

# Copenhagen Business School

## Master's Thesis

MSc in Financial Strategic Management & Applied Economics and Finance

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### Investigating the beta anomaly on the Oslo Stock Exchange: A cross-sectional analysis

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# Abstract

We investigate the ability of the CAPM, the Fama-French three-factor model, and the Carhart four-factor model to describe the average monthly stock returns on the Oslo Stock Exchange in the period 2009 – 2017. In addition, we test whether the observed beta anomaly on the Oslo Stock Exchange can be attributed constrained investors in terms of leverage and margin requirements, or if the beta anomaly can be attributed a demand for lottery-like stocks. Using a sample free of survivorship bias, we construct a market factor and factors related to firm size, book-to-market-value of equity, momentum, margin- and leverage constrained investors, and the demand for lottery stocks. We apply the portfolio sorts approach, as well as Fama and MacBeth (1973) regressions on portfolios formed on firm characteristics to estimate factor exposures and risk premia.

From our portfolio sorts approach we find no systematic pattern in the excess returns on portfolios sorted on firm characteristics. The CAPM estimates insignificant market risk premia across all portfolio sorts. We find that the three-factor model increases the predictability of excess stock returns on the Oslo Stock Exchange, and produces a significant market risk premium and a SMB risk premium for portfolios sorted on size. Surprisingly, both risk premia are estimated to be negative. The three-factor model has trouble explaining the returns on portfolios sorted on other firm characteristics. The inclusion of a momentum factor does not improve the predictability significantly, and we conclude that the estimated factor models explain the cross-section of excess returns on the Oslo Stock Exchange to a limited extent.

We find that constrained investors in terms of leverage and margin requirements are a likely contributor to the beta anomaly on the Oslo Stock Exchange. Further, we find no evidence that a demand for lottery stocks contribute to the beta anomaly. This coincides with the lottery stock hypothesis, given the low ownership share of private investors on the Oslo Stock Exchange.

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# 1. INTRODUCTION

## 1.1 BACKGROUND AND MOTIVATION

Asset pricing theory aims to explain why some assets pay higher average returns than other. What might seem like a relatively trivial question has dominated financial literature for decades, and has split research into opposing schools of thought. First, there are approaches that originate from the consumption-based model, asserting that stocks whose returns have a negative correlation with the marginal utility of consumption must offer higher expected returns to investors. Typically, these models attempt to identify common sources of risk, formally known as systematic risk factors, which correlate with asset returns. In contrast, advocates for the behavioral paradigm argue that asset returns cannot simply be explained by a set of risk factors. Rather, asset returns are determined in the marketplace by cognitively biased investors who make irrational decisions.

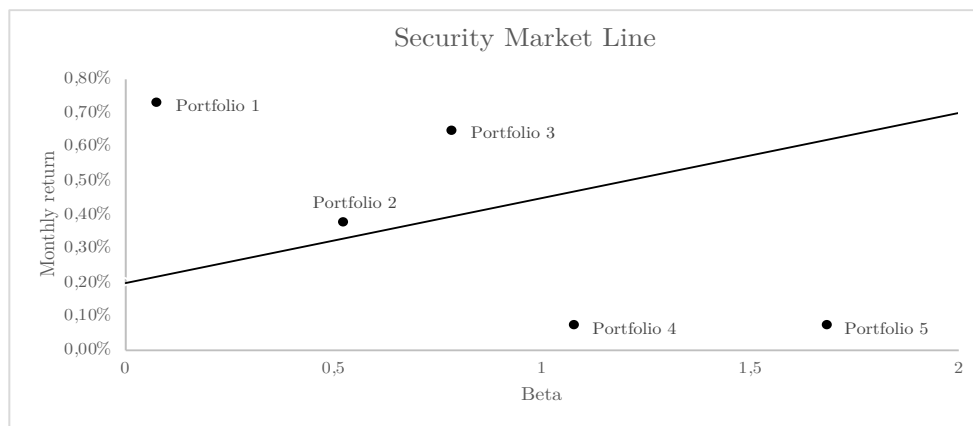
The capital asset pricing model (CAPM) is arguably the most prominent and important financial model taught in finance courses, and is central in the systematic risk-based paradigm. The CAPM postulates that the excess return of a well-diversified portfolio is a function of its covariance with the market portfolio, i.e., its systematic risk. In spite of its prominence, early empirical research found the CAPM to be fundamentally flawed:

Already in the early 1970's, Jensen, Black, and Sholes (1973) found that the security market line was too flat. Furthermore, empirical evidence using US data showed that high-beta stocks underperform low-beta stocks on a risk-adjusted basis (Fama & French, 1992). These empirical findings contradict the central prediction of the CAPM, and the phenomenon has been coined “the beta anomaly”.

The beta anomaly is not confined to the United States. Frazzini and Pedersen (2014) show that the beta anomaly exists in 19 developed markets in the period from 1989 to 2012. In 2018, Finn Øystein Bergh wrote an article in the Norwegian financial newspaper *Dagens Næringsliv*, where he argued that low-beta stocks on the Oslo Stock Exchange (OSE) had provided an annual return of 19.2% versus -8% for high-beta stocks in the period 2001 - 2017 (Bergh, 2018). This article serves as the main motivation for this thesis. In the early stages of this project, we collected data from the OSE in the period 2007 –

2017. Subsequently, we constructed five portfolios sorted on market beta to see whether we could find a similar relationship between historical returns and beta. Figure 1 shows average monthly return from the five portfolios in the period 2007 – 2017 plotted against the predicted return according to CAPM, as illustrated by the Security Market Line (SML)<sup>1</sup> The figure indeed confirms that the beta anomaly was present on the OSE in the period. As a result, the motivation for this thesis is to identify the factors that explain the cross-section of returns on the OSE. Furthermore, the thesis will aim to find an explanation for the negative relationship between systematic market risk and returns on the Norwegian stock market.

**FIGURE 1** – MONTHLY RETURNS FROM BETA-SORTED PORTFOLIOS VS. SML (2007 – 2017)



Recently, two influential papers have attempted to explain the beta anomaly. In 2014, Frazzini and Pedersen (2014) presented evidence that lower risk-adjusted returns for high-beta stocks can be explained by the fact that investors are constrained in terms of leverage and margin requirements. Specifically, constrained investors purchase assets riskier than would be optimal to achieve higher returns. They argue that the tilt towards high-beta assets results in a lower risk-adjusted return for these assets compared to low-beta assets. By constructing a betting against beta (BAB) factor, defined as a portfolio that holds low-beta assets, leveraged to a beta of one, and that shorts high-beta assets, de-levered to a beta of one, they achieve positive abnormal returns. With reference to the

<sup>1</sup> For a detailed description of our data sample, see Section 4 – Data. For description of the portfolio construction, see Section 5 - Methodology

Arbitrage Pricing Theory (APT), they argue that the prevailing arbitrage returns imply that the beta anomaly is a consequence of constrained investors.

In a more recent paper, Bali, Brown, Murray, and Tang (2017) attribute the beta anomaly to investors' demand for stocks with lottery-like payoffs, referred to as "lottery stocks". Bali et al. (2017) proxy the lottery demand by MAX, defined as the average of the five highest daily returns of the given stock in a given month. They demonstrate that the abnormal returns of a long-short beta portfolio, similar to that of Frazzini and Pedersen (2014), are no longer significant when the portfolio is constrained to be neutral to MAX.

The purpose of this thesis is two-fold: first, we aim to investigate whether the cross-section of returns in the period 2007 – 2017 can be explained by using standard asset pricing models. However, as argued in subsection 6.2.2, we choose to exclude the 2007 – 2008 period from our sample period based on our findings in the portfolio sorts approach. We will estimate systematic risk factors on the OSE using three asset pricing models: the CAPM, the Fama and French three-factor model (Fama & French, 1992), and the Carhart four-factor model (Carhart, 1997). Second, we will investigate whether the beta-anomaly on the OSE can be attributed to leverage and margin constraints, following Frazzini and Pedersen (2014), and/or whether the beta anomaly on the OSE can be attributed a demand for lottery-like stocks, following Bali et al. (2017).

The abovementioned is synthesized into the following research questions:

1. *To what extent does the CAPM explain the cross-section of returns on the OSE in the period 2009 – 2017?*
2. *To what extent does the Fama-French three-factor model explain the cross-section of returns on the OSE in the period 2009 – 2017?*
3. *To what extent does the Carhart four-factor model explain the cross-section of returns on the OSE in the period 2009 – 2017?*
4. *Can the beta anomaly be attributed to leverage and margin constraints with investors, or can it be attributed to a demand for stocks with lottery-like returns?*

## 1.2 DELIMITATIONS

In their 1992 paper, Fama and French (1992) constructed the SMB factor based on the empirical evidence that stocks with a small market capitalization provide higher risk-adjusted returns than stocks with a large market capitalization. Similarly, the HML factor was constructed based on the empirical evidence that stocks with high book-to-market values of equity (BM) provide higher risk-adjusted returns than stocks with low book-to-market values of equity. They argue that the factors represent unidentified systematic risks. However, there has been, and still is, considerable debate whether they represent systematic risk factors, or whether the difference in risk-adjusted returns can be attributed to investors' irrational behavior, as proposed by proponents of the behavioral paradigm. In this thesis, we will not discuss whether the factors do in fact represent systematic risks. The factors are merely constructed in an attempt to explain the cross-section of returns on the OSE. Hence, when we refer to “risk-factors”, we do not ascribe the returns from the factors as a function of some systematic risk.

Our sample does not include every stock on the OSE in the period 2007 – 2017. The reason for this is that various filter criteria are applied to the raw data before entering our data sample. For example, penny stocks, defined as stocks trading below NOK 5, and stocks with less than 50 yearly trading days, are excluded. For a detailed explanation of the sample construction, see Section 4 – Data.

## 2 LITERATURE REVIEW

### 2.1 THE CONSUMPTION-BASED MODEL

Asset pricing theories attempt to explain the prices or values of claims to uncertain payments. Since a low price implies a high rate of return, one may also think of the theory as explaining why some assets pay a higher return than others. All asset pricing theory stems from the concept that an asset's price is equal to its discounted payoff. With this in mind, there are two distinct approaches to which assets are priced: *absolute pricing* and *relative pricing*. Absolute pricing bases prices on economic theory, and use fundamentals such as exposure towards macroeconomic risk and references of economic agents to find an asset's price, irrespective of the price of other assets. Relative pricing, on the other hand, uses the price of other assets as a foundation for valuation. A typical relative pricing model is the Black-Scholes option pricing model that determines an option's price, given the price of an underlying stock. The CAPM and succeeding factor models, on the other hand, are paradigms of the absolute pricing approach.

Factor pricing models aim to explain risk premia that can be observed in the market. The models can be explained by deriving insights from the consumption-based model, which asserts that investors have an increasing marginal utility with consumption, but at a decreasing rate<sup>2</sup>. Consequently, rational investors have a higher marginal utility of consumption in bad times than in good times. To maximize their utility, investors are drawn towards stocks that perform well in bad times, i.e., low beta stocks, increasing the price and lowering returns. Conversely, the price for stocks that have low returns in bad times, i.e., high beta stocks, will fall and yield higher returns. Since marginal utility is high when consumption is low, one expects that assets that covary with the market have lower prices and thereby a higher market risk premium.

The intuition behind factor models, and how they can predict stock market returns, can be explained by the investors' *utility function* and the *basic pricing equation*. To figure out the value of any stream of cash flows, one must determine what the cash flow is worth to a typical investor. Investors' utility function is defined over current and future values of

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<sup>2</sup>The following section is taken from Cochrane (2009), chapter 1 - Consumption-based model and overview.

consumption. At each point in time, an investor faces a trade-off between immediate consumption and investing for future consumption:

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})] \quad (1)$$

Where  $U$  denotes the utility to the investor,  $c_t$  and  $c_{t+1}$  is consumption at time  $t$  and  $t + 1$ , respectively. The utility function is concave and captures the fundamental desire for consumption and the fact that consumption in  $t + 1$  is uncertain. The beta is a subjective discount factor that captures investors' impatience. Since investors face a trade-off between consumption and investment, investors will choose the optimal level of consumption and investment by maximizing Equation 1, subject to the budget constraint that increased consumption today reduces consumption tomorrow, and vice versa. Maximizing the utility function in Equation 1, subject to the budget constraint, yields the first-order condition for optimal consumption and portfolio choice:

$$p_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right] \quad (2)$$

Equation 2 is the basic pricing equation. Given the payoff,  $x_t + 1$  and the investor's optimal consumption choice,  $c_t, c_t + 1$ , it gives you the expected market price,  $p_t$ .

Equation 2 is often broken up to define the stochastic discount factor,  $m_t + 1$ :

$$m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)} \quad (3)$$

The stochastic discount factor is the stochastic variable that satisfies the basic pricing equation. Thus, according to the consumption-based model, there is one single stochastic discount factor that prices all assets. Accordingly, this satisfies the law of one price and implies a market free of arbitrage.

Substituting the stochastic discount factor into Equation 3, we get that a security's price can be expressed as the expected payoff from the given security, discounted with some discount factor, accounting for the rate at which the investor is willing to substitute consumption at time  $t + 1$  for consumption at time  $t$ :

$$p_t = E_t(m_{t+1}x_{t+1}) \quad (4)$$

Recognizing that the gross rate of return on a risky asset is defined as:

$$1 + R_{t+1} \equiv \frac{x_{t+1}}{p_t} \quad (5)$$

The price function can be divided by  $p_t$ , to obtain the Euler equation:

$$1 = E_t(m_{t+1}R_{t+1}) \quad (6)$$

The Euler equation clearly shows that the returns of risky assets and consumption (through the ratio of marginal utilities) are related. Since both the stochastic discount factor and the gross rate of return are stochastic variables, the Euler equation can be rewritten<sup>3 4</sup>:

$$1 + E_t(m_{t+1})E_t(R_{t+1}) + Cov(m_{t+1}, R_{t+1}) \quad (7)$$

By considering a risk-free asset instead of a risky asset, Equation 7 can be rewritten. Since the risk-free asset offers certain returns, i.e., returns that are uncorrelated with consumption, the covariance between the return of the stochastic discount factor and the return of a risk-free asset will be zero. Hence:

$$1 = E_t(m_{t+1})E_t(R_{f,t+1}) \quad (8)$$

$$1 + R_{f,t+1} = \frac{1}{E_t(m_{t+1})} \quad (9)$$

By using the expression for the gross rate of return on a risk-free asset in Equation 7, and a simplified notation for a clear distinction between the risky and the risk-free asset, we get that:

$$E(R_i) - R_f = -R_f Cov(m, R_i) \quad (10)$$

$$E(R_i) - R_f = -\frac{1}{E(m)} Cov(m, R_i) \quad (11)$$

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<sup>3</sup> See Appendix 1 for details.

<sup>4</sup> The following section is taken from Cochrane (2009), chapter 6 – Relation between discount factors, betas, and mean-variance frontiers.

$$R(R_i) - R_f = -\frac{Cov[u'(c_{t+1}), R_{i,t+1}]}{E[u'(c_{t+1})]} \quad (12)$$

Equation 12 illustrates that the expected excess returns of assets will differ only because the covariance between of their respective returns with the marginal utility of consumption differs (Wälti, 2007). An asset whose return has a negative covariance with the marginal utility of consumption, and hence, a positive covariance with consumption, due to diminishing marginal utility, must necessarily offer higher expected returns for investors to be willing to hold this asset.

## 2.2 LINEAR FACTOR MODELS

In the consumption-based model, the stochastic discount factor is tied to the investors' marginal utility associated with changes in consumption. However, these models have proved not to work well in practice, and the stochastic discount factor has been tied to other types of data.<sup>5</sup> Linear factor models replace the consumption-based expression for marginal utility growth with a linear model and can be derived from Equation 6. Solving for the expected return, multiplying and dividing by  $var(m)$  and defining  $\alpha \equiv \frac{1}{E_t(m)}$ , one derives the expected return-beta representation:

$$E(R_i) = \frac{1}{E(m)} - \frac{Cov(m, R^i)}{E(m)} \quad (13)$$

$$E(R_i) = \alpha + \left( \frac{Cov(m, R^i)}{var(m)} \right) \left( -\frac{var(m)}{E(m)} \right) \quad (14)$$

$$E(R_i) = \alpha + \beta_{i,m} \lambda_m \quad (15)$$

As seen from Equation 15, factor models replace the consumption-based expression for marginal utility with a linear model. Here,  $\beta_{i,m}$  is to be interpreted as the quantity of risk

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<sup>5</sup> The following section is taken from Cochrane (2009), chapter 9 – Factor pricing models



in each asset, and  $\lambda_m$  is the risk premium. Alternatively, one may operate with a multiple-beta model, in which the betas are multiple regression coefficients:

$$m_{t+1} = \alpha + \beta' f_{t+1} \quad (16)$$

Where  $f_{t+1}$  represent multiple factors. For the models to be effective, the factors must be good proxies for aggregating marginal utility growth, so that:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx \alpha + \beta' f_{t+1} \quad (17)$$

For the factor models to aggregate marginal utility growth with investors, satisfying the expression above, the factors must represent certain states in the economy in which investors are willing to trade off average return in their portfolios to perform well in "bad states" of the economy. Positive expected returns are associated with positive correlation with consumption growth, and hence a negative correlation with marginal utility growth, due to diminishing marginal utility of consumption. Thus, we expect  $\lambda > 0$ , from Equation 15. Although factors such as market returns, size or ratio of market-to-book value of equity do not measure states of the economy directly, they are factors that, according to the consumption-based model, aggregate the marginal utility growth of investors. Hence, as illustrated, all factor models are derived as specializations of the consumption-based model.

Factor models and the identification of appropriate factors that explain the cross-section of returns have received significant attention from financial researchers, and consequently, been subject to considerable debate. The following section will provide a broad overview of the development of factor models, as well as the theoretical underpinnings and empirical findings of the models leading to the development of the four-factor model.

### 2.3 DEVELOPMENT OF FACTOR MODELS

Drawing on the central insights from Markowitz (1952), who asserts that investors are inherently risk-averse and will maximize their expected return for a given level of risk, the capital asset pricing model (CAPM), developed by Treynor (1961, 1962), Sharpe (1964) and Lintner (1965a), became the fundamental framework for explaining returns across

assets. Founded on a set of relatively restricting assumptions regarding market efficiency and investor rationality, the model attempts to explain stock market returns as a function of individual stocks' systematic risk. In early empirical tests of the CAPM, Lintner (1965b) and Douglas (1968) indeed confirmed a strong positive relationship between the cross-section of average asset returns and market beta. However, imprecise measurements of market betas and inference problems related to the regression residuals, lead to improved testing methodologies proposed by Jensen, Black, and Scholes (1972) and Fama and MacBeth (1973). Also, subsequent empirical testing of Sharpe-Lintner version of the CAPM found the positive relationship between beta and average returns to be "too flat" (Fama & French, 2004).

Starting in the late 1970s, the revised version of the CAPM was criticized by researchers, challenging the fundamental premise that much of the variation in expected return is only attributable to market beta. Basu (1977) showed that when stocks sorted on earnings-price ratios, future returns on high E/P-ratios are higher than predicted by the CAPM. Subsequent studies found that fundamental variables, such as size (Banz, 1981), debt-equity ratios (Bhandari, 1988), and book-to-market equity ratios (Stattman, 1980; Rosenberg, Reid & Lanstein, 1985) explain significant variation in average returns not captured by their market betas. Furthermore, Jegadeesh and Titman (1993) challenge the efficient market hypothesis in their evidence that stocks with high short-term previous returns tend to have higher future returns, and vice versa, as naïve investors extrapolate past trends into the future.

The synthesis of the evidence on the empirical problems of the CAPM culminates in the three-factor model, proposed by Fama and French (1992). In addition to the market factor used in the original CAPM, a size and a value factor are used to explain the cross-section of stock market returns. The rationale behind the model extension is that the two factors reflect unidentified variables that produce undiversifiable risks that are not captured by the market return (Fama & French, 2004). Empirical testing of the model by Fama and French (1993, 1996) find that the model captures much of the variation in average return for portfolios formed on size, book-to-market value of equity and other price ratios that cause problems for the CAPM (Fama & French, 2004). As such, the three-factor model is

widely recognized and is used in empirical research that requires a model of expected returns.

Whereas the size and value factors are argued to reflect unknown systematic risks, the momentum effect of Jegadeesh and Titman (1993) was left unexplained and unaccounted for. As a response to this, Carhart (1997) proposed a four-factor model, which includes a momentum factor. The four-factor model noticeably reduced the average pricing errors relative to both the CAPM and the three-factor model. Although the four-factor model was proposed as an improved methodology for assessing mutual fund performance, the model has been used extensively to explain the cross-section of average returns on local stock markets and globally (Chui, Wei, & Titman, 2000; Fama & French, 2008). The following sections will provide a detailed review of relevant literature.

### 2.3.1 PORTFOLIO SELECTION THEORY

The CAPM and subsequent factor models build on the model of portfolio choice developed by Harry Markowitz (1952). Although its concepts have been criticized for capturing the reality only poorly, no other model for optimal portfolio choice has been widely accepted (Krause, 2001). Fundamental to Markowitz's portfolio theory is the mean-variance criterion, asserting that optimal portfolio selection depends only on two moments of the distribution of outcomes, namely, the mean (expected return) and variance (risk). The mean-variance criterion is defined as:

$$a_i \succcurlyeq a_j \iff \begin{cases} \text{Var}[a_i] < \text{Var}[a_j] & \text{and} & E(a_i) \geq E(a_j) \\ \text{or} \\ \text{Var}[a_i] \leq \text{Var}[a_j] & \text{and} & E[a_i] > E[a_j] \end{cases} \quad (18)$$

Comparing two portfolios,  $a_i$  and  $a_j$ , a necessary condition is that one would prefer portfolio  $a_i$  over  $a_j$  when its variance is lower, and its expected return is equal or higher. The other necessary condition is that one would prefer portfolio  $a_i$  over  $a_j$  is when its variance is equal or lower, and its expected return is higher. Hence, the model assumes investors are risk-averse, and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios, so that the portfolios minimize the variance of

portfolio return, given the expected return, and maximize expected return, given the variance (Fama & French, 2004).

To see how optimal portfolio selection is determined from a portfolio's expected return and variance, consider a portfolio with  $N$  different assets where  $R_i$  is the return on the  $i_{th}$  asset. Let  $\mu_i$  and  $\sigma_i$  be the mean and variance, and let  $\sigma_{i,j}$  ( $=\rho_{i,j}\sigma_i\sigma_j$ ) be the covariance between  $R_i$  and  $R_j$ . Suppose that the relative value of the portfolio invested in asset  $i$  is  $x_i$ . If  $R$  is the return on the portfolio as a whole, then:

$$\mu = E[R] = \sum_{i=1}^n x_i \mu_i \quad (19)$$

$$\sigma^2 = Var[R] = \sum_{i=1}^n x_i^2 \sigma_i^2 = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \quad (20)$$

$$\sum_{i=1}^n x_i = 1 \quad (21)$$

$$x_i \geq 0, i = 1, 2, \dots, n \quad (22)$$

A critical observation is that as the portfolio variance from Equation 20, unlike the expected return of the portfolio from Equation 19, is not a weighted average of the individual asset values. Furthermore, it is assumed that the relative value of the portfolio invested in asset  $x_i$  is larger than zero for all assets, implying no shorting of assets. Given the expected return and variance of asset  $i$  and  $j$ , respectively, investors are now faced with an opportunity set of risky assets, depending on the relative portfolio weights (Bodie, Kane, & Marcus, 2014, p. 217). Subject to the mean-variance criterion, however, investors will only hold the mean-variance-efficient portfolio corresponding to his or her level of risk tolerance<sup>6</sup>

Other than the mean-variance criterion for optimal portfolio selection, Markowitz (1952) also demonstrated the importance of diversification. By combining assets that have a non-perfect correlation, one may achieve higher risk-adjusted returns. This can be seen from

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<sup>6</sup> See Appendix 2 for a graphical representation.

Equation 20, where a lower covariance between assets, ceteris paribus, result in a lower portfolio variance. In the extreme case where the returns of two assets are perfectly uncorrelated, the portfolio variance is simply the weighted average of the assets' variance. To formally illustrate the diversification benefits, consider the naïve strategy in which an equally weighted portfolio is constructed, such that  $w_i = 1/n$  for each security. From Equation 20 we get that:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n \frac{1}{n} \sigma_i^2 + \sum_{j=1}^n \sum_{i=1}^n \frac{1}{n^2} \text{Cov}(x_i, x_j) \quad (23)$$

Defining the average variance and average covariance as:

$$\bar{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n \sigma_i^2 \quad (24)$$

$$\overline{\text{Cov}} = \frac{1}{n(n-1)} \sum_{j=1}^n \sum_{i=1}^n \text{Cov}(r_i, r_j) \quad (25)$$

The portfolio variance can be expressed as:

$$\sigma_p^2 = \frac{1}{n} \bar{\sigma}^2 + \frac{n-1}{n} \overline{\text{Cov}} \quad (26)$$

As seen from Equation 26, when the average covariance between assets returns is zero, portfolio variance can be driven to zero as  $n$  increases. More realistically, when assets have a positive correlation, diversification reduces portfolio variance, although not completely due to the systematic risk. In the case of perfect correlation, there will be no diversification effect whatsoever, as all risk in the portfolio will be systematic (Bodie, Kane, & Marcus, 2014, p. 227).

### 2.3.2 SINGLE-FACTOR MODEL - CAPM

#### 2.3.2.1 THEORETICAL FOUNDATION

The CAPM builds on the model of portfolio choice by Markowitz (1952), as Sharpe (1964) and Lintner (1965a) adds two highly restrictive, but key assumptions to the original

model<sup>7</sup>. (1) Given the market prices,  $P_{i,t-1}$ , investors agree on the joint distribution of asset returns from  $t - 1$  to  $t$ . (2) There are unrestricted borrowing and lending at the risk-free rate (Fama & French, 2004). According to the model of portfolio choice, all portfolios on the mean-variance efficient frontier are efficient, as they maximize expected return for a given level of risk. However, with the possibility of risk-free borrowing and lending, all mean-variance-efficient portfolios are combinations of the optimal risky portfolio and the risk-free rate. The optimal risky portfolio, commonly referred to as the tangency portfolio, is defined as the portfolio that maximizes the excess expected return per unit of risk, known as the Sharpe ratio. By holding various combinations of the tangency portfolio and the risk-free rate, investors can lever and de-lever their portfolios depending on their risk appetite, holding the Sharpe-ratio constant.

With complete agreement about the distributions of returns, all investors necessarily combine the same risky portfolio with the risk-free rate. In short, the CAPM assumptions imply that the market portfolio,  $M$ , must be the portfolio where the efficient frontier tangents the minimum variance frontier, if the market is to clear<sup>8</sup>. Hence, the optimal portfolio must be the value-weighted market portfolio of risky assets.

