MULTI-FACTOR MODELS AND FACTOR TIMING

A dynamic multi-factor portfolio approach

Master's thesis

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Abstract

In the search for alpha and higher risk-adjusted returns, we have focused on two approaches popular among practitioners and academics: factor investing and macro indicators. Factor investing was made popular when Fama and French (1993) presented their famous three-factor model. Since then, all kinds of different factors have been explored and proven to provide significant alpha or describe returns not captured by the initial three-factor model. In a similar manner, different macroeconomic indicators have been documented to move together with, or have prediction power over the stock market. A relationship between some of the factor returns and the general stock market return has also been proven, but few have explored the direct relationship between the factor returns and macro indicators.

We build multi-factor models consisting of seven different established factors: size, value, profitability, investment, momentum, quality and betting against beta. Using the factors, we build three different multi-factor models. The first and simplest model is an equally weighted portfolio of all the factors. The second model is a dynamic approach to Markowitz' (1952) mean-variance model with a rolling sample window. A response to some observable macroeconomic state variables, namely the short-term interest rate, term spread and dividend yield, is implemented following the methodology of Brandt and Santa-Clara (2006) to the third model.

For the U.S. market in the period 1978 - 2017, we test whether a multi-factor portfolio in fact is better than any single-factor portfolio, and further if the timing strategies of the factor allocation improves the returns even more in terms of Sharpe ratio and market neutrality. We find that when going from a single-factor portfolio consisting of the best performing factor to the equally weighted portfolio, the Sharpe ratio increases by 23%, while the dynamic mean-variance model increases the Sharpe ratio with further 31%. Adding state variables to the dynamic approach, we achieve a Sharpe ratio of 1.64, which is a further 9% increase. This is done by fitting the model for the best combination of covariance matrix shrinkage and the length of the rolling window for both of the mean-variance models. By performing empirical analysis of the model performances, we find the improvements to be statistical significant for our in-sample data.

Applying the pre-determined variables from the fitted U.S. models to our out-of-sample markets, Europe and Japan, we test whether the models are applicable to the other markets and thus work as trading strategies. We find the U.S. models to work similarly in Europe, but not in Japan. Here we find the equally weighted portfolio to be the best performing portfolio. This is because the equally weighted portfolio still benefits from the diversification, but the dynamic models fail to time the factor exposure. As an explanation to the failed factor timing, we argue the results are due to poor consistency in returns among the different factors which leaves us with no exploitable trends. The poor consistency in factor returns is documented by running a factor momentum strategy.

Contents

1 Introduction

Our thesis is inspired by the course Hedge Fund Strategies, taught by Lasse H. Pedersen and Niklas Kohl at Copenhagen Business School. We decided to merge two topics of our interest into this thesis. These topics are trading strategies and factor investing.

Investopedia defines a trading strategy as "*a set of objective rules designating the conditions that must be met for trade entries and exits to occur. A trading strategy includes specifications for trade entries, including trade filters and triggers, as well as rules for trade exits, money management, timeframes, order types, etc. A trading strategy, if based on quantifiable specifications, can be analyzed on historical data to project the future performance of the strategy*".[1] Trading strategies have been in focus of both academia and practitioners. Practitioners were simply looking for a system that allow them to make decent returns while they can eliminate the emotional factor from their investment strategies, while academia was rather curious about testing theories of whether the efficient market hypothesis holds. Nowadays there are hedge funds, most famous of them is AQR, that are conducting deep academic research while setting up specific funds that will implement a strategy built on these research and findings.

The second focus of our thesis is factor investing. Factor investing was considered part of only academia until two decades ago, when the first quantitative trading desks were set up by the large investment banks. Factor models are econometric models that tries to describe the return on a financial asset through a linear dependence on multiple factors. There are many definitions of factor investing, but it is common to refer to it as instead of building portfolios based on the traditional methods of starting the allocation on the asset class level (for example, 60/40 equity-fixed income allocation), a specific risk factor that is supposed to produce some kind of excess return is chosen, and then the capital is allocated through asset classes to best optimize the portfolio.

The existence of style factors and their ability to make returns not captured by the market beta, has since the publishing of the three-factor model by Fama and French in 1993 been subject to a significant amount of research. Macro-factors and their ability to forecast market returns have also been profoundly documented. In this thesis, we will see if implementing macro-factors to our trading strategy can help us time the factor exposure in a monthly rebalanced portfolio, and yield a higher

1 Investopedia (2017, July 7) Trading Strategy. Retrieved from Investopedia: http://www.investopedia.com/terms/t/trading-strategy.asp

return compared to a single-factor portfolio or an equally weighted factor portfolio. We also implement this into a practical trading strategy. Our thesis depends on the paper of Brandt and Santa-Clara (2006) who introduced a way to expand the mean-variance framework of Markowitz (1952) to include a response macroeconomic state variables.

1.1 Research Questions

1.1.1 Main Questions

- Can a mean-variance investor benefit from expanding a single-factor portfolio to a multi-factor portfolio?
- Further, can the multi-factor portfolio benefit from implementing a response to some observable state variables?

1.1.2 Sub Questions

- How is the factor allocation in a multi-factor portfolio affected by the dynamic strategies?
- Is the performance of the portfolios dependent on the market performance?
- Are our findings consistent among the tested markets?
- Why is a given model successful/ unsuccessful in the given market?
- Can we build a trading strategy based on the in-sample data of the U.S. market to make a risk-adjusted profit on the out-of-sample data for the European and Japanese market?

1.2 Structure

1. Introduction
2. Literature review
    - Introduction to the factors and the theory behind multi-factor models
    - Introduction to the state variables
3. Methodology
    - Data gathering
    - Model creation
    - Shrinking of the covariance matrix
    - Performance measures
    - Chi-squared test
4. In-sample analysis
    - We fit and expand our U.S. model gradually and analyze the performance of each step:
        i. Individual factors

    ii. Equally weighted portfolio
    iii. Mean-variance portfolio
    iv. Mean-variance portfolio with state variables
5. Out-of-sample analysis
    - We test if the fitted model for the U.S. data is applicable to our out-of-sample markets, Europe and Japan
6. Summary and conclusion

1.3 Delimitations and Assumption

We have chosen to focus our research on the topic in hand, namely the timing of the factor portfolios. This means that for topics outside of what we consider our contribution to the literature, we rely on already available data. The main implication of this, is that we use factor returns of premade factor portfolios available from the Kenneth French website and AQR capital data library, rather than creating the portfolios our self. We think that creating the factor portfolios our self, do not contribute towards answering the problem statements and our concerns with building the portfolios from scratch are that it is a tedious process and we consider the data available to be more reliable. We are fully aware that the portfolios are possible to build from scratch by our self with the databases available following the methodology of Fama and French (1992; 1993).

If not anything else is stated, we assume the six conditions for a perfect market given by Merton (1973, pp. 868-869). Similar assumptions are made in the reviewed literature for the thesis:

- "All assets have limited liability.
- There are no transaction costs, taxes, or problems with divisibilities of assets.
- There are a sufficient number of investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price.
- The capital market is always in equilibrium (i.e., there is no trading at non-equilibrium prices).
- There exists an exchange market for borrowing and lending at the same rate of interest.
- Short-sales of all assets, with full use of the proceeds, is allowed."

Note that more assumptions will be made throughout the thesis and they will be explained as we introduce them.

2 Literature Review

2.1 Modern Portfolio Theory and Factors Models

Before going on introducing the different factors, we will start off with some basic risk measures and explaining the mean-variance framework of Harry Markowitz (1952) and how it has developed to multi-factor models. His mean-variance framework is also the ground pillar the models we will use for most of our research.

2.1.1 Risk and Return

Financial theory assumes a tradeoff between risk and return: the greater the risk, the higher return can be realized. As the realized return cannot be known up front, the tradeoff is actually between risk and expected return. The expected value is defined as the average of the possible returns, where the weight applied to a particular return equals the probability of that return to occur. By a subjective assessment or estimates based on historical data, the possible returns and their probabilities can be estimated (Hull, 2015). Likewise, the expected return of an investment portfolio can be obtained by calculating the weights of the different assets and their expected returns.

A common risk measure is the variance which is given by:

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(R_i - E(r_i)\right)^2}{n-1} \tag{1}$$

Where R_i is the return of the ith observation for a single asset, n is the number of observations. E denotes the expected value, so $E(R)$ becomes the expected return or the average return.

Another risk measure is the standard deviation (SD or σ), which is the square root of the variance. Often, the standard deviation is more convenient as it is expressed in the same unit as the means, while the variance is expressed in squared units.

The standard deviation of the return of a portfolio depends on the standard deviation of the return of the underlying investment objects (assets), their correlation coefficients and the fraction of the portfolio invested in each asset. If we have a portfolio of two assets, the standard deviation of the portfolio is given by:

$$\sigma_P = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2\rho x_1 x_2 \sigma_1 \sigma_2} \tag{2}$$

Where x_1 and x_2 are the weights in asset 1 and 2 respectively, which sums up to 1. σ_1 and σ_2 are the standard deviations of the return of the assets, and ρ is the correlation between the two.

The correlation coefficient of the two assets, measures to which extent there is a linear relationship between them. This value is between -1 and 1. A high positive value means that if one of the returns of one asset is increasing, the other asset tends to increase too. For a negative value, the opposite is true. Note that a correlation coefficient of zero does not translate into the variables being independent of each other, as they still can be dependent on a non-linear manner.

One way to calculate the correlation is to go through the covariance, which is given by:

$$Cov(1,2) = \frac{\sum_{i=1}^{n}\left(\left(R_{1,i} - E(R_1)\right)\left(R_{2,i} - E(R_2)\right)\right)}{n-1} \tag{3}$$

Using the covariance, the correlation is given by:

$$\rho(1,2) = \frac{Cov(1,2)}{\sigma_1 \sigma_2} \tag{4}$$

2.1.2 Markowitz's Mean-Variance Portfolio Theory

The framework of the modern portfolio theory was laid by Markowitz (1952, 1959) when he introduced the mean-variance model: in a single period, an investor will maximize expected return given an acceptable level of risk (or vice versa). The risk is measured by the variance or standard deviation of the return of the portfolio. As we saw above, this risk or volatility of a portfolio is dependent on the correlation of the assets in the portfolio. The investor can reduce exposure to individual asset's risk (idiosyncratic risk) by building a portfolio of securities that are not perfectly correlated (diversifying). By plotting every possible combination of assets weights in a portfolio, the efficient frontier can be drawn from the upper half of the hyperbola created. With the ability to leverage the position, the capital allocation line defines the best possible risk-adjusted return by combining the risk-free rate and tangency portfolio.

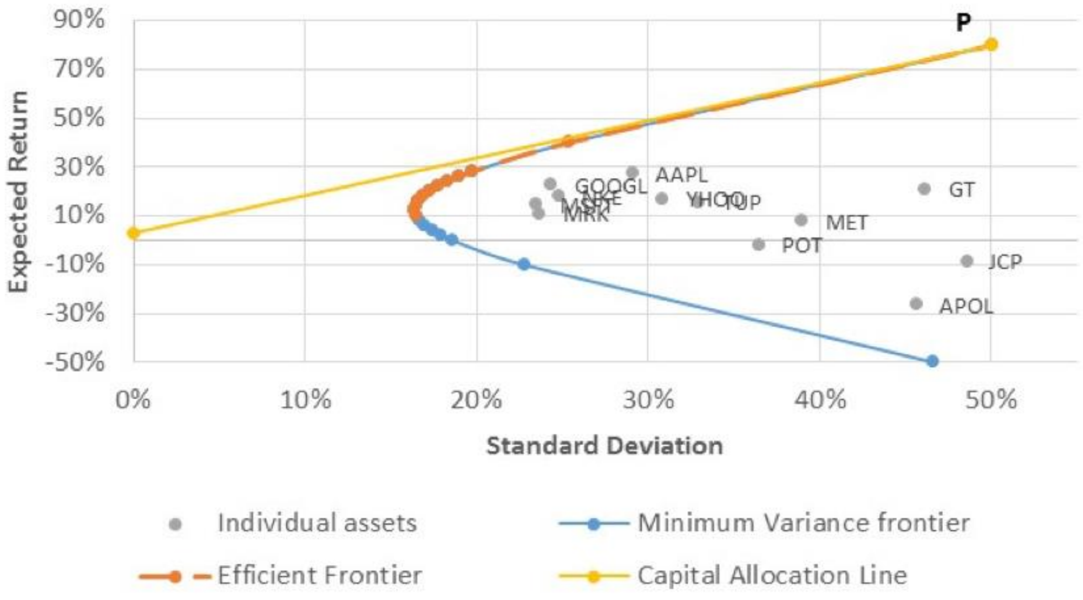

Figure 1: Markowitz' efficient frontier. Source: https://financetrainingcourse.com/education/2016/11/portfolio-management-capital-asset-pricing-model-capm/

From Figure 1 we see the benefits of diversifying by the fact that a higher expected return can be achieved by taking the same risk by moving along the capital allocation line compared to any individual asset. The idiosyncratic risk is given by the difference in standard deviation between the individual assets and the capital allocation line.

The aim of the model is to choose the optimal weight of various assets in a portfolio. The optimal weight is achieved by finding the combination of assets that yields the minimum volatility with expected return above the acceptable baseline. The mean-variance approach exploits the different covariations between random assets, which describe to which degree two assets deviates from their expected values in a similar way. A negative covariance would indicate the two assets deviates in different direction. If we make a portfolio of the two assets, the goal is to make the opposite deviations neutralize each other, making the return less volatile while maintaining the same expected return. With an extended amount of assets in a portfolio, the mean-variance approach exploits the correlation of all the assets by creating a matrix of all the covariances between assets in a portfolio.

2.1.2.1 Example Using Matrix Calculation

We will now go on explaining the optimization problem of Markowitz using matrixes inspired by the Markowitz Mean-Variance Portfolio Theory paper (Markowitz, 1952).

Let r_i be the random variable associated with the rate of return for asset i, for $i = 1,2, \ldots, n$, and define the random vector

$$y = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

We set $\mu_i = E(r_i)$, $m = (\mu_1, \mu_2, \dots, \mu_n)^\top$,[2] and $cov(y) = \Sigma$. Then let $x = (x_1, x_2, \dots, x_n)^\top$ be a set of weights associated with a portfolio. The rate of return of the portfolio is then $r = \sum_{i=1}^{n} r_i x_i$ which is also random with mean (expected return) $m^\top x$ and variance $x^\top \Sigma x$. If $\mu_b$is the acceptable baseline of acceptable return, the Markowitz framework gives an optimal portfolio solving the following quadratic program:

$$Minimize\ \left[\frac{1}{2} x^\top \Sigma x\right]$$

Subject to: (5)

$$m^\top x \geq \mu_b$$

$$\vec{1}x = 1$$

Where $\vec{1}$ is a horizontal vector where all values equal 1.

As we can see, this is a one period optimization problem. We will build further on this framework in 3.2 when we expand the asset space to include state variables. The finalized model will be an important part of our research.

2.1.3 Capital Asset Pricing Model

Based on the mean-variance model of Markowitz, Sharpe (1964), Lintner (1965a, 1965b) and Mossin (1966) independently developed what we know as the capital asset pricing model (CAPM). Black (1972) adds to the CAPM by allowing for unlimited long or short positions in the risky assets. Instead of using the CAL, CAPM introduces the capital market line, which is a combination of risk-free assets and the market portfolio consisting of all risky assets in the market. The security's sensitivity to the market is the systematic risk which is non-diversifiable, meaning investors are compensated for taking on this risk. Diversifying the idiosyncratic risk would leave the expected return of a portfolio to a linear function of its tendency to move with the market portfolio. This linear tendency is given by the beta coefficient. According to the CAPM, the expected return of an asset is

[2] ⊤ indicates a transposed matrix or vector

$$E(R_i) = E(R^f) + \beta_i \left(E(R_m) - E(R^f) \right) \tag{6}$$

Where $\mathrm{E}(\mathrm{R_i})$ is the expected return on security i, $\mathrm{E}(\mathrm{R^f})$ is the expected risk-free return, β_i is the market beta for security i and $\mathrm{E}(\mathrm{R_m})$ is the expected market return.

The model has a poor empirical record and has been proven wrong in a lot of papers. Maybe the most obvious rejection of the model was done by Jensen Black and Scholes (1972) who proved that assets with lower beta on average earn a higher return than high beta assets. The model is criticized for being subject to a lot of assumptions, especially homogenous expectations and the single-period nature for the model. Nevertheless, the model has made a huge impact on financial academia and in practice due to its simplicity.

2.1.4 Arbitrage Pricing Theory

As a more flexible alternative to the CAPM, Ross (1976) introduced the arbitrage pricing theory (APT). Instead of assuming the return of a security only depends on the market factor, the APT assumes there is a linear relationship of various macro- and market factors and the asset price. The asset price is a result of the exposure to these factors and the risk premium of the factors. Using the model to find misprices securities, arbitrageurs will go long in the undervalued securities and shorting the overpriced securities to make a portfolio that has a net exposure of zero to the factors included in the model. By holding the portfolio until the asset prices converges to their implied value, a risk-free (other than firm-specific risk) profit can be made. Because arbitrage opportunities are quickly exploited, the prices will soon be brought back in line.

2.1.5 Intertemporal Capital Asset Pricing Model

Merton (1973) developed the intertemporal capital asset pricing model (ICAPM) as a utility maximization problem of lifetime consumption by an arbitrary number of investors who can trade continuously in time. Expanding the CAPM to a continuous model, the ICAPM recognizes that investment risks and opportunities might change over time, resulting in periods where an investor may wish to hedge. As a result, current demands are affected by the possibility of uncertain changes in the future investment opportunities. This means the hedging itself might result in a higher equilibrium prize for the security.

Both the ATP and ICAPM leaves the number of factors to be included for the analyst to decide, and the factors are likely to change with time and differ among markets. Maio and Santa-Clara (2012, p. 587) identify three conditions of a multi-factor model to be justifiable by the ICAPM:

- "First, the candidates for ICAPM state variables [here: factors] must forecast the first or second moments of stock market returns.
- Second, and most importantly, the state variables should forecast changes in investment opportunities with the same sign as its innovation prices the cross-section. Specifically, if a given state variable forecasts positive expected returns it should earn a positive risk price in the cross-sectional test of the respective multifactor model.
- The third restriction associated with the ICAPM is that the market (covariance) risk price estimated from the cross-sectional tests must be economically plausible as an estimate of the coefficient of relative risk aversion of a representative investor."

2.2 Asset Pricing Theories

Before going on reviewing the different factors and theories of their existence and abnormal performance, two views of the financial market and how asset prices are determined, should be clarified. Fama and French (1992) distinguish what they call "rational asset-pricing stories" from "irrational asset-pricing stories".

The irrational asset-pricing theory explains the existence of the investment factors as irrationality and behavioral biases among investors, resulting in under- and overreactions in the market, which eventually will be corrected. This justifies that excess market return can be achieved by exploiting systematic misbehavior among investors without taking on more risk.

According to the strong-form of market efficiency, investors cannot earn consistent excess market return over a long period of time. Following the efficient market hypothesis (EMH), the investors are considered to be fully rational and the stock prices to reflect all the available information in the market. Hence an arbitrage opportunity will soon be exploited by all the rational investors and made obsolete. In the light of the EMH, Fama and French (1992) argues that if all stocks are constrained by the Fama-MacBeth intercept (Fama and MacBeth, 1973), which impose a linear factor structure on returns and expected returns that is consistent with the multi-factor asset-pricing models of Merton (1973) and Ross (1976), the asset prices are regarded as rational. If the asset pricing is rational, the factors are in fact a proxy for risk, and thus implies that portfolio performance can be evaluated by comparing the average returns to other portfolios with the same risk characteristics. According to the rational-asset pricing theory, the existence of factors premiums is explained as a compensation for investors to take on a systematic underlying risk, also called risk premia.

Fama (1970) emphasizes that all economic models are incomplete descriptions of systematic patterns in average returns during any sample period. As a result, all models are subject to a "bad-model problem" or "joint hypothesis problem" when testing for efficiency. The bad-model problem becomes more prevalent as the horizon of the model increases and return significant abnormal returns. As a result, the market efficiency is per se not testable.

2.3 Factors

With the APT and ICAPM as base models, a wide variety of factors and multi-factor models have been explored and documented to have a predictive power of the stock market. We have chosen seven factors which are among the most well documented factors: size, value, profitability, investment, momentum, quality and betting against beta. The history and existence of the factors will now be explained before we are using them in our portfolios later on in the thesis.

2.3.1 Size

Banz (1981) was the first to refer to the size effect when he showed that between 1936 and 1975, small firms delivered higher risk-adjusted returns compared to large firms on the NYSE. Later, Fama and French (1993) applied the size effect in their three-factor model as the SMB (small minus big) factor.[3] Both papers define size as the market value of equity, also called market capitalization, calculated by taking the number of outstanding shares times the closing price per share.

2.3.1.1 Explanation

Banz (1981) ends his paper by saying there is no theoretical foundation for the size effect. He goes on saying that the size effect might just be a proxy for one or more unobservable risk factors correlated with size. Both Liu (2006) and Amihud (2002) suggest the size factor is a proxy for liquidity risk as illiquidity affects small firms' stocks the most. Vassalou and Xing (2004) link the size effect to the default rate risk of small firms. Common for the theories, is that all of them link the performance of the small-cap stocks to a compensation for an underlying fundamental risk, as according to the efficient market hypothesis.

Investigating size and macroeconomic factors, Liew and Vassalou (2000) find that HML (see 2.3.2) and SMB contain information about future GDP growth. Chan and Chen (1991) identify most of the small firms as what they call marginal firms. The marginal firms perform poorly, have high financial leverage and use cash inefficiently. They argue that prices of marginal firms tend to be more sensitive to changes

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[3] See 2.4 for the explanation on how the factor is created.

in the economy and the firms are more likely to default in an economic recession. As according to the EMH, investing in marginal firms is compensated by a risk premium.

Zhang (2006) finds results that firm size, among five other factors, proxies for information uncertainty. He provides evidence that the initial market reaction to new public information is incomplete. The degree of incompleteness of the market reaction increases with the level of information uncertainty, which is higher for smaller firms with more volatile fundamentals. As a result of the uncertainty, the market underreact to new public information and news predicting the future returns of the companies. He claims this failure to exploit public information is due to behavioral biases and a sign that the firm size behaves more like a proxy for information uncertainty than a common risk factor, inconsistent with the notion that size is a proxy for an underlying risk premium.

With the size premium having a cyclical nature, the premium might not be prevalent at all times. Fama and French (2012) find the SMB premium to be close to zero in all tested markets (North America, Europe, Japan and Asia Pacific) in the period 1991 to 2010.

2.3.2 Value

Graham and Dodd's "Security Analysis" (1934) introduced the value perspective of investing by emphasizing that market factors can be psychological and manipulative, so the market price should be put up against the fundamental value factors (balance sheet numbers) of a company. The fundamental value element acts as a proxy for the expected future performance of the company. Divided by the price or market value of the company, the value factor captures the expected future performance of a company relative to its assessed value.

Several different balance sheet posts have been applied as an approach to the fundamental value factor implied by Graham and Dodd. Basu (1977) uses earnings-to-price (E/P) as a dimension for value and conclude that high E/P firms on average earn higher risk-adjusted returns compared to low E/P firms using data from the NYSE.

Fama and French (1992) refers to Stattman (1980)[4] and Rosenberg, Reid & Lanstein (1985) as the first papers to use a firm's book value of common equity (B) to its market value (M), called book-to-market ratio, for the value factor. Rosenberg et al. find that a strategy, which buys stocks with a high book-to-market ratio and sells stocks with a low book-to-market ratio, achieves a significant excess market

[4] Stattman, D., (1980). Book values and stock returns, The Chicago MBA: A Journal of Selected Papers 4, 25-45. (Not publicly available)

return in the U.S. market. Chan, Hamao and Lakonishok (1991) find the book-to-market ratio to have a strong role in explaining the cross-section of returns in Japan.

Fama and French (1992) include both earnings-to-price- and the book-to-market ratio when exploring factors that can describe the average returns of the U.S. market. They find when size and book-to-market are included in the regression together with the E/P factor, the E/P dummy becomes small and insignificant. Further they say the average size of high E/P firms is small, and thus size better catches the returns. As a result, Fama and French (1993) find the E/P factor redundant and instead include the book-to-market factor, HML (high minus low), as the value factor in their three-factor model.[5]

2.3.2.1 Explanation

Fama and French (1995) show a link between firms with high book-to-market ratio and persistent poor earnings. According to Chen and Zhang (1998), high book-to-market firms also qualify as marginal firms (just as small firms). The marginal firms are more sensitive to economic factors and face an uncertain future, making them riskier to invest in. Because marginal firms still enjoy the benefits of a rapidly expanding economy in emerging markets, the risk premium of marginal firms is higher in mature markets like U.S. and Japan (Europe is not included in the research paper). Testing on the international stock market, Fama and French (2012) find the value premium to be larger for small stock in all tested markets except from Japan.

Further studies by Zhang (2005) provide a framework of rationalizing the cross-section of returns and value: as the value firms have tied up more unproductive capital during bad times compared to growth firms (low book-to-market), and in good times, raising capital is relative easy for the growth firms, there is a dispersion of risk between growth and value firms. Accordingly, the value premium is the compensation for taking on this risk with the premium being higher in bad times.

According to the semi-strong form of the efficient market hypothesis, Basu (1977) (here: P/E) argues that the publicly available information is not instantly impounded in the market prices, making it possible to make an excess risk-adjusted return.

Lakonishok, Shleifer and Vishny (1994) reject that value stocks (here: high E/P, B/M and CF/P) are fundamentally riskier and the higher average return on value stocks is just a compensation for taking on this risk. They link the performance of value stocks to behavioral theories where the investors have limited rationality: investors tend to get overexcited by stocks that have performed well in the past

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[5] See 2.4 for the explanation on how the factor is created.

and buy them so they become overpriced (glamour stocks). At the same time, they sell stocks that have performed poorly and these stocks become underpriced (value stocks). According to this theory, investors being systematically wrong about the future growth of stocks, explains the higher returns to value stocks while still not being any riskier than glamour strategies.

Chan et al. (2000) show that the U.S. value premium disappeared in the late 1990s and argue that the shift because investors got too excited about value stocks

2.3.3 Profitability and Investment

In 2015, Fama and Frech expanded their initial three-factor model to include two more factors, namely investment and profitability. According to their paper, the extension was a response to Novy-Marx (2013) and Titman, Wei & Xie (2004) among others, who argued that their initial three-factor model was incomplete because it lacked variation in returns related to profitability and investment.

By applying clean surplus accounting rather than expected dividends, and dividing by book value of equity on the Miller and Modigliani's (1961) divided discount model, Fama and French (2006) present an equation on the relationship between expected return, expected profitability, expected investment and B/M:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{7}$$

Where $Y_{t+\tau}$ is the total equity earnings for period $t + \tau$ and $dB_{t+\tau}$ is the change in total book equity over the period. The equation makes three statements regarding the factors and the expected returns. First, a higher book-to-market ratio implies a higher expected return everything else kept equal (as in the Fama-French three-factor model). Secondly, the equation states that higher expected earnings imply a higher expected return everything else kept equal. Finally, everything else kept equal, higher expected growth in investments (given by book equity) implies a lower expected return.

Despite the predicted relationship of expected profitability and investment effects on average returns by the equation, Fama and Frech's empirical results were inconclusive with the proxies used in the paper.[6]

[6] With B_t/M_t, D_t/B_t and to some extent lagged asset growth, having the explanatory power of future investment (in terms of assets growth) and lagged current earnings having the strongest forecasting power of profitability, all accounting variables based on a per share basis.

2.3.3.1 Investment

Early papers on investment include Fairfield, Whisenant and Yohn (2003) and Titman et al. among others. Faifield et al. find that after controlling for current profitability, both growth in accruals and long-term net operating assets have negative associated one-year-ahead return on assets. Titman et al. use the firm's capital expenditure scaled by it sales in year $t-1$ divided by the average capital expenditures of the three previous years as a measure for capital investment. Their results also verify a negative relationship between abnormal capital investments and future stock returns.

When revisiting the Fama and French (2006) paper, Aharoni, Grundy and Zeng (2013) find that the empirical failure of the paper was due to the use of per share accounting variables rather than firm based accounting. They argue that investments can be proxied either by asset growth or equity growth, but share issuance and repurchase will change the number of shares outstanding, and the per-share growth can differ from the firm level growth. Once they adjust the accounting data to be on the firm level, their findings support the insight of the Fama and French paper.

By applying the firm based accounting suggested by Aharoni et al., Fama and French (2015) find that the investment variable can be defined by expected growth of both book equity and assets, but using assets produce a larger spread in average returns. They argue this might imply the lagged growth of assets is a better proxy for the infinite sum of expected future growth in book equity in (7) than the lagged growth in book equity. For that reason, they apply the total asset growth rate between the fiscal year ending in $t-2$ and $t-1$ as the investment factor. By forming portfolios based on this definition of investment after sorting by size,[7] the paper concludes a negative relationship between abnormal investments and stock returns given by the conservative minus aggressive investment factor, CMA.

2.3.3.2 Profitability

Ball and Brown (1968) show that earnings, defined as bottom line net income excluding extraordinary items can predict the cross section of average returns.[8] However, subsequent research argues that earnings explain little when adding size and book-to-market. Years later, Novy-Marx (2013) find that another measure, the gross profit (revenues minus costs of goods sold) to book equity ratio of a firm, has the same power as book-to-market in predicting the cross-section of average return. Earlier, Fama

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[7] See 2.4 for explanation on size sorting.

[8] Other early studies include Haugen and Baker (1996), Piotroski (2000), Cohen, Gompers, and Vuolteenaho (2002), and Griffin and Lemmon (2002)

and French (2006) used current earnings as a proxy for future profitability, but concluded this might not be the best proxy for expected profitability. Novy-Marx argues that the current earnings post is too far down the income statement and is influenced by other variables, moving the measure away from true economic profitability. For instance, a company with large investments in research and development might have high future profitability, but low current earnings. Influenced by this, Fama and French (2015) uses revenues minus cost of goods sold (COGS), minus selling, general, and administrative expenses (SGA), minus interest expense all divided by book equity for their profitability measure:

$$Profitability = \frac{(Revenue - COGS - SGA - Interest\ Expense)}{B} \quad (8)$$

Incorporated into their five factor model as robust minus weak profitability (RMW), their expanded five-factor model explains 71% - 94% of the expected returns among the factor mimicking portfolios.

