support of a longer retirement than to one’s wish to enjoy more or less leisure due to a different length of lifecycle. The implication then, is that it is on the *level* of consumption-leisure that the main focus should be placed - and not its intertemporal allocation rate - when considering shifts in longevity and retirement age.

5.6. Realistic mortality profile

The mortality profile featured in the model is a classic Blanchard-Yaari style perpetual youth model where agents face the same mortality risk at all ages. We apply a mandatory retirement age, making agents’ proximity to retirement age an important aspect when making consumption-savings decisions.

In order to portray a fair picture of the size of the backlash effect, we calibrated the baseline to fit a dependency ratio that is close to those observed in reality, see more details in section 4.

This does however imply that the age structure of society, both before and after the retirement age is not realistic. Agents face a higher probability of dying before retirement, and once retired they live longer than observed. The existence of actuarial notes mitigates the mortality risk for savings, but pension transfers are not subject to any such mitigation. These factors affect the absolute size of the backlash effect, but should not change our main results: Namely, the existence of the backlash effect and the fact that the set-up of the pension scheme matters.

5.7. Age-dependent wage profiles

The model features a flat wage-profile, implying that agents' productivity is constant over their life cycle. The fact is that young and old workers are not perfect substitutes in reality. Younger workers might adapt better to new production technologies while older workers have more experience. By applying age dependent wages, the labour supply over the lifecycle would feature a hump shape where agents work more during the ages where they are relatively more productive, see equation (10). This in turn would have an interesting impact on the backlash effect. As labour supply is increased on the extensive margin, those extra years available to the agent are now less productive compared to the case with constant wages.

This implies two things; first, that the size of the backlash effect would be smaller, since their lifetime earnings potential is increased to a lesser extent; Second, the backlash effect would, in absolute terms, be more pronounced at specific ages due to the hump shape of the labour supply. This extension would then invite analysis on how different age groups respond to retirement reform.

5.8. Endogenous retirement decisions

Agents in this model are forced out of the labour force at a specific age, at which they might want to work for more years. As noted in the introduction, the connection between the official and actual retirement age is complex and explained by both social and financial factors. An interesting extension to this model would be to allow for endogenous retirement. Analysing both the intensive and extensive margin of retirement decisions requires a utility function featuring a distinctive mechanism for intensive and extensive labour supply decisions. This could, for example, be done by applying disutility of work similar to the one currently featured in the model, coupled with extra utility from being retired (see for example: Kalemli-Ozcan and Weil (2010)). The lifetime utility function could be re-written as:

$$u(a) = \int_0^\infty \varepsilon \ln\big(c(a)\big)\, e^{-a(\rho+\beta)} da + \int_0^g \Big(\ln\big(1 - l(a)\big)\,(1-\varepsilon) - \gamma\Big)\, e^{-a(\rho+\beta)} da$$

Where γ is disutility from being a part of the labour force. The agent would then maximize his lifetime utility based on his consumption profile $c(a)$, intensive labour supply $l(a)$ and time of retirement g. This would surely allow for endogenous retirement on both intensive and extensive margins, but also invite a host of questions about the set-up of the pension scheme - and how the pension age would be tied to the retirement age. In order to provide a realistic response to the actual retirement age when the official retirement age increases, as traced in the introduction, some explicit link between the two has to be established. We leave this to future work.

5.9. Proportional pension contributions

Applying pension contributions proportional to the labour income of the agents is another interesting extension. This would establish a link between intensive margin labour supply decisions of the agents and the pension scheme. These effects would fundamentally change the dynamics of the model: First, equation (10) would change to include proportional contributions which would act like taxation; Second, the pension system balance would include aggregate labour supply in addition to the demographic parameters.

The representative agent would therefore make labour supply decisions, which would in turn affect the balance of the pension scheme, implying either a change in the contribution or benefits rates, which would then impact the agent's labour supply choice. This dynamic would lead to changes in the Euler equation of labour supply, affecting the shape of the intensive margin of labour supply.

Having an impact on the intensive margin of labour supply, proportional pension contributions would deepen the sophistication of the analysis of the backlash effect. The ability to separate the backlash effect into income and substitution effects is one possible avenue this extension could provide the analysis with. We leave this to future work.

5.10. Decreasing returns to labour

In the analysis, constant returns to scale of labour supply was assumed. Production, Y, is assumed to be a function of labour supply, L, and the economy's technology level, ω, which was assumed to be exogenous and potentially time varying. Perfect markets imply that wages are set equal to the technology level, $w = \omega$. This set-up of the labour market implies that wages do not impact an agent's decisions on labour market participation.

By introducing decreasing (or increasing) returns to scale, the agent's wages would react to changes in the aggregate labour supply, both on the intensive and extensive margins. This implies that in a DRS setting, the backlash effect would become greater, since as labour supply drops on the intensive margin, the wages per hour worked would increase, causing agents less financial loss from reducing their working hours. The opposite would hold true in the increasing returns to scale case.

6. Concluding remarks

We specify a model of leisure-consumption choices that confirms our insights about retirement reform. This is a reform that aims at increasing the labour supply on the extensive margin, and how this will have the unintended effect of decreasing labour supply on the intensive margin. This was done in a steady state setting of dynamic efficiency with an unfunded pension system, where backlash effects were shown to exist in an analytical setting for DB and FF pension schemes.

We have shown that the type of mandatory pension scheme matters a great deal in the context of either increased longevity or an increase in the retirement age. The calibration exercises demonstrated how important the design of the pension system is when implementing retirement age increases. A feature as simple as DC and DB can make a difference to the decision-making process of agents when choosing the intensity of labour supply throughout their lives. Basically, whoever pays the "longevity bill" matters for how agents react to retirement age increases. In this case, the existence and type of pension scheme can either reduce or increase the unintended effects of increased extensive labour supply on intensive labour supply - or as we call it, the backlash effect.

Through our calibration exercise, we found that the effects under a FF pension regime were in line with our insights about labour intensity. In the FF case, changes in labour intensity were driven by anticipated length of the working life and retirement. By adding a pension system, where pension benefits were contemporaneously covered by contributions by the working population, another layer of complexity arose. The effects of retirement reform on labour supply increased substantially in the DB case compared to the FF case, leading to a relatively large drop in labour supply on the intensive margin as a result of an increase on the extensive margin. But in the case of DC, the opposite situation occurred, where the *backlash* effects of pension reform were smaller than in the FF case. These differences indeed stem from the existence of an unfunded pension scheme, its value to individuals, and how that value changes when the retirement age is raised.

We confirmed the existence of backlash effects under different calibration exercises. Our results are generally robust to changes in leisure preferences. In terms of the interplay between interest rates and subjective discount rates, a key relationship for our results, we conclude that debtor nations, where individuals have stronger subjective discount rates compared to the world interest rate, have more pronounced backlash effects. A one third increase (decrease) to the subjective discount rate (interest rate) yields two to three times stronger backlash effects. In the case of an even more relative patient economy, making it more of a creditor nation compared to the baseline, the backlash effects tend to be mitigated.

The lesson to take from this study is that pension rebalancing reforms aimed at either indexing retirement age increases in a one-for-one manner with respect to increases in life expectancy or increasing the retirement age to preserve the same old-age dependency ratio, may fall short from accomplishing their goals at steady state. Increases to the retirement age must be more-than-proportional with respect to increases in longevity and need to account for the type of pension system in place.

For countries running DB unfunded schemes, increases to the retirement age should most likely be much stronger relative to expected longevity increases, when compared to other pension system types. Complimentary pension reforms should therefore be explored. The situation looks much more benign for DC pension systems, where retirement age increases seem to create relatively weak backlash effects.

This pension system type is much more robust to retirement age increases where labour supply responses are concerned, having a lesser need of complementary pension reform.

Given the results presented here, a further look into these backlash results is warranted, where other features should be considered. Future work should consider features such as endogenous retirement decisions, realistic mortality and wage profiles and proportional taxation. We acknowledge that the addition of these features would provide a more accurate measurement of the backlash effects. The question of economic transition to the new steady state is also a relevant one, which this paper does not answer. Given the nature of inter-generational transfers that an unfunded pension system implies, the potential for divergence upon a demographic shock, plus retirement age reform should be investigated.

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Chapter 4

The Residual Group in Denmark: Who is at risk?

The Residual Group in Denmark: Who is at risk?*

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Abstract

For the past decades, Denmark has fostered the funded component of its pension system via quasi-mandatory labor-market pension schemes alongside the strengthening of old-age public insurance in the form of means-tested unfunded supplementary benefits. Within this institutional setting, which requires higher individual responsibility to save for retirement, the group of individuals that are eligible to the full public pension supplement (*folkepensionstillæg*) is analyzed, a group termed the "residual group" in Denmark. This pension supplement is means-tested, making this group the collection of individuals with the lowest pension savings in Denmark and that can become a fiscal pressure to the public pillar of the Danish pension system. We investigate the factors associated with higher or lower risk of belonging to this group. We then quantify that risk for cohorts that will reach retirement in the next decade and provide a framework that can be used to measure what level of means-tested pension expenditures the government could expect to pay out in the future.

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1. Introduction

In a world where human longevity is increasing, the issue of properly saving for retirement is one of great importance. In this context, pension systems across the globe work towards helping people reach retirement with enough resources to enjoy their golden years. The Danish pension system has been successful along these lines as evidenced by cross-country evaluations and comparisons, such as the Melbourne Mercer Global Pension Index (Mercer, 2018). The Danish pension system has consistently been placed 1st since it became part of the index, only to lose the top score to the Dutch pension system in 2018.

Part of the Danish success has to do with the implementation of the 2nd pillar of quasi-mandatory, employer-made pension contributions since the early 1990's. This reform has been successful in providing significant income at retirement for individuals. In particular, this success can be explained by the automatic nature of employer-made contributions. Estimates in Chetty et al. (2014) point to a proportion of "passive" savers in Denmark in the order of 85% of the population, meaning that being automatically enrolled to contribute does not necessarily imply a crowd-out in otherwise voluntarily-made contributions to most of the Danish population. Overall, the Danish pension system has been able to achieve low poverty among the retired (see OECD 2017a, 2017b, 2019), as well as comparatively good replacement rates across the income distribution.

Nonetheless, there exists a group of individuals that are characterized by low retirement savings, to the point that they become eligible for full pension supplements and therefore fully dependent on the public pillar of the Danish pension system. This group of individuals has been labeled "the residual group" (the literal translation of *restgruppen* in Danish). In an ideal insurance scheme, these individuals would be the ones affected by exogenous adverse shocks, like rare diseases, which translate into an inability to save for retirement. However, such poverty insurance can also induce free-riding behavior or moral hazard issues, where people with the capacity to save don't do it due to the saliency of pension supplements in the public pillar.

In this study we propose a framework to analyze how large the residual group is in Denmark and what are the personal characteristics that affect the probability of being part of that group. This approach is based on assessing the probability of individuals becoming dependent on pension supplements at retirement, where individual characteristics help us understand what types of individuals are at higher risk. We focus only on absolute measurements of low savings, as opposed to relative measurements that make use of distributional moments. Our definition of low saving in this study is related to saving behavior throughout the life cycle that translates into being entitled to *full* public pension supplement (*folkepensionstillaeg* in Danish) at retirement.

This definition has a governmental perspective, since the benchmark for low savings is becoming fully dependent on the state pension system. An advantage of this definition is that the assignment of this pension supplement to an individual is determined objectively by law according specific income-limit rules. This enables a precise definition of absolute thresholds of what pension contributions/wealth to aim for such that an individual does not become fully dependent from the government at retirement.[1] This concrete threshold for being characterized as a "low-saver" (or person with low savings) allows for techniques that can be employed to clearly estimate the size of this group and its characteristics in a perspective of interest to policymakers.

Previous studies have focused on measurements of the residual group that are non-stochastic and more descriptive, like SFI (2014) and ATP (2015). Previous approaches have based their analysis on classifying currently working individuals according to some criteria that is conceptualized to be an indicator of whether these individuals *will become* entitled to the full pension supplement (i.e. the group of low savers) upon retirement. Even though this method can reveal distributional characteristics of individuals in the residual group and estimates on its size, it does so independently across each characteristic. For example, it may be that immigrants are overrepresented in the residual group among the currently working; however, this could be driven by income or market attachment, and therefore being an immigrant may not be a significant determinant of belonging to the residual group after controlling for other characteristics.

Moreover, these studies generally rely on classifying low-saving behavior with classifiers that can be sensitive to certain dimensions. For example, defining low-saving individuals as not contributing enough into one's pension accounts will likely yield different measurements depending on the age range. During an individual's peak earnings' age, it is arguably easier to contribute higher amounts when compared to very young or old ages, especially in the presence of (quasi-) mandatory occupational pensions.

Our approach takes into account important individual characteristics, instead of just looking at distributional descriptions of each characteristic based on a univariate classifier estimate. First, we estimate a Probit model to explain the probability of belonging to the residual group at retirement, conditional on a set of relevant individuals' demographic and financial characteristics. We find that females, immigrants, individuals with less than a university degree, and single individuals are the most at risk. In terms of work sector, the sectors most at risk are manufacturing, mining and quarrying, accommodation and food services, construction, wholesale, retail and vehicle repair. In terms of labor status, people most at risk are those receiving cash benefits and the self-employed. In terms of risk group composition, most of those in the high-risk group are individuals with low labor market attachment. A rather high portion of employees

1 A drawback of this definition is that it may include individuals that could save enough to avoid becoming fully dependent from the government. Nonetheless, they fail to save the minimum and become totally dependent of the public pension scheme. The identification of such individuals is a relevant question for governments that requires the modelling of moral hazard as well as a subjective benchmark of what people should consume and save. This is an issue that is beyond the scope of this study and is part of our future research agenda.

are also part of this high-risk group. Setting up a truly mandatory occupational pension system would capture these employees and reduce the size of the residual group, if automatic contribution rates are set high enough.

Second, we predict the probability of belonging to the residual group in cohorts that will retire in the next decade or so. The estimation of a probability distribution of belonging to the residual group allows the identification of characteristics that are associated with higher or lower risk of arriving at retirement with low savings. In our framework, we separate the probability distribution of belonging to the residual group into 3 sections, two extremes representing very low and very high risk, and a middle group whose residual group membership can go either way. We then identify individual characteristics of those most at risk and those that are on the fence. This categorization allows us to identify the type of individuals that will most likely never (always) enter the residual group and understand the structural factors associated with the high likelihood of belonging to this group. Immigrants are overrepresented in the higher risk group. Low education is a relevant determinant of residual group membership as well. Very little individuals with a university degree (short- or long-cycle) are a part of the high-risk group. A very prominent educational group in Denmark, those with vocational education (as well as normal secondary education), compose almost half of the low risk group and roughly 60-65% of the middle risk group. Furthermore, singles are overrepresented in the high-risk group. Among those single, the proportion of divorcees and widowers does not significantly change across risk. Employees are overrepresented at low and middle risk, though there is still a relatively large proportion of employees in the high-risk group. This likely stems from the quasi-mandatory occupational pensions, where these are not completely widespread, even among the group of employees.

The paper is structured as follows: in section 2, a historical perspective is given on the Danish pension system and how it developed over time, as well as key details of how it works nowadays. Section 3 describes the data used in this paper. Section 4 details our methodology, the outcome of interest, how it is identified, and the econometric methods utilized. Section 5 reveals our main results, starting with residual group size estimations using stock-flow threshold approaches. We then explain our risk-based framework, revealing the size of the residual group among different age-cohorts and characteristics associated with a higher risk of membership. Section 6 discusses our results further, contextualizing the predictive power of our framework against the preexisting threshold techniques, and exploring different dimensions of our results. Section 7 concludes.

2. Historical context and institutional setting

2.1. Brief development history of the Danish pension system

We start by providing context to the Danish pension system in terms of how it developed from its inception up to 2016. The following history brief is heavily based in Goul Andersen (2011), which contains an excellent and detailed overview of the Danish pension system.

The current Danish pension system is quite mixed in terms of its multi-pillar system. Making use of the World Bank's 1994 pension classification, the current average Danish individual is/will be dependent on both 1^{st} pillar and 2^{nd} pillar for retirement income. Both pillars have relevance in the old-age income structure, thus making the Danish pension system quite mixed in terms of funded or unfunded financing of pension income. The current institutional setting of the Danish pension system is still relatively young, a system that is still consolidating over time, especially in terms of its 2^{nd} pillar.

The first legal appearance of a pension system in Denmark came with the policy package "*alderomsunderstøttelsen*" in 1891, which focused on supporting those in need at the age of 60 or older, requiring citizenship and residence in Denmark. The pension amount was established via discretionary evaluation and was financed by taxation. Roughly 30% of people were covered around the inception of this pension system (Jørgensen, 2007). This system was then further reformed in 1922 with the policy package "*aldersrenten*". In it, entitlement requirements and benefits were further standardized across the country, moving away from discretionary evaluation, as well as setting the pension age at 65 years of age.

In 1957 a new pension system was implemented with the bill "*folkepension*", the name in use nowadays. A major shift in this new pension system was the universality of entitlement, though the benefits were means-tested, and a pension age of 67 years. Between the years 1956 and 1970, several other changes were introduced, but the last large reform to the pension system came in 1964 where a flat-rate unfunded pension benefit was introduced alongside a smaller means-tested benefit. The layout of the public pension system has not suffered major changes since, though the trajectory of means-testing regulation and benefit sizes has with the spreading of occupational pension schemes in Denmark.

A major change of direction within the Danish pension system came with the proliferation of occupational pension schemes. Such schemes have existed since the beginning of the 20^{th} century (Jørgensen, 2007), but their size at the macro level was relatively small. In 1984, the government declared support, or a wish for widespread occupational pensions. Starting with the Danish Confederation of Trade Unions working towards implementing occupational schemes for its members in the second half of the 80's, the adoption of such schemes expanded rapidly during the 90's. Though older pre-existing schemes could have contribution rates in the range of 12-18%, initial contribution rates in these new 90's schemes were small, being for example 0.9% within many unions (Jørgensen, 2007). Over time, the contribution rates on newer schemes have converged to the rates seen in older schemes.

Given the development of occupational schemes, becoming the *de facto* 2nd pillar in the Danish pension system topology, the role of the 1st pillar morphed as a result. The 1st pillar became much more targeted at alleviating old-age poverty situations, distributing more means-tested benefits relative to the universal flat-rate benefit (Goul Andersen, 2011). Under the "old" pension system, means-tested benefits were small, and most were eligible to it. After a tax reform in 1994, these means-tested benefits became roughly in line with the flat-rate benefit the state gives to every eligible individual. As Goul Andersen (2011) frames it, even though the 1st pillar has become largely means-tested, it "does not mean that the system has become residual". The pension complex must be seen as a whole, where all pension pillars are both complements and substitutes of each other.

It is important to note that these occupational schemes were not born out of legislation, but instead created out of labor market negotiations, with contributions being managed, not by a central institution, but by a privately organized market. This means that employees are usually contractually "forced" into these schemes, leaving self-employed, unemployed or low labor-market attachment-type individuals at risk of not setting up/contributing into pension saving schemes voluntarily. Nonetheless, the proportion of non-zero contributions to funded pension schemes in 2016 (employer-made or voluntarily made) is quite high. Among all individuals aged between 25 and 64 in 2016, 25% had zero pension contributions. But among those in an employer-employee relationship, only 7.6% had zero pension contributions. In terms of accumulated pension assets, these have grown from 81% of GDP in 1993 to 199% of GDP in 2015 (Danish Ministry of Taxation, 2017). This exhibits tremendous growth in the 2nd and 3rd pension pillars. Since the schemes are quasi-mandatory, the question remains if individuals are actively ensuring appropriate saving behavior adjusted for income level, or if generalized passive behavior occurs with, for example, individuals assuming that the state is ensuring an appropriate level of income at retirement with no further individual action.

2.2. Danish Pension System in 2016

We now describe the main and relevant features of the Danish pension system as it was in 2016, the year our main results are based on. We start by exhibiting a schematic summarizing the main components of the Danish pension system over a 3-pillar layout (see Goul Andersen, 2011, for a more detailed description):

Table 1 – Danish Pension System Structure

<table>
<tr><td>Pillar 1</td><td>Pillar 2</td><td>Pillar 3</td></tr>
<tr><td>Universal Flat-rate*</td><td>Quasi-mandatory occupational pensions</td><td>Personal Pensions</td></tr>
<tr><td>Public Pension Supplement**
(folkepensionstillæg)</td><td colspan="2" rowspan="2">$\begin{cases} Life\ annuities \\ Non-life\ annuities \\ Age\ pension\ (lump-sum) \end{cases}$</td></tr>
<tr><td>Other Individual Supplements ***
(housing, heating, personal, health related transfers)</td></tr>
<tr><td>ATP</td><td></td><td></td></tr>
</table>

Note: based on Goul Andersen (2011); * Means-tested against labor income only (relatively high ceiling); ** Means-tested over most taxable income that does not stem from 1st pillar transfers; *** More targeted/idiosyncratic means-testing.

In a 3-pillar scheme, the 1st pillar of the Danish pension system represents unfunded benefits whose accessibility is based on a residence criterion, and not citizenship. For those with less than 40 years of residence, a reduction is applied to benefits. Certain benefits have different rules applying to singles than to couples. Couples are classified as people that effectively share an economy. This means that even if two people are married, they may be considered single by the state if those two people are not effectively sharing an economy (for example, if they are separated, living separately, yet not divorced). The rules can also depend on whether a person is in a couple where the partner is a pensioner or not. The 2nd pillar is comprised of occupational pensions that are quasi-mandatory in nature, with the exception of the ATP scheme, which is mandatory for most working-age individuals in Denmark. The 3rd pillar is comprised of voluntarily accumulated pension assets.

