Applying Options Pricing in Valuing Real Estate Developments

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Resumé

Dette speciale beskæftiger sig med realoptioner som værdiansættelsesmetode for ejendomsprojekter. Det søges undersøgt hvilke udfordringer en bredere anvendelse af realoptionsværdiansættelse står overfor i forhold til DCF-modellen. Dette gøres gennem en case, hvor realoptionsmetoden anvendes på casen Greve Distributions Center samt en diskussion af den akademiske litteratur, der beskæftiger sig med realoptioner og dens anvendelse på ejendomsmarkedet.

Der findes flere konkurrerende værdiansættelsesmetoder inden for finansiering generelt og ejendomsinvesteringer specifikt. Det etableres, at DCF-metoden er en af de mest udbredte metoder, hvorimod realoptioner stort set ikke anvendes af praktikere inden for ejendomsbranchen.

Ejendomsbranchens struktur gennemgås med fokus på splittet mellem aktiv- og lejemarkeder samt de forskellige ejendomstyper.

Herefter analyseres styrker og svagheder for både den traditionelt anvendte DCF-metode samt realoptionsmetoden. I praksis svækkes DCF-metoden ved at blive anvendt i en simplificeret udgave med antagelser så som en konstant diskonteringsrente. Derudover er det en svaghed, at DCF-modellen bruger punktestimater til at repræsentere et interval af mulige input og output. Realoptionsmodeller kan tage højde for nogle af disse svagheder, ved at kvantificere værdien af fleksibiliteten af indbyggede optioner. Metoden bygger dog på input, som i praksis kan være svære at estimere f.eks. volatiliteten for det enkelte aktiv. Derudover kræver modellen en høj grad af matematisk samt finansiel forståelse.

I specialet præsenteres et framework, der kan identificere og skabe overblik over hvilke ejendomsprojekter, hvor realoptioner som værdiansættelsesmetode kan tilføre en merværdi i forhold til traditionelle metoder. Dette gøres med udgangspunkt i, om udnyttelsen af optionen kan forskydes tidsmæssigt, samt om der er mulighed for fysiske ændringer af ejendommen.

Til slut identificeres hvilke faktorer der skal være til stede i en organisation for at kunne implementere realoptioner med succes samt den generelle udbredelse af metoden inden for ejendomsmarkedet.

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1. Introduction

Real estate is an inherently interdisciplinary field and the description and definition of it will depend on whether you ask an architect, engineer, lawyer, economist or financial analyst to name a few. The architect will impress on you the beauty and how it affects the people who inhabit it. An engineer might marvel at the way the loads have been supported. A lawyer can speak to the rights and obligations. Finally, an economist and financial analyst will look at the supply, demand and cash flows in the market and of the individual property.

In the development of new real estate or the redevelopment of existing structures, we are faced with many options on how to proceed. Again, the architect will contemplate how different design will affect the esthetics and the engineer will make the sure the building stays erect. The lawyer should evaluate whether the potential choices fall within the law. These decisions will also have financial implications and it is the effect of these implications on valuations that we want to further explore in this thesis.

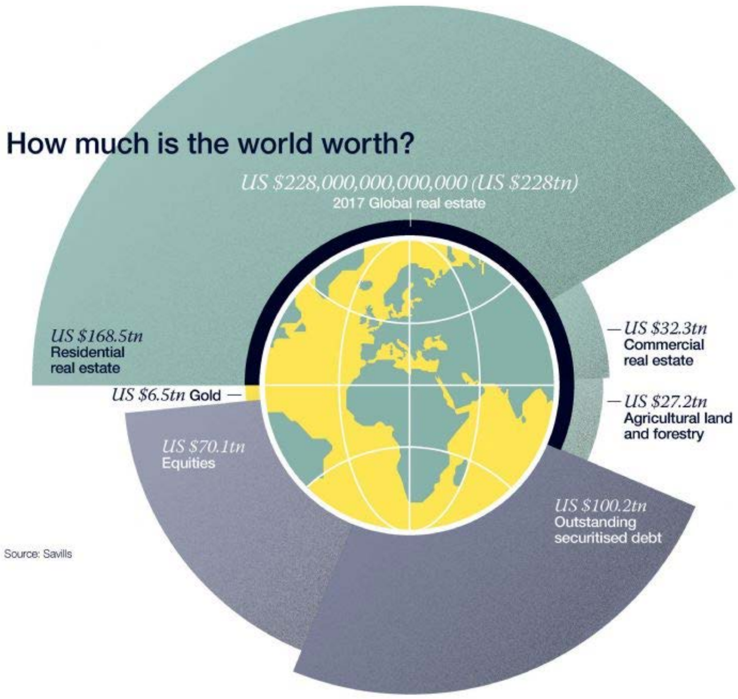

Figure 1 Real Estate Asset Value Compared (Savills 2017).

The asset market to which real estate valuation methodologies will apply makes up a sizable share of the global economy with an estimated asset value of US$ 228 trillion, which is 2.8x global GDP. Global real estate is a more valuable asset class than equity and securitized debt combined, which together amount to US$170 trillion. Another comparison is the value of all the gold ever mined throughout history, which pales into even greater insignificance at just US$6.5 trillion. (Savills 2017)

It is widely held that traditional valuation methodologies fail to take into account the value that real estate owners of have in the form of options (E.g. Trigeorgis 1993; Dixit and Pindyck 1995; Sirmans 1997; Oppenheimer 2002). However, no clear practical modelling solution has emerged as is evident by the continuing domination of the same criticized valuation methodologies.

One valuation methodology that could accommodate this criticism is that of real options analysis. Titman (1985) was among the first to apply the theory to real estate valuation, however, despite three decades passing, we observe little sign of practical adoption of the theory. The explanation of this can lie in practitioners not feeling a need for a change in methodology, mathematical complexity or other shortcomings of real options pricing. Through an application of the valuation method and reading of the literature, we attempt to understanding the challenges faced in attempting an implementation of real options analysis.

The remainder of this thesis is structured as follows. In the succeeding sections, the guiding problem statement is presented, the delimitations are outlined and our methodological considerations are described. Following this, the section four will introduce the real estate asset class including a brief overview of investment methodologies before the two main methodologies – discounted cash flows and real option valuation theory – are reviewed in further detail. Section seven and eight present and discuss the case study respectively. Section nine concludes the thesis.

2. Research Question

An approach to real estate valuation with a higher predictive power would be of competitive advantage to any investor who successfully implemented it as it would allow for the discovery of undervalued investment opportunities.

The main research question is:

> **What are the obstacles for wider adoption of real options pricing in real estate valuations in competition with the DCF model?**

This overall question will be answered through a number of sub questions:

- How is the real estate market structured?
- How does real option theory value real estate assets?
- How does the discounted cash flow model compare to real options pricing?
- In what type of real estate investments does the real options approach add value?
- What would be the characteristics of a more adoptable model that still took optionality into account?

2.1. Delimitation

While there are numerus valuation methods we will in this thesis focus on real options analysis and discounted cash flows (DCF). The DCF method is chosen as the comparison due to its wide usage in the industry. We have chosen to focus our attention on just two models as so not to spread our attention too thin as well as due to the domination of the DCF method in the literature.

While as will be described there are many options available in a real estate development project, the focus of this thesis is on the option to delay phases of a development project. Many of the conclusions will still be applicable to other types of options as the basic modelling is the same.

We are excluding game theory in this thesis, despite the fact that this academic field could shed light on how different competitors interact in the real estate development industry and how real options can be used for addressing optimal strategies for real estate developers. However, this would be too comprehensive to include in this thesis as it could easily be a topic on its own. Furthermore, we have chosen not to include the Monte Carlo simulation theory and apply it in a valuation approach as we believe that it would cloud the overall objective of identifying the obstacles for a wider adaptation of real options in practice, since it would add additional complexity in an already fairly complex field.

3. Methodology

Understanding of the lacking practical adoption of real option valuation methodology to real estate is first of all a practical endeavor but deeply grounded in the academic literature, which over the past three decades has developed the theoretical foundation.

Through the application of real options theory to a real-life case, the practical feasibility of valuation method on real estate developments is studied first hand. Though the case study is unique by definition it is implicitly theoretical since it is done not out of interest in the uniqueness of the selected case but because the case is assumed representative and will enable a better understanding of the theory (Eidlin 2012). The case ensure that the discussion of the literature is firmly grounded in the practical experience.

The case study is focused on understanding common patterns of theory in the context of application for the purpose of furthering understand of the valuation methods practical implementation. Thus, the case serves as an instrument to explore the specific valuation methods in practice and the case itself is therefore not of particular interest. Consequently, little time will be spent on understanding the broader context of the investment and no interest will be taken in the opinions of stakeholders or problems faced by the developer. On the other hand, the case's role as an instrument is to replicate the information shortages of practitioners. (Eriksson and Kovalainen 2012)

Followingly, the main interest lies in understanding the application of theory and not understanding the case used. For this reason, the thesis makes no attempt to determine the true or fair value of real estate developments projects as it is not of interest. This would be fitting in a positivistic view of the economic and financial science but as it has been argued many years ago, the positivistic view may today truly belong in the historic annals of the philosophy of science (Caldwell 1980).

Advancing methodologies intent on estimating price has an obvious positivistic flair in terms of the ontological question of real estate value. However, our use of the case study and focus on theory adoption is more congruent with the epistemology and methodology of realism (Perry, Riege, and Brown 1999). This follows since our understanding of a real options valuation methodology will partly be understood through an application of the method. Thus, we are participants within the subject being investigated. Further, the reality of the applicability of real option theory to real estate valuation will be constructed by each practitioner and any external reality cannot be viewed independently of the researcher or practitioner. Finally, it can be questioned whether the value of real estate can be ascertained in a positivistic sense since the numerical value is first arrived at in the interplay between two or more parties.

Generalizability does to some degree depend on this ontological position underpinning of the case study and it is likely that none will be fully convincing (Moriceau 2012). This will not preclude an attempt form being made in the following. The case study can be deemed methodologically successful if it satisfies the three tenets of qualitative research: describing, understanding and explaining (Yin 2014). As the investment objectives, participants and processes among the subjects by whom the real options valuation method would be applied are relatively uniform, we find that one case study is sufficient to offer a degree of generalizability. While we only conduct one case study ourselves, it fits into a larger body of research that together can form a basis for a broader conclusion. Further, as we do not aim to generalize our understanding onto a population but instead onto the theory, the need for a large sample is less pertinent (Yin 2006).

The selection of case study in the extensive case study approach follows the logic of easy replication as to make the process more representative. The selection of case study is also influenced by pragmatic considerations as quantitative data that would be available internally to investment firms had to be available to us such not limit the case study further than the actual situation would have done. The case study will be based on public quantitative data from the current logistics development of Greve Distribution Center as presented in section 7.

In relation to real option theory it also seems appropriate to touch on the performative effect of the Black-Scholes-Merton (BSM) theorem. As Watson (2007) outlines, an increased number of scholars point to this capacity of economic theory to have a performative effect on economic reality, which seems to conflict with the external reality of positivism. In other words, the proposal of and subsequent real-world application of the BSM theorem created a reality that increasingly became to resemble that proposed by the theory. The strong performativity of the BSM theorem is supported by speed with which the options market converged on the BSM price. Thus, the BSM theorem did not describe the world as it was but offered a normative guide to how it ought to be. Given this reflexivity we find the investigation of the price itself futile and focus our intention on the interplay between theory and practice.

4. The Real Estate Market

In many ways there is no such thing as a real estate market. There are many real estate markets. Real estate is a very segmented asset class due to the immobility and conversion costs (e.g. time, financial, opportunity cost, zoning permitting) between asset types. Further, there is both a real estate market for space and a real estate market for assets. This is in contrast to other markets such as the financial markets (an investment manager can easily choose to move his money from one stock to a bond but a NYC law firm is not interested in the space market of Hong Kong) or commodity markets for things such as flour, which is nationally integrated and easy to move.

This leads to the law of one price not being applicable within real estate as it is only applicable if the market is well integrated. The same exact office building in Copenhagen can easily have a different price than the same structure erected in Aarhus. Thus, the actual product consumed by users in the real estate market and invested in by owners is the combination of physical structure and geospatial placement. This results in restricted competition as it is not enough to have an idea for the physical product – users also need the location to be part of the unified consumption bundle, which limits supply.

Real estate is the largest investable asset class in the world and just as equities vary greatly, so does real estate. In this chapter, we will introduce the real estate market or markets because there are two. The real estate market can be split into the interconnected space market and asset market. Further, we will describe the different property types, investment strategies and players in the real estate industry. Finally, we will give a brief overview of the Danish real estate market.

4.1. The Space and Asset Markets

There are two markets relevant to the study of real estate investment. The space market is the most fundamental as it is the market for the physical real estate whereas the asset market is the market for the financial asset. The space market is also known as the usage market and determines the cash flows that a property is able to generate. On the demand-side are individuals, households, firms etc. all with the purpose of using the physical space for consumption or production and on the supply-side are real estate owners.

The product offered in the space market is heterogenous and immobile. This makes the space market highly segmented in terms of both location and type. To change a property's type is not impossible, as exemplified by the current trend in office to residential in Central Business Districts (CBDs) such as Copenhagen, but it is an expensive endeavor often requiring political backing to change land use regulations. The segmentation of the space market allows for price differences between different locations and types since the products are not substitutes.

The asset market on the other hand is much less segmented as the demand is for the financial asset i.e. the claim on future cash flow. Thus, since cash is fungible ceteris paribus an investor will not care if the cash is net rent from a retail property, an office property or a dividend from stock ownership.

The asset market it is an equally important market because it affects the supply side in the space market and through pricing affects the flow of financial capital to and from real estate. The fungibility of cash and integrated nature of the asset market implies that geography and property type matter less than in the space market. In the asset market properties differing in type and location can have the same returns if perceived risk and growth potential are equal. In the context of this thesis on real estate valuation the asset market is more directly relevant.

The property asset market is part of the larger capital market. Compared with financial assets such as bonds and stocks, direct property investments have low liquidity as the search costs are high i.e. takes time for buyers and sellers to find each other. Further, direct real estate ownership has lower informational efficiency as prices are difficult to observe and it takes time for news to be reflected in prices, since each asset is thinly traded and heterogenous, so comparable prices are never perfect.

Broadly speaking the capital market can be divided into four categories based on two distinguishing features. Firstly, whether they are public or private. Public markets trade homogeneous units of financial assets between many buyers and sellers. Consequently, there is typically a high level of informational efficiency. In contrast, private markets trade in heterogenous assets that are often traded whole (e.g. a whole company) and the search process adds to the transaction cost and decreases liquidity. The average transaction size tends to be larger in private markets (Geltner et al. 2013).

The other feature is whether the asset is debt or equity. Debt being a senior claim on cash flows with an infinite lifetime and predetermined rate of return during its lifetime. On the other hand, equity is the claim on all residual cash flows and has an infinite life.

	Public Market	Private Market
Equity	• Real Estate Stocks	• Direct real estate • Private equity
Debt	• Mortgage backed securities	• Bank loans

Figure 2 Types of real estate assets. Based on Geltner et al. (2013).

There are financial real estate assets in each of these for categories and in all of them it is relevant to be able to value the underlying property. Publicly traded real estate companies inhabit the public equity market and mortgage backed securities is a real estate investment in public debt markets. Private debt markets also allow for real estate investments such as banks balance sheet loans secured by private property (e.g. financing over the 80% allowed for residential listed mortgages). Finally, private equity markets hold the purest form with direct property investment as well as real estate private equity.

4.2. Valuation Methodologies in Practice

We have already begun referencing values of real estate in describing it, so there is clearly a need to investigate how best to arrive at real estate values. This is a more complicated endeavor than in other asset classes such as the large public markets for stocks and bond. In those, there are generally speaking publicly observable prices from frequent transactions of homogenous assets. Real estate on the other hand is a heterogenous and illiquid asset with transactions happening privately thereby reducing transparence. While it is harder to arrive at a value of real estate it is still often required such as for financial statements, taxes, inheritance and expropriation. It is therefore also no surprise that 68% of respondents in one survey of real estate professionals found “pricing/valuation” to be the key real estate market issue in 2016 (Deloitte 2016).

A valid valuation method should reflect the underlying fundamentals at the time of the valuation for it to be a best estimate of the price a property would command if a transaction took place between two arm’s length parties at this moment. Thus, *price* being an actual exchange in the market place, whereas the *market value* is an estimation of a hypothetical case of an actual exchange at the given time (Pagourtzi et al. 2003).

The valuation approaches included by Deloitte (2016) at their 2016 real estate valuation conference were grouped into market-, income- and cost approaches. The market approach entails finding similar property transactions and adjusting for any remaining differences. The value of property in question is then determined by applying for example the comparable(s) square meter price or net income multiplier to the property being valued. Using the net income multiplier straddles the distinction between market- and income approaches as it is based on comparable transaction in the market but also on income.

The income approach is a group of methods including the net income multiplier also known as the income capitalization method. The capitalization rate is arrived by dividing the net operating income by property price/value. The net income multiplier is merely one divided by the capitalization rate. Another income approach is the residual value method, which is often used for development projects as the present value of the costs to complete are subtracted from the present value of the completed project.

The discounted cash flow method (DCF) is likely the most common in real estate investments (Mintah et al. 2018) and is also an income approach. As Geltner et al. (2013 p. 204) write, the “DCF is probably the single most important quantification procedure in microlevel real estate investment analysis” and just as his textbook focuses on this method, so do many of the other common textbooks in the field (E.g. Poorvu and Cruikshank 1999; Brueggeman and Fisher 2015; Floyd and Allen 2015).

The final approach discussed by Deloitte is the cost approach, which can be used if there is no market activity producing comparable transactions nor any income directly tied to the property. It involves estimating the depreciated replacement cost of the structure and adding land value. This approach is the least used as there is no guarantee that the depreciated cost is representative of the market price.

A survey by KPMG (2017) of Australian investment professionals across asset classes finds that the market approach is the most common with it being used *always* or *often used* by 90% of respondents. The DCF is the second most common approach with about 70%. Due to the lower transparency in real estate it would not be surprising that the figures for real estate would yield a flip between the two top spots. A similar result is arrived at in a survey by Valueonshore Advisors (2017).

Pagourtzi et al. (2003) have written an often-cited academic article[1] on valuation methods in real estate, which includes a long range of more or less practical methods split between traditional, which includes the previously mentioned, and advanced methods, which are for example artificial neural network, fuzzy logic and hedonic pricing models. Notably absent from their review article in our context is the real options methodology.

The optionality presented to stakeholders in real estate investments is scantly represented in the dominating methodologies as evident from the above. Titman (1985) was among the first to apply the options valuation methodologies known from financial options to the real estate sphere. However, despite 30 years passing the real options approach has yet to gain a foothold in the practitioners’ toolbox as evident by the surveys cited above as well as a survey by Bennouna, Meredith and Marchant (2010) among the 500 largest firms in Canada of which finds that only 8% of these use real options in valuations. This is consistent with a European survey, which finds that real option valuation

[1] 103 citations according to the articles Scopus record as of September 2018 with a Field-Weighted Citation Impact of 3.82 with 1.00 being the expected level.

is rarely used by practitioners regardless of educational achievement or experience. This suggests that method choice is largely driven by peers and not curriculum (Bancel and Mittoo 2014).

Further, textbooks have not allocated many pages to the method. Geltner et. al is probably with most pages dedicated to real options among the widely used textbooks as they spend two chapters at the end of the book. In contrast, Brueggeman and Fisher (2015) only dedicate two pages. Geltner et al. (2013) also blankly admit the methodologies in the advanced sections of their textbook are not widely used explicitly, however, they argue that successful developers and investors must be using them implicitly due to its grounding in market based economics and the market equilibrium. The curriculum of the CFA institute also focuses on DCF and relative valuation methodologies (Bancel and Mittoo 2014) just as a relatively recent valuations booklet from the Danish Association of Chartered Real Estate Agents and the Danish Property Federation makes no mention of options (2013).

