Liquidity and asset prices

A study of the relationship between market liquidity and firm liquidity using Acharya & Pedersen's Liquidity-adjusted Capital Asset Pricing Model

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Abstract

This thesis investigates the role of firm liquidity in asset pricing and how well time-varying corporate events are included in commonly used liquidity measures such as Amihud's illiquidity (ILLIQ) [2] proxy and asset pricing models such as Acharya and Pedersen's Liquidity-adjusted Capital Asset Pricing Model (L-CAPM) [1]. The subject is motivated by the L-CAPMs inherent methodological focus on static measures of market liquidity used for portfolio pricing which are likely to disregard some aspects of time-varying corporate events relevant for individual securities. An event study was setup to investigate the effects of Facebook's share repurchase and McDonald's debt issue on liquidity, measured by the ILLIQ proxy, and asset prices, measured by the L-CAPM. While the ILLIQ proxy significantly captures a price impact on event day, the level of abnormal firm illiquidity is variant throughout the event window and susceptible to appropriate measures of average illiquidity. The results also show that the liquidity impact of the events is not significantly captured by the L-CAPM, in addition to the model being slightly outperformed by the standard CAPM. The findings suggest that the L-CAPM struggles to capture the firm liquidity effects of corporate events, possibly due to the model's reliance on static measures of market liquidity on portfolio level rather than considering the effects of time-varying firm liquidity events for individual securities. However, the simplistic setup of the event study has limitations, which only allow for narrow interpretation the relationship between market liquidity and firm liquidity and asset prices. Further research is needed on robust liquidity proxies designed to capture firm liquidity, the additive nature of illiquidity costs and possible modifications on asset pricing models to better account for the time variance of liquidity.

Keywords: liquidity, illiquidity, illiquidity proxies, liquidity risk, market liquidity, firm liquidity, liquidity adjusted capital asset pricing model, capital asset pricing model

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Introduction

In 2005, Acharya and Pedersen (hereafter *the authors*) presented the Liquidity-adjusted Capital Asset Pricing Model (L-CAPM) in their paper *Asset Pricing with Liquidity Risk* [1]. Until then, various models were used to price assets, such as the Capital Asset Pricing Model (CAPM), building on the assumption that investors can trade all financial assets at any point in time without transaction costs [48]. These assumptions allow for limited interpretation of *market frictions*, including measurement of liquidity risk, "the risk that a security will be more illiquid when its owner needs to sell it in the future" [10, p. ix], when pricing assets. With the L-CAPM, the authors manage to provide a framework for measuring how *market liquidity* affects asset prices [1]. However, the authors' attempts to include firm-specific liquidity aspects in the model are limited, despite theoretical as well as empirical findings underlining the importance of the effects of corporate financial strategies on *firm liquidity*, defined as the liquidity of a firm's issued securities. Since liquidity is a rather ambiguous subject, the L-CAPMs emphasis on market liquidity warrants a discussion on how firm liquidity is considered when using the model to estimate asset prices. In light of this misalignment between market liquidity and firm liquidity (hereafter *market and firm liquidity*) in the L-CAPM, a further analysis of the associations and interconnections of the two aspects is needed in order to improve the understanding of the implications of liquidity on asset prices.

1.1 Importance, relevance and motivation

The vast literature on liquidity generally identifies that liquidity is priced [3] and therefore affects securities' prices [10], playing a major role in how investors allocate their portfolios [9], since *illiquidity*, defined as the lack of liquidity, can force a seller to accept considerable discounts from fair market value in order to achieve his financial goals, as well as forcing the buyer to pay a premium if he wishes to obtain an illiquid asset in haste [48].

The authors' L-CAPM builds on this general theory in an attempt to create the aforementioned unified framework for asset pricing. The authors' findings have numerous implications for the role of liquidity risk in asset pricing, with the definition of three different types of liquidity risk with varying characteristics and effects, with the majority of the explanatory power on cross-sectional asset returns stemming from the covariance between an asset's illiquidity and market return which has not been previously studied in literature [1]. One of the main findings is named *the flight to liquidity*, as illiquid stocks have high values of all three different liquidity risk types, and are therefore more liquidity risky. The authors also note that the effect of liquidity risk must increase when investors trade more frequently, which implies that different periods as well as different sources of liquidity might have a differing effect on trading recurrences, such as times of market turbulence as well as asymmetric information, taxes and institutional effects.

The findings provide valuable insight into the role of market liquidity in asset pricing, with firm liquidity playing a secondary role in the authors' analysis. However, the general theory that liquidity is priced is not only associated with market liquidity but also firm liquidity. Market liquidity can stem from many sources such as exogenous trading costs, demand pressure and inventory risk, asymmetric information and search frictions [13][9][7], which can differ in influence and repercussions depending on firms' financial strategies, resulting in possibly different aggregate effects on market liquidity. This is entirely plausible since the theory of liquidity and asset pricing implies that the liquidity of a firm's own securities affects its firm value, which suggests that a firm has an incentive to maintain a financial strategy that positively affects the liquidity of its issued securities [3]. This implication further suggests that an asset's price is not only reliant on generic market liquidity but also its underlying firm liquidity, an area less explored by the L-CAPM.

The authors' findings also provide insight into underlying elements of liquidity risk which are of interest to this thesis. As liquidity is not an observable variable there exist various incomplete measures, or *proxies*, for illiquidity. Levels of illiquidity are often measured as direct transaction costs such as fees, commissions and tax payments related to the administration of a purchase or sale of an asset [48]. However, the authors use a normalised version of Amihud's illiquidity proxy (hereafter the *ILLIQ proxy*) which indicates the cost of selling an asset [2]. The authors identify the proxy as "a noisy measure of illiquidity" [1, p. 387] and recognise that there exist numerous different liquidity proxies, for example the bid-ask spread which might yield different results. The methodology of measuring illiquidity has received considerable academical interest [5][2][39][43] but to my knowledge the application of proxies to investigate firm liquidity aspects by specifically applying them to the L-CAPM is limited.

Liquidity is also time varying, as times of market turbulence can have major effects on an asset's liquidity, as seen in the financial crisis in 2008 where mortgage-backed securities and money market funds investing in commercial paper were considered illiquid almost overnight [48]. The authors identify the L-CAPM as simplistic and suggestive and due to its strict assumptions on the short longevity of investors' lifespan, it does not allow for exhaustive analysis of the "*time-variation of liquidity*", which is one of the areas they suggest for future research [1, p. 405]. This time-variation has been partly investigated, specifically with focus on the associations of market liquidity and funding liquidity and how their interactions can result in liquidity spirals and liquidity crises [27]. However, empirical work on this element has mostly improved our understanding of market liquidity, with less focus on underlying firm liquidity.

1.2 Problem Formulation

The general objective of the thesis is to explore the relationship between market and firm liquidity in order to provide better insight into how firm liquidity affects asset pricing. In order to do so, I attempt to explore some of the authors' general findings of the L-CAPM framework on market liquidity and asset pricing from an individual investor's perspective and apply them to firms' characteristics in order to gain better understanding of the role of firm liquidity when pricing assets using the L-CAPM. I investigate three major aspects of the model as relevant for firm liquidity and asset pricing: corporate events, the $ILLIQ$ proxy and the time-variance of liquidity.

1.2.1 Research question

In light of the problem formulation, the main research question of the thesis is:

- What are the effects of time-varying corporate events on liquidity, measured by the $ILLIQ$ proxy, and asset pricing, measured by the L-CAPM?

1.2.2 Methodology

To answer the research question I first thoroughly examine the L-CAPMs theoretical framework, assumptions, mathematical setup, limitations, empirical findings and its implications for market liquidity and asset pricing. I then provide an overview over numerous corporate actions that are likely to impact firm liquidity and consider the relationship between firm liquidity and the underlying sources of market liquidity. Lastly, I conduct an event study measuring how two separate corporate events, share repurchase and debt issue, impact the firms' liquidity and asset prices.

1.3 Delimitations

As the literature on liquidity is rather vast, the scope of the analysis is narrow in order to maintain coherence throughout the thesis. The analysis of market liquidity and asset prices primarily builds on L-CAPMs theoretical framework [1] since it is arguably the most established methodology available for examination of the dependence of market liquidity and asset prices. The analysis of firm liquidity is based on the framework surrounding financial management implications for liquidity and asset prices. I argue that the framework is relevant for the L-CAPM since it originated, alongside the L-CAPM, from the same original theory of the pricing of liquidity [3][6][7]. An event study methodology was chosen due to its ability to define specific corporate actions as individual events [28], and measure the effect of said events on illiquidity and asset prices, which is likely to effectively demonstrate the time-variance of liquidity. By doing this, I attempt to gain insight into how time-variant events that are designed to impact firm liquidity are captured by the $ILLIQ$ proxy, the most common measure used to estimate market liquidity. Also, the $ILLIQ$ proxy is applied to asset pricing models, L-CAPM and CAPM, which are most commonly used to price portfolios as opposed to individual assets.

In light of this, I maintain a limited focus on certain aspects that, while likely to provide insight into the general attributes of liquidity, such as funding liquidity, asset classes, biases related to illiquid asset reporting and empirical findings from periods of specific liquidity crises, they provide limited insights into the specific topic of this thesis. The thesis also does not touch upon aspects of liquidity related to financial institutions, such as the role of regulatory authorities and financial institutions or the use of relevant models for liquidity risk management, for example Value at Risk models.

1.4 Structure

Following the first chapter, *Introduction*, the second chapter, *Literature Review*, provides an overview of relevant literary work in the area of market liquidity, firm liquidity and the pricing of liquidity. The third chapter, *The Liquidity-adjusted Capital Asset Pricing Model*, presents the theoretical framework surrounding the L-CAPM and identifies its assumptions, setup, implications, limitations and empirical findings relevant for market liquidity and asset pricing. The fourth chapter, *Liquidity - sources and characteristics*, formally defines market and firm liquidity, with sources of market illiquidity presented along with corporate actions that affect firm liquidity. The fifth chapter, *Event study*, presents the general event study methodology along with a presentation of the relevant research design, objectives, data, hypotheses as well as the estimation and testing procedure, before analysing the impact of Facebook's share repurchase and McDonald's debt issue on the firms' liquidity, measured by the $ILLIQ$ proxy, and asset price, measured by the L-CAPM and CAPM. The sixth chapter, *Discussion*, analyses the implications of the event study findings on the ILLIQ proxy and L-CAPM with the goal of answering the research question and provide insight into the gap between market and firm liquidity. Lastly, the thesis finishes off with the *Conclusion*, where the most important findings of the thesis are summarised and relevant areas are identified for future research.

2

Literature Review

As noted in the delimitations, the fundamentals of the thesis are founded on the general theory, strengthened with significant empirical findings, that liquidity is in fact priced [3][6][7]. The theory originated in 1986 with Amihud and Mendelson's research on the role of liquidity costs and their effect on bid-ask spreads [3]. It sparked considerable interest in the area of liquidity and asset pricing and was follow by numerous academical papers on the subject. These academic papers, their theories and empirical findings, were largely summarised in the 2006 paper *Liquidity and Asset Prices* by Amihud, Mendelson and Pedersen [9]. Also, the book *Market liquidity* by Amihud, Mendelson and Pedersen, published in 2013, provides a more formal and exhaustive overview over relevant papers and research on the areas of liquidity costs and asset pricing, liquidity risks and liquidity crises [10], including the associations of market and funding liquidity and their role in liquidity crises [27][12]. While the literature surrounding the rather generically defined foundation is vast, the relevant literature for this thesis consists of two main prospects within the fundamental theory.

2.1 Pricing of liquidity

Pastor and Lambaugh demonstrated the cross-sectional relation between expected stock returns and fluctuations in aggregate liquidity along with empirical findings in 2003 [52], and Acharya and Pedersen introduced the L-CAPM in an attempt to create a framework for said relation two years later [1]. The L-CAPM is an augmented version of the standard Capital Asset Pricing Model (CAPM), which was derived independently by Treynor [61], Sharpe [58], Lintner [41] and Mossin [47], who all built on Markowitz' Modern Portfolio Theory published in 1952 [45]. Amihud, Mendelson and Pedersen provide a literature overview of the relationship between liquidity and asset prices encapsulated by the L-CAPM, noting that if one of the most important assumptions of the CAPM, *frictionless markets*, is broken, the "building blocks of standard asset pricing are shaken" [9, p. 275]. By partly reflecting these frictions, the L-CAPM manages to provide a "unified framework" for "how liquidity risk may affect asset prices" [1, p. 375] which makes it an applicable option for empirical testing in common securities markets. Today, the L-CAPM is an acclaimed method of choice for both academics and investors, although there exist numerous other liquidity models which provide explanations, in part, of the relationship between liquidity and asset returns. For example, Duffie, Garleanu and Pedersen find that search frictions in over-the-counter (OTC) markets can increase the bid-ask spread of assets which increases the asset's liquidity premium [33]. Also, Grossman and Miller built an inventory risk model to price assets which includes no transaction costs and therefore provides a limited picture of the explicit effect of liquidity on an asset's required return [38].

The L-CAPM has provided strong empirical results, with Acharya and Pedersen [1] providing significant empirical evidence for the relationship between an asset's liquidity and its returns for the United States (US) market and Lee likewise for global financial markets [40]. Similar findings have also been observed in individual markets such as the Greek market [50], although research on the role of L-CAPM in emerging markets is lacking in general [40]. One of the major areas of vulnerability of the L-CAPM is the liquidity measurement, with Acharya and Pedersen identifying the $ILLIQ$ proxy, originally defined by Amihud [5], as a "noisy measure of illiquidity" [1, p. 387] and Amihud noting that there are better measures of illiquidity available, such as the bid-ask spread, market impact or probability of information trading [2]. Kim and Lee test the L-CAPM using different liquidity measures and note that "empirical results are sensitive to the liquidity measure used" [39, p. 112].

2.2 Corporate events that impact liquidity

Building on implications of their original theory that liquidity is priced [3], Amihud and Mendelson outline several firm-specific liquidity-increasing actions in their 1991 paper, *Liquidity and Asset Prices: Financial Management Implications* [6], which have received increased attention for the last quarter century. Amihud and Mendelson summarise the latest empirical findings in relation to these actions in their 2012 paper, *Liquidity, the Value of the Firm, and Corporate Finance** [7].

The event study conducted in this thesis focuses on the actions related to capital structure and payout methods. Lipson and Mortal provide evidence that the more liquid a firm's stock is, the more likely the firm will issue equity as opposed to debt when financing its investments, and that firms with more liquid stocks tend to have lower levels of leverage [42]. Bharath, Pasquariello and Wu find that general increases in a stock's illiquidity can be associated with increases in leverage [18]. Firms therefore must measure the benefits of debt, such as tax benefits, against the costs of the debt, not only in terms of capital expenditures but also the increase in illiquidity costs. Banerjee, Gatchev and Spindt find that dividend payments are more likely for illiquid stocks as they tend to have high transaction costs [16], whereas Brockman, Howe and Mortal show that repurchases are more likely to be used to distribute excess cash by firms with more liquid stocks [23].

Amihud and Mendelson note numerous other actions as well. They show that bid-ask spreads are significantly lower for firms listed on the New York Stock Exchange compared to standard over-the-counter dealer markets [6]. A more controlled experiment by Amihud, Mendelson and Lauterbach, where a small sample of stocks was moved overnight to a more liquid trading system, showed that the firms were subject to a 6% average increase in firm value after the move [8], which implies that the increased liquidity of listing securities on an exchange is positively correlated with firm value. By increasing the investor base a firm can also increase the liquidity of its issued securities. Muscarella and Vetsuypens demonstrate that stock splits for foreign stocks in the US market significantly increase both the stock's price as well as its trading volume for smaller trades [49]. Also, Amihud and Mendelson find that by reducing minimum trading units from 1,000 shares to 100 shares for 66 firms operating in the Japanese market in the 1990s, the amount of investors subsequently doubled, resulting in an increase for both average stock liquidity and stock prices by around 6% [11]. Botosan demonstrates that firms that disclose more detailed information about their operations in reports tend to have a lower cost of equity [20]. Also, Raman and Tripathy found significant decreases in illiquidity, through the bid-ask spread, for firms in the oil and gas industry that voluntarily published information on the value of their reserves on a regular basis [56]. Lastly, Dass, Nanda and Xiao demonstrate that a 10% increase in a firm's Research and Development (R&D) costs were associated with a 7.4% decrease in the $ILLIQ$ proxy [32].

