11th of September

2017

A comparative analysis of the one-factor Gaussian copula and the onefactor Student t copula applied to synthetic CDO valuation

- A financial crisis perspective



Master's Thesis - Cand. merc. Finance and Investments

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Character Count: 169,899 / 80 pages

Advisor: Søren Plesner



Executive summary

In the aftermath of the financial crisis of 2007-2009, the Gaussian copula has received a lot of negative attention concerning its role in the valuation process of Collateralized Debt Obligations (CDOs). Specifically, by generating the portfolio loss distribution, the application of this model allowed its users to quantify the risk in the portfolio of assets underlying the CDO. However, because of its underlying distributional assumption, the Gaussian copula critically underestimates the probability of many simultaneous defaults, and thus the risk associated with the most senior CDO tranches.

This Master's Thesis considers the use of the one-factor Gaussian copula applied to synthetic CDO valuation, and compares it to an alternative represented by the one-factor Student t copula. Through a semi-analytical implementation of the one-factor Gaussian copula, and a Monte Carlo implementation of the one-factor Student t copula, this thesis finds that the Student t copula, due to its fatter-tailed nature, distributes the relative risk between tranches somewhat differently. In general terms, the one-factor Student t copula assigns more risk to the most senior tranche in the CDO and less risk to the equity tranche. In spite of this difference, no immediate improvement is obtained in relation to matching the observed tranched ITraxx Europe quotes, and thus in the ability to simultaneously price all synthetic CDO tranches correctly.

Through a thorough sensitivity analysis of both models, the researcher finds it unlikely that the specifics of the Gaussian copula played a significant role in the escalation of the financial crisis. The similarities between the behavior of the two models under varying input assumptions indicate that no major improvement would have been obtained, had the one-factor Student t copula been applied as the market-accepted valuation model. Instead, it appears that other underlying factors within the financial industry were the main contributors to the outbreak of the crisis, and that math represented by the Gaussian copula, was used as an excuse to justify some of the unhealthy and immoral behavior that occurred across the financial industry. This, both in relation to some very concerning moral hazard problems within financial institutions, the inevitable conflict of interest that exists in the business model of the credit rating agencies, and the general overconfidence, which dominated across the financial industry during this period.

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1. Introduction

Approaching the 10-year anniversary of the Global Financial Crisis ("the crisis" or "the financial crisis"), it has become clear to most that asset securitization played a significant role in the escalation of what many consider the worst financial crisis since the Great Depression (Stewart, 2008). In its most general form, asset securitization allows the originator to transfer risk from its own balance sheet to another counterparty, which is done by pooling the desired assets and selling them off to a so-called Special Purpose Vehicle (Jobst, 2008). In isolation, such a transaction does not appear to be of particularly harmful nature. However, as it is often the case in today's world, one cannot consider such an instrument to be independent from the environment in which it is applied.

The years leading up to the outbreak of the crisis were dominated by a heavy increase in both the dollar amount of outstanding securities as well as the complexity of the instruments traded in the market. One of the most notorious instruments within this category was, and still is, collateralized debt obligations, usually referred to as CDOs. A CDO consists of a number of securities with different levels of seniority and corresponding spreads, all backed by the cash flows from the same underlying reference portfolio of debt instruments. The cash flows generated from the reference portfolio are then distributed to CDO investors, based on the seniority of their claims (Bomfim, 2005). Following the crisis, headlines like "The CDOs That Destroyed AIG" (Fiderer, 2011) and "A Wall Street Invention Let the Crisis Mutate" (Nocera, 2010) have painted a picture of CDOs as one of the main contributors to the escalation of the financial crisis. Articles like the above, typically refer to the so-called synthetic CDOs, which are products that are similar to the cash CDOs described above, the only difference being that the reference portfolio consists of credit default swaps (Gibson, 2004). As payments to CDO investors depend on the cash flows generated from the reference portfolio, they need to consider the probability that some of the reference entities default on their payments, and even more importantly, the probability that several of the reference entities default simultaneously. In other words, a well-prepared synthetic CDO investor should be aware of the default correlation between the assets in the reference portfolio, and thus the corresponding loss distribution function (Bomfim, 2005).

A natural consequence of increasing complexity in the products traded in the financial markets is an increase in the complexity of the models used to value them. In a study from 2014, Donald MacKenzie and Taylor Spears investigate the use of the Gaussian copula model for CDO valuation in investment banks and credit rating agencies. Here they find that a specific semi-analytical version of this model was accepted as the general market model in derivatives departments in investment banks, as well as in all major credit rating agencies. However, Master's Thesis

following the crisis, this model has been criticized heavily by several people in and around the financial industry, some even going as far naming it *"The formula that killed Wall Street"* (Salmon, 2012). As indicated by the name, the Gaussian copula is based on the well-known Gaussian distribution, which as it turns out, can be said to be both its greatest strength and weakness. This assumption of multivariate Gaussianity made the model appealing to practitioners, which in turn facilitated the adoption of the model across the financial industry (MacKenzie & Spears, 2014). However, in retrospect the same assumption has been subject to enormous amounts of criticism, mainly evolving around the fact that this assumption means that the Gaussian copula completely ignores tail dependence. In non-statistical terms, this means that the model does not account for dependence between extreme values, i.e. the possibility that large crashes, or peaks, can be correlated across assets, which is in fact often indicated from empirical data (Oh & Patton, 2015). Considering the events that occurred around the financial crisis, this characteristic appears particularly unfortunate, as the instability around the crisis meant that the ability to model tail dependence turned out to be highly relevant.

This thesis sets out to investigate the role of copula models in the financial crisis, with a specific focus on the use of the Gaussian copula and Student t copula in synthetic CDO valuation. The investigation will evolve around some of the short-comings of the Gaussian copula model, and the potential implications the use of this model had on the escalation of the financial crisis. Furthermore, the application of the Gaussian copula will be put in perspective to the potential improvements in synthetic CDO valuation, which could have been achieved had the Student t copula model been applied instead. In other words, the author of this thesis poses the question of whether the financial crisis would have developed any differently, had the Student t copula model been applied to synthetic CDO valuation instead of the Gaussian copula. The thesis seeks to answer this question through a model specific investigation of the relevant models, and considers the findings of this analysis in relation to the financial crisis.

1.1 Research question

The purpose of this Master's Thesis is to investigate the role of the Gaussian copula model in the financial crisis. The thesis is focused around on a comparative analysis of the performance of the Gaussian copula and the Student t copula, and furthermore aims to consider the potential implications of using one opposed to the other in relation to the development of the financial crisis. Conducting this investigation, the aim is to answer the following research question: What were the implications of the use of the Gaussian copula model in synthetic CDO valuation for the escalation of the financial crisis, and would anything have been different had the Student t copula been applied instead?

In answering this question, the author seeks to answer the following sub-questions:

- How is the Gaussian copula model applied to synthetic CDO valuation and how does it perform compared to market prices?
- What are the short-comings of the Gaussian copula when applied to synthetic CDO valuation, and what implications could these short-comings have for investors?
- How can the Student t copula model be applied to synthetic CDO valuation, and how does it perform compared to the Gaussian copula in relation to market prices?
- Could using the Student t copula instead of the Gaussian copula for synthetic CDO valuation have changed anything in the development of the financial crisis?

These questions will be answered through a theoretical analysis of the relevant models, as well as an empirical model-oriented analysis of the performance of both models compared to market data at different points in time. However, as the primary focus is to investigate the difference between the characteristics of the two models, the empirical data is primarily used to illustrate model behavior under various assumptions. The empirical analysis of this thesis will be conducted on multiple series of the tranched ITraxx Europe with 5-year maturity on data points representing different market conditions.

1.2 Structure

The thesis is structured in the following way. Chapter 1 lays out the problem statement and accounts for the limitations of the thesis. Moreover, chapter 1 also introduces the market for credit derivatives, as financial instruments within this market will be the focus moving forward. Chapter 2 provides a theoretical introduction to the components of synthetic CDO valuation. This includes the main approaches to credit risk modelling, and modelling of the loss distribution using copula theory. Chapter 2 also gives an overview of relevant existing literature within copula theory and CDO valuation. Chapter 3 introduces CDS indices and the selected empirical data, which lays the foundation for the model-oriented analysis conducted in chapter 4. Furthermore, chapter 3 accounts for the general research approach applied in this thesis.

In chapter 4, the introduced one-factor Gaussian and one-factor Student t copula models are applied to synthetic CDO valuation, and a sensitivity analysis is conducted on the results found using the two model approaches. The estimated tranche spreads computed under various input assumptions are then compared and used to infer if any improvement can be achieved, when using the Student t copula model as an alternative to the Gaussian. Chapter 5 contains a discussion on the results found in chapter 4, as well as a discussion on other factors that might have influenced in the escalation of the financial crisis. Finally, chapter 6 concludes and answers the overall research question, and chapter 7 takes a brief look ahead.

1.3 Delimitations

When investigating a topic as wide and complex as a global financial crisis, many factors can undoubtedly be examined from a variety of different angles and perspectives. Consequently, a clear delimitation of the areas covered within this thesis is important to secure a match between the expectations of the readers, and the intentions of the author.

As stated in the problem statement presented in section 1.2, this thesis takes a very narrow starting point in the Gaussian copula, and the implications of its use for synthetic CDO valuation, and compares this with the potential alternative in the Student t copula. This narrow focus has been consciously chosen, acknowledging that many other interesting copula model alternatives exist in the literature. These models, however, will only be briefly touched upon as a detailed discussion is outside the scope of this thesis. This delimitation includes other families of copula models as well as extensions to the models considered, including stochastic input variables such as those related to stochastic correlation, stochastic recovery rate, or stochastic interest rates.

The analysis conducted in the thesis, should not be considered a classical empirical analysis, since the purpose is not to prove certain phenomena in the data. Rather the focus of the analysis is to illustrate the behavior and the characteristics of the models considered, and discuss these in the context of the crisis. Consequently, the quality and the specifics of the applied data is of lower importance than it would be in the case in a classic empirical analysis, as the data only functions as a means towards understanding the models. For comparability purposes, a maturity of 5 years is assumed for all calculations and model quotes. Only the specific use of copula models for synthetic CDO valuation is considered in this thesis. Due to the complexity of the models included, a certain level of mathematical sophistication is required to understand both their characteristics and implementation. However, the full mathematical derivations underlying the models are outside the scope of this thesis, as the purpose is not to contribute with mathematical advances within the field, but more so to investigate the use and practical implementation of the models in relation to the crisis. Thus, the thesis will only introduce mathematical content to the extent that is required to understand the characteristics of the models and their implementation.

As mentioned, a very narrow focus on the two mentioned copula models has been chosen in this thesis. However, the author acknowledges that many other factors did undoubtedly also contribute to the escalation of the financial crisis. No detailed account of all these factors will, however, be given here, as this is simply not possible within the scope of this thesis. However, some of them will be discussed in relation to the results obtained through the model analysis, as this will provide a better understanding of the role of the models in relation to other factors.

Regulatory details concerning defaults, issuance of credit derivatives, as well as taxations issues related to credit derivatives will not be included in this thesis.

1.4 The market for credit derivatives

To properly set the scene for the remainder of the thesis, it is necessary to introduce the context in which it is written. Thus, the following section will introduce the market for credit derivatives, and outline the development this market has undergone in recent decades. Of the many different derivatives traded in this market, two of the most well-known are credit default swaps (CDS) and collateralized debt obligations (CDO). As these are also the products that are most relevant to this thesis, the following introduction to the market for credit derivatives will mainly evolve around these products. However, the specific characteristics of the particular products will be introduced in more detail in section 2.1.

The market for credit derivatives has existed for several decades. However, much has changed since the early beginning in the 1990s. The origin of credit derivatives can be found in the financial innovations, which took place in banks at this point in time. Basically, banks needed at method that would allow them to extend more credit to their most important clients, towards whom the limit of credit exposure had already been reached. The solution was to sell this credit risk to another financial institution as this allowed the originating bank to keep

increasing the credit volume towards the clients, without increasing risk on their own balance sheet (Dufey & Rehm, 2000). Even though many things have happened since these original credit risk transaction, one thing remains the same, a credit derivative carries credit risk.

An important development, which has catalyzed both the size and the extent of the credit derivatives market, was the formulation of the 1999 ISDA Credit Derivative Definitions, which essentially worked as a standardization of the terms under which transactions were made, e.g. by defining credit events (Rule, 2001). These standardized contracts must be regarded as a risk reducing alternative to the bilateral negotiations that otherwise occurred prior to each individual trade (Rule, 2001).

1.4.1 Market development

As mentioned, a lot has happened in the market for credit derivatives in recent decades. To give a better understanding on how the size of this market has evolved in the years before, during and after the crisis, this section will give a numerical and graphical presentation of this development. Figure 1.1, describes the total notional amount outstanding of CDS contracts in the OTC market in the period from 2004 to 2016.



Figure 1.1: Notional amount outstanding CDS contracts, USD Billion, 2004-2016. Source: Own contribution - data retrieved from BIS

From figure 1.1 it becomes evident that the years leading up to the crisis were dominated by an explosive growth in the notional amount of outstanding CDS contracts. The graph clearly shows that a peak occurs around the

second half of 2007, which also approximately marks the outbreak of the crisis. After this, the impact of the crisis can be observed as the sudden decrease in the CDS market that occurred in the following years.

Focusing specifically on CDO issuance, a similar pattern shows some very prosperous years leading up to the financial crisis, followed by a very drastic downturn. From figure 1.2, an increase in the yearly CDO issuance from around USD 25,000 million in 2001 to more than USD 130,000 million in 2008 can be observed. Following the outbreak of the crisis the yearly CDO issuance decreased drastically from its peak in 2008 to around USD 12,000 million in 2013.



Figure 1.2: CDO issuance, Europe, USD Million, 1996-2016 Source: Own contribution – data retrieved from SIFMA

The main drivers behind this explosive growth in credit derivatives issuances will be discussed further later in the thesis. For now, the key takeaway from this brief introduction to the market for credit derivatives is the explosive expansion that the market underwent, and how this came to an immediate stop at the outbreak of the crisis.

1.4.2 Market participants

The participants in the credit derivatives market can generally be separated into three groups, end-buyers of protection, end-sellers of protection, and intermediaries (Rule, 2001). End-buyers of protection typically wish to hedge the credit risk they are exposed to, while end-sellers of protection typically wish to take on risk in order to diversify their existing portfolio. The intermediaries typically provide liquidity and assemble and manage structured finance products (Rule, 2001). Common for the participating groups on both sides of the deal is that

they mainly consist of banks. This is especially the case for the protection buyers, where banks, by far is the most dominating player. Other large players on the buyer side are securities houses, large corporates, and insurance companies. Looking at the protection sellers, banks still represent the largest group, however, on this side of the deal, insurance companies are also a very dominant player alongside securities houses (Rule, 2001).

2. Theoretical Background

Before moving on to the empirical analysis, a theoretical introduction to CDOs must be given, as a full understanding of these financial instruments is a necessity for understanding the methods used in the valuation process. Furthermore, due to the complexity of the valuation process of synthetic CDOs, a theoretical introduction to the different elements in the valuation process must be given, as this will allow the reader to better understand the actual valuation conducted in chapter 4. In addition to this, a thorough theoretical introduction to the entire valuation process is a necessity for identifying the influence of copula models in the valuation of these financial products. Due to the magnitude of different theoretical strings required in the valuation process, the various theories and their origin will only be accounted for to the extent necessary for understanding the remaining sections of the thesis. For further details and full mathematical derivations, interested readers are referred to the relevant literature.

Specifically, this chapter is structured in the following way. Section 2.1 introduces the characteristics and the structure of CDOs, as well as their main applications. The purpose of this section is to provide the reader with a basic understanding of the mechanics of these financial products, and what the basis for their existence is in today's financial markets. This understanding will be required to recognize the role of copula models in the valuation of these product. In continuation of this, section 2.2 will focus on two common approaches to credit risk modelling, namely the structural approach and the reduced-form approach. The approaches discussed in this section will lay the foundation for the valuation method applied later in the thesis, and understanding these two approaches in their most general form is therefore a prerequisite for understanding the method applied later in the thesis. Section 2.3 introduces the loss distribution and explains how one can move from the independent default probabilities, modelled using either the structural or reduced form approach, to the joint distribution for a given portfolio of assets. This section is essential as this is the point in the valuation process where copula models are applied with the purpose of capturing the dependence structure between multiple assets in a portfolio. The last section of the theoretical introduction ties everything together, and illustrates the role of the loss distribution in the calculation of the fair spread for each synthetic CDO tranche.

Following the theoretical introduction to synthetic CDO pricing, relevant existing literature on copula models and CDO valuation will be reviewed, as this will help clarify where the contributions of the thesis fit within this specific field of research. To give a comprehensive understanding of the development of CDO valuation methods, a wide

selection of research is considered, as this will enable both the author and reader to position the contributions of this thesis in relation to the existing literature.

2.1 The Collateralized Debt Obligation (CDO)

Before turning to the description of the synthetic CDO, which is the focus of this thesis, the dynamics of the original cash CDO will be introduced. The cash CDO is a forerunner for the synthetic CDO, which was originally created to mimic the cash flows of a cash CDO. A CDO is categorized as a credit derivative as it, as all other credit derivatives, allows market participants to transfer credit risk between one another (Bomfim, 2005). The structure and characteristics of both cash and synthetic CDOs will be introduced in the following sections.

2.1.1 The Cash CDO

As previously mentioned, a cash CDO consists of a number of securities with claims of different seniority towards the cash flows generated by the assets in the underlying portfolio. These different levels of seniority mean that investors with the most senior claims, often referred to as the senior tranche holders, must be fully compensated before any junior tranche investors receive any of the cash flows that they are entitled to (Bomfim, 2005). The implications of offering tranches with different levels of seniority can be illustrated using a simple example. Consider a CDO issuer, typically a so-called Special Purpose Vehicle (SPV) that buys a portfolio of loans with a face value of USD 1 billion. The SPV finances the purchase of the loan portfolio by issuing notes that are backed by the cash flows generated by the underlying portfolio. If we assume that payments for both the underlying loans and the notes issued by the SPV occur with similar intensity, e.g. quarterly or monthly payments, the issuer simply passes on the payments received on the loans to the CDO investors, according to the seniority of the notes. Assuming that the CDO in this example consists of three tranches, a senior tranche with a face value of USD 850 million, a mezzanine tranche with face value USD 100 million, and an equity tranche with a face value of USD 50 million, the structure of the CDO can be illustrated as in figure 2.1.



Figure 2.1: Simple cash CDO. The figure illustrates the directions of cashflows after the purchase of the loan portfolio has occurred. Source: Own contribution with inspiration from Bomfim (2005)

This structure means that equity tranche holders are subject to the highest level of credit risk, as investors in this tranche will be the first to incur a loss in the event that borrowers in the underlying portfolio default on their loan payments. For this reason, investors in the equity tranche are compensated by receiving significantly higher spreads, compared to investors in the more senior tranches.

2.1.2 The Synthetic CDO

Before continuing to the introduction of the synthetic CDO, a brief introduction to credit default swaps will have to be given. In short, a credit default swap (CDS) is an agreement between two parties, a protection seller and a protection buyer. In a CDS, the protection buyer agrees to making periodic payments to the protection seller, who in turn, commits to covering any losses that protection buyer might have on the reference entity (Bomfim, 2005). Thus, a CDS can be used to transfer credit risk without transferring the ownership of the actual asset. As mentioned earlier, CDSs play a significant role in the structure of a synthetic CDO.

The main difference between a synthetic CDO and the cash CDO described in section 2.1.1, is the fact that the reference portfolio in a synthetic CDO consists of credit default swaps (Gibson, 2004). Figure 2.2 illustrates the structure of a simple synthetic CDO.



Figure 2.2: Simple unfunded synthetic CDO illustrating the transfer of credit risk from the sponsoring bank and all the way to the synthetic CDO investors. Source: Own contribution with inspiration from Bomfim (2005)

As shown in figure 2.2, the SPV sells protection on losses in the portfolio of reference assets owned by the sponsoring bank. As it was the case in the cash CDO, the SPV issues notes with different seniority, effectively buying protection on losses in the reference portfolio from synthetic CDO investors (Gibson, 2004). The tranche structure of a synthetic CDO allows investors to sell protection on a specific fraction of losses in the reference portfolio, according to their own risk-return preferences. In the example illustrated in figure 2.2, equity investors are committed to covering losses between 0% and 5% of the value of the reference portfolio, mezzanine investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses between 5% and 15%, and senior investors are committed to covering losses from the first default in the reference portfolio, while senior tranche investors are unlikely to incur any loss at all. Consequently, investors in the equity tranche receive a significantly higher spread from the issuer than it is the case for investors in the senior tranche of the synthetic CDO.

The specific setup of the synthetic CDO deal depends on whether the CDO tranches are "funded" or "unfunded". In the "funded" version, the CDO investor pays the entire notional of the tranche at the time of issuance. The funds paid by the investor are typically invested in low-risk securities by the CDO issuer. As defaults occur in the reference portfolio, the principal of the tranche is written down by an amount corresponding to these defaults. In the "unfunded" synthetic CDO, the issuer has to rely on the creditworthiness of the investors as no payments are made at the time of issuance, but only when defaults occur (Gibson, 2004).

2.1.3 CDO applications

After having established the general structure and dynamics of a CDO deal, the attention can now be switched towards understanding the motivation behind the use of these financial instruments. As illustrated in the introduction to the market for credit derivatives, the years leading up to the outbreak of the crisis were dominated by explosive growth in both CDS and CDO issuance. The following section will clarify some of the incentives behind this explosive growth before the crisis.

The underlying motivation for CDO issuance can generally be separated into two groups, those based on balance sheet considerations and those based on arbitrage speculations (Garcia & Goossens, 2010). Considering the CDOs issued due to balance sheet considerations, the securitization here happens mainly with the purpose of either gaining capital relief, increasing the liquidity of the assets in the underlying portfolio, or transferring risk off the balance sheet. Through such activities, financial institutions had a way of reducing some of the capital requirements that were forced upon them by the Basel Accords. In the arbitrage CDO, the issuer attempts to exploit a possible difference between the yield received on the underlying assets and the cost of funding them. Basically, this means that the CDO issuer tries to benefit from paying a lower fee to CDO investors than they pay on the underlying reference entities, as this would allow them to pocket risk-free profits (Garcia & Goossens, 2010).

During the years leading up to the crisis, the increasing popularity of the synthetic CDO compared to the cash CDO, can probably be attributed to the fact that no legal transfer of assets takes place between originator and SPV in the synthetic CDO. This characteristic simplifies the process of the deal significantly compared to that of a cash CDO, where a so-called *true sale* occurs between originator and SPV. By circumventing the classification as a *true sale*, a number of legal issues regarding the transfer of assets are avoided. Without going into legislative details, some of the main issues that are avoided in the process are approval of the sale from obligor, and consolidation (Garcia & Goossens, 2010). Another major benefit of the synthetic CDO relates to funding the deal. In a cash CDO, the issuer must raise cash to buy the reference portfolio from the originator. This is opposed to in a synthetic CDO, where the issuer only needs to raise cash in the event of defaults in the reference portfolio. This is due to the fact that the underlying portfolio in the synthetic CDO consists of credit default swaps on the reference entities.

In the years leading up to the crisis, CDOs were especially used to take advantage of market mispricing. This mispricing was mainly caused by the credit rating agencies' inability to correctly rate these structured finance

products. Following the crisis, credit rating agencies have been heavily criticized for these incorrect ratings, which were mainly caused by the complexity of the CDOs, and the obvious conflict of interest that exists in the business model of credit rating agencies (Jarrow, 2011). This conflict of interest and the pre-crisis financial environment will be discussed further in chapter 5.

2.2 Credit Risk Modelling

In section 2.1 it is shown that the payoff to CDO investors is highly dependent on the probability of default of the assets in the reference portfolio. In other words, in assessing potential investment opportunities CDO investors should be highly concerned with the credit risk of these reference entities as this will be the main determinant of their final payoff. In the extensive literature on credit risk modelling, researchers usually distinguish between the structural approach and the reduced-form approach. As these approaches function as a general foundation for the models applied later in this thesis, they will be introduced in the following.

2.2.1 The Structural Approach to Default Modelling

The idea behind the structural approach to credit risk modelling is that obligor's ability to honor its obligations is connected to the total asset value of same obligor. The main assumption within this framework is that a firm goes bankrupt if the asset value of the firm falls below a given barrier (Giesecke, 2004). Different variations of the position of this default barrier exists across models, however, the intuition behind the structural approach can be illustrated using the well-known Black-Scholes-Merton model framework. Even though this model will not be applied directly in the thesis, the intuition behind this approach can be considered foundation for the models applied in chapter 4.

Assume a company that is financed through equity and a zero-coupon bond with face value K and maturity T. In the Black-Scholes-Merton model, default occurs if bond issuer's total asset value is below the face value of the debt at maturity (Duffie & Singleton, 2003). Furthermore, it is assumed that the market value of the firm's assets follows a log-normal diffusion process:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t, \ V_0 > 0, \tag{2.1}$$

where μ is a drift parameter, $\sigma > 0$ is a volatility parameter, and W is a standard Brownian motion. Through the application of Ito's Lemma, it can be showed that:

(2.2)

$$V_t = V_0 * e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}$$
 (Giesecke, 2004)

ASSET VALUE DISTANCE TO QUIT PRICE DEFAULT BOOK LIABILITIES NOW TIME т

The intuition behind the Black-Scholes-Merton model is illustrated in figure 2.3.

Figure 2.3: The intuition behind the Black-Scholes-Merton framework to credit risk modelling. Source: By inspiration from Duffie and Singleton (2003).

As illustrated in figure 2.3, the default probability of the firm can be obtained from the probability distribution function at time T. Since we have that W_T is normally distributed with a mean of zero and variance T, the default probability p(T) can be written as:

$$p(T) = P[V_T < K] = P[\sigma W_T < Log \ L - mT] = \Phi\left(\frac{Log \ L - mT}{\sigma\sqrt{T}}\right)$$
(Giesecke, 2004) (2.3)

Where $m = \mu - \frac{1}{2}\sigma^2$, $L = \frac{K}{V_0}$ is the initial leverage ratio, and Φ is the standard normal distribution function. Writing this out gives:

$$p(T) = \Phi\left(\frac{\log \frac{K}{V_0} - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right)$$
(2.4)



In the event of default, the firm's debtholders will gain ownership of the company but will overall lose $K - V_T$. If no default occurs, debtholders will receive the promised payment of K, and equity holders will receive $V_T - K$. Thus, the value of the bond at time T can be written as:

$$B_T^T = \min(K, V_T) = K - \max(0, K - V_T),$$
(2.5)

which turns out to be equivalent to the payoff generated from a portfolio consisting of a short European put option on the company's assets with strike K and maturity T, and a default free loan with a face value of K and maturity T (Giesecke, 2004)

Using a similar argument, the value of equity at time T can be written as a call option on the company's assets with strike K and maturity T:

$$E_T = \max(0, V_T - K) \tag{2.6}$$

Valuing equity and debt using options theory is admittedly what the Black-Scholes-Merton framework is most commonly known for. However, as the details of this part of the framework have no immediate relevance to the remainder of this thesis, it will not be explained any further. Interested readers are referred to Merton (1974) for a more elaborate presentation.

As mentioned in the beginning of the section, the Black-Scholes-Merton model is only one of many that fit under the structural approach to default modelling. Since the publication of the original Black-Scholes-Merton model in 1974, many alternatives and extensions to the original model have emerged. Opposed to the BSM model, some of these models allow default to occur before the maturity of the debt, and at a different barrier than the face value of the debt. However, as the intuition behind these models is similar to in the BSM model, they will not be covered in the thesis. For more details on some of these extensions, see among others, Black & Cox (1976) and Longstaff & Schwartz (1995).

As the models applied in this thesis are mainly based on the structural approach to default modelling, the intuition behind this approach is the foundation for understanding the analysis conducted in chapter 4 of the thesis. However, as the intuition behind the reduced-from approach also contributes to some of the underlying assumptions, this approach will be introduced in the following.

2.2.2 The Reduced-Form Approach to Default Modelling

Opposed to the structural approach, the reduced-form approach to default modelling does not assume a direct connection between a firm's asset value and the probability of default. Instead, this approach models default as an exogenous event that occurs at unpredictable times (Bomfim, 2005). This means that default is modelled as a stochastic process calibrated from observable data such as historic or current market prices. Generally, two types of models are discussed under this approach to default modelling, intensity-based and ratings-based models. With the purpose of this thesis in mind, only the intensity-based models will be discussed in the following.

As indicated by the name, intensity-based models are based on modelling the arrival intensity of defaults as a stochastic process. In its simplest form, default is defined as the first jump in a Poisson process happening with a constant intensity λ . According to Schönbucher (2003), we can assume that the probability of a jump in the next small time interval Δt is proportional to Δt . This can be written as:

$$P(N(t + \Delta t) - N(t) = 1) = \lambda \Delta t$$
(2.7)

Moreover, if we assume that the probability of more than one default occurring during a very short time interval is practically zero, and that the number of defaults occurring in nonoverlapping time periods are independent, then the probability of no defaults can be written as:

$$P(N(t + \Delta t) - N(t) = 0) = 1 - \lambda \Delta t$$
(2.8)

Similarly, the probability of no defaults occurring within two time-intervals can be written as:

$$P(N(t + 2\Delta t) - N(t) = 0) = (1 - \lambda \Delta t)^{2}$$
(2.9)

According to Schönbucher (2003), the probability of no defaults during the entire interval [t, T], after dividing it into n intervals, so that $\Delta t = \frac{T-t}{n}$, can be written as:

$$P(N(T) = N(t) = (1 - \Delta t\lambda)^n = \left(1 - \frac{1}{n} * (T - t)\lambda\right)^n$$
(2.10)

As
$$\left(1+\frac{x}{n}\right)^n \to e^x$$
 when $n \to \infty$, the above expression converges to:
 $P(N(T) = N(t)) \to e^{-(T-t)\lambda}$
(2.11)

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Similar to before, this corresponds to the following expression denoting the probability that at least one default occurs within the period:

$$P(N(T) = 1) = 1 - e^{-(T-t)\lambda}$$
(2.12)

Since its introduction, several extensions to this approach has been published. A natural extension relates to the assumption of a constant intensity of the jump process. Among others, Duffie & Singleton (2003) describes the implementation of both a deterministic time-varying intensity as well as a stochastic intensity. For more details on this, see Duffie & Singleton (2003).

2.2.3 Summary – Credit risk modelling

In the above sections, two general approaches to credit risk modelling have been presented, the structural approach and the reduced-form approach. In the structural approach, the probability of default is directly connected to the asset value of the firm, whereas the reduced-form approach considers default as an exogenous event occurring at unpredictable times. However, common for both is that they only describe the probability of default for a single asset, meaning that they are insufficient when assessing the credit risk of a portfolio of assets, as some degree of correlation will have to be included in such assessment. This particular issue will be addressed in the next section.

2.3 The Loss Distribution

In continuation of the approaches introduced for modelling the probability of default for a single firm, this section will focus on the distribution of losses of a large portfolio of assets, such as the one underlying a synthetic CDO. Using either one of the approaches introduced in the above, investors can create the marginal distribution of losses for each asset in the portfolio. This, however, is not sufficient when assessing the full credit risk of a given reference portfolio. Assuming independence between defaults in a large portfolio of assets is unrealistic, which is why investors require a method for evaluating the joint default behavior of the reference entities. A way of addressing this issue is through the application of copula theory as a tool for modelling the dependence structure between entities in a given portfolio.

2.3.1 Copula Theory

In this section, the notion of copulas will be introduced both in a general sense and in terms directly applicable to the use of the specific copulas in this thesis. In general terms, copula models are used to describe the dependence between multiple random variables (Malevergn & Sornette, 2006). In terms of financial markets, these variables could typically be equity returns, or as in this thesis, the value of given assets at different points in time. More formally, a copula function can be defined by the following properties:

Definition:

A function C: $[0,1]^n \rightarrow [0.1]$ is a n-copula if it enjoys the following properties:

- $\forall u \in [0,1], C(1, ..., 1, u, 1, ..., 1) = u$
- $\forall u \in [0,1], C(u_1 \dots, u_n) = 0$ if at least one of the u_i 's equals zero
- C is grounded and n-increasing (Malevergn & Sornette, 2006)

In essence, a copula function is a multivariate distribution with uniform marginals and with support within the interval $[0,1]^n$.

Copula functions are relevant for modelling the loss distribution of a portfolio of assets, as they allow for a way to link univariate marginal distributions to their full multivariate distribution (Li, 2000). For n uniform random variables, $U_1, U_2, ..., U_m$, the joint distribution function C can be defined as:

$$C(u_1, u_2, \dots, u_n, \rho) = \Pr(U_1 \le u_1, U_2 \le u_2, \dots, U_n \le u_n)$$
(2.13)

where ρ is a dependence parameter and C can be considered a copula function (Li, 2000).

In 1959, Abe Sklar proved that if $F(x_1, x_2, ..., x_n)$ is a joint multivariate distribution function with continuous marginal distributions $F_1, F_2, ..., F_n$, there exist a copula function $C(u_1, u_2, ..., u_n)$ such that

$$F(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n) \text{ (Sklar, 1973)}$$
(2.14)

Therefore, the copula function allows for a way to combine a set of marginal distributions to form a joint multivariate distribution. The copula function describes the dependence structure between these marginal distributions. Moreover, as Sklar's theorem states that the dependence structure and marginal distributions can be separated completely, it is possible to apply the dependence structure for one set of dependent random variables, to a different set of random variables with different marginal distributions (Schönbucher, 2003). Only

copula functions allow for the separate study of marginal distributions and of the dependence, as they allow for clear distinction between the information held in each component.

Several different copula functions have been used in biostatistics, physics, and actuarial sciences long before they were introduced to finance by David X. Li in his now infamous paper *"On Default Correlation: A Copula Function Approach" (2000)*. However, as the focus of this thesis is limited to the comparison between the use of two different factor copulas, namely the one-factor Gaussian copula and the one-factor Student t copula, the specifics of these two models will be investigated further in the next sections. Other copula model alternatives will be discussed as part of the literature review in section 2.5.

2.3.2 Copula models

In the following section, the Gaussian copula and the Student t copula will be presented. Both models belong to the group of copulas derived from elliptical distributions. First, the general form of both models is presented, before the actual factor models applied in chapter 4 are introduced.

2.3.2.1 Elliptical Copulas

As indicated from the name, elliptical copulas have their foundation in multivariate elliptical distributions (Malevergn & Sornette, 2006). Two of the most widely used distributions within this family of probability distributions are the Gaussian and the Student t distribution, and these are exactly the two distributions that form the basis for the copula models applied in this thesis. Generally speaking, these two copula models are quite similar, however, as it is discussed in a later section, their behavior in relation to the dependence between extreme values can be very different.

The Gaussian copula:

The Gaussian copula is based on the multivariate Gaussian distribution. The random vector $\mathbf{X} = (X_1, ..., X_n)$ is multivariate normal if the following two properties hold:

- The univariate margins F_1, F_2, \ldots, F_n follows a Gaussian distribution
- The dependence structure between these margins can be described by the Gaussian copula:

$$C_{\rho,n}(u_1, \dots, u_n) = \Phi_{\rho,n} \big(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n) \big),$$
(2.15)

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where $\Phi_{\rho,n}$ denotes the n-dimensional standard Gaussian distribution, which is completely determined by correlation matrix ρ , and Φ denotes the standard normal cumulative distribution (Malevergn & Sornette, 2006).

The Student t copula:

The Student t copula is derived from the multivariate Student t distribution. It can be expressed as:

$$C_{n,\rho,\nu}(u_1,\dots,u_n) = \mathcal{T}_{n,\rho,\nu}\left(\mathcal{T}_{\nu}^{-1}(u_1),\dots,\mathcal{T}_{\nu}^{-1}(u_n)\right),\tag{2.16}$$

where $T_{n,\rho,v}$ is an n-dimensional Student t distribution with v degrees of freedom and shape matrix ρ . T_v denotes the univariate Student t distribution with v degrees of freedom (Malevergn & Sornette, 2006).

Since it is the case for the underlying distributions that the Student t distribution goes towards the Gaussian distribution, when v goes to infinity, the same characteristics is transferred to the corresponding copula models. Thus, when v goes to infinity, the Student t copula model approaches the Gaussian copula (Malevergn & Sornette, 2006). However, a key difference that will play an important role later in the thesis is the level of tail dependence in the Student t copula. In particular, this characteristic makes this model better suited for modelling situations of financial instability than its Gaussian copula in the chapter 4. The concept of tail dependence will be introduced further after the introduction of the applied factor models.

2.3.2.2 Factor models

A subclass of copula models referred to as factor copulas have been widely used in the financial industry. This type of model allows for a higher level of computational tractability, which is why it has been preferred over the entirely simulation based models in the industry (Cousin & Laurent, 2009). MacKenzie and Spears (2014), describe how this semi-analytical version of the Gaussian copula was used by the majority of financial market participants in the time leading up to the crisis.

In factor copulas, the dependence structure of default times across assets is assumed to follow a factor structure, meaning that the dependence structure is driven by latent variables V_1, \ldots, V_n , where V_i in turn depends on a common risk factor, M, and an idiosyncratic risk factor, Z_i . Within the group of factor copulas, the additive factor copulas have been used extensively in relation to pricing of synthetic CDO tranches. As indicated from the name, in this specific family of factor copulas the latent variable V_i is assumed to follow an additive function (Cousin & Laurent, 2009). The one-factor Gaussian copula and the one-factor Student t copula applied in chapter 4 both belong to the family of additive factor copulas. Both models will be introduced in the next sections.

2.3.3 One-factor Gaussian Copula

As mentioned in chapter 1, the one-factor Gaussian copula was broadly accepted as the market model for synthetic CDO valuation (MacKenzie & Spears, 2014). Essentially, this version of the Gaussian copula can be considered a special case of the original Gaussian copula introduced by David Li (2000). This semi-analytical version introduced by among others, Jean-Paul Laurent & Jon Gregory (2003), was accepted across the industry in the time leading up to the financial crisis, mainly due to the significant reduction in computation time that could be achieved. This allowed traders and other users of the model to react much faster than it had otherwise been the case (MacKenzie & Spears, 2014). In the literature, several different implementation methods are applied, however, this thesis adopts the approach described by Gibson (2004) and Hull & White (2004), and combines this with the approach described in Schönbucher (2003).

Consider a portfolio of *N* credits described by the following parameters:

 A_i : Notional amount of each credit i

 R_i : Recovery rate of credit *i*

 t_i :The time of default of company i

 $q_i(t)$:The cumulative risk-neutral probability that company i will default before time t, i.e. the probability that $t_i \leq t$

 $S_i(t) = 1 - q_i(t)$: The risk-neutral probability that company *i* will survive beyond time *t*, i.e. the probability that $t_i \ge t$

The general assumption regarding this model is that creditworthiness of a given company depends on its normalized asset value x_i . Thus, default occurs when x_i falls below \bar{x}_i , which is a specified default threshold. The normalized asset value $x_i(1 \le i \le N)$ is defined as:

$$x_i = a_i M + \sqrt{1 - a_i^2} Z_i$$
 (2.17)

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where x_i , M and Z_i all have independent standard normal distributions, F_i , G and H_i , and $-1 \le a_i \le 1$. In this expression, M represents a common factor that affects the probability of default for all companies in the economy, and can therefore be perceived as an economy-wide factor such as the business cycle or interest rate levels. Conversely, Z_i represents an idiosyncratic factor that only affects the probability of default of the individual company. Finally, a_i specifies the factor loading, and $a_i a_j$ denotes the correlation between asset value for company i and company j. The default threshold \bar{x}_i is defined as:

$$\bar{x}_i = F_i^{-1}(q_i(t)),$$
(2.18)

where F_i^{-1} denotes the inverse of the standard normal cumulative distribution function.

According to Hull & White (2004), the copula model implies that x_i is mapped to t_i using a percentile-topercentile transformation. This means that the conditional default probability $q_i(t < t_i|M) = P(x_i < \bar{x}_i|M)$ can be written as:

$$q_i(t|M) = H_i\left(\frac{\bar{x}_i - a_i M}{\sqrt{1 - a_i^2}}\right)$$
(2.19)

The main point of formulating the probability of default, conditional on the common factor M, is the fact that this leaves the idiosyncratic factor as the only driver of asset value. Consequently, both firm value and defaults are now independent across entities as they, conditional on a given realization of the common factor M, are only affected by the firm-specific factor Z_i , which by definition is uncorrelated across entities.

The loss distribution is a discrete process as each credit either incurs no loss or a loss of $A_i(1 - R_i)$. Assuming that A_i and R_i are equal for every credit in the reference portfolio, the number of discrete values must equal N + 1. Furthermore, this assumption means one can go from the distribution of the number of defaults to the loss distribution, simply by multiplying the number of defaults by A(1 - R).

The conditional distribution of the number of defaults can be computed using the recursion algorithm applied by Gibson (2004). The probability of exactly l defaults by time t, conditional on M, in a reference portfolio consisting of K credits can be written as:

$$p^{K}(l,t|M) \quad l = 0, \dots, K$$
 (2.20)

By adding one credit with a conditional default probability of $q_{K+1}(t|M)$, the default distribution for the new reference portfolio of K + 1 credits is now:

$$p^{K+1}(0,t|M) = p^{K}(0,t|M)(1 - q_{K+1}(t|M))$$
(2.21)

$$p^{K+1}(l,t|M) = p^{K}(l,t|M)(1 - q_{K+1}(t|M)) + p^{K}(l-1,t|M)q_{K+1}(t|M) \quad l = 1, \dots, K$$
(2.22)

$$p^{K+1}(K+1,t|M) = p^{K}(K,t|M)q_{K+1}(t|M)$$
(2.23)

By continuing to add credits until K = N, this recursion can be used to find the default distribution for the reference portfolio of N credits.

However, as it in the context of this thesis is assumed that all reference entities have equal default probabilities, and that the pairwise asset correlation is equal across all assets in the portfolio, the conditional default probability will be equal for all entities in the reference portfolio. This means that instead of the above described recursion algorithm, the conditional probability distribution of the number of defaults in the reference portfolio can be found using the binomial distribution (Schönbucher, 2003).The conditional default probability of exactly l defaults by time t in the entire portfolio can then be written as:

$$p(l,t|M) = \frac{N!}{(N-l)!l!} * q_i(t|M) * \left(1 - q_i(t|M)\right)^{N-l},$$
(2.24)

where N is the number of credits in the portfolio, l denotes the number of defaults in the reference portfolio, and $q_i(t|M)$ is the risk-neutral conditional default probability for each underlying reference entity.