4.3. Real Estate Asset Market Participants

Real estate and land owners are a heterogenous group ranging from one the world's largest land owner The Crown, who holds trust over 6,600 million acres of land, which is about a sixth of the worlds non-ocean surface to individual home owners all over the world (McEnery 2011). Even focusing on professional owners of real estate, who purchase real estate to profit from the types of owners still range from users for whom real estate is a factor of production to short-term fix and flippers and institutional investors, who purchase real estate for the long-term cashflows. However, our focus will be on the investors for whom real estate is purchased to be owned for a period of years around 10 years and then sold again as the perpetual ownership of institutional investors muddles the picture. The investors, who invest for a holding period of approximately 10 years are often private equity funds, and are some of the most financially sophisticated alongside the institutional investors.

It is increasingly common place that ownership and usage of properties are separated. The investor performs a valuation a valuation based on the stream of cash flows that the property will provide, whereas an owner-occupier will view the property as a means of production and assign its worth base on its contribution. However, the two owner-types act for the most part in the same market and the market value will be the same for both. (Pagourtzi et al. 2003)

4.4. Property Types

There are a number of property types, also called segments, within real estate investing with distinct characteristics that affect how value is created. Overall, the types can be grouped in residential and nonresidential. Residential includes single-family houses and multifamily properties and they can be either by owner-occupied, co-operative or rental properties. Although hotels and motels can be thought of as providing residence, they are not considered part of residential as the housing they provide is mostly temporary and the economic forces that act upon their demand and supply are different than from those affecting single- and multifamily housing.

Nonresidential properties are typically broken down in six major categories: office, retail, industrial, hotel, recreational and institutional. These can be further subdivided as for example retail both contains strip malls, stand-alone grocery stores and large shopping centers. A 200 sqm store in a small town and a 100,000 sqm mall are both retail and it may seem odd to equate the two as they on the face of it have very little in common. However, they share nomenclature and are affected by the same value drivers, whereby it makes sense to treat them together. Thus, both across but also within each property type we see large variation.

Land could also be added to the list of segments; however, it is the (potential) use that determines its value. Based on zoning laws it will often be possible to place land into one of the segments, such that a value can be determined.

In the following, the four main investment segments residential, office, retail and industrial will be introduced and put into Danish context as well as described through the lens of an investors intent on valuing the market fundamentals for a particular investment opportunity. These characteristics are:

- Structure and its quality
- Location
- Value drivers (Supply and demand)

Most of these characteristics are examined from both a "macro" and a "mirco" point of view. On the macro level, the analysis focuses on the economics, demographic and sociological trends that affect the aggregate demand and supply of a given property types. At the micro level, the focus shifts to the issues that affect the particular property and how it will successfully attract tenants. This can range from zoning rules and local transportation infrastructure down to the layout of the individual units.

4.4.1. Residential

Residential properties serve a uniform need across the globe; however, as a segment there is big difference in how it is researched and defined by market actors in different geographies. This is one of the reasons why it is difficult to aggregate global statistics for the residential market. In some geographies, only income-producing properties are included when calculating trading volumes and property stock, whereas in other markets owner-occupied residential is also included. Further, there are traps in the nomenclature to the untrained eye. For example, in the United States *apartments* is often used to denote residential, multi-family income-producing real estate, whereas to for example a northern European investor this word does not differentiate the product from owner-occupied apartments. These are, however, usually called *condominiums* in the United States (Peiser and Hamilton 2012).

Globally the residential market made up 75% of property value as of 2015 according to Savills (2016) but this real estate wealth is not evenly spread out across the globe's approximately 2 billion households. There is a substantial western skew as North America for example only accounts for 7% of the world's population but 22% of residential property value. Similarly, both Europe and to a lesser degree China and Hong Kong have a disproportionally larger share of residential real estate wealth.

In Denmark, residential real estate is an important segment with a big presence of property funds and other professional owners. It consistently ranks as the largest segment when measured by transactions volume, where it since 2012 has measured in at between 31% and 42% (RED 2018). This volume is about double the segments share of the global transaction volume (Savills 2016). In 2017, its 42% of transaction volume equated to 38.2 bnDKK worth of residential real estate trading hands with 45% of that being in Copenhagen (RED 2018). The investor interest in Danish residential real estate is also reflective of the large demand for rental units as a third of Danish households rent their dwelling, which is the second highest in Europe only behind Germany (Deloitte 2017).

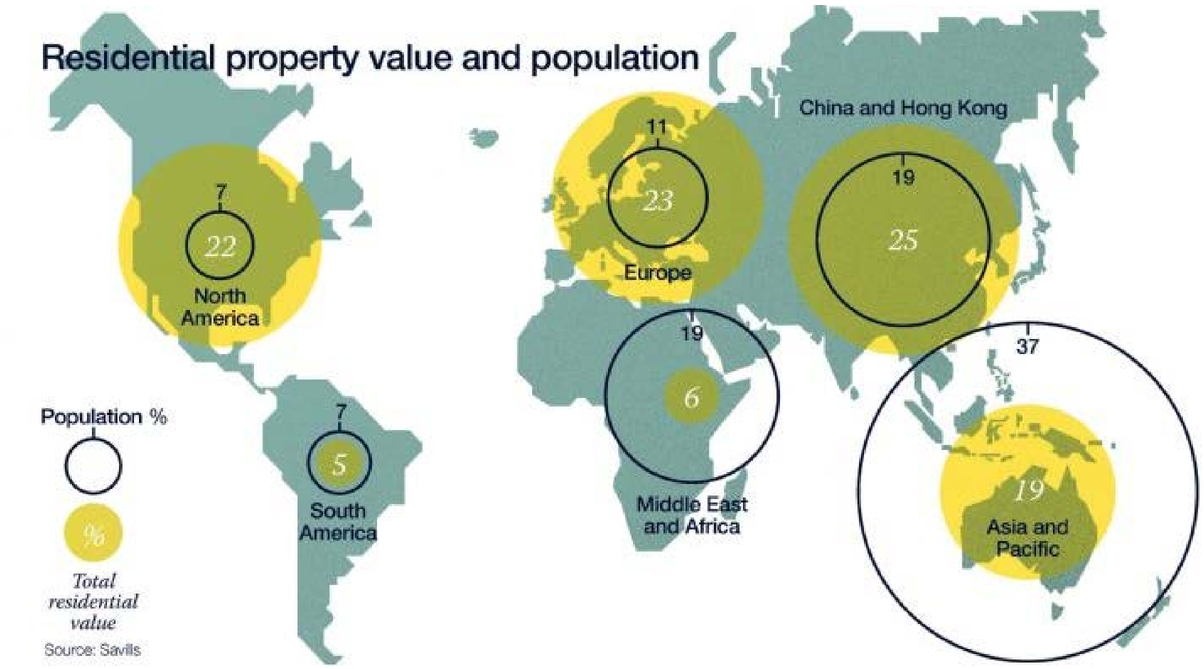

Figure 3 Residential property values and population both as percent. (Savills 2017)

Compared to other property types, residential real estate provides a relatively stable income stream with easy to forecast future capital needs. Further, the segment has a low risk of obsolescence and well-kept-up residential properties have as a segment a lower downside risk for investors than most other property types (Poorvu and Cruikshank 1999).

Structure and Quality

A residential property is assigned a quality rating (Class A, B or C) in accordance with the quality of its physical characteristics and location in context of the local market definition of classes. There is no universally accepted definition of what each class entails. Further, assignment of class can be strategic as evident by how seller and potential buyer often rate the property differently. E.g. seller will rate a property as Class A but the potential buyer will say it is only Class B to try and drive price down.

In general, Class A refers to a newer, prime location building that offers high level of amenities (relative to the market norm). A class B property can then either be a new building in a secondary location or an older building in a prime location. It is important to note that the usage is relative and thus structures cannot be compared across markets based on their class. However, they do share similarities as they class A for example represents the best a market has to offer and thus from an asset market perspective share certain characteristics for example relating to what can be expected in terms of vacancies and relative rent level. Most institutional investors, are only willing to invest in class A properties, which explains the term “invest-grade property” (not to be confused with investable property); however, standards tend to slide in an overheated investment market.

Another way to distinguish different residential properties are through their size and layout. Apartments are generally labelled as either low-rise, mid-rise or high-rise, which again like class is very context dependent. In most markets these physical characteristics of the structure are limited by zoning rules, which can specify such things as maximum building heights and plot density.

Location

As the apocryphal real estate adage has it, the three most important characteristics of real estate are location, location, location. Location is often described using the terms *prime, secondary* and *tertiary.* The location criteria at play are at a macro level population and economic growth in the region. Again, the effect is context depended and multifaceted. For example, population and economic growth has had less of an impact on the rental apartment market in the U.S. South as land is abound land and home-ownership has always been relatively affordable. In contrast the U.S. West exemplified by

California has roughly the same population growth and a strong economy[2] but the sky-high home prices keep the demand for apartments strong (Poorvu and Cruikshank 1999). Microlevel determinants of location is everything from neighborhood reputation and quality of school district to proximity of transportation and amenities.

Value drivers

The basic value driver is the mismatch between supply and demand, which in residential real estate breaks down to demographics and construction (past and present). Demographic trends such as population growth, household formations and the size of prime renter age groups are key determinants of demand. Further, the substation effect plays a role for rental units as the attractiveness of owner-occupied will influence demand for rentals. Homeownership becomes ceteris paribus more attractive as economic conditions improve, mortgage rates fall or the affordability of homes increases. There is no clear, single path to all of these factors as they are interdependent. For example, improved economic conditions could lead to higher central back interest and thus higher mortgage rates or lower mortgage rates driving more demand and thus lowering affordability. Migration can further muddle the picture as rising income will often increase immigration, and since newcomers typically start in rental, this will drive demand for rental units and not homeownership. Consequently, it is also important to determine if population growth is organic or from immigration.

Another less predictable determinant of demand is changes in consumer preferences. The desire of couples to continue living in the city while rising kids as opposed to heading for more space in the suburbs will induce a shift apartment demand in cities towards larger units. Empty-nesters downsizing, an increased interest in communal living or lifestyle renting are also all consumer preference trends that can be immensely profitable to notice early.

The supply of residential real estate is driven by vacancy and absorption rates. However, it is important not only to look at the headline vacancy rate and build if it is low. The rate can hide a lot of variance between different product types if there is a large disconnect between what the market offers and what consumers demand. E.g. the headline vacancy rate may be high in a warm location but looking closer could reveal that all the vacant units have no air conditioning whereas units with this amenity is in high demand.

[2] The hypothetical country of California has since 2013 risen from being the world's eights to now fifth largest economy (Cooper 2018; Garosi and Sisney 2014)

Further, it is important to look at greatest need and not vacancy rate. A low vacancy rate can of course point to a high need for housing but if demand is trending downwards the future need is not going to be present. Enter the absorption rate, which attempts to quantity the how many units the market will absorbed over the coming year. Combined with knowledge of current stock and project pipeline one can arrive at the number of months it will take to absorb the existing and planned inventory, which is a good metric to gauge the market need. This measure is of course subject to more estimation error and uncertainty than the vacancy rate, which is a measure of the current state and not future development.

Restrictions facing supply is also likely to drive up price. Zoning laws and difficult approval processes are prevalent examples of this. The local zoning laws are for example in the United States and Northern Europe largely driven by local politicians, who are elected by the current residents, where there is often a large block of home owners, who have an interest in limiting supply to keep prices high. This is currently exemplified in the vocal NIMBY/YIMBY movements (not/yes in my back yard) against increased density in land supply restricted areas.

4.4.2. Office

The office market is highly cyclical and has historically been the most volatile sector of the cyclical real estate industry (Poorvu and Cruikshank 1999; Peiser and Hamilton 2012). In Denmark, the segment has historically been dominated by institutional investors seeking stable, long-term cash flows to counter their long-term obligations. A shift is occurring with more real estate companies and funds being active in the office segment (Jørgensen and Wejp-Olsen 2017). In addition, Denmark's largest public real estate company Jeudan is a larger player in the Copenhagen office market. Compared with residential real estate, the counterparty is often more competent, which can lead to a more complex negotiation process but also less tenant handholding.

Office takes up the largest share of global real estate transaction volume at 36% (2015); however, it has fallen from a pre-crisis level of 42% in 2008. The void left by office has been filed by residential which grew from 10% to 18% in the same period. The drop is attributed to a falling institutional appetite and changes in working places space usage (Savills 2016).

Structure and Quality

Just as residential properties, office tends to be classified by the indistinctly defined classes. For example, in the Baltimore office market, a Class A office refers to a large building that is less than 25 years old, in a prime location and with first-class tenants. In contrast, a Cincinnati Class A office building differs in that it is less than 10 years old. Rehabilitation and tenant improvements will confuse the determination as some will not reclassify e.g. Class B to A based on a renovation. (Poorvu and Cruikshank 1999)

Office buildings, again like residential, are grouped based on highest. Thus, they are either high-, mid- or low-rise structures. The CBD of a city will often see a higher density translating into higher buildings. Although this is not always the case since some CBDs are located in historic areas that have strict height restrictions. Taller buildings generally have higher square meter construction costs, which is what makes suburban office parks attractive as they are less expensive to construct on land that is often also less expensive allowing for a lower rent. Many suburban office parks have space for expansion such that they can be built in phases to ensure that supply and demand stay in balance. In dense CBDs this kind of flexibility is rarely available except for costly vertical expansion.

Finally, the layout of the floorspace is influenced by changing trends and an office with considerable interior space is less flexible assuming natural lighting is required. E.g. private office can be fit out along the exterior walls leaving large interior spaces without natural lighting.

Location

The location choice for office real estate must balance the desires of different employees with the cost of space. According to a survey by the real estate services firm Savills and the British Council for Offices (2016), found that over half the respondents preferred a city center location with the preference being stronger in younger age brackets.

Value drivers

The demand for office space is driven by job growth in within sectors such as technology, professional services and others, where work is performed in an office. More precisely, the demand would be driven by the projected job growth since firms must secure space for new employees before they begin working. Another demand driver is the trend in square meters allocated per person. Increasingly, tenants want open floor plans to accommodate increased density.

Finally, there are macro catalysts the can fundamentally change the demand. One such example is the shift from manufacturing to service jobs in many western countries. It still seams unclear how the changes in technology will change the overall demand for office space but as the desktop computer has been replaced by laptops for many workers accommodating them has been become more flexible with for example hotdesking becoming more common.

Supply of office space is driven by the absorption rate, barriers to entry such as zoning and regulatory approvals (e.g. building permits) and the effect of other nearby markets such as when as when suburban locations increases in price the relative attractiveness of relocating to CBD will increase ceteris paribus. (Poorvu and Cruikshank 1999)

4.4.3. Industrial

Industrial covers a diverse set of properties related to both production and distribution of physical products. Most manufacturing facilities are highly specialized and as a consequence they tend to be owner-occupied. Investable industrial real estate is therefore more commonly logistics properties, which are less specialized. However, with the advent of automation technology, warehouses become more specialized. In the most extreme case is the automated clad rack warehouse, where the racks are serviced by robots, the system is customized to a specific tenant that re-leasing will pose a bigger challenge than traditional warehouses. The other types of industrial properties are R&D facilities and flex space/showrooms. These make up a small portion of the industrial real estate universe (Poorvu and Cruikshank 1999). Self-storage is also sometimes included as industrial but they share many similarities with retail and residential due to the very different end users.

The transaction volume in industrial has over the past decade made up around 10% globally (Savills 2016). In Denmark, this has been a bit lower but on an increasing trajectory with 4% in 2014 growing to 10% last year dominated by investment managers such as the private equity firms NREP, NIAM and Blackstone (RED 2018).

Location

Location I critical to all industrial tenants has they share a need for access to transportation. For the logistics subsegment especially, transportation is part of the service they are selling. Ceteris paribus, the location with the easiest access to the most extensive transportation network will enjoy the highest demand.

The important modes of transportation depend on what is demanded in the area. For example, in 2016 90% of inland freight transportation in the UK was by road but this accounted for less than a quarter in Latvia, where rail dominates with a 77% share (Eurostat 2018).

As e-commerce grows and sellers compete on delivery times, last-mile logistics facilities will become increasingly important in order to quickly delivery products to customers. This could lead to a demand for warehouses closer to urban centers on more expensive land.

Finally, agglomeration forces will lead to industrial properties collocating as tenants desire to locate near firms they do business with. This is especially true for time-sensitive products such as perishable foods or manufacturing and technical research universities wanting to benefit from knowledge spillovers generated by both. (Peiser and Hamilton 2012)

Structure and Quality

Industrial is not typically described using the class system know from apartments and offices. Instead they are either grouped by their age, e.g. "obsolete", "older", "newer" space, or using the terminology tertiary, secondary and prime, which closely corresponds to the age and also location of the property as known from the class nomenclature and common within logistics.

Industrial has historically been considered simply "roods and parking lots" (Poorvu and Cruikshank 1999), but now the demand has become more detailed in their requirements. For the logistics, truck parking, turn space or access to drive around the property and the number of loading docks are criteria on which the properties are evaluated.

Value drivers

Demand for industrial space is closely tied to the strength of the economy and the subsegment logistics also to international trade. However, GDP growth is not a perfect predictor as industrial demand has structural shifts can change the relationship as it happened with the advent of just-in-time production reducing overall inventory levels and demand for warehouses (Poorvu and Cruikshank 1999).

The supply of industrial real estate is very responsive to increases in demand as development time is low. A developer seeking to build a warehouse can do so in around one year including obtaining a building permit, whereas office or residential project will take multiple years. High vacancy will result in developers holding back on speculative development (i.e. building before a tenant is found). Development to suit a precontracted tenant carries much less risk and will still occur in a market with high vacancy (there could be a structural mismatch) but demand for this will likely also fall coincidently.

4.4.4. Retail

More than any of the other segments retail returns are dependent on the performance of the tenants and the landlord plays a large role in this with determining the proper tenant mix. Developers and owners will face tough negotiators from large national chains, who come prepared with a very good understanding of consumers and location dynamics. They will often be able to negotiate a very favorable lease. The property owner can leverage securing such an anchor tenant in attracting smaller tenants. The interdependence of tenants can also be seen in the downwards spiral phenomenon, where stores close because other stores in the mall are closing.

As of 2015, retail made up 20% of global transaction volume (Savills 2016). The same year in Denmark has retail providing 17% of volume and more recent numbers from 2017 show the share risen to 19% (RED 2018). Another common retail size metric is per capita retail space. This metric reveals large international differences. The U.S. has the world's most retail space per capita at 2,000 sqm per capita compared with 39 sqm in China and 926 sqm in Norway. Denmark and Sweden both have around 530 sqm per capita. (Richter 2017)

Location

A good retail location is very visible and highly trafficked. Retail is about physical interactions, thus a good site has as many consumers close by as possible, which is why current and projected population density is a key determinant for the quality of a location. Household income is also an important metric and in some geographies the availability of public transit. Finally, the fewer other sites in the area where potential competitors could be developed will reduce the risk of increased competition.

Structure and Quality

Retail is grouped by type of center, which is mostly a function of how size of the mall and of the area from which it draws customers. These two are typically highly correlated. The smallest is the stand-alone retail asset, which is often a grocery store either in a residential building or also physically a single tenant structure. Next comes varying sizes of a community or neighborhood malls and in the large end are the regional malls that attract visitors from an entire region. A number of special categories fall outside this general classification such as outlet and theme malls. (Poorvu and Cruikshank 1999)

Another way of categorizing retail property is between commodity shopping and emotional shopping. The former is where the primary purpose is the delivery of goods and services, which are consumed on a regular basis. These are primarily commodity goods purchased without much emotional investment and thus price and convenience drive the purchasing decision. The latter emphasizes emotional feelings attained from the shopping experience's combination of place and goods. These purchases are often optional and made from discretionary funds. (Peiser and Hamilton 2012)

The quality of a retail property is judged by the tenants. As opposed to other property types the tenants of retail are very visible and send single about what kind of property it is. Especially the large chains do very thorough market research before they choose a site. Thus, it has a large signaling effect to attract these tenants with positive spillover effect on attracting subsequent tenants. The reverse is also true, if a mall loses its anchor tenant, it will often be very difficult to attract new tenants.

