Real Option Valuation of High Growth Tech Firms

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Abstract

This study provides an improved valuation approach for high growth tech firms. We identify the shortcomings of traditional valuation methods and discuss alternative valuation concepts by exploring real option theory. By applying fundamental real option techniques, we expand current valuation theory and present a new framework for company valuation: the Extended Schwartz-Moon model. We demonstrate the relevance and validation of the Extended Schwartz-Moon model by applying it to Spotify in parallel to its listing. The case study provides evidence of that high growth tech firms deserve their apparent high valuation if experiencing high enough growth rate in key variables. The improved model thus contributes to firm valuation, both academically and practically.

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1 Introduction

This introductory chapter aims to present the background of the study and demonstrate its importance and relevance in today's financial markets. The chapter includes purpose and problem statement, as well as a motivation for how we approach the problem. Finally, we present limitations and the outline of the study.

1.1 Background

Digitalisation, globalisation and the use of internet are all characteristics of the modern business environment. In this new digitalised world, settings are more uncertain compared to the pre-internet era since information, rapidly travelling around the globe, can result in instant changes. At the same time, this creates a possibility for firms to grow and gain grounds globally tremendously fast. Alongside the digital globalisation, a new type of firm is emerging; disruptive high growth tech firms which business models have its basis in the internet and are highly dependent on flows of information (Manyika et al., 2016). These firms, such as Facebook, Snapchat, Uber, Airbnb and Spotify grow rapidly (S&P Capital IQ, 2018) by using technological solutions and offering innovative products and services. The successful firms create new markets, change the way to do business within their respective industry and receive attention in the media for their rapid growth and influential offerings. Their high valuations are also regularly discussed (Manyika et al., 2016); one example is Facebook that went public in 2012 with the third highest IPO valuation in the US history (Geron, 2012). Additionally, Snapchat went public in 2017 at a valuation of USD 24 billion, while making a loss of USD 514 million the same year (Snap Inc., 2017; Balakrishnan and Picker, 2017). The industry recognises Snapchat's listing, which was double and fourfold as expensive as Facebook and Google's listings respectively, as one the most expensive tech IPOs in history (Shen, 2017). Most recently, Spotify went public through a direct listing at a valuation of approximately USD 27 billion, while also making millions of losses (NYSE, 2018; Spotify Technology S.A., 2018). Currently, there exists a debate regarding the valuations of this new type of tech firms and whether they are possible to justify (The Economist, 2017a).

The private firms that succeed with their disruptive business ideas are historically not experiencing any problems with raising additional capital (The Economist, 2017b). However, as many of the investors in high growth tech firms are employees, institutions and funds, these firms eventually aim for giving their investors an easy way to liquidate. On this basis, direct listings, recently initiated by Spotify, might be trendsetting going forward; high growth tech firms commonly conduct their businesses in controversial and ground-breaking ways and may follow Spotify's path (The Economist, 2018). Without need of additional capital, firms have incentives to avoid going through the traditional initial public offering (IPO) process with expensive bills and long lock-up periods.

In a direct listing, firms switch their private shares to public at the listing day, which implicates that no initial share price is set; instead, the matching of buy and sell orders received from investors prior to the listing day determine the price. This however comes at a high risk as it can result in a volatile share price during an initial period due to lack of liquidity, potentially resulting in abnormal pricing. Additionally, no investment bank guarantees a floor of the price through underwriting of shares, which also make the pricing increasingly volatile. Since no investment bank set the IPO price, investors do not receive guidance regarding a price range. Thus, the investors have no expert opinion to base their buy and sell decision on (The Economist, 2018).

A problematic part of valuing this new type of firm, which often experience negative cash flows, is the lack of an established valuation model that correctly captures the growth opportunities these firms face. The most common valuation model to apply in the industry is the discounted cash flow (DCF) model (Penman, 2013), which fails in several aspects when it comes to valuing high growth tech firms. As analysts recognise this, they commonly justify a higher valuation by using multiple and comparable valuation methods, which provide a market-based valuation of the company rather than the intrinsic value. As the concept of this new type of tech company is relatively new and the number of high growth tech firms is still small, finding peers and comparable transactions is likely a difficult task. On this basis, the valuation of these firms is as much art as science and depends to a certain extent on the possible value you can sell a firm for than finding the intrinsic value (Carey, 2016).

1.2 Purpose and Contribution

We identify an absence of suitable models for valuing this emerging type of tech firm and recognise a need for an improved valuation model determining the intrinsic value of these firms since they continuously get more common. A way of finding the correct value of this type of firm lies in the interest of all parties operating in the financial markets. It can possibly justify the high valuations that high growth tech firms receive and thereby counteract speculations regarding overvalued firms, but also question if investors overrate the values of these firms. Additionally, an applicable valuation method for high growth tech firms is increasingly important for direct listings in the future, as investors lack initial shares prices to base their investment decisions on.

Academia presents supplementary frameworks based on real option theory that can work as substitutes to the DCF model when it fails to capture the complete value. The theory of real options is suitable for addressing problems such as fast changes, managerial flexibility and unpredictability that characterise the new business environment (Schwartz and Moon, 2000; 2001). However, we assess that real options are both hard and uncommon to apply in practical firm valuations, even though there exist good examples in the literature. Additionally, there are no recent attempts for applying the theory to high growth tech firms. It is therefore relevant to evaluate whether real options are suitable for valuing these firms and how the theory possibly overcomes the setbacks with the traditional valuation models.

For clarification, we define the following characteristics for high growth tech firms:

- *Early stage firms, commonly experiencing negative cash flows*
- *Rapid growth*
- *Gain their main value from user subscriptions and/or their user bases*
- *Technological product or service*
- *Operating in dynamic markets with high uncertainty*

Due to the increasing importance of valuing high growth tech firms, the purpose of this thesis is to propose a method for appropriately finding the intrinsic value of these firms. We hope to reconcile academia and practice and reach consensus of a suitable valuation method by presenting a model that is easily applicable in practice, whilst having a stable theoretical base.

To demonstrate the important contribution of a new valuation method, we perform a case study on the latest listing of a high growth tech firm: Spotify. We conduct this study in parallel to the listing process and hope to contribute with both an intrinsic value of a firm and a practical demonstration of how to value these firms in the future.

1.2.1 Problem Statement

To accomplish the purpose of this study, we form the following main research question:

- *What is a suitable valuation method to determine the intrinsic value of high growth tech firms?*

Additionally, to provide an answer to the research question, we in essence answer a secondary research question:

- *Why are traditional valuation methods not suitable for high growth tech firms and how can real options improve their shortcomings?*

1.3 Research Approach

In order to fulfill the purpose of this study and provide an answer to the research question, we conduct an inductive research approach, aiming to contribute with new insights within the theory of company valuation by using both existing academic theory and an empirical illustration in form of a case study. A prior hypothesis does not restrain this approach (O'Boyle et al., 2017) and we aspire to have an explorative orientation when investigating the problem, thus the inductive research approach is suitable for this study.

In order to tackle the challenge of valuing high growth tech firms, we initially consult current theory and its possible limitations or contributions concerning the problem. We evaluate traditional valuation models that practitioners commonly apply and conduct a detailed literature review on the field of real options. We start with an assessment of the development of the theory of real options in order to reach an understanding regarding real option characteristics and how the theory can contribute to the valuation of high growth tech firms. We further examine relevant fundamental principles and explore how recent research use these principles in valuation models.

To conduct an in-depth investigation that allows for exploring new situations and understandings of complex problems, it is common to conduct a case study research approach. There are no specific requirements for case studies and it is therefore possible to design the method according to the situation in question. However, it requires us to continuously address choices made in order to create a well thought-out and transparent study (Meyer, 2001). We decide to use Spotify for the case study as the company well qualifies as a representative for high growth tech firms by fulfilling the characteristics. This increases the possibility of generalising the applicability of the suggested method and replicate the emergent model to all high growth tech firms.

1.4 Limitations of the Study

We start with limiting this study to the area of real options, mostly due to its strong academic fundamentals and principles. There may exist other methods as substitutes to the traditional models that we do not consider. Nonetheless, we believe that the fundamental theories that real options build on can bring important contributions to the empirical valuation scene and we thereby focus our attention to this field.

We further limit our study to valuing one high growth tech firm since we believe this is the best approach when taking the aim of the study into account. Using multiple cases would be preferable in order to increase the generalisability of the study (Meyer, 2001). However, due to the limit of scope of this study, we decide to thoroughly investigate only one firm since we assess that meticulousness of one case is of higher importance than generalisability at this stage.

1.5 Disposition

We divide this study into four main parts: a description of traditional valuation methods, a presentation of real option theory, an identification of the preferred valuation method and a practical case study. Additionally, we provide two finalising sections: discussion and analysis as well as conclusion.

Traditional valuation methods

The study starts with a presentation of three traditional valuation methods that practitioners commonly apply: the DCF model, the comparable company analysis and the precedent transaction analysis. In addition to describing the techniques, we outline the main shortcomings of each method and specify why these methods are inappropriate for valuing high growth tech firms.

Real options

In the second section, we aim to investigate if real options can overcome the shortcomings of the traditional valuation methods in order to assess whether the theory is useful for valuing high growth tech firms. We present fundamental models and techniques within the area with the goal of investigating and illustrating the theory's possible contribution to the valuation of high growth tech firms. We also present two specific company valuation models: the Schwartz-Moon model and models using real options as add-on components to the traditional DCF model.

Identification of Valuation Method

In the third section, we recognise the advantages and shortcomings of presented valuation methods in order to identify a suitable valuation approach for high growth tech firms.

Case study

In the forth section, we intend to apply the knowledge and techniques developed in the previous sections to a real-world case. We present the valuation method we believe is suitable for valuing high growth tech firms: the Extended Schwartz-Moon model. We apply this approach to Spotify and thus continue with a presentation of the firm and the industry. Finally, we present the result and conduct a sensitivity analysis.

Analysis and discussion

In the fifth section, we discusses and analyses the results and findings of the study.

Conclusions

The concluding section provides final remarks and further identifies limitations as well as suggestions for future research.

2 Traditional Valuation Methods

In this section, we present traditional valuation methods that practitioners commonly use for firm valuations. We further discuss shortcomings with these methods in regard to high growth tech firms.

2.1 The Discounted Cash Flow Model

One of the most well-known models within capital budgeting decisions and firm valuation is the DCF model. The finance industry uses the model widely, presumably because of its easy concept and independence of accounting rules (Penman, 2013).

The DCF model applies net present value (NPV) techniques and calculates the intrinsic value of a firm. The method estimates future free cash flows (FCFs) as well as a terminal value and applies discounting techniques to acquire total firm value. It commonly risk-adjusts the FCFs in the denominator, using a firm-specific discount rate, ρ_w, that originates from the weighted average cost of capital (WACC), denoted with subscript w, of the firm. The resulting valuation formula is (Penman, 2013):

$$EV_0 = \frac{FCF_1}{\rho_w} + \frac{FCF_2}{\rho_w^2} + ... + \frac{FCF_T}{\rho_w^T} + \frac{TV_T}{\rho_w^T} \tag{1}$$

where $FCF_t = C_t - I_t$. The model adjusts cash flows from operations downwards for investments and the resulting FCFs is thereby the amount available for investors after making necessary investments in order to grow the business. In other words, the model treats investments as value losses since they reduces the cash available for investors. Investments in earlier years decrease FCFs but increase the cash from operations in the future (Brealey et al., 2011).

To make predictions about future FCFs, it is necessary to use historical financial data of the firm. There is no consensus regarding how many forecasting periods to use in the model, but a general rule is to set the terminal value from the point in time where it is reasonable to assume that the firm continues to grow at a constant long-run growth rate. The forecasts can have its basis in both historical patterns and analysts' forecasts, but the model requires precise estimations of each proceeding FCF (Penman, 2013). In practice, predictions are difficult, especially for start-ups with both limited historical data and publicly available information. Normally, high growth tech firms have negative FCFs in early years, which also obstruct the forecasts. The future is often highly uncertain for these disruptive firms since the industries can be both new and changing. This results in aggravation of the forecasting of precise FCFs.

The terminal value component consists of the value generated after the explicit forecast period, starting in time T. By adding this perpetual component, the model does not require additional explicit forecasts of FCFs, which estimation is even harder farther ahead in time. The terminal value follows the Gordon growth formula (Penman, 2013):

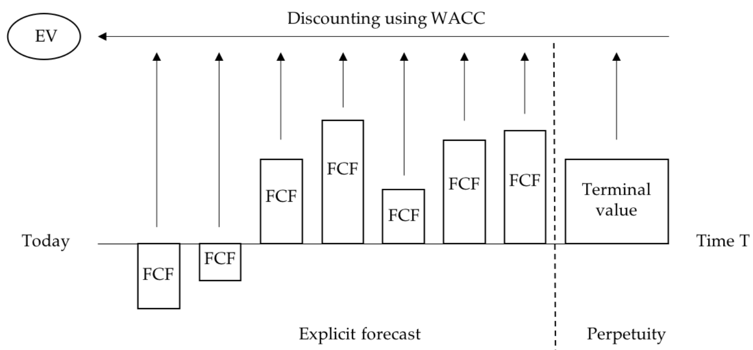
$$TV_T = \frac{FCF_T(1+g)}{\rho_w - g} \tag{2}$$

where g is the long-term growth rate of the firm. The growth rate is a crucial parameter in the model since most of the value in general comes from the terminal value. By increasing the long-run growth rate marginally, the value of the firm can increase substantially. Analysts that subjectively estimate this rate can therefore easily affect and adjust the valuation (Penman, 2013). A common growth rate to apply is the long-term growth rate of gross domestic product (GDP) in the industry together with inflation (Koller et al., 2005).