There are four major components in the 1st pillar:

- First, a universal flat-rate benefit (*folkepension*):
  - Pays out a maximum yearly amount of 72.756kr.
  - This income is attributed to each individual, irrespective of marital/couple status.
  - It is only means-tested against labor income.
  - Any labor income (and labor income alone) above 310.000kr. starts to reduce the flat-rate benefit, and when that income reaches 546.500kr. the flat-rate benefit is not paid out anymore.
- Second, the public pension supplement (*folkepensionstillæg*):
  - Pays out a maximum yearly amount of 76.788kr. if single and 37.632kr. if in couple (each).
  - It is means-tested against all taxable income, not counting the public pension amount (though including ATP income). Notably, some items do not count for the means-

testing, such as other general wealth and housing wealth. Certain types of pension accounts do not count as well, such as *kapitalpension* and *aldersopsparing*[2].

  - Means-testing is dependent on couple status.
  - For singles: Full pension supplement is paid out if privately earned income is below 68.400kr., and the supplement is no longer paid if the same income is above 316.900kr.
  - For couples:
    - Partner is not retired: a deduction up to a maximum of 107.050kr. is allowed from the working partner's labor income and a deduction of 60.000kr. is allowed on labor income from the retiree partner. After the deduction, the couple's total income cannot exceed 137.300kr. for the *full* pension supplement to be paid out. Entitlement to any supplement stops with combined income above 254.900kr.
    - Partner is also retired: a deduction of 60.000kr. is allowed on own labor income. After the deduction, the couple's total privately earned income cannot exceed 137.300kr. for the *full* pension supplement to be paid out. Entitlement to any supplement stops with combined income above 372.600kr.
- Third: Other Individual Supplements
  - Generally, these supplements are targeted at the poorest individuals.
  - Usually means-tested against both income and wealth.
  - Supplements attributed according to specific life expense needs (health, house heating, housing).
  - Certain supplements can be given over idiosyncratic evaluation.
- Fourth: ATP
  - Fully-funded defined-contribution scheme.
  - Contribution amount is flat across income rates, only adjusted for work-hours.
  - Contribution amount for full-time work is 3.408kr.
  - Mandatory for employees (with more than 9 hours of weekly labor). Also mandatory for unemployed under unemployment insurance (*a-kasse*).
  - Possible for self-employed to participate in scheme voluntarily.
  - Benefits setup as a guaranteed deferred annuity.
  - Usually described as 1st pillar benefit, though technically can be considered a part of 2nd pillar.

For the 2nd and 3rd pillars, the pension taxation regime is ETT for most pension products (see below). Capital gains are taxed at 15,3%, whereas out-payments are either taxed at 40% for lump-sum payouts, or

2 Contributions into these two types of pension accounts were generally not tax deductible (up to certain law-defined limits under which deductions were allowed).

they pay normal personal income tax otherwise (Goul Andersen, 2016). There are three main types of pension products:

- Life annuities (*livrente*):
  - Contributions are tax deductible.
  - Payouts are taxed.
  - Pays amount until death.
  - Count towards means-testing of public pension supplement.
- Non-life annuities (*ratepension*):
  - Contributions are tax deductible up to a limit (52.400kr.).
  - Payouts are taxed.
  - Can be paid out over a period of 10 to 30 years.
  - Count towards means-testing of public pension supplement.
- Age pension (*aldersopsparing*):
  - Substituted *kapitalpension* in 2013, which was a similar, more generous scheme.
  - Contributions are not tax deductible.
  - Payouts are tax-free.
  - Typically pays out in lump-sum.
  - Does not count towards means-testing of public pension supplement.

There is also the possibility to become an early retiree under two different systems. First, the state can award early retirement benefits to people at any age, if these individuals have reduced or no capacity to work. These benefits are in line with unemployment benefits, and not public pension benefits. Concurrent work is allowed up to a certain income threshold. Second, there exists a voluntary early retirement scheme, *efterløn*, where individuals can choose to contribute into the scheme for 30 years through their unemployment insurance company (*a-kasse*) during their work years. This allows them to receive benefits (in line with unemployment benefits) just before the official pension age in a situation of unemployment. In order to receive these benefits, individuals must have had a sufficient degree of employment in the years leading to early *efterløn* retirement. Moreover, the benefit rates are downwards adjusted against pension wealth. This makes *efterløn* disadvantageous for people with occupational/private pension accounts.

Recall that our definition of under-saving behavior in this paper are those individuals that are/will become entitled to the *full* public pension supplement (*folkepensionstillæg*) among those at or over the official state pension age. We ignore all other means-tested benefits as they are generally given to the very bottom of individuals over the income distribution, at the same time that they are heavily targeted towards specific types of necessities/expenditures retirees face at an individual level. The *folkepensionstillæg* (public pension supplement henceforth) is instead chosen due to its broader eligibility base and simplified rules, as the only means-testing criteria is defined over income. Overall, focusing on individuals with a low amount of pension savings allows for a simple, tractable definition of this group, though it does not allow for detecting under-saving behavior in a consumption-smoothing perspective.

2.3. The residual group in pensions – previous measurements

The residual group problem in Denmark has been measured in previous reports. ATP (2015) summarizes the key findings of some of these past reports. As established before, we focus only on absolute measurements of the residual group using threshold techniques. Our starting point is based on two studies that perform such measurements: ATP (2015) and SFI (2014). Their residual group size estimations are summarized in the following table:

Table 2 - key results from previous studies

Study	Measuring	Criteria	Population	Result	
				Proportion	Number of individuals
SFI (2014)	Pension wealth	Pension wealth lower than 1.088.500kr. (2011 DKK)	64 year-olds in 2012	46%	34557
	Pension contributions	Pension contributions lower than 100000kr over 5 years (2011 DKK, excluding ATP fund)	30-59 year-olds in 2011 from 2007 to 2011	32%	720746
ATP (2015)	Pension contributions	Average yearly pension contributions lower than 23240kr over 10 years (2014 DKK)	30-59 year-olds in 2012 from 2003 to 2012	35%	788316

The numbers reported above show a sizeable residual group. Recall that these individuals are estimated to receive the *full* public pension supplement, something that is, in its principle, to be given to the most disadvantaged within the Danish society. Coupled with the past decades' push to implement stronger 2nd pillar occupational pensions, making the individual more reliant on own pension savings, the issue of under-saving has become a highlight in the Danish social discussion. Given that these reports measure the residual group's size in the range of 32 to 46% of working age individuals at the same time that aggregate pension assets as a proportion of GDP have reached record high levels of around 200% in 2016, questions of moral hazard, free-riding behavior and institutional shortcomings have been raised.

One should note that these results only focus on those currently working or people about to retire[3]. This brings uncertainty into the estimation of the residual group size, since proxy measurements are being used to estimate who will become entitled to the full pension supplement at retirement. It should be noted that the pension wealth technique in SFI (2014) should be the most reliable one, since it is measuring pension wealth just before the official pension age, after all the saving decisions have been made. In this paper, we

3 Note that some of the reports referenced above include analysis of the currently retired, but not with absolute measurements based on thresholds for receiving the full pension supplement, the measure we focus on in this paper.

also perform residual group size estimation with the same proxy methods of the two reports referred to in the table above. However, we go beyond and also look at the currently retired and evaluate if they are eligible to the full pension supplement through direct income assessment against the legal conditions they face for receiving the supplement. Since this is a virtually certain (apart measurement error) classification of residual group membership, we use it in our risk-based framework, removing residual group membership uncertainty from our risk classification. We then evaluate how individual characteristics affect the probability of being part of the residual group and use these predictions to forecast the probability of ending up in the residual group in the future among those current working.

Furthermore, these reports only consider saving performed into savings accounts that are earmarked for retirement. This means that every working age individual that is saving into stocks, bonds and/or other liquid/illiquid assets, instead of pension accounts, is more likely to be falsely identified as part of the residual group, even though they might be relatively wealthy overall. We explore how the inclusion of such wealth items alters the residual group size. Related to the previous point is saving through housing wealth, which can also be used to supplement retirement savings via partial alienation of housing equity. More than half of the Danish population has housing assets and they can have a relatively large size in the balance sheets of many Danish households (Leth-Petersen and Andersen, 2019), such that it is an important dimension to consider.

3. Data

The present analysis is performed using administrative data from the Danish statistics agency *Danmarks Statistik* (DST henceforth). We use several registers linked by personal identification numbers for every Danish resident between the ages of 20 and 100 present in these registers, though more restricted age cohorts are used in specific contexts throughout the paper. We use registers with information on personal characteristics, such as sex, age, household size, relationship status, immigration, education level, labor status, work sector, official retirement status, income, assets, liabilities and pension contributions.

Descriptive statistics will span the period 2002-2016, whereas our econometric methods will focus on the year 2016 in the dependent variable and span the period 2000-2016 in the regressors[4]. In terms of wealth data, there exists two different sources. There are long running assets and liabilities measurements that exist since 1995[5] based on official sources of information from the state's property register, reports from financial institutions and the tax authority. Remarkably, these long-running wealth measurements do not include pension wealth. Recently, DST introduced new wealth data of higher quality, but it only exists

4 We do not use panel data methods for the regressions. We use cross-sectional methods based on the year 2016, but regressors may be measurements made on the relevant population of individuals in 2016 when they were younger.

5 There is a significant break in this series between the years of 1996 and 1997, since 1996 was the last year Denmark had a wealth tax. The break is characterized by a change in how the value of certain stocks is observed by the state, which brings a change to the observed holdings of stock wealth.

from 2014. This new data does include pension wealth. We do not use both sources interchangeably. Overall, as Leth-Petersen and Andersen (2019) argue, this tax return data is regarded as highly reliable.

Immigrant status is defined by country of origin not being Denmark. Education level is divided by 3 categories: first, primary school or lower; second, secondary or any vocational schooling; third, any university or equivalent institution's degree (short or longer cycle). Those categorized as "married" are all individuals legally married, "unmarried" are individuals never having been legally married, "widow" and "divorced" are self-explanatory where the individual has not legally re-married, and "couple" is a dummy that does not follow the legal marital status but instead categorize a couple as two individuals actually sharing an economy. This means that two people that are legally married may not be considered a couple if they are officially or unofficially separated[6], meaning, they are not sharing an economy. In the eyes of the pension institution, it is the actual situation that matters on whether an individual is under couple's or single's rules, and not the legal marital status. More generally, two individuals are marked as a couple considering the following dimensions: if they live at the same address, if they could potentially get married or are legally married, if their age gap is not bigger than 15 years, if they are not close family to each other, and if they have children together (not a necessary condition).

As for work sector, it is categorized loosely around the highest level of aggregation of NACE codes. An individual is designated into a work sector if it is the biggest source of employment in a given year, so some smaller variance may exist with employment at other sectors. We also aggregate people into a placeholder sector dummy, people to which work sector is not relevant (for example early retirees and retirees) or if a main work sector could not be defined (low sector attachment). In terms of labor status, it again represents the status most prominent in a given year. We categorize individuals into employed, self-employed, unemployed (but ready to work), receivers of cash benefits (*kontanthjælp*), receivers of sickness/parental leave and labor market-related student benefits, students, early retiree in public scheme (social insurance on inability to work), early retiree in *efterløn* scheme (voluntary early retirement scheme, available only at age 60), retiree, and finally, a dummy for individuals that could not be categorized (ie. non-students with generally low income from all sources).

In terms of the components of income, wealth and pension contributions we use, income measurements include income from labor activities, plus financial income, public transfers and other residual income (observed income that cannot be characterized into the former categories, such as child support payments). Two measures of aggregated income are used. One is gross income, which is total income before tax, the other is disposable income as total income after tax plus the rental value of owned property minus deductions of interest expenses.

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6 Officially separated is a status that is legally defined, a status between marriage and divorce.

For wealth data, we use total assets and total liabilities for the long series starting in 2002 but use disaggregated wealth components for the high-quality wealth data starting in 2014. This disaggregated data is comprised of financial assets, real assets (buildings and cars), pension assets, mortgage debt, and all other debt items (ie. bank debt, student loans, debt to the state and the like). In terms of pension contributions, we have data since the year 2000. We have disaggregated data for contributions made by an employer[7] and voluntarily made. We also observe pension wealth and pension contributions made into pension accounts of a type that does not count towards the means-testing of the public pension supplement, pension accounts of the type *kapitalpension* and *aldersopsparing*. These contributions/pension accounts are not used in estimating the residual group size and in the regression models.

Moreover, when plotting distributions of income and wealth and income quantiles, we utilize equivalized measures across households (as opposed to individual measures) to account for economies of scale in sharing an economic life between household members. We use the square root rule (otherwise known as the LIS equivalence scale), in line with recent OECD tradition (OECD 2015, 2019). This rule distributes household income to each adult member in the following way:

$$\text{Equivalized flow/stock} = \frac{\text{Total household income/stock}}{\sqrt{\text{number of individuals in household}}} \tag{1}$$

Full summary statistics can be found in Appendix A, Table A1 for our population of individuals aged 25-101 in the year 2016. Below, we show selected key statistics:

Table 3 – Summary Statistics

	Mean	SD
Male (%)	49.1	50.0
Immigrant (%)	12.0	32.5
Age	53.1	17.0
Couple (%)	64.7	47.8
Education level obtained		
Primary or less (%)	24.1	42.8
Secondary and equivalent (%)	41.0	49.2
Tertiary (%)	32.7	46.9
Disposable Income	248.3	819.1
Disposable Income (equivalized)	283.4	679.0
Total Net Wealth (equivalized)	2082.4	10800.0
Under poverty line (ages 25-101) (%)	8.1	
Number of individuals aged 25-100	4074652	

Note: Financial items are displayed in 1000's, 2016 DKK; income poverty line defined as 50% of equivalized median income

The proportion of immigrants stands at 12%, which is a significant percentage. The majority of individuals are in a couple, at roughly 65% of the population. Most people hold at most secondary

7 We do not have data on pension contributions made into the ATP scheme. The full-time employment contribution amounts to roughly 3400kr. a year. Not including these pension contributions will bias our threshold-based residual group size estimate. It will not affect our results under the risk-based framework, with the exception of our estimate for pension contributions made by employer.

education or equivalent (such as vocational training). The proportion of individuals with primary schooling or less is still rather high at 24%. Across each individual, average disposable income stands at 248.000DKK. Accounting for economies-of-scale in the household, the equivalized measure raises that number to 283.000DKK. In terms of poverty, 8.1% of individuals are technically income-poor, as defined by 50% of the equivalized median income.

4. Empirical strategy

4.1. Empirical framework

In order to develop an accurate risk approach, we start by only focusing on the currently retired, since we know with certainty (apart measurement error) who is entitled to the full pension supplement. This removes an extra layer of uncertainty of residual group classifiers based on the currently working, as these estimates are exactly only estimating the group size. Next, we fit a probability model with selected regressors (see Appendix C for a list). Any relevant information we have on all individuals is then used to draw out risk factors related to belonging to the residual group.

Technically, the Probit model being fitted is of the form:

$$p_i = \mathrm{prob}[RG_i = 1|X] = \Phi(x_i'\beta)$$

Here, p_i denotes the probability of belonging to the residual group for individual i, RG_i is the dummy equal to 1 if individual i belongs to the residual group, and Φ represents the Normal cumulative distribution. Probability has a relationship with individual characteristics through a Normal cumulative distribution transform applied over a linear combination of individual characteristics in X. Given the model's structure, in order to present how changes in explanatory variables are reflected in changes to probability, one must derive marginal effects. These are derived differently on whether the explanatory variable is continuous or dichotomous. For continuous variables, the marginal effect is computed as:

$$\frac{\partial p_i}{\partial x_{ik}} = \phi(x_i'\beta)\beta_k$$

For dummy variables it is determined as:

$$\Delta p_i = \Phi\big(\bar{x}_i'|_{dummy=1}\beta\big) - \Phi\big(\bar{x}_i'|_{dummy=0}\beta\big)$$

We specify different Probit models with mutable regressors measured at different past age-points for the *new* retirees that become 65/66 years old in 2016[8]. Note that including higher ages in the analysis reduces how long back the analysis can be performed. Hence, to keep each Probit model consistent over ages, we refrain from mixing different age-cohorts. As such, we only look at the two earliest ages 65 and

8 In first year under pension age, where no deferment of official retirement took place. More than 90% of individuals retire at these ages. By retirement, we mean becoming an old-age pensioner, even if the individual was previously under the public or the voluntary early retirement schemes.

66, that can be observed in the data in 2016. We go as far back as the data allows us. This means that we formulate 13 Probit models performed with mutable regressors at ages 52-64 onto new retirees that became 65 or 66 in 2016. It also means that we can no longer use high-quality wealth data, since we need to look back in time beyond 2014 (the year at which high quality wealth data becomes available). This measure is substituted with the lower quality measure of total net wealth[9].

We then project the resulting Probit equations onto individuals that become 52, 53, …, 64 in 2016, thus obtaining forecasts for residual group size within this age-range. This offers us a distribution of forecasted probabilities for individuals 15 years away from reaching the public pension age of becoming entitled to the full pension supplement at retirement. If data is available over time, this method also allows to keep track of how the risk distribution evolves over time. This is useful information, since it allows us to identify main characteristics at different risk levels of becoming dependent on pension supplements at retirement.

Table 4 below summarizes the types of regressors used in the Probit models (see Appendix C for a complete list):

Table 4 - Regressors used in the Probit models

Variables	Definition
Basic Characteristics	Sex, number of children, immigrant, education, marital status in a given year
Work Sector	Predominant work sector in a given year
Labor Status	Predominant labor status in a given year
Wealth measures	Measured holdings of assets and liabilities in a given year
Income measures	Measured income flows in a given year
Pension contribution measures	Amount of contributions into means-tested pension accounts in a given year

First, we consider relevant personal characteristics that can impact saving behavior, such as gender, immigration status, education level, number of children, being in a couple or widower or divorced. Both SFI (2014) and ATP (2015) show that labor status and work sector matter in how individuals save for retirement. As such, we include regressors for different work sectors, including one dummy for when work sector does not apply or could not be determined (it does not apply when an individual is mostly not, or not at all, working). In the same way, we include dummies for labor status, whether a person is unemployed, self-employed, receiving poverty alleviating cash benefits, in early retirement discerned for the public and voluntary[10] schemes, and a category for those that couldn't be categorized (basically, those with virtually no attachment to the labor force).

9 This situation is less than ideal, since the lower quality wealth measurement does not include pension assets. Other asset items are included in the model, nonetheless. Later on, we do show how the Probit model behaves if we use the high-quality wealth data for the model at age of 64 (in 2014 or 2015, when the new retirees in 2016 respectively become 65 or 66 years old).

10 Note that the voluntary scheme is only available when an individual turns 60. As such, all the Probit equations for ages 52-59 do not include this dummy variable.

We then include as regressors a series of relevant financial measurements, such as total assets (excluding pension assets, including housing assets) and total liabilities, primary income, public transfers, financial income, residual income (including income items that cannot be classified in the former categories), as well as pension contributions made automatically by an employer in the behalf of the employee and ones voluntarily made. Assets and liabilities tell us how much money has been put aside, the disaggregated income regressors help control for the income level from labor activities versus how much comes from public transfers, financial returns and other types of income. And finally, pension contributions are an obvious addition to the model to measure the level of contributions versus how it contributes to not be in the residual group, separated for employer-made contributions where the employee has little short-term say (the employee might have negotiation power upon labor contract negotiation, but little after that contract is signed) and voluntarily made contributions, measuring active decisions towards savings.

Against the dummies included in the Probit models, we have a baseline individual that is female, non-immigrant, university educated, not in a couple, not a widower, not a divorcee, working in the public administration, and not self-employed. Furthermore, we *winsorize* all non-categorical variables at the 1th and 99th percentiles (only at the 99th percentile for variables bounded below at 0) to deal with the impact of outliers on our results and improve the stability of the Probit estimation procedure[11].

Moreover, we also analyze individuals aged 25-44 using a similar Probit model, with the exception that projections and forecasts will not be made. Since the Probit models will be made over working-age individuals, the dependent variable will be a dummy variable equal to 1 if an individual is part of the residual group of those that are not accumulating enough pension wealth. We then obtain average marginal effects and observe how individual characteristics impact the probability of belonging to the residual group.

4.2. Identification of residual group members

In order to assess the risk of becoming dependent on pension supplements, we first need to identify those currently retired that are eligible to it. In the Probit models, the outcome variable of analysis is binary in terms of residual group membership. The residual group is the group of individuals that are, or will be, entitled to the full public pension supplement. A member of this group is coded as 1 in the binary outcome variable and 0 otherwise. When determining group membership, it is important to account for two dimensions: one is co-habiting couples sharing economy vs. being single; the other is considering only pension contributions or pension wealth that counts towards the means-test of the public pension supplement. We account for these dimensions. When determining residual group membership for the currently retired, we do not consider early retirees as “retired”. Our definition of “retiree” is officially being retired under the public pension age (or *folkepensionsalder* in Danish). It is the case that one may still work

11 The Probit model performed for those aged 64 using high quality pension wealth data required winsorization at percentiles 2 and 98, and 0 and 98 for those variables bound at zero, due to computational issues with outliers.

while being under official retirement at public pension age. In 2016, the income limit over which one stops receiving the full public pension supplement was 68.400kr for singles and 137.300kr for couples[12].