After all conditional distributions have been calculated, the unconditional default distribution can be written as the following integral:

$$p(l,t) = \int_{-\infty}^{\infty} p^N(l,t|M)g(M)dM$$
(2.25)

where g is the probability density of M. This integral can be considered as an average of the conditional distributions weighted by the density function of the common factor M. Mathematically, this expression can be calculated using numerical integration. In this thesis, this numerical integration is implemented through the use of the trapezoidal rule.

2.3.4 One-factor Student t Copula

Although the one-factor Gaussian copula was accepted as the market model by the majority of market participants, researchers both before and after the crisis have heavily criticized the widespread use of this model for synthetic CDO valuation. The details of this criticism will be elaborated when discussing the results of the model analysis. However, in order to understand the purpose of introducing the one-factor Student t copula as an alternative model, the general basis for the criticism will be presented. The main area of criticism related to the one-factor Gaussian copula model presented in the previous section, is the distributional assumption underlying the model. The underlying assumption of multivariate Gaussianity results in a lack of tail dependence in the copula model. Specifically, this means that the Gaussian copula model is unable to price all CDO tranches simultaneously, which is mainly due to the occurrence of the so-called correlation smile created from the varying implied tranche correlations (Goegebeur, Hoedemakers, & Tistaert, 2007). Therefore, introducing an alternative model, which better incorporates tail dependence will allow the researcher to compare the results found under this different distributional assumption. The Student t distribution distinguishes itself from the Gaussian in the sense that it has "fatter tails", meaning that it accounts for tail dependence. Compared to the one-factor Gaussian copula presented in section 2.3.3, multiple modifications will have to be made in order to include the change in distributional assumption. In the one-factor Student t copula, the following factor model forms the basis for the calculations:

$$y_i = a_i M + \sqrt{1 - a_i^2} Z_i$$
 (2.26)

Where y, M and Z_i all have independent standard normal distributions, F_i , G and H_i , and $-1 \le a_i \le 1$.

However, where the output of this model could be compared directly to the default threshold in the Gaussian case, the following transformation is necessary to compute the asset value of the company in the one-factor Student t model. According to Goegebeur et. al (2007) and Greenberg, Mashal, Naldi, & Schloegl (2004), x_i follows a Student t distribution with v degrees of freedom if:

$$x_i = \sqrt{\frac{\nu}{W}} y_i \tag{2.27}$$

where x_i again denotes the normalized asset value of company i and W follows a χ_v^2 distribution with v degrees of freedom and is independent of y_1, y_2, \dots, y_n .

Combining the two expressions therefore gives the following factor model:

$$x_i = \left(a_i M + \sqrt{1 - a_i^2} Z_i\right) * \sqrt{\frac{\nu}{w}}$$
(2.28)

The threshold for default, \bar{x}_i , can in this setup be expressed as:

$$\bar{x}_i = t_v^{-1}(q_i(t)),$$
 (2.29)

where t_v^{-1} denotes the inverse of the cumulative distribution function for the Student t distribution with v degrees of freedom.

Contrary to the semi-analytical implementation of the one-factor Gaussian copula presented in section 2.3.3, the one-factor Student t copula will be implemented through the use of Monte Carlo simulation. The specifics of this implementation approach will be introduced next.

2.3.4.1 Monte Carlo implementation

The Monte Carlo implementation of the one-factor Student t copula can be explained in the following steps.

- 1. The random independent drawings from both the normal distribution and the χ^2 distribution are made. For the χ^2 distribution, 4 degrees of freedom is chosen. For every simulation, the market factor M is defined as 1 standard normal variable common for all entities within the individual simulation. The idiosyncratic factor Z_i is represented by an individual standard normal variable specific to each firm in the simulation (here 125 firms). The sampled χ^2_{ν} variable is in the same way as the market factor M, common for all entities within the individual simulation. Thus, for every simulation step, 1 occurrence of the market factor is drawn, 1 occurrence of the χ^2_{ν} variable is drawn, and 125 occurrences of the idiosyncratic factor is drawn.
- Based on the values drawn in 1., the normalized asset value of company *i* is calculated using equation
 2.28. For every simulation 125 asset values are calculated. In practice, this results in a 100000 x 125 matrix of simulated asset values as the process of calculating the normalized asset value of company *i* is repeated 100.000 times.
- 3. Each of the calculated normalized asset values are then compared to the default threshold $\bar{x}_i = t_v^{-1}(q_i(t))$ at each of the 20 timesteps, and information about whether x_i is above or below the

threshold at that given time is stored. If $x_i < t_v^{-1}(q_i(t))$, a value of 1 is assigned, whereas if $x_i \ge t_v^{-1}(q_i(t))$ a value of 0 is stored.

- 4. The algorithm then counts how many of the 125 credits that have defaulted at each of the 20 time steps.
- 5. Finally, the loss distribution is constructed by calculating the probability of experiencing l defaults at each individual time step.

The reasoning behind the simulation can be illustrated by the following simplified example:

	Values generated from one-factor Student t <u>copula</u>					Default		
Simulations	Asset 1	Asset 2	Asset 3	Default threshold	Asset 1	Asset 2	Asset 3	Total
1	-0.08878365	-0.651036761	0.294934815	-2.98389	0	0	0	0
2	-3.06287368	-0.441868182	-2.410066085	-2.98389	1	0	0	1
3	0.35834343	1.396235915	-0.528660725	-2.98389	0	0	0	0
4	-6.42518596	-0.369945628	-0.777880347	-2.98389	1	0	0	1
5	0.3493049	-0.732539685	0.510320087	-2.98389	0	0	0	0
6	-0.27877405	-1.886157019	-3.951895129	-2.98389	0	0	1	1
7	1.1849979	-0.686559375	0.281363638	-2.98389	0	0	0	0
8	-0.43747062	0.020495039	0.38574815	-2.98389	0	0	0	0
9	-5.93817333	-4.494381026	-1.372588766	-2.98389	1	1	0	2
10	-0.81360068	0.463192565	-2.333250097	-2.98389	0	0	0	0

Table 2.4: Simplified illustration of the counting process within the applied Monte Carlo simulation. The results are generated for t=5, ahazard rate of 0.41%, 4 degrees of freedom, and an assumed asset correlation of 0.3.Source: Own contribution.

In the above example, the value of 3 individual assets are simulated 10 times. Every time an asset value falls below the common default threshold, this information is stored and used to construct the loss distribution. In the above, the probability of experiencing 0 defaults at time 5 in this very simplified example is therefore 6/10, which is equal to 60%. In the actual implementation, 125 asset values are simulated and compared to the default threshold at every timestep as it has been done for t = 5 in the above example. This process is then repeated 100.000 times.

2.3.5 Tail dependence

To understand the reasoning behind selecting the Student t copula as an alternative to the Gaussian copula, the notion of tail dependence will be introduced briefly. However, with the scope of this thesis in mind, only a very mathematically-light introduction will be given. For the full mathematical details, see McNeil, Frey, & Embrechts

(2005). In general terms, tail dependence describes the level of extremal dependence between random variables (Ferreira & Ferreira, 2012). In other words, the term refers to the ability to model the clustering of defaults that often occurs between firms during economically challenging times. According to (Donnelly & Embrechts, 2010), the coefficient of upper tail dependence is equal to 0 in the Gaussian copula, when correlation ρ <1, and equal to 1 if $\rho = 1$. Due to the symmetry of the Gaussian distribution, this conceptually means that extreme events occur independently in both tails of the distribution function. In the context of this thesis, this means that default times occur independently in the tails. Clearly, this is an undesirable characteristic when attempting to model the clustering of defaults in a given portfolio.

In the Student t copula, both the upper and lower coefficient of tail dependence can be expressed as:

$$2t_{\nu+1}\left(-\sqrt{\nu+1}*\sqrt{\frac{(1-\rho^2)}{1+\rho^2}}\right)$$
(2.30)

The fact that the Student t copula exhibits upper (and lower) tail dependence, makes it theoretically more suitable for modelling defaults in a portfolio of assets. Specifically, the amount of tail dependence included in the model depends on the number of degrees of freedom v, as the Student t distribution approaches the Gaussian distribution when v approaches infinity. As will be seen later in the chapter, several other copula models exhibit tail dependence as well. These will be introduced further in section 2.5.

2.3.6 Summary – Loss Distribution

In this thesis copula theory is applied in relation to the construction of the portfolio loss distribution, which represents a key component in synthetic CDO valuation. As the copula models describe the dependence structure between the marginal distributions in a given portfolio, the use of copula theory allows the researcher to combine a set of marginal distributions to form a joint multivariate distribution. Specifically, one-factor versions of the Gaussian copula and the Student t copula will be implemented semi-analytically and through Monte Carlo simulation, respectively. As it has now been introduced how the loss distribution for the reference portfolio is constructed, the next section of the thesis will describe how this loss distribution contributes, when determining the fair spread paid on a given synthetic CDO tranche.

2.4 Pricing synthetic CDO tranches

As mentioned in section 2.1, the individual tranches in a synthetic CDO is defined by its attachment and detachment point. Concretely, the position of these points specifies which interval of losses in the underlying portfolio, investors in the tranche can be held accountable for. In continuation of the thorough theoretical introduction to loss distribution modeling, the next section will outline how synthetic CDO tranches are valued given a certain loss distribution. In line with the Gaussian copula framework introduced in section 2.3.3, this section will also be based on the approach described by (Gibson, 2004).

In terms of valuation, each synthetic CDO tranche can be considered similar to a swap contract as investors receive spread payments from the CDO issuer, while at the same time are obligated to pay the issuer in case defaults in the underlying portfolio affect the tranche in which they have invested (Gibson, 2004). In literature, these payment streams are referred to as the "fee leg" and "contingent leg", respectively. Assuming that payments in both directions occur on periodic payment dates, the value of the tranche is equal to the difference between the expected present value of the fee leg and the expected present value of the contingent leg. This provides the value seen from the perspective of the investor.

To identify the expected difference between the two payment streams, the expected default loss on the tranche up to payment date T_i must be defined:

$$EL_{i} = \sum_{l=0}^{n} p(l, T_{i}) \max(\min(lA(1-R), H) - L, 0)$$
(2.31)

where $p(l, T_i)$ is the default probability distribution found using the chosen copula model, A(1 - R) is equal to the loss in case of default, L denotes the attachment point of the tranche, and H denotes the detachment point of the tranche.

With the expected default loss defined, the expected present value of the contingent leg can be found as:

$$Contingent = \sum_{i=1}^{n} D_i (EL_i - EL_{i-1})$$
(2.32)

where D_i denotes the risk-free discount factor corresponding to T_i .

Moreover, the expected present value of the fee leg can be defined as:

$$Fee = s \sum_{i=1}^{n} D_i \Delta_i ((H-L) - EL_i)$$

$$(2.33)$$

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where $\Delta_i \approx T_i - T_{i-1}$ and *s* is the annual spread received by the investor.

The mark-to-market value can then be defined as:

$$MTM = Fee - Contingent \tag{2.34}$$

As the fair spread of the tranche is defined as the spread that makes the contingent leg equal to the fee leg, this spread can be calculated as:

$$s_{fair} = \frac{\sum_{i=1}^{n} D_i (EL_i - EL_{i-1})}{\sum_{i=1}^{n} D_i \Delta_i ((H-L) - EL_i)}$$
(2.35)

This calculation is repeated for all tranches in the synthetic CDO by varying the attachment and detachment point in the above equations. As a result, separate fair spreads are calculated for each tranche representing the level of risk corresponding to an investment in the given tranche.

As the role of the copula model has now been illustrated, both in the generation of the loss distribution, and ultimately in the determination the fair spread for the tranches in a synthetic CDO, this theoretical background section will be concluded with a review of the findings of other researchers in their application of copula models for CDO valuation.

2.5 Review of other relevant literature

From the theoretical introduction to the process of pricing synthetic CDO tranches provided, it becomes evident that the choice of copula model, and the assumptions related to this model, are the most critical steps in the valuation process. This has resulted in a very large selection of models available for synthetic CDO valuation with various levels of mathematical complexity. In this section, some of the most relevant findings based on these models and model extensions will be discussed. The purpose of this section is to define the broad theoretical field, and thereby allow the researcher to identify the gap in literature, which this thesis will contribute to fill. It will, furthermore, provide the reader with a foundation for evaluating this contribution in relation to the extensive research that has already been conducted within this field. The focus in this section will be to discuss the conclusions drawn by other researchers through their application of copula models.

Gaussian copula

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As discussed earlier in this thesis, consensus has been established around the fact that the Gaussian copula, despite its very widespread use, has several clear drawbacks. Goegebeur et al. (2007) mention the so-called implied correlation smile as one of the main issues that must be considered when applying the market-accepted Gaussian copula model. More specifically, the implied correlation smile can be described as the fact that the Gaussian copula fails to simultaneously match the prices of all CDO tranches. Goegebeur et al. (2007) argue that this phenomenon has its foundation in the lack of tail dependence that characterizes the Gaussian copula through its underlying distributional assumptions. Thus, in this paper the researchers suggest incorporating more tail dependence as a way of improving model fit to market prices. This view is supported by a study conducted by Arturo Cifuentes & Georgis Katsaros (2007). In this study, the authors find different implied correlation values depending on which CDO tranche is considered. More precisely, they conclude that the implied correlation is higher in the most junior and most senior tranches in the CDO, whereas it is lower for the intermediate tranches. Thus, the term *"correlation smile"*.

Student t copula

As suggested in Goegebeur et al. (2007), a possible solution to the implied correlation smile issue would be to introduce a valuation model that better incorporates tail dependence. For this reason, multiple authors have considered the Student t copula as a feasible alternative to the one-factor Gaussian copula. Among others, Embrechts, Lindskog, & Mcneil (2003) consider the Student t copula in relation to credit risk modelling and suggest that it potentially could be more suitable for credit risk analysis than the Gaussian version. This hypothesis is based on an investigation of tail dependence in a bivariate t copula. Embrechts et al. (2003) argue that positive coefficients of both upper and lower tail dependence will allow the Student t copula to better capture the extreme joint tail observations, which is evident in the financial data used in their study.

Contrary to the above hypothesis of Embrechts et al. (2003), 2004 Greenberg et al. (2004) conclude that the Student t copula does not prove to be a better alternative than the Gaussian copula when it comes to replicating the market prices of Trac-X Series II tranches. However, worth mentioning in relation to this conclusion is that the analysis conducted within this study, is only based on empirical data from a single day, which clearly makes the results problematic to generalize from. As a consequence of the mixed results obtained through the application of the Student t copula, a variety of other copulas have been suggested as possible alternatives. The characteristics and results obtained through the use of some of these models will be discussed in the following.

Double-t copula

In a research paper from 2004, John Hull & Alan White present a different extension to the original one-factor Gaussian copula, which they refer to as the double-t copula. The difference between their suggested model and the original factor copula model can be found in the distributional assumptions of both model factors. Assuming that the factor model can be described similar to what was done in section 2.3.3, we see that the asset value of company *i* is:

$$x_i = a_i M + \sqrt{1 - a_i^2} Z_i$$

However, different from the presentation of the Gaussian copula in section 2.3.3, both the common factor M and the idiosyncratic factor Z_i are now assumed to follow a Student t distribution with v degrees of freedom instead of a standard Gaussian distribution. As previously mentioned, the Student t distribution converges to the Gaussian distribution as the number of degrees of freedom increase, and on the contrary, the lower the v, the heavier the tails of the Student t distribution. In their study, Hull and White (2004) find that by assuming that both factors follow a Student t distribution with 4 degrees of freedom, the double-t copula fits the ITraxx and CDX data reasonably well. Consequently, the authors conclude that using heavier tailed distributions for both market factor and idiosyncratic factor, significantly improves this factor model's ability to fit market data.

A variety of other factor models based on different distributional assumptions have been introduced. Generally, one can distinguish between these models by looking at the distributional assumptions for the factors included in the model. Relevant examples of other factor copulas are the NIG copula presented by Guegan & Houdain, (2005), and the double-NIG copula introduced by Kalemanova, Schmid, & Werner (2005). For more details on these, see respective articles.

Stochastic correlation models

Another relevant branch of extensions from the Gaussian copula model is the class of models that dismiss the assumption of a constant correlation parameter. In the stochastic correlation models, the dependence parameter ρ is assumed to follow a stochastic process. According to Cousins & Laurent (2008), the latent variables in such a model can be expressed as:
$$V_{i} = \tilde{\rho}_{i}V + \sqrt{1 - \tilde{\rho}_{i}^{2}\bar{V}_{i}}, \quad i = 1, ..., n,$$
(2.36)

where V and \bar{V}_i , i = 1, ..., n are independent standard Gaussian random variables, and $\tilde{\rho}_i$, i = 1, ..., n are identically distributed random variables in the interval [0,1]. In a study from 2007, Gregory, Burtschell & Laurent found that a three states stochastic correlation model was able to fit market quotes of CDO tranches well. As the parameter $\tilde{\rho}_i$ in their model is expressed as:

$$\tilde{\rho}_i = (1 - B_s)(1 - B_i)\rho + B_s \tag{2.37}$$

Their total factor model can be expressed as:

$$V_i = \left((1 - B_s)(1 - B_i)\rho + B_s \right) V + \left((1 - B_s) \left((1 - B_i)\sqrt{1 - \rho^2} + B_i \right) \right) \bar{V}_i , i = 1, \dots, n,$$
(2.38)

where $\bar{V}_1, \ldots, \bar{V}_n, V, B_s, B_1, \ldots, B_n$ are independent, $\bar{V}_1, \ldots, \bar{V}_n, V$ are standard Gaussian random variables, B_s, B_1, \ldots, B_n are Bernoulli random variables and $0 \le \rho \le 1$. As they assume that $q = Q(B_i = 1)$ and $q_s = Q(B_s = 1)$, they get a discrete marginal distribution for $\tilde{\rho}$, which can take three possible values. These being; the value of 0 with probability $q(1 - q_s)$, the value ρ with probability $(1 - q)(1 - q_s)$, and the value 1 with probability q_s . By calibrating the model to market quotes, the authors were able to simultaneously fit the model to the observed tranche quotes from both ITraxx and CDX in a satisfactory manner. For more details on stochastic correlation models, see among others Gregory, Burtschell , & Laurent (2007).

Archimedean copulas

Archimedean copulas are often considered as a direct alternative to the Gaussian approach. As the models within this family are fundamentally exchangeable, they also allow for a factor representation (Cousin & Laurent, 2009). Specifically, each copula can be connected to a positive random factor V with inverse Laplace transform denoted by $\varphi(.)$ and correspondingly, Laplace transform denoted $\varphi^{-1}(.)$. The latent variable can then be expressed in the following way:

$$V_i = \varphi^{-1} \left(\frac{-\ln \overline{V}_i}{V} \right), i = 1, \dots, n$$
(2.39)

where \overline{V}_i , i = 1, ..., n are independent uniform random variables, and the joint distribution of the random vector $(V_1, ..., V_n)$ follows a φ -Archimedian copula, which can be expressed as:

$$C_n(v_1, ..., v_n) = P(V_1 \le v_1, ..., V_n \le v_n) = \varphi^{-1}(\varphi(v_1) + \dots + \varphi(v_n))$$
 (Cousin & Laurent, 2009) (2.40)

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A variety of Archimedean copula models exist in the literature, but most re-known are probably the Clayton, Frank, and Gumbel copula. The variation between these models can be attributed to the generator φ , which for each model can be expressed as:

Clayton:
$$\varphi(t) = t^{-\theta} - 1, \ \theta \ge 0$$
 (2.41)

Gumbel:
$$\varphi(t) = (-\ln(t))^{\theta}, \ \theta \ge 1$$
 (2.42)

Frank:
$$\varphi(t) = -\ln\left[\frac{1-e^{-\theta t}}{1-e^{-\theta}}\right], \ \theta \in \mathbb{R}$$
 (Cousin & Laurent, 2009) (2.43)

Several authors have considered the use of the Archimedean copula models in relation to credit risk analysis. In a study from 2014, Gabriel Gaiduchevici calibrates several Archimedean copula models to replicate ITraxx Europe Series 15 tranche quotes. In his study Gaiduchevici (2014) finds that a two-parameter Gumbel copula performed best among the included Archimedean copulas. In this study, Gaiduchevici argues that this result is in line with the fact that the Gumbel copula exhibits upper tail dependence, meaning that it is more suitable when describing outcomes that simultaneously produce upper tail values. In addition, Gaiduchevici concludes that the structure of tail dependence is of higher importance than the specific parameter values calibrated from market quotes. For the Clayton copula, Gaiduchevici finds that this model has a tendency to underprice junior tranches while overpricing the more senior tranches. Thus, deeming this copula unsuitable for the context described in his research paper. Worst, however, was the performance of the Frank copula, which due to its lack of tail dependence, was unable to properly fit market quotes.

2.5.1 Summary – Literature review:

In short, multiple alternatives to the conventional Gaussian copula exist in literature. Whether these are extensions within the family of elliptical copulas, or belonging to the Archimedean copulas, the key term identified for a successful fit to market quotes is tail dependence. Across the research described in the above, it appears that the fit between the individual model and market quotes is dependent on whether the applied copula model includes a sufficient level of tail dependence. Consequently, comparing the Gaussian copula to a heavier-tailed alternative is not where this thesis separates itself from the existing literature. Instead, the contribution of this thesis arises from the context in which the performance of these two models is discussed. Opposed to focusing solely on the mathematical properties of the models considered, as many other researchers have done, this thesis considers the practical use of the two models in relation to the escalation of the financial crisis. Concretely, the thesis posed the question of whether using one model opposed to the other could have

changed anything in relation to the crisis, and compares this effect of a different synthetic CDO valuation method, to some of the other main contributors existing in the financial system.

3. Data

Before continuing to the model-oriented analysis and the application of the theories introduced in the previous section, a description of the data used for the analysis will be given. Building on the general introduction to the market for credit derivatives, which was given in chapter 1, this section will focus on specific details concerning the part of the market considered in this thesis.

3.1 Credit indices

As the market for credit derivatives has evolved, the standardization of the transactions and the terms under which they are made, has paved the way for the introduction of standardized credit indices. These indices track the CDS spreads on corporate bonds for a given portfolio of companies. The analysis conducted in this thesis, which as indicated in the above is more of an analysis of the introduced models than it is an empirical study, will nevertheless compare the calculated model estimates to quoted prices from one of the credit indices. The index that will provide the benchmark for model assessment is the ITraxx Europe, which is owned by Markit. This index consists of 125 equally-weighted European names that represent the 125 most liquid names in Europe. To ensure that this remains true, a new series of the index is introduced every 6 months (Markit, 2014). A list of the 125 names included in the ITraxx Europe series 27 can be found in appendix 1. The characteristic making this index relevant to the analysis conducted in this thesis, is the existence of the tranced version of the index. As it is the case in a synthetic CDO, this feature allows investors to obtain exposure to a specific range of the index loss distribution. As always, the size of the tranches is defined by the relevant attachment and detachment points. Currently, the ITraxx Europe index is composed of the tranches 0-3%, 3-6%, 6-12%, 12-100% (Bloomberg). However, this tranche structure has changed during recent years, meaning that some of the dates considered in this thesis, will have a tranche structure slightly different from the current.

3.2 Chosen data points

To give an impression of the performance of both models under different economic conditions, the empirical analysis will consider market quotes at different points in time. Doing this will allow for the opportunity to infer whether one of the models perform better or worse under different economic conditions, as one must expect that the economic conditions to some extent should be incorporated in the observed market quotes. However,

it should be mentioned that both attachment and detachment points for the tranched ITraxx Europe have changed through the years, which as a result makes a direct comparison of the tranche quotes problematic.

The specific dates considered are April 3, 2007, November 13, 2008, and May 9, 2017. As it has proven rather difficult to obtain historical daily tranche quotes for the ITraxx Europe, secondary sources have been used to obtain market quotes for April 3, 2007 and November 13, 2008. For May 9, 2017 market quotes from Bloomberg has been used as benchmark for comparison. The tranche quotes forming the basis for the model assessment is summarized below:

Date (03-04-2007	Date :	13-11-2008	Date 09-05-2017	
Tranche	Market quote	Tranche Market quote		Tranche	Market quote
0-3%	815	0-3%	N/A	0-3%	989
3-6%	59	3-6%	950	3-6%	181
6-9%	14	6-9%	492	6-12%	77
6-12%	6	9-12%	260	12-100%	27
12-22%	3	12-22%	105	-	-
-	-	22-100%	52	-	-

 Table 3.1: Converted market quotes from April 3, 2007, November 13, 2008, and May 9, 2017.

Sources: Bloomberg, Justesen (2009), and Hull & White (2004)

All tranche quotes in table 3.1 are quarterly spreads denoted in basis points. This quotation style is different from the conventional style used in Bloomberg, where both the equity tranche and the junior mezzanine tranche are quoted as a percentage upfront payment of the notional amount, plus a running spread of 100 basis points. Similar to the tranche attachment and detachment points, the used quotation style has changed through the years, which complicates comparisons across different time periods further. Consequently, all tranche quotes have been converted to the conventional quarterly spread quotes, which are generally used for the more senior tranches. For the most recent of the dates considered, this has been done using Markit's own converter (Markit, 2017). However, as Markit's converter relies on present information about the interest rate environment and the specific ITraxx series, this tool cannot be used for the former dates as the required underlying information is unobtainable. Therefore, for April 3, 2007, this thesis relies on the conversion conducted by the secondary source, and for November 13, 2008, the equity tranche quote will be omitted as no converted data can be obtained for this tranche.

3.3 Research approach

As mentioned earlier, the empirical data presented in the above will function as a benchmark for the estimated values calculated using the one-factor Gaussian copula and the Student t copula, respectively. In relation to this, it is important to stress that the focus of the analysis is model behavior rather than it is the empirical data. Thus, the quoted market prices only play the role of benchmark values around which the estimated model values will be positioned depending on the underlying assumptions. Therefore, the following analysis is more of an analysis of model behavior than it is an empirical study. This approach is chosen to support the examination of the research question by putting the use of these models into perspective, and considering what implications their use had, or could have had, on the financial crisis. To be able to do so, a deep understanding of the performance and sensitivity of the models is required.

In order to compare both models' ability to price all tranches simultaneously, a calibration to the equity quote will be carried out at each data point. Practically, this means that the Gaussian model will be forced to fit the quoted price on the equity tranche by backing out the implied correlation for this tranche. The purpose of this approach is to test each model's ability to price all other tranches by applying the implied correlation level for the equity tranche. This will help enable an identification of the shortcomings of each model, and support the assessment of potential differences between the two models.

The information gained through the assessment of both copula models, will form the basis for a discussion about whether anything would have been different, had the Student t copula been applied as the market standard instead of the Gaussian. The application of the models, and the credit derivatives market in general, must be considered through the lens of the financial crisis. Doing this will help identify the role of this market in the escalation of the crisis, and form the basis for a discussion about how other factors might have contributed as well.

3.4 Summary – Data

In short, the described ITraxx Europe data will form the basis for a comparative analysis of the one-factor Gaussian copula and the one-factor Student t copula. The focus of this analysis will be the variation in model behavior under different input assumptions as well as the models' ability to match the observed market quotes. Thus, as the empirical data is merely used as tool for model evaluation, the use of secondary sources is considered to be acceptable.

4. Synthetic CDO Valuation

In the following, the valuation of the synthetic CDO tranches will be conducted. As introduced in the theoretical chapter of the thesis, the application of both copula models requires a number of input assumptions. Consequently, a discussion concerning these assumptions is necessary, as they ultimately will determine the estimated tranche spreads. Through the estimation of tranche spreads, the two models' differences and similarities should be identifiable, and their relative performance in relation to the observed market quotes should become clear. The importance of the input assumptions will be investigated further in the sensitivity analysis that follows the initial valuation process for each copula model. Through this analysis, the importance of some of the main assumptions as well as the sensitivity of the calculated tranche spreads, should be identifiable.

4.1 Assumptions

In order to apply the introduced copula models, the following inputs have to be specified:

- 1. The number of reference entities in the reference portfolio (N)
- 2. The notional amount on each individual entity
- 3. The pairwise asset correlation between all entities in the reference portfolio
- 4. The recovery rate on each individual entity in case of default
- 5. The CDS-spread on the reference entities
- 6. The time horizon, t
- 7. The risk-free rate
- 8. Tranche attachment and detachment points

In the following, each assumption will be discussed before moving on to the actual valuation process.

<u>1.The number of reference entities</u>: As mentioned in chapter 3, the ITraxx Europe index contains the 125 most liquid credit default swaps in Europe.

<u>2. Notional amount</u>: The notional amount of each entity is assumed to be EUR 1 million.

<u>3. Pairwise asset correlation</u>: The pairwise asset correlation will initially be assumed equal to 0.3. However, this value will be calibrated to the equity tranche quotes after the initial implementation of the one-factor Gaussian copula model, and this implied correlation forms the basis for the pricing of the other tranches.

<u>4. Recovery rate</u>: A recovery rate of 40% is assumed for all reference entities. According to the literature, this is market standard. See among others, Hull & White (2003), Kalemanova et al. (2005), and (Burtschell et al.(2009).

<u>5. The CDS-spread</u>: The CDS-spreads for the relevant untranched ITraxx series are used to calculate the implied hazard rates at the three different points in time. In line with Hull (2015), the implied hazard rate can be derived from the observed spread in the following way. The relevant data for each of the investigated dates is summarized in the following table:

Date	April 3, 2007	November 13, 2008	May 9, 2017	
Itraxx index spread	24.6 bps	138 bps	55 bps	
Implied hazard rate	0.246%/(1-40%)=0.41%	1.38%/(1-40%)=2.30%	0.55%/(1-40%)=0.92%	

Table 4.1: The implied hazard rates derived from the ITraxx CDS spreads at the considered dates.Source: Bloomberg

<u>6. The time horizon, t</u>: For all dates, a maturity of 5 years is assumed. This assumption has been made to secure consistency across the three dates considered as the focus of the analysis is model behavior rather than empirical data.

<u>7. The risk-free rate</u>: As a proxy for the risk-free rate, the German generic 10-year government bond is applied in this thesis. This proxy has been chosen for multiple reasons. First, this bond is based on the Euro, which is also the case for index considered. Second, historical data on this bond goes back almost 30 years, meaning that a relatively accurate estimate of this yield can be obtained for all dates considered in the analysis. However, as Bloomberg's data frequency decreases, when moving longer into the past, the rates for the specific dates considered, must be approximated based on the closest available data points. Especially, for the earliest of the three dates, this is an issue, as only quarterly data is available for this point in time. In this case, the specific value is estimated as an average of the closest available quotes. The applied risk-free rates are summarized in the table below:

Date April 3, 2007		November 13, 2008	May 9, 2017
Risk-free rate	4.15%	3.67%	0.43%

Table 4.2: The German generic 10-year government bond at the considered dates.

Source: Bloomberg

<u>8. Attachment and detachment points</u>: As mentioned in section 3.1, there has been a change in the attachment and detachment points on the ITraxx Europe during the period covered in this study. Therefore, different

attachment and detachment points have been used corresponding to the dates analyzed. For date 1 and 2, the tranched version of the ITraxx was structured as: 0-3%, 3-6%, 6-9%, 9-12%, 12-22%, 22-100%. For the latest date, the tranches are structured as: 0-3%, 3-6%, 6-12%, 12-100%.

4.2 One-factor Gaussian copula estimation

In the following, the estimated tranche quotes at each point in time will be calculated using the one-factor Gaussian copula model introduced in section 2.3.3. Intermediate results and calculations will be presented in order to give the reader the opportunity to follow this rather complex estimation process. In this thesis, all models are implemented through R programming, meaning that every step of the valuation process has been manually coded by the author. Even though some standardized software does exist, the manual approach has been chosen by the author to ensure that every step in the process is implemented with the desired method. Standardized software does not allow for the same transparency in the process as it is the case when the model is built from scratch. For the R code used for the one-factor Gaussian copula, see appendix 2.

As mentioned in the theoretical introduction to the one-factor Gaussian copula, the conditional default probabilities under the common factor M can be calculated first. Through numerical integration using the trapezoidal rule, the unconditional default probabilities can be calculated at each point in time. In other words, the loss distribution can be obtained. The following computational example will be based on the calculation of the spread of the tranche covering losses from 3-6% on April 3, 2007.

Table 4.3 below shows the conditional probability distributions for some given values of M at time t = 1. Since this table is only inserted for illustrative purposes, only the first 10 rows are included. In the actual implementation, a full 126 rows exist and the steps between the values for the common factor, M, are only 0.05. Thus, the true table is significantly larger than the illustration below.

	Conditional default probabilities								
# of defaults	Common factor, M								
	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
o	1.70%	15.75%	46.51%	74.96%	90.63%	97.00%	99.15%	99.79%	99.95%
1	7.05%	29.33%	35.71%	21.63%	8.92%	2.95%	0.84%	0.21%	0.05%
2	14.48%	27.09%	13.60%	3.10%	0.44%	0.04%	0.00%	0.00%	0.00%
3	19.66%	16.54%	3.42%	0.29%	0.01%	0.00%	0.00%	0.00%	0.00%
4	19.86%	7.52%	0.64%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
5	15.91%	2.71%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6	10.54%	0.81%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7	5.94%	0.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
8	2.90%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
9	1.25%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.48%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 4.3: Illustrative example of conditional default probabilities for values of M in interval -2 to +2 at time t=1.

To illustrate how these conditional default probabilities can be calculated, the example of the probability of experiencing exactly 1 default in the portfolio, given that the common factor m=-1 can be considered. This value is calculated in the following way:

First, the conditional default probability for each individual underlying entity can be calculated using equation (2.19) from section 2.3.3:

$$q_i(t|M) = H_i\left(\frac{-2.644 - (\sqrt{0.3} * -1)}{\sqrt{1 - 0.3}}\right) = 0.00610$$

The conditional default probability of exactly 1 default by time t=1 in the entire portfolio can then be calculated by applying equation (2.24):

$$P(l,t|M) = \frac{125!}{(125-1)!\,1!} * 0.00610 * (1 - 0.00610)^{125-1} = 35.71\%$$

The value calculated in this example has been highlighted in table 4.3.

Through numerical integration over the common factor M, implemented through the trapezoidal rule, the loss distribution can be calculated. The loss distribution presents the probability distribution for the number of defaults at each payment date within the maturity of the synthetic CDO. Figure 4.5, illustrates the loss distribution at the last payment date, t=5. For illustrative purposes, the loss distribution for other correlation inputs are included as well. Similar graph can be generated for all 20 time steps within the maturity of the synthetic CDO.



Figure 4.5: The portfolio loss distribution at t=5 for varying correlation assumptions

From figure 4.5, it is clear that the underlying correlation assumptions play a significant role in the generation of the loss distribution, and thus in the valuation process continued below. Based on the calculated loss distribution, the fair spread of each tranche can be calculated. Again, using the tranche covering losses from 3-6% as an example, the expected default loss at each relevant point in time can be calculated using equation (2.31):

$$EL_{i} = \sum_{l=0}^{125} p(l, T_{i}) \max(\min(l * 1.000.000 * (1 - 40\%), 6\% * 125.000.000) - 3\% * 125.000.000)$$

Using the expected default loss, the fair spread of the tranche can be calculated by applying equation (2.32) through (2.35) from section 2.4. Table 4.6 illustrates the calculation of the fair spread for this tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
М	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	2,027	2,006	2,027	2,006	927,322 s
0.5	2	0.9795	7,002	6,858	4,975	4,873	916,533 s
0.75	3	0.9694	14,095	13,663	7,093	6,876	905,354 s
1	4	0.9593	22,860	21,930	8,764	8,408	893,907 s
1.25	5	0.9494	32,997	31,329	10,137	9,625	882,275 s
1.5	6	0.9396	44,290	41,617	11,293	10,611	870,516 s
1.75	7	0.9299	56,570	52,607	12,280	11,420	858,676 s
2	8	0.9204	69,704	64,152	13,134	12,088	846,791 s
2.25	9	0.9109	83,583	76,132	13,879	12,641	834,891 s
2.5	10	0.9015	98,114	88,445	14,532	13,100	822,999 s
2.75	11	0.8921	113,221	101,010	15,107	13,478	811,135 s
3	12	0.8829	128,837	113,755	15,616	13,788	799,316 s
3.25	13	0.8738	144,904	126,621	16,067	14,040	787,556 s
3.5	14	0.8648	161,372	139,556	16,468	14,241	775,867 s
3.75	15	0.8559	178,196	152,515	16,824	14,399	764,259 s
4	16	0.8470	195,337	165,460	17,141	14,519	752,741 s
4.25	17	0.8383	212,760	178,357	17,422	14,605	741,320 s
4.5	18	0.8297	230,432	191,178	17,673	14,662	730,003 s
4.75	19	0.8211	248,327	203,898	17,894	14,693	718,796 s
5	20	0.8126	266,417	216,494	18,091	14,701	707,701 s
Sums						234,774	16,347,957 s
Fair spread							143.61 bps

Table 4.6: Calculation of fair spread of 3-6% tranche.

Ultimately, the fair spread of the tranche is calculated as:

 $Fair \ spread = \frac{234,774}{16,347,957} = 143.6 \ bps$

Comparing this to the quoted spread of 59 basis points, it is clear that at least either the model or the input assumptions does a poor job of matching the quoted spread for this tranche.

By repeating the same procedure for each of the three dates and for all CDO tranches, the remaining results of the one-factor Gaussian copula can be calculated in a similar way. However, in order to compare the performance of the Gaussian and the Student t copula, the model is instead calibrated to the available market quote for the equity tranche at each point in time. This approach allows for a direct comparison between the two models' ability to price all tranches simultaneously. The implied correlation corresponding to the market quote for the equity tranche has been found through an iterative approach. As mentioned in section 3.2, no converted equity quote is available for November 13, 2008, which means that calibration cannot be conducted in the same way for this date. Consequently, only equity calibration results for April 3, 2007 and May 9, 2017 will be presented below. The outcome of the calibration process is summarized in table 4.7:

Equity tranche calibration									
03-04	-2007	09-05-2017							
Correlation	Model quote	Correlation	Model quote						
0.15	861	0.2	1542						
0.175	831	0.3	1402						
0.185	819	0.45	1033						
0.186	817	0.471	989						
0.187	815	0.475	981						
0.2	771.8	0.5	931						
0.3	684.2	0.6 844							
Market	815	Market	989						

Table 4.7: Calibration results for one-factor Gaussian copula model for April 3, 2007 and May 9, 2017.

In table 4.7, the implied correlations for date 1 and 3 have been marked in green. The implied correlations are 18.7% and 47.1% for April 3, 2007 and May 9, 2017 respectively. In other words, the correlation factor that makes the one-factor Gaussian copula fit the equity tranche quote on April 3, 2007 is 18.7% and on May 9, 2017 is 47.1%. The loss distributions generated under these correlations assumptions can be found in appendix 4-5. Individually, the implied correlation for the equity tranche does not provide much information about the model's ability to price each tranche correctly. However, when applied to the pricing model for all other tranches on the given date, it will become evident how well the one-factor Gaussian copula is able to fit market quotes in general. In the table 4.8-4.9 below, the remaining tranches have been priced from the implied correlations from table 4.7. The fair spread calculation table for all tranches for April 3, 2007 can be found in appendix 6.

	Estimation results 03-04-2007								
Tranche	Estimated spread	Market quote	Error	Comment					
0-3%	815	815	0.00%	Calibrated					
3-6%	110	59	86.44%	Overestimate					
6-9%	26	14	85.71%	Overestimate					
9-12%	7	6	16.67%	Overestimate					
12-22%	1	3	-66.67%	Underestimate					
22-100%	0	N/A	-	-					

 Table 4.8: Estimation results for April 3, 2007 based on implied equity tranche correlation.

	Estimation results 09-05-2017								
Tranche	Tranche Estimated spread Market quote Error Comment								
0-3%	989	989	0.00%	Calibrated					
3-6%	392	181	116.57%	Overestimate					
6-12%	188	77	144.16%	Overestimate					
12-100%	11	27	-59.26%	Underestimate					

 Table 4.9: Estimation results for May 5, 2017 based on implied equity tranche correlation.

Looking at the results from the two dates considered, a relatively clear pattern can be identified. From the above, it seems that the one-factor Gaussian calibrated to the equity tranche quote tends to overestimate the spread on the tranches covering losses from 3-12%, while underestimating the spread on the most senior tranches covering losses above 12% of the reference portfolio. This result is similar across the two dates. Thus, it appears that the implied correlation at which the different tranches are traded varies significantly across tranches.

As mentioned earlier, converted data for November 13, 2008 has proved to be unobtainable. However, as this date represent the environment during the financial crisis, important insights can still be gained from the available market quotes, and through an assessment of how well the one-factor Gaussian copula fits the available quotes at this point in time. By calibrating the one-factor Gaussian copula model to the lowest available market quote, the following result appears:

	Estimation results 13-11-2008							
Tranche	Estimated spread	Market quote	Error	Comment				
0-3%	-	N/A	-	-				
3-6%	950	950	0%	Calibrated				
6-9%	638	492	30%	Overestimate				
9-12%	465	260	79%	Overestimate				
12-22%	267	105	155%	Overestimate				
22-100%	23	52	-56%	Underestimat				

Table 4.10: Estimation results for November 13, 2008 based on implied correlation derived from 3-6% tranche.

Common for all three dates is that the estimated spreads are relatively far from the available market quotes. In addition, it seems that there is no significant difference in model performance across the three dates. From this, it is clear that each tranche trades at very different implied correlations, assuming that the one-factor Gaussian copula can be considered as the standard market model. This is independent from whether the model is calibrated to the equity quote or the 3-6% tranche quote. In other words, it seems that the one-factor Gaussian copula is insufficient for pricing all tranches simultaneously, as different correlation assumptions are necessary to make each tranche fit the quoted spreads. Exactly how different the underlying correlation assumptions are for the individual tranche quotes, is illustrated in the following implied correlation plot for April 3, 2007:



Figure 4.11: Implied correlations for all tranches at April 3, 2007. The shape of this graph forms the so-called "correlation smile".