2.3.3.3 Explanation

2.3.3.3.1 Investment

McConnell and Muscarella (1985) find that the announcement of a planned increase in capital investment is associated with a positive stock return. However, Loughran and Ritter (1995) among others, find the choices of financing related to the investments, including both initial public offerings and seasoned equity offerings, generally result in negative stock returns. Ikenberry, Lakonishok and Vermaelen (1995) also find the announcement associated with decreased investment, such as share repurchases, generally result in positive abnormal stock returns of a four year buy-and-hold strategy. This can be connected to findings of Lamont and Stein (2006) that firms are more likely to issue shares (i.e. finance investments) when the firm's market-to-book value is higher than its historical value and most likely overvalued. Knowing this, the underperformance of share issuing companies can be explained by the investors starting to question the fundamental value of the firm and why the firm is issuing shares at the given moment. Following the De Bondt and Thaler's (1985) return reversal theory, the stock price of the past "winners" will be outperformed by the past "losers" within a period of 3-5 years, consistent with the market overreaction hypothesis.[9]

[9] "Winners" being the stock with the highest growth in share price and losers being the stocks with the "lowest" (or negative) growth. See 2.3.4 on momentum for more on the topic.

Titman et al. (2004) find the negative performance of investment firms to be independent of the long-term reversal and seasoned offering anomalies. Instead, they find evidence consistent with their hypothesis of unfavorable information: the individuals who manage the firms that invest the most, are more likely invest more than other managers due to overinvestment rather than because of better investment opportunities. An abnormal performance around earnings announcements also suggests investors incorrectly assess the empire building tendencies among the managers.

Applying the insight of the rational asset pricing model, Berk, Green and Naik (1999) explain the lower average returns on aggressive investments by a decrease in the firm's risk premium. They claim the investing firms usually have discovered valuable investment opportunities. An investment with low systematic risk is attractive for the firm and will lead to an increase of the firm value, but as a result, the systematic risk of the cash flow of the subsequent periods will decrease. Overall, this leads to lower average returns.

Fama and French (2015) find the average rate of investment to be significantly higher among microcaps compared to megacaps.

2.3.3.3.2 Profitability

Novy-Marx (2013) explains the existence of the profitability factor by comparing it to the value factor: while the value strategies finance acquisition of inexpensive assets by selling expensive assets, profitability strategies finance the acquisition of productive assets by selling unproductive assets. Further he says that investors require a higher rate of return for risky firms, and because of that, risky firms constantly have a higher book-to-market value compared to firms where the investors require lower returns. Similar arguments suggest that firms with productive assets should yield higher returns than firms with unproductive assets on average.

Even though the two factors share a lot of the same philosophy, their characteristic and covariances are very different. Novy-Marx finds that profitable firms earn significantly higher average return than unprofitable firms, despite on average having a lower book-to-market and higher market capitalization. As the strategies based on profitability are growth strategies, they provide a great hedge for value strategies.

Wang and Yu (2013) find the profitability premium to exist primarily among firms with high arbitrage cost or high information uncertainty like firms with smaller capitalization, younger age, higher return volatility and less analyst coverage. Because of the arbitrage limits, the investors are more likely to underreact to current profitability news about the firms, and hence high profitability firms are relatively underpriced. The profitability premium emerges due to the correction of the high return on equity firms in the subsequent periods.

Sun, Wei and Xie (2014) test the findings of Wang and Yu in more than 40 markets. They test two hypotheses of the profitability premiums existence: is the existence of the premium due to rational explanations or is the existence of the premium due to behavioral biases. The behavioral bias theory is the one of Wang and Yu as explained above. The rational theory hypothesis is the investment-based asset pricing theory (or q-theory) by Cochrane (1991; 1996), which is further developed by Zhang (2005) and more. The theory says that the marginal cost of investments (investment friction) will steepen the relation between expected returns and firm investment. For firms facing a lower level of investment friction, the net present value of the investment will be higher and the positive relation between profitability and expected return should be stronger (Li and Zhang, 2010). This friction varies a lot among the international firms tested with the firms in the developed countries having more easy access to capital and lower limits for arbitrage than the less developed countries. Their findings are in favor of the q-hypothesis and the mispricing hypothesis is rejected.

Wang and Yu (2013) also test for the investment premium being a result of underlying macro risk, but reject the hypothesis as the profitability is not larger on announcement days than non-announcement days of macro news.

2.3.4 Momentum

The momentum factor emphasizes that stocks, which recently have performed the best, are also expected to perform best in the future in terms of stock prices. Jegadeesh and Titman (1993) were the first to document an excess risk-adjusted return when buying winners and selling losers over a 3 to 12 month holding period. Carhart (1997) argues that the 12-month momentum is the most robust, meaning stocks are sorted by the past 12-month performance excluding the current month. Expanding the Fama-French three-factor model, he adds momentum as a fourth factor. This model proves momentum forecasts returns not earlier captured by value or size. Fama and French (2012) use the same approach for their momentum strategy when they test the four-factor model on international stock returns. They find the model to be significant in North America, Europe and Asia Pacific, but not Japan. Further, they find the spread in momentum returns to decrease from smaller to larger stocks.

De Bondt and Thaler (1985) find that a three year buy and strategy of earlier losers, on average returns 19.6% above the market return[10], while the winner portfolio returns about 5% below the market return over the same period. They find the reversal to exist until as long as five years after portfolio formation.

Fama and French (2015) left momentum out when they expanded their initial three-factor model due to momentum being largely independent of the other factors. Asness (2014) makes a case for expanding the five-factor model of Fama and French further to include momentum. He argues that when the goal is not to explain the cross-section of portfolio returns, but rather to form the best portfolio to invest in, the high and statistically significant intercept of momentum makes it a natural extension of the five-factor model.

2.3.4.1 Explanation

Many regard the momentum effect as one of the most critical evidences against the efficient market hypothesis as it does not proxy for any obvious underlying risk factors.

The momentum reversal documented by De Bondt and Thaler indicates a correction in the long run, which implies that the momentum in the first case is due to an overreaction. Daniel, Hirshleifer and Subrahmanyam (1998) present a theory of the over- and underreaction being a subject of two behavioral biases: the overconfidence among investors of private information and self-attribution. The theory implies investors overreact to private information and underreact to public information signals. The momentum starts with private investors being overconfident in their own information and overreacts to it. The overreaction continues as good news arrives and the public signals of an overreaction are neglected, resulting in a return momentum. They find the short-term positive autocorrelation can be a result of continuing overreaction, which is followed by a long run correction. Hence, the short run positive autocorrelation and the long-term negative autocorrelation are complimentary evidences.

Momentum has in several papers been proven to be procyclical, meaning momentum premium is low when market volatility is high and market return is negative (Stiver & Sun, 2010; and Daniel & Moskowitz, 2016). This can be explained by a state of panic occurring as the market declines and volatility increases, and the (over)confidence among investors is reduced. Bird and Whitaker (2004) find evidence for a value/momentum cycle. Supported by Swaminathan and Lee (2000) they suggest expensive losers are early into their negative momentum cycle, while cheap losing stocks are late into

[10] Sample period January 1933 – December 1980

their negative momentum cycle and will soon turn around and give higher returns. Asness (1997) also find the negative correlation and conclude a value portfolio will include to some extent "losers" and a momentum portfolio to include firms with a low book-to-market ratio.

Fama (1998) argues that the reason why the market underreacts in some circumstances and overreacts in others is a result of the investors taking chances. In the end, underreaction is just as common as overreaction he claims, with the long-run expected returns of the anomalies being zero. Arguing the anomalies are a result of bets taken by the investors, he claims the under- and overreactions still are consistent with the efficient market hypothesis.

Chordia and Shivakumar (2002) find that profits related to momentum strategies can be explained by a set of lagged macroeconomic variables (short-term interest rate, term spread, default spread and dividend yield). When adjusting for the predictability of stock returns based on the macroeconomic variables, they find the payoff of the momentum strategy to disappear. They claim their findings provide a plausible explanation for the momentum profit in terms of rational pricing theories, as the momentum return is a compensation of the underlying time-varying return risk.

2.3.5 Quality Minus Junk

The quality minus junk (QMJ) factor was first documented by Asness, Frazzini and Pedersen (2013). Asness et al. define quality as characteristics investors should be looking for when deciding which companies to buy or sell. The firms that possess higher quality characteristics are shown to deliver historically high risk-adjusted returns and they are expected to do so in the future. Compared to high quality stocks, their low quality or junk counterparts deliver lower risk-adjusted returns.

The stock characteristics that the QMJ factor is built on are identified from the Gordon Growth Model. Rewriting the Gordon Growth formula to express the price to book value, they get:

$$\frac{P}{B} = \frac{profitability * payout\ ratio}{required\ return - growth} \tag{9}$$

To be able to evaluate and quantify each of the individual characteristics, Asness et al. consider different financial measures and ratios. These measures are as follows:

- Profitability: profitability is measured as the profit per unit of book value. Ceteris paribus, investors should pay a higher price for a more profitable company. The measurements are done several ways, including gross profits, margins, earnings, accruals and cash flows and focus on the rank of the individual stocks in these measurements.

- Growth: investors should also pay a higher price for companies with growing profits. The growth is measured by a five-year growth in each of the profitability measures listed above.
- Safety: when considering safety, investors are supposed to pay a higher price for a safer company. Safer companies are defined by both return-based measures, including lower beta and volatility and by fundamental-based measures (e.g., low volatility of profitability, low credit risk, low leverage).
- Payout: payout is the fraction of profits that are paid out to shareholders. A higher payout ratio does not necessary produces a higher stock price because it can easily lead to lower future profitability, but on the other hand, a higher payout ratio should yield a higher stock price if all other factors are hold constant.

"For the market to rationally put a price on these quality characteristics, they need to be measured in advance and predict future quality characteristics, that is, they need to be persistent. We show that this is indeed the case; profitable, growing, safe and high-payout stocks continue on average to display these characteristics over the following five or ten years". (Asness et al., 2013, p. 4)

2.3.5.1 Explanation

On average, high quality stocks have a higher price, but still lower than what is reasonable. This implies that either the stocks labeled as low-quality are more expensive, or that the stocks labeled as high-quality are cheaper than the model would imply. This phenomenon makes it possible to realize a positive risk-adjusted return on the QMJ factor, where a portfolio is constructed by longing the stocks that are labeled high-quality and short their low-quality counterparts, meaning a market neutral exposure is maintained.

Asness et al. also find that the premium investors pay for high quality stocks is not constant, but rather varies over time. For example, right before the dotcom-bubble burst and the global financial crisis of 2007-2009, the premium on high quality stocks was particularly low. But following the business cycle peaks, the price of quality increased. This positive convexity related to recessions is because of a known phenomenon called flight to quality, which is captured by QMJ's negative market exposure. Flight to quality is a result of investors seeking higher quality stocks during recessions. This turns the prices of unprofitable stocks to drop more than the prices of profitable stocks, even adjusting for their betas, which leads to the strong performance of the QMJ portfolio in down markets.

According to Asness et al., in a forward-looking rational market, stock prices should be related to future quality characteristics. The predictability of quality is perfectly consistent with the efficient market hypothesis as the prices should reflect quality which itself is not unpredictable, but the stock returns are.

2.3.6 Betting Against Beta

According to the CAPM, a higher beta stock should yield a higher excess return in the long run. Jensen, Black and Scholes (1972) did one of the most obvious rejections of the CAPM model when they proved that low beta stocks on average perform better than the high beta stocks. Contrary to the CAPM, which assumes all securities to have a linear relationship between beta and excess return on the security with an intercept of zero, they find that low beta stocks tend to have a positive alpha and high beta stock tend to have a negative alpha.

Frazzini and Pedersen (2014) created the Betting against beta (BAB) factor as a practical way to exploit the findings of Black et al. They formed a market neutral betting against beta portfolio that holds low beta assets, leverages them up to a beta of one, while simultaneously shorting high beta assets, deleveraging them to a beta of one. By offsetting the position in the risk-free asset, they make the position self-financed. The model predicts that BAB factor has a positive average return and the return increases ex-ante the tightness of constraints and in the spread in betas between high- and low beta securities.

Constructing the BAB factor on the U.S. equity market yields a Sharp Ratio of 0.78 between 1926 and March 2012. In perspective, this means that BAB realizes a Sharpe ratio two times higher than the value effect and 40% higher than the momentum effect for the same period. Regressing the BAB returns on the four factor model of Carhart, the BAB portfolio achieves a monthly beta of 0.55% for the U.S. market and 0.33% for international equities.

2.3.6.1 Explanation

In practice, several market participants including individual investors, mutual funds and pension funds have no or only strict constrained access to leverage, mostly for regulatory reasons. The result is that these constrained investors will overweight risky assets in their portfolios to the less risky ones to achieve the desired return. The higher demand for risky assets in the CAPM framework will increase their prices and decrease their expected return in exchange compared to the low-beta assets. Frazzini and Pedersen observe a high demand for exchange-traded funds with embedded leverage that provides evidence that investors in fact cannot use leverage directly. They also find the beta spread to increase in periods of funding tightness, defined as the spread between three-month Eurodollar LIBOR and the three-month U.S. Treasuries rate (TED spread), as the access to leverage is reduced.

2.4 Factor Creation

All of our factor data is acquired from either the Kenneth French website or from the AQR data library. Data on size, value, profitability and investment returns is from the Fama-French five-factor dataset. Momentum is downloaded from the Fama-French momentum dataset, while betting against beta and quality data is downloaded from two separate datasets from the AQR website. All factors except betting against beta are based on the 2x3 portfolio sorting introduced by Fama and French (1993).

If we consider the example of the HML factor, the asset universe is first divided in two by size. The size breakpoint for the U.S. market is the NYSE median market cap, and for other markets, the universe is the split between the top 90% and the bottom 10% of the market cap (Kenneth French webpage). The portfolio now consists of a "Small" portfolio and a "Big" portfolio. From here, they again split each of the two size portfolios in three more portfolios based on the book-to-market ratio, creating a 2x3 portfolio sorting. All portfolios are value-weighted, meaning the weights of a company in a portfolio is given by the market cap of the firm compared to the entire market cap of the portfolio. The weights are rebalanced monthly. The breakpoints for the value sorting are the 70th and 30th percentiles points of the book-to-market ratio for each region. The same breakpoints are used for profitability, investment and momentum. For the U.S. market, the breakpoints are decided by the NYSE breakpoint. The three sorting on each size is called value, neutral and growth and the universe is now arranged into six value-weighted portfolios:

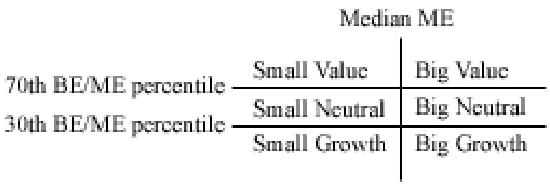

Figure 2: arrangement of the value-weighted portfolios. Source: Kenneth French website[11]

By going long the "Small Value" and "Big Value", and shorting the corresponding growth portfolios, the position is self-financing. The factor return is calculated by the following equation:

$$HML = \frac{1}{2}(Small\ Value + Big\ Value) - \frac{1}{2}(Small\ Growth + Big\ Growth)$$

(10): calculation of HML factor returns. Source: Kenneth French website[12]

[11] Fama, E. and French, K. (2017, March 26) Six Portfolios. Retrieved from the Kenneth French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/six_portfolios.html

[12] Fama, E. and French, K. (2017, March 26) Fama-French 5-Factors 2x3. Retrieved from the Kenneth French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html

Where each of the group names represent the return of the group. As we can see, the "Small Neutral" and "Big Neutral" is excluded from the factor formation.

This approach is applied to all factors by first splitting the portfolio in half by the market cap and then again split each portfolio in three more parts given by the decided breakpoint for the given factor. The exception is the size and betting against beta. The BAB factor skips the size sorting by going straight to dividing the portfolio by the median value of the ranked betas. The size factor is created by the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios from the sorting of the Fama-French five-factor model:

$$SMB_{(B/M)} = \frac{1}{3}(Small\,Value + Small\,Neutral + Small\,Growth)$$

$$-\frac{1}{3}(Big\,Value + Big\,Neutral + Big\,Browth)$$

$$SMB_{(OP)} = \frac{1}{3}(Small\,Robust + Small\,Neutral + Small\,Weak)$$

$$-\frac{1}{3}(Big\,Robust + Big\,Neutral + Big\,Weak)$$

$$SMB_{(INV)} = \frac{1}{3}(Small\,Consercative + Small\,Neutral + Small\,Aggressive)$$
$$-\frac{1}{3}(Big\,Conservative + Big\,Neutral + Big\,Aggressive)$$

$$SMB = \frac{1}{3}(SMB_{(B/M)} + SMB_{(OP)} + SMB_{(INV)})$$

(11): creation of SMB factor. Source: Kenneth French website[13]

All factors except size and BAB are based on balance sheet numbers from the annual report, meaning the numbers are lagged according to Fama and French (1993) with a conservative lag of minimum six months after the fiscal yearend, e.g. if the fiscal year is ending December 31st year $t-1$, the accounting data will be incorporated into the model from June 31st year t and for the next 12 months. This is to ensure the accounting numbers are publicly available before they are incorporated into a trading strategy. Assnes, Frazzini and Pedersen (2013) add a six-month lag similar to Fama and French, but

[13] Fama, E. and French, K. (2017, March 26) Fama-French 5-Factors 2x3. Retrieved from the Kenneth French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html

they align all the accounting variables available at any point in the calendar year $t-1$, to be included in the end of June in calendar year t. The same goes for Frazzini and Perdersen (2014)

The quality factor is created in the same matter as the value factor with the median of the NYSE market cap being the breakpoint for the U.S. size, but for international securities the breakpoint is the 80th percentile for the given country which according to Assnes, Frazzini and Pedersen (2013) is approximately the same as the NYSE breakpoints.

2.5 Macroeconomic Indicators

Stock prices are commonly believed to be sensitive to economic news and react to external forces like macroeconomic changes. Knowing the current and future state of the economy is a key insight to predict the future movement of the stock prices. A systematic variable that can describe the state of the economy is called a state variable. The state variables are a result of external factors but also endogenous to some extent. This means a change in one variable will lead to change in another variable, impacting the economy in different directions. This makes the economy hard to predict and forecasting the market is far from exact science. Still, some key indicators are assumed to contain information about the future state of the economy.

The relationship between stock returns and economic market conditions can be supported by the literature. For example, the dividend discount model values a company's stock priced based on the sum of its expected future dividend payments, discounted by their present value:

$$P = \frac{D_0(1+g)}{r-g} \qquad (12)$$

Where P is the current stock price, D_0 is the value of this year's dividend, g is the expected perpetual growth for the dividend, and r is the constant cost of equity, or the discount rate for the company. In this simple model there are at least four fundamental sources of value; expected cash flow, the term structure of interest rates, the expected market risk premium and the expected exposure to this market risk.

Backed by a wide selection of research papers and influenced by the dividend discount model, we have found some state variables that have a direct relationship to the variables in the model, and are easy to implement: the short-term interest rate, the term spread and dividend yield of the aggregated stock market. The short-term interest rate and term spread are based on the yield on government bonds, which are easily available in real time. Dividends are usually announced in good time prior to the payout, so dividend yield data is also available before the pay-outs takes place. This means that in the end of a period, all the data needed for making stock allocations for the next period, is available.

Another indication of how the stock market is performing is the business cycle, which refers to fluctuation of the real gross domestic product (GDP) along its long-term growth rate. As defined by the OECD (2002): "*Gross domestic product is an aggregate measure of production equal to the sum of the gross values added of all resident institutional units engaged in production (plus any taxes, and minus any subsidies, on products not included in the value of their outputs*.". [14] A problem using the business cycle to predict stock returns is that that it is set ex post. According to the National Bureau of Economic Research (NBER), a recession is declared 6 – 21 months after the recession have in fact started. Instead, using the short-term interest rate, term spread and dividend yield as a forecasting variables of the business cycle, we can get a good indication on which state the economy is in, and hence which factors are supposed to perform the best by looking at earlier performance along the business cycle.

2.5.1 Short-Term Interest Rate

The short-term interest rate is a tool of the monetary policy to influence the market. A low interest rate is intended to increase consumption and investments as it is cheaper to borrow money and less attractive to save. This makes a low interest rate desirable for the government during a recession to stimulate the economy.

Usually a one or three months zero coupon bond issued by the government is used as a proxy for the short-term interest rate. As the bond is backed by the government, the government can in worst case print more money to meet its debt obligations and is by the definition risk-free. A short-term bond backed by the U.S. government is called a Treasury bill or T-bill. The risk-free rate is then the yield to maturity (YTM) on the bond, calculated by the following formula:

$$YTM = \sqrt[T]{\frac{Face\ Value}{Present\ Value}} - 1 \qquad (13)$$

For the annual YTM, T denotes the time to maturity of the bond in years.

As explained, the yield on the government backed bonds is not directly set by the government, but the market. The market assesses the risk of the bond investment and set the price accordingly. Included

[14] OECD (2017, June 7) Gross Domestic Product (GDP). Retrieved from the OECD website: https://stats.oecd.org/glossary/detail.asp?ID=1163

in the risk calculation are the macroeconomic conditions, the government's monetary policy and the overall supply and demand for the government bonds. What makes the short-term government bond a proxy for the interest rate is the high correlation with the interbank rate. Because the bond is competing with bank deposits as an item for short-term savings, they move closely together. The deposit rate is again heavily influenced by the overnight rate set by the government to provide domestic banks and credit institutions with overnight funds.

In his book "The Theory of Interest", Fisher (1930) breaks down the one period interest rate, TB_t, observed at time t, into the expected real return of period $t+1$, $E(R_{t+1})$, and the expected inflation rate, $E(I_{t+1})$, which is given by the equation:

$$TB_t = E(R_{t+1}) + E(I_{t+1}) \tag{14}$$

Fama and Gibbons (1982) later confirm the relationship and find the variation in interest rates to be dominated by variation in expected inflation, and to some extent the expected real returns. They also find the expected real returns on Treasury bills to be correlated with the expected return on common stocks. Both equation (14) and according to money demand theory predicts that when controlling for variation in nominal money growth, inflation and real economic growth is negatively partial correlated. Reformulating the equation, they get:

$$I_{t+1} = TB_t - E(R_{t+1}) + \eta_t \tag{15}$$

Where η is the unexpected inflation. Meaning the inflation of a period is given by the Treasury bill rate of the previous period minus the expected real return for the period and the unexpected inflation.

Looking back at equation (12), the (14) discount rate is an average of rates over time and changes with both the current interest rate and the term spread over different maturities. According to Chen, Roll and Ross (1986) the unanticipated changes in the risk-free rate will therefor change the current value of the expected cash flows and hence the present value of the company. The discount rate is also dependent on the risk premium, which on the demand side changes along with changes in the indirect marginal utility of real wealth.

2.5.2 Term Spread

The term spread is defined as the long-term interest rate minus the short-term interest rate. Commonly, the yield to maturity of a no dividend paying government bond with the same maturity date is used as a proxy for the interest rate. Most papers consider the ten-year and three-month spread, but as our short-term interest rate is based on the yield of a one-month government bond, the term spread is calculated as the yield on a ten year zero coupon issued by the government minus a the yield on a bond with one year to maturity.

Chen (1991) find that the short-term rate and the term spread can forecast future growth rates of the GNP (gross national product) and consumption, implying a high current spread indicates positive future growth and vice versa. He uses Friedman's (1957) permanent income hypothesis to explain that if the output of the economy is expected to be high in the future, individuals desire to smooth consumption by attempting to borrow against the future increase in production. The increase in borrowings increases the current consumer spending and the currency in circulation. As a result, the interest rate is usually increased to limit the money supply and keep inflation on the target level. Further, he says the excess market return is positively correlated to the expected future growth and negatively correlated with the recent economic growth.

The findings of Chen (1991) can be related to the earlier findings Fama and French (1989) who track the returns on common stocks and long-term bonds along the movements in the business cycle supplied by the NBER.[15] Fama and French find the term spread to be low around business cycle peaks and high around local minimums. As Fama (1990) explains it, this is because the long-term rates tend to rise less than the short-term rates during business expansions and fall less during contradictions. They present one explanation: when the business conditions are poor, the income is low and the expected returns on bonds and stocks must be high encourage a substitution from consumption to investment. Further, Fama and French (1989) also find the term spread to track the term premium of stock and bonds.

Zhang, Hopkins, Satchell and Schwob (2009) argues that the term spread proxies for asset duration risk: if a company has a mismatch in the duration of cash flows where the company has more long-term liabilities than assets, an increase in the term spread will increase the value of the liabilities more than the assets. Accordingly, an increase in the term spread results in a higher risk premium for the duration risk.

2.5.3 Dividend Yield

Looking at the dividend discount model again, the dividend itself is an obvious variable for the stock price. The dividend is the distribution of a company's earnings to the shareholders, representing the

[15] Fama and French (1989) uses the yield of Aaa-rated corporate bonds by Moody's as an alternative to the 10-year Treasury bill yield.

shareholder's cash flow. The dividend yield is the anticipated dividend per share for the next 12 month divided by the current share price. By doing this for an index representing the stock market for the desired region, like S&P 500 for the U.S. and NIKKEI 225 for Japan, the dividend yield of the indices proxies for the dividend yield for all the common stocks of the entire region.

Fama and French (1989) find that the dividend yield and the default rate track similar components of returns as they are highly correlated (0.75 for 1941 – 1987 in the U.S. market) with high values during periods of persistent poor performance of the business cycle and low during strong periods. For each state variable we include in our model, the asset space will be extended by the number of factors in the (as explained in the methodology, 3.2) meaning the optimization process will take a lot more time. Because of this, we are keeping the numbers of state variables to a minimum. As a result, we exclude the default spread from our thesis even though the term spread is documented to have a forecasting effect on the market. Our choice to go with the dividend yield is because the variable is built from components from the same market as we are trying to predict, the stock market, rather than the default spread which is built with components from the bond market.

Further, Fama and French (1989) explain the intuition of the forecasting power by the terms of the efficient-market hypothesis: when discount rates and expected returns are high, stock prices are low compared to dividends, and vice versa. They also derive an explanation that the dividend yield can be due to irrational bubbles in the stock prices, and as a result dividend yields and expected returns are high when prices are temporarily irrationally low and vice versa.

Campbell and Shiller (1988) break down the log dividend yield into three components attributable to: expected future growth in dividends, expected future discount rates and unexplained factors, and test for relations. They find the log dividend yield to move with expected future growth in dividend and unexplained variation. Using short-term interest rates and as a measure for discount rates, they find neither short-term rates nor real stock returns to be Granger caused by log dividend yield. Combined with an exogenously determined ARMA model, Binsbergen and Koijen (2010) find the dividend yield to be good predictors for future returns and future dividend growth rates, contrary to the finding of Campbell and Shiller.

2.6 Summary and Portfolio Implication

So far we have reviewed the origin and the development of some of the most established factors which we will apply in our model. We have also described how the factors move, both together with each other and along with the the market. The state variables have been introduces to gain us insight about the current state of the economy. Next, we are evaluating how the findings in the literature review can affect our model and portfolio creation.

2.6.1 Factor Portfolio

Data shows that the factors returns can totally vanish for long periods like the size premium in all tested regions between 1991 and 2011 in Fama and French (2012). Some of the factors are also known to have different historical correlations with each other, suggesting diversification benefit among the factors can be exploited in a multi-factor portfolio.

Documented correlations:

- Profitability and value: uncorrelated (see 2.3.3)
- Momentum and value: negatively correlated (see 2.3.4)

While negatively correlated and uncorrelated factors can all be added to a portfolio for diversification purposes, only the factor with the highest risk-adjusted return should be held if the factors are highly correlated.

2.6.2 Forecasting of Factor Returns

All the factors are found to perform well historically and yield a significant alpha when regressed on the market and other factors, at least for some periods. Still, the factors do not yield positive returns at all times and will perform poorly in periods. In the reviewed literature, the following factor performance within the business cycle has been documented:

- Value: countercyclical (see 2.3.2)
- Momentum: procyclical (see 2.3.4)
- Quality: countercyclical (see 2.3.4)

Knowing these periods ex ante will make the investor change the weighting to the factors with a higher expected return for given state of the economy. But knowing what state the economy is currently in, is as explain hard to do. Instead the state variables give an indication of the current state of the economy:

- Short-term interest rate: high around business cycle peaks and low around troughs (see 2.5.1)
- Term spread: low around business cycle peaks and high around troughs (see 2.5.2)
- Dividend yield: low around business cycle peaks and high around troughs (see 2.5.3)

2.7 Practical Implications

The assumption of a perfect market is a great simplification of the real market, but is necessary to bring foresight to our model and is often used when working on trading strategies on a theoretical level. To our knowledge, all papers considered in this thesis make similar assumptions.

Especially two of the assumptions are rather extreme compared to the real life situation, and will have to be considered when engaging the portfolios in the real world, namely transaction costs and margin requirements. We will now discuss the practical implications by bringing the theoretical portfolios to life, but after that continue with the assumption of a perfect market.

2.7.1 Short Sales

A major assumptions of the thesis is that we can short sell one end of portfolio and use all the proceeds to finance the long position, resulting in a self-financed portfolio. Technically, a shorted stock is a stock that is borrowed from the owner with the promise to buy it back later. By selling the borrowed stock, the hope is for the stock to drop in price so it can be bought back for a lower price, but there is also a risk that the stock price will increase in price, so it has to be bought back for a higher price than what it was sold it for. Theoretically, the stock price can increase infinitely and the same happens to the downside of the short sale. To reduce risk of the short seller not being able to buy back the stock after the short sale, the short seller is usually required to keep all the proceeds plus a margin on a short sale account. In this case, the short seller is unable to use the proceeds to finance a long position.

For example, Regulation T by the Federal Reserve Board requires all short sellers to post a 50% margin, called the Fed Call, plus the initial sales proceeds on a short sale account. [16] By taking a long-short position without this requirement, an investor could invest as much as he pleases without having any initial funds. But in practice the investor must be able to cover the long position as he cannot use the proceeds of the short sale and also post the 50% margin, so in fact he must have funds equal to 1.5 times the position he is taking. On top of this, the investor must pay interest as the short position can be considered a loan, and also be able to cover possible margin calls if the value of the shorted stocks increases.

[16] U.S. Government Publishing Office (2017, May 27) Electronic Code of Federal Regulations. Retrieved from GPO's Federal Digital System: https://www.ecfr.gov/cgi-bin/text-idx?SID=ec9173a10a3853ca9a688fc486468545&mc=true&node=se12.3.220_112&rgn=div8

Other implications regarding short sales includes the availability of stocks to be shorted, especially small cap stocks which are usually less liquid, and the horizon of a short position as the lender can call back the stock.

2.7.2 Transaction Costs

Another major implication is the transaction costs. The transaction cost includes the commission to the broker and the spread between the bid and the ask price. After subtracting for transaction costs, the results would have been lower than the returns achieved in this thesis.

The bid-ask spread depends on the liquidity of the stock. A large cap stock is usually traded more frequently and will have a lower spread than small cap stocks. Numbers from Vanguard says their 30-day average bid-ask spread (as of 26.05.2017) is 0.02% for the Vanguard Large-Cap ETF and 0.03% for Vanguard Small-Cap ETF.[17] The funds seeks to track the performance of the CRSP indices, which includes NYSE, NASDAQ and ARCA. For less liquid stocks, a high buy or sell order is also likely to affect the bid-ask spread which is hard to predict without performing the actual transactions.