The identification for the currently retired is quite accurate since the data is granular enough to identify items of income that count towards the means-test of the pension supplement. There are specific rules for different items of income, and we account for these in terms of labor income, income from private pensions that count towards the means-testing (private pension income from *aldersopsparing* or *kapitalpension* not included), financial income and other residual income[13]. We also have information on whether individuals are tagged as a couple or not (as measured by DST matching criteria). It is important to consider that, in the perspective of the public pension institution, a couple is not necessarily the same as two legally married individuals. Being a couple, in determining which set of rules to apply, is dependent on whether two individuals share an economy. So, if there is a married couple that are even informally separated, having separated economies, each of the married individuals will be considered as "actual single" if each are not subsequently in another relationship with a shared economy.

We also replicate the results in SFI (2014) and ATP (2015) and expand their residual group methodology. Given the different sources of data in those reports and the present paper, we start by contextualizing those reports to our data and verifying how their methodology behaves under different dimensions such as over cohorts, over time and over income quantiles. The identification of the residual group over those individuals currently working is done according to pension contribution or pension wealth thresholds, determined in a way such that an individual will likely be entitled to the full pension supplement at retirement. This is possible to define under certain assumptions since the legal rules around the pension supplement have themselves well-specified income limits in its means-testing.

For defining pension contribution and pension wealth thresholds, we simulate all pension wealth as if it will all be converted to a guaranteed deferred annuity type of instrument (in similar fashion to SFI 2014). For pension contributions, we assume that pension contributions start at the age of 25 until the age 65 (40 years of contributions), that these are flat along the contributory career (in real terms), with a real return of 1%. For pension wealth thresholds, we also assume a 1% real return on the annuity assets and a remaining average lifetime in retirement of 20 years (matching the Danish population, see SFI 2015). Assuming a 1% real return might be on the lower end of possibilities, but given that this return refers to pension assets

12 These income limits only apply to certain types of income, and different types of income can have allowances associated with them. Certain income types from spouses (as in, the other individual in a couple) also count towards the means testing of the pension supplement, and the rules differ on whether the spouse is also retired or still officially not retired. See section 2 for more details.

13 There is of course a certain degree of uncertainty in measuring items of income that count towards the means-testing of pension supplements, stemming from items of income in the "residual income" variable that aggregates certain types of income that do not fall under the other categories of gross income disaggregation. We include "residual income" in the means-testing, and certain items there may not actually count for the means-testing of supplements. We do not expect this effect to be relevant, as the residual income average observed in Table A1 in appendix is really low.

and its return after pension administration costs, we find it reasonable as a baseline assumption. The formulas for the wealth threshold and average pension contribution are:

$$\text{Pension wealth} = \text{Yearly withdrawal} \times \frac{1-(1+r)^{-n}}{r} \tag{2}$$

$$\text{Yearly contribution} = \text{Pension wealth} \times \frac{r}{(1+r)^{n+1}-(1+r)} \tag{3}$$

The pension wealth threshold that an individual must have at retirement is derived in equation (2). It is found by substituting in the real return on remaining assets in the annuity, number of years in retirement and the yearly income withdrawal at which point the public pension supplement starts to be reduced. We also use wealth thresholds for each age, with these thresholds being defined by the natural accumulation progression implied by equation (3). The natural accumulation progression is dependent on the yearly contribution into the pension account, which, again, is flat across time. By inserting the amount of pension wealth required at retirement, the return on the pension account and the number of contributory years, one finds the average pension contribution required. Then, one can derive pension wealth thresholds at each age point as implied by the average pension contribution found before. The exact age-dependent wealth thresholds are shown in Appendix B. This amount is different for singles than it is for couples, and we account for that difference.

Finally, for estimation of residual group size and probabilities, we utilize household-level flows/stocks but not equivalized. This is because the determination of residual group hinges on specific monetary values, and not a measure of well-being (as equivalized measures correct for). We implicitly assume that current couples will remain a couple for the rest of their lives, planning their lives as such, and that total household income and wealth is pooled among all household heads equally (always one minimum or two individuals maximum).

Overall the residual group definitions we employ are summarized in the table below:

Table 5 - Residual group definitions

Residual group	Definition	Perspective	Population
Definition 1	Privately earned income below 68.400kr for singles and 137.300kr for couples	Actual Entitlement	Retirees Years: 2002-2016
Definition 2	Pension wealth below threshold according to formulas (2) and (3) adjusted for couples and singles	Stock	Working-age Years: 2014-2016
Definition 3	Pension contributions below 25000kr for singles and 50000kr for couples as determined by formula (3)	Flow	Working-age Years: 2002-2016

Briefly summarizing the choices above, definition 1 allows us to obtain an accurate measurement of the residual group, since it is only applied to the already retired; definition 2 checks that individuals have enough pension wealth accumulated; and definition 3 checks that individuals are saving at an enough rate into pension accounts.

For each definition, we apply a broad age range and a restricted age range to perform an extra check of how much the age range drives results. Table 6 below reveals the age ranges applied:

Table 6 – Age ranges considered in measuring residual group

Measure	Range type	Age range
Definition 1	Broad	(All retirees) 65-100
	Restricted	(New retirees only) 65/66
Definition 2	Broad	25-64
	Restricted	(SFI 2014) 64
Definition 3	Broad	25-64
	Restricted	(ATP 2015) 30-59

Note that definition 2 is inspired by the report SFI (2014), but whereas they only focus on 64 years old (see Table 2), we generalize the pension wealth threshold technique they employ to all ages. Similarly, Definition 3 is inspired by both SFI (2014) and ATP (2015), who consider only those aged 30-59, and we also expand and explore the age dimension in how it impacts residual group size measurements. Conceptually, restricting the age range is expected to change the estimated residual group size. At younger ages, lower labor market attachment might correlate with a lower intensity of pension contributions/wealth accumulation. The same goes for cohorts just about to hit the public pension age. As for retirees, those surviving well into retirement are also more likely to run down their wealth to a point where they end up becoming eligible to the full pension supplement.

Specifically, Definition 3 will be the most vulnerable one, since the flow intensity of saving is likely to be, on average, weaker at the beginning and end of the working period of life. This is why ATP (2015) focuses on measuring pension contributions in the middle of the working period of life. A potential problem here is that, taking pension contributions in this middle period as representative of saving behavior across the entire working life can lead to under-estimation of residual group size. We expect this definition to not be stable over different age-cohorts.

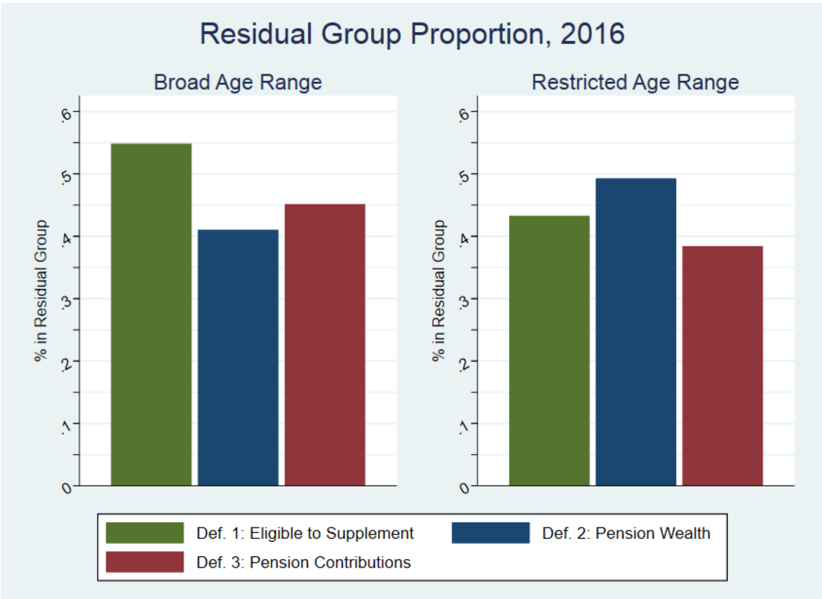
Definition 2 should be more robust than Definition 3 since this definition is based on stock measurements, and stocks are "sticky" by nature as opposed to flows. It checks if someone is on track to save appropriately, even if pension contributions have been unstable across years but have a high enough average contribution. Since a certain threshold of pension wealth will ensure that a person is not eligible to the full pension supplement, Definition 2 will likely produce an accurate estimate if measured among those about to retire. There is one vulnerability in it, in that it says nothing about future saving behavior, if it is measured earlier in one's life. SFI (2014) dealt with this problem by only looking at 64 year-olds, though we expand their analysis to the full age range (25-64) and verify the stability of estimates over age cohorts (among other dimensions).

5. Results

5.1. Direct measurements of residual group size – threshold methods

We now turn to evaluate the residual group size according to the threshold techniques described in Section 4. We focus on Definitions 2 and 3 as defined in Table 5 and apply each to two different age ranges, specified in Table 6. The estimated residual group sizes from Definitions 2 and 3 are then contextualized with the residual group size measurement in Definition 1. The estimates for residual group size for both the broad and restricted age range are revealed in Figure 1 below:

Figure 1 – Residual group size, broad and restricted age ranges

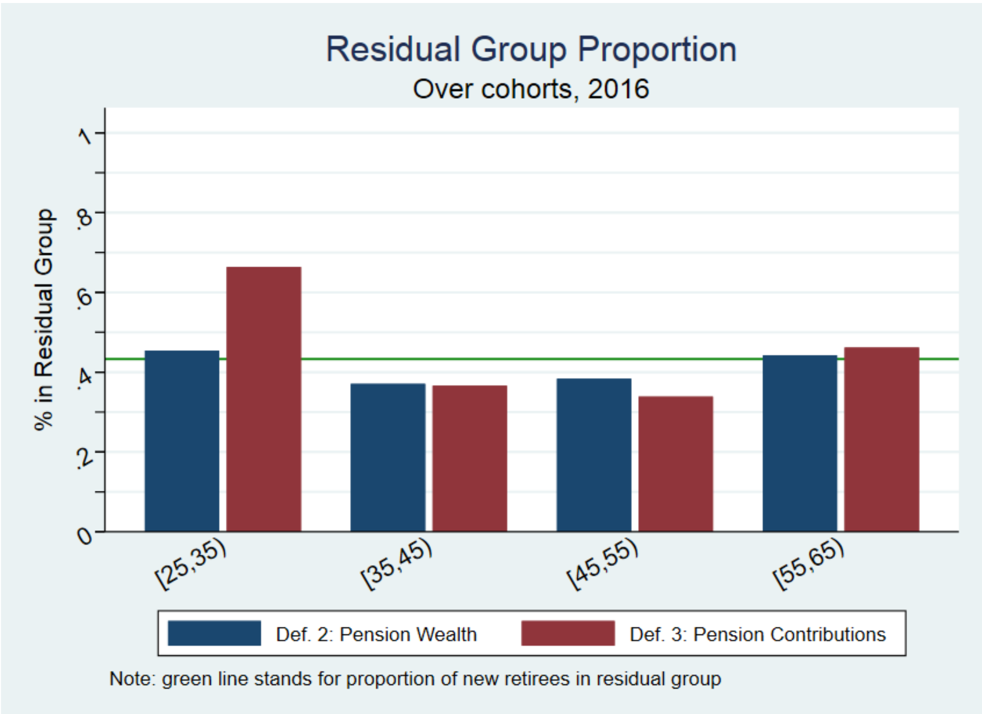


First, the current residual group size, as measured by Definition 1, is markedly bigger when considering all retirees (broad age) as opposed to newly retirees (restricted age), going from 54.8% down to 43.4%. This is, at least to a certain extent, expected, as people consume their pension wealth over time when retired. At some point, individuals may consume their pension wealth to the point where they become entitled to pension supplements. One way this can take place is due to the existence of non-life annuity-type retirement income, such as *ratepension* type of products with a payout period of 10 to 30 years. If individuals survive longer than the outpayment period of their annuities, they will suffer an income drop if their consumption during that period was in line with the higher income stemming from these non-life annuities.

Second, Definitions 2 and 3 also suffer a marked change when considering the different age ranges, revealing their dependence on the age considered. For the restricted age range, Definition 2 measures the residual group size at 49.7% and Definition 3 at 38.5%. Thus, our results do not exactly replicate the reports SFI (2014) and ATP (2015), with our estimates being slightly increased. When expanding the measurements to all those aged 25-64, Definition 2 lowers to 40.9% and Definition 3 increases to 45.3%. Definitions 2 and 3 seem to better explain the residual group among those newly retired, compared to the average eligibility rate observed within the entire pool of retirees. This dependence on the ages considered is a cause for concern over the stability of the estimates given by definitions 2 and 3.

A few reasons for the differences between broad and restricted age measurements can be hypothesized. For one, people do not save smoothly over their working period of life, and the definitions 2 and 3 assume so. As such, if the average individual displays strong saving behavior during peak earning years (say, around the age of 40-55), they can then show lower saving behavior at other ages and still save enough so that they do not become entitled to pension supplements. In short, timing can matter, as Figure 2 below shows. Another reason is average behavior changing over time, in the sense that different cohorts might be saving more or less on average, which can impact the measured residual group size at different age-points. For these reasons, we analyze all the definitions below in two different perspectives: over age cohorts and over time. Overall, definitions 2 and 3 seem to underestimate the residual group size over the broad age range.

Figure 2 - Residual group over cohorts, broad age

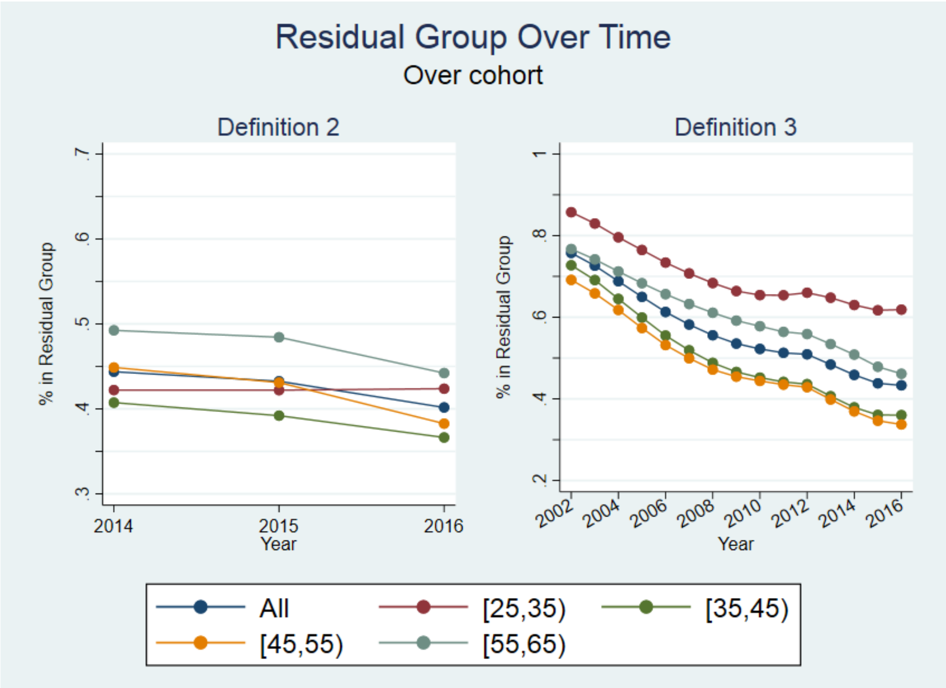


Starting with how the definitions perform over different cohorts, Definition 2 is rather stable over cohorts. It seems that younger people do not start saving that early, and older individuals about to retire also seem to be more vulnerable on average when compared to middle aged cohorts. Older cohorts seem to be more entitled to the full pension supplement compared to middle aged cohorts. It may be that individuals are saving strongly during their top earning years, thus accumulating pension wealth above the threshold in that period, but then reducing pension contributions in the following years to the point where pension assets fall below the threshold.

The same pattern is observed for pension contributions measured in definition 3. Including age periods where lower pension contributions are likely to take place, before labor market status largely stabilizes, (ages 25-29) and just before retirement, where labor market participation is also more unstable (ages 60-64), increases the estimate of residual group size. Interestingly, definition 2 is somewhat resilient to the different age-cohorts when compared to the performance of definition 3. Note that the very high proportion of individuals flagged by definition 2 in the cohort aged 25-34 remains high (around 60%) even if students are excluded from it.

Now turning to the time dimension, it is possible to plot residual group size under definition 2 over the period 2014-2016 and definition 3 over time for the period 2002-2016[14]. We perform these measurements assuming that all individuals will retire under 2016 rules.

Figure 3 – Residual Group Size Over Time



Starting with the estimates in Figure 3 under definition 2, the wealth-based measurement, a downwards trend is observed over the years 2014-2016 for residual group membership over all cohorts with the

14 We cannot show it for the years 2000 and 2001 due to data limits on pension contributions and retirement age.

exception of the youngest cohort aged 25-34. This downwards trend is rather strong, seeing drops in the order of almost 5 percentage points. Remarkably, the youngest cohort does not share this downwards trend, revealing that younger individuals are lagging behind in the overall trend of higher pension assets in Denmark. Either the occupational pension system is already matured within the youngest cohort, meaning that a very broad base of these young individuals is already contributing to a high degree, or a worrying new under-saving trend is dominating behavior within the youngest cohort.

As for definition 3, the pension contribution-based measurement, its progression over time also displays a decreasing trend for the residual group size. In terms of cohort behavior, it is in line with definition 2, since the youngest cohort displays the largest proportion of low contributing individuals. As expected, people around their peak earnings age (say, 35-54) are the ones contributing the most. In line with the situation in Definition 2, a downwards under-contributing trend observed in older cohorts is not seen among the youngest cohort. Whereas older cohorts' under-contributing patterns are still exhibiting a downwards trend, the proportion of under-contributors within the youngest cohort has nearly flattened since 2009, a year of economic crisis where the unemployment rate nearly doubled in Denmark (OECD, 2017c). Since the under-contributing rate among the youngest cohort is far above that of all older cohorts, it shows that the flattening of the residual group size under Definition 2 for the youngest cohort is likely to stem from a worrying new trend of under-saving, and not a full-maturation of pension contributions among the youngest cohort.

Another important dimension in terms of under-saving behavior relates to those individuals that simply cannot save enough due to low disposable income, otherwise known as hand-to-mouth type of individuals. These individuals can be thought of as "legitimate" residual group members, since means-tested public pension supplements are exactly meant to curb poverty situations at retirement among the most income-disadvantaged ones. In order to observe how the residual group behaves across the distribution of income, we show the estimated residual group size for each equivalized disposable income quintile of 2016 for the broad age range:

Figure 4 – Residual Group Size Over Income Quintiles

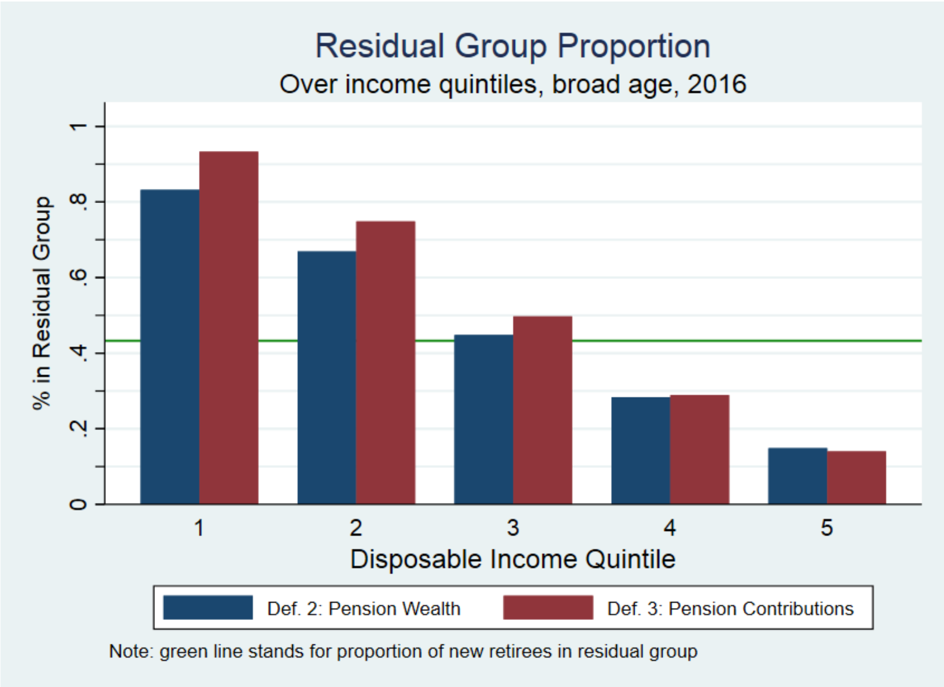


Figure 4 reveals a decreasing residual group size over income quintiles, as expected. What is perhaps less expected is that, even at median income, we observe residual group sizes of around 45-50%, a value in line with the average estimated residual group size for the entire population of individuals aged 25-64. This means that the distribution of individuals estimated to be in the residual group under Definitions 2 and 3 is rather centered. To contextualize the meaning of this residual group size at median income, note that a 12% pension contribution rate out of the median income will roughly suffice to comply with the residual group definition 3. So, on average, 45-50% of individuals at the median Danish income distribution are saving less than 12% of their disposable income. Even less expected is the positive estimate for the residual group size in definitions 2 and 3 at the top quintile. The estimated residual group size among the highest earners in Denmark is around 14% of those individuals. This is a surprisingly high degree of under-saving among those earning the most.

The reason for these high residual group sizes at the top quintile likely stems from the fact that, first, definitions 2 and 3 only look at savings earmarked as pension saving, though it may be that it is showing actual under-saving behavior. A possibility is that many households (and especially at higher incomes) are saving for retirement thorough other asset types. As such, just looking at pension assets might distort the estimate of residual group size at higher income quantiles. Second, not all types of income count towards the means-testing of the pension supplement. Especially so, non-pension assets are not considered in the means-testing for the public pension supplement, only the financial income generated by it. It may be that wealthy people may manage to structure their wealth holdings in a way that still enables them to receive the full pension supplement, for example by holding wealth in money accounts or low yielding/risk financial instruments. Third, it is possible to combine some work income while retired and still be eligible to the supplement, especially if one's spouse is still not a retiree. As such, supplement-eligible *households* might have relatively high income but still "optimize" their income sources and remain eligible to the pension supplement. This is evidence that a potential improvement to the means testing system could take place, though not without pervasive incentive effects compared to the current status quo.