Figure 4.11 shows that the most and least risky tranches trade at a significantly higher implied correlation values than it is the case for the intermediate tranches. In the literature, this phenomenon is typically referred to as the correlation smile, simply due to the appearance of the graph connecting the points between the implied correlation of each tranche. The occurrence of this phenomenon, when applying the Gaussian copula for synthetic CDO valuation, is supported by multiple other researchers. Among others, Amato & Gyntelberg (2005) find a similar result when applying the model to the CDX.NA.IG index. However, proving the existence of the phenomenon in the data considered in this thesis allows for a more direct comparison between the one-factor Gaussian copula and possible alternatives. Several possible explanations behind the occurrence of the model, Amato & Gyntelberg (2005) suggest that the smile reflects uncertainty between market participants, concerning how to correctly model credit risk correlation. If this is indeed the case, it would mean that the tranches that are most sensitive to correlation changes, the equity and the most senior tranche, would have a larger model risk premium embedded in their pricing.

In short, the valuation conducted in this section shows that the one-factor Gaussian copula is insufficient for matching the market quotes of the tranched ITraxx index. However, as a number of model assumptions are required in the application of the model, the next section will consist of a sensitivity analysis targeting several of these model assumptions.

4.3 One-factor Gaussian copula sensitivity analysis

In this section, a sensitivity analysis of some of the main model assumptions will be conducted. The purpose of this section is to provide the reader with an understanding of the importance of some of the key assumptions, which every investor is required to make when valuing synthetic CDOs within this model setup. A full understanding of the sensitivity of model estimates is required in order to understand the use, and potential misuse, of the Gaussian copula in the crisis, and the implications that this could have had for its escalation. The implications of model assumptions will play an important part of the discussion in chapter 5.

In the following, the sensitivity of model estimates towards three model inputs will be emphasized. The assumptions considered are the pairwise asset correlation in the reference portfolio, the recovery rate assumption, and the sensitivity towards changes in the underlying CDS spreads.

4.3.1 Correlation sensitivity



Table 4.12 displays the sensitivity of the fair spread of each tranche to varying asset correlation assumptions.

Figure 4.12: The correlation sensitivity of synthetic CDO tranches. Note that a different axis (left) is applied for the 0-3% tranche.

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By varying the asset correlation assumption, while keeping all other inputs fixed, several observations can be made regarding the sensitivity of the fair spread of each tranche. For the equity tranche, it is clear that the fair spread is a strictly decreasing function of the pairwise asset correlation. The higher the asset correlation, the lower the risk of the equity tranche. Intuitively, this result can be explained from the fact that a higher asset correlation implies an increasing probability of the extreme case of 0 defaults in the portfolio, and thereby no loss to the equity tranche investors. Conversely, an increasing asset correlation also implies a greater likelihood that many assets will default simultaneously, meaning that the equity and mezzanine tranches will be wiped out completely, and investors even in the most senior tranche will incur a loss. However, as equity tranche investors gain more in the first scenario than they lose from the second, the spread of this tranche is a negative function of correlation.

The opposite is the case for investors in the 22-100% tranche, meaning that the spread of this tranche is an increasing function of the asset correlation. Again, this makes sense intuitively as increasing asset correlation means that the probability of incurring a loss for investors in the 22-100% tranche increases correspondingly. Thus, the 22-100% tranche becomes increasingly risker for higher asset correlations.

For the remaining intermediate tranches, the sensitivity towards correlation changes is not as straightforward. In figure 4.12, this can be observed from the fact that the 3-6%, 6-9%, and 9-12% displays concave-looking graphs, meaning that identical fair spread values can be generated from multiple correlation inputs. In other words, this means that these tranches have more than one implied correlation value for some range of the possible fair spreads. For instance, in the 3-6% tranche, a fair spread of approx. 142 bps can be generated for correlation values of approx. 0.3 and 0.7. In addition, it appears that the more senior the attachment point of the intermediate tranches, the less sensitive the tranche is to correlation changes. It seems that the less the credit enhancement, the earlier the function for the fair spread of the intermediate tranche reaches its maximum. By comparing the 3-6% and 9-12% tranche, it is clear that the curve for the 3-6% tranche increases rapidly for correlations less than 0.5, before it reaches its maximum, and starts decreasing as the correlation continuous towards 1. For the 9-12% tranche, this does not happen before a correlation of approximately 0.8 is reached.

4.3.2 Sensitivity to recovery rate assumption

By including the recovery rate assumption in the sensitivity analysis, an impression of the tranche sensitivity towards varying recovery rate assumptions can be achieved. Figure 4.13, illustrates the sensitivity of the calculated fair spreads for various combinations of asset correlation and recovery rate assumptions.





Figure 4.13: Fair spread sensitivity towards changing correlation and recovery rate assumptions for 0-3% tranche and 22-100% tranche at April 3, 2007 in one-factor Gaussian copula.

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Figure 4.13 compares the sensitivity of the 0-3% tranche with the sensitivity of the 22-100% tranche. As described in section 4.3.1, the correlation effect on these two tranches is directly opposite, as the increasing probability of extreme values affects each tranche differently. Thus, the higher the correlation, the lower the spread on the equity tranche, and conversely, the higher the correlation, the higher the spread paid on the 22-100%. Furthermore, from figure 4.13 it is evident that the fair spreads in both tranches are increasing for decreasing recovery rate assumptions. Intuitively this makes sense as decreasing recovery rates implicitly means that tranche investors take on more risk, when a smaller fraction of the notional is recovered in case of default. However, although the general effect of recovery rate changes is similar for both tranches, the magnitude of the changes is highly dependent on both correlation assumption, and the tranche considered. From the surface plot of the 0-3% tranche, it becomes apparent that the sensitivity towards recovery rate changes is highest for low correlation values. Intuitively, this result can be explained from the fact that the recovery rate naturally only influences the generation of the fair spread in a situation where defaults are likely to occur, which for the equity tranche is most likely to happen when correlation is low. Oppositely, the surface plot for the 22-100% tranche shows that the recovery rate only affects the fair spread when correlation exceeds 40%. Again, the reason for this is that for correlation assumptions below 40%, the probability of a loss in this tranche is so small that recovery rate assumptions become practically unimportant.

4.3.3 Changing credit spreads

As illustrated in the above, the amount of risk associated with an investment in the different synthetic CDO tranches varies significantly. Apart from the risk of using incorrect or unrealistic input assumptions, such as incorrect asset correlations, CDO investors must also be aware of the risk of changing CDS spreads in the reference portfolio. Possible changes in the fair value of a given synthetic CDO tranche is relevant both to investors that do not intend to hold their investment until maturity, as well as other financial institutions that are subject to mark-to-market accounting rules. The specific implications of mark-to-market accounting will be discussed further in chapter 5. In the following, the sensitivity of the fair value to changes in the underlying CDS spreads will be investigated. Comparing the sensitivity across the different tranches will provide an indication of the amount of risk associated with investments in these tranches. Specifically, the impact of a shock of 10 basis points to the CDS spreads in the underlying portfolio, will be the focus of this section.

As presented in section 2.4 the mark-to-market value of a synthetic CDO tranche is defined as:

$$MTM = s \sum_{i=1}^{n} D_i \Delta_i ((H-L) - EL_i) - \sum_{i=1}^{n} D_i (EL_i - EL_{i-1})$$
(4.1)

or

$$MTM = Fee - Contingent \tag{4.2}$$

As mentioned earlier, the fair spread calculated in the previous sections, corresponds to a MTM value of 0.

However, as changes in the underlying CDS spreads implicitly corresponds to changes in the default probability of the individual reference entities, a new loss distribution will have to be calculated in order to implement the shock to the underlying CDS spreads. Thus, by keeping the initial fair spread fixed, but applying the new loss distribution for the generation of everything else in equation (4.1), the change in mark-to-market value can be determined. Table 4.14, summarizes the result of a 10bps negative and positive shock, respectively.

Tranche		Principal		-10 bps	% of principal		+10 bps	% of principal
0-3%	€	3,750,000.00	€	500,275.42	13.34%	€	-417,399.82	-11.13%
3-6%	€	3,750,000.00	€	116,838.70	3.12%	€	-148,031.39	-3.95%
6-9%	€	3,750,000.00	€	31,591.54	0.84%	€	-50,975.62	-1.36%
9-12%	€	3,750,000.00	€	9,502.05	0.25%	€	-18,284.93	-0.49%
12-22%	€	12,500,000.00	€	4,389.64	0.04%	€	-10,274.07	-0.08%
22-100%	€	97,500,000.00	€	107.69	0.00011%	€	-366.57	-0.00038%

Table 4.14: The impact of a 10 bps shock to the underlying CDS spreads on the fair value of synthetic CDO tranches.

From table 4.14, it is clear that an increase in the underlying CDS spreads has a negative impact on the MTM value for all tranches, while a decrease in the underlying CDS spreads causes MTM values to increase. Intuitively, this can be explained by the fact that increasing CDS spreads implicitly corresponds to increasing default probabilities in the underlying portfolio, and vice versa. Furthermore, it can be observed that that the equity tranche carries the majority of the risk of changing underlying CDS spreads. Considering the case of a 10bps increase in the underlying CDS spreads, the MTM value of the equity tranche decreases by EUR 417,399, which corresponds to 11.13% of the total tranche principal. Comparing this to the EUR 366 decrease in MTM value of the 22-100% tranche, it is clear that it is primarily the holders of the equity tranche that are exposed to the risk of changing CDS spreads, despite the fact that it only constitutes 3% of the total CDO notional. The 3-6% tranche has a decrease in MTM value, which corresponds to 3.95% of the tranche principal. Thus, out of the total impact that this shock has on the entire CDO, almost all the risk is carried by the tranches with no, or very little credit

enhancement. Naturally, this is important to consider for potential synthetic CDO investors as changing market conditions, here reflected by changing CDS spreads, is an important factor in any investment decision.

4.3.4 Summary – One-factor Gaussian CDO valuation

In short, the calibration of the one-factor Gaussian copula to the market equity tranche quote proved the existence of the so-called correlation smile. Specifically, this phenomenon means that the model is unable to price all tranches simultaneously, as the outer tranches are traded at higher implied correlations than it is the case for the intermediate tranches. For the estimated fair spreads, this means that the model compared to market quotes, overestimates the fair spread for the intermediate tranches the fair spread for the intermediate tranches, while it underestimates the fair spread for the most senior tranche in the synthetic CDO. Mainly, this is mispricing is due to the lack of tail dependence in the Gaussian copula, which ultimately results in the probability of extreme values being too small.

The sensitivity analysis illustrated the importance of the applied input assumptions in relation to the estimated fair spreads for the 0-3% tranche and the 22-100% tranche in the synthetic CDO. Specifically, it was shown how increasing correlation assumptions had directly opposite effects on the two tranches, as the fair spread of the equity tranche was decreasing with correlation, while the fair spread of the most senior tranche was increasing. For the tranches in between, the effect of increasing correlation was not as straightforward as it depended on the range within this increase in correlation occurred. In addition, by including the recovery rate assumption in the sensitivity analysis, it became clear that the estimated spreads were most sensitive to recovery rate changes, in the range of correlation values that generated the highest fair spread values for the given tranche. The substantial amount of risk associated with investments in the 0-3% tranche was also made evident through the analysis of fair value sensitivity towards changes in the underlying CDS spreads. Here it was shown that almost all the risk associated with changes in the reference portfolio, was carried by the 0-3% and 3-6% tranche, whereas the MTM value of the more senior tranches were almost immune to small changes in default probabilities in the reference portfolio.

4.4 One-factor Student t copula estimation

As described in the theoretical introduction in section 2.3.4, the Student t valuation process will be implemented through the use of Monte Carlo simulation. The simulation of the loss distribution and the valuation of the tranches at each date will be based on the same assumptions as described in section 4.1. However, one additional

assumption will have to be made regarding the applied Student t distribution. In the following, the asset value of each entity is assumed to follow a Student t distribution with 4 degrees of freedom. As mentioned earlier in the thesis, the t distribution moves towards the Gaussian when the number of degrees of freedom goes towards infinity. Therefore, 4 degrees of freedom has been chosen to insure an appropriate amount of tail dependence in the applied distribution.

The simulation of the loss distribution will be carried out through the steps described in section 2.3.4. The actual pricing of the tranches is done in the same way as in the Gaussian case, meaning that the only difference between the two approaches is the modelling of the portfolio loss distribution.

To be able to directly compare the two models discussed, the estimation carried out through the Student t copula model will be based on the implied correlation for the equity tranche found in section 4.2. This correlation input has been chosen since it will allow for a direct comparison between each model's ability to price all tranches simultaneously. In other words, providing the Student t copula model with the exact same inputs as the Gaussian copula will allow us to identify if any significant differences exist between the models' pricing abilities. Eventually, the insights gained about potential differences and similarities between the models will provide the basis for inference about the role of the Gaussian copula in the escalation of the global financial crisis, and if anything would have been different had the Student t copula been applied instead.

Applying the inputs described in section 4.1 and the simulation approach introduced in section 2.3.4, the estimated tranche spreads for each date can be generated. As the estimates are based on 100.000 simulations at each given time step, the underlying computations for the generated estimates were demanding. The underlying code for the simulation algorithm, which again was implemented through R programming, can be found in appendix 3. As the valuation process following the generation of the loss distribution is equal to the Gaussian case in section 4.2, the individual steps will not be presented in this section. However, some of the generated output from the simulation are included below for illustrative purposes.

Below an illustration of the loss distribution at time t = 5 with a correlation of 18.7% for April 3, 2007 is provided. For illustrative purposes, loss distributions for other correlation inputs are included as well. For the full loss distribution for the correlation value of 18.7%, see appendix 7.



Figure 4.15: Loss distribution for varying asset correlations in Student t copula for time t=5.

Based on the full loss distribution obtained through the Monte Carlo simulation, the fair spread of each tranche is again calculated using equation (2.31) through (2.35). An illustration of the fair spread calculation for the 3-6% tranche is provided below. For a similar description for the other tranches, see appendix 9.

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	13398	13260	13398	13260	924509 s
0.5	2	0.9795	26901	26349	13503	13226	911660 s
0.75	3	0.9694	41198	39935	14297	13858	898786 s
1	4	0.9593	56096	53815	14898	14292	885936 s
1.25	5	0.9494	70967	67379	14871	14119	873262 s
1.5	6	0.9396	86360	81148	15393	14464	860633 s
1.75	7	0.9299	102354	95184	15995	14874	848032 s
2	8	0.9204	118295	108872	15941	14671	835611 s
2.25	9	0.9109	133626	121713	15332	13965	823495 s
2.5	10	0.9015	149108	134413	15482	13956	811507 s
2.75	11	0.8921	164829	147052	15722	14026	799624 s
3	12	0.8829	180818	159651	15989	14117	787842 s
3.25	13	0.8738	196739	171915	15921	13912	776232 s
3.5	14	0.8648	213008	184210	16269	14070	764703 s
3.75	15	0.8559	228923	195930	15915	13621	753405 s
4	16	0.8470	244535	207132	15612	13224	742323 s
4.25	17	0.8383	260220	218143	15686	13149	731374 s
4.5	18	0.8297	276213	229160	15993	13269	720508 s
4.75	19	0.8211	292158	239887	15945	13092	709798 s
5	20	0.8126	307721	250058	15563	12646	699310 s
Sums						275811	16158551 s
Fair spread							170.7 bps

 Table 4.16: Calculation of fair spread for 3-6% tranche in one-factor Student t copula on April 3, 2007.

The estimated tranche spreads in the one-factor Student copula, given the implied equity correlation derived in the one-factor Gaussian copula in section 4.2, are presented below:

	Estimation results 03-04-2007									
Tranche Student t Gaussian Market										
0-3%	461	815	815							
3-6%	171	110	59							
6-9%	93	26	14							
9-12%	55	7	6							
12-22%	22	1	3							
22-100%	1	0	N/A							

Table 4.17: Tranche estimates generated from the one-factor Student t copula model on April 3, 2007. Fair spread values are based on

 the implied equity correlation value derived through the one-factor Gaussian copula in section 4.2

Estimation results 09-05-2017					
Tranche	Student t	Gaussian	Market		
0-3%	679	989	989		
3-6%	339	392	181		
6-12%	199	188	77		
12-100%	19	11	27		

Table 4.18: Tranche estimates generated from the one-factor Student t copula model on May 9, 2017. Fair spread values are based on

 the implied equity correlation value derived through the one-factor Gaussian copula in section 4.2

From the tables presented in the above, it is evident that given the same inputs as the Gaussian copula, different results are generated when applying the one-factor Student t copula. From the two analyzed dates, it appears that based on the same input, the Student t copula generates a lower spread for the equity tranche, but generally a higher spread for all other tranches. Using the Gaussian based input from section 4.2, it turns out that the estimates generated by the Student t copula deviates more from the market quotes. However, as the Student t model has not been independently calibrated to market quotes, these tables do not provide a fair illustration of Student t copula's ability to match these quotes. Instead, the tables can be used as a tool towards understanding how the two models price the individual tranches in relation to each other. Based on these two dates, it appears that the Student t distributes the risk somewhat differently than the Gaussian copula. More specifically, the fact that the Student t distribution has positive tail dependence means that the likelihood of many defaults increases in the loss distribution, which ultimately is reflected in the higher estimated spreads for the more senior tranches, and a lower spread for the equity tranche. Thus, compared to the Gaussian copula, the Student t copula redistributes some of the perceived risk from the equity tranche to the more senior tranches.

To gain a fair picture of the Student t copula's ability to price all tranches simultaneously, this model can be calibrated to the market quote for the equity tranche in the same way as it was done in section 4.2. Again, using an iterative approach the calibration of the Student t copula for date 1 and 3 yields the following implied correlations:

Equity tranche calibration					
03-04-2007		09-05-2017			
Correlation	Estimated spread	Correlation	Estimated spread		
0.187	461	0.471	679		
0.1	513	0.23	970.1		
0.01	565	0.215	982.4		
0	572	0.213	989		
Unable to match quote		0.21	1006		
		0.2	1018		
Market	815	Market	989		

Table 4.19: Calibration results for one-factor Student t copula for April 3, 2007 and May 5, 2017.

From table 4.19, it can be observed that the one-factor Student t copula with v = 4 is unable to match the equity tranche quote at date 1, whereas the implied correlation at date 3 is 0.213. Even with a correlation of 0, the model is not even close to matching the market quote of 815 observed on April 3, 2007. According to Burtschell, Gregory, & Laurent (2009), who experienced a similar issue in their paper, this can be explained from the fact that even with a correlation of 0, there is still too much tail dependence in the Student t model. As the level of tail dependence is a result of the number of degrees of freedom assumed in the model, a possible solution could be to increase this number from v = 4. This would make the Student t model go towards the Gaussian, and thereby lose tail dependence. However, as the rationale for considering the Student t copula is to include tail dependence in the valuation process, the number of degrees of freedom should not be raised too much. Nevertheless, this leaves May 9, 2017 as the only date applicable for assessing the Student t copula's ability to price all synthetic CDO tranches simultaneously. Using the implied correlation for the equity tranche to price the remaining tranches at this date, yields the following result:

Tranche	Estimated spread	Market quote	Comment
0-3%	989	989	Calibrated
3-6%	413	181	Overestimate
6-12%	193	77	Overestimate
12-100%	10	27	Underestimate

 Table 4.20: Tranche estimates from one-factor Student t copula model on May 9, 2017 based on implied equity correlation value derived using the one-factor Student t copula.

From table 4.20, it is evident that the Student t model does not accurately match the other tranche quotes, when using the implied equity correlation as input. Similar to the Gaussian, when calibrated to the equity tranche quote, the Student t copula overestimates the spreads on the intermediate tranches, while underestimating the spread on the most senior tranche. However, comparing the two implied correlations for the equity tranche across the two models, some differences still exists between the two models.

With a calculated implied correlation of 0.471 in the Gaussian model, and a corresponding value 0.213 in the Student t model, it is apparent that given the same correlation input, these two models price synthetic CDO tranches quite differently. Investigating this difference more closely will be the topic of the next section.

4.5 One-factor Student t sensitivity analysis

To fully understand the behavior and characteristics of the Student t copula compared to the Gaussian, a sensitivity analysis will be conducted in the same way as in section 4.3.2. By varying the correlation and recovery rate assumptions, a surface plot representing the Student t copula can be produced. As mentioned earlier, the main argument for investigating the Student t copula as an alternative to the Gaussian, is the fact that the Student t copula includes tail dependence. Theoretically, this should increase the probability of experiencing the extreme scenario of either no defaults or of many simultaneous defaults. In line with this argument, the surface plots presented below describe the spread sensitivity of the outer tranches, as these are the part of the synthetic CDO that are most highly exposed to the occurrence of these extreme scenarios. For comparability purposes, the surface plots are produced for April 3, 2007. Again, the plots are generated from 100.000 simulations at every correlation input. Since these plots are based on simulations, some level of uncertainty is connected to the results. However, a reliable impression of the behavior of the model can still be gained through the analysis.





Figure 4.21: Fair spread sensitivity towards changing correlation and recovery rate assumptions for 0-3% tranche and 22-100% tranche on April 3, 2007 in one-factor Student t copula.

By comparing these plots to the ones produced in section 4.3, several interesting observations can be made. First, looking at the equity tranche, it can be observed that the one-factor Student t copula generally prices the equity tranche lower than the Gaussian copula. This observation is intuitively aligned with the fact that the Student t distribution has above zero tail dependence, meaning that the extreme case of 0 defaults is assigned a higher likelihood in this model than in the Gaussian. This can be observed by comparing the top rows of appendix 4 and 7, as they represent the probability of experiencing 0 defaults at every time step during the lifetime of the COD. This has been done in table 4.22.



Figure 4.22: The probability of 0 defaults at each timestep for correlation of 0.187 in both Gaussian and Student t model.

As a result, the equity tranche is perceived as less risky in the Student t model, which means that the estimated spread is lower for any given combination of correlation and recovery rate. The lower general risk assigned to the equity tranche in the Student t copula also makes the plot appear less steep than the Gaussian, indicating that the estimated spreads are less sensitive to varying correlation and recovery rate inputs. This is especially the case for the low correlation levels, which results in the peak being much higher and steeper in the Gaussian model.

Second, considering instead the plot describing the sensitivity of the most senior tranche in the synthetic CDO, a different pattern can be observed. Here, it appears that the Student t model generally estimates a higher spread for the given correlation and recovery rate combination, except for when correlation reaches 1. Similar to the explanation for the equity tranche, the fact that the Student t copula perceives the secured super senior tranche to be riskier than the Gaussian copula, can be explained by its "fatter" tails. This tail dependence means that the extreme scenario of many simultaneous defaults is more likely in this model. As this characteristic puts investors in the most senior tranche at a higher risk, the estimated fair spread is correspondingly higher.

However, due to the general similarity in the shape of the surface plots for the two models considered, another interesting observation can be made. Fixing the recovery rate assumption at 40%, which has been the case throughout the analysis, it appears that applying the Student t copula instead of the Gaussian, approximately corresponds to increasing the correlation coefficient in the Gaussian copula. For the equity tranche, this will

decrease the calculated spread, and increase the estimated spread for the 22-100% tranche. This can be observed from the graphs below.



Figure 4.23: Correlation sensitivity of fair spread of 0-3% tranche in one-factor Gaussian copula and one-factor Student t copula.

Recovery rate kept fixed at 40%.



Figure 4.24: Correlation sensitivity of fair spread of 22-100% tranche in one-factor Gaussian copula and one-factor Student t copula. Recovery rate kept fixed at 40%.

The above graphs show that except for the highest correlation values $\sim(0.95 - 1)$ it approximately holds true that using the Student t copula instead of the Gaussian corresponds to increasing the correlation input in the Gaussian copula. Of course, due to the non-linearity of the illustrated graphs, the magnitude of this increase varies depending on the correlation level. However, the resemblance in the shape of the graphs indicates that within each model, the relative impact of correlation changes is quite similar. This finding corresponds to the fact that no obvious improvement was achieved in relation to simultaneously pricing all synthetic CDO tranches correctly in the Student t copula. Both models appear to have similar issues when pricing the remaining synthetic CDO tranches based on the implied equity correlation.

4.6 Summary – One-factor Student t estimation

In short, through the Monte Carlo implementation of the Student t copula multiple things can be concluded. Using the calibrated implied equity correlation from the Gaussian copula, it was evident that for this particular input value, the Student t copula distributed the risk rather differently between the synthetic CDO tranches. Specifically, less risk was assigned to the equity tranche, and more risk assigned to the more senior tranches. Consequently, a lower fair spread was generated for the equity tranche and a higher spread for the senior tranches.

In the calibration of the Student t copula to the market equity tranche quote, difficulties occurred due to the heavy-tailed nature of a Student t distribution with 4 degrees of freedom. However, even in the case where a successful result was obtained through the calibration, no particular improvement could be identified in relation to simultaneously pricing all synthetic CDO tranches.

Last, through the sensitivity analysis it was clear that compared to the Gaussian copula, the relative tranche risk was distributed differently in the Student t copula. More precisely, it could be observed that for a given value of correlation and recovery rate, the Student t copula generated a lower equity tranche spread, and a higher senior tranche spread than the Gaussian copula. However, the sensitivity to input changes within the Student t copula appeared to be quite similar to the picture gained through the sensitivity analysis of the Gaussian copula. This is in line with the lack of improvement in relation to pricing all tranches simultaneously. Thus, it appears that using the Student t copula instead of the Gaussian mostly has a "level" effect on the estimated fair spreads, but no significant improvement is obtained regarding the relative pricing of the synthetic CDO tranches.

4.7 Summary – Synthetic CDO valuation

In summary, several things can be concluded about the use of the one-factor Gaussian copula and one-factor Student t copula for synthetic CDO valuation. First, it became clear that neither of the models showed particularly satisfying results in relation to pricing all synthetic CDO tranches simultaneously. When calibrated to the market Master's Thesis

equity tranche quotes, both models overpriced the intermediate tranches but underpriced the most senior tranche in the CDO. In the Gaussian section, the occurrence of the so-called correlation smile was argued to be the main contributor to the mispricing. Here, it was shown that the outer tranches typically trade at lower implied correlations than it is the case for the intermediate tranches. In addition, it was shown just how sensitive the MTM value of the most junior CDO tranches is to changes in the underlying CDS spread. From a 10bps change in the CDS spreads of the reference entities, the equity tranche lost approx. 11% of its fair value, proving just how easily investors in such a tranche can lose substantial amounts of value due to only minor changes in the underlying market conditions.

Second, through a sensitivity analysis of both models it was evident that the estimated spreads are highly sensitive to recovery rate and correlation assumptions, which underlines the importance of carefully choosing these input variables, when applying either model. In addition, by comparing the surface plots of the 0-3% and 22-100% tranche produced with both models, it was concluded that the Student t copula generally estimates a lower fair spread for the 0-3% tranche, while it on the other hand estimates a higher spread for the 22-100% tranche, given the same correlation input. This finding is in line with the Student t copula's higher level of tail dependence, as this characteristic increases the probability of extreme scenarios in both ends of the distribution.

Finally, based on the similarity in the shape of the surface plots it can be said that applying the Student t copula roughly corresponds to increasing the correlation level in the Gaussian copula. This would have the above described effect of decreasing the 0-3% tranche spread, while increasing the 22-100% tranche spread. However, considering a scenario where the users of the models build their expectations on a given correlation assumption, quite different results will be achieved by using one opposed to the other. As a result, investors' impression of risk associated with a given tranche will vary correspondingly. This along with other implications of the use of Gaussian copula model, and the Student t copula as an alternative, will be discussed next.

5. Discussion

In continuation of the model oriented analysis conducted in chapter 4, the purpose of this section is to relate the model specific conclusions to the financial crisis. This will be done by considering the impact of applying an incorrect model, compared to some of the other factors that might have contributed to the eruption of the crisis. As mentioned earlier in the thesis, the Gaussian copula has been labeled as a main contributor to the financial crisis, and some have even gone as far as naming it "the formula that killed Wall Street" (Salmon, 2012) or "the formula from Hell" (Lee, 2009). It has therefore been speculated whether the use of an alternative model could have mitigated the impact of inaccurate CDO pricing on the global financial crisis. However, from the analysis conducted in chapter 4 it appears that specifically accusing the Gaussian copula, and the mathematician behind it, of being main contributors to the outbreak of crisis cannot at all be justified.

5.1 Did the Gaussian copula really kill Wall Street?

Although the analysis in the previous section confirms the short-comings of the Gaussian copula, it also proves that the underlying assumptions are what ultimately determines the estimated spreads. In addition, the comparison between the performance of the Gaussian copula and the Student t copula showed that even when correcting for the Gaussian's main flaw, the lack of tail dependence, no immediate improvement in the estimation results was obtained. Consequently, the analysis showed that using the Student t copula as the market accepted model instead of the Gaussian, probably would not have resulted in radical changes for its applicants, as no real improvement was achieved in the ability to price all tranches simultaneously.

However, a scenario exists in which you could argue that valuable information could have been gained through the use of this alternative model. This is so, as the higher estimated spreads generated for the most senior tranche in the Student t copula, could have provided investors with some kind of warning sign about the risk associated with the use of the Gaussian copula. Had investors directly compared the estimated 22-100% tranche spreads across the two models, it could potentially have provided an indication about the riskiness of this tranche, and that this riskiness might be worth investigating in more detail. In other words, the alternative Student t copula could have been used as a way of stress-testing the results generated in the Gaussian copula.

For instance, returning to figure 4.23 one could argue that if an investor had run both models with the same correlation input, say 0.6, he would have discovered that where the Gaussian copula estimates a fair spread of

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approximately 2 bps, the Student t copula estimates a fair spread of more than 5 bps. Despite this being very small values, relatively speaking this is still a considerable difference, as investors typically have had extremely large positions in senior CDO tranches due to a very low level of perceived risk. Revisiting also table 4.17 and 4.18, this result also shines through for the correlation input applied in this thesis. For date 1, the Gaussian copula estimates a spread for 1 bps and 0 bps for the 12-22% tranche and 22-100% tranche, respectively. In comparison, the Student t copula estimates a spread of 22 bps and 1 bps for the same tranches. For date 3, the Gaussian copula estimates a fair spread of 11 bps for the 12-100% tranche, and the Student t copula generates a spread of 19 bps. Considering these results, it can be inferred that the Student t copula generates a considerably higher spread for the most senior synthetic CDO tranches. However, from the very low estimated spread for the 22-100% tranche at date 1, it is clear that the Student t copula is in no way an optimal model either. Again, despite this being small values, a considerable relative difference exists between the spreads generated through each model. As investors typically have had extremely large positions in the senior CDO tranches, it is imaginable how using one model opposed to the other could end up being worth millions of dollars.

This being said, the sensitivity analysis conducted on both models perfectly illustrated that the estimated spreads are highly dependent on both correlation and recovery rate assumptions. Within science it is commonly known that no model is any better than its underlying assumptions. As the majority of all other mathematically based financial models, most of the applied copula model inputs are based on historical data. However, as it turned out, many of the things that happened before and during the financial crisis were to some extent considered to be unprecedented, or at least many people had forgotten that things could go any differently.

A perfect example of markets being blinded by the most recent trends and developments, was the situation in the US housing market in the years leading up to the crisis. Figure 5.1 shows the U.S National Case-Shiller index from the period 1995-2012. This index, originally developed by economists Robert Shiller and Karl Case, tracks US housing prices on a national level.



Figure 5.1: The S&P Corelogic Case-Shiller US National index from 1995-2012 Source: Own contribution. Data retrieved from S&P Down Jones.

As it can be seen from figure 5.1, the US housing market increased significantly in the years 1995 to 2007. The prosperous environment in the housing market combined with the explosion in the so-called subprime mortgages, turned out to be a very dangerous cocktail. The increasing housing prices allowed borrowers to continuously refinance their loans, thereby avoiding the substantial interest rate increases that often kicked in after the first 2-3 years, at the expiration of the teaser period. However, as the bubble burst in 2007, they no longer had this opportunity and were therefore forced to pay the high interest rates that were required in the adjustable rate mortgages (ARMs) after the end of the teaser period (Jarrow, 2011). As many of the subprime borrowers had very bad credit ratings, they were unable to meet these payments and had to default on their loans. Assuming that a number of these subprime loans had been pooled together with the purpose of issuing securities, this would mean that the default correlation between the loans in such a pool would increase significantly. In relation to CDOs and the Gaussian copula, this would mean that the market implied correlation would drastically increase, among other things causing the market spread for the most senior tranche to increase as well.

The point of the above example is to illustrate that investors cannot rely solely on historical data and events, as it is in fact the expectation in the market that ultimately will determine the value of any product traded, and that this expectation can change in a matter of seconds. In the example above, no one, or at least very few, expected the housing market to crash until it did, which caused market expectations to change almost immediately. Consequently, it seems displaced to accuse the Gaussian copula model, or any other copula model for that matter, of being the main contributor to the escalation of the financial crisis. As it is the case for any other mathematical model, all the Gaussian copula model did was to provide the users with the best possible estimate, given the inputs it was provided. Thus, having used the Student t copula instead of the Gaussian probably would not have changed very much as any spread value estimated through this model is just as sensitive to the inputs as it is the case for the Gaussian. In a research paper from 2010, Catherine Donnelly & Paul Embrechts support the view presented in this section with the following phrase: *"A copula does not model economic reality but is a mathematical structure which fits historical data"* (Donnelly & Embrechts, 2010, p.15).

5.2 The impact of mark-to-market accounting

From the introduction to credit derivatives markets given in section 1.4, it became clear that the years preceding the financial crisis were dominated by explosive growth in both CDS and CDO issuances, as well as the outstanding notional amounts of these products. This development was primarily driven by the search for arbitrage profits, capital relief by the issuers, and high investor demand. For many financial institutions, the combination between inaccurate pricing and risk assessment of credit derivatives resulting from the outspread use of the Gaussian copula, and the concept of mark-to-market accounting, turned out to be extremely dangerous.

In essence, mark-to-market accounting implies that a firm's balance sheet reflects market value of both assets and liabilities. More specifically, this is secured by reevaluating the assets quarterly, according to the price that could be obtained if sold under the current market conditions (Pozen, 2009). This approach makes the acquisition price of the asset unimportant, meaning that it does not leave room for over-optimistic valuations based on historical values. During the financial crisis, many actors within the financial industry argued that mark-to-market accounting drove the value of many credit derivatives below their true value, mainly due to a high level of uncertainty and illiquidity in the market. This pushed some financial institutions into insolvency, and thereby forced them to sell their assets at fire-sales prices, which ultimately resulted in the value of similar assets decreasing even further, as they in turn were market to these fire-sales prices (Pozen, 2009). Thus, mark-tomarket accounting forces financial institutions to take loses on the books at a running basis.

Considering mark-to-market accounting in relation to the findings in chapter 4, it is imaginable how some financial institutions could find themselves in serious trouble, given an incorrect pricing and risk-assessment of synthetic CDO tranches. For CDO tranche holders, table 4.14 illustrated the loss that investors would be forced to take on their books, due to the changes in the underlying market conditions. In the analysis in chapter 4, the

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investigated loss in MTM value was caused by changes in the underlying CDS spreads. However, during the financial crisis much talk evolved around the fact that many financial institutions were forced to take large losses on their balance sheet, mainly due the very illiquid CDO market that existed at this time. Returning to the observed market quotes presented in table 3.1, major changes occurred in the perceived riskiness of the tranched ITraxx Europe from April 3, 2007 to November 13, 2008. During this period, the quoted spread for the 3-6% tranche increased from 59 bps to 950 bps. This explosive increase in the perceived riskiness provides a clear indication of radically different underlying economic conditions on November 13, 2008. Considering this, it is imaginable how some financial institutions could not withstand incorporating such radical changes in the valuation of their balance sheet assets during such a short period of time.

Following the financial crisis, it seems that math, represented by the Gaussian copula, has been used as a scapegoat by many within the financial industry. However, as the analysis conducted in this thesis shows, it appears that the source of the problems should not be found in the mathematical details of this model, but rather in the behavior and assumptions underlying the model. As a result, the remainder of this chapter will evolve around a discussion of the underlying behavior, which brought numerous financial institutions to their knees. Naturally, the scope of the thesis means that the topics discussed in the following can only be treated at a rather general level. However, gaining an overview of some of the other factors in the financial system, which might have contributed to the escalation of the crisis, will help understanding why a mathematical model cannot be blamed individually.

5.3 The financial institutions

In their study from 2014, Donald MacKenzie and Taylor Spears document the widespread use of the Gaussian copula in the pre-crisis years across the financial sector. Among other things, this study finds that many actors were fully aware of some of the short-comings and dangerous simplifications embedded in the model, but kept on using it nonetheless. Whether this was for interbank communicative purposes, or because it might have justified certain behavior in the financial institutions, it is evident that the model was not considered and trusted naively as a correct representation of reality. Consequently, it seems difficult to argue that the specifics of the Gaussian copula, can be blamed for the eruption of a global financial crisis, as market agents knowingly used it in despite its now documented short-comings. Instead, it appears that other factors, possibly combined with the use of the Gaussian copula, carries the majority of the blame for how things evolved. These factors and their role within financial institutions will be discussed in the following.
5.3.1 Moral Hazard in financial institutions

"A moral hazard is where one party is responsible for the interests of another, but has an incentive to put his or her own interests first" (Dowd, 2009). In the years following the financial crisis, much talk has evolved around the excessive risk-taking that occurred in many financial institutions in the years leading up to the crisis, as well as the highly disputed government bailouts that were carried out after all the losses started raining down. Many argue that much of this excessive risk-taking has its basis in moral hazard problems across the financial industry, and that the key to well-functioning financial markets is to properly control these problems (Dowd, 2009).

5.3.1.1 The originate-to-distribute model

In relation to the financial crisis, an obvious example of a moral hazard problem is the so-called originate-todistribute mortgage origination model. As indicated from its name, this model builds on the mortgage securitization process, which has received a lot of criticism following the crisis. Specifically, the originate-todistribute model allows the mortgage broker to sell the originated loans to third parties. Typically this would either be to government sponsored enterprises such as Federal National Mortgage Association (Fannie Mae) and The Federal Home Loan Mortgage Corporation (Freddie Mac), or to credit derivatives issuers such as investment banks (Jarrow, 2011). In this process, the mortgage broker receives the loan principal plus a fee to cover the origination process. The next step is then that hundreds and sometimes even thousands of these loans are pooled together to form mortgage-backed-securities (MBS) that are ultimately sold off to investors (Jarrow, 2011). This loan origination process should be compared to the classic originate-to-hold model, where the mortgage originator keeps the loan on its own balance sheet, and thereby also keeps the risk associated with the loan. The point of the example is that as the distance between the originators and the ultimate holders of risk increase from the original originate-to-hold model, the incentives of the originator to complete a thorough screening and due diligence process, before originating a loan, decreases significantly (Purnanandam, 2011). As the loan originators carry no risk in relation to potential defaults, having received the full principal for the loan through the sale, a clear incentive problem arises. Among others, Purnanandam (2011) finds that this results in the origination of poor quality loans given to borrowers, who never should have received them in the first place. Even the investment banks buying the mortgages from the originators would not care to do their own due diligence, since they only played the role of an intermediary in this process, as long as demand for the issued credit derivatives remained. The lack of incentive from the investment banks to do their own due diligence meant that the incentive problem, concerning the mortgage brokers, was practically being passed on to the final investors in the mortgage-backed-securities.

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In relation to the synthetic CDOs considered in this thesis, the originate-to-distribute model is relevant to discuss as it became common for banks to apply this lending model to the origination of corporate loans, in the same way as it had been done for mortgages, credit card credits, student loans etc. (Bord & Santos, 2012). Thus, in the years preceding the crisis, many of the loans underlying synthetic CDO reference portfolios were originated in this fashion, which meant that no or very little due diligence had been conducted by the originating banks (Bord & Santos, 2012). According to Gibson (2004), issuers of synthetic CDOs often start out in a position, where they pay out more to tranche investors than they receive from the protection sold on the reference portfolio. However, during the lifetime of the CDO, this negative carry is expected to disappear as defaults occur in the reference portfolio, and the principal of the "expensive" equity tranche starts to diminish. In practice, this means that issuers are dependent on the occurrence of defaults in the reference portfolio, and one could therefore speculate whether banks deliberately choose to include "bad assets" when issuing a given CDO. Generally, the lack of incentive from security issuers to carefully screen the assets included in the underlying portfolio, meant that investors had to rely on credit rating agencies to correctly assess the risk of these securitized products. This reliance on credit rating agencies will be discussed further in section 5.4

5.3.1.2 Agency problems

A different, but equally relevant area within the financial industry, which cannot be ignored when discussing unhealthy behavior related to moral hazard problems, is the misalignment between the short-term incentives in compensation structures for the management of financial institutions, and the interests of the shareholders. Fundamentally, the issue arises from the fact that bankers, hedge-fund managers etc., use other people's money for their deals. In the same was as investment bankers use their banks' capital, hedge fund and private equity managers invest their backers' funds. The combination between the fact that their own personal exposure is often very limited, or even non-existent, and the short-term bonus structure within these institutions, results in what Robert Preston (2008) refers to as the "Greed Game". Typically, investment fund managers receive a significant part of their compensation through an annual bonus, which is based on the short-term trading performance (Jarrow, 2011). However, the basis for the misalignment of interests is the fact that this bonus, after being paid, cannot be rolled back in case of bad results in the following years. In his article from 2008, Robert Preston provides an example of what he refers to as the "Greed Game", a game which he describes as *"heads they win, tails everyone else lose"* (Preston, 2008). According to this article, hedge-fund managers could receive as much as 20% of the gains made from the funds invested, plus an annual management fee of 2% of the value of the funds. If, however, no gains were made during the year, but instead a substantial loss occurred,

there would be no sharing of losses, as only the backers would be affected. Simply speaking, this structure would mean that in a situation where the fund had made a gain of USD 500 million over the past three years, but then incurred a loss of USD 500 million in the next year, management would have been rewarded a bonus of USD 100 million during this period, while the backers would be left with a total loss of USD 100 million.

The fact that this very advantageous remuneration system in hedge funds and private equity funds obviously quickly became attractive for investment banking employees, ultimately forced the investment banks to introduce similar packages for their top employees (Preston, 2008). Again, introducing this bonus structure in investment banks ultimately meant that top bankers had the chance of benefitting heavily from the deals they made using the bank's capital, without risking any of their own wealth in case they turned out unsuccessful. With this payment structure in both investment banks, hedge funds, and private equity funds, managers across the industry in charge of trillions of dollars were encouraged to take on tremendous amounts of risk with the aim of making high short-term profits. Consequently, financial products that generated high short-term yields were in very high demand.