As technology improves the flow of information and communication, a natural consequence is that it becomes easier for a buyer to find a seller and vice versa. Naturally, this increases the liquidity and hence reduces the bid-ask spread. Figure 1 in Jones (2002) shows that the bid-ask spread on Dow Jones stocks decreased by almost 2/3 between 1990 and 2000. The variation in transaction cost makes implementing transaction cost to the back testing of a trading strategy, hard to do.

[17] Vanguard (2017, May 27) Bid-ask Spread. Retrieved from Vangurd: https://advisors.vanguard.com/VGApp/iip/site/advisor/investments/bidaskspread

3 Methodology

In the methodology part, we will go through the entire process from data gathering to building the models. Relevant theory for the models, which have not been discussed in the literature review, will also be included to motivate our decisions.

3.1 Data Gathering

Our final month of factor data is January 2017. The factor return data is usually posted the month after, but could also be retrieved the same day by building the portfolios our self, based on the available market data. For the U.S. and Japan, the availability of data on the dividend yield limits our data set to a starting point of January 1973 for the U.S. and January 1991 for Japan. The availability of momentum return data limits our Europe sample to the starting point of November 1990

3.1.1 Regions

Our thesis is concerned with the stock markets of three regions: U.S., Japan and Europe. The U.S. and Japan markets are self-explanatory as they are individual countries, but the European market needs some more explanation. The European region is defined by countries included in the Fama and Frecnh (2012) paper where they apply their three-factor model plus momentum on the international stock market. This same market definition is also used for the data on quality and betting against beta. Included in the European region are Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Ireland, Italy, Netherland, Norway, Portugal and Sweden. As we see, the sample is mostly concerned with the Western European countries. According to Wikipedia[18], this sample includes the 15 biggest economies in the European Union (EU) excluding Poland, but including Norway and Switzerland. This suggests macroeconomic indicators for the EU should be good indicators for our European region.

The choice of our three markets is simply because of the limited market data available on the Kenneth French website. While AQR supplies data for each individual country in the European region, the Kenneth French website only provides data for the aggregated market. The website does include a global market, but this is left out of the thesis as the state variables for a global market has limited precision and is hard to calculate as the global portfolio is in fact not global, but consisting only of the

[18] Wikipedia (2017, April 18) Economy of the European Union. Retrieved from Wikipedia: https://en.wikipedia.org/wiki/Economy_of_the_European_Union

European, North-American and Japanese portfolio. The North-American portfolio is left out as it is too similar to the U.S. portfolio and provides little added value.

3.1.2 Factors

For the definitions of the individual factors, see the literature review, chapter 2.3.

3.1.2.1 Fama-French Five Factor Model and Momentum

For size, value, profitability and investment we use the same dataset as the Fama and French (2015) paper on the five-factor model. Momentum data is downloaded from a separate size and momentum dataset. The data is publicly available on the Kenneth French website and updated monthly. The data includes all common stocks from the CRSP and Compustat database for the U.S. market (filtered for share type 10 and 11), represented by American companies listed on NYSE, NASDAQ and Arca. Financial data is pulled from the merged CRSP/Compustat database. For international stock return, the Bloomberg database is used. The factor data on the U.S. market from the Kenneth French website goes back to July 1963 and for Europe and Japan, the factor returns are available from the end of July 1990 for the five-factor model data and November 1990 for momentum.

3.1.2.2 Quality and Betting Against Beta

For the U.S., the BAB factor returns goes back to the end of January 1931 and July 1957 for quality returns. To create the factor returns, data from the merged CRSP/XpressFeed database is used. The XpressFeed Global database is used for the global dataset of common stocks (issue code 0). From the AQR website, factor returns back to February 1989 is available for betting against beta and January 1986 for quality minus junk for Europe and Japan.

3.1.3 State Variables

For the definition of the state variables, see the literature review, chapter 2.5.

3.1.3.1 Short-Term Interest Rate

We use the risk-free rate provided in the Fama-French datasets as our short-term interest rates. This short-term rate is based on the one-month Treasury bill provided from by Ibbotson and Associates, INC for the U.S. market and for Europe and Japan the data on the one-month government bonds is provided by Bloomberg.

3.1.3.2 10-Year Government Bond

We obtained the YTM of the 10-year government bond for the U.S. from the data library of the Federal Reserve Bank of St. Louis.[19] For Japan, the YTM is pulled from Datastram's 10-year government bond index for Japan (time series: BMJP10Y). For Europe, we use the 10-year government benchmark index for the euro area provided by the European Central Bank.[20] Note that this index is for the euro area and not for our defined European market. Still, both markets are mainly concerned with the Western- and Central European countries and should not deviate too much from each other.

3.1.3.3 Dividend Yield

The U.S. dividend yield is given by the S&P 500 composite index provided the S&PCOMP times series from Datastream. For Japan, the dividend yield is acquired from the Nikkei 500 index provided by the JAPA500 time series from Datastream. The dividend yield for Europe is taken from the EU equity index provided by Datastream and the time series TOTMKEU. Note that also here, the index is for the EU and not exactly the same as our definition of Europe.

According to Datastram Navigator, the dividend yield of a share is the anticipated dividend per share for the coming year, divided by the current stock price. The anticipated dividend payment is derived by multiplying the latest payment with the frequency of payment. For example, if the latest dividend payment was a quarterly payment, this payment is then multiplied by 4 to get the expected payments for the next 12 months.

3.2 Portfolio Optimization Conditional on State Variables

Brandt and Santa-Clara (2006) present an approach to implement timing of assets in a dynamic trading strategy. The portfolio consists of assets that are conditional on state variables that capture the first and second moments of asset the returns. By exploiting the predictability of the future asset returns given by the state variables, they present a way to time a dynamic strategy by including the state variables to affect the asset allocation in the portfolio. While other numerical solution methods to a dynamic portfolio includes solving partial differential equations and Monte Carlo simulations, Brandt and Santa-Clara (2006) Present a model that can be implemented just as easy as the static Markowitz model. Their model is an approximation to a dynamic model by a fixed combination of mechanically

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[19] Federal Reserve Bank of St. Louis (2017, March 2) 10-Year Treasury Constant Maturity Rate. Retrieved from Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/GS10

[20] European Central Bank (2017, March 2) Euro Area 10-Year Government Benchmark Bond Yield. Retrieved from the European Central Bank: http://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=143.FM.M.U2.EUR.4F.BB.U2_10Y.YLD

managed portfolios, which is very fit for our purpose, as our portfolio is not continuous, but is instead rebalanced on a monthly basis.

3.2.1 Single-Period Problem

The two next sections are based on the paper of Brandt and Santa-Clara (2006) with some personal interpretations and most equations in the first section are taken from there.

Consider an investor who wishes to maximize the conditional expected value of a quadratic utility function over next period's wealth, W_{t+1}:

$$max\, E_t \left[W_{t+1} - \frac{b_t}{2} W_{t+1}^2 \right] \tag{16}$$

Where $0 < b_t < \frac{2}{W_{t+1}}$, so the marginal utility of wealth remains positive. Now consider that the investor invests the initial wealth in a portfolio consisting of risky assets and the wealth at $t+1$ is dependent on the return of the portfolio and the risk-free rate. The wealth at $t+1$ is now given by:

$$W_{t+1} = W_t (R_t^f + r_{t+1}^p) \tag{17}$$

Putting equation (16) and (17) together and simplifying, we obtain:

$$maxE_t \left[cte + r_{t+1}^p - \frac{b_t W_t}{2(1 - b_t W_t R_t^f)} \left(r_{t+1}^p \right)^2 \right] \tag{18}$$

Where cte contains terms that are constant given the information available at time t. For simplicity we ignore the constant term and the problem can be rewritten as

$$max\, E_t \left[r_{t+1}^p - \frac{\gamma}{2} \left(r_{t+1}^p \right)^2 \right] \tag{19}$$

Where γ is a positive constant representing the risk appetite. Further, the weights of the risky assets in the portfolio at time t is given by vector x_t and $r_{t+1}$is the vector of the excess return of the N risky assets. The optimization problem then becomes:

$$\max_{x_t} E_t \left[x_t^\top r_{t+1} - \frac{\gamma}{2} x_t^\top r_{t+1} r_{t+1}^\top x_t \right] \tag{20}$$

x_t^{T} is the transposed vector of x_t. Because the model is concerned with excess returns, we assume the rest of the portfolio is invested in risk-free assets with return R_t^f.

In the case of the returns being independent and identical distributed the optimal asset weights are constant over time, that is $x_t = x$, we can replace the conditional expectation with an unconditional expectation and solve using the Lagrange method:

$$x = \frac{1}{\gamma} E[r_{t+1} r_{t+1}^{\mathsf{T}}]^{-1} E[r_{t+1}] \tag{21}$$

This is the known Markowitz solution and can be implemented to our problem in practice by replacing the population moments by sample averages.

$$x = \frac{1}{\gamma} \left[\sum_{t=1}^{T-1} r_{t+1} r_{t+1}^{\mathsf{T}} \right]^{-1} \left[\sum_{t=1}^{T-1} r_{t+1} \right] \tag{22}$$

Written another way:

$$x = \frac{1}{\gamma} Var(r_{t+1})^{-1} E(r_{t+1}) \tag{23}$$

Ferson and Siegel (2001) find that an unconditional mean-variance optimized portfolio can reduce risk by incorporating a response to extreme signals in the market. If the state variables capture the first and second moment of the asset returns, one can identify the effect on the asset return from each state variable by modelling the conditional means, variance and covariance of the returns as a function of the state variables, and then derive the optimal assets weights as a function of state variables. By focusing on the asset weights, rather than the conditional return distribution, Brandt and Santa-Clara (2006) present an approach to an approximation of the Ferson and Siegel framework, as the approach of Ferson and Siegel suffer from difficulty in modelling the conditional covariance with state variables and a lot of parameters to be estimated. In the approach of Brandt and Santa-Clara, the optimal portfolio weights are a linear function of the observed state variables:

$$x_t = \theta z_t \tag{24}$$

Where z_t is the state variable vector consisting of K state variables and the first variable is generally a constant. θ is a $N * K$ matrix of coefficients. The problem now becomes

$$\max_{\theta} E_t \left[(\theta z_t)^\top r_{t+1} - \frac{\gamma}{2} (\theta z_t)_t^\top r_{t+1} r_{t+1}^\top (\theta z_t) \right] \tag{25}$$

From linear algebra we get

$$(\theta z_t)^\top r_{t+1} = z_t^\top \theta^\top r_{t+1} = vec(\theta)^\top (z_t \otimes r_{t+1}) \tag{26}$$

Where $vec(\theta)$ piles up the columns of matrix θ into a vector and $\otimes$ is the Kronecker product of two matrices. We can now write

$$\tilde{x} = vec(\theta) \tag{27}$$

$$\tilde{r}_{t+1} = z_t \otimes r_{t+1} \tag{28}$$

The problem can now be written as

$$\max_{\tilde{x}} E_t \left[\tilde{x}^\top \tilde{r}_{t+1} - \frac{\gamma}{2} \tilde{x}^\top \tilde{r}_{t+1} \tilde{r}_{t+1}^\top \tilde{x} \right] \tag{29}$$

Because the same $\tilde{x}$ maximizes the conditional expected utility at all dates, t, it also maximizes the unconditional expected utility:

$$\max_{\tilde{x}} E_t \left[\tilde{x}^\top \tilde{r}_{t+1} - \frac{\gamma}{2} \tilde{x}^\top \tilde{r}_{t+1} \tilde{r}_{t+1}^\top \tilde{x} \right] \tag{30}$$

Correspondingly, the problem now is to find the unconditional portfolio weights $\tilde{x}$ for the expanded set of $(N * K)$ risky assets with returns $\tilde{r}_{t+1}$. Each part of the expanded set of risky assets can be interpreted as a managed portfolio which invests in a single basis asset that is proportional to the value of one of the state variables. Following equation (21), we now get the solution to the portfolio optimization problem:

$$\tilde{x} = \frac{1}{\gamma} E[\tilde{r}_{t+1} \tilde{r}_{t+1}^\top]^{-1} E[\tilde{r}_{t+1}]$$

$$= \frac{1}{\gamma} E[(z_t z_t^\top) \otimes (r_{t+1} r_{t+1}^\top)]^{-1} E[z_t \otimes r_{t+1}] \tag{31}$$

Once again we implement it by replacing the population moments by sample averages

$$\tilde{x} = \frac{1}{\gamma} \left[\sum_{t=0}^{T} (z_t z_t^\top) \otimes (r_{t+1} r_{t+1}^\top) \right]^{-1} \left[\sum_{t=0}^{T} z_t \otimes r_{t+1} \right] \tag{32}$$

which again can be written as

$$\tilde{x} = \frac{1}{\gamma} Var(\tilde{r}_{t+1})^{-1} E(\tilde{r}_{t+1}) \tag{33}$$

Based on equation (32), we can calculate weights of each of the assets by adding the corresponding products of elements of $\tilde{x}$ and z_t. Note that the solution (32) depends only on the data and hence does not require any assumptions about the distribution of returns apart from stationarity.

3.2.2 Illustrative Example

The original model of Brandt and Santa-Clara expands the asset space by multiplying each factor by each of the state variables. For the historical data sample, this is done by multiplying the asset return of the beginning of time t with the state variables at the beginning of time $t-1$. But when allocating weights, the static solution is multiplied by the state variables as time t (see equation (29)). Instead of doing this, we run two versions of the model, with one version using a one period lag in the state variables also when allocating weighs, and another version with no lags in both the sample and when allocating weights. The reason for doing this is to see if the lagged state variables better predict the stock returns than no lag.

Now consider a situation where the risky assets are different factor mimicking portfolios and the state variables are macroeconomic factors. Let's say we have two assets (s_1, s_2), two state variables and a time series of four months of data. The time series has following observations of the historical excess returns:

$$\begin{bmatrix} r_1^{s_1} & r_1^{s_2} \\ r_2^{s_1} & r_2^{s_2} \\ r_3^{s_1} & r_3^{s_2} \\ r_4^{s_1} & r_4^{s_2} \end{bmatrix} \tag{34}$$

Correspondingly to the one period lag method, we have the vector of the state variables with the first variable being a constant i.e. 1: $z_t = (1, z_t^1, z_t^2)$. As mentioned, the state variables are known in the beginning of each period. The matrix of the state variables in the time series becomes

$$\begin{bmatrix} 1 & z_0^1 & z_0^2 \\ 1 & z_1^1 & z_1^2 \\ 1 & z_2^1 & z_2^2 \\ 1 & z_3^1 & z_3^2 \end{bmatrix} \tag{35}$$

We expand the matrix of returns according to (28):

$$\begin{bmatrix} r_1^{S_1} & r_1^{S_2} & z_0^1 r_1^{S_1} & z_0^2 r_1^{S_1} & z_0^1 r_1^{S_2} & z_0^2 r_1^{S_2} \\ r_2^{S_1} & r_2^{S_2} & z_1^1 r_2^{S_1} & z_1^2 r_2^{S_1} & z_1^1 r_2^{S_2} & z_1^2 r_2^{S_2} \\ r_3^{S_1} & r_3^{S_2} & z_2^1 r_3^{S_1} & z_2^2 r_3^{S_1} & z_2^1 r_3^{S_2} & z_2^2 r_3^{S_2} \\ r_4^{S_1} & r_4^{S_2} & z_3^1 r_4^{S_1} & z_3^2 r_4^{S_1} & z_3^1 r_4^{S_2} & z_3^2 r_4^{S_2} \end{bmatrix} \tag{36}$$

From here we can calculate the optimal static portfolio weights of the expanded set of assets calculated by the Markowitz solution (33) on the matrix of returns of the expanded asset set (36). The covariance matrix and the vector of means take into account both the covariances among returns and between returns and lagged state variables. The static solution is $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3,\ \tilde{x}_4, \tilde{x}_5, \tilde{x}_6)$ which corresponds to the 2 assets and the 4 portfolios of the extended assets space. By (36), multiplying the weights in the extended asset space with the state variables, we get:

$$\begin{bmatrix} x_t^{S_1} \\ x_t^{S_2} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 + \tilde{x}_3 z_{t-1}^1 + \tilde{x}_4 z_{t-1}^2 \\ \tilde{x}_2 + \tilde{x}_5 z_{t-1}^1 + \tilde{x}_6 z_{t-1}^2 \end{bmatrix} \tag{37}$$

We also run our model with no lags in the state variables, meaning the expanded matrix of returns is given by:

$$\begin{bmatrix} r_1^{S_1} & r_1^{S_2} & z_1^1 r_1^{S_1} & z_1^2 r_1^{S_1} & z_1^1 r_1^{S_2} & z_1^2 r_1^{S_2} \\ r_2^{S_1} & r_2^{S_2} & z_2^1 r_2^{S_1} & z_2^2 r_2^{S_1} & z_2^1 r_2^{S_2} & z_2^2 r_2^{S_2} \\ r_3^{S_1} & r_3^{S_2} & z_3^1 r_3^{S_1} & z_3^2 r_3^{S_1} & z_3^1 r_3^{S_2} & z_3^2 r_3^{S_2} \\ r_4^{S_1} & r_4^{S_2} & z_4^1 r_4^{S_1} & z_4^2 r_4^{S_1} & z_4^1 r_4^{S_2} & z_4^2 r_4^{S_2} \end{bmatrix} \tag{38}$$

And the weights invested in the stocks is then given by

$$\begin{bmatrix} x_t^{S_1} \\ x_t^{S_2} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 + \tilde{x}_3 z_t^1 + \tilde{x}_4 z_t^2 \\ \tilde{x}_2 + \tilde{x}_5 z_t^1 + \tilde{x}_6 z_t^2 \end{bmatrix} \tag{39}$$

3.2.3 Implementing the Model to Our Data Set

Going from (16) to (30) we see the problem goes from maximizing the conditional expected value of a quadratic utility function over next period's wealth, to maximize the expected portfolio return conditional on expected value of a quadratic utility function over next period's return for the augmented asset space, where the utility now is dependent on the risk appetite. Rather than maximizing expected return for a given risk appetite as in equation (30), we set a target return and minimize the risk for the given return. From there we let the Excel Solver find the optimal weights with the assumptions discussed in chapter 3.5: Model Creation.

3.2.4 Other Appliances of the Model

As the framework of Brandt and Santa-Clara (2006) for building a portfolio by augmenting the asset space plays such an important role in our model, we would like to justify the use of this model as it is not commonly referred to in the literature. Nevertheless, the paper has of today (20.04.2017) been cited by 160 publications according to Google Scholar.

The whole model is rendered in "Handbook of Financial Econometrics: Volume 1- Tools and Techniques" (2009) as an alternative econometric approach to parametric portfolio weights, with one of the editors being the Nobel winning professor Lars Peter Hansen.

Basak and Chabakauri (2010) derive a dynamic mean-variance asset allocation model and compare their model to the one of Brandt and Santa-Clara among others. Here they argue their approach better represent the "*fully explicit closed-form solution for specific stochastic environment opportunity set as in the continuous-time formulation*" (Basak and Chabakauri, 2010 p. 3001) compared to the assumptions made the Brandt and Santa-Clara paper. However, this is not a concern for us as we operate in discrete time with monthly rebalancing.

3.3 Shrinking of Covariance Matrix

A sticky point in the Markowitz's mean-variance framework and the augmented asset space of Brandt and Santa-Clara is the use of the sample covariance matrixes of the historical returns to calculate the risk of the portfolio. According to Ledoit and Wolf (2003) who cite Jobson and Korkie (1980), there are some well-documented problems with relying on the covariance matrix. The problem becomes more severe when the number of assets in the portfolio is significantly larger than the sample size, and the sample covariance matrix becomes estimated with a lot of error. The covariance-matrix then tend to take on extreme values because of extreme amount of errors, rather than what is "the truth". Ledoit and Wolf further say that the optimization solutions tends to latch on the extreme amounts and place its biggest bets on those coefficients witch is most extremely unreliable.

Kwan (2011) goes on saying a covariance matrix estimated with insufficient observations will not be positive definite. A positive definite matrix is always invertible, but not vice versa. For a covariance matrix to be acceptable for portfolio analysis, it must be positive definite according to Kwan. On the other side, a high frequency of observations, such as daily observations, tends to be noisier and thus affecting the quality of the estimated covariance matrix.

To reduce extreme positive errors in the covariance matrix, the errors should to be pulled down. Similarly, the extreme negative values need to be pulled up closer to zero. We do this by shrinking the covariance matrix according to Kwan (2011):

$$\hat{C} = (1 - \lambda)\Sigma + \lambda\widehat{D} \tag{40}$$

Where $\hat{C}$ is the shrunk covariance matrix, Σ is the sample covariance matrix and $\widehat{D}$ is the diagonal of the sample covariance matrix going from north-west to south-east. This diagonal matrix corresponds to the sample variance of the underlying assets. The equation then indicates that the individual variance is the same regardless of λ (lambda) and the shrinkage approach only pertains the covariances. λ is a value between 1 and 0, where a value of 0 leaves us with the initial sample covariance matrix, and a value of 1 result in all covariances to equal 0. In this extreme case we are only left with the individual variances.

The big question is how much the covariance matrix should be shrunk. According to Ledoit and Wolf, correct shrinking can result in significant improvements of the realized stock returns. In our case, we will test our in-sample U.S. model with a covariance matrix where all the different values of lambda are run with intervals of 0.1. This is done until we find the lambda that yields the highest Sharpe ratio. In other words, we fit the covariance matrix to our model. To exclude any data mining biases, the model is only fit to the U.S. sample and then the same shrinkage is applied as it is to Europe and Japan. This way we see if our model is also applicable to the out-of-sample data.

3.4 Illustrative Example

For a step by step example of the process to get from the observation of factor returns and state variables to weight allocation for the one period lag version of the model, readers are encouraged to see Appendix 8.1: Step by Step Example of SVMVM, or look at the attached spreadsheets.

3.5 Model Creation

3.5.1 Set-Up

We start out with seven time series of the different factor returns and the state variables for the same period. From here we create three different multi-factor portfolios. The simplest one is the equally weighted portfolio, which is rebalanced monthly to have the same weights in all the factor portfolios. Following the framework of Brandt and Santa-Clara, we create a dynamic multi-factor model dependent on the state variables. We also run the same dynamic model without state variables. For the two dynamic models, we test for different combinations of rolling windows and shrinkage.

First, the rolling window of the model and shrinkage must be set. We define rolling window as the number of monthly observed factor returns that the model will use to calculate the sample covariance matrix and the estimated factor return. We test for rolling windows between one and five years with an interval of one year. The shrinkage is set according to 3.3.

For the decided window and lambda, we set the Excel Solver to allocate weights to the different factors by creating a portfolio that minimizes the volatility for a given target return. In portfolio optimization problems, the model can choose to either maximize the expected target return with a given variance, or pick a variance where the expected target return is maximized. We have chosen the first approach after a lot of trial and error. We experienced in some cases when having a target variance, the model placed some especially heavy weights on factors with a recent low volatility. This introduced a lot of dependence on the specific factor to perform well, and did in some cases fail, resulting in big losses. This could also be avoided by introducing absolute weight constrains, but instead we chose to go with minimizing the volatility. In this case the model is satisfied when the target return is reached and it places no unnecessary high weights after that.

The given weights are the weights we should have held in the beginning of the period of the rolling window to achieve the highest risk-adjusted return by the end of the period. We assume that these positions are the best approach for what we should hold for the next period. This process is repeated for each month by including the most recent data and leaving out the oldest, keeping our rolling window constant. For the mean-variance approach with the extended asset space (state variables), the model to also base the allocation of factors according to the state variables observed at the current or previous period (depending on if we add a lag or not).

3.5.2 Constraints

We allow our model to go both long and short in the factor portfolios. Because all of our factors consist of one long portfolio with a counterpart shorted by the same amount, the sum of weights within a factor portfolio is always zero. Shorting the factor would then mean to go long in the counterpart that is usually shorted and shorting the long portfolio. As discussed in the literature review, all the factors have expected positive return for a long holding period, but are known to perform poorly for smaller periods. With a recent poor performance, a factor might have a negative average return for the given rolling window. This means that the expected return for the next period is also negative for the factor, and we can expect the model will allocate a negative weight to the factor.

We set no limit to the absolute weights of the position and leave the allocation up to the performance of the factors and the properties of the mean-variance framework. A constrain on the weighs would force the model to make unfavorable allocations, as it has to abandon the factor that would have delivered the highest expected risk-adjusted return. On the other side, not limiting the weight on a single-factor, increases the possible downside if the factor would realize a return opposite of the expected value calculated in the model. After several runs with the model, we observe that the model never places any extreme weights and hence we leave the allocation entirely to the model. This is good because for the augmented asset space model we optimize for an extended asset space of 28 assets, which is done by first observing the state variables and then we assign the weights to our 7 asset, making constraining the factor weights hard to do.

3.5.3 Optimization

The challenge of an optimization problems is to find global maximum or minimum within the constraints of the problem. It is often impossible to be completely certain that the minimum or optimum you end up with, in fact is the global optimal solution and not a local one.

In our case, the standard mean-variance model consists of a portfolio of seven factors which delivers a return given by the factor weights times the factor returns. This means there is a linear tradeoff between weights and returns. According to the optimization problem of Markowitz, equation (5), we are minimizing the portfolio variance which is given by a quadratic function. The quadratic optimization problem of Markowitz is known to be a convex function (Cornuejols and Tütüncü, 2006). A convex function makes the optimization problem easier than the general case because by the property of the function, any local minimum must be a global minimum. By running the model numerous times with different target returns, a consistent Sharpe ratio is an indicator we indeed have found the global maximum.

For our extended model, the model increases with the number of factors times the number of state variables, leaving us with an optimization problem of 28 assets (7+3*7). The initial model with a linear relationship between weights and factor returns is now extended as the weights of the extended asset space is dependent on the state variables and we can no longer assume the model is linear. According to Brandt & Santa-Clara and Ferson & Siegel, the optimal portfolio weights are approximately linear in the expected returns for an extended range of the state variables around their unconditional means when the returns are homoscedastic. Therefore, the portfolio weights will also be linear in the state variables if the expected returns are linear in the state variables. This assumption of homoscedasticity in stock returns has been proven several times not to be the case, for example, Schwert and Seigun (1990) find the heteroskedasticity to be a pervasive phenomenon in the U.S. stock market between 1927 and 1986, meaning we abandon the assumption of a linear relationship. This results in a significant increase in the processing time and we now also face the problem that the located minimum is just the local minimum rather than the global. Running the model several times and ending up with the same results, makes us more confident we in fact have found the global minimum.

One concern is that because we do not allow to take positions in the risk-free rate, we cannot lever the positions along the tangency portfolio, so our returns are limited to be only be located along the efficient frontier. But contrary to the original Markowitz portfolio optimization, our portfolios are self-financed and we do not have the constraint of all the weights having to add up to the sum of one. We think that because of this, the result of the optimization problem will be located on the tangency line. We will address this problem later by running the optimization for different target returns for our sample and see if the achieved Sharpe ration is the same for the portfolios (see 4.3.5). If it does, this indicates the weights are scaled along the tangency line.

3.5.4 Lags

According to Brandt and Santa-Clara, we assume that the current state variables have forecasting power over the next period's return. No lags means that the returns and state variables are observed immediately before the last trading day of period $t-1$ ends, and the observations are used to place weights in the beginning of the first trading day of period t. Theoretically, trading on yesterday's closing price and macro indicators is possible as all data is available, but might be considered unrealistic. Still, the method is often applied in academia.

We also run the model with a lag in the state variables to see if our portfolios benefit from having a slower response to the state variables. The reason to test for a slower response to the state variables in our portfolios can be proposed by a slow reaction in the market prices. Because we see no reasons why a lag in the factor returns should be a better predictor for the next period's factor returns, there will be no lag in the returns for the two occasions.

3.5.5 Rolling Windows

We limit our rolling windows to go from one to five years. This is because shorter rolling windows are expected to yield the best results as the more recent return is a better indication for the return of the next period. Also for our out-of-sample markets, the sample size is much lower than for the U.S., resulting in fewer observations for our regressions. Just as the lambda for the shrinking, the rolling window is also fit to model to find the rolling window yielding the best result. This means that the rolling window we find in the U.S. market will be used for our Europe and Japan sample.

3.6 Portfolio Performance Measures

When evaluating our models, some measures for the overall portfolio performance should be discussed. The "Efficiently Inefficient" book by Pedersen (2015) inspires this chapter, and the equations from the "Evaluating Trading Strategies" chapter are used.

3.6.1 Alpha and Beta

The most basic measure of a trading strategy is the return, R_t, for a given period, t. For a better understanding of the return, the return is often separated into alpha and beta. The alpha and beta is determined by the ordinary least squares regression model (OLS). The OLS is a linear regression model, which minimizes the sum of the squares of the difference between the observed responses (portfolio return) and the value predicted by the linear function of the explanatory variables (factor returns). Assumptions for the OLS regression to deliver the best result in terms of lowest variance compared to other unbiased estimators, are given by the Gauss-Markov theorem: the errors must have an expected value of zero, be uncorrelated and have equal variances. As the OLS estimator is generally applied in the academia of stock returns and factor research, we make no further tests to check whether the OLS regression is the best linear unbiased estimator or not.

When regressing the return on a factor, the beta describes the linear relationship between the return and the variable, while the alpha is the intercept of the linear function. The most basic analysis is to regress the portfolio return excess of the risk-free rate (R^f), $r_t^e = R_t - R^f$, on the excess return of the market, $r_t^{M,e}$:

$$r_t^e = \alpha + \beta r_t^{M,e} + \varepsilon_t \tag{41}$$

The regression then minimizes the sum of squared residuals (SSR) given by:

$$SSR = \sum_{t=1}^{n} (\varepsilon_t)^2 = \sum_{t=1}^{n} (r_t^e - (\alpha + \beta r_t^{M,e})^2 \qquad (42)$$

Here, β is the market beta, α is the alpha and n is the number of observations. Note that we use r (small r) to describe excess returns. A beta of 0.5 means that if market goes up by 10%, the portfolio goes up by $0,5 * 10\% = 5\%$, everything else being equal. The idiosyncratic risk, ε_t, is the risk exposure uncorrelated with the overall market risk with a mean of zero. For this regression, the alpha and market beta are the same as values applied in the CAPM.

Many hedge funds aim to be market neutral, meaning $\beta = 0$. In this case the return of the strategy is independent of the market return and the fund has an equally good chance to make positive returns in both bull and bear markets. The alpha measures the returns beyond the market return. Clearly a high alpha is desirable. The CAPM predicts that all returns can be captured by the market beta and hence an alpha of zero. A positive and significant alpha therefore implies a rejection of the CAPM.