Given the results in Figures 4, we now look at balance sheet composition of those in the residual group and those not in it, as a proportion of disposable income. Because of this weighting of financial components, we start by showing the average disposable income for definitions 2 and 3 over those that are members and those that are not:

Figure 5 – Average disposable income over residual group membership

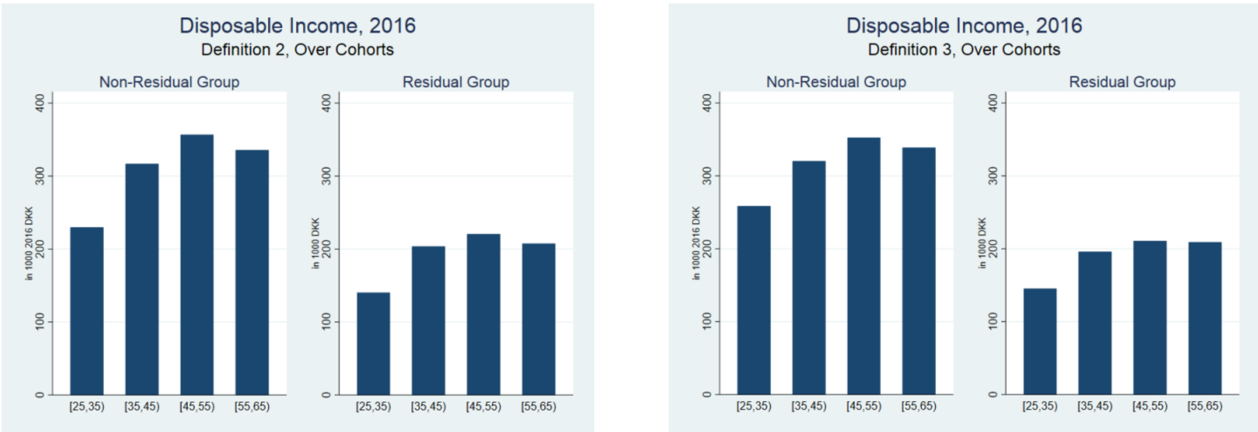
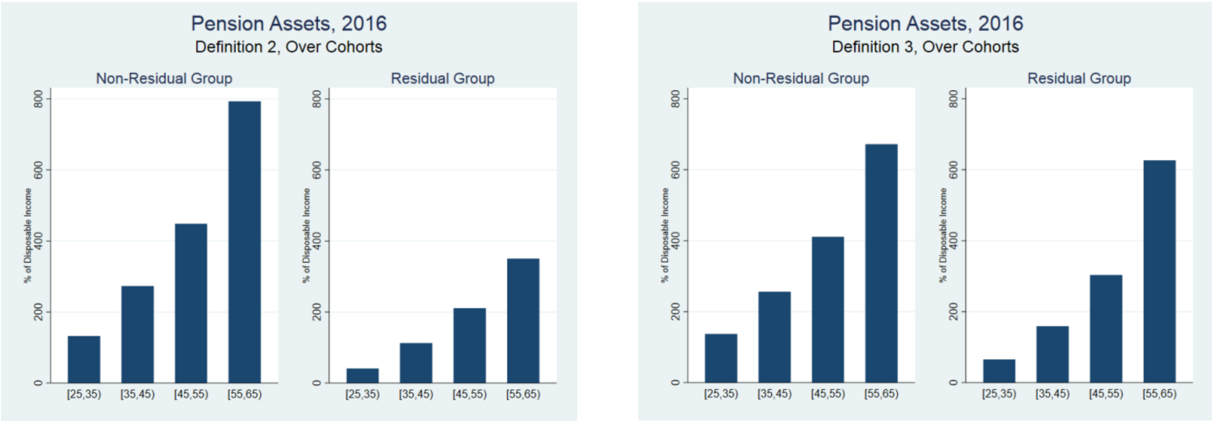


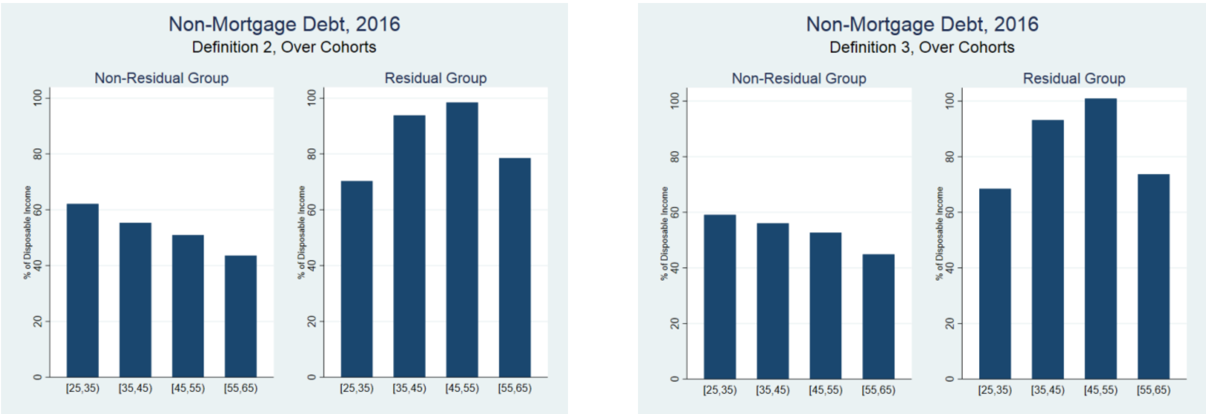
Figure 5 shows that both definitions 2 and 3 capture residual group members that are, on average, obtaining much less income compared to their non-residual group counterparts. Both income trajectories across cohorts have the usual hump-shape, but the residual group is indeed characterized by much lower peak earnings, averaging around 350.000kr. among those who save enough versus average peak earnings of just over 200.000kr. for individuals in the residual group. The former group attains their peak earnings at ages 45-54 whereas residual group members attain their peak one cohort before, between the ages 35-44.

Figure 6 – Pension Assets over residual group membership



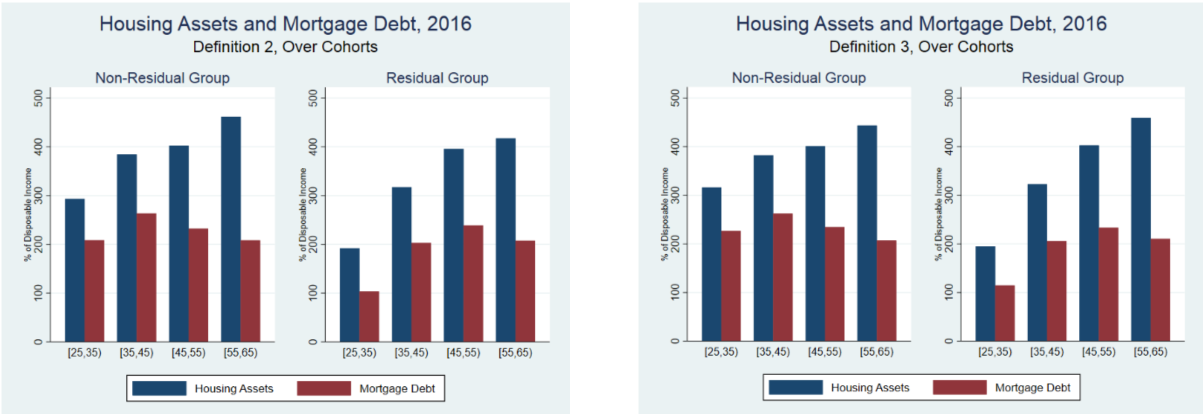
Turning to the accumulation rate of pension assets, the left-hand side of Figure 6 shows that, just before retirement, those outside the residual group under definition 2 accumulate pension assets to an amount of 800% of disposable income, whereas their residual group counterparts accumulate less than 400% of their disposable income, on average. Though Definition 2 successfully captures saving differences between members and non-members, Definition 3 fails to account for diverging saving behavior among those in the residual group versus those not in it. especially at the age cohort 55-64. There, both residual group members and non-members accumulate pension assets to the order of 800% of disposable income. This reveals, again, the sensitivity of Definition 3 to the age range of measurement. Pension contributions simply have high variance over age.

Figure 7 – Non-mortgage Debt over residual group membership



Focusing now on the negative side of the balance sheet, the distribution of non-mortgage liabilities over age cohorts reveals an interesting relationship. For both Definitions, members of the residual group have more debt in proportion to their disposable income, and it increases among older cohorts. This could be an indication of under-saving behavior, where households reveal a proclivity towards over-consumption or it refers to households that are hit with adverse economic shocks, lowering income and increasing relative indebtedness as a result.

Figure 8 - Housing Assets and Mortgage Debt over residual group membership



Lastly, we show the accumulation structure of housing assets and corresponding mortgage debt. One reason under-saving could be taking place is due to choices of purchasing housing assets at the cost of contributing less to pension savings. If this were the case, one would expect residual group members to hold relatively more housing assets/higher mortgage debt compared to those not in the residual group. Figure 8 above shows no evidence of this. It does not seem to be the case that, for the average residual group member, housing is causing under-saving, as both groups have an average housing wealth of around 400% of disposable income (this includes individuals with no housing assets). It should be noted that mortgage debt may be impacting saving rates at the age cohort 45-54 for residual group members, since the hump-shape distribution of mortgage debt peaks one cohort later for those in the residual group. Given the early flattening of disposable income for the average residual group member, mortgage debt repayment seems to be slower for these individuals, which may impact saving behavior well into older age.

Overall, the results stemming from threshold techniques showed us that there is a general downwards trend in the residual group in Denmark. It also showed us some evidence that under-saving is likely occurring among individuals that could otherwise save sufficiently to not be members of the residual group. We also saw that residual group size estimates from pension contributions are sensitive to the age cohorts considered, and its performance becomes uncertain as a result. Definition 2 showed better stability in its estimates and resulting group characteristics. Nonetheless, these threshold techniques do not provide any information in the risk dimension, nor they control for individual characteristics other than the level of pension assets or pension contributions used in the dichotomous characterization of the residual group. For this, we now apply a Probit model for individuals aged 25-34 and 35-44 in the next sub-section.

5.2. Determinants of being in the residual group among younger workers

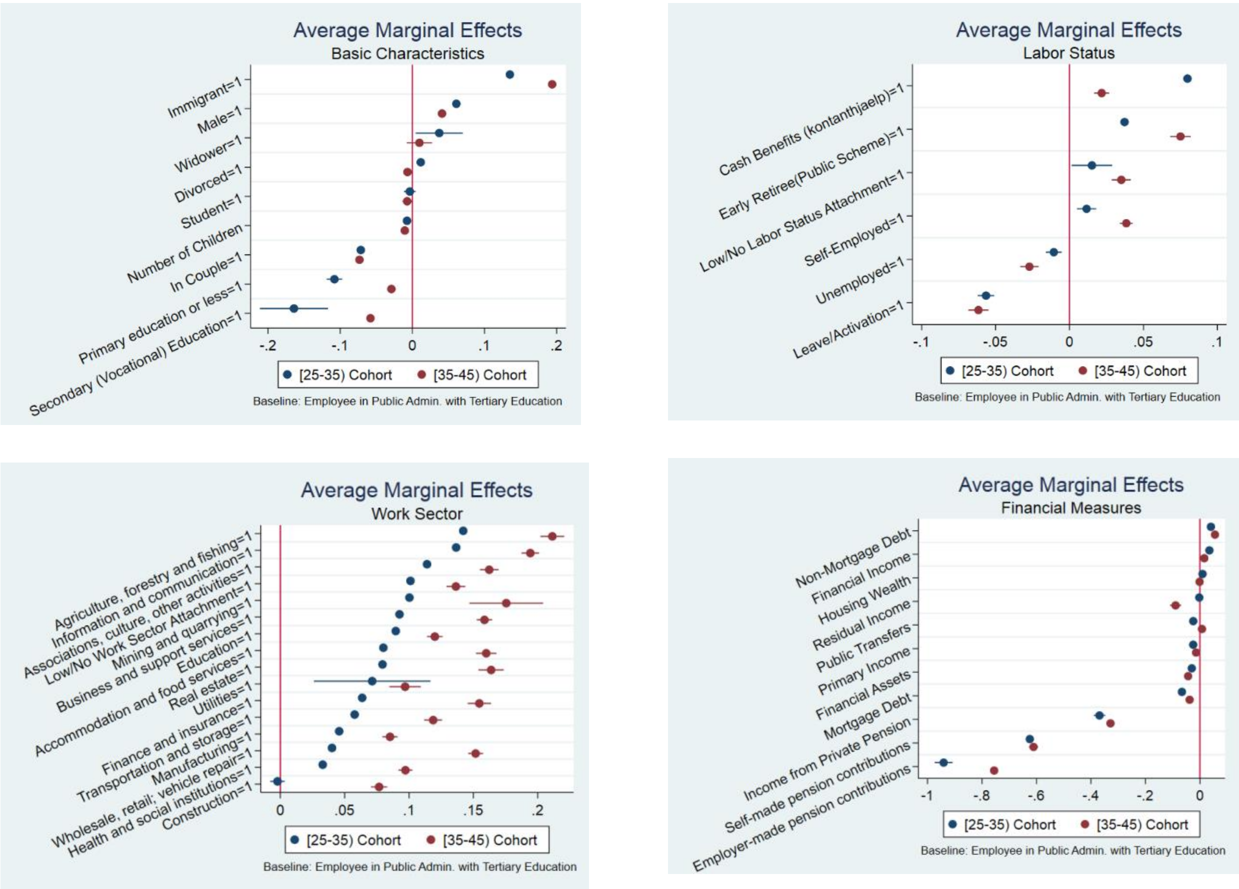
The previous section analyzed working-age individuals and their saving behavior. Using the threshold techniques, based on pension wealth and contribution measurements, we identified those working-age individuals that are on a saving path such that they will likely end up becoming dependent on the full public pension supplement at retirement.

Having identified the residual group, we can now measure how individual characteristics influence the probability that an individual find itself on an under-saving path. For this, we apply a Probit model, regressing a vector of individual characteristics (see Appendix C for the list of regressors) on whether the individual is a member of the residual group under definition 2. We use definition 2 since the analysis in the previous sub-section 5.1 indicates that it is superior to Definition 3, being more stable across age-cohorts and far superior at identifying the accumulation of pension wealth.

We display the average marginal effects for two different age-groups (see Appendix H for the coefficient tables). First, the youngest cohort in our sample, aged 25-34. This group can be conceptualized as still having low labor market stability, something that is observable in Figure 3 where this age cohort is the one that contributes the least into pension accounts above the 25.000kr. threshold. Given how widespread employer-made contributions are among the employed, the conclusion is that these younger individuals are either still studying, have low employability, or the work contracts offered to them do not typically include employer-made pension contributions. The second age-cohort we consider is the one between the ages of 35-44. Turning to figure 3 again, we see that this age group is roughly tied with the age group 45-54 in making the most contributions above the threshold. This is in stark contrast with the age cohort 25-34.

The average marginal effects are now displayed for the two age groups 25-34 and 35-44. These marginal effects tell us how different characteristics impact the probability of not accumulating enough pension wealth above a certain threshold, or in other words, the probability of being part of the residual group under definition 2 (see Appendix B for the age-specific thresholds):

Figure 9 – Marginal effects at age groups 25-34 and 35-44



Starting with the top left quadrant of Figure 9, it shows the average marginal effects for basic characteristics of individuals. Overall, all margins are significant with very few exceptions, though not all characteristics have significant impacts in size. The characteristics yielding the most impact are being an immigrant, male, being in a couple, and having lower education than a university degree.

On average, being an immigrant is associated with an increase in the probability of being in the residual group of 14pp. for those aged 25-34. This number increases to almost 20pp. for those aged 35-44. Since the regression equation controls for labor status, income, being self-employed, and other relevant dimensions, the explanation for this positive effect of being immigrant can be interpreted (at least in part) as low commitment to the Danish pension system. It can be that immigrants avoid contributing to the Danish pension system, by for example contributing to their foreign social security systems or saving in foreign accounts. In the data, some foreign assets are captured, but contributions to foreign social security systems are not.

Being in a couple is associated with a lower probability of being in the residual group for both age groups equally. Since classifying people as being part of the residual group or not is done at the household level, this result is not surprising since being in a couple provides a type of insurance against low contributions into pension accounts.

Turning to the marginal effects related to education level, it is seen that not obtaining a university degree is associated with markedly lower probabilities of being in the residual group. This result seems counter-intuitive, and the explanation lies with the following: those that do not pursue further education start working earlier. This means that they accumulate pension wealth at an earlier age, compared to those that are still studying at the age of 25 and above. Even if the low education group makes smaller contributions to pension accounts on average (which they do), starting work at an early age is associated with a lower probability of being in the residual group for the age-cohort 25-34. Since the low education group can expect a lower wage profile moving forward in time, this effect should start to disappear for older cohorts. This is seen in figure 9, where the marginal effects for the low education dummies is more than halved for those aged 35-44.

Now focusing on the bottom left quadrant of figure 9, showing the average marginal effects associated with the work sector the individual is the most associated with throughout a given year, considering that the baseline case is an individual that works for the public administration (a sector with strong, widespread mandatory pension contributions), it is no surprise that being in any other sector than the public administration is associated with a higher probability of being in the residual group. The sectors with the largest probability margins are:

- Agriculture, forestry and fishing;
- Information and communication;
- Associations, culture and other activities;
- Low/No work sector attachment;
- Mining and quarrying;
- Accommodation and food services;
- Real estate.

These sectors are the ones most at risk among those aged 25-44, increasing the probability of being in the residual group in the range of 10-20pp.

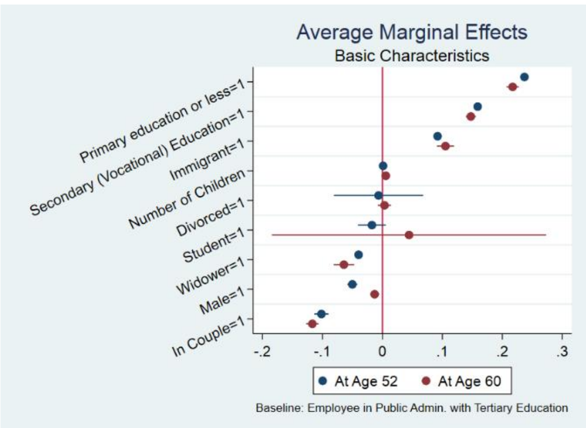
The top right quadrant of figure 9 shows the marginal effects in terms of labor status. The biggest effects are seen on those that are receiving cash benefits in the form of *kontanthjælp* (a poverty-alleviating transfer for those that cannot sustain themselves) and being an early retiree (so, those that lost their ability to work). Receiving cash benefits at a younger age has margins of around 8pp. For those aged 35-44, this margin reduces to almost 0. Bad outcomes earlier in life are thus more connected with low pension savings, as expected. Losing the ability to work penalizes those in the older age group, 35-44, by a positive margin of around 7pp.

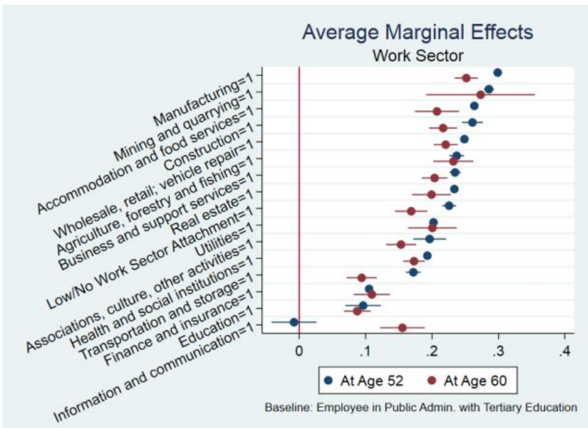
Finally, the bottom right quadrant of figure 9 shows the marginal effects for financial measurements. All stock values were normalized by 1.000.000, and flow values by 100.000. Given this scaling, all regressors are relatively small, with two exceptions: income from private pension funds and pension contributions. In terms of income from private pension funds, people receiving this type of income are likely for the most part those that inherited the stock of wealth from spouses or ascendants, given the age of the recipients. Given this type of event, these individuals are then inheriting pension wealth. This explains the negative drop in the expected probability of being in the residual group of around 37pp. per 100.000kr. of income received from pension accounts. For pension contributions, each 100.000kr. of pension contributions made by an employer in a year reduces the probability of being in the residual group almost entirely down to 0. This is not surprising, since 100.000kr. of contributions is a relatively large sum. As for pension contributions made voluntarily by the individual, it reduces the probability by around 60pp., which is also a very large number, though markedly lower compared to contributions made by an employer. This reveals a key detail: the spread of occupational pensions throughout the Danish population is the key driver of higher pension wealth.