Koller et al. (2005) estimate that 56% to 125% of high growth tech firms' value originates from the terminal value, which they partly explain by the model's treatment of capital expenditures as value losses. The fact that the majority of the value derives from the terminal component illustrates that it is possible to question whether the DCF is suitable for valuing these firms.

Figure 1 summarises the valuation process of how to obtain the enterprise value (EV) of the firm.

Figure 1: The DCF Model



The figure provides an illustration of the DCF model's procedure, which involves estimations of FCFs and terminal value, as well as discounting of the components to present time by using the firm-specific WACC. This yields the EV of the firm.

2.1.1 Weighted Average Cost of Capital

A common way to estimate the firm's cost of capital, the discount rate, is by using the WACC. The WACC is a composition of the cost of equity and the cost of debt to create consistency between the components of the WACC and the FCFs. The formula for the after-tax WACC is (Brealey et al., 2011):

$$\rho_w = \frac{E}{(E+D)} r_E + \frac{D}{(E+D)} r_D (1 - \tau_C) \tag{3}$$

where E is the market value of equity, D is market value of debt and τ_C is the tax rate. Koller et al. (2005) argue that the yield on the firm's long-term debt gives an estimation of the cost of debt, r_D. For determining the cost of equity, r_E, a common method is the capital asset pricing model (CAPM) as it estimates the expected return on the firm's stock (Koller et al., 2005; Brealey et al., 2011). The CAPM requires identification of the risk-free rate, the firm's beta as well as the market risk premium. The resulting formula for the cost of equity is:

$$r_E = r_f + \beta_E(r_M - r_f) \tag{4}$$

where r_f is the risk-free rate, β_E is the beta of the firm and r_M is the market return. The beta of the firm has its base in the covariance between the market return and the return of the firm. If valuing private a firm, this parameter is problematic to estimate as it is impossible to imply it from the stock of the firm. In that case, the model suggests using a publicly traded twin security as a proxy. This is in many cases hard to obtain as well, as there might be a lack of identical firms trading in the market. As a result, the CAPM is difficult to apply (Koller et al., 2005; Brealey et al., 2011). Additionally, a private firm may be subject to more risks than the just systematic market risk, and the CAPM, not accounting for these, generally works improperly for a private firm (Fabiozzo, 2016). Hull (2012) suggests several ways to estimate the risk-free rate, including using long-term government bonds or swap rates.

2.1.2 Shortcomings

Even though the industry regularly uses the DCF model, academics have long criticised its suitability under certain circumstances. Koller et al. (2005) emphasise the efficiency of the model, but further point out that its result depends heavily on subjective underlying forecasts. When these are hard to estimate, the valuation becomes less precise. MacMillan and Van Putten (2004) and Schulmerich (2010) share this view and argue that the DCF model only works well if a firm's cash flows are predictable and forecasts are not uncertain. When it is neccessary to base the future of a firm on numerous subjective assumptions, the DCF model can lead to inaccurate and unreliable results. Additionally, for a firm that is subject to high uncertainty, the method adjusts for risk by assuming a higher discount rate, which normally results in undervaluation. Since high growth tech firms are disruptive innovators, there is high uncertainty regarding their future cash flows. This indicates that the model is especially hard to apply on these firms and thus may result in inaccurate results.

Furthermore, in a highly uncertainty and unpredictable environment, managerial flexibility is of high importance (Trigeorgis, 1996). MacMillan et al. (2006) argue that the DCF model cannot properly value investments in this setting as the model ignores managerial flexibility. The managers ability to adjust their decisions contingent on additional information received as time passes is not possible to accurately value by using the model. For high growth tech firms and start-ups, this flexibility is of extra importance. If introducing a new product or disrupting an existing industry with digital solutions, it is necessary to adjust investments and strategies when receiving more information regarding outcomes and competitors. Since the DCF model ignores this flexibility, it likely leads to undervaluation of this type of firm.

Amram and Kulatilaka (2000) argue that the DCF model is not the best approach for a firm having negative cash flows as the main part of the value originates from the terminal value. Negative cash flows are a common feature for high growth tech firms that typically invest heavily in an early stage to have the opportunity to realise their potential in the future. In order to value these type of firms, Amram and Kulatilaka (2000) argue that there is a need for other methods than the DCF model.

In conclusion, high growth tech firms have many of the characteristics that the traditional DCF model cannot take into account. These disruptive firms change rapidly, making their future increasingly unpredictable, which obstructs the precision of forecasts of future FCFs. In addition, the majority of the value originates from the terminal value. In order to correctly value these firms, it is also important to take managerial flexibility into account, which the model fails to do.

2.2 Comparable Company and Precedent Transaction Analysis

The comparable company valuation and the precedent transaction valuation approaches are market-based valuation methods with basis in the arbitrage pricing principle, claiming that substitute investments should trade at the same price (Meitner, 2006). The valuation models are especially valuable as complements to the DCF model and may increase the accuracy of the valuation of a company but can also independently value a firm. If applying the methods correctly, they enable comparison between industry peers and precedent transactions, and thereby assess whether a strategic position of a firm should be better or worse than that of the comparable object. This knowledge conveys value when there is high uncertainty regarding estimations of FCFs (Koller et al., 2005).

In a comparable company analysis, it is necessary to identify a peer group of firms with common characteristics and calculate the average of specified key ratios for these firms to conduct the valuation. Common multiple ratios in theoretical descriptions are price per earnings (P/E) and price per sales (P/sales) (Saputro and Hartono, 2017; Brealey et al., 2011; Koller et al., 2005). On the contrary, practitioners commonly use enterprise value multiples of earnings before interest and taxes (EBIT) and sales, such as EV/EBIT and EV/sales, as these metrics are independent of capital structure (Trainer, 2016). A precedent transaction analysis identifies the same multiples but uses previous similar transactions instead of comparable firms.

The most crucial part of a comparable company or precedent transaction valuation is the identification a suitable peer group. Firms in general differ in several ways, such as business model, financial characteristics, drivers of performance, capital structure and risks; finding identical or very similar firms can therefore be challenging. Using an industry average for the analysis ignore these important firm specific factors but can give a good starting point for identifying suitable comparable firms. After establishing the multiples, it is essential to understand why the multiples are different from firm to firm. This yields an understanding of competitive advantages, efficiency and economics of scale across firms. When obtaining this understanding, it is feasible to estimate the multiple a firm should trade at in comparison to other firms (Koller at al., 2005).

Furthermore, it may be challenging to distinguish which multiple to choose for a particular valuation, since different characteristics affect multiples differently. For example, capital structure and non-operating gains and losses affect the P/E multiple and possibly make it artificially high. If the future of a firm is highly uncertain, it is preferable to switch to a multiple based on sales or earnings before interest, taxes, depreciation and amortisation (EBITDA), if positive. Otherwise, only including firms with positive profits in the peer group will incorrectly affect the multiple, resulting in overestimated values. Important to recognise is however that the farther down the income statement, the more similarities should exist between a firm and its peers (Koller et al., 2005; Trainer, 2016; Pearl and Rosenbaum, 2009).

2.2.1 Shortcomings

One distinct disadvantage with the comparable valuation approaches is the difficulty of finding peer firms and similar precedent transactions. Kim and Ritter (1999) emphasise that the variation can be large enough to justify almost any price when using P/E multiples from comparable transactions to value a firm going public. We assess this as a problem especially for high growth tech firms in the start-up scene. A high multiple in one transaction can justify similar valuations for other firms, without having substantial basis in valuation theory. Kim and Ritter (1999) further identify that practitioners using the comparable method have room for manipulation of the valuation. If finding a hyped deal, they have the opportunity to pick high multiples in order to justify a high price. This could possibly drive up prices in general and potentially create overpriced industries. Kim and Ritter (1999) also find that young firms going public have substantial valuation errors when applying the techniques since valuable growth options likely exist, which are difficult to capture in the current valuation. For high growth tech firms, this is a common feature.

In summary, we thus consider that the comparable valuation methods, that work best in combination with the DCF model, on their own would provide speculative values for high growth tech firms since the identification of peers and precedent transactions tends to be especially challenging for this type of firm. With these shortcomings in mind, we believe that comparable valuations are of little use in this context and further assess that instead of relying on subjectively chosen multiples, there exists a need for finding a rigid valuation method with reliance on the intrinsic value.

3 Real Options

The following section aims to give an overview of the development of the real option theory and the previous research within the area. Further, the section presents fundamental principles and pricing methods within the subject, and how to apply these in company valuation.

3.1 Introduction to Real Option Theory

The foundation of real options closely relates to the theory of financial options. In the 1970s, Black and Scholes (1973) and Merton (1973) pioneer financial option pricing with their path-breaking papers "The Pricing of Options and Corporate Liabilities" and "Theory of Rational Option Pricing". They thereby form the basis for the quantitative roots of real options and introduce the development of the Black-Scholes model, the first widely used pricing formula for determining the fair value of a European option, which is only exercisable at the expiration date. Cox, Ross and Rubenstein (1979) further develop a simplified pricing model for financial options. Their binomial approach to derive the options fair value, based on arbitrage pricing, provides a valuation method for discrete time, in which the Black-Scholes model works as the limiting case. Following the publication of these two fundamental theories, the development in academia continues with more specific, financial options models. For example, Margrabe (1978) provides an extension of Black and Scholes's (1973) work by developing a framework for valuing an option exchanging one risky asset for another. Geske (1978) further presents a theory for pricing options on options, so called compound options, based on the assumption that the variance of the rate of return, in comparison to the Black-Scholes model, is not constant but rather a function of the level of the stock price. Carr (1988) integrates the work of Margrabe (1978) and Geske (1978) and develops a valuation formula for a "compound exchange option", which at exercise delivers one asset in exchange for another.

Further, Cox and Ross (1976) play important roles in the advance of new, highly important disciplines, which originate from the well-known Modigliani-Miller theorems and the derivation of the Black-Scholes model from 1973. In contrast to Black and Scholes (1973), assuming that the underlying asset follows the log-normal diffusion process, Cox and Ross (1976) contribute to academia by presenting an option pricing formula that directly connects to other stochastic underlying processes. They develop the concept of risk-neutral valuation by showing that it is possible to replicate an option using traded assets and further show that the price of the option is independent of investors' preferences and capital-market equilibrium. This enables applications of the important risk-neutrality assumption, implicating that the discounting of the future cash flows uses the risk-free interest rate, independent of the riskiness of the asset.

Myers coins the term real options in 1977 when publishing the article "Determinants of Corporate Borrowing", in which he investigates why tax advantage of debt does not result in all firms borrowing "as much as possible" in order to exploit the tax shield it generates. He describes firms as ongoing concerns that make investments in projects with positive NPV; a part of the value of a firm thus originates from the option to make these investments and this value depends on whether it is optimal to exercise the option. Myers (1977) further states that the exercise decision is contingent on the financing opportunities available and that a firm with risky debt might pass on investments that could increase the value of the firm. Hence, the decision of raising debt relies on the trade-off between the tax advantage and the future investment strategy, yielding an explanation for why firms do not fully exploit the available tax-shield. Thereby, the value of a firm is the sum of the market value of assets already in place and the NPV of the firm's option to make future investments. The standard discounted cash flow techniques understate this value since these methods ignore the value of future growth. Thus, Myers (1977) emphasises growth options, which the traditional models ignore, to be a crucial part in company valuation.

Another important contribution to the theory of real options is Mason and Merton's (1985) application of risk-neutrality concepts similar to Cox and Ross's (1976) ideas. In their article, "The Role of Contingent Claims Analysis in Corporate Finance", they use contingent claims analysis (CCA) to value an asset which payoff depends on the value of another asset, such as an option. The technique is based on the fact that a firm's debt and equity can imitate the payoff of options and they emphasise that CCA is a well-suited method for valuing managerial flexibility in projects. Mason and Merton (1985) further describe that CCA enables valuation of non-traded real assets by identifying a twin security, i.e. a perfectly correlated traded asset. According to the arbitrage argument, the price of an option on a non-traded security must be the same as the price of an option on an identical traded security. Thus, it is possible to value options on non-traded assets as well, which is an important finding as real options usually are non-traded. Mason and Merton (1985) thereby make the important contribution that it is possible to value real and financial options similarly since the interest lies in pricing real options as if they trade in the market.

The development of the real option theory continues further with a focus on quantitative techniques and closed-form solutions for specific types of real options, referred to as analytical methods. McDonald and Siegel (1985) use real options theorems to value a firm that has a costless possibility of temporarily shutting down production if the variable cost exceeds the revenue from a product. Kester (1984) investigates a firm's growth possibilities by using real growth options, similar to the theory Myers (1977) describes. In order to make the real option theory improvingly applicable to actual budgeting decisions, Trigeorgis (1993a) investigates more complex and integrated options. He recognises that in practice, firms have interacting options to defer, abandon, contract, expand or switch. Interactive options may significantly differ in value together and separately and his findings make it possible to more precisely value options in practical situations. Kogut and Kulatilaka (1994) value the flexibility that exist in multinational corporations to coordinate plants and production globally. They recognise that the ability to switch and organise recourses internationally creates real option value.