By regressing the excess return of a strategy on a multi-factor model like the Fama-French three- and five-factor model or the four-factor model of Carhart, we can check if the alpha not captured by the market beta is captured by other factor returns. For example, if we regress on the three-factor model of Fama and French, the return would be captured by the following equation

$$r_t^e = \alpha + \beta^M r_t^{M,e} + \beta^{HML} r_t^{HML} + \beta^{SMB} r_t^{SMB} + \varepsilon_t \qquad (43)$$

Where r_t^{HML} and r_t^{SMB} is the return of the HML and SMB strategies and β^{HML} and β^{SMB} measures the tilt towards the strategies. In this case, a positive alpha is a return not described by the market return, size factor or the value factor. Expanding the regression to our own eight-factor model, the following equation describes the return of our models

$$\begin{aligned} r_t^e = \alpha + \beta^M r_t^{M,e} + \beta^{HML} r_t^{HML} + \beta^{SMB} r_t^{SMB} + \beta^{WML} r_t^{WML} + \beta^{RMW} r_t^{RMW} \\ + \beta^{CMA} r_t^{CMA} + \beta^{QMJ} r_t^{QMJ} + \beta^{BAB} r_t^{BAB} + \varepsilon_t \end{aligned} \qquad (44)$$

A positive alpha here would indicate the portfolio creates a return not describes by any of the seven factor returns, nor the market. To test if the alpha is just a result of luck, we look at the significance of the variables. By dividing the alpha value on the standard deviation (equation (1) and (2)), we get the t-statistic of the alpha. The higher the t-static is, the surer can we be the alpha is different from zero. A t-statistic of 1.96 means we are 95% sure the alpha is different from zero, which we consider to be statistically significant.

3.6.2 R-squared

R-squared measures the relationship between a portfolio and its benchmark. In our case, R-squared tells us how good the different multi-factor models are able to explain the returns of our portfolio. Technically, the R-squared indicates the variance of the dependent variable that is predicted able from the chosen multi-factor model. An R-squared value of 0 indicates that the model does not explain any of the variability of the portfolio around its mean, while a value of 1 would indicate the model explains all the variability of the portfolio around its mean. The general definition of R-squared is given by

$$R^2 = 1 - \frac{SSR}{SST} \tag{45}$$

Where SSR is the sum of squared residuals by equation (42) and SST is the total sum of squares given by:

$$SST = \sum_{t=1}^{n} \left(r_t - E(r_t)\right)^2 \tag{46}$$

This is proportional to the variance.

3.6.3 Sharpe Ratio

While the alpha expresses the size of a return excess of the regressed model, the alpha does not say to what risk. For instance, the alpha can be increased by scaling up the strategy. Increasing the positions naturally increases the risk too, but this is not captured by the alpha measure. To deal with this issue the return have to be measured by a risk-reward ratio. The Sharpe Ratio (SR) such a ratio which measures the risk-adjusted return:

$$SR = \frac{E(R - R^f)}{\sigma(R - R^f)} \tag{47}$$

Where $\sigma(R - R^f)$ is the standard deviation of the excess return. A high Sharpe ratio is desirable as this indicates a high return compared to underlying risk. For a mean-variance orientated investor which we consider in this thesis, the goal is always to achieve the highest Sharpe ratio possible. Without any limitations to leverage, the position that gives the highest Sharpe ratio can be levered up or down to achieved the desired return or standard deviation. Throughout the thesis, we will always consider a yearly Sharpe ratio.

3.6.4 Cumulative Return and High Water Mark

To visualize the return of a portfolio, the cumulative return is drawn. The cumulative return shows the return of a buy and hold strategy at time t if the investor invested 1 unit of capital at t_0. This might seem a little strange as the portfolio is self-financing and no initial capital is required for the investment. To make it more intuitive, think of the investor placing 1 unit of capital in the long position, 1 unit of capital in the short position and also 1 unit of capital in risk-free assets. This way the investor makes the spread of the long-short position and the risk-free rate.

Together with cumulative return, we also include the high water mark (HWM) which shows the highest cumulative return achieved in the past.

The cumulative return is an exponential function where each tick mark on the graph is the new portfolio value multiplied by the previous tick mark. This means the function is accelerating and the graph is becoming steeper as time passes. A graph like this is not very informative as the latest returns have bigger effects than the earlier returns. Because of this, we plot the cumulative returns and HWM in a logarithmic scale with 10 as base value. This means the scale is given by an exponential function of 10. The first step is then going from 0 to 10 while the next step is going from 10 to 100 for the same distance. This turns the cumulative returns to look like a linear function.

3.6.5 Drawdown

When the current cumulative return is lower than the high water mark, the drawdown (DD) gives us the cumulative loss since the loss started. This is an important risk measure for a portfolio and a good way to see in which periods the losses are most severe. The drawdown is given by

$$DD_t = (HWM_t - P_t)/HWM_t \tag{48}$$

Where P_t is the cumulative return at time t. If the cumulative return is at its peak, meaning $P = HWM$, the drawdown is 0.

3.7 Chi-Squared Test

Inspired by Zhang et al. (2009), we test whether or not the factors are independent from the state variables. This is done by using a chi-squared independence test. Following the methodology of the paper, we divide the all the variables in a time series in three groups: one group of the top 25%, one for the middle 50% and one for the bottom 25%.

Comparing the three groups for one state variable and one factor, we build a 3*3 coincidence matrix, which sums all the different combination the three groups of the two variables occur together during the sample period. We also build a contingency table, which shows the expected distribution of the coincidence matrix by the assumption of the returns being independent and normally distributed:

	Top 25%	Mid 50%	Bottom 25%	Sum
Top 25%	6.25 %	12.50 %	6.25 %	25.00 %
Mid 50%	12.50 %	25.00 %	12.50 %	50.00 %
Bottom 25%	6.25 %	12.50 %	6.25 %	25.00 %
Sum	25.00 %	50.00 %	25.00 %	100.00 %

Table 1: expected distributions of the coincidence matrix. Source: own creation

As an example, the top 25% results of the datasets are expected to appear the same time in 6.25% of the cases (25%*25%).

The chi-squared test measures the sum of squared errors between the expected distribution and the realized distribution. The expected distribution is a result of the assumption of independent and normally distributed data, which according to the central limit theorem is in most cases true. With this, the null hypothesis of the two datasets being independent is either rejected or accepted. By rejecting the null hypothesis, the alternative hypothesis of a statistical relationship between the datasets is accepted.

The following equation gives us the chi-squared value:

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \tag{49}$$

Where O_{ij} is the observed value at the *i*-th row, *j*-th column, and E_{ij} is the expected value of the same observation.

The significance of the chi-squared value is dependent on the degrees of freedom, which is given by:

$$(number\ of\ rows - 1)(number\ of\ columns - 1) \tag{50}$$

This means that in our 3*3 matrix, the degrees of freedom is then 4 with the following chi-squared value needed to accept the null hypothesis for a given confidence interval:

Confidence level	100.0 %	99.9 %	99.5 %	99.0 %	97.5 %	95.0 %	90.0 %	85.0 %	80.0 %
Chi-squared value	0.064	0.091	0.207	0.297	0.484	0.711	1.064	1.366	1.649

Table 2: chi-squared values and corresponding confidence intervals. Source: https://faculty.elgin.edu/dkernler/statistics/ch09/9-3.html (visited 24.05.2017), independent analysis

Zhang et al. use a 90% confidence interval, meaning a chi-squared value of 1,064 or lower is needed to accept the null hypothesis of a independent distribution.

4 In-Sample Analysis

In this section, we start off the research and we will perform an in-sample analysis to look for answers to our research questions based on the U.S. market. In the in-sample analysis, we wish to improve a single-factor portfolio by expanding to a multifactor portfolio, testing various parameters and variables. After finishing the in-sample analysis and conclude the findings, we will implement the model with the most promising parameters on the European and Japanese markets and conduct an out-of-sample test in chapter 5 to see if the in-sample specifications yield the same results on other developed markets. The analysis part of our in-sample test will be divided into the following subparts:

4.1. We analyses and evaluate the well-researched and documented factors as individual investment opportunities for the entire sample period. Implementing performance measurements, an environment is created where we are later able to evaluate the various models that we will introduce in the coming parts and combined factor portfolio strategies against the individual factors.

4.2. We start constructing a multi-factor portfolio of the different factors. Our starting point is an equally weighted portfolio consisting of the seven individual factors. We extend the analysis of chapter 4.1 to also include the new portfolio. Constructing an equally weighted portfolio, we look for evidence for a positive diversification effect that is easily available to investors.

4.3. We implement a dynamic approach to the mean-variance portfolio optimization of Markowitz with different rolling windows and use it to optimize the factor allocation in the multi-factor portfolio, in order to achieve a better risk-return profile than we achieved in the previous parts.

4.4. We focus on the performance of the factors in the different phases of the economic cycle. We do not wish to use any analytical method to determine when the different cycles peak, end or begin. Rather, we take the officially published dates from the National Bureau of Economic Research and analyses the behavior of the factors and the state variables.

4.5. We implement the macroeconomic state variables to the optimization problem of 4.3 and aim to achieve a better timing of the factors. In this part, we are comparing our results with the results of 4.2 and 4.3 looking for evidence that supports the market timing ability of our state variable mean-variance model.

4.6. We conclude our results and findings for the in-sample analysis.

4.1 Summary Statistics of Individual Factors

Going from January 1973 to January 2017, our U.S. sample consists of 529 periods of observation. Note that as we build our model, which is dependent on various rolling windows, some of the early months will only be observed and no portfolio weights will be placed.

4.1.1 Returns

The following table shows the performance of the different factors for our entire sample period.

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Ann .Avg. Ret.	6.36 %	2.77 %	4.67 %	3.16 %	4.20 %	7.84 %	4.55 %	10.48 %
Ann. SD	15.82 %	10.53 %	10.18 %	8.16 %	6.83 %	15.37 %	8.61 %	11.72 %
Ann. SR	0.40	0.26	0.46	0.39	0.62	0.51	0.53	0.89
Median	0.88 %	0.10 %	0.30 %	0.26 %	0.22 %	0.74 %	0.33 %	1.10 %
Max	16.10 %	18.73 %	12.91 %	13.52 %	9.55 %	18.38 %	12.70 %	13.06 %
Min.	-23.24 %	-15.28 %	-11.25 %	-19.11 %	-6.88 %	-34.58 %	-10.27 %	-15.13 %
Max DD	50.39 %	31.24 %	37.07 %	37.47 %	12.76 %	57.53 %	28.80 %	47.03 %
# Positive	309	277	284	292	292	332	304	352
# Negative	220	247	244	237	237	197	225	177
Avg. Positive	3.49 %	2.34 %	2.37 %	1.66 %	1.68 %	2.92 %	1.89 %	2.57 %
Avg. Negative	-3.62 %	-2.13 %	-1.91 %	-1.45 %	-1.29 %	-3.16 %	-1.67 %	-2.50 %
Skewness	-0.54	0.38	0.04	-0.38	0.36	-1.39	0.18	-0.66
Kurtosis	2.02	4.24	2.14	12.97	1.87	10.56	3.08	3.40

Table 3: performance of the individual factors. Avg. Ret., SD and SR are annual numbers, the rest is monthly. Source: own creation

The conditional color formatting is applied to easier spot the different values. A solid green indicates the highest value, while the red indicates the lowest value. The scale from green to red indicates everything in between: a light green to yellow indicates the value belongs to the upper half of the values and an orange color means the value are among the lowest. We will continue to use the color formatting throughout the thesis.

As we can see, the factors are performing very differently from one another. Not surprisingly, the SMB factor performs the worst of its peers as the value premium has been reported to slowly disappear over the last decades (Fama and French, 2012). One might argue for removing the factor at this point in our research, especially since the sample of Japan and Europe only goes back to the early 90's, but because the factor is known to appear in cycles, we still think it should be included. In addition, due to the properties of the mean-variance approach, a recent low average return will only result in small weights or shorting of the factor and should not reduce the performance of the portfolio. It might also be correlated with other factors, which can make it relevant for diversification purposes. The table reports the performance of the factors for the whole dataset used, which does not necessarily mean that there are no shorter or longer periods in the sample when the SMB factor is superior to its peers.

We see that BAB and WML have the highest average monthly returns, but BAB and CMA have the highest Sharpe ratios, with values of 0.89 and 0.62. Without considering the correlations, the mean-variance approach favor the assets with the highest risk-adjusted return, thus we expect BAB and CMA to have the highest average weight in the portfolio. The individual factor performance together with the correlation will be important to further understand how the models behave and allocate weights.

4.1.2 Correlation Matrix

To see how the different factors move together, we present the correlation matrix:

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Mkt-RF	1.00							
SMB	0.24	1.00						
HML	-0.28	-0.07	1.00					
RMW	-0.25	-0.38	0.15	1.00				
CMA	-0.39	-0.04	0.69	0.05	1.00			
WML	-0.14	-0.03	-0.18	0.09	0.03	1.00		
QMJ	-0.51	-0.49	-0.01	0.77	0.08	0.25	1.00	
BAB	-0.11	-0.00	0.33	0.33	0.30	0.20	0.20	1.00

Table 4: correlation matrix of the factors and the market return. Source: own creation

Just as the past returns play an important role in the mean-variance framework, the correlation between factors is also important when deciding the weights. Analyzing the correlation matrix for the entire sample period, we see some factor being highly positively and negatively correlated and other factors being almost completely uncorrelated. Two negatively correlated factors implies that when one factor yields a negative return for a given month, the other factor is supposed to yield on average a positive return. These factors should benefit the portfolio in a diversification purpose.

Some of the factors have quite high negative correlation, like QMJ and SMB, and QMJ and the market return. A negative market correlation suggests the factor returns should move the opposite direction of the market return. As we can see, momentum (WML) and value (HML) are negatively correlated with a correlation of -0.18 which has been documented by several papers (see 2.3.4). In the literature review, we mentioned that Novy-Marx (2012) says the profitability premium and the value premium both are a result of growth strategies, but are still uncorrelated. We find this to be true also for our sample, with a correlation of 0.08, which is close to being uncorrelated. As the two strategies are said to be a good hedge for each other, we might expect the allocation to the two factors to increase and decrease at the same time. Overall, we find the correlations to be low and positive on average.

4.1.3 Multi-Factor Model Regressions

Next, we investigate whether individual factors add any value in a multi-factor model by regressing the factor returns on the market return (CAPM), the three- and five-factor model of Fama and French (FF3 and FF5), and the four-factor model of Carhart (C4):

	CAPM					FF3			C4			FF5		
	Alpha	t-stat	R^2	Beta	t-stat	Alpha	t-stat	R^2	Alpha	t-stat	R^2	Alpha	t-stat	R^2
SMB	0.15 %	1.13	0.06	0.16	5.69									
HML	0.48 %	3.92	0.08	-0.18	-6.69									
RMW	0.33 %	3.33	0.06	-0.13	-6.00	0.34 %	3.54	0.18	0.30 %	3.08	0.19			
CMA	0.44 %	5.55	0.16	-0.17	-9.85	0.23 %	3.79	0.52	0.19 %	3.01	0.53			
WML	0.73 %	3.79	0.02	-0.14	-3.36	0.90 %	4.74	0.07				0.71 %	3.67	0.11
QMJ	0.53 %	5.62	0.26	-0.28	-13.59	0.64 %	7.66	0.43	0.57 %	6.73	0.45	0.38 %	7.02	0.78
BAB	0.92 %	6.22	0.01	-0.08	-2.53	0.73 %	5.13	0.11	0.54 %	3.85	0.18	0.48 %	3.48	0.23

Table 5: results of the monthly factor returns regressed on different factor models. Source: own creation

We see that except from SMB, all factors produce a significant alpha when regressed on the different multi-factor models and that WML and BAB are producing the highest alpha. Not surprisingly, the alpha decreases most of the times when more factors are added, but it remains significant in all cases except for SMB. This can be understood as the newly added factor(s) to the model is able to capture a part of the return that none of the initial factors were able to do. The significant alphas implies that our extended seven-factor portfolio might be able to produce returns not captured by the other multi-factor models.

The R-squared values, which indicates how much the given multi-factor regression model is able to capture of the individual factor returns, are increasing as we introduce more factors to the models. This supports our theory that the multi-factor models' ability to explain factor returns is increasing as we introduce more factors. The highest R-squared value is achieved by the Fama-French five factor model and QMJ, with a value of 0.78. This indicates most of the QMJ return can be captured by the factors included in the model. Still, we see a high and significant alpha.

With all factors consisting of a long-short portfolio with a random distribution of market betas among the firms in each position, we expect the factor returns to be close to market neutral. We see that most of the factors have market beta values close to zero which confirm our expectations. Except SMB, all factors have a negative market beta. This come a no surprise as it is similar to the correlations of the factors and the market return. Especially QMJ has a highly negative and significant market beta with a value of -0.28. This can maybe be explained by the flight to quality phenomenon during recessions as described in the literature review. Still, with a monthly average excess return in the market of 0.53%, a beta of –0.28 has little effect when compared to QMJ's alpha of 0.53%. The fact that the BAB factor has the market beta closest to zero is also as expected, considering the strategy creates a portfolio with a historical market beta of 0.

SMB’s significant market beta and insignificant alpha suggest that the factor doesn’t produce a return that cannot be explained by the CAPM. With the only factor having a positive market beta, the SMB factor should be included in a portfolio to compensate for all the negative market betas when building a market neutral portfolio.

4.2 Equally Weighted Portfolio

4.2.1 Portfolio Performance

In order to examine whether it is reasonable to build a portfolio of different factors, an equally weighted portfolio (EWP) is first formed with all the seven factors mentioned earlier, allocating 14.29% weight to each factor. Note, that the market factor is left out from the equally weighted portfolio. This is done for two reasons: for the first, we are looking for a strategy that beats the market and not follows it, and secondly, all the factors are considered market neutral, which is a desired feature by investors. By building a portfolio that consist of only market neutral elements, we are able to keep this beneficial effect.

By creating a portfolio of different factors, our main goal is to reap the benefits of the “only free dinner in finance”, the diversification effect. In the previous chapter, we concluded that in general, factors are positively correlated to each other, but the correlation is still relatively small which indicates that diversification benefits exist and should be able to be exploited.

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB	EWP
Ann. Avg. Ret	6.36 %	2.77 %	4.67 %	3.16 %	4.20 %	7.84 %	4.55 %	10.48 %	5.38 %
Ann. SD	15.82 %	10.53 %	10.18 %	8.16 %	6.83 %	15.37 %	8.61 %	11.72 %	4.89 %
Ann. SR	0.40	0.26	0.46	0.39	0.62	0.51	0.53	0.89	1.10
Median	0.88 %	0.10 %	0.30 %	0.26 %	0.22 %	0.74 %	0.33 %	1.10 %	0.41 %
Max	16.10 %	18.73 %	12.91 %	13.52 %	9.55 %	18.38 %	12.70 %	13.06 %	9.73 %
Min.	-23.24 %	-15.28 %	-11.25 %	-19.11 %	-6.88 %	-34.58 %	-10.27 %	-15.13 %	-7.96 %
Max DD	50.39 %	31.24 %	37.07 %	37.47 %	12.76 %	57.53 %	28.80 %	47.03 %	11.42 %
# Positive	309	277	284	292	292	332	304	352	375
# Negative	220	247	244	237	237	197	225	177	154
Avg. Positive	3.49 %	2.34 %	2.37 %	1.66 %	1.68 %	2.92 %	1.89 %	2.57 %	1.01 %
Avg. Negative	-3.62 %	-2.13 %	-1.91 %	-1.45 %	-1.29 %	-3.16 %	-1.67 %	-2.50 %	-0.92 %
Skewness	-0.54	0.38	0.04	-0.38	0.36	-1.39	0.18	-0.66	0.21
Kurtosis	2.02	4.24	2.14	12.97	1.87	10.56	3.08	3.40	9.35

Table 6: summary statistics of the individual factors and the equally weighted portfolio. Source: own creation

The equally weighted portfolio achieves a yearly average return of 5.38% with a lower observed standard deviation than any of the individual factors on their own. This results in a Sharpe ratio of 1.10, which is a 23% improvement compared to the Sharpe ratio of the best performing individual factor (BAB). Already at this point the benefit of diversification is evident as we now have a portfolio with the average return of all the factors, but a standard deviation that is lower than any of the factors. Compared to the market, our equally weighted portfolio achieves lower returns, but when adjusting for risk, the EWP portfolio provides the highest Sharpe ratio.

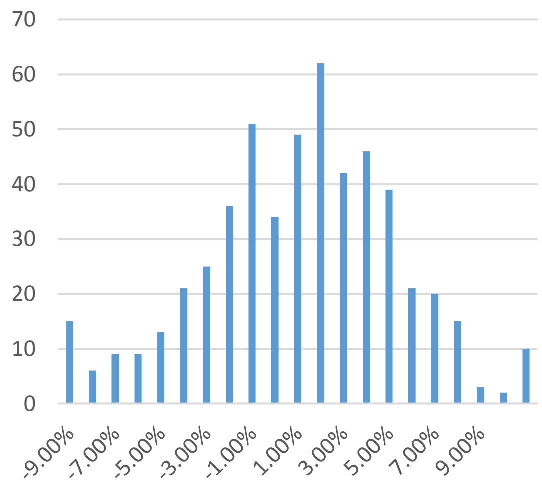

Figure 3: histogram of the excess market return. Source: own creation

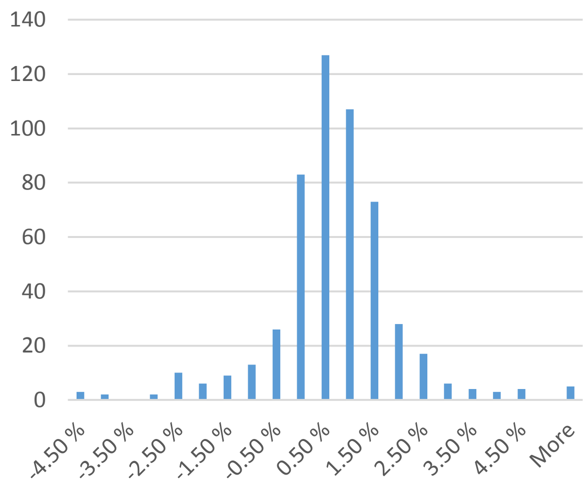

Figure 4: histogram of the excess return of the EWP. Source: own creation

In the graphs above, we can see the histograms of the market portfolio and the EWP return. Note that the step sizes are different. In the sample, market returns are skewed to the left, with a negative value of -0.54 while the EWP skewed to the right with a value of 0.21. Kurtosis is also notably different in the two examined portfolios. The EWP has an extremely high kurtosis value of 9.35 which is known as a leptokurtic distribution. A leptokurtic distribution is associated with high peaks and corresponding fat tails, meaning the distribution is more clustered around the mean. In this case we see that the distribution around the mean leads to a lower standard deviation, even though there is no direct relationship between kurtosis and standard deviation.

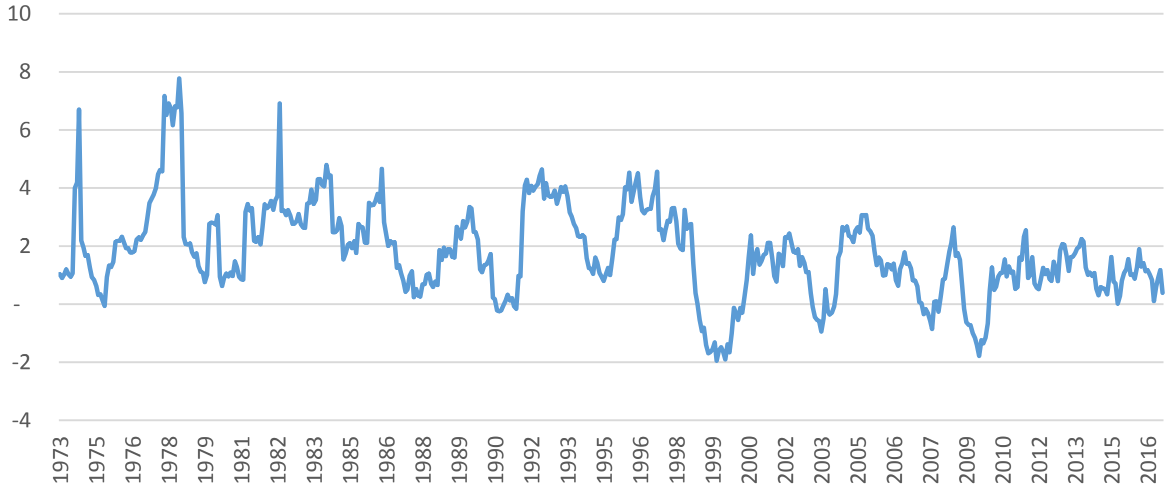

Figure 5: one-year rolling Sharpe ratio of the EWP. Source: own creation

Running a one-year rolling Sharpe ratio on the equally weighted portfolio, we see that the Sharpe ratio in the earliest years of the data set moves in a higher range compared to the two last decades. We can observe periods when the portfolio performs extremely well, especially the period between 1973 and 1997, where there is only three short periods observable where the Sharpe ratio was negative.

It is worth noting that we cannot see as high Sharpe ratios in the past decades as we do in the seventies and eighties. This can also give evidence to what the market practitioners report: as the different factor strategies get "crowded" over time, their positive effect might slowly evaporate (AQR, 2015).[21]

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[21] *AQR. (2015, August 31) How Can a Strategy Still Work If Everyone Knows About It? Retrieved from the AQR website: https://www.aqr.com/cliffs-perspective/how-can-a-strategy-still-work-if-everyone-knows-about-it*

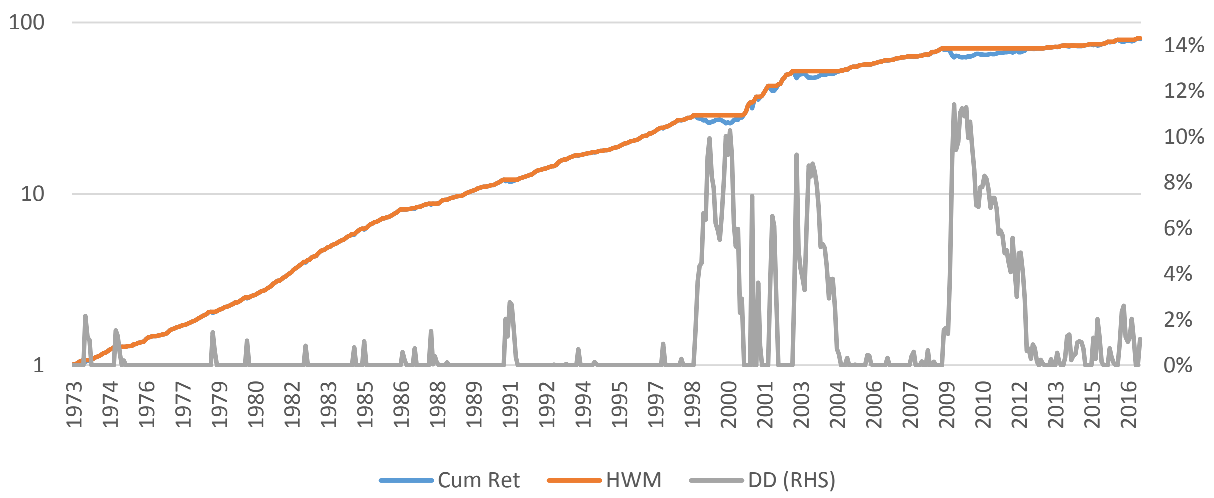

Figure 6: cumulative returns with HWM and DD for the equally weighted portfolio, logarithmic scale for LHS. Source: own creation

Analyzing the cumulative return and drawdowns of our equally weighted portfolio, we see it achieves steady returns for the first 25 years without any major drawdowns, but during the next 20 years the drawdowns reach 10% at four points in time, distributed at two drawdown periods which are located in the two last recessions of the American economy (see 4.4.1). The recovery period is also very long for all of the three biggest drawdowns, and the portfolio uses from two to three and a half year to get back to the high watermark. By expanding our model to time the factor weights, we are hoping to identify the recessions and reduce the drawdowns in these periods. Ideally the drawdowns will be equally distributed along the sample period. A reduction in drawdowns during recessions will imply that the model is less prone to be effected by market movements. Still, a maximum drawdown of 11% must be considered favorable compared to the S&P 500 which suffered a drawdown of 56.8% between October 2007 and March 2009.[22]

[22] Wikipedia (2017, May 11) United States bear market of 2007–09. Retrieved from Wikipedia: https://en.wikipedia.org/wiki/United_States_bear_market_of_2007%E2%80%9309

4.2.2 Multi-Factor Model Regressions

	CAPM			FF3			C4			FF5		
	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2
Alpha	0.51 %	8.92	0.14	0.41 %	7.79	0.32	0.23 %	6.19	0.68	0.22 %	5.65	0.63
Mkt-RF	-0.12	-9.39		-0.09	-7.79		-0.05	-6.21		-0.04	-4.04	
SMB				0.07	4.04		0.07	5.79		0.15	11.00	
HML				0.20	10.83		0.27	20.92		0.05	2.89	
RMW										0.35	19.50	
CMA										0.29	10.31	
WML							0.20	24.20				

Table 7: excess monthly returns of the equally weighted portfolio regressed on the different multifactor models. Source: own creation

When regressing the monthly portfolio returns on the market return, we see that we now have created a portfolio with an alpha of 0.51%, which is lower than alpha of the WML, BAB and QMJ factor returns. The market beta of the equally weighted portfolio is the average of all the market betas, making it the beta second closest to zero after BAB.

As we add more factors to the regression, we see that the alpha decreases, but is still significant for all the multifactor models. This implies the equally weighted portfolio is able to create a return that the other multifactor models are able to explain entirely. The R-squared values also show this.

We left out the eight-factor regression as the portfolio has a beta equal the weight for each of the individual factors, 0.1429 an alpha of 0 and an R-squared value of 1. Our goal is to create a portfolio which returns a significant and positive alpha when regressed on the eight-factor regression model. If we achieve this, it would imply that the timing of the factors creates a return on top of the factor returns themselves. The equally weighted portfolio will serve as a benchmark portfolio for further analysis of our other portfolios.

4.3 Mean-Variance Model (MVM)

4.3.1 Parameter Selection

In the previous chapter, we showed that a very simple equally weighted portfolio is superior to any of the individual factors. By implementing the mean-variance framework to our portfolio formation as explained in the methodology, our goal is to exploit the more recent correlations and the current performance of the individual factors to improve the model further.