The Sharpe-Lintner version of the CAPM turns the algebraic statement on asset weights in mean-variance-efficient portfolios, into a testable prediction about the relationship between risk and expected return (Fama & French, 2004). Specifically, the Sharpe-Lintner CAPM asserts that:

$$E(R_i) = r_f + \beta_i[E(r_m - r_f)] \quad (27)$$

Where  $E[r_i]$  is the expected return on the  $i_{th}$  asset,  $r_f$  is the risk-free rate of return,  $E[r_m]$  is the expected return of the market portfolio, and  $\beta_i$  identifies the exposure of a given security to the market.  $\beta_i$  is defined as:

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} \quad (28)$$

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<sup>7</sup> See Appendix 3 for assumptions of the CAPM

<sup>8</sup> See Appendix 4 for a graphical representation.

The CAPM asserts that the expected return of a given asset is equal to risk-free rate, plus a risk market premium. The risk premium is the asset's market beta times the premium per unit of beta risk. Hence, with reference to the consumption-based model and Equation 17, assets' sensitivity to the changes in the excess return of the market portfolio represents a state where investors are willing to give up expected return for increased marginal utility of consumption.

#### 2.3.2.2 DEVELOPMENTS IN METHODOLOGY AND EMPIRICAL RESULTS

The early cross-sectional regression tests focus on the Sharpe-Lintner model's predictions about the intercept and slope in the relation between expected return and market beta. The first methodologically satisfactory testing of the CAPM was proposed by Jensen, Black, and Scholes (1972). Tests before this involved regressing the cross-section of returns on estimates of individual asset betas. However, it quickly became apparent that beta estimates for individual securities are imprecise, creating measurement errors when they are used to explain average returns. Blume (1970), Friend and Blume (1970), and Jensen, Black and Scholes (1972) recognized the benefits of working with diversified portfolios, rather than individual securities, to reduce the measurement error. The methodology is based on a two-stage model starting with a time-series regression:

$$R_{i,t} - r_f = \alpha_i + \beta_i(R_m - r_f) + \epsilon_{i,t} \quad (29)$$

In a first pass, Equation 29 is run as a time-series of each stock's monthly excess returns against the excess market return in the same month. Jensen, Black, and Scholes (1972) formed ten decile portfolios from the lowest beta to the highest beta stocks. Necessarily, if the CAPM explains individual asset returns, it also explains portfolio returns. Since the CAPM asserts that the market risk premium fully explains assets' excess returns, a well-specified model implies an intercept ( $\alpha_i$ ) indistinguishable from zero, as well as  $E(\epsilon_i) = 0$  and  $Cov(\epsilon_i, R_m) = 0$ .

The second pass regression was run as a single cross-section of the excess portfolio returns on the portfolio estimates from Equation 29. Hence:

$$(R_p - r_f) = \gamma_0 + \gamma_1\beta_o + \epsilon_p \quad (30)$$

The model predicts that the intercept in these regressions is the risk-free rate, and the coefficient on beta is the expected return on the market in excess of the risk-free rate. Thus, if Equation 29 is well-specified, the intercept  $\gamma_0$  should be statistically indistinguishable from zero, and the coefficient  $\gamma_0$  on  $\beta_P$ 's should identify the excess market return,  $(R_m - r_f)$ .

Interestingly, Jensen, Black, and Scholes (1972), along with Douglas (1968), Miller and Scholes (1972), Blume and Friend (1973) and Fama and MacBeth (1973), consistently found negative intercepts for the high-beta portfolios, and positive intercepts for the low-beta portfolios in the time-series regressions. In the cross-sectional regressions, the intercept was found to be positive, and the slope too low relative the CAPM's predictions. Both pass regressions thus contradicted the CAPM. The lack of empirical evidence for the CAPM has significant implications; if the market does not appropriately reward systematic market risk, markets cannot be held rational. Given the validity of the results, no investor would invest in high-beta stocks, but rather invest it in the lowest beta stocks and lever the portfolio using the risk-free rate to achieve a similar market exposure (Black, 1993). In recent years, the contradicting relationship between beta and expected return has been dubbed the "beta anomaly".

As an attempt to explain the beta anomaly, Black (1972) develops an alternative version of the Sharpe-Lintner CAPM. He argues that the assumption of unrestricted borrowing and lending is unrealistic, and shows that the market portfolio is mean-variance efficient when one assumes unrestricted short sales of risky assets. Under the new assumption, investors can short assets to obtain a zero-beta portfolio, i.e., a portfolio that is unaffected by market portfolio, similarly as with the risk-free rate, but with a higher return. Formally, the CAPM can be rewritten:

$$E(R_i) = E(R_z) + \beta_i[E(R_m) - E(R_z)] \quad (31)$$

Where  $R_z$  is postulated as representing the return on a portfolio that has zero covariance with the return on the market portfolio. Black's version of the CAPM potentially solves the issues in the early cross-sectional tests, since  $R_z > r_f$  would adjust the intercepts and explain the lower slopes of the cross-sectional regressions (Dempsey, 2013). Following the



cross-sectional methodology of Black, Jensen, and Scholes, the first pass time-series regression equation can be rewritten to express the excess return:

$$R_{i,t} = R_z + \beta_i(R_m - R_z) + \epsilon_i \quad (32)$$

$$R_i - r_f = (R_z - r_f)(1 - \beta_i) + \beta_i(R_m - r_f) + \epsilon_i \quad (33)$$

As seen from Equation 33, the first pass regression prediction of  $a_P$  is now consistent with a higher intercept for high-beta assets and vice versa for low-beta assets. Additionally, the second pass cross-section regression now predicts:

$$\gamma_0 = (R_z - r_f) \text{ and } \gamma_1 = R_m - R_z \quad (34)$$

which is consistent with a positive intercept and a slope that understates the excess market return. As a result, early empirical testing provided a consensus that Black's version of the CAPM gave a good description of expected returns (Fama & French, 2004).

In their highly influential article, Fama and MacBeth (1973) provided a methodological framework that built on the two pass methodology of Black, Jensen, and Scholes (1972). Instead of performing a single time-series regression on each asset before sorting into portfolios, they estimate month-by-month regressions of monthly returns on betas, often referred to as "rolling regressions". Researchers had observed common sources of variation in the regression residuals, i.e., common sources of unsystematic risk, such as industry effects. The Fama and MacBeth methodology addresses the inference problem, as the residual correlations are captured via repeated sampling of the regression coefficients (Fama & French, 2004). A detailed description of the Fama and MacBeth (1973) methodology can be found in Section 5 – Methodology.

#### 2.3.2.3 EMPIRICAL TESTING OF THE CAPM ON THE OSLO STOCK EXCHANGE

To our knowledge, few comprehensive studies have conducted empirical testing of the CAPM on OSE. Næs, Skjeltorp, and Ødegaard (2007) and Ødegaard (2016a) have performed extensive empirical testing of single- and multifactor models on the OSE. Using data from 1980 – 2006, Næs, Skjeltorp, and Ødegaard (2007) estimate the CAPM on beta-sorted portfolios, as well as portfolios sorted on anomalies variables, such as market

capitalization, relative spread, the book-to-market value of equity, and momentum following the methodology of Fama and MacBeth (1973). The results from the first pass time-series regressions show some consistency with prior empirical tests, mentioned in subsection 2.3.2.2, as the intercepts are negative for the high-beta portfolios and positive for the low-beta portfolios. However, neither of the intercepts except one are distinguishable from zero. The second pass cross-sectional regressions indicate a significant risk premium for the beta portfolios with an insignificant intercept, concluding that the CAPM is a relatively well-specified model explaining the excess returns of beta sorted portfolios. However, the model does not perform well on portfolios sorted on market capitalization, liquidity, or market-to-book equity.

Ødegaard (2016a) uses a similar methodology as Næs, Skjeltorp, and Ødegaard (2007), but with an updated dataset from 1980-2016. The second pass cross-sectional regressions on beta-sorted portfolios show an insignificant alpha term, but also an insignificant coefficient on the risk premium at the 5% level.

A recent Master's thesis (Korneliussen & Rasmussen, 2014) use the similar methodology, but monthly data from 1991-2010, and finds significant risk premia on portfolios sorted on beta, the book-to-market value of equity, and momentum. However, the intercepts for all portfolios are highly significant. Hence, the thesis concludes that the CAPM is an inadequate model for the Norwegian stock market.

#### 2.3.2.4 CRITIQUE OF THE CAPM

Despite the fact that Black's version of the CAPM performed well in explaining excess average returns on beta-sorted portfolios, the CAPM rests on unrealistic assumptions. First, the assumption that all investors have homogeneous expectations in regards to expected returns and variances is untrue. Second, unrestricted borrowing and lending at the same risk-free rate in the Sharpe-Lintner version of the CAPM is a simplification of reality. Unrestricted shorting of risky assets in Sharpe's version is not less of a simplification. Perhaps the most famous critique of the CAPM was promoted in Roll's (1977) influential article, where he argues that any proxy of the market portfolio is inadequate, as the true market portfolio would include every asset in every market, including commodities, collectibles, and human capital.

Many economic models are founded on simplistic, and even unrealistic assumptions. Thus, it was not until researchers started regressing certain factors against portfolios sorted firm characteristics that it became evident that much of the variation in expected stock returns was unrelated to market beta (Fama & French, 2004). Basu (1977) was the first to show that when portfolios are sorted on earnings-price ratios, as opposed to market beta, future returns on higher E/P stocks are higher than predicted with CAPM. Banz (1981) sorts portfolios on market capitalization and find that average returns on small stocks are higher than predicted by CAPM. Statman (1980) and Rosenberg, Reid, and Lanstein (1985) proves that stocks with high book-to-market equity ratios have high average returns that are not captured by their betas. Bhandari (1988) finds that portfolios sorted on debt-equity ratios are associated with returns higher than explained by CAPM. These CAPM anomalies (referred simply to as anomalies henceforth) led to a general acknowledgment among researchers and practitioners that the CAPM was empirically flawed. The failure of the CAPM led to a consensus that there are factors other than the market factor that drives average asset returns.

### 2.3.3 MULTI-FACTOR MODELS - ARBITRAGE PRICING THEORY

In an attempt to provide a model that describes asset's expected returns as a function of factors other than the market factor, Ross (1976) developed the Arbitrage Pricing Model (APT). The APT is an equilibrium model relying on the law of one price and no arbitrage, and is based on the idea that an assets' return can be predicted from the several common risk factors (Szylar, 2013). The CAPM postulates that all investors will hold the market portfolio due to the relationship between risk and return. APT, on the other hand, postulates that asset prices, and therefore also asset returns, are set by market participants immediately responding to arbitrage opportunities. The APT asserts that an asset's excess returns can be described as:

$$R_i = E(R_i) + \sum \beta_{i,k} F_k + \epsilon_i \quad (35)$$

Where  $E(R_i)$  is the expected excess return,  $F_i$  are factors,  $\beta_{i,k}$  is the factor loading or measure of the sensitivity of the  $i_{th}$  asset to factor  $k$ , and  $\epsilon_i$  represents the residuals, i.e., the unsystematic risk. As seen from Equation 35, the realized excess return of an asset is

its expected excess return, plus the sensitivity to some unanticipated factors. Unlike the CAPM, the APT does not specify which factors that affect the assets' returns and gives no guidance where to look for such factors. However, the influence of a factor cannot be diversifiable, and hence, they must be systematic risk factors that affect a large number of assets (Szyilar, 2013).

#### 2.3.4 FAMA-FRENCH THREE-FACTOR MODEL

In 1992, Fama and French (1992) realized the necessity of retaining a risk-based model for asset pricing. In the absence of such a model, the market could not be deemed rational. Fama and French (1992) argue that many of the observed anomalies in reference to CAPM are systematic risk factors that could be captured in a three-factor model, based on the APT-framework of Ross (1973). Specifically, the model asserts that the return on a portfolio in excess of the risk-free rate is explained by the sensitivity of its returns to three factors: (a) the excess return on a broad market portfolio; (b) the difference between the return on a portfolio of small firm stocks and the return on a portfolio of large firm stocks, denoted as the SMB (small minus big) factor, and (c) the difference between the return on a portfolio of high-book to-market stocks and the return on a portfolio of low-book-to-market stocks, denoted as the HML (high minus low) factor. Thus:

$$E(R_i) - r_f = b_i[E(R_m) - r_f] + s_i E(R_{SMB}) + h_i(R_{HML}) \quad (36)$$

Where  $E(R_m) - r_f$ ,  $E(R_{SMB})$  and  $E(R_{HML})$  are the expected risk premia, and the factor loadings,  $b_i$ ,  $s_i$  and  $h_i$ , are the slopes of the times-series regression:

$$R_i - r_f = \alpha_i + b_i(R_m - r_f) + s_i R_{SMB} + h_i R_{HML} + \epsilon_i \quad (37)$$

In their study, Fama & French (1993) perform time-series regressions on 25 double-sorted portfolios depending on the assets' size and book-to-market ratios. To illustrate the model's performance improvement, they first perform the CAPM regression from Equation 29 and find that the market factor explains (as measured by the  $R^2$ -statistic) between 0.61 and 0.92 percent of the portfolios' excess returns. After including the SMB and the HML factor, using Equation 37, they find that the factors combined explain between 0.83 and 0.97 percent of the variation in the portfolios' excess returns. Furthermore, the intercepts

of the three-factor model are comparatively lower, most being insignificant. Similar results are reproduced in Fama and French (1996) and Fama and French (1998), where the latter investigate international markets to exclude the possibility of sampling error and to prove generalization. Interestingly, however, Fama and French (1992) find that the beta anomaly is present in the multi-factor context as well. In fact, they find that the relationship between beta and expected return is even flatter after controlling for size and book-to-market characteristics. In spite of this, the empirical success of the three-factor model has led to the model being widely used in empirical research that requires a model of expected returns and is generally preferred over CAPM (Fama & French, 2004).

Regardless of its ability to explain stock returns, the empirical motivation of the three-factor model is not all clear. As opposed to the market factor, the SMB and HML factors are not motivated by predictions of about state variables of concern to investors. Fama and French (1993) are convinced the SMB and HML are proxies for yet unknown more-fundamental variables but fail to identify what those are. They do, however, point out that firms with high book-to-market equity ratio are more likely to be in financial distress, and small stocks may be more sensitive to changes in business conditions (Bodie, Kane, & Marcus, 2014). From a theoretical perspective, the uncertainty in regards to the fundamental source of systematic risk is the main shortcoming of the three-factor model (Fama and French, 2004).

The inability to identify the source of the systematic risk implies that one cannot rule out the possibility that the factors merely capture irrational market behavior. This is the view of the behavioralists, who argue that sorting firms on book-to-market ratios exposes investor overreaction to good and bad times. Proponents of this view are De Bondt and Thaler (1985), who find evidence supporting what Basu (1977) refers to as the "price-ratio" hypothesis: that stocks with a low P/E earn larger risk-adjusted returns than high P/E stocks, with the explanation that that firms with very low P/E are thought to be temporarily undervalued by the market, and subsequently yield higher returns. Lakonishok, Shleifer, and Vishny (1994) extend this notion, and find it likely that the higher risk-adjusted returns associated high-BM stocks compared to low-BM stocks, is a result of investors consistently overestimating the future growth rate of low-BM stocks,

referred to "glamour stocks", relative to high-BM stocks, referred to as "value-stocks" (Lakonishok, Shleifer, & Vishny, 1994). Whether value strategies have produced higher returns because they are relatively underpriced or because value stocks underperform in bad states of the world when the marginal utility of consumption is high (i.e., stocks with higher systematic risk), is difficult to test. When empirical tests reject the CAPM or the three-factor model, one cannot say whether the problem is the assumption that prices are rational, as expressed by the behavioralists, or if the assumptions of the models are violated (Fama & French, 2004).

Given the publicity of the BM anomaly, one would expect the higher risk-adjusted returns to be "arbitraged away", if it does not represent a systematic risk. However, Shleifer and Vishny (1997) show that the volatility of a long-short portfolio based on BM can be high enough to deter arbitrage activity. To maximize trading profits, the investor must trade the most volatile stocks because the BM anomaly is largest for stocks with the highest idiosyncratic volatility. Furthermore, the BM anomaly only offer stable and higher risk-adjusted returns over a holding period of three to five years. Due to the high idiosyncratic volatility and the long holding periods required to earn arbitrage returns, Shleifer and Vishny (1997) argue that the BM anomaly will not fully be arbitrated away.

#### 2.3.5 CARHART FOUR-FACTOR MODEL

Based on the abovementioned arguments, proponents of risk-based asset pricing models have been able to justify the rational market expectation. However, they have trouble explaining the "momentum effect". In their influential article, Jegadeesh and Titman (1993) provide evidence that simple strategies where stocks are ranked based on their past 3-12 months cumulative return, predict relative performance over the next 3-12 months. That is, recent winners will continue to be winners over the next 3-12 months, and recent losers will continue to be losers over the next 3-12 months. They find that the most successful strategy selects stocks based on their returns over the previous 12 months, and then hold the portfolio for three months. After 12 months, mean reversion becomes strong, and the portfolios experience negative abnormal returns (Jegadeesh & Titman, 1993).

Based on the findings in Jegadeesh and Titman (1993), Carhart (1997) constructs a four-factor model by extending the Fama – French three-factor model with an additional factor,

PR1YR, that captures the momentum effect. Jegadeesh (1990) and Lo and MacKinlay (1990) demonstrate how there is a significant one-month reversal in returns. Hence, the factor PR1YR is constructed as average return from the stocks with the highest 30 percent eleven-month returns lagged one month, minus the equally-weighted average of firms with the lowest 30 percent eleven-month returns lagged one month (Carhart, 1997). The four-factor model can be expressed as:

$$E(R_i) - r_f = b_i[E(R_m) - r_f] + s_i E(R_{SMB}) + h_i E(HML) + p_i E(PR1YR) \quad (38)$$

Where PR1YR is the difference in the average return of past winners and losers, and the other factors and factor loadings is the same as in Equation 36. The time-series equation is therefore:

$$R_i - r_f = \alpha + \beta_i(R_m - r_f) + s_i SMB + h_i HML + p_i PR1YR + \epsilon_i \quad (39)$$

Carhart (1997) tests both the CAPM and his four-factor model on ten portfolios ranked on the cumulative one-year returns, lagged one month. He finds that the CAPM does not explain the excess return from the portfolios, while the four-factor model accounts for almost all of the cross-sectional variation in the expected return (Carhart, 1997).

The literature on the momentum effect is vast. Lakonishok, Shleifer, Thaler, and Vishny (1991) attribute the effect to what they refer to as "window dressing". This is when fund managers get rid of underperforming stocks prior presentation of progress reports on their portfolios to clients, to avoid defending a stock's presence in the portfolio. Because fund managers typically are evaluated against a benchmark index, they may alter their portfolios at the end of given period to impress sponsors. Lakonishok, Shleifer, and Vishny (1992) argue that this helps to sustain the momentum effect.

Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1988) and Hong and Stein (1999) present behavioral models based on the idea that momentum profits arise because of inherent biases in the way that investors interpret information. Grundy and Martin (2001) show how the profitability of momentum strategies reflects momentum in the idiosyncratic component of returns. George and Hwang (2004) state that a significant portion of the momentum effect can be obtained by using the 52-week high

price. Irrespective of the fundamental cause of the momentum effect, it represents a substantial threat to the notion of market rationality and efficiency, as the efficient market hypothesis asserts that in the weak form of market efficiency, past price movements do not affect stock prices (Fama, 1965a, 1995; Samuelson (1965)). The extent to which behavioral financial theories have become increasingly popular for explaining stock market phenomena, as well as the broad adoption of Carhart's four-factor model in explaining asset's expected returns, indicate that the market may not be assumed to be entirely rational.

### 2.3.6 EMPIRICAL RESEARCH EXPLAINING THE BETA ANOMALY

The beta anomaly, first documented by Jensen, Black, and Scholes (1972), and later confirmed in multi-factor models (Reinganum, 1981; Stambaugh, 1982; Lakonishok & Shapiro, 1986; Fama & French, 1993, 1996), has been shown renewed interest by researchers in recent years. Research by Black (1993), Haugen and Baker (1991; 1996), Falkenstein (1994), and more recently, Clarke de Silva and Thorley (2010) and Baker, Bradley, and Wurgler (2011) have found evidence of a negative relation between market beta and expected return. Using US data from 1968-2012, Baker, Bradley, and Taliaferro (2014) find that a \$1.00 investment in a low-beta sorted portfolio in 1968 compounds to \$81.66. The same investment in a high-beta portfolio compounds to only \$9.76.

A growing number of papers have attempted to explain the beta anomaly, and explanations have emphasized a combination of behavioral demand and limits to arbitrage, including limited borrowing capacity and the delegation of stock selection (Baker, Bradley, and Taliaferro, 2014). In a recent influential article, Frazzini and Pedersen (2014) present evidence that the lower risk-adjusted returns for high-beta stocks can be explained by the fact that investors are constrained in regards to leverage and margin requirements. Instead of buying low-beta assets and subsequently leveraging the portfolio to obtain a higher return, constrained investors purchase assets riskier than would be optimal to achieve higher returns. They argue that the tilt towards high-beta assets suggests that high-beta assets require a lower risk-adjusted return than low-beta assets. By constructing a betting against beta (BAB) factor, defined as a portfolio that holds low-beta assets, leveraged to a



beta of one, and that shorts high-beta assets, de-levered to a beta of one, they obtain abnormal returns.

Bali, et al. (2017) attribute the beta anomaly to investors' demand for stocks with lottery-like payoffs, referred to as "lottery stocks". Analogous to actual lotteries, investors may be fully aware that stocks with lottery-like characteristics have negative expected payoffs, but nonetheless, they may exhibit a preference for lottery stocks, as a remote chance of winning can be considered better than no chance of winning at all (Kumar, 2009). Hence, the demand for lottery stocks is consistent with the cumulative prospect theory of Tversky and Kahneman (1992), asserting that people make decisions based on the potential value of losses and gains, rather than the outcome.

Specifically, the rationale of Bali et al. (2017) is that lottery investors generate demand for stocks with high probabilities of large short-term up movements in the stock price. Such stocks generally have a higher covariance with the overall market, and hence a relatively large market beta. The lottery demand for such stocks puts a disproportionately large price pressure on high-beta stocks relative to low-beta stocks, resulting in lower future returns. This price pressure generates a positive alpha and a slope less than the market risk premium for the SML. Hence, their hypothesis disregard Black's (1972) version of the CAPM, as well as Frazzini and Pedersen's (2014) betting against beta theory.

Bali et al. (2017) proxy the lottery demand with the factor MAX, defined as the average of the five highest daily returns of a given stock in a given month. They demonstrate that the abnormal returns of the long-short beta portfolio, similar to that of Frazzini and Pedersen (2014), are no longer significant when the portfolio is constrained to be neutral to MAX. They also use time-series regressions that indicate a positive and significant relation between beta and stock returns when MAX is included in a four-factor model.

Furthermore, they create a factor, FMAX, by first sorting stocks into two portfolios based on market capitalization, before independently sorting all stocks in the sample based on ascending sort of MAX, generating a total of 6 portfolios. By including the FMAX factor in the regressions, they find that the BAB factor no longer generates positive abnormal returns, as the returns generated by the BAB is captured by FMAX in the factor model.

Interestingly, they observe that the lottery demand is only prominent among private investors but not among institutional investors.

### 3 CHARACTERISTICS OF THE OSLO STOCK EXCHANGE

In this section, we give the reader a brief introduction to the Oslo Stock Exchange (OSE) by providing some descriptive measures. The empirical results in this thesis are directly affected by the characteristics of the stocks listed on OSE, and we therefore believe that interpretation of our results requires a contextual understanding.

#### 3.1 DEVELOPMENT OF THE OSLO STOCK EXCHANGE

The OSE has grown rapidly during the last decades. Figure 2 shows the market value of the Oslo All-Share Index (OSEAX) from 1996 to April 2018. In 1996, the total market value of the listed stocks was 275 billion NOK. On April 17th, 2018, the total market value is 2.461 billion NOK. The average company market value has risen from 1.96 billion NOK in 1996, to 12 billion in 2018. From 2003 to 2007, the market capitalization on the OSE increased by a factor of five, reaching a total market capitalization of 2.200 billion NOK in July 2007. However, as with other stock markets, the OSE was hit hard during the financial crisis. Compared to the late 2007-early 2008 market capitalization, the OSE lost more than half of its value in 2008. Despite relatively volatile market conditions in the years succeeding the crisis, the OSE has gained significant value in the last decade, and the capitalization reached pre-crisis levels late 2017.

**Figure 2** - Market value of the OESEAX (Oslo All-Share Index) 1996 - 2007



(Oslo Stock Exchange Information Services, 2018)

To judge the relative importance of the stock market in the Norwegian economy, Figure 3 shows the total market value of companies at the OSE as a fraction of annual GDP of Norway. Between 1996 and 2003, the market capitalization of the OSEAX relative to the GDP was relatively stable at around 40%. However, in the years leading up to the crisis, the relative market capitalization of the OSE to GDP rose to approximately 90%. In 2008, this number fell to 36%, but has since then increased to 75% in 2017. To provide some context, Table 1 shows the same ratio for comparable countries at year-end in 2017. Since the Norwegian GDP is heavily affected by revenues from oil-related activities, the measure has been calculated without taking this activity into account. As from the Table 1, the Norwegian stock market relative to GDP is significantly smaller than for comparable countries, even after we adjust for oil-related activities.



(Oslo Stock Exchange Information Services, 2018; SSB, 2018)

**Table 1** - Comparison of market value-to-GDP

Country	Market value-to-GDP
Norway	74.67 %
Norway (less oil sector)	87.33 %
Sweden	144.30 %
Denmark	126.33 %
Finland	99.82 %
US	139.69 %
UK	109.72 %

(SSB, 2018; Ceicdata.com, 2017)

### 3.2 COMPANY SIZE AND SECTOR ALLOCATION

A distinct feature of the OSE is that it has always been dominated by a few, large companies. In 1980, Norsk Hydro constituted in excess of 50% of the total market capitalization at OSE. However, this percentage gradually declined during the 1990's and in the beginning of the new millennium (Ødegaard, 2015). In 2001, the national oil and gas company Statoil went public, and became the largest company on OSE measured by market capitalization, constituting 23.7% of the total market capitalization. In 2006, Statoil, Norsk Hydro, Telenor and DNB were the four largest companies on OSE, and constituted an aggregate of 60.8% of the total market capitalization (Ødegaard, 2015). As of April 15th 2018, their combined market capitalization has fallen to 48.3%.