The analytical models provide closed-form solution for specific, simplified empirical capital budgeting problems. However, the methods are specific and challenging to apply to more complex options. All models require composition of the partial differential equation with the underlying stochastic process. Therefore, if valuing multiple, interacting real options, there is a need for other methods. For such situations, there commonly exist a lack of analytical solutions and difficulties in deriving partial differential equations. However, the development of several numerical techniques, many based on risk-neutral valuation, makes it possible to handle this type of option valuation. It is possible to divide these numerical valuation techniques into two subgroups: (i) direct approximation of underlying stochastic processes, and (ii) approximation of the resulting partial differential equation (Trigeorgis, 1993b). The first category, including for example the important Monte Carlo simulation method developed by Boyle (1976), is generally more intuitive. The Monte Carlo simulation process, involving the risk-neutrality assumptions, determines an option's price by simulating returns on an underlying asset. According to Boyle (1976), this method has a clear advantage in specialised situations, for example when continuous and jump processes (a mixture of stochastic processes) at the same time generate the underlying asset's return, which in general give rise to a mixed partial differential equation. Two other important methods are the binomial lattice approach, that Cox, Ross and Rubenstein (1979) develop, and the log-transformed binomial method, that Trigeorgis (1991) introduces. The binomial lattice allows for modelling of possible future paths of projects and can thereby take contingent decisions into account. Trigeorgis's (1991) model allows for valuation of investments with multiple types of real options and captures the interaction between them. Examples of approaches that approximate the stochastic differential equation are numerical integration as well as explicit and implicit difference methods.

A shortcoming of the real option pricing methods before the 1990s is the weak connection between academia and practice. Trigeorgis creates the first link between academia and practice in 1996 when publishing the book "Real Options". Similarly, Copeland and Antikarov (2003) present a guide for real options valuation in practice aimed for corporate finance professionals, which compared to Trigeorgis (1996), takes a less quantitative approach.

Schwartz and Moon (2000) make an important contribution with their model aimed for valuing internet firms, such as eBay and Amazon, by using capital budgeting techniques and real option theory. They justify the apparent high valuations of internet firms at the time of the dot-com bubble with high growth in revenues in combination with high volatility in key variables. The model includes two stochastic variables, revenues and growth in revenues, and by conducting Monte Carlo simulations, they model random future paths of a firm to capture the uncertainty in these variables. Schwartz and Moon (2000) apply their model to Amazon and estimate the value of the firm by discounting the cash flows under the equivalent martingale measure, implying the possibility of using the risk-free rate for discounting. In 2001, Schwartz and Moon present an improved model in which they advance a firm's cost structure by introducing an additional stochastic variable: variables costs. The new variable captures the uncertainty regarding competition and future prices, which are crucial factors for firms the model aims to value. Klobucnik and Sievers (2013) are the ones to firstly apply Schwartz and Moon's (2001) model cross-sectional using around 30,000 US firm quarterly observations. They conclude that the model is suitable for high growth tech firms and that it yields accurate results compared to sales multiples. The key takeaway from the paper by Klobucnik and Sievers (2013) is however that the Schwartz-Moon model is able to identify over- and undervalued markets, which makes it useful when forming trading strategies and detecting pricing bubbles.

Another development within the field of real options is valuation models using the DCF model and real options as complements. MacMillan and Van Putten (2004) develop a framework in which real options work as add-on components to the DCF model and thereby captures the high reward potential in growth options and managerial flexibility that the DCF model by its own fails to recognise. Similarly, Trigeorgis and Ioulianou (2013) suggest changing the terminal value calculations in the traditional DCF model to a real option-based term since this better captures the growth opportunities facing firms.

3.2 Classifications of Real Options

In order to simplify the identification and analysis of a company's operating and strategic flexibility and available real options, Kodukula and Papudesu (2006) and Copeland and Antikarov (2001) divide real options into two main types; simple and advanced options. Koller et al. (2005) do not make the same distinction, but present similar option types. Simple options give the holder the right but not the obligation to abandon, expand, contract or defer a project and thus provide the holder with potential upside while limiting the downside. Advanced real options deal with compound and switching options, as well as rainbow options. Trigeorgis (1993b; 2005) further classify corporate growth options and multiple interacting options into the category. Table 1 and Table 2 summarise the different classifications within simple and advanced options respectively.

Table 1: Simple Options

Option to abandon	This option shares the same characteristics as a put option on a stock and is useful if a project performs poorly. The exercise value is the expected liquidation value of the project and the option becomes valuable in case the project value falls below the liquidation value.
Option to expand or contract	The option to expand provides the opportunity to scale a project and is comparable to a call option on a stock. The option holder has the flexibility, but not the obligation, to make follow-on investments or accelerate the utilisation of resources. Conversely, the option to contract scalability is equivalent to a put option on a stock, and the holder of such option thus has the opportunity to scale down operations.
Option to defer	This option is comparable to a call option on a stock. By paying beforehand, the owner has the opportunity to postpone the usage of the option until the timing fits. The cost of using the options is equal to the exercise price and the owner exercises the option if the opportunity cost of deferring is too high.

Sources: Kodukula and Papudesu (2006), Copeland and Antikarov (2001) and Koller et al. (2005). The table provides an overview as well as explanations of different simple options.

Table 2: Advanced Options

Option to switch	This option allows the holder to switch back and forth among alternatives, for example changing output mix of a production facility or exiting and re-entering an industry.
Compound option	This option is an option on options and the holder generates new options when exercising existing ones. A compound option is either simultaneous or sequential. The first type constitutes options that are possible to use in parallel. The latter refers to staged investments, implicating that the option holder in each stage has the opportunity to continue investing in the project or to abandon the project.
Rainbow option	A rainbow option depends on several underlying assets and multiple sources of uncertainty affect its value. Many real-world applications require a real option valuation method that can handle this type of option, since it covers a wide and important range of decisions. Practical examples of the rainbow option refer to research and development (R&D) and new product developments.
Multiple interacting option	This option contains a collection of different options. In combination, the value of the collection may differ from the sum of the options, which is why owners of the options should consider them as one multiple interactive option rather than single options.
Corporate growth option	A growth option gives the possibility to unlock future opportunities and is a type of expansion option. In general, early investments such as R&D or a strategic acquisition can link interrelated projects together, resulting in for example accesses to new markets or strengthening of strategic positions. The value originates from future growth opportunities instead of direct cash flows, making the structure similar to compound options. This option is common in strategic industries, such as high tech or R&D heavy industries.

Sources: Kodukula and Papudesu (2006), Copeland and Antikarov (2001), Koller et al. (2005) and Trigeorgis (1993b; 2005). The table provides an overview as well as explanations of different advanced options.

3.3 Fundamental Real Option Theory

To give an improved understanding of how company valuation techniques can incorporate real options, we investigate fundamental methods for option pricing. As real and financial options closely relate, most of the presented theory is applicable to both option types. However, our focus throughout the section is to give a full representation of the real option theory.

3.3.1 The Black-Scholes Model

The Black-Scholes model has its basis in the concept of no arbitrage, meaning that two portfolios or assets with the same payoff should have identical prices in order for the market to be arbitrage free. The logic is that, if there exist arbitrage opportunities, investors take positions to exploit these, resulting in the opportunities being "arbitraged away". The model is valid under the following conditions (Black and Scholes, 1973; Hull, 2012):

- *There are no transaction costs or differential taxes*
- *Borrowing and lending, at the same rate of interest, are unrestricted*
- *The short-term risk-free rate of interest is known and constant through time*
- *Short sales, with full use of proceeds, are unrestricted*
- *Trading takes place continuously in time*
- *The movement of the stock price can be described by a diffusion-type process*

Below follow the abbreviations used in the model (Black and Scholes, 1973; Hull, 2012):

α	Expected return of the underlying stock per unit time
σ	Standard deviation of the return of the stock per unit time
dz	Gauss-Wiener process
C	The price of the call option
t	A general time
T	Time to expiration
r	Risk-free interest rate
K	Exercise price
S	Price of underlying asset

The stock price of the underlying asset follows the process (Black and Scholes, 1973):

$$dS = \alpha S dt + \sigma dz \tag{5}$$

The derivation of the model originates from the partial differential equation (Hull, 2012):

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC = 0 \tag{6}$$

The formula implies that it is possible to perfectly hedge the option by buying and selling the underlying asset. The equation has several solutions and to obtain the pricing formula that Black and Scholes (1973) derive, it is necessary to set specific boundary conditions. For a call options, these conditions are:

$$C(S, 0) = max(S - K, 0) \tag{7}$$

$$C(0, T) = 0 \tag{8}$$

$$\frac{C(S, T)}{S} \rightarrow 1 \text{ as S} \rightarrow \infty \tag{9}$$

The partial differential equation in combination with the boundary conditions generates the perfect hedge of the option, implying what price the option must have. Thereby, the resulting pricing formula for call options is:

$$C(S, T, K) = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{10}$$

where $N(\bullet)$ is the cumulative normal distribution function and:

$$d_1 = \frac{log(\frac{S_0}{K}) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{11}$$

$$d_2 = \frac{log(\frac{S_0}{K}) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{12}$$

One explanation for the success of the model is that all input parameters are directly observable in the market, yielding an easily applicable model to European options. The formula uses the important principles of risk-neutral valuation when applying the partial differential equation, which yields prices that are independent of investors' unobservable risk preferences. Important to notice is however that the model only is applicable to European options since it cannot account for possible early exercise of American options, which are exercisable at any time (Hull, 2012).

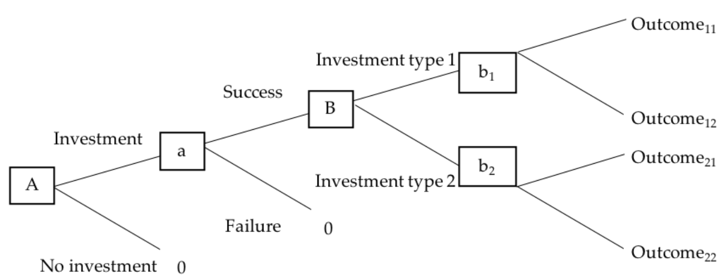
In some cases, it is feasible to value real options by using the Black-Scholes model. Borison and Triantis (2001) state that the model can give an approximate solution to simple real options with similar payoff structure as financial options. These real options only include one investment decision, at a certain point in time, and can for example be an option to defer an investment (call) and an option to abandon a project (put). Arnold (2014) supports this view, pointing out that the model works for simple options that do not interact with other options. However, the input parameters are different; the exercise price is the investment and the features of the investment replace the characteristics of the underlying stock. It is also important that the assumptions of the Black-Scholes model are valid, which is rarely the case in practice for real options. Real options are generally more complex, resulting in the model being inapplicable. Because of these shortcomings, it is not common to use the model in the area of real options, but it still provides important insights and path-breaking findings that are important when investigating pricing models for real options (Arnold, 2014; Borison and Triantis, 2001).

3.3.2 Decision Tree Analysis

Decision tree analysis (DTA) is a technique that captures value arising from managerial flexibility. It is especially suitable for projects in which there is a need of taking multiple contingent decisions in particular points in time into consideration and when uncertainty decreases over time (Trigeorgis, 1996). Instead of using a fixed pre-determined path, as done in the DCF model, DTA allows the future to consist of many possible outcomes and is thus able to capture flexibility.

The method models different paths using a contingent decision tree, where each node reflects available decisions, their costs or payoffs and the different outcomes resulting from each decision. The final nodes display all possible finite values of the project, dependent on the chosen paths. Additionally, the model assigns physical probabilities to each possible path. The discount factor, depending on the riskiness of the project, discounts all the modelled expected cash flows, which yields an expected value of each path. Decision makers can finally intuitively choose the path with the highest NVP. Figure 2 gives an illustrative example of a DTA process.

Figure 2: Decision Tree



The figure provides an illustrative example of a typical project in which it is possible to take multiple investment decisions, represented by A and B, into account. The outcomes are contingent on the decisions and the value of the project thereby depends on the chosen path.

Furthermore, Papudesu and Kodukula (2006) present DTA as a substitute to the DCF model since it creates a complex setup, more similar to real cases. They emphasise, in accordance with Schulmerich (2010), the importance of the model's possibility to take flexibility of managers' actions into account when valuing a project. This increases the accuracy of the valuation compared to a traditional NPV analysis of projects and firms.

According to Schulmerich (2010) and Trigeorgis (1996), DTA is further useful when valuing simpler real options. In this case, the payoff in each node depends on both the expected value from that decision and the possible real option value. For example, in case of an abandonment option, the decision in each node depends on both the value of continuing the project and the possible salvage value of abandoning the project: the holder of the option chooses the alternative with the highest expected payoff. In this case, the value of the option originates from the fact that managers can abandon the project if the conditions turn out to be worse than originally estimated. In this case, DTA yields two different NPVs for the project; static NPV and strategic NPV. The former NPV is the value from the project without considering the real option, while the latter is the NPV of also including the real option, consisting of the value of the choice between continuing or abandoning the project and receiving the salvage value. The difference between the two gives the option premium.

$$\textit{Value of real option} = \textit{option premium}$$

$$\textit{Option premium} = \textit{strategic NPV - static NPV}$$

For many projects, especially high-risk projects with uncertain cash flows, the NPV may be low or even negative. Because of this, managers might pass on value-adding projects. High growth tech firms usually have negative cash flows in earlier years, while also having a huge growth potential. Only considering the static NPV results in considerable undervaluation of these firms, which illustrates the importance of real options in valuation schemes (Schulmerich, 2010).