Retail structures have changed with changes in business practices. The advent of just-in-time inventory management as reduced the need for inventory at local storage. It remains to be seen how increased online shopping will affect this. If stores become an extension of the retailers supply chain with pickups of online purchases and last-mile deliveries, it will change what tenants need from their physical locations. On the other hand, brick-and-mortar stores now more than ever function as showrooms, which translates into a need for consumer facing floor space and not inventory space.

Value drivers

Demand for retail properties is related to demand for goods. Thus, it is like demand for retail goods dependent on household income, distribution of wealth (people spend a higher percentage of their income in the lower income brackets), spending patterns and increases in the nearby population.

An obvious impact on retail spending patterns is the growth of online shopping and the changes to supply-and-demand dynamics induced by this shift leaving selling durable goods vulnerable and perishable foods seem to be following shortly with the advent of online grocers. Instead spaces for experiential retail such as dining, activities as well as health, beauty and fitness facilities. experiential retail's share of consumer retail spending has grown from 24% in 2006 to 39% in 2016 and visitors to malls with a strong experiential offering are there for about twice as long and spend almost three times as much compared with mall without a strong experiential offering (Colliers and GlobalData 2017).

5. Valuation Using Discounted Cash Flows in Real Estate

In order to use the discounted cash flow approach as an investment decision tool, one must understand the concepts of compound interest and cash inflow and cash outflow that is likely to result from a particular investment decision. The use of compound interest can be traced back 4400 years to Sumerian civilization (Muroi 2015). Back then it was not used to calculate cash implications on investments due to its nature of uncertainty. The use was merely focused on loans and life insurance where the future cash flow is known or the probability of it can be calculated on historical data. The discounted cash flow approach used to value investments came much later and was first used by engineers and economist to value fixed assets. The discounted cash technique was not applied to nonfinancial investments until the nineteenth century. It is believed that this was not only due to the difficulties of forecasting the relevant cash flow, but also the small size of investments. In order for the method to be useful, the capital outlay in exchange for an uncertain higher cash inflow in the future, needed to be of a certain magnitude. This magnitude came with the location aspect of building the railways in the US. The American civil engineer A.M. Wellington working on this problem was forced to address some of the ideas behind modern capital expenditure analysis. He had to consider the probability of a rapid increase traffic in order to justify the present expenditure on building the railroads in 1887 (Parker 1968).

Today we still have the same difficulties that A.M Wellington had, when we need choose the location and development of a property. The need to justify present expenditures in return for a future return is based on the demand from tenants and contains the same methodology problems. The discounted cash flow approach has since A.M Wellington's work on railways been refined and is now used to value everything from where a future potential cash flow is involved including real estate investments.

The DCF approach is one of multiple real estate valuation methods available to actors in the real estate investment market. One of its strength is that the valuation is based on the first principles of cash flows accrued to the owner in contrast with a comparable valuation methodology. The DCF methodology is therefore intrinsically linked to the business activity and economic value provided by the physical space. It ties together the two real estate markets in that it capitalizes the space market evaluation of the property as well as the horizon value using the cap rate, which reflects the value of a currency unit of net cash inflow to investors in the market. (Pagourtzi et al. 2003)

5.1. The Fundamentals Behind Present Value

We know from basic economic theory that a dollar today is worth more than a dollar tomorrow. This is due to the concept of time value of money, which assumes your dollar today is worth more because of various factors such as interest rates and inflation. Inflation is a term in economics which describes the general level of rising prices for goods and services, and this is devaluing the dollar in the future, because it does not give you the same purchase power as a dollar today. By having the dollar today, gives us the opportunity to invest it. In the average market we would be able to receive an interest and therefore increase the value of the current dollar.

Due to the above factors mentioned we need to know what the future cash flow would be amount to in present value as basis for decision purposes. The DCF model makes use of the net present value (NPV) investment decision rule as a tool to determine to go forward with an investment or not. The NPV of an investment project is defined as the present dollar value of what being is being obtained minus the present dollar value of what is being given up. (Geltner et al. 2013)

The present value is calculated of a future cash flow

$$Present\ value = PV = \frac{C_t}{(1+r)^t}$$

- → C_t is the expected cash flow to be received in the future
- → t is the amount of years before you receive the cash flow
- → r is the interest rate
- → $(1+ r)^t$ this expression is called the discount factor. This measures the present value of the cash flow received in year t.

As an example, take that the interest rate is 5% and you will receive a dollar in two years, then the value of that dollar would be:

$$PV\ of\ cash\ received = \frac{1}{(1+5\%)^2} = 0.907$$

This means that the value of the received dollar in two years is only worth 0.9070 dollars today, and therefore an investor should not pay more than this for a dollar in two years. In order to determine the NPV of a project this would also be done to the cash outflow. As an example, you are able to pay 0.9200 in one year to receive and in return receive a dollar in two year two years, then the NPV would be calculated as the following.

$$PV\ of\ cash\ paid = \frac{0.9200}{(1+5\%)^1} = 0.876$$

$$NPV = PV\ of\ cash\ received - PV\ of\ cash\ paid = 0.907 - 0.876 = 0.031$$

As a rule of thumb we accept all investment projects in real estate with a positive NPV or a NPV equal to zero, because these are projects that meet or exceed our required return for the investment (Geltner et al. 2013). In the above example 5% is used as the interest rate, which goes into the discount factor. In real estate this factor is a combination of a lot of things and can be relatively difficult to determine. This is addressed further in section 5.2.5.

5.2. The Fundamental Theory Behind the DCF Approach

In the following section we present the discounted cash flow approach in a practical way. We present the mathematical formula for calculating the DCF, but more importantly we identify the key variables that goes into the model and how to compute them. Furthermore, we touch upon some of the modifications that practitioners use which deviates from the theoretical DCF methodology.

The discounted cash flow approach consists of mainly three main steps:

- Forecast the future cash flows from the investment
- Determine the total required return for the project
- Discount the cash flows to present value at the determined required return rate for the project

The value of a property

$$V = \frac{E_0\{CF_1\}}{(1+E_0\{r\}} + \frac{E_0\{CF_2\}}{(1+E_0\{r\}^2} + \cdots + \frac{E_0\{CF_{T-1}\}}{(1+E_0\{r\}^{T-1}} + \frac{E_0\{CF_T\}}{(1+E_0\{r\}^T}$$

$$CF_t = Net\ cash\ flow\ generated\ by\ the\ property\ in\ period\ t$$

$$E\{r\} = Expected\ average\ multiperiod\ return\ per\ period\ as\ of\ time\ zero$$

T= The terminal period in the expected investment holding period, such that CF_T also includes the resale value of the property at that time, in addition to normal operating cash flow.

5.2.1. Forecasting the Cash Flow from the Investment

In the real estate industry, a document that lays out a projection for a future cash flow for a property is called a proforma. The cash flow in the proforma is used in the DCF valuation model. However, forecasting of the future cash flow that could arise from an investment is always something practitioners have had as challenge. In some cases, the future cash flow is easy to calculate with a high degree of certainty, this is the case with bonds, lease contracts etc. where the future cash flow is predetermined and fixed. However, this is not always the case when doing real estate development projects, where a numerous of uncertain factors determine the future cash flow such as vacancy rate, rent prices, construction cost, operating expenses etc. Hence, determining the future cash flow from a real estate development project can be a challenging. When applying the DCF model to a real estate we divide the cash flow into two main categories, the operating cash flow and the reversion cash flow. The operating cash flow consist of cash inflow less cash outflow that arises from operating the building.

5.2.2. Cash Outflow

When investment managers deal with this part of the valuation part in practice, the data available for a case under analysis increases over time. This means that, in the early phase of validating a project the most reliable data for budgeting the cash outflow is not available. An example of this is the construction cost. Here investment managers would get an estimate of the construction cost performed by a consulting firm. With the given estimate for construction cost and all the other estimations made for factors impacting the cash flow, the investment manager validates the case and makes the decision to progress forward with or stop it. If the case makes it to the next phase, then the case will undergo competitive bidding for the construction cost, and the winning bid will then become the new and more exact estimate of the construction cost, which is then used in the DCF model.

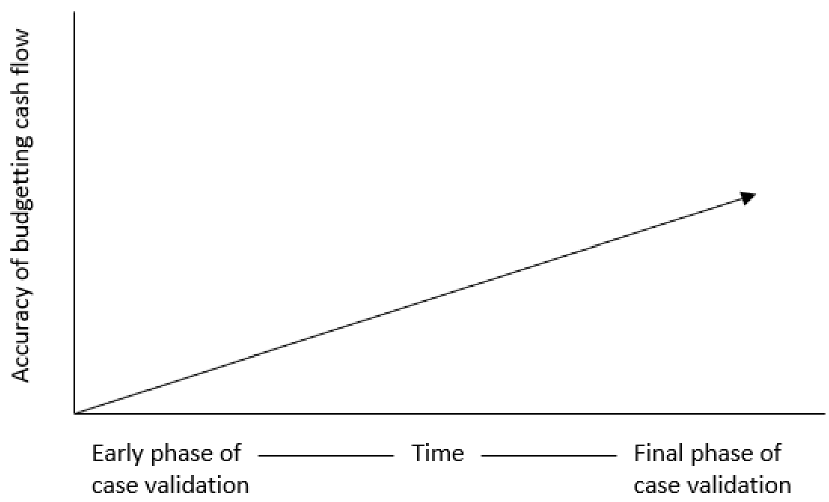

Figure 4 Development of forecasting cash flow

Obviously, a lot of other costs than the construction costs goes into the cash outflow of a real estate development project. In practice a method called the bottom up approach is used for determining the total cash outflow. The method is fairly simple, all costs that goes into the project are stacked on top of each other starting with the construction cost as a base. Some costs are fixed and some are variable. Taxes is a fixed cost, which is embedded in the project and must be accounted for. Cost that are variable could include, maintenance, internet, staff salary, surveillance etc. The total sum of all these costs is then calculated in order to find the total cash outflow.

5.2.3. Cash Inflow

In a real estate development project there are two main cash flows which makes up almost all the cash inflow. First is the rent, which is the continuous cash flow which is received throughout the entire holding period of the building. Second, is the cash received from exiting the project by selling the building. The cash flow received from rent can be relatively easy or difficult to estimate depending on the project. Take a logistic building with one tenant as an example, here the sole rent for the project is depending on one tenant. Investment managers would then be able to make an estimate of the rent, based on a thorough market analysis looking into the current rent level in the market, location, supply and demand etc. however even if the investment managers is able to estimate this fairly accurate, it is important to remember the increased risk, which is involved when you sole rent cash flow is based on one tenant, something that must be accounted for when calculating the discount rate, which address further in section 5.2.5.

A project where this is not the case is in the residential segment where you typically have multiple tenants in the same building. The method used for calculating the rent/square meter is the same as in the logistic project example mentioned before. However, when the cash flow of a project is based on multiple tenants estimating the vacancy rate is really important because it impacts the cash flow. The term vacancy rate is the percentage of all available units in a rental property that are vacant or unoccupied at a particular time. Because the vacancy rate might change over time it is important that it is applied to all the cash received in the future on an individual level. A vacancy rate of 0 percent would mean that there are no available units in the building, this might seem like the ideal situation, but some professional would see this as a strong indicator of the price being too low (Morris Invest 2017), and they would rather aim for a vacancy rate of 4-5% which could be explained by a normal distribution of tenant turnover. To estimate what the exact vacancy rate is going to be, on a specific point of time, can be fairly challenging especially in the development of a building with no historical data available. Investment managers would need to perform a thorough analysis of the market in order to estimate the vacancy rate based on specific rent prices. An important risk factors that such analysis would also need to account for is the health of the local economy and job stability. In an area where there is one central employer the vacancy rate could spike if the employer decides to move its operation elsewhere, due to the likely increase in the unemployment rate in the local community as a result. The real estate industry is no different from other industries and therefore also affected by the health of the local economy. A higher disposable income is likely to result in higher rent prices etc.

5.2.4. The Reversion Cash Flow

In contrast to the operating cash flows which is the result of cash generated from the normal operation of the property, the reversion cash flow is the word used in real estate to describe the cash generated from the sale the entire or portion of the property. Usually the entire project is sold at once, which makes the reversion cash flow only to occur in the last period in the DCF model. The forecasted reversion cash flow consists of the expected resale price of the property at the expected point of time in the future – the selling expenses such as brokers fees and transaction costs. The reversion cash flow often accounts for a large portion of the cash flow generated from a property although this I depending on the holding period. In fact, the present value of the reversion cash flow, commonly accounts for more than 30% of the total present value of a property when doing a 10-year DCF valuation. However, it is at most important to include the reversion cash flow in the proforma which is basis for the DCF valuation. Finding the expected resale price of a property numerous years into the future is for sure difficult. Practitioners have even made up a word for this exercise calling it crystal ball gazing (Geltner et al. 2013).

It is important to remember that the discounted cash flow model does not expect 100% accurate cash flow estimations, because the risk of not being able to estimate the cash flow without error should be embedded in the discount rate. In practice the most widely used method for forecasting the resale price of a property is to apply a direct capitalization rate to the end of the proforma projection period. Analyst tend to project the net operating income for one year beyond the proforma period and applies the capitalization rate to it, in order to come up with an exact number. Now this method leaves us with the issue of figuring out an appropriate capitalization rate, which is also known as the going-out cap rate or terminal cap rate. The method entails the underlying assumption, that the property is only worth something if it has a positive net operating income. Determining the prudent capitalization rate is often done by applying the same capitalization rate or a slightly higher than the one used for the purchase price. If this approach is used and the net operating income is deemed realistic then this method would amount to a fair guess for the resale price of a property (Geltner et al. 2013). Another and simpler method for forecasting the resale price is simply to make an extrapolation of the current purchase price. By using this method, you are able to disregard the underlying assumption that the value of the property is only worth something, if its projected net operating income is positive. Both methods are subject to shortfall if the future market conditions are not accounted for. Bubbles can occur in the real estate industry and impact market cap rates significantly negative and positive on a short time horizon, which both methods are be challenged by.

5.2.5. Determination of the Discount Rate

The discount rate is the factor in the discounted cash flow model that converts the future cash flow into their present value equivalents. Various factors need to be accounted for when doing this exercise some of which have been mentioned earlier in the section 5.2. Here we talked about the time value of money which accounts for increasing prices and interest rates. The combination of these factors amounts to what is labeled in the discounted cash flow model as the risk-free interest. The found risk-free interest is a macroeconomic factor that only changes with time. Therefore, this factor should be the same for all projects with the same time horizon. However, what does change from project to project is the risk premium component in the formula which essentially match the discount rate to the risk of the project.

$$r = r_f + \ Risk\ Premium$$

As the r is the factor which you discount the future cash flow with in order to find its present value, means that a higher risk premium should be applied for projects with risky cash flows. We mentioned earlier that there are different segments such as, residential, office industrial and retail. On a general level the segments have different risk profiles. This is important to consider when determining the risk premium for a specific project. For instance, the industrial segment might bear a significant higher risk than the residential segment in Denmark. This is due to the fact that the rent from an industrial property often depends on one tenant, in contrary a residential property will depend on multiple tenants, hence the default risk of the tenants is based on more people and therefore contributing a lower risk. This means that by default that the risk assumed when developing a logistic property is higher than the risk assumed when developing a residential property which should in general amount to a higher risk premium required for logistic properties. However, even though that general trends in the market tells the story that some segment bears more risk than others, it is at most important to evaluate the specific project, because relevant risk factors could exist that either minimizes or increases the risk. Examples of these factors could be location, long-term lease agreements, time horizon, condition etc. All of these factors are something an investment manager needs to account for when determining the risk premium. One thing is accounting for the factors, another is that in theory these factors must be applied on a specific cash flow level basis, meaning that the discount rate should be determined for each cash flow. In order to demonstrate this, we use the example of interlease vs. intralease as a factor and apply it to the cash flow of a project.

If a real estate developer takes on a project on the behalf of a tenant, who wishes to enter a long-term lease agreement with the developer before the developer has committed to the project, then the risk assumed would be lower. This is due to the fact, that the real estate developer would have minimized his market exposure because the real estate developer now has secured his cash flow by contract. However, this does not mean that the real estate developer has removed his risk completely, because he is still exposed to the risk of the tenant defaulting the contractual lease obligation. When the leasing agreement expire the real estate developer will face market exposure again, hence a higher risk premium should be required for the following cash flow. However, in theory this does not mean that the rest of the entire cash flow should be subject to this risk premium. This is due to the fact, that we expect that the real estate developer is able to enter a new long-term lease agreement in the following year, and thereby only be exposed to the market risk in one year. In practice the interlease discount rate would be applied to all cash flows that is not subject to a lease agreement. By doing so it creates a hidden discount on the project, which will benefit a potential buyer.

The following numerical example serves to clarify what this means for the DCF valuation and how it impacts the operating cash flow. Imagine a project having a holding period of 10 years with a lease agreement of 5 years. The rent level is set to be 300,000 dollars, interlease discount rate for cash flow exposed to market risk are set at 14% and the intralease discount rate for the tenant to default his lease obligation is set to be 6%.

Year	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Operating cash flow	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000
Discount rate applied	6%	6%	6%	6%	6%	**14%**	6%	6%	6%	6%
Discount factor	0,9434	0,8900	0,8396	0,7921	0,7473	0,4556	0,6651	0,6274	0,5919	0,5584
PV Operating Cash Flow	283.019	266.999	251.886	237.628	224.177	136.676	199.517	188.224	177.570	167.518
Total PV OCF	**2.133.214**									

Figure 5 Example 1 of interlease and intralease

Year	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Operating cash flow	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000
Discount rate applied	6%	6%	6%	6%	6%	**14%**	14%	14%	14%	14%
Discount factor	0,9434	0,8900	0,8396	0,7921	0,7473	0,4556	0,3996	0,3506	0,3075	0,2697
PV Operating Cash Flow	283.019	266.999	251.886	237.628	224.177	136.676	119.891	105.168	92.252	80.923
Total PV OCF	**1.798.620**									

Figure 6 Example 2 of interlease and intralease

As the two tables show the theoretical right method values the operating cash flow in total to 2.133.214 USD versus 1.798.620 USD. As a result, not using the theoretical right methodology creates a difference of 334.594 USD between the two valuation methods. The difference could be conceived as a hidden discount for a potential buyer. However, neither of these two methods is likely to be the ones used by practitioners.

5.3. The Internal Rate of Return

The theoretical discounted cash flow model has not found its use in practice. In fact, practitioners typically do not apply cash flow specific discount rates. The real estate industry has more or less modified the approach of using discount rates and instead uses a blended internal rate of return method (Geltner et al. 2013). The internal rate of return is a metric that is used to estimate profitability of investments. The IRR approach determines the one single discount rate, that if applied to the entire cash flow of a given project will result in the project being a zero NPV project. A justification for this simplification of the method is that properties with multiple tenants would be extremely tedious and time consuming to analyze each cash flow in the theoretical right manner.

Applying a blended internal rate of return is a shortcut, practitioners in the real estate development industry uses, but can also make sense from a theoretical point of view if the property has a specific pattern of lease expirations. Investment managers uses the internal rate of return as a tool to see if it is realistic that a property is able to generate the required return that they strive for. However, the problem with applying the IRR is how to estimate a given hurdle rate for a project. At what IRR is investment managers willing to accept a given project, and what is this decision based on. Ideally you would compare the IRR with projects with similar characteristics in the market, in order to determine if the IRR is high enough for that particular type of project. This is often not possible, as it is likely that data for a comparable project is unavailable. Therefore, it often comes down to the individual investment manager, as he is to base his decision of whether or not to invest on his experience and gut feeling. To illustrate this let us take the example from before and add a purchase price of 7,000,000 USD in 2018 and a selling price of 7,600,000 in 2028.