For firms investing in investment-grade bonds, the AAA-rated ABS, CDOs, and synthetic CDOs provided significantly higher yielded alternatives to AAA-rated Treasuries. Judging from the ratings, of course, no higher risk should be associated with investing in these products opposed to the Treasuries. However, according to Jarrow (2011), no one in the industry really believed this to be the case, but motivated by their short-term incentives, many chose to trust the ratings assigned by the credit rating agencies, as they did not want to spend resources doing their own due diligence. Of course, as this goal of short-term profit maximization is not in the interest of the shareholders, a clear misalignment existed between those providing the money and those managing the money. The fact that these structured finance products were assigned a AAA-rating by the rating agencies, allowed investment managers to increase the risk of their investments, without doing so on paper. However, as most people now know, these ratings turned out to be wildly wrong. Consequently, many have questioned the credibility of the credit rating agencies, and their allegedly neutral role as market "watchdogs". The role of the credit rating agencies will be discussed in the next section.

5.4 Credit rating agencies

In the US, credit rating agencies have been assigned the role of "national statistical rating organizations" by the US Securities and Exchange Commission (SEC). In this role, credit rating agencies are relied on, both by investors

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and regulators, to create unbiased and accurate ratings on debt instrument issued and traded in the financial markets (Jarrow, 2011).

For investors, credit ratings help determine and manage risk in a portfolio, while regulators rely on the ratings when setting up restrictions in relation to risk-taking, e.g. for pension funds. Essentially, credit rating agencies help limit the extent of asymmetric information between the issuer and investors in financial debt instruments. In order for the above mentioned agents to make decisions on an informed basis, complete reliability of the given credit ratings is required. However, following the outbreak of the financial crisis, the massive downgrades of structured debt products tells a rather different story. In December 2007, Moody's estimated that more than USD 45 billion worth of CDOs had gone into default, and that more than 2000 downgrades had been carried out in the previous month alone (Davies, 2007). Naturally, the substantial amounts of defaults and downgrades, of otherwise AAA-rated structured debt products, poses the question of how all major credit rating agencies could be so wrong in their judgement of these products. In the literature, two main causes are usually given, this being poor modelling skills by the credit rating agencies, and the inevitable conflict of interest that is present in the business model of every credit rating agency.

Coval, Jurek & Stafford (2008) argue that the ability to correctly rate corporate bonds, does not necessarily transfer to evaluating the risks of structured credit products. Due to the heavy growth in the market for credit derivatives, credit rating agencies were forced to develop new capabilities in relation to estimating the default correlation between assets, something they had never done when rating the usual single-name products. In addition, Coval et al. (2008) argue that credit ratings include nothing about systematic risk exposure, as they only account for the cash flow risk associated with default probabilities. Specifically, the systematic risk exposure is important to account for as structured finance creates distortions in the systematic risk exposures of securities, compared to single-name corporate bonds (Coval, Jurek, & Stafford, 2008).

According to MacKenzie & Spears (2014), a number of simplifying assumptions were used in relation to the application of the Gaussian copula in credit rating agencies. Especially, the simplifications were made in connection to the assumed asset correlation underlying the various types of CDOs. As it turned out, and as shown in chapter 4 of this thesis, the sensitivity towards the asset correlation assumption has proved to be extremely critical for the fair spreads generated through the model.

Most heavily criticized, however, has been the inevitable conflict of interest that exist in the business model of the credit rating agencies. This conflict originates from the fact that the rating agencies are paid to conduct the

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rating, by the issuer of the product being rated. According to Jarrow (2011), the payment to the rating agency takes place at a continuous basis, where the credit rating agencies provide continuous credit assessments in return. Thus, the future business of the rating agencies is highly dependent on a prosperous long-term relationship with some of the larger securities issuers. Naturally, such relationship is most easily achieved by providing the issuer with the desired rating, or alternatively specific information about how the product should be structured in order to achieve this particular rating. According to MacKenzie and Spears (2014), issuers had the opportunity to download CDO valuation models directly from the credit rating agencies. Essentially, this allowed issuers to tailor the underlying pool of assets such that a CDO with a sufficiently large AAA-rated tranche could be constructed from an underlying pool of the highest yielding assets possible. In other words, the use of the Gaussian copula in credit rating agencies allowed CDO issuers to maximize profits by pooling the riskiest possible assets that would still result in a sufficiently large AAA-rated tranche.

5.5 Summary – Discussion

In summary, it seems fair to conclude that it was not the Gaussian copula, which specifically "killed Wall Street". Through the comparison between this model and the alternative Student t copula, it became apparent that the only difference between the two was the fact that given the same input assumptions, the Student t copula generated a higher spread for the most senior CDO tranches and a lower spread for the equity tranche. However, no immediate improvement was achieved in the ability to simultaneously price all synthetic CDO tranches, which refers to the correct relative pricing between the synthetic CDO tranches. Thus, the only situation in which important knowledge could have been gained through the use of the Student t copula, was in the event that the user had provided both models with the same underlying assumptions, and thereby used the Student t copula as a way of stress-testing the spreads generated for the outer tranches. Had this method been applied, users could have been provided with an indication that the risk of the most senior CDO tranches was worth further investigation.

However, as the sensitivity analysis from chapter 4 shows, the estimated spreads in both models are very sensitive to the underlying assumptions. Therefore, it appears that more than it is the specifics of the model applied, it is the nature of the underlying assumptions that determine the fair spread generated through the copula model. In the context of the financial crisis, the fact that the majority of model inputs were based on historical data means that they in no way accounted for the risk that the future could be any different than the

past. Thus, as market conditions radically changed in the years surrounding the financial crisis, model estimates based on historical market conditions proved useless.

Based on the above, it is argued that the surrounding structures, rather than the specific use of the Gaussian copula model, was a more direct contributor to the escalation of the financial crisis. This specifically relates to the combination of the rather unrealistic assumptions about continuous growth and prosperity, and the fact that the financial system in general was packed with moral hazard problems and conflict of interests. It is especially the moral hazard problems connected to the securitization process in financial institutions, and the inability and unwillingness of the credit rating agencies to correctly rate these structured finance products, which is argued to have played a significant role in the crisis. In addition, studies from around the industry shows that market participants knowingly used the Gaussian copula, well-aware of its short-comings. Thus, it appears that more than having caused the crisis, math represented by the Gaussian copula, was used as a method to justify certain behavior, both in financial institutions and credit rating agencies.

6. Conclusion

In this thesis, the use of the one-factor Gaussian copula model for synthetic CDO valuation has been investigated. The motivation for this investigation was the heavy criticism that the Gaussian copula has received following the financial crisis. Headlines such as "the formula that killed Wall Street" and "the formula from Hell" paint a picture of a model, which many consider as one of the main contributors to the escalation of the financial crisis. The investigation has been conducted through a model-oriented comparison between the characteristics of the onefactor Gaussian copula and the one-factor Student t copula. The one-factor Student t copula is included in the analysis with the purpose of correcting the heavily criticized main flaw in the Gaussian copula, namely the lack of tail dependence.

Through a calibration of a semi-analytical version of the Gaussian copula to the market equity tranche quote, the implied correlation was derived and used as a foundation for the valuation of the remaining synthetic CDO tranches. For both dates considered here, the one-factor Gaussian copula showed a clear tendency towards overestimating the fair spreads on the intermediate tranches, while underestimating the fair spread on the most senior synthetic CDO tranches, if compared to market quotes. Through an analysis of the implied market correlations for all tranches, this relative mispricing of tranches was explained from the occurrence of the so-called "implied correlation smile", which describes the fact that the outer tranches of the CDO trade at significantly higher implied correlations than it is the case for the intermediate tranches. The implied correlation smile in the Gaussian copula is argued to be a result of the lack of tail dependence in the model.

Through the application of the Gaussian implied equity correlation to the valuation of all tranches in the onefactor Student t copula, the researcher was able to directly compare the behavior of each model given the exact same input assumptions. However, different from the implementation of the Gaussian copula, the Student t copula was implemented using Monte Carlo simulation. Here it was found that given the same assumptions, the one-factor Student t copula distributed the relative risk between the tranches somewhat differently. Compared to the Gaussian, the Student t copula generated a significantly higher fair spread for the senior tranches, while it generated a significantly lower spread for the equity tranche. However, when considering the ability to price all synthetic CDO tranches simultaneously, no immediate improvement was achieved through the use of this alternative model. As was the case for the first model, when calibrated to the market quote for the equity tranche, the Student t copula overestimated the fair spread of the intermediate tranches, whereas it underestimated the fair spread of the most senior CDO tranche. Master's Thesis

Through a sensitivity analysis of both models, it became evident just how sensitive the estimated spreads are to recovery rate and correlation assumptions. Thus, underlined through this analysis is the importance of carefully choosing these input variables, when applying either model. In addition, by comparing the surface plots produced in this analysis, it proved to be a general result that the Student t copula estimated a higher fair spread for the most senior synthetic CDO tranche, while it estimated a lower spread for the equity tranche, given the same underlying assumptions. The researcher finds this to be in line with the higher level of tail dependence in the Student t copula model, as this characteristic increases the probability of extreme scenarios in both ends of the loss distribution. In continuation of this, the researcher concludes that applying the Student t copula instead of the Gaussian copula approximately corresponds to increasing the correlation level in the Gaussian copula, as this would have a similar effect on the two tranches mentioned.

Finally, due to the similarities found between the two copula models, the researcher concludes that no radical changes would have occurred, had the Student t copula model been applied as market standard instead of the Gaussian. However, the higher fair spread for the most senior tranche generated through the Student t copula could potentially have provided its users with an indication that this tranche might not be as secure as otherwise communicated through its rating. Potentially, this could have limited the exposure to this tranche marginally, but in the big picture it probably would not have changed much. It was evident from the sensitivity analysis that the generated spreads in both models are highly sensitive to the underlying assumptions, meaning that the main focus should be on these, and the terms under which the model is applied, rather than the model itself. In the years preceding the crisis, a general attitude of overconfidence existed throughout financial markets. This overconfidence naturally influenced the assumptions and estimates made by market agents, as many saw no reason for the future being any different from the past. However, this proved to be an inarguably wrong assumption, and losses started raining down across the financial industry. As a result, a lot of the unhealthy and immoral behavior that took place throughout the industry was exposed to the public during this period. Thus, it is argued that more so than it was the short-comings of the Gaussian copula that played a significant role in the escalation of the crisis, it was the assumptions and behavior underlying the use of the model that should be considered as the main contributor. This is both in relation to the moral hazard problems that facilitated excessive risk-taking within financial institutions, and the inevitable conflict of interest that existed, and still does exist, within the business model of the credit rating agencies. Thus, this thesis shares the opinion expressed by Donnelly and Embrechts (2010) in the following quote: "For me, this (blaming the Gaussian copula for the crisis) is akin to blaming Einstein's $E=mc^2$ formula for the destruction wrecked by the atomic bomb" (p.24).

7. Future perspectives

"In a Blast From a Financial Crisis Past, Synthetic CDOs are back" (Whittall & Bird, 2017). Thus, sounded the headline on The Wall Street Journal website just a few weeks ago. Within the past few years, similar headlines have emerged across the financial press, most of them signaling a general concern related to the return of securitization, and structured finance products such as the CDO.

In its purest form, securitization plays an important facilitating role in the economy, as it allows for the transfer of credit risk and asset exposure across the financial industry. Among other things, this allows for improved access to funds, greater efficiency in loan origination, and improved possibilities for portfolio diversification for investors (Kiff, Jobst, Kisser, & Scarlata, 2009)

However, in the light of the recent financial crisis, it is clear that some boundaries will have to be set, in order to avoid a repetition of the mutation that occurred in this market in the years preceding the financial crisis. In this context, several regulative initiatives have already been introduced, with the most commonly known being the "Basel III" and the "Dodd-Frank Act". However, whether the most optimal way to avoid a repetition, is to wrap the financial institutions in a thick layer of capital and liquidity requirements, only time will tell. For now, one could speculate, whether it should simply be up to the management and the board of directors to ensure the long-term existence of the firm, as it is usually the rule in the world of capitalism. This, however, brings back the "too big to fail" issue, where the bankruptcy of large corporations is considered to have such a large negative impact on the general economy that it is an event that cannot be allowed to occur.

Looking ahead, the future development of the global financial markets must be considered dependent on whether the lessons learned from the financial crisis will remain in the conscience of market participants, or if they yet again will be blinded by greed and high growth rates. Seen in relation to the finding of this thesis, one can only speculate which mathematical model or financial innovation that will be held responsible the next time around.

8. References

Amato, J. D., & Gyntelberg, J. (2005). CDS index tranches and the pricing of credit risk correlations. *BIS Quarterly Review*, (March), 73–87.

Black, F., & Cox, J. (1976). Valueing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, *31(2)*, 351–367.

Bomfim, A. N. (2005). Understanding Credit Derivatives and Related Instruments. Elsevier Academic Press

Bord, V. M., & Santos, J. A. C. (2012). The Rise of the Originate-To-Distribute Model and the Role of Banks in Financial Intermediation. *Economic Policy Review (19320426), 18(2),* 21–34.

Bloomberg. (2017). ITraxx Europe tranche data. Available on 5th of May, 2017.

Burtschell, X., Gregory, J., & Laurent, J.-P. (2009). A Comparative Analysis of CDO Pricing Models under the Factor Copula Framework. *The Journal of Derivatives*, *16(4)*, 9–37.

Cifuentes, A., & Katsaros, G. (2007). The One-Factor Gaussian Copula Applied To CDOs: Just Say NO (Or, If You See A Correlation Smile, She Is Laughing At Your "Results"). *Journal of Structured Finance*, *13(3)*, 60–71.

Cousin, A., & Laurent, J. (2009). An overview of factor models for pricing CDO tranches. *In Frontiers In Quantitative Finance*: Volatility and Credit Risk Modeling (pp. 185–212). WILEY FINANCE.

Coval, J., Jurek, J., & Stafford, E. (2008). "Re-Examining The Role of Rating Agencies: Lessons From Structured Finance. *Journal of Economic Perspectives, (January),* 1–23.

Davies, P. (2007, December 13). CDO downgrades break new records. Financial Times. Retrieved from https://www.ft.com/content/03de1234-a8e7-11dc-ad9e-0000779fd2ac. Available 10th of April, 2017.

Donnelly, C., & Embrechts, P. (2010). The devil is in the tails: actuarial mathematics and the subprime mortgage crisis. *ASTIN Bulletin*, *4*(1), 1–33.

Dowd, K. (2009). Moral Hazard and the Financial Crisis. Cato Journal, 29, 141–166.

Dufey, G., & Rehm, F. (2000). An Introduction to Credit Derivatives. *University of Michigan Business School Working Paper No. 00–013*. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=249155

Duffie, D., & Singleton, K. J. (2003). *Credit risk: Pricing, management, and measurement*. Princeton series in finance.

Embrechts, P., Lindskog, F., & Mcneil, A. (2003). Ch.8 Modelling dependence with copulas and applications to risk management. *Handbook of Heavy Tailed Distributions in Finance*, 329–384.

Ferreira, H., & Ferreira, M. (2012). Tail dependence between order statistics. *Journal of Multivariate Analysis, 105(1),* 176–192.

Fiderer, D. (2011). The CDOs That Destroyed AIG: The Big Short Doesn't Quite Reveal What They Knew and When They Knew It. HuffPost. Retrieved from http://www.huffingtonpost.com/david-fiderer/the-cdos-that-destroyed-a_b_499875.html. Available 25th of April, 2017.

Gaiduchevici, G. (2014). Post-crisis CDO Valuation with Archimedean Copulas. *Procedia Economics and Finance*, *15(0)*, 19–26.

Garcia, J., & Goossens, S. (2010). The Art of Credit Derivatives. WILEY FINANCE.

Gibson, M. S. (2004). Understanding the Risk of Synthetic CDOs. FEDS Working Paper No. 2004-36. Available at SSRN Electronic Journal. Retrieved from http://doi.org/10.2139/ssrn.596442

Giesecke, K. (2004). Credit risk modeling and valuation: An introduction. *Available at SSRN 479323, (June), 1–* 40. From: http://doi.org/http://dx.doi.org/10.2139/ssrn.479323

Goegebeur, Y., Hoedemakers, T., & Tistaert, J. (2007). Synthetic CDO Pricing Using the Student t Factor Model with Random Recovery. *In Third Brazilian Conference on Statistical Modelling in Insurance and Finance*.

Greenberg, A., Mashal, R., Naldi, M., & Schloegl, L. (2004). Tuning correlation and tail risk to the market prices of liquid tranches. *Quantitative Research Quarterly*, (March).

Gregory, X., Burtschell J, & Laurent, J. (2007). Beyond the Gaussian Copula: Stochastic and Local Correlation. *Journal of Credit Risk*, *3*(*1*), 31–62.

Guegan, D., & Houdain, J. (2005). Collateralized Debt Obligations Pricing and Actor Models: A New Methodology using Normal Inverse Gaussian Distributions. *Papersssrncom*, 33(1), 1–29.

Hull, J. C. (2015). Options, Futures and Other Derivatives. Pearson.

Hull, J. C., & White, A. D. (2004). Valuation of a CDO and an n -th to Default CDS Without Monte Carlo Simulation. *Journal of Derivatives*, *12*(*2*), 8–23.

Jarrow, R. A. (2011). The Role of ABS, CDS and CDOs in the Credit Crisis and the Economy. *In Rethinking the Financial Crisis* (Vol. 202, pp. 210–235).

Jobst, A. (2008). What Is Securitization. Finance and Development 45(3), 48–49.

Justesen, J. (2009). *Prisfastsættelse af CDO ved Gaussian copula og Student t copula*. Copenhagen Business School.

Kalemanova, A., Schmid, B., & Werner, R. (2005). The Normal inverse Gaussian distribution for synthetic CDO pricing. *Working paper*.

Kiff, J., Jobst, A., Kisser, M., & Scarlata, J. (2009). Restarting securitization markets: policy proposals and pitfalls. *Global Financial Stability Report, (October),* Chapter 2.

Laurent, J., & Gregory, J. (2003). Basket Default Swaps, CDO's and Factor Copulas. *Journal of Risk,* 7(September), 1–21.

Li, D. X. (2000). On Default Correlation : A Copula Function Approach. *The Risk Metrics Group, Working Paper(99),* 31. Retrieved from http://doi.org/10.2139/ssrn.187289. Available 5th of February, 2017.

Longstaff, F. A., & Schwartz, E. S. (1995). A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *The Journal of Finance*, *50(3)*, 789–819.

MacKenzie, D., & Spears, T. (2014). "The formula that killed Wall Street": The Gaussian copula and modelling practices in investment banking. *Social Studies of Science*, *44*(*3*), 393–417.

Malevergn, Y., & Sornette, D. (2006). *Extreme financial risks: From dependence to risk management*. Springer-Verlag Berlin Heidelberg.

Markit. (2014). *Markit Credit Indices A Primer*. Markit Group. Retrieved from http://content.markitcdn.com/corporate/Company/Files/DownloadFiles?CMSID=577e364482314b31b158ae2c 2cecc89d. Available 10th of April, 2017.

Markit. (2017). *Markit Converter*. Markit Group. Retrieved from http://www.markit.com/converter.jsp. Available 5th of May, 2017.

McNeil, A. J., Frey, R., & Embrechts, P. (2005). Quantitative risk management: Concepts, techniques and tools. *Risk Management*, *101(476)*, 30.

Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance, 29(2),* 449.

Nocera, J. (2010, April 16). A Wall Street Invention Let the Crisis Mutate. *The New York Times*. Retrieved from http://www.nytimes.com/2010/04/17/business/17nocera.html. Available 15th of April 2017.

Oh, D. H., & Patton, A. J. (2015). Modelling Dependence in High Dimensions with Factor Copulas. *Journal of Business & Economic Statistics*, 37(1).

Pozen, R. C. (2009, November). Is It Fair to Blame Fair Value Accounting for the Financial Crisis? *Harvard Business Review*. Retrieved from https://hbr.org/2009/11/is-it-fair-to-blame-fair-value-accounting-for-the-financial-crisis. Available 20th of March, 2017.

Preston, R. (2008). *We lose in the Greed Game*. Retrieved from http://www.bbc.co.uk/blogs/thereporters/robertpeston/2008/03/we_lose_in_greed_game.html. Available 3rd of April, 2017.

Purnanandam, A. (2011). Originate-to-distribute model and the subprime mortgage crisis. *Review of Financial Studies, 24(6),* 1881–1915.

Rule, D. (2001). The credit derivatives market : its development and possible implications. *Financial Stability Review, Bank of England, (June)*, 117–140.

Salmon, F. (2012). The formula that killed Wall Street. Significance, 9(1), 16–20.

Schönbucher, P. J. (2003). *Credit derivatives pricing models: models, pricing, and implementation*. WILEY FINANCE.

Sklar, A. (1973). Random variables, joint distribution functions, and copulas. *Kybernetika*, 9(6), 449–460.

Stewart, H. (2008). *We are in the worst financial crisis since Depression, says IMF*. Retrieved from https://www.theguardian.com/business/2008/apr/10/useconomy.subprimecrisis. Available 20th of February, 2017.

Whittall, C., & Bird, M. (2017, August 28). In a Blast From a Financial Crisis Past, Synthetic CDOs are back. *The Wall Street Journal*. Retrieved from https://www.wsj.com/articles/in-a-blast-from-a-financial-crisis-past-synthetic-cdos-are-back-1503912601. Available 30th of August, 2017.

9. Appendices

9.1 - Appendix 1: Markit ITraxx Europe Series 27 Membership List

Sector	Markit Ticker	Markit Long Name
Autos & Industrials	AIGROU	Airbus Group SE
Autos & Industrials	AKZO	Akzo Nobel N.V.
Autos & Industrials	ALSTOM	ALSTOM
Autos & Industrials	ATSPA	ATLANTIA S.P.A.
Autos & Industrials	BAPLC	BAE SYSTEMS PLC
Autos & Industrials	BASFSE	BASF SE
Autos & Industrials	BMW	Bayerische Motoren Werke Aktiengesellschaft
Autos & Industrials	BOUY	BOUYGUES
Autos & Industrials	BYIF	Bayer Aktiengesellschaft
Autos & Industrials	COMPFIAC	Compagnie Financiere Michelin SCmA
Autos & Industrials	CONTI	Continental Aktiengesellschaft
Autos & Industrials	DAMLR	Daimler AG
Autos & Industrials	GLCORE	Glencore International AG
Autos & Industrials	GSK	GLAXOSMITHKLINE PLC
Autos & Industrials	HEI	HeidelbergCement AG
Autos & Industrials	KDSM	Koninklijke DSM N.V.
Autos & Industrials	LAFALTD	LafargeHolcim Ltd
Autos & Industrials	PNL	PostNL N.V.
Autos & Industrials	RENAUL	RENAULT
Autos & Industrials	ROLLS	ROLLS-ROYCE PLC
Autos & Industrials	SANFI	SANOFI
Autos & Industrials	SIEM	Siemens Aktiengesellschaft
Autos & Industrials	SOLVAY	Solvay
Autos & Industrials	STGOBN	COMPAGNIE DE SAINT-GOBAIN
Autos & Industrials	UPMKYM	UPM-Kymmene Oyj
Autos & Industrials	VINCI	VINCI
Autos & Industrials	VLOF	VALEO
Autos & Industrials	VLVY	Aktiebolaget Volvo
Autos & Industrials	VW	VOLKSWAGEN AKTIENGESELLSCHAFT
Autos & Industrials	WENL	WENDEL
Consumers	ACCOR	ACCOR
Consumers	AHOLD	Koninklijke Ahold N.V.
Consumers	ANHEUIN	Anheuser-Busch InBev
Consumers	AUCHHOL	Auchan Holding
Consumers	AYLL	SAFEWAY LIMITED
Consumers	BATSLN	BRITISH AMERICAN TOBACCO p.l.c.
Consumers	BRYBDC-Brew	CARLSBERG BREWERIES A/S
Consumers	CARR	Carrefour
Consumers	DANONE	DANONE
Consumers	DIAG	
Consumers	ELILX	Aktiebolaget Electrolux
Consumers	EXPGRL-EXPFIN	EXPERIAN FINANCE PLC
Consumers	HEIANA	
Consumers	HENAGK	Henkel AG & Co. KGaA
Consumers		IMPERIAL BRANDS PLC

Sector	Markit Ticker	Markit Long Name
Consumers	KERIAA	Kering
Consumers	KONIPHI	Koninklijke Philips N.V.
Consumers	MKS-M+SPlc	MARKS AND SPENCER p.l.c.
Consumers	MOET	LVMH MOET HENNESSY LOUIS VUITTON
Consumers	NESTLE	Nestle S.A.
Consumers	NXT	NEXT PLC
Consumers	PERNOD	PERNOD RICARD
Consumers	SUEDAG	Suedzucker AG
Consumers	TATELN	TATE & LYLE PUBLIC LIMITED COMPANY
Consumers	ULVR	Unilever N.V.
Energy	BAD	EnBW Energie Baden-Wuerttemberg AG
Energy	BPLN	BP P.L.C.
Energy	CENTRI	Centrica plc
Energy	EDF	Electricite de France
Energy	ENEL	ENEL S.P.A.
Energy	ENGIEAA	ENGIE
Energy	ENI	ENI S.P.A.
Energy	EONSE	E.ON SE
Energy	FORTUM	Fortum Oyj
Energy	GASSM	GAS NATURAL SDG, S.A.
Energy	IBERDU	Iberdrola, S.A.
Energy	NGP	NATIONAL GRID PLC
Energy	RDSPLC	ROYAL DUTCH SHELL PLC
Energy	SSEP	SSE PLC
Energy	STOL	STATOIL ASA
Energy	TECFPN	TECHNIP
Energy	TOTALN	TOTAL SA
Energy	UU	UNITED UTILITIES PLC
Energy	VATTAB	VATTENFALL AB
Energy	VEOLIA	VEOLIA ENVIRONNEMENT
Financials	ACAFP	CREDIT AGRICOLE SA
Financials	AEGON	Aegon N.V.
Financials	ALZSE	Allianz SE
Financials	ASSGEN	ASSICURAZIONI GENERALI - SOCIETA PER AZIONI
Financials	AVLN	AVIVA PLC
Financials	AXAF	AXA
Financials	BACR-Bank	BARCLAYS BANK PLC
Financials	BACRED	MEDIOBANCA BANCA DI CREDITO FINANZIARIO SOCIETA PER AZIONI
Financials	BBVSM	BANCO BILBAO VIZCAYA ARGENTARIA, SOCIEDAD ANONIMA
Financials	BNP	BNP PARIBAS
Financials	CMZB	COMMERZBANK Aktiengesellschaft
Financials	COOERAB	Cooeperatieve Rabobank U.A.
Financials	CSGAG	Credit Suisse Group AG
Financials	DANBNK	DANSKE BANK A/S
Financials	DB	DEUTSCHE BANK AKTIENGESELLSCHAFT
Financials	HANNRUE	Hannover Rueck SE
Financials	HSBC-HSBCBank	HSBC BANK PLC
Financials	INTNED-BankNV	ING Bank N.V.

Sector	Markit Ticker	Markit Long Name
Financials	LLOYDBA	LLOYDS BANK PLC
Financials	MUNRE	Muenchener Rueckversicherungs-Gesellschaft
		Aktiengesellschaft in Muenchen
Financials	PRUFIN	PRUDENTIAL PUBLIC LIMITED COMPANY
Financials	RBOS-RBOSplc	The Royal Bank of Scotland public limited company
Financials	SANPAO	INTESA SANPAOLO SPA
Financials	SANTNDR	BANCO SANTANDER, S.A.
Financials	SOCGEN	SOCIETE GENERALE
Financials	STAN-Bank	STANDARD CHARTERED BANK
Financials	SWREL	Swiss Reinsurance Company Ltd
Financials	UBS	UBS AG
Financials	USPA	UNICREDIT, SOCIETA PER AZIONI
Financials	ZINCO	Zurich Insurance Company Ltd
TMT	BERTSE	Bertelsmann SE & Co. KGaA
TMT	BRITEL-BritTel	BRITISH TELECOMMUNICATIONS public limited company
TMT	CAPP	CAP GEMINI
TMT	DT	DEUTSCHE TELEKOM AG
TMT	ITV	ITV PLC
TMT	KPN	Koninklijke KPN N.V.
TMT	ORANAA	Orange
TMT	PSON	PEARSON plc
TMT	PUBFP	PUBLICIS GROUPE SA
TMT	RELXPLC	RELX PLC
TMT	SKYPLC	Sky PLC
TMT	TELDAN	TDC A/S
TMT	TELEFO	TELEFONICA, S.A.
TMT	TELICOM	Telia Company AB
TMT	TELNOR	TELENOR ASA
TMT	TKA	Telekom Austria Aktiengesellschaft
TMT	VIVNDI	Vivendi
TMT	VOD	VODAFONE GROUP PUBLIC LIMITED COMPANY
TMT	WOLKLU	Wolters Kluwer N.V.
TMT	WPPGRP-2005	WPP 2005 LIMITED

9.2 - Appendix 2: R-code for generation one-factor Gaussian loss distribution

R-code for generation of loss distribution in one-factor Gaussian copula implemented as described in the theoretical introduction in section 2.3.3, and the synthetic CDO valuation in section 4.2. Below the code for the calculation of the loss distribution at t=0.25 on April 3, 2007 is shown. Note that this code has to be run for t's between 0.25 and 5 in order to generate the full loss distribution:

#Full code for one-factor Gaussian copula:

```
#Definitions:
Hazard_rate=0.0041
LoadingFactor=0.187
N=125
t=0.25
#calculation of default rate:
Default_rate=(1-exp(-Hazard_rate*t))
#calculation of default threshold:
threshold=qnorm(Default_rate)
#Conditional default probabilities:
seqM=c(seq(-5,5,by=0.05))
storageInd=matrix(NA,nrow=126,ncol=201)
storageconProb=matrix(NA,nrow=126,ncol=201)
vecT=seq(0,5,by=0.25)
for(i in 1:201){
 for(l in 1:126){
  Ind_default_prob=pnorm((threshold-sqrt(LoadingFactor)*seqM[i])/(sqrt(1-sqrt(LoadingFactor)^2)))
  storageInd[l,i]=Ind_default_prob
  storageconProb[l,i]=dbinom(l-1,N,storageInd[l,i])
 }
}
# Unconditional default probabilities:
minM=-5
maxM=5
stepM=0.05
seqM=c(seq(-5,5,by=0.05))
storageInd2=matrix(NA,nrow=126,ncol=201)
storageTrap=matrix(NA,nrow=126,ncol=201)
```

```
for(d in 1:126){
 for(k in 1:201){
  Ind_default_prob2=pnorm((threshold-sqrt(LoadingFactor)*seqM[k])/(sqrt(1-sqrt(LoadingFactor)^2)))
  storageInd2[d,k]=Ind_default_prob2
  trapRes=dbinom(d-1,N,storageInd2[d,k])*dnorm(seqM[k],0,1)
  storageTrap[d,k]=trapRes
 }
}
trapRes1=matrix(NA,nrow=126,ncol=201)
UnconProbInt=matrix(NA,nrow=126,ncol=1)
for(a in 1:126){
 for (e in 1:200){
  trapRes1[a,e]=(storageTrap[a,e]+storageTrap[a,e+1])/2
 }
}
UnconProbInt=as.matrix(rowSums(trapRes1,na.rm =TRUE,dims=1))
UnconProbFinal=UnconProbInt*stepM
```

9.3 - Appendix 3: R-code for generation of one-factor Student t loss distribution

R-code for generation of loss distribution in one-factor Student t copula implemented as described in the theoretical introduction in section 2.3.4, and the synthetic CDO valuation in section 4.4. Below the code for the calculation of the loss distribution based on the implied equity correlation for April 3, 2007 is shown. Note that this code has to be run for t's between 0.25 and 5 in order to generate the full loss distribution:

#Full code for one-factor Student t copula:

correlationT=0.187 a=sqrt(correlationT) df=4 n=125 s=100000 HazardTrate=0.0041 storex=matrix(NA,100000,125) storegamma=matrix(NA,100000,125) storenorm1=matrix(NA,100000,1) storenorm2=matrix(NA,100000,125) for (s in 1:100000){

randomchi=rchisq(1,df,0) storechi[s,1]=randomchi norm1=rnorm(1,0,1) storenorm1[s,1]=norm1

for (n in 1:125){

norm2=rnorm(1,0,1)

storenorm2[s,n]=norm2

 $x_i = (a*storenorm1[s,1]*(sqrt(df/storechi[s,1]))) + (sqrt(1-a^2)*storenorm2[s,n]*(sqrt(df/storechi[s,1]))) + (sqrt(df/storechi[s,1])) +$

storex[s,n]=x_i

}

}

The code below is repeated 20 times for all t's from 0.25 to 5. Below is an illustration of the first two runs. The only changing variable in the remaining 18 runs is t.

t1=0.25

defaualtRateT1=(1-exp(-HazardTrate*t1))

DefThresholdT1=qt(defaualtRateT1,df)

storelf1=matrix(NA,100000,125)

for (s in 1:100000){

for(n in 1:125){

valueD1=ifelse(storex[s,n]<DefThresholdT1,1,0)

storelf1[s,n]=valueD1

}

}

count1=as.matrix(rowSums(storelf1,na.rm=TRUE,dims=1))
dist1=as.matrix(table(count1))

t2=0.5

defaualtRateT2=(1-exp(-HazardTrate*t2)) DefThresholdT2=qt(defaualtRateT2,df) storelf2=matrix(NA,100000,125)

for (s in 1:100000){
 for(n in 1:125){
 valueD2=ifelse(storex[s,n]<DefThresholdT2,1,0)
 storelf2[s,n]=valueD2</pre>

}

}

count2=as.matrix(rowSums(storelf2,na.rm=TRUE,dims=1))
dist2=as.matrix(table(count2))

After all 20 runs have been completed, the full loss distribution is found by combining the distributions found in the individual runs. This is done below:

new1=as.matrix(table(factor(count1,levels=0:125))) new2=as.matrix(table(factor(count2,levels=0:125))) new3=as.matrix(table(factor(count3,levels=0:125))) new4=as.matrix(table(factor(count4,levels=0:125))) new5=as.matrix(table(factor(count5,levels=0:125))) new6=as.matrix(table(factor(count6,levels=0:125))) new7=as.matrix(table(factor(count7,levels=0:125))) new8=as.matrix(table(factor(count8,levels=0:125))) new9=as.matrix(table(factor(count9,levels=0:125))) new10=as.matrix(table(factor(count10,levels=0:125))) new11=as.matrix(table(factor(count11,levels=0:125))) new12=as.matrix(table(factor(count12,levels=0:125))) new13=as.matrix(table(factor(count13,levels=0:125))) new14=as.matrix(table(factor(count14,levels=0:125))) new15=as.matrix(table(factor(count15,levels=0:125))) new16=as.matrix(table(factor(count16,levels=0:125))) new17=as.matrix(table(factor(count17,levels=0:125))) new18=as.matrix(table(factor(count18,levels=0:125))) new19=as.matrix(table(factor(count19,levels=0:125))) new20=as.matrix(table(factor(count20,levels=0:125)))

LD=cbindX(new1,new2,new3,new4,new5,new6,new7,new8,new9,new10,new11,new12,new13,new14,new15,new16,new17,new18,new20)

9.4 - Appendix 4: Loss distribution for correlation value of 0.187 in one-factor Gaussian

Full loss distribution for calibrated correlation value of 0.187 in the one-factor Gaussian copula for April 3, 2007.