Even though DTA is able to capture the value of flexibility of investments by modelling discrete points in time for managerial decisions by using option payoffs, the method also contains significant shortcoming. Since the model requires modelling of expected cash flows from each decision, it is difficult to apply practically when decisions and scenarios are uncertain or complex. The method further requires physical probabilities for every different outcome, which are unobservable and therefore subjectively estimated. This results in sensitive valuations that are easy to influence (Schulmerich, 2010; Trigeorgis, 1996; Papudesu and Kodukula, 2006).

Another limitation with DTA is that the method assumes "high" and "low", and possibly "median" outcome values from each decision. These values are hard to accurately estimate and simplifications of practice since there possibly exist a continuous number of possible outcomes. The model also has discrete time steps, which is not a precise reflection of the real world, where managers can take decisions on continuous basis rather than at a predetermined time (Trigeorgis, 1996).

Schulmerich (2010) also points out that the model assumes a constant discount factor, even though the uncertainty in reality decreases as time passes and the project reveals more information. The discount factor is hard to estimate accurately and affect the result extensively. A decreasing discount rate across the tree can partially solve this problem, but Schulmerich (2010) still emphasises that there exists no consensus regarding how to find an appropriate discount rate. The model is therefore hard to implement on practical complex cases. High growth tech firms commonly experience high uncertainty regarding the future, which results in a high discount rate if applying DTA in the valuation, even though these firms might deserve a lower discount rate as time passes and uncertainty decreases. Discounting all cash flows with a constantly high discount rate thus results in undervaluation of these firms.

3.3.3 Contingent Claims Analysis

DTA is complicated to apply even in simple situations, as it requires subjective estimates of probabilities, and does not incorporate a varying discount factor. Schulmerich (2010) presents CCA, based on the suggestions made by Mason and Merton (1985), as an improved method that can solve these shortcomings. CCA converts physical probabilities into risk-adjusted probabilities, allowing for the usage of a constant, risk-free interest rate as discount rate, independently of projects' risk structure.

CCA originates from financial option theory and has its base in fundamental principles, such as no arbitrage, and is applicable to real options. This, the method enables pricing of an option by replicating its payoff and risk characteristic by using an identical security and risk-free bonds. This makes CCA applicable to both private and trading assets (Dixit and Pindyck, 1994). Since identifal payoffs should yield the same price according to no arbitrage, it is possible to find the value of the option from the formed replicating portfolio (Trigeorgis, 1996).

The pricing techniques of CCA use the following abbrevations (Schulmerich, 2010; Trigeorgis, 1996):

F	Contingent claim
V_i	Value of n underlying assets or state variables
α_i	Expected growth rate of V_i
σ_i	Standard deviation of V_i
dz_i	Differentials of Brownian motion processes (mean 0 and variance dt)
t	Time
μ_j	Return offered by F_j
s_i	The component of asset F_j's standard deviation attributed to variable V_i
r	Risk-free return

The underlying assets follow a diffusion process:

$$\frac{dV_i}{V_i} = \alpha_i dt + \sigma_i dz_i \tag{13}$$

Apart from the assumptions in the Black-Scholes model, CCA also assumes that the financial markets are complete with $n+1$ traded assets, which prices, F_j, depend on the prices of the underlying assets, V_i, and time, t. In a complete market, it is feasible to replicate all contingent assets. Equation 14 defines the diffusion process, found by using the extended form of Ito's lemma:

$$\frac{dF_j}{F_j} = \mu_j dt + \sum\nolimits_i s_i dz_i \tag{14}$$

where

$$\mu_j = \frac{\frac{1}{2}\sum_{i,k} \rho_{ik}\sigma_i\sigma_k V_i V_k \frac{\partial^2 F}{\partial V_i \partial V_k} + \sum_i \alpha_i V_i \frac{\partial F}{\partial V_i} - \frac{\partial F}{\partial \tau}}{F_j} \tag{15}$$

$$s_i = \left(\frac{\frac{\partial F}{\partial V_i} V_i}{F_j}\right)\sigma_i \tag{16}$$

By investing a weight, ω_j, in each security, F_j, it is possible to create a risk-free hedge portfolio over any interval dt. How much to invest in each asset depends on the stochastic part of the equation, $(\frac{dF_j}{F_j})$, and the goal is to eliminate this part in order to make the portfolio free of risk. Since a risk-free portfolio should only earn the risk-free rate according to no arbitrage, it results in the asset pricing model:

$$\mu_j - r = \sum\nolimits_i \lambda_i s_i \tag{17}$$

where $\lambda_i = (\mu_j - r)/\sigma_i$ is the market price of risk. As real options do not trade in the market, this is found by using an identical traded security.

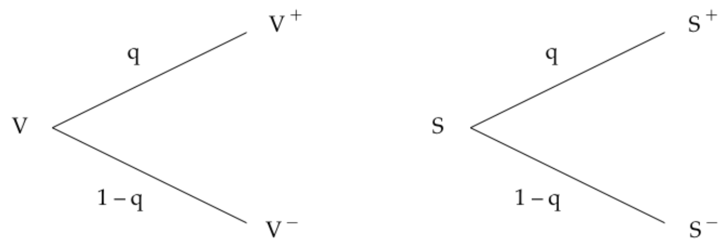
Finally, by substituting Equation 15 and Equation 16 into Equation 17 it is possible to obtain the fundamental partial differential equation for CCA:

$$\frac{1}{2}\sum\nolimits_{i,k} \rho_{ik}\sigma_i\sigma_k V_i V_k \frac{\partial^2 F}{\partial V_i \partial V_k} + \sum\nolimits_i (\alpha_i - \lambda_i\sigma_i) V_i \frac{\partial F}{\partial V_i} - \frac{\partial F}{\partial \tau} - rF + d = 0 \tag{18}$$

where d is the total payoff from the option before exercise. Similar to the Black-Scholes model, it is necessary to impose particular boundary conditions in order to value different real options. The important take away for this derivation is that it is possible to value any contingent claim, with a value dependent only on n underlying assets and time t, by using this partial differential equation (Trigeorgis, 1996).

Furthermore, for an illustrative purpose, we present a one-period case, assuming that a project, V, and its twin security, S, follow the paths that Figure 3 displays. q and $1 - q$ reflect the physical probabilities (Schulmerich, 2010). It is of high importance to identify a twin security with the same risk structure as the project when applying CCA. This is often a challenging task, especially when applying CCA to real options as only limited data is available (Schulmerich, 2010).

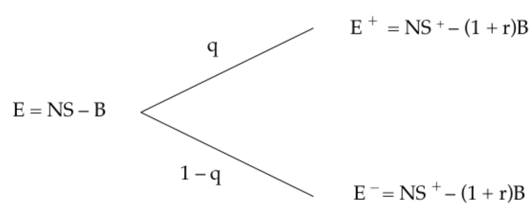
Figure 3: Project and Twin Security



Source: Schulmerich (2010). The figure illustrates the payoff structure of a project, V, and its twin security, S. q and $1 - q$ *represent physical probabilities.*

The pricing of a contingent claim requires replication of the equity value, E, which should perfectly correlate with the movements of both the project and the twin security. To conduct the replication, it is necessary to invest N shares in the twin security financed by shorting B amount in the risk-free rate. The replicating portfolio should yield exactly the same payoff as the real option, independently of what state of the world that becomes true (Schulmerich, 2010). Figure 4 illustrates the replicating portfolio.

Figure 4: The Replicating Portfolio



Source: Schulmerich (2010). The figure illustrates the payoff structure of the replicating portfolio formed by investing N shares in the twin security and borrowing B amount of the risk-free rate.

To recognise how many shares of the twin security to buy and how many risk-free bonds to short, it is crucial to solve for n and B. This gives the solution of the replicating portfolio. The equations follow:

$$n = \frac{E^+ - E^-}{S^+ - S^-} \tag{19}$$

$$B = \frac{E^+ S^- - E^- S^+}{(S^+ - S^-)(1+r)} = \frac{NS^- - E^-}{1+r} \tag{20}$$

$$E = \frac{pE^+ + (1-p)E^-}{1+r} \tag{21}$$

where p and $(1-p)$ are risk-neutral probabilities. The equation for p is:

$$p = \frac{(1+r)S - S^-}{S^+ - S^-} = \frac{(1+r) - d}{u - d} \tag{22}$$

where

$$u = e^{\sigma\sqrt{\Delta T}} \tag{23}$$

$$d = \frac{1}{u} \tag{24}$$

where u and d represent the up and down movements. The method eliminates the risk related to project when forming a replicating portfolio since the strategy ensures cash flows independently of the state of the world. This enables switching from physical to risk-neutral probabilities, which are higher for unfavourable states and lower for favourable states. By doing this, CCA looks at the world as if all agents are risk-neutral and therefore does not require any return above the risk-free rate, which is commonly referred to as the certainty equivalent approach (Schulmerich, 2010). Resulting, the risk-neutral probabilities depend only on the risk-free rate and the up and down movements and thus allow for discounting using the risk-free rate since cash flows under the certainty equivalent measure are independent of risk. Because of this, the present value of the cash flows are the same under both measures, even if applying different discount rates (Copeland and Antikarov, 2003). The process eliminates the problem of finding the correct discount factor and is therefore an important contribution of CCA.

Another clear advantage with CCA is the possibility of identifying the value of a real option without recognising specific cash flows, which makes CCA theoretically preferred over DTA. However, the model is not applicable if the required input parameters are unavailable. It is for example difficult to identify an identical security trading in the market; when valuing a private firm in a new industry, this is commonly challenging to find (Koller et al., 2005).

3.3.4 Binomial Lattice

The binomial lattice approach is a numerical real option valuation method that approximate the underlying stochastic process. The approximation uses binomial or trinomial trees and has its starting point in the current value. In comparison to a Monto Carlo approach, presented in the next section, the binomial lattice approach is more suitable for valuing American options since it easily can take early exercise into account (Schulmerich, 2010).

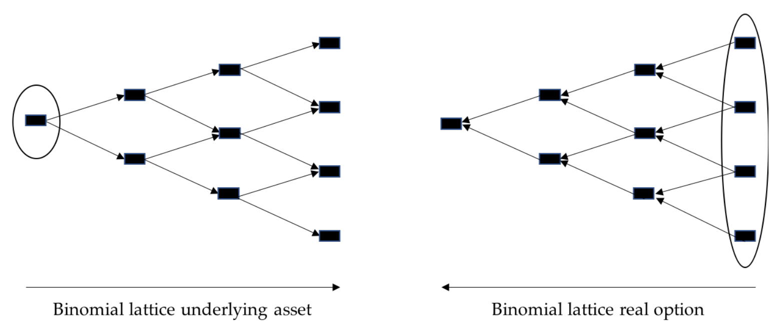
Cox, Ross and Rubenstein's paper from 1979, "Option Pricing: A Simplified Approach", is as an important building block for the lattice approach. Having its basis in Cox and Ross's earlier work that introduce option pricing based on replicating portfolios and risk-neutral valuation, their binomial tree approach is a breakthrough in option pricing in discrete time and today the most classical tool for option pricing of this type.

The model's derivation is almost identical to the ones for CCA in Section 3.3.3 Contingent Claims Analysis. Thus, we do not go into detail regarding its derivations. The main difference is however that Cox, Ross and Rubenstein (1979) use continuously compounded rates rather than discrete. Hence, the risk-free return, r, exchanges for e^r. Additionally, n shares of the underlying stock and a Δt-year bond B create the replicating portfolio E.

Further, in order to allow for stochastic risk-free interest rates, e.g. rates depending on a varying rather than flat term structure, it is possible to modify the model of Cox, Ross and Rubenstein's (1979). The modified model, in comparison to the basic model, thus has a time subscript on the risk-free rate and the risk-neutral probabilities (Schulmerich, 2010).

Baduns (2013) intuitively explains how to apply the binomial lattice approach. He argues that for a simple real option, it is necessary to model one lattice for the underlying asset, or a twin security, and one for the option itself. Risk-neutral probabilities and the up and down movements provides the first lattice, while backwards induction derives the latter, which Figure 5 illustrates.

Figure 5: Binomial Lattices



Source: Baduns (2013). The figure gives an illustration of binomial lattices for both an underlying asset and a real option.

Strengths of the binomial lattice model are its simple intuition and flexibility when it comes to managing different stochastic processes and multiple options. However, the method yields the option value for only one single underlying asset, thus it is necessary to carry out the pricing procedure several times when there exist many starting values. The model is in other words time consuming, especially when determining whole option value distributions (Schulmerich, 2010). In order to find an efficient model, applicable on complex, potentially interacting investments, Trigeorgis (1991) develops an extension of Cox, Ross and Rubenstein's (1979) model: the log-transformed binomial tree approach. Trigeorgis's (1991) method thus allows for valuing projects involving multiple real options and tries to reach consistency between the discrete and the continuous time approach.