<b>Table 2</b> Sector allocation on the OSEAX			
Sector	Relative size	Number of companies	Average market value (millions)
Consumer Discretionary	2.85 %	9	8,277.64
Consumer Staples	11.77 %	11	27,917.93
Energy	37.46 %	50	19,550.53
Equity Certificate	2.59 %	21	3,221.45
Financials	14.92 %	14	27,803.27
Health Care	0.34 %	8	1,097.25
Industrials	6.77 %	34	5,193.15
Information Technology	2.48 %	25	2,590.18
Materials	8.41 %	8	27,445.06
Real Estate	1.76 %	6	7,665.34
Telecommunications	9.92 %	2	129,425.76
Utilities	0.73 %	3	6332.31

(Oslo Børs, 2018c)

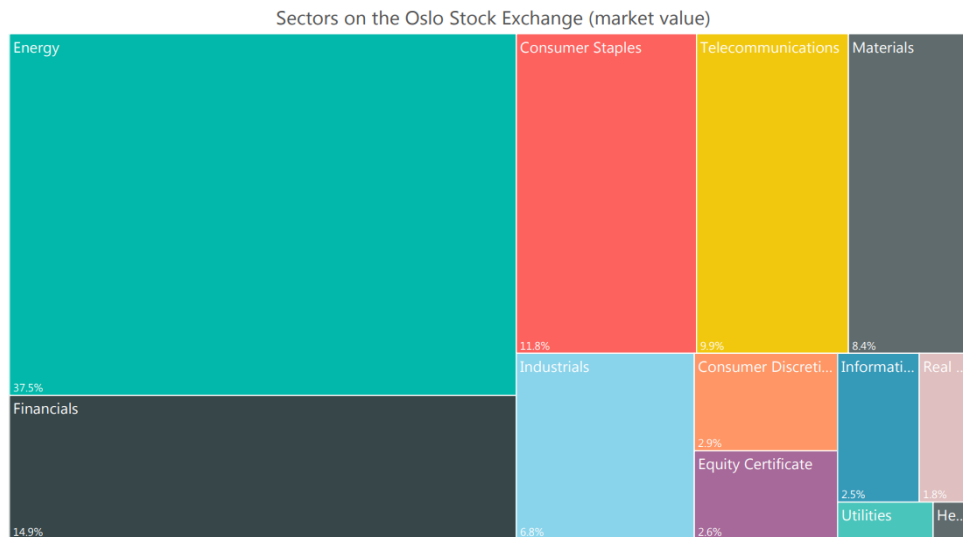
The sector allocation<sup>9</sup> on the OSE has changed significantly during the last decades. In terms of number of companies, the industrial and financial sector dominated the OSE up until 1990 (Ødegaard, 2015). Since then, there has been a significant increase in energy companies (50) and information technology companies (25), as seen from Table 2.

Considering the market capitalization, it can be seen from Figure 4 that the energy sector

<sup>9</sup> The sector allocation follows the CIGS standard (Global Industry Classification Standard) developed by Morgan Stanley Capital International (MSCI) and Standard & Poor.

is highly represented on the OSE with a weight of 37.5% as of April 30th, 2018. The vast majority of these companies are engaged in various parts of the oil and gas value chain. The financial sector (the sector excludes local savings banks issuing equity certificates, as indicated by Table 2) is the second largest sector in terms of market value, constituting almost 15% of the value on OSE. Interestingly, whereas the information technology sector is highly represented in terms of numbers of companies, the average market capitalization is only 2.5 billion, which is only bigger than real estate, utilities, and healthcare. Note that in the telecommunications sector, the average market capitalization is biased by Telenor being one of the two companies in the sector.

**FIGURE 4** – SECTORS ON THE OSLO STOCK EXCHANGE MEASURED BY MARKET VALUE



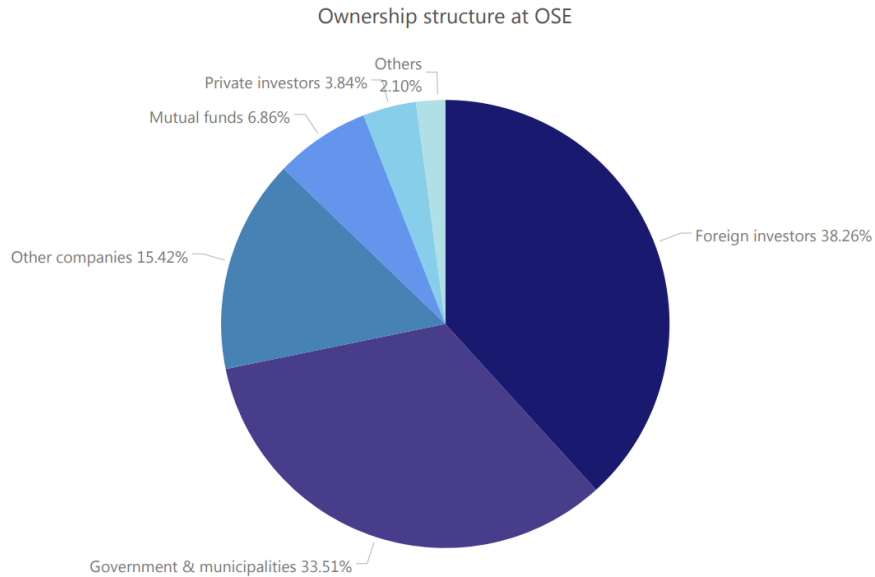
(Oslo Børs, 2018b; Oslo Børs, 2018c)

### 3.3 INSTITUTIONAL VS. HOUSEHOLD OWNERSHIP

There are some characteristics in regards to the ownership structure that distinguishes the OSE from other comparable stock exchanges. Notably, as seen from the Figure 5, the government and municipal ownership in listed stocks is high at 33.5%. Foreign investors currently own 33.2%, which is significant, but is in line with other comparable European stock markets (Morrow Sodali, 2014; Deutsche Bundesbank, 2014; "Share capital decreased", 2017; Danmarks Nationalbank, 2017). More interestingly, compared to other Nordic stock exchanges, Norwegian households' ownership percentage of listed equities is

low at 3.84%. As a comparison, Danish and Swedish households' ownership percentage were 12% and 11%, respectively in 2017 (Danmarks Nationalbank, 2017; "Share capital decreased", 2017). In the US, this number is estimated to be 37%, as of 2016 (Bryan, 2016).

**FIGURE 5** – OWNERSHIP STRUCTURE AT THE OSLO STOCK EXCHANGE



(Oslo Børs, 2018d)

## 4 DATA

In this section, we give a detailed description of our data sample. We begin by describing the data sources used in the data collection and provide an overview of the raw data used to construct our sample. Numerous adjustments to the raw data have been made and will be described in detail.

### 4.1 DATA SOURCE SELECTION

The prevalence of studies applying multi-factor models has resulted in a large number of publicly available datasets across different stock exchanges. For example, Kenneth French's database contains factor-based data for numerous factor models. The return data is available for both US, European, Asian and other global stock markets (French, 2018). However, the database does not provide raw data, and it is therefore not possible to

extract data on securities listed on OSE. As a result, the data is not applicable to our study.

More relevant to this paper, we find the CCRS-DFB Risk Factor Database (2014), which contains factor-based data on OSE from 1987 to 2012. Similarly, Ødegaard (2016b) provides extensive data on factors for the OSE from 1980 to 2016. An obvious problem regarding these datasets is their timespan, which does not match our period of interest. Further, the datasets do not include all the factors relevant to this paper, such as BAB and MAX. A possible solution to this problem is to apply the existing factors and supplement the dataset with factors constructed from other data sources. However, the underlying assumptions for factor construction can vary considerably between different datasets, and consequently, the choice of data provider can lead to different model outcomes (Brückner, Lehmann, Schmidt, & Stehle, 2015). Furthermore, examining existing datasets that have already been used for empirical testing would result in a limited contribution to the existing literature. As such, we conclude that it is most beneficial for our study to use one single data provider and to construct the factors from raw data. This ensures that the same set of assumptions are used for each factor at each time-period.

## 4.2 SAMPLE CONSTRUCTION

Our primary data source is Wharton Research Data Services. We download two datasets from the Compustat global database, one containing all available daily observations of security data and one containing all available accounting data on OSE. Because our beta estimates require 36 lagged values of monthly returns, the security data ranges from December 2003 to December 2017. The accounting sample only requires one lagged value of annually reported data and starts one year prior to our time-period of interest in 2006. An overview of the variables in the datasets can be found in Table 3.

<b>Table 3</b>	Overview and description of variables used for sample construction	
	Variable	Content
<b>Common ID Variables</b>		
	datadate	Date
	conm	Company name
	gvkey	Global Company Key
	exchg	Stock exchange code
	fic	Incorporation country code
<b>Security Variables</b>		
	prcd	Daily Close Price
	iid	Issue ID
	Currcd	Currency Code Daily
	ajexdi	Adjustment factor
	cshoc	Shares Outstanding
	cshtrd	Trading Volume Daily
	sic	Standard Industry Classification Code
	tpci	Issue Type Daily
<b>Accounting variables</b>		
	Currcd	Currency code
	fyear	Fiscal Year
	fyr	Fiscal Year-end Month
	at	Total Assets
	seq	Stockholders' Equity
	it	Total Liabilities
	txditc	Deferred Taxes and Investment Tax Credit

After extracting end-of-month observations, the security dataset has 35,509 monthly observations on 403 different companies, while the accounting dataset has 3,217 yearly observations on 341 different companies. The dataset is free from survivorship-bias, since it includes all stocks, both active and inactive, during the period. To make the data usable for our analysis, we adjust the datasets by adding filters and calculate new variables from the raw data. After the filters are applied, the two files are merged, ensuring that we only keep companies that are present in both datasets. As seen from Table 4, the filters significantly reduce the number of observations. However, the adjustments will reduce the risk of biases in our dataset, and filtering of the raw data is considered to be beneficial to our analysis. All of the appropriate adjustments will be further discussed in the following subsection.



**Table 4** Filtering, adjustments and merging of the datasets

<i>Panel A: Security data</i>	Observations		Companies
	<i>number</i>	<i>diff</i>	<i>number</i>
Compustat file	35,509		403
Day > 25	34,402	-1,107	400
No financial firms	30,364	-4,038	343
Trading days $\geq 50$	28,800	-1,554	332
Return calculation	27,614	-1,186	332
Price > 5 NOK and market cap > 30 mill NOK	21,420	-6,194	319
Exclusion of zero-returns	20,838	-582	318
<hr/>			
<i>Panel B: Accounting data</i>	Observations		Companies
	<i>number</i>	<i>diff</i>	<i>number</i>
Compustat file	3,217		341
Omit BE $\leq 0$	3,144	-73	340
<hr/>			
<i>Panel C: Merged file</i>	Observations		Companies
	<i>number</i>	<i>diff</i>	<i>number</i>
Merged with $ME_t$	1,713		291
Merged with ME $\tau - 1$	1,625	-88	275
Match with return data	15,246		252

#### 4.2.1 EXTRACT END-OF-MONTH OBSERVATIONS

The security data in Compustat's global database is only available in daily observations. We will follow the general practice of testing multi-factors on monthly stock returns and convert the data by subtracting the end-of-month observations. The observations are identified by constructing a variable *Last day*, LD, that is equal to the difference between the numeric value of  $Month_\tau$  and  $Month_{\tau+1}$ . The variable is subjected to the constraint  $Company_{month_t} = company_{month_{t+1}}$ . By creating a subset of our security data where  $LD \neq 0$ , we are left with the last observation of each month for every company. Our dataset contains several cases where the last observation for a company in a given month deviates considerably from the previous official trading day that month. The missing values occur whenever a company is not traded during the last days of the month. As we later will

calculate monthly returns, keeping mid-month or start-of-month observations would leave us with different return intervals, which are incomparable to other security returns, as well as the risk-free investment. Because the inclusion of such returns could bias our results, we will restrict our dataset only to include end-of-month observations. Nevertheless, because missing prices might be due to national holidays or insufficient reporting, we will allow our observations to deviate to some extent from the last trading day of the month. More precisely, we accept every observation that falls on the 26th or later. We see a clear break in our observations on this day of the month, and the filter thus ensures relatively equal return intervals without excluding a large number of observations. From Table 4, it can be seen that the filter reduces the number of observations by 1,107 and the number of companies by three.

#### 4.2.2 PENNY STOCKS AND MICRO-CAPS

Securities of extremely low value, referred to as penny stocks henceforth, can be a source of bias in our returns. The intuition is that even a small movement in the price of such securities can result in very high returns. This is misleading when the returns reflect minimal price fluctuations. The OSE already has rules stating that a company with a consistent share price below 1 NOK over a period of six months will be delisted (Oslo Børs, 2018). However, we will apply a more conservative filter criterion by excluding all stocks with a share price of less than 5 NOK. We will also disregard securities with a market capitalization of less than 30 million NOK, referred to as microcaps. Exclusion of both penny stocks and micro-caps is standard in financial literature. Table 4 shows that 6.194 observations and 13 companies are excluded when the penny stock filter is applied to the dataset.

#### 4.2.3 OPERATIONAL VS. FINANCIAL FIRMS

When we construct factors that aim to explain the relationship between different risk factors and security returns, it is essential that the measured risk is directly comparable across all companies. Taking the above into consideration, Fama and French (1992) exclude all financial firms from their analysis, arguing that the leverage of non-financial firms and financial firms are incomparable. For non-financial firms, a high leverage level typically translates into a state of financial distress. Contrary to non-financial firms, a high

leverage is normal for financial firms. Hence, including financial firms in our data sample may bias our estimates. Barber and Lyon (1997) test a hold-out sample of financial firms' relation to firm size, book-to-market ratios, and security returns. They document that financial and non-financial firms have very similar return patterns. As such, we have decided to include most financial firms in our dataset. We will, however, exclude all equity certificates issued by the savings banks. The main difference between equity certificates and ordinary shares lies in the owners' rights to the bank's assets and influence over the bank's governing bodies. Profits are distributed proportionally on the basis of ownership stake and the bank's other capital. At a limited company, losses hit shareholder's equity directly. At a saving bank, losses are first absorbed by the primary capital and equalization reserve, and the equity certificate capital is at risk only if the primary capital is exhausted (Sparebankforeningen, 2015). As a result, the equity in a savings bank is not distributed symmetrically with respect to profits and losses. Because the risk characteristics of the equity certificates differ from ordinary shares, we will disregard these going forward. All other financial firms are included in the sample.

#### 4.2.4 ILLIQUID STOCKS

Our dataset contains a number of stocks that are rarely traded. These illiquid stocks cannot easily be sold without a loss in value, translating into a higher risk for illiquid stocks, compared to liquid stocks. As a result, the future returns of illiquid stocks have been hypothesized to be higher (Amihud, 2002). The hypothesis is backed up by several studies. For example, Marcelo and Quirós (2005) find that expected market illiquidity positively affects ex-ante stock returns on the Spanish stock market. To omit illiquidity risk from his dataset, Ødegaard (2015) adds a filter excluding observations associated with fewer than 20 yearly trading days. We will be more conservative and construct a filter that excludes observations with less than 50 trading days, in line with more recent studies of the OSE (Korneliussen & Rasmussen, 2014). The filter is constructed by first making a new binary variable,  $trade_t$ , which equals 1 if the security was traded at  $day_t$  and 0 if it did not occur any trade that day. Next, the trading days are summed by company and year. Lastly, we subset the security data by excluding observations associated with fewer

than 50 yearly trading days. From Table 4, we can see that 1,554 observations and 11 companies are excluded from the sample.

#### 4.2.5 COMMON AND PREFERRED STOCK

In Norway, preferred stocks are rare but we do find some companies with class A and class B shares. After reading the annual reports of these companies, we find that they define the security classes differently. The majority of the class A shares are referred to as common shares, while most of the class B shares are described as either a series of common stocks or as shares with restricted dividend and/or voting rights. However, in some cases, class B shares are classified as common stocks, while class A shares are subjected to restrictions. Further, Compustat defines all Norwegian securities as common stock. The combination of ambiguous share classifications and the lack of information makes it difficult to establish a clear exclusion rule. Going forward, we will therefore treat all stocks as common stock.

#### 4.2.6 CALCULATION OF RETURNS

Compustat does not provide return calculations in its global database. Therefore, we calculate the returns using a price variable and an adjustment factor variable. From the moment a company is listed, corporate actions can be issued by the company that affect the stock price. However, this does not represent a change in the fundamental value of the company. An example of a corporate action is stock splits, where a company divides its existing shares into multiple shares to boost the liquidity of the stock. This would decrease the value of each share, and an unadjusted price would result in substantial negative returns. The adjustment factor adjusts for the stocks splits, providing investors with a more accurate evaluation of a stock's return. Similarly, the adjustment factor also adjusts stock prices for dividends. The adjustment factor takes a value of 1 if no dividends or splits have occurred and a value of  $> 0$  in the years leading up to a capital movement, if any are present in the time series. The adjusted price can be written as:

$$adj. P = \left( \frac{p_t^i}{adj_t^i} \right)$$

Another concern in the return calculation is whether to use simple (arithmetic) or logarithmic returns. While logarithmic returns have several beneficial properties, such as

time additiveness, the method is not always appropriate. Most relevant to this paper, logarithmic returns are not asset additive. That is, the weighted average of logarithmic returns of individual stocks in a portfolio is not equal to the portfolio return. Simple returns, on the other hand, are asset additive but not time additive. Our desired variables will be the weighted return of a large number of securities, and as such, we will compute the returns as simple returns. The adjusted returns can be written as:

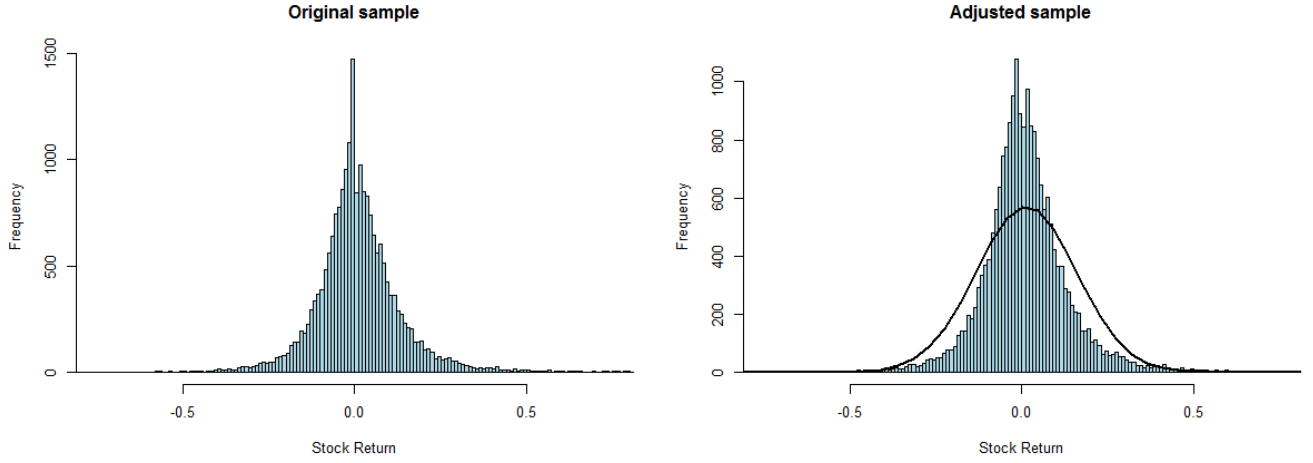
$$r_t^i = \frac{adj. P_t^i - adj. P_{t-1}^i}{adj. P_{t-1}^i}$$

In cases where an adjusted price observation is missing, a return calculation is not computable. These return observations will be excluded from our sample, decreasing the number of observations by 1,186 as seen from Table 4. There is, however, no reduction in the number of companies in the sample.

#### 4.2.7 EXCLUSION OF ZERO-RETURN OBSERVATIONS

The distribution of returns on OSE from December 2003 to December 2017 is shown in Figure 6. Immediately, we see a disproportionately high number of stocks with returns precisely equal to 0.00%. A return of 0.00% would only occur if the adjusted price of the stock did not move during the return interval. A non-changing price could be due to a low trading volume from illiquid stocks. However, the returns are calculated after the implementation of a trading filter, and we would therefore not expect such a high percentage of zero-returns. Looking through our data sample, we find that observations associated with zero-returns are also missing other variable values. Additionally, when we compare a random sample of zero-returns with corresponding returns from other data sources, we find that the vast majority of returns are in fact different from zero. It is therefore our understanding that the majority of the zero-returns stems from insufficient reporting. The inclusion of these returns would bias our returns towards zero, and we argue that although the exclusion of zero-returns would affect some returns that are in fact not due to insufficient reporting, the negative consequences of including the returns are far greater than the consequences of excluding them. Figure 6 shows the distribution of returns before and after we remove all zero-returns.

**FIGURE 6** – ORIGINAL VS. ADJUSTED SAMPLE DISTRIBUTION OF RETURNS



The black line in the return distribution of the adjusted sample shows a normal distribution for a sample with the same mean and standard deviation as the adjusted sample. Both samples have a bell-shaped distribution, but the adjusted sample has a higher kurtosis, and is somewhat skewed. Not surprisingly, the Jarque-Bera test rejects the null hypothesis of normally distributed returns, with a P-value of  $< 2.2\text{e-}16$ .

---

Table 5 shows the distribution characteristics of both the original and adjusted sample. Because the difference between the samples is the exclusion of zero-returns, the only key figure we would expect to change significantly is the mean. As seen from the table, the original and the adjusted sample have similar characteristics, and as expected, the adjusted sample has a slightly higher mean.

The max return observation of almost 360 percent raises some questions in regards to outliers' effect on our results. From one perspective, the outliers could potentially distort the analysis and bias our estimates upwards, decreasing the predictive power of our models. On the other hand, asset pricing models should optimally be able to price all assets in the economy, and investors might be especially interested in the extreme cases. Additionally, the exclusion of these returns could potentially bias our MAX-factor, which is based on the extreme daily returns. Lastly, we consider the tails of the distribution to be relatively thin and the outliers' effect on the overall conclusion to be minimal. Taking the

aforementioned into consideration, we decide not to exclude any of the extreme values in this paper.

<b>Table 5</b>	Comparison of the characteristics of the sample distribution of returns	
	Original sample	Adjusted sample
Min	-90.43 %	-90.43 %
Mean	1.23 %	1.27 %
Max	357.97 %	357.97 %
Std	14.37 %	14.57 %
Kurtosis	61.5	59.74
Skewness	3.74	3.68

#### 4.2.8 FIRMS WITH NEGATIVE BOOK VALUE OF EQUITY

We find 73 observations in our sample with a book value of equity (BE) below or equal to zero. Practically speaking, a firm's limited liability structure imply that shareholders' equity can never be negative, making negative values of BE challenging to interpret. Consequently, many practitioners exclude negative BE-firms, arguing that they have a high default risk (Brown & Li, 2008). In line with Fama and French (1992), we choose to exclude negative BE-observations from our sample, reducing it with one firm and 73 observations.

#### 4.2.9 EXCHANGE RATES

Since our security data only contains stocks listed on OSE, all asset prices from Compustat are reported in NOK. Compustat's accounting data, on the other hand, is denoted in the local currency of the country where each firm has its primary operations. Thus, our accounting data comes with fiscal numbers reported in 10 different currencies, listed in Table 6.

Table 6	List of currencies used to report accounting data	
Currency code	Currency	
AUD	Australian Dollar	
AED	United Arab Emirates Dirham	
CAD	Canadian Dollar	
DKK	Danish Krone	
EUR	Euro	
GBP	British Pound	
INR	Indian Rupee	
NOK	Norwegian Krone	
SEK	Swedish Krone	
USD	US dollar	

To make the accounting data comparable, we convert the foreign currencies to NOK by matching the reported numbers with the end-of-month exchange rates obtained from Thomson Reuters Datastream database. If the exchange rate between a given currency and NOK is not available in Datastream, we will first convert the data into USD before we convert it to NOK.

#### 4.2.10 MARKET PORTFOLIO AND THE RISK-FREE RATE

We use Oslo Stock Exchange All-Share Index (OSEAX) as a proxy for the market factor. The OSEAX is the value-weighted portfolio consisting of all shares listed on OSE. The monthly Return Index (RI) is obtained from Datastream. Similarly to our adjusted stock prices, the index is adjusted for stock splits and dividends. In line with previous research by Ødegaard (2016b), we use the 30-days Norwegian Interbank Offered Rate (NIBOR) as a proxy for the monthly risk-free return. Since the interest rate is reported as the effective annual interest rate, we divide it by 12 to get the monthly risk-free rate.



## 5 METHODOLOGY

### 5.1 IDENTIFYING ANOMALIES

Two approaches are commonly used to identify CAPM anomalies. The first approach examines returns on sets of portfolios formed from sorts on anomaly variables. The second approach use anomaly variables to explain the cross-section of average returns, as done in Fama and MacBeth (1973). In this thesis, we apply both methodologies to investigate what drives the average returns on the OSE. The first part of the methodology section will explain the fundamental difference between the two approaches, as well as our rationale for applying both.

#### 5.1.1 PORTFOLIO SORTS APPROACH

Portfolio sorts are now the dominant approach in finance to establish and test for systematic cross-sectional patterns in expected stock returns related to firm or stock characteristics (Timmermann, 2007). The standard approach is to sort stocks into multiple portfolios at some formation date and study the patterns emerging in the average returns over the subsequent holding period, going from the “low” end to the “high” end of the portfolios ranked on the variable(s) of interest. The main benefit of the portfolio sorts approach is that it provides a simple picture of how returns vary across the spectrum of the anomaly variable (Fama & French, 2008).

Although the portfolio sorts approach is a powerful methodology for observing the relationship between expected returns and anomaly variables, the approach have a main shortcoming. The approach is inconvenient for drawing inferences about which variables have unique information about average returns. For example, suppose that stocks are sorted into three portfolios depending on their market capitalization, where “small”, “medium”, and “large” stocks have the expected returns 1%, 1.2% and 0.8%, respectively, per month. It is common to focus on the "hedge portfolio" return obtained from a long-short position in the two extreme quantile portfolios, in this case the small and large portfolio. For creating trading strategies, assuming that a long-short position in the portfolios is possible, the methodology is useful. However, from the abovementioned example, a comparison only between the top and the bottom portfolio would lead to the

conclusion that small firms earn a higher return than large firms, ignoring the middle portfolio. Furthermore, even when a monotonic relationship between the portfolios and returns is detected, the portfolio sorts approach does not allow for testing of the cross-sectional pattern in expected returns.

However, the simplicity of the portfolio sorts approach enables us to get an overview of the relationship between a stock's characteristics and its expected returns. We will therefore apply the portfolio sorts approach as a preliminary analysis.

### 5.1.2 CROSS-SECTIONAL REGRESSIONS

After having established the relationship between portfolios sorted on anomaly variables and their expected returns in the preliminary analysis, we conduct formal testing using time-series- and cross-sectional regressions. As briefly discussed in the literature review Section 2.3.2.2, Jensen, Black, and Scholes (1972) and Fama and MacBeth (1973) propose two distinct approaches for cross-sectional testing. In financial research, the Fama and MacBeth (1973) methodology is generally considered superior, as potential cross-sectional dependence in the regression residuals are mitigated by allowing estimations of beta to vary. Hence, we apply this methodology in the second part of our analysis.