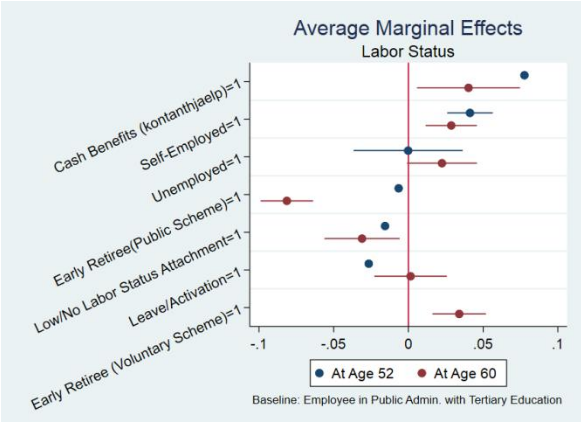
5.3. Determinants of being in the rest group – a risk approach

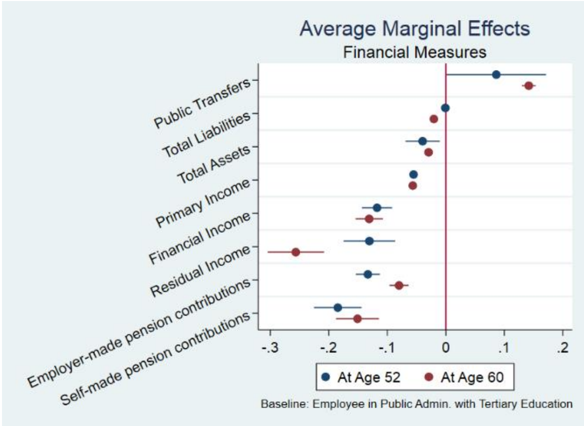
We now turn to an evaluation of the residual group using a different approach than the one seen in the previous sub-section. We investigate the determinants of belonging to the residual group according to a probabilistic model. For this, we use a Probit model as described in Section 4, focusing on definition 1. We regress the Definition 1 residual group classifier onto the regressors specified in Table 4 for the newly retired aged 65 or 66. We fit the model for sets of regressors measured from age 52 to age 64. Appendix D shows the regression tables for ages 52, 58 and 64 with Probit coefficients (appendix E also shows the block-wise addition of variables and the associated Wald tables). The figures below summarize the average marginal effects for ages 52 and 60:

Figure 10 – Marginal effects at ages 52 and 60









Above, we separate marginal effects according to the same regressor structure presented before. Starting with basic characteristics at the top left quadrant of Figure 10, being male, non-immigrant, having tertiary education and being in a couple are all characteristics associated with a lesser probability of being dependent on pension supplements at retirement. It should be underlined that, in terms of basic personal characteristics, the strongest effects can be seen from education, where those that have secondary education or less have an increased risk of belonging to the residual group in the range of 15 to 24pp. Recall that these marginal effects within education levels are controlled for all other personal characteristics. That means that education itself is a strong driver of under-saving behavior. The idea is not extreme, as education should somewhat be a good proxy for financial knowledge.

In terms of work sector, the baseline category is working for the public administration. This baseline was chosen because it generally has strong implementation of occupational pensions. This is reflected in that, not being in the public administration is associated with a higher probability of belonging to the residual group, as Figure 10 (bottom left quadrant) shows. The smallest impacts to the probability of belonging to the residual group are observed among workers in the education and finance and insurance sectors, where marginal effects are in the order of 10pp. The biggest impacts are seen among those working in:

- Manufacturing;
- Mining and quarrying;
- Accommodation and food services;
- Construction;
- Wholesale, retail;
- Vehicle repair.

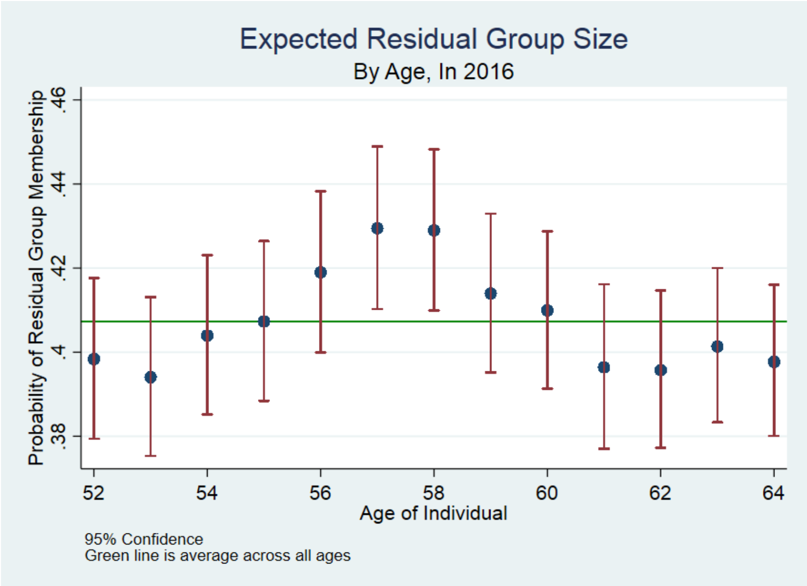
Margins for these sectors are in the range of 25 to 30pp at age 52. These sectors do not exactly match the ones found under the Probit models of those aged 25-44 in sub-section 5.2. The sectors in common are mining and quarrying and accommodation and food services. Most sectors have increased average risk of being part of the residual group upwards of 20 percentage points. Again, we underline that this increased risk comes after controlling for things like income level. It is not income that is driving this result, but the prevalent occupational pension contractual setup within these sectors coupled with low active saving decisions.

Looking at the top right quadrant of Figure 10, being unemployed with unemployment insurance does not reveal a significant increase in risk but being dependent on cash transfers (*kontanthjælp* in Danish) does, increasing risk by roughly 8 pp. at age 52, dropping to 4 pp. at age 60. Being self-employed is not largely associated with an increased risk of belonging to the residual group. Belonging to this labor category is associated with an increase in the probability of being in the residual group of 3 to 4pp.. SFI (2014) and ATP (2015) report that the self-employed group has low average pension contributions. According to that metric, one could be inclined to say that the self-employed are at higher risk of belonging to the residual group. But with the probabilistic approach, which accounts for income and wealth, being self-employed is not inherently associated with a markedly higher risk of becoming dependent on pension supplements. The prevalence of self-employed people in the residual group is thus driven by other events, such as low income, than the institutional setting they face in terms of saving for retirement. Surprisingly, becoming an early retiree under the public insurance scheme is not associated with an increase in probability of being in the residual group, even though those that become early retirees under the public scheme can only do so if their work capacity is reduced. Becoming an early retiree at age 60 is associated with a negative marginal effect of 8pp. Though entering the *voluntary* early retirement scheme *efterløn* is also associated with increased risk in the order of 3pp.

In terms of financial measurements, the only item associated with increased risk of under-saving naturally is being dependent on public transfers. Those that actively save in other financial assets are also less at risk, where for each 100.000kr. in financial income decreases the risk of belonging to the residual group by around 12pp. Similarly, for each 100.000kr. extra in primary income decreases residual group risk by roughly 6pp. In terms of pension contributions, the marginal effects are similar between contributions made by the employer and ones voluntarily made. Contributions made earlier in life have stronger marginal effects, where contributions made at age 52 reduce risk by 13-19pp. for every 100.000kr. of contributions, but contributions made at age 60 only decrease risk in the order of 8-16pp. In terms of automatic vs. voluntarily made contributions, the latter is associated with stronger decreases to the probability of being in the residual group.

We now project each Probit model, derived for characteristics recorded at ages 52-64 of the currently newly retired in 2016 onto individuals aged 52-64 in 2016. This yields an estimated probability curve for each age-cohort. The average probability for each age-cohort is then the estimated residual group size. We plot the average probability for each age-cohort below:

Figure 11 – Forecasts of residual group size by age cohort



The expected residual group size for those aged 52-64 in 2016 is 40.7% (the green line in Figure 11). This contrasts with the residual group size among new retires aged 65-66 in 2016 of 43.4%. Thus, a drop in the residual group size is expected. But the analysis reveals an interesting characteristic beyond the mean probability. The image also reveals an interesting "carousel" shape to the evolution of the expected residual group size over each age-cohort. Interestingly, the age-cohorts that have an expected residual group size above the overall average (the green horizontal line) are exactly the cohorts aged 47-53 in 2009, the year where the unemployment rate nearly doubled in Denmark after the 2008 financial crisis hit. Coupling with the fact that the age cohort 45-54 is the one making the most pension contributions among all age cohorts (see Figure 3, under Definition 3, yellow line), our risk model picks up exactly that, an increased risk of belonging to the residual group due to the economic crisis. For all other ages, the impact of the economic crisis wasn't so felt, as all other age points are below the overall average.

One, the risk model shows here its power in considering each age cohort idiosyncratically. Second, given that the increased expected residual group size for those aged 57 and 58 in 2016 is statistically significantly different from the overall mean, the government should expect a slight increase in costs with pension supplements when these individuals retire.

Another important question is in regard to the shape of the probability forecast distribution. Is it the case that most individuals bundle at the middle, or are most people bundling at the extremes? In the former case, policy directed at increasing saving is likely to be more effective at tilting individuals to lower risk groups. But if most individuals are bundling at the extremes, more structural, deep policy reforms will likely be required to shift the residual group paradigm.

Figure 12 – Forecast Probability distribution over age

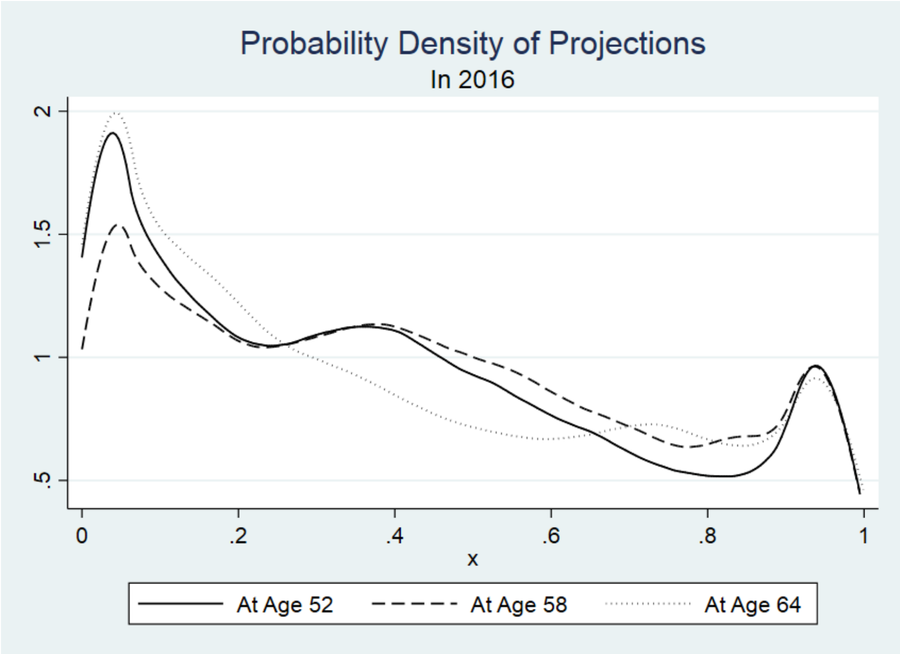


Figure 12 shows the probability forecast distribution density at three age points, 52, 58 and 64. The distribution at all ages can be found in Appendix G. From age 52 to age 64, the distribution of risk of belonging to the residual group flattens out in the middle. This makes sense, since at ages far away from the state pension age, there is still a higher degree of uncertainty on whether an individual will receive a full pension supplement. Even though the average expected residual group size is relatively stable around 40%, as shown in Figure 11 above, many more people come from the middle point of the distribution to the lower risk side than to the higher risk side of it, as Figure 12 reveals. Note that the cohort aged 58 is the one at the top of the “carousel” shape in Figure 11. This is reflected in the distribution density, where less individuals are bundled towards probability 0, and more accumulate over the probability range of 40-90%.

In terms of goodness-of-fit, we focus on fit measurements that are focused on the classification power of the model. Here, note that our outcome variable of interest, eligibility for pension supplements at retirement (Definition 1), is relatively balanced between negative and positive outcomes. As such, measuring goodness-of-fit with the area under the ROC curve and accuracy/precision/sensitivity should properly inform us on the classification power of the model. At an age closer to the retirement age, the fit is expected to improve since there is less uncertainty in terms of how individual characteristics determine residual group membership risk. We obtain an area under ROC curve of 0.815 and an accuracy of 73.6% for our model at age 52, both values revealing good classification power for our most age-distant projection model. For the model at age 64, the area under the ROC curve improves to 0.858 and an accuracy of 78.4% for our age 64 model. See Appendix F for few other goodness-of-fit measures.

Having the probability forecast distribution, we can now evaluate the types of individuals at risk of belonging to the residual group at retirement. We start by splitting the distribution equally over three regions: individuals with probabilities lower than 33.3% are labeled as “Low Risk”, between 33.3% and 66.7% are labeled “Mid Risk” and between 66.7% and 100% are labeled “High Risk”. We then reveal group characteristics across these three groups.

Figure 13 - Gender

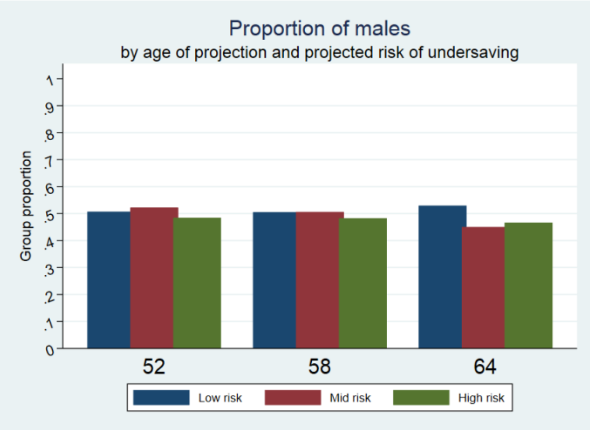
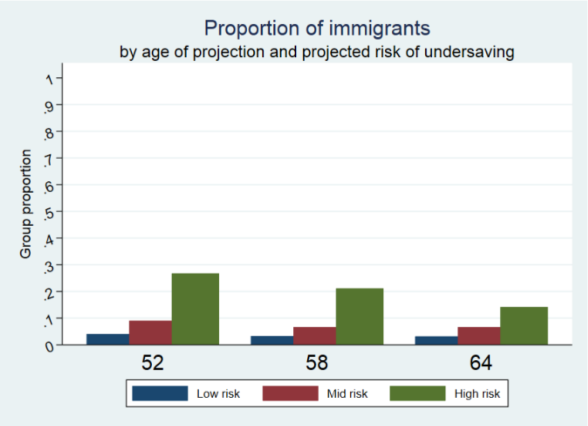


Figure 14 – Immigration status



We start by showing gender over each risk group. Figure 13 reveals quite a balanced result in terms of proportions of females over each risk group, though for the age group 64, higher risk groups have proportionally more women in them in a 45-55 ratio to men. Figure 14 shows that immigrants are overrepresented in the higher risk group and underrepresented at lower risk. Recalling that the size of immigrants in Denmark stands at around 12% of the population, immigrants compose roughly the same proportion of the high-risk group among those aged 64 in 2016, increasing to nearly 30% for those aged 52 in 2016. This increasing trend is consistent across all projected ages. Thus, this can be an indication of a trend of immigrants being amongst the lowest savers in Denmark.

In terms of education, the marginal effects already indicated that low education is a relevant determinant of residual group membership. This result is further confirmed in Figure 15, where half of the group at high risk is comprised of individuals with primary or less education. Very little individuals with a university degree (short- or long-cycle) are a part of the high-risk group. A very prominent educational group in Denmark, those with vocational education (as well as normal secondary education), compose almost half of the low risk group and roughly 60-65% of the middle risk group. These individuals are thus prime candidates for targeted action, as most of them are on the fence between becoming part of the residual group and not. Figure 16 shows that couples are overwhelmingly represented in the low risk group. On the other side of that coin, singles are overrepresented in the high-risk group. Among those single, the proportion of divorcees and widowers does not significantly change across risk. Overall, one can hypothesize that being in a couple offers an important insurance mechanism between partners, or similarly, higher economies of scale in sharing a life in terms of time and money spent on certain activities that benefit both partners in the couple.

Figure 15 -Education

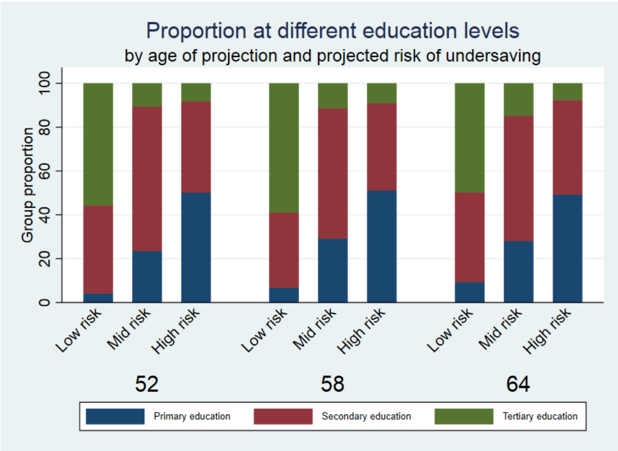


Figure 16 – Couple/Marital Status

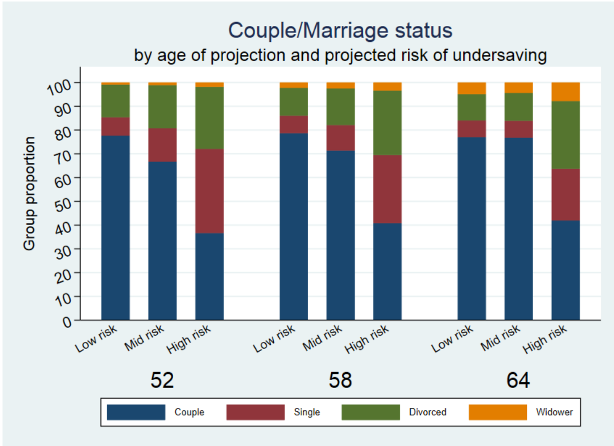
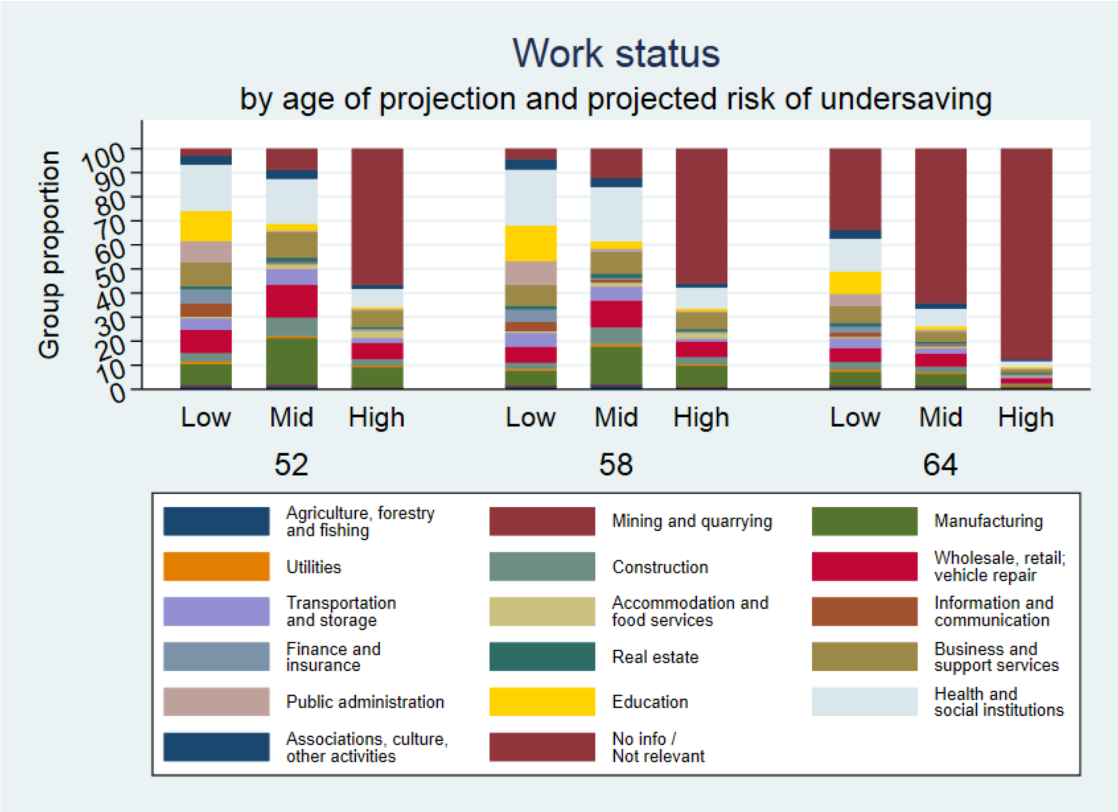


Figure 17 below shows work sectors by proportional representation over each risk group. The main takeaway is that the top five work sectors that most represent the higher risk group are:

- Individuals with low work sector attachment or early retired individuals
- Manufacturing
- Health and social institutions
- Business and support services
- Wholesale, retail, vehicle repair

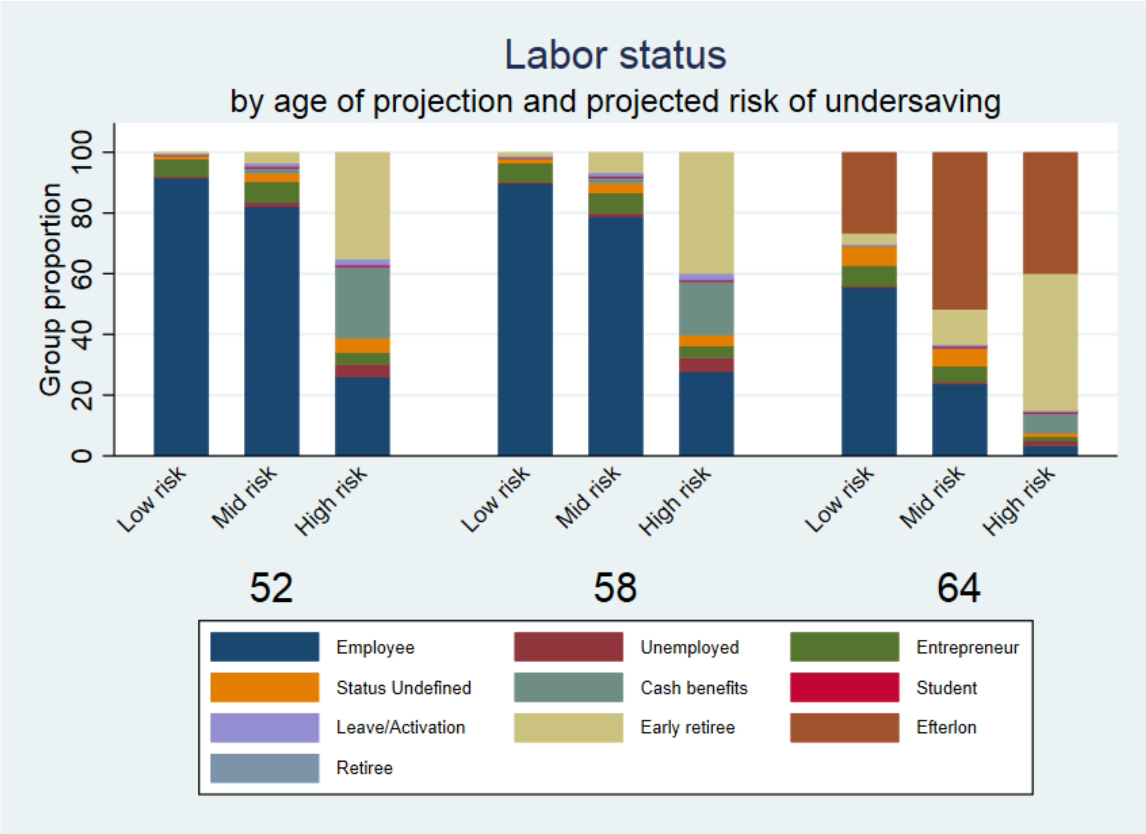
These work sectors do not exactly correspond to the most part with the results seen with the marginal effects. It should be noted that even if a work sector has strong representation in the higher risk group, it can also have strong representation in the other risk groups. The sector "Health and social institutions" is an example of this, where it is actually more represented in lower risk groups compared to the high one. This observation is congruent with what was found before, in that the sector "Health and social institutions" actually had one of the smallest marginal effects. Thus, care is warranted when drawing conclusions out of group proportions. The risk approach, and the associated marginal effects control for the entire set of individual characteristics.