Similar to CCA, a shortcoming with the binomial lattices is the difficulty of identifying a traded twin security. Both Copeland and Anikarov (2003) and Dixit and Pindyck (1994) point out that projects with an aim to develop new products, such as R&D ventures, are hard to replicate since they are unrelated to existing assets. In that case, the assumption that an asset is possible to span in the market does not hold. To overcome this issue, Copeland and Antikarov (2003) suggest applying the Marketed Asset Disclaimer (MAD) assumption. They argue that the NPV of the project itself can replace the twin security when infeasible since the project and the real option should perfectly correlate. The MAD assumption is especially valuable for the high growth tech firms since they commonly introduce new innovative products that are not possible to find traded in the market.

3.3.5 Monte Carlo Analysis

Monte Carlo is a numerical method that directly approximates an option's underlying stochastic process and relies on the risk-neutrality concept (Geske and Shastri, 1985). Equation 25 presents the stochastic differential equation that describes the path of the underlying asset for time $t \geq 0$, S_t. α is the instantaneous return and σ the instantaneous standard deviation of the underlying asset. The random variable dz_t is normally distributed and has a variance of dt (Schulmerich, 2010).

$$dS_t = \alpha S_t dt + \sigma S_t dz_t \tag{25}$$

The goal of the Monte Carlo simulation is to create sample paths for the underlying asset for each point in time t for a specific time interval up to time T, with N subintervals with the step size of Δ_s=T/N. The method calculates the path iteratively from a starting value, S_0 . One possible function for the iterations is the Euler scheme:

$$S_{i+1} = S_i + \alpha S_i \Delta_s + \sigma S_i \Delta z_i \quad i = 0,\ 1,\ \ldots,\ N \tag{26}$$

When simulating the path, the subsequent steps that the process requires to price the option depends on both option type and its pricing function. For example, a European call with maturity T and exercise price X uses the pricing function: $P_j = max(S_T - X, 0)$, where index j reflects the price of the $j^t h$ simulated path. The method is in other words conducting the simulation A times independently and thereby generates as many number of prices, P_j. The mean of the prices constitutes the option valuation at time T:

$$z = \sum_{j=1}^{A} P_j \tag{27}$$

The present value of z under the risk-neutral measure, ze^{-rT}, finally gives the Monte Carlo option price (Schulmerich, 2010).

The aim of the Monte Carlo simulation is traditionally to price European options since it suits "one-off" situations (Boyle, 1976). If applying Monte Carlo to American options, it requires N lognormal distribution approximations, compared to only one at time T for the European options. Therefore, the method requires a full set of simulated paths conditional on the starting point. This enables pricing of multiple options with different characteristics but at the same time, it makes the Monte Carlo process less efficient (Geske and Shastri, 1985). However, there exists attempts for overcoming this shortcoming, which the improvement of Monte Carlo for American options presented by Longstaff and Schwartz (2001) is an example of.

Advantages of using Monte Carlo are its simplicity and its ability to handle different complex processes of the underlying asset, involving for example continuous and jump processes at the same time. This type of process combination results in mixed partial differential equations, which without the Monte Carlo approach are difficult to solve. The method can also value many types of real options and allows for flexibility regarding the distribution that generates the returns of the underlying asset. This is possible because there is no requirement for closed form solutions for the underlying asset (Boyle, 1976). Furthermore, the Monte Carlo approach is useful when valuing real options which decisions are contingent over time, as the method takes path-dependency into account (Borison and Triantis, 2001).

3.4 Real Options in Company Valuation

In the previous sections, we presented the most well-known methods and models for option pricing, with a focus on real options. In this section, we present company valuation models that use these basic principles, with a focus on their applicability to high growth tech firms.

3.4.1 The Schwartz-Moon Model

Schwartz and Moon (2000; 2001) develop a pricing model with basis in real option theory in order to price internet firms. When writing their article, internet firms received a lot of attention for their apparent high valuations. The aim of the model was thus to capture the high growth potential these firms experience that traditional models fails to count for. Schwartz and Moon (2000) show that high valuations are justifiable in case firms experience high enough growth rates and volatilities in key variables. An important assumption in the model is that the exceptional high growth rates of internet firms converge towards a long-run industry level, partly due to competition. A clear advantage with the model, in comparison to the DCF model, is its independence of subjective forecasts. Instead, the Schwartz-Moon model rely on current characteristics and long-term levels and simulates the paths in between by using the Monte Carlo approach, described in Section 3.3.5 Monte Carlo Analysis. Additionally, the model applies the concept of the certainty equivalent and thereby adjust the cash flows for risk before discounting with the risk-free rate.

The model developed in 2000 has two sources of uncertainty: changes in revenues, $R(t)$, and the expected growth rate in revenues, $\mu(t)$. In the revisited model from 2001, Schwartz and Moon introduce an additional stochastic variable: variable cost fraction, $\gamma(t)$. The variable cost variable aims for capturing that most internet firms are not making a profit in the beginning, hence the costs should be able to decrease over time in order for firms to become profitable. All stochastic variables follow a mean reverting process. Additionally, the model has three deterministic, path dependent variables: available cash, $X(t)$, loss-carry-forward, $L(t)$, and property, plant and equipment, $PPE(t)$.

Schwartz and Moon (2000; 2001) initially derive the model in continuous time and later approximate it in discrete time. The dynamics of a firm's revenues follows the stochastic differential equation:

$$\frac{dR(t)}{R(t)} = \mu(t)dt + \sigma(t)dz_1 \tag{28}$$

where $\mu(t)$ is the expected growth rate in revenue at time t (the drift), $\sigma(t)$ is its volatility and z_1 captures unexpected changes in the growth rate and follows a Brownian motion process.

The expected growth rate converges to the common long-run industry level, by stochastically following:

$$d\mu(t) = \kappa(\bar{\mu} - \mu(t))dt + \eta(t)dz_2 \tag{29}$$

where $\eta(t)$ is the volatility of expected growth in revenues and κ is the mean-reversion coefficient of the stochastic process, affecting how fast the firm converges to the long-term growth level. Schwartz and Moon (2000; 2001) define the half-life of any deviations from the long-run growth rate as $ln(2)/\kappa$. Equation 29 further reveals that the initially high growth rates that internet firms experience stochastically decrease towards a long-run industry standard, $\bar{\mu}$. The revenue dynamics enable these firms to sustain high growth in the near future while at the same time stochastically mean reverting towards more reasonable levels in the long-run. As previously mentioned, Schwartz and Moon (2000; 2001) justify this process by assuming that in the long-run, firms cannot grow faster than the overall industry or economy.

The unanticipated changes in revenues also converge to a deterministic long-run average, $\bar{\sigma}$, while the drift converge deterministically to zero.

$$d\sigma(t) = \kappa_1(\bar{\sigma} - \sigma(t))dt \tag{30}$$

$$d\eta(t) = -\kappa_2 \eta(t) dt \tag{31}$$

To determine costs, Schwartz and Moon (2001) use Equation 32 that captures uncertainty regarding factors such as future prices, competitors and technological developments, which all affect the costs of the firm.

$$Cost(t) = \gamma(t) R(t) + F \tag{32}$$

F reflects fixed costs and $\gamma(t)$ determines variable costs, estimated as a fraction of revenue. This new feature is of particular importance for early stage companies operating in new industries or introducing new products or services since their costs are hard to predict. The dynamics also capture that unprofitable firms need to improve their cost structures in order to become profitable and sustain operations in the long-run.

Variable costs follow the stochastic differential equation:

$$d\gamma(t) = \kappa_3(\bar{\gamma} - \gamma(t))dt + \varphi(t) dz_3 \tag{33}$$

where κ_3 represents the mean-reversion coefficient for variable costs and $\varphi(t)$ is the volatility of variable costs. The half-life of the deviations is still $ln(2)/\kappa_3$, and reflects how fast the variable costs convert to the long-run level, $\bar{\gamma}$. The unanticipated changes converge to the long-run level, $\bar{\varphi}$, by following:

$$d\varphi(t) = \kappa_4(\bar{\varphi} - \varphi(t))dt \tag{34}$$

The Brownian motions, z_1, z_2 and z_3 may all correlate:

$$dz_1 dz_2 = \rho_{12} dt \tag{35}$$

$$dz_1 dz_3 = \rho_{13} dt \tag{36}$$

$$dz_2 dz_3 = \rho_{23} dt \tag{37}$$

This reveals that the drift and the unanticipated changes in the growth rate of revenues may correlate and that both revenue and growth rates in revenue may correlate with unanticipated changes in variables costs in the model.

The following equation gives the net income after tax:

$$Y(t) = (R(t) - Cost(t) - Dep(t))(1 - \tau_C) \tag{38}$$

where τ_C is the tax rate of the firm and $Dep(t)$ is the depreciation rate. If net income is negative or if the firm has accumulated loss-carry-forward, the tax is not applicable. Hence, the loss-carry-forward is central for net income and follows the dynamics below:

$$dL(t) = -Y(t)dt \quad \textit{if L(t) > 0} \tag{39}$$

$$dL(t) = Max[-Y(t)dt, 0] \quad \textit{if L(t) = 0} \tag{40}$$

Equation 39 and Equation 40 demonstrate that the loss-carry forward increases if the firm makes a loss and is otherwise unaffected. The loss-carry forward is an important feature particularly for early stage firms since they commonly make a loss in their early years. By including these dynamics, the model captures how firms can decrease their taxes by using accumulated loss-carry-forward.

Moreover, the accumulated property, plant and equipment is dependent on capital expenditures as well as depreciation and follows the dynamics:

$$dP(t) = [Capex(t) - Dep(t)]dt \tag{41}$$

The level of the item property, plant and equipment determines depreciation and revenue govern capital expenditures:

$$Capex(t) = CX(t) \quad for\ t \leq \bar{t} \tag{42}$$

$$Capex(t) = CR * R(t) \quad for\ t \geq \bar{t} \tag{43}$$

$$Dep(t) = DR * P(t) \tag{44}$$

where $CX(t)$ is planned capital expenditures and CR is capital expenditures as fraction of revenue, $R(t)$. Planned capital expenditures is known up to a specific time period, $\bar{t}$, and afterwards constitutes a fraction of revenue. DR represents depreciation as a fraction of property, plant and equipment.

Taking all derived dynamics together, the amount of cash available develops according to:

$$dX(t) = [rX(t) + Y(t) + Dep(t) - Capex(t)]dt \tag{45}$$

The cash available earns untaxed interest, which makes the valuation independent of when cash flow allocations occur to the owners. The risk-free rate accumulates the cash flows up to time T, when the owners receive the remaining cash and the firm no longer grows at the exceptional high rate. This yields the same result as discounting the cash flows under the equivalent martingale measure at the time they occur. Further, the model assumes that the firms does not pay any dividends.

Furthermore, the model accounts for that the firm can reach a negative level of cash without going bankrupt by taking new financing opportunities into consideration. To capture this, Schwartz and Moon (2001) assume that the firm can reach a pre-decided negative level of cash, X^*, without going bankrupt. If reaching a level beneath this, the firm goes bankrupt.

The firm value in the Schwartz-Moon model consists of two components, the outstanding cash balance at time T and an on-going part, commonly referred to as terminal value. A multiple, M, of EBITDA determines the last part, as commonly done among professionals (Schwartz and Moon, 2000; 2001). The function for the present value of the firm is:

$$V(0) = E_Q[X(T) + M(R(T) - Cost(T))]e^{-rT} \tag{46}$$

where E_Q is the equivalent martingale measure, indicating that the model uses the risk-neutral measure to discount the expected value of the firm.

The model simulates available revenues and costs up to the pre-determined time period, T. The simulated values, dependent on their previous paths, determine the value of the firm for every given simulation. Hence, the procedure yields as many firm values as simulations and the average of these represents the value of the firm.

As previously mentioned, the model constitutes three stochastic variables: changes in revenue, expected rate of growth in revenues and variable costs. The revenue changes are the only uncertainty generating a risk premium, while the other two stochastic processes have identical true and risk-adjusted processes. The resulting risk-adjusted process for the change in revenues is:

$$\frac{dR(t)}{R(t)} = [\mu(t) - \lambda(t)]dt + \sigma(t)dz_1 \tag{47}$$

where $\lambda(t)$ is the time-dependent risk-adjustment. Schwartz and Moon (2001) infer the risk premium from the beta of the firm's stock by using Equation 48 and Equation 49.

$$\lambda(t) = \beta_R(r_M - r) \tag{48}$$

$$\beta_S = \frac{RS_R}{S}\frac{\lambda(t)}{r_M - r} \tag{49}$$

where β_R is the beta for revenues, β_S is the beta for the stock, S_R is the derivation of the stock price with respect to revenues and r_M is the return of the market portfolio.

Schwartz and Moon (2001) further set $\lambda(t)$ to $\bar{\lambda}\sigma(t)$ when implementing the model with the motivation that the beta of revenues and the risk premium is proportional and the volatility of growth in revenues changes with time.

In order to calculate the price of the firm at per share basis, Schwartz and Moon (2001) adjust the number of outstanding shares for employee stock options and convertible bonds. If the firm does not go bankrupt, these types of shares adjust to common shares. To get a fair estimation of the transactions with shareholders, it is necessary to estimate the cash flows available to shareholders after the conversion since this increases the total value of the firm. Schwartz and Moon (2001) calculate this by adding the payments from the exercise price of the share options and subtracting the principal and coupon payments of the bonds.