This thesis focuses on *bridging the gap* between the two prospects, examining empirically how corporate events are captured by measurements used in liquidity pricing models, more specifically the $ILLIQ$ proxy and the L-CAPM. While some of the empirical research on the L-CAPM previously mentioned relies on individual stocks [40][50], the majority utilises portfolios [1][39][52]. Relevant empirical research focuses on an extended horizon when estimating a general level of liquidity and its effect on asset returns. Even less focus is on measuring specific events using illiquidity proxies and examining the implications for asset pricing models such as the L-CAPM, as done by Cao and Petrasek [29] where they examine the effects of the liquidity risk betas in the L-CAPM on asset returns during liquidity crises. While their study utilises event study methodology, the $ILLIQ$ proxy and the L-CAPM, it investigates the role of liquidity crises and not corporate events that can impact liquidity. While Amihud and Mendelson [7] acknowledge a number of empirical studies examining the effects of corporate events on liquidity, none of them explicitly investigate an individual firm's corporate event and its implications for the L-CAPM as captured by the $ILLIQ$ proxy. This is where I attempt to provide a preliminary insight into that relationship and provide opportunities for future research which could further expand the understanding of the associations between an asset's liquidity and its expected return.

3

The Liquidity-adjusted Capital Asset Pricing Model

In order to understand the L-CAPM one must be familiar with the basic intuition and setup of the standard CAPM. Imagine a one-period market populated by investors that can invest in both risky assets, called the *market portfolio*, and risk free assets. The CAPM describing the market equilibrium is based on the assumptions that a risk free asset does in fact exist in the economy and that investors agree on its return rate, have mean-variance preferences and are not subject to any investment constraints when allocating assets in their portfolios as well as all agreeing on the setup of the efficient frontier of risky assets. Given these assumptions, investors only have the possibility of investing in some combination of risky assets, called the *tangency portfolio*, and risk free assets. When the market is in equilibrium, both the supply and demand as well as the expected return of assets in the market must be aligned. Given the strict assumptions in the model, the only option for investors is to optimise their portfolios by investing in all risky assets available in the market. This means that the tangency portfolio and market portfolio are aligned in market equilibrium, which allows for the definition of the expected return of an asset available in the market according to the CAPM, and its risk premium respectively [48]:

$$r_i = r_f + \beta_i(r_m - r_f) \tag{3.1}$$

$$r_i - r_f = \beta_i(r_m - r_f), \tag{3.2}$$

where r_i is the return for asset i, r_f is the risk free return and r_m is the market portfolio return. β_i is the market beta for asset i, defined as:

$$\beta_i = \frac{cov[r_i, r_m]}{var[rm]}, \tag{3.3}$$

with $cov[r_i, r_m]$ as the covariance of the return of asset i and the market portfolio return and $var[r_m]$ as the variance of the market portfolio return. β_i, as defined in Formula 3.3, demonstrates how susceptible the return of asset i is to the return of the market. For example, we see that if the market return increases, the return of asset i is bound to increase as well, but only by the ratio of β_i. The definition of β is a key aspect of the CAPM and the main area of development in the L-CAPM, since β represents an asset's sensitivity to market risk. The intuition behind β indicates that assets that are highly correlated with the market in terms of returns are rather unattractive, since they have a high β and move alongside developments in market return. Therefore, assets with high β provide limited assurance against market downturns, whereas the opposite applies for assets with low β. The L-CAPM attempts to enhance the CAPM model by further defining certain aspects of β in relation to liquidity risks [48].

3.1 Assumptions

The L-CAPM defines how a security's required return is reliant on both the expected liquidity of the security as well as its covariance with market liquidity. In order to do so, the model's authors make a number of assumptions.

3.1.1 Investors

The model is based on a notion of overlapping investor generations, where new investors, N, are born at time t where $t \in ..., -2, -1, 0, 1, 2, ...$ but only live and invest for two periods as each investor has a given endowment at time t, and no other income, which they then trade in t and $t+1$. Investors derive utility from the consumption, x_{t+1}, on their investments as they must sell all securities in $t+1$. As investors are risk averse, the utility of their consumption can be represented by the expected utility function, $U(W)$ =$\mathrm{E}_t$exp(-$\mathrm{A}^n x_{t+1}$). When the L-CAPM is in equilibrium, investors invest and consume as to maximise their expected utility. They take prices as given, which implies that the market is *clear*, meaning that market prices are apparent. An investor has the "constant absolute risk aversion A^n" [1, p. 378] which implies that with increasing wealth the investor will hold the same dollar amounts in the assets he has invested in. In order to demonstrate the characteristics of the absolute risk aversion (ARA) in the utility function I apply the Arrow-Pratt risk aversion method [48]:

$$ARA(W) = -\frac{u^{''}(W)}{u^{'}(W)}, \tag{3.4}$$

with $u^{''}(W)$ and $u^{'}(W)$ as:

$$u^{''}(W) = -A^{2n} exp(-A^n x_{t+1}) \tag{3.5}$$

$$u^{'}(W) = A^n exp(-A^n x_{t+1}), \tag{3.6}$$

so that by dividing $-\frac{u^{''}(W)}{u^{'}(W)}$ I get:

$$U(W) = -\frac{u^{''}(W)}{u^{'}(W)} = -\frac{-A^{2n}}{A^n} = A^{2n-n} = A^n. \tag{3.7}$$

Formula 3.7 shows that the ARA is independent of the endowment level of the investor, implying that investors will invest in securities based on their characteristics as opposed to their own endowment levels. Hence, the negative exponential utility function used in the

L-CAPM is in fact a constant absolute risk aversion (CARA) utility function as noted by the authors.

3.1.2 The market

Investors operate in a market with I securities, indexed as $i = 1, ..., I$ where investors can buy a total of S^i shares of security i, which pays a dividend of D_t^i at time t, at the ex-dividend share price of P_t^i which is subject to an illiquidity cost of C_t^i. Note that the illiquidity cost and dividend are assumed to be random variables. Furthermore, the illiquidity cost represents the cost of selling one share of a security i, so investors must sell at $P_t^i - C_t^i$. When optimising their portfolios, investors can borrow and lend at a risk free return of $r^f > 1$ but short selling is not allowed. As the illiquidity cost is a random variable it is the main generator of liquidity risk in the model. More specifically, C_t^i and D_t^i and are defined as autoregressive processes of order one, $AR(1)$:

$$C_t^i = \overline{C} + p^C(C_{t-1} - \overline{C}) + \eta_t \tag{3.8}$$

$$D_t^i = \overline{D} + p^D(D_{t-1} - \overline{D}) + \varepsilon_t, \tag{3.9}$$

with $\overline{C}$ and $\overline{D}$ as positive real vectors, $\overline{C}, \overline{D} = \mathbb{R}_+^1$ and $p^C, p^D \in [0, 1]$. The processes η_t and ε_t are independent identically distributed (i.i.d.) normal processes with mean zero, $E(\eta_t) = E(\varepsilon_t) = 0$ and variance-covariance matrices, $var(\eta_t) = \sum^C$, $var(\varepsilon_t) = \sum^D$ and $E(\varepsilon_t \eta_t^\Gamma) = \sum^{CD}$. I note that these $AR(1)$ processes resemble a random walk process [28]:

$$P_t = \mu + P_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim i.i.d.(0, \sigma^2), \tag{3.10}$$

where P is a random process, μ is a positive constant and ε is an i.i.d. process identical to the one defined in the AR(1) processes for C_t^i and D_t^i. This resemblance between Formula 3.10 and Formulas 3.8 and 3.9 respectively, demonstrates that the assumption of illiquidity cost being in fact random is adhered to.

3.2 Model equilibrium

The model shows how an asset's expected gross return [1, p. 380],

$$r_t^i = \frac{D_t^i + P_t^i}{P_{t-1}^i} \tag{3.11}$$

depends on its relative illiquidity cost,

$$c_t^i = \frac{C_t^i}{P_{t-1}^i} \tag{3.12}$$

on the market return,

$$r_t^M = \frac{\sum_i s^i (D_t^i + P_t^i)}{\sum_i s^i P_{t-1}^i} \tag{3.13}$$

and on the relative market illiquidity,

$$c_t^M = \frac{\sum_i s^i c_t^i}{\sum_i s^i P_{t-1}^i}. \tag{3.14}$$

In order to determine equilibrium prices the authors assume an economy with an asset paying a dividend of $D_t^i - C_t^i$, but no illiquidity cost so that $C_t^i = 0$. In this imagined economy, one can state that the standard CAPM holds because there is no illiquidity cost so that there are no frictions in the market. The authors further claim that equilibrium prices must be the same in an actual economy with illiquidity cost, or frictions, as in the imagined economy, due to two factors: [1, p. 380-381]

1. Net return on a long position is the same in both economies, since no illiquidity cost is present at this point.

2. As investors in the imagined economy hold a long position in a market portfolio, or risky assets, and a (long or short) position in a risk-free asset, the authors conclude that an equilibrium of that kind also fulfills assumptions in the actual economy, and is therefore also feasible and optimal in that economy due to short-selling constraints.

Using these arguments, the authors show that the CAPM in the imagined economy, where there are no illiquidity costs, can be interpreted as CAPM for net returns in an actual economy, where illiquidity costs create frictions. This brings them to the first, and main, proposition of the model, which shows that the L-CAPM for gross returns can be deduced by rewriting the standard CAPM for net returns.

Proposition 1 shows that when the model is in equilibrium, the conditional expected net return of security i is:

$$E_t(r^i_{t+1} - c^i_{t+1}) = r^f + \lambda_t \frac{cov_t(r^i_{t+1} - c^i_{t+1}, r^M_{t+1} - c^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})}, \tag{3.15}$$

whereas $\lambda_t = E_t(r^M_{t+1} - c^M_{t+1} - r^f)$ is defined as the risk premium. Formula 3.15 shows the expected net return as the relative illiquidity cost has been subtracted from the gross return. However, measuring an asset's net return is difficult in practice and depends on the investor's holding period which is simplified and exogenous in the L-CAPM but variable in actual market economies. Therefore, the authors define the gross returns by using formulas for c^i_t, r^M_t and c^M_t so that:

$$\begin{aligned} E_t(r^i_{t+1}) = r^f + E_t(c^i_{t+1}) + \lambda_t \frac{cov_t(r^i_{t+1}, r^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} + \lambda_t \frac{cov_t(c^i_{t+1}, c^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} \\ -\lambda_t \frac{cov_t(r^i_{t+1}, c^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})} - \lambda_t \frac{cov_t(c^i_{t+1}, r^M_{t+1})}{var_t(r^M_{t+1} - c^M_{t+1})}. \end{aligned} \tag{3.16}$$

Formula 3.16 clearly still only has one risk premium, λ_t, which enables empirical testing of the model without adding factors and increasing the degrees of freedom from the standard CAPM. The formula partly states that the expected future return of an asset is positively reliant on the risk free rate, the expected future relative illiquidity cost and the asset's expected future risk premium, which is in line with the theory of the pricing of liquidity [4]. However, the distinctive feature of the L-CAPM model are the three liquidity risks which play a large role in deciding an asset's expected return by directly influencing an asset's risk premium. The risks are all relative to the variance of the difference between future market return and future market illiquidity, as shown by $var_t(r^M_{t+1} - c^M_{t+1})$. These three covariances provide a sort of "characterisation of the liquidity risk of a security" [1, p. 383].

While Proposition 1 is the main area of analysis for this thesis, it is important to note that the authors also study how asset returns are predictable based on their illiquidity and how returns and illiquidity move together. The intuition behind the additional propositions, named Proposition 2 and Proposition 3 respectively, are somewhat included in the main proposition of the L-CAPM paper, but explicitly defined and outlined in detail by the authors. ***Proposition 2*** states that if liquidity is persistent, $p^C > 0$, and an investor holds a portfolio, $q \in \mathbb{R}^I$, with $E_t(P^q_{t+1} + D^q_{t+1}) > p^C P^q_t$, the expected return increases with illiquidity [1, p. 384]:

$$\frac{\partial}{\partial C^q_t} E_t(r^q_{t+1} - r^f) > 0. \tag{3.17}$$

Formula 3.17 shows the first order condition of L-CAPMs conditional expected return with relation to the illiquidity cost of the portfolio, C^q_t. With more illiquidity, the expected risk return of the portfolio is positive, which implies that the return must increase as well. More specifically, the expected risk premium of the portfolio (the expected return subtracted by the risk free rate) increases with increased illiquidity. This implies that due to the persistence of liquidity, an asset's illiquidity *today* can somewhat predict an asset's return *tomorrow*. Building on the second proposition, ***Proposition 3*** further defines the covariance between an asset's illiquidity and its return. Due to the predictability of liquidity, it is apparent that returns and illiquidity have a negative covariance. As for Proposition 2, if an investor holds a portfolio, $q \in \mathbb{R}^I$, with $p^C(r^f q^T \sum^{CD} q + (r^f - p^D) q^T \sum^C q) > (r^f)^2 q^T \sum^{CD} q$, the return is low when illiquidity increases [1, p. 384]:

$$cov_t(c^q_{t+1}, r^q_{t+1}) < 0 \tag{3.18}$$

Formula 3.18 shows that if liquidity is persistent and the correlation of dividends and illiquidity costs are low, the covariance between the expected illiquidity cost of the portfolio and its expected return is negative.

3.3 Three liquidity risks

The authors show that the model outperforms the standard CAPM largely by capturing three different forms of liquidity risk when explaining expected returns.

1. **Asset's liquidity and market liquidity**, $cov_t(c^i_{t+1}, c^M_{t+1})$. The covariance between an asset's illiquidity cost and market illiquidity positively affects an asset's risk premium, so the required return increases if the covariance increases since the risk premium is increasing. This implies that with a high covariance, an asset's liquidity is analogous with market liquidity. This in turn increases the risk premium of the asset, since it is risky for investors to hold an asset that moves so similarly with the market because of the difficulty selling it during market turbulence. In other words, a high covariance implies that if the market is illiquid, the asset is illiquid as well. In order to justify for this increase in risk, investors demand a higher return on the asset as they must be compensated for the added risk of holding an illiquid asset when the market is illiquid.

2. **Asset's return and market liquidity**, $cov_t(r^i_{t+1}, c^M_{t+1})$. The covariance between an asset's gross return and market illiquidity negatively affects an asset's risk premium, so the required return decreases if the covariance increases since the risk premium is decreasing. This implies that if an asset has high returns in times of market illiquidity, investors are willing to accept a lower return for said asset. The aspect of having an asset that delivers high returns in market turmoil is a key strength for

an investor's portfolio, which results in a decrease in the risk premium. This is in line with empirical findings from Pástor and Stambaugh who show that returns for assets which are highly sensitive to market liquidity are higher than for assets with low sensitivity [52].

3. **Asset's liquidity and market returns**, $cov_t(c^i_{t+1}, r^M_{t+1})$. The covariance between an asset's illiquidity cost and market return negatively affects an asset's risk premium, so the required return decreases if the covariance increases since the risk premium is decreasing. This implies that if an asset is liquid in times of market downturn, investors are willing to accept a lower return for said asset. Assets with high liquidity are generally considered very attractive since they offer investors the possibility of redeeming their investment without much difficulties or delays. Being able to do that in times of market downturn is an even more attractive prospect since investors could then find buyers in uninspiring market circumstances with low margins. Investors are willing to request lower returns for assets with these attributes. Interestingly, this specific liquidity risk had minimal empirical and academic considerations when the paper was issued.