Year		2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Operating cash flow		300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000
Purchase/ sell of property	- 7.000.000										7.600.000
Net operating income	- 7.000.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	300.000	7.900.000
IRR	4,97%										

Figure 7 Example of calculating IRR

In the above table the IRR has been determined as 4.97% for the project. Investment managers would be faced with the question *is almost 5% is that an appropriate return for project?* It might or it might not, that all depends on the risk embedded in this project, the opportunity cost of capital and the required return from the investors. This approach is loved by practitioners because it is easy and fast to apply, but it is not the silver lining as investment managers still have to consider if the return is matching the risk and the required return for the investors.

6. Real Options as an Alternative Valuation Approach

It is now widely recognized that DCF approach to valuations does not adequately capture the flexibility held by management to revise decision in response to unexpected developments. Traditional cash flow approaches make the implicit assumption that once a decision is made the scenario will run its course with management passively along for the ride. In the real-world management will be there to react to changes in market conditions. Thus, as new information arrives and uncertainty about future cash flows are gradually resolved, management has the option to alter its operating strategy (Trigeorgis 1993).

The financial option model of land prices, which focuses on the uncertain value of the future V as a call option allows the holder to benefit from the upside risk in the underlaying asset without exposure to the potential downside. The analogy to financial options allowing the option to purchase a given security upon the payment of an exercise price is clear.

Many investors want to retain land and wait for a more favorable opportunity to invest in its usage. It is therefore useful to be able to estimates the value of the undeveloped land where the future prices of building units are uncertain. Titman (1985) is among the first to adapt to real estate the option pricing methods introduced by Black and Scholes (1973) as well as Merton (1973). Titman approaches this from the angle of valuing the land. However, since land value depends on what can be built on it, this is also a valuation of the potential future structure less construction cost as it follows from Ricardo's law of rent (Ricardo 1817). The vacant land is viewed as a call option, the construction costs as the strike price, and determined the vacant land's value through a combination of construction cost and government bonds. Titman's article is motivated by the large number of underutilized plots in cities that exist for long periods of time despite them being legally and physically developable. The option model suggests that the option characteristic of the vacant land in the presence of uncertain future building prices, which could be built on the land, explain this phenomenon.

The existence of underutilized land inside cities could be explained by non-financial use i.e. consumption benefits for the owner, which a new owner would not enjoy. This could for example be esthetic or sentimental value. While this may explain some cases, it seems unlikely to explain all instances such as the surface parking lots seen the Chicago Loop or – tough less common – in the old parts of Copenhagen[3].

[3] E.g. along S State St and S Wabash Ave in Chicago or on Ny Østergade (by Montmartre) in Copenhagen.

Titman provides a strong intuitive model for understanding why it can be rational to postpone a positive NPV investment. If there is a lot of uncertainty about real estate prices in the future, then the option to select the type of building in the future is very valuable. In this scenario it is less attractive to decide as to the usage of the land now instead benefiting from the future potential appreciation.

Titman highlights how the option valuation method has explanatory power when it comes to land use policy as for example height restrictions intended to limit growth in an area will reduce the uncertainty surrounding the optimal building heights, thus spurring more development and thus more growth in an area where policy makers attempted to limit growth.

In an article not specific to real estate, McDonald and Siegel (1986) look at how the irreversibility of investment decisions affect the optimal decision rule. It is framed as the asymmetry between the irreversibility of deciding to build and the reversibility of deciding to wait under the assumption of an infinitely lived option. Reversibility can be seen understood as the speed of depreciation with quickly depreciating more reversable. They find that that timing considerations are quantitatively important. The usual investment rule to invest if NPV is positive is only valid if the variance of future gains and costs are zero or if the expected growth rate of the PV is negative infinity. Otherwise it can easily lead to the loss of 10 – 20% loss of the projects value. In other cases, they argue that it is optimal to wait and invest when the PV of the investment is twice the investment cost.

Another explanatory model is the dynamic deterministic model of land speculation (Geltner 1989). It shows how the expected growth in future rents could make it optimal to delay construction even though it would be possible to construct at positive NPV structure at the given time. To simply illustrate the dynamic deterministic model, suppose that 100 single-family houses could be constructed today on a certain parcel of land for the cost of K(0)= 80 million and that they would sell for V(0)= 100 million. This would give a profit today of V(0) – K(0) = 20 million. However, we know that in 10 years, growth in the area (or better transportation) means that it would be commercially viable to instead build 500 condominium units on the same parcel of land. These would be worth V(10) = 400 million and cost K(10) = 300 million. The interest rate is 2%. In present terms, the profit would be $[V(10) – K(10)]/1.02^{10}$ = 82 million if we wait and develop in ten years. This assumes that demand growth is such that it does not make economic sense to develop the condominiums sooner than in 10 years.

This shows how it can be optimal to hold land despite there being a positive NPV project that could be built at the given time. A weakness of the dynamic deterministic model is that it assumes that the landowner/developer is gifted with perfect foresight of the future path of building values. While this is obviously unrealistic in ignoring uncertainty, the point about the influence of landowner expectations is an important one.

Following in the path of Titman, Williams (1991, 1993) looks at real estate development as an option. The value of this option depends on the stochastic development of construction cost and operating revenues through time. Williams adds a stochastic carrying cost of land to his model and then finds the optimal date of abandonment. He also finds the optimal density of development and resulting market values of developed and undeveloped properties. Williams also examines the interplay between different option holders under imperfect competition. This is among the differences from financial options in that the exercising of an options develops a property will affect the aggregate supply of the asset and thereby the price in equilibrium of the output from each developed asset. Further, if contractors have increasing costs or limited capacities, which are realistic assumptions, then the cost of development depends on the aggregate demand for development. This not only alters each developer's optimal exercise policy but makes impossible the simultaneous exercise of all outstanding options. In contrast, exercising a financial option will not affect other option holders' ability to exercise.

The real estate development option differs in other important ways from the financial options traded on exchanges. Subject to legal and physical limitations, the landowner has more than a binary choice to exercise or not to exercise the option. The owner can select the scale and density of development. Also, the value of the option does not only depend on the change in the value of the underlying asset but also on the change in strike price i.e. change in the development cost. Additionally, the property development option typically has an infinite maturity whereas financial options have an expiration.

6.1. Understanding Real Options Conceptually

In this section we try take the theory of real option just described and put it into a more conceptually understandable context using simplified examples and touch upon the different option types, which real estate development can have embedded. The valuation process in real estate development involves a series of forecasts of variables such as cash flow, discount rates, capitalization rates etc. with the intention that it can provide us with an unbiased and realistic projection of the potentially uncertain future price of a property. In the previous section regarding the discounted cash flow valuation methodology we concluded that is uses a statically approach. An approach such as this provides little way of actively responding to changes in market conditions. However, some real estate development projects have embedded a flexibility enabling the project allowing it to respond accordingly to favorable changes in market conditions in the future in order to maximize profit, or to avoid potentially negative outcome and minimize losses. This flexibility embedded in a real estate development project make the asset inherently more valuable. The flexibility provides investment managers with the option to respond to volatility in the market conditions, and as similar to financial assets, the greater potential volatility of future results there is, the more valuable the flexibility is.

Flexibility in real estate development comes from the ability to change the nature or course of a project as future events are revealed. These opportunities are referred to as 'real options' and are the right, but not the obligation, to different actions in capital budgeting in response to future knowledge or events.

To clarify why options can be valuable and why the justification can be made to asses them in Real estate valuation let us begin this section of with an example.

Imagine the following scenario:

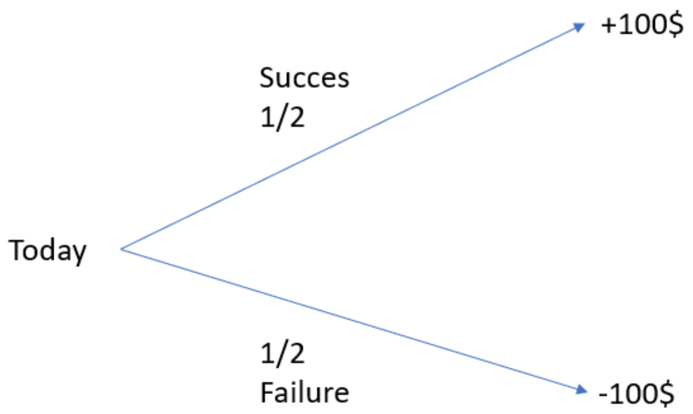

Figure 8 Real option outcome example 1

The listed scenario has a 50% chance of success and 50% chance of failure. The Expected value of this example is 0, which makes it an unattractive investment. Now say that we are able to brake this investment down into two steps. By doing this we state that there is an abandonment option embedded in this investment. We therefore start the investment of by either receiving $20 or losing $20. In the case of losing the $20 we stop the investment an minimizes our total loss to -$20. On the other hand, if we end up receiving the $20, we continue with the investment. In addition, we then end up by either receiving an additional $80 or losing $100. The broken-down example has the same 50% chance of success or failure as previous, but by adding the abandonment option the expected value of the investment becomes profitable. The expected value of investment now amounts to $13.33 instead of $40. Hence the value of the embedded abandonment option is $13.3

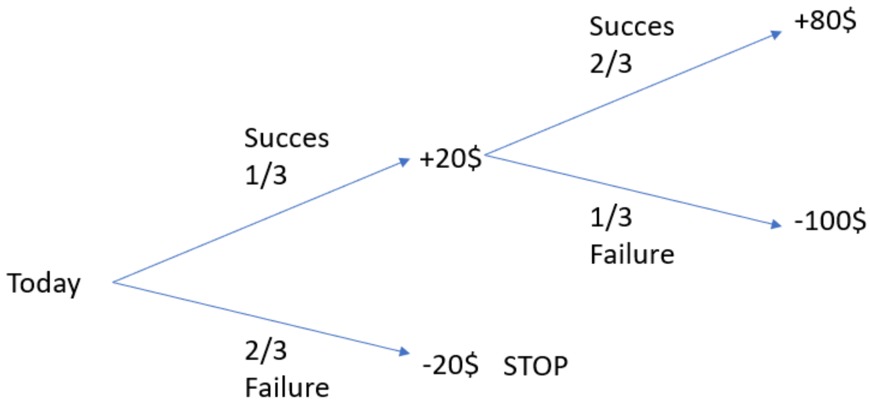

Figure 9 Real option outcome example 2

A statically model such as the traditional discounted cash flow approach ignores the value of this option. Therefore, it might be prudent for investment managers to apply a method that can account for the flexibility embedded in a project. The real options analysis does this effectively and could be used as an alternative approach to the discounted cash flow method which is probably the single most important quantification procedure in micro-level real estate investment analysis (Geltner et al. 2013).

6.2. Description of Real Options

Before we are able to comprehend how real options can be used in practice to valuate real estate development projects, a brief walkthrough of the basic concept of options is needed. In finance an option is defined as "the right without obligation to obtain something of value upon the payment or giving up something else of value" (Geltner et al. 2013, 707).

A person owning an option has the right to exercise his option or not. By exercising the option, the owner simply activates the right owned. There are two main types of options and two categories:

Option Types	
Call	The owner of a call option has the right but not the obligation to purchase an underlying asset at a specified price, which is also referred to as the strike price, for a certain period of time.
Put	The owner of a put option has the right but not the obligation to sell an underlying asset at a specified price, which is also referred to as the strike price for a certain period of time.

Option Categories	
American	If the option can be exercised at any time prior to its expiration date or maturity, then the option is defined as an American option.
European	If the option can only be exercised on its expiration date and not before, then the option is defined as a European option.

To clarify the different type and categories of options, we use an example with a stock as the underlying asset. If you own the right to purchase a stock for a $100 and you have the right to exercise that option at any time before its expiration date, then such option would be defined as an American call option. If the stock price increases to >$100, then you would exercise your option at the strike price of $100, and make the difference in profit. However, if the price drop <$100, then the option would not be exercised, because exercising the option would amount to a loss. This scenario would be the complete opposite if the option owned was an American put option. In such a scenario profit could be made if the stock price dropped to <$100, exercising such option would mean that you could buy the stock for less than a $100 and sell it at the strike price of a $100, and thereby securing a profit.

The term "real option" refers to the study of options whose underlying assets (that is either what is obtained or what is given up on the exercise of the option are real assets (i.e., physical capital as opposed to purely financial assets (Geltner et al. 2013). Hence, a real estate asset qualifies as being something physical.

In order to put this into a real estate development context the concept option valuation theory is applied to a piece of land itself. By doing this we can directly reflect the nature of the real estate system and the relation between land value, time and developments on land. In the following section we use a call option model of land value. Owning a piece of land can be qualified as an American call option with perpetuity as its maturity, because it gives you the right but not the obligation to start a development project. Because of this the landowner is also given a second option, the option to wait and postpone the development project. We have developed the following example to stress the importance of this aspect and what it means in practice when doing a real option valuation.

Imagine a landowner having the following scenario: He can build a property now and know with 100% certainty that he would be able to get $42m for the property with the building and it would cost him 30 million dollars to build it. This scenario amounts to $NPV = 42 - 10 = \$12m$. Now remember that the landowner has the option to postpone the project to next year. We can´t know anything with a

100% certainty in the future, so following scenario is likely to happen: Construction cost increases with 4% and then cost $construction\ cost: 30 * 1.04 = \$31.20\ m$, there is a 25% probability that the value of the developed property next year will be $\$25m$ and a 75% probability that the value will be $\$75m$. The NPV of the different scenarios then follows $25\%\ NPV = 25 - 31.20 = -\$6.20m$ and $75\%\ NPV = 52 - 31.20 = \$ < 20.80m$. Due to the fact that we would not exercise our option with negative NPV, the following expected value of the developed property can be calculated as; $25\% * 0 + 75\% * 20.80 = \$15.60m$. However, even though the $\$15.60m > \$12m$, we need to remember to find its value today in order to compare the two numbers. Real estate development can be classified as quite risky and we therefore use a discount rate of 15% in order to account for the things mentioned in the earlier section regarding discount rates.

This gives a present value of $\frac{15{,}60}{1{,}15} = \$13.57m$ with the scenario of postponing the real estate development project until next year. By doing this exercise we are able to determine the option premium that is embedded with owning the property as $13.57 - 12 = \$1.57m$.

Values in $ millions	Today	Next year	
Properbility	100%	25%	75%
Construction costs	30	31,20	31,20
Value of developed property	42	25	52
NPV of excercise	12	-6,20	20,80
Actions		Don´t build	Build
Expected value of built property (defined as the properbility x outcome)	12	15,60	
Discount rate 15%	12	13,57	
Present value today of option	**1,57**		

Figure 10 Illustration of option value

The above example emphasizes the potential value that options might have in real estate development projects. However, as just shown in the above example it is not only put and call option which may be applicable to real estate development projects.

6.2.1. Types of Options

There are different types of options that are relevant for real estate and while each is an option they differ in when they are available for exercise. Some options will change the physical character of the property while others change the way it is used. Common for them all is the flexibility allowed the option holder.

Even though that many different options might be embedded in a real estate development project, it is not necessarily that all options are valuable. In order for options to be valuable they need to have a degree of exclusiveness. This is something that naturally would be likely for landowners, but exclusiveness on its own does not imply a high value by itself. Imagine having the right to buy a property for a specific price which today is fixed as at market value and the option is exclusive to you. If the market price fluctuates around the strike price of the option, then the value of the option is close to zero. In the commonly used valuation methods a low volatility would equal a lower risk, hence a higher value. Real options flips this understanding on its head by correlating volatility with a higher option value. This is because a real option is fixing the potential loss of any negative outcomes to a certain amount, while keeping an infinite upside for positive outcomes. On that note, it is important to identify the options embedded in a project and whether they add significant value before using an option pricing model that can account for the value. This is in order to avoid unnecessary complexity and burdensome work without it adding any value in relation to the simplified discounted cash flow methodology, and is something we explore further in section 8.6.

The options that might be relevant for a real estate development projects are described in the following based on non-real estate specific overview by Trigeorgis (1993) as well as Lucius's real estate specific discussion (Lucius 2001).

At the highest level real estate options can be divided between portfolio level and asset level options. Portfolio options are strategic and are exercised at the highest management level. The knowledge transfer option is an option to implement the knowledge generated from running projects into new ventures. Another portfolio option is the synergetic properties option, which is the option of acquiring more properties of similar nature in order to benefit from the increased returns to scale. While these portfolio level options are interesting, they are beyond the scope of this thesis and will not be discussed further.

The other group of options, which are relevant to this thesis, are asset level options. Each of these are explained in more detail below.

Demolition option
The property owner almost always holds the option to demolish the structure, which can convert a property into a vacant plot and thus a new form of option will be created i.e. the option to build. However, due to the high fixed cost of real estate it often cannot pay as the cost of demolition and subsequent construction is to high compared with the potential rent. An example of when the option is not available is when the building is listed as a historic landmark.

Abandonment option:
This is the right to exit a real estate project either by selling or less commonly literal abandonment as seen during the financial crises in place such as Detroit. Going forward with a real estate development project might not always be profitable or fit within the portfolio desires of the current owner when the risk of the project and selling price of the land is accounted for.

Shutdown option
This options ties to vacancy in that a property owner has the option to shutdown parts or all of a property if it is vacant. This could for example be closing of a section of a larger mall if it was not leased thereby reducing operating costs as well as reducing the appearance of a dead mall.

Up-sizing option:
If a project is built into more phases, investment managers can choose to upscale a project if the first phase is deemed a success. By doing so can create additional value to a project and in the end maximize the return of the real estate development project.

Delay/waiting option:
As shown in the previous example it might be prudent to wait with the development of a project if key factors in the business is uncertain such as rent prices, construction costs or the price of the property and the outlook for these factors seems more favorable in the future, then investment managers may be able to obtain a higher return by postponing a project.

Stage/phasing option:
The ability to split a project into multiple stages will decrease the risk as future stages can be delayed until marginal cost is higher than marginal revenue. During previous stages, knowledge will be acquired that may allow optimization of subsequent stages.

If the project is built in phases more accurate calculation on the return can be done based on the exact cost the initial phase. Based on this knowledge investment managers are able to make use of other suitable options such as the abandonment option, design option and delay option.

Design option:
This refers to the optionality real estate developers has to pick and choose different design solutions. For instance, developers can switch the different materials used for a property in order to make it more cost effective by lowering the maintenance cost or construction cost. On the contrary a switch towards more expensive materials may also be prudent, if the design generated results in higher rent prices.

Usage option:
This option is also referred to as the switching option in the real estate industry. If flexibility is embedded in the project in its initial design, then real estate real estate managers has the option to react to the uncertainty in the future for different markets. If the property currently serves as a hotel and the competition increases it might be favorable to make a conversion (switch) to condominiums if the outlook for that market is better. Another example, which has been exercised in for example Copenhagen in large scale recently is the conversion of office to residential.

Compound options
Options do not need to come alone. As alluded to in the examples above, some options generate new options e.g. by using phasing and learning from the first phase gives the option to change subsequent phases. Phasing can in itself be seen as a bundle of waiting options with different waiting periods with the exercise of the first giving access to the next.