The benefit of using cross-sectional regressions is that the regression slopes provide direct estimates of marginal effects from the anomaly variable on the expected returns.

Furthermore, simple diagnostics on the regression residuals allow us to judge whether the relationships between anomaly variables and expected returns, implied by the regression slopes, are apparent across the full range of variables (Fama & French, 2008). Thus, the cross-sectional regressions enable us to test whether deviations from monotonic patterns in the portfolio sorts approach constitute significant evidence against monotonicity.

Furthermore, having estimated the relevant factor exposures (coefficient estimates), the methodology allows for direct estimates of the magnitude of the factors.

The Fama and MacBeth (1973) methodology is a two pass procedure. In the first pass regression, each portfolio's return is regressed against one or more factors to determine the magnitude of the factor exposure. For  $n$  portfolio returns and  $m$  factors, the factor

exposures,  $\beta_{n,F_m}$ , are obtained by calculating  $n$  regressions on  $m$  factors. The number of regressions equals the number of portfolios one is testing:

$$\begin{aligned} R_{1,t} &= \alpha_1 + \beta_{1,F_1} F_{1,t} + \beta_{1,F_2} F_{2,t} + \cdots + \beta_{1,F_m} F_{m,t} + \epsilon_{1,t} \\ R_{2,t} &= \alpha_2 + \beta_{2,F_1} F_{1,t} + \beta_{2,F_2} F_{2,t} + \cdots + \beta_{2,F_m} F_{m,t} + \epsilon_{2,t} \\ &\vdots \\ R_{n,t} &= \alpha_n + \beta_{n,F_1} F_{1,t} + \beta_{n,F_2} F_{2,t} + \cdots + \beta_{n,F_m} F_{m,t} + \epsilon_{n,t} \end{aligned} \quad (40)$$

The time-series regressions estimate to what extent each portfolio's return is affected by each factor. The estimated factor exposures from the first pass regressions are then used in the second pass regressions to calculate the factor risk premia.

The second step is to compute  $T$  cross-sectional regressions on the returns of  $n$  portfolios, using the estimated factor exposures,  $\hat{\beta}_i F_m$ , from the first step as explanatory variables. The goal of the second pass regressions is to establish whether a larger factor exposure leads to a higher return. The cross-sectional regressions can be written as:

$$\begin{aligned} R_{i,1} &= a_1 + \gamma_{1,1} \hat{\beta}_{i,F_1} + \gamma_{1,2} \hat{\beta}_{i,F_2} + \cdots + \gamma_{1,m} \hat{\beta}_{i,F_m} + \epsilon_{i,1} \\ R_{i,2} &= a_2 + \gamma_{2,1} \hat{\beta}_{i,F_1} + \gamma_{2,2} \hat{\beta}_{i,F_2} + \cdots + \gamma_{2,m} \hat{\beta}_{i,F_m} + \epsilon_{i,2} \\ &\vdots \\ R_{i,T} &= a_T + \gamma_{n,1} \hat{\beta}_{i,F_1} + \gamma_{n,2} \hat{\beta}_{i,F_2} + \cdots + \gamma_{n,m} \hat{\beta}_{i,F_m} + \epsilon_{i,T} \end{aligned} \quad (41)$$

As seen from Equation 41, the beta estimates found in Equation 40,  $\hat{\beta}_{i,F_m}$ , remain constant, while the portfolio returns vary in each time period,  $T$ . The  $\gamma_{i,T}$  terms are the regression coefficients. Fama and MacBeth (1973) suggest that the estimated intercept and factor risk premium for a given factor is the average of the cross-sectional regression estimates:

$$\hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_{i,t}, \quad \hat{\gamma} = \frac{1}{T} \sum_{t=1}^T \hat{\gamma}_t \quad (42)$$

Furthermore, they suggest using the time-series variance of the coefficients from the cross-sectional regressions as standard errors to test the statistical significance of the coefficients:

$$\sigma^2(\hat{\alpha}_i) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\alpha}_{i,t} - \hat{\alpha}_i)^2, \quad \sigma^2(\hat{\gamma}) = \frac{1}{T} \sum_{t=1}^T (\hat{\gamma}_t - \hat{\gamma})^2 \quad (43)$$

## 5.2 CONSTRUCTION OF TESTING PORTFOLIOS

In the following section, we provide definitions of the sorting variables, as well as an overview of the portfolio dynamics in terms of portfolio construction and rebalancing.

### 5.2.1 SORTING OF TESTING PORTFOLIOS

Widespread in financial research is to sort stocks into portfolios, as opposed to conduct tests on individual stocks. As previously mentioned in the literature review Section 2.3.2.2, beta estimates on portfolios are considered more precise than estimates on individual securities. Furthermore, double sorts, and more recently, triple sorts on firm characteristics are frequently used in financial literature. The benefit of multiple sorting procedures is the ability to isolate the effect of one characteristic from others, as this provides portfolios consisting of stocks with similar characteristics. The drawback from sorting on multiple firm characteristics is that the number of portfolios increase exponentially for each sorting, significantly reducing the number of stocks in each portfolio. Considering the restricted number of stocks in our sample, we stick with single sorted testing portfolios, although we are aware that results might be biased in the presence of factor correlation.

### 5.2.2 SORTING VARIABLES

#### 5.2.2.1 BETA

Beta is defined as the covariance between the return of the  $i_{th}$  security and the return of the market portfolio, divided by variance of the market portfolio:

$$\beta_{it} = \frac{Cov(R_{it}, R_{mt})}{Var(R_{mt})} \quad (44)$$

There are several considerations that must be made when we estimate beta. First, the frequency at which the returns are measured must be specified. Second, the time period over which betas are estimated must be established. Finally, the market portfolio must be defined.

### *1) Specifying the return frequency*

Specifying the frequency of which of returns are calculated in the estimation of beta is of importance, as a stock's beta varies across return frequencies. Lately, measuring a security's instantaneous and time-varying riskiness has increased in importance, due to the increase in high-frequency trading and the reduction in investment horizons (Gilbert, Hrdlica, Kalodimos, and Siegel, 2014). Hence, literature has explicitly derived the potential for using more finely sampled realized returns to measure beta (Andersen et al., 2006). Return frequencies of only 30 minutes as proposed by Cenesizoglu, Liu, Reeves, and Wu (2014) contrasts earlier papers, where beta estimations are based on monthly return frequencies (Fama & French, 1992, Fama & MacBeth, 1973). A relatively recent paper by Gilbert et al. (2014) investigates the difference between high frequency (daily) and low frequency (quarterly) market betas. Their research shows that the frequency dependence of betas is associated with firm- and industry-level proxies of opacity. Here, opacity should be understood as the uncertainty about the effect of systematic news on firm value. Specifically, opaque firms have high-frequency betas that are smaller than their low-frequency betas, while the opposite is true for transparent firms. They conclude that asset pricing models that might be appropriate at low frequencies will not price assets correctly when applied at high frequencies, as the effect of opacity-induced uncertainty is not captured by betas.

Bearing this in mind, the market betas are calculated using monthly returns as opposed to daily returns to avoid the downward bias in beta estimates, which coincides with the earlier asset pricing papers. Using longer return frequencies such as quarterly returns would significantly reduce the number of observations in the beta estimations, and is therefore found to be suboptimal compared to using monthly returns.

### *2) Specifying the time frame for beta calculation*

Beta estimates will vary depending on the time frame of the beta calculation. Specification of an appropriate time frame is therefore of importance and involves a tradeoff: by going further back in time, we get the advantage of getting more observations in the estimation. This could however be offset by the fact that firms have changed its characteristics in

terms of business mix and leverage (Damodaran, 1999). Critically, increasing the time frame will reduce the number of beta estimates in our sample, as this requires additional historical returns that may not be available. Taking this into account, we argue that an adequate time frame for beta estimation is 36 months. Furthermore, we estimate betas every month using rolling regressions to mitigate potential cross-sectional dependence in the regression residuals.

### 3) *Specifying the market portfolio*

In practice, no indices are adequate as proxies for the market portfolio (see Roll's critique in literature review Section 2.3.2.4). Optimally, our proxy for the market portfolio should include all fixed income and real assets. However, in line with previous academic research, we choose an index composed of all the listed shares on the OSE. Specifically, we use the return from the OSEAX, which is a value-weighted index adjusted for corporate actions.

To conclude, betas are estimated every month from monthly observations based on three years (36 months) of data using the excess returns from the OSEAX as a proxy for the market portfolio.

#### 5.2.2.2 SIZE

Size is defined as the market capitalization of the  $i_{th}$  firm at the end of each June of year  $t$ :

$$market\ capitalization_{i,t} = share\ price_{i,t} \times no.\ shares\ outstanding_{i,t} \quad (45)$$

#### 5.2.2.3 BOOK-TO-MARKET VALUE OF EQUITY

Book-to-market value (BM) of equity is defined as the ratio of a firm's book value of equity at the end of fiscal year  $t - 1$  and its market value of equity at the end of December  $t - 1$ :

$$\frac{BE}{ME_t} = \frac{book\ value\ of\ equity_{t-1}}{market\ value\ of\ equity_{Dec-1}} \quad (46)$$

Where the market value of equity is the same as market capitalization as in Equation 45 and the book value of equity is calculated as:

$$\text{Book value of equity}_t = \text{stockholders' equity}_t + \text{deferred taxes}_t - \text{book value of preferred stock}_t$$

The use of December market equity is objectionable for firms that do not end their fiscal year in December, because the accounting variable in the numerator of a ratio does not correspond with the market value in the denominator. However, Fama and French (1992) find that excluding stocks that do not end their fiscal year in December result in similar results as when they are included. Since most listed companies on the OSE end their fiscal year in December, we assume that inclusion of stocks with irregular fiscal years will not significantly alter our results. Hence, we make no exclusions or adjustments on the basis of this.

#### 5.2.2.4 MOMENTUM

A stock's momentum is defined as the cumulative return from month  $t - 12$  to  $t - 2$ . The momentum factor is measured as the cumulative return from May in the previous year, to June in the current year:

$$\text{momentum}_t = \frac{P_{t-1} - P_{t-13}}{P_{t-13}} \quad (47)$$

#### 5.2.2.5 MAX

MAX is defined as the average of the five highest daily returns in month  $t - 1$  for a given stock. The variable is constructed to capture the demand for lottery stocks.

### 5.2.3 PORTFOLIO DYNAMICS

Rebalancing of the portfolios occur either monthly or yearly, depending on the nature of the sorting variable. The portfolios sorted on size, value and momentum are rebalanced on a yearly basis, while the portfolios sorted on beta and MAX are rebalanced monthly.

#### 5.2.3.1 YEARLY REBALANCING

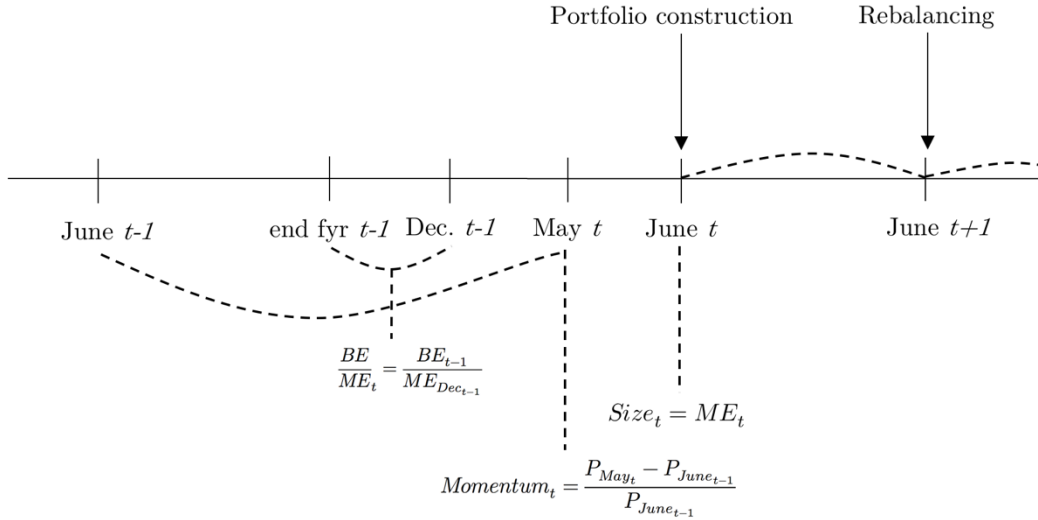
The dynamics of the yearly rebalanced portfolios are illustrated in Figure 7. The portfolios sorted on BM are based on a company's book value of equity from the previous fiscal year and the market value of equity at the end of December  $t - 1$ . The reason for the time lag between measuring the ratio of book-to-market equity and portfolio rebalancing is to avoid using information that may not have been publicly available at the time, known as "look-

ahead bias". Portfolios sorted on size, on the other hand, use the market value of equity at the end of June in year  $t$ .

Portfolios sorted on the 12-month price momentum use the price 13 months and 1 month prior to the portfolio rebalancing date. As mentioned in literature review Section 2.3.5., momentum is lagged one period to avoid the reversion of returns in  $t - 1$ , which is standard in financial literature. Hence, portfolios sorted on the 12-month momentum is based on its price from June from the previous year,  $t - 1$ , and the price from May in year  $t$  to calculate the cumulative return over the period.

At the end of June each year, five portfolios for each sorting variable is constructed. The excess return from July to June in  $t + 1$  is tracked, before the portfolios again are rebalanced into five new portfolios.

**FIGURE 7** – CONSTRUCTION OF PORTFOLIOS SORTED ON BM, SIZE, AND MOMENTUM

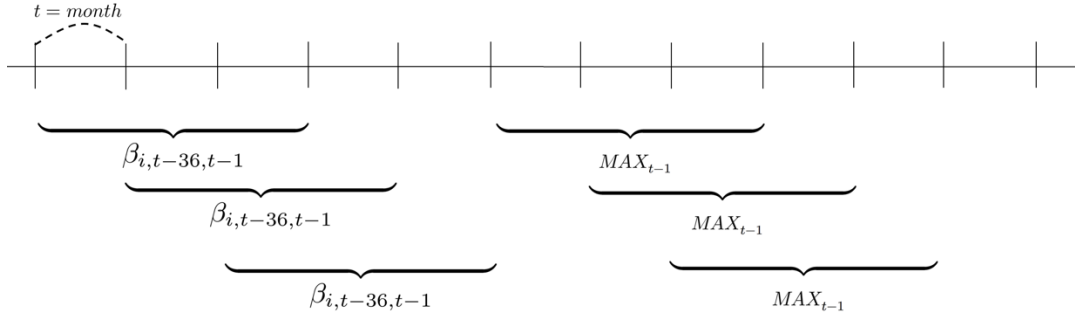


#### 5.2.3.2 MONTHLY REBALANCING

The portfolios sorted on beta and MAX are rebalanced monthly, and the portfolio dynamics are illustrated in Figure 8. At the end of each month, five portfolios for each of the two sorting variables are constructed. The returns of the portfolios are subsequently tracked, before the portfolios again are rebalanced into 5 new portfolios.



**FIGURE 8** – CONSTRUCTION OF PORTFOLIOS SORTED BETA AND MAX



### 5.2.3.3 PORTFOLIO WEIGHTING

It is common to form equally-weighted quantile portfolios on the variable of interest. A potential problem with equally-weighted portfolios occurs if the distribution of the stocks' market capitalization is positively skewed. With a positively skewed sample, the average returns will be dominated by smaller stocks (Fama & French, 2008). A potential solution to this problem is to use value-weighted portfolio returns. However, a similar issue would occur here, if the distribution of the stocks' market capitalization is negatively skewed. With a negatively skewed sample, the average returns will be dominated by larger stocks.

A double sorting procedure using the market capitalization along with a second firm characteristic would allow us examine the average returns from separate sorts of small and big stocks. However, as discussed in the methodology subsection 5.2.1, we have chosen not to do so because the average number of stocks in each portfolio would be too low. In subsection 3.2, we found that the Norwegian stock market is dominated by a few large stocks. We therefore argue that sorting the testing portfolios based on their market capitalization would neglect the small sized stocks to such an extent that it would not serve the purpose of this thesis. While aware of the pitfalls of sorting the testing portfolios on an equally-weighted basis, we believe that this allows for a better understanding of what drives the returns on the OSE. The expected returns of the testing portfolios are therefore calculated on an equally-weighted basis:

$$r_{P,t} = \frac{1}{n} \sum_{i=1}^n (r_{i,t} - r_f) \quad (48)$$

### 5.3 CONSTRUCTION OF FACTOR MIMICKING PORTFOLIOS

Having constructed the portfolios whose returns are to be explained, we now turn to the factor-mimicking portfolios. In addition to the market factor outlined in the previous section, we construct four additional factors.

#### 5.3.1 SMB AND HML

The construction of SMB and HML follows the methodology of Fama and French (1992). We create six portfolios sorted on the size and BM, as they are defined in subsection 5.2.2.2 and 5.2.2.3.

In June of each year  $t$ , all stocks in our filtered sample are allocated into two portfolios, small (S) or big (B), based on whether their market capitalization is higher or lower than the median market capitalization.

Independently, the stocks are also allocated into three BM portfolios based on the breakpoints for the bottom 30% (Low), middle 40% (Medium), and top 30% (High). Similarly to the testing portfolios sorted BM from the previous section, the book value of equity is measured at the fiscal year ending in the calendar year  $t - 1$ . The market capitalization is correspondingly measured in December  $t - 1$ . Firms with negative BE are excluded from the sort.

Six portfolios are defined from the intersection of the two size portfolios and the three value portfolios, as seen from Figure 9. For example, the stocks allocated to the B/H portfolio have a BM in the top 30% and a market capitalization higher than the median. The six portfolios serve as the basis for the factor mimicking portfolios. The monthly value-weighted returns are calculated for each of the six portfolios from July in year  $t$  to June in year  $t + 1$ , before the portfolios are rebalanced. If companies are delisted during the year, the total number of stocks in each portfolio will vary over the period. If companies are listed after June in year  $t$ , they will not be included before portfolio rebalancing in June year  $t + 1$ . The SMB factor mimicking portfolio is calculated on a monthly basis, defined as the difference between the average value-weighted returns of the three small-stock portfolios and the average value-weighted returns of the three big-stock portfolios, as seen from Equation 49. Likewise, the HML factor mimicking portfolio is

calculated as the difference between the average value weighted returns from the two high-BM portfolios and the average value-weighted returns from the two low-BM portfolios, as seen from Equation 50.

**FIGURE 9** – CONSTRUCTION OF THE SMB AND HML FACTOR

		BE/ME		
		H	M	L
Size	Small	S/H	S/M	S/L
	Big	B/H	B/M	B/L

$$SMB = (S/H + S/M + S/L)/3 - (B/H + B/M + B/L)/3 \quad (49)$$

$$HML = (S/H + B/H)/2 - (S/L + B/L)/2 \quad (50)$$

### 5.3.2 PR1YR

We follow Carhart (1997) when we construct the PR1YR factor, which captures the one-year momentum anomaly documented by Jegadeesh and Titman (1993). The PR1YR is the return on a value-weighted, zero-investment, factor mimicking portfolio. Each month,  $t$ , we divide stocks into portfolios based on the 30<sup>th</sup> and 70<sup>th</sup> percentiles of the lagged momentum returns of all the stocks in our sample. Hence, for portfolios formed at the end of month  $t - 1$ , we calculate the stock's cumulative return for month  $t - 12$  to month  $t - 2$ , to avoid the reversal in short-term returns in  $t - 1$ , as discussed in the literature review section 2.3.5. To be included in the portfolio for month  $t$ , a stock must therefore have a price at the end of month  $t - 13$ , as well as consecutive monthly returns from  $t - 12$  to  $t - 2$ , subject to our filter criterion.

For both the top and bottom quantile portfolio sorted on their lagged cumulative return, the value-weighted average portfolio return is calculated in month  $t$ . Since the PR1YR factor mimicking portfolio is defined as long position in the previous winners and a short position in the previous losers, the average return from the momentum strategy is calculated as the average value-weighted return of the winner portfolio minus the average value-weighted return of the loser portfolio, each month. At the end of month  $t$ , the portfolios are rebalanced using the same procedure.

### 5.3.3 BETTING AGAINST BETA

To investigate whether the presence of the beta anomaly on the OSE can be attributed funding frictions, we create a betting against beta (BAB) factor, following Frazzini and Pedersen (2014). A BAB factor is a portfolio that holds low-beta assets, leveraged to a beta of 1, and that shorts high beta assets, de-levered to a beta of 1. In effect, the portfolio is market neutral with a beta of zero.

To construct the BAB factor, we run monthly rolling regressions using 36 months of data to obtain beta estimates for each stock for each month, as outlined in the methodology section 5.2.2.1. This is the same procedure used in the paper by Frazzini and Pedersen (2014), although they estimate their betas using daily return frequencies, and not monthly (see the methodology section 5.2.2.1 for justification). To reduce the influence of outliers, we shrink the time-series estimate of beta ( $\hat{\beta}_i^{TS}$ ), toward the cross-sectional mean ( $\beta^{XS}$ ):

$$\hat{\beta}_i = w_i(\hat{\beta}_i^{TS} + (1 - w_i)\beta^{XS}) \quad (51)$$

We set  $w = 0.6$  and  $\beta^{XS} = 1$  for all periods and across all assets, following Frazzini and Pedersen (2014). The shrinkage factor does not affect how securities are sorted into portfolios, since the common shrinkage does not change the ranks of the security betas. However, the amount of shrinkage affects the construction of the BAB portfolios since the estimated betas are used to scale the long and short sides of the portfolios, as seen from Equation 55

To construct each portfolio, all securities are ranked in ascending order on the basis of their estimated shrunken betas. The ranked securities are assigned to one of two portfolios: low-beta and high-beta. The low (high) beta portfolio is comprised of all stocks with a beta below (above) the median of the ascending beta estimates. In each portfolio, securities are weighted by the rank of their betas. Specifically, this means that the lower-beta securities have larger weights in the low-beta portfolio and higher-beta securities have larger weights in the high-beta portfolio. The portfolios are rebalanced every month.

Formally, let  $v$  denote the  $n \times 1$  vector of beta ranks  $v_i = \text{rank}(\beta_{it})$  at portfolio formation, and let  $\bar{v} = \frac{1'_n v}{n}$  be the average rank of all the assets where  $n$  is the total number of assets

and  $1_n$  is a vector of ones with the dimension  $n \times 1$ . Given these definitions, the vectors of individual asset weights can be written as:

$$w_L = -\frac{1}{k} \min(0, v - \bar{v}) \quad (52)$$

$$w_H = \frac{1}{k} \max(0, v - \bar{v}) \quad (53)$$

Where  $k$  is a normalizing constant that assures that the sum of weights in each portfolio is equal to one, hence  $k = \frac{1'_n |v - \bar{v}|}{2}$ .

For matters of simplification, consider the following example. In the case of  $n = 7$  and after ordering the betas so that  $\beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_5 < \beta_6 < \beta_7$ , the variables take the following values:

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ 5 \\ 7 \end{bmatrix}, \quad \bar{v} = \frac{\sum_{i=1}^7 v_i}{7} = 4, \quad k = \frac{\sum_{i=1}^7 |v_i - \bar{v}|}{2} = 6, \quad \mathbf{w}_L = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{w}_H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/3 \\ 1/6 \\ 1/2 \end{bmatrix}$$

The process ensures that both the low-beta portfolio and the high-beta portfolio is weighted to have betas equal to one with the following characteristics:

$$r_{t+1}^L = r'_{t+1} w_L, \quad r_{t+1}^H = r'_{t+1} w_H, \quad \beta_t^L = \beta'_t w_L, \quad \text{and} \quad \beta_t^H = \beta'_t w_H \quad (54)$$

Having constructed the two portfolios, the BAB factor is created as a long-short combinations of the portfolios – the low beta portfolio is the long position while the high beta portfolio is sold short. Since both portfolios have a beta of one, the BAB factor is market neutral. The BAB factor is calculated as the average excess return from the low-beta portfolio minus the average excess return of the high-beta portfolio:

$$r_{t+1}^{BAB} = \frac{1}{\beta_{t+1}^L} (r_{t+1}^L - r^f) - \frac{1}{\beta_{t+1}^H} (r_{t+1}^H - r^f) \quad (55)$$

### 5.3.4 FMAX

To investigate the contrarian hypothesis that the prevalence of the beta anomaly on the OSE can be attributed the demand for lottery stocks, we create the factor FMAX, following Bali et al. (2017).

At the end of each month  $t$ , we sort all stocks into two groups based on the market capitalization, with the median as the breakpoint. Subsequently, we sort all stocks in our sample into three groups based on an ascending sort of MAX, as defined in the methodology section 5.2.2.5. Using the same factor creation technique as with the HML and SMB, we construct six portfolios in the intersection of the two market capitalization-based groups and the three MAX groups. As seen from Figure 10 and Equation 56, the FMAX factor return in month  $t + 1$  is the average return of the two value-weighted high-MAX portfolios minus the two value-weighted low-MAX portfolios. As such, the FMAX factor portfolio is designed to capture returns associated with lottery demand, while maintaining neutrality to market capitalization.

**FIGURE 10** – CONSTRUCTION OF THE FMAX FACTOR

		MAX		
		Low	Neutral	High
Size	Small	S/L	S/N	S/H
	Big	B/L	B/N	B/H

$$FMAX = (S/H + B/H)/2 - (S/L + B/L)/2 \quad (56)$$

## 6 ANALYSIS

In this section we present the empirical evidence from the investigation of OSE. As mentioned in the methodology section, we begin by presenting our findings from the portfolio sorts approach. This involves analyzing the average returns from portfolios sorted firm characteristics to get an overview of what drives the returns on OSE. Furthermore, we present the excess returns associated with a long/short trading strategy in the top and bottom quintile portfolios and its statistical significance. From here, we move on to the time-series regressions and Fama and MacBeth (1973) cross-sectional regressions. Here, we estimate the direct marginal effects from our constructed factors on the portfolio sorts, as well as the estimated size of the risk premia. Finally, we analyze to whether the beta anomaly on the Norwegian stock market can attributed leverage and margin constraints with investors, as postulated by Frazzini and Pedersen (2014) or a demand for lottery stocks, as postulated by Bali et al. (2017). For all the statistical tests in the analysis, we operate with a significance level of 5%.

### 6.1 PORTFOLIO SORTS

#### 6.1.1 PORTFOLIOS SORTED ON BETA

At the end of each month  $t$ , we form five portfolios sorted on the individual securities' beta estimates. The portfolio return for each quintile portfolio is subsequently tracked, before the portfolios are rebalanced at the end of the month with updated beta estimates.