3.4 Empirical analysis

3.4.1 Unconditional version

In order to test the L-CAPM empirically, the authors derive an unconditional version of the model:

$$E(r^i_t - r^f_{t+1}) = E(c^i_i) + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i}, \tag{3.19}$$

which shows that the expected return of an asset, net of the risk free interest rate in the market, is reliant on the expected illiquidity cost of the asset, risk premium and liquidity risks. The authors use observations at t and $t-1$ in their empirical analysis and replace the three liquidity risks and the market beta, or the covariance between an asset's return and market return, with betas. The risk premium is therefore changed accordingly:

$$\lambda = E(\lambda_t) = E_t(r_t^M - c_t^M - r^f), \tag{3.20}$$

with the betas defined as:

$$\beta^{1i} = \frac{cov_t(r_t^i, r_t^M - E_{t-1}(r_t^M))}{var_t(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)])} \tag{3.21}$$

$$\beta^{2i} = \frac{cov_t(c_t^i - E_{t-1}(c_t^i), c_t^M - E_{t-1}(c_t^M))}{var_t(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)])} \tag{3.22}$$

$$\beta^{3i} = \frac{cov_t(r_t^i, c_t^M - E_{t-1}(c_t^M))}{var_t(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)])} \tag{3.23}$$

$$\beta^{4i} = \frac{cov_t(c_t^i - E_{t-1}(c_t^i), r_t^M - E_{t-1}(r_t^M))}{var_t(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)])}. \tag{3.24}$$

3.4.2 Test process

For each month t the authors estimate the illiquidity measure c_t^i for each security i. The return and illiquidity measures are also measured for both a market portfolio as well as 25 test portfolios, sorted by measures of illiquidity, illiquidity-variance, size and book-to-market ratio. Innovations in illiquidity, $c_t^p - E_{t-1}(c_t^p)$, are estimated for both market and test portfolios and used, along with returns, to measure the liquidity betas. Cross-sectional regressions and robustness tests are done to measure the empirical fit of the L-CAPM.

3.4.3 Illiquidity measure

The authors use the $ILLIQ$ proxy to measure illiquidity [2]:

$$ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{|R_{td}^i|}{V_{td}^i}, \tag{3.25}$$

with $|R_{td}^i|$ and V_{td}^i as the absolute return and trading dollar volume in millions respectively and $Days_t^i$ as the number of valid observation days per month t for security i. Formula 3.25 shows that an asset is illiquid if the proxy is high. If the return is high compared to a low volume, the $\frac{|R_{td}^i|}{V_{td}^i}$ ratio is high, which results in a higher $ILLIQ$ proxy. This suggests that a stock's price is highly influential to a low trading volume, which is an indicator of illiquidity. The authors define liquidity cost as the cost of selling an asset while the $ILLIQ$ proxy is defined as "the daily ratio of absolute stock return to its dollar volume, averaged over some period" and "can be interpreted as the daily price response associated with one dollar of trading volume, thus serving as a rough measure of price impact" [2, p. 32]. The $ILLIQ$ proxy is considered very accessible for empirical testing, as it does not require micro structural data like many other proxies and can therefore provide considerable benefits for testing both in terms of more security types as well as long time horizons. The authors identify two issues with using the $ILLIQ$ proxy.

1. The $ILLIQ$ proxy measures cost in percentages while the L-CAPM measures the cost in dollars, per dollar invested. Hence, $ILLIQ$ is a non-stationary process.
2. The $ILLIQ$ proxy is not a direct measure of the cost of a trade.

The $ILLIQ$ proxy is therefore adjusted:

$$c_t^i = min(0.25 + 0.30 ILLIQ_t^i P_{t-1}^M, 30.00), \tag{3.26}$$

with P_{t-1}^M as the capitalisation ratio of of the market portfolio at $t-1$. By adding the capitalisation ratio the authors manage to transform c_t^i into a stationary process[1]. The coefficients 0.25 and 0.30 are chosen to match the actual observed "difference between the transaction prices and the midpoint of the prevailing bid-ask midpoint" [30, p. 168]. The maximum selling cost is set at 30% to exclude outliers.

3.4.4 Portfolio setup

The authors use portfolios instead of individual stocks when empirically testing in order to reduce the effect of their noisy measure of illiquidity, characterised as follows[2]:

1. **Market portfolio**, which represents the normal market return on a monthly basis in the period 1 July 1962 to 31 December 1999.

2. **Illiquidity portfolios**, a total of 25 portfolios representing the market illiquidity on a yearly basis from 1964 to 1999.

3. **Illiquidity variance portfolios**, a total of 25 portfolios representing the market illiquidity variance on a yearly basis from 1964 to 1999.

The authors use data on daily returns and volume from the Center for Research in Security Prices (CRSP) for shares listed on New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). For the illiquidity portfolios, the annual average illiquidity for each stock is measured using the $ILLIQ$ proxy noted in Formula 3.26 and then sorted,

[1]By making the process stationary the authors attempt to make sure that assumptions, in the L-CAPM as well as the generalised method of moments when setting up the model estimators, are upheld when performing the empirical tests using the $ILLIQ$ proxy.

[2]The market portfolio only includes stocks priced between USD 5 and USD 1000 at the beginning of each month. Stock data for return and trading volume must be available for a minimum of 15 days each month. Both illiquidity portfolios and illiquidity-variance portfolios only include stocks priced between USD 5 and USD 1000 at the beginning of each year with stock data for return and trading volume available for a minimum of 100 observations days in year y-1.

based on the level of illiquidity in $y-1$, into 25 portfolios, $p \in 1, 2, 3, ..., 25$. The illiquidity-variance portfolios are set up in a similar manner, but the stocks are sorted into portfolios based on the standard deviation of daily illiquidity in $y - 1$. The portfolio return is calculated as:

$$r_t^p = \sum_{iinp} w_t^{ip} r_t^i, \tag{3.27}$$

with r_t^i as the return of asset i at time t and w_t^{ip} as the weight of asset i in portfolio p at time t. The normalised illiquidity of the portfolio is calculated similarly:

$$c_t^p = \sum_{iinp} w_t^{ip} c_t^i, \tag{3.28}$$

with the weight calculated as before but with c_t^i as the illiquidity proxy for asset i at time t.

3.4.5 Autocorrelation of market illiquidity

Due to the high autocorrelation of market illiquidity the authors measure *innovations in illiquidity* instead:

$$\triangle c^p = c_t^p - E_{t-1}(c_t^p), \tag{3.29}$$

which can be interpreted as the difference between the actual illiquidity of a portfolio at time t and the expected future illiquidity at time $t-1$. While the steps taken to reach a version of the $ILLIQ$ proxy applicable for illiquidity innovations are not clarified by the authors, I define c_t^p from Formula 3.29 as $ILLIQ_t^p$ for differentiation and transfer the 0.25 factor from Formula 3.26 so that:

$$c_t^i = min(0.30 ILLIQ_t^i P_{t-1}^M, 30.00 - 0.25), \tag{3.30}$$

and then divide with 0.30 and P^M_{t-1} in order to separate $ILLIQ^i_t$:

$$c^i_t = min(ILLIQ^i_t, \frac{30.00 - 0.25}{0.30P^M_{t-1}}). \tag{3.31}$$

This demonstrates the steps taken by the authors when defining the $ILLIQ$ proxy for portfolio p as opposed to single asset i:

$$\overline{ILLIQ^p_t} = \sum_{iinp} w^{ip}_t min(ILLIQ^i_t \frac{30.00 - 0.25}{0.30P^M_{t-1}}), \tag{3.32}$$

so illiquidity innovations can be measured as noted in Formula 3.29, with $c^p = \overline{ILLIQ^p}$:

$$\triangle\overline{ILLIQ^p} = \overline{ILLIQ^p_t} - E_{t-1}(\overline{ILLIQ^p_t}). \tag{3.33}$$

In order to predict the market illiquidity innovations the authors perform a regression:

$$c^M_t = a_0 + a_1(c^M_{t-1}) + a_2(c^M_{t-2}) + c^M - E_{t-1}(c^M_t), \tag{3.34}$$

with the slight adjustment to c^M so that the capitalisation ratio is fixed at P^M_{t-1} in order to assure measurement of actual illiquidity innovations as opposed to changes in P^M:

$$\begin{aligned}(0.25 + 0.30\overline{ILLIQ^M_t}P^M_{t-1}) = a_0 + a_1(0.25 + 0.30\overline{ILLIQ^M_{t-1}}P^M_{t-1})\\ +a_2(0.25 + 0.30\overline{ILLIQ^M_{t-2}}P^M_{t-1}) + \mu_t,\end{aligned} \tag{3.35}$$

with $\mu_t = c^M_t - E_{t-1}(c^M_t)$ as the market illiquidity innovation the authors wish to measure. They note that the autocorrelation of the innovations is low, or -0.03, and appears stationary after visual inspection. The authors end up using the $AR(2)$ specification for the series since it has the highest R^2 of 78%. A high level of autocorrelation can have a biased effect on the estimation of illiquidity, since it suggests that a relationship exists between values at time t and its previous values at $t_{t-1}, t_{t-2}, ...$ [24]. By using the illiquidity innovations instead of the actual illiquidity proxy, the authors manage to measure illiquidity using a stationary series with low autocorrelation which increases the explanatory capabilities of their empirical analysis.

3.4.6 Biases

In addition to the noisy illiquidity proxy the authors adjust the observed asset returns, r_t^i, for delistings to minimise the survivorship bias as described by Ang [13]. Although they use value-weighted portfolios for robustness testing, the main empirical results are based on an equal-weighted market portfolio in order to minimise the over-representation of liquid equities, since other securities account for the majority of actual wealth, for example private equity, real estate and other financial wealth [1].

3.4.7 Liquidity risk findings

The next step is to compute the traditional market beta, β^{1p}, as well as the value-weighted[3] portfolio liquidity risk betas, β^{2p}, β^{3p} and β^{4p} used in the unconditional L-CAPM defined in Formulas 3.22, 3.24 and 3.25, using previously defined market return time series, illiquidity innovations $\triangle c^p = c_t^p - E_{t-1}(c_t^p)$, as described in Formula 3.29, and the market illiquidity innovations based on the equal-weighted market portfolio, μ_t, as described in Formula 3.35.

The properties listed in Table 3.1 demonstrate that the average illiquidity increases from the first portfolio to the last, which is intuitive since the earlier formed portfolios are composed of fewer illiquid assets than the latter portfolios, as shown in column 6. The earlier portfolios which include assets with high average illiquidity tend to have higher volatility of returns, lower turnover and smaller market capitalisation, as shown in columns 9, 10 and 11. Understandably, the standard deviation of portfolio illiquidity innovations in column 7 increases along with the average illiquidity. Perhaps most importantly, portfolios consisting of a higher amount of illiquid assets have higher values of all three liquidity risk betas in columns 3, 4 and 5. The majority of these findings are statistically significant at

[3] The authors use both equal- and value-weighted portfolios when computing the betas but only report the value-weighted portfolio results, although "similar conclusions are drawn from examining the properties of equal-weighted illiquidity portfolios" [1, p. 390]

either 5% or 1% confidence levels.

Table 3.1: Properties of illiquidity portfolios

The table lists the properties of the illiquidity portfolios with t-statistics in parantheses and includes the beta values, average illiquidity, $E(c^p)$, illiquidity innovations of the portfolios, $\sigma(\triangle c^p)$, average excess return, $E(r^{e,p})$, average standard deviation of daily returns, $\sigma(r^p)$, turnover rate, (trn), and the market capitalisation, $bn\$$ of the portfolios[1, p. 391].

Portfolio #	β^{1p} (*100)	β^{2p} (*100)	β^{3p} (*100)	β^{4p} (*100)	$E(c^p)$ (%)	$\sigma(\triangle c^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	trn (%)	$Size$ (bn$)
1	55.10 (14.54)	0.00 (0.08)	-0.80 (-5.90)	-0.00 (-0.10)	0.25	0.00	0.48	1.43	3.25	12.50
3	66.70 (16.32)	0.00 (0.58)	-1.05 (-7.14)	-0.03 (-0.62)	0.26	0.00	0.39	1.64	4.19	2.26
5	74.67 (20.44)	0.00 (1.27)	-1.24 (-7.43)	-0.07 (-1.36)	0.27	0.01	0.60	1.74	4.17	1.20
7	76.25 (20.63)	0.00 (2.18)	-1.27 (-7.49)	-0.10 (-2.03)	0.29	0.01	0.57	1.83	4.14	0.74
9	81.93 (33.25)	0.01 (3.79)	-1.37 (-8.00)	-0.18 (-3.74)	0.32	0.02	0.71	1.86	3.82	0.48
11	84.59 (34.21)	0.01 (5.07)	-1.41 (-7.94)	-0.33 (-5.85)	0.36	0.04	0.73	1.94	3.87	0.33
13	85.29 (34.15)	0.01 (6.84)	-1.47 (-8.01)	-0.40 (-7.46)	0.43	0.05	0.77	1.99	3.47	0.24
15	88.99 (42.88)	0.02 (6.87)	-1.61 (-8.35)	-0.70 (-8.45)	0.53	0.08	0.85	2.04	3.20	0.17
17	87.89 (27.54)	0.04 (8.16)	-1.59 (-8.18)	-0.98 (-9.30)	0.71	0.13	0.80	2.11	2.96	0.13
19	87.50 (40.74)	0.05 (7.63)	-1.58 (-8.75)	-1.53 (-8.77)	1.01	0.21	0.83	2.13	2.68	0.09
21	92.73 (37.85)	0.09 (7.33)	-1.69 (-8.34)	-2.10 (-6.11)	1.61	0.34	1.13	2.28	2.97	0.06
23	94.76 (39.71)	0.19 (6.85)	-1.71 (-8.68)	-3.35 (-5.91)	3.02	0.62	1.12	2.57	2.75	0.04
25	84.54 (20.86)	0.42 (6.40)	-1.69 (-8.23)	-4.52 (-3.35)	8.83	1.46	1.10	2.87	2.60	0.02

The illiquidity innovations, $\triangle c^p$, and volatility of returns, $\sigma(r^p)$, are not the only causes of liquidity risk. As shown in Table 3.2, there is clear correlation between the different betas as well as market liquidity risk, β^{1p}. Interestingly, these correlations are not bound to portfolios but also exist on individual asset levels. As shown in column 3, β^{2p}, or the covariance between portfolio illiquidity and market illiquidity, is positively correlated with market liquidity risk. Columns 4 and 5 show that β^{3p}, the covariance between portfolio return and market illiquidity, as well as β^{4p}, the covariance between portfolio illiquidity and market return, are negatively correlated with market liquidity risk. While the correlations are in line with the intuition behind the different liquidity risks previously defined, they make it more difficult to demonstrate the separate effects of the individual liquidity risk betas on asset returns.

Table 3.2: Beta correlations

The table shows the correlation between betas for the 25 value weighted portfolios as well as individual assets, shown in parentheses.

	β^{1p}	β^{2p}	β^{3p}	β^{4p}
β^{1p}	1.000 (1.000)	0.441 (0.020)	-0.972 (-0.685)	-0.628 (-0.164)
β^{2p}		1.000 (1.000)	-0.573 (-0.072)	-0.941 (-0.270)
β^{3p}			1.000 (1.000)	0.726 (0.192)
β^{4p}				1.000 (1.000)

In light of the previously described characteristics of the liquidity risks the findings demonstrate that illiquid assets are highly coherent with market liquidity, $\beta^{2p} = cov_t(c^i_{t+1}, c^M_{t+1})$, have returns which are highly coherent with market liquidity, $\beta^{3p} = cov_t(r^i_{t+1}, c^M_{t+1})$, and have liquidity that is highly coherent with the return of the market $\beta^{4p} = cov_t(c^i_{t+1}, r^M_{t+1})$. In other words, an asset that is considered illiquid according to a chosen proxy, moves more often than not in line with market liquidity which implies that an illiquid asset is more likely to be illiquid when the market itself is illiquid. The returns of an illiquid asset are also more often than not sensitive to liquidity in the market, as the market liquidity can have direct effects on the return of the asset. An asset is more likely to have lower returns in turbulent markets and higher returns in stable markets. Finally, the liquidity of an asset is dependent on the return of the market. An asset is more likely to be illiquid in times of market downturn compared to a booming market.

3.5 Liquidity risk and asset returns

By calculating the betas the authors manage to better grasp the relationship between liquidity and liquidity risk. However, in order to comprehend the effect liquidity risk has on an asset's expected return they perform cross-sectional regressions on the portfolios.

3.5.1 Estimators

The authors use the generalised method of moments (GMM) when selecting parameters to fit the model for empirical testing. The GMM offers no single solution for parameter selection, but rather a large variety of options that allow for a high grade of flexibility. "GMM makes it easy to estimate and evaluate nonlinear models" [31, p. 169] and is a "natural fit for a discount factor formulation of asset pricing theories" [31, p. 187], partly due to the fact that the method is based on sample observations instead of the actual population. The GMMs underlying goal is to select parameters which minimise the weighted sum of squared pricing errors, similarly to the ordinary least squares (OLS) method. This is further indicated by the fact that the authors get the same estimators using either the GMM or OLS method. The main reason for selecting GMM over OLS are the distribution assumptions. OLS assumes i.i.d. observations while GMM does not require any assumptions on the full shape of the distribution function of the observations at hand. This provides flexibility for the GMM and allows for inclusion of misspecifications in the distribution theory. In other words, even though the authors do not know the full nature of the distribution of their dataset they still want to perform some sort of time series regression as part of their empirical analysis, and the GMM allows them to do that without making any blind assumptions.