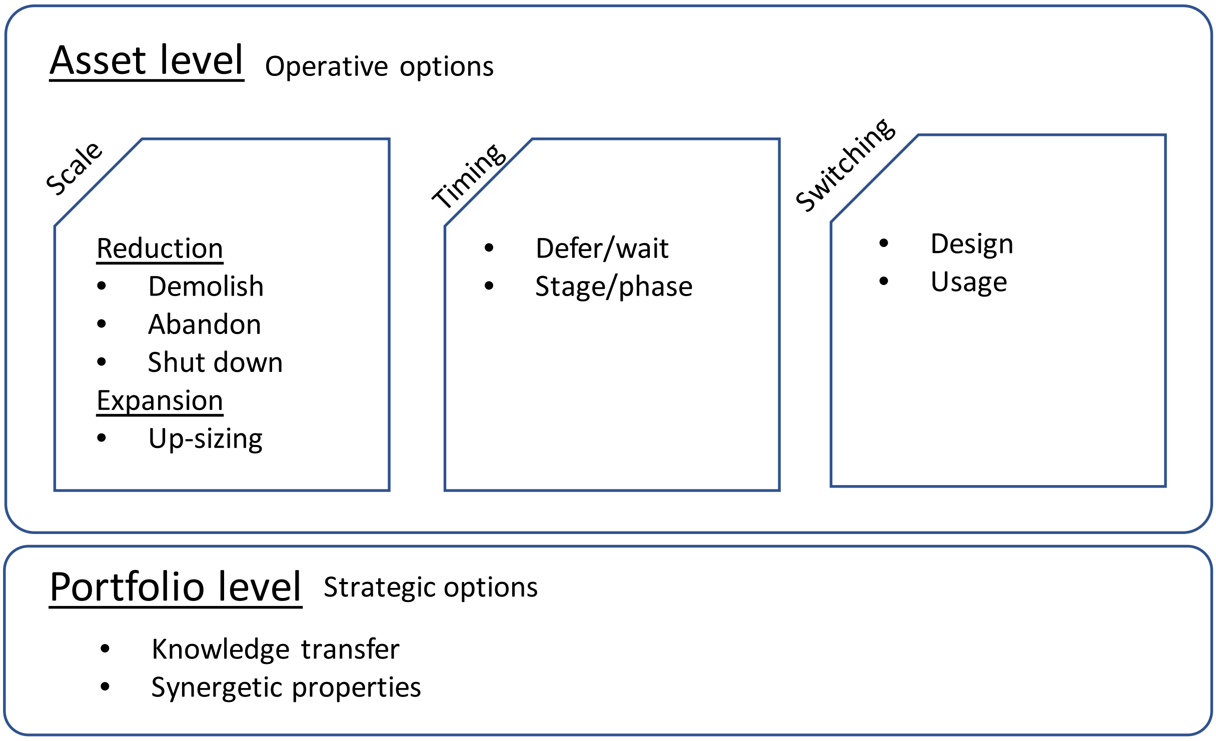

Figure 11 Overview of option types

6.3. The Different Real Option Methods

The previous used example is an extreme simplification of a real option method that illustrates the value of an option. This methodology is to simplified and cannot be used on any practical case in the real estate industry but other and more sophisticated models can. Through the years many different real option valuation models have been developed, some which are more applicable to the real estate development market. In this thesis we will focus on two main real option valuation methodologies being defined as the economic approach and the engineering approach.

6.3.1. The Economic Approach

What in this thesis is referred to as the economic approach builds on the fundamental acknowledged Black-Scholes formula used for valuation of European call options. The formula was developed back in 1970s by Fischer Black, Myron Scholes and Robert Merton whom also received a noble prize award for their work. What differentiate their model from other partial differential methodologies is that it incorporated the rule of no arbitrage. Even though the Black-Scholes formula has gained wide acceptance in the financial world to price options and real options, the formula has it´s shortfalls if applied to real estate. Remember that we earlier in this thesis classified a piece of land as being an American call option in perpetuity. The Black-Scholes formula requires a fixed decision date, hence European options, which makes it incapable of valuing complex real estate development projects by itself. Furthermore, the Black-Scholes formula cannot in its original form handle dividend payments which rent payments would be regarded as in real estate.

6.3.2. Binomial Option Valuation Method

This approach builds on the Black-Scholes formula and recognizes the strengths and weaknesses behind the formula. The Binomial option valuation model was developed and first introduced in 1979 by Cox, Ross and Rubenstein (1979). The method uses an illustrative and visual approach which recognizes the flexibility that is embedded in some real estate projects through a value tree. The value tree gives investment managers an intuitive understanding of the intermediate value of the option in the holding time of the real estate project. One of the key assumptions behind the model is that it uses risk neutrality as the Black-Scholes formula which enables the model to compute a true economic value of a project and thereby setting aside one of the usual challenging things for investment managers to determine in the traditional theoretical discounted cash flow method being the risk adjusted discount rates. tool, which enables them However, the method does not come without its challenges one of them being that it also requires a fixed end date. The fact that a piece of land could be qualified as an American Call option in perpetuity challenges the model, this can be overcome be creating a fearsome big model that it tries to account for a realistic perpetuity in a finite model.

The binomial tree included in this method consists of “up” and “down” values for the underlying asset for each point in time through the holding period of the project. In order to compute those values, the following variables and input are needed for a given project:

→ $V_{i,j}$: Value of the underlying asset at period j, with I representing the total number of down outcomes out of j periods
→ K_j: Construction cost at period i, corresponding to V at the same period (Here instantaneous construction is assumed, something we later modify)
→ $C_{i,j}$: Value of the option (land price) at period i, with j representing the total number of down outcomes (corresponding to the movement of V) out of i periods
→ PV_t{n]: Present value of n as of period t
→ E_t[n]: Expected value of n as of period t
→ R_V: Expected annual total return on investment in the underlying asset (assumption of constant expected return (r_v) as well as a constant volatility sigma V, through the holding period of the option)
→ Y_v: Annual net rental income cash payout (yield) as a fraction of current building value
→ G_V: Expected annual growth rate in value of underlying asset defined by the equation $G_v + 1 = \frac{1+r_v}{1+y_v}$
→ P: Probability of the up outcome in each period, the probability for down values is determined by 1-p
→ σ_V: Expected annual volatility of returns on the specific underlying asset
→ R_f: Risk-free-interest
→ G_K: Expected annual growth rate in the construction cost (A constant growth in construction cost I assumed and is uninflected by the up & down outcomes)
→ Y_k: Construction cost yield defined as $g_k + 1 = \frac{1+r_f}{1+y_k}$

Below Is a figure that visually presents how binomial value tree is created in order to gain a fundamental understanding of what happens before we dive into the underlying mathematics behind the methodology. The figure is simplified by using a holding period of a project of four years, meaning it does not try to mediate the challenges regarding the option perpetuity. Afterwards we present the binomial tree with a mathematical approach.

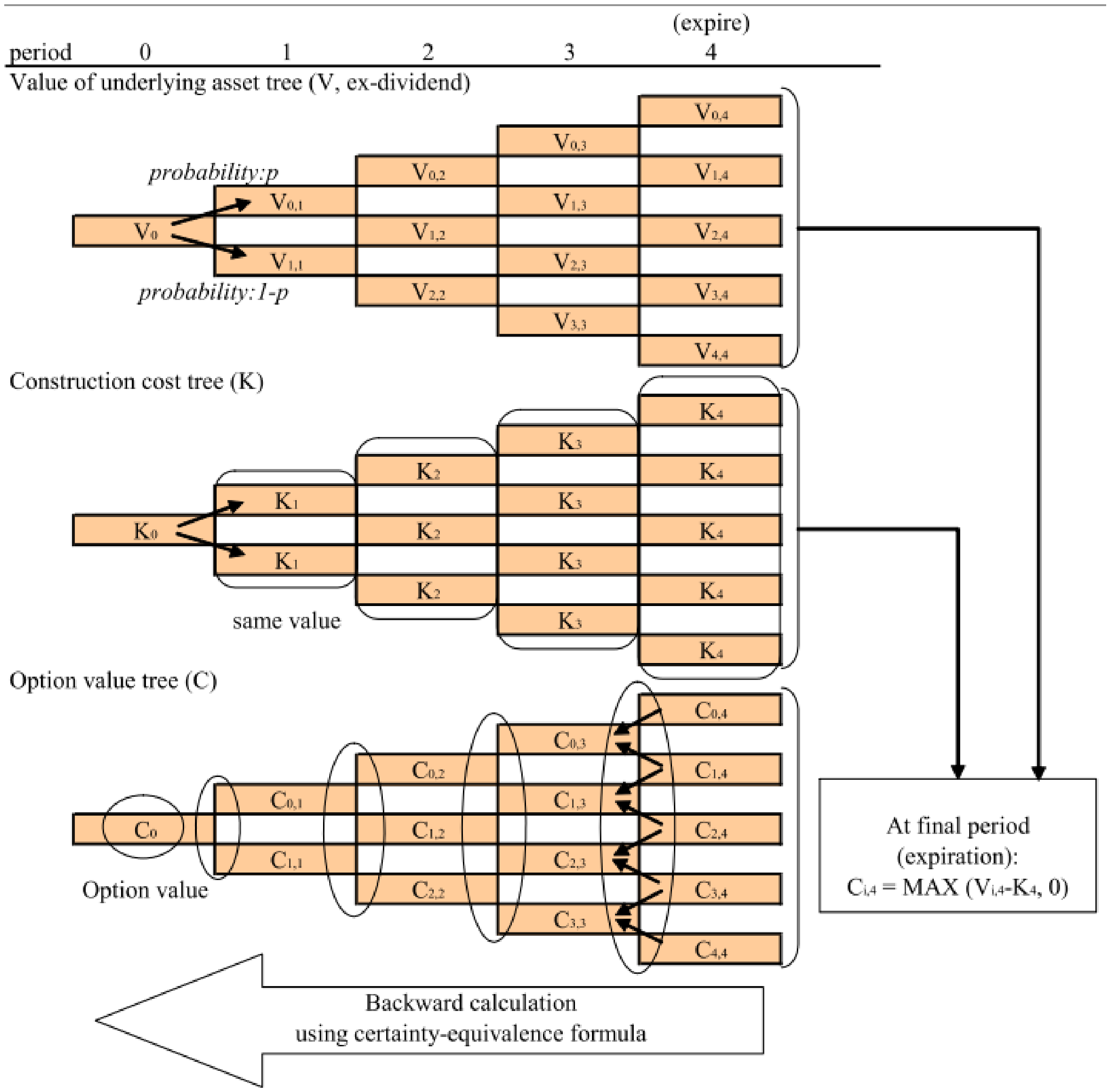

Figure 12 (Mansunaga 2007)

In order to create the Binomial value tree, one must know the value of the build property at t0 along as well as the construction cost of it at t_0. Furthermore, investment managers most estimate the following factors for a given project:

$$r_V = Expected\ annual\ total\ return\ on\ investment\ of\ the\ completed\ building$$

$$Y_V = Annual\ net\ rental\ income\ cash\ payout\ (yield)\ as\ fraction\ of\ current\ building\ value$$

$$\sigma_V = Expected\ annual\ volatility\ of\ returns\ on\ the\ specifik\ building$$

Note that the σ_V must account for both the idiosyncratic risk subject to the specific building along with the general market volatility in the real estate market. Furthermore, it must be noted that estimation of expected annual return and annual net rental income serves as the assumption for computing the expected annual growth rate in vale of the specific building through the formula:

$$g_V = \frac{1 + r_V}{1 + Y_v} - 1$$

With the above information it is now possible to compute the value of "Up notch" and "Down notch" for each specific point in time in the Binomial tree through the following equations:

$$\text{"Up notch": } V_1^{up} = V_0 x(1 + \sigma_V)/(1 + y_V)$$

$$\text{"Down notch" } V_1^{down} = V_0/(1 + \sigma_V)/(1 + y_V)$$

The probability for each "Up" and "Down" notch is determined through the following equation:

$$P_{up} = ((1 + r_V) - \frac{1}{1 + \sigma_V})/((1 + \sigma_V) - 1/(+\sigma_V))$$

$$P_{down} = 1 - p_{up}$$

Next step is calculating the development in the construction cost. For simplicity we have decided to use the assumption that the construction cost growths with a constant rate being G_k. The construction cost can be calculated through the following equation for each period:

$$K_{t0+x} = K_{t0} * (1 + G_k)^{t_x}$$

Valuating a piece of land as an American option where it is possible for the developer two exercise the obligation at any time prior to the expiration date is not completely straight forward. The complete formula for doing this valuation in the binomial world consists of two parts, which are embedded in an algorithm.

The first step of the algorithm consists of determining the option value at the expiration date, which can be regarded as the process of valuing a European option. To do so we use the following formula:

$$C_{iT} = Max\left(V_{i,T} - K_T, 0\right)$$

In the simplified example we only use a time span of 4 years equaling T=4. The formula provides us with the maximum value of the option under these assumptions and is used in the next step where the found value is used as a starting point for computing the values in the periods beforehand. The option value for all stages in the example is then calculated as followed:

$$C_{i,t} = Max \left\{ \begin{array}{c} V_{i,t} - K_t, \\ \dfrac{\left(p * C_{i,t+1} + (1-p) * C_{i+1,j+1}\right) - \left(C_{i,t+1} - C_{i+1,j+1}\right) * \dfrac{r_V - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))}}{1 + r_f} \end{array} \right\}$$

The Max value Is either determined as the first or last part of the above equation, depending on which part compute the highest value. However, we will later in this thesis introduce the concept of construction time and integrate that into the above formulas, so that it is capable of supporting a real estate development case such as the one under analysis in this thesis.

6.3.3. The Samuelson-McKean formula

The above-mentioned Binomial option valuation approach has important merits and is capable of capturing some of the valuable options embedded in a real estate project. However, it is subject to the significant weakness, that it is incapable of capturing and accounting for perpetuity options. The real estate development industry does have projects where options with the characteristics of perpetuity is embedded. Like the example mentioned earlier of landownership that could be considered as an American call option in perpetuity. In order to overcome this hurdle and compute the value of perpetuity options Geltner et. Al (2014) recommends the use of Samuelson-McKean's formula. The formula was presented in 1965 by Paul Samuelson and Henry McKean with their work on pricing perpetual American warrants. However, even though Geltner advices the application of formula to real estate development project, it does have its weaknesses. The formula is built on the assumption of instantaneous construction, meaning that it ignores the time of construction. This assumption does not reflect the practice in the real estate development industry, and is therefore to be regarded as unrealistic. Furthermore, the Samuelson-McKean formula assumes riskless construction cost that grow deterministically at a constant rate through time.

The formula functions as a closed formula like the Black-Scholes formula. The formula requires three parameter values as inputs, which describe the underlying real estate assets current cash yield rate, the volatility of the property value and "construction cost yield" which is the difference between opportunity cost of capital of construction cost cash flow and the expected growth rate in construction cost.

$$Y_k = r_f - g_k (risk\ free\ interest\ rate - growth\ rate\ in\ construction\ cost)$$

$$Y_V = Current\ cash\ yield\ rate\ of\ property$$

$$O_v = The\ volatility\ in\ the\ market\ value\ of\ the\ property$$

With value the above three parameters we are capable of defining the option elasticity labelled as the value n, by using the following formula:

$$n = \left\{ y_v - y_k + \frac{\sigma_V^2}{2} + \left[\left(y_k - y_v - \frac{\sigma_V^2}{2} \right)^2 + 2y_k * \sigma_V^2 \right]^{1/2} \right\} / \sigma_V^2$$

n provides you with a percentage of change in option value when the underlying asset is subject to a 1% change value.

Before we are capable of calculating the vacant land value under the Samuelson-McKean formula one must determine the hurdle rate also referred to as the critical value. The hurdle rate determines at which value a developed property should remain undeveloped for the time being and above which value one should start the development immediately, hence exercise the option. In order to calculate the hurdle rate labelled as V^*, one must determine the following inputs:

$$K_0 = Construction\ cost\ of\ building\ the\ property\ (Excluding\ land\ value)$$
$$n = The\ option\ elasticity$$

The hurdle rate is given by the following formula:

$$V^* = K_0 * \frac{n}{n-1}$$

Now we can compute the value of the vacant land under the assumptions of Samuelson-McKean labeled as C0. The following inputs are needed in order to do so:

$$V_0 = Current\ value\ of\ build\ property$$
$$K_0 = Construction\ cost\ of\ building\ the\ property$$
$$V^* = The\ hurdle\ rate\ (critical\ value)$$
$$n = Option\ elasticity$$

C0 is calculated using the following formula:

$$C_0 = (V^* - K_0) * \left(\frac{V_o}{V^*}\right)^n$$

The key issue of the before presented formulas is the assumption of instantaneous construction time and disregarding the time of construction. However, it is possible to make adjustment to the original model, so that it is capable of capturing a more realistic view on the construction time.

In order to account for the construction time, we label the construction time as “CT”. Suppose we are exercising the option at time t then we receive the completed building at (t+CT). Therefore, in order to be capable of making the decision of whether or not to exercise the option of building the future of value of the building must be discounted to the present value of time t using the risk-adjusted discount rate for the underlying asset. This can be done by using the following formula:

$$PV_t[V_{t+CT}] = \frac{E_t[V_{t+CT}]}{(1+r_v)^{CT}} = \frac{V_t(1+g_v)^{CT}}{(1+r_v)^{CT}} = \frac{V_t\frac{(1+r_v)^{CT}}{(1+y_v)^{CT}}}{(1+r_v)^{CT}} = \frac{V_t}{(1+y_v)^{CT}}$$

The same considerations should be made in relation to the time of payment for the construction costs. Geltner makes the rough assumption that the construction cost is paid at the time of completion of the project and that It should be calculated using the same approach as for the value of the building at the exercise time but instead of using a risk-adjusted discounted rate, the risk-free rate is used as the discount rate in the following formula:

$$PV_t[K_{t+CT}] = \frac{E_t[K_{t+CT}]}{\left(1+r_f\right)^{CT}} = \frac{K_t(1+g_k)^{CT}}{(1+r_v)^{CT}} = \frac{K_t\frac{\left(1+r_f\right)^{CT}}{(1+y_k)^{CT}}}{\left(1+r_f\right)^{CT}} = \frac{K_t}{(1+y_k)^{CT}}$$

7. Case: Greve Distribution Center

To investigate the potential use of real options valuation methods in practice, the Greve Distributions center will serve as a case to examine how these concepts can be applied to a conceived and executed in a real estate development context and how it differs from the traditional discounted cash flow approach.

The information presented in this chapter is gathered from the following sources: Byggefakta, CBRE and BiQ.

7.1. Project Description

The Greve Distributions center is here after referred to as GDC. The GDC is a newly established distribution center with storage and logistic facilities is located close to the E47 highway. According to the developers is the location of the property in Greve in the greater Copenhagen area a “Hot Spot”, where more than half of the Danish population can be reached with an hour by car. The construction of the property began in January 2018 and is expected finished in December 2019.

Figure 13 Greve Distribution Center 1

7.1.1. Site Characteristics

The site is located at Kildebrøndevej 44 2670 Greve the plot area is 230.00 square meters. Prior to the build of GDC the land has been used as an orchard, market garden and plant nursery. The plot is located next to the E27 highway also known as the Køge Bugt highway enabling transportation to the Copenhagen airport in 25 minutes and reaching Jutland, Copenhagen and southern Sweden all within 1 hour. The land was purchased the 10/2-2017 by the MG Real Estate group for 20.700.000 DKK.

7.1.2. Building Specifications

The GDC is expected to be approximately 110.000 square meters logistic, storage and distribution property. The square meters are supposed to be divided into 11 warehouses. The property is planned to consist of four different types of warehouses.

Warehouse Type 1:

This type is expected to be the biggest type of warehouse in the GDC consisting of a total of 9.797 square meters which is divided into 9.279 storage space on the floor level and 518 of office space on a first floor. The GDC ensures an efficient unloading and loading process of goods with 13 docks. It is planed that the GDC will consist of 8 warehouse type 1 buildings.

Warehouse Type 2:

This type of warehouse is expected to be consisting of a total of 7.637 square meters which is divided into 7.119 storage space on the floor level and 518 of office space on a first floor. The GDC ensures an efficient unloading and loading process of goods with 10 docks. It is planed that the GDC will consist of 1 warehouse type 2 building.

Warehouse Type 3:

This type of warehouse is expected to be consisting of a total of 7.715 square meters which is divided into 7.197 storage space on the floor level and 518 of office space on a first floor. The GDC ensures an efficient unloading and loading process of goods with 10 docks. It is planed that the GDC will consist of 1 warehouse type 3 building.

Warehouse Type 4:

This type of warehouse is expected to be consisting of a total of 7.434 square meters which is divided into 6.916 storage space on the floor level and 518 of office space on a first floor. The GDC ensures an efficient unloading and loading process of goods with 10 docks. It is planed that the GDC will consist of 1 warehouse type 4 building.