Table 7 shows the average monthly returns over the time period 2007 – 2017 for portfolios sorted on beta, where portfolio 1 is the low-beta portfolio and portfolio 5 is the high-beta portfolio. To illustrate the effect of large stocks, Panel A and Panel B shows the equally-weighted and value-weighted portfolio returns, respectively. The average beta of the two extreme portfolios varies significantly when we compare the equally-weighted and the value-weighted portfolios: Portfolio 1 has an average beta of 0.08 versus 0.21 and portfolio 5 has an average beta of 1.69 versus 1.48. Thus, the larger stocks in terms of market capitalization allocated in the low-beta portfolios typically have a higher beta than the smaller stocks. Conversely, the larger stocks allocated in the high-beta portfolios typically have a lower beta than the smaller stocks.

No monotonic relationship can be observed between the average portfolio beta and average monthly excess return, regardless of whether the returns are equally-weighted or value-weighted. This also applies when we consider the median return. Considering the equally-weighted returns in Panel A, the low-beta portfolio yields an average monthly excess return of 0.53%, compared to a return of -0.13% for the high-beta portfolio. The returns from all the portfolios move in opposite directions with increasing average betas, indicating no obvious relationship between systematic market risk and return. The value-weighted portfolios exhibit the same pattern in returns; the small-beta portfolio yields an average monthly excess return of 1.24%, compared to a return of 0.6% in the high-beta portfolio. Based on our data, there seem to be no relationship between securities' systematic market risk and return, which contradicts the CAPM. Hence, we expect that the CAPM will explain little of the variation in portfolio returns in the cross-sectional regressions later in the analysis.

Table 8 shows the average monthly excess returns per period for portfolio 1 and 5, as well as the returns from a long high-beta/short low-beta trading strategy and its associated t-values. Unsurprisingly, both the equally weighted and the value-weighted returns for the extreme portfolios are highly negative in the 2007 - 2008 period, due to the financial crisis. For both the equally-weighted and value-weighted portfolios, the long/short portfolio yields positive returns from 2007 – 2010. However, the returns are not found to be statistically significant. In the 2011 - 2016 period, the long/short portfolios yield negative excess returns, although statistically insignificant at the 5% level for both the value-weighted and equally-weighted portfolio. The beta anomaly is present when high-beta stocks underperform low-beta stocks on a risk-adjusted basis. The results from Table 7 and Table 8 indicate that investors in high-beta stocks are not adequately compensated for the systematic market risk they undertake. Hence, the beta-anomaly seems to be present in our sample of Norwegian stocks.

The dominance of a few, large stocks, as documented in the characteristics of the OSE Section 3.2, is likely to have significant impact on our results. Going forward, we sort portfolios on an equally-weighted basis, and hence report equally-weighted returns. As discussed in the methodology Section 5.2.3.3, there are both pros and cons associated with



using value-weighted and equally-weighted returns. Although many of the small stocks will be assigned higher weights than their market capitalization imply, we argue that equally-weighted portfolios will provide a better understanding of what drives the returns on the OSE. For the interested reader, value-weighted portfolios and their associated returns can be found in Appendix 5 – 8.

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**Table 7** Monthly excess returns from portfolios sorted on beta (2007 - 2017)

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<b>Panel A</b>		Equally-weighted portfolios							
Portfolio	Average beta	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	0.08	0.53	(4.43)	-21.01	0.52	11.38	11	15	21
Portfolio 2	0.53	0.17	(5.89)	-26.18	0.55	18.65	10	15	20
Portfolio 3	0.79	0.45	(6.04)	-28.29	0.74	14.42	10	15	20
Portfolio 4	1.08	0.04	(6.45)	-28.94	0.17	14.70	10	15	20
Portfolio 5	1.69	-0.13	(8.20)	-33.92	-0.18	18.23	11	15	21

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<b>Panel B</b>		Value-weighted portfolios							
Portfolio	Average beta	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	0.21	1.24	(5.96)	-18.77	1.42	20.95	11	15	21
Portfolio 2	0.59	0.40	(7.64)	-37.95	0.95	34.76	10	15	20
Portfolio 3	0.77	0.36	(5.74)	-25.53	0.88	15.05	10	15	20
Portfolio 4	1.13	0.98	(6.86)	-29.12	1.04	22.51	10	15	20
Portfolio 5	1.48	0.60	(8.04)	-37.99	0.35	20.30	11	15	21

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Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Beta estimates are calculated every month using 36 months of historical returns. Historical returns from 2005 - 2015 are used in beta calculation.

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**Table 8** Monthly excess return from a long-short position in beta portfolios (2007 – 2017)

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Year	Equally-weighted portfolios				Value-weighted portfolios			
	PF 1	PF 5	Diff	t: Diff = 0	PF 1	PF 5	Diff	t: Diff = 0
2007 - 2008	-3.62	-3.52	0.10	(0.07)	-2.39	-2.16	0.23	(0.11)
2009 - 2010	2.78	4.10	1.31	(0.93)	2.49	4.49	2.00	(1.16)
2011 - 2012	0.98	-1.55	-2.53	(-1.88)*	1.47	-0.92	-2.39	(-2.00)*
2013 - 2014	1.02	-0.74	-1.76	(-1.94)*	2.34	0.75	-1.59	(-1.56)
2015 - 2016	1.59	0.56	-1.03	(-0.79)	2.75	0.60	-2.15	(-1.64)
2017	0.28	0.90	0.62	(0.27)	0.33	1.08	0.75	(0.42)
Full sample	0.53	-0.13	-0.66	(-1.15)	1.24	0.60	-0.64	(-1.00)

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

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### 6.1.2 PORTFOLIOS SORTED ON SIZE

Banz (1981) was the first to document that firms with a smaller market capitalization offer higher risk-adjusted returns than those with a higher market capitalization. Fama and French (1993) argue that this is due to a common source of risk. To investigate whether the stocks at the OSE exhibit similar return characteristics, we sort stocks into portfolios based on their market capitalization, as described in the methodology section 5.2.2.2

Table 9 shows the average monthly excess returns on portfolios sorted on size. The average size of the stocks in portfolio 5 is almost 83 billion NOK, compared to only 438 million NOK for the stocks in portfolio 1. This emphasizes that there are a few large stocks that dominate the OSE. There is no monotonic relationship between the average size and the returns, as portfolio 2 provides a significantly lower return than portfolio 3.

Table 10 shows the average monthly excess return for portfolio 1 and 5 per period, as well as the returns from long big-size/short small-size stocks. For the full period, the trading strategy would yield negative excess returns. However, none of the t-statistics indicates that the returns from the trading strategy is statistically distinguishable from zero. From Table 10, we see that the low monthly returns to a large extent can be attributed the period 2007 – 2008.

**Table 9** Monthly excess returns from portfolios sorted on size (2007 - 2017)

Portfolio	Average size (millions)	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	438.07	0.81	(6.61)	-22.06	0.26	36.34	6	14	20
Portfolio 2	1,458.22	-0.45	(6.61)	-27.33	-0.44	21.16	7	13	20
Portfolio 3	3,971.08	0.16	(5.82)	-25.87	0.61	17.59	10	14	19
Portfolio 4	9,248.04	0.10	(6.18)	-32.16	0.22	18.54	9	14	20
Portfolio 5	82,849.73	0.11	(7.07)	-29.66	0.07	18.04	9	15	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based the market capitalization at the end of June each year, and the monthly returns are subsequently tracked, before the portfolios are rebalanced in June  $t + 1$ . The returns are equally-weighted.

**Table 10** Monthly excess returns from a long-short position in size portfolios (2007 – 2017)

Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0
2007 - 2008	-4.91	-5.24	-0.33	(-0.20)
2009 - 2010	4.01	3.11	-0.91	(-0.36)
2011 - 2012	0.91	-0.27	-1.18	(-1.06)
2013 - 2014	0.74	0.08	-0.66	(-0.77)
2015 - 2016	1.62	0.87	-0.75	(-0.87)
2017	1.31	1.39	0.08	(-0.06)
Full sample	0.81	0.11	-0.70	(-1.12)

Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

### 6.1.3 PORTFOLIOS SORTED ON BM

The value anomaly has been found in numerous studies, and asserts that stocks with a high ratio of book-to-market value of equity offer higher risk-adjusted returns than stocks with a low ratio book-to-market value of equity. To investigate whether high-BM stocks are associated with higher excess returns, we construct 5 portfolios sorted on BM, as described in the methodology section 5.2.2.3

Table 11 shows the average monthly excess returns on 5 portfolios sorted on BM. The average BM spans from 0.25 for the lowest BM portfolio to 2.13 for the highest BM

portfolio. As seen from the table, no monotonic relationship can be observed between BM and average returns: portfolio 1 has an average monthly excess return of 0.01%, and the subsequent portfolio returns move in opposite directions with ascending values of BM.

Table 12 shows the average monthly excess return on the high BM portfolios and low BM portfolios per period, as well as the average monthly excess returns from a long high-BM – short low-BM trading strategy and its associated t-statistic. There does not seem to be systematic pattern of positive or negative returns, which contradicts the value anomaly. However, the returns are not found to be statistically significant. Based on our findings summarized in Table 11 and Table 12, we do not expect that the HML factor will explain a significant portion of the variation in the cross-section of returns. Again, we see that the low returns can be attributed the crisis period from 2007 to 2008.

<b>Table 11</b>		Excess monthly returns from portfolios sorted on BM (2007 - 2017)							
Portfolio	Average BM	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	0.25	0.01	(6.32)	-31.36	0.11	16.92	10	14	20
Portfolio 2	0.48	0.62	(6.00)	-29.70	1.03	14.74	8	14	20
Portfolio 3	0.74	0.17	(5.36)	-22.87	0.46	13.39	8	14	19
Portfolio 4	1.08	0.04	(6.29)	-28.71	0.39	17.68	10	14	20
Portfolio 5	2.13	-0.17	(7.11)	-21.14	-0.24	22.85	4	14	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the ratio of the book value of equity at fiscal yearend and the market capitalization at December  $t - 1$ , at the end of June in year  $t$ . The monthly returns are subsequently tracked, before the portfolios are rebalanced in June  $t + 1$ . The returns are equally-weighted.

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**Table 12**      Monthly excess returns from a long-short position in BM portfolios (2007 – 2017)

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Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0
2007 - 2008	-5.61	-6.61	-1.00	(-0.73)
2009 - 2010	2.70	3.25	0.55	(0.44)
2011 - 2012	-0.56	-0.30	0.26	(0.27)
2013 - 2014	0.93	0.54	-0.39	(-0.48)
2015 - 2016	0.99	-0.10	-1.09	(-0.87)
2017	0.37	1.38	1.01	(0.75)
Full sample	0.01	-0.17	-0.18	(-0.37)

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

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#### 6.1.4 PORTFOLIOS SORTED ON MOMENTUM

Studies have shown that stocks that have performed well over the past 12 months offer higher returns than stocks that have performed poorly over the same period. To investigate whether the momentum effect is prevalent among the stocks on the OSE, we construct 5 portfolios based on the 12-month cumulative return, as described in the methodology section 5.2.2.4.

Table 13 shows the average monthly excess returns on portfolios sorted on momentum, where portfolio 1 contains stocks that have performed poorly over the last year, and portfolio 5 contains stocks that have the highest performance over the same period. No monotonic relationship can be observed between the momentum portfolios and average monthly excess returns, as the returns from portfolio 3 is lower than portfolio 2.

As seen from Table 14, a momentum effect cannot be found on the OSE: A trading strategy where one shorts the portfolio containing the worst-performing stocks and takes a long position in a portfolio containing the best-performing stocks over the past year, have provided an average monthly excess return of 0.96% from 2007 – 2017. However, this result is significant at the 10% level, which is below our significance threshold. In the 2007 – 2008 and 2011 – 2012 period, the trading strategy yields negative returns, although not statistically significant. Based on our findings summarized in Table 14 and Table 13, we

do not expect that the factor PR1YR will capture a large proportion of the variation in the cross-section of returns.

**Table 13** Monthly excess returns from portfolios sorted on momentum (2007 - 2017)

Portfolio	Average cum. return	Portfolio returns					Stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	-16.66	-0.20	(7.31)	-22.27	-0.11	15.53	7	14	20
Portfolio 2	-2.53	0.03	(5.67)	-26.16	0.09	14.39	8	14	20
Portfolio 3	5.42	-0.06	(5.89)	-26.74	0.43	14.25	10	15	19
Portfolio 4	15.80	0.17	(6.01)	-25.68	0.79	20.30	8	14	20
Portfolio 5	54.20	0.76	(6.90)	-32.73	1.37	20.73	9	14	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the cumulative return from  $t - 12$  to  $t - 2$ , where  $t$  refers to the first month holding the portfolio (July). The monthly returns are subsequently tracked, before the portfolios are rebalanced in June the following year. The returns are equally-weighted.

**Table 14** Monthly excess returns from a long-short position in momentum portfolios (2007 - 2017)

Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0
2007 - 2008	-4.92	-6.33	-1.41	(-1.06)
2009 - 2010	2.96	4.04	1.07	(0.79)
2011 - 2012	0.40	-0.03	-0.42	(-0.42)
2013 - 2014	0.22	1.21	0.99	(1.06)
2015 - 2016	-0.05	2.54	2.59	(1.86)*
2017	-1.80	1.96	3.76	(1.62)
Full sample	-0.20	0.76	0.96	(1.77)*

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

### 6.1.5 PORTFOLIOS SORTED ON MAX

Bali et al. (2017) argue that the beta anomaly can be explained by private investors' demand for lottery-like stocks. To get an impression of how lottery stocks perform, we first sort stocks into quintile portfolios based on the average 5-day average of the previous month's highest returns, MAX, as described in the methodology Section 5.2.2.5

Table 15 shows the average monthly excess return on portfolios sorted on MAX. Portfolio 1 contains stocks that have provided the lowest MAX with 1.65%, and portfolio 5 contains the stocks that have provided the highest MAX with 8.91%. No monotonic relationship can be observed between the portfolios sorted on MAX and their subsequent monthly returns, as the monthly returns change directions for ascending levels MAX.

Table 16 shows the average monthly return on the two extreme quintile portfolios per period, as well as the average return for a trading strategy, where a long position is taken in the high-MAX portfolio and the short position is taken in the low-MAX portfolio and its corresponding t-statistic. There seems to be no systematic pattern in the returns for the long/short trading strategy across the periods, and neither of the returns are distinguishable from zero. For the full period, the average monthly excess return is slightly positive, again contradicting the lottery-stock hypothesis, despite not being statistically significant.

<b>Table 15</b> Monthly excess returns from portfolios sorted on MAX (2007 - 2017)									
Portfolio	Avg. High return	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	1.65	0.31	(5.02)	-22.81	0.60	14.33	11	15	21
Portfolio 2	2.82	0.05	(6.04)	-24.66	0.15	21.18	10	15	20
Portfolio 3	3.72	0.12	(6.51)	-30.41	0.36	20.28	10	15	20
Portfolio 4	4.97	0.21	(6.42)	-28.06	0.19	17.51	10	15	20
Portfolio 5	8.91	0.37	(6.75)	-28.40	0.63	18.41	11	15	21

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the highest average 5-day return. The return in the subsequent month is calculated, before the portfolios are rebalanced at the end of the next month. The returns are equally-weighted.

<b>Table 16</b> Monthly excess returns from a long-short position in MAX portfolios (2007 – 2017)				
Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0
2007 - 2008	-2.96	-3.36	-0.40	(-0.41)
2009 - 2010	3.66	2.39	-1.27	(-0.92)
2011 - 2012	-1.12	0.82	1.94	(1.72)*
2013 - 2014	0.29	1.03	0.74	(0.86)
2015 - 2016	1.06	1.23	0.17	(0.17)
2017	1.54	-0.12	-1.66	(-1.40)
Full sample	0.31	0.37	0.06	(0.14)

Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

## 6.2 FAMA AND MACBETH REGRESSIONS

In the following section, we will present the results from our time-series- and cross-sectional regressions, using the Fama and MacBeth (1973) methodology. We start by estimating the CAPM for portfolios sorted on firm characteristics, before we move on to the Fama-French three-factor model, and the Carhart four-factor model. The asset pricing models are estimated on the same equally-weighted portfolios as in the previous section, namely, portfolios sorted on beta, size, value, momentum and MAX. However, although not included in the thesis, we have also estimated all models on a value-weighted basis, which the interested reader can find in the Appendix 10 - 33. In the last section, we test whether the beta anomaly can be attributed to margin- and leverage constrained investors or a demand for lottery stocks.

### 6.2.1 FACTOR CHARACTERISTICS

Summary statistics for the factor mimicking portfolios are reported in Table 17. All factors have provided positive monthly excess returns over the period. The SMB factor provides the highest monthly returns of 1.30% monthly, and they are highly significant.

Interestingly, the FMAX factor provides significant positive returns of 0.99%, contradictory to the theory, indicating that the lottery stock effect is not prevalent on the OSE.



The cross-correlation between the factors imply that multicollinearity will not substantially affect the estimated factor loadings: the highest cross-correlation is found between the market factor, MKT, and FMAX and is 0.36. This indicate that the factors, if found to be statistically significant across assets, will not explain the same variation in returns.

Table 17 Factor returns and cross-correlations (2007 - 2017)									
	Returns			Cross-correlations					
	Mean	(Std.)	t-test: Diff = 0	SMB	HML	MKT	PR1YR	FMAX	BAB
SMB	1.30	3.67	4.07***	1.00	0.33	-0.18	-0.09	0.18	0.28
HML	0.40	5.62	0.82	0.33	1.00	-0.12	-0.09	0.07	0.11
MKT	0.25	5.81	0.49	-0.18	-0.12	1.00	-0.24	0.36	0.05
PR1YR	0.31	7.13	0.50	-0.09	-0.09	-0.24	1.00	-0.27	0.09
FMAX	0.99	5.19	2.20**	0.18	0.07	0.36	-0.27	1.00	-0.17
BAB	0.91	5.78	1.81*	0.28	0.11	0.05	0.09	-0.17	1.00

Returns are percentage monthly excess returns. Returns are calculated as simple value-weighted returns and are not annualized. SMB and HML are the Fama and French (1992) factor that mimic size and value. MKT is the excess returns from the OSEAX-index. PR1YR is Carhart (1997) factor that mimic a 12-month momentum pricing strategy. FMAX is the Bali et al. (2017) representing the demand for lottery-like stocks. BAB is the betting against beta factor proposed by Frazzini and Pedersen (2014). The right-hand side of the table show the cross-correlations between the factors over the whole period. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*"

### 6.2.2 EXCLUSION OF 2007 – 2008 OBSERVATIONS

From the previous section, where we analyzed the returns from portfolio sorts, we saw that the extreme portfolios provided substantial negative returns during in the period 2007 – 2008. Of course, this was due to the financial crisis that hit global markets. The period was characterized by extreme risk aversion and the lack of liquidity, and we therefore expect the factor portfolios to behave differently than during the bull-market from 2009 until present. To illustrate this, Table 18 shows the same summary statistics as Table 17, but from 2009 – 2017. When excluding the crisis years, the returns from the MKT and BAB factor become statistically significant at the 5% level. The factor cross-correlations do not change significantly when comparing the two periods.

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**Table 18** Factor returns and cross-correlations (2009 - 2017)

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	Returns			Cross-correlations					
	Mean	(Std.)	t-test: Diff = 0	SMB	HML	MARKET	PR1YR	MAX	BAB
SMB	1.50	(3.62)	4.32***	1.00	0.34	-0.15	-0.06	0.16	0.26
HML	0.52	(5.55)	0.97	0.34	1.00	0.07	-0.12	0.15	0.08
MKT	0.98	(4.08)	2.49***	-0.15	0.07	1.00	-0.26	0.40	-0.08
PR1YR	0.34	(7.25)	0.49	-0.06	-0.12	-0.26	1.00	-0.23	0.08
MAX	1.08	(5.22)	2.15**	0.16	0.15	0.40	-0.23	1.00	-0.21
BAB	1.76	(5.72)	3.19***	0.26	0.08	-0.08	0.08	-0.21	1.00

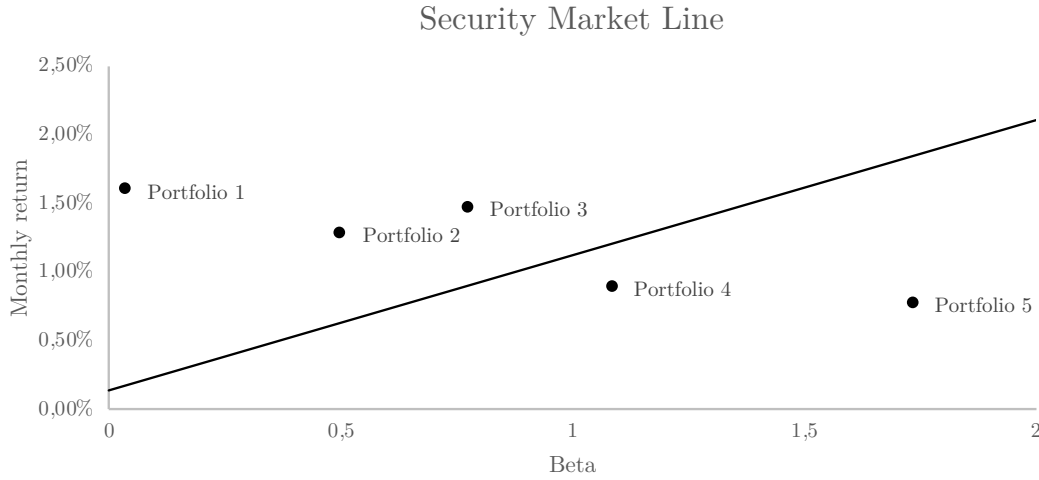
The table is the same as Table 17, but calculated for the period 2009 – 2017. Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

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We realize that due to our relatively short sample period, returns from 2007 – 2008 period are likely to affect our analysis to such the extent that it becomes difficult specify a model that adequately captures the variation in portfolio returns. Based on the abovementioned, we have chosen to exclude the first two years of our sample period when performing the cross-sectional analysis. We believe that the loss of observations can be justified by the increased reliability of our results. Hence, all our cross-sectional regression will use historical data from the period 2009 – 2017.

Since we now use historical data from 2009 – 2017, we examine whether the beta anomaly is present in the new data sample. In Figure 11, the average monthly return from five equally-weighted portfolios sorted on beta is shown together with the security market line (SML). The figure shows that portfolio 1, 2 and 3 provide returns substantially higher than predicted by the CAPM. Conversely, portfolio 4 and 5 provide returns substantially lower than predicted by the CAPM . In other words, for the high-beta portfolios, investors are not adequately compensated for the systematic risk they undertake, and the beta anomaly is evidently present on the OSE between 2009 and 2017. Hence, our motivation to identify other systematic risk factors and the origin of the beta anomaly persevere.

**FIGURE 11 - MONTHLY RETURNS OF BETA-SORTED PORTFOLIOS VS. SML (2009 – 2017)**



### 6.2.3 ESTIMATION OF THE CAPM

We begin by estimating the CAPM parameters to assess whether the excess market return alone can explain the cross-section of returns in our sample, and whether the market risk premium has been priced on the OSE. As explained in the methodology section, we apply the Fama and MacBeth (1973) methodology. First, we run five time-series regressions from 2009 – 2017 on the five portfolios sorted on firm characteristics. Next, the beta estimates obtained from the time-series regressions are used as explanatory variables in 108 monthly cross-sectional regressions, to obtain the estimated risk premia. The model is considered to be well-specified if the estimated intercept from the cross-sectional regression is statistically indistinguishable from zero, and the risk premium is statistically significant. All the time-series regression output can be found in the Appendices. However, since the second step cross-sectional regressions amount to a total 540 outputs, we decided not to include these in the Appendix.

#### 6.2.3.1 ESTIMATION OF THE CAPM FOR BETA-SORTED PORTFOLIOS

Table 19 shows the parameter estimations from the CAPM regressions on five beta-sorted portfolios in the period 2009 – 2017. Panel A shows the intercepts and factor exposures for the portfolios sorted on beta, and Panel B shows the average coefficients from the cross-sectional regression, equivalent to the estimated risk premia. The full regression output can be found in Appendix 9, along with the same regressions with value-weighted portfolios in Appendix 10. Concentrating on Panel A, it can be seen that one of the alpha estimates are

significantly different from zero at the 5% level. As expected, the average market factor exposure increases monotonically 0.46 for portfolio 1 to 1.41 for portfolio 5. The excess market return is only able to explain 0.28% of the variation of the lowest beta portfolio, but increases for portfolios with ascending betas.

Estimation of the risk premia can be seen from Panel B. Clearly, the model is not well-specified, as the average alpha estimate from the cross-sectional regressions is statistically significant at the 1% level. The market risk premium is found to be negative, but not statistically significant.

Table 19 Estimation of the CAPM for portfolios sorted on beta (2009 - 2017)			
Panel A		Time-series regression exposure estimates	
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	Adjusted $R^2$
Low beta	0.010***	0.464***	0.281
2	0.004	0.783***	0.442
3	0.004	0.953***	0.595
4	-0.003	1.106***	0.631
High beta	-0.008*	1.411***	0.616

Panel A shows the results from the time-series regression using Equation 40. The portfolios are pre-sorted on 36-month beta estimates and rebalanced monthly. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted R2 is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in  $R^2$  with increasing independent variables.

Significance codes: P < 0.01 "\*\*\*", P < 0.05 "\*\*", P < 0.1 "\*" .

Panel B Estimation of risk premia		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0190	(3.80)***
$\hat{\gamma}_{MKT}$	-0.0090	(-1.39)

Panel B shows the estimated risk premia for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level.

### 6.2.3.2 ESTIMATION OF THE CAPM FOR BM- AND SIZE-SORTED PORTFOLIOS

We now turn to estimating CAPM on portfolios sorted on firm characteristics other than beta. Table 20 summarizes the regression output from the first-pass time-series regression and second-pass cross-sectional regression for portfolios sorted on both BM and size. The regression output can be found in Appendix 11 and 13. For all portfolios sorted on BM and size, the market factor exposures from the time-series regressions are statistically significant, and with good dispersion. The intercepts are statistically indistinguishable from zero for all BM portfolios, except for portfolio 2. As seen from the adjusted  $R^2$ s, the market risk factor is able to explain between 0.46% and 0.57% of the portfolio returns, depending on the portfolio. For the size-sorted portfolios, the market factor exposure increases monotonically with size, indicating that the larger-sized portfolios contain stocks with higher betas. Portfolio 1 has an alpha significantly different from zero, indicating that the CAPM is inadequate for explaining the portfolio returns. Furthermore, the adjusted  $R^2$  increases with portfolio size.

Turning to Panel B and the estimated risk premia, the average cross-sectional intercept is indistinguishable from zero for the portfolios sorted on BM. However, the market factor is estimated to -0.0025 and is not statistically significant, indicating that market risk is not a priced factor, contradictory to the CAPM. For the size-sorted portfolios, the average alpha is statistically significant at the 10% level, and therefore not considered to be a priced risk factor. Furthermore, the market risk premium is not found to be significant.