3.5.2 Standard errors

Similarly to the issues surrounding the measurement of market illiquidity, the standard errors also tend to have a high degree of autocorrelation. The GMM framework includes evaluations of standard errors that correct for both autocorrelation and the conditional heteroscedasticity of errors. The authors use the Newey-West estimator for two lags which is in line with AR(2) when computing the $ILLIQ$ proxy for the betas. While not explained by the authors, I speculate that the reason they choose the GMM method to calculate the standard errors as opposed to other methods is the method's clear agility. For example, by using the maximum likelihood (ML) method to estimate parameters the authors would be bound to use OLS to estimate the standard errors, which can be significantly lacking since OLS has limited ability for adjustments. The GMM allows for an estimation of parameters similarly to OLS but without making too many assumptions, and provides the ability to adjust the standard errors for non-i.i.d. observations [31].

3.5.3 Constrained L-CAPM

The authors define a *net beta*, defined as a combination of the market beta and the three liquidity risk betas, in order to account for the same risk premium, λ, noted in Formula 3.19. The net beta is:

$$\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}. \tag{3.36}$$

The unconditional L-CAPM therefore goes from:

$$E(r_t^p - r_t^f) = E(c_t^p) + \lambda\beta^{1p} + \lambda\beta^{2p} - \lambda\beta^{3p} - \lambda\beta^{4p}, \tag{3.37}$$

to the following formulation, using $\beta^{net,p}$:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda\beta^{net,p}. \tag{3.38}$$

However, Formula 3.38 does not differentiate between the market beta and the liquidity risk betas. The authors therefore also separate the two for empirical testing, using λ and λ^1:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda \beta^{net,p}. \tag{3.39}$$

The authors note that the methodology used in the empirical testing has two limitations:

1. The model implies that the intercept is $\alpha = 0$, while the authors allow for $\alpha \neq 0$.

2. The model implies that $\kappa = 1$ because investors are only exposed to illiquidity exactly once in each period. This is of course not the case in the actual economy, so the authors define $\kappa = 0.034$, which is the average monthly turnover of assets in their sample, which indicates an investor holding period of $\frac{1}{0.034} \approx 29$ months. This allows them to scale the expected illiquidity, $E(c_t^p)$, with time since it is calculated as the average illiquidity of the portfolios and does not allow for summarisation. By doing this, the authors hope to adjust for the difference between their sample period and the actual investor holding period.

3.6 Results

Table 3.3 shows the authors' main findings, divided into two sections for illiquidity portfolios and illiquidity-variance portfolios. They conduct eight separate regressions while controlling for various parameters as listed in Table 3.4.

Table 3.3: Regression findings for illiquidity portfolios and illiquidity-variance portfolios
The table lists the coefficients, and their t-statistics in parantheses, of cross-sectional regressions of the L-CAPM for 25 value-weighted portfolios and an equal-weighted market portfolio. The table lists the regression constant, average illiquidity, $E(c^p)$, beta values and the R^2 value with $adjR^2$ values in parantheses[1, p. 395].

Portfolio #	Constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
Illiquidity portfolios								
1	-0.556	0.034					1.512	0.732
	(-1.450)	(-)					(2.806)	(0.732)
2	-0.512	0.042					1.449	0.825
	(-1.482)	(2.210)					(2.532)	(0.809)
3	-0.788		1.891					0.653
	(-1.910)		(3.198)					(0.638)
4	-0.333	0.034	-3.181				4.334	0.843
	(-0.913)	(-)	(-0.998)				(1.102)	(0.836)
5	0.005	-0.032	-13.223				13.767	0.878
	(0.013)	(-0.806)	(-1.969)				(2.080)	(0.861)
6	-0.160		-8.322				9.164	0.870
	(-0.447)		(-2.681)				(3.016)	(0.858)
7	-0.089	0.034	0.992	-153.369	7.112	-17.583		0.881
	(-0.219)	(-)	(0.743)	(-1.287)	(0.402)	(-1.753)		(0.865)
8	-0.089	0.033	0.992	-151.152	7.087	-17.542		0.881
	(-0.157)	(0.166)	(0.468)	(-0.280)	(0.086)	(-1.130)		(0.850)
σ (illiquidity) portfolios								
1	-0.528	0.035					1.471	0.865
	(-1.419)	(-)					(2.817)	(0.865)
2	-0.363	0.062					1.243	0.886
	(-1.070)	(2.433)					(2.240)	(0.875)
3	-0.827		1.923					0.726
	(-2.207)		(3.322)					(0.714)
4	-0.014	0.035	-7.113				7.772	0.917
	(-0.037)	(-)	(-1.939)				(2.615)	(0.914)
5	0.094	0.007	-11.013				11.467	0.924
	(0.235)	(0.158)	(-2.080)				(2.480)	(0.914)
6	0.119		-11.914				12.320	0.924
	(0.305)		(-2.413)				(2.608)	(0.917)
7	0.464	0.035	-1.105	-83.690	-74.538	-14.560		0.940
	(0.913)	(-)	(-0.728)	(-0.663)	(-1.175)	(-1.662)		(0.931)
8	0.459	0.148	-1.125	-390.588	-73.552	-21.688		0.942
	(0.565)	(0.140)	(-0.485)	(-0.140)	(-1.943)	(-0.335)		(0.927)

A test of line 1 in Table 3.4 shows promising results which are supportive of the L-CAPM, with α as insignificant and $\lambda > 0$ significant at the 1% confidence level for both illiquidity and illiquidity-variance portfolios. The explanatory power is relatively high with $adjR^2 = 0.732$, compared to an $adjR^2 = 0.638$ for the standard CAPM which the authors also use to estimate the expected return of the portfolios in line 3. Interestingly, while both models have a significant $\lambda > 0$ at the 1% confidence level, the market beta, β^{1p}, used in the CAPM model has a slightly higher t-statistic than the $\beta^{net,p}$ used for the L-CAPM, which means that the authors can reject the hypothesis that $\lambda = 0$ with slightly more strength for the standard CAPM than the L-CAPM. Also, by defining κ as a free parameter in line 2 and 5 the explanatory power of the L-CAPM slightly increases, with the $adjR^2 = 0.732$ increasing to $adjR^2 = 0.809$ for illiquidity portfolios and from $adjR^2 = 0.865$ to $adjR^2 = 0.875$ for illiquidity-variance portfolios. This is perhaps to be expected as a fixed parameter of $\kappa = 0.034$ in line 1 is bound to have less explanatory power on the expected return due to its rigidity.

Table 3.4: Description of regressions conducted in Table 3.3
The table describes the formulas and parameter controls used in regressions in Table 3.3.

Line #	Description
1	L-CAPM in Formula 3.34, using $\beta^{net,p}$
2	L-CAPM in Formula 3.34 with κ as a free parameter
3	Standard CAPM in Formula 3.2
4	L-CAPM in Formula 3.35, using β^{1p} and $\beta^{net,p}$
5	L-CAPM in Formula 3.35 with κ as a free parameter
6	L-CAPM in Formula 3.35 with $\kappa = 0$
7	L-CAPM in Formula 3.36 with different risk premium λ^i
8	L-CAPM in Formula 3.36 with κ as a free parameter

The test of the constrained L-CAPM in lines 1 and 2 does not differentiate between the market beta and liquidity risk betas for the L-CAPM. A test with the two factors separated is performed in line 4 in order to perform an empirical analysis of the actual effect of the three liquidity risks on asset returns. However, the results are mixed. With a calibrated $\kappa = 0.034$ the $\beta^{net,p}$ is insignificant for illiquidity portfolios but significant for illiquidity-variance portfolios. However, by defining κ, both as a free parameter in line 5 and as $\kappa = 0$ in line 6, the $\beta^{net,p}$ is significant for both portfolio types.

The authors also restrict the premiums so that they are alike, $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$ resulting in an adjusted L-CAPM:

$$E(r_t^p - r_t^f) = \alpha + (c_t^p) + \lambda^1\beta^{1p} + \lambda^2\beta^{2p} + \lambda^3\beta^{3p} + \lambda^4\beta^{4p} \tag{3.40}$$

When testing Formula 3.40 in lines 7 and 8 the authors do not find significant results due to the high collinearity between estimators, both when calibrating $\kappa = 0.034$ and when defining κ as a free parameter.

3.6.1 Economic interpretation

In order to estimate the effect of liquidity risk on expected returns the authors compute the difference between the betas by comparing the expected returns per year between portfolio 1 and portfolio 25, since portfolio 1 should reflect liquid assets in the market and portfolio 25 should reflect illiquid assets in the market. Since the authors conduct several tests on the L-CAPM model with varying results they use the structured model tested in line 1 of Table 3.4, with κ calibrated at $\kappa = 0.034$ and the resulting $\lambda = 1.512$ estimator for $\beta^{net,p}$.

Table 3.5: Combined effect of liquidity risk on expected returns

Beta	**Notation**	**Results**
β^2	$\lambda(\beta^{2,p_{25}} - \beta^{2,p_1})12$	0.08%
β^3	$\lambda(\beta^{3,p_{25}} - \beta^{3,p_1})12$	0.16%
β^4	$\lambda(\beta^{4,p_{25}} - \beta^{4,p_1})12$	0.82%
	Total	**1.10%**

As noted in Table 3.5, the total effect of liquidity risk on an asset's expected returns is 1.10% with a 95% confidence interval (CI) of $[0.24\%, 1.88\%]$, and therefore significantly different from zero. In addition to this, the expected illiquidity, $E(c)$, difference between portfolios 1 and 25, is 3.50%, so the combined effect of both expected illiquidity and liquidity risks on expected return is 4.60% on a yearly basis. Perhaps the most interesting aspect is the vast difference between the effect of β^4 and β^2 and β^3, which implies that the covariation between an asset's illiquidity to market returns has the largest impact on expected returns in relation to liquidity risk. As previously noted, this covariance has not been studied before.

3.6.2 Robustness

In order to identify whether their results are robust or not, the authors investigate the difference in findings between portfolio weights, comparing equal-weighted and value-weighted illiquidity portfolios as well as market portfolio.

Table 3.6: Portfolio weights overview

Test type	**Illiquidity portfolio weights**	**Market portfolio weights**
Original	Value	Equal
Robustness 1	Equal	Equal
Robustness 2	Value	Value

Table 3.6 shows a summary of the weights of each portfolio, for both illiquidity portfolios as well as the market portfolio. The results are noted in Appendix 1, but in short the robustness test results are positive for the constrained L-CAPM, but mixed when adding parameter restrictions. Both robustness test outperform the standard CAPM as in the original test results, however the estimator is borderline significant at the 5% significance level when testing equal-weighted portfolios but not significant at the 5% significance level when testing value-weighted portfolios. Interestingly, the explanatory power of the value-weighted portfolios is vastly lower than in the original tests, with $AdjR^2 = 0.486$ for the constrained L-CAPM compared to $AdjR^2 = 0.825$ using equal weights, which might indicate that value-weights are too concentrated on asset types that do not represent the actual assets allocated in investor portfolios, as suggested by both Acharya and Pedersen [1] and Ang [13].

Table 3.7: Specification test null-hypotheses overview

Test #	**Null- hypothesis**
1	$H_{01} : \alpha = 0$
2	$H_{02} : \lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$
3	$H_{03} : \alpha = 0$
4	$H_{04} : K = \kappa$
5	$H_{05} : \lambda_t = E_t(r_t^M - \kappa c_t^M - r^f)$,
6	$H_{06} : \lambda_t = E_t(r_t^M - r^f)$
7	$H_{07} : \varepsilon_t = 0$

The specification tests noted in Table 3.7 provide further suggestion that the L-CAPM is more robust than the standard CAPM. First, the authors fail to reject H_{01} that the intercept is equal to zero for the constrained L-CAPM model while they reject it for the CAPM. While it is rather counter-intuitive to imagine that not rejecting a null hypothesis can be a supportive result for the L-CAPM, one must realise that a failure to reject that the intercept in the L-CAPM is zero is in line with the theoretical assumptions of the model, and the same rationale applies for the remaining specification tests.

Second, the authors fail to reject H_{02} for the L-CAPM that the estimators for the betas are equal, H_{03} that the intercept is equal to zero as well as H_{04} that K is calibrated as κ.

Third, the authors test the restriction that the risk premium in the model should equal the expected net market return subtracted by the risk free rate. They know that the risk premium is $\lambda_t = E_t(r_t^M - c_t^M - r^f)$ and they test whether it holds by measuring if the estimated risk premium, λ_t, is equal to the sample's average net market return subtracted by the risk free rate. They fail to reject H_{05} at the 5% significance level, which is supportive of the model. In contrast to this, they reject H_{06} for the CAPM that the risk premium equals the return of the market net of the risk free rate, which is not supportive of the CAPM model.

Finally, the authors fail to reject H_{07} for the L-CAPM that the model's average pricing error is zero but reject it for the CAPM at the 5% significance level. While this applies to the standard illiquidity portfolios built for the model, the results for the variance-illiquidity (sigma) portfolios is similar, albeit with a significance level of 10%.

The robustness tests consisting of various portfolio selections and significance tests suggest that the L-CAPM has a higher explanatory power and is more receptive to changes on parameters, variables and assumptions than the CAPM. The significance tests show that null-hypotheses based on assumptions and implications of the L-CAPM are more likely to hold under stringent statistical testing than the CAPM.

3.7 Summary

The L-CAPM demonstrates, in simplistic terms, that an asset's required excess return consists of an illiquidity cost (reflecting transaction costs, broadly interpreted), and four terms reflecting liquidity risk, whereas one of the terms reflects the standard market beta used in the CAPM. The other three terms are noted using betas and reflect the covariances between asset illiquidity or returns and market illiquidity or returns. The first term shows that the required excess return of an asset increases with the covariance between the asset's illiquidity and the market's illiquidity, implying that assets that are liquid in illiquid markets are attractive, hence their low expected excess returns. The second term shows that an asset's required excess return decreases with the covariance between the asset's actual return and the market's illiquidity, implying that an asset with high actual returns in illiquid markets is also attractive so investors require lower returns and higher prices for said assets. The last term shows that an asset's required excess return decreases with the covariance between the asset's illiquidity and the market's returns, implying that liquid assets in struggling low-return markets are attractive which justifies lower required returns and higher prices.

Furthermore, the L-CAPM significantly outperforms the CAPM when it comes to empirical tests, both in terms of explaining expected returns as well as testing for robustness and significance. However, the L-CAPM struggles when sorting illiquidity portfolios by size, adding constraints on the risk premium and holding period, collinearity and when estimating the liquidity risk terms, or betas, separately. The author's findings show that illiquidity cost explains 3.5% of cross-sectional returns while liquidity risk explains 1.1% of cross-sectional returns, with a large majority, or around 80%, of the explanatory power coming from the third term, the previously unrecognised relationship between an asset's required excess returns and covariance between an asset's illiquidity and the return of the market. In spite of the authors' relatively strong empirical results they do identify the L-CAPM as simplistic, resulting in empirical corrections of various components, such as calibrating the investor's holding period to represent monthly turnover rate of securities while it was originally based on the life-term of investors and did therefore not adequately account for liquidity's time-variation. The authors also assume that investors are subject to higher illiquidity when trading more frequently, since they would not only have to spend time but also suffer additional trading costs when trading often. These effects would then appear through higher levels of illiquidity. However, these illiquidity levels are largely variant between different sources of illiquidity and asset categories as well. While the model does sort portfolios based on equal weights and value weights in order to account for different securities, it does not incorporate aspects of illiquidity sources or different types of securities due to measurement difficulties.

4

Liquidity - sources and characteristics

The liquidity issues discussed in this thesis can be viewed as an attempt to move from traditional economic issues, a topic which has received increased attention after the fundamental pillars of traditional economics were challenged in the aftermath of the financial crisis in 2008 [10]. Specifically, firms that had difficulties financing themselves due to liquidity constraints had to alter their investment strategies, which directly opposes Modigliani-Miller's proposition that firms' investment decision making is not reliant on the way they finance themselves. This underlines the notion that firms actually play a role in influencing the liquidity of their issued securities. It has also long been clear that assumptions on *perfect* frictionless markets, without information or transactions costs, could not be farther from the truth in actual markets, and it is these frictions that actually play a large role in liquidity levels in markets. Modern asset pricing models such as the L-CAPM have attempted to reflect this theoretical shift in its assumptions.