In addition to the warehouses there is a need for parking lots, which will also be developed in order to support the number of employees working in the warehouses. Furthermore, the MG Real Estate have outlined plans for a water drain system located behind the warehouses securing the facility against flooding. The GDC is built to support the modern needs and requirements for pharmaceutical, e-commerce and food companies. It is expected that GDC will serve as a distribution hub for 6-7 companies with different demands for space.

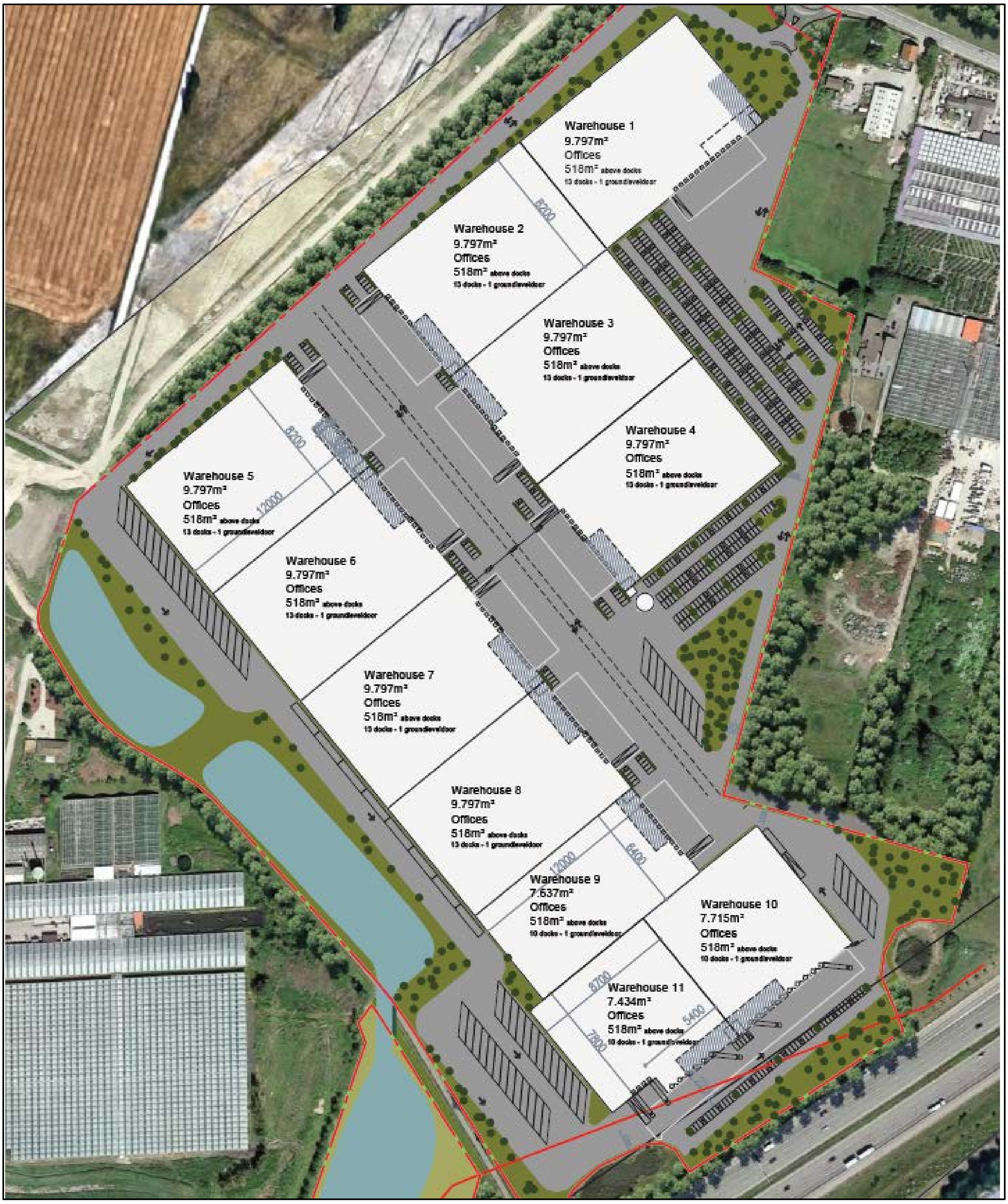

Figure 14 Greve Distribution Center 2

7.1.3. Cash Flow from the GDC

The MG Real Estate Group have estimated the operating cash flow for the GDC property. However, the reversion cash flow has not been estimated by the developer. Neither has the MG Real Estate Group disclosed their holding period of the GDC. Therefore, we are estimating both of these factors in order to value the GDC as an investment through the DCF and real option valuation methodologies.

7.1.4. Phasing Option Embedded in the Project

Figure 15 Greve Distribution Center 3

Phase 1:

In the first phase it is planned to build 4 warehouses of type 1 which amount to a total of Approximately 40.000 square meters. The developer MG Real Estate initiated the construction of phase 1 the 1/1-2018 and is expected to be finished the 31/12-18.

Phase 2:

In the second phase it is planned to build the remaining warehouses consisting of 4 type 1, 1 type 2, 1 type 3 and 1 type 4 amounting to a total of approximately 60.000 square meters. The second phase is expected to be initiated 1/1-2019 and is expected to be finished the 31/12-2019.

7.2. Valuing GDC Using the Binomial Approach

In this section we will go through our binomial valuation of the GDC step by step. As mentioned previously the GDC project is expected to be built in 2 phases creating an abandonment option embedded in the project. This abandonment option is then valued using the binomial approach and Samuelson-McKean approach among with the valuation of the entire project. However, in order for us to be capable of valuing the abandonment option one must know the alternative scenario to GDC project. Therefore, we start this section of with introducing the alternative scenario of a small warehouse.

There will always be some uncertainty related to estimating the demand and rent prices for a given area and building. When valuing the abandonment option, it is therefore a necessity to assume a minimum scenario for these factors. In this thesis we assume that the land on Kildebrøndevej 44 2670 Greve always will be subject to the demand and capable of supporting rent prices of a small warehouse based on its location. The following factors is estimated for the abandonment option of a small warehouse.

Total square meters	10.000
Rent price pr. Square meter	600 DKK
Operating expenses pr. Square meter	10 DKK
Construction cost pr. Square meter	6.500 DKK
Total construction cost = K_0	65.000.000 DKK
Construction time	1 year

We have estimated the value of the small warehouse by using a cap rate of 5.75% and applied it to the annual Net operating income generated by the small warehouse. The cap rate used is estimated logistic properties located in the Copenhagen area by RED who acts as a commercial broker on the Danish real estate market(RED 2018). The small warehouse is valued using the following formula:

$$V_0 = \frac{10.000 * (600 - 10)}{5.75\%} = 102.61\ mDKK$$

The construction cost is calculated by using the same construction cost of 6,500 DKK pr. Square meter and methodology for both projects. Before we dive any further into the possible scenarios of outcomes for the development projects we need to outline that the option for exercising phase 2 is first available after phase 1 has been built. The following figure outlines the process and possible development of the land.

The operating cost is based on the assumption that the lease is triple net with substantially all costs paid by the occupier such as taxes, insurance and maintenance.

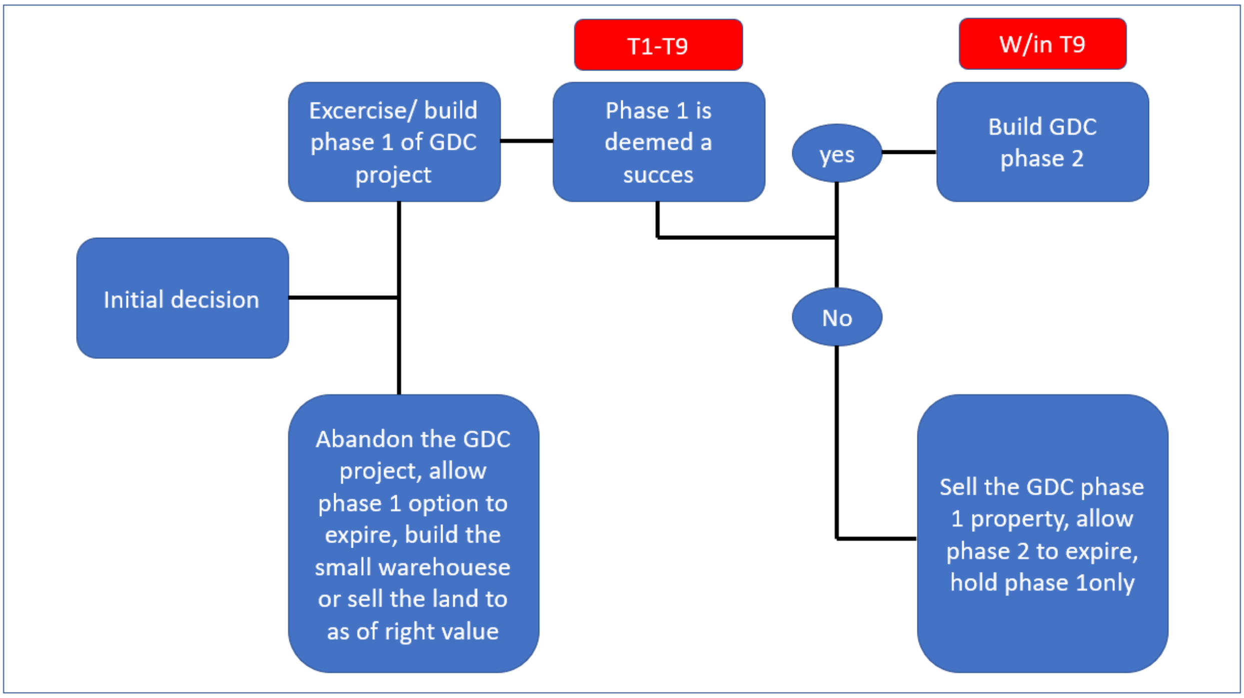

Figure 16 The possible scenarios for development of GDC

One of the downsides by using the binomial method to value the GDC project is that the option needs an expiration date. In the interest of simplicity, we assume that the investor fund in the project has lifetime of 10-years. Furthermore, a construction time of 1 year is assumed for both phases of the GDC project. Due to these assumptions the option of building either phase 1 or 2 expires after 9 years. This also means that the option to exercise phase 2 expires if phase 1 is not build within 8 years.

The characteristics of phase 1 and phase 2 of the GDC project is listed below:

Phase 1

Total square meters	40.000
Rent price pr. Square meter	600 DKK
Operating expenses pr. Square meter	10 DKK
Construction cost pr. Square meter	6.500 DKK
Total construction cost = K_0	260.000.000 DKK
Construction time	1 year

$$V_0 = \frac{40.000 * (600 - 10)}{5.75\%} = 410{,}43\ mDKK$$

Phase 2

Total square meters	60.000
Rent price pr. Square meter	600 DKK
Operating expenses pr. Square meter	10 DKK
Construction cost pr. Square meter	6.500 DKK
Total construction cost	390.000.000 DKK
Construction time	1 year

$$V_0 = \frac{60.000 * (600 - 10)}{5.75\%} = 615{,}65\ mDKK$$

In order to be capable of applying the binomial approach to the GDC project and the small warehouse we need to estimate the following factors:

Volatility for the properties (σ)
The data for single asset volatility is not available in Denmark to our knowledge as it would require multiple sale prices for individual properties. Furthermore, the case used in this thesis is a development project and therefore naturally no historic sale prices is available. We have therefore chosen to apply a volatility of 15% which is calculated by Geltner for properties (Geltner et al. 2013).

risk-free rate (r_f)
It is in general assumed that the 10-year Danish government bond can be characterized as risk-free and therefore can serve as the risk-free rate in our example. The risk-free rate of 0.375% is based on an average of the last 12 months observed efficient yield for the 10-year Danish government bond (Statistics Denmark 2018a).

Growth in construction cost (gK)
Byggefakta is supplying Statistics Denmark with a construction price index for the construction industry in Denmark. 2.2% is used as the growth in construction cost as it represents the increase in prices for renovation and maintenance for 2016-2017 (Statistics Denmark 2018b).

Underlying asset cash yield (yV)
As established earlier, we use a cap rate of 5.75% for the return of underlying asset, as this is the prevailing cap rate for Copenhagen logistics as of 2018 Q2 according the commercial brokerage firm RED (2018).

Underlying asset total return (rV)
In order to be consistent in our approach we base our estimation on historical numbers following the suggested return of 7.20% used, estimated by the Danish Property Federation[4] which is the trade industry association (Ejendomsforeningen Danmark 2018).

By now we have established that the GDC project can be characterized as a compound option because the option to build phase 2 relies on the exercising of phase 1. We also established that both of these options have an expiration date due to the simplified assumption of 10-years lifetime of fund ownership in order to be in accordance with the binomial approach. However, the abandonment option is not subject to this assumption making the option to build the small warehouse perpetual but only the one-year time to build. Therefore, we start of by valuating the as of right now value of the abandonment option by using the Samuelson-McKean formula previously described in the section 6.3.3.

$$y_k = \frac{1+r_f}{1+gK} - 1 = \frac{1+0.375\%}{1+2{,}20\%} - 1 = -1.79\%$$

$$n = \frac{\left\{ y_v - y_k + \frac{\sigma_V^2}{2} + \left[\left(y_k - y_v - \frac{\sigma_V^2}{2} \right)^2 + 2y_k * \sigma_V^2 \right]^{\frac{1}{2}} \right\}}{\sigma_V^2}$$

$$= \frac{\left\{ 5.75\% - (-1.79\%) + \frac{15\%^2}{2} + \left[\left(-1.79\% - 5.75\% - \frac{15\%}{2} \right)^2 + 2 * -1.79\% * 15\%^2 \right]^{\frac{1}{2}} \right\}}{15\%^2}$$

$$= 7.49$$

$$V^* = K_o * \frac{1+gK}{1+r_f} * \frac{n}{n-1} = 65 * \frac{1+2.2\%}{1+0.375\%} * \frac{7.49}{7.49-1} = 76.39$$

With the above calculated factors, we are now capable of calculating the as of right now option value for building the small warehouse using the following formula:

$$c_0 = \begin{cases} (V^* - K_0/(1+y_K)) * (\frac{\frac{V_0}{1+y_V}}{V^*})^n, if\ V_0/(1+y_V) \leq V^* \\ \frac{V_0}{1+y_V} - \frac{K_0}{1+y_K}, otherwise \end{cases}$$

[4] Danish: *Ejendomsforeningen Danmark*

$$\frac{V_0}{1+y_V} \leq V^* = \frac{102.61}{1+5.75\%} = 97.03$$

$$V^* = K_o * \frac{1+gK}{1+r_f} * \frac{n}{n-1} = 65 * \frac{1+2.2\%}{1+0.375\%} * \frac{7.49}{7.49-1} = 76.39$$

$$97.03 > 76.39 \; stating \; that \; C_0 \; is \; given \; by \; \frac{V_0}{1+y_V} - \frac{K_0}{1+y_K}$$

$$C_0 = \frac{102.61}{1+5.75\%} - \frac{65}{1+(-1.79\%)} = 30.85 \; mDKK$$

In order to apply this procedure to future stages, we need to identify the probability for "Up" and "Down" notches. The abandonment option introduced as a small warehouse is a logistic property and given the GDC project also is a logistic property we use the same input for both projects, when calculating the probability for "Up" and "Down" notches. This naturally results in single probability applied for all projects, which is given by the following formula:

$$P_{up} = ((1+r_V) - \frac{1}{1+\sigma_V})/((1+\sigma_V) - 1/(+\sigma_V))$$

$$= ((1+7.2\%) - \frac{1}{1+15\%})/((1+15\%) - 1/(1+15\%)) = 0.7219$$

$$P_{down} = 1 - p_{up} = 1 - 0.7219 = 0.2781$$

The following tables provide an overview of the possible future scenarios for the small warehouse project at different stages equivalent to the 10-year assumed lifetime of the fund:

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9	10
Small Warehouse Expected Val. (as if new):		104.02	105.44	106.89	108.35	109.84	111.34	112.87	114.42	115.99	117.58
"down" moves ("*i*"):	**Small Warehouse Value Tree (as if new):**										
0	102.61	111.58	121.34	131.96	143.50	156.05	169.70	184.55	200.69	218.24	237.33
1		84.37	91.75	99.78	108.51	118.00	128.32	139.54	151.75	165.02	179.46
2			69.38	75.45	82.05	89.22	97.03	105.52	114.74	124.78	135.70
3				57.05	62.04	67.47	73.37	79.78	86.76	94.35	102.61
4					46.91	51.01	55.48	60.33	65.61	71.34	77.58
5						38.57	41.95	45.62	49.61	53.95	58.67
6							31.72	34.49	37.51	40.79	44.36
7								26.08	28.36	30.84	33.54
8									21.45	23.32	25.36
9										17.64	19.18
10											14.50

Figure 17 Value tree of the Small Warehouse

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):	**Small Warehouse Construction Cost Tree**:										
0	65.00	66.43	67.89	69.39	70.91	72.47	74.07	75.70	77.36	79.06	80.80
1		66.43	67.89	69.39	70.91	72.47	74.07	75.70	77.36	79.06	80.80
2			67.89	69.39	70.91	72.47	74.07	75.70	77.36	79.06	80.80
3				69.39	70.91	72.47	74.07	75.70	77.36	79.06	80.80
4					70.91	72.47	74.07	75.70	77.36	79.06	80.80
5						72.47	74.07	75.70	77.36	79.06	80.80
6							74.07	75.70	77.36	79.06	80.80
7								75.70	77.36	79.06	80.80
8									77.36	79.06	80.80
9										79.06	80.80
10											80.80

Figure 18 Construction tree of the Small Warehouse

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):	**Small Warehouse Hurdle Value Tree (Samuelson-McKean, reflecting 1 yr time-to-build):**										
0	76.39	78.07	79.78	81.54	83.33	85.17	87.04	88.95	90.91	92.91	94.95
1		78.07	79.78	81.54	83.33	85.17	87.04	88.95	90.91	92.91	94.95
2			79.78	81.54	83.33	85.17	87.04	88.95	90.91	92.91	94.95
3				81.54	83.33	85.17	87.04	88.95	90.91	92.91	94.95
4					83.33	85.17	87.04	88.95	90.91	92.91	94.95
5						85.17	87.04	88.95	90.91	92.91	94.95
6							87.04	88.95	90.91	92.91	94.95
7								88.95	90.91	92.91	94.95
8									90.91	92.91	94.95
9										92.91	94.95
10											94.95

Figure 19 Hurdle rate for the Small Warehouse

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):	**Small Warehouse Land Value Tree (Samuelson-McKean, reflecting 1 yr time-to-build):**										
0	30.85	37.88	45.62	54.14	63.50	73.78	85.06	97.44	111.01	125.88	142.16
1		12.15	17.64	23.71	30.41	37.79	45.93	54.88	64.73	75.55	87.43
2			2.46	4.01	6.52	10.61	16.34	22.71	29.74	37.50	46.05
3				0.49	0.80	1.31	2.13	3.46	5.63	9.16	14.76
4					0.10	0.16	0.26	0.43	0.69	1.13	1.84
5						0.02	0.03	0.05	0.09	0.14	0.23
6							0.00	0.01	0.01	0.02	0.03
7								0.00	0.00	0.00	0.00
8									0.00	0.00	0.00
9										0.00	0.00
10											0.00

Figure 20 Land value tree of Small Warehouse

What is worth noticing here is that the small warehouse project value is > than the hurdle rate (critical value), implying that the option to build the small warehouse should be exercised right away. However, it is important to recall that this would depend on whether or not it would be favorable to exercise phase 1 of the GDC project as well. In order to assess this decision further we create a binomial tree consisting using the methodology described in section 8.2.2.

We start of by developing a binomial value and construction tree for phase 1 applying the same methodology used for the small warehouse project earlier.