Defaults	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
0	9.02E-01	8.29E-01	7.68E-01	7.16E-01	6.70E-01	6.30E-01	5.93E-01	5.61E-01	5.31E-01	5.03E-01	4.78E-01	4.54E-01	4.33E-01	4.13E-01	3.94E-01	3.76E-01	3.60E-01	3.45E-01	3.30E-01	3.17E-01
1	7.75E-02	1.22E-01	1.52E-01	1.74E-01	1.90E-01	2.02E-01	2.10E-01	2.16E-01	2.20E-01	2.23E-01	2.25E-01	2.26E-01	2.25E-01	2.25E-01	2.23E-01	2.22E-01	2.20E-01	2.17E-01	2.15E-01	2.12E-01
2	1.40E-02	3.04E-02	4.55E-02	5.89E-02	7.06E-02	8.08E-02	8.98E-02	9.76E-02	1.04E-01	1.10E-01	1.15E-01	1.20E-01	1.23E-01	1.27E-01	1.29E-01	1.32E-01	1.34E-01	1.35E-01	1.36E-01	1.37E-01
3	3.78E-03	1.02E-02	1.74E-02	2.45E-02	3.14E-02	3.80E-02	4.41E-02	4.99E-02	5.52E-02	6.01E-02	6.47E-02	6.89E-02	7.27E-02	7.62E-02	7.94E-02	8.23E-02	8.50E-02	8.74E-02	8.96E-02	9.15E-02
4	1.30E-03	4.14E-03	7.71E-03	1.16E-02	1.57E-02	1.98E-02	2.38E-02	2.78E-02	3.16E-02	3.53E-02	3.88E-02	4.21E-02	4.53E-02	4.83E-02	5.11E-02	5.38E-02	5.63E-02	5.86E-02	6.09E-02	6.29E-02
5	5.21E-04	1.89E-03	3.80E-03	6.06E-03	8.52E-03	1.11E-02	1.38E-02	1.65E-02	1.92E-02	2.18E-02	2.44E-02	2.69E-02	2.94E-02	3.18E-02	3.41E-02	3.64E-02	3.85E-02	4.06E-02	4.25E-02	4.44E-02
6	2.33E-04	9.45E-04	2.02E-03	3.37E-03	4.91E-03	6.59E-03	8.38E-03	1.02E-02	1.21E-02	1.40E-02	1.60E-02	1.79E-02	1.98E-02	2.16E-02	2.35E-02	2.53E-02	2.70E-02	2.88E-02	3.04E-02	3.21E-02
7	1.13E-04	5.03E-04	1.14E-03	1.97E-03	2.96E-03	4.08E-03	5.30E-03	6.59E-03	7.94E-03	9.33E-03	1.08E-02	1.22E-02	1.36E-02	1.51E-02	1.65E-02	1.80E-02	1.94E-02	2.08E-02	2.22E-02	2.36E-02
8	5.84E-05	2.82E-04	6.68E-04	1.20E-03	1.85E-03	2.61E-03	3.45E-03	4.37E-03	5.34E-03	6.36E-03	7.42E-03	8.51E-03	9.62E-03	1.08E-02	1.19E-02	1.30E-02	1.42E-02	1.53E-02	1.64E-02	1.76E-02
9	3 17E-05	1.64E-04	4 07E-04	7 54E-04	1 19E-03	1 72E-03	2 31E-03	2 97E-03	3 68E-03	4 43E-03	5 23E-03	6.06E-03	6 91F-03	7 79E-03	8 68E-03	9 59E-03	1.05E-02	1 14E-02	1 24E-02	1 33E-02
10	1.795-05	9.905-05	2 55E-04	4 865-04	7 88F-04	1.155-03	1.585-03	2.055-03	2 58E-03	3 145-03	3 74E-03	4 38E-03	5.04E-03	5 72E-03	6.43E-03	7 155-03	7.89E-03	8.63E-03	0 30F-03	1.025-02
11	1.046-05	6 145.05	1.645-04	2 215.04	5 215.04	7.015-04	1 105-02	1.455.02	1.945.02	2 265.02	2 725.02	2 215.02	2 775.02	4 765-02	4 925.02	5 205-02	5 005.02	6 505.02	7 715.02	7 845.02
12	6 265.06	2 005.05	1.095.04	2 165-04	2 645.04	5 515.04	7 755-04	1.025.02	1 225.02	1 655.02	2.005.02	2 285.02	2 785.02	2 205-02	2 655.02	4 115-02	4 505.02	5.085.02	5 5 95.02	6 105-02
12	2.845.06	3.502-05	7.315.05	1.495.04	3.040.04	2.805.04	F F F F 04	7.405.04	0.705.04	1.032-03	1.405.02	1 785 02	2.700-03	3.435.03	3.032-03	3.165.03	2.545.02	2.045.02	4 265 02	4 705 02
13	3.046-00	2.350-05	4.805.05	1.405-04	1.705.04	3.09E-04	4.035.04	7.49E-04	9.70E-04	0.075.04	1.495-05	1.765-03	2.100-03	1.965.02	2.795-03	3.100-03	3.346.03	3.946-03	4.505-05	4.700-03
14	2.412-00	1.000-005	4.092-05	7.305.05	1.795-04	2.795-04	4.022-04	3.400-04	7.172-04	9.072-04	0.455.04	1.555-05	1.002-05	1.005-03	2.132-03	2.450-05	2.702-05	3.082-03	3.422.03	3.772-03
15	0.005.07	7.555.05	3.372-03	7.200-05	0.235.05	2.022-04	2.946-04	4.032-04	3.546-04	0.022-04	0.402-04	7.005.04	0.455.04	1.446-05	1.072-03	1.912-05	2.102-05	2.455-05	2.712-03	2.995-05
16	9.90E-07	7.56E-06	2.35E-05	5.12E-05	9.21E-05	1.4/E-04	2.17E-04	3.02E+04	4.02E-04	5.16E-04	6.45E-04	7.89E-04	9.465-04	1.12E+03	1.30E-03	1.50E+03	1.70E-03	1.92E-03	2.15E-03	2.39E-03
17	6.45E-07	5.19E-06	1.65E-05	3.67E-05	6.70E-05	1.08E-04	1.62E-04	2.27E-04	3.04E-04	3.94E-04	4.96E-04	6.09E-04	7.35E-04	8.72E-04	1.02E-03	1.18E-03	1.35E-03	1.53E-03	1.72E-03	1.92E-03
18	4.22E-07	3.61E-06	1.18E-05	2.66E-05	4.92E-05	8.05E-05	1.21E-04	1.72E-04	2.32E-04	3.02E-04	3.83E-04	4.74E-04	5.74E-04	6.85E-04	8.05E-04	9.35E-04	1.07E-03	1.22E-03	1.38E-03	1.54E-03
19	2.76E-07	2.53E-06	8.44E-06	1.94E-05	3.64E-05	6.02E-05	9.15E-05	1.31E-04	1.78E-04	2.33E-04	2.97E-04	3.70E-04	4.51E-04	5.40E-04	6.38E-04	7.44E-04	8.58E-04	9.80E-04	1.11E-03	1.25E-03
20	1.80E-07	1.79E-06	6.11E-06	1.42E-05	2.71E-05	4.53E-05	6.95E-05	1.00E-04	1.37E-04	1.81E-04	2.32E-04	2.90E-04	3.56E-04	4.28E-04	5.08E-04	5.95E-04	6.89E-04	7.90E-04	8.97E-04	1.01E-03
21	1.16E-07	1.27E-06	4.45E-06	1.05E-05	2.03E-05	3.43E-05	5.30E-05	7.69E-05	1.06E-04	1.41E-04	1.82E-04	2.29E-04	2.82E-04	3.41E-04	4.06E-04	4.77E-04	5.55E-04	6.38E-04	7.28E-04	8.23E-04
22	7.38E-08	9.08E-07	3.26E-06	7.85E-06	1.53E-05	2.61E-05	4.07E-05	5.95E-05	8.27E-05	1.11E-04	1.43E-04	1.81E-04	2.24E-04	2.72E-04	3.26E-04	3.84E-04	4.48E-04	5.17E-04	5.92E-04	6.72E-04
23	4.62E-08	6.52E-07	2.40E-06	5.87E-06	1.16E-05	1.99E-05	3.14E-05	4.62E-05	6.46E-05	8.69E-05	1.13E-04	1.44E-04	1.79E-04	2.18E-04	2.62E-04	3.10E-04	3.63E-04	4.21E-04	4.83E-04	5.50E-04
24	2.83E-08	4.69E-07	1.78E-06	4.42E-06	8.80E-06	1.53E-05	2.43E-05	3.60E-05	5.07E-05	6.85E-05	8.98E-05	1.15E-04	1.43E-04	1.75E-04	2.11E-04	2.51E-04	2.95E-04	3.43E-04	3.95E-04	4.51E-04
25	1.69E-08	3.37E-07	1.33E-06	3.34E-06	6.72E-06	1.18E-05	1.89E-05	2.81E-05	3.99E-05	5.42E-05	7.14E-05	9.16E-05	1.15E-04	1.41E-04	1.71E-04	2.04E-04	2.40E-04	2.80E-04	3.24E-04	3.71E-04
26	9.84E-09	2.42E-07	9.90E-07	2.53E-06	5.15E-06	9.13E-06	1.47E-05	2.21E-05	3.15E-05	4.30E-05	5.70E-05	7.34E-05	9.24E-05	1.14E-04	1.39E-04	1.66E-04	1.96E-04	2.30E-04	2.66E-04	3.05E-04
27	5.56E-09	1.72E-07	7.41E-07	1.93E-06	3.97E-06	7.09E-06	1.15E-05	1.74E-05	2.49E-05	3.43E-05	4.56E-05	5.89E-05	7.45E-05	9.24E-05	1.13E-04	1.35E-04	1.61E-04	1.88E-04	2.19E-04	2.52E-04
28	3.05E-09	1.22E-07	5.56E-07	1.47E-06	3.06E-06	5.52E-06	9.01E-06	1.37E-05	1.98E-05	2.73E-05	3.65E-05	4.74E-05	6.02E-05	7.49E-05	9.17E-05	1.11E-04	1.32E-04	1.55E-04	1.80E-04	2.08E-04
29	1.62E-09	8.48E-08	4.17E-07	1.13E-06	2.37E-06	4.31E-06	7.09E-06	1.09E-05	1.57E-05	2.19E-05	2.93E-05	3.83E-05	4.88E-05	6.09E-05	7.48E-05	9.05E-05	1.08E-04	1.27E-04	1.49E-04	1.72E-04
30	8.37E-10	5.83E-08	3.12E-07	8.65E-07	1.84E-06	3.37E-06	5.59E-06	8.61E-06	1.25E-05	1.75E-05	2.36E-05	3.09E-05	3.96E-05	4.96E-05	6.11E-05	7.42E-05	8.88E-05	1.05E-04	1.23E-04	1.43E-04
31	4.17E-10	3.94E-08	2.33E-07	6.64E-07	1.43E-06	2.65E-06	4.41E-06	6.84E-06	1.00E-05	1.41E-05	1.90E-05	2.51E-05	3.22E-05	4.05E-05	5.00E-05	6.09E-05	7.31E-05	8.68E-05	1.02E-04	1.19E-04
32	2.01E-10	2.61E-08	1.72E-07	5.11E-07	1.12E-06	2.08E-06	3.49E-06	5.45E-06	8.03E-06	1.13E-05	1.54E-05	2.03E-05	2.62E-05	3.31E-05	4.10E-05	5.01E-05	6.03E-05	7.18E-05	8.45E-05	9.86E-05
33	9.39E-11	1.70E-08	1.27E-07	3.92E-07	8.72E-07	1.64E-06	2.77E-06	4.35E-06	6.44E-06	9.12E-06	1.25E-05	1.65E-05	2.14E-05	2.70E-05	3.36E-05	4.12E-05	4.98E-05	5.94E-05	7.01E-05	8.20E-05
34	4.23E-11	1.08E-08	9.22E-08	3.00E-07	6.81E-07	1.29E-06	2.20E-06	3.47E-06	5.17E-06	7.35E-06	1.01E-05	1.34E-05	1.74E-05	2.22E-05	2.76E-05	3.40E-05	4.11E-05	4.92E-05	5.83E-05	6.83E-05
35	1.85E-11	6.65E-09	6.62E-08	2.29E-07	5.32E-07	1.02E-06	1.75E-06	2.78E-06	4.16E-06	5.94E-06	8.18E-06	1.09E-05	1.42E-05	1.82E-05	2.27E-05	2.80E-05	3.40E-05	4.08E-05	4.84E-05	5.69E-05
36	7.79E-12	4.01E-09	4.68E-08	1.74E-07	4.15E-07	8.08E-07	1.40E-06	2.23E-06	3.35E-06	4.81E-06	6.65E-06	8.91E-06	1.17E-05	1.49E-05	1.87E-05	2.31E-05	2.82E-05	3.39E-05	4.03E-05	4.75E-05
37	3.18E-12	2.35E-09	3.26E-08	1.31E-07	3.24E-07	6.39E-07	1.11E-06	1.79E-06	2.70E-06	3.89E-06	5.40E-06	7.27E-06	9.54E-06	1.22E-05	1.54E-05	1.91E-05	2.33E-05	2.81E-05	3.36E-05	3.96E-05
38	1.25E-12	1.34E-09	2.22E-08	9.77E-08	2.51E-07	5.05E-07	8.88E-07	1.43E-06	2.18E-06	3.15E-06	4.40E-06	5.94E-06	7.82E-06	1.01E-05	1.27E-05	1.58E-05	1.93E-05	2.34E-05	2.80E-05	3.31E-05
39	4.78E-13	7.44E-10	1.48E-08	7.20E-08	1.94E-07	3.99E-07	7.09E-07	1.15E-06	1.76E-06	2.56E-06	3.58E-06	4.86E-06	6.41E-06	8.28E-06	1.05E-05	1.31E-05	1.60E-05	1.95E-05	2.33E-05	2.76E-05
40	1.76E-13	4.01E-10	9.70E-09	5.23E-08	1.49E-07	3.14E-07	5.66E-07	9.27E-07	1.42E-06	2.08E-06	2.92E-06	3.97E-06	5.26E-06	6.82E-06	8.66E-06	1.08E-05	1.33E-05	1.62E-05	1.94E-05	2.31E-05
41	6.30E-14	2.10E-10	6.19E-09	3.74E-08	1.13E-07	2.47E-07	4.51E-07	7.45E-07	1.15E-06	1.69E-06	2.38E-06	3.25E-06	4.32E-06	5.61E-06	7.15E-06	8.96E-06	1.11E-05	1.35E-05	1.62E-05	1.93E-05
42	2.18E-14	1.07E-10	3.86E-09	2.63E-08	8.53E-08	1.93E-07	3.59E-07	5.99E-07	9.30E-07	1.37E-06	1.94E-06	2.66E-06	3.55E-06	4.62E-06	5.91E-06	7.42E-06	9.18E-06	1.12E-05	1.35E-05	1.61E-05
43	7.27E-15	5.26E-11	2.34E-09	1.81E-08	6.35E-08	1.49E-07	2.85E-07	4.81E-07	7.52E-07	1.11E-06	1.58E-06	2.18E-06	2.92E-06	3.81E-06	4.88E-06	6.15E-06	7.63E-06	9.34E-06	1.13E-05	1.35E-05
44	2.35E-15	2.52E-11	1.39E-09	1.22E-08	4.65E-08	1.15E-07	2.25E-07	3.86E-07	6.08E-07	9.06E-07	1.29E-06	1.79E-06	2.40E-06	3.14E-06	4.04E-06	5.10E-06	6.34E-06	7.78E-06	9.42E-06	1.13E-05
45	7.38E-16	1.17E-11	7.99E-10	8.08E-09	3.36E-08	8.75E-08	1.77E-07	3.08E-07	4.91E-07	7.36E-07	1.06E-06	1.46E-06	1.97E-06	2.59E-06	3.34E-06	4.23E-06	5.27E-06	6.48E-06	7.87E-06	9.45E-06
46	2 24E-16	5 26E-12	4 48F-10	5 22E-09	2 38E-08	6 59E-08	1 38E-07	2 46E-07	3 96E-07	5 98E-07	8 62E-07	1 20E-06	1.62E-06	2 14E-06	2 76E-06	3 50E-06	4 38E-06	5 39E-06	6 57E-06	7 91E-06
40	6 58E-17	2 30E-12	2 44E-10	3 29E-09	1.65E-08	4 89E-08	1.07E-07	1 95E-07	3 18E-07	4 85E-07	7 03E-07	9.82E-07	1 33E-06	1 76E-06	2 28E-06	2 90E-06	3.64E-06	4 49E-06	5.48E-06	6.61E-06
40	1.87E-17	9 74F-13	1 30E-10	2.025-09	1.135-08	3.575-08	8 16E-08	1.535-07	2 55E-07	3.935.07	5 73E-07	8 04E-07	1.095-06	1.455-06	1.895-06	2.415-06	3.025-06	3 745-06	4 57E-06	5.535-06
40	5.17E-18	4.015-13	6.68E-11	1 215-09	7.515-09	2 57E-08	6.17E-08	1.20E-07	2.04E-07	3.175.07	4.67E-07	6 58E 07	8 99F-07	1.20E-06	1.565-06	1.995-06	2.515-06	3 115-06	3.825-06	4.625-06
49	1 295.19	1.605.12	2 245.11	7.095-10	4 005-00	1 915.09	4.605-08	0.275.08	1.615.07	2 555.07	2 705-07	5 285.07	7 295.07	0.865.07	1.305.06	1.655.06	2.095.06	2 505.06	2 195.06	2 865.06
50	2 575-10	6 105-14	1.625.11	4.025-10	2 125.00	1.255.08	2 295.09	7.005.08	1.275.07	2.045.07	2.075.07	4 205.07	6.065.07	9 175.07	1.065.06	1.275-06	1 775.06	2.552.00	2 655.06	2 225.06
51	8.075-20	2 225-14	7.605.17	2 225.10	1.045.00	8.475.00	2.445-08	5 255.08	0.905.09	1.625.07	2 495-07	2 5 8 5.07	4.065.07	6.685.07	8 705.07	1 125.06	1.445.06	1 705-06	2 215.06	2 705-06
52	2 185.20	8 465.15	3 525.12	1 205-10	1 175.00	5.615.00	1 775.09	3 085.09	7.625-08	1 205.07	1 995-07	2 905-07	4 065-07	5 405.07	7 755.07	0.375-07	1 105.06	1 405-06	1.845.06	2.755.06
55	5 14F-21	2 99F-15	1.58E-12	6 205-11	6.03E-10	3.635.00	1.205-09	2 925-09	5.80E-09	1.016.07	1.595-07	2.356.07	3.315.07	4 505-07	5.97E-07	7 755-07	9.885-07	1.745.06	1.545.06	1.885.06
54	1 175-21	1.025-15	6.875.12	3 715.17	3 005-10	2 205-00	8 145.00	2 105.09	4 355-08	7 705.09	1.265.07	1.805.07	2 605-07	3.685.07	4 915-07	6.405-07	8 185.07	1.025-06	1.285.06	1.575-06
55	2.605-22	3.415-16	2.87E-12	1.505.11	2 235-10	1.415-00	5.416-09	1.485-09	3.225-08	5.955.08	0.855-09	1.516.07	2.030-07	3.015.07	4.03E-07	5.27E-07	6.76E-07	8.54E-07	1.065.06	1.315.06
50	5 505-22	1 105.16	1 175.12	7.645.17	1 225-10	8 505-10	3 575.00	1.025.09	2 345-02	4 405.00	7.645-09	1 105.07	1 755.07	2 445.07	3 305-07	4 345-07	5 505-07	7.075-07	8 825.07	1.005-06
5/	3.596-23	2 445 17	1.1/2-13	2 5 75 1 7	6.465.11	4.085.10	3.325.00	6.025.00	1.665.00	3 325 00	7.04E-08	0.355.05	1.205.07	1.075.07	3.500-07	2.565.07	4.615.07	7.07E-07	7 225 07	0.045.07
50	2 255 24	1.045.17	1.705.1.4	1.625.12	2 225 11	7.905-10	1 295 00	4 595.00	1.165.00	3.335.08	4.425.00	7.355.00	1.105.07	1.9/2-0/	2.095-07	2.015.07	2 705 07	4.825.07	6.075.07	7.515.07
59	4.605.35	2.075.10	6.675.15	7.175.17	1.675.11	1.595.10	9.245.10	7.305.09	7.065.00	1.745.00	3 205 00	7.230-08	9.625.00	1.305-07	1.765.07	2.910-07	3.195-07	7.000-07	5.025.07	6 325 07
60	4.60E-25	3.U/E-18	0.0/E-15	7.17E-13	1.0/E-11	1.58E-10	0.34E-10	2.901-09	7.90E-09	1.74E-08	3.291-08	3.55E-U8	6.635-08	1.201-07	1./0E-U/	2.3/E-U/	5.10E-07	3.98E-07	3.02E-07	0.23E-07
61	6./3E-26	0.77E-19	2.42E-15	3.U/E-13	0.15E-12	0.546-11	4.91E-10	1.8/1-09	3.32E-09	1.22E-U8	2.40E-08	4.195-08	0.0/E-08	9.951-08	1.412-07	1.925-07	2.535-07	3.2/E-0/	4.14E-07	5.10E-07
62	1.012-26	2.43E-19	0.50E-16	1.28E-13	5.80E-12	4.49E-11	2.81E-10	1.15E-09	5.48E-09	6.42E-09	1.72E-08	3.1112-08	3.09E-08	7.70E-U8	1.12E-U/	1.54E-07	2.065-07	2.0/E-0/	3.40E-07	4.20E-U/
63	2.80E-27	0.53E-20	2.90E-16	5.1/E-14	1.78E-12	2.30E-11	1.57E-10	0.92E-10	2.22E-09	5.6/E-09	1.21E-08	2.2/E-08	3.83E-08	5.98E-08	8.78E-08	1.23E-07	1.00E-07	2.18E-07	2.79E-07	3.51E-07
64	4.95E-28	1.70E-20	9.60E-17	2.03E-14	7.95E-13	1.14E-11	8.52E-11	4.04E-10	1.39E-09	3.73E-09	8.34E-09	1.62E-08	2.83E-08	4.54E-08	0.81E-08	9.70E-08	1.33E-07	1./6E-07	2.2/E-07	2.88E-07
65	8.30E-29	4.30E-21	3.09E-17	7.72E-15	3.45E-13	5.51E-12	4.50E-11	2.30E-10	8.41E-10	2.39E-09	5.62E-09	1.14E-08	2.06E-08	3.39E-08	5.21E-08	7.5/E-08	1.05E-07	1.41E-07	1.84E-07	2.35E-07
66	1.35E-29	1.05E-21	9.61E-18	2.85E-15	1.46E-13	2.59E-12	2.31E-11	1.28E-10	4.98E-10	1.50E-09	3.70E-09	7.82E-09	1.46E-08	2.49E-08	3.93E-08	5.83E-08	8.25E-08	1.12E-07	1.48E-07	1.91E-07
67	2.13E-30	2.50E-22	2.91E-18	1.02E-15	5.96E-14	1.18E-12	1.15E-11	6.88E-11	2.87E-10	9.15E-10	2.38E-09	5.25E-09	1.02E-08	1.80E-08	2.91E-08	4.43E-08	6.39E-08	8.83E-08	1.18E-07	1.54E-07
68	3.25E-31	5.77E-23	8.52E-19	3.56E-16	2.37E-14	5.22E-13	5.59E-12	3.61E-11	1.61E-10	5.45E-10	1.49E-09	3.44E-09	6.97E-09	1.27E-08	2.12E-08	3.31E-08	4.87E-08	6.86E-08	9.31E-08	1.23E-07
69	4.80E-32	1.29E-23	2.42E-19	1.20E-16	9.13E-15	2.25E-13	2.63E-12	1.84E-11	8.78E-11	3.16E-10	9.09E-10	2.20E-09	4.65E-09	8.78E-09	1.51E-08	2.43E-08	3.66E-08	5.26E-08	7.26E-08	9.69E-08
70	6.88E-33	2.79E-24	6.67E-20	3.93E-17	3.41E-15	9.37E-14	1.20E-12	9.09E-12	4.66E-11	1.78E-10	5.41E-10	1.38E-09	3.03E-09	5.94E-09	1.06E-08	1.75E-08	2.70E-08	3.97E-08	5.58E-08	7.57E-08
71	9.56E-34	5.85E-25	1.78E-20	1.25E-17	1.24E-15	3.79E-14	5.34E-13	4.37E-12	2.40E-11	9.75E-11	3.13E-10	8.36E-10	1.92E-09	3.92E-09	7.24E-09	1.23E-08	1.96E-08	2.94E-08	4.22E-08	5.83E-08
72	1.29E-34	1.19E-25	4.61E-21	3.84E-18	4.36E-16	1.49E-14	2.30E-13	2.04E-12	1.20E-11	5.20E-11	1.77E-10	4.95E-10	1.19E-09	2.53E-09	4.84E-09	8.50E-09	1.39E-08	2.14E-08	3.14E-08	4.42E-08
73	1.68E-35	2.34E-26	1.16E-21	1.14E-18	1.49E-16	5.67E-15	9.62E-14	9.25E-13	5.85E-12	2.69E-11	9.67E-11	2.85E-10	7.19E-10	1.59E-09	3.15E-09	5.73E-09	9.66E-09	1.53E-08	2.30E-08	3.30E-08
74	2.11E-36	4.46E-27	2.81E-22	3.30E-19	4.91E-17	2.09E-15	3.90E-14	4.06E-13	2.76E-12	1.35E-11	5.15E-11	1.60E-10	4.22E-10	9.72E-10	2.01E-09	3.77E-09	6.56E-09	1.07E-08	1.64E-08	2.42E-08
75	2.58E-37	8.23E-28	6.61E-23	9.24E-20	1.57E-17	7.48E-16	1.53E-14	1.73E-13	1.26E-12	6.61E-12	2.66E-11	8.71E-11	2.41E-10	5.79E-10	1.24E-09	2.42E-09	4.35E-09	7.29E-09	1.15E-08	1.73E-08

76	3.05E-38	1.47E-28	1.51E-23	2.50E-20	4.88E-18	2.59E-16	5.84E-15	7.16E-14	5.61E-13	3.13E-12	1.34E-11	4.61E-11	1.34E-10	3.36E-10	7.49E-10	1.51E-09	2.81E-09	4.86E-09	7.90E-09	1.22E-08
77	3.50E-39	2.54E-29	3.32E-24	6.57E-21	1.46E-18	8.71E-17	2.15E-15	2.87E-14	2.42E-13	1.44E-12	6.50E-12	2.36E-11	7.19E-11	1.89E-10	4.39E-10	9.22E-10	1.77E-09	3.16E-09	5.28E-09	8.36E-09
78	3.87E-40	4.26E-30	7.09E-25	1.67E-21	4.26E-19	2.83E-17	7.70E-16	1.11E-14	1.01E-13	6.40E-13	3.07E-12	1.18E-11	3.76E-11	1.03E-10	2.50E-10	5.46E-10	1.09E-09	2.00E-09	3.45E-09	5.61E-09
79	4.14E-41	6.89E-31	1.46E-25	4.10E-22	1.20E-19	8.91E-18	2.66E-16	4.18E-15	4.08E-14	2.76E-13	1.41E-12	5.69E-12	1.91E-11	5.49E-11	1.39E-10	3.14E-10	6.47E-10	1.23E-09	2.19E-09	3.66E-09
80	4.29E-42	1.08E-31	2.92E-26	9.74E-23	3.27E-20	2.71E-18	8.92E-17	1.52E-15	1.60E-14	1.15E-13	6.24E-13	2.66E-12	9.39E-12	2.83E-11	7.45E-11	1.75E-10	3.75E-10	7.39E-10	1.35E-09	2.33E-09
81	4.29E-43	1.63E-32	5.65E-27	2.24E-23	8.60E-21	7.99E-19	2.89E-17	5.36E-16	6.05E-15	4.67E-14	2.68E-13	1.21E-12	4.48E-12	1.41E-11	3.88E-11	9.51E-11	2.11E-10	4.30E-10	8.14E-10	1.45E-09
82	4.14E-44	2.39E-33	1.05E-27	4.97E-24	2.19E-21	2.28E-19	9.06E-18	1.82E-16	2.22E-15	1.83E-14	1.11E-13	5.30E-13	2.07E-12	6.83E-12	1.96E-11	5.00E-11	1.15E-10	2.43E-10	4.75E-10	8.70E-10
83	3.87E-45	3.37E-34	1.90E-28	1.07E-24	5.38E-22	6.27E-20	2.74E-18	6.00E-17	7.85E-16	6.93E-15	4.48E-14	2.25E-13	9.24E-13	3.20E-12	9.59E-12	2.55E-11	6.09E-11	1.33E-10	2.69E-10	5.09E-10
84	3.48E-46	4.60E-35	3.31E-29	2.21E-25	1.28E-22	1.67E-20	8.03E-19	1.91E-17	2.69E-16	2.54E-15	1.74E-14	9.26E-14	4.00E-13	1.45E-12	4.54E-12	1.26E-11	3.12E-11	7.08E-11	1.48E-10	2.89E-10
85	3.03E-47	6.06E-36	5.55E-30	4.43E-26	2.94E-23	4.28E-21	2.27E-19	5.87E-18	8.90E-17	8.97E-16	6.54E-15	3.68E-14	1.67E-13	6.36E-13	2.08E-12	6.00E-12	1.55E-11	3.64E-11	7.88E-11	1.59E-10
86	2.54E-48	7.69E-37	9.00E-31	8.55E-27	6.50E-24	1.06E-21	6.19E-20	1.74E-18	2.84E-17	3.06E-16	2.37E-15	1.41E-14	6.75E-14	2.69E-13	9.22E-13	2.77E-12	7.44E-12	1.81E-11	4.06E-11	8.45E-11
87	2.05E-49	9.41E-38	1.41E-31	1.59E-27	1.39E-24	2.53E-22	1.63E-20	4.98E-19	8.77E-18	1.01E-16	8.32E-16	5.23E-15	2.63E-14	1.10E-13	3.94E-13	1.24E-12	3.45E-12	8.73E-12	2.03E-11	4.35E-11
88	1.60E-50	1.11E-38	2.12E-32	2.86E-28	2.85E-25	5.84E-23	4.13E-21	1.37E-19	2.61E-18	3.21E-17	2.81E-16	1.87E-15	9.91E-15	4.35E-14	1.63E-13	5.32E-13	1.55E-12	4.06E-12	9.76E-12	2.17E-11
89	1.20E-51	1.26E-39	3.08E-33	4.94E-29	5.66E-26	1.30E-23	1.01E-21	3.65E-20	7.47E-19	9.84E-18	9.16E-17	6.44E-16	3.60E-15	1.66E-14	6.49E-14	2.21E-13	6.69E-13	1.82E-12	4.54E-12	1.04E-11
90	8.67E-53	1.38E-40	4.30E-34	8.21E-30	1.08E-26	2.77E-24	2.38E-22	9.35E-21	2.06E-19	2.90E-18	2.88E-17	2.14E-16	1.26E-15	6.09E-15	2.49E-14	8.87E-14	2.79E-13	7.90E-13	2.04E-12	4.84E-12
91	6.02E-54	1.45E-41	5.77E-35	1.32E-30	1.98E-27	5.70E-25	5.38E-23	2.30E-21	5.47E-20	8.25E-19	8.69E-18	6.84E-17	4.24E-16	2.15E-15	9.23E-15	3.42E-14	1.12E-13	3.30E-13	8.82E-13	2.17E-12
92	4.02E-55	1.47E-42	7.45E-36	2.02E-31	3.50E-28	1.13E-25	1.17E-23	5.46E-22	1.40E-20	2.26E-19	2.53E-18	2.10E-17	1.37E-16	7.32E-16	3.29E-15	1.27E-14	4.34E-14	1.33E-13	3.68E-13	9.36E-13
93	2.57E-56	1.42E-43	9.24E-37	2.99E-32	5.94E-29	2.14E-26	2.45E-24	1.24E-22	3.43E-21	5.93E-20	7.06E-19	6.22E-18	4.28E-17	2.40E-16	1.13E-15	4.55E-15	1.62E-14	5.13E-14	1.47E-13	3.88E-13
94	1.58E-57	1.33E-44	1.10E-37	4.24E-33	9.67E-30	3.90E-27	4.93E-25	2.72E-23	8.09E-22	1.50E-20	1.89E-19	1.77E-18	1.28E-17	7.54E-17	3.71E-16	1.56E-15	5.79E-15	1.91E-14	5.69E-14	1.55E-13
95	9.34E-59	1.19E-45	1.26E-38	5.78E-34	1.51E-30	6.82E-28	9.50E-26	5.70E-24	1.83E-22	3.62E-21	4.88E-20	4.82E-19	3.68E-18	2.27E-17	1.17E-16	5.16E-16	1.99E-15	6.81E-15	2.11E-14	5.95E-14
96	5.27E-60	1.01E-46	1.37E-39	7.53E-35	2.26E-31	1.14E-28	1.75E-26	1.15E-24	3.97E-23	8.39E-22	1.20E-20	1.26E-19	1.01E-18	6.58E-18	3.55E-17	1.63E-16	6.55E-16	2.33E-15	7.49E-15	2.19E-14
97	2.85E-61	8.31E-48	1.44E-40	9.39E-36	3.24E-32	1.83E-29	3.10E-27	2.21E-25	8.23E-24	1.86E-22	2.84E-21	3.15E-20	2.68E-19	1.82E-18	1.03E-17	4.95E-17	2.07E-16	7.66E-16	2.55E-15	7.72E-15
98	1.47E-62	6.50E-49	1.44E-41	1.12E-36	4.43E-33	2.81E-30	5.24E-28	4.06E-26	1.63E-24	3.96E-23	6.43E-22	7.54E-21	6.75E-20	4.83E-19	2.86E-18	1.43E-17	6.25E-17	2.41E-16	8.31E-16	2.61E-15
99	7.25E-64	4.86E-50	1.37E-42	1.28E-37	5.79E-34	4.12E-31	8.46E-29	7.13E-27	3.10E-25	8.04E-24	1.39E-22	1.72E-21	1.63E-20	1.22E-19	7.58E-19	3.97E-18	1.80E-17	7.23E-17	2.59E-16	8.42E-16
100	3.41E-65	3.46E-51	1.25E-43	1.39E-38	7.22E-35	5.76E-32	1.30E-29	1.20E-27	5.60E-26	1.56E-24	2.86E-23	3.76E-22	3.75E-21	2.96E-20	1.92E-19	1.05E-18	4.97E-18	2.07E-17	7.71E-17	2.60E-16
101	1.53E-66	2.35E-52	1.09E-44	1.43E-39	8.57E-36	7.66E-33	1.91E-30	1.91E-28	9.64E-27	2.87E-25	5.62E-24	7.82E-23	8.21E-22	6.82E-21	4.63E-20	2.65E-19	1.31E-18	5.66E-18	2.19E-17	7.63E-17
102	6.50E-68	1.52E-53	8.95E-46	1.41E-40	9.68E-37	9.70E-34	2.66E-31	2.90E-29	1.58E-27	5.04E-26	1.05E-24	1.55E-23	1.71E-22	1.49E-21	1.06E-20	6.36E-20	3.27E-19	1.47E-18	5.90E-18	2.13E-17
103	2.62E-69	9.29E-55	7.01E-47	1.32E-41	1.04E-37	1.17E-34	3.53E-32	4.18E-30	2.46E-28	8.39E-27	1.86E-25	2.90E-24	3.39E-23	3.11E-22	2.32E-21	1.45E-20	7.76E-20	3.64E-19	1.51E-18	5.67E-18
104	1.00E-70	5.39E-56	5.20E-48	1.16E-42	1.05E-38	1.33E-35	4.43E-33	5.72E-31	3.62E-29	1.32E-27	3.13E-26	5.17E-25	6.37E-24	6.14E-23	4.80E-22	3.13E-21	1.75E-20	8.52E-20	3.68E-19	1.43E-18
105	3.63E-72	2.96E-57	3.65E-49	9.75E-44	1.01E-39	1.43E-36	5.26E-34	7.39E-32	5.05E-30	1.98E-28	4.97E-27	8.71E-26	1.13E-24	1.15E-23	9.39E-23	6.40E-22	3.73E-21	1.89E-20	8.48E-20	3.41E-19
106	1.24E-73	1.53E-58	2.41E-50	7.69E-45	9.18E-41	1.45E-37	5.89E-35	9.02E-33	6.65E-31	2.79E-29	7.46E-28	1.38E-26	1.90E-25	2.02E-24	1.73E-23	1.24E-22	7.51E-22	3.96E-21	1.84E-20	7.70E-20
107	3.99E-75	7.46E-60	1.50E-51	5.72E-46	7.83E-42	1.39E-38	6.21E-36	1.04E-33	8.25E-32	3.70E-30	1.05E-28	2.07E-27	3.00E-26	3.36E-25	3.02E-24	2.25E-23	1.42E-22	7.81E-22	3.78E-21	1.63E-20
108	1.20E-76	3.41E-61	8.80E-53	3.99E-47	6.28E-43	1.25E-39	6.15E-37	1.12E-34	9.60E-33	4.62E-31	1.40E-29	2.92E-28	4.45E-27	5.24E-26	4.93E-25	3.84E-24	2.53E-23	1.45E-22	7.27E-22	3.26E-21
109	3.40E-78	1.46E-62	4.82E-54	2.61E-48	4.71E-44	1.05E-40	5.71E-38	1.13E-35	1.05E-33	5.39E-32	1.74E-30	3.84E-29	6.18E-28	7.65E-27	7.55E-26	6.14E-25	4.23E-24	2.51E-23	1.31E-22	6.09E-22
110	8.94E-80	5.84E-64	2.46E-55	1.59E-49	3.30E-45	8.23E-42	4.94E-39	1.06E-36	1.06E-34	5.87E-33	2.02E-31	4.72E-30	8.02E-29	1.04E-27	1.08E-26	9.17E-26	6.58E-25	4.07E-24	2.20E-23	1.06E-22
111	2.19E-81	2.17E-65	1.17E-56	8.99E-51	2.14E-46	6.00E-43	3.97E-40	9.31E-38	1.01E-35	5.94E-34	2.17E-32	5.39E-31	9.66E-30	1.32E-28	1.43E-27	1.27E-26	9.53E-26	6.13E-25	3.45E-24	1.72E-23
112	4.95E-83	7.44E-67	5.13E-58	4.71E-52	1.29E-47	4.05E-44	2.95E-41	7.55E-39	8.79E-37	5.57E-35	2.17E-33	5.69E-32	1.08E-30	1.55E-29	1.76E-28	1.64E-27	1.28E-26	8.54E-26	4.99E-25	2.59E-24
113	1.03E-84	2.35E-68	2.07E-59	2.27E-53	7.14E-49	2.51E-45	2.02E-42	5.63E-40	7.08E-38	4.80E-36	1.99E-34	5.54E-33	1.11E-31	1.67E-30	1.99E-29	1.94E-28	1.57E-27	1.10E-26	6.66E-26	3.58E-25
114	1.97E-86	6.80E-70	7.67E-61	1.00E-54	3.62E-50	1.43E-46	1.27E-43	3.84E-41	5.22E-39	3.79E-37	1.68E-35	4.94E-34	1.04E-32	1.65E-31	2.06E-30	2.10E-29	1.78E-28	1.29E-27	8.14E-27	4.54E-26
115	3.41E-88	1.79E-71	2.58E-62	4.02E-56	1.67E-51	7.38E-48	7.22E-45	2.39E-42	3.49E-40	2.72E-38	1.28E-36	4.00E-35	8.89E-34	1.48E-32	1.94E-31	2.06E-30	1.83E-29	1.38E-28	9.04E-28	5.22E-27
116	5.32E-90	4.23E-73	7.81E-64	1.45E-57	6.92E-53	3.44E-49	3.71E-46	1.33E-43	2.11E-41	1.76E-39	8.82E-38	2.92E-36	6.85E-35	1.20E-33	1.65E-32	1.83E-31	1.69E-30	1.33E-29	9.05E-29	5.43E-28
117	7.41E-92	8.94E-75	2.11E-65	4.69E-59	2.57E-54	1.43E-50	1.70E-47	6.67E-45	1.14E-42	1.02E-40	5.43E-39	1.90E-37	4.71E-36	8.70E-35	1.25E-33	1.45E-32	1.40E-31	1.14E-30	8.09E-30	5.03E-29
118	9.10E-94	1.67E-76	5.03E-67	1.33E-60	8.39E-56	5.24E-52	6.87E-49	2.94E-46	5.41E-44	5.18E-42	2.94E-40	1.09E-38	2.86E-37	5.55E-36	8.38E-35	1.02E-33	1.02E-32	8.68E-32	6.38E-31	4.11E-30
119	9.70E-96	2.69E-78	1.04E-68	3.29E-62	2.38E-57	1.66E-53	2.41E-50	1.12E-47	2.23E-45	2.29E-43	1.39E-41	5.46E-40	1.50E-38	3.07E-37	4.86E-36	6.17E-35	6.46E-34	5.72E-33	4.37E-32	2.92E-31
120	8.79E-98	3.70E-80	1.83E-70	6.91E-64	5.73E-59	4.50E-55	7.18E-52	3.64E-49	7.82E-47	8.60E-45	5.54E-43	2.31E-41	6.74E-40	1.45E-38	2.40E-37	3.18E-36	3.48E-35	3.21E-34	2.54E-33	1.76E-32
121	6.58E-100	4.21E-82	2.66E-72	1.20E-65	1.14E-60	1.01E-56	1.77E-53	9.78E-51	2.27E-48	2.67E-46	1.83E-44	8.12E-43	2.49E-41	5.63E-40	9.79E-39	1.36E-37	1.55E-36	1.49E-35	1.22E-34	8.81E-34
122	3.91E-102	3.79E-84	3.07E-74	1.65E-67	1.81E-62	1.78E-58	3.46E-55	2.09E-52	5.22E-50	6.58E-48	4.82E-46	2.26E-44	7.32E-43	1.74E-41	3.17E-40	4.60E-39	5.47E-38	5.46E-37	4.68E-36	3.49E-35
123	1.73E-104	2.54E-86	2.63E-76	1.69E-69	2.12E-64	2.35E-60	5.04E-57	3.31E-54	8.93E-52	1.21E-49	9.41E-48	4.68E-46	1.60E-44	4.00E-43	7.65E-42	1.16E-40	1.44E-39	1.50E-38	1.33E-37	1.03E-36
124	5.05E-107	1.13E-88	1.49E-78	1.14E-71	1.65E-66	2.05E-62	4.85E-59	3.47E-56	1.01E-53	1.47E-51	1.22E-49	6.41E-48	2.32E-46	6.09E-45	1.22E-43	1.93E-42	2.50E-41	2.71E-40	2.50E-39	2.01E-38
125	7.32E-110	2.48E-91	4.20E-81	3.84E-74	6.38E-69	8.89E-65	2.32E-61	1.81E-58	5.68E-56	8.82E-54	7.80E-52	4.36E-50	1.66E-48	4.59E-47	9.65E-46	1.60E-44	2.16E-43	2.43E-42	2.34E-41	1.95E-40

9.5 - Appendix 5: Loss distribution for correlation value of 0.471 in one-factor Gaussian

Full loss distribution for calibrated correlation value of 0.471 in the one-factor Gaussian copula for May 9, 2017.