**Table 20** Estimation of the CAPM for portfolios sorted on BM and size (2009 - 2017)

Panel A				Factor exposures for size			
Factor exposures for BM							
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	Adjusted $R^2$	Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	Adjusted $R^2$
Low BM	0.001	0.897***	0.564	Low size	0.011**	0.627***	0.168
2	0.007**	0.911***	0.565	2	-0.002	0.778***	0.321
3	0.001	0.774***	0.506	3	0.002	0.825***	0.443
4	-0.002	1.001***	0.570	4	0.000	0.979***	0.644
High BM	-0.001	1.063***	0.458	High size	-0.003	1.298***	0.840

Panel A shows the results from the time-series regression using Equation 40. In the left side of the panel, portfolios pre-sorted on BM are used as dependent variable in the time-series. In the right side of the panel, portfolios pre-sorted on size are used as dependent variable in the time-series regression. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted  $R^2$  is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in  $R^2$  with increasing independent variables. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

Panel B					
Sort: BM			Sort: size		
Coefficient	Risk premium	T-statistic	Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0126	(0.84)	$\hat{\alpha}$	0.0158	(1.76)*
$\hat{\gamma}_{MKT}$	-0.0025	(-0.15)	$\hat{\gamma}_{MKT}$	-0.0059	(-0.64)

Panel B shows the estimated risk premium for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level.

### 6.2.3.3 ESTIMATION OF THE CAPM FOR MOMENTUM- AND MAX-SORTED PORTFOLIOS

Finally, we estimate the CAPM on portfolios sorted on momentum and MAX. Table 21 summarizes the results from both regressions, and the full regression output can be found in Appendix 15 and 17. Similarly, as with the CAPM estimation on the other characteristics, the market factor exposures are found to be significantly different from zero for all momentum- and MAX-sorted portfolios. For the portfolios sorted on momentum, the alpha for portfolio 5 is significantly different from zero, indicating a

wrongly specified model. For the portfolios sorted on MAX, on the other hand, there are no significant intercepts.

Considering the estimated risk premia for the portfolios sorted on momentum from Panel B, market risk is not a priced factor, and the significant alpha suggests that a considerable part of the cross-sectional returns remain unexplained. For MAX, both the alpha and the market risk premium is found to be insignificant.

**Table 21** Estimation of the CAPM for portfolios sorted on momentum and MAX (2009 - 2017)

Panel A				Factor exposures for MAX			
Factor exposures for momentum							
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	Adjusted $R^2$	Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	Adjusted $R^2$
Low mom	-0.006	1.246***	0.553	Low MAX	0.003	0.728***	0.461
2	-0.001	0.885***	0.597	2	-0.001	1.003***	0.625
3	-0.001	0.793***	0.451	3	0.002	0.974***	0.541
4	0.002	0.919***	0.606	4	0.000	1.032***	0.560
High mom	0.011**	0.859***	0.401	High MAX	0.002	0.974***	0.487

Panel A shows the results from the time-series regression using Equation 40. In the left side of the panel, portfolios pre-sorted on momentum are used as dependent variable in the time-series. In the right side of the panel, portfolios pre-sorted on MAX are used as dependent variable in the time-series regression. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted R2 is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in R2 with increasing independent variables. Significance codes: P < 0.01 "\*\*\*\*", P < 0.05 "\*\*\*", P < 0.1 "\*\*" .

Panel B					
Sort: momentum			Sort: MAX		
Coefficient	Risk premium	T-statistic	Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0218	(2.03)**	$\hat{\alpha}$	0.0105	(0.99)
$\hat{\gamma}_{MKT}$	-0.0122	(-1.02)	$\hat{\gamma}_{MKT}$	0.0001	(0.01)

Panel B shows the estimated risk premium for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level.

#### 6.2.4 SUMMARY OF THE CAPM ESTIMATIONS

Table 22 summarizes our findings from the estimation of the CAPM on portfolios sorted on various characteristics. As earlier mentioned, the model is well-specified if the alpha terms are insignificant at the 5% level, and the estimated market risk premium is statistically significant. From Table 22 it can be seen that the alpha term is only indistinguishable from zero at the 5% level for portfolios sorted on BM, MAX, and size. In terms of the market risk premium, the coefficients indicate that investors are negatively compensated for market risk exposure, which supports our previous findings that the beta anomaly is present on the OSE. Importantly, however, the market risk premium is not a priced risk factor across assets, as none of the coefficients are statistically significant.

To conclude, our findings indicate that the CAPM is an inadequate model for explaining the cross-section of returns on the OSE in the period 2009 – 2017.

<b>Table 22</b>		Summary of intercept and risk premia from the CAPM estimations (2009 – 2017)		
Portfolio sorts	$\hat{\alpha}$	T-statistic	$\hat{\gamma}_{MKT}$	T-statistic
Beta	0.0190	(3.80)***	-0.0090	(-1.39)
BM	0.0126	(0.84)	-0.0025	(-0.15)
Size	0.0158	(1.76)*	-0.0059	(-0.64)
Momentum	0.0218	(2.03)**	-0.0122	(-1.02)
MAX	0.0105	(0.99)	0.0001	(0.01)

#### 6.2.5 ESTIMATION OF THE FAMA-FRENCH THREE-FACTOR MODEL

Next, we will extend the CAPM to include the SMB and HML factors, as proposed by Fama and French (1992). Previous studies on both international markets and the OSE have concluded that the three-factor model describes more of the variation in excess stock returns. The following section will test whether this is true for the OSE in the period 2009 - 2017. Similarly as with the CAPM estimations, we follow the Fama and Macbeth (1973) methodology with two step regressions to assess the factor premiums for portfolios sorted on five different criteria.



### 6.2.5.1 ESTIMATION OF THE THREE-FACTOR MODEL FOR BETA-SORTED PORTFOLIOS

Table 23 summarizes the estimation of the three-factor model on portfolios sorted on beta. The full regression output can be found in Appendix 19. The output in Panel A from Table 23 shows that the market factor exposures are significant and, not surprisingly, increasing in beta. Similarly, the SMB-exposures are significant for all beta-sorted portfolios. The SMB exposures are relatively similar across all portfolios, pointing towards an even size-distribution. Beta-portfolio 1, 2, 3 and 4 all have insignificant exposures to HML and do not seem to be dominated by either high-BM or low-BM stocks. Portfolio 5, on the other hand, has significant and positive HML exposure, indicating an overweight of high-BM stocks. The adjusted  $R^2$ s are monotonically increasing in beta and, generally speaking, marginally higher than for the CAPM estimation on beta-sorted portfolios. The intercepts are significant for portfolio 4 and 5 and generally higher for the low beta portfolios. However, we do not find that any of the factor exposures explain the variation in excess returns, translating into insignificant risk premia for all factor exposures, including alpha, as seen from Panel B.

<b>Table 23</b>		Estimation of the three-factor model for portfolios sorted on beta (2009 - 2017)			
<b>Panel A</b>		Factor exposures for portfolios sorted on beta			
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	Adjusted R2
Low beta	0.003	0.518***	0.421***	0.028	0.470
2	-0.004	0.845***	0.471***	0.009	0.562
3	-0.004	1.023***	0.478***	-0.065	0.694
4	-0.010***	1.160***	0.404***	0.001	0.691
High beta	-0.015***	1.445***	0.388***	0.181**	0.684

Panel A shows the results from the time-series regression using Equation 40. The portfolios are pre-sorted on 36-month beta estimates and rebalanced monthly. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted R2 is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in R2 with increasing independent variables.

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

<b>Panel B</b> Estimation of risk premia		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0038	(0.16)
$\hat{\gamma}_{MKT}$	-0.0075	(-1.21)
$\hat{\gamma}_{SMB}$	0.0328	(0.62)
$\hat{\gamma}_{HML}$	0.0002	(0.01)

Panel B shows the estimated risk premium for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level.

#### 6.2.5.2 ESTIMATION OF THE THREE-FACTOR MODEL FOR BM- AND SIZE-SORTED PORTFOLIOS

Table 24 summarizes the estimation of the three-factor model on portfolios sorted on BM and size. The full regression output can be seen in Appendix 21 and 23. As seen from Panel A, the market risk exposures and the exposures to SMB are significant on 1% level for the BM-sorted portfolios. The companies on OSE seem to be relatively equally distributed across the BM-portfolios, with respect to size. The HML exposures are significant for all portfolios, except portfolio 2, and almost monotonically increasing for the BM-sorted portfolios. Interestingly, the regressions produces significant alphas for portfolio 4 and 5 when we include SMB and HML. Further, we find the adjusted  $R^2$ s to be relatively high for all portfolios, ranging from 0.62 to 0.69, which is clearly higher than the corresponding values for the CAPM estimations. Nevertheless, we do not find any of the factor risk premia to be significantly different from zero when sorted on BM, as seen from Panel C.

As seen from Panel B, the market factor exposure is significant for all size-sorted portfolios, and the exposure is almost monotonically increasing. In other words, large-size portfolios have an overweight of high-beta stocks, while the low-size portfolios have an overweight of low-beta stocks. We also find the SMB-factor to be significant on a 1% level for all portfolios, except for portfolio 5. Unsurprisingly, the exposures are monotonically decreasing with size. Only one portfolio has significant exposures to HML above the 5%

level, namely portfolio 3. The insignificant exposures of the remaining portfolios indicate that size-portfolios are without clear BM-characteristics. Further, we only find two significant intercepts in portfolio 2 and portfolio 3. The adjusted  $R^2$ s are significantly higher than for the CAPM estimations and monotonically increasing with size. This is not surprising, given that the constructed factors are value-weighted and that the large stocks therefore will dominate them.

As seen from Panel C, the estimated risk premia are found to be significant for both the market factor and SMB. Further, we estimate a significant alpha risk premium.

Surprisingly, the risk premium for SMB is found to be negative and significant for size-sorted portfolios, which is contradictory to the theory. Still, compared to the CAPM, the three-factor model with portfolios sorted on size seems to be a better fit.

<b>Table 24</b>		Estimation of the three-factor model for portfolios sorted on BM and size (2009 - 2017)			
<b>Panel A</b>		Factor exposures for portfolios sorted on BM			
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	Adjusted $R^2$
Low BM	-0.004	0.957***	0.350***	-0.134**	0.619
2	0.000	0.979***	0.444***	-0.094*	0.650
3	-0.005	0.839***	0.377***	-0.142***	0.585
4	-0.008**	1.026***	0.331***	0.197***	0.686
High BM	-0.012***	1.113***	0.578***	0.264***	0.664

Panel B		Factor exposures for portfolios sorted on size (2009 - 2017)			
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	Adjusted $R^2$
Low size	0.000	0.734***	0.717***	-0.107	0.313
2	-0.016***	0.905***	0.860***	-0.120*	0.59
3	-0.008**	0.880***	0.527***	0.157***	0.655
4	-0.004	1.012***	0.246***	-0.004	0.669
High size	-0.002	1.284***	-0.058	0.067	0.841

Panel A and B shows the results from the time-series regression using Equation 40. In Panel A, portfolios pre-sorted on BM are used as dependent variable in the time-series. In Panel B, portfolios pre-sorted on size are used as dependent variable in the time-series regression. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted R2 is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in R2 with increasing independent variables. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

Panel C		Risk premia for portfolios sorted on BM and size			
Sort: BM			Sort: size		
Coefficient	Risk premium	T-statistic	Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	-0.0235	(-0.87)	$\hat{\alpha}$	0.0573	(2.59)**
$\hat{\gamma}_{MKT}$	0.0313	(1.03)	$\hat{\gamma}_{MKT}$	-0.0385	(-2.05)**
$\hat{\gamma}_{SMB}$	0.0080	(0.47)	$\hat{\gamma}_{SMB}$	-0.0213	(-2.05)**
$\hat{\gamma}_{HML}$	-0.0217	(-1.50)	$\hat{\gamma}_{HML}$	-0.0164	(-0.93)

Panel C shows the estimated risk premium for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

### 6.2.5.3 ESTIMATION OF THE THREE-FACTOR MODEL FOR MOMENTUM- AND MAX-SORTED PORTFOLIOS

Table 25 summarizes the estimation of the three-factor model on portfolios sorted on momentum and MAX. The regression output can be found from Appendix 25 and 27. As seen from Panel A, all momentum portfolios have a significant exposure to the market factor and SMB in the three-factor model. The market capitalization of the stocks seems to

be equally distributed across the portfolios, except for portfolio 4, which has a surprisingly low exposure to SMB. The model estimates three significant intercepts and the adjusted  $R^2$ s are higher for all portfolios compared to the CAPM estimations.

As seen from Panel C, the market factor risk premium and the risk premium for SMB are not significant in the three-factor model. However, the market does compensate for exposure to the HML factor. Lastly, we find the alpha risk premium to be positive and significant.

Panel B shows the factor exposures of portfolios sorted on MAX. Similar to the CAPM estimation on MAX-sorted portfolios, we find the exposures to the market factor to be significantly different from zero for all portfolios in the three-factor models. All portfolios have a significant exposure to SMB, but we do not find any of the portfolios to have a significant exposure to HML. We estimate one significant intercept and the adjusted  $R^2$ s have increased significantly from the CAPM estimation. However, we do not find any significant risk premia for portfolios sorted on MAX.

<b>Table 25</b>		Estimation of the three-factor model for portfolios sorted on momentum and MAX (2009 - 2017)			
<b>Panel A</b>		Factor exposures for portfolios sorted on momentum			
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	Adjusted R2
Low mom	-0.015***	1.314***	0.521***	0.023	0.625
2	-0.007**	0.939***	0.392***	-0.014	0.679
3	-0.008**	0.852***	0.432***	-0.009	0.544
4	-0.001	0.942***	0.174**	-0.000	0.615
High mom	0.002	0.926***	0.547***	0.065	0.541

Panel B		Factor exposures for portfolios sorted on MAX			
Portfolio	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	Adjusted R2
Low MAX	-0.003	0.785***	0.386***	-0.054	0.543
2	-0.007**	1.048***	0.393***	0.073	0.716
3	-0.006*	1.032***	0.470***	0.042	0.645
4	-0.007*	1.086***	0.438***	0.043	0.643
High MAX	-0.006	1.034***	0.488***	0.052	0.588

Panel A and B shows the results from the time-series regression using Equation 40. In Panel A, portfolios pre-sorted on momentum are used as dependent variable in the time-series regression. In Panel B, portfolios pre-sorted on MAX are used as dependent variable in the time-series regression. Constants that are significantly different from zero at a 5% level indicate a wrongly specified model. The adjusted R2 is an indicator of the goodness of fit, and is the proportion of the variance in the dependent variable that is predictable from the independent variable(s), adjusted for the increase in R2 with increasing independent variables. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

Panel C		Risk premia for portfolios sorted on momentum and MAX			
Sort: momentum			Sort: MAX		
Coefficient	Risk premium	T-statistic	Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0277	(2.26)**	$\hat{\alpha}$	0.0021	(0.08)
$\hat{\gamma}_{MKT}$	-0.0154	(-1.30)	$\hat{\gamma}_{MKT}$	-0.0033	(-0.09)
$\hat{\gamma}_{SMB}$	-0.0105	(-0.82)	$\hat{\gamma}_{SMB}$	0.0275	(0.64)
$\hat{\gamma}_{HML}$	0.1784	(2.76)***	$\hat{\gamma}_{HML}$	-0.0045	(-0.05)

Panel C shows the estimated risk premium for the intercept and each risk factor using Equation 41. If the model is true, the intercept,  $\hat{\alpha}$ , is zero. The factor,  $\hat{\gamma}_i$ , is priced if it is statistically significant at the 5% level. Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*\*",  $P < 0.1$  "\*\*" .

## 6.2.6 COMPARISON OF THE THREE-FACTOR MODEL TO THE CAPM

Table 26 compares the risk premia found in the CAPM and three-factor model. The table shows that exposures to the market factor are uncompensated for all sorts in the CAPM. Similarly, all three-factor models report insignificant risk premia for the market factor. The CAPM estimates significant alphas for portfolios sorted on beta and momentum, but insignificant alphas for the remaining portfolios. The results indicate ambiguous return characteristics and that the market risk factors are unable to explain the variation in

returns for the portfolios. When we include SMB and HML as explanatory variables, we see that the three-factor model estimates significant risk premia for some portfolio sorts. Interestingly, the alpha of the size-sorted portfolios becomes significant and the alpha of the beta-sorted portfolios becomes insignificant. In total, our model still has problems explaining the variation in returns for most portfolio sorts, and hence, we cannot claim a good model specification. Nevertheless, taking the above into consideration, we argue that the three-factor model explain the variation in excess stock returns on OSE better than the CAPM model.

<b>Table 26</b>		Comparison between estimated risk premia for CAPM and three-factor model					
		Three-factor model				CAPM	
Portfolio sorts		$\hat{\alpha}$	$\hat{\gamma}_{MKT}$	$\hat{\gamma}_{SMB}$	$\hat{\gamma}_{HML}$	$\hat{\alpha}$	$\hat{\gamma}_{MKT}$
Beta		0.0038	-0.0075	0.0328	0.0002	0.0190	-0.0090
t-stat		(0.16)	(-1.21)	(0.62)	(0.01)	(3.80)***	(-1.39)
BM		-0.0235	0.0313	0.0080	-0.0217	0.0126	-0.0025
t-stat		(-0.87)	(1.03)	(0.47)	(-1.50)	(0.84)	(-0.15)
Size		0.0573	-0.0385	-0.0213	-0.0164	0.0158	-0.0059
t-stat		(2.59)**	(-2.05)**	(-2.05)**	(-0.93)	(1.76)*	(-0.64)
MOM		0.0277	-0.0154	-0.0105	0.1784	0.0218	-0.0122
t-stat		(2.26)**	(-1.30)	-0.82	(2.76)***	(2.03)**	(-1.02)
MAX		0.0021	-0.0033	0.0275	-0.0045	0.0105	0.0001
t-stat		(0.08)	(-0.09)	(0.64)	(-0.05)	(0.99)	(0.01)

#### 6.2.7 ESTIMATION OF THE FOUR – FACTOR MODEL

We will now expand our model to include Carhart's momentum factor, PR1YR, to evaluate whether the momentum factor is a priced risk factor on the OSE. The methodology is the same as in the previous sections, where we sort on beta, size, BM, momentum and MAX and run two step regressions according to Fama and MacBeth (1973). We will not show the regression output, but the statistics for the estimated risk premia are summarized in Table 27 and compared to the three-factor risk premia. For the interested reader, the full regression output from the first step regressions can be found in Appendix 29 – 33.

As with the three-factor model, the first-step regressions for all sorts have significant exposures to the market factor. Further, all portfolios except size-portfolio 4 and momentum-portfolio 4 have significant exposures to SMB. Considering the HML-factor, we find significant exposures for all BM-sorted portfolios, except for portfolio 2. We also find significant exposures for size-portfolio 3 and beta-portfolio 5, while significant exposures for momentum and max-sorted portfolios are absent in our sample. PR1YR exposures are significant for momentum-portfolio 1 and 5, and MAX-portfolio 3. Lastly, we still estimate several significant intercepts across all sorts. The adjusted  $R^2$ s are not significantly different from that of the three-factor model.

Table 27 shows the estimated risk premia for the three-factor regressions and four-factor regressions. Comparing the alphas of the two models, we notice that the alpha of the portfolios sorted on momentum and size turns insignificant when we include the momentum factor. In fact, we have no significant alpha risk premia above the 5% level when we extend the model with PR1YR. Further, we find the PR1YR factor risk premia to be insignificant for all portfolio-sorts. Additionally, SMB and market factor exposures are no longer compensated with size-sorted portfolios in the four-factor model. Momentum-sorted portfolios have a significant HML-risk premium when we include PR1YR, and the MAX-sorted portfolios are still not compensated for any of the risk exposures.

It is difficult to determine the best model. On the one hand, the four-factor model has no significant alphas. On the other hand, we only find one significant risk premium when we extend the model. Because the contribution of the PR1YR factor is close to non-existent and the risk premia no longer explains the variation in excess returns across sorted portfolios when we include the factor, it is our understanding that the three-factor model is slightly better. However, as noted in the evaluation of the three-factor model, neither of the model specifications we have presented so far are able to explain the returns to a satisfactory level.



**Table 27**

Comparison between estimated risk premia for three- and four-factor model

Sorts	Four-factor model					Three-factor models			
	$\hat{\alpha}$	$\hat{\gamma}_{MKT}$	$\hat{\gamma}_{SMB}$	$\hat{\gamma}_{HML}$	$\hat{\gamma}_{PR1YR}$	$\hat{\alpha}$	$\hat{\gamma}_{MKT}$	$\hat{\gamma}_{SMB}$	$\hat{\gamma}_{HML}$
Beta	-0.0177	-0.0115	0.0908	0.0010	-0.0651	0.0038	-0.0075	0.0328	0.0002
t-stat	(-0.51)	(-1.70)*	(1.08)	(0.04)	(-0.90)	(0.16)	(-1.21)	(0.62)	(0.01)
BM	-0.0668	0.0832	-0.0092	-0.0275	0.0390	-0.0235	0.0313	0.0080	-0.0217
t-stat	(-1.80)*	(1.89)*	(-0.43)	(-1.84)*	(1.16)	(-0.87)	(1.03)	(0.47)	(-1.50)
Size	0.0216	-0.0110	0.0006	-0.0556	-0.1283	0.0573	-0.0385	-0.0213	-0.0164
t-stat	(0.89)	(-0.57)	(0.04)	(-1.48)	(-1.28)	(2.59)**	(-2.05)**	(-2.05)**	(-0.93)
Mom	0.0556	-0.0519	0.0050	0.2770	-0.0772	0.0277	-0.0154	-0.0105	0.1784
t-stat	(1.56)	(-1.20)	(0.28)	(1.97)**	(-1.01)	(2.26)**	(-1.30)	-0.82	(2.76)***
MAX	0.0028	-0.0042	0.0280	-0.0047	-0.0038	0.0021	-0.0033	0.0275	-0.0045
t-stat	(0.10)	(-0.11)	(0.65)	(-0.05)	(-0.17)	(0.08)	(-0.09)	(0.64)	(-0.05)

### 6.2.8 BETTING AGAINST BETA

In this section, we will test whether the beta anomaly on OSE can be explained by Frazzini and Pedersen's (2014) betting against beta factor. As previously mentioned, Frazzini and Pedersen (2014) argue that the beta anomaly is attributable to leverage and margin constraints with investors. They theorize that investors with limited access to leverage tend to overweigh risky securities in their portfolio, pushing prices of high-beta stocks upwards and reducing future returns. In our analysis, we will first compare alphas from regressions specified by the CAPM, the three-factor model, and the four-factor model on beta-sorted portfolios. Next we will compare the sharp ratios of the portfolios and conclude whether the capital market line (CML) holds on the OSE. Lastly, we will run the same regressions on the BAB-factor and examine if unconstrained investors can bet against beta and earn abnormal returns.

Table 28	Alpha estimates for portfolios sorted on five beta-sorted portfolios and the BAB-factor					
	Portfolios sorted on beta and BAB					
	1	2	3	4	5	BAB
Excess Return	1.45	1.13	1.32	0.74	0.63	1.76
CAPM $\alpha$	0.010***	0.004	0.004	-0.003	-0.008*	0.019***
3-factor $\alpha$	0.003	-0.004	-0.004	-0.010***	-0.015***	0.012**
4-factor $\alpha$	0.003	-0.005	-0.004	-0.010***	-0.014***	0.011**
Std.	3.53	4.78	5.03	5.67	7.32	5.72
Sharpe ratio	0.41	0.24	0.26	0.13	0.09	0.31

In Table 28, we have summarized the Sharpe ratios and the estimated alphas from the CAPM-, the three-factor model-, and the four-factor model regressions on five beta-sorted portfolios, as well as the BAB factor. The table shows almost monotonically decreasing Sharpe ratios with beta, confirming that low-beta stocks earn higher risk adjusted returns than high-beta stocks. The findings does not only confirm the relative flatness of the CML on OSE; the CML appears to have a negative slope. We can see that the alphas for all regressions are almost monotonically decreasing with beta, although they are not all statistically significant. We should be cautious when we draw inferences from insignificant alphas. Nevertheless, the low-beta stocks are generally found to have alphas that are indistinguishable from zero and the high-beta stocks have increasingly negative and significant alphas. In other words, having accounted for the systematic risk exposures, the high-beta stocks experience significant negative abnormal returns, while the low-beta stocks experience insignificant abnormal returns. Our findings therefore indicate that the high-beta portfolios earn lower risk-adjusted returns.

The far right column in Table 28 reports the excess return and alphas for the BAB factor. That is, a long leveraged low-beta stocks and short de-leveraged high-beta stocks portfolio constituting a market neutral portfolio. The portfolio earns significant CAPM abnormal returns of 1.9% on average. Further, the portfolio earns significant abnormal returns of 1.2% and 1.1% for the three-factor and four-factor model, respectively. The results imply that an unconstrained investor could exploit the arbitrage opportunity and earn positive

returns on a market neutral portfolio. Thus, if agents on OSE were unconstrained with respect to leverage and margin, we would expect that the abnormal returns would be arbitrated away and our alphas to be insignificant. Hence, our findings indicate that leverage and margin constraints can be the source of the beta-anomaly at OSE.

#### 6.2.9 LOTTERY STOCK DEMAND

Bali et al. (2017) argue that the demand for lottery stocks explains the beta anomaly. The theory states that investors generate demand for stocks with high probabilities of large short-term up moves in the stock price, putting a price pressure on high-beta stocks and reducing future returns. To test this hypothesis on the OSE, we conduct a bivariate portfolio analysis where we first sort stocks on MAX and subsequently on beta. This is done on a monthly basis and the monthly excess returns of the portfolios are compared. By double sorting stocks into bivariate portfolios, we control for MAX-characteristics, which allows us to examine the relationship between excess returns and betas without the influence of a demand for lottery stocks. Hence, if the demand for lottery stocks triggers the beta anomaly, we would expect beta portfolios within each MAX portfolio to have increasing excess returns in beta. Next, we will extend the four-factor model to include the FMAX factor, and regress it on the BAB factor. If the demand for lottery stocks explains the abnormal returns achieved by the BAB-factor in the four-factor model, the BAB factor should have a significant exposure to the FMAX-factor and the alpha of the extended four-factor model should be insignificant.

Table 29 summarizes the average monthly excess return for each double sorted portfolio from 2009 to 2017. The table shows that portfolios sorted on beta are without any clear return characteristics. The low-beta portfolios still earn higher returns compared to the high-beta portfolios on average. In fact, the average return is almost monotonically decreasing in beta. Clearly, the beta anomaly is still persistent after controlling for MAX-characteristics. As explained in the methodology section 5.2.1, a double sorting procedure result in a limited number of stocks in each portfolio. As a result, the minimum number of companies for our double sorted portfolios is only three companies, and we are therefore cautious to draw any inferences on the basis of the bivariate analysis.