Figure 17 – Work Sector Over Risk



As for labor status and risk shown in Figure 18, employees are overrepresented at low and middle risk. This likely stems from the quasi-mandatory occupational pensions. At high risk and at age 52, early retirees dominate the group. Nonetheless, employees still comprise just over 20% of the highest risk group. This shows a combination of three possible mechanisms at play: occupational pensions are not being completely spread among all employees; these high-risk employees have a very unstable relationship with the labor market over time; or the contribution rates set in the contracts of these high risk employees is not high enough. At age 64, just before retirement, the high-risk group is almost completely dominated by individuals in both early retirement schemes, public and *efterløn*. This is, again, likely capturing bad outcomes in life in terms of either capacity to work, or susceptibility to unemployment (as *efterløn* is conditional on being unemployed). Note that individuals in the *efterløn* scheme also have strong presence in the middle and low risk groups, so that the scheme is not fundamentally associated with low saving behavior, on average.

Figure 18 – Labor Status Over Risk



6. Discussion

6.1. Risk-model performance versus threshold techniques

Since Definition 1 is a certain measurement, the performance of definitions 2 and 3 can be measured by tabulating them against the certain outcome in Definition 1. To achieve this, we register residual group membership for Definitions 2 at age 64 for those newly retired in 2016 aged 65/66, the definition used in SFI (2014). For Definition 3, based on pension contributions only, we have data spanning 2002-2016 (we lose 2000 and 2001 since we are averaging 3 years of past contributions). This means that we can register contributions of the currently newly retired in 2016 when they were 52 years old and forward. We limit the age range to 52-59 years of age as in ATP (2015). The prediction quality is then averaged over each prediction quality over the age range.

The following table summarizes the performance of the residual group definitions in Table 5 vs. the performance of the Probit risk model:

Table 7 – Accuracy comparison between methods

	Measurement	Rate
Risk model (At ages 52-64)	Wrong predictions	**24.1%**
	False positives	10.8%
	False negatives	13.3%
Risk model (At age 64)	Wrong predictions	**21.7%**
	False positives	9.7%
	False negatives	12.0%
Definition 2 (At age 64, SFI 2014)	Wrong predictions	**20.8%**
	False positives	16.2%
	False negatives	4.6%
Definition 3 (At ages 52-59, ATP 2015)*	Wrong predictions	**42.2%**
	False positives	39.0%
	False negatives	3.2%

Note: We do not have data to replicate the range in ATP (2015) of 30-59.

We see that the classification power of our risk approach is slightly inferior to the performance of definition 2. Definition 3 displays the worst performance of all. Over the age range 52-64, the risk model predicts wrongly 24.1% of the time. Akin to the age range considered in SFI (2014), if we restrict the prediction to the age of highest certainty, age 64, the risk model's wrong predictions drop to 21.7%. This value is contrasted with 20.8% of wrong predictions with Definition 2 and a much higher value of 42.2% for Definition 3. These results are in line with our conclusions on the instability of Definition 3 as opposed to Definition 2. Also important is that false negatives and false positives are balanced with the risk approach, something that is not observed in Definitions 2 and 3. These two definitions tends to overclassify individuals as members of the residual group. They wrongly capture individuals that save enough, though irregularly.

6.2. Saving outside of pension accounts

We now develop further Definition 2 in Table 5 by re-considering what types of wealth count for the wealth threshold. The idea is to explore whether people are saving in other accounts that are not earmarked for retirement. For this, we develop three new definitions: One that considers the net financial position, excluding housing wealth and mortgage debt; Another that adds to this 20% of net housing wealth, where the idea is that people may partially downgrade their housing situation at retirement when, for example, a bigger house in the city area makes less sense; Lastly, a definition that adds 100% of net housing wealth instead of 20%, to evaluate the situation where a household taps into their housing wealth completely for consumption purposes through, for example, a reverse mortgage.

The definitions are laid in table 8 below:

Table 8 – Definitions over broad wealth measurements

Definition 5	Same as Definition 2, but wealth is pension and net financial wealth (excluding mortgage debt)	Stock	Non-students, non-retirees Years: 2014-2016
Definition 6	Same as Definition 2, but wealth is pension, net financial (excl. mortgage debt) and 20% of net housing wealth	Stock	Non-students, non-retirees Years: 2014-2016
Definition 7	Same as Definition 2, but wealth is pension, net financial (excl. mortgage debt) and 100% of net housing wealth	Stock	Non-students, non-retirees Years: 2014-2016

The assumption being made here is that these extra items of wealth are equivalent to pension savings in that they also get annuitized at retirement. This would mean that both the investment income generated by the stock and capital withdrawals from the stock would count to the means-testing of pension supplements. In reality, wealth stocks do not count to the means-testing, only the income generated by it. For housing, the capital gains from selling can count to the means-testing, but not uniformly over the retirement period, only in the year the sell is made. As such, we are performing a policy experiment of considering what would happen if all these extra items of wealth were totally considered in the means-testing of the pension supplement. This also shows us a proxy measurement of under-saving, given that we are considering the entire household balance sheet. If higher income households are below the wealth threshold implied by equations (2) and (3) across all asset types, that is strong evidence for under-saving behavior.

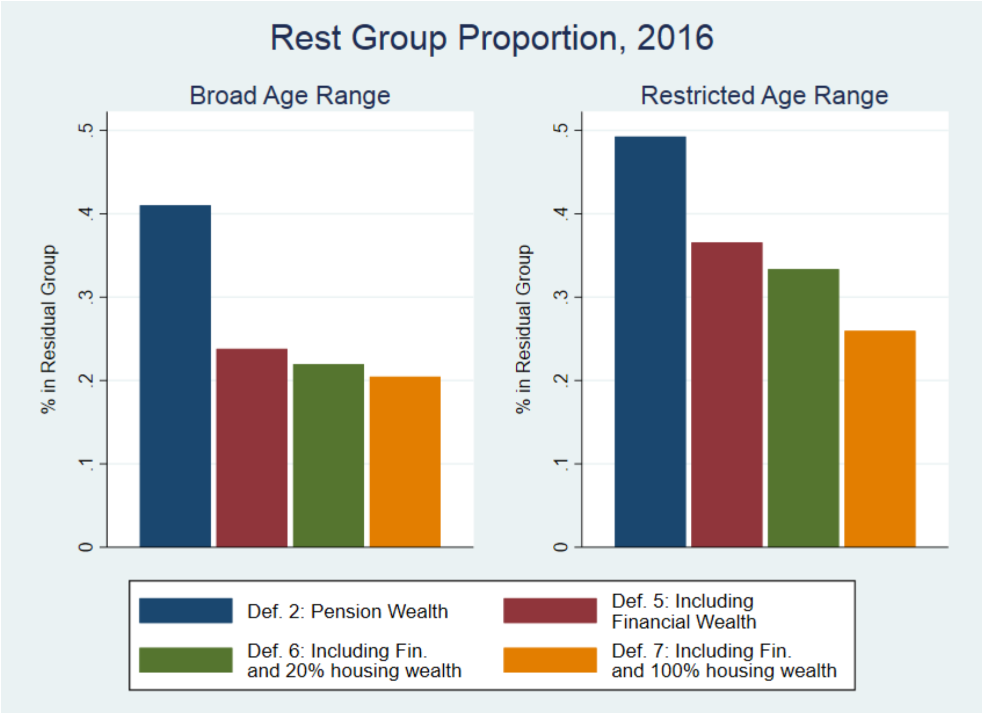
Introducing non-pension wealth into the means-testing of pension supplements would introduce a few significant incentives into the saving behavior of individuals:

1. It would add incentives to save more through pension accounts as opposed to non-pension ones, since it would raise the effective tax rate for those individuals around eligibility to receive pension supplements. For these individuals, non-pension savings would effectively get double-taxed, since private savings are generally accumulated from after-tax income.
2. It would re-direct resources to those in real need, since the entire balance sheet is now considered to evaluate supplementation necessity of old-age income.
3. Defining such means-testing would be technically difficult as well as potentially unpopular. When defining how much of non-pension wealth to consider in the means-testing, a part of the answer should consider that not all wealth is expected to be consumed by its holder, but given in the form of bequests to descendants/spouse. A solution would be to create an allowance under which *select* wealth holdings would not count for means-testing. Choosing the allowance amount and asset types that do not count to the means-testing is the technically difficult part, where a real solution would likely be a discretionary political choice.
4. One would need to specify how debt items would count in the means-testing, along with asset items. Care should be taken here, as considering net wealth instead of assets only would

introduce incentives to do leveraged purchases, the most obvious example being housing purchases with a mortgage.

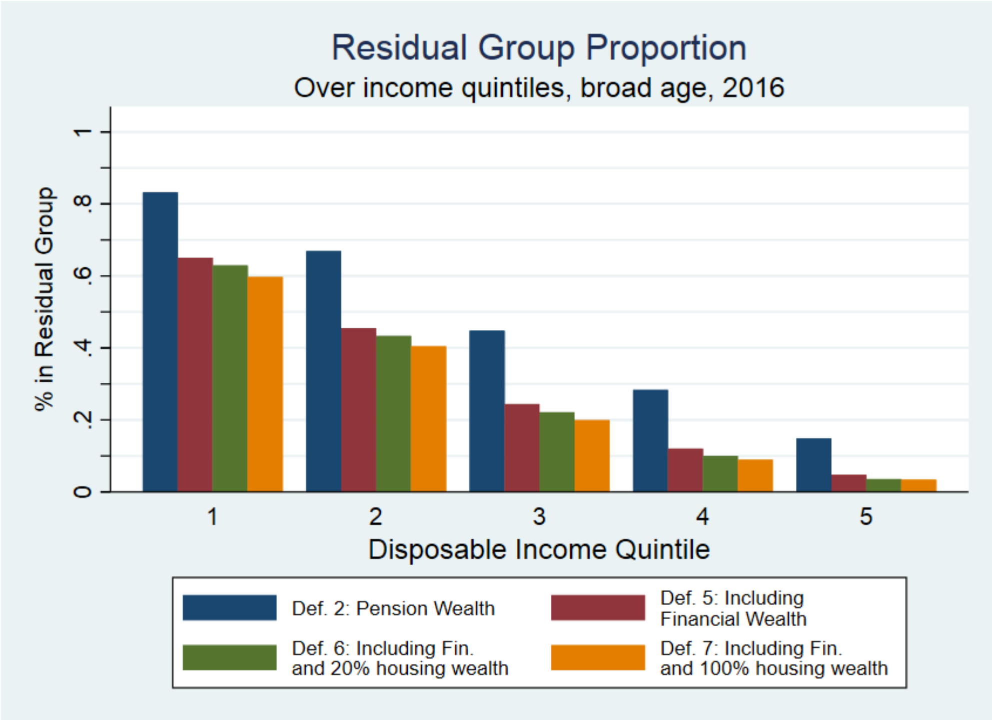
Figure 19 below reveals residual group size under the new definitions 5-7 (against definition 2):

Figure 19 – Residual group size estimates under broader wealth



If non-pension financial wealth items were to be completely considered in the means-testing of the pension supplement, the estimated residual group size would drop from a range of 40-50% to a range of 23-36%, a very significant drop. This is evidence that at least some people are saving throughout their lives into non-pension accounts between the ages 25-64, but also carrying those savings into retirement up to age 64, as shown in the right-hand side of Figure 19. When 20% of net housing wealth is considered, the residual group size lowers even further to a range of 21-33%. Even though households have relatively high mortgage debt, it is accompanied by even bigger housing wealth on average. The same is observed for definition 7. At age 64, if the average household applies a reverse mortgage on the entire housing equity, the estimated residual group size drops to 26%, where the housing wealth is alone responsible for removing 10pp. of individuals out of the residual group. Thus, access to financial devices, such as reverse mortgages, could potentially have a tremendous impact in providing extra income at retirement, including those eligible to the full public pension supplement.

Figure 20 – Residual group with broader wealth over income quintiles



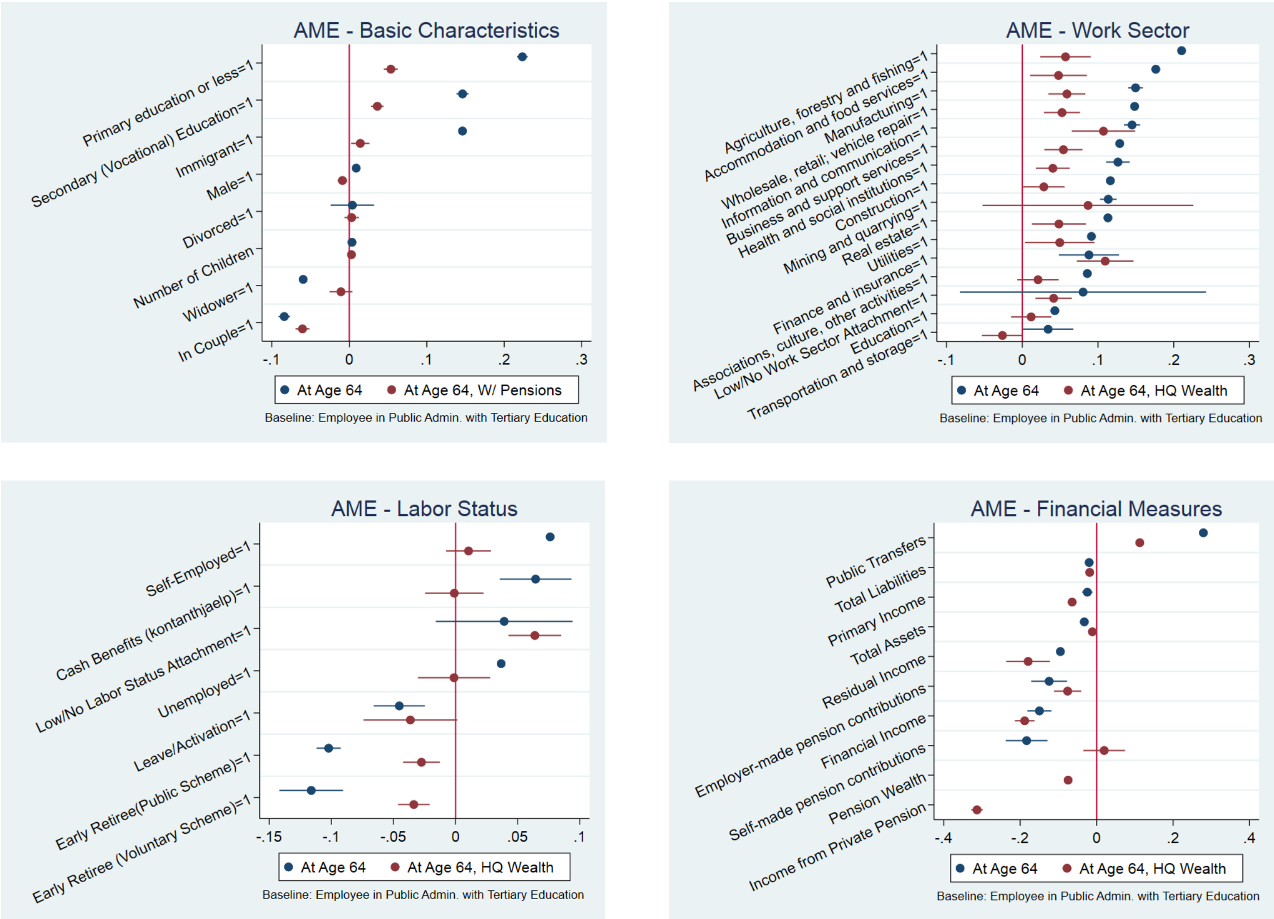
The question remains on how these new definitions perform over disposable income quintiles. Since we are virtually considering the entire balance sheet of households, and given that a 10% saving rate is enough for the average household to not be considered part of the residual group for a contributory career of 40 years, individuals that are still labeled as part of the residual group under definitions 5-7, for the 3rd quintile and up, are likely to be under-savers. Using this "proxy", the degree of under-saving around the median income in Figure 20 is around 20%. Around the 4th quintile of income, this degree drops to around 10%, and at the very top of the income distribution around 4% of individuals seem to be under-saving to the point where they will become entitled to pension supplements. Thus, under-saving behavior on the top half of the income distribution is significantly high, given that our definition of under-saving is being eligible to full pension supplements. This means that higher income individuals just before retirement will suffer a tremendous income shock upon retirement, coming to living conditions associated with the very bottom of the income distribution.

6.3. Including pension wealth in risk model

In developing the risk approach in Section 5, we were constrained by data availability where pension asset data was not available. We instead used measures of total assets and liabilities that did not include pension assets. The measure of pension assets is a key regressor in the risk modelling we develop in this paper. As a robustness check, we can, first, evaluate the margins resulting from the risk model at age 64 by adding a pension wealth regressor and compare it with the margins observed in Section 5, and second, showcase how this model performs in estimating the probability of belonging to the residual group for those aged 64 in 2016.

The estimation results are summarized in Appendix D, alongside the tabled results for the Probit models developed in Section 5. The margins are shown in Figure 21 below:

Figure 21 – Average Marginal Effects, LQ vs. HQ wealth data



As can be observed in Figure 21 above, adding pension wealth to the model can have some significant effects on marginal effects, compared to the model without it. It reduces the effects stemming from being an immigrant and from education. Low education is still associated with a higher probability of belonging to the residual group, but now only bringing an increase of around 5-6pp. Being an immigrant is no longer inherently associated with a significant increase in the probability of being in the residual group. In terms of the work sector, all the marginal effects are reduced across the board, where most sectors now have marginal effects of around 3-6pp. For labor status, controlling for pension wealth turns the average marginal effects of being self-employed, on cash benefits, having low labor market attachment and unemployed insignificant and close to 0. In terms of financial marginal effects, the inclusion of pension wealth reduces the marginal effect from self-made pension contributions, though the one for employer-made contributions remain relatively the same. It seems that, at age 64, voluntarily contributing to a pension account is irrelevant for the determination of residual group membership after controlling for the pension stock one holds. The marginal effect for employer-made contributions is not reduced, remaining at around negative 8 percentage points for each 100.000kr. in contributions. This regressor is also capturing the fact that individuals with this type of pension contribution are still working at the age of 64, hence its non-zero effect.

Another dimension that can be impacted by the model change is prediction power. Comparing goodness-of-fit focused on classification power between the models in Section 5 at age 64 and the model described in this section, the Pseudo-R2 marginally improved from 0.319 to 0.476, the area under the ROC curve suffered a similar improvement from 0.86 to 0.93, and the accuracy also improved from 78.3% to 84.5%. In Section 5's model, the percentage of false positives and negatives was respectively 9.7 and 12.0%. Under the current model, these numbers become 7.3 and 8.2%. Thus, the model became better at avoiding false negatives and false positives. This difference likely stems from controlling for pension assets, in that individuals with low labor market attachment and labor income at age 64 could be classified as a residual group member with Section 5's model, though that individual could already have accumulated enough (pension) assets, loosing eligibility to pension supplements. The model presented here controls for this type of situations.