These challenges, or perhaps *contradictions*, suggest that it is not only market liquidity, for example transaction costs, narrowly or broadly interpreted, that play a role in an asset's liquidity level but also the underlying firm liquidity. While market liquidity is the main subject of consideration for most investors, especially since it is the topic of most financial models, it is clear that liquidity issues related to underlying firms that issue the securities investors can invest in are far from trivial. The level of inclusion of firm specific liquidity issues in market liquidity measurements is ambiguous according to current literature which further underlines the importance that investors are aware of the effects of firm liquidity as well as market liquidity when implementing their investment strategies.

4.1 Market liquidity

4.1.1 General

The concept of liquidity is elusive. It is a concept that is very difficult to formally define and characterise, yet it appears very clearly when you have the opportunity to observe it. In light of this predicament, there does not exist a single agreed upon definition, but I use Amihud, Mendelson and Pedersen's definitions of key concepts regarding liquidity [10, p. ix] in order to provide clarity and cohesion in this chapter. **Liquidity** is characterised as the ability of buying or selling large amounts of an asset at a low cost. **Illiqudity** is characterised as a large difference between the buy (bid) and sale price of an asset, if buying or selling a large quantity of an asset has an extensive effect on the asset's price movement or when it is time consuming to sell an asset. **Liquidity risk** is characterised as the possibility of an asset being more illiquid when its owner needs to sell it in the future.

4.1.2 Sources of illiquidity

Illiquidity costs have various implications. For example, they can lead to *clientele effects and participation costs* [7], which characterises the costs of entering and participating in a market, as many markets are difficult to navigate and have barriers in relation to capital, investor type or location. *Funding constraints* [62] characterise the implications of credit on investments, since many investors are leveraged in most markets, and when buying large amounts of listed equity or real estate, a large amount of capital is required, which can lead to more leveraged positions. If investors' credit lines suffer a drawback it can result in less trading on the market due to limited funding.

In the authors' empirical analysis of the L-CAPM, $ILLIQ$ is assumed to be an *umbrella* of sorts over various different selling costs (since illiquidity is defined as the cost of selling in the model), such as broker fees, bid-ask spreads, market impact and search costs. They assume that $ILLIQ$ is a "valid instrument for the costs of selling, broadly interpreted" [1, p. 386]. The authors explain that with their broad interpretation of $ILLIQ$ they manage to include not only transaction costs such as broker fees and bid-ask spreads but also real costs, for example trade delays and search costs. In order to fully comprehend this broad interpretation I investigate the most common sources of illiquidity, which should intuitively be included in the authors' interpretation of $ILLIQ$. Amihud, Mendelson and Pedersen provide an overview over these main sources of illiquidity [9] which have been covered extensively in theoretical literature[1].

1. **Exogenous trading costs** are direct fees related to trading, for example commissions, exchange fees and taxes. This also includes, in certain markets, due diligence costs and fees to consultants, lawyers and accountants, which can weigh heavily in certain transaction types.

2. **Demand pressure and inventory risk**, where a seller is eager to offload his position but no suitable buyers are operating in the market since not all market participants are present at all times, which creates demand pressure. In that position, a market maker might enter a buy for the position in hopes of offloading it at a later point, resulting in inventory risk while he holds the position.

[1] Amihud and Mendelson [6] define four different components of illiquidity costs or bid-ask spreads, market impact costs, delay and search costs and direct transaction costs, which is in line with the authors' definition of costs of selling, measured by the $ILLIQ$ proxy [1]. In addition to these items, Vayanos and Wang define clientele effects and participation costs, asymmetric information and funding constraints as sources of illiquidity in asset markets [62].

3. **Asymmetric information** characterises the difference of levels of knowledge between investors. For example, a seller might worry that a buyer is purchasing stock because he has information about possible future price increases, and the buyer might worry that a seller is trying to offload stock because he has information on the poor status of the firm's operations. This might result in unwillingness from investors to enter buys or sales.

4. **Search frictions** occur because of the time and effort spent in matching buyers and sellers and can be difficult with block trades or in illiquid market types, for example corporate real estate and complicated credit products. This cost represents the investor's opportunity cost of waiting to trade because of their search for an optimal price.

While the effect of firm-specific liquidity issues are bound to be included in these sources, at least indirectly when considering information asymmetries between underlying firms and investors, common illiquidity measures do not attempt to explicitly capture the full effect of firm-specific aspects that might have an effect on the market. It is therefore likely that these well researched sources of illiquidity provide limited insight into the direct effects of firm-specific liquidity issues on market liquidity.

4.2 Firm liquidity

4.2.1 General

While liquidity is certainly relevant from an investor's perspective when deciding their preferred portfolio characteristics, liquidity is also a key aspect in deciding how a firm runs its business. For a firm, accounting liquidity can severely affect its profitability, value and in some cases even cause solvency. A firm's short-term accounting liquidity relates to its ability to satisfy operational obligations such as salary, production and rent payments. A firm's long-term accounting liquidity relates to its ability to satisfy financial health obligations such as building new production sites and research and development of new products or services [55]. Failure to meet these obligations can lead to illiquidity which can severely hinder a firm's utilisation of profitable opportunities, for example by limiting management's operational options or leading to suspension of short- or long term payments to creditholders [55]. This illiquidity can in turn have negative effects on the value of the firm by limiting its cash flows. This can be considered as directly related to the concept of *fundamental risk* in corporate finance theory, which is the risk investors are exposed to because of how a firm conducts its business, both in relation to investments and operations as well as financing and leverage [54].

4.2.2 Corporate events that impact liquidity

Amihud and Mendelson argue that in addition to corporate finance theory which, amongst other things, focuses on accounting liquidity issues, the liquidity of a firm's *own securities* plays a large role in determining firm value [5][7]. They base this argument on their findings that portfolio returns increase with an increase in the bid-ask spread [3]. They suggest that this relationship provides an incentive for firms to reduce their opportunity cost of capital by increasing the liquidity of their securities. This implies that firms can maintain a financial strategy that increases their liquidity which results in higher firm value, which further suggests that firm liquidity issues can be transcribed into market liquidity. By doing this, firms can increase their market value by increasing the liquidity of their issued securities [5]. Amihud and Mendelson further note that this strong incentive can explain the investments firms make in corporate actions that increase liquidity such as public offerings, limited liabilities and exchange listings. In their 1991 paper, *Liquidity and Asset Prices: Financial Management Implications*, Amihud and Mendelson outline several firm-specific liquidity-increasing actions [6] which have received increased attention for the last quarter century. Amihud and Mendelson summarise empirical findings in relation to these actions in their 2012 paper, *Liquidity, the Value of the Firm, and Corporate Finance** [7]:

1. **Increasing the liquidity of a firm's issued securities**, for example where a firm lists its securities on an organised exchange which provides more active trading and increases the depth of the market it operates in. Firms also have other liquidity-increasing options when listing their securities on an exchange. For example, an **underwriter** during an initial public offering (IPO) offers stability in the IPO price and acts as market maker for the firm's initial period in the market, offering liquidity to investors that the firm can not provide by itself. However, these actions are not limited to listings on an exchange. A firm can also **go public**, where it moves from a closed corporation to a public corporation which entails that transfer of its securities are made possible with minimal restrictions and costs when compared to closed corporations. A firm can also assign **limited liability** to their issued securities which facilitates a liquid secondary market as investors can trade the securities without being subject to excessive liabilities.

2. **Capital structure**, as firms are usually subject to lower transaction costs than investors, which allows firms to lessen some of the illiquidity costs faced by investors by increasing their debt, for example by corporate borrowing or issuing corporate bonds. However, when transaction costs become high enough, the information asymmetry, between *informed* individual investors and *less-informed* investors and market makers, increases with the costs of issuing debt. This implies that "an increase in corporate leverage has the potential to cause a reduction in the liquidity of the company's stock and, along with it, an increase in its cost of equity capital" [7, p. 23].

3. **Security design**, where firms issue unambiguous securities such as corporate bonds

or stocks in its equity as opposed to complex contracts and customised direct trading agreements, often observed in closed corporations. By issuing multiple different security types a firm risks fragmentation of its investor base which can lead to an increase in illiquidity. This standardisation of securities minimises that risk.

4. **Payout policy**, where, in markets with high transaction costs, dividends play a role in providing investors with increased liquidity. This implies that dividend payments are more likely for illiquid stocks as they tend to have high transaction costs. For stock repurchases however, the transaction costs are lower since firms themselves are repurchasing the shares from investors directly. Also, repurchases tend to signal to investors that the firm has positive private information about their future prospects. Since this suggests that dividends are more likely to be used by firms with illiquid stock, stock repurchases should rather be used to distribute cash to investors and increase liquidity.

5. **Investor base increase**, where firms offer lower nominals of their securities to investors. For example, a firm can offer 10,000,000 shares at 1 USD per share as opposed to 100,000 shares at 100 USD per share, in order to service a greater population of investors, and minimise block trading, which minimises the bid-ask spread and leads to an increase in liquidity. By offering smaller nominal values, for example with the use of stock splits, firms open doors for smaller less-informed individual investors which are more likely to trade without private information, which reduces informational asymmetry and therefore illiquidity costs.

6. **Disclosure of information and analyst following**, where firms can make inside information such as financial reports, organisational and management changes, public in order to decrease information asymmetries in the market, which lowers the bid-ask spread of its issued securities and leads to increased liquidity.

7. **Innovation**, where financially constrained firms with significant growth opportuni-

ties are more likely to attempt to increase their stock's liquidity in order to decrease their cost of capital since equity is relatively more valuable for them compared to larger, more established firms. Firms can maintain high levels of R&D costs to signal to investors that they have significant growth opportunities.

Amihud and Mendelson note that these actions of course come with a cost. For example, underwriting and administration costs of going public or listing on organised exchanges are usually high which can have an excessive negative effect on a firm's short term cash flow. These costs, somewhat variable between the actions chosen, must be considered when pursuing these liquidity increasing corporate actions.

4.3 Summary

A firm's financial strategy affects its inherent liquidity issues, which in turn affect the liquidity of the securities it issues [7]. Since markets largely consist of securities issued by firms, firm liquidity is bound to be transcribed into market liquidity to some degree. However, this transcription is not fully analogous since market liquidity stems from multiple sources [7], many of which are foreign to firm liquidity. For example, a firm's corporate policy has a limited effect on direct trading costs such as a broker's commission or market turbulence which can cause illiquidity, demonstrating the influence of the time-variance of liquidity. Measures on illiquidity, such as the $ILLIQ$ proxy used in the L-CAPM, measure market illiquidity from a market's perspective, based on the return and trading volume of firms' issued securities. This suggests that commonly used measures on illiquidity do not explicitly include all aspects of a firm's underlying liquidity issues in their estimations.

5

Event study

An event study measures the effect of a specified event on the value of a firm's issued securities [28]. Most often, the effect is studied through price changes of a certain class of securities of the firm, commonly equity. This is made possible by the assumption that markets are efficient, so that information made available through an event should be incorporated immediately in market prices. This allows for measurement of an observed effect in a short period of time. Event studies can be applied to various fields, for example earnings announcements, issues of debt and equity, implementation of new regulations in law and to assess damages in insurance and liability cases [28]. The benefit of additional control is added by applying an event study to an empirical analysis as it allows limitation, control or exclusion of factors outside the event's estimated effect, such as general stock market movements or other events happening simultaneously. This allows for better identification of correlation between the variables that are subject to investigation.

5.1 Methodology

Although event studies have existed for just over eighty years since Dolley published his examination of price effects of stock splits in 1933, the methodology has often been modified in order to provide more sophisticated studies. The fundamental methodology used today was introduced in two separate papers just under fifty years ago, by Ball and Brown in 1968 [15] and Fama, Fisher, Jensen and Roll in 1969 [37]. Using this fundamental methodology, Campbell, Lo and MacKinlay define seven steps which are applied to this event study [28] as outlined in Appendix 2.

5.2 Research design

In this thesis I conduct two analogous event studies analysing two separate corporate events for two separate non-financial firms which are likely to have considerable impact on their firm liquidity. The studies attempt to demonstrate the importance of corporate actions used to affect the liquidity of a firm's issued securities and to which degree those changes appear in the securities' pricing over the time of the events, also providing valuable insight into the time-variance aspects of liquidity.

5.2.1 Objectives

To summarise, by conducting these event studies I attempt to demonstrate the following:

1. **The effect of corporate events on liquidity and asset prices**, as firms can affect the liquidity of their issued claims in order to affect their market value [7]. The events should therefore not only appear in changes in liquidity levels but also asset pricing models such as the L-CAPM.

2. **The time variance of liquidity**, demonstrating that firm liquidity is in fact time-variant and is likely to play an important role in the pricing of assets. The firm liquidity should be *dynamic* during the event window and influenced by the event

information entering the market.

3. **The applicability of the *ILLIQ* proxy to capture firm-specific events**, as the authors use portfolios rather than individual firms to evade the noisiness of the *ILLIQ* proxy [1]. The event studies should demonstrate to what effect the *ILLIQ* proxy manages to capture said events for individual firms.

5.2.2 Event background

On 18 November 2016 **Facebook Inc.** (hereafter *FB*) announced their decision to repurchase up to $6*bn* of its Class A common stock [36], the first repurchase in the history of the firm. According to FBs third quarterly report in 2016 they held around 26 billion in cash, cash equivalents and marketable securities on 30 September 2016 which were used for the repurchase [35]. According to Amihud and Mendelson, share repurchases are done by firms with more liquidity as opposed to firms that pay out dividends. The repurchase also plays a role in signalling to FBs shareholders that the firm is positive regarding its future profitability prospects.

On 2 December 2015 **Macdonald's Corporation** (hereafter *MCD*) announced their decision to issue debt amounting to $6*bn* in five separate medium term notes [44], the largest debt issue in the history of the firm [34]. The issue was split into five separate bonds with different principal amounts, coupon rates and maturity dates. While the announcement took place on 2 December 2015, I define the event day as 4 December 2015, which is the day the registration statements, including the prospectus, for the bonds were published. MCD recommended that investors read the registration statements and prospectuses before they invest [44], since the documents included important information in relation to the issue which further defined the nature of reaction by market participants. Along with the debt issue MCD published a statement stating that the firm was "taking advantage of favorable conditions and rates" [34, p. 1] where they aimed at distributing \$10*bn* to shareholders before the end of 2016, with the majority of the distribution originating from the debt issue, without specifying exactly how the distribution would occur. Ratings agencies Moody's Investors Service [46] and Standard and Poor's [60] both lowered MCDs credit rating by one level after the announcement, criticising their decision to distribute cash to shareholders by issuing debt. According to Amihud and Mendelson, an increase in leverage can decrease the liquidity of a firm's stock, whereas firms with more liquid stocks usually are subject to lower leverage levels and firms that are considered liquid tend to finance themselves by issuing equity rather than debt.

5.2.3 Proxy

The $ILLIQ$ proxy is used to define the illiquidity of an asset in the L-CAPM, with illiquidity defined as "the cost of selling, broadly interpreted" [1, p. 386]. This broad assumption implies that the $ILLIQ$ proxy used in the L-CAPM model should represent selling costs, for example transaction costs such as broker fees, bid-ask spreads, market impact or search costs, as opposed to other liquidity measures such as volume-based measures, price-impact measures or market impact measures [57]. The bid-ask spread is recognised as an appropriate method of measuring the liquidity of an asset. The spread represents the difference between the buying and selling price quoted by a broker in a securities market, such as a market maker or other intermediary depending on the market setup. These quotes however only represent a buy or sale of a limited quantity of a given security, as the selling price increases and buying price decreases for high quantities, due to the higher impact of the transaction to the market. Since data on the bid-ask spread is commonly sparse, proxies such as the $ILLIQ$ proxy are often used to estimate transaction costs instead. In fact, numerous different liquidity proxies, including the $ILLIQ$ proxy, are known to be strongly positively correlated with the bid-ask spread [5], which implies that the proxy should reflect the transaction cost appropriately.

5.3 Event study setup

When conducting the event study I follow the steps previously described by Campbell, Lo and Mackinlay [28].