<b>Table 29</b> Average return for portfolios sorted on beta and MAX (2009 - 2017)					
	Low MAX	2	3	4	High MAX
Low beta	1.54	0.88	1.04	0.86	1.68
2	0.82	0.80	1.12	1.49	1.12
3	2.29	0.89	1.22	1.39	0.78
4	0.31	0.83	1.52	0.96	0.74
High beta	-0.34	0.57	0.19	-0.10	1.19

In order to evaluate the effect of lottery stock demand without being subjected to the biases that might occur due to small portfolio sizes, we will apply the factor models and use the BAB factor as the dependent variable. The factor exposures are presented in Table 30. As seen from the far-right column, the BAB factor has a significant exposure to FMAX. However, the portfolio still earns significant abnormal returns of 1.2%. As earlier mentioned, the persistence of the significant alpha term indicates that the demand for lottery stocks does not explain the abnormal returns from the BAB factor in the four-factor model. Consequently, we cannot conclude that lottery demand explains the beta anomaly on Oslo Stock Exchange. This is, however, not surprising, as Bali et al. (2017) found private and institutional investors have different behavioral characteristics, and that the demand for lottery stocks is only present for private investors. In Section 3.3, we found that private investors only hold 3.84% of the total value on the OSE. Thus, if we had found that the demand for lottery stocks is a determinant of excess returns on OSE, the results would have pointed towards somewhat irrational trading from larger institutional investors. Furthermore, from the analysis of portfolio sorts, we found that high MAX-stocks have significant and positive returns the following month. As such, if lottery demand is tilted towards high-beta stocks, we would expect high beta stocks to have larger excess returns compared to low-beta stocks and the beta anomaly to be non-existent.

Table 30		Exposure estimates for BAB-factor				
Model	$\alpha$	$\beta_{MKT}$	$\beta_{SML}$	$\beta_{HML}$	$\beta_{PR1YR}$	$\beta_{FMAX}$
CAPM	0.019***	-0.112				
Three-factor model	0.012**	-0.057	0.406**	-0.006		
Four-factor model	0.011**	-0.022	0.417**	0.001	0.074	
Four-factor model FMAX	0.012**	0.142	0.510***	0.015	0.05	-0.320***

### 6.2.10 SUB-CONCLUSION BAB vs. MAX

From analysis, we found that the BAB factor produces abnormal CAPM, three-factor and four-factor returns. The results imply that an unconstrained investor could exploit the arbitrage opportunity and earn positive returns on a market neutral portfolio. Thus, if agents on OSE were unconstrained with respect to leverage and margin, the abnormal returns from the BAB factor would be arbitrated away, and our results would be insignificant. Hence, our findings indicate that the beta anomaly at OSE can be attributed to leverage and margin constraints with investors. To see whether the presence of a lottery stock demand could explain the beta anomaly on the OSE, we performed a bivariate analysis and the extended four-factor model to include the FMAX factor. Our results showed no indications of lottery stock demand on OSE, and thus we reject the hypothesis that a demand for lottery stocks drives the beta anomaly.

## 7 CONCLUSION

In this thesis, we examine to what extent the CAPM, the Fama-French three-factor model, and the Carhart four-factor model explain the cross-section of returns on OSE in the period 2009 – 2017. We find that all of our constructed portfolios have significant exposures to the market factor in the CAPM estimations. However, the market risk premia are not significantly different from zero for any portfolio sort, and we conclude that the CAPM is not able to explain the variation in returns at OSE. Further, we find that the SMB factor and the HML factor increase the predictability of the model. The Fama-French three-factor model estimates a significant market risk premium and SMB risk premium for portfolios sorted on size, and a significant HML risk premium for portfolios sorted on momentum. Surprisingly, the market- and SMB risk premia are negative for

portfolios on size, contradicting Fama and French (1992). We conclude that the three-factor model explains the variation in returns on the OSE poorly, but that the model specification is an improvement from the CAPM. We also find that the four-factor model contributes minimally to our analysis. Despite the fact that we estimate insignificant alpha risk premia for all portfolio sorts, the four-factor model only produce one significant risk factor. We therefore conclude that the three-factor model has the best fit, but that the models explain the variation in returns on OSE to a limited extent.

Next, we examine whether the beta anomaly can be attributed to leverage and margin constraints with investors or a demand for lottery stocks. After running the CAPM, the three-factor model and the four-factor model on a market neutral portfolio, we find that the beta anomaly can be attributed to leverage and margin constraint investors, who overweight high-beta stocks in their portfolio in order to increase their risk exposure. Further, we find that the demand for lottery stocks is absent on OSE, and that the BAB-factor still earns significant abnormal returns when we control for the MAX-factor.

## 7.1 IMPLICATIONS

### 7.1.1 FACTOR INVESTING

Our results deviate significantly from previous research conducted on the OSE. In part, this is due to choice of data sources and the filter criteria applied on the raw data, which is based on subjective considerations. Mainly, however, our results deviate from prior research due the time-period over which we estimate the various risk premia. This emphasizes that the correlation between asset returns and risk factors are indeed not constant over time. For example, contrary to our findings, Ødegaard (2016a) finds that when portfolios are sorted on size, HML is highly statistically significant in the period 1980 – 1992. Despite constructing the BAB factor slightly different than we do in this paper, Korneliussen and Rasmussen (2014) find no evidence that constrained investors in terms of leverage and margin is a factor affecting the cross-section of average stock market returns on the OSE, using data from 1991 to 2010. Using global data, the Norwegian insurance company Storebrand shows how factors such as value, size, and momentum have provided both higher and lower excess returns, compared to a global index, over various periods

(Storebrand, 2016). They emphasize that the value factor, in particular, (defined as the Fama-French BE factor), has been performing poorly in the period 2010 – 2016.

In recent years, systematic risk factors have received increased attention following the rise of so-called «smart beta» funds (Thompson, 2017). As hybrid between active and passive investment management, these mutual funds take a passive strategy, but use complex algorithms to track one or more factors in an attempt to generate higher returns. The passive investment component in smart-beta funds provides a cost-effective alternative to actively managed funds, and has subsequently become increasingly popular, also in Norway. According to Morningstar, Storebrand's smart beta fund were the fourth most popular mutual fund Norway in 2017 (Furuseth, 2018). Despite the fact that all the smart beta funds on the Norwegian market are highly diversified with a global scope, we find it likely that the current hype of factor investing is likely to influence private investors to follow similar strategies on the local market. As such, our empirical findings are highly relevant. However, we argue that the key take-away for private investors is that the dynamic nature of the correlations on the stock market imply that the factors that have provided significant returns in the past, will not necessarily do so in subsequent periods.

#### 7.1.2 THE COST OF EQUITY CAPITAL

Major corporate decisions, such as capital budgeting decisions and M&A activities, involves calculating the cost of equity capital. The finance literature defines the cost of equity capital as the expected return on a company's stock, and standard in many corporate finance books is to estimate the cost of equity capital using the Sharpe-Lintner version of the CAPM. With the substantial attention the model receives in business schools, it is reasonable to assume that the model is used relatively uncritically by practitioners in various corporate settings as well. This is remarkable, given the lack of empirical evidence in support of the model. Our findings from the OSE does not only prove that the SML is too flat, as found in empirical research, but also that there is a negative relationship between systematic market risk and average return. Naturally, this has implications for the estimations cost of equity capital. Our findings emphasize that the CAPM cannot be used uncritically to calculate the cost of equity capital. According to PWC, practitioners in Norway used an average market risk premium of 5% in 2017 (PWC,

2017). Our findings from the estimated three-factor model suggests that the market risk premium was indistinguishable from zero for all portfolios sorts, except for size-sorted portfolios. Here, the market risk premium was estimated to -3.8%. Although the alpha is found to be significant, it seems clear that the market risk premium is greatly overestimated by practitioners in Norway.

## 8 LIMITATIONS

We will now discuss the most prominent limitations of our thesis. The limitations regard decisions that were made with respect to the model specifications and significance tests.

In this paper, we work extensively with panel data. Because panel data residuals might be correlated across firms and time, estimated OLS standard errors are potentially biased (Petersen, 2009). In order to deal with the potential bias, we have used the Fama and MacBeth (1973) procedure. The procedure assumes that the estimated coefficients are independent of each other. However, this is not true when there are firm effects in the data, and the variance of our estimates will therefore be downward biased (Cochrane, 2009). Hence, our estimates might be found to be statistically significant when, in fact, the assumed relation is insignificant. As such, the robustness of our findings would be improved by applying alternative error measurements.

In our model estimations, we have controlled for variables that are known to predict the cross section of future stock returns. More specifically, we have controlled for firm characteristics, such as, market capitalization, BM, momentum and MAX. To some extent, we have also controlled for liquidity by applying a trading volume filter. However, we have not included measures of stock sensitivity to aggregate funding liquidity factors, as proposed by Frazzini and Pedersen (2014). Further, we have not included measures of risk, including coskewness, total skewness, downside beta and tail beta. The mentioned control variables and stock returns are found to have a significant relationship, and disregarding them could potentially distort our results and make estimates less significant.



## 9 FUTURE RESEARCH

The results in this thesis show that the standard CAPM, the Fama and French (1992) three-factor and the Carhart (1997) four-factor model have problems explaining the variation in excess returns on the OSE in the period 2009 - 2007. The models suggest that size, BM and momentum are additional risk factors and that investors are compensated for their exposures to these. Nevertheless, our model estimations showed that, in most cases, investors are not compensated proportionally to their exposures. These results indicate that there are other risk factors on the OSE that determine the cross-section of returns. For example, we would expect the energy sector, with a 37,5% weight on OSE, to have a significant exposure to the oil price. It is reasonable to assume that investors holding these stocks are compensated for this exposure. We therefore encourage future studies to include factors that capture industry-specific risk factors that might explain more of the variation in returns on OSE.

Our findings also proves that the demand for lottery stocks is unable to explain the beta anomaly at OSE. We concluded that the findings are inconsistent with Bali et al. (2017) and that the absence of lottery stock demand might be due to the low fraction of private investors on the OSE. However, we encourage future research papers to examine whether similar trading patterns are found with institutional investors. From our analysis of the BAB factor, we found that investors with leverage and margin constraints overweigh high-beta stocks as an alternative to leveraging low-beta stocks, in order to increase their risk exposure. However, this does not necessarily imply that constrained investors are the only driver of the beta anomaly. For example, Christoffersen and Simutin (2017) find that pension plan sponsors rely heavily on benchmarking as a defensible mechanism in deciding which funds to keep and remove from the plan. Hence, fund managers alter their behavior and focus on beating the benchmark. In their pursuit to outperform the benchmark, leveraged-constrained fund managers overweight their portfolio with high-beta stocks because it will yield in expectation a return that is more likely to outperform the benchmark (Christoffersen & Simutin, 2017). Such findings indicate that investors are more risk-seeking than the CAPM assumes, and should encourage future research papers

to identify specific behavioral aspects of institutional investors that might drive the beta anomaly on the OSE.

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# APPENDICES

## Appendix 1 – Product of two stochastic variables

For any two variables  $X$  and  $Y$  with  $E(X) = \mu_X$  and  $E(Y) = \mu_Y$ , the covariance between  $X$  and  $Y$  is defined as:

$$Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

Hence,

$$Cov(X, Y) = E(XY - \mu_X Y - \mu_Y X + \mu_X \mu_Y)$$

$$Cov(X, Y) = E(XY - \mu_X \mu_Y - \mu_Y \mu_X + \mu_X \mu_Y)$$

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

Since

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

Or

$$E(XY) = E(X)E(Y) + Cov(X, Y)$$

The Euler equation

$$1 = E_t(m_{t+1}R_{t+1})$$

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1}$$

Implies

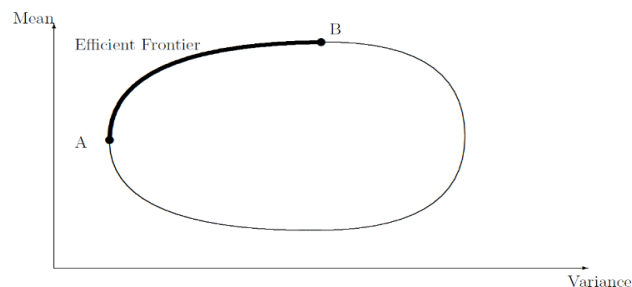
$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1} + Cov\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}, R_{t+1}\right)$$

$$1 = E_t(m_{t+1})E(R_{t+1}) + Cov(m_{t+1}, R_{t+1})$$

## Appendix 2 – Markowitz’ Optimal portfolio choice

Appendix figure 1 distinguish the “efficient frontier” of mean-variance efficient portfolios from other alternative portfolio sets. For any portfolio on the efficient frontier, investors maximize the expected return (mean) for the given level of risk (variance), in accordance with the mean-variance criterion. Hence, for any of the portfolios *not* on the efficient frontier, investors can always achieve a higher expected return per unit of risk, or a lower risk per unit of expected return. Assuming rational investors, only portfolios on the efficient frontier will be held.

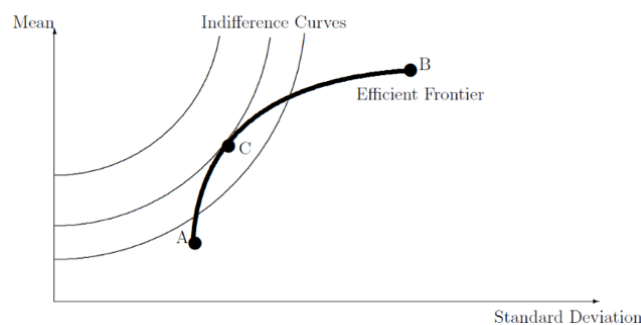
**APPENDIX FIGURE 1 – THE EFFICIENT FRONTIER**



(Krause., 2001, p. 26)

Appendix figure 2 illustrates how the investors’ optimal portfolio is determined. The mean-variance criterion is not optimal in general, because it does not reflect investors’ level of risk aversion. The optimal risky portfolio choice for rational investors are located on the point where the efficient frontier is tangential to his or hers indifference curve, as illustrated by portfolio C. Portfolio A and B are optimal risky portfolios for investors exhibiting a lower and higher aversion to risk, respectively.

**APPENDIX FIGURE 2 – DETERMINATION OF INVESTORS’ OPTIMAL PORTFOLIO**



(Krause, 2001, p. 29)

## Appendix 3 – Assumptions of the CAPM

1. One-period investment horizon.
2. Rational, risk-averse investors.
3. There are no taxes.
4. There are no transaction costs and inflation.
5. All assets are infinitely divisible.
6. Free flow and instant availability of information.
7. There are many investors on the market.
8. All assets are marketable
9. *Unlimited borrowing and lending is allowed at a risk-free rate that is the same for all investors.*
10. *All investors have homogenous expectations about expected returns, variances, and covariances of assets*

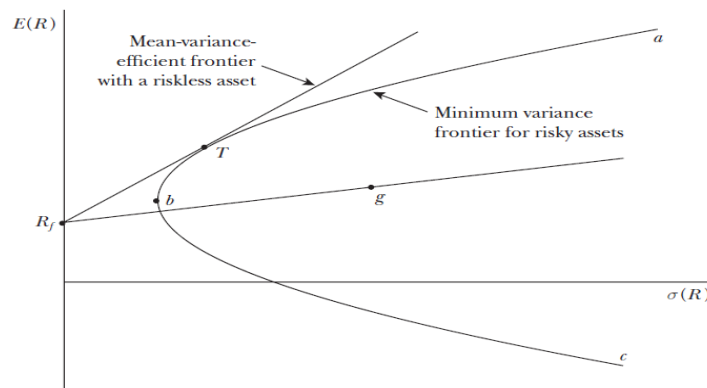
(Szylar, 2013, p. 101)

Assumption 1-8 applies to Markowitz' original model of portfolio choice, whilst assumption 9 and 10 were included to develop the CAPM.

## Appendix 4 – Optimal Portfolio Choice under CAPM

Appendix figure 3 illustrates how the assumptions of homogenous expectations and unlimited borrowing and lending leads to a new mean-variance efficient frontier. Consider an investor whose utility function tangents the “old” mean-variance efficient frontier at portfolio  $a$ , implying a relatively low aversion to risk. By holding a combination of the market portfolio,  $T$ , and borrowed funds at the risk-free rate, the investor can obtain a higher expected return than portfolio  $a$ , for the same level of risk. Since all investors have the same expectations of the distribution of expected return and variance, all investors will hold the market portfolio.

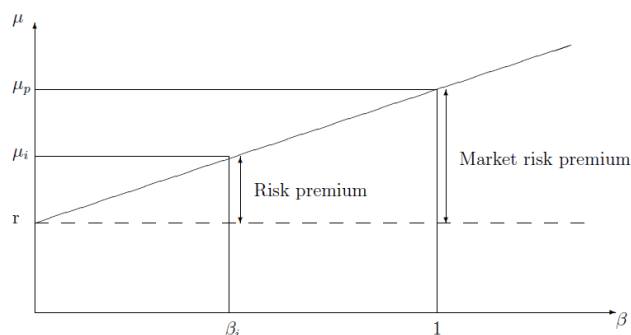
**APPENDIX FIGURE 3 – MEAN-VARIANCE OPTIMAL PORTFOLIO WITH HOMOGENOUS EXPECTATIONS AND UNLIMITED SHORTING AND LENDING**



(Fama & French, 2004, p. 27)

Seen from Appendix figure 4, the beta of the market portfolio has a beta of 1, and there is a linear relation between the expected return and the relative risk of an asset. For the risk an investor takes he is compensated a risk premium:  $(\mu_p - r)/\beta_i$

**APPENDIX FIGURE 3 – RELATIONSHIP BETWEEN BETA AND EXPECTED RETURN**



(Krause, 2001, p. 45)

## Appendix 5 – Value-weighted returns from portfolios sorted on size (2007 – 2017)

<b>Appendix table 1</b>		Excess monthly returns from portfolios sorted on size (2007 - 2017)							
Portfolio	Average size (millions)	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	438.07	2.26	(7.74)	-22.11	1.62	31.49	6	14	20
Portfolio 2	1,458.22	0.90	(6.82)	-25.18	0.54	26.16	7	13	20
Portfolio 3	3,971.08	1.24	(5.55)	-23.87	1.71	18.77	10	14	19
Portfolio 4	9,248.04	0.84	(5.91)	-29.27	0.91	14.44	9	14	20
Portfolio 5	82,849.73	0.41	(5.65)	-21.98	0.72	13.57	9	15	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based the market capitalization at the end of June each year, and the monthly returns are subsequently tracked, before the portfolios are rebalanced in June  $t + 1$ . The returns are value-weighted.

<b>Appendix table 2</b>		Monthly excess returns from a long-short position in size portfolios			
Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0	
2007 - 2008	-4.34	-3.50	0.84	(0.46)	
2009 - 2010	4.94	2.51	-2.43	(-1.36)	
2011 - 2012	1.73	-0.07	-1.80	(-1.49)	
2013 - 2014	1.93	0.77	-1.15	(-1.27)	
2015 - 2016	4.07	0.83	-3.24	(-3.06)***	
2017	4.90	1.43	-3.47	(-1.29)	
Full sample	2.26	0.41	-1.85	(-3.04)***	

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .



## Appendix 6 – Value-weighted returns from portfolios sorted on BM (2007 – 2017)

<b>Appendix table 3</b>		Excess monthly returns from portfolios sorted on BM (2007 - 2017)							
Portfolio	Average BM	Portfolio returns					Number of stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	0.31	0.29	(7.25)	-32.24	0.93	20.21	10	14	20
Portfolio 2	0.56	0.39	(7.41)	-33.73	0.50	21.42	8	14	20
Portfolio 3	0.72	1.16	(5.57)	-18.76	1.79	20.13	8	14	19
Portfolio 4	0.95	0.45	(8.05)	-30.99	0.32	24.84	10	14	20
Portfolio 5	1.61	1.11	(8.70)	-22.63	0.90	37.73	4	14	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the ratio of the book value of equity at fiscal yearend and the market capitalization at December  $t - 1$ , at the end of June in year  $t$ . The monthly returns are subsequently tracked, before the portfolios are rebalanced in June  $t + 1$ . The returns are value-weighted.

<b>Appendix table 4</b>		Monthly excess returns from a long-short position in BM portfolios			
Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0	
2007 - 2008	-4.30	-4.65	-0.35	(-0.19)	
2009 - 2010	2.78	5.53	2.74	(1.26)	
2011 - 2012	0.00	0.32	0.32	(0.21)	
2013 - 2014	0.81	1.44	0.63	(0.62)	
2015 - 2016	0.12	1.07	0.94	(0.55)	
2017	2.07	1.93	-0.14	(-0.12)	
Full sample	0.29	1.11	0.82	(1.18)	

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

## Appendix 7 – Value-weighted returns from portfolios sorted on momentum (2007 – 2017)

Appendix table 5		Monthly excess returns from portfolios sorted on momentum (2007 - 2017)							
Portfolio	Average cum. return	Portfolio returns					Stocks		
		Mean	(Std)	Min	Med	Max	Min	Med	Max
Portfolio 1	-9.90	0.20	(8.72)	-29.58	0.52	23.04	7	14	20
Portfolio 2	7.33	1.10	(6.13)	-24.11	1.28	16.47	8	14	20
Portfolio 3	10.32	0.40	(6.76)	-23.24	0.48	16.61	10	15	19
Portfolio 4	13.92	-0.02	(7.11)	-29.42	0.12	25.00	8	14	20
Portfolio 5	36.58	1.65	(6.99)	-21.27	2.20	16.54	9	14	20

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the cumulative return from  $t - 12$  to  $t - 2$ , where  $t$  refers to the first month holding the portfolio (July). The monthly returns are subsequently tracked, before the portfolios are rebalanced in June the following year. The returns are value-weighted.

Appendix table 6		Monthly excess returns from a long-short position in momentum portfolios			
Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0	
2007 - 2008	-4.05	-2.32	1.73	(1.09)	
2009 - 2010	2.86	4.78	1.92	(1.44)	
2011 - 2012	0.59	-0.14	-0.73	(-0.52)	
2013 - 2014	-0.68	3.49	4.17	(2.13)**	
2015 - 2016	0.83	1.61	0.79	(0.37)	
2017	0.96	1.36	0.40	(0.17)	
Full sample	0.20	1.65	1.46	(1.98)**	

Significance codes:  $P < 0.01$  "\*\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

## Appendix 8 – Value-weighted returns from portfolios sorted on MAX (2007 – 2017)

Appendix tale 7		Monthly excess returns from portfolios sorted on MAX (2007 - 2017)							
Portfolio	Avg. High return	Portfolio returns		Min	Med	Max	Number of stocks		
		Mean	(Std)				Min	Med	Max
Portfolio 1	1.80	0.55	(5.30)	-20.16	0.47	16.24	11	15	21
Portfolio 2	2.77	0.46	(6.48)	-19.18	0.51	21.35	10	15	20
Portfolio 3	3.65	0.25	(7.50)	-34.11	0.76	18.64	10	15	20
Portfolio 4	4.73	0.61	(7.39)	-26.68	1.23	22.55	10	15	20
Portfolio 5	7.71	1.44	(8.30)	-26.63	1.81	20.45	11	15	21

Returns are percentage monthly excess returns. Returns are calculated as simple returns and are not annualized. Portfolios are sorted based on the highest average 5-day return. The return in the subsequent month is calculated, before the portfolios are rebalanced at the end of the next month. The returns are value-weighted.

Appendix table 8		Monthly excess returns from a long-short position in MAX portfolios			
Year	Portfolio 1	Portfolio 5	Diff	t-test: Diff = 0	
2007 - 2008	-1.70	-2.83	-1.13	(-0.85)	
2009 - 2010	2.01	3.99	1.99	(1.27)	
2011 - 2012	-0.74	0.79	1.53	(1.10)	
2013 - 2014	0.96	2.27	1.31	(1.07)	
2015 - 2016	1.77	2.87	1.09	(0.67)	
2017	1.49	1.61	0.12	(0.11)	
Full sample	0.55	1.44	0.88	(1.50)	

Significance codes:  $P < 0.01$  "\*\*\*",  $P < 0.05$  "\*\*",  $P < 0.1$  "\*" .