Defaults	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
0	8.97E-01	8.33E-01	7.82E-01	7.40E-01	7.03E-01	6.70E-01	6.40E-01	6.13E-01	5.89E-01	5.66E-01	5.45E-01	5.25E-01	5.07E-01	4.90E-01	4.74E-01	4.58E-01	4.44E-01	4.30E-01	4.17E-01	4.05E-01
1	5.56E-02	7.96E-02	9.50E-02	1.06E-01	1.14E-01	1.20E-01	1.25E-01	1.28E-01	1.31E-01	1.33E-01	1.34E-01	1.35E-01	1.36E-01	1.36E-01	1.36E-01	1.36E-01	1.36E-01	1.35E-01	1.35E-01	1.34E-01
2	1.82E-02	2.98E-02	3.84E-02	4.53E-02	5.08E-02	5.54E-02	5.92E-02	6.25E-02	6.52E-02	6.76E-02	6.96E-02	7.13E-02	7.28E-02	7.40E-02	7.51E-02	7.60E-02	7.67E-02	7.73E-02	7.78E-02	7.82E-02
3	8.95E-03	1.58E-02	2.13E-02	2.58E-02	2.98E-02	3.31E-02	3.61E-02	3.86E-02	4.09E-02	4.29E-02	4.47E-02	4.63E-02	4.77E-02	4.90E-02	5.01E-02	5.11E-02	5.20E-02	5.28E-02	5.35E-02	5.41E-02
4	5.24E-03	9.74E-03	1.36E-02	1.69E-02	1.98E-02	2.24E-02	2.47E-02	2.68E-02	2.86E-02	3.03E-02	3.18E-02	3.32E-02	3.45E-02	3.56E-02	3.67E-02	3.76E-02	3.85E-02	3.93E-02	4.00E-02	4.06E-02
5	3.40E-03	6.59E-03	9.41E-03	1.19E-02	1.42E-02	1.62E-02	1.81E-02	1.98E-02	2.13E-02	2.28E-02	2.41E-02	2.53E-02	2.64E-02	2.74E-02	2.84E-02	2.92E-02	3.00E-02	3.08E-02	3.15E-02	3.21E-02
6	2.36E-03	4.72E-03	6.89E-03	8.87E-03	1.07E-02	1.23E-02	1.39E-02	1.53E-02	1.66E-02	1.78E-02	1.90E-02	2.00E-02	2.10E-02	2.19E-02	2.27E-02	2.35E-02	2.43E-02	2.50E-02	2.56E-02	2.62E-02
7	1.71E-03	3.53E-03	5.24E-03	6.84E-03	8.32E-03	9.70E-03	1.10E-02	1.22E-02	1.33E-02	1.44E-02	1.54E-02	1.63E-02	1.72E-02	1.80E-02	1.87E-02	1.95E-02	2.01E-02	2.08E-02	2.14E-02	2.19E-02
8	1.28E-03	2.72E-03	4.11E-03	5.42E-03	6.66E-03	7.82E-03	8.92E-03	9.95E-03	1.09E-02	1.18E-02	1.27E-02	1.35E-02	1.43E-02	1.50E-02	1.57E-02	1.64E-02	1.70E-02	1.76E-02	1.82E-02	1.87E-02
9	9.90E-04	2.15E-03	3.29E-03	4.39E-03	5.43E-03	6.43E-03	7.37E-03	8.27E-03	9.12E-03	9.93E-03	1.07E-02	1.14E-02	1.21E-02	1.28E-02	1.34E-02	1.40E-02	1.46E-02	1.51E-02	1.57E-02	1.61E-02
10	7.80E-04	1.73E-03	2.68E-03	3.61E-03	4.51E-03	5.37E-03	6.19E-03	6.98E-03	7.73E-03	8.45E-03	9.14E-03	9.79E-03	1.04E-02	1.10E-02	1.16E-02	1.21E-02	1.27E-02	1.32E-02	1.37E-02	1.41E-02
11	6.25E-04	1.41E-03	2.22E-03	3.01E-03	3.79E-03	4.54E-03	5.26E-03	5.95E-03	6.62E-03	7.27E-03	7.88E-03	8.48E-03	9.05E-03	9.59E-03	1.01E-02	1.06E-02	1.11E-02	1.16E-02	1.20E-02	1.24E-02
12	5.08E-04	1.17E-03	1.86E-03	2.54E-03	3.22E-03	3.88E-03	4.52E-03	5.13E-03	5.73E-03	6.31E-03	6.87E-03	7.40E-03	7.92E-03	8.42E-03	8.91E-03	9.37E-03	9.82E-03	1.02E-02	1.07E-02	1.11E-02
13	4.19E-04	9.80E-04	1.57E-03	2.17E-03	2.76E-03	3.34E-03	3.91E-03	4.46E-03	5.00E-03	5.52E-03	6.03E-03	6.52E-03	6.99E-03	7.45E-03	7.89E-03	8.32E-03	8.74E-03	9.14E-03	9.53E-03	9.90E-03
14	3.48E-04	8.29E-04	1.34E-03	1.87E-03	2.39E-03	2.91E-03	3.41E-03	3.91E-03	4.40E-03	4.87E-03	5.33E-03	5.78E-03	6.21E-03	6.63E-03	7.04E-03	7.44E-03	7.82E-03	8.20E-03	8.56E-03	8.91E-03
15	2.92E-04	7.07E-04	1.16E-03	1.62E-03	2.08E-03	2.54E-03	3.00E-03	3.45E-03	3.89E-03	4.32E-03	4.74E-03	5.15E-03	5.55E-03	5.94E-03	6.32E-03	6.69E-03	7.04E-03	7.39E-03	7.73E-03	8.06E-03
16	2.47E-04	6.07E-04	1.00E-03	1.41E-03	1.82E-03	2.24E-03	2.65E-03	3.06E-03	3.46E-03	3.85E-03	4.24E-03	4.61E-03	4.98E-03	5.34E-03	5.69E-03	6.04E-03	6.37E-03	6.70E-03	7.01E-03	7.32E-03
17	2.11E-04	5.24E-04	8.72E-04	1.24E-03	1.61E-03	1.98E-03	2.35E-03	2.72E-03	3.09E-03	3.45E-03	3.80E-03	4.15E-03	4.49E-03	4.83E-03	5.15E-03	5.47E-03	5.78E-03	6.09E-03	6.39E-03	6.68E-03
18	1.81E-04	4.56E-04	7.64E-04	1.09E-03	1.42E-03	1.76E-03	2.10E-03	2.44E-03	2.77E-03	3.10E-03	3.43E-03	3.75E-03	4.07E-03	4.38E-03	4.68E-03	4.98E-03	5.27E-03	5.56E-03	5.84E-03	6.11E-03
19	1.56E-04	3.98E-04	6.73E-04	9.65E-04	1.27E-03	1.57E-03	1.88E-03	2.19E-03	2.50E-03	2.80E-03	3.10E-03	3.40E-03	3.69E-03	3.98E-03	4.27E-03	4.55E-03	4.82E-03	5.09E-03	5.35E-03	5.61E-03
20	1.35E-04	3.49E-04	5.95E-04	8.58E-04	1.13E-03	1.41E-03	1.69E-03	1.97E-03	2.26E-03	2.54E-03	2.82E-03	3.09E-03	3.37E-03	3.64E-03	3.90E-03	4.16E-03	4.42E-03	4.67E-03	4.92E-03	5.16E-03
21	1.18E-04	3.08E-04	5.28E-04	7.66E-04	1.01E-03	1.27E-03	1.53E-03	1.79E-03	2.05E-03	2.31E-03	2.57E-03	2.82E-03	3.08E-03	3.33E-03	3.58E-03	3.82E-03	4.06E-03	4.30E-03	4.54E-03	4.77E-03
22	1.03E-04	2.72E-04	4.70E-04	6.85E-04	9.11E-04	1.14E-03	1.38E-03	1.62E-03	1.86E-03	2.10E-03	2.34E-03	2.58E-03	2.82E-03	3.05E-03	3.29E-03	3.52E-03	3.74E-03	3.97E-03	4.19E-03	4.41E-03
23	9.04E-05	2.42E-04	4.20E-04	6.16E-04	8.22E-04	1.03E-03	1.25E-03	1.47E-03	1.70E-03	1.92E-03	2.14E-03	2.37E-03	2.59E-03	2.81E-03	3.03E-03	3.24E-03	3.46E-03	3.67E-03	3.88E-03	4.09E-03
24	7.96E-05	2.15E-04	3.77E-04	5.54E-04	7.43E-04	9.39E-04	1.14E-03	1.34E-03	1.55E-03	1.76E-03	1.97E-03	2.18E-03	2.38E-03	2.59E-03	2.80E-03	3.00E-03	3.20E-03	3.40E-03	3.60E-03	3.79E-03
25	7.03E-05	1.92E-04	3.38E-04	5.00E-04	6.73E-04	8.53E-04	1.04E-03	1.23E-03	1.42E-03	1.61E-03	1.81E-03	2.00E-03	2.20E-03	2.39E-03	2.59E-03	2.78E-03	2.97E-03	3.16E-03	3.34E-03	3.53E-03
26	6.23E-05	1.72E-04	3.05E-04	4.53E-04	6.11E-04	7.78E-04	9.49E-04	1.12E-03	1.30E-03	1.48E-03	1.67E-03	1.85E-03	2.03E-03	2.21E-03	2.40E-03	2.58E-03	2.76E-03	2.94E-03	3.11E-03	3.29E-03
27	5.54E-05	1.54E-04	2.75E-04	4.11E-04	5.57E-04	7.10E-04	8.69E-04	1.03E-03	1.20E-03	1.37E-03	1.54E-03	1.71E-03	1.88E-03	2.05E-03	2.22E-03	2.39E-03	2.56E-03	2.73E-03	2.90E-03	3.07E-03
28	4.93E-05	1.39E-04	2.49E-04	3.73E-04	5.08E-04	6.49E-04	7.97E-04	9.49E-04	1.10E-03	1.26E-03	1.42E-03	1.58E-03	1./4E-03	1.90E-03	2.07E-03	2.23E-03	2.39E-03	2.55E-03	2.71E-03	2.8/E-03
29	4.40E-05	1.25E-04	2.26E-04	3.40E-04	4.64E-04	5.95E-04	7.32E-04	8.73E-04	1.02E-03	1.17E-03	1.31E-03	1.4/E-03	1.62E-03	1.77E-03	1.92E-03	2.08E-03	2.23E-03	2.38E-03	2.53E-03	2.68E-03
30	3.94E-05	1.13E-04	2.05E-04	3.10E-04	4.24E+04	5.46E-04	6.73E-04	8.05E-04	9.40E-04	1.08E-03	1.22E-03	1.365-03	1.50E-03	1.65E-03	1.79E-03	1.94E-03	2.08E+03	2.23E-03	2.37E-03	2.51E-03
31	3.53E-05	1.02E-04	1.865-04	2.83E-04	3.89E-04	5.02E-04	6.20E-04	7.43E-04	8.70E+04	9.99E-04	1.13E-03	1.265-03	1.40E-03	1.54E-03	1.67E-03	1.81E-03	1.955-03	2.08E-03	2.22E-03	2.36E-03
32	3.172-03	9.235-05	1.702-04	2.395-04	3.375-04	4.022-04	5.722-04	6.265.04	2.00E-04	9.272-04	0.785.04	1.105.03	1.305-03	1.450-05	1.305-03	1.095-05	1.022-05	1.935-03	1.065.03	2.212-03
33	2.032-03	7.635.05	1.335-04	2.375-04	3.200-04	3.035.04	4.805.04	0.300-04	6.035.04	8.01E-04	9.765-04	1.102-03	1.145.03	1.346.03	1.402-05	1.305-03	1.712-05	1.050-05	1.902-03	2.065.03
34	2.375-05	7.03E-05	1.305.04	2.100-04	3.022-04	3.935-04	4.092-04	5.095-04	6.445.04	7.455.04	9.102-04	0.545.04	1.146-03	1.250-05	1.375-03	1.405-03	1.002-03	1.625.02	1.040-03	1.902-03
35	2.522-05	6.225.05	1.105.04	1.845.04	2.765-04	2 255.04	4.355-04	5.085.04	5.005.04	6.045.04	7.025.04	9.546-04	0.025-04	1.1/2-05	1.205-03	1.215.02	1.415.02	1.525-02	1.625.02	1.745.02
30	1 805.05	5 775.05	1.005-04	1.605.04	2.376.04	2 105-04	2 805-04	4 725.04	5.595.04	6.475.04	7 205-04	8 225.04	0.205-04	1.025.02	1 125.02	1.372-03	1 225.02	1.425.02	1.525.02	1.645.02
38	1.725-05	5.27E-05	9.995-05	1.565-04	2 195-04	2 88F-04	3.615-04	4 395-04	5.20E-04	6.04E-04	6 91F-04	7 80E-04	8 71F-04	9.635-04	1.065-03	1.155-03	1.255-03	1.355-03	1.445-03	1.545-03
30	1.565-05	4.825-05	9 185-05	1.445-04	2.025-04	2.675-04	3 365-04	4.085-04	4.855-04	5.64E-04	6.46E-04	7.305-04	8 165-04	9.045-04	9.945-04	1.085-03	1.185-03	1.275-03	1.365-03	1.465-03
40	1 41E-05	4 41E-05	8 44E-05	1 33E-04	1.87E-04	2 47E-04	3 12E-04	3.80F-04	4 52E-04	5.27E-04	6.05E-04	6.84E-04	7.66E-04	8 49E-04	9 34E-04	1.02E-03	1 11E-03	1.20E-03	1.29E-03	1 38E-03
41	1.28E-05	4.04E-05	7.77E-05	1.23E-04	1.74E-04	2.30E-04	2.90E-04	3.55E-04	4.22E-04	4.93E-04	5.66E-04	6.42E-04	7.19E-04	7.98E-04	8.79E-04	9.61E-04	1.04E-03	1.13E-03	1.21E-03	1.30E-03
42	1.17E-05	3.70E-05	7.15E-05	1.13E-04	1.61E-04	2.13E-04	2.70E-04	3.31E-04	3.95E-04	4.61E-04	5.30E-04	6.02E-04	6.75E-04	7.50E-04	8.27E-04	9.05E-04	9.85E-04	1.07E-03	1.15E-03	1.23E-03
43	1.06E-05	3.39E-05	6.59E-05	1.05E-04	1.49E-04	1.98E-04	2.52E-04	3.09E-04	3.69E-04	4.32E-04	4.97E-04	5.65E-04	6.35E-04	7.06E-04	7.79E-04	8.53E-04	9.29E-04	1.01E-03	1.08E-03	1.16E-03
44	9.68E-06	3.12E-05	6.08E-05	9.70E-05	1.38E-04	1.85E-04	2.35E-04	2.88E-04	3.45E-04	4.05E-04	4.66E-04	5.31E-04	5.97E-04	6.65E-04	7.34E-04	8.05E-04	8.77E-04	9.50E-04	1.02E-03	1.10E-03
45	8.82E-06	2.86E-05	5.61E-05	8.98E-05	1.29E-04	1.72E-04	2.19E-04	2.69E-04	3.23E-04	3.79E-04	4.38E-04	4.99E-04	5.61E-04	6.26E-04	6.92E-04	7.59E-04	8.28E-04	8.98E-04	9.69E-04	1.04E-03
46	8.05E-06	2.63E-05	5.18E-05	8.32E-05	1.20E-04	1.60E-04	2.04E-04	2.52E-04	3.02E-04	3.55E-04	4.11E-04	4.69E-04	5.28E-04	5.89E-04	6.52E-04	7.17E-04	7.82E-04	8.49E-04	9.17E-04	9.86E-04
47	7.34E-06	2.42E-05	4.79E-05	7.71E-05	1.11E-04	1.49E-04	1.91E-04	2.36E-04	2.83E-04	3.33E-04	3.86E-04	4.41E-04	4.97E-04	5.56E-04	6.15E-04	6.77E-04	7.39E-04	8.03E-04	8.68E-04	9.34E-04
48	6.71E-06	2.23E-05	4.43E-05	7.15E-05	1.03E-04	1.39E-04	1.78E-04	2.20E-04	2.65E-04	3.13E-04	3.63E-04	4.15E-04	4.68E-04	5.24E-04	5.81E-04	6.39E-04	6.99E-04	7.60E-04	8.22E-04	8.85E-04
49	6.13E-06	2.05E-05	4.09E-05	6.64E-05	9.61E-05	1.30E-04	1.66E-04	2.06E-04	2.49E-04	2.94E-04	3.41E-04	3.90E-04	4.41E-04	4.94E-04	5.48E-04	6.04E-04	6.61E-04	7.19E-04	7.79E-04	8.39E-04
50	5.60E-06	1.89E-05	3.79E-05	6.16E-05	8.95E-05	1.21E-04	1.56E-04	1.93E-04	2.33E-04	2.76E-04	3.20E-04	3.67E-04	4.16E-04	4.66E-04	5.18E-04	5.71E-04	6.25E-04	6.81E-04	7.38E-04	7.96E-04
51	5.13E-06	1.74E-05	3.51E-05	5.72E-05	8.33E-05	1.13E-04	1.45E-04	1.81E-04	2.19E-04	2.59E-04	3.01E-04	3.46E-04	3.92E-04	4.40E-04	4.89E-04	5.40E-04	5.92E-04	6.45E-04	6.99E-04	7.55E-04
52	4.69E-06	1.60E-05	3.25E-05	5.32E-05	7.76E-05	1.05E-04	1.36E-04	1.69E-04	2.05E-04	2.43E-04	2.84E-04	3.26E-04	3.69E-04	4.15E-04	4.62E-04	5.10E-04	5.60E-04	6.11E-04	6.63E-04	7.16E-04
53	4.29E-06	1.48E-05	3.01E-05	4.94E-05	7.23E-05	9.83E-05	1.27E-04	1.59E-04	1.93E-04	2.29E-04	2.67E-04	3.07E-04	3.48E-04	3.92E-04	4.36E-04	4.82E-04	5.30E-04	5.79E-04	6.28E-04	6.79E-04
54	3.93E-06	1.36E-05	2.78E-05	4.59E-05	6.74E-05	9.18E-05	1.19E-04	1.49E-04	1.81E-04	2.15E-04	2.51E-04	2.89E-04	3.29E-04	3.70E-04	4.12E-04	4.56E-04	5.02E-04	5.48E-04	5.96E-04	6.44E-04
55	3.60E-06	1.26E-05	2.58E-05	4.27E-05	6.28E-05	8.58E-05	1.11E-04	1.39E-04	1.70E-04	2.02E-04	2.36E-04	2.72E-04	3.10E-04	3.49E-04	3.90E-04	4.32E-04	4.75E-04	5.19E-04	5.65E-04	6.12E-04
56	3.30E-06	1.16E-05	2.39E-05	3.97E-05	5.85E-05	8.01E-05	1.04E-04	1.31E-04	1.59E-04	1.90E-04	2.22E-04	2.57E-04	2.92E-04	3.30E-04	3.68E-04	4.09E-04	4.50E-04	4.92E-04	5.36E-04	5.80E-04
57	3.02E-06	1.07E-05	2.22E-05	3.69E-05	5.46E-05	7.49E-05	9.76E-05	1.23E-04	1.50E-04	1.79E-04	2.09E-04	2.42E-04	2.76E-04	3.11E-04	3.48E-04	3.87E-04	4.26E-04	4.67E-04	5.08E-04	5.51E-04
58	2.77E-06	9.88E-06	2.05E-05	3.43E-05	\$.09E-05	7.00E-05	9.14E-05	1.15E-04	1.41E-04	1.68E-04	1.97E-04	2.28E-04	2.60E-04	2.94E-04	3.29E-04	3.66E-04	4.03E-04	4.42E-04	4.82E-04	5.23E-04
59	2.54E-06	9.12E-06	1.91E-05	3.19E-05	4./5E-05	0.54E-05	8.56E-05	1.08E-04	1.32E-04	1.58E-04	1.86E-04	2.15E-04	2.46E-04	2.78E-04	3.11E-04	3.46E-04	3.82E-04	4.19E-04	4.57E-04	4.9/E-04
60	2.33E-06	8.42E-06	1.77E-05	2.97E-05	4.43E-05	6.11E-05	8.01E-05	1.01E-04	1.24E-04	1.49E-04	1.75E-04	2.03E-04	2.32E-04	2.63E-04	2.95E-04	3.28E-04	3.62E-04	3.97E-04	4.34E-04	4./IE-04
61	2.14E-06	7.78E-06	1.64E-05	2.765-05	4.13E-05	5.71E-05	7.51E-05	9.49E-05	1.17E-04	1.40E-04	1.65E-04	1.916-04	2.19E-04	2.48E-04	2.79E+04	3.10E-04	3.43E-04	3.77E-04	4.12E-04	4.48E-04
62	1.905-00	6.62E-05	1.52E-05	2.375-05	2.005-05	3.54E-U5	7.USE-US	0.902-05	1.09E-04	1.525-04	1.332-04	1.0UE-04	2.0/E-04	2.54E-04	2.05E-04	2.945-04	3.23E-04	3.37E-04	3.912-04	4.232-04
64	1.655.05	6.125-06	1,410-05	2.395-05	3.395-05	4.995-05	6 165.05	7 925.05	0.665-05	1.165.04	1.405-04	1.605-04	1.955-04	2.216-04	2.495-04	2.705-04	2.015.04	2 216-04	2.525.04	2 825.04
65	1.032-00	5.655-06	1.215.05	2.225.05	3 125.05	4 365.05	5 775.05	7 345.05	9.005-05	1.005-04	1.305-04	1.515.04	1.045-04	1.095-04	2.335.04	2.035-04	2.315-04	3.045-04	3 375-04	3.645-04
66	1.315-00	5.225.06	1.126.05	1.925-05	2 915.05	4.075-05	5.405-05	6.89F-05	8.52E-05	1.03E-04	1.225-04	1.425.04	1.645-04	1.50E-04	2.2.55-04	2.355.04	2.705-04	2.88F-04	3.165.04	3.45E-04
67	1.275-06	4.815-06	1.045-05	1.705-05	2.512-05	3.816.05	5.065-05	6.465-05	8.00F-05	9.675-05	1.15E-04	1.345-04	1.555-04	1.76E-04	1.995-04	2.335-04	2.485-04	2.735-04	3.005-04	3.785-04
68	1 165-06	4 445-06	9.655-06	1.665-05	2 535.05	3 565-05	4 735-05	6.055-05	7 515-05	9 105-05	1.085-04	1.265-04	1.465-04	1.665-04	1.885-04	2 115-04	2 345-04	2 59F-04	2 84F-04	3 11 5-04
69	1.065-06	4 105-06	8.94F-06	1.555.05	2.3365.05	3.302-03	4.435-05	5.675.05	7.055-05	8.555-05	1.025-04	1 195.04	1.385-04	1.575-04	1.785-04	1.005.04	2.275-04	2.45E-04	2.705-04	2.955-04
70	9.75E-07	3 78E-06	8 28E-06	1.44E-05	2 20E-05	3 10E-05	4 14E-05	5 32E-05	6.62E-05	8.04E-05	9.57E-05	1 12E-04	1 30E-04	1.48E-04	1.68E-04	1.89E-04	2 10F-04	2 32E-04	2 56E-04	2 80F-04
71	8.93F-07	3.49F-06	7.67F-06	1.34F-05	2.05F-05	2.90F-05	3.88F-05	4.98F-05	6.21F-05	7.55F-05	9.00F-05	1.06F-04	1.22F-04	1.40F-04	1.59F-04	1.78F-04	1.99F-04	2.20F-04	2.42F-04	2.65F-04
72	8.17E-07	3.22E-06	7.11E-06	1.24E-05	1.91E-05	2.70E-05	3.63E-05	4.67E-05	5.82E-05	7.09E-05	8.47E-05	9.95E-05	1.15E-04	1.32E-04	1.50E-04	1.68E-04	1.88E-04	2.08E-04	2.30E-04	2.52E-04
73	7.48E-07	2.96E-06	6.58E-06	1.15E-05	1.78E-05	2.52E-05	3.39E-05	4.37E-05	5.46E-05	6.66E-05	7.96E-05	9.37E-05	1.09E-04	1.25E-04	1.41E-04	1.59E-04	1.78E-04	1.97E-04	2.18E-04	2.39E-04
74	6.84E-07	2.73E-06	6.09E-06	1.07E-05	1.65E-05	2.35E-05	3.17E-05	4.09E-05	5.12E-05	6.25E-05	7.49E-05	8.81E-05	1.02E-04	1.18E-04	1.34E-04	1.50E-04	1.68E-04	1.87E-04	2.06E-04	2.26E-04
75	6.25E-07	2.51E-06	5.63E-06	9.93E-06	1.54E-05	2.20E-05	2.96E-05	3.83E-05	4.80E-05	5.87E-05	7.03E-05	8.29E-05	9.64E-05	1.11E-04	1.26E-04	1.42E-04	1.59E-04	1.77E-04	1.95E-04	2.14E-04

76	5.72E-07	2.31E-06	5.21E-06	9.21E-06	1.43E-05	2.05E-05	2.76E-05	3.58E-05	4.50E-05	5.51E-05	6.61E-05	7.80E-05	9.08E-05	1.04E-04	1.19E-04	1.34E-04	1.50E-04	1.67E-04	1.85E-04	2.03E-04
77	5.22E-07	2.13E-06	4.81E-06	8.54E-06	1.33E-05	1.91E-05	2.58E-05	3.35E-05	4.21E-05	5.16E-05	6.20E-05	7.33E-05	8.54E-05	9.84E-05	1.12E-04	1.27E-04	1.42E-04	1.58E-04	1.75E-04	1.92E-04
78	4.77E-07	1.96E-06	4.44E-06	7.92E-06	1.24E-05	1.78E-05	2.41E-05	3.13E-05	3.94E-05	4.84E-05	5.82E-05	6.89E-05	8.04E-05	9.26E-05	1.06E-04	1.19E-04	1.34E-04	1.49E-04	1.65E-04	1.82E-04
79	4.35E-07	1.80E-06	4.10E-06	7.34E-06	1.15E-05	1.65E-05	2.25E-05	2.92E-05	3.69E-05	4.54E-05	5.46E-05	6.47E-05	7.56E-05	8.72E-05	9.96E-05	1.13E-04	1.26E-04	1.41E-04	1.56E-04	1.72E-04
80	3.97E-07	1.65E-06	3.79E-06	6.79E-06	1.07E-05	1.54E-05	2.09E-05	2.73E-05	3.45E-05	4.25E-05	5.12E-05	6.08E-05	7.10E-05	8.20E-05	9.38E-05	1.06E-04	1.19E-04	1.33E-04	1.48E-04	1.63E-04
81	3.61E-07	1.52E-06	3.49E-06	6.28E-06	9.89E-06	1.43E-05	1.95E-05	2.55E-05	3.22E-05	3.98E-05	4.80E-05	5.70E-05	6.67E-05	7.72E-05	8.83E-05	1.00E-04	1.13E-04	1.26E-04	1.39E-04	1.54E-04
82	3.29E-07	1.39E-06	3.22E-06	5.81E-06	9.17E-06	1.33E-05	1.82E-05	2.38E-05	3.01E-05	3.72E-05	4.50E-05	5.35E-05	6.27E-05	7.25E-05	8.31E-05	9.42E-05	1.06E-04	1.19E-04	1.32E-04	1.45E-04
83	2.99E-07	1.28E-06	2.96E-06	5.37E-06	8.49E-06	1.23E-05	1.69E-05	2.22E-05	2.81E-05	3.48E-05	4.21E-05	5.01E-05	5.88E-05	6.81E-05	7.81E-05	8.87E-05	9.99E-05	1.12E-04	1.24E-04	1.37E-04
84	2.72E-07	1.17E-06	2.73E-06	4.96E-06	7.87E-06	1.15E-05	1.57E-05	2.06E-05	2.62E-05	3.25E-05	3.94E-05	4.70E-05	5.51E-05	6.40E-05	7.34E-05	8.34E-05	9.41E-05	1.05E-04	1.17E-04	1.30E-04
85	2.47E-07	1.07E-06	2.51E-06	4.57E-06	7.28E-06	1.06E-05	1.46E-05	1.92E-05	2.45E-05	3.03E-05	3.68E-05	4.40E-05	5.17E-05	6.00E-05	6.89E-05	7.84E-05	8.85E-05	9.92E-05	1.10E-04	1.22E-04
86	2.24E-07	9.77E-07	2.30E-06	4.22E-06	6.73E-06	9.84E-06	1.36E-05	1.79E-05	2.28E-05	2.83E-05	3.44E-05	4.11E-05	4.84E-05	5.63E-05	6.47E-05	7.37E-05	8.33E-05	9.34E-05	1.04E-04	1.15E-04
87	2.03E-07	8.92E-07	2.11E-06	3.88E-06	6.21E-06	9.11E-06	1.26E-05	1.66E-05	2.12E-05	2.64E-05	3.21E-05	3.84E-05	4.53E-05	5.27E-05	6.07E-05	6.92E-05	7.82E-05	8.78E-05	9.79E-05	1.09E-04
88	1.84E-07	8.14E-07	1.94E-06	3.57E-06	5.73E-06	8.43E-06	1.17E-05	1.54E-05	1.97E-05	2.46E-05	3.00E-05	3.59E-05	4.24E-05	4.93E-05	5.69E-05	6.49E-05	7.35E-05	8.25E-05	9.21E-05	1.02E-04
89	1.66E-07	7.42E-07	1.77E-06	3.28E-06	5.28E-06	7.78E-06	1.08E-05	1.43E-05	1.83E-05	2.29E-05	2.79E-05	3.35E-05	3.96E-05	4.61E-05	5.32E-05	6.08E-05	6.89E-05	7.75E-05	8.66E-05	9.61E-05
90	1.49E-07	6.75E-07	1.62E-06	3.01E-06	4.87E-06	7.19E-06	9.98E-06	1.33E-05	1.70E-05	2.13E-05	2.60E-05	3.12E-05	3.69E-05	4.31E-05	4.98E-05	5.70E-05	6.46E-05	7.27E-05	8.13E-05	9.03E-05
91	1.34E-07	6.14E-07	1.48E-06	2.76E-06	4.47E-06	6.63E-06	9.22E-06	1.23E-05	1.58E-05	1.98E-05	2.42E-05	2.91E-05	3.44E-05	4.03E-05	4.65E-05	5.33E-05	6.05E-05	6.81E-05	7.63E-05	8.48E-05
92	1.20E-07	5.57E-07	1.35E-06	2.53E-06	4.11E-06	6.10E-06	8.51E-06	1.14E-05	1.46E-05	1.83E-05	2.25E-05	2.71E-05	3.21E-05	3.76E-05	4.35E-05	4.98E-05	5.66E-05	6.38E-05	7.15E-05	7.96E-05
93	1.07E-07	5.05E-07	1.23E-06	2.31E-06	3.77E-06	5.61E-06	7.85E-06	1.05E-05	1.35E-05	1.70E-05	2.09E-05	2.51E-05	2.99E-05	3.50E-05	4.05E-05	4.65E-05	5.29E-05	5.97E-05	6.69E-05	7.46E-05
94	9.42E-08	4.57E-07	1.12E-06	2.11E-06	3.45E-06	5.15E-06	7.23E-06	9.67E-06	1.25E-05	1.57E-05	1.93E-05	2.33E-05	2.77E-05	3.26E-05	3.78E-05	4.34E-05	4.94E-05	5.58E-05	6.26E-05	6.98E-05
95	8.25E-08	4.13E-07	1.02E-06	1.93E-06	3.16E-06	4.73E-06	6.64E-06	8.91E-06	1.15E-05	1.45E-05	1.79E-05	2.16E-05	2.58E-05	3.03E-05	3.51E-05	4.04E-05	4.60E-05	5.21E-05	5.85E-05	6.53E-05
96	7.15E-08	3.72E-07	9.23E-07	1.76E-06	2.89E-06	4.33E-06	6.10E-06	8.19E-06	1.06E-05	1.34E-05	1.65E-05	2.00E-05	2.39E-05	2.81E-05	3.26E-05	3.76E-05	4.29E-05	4.85E-05	5.46E-05	6.10E-05
97	6.10E-08	3.35E-07	8.35E-07	1.59E-06	2.63E-06	3.96E-06	5.59E-06	7.53E-06	9.78E-06	1.24E-05	1.53E-05	1.85E-05	2.21E-05	2.60E-05	3.03E-05	3.49E-05	3.99E-05	4.52E-05	5.08E-05	5.69E-05
98	5.12E-08	3.01E-07	7.55E-07	1.45E-06	2.39E-06	3.61E-06	5.11E-06	6.90E-06	8.98E-06	1.14E-05	1.41E-05	1.71E-05	2.04E-05	2.41E-05	2.81E-05	3.24E-05	3.70E-05	4.20E-05	4.73E-05	5.30E-05
99	4.20E-08	2.70E-07	6.80E-07	1.31E-06	2.17E-06	3.29E-06	4.67E-06	6.31E-06	8.24E-06	1.04E-05	1.29E-05	1.57E-05	1.88E-05	2.22E-05	2.60E-05	3.00E-05	3.43E-05	3.90E-05	4.39E-05	4.92E-05
100	3.37E-08	2.41E-07	6.12E-07	1.18E-06	1.97E-06	2.99E-06	4.25E-06	5.76E-06	7.54E-06	9.58E-06	1.19E-05	1.45E-05	1.74E-05	2.05E-05	2.40E-05	2.77E-05	3.18E-05	3.61E-05	4.08E-05	4.57E-05
101	2.63E-08	2.14E-07	5.49E-07	1.07E-06	1.78E-06	2.71E-06	3.87E-06	5.25E-06	6.88E-06	8.76E-06	1.09E-05	1.33E-05	1.59E-05	1.89E-05	2.21E-05	2.56E-05	2.93E-05	3.34E-05	3.77E-05	4.24E-05
102	1.99E-08	1.90E-07	4.91E-07	9.57E-07	1.61E-06	2.45E-06	3.51E-06	4.77E-06	6.27E-06	7.99E-06	9.96E-06	1.22E-05	1.46E-05	1.73E-05	2.03E-05	2.35E-05	2.70E-05	3.08E-05	3.49E-05	3.92E-05
103	1.45E-08	1.67E-07	4.38E-07	8.58E-07	1.45E-06	2.21E-06	3.17E-06	4.33E-06	5.70E-06	7.28E-06	9.08E-06	1.11E-05	1.34E-05	1.59E-05	1.86E-05	2.16E-05	2.49E-05	2.84E-05	3.21E-05	3.62E-05
104	1.02E-08	1.46E-07	3.89E-07	7.66E-07	1.30E-06	1.99E-06	2.86E-06	3.92E-06	5.16E-06	6.61E-06	8.26E-06	1.01E-05	1.22E-05	1.45E-05	1.71E-05	1.98E-05	2.28E-05	2.61E-05	2.96E-05	3.33E-05
105	6.88E-09	1.26E-07	3.44E-07	6.82E-07	1.16E-06	1.79E-06	2.57E-06	3.53E-06	4.66E-06	5.98E-06	7.50E-06	9.20E-06	1.11E-05	1.32E-05	1.56E-05	1.81E-05	2.09E-05	2.39E-05	2.71E-05	3.06E-05
106	4.43E-09	1.07E-07	3.04E-07	6.05E-07	1.03E-06	1.60E-06	2.31E-06	3.17E-06	4.20E-06	5.40E-06	6.78E-06	8.34E-06	1.01E-05	1.20E-05	1.42E-05	1.65E-05	1.91E-05	2.18E-05	2.48E-05	2.80E-05
107	2.71E-09	8.94E-08	2.66E-07	5.35E-07	9.15E-07	1.42E-06	2.06E-06	2.84E-06	3.77E-06	4.86E-06	6.11E-06	7.53E-06	9.12E-06	1.09E-05	1.29E-05	1.50E-05	1.73E-05	1.99E-05	2.26E-05	2.56E-05
108	1.58E-09	7.28E-08	2.32E-07	4.70E-07	8.09E-07	1.26E-06	1.83E-06	2.53E-06	3.37E-06	4.35E-06	5.48E-06	6.77E-06	8.22E-06	9.84E-06	1.16E-05	1.36E-05	1.57E-05	1.81E-05	2.06E-05	2.33E-05
109	8.64E-10	5.75E-08	1.99E-07	4.11E-07	7.11E-07	1.11E-06	1.62E-06	2.25E-06	3.00E-06	3.88E-06	4.90E-06	6.06E-06	7.38E-06	8.84E-06	1.05E-05	1.23E-05	1.42E-05	1.63E-05	1.86E-05	2.11E-05
110	4.45E-10	4.38E-08	1.69E-07	3.57E-07	6.21E-07	9.75E-07	1.43E-06	1.99E-06	2.66E-06	3.45E-06	4.36E-06	5.41E-06	6.59E-06	7.91E-06	9.38E-06	1.10E-05	1.28E-05	1.47E-05	1.68E-05	1.91E-05
111	2.15E-10	3.19E-08	1.41E-07	3.07E-07	5.40E-07	8.51E-07	1.25E-06	1.74E-06	2.34E-06	3.04E-06	3.86E-06	4.80E-06	5.86E-06	7.04E-06	8.37E-06	9.83E-06	1.14E-05	1.32E-05	1.51E-05	1.71E-05
112	9.65E-11	2.20E-08	1.15E-07	2.62E-07	4.65E-07	7.37E-07	1.09E-06	1.52E-06	2.05E-06	2.67E-06	3.40E-06	4.23E-06	5.17E-06	6.24E-06	7.42E-06	8.73E-06	1.02E-05	1.17E-05	1.35E-05	1.53E-05
113	4.01E-11	1.43E-08	9.00E-08	2.19E-07	3.97E-07	6.34E-07	9.39E-07	1.32E-06	1.78E-06	2.33E-06	2.97E-06	3.70E-06	4.54E-06	5.48E-06	6.54E-06	7.71E-06	8.99E-06	1.04E-05	1.19E-05	1.36E-05
114	1.54E-11	8.70E-09	6.76E-08	1.79E-07	3.34E-07	5.39E-07	8.03E-07	1.13E-06	1.53E-06	2.01E-06	2.57E-06	3.22E-06	3.95E-06	4.79E-06	5.72E-06	6.75E-06	7.89E-06	9.15E-06	1.05E-05	1.20E-05
115	5.37E-12	4.88E-09	4.80E-08	1.42E-07	2.76E-07	4.53E-07	6.80E-07	9.64E-07	1.31E-06	1.72E-06	2.21E-06	2.77E-06	3.41E-06	4.14E-06	4.96E-06	5.87E-06	6.87E-06	7.98E-06	9.19E-06	1.05E-05
116	1.70E-12	2.51E-09	3.18E-08	1.07E-07	2.22E-07	3.74E-07	5.68E-07	8.10E-07	1.11E-06	1.46E-06	1.88E-06	2.36E-06	2.92E-06	3.55E-06	4.25E-06	5.05E-06	5.92E-06	6.89E-06	7.95E-06	9.10E-06
117	4.81E-13	1.17E-09	1.94E-08	7.64E-08	1.72E-07	3.01E-07	4.65E-07	6.70E-07	9.19E-07	1.22E-06	1.57E-06	1.99E-06	2.46E-06	3.00E-06	3.61E-06	4.29E-06	5.04E-06	5.88E-06	6.80E-06	7.80E-06
118	1.21E-13	4.84E-10	1.07E-08	5.03E-08	1.25E-07	2.32E-07	3.70E-07	5.41E-07	7.50E-07	1.00E-06	1.30E-06	1.64E-06	2.04E-06	2.50E-06	3.01E-06	3.59E-06	4.23E-06	4.94E-06	5.73E-06	6.59E-06
119	2.63E-14	1.76E-10	5.27E-09	3.00E-08	8.44E-08	1.69E-07	2.82E-07	4.23E-07	5.96E-07	8.02E-07	1.05E-06	1.33E-06	1.66E-06	2.04E-06	2.47E-06	2.95E-06	3.49E-06	4.08E-06	4.74E-06	5.47E-06
120	4.89E-15	5.48E-11	2.24E-09	1.57E-08	5.11E-08	1.13E-07	2.00E-07	3.13E-07	4.53E-07	6.20E-07	8.18E-07	1.05E-06	1.32E-06	1.62E-06	1.97E-06	2.37E-06	2.81E-06	3.30E-06	3.84E-06	4.43E-06
121	7.55E-16	1.43E-11	8.01E-10	7.02E-09	2.68E-08	6.65E-08	1.28E-07	2.13E-07	3.20E-07	4.51E-07	6.07E-07	7.90E-07	1.00E-06	1.24E-06	1.52E-06	1.83E-06	2.18E-06	2.57E-06	3.01E-06	3.49E-06
122	9.26E-17	2.96E-12	2.31E-10	2.55E-09	1.16E-08	3.30E-08	7.05E-08	1.26E-07	2.02E-07	2.97E-07	4.12E-07	5.50E-07	7.10E-07	8.95E-07	1.11E-06	1.34E-06	1.61E-06	1.91E-06	2.25E-06	2.62E-06
123	8.47E-18	4.61E-13	5.01E-11	7.07E-10	3.88E-09	1.28E-08	3.07E-08	6.06E-08	1.04E-07	1.64E-07	2.39E-07	3.31E-07	4.42E-07	5.71E-07	7.21E-07	8.91E-07	1.08E-06	1.30E-06	1.54E-06	1.81E-06
124	5.14E-19	4.77E-14	7.27E-12	1.32E-10	8.81E-10	3.40E-09	9.35E-09	2.06E-08	3.90E-08	6.62E-08	1.03E-07	1.52E-07	2.13E-07	2.87E-07	3.75E-07	4.78E-07	5.97E-07	7.32E-07	8.85E-07	1.06E-06
125	1.55E-20	2.46E-15	5.28E-13	1.24E-11	1.02E-10	4.64E-10	1.48E-09	3.69E-09	7.77E-09	1.45E-08	2.46E-08	3.89E-08	5.82E-08	8.33E-08	1.15E-07	1.53E-07	2.00E-07	2.55E-07	3.19E-07	3.93E-07

9.6 - Appendix 6: Fair spread calculation tables in Gaussian and correlation equal to 0.187

Fair value calculation tables for all tranches for the calibrated correlation value of 0.187 for April 3, 2007.

0-3% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	76522	75732	76522	75732	908891 s
0.5	2	0.9795	151841	148723	75320	73773	881067 s
0.75	3	0.9694	225646	218731	73805	71543	854087 s
1	4	0.9593	297821	285714	72175	69241	827961 s
1.25	5	0.9494	368322	349703	70502	66937	802681 s
1.5	6	0.9396	437145	410762	68823	64669	778229 s
1.75	7	0.9299	504304	468977	67159	62454	754583 s
2	8	0.9204	569824	524439	65521	60302	731720 s
2.25	9	0.9109	633740	577244	63916	58218	709613 s
2.5	10	0.9015	696089	627490	62349	56204	688238 s
2.75	11	0.8921	756910	675274	60821	54261	667569 s
3	12	0.8829	816244	720693	59334	52388	647581 s
3.25	13	0.8738	874132	763838	57888	50584	628252 s
3.5	14	0.8648	930614	804801	56483	48847	609555 s
3.75	15	0.8559	985732	843669	55118	47174	591470 s
4	16	0.8470	1039525	880526	53793	45565	573974 s
4.25	17	0.8383	1092031	915453	52506	44016	557046 s
4.5	18	0.8297	1143288	948530	51257	42525	540665 s
4.75	19	0.8211	1193332	979831	50044	41091	524812 s
5	20	0.8126	1242199	1009427	48867	39710	509468 s
Sums						1125234	13787464 s
Fair spread							816 bps

3-6% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	296	293	296	293	927751 s
0.5	2	0.9795	1618	1585	1323	1295	917851 s
0.75	3	0.9694	4193	4064	2574	2496	907754 s
1	4	0.9593	8067	7739	3874	3717	897455 s
1.25	5	0.9494	13227	12558	5160	4899	886967 s
1.5	6	0.9396	19633	18448	6405	6019	876308 s
1.75	7	0.9299	27230	25323	7597	7065	865497 s
2	8	0.9204	35961	33096	8731	8035	854555 s
2.25	9	0.9109	45763	41683	9803	8929	843503 s
2.5	10	0.9015	56577	51001	10814	9748	832360 s
2.75	11	0.8921	68343	60972	11766	10497	821144 s
3	12	0.8829	81003	71520	12660	11178	809875 s
3.25	13	0.8738	94502	82578	13499	11796	798567 s
3.5	14	0.8648	108787	94080	14285	12354	787236 s
3.75	15	0.8559	123808	105965	15021	12856	775896 s
4	16	0.8470	139517	118178	15709	13306	764561 s
4.25	17	0.8383	155869	130666	16352	13708	753243 s
4.5	18	0.8297	172821	143381	16952	14064	741953 s
4.75	19	0.8211	190333	156280	17511	14378	730700 s
5	20	0.8126	208365	169320	18032	14653	719495 s
Sums						181287	16512670 s
Fair spread							110 bps

6-9% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	14	14	14	14	927820 s
0.5	2	0.9795	114	112	100	98	918219 s
0.75	3	0.9694	368	356	254	246	908681 s
1	4	0.9593	825	791	457	439	899192 s
1.25	5	0.9494	1524	1447	699	664	889745 s
1.5	6	0.9396	2493	2343	969	911	880334 s
1.75	7	0.9299	3755	3492	1262	1174	870955 s
2	8	0.9204	5327	4903	1572	1447	861603 s
2.25	9	0.9109	7222	6578	1894	1726	852279 s
2.5	10	0.9015	9448	8517	2226	2007	842981 s
2.75	11	0.8921	12013	10717	2565	2288	833708 s
3	12	0.8829	14920	13174	2908	2567	824461 s
3.25	13	0.8738	18173	15880	3253	2843	815241 s
3.5	14	0.8648	21773	18829	3599	3113	806048 s
3.75	15	0.8559	25718	22011	3945	3377	796885 s
4	16	0.8470	30007	25417	4289	3633	787751 s
4.25	17	0.8383	34638	29038	4631	3882	778650 s
4.5	18	0.8297	39608	32861	4970	4123	769583 s
4.75	19	0.8211	44913	36878	5305	4356	760551 s
5	20	0.8126	50548	41076	5635	4579	751556 s
Sums						43485	16776244 s
Fair spread							26 bps

9-12% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	1	1	1	1	927824 s
0.5	2	0.9795	13	12	12	11	918244 s
0.75	3	0.9694	49	48	37	36	908758 s
1	4	0.9593	124	119	75	72	899360 s
1.25	5	0.9494	251	238	126	120	890048 s
1.5	6	0.9396	440	414	189	178	880817 s
1.75	7	0.9299	704	654	263	245	871664 s
2	8	0.9204	1051	967	347	320	862587 s
2.25	9	0.9109	1491	1358	440	401	853584 s
2.5	10	0.9015	2032	1832	541	488	844652 s
2.75	11	0.8921	2681	2392	649	579	835789 s
3	12	0.8829	3446	3043	765	675	826994 s
3.25	13	0.8738	4331	3785	886	774	818265 s
3.5	14	0.8648	5343	4621	1012	875	809600 s
3.75	15	0.8559	6487	5552	1143	978	801000 s
4	16	0.8470	7765	6577	1279	1083	792461 s
4.25	17	0.8383	9183	7698	1418	1189	783985 s
4.5	18	0.8297	10744	8913	1561	1295	775570 s
4.75	19	0.8211	12450	10222	1706	1401	767214 s
5	20	0.8126	14304	11623	1854	1507	758919 s
Sums						12227	16827335 s
Fair spread							7 bps

12-22% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	0	0	0	0	3092746 s
0.5	2	0.9795	1	1	1	1	3060824 s
0.75	3	0.9694	9	8	7	7	3029230 s
1	4	0.9593	26	25	18	17	2997960 s
1.25	5	0.9494	60	57	34	32	2967009 s
1.5	6	0.9396	115	108	55	52	2936373 s
1.75	7	0.9299	197	183	82	76	2906046 s
2	8	0.9204	311	286	114	105	2876026 s
2.25	9	0.9109	464	423	153	139	2846306 s
2.5	10	0.9015	661	596	197	178	2816884 s
2.75	11	0.8921	908	810	247	220	2787755 s
3	12	0.8829	1211	1069	303	267	2758915 s
3.25	13	0.8738	1575	1376	364	318	2730360 s
3.5	14	0.8648	2005	1734	430	372	2702085 s
3.75	15	0.8559	2507	2146	502	430	2674089 s
4	16	0.8470	3087	2615	580	491	2646366 s
4.25	17	0.8383	3749	3143	662	555	2618913 s
4.5	18	0.8297	4498	3732	749	622	2591727 s
4.75	19	0.8211	5339	4384	841	691	2564804 s
5	20	0.8126	6277	5101	938	762	2538141 s
Sums						5335	56142560 s
Fair spread							1 bps

22-100% tranche:

			Cummulative Loss	PV of Cummulative	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	Loss until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	0	0	0	0	3092746 s
0.5	2	0.9795	0	0	0	0	3060824 s
0.75	3	0.9694	0	0	0	0	3029232 s
1	4	0.9593	0	0	0	0	2997967 s
1.25	5	0.9494	0	0	0	0	2967024 s
1.5	6	0.9396	0	0	0	0	2936400 s
1.75	7	0.9299	1	1	1	1	2906092 s
2	8	0.9204	2	2	1	1	2876097 s
2.25	9	0.9109	4	3	2	2	2846411 s
2.5	10	0.9015	6	6	3	2	2817032 s
2.75	11	0.8921	10	9	4	3	2787955 s
3	12	0.8829	16	14	5	5	2759179 s
3.25	13	0.8738	22	20	7	6	2730699 s
3.5	14	0.8648	31	27	9	8	2702512 s
3.75	15	0.8559	42	36	11	9	2674616 s
4	16	0.8470	56	47	13	11	2647008 s
4.25	17	0.8383	72	60	16	14	2619684 s
4.5	18	0.8297	91	76	20	16	2592641 s
4.75	19	0.8211	115	94	23	19	2565876 s
5	20	0.8126	142	115	27	22	2539388 s
Sums						119	56149382 s
Fair spread							0 bps

9.7 - Appendix 7: Loss distribution generated in Student t with correlation equal to 0.187

Full loss distribution generated by applying the implied equity correlation of 0.187 in the one-factor Student t copula model for April 3, 2007.