5.3.1 Event definition

I investigate FBs share repurchase and MCDs debt issue by analysing the impact of these corporate actions on the liquidity of their shares, through the $ILLIQ$ proxy, of their issued equities with FBs shares listed on the Nasdaq Stock Market (NASDAQ) and MCDs shares listed on the New York Stock Exchange (NYSE).

The event is defined as the announcement day of the official approvals, including all relevant documentation, from the United States Securities Exchange Commission (SEC) but I also analyse the effect of the announcements on the days surrounding the approvals. By including a prolonged event window in the assessment I attempt to identify information leakages before the event as well as minimising the ramifications of the time difference between the SEC announcement and trading periods at both NASDAQ and NYSE respectively, making it more likely that the effects of the announcements will be observed [28].

5.3.2 Selection criteria

Data on SP-500, risk free rate[1], FB and MCDs daily closing price, available in Appendix 3, was adjusted for dividends, stock splits and trading volume and collected from Thomson Reuters' Datastream for the periods 1 January 2015 to 31 December 2015 (MCD) and 1 January 2016 to 31 December 2016 (FB).The data was also adjusted to include only business days where NASDAQ and NYSE were open for trading.

5.3.3 Normal and abnormal liquidity

Andrew, Campbell and Mackinlay [28] note that standard event studies investigate the general effect of an event on a security's returns, while the event studies conducted in this thesis primarily investigate the effect on a security's liquidity, through the $ILLIQ$ proxy. While I focus on the events' effects on liquidity, I do apply the $ILLIQ$ proxy to asset pricing models to see if the liquidity changes have an effect on the firms' estimated returns. By using this methodology I attempt to capture the liquidity effect of the events separately rather than just capturing the general effect of the events on the asset prices.

I therefore slightly alter the definition on normal and abnormal returns so they refer to liquidity instead. The *normal liquidity* is the expected liquidity if the event would not occur while the *abnormal liquidity* is the observed *ex post* liquidity of the security investigated during the event window minus the normal liquidity of the security. The $ILLIQ$ proxy previously defined for the L-CAPM is used to estimate the illiquidity of the firm's stock, both in terms of normal as well as abnormal liquidity as previously described. The proxy is adjusted to represent daily data as opposed to months in the authors' empirical study of the L-CAPM.

[1]The SP-500 was used as a proxy for the market return and the 3-month US treasury bill as the risk free rate

5.3.4 Estimation procedure

While there exists no general consensus on the appropriate length of estimation windows, longer estimation windows are more likely to result in less biased estimators as possible measurement errors will be minimised with the increased observations. However, longer estimation windows are also more likely to be subject to *clustering*, where the window overlaps with other events or *unusual* market movements which could result in biased estimates of the variance of abnormal illiquidity [28]. In order to control for possible clustering effects the abnormal illiquidity measures will not be aggregated [28]. Empirical results have shown that the estimation window length has a significant effect on the magnitude of abnormal illiquidity, but that aspects in relation to market maturity, industries and country-specific as well as firm-specific unusual movement are likely to create noise during shorter estimation periods [51]. MCD is a well established *blue-chip* firm which has been listed on a public exchange since 1965 and FB is one of the biggest firms in the information-technology industry. Both firms are a part of the SP-500 index, meaning that they are leaders in their markets in terms of market capitalisation. The illiquidity of the firms is therefore estimated over a relatively long period, or 200 trading days before the event in accordance with a comparable event study by Boehmer, Masumeci and Poulsen [19].

The normal illiquidity is a simple average of daily illiquidity over the estimation window, a methodology similar to the *constant mean return model*, which uses the mean return of a security over an estimation window to estimate its expected return over the event window [28]. While this method is perhaps not very sophisticated, empirical findings suggest that, in terms of returns, the method often delivers results similar to those of more sophisticated models [25][26].

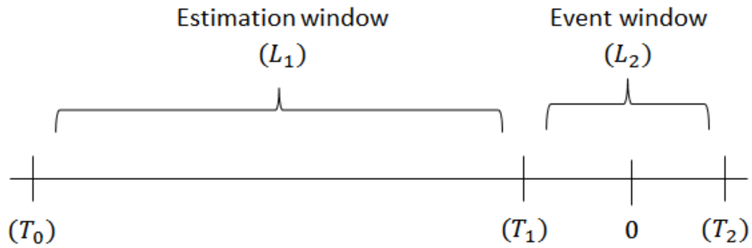

Figure 5.1: Event study timeline

Figure 5.1 shows the timeline for the event study with $\tau = 0$ defined as the event date, $L_1 = T_1 - T_0$ is the length of the estimation window where normal illiquidity of the firms is estimated and $L_2 = T_2 - T_1$ is the length of the event window where the abnormal illiquidity is measured. While the normal illiquidity is measured over a 200 day period, $L_1 = 200$, the length of the event window is 11 days, $L_2 = 11$, with 5 days before the event itself and 5 days after the event. By including 5 days before the event I mitigate the possibility of biased estimates of normal illiquidity during the estimation window:

$$ILLIQ^i_{L_1} = \frac{1}{Days^i_{L_1}} \sum_{d=1}^{Days^i_{L_1}} \frac{|R^i_d|}{VOLD^i_d}, \tag{5.1}$$

with i as one share in the stock of the firms, FB and MCD respectively, $VOLD^i_d$ as the volatility and $L_1 = T_1 - T_0$ as the 200 day estimation window with $T_0 = -206$ and $T_1 = -6$. I calculate the average illiquidity of each firm by summarising the daily return divided by the daily trading volume for day 1 to day 200, $d = [1, ..., 200]$, and dividing with the total length of the estimation window, or 200 days. The normal illiquidity amounts to 0.0382% for FB and 0.1279% for MCD respectively. The abnormal illiquidity is then calculated using the same method, but on a daily basis so that the measure can be further simplified:

$$AILLIQ^i_{L_2} = \frac{|R^i_\tau|}{VOLD^i_\tau} - ILLIQ^i_{L_1}, \text{ for } \tau \in T_1 + 1, T_1 + 2, ..., T_2, \tag{5.2}$$

where the 11 day event window is $L_2 = T_2 - T_1$. The abnormal illiquidity is found by calculating the illiquidity of the separate firms on a daily basis during the event window, L_2, and subtracting the expected illiquidity calculated in the estimation window, L_1. The abnormal illiquidity is calculated for $\tau = T_1{+}1, T_1{+}2, ..., T_2$ but when defining the empirical results I note that the event window ranges from day -5 to day 5, $\tau = [-5, ..., 5]$ since I refer to the event day as day 0, $\tau = 0$. The sample variance of the abnormal illiquidity is [59]:

$$\sigma^2 = \frac{1}{L_1 - 1} \sum_{\tau=T_1+1}^{T_2} (AILLIQ_\tau^2), \tag{5.3}$$

as dividing the total squared abnormal illiquidity by $L_1 - 1$ as opposed to L_1 "ensures that the sample variance is an unbiased estimate of the unknown population variance" [48, p. 62].

5.3.5 Testing procedure

When defining the hypotheses I acknowledge the three objectives of the event study as noted in clause 5.2.1, to investigate the effect of the FB share repurchase and MCD debt issue on the firms' stock liquidity and asset prices, how this effect differentiates during the time period in the event window and the applicability of the $ILLIQ$ proxy used in the L-CAPM to capture these effects.

To attain the first objective, I wish to enquire if investors perceive these corporate actions to be both important and relevant for the underlying liquidity of the firm. Theory indicates that more liquid firms conduct share repurchases compared to dividend payments, and an increase in debt should result in decreased liquidity over a longer horizon. However, the event study is not designed to measure this directly, but rather the direct effect of the event itself on liquidity, which will allow for speculation on the long term effect of the financial decisions on firm liquidity. It is clear that most of the corporate actions meant to impact firm liquidity come at a cost which firms have to include in their assessment when making financial management decisions [7]. In addition to direct trading costs, "open market repurchases increase the fraction of informed traders in the market...[which] increases the bid-ask spread" [17, p. 66]. Since the share repurchase and debt issue are costly options for the firms, the illiquidity of the shares should increase during the events. This increase should result in an increase in the asset's required returns since investors must be compensated for the increase in illiquidity, supported by empirical findings for share repurchases [17][21][22][23] and debt issues [42][18].

To attain the second objective, I recall the difference between Acharya and Pedersen's L-CAPM whereas "liquidity varies randomly in time" [1, p. 376] and the implications of their results that "empirically, liquidity is time-varying" [1, p. 383], an area they recommend for further research. It is perhaps no surprise that liquidity is in fact time-varying in actual markets but by performing the event study I wish to provide insight into the relationship between the time-variance of firm liquidity and market liquidity. If the relationship between the firms' liquidity and market liquidity is in fact time-varying the corporate events should be clearly conveyed into the market in an efficient manner. This assumption implies that FBs stock repurchase and MCDs debt issue should have a clear effect on the liquidity of the firms' stocks and appear effectively after the event.

To attain the third and final objective I wish to demonstrate the applicability of the $ILLIQ$ proxy when measuring firm liquidity. Various limitations of the proxy have been identified, but as it is the most popular option among investors and scholars to measure market liquidity I wish to test to which degree the proxy manages to capture the events on a firm level. Since the events should have a clear effect on the underlying liquidity of the firms, the $ILLIQ$ proxy should capture them effectively.

In light of these objectives I expect to observe positive abnormal illiquidity on event day which leads me to the definitions of the hypotheses as well as the alternative hypotheses, for FB and MCD respectively:

$$H_{01_{FB}} : AILLIQ^{FB}_{\tau=0} = 0 \tag{5.4}$$

$$H_{a1_{FB}} : AILLIQ^{FB}_{\tau=0} > 0 \tag{5.5}$$

$$H_{02_{MCD}} : AILLIQ^{MCD}_{\tau=0} = 0 \tag{5.6}$$

$$H_{a2_{MCD}} : AILLIQ^{MCD}_{\tau=0} > 0, \tag{5.7}$$

since any effect should be observable directly after the event on event day ($\tau = 0$). The hypotheses are tested separately using a one, five and ten percent significance levels. I assume a standard t-distribution so that I can form the following test statistics similarly for both firms [59]:

$$\theta_1^{FB} = \frac{AILLIQ^{FB}_{\tau=0}}{\sqrt{\sigma^2}} \tag{5.8}$$

$$\theta_2^{MCD} = \frac{AILLIQ^{MCD}_{\tau=0}}{\sqrt{\sigma^2}}, \tag{5.9}$$

which is asymptotically normally distributed with mean zero and variance of one under the null hypotheses.

5.3.6 Estimated returns

In order to be able to fully analyse the implications of the events on the liquidity of the firms I apply the $ILLIQ$ proxy to the L-CAPM model. By doing this I attempt to identify to which degree the changes in the $ILLIQ$ proxy impact the securities' required returns as defined by the L-CAPM. The estimation and testing procedure is similar to the firms' abnormal illiquidity procedure, with the abnormal returns (hereafter AR) defined similarly to Formula 3.19:

$$AR^i_\tau = E(r^i_t - r^f_{t+1}) - E(c^i_i) - \lambda\beta^{1i} - \lambda\beta^{2i} + \lambda\beta^{3i} + \lambda\beta^{4i}, \tag{5.10}$$

the sample variance in accordance with Formula 5.3:

$$\sigma^2 = \frac{1}{L_1 - 1} \sum_{\tau=T_0+1}^{T_1} (AR_\tau^2), \tag{5.11}$$

and test statistics in accordance with Formulas 5.8 and 5.9:

$$\theta_3^{FB} = \frac{AR_{\tau=0}^{FB}}{\sqrt{\sigma^2}} \tag{5.12}$$

$$\theta_4^{MCD} = \frac{AR_{\tau=0}^{MCD}}{\sqrt{\sigma^2}}, \tag{5.13}$$

which are used to test the hypotheses in accordance with Formulas 5.4 - 5.7:

$$H_{03_{FB}} : AR_{\tau=0}^{FB} = 0 \tag{5.14}$$

$$H_{a3_{FB}} : AR_{\tau=0}^{FB} > 0 \tag{5.15}$$

$$H_{04_{MCD}} : AR_{\tau=0}^{MCD} = 0 \tag{5.16}$$

$$H_{a4_{MCD}} : AR_{\tau=0}^{MCD} > 0. \tag{5.17}$$

5.4 Results

5.4.1 ILLIQ results

By calculating the abnormal illiquidity of the firms during the event window, given the normal illiquidity calculated during the 200 day estimation window, I find that the abnormal illiquidity of FB on event day is $\tau_0 = 0.0764\%$ and is significantly different from zero given a 1% significance level. I therefore strongly reject the null-hypothesis $H_{01_{FB}}$ that the abnormal illiquidity of FB is equal to zero on event day. For MCD, I find that the abnormal illiquidity on event day is $\tau_0 = 0.1907\%$, which is significantly different from zero given a 5% significance level. I therefore reject the null-hypothesis $H_{01_{MCD}}$ that the abnormal illiquidity of MCD is equal to zero on event day.

Table 5.1: Event study results

The table presents the abnormal illiquidity (AILLIQ) of the firms during the event window, where t-values are presented to the right and correspond to a one-sided test. AILLIQ coefficients significant at the 10%, 5% or 1% level are marked with one, two or three asterisks, respectively.

	Facebook		McDonald's	
τ	A-ILLIQ	t-value	A-ILLIQ	t-value
-5	0.0262%	1.0093	0.1016%	1.1902
-4	0.0149%	0.5736	-0.1201%	-1.4066*
-3	-0.0156%	-0.5995	-0.0802%	-0.9388
-2	0.0288%	1.1087	-0.0158%	-0.1855
-1	-0.0096%	-0.3714	-0.0836%	-0.9796
0	0.0764%	2.9413***	0.1907%	2.2336**
1	-0.0288%	-1.1079	-0.0885%	-1.0368
2	-0.0051%	-0.1973	-0.1083%	-1.2679
3	0.0057%	0.2212	-0.1222%	-1.4317*
4	-0.0368%	-1.4185*	-0.0273%	-0.3197
5	-0.0180%	-0.6929	0.0387%	0.4533

Table 5.1 shows that neither firm has significant results on the day after the events ($\tau = 1$). In both cases the effect of the share repurchase for FB and debt issue for MCD appears to be essentially admitted to the market on event day itself, since the results on $\tau = 0$ are significant. Interestingly, I find significant observations on a 10% significance level for both firms after the event, on $\tau = 4$ for FB and $\tau = 3$ for MCD. This can suggest that the impact of at least some of the aspects of the event are delayed and therefore not fully integrated into the $ILLIQ$ proxy on event day, albeit at a high significance level. Also, I find significant observations on a 10% significance level for MCD before the event, at $\tau = -4$. This can suggest that some degree of information leakage on MCDs debt issue occurred prior to event day.

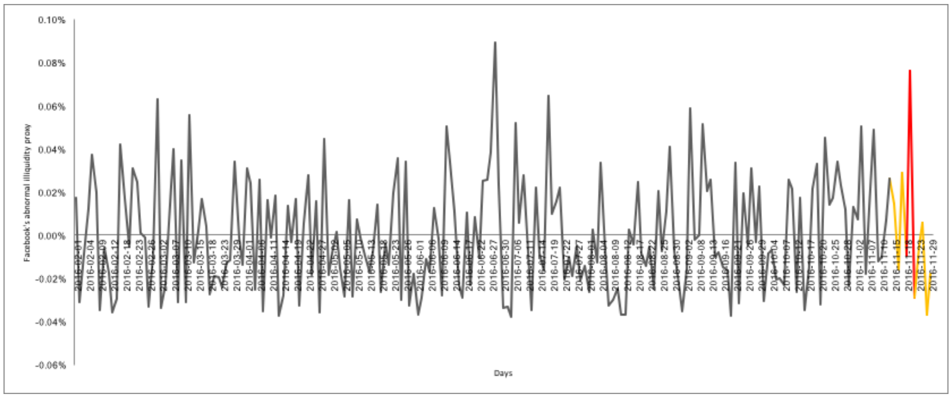

Figure 5.2: Facebook's abnormal illiquidity (AILLIQ) proxy
Facebook's abnormal illiquidity (AILLIQ) proxy over the 200 day estimation window (gray) as well as the 11 day event window, whereas the red area accounts for the event day, $\tau = 0$, and the amber area accounts for the remaining days in the event window both before and after the event.

Figure 5.2 shows the visual effect of the share repurchase on FBs $AILLIQ$ proxy on event day. The event, marked red, has the largest impact on the $AILLIQ$ proxy outside of a single event on 28 June 2016 specifically when compared to changes during the estimation window.