Rolling	Lag ret.	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t
Window	Lambda	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	AVG	4.72 %	5.79 %	5.96 %	6.02 %	6.07 %	6.10 %	6.13 %	6.14 %	6.15 %	6.15 %	6.12 %
1Y	SD	4.64 %	3.92 %	3.94 %	4.00 %	4.06 %	4.13 %	4.19 %	4.26 %	4.34 %	4.44 %	4.57 %
	SR	1.02	1.48	1.51	1.51	1.49	1.48	1.46	1.44	1.42	1.38	1.34
	AVG	5.38 %	5.81 %	5.91 %	5.93 %	5.92 %	5.89 %	5.84 %	5.77 %	5.68 %	5.56 %	5.41 %
2Y	SD	5.04 %	4.85 %	4.84 %	4.86 %	4.91 %	4.97 %	5.05 %	5.15 %	5.27 %	5.42 %	5.63 %
	SR	1.07	1.20	1.22	1.22	1.21	1.18	1.16	1.12	1.08	1.03	0.96
	AVG	6.70 %	6.83 %	6.89 %	6.91 %	6.89 %	6.87 %	6.82 %	6.76 %	6.68 %	6.58 %	6.45 %
3Y	SD	5.57 %	5.51 %	5.55 %	5.61 %	5.69 %	5.78 %	5.88 %	6.00 %	6.15 %	6.33 %	6.58 %
	SR	1.20	1.24	1.24	1.23	1.21	1.19	1.16	1.13	1.09	1.04	0.98
	AVG	6.32 %	6.61 %	6.68 %	6.70 %	6.69 %	6.66 %	6.61 %	6.54 %	6.46 %	6.34 %	6.19 %
4Y	SD	5.58 %	5.59 %	5.62 %	5.66 %	5.73 %	5.81 %	5.91 %	6.03 %	6.19 %	6.40 %	6.68 %
	SR	1.13	1.18	1.19	1.18	1.17	1.15	1.12	1.08	1.04	0.99	0.93
	AVG	5.62 %	6.06 %	6.16 %	6.19 %	6.18 %	6.15 %	6.11 %	6.05 %	5.98 %	5.90 %	5.80 %
5Y	SD	5.99 %	6.10 %	6.20 %	6.30 %	6.40 %	6.50 %	6.62 %	6.75 %	6.91 %	7.11 %	7.37 %
	SR	0.94	0.99	0.99	0.98	0.97	0.95	0.92	0.90	0.87	0.83	0.79

Table 8: annual returns of the different MVM portfolios for the U.S. Source: own creation

To construct the portfolios in the table above, we implemented the methods covered in details in parts 0 (Mean-Variance Framework), 3.3 (Shrinking of Covariance Matrix) and 3.5 (Model Creation). The target return is set to 10% and will continue to be so throughout this thesis, but our results are in fact indifferent of the target return. When the target return is changed, the positions are levered up or down, keeping the same relative weights. The Sharpe ratio would be the same for all target returns while the alpha would increase or decrease with by the proportion we adjust the target return with. This can be explained by that the fact that the returns are linearly related to the change in risk given by the properties of the mean-variance model. More on this in chapter 4.3.5.

In the mean-variance model, we assume that at the end of period t, we can observe the factor returns for the period and based on this, allocate the weights according to the model for period $t + 1$, resulting in a zero period lag. The model is then tested for all possible combinations of rolling windows and shrinkage. A change in rolling window will change the size of the data sample for each optimization process, while a change in lambda calibrates the covariance matrix for a given sample size. For each row, the rolling window increase by one year, while for each column the lambda increases by 0.1, going from 0 to 1. By running all the possible combinations, we are looking for the portfolio with the highest risk-adjusted return. This is achieved with a rolling window of one year and a lambda of 0.2. With a Sharpe ratio of 1.51, it is a 37% increase compared to the equally weighted portfolio with a Sharpe ratio of 1.10.

We see that the most significant improvements of introducing shrinkage occur for the models with the shortest rolling window, especially the one- and two-year rolling windows. For instance, we see an increase of 0.49 in the Sharpe ratio by going from lambda 0 to 0.2 for the one-year rolling window. This is according to theory of chapter 3.3, as the estimation errors should be higher for the models with a smaller sample size used for the covariance matrix calculation.

The outperformance of the one-year rolling window is also not surprising because a smaller rolling window means that the recent returns have a higher impact on the factor allocation. These recent returns should be a better estimation for the next period's returns, than the average of a longer sample.

Even though the main focus of our thesis is to find the best possible strategy among the available opportunities, it is worth noting that all 50 models presented in Table 8 produce positive return and a large portion of them turned out to achieve a higher risk-adjusted return than the EWP constructed in the previous section. This clearly implies that an actively managed and rebalanced portfolio utilizing the Markowitz framework could be used in the past to beat a passive equal weighted factor strategy.

We will continue to analyze the one-year rolling window model with a lambda of 0.2, as this is what a mean-variance investor would be most satisfied with considering the highest achieved Sharpe ratio among the tested models. From now on, this is the model referred to when we talk about the mean-variance model or the MVM of the U.S.

4.3.2 Portfolio Performance

	EWP	MVM
Ann. Avg. Ret	5.41%	5.96%
Ann. SD	4.91%	3.94%
Ann. SR	1.10	1.51
Median	0.42%	0.49%
Max	9.73%	4.56%
Min.	-7.96%	-4.08%
Max DD	11.42%	5.42%
# Positive	365	363
# Negative	152	154
Avg. Positive	1.02%	1.04%
Avg. Negative	-0.91%	-0.79%
Skewness	0.23	0.08
Kurtosis	9.38	1.24

Table 9: summary statistics of the EWP and MVM. Source: own creation.[23]

Just by observing the Sharpe ratios in Table 9, we can see that the new model is superior to the EWP. The selected MVM portfolio achieves a 0.55% (percent points, pp) higher average annual return and a 0.97% (pp) lower annual standard deviation compared to the EWP which results in an increase of 0.41 in Sharpe ratio. We can see a further decline in the extreme values which points toward a more stable long-term return. While the number of months of positive return slightly decreases, the average loss on losing days also decreases by 0.12% (pp). Note that the kurtosis is in fact the excess kurtosis.

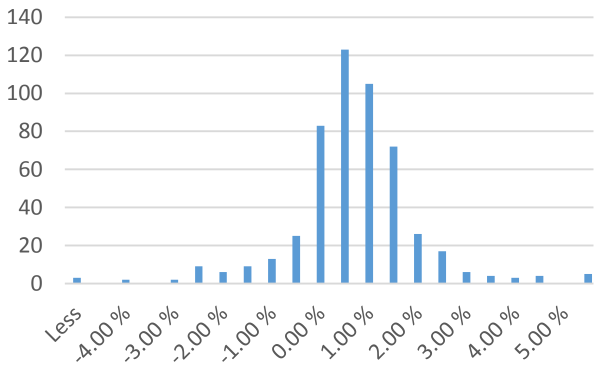

Figure 7: histogram of the excess returns of the EWP. Source: own creation

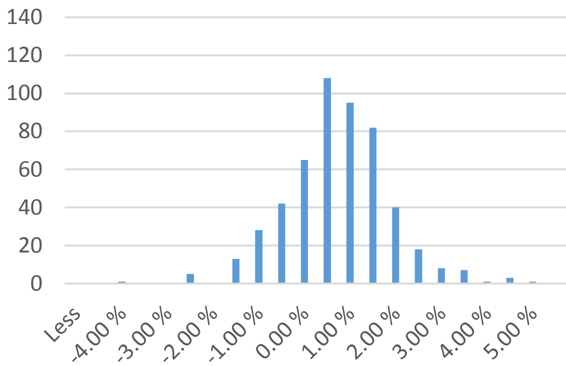

Figure 8: histogram of the excess returns of the MVM. Source: own creation

We already introduced the histogram of the EWP in the previous chapter, but instead of comparing it to the market, we compare it to the MVM. We can see that with the MVM, we are able to eliminate the heavy tails, but the distribution is not as centered around the mean as the EWP. This is also reflected in the low kurtosis of the MVM compared to the EWP (1.24 vs 9.38). But because of the big outliers of the EWP, the MVM achieves a lower standard deviation.

[23]Note that the performance measures values of the EWP changed slightly compared to the one reported in the previous Chapter. The reason behind is that due to the 1 year rolling window period of MVM. We adjusted the performance period of the EWP by cutting the observations of the first year.

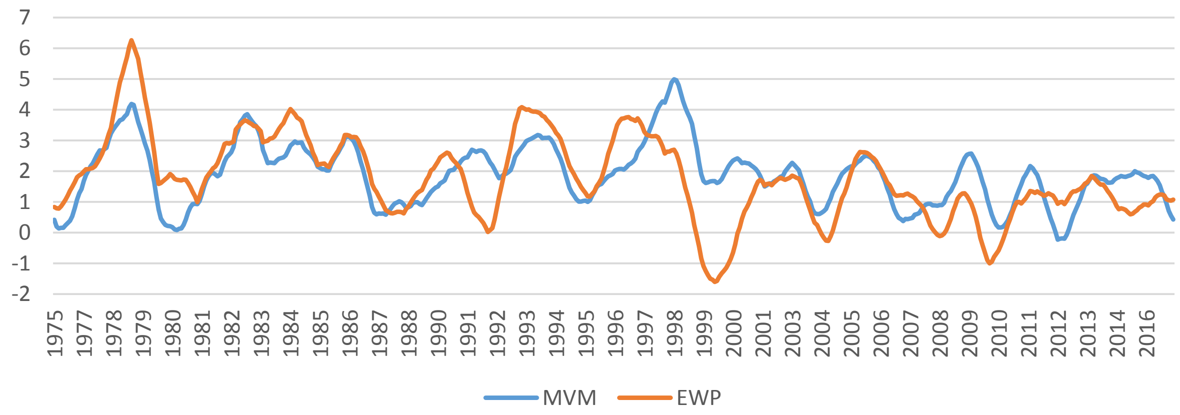

Figure 9: one-year average of the one-year rolling Sharpe ratio of the EWP and MVM. Source: own creation

Figure 9 shows the one-year average of the one-year rolling Sharpe ratio of the EWP and MVM. For the EWP, we see three notable periods where the Sharpe ratio drops below the local average: the Indian economic crisis in 1991-1992, the various global economic crisis in 1998-2001 and the sub-prime crisis in 2007-2008. During the period of the Indian crisis, the MVM produces a positive Sharpe ratio in the territory of 2-4 while the EWP fall to 0. We can observe similar results in 1991-1992 where the EWP produces risk-adjusted returns close to zero, while the rolling Sharpe ratio of the MVM remains around 2 during the period. During the period of the latest financial crisis, we also see a favorable rolling Sharpe ratio for the MVM. Even though it also reaches a negative value, the MVM remains in the positive territory significantly longer and also recovers faster.

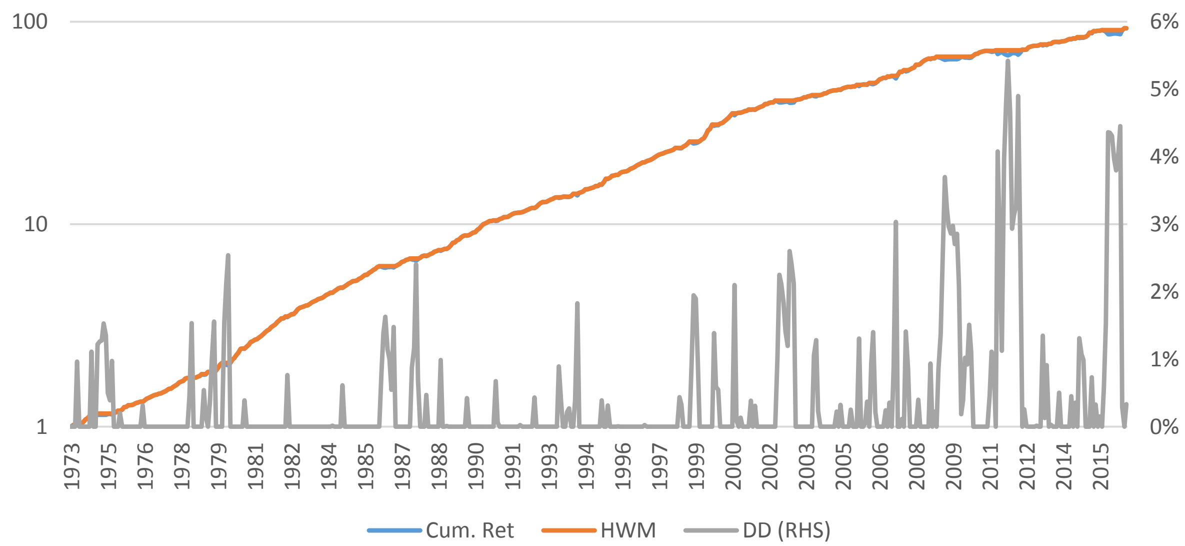

Figure 10: cumulative return with HWM and DD of the MVM, logarithmic scale for LHS. Source: own creation

Comparing the equally weighted portfolio and the mean-variance portfolio we see the MVM has a maximum drawdown that is less than half of the equally weighted portfolio's, as we saw in Figure 6. The equally weighted portfolio experiences all of the major drawdowns around the two latest recessions, while the major drawdowns of the MVM are allocated outside of recessions. We also see a more even distribution of drawdowns along the entire sample period for the MVM. This implies the portfolio is less subject to changes in the market. Still, we see that the major drawdowns are occurring in the last decade, which means large drawdowns are also likely to occur in the near future. A major improvement is that all drawdowns recover within a year, compared to three and a half year for the equally weighted portfolio.

So far, we see the MVM does a good job in creating a market neutral portfolio and achieves a higher risk-adjusted return. By including the state variables, we are hoping to reduce the latest drawdowns and make the allocation more evenly distributed across the period. Next, we will investigate the allocation among the individual factors to see how the mean-variance portfolio differs from the equally weighted portfolio.

4.3.3 Weights

	SMB	HML	RMW	CMA	WML	QMJ	BAB	Sum
Avg.	0.05	0.02	0.09	0.07	0.02	0.13	0.17	0.55
Norm. Avg	0.09	0.03	0.16	0.13	0.04	0.24	0.31	1.00
SD	0.14	0.25	0.26	0.31	0.14	0.19	0.20	
Max	0.58	1.50	0.98	1.36	0.54	0.81	1.06	
Min.	-0.43	-1.07	-0.84	-0.74	-0.45	-0.39	-0.26	
Abs. Avg.	0.11	0.16	0.20	0.24	0.10	0.18	0.20	1.20
Abs. Norm. Avg	0.09	0.13	0.17	0.20	0.09	0.15	0.17	1.00

Table 10: factor weights of the MVM. Source: own creation

Because of the possibilities to go both long and short, analyzing the average weights gains us little insight; if an investor short a portfolio with the same amount as he goes long, the average weights will be 0 even though he has been allocating weights to the portfolio. Instead, we use the absolute weights (abs.) to see the total average of both the long and short positions made.

For all the factors, we see a positive average weight, meaning the portfolio on average goes more long than short in all of the factors. From the equally weighted portfolio we remember the constant weight was 0.143 for each factor, which sum up to 1. To make the portfolios more comparable, we have normalized the weights (Norm. Avg.) to also sum up to 1. By comparing the portfolios, we see an increase in average weights of BAB, QMJ and RMW. The increase in BAB and QMJ can be explained by the fact that they are among the top three performing factors, but the high allocation to RMW is less obvious. One explanation can be the low correlation with CMA which adds a diversification benefit to the portfolio, as the two factors have almost the same average weight.

If a factor has a low average weight and a higher absolute average, we know that the factor is shorted frequently. Examples of frequently shorted factors are HML and WML, which we can see doubles in value when going from normalized average weight to the absolute normalized average. This is not strange as the HML factor is known to have performed poorly the last decades. As we can see from Table 3, the WML factor has the second highest average annual return of all the factors, but also the highest standard deviation and the highest average negative return. We know from the literature review that the WML factor is known to perform poorly in recessions, which makes is attractive to short in periods. But as both factors have a positive average return, the average weights are as expected positive.

With 1.5 being the highest absolute value among the maximum and minimum weights, we see that the model does not take any large positions which puts a lot of risk on the return of a individual factor. This also proves our point that introducing a weight constriction to the model, is not necessary as properties of the model is enough to adjust the weight within reasonable amounts.

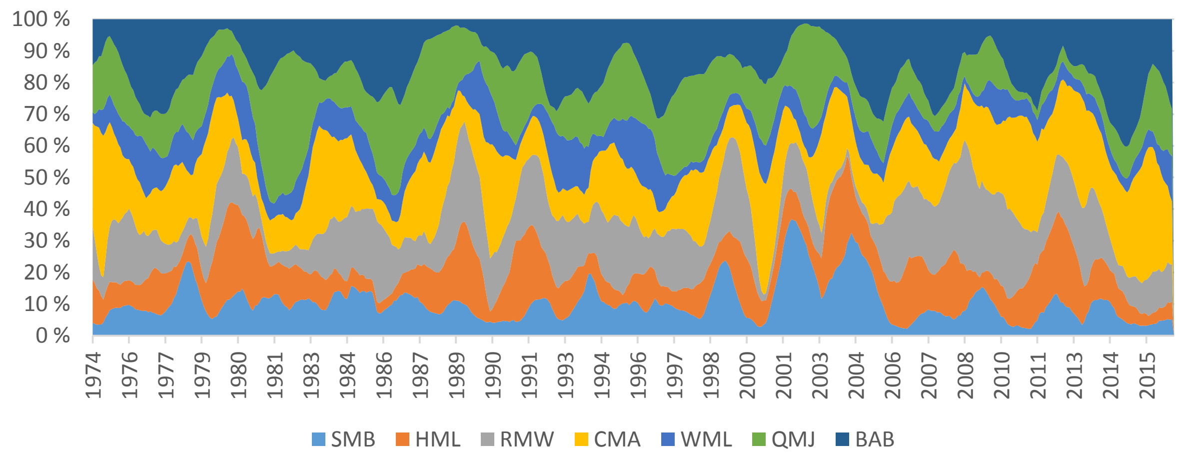

Figure 11: one-year moving average of absolute factor weights for the MVM. Source: own creation

The figure above graphs the one-year moving average of the absolute relative weights of factors in the MVM. It clearly shows that the weights changes as time passes and that the portfolio is diversified at all times.

4.3.4 Multi-Factor Model Regressions

	CAPM			FF3			C4			FF5			8FM		
	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2
Alpha	0.51 %	10.15	0.01	0.50 %	9.95	0.03	0.40 %	8.76	0.07	0.45 %	8.73	0.25	0.36 %	7.48	0.26
Mkt-RF	-0.02	-2.08		-0.03	-2.89		-0.01	-0.86		-0.01	-1.17		0.01	0.68	
SMB				0.06	3.74		0.05	3.66		0.07	3.91		0.06	3.62	
HML				-0.01	-0.49		0.04	2.56		-0.07	-3.23		0.02	0.79	
RMW										0.04	1.83		-0.03	-0.87	
CMA										0.15	4.15		0.07	2.28	
WML							0.12	11.98					0.12	10.58	
QMJ													0.05	1.44	
BAB													-0.00	-0.32	

Table 11: excess returns of the MVM regressed on the different multi-factor models. Source: own creation

Looking at Table 11, we see that the market beta of our portfolio decreases as we add more factors to our regression model and becomes insignificant when regressed on the four-, five-, and eight-factor (8FM) models. Even when regressed on the CAPM, the market beta is initially very small with a value of –0.02. This is a significant decrease from the equally weighted portfolio, and we are close to having a completely market neutral portfolio. When looking at the beta values when regressed on our eight-factor model, we see the portfolio has a significant beta for SMB, CMA and WML, with the WML beta being the highest with a value of 0.12. This is interesting because WML had the lowest average absolute weights in chapter 4.3.3. The reason must lie in the high correlation with QMJ and BAB.

The portfolio now has a significant alpha when regressed on the eight-factor model, which implies that some of the return of the MVM cannot be attributed to the factor performance. The significant alpha must then be a result of the timing that comes with the mean-variance framework. We also see that the R-squared value is increasing for each step, but is still only 0.26 for our eight-factor model, which support the significant alpha and the explanation that only parts for the MVM return can be explained by the eight factors alone.

4.3.5 Control of the Tangency Portfolio

To make sure the solution to our optimization problem is actually located on the tangency portfolio and not on the hyperbola as we addressed chapter 3.5.3, we will run the model with a 5% target return and see how it compares to the same model with a target return of 10%. If the Sharpe ratios are the same, it means that the relative weights are the same and are only being scaled up and down along the tangency line of the best possible weight combination.

Rolling Window	Lag ret. Lambda	0t 0	0t 0.1	0t 0.2	0t 0.3	0t 0.4	0t 0.5	0t 0.6	0t 0.7	0t 0.8	0t 0.9	0t 1
	AVG	2.35 %	2.90 %	2.98 %	3.02 %	3.04 %	3.06 %	3.07 %	3.07 %	3.08 %	3.07 %	3.06 %
1Y	SD	2.32 %	1.96 %	1.97 %	2.00 %	2.03 %	2.06 %	2.10 %	2.13 %	2.17 %	2.22 %	2.29 %
	SR	1.02	1.48	1.51	1.51	1.50	1.48	1.46	1.44	1.42	1.38	1.34

Table 12: MVM performance with a target return of 5%. Source: own creation

Comparing Table 12 with Table 8, we see that for the one-year rolling window, the Sharpe ratios are the same for both target returns. The average returns are reduced by the half and the same is the standard deviation. This indicates that the results in Table 8 in fact are located on the tangency line, and that the portfolios created really are the most efficient ones.

4.4 Business Cycle Performance

Before finalizing our model by including state variables to our mean-variance model, a relationship between the factor returns and the state variables will be established. To do this, we will look at the factor performance and state variables in relation to the business cycle and also perform a chi-squared test.

As covered in the literature review, some of our factors are documented to be effected by the business cycle. A relationship between the entire U.S. stock market and business cycle is also documented by Hamilton and Lin (1996) who find economic recessions to account for 60% of the variance in stock returns between 1965 and 1993. To identify whether the economy is currently in a recession or expansion is hard to do, and it can often not be decided until many months after a peak or trough have occurred. It is important to understand that we are not trying to identify these peaks and troughs with our state variables directly, but we are rather using them to show that both the factor returns and the state variables are changing as the business cycle is changing. This is to better understand the relationship between the state variables and the factor returns.

4.4.1 Recessions and Expansion

Instead of trying to use any analytical method to determine when the various business cycles end or begin, we use the “officially” published dates from the National Bureau of Economic Research, which publishes the U.S. business cycle data regularly. The most recent decision of the business cycle dating committee took place in September 2010, when they announced that the latest economic expansion began in June 2009. According to the National Bureau of Economic Research, the following dates mark the contractions and expansions during our sample period:

Peak	Trough	Contraction	Expansion	Description
November 1973	March 1975	16	36	Stagflation, oil crisis, stock market crash
January 1980	July 1980	6	58	W-shaped recession
July 1981	November 1982	16	12	Tight monetary policy after the energy crisis of 1979
July 1990	March 1991	8	92	Inflation, increased interest rates, oil price shock, consumer pessimism, weakened economy
March 2001	November 2001	8	120	Dot-com bubble
December 2007	June 2009	18	73	Subprime mortgage crisis
Average		12	65,17	

Table 13: business cycle reference dates. Values in months. Source: The National Bureau of Economic Research (2017, July 1) Retrieved from NBER: http://www.nber.org/cycles/cyclesmain.html; Wikipedia (2017, July 1) List of recessions in the United States. Retrieved from Wikipedia: https://en.wikipedia.org/wiki/List_of_recessions_in_the_United_States, independent analysis

4.4.2 Factor Characteristics in Recession and Expansion

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Ann. Avg. Ret.	8.72 %	2.41 %	4.72 %	2.62 %	3.67 %	8.49 %	3.52 %	11.93 %
Ann. SD	14.31 %	10.24 %	9.69 %	8.17 %	6.58 %	13.71 %	8.20 %	11.10 %
Ann. SR	0.61	0.24	0.49	0.32	0.56	0.62	0.43	1.07
Max	12.47 %	18.73 %	12.91 %	13.52 %	9.55 %	18.38 %	12.70 %	13.06 %
Min.	-23.24 %	-15.28 %	-10.49 %	-19.11 %	-6.88 %	-25.01 %	-10.27 %	-15.13 %
Median	1.03 %	0.07 %	0.28 %	0.23 %	0.20 %	0.70 %	0.30 %	1.21 %
# Positive	278	239	245	250	253	289	260	319
# Negative	181	216	213	209	206	170	199	140
Avg. Positive	3.29 %	2.26 %	2.28 %	1.61 %	1.58 %	2.76 %	1.75 %	2.45 %
Avg. Negative	-3.21 %	-2.08 %	-1.78 %	-1.44 %	-1.25 %	-2.78 %	-1.61 %	-2.33 %
Skewness	-0.70	0.31	0.26	-0.49	0.41	-0.56	0.19	-0.89
Kurtosis	2.84	4.89	2.43	14.77	2.53	6.18	3.83	4.86

Table 14: summary statistic of factor returns during expansion. Source: own creation

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Ann. Avg. Ret.	-9.07 %	5.11 %	4.39 %	6.69 %	7.70 %	3.62 %	11.36 %	1.01 %
Ann. SD	22.96 %	12.26 %	13.12 %	8.16 %	8.37 %	18.01 %	9.93 %	14.21 %
Ann. SR	-0.39	0.42	0.33	0.82	0.92	0.20	1.14	0.07
Max	16.10 %	12.80 %	8.22 %	7.51 %	6.57 %	12.45 %	9.02 %	11.87 %
Min.	-17.23 %	-6.94 %	-11.25 %	-4.40 %	-4.74 %	-13.86 %	-6.16 %	-9.18 %
Median	-1.08 %	0.34 %	0.39 %	0.32 %	0.36 %	1.66 %	1.07 %	-0.31 %
# Positive	28	36	38	41	38	42	43	31
# Negative	39	30	29	26	29	25	24	36
Avg. Positive	5.28 %	2.76 %	2.84 %	2.00 %	2.40 %	3.97 %	2.75 %	3.42 %
Avg. Negative	-5.56 %	-2.54 %	-2.92 %	-1.57 %	-1.53 %	-4.16 %	-1.70 %	-3.00 %
Skewness	0.20	0.66	-0.59	0.32	0.02	-0.52	0.34	0.29
Kurtosis	-0.04	2.00	0.84	0.31	-0.34	0.62	0.64	0.27

Table 15: summary statistic of factor returns during recession. Source: own creation

Looking at the performance of the individual factors during expansion and recession periods, we see that only the market return is negative during the recessions, while every factor remains in the positive territory, which leads to a positive Sharpe ratio as well. This is accordingly to the low CAPM beta values and significant alphas (except SMB) from Table 5, which means they are expected to produce positive returns in all market conditions.

During the recession, the standard deviation of the observable returns is larger in than in the expansion period, except for RMW where it is almost the same. The QMJ factor has among the lowest risk-adjusted returns in the expansion period, while during the recession it produces the highest risk-adjusted return. This can be related to the “flight to quality” phenomenon discussed in the literature review. The opposite is true for the BAB factor, which goes from being the worst performing factor during recessions from the best performing one during expansions. It is also worth noting that CWA and RMW produce a significantly higher risk-adjusted return in recessions than they do in the expansions months, which suggests that the optimization model will allocate larger weights to these factors during recession periods. We find no evidence of the countercyclicality of HML documented by Zhang (2005), but this is probably because of our short sample and the fact that the value premium has disappeared for half of the period.

4.4.3 Covariance matrix in recession and expansion

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Mkt-RF	1.00							
SMB	0.21	1.00						
HML	-0.33	-0.15	1.00					
RMW	-0.27	-0.43	0.25	1.00				
CMA	-0.34	-0.03	0.72	0.09	1.00			
WML	-0.01	0.08	-0.18	0.06	-0.08	1.00		
QMJ	-0.49	-0.51	0.10	0.79	0.07	0.14	1.00	
BAB	-0.19	-0.01	0.46	0.41	0.36	0.18	0.27	1.00

Table 16: correlation matrix of factor returns during expansion

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Mkt-RF	1.00							
SMB	0.36	1.00						
HML	-0.15	0.26	1.00					
RMW	-0.20	-0.16	-0.30	1.00				
CMA	-0.58	-0.10	0.57	-0.16	1.00			
WML	-0.47	-0.38	-0.17	0.23	0.35	1.00		
QMJ	-0.56	-0.44	-0.41	0.71	0.09	0.58	1.00	
BAB	0.09	0.04	-0.13	-0.01	0.09	0.27	0.00	1.00

Table 17: correlation matrix of factor returns during recession

The two tables above show the covariance matrixes for the expansion and recession periods and the most notable differences are highlighted. The correlations among the different factors are not constants but functions of the current market conditions and as we can see, they might change significantly when the market turns. Investors that are not aware of this effect might be exposed to larger risk in their portfolio than they initially wish, due to the changing diversification effects.

It is worth noting some specific changes between the expansion and recession periods in the covariance matrix tables. We can draw similar conclusions as we did in Table 15 related to the BAB factor: when we run a covariance matrix on the months that are associated with a recession, we end up with a slightly positive correlation to the market factor. As such, this makes the BAB factor less attractive in the recession periods as in expansion. The SMB factor also stays positively correlated to the market during recession, which we would rather associate with lower expected weights in the optimization models during months associated with recession.

Results of the previous tables suggest that in various market conditions, different factors perform better and they have changing diversification effects. Naturally, the return of the factors vary a lot during the observed periods and investors should rather expect to earn the returns on the long run.

4.4.4 State Variables

The relationship between our state variables and the business cycle has been widely documented. From the website of the Federal Reserve Bank of St. Louis we get the peaks and troughs in the business cycle defined by OECD and the NBER. The point where the gray area starts is the peak of a business cycle and the end of the gray area defines the trough. Hence the grey area defines the recessions and the white area defines expansion of the economy.

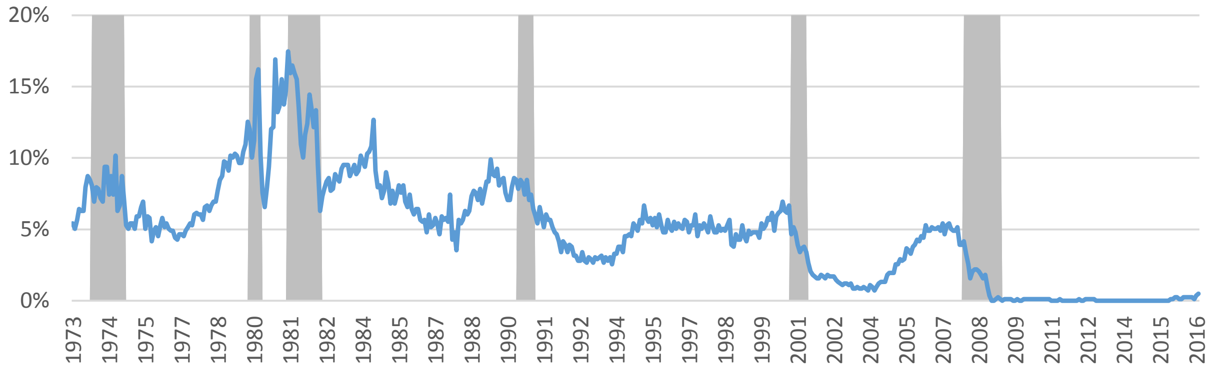

Figure 12: annualized one-month T-bill yield and the business cycle, the gray area marks recessions. Source: own creation

For the one-month T-bill yield, we see the yield is always decreasing during recessions, and usually with an increase prior to the recession. Except from the large fall in the yield curve between 1984 and 1985, this is true for all the major decreases in our sample. As our model looks back at least one year of data, a constant decrease in the short-term interest rate for the last months, should be a good indicator for a recession. The problem is that the current risk-free rate is close to zero, so any major decreases in the interest rate in the future is unlikely. Another problem is that the short-term interest in some occasions continue to fall even though the recession has reached its trough. This makes the recovery of the economy hard to identify. As a result, the interest yield should be evaluated together with other state variables to provide information on where the economy is heading.