7. Conclusion

In this paper, we analyzed individuals in Denmark with low savings as measured by those that are, or will likely become entitled to the full public pension supplement *folkepensionstillæg*. This group is termed the residual group in pensions in Denmark. We utilize two frameworks to analyze this issue. The first is based on threshold techniques for pension contributions and pension wealth, inspired by SFI (2014) and ATP (2015). The second is a framework based on a probabilistic approach, where characteristics associated with residual group "membership", registered among those currently retired, is used to forecast the risk associated with belonging to the under-saving group among those in working age. We find an expected residual group size of 40 to 45% of those aged 25-64 in 2016 as measured by threshold techniques with pension contributions and pension wealth, respectively. Using the risk approach, we estimate the residual group size to be almost 41%.

Furthermore, we identify the types of individuals at risk of belonging to the residual group. Overall, females, immigrants, individuals with less than a university degree, and single individuals are the most at risk. The insurance dimension between partners in a couple seems to be relevant for explaining residual group membership. This could be interplaying with gender, where, if females earn less income on average, they will be more likely to be part of the residual group, and more so if they remain single. The system currently characterizes individuals as being in a couple if they share an economy, it is not based on official marital status. Separations between two individuals with different income histories can imply that one of the partners is left in a precarious situation, since unofficial couple separation is not regulated by law in terms of post-separation transfers. Another level of intervention is to increase financial education among those with lower education. For example, exposing individuals at a young age to knowledge on how the pension system works and the problematic of saving for retirement could be an effective strategy since the middle and high-risk groups are dominated by people with secondary education or equivalent. This is of course easier said than done, but the point being made here is that any financial education that takes place after the secondary education level will not reach most of its intended audience.

In terms of work sector, the sectors most at risk are:

- Manufacturing;
- Mining and quarrying;
- Accommodation and food services;
- Construction;
- Wholesale, retail;
- Vehicle repair.

Three possibilities emerge: One, these sectors have contract/job types that do not allow for setting up occupational pensions; Two, there is generally no culture of adding occupational pensions to labor contracts; Three, there are occupational pensions setup in labor contracts, but contribution rates are too low. In this last type of situation, a pervasive effect can happen, where an individual might know that the employer is making pension contributions, but the employee might not be aware that these contributions will not be enough to ensure proper income at retirement.

In terms of labor status, people most at risk are those receiving cash benefits and the self-employed. In terms of risk group composition, most of those in the high-risk group are individuals with low labor market attachment. As such, policies that aim at increasing labor market participation would be a good strategy to reduce the size of the residual group. A rather high portion of employees are also part of this high-risk group. Setting up a truly mandatory occupational pension system would capture these employees and reduce the size of the residual group, if automatic contribution rates are set high enough.

We show that our risk-based approach performs better in terms of its classification accuracy when pension assets are included in the model. In general, the risk approach offers a platform for policymakers and the like to inform and target policy directed at stimulating saving behavior, and as an early warning system for incoming generations of potential under-savers. We also considered other asset types as alternative sources of income at retirement. Namely, considering the full consumption of housing equity during retirement via, for example, a reverse mortgage can have a significant impact to the estimated residual group size, if that wealth was to be means-tested for the pension supplement. When looking at the entire balance sheet of households, we see some evidence of under-saving behavior among the top half of the income distribution of those aged 64 of 4 to 20% of this high-income group.

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Appendix A

Below, summary statistics can be found for our population in the year 2016. Details on the meaning of variables can be found in the data section. Note that this table reports values in 2016. Monetary values are de-scaled by 1,000 and displayed in 2016 DKK.

Table A1: Summary statistics

	Mean	SD
Male (%)	49.1	50.0
Immigrant (%)	12.0	32.5
Age	53.1	17.0
Education level obtained		
Primary or less (%)	24.1	42.8
Secondary and equivalent (%)	41.0	49.2
Tertiary (%)	32.7	46.9
Kids	1.742	1.149
Unmarried (%)	27.7	44.7
Married (%)	52.4	49.9
Divorced (%)	12.7	33.3
Widower (%)	7.2	25.9
Couple (%)	64.7	47.8
Main work sector (%) – among non-retirees		
1: Agriculture, forestry and fishing	1.5	12.2
2: Mining and quarrying	0.1	3.8
3: Manufacturing	9.3	29.1
4: Utilities	0.7	8.3
5: Construction	4.8	21.3
6: Wholesale, retail; vehicle repair	10.2	30.3
7: Transportation and storage	3.9	19.3
8: Accommodation and food services	2.3	15.0
9: Information and communication	3.3	17.8
10: Finance and insurance	2.5	15.6
11: Real estate	1.2	10.7
12: Business and support services	9.6	29.5
13: Public administration	4.4	20.6
14: Education	7.4	26.1
15: Health and social institutions	16.5	37.1
16: Associations, culture, other activities	3.2	17.7
17: Not relevant	19.1	39.3
Employee	52.0	50.0
Unemployed	1.2	11.1
Self-employed	3.8	19.2
Cash benefits (kontanthjaelp)	4.3	20.3
Student	2.7	16.1
Sick/parental leave and education activation	1.0	10.0
Early retirement (public)	5.3	22.3
Early retirement (efterlon scheme)	1.9	13.5
Retiree	25.1	43.4

Undefined main status	2.9	16.7
Disposable Income	248.3	819.1
Disposable Income (equivalized)	283.4	679.0
Gross Income	347.7	1574.0
Gross Income (equivalized)	395.7	1215.3
Primary income (equivalized)	267.6	1018.0
Salary income (equivalized)	247.5	309.2
Self-employment income (equivalized)	18.3	192.1
Honorarium income (equivalized)	1.8	950.0
Public transfers (equivalized)	77.4	76.0
Private pension income (equivalized)	31.2	82.1
Financial income (equivalized)	17.6	631.1
Residual income (equivalized)	1.9	149.2
Total Net Wealth (equivalized)	2082.4	10800.0
Pension wealth (equivalized)	1208.4	1725.6
Housing wealth (equivalized)	1140.6	3224.2
Mortgage debt (equivalized)	542.1	1658.0
Financial Wealth (equivalized)	370.9	10000.0
Other debts (equivalized)	152.1	1090.0
Car wealth (equivalized)	56.7	68.0
Pension contributions		
Employer-made	32.2	47.2
Voluntarily-made	3.0	24.2
Zero employer-made contributions (%)	30.0	
Zero voluntarily-made contributions (%)	86.7	
Housing wealth / Disposable income (%)	402%	
Mortgage debt / Disposable income (%)	191%	
Pension wealth / Disposable income (%)	426%	
Under poverty line (ages 25-101) (%)	8.1	
Number of individuals aged 25-100	4074652	
Number of families	2773839	

Note: Financial items are displayed in 1000's, 2016 DKK; poverty line defined as 50% of equivalized median income/assets as a past three-year average.

Appendix B – Wealth thresholds

These are the thresholds used in definitions 2, 4, 5, 6 and 7. They are derived as the natural progression implied by equation (3) in the text. The thresholds are derived for both singles and couples. We do not use equivalized wealth since these thresholds are being checked against legally defined thresholds in the means-test for pension supplement eligibility.

Table B1

Age	Singles	Couples	Age	Singles	Couples
26	25249	50682	46	586759	1177807
27	50750	101871	47	617875	1240267
28	76506	153571	48	649302	1303351
29	102520	205789	49	681044	1367067
30	128794	258529	50	713103	1431419
31	155330	311796	51	745483	1496415
32	182132	365596	52	778186	1562061
33	209202	419934	53	811217	1628364
34	236543	474815	54	844578	1695329
35	264157	530245	55	878272	1762964
36	292047	586229	56	912304	1831276
37	320216	642773	57	946675	1900271
38	348667	699883	58	981391	1969955
39	377402	757564	59	1016453	2040337
40	406425	815821	60	1051866	2111422
41	435738	874661	61	1087634	2183218
42	465344	934090	62	1123759	2255732
43	495246	994113	63	1160245	2328971
44	525447	1054736	64	1197096	2402943
45	555951	1115965	65	1234316	2477654

Appendix C – Regressors used

The following table shows the entire list of regressors used in the Probit models. Note that the variable for the voluntary early retirement scheme only becomes available at the age of 60, so that it is included in the model at ages 60 and over.

Table C1: Regressors used in Probit models

Grouping	Variables	Definition
Basic Characteristics	MALE	Dummy for sex being male
	IMMIGRANT	Dummy for not having Danish origins
	BASIC_EDUCATION	Dummy for having primary schooling or less
	MIDDLE_EDUCATION	Dummy for having secondary or equivalent education
	KIDS	Number of children
	COUPLE_xx	Being in a couple (economy sharing) at age xx
	WIDOW_xx	Widower not legally re-married (can be in a couple) at age xx
	DIVORCED_xx	Divorcee not legally re-married (can be in a couple) at age xx
Work Sector	WSECTOR_1_xx	Dummy if in Agriculture, forestry and fishing at age xx
	WSECTOR_2_xx	Dummy if in Mining and quarrying at age xx
	WSECTOR_3_xx	Dummy if in Manufacturing at age xx
	WSECTOR_4_xx	Dummy if in Utilities at age xx
	WSECTOR_5_xx	Dummy if in Construction at age xx
	WSECTOR_6_xx	Dummy if in Wholesale, retail; vehicle repair at age xx
	WSECTOR_7_xx	Dummy if in Transportation and storage at age xx
	WSECTOR_8_xx	Dummy if in Accommodation and food services at age xx
	WSECTOR_9_xx	Dummy if in Information and communication at age xx
	WSECTOR_10_xx	Dummy if in Finance and insurance at age xx
	WSECTOR_11_xx	Dummy if in Real estate at age xx
	WSECTOR_12_xx	Dummy if in Business and support services at age xx
	WSECTOR_14_xx	Dummy if in Education at age xx
	WSECTOR_15_xx	Dummy if in Health and social institutions at age xx
	WSECTOR_16_xx	Dummy if in Associations, culture, other activities at age xx
	WSECTOR_17_xx	Dummy if in Not relevant at age xx
Labor Status	UNEMP_xx	Dummy if Unemployed at age xx
	ENTREP_xx	Dummy if Self-employed at age xx
	CASHBENEFITS_xx	Dummy if Cash benefits (*kontanthjaelp*) at age xx
	STUDENT_xx	Dummy if Student at age xx
	SICK_EDUC_PARENTAL_xx	Dummy if Sick/parental leave and educ. activation at age xx
	EARLY_RETIREE_xx	Dummy if Early retirement (public) at age xx
	EARLY_RETIREE_EL_xx	Dummy if Early retirement (*efterlon* scheme) at age xx
	STATUS_UNDEFINED_xx	Dummy if Undefined main status at age xx
Wealth	RE_ASSETS_xx	Assets value in millions in 2016 DKK at age xx
	RE_LIABILITIES_xx	Liabilities value in millions in 2016 DKK at age xx
	RE_PENSION_WEALTH_xx	Pension assets that count to means-testing in 2016 DKK at age xx
Income	RE_PRIM_INC_xx	Primary income in 100.000 in 2016 DKK at age xx
	RE_PUB_TRANSF_xx	Public transfers in 100.000 in 2016 DKK at age xx
	RE_FIN_INC_xx	Financial income in 100.000 in 2016 DKK at age xx
	RE_RESID_INC_xx	Residual income in 100.000 in 2016 DKK at age xx
Pension contributions	RE_PCONT_OP_SUP_xx	Employer-made pension contributions in 100.000 in 2016 DKK at age xx
	RE_PCONT_PRIV_SUP_xx	Self-made pension contributions in 100.000 in 2016 DKK at age xx

Note: for regressions over the younger cohorts aged 25-34, these regressors are simply measured at their respective age in 2016.

Appendix D – Regression tables for projections

The Probit coefficient table follows for ages 52, 58, and 64. The last column is the same model as for age 64 in column 3, except that the pension wealth regressor is added to the model.

Probit coefficients for projections, exemplary ages

	(1) Age 52	(2) Age 58	(3) Age 64	(4) Age 64, HQ wealth
Definition 1				
Male=1	-0.184***	-0.144***	-0.0745***	-0.0432**
	(0.0135)	(0.0139)	(0.0149)	(0.0151)
Immigrant=1	0.256***	0.201***	0.180***	0.0691*
	(0.0275)	(0.0274)	(0.0292)	(0.0292)
Primary education or less=1	0.716***	0.629***	0.371***	0.257***
	(0.0186)	(0.0191)	(0.0213)	(0.0218)
Secondary (Vocational) Education=1	0.525***	0.469***	0.262***	0.177***
	(0.0167)	(0.0171)	(0.0197)	(0.0203)
In Couple=1	-0.311***	-0.314***	-0.290***	-0.291***
	(0.0179)	(0.0185)	(0.0221)	(0.0219)
Widower=1	0.0741	-0.0179	-0.0158	-0.0526
	(0.0473)	(0.0383)	(0.0373)	(0.0369)
Divorced=1	-0.0193	0.0122	0.0360	0.0148
	(0.0194)	(0.0205)	(0.0235)	(0.0234)
Number of Children	0.00157	0.0144*	0.0173*	0.0136*
	(0.00605)	(0.00621)	(0.00672)	(0.00674)
Agriculture, forestry and fishing=1	0.772***	0.681***	0.512***	0.280***
	(0.0511)	(0.0584)	(0.0849)	(0.0847)
Mining and quarrying=1	1.038***	0.950***	0.553	0.429
	(0.174)	(0.160)	(0.333)	(0.359)
Manufacturing=1	1.033***	0.938***	0.486***	0.290***

	(0.0312)	(0.0335)	(0.0609)	(0.0623)
Utilities=1	0.677***	0.700***	0.390***	0.243*
	(0.0812)	(0.0697)	(0.114)	(0.116)
Construction=1	0.908***	0.852***	0.333***	0.139*
	(0.0374)	(0.0390)	(0.0675)	(0.0686)
Wholesale, retail; vehicle repair=1	0.859***	0.790***	0.441***	0.258***
	(0.0330)	(0.0345)	(0.0591)	(0.0605)
Transportation and storage=1	0.592***	0.365***	0.0343	-0.128
	(0.0390)	(0.0397)	(0.0661)	(0.0671)
Accommodation and food services=1	0.923***	0.748***	0.427***	0.234*
	(0.0600)	(0.0620)	(0.0931)	(0.0945)
Information and communication=1	-0.0176	0.596***	0.610***	0.533***
	(0.0559)	(0.0581)	(0.106)	(0.110)
Finance and insurance=1	0.375***	0.361***	0.526***	0.546***
	(0.0463)	(0.0491)	(0.0959)	(0.0990)
Real estate=1	0.797***	0.673***	0.399***	0.237**
	(0.0567)	(0.0566)	(0.0885)	(0.0899)
Business and support services=1	0.816***	0.755***	0.417***	0.267***
	(0.0371)	(0.0364)	(0.0620)	(0.0640)
Education=1	0.322***	0.257***	0.132*	0.0570
	(0.0348)	(0.0365)	(0.0642)	(0.0666)
Health and social institutions=1	0.663***	0.575***	0.347***	0.198***
	(0.0304)	(0.0318)	(0.0556)	(0.0569)
Associations, culture, other activities=1	0.671***	0.506***	0.249***	0.101
	(0.0378)	(0.0422)	(0.0671)	(0.0683)
Low/No Work Sector	0.756***	0.597***	0.330***	0.196***

Attachment=1				
	(0.0433)	(0.0484)	(0.0562)	(0.0567)
Unemployed=1	-0.0435	0.234***	-0.0212	-0.00639
	(0.0411)	(0.0576)	(0.0737)	(0.0723)
Self-Employed=1	0.131***	0.0750*	0.0925*	0.0506
	(0.0274)	(0.0304)	(0.0441)	(0.0451)
Cash Benefits (kontanthjaelp)=1	0.178**	-0.0195	0.00251	-0.00520
	(0.0634)	(0.0613)	(0.0605)	(0.0586)
Student=1	-0.130	-0.317	0.717	0.897
	(0.130)	(0.234)	(0.416)	(0.503)
Leave/Activation =1	-0.102	-0.0871	-0.206*	-0.178
	(0.0616)	(0.0624)	(0.0945)	(0.0938)
Early Retiree(Public Scheme)=1	0.141**	-0.0126	-0.103**	-0.135***
	(0.0429)	(0.0455)	(0.0374)	(0.0377)
Early Retiree (Voluntary Scheme)=1			-0.234***	-0.166***
			(0.0319)	(0.0323)
Low/No Labor Status Attachment=1	-0.0361	-0.112*	0.263***	0.313***
	(0.0521)	(0.0533)	(0.0524)	(0.0535)
Total Assets	-0.130***	-0.0867***	-0.0853***	-0.0543***
	(0.0112)	(0.00761)	(0.0101)	(0.0101)
Total Liabilities	-0.0294	-0.0667***	-0.0817***	-0.0888***
	(0.0158)	(0.0140)	(0.0172)	(0.0175)
Primary Income	-0.215***	-0.288***	-0.400***	-0.313***
	(0.00659)	(0.00887)	(0.0154)	(0.0155)
Public Transfers	0.299***	0.388***	0.498***	0.550***
	(0.0214)	(0.0242)	(0.0266)	(0.0266)
Financial Income	-0.581***	-0.507***	-1.045***	-0.917***

	(0.0555)	(0.0454)	(0.0648)	(0.0655)
Income from Private Pension	-1.779***	-1.703***	-2.033***	-1.525***
	(0.0527)	(0.0368)	(0.0378)	(0.0413)
Residual Income	-0.501***	-0.913***	-0.836***	-0.873***
	(0.131)	(0.133)	(0.144)	(0.143)
Employer-made pension contributions	-0.483***	-0.324***	-0.684***	-0.370***
	(0.0372)	(0.0295)	(0.0781)	(0.0882)
Self-made pension contributions	-0.691***	-0.372***	-0.332*	0.0962
	(0.0792)	(0.0604)	(0.133)	(0.135)
Pension Wealth				-0.364***
				(0.0169)
Constant	-0.147***	0.183***	0.474***	0.736***
	(0.0429)	(0.0490)	(0.0733)	(0.0757)
Observations	59606	59899	60289	60289
Pseudo R^2	0.263	0.311	0.462	0.476
AIC	60145.5	56478.3	44471.1	43259.6
BIC	60514.3	56847.3	44849.4	43646.9
df_m	40	40	41	42
chi2	13605.5	13575.8	13282.0	12029.1
p	0	0	0	0

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix E – Block-wise addition of variables and their significance

Here, a nested testing of regressors is shown only for age 52:

	(1) Definition 1	(2) Definition 1	(3) Definition 1	(4) Definition 1	(5) Definition 1	(6) Definition 1
Pension-aged individuals receiving full pension supplement						
Male	-0.0699***	-0.0732***	-0.0780***	-0.0782***	-0.165***	-0.171***
	(0.0110)	(0.0116)	(0.0116)	(0.0117)	(0.0133)	(0.0133)
Immigrant	0.711***	0.388***	0.384***	0.362***	0.326***	0.308***
	(0.0247)	(0.0258)	(0.0258)	(0.0260)	(0.0263)	(0.0268)
Primary education or less	1.222***	0.920***	0.900***	0.882***	0.767***	0.771***
	(0.0155)	(0.0168)	(0.0168)	(0.0170)	(0.0183)	(0.0184)
Secondary (Vocational) Education	0.774***	0.660***	0.642***	0.635***	0.553***	0.555***
	(0.0140)	(0.0150)	(0.0151)	(0.0151)	(0.0166)	(0.0166)
In Couple	-0.513***	-0.336***	-0.328***	-0.321***	-0.339***	-0.339***
	(0.0158)	(0.0172)	(0.0172)	(0.0173)	(0.0175)	(0.0175)
Widower	-0.231***	-0.169***	-0.166***	-0.136**	-0.139**	-0.137**
	(0.0395)	(0.0429)	(0.0427)	(0.0432)	(0.0440)	(0.0440)
Divorced	0.0766***	0.0108	0.00473	-0.0149	-0.0179	-0.0222
	(0.0175)	(0.0186)	(0.0187)	(0.0188)	(0.0190)	(0.0191)
Number of Children	-0.0160**	0.00777	0.00694	0.00997	0.00544	0.00377
	(0.00541)	(0.00575)	(0.00576)	(0.00583)	(0.00591)	(0.00595)
Primary Income		-0.239***	-0.207***	-0.204***	-0.189***	-0.188***
		(0.00588)	(0.00612)	(0.00632)	(0.00646)	(0.00661)
Public Transfers		0.296***	0.307***	0.284***	0.291***	0.293***
		(0.0164)	(0.0165)	(0.0169)	(0.0178)	(0.0207)