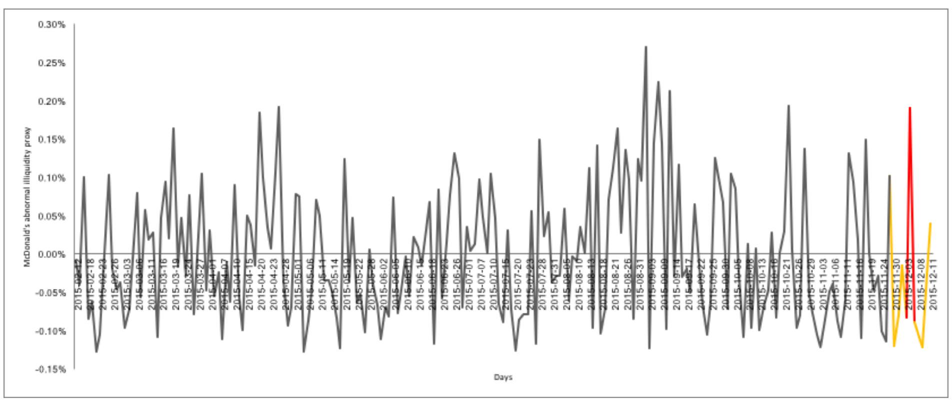

Figure 5.3: McDonald's abnormal illiquidity (AILLIQ) proxy.
McDonald's abnormal illiquidity (AILLIQ) proxy over the 200 day estimation window (gray) as well as the 11 day event window, whereas the red area accounts for the event day, $\tau = 0$, and the amber area accounts for the remaining days in the event window both before and after the event.

Figure 5.3 shows the visual effect of the debt issue on MCDs $AILLIQ$ proxy on event day.

The illiquidity effect on the event day, marked red, and the event window, market amber, appears inferior when compared to effects observed in the estimation window, specifically during end of August and beginning of September 2015.

5.4.2 L-CAPM results

By applying the $ILLIQ$ proxy to the L-CAPM model to estimate the AR of the firms during the event window, I find that the AR on event day are $\tau_0 = 3.3101\%$ for FB and $\tau_0 = 2.0859\%$ for MCD. However, neither of the results are statistically significant using standard significance levels as demonstrated in Table 5.4 for FB and Table 5.5 for MCD. I therefore fail to reject the null-hypotheses, $H_{03_{FB}}$ and $H_{04_{MCD}}$, that the AR for FB or MCD are equal to zero on event day. Also, no significant AR are observed at any time during the event window for either firm.

5.5 Robustness tests

5.5.1 ILLIQ robustness tests

For robustness testing, the event study was recreated using normal illiquidity measures over different periods, or 150, 100 and 60 [19] business days respectively. Between 60 and 200 days were selected since while Boehmer, Masumeci and Poulsen use 200 business days for estimation in their event study on event-induced variance [19], Mackinlay identifies 120 days as an example to estimate parameters [28] and Cao and Petrasek use 60 days in their event study for liquidity in stock returns [29]. Performing the event study using different estimation windows between 60 and 200 days, similar to what other empirical work has utilised, provides further confidence in the analysis.

Table 5.2: Facebook's robustness test results

The table presents the one-sided t-value tests of the abnormal illiquidity (AILLIQ) of FB during the event window using three different estimation window lengths for robustness to estimate the normal AILLIQ, or 150, 100 and 60 days. *AILLIQ* coefficients significant at the 10%, 5% or 1% level are marked with one, two or three asterisks, respectively.

	Facebook		
τ	d=150	d=100	d=60
-5	1.0219	0.8931	0.9130
-4	0.5853	0.4571	0.4621
-3	-0.5905	-0.6502	-0.7522
-2	1.1216	0.9885	1.0160
-1	-0.3620	-0.4315	-0.5161
0	2.9585***	2.7466***	2.9129***
1	-1.1002	-1.1380	-1.2785
2	-0.1875	-0.2644	-0.3359
3	0.2320	0.1370	0.0973
4	-1.4115*	-1.4359*	-1.5999*
5	-0.6842	-0.7398	-0.8489

Table 5.2 shows that FBs results weaken slightly with a 100 day estimation window, but remain significant at the 1% confidence level. The results on event day, $\tau = 0$, are significant given a 1% level and $\tau = 4$ given a 10% level in all estimation windows.

Table 5.3: McDonald's robustness test results

The table presents the one-sided t-value tests of the abnormal illiquidity (AILLIQ) of MCD during the event window using three different estimation window lengths for robustness to estimate the normal AILLIQ, or 150, 100 and 60 days. AILLIQ coefficients significant at the 10%, 5% or 1% level are marked with one, two or three asterisks, respectively.

	McDonald's		
τ	d=150	d=100	d=60
-5	1.1121	1.0354	1.1280
-4	-1.3170*	-1.3039*	-1.1905
-3	-0.8794	-0.8825	-0.7728
-2	-0.1747	-0.2039	-0.1002
-1	-0.9176	-0.9192	-0.8092
0	2.0882**	1.9753**	2.0596**
1	-0.9711	-0.9708	-0.8603
2	-1.1873	-1.1790	-1.0666
3	-1.3405*	-1.3265*	-1.2129
4	-0.3002	-0.3248	-0.2200
5	0.4228	0.3715	0.4701

Table 5.3 shows that the significance of the results for MCD decreases with shorter estimation windows. The observation on event day, $\tau = 0$, is slightly weakened but still significant given a 5% level using 200, 150, 100 and 60 day estimation windows. However, while observations on $\tau = -4$ and $\tau = 3$ are significant given a 10% level using 200, 150 and 100 day estimation windows, they are not significant using a shorter, 60 day, estimation window.

5.5.2 L-CAPM robustness tests

For robustness testing on the L-CAPMs estimated returns the $AILLIQ$ proxy was applied to the CAPM model with the purpose of comparing results from both models and seeing if the $ILLIQ$ proxy has some explanatory power for the AR of the events.

Table 5.4: Facebook's L-CAPM and CAPM comparison

The table presents FBs estimated abnormal returns (AR) and one-sided t-value tests using L-CAPM and CAPM during the event window.

	Facebook			
	L-CAPM		CAPM	
τ	A-ILLIQ	t-value	A-ILLIQ	t-value
-5	-3.7967%	-1.2522	-3.7253%	-1.3124
-4	1.1906%	0.3927	1.2441%	0.4385
-3	-1.1197%	-0.3693	-0.9914%	-0.3495
-2	0.7293%	0.2405	0.7965%	0.2808
-1	-1.0514%	-0.3467	-0.8640%	-0.3045
0	3.3101%	1.0917	3.4249%	1.2072
1	-0.6493%	-0.2141	-0.6392%	-0.2253
2	-0.9150%	-0.3018	-0.8810%	-0.3105
3	-0.8761%	-0.2889	-0.8318%	-0.2932
4	-0.4714%	-0.1555	-0.1212%	-0.0427
5	-0.0048%	-0.0016	0.0162%	0.0057

Table 5.4 shows that the CAPM estimates AR for FB of $AR_{\tau_0} = 3.4249\%$ on event day with higher significance than the L-CAPM model. However, while the CAPMs results are slightly stronger than for the L-CAPM, they are also not statistically significant using

standard significance levels.

Table 5.5: McDonald's L-CAPM and CAPM comparison
The table presents MCDs estimated abnormal returns (AR) and one-sided t-value tests using L-CAPM and CAPM during the event window.

	McDonald's			
	L-CAPM		CAPM	
τ	A-ILLIQ	t-value	A-ILLIQ	t-value
-5	0.2220%	0.0897	0.4586%	0.1828
-4	-0.2652%	-0.1072	-0.2730%	-0.1088
-3	0.0790%	0.0319	0.0725%	0.0289
-2	-0.8746%	-0.3535	-0.8196%	-0.3268
-1	-0.4242%	-0.1715	-0.4588%	-0.1829
0	2.0859%	0.8431	2.2891%	0.9126
1	-0.1001%	-0.0405	-0.0870%	-0.0347
2	-0.1580%	-0.0639	-0.1627%	-0.0649
3	-0.1724%	-0.0697	-0.1997%	-0.0796
4	0.1883%	0.0761	0.2899%	0.1156
5	-1.1771%	-0.4758	-1.1215%	-0.4471

Table 5.5 shows that the CAPM estimates AR for MCD of $AR_{\tau_0} = 2.2891\%$ on event day with higher significance than the L-CAPM model. However, while the CAPMs results are slightly stronger than for the L-CAPM they are also not statistically significant using standard significance levels.

5.6 Summary

FBs share repurchase results in significant abnormal illiquidity of $\tau_0 = 0.0764\%$, as measured by the $AILLIQ$ proxy, and AR of 3.3101% on event day, as estimated by the L-CAPM. MCDs debt issue results in significant abnormal illiquidity of $\tau_0 = 0.1907\%$, as measured by the $AILLIQ$ proxy, and AR of 2.0859%. For comparison, actual AR amounted to 3.5189% for FB and 2.2280% for MCD on event day respectively. However, neither of the estimated AR are significant using standard significance levels, and the significance of the results were higher for the CAPM than the L-CAPM. No significant AR were estimated during the event window for either firm, neither by using the CAPM nor L-CAPM. Also, weak evidence is found for abnormal illiquidity before MCDs debt issue as well as after the events for both firms, but robustness tests show that the $AILLIQ$ proxy is reliant on proper estimation window lengths used for estimating the average $ILLIQ$ of both firms.

6

Discussion

The general objective of the thesis is to investigate the associations of market and firm liquidity on the L-CAPM in order to provide better insight into how firm liquidity affects asset pricing as seen through commonly used asset pricing models. In order to do so I have conducted an event study analysing the effects of two time-varying corporate events on the ILLIQ proxy and applied it to the L-CAPM. I discuss the implications of the event study findings in the following chapter, building on my previous analysis of L-CAPMs implications, limitations and empirical findings, formal definitions of market and firm liquidity, sources of market illiquidity and corporate actions that impact firm liquidity.

The findings imply that FBs share repurchase and MCDs debt issue have a strong abnormal illiquidity increasing effect on the firms' shares on event day which is captured by the $AILLIQ$ proxy. Weak evidence is also found for increases in abnormal illiquidity after the events for both firms as well as before MCDs debt issue, suggesting possible information leakages and insufficient implementation of new information into the market. This suggests that it is important to include effects of the time-variance of liquidity when estimating asset prices, since liquidity is likely to change over time depending on new and private information entering the market. The events also have an effect on the observed AR of the firms. However, the increase in firm illiquidity is not directly transformed into estimated AR for either firm, since significant AR using the L-CAPM can not be observed during the event window. Interestingly, the CAPM slightly outperforms the L-CAPM for both firms in terms of statistical significance, which suggests that the firms' liquidity levels and risks do not offer adequate explanatory value for the pricing of the individual securities. However, the CAPMs estimation is also not statistically significant using traditional significance levels. While the event study is obviously simplistic and has limitations, the findings suggest that asset pricing models frequently used by investors, such as the L-CAPM, do not effectively capture events that impact firm liquidity. This further suggests that future research is needed on illiquidity proxies that efficiently measure firm liquidity which can be used to improve asset pricing models so that they are more accessible to capture at least some of the time-varying effects of illiquidity. In practice, investors need to be aware of a firm's corporate strategy and setup in order to gain sufficient insight into the value of a firm and how it transcribes into its market valuation. This is likely to mitigate the risk of investors reaching biased asset prices using an asset pricing model based solely on market liquidity which is unlikely to include all aspects and implications of firm liquidity.

6.1 Implications for the ILLIQ proxy

The $ILLIQ$ proxy measures a stock's daily ratio of absolute return with its trading volume, which can be interpreted as the stock's price response associated with changes in its trading volume. The intuition behind the proxy is that a stock is less liquid if a change in its trading volume generates an even larger change in its returns. For example, if a stock's trading increases while its return remains unchanged, the illiquidity of the stock will decrease. In line with this design, the proxy efficiently captures the price impact of the events in question, since both events result in both higher returns and increases in trading volume, resulting in a higher $ILLIQ$ proxy value.

The $ILLIQ$ proxy's reliance on data that is readily available in common securities' markets over a prolonged period, a stock's return and trading volume, has helped it become one of the most popular choices of liquidity measures [43] since its presentation. However, the simplistic nature of the proxy which enables its renowned accessibility does not allow for particularly intricate interpretation of the underlying economics behind the events. According to Amihud and Mendelson's original hypothesis, backed by empirical findings, returns increase in illiquidity [3] since investors need to be compensated for higher transaction costs. Amihud also suggests that the same hypothesis must apply to excess returns, as they partly represent an illiquidity premium [2]. The limitations of the $ILLIQ$ proxy make it difficult to investigate this well-recorded correlation between illiquidity and returns without applying it to a pricing model. Interestingly, there is a large difference between the significance of the L-CAPMs findings and the findings of the $AILLIQ$ proxy, since the model does not estimate significant AR on event day while the $AILLIQ$ proxy's findings are clearly significant. This may suggest that while the events have a clear price impact on the shares' illiquidity levels, that impact is not convincingly transcribed into the actual price estimation of the securities.

The significant $AILLIQ$ proxy findings for both firms after the event suggest that some of the information might have been infused into the market outside of the event day itself. Investors are subject to time difference which might affect when they receive the information and invest in the market [28], which might create a lag as information is not fully incorporated into the market as soon as possible after the event. However, the findings occur on event day 3 for MCD and event day 4 for FB, which can not be explained by the simple time difference between markets. A possible explanation for this might be that further analysis of the events might prompt an increase in buys or sales for investors, triggering an increase in the stocks' return and trading volume which is then captured by the $AILLIQ$ proxy. However, given no estimated AR, the $AILLIQ$ proxy's high significance level of 10% and the lack of significance of the $AILLIQ$ proxy findings for MCD after the robustness tests, the findings are weak and only allow for inconclusive speculation as opposed to concrete results.

The significant $AILLIQ$ proxy findings for MCD before the event suggests possible information leaks in relation to the event, where investors or insiders can act upon the information on the debt issue before it is announced to the market. However, as with the findings after both events, the high significance level of 10% and poor robustness test performance suggest that the findings are rather weak and do not offer sufficient basis for conclusive interpretation of the results. However, while inconclusive, the significant findings before MCDs debt issue as well as after both events, along with the clear impact of the events on event day, further suggest that firm liquidity is very much reliant on corporate information and variant through time.

While the $AILLIQ$ proxy captures the effects of the events on $\tau = 0$, the proxy displays its fragility when faced with short estimation windows for MCD. The proxy provides significant, albeit slightly weaker, results on event day given shorter estimation windows for both firms as well as after FBs share repurchase. However, the sudden lack of significant results for MCD before and after the event day, at $\tau = -4$ and $\tau = 3$, using a 60 day estimation window, are not supportive of the results using longer estimation windows. This fragility suggests that the $AILLIQ$ proxy is reliant on the estimation method of normal illiquidity and can provide conflicting results using inappropriate estimation windows.

In general terms, the L-CAPMs theoretical setup implies that an asset's expected return depends on both its liquidity level and liquidity risk, which is supported by the authors' previous empirical findings, with the estimated combined effect of both aspects being 4.6% in terms of expected returns [1]. However, the model does not attempt to differentiate between the underlying sources of liquidity, but rather to capture its combined effect as demonstrated by the authors' general interpretation that illiquidity costs broadly represent transaction costs. This broad interpretation plays a key role in defining one of the main aspects of the model, the liquidity risks, through the levels of covariances between an asset's and the market's returns and general liquidity levels. The main sources of illiquidity, exogenous transaction costs, demand pressure and inventory risk, asymmetric information and search frictions [9], have highly contrasting characteristics which suggests that their effects on liquidity are distinctive in nature. None of these sources explicitly include a financial manager's decision making as a possible influence on liquidity. The event study findings demonstrate how changes in firm liquidity are only partly captured by the model, possibly due to the aforementioned issues. The event study findings are in line with significant empirical findings, that "some corporate events may change the liquidity of the firm's stock and, if liquidity is priced, they can affect the stock price" [9, p. 320]. However, due to the simplicity of the $ILLIQ$ proxy it is difficult to define the definitive interconnections between the different sources of illiquidity and firm liquidity in order to map how they are linked to a firm's complete financial strategy and interact with market liquidity.