Furthermore, Schwartz and Moon (2000; 2001) describe how to solve the model using discrete time since the input data, collected from quarterly or annual reports, is in discrete time. In the application, they also assume that all mean-revering coefficients, κ, are equal. The resulting risk-adjusted processes in discrete time are:

$$R(t+\Delta t) = R(t)e^{(\mu(t)-\bar{\lambda}\sigma(t)-\frac{\sigma(t)^2}{2})\Delta t+\sigma(t)\sqrt{\Delta t}\varepsilon_1} \tag{50}$$

$$\mu(t+\Delta t) = e^{-\kappa\Delta t}\mu(t) + (1-e^{-\kappa\Delta t})\bar{\mu} + \sqrt{\frac{1-e^{-2\kappa\Delta t}}{2\kappa}}\eta(t)\varepsilon_2 \tag{51}$$

$$\gamma(t+\Delta t) = e^{-\kappa\Delta t}\gamma(t) + (1-e^{-\kappa\Delta t})\bar{\gamma} + \sqrt{\frac{1-e^{-2\kappa\Delta t}}{2\kappa}}\varphi(t)\varepsilon_3 \tag{52}$$

where

$$\sigma(t) = \sigma_0 e^{-\kappa t} + \bar{\sigma}(1-e^{-\kappa t}) \tag{53}$$

$$\eta(t) = \eta_0 e^{-\kappa t} \tag{54}$$

$$\varphi(t) = \varphi_0 e^{-\kappa t} + \bar{\varphi}(1 - e^{-\kappa t}) \tag{55}$$

The variables ε_1, ε_2 and ε_3 are standard correlated normal variates.

In the long-run, the revenue dynamics converge to:

$$\frac{dR(\infty)}{R(\infty)} = \bar{\mu}dt + \bar{\sigma}dz_1 \tag{56}$$

As previously mentioned, Schwartz and Moon (2000; 2001) estimate the stochastic processes by using Monte Carlo simulations since it can capture the path-dependency of the variables. This enables estimations of all possible paths of the firm, taking uncertainty regarding the cost structure and the growth in revenues into account.

In the case of valuing high growth tech firms, the Schwartz-Moon model seems highly appropriate mainly because it enables modelling of many different scenarios. The process enables the valuation of these firms without the need of specifically estimating unpredictable and uncertain cash flows. It also captures that the path for these firms are hard to know beforehand, which is one of the reasons for why the DCF model is inappropriate. High growth tech firms face a high up-side, but at the same time a substantial risk of going bankrupt. By incorporating both the potential upside and the high risk, the model can capture these features in a way the DCF model fails to do. Additionally, high growth tech firms commonly rely on improving their cost structure while revenues rapidly grow in order to become profitable in the future. The Schwartz-Moon model captures both features by stochastically estimating growth in revenues and the variable costs fraction.

3.4.1.1 Estimations of Parameters

The model, as most valuation models, is highly dependent on the input parameters. It is therefore necessary to estimate them carefully. This section presents the methods that Schwartz and Moon (2000; 2001) suggest, together with estimation procedures in other applications of the model.

Revenue and Growth in Revenues Dynamics

The income statement of the firm in question identifies initial revenue, R_0, which is the starting point of the simulation of the stochastic revenue process. With the purpose of estimating the initial volatility of revenues, σ_0, Schwartz and Moon (2000; 2001) suggest calculating the change in revenues over the past recent years and use the standard deviation of these changes as a proxy. Klobucnik and Sievers (2013) use the same approach. This method is problematic if valuing start-ups or private firms since these firms commonly have limited amount of historical data publicly available.

To obtain the long-term volatility of revenues, $\bar{\sigma}$, Schwartz and Moon (2001), Klobucnik and Sievers (2013) and Doffou (2015) assume that the initial volatility is halved in the long-run. Schwartz and Moon (2000) instead suggest estimating the variable from a mature company operating in the same industry. This is problematic if the industry is fairly new and there are no mature companies established yet.

Growth rate in revenues is the second stochastic process that the Schwartz-Moon model simulates. Schwartz and Moon (2000; 2001) estimate the initial expected growth rate, μ_0, by using both historical growth rates and analysts' forecasts for the following year. Klobucnik and Sievers (2013) address that forecasts are hard to find when valuing young, private tech firms and that the method is not applicable when there is a limitation of available information. Instead, they rely solely on past income statements for the calculation of the initial growth rate, even though it might not represent the best proxy for the future. Additionally, Trueman et al. (2001) show that analysts' forecasts highly depend on already realised revenues, hence the importance of analysts' reports is low for estimating future growth.

The long-term growth in revenues, $\bar{\mu}$, is a critical parameter. Schwartz and Moon (2000) estimate the parameter by looking at a stable company in the same industry. As mentioned before, this is difficult in younger industries containing few mature companies. Klobucnik and Sievers (2013) instead suggest the usage of the long-term average annual inflation rate. Another common long-run growth level in valuation models is nominal GDP growth (Koller et al., 2005).

For the initial volatility of expected growth rate in revenues, η_0, Schwartz and Moon (2000; 2001) and Doffou (2015) suggest estimating the variable from the stock price of the firm. Clearly, this method is not applicable when valuing a private firm. Therefore, Klobucnik and Sievers (2013) propose using an AR(1) regression on the growth rates in revenues, for which the standard deviation of the residuals equals the initial volatility. As before, this is problematic if there is only limited data available.

The long-term volatility in growth rate of revenues, $\bar{\eta}$, converges to 0 according to Schwartz and Moon (2000; 2001). When the firm reaches the constant long-run growth level at time T, there is no volatility as no uncertainty exposes future growth.

Variable Costs Dynamics

The variable costs fraction is the last stochastic variable in the model. Schwartz and Moon (2001) suggest usage of a regression of cash costs on revenues in order to determine both initial variable costs fraction, γ_0, and fixed costs, F. The regression intercept reflects the fixed costs while the regression beta represents the variable costs, expressed as a fraction of revenue. Klobucnik and Sievers (2013) oppose this method since it may lead to unrealistic estimates. For young tech firms making losses, the regression commonly yields a negative intercept, suggesting negative fixed costs, and an extreme slope. In these situations, Klobucnik and Sievers (2013) suggest calculating the average of the sum of variable costs and fixed costs as a fraction of revenue over the preceding years. This simplification relies on the assumption of that fixed costs grow proportional to sales, which they acknowledge as a weakness, but still an improved estimation compared to assuming independence from growth as Schwartz and Moon (2001) do.

For the long-term variable costs ratio, $\bar{\gamma}$, Klobucnik and Sievers (2013) calculate the median of variable costs fraction over a long time-period for mature firms within the same industry. As previously discussed, this approach is problematic when an industry absences mature firms. Schwartz and Moon (2001) assume an unchanged variable costs fraction in the long-run. This is not a realistic assumption for early stage firms that often are unprofitable in the beginning and dependent on increasing its efficiency going forward.

For the initial volatility of variable costs, φ_0, Schwartz and Moon (2001) use the standard deviation from the regression of costs on revenues. This approach is suitable only if the regression yields reasonable results. Klobucnik and Sievers (2013) thus use the standard deviations of the residuals from the AR(1) regression on the cost ratios used to obtain the initial variable costs.

For the long-term volatility of variable costs, $\bar{\varphi}$, both Schwartz and Moon (2001) and Doffou (2015) assume that the initial volatility decreases to half in the long run. Klobucnik and Sievers (2013) use industry medians.

Speed of adjustment, half-life and correlations

The speed of adjustment parameter, κ, reveals how fast the mean-revering processes revert to their long-run levels. Schwartz and Moon (2001) assume that all three mean-revering processes have the same speed of adjustment coefficient. To obtain the variable, they use the half-life of the deviations. Doffou (2015) argue that the half-life depends on the competitiveness of the industry and that higher competitiveness results in a lower half-life and thereby a higher mean-reversion coefficient.

Schwartz and Moon (2001) and Klobucnik and Sievers (2013) further assume that the three stochastic processes do not correlate.

Risk parameters

In order to use the risk-free rate for discounting the cash flow, it is necessary to risk adjust the cash flows in the numerator. Schwartz and Moon (2001) assume in line with CAPM that only the covariance between revenues and the return of the market deserves risk compensation. They assume that the other stochastic processes are orthogonal to the market and thereby not deserving risk premiums, thus the true and risk-adjusted processes are the same. Schwartz and Moon (2001) and Doffou (2015) imply the market price of risk from the beta of the stock. On the contrary, Klobucnik and Sievers (2013) associate a risk premium to all three stochastic processes to correct for uncertainty, which may better capture the total risk for private firms. They obtain the risk premiums by using the covariance between market return and each stochastic process to estimate the premiums. Klobucnik and Sievers (2013) use the return of the Nasdaq Composite Index over a specified period as the return on the market since they value American firms.

Simulations
To fully apply the model, Schwartz and Moon (2000) use 100,000 simulations of the stochastic variables for a period of 25 years with quarterly steps. They point out that the decision of the time increment depends on the availability of data. At the end of the time horizon, they use an EBITDA multiple of 10 to capture the value of the firm going forward from this point. In the revisited model from 2001, Schwartz and Moon (2001) instead use yearly steps and 10,000 simulations for a period of 10 years. Again, to capture the terminal value, they use the same EBITDA multiple. Klobucnik and Sievers (2013) also use an EBITDA multiple of 10 at the horizon of 25 years.

3.4.1.2 Shortcomings of the Schwartz-Moon Model

One shortcoming of the model is that it requires estimations of numerous input parameters, all demanding specific approaches. This disadvantage is especially distinct for firms with limited public information or firms that operate in new industries. Thus, the model crucially depends on many different variables that commonly are hard to correctly estimate. At the same time, the Schwartz-Moon model provides a great advantage by only requiring estimations of starting points and long-run levels. The model simulates intermediate estimations, compared to the DCF model that requires subjective estimations of all intermediate cash flows.

Furthermore, Schwartz and Moon (2000; 2001) identify several critical parameters for the valuation result. As these variables have great impact on the value, estimating them incorrectly can cause substantial valuation errors. It is therefore crucial to critically evaluate the result by a thorough sensitivity analysis.

Yet another shortcoming is the model's dependency on volatility changes. Increasing volatility of growth rate in revenues increases the value of the firm. Similarly, a higher volatility of variable costs, holding everything else constant, results in higher firm value, even though the uncertainty regarding profitability increases substantially and can potentially result in bankruptcy. This is problematic since higher unpredictability in costs should not increase the value of the firm to the same extent as growth in revenue. In the case of growth rate in revenues, increased volatility does not result in as devastating downsides and is therefore the high potential in this variable deserve compensation in form of a higher firm value.

Doffou (2015) applies the Schwartz-Moon model to five major internet firms as of June 5, 2014. His result shows that the model yields prices approximately 57% to 63% below the market price, indicating that the model is not able to capture all the value in these firms. It is however important to remember that investors' beliefs can affect market prices, and under-pricing may not always indicate a faulty model.

Additionally, when Schwartz and Moon (2000) apply their model to Amazon, their estimated revenues for the future are far below the actual outcome for the firm. Their obtained share price of USD 12.42 is substantially lower than the market price of USD 76.13. In Schwartz and Moon's (2001) application of the model to eBay in 2001, the market price is 75% higher than the implied model price. The reason for this huge difference is either that the market is overvaluing these types of stocks, or that the model is not successful in capturing the value of these firms.

3.4.2 Real Options as Add-on Components

An alternative way of valuing high growth tech firms is by using real options as add-on components to the DCF model to capture the additional value that managerial flexibility creates, and traditional models fail to capture. In academia, it is common to describe these methods in terms of project valuation. However, these approaches are easily applicable to company valuation as well, as a company works as a collection of different projects.

The traditional real option valuation approaches generally treat real options as substitutes for the DCF model. On the other hand, Trigeorgis and Ioulianou (2013) develop a new approach of how to incorporate real options into the rigid DCF model by modelling the growth opportunities of firms as real options. They suggest changing the constant growth rate in the terminal value to an option based "present value of growth opportunities" (PVGO), since it better captures a company's growth prospects. Most of the value in the DCF model originates from the terminal part, which means that the analyst should have high confidence regarding the estimated long-term growth rate. Using PVGO instead make this variable less sensitive to subjective estimates. Similarly, Smit and Moratis (2010) suggest that a company's market value consist of the NPV of its future cash flows and the present value of its growth options, as Equation 57 reveals. The first variable, PV relates to current assets and the latter to future strategic value.

$$MV = PV + PVGO \tag{57}$$

To illustrate how to practically apply the model, Trigeorgis and Ioulianou (2013) use a real-world case: the high growth tech company EchoStar Communication Corporation. They use the binomial lattice approach to value the PVGO of three concrete expansion options. They further compare the total company value to the value extracted from the DCF model. Their investigation suggests that the DCF undervalues the company due to its ignorance of flexibility. Another reason for the undervaluation is that high growth tech firms experience higher uncertainty and therefore use a higher discount rate, resulting in a lower valuation. The method by Trigeorgis and Ioulianou (2013) is similar to the Schwartz-Moon model in the sense that both models aim to capture potential growth that the DCF model fails to value, even if applying different methods.