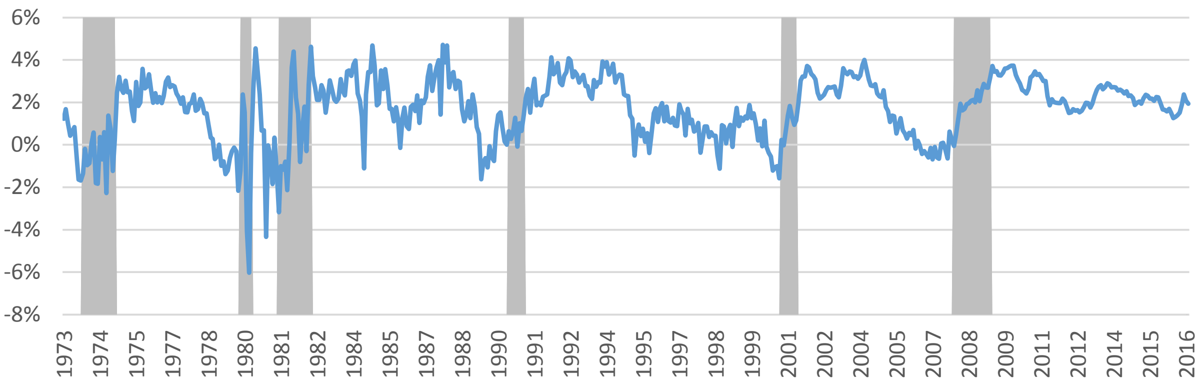

Figure 13: the term spread and the business cycle, the gray area marks recessions. Source: own creation

Contrary to the interest yield, the term spread is increasing during recessions. We see that the increase in the term spread seems to stop as a recession is over, making the trough easy to observe as the spread flattens or decreases.

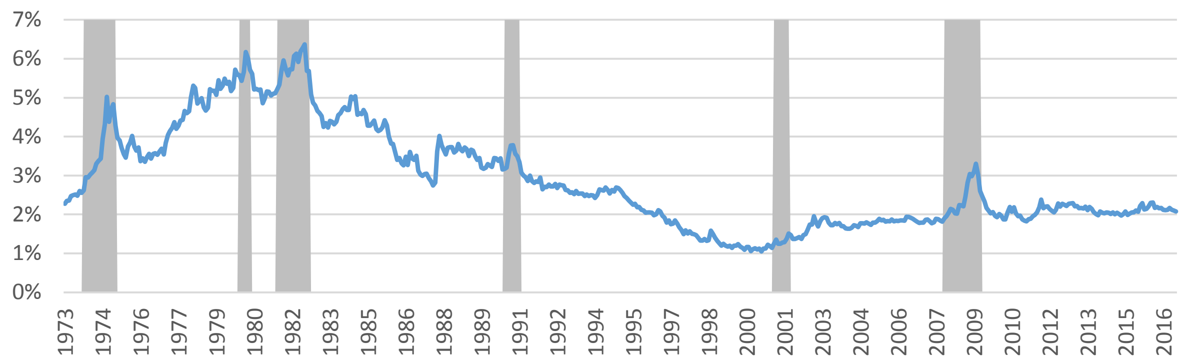

Figure 14: the dividend yield and the business cycle, the gray area marks recessions. Source: own creation

The dividend yield seems to form local peaks in the middle of a recession, but with a small tilt to the end of the recession. If you earlier have attributed changes in the term spread and the risk-free rate to an ongoing recession, a sudden rise and fall in the dividend yield could indicate when you are close to the end recession.

As we can see, our chosen state variables should be good indicators for which state the economy is currently in. A few months of observation of the state variables are needed to be certain of the fact, but for the last six recessions, the average duration was 12 months, implying our model should have time to account for the recession before it is over. It is also important to understand that our model do not look at the combination of state variables, but as explained in the methodology, allocates weights by observing individual factor performances related to a single state variable. Still, we see each of the state variables changing significantly during a recession, which will hopefully result in a change in factor weights in our state variable model.

4.4.5 Discrete State Analysis

Running the chi-squared test as explained in the methodology (chapter 3.7) by comparing the realized distribution between to variables to the expected distribution by the assumption of normalized distribution, we end up with the following p-values

	SMB	HML	RMW	CMA	WML	QMJ	BAB
1M T-Bill	0.58	0.91	0.66	0.19	0.01	0.21	0.42
Term Spread	0.04	0.04	0.85	0.74	0.42	0.01	0.03
S&P500 Div	0.03	0.29	0.03	0.01	0.06	0.42	0.01

Table 18: p-values of the chi-squared tests. Source: own creation

With a confidence level of 10%, a p-value below 0.1 means we reject the null hypothesis of the relationship of the chosen factor and state variables to be independent, resulting in the alternative hypothesis of an existing statistical relationship between the two time series, being accepted. The rejected null hypothesizes are highlighted. A lower p-value indicates a stronger relationship than a high P-value.

The table shows that all of the factors have a significant relationship with at least one of the state variables. This indicates the state variables might benefit the timing of the factors. Maybe the most surprising finding is that the one-month T-bill only has a relationship with one of the factors, the momentum factor, as the T-Bill have a strong effect on the stock market as discussed in chapter 2.5.1.

4.4.6 Factor Returns

The graph of the 12-moth rolling average return along the business cycle of the individual factors are located in the appendix. Most factors seem to be generally untouched by the first recessions, but for the two last recessions of 1900-1991 and 2007-2009, we see a drop in return for most of the factors.

4.5 State Variable Mean-Variance Model (SVMVM)

In 4.3, we tested the different rolling windows for the mean-variance model and applied shrinkage. The one-year rolling window with a lambda of 0.2 turned out to yield the highest Sharpe ratio with a value of 1.51. In this chapter, we will continue to explore the different combinations of lambda and rolling windows to see if the risk-adjusted return can be improved further by implementing state variables according to Brandt and Santa-Clara (2006), as explained in the methodology section (3.2). The model is still set to reach a target return of 10% while minimizing the variance. We allow the model to go either long or short in each factor with unlimited weights.

4.5.1 Parameter Selection

In subchapter 4.3.1 we concluded that the one-year rolling window works best for the mean-variance model without state variables. As we expect the recent performance of the factors to still be the most important determinant when allocating weights even after the state variables are introduced to our model, we would expect this one-year rolling window to be the best for this model as well. But on the other side, a longer rolling window should contain more information about the correlation of our state variables and factors returns, which might make a longer rolling to perform better than the one-year rolling window. Because of this, we will also test the new model for the five different rolling windows.

We will apply shrinkage in the same way as explained in 3.3 and 4.3.1. Different from the mean-variance model, we will also run the model for two different lags for the state variables as explained in 3.5.3; one model with a one period lag in the state variables and one model without lags.

Running the model with rolling windows between one and five years and no lags to the state variables, we get the following returns by applying shrinkage with intervals of 0.1:

Rolling window	Lag ret.	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t
	Lag SV.	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t
	Lambda	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1Y	AVG	3.59 %	4.89 %	5.28 %	5.33 %	5.38 %	5.42 %	5.51 %	5.65 %	5.69 %	5.74 %	5.83 %
	SD	4.75 %	3.67 %	3.63 %	3.65 %	3.62 %	3.64 %	3.70 %	3.75 %	3.87 %	3.98 %	4.28 %
	SR	0.76	1.33	1.46	1.46	1.49	1.49	1.49	1.51	1.47	1.44	1.36
2Y	AVG	4.79 %	4.91 %	4.94 %	5.04 %	5.24 %	5.33 %	5.34 %	5.35 %	5.29 %	5.28 %	5.07 %
	SD	4.88 %	4.35 %	4.28 %	4.29 %	4.26 %	4.29 %	4.34 %	4.41 %	4.51 %	4.73 %	5.20 %
	SR	0.98	1.13	1.15	1.17	1.23	1.24	1.23	1.21	1.17	1.12	0.98
3Y	AVG	5.93 %	5.43 %	5.64 %	5.72 %	5.78 %	6.01 %	6.18 %	6.22 %	6.25 %	6.26 %	6.04 %
	SD	5.82 %	4.82 %	4.79 %	4.72 %	4.76 %	4.80 %	4.92 %	5.03 %	5.15 %	5.45 %	6.00 %
	SR	1.02	1.13	1.18	1.21	1.21	1.25	1.26	1.24	1.21	1.15	1.01
4Y	AVG	4.99 %	5.38 %	5.43 %	5.57 %	5.68 %	5.78 %	5.72 %	5.81 %	5.91 %	5.77 %	5.44 %
	SD	5.36 %	5.05 %	4.92 %	4.84 %	4.83 %	4.91 %	4.89 %	4.98 %	5.05 %	5.28 %	5.78 %
	SR	0.93	1.07	1.10	1.15	1.18	1.18	1.17	1.17	1.17	1.09	0.94
5Y	AVG	5.17 %	4.89 %	5.07 %	5.20 %	5.14 %	5.26 %	5.38 %	5.43 %	5.42 %	5.26 %	5.05 %
	SD	5.25 %	4.90 %	5.00 %	4.96 %	5.07 %	5.16 %	5.24 %	5.36 %	5.47 %	5.71 %	6.25 %
	SR	0.98	1.00	1.01	1.05	1.01	1.02	1.03	1.01	0.99	0.92	0.81

Table 19: mean-variance model with state variables with state variables lagged one period. Source: own creation

Looking at Table 19, we see the highest Sharpe ratio is achieved by the one-year rolling window and a lambda of 0.7. We see the highest Sharpe ratios are achieved with higher lambdas compared to the model without state variables. The Sharpe ratio of 1.51 the same as the model without state variables and not an improvement of the risk-adjusted returns. We go on adding a lag to the state variables:

Rolling window	Lag ret.	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t	0t
	Lag SV.	1t	1t	1t	1t	1t	1t	1t	1t	1t	1t	1t
	Lambda	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1Y	AVG	3.72 %	5.29 %	5.67 %	5.81 %	5.81 %	5.81 %	5.81 %	5.93 %	5.97 %	6.00 %	6.04 %
	SD	4.72 %	3.68 %	3.59 %	3.58 %	3.55 %	3.56 %	3.66 %	3.66 %	3.74 %	3.86 %	4.16 %
	SR	0.79	1.44	1.58	1.62	1.64	1.63	1.59	1.62	1.60	1.55	1.45
2Y	AVG	5.47 %	5.59 %	5.63 %	5.66 %	5.65 %	5.65 %	5.61 %	5.64 %	5.73 %	5.56 %	5.21 %
	SD	4.87 %	4.30 %	4.29 %	4.19 %	4.18 %	4.21 %	4.35 %	4.37 %	4.52 %	4.71 %	5.18 %
	SR	1.12	1.30	1.31	1.35	1.35	1.34	1.29	1.29	1.27	1.18	1.01
3Y	AVG	5.64 %	6.13 %	6.28 %	6.32 %	6.21 %	6.33 %	6.42 %	6.44 %	6.43 %	6.41 %	6.13 %
	SD	5.46 %	4.91 %	4.81 %	4.83 %	4.85 %	4.89 %	4.95 %	5.08 %	5.27 %	5.50 %	6.06 %
	SR	1.03	1.25	1.31	1.31	1.28	1.29	1.30	1.27	1.22	1.17	1.01
4Y	AVG	5.18 %	5.78 %	5.91 %	5.93 %	5.93 %	5.97 %	6.05 %	6.04 %	5.99 %	5.84 %	5.50 %
	SD	5.37 %	4.95 %	4.81 %	4.76 %	4.78 %	4.86 %	4.87 %	4.94 %	5.03 %	5.23 %	5.75 %
	SR	0.96	1.17	1.23	1.25	1.24	1.23	1.24	1.22	1.19	1.12	0.96
5Y	AVG	4.85 %	5.21 %	5.43 %	5.44 %	5.45 %	5.47 %	5.55 %	5.56 %	5.42 %	5.30 %	5.02 %
	SD	5.17 %	4.84 %	4.90 %	4.99 %	5.08 %	5.15 %	5.27 %	5.36 %	5.49 %	5.74 %	6.25 %
	SR	0.94	1.08	1.11	1.09	1.07	1.06	1.05	1.04	0.99	0.92	0.80

Table 20: mean-variance model with state variables with state variables lagged two periods. Source: own creation

The highest Sharpe ratio is now achieved with a one-year rolling window and a lambda of 0.4. With a value of 1.64, it is an 8% improvement from both the no state variables model and the state variable model with no lag, both with Sharpe ratios of 1.51.

Table 20 shows that a higher lambda gives the best result, compared to the no state variable model (0.4 vs 0.2). This is not surprising as the covariance matrix is now expanded to a 28*28 matrix while the sample size of the one-year rolling window is still 12 months, which means the covariance matrix calculations should contain more errors as discussed in 3.3.

The fact that lagging the state variables with one period performs better in almost all the individual instances compared to the one period lagged state variable model, must be because the changes of the state variables and the economy, do not affect the stock market and the factors immediately, but rather takes some time.

We also see that for all our models, even though the target return is set to 10%, we only achieve a return around 6%. This means that on average, the past top performing factors delivers a lower return or the past worst performing factors delivers a higher return compared their past one-year average (given a one-year rolling window). This error can be related to the estimation error of the covariance matrix which we improved by applying shrinkage. Shrinkage can also be applied to the returns to enhance the prediction accuracy, but is a lot more demanding to implement and out of the scope for this thesis. The reason we leave this out is because we are considering only the risk-adjusted return, which will be the same as long the relative weights are unchanged. Shrinking the estimators is more relevant for real-life appliances where the investors also consider a target return and predictability of the returns. Interested readers are suggested to look up LASSO regression analysis.

We will continue referring to the one-year rolling window with lambda of 0.4 and a one period lag in state variables as the state variable mean-variance model or the SVMVM.

Lambda	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1Y	-0.23	-0.04	0.07	0.12	0.14	0.15	0.12	0.18	0.18	0.17	0.11
2Y	0.06	0.10	0.09	0.13	0.14	0.16	0.13	0.17	0.19	0.15	0.05
3Y	-0.17	0.01	0.06	0.08	0.07	0.11	0.14	0.14	0.13	0.13	0.03
4Y	-0.17	-0.02	0.04	0.06	0.07	0.08	0.12	0.14	0.15	0.13	0.03
5Y	0.00	0.08	0.12	0.11	0.10	0.12	0.13	0.14	0.12	0.09	0.02

Table 21: the increase in Sharpe ratios by going from MVM to SVMVM and. Source: own creation

Looking at Table 21, we see the different changes in Sharpe ratio by going from the MVM to SVMVM for each combination of shrinkage and rolling window. The table shows that with a few exceptions for lambda values of 0 and 0.1, the Sharpe ratio improves by adding state variables to the model. Because MVM and SVMVM use a one-year rolling window, we see the Sharpe ratio would actually decrease by moving to the SVMVM if we did not include the covariance matrix shrinkage. Generally, the best improvements in the Sharpe ratio is achieved within the higher the lambda values. We interpret this as a proof that adding the state variables increases the estimation errors of the covariance matrix and thus shrinkage should be added to the model to achieve the best results.

4.5.2 Portfolio Performance

	EWP	MVM	SVMVM
Ann. Avg. Ret	5.41%	5.95%	5.81%
Ann. SD	4.92%	3.94%	3.55%
Ann. SR	1.10	1.51	1.64
Median	0.42%	0.48%	0.47%
Max	9.73%	4.56%	4.00%
Min.	-7.96%	-4.08%	-3.54%
Max DD	11.42%	5.42%	5.60%
# Positive	364	362	361
# Negative	152	154	155
Avg. Positive	1.02%	1.04%	0.98%
Avg. Negative	-0.91%	-0.79%	-0.67%
Skewness	0.23	0.08	0.11
Kurtosis	9.36	1.24	0.95

Table 22: summary statistics of the EWP, MVM and SVMVM[24]. Source: own creation

With the selected rolling window and lambda parameters from the last chapter, we are able to produce a strategy that is superior to the MVM. The difference in Sharpe ratio can only be described due to the timing effect which we were able to utilize through the state variables described in the previous chapters. The achieved annual average return slightly decreases, but the portfolio yields the return by taking lower risk, hence an increase in the Sharpe ratio. We see a further decrease in the extreme measures and once again a lower maximum and higher minimum returns were achieved compared to the previous model.

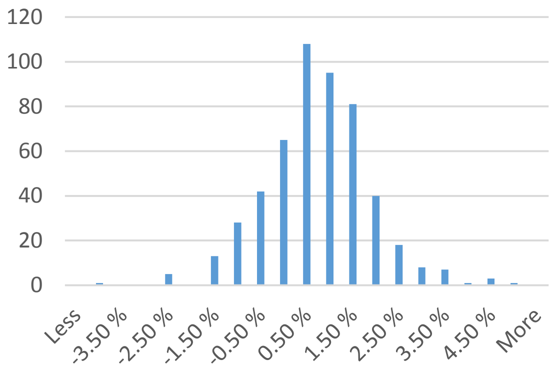

Figure 15: histogram of the excess returns of the MVM. Source: own creation

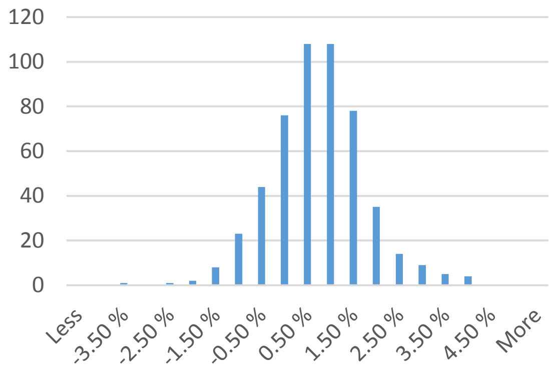

Figure 16: histogram of the excess returns of the SVMVM. Source: own creation

The difference between the monthly distribution of SVMVM and MVM is not as notable as between Figure 7 and Figure 8 (EWP vs MVM). SVMVM has slightly slimmer tails compared to the MVM, resulting in a lower kurtosis.

[24] Note that the performance measures values of the EWP changed slightly compared to the one reported in the Chapter 4.2. The reason behind is that due to the one-year rolling window period of MVM. We adjusted the performance period of the EWP by cutting the observations of the first year.

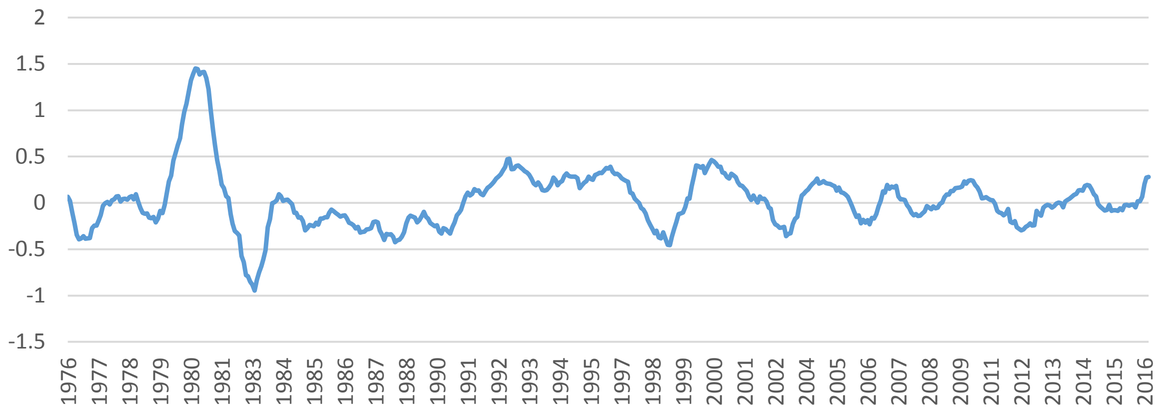

Figure 17: one-year average of the one year rolling Sharpe ratio spread between MVM and SVMVM. Source: own creation.

Compared to Figure 9 in 4.3.2, we decided to report the spread on the one-year average of the one-year rolling Sharpe ratio of instead of the individual rolling Sharpe ratios of the two strategies. This is done to better see the compared performance on visualizing the risk-adjusted. Negative values mean that the MVM performs better than the SVMVM. In the earlier years of the dataset, the SVMVM is clearly the superior strategy to the MVM. However, after 2000 it seems like the timing effect starts to disappear and none of the models are clearly superior to the other. Before 2000, there are long periods where each strategy performed better than the other, but after 2000, the Sharpe ratio spread is rapidly changing between being in favor of the MVM and SVMVM.

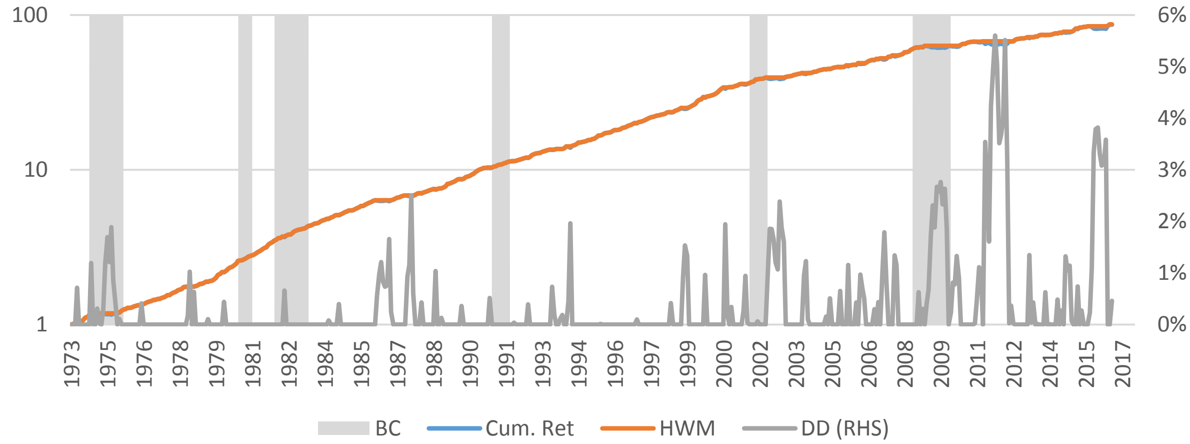

Figure 18: cumulative return with HWM and DD of the SVMVM, logarithmic scale for LHS. Source: own creation

As we can see from Figure 18, the drawdowns are very similar to what we achieved with the model without the state variables in Figure 10. The maximum drawdown is almost the same and we have the highest drawdowns at the end of the sample period outside of recessions, just like before. But while the MVM had four drawdowns above 3%, we have now reduced this to only two drawdowns which are located outside any of the recessions. There is also a general decrease in drawdowns for the earlier parts of the sample, compared to the mean-variance model without state variables. No major drawdowns during recessions is a good indication the portfolio is market neutral, and losses are fewer and lower than before. We will further asses the market neutrality of the portfolio in Chapter 4.5.4. Looking back at the individual factor performance in the appendix, commented in chapter 4.3.5, we saw a lot of the factors becoming more volatile around 2001. Surprisingly the model seems to good job here by having a maximum drawdown of around 2%.

Before 1999, we observe longer periods where the strategy barely loses any money. We can also see from the graph that after 1999 the frequency of the drawdowns increases. This might be because of what we mentioned earlier about more investors starting to follow the various factor strategies and as a result the factor returns became more volatile and less effective in terms of Sharpe ratio. It is no longer only the highly quantitate funds that invest into these factors, but it has also become available to retail investors through mutual funds focusing solely on one single or multiple factor strategies.

4.5.3 Weights

	SMB	HML	RMW	CMA	WML	QMJ	BAB	Sum
Avg.	0.05	0.01	0.07	0.08	0.02	0.12	0.15	0.50
Norm. Avg.	0.10	0.03	0.14	0.15	0.03	0.23	0.31	1.00
SD	0.13	0.20	0.22	0.26	0.12	0.18	0.19	
Max	0.56	0.83	0.99	1.32	0.51	0.70	0.76	
Min.	-0.39	-0.90	-0.92	-0.69	-0.43	-0.40	-0.34	
Abs. Avg.	0.11	0.14	0.17	0.21	0.09	0.16	0.20	1.08
Abs. Norm. Avg.	0.10	0.13	0.16	0.19	0.09	0.15	0.18	1.00

Table 23: weights allocation among factors for the SVMSM. Source: own creation

Compared to the weights of the MVM in Table 10, we see the sum of the average of absolute returns have slightly decreased, meaning the portfolio on average makes positions closer to 0. This is also implied by the reduction of the maximum and minimum values. The smaller positions explain the slightly lover return of the SVMVM. Normalizing the absolute average, we see that the portfolio is has almost the same relative weights among the factors compared to the MVM, with CMA and BAB being the heaviest weighted.

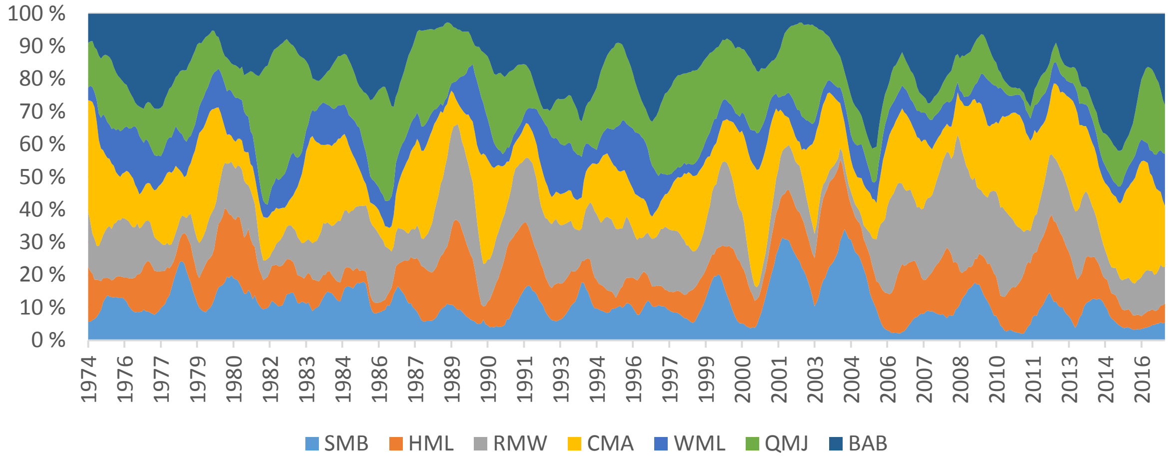

Figure 19: one-year moving average of absolute factor weights for the SVMVM. Source: own creation

Comparing the one-year moving average of absolute factor weights there are not any major deviation from the MVM which implies that adding the state variables to our mean-variance model only slightly alters the weight allocation. But as we saw in subchapter 0, the difference is enough to reduce the standard deviation.

4.5.4 Multi-Factor Model Regressions

	CAPM			FF3			C4			FF5			8FM		
	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2
Alpha	0.50 %	11.19	0.02	0.49 %	10.75	0.05	0.40 %	9.59	0.22	0.43 %	9.40	0.09	0.35 %	7.95	0.24
Mkt-RF	-0.03	-3.46		-0.04	-3.67		-0.02	-1.88		-0.02	-1.78		0.00	0.31	
SMB				0.05	3.42		0.04	3.13		0.06	3.94		0.06	3.74	
HML				0.02	1.29		0.06	4.06		-0.04	-1.97		0.04	1.77	
RMW										0.06	2.65		-0.02	-0.67	
CMA										0.13	4.21		0.07	2.40	
WML							0.10	10.63					0.09	9.06	
QMJ													0.07	2.02	
BAB													-0.00	-0.20	

Table 24: excess returns of the SVMVM regressed on the different multi-factor models. Source: own creation

Table 24 shows that when the portfolio is regressed on the CAPM, we get a high and significant alpha, suggesting the return of the portfolio creates a return not dependent on the market. This is also reflected in the low, but significant market beta of the CAPM. The market beta becomes insignificant on a 95% confidence level when we regress on the entire eight-factor portfolio. Just as the MVM, the WML is still the highest and most significant factor. Other significant factors are SMB, CMA and QMJ. Also here, the BAB is insignificant even though it has the second highest average absolute weights. The higher R-squared for C4 and 8FM together with the high beta, indicates WML is the factor that explains most of the portfolio returns. Also in this portfolio, WML is the lowest weighted portfolio, which means the high beta and R-squared value must be because of the high correlation with BAB and QMJ.

Compared to the MVM, we see that the alpha of the eight-factor model actually decreases with 0.01% per month, which indicates that the state variables does not creates a higher return that cannot be described by the state variables. To investigate if including the state variables creates a return

4.5.5 Empirical Evidence on Factor Timing

Comparing the alphas of two portfolios with different returns and standard deviation makes no sense as the alpha can be modified by scaling the asset weights: by increasing all the weights in a portfolio while still keeping the same relative weights, the alpha increases even though the risk-adjusted return remains the same. To conduct further empirical research of the performance of the SVMVM compared to the two other multi-factor models, we make the models comparable by scaling the achieved monthly returns of the SVMVM so it achieves the same standard deviation as the two other models. This allows us to research the added value of the factor timing in the model.

	Scaled SVMVM on EWP	Scaled SVMVM on MVM
Monthly alpha	0.49 %	0.08 %
Annual alpha	5.91 %	1.01 %
t-stat	8.16	3.79
P value	0.0000	0.0002

Table 25: scaled SVMVM returns regressed on the EWP and the MVM. Source: own creation

Regressing the scaled SVMVM model on both the EWP and the MVM, we get the results summarized above. We see that the SVMVM generates a monthly alpha of 0.49% which is confident on a 99% level when regressed on the EWP. What is even more relevant to our research is the second column that shows the scaled SVMVM regressed on the MVM. A monthly alpha of 0.08% is observed, which is significant within a 97% confidence level. This alpha is solely due to the added timing effect of the state variables.

Next, we test whether the difference of means between the scaled SVMVM and the MVM is significantly different from zero.