Financial Income	-0.696***	-0.583***	-0.372***	-0.411***	-0.400***
	(0.0443)	(0.0447)	(0.0451)	(0.0450)	(0.0448)
Residual Income	-0.483***	-0.435***	-0.413***	-0.444***	-0.444***
	(0.0762)	(0.0763)	(0.0766)	(0.0765)	(0.0767)
Employer-made pension contributions		-0.469***	-0.473***	-0.470***	-0.453***
		(0.0341)	(0.0350)	(0.0358)	(0.0358)
Self-made pension contributions		-0.724***	-0.509***	-0.591***	-0.628***
		(0.0692)	(0.0692)	(0.0706)	(0.0709)
Total Assets			-0.125***	-0.132***	-0.135***
			(0.0107)	(0.0109)	(0.0109)
Total Liabilities			0.00984	0.00413	-0.00280
			(0.0142)	(0.0144)	(0.0144)
Agriculture, forestry and fishing				0.885***	0.818***
				(0.0495)	(0.0510)
Mining and quarrying				1.004***	1.005***
				(0.173)	(0.174)
Manufacturing				1.042***	1.039***
				(0.0310)	(0.0310)
Utilities				0.690***	0.691***
				(0.0812)	(0.0812)
Construction				0.928***	0.909***
				(0.0368)	(0.0370)
Wholesale, retail; vehicle repair				0.881***	0.865***
				(0.0326)	(0.0327)
Transportation and storage				0.598***	0.586***
				(0.0385)	(0.0386)
Accommodatio				0.946***	0.915***

n and food services		
	(0.0587)	(0.0591)
Information and communication	-0.0241	-0.0269
	(0.0557)	(0.0558)
Finance and insurance	0.359***	0.356***
	(0.0459)	(0.0459)
Real estate	0.821***	0.805***
	(0.0566)	(0.0565)
Business and support services	0.827***	0.813***
	(0.0367)	(0.0368)
Education	0.329***	0.329***
	(0.0345)	(0.0345)
Health and social institutions	0.683***	0.679***
	(0.0302)	(0.0302)
Associations, culture, other activities	0.686***	0.674***
	(0.0373)	(0.0374)
Low/No Work Sector Attachment	0.746***	0.740***
	(0.0342)	(0.0422)
Unemployed		-0.000546
		(0.0404)
Self-Employed		0.139***
		(0.0270)
Cash Benefits (kontanthjaelp)		0.261***
		(0.0622)
Student		-0.0600
		(0.130)
Leave/Activation		-0.0908

						(0.0597)
Early Retiree (Public Scheme)						-0.0223
						(0.0405)
Low/No Labor Status Attachment						-0.0533
						(0.0512)
Constant	-0.472***	0.211***	0.201***	0.286***	-0.316***	-0.313***
	(0.0188)	(0.0304)	(0.0306)	(0.0320)	(0.0424)	(0.0429)
Observations	59608	59606	59606	59606	59606	59606
Pseudo R^2	0.112	0.207	0.211	0.216	0.241	0.242
AIC	72392.2	64662.7	64281.6	63953.7	61897.5	61846.0
BIC	72473.1	64779.6	64416.5	64106.7	62194.3	62205.9
df_m	8	12	14	16	32	39
chi2	8209.9	11373.5	11153.8	11090.4	12916.2	12909.7
p	0	0	0	0	0	0

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The corresponding Wald table:

Block	Wald Chi2	df	Pr > F
1	8208.38	8	0.0000
2	5540.54	4	0.0000
3	301.76	2	0.0000
4	224.97	2	0.0000
5	1995.33	16	0.0000
6	60.72	7	0.0000

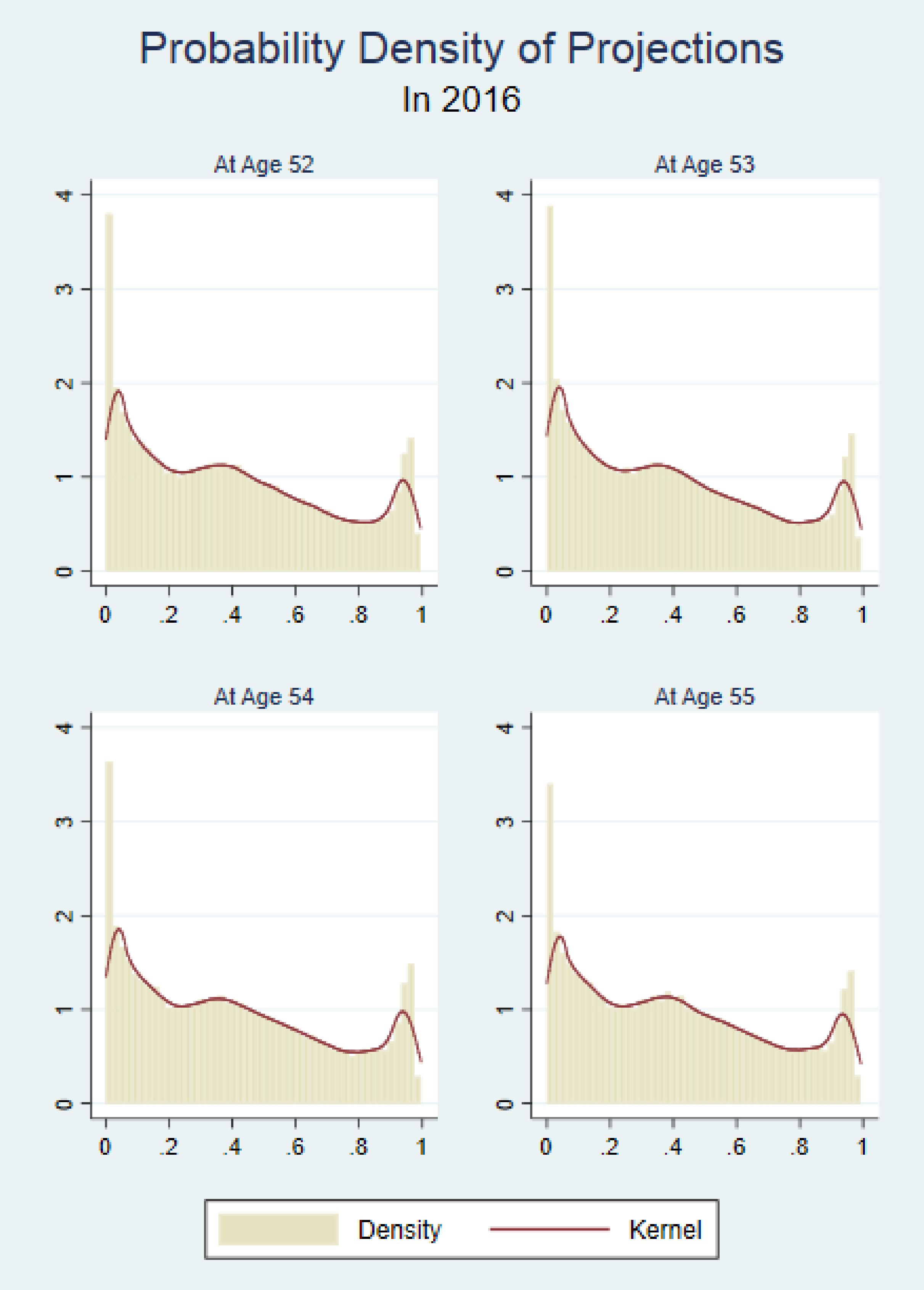
Appendix F – Fit of projections

The table below shows the goodness-of-fit with a perspective of prediction (whenever relevant, a 50% cutoff point is used):

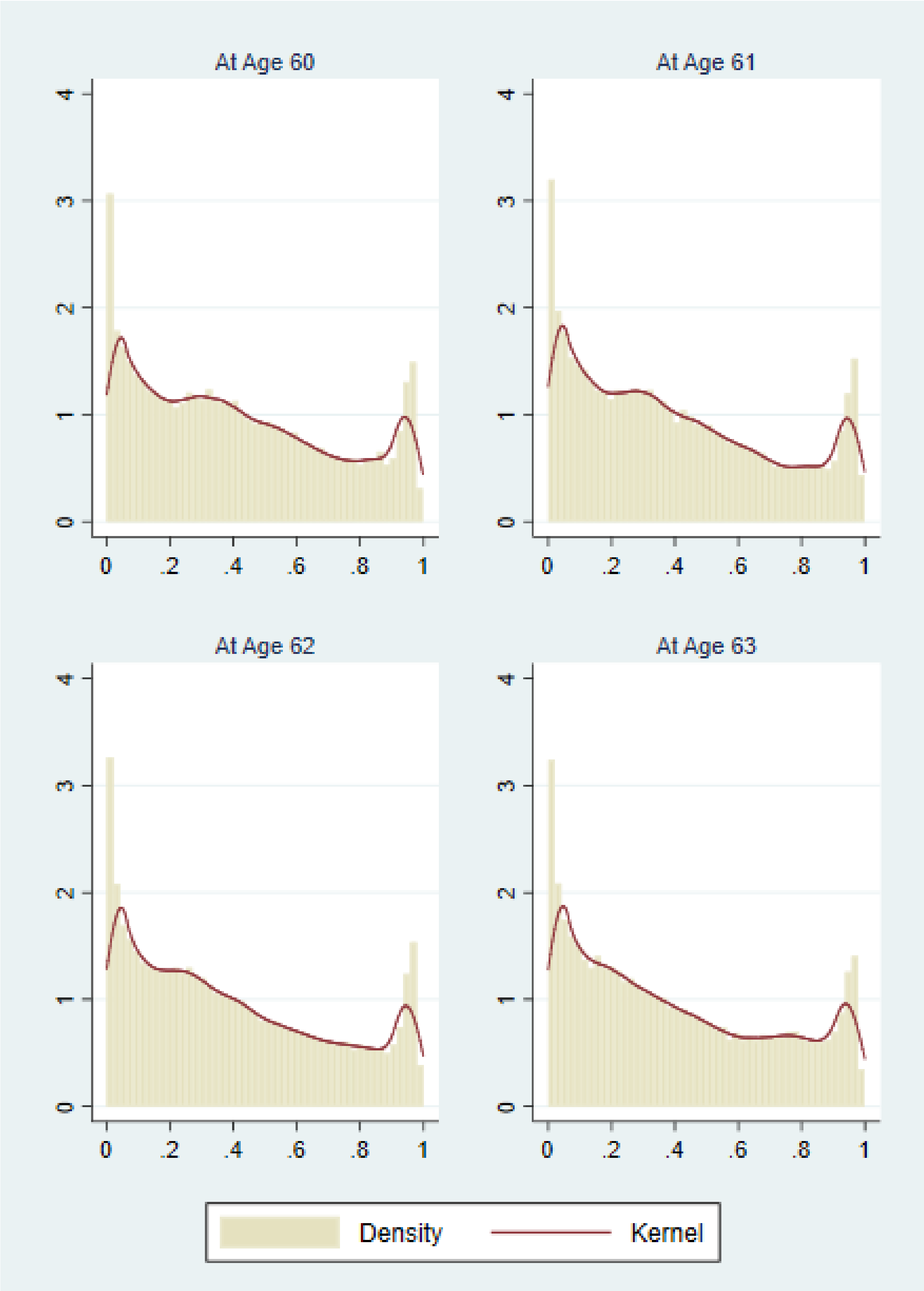
Measure	At age	Value
McFadden's Pseudo R2	52	0.242
	64	0.319
Area under ROC curve	52	0.815
	64	0.858
Count R2 (Accuracy)	52	73.55%
	64	78.4%
Adjusted count R2	52	0.386
	64	0.500
False positives	52	11.8%
	64	9.6%
False negatives	52	14.6%
	64	12.0%

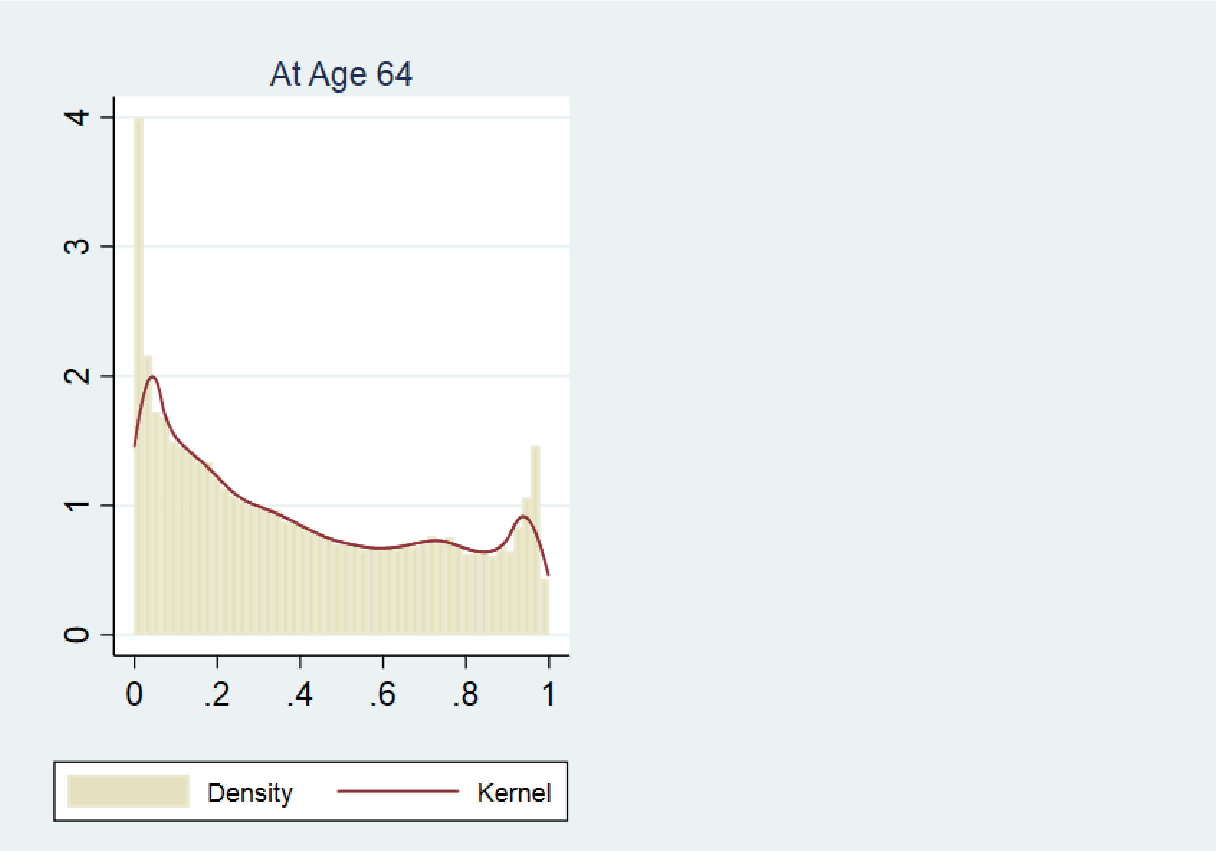
These goodness-of-fit results show that the fit at both the age 52 and age 64 models is good. Even though McFadden's Pseudo R2 is relatively low at age 52, the classifying performance is still elevated with an accuracy of 73.5%. One should take into account that we are predicting an outcome separated by 13 years between the measurement year and the year where the outcome is registered. Adjusted count R2 measures the proportion of correct predictions compared to a baseline model of always guessing the most common binary outcome. It is 0.386 at age 52 and 0.5 at age 64.

Appendix G – Probability forecast distribution over ages




At Age 56
At Age 57
At Age 58
At Age 59
0
1
2
3
0
.2
.4
.6
.8
1
Density
Kernel


At Age 60
At Age 61
At Age 62
At Age 63
0
1
2
3
4
0
.2
.4
.6
.8
1
Density
Kernel


At Age 64
4
3
2
1
0
0
.2
.4
.6
.8
1
Density
Kernel

Appendix H – Regression tables for analysis of young individuals

Probit coefficients and margins for the young cohorts

	(1) 25-34 Cohort Coefficients	(2) 25-34 Cohort Margins	(3) 35-44 Cohort Coefficients	(4) 35-44 Cohort Margins
Male=1	0.264***	0.0610***	0.194***	0.0411***
	(0.00447)	(0.00103)	(0.00437)	(0.000924)
Immigrant=1	0.559***	0.135***	0.814***	0.194***
	(0.00531)	(0.00132)	(0.00557)	(0.00142)
Primary education or less=1	-0.495***	-0.108***	-0.140***	-0.0291***
	(0.00645)	(0.00131)	(0.00643)	(0.00129)
Secondary (Vocational) Education=1	-0.698***	-0.164***	-0.274***	-0.0581***
	(0.00494)	(0.00113)	(0.00473)	(0.000995)
In Couple=1	-0.301***	-0.0715***	-0.331***	-0.0734***
	(0.00433)	(0.00106)	(0.00487)	(0.00112)
Widower=1	0.162	0.0373	0.0457	0.00975
	(0.1000)	(0.0231)	(0.0419)	(0.00897)
Divorced=1	0.0504***	0.0116***	-0.0322***	-0.00679***
	(0.0132)	(0.00304)	(0.00657)	(0.00138)
Number of Children	-0.0323***	-0.00742***	-0.0495***	-0.0105***
	(0.00191)	(0.000438)	(0.00199)	(0.000421)
Agriculture, forestry and fishing=1	0.615***	0.142***	0.930***	0.211***
	(0.0192)	(0.00438)	(0.0205)	(0.00473)
Mining and quarrying=1	0.434***	0.100***	0.778***	0.176***
	(0.0727)	(0.0167)	(0.0629)	(0.0146)
Manufacturing=1	0.199***	0.0457***	0.394***	0.0852***
	(0.0138)	(0.00316)	(0.0138)	(0.00301)

Utilities=1	0.309***	0.0713***	0.439***	0.0969***
	(0.0304)	(0.00701)	(0.0274)	(0.00625)
Construction=1	-0.00976	-0.00224	0.352***	0.0768***
	(0.0145)	(0.00334)	(0.0146)	(0.00323)
Wholesale, retail; vehicle repair=1	0.175***	0.0401***	0.689***	0.152***
	(0.0128)	(0.00292)	(0.0133)	(0.00295)
Transportation and storage=1	0.250***	0.0577***	0.535***	0.119***
	(0.0161)	(0.00371)	(0.0156)	(0.00356)
Accommodation and food services=1	0.345***	0.0800***	0.707***	0.160***
	(0.0152)	(0.00354)	(0.0174)	(0.00406)
Information and communication=1	0.592***	0.137***	0.865***	0.194***
	(0.0151)	(0.00343)	(0.0156)	(0.00352)
Finance and insurance=1	0.276***	0.0636***	0.693***	0.155***
	(0.0211)	(0.00485)	(0.0204)	(0.00464)
Real estate=1	0.344***	0.0794***	0.728***	0.164***
	(0.0245)	(0.00565)	(0.0218)	(0.00504)
Business and support services=1	0.400***	0.0926***	0.711***	0.159***
	(0.0128)	(0.00295)	(0.0135)	(0.00306)
Education=1	0.390***	0.0897***	0.551***	0.120***
	(0.0136)	(0.00309)	(0.0141)	(0.00309)
Health and social institutions=1	0.144***	0.0329***	0.455***	0.0972***
	(0.0124)	(0.00282)	(0.0130)	(0.00277)
Associations, culture, other activities=1	0.493***	0.114***	0.723***	0.162***
	(0.0158)	(0.00363)	(0.0161)	(0.00369)
Low/No Work Sector Attachment=1	0.418***	0.101***	0.584***	0.136***
	(0.0135)	(0.00338)	(0.0149)	(0.00373)

Unemployed=1	-0.0463**	-0.0106**	-0.130***	-0.0271***
	(0.0143)	(0.00327)	(0.0156)	(0.00319)
Self-Employed=1	0.0502***	0.0116***	0.177***	0.0384***
	(0.0137)	(0.00317)	(0.0101)	(0.00223)
Cash Benefits (kontanthjaelp)=1	0.340***	0.0798***	0.101***	0.0217***
	(0.0118)	(0.00283)	(0.0119)	(0.00261)
Student=1	-0.0154*	-0.00354*	-0.0341*	-0.00721*
	(0.00760)	(0.00174)	(0.0156)	(0.00327)
Leave/Activation =1	-0.248***	-0.0565***	-0.303***	-0.0616***
	(0.0129)	(0.00289)	(0.0180)	(0.00350)
Early Retiree(Public Scheme)=1	0.161***	0.0372***	0.339***	0.0750***
	(0.0176)	(0.00410)	(0.0155)	(0.00355)
Low/No Labor Status Attachment=1	0.0657***	0.0152***	0.162***	0.0350***
	(0.0117)	(0.00272)	(0.0148)	(0.00328)
Financial Assets	-0.129***	-0.0297***	-0.204***	-0.0431***
	(0.0152)	(0.00349)	(0.00924)	(0.00196)
Housing Wealth	0.0425***	0.00977***	-0.00460	-0.000974
	(0.00575)	(0.00132)	(0.00379)	(0.000804)
Mortgage Debt	-0.287***	-0.0660***	-0.177***	-0.0376***
	(0.00872)	(0.00200)	(0.00581)	(0.00123)
Non-Mortgage Debt	0.176***	0.0404***	0.261***	0.0554***
	(0.0124)	(0.00286)	(0.00883)	(0.00187)
Primary Income	-0.107***	-0.0246***	-0.0656***	-0.0139***
	(0.00279)	(0.000637)	(0.00217)	(0.000457)
Public Transfers	-0.107***	-0.0245***	0.0357***	0.00756***
	(0.00616)	(0.00142)	(0.00631)	(0.00134)
Financial Income	0.151***	0.0348***	0.0763***	0.0162***

	(0.0238)	(0.00548)	(0.0124)	(0.00263)
Income from Private Pension	-1.603***	-0.369***	-1.549***	-0.328***
	(0.105)	(0.0242)	(0.0362)	(0.00759)
Residual Income	-0.00977	-0.00225	-0.421***	-0.0893***
	(0.0465)	(0.0107)	(0.0497)	(0.0105)
Employer-made pension contributions	-4.091***	-0.941***	-3.564***	-0.755***
	(0.0200)	(0.00396)	(0.0171)	(0.00303)
Self-made pension contributions	-2.716***	-0.624***	-2.881***	-0.611***
	(0.0735)	(0.0168)	(0.0403)	(0.00839)
Constant	1.059***		0.758***	
	(0.0149)		(0.0157)	
Observations	620261	620261	728184	728184
Pseudo R^2	0.405		0.429	
AIC	508545.9	.	548774.8	.
BIC	509033.4	.	549269.3	.
df_m	42		42	
chi2	193163.6		197460.2	
p	0		0	

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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