Furthermore, MacMillan and Van Putten (2004) claim that the DCF model and real options should work complementary to each other rather than mutually exclusive. They argue that the total project value is the sum of the NPV and the value of managerial flexibility. They thereby believe that managers should use real options as add-on components to the DCF model in order to value projects properly. This method captures the NPV of the project and the impact of positive potential uncertainty by the DCF and real options respectively. The contribution of each respective part depends on the project's uncertainty. In case the uncertainty is high and the NPV of the project's cash flows is negative, almost the total project value constitutes real option value. MacMillan and Van

Putten (2004) argue that if all the value originates from the option part and the DCF value is negative, managers should generally avoid the project since it is highly uncertain. However, it is important to remember that these projects still can have high NPVs and therefore can add value to the firm. It is thus reasonable to question making investments decisions solely based on this rule of thumb.

Baduns (2013) similarly recognises that real options should complement the DCF model. He uses risk-neutral valuation and presents two approaches for "risk stripping" a security; either adjust the cash flows, as done in the Schwartz-Moon model, or use risk-adjusted probabilities. For the latter, Baduns (2013) recalculates physical probabilities as risk-neutral probabilities, which enables discounting with the risk-free rate. We present this method in detail in Section 3.3.3 Contingent Claims Analysis. Baduns (2013) further uses binomial lattices to solve for the value of the real option.

Baduns (2013) suggests estimating the starting value of the underlying asset using traditional NPV techniques, which is the basis for modelling the binomial lattice of the underlying asset. The binomial lattice of the underlying asset is in turn the basis for the option value, which uses backwards induction to identify the value. Baduns (2013) refers to the resulting total project value as eNPV and the difference between the static NPV and the eNPV as the real option value. The method that Baduns (2013) provides is consistent with Copeland and Antikarov's (2003) MAD assumption. However, this assumption crucially depends on NPV techniques to correctly capture the value of the underlying asset, which is possible to question. One possible way to improve this is by using the certainty equivalent approach, which eliminates subjective estimations of probabilities and project specific discount rates.

Furthermore, the method MacMillan and Van Putten (2004) and Baduns (2013) present requires identification of specific real options within the firm. If valuing private firms with limited publicly available information, real options may be difficult to identify and the model may thus be more suitable for project valuations carried out by operating managers rather than third-party analysts. One can also question using the DCF model as a base, as it is hard to estimate FCFs with precision for young and fast-growing firms operating in uncertain industries.

3.4.2.1 The Certainty Equivalent Approach

One way to develop NPV techniques further is by using the certainty equivalent approach to calculating the base value. The PV obtained from the two methods are the same; the methods only differ in how to adjust the cash flows for risk. Investors are therefore indifferent between predicted risky cash flows and their certainty equivalent, which is the certain cash flow received if having no risk. The method enables discounting using the risk-free rate instead of a risk-adjusted project specific discount rate (Copeland and Antikarov, 2003).

Copeland and Antikarov (2003) argue that the certainty equivalent approach is suitable for real option valuation using the binomial lattice approach. They recognise that the risk premium in the traditional CAPM model is $\lambda = [E(r_m) - r]/\sigma_m^2$, where σ_m^2 is the variance of the market return, and defines the present value using the certainty equivalent as:

$$PV = \frac{E(FCF) - \lambda COV(FCF, r_m)}{1 + r} \tag{58}$$

where r_m is the return on the market portfolio and r is the risk-free rate. Hence, the method adjusts the expected FCFs for risk in form of the covariance between the FCFs and the rate of return on the market. Using this approach, there is no longer a need for estimating individual discount rates depending on the riskiness of the project. The method instead risk-adjusts in the numerator and enables discounting with the risk-free rate (Copeland and Antikarov, 2003).

A main assumption of the CAPM is that the only risk investors receive compensation for is the systematic risk since it is unavoidable, even if holding a well-diversified portfolio. The method can therefore account for the fact that assets with positive payoffs in bad market states can make a portfolio more valuable. However, important to recognise is that there are methods opposing this assumption that highlight the importance of considering other factors as well when adjusting for risk. This is especially relevant for investors binding a major part of their total capital to private investments, as it restricts their ability to form well diversified portfolios (Fabiozzo, 2016).

4 Identification of Valuation Method

In this section, we recognise the advantages and shortcomings of previously presented valuation methods in order to identify a suitable valuation approach for high growth tech firms.

4.1 Valuation Method Specification

We aim to find a valuation approach applicable to high growth tech firms operating in a fast changing and highly uncertain market that captures both growth prospect and future strategic options. There is evidence that the DCF is not suitable for valuing this type of firm and we thus investigate the presented company valuation approaches within real options in order to find the most appropriate method.

Since the future of high growth tech firms is highly uncertain and hard to predict, we assess that real options as add-on components to the DCF model cannot capture the firm value correctly. Even though add-on components can value flexibility and possible future growth opportunities, for example through a PVGO terminal component, the base value is still hard to determine correctly. The DCF model restricts the forecasts to one path and in addition discounts the values at a high discount rate, generally resulting in undervaluation of high growth tech firms. Hence, we identify that this method is insufficient in valuing these firms, even though it improves the traditional DCF model.

The Schwartz-Moon model captures high uncertainty and high growth and is appropriate for early stage companies since it enables modelling of many different scenarios. It utilises Monte Carlo simulations for the stochastic variables that can handle complex processes and path dependency. A clear advantage of the model in comparison to the DCF model is thus its independence of subjective forecasts of intermediate values. We consider estimations and forecasts of starting points and long run levels reasonable to estimate, even if some variables are critical and may be hard to find. On this basis, we believe that the Schwartz-Moon model is the most appropriate valuation model to apply to our target companies and we use its framework as basis for our valuation model.

However, as identified in Section 3.4.1.2 Shortcomings the Schwartz-Moon Model, empirical tests of the model yield a low price compared to the market price. With base in real option theory, we identify that the model's exclusion of strategic options can explain the significant difference in prices between the model and the market. For example, Schwartz and Moon (2000) value Amazon lower than the market in 2001, when the firm operated mainly within the online book market. Today, the company offers a much broader product base due to its entrance into many new markets in the previous years. This indicates that even though the model captures the uncertainty regarding the development of current assets, it misses the fact that these companies can potentially expand into new markets. We estimate this aspect to be additionally problematic for high growth tech firms that operate in a continuously changing environment, and thus have many opportunities to expand into new operating areas. On this basis, we suggest using the Schwartz-Moon model together with real option add-on components. This approach captures the total value of the company in question and thereby increase the precision of the valuation.

For the potential add-on components, we consider using Monte Carlo estimations or the binomial lattice approach. Advantages of the Monte Carlo approach is its simplicity and that it can handle complex processes. It is however restrictively applicable when it comes to options that are exercisable in many periods since it requires simulations of many different paths. It is thus problematic to apply to interacting options, which are common in practical situations and possible to exercise at any time. On the contrary, the binomial lattice approach is intuitive and can capture flexibility of advanced, interacting options, which neither the traditional NPV methods nor the Monte Carlo approach are appropriate for. One challenge with this approach is that new products and services in general are unrelated to existing assets and therefore hard to replicate. The MAD assumption on the other hand provides an easily applicable solution to this matter. Another shortcoming with the lattice approach is that it requires forecasts of specific outcomes, cash flows as well as real option features. This is difficult to estimate if valuing complex situations but should be reasonably straightforward when considering the strategic value as one or several stand-alone projects available for firms. The precision of the value added depends on the information provided and potential add-on components thereby differ in ease of applicability. With this in mind, and mainly due to the fact that the binomial lattice can model different paths and take advanced, interactive options into account, we find the binomial lattice approach most appropriate for capturing strategic value in the case of high growth tech firms.

4.1.1 The Extended Schwartz-Moon Model

Based on the above discussion, we conclude that the following formula governs the valuation of high growth tech firms:

$$V(0) = E_Q[X(T) + M * (R(T) - Cost(T))]e^{-rT} + E_Q[ROV] \tag{59}$$

The first term reflects the Schwartz-Moon model and ROV represents the value originating from real options estimated by the binomial lattice approach with the MAD assumption.

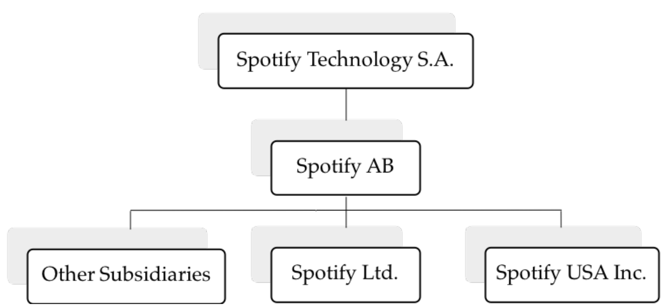
5 Case Study

The goal of this section is to use the presented theory to value a high growth tech firm: Spotify. In order to estimate input variables and identify possible strategic options for the company, we conduct an extensive analysis of the firm and its industry before applying the model. Finally, we value the company and perform a sensitivity analysis of the result.

5.1 Introduction to Spotify

Spotify AB is a Swedish private limited liability company founded in 2006 by Daniel Ek and Martin Lorentzon, operating in the online music streaming industry. The holding company, Spotify Technology S.A. is based in Luxemburg and operates entirely through its subsidiaries. The Swedish unit, Spotify AB, accounts for the main business operations and is the parent company of Spotify Ltd., Spotify USA Inc, as illustrated in Figure 6. and the remaining subsidiaries. Further throughout this paper, we refer to the group as "Spotify".

Figure 6: Spotify's Organisational Structure

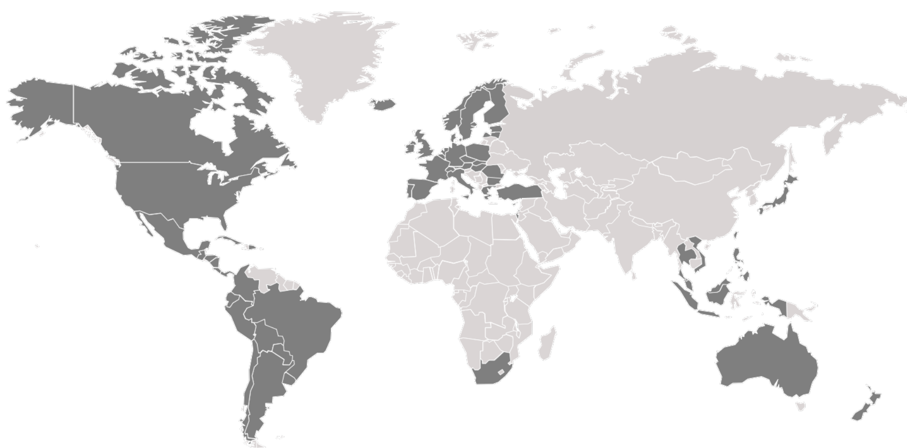


Source: Spotify Technology S.A. (2018). The figure illustrates the company structure of Spotify. Spotify Technology S.A. is the sole owner of Spotify AB, which in turn is the parent company of all remaining subsidiaries.

The company provides an online streaming platform offering more than 35 million tracks. As of December 2017, the company had over 159 monthly active unique streamers (MAUs) and 71 million premium subscribers (Spotify Technology S.A., 2018). The number of premium users rapidly grows over time; in March 2017, the number for example amounted to 50 million (Music Business Worldwide, 2018).

Spotify offers its services in 65 countries; US, Brazil, UK, Mexico and Germany are the largest markets in terms of users. In the home market Sweden, the company has a market penetration of over 50%. The well-established position in Sweden is possibly due to a first-mover advantage and analysts are not expecting the company to reach such a high penetration in other markets (Spotify, 2018a; GP Bullhound, 2017; Carlsson, 2018b). Figure 7 summarises Spotify's geographical presence.

Figure 7: Spotify's Geographical Presence



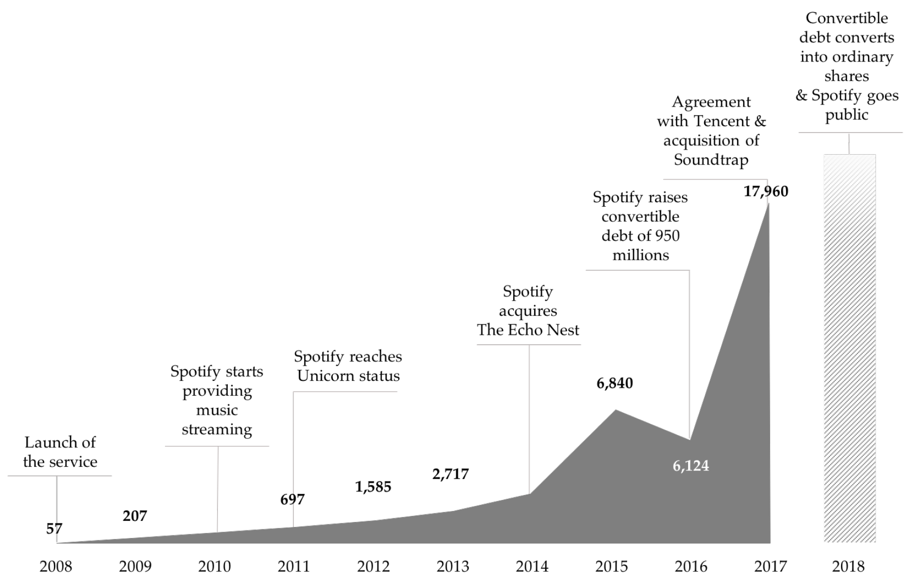
Source: Spotify (2018a). The figure illustrates where Spotify offers its services, represented by the dark area. In total, Spotify's services are available in 65 countries.