	Scaled SVMVM	*MVM*
Mean	0.54 %	0.50 %
Variance	0.01 %	0.01 %
Observations	516	516
Pearson Correlation	0.91	
Hypothesized Mean Diff.	0	
df	515	
t-stat	1.99	
P(T<=t) one-tail	0.02	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.05	
t Critical two-tail	1.96	

Table 26: results of the paired t-test of the scaled SVMVM and MVM. Source: own creation

We conducted a paired t-test and test to see whether the mean of the difference between the two time series is different from zero. The null hypothesis is that the mean difference is zero (x1-x2 = 0). The alternative hypothesis is that the as the mean difference is not equal to zero, in other words, the means are not the same.

Since the observed t-statistics, 1.99, is higher than the critical value of the two-tailed test, 1.96, we reject the null hypothesis with a 95% confidence. This results in an acceptance of the alternative hypothesis of the different in means being different from zero.

4.6 Conclusion of the In-Sample Analysis

The first portfolio we built is an equally weighted portfolio, which shows that the simplest form of a multi-factor portfolio performs better than any single-factor portfolio on its own. The portfolio can easily be implemented and only requires a monthly rebalancing to keep the weights constant. Because the EWP return is just the average of all the factor returns, the outperformance is a result of the diversification benefits by exploiting the correlations between the individual factors, which leads to lower volatility and hence a lower Sharpe ratio.

Assigning weights based on a mean-variance model with a specified rolling window (MVM), means that the diversification benefit due to the correlations is still exploited, but we see that the average weights shifts towards the factors with the highest Sharpe ratios. By testing different variations of rolling windows, we find the one-year rolling window to produce the highest risk-adjusted return for the model, which suggests that the last twelve months are the most relevant to forecast returns for the next month. The achieved risk-adjusted return is higher than return achieved by the equally weighted portfolio. A significant alpha when regressed on the eight-factor model indicates that the timing of the mean-variance model adds a positive return unexplained by the factors.

By running the chi-squared test, we establish a relationship between at least one of the state variables and each of the factors. This shows there is a connection between state variables and factor returns which can possibly be exploited in a factor timing strategy.

By including a response to the state variables following the approach of Brandt and Santa-Clara (2006), combined with a rolling window of sample data and covariance matrix shrinking according to Kwan (2011), we are able to select specific parameters of the various examined variables to form strategies that achieve a higher Sharpe ratio compared to the equally weighted portfolio and MVM. Even though the average relative weights among the factors are almost identical between the two models, the outperformance of the SVMVM means that the state variables improve the timing of the factor weights compared to the MVM. This is also confirmed by the significant difference from the paired t-test for the scaled SVMVM return to match the MVM.

It is important to remember our two dynamic models have been built on in-sample data. This means that all the variables in the models, both the rolling window and lambda have been tested for every possible combination and we have preceded with combination of variables that give us the best results. This is clearly subject to data mining bias and look-ahead bias. At the beginning of our sample period, we do not have the information needed to make the trading strategy we now have worked out by looking on the historical data. Rather than splitting up the dataset and run an in-sample and out-of-sample test on the U.S. market, we have used the entire sample to look for proof of the possibility to time the factors. The in-sample test results shown that it is in fact possible to exploit available state variables to produce a higher return than a regular mean-variance model or an equally weighted portfolio.

We know the results would be more impressive if we based our model on only half of the sample and used the rest of the sample for an out-of-sample test, but when we first started with the thesis our main concern was to look for proof that it is in fact possible to time the factor allocation based on state variables. As we found out this was in fact possible, we wanted to expand the model to a set of out-of-sample data. Because of this, we will in the next chapter see if our findings can be turned into a trading strategy by applying the same unadjusted model to the European and Japanese markets. Also, we are looking for evidence that the SVMVM produces a better risk-adjusted return than EWP or MVM. One concern is that if the markets are correlated, our in-sample test for the U.S. market also is an in-sample test for the other markets, making the data mining bias and look-ahead bias present to our other markets as well.

5 Out-of-Sample Analysis

By running a briefer version of the same analysis we did on the U.S. market on Europe and Japan, we are testing whether the models based on the in-sample data are also applicable for our out-of-sample data. This means that only the parameters determined in the last chapter are used. For the mean-variance model this means a rolling window of one year and a lambda of 0.2. For the state variable mean-variance model, the rolling window is also here set to one year and the lambda to 0.4. The state variables are lagged with one period while the returns are lagged zero periods.

5.1 Comparing the Markets

Contrary to the U.S. sample which runs from January 1973 to January 2017, the Europe sample runs from November 1990 and the Japan sample from January 1991, both ending January 2017.

Considering that in the U.S. market, we were able to build a superior active strategy to the EWP and the single-factor portfolios, we hope to see the SVMVM to outperform the EWP in both Europe and Japan as well. However, noting that these markets are less mature and less liquid, we might observe different results. Before analyzing the portfolio returns for Europe and Japan, we will compare the market movements of the different regions.

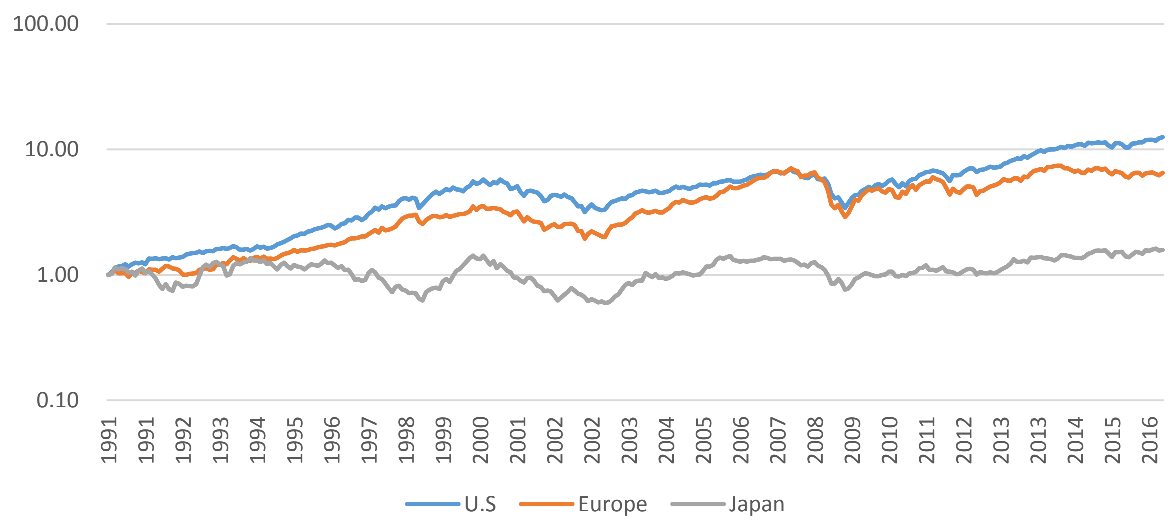

Figure 20: market returns among the different markets all normalized to a starting value of 1. Logarithmic scale. Source: own creation

Looking at the different market returns in Figure 20, we see a close movement of the European and U.S. market returns for the entire period. The high correlation of the U.S. and the European market might introduce a bias to our out-of-sample test if the data we use for the in-sample data is correlated to the out-of-sample data. This means that as we fit our model to the in-sample data, we also indirectly fit it to the out-of-sample data of Europe. After all, we seek to create market neutral portfolio, so the correlation of the market returns only explains a small part of how we can expect the model to work between different data sets. In the end, an analysis of the individual factor performance tells us how similar the markets really are. To investigate this, we check the correlations among the factor return of the different markets:

	U.S.							
	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB
Europe	0.80	0.28	0.58	0.21	0.49	0.75	0.62	0.47
Japan	0.44	0.08	0.40	-0.05	0.18	0.44	0.29	0.11

Table 27: correlation among the European and Japanese to the U.S. factor returns. Source: own creation

Looking at the correlations, we see that for both Europe and Japan, the highest correlation is between the market returns, while the WML factor share the same or almost the same correlation as the market return. The correlation between the out-of-sample market returns and U.S. shows a correlation for Europe of 0.80 and almost the half for Japan. We remember that the SVMVM strategy for the U.S. favored CMA and BAB factors in term of average absolute weights. A high correlation among those factors is then what we should be concerned about for the models to be identical. For Japan, the correlation for the two factors across the market is 0.18 for CMA and 0.11 for BAB. We consider this to fairly low and doesn't introduce a bias to our Japan model. For Europe the numbers are higher with correlations of 0.49 and 0.47 for CMA and BAB respectively. This might introduce a bias to our model and we will compare the factor performances of Europe and U.S. further in the next chapter. Also note that the correlations are based on the period January 1991 to January 2017, while the model is fit to the U.S. time series going back to 1973, which means that almost half of the fitting is done to numbers unrelated to the time series of Europe and Japan.

5.2 Europe

Starting with Europe, we saw in the last chapter that the correlation of factor returns with the U.S. was somewhat high for most factors in the period 1991 to 2017. We will now go deeper into the performance of the individual factors and the models and compare the results of Europe to the results of U.S. when using the entire sample.

5.2.1 Factor and Portfolio Performance

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB	EWP	MVM	SVMVM
Ann. Avg. Ret	6.04 %	0.85 %	4.30 %	4.75 %	2.53 %	11.13 %	4.99 %	10.19 %	5.64 %	6.16 %	6.15 %
Ann. SD	16.94 %	7.65 %	8.43 %	5.28 %	6.43 %	13.92 %	8.29 %	11.83 %	4.77 %	4.18 %	3.81 %
Ann. SR	0.36	0.11	0.51	0.90	0.39	0.80	0.60	0.86	1.18	1.47	1.61
Median	0.75 %	0.15 %	0.35 %	0.46 %	0.05 %	1.16 %	0.36 %	0.65 %	0.41 %	0.55 %	0.49 %
Max	13.84 %	8.74 %	11.39 %	5.71 %	8.70 %	13.66 %	10.29 %	14.33 %	5.97 %	6.42 %	5.60 %
Min.	-22.14 %	-7.40 %	-9.13 %	-4.78 %	-7.13 %	-26.20 %	-11.07 %	-11.72 %	-5.91 %	-2.89 %	-2.76 %
Max DD	59.17 %	37.74 %	33.50 %	13.57 %	36.57 %	45.65 %	23.09 %	34.26 %	10.39 %	5.93 %	5.32 %
# Positive	178	165	182	196	160	212	180	196	215	211	211
# Negative	136	150	132	118	154	101	135	119	87	91	91
Avg. Positive	3.73 %	1.69 %	1.82 %	1.31 %	1.46 %	2.86 %	1.90 %	2.75 %	1.02 %	1.07 %	1.02 %
Avg. Negative	-3.72 %	-1.72 %	-1.66 %	-1.12 %	-1.09 %	-3.10 %	-1.56 %	-2.27 %	-0.90 %	-0.78 %	-0.67 %
Skewness	-0.56	-0.06	0.38	-0.27	0.41	-1.28	-0.13	-0.08	-0.14	0.45	0.45
Kurtosis	1.74	0.83	2.96	0.62	3.37	7.57	2.95	1.78	5.27	2.45	2.03

Table 28: summary statistics of individual factors and portfolios. Source: own creation

Looking at Table 28, we see the three best factor performances are by RMW, BAB and WML with Sharpe ratios of 0.90, 0.86 and 0.80 respectively. This is different from the U.S. where BAB, CMA and QMJ were the best performing factors (Table 3).The SMB factor delivers the lowest Sharpe ratio for both markets. While the CMA factor is the second highest performing factor for the U.S., the factor is the second worst performing factor for Europe. RMW, which is the second worst performing factor in the U.S., is as mentioned earlier the highest performing factor of the European market. The fact that the factors are performing differently in the two markets, makes the performance of our models more interesting as they work in two different markets with different properties. We can also be more confident that the models are not indirectly fit to the European market because of the high correlation.

We see that just like the U.S. market, a gradual improvement in the Sharpe ratio of the portfolios is achieved: the equally weighted portfolio performs better than any of the individual factors, the mean-variance model performs better than the equally weights portfolio, and including the state variables results in the highest Sharpe ratio of them all. The EWP improves the Sharpe ratio compared to the best individual factor by 31%, the MVM improves by 25% compared to the EWP and the SVMVM improves the Sharpe ratio by 9% compared to the MVM. For the U.S., the improvements were 23%, 37% and 8% respectively. For both markets, the smallest improvement is when adding state variables to the mean-variance model.

Going from the equally weighted portfolio to the MVM, we see the number of positive and negative returns are almost the same, but the average positive value is higher and the average negative value is closer to zero. Similarly, the maximum return is higher and the minimum is closer to zero for the

MVM. This indicates that the entire distribution of returns is moved further to the right while also delivering less volatile returns. Going from MVM to SVMVM, we see the average returns are the same, and the distribution of positive and negative returns is also the same, with both the average positive return and maximum return actually being higher for the MVM. The outperformance of the SVMVM is then because of a more persistent return, resulting in a lower standard deviation.

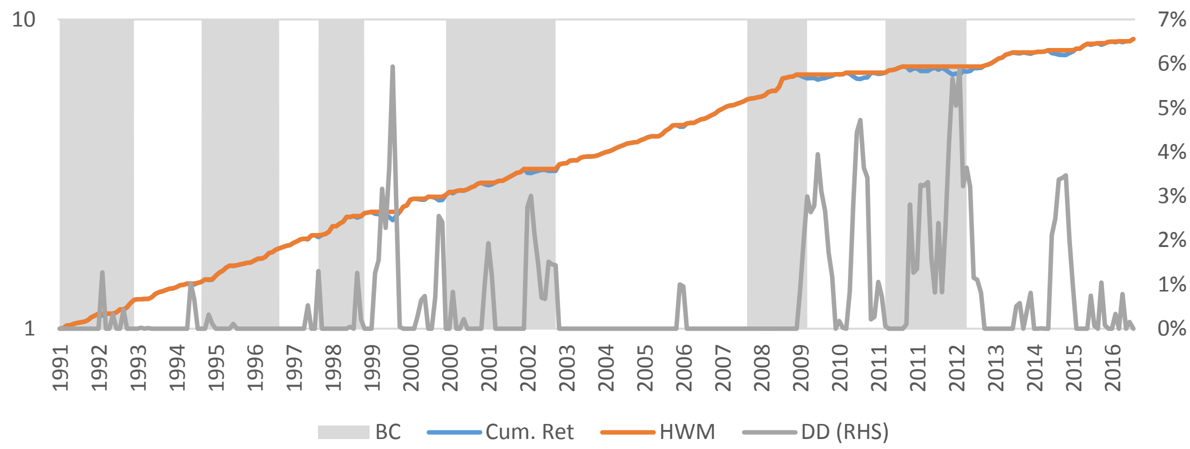

Figure 21: cumulative returns, HWM and DD for SVMVM. Logarithmic scale for LHS. The gray area marks a recession in the economy. Source: own creation

From Figure 21, we see the drawdowns go above 5% at two points. The first major drawdown recovers within a year (1999), while the second time, the drawdown takes two years to recover (2011-2013). Looking at the recessions market by the grey areas[25], we see recessions are more frequent in Europe than in the U.S. over the same period. We also see that most of the drawdowns are happening outside of recessions, except for the last recession where we see a drawdown lasting for almost the entire period.

Comparing the drawdowns of Figure 21 and Figure 18 (U.S. SVMVM) we see the same model for the U.S. market do not have the large drawdown around 1999, but both of the markets have a drawdown around 2012. The interesting thing is that this is not a recession period for the U.S. economy, suggesting the drawdown for Europe might not be a result of a recession in the economy. The frequency of drawdowns is also lower for the European market than the U.S. This indicates our European portfolio is close to be market neutral and also delivers a more consequent positive return compared to the U.S. over the same period.

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[25] Federal Reserve Bank of St. Louis (2017, June 17) OECD Recession Based Indicators for the Euro Area. Retrieved from the Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/series/EUROREC

5.2.2 Weights

	SMB	HML	RMW	CMA	WML	QMJ	BAB	Sum
Avg.	-0.01	0.18	0.14	0.03	0.09	0.09	0.08	0.59
Norm. Avg.	-0.01	0.30	0.23	0.05	0.15	0.15	0.13	1.00
SD	0.16	0.24	0.22	0.27	0.14	0.21	0.15	
Max	0.46	0.78	0.84	0.88	0.79	1.10	0.68	
Min.	-0.48	-0.58	-0.59	-1.06	-0.29	-0.56	-0.47	
Asb. Avg	0.12	0.25	0.21	0.19	0.13	0.16	0.13	1.19
Norm. Abs. Avg.	0.10	0.21	0.18	0.16	0.11	0.14	0.11	1.00

Table 29: weight allocation among factors of the SVMVM. Source: own creation

Except for RMW, we see that the top performing factors (WML and BAB) have absolute weights below average. We also see that the portfolio goes more short than long in SMB. The highest absolute value is 1.10, which also in this case shows that a weight restriction is not needed. Compared to the U.S. state variable model (Table 23), the weight allocation is very similar except BAB is on average heavier weighted in the U.S. model on the expense of HML, which is heavier weights in the European model. Considering how different the factors perform in the two markets, this is unexpected.

5.2.3 Multi-Factor Model Regressions

	CAPM			FF3			FF5			C4			8FM		
	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2
Alpha	0.53 %	8.49	0.03	0.53 %	8.33	0.03	0.42 %	6.29	0.24	0.37 %	6.22	0.10	0.33 %	5.25	0.25
Mkt-RF	-0.04	-3.12		-0.04	-2.84		-0.01	-0.61		-0.01	-0.64		0.01	0.56	
SMB				0.03	0.97		0.04	1.49		0.02	0.68		0.03	0.83	
HML				-0.00	-0.06		0.01	0.33		0.05	2.22		0.04	1.27	
RMW							0.19	3.96					0.07	1.38	
CMA							0.09	2.12					0.06	1.47	
WML										0.13	8.86		0.12	7.05	
QMJ													0.02	0.41	
BAB													-0.00	-0.03	

Table 30: excess returns of SVMVM regressed on different factor models. Source: own creation

Table 30 shows the European mean-variance model with the state variables regressed on the different state variable models. As we can see, the portfolio has a low and negative market beta in the CAPM and the Fama-French three-factor model, but the beta becomes insignificant for the other models. We can interpret this as the portfolio is still slightly negatively dependent on the market, but the dependence is now captured by the other factors. The low and negative market beta also support the hypothesis that the drawdown in Figure 18 was not due to a recession in the economy. In the eight-factor model, only WML has a significant beta with a value of 0.12. WML also had the highest factor beta for the U.S. market, which is interesting because it among the lowest weighted factors on average in both portfolios. Especially for the U.S. is this interesting as WML is not among the top performing factors. The alpha is 0.33% in Europe and 0.35% for the U.S. The t-stat of the U.S. model is higher, which might be because of the longer sample period.

5.2.4 Empirical Evidence on Factor Timing

In this chapter, we are reproducing the same analyses as in 4.5.5, but with the European data.

	Scaled SVMVM on EWP	Scaled SVMVM on MVM
Monthly alpha	0.44 %	0.06 %
Annual alpha	5.27 %	0.78 %
t-stat	5.80	3.35
P value	0.0000	0.0000

Table 31: scaled SVMVM returns regressed on the EWP and the MVM. Source: own creation

Just like earlier, the scaled SVMVM is regressed on both the EWP and MVM. The SVMVM strategy regressed on the EWP portfolio generates a monthly alpha of 0.44%, just like what we observed in the U.S. Both the t-statistic and p-value shows that our results are significant on a 99% level for Europe as well.

As we are investigating the timing effect, we once again regress the scaled SVMVM on the MVM strategy. A monthly alpha of 0.06% is observed with also is confident on a 99% level. Since on this dataset we run an out-of-sample analysis, the results are especially compelling, even though the excess returns and t-statistics are slightly lower than the observations on the U.S. market.

	Scaled SVMVM	*MVM*
Mean	0.56 %	0.51 %
Variance	0.01 %	0.01 %
Observations	302	302
Pearson Correlation	0.97	
Hypothesized Mean Diff.	0	
df	301	
t-stat	2.66	
P(T<=t) one-tail	0.00	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.01	
t Critical two-tail	1.97	

Table 32: results of the paired t-test of the scaled SVMVM and MVM. Source: own creation

We once again conducted a paired t-test to analyze whether the difference between the means are statistically significant. Similarly to 4.5.5, we define the null hypothesis as the means being the same. The alternative hypothesis is defined as the mean difference is not equal to zero, in other words, the means are not the same.

Since the observed t-statistics, 2.66, is higher than the critical two tailed T statistics 1.97, we can reject the null hypothesis with 95% confidence and accept the alternative hypothesis of the means not being the same.

5.3 Japan

5.3.1 Factor and Portfolio Performance

	Mkt-RF	SMB	HML	RMW	CMA	WML	QMJ	BAB	EWP	MVM	SVMVM
Ann. Avg. Ret	1.11 %	1.76 %	4.56 %	1.10 %	0.89 %	1.21 %	2.56 %	3.68 %	2.44 %	2.12 %	2.01 %
Ann. SD	19.30 %	11.33 %	10.06 %	7.52 %	8.39 %	15.59 %	10.70 %	14.43 %	4.92 %	5.10 %	4.85 %
Ann. SR	0.06	0.16	0.45	0.15	0.11	0.08	0.24	0.25	0.50	0.42	0.41
Median	0.07 %	0.26 %	0.29 %	0.10 %	0.15 %	0.41 %	0.25 %	0.37 %	0.22 %	0.20 %	0.14 %
Max	16.88 %	13.10 %	10.94 %	9.09 %	7.40 %	15.06 %	6.56 %	15.07 %	4.60 %	5.96 %	5.22 %
Min.	-16.22 %	-11.56 %	-13.70 %	-7.83 %	-12.98 %	-20.06 %	-9.83 %	-11.62 %	-6.52 %	-5.87 %	-4.97 %
Max DD	58.13 %	49.27 %	43.71 %	38.15 %	54.84 %	43.00 %	43.00 %	48.77 %	11.79 %	14.76 %	12.82 %
# Positive	158	171	175	164	167	171	164	170	179	177	176
# Negative	155	141	138	147	146	142	149	143	121	123	124
Avg. Positive	4.41 %	2.37 %	2.22 %	1.61 %	1.64 %	2.98 %	2.50 %	3.24 %	1.05 %	1.09 %	1.02 %
Avg. Negative	-4.30 %	-2.55 %	-1.96 %	-1.60 %	-1.72 %	-3.36 %	-2.31 %	-3.19 %	-1.06 %	-1.13 %	-1.04 %
Skewness	0.28	0.12	-0.16	0.04	-0.78	-0.45	-0.39	0.17	-0.68	-0.20	0.20
Kurtosis	0.44	1.83	2.30	1.95	4.31	2.65	0.44	0.81	2.96	1.79	1.71

Table 33: summary statistics of individual factors and portfolios. Source: own creation

In Table 33, we have the summary statistics of individual factors and the multi-factor models for Japan. Compared to the factor returns of the U.S. and Europe, all the factors perform significantly worse in the Japanese market. We see both lower average returns and more volatile returns for almost all the factors. Only HML delivers a Sharpe ratio higher than 0.26, which for the U.S. was the lowest Sharpe ration among all the factors. Europe only had one factor with a Sharpe lower than 0.39. Nevertheless, all the factors perform better than the market, which is not the case for the U.S. and Europe where both markets have two factors delivering a lower Sharpe ratio.

Going from the best performing factor, HML, to the equally weighted portfolio, we see a 9% increase in the Sharpe ratio, but as opposed to the U.S. and Europe, the equally weighted portfolio performs better than the two mean-variance models (MVM and SVMVM). We also see the HML factor portfolio is superior to the MVM and SVMVM strategies. Because the factor timing models applied in the U.S.

and Europe do not seem to work in Japan, we will investigate the reason for this and try to find any requirements for our models to work.

We know that by the properties of the Markowitz mean-variance framework and our models, that the weights placed in each factor is dependent on the average return of the factors over the last 12 months along with the correlations and volatility. Because of the historical sample, the models are dependent on the recent factor returns to contain information about the future performance of the factors. Hence the factors must continue to perform similarly in the preceding month. Looking at the high standard deviation of returns, it is clear that the individual factor returns are moving within a big range and can deviate a lot from their historical average. A factor is more likely to get a bigger allocation when it has had a recent high positive average deviation from its mean. In the long run, this positive deviation must be corrected by a below average return. These deviations and corrections will be more extreme when the standard deviations are higher. Changing returns also means changing correlations, which makes the diversification less successful. It turns out the returns are not as stable as our models require.

The high unpredictability of the factor returns and the extreme corrections are also reflected in the return of the MVM: just like U.S. and Europe, the target return is set to 10%, but as we can see, the realized annual return is around 2%. If the factors would continue with the exact same movements the next month as in the past, there would be a perfect foresight and the realized return would also been 10%. We interpret the low realized return as the past 12 months' factor returns contain little information of next month's return.

We would like to present the theory that the outperformance of the equally weighted portfolio to the mean-variance portfolios must be because of the volatile and unpredictable factor returns, so the factors that have performed the best the past year, are not as likely to perform as well in the following month, as compared to U.S. and Europe. Because of this, the mean-variance approach to exploit the recent performance and correlation of the factors, is not effective. We see that keeping the weights equal and constant yields a higher Sharpe ratio because of diversification benefits and no failed attempts to forecast the best performing factors for the next month. This theory will be investigated throughout the chapter. If this is true, it can seem like a reasonable explanation why the WML factor is unsuccessful in the Japanese market too, as the performance of the momentum factor is dependent on the top performing firms of the past 12 months to continue to perform well, while the underperforming firms continue to underperform. The failure of momentum in Japan is also documented by Fama and French (2012) as mentioned in the literature review for the momentum factor.

We also see that for the mean-variance models, the model with state variables actually performs slightly worse than the model without state variables. This implies that the state variables add no forecasting abilities over the factor returns. In this chapter, we do not analyze performance of the SVMVM portfolio or empirical evidence on factor timing as we did in Chapter 4.5 and 5.2, but rather we are looking for an answer to why the model fail to produce favorable risk-adjusted returns compared to the EWP for Japan.

5.3.2 Factor Momentum

Just as the price momentum strategy (WML) is dependent on the individual firms to continue performing similar in the future as in the present, the performance of our model is relying on that the best performing factors, which the model will long, will continue to produce a positive risk-adjusted return the next month while the worst performing factors, which the model will short (or take no position), keep on yielding low risk-adjusted returns. As stated in the last chapter, we think the underperformance of our model in Japan is because the factors do not perform the same in the next months compared to the last 12 months: the past highest performing factors are not performing as well in the next period and the worst performing factors are not performing as bad. To check this hypothesis, we build a factor momentum model. The model is very similar to the price momentum of Carhart (and Fama and French, 2012, WML); we rank the factors by the performance of the past 12 months and go long in the two factors that have the highest average returns over the period, while shorting the two worst performing factors. The only difference is that Carhart trails only 11 months of returns and lag the returns one month while we use 12 months and no lag. This is because that is how our models are built. All positions consist of the same absolute weights, leaving us with a self-financed portfolio. We get the following results:

	U.S.	Europe	Japan
Avg. Ret.	18.31 %	17.29 %	8.52 %
SD	27.55 %	24.16 %	31.29 %
SR	0.66	0.72	0.27

Table 34: returns of the factor momentum strategy. Source: own creation

	U.S.	Europe	Japan
Avg. Ret.	7.84 %	11.13 %	1.21 %
SD	15.37 %	13.92 %	15.59 %
SR	0.51	0.80	0.08

Table 35: returns of the price momentum strategy (WML). Source: own creation

Comparing the factor momentum and the price momentum strategies, we see the performances rank the same among the three markets in our sample. The factor momentum strategy of U.S. and Europe creates a Sharpe ratio which is more than the double of the Sharpe ratio of Japan. Further, we see that both the price and the factor momentum performs poorly in Japan. Looking on the ranking, it might seem that a poor performance in price momentum can be related to a poor performance of factor momentum which is important for our mean-variance strategy to work. We test this further by calculating the correlations among the two momentum strategies in each market and the MVM. The reason we test the MVM is because the weights placed are only based on past factor performance, leaving out the state variables.

	U.S.	Europe	Japan
Factor mom. vs. MVM	0.59	0.68	0.58
WML vs. MVM	0.46	0.50	0.41
Factor mom. vs. WML	0.62	0.75	0.62

Table 36: correlation of the different momentum strategies and MVM returns in the markets. Source: own creation

Looking at Table 36, we find the correlation of price and factor momentum to be high in all markets. Comparing it to the factor correlation of the U.S. market in Table 4, we see the achieved correlations are a lot higher than most factor correlations. We also find a high correlation between the factor momentum strategy and the return of the mean-variance model in all the markets. This correlation is higher than the price momentum and the MVM return.

Rationally, the performance of the factor momentum strategy seems like a reasonable explanation for why our model works in U.S. and Europe, while it fails in Japan as we explained above. Empirically, this can be supported by the high correlation of factor momentum and the MVM performance. The medium-high correlation between the WML portfolio and the MVM, makes WML a possible indicator for how good our two factor timing strategies work in a market. It is also interesting that the WML factor has achieved the highest beta values and the highest t-statistics for both U.S. and Europe in all the multi-factor model regressions. For the U.S., WML was the most significant factor in our eight-factor model regression but also the least average absolute weighted factor. In Europe, WML was the second lowest absolute weighted portfolio but had the only significant beta for the eight factor model regression.

From this, we can conclude that a factor momentum strategy that uses the last 12 months of data to form a long-short portfolio, turns out to be extremely unsuccessful in Japan. This might indicate that our model, which uses the last 12 months' factor returns and macroeconomic indicators to achieve a desired target return, might turn out to be unsuccessful simply by the fact that there are no existing trends in the given market that can be exploited.

Further, we documented that the WML factor is highly correlated to the factor momentum, which we think is a good performance measure for this exploitable trend. Next, we will investigate the performance of WML for Japan.

5.3.3 Weights

	SMB	HML	RMW	CMA	WML	QMJ	BAB	Sum
Avg.	0.02	0.07	0.10	0.16	-0.04	0.09	0.01	0.40
Norm. Avg.	0.04	0.17	0.25	0.40	-0.10	0.22	0.03	1.00
Max	0.78	1.09	1.61	2.09	0.89	0.56	0.74	
Min.	-0.81	-1.49	-0.96	-1.15	-0.60	-0.79	-0.46	
Asb. Avg	0.17	0.22	0.25	0.29	0.16	0.17	0.16	1.42
Norm. Abs. Avg.	0.12	0.15	0.17	0.21	0.11	0.12	0.11	1.00

Table 37: factor weights of the MVM portfolio. Source: own creation

As we can see, also for Japan is the WML portfolio the second least absolute weighted portfolio on average and it is also shorted more than taken a long position. This is not surprising as the portfolio is the worst performing factor. Still, there are no big outliers, and none of the factors have any extreme absolute averages. We will now see if the WML factor is significant in a multi-factor regression even when being the lowest weighted factor.