Defaults	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
0	9 74F-01	9.51E-01	9 29F-01	9.07E-01	8.86E-01	8 66F-01	8.47F-01	8 29E-01	8 12F-01	7 94F-01	7 77E-01	7 60F-01	7.43E-01	7 28E-01	7 12F-01	6 98E-01	6.84E-01	6 70E-01	6 56E-01	6.43E-01
1	1.055.03	1 805 02	3.605.03	2 455 02	4 175 02	4 845 02	5.335.03	5 995 02	6 375 03	6.035.03	7.395.03	7.002.01	8 365 03	0 5 05 03	0.005.03	0.335.03	0.505.03	0.765.02	0.065.02	1.035.01
1	1.05E-02	1.89E-02	2.69E-02	3.45E-02	4.1/E-02	4.84E-02	5.33E-02	5.886-02	6.37E-02	6.92E-02	7.38E-02	7.81E-02	8.20E-02	8.588-02	9.00E-02	9.32E-02	9.50E-02	9.78E-02	9.962-02	1.03E-01
2	4.22E-03	8.20E-03	1.14E-02	1.4/E-02	1.81E-02	2.09E-02	2.41E-02	2.64E-02	2.88E-02	3.13E-02	3.38E-02	3.65E-02	3.93E-02	4.1/E-02	4.32E-02	4.53E-02	4.78E-02	4.88E-02	5.08E-02	5.21E-02
3	2.29E-03	4.81E-03	6.67E-03	8.83E-03	1.07E-02	1.28E-02	1.46E-02	1.63E-02	1.75E-02	1.91E-02	2.09E-02	2.29E-02	2.43E-02	2.57E-02	2.72E-02	2.88E-02	3.07E-02	3.24E-02	3.34E-02	3.50E-02
4	1.58E-03	2.99E-03	4.72E-03	5.95E-03	7.48E-03	8.78E-03	1.02E-02	1.16E-02	1.29E-02	1.39E-02	1.48E-02	1.58E-02	1.67E-02	1.76E-02	1.92E-02	2.01E-02	2.11E-02	2.27E-02	2.39E-02	2.45E-02
5	1.24E-03	2.42E-03	3.69E-03	4.63E-03	5.38E-03	6.21E-03	7.00E-03	8.29E-03	9.22E-03	1.03E-02	1.13E-02	1.20E-02	1.28E-02	1.40E-02	1.44E-02	1.48E-02	1.55E-02	1.65E-02	1.79E-02	1.91E-02
6	8.40E-04	1.78E-03	2.62E-03	2.99E-03	4.24E-03	5.06E-03	5.68E-03	6.49E-03	7.34E-03	8.34E-03	9.02E-03	9.42E-03	9.60E-03	1.05E-02	1.14E-02	1.24E-02	1.31E-02	1.30E-02	1.36E-02	1.46E-02
7	5.50E-04	1.27E-03	1.81E-03	2.88E-03	3.34E-03	4.28E-03	4.66E-03	5.08E-03	5.71E-03	6.17E-03	6.69E-03	7.78E-03	8.38E-03	9.08E-03	9.36E-03	1.04E-02	1.05E-02	1.13E-02	1.17E-02	1.16E-02
8	6 20F-04	1 11E-03	1.63E-03	2 49F-03	2 88E-03	3 13F-03	3 78F-03	4 54E-03	5 07F-03	5.44F-03	5.89F-03	6 34F-03	6 90F-03	6.82E-03	7.66F-03	7 83E-03	8 58F-03	9 31F-03	9.76F-03	1 02F-02
0	4 50E-04	9.005-04	1 225-03	1 905-03	2.495-02	2 975-03	3 375-03	3 495-03	4 18E-03	4.63E-03	5.02E-03	5.34E-03	6 14E-03	6.695-03	6.47E-03	6.625-03	7 225-03	7 375-03	7.825-03	8 655-03
10	3.705.04	0.000 04	4 205 03	4.355.03	4.005 00	2.375.03	3.572.03	3.435.03	4.102.03	4.035 03	5.022 05	4.755.00	5.045.03	5.005.00	5.305.03	5.005.00	6.405.03	6.075.03	7.052.05	7.000 00
10	3.70E-04	8.00E-04	1.29E-03	1.35E-03	1.902-03	2.37E-03	3.13E-03	3.42E-U3	3.42E-03	4.01E-03	4.44E-03	4.75E-U3	5.04E-03	5.082-03	5.79E-03	5.982-03	0.48E-U3	6.97E-03	7.36E-U3	7.33E-03
11	2.20E-04	7.10E-04	8.60E-04	1.20E-03	1.64E-03	2.21E-03	2.33E-03	2.50E-03	2.98E-03	3.41E-03	3.76E-03	4.11E-03	4.65E-03	5.18E-03	5.45E-03	5.83E-03	5.92E-03	5.84E-03	5.68E-03	6.06E-03
12	3.00E-04	4.40E-04	9.00E-04	1.15E-03	1.26E-03	1.74E-03	2.06E-03	2.29E-03	2.84E-03	2.83E-03	3.10E-03	3.55E-03	3.67E-03	4.07E-03	4.56E-03	5.02E-03	4.93E-03	5.24E-03	5.67E-03	6.07E-03
13	1.50E-04	4.10E-04	6.70E-04	1.01E-03	1.20E-03	1.41E-03	1.82E-03	2.28E-03	2.40E-03	2.37E-03	2.77E-03	3.04E-03	2.97E-03	3.75E-03	3.90E-03	4.04E-03	4.37E-03	4.90E-03	4.91E-03	5.05E-03
14	2.20E-04	3.60E-04	6.60E-04	9.50E-04	1.08E-03	1.23E-03	1.62E-03	2.02E-03	2.16E-03	2.62E-03	2.73E-03	2.63E-03	3.19E-03	3.03E-03	3.44E-03	3.50E-03	3.93E-03	4.25E-03	5.00E-03	5.03E-03
15	2.10E-04	3.70E-04	5.70E-04	6.30E-04	9.30E-04	1.03E-03	1.26E-03	1.67E-03	1.65E-03	2.12E-03	2.28E-03	2.75E-03	2.88E-03	2.86E-03	2.81E-03	3.34E-03	3.53E-03	3.61E-03	3.82E-03	4.21E-03
16	2.30E-04	3.30E-04	4.00E-04	6.70E-04	8.50E-04	1.01E-03	1.08E-03	1.46E-03	1.63E-03	1.69E-03	2.07E-03	2.25E-03	2.44E-03	2.77E-03	2.85E-03	2.80E-03	2.91E-03	3.20E-03	3.39E-03	3.47E-03
17	1.50E-04	2.60E-04	4.70E-04	5.90E-04	8.50E-04	8.80E-04	1.03E-03	1.06E-03	1.51E-03	1.41E-03	1.45E-03	1.80E-03	2.02E-03	2.22E-03	2.53E-03	2.76E-03	2.82E-03	2.81E-03	2.84E-03	3.13E-03
18	1 50F-04	2 80F-04	3 90F-04	4 00F-04	6.60E-04	8 10F-04	9.60F-04	9 80F-04	1 15E-03	1 78F-03	1 73E-03	1 98F-03	1 97F-03	2 15E-03	2 30F-03	2 60E-03	2 74F-03	2 97F-03	2 97F-03	2 92F-03
10	1 305 04	1 705 04	3.605.04	E EOE 04	4 505 04	8 305 04	8 605 04	1.025.02	1 105 03	1 305 03	1 455 03	1 545 02	1 865 03	1.025.02	1 035 03	2 105 02	3 225 03	3 335 03	3 805 03	2.055.02
15	1.302-04	1.702-04	3.002-04	3.305-04	4.302-04	8.302-04	8.002-04	1.032-03	1.192-03	1.302-03	1.432-03	1.346-03	1.802-03	1.552-03	2.035.03	2.102-03	2.332-03	2.322-03	2.895-03	3.032-03
20	8.002-05	2.20E-04	3.00E-04	3.30E-04	4.908-04	5.90E-04	8.00E-04	9.806-04	1.01E-03	1.12E-03	1.476-03	1.202-03	1.402-03	1.776-03	2.02E-03	1.988-03	2.28E-03	2.33E-03	2.292-03	2.50E-U3
21	9.00E-05	2.40E-04	2.50E-04	4.30E-04	6.40E-04	5.80E-04	6.70E-04	8.30E-04	8.70E-04	1.02E-03	1.14E-03	1.39E-03	1.31E-03	1.41E-03	1.63E-03	1.86E-03	1.81E-03	2.07E-03	2.09E-03	2.33E-03
22	6.00E-05	1.60E-04	3.00E-04	3.60E-04	4.20E-04	5.20E-04	6.70E-04	6.90E-04	7.90E-04	7.90E-04	9.90E-04	1.16E-03	1.34E-03	1.37E-03	1.55E-03	1.59E-03	1.89E-03	1.80E-03	1.90E-03	1.94E-03
23	1.40E-04	1.50E-04	1.70E-04	4.20E-04	4.10E-04	5.50E-04	5.50E-04	6.40E-04	8.30E-04	8.70E-04	7.20E-04	1.05E-03	1.28E-03	1.37E-03	1.45E-03	1.57E-03	1.59E-03	1.97E-03	2.06E-03	2.03E-03
24	7.00E-05	1.20E-04	2.60E-04	3.70E-04	3.60E-04	4.70E-04	5.10E-04	5.90E-04	6.40E-04	8.50E-04	9.50E-04	7.80E-04	7.60E-04	1.11E-03	1.24E-03	1.48E-03	1.48E-03	1.42E-03	1.67E-03	1.90E-03
25	5.00E-05	1.30E-04	2.40E-04	2.50E-04	4.30E-04	4.60E-04	5.00E-04	5.90E-04	6.60E-04	6.70E-04	8.90E-04	8.20E-04	9.40E-04	9.30E-04	1.07E-03	1.15E-03	1.43E-03	1.52E-03	1.49E-03	1.56E-03
26	8.00E-05	1.20E-04	1.60E-04	2.70E-04	2.90E-04	4.90E-04	5.10E-04	5.10E-04	6.50E-04	5.90E-04	7.70E-04	8.20E-04	8.90E-04	8.80E-04	1.01E-03	1.07E-03	1.24E-03	1.46E-03	1.44E-03	1.56E-03
27	5.00E-05	1.40E-04	1.80E-04	2.60E-04	2.50E-04	3.20E-04	3.70E-04	4.00E-04	5.60E-04	7.50E-04	7.80E-04	9.40E-04	8.00E-04	9.20E-04	8.40E-04	9.00E-04	8.40E-04	1.04E-03	1.29E-03	1.26E-03
28	5.00E-05	1.00E-04	6.00E-05	2.10E-04	3.20E-04	3.80E-04	4.90E-04	5.30E-04	4.30E-04	5.40E-04	6.10E-04	6.80E-04	8.90E-04	8.50E-04	9.80E-04	9.90E-04	1.02E-03	1.07E-03	1.18E-03	1.10E-03
29	4 00F-05	9.00E-05	1 40F-04	1 40F-04	2 70F-04	3 00F-04	3.00F-04	4 50F-04	4 40F-04	5 20F-04	5.00F-04	7 00F-04	6 80F-04	6 90F-04	7 40F-04	8 50E-04	8 60F-04	1.01E-03	1 20F-03	1 39E-03
20	4 005 05	1 205 04	1 105 04	1.005.04	2 405 04	2 605 04	3 805 04	2 605 04	E 00E 04	4 005 04	E 80E 04	5 705 04	6.005.04	8 505 04	7.005.04	7 505 04	0.405.04	8 005 04	0.405.04	1 025 02
30	4.002-03	1.302-04	1.102-04	1.502-04	2.402-04	2.002-04	3.802-04	5.000-04	3.002-04	4.505-04	3.802-04	5.702-04	0.00E-04	6.305-04	7.00E-04	7.302-04	5.402-04	0.005.04	3.402-04	1.020-03
31	5.00E-05	1.40E-04	1.20E-04	6.00E-05	1.008-04	2.10E-04	3.80E-04	5.10E-04	4.00E-04	4.50E-04	4.00E-04	5.90E-04	7.20E-04	6.70E-04	6.70E-04	5.00E-04	6.80E-04	8.00E-04	7.70E-04	1.046-03
32	4.00E-05	1.10E-04	1.10E-04	1.80E-04	2.60E-04	2.10E-04	2.90E-04	3.80E-04	4.60E-04	4.50E-04	5.30E-04	4.00E-04	7.00E-04	5.40E-04	7.90E-04	8.30E-04	6.10E-04	6.50E-04	6.70E-04	7.70E-04
33	1.00E-05	9.00E-05	1.00E-04	1.40E-04	1.90E-04	1.90E-04	1.70E-04	2.80E-04	3.80E-04	3.80E-04	3.60E-04	5.00E-04	3.60E-04	5.90E-04	6.10E-04	6.80E-04	7.30E-04	7.50E-04	7.40E-04	7.40E-04
34	2.00E-05	5.00E-05	1.10E-04	1.00E-04	1.40E-04	3.10E-04	2.10E-04	2.40E-04	3.90E-04	4.00E-04	4.80E-04	4.10E-04	4.80E-04	5.80E-04	5.40E-04	6.90E-04	7.80E-04	8.10E-04	7.10E-04	6.60E-04
35	4.00E-05	4.00E-05	1.00E-04	1.10E-04	1.00E-04	1.70E-04	2.40E-04	2.10E-04	2.70E-04	3.50E-04	3.80E-04	4.20E-04	3.90E-04	4.90E-04	5.70E-04	5.30E-04	6.40E-04	5.40E-04	7.50E-04	7.60E-04
36	1.00E-05	4.00E-05	1.60E-04	1.00E-04	1.30E-04	1.30E-04	1.80E-04	2.80E-04	2.10E-04	3.10E-04	3.40E-04	4.30E-04	4.10E-04	4.60E-04	4.90E-04	6.30E-04	6.70E-04	7.50E-04	6.30E-04	6.70E-04
37	2.00E-05	3.00E-05	1.40E-04	1.40E-04	1.20E-04	1.30E-04	2.30E-04	2.30E-04	2.60E-04	3.00E-04	3.50E-04	3.40E-04	4.30E-04	4.20E-04	5.00E-04	5.70E-04	5.50E-04	5.10E-04	7.40E-04	6.70E-04
38	4.00E-05	9.00E-05	6.00E-05	1.00E-04	1.20E-04	1.50E-04	1.80E-04	1.40E-04	2.20E-04	2.30E-04	3.20E-04	2.80E-04	3.90E-04	3.00E-04	3.70E-04	3.80E-04	5.20E-04	5.50E-04	5.90E-04	6.80E-04
39	2.00E-05	2.00E-05	9.00E-05	1.50E-04	1.30E-04	6.00E-05	1.00E-04	1.60E-04	1.90E-04	1.80E-04	3.00E-04	3.60E-04	2.30E-04	3.60E-04	3.30E-04	3.80E-04	3.60E-04	5.50E-04	4.70E-04	5.30E-04
40	0.00E+00	7.00E-05	3.00E-05	8.00E-05	1.10E-04	1.20E-04	1.40E-04	1.60E-04	2.00E-04	2.90E-04	2.10E-04	3.10E-04	3.40E-04	3.80E-04	3.80E-04	3.80E-04	4.40E-04	3.10E-04	4.70E-04	5.20E-04
41	0.00F+00	1.00E-05	7.00F-05	1 30F-04	1 40F-04	1 10F-04	1 20F-04	1 90F-04	2 50F-04	1 70F-04	1 90F-04	3 00F-04	2 80F-04	2 60F-04	2 90F-04	3 50E-04	4 10F-04	5 00F-04	4 30F-04	4 90F-04
12	2.005-05	2.005-05	9.005-05	7.00E-05	1.005-04	1 205-04	1.005-04	9.005-05	1 205-04	2.405-04	1 905-04	2 10E-04	2 905-04	3 305-04	3 30E-04	3 605-04	2 20E-04	4 105-04	4 20E-04	3 805-04
42	1.005.05	2.000 05	3.005.05	1.005.04	0.005.05	1.405.04	1.000.04	1 705 04	1.205.04	1.405.04	1.005.04	1.605.04	2.305.04	3.405.04	3.005.04	3.005.04	3.502.04	3 305 04	4.202.04	4 305 04
44	0.005.00	2.000 05	2.000 05	7.000.04	4.005.04	1.405.04	1.205.04	4.205.04	4 205 04	1.400 04	2.305.04	4.505.04	4.005.04	2.400.04	2.502.04	3.000.04	2.002.04	3.200.04	3.005.04	4.305.04
44	0.00E+00	3.00E-05	2.00E-05	7.00E-05	1.80E-04	1.40E-04	1.30E-04	1.20E-04	1.20E-04	1.90E-04	2.70E-04	1.50E-04	1.80E-04	2.80E-04	2.60E-04	2.60E-04	3.80E-04	3.70E-04	2.60E-04	4.20E-04
45	2.00E-05	3.00E-05	6.00E-05	5.00E-05	8.00E-05	1.20E-04	1.80E-04	1.50E-04	1.30E-04	1.20E-04	1.00E-04	2.30E-04	1.70E-04	2.10E-04	2.40E-04	2.60E-04	3.10E-04	3.80E-04	3.40E-04	3.70E-04
46	0.00E+00	3.00E-05	2.00E-05	4.00E-05	7.00E-05	1.10E-04	1.20E-04	1.30E-04	2.00E-04	1.20E-04	1.70E-04	2.10E-04	2.40E-04	1.90E-04	2.80E-04	2.90E-04	3.00E-04	2.00E-04	3.30E-04	2.80E-04
47	0.00E+00	2.00E-05	2.00E-05	7.00E-05	6.00E-05	1.10E-04	1.20E-04	1.50E-04	1.30E-04	1.60E-04	1.70E-04	1.70E-04	2.00E-04	2.00E-04	2.20E-04	2.00E-04	2.30E-04	3.00E-04	3.00E-04	3.30E-04
48	1.00E-05	2.00E-05	7.00E-05	4.00E-05	7.00E-05	1.10E-04	6.00E-05	1.20E-04	9.00E-05	1.30E-04	1.20E-04	1.50E-04	1.70E-04	1.60E-04	1.80E-04	2.90E-04	1.80E-04	2.30E-04	2.50E-04	3.00E-04
49	3.00E-05	1.00E-05	2.00E-05	3.00E-05	5.00E-05	5.00E-05	1.80E-04	1.00E-04	1.20E-04	1.30E-04	1.40E-04	1.10E-04	1.50E-04	2.00E-04	1.50E-04	1.40E-04	2.20E-04	2.40E-04	2.50E-04	2.40E-04
50	3.00E-05	0.00E+00	2.00E-05	5.00E-05	5.00E-05	6.00E-05	7.00E-05	9.00E-05	1.20E-04	1.50E-04	1.40E-04	1.20E-04	1.70E-04	2.00E-04	2.30E-04	1.90E-04	2.50E-04	2.20E-04	2.50E-04	2.10E-04
51	1.00E-05	2.00E-05	1.00E-05	1.00E-05	4.00E-05	7.00E-05	7.00E-05	7.00E-05	1.30E-04	1.00E-04	1.50E-04	1.20E-04	1.30E-04	1.40E-04	1.90E-04	2.00E-04	2.00E-04	1.90E-04	2.80E-04	3.20E-04
52	1.00E-05	1.00E-05	3.00E-05	7.00E-05	4.00E-05	5.00E-05	4.00E-05	1.40E-04	9.00E-05	1.30E-04	1.20E-04	2.00E-04	1.30E-04	1.60E-04	1.30E-04	1.80E-04	1.40E-04	2.00E-04	1.40E-04	2.20E-04
53	2.00E-05	2.00E-05	0.00E+00	2.00E-05	6.00E-05	4.00E-05	9.00E-05	5.00E-05	7.00E-05	1.10E-04	9.00E-05	1.30E-04	2.30E-04	1.10E-04	1.10E-04	1.00E-04	1.10E-04	2.10E-04	1.90E-04	1.90E-04
54	0.00E+00	1.00E-05	4.00E-05	1.00E-05	6.00E-05	6.00E-05	6.00E-05	3.00E-05	1.20E-04	8.00E-05	1.60E-04	1.20E-04	7.00E-05	1.70E-04	1.90E-04	1.50E-04	1.70E-04	1.80E-04	2.10E-04	1.90E-04
55	0.00E+00	2.00E-05	0.00E+00	3.00E-05	2.00E-05	4.00E-05	6.00E-05	8.00E-05	8.00E-05	7.00E-05	8.00E-05	1.40E-04	1.30E-04	1.50E-04	1.40E-04	2.20E-04	2.10E-04	1.30E-04	1.50E-04	1.40E-04
56	0.00F+00	0.00F+00	3.00F-05	1.00F-05	1,00F-05	6,00F-05	5.00F-05	8,00F-05	3,00F-05	1,10F-04	8.00F-05	8.00F-05	1.60F-04	1.80F-04	1.90F-04	1.50F-04	1.50F-04	1.30F-04	1.50F-04	1.70F-04
57	0.00E+00	0.00E+00	1.00E-05	2 00E-05	3.00F-05	3.00F-05	4.00E-05	6.00E-05	1 20F-04	9.00F-05	6.00E-05	7.00E-05	3.00E-05	1.105-04	1 30F-04	1 50E-04	1.40E-04	2 10F-04	1.40E-04	1.40E-04
58	0.005.00	2 005 05	1,005,05	4 000 00	2 005 05	0.005.00	3 005 05	5.005.05	3 000 07	6,005,05	8 005 07	7.005.05	8 000 00	6.005.07	1 105 07	1 605 04	1.405.04	1 105 04	1 705 04	1 705 04
50	0.005.000	2.000-05	0.005+00	1.005.05	2.005-05	2.005.05	5.005.05	5.00E-05	5.005.05	6.005.05	1.005.04	1.105.01	1 205 04	1.005.04	8.005.05	0.005.05	1.405-04	1.100-04	1.405.04	1.505.04
55	0.000000	0.000000	0.002+00	1.000-05	2.008-05	2.000-05	3.00E-05	0.002-05	3.002-05	0.002-05	1.002-04	1.102-04	1.50E-04	1.008-04	0.UUE-US	9.00E-05	1.7UE-U4	1.308-04	1.402-04	1.50E-04
60	U.UUE+U0	2.00E-05	3.00E-05	1.00E-05	2.UUE-05	2.00E-05	0.00E+00	5.00E-05	9.00E-05	6.UUE-05	9.00E-05	8.00E-05	9.00E-05	6.00E-05	1.10E-04	1.30E-04	1.40E-04	1.80E-04	1.60E-04	1.40E-04
61	1.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	3.00E-05	5.00E-05	2.00E-05	5.00E-05	6.00E-05	4.00E-05	1.30E-04	9.00E-05	7.00E-05	6.00E-05	8.00E-05	8.00E-05	1.20E-04	1.50E-04	1.70E-04
62	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	3.00E-05	0.00E+00	2.00E-05	3.00E-05	8.00E-05	6.00E-05	4.00E-05	4.00E-05	1.10E-04	9.00E-05	3.00E-05	1.00E-04	8.00E-05	1.40E-04	1.70E-04
63	0.00E+00	0.00E+00	0.00E+00	1.00E-05	5.00E-05	3.00E-05	1.00E-05	3.00E-05	1.00E-05	1.00E-05	8.00E-05	4.00E-05	9.00E-05	9.00E-05	9.00E-05	1.10E-04	6.00E-05	7.00E-05	6.00E-05	1.40E-04
64	0.00E+00	1.00E-05	2.00E-05	0.00E+00	1.00E-05	4.00E-05	4.00E-05	1.00E-05	3.00E-05	2.00E-05	2.00E-05	4.00E-05	4.00E-05	3.00E-05	2.00E-05	7.00E-05	8.00E-05	9.00E-05	1.10E-04	4.00E-05
65	1.00E-05	0.00E+00	0.00E+00	2.00E-05	0.00E+00	1.00E-05	0.00E+00	1.00E-05	2.00E-05	5.00E-05	5.00E-05	7.00E-05	8.00E-05	9.00E-05	1.00E-04	8.00E-05	9.00E-05	9.00E-05	8.00E-05	7.00E-05
66	0.00E+00	1.00E-05	1.00E-05	2.00E-05	1.00E-05	1.00E-05	6.00E-05	5.00E-05	1.00E-05	1.00E-05	3.00E-05	4.00E-05	3.00E-05	7.00E-05	1.00E-04	9.00E-05	6.00E-05	9.00E-05	9.00E-05	1.10E-04
67	0.00E+00	1.00E-05	2.00E-05	2.00E-05	2.00E-05	1.00E-05	1.00E-05	1.00E-05	3.00E-05	1.00E-05	3.00E-05	2.00E-05	5.00E-05	2.00E-05	5.00E-05	3.00E-05	7.00E-05	8.00E-05	7.00E-05	1.00E-04
68	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	3.00E-05	2.00E-05	2.00E-05	2.00E-05	3.00E-05	1.00E-05	1.00E-05	3.00E-05	3.00E-05	3.00E-05	5.00E-05	4.00E-05	4.00E-05	7.00E-05	3.00E-05
69	1.00E-05	0.00E+00	0.00E+00	2.00E-05	0.00E+00	0.00E+00	1.00E-05	3.00E-05	3.00E-05	2.00E-05	1.00E-05	3.00E-05	2.00E-05	4.00E-05	3.00E-05	5.00E-05	7.00E-05	4.00E-05	3.00E-05	1.00E-04
70	0.00F+00	0.00F+00	1.00F-05	0.00F+00	2,00F-05	1,00F-05	3.00F-05	3.00F-05	3,00F-05	2.00F-05	3.00F-05	4.00F-05	3.00F-05	4.00F-05	5.00F-05	4.00F-05	5.00F-05	6.00F-05	7.00F-05	5.00F-05
71	0.000.000	1.005-00	1.005-05	2.005.00	1.005.05	0.005+00	0.000-03	1.005-05	1.005.05	3.005-05	0.005-00	0.005400	2.005-05	1.005-05	2.005-05	5.00E.05	4.005-05	0.00E-05	2.005-05	4 00E-05
72	1.005.05	2.005.05	0.005-05	0.005-05	1.005.05	0.000100	0.005.00	1.005.05	£ 000 07	3.005.05	5.00E+00	2.005.05	2.000-03	2.005.05	4.005.05	4.005 OF	4.005.05	1.005.05	2.000-00	
72	1.002-05	3.002-05	0.002+00	0.000	1.002-05	0.002+00	0.005.05	1.008-05	5.00E-05	2.005-05	5.UUE-05	3.002-05	2.002-05	3.002-05	4.002-05	4.002-05	4.002-05	1.008-05	7.002-05	5.002-05
/3	0.00E+00	U.UUE+00	U.UUE+00	1.00E-05	U.UUE+00	4.00E-05	2.00E-05	0.00E+00	U.UUE+00	2.00E-05	3.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	2.00E-05	4.00E-05	3.00E-05	5.00E-05	4.00E-05
74	0.00E+00	0.00E+00	0.00E+00	1.00E-05	2.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	3.00E-05	2.00E-05	5.00E-05	2.00E-05	3.00E-05	2.00E-05	2.00E-05	3.00E-05	6.00E-05	6.00E-05	4.00E-05
75	0.00E+00	1.00E-05	2.00E-05	0.00E+00	2.00E-05	1.00E-05	1.00E-05	1.00E-05	0.00E+00	2.00E-05	2.00E-05	2.00E-05	3.00E-05	3.00E-05	2.00E-05	0.00E+00	0.00E+00	1.00E-05	2.00E-05	6.00E-05

76	0.00E+00	0.00E+00	2.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	2.00E-05	0.00E+00	0.00E+00	2.00E-05	2.00E-05	2.00E-05	1.00E-05	2.00E-05	3.00E-05	2.00E-05	1.00E-05	2.00E-05	2.00E-05
77	1.00E-05	0.00E+00	2.00E-05	2.00E-05	1.00E-05	1.00E-05	2.00E-05	1.00E-05	2.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	3.00E-05	2.00E-05	2.00E-05	1.00E-05	2.00E-05
78	1.00E-05	0.00E+00	0.00E+00	1.00E-05	0.00E+00	3.00E-05	1.00E-05	1.00E-05	2.00E-05	1.00E-05	0.00E+00	2.00E-05	4.00E-05	3.00E-05	4.00E-05	2.00E-05	3.00E-05	3.00E-05	3.00E-05	5.00E-05
79	0.00E+00	0.00E+00	0.00E+00	2.00E-05	1.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	3.00E-05	1.00E-05	3.00E-05	2.00E-05	2.00E-05	2.00E-05	1.00E-05	2.00E-05	2.00E-05	1.00E-05
80	0.00E+00	1.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	0.00E+00	1.00E-05	1.00E-05	2.00E-05	0.00E+00	0.00E+00	0.00E+00	3.00E-05	2.00E-05	2.00E-05	2.00E-05	0.00E+00	1.00E-05	2.00E-05
81	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E-05	0.00E+00	2.00E-05	1.00E-05	2.00E-05	2.00E-05	1.00E-05	2.00E-05	0.00E+00	0.00E+00	1.00E-05	2.00E-05	3.00E-05	4.00E-05	3.00E-05	3.00E-05
82	0.00F+00	0.00F+00	0.005+00	0.00F+00	0.00E+00	1.00F-05	0.00F+00	1.00E-05	0.00F+00	1.00E-05	3.00E-05	2.00E-05	2.00E-05	2.00E-05	2 00E-05	1.00E-05	0.00F+00	1.00E-05	1.00E-05	0.00F+00
83	0.005+00	0.005+00	0.005+00	0.00E+00	1.005-05	1.005-05	2.00E-05	2.00E-05	1.005-05	0.005+00	1.005-05	2.00E-05	2.005-05	2.005-05	0.005+00	2.005-05	3.005-05	2.006-05	2.00E-05	3 00E-05
84	0.00E+00	0.00E+00	0.005+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	1.00E-05	2.00E-05	2.00E-05	3.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	0.005+00
85	0.005+00	1.005-05	1.005-05	0.005+00	0.005+00	2.005-05	0.005+00	1.005-05	0.005+00	1.005-05	1.005-05	0.006+00	0.005+00	1.005-05	1.005-05	2.005-05	2.005-05	1.005-05	2.005-05	2.005-05
86	0.005+00	0.005+00	0.005+00	0.000+00	0.0000-000	0.005+00	1.005-05	1.00E-05	2.005-05	2.00E-05	1.005-05	0.000+00	0.000+00	0.005+00	0.005+00	0.005+00	1.005-05	3.00E-05	2.000.05	2.00E-05
00	0.005.00	0.005+00	0.005.00	0.005.00	0.005.00	1.005.05	1.005.05	0.005+00	1.005.05	0.005.00	0.005.00	2.005.05	2.005.05	3.005.05	4 005 05	2.005.05	2.005.05	1.005.05	0.005.00	0.005.00
00	0.000+00	0.000000	0.005+00	0.005+00	0.000000	0.005+00	1.005.05	2.005.05	0.005+00	0.000+00	1.005.05	1.005.05	0.005+00	0.005-00	4.002-03	3.005-05	1.005.05	2.005.05	2.005.05	1.005.05
00	0.002+00	0.000-00	4.005.05	0.000-000	0.000000	0.000+00	0.005.00	2.000-03	0.000+00	4.005.05	0.005-00	1.002-03	0.000+00	0.002+00	0.002400	2.000-03	2.005.05	2.002-03	3.000-03	1.000-03
89	0.00E+00	0.000+00	1.00E-05	1.002-05	0.000000	0.00E+00	0.000+00	0.000000	0.000+00	1.00E-05	0.002+00	0.000+00	1.002-05	0.002+00	1.002-05	1.00E-05	3.00E-05	1.00E-05	2.00E-05	1.002-05
90	0.00E+00	0.000+00	0.000000	0.000+00	1.002-05	0.00E+00	0.000+00	0.000000	2.002-05	2.00E-05	3.002-05	2.00E-05	2.002-05	1.00E-05	1.002-05	1.002-05	0.000+00	1.00E-05	0.00E+00	3.002-05
91	0.002+00	0.00E+00	0.002+00	0.002+00	0.000+00	1.002-05	1.00E-05	0.000+00	0.002+00	0.000+00	0.00E+00	1.00E-05	0.00E+00	0.002+00	0.00E+00	0.002+00	1.00E-05	1.00E-05	2.00E-05	1.002-05
92	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	0.00E+00	1.00E-05	2.00E-05	2.00E-05	1.00E-05	0.00E+00	1.00E-05	1.00E-05	2.00E-05
93	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00
94	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05								
95	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	2.00E-05	2.00E-05	0.00E+00
96	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	0.00E+00	2.00E-05
97	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	0.00E+00						
98	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	0.00E+00	1.00E-05	0.00E+00
99	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05
100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	0.00E+00	1.00E-05								
101	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00								
102	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00							
103	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
104	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	0.00E+00	0.00E+00
105	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
106	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05
107	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
108	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
109	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
110	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
111	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
112	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
113	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
114	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
115	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
116	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
117	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
118	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
119	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
120	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
121	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
122	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
123	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
124	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
125	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

9.8 - Appendix 8: Loss distribution generated in Student t with correlation equal to 0.471

Full loss distribution generated by applying the implied equity correlation of 0.471 in the one-factor Student t copula model for May 5, 2017.