5.1.1 Milestones

Figure 8 displays an overview of Spotify's historical milestones, in addition to past valuations based on private transactions[1].

[1] Converted into EUR from SEK using the average exchange rate for each year (The Swedish Central Bank, 2018).

Figure 8: Overview of Spotify's History



Source: Carlsson (2018d). All numbers are in EUR million. The figure shows historical valuations of Spotify based on unofficial transactions together with important events. It reveals that the company experienced rapid growth in the most recent years, even though there was a smaller decrease in value between 2015 and 2016. In the latest financing round, the value of the company increased substantially.

Two years after the foundation, Spotify launched its services and made it available for public use. At that point, Spotify provided a downloading service through which you could create your own music library. As the business grew and Spotify developed leverage over music providers, the company could eventually, in 2010, offer streaming of music (Halliday, 2010).

During the first years, Spotify crucially relied on private funding. In 2011, Spotify raised additional capital at a valuation of USD 1 billion and reached the status of a Unicorn (Sorkin and Rusli, 2011). The same year, Spotify launched its service in the US and reached 1 million paying subscribers. In 2012, the number of paying subscribers increased fivefold and in September the company reached a valuation of EUR 1,585 million[2] (Halliday, 2011; Carlsson, 2018d).

[2]Converted into EUR from SEK using the average exchange rate for each year (The Swedish Central Bank, 2018).

In 2013, Spotify made its first acquisition by buying the Sweden-based music application software company Tunigo AB (Mergermarket, 2018). During the following years, the company continued to acquire related firms in the industry in order to sustain high growth. In 2014, Spotify bought The Echo Nest, a music intelligence platform. The Echo Nest became an important partner for Spotify concerning selection and recommendation of music to the customers. This gave Spotify an important advantage that could distinguish them from competitors by providing its listeners with music especially selected for them (Spotify, 2014). In 2017, Spotify acquired the Sweden based online music studio app Soundtrap, signalling that it further ahead aims to expand into music production (Reuters, 2017). However, there is still no official information that the company plans to enter this market.

In 2016, Spotify raised EUR 25 million from the Swedish pension company AMF and EUR 885 million[3] in convertible debt. The latter gave the debt holders the right to convert the debt at discount in an IPO (Spotify Technology S.A., 2016; Carlsson, 2018b; Carlsson, 2018a). During 2017, the company further entered an agreement with the Chinese company Tencent Holdings (further referred to as Tencent) in order to settle some of its outstanding debt. The company made several agreements in 2017 and early 2018 to exchange convertible notes for ordinary shares (Spotify, 2018a). As of January 2018, Spotify no longer had convertible debt outstanding (Spotify Technology S.A., 2018).

Other important milestones for Spotify in 2017 was securing important licencing deals with the major record labels and starting a collaboration with Tencent. As Tencent dominates the music streaming market in China, the collaboration brought Spotify closer to entering the huge Chinese market, in which Spotify has no current operations (Cervenka, 2018).

On April 3, 2018, Spotify went public through a direct listing on the New York Stock Exchange. By choosing this path, Spotify did not raise additional capital through ordinary roadshows and got no support from investment banks in form of underwriting. The unusual process involved a higher valuation risk since

[3]Converted into EUR from USD 1,000 million by using the average exchange rate for 2017 (ECB, 2018b).

the market determined the initial price through buy and sell orders. As projected, the price set by the market at the date of listing was subject to both volatility and low liquidity during the first days (Moyer, 2018; Levine, 2018).

The speculative reason for why Spotify chose a direct listing is that the company was in no need of additional funds. Instead, the purpose of the listing was likely to offer the current investors a way to sell their shares and cash in their profits. The direct listing involved no usual lockup period for existing investors, with exception for the latest investor Tencent, thus the investors could directly sell their shares on the market (Carlsson, 2018d; The Economist, 2018).

5.1.2 Product Offering and Business Model

Spotify's provides its services to both premium subscribers and ad-supported users. The first customer group pays a subscription fee while the latter listens for free with interruption of advertisements. Spotify's view is that its unique business model is a key factor to reach scale and as such critical for the company's success (Spotify Technology S.A., 2018). So far, premium subscribers use Spotify's services three times more than non-paying customers, which the revenue split reflects; revenue generated from premium subscribers in 2017 was approximately 90%. An aim is to increase the share of premium subscribers, but Spotify invests heavily in both sides of the business model as they complement each other. According to the company, the ad-supported service unit attracts users and channels them toward a premium subscription, today driving approximately 60% of the increase in premium subscribers. Another way to attract new customers is through offering of beneficial listening subscriptions such as "Family Plan" and "Student Plan" (Spotify Technology S.A., 2018).

Spotify offers access to over 35 million tracks and a great deal of freedom, especially for premium users that have the opportunity to listen to music both online and offline, on and across all types of devices. So far, offering the possibility to stream music however comes at a high cost; the company pays approximately 80% of revenue in royalties to artists, music labels and publishers (Spotify Technology S.A., 2018).

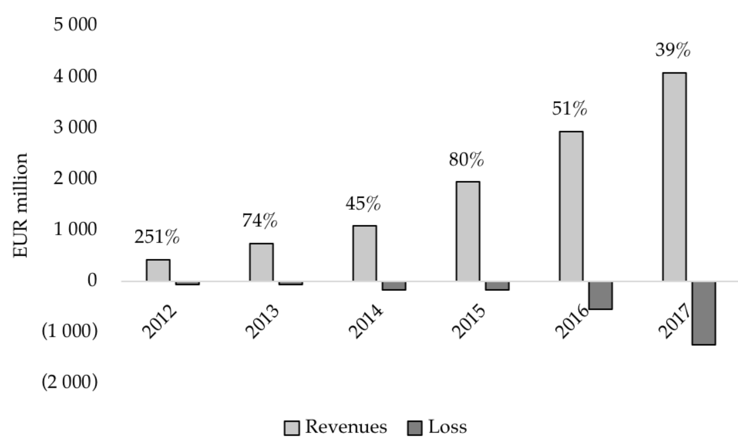
Furthermore, Spotify creates advantages from data collection and the benefits that come with it. By collecting unique user data, the company can differentiate and personalise every user's experience, which is an important competitive advantage. For example, Spotify offers personalised algorithm-generated playlists such as "Discover weekly", "Daily Mix" and "Release Radar" and playlists for specific moods and activities for each individual user. Additionally, Spotify offers podcasts, lyrics and customised videos. By continuously redefining the personalised services, Spotify avoids a high churn rate and continues its expansion through new customer attraction (Spotify Technology S.A., 2018).

According to Spotify, a crucial part of its business model is to be an important partner for artists, making their music easier to discover, and to connect artists to audiences around the world. By providing a large stage for artists, where they can interact with fans, personalise profiles, videos and releases as well as access data and analytics of their fan bases, Spotify aims to provide greater benefits for the creators (Spotify Technology S.A., 2018). The greater benefits, the higher is the demand for being present at Spotify's platform. This is of high importance since it secures Spotify's ability to provide a wide ranged, updated and unique service offering, but also because it gains ground for a more profitable business model, having potential of lowering cost of goods sold (COGS) (Spotify, 2018b).

5.1.3 Financial Overview

Spotify experiences a remarkably fast growth in revenues, as Figure 9 illustrates. The growth rate in 2017 is 39% and the compounded annual growth rate since launch of the services is 116%. Despite the rapid growth, and its 10 years in the business, Spotify still does not make a profit; the loss more than doubled in 2017 in comparison to the year before (Spotify Technology S.A., 2018).

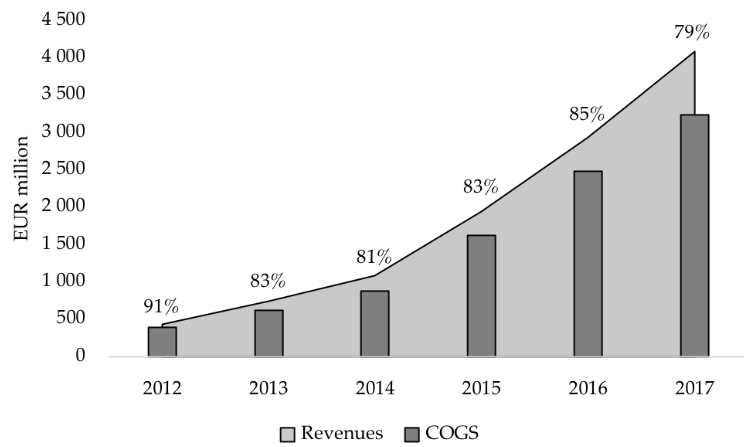
Figure 9: Revenue and Loss Development from 2012 to 2017



Source: Spotify Technology S.A. (2012-2016, 2018). The bars represent revenues and net losses. In 2017, Spotify's revenue amounts to EUR 4,090 million, reflecting a 39% revenue growth from 2016. The compounded annual growth rate between 2012 and 2017 is 57%. The loss in 2017 is EUR 1,235 million.

Further, Figure 10 displays the relation between revenue and COGS. It illustrates that COGS grows at almost the same pace as revenue in the past years. COGS constitutes predominantly of royalty and distribution costs, which currently makes up 80% of revenue (Carlsson, 2018d). It was not until 2017, when Spotify re-negotiated its agreements with record labels that COGS finally decreased marginally, by 6%, in relation to revenue (Spotify Technology S.A., 2018).

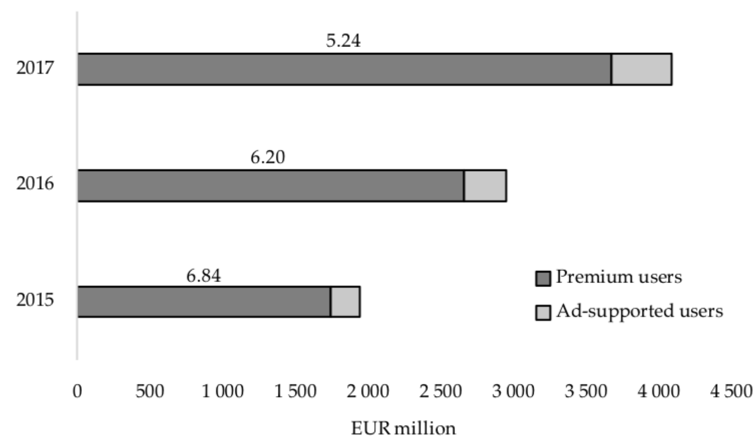
Figure 10: COGS as a Fraction of Revenue from 2012 to 2017



Source: Spotify Technology S.A. (2012-2016, 2018). The figure illustrates how revenue and COGS develop over time. The percentage numbers represent the cost fractions of revenue. As a result of re-negotiations with record labels, the fraction decreased by 6% in 2017.

Figure 11 reveals that Spotify earns the majority of its revenue from premium users that pay a subscription fee each month; the ad-supported service only accounts for approximately 10% of revenue. The average revenue per premium user decreases over time, which Spotify explains with its new family and student discount packages (Spotify Technology S.A., 2018).

Figure 11: Revenue by User Group

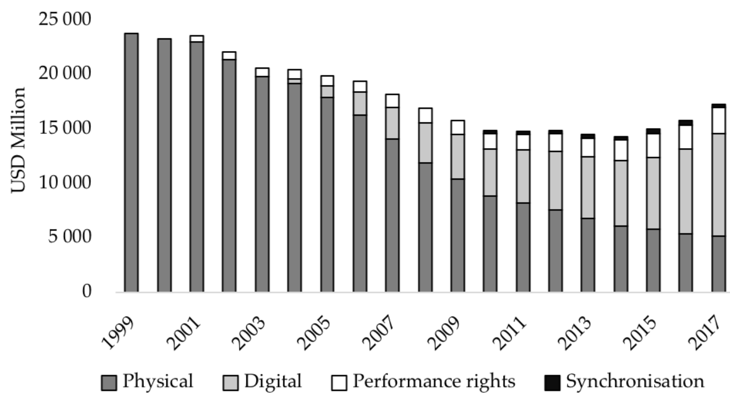


Source: Spotify Technology S.A. (2015-2016; 2018). The bars represent revenue per user group, indicating that premium users account for 90% of revenue. The centred numbers reflect average revenue per premium user, which is decreasing each year between 2015 and 2017.

One recognisable solution for Spotify to become profitable is to further decrease COGS as to increase margins; at the moment, the gross margin is not substantial enough to cover the remaining costs. However, this is not an easy task since laws regulate the main building block of COGS through royalty and distribution agreements (Spotify Technology S.A., 2018). Another solution may be to reverse the trend of decreasing revenues per user, as premium users are the main source of income.

As Figure 12 displays, the global recorded music industry is growing after many years of lost revenues; the loss between 1999 to 2014 was 40%. In 2017, total revenues amount to USD 17,300 million, where the digital music industry revenue, mainly generated from music streaming, measures to USD 9,400 million. Music streaming this accounts for approximately 54% of the total market (IFPI, 2018).