5.3.4 Multi-Factor Model Regressions

	CAPM			FF3			C4			FF5			8FM		
	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2	Coeff.	t-stat	R^2
Alpha	0.19 %	2.19	0.01	0.19 %	2.26	0.01	0.15 %	1.89	0.18	0.17 %	1.94	0.03	0.14 %	1.82	0.19
Mkt-RF	-0.03	-1.85		-0.03	-1.91		-0.01	-0.48		-0.02	-1.06		-0.02	-0.94	
SMB				-0.00	-0.17		0.00	0.13		0.00	0.13		-0.01	-0.22	
HML				-0.02	-0.64		0.05	1.80		0.02	0.47		0.07	2.19	
RMW										0.10	1.82		0.08	1.42	
CMA										-0.01	-0.12		-0.01	-0.11	
WML							0.14	7.77					0.14	7.31	
QMJ													-0.06	-1.35	
BAB													0.01	0.49	

Table 38: regression results if the MVM. Source: own creation

We see the same is true for Japan as our other markets: no matter the weights and performance of the WML portfolio, it always achieves a significant positive beta and increases the R-squared value. The positive beta means that the portfolio performs better when the WML factor performs better. This is interesting considering the model goes more short than long in WML, which would imply the opposite. The explanation power of WML must lie in WML's correlation with the other factors. Going from the Fama-French three-factor model to the four-factor model of Carhart, the R-squared increases from 0.01 to 0.18. For both the Carhart model and the eight-factor model, WML has the only significant beta with a value 0.14 and a t-statistic above 7.

5.3.5 Correlations

	SMB	HML	RMW	CMA	WML	QMJ	BAB
SMB	1.00						
HML	0.11	1.00					
RMW	-0.19	-0.37	1.00				
CMA	0.23	0.55	-0.68	1.00			
WML	-0.12	-0.28	0.32	-0.25	1.00		
QMJ	-0.26	-0.04	0.47	-0.22	0.33	1.00	
BAB	0.24	-0.06	0.07	-0.07	0.26	0.44	1.00

Table 39: correlations of the factors. Source: own creation

As the WML factor is the only significant factor in the eight-factor model (Table 38), and has the lowest average absolute weight in the SVMVM portfolio, there must be a correlation with the other factors explaining the results. For instance, we see a correlation of -0.25 with CMA and 0.32 with RMW, which are the two heaviest weights factors.

5.4 Conclusion of the Out-of-Sample Analysis

In the out-of-sample analysis part of our research, we test the EWP as well as the MVM and SVMVM with the predetermined parameter from the U.S. sample on Europe and Japan. This means a lambda of 0.2 for the MVM and lambda of 0.4 for the SVMVM, both with a one-year rolling window and state variables lagged one period.

Comparing the market returns, we document a high correlation between the U.S. and the European stock market. For Japan, the correlation with the U.S. market is much lower. We find all the factors to have the same or a lower correlation across the different markets compared to the market return. The high correlation among some of the factors between Europe and the U.S. raise a concern that the by fitting the models to the U.S. market, we also fit the models indirectly to the European market. By comparing the factor returns for U.S. and Europe further, we see that the returns are in fact very different and we can be more confident that the bias is not too extensive. The reason why the two analyses yield different results is because the correlation analysis only uses data from 1991, while in fact the model is fitted to the entire U.S. sample going back to 1973.

We find that for the European market, the SVMVM portfolio with the given parameters outperform both the EWP and the MVM, which again outperform any of the individual factor portfolios. For Japan, SVMVM and MVM fail to outperform both the passive EWP and HML, which is the best performing factor. As the EWP will outperform any single-factor portfolio, we conclude that a mean-variance investor can benefit from holding a multi-factor portfolio in every market because of the benefit of diversification it introduces. However, the fact that the timing of the dynamic portfolios only works for the European market, shows that the U.S. models are applicable to other markets, but not every market.

Analyzing the results of Europe, we see many of the same properties of the SVMVM as in the U.S.; the weight allocation is very similar except BAB is on average heavier weighted in the U.S. model on the expense of HML, which is heavier weights in the European model. The model achieves smaller and less frequent drawdowns compared to the U.S. For the eight-factor model regression, the only significant factor is the WML factor.

Similar to the in-sample analysis on the U.S., we run a paired t-test for a scaled SVMVM result to the MVM on Europe. We find empirical evidence for the factor timing effect with a significant monthly alpha of 0.08%. This also leave us with a significant non-zero difference between the SVMVM and the MVM returns. Even though the difference is marginal, we find this compelling, considering that for the European market we conducted an out-of-sample analysis and not just looking for the best possible parameters as we did for the U.S. market.

By the properties of our mean-variance models, we know that the performances is dependent on the factors to continue performing similar in the preceding month as in the past 12 months. As an explanation to the failed factor timing, we argue the results are due to poor consistency in returns among the different factors which leaves us with no exploitable trends. To compare the consistency of the factor performances, we test a factor momentum strategy for all the markets and find the strategy to perform a lot worse in Japan. We also find the strategy to be highly correlated with the price momentum strategy (WML) which is the worst performing factor in Japan. The WML factor is the only factor that achieves a positive and significant beta in an eight-factor regression model of the MVM returns for Japan, while at the same time being shorted average. This means its explanation power lies in the correlation with the other factors.

6 Summary

6.1 Conclusion

We see that across all the markets, an equally weighted portfolio of all the factors (size, value, profitability, investment, momentum, quality and betting against beta), always outperform any single-factor portfolio. Going from a single-factor portfolio consisting of the best performing factor in the market to an EWP, we see a 23%, 31% and 9% improvement of the Sharpe ratio in the U.S., European and Japanese market respectively. The equally weighted portfolio achieves a return, which is the average return of all the individual factors. Because of the correlation among the factors, the portfolio achieves a lower standard deviation and hence a higher risk-adjusted return. This means that there is a diversification effect which is easy to exploit in all the tested markets.

Fitting the mean-variance model with and without state variables (SVMVM and MVM) to the U.S. market, we find the MVM to achieve a higher risk-adjusted return compared to the EWP. The weight allocation is now changing for each period dependent on the recent factor return, variance and correlation. Adding the state variables increases the risk-adjusted return further. For both models, the one-year rolling window performs the best. We also see that the results are improved by applying shrinkage to the covariance matrix (lambda value of 0.2 for MVM and 0.4 for SVMVM). When regressing the return of the models on the eight-factor model, we achieve a significant alpha and an insignificant market beta, implying that the factor timing creates a return not explained by the other factors and that the portfolio performances is independent of the market performance. Especially for the SVMVM, we find the drawdowns to occur outside of recessions, which supports the insignificant market beta. We see that WML has the highest and most significant beta for both MVM and SVMVM.

From the U.S. sample, we conclude that a risk-adverse investor will benefit by including any of the three multi-factor portfolios, compared to a single-factor portfolio. We also see that the highest risk-adjusted return is achieved by implementing a response to some observable state variables (short-term interest rate, term spread and dividend yield) for our dynamic strategies.

Comparing U.S. and Europe, we see that most of the factors are highly correlated between 1991 and 2017. Still, the U.S. models are fit to a sample almost twice as long as the European sample and we see the factors are performing differently when comparing the performance for their entire sample periods. Applying the same models as fitted to the U.S. sample, both mean-variance models are performing better than the EWP, and the SVMVM is performing better than the MVM, just like the U.S. By running regressions on these portfolios, we find the scaled SVMVM return to be significantly higher than the MVM. This shows that the models are applicable to the out-of-sample data from Europe, and thus work as trading strategies.

For Japan, the EWP is the only multi-factor model that performs better than the best performing individual factor. We find no other explanation to this than failed attempts to time the factor allocation for the two dynamic models. To test this conclusion, we develop a factor momentum strategy and use this to test for exploitable trends in the different markets. The strategy turns out to be highly correlated with the MVM return for all of the markets and performs poorly in Japan.

6.2 Further Studies

6.2.1 Analyze Additional Markets

It would be interesting to apply the models of this thesis to more than the three markets investigated. Three markets are not enough to reach a general conclusion of our models applicability and to test if our findings of a relationship between the model performance and factor momentum performance. As explained in the data section of the methodology, the available data on the Kenneth French website is limited to only three relevant markets. Extending the markets then requires building the portfolios for the desired markets from scratch or find alternative datasets. AQR supplies SMB, HML, WML, BAB and QMJ data for individual countries, which can be used to test our model for an expanded selection of markets, but with a smaller selection of factor portfolios.

6.2.2 Include More State Variables (Macroeconomic Factors)

There are many more macroeconomic indicators that have been proved to have some kind of relationship with the market return. It would be interesting to add more state variables and see if they could improve the risk-adjusted return of the SVMVM compared to the MVM, which would imply improved factor timing by the state variables. There are generally two concerns by adding more state variables to the SVMVM: calculation time and availability of data. Adding one more state variables means that the extended asset space increases by the number of factors, which increases the calculation time of the model significantly. Also, macroeconomic indicators like GDP, employment rate and inflation would be interest to add, but the numbers are usually not available instantly and a time lag must be added to the values to make sure they were available by the time of the asset allocation.

6.2.3 Shrink Returns

To make the realized returns closer to the target return, shrinkage of the returns can be applied. We do not expect the shrinking to have an effect of the Sharpe ratios of the portfolios, as the returns seem to be located along the tangency line (see 4.5.1). Still, it is relevant for a real life setting, where the investor is not only concerned about the risk-adjusted returns, but also the realized returns. Interested readers are suggested to look up LASSO regression analysis.

6.2.4 Consider Trading Costs

As the portfolios are rebalanced monthly and new long and short positions are taken, there are a lot of buy and sell transactions being made between two periods. We do not see the changing positions as our data is only for the factor portfolios, and not the underlying stocks, but if the portfolios where created from scratch, it would be interesting to include an approximation to the transaction costs for the rebalancing to see if our models could still beat the market or other factor returns that can be achieved through index funds. When transaction costs are included, it would also be interesting to test a quarterly rebalancing strategy to reduce the transaction costs.

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7.2 Databases

AQR: https://www.aqr.com/library/data-sets

Datastream (Thomson Reuters): access through CBS computers

FRED: https://fred.stlouisfed.org/categories/32262

Kenneth French website:

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

8 Appendix

8.1 Step by Step Example of SVMVM

A numerical and illustrative example of our mean-variance model with state variables is provided to clarify any uncertainties about our model. This example is with a one year rolling window, and the factors are limited to SMB and HML.

8.1.1 Expanding the Asset space

First step is to expand the asset space according to (28).

	Factor Return		State Variables			Factor return		z1		z2		z2	
Period	SMB	HML	T-Bill	Term	Div								
	r1	r2	z1	z2	z3	r1	r2	r1	r2	r1	r2	r1	r2
0	-2,80%	2,70%	5,41%	1,23%	2,27%								
1	-4,02%	1,70%	5,03%	1,68%	2,35%	-4,02%	1,70%	0,22%	0,09%	-0,05%	0,02%	-0,09%	0,04%
2	-2,36%	2,83%	5,66%	1,01%	2,36%	-2,36%	2,83%	-0,12%	0,14%	-0,04%	0,05%	-0,06%	0,07%
3	-3,01%	5,69%	6,42%	0,43%	2,47%	-3,01%	5,69%	-0,17%	0,32%	-0,03%	0,06%	-0,07%	0,13%
4	-5,99%	0,21%	6,29%	0,61%	2,50%	-5,99%	0,21%	-0,38%	0,01%	-0,03%	0,00%	-0,15%	0,01%
5	-2,54%	1,42%	6,29%	0,84%	2,51%	-2,54%	1,42%	-0,16%	0,09%	-0,02%	0,01%	-0,06%	0,04%
6	7,25%	-5,18%	7,96%	-0,56%	2,48%	7,25%	-5,18%	0,46%	-0,33%	0,06%	-0,04%	0,18%	-0,13%
7	-1,76%	1,19%	8,73%	-1,64%	2,60%	-1,76%	1,19%	-0,14%	0,09%	0,01%	-0,01%	-0,04%	0,03%
8	3,54%	2,11%	8,47%	-1,68%	2,55%	3,54%	2,11%	0,31%	0,18%	-0,06%	-0,03%	0,09%	0,05%
9	-0,21%	1,74%	8,08%	-1,35%	2,62%	-0,21%	1,74%	-0,02%	0,15%	0,00%	-0,03%	-0,01%	0,04%
10	-7,30%	4,01%	6,93%	-0,19%	2,96%	-7,30%	4,01%	-0,59%	0,32%	0,10%	-0,05%	-0,19%	0,11%
11	-4,63%	4,24%	7,96%	-0,97%	2,95%	-4,63%	4,24%	-0,32%	0,29%	0,01%	-0,01%	-0,14%	0,13%
12	10,50%	5,86%	7,83%	-0,87%	3,02%	10,50%	5,86%	0,84%	0,47%	-0,10%	-0,06%	0,31%	0,17%

Appendix Table 1: expanding the asset space example. Source: own creation.

8.1.2 Covariance Matrix

We create a covariance matrix based on the 12 observations.

Covariance Matrix

		Factor return		z1		z2		z2	
		r1	r2	r1	r2	r1	r2	r1	r2
Factor return	r1	0,002719454	-0,00046187	0,000196638	-2,2705E-05	-8,63639E-06	-8,1242E-06	7,30439E-05	-9,8342E-06
	r2	-0,00046187	0,001245508	-2,2705E-05	8,53942E-05	-8,12417E-06	1,41777E-07	-9,8342E-06	3,28118E-05
z1	r1	0,000196638	-2,2705E-05	1,44741E-05	-9,5997E-07	-8,72978E-07	-5,2766E-07	5,3244E-06	-4,3345E-07
	r2	-2,2705E-05	8,53942E-05	-9,5997E-07	6,00409E-06	-5,27663E-07	-1,3047E-07	-4,3345E-07	2,27417E-06
z2	r1	-8,6364E-06	-8,1242E-06	-8,7298E-07	-5,2766E-07	2,76123E-07	-4,4079E-08	-2,724E-07	-2,1623E-07
	r2	-8,1242E-06	1,41777E-07	-5,2766E-07	-1,3047E-07	-4,40793E-08	1,35436E-07	-2,1623E-07	-1,9182E-08
z2	r1	7,30439E-05	-9,8342E-06	5,3244E-06	-4,3345E-07	-2,72396E-07	-2,1623E-07	1,97666E-06	-1,9712E-07
	r2	-9,8342E-06	3,28118E-05	-4,3345E-07	2,27417E-06	-2,16232E-07	-1,9182E-08	-1,9712E-07	8,72156E-07

Appendix Table 2: covariance matrix example. Source: own creation

8.1.3 Shrinkage and Solver

Next, we shrink the covariance matrix according to (40) and run the solver to achieve target return with minimal standard deviation by assigning weights to the expanded asset space.

Shrink Int	0,400

Covariance Matrix after Shrinkage

		Factor return		z1		z2		z2	
		r1	r2	r1	r2	r1	r2	r1	r2
Factor return	r1	0,002719454	-0,00027712	0,000117983	-1,3623E-05	-5,18183E-06	-4,8745E-06	4,38263E-05	-5,9005E-06
	r2	-0,00027712	0,001245508	-1,3623E-05	5,12365E-05	-4,8745E-06	8,50659E-08	-5,9005E-06	1,96871E-05
z1	r1	0,000117983	-1,3623E-05	1,44741E-05	-5,7598E-07	-5,23787E-07	-3,166E-07	3,19464E-06	-2,6007E-07
	r2	-1,3623E-05	5,12365E-05	-5,7598E-07	6,00409E-06	-3,16598E-07	-7,8284E-08	-2,6007E-07	1,3645E-06
z2	r1	-5,1818E-06	-4,8745E-06	-5,2379E-07	-3,166E-07	2,76123E-07	-2,6448E-08	-1,6344E-07	-1,2974E-07
	r2	-4,8745E-06	8,50659E-08	-3,166E-07	-7,8284E-08	-2,64476E-08	1,35436E-07	-1,2974E-07	-1,1509E-08
z2	r1	4,38263E-05	-5,9005E-06	3,19464E-06	-2,6007E-07	-1,63437E-07	-1,2974E-07	1,97666E-06	-1,1827E-07
	r2	-5,9005E-06	1,96871E-05	-2,6007E-07	1,3645E-06	-1,29739E-07	-1,1509E-08	-1,1827E-07	8,72156E-07

Observation Average

-0,88%	2,15%	-0,04%	0,15%	-0,01%	-0,01%	-0,02%	0,06%

Weights

-0,17	1,39	0,00	24,27	-3,76	-0,30	-4,70	52,68

Return	10,00%
SD	6,61%
Target Ret	10,00%

Appendix Table 3: calculating return and standard deviation example. Source: own creation

8.1.4 Calculating Factor Weights

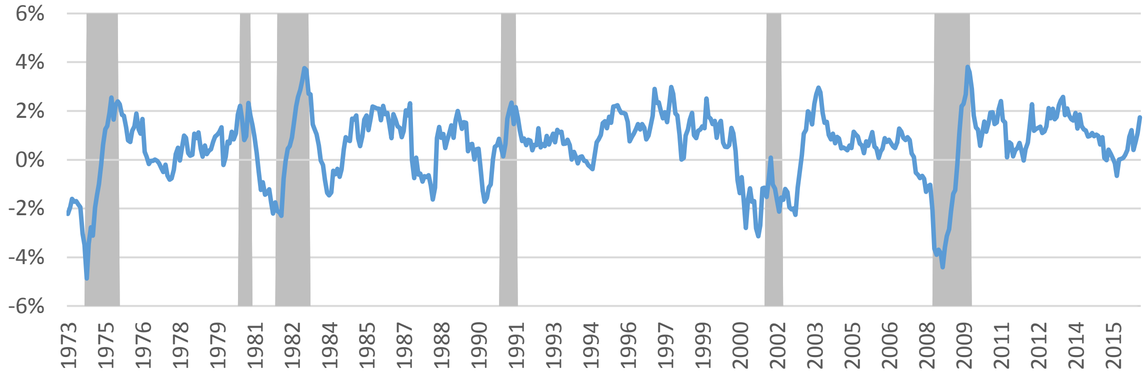
Next we observe the state variables and assign the factor weights based on the weights for the expanded asset space and the state variables. The return is given by the factor returns and their respective weights.

	Factor Returns		State Variables			Factor weights			Weights of Augmented Asset Space							
									Factor return		z1		z2		z2	
Period	SMB	HML	T-Bill	Term	Div	SMB	HML	Return								
	r1	r2	z1	z2	z3				r1	r2	r1	r2	r1	r2	r1	r2
0	-2,80%	2,70%	5,41%	1,23%	2,27%											
1	-4,02%	1,70%	5,03%	1,68%	2,35%											
2	-2,36%	2,83%	5,66%	1,01%	2,36%											
3	-3,01%	5,69%	6,42%	0,43%	2,47%											
4	-5,99%	0,21%	6,29%	0,61%	2,50%											
5	-2,54%	1,42%	6,29%	0,84%	2,51%											
6	7,25%	-5,18%	7,96%	-0,56%	2,48%											
7	-1,76%	1,19%	8,73%	-1,64%	2,60%											
8	3,54%	2,11%	8,47%	-1,68%	2,55%											
9	-0,21%	1,74%	8,08%	-1,35%	2,62%											
10	-7,30%	4,01%	6,93%	-0,19%	2,96%											
11	-4,63%	4,24%	7,96%	-0,97%	2,95%											
12	10,50%	5,86%	7,83%	-0,87%	3,02%	-0,27	4,88		-0,17	1,39	0,00	24,27	-3,76	-0,30	-4,70	52,68
13	0,16%	2,55%	7,19%	0,02%	3,07%			12,40%								

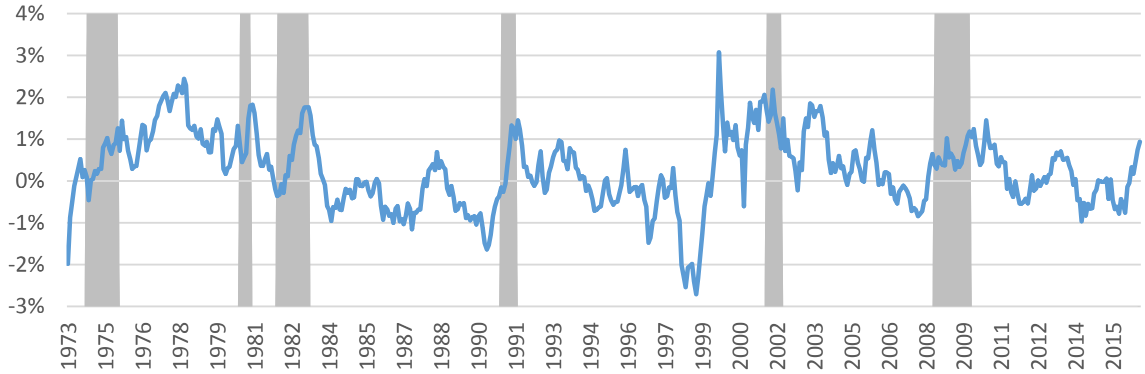
Appendix Table 4: going from weights of expanded asset space to factor weights. Source: own creation

The process is repeated for all the periods in our samples.

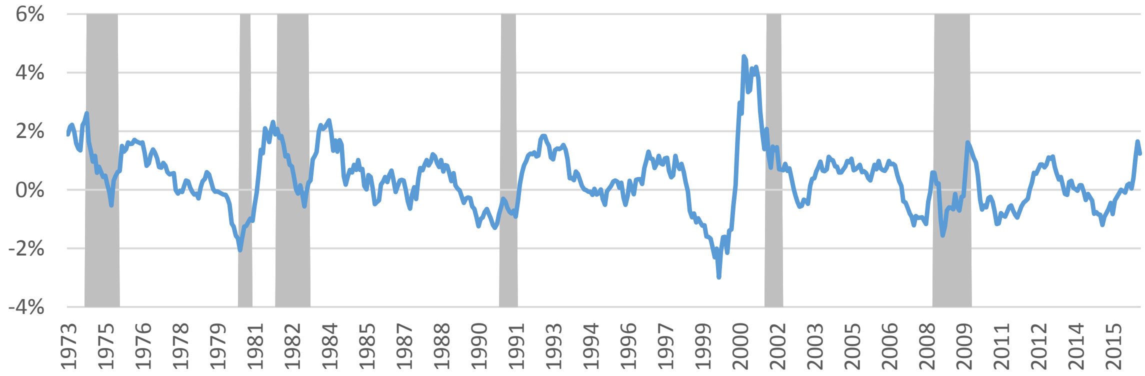
8.2 Factor Returns



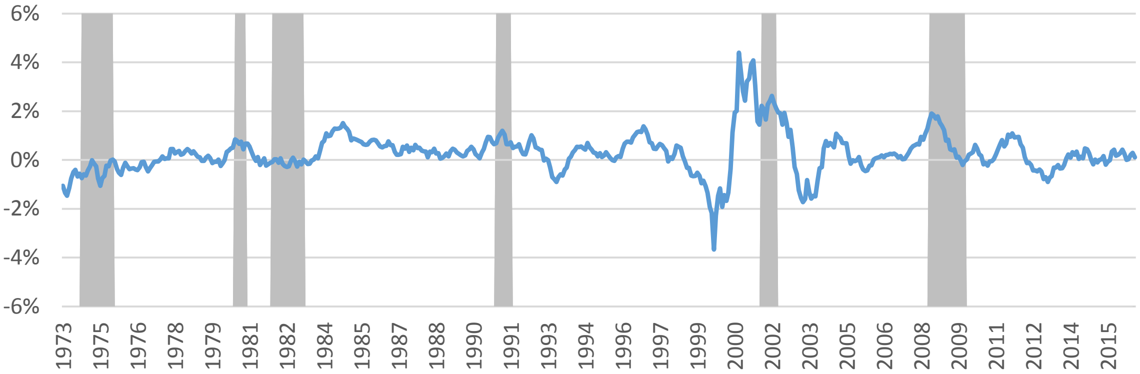
Appendix Figure 1: 12 month rolling average excess market return



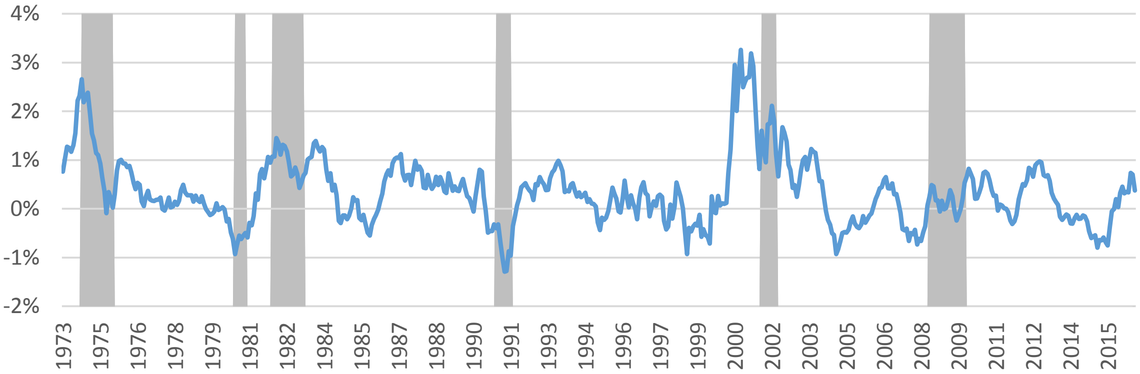
Appendix Figure 2: 12 month rolling average SMB return



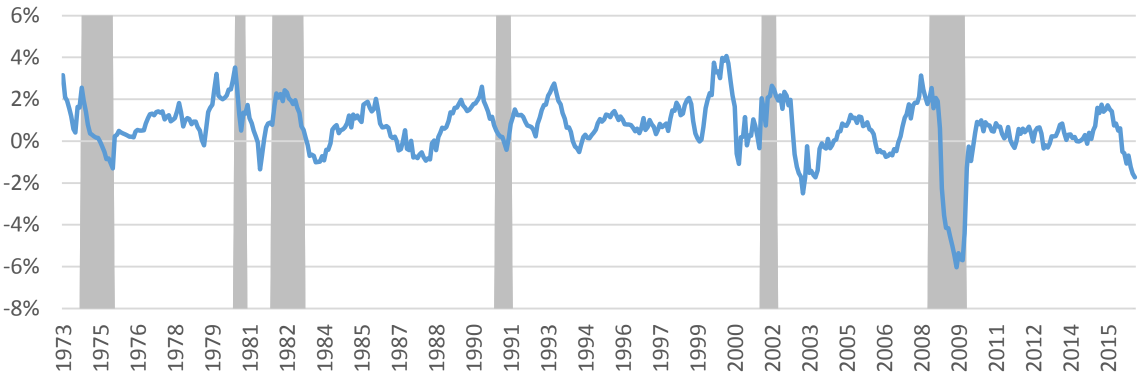
Appendix Figure 3: 12 month rolling average HML return



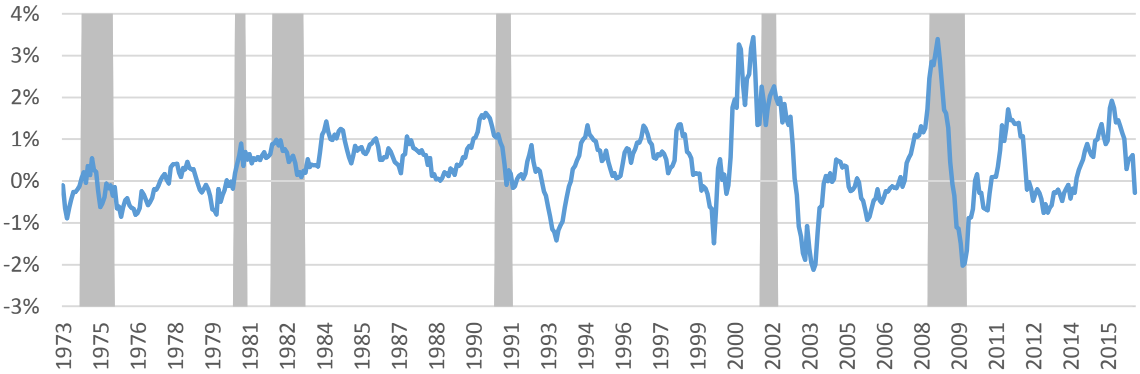
Appendix Figure 4: 12 month rolling average RMW return



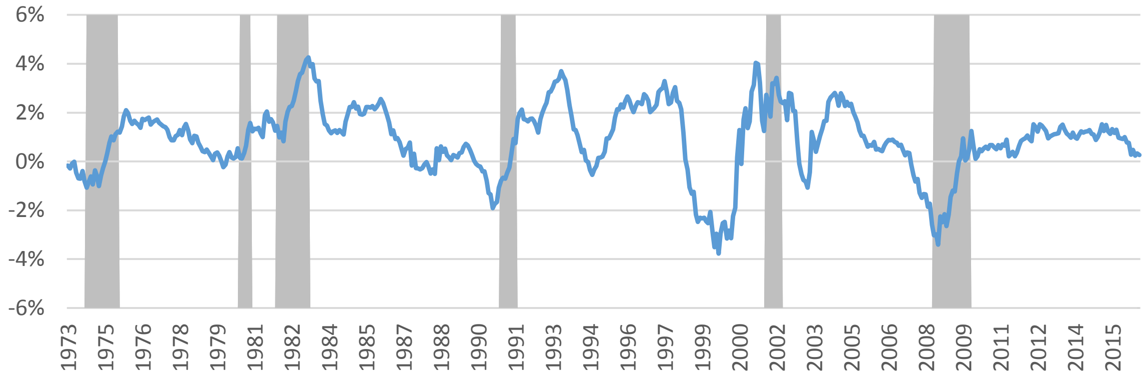
Appendix Figure 5: 12 month rolling average CMA return



Appendix Figure 6: 12 month rolling average WML return



Appendix Figure 7: 12 month rolling average QMJ return



Appendix Figure 8: 12 month rolling average BAB return

8.3 Regression Results for Chapter 4.5.5

Regression Statistics	
Multiple R	0.9132
R Square	0.8340
Adjusted R Square	0.8337
Standard Error	0.0046
Observations	516.0000

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.0000	0.0556	0.0556	2,582.4247	0.0000
Residual	514.0000	0.0111	0.0000		
Total	515.0000	0.0667			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0008	0.0002	3.7917	0.0002	0.0004	0.0013	0.0004	0.0013
X Variable 1	0.9132	0.0180	50.8176	0.0000	0.8779	0.9485	0.8779	0.9485

Appendix Table 5: scaled SVMVM return regressed on the MVM

Regression Statistics	
Multiple R	0.3945
R Square	0.1557
Adjusted R Square	0.1540
Standard Error	0.0131
Observations	516.0000

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.0000	0.0162	0.0162	94.7598	0.0000
Residual	514.0000	0.0876	0.0002		
Total	515.0000	0.1038			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0049	0.0006	8.1622	0.0000	0.0037	0.0061	0.0037	0.0061
X Variable 1	0.3945	0.0405	9.7345	0.0000	0.3149	0.4742	0.3149	0.4742

Appendix Table 6: scaled SVMVM returns regressed on the EWP