Defaults	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
0	9.64E-01	9.33E-01	9.04E-01	8.77E-01	8.52E-01	8.27E-01	8.03E-01	7.81E-01	7.59E-01	7.39E-01	7.20E-01	7.01E-01	6.83E-01	6.65E-01	6.48E-01	6.32E-01	6.16E-01	6.01E-01	5.85E-01	5.71E-01
1	1 26E-02	2 15E-02	2 97E-02	3 74F-02	4 34E-02	4 95E-02	5 55E-02	6.03E-02	6 53E-02	6 86F-02	7 22E-02	7.61E-02	7 86F-02	8 23E-02	8 45E-02	8 57E-02	8 89E-02	9.02E-02	9 22E-02	9 41 E-02
-	5.085-02	0.775-02	1 215.02	1 665.02	2.005-02	2 285-02	2 525.02	2 705.02	2 1 85.02	2 405-02	2 665-02	2 755-02	2 085-02	4 125.02	4 215.02	4.475.02	4 585.02	4 815.02	4.965-02	5.005-02
-	3.002.03	5.000	0.105.02	1.002.02	1.205.02	1.475.02	1.000 02	1.705.02	1.025.02	3,305,02	3.002 02	3.735.02	3.502.02	2,745,02	2,005,02	2,005,02	3.075.02	2.125.02	2,225,02	3.335.02
3	5.00E-03	3.000-03	8.19E-05	1.07E-02	1.28E-02	1.47E-02	1.03E-02	1.795-02	1.926-02	2.20E-02	2.366-02	2.32E+02	2.60E-02	2.74E-02	2.885-02	2.992-02	3.07E-02	3.13E-02	3.23E-02	5.55E-02
4	2.14E-03	4.00E-03	5.48E-03	7.23E-03	8.97E-03	1.08E+02	1.19E-02	1.32E-02	1.41E+02	1.51E-02	1.61E-02	1.81E-02	1.92E-02	2.00E+02	2.09E-02	2.29E+02	2.38E-02	2.38E-02	2.44E-02	2.50E-02
5	1.81E-03	2.97E-03	4.37E-03	5.71E-03	6.44E-03	8.22E-03	9.15E-03	1.07E-02	1.11E-02	1.16E-02	1.32E-02	1.39E-02	1.53E-02	1.64E-02	1.68E-02	1.70E-02	1.81E-02	1.93E-02	2.01E-02	2.10E-02
6	1.27E-03	2.39E-03	3.81E-03	4.28E-03	5.58E-03	6.28E-03	7.37E-03	7.59E-03	9.06E-03	9.78E-03	1.03E-02	1.07E-02	1.16E-02	1.23E-02	1.34E-02	1.39E-02	1.44E-02	1.52E-02	1.57E-02	1.64E-02
7	9.40E-04	1.97E-03	2.95E-03	3.52E-03	4.38E-03	5.39E-03	6.22E-03	7.20E-03	7.26E-03	7.94E-03	8.47E-03	9.65E-03	9.83E-03	1.07E-02	1.13E-02	1.19E-02	1.23E-02	1.27E-02	1.35E-02	1.35E-02
8	8.40E-04	1.70E-03	2.42E-03	2.97E-03	3.79E-03	4.40E-03	5.36E-03	5.82E-03	6.36E-03	6.95E-03	7.25E-03	7.20E-03	8.10E-03	8.52E-03	8.98E-03	1.04E-02	1.15E-02	1.14E-02	1.17E-02	1.24E-02
9	7.10E-04	1.14E-03	2.17E-03	2.62E-03	3.32E-03	3.76E-03	4.11E-03	4.99E-03	5.57E-03	5.79E-03	6.36E-03	7.01E-03	7.16E-03	7.85E-03	8.21E-03	8.47E-03	8.67E-03	9.56E-03	1.01E-02	1.10E-02
10	5.30E-04	1.44E-03	1.69E-03	2.48E-03	2.68E-03	3.57E-03	3.69E-03	4.51E-03	5.01E-03	5.58E-03	6.07E-03	6.03E-03	6.57E-03	6.81E-03	7.03E-03	7.33E-03	8.18E-03	8.71E-03	9.12E-03	9.19E-03
11	4 90F-04	1 14F-03	1.35E-03	2 31E-03	2 21E-03	2.62E-03	3 59F-03	3.62E-03	4 74F-03	4 80E-03	4 77F=03	5.82E-03	5 72E-03	6 14E-03	6 79F-03	6 76F-03	7 21E-03	7 78F-03	7 77E-03	8 36F-03
12	4.605-04	1.175.02	1.245.02	1.975-02	2.255.02	2 515-02	2.015.02	2.615.02	2 795.02	4.425-02	4 825-02	4.975-02	5.285.02	5 225-02	5.615-02	5.085-02	6.005-03	6 565-02	7.425.02	7.416-02
12	4.002-04	7.005.04	1.346-03	1.672.03	2.250-05	2.512-05	3.005.03	3.105.03	3.7 92-03	3,005,03	4.022-03	4.972-03	5.200-03	5.222.03	5.075.00	5.902-03	5.005.03	0.302-03	C 205 02	6.505.00
13	4.50E-04	7.80E-04	1.39E-03	1.61E-03	2.05E-03	2.41E-03	2.89E-03	3.10E-03	3.44E-03	3.88E-03	3.99E-03	4.48E-03	5.05E-03	5.31E-03	5.27E-03	5.93E-03	5.88E-03	6.21E-03	0.20E-U3	0.58E-03
14	3.00E-04	8.30E-04	1.10E-03	1.27E-03	1.89E-03	2.09E-03	2.18E-03	2.78E-03	2.97E-03	3.26E-03	3.87E-03	4.02E-03	4.22E-03	4.89E-03	4.97E-03	5.21E-03	5.47E-03	5.23E-03	5.90E-03	6.18E-03
15	2.20E-04	6.50E-04	9.60E-04	1.20E-03	1.71E-03	1.95E-03	2.24E-03	2.46E-03	2.83E-03	3.31E-03	3.65E-03	3.51E-03	3.99E-03	4.16E-03	4.59E-03	4.83E-03	5.47E-03	5.39E-03	5.39E-03	5.60E-03
16	3.30E-04	5.80E-04	1.07E-03	1.40E-03	1.55E-03	1.76E-03	2.08E-03	2.29E-03	2.83E-03	2.87E-03	3.21E-03	3.47E-03	3.61E-03	4.00E-03	4.49E-03	4.12E-03	4.35E-03	5.09E-03	4.76E-03	5.17E-03
17	2.50E-04	5.20E-04	7.40E-04	1.16E-03	1.25E-03	1.73E-03	1.85E-03	2.06E-03	2.27E-03	2.72E-03	3.03E-03	3.45E-03	3.51E-03	3.54E-03	3.81E-03	4.09E-03	4.44E-03	4.19E-03	5.02E-03	4.71E-03
18	3.50E-04	5.10E-04	8.80E-04	1.13E-03	1.36E-03	1.39E-03	1.87E-03	1.93E-03	1.99E-03	2.39E-03	2.70E-03	3.15E-03	3.15E-03	3.32E-03	3.36E-03	3.96E-03	3.92E-03	4.38E-03	4.37E-03	4.78E-03
19	2.80E-04	4.90E-04	7.40E-04	8.40E-04	1.38E-03	1.57E-03	1.50E-03	1.89E-03	1.98E-03	2.25E-03	2.32E-03	2.59E-03	3.02E-03	2.92E-03	3.52E-03	3.48E-03	3.88E-03	3.96E-03	4.21E-03	4.53E-03
20	2.10E-04	3.70E-04	6.20E-04	8.50E-04	1.12E-03	1.52E-03	1.61E-03	1.69E-03	1.88E-03	2.07E-03	2.31E-03	2.39E-03	2.94E-03	2.79E-03	3.18E-03	3.65E-03	3.49E-03	3.59E-03	3.84E-03	3.91E-03
21	2 00F-04	4 20F-04	6 20E-04	7 90F-04	8 70F-04	1.08E-03	1 38F-03	1.655-03	1.69F-03	1.965-03	2 37F=03	2 215-03	2 58E-03	2 96F-03	2 88F-03	2 91F-03	3 26E-03	3 36F-03	3 36F-03	3 69F-03
22	1 50E-04	3.60F-04	6.60E-04	8 30E-04	1.09E-03	1.00E-03	1 38E-03	1.435-03	1.78E-03	1.76E-03	1.86E-03	2.665-03	2 23E-03	2.68E-03	2 57E-03	2.915-03	2 99F-03	3 355-03	3.415-03	3.655-03
22	1.305.04	3.005.04	4 705 04	6.005.04	0.705.04	1.015.02	1.302.03	1.205.02	1.655.03	1.565.03	1.735.03	1.075.03	2.615.02	2.002.03	2.572.03	2.512.03	2.552.05	2.075.02	2 285 02	2 225 02
25	7.005.05	3.005-04	4.705-04	6.90E-04	9.70E-04	1.012-03	1.216-03	1.595-03	1.03E-03	1.332-03	1.735-03	1.972-03	2.010-03	2.386-03	2.020-03	2.000-03	2.776-03	2.975-03	3.200-03	3.325-03
24	7.00E-05	2.80E-04	5.20E-04	6.10E-04	7.60E+04	1.08E-03	1.15E-03	1.50E-03	1.47E-03	1.48E-03	1.67E-03	1.69E-03	2.04E-03	2.36E-03	2.79E=03	2.42E-03	2.43E-03	2.81E-03	3.10E-03	3.20E-03
25	1.00E-04	3.30E-04	3.80E-04	6.40E-04	6.70E-04	1.04E-03	1.07E-03	1.20E-03	1.18E-03	1.43E-03	1.45E-03	1.72E-03	1.63E-03	2.04E-03	2.17E-03	2.46E-03	2.51E-03	2.40E-03	2.51E-03	2.95E-03
26	1.70E-04	2.20E-04	4.20E-04	5.10E-04	6.30E-04	9.10E-04	9.90E-04	1.16E-03	1.45E-03	1.59E-03	1.75E-03	1.58E-03	1.87E-03	2.17E-03	2.02E-03	2.31E-03	2.70E-03	2.70E-03	2.66E-03	2.64E-03
27	9.00E-05	3.10E-04	3.80E-04	6.60E-04	7.40E-04	8.10E-04	9.00E-04	1.04E-03	1.19E-03	1.52E-03	1.35E-03	1.52E-03	1.72E-03	1.77E-03	2.28E-03	2.36E-03	2.38E-03	2.54E-03	2.85E-03	2.80E-03
28	1.40E-04	2.70E-04	4.10E-04	5.20E-04	6.30E-04	6.70E-04	9.70E-04	1.20E-03	1.22E-03	1.32E-03	1.52E-03	1.72E-03	1.70E-03	1.77E-03	1.68E-03	2.04E-03	2.17E-03	2.32E-03	2.37E-03	2.57E-03
29	1.00E-04	1.80E-04	3.80E-04	5.30E-04	6.10E-04	5.90E-04	9.60E-04	7.70E-04	1.04E-03	1.24E-03	1.35E-03	1.47E-03	1.55E-03	1.70E-03	1.75E-03	1.95E-03	1.97E-03	2.32E-03	2.31E-03	2.37E-03
30	1.20E-04	2.30E-04	4.10E-04	5.50E-04	5.00E-04	6.80E-04	8.40E-04	1.13E-03	1.16E-03	1.15E-03	1.27E-03	1.38E-03	1.56E-03	1.59E-03	1.73E-03	1.65E-03	1.88E-03	1.79E-03	2.01E-03	2.57E-03
31	1.60E-04	2.50E-04	3.10E-04	4.00E-04	6.60E-04	6.60E-04	6.50E-04	8.90E-04	1.06E-03	1.20E-03	1.26E-03	1.35E-03	1.25E-03	1.42E-03	1.68E-03	1.78E-03	1.82E-03	1.85E-03	2.13E-03	2.00E-03
32	1.50E-04	1.10E-04	3.00E-04	3.00E-04	5.10E-04	6.00E-04	5.70E-04	9.00E-04	1.03E-03	1.02E-03	1.21E-03	1.14E-03	1.47E-03	1.35E-03	1.47E-03	1.60E-03	1.82E-03	1.89E-03	1.77E-03	1.77E-03
33	6.00E-05	1.80E-04	3.80E-04	4.10E-04	4.50E-04	6.00E-04	6.50E-04	6.40E-04	9.20E-04	1.06E-03	1.20E-03	1.38E-03	1.19E-03	1.46E-03	1.48E-03	1.53E-03	1.72E-03	1.79E-03	1.98E-03	1.93E-03
34	1.00F-04	5.00E-05	2 10E-04	3 20F-04	5 30F-04	6 00F-04	8 30F-04	8 20E-04	8 10F-04	9 10E-04	1.00F-03	1 18F-03	1.09E-03	1 39E-03	1.60E-03	1.66F-03	1.88E-03	1 79E-03	1 78F-03	1 76F-03
25	1 105-04	2.005-04	1.405-04	4 205-04	5.605-04	5 505-04	5.005-04	6.405-04	7.405-04	9.405-04	1 105-02	1.055-02	1 225.02	1 165-02	1.275-02	1 525.02	1.485-02	1 705-02	1.875.02	2 115-02
20	5.005-05	1.805-04	1.705-04	2 905-04	3.805-04	5 105-04	6.205-04	6 205-04	7 705-04	9.805-04	9.805-04	1.105-03	1.205-02	1.105-03	1.245-02	1.41E-02	1.475-03	1.705-02	1.525.02	1.615-02
30	1.705.04	1.000-04	1.700-04	3.505.04	5.000-04	5.205.04	5.505.04	0.302-04	7.702-04	9.002-04	9.002-04	1.005.00	1.200-03	1.105.03	1.346-03	1.905.03	1.472-03	1.702-03	1.552-05	1.012-03
5/	1.70E-04	1.105-04	1.505-04	3.302-04	3.10E-04	5.200-04	5.30E-04	0.70E-04	7.10E-04	6.40E-04	9.10E-04	1.085-03	1.175-03	1.192-03	1.135-03	1.296-03	1.486-03	1.450-05	1.022-03	1.005-03
38	3.00E-03	1.506-04	1.306-04	2.902-04	4.402-04	5.70E-04	3.00E-04	5.70E-04	0.80E-04	0.90E-04	8.30E+04	1.085-03	1.146.03	1.51E-05	1.235-05	1.535-05	1.32E-03	1.376-05	1.712-03	1.000-05
39	5.00E-05	1.20E-04	2.00E-04	3.30E-04	3.60E-04	3.50E-04	5.50E-04	6.10E-04	6.70E-04	6.60E-04	7.90E-04	1.07E-03	1.14E-03	9.50E-04	1.15E-03	1.15E-03	1.28E-03	1.61E-03	1.46E-03	1.65E-03
40	5.00E-05	2.30E-04	2.10E-04	1.80E-04	3.50E-04	4.60E-04	5.40E-04	5.90E-04	6.90E-04	8.90E-04	9.60E-04	7.10E-04	9.70E-04	1.24E-03	9.30E-04	1.06E-03	1.16E-03	1.24E-03	1.39E-03	1.33E-03
41	4.00E-05	1.10E-04	2.10E-04	2.40E-04	3.20E-04	4.70E-04	5.20E-04	5.20E-04	6.30E-04	6.30E-04	6.80E-04	7.30E-04	9.50E-04	9.70E-04	1.24E-03	1.19E-03	1.09E-03	1.15E-03	1.36E-03	1.54E-03
42	9.00E-05	2.00E-04	1.70E-04	1.70E-04	2.80E-04	4.80E-04	4.10E-04	4.60E-04	5.60E-04	6.60E-04	7.30E-04	7.60E-04	8.10E-04	1.10E-03	1.17E-03	1.24E-03	1.13E-03	1.24E-03	1.26E-03	1.45E-03
43	8.00E-05	1.90E-04	1.70E-04	2.10E-04	3.10E-04	3.30E-04	3.90E-04	5.00E-04	5.40E-04	6.90E-04	7.00E-04	7.70E-04	7.70E-04	8.30E-04	1.09E-03	1.03E-03	1.15E-03	1.07E-03	1.29E-03	1.32E-03
44	8.00E-05	1.80E-04	1.60E-04	2.10E-04	3.10E-04	4.30E-04	4.30E-04	5.60E-04	5.10E-04	4.90E-04	5.90E-04	7.40E-04	1.00E-03	7.00E-04	7.80E-04	9.20E-04	1.18E-03	1.08E-03	1.16E-03	1.11E-03
45	5.00E-05	5.00E-05	1.80E-04	1.80E-04	2.90E-04	3.10E-04	4.40E-04	4.80E-04	4.40E-04	5.60E-04	6.90E-04	6.00E-04	6.90E-04	9.10E-04	8.50E-04	1.06E-03	1.10E-03	1.04E-03	1.22E-03	1.27E-03
46	8.00E-05	7.00E-05	1.80E-04	1.10E-04	2.20E-04	2.80E-04	3.80E-04	4.10E-04	3.30E-04	4.60E-04	5.80E-04	7.40E-04	7.50E-04	9.00E-04	8.70E-04	9.10E-04	8.90E-04	1.13E-03	1.08E-03	1.24E-03
47	5.00E-05	4.00E-05	1.70E-04	1.90E-04	1.90E-04	3.20E-04	4.00E-04	3.10E-04	5.40E-04	5.40E-04	5.60E-04	7.10E-04	6.40E-04	6.70E-04	8.30E-04	8.80E-04	9.60E-04	8.40E-04	9.60E-04	1.21E-03
48	2.00E-05	9.00E-05	2.40E-04	1.90E-04	1.70E-04	3.40E-04	3.50E-04	3.40E-04	4.20E-04	4.20E-04	5.30E-04	5.50E-04	5.50E-04	7.90E-04	7.10E-04	9.10E-04	9.50E-04	1.01E-03	8.80E-04	9.30E-04
49	5 00E-05	8 00E-05	1.60F-04	1.60F-04	2 30E-04	3 40F-04	4 20F-04	3 30E-04	5 50E-04	4 90F-04	4 70F-04	4 90F-04	6 30F-04	5.80F-04	8 80F-04	8 10F-04	1.09E-03	1 11E-03	9 90F-04	9 90F-04
50	6.00E-05	6.00E-05	2 00F-04	1 70E-04	2 20E-04	2 70E-04	3.80E-04	4 40F-04	4 00F-04	4 20E-04	3 90F-04	5 30E-04	5.60E-04	8 20E-04	6.80F=04	7.60E+04	7 50E-04	9.40F-04	1.065-03	8 30F-04
50	4.005.05	1 105 04	1.605.04	2 205 04	1 705 04	2.505.04	3.405.04	4.605.04	4.105.04	4.005.04	6 105 04	5.502.01	5.002.01	5.10E 04	6.005.04	7.105.04	7.705.04	0.205.04	1.075.02	0.305.04
51	1.005-04	1.205-04	7.005-05	2.105-04	1.705-04	2 105-04	2.205-04	2 805-04	4.105-04	4.405-04	5.405-04	5 205-04	6 705-04	6.005-04	6.805-04	7.105-04	7.405-04	7 705-04	0.005-04	1 125-02
52	4.005.05	0.005.05	1.005.04	2.105.04	3.405.04	1.805.04	2.202.01	4 705 04	3.005.04	5 305 04	4 505 04	4.605.04	6 405 04	E 40E 04	6.005.04	6 705 04	7.705.04	7 205 04	7 205 04	0.105.04
55	4.002-03	9.00E-03	1.005-04	2.102-04	2.40E-04	1.005.04	2.000-04	4.70E-04	2.900-04	3.20E-04	4.305-04	4.002-04	0.400-04	5.402-04	0.00E+04	6.70E-04	7.70E-04	7.50E-04	7.502-04	9.105-04
54	2.000-03	7.00E-03	1.105-04	9.00E-03	1.70E-04	1.005-04	2.80E-04	2.000-04	4.302-04	4.20E-04	4.30E+04	3.30E-04	3.00E-04	3.20E-04	0.202-04	0.002-04	0.10E-04	0.30E-04	8.00E-04	1.106-05
55	8.00E-05	6.00E-05	7.00E-05	2.30E-04	1.30E-04	3.10E-04	3.10E-04	4.10E-04	4.50E-04	5.00E-04	4.30E-04	5.40E-04	4.80E-04	6.50E-04	4.90E-04	4.70E-04	7.30E-04	7.80E-04	8.70E-04	8.00E-04
56	1.00E-05	8.00E-05	1.10E-04	1.30E-04	1.50E-04	2.00E-04	1.60E-04	3.20E-04	2.90E-04	3.10E-04	4.20E-04	5.60E-04	5.50E-04	5.90E-04	6.40E-04	7.70E-04	6.60E-04	5.70E-04	6.90E-04	6.90E-04
57	3.00E-05	8.00E-05	7.00E-05	1.70E-04	1.50E-04	1.50E-04	2.80E-04	3.00E-04	4.10E-04	3.60E-04	3.50E-04	3.20E-04	5.90E-04	6.70E-04	5.90E-04	6.30E-04	6.00E-04	7.90E-04	7.00E-04	8.60E-04
58	2.00E-05	6.00E-05	1.60E-04	1.70E-04	1.70E-04	1.40E-04	2.60E-04	2.40E-04	3.10E-04	3.90E-04	3.60E-04	4.10E-04	4.50E-04	5.30E-04	7.20E-04	5.90E-04	5.60E-04	6.20E-04	7.70E-04	7.50E-04
59	4.00E-05	5.00E-05	8.00E-05	1.60E-04	1.30E-04	2.50E-04	1.80E-04	2.40E-04	3.80E-04	3.00E-04	4.10E-04	3.20E-04	4.00E-04	4.60E-04	5.00E-04	7.50E-04	7.50E-04	6.80E-04	5.40E-04	7.20E-04
60	3.00E-05	8.00E-05	9.00E-05	1.50E-04	2.30E-04	1.30E-04	2.20E-04	2.90E-04	2.90E-04	3.40E-04	4.90E-04	4.70E-04	3.00E-04	4.60E-04	4.90E-04	5.50E-04	6.70E-04	6.50E-04	7.80E-04	5.60E-04
61	4.00E-05	6.00E-05	1.40E-04	1.10E-04	2.10E-04	1.70E-04	2.30E-04	2.80E-04	3.10E-04	3.20E-04	4.70E-04	4.40E-04	3.90E-04	4.60E-04	4.90E-04	5.20E-04	7.00E-04	5.70E-04	6.20E-04	6.90E-04
62	3.00E-05	5.00E-05	9.00E-05	1.30E-04	1.40E-04	1.30E-04	1.50E-04	2.30E-04	2.80E-04	2.80E-04	3.20E-04	3.10E-04	3.60E-04	3.60E-04	5.10E-04	5.30E-04	5.40E-04	7.00E-04	6.60E-04	7.30E-04
63	3.00E-05	8.00E-05	1.10E-04	1.10E-04	1.80E-04	1.50E-04	1.30E-04	1.10E-04	2.80E-04	4.60E-04	3.00E-04	5.90E-04	4.10E-04	4.20E-04	4.70E-04	3.90E-04	4.10E-04	6.10E-04	6.90E-04	6.50E-04
64	4.00E-05	7.00E-05	8.00E-05	8.00E-05	1.10E-04	1.80E-04	1.20E-04	1.90E-04	1.90E-04	3.00E-04	2.30E-04	3.20E-04	3.50E-04	3.50E-04	4.00E-04	4.80E-04	5.00E-04	5.20E-04	6.30E-04	7.40E-04
65	2.00E-05	2.00E-05	7.00E-05	1.30E-04	1.80E-04	2.70E-04	2.00E-04	2.10E-04	2.60E-04	2.20E-04	2.60E-04	3.50E-04	4.40E-04	4.10E-04	3.40E-04	5.10E-04	6.00E-04	5.90E-04	5.30E-04	6.40E-04
66	2 00E-05	4 00E-05	1.00F-04	1 00F-04	1 40F-04	1 50F-04	1 10F-04	2 00F-04	2 20E-04	1.80F-04	3 50F-04	3 40F-04	4 00F-04	3 90F-04	4 20F-04	3 20F-04	4 20F-04	4 00F-04	4 40F-04	4 60F-04
67	2 005-05	7 00F-05	4 005-05	1 90F-04	1 30F-04	1 50F-04	2 40F-04	1 90F-04	2 30F-04	3 105-04	3 50F-04	2 90F-04	4 30F-04	3 40F-04	4 70F-04	4 20F-04	4 10F-04	5 70F-04	7 00F-04	6 20F-04
69	2.005.05	2.005.05	7 005 05	0.005.05	1 205 07	1.005.04	1.605.04	2.005.04	2 105 04	2 105 07	2 205 07	2 205 04	2 805 04	4.405.04	2.605.04	4 605 04	2 205 07	4 605 04	4 705 04	5 705 04
00	2.00E-05	5.00E-05	1.00E-05	9.00E-05	1.5UE-04	1.00E-04	1.605.04	2.00E-04	2.10E-04	2.10E-04	2.201-04	2.5UE-04	3.40E-04	4.4UE-U4	5.0UE-04	4.005-04	3.30E-04	4.00E-04	4./UE-04	5.7UE-04
69	1.00E-05	0.UUE-US	4.00E-05	0.00E-05	1.4UE-04	1.90E-04	1.00E-04	1.00E-04	1.40E-04	2.50E-04	2.905-04	2./UE-U4	5.40E-04	5.5UE+U4	4.7 UE-04	4.00E-04	4.70E-04	4.70E-04	3.4UE-U4	5.50E-04
70	1.00E-05	1.00E-05	0.00E-05	9.00E-05	1.00E-04	1.70E-04	1.10E-04	1.20E-04	2.50E-04	2.50E-04	2.40E-04	2.40E-04	2.80E-04	3.40E-04	5.80E-04	4.10E-04	4.50E-04	3.2UE-04	3.00E-04	5.50E-04
71	0.00E+00	2.00E-05	3.00E-05	8.00E-05	1.80E-04	8.00E-05	1.40E-04	1.90E-04	2.20E-04	1.80E-04	2.00E-04	3.00E-04	3.40E-04	3.00E-04	3.60E-04	4.90E-04	4.20E-04	4.00E-04	3.70E-04	4.50E-04
72	0.00E+00	5.00E-05	1.00E-04	8.00E-05	1.30E-04	8.00E-05	1.60E-04	9.00E-05	1.00E-04	2.50E-04	3.10E-04	2.80E-04	2.30E-04	2.70E-04	2.30E-04	3.60E-04	4.30E-04	4.30E-04	4.50E-04	3.30E-04
73	0.00E+00	5.00E-05	4.00E-05	7.00E-05	9.00E-05	1.50E-04	1.30E-04	1.70E-04	2.10E-04	2.10E-04	1.90E-04	1.80E-04	3.30E-04	4.10E-04	3.40E-04	2.60E-04	4.50E-04	4.60E-04	4.50E-04	4.80E-04
74	1.00E-05	3.00E-05	1.10E-04	1.00E-04	1.20E-04	1.60E-04	1.60E-04	1.90E-04	1.10E-04	1.70E-04	2.40E-04	2.90E-04	1.70E-04	3.00E-04	3.30E-04	2.40E-04	3.80E-04	4.10E-04	4.20E-04	4.90E-04
75	0.00E+00	6.00E-05	5.00E-05	7.00E-05	8.00E-05	1.30E-04	2.00E-04	1.50E-04	1.00E-04	1.70E-04	2.10E-04	2.80E-04	2.40E-04	2.90E-04	2.90E-04	3.40E-04	2.60E-04	3.60E-04	4.20E-04	4.00E-04

76	0.00E+00	4.00E-05	2.00E-05	3.00E-05	1.20E-04	1.40E-04	1.50E-04	1.90E-04	1.50E-04	1.70E-04	2.30E-04	2.10E-04	2.00E-04	1.80E-04	2.90E-04	4.10E-04	3.20E-04	4.90E-04	4.40E-04	3.70E-04
77	2.00E-05	4.00E-05	8.00E-05	4.00E-05	1.10E-04	1.50E-04	1.60E-04	1.60E-04	2.20E-04	1.60E-04	1.50E-04	2.20E-04	2.60E-04	2.80E-04	2.50E-04	2.70E-04	2.80E-04	2.70E-04	4.10E-04	4.70E-04
78	2.00E-05	3.00E-05	4.00E-05	1.10E-04	1.10E-04	1.20E-04	1.00E-04	1.40E-04	1.30E-04	1.70E-04	2.00E-04	2.90E-04	2.10E-04	1.90E-04	2.90E-04	3.20E-04	2.70E-04	2.60E-04	2.40E-04	3.10E-04
79	1.00E-05	4.00E-05	2.00E-05	7.00E-05	5.00E-05	1.20E-04	2.00E-04	1.30E-04	1.50E-04	1.10E-04	1.70E-04	2.20E-04	3.10E-04	2.50E-04	2.40E-04	2.00E-04	4.20E-04	2.70E-04	3.50E-04	3.50E-04
80	1.00E-05	1.00E-05	4.00E-05	8.00E-05	7.00E-05	1.60E-04	1.20E-04	1.30E-04	1.40E-04	2.00E-04	1.60E-04	1.90E-04	2.60E-04	2.70E-04	2.30E-04	2.90E-04	3.40E-04	3.80E-04	3.00E-04	3.50E-04
81	0.00E+00	1.00E-05	3.00E-05	5.00E-05	1.10E-04	1.30E-04	1.40E-04	1.80E-04	1.70E-04	1.20E-04	1.20E-04	9.00E-05	1.90E-04	2.80E-04	3.10E-04	3.20E-04	3.00E-04	3.40E-04	3.70E-04	3.10E-04
82	0.00E+00	3.00E-05	8.00E-05	4.00E-05	1.00E-04	1.00E-04	1.40E-04	1.60E-04	2.20E-04	1.50E-04	1.60E-04	1.60E-04	2.30E-04	2.30E-04	2.30E-04	2.80E-04	2.20E-04	3.30E-04	3.50E-04	3.80E-04
83	3.00E-05	0.00E+00	3.00E-05	4.00E-05	3.00E-05	1.40E-04	1.30E-04	2.10E-04	1.80E-04	1.30E-04	1.80E-04	1.70E-04	1.90E-04	1.70E-04	2.10E-04	1.90E-04	2.40E-04	2.70E-04	2.80E-04	4.00E-04
84	1.00E-05	0.00E+00	1.00E-05	3.00E-05	5.00E-05	6.00E-05	1.10E-04	1.10E-04	1.20E-04	1.70E-04	1.40E-04	1.70E-04	1.30E-04	3.10E-04	2.10E-04	2.60E-04	3.10E-04	3.20E-04	4.50E-04	3.80E-04
85	1.00E-05	2.00E-05	7.00E-05	2.00E-05	6.00E-05	4.00E-05	1.20E-04	1.40E-04	1.50E-04	1.90E-04	1.80E-04	1.60E-04	1.60E-04	1.80E-04	2.70E-04	2.40E-04	2.50E-04	3.30E-04	3.10E-04	4.00E-04
86	1.00E-05	2.00E-05	2.00E-05	7.00E-05	3.00E-05	8.00E-05	1.40E-04	1.60E-04	1.70E-04	2.00E-04	1.00E-04	1.50E-04	1.50E-04	2.10E-04	2.60E-04	2.20E-04	2.00E-04	2.40E-04	2.80E-04	2.60E-04
87	0.00E+00	2.00E-05	2.00E-05	7.00E-05	6.00E-05	7.00E-05	1.10E-04	1.30E-04	1.60E-04	1.60E-04	2.30E-04	1.50E-04	1.70E-04	1.10E-04	2.70E-04	3.10E-04	3.00E-04	1.90E-04	2.30E-04	2.10E-04
88	2.00E-05	4.00E-05	4.00E-05	3.00E-05	5.00E-05	5.00E-05	7.00E-05	9.00E-05	1.50E-04	1.40E-04	2.40E-04	1.40E-04	1.20E-04	1.70E-04	1.50E-04	2.00E-04	2.40E-04	2.20E-04	2.10E-04	2.90E-04
89	1.00E-05	1.00E-05	2.00E-05	2.00E-05	6.00E-05	5.00E-05	7.00E-05	1.10E-04	1.70E-04	2.50E-04	1.50E-04	2.30E-04	1.90E-04	1.70E-04	1.60E-04	1.80E-04	1.40E-04	2.80E-04	3.00E-04	2.90E-04
90	0.00E+00	0.00E+00	2.00E-05	4.00E-05	7.00E-05	7.00E-05	6.00E-05	9.00E-05	1.30E-04	1.10E-04	2.00E-04	2.20E-04	2.00E-04	1.20E-04	1.30E-04	1.70E-04	2.20E-04	2.10E-04	2.70E-04	3.10E-04
91	0.00E+00	3.00E-05	3.00E-05	7.00E-05	3.00E-05	6.00E-05	5.00E-05	1.00E-04	1.10E-04	1.90E-04	1.40E-04	2.20E-04	1.50E-04	1.70E-04	1.60E-04	1.50E-04	2.30E-04	1.80E-04	1.90E-04	1.90E-04
92	1.00E-05	0.00E+00	2.00E-05	3.00E-05	4.00E-05	5.00E-05	9.00E-05	5.00E-05	4.00E-05	1.20E-04	1.10E-04	1.60E-04	2.10E-04	1.90E-04	1.40E-04	1.90E-04	2.20E-04	1.80E-04	2.40E-04	2.70E-04
93	1.00E-05	1.00E-05	2.00E-05	2.00E-05	5.00E-05	5.00E-05	5.00E-05	1.00E-04	8.00E-05	7.00E-05	1.70E-04	9.00E-05	1.60E-04	1.80E-04	2.00E-04	1.80E-04	1.30E-04	2.30E-04	1.40E-04	1.40E-04
94	1.00E-05	2.00E-05	3.00E-05	2.00E-05	2.00E-05	5.00E-05	4.00E-05	5.00E-05	1.30E-04	1.20E-04	1.90E-04	2.00E-04	1.60E-04	2.10E-04	2.10E-04	1.80E-04	1.70E-04	2.10E-04	2.50E-04	2.20E-04
95	0.00E+00	1.00E-05	0.00E+00	2.00E-05	4.00E-05	1.00E-05	6.00E-05	6.00E-05	9.00E-05	6.00E-05	8.00E-05	1.80E-04	1.90E-04	1.40E-04	1.70E-04	1.70E-04	2.20E-04	1.90E-04	1.60E-04	2.20E-04
96	0.00E+00	0.00E+00	3.00E-05	0.00E+00	5.00E-05	3.00E-05	4.00E-05	4.00E-05	8.00E-05	9.00E-05	1.10E-04	1.30E-04	1.70E-04	1.60E-04	1.10E-04	1.90E-04	1.50E-04	2.00E-04	2.50E-04	2.40E-04
97	0.00E+00	4.00E-05	2.00E-05	4.00E-05	4.00E-05	3.00E-05	5.00E-05	1.00E-04	2.00E-05	1.40E-04	6.00E-05	1.00E-04	1.80E-04	2.10E-04	2.20E-04	1.50E-04	2.10E-04	1.50E-04	1.80E-04	2.10E-04
98	2.00E-05	0.00E+00	1.00E-05	4.00E-05	3.00E-05	5.00E-05	1.00E-05	6.00E-05	6.00E-05	7.00E-05	1.30E-04	1.10E-04	1.20E-04	2.10E-04	1.80E-04	2.30E-04	1.90E-04	1.90E-04	1.60E-04	1.80E-04
99	0.00E+00	0.00E+00	3.00E-05	3.00E-05	2.00E-05	4.00E-05	6.00E-05	4.00E-05	7.00E-05	5.00E-05	9.00E-05	1.00E-04	1.30E-04	1.30E-04	2.20E-04	1.10E-04	1.60E-04	1.80E-04	1.80E-04	1.70E-04
100	2.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	5.00E-05	3.00E-05	4.00E-05	5.00E-05	8.00E-05	7.00E-05	5.00E-05	9.00E-05	1.10E-04	1.20E-04	2.30E-04	2.40E-04	2.10E-04	1.50E-04	1.60E-04
101	0.00E+00	0.00E+00	0.00E+00	2.00E-05	3.00E-05	6.00E-05	4.00E-05	2.00E-05	8.00E-05	6.00E-05	1.10E-04	8.00E-05	8.00E-05	1.50E-04	1.10E-04	1.10E-04	1.20E-04	2.20E-04	2.40E-04	2.80E-04
102	0.00E+00	1.00E-05	0.00E+00	1.00E-05	1.00E-05	3.00E-05	6.00E-05	5.00E-05	5.00E-05	4.00E-05	5.00E-05	1.30E-04	1.00E-04	1.00E-04	2.10E-04	2.10E-04	1.90E-04	1.80E-04	2.10E-04	1.40E-04
103	0.00E+00	0.00E+00	3.00E-05	3.00E-05	1.00E-05	1.00E-05	5.00E-05	5.00E-05	5.00E-05	4.00E-05	5.00E-05	7.00E-05	1.00E-04	7.00E-05	9.00E-05	1.50E-04	1.70E-04	1.20E-04	2.00E-04	2.70E-04
104	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.00E-05	1.00E-05	2.00E-05	3.00E-05	3.00E-05	9.00E-05	8.00E-05	6.00E-05	1.00E-04	9.00E-05	8.00E-05	7.00E-05	1.20E-04	2.00E-04	1.10E-04	1.10E-04
105	1.00E-05	0.00E+00	0.00E+00	3.00E-05	0.00E+00	2.00E-05	3.00E-05	4.00E-05	3.00E-05	5.00E-05	3.00E-05	9.00E-05	7.00E-05	9.00E-05	6.00E-05	9.00E-05	1.20E-04	1.10E-04	2.00E-04	1.90E-04
106	0.00E+00	2.00E-05	1.00E-05	1.00E-05	4.00E-05	0.00E+00	1.00E-05	2.00E-05	0.00E+00	4.00E-05	8.00E-05	4.00E-05	7.00E-05	1.00E-04	1.00E-04	1.00E-04	9.00E-05	1.20E-04	1.30E-04	1.40E-04
107	0.00E+00	2.00E-05	0.00E+00	0.00E+00	1.00E-05	3.00E-05	1.00E-05	1.00E-05	4.00E-05	1.00E-05	3.00E-05	4.00E-05	4.00E-05	8.00E-05	9.00E-05	1.30E-04	8.00E-05	7.00E-05	8.00E-05	1.60E-04
108	0.00E+00	0.00E+00	1.00E-05	0.00E+00	2.00E-05	0.00E+00	3.00E-05	2.00E-05	5.00E-05	3.00E-05	3.00E-05	6.00E-05	6.00E-05	4.00E-05	1.00E-04	7.00E-05	1.40E-04	9.00E-05	6.00E-05	8.00E-05
109	0.00E+00	0.00E+00	1.00E-05	1.00E-05	2.00E-05	3.00E-05	2.00E-05	5.00E-05	3.00E-05	3.00E-05	6.00E-05	6.00E-05	8.00E-05	8.00E-05	5.00E-05	9.00E-05	9.00E-05	1.20E-04	1.30E-04	9.00E-05
110	0.00E+00	2.00E-05	1.00E-05	2.00E-05	1.00E-05	2.00E-05	1.00E-05	2.00E-05	3.00E-05	4.00E-05	2.00E-05	3.00E-05	3.00E-05	5.00E-05	8.00E-05	8.00E-05	8.00E-05	1.30E-04	1.50E-04	1.50E-04
111	0.00E+00	0.00E+00	2.00E-05	2.00E-05	3.00E-05	2.00E-05	1.00E-05	1.00E-05	2.00E-05	5.00E-05	3.00E-05	4.00E-05	5.00E-05	5.00E-05	5.00E-05	5.00E-05	5.00E-05	7.00E-05	7.00E-05	1.20E-04
112	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	3.00E-05	4.00E-05	3.00E-05	2.00E-05	0.00E+00	3.00E-05	2.00E-05	2.00E-05	2.00E-05	5.00E-05	6.00E-05	9.00E-05	9.00E-05	9.00E-05	6.00E-05
113	0.00E+00	0.00E+00	0.00E+00	3.00E-05	1.00E-05	2.00E-05	3.00E-05	2.00E-05	5.00E-05	3.00E-05	3.00E-05	4.00E-05	5.00E-05	2.00E-05	2.00E-05	3.00E-05	4.00E-05	5.00E-05	1.00E-04	1.00E-04
114	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E-05	3.00E-05	2.00E-05	2.00E-05	1.00E-05	6.00E-05	4.00E-05	3.00E-05	4.00E-05	8.00E-05	5.00E-05	4.00E-05	2.00E-05	4.00E-05	4.00E-05	8.00E-05
115	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	4.00E-05	5.00E-05	3.00E-05	2.00E-05	2.00E-05	4.00E-05	3.00E-05	4.00E-05	5.00E-05	5.00E-05	7.00E-05	6.00E-05	4.00E-05	4.00E-05
116	0.00E+00	2.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	1.00E-05	3.00E-05	2.00E-05	3.00E-05	4.00E-05	4.00E-05	4.00E-05	4.00E-05	4.00E-05	3.00E-05	5.00E-05	9.00E-05	7.00E-05
117	1.00E-05	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	1.00E-05	2.00E-05	2.00E-05	1.00E-05	3.00E-05	3.00E-05	5.00E-05	6.00E-05	6.00E-05	4.00E-05	3.00E-05	7.00E-05
118	0.00E+00	0.00E+00	1.00E-05	2.00E-05	1.00E-05	0.00E+00	0.00E+00	1.00E-05	0.00E+00	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	1.00E-05	3.00E-05	3.00E-05	5.00E-05	4.00E-05	2.00E-05
119	0.00E+00	0.00E+00	1.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	2.00E-05	1.00E-05	0.00E+00	2.00E-05	2.00E-05	2.00E-05	3.00E-05	3.00E-05	1.00E-05	1.00E-05	2.00E-05	3.00E-05	4.00E-05
120	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	2.00E-05	1.00E-05	3.00E-05	3.00E-05	2.00E-05	1.00E-05	3.00E-05	4.00E-05	3.00E-05	4.00E-05	5.00E-05
121	0.00E+00	0.00E+00	0.00E+00	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E-05	2.00E-05	3.00E-05	2.00E-05	2.00E-05	3.00E-05	4.00E-05	4.00E-05	5.00E-05	6.00E-05	5.00E-05	4.00E-05
122	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	0.00E+00	1.00E-05	1.00E-05	1.00E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-05	2.00E-05
123	0.00E+00	1.00E-05	1.00E-05	1.00E-05	1.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	1.00E-05	1.00E-05									
124	0.00E+00	1.00E-05	1.00E-05																	
125	0.00E+00																			

9.9 - Appendix 9: Fair spread calculation tables in Student t and correlation equal to 0.187

Fair spread calculation table for all tranches in one-factor Student t copula for implied equity correlation of 0.187 (generated from one-factor Gaussian copula). Similar to the one presented for the 3-6% tranche in table 4.16 in section 4.4 of the thesis.

0-3% tranche:

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	44691	44230	44691	44230	916766 s
0.5	2	0.9795	88632	86812	43941	43039	896544 s
0.75	3	0.9694	130793	126784	42161	40868	877074 s
1	4	0.9593	172901	165872	42108	40396	857922 s
1.25	5	0.9494	215126	204250	42225	40090	839044 s
1.5	6	0.9396	256292	240824	41166	38682	820714 s
1.75	7	0.9299	295599	274892	39308	36554	803105 s
2	8	0.9204	335045	308359	39446	36304	785740 s
2.25	9	0.9109	373038	339782	37994	34606	768978 s
2.5	10	0.9015	411377	370836	38339	34560	752401 s
2.75	11	0.8921	448629	400243	37253	33235	736327 s
3	12	0.8829	486251	429329	37622	33217	720422 s
3.25	13	0.8738	522497	456571	36246	31673	705068 s
3.5	14	0.8648	558818	483269	36321	31411	689938 s
3.75	15	0.8559	594020	508409	35202	30129	675285 s
4	16	0.8470	628922	532726	34902	29564	660924 s
4.25	17	0.8383	663399	556130	34478	28903	646877 s
4.5	18	0.8297	697542	578717	34143	28327	633119 s
4.75	19	0.8211	732228	601224	34686	28480	619464 s
5	20	0.8126	765881	622365	33653	27346	606234 s
Sums						691613	15011947 s
Fair spread							460.7 bps

6-9% tranche:

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	7041	6968	7041	6968	926082 s
0.5	2	0.9795	14121	13831	7080	6935	914790 s
0.75	3	0.9694	21396	20740	7275	7052	903585 s
1	4	0.9593	29111	27927	7715	7401	892408 s
1.25	5	0.9494	36998	35127	7887	7488	881325 s
1.5	6	0.9396	44970	42256	7973	7491	870356 s
1.75	7	0.9299	52911	49205	7941	7385	859527 s
2	8	0.9204	61419	56527	8508	7830	848697 s
2.25	9	0.9109	69909	63677	8490	7733	838004 s
2.5	10	0.9015	78992	71207	9083	8187	827308 s
2.75	11	0.8921	87767	78301	8775	7829	816812 s
3	12	0.8829	96536	85235	8769	7742	806446 s
3.25	13	0.8738	105486	92176	8951	7821	796167 s
3.5	14	0.8648	114611	99116	9125	7891	785977 s
3.75	15	0.8559	123725	105893	9114	7800	775914 s
4	16	0.8470	133209	112834	9485	8034	765897 s
4.25	17	0.8383	142898	119791	9689	8122	755962 s
4.5	18	0.8297	152081	126174	9183	7619	746254 s
4.75	19	0.8211	161729	132793	9648	7922	736572 s
5	20	0.8126	171231	139145	9503	7722	727039 s
Sums						152973	16475122 s
Fair spread							92.9 bps

9-12% tranche:

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
M	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	3858	3818	3858	3818	926869 s
0.5	2	0.9795	8412	8239	4554	4460	916188 s
0.75	3	0.9694	12474	12092	4062	3938	905747 s
1	4	0.9593	17312	16608	4838	4641	895238 s
1.25	5	0.9494	21923	20814	4611	4378	884904 s
1.5	6	0.9396	26697	25086	4775	4486	874649 s
1.75	7	0.9299	31344	29148	4647	4321	864541 s
2	8	0.9204	36246	33359	4902	4512	854489 s
2.25	9	0.9109	41271	37592	5025	4577	844526 s
2.5	10	0.9015	46049	41510	4778	4307	834732 s
2.75	11	0.8921	51506	45950	5457	4868	824900 s
3	12	0.8829	56537	49918	5031	4442	815275 s
3.25	13	0.8738	61701	53916	5165	4513	805732 s
3.5	14	0.8648	67145	58067	5444	4708	796239 s
3.75	15	0.8559	72644	62174	5499	4706	786844 s
4	16	0.8470	78327	66347	5684	4814	777519 s
4.25	17	0.8383	84218	70600	5891	4938	768260 s
4.5	18	0.8297	90497	75081	6279	5209	759028 s
4.75	19	0.8211	96254	79033	5757	4727	750012 s
5	20	0.8126	102180	83033	5927	4816	741067 s
Sums						91180	16626757 s
Fair spread							54.8 bps

12-22% tranche:

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	4650	4602	4650	4602	3091595 s
0.5	2	0.9795	10132	9924	5482	5369	3058343 s
0.75	3	0.9694	16091	15598	5959	5776	3025333 s
1	4	0.9593	22095	21197	6004	5760	2992667 s
1.25	5	0.9494	28601	27155	6506	6177	2960235 s
1.5	6	0.9396	34482	32401	5881	5526	2928300 s
1.75	7	0.9299	40943	38075	6461	6008	2896573 s
2	8	0.9204	47483	43701	6540	6019	2865172 s
2.25	9	0.9109	54157	49329	6674	6079	2834080 s
2.5	10	0.9015	60858	54860	6701	6041	2803318 s
2.75	11	0.8921	67839	60522	6981	6228	2772827 s
3	12	0.8829	75051	66265	7212	6368	2742616 s
3.25	13	0.8738	82335	71946	7284	6365	2712717 s
3.5	14	0.8648	89462	77367	7127	6163	2683177 s
3.75	15	0.8559	96661	82730	7199	6161	2653943 s
4	16	0.8470	104285	88334	7624	6458	2624936 s
4.25	17	0.8383	111963	93859	7678	6436	2596234 s
4.5	18	0.8297	119631	99252	7668	6362	2567847 s
4.75	19	0.8211	127364	104577	7733	6349	2539756 s
5	20	0.8126	135237	109895	7873	6398	2511943 s
Sums						120647	55861611 s
Fair spread							21.6 bps

22-100% tranche:

			Cummulative Loss	PV of Cummulative Loss	Loss payment/writedown	PV of Loss	PV of expected
м	Time(i)	Discount factor	until T(i) EL(i)	until T(i) EL(i)	(EL(i)-EL(i-1)	payments	spread payments
0.25	1	0.9897	1278	1265	1278	1265	24123101 s
0.5	2	0.9795	2498	2447	1220	1195	23873818 s
0.75	3	0.9694	3895	3776	1397	1354	23627069 s
1	4	0.9593	5163	4953	1268	1216	23382902 s
1.25	5	0.9494	6613	6279	1450	1377	23141214 s
1.5	6	0.9396	8214	7718	1601	1504	22901989 s
1.75	7	0.9299	9751	9068	1537	1429	22665252 s
2	8	0.9204	11407	10498	1656	1524	22430935 s
2.25	9	0.9109	13241	12061	1834	1671	22198999 s
2.5	10	0.9015	15072	13587	1831	1651	21969462 s
2.75	11	0.8921	17103	15258	2031	1812	21742255 s
3	12	0.8829	18957	16738	1854	1637	21517436 s
3.25	13	0.8738	20823	18196	1866	1631	21294939 s
3.5	14	0.8648	22744	19669	1921	1661	21074731 s
3.75	15	0.8559	24797	21223	2053	1757	20856771 s
4	16	0.8470	26803	22703	2006	1699	20641076 s
4.25	17	0.8383	28725	24080	1922	1611	20427629 s
4.5	18	0.8297	30843	25589	2118	1757	20216349 s
4.75	19	0.8211	32842	26966	1999	1641	20007278 s
5	20	0.8126	35139	28554	2297	1867	19800309 s
Sums						31259	437893514 s
Fair spread							0.7 bps