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9	10
Phase 2 Expected Values:		624.09	632.65	641.33	650.12	659.03	668.07	677.23	686.52	695.93	705.47
"down" moves ("*i*"):	**Phase 2 Underlying Asset Value Tree (as if new):**										
0	615.65	669.50	728.07	791.75	861.00	936.32	1,018.22	1,107.28	1,204.13	1,309.46	1,424.00
1		506.24	550.52	598.68	651.04	707.99	769.92	837.26	910.50	990.14	1,076.75
2			416.27	452.69	492.28	535.34	582.17	633.09	688.47	748.69	814.18
3				342.30	372.24	404.80	440.20	478.71	520.58	566.12	615.63
4					281.46	306.08	332.86	361.97	393.63	428.07	465.51
5						231.44	251.69	273.70	297.64	323.68	351.99
6							190.31	206.96	225.06	244.75	266.16
7								156.49	170.18	185.06	201.25
8									128.68	139.94	152.18
9										105.81	115.07
10											87.01

Figure 21 Value tree of Phase 2 underlying Asset

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("i"):	**Phase 2 Construction Cost Tree:**										
0	390.00	398.58	407.35	416.31	425.47	434.83	444.40	454.17	464.16	474.38	484.81
1		398.58	407.35	416.31	425.47	434.83	444.40	454.17	464.16	474.38	484.81
2			407.35	416.31	425.47	434.83	444.40	454.17	464.16	474.38	484.81
3				416.31	425.47	434.83	444.40	454.17	464.16	474.38	484.81
4					425.47	434.83	444.40	454.17	464.16	474.38	484.81
5						434.83	444.40	454.17	464.16	474.38	484.81
6							444.40	454.17	464.16	474.38	484.81
7								454.17	464.16	474.38	484.81
8									464.16	474.38	484.81
9										474.38	484.81
10											484.81

Figure 22 Construction tree of Phase 2 underlying Asset

$$C_{i,j} = Max \left\{ \begin{array}{c} \frac{V_{i,j}}{(1+y_V)^1} - \frac{K_{i,j}}{((1+y_k)^1}, \\ \frac{\left(pC_{i,j+1} + (1-p)C_{i,j+1}\right) - (C_{i,j+1} - C_{i+1,j+1})[\frac{r_V - r_f}{(1+\sigma\sqrt{T/n}) - 1/(1+\sigma\sqrt{T/n})}}{1+r_f} \end{array} \right\}$$

$$C_{0,0} Max =$$

$$\left\{ \begin{array}{c} \frac{615.65}{(1+5.75\%)^1} - \frac{390}{(1+(1{,}79\%)^1}, \\ \frac{\left((0.7219) * 227.27 + (0.2781) * 72.89\right) - (227.27 - 72.89)[\frac{7.2\% - 0.375\%}{(1+15\%) - 1/(1+15\%)}}{1+0.375\%} \end{array} \right\}$$

$$Max = \{185.09{,}146.211\} = 185.09\ mDKK$$

The value of the phase 2 option is valued to be 185.09 mDKK t0. However, we are dealing with a compound option meaning that the value of 185.09 at t0 is an imaginary value that does not exist because phase 2 is depended on the built of phase 1. Therefore, an option value for phase 2 can at the earliest occur at t1, if the option to build phase 1 is exercised at t0.

In order to value phase 2 option at t0 we must account for the lag is embedded with the construction time of building phase 1. By doing so we are able to establish the option value of phase 2 that can be obtained by exercising the option to build phase 1 at t1. Due to the construction time of 1-year we must identify the value of the phase 2 option in the “up” and “down” notch scenario at time 1, this is done using the same approach we applied for when we found the 185.09 mDKK at t0:

This is actual equivalent to the second part of the equation listed before, and the value is therefore computed to be 146.211 mDKK. This is the stand-alone value of phase 2 presents value at t0 if the option to build phase 1 is exercise at t0. Now that we have identified the value of the last part of the compound option the next step is to value the entire compound option of building the GDC project.

In order to do so we need to compute a binomial value tree for the phase 1 value and construction cost at all stages of the 10-year lifetime of the fund. This is done using the same methodology as we used for the small warehouse and phase 2.

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("*i*"):	**Value of Option on Phase 2, reflecting 1-yr time-to-build:**									Opt Expires
0	185.09	227.27	273.72	324.82	380.98	442.67	510.38	584.64	666.06	755.26
1		72.89	105.83	142.24	182.44	226.76	275.58	329.31	388.39	453.30
2			7.98	16.07	32.31	63.50	98.04	136.24	178.43	224.98
3				0.61	1.29	2.70	5.67	11.89	24.95	52.33
4					-	-	-	-	-	-
5						-	-	-	-	-
6							-	-	-	-
7								-	-	-
8									-	-
9										-

Figure 23 Value tree of phase 2 option

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("*i*"):	**PV of 1 period delayed receipt of Phase 2 option value:**									
0	146.21	185.47	228.75	276.40	328.84	386.48	449.80	519.30	595.55	-
1		54.60	76.16	103.76	141.09	182.31	227.77	277.85	332.98	-
2			7.98	16.07	31.67	49.68	71.12	98.02	134.44	-
3				0.61	1.29	2.70	5.67	11.89	24.95	-
4					-	-	-	-	-	-
5						-	-	-	-	-
6							-	-	-	-
7								-	-	-
8									-	-
9										-
Note: Phase 2 option has no value in year 9 as it can first then be exercised after phase 1 is completed by year 10, which is the fund end year and thus no time for construction.										

Figure 24 Value tree of phase 2 option at t0

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
Phase 1 Expected Values:		416.06	421.77	427.55	433.41	439.36	445.38	451.49	457.68	463.95	470.31
"down" moves ("i"):	**Phase 1 Underlying Asset Value Tree (as if new):**										
0	410.43	446.34	485.38	527.83	574.00	624.21	678.81	738.19	802.76	872.97	949.33
1		337.49	367.01	399.12	434.03	471.99	513.28	558.18	607.00	660.09	717.83
2			277.52	301.79	328.19	356.89	388.11	422.06	458.98	499.13	542.78
3				228.20	248.16	269.86	293.47	319.14	347.05	377.41	410.42
4					187.64	204.06	221.90	241.31	262.42	285.38	310.34
5						154.30	167.79	182.47	198.43	215.79	234.66
6							126.87	137.97	150.04	163.17	177.44
7								104.33	113.45	123.38	134.17
8									85.79	93.29	101.45
9										70.54	76.71
10											58.00

Figure 25 Value tree of Phase 1 Underlying Asset

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("i"):	**Phase 1 Construction Cost Tree:**										
0	260.00	265.72	271.57	277.54	283.65	289.89	296.26	302.78	309.44	316.25	323.21
1		265.72	271.57	277.54	283.65	289.89	296.26	302.78	309.44	316.25	323.21
2			271.57	277.54	283.65	289.89	296.26	302.78	309.44	316.25	323.21
3				277.54	283.65	289.89	296.26	302.78	309.44	316.25	323.21
4					283.65	289.89	296.26	302.78	309.44	316.25	323.21
5						289.89	296.26	302.78	309.44	316.25	323.21
6							296.26	302.78	309.44	316.25	323.21
7								302.78	309.44	316.25	323.21
8									309.44	316.25	323.21
9										316.25	323.21
10											323.21

Figure 26 Construction tree of Phase 1 Underlying Asset

We can express the value of immediate exercise of the GDC phase 1 option + the option of building phase 2 that is obtained in relation to both their respective construction time by the following formula:

$$Cx = Max\ \{As\ of\ right\ land\ value\ x, PVx\ [V_x - K_x\} + PVx[Ph.2\ Opt_{t+1}], PV_t[C_{t+1}]$$

With this established we can compute the value of any stage using the following formula:

$$C_t = Max\ \{As\ of\ right\ land\ value_t\ , PV_t\ [V_{t+1} - K_{t+1}\} + PV_t[Ph.2\ Opt_{t+1}], PV_t[C_{t+1}]$$

$$C_{0,0} =$$

$$Max\left\{\begin{array}{c} 30.85, \\ \dfrac{410.43}{(1+5.75\%)^1} - \dfrac{265.72}{(1+0.375\%)^1} + 146.211, \\ \dfrac{\left((0.7219)*336.99 + (0.2781)*103.19\right) - (336.99 - 103.19)[\dfrac{7.2\% - 0.375\%}{(1+15\%) - \dfrac{1}{1+15\%}}}{1+0.375\%} \end{array}\right\}$$

$$C_{0,0} = Max\,\{30.85, 269.60, 214.25\} = 269.60\; mDKK$$

By now we have calculated the value of the compound option to build the GDC project that the MG Group is currently in possession of and still account for the abandonment option of the perpetual assumed small warehouse which can be built by anyone and at any time in the future, the option to wait and exercise both options at a later point in time or on a stand-alone basis.

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("*i*"):	**Value of Option on Phase I , reflecting 1-yr time-to-build:**									Opt Expires
0	269.60	336.99	411.23	492.95	582.83	681.59	790.05	909.07	1,039.59	503.51
1		103.19	146.71	198.59	262.71	333.48	411.48	497.39	591.91	302.20
2			13.21	26.51	53.21	92.01	136.48	188.84	253.39	149.99
3				1.10	2.20	4.45	9.04	18.51	38.06	34.89
4					0.10	0.16	0.26	0.43	0.69	1.13
5						0.02	0.03	0.05	0.09	0.14
6							0.00	0.01	0.01	0.02
7								0.00	0.00	0.00
8									0.00	0.00
9										0.00

Figure 27 Value tree of Option land value Phase 1

Year ("*j*"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("*i*"):	**1st Phase Optimal Exercise**:									
0	exer	exer	exer	exer	exer	exer	exer	exer	exer	exer
1		exer	exer	exer	exer	exer	exer	exer	exer	exer
2			hold	hold	exer	exer	exer	exer	exer	exer
3				hold	hold	hold	hold	hold	exer	exer
4					hold	hold	hold	hold	hold	sell
5						hold	hold	hold	hold	sell
6							hold	hold	hold	sell
7								hold	hold	sell
8									hold	sell
9										sell

Figure 28 Hold exercice option tree for Phase 1

Year ("j"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("i"):	**Contingency Probabilities:**									
0	100.00%	72.19%	52.11%	37.61%	27.15%	19.60%	14.15%	10.21%	7.37%	5.32%
1		27.81%	40.16%	43.48%	41.85%	37.76%	32.71%	27.55%	22.73%	18.46%
2			7.74%	16.75%	24.19%	29.10%	31.51%	31.84%	30.65%	28.44%
3				2.15%	6.21%	11.21%	16.19%	20.45%	23.62%	25.57%
4					0.60%	2.16%	4.68%	7.88%	11.38%	14.78%
5						0.17%	0.72%	1.82%	3.51%	5.70%
6							0.05%	0.23%	0.68%	1.46%
7								0.01%	0.07%	0.24%
8									0.00%	0.02%
9										0.00%

Figure 29 Contingency tree for possible outcomes

7.2.1. Calculation of the opportunity cost of capital embedded in the GDC project:

By now we have shown you how the real option pricing methodology used in this thesis is calculated, but because the methodology is based on economic theory such as the opportunity cost of capital it enables us to quantify the opportunity cost of capital embedded in any project including this one. Therefore, In the efforts of being capable doing a valid and interesting discussion later in this thesis on the different valuation methodologies, we exploit the advantage that the methodology has, which allows us to compute the opportunity cost of capital that is embedded in the GDC case at any point in time. This can be done by using the following formula:

$$C_i,j = \frac{CEQ[C_{j+1}]}{1+r_f} = \frac{E_j[C_{j+1}]}{1+r_f+E[RP_{Ci,j}} = \frac{E_j[C_{j+1}]}{1+OCC_{i,j}} \rightarrow 1+OCC_{i,j} = (1+r_f) * \frac{E_j[C_{j+1}]}{CEQ[C_{j+1}]}$$

$$1+OCC_{0,0} = (1+0{,}375\%) * \frac{0.7219*336.99+0.2781*103.19}{(0.7219*336.99+0.2781*103.19-(336.99-103.19))*\left[\frac{7.2\%-0.375\%}{(1+15\%)-1/(1+15\%}\right]}$$

$$OCC_{0,0} = 1.00375 * \frac{271.96}{215.058} = 1.26932 \rightarrow 26.932\%$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9
"down" moves ("i"):	**GDC OCC:**									
0	26.93%	24.02%	21.56%	17.86%	16.35%	15.16%	14.19%	13.39%	12.71%	12.14%
1		42.68%	38.99%	32.88%	28.50%	24.08%	21.00%	18.77%	17.06%	15.73%
2			47.19%	39.83%	39.83%	39.26%	37.48%	34.68%	30.84%	26.09%
3				39.83%	39.83%	39.83%	39.83%	39.83%	39.83%	39.72%
4					39.83%	39.83%	39.83%	39.83%	39.83%	39.83%
5						39.83%	39.83%	39.83%	39.83%	39.83%
6							39.83%	39.83%	39.83%	39.83%
7								39.83%	39.83%	39.83%
8									39.83%	39.83%
9										39.83%

Figure 30 Opportunity Cost of Capital tree

We can now use the opportunity cost of capital calculated to quantify the amount of risk there is embedded in the GDC project relative to the risk premium required for already built properties in general by using the following formula:

$$\frac{OCC\ of\ GDC - r_f}{r_v - r_f} = \frac{26.93\% - 0.375\%}{7.2\% - 0.375\%} = 3.89$$

The GDC development project is an investment with an embedded 3.89 times higher risk than compared to an unlevered investment in a completed property. It is important to notice that the higher risk embedded in the project is in relation with a higher return than the average property market. In order to carry out our analysis of the GDC project fully, we also analyze the project by applying a discounted cash flow valuation approach.

7.3. Valuing Case using DCF

The discounted cash flow approach is used in this thesis to value the GDC project. This is done so that we are capable of comparing the two methodologies and identify the strength and weaknesses behind both models in the discussion section. In order to make the cases comparable the assumption is made that the MG Group would sell of the phase 1 building and phase 2 building of the project as soon as it is developed. Due to this assumption, we are only valuing the present value of the building, and thereby avoiding any intermediate holding period that would create a net operating income, making it comparable with the real option pricing approach used in this thesis. Furthermore, we assume that the building of phase 1 and phase 2 is initiated right away meaning that the phase 1 building would be completed in year t1, and the phase 2 building would be completed in year t2.

In discounted cash flow approach uses the same cap rate found and the applied in the options approach of 5.75% (y_V). Theoretical this number should be same, because the cap rate is determined by the market and not the valuation methodology used. The same case is made in relation to the estimated net operating income estimated as 590 DKK pr. Square meter for the phase 1 building and phase 2 building and all other parameters that is not depended on the valuation methodology.

The key component in the discounted cash flow approach, which would vary depending on the valuation methodology is the discount rate applied. One could argue that the discount rate should be equivalent to the opportunity cost of capital identified in the previous section, but this is likely not to be the case in practice as it would require doing the rigorously option pricing methodology performed in this thesis in order to estimate that number. Geltner states that the discount rate used for real estate development projects back that was used back in the 2000´s was around 20% p.a. His justification for this number is fairly week as they suspect this is due to the number being nice and round and in consistent with the conventional wisdom for required returns (Geltner et al. 2013). However, recall that we stated earlier in this thesis in section 5.2.5 that the discount rate should be calculated for each cash flow in accordance with its individual embedded risk. If we were to follow that theoretical logic, we would likely not apply the same discount rate used for calculating the value of the phase 1 building and phase 2 building. Numerous reasons support this point. First there is the time to build. The phase 1 building could be completed within a year versus the phase 2 building can at the earliest be completed in 2 years. In general, it can be assumed that uncertainty increases the further we look out in time. As a result, our predictability of estimating the actual value of the phase 1 building at t1 is higher than for the phase 2 building at t2. This implies that a lower discount rate should be applied when estimating the value of the phase 1 building.

Secondly there is the size difference of the two buildings. Recall that the phase 1 building consisted of a total of 40.000 square meters and the phase 2 building consists of 60.000 square meters. This impacts the vacancy risk associated with the building. Naturally the vacancy risk increases with the number of square meters in a building, due to the need of a higher market demand in order for the building to be fully rented. This also implies that the discount rate used when estimating the value of the property today should be lower for the phase 1 building.

Estimation of the appropriate discount rate for each building. One approach to do this could be the bottom-up- approach. Here the risk-free rate is identified as the base rate for the discount rate, this could be equivalent to the already established discount rate used in this thesis as the 10-year Danish government bond effective yield. The next step is to identify all the different factors that could potentially impact the net operating profit of the building in a negative way, and assign a risk premium for each factor. These factors could be things such as legislation, vacancy risk, liquidity risk, business risk and management risk etc. Once the risk premiums for all the relevant factors are identified, all the risk premiums are added on top of the identified base rate and sum amounts to the appropriate discount rate. However, it is immensely challenging to estimate the risk premium for each factor making the approach difficult to apply. Another approach is looking towards the market and use the discount rates for similar projects. Due to the real estate industry being of somewhat private and closed, data availability is likely not be the case making the approach near impossible. One approach we have not touched up is the one asking a more experienced person for a discount rate and trust that their gut feeling is better than yours. David Geltner is such a person and therefore we trust that the appropriate discount rate for real estate development projects is 20% as he suggested. However, we modify this by making it an average discount rate for both projects by using a discount rate of 15% for the phase 1 building and 25% for the phase 2 building. This ensures that we are in accordance with the theoretical understanding that the risk is lover for the phase 1 building, which should be reflected in the discount rate.

The Traditional DCF			
Time	0	1	2
Discount rate		15%	25%
Land price	269.60		
Phase 1 Building			
Value		416.06	
Construction cost		(265.72)	
Phase 2 Building			
Value			632.65
Construction cost			(407.35)
CF	(269.60)	150.34	225.30
PV	(269.60)	130.73	144.19
NPV total	5.32		
IRR	23.46%		

Figure 31 DCF calculation of GDC

The above figure shows an NPV of 5.32 mDKK and an IRR of 23.46%. In the calculation we have used the land option value calculated using the real option pricing approach. Even though the found land value is used the DCF methodology still gives an NPV >0, which indicates that the applied discount rates of 15% and 25% is to low resulting in an overestimate in value, when compared to the embedded OCC of 26.93%. Obviously, this is looked at from an economical theoretical point of view assuming market equilibrium as in the real option pricing methodology. However, in the real world the assumption of an efficient market in the real estate development industry is most likely not the case as land is not purchased at it´s given option price.

8. Discussion and Implementation

In the following discussion we try to outline and discuss the shortcomings and strengths of the discounted cash flow method as well as the real option pricing methodology used to examine the GDC case. Furthermore, we discuss in which cases a real option pricing approach is suitable and why the use of it is absent in commercial practice, even though theorist have deemed it to have a higher explanatory power. Finally, we will discuss the future of real option pricing methodologies in the real estate market and the alterations that could be made in order to secure its presence in a competitive landscape of valuation methodologies.

8.1. Shortcomings of the Discounted Cash Flow Method

A longstanding criticism of the DCF method is how it ignores the strategic options available to management (Myers 1984). Dixit and Pindyck (1995) further this argument saying that the traditional cash flow approach to investment decision making is flawed as it assumes that strategic decision-making cannot be deferred i.e. if a company does not invest now, it will lose the opportunity forever. The uncertainty in the DCF input values will also result in uncertainty into the DCF output.

In addition to Dixit and Pindyck's criticism of the DCF model, French and Gabrielli (2004) point to how the DCF ignores that uncertainty is probability based and they cite Gerald Roderick Brown: "In those situations where a single value can be misleading it has been suggested that a range of values might be more meaningful" (Brown 1991 p. 63 in French and Gabrielli 2004).

It may be valuable to delay irreversible investment decisions to obtain additional information to guide the decision-making. This way the option to choose a better investment in the future is retained.