## Appendix 9 – CAPM regression on equally-weighted portfolios sorted on beta (2009 – 2017)

Appendix table 9		Factor exposures for CAPM on portfolios sorted on beta (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on beta				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.464*** (0.071)	0.783*** (0.085)	0.953*** (0.076)	1.106*** (0.082)	1.411*** (0.107)
	Alpha	0.010*** (0.003)	0.004 (0.004)	0.004 (0.003)	-0.003 (0.003)	-0.008* (0.004)
Observations		108	108	108	108	108
R2		0.288	0.447	0.599	0.635	0.62
Adjusted R2		0.281	0.442	0.595	0.631	0.616
Residual Std. Error. (df = 129)		0.03	0.036	0.032	0.034	0.045
F Statistic (df = 1; 129)		42.860***	85.745***	158.322***	184.076***	172.982***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 10 – CAPM regression on value-weighted portfolios sorted on beta (2009 – 2017)

Appendix table 10		Factor exposures for CAPM on portfolios sorted on beta (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on beta				
		(1)	(2)	(3)	(4)	(5)
MKT		0.557*** (0.106)	0.871*** (0.106)	0.983*** (0.079)	1.236*** (0.067)	1.474*** (0.078)
Alpha		0.015*** (0.004)	0.007 (0.004)	0.000 (0.003)	0.005* (0.003)	-0.002 (0.003)
Observations		108	108	108	108	108
R2		0.207	0.389	0.596	0.760	0.771
Adjusted R2		0.199	0.383	0.592	0.758	0.769
Residual Std. Error. (df = 129)		0.045	0.045	0.033	0.028	0.033
F Statistic (df = 1; 129)		27.641***	67.347***	156.109***	335.924***	357.024***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Risk premium		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0215	(3.41)***
$\hat{\gamma}_{MKT}$	-0.0064	(-0.95)

# Appendix 11 – CAPM regression on equally-weighted portfolios sorted on BM (2009 – 2017)

Appendix table 11		Factor exposures for CAPM on portfolios sorted on BM (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on BM				
		(1)	(2)	(3)	(4)	(5)
MKT		0.897*** (0.076)	0.911*** (0.077)	0.774*** (0.074)	1.001*** (0.084)	1.063*** (0.111)
Alpha		0.001 (0.003)	0.007** (0.003)	0.001 (0.003)	-0.002 (0.004)	-0.001 (0.005)
Observations		108	108	108	108	108
R2		0.568	0.569	0.511	0.574	0.463
Adjusted R2		0.564	0.565	0.506	0.570	0.458
Residual Std. Error. (df = 129)		0.032	0.033	0.031	0.035	0.047
F Statistic (df = 1; 129)		139.314***	139.795***	110.710***	142.803***	91.500***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 12 – CAPM regression on value-weighted portfolios sorted on BM (2009 – 2017)

Appendix table 12		Factor exposures for CAPM on portfolios sorted on BM (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on BM				
		(1)	(2)	(3)	(4)	(5)
MKT		1.033*** (0.091)	1.118*** (0.082)	0.894*** (0.069)	1.468*** (0.089)	1.234*** (0.153)
Alpha		0.00 (0.004)	0.001 (0.003)	0.005* (0.003)	-0.002 (0.004)	0.009 (0.006)
Observations		108	108	108	108	108
R2		0.549	0.639	0.612	0.722	0.381
Adjusted R2		0.545	0.635	0.608	0.719	0.375
Residual Std. Error. (df = 129)		0.038	0.034	0.029	0.037	0.065
F Statistic (df = 1; 129)		129.282***	187.335***	166.971***	274.685***	65.211***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Risk premium		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0029	(0.28)
$\hat{\gamma}_{MKT}$	0.0107	(1.04)

# Appendix 13 – CAPM regression on equally-weighted portfolios sorted on size (2009 – 2017)

Appendix table 13		Factor exposures for CAPM on portfolios sorted on size (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on size				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.627*** (0.132)	0.778*** (0.108)	0.825*** (0.089)	0.979*** (0.070)	1.298*** (0.055)
	Alpha	0.011** (0.006)	-0.002 (0.005)	0.002 (0.004)	0.0002 (0.003)	-0.003 (0.002)
Observations		108	108	108	108	108
R2		0.175	0.327	0.449	0.647	0.842
Adjusted R2		0.168	0.321	0.443	0.644	0.840
Residual Std. Error. (df = 129)		0.056	0.046	0.038	0.030	0.023
F Statistic (df = 1; 129)		22.540***	51.587***	86.233***	194.614***	562.867***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		



## Appendix 14 – CAPM regression on value-weighted portfolios sorted on size (2009 – 2017)

Appendix table 14		Factor exposures for CAPM on portfolios sorted on size (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on size				
		(1)	(2)	(3)	(4)	(5)
MKT		0.864*** (0.153)	0.829*** (0.120)	0.814*** (0.086)	0.923*** (0.071)	1.061*** (0.024)
Alpha		0.025*** (0.006)	0.010* (0.005)	0.012*** (0.004)	0.007** (0.003)	0.002 (0.001)
Observations		108	108	108	108	108
R2		0.232	0.312	0.458	0.617	0.948
Adjusted R2		0.225	0.306	0.452	0.614	0.947
Residual Std. Error. (df = 129)		0.064	0.051	0.036	0.030	0.010
F Statistic (df = 1; 129)		32.009***	48.108***	89.401***	170.951***	1,931.401***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Risk premium		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	-0.0480	(-3.01)***
$\hat{\gamma}_{MKT}$	0.0627	(3.94)***

## Appendix 15 – CAPM regression on equally-weighted portfolios sorted on momentum (2009 – 2017)

Appendix table 15	Factor exposures for CAPM on portfolios sorted on momentum (2009 - 2017)				
	<i>Dependent variable:</i>				
	Portfolios sorted on momentum				
	(1)	(2)	(3)	(4)	(5)
MKT	1.246*** (0.108)	0.885*** (0.070)	0.793*** (0.084)	0.919*** (0.072)	0.859*** (0.101)
Alpha	-0.006 (0.005)	-0.005 (0.003)	-0.001 (0.004)	0.002 (0.003)	0.011** (0.004)
Observations	108	108	108	108	108
R2	0.557	0.601	0.456	0.609	0.406
Adjusted R2	0.553	0.597	0.451	0.606	0.401
Residual Std. Error. (df = 129)	0.046	0.030	0.036	0.030	0.043
F Statistic (df = 1; 129)	133.411***	159.495***	88.747***	165.252***	72.571***
Significance codes	*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 16 – CAPM regression on value-weighted portfolios sorted on momentum (2009 – 2017)

Appendix table 16		Factor exposures for CAPM on portfolios sorted on momentum (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on momentum				
		(1)	(2)	(3)	(4)	(5)
MKT		1.417*** (0.126)	0.971*** (0.073)	1.057*** (0.078)	1.271*** (0.070)	0.938*** (0.119)
Alpha		-0.005 (0.005)	0.009*** (0.003)	0.002 (0.003)	-0.005* (0.003)	0.014*** (0.005)
Observations		108	108	108	108	108
R2		0.544	0.624	0.635	0.756	0.368
Adjusted R2		0.539	0.620	0.632	0.754	0.362
Residual Std. Error. (df = 129)		0.053	0.031	0.033	0.030	0.050
F Statistic (df = 1; 129)		126.320***	175.688***	184.503***	328.417***	61.785***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Risk premium		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	-0.0281	(-2.19)**
$\hat{\gamma}_{MKT}$	0.458	(3.34)***

## Appendix 17 – CAPM regression on equally-weighted portfolios sorted on MAX (2009 – 2017)

Appendix table 17	Factor exposures for CAPM on portfolios sorted on MAX (2009 - 2017)				
	<i>Dependent variable:</i>				
	Portfolios sorted on MAX				
	(1)	(2)	(3)	(4)	(5)
MKT	0.728*** (0.076)	1.003*** (0.075)	0.974*** (0.086)	1.032*** (0.088)	0.974*** (0.096)
Alpha	0.003 (0.003)	-0.001 (0.003)	0.002 (0.004)	0.000 (0.004)	0.002 (0.004)
Observations	108	108	108	108	108
R2	0.466	0.628	0.546	0.564	0.491
Adjusted R2	0.461	0.625	0.541	0.560	0.487
Residual Std. Error. (df = 129)	0.032	0.032	0.036	0.037	0.041
F Statistic (df = 1; 129)	92.509***	179.319***	127.319***	137.385***	102.427***
Significance codes	*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 18 – CAPM regression on value-weighted portfolios sorted on MAX (2009 – 2017)

Appendix table 18		Factor exposures for CAPM on portfolios sorted on MAX (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on MAX				
		(1)	(2)	(3)	(4)	(5)
MKT		0.780*** (0.075)	1.125*** (0.086)	1.137*** (0.089)	1.242*** (0.096)	1.420*** (0.114)
Alpha		0.003 (0.003)	0.001 (0.004)	0.002 (0.004)	0.002 (0.004)	0.010** (0.005)
Observations		108	108	108	108	108
R2		0.506	0.619	0.608	0.612	0.593
Adjusted R2		0.501	0.616	0.604	0.609	0.589
Residual Std. Error. (df = 129)		0.032	0.036	0.037	0.041	0.048
F Statistic (df = 1; 129)		108.431***	172.491***	164.364***	167.418***	154.276***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Risk premium		
Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0186	(1.89)*
$\hat{\gamma}_{MKT}$	-0.0067	(-0.68)

## Appendix 19 – Three-factor regression on equally-weighted portfolios sorted on beta (2009 – 2017)

Appendix table 19		Factor exposures for three-factor model on portfolios sorted on beta (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on beta				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.518*** (0.062)	0.845*** (0.076)	1.023*** (0.067)	1.160*** (0.076)	1.445*** (0.099)
	SMB	0.421*** (0.074)	0.471*** (0.091)	0.478*** (0.080)	0.404*** (0.091)	0.388*** (0.119)
	HML	0.028 (0.048)	0.009 (0.059)	-0.065 (0.052)	0.001 (0.059)	0.181** (0.077)
	Alpha	0.003 (0.003)	-0.004 (0.003)	-0.004 (0.003)	-0.010*** (0.003)	-0.015*** (0.004)
Observations		108	108	108	108	108
R2		0.485	0.574	0.703	0.700	0.693
Adjusted R2		0.470	0.562	0.694	0.691	0.684
Residual Std. Error. (df = 129)		0.026	0.032	0.028	0.032	0.041
F Statistic (df = 1; 129)		32.643***	46.699***	81.940***	80.741***	78.220***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 20 – Three-factor regression on value-weighted portfolios sorted on beta (2009 – 2017)

Appendix table 20		Factor exposures for three-factor model on portfolios sorted on beta (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on beta				
		(1)	(2)	(3)	(4)	(5)
MKT		0.603*** (0.104)	0.856*** (0.105)	0.995*** (0.081)	1.247*** (0.069)	1.466*** (0.079)
SMB		0.355*** (0.125)	0.047 (0.125)	0.030 (0.096)	0.019 (0.082)	0.029 (0.094)
HML		0.010 (0.081)	0.218*** (0.081)	-0.074 (0.062)	-0.086 (0.053)	0.118* (0.061)
Alpha		0.009* (0.005)	0.005 (0.005)	-0.002 (0.004)	0.005* (0.003)	-0.003 (0.004)
Observations		108	108	108	108	108
R2		0.274	0.438	0.601	0.766	0.781
Adjusted R2		0.253	0.422	0.589	0.760	0.775
Residual Std. Error. (df = 129)		0.043	0.043	0.033	0.028	0.033
F Statistic (df = 1; 129)		13.056***	27.050***	52.215***	113.729***	123.919***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0117	(1.17)
$\hat{\gamma}_{MKT}$	0.0010	(0.11)
$\hat{\gamma}_{SMB}$	0.0228	(1.15)
$\hat{\gamma}_{HML}$	0.0006	(0.04)

## Appendix 21 – Three-factor regression on equally-weighted portfolios sorted on BM (2009 – 2017)

Appendix table 21		Factor exposures for three-factor model on portfolios sorted on BM (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on BM				
		(1)	(2)	(3)	(4)	(5)
MKT		0.957*** (0.072)	0.979*** (0.070)	0.839*** (0.069)	1.026*** (0.073)	1.113*** (0.089)
SMB		0.350*** (0.087)	0.444*** (0.084)	0.377*** (0.082)	0.331*** (0.087)	0.578*** (0.107)
HML		-0.134** (0.056)	-0.094* (0.054)	-0.142*** (0.053)	0.197*** (0.057)	0.264*** (0.069)
Alpha		-0.004 (0.003)	0.000 (0.003)	-0.005 (0.003)	-0.008** (0.003)	-0.012*** (0.004)
Observations		108	108	108	108	108
R2		0.630	0.660	0.597	0.694	0.673
Adjusted R2		0.619	0.650	0.585	0.686	0.664
Residual Std. Error. (df = 129)		0.030	0.029	0.028	0.030	0.037
F Statistic (df = 1; 129)		58.980***	67.249***	51.363***	78.759***	71.455***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		



## Appendix 22 – Three-factor regression on value-weighted portfolios sorted on BM (2009 – 2017)

Appendix table 22		Factor exposures for three-factor model on portfolios sorted on BM (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on BM				
		(1)	(2)	(3)	(4)	(5)
MKT		1.060*** (0.085)	1.157*** (0.079)	0.878*** (0.071)	1.448*** (0.087)	1.163*** (0.102)
SMB		-0.016 (0.101)	0.126 (0.095)	-0.068 (0.085)	0.001 (0.104)	0.109 (0.122)
HML		-0.292*** (0.065)	-0.224*** (0.061)	0.065 (0.055)	0.207*** (0.067)	0.856*** (0.079)
Alpha		0.002 (0.004)	0.002 (0.004)	0.006* (0.003)	-0.003 (0.004)	0.003 (0.005)
Observations		108	108	108	108	108
R2		0.632	0.680	0.618	0.748	0.739
Adjusted R2		0.622	0.671	0.606	0.741	0.732
Residual Std. Error. (df = 129)		0.035	0.033	0.029	0.036	0.042
F Statistic (df = 1; 129)		59.608***	73.610***	55.965***	102.880***	98.303***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0196	(1.75)*
$\hat{\gamma}_{MKT}$	-0.0061	(-0.56)
$\hat{\gamma}_{SMB}$	0.0112	(0.48)
$\hat{\gamma}_{HML}$	0.0081	(1.33)

## Appendix 23 – Three-factor regression on equally-weighted portfolios sorted on size (2009 – 2017)

Appendix table 23		Factor exposures for three-factor model on portfolios sorted on size (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on size				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.734*** (0.123)	0.905*** (0.086)	0.880*** (0.071)	1.012*** (0.069)	1.284*** (0.056)
	SMB	0.717*** (0.146)	0.860*** (0.103)	0.527*** (0.085)	0.246*** (0.083)	-0.058 (0.067)
	HML	-0.107 (0.095)	-0.120* (0.066)	0.157*** (0.055)	-0.004 (0.053)	0.067 (0.043)
	Alpha	0.0002 (0.006)	-0.016*** (0.004)	-0.008** (0.003)	-0.004 (0.003)	-0.002 (0.003)
Observations		108	108	108	108	108
R2		0.332	0.602	0.665	0.678	0.845
Adjusted R2		0.313	0.590	0.655	0.669	0.841
Residual Std. Error. (df = 129)		0.051	0.036	0.030	0.029	0.023
F Statistic (df = 1; 129)		17.230***	52.384***	68.827***	73.067***	189.431***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 24 – Three-factor regression on value-weighted portfolios sorted on size (2009 – 2017)

Appendix table 24		Factor exposures for three-factor model on portfolios sorted on size (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on size				
		(1)	(2)	(3)	(4)	(5)
MKT		0.980*** (0.139)	0.970*** (0.094)	0.856*** (0.073)	0.959*** (0.069)	1.043*** (0.023)
SMB		0.850*** (0.166)	0.959*** (0.112)	0.437*** (0.087)	0.264*** (0.083)	-0.111*** (0.028)
HML		-0.027 (0.108)	-0.131* (0.073)	0.166*** (0.056)	-0.010 (0.053)	0.039** (0.018)
Alpha		0.011* (0.006)	-0.005 (0.004)	0.004 (0.003)	0.003 (0.003)	0.002* (0.001)
Observations		108	108	108	108	108
R2		0.399	0.600	0.635	0.655	0.955
Adjusted R2		0.382	0.588	0.624	0.645	0.954
Residual Std. Error. (df = 129)		0.058	0.039	0.030	0.029	0.010
F Statistic (df = 1; 129)		23.014***	51.945***	60.296***	65.685***	739.712***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	-0.0987	(1.50)
$\hat{\gamma}_{MKT}$	0.1072	(1.68)
$\hat{\gamma}_{SMB}$	0.0302	(2.78)
$\hat{\gamma}_{HML}$	0.0915	(2.12)

## Appendix 25 – Three-factor regression on equally-weighted portfolios sorted on momentum (2009 – 2017)

Appendix table 25		Factor exposures for three-factor model on portfolios sorted on momentum (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on momentum				
		(1)	(2)	(3)	(4)	(5)
MKT		1.314*** (0.101)	0.939*** (0.064)	0.852*** (0.078)	0.942*** (0.072)	0.926*** (0.090)
SMB		0.521*** (0.121)	0.392*** (0.076)	0.432*** (0.094)	0.174** (0.086)	0.547*** (0.108)
HML		0.023 (0.078)	-0.014 (0.049)	-0.009 (0.060)	0.000 (0.056)	0.065 (0.056)
Alpha		-0.015*** (0.005)	-0.007** (0.003)	-0.008** (0.004)	-0.001 (0.003)	0.002 (0.004)
Observations		108	108	108	108	108
R2		0.636	0.688	0.557	0.626	0.553
Adjusted R2		0.625	0.679	0.544	0.615	0.541
Residual Std. Error. (df = 129)		0.042	0.026	0.032	0.03	0.037
F Statistic (df = 1; 129)		60.545***	76.386***	43.632***	58.039***	42.972***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 26 – Three-factor regression on value-weighted portfolios sorted on momentum (2009 – 2017)

Appendix table 26		Factor exposures for three-factor model on portfolios sorted on momentum (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on momentum				
		(1)	(2)	(3)	(4)	(5)
MARKET		1.441*** (0.128)	0.959*** (0.075)	1.080*** (0.079)	1.272*** (0.072)	0.942*** (0.122)
SMB		0.222 (0.153)	-0.027 (0.089)	0.168* (0.094)	-0.018 (0.086)	0.095 (0.145)
HML		0.057 (0.099)	0.083 (0.058)	-0.011 (0.061)	-0.034 (0.056)	0.092 (0.094)
Alpha		-0.009 (0.006)	0.009*** (0.003)	-0.001 (0.004)	-0.005 (0.003)	0.012** (0.005)
Observations		108	108	108	108	108
R2		0.558	0.631	0.647	0.757	0.381
Adjusted R2		0.546	0.621	0.636	0.750	0.363
Residual Std. Error. (df = 129)		0.053	0.031	0.033	0.030	0.050
F Statistic (df = 1; 129)		43.847***	59.330***	63.433***	108.195***	21.318***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	0.0360	(2.20)**
$\hat{\gamma}_{MKT}$	-0.0215	(-1.45)
$\hat{\gamma}_{SMB}$	0.0045	(0.26)
$\hat{\gamma}_{HML}$	0.0597	(1.82)*

## Appendix 27 – Three-factor regression on equally-weighted portfolios sorted on MAX (2009 – 2017)

Appendix table 27		Factor exposures for three-factor model on portfolios sorted on MAX (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on MAX				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.785*** (0.071)	1.048*** (0.067)	1.032*** (0.078)	1.086*** (0.081)	1.034*** (0.088)
	SMB	0.386*** (0.085)	0.393*** (0.080)	0.470*** (0.093)	0.438*** (0.097)	0.488*** (0.097)
	HML	-0.054 (0.055)	0.073 (0.051)	0.042 (0.060)	0.043 (0.063)	0.052 (0.068)
	Alpha	-0.003 (0.003)	-0.007** (0.003)	-0.006* (0.003)	-0.007* (0.004)	-0.006 (0.004)
Observations		108	108	108	108	108
R2		0.556	0.724	0.655	0.653	0.6
Adjusted R2		0.543	0.716	0.645	0.643	0.588
Residual Std. Error. (df = 129)		0.029	0.028	0.032	0.034	0.036
F Statistic (df = 1; 129)		43.409***	90.756***	65.708***	65.160***	51.976***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 28 – Three-factor regression on value-weighted portfolios sorted on MAX (2009 – 2017)

Appendix table 28	Factor exposures for three-factor model on portfolios sorted on MAX (2009 - 2017)				
	<i>Dependent variable:</i>				
	Portfolios sorted on MAX				
	(1)	(2)	(3)	(4)	(5)
MKT	0.797*** (0.076)	1.112*** (0.085)	1.140*** (0.091)	1.244*** (0.097)	1.444*** (0.117)
SMB	0.055 (0.091)	0.021 (0.102)	0.065 (0.108)	0.096 (0.116)	0.187 (0.139)
HML	-0.097 (0.059)	0.159** (0.066)	0.055 (0.070)	0.110 (0.075)	0.015 (0.090)
Alpha	0.002 (0.003)	-0.001 (0.004)	0.001 (0.004)	0.005 (0.004)	0.007 (0.005)
Observations	108	108	108	108	108
R2	0.518	0.644	0.613	0.627	0.601
Adjusted R2	0.504	0.633	0.602	0.617	0.590
Residual Std. Error. (df = 129)	0.032	0.035	0.038	0.040	0.048
F Statistic (df = 1; 129)	37.308***	62.619***	55.018***	58.383***	52.316***
Significance codes	*p<0.1;	**p<0.05;	***p<0.01		

Coefficient	Risk premium	T-statistic
$\hat{\alpha}$	-0.0085	(-0.19)
$\hat{\gamma}_{MKT}$	0.0195	(0.33)
$\hat{\gamma}_{SMB}$	0.0210	(0.10)
$\hat{\gamma}_{HML}$	-0.0224	(-0.22)

## Appendix 29 – four-factor regression on equally-weighted portfolios sorted on beta (2009 – 2017)

Appendix table 29		Factor exposures for four-factor model on portfolios sorted on beta (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on beta				
		(1)	(2)	(3)	(4)	(5)
MKT		0.519*** (0.065)	0.879*** (0.079)	1.029*** (0.070)	1.145*** (0.079)	1.403*** (0.102)
SMB		0.422*** (0.075)	0.481*** (0.091)	0.480*** (0.081)	0.399*** (0.091)	0.375*** (0.118)
HML		0.028 (0.048)	0.016 (0.059)	-0.064 (0.052)	-0.002 (0.059)	0.172** (0.077)
PR1YR		0.003 (0.036)	0.071 (0.044)	0.012 (0.039)	-0.03 (0.044)	-0.089 (0.057)
Alpha		0.003 (0.003)	-0.005 (0.003)	-0.004 (0.003)	-0.010*** (0.003)	-0.014*** (0.004)
Observations		108	108	108	108	108
R2		0.485	0.584	0.703	0.701	0.7
Adjusted R2		0.465	0.568	0.691	0.689	0.688
Residual Std. Error. (df = 129)		0.026	0.031	0.028	0.032	0.041
F Statistic (df = 1; 129)		24.250***	36.208***	60.943***	60.371***	60.097***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		



# Appendix 30 – four-factor regression on equally-weighted portfolios sorted on BM (2009 – 2017)

Appendix table 30		Factor exposures for four-factor model on portfolios sorted on BM (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on BM				
		(1)	(2)	(3)	(4)	(5)
MKT		0.932*** (0.075)	0.997*** (0.073)	0.871*** (0.071)	1.013*** (0.076)	1.086*** (0.093)
SMB		0.342*** (0.087)	0.450*** (0.084)	0.386*** (0.082)	0.327*** (0.088)	0.569*** (0.107)
HML		-0.140** (0.056)	-0.09 (0.055)	-0.135** (0.053)	0.194*** (0.057)	0.258*** (0.069)
PR1YR		-0.052 (0.042)	0.038 (0.041)	0.068* (0.039)	-0.027 (0.042)	-0.057 (0.051)
Alpha		-0.004 (0.003)	-0.0001 (0.003)	-0.005* (0.003)	-0.008** (0.003)	-0.011*** (0.004)
Observations		108	108	108	108	108
R2		0.635	0.663	0.608	0.696	0.677
Adjusted R2		0.621	0.650	0.593	0.684	0.665
Residual Std. Error. (df = 129)		0.03	0.029	0.028	0.030	0.037
F Statistic (df = 1; 129)		44.861***	50.600***	39.985***	58.840***	54.014***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

# Appendix 31 – four-factor regression on equally-weighted portfolios sorted on size (2009 – 2017)

Appendix table 31		Factor exposures for four-factor model on portfolios sorted on size (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on size				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.742*** (0.128)	0.951*** (0.088)	0.859*** (0.074)	1.016*** (0.072)	1.262*** (0.058)
	SMB	0.720*** (0.147)	0.874*** (0.102)	0.521*** (0.085)	0.247*** (0.083)	-0.065 (0.066)
	HML	-0.105 (0.095)	-0.111* (0.066)	0.153*** (0.055)	-0.003 (0.054)	0.062 (0.043)
	PRIYR	0.016 (0.071)	0.097* (0.049)	-0.044 (0.041)	0.008 (0.040)	-0.045 (0.032)
	Alpha	0.00004 (0.006)	-0.017*** (0.004)	-0.007** (0.003)	-0.004 (0.003)	-0.002 (0.003)
Observations		108	108	108	108	108
R2		0.332	0.616	0.669	0.678	0.848
Adjusted R2		0.306	0.601	0.656	0.666	0.842
Residual Std. Error. (df = 129)		0.051	0.035	0.03	0.029	0.023
F Statistic (df = 1; 129)		12.818***	41.371***	51.983***	54.301***	143.863***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 32 – four-factor regression on equally-weighted portfolios sorted on momentum (2009 – 2017)

Appendix table 32		Factor exposures for four-factor model on portfolios sorted on momentum (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on momentum				
		(1)	(2)	(3)	(4)	(5)
	MKT	1.253*** (0.103)	0.919*** (0.066)	0.878*** (0.081)	0.920*** (0.075)	0.983*** (0.091)
	SMB	0.503*** (0.119)	0.386*** (0.076)	0.440*** (0.094)	0.168* (0.086)	0.564*** (0.106)
	HML	0.011 (0.077)	-0.018 (0.049)	-0.004 (0.061)	-0.005 (0.056)	0.077 (0.068)
	PR1YR	-0.127** (0.057)	-0.043 (0.037)	0.055 (0.045)	-0.047 (0.042)	0.121** (0.051)
	Alpha	-0.014*** (0.004)	-0.006** (0.003)	-0.008** (0.004)	-0.0003 (0.003)	0.0005 (0.004)
Observations		108	108	108	108	108
R2		0.652	0.692	0.564	0.631	0.577
Adjusted R2		0.639	0.68	0.547	0.616	0.56
Residual Std. Error. (df = 129)		0.041	0.026	0.032	0.03	0.037
F Statistic (df = 1; 129)		48.344***	57.831***	33.252***	43.981***	35.067***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

# Appendix 33 – four-factor regression on equally-weighted portfolios sorted on MAX (2009 – 2017)

Appendix table 33		Factor exposures for four-factor model on portfolios sorted on FMAX (2009 - 2017)				
		<i>Dependent variable:</i>				
		Portfolios sorted on FMAX				
		(1)	(2)	(3)	(4)	(5)
	MKT	0.817*** (0.073)	1.030*** (0.069)	1.079*** (0.079)	1.050*** (0.083)	0.991*** (0.090)
	SMB	0.396*** (0.084)	0.388*** (0.080)	0.484*** (0.091)	0.428*** (0.096)	0.475*** (0.104)
	HML	-0.047 (0.055)	0.069 (0.052)	0.052 (0.059)	0.035 (0.062)	0.043 (0.067)
	PR1YR	0.069* (0.041)	-0.038 (0.038)	0.099** (0.044)	-0.077 (0.046)	-0.090* (0.050)
	Alpha	-0.004 (0.003)	-0.007** (0.003)	-0.007** (0.003)	-0.006* (0.004)	-0.005 (0.004)
Observations		108	108	108	108	108
R2		0.568	0.726	0.671	0.662	0.612
Adjusted R2		0.551	0.716	0.658	0.649	0.597
Residual Std. Error. (df = 129)		0.029	0.028	0.031	0.033	0.036
F Statistic (df = 1; 129)		33.851***	68.297***	52.490***	50.372***	40.636***
Significance codes		*p<0.1;	**p<0.05;	***p<0.01		

## Appendix 34 – All model regressions on the BAB factor

Appendix table 34	BAB-factor regressed against CAPM, the three-factor model, and the four-factor model			
	<i>Dependent variable:</i>			
	BAB			
	(1)	(2)	(3)	(4)
MKT	-0.112 (0.136)	-0.057 (0.135)	-0.022 (0.140)	0.142 (0.148)
SMB		0.406** (0.162)	0.417** (0.162)	0.510*** (0.160)
HML		-0.006 (0.105)	0.001 (0.105)	0.015 (0.102)
PRIYR			0.074 (0.078)	0.050 (0.076)
MAX				-0.320*** (0.114)
Alpha	0.019*** (0.006)	0.012** (0.006)	0.011** (0.006)	0.012** (0.006)
Observations	108	108	108	108
R2	0.006	0.07	0.078	0.144
Adjusted R2	-0.003	0.043	0.042	0.102
Residual Std. Error.	0.057 (df = 106)	0.056 (df = 104)	0.056 (df = 103)	0.054 (df = 102)
F Statistic (df = 1; 129)	0.683 (df = 1; 106)	2.605* (df = 3; 104)	2.179 (df = 4; 103)	3.438*** (df = 5; 102)
Significance codes	*p<0.1;	**p<0.05;	***p<0.01	