6.2 Implications for the L-CAPM

The L-CAPM does convert the increase of abnormal illiquidity into an estimated increase in AR. This is in line with the observed impact of the event, with the actual AR for the firms being 3.5189% for FB and 2.2280% for MCD. However, the L-CAPMs results are not significant, with p-values over the 10% significance level for both firms. Also, the CAPM slightly outperforms the L-CAPM when estimating AR for both firms. However, while slightly stronger, the CAPMs results are not statistically significant either, using a 10% significance level. The strong capture of the events by the $AILLIQ$ proxy, the lack of significance of L-CAPMs results, the actual increase in returns and the slight outperformance by the CAPM suggests that the L-CAPM does not sufficiently capture the liquidity effects of individual corporate events in its asset pricing methodology. This brings to attention the authors' decision to measure the effects of liquidity over a long horizon on a portfolio level.

Due to the noisiness of the $ILLIQ$ proxy, it is customary to investigate empirical correlations between portfolios, as opposed to individual firms, when pricing assets. This is specifically noted by Acharya and Pedersen in their empirical analysis of the L-CAPM, where "the noise is reduced by considering portfolios rather than individual stocks" [1, p. 387], an opinion maintained by Pedersen during a personal email correspondence with me where he stated that the "ILLIQ is noisy even at the level of the whole market and super noisy at the level of each individual stock. Most if not all measures of liquidity are imperfect." [53, p. 1]. This methodology however is not necessarily likely to lead to smaller standard errors for beta estimates, and the aggregation of stocks into portfolios can "lose information by reducing the cross-sectional dispersion of the betas" [14, p. 32]. By investigating portfolios rather than individual firms it is fair to consider the possibility that scholars and investors might ignore corporate events which are directly linked to the liquidity of a firm's issued securities, since its effects are bound to be stifled by the remaining assets in the portfolio. The L-CAPMs empirical findings are implicitly composed of the general concept of market liquidity since the ILLIQ proxy does not explicitly include effects of the underlying sources of market liquidity on asset prices, including firm liquidity, since portfolios were used as opposed to individual securities. Also, the model's estimation of liquidity over such a long horizon provides a static insight into the associations of liquidity on asset prices and does not provide sufficiently meaningful insight into the time-variance of liquidity.

This methodology exposes two major components of the L-CAPM that are considered in the event study setup. The event study demonstrates that the $AILLIQ$ proxy is in fact capable of capturing the price impact of individual corporate events which are largely variant through time. This would have proved extremely difficult if the firms' shares were being estimated as part of a portfolio rather than individual shares. The effect of the increase in illiquidity on asset returns is not captured by the L-CAPM which raises concerns over investors' use of asset pricing models. This knowledge might provide investors with the possibility of optimising their investment strategies in the future in order to capture attractive liquidity effects of corporate events, for example by using asset pricing models modified to reflect underlying aspects of firm liquidity or by using proxies more suited to measuring the time-variance of liquidity.

The authors use a common methodology of setting up portfolios as opposed to individual firms due to noisiness when estimating asset returns [1]. When interpreting the results, the portfolio returns are then used as a sort of proxy for the returns of individual assets. In effect, the empirical tests performed on portfolios are used to interpret effects and implications for individual stocks. For example, one of the authors' main findings when testing the L-CAPM was that "illiquid stocks also have high liquidity risk" [1, 391]. This finding is based on the performance of portfolios consisting of illiquid individual stocks and is later used to signify effects for each of the individual stocks as opposed to general findings applied to the portfolio as a unit. This can clearly be seen when the authors further explain the above mentioned findings that "a stock, which is illiquid in absolute terms..., also tends to have a lot of commonality in liquidity with the market..." [1, 392]. This clearly shows that the authors apply portfolio findings to individual stocks. However, the interrelationship between the two is not necessarily this clear-cut. In the authors' case, where the general role of liquidity is investigated over a long horizon, it is fully possible that the findings can be applied to both portfolios, representing a number of firms with issued securities that have similar characteristics operating in a market, and the individual underlying firms. However, I argue that when effects of financial management and the time-variance of liquidity are included, the relationship becomes more convoluted as suggested by the event study findings.

The act of considering portfolios rather than individual assets might also actually increase the illiquidity costs for investors. According to the acclaimed Modern Portfolio Theory (MPT), investors should diversify their assets into portfolios in order to maximise their expected returns given a minimised level of risk [45]. While the MPT applies for risk it does not necessarily apply for liquidity costs. Investors have limited options to avoid liquidity costs related to assets and are unable to eliminate or minimise them by building portfolios. Rather, building portfolios with a high number of assets would increase the liquidity cost of the portfolio since they add up for each asset and do not cancel each other out [5]. As shown in Formula 3.28 and in accordance with standard procedure, the L-CAPM weighs an asset's relative liquidity cost when setting up portfolios, similar as when defining portfolio returns. While theoretically correct, this methodology does not adequately reflect the additive nature of liquidity costs [5] and is likely to underestimate the effect of such costs on portfolios.

These findings also have direct implications for investors using the L-CAPM for asset pricing. It is likely that using the L-CAPM for portfolio optimisation is more suitable for long term investment strategies focusing on portfolios in markets where liquidity costs are low. This is supported by the author's findings for the L-CAPM where they assess the effect of liquidity on portfolio prices rather than asset prices over a period of months. Also, if the liquidity costs are low the effect of combining assets into a portfolio is minimal. For investors hoping to create *alpha* by investing actively in individual assets in markets where liquidity costs are high and vary a lot over time the L-CAPM might not provide appropriate estimation of the effect of liquidity costs on the assets' prices. This is supported by the event study findings which show that the ILLIQ proxy manages to capture the clear price impact of the corporate actions but the increase in illiquidity does not translate into abnormal returns when using the L-CAPM.

6.3 Economic interpretation of the event study findings

Amihud and Mendelson's financial management theory [7] states that more liquid firms tend to partake in share repurchases rather than dividend payments [23]. The event study does not attempt to directly engage that statement but rather the intuition behind it, demonstrating that the corporate events have a clear effect on firm liquidity and do result in increased AR. However, this increase in abnormal illiquidity is not sufficiently captured by the L-CAPM or CAPM when estimating returns.

The positive nature of FBs announcement to repurchase shares is likely to interest investors who then wish to purchase more shares in the firm. This increase in demand then pushes the price of the stock upwards, resulting in increased returns. Since the price increases more than the stock's trading volume, the illiquidity increases, because investors must pay a premium in order to enter buys. A similar interpretation can be applied to MCDs debt issue, but it is likely that the event has both positive and negative implications for the stock's long term illiquidity and return. In principal, increased leverage should increase the risk of a firm's equity and therefore increase a stock's illiquidity [7][18]. However, MCD argued that the firm was simply utilising favorable conditions for debt financing at the time of the announcement and that its cash flows were sufficiently strong to cover obligations obtained by the increased debt. MCD also announced its plan to return $\$10bn$, much of it backed by the newly issued debt, to its investors in the coming year after the event [34]. It is therefore likely that the event's long term effect on illiquidity is somewhat affected by both negative implications of the increased debt as well as the positive implications of increased payments to shareholders, which can not be separated using the $ILLIQ$ proxy.

Most importantly, the changes to MCDs capital structure are also likely to impact its equity cost of capital. The added debt increases the risk of its equity which makes the firm more susceptible to new information, creating a misalignment between informed and less-informed investors. This increased asymmetry can lead to both ”wider bid-ask spreads and higher price-impact costs” [7, p. 23]. MCDs debt issue is therefore likely to have negative long-term effects on its firm liquidity.

6.4 Limitations

The general limitation of biases when reporting illiquid asset returns applies to the event study as the methodology used does not attempt to control for them. In general terms, ”reported illiquid asset returns are not returns” [13, p. 9] since survivorship and selection bias as well as infrequent sampling can lead to inadequate estimation of illiquidity when estimating returns. The event studies use stock data from two firms that operate in liquid equities' markets in the US, which minimises the impact of survivorship and selection bias. However, I do not attempt to account for the frequency of trading, as the authors do when measuring illiquidity innovations noted in Formula 3.34 or by desmoothing observed returns as suggested by Ang [13].

The study measures two specific events for two individual firms over a relatively short time period which does not provide an inadequate estimation of the average illiquidity of the firms over a longer horizon. This simplistic estimation is suitable to measure the price impact of the events but it does not provide definitive reasoning supporting Amihud and Mendelson's Financial Management theory. In order to do so, a more sophisticated event study would have to be considered, which empirically compares the differences between the liquidity levels of individual firms conducting either dividend payments or share repurchases, while simultaneously controlling for various factors such as time-frames and market area that might bias the study. Therefore, the conclusions drawn from the event study findings are narrow and can only be attributed to the theory of liquidity and asset pricing in order to speculatively discuss the events' long term implications for the firms' comparable liquidity levels and prices.

While the $ILLIQ$ proxy manages to clearly capture the price impact of the events, robustness tests using the CAPM compared to the L-CAPM do not show significant estimates of AR for either firm. Robustness tests also demonstrate that the proxy is suscept to the estimation methodology used to estimate average illiquidity. The significant effects observed outside of the event day are too weak to justify anything outside a speculative discussion on a possible information leakage prior to MCDs debt issue and slow integration of information for both firms after the event.

It is also important to note that while measures on market illiquidity are notoriously noisy, research on possible measures for firm illiquidity are limited. Financial statement analysis, such as financial ratios, are of course not directly applicable to firm liquidity but might capture effects of some corporate actions that have an indirect effect on a firm's balance sheet. However, these measures are rather naive and backwards looking. Due to these limitations it is difficult to identify other measures than market liquidity proxies to correctly reflect changes in firm liquidity which can be used in asset pricing models. This represses the possibilities of the event study methodology when used to measure firm liquidity.

7

Conclusion

FBs share repurchase and MCDs debt issue have a significant effect on the firms' abnormal illiquidity and increase their abnormal stock returns on event day. The $ILLIQ$ proxy efficiently measures the price impact of the events, but is susceptible to precise estimations of normal illiquidity in order to provide coherent estimates. The firms' illiquidity is far from static and in fact largely variant throughout the event window, but only provides weak evidence of information leakages and delayed incorporation of information into the market.

The L-CAPM fails to both capture the event and outperform the CAPM when estimating AR. However, neither model manages to efficiently estimate the events' impact since their findings are not significant. L-CAPMs failure to outperform the CAPM suggests that the model's inclusion of liquidity costs and liquidity risks does not have explanatory power on asset prices, when measuring time-varying as opposed to static liquidity, and firm liquidity as opposed to market liquidity. This further suggests that asset pricing models struggle to capture the liquidity effects of corporate events on returns, which can be due to many reasons, for example the inherent setup of the L-CAPM, including its focal point on market liquidity and aggregation of shares into portfolios.

To summarise, the findings demonstrate that time-varying corporate events have a clear price impact on firm liquidity which are not captured by commonly used asset pricing models. This suggests a sort of *disconnection* between aspects related to underlying time-variant firm liquidity and asset pricing models, possibly since models such as the L-CAPM estimate the effect of static market liquidity on portfolio prices with less focus on the aforementioned firm liquidity aspects. Complete trust should therefore not be given to the L-CAPM when pricing assets while it remains subject to misalignment between market and firm liquidity aspects. Instead, the model should be used as one of many inputs into a more coherent price valuation of an asset. When estimating asset prices, investors must also be aware of the corporate strategy of the firm they are investing in. This includes aspects such as a firm's business plan, future developments, market segmentation and industry developments, which are likely to impact firm liquidity. Only by analysing a firm's corporate strategy can an investor put its corporate events into perspective, which is likely to aid his estimation of the firm's value based on standard asset pricing models such as the L-CAPM.

Future research is needed on illiquidity proxies that efficiently measure firm liquidity which can be used to improve asset pricing models so that they are able to capture at least some of the time varying effects of illiquidity. Given sufficient data quality, further investigation of the relationship between firm liquidity and asset prices using the L-CAPM and corporate events that can affect firm liquidity other than those used in this event study, such as information disclosures and investor base increases, would provide a more thorough base for the subject of this thesis. Research focusing on the modification of the L-CAPM in order to measure time-variant liquidity as opposed to static liquidity might provide more appropriate estimates of asset prices. An example of this is an estimation of individual firm stock prices using the L-CAPM and intra-day data, which has been made possible in the last decade with the increase of technological capabilities and data availability. Also, performing a similar event study on portfolios and comparing it to the illiquidity measures of the portfolios' underlying assets could provide additional insight into the possible additive nature of liquidity costs and provide great value to investors utilising liquidity aspects in their investment strategies.

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Appendices

Appendix 1

Table A.1: Portfolio weights

The table shows a summary of the weights of each portfolio, for both illiquidity portfolios as well as the market portfolio.

Portfolio #	Constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
Illiquidity portfolios								
1	-0.391	0.046					1.115	0.825
	(-.889)	(-)					(1.997)	(0.825)
2	-0.299	0.062					0.996	0.846
	(-0.737)	(3.878)					(4.848)	(0.832)
3	-0.530		1.374					0.350
	(-1.082)		(2.085)					(0.322)
4	-0.088	0.046	-2.699				3.395	0.879
	(-0.249)	(-)	(-1.441)				(1.782)	(0.873)
5	0.105	0.008	-6.392				6.800	0.901
	(0.296)	(0.318)	(-2.238)				(2.427)	(0.886)
6	0.143		-7.115				7.467	0.900
	(0.397)		(-3.623)				(3.871)	(0.891)
7	-0.132	0.046	1.568	-141.416	47.823	-12.784		0.911
	(-0.633)	(-)	(1.295)	(-1.032)	(0.469)	(-1.553)		(0.898)
8	-0.053	0.117	1.207	-346.547	33.043	-17.356		0.913
	(-0.060)	(0.837)	(0.343)	(-0.796)	(0.186)	(-0.981)		(0.890)
σ (illiquidity) portfolios								
1	-1.938	0.034					2.495	0.486
	(-1.203)	(-)					(1.627)	(0.486)
2	-2.059	0.081					2.556	0.642
	(-1.755)	(2.755)					(2.107)	(0.609)
3	0.700		0.062					0.000
	(0.272)		(0.025)					(-0.043)
4	-1.536	0.034	-6.070				8.099	0.754
	(-2.033)	(-)	(-1.540)				(2.040)	(0.743)
5	-0.583	-0.076	-16.226				17.333	0.841
	(-0.718)	(-0.902)	(-2.978)				(3.543)	(0.819)
6	-1.241		-9.210				10.954	0.800
	(-1.271)		(-2.733)				(3.183)	(0.781)
7	-0.301	0.034	0.363	-4494.924	-370.840	-26.044		0.850
	(-0.285)	(-)	(0.268)	(-1.060)	(-0.806)	(-1.366)		(0.828)
8	0.039	-0.056	0.015	-116.450	-405.451	-13.135		0.865
	(0.031)	(-0.140)	(0.007)	(-0.010)	(-0.413)	(-0.270)		(0.829)

Appendix 2

Campbell, Lo and MacKinlay's recommended seven step methodology when conducting an event study [28].

1. **Event definition**, where the period relevant for the occurrence of the event is chosen. This window can vary between intraday and interday periods depending on the type of event.

2. **Selection criteria**, where the data used for the event study is selected and possible limitations are identified. The data should represent the firm or industry of research based on its criteria, such as market capitalisation or industry representations.

3. **Normal and abnormal return**, where the *normal* return is the "return that would be expected if the event did not take place" [28, p. 151] and the *abnormal* return is the "actual *ex post* return of the security over the event window minus the normal return of the firm over the event window" [28, p. 151].

4. **Estimation procedure**, where most often the period prior to the event window is chosen in order to estimate parameters in a model used to measure the effect of an event. The event window must not overlap with the estimation window in order to not risk biased estimates.

5. **Testing procedure**, where the testing framework for the abnormal returns is defined, such as null-hypotheses and techniques used for aggregating the abnormal returns, if applicable.

6. **Empirical results**, where the numerical and statistical results of the event study are clearly presented.

7. **Interpretation and conclusions**, where the numerical and statistical results are interpreted in order to provide meaningful facts, actionable insights and valuable

conclusions.

Appendix 3

See xlsx file ”Event study final.xlsx” uploaded separately.

Blue sheet is the final table output used in the thesis. Green sheets are event studies and robustness tests for FB used in the thesis. Orange sheets are event studies and robustness tests for MCD used in the thesis. Red sheets are massaged data input sheets for FB, MCD, S&P-500 and the risk free rate. Black sheets are raw data inputs sheets.