DCF models have been criticized for needing uncertain discount rates and subjective estimates of future cash flows (Oppenheimer 2002). It seems very likely that non-rigorous discount rates are widely used by some practitioners, who determine it largely based on gut feeling (Geltner et al. 2013). The assumption made about future discount rates will often be that it is stable over time despite the interlease vs. intralease dilemma discussed previously. Further, discount rate stability would historically be an anomaly (Sirmans 1997).

Similarly, Hodder and Riggs (1985) point out how the actual usage of the DCF model is wrong as the discount should fall over time as the riskiness of the project is reduced as the project becomes operational and management learns more about the endeavor in practice. This is, however, not a weakness of the DCF model but of the practitioners using it in an overly simplistic way.

The importance of the discount rates is for example evident from the finding by Geltner and Mei (1995) that most of the change in real estate values come from changes in the discount rate and not changes in the operating cash flows.

An easy out is to abandon the discount rate and traditional NPV rule to instead use the IRR for decision making. However, without applying a rigorous hurdle rate in the evaluation of the IRR the outcome becomes equally weak i.e. it will be based on a gut feeling of whether the IRR is high enough to satisfy the decision-maker. This gut-based decision-making can for example be based on a fund manager who has promised a return to investors, which can result in rejecting positive NPV projects in a time with falling opportunity costs of capital will have changed what investors would now have been satisfied with. The fund manager will in this scenario have a hard time placing the capital raised.

In addition, DCF models do not incorporate valuations of implicit options imbedded in capital projects (Oppenheimer 2002). As seen the case, it is possible to apply the DCF model to a phased investment project but the timeline will be deterministic and ignore the embedded option and thus the active role through which management can create value.

It is not uncommon in comparisons between the DCF and real option approach that the authors write of the optionality completely (E.g. Mintah et al. 2018) when discussing their DCF model as if the optionality disappears from the investors decision space when it is not valued. It therefore seems necessary to point out that investors may knowingly hold options despite not conducting a valuation that financially takes this into account. Thus, while it is true that the option is not accounted for explicitly in the financial valuation it is often accounted for in the investment decision material (e.g. investment memorandum).

8.2. Empirical Testing of Option Models

Geltner (1989) argues that the theoretical underpinnings that give options such power in financial securities do not exist for land. Without strong theory to give us confidence in the model it becomes paramount to justify the model with empirical validity.

The first research to do large-scale testing of option-pricing models empirically within real estate is Quigg (1993), who uses it to value a sample of 2,700 transacted land parcels in Seattle. She compares both the intrinsic value and the option value with the transaction price and finds the option-pricing model to have more explanatory power i.e. it is better at approximating actual transaction prices. The paper finds a mean option premium over intrinsic value of 6% in the range 1% to 30% across the sample with all subsamples by year and property type having a positive option premium. In other words, in all subsamples the land was valued higher with the option model than with the intrinsic model that assumes immediate construction and calculates value as the residual of price and cost.

It is speculated that this option premia represents a lower bound as the sample considers urban land in a city under expansion with tight growth controls. Higher premia would be expected in an area with little new construction as the value of the land would mostly be an option for construction far out in the future, whereas many of the options in Seattle would be "in the money" in this sample period. Another contribution of the paper is the estimation that the standard deviation of individual asset prices is in the range 18 to 28% with no statistically significant difference between property types. This reflects the 70's Seattle and is not representative for real estate in general but is one of few studies finding asset level standard deviations needed for option valuations.

Largely replicating Quigg's methodology but with slight adjustments, Sing and Patel (2001) attempt to estimate waiting option premia for the United Kingdom. Their study is based on 2,286 transactions collected between 1984 – 97. Using a contingent claim option valuation method, they found the waiting option premia to be 29% for office, 26% for industrial and 16% for retail. These estimates are substantially higher than Quigg's. The authors offer two potential explanations. First, unlike Quigg they were not able to consider the heterogenous characteristics of the hypothetical building. Second, the authors use developed land and assume that the properties built are developed optimally. Thus, the premia may also represent pay-off associated with development risk. These estimates can thus be regarded as the upper bound for timing options.

A newer large sample empirical study from Grovenstein, Kau, and Munneke (2011) uses a 2,870 transactions sample of Chicago properties and vacant land transacted between 1986 and 1993. Following the methodology of Quigg, they find an average delay premium of 6.6% across property types which is slightly higher than Quigg but unlike Quigg there is a difference between the property types.

Guma et al. (2009) used four case studies to examine the potential of vertical expansion of corporate real estate buildings. They find the option valuable in the specific cases as it offers additional upsides that the addition of off-site square meters cannot offer e.g. the value of keeping all employees located together or the value of avoiding a move. However, the study did not dive into the difficulty involved with determining the relevant input need for the option analysis performed.

8.3. Shortcomings of Real Options Valuation

Geltner (1989) has addressed desire to move real options valuation from the academic realm and into the world of practical application. For the option model to be as realistic for real estate as it is in the stock market, long and short positions in fractional shares would have to be traded in both the land in question and the building that has yet to be built with little transactions cost and at frequent trading intervals. However, this is obviously not the case. One fundamental reason is that the underlying asset does not yet exist.

Despite the first empirical testing taking place 25 years ago (Quigg 1993), real option valuation still has yet to experience serious practical adoption as we have described previously (see section 4.2). One argument made against applying real options theory as a practical valuation tool for real estate is that the proposed complex pricing model has underlying assumptions that add uncertainty themselves (Oppenheimer 2002).

We find this to be true in our experience applying options pricing. An example of this is the uncertainty surrounding the volatility used in the model. It is important to remember that the volatility needed in the model is that of a single-asset and not of a portfolio or index as is most easily observed since single assets are traded infrequently. The single-asset volatility includes the idiosyncratic risk of the individual property and is therefore larger than the volatility typically measured for a portfolio or index. However, finding an appropriate single-asset volatility requires finding a property that is comparable i.e. it must share property type, class and location. Further, we then need to have knowledge of transactions with the asset in order to determine the single-asset volatility. The difficulties are reflected in the lack of published data and academic literature on the subject of idiosyncratic real estate asset level risk (Sagi 2016).

This input uncertainty is also point to by Oppenheimer (2002) in his commentary on the methodology. While the financial input needed for the DCF method also carries uncertainty, the addition of option parameters such as volatility only heightens the uncertainty.

Further, options models are mathematically complex when compared to the approach taking by the DCF method. The probability analysis and differential equations of the BSM model and the binomial options pricing method make less accessible and more difficult to communicate to stakeholders, who are less well versed in the advanced mathematics and economic theory. (Oppenheimer 2002; Mintah et al. 2018)

An important issue for valuation professionals is that the information prepared for a client is clear and unambiguous. All stakeholders must be able to understand the terminology used and the decisionmakers must be able to act upon the information received knowing which actions can be supported by the valuation and its methodology (Pagourtzi et al. 2003).

8.4. Option Model Detail Level

Compared with the theoretically all-encompassing model, many simplifying assumptions are made to avoid a model too complex to function in practice. However, the question will be where to draw the line between necessary and superfluous.

In the methodology applied in the GDC case as well as many published methodologies the risk-free rate is assumed to be constant over the entire time span of the project (Oppenheimer 2002). This does not only apply to the risk-free rate but the volatility is also assumed to be constant and known in both the GDC case as well as many other published methodologies.

Furthermore, the value of options may depend on each other thus requiring the value of one option to calculate that of another option (Oppenheimer 2002). E.g. the value of an up-sizing option may dependent on the exercise of a usage option as the exercise of the usage option may increase the value of the up-sizing option. This would in the extreme result in a complex and interdependent matrix of options.

8.4.1. Construction Cost

Another complexity that should be added to arrive at a more theoretically complete model regards the construction cost as considered by Geltner et al. (2013). The complexity comes if we want to relax the assumption that the project is paid for in a lump sum at the project competition. In practice the construction cost is paid in rates through the entire construction time consisting of an upfront payment and payments due through the completion of the build. With the approach suggested by Geltner you would tend to underestimate construction cost. In contrary if the entire payment of the construction cost is assumed to happen at start time of exercise one would overestimate the construction cost. In order to overcome this challenge investment managers could modify the approach suggested by applying an extension that balances out this issue. This extension could consist of dividing the construction cost into two. An upfront payment of 50% of the total construction cost and 50% on completion of the property. By doing so we are not extremely overestimating or extremely underestimating the construction cost, but is somewhere in between. The extension could be formulated mathematical as follow:

$$PV_t[K_{t+CT}] = \frac{1}{2}K_t + \frac{\frac{1}{2}K_t}{(1+y_k)^{CT}}$$

Alterations such as this one could be made for an infinite number of factors and hand tailored for a specific real estate case. All alterations would be possible to integrate in the real option pricing methodology unless they are contrary. However, even though it might make sense from a theoretical point of view to rigorously modify a real option model for a specific case, doing so is burdensome work and adds additional complexity to a methodology which is already criticized for its complexity.

8.5. Overcoming the Challenges to Real Option Valuations

Different attempts have been made to overcome the challenges faced by real options in valuing real estate. Some attempts have been made to capture the inherent uncertainty using Monte Carlo simulation based on distributions of input and thereby producing a distribution of outcomes.

Guma et al. (2009) and Geltner and De Neufville (2012) use Monte Carlo simulations combined with binomial lattices requiring the computation of volatility, which as already mentioned is difficult to quantify reliably. In contrast, Mintah, Higgins, Callanan and Wakefield (2018) suggest an approach using a newly developed but practically adaptable fuzzy payoff method (FPOM) with scenario planning and familiar DCF inputs. Thus, they use an approach that does not involve the assignment of probabilities, use of Brownian motion or computation of volatility to represent the different uncertainties.

Mintah et al. demonstrate this on a residential development project in Australia that is horizontally phased. An advantage of this approach compared with the binomial option pricing method as applied in this thesis is in using a scenario planning approach which does not require the computation of volatility, thereby simplifying application. Further, scenario planning is a familiar method of representing uncertainties in the real estate industry; thus, analysts and other stakeholders may be better able to relate to this method.

The FPOM uses the fuzzy set theory to treat uncertainty and compute real option values from a payoff distribution of NPVs generated from Monte Carlo simulation. Three scenarios are projected: minimum, most likely and maximum. In this triangular payoff distribution, the most likely scenario is given a complete membership (i.e. value of 1), the minimum and maximum scenarios are given complete non-membership (i.e. value of 0) and other scenarios between have intermediate degrees of membership.

When compared with a DCF valuation of the case, the FPOM finds a higher value of the project in line with expectation as the value of the optionality is accounted for in the FPOM and developers are assumed to proceed with phases that are profitable and abandon phases that are unprofitable. Overall, the FPOM appears to be a better candidate for practical implementation as it is less mathematically complex than the binominal options pricing lattice and the Samuelson-McKean formula. Many analysts will probably want a basic understanding of the fuzzy logic on which it is built, but extending logic from being binary to being a range, should be a surmountable challenge. Further, practitioners are familiar with scenario analysis, so an implementation of FPOM will be less of a departure from what stakeholders are currently using.

8.6. Types of Investments Suitable for Real Options

Grovenstein, Kau, and Munneke (2011) find the option premium to vary widely between property types. This goes to indicate that there is more to gain from adopting a real option method on some property types over others. The light manufacturing category has the lowest option premium at 1.22% in contrast with the highest in warehouses reaching 11.29%. Thus, with a lower premium in light manufacturing there is less of a potential upside missed by not using the real options methodology when doing the valuation. The types most suitable of using real option valuation are naturally the types were the owner has the most options for revenue and cost as if these are fixed the value will match that of the DCF.

Property type is not the only determining factor of option valuation method applicability. The ability to divide a property into multiple stages of development will increase the option premium, however, this does not correlate 1:1 with property type. For example, while a strip mall with enough vacant land on the plot will be relatively easy to expand with more retail space if demand is present, the expansion of an indoor mall structure will require changes of the exterior making the option more expensive to exercise thus making its value lower.

As the above example also touches upon, many options require vacant space on the plot. While warehouses as a property type is found to have the highest option premium this is only the case if the physical and zoning characteristics of the plot allows for it.

Further, an option only has value to the owner if the capital to exercise it is available. While the easiest source of capital is from oneself it should be possible to realize the option value in the market place even if the investor herself is not able to finance the exercise.

The likelihood of options being presented and thereby real option valuation adding value as a methodological choice over the discounted cash flow method can be assessed on the basis of two questions. First whether the property allows for significant physical alterations and second whether the exercising of the option can be shifted temporally. If the neither is the case, then the only potential option is a change of usage although this itself will be limited as most changes will require physical alterations thus establishing apartment front doors and common access space when converting from office to apartment.

If intertemporal shifts are possible it opens up for the additional potential presence of waiting and staging options as these options require flexibility in terms of time. Thus, if there is a contractual obligation to develop immediately or architectural choices or zoning does not make staging possible, then there will not be temporal flexibility and neither option will be available.

Many options require physical alterations. Even without temporal flexibility, scale and construction options are likely to be added to usage changes as the option types present. Finally, if both physical alteration and temporal flexibility is present the whole suit of options may be available. We can from this deduce that the likelihood for the presence of options being highest in this final quadrant as seen in Figure 32. This can be used as a tool to quickly gauge whether it will be fruitful to apply a real options methodology and as well as where the methodology is likely to first gain widespread usage.

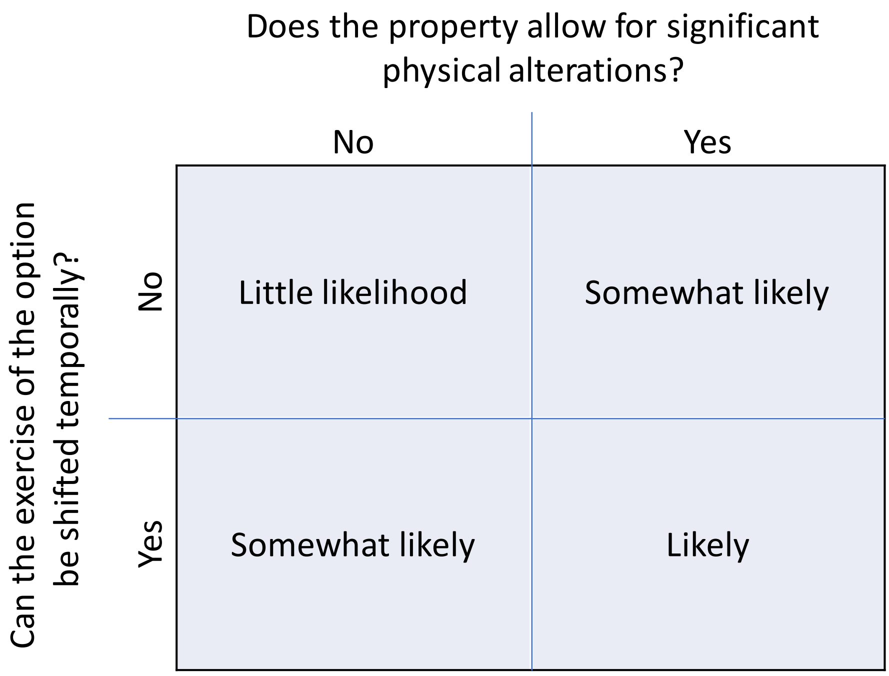

Figure 32 Likelihood of option availability

8.7. Considerations on the Implementation of Options Pricing in Decision-Making

In order for a company to successfully implement real option valuation for real estate valuation a number of conditions must be present. Firstly, it must be an organization that has adequately sophisticated employees within financial theory and mathematics. Furthermore, these employees would need to be able to convert this knowledge into complex option models in programs such as the commonly used Excel. Senior management must also buy into the need for optional valuation methodology as juniors will not allocate time into development of real option valuation skills and models without this. It is likely a hurdle that senior management is often the furthest removed from the educational system and thus from exposure to new theoretical developments. An incentive that could increase senior managements interest in real options valuation would be if external capital sources showed interest. However, these organizations themselves would face the same obstacles in implementing a real options valuation methodology into their standard operating procedures.

One important party for speeding up the adoption of the methodology is banks. If the source for debt capital showed recognition of the methodology it would lead to investors being more willing to adopt it. The option valuation method will allow for more debt in the capital structure as the positive option premium will increase the valuation and thus how much a loan-to-value restriction will allow (cf. section 8.2 and e.g. Quigg 1993).

Once the above resources are present in an organization further steps can be taken in implementing real option valuation, starting off with identifying the cases which is most likely to have significant option value embedded to make the impact sizeable and thus noticeable. We suggest that that the framework developed in this thesis for identifying option value in real estate Figure 32 can be used for this process. This should provide the organization with a rough overview of cases that might be applicable for real option valuation and therefore subject to a deeper analysis identifying possible valuable options.

The likelihood of all these mentioned conditions is meet seems to continue to be low as there has been significant talk about the practical implementation for around three decades without the methodology gaining widespread usage. The challenges identified by Lucius in 2001 is still highly pertinent: "As promising as the real options approach appears in the field of real estate research and as convincing as the academic findings may be, the challenge lies in the transfer to practical application in the field of investment valuation" (Lucius 2001, 78).

The experience from applying real option valuation to the GDC case supports the challenge outlined by Lucius (2001) and Oppenheimer (2002). The option value is found but there are many overly simplifying assumptions such as expense timing and the only option accounted for is phasing and not the other options such as expansion that may be part of the projects option bundle.

9. Conclusion

We identified the division of real estate into the asset and space markets with further subdivision of the asset market into the major property types retail, residential, office and industrial and compared the valuation of such an asset based on the DCF and a real option approaches to the case asset Greve Distribution Center.

We have identified how practitioners have simplified the DCF model in ways that reduce its theoretical correctness. The biggest simplification is how one single discount rate is often applied to all periods. Avoiding the often-criticized determination of a discount rate leads in many cases to adopting an IRR based decision-rule, which eliminates the use of a discount rate all together. However, the decision-rule will be based on an often unstated IRR hurdle, which is just another form of discount rate determination. Further, the DCF model does not consider the flexibility embedded in almost all real estate development projects and standing assets. Real option analysis improves on this; however, it adds additional complexity.

It comes as no surprise that real estate practitioners are conservative in their approach to investment analysis. This seems to be rather reflective of the industry in general. However, it is important for real estate decision-makers to understand the volatile nature of the market they operate in and appreciate that this presents both risks and opportunities that require flexibility as events unfold after the initial investment decision has been made. The flexibility to phase, delay, expand, switch usage or abandon a project should be considered in the financial analysis for it to be complete as this has the potential to mitigate downside risks while retaining the upside potential.

An obstacle of the adoption of the real option pricing model is first and foremost the quantification of variables such as volatility and the risk-free rate. Further, the complexity induced by the number of elements in the option bundle and the complexity of the model itself stand in the way of adoption. The adoption would likely move faster with widespread senior management buy-in, which could be initiated by external stakeholder (e.g. debt capital providers) recognition of the models benefits.

We developed a framework capable of estimating the likelihood for a real estate project to have options of significant value embedded. The framework is based on the identified drivers for significant option value as time and physical space. These drivers are structured as two questions: (1) *Does the property allow for physical alterations?* (2) C*an the exercise of the option be shifted temporally?* If this is the case for a real estate project then it is most likely a case, which is suitable for real option valuation.

Looking beyond the Samuelson-McKean and binomial models for real option valuation, we suggest a model which is simpler to convey and understand mathematically would make adoption more likely. This is for example the case with simulation-based Monte Carlo approaches that require the same inputs as the familiar DCF model. One such model is the fuzzy payoff method, which is based on fuzzy logic and scenario planning. The latter being familiar to most analysts and the former being a relatively surmountable challenge to analytically inclined finance employees. Further analysis and promotion of the model would be needed to see whether this would be the case.

The theoretically more correct model is still struggling to gain practical application 25 years after the first large scale empirical test showing the models superior explanatory power (Quigg 1993). As a result of the challenges of quantification of inputs such as variance, the need for mathematically sophisticated stakeholders and complexity of the many options contained in the property option bundle, we do not predict an imminent or widespread change in the adaptation rate of the real option valuation methodology.

10. References

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11. List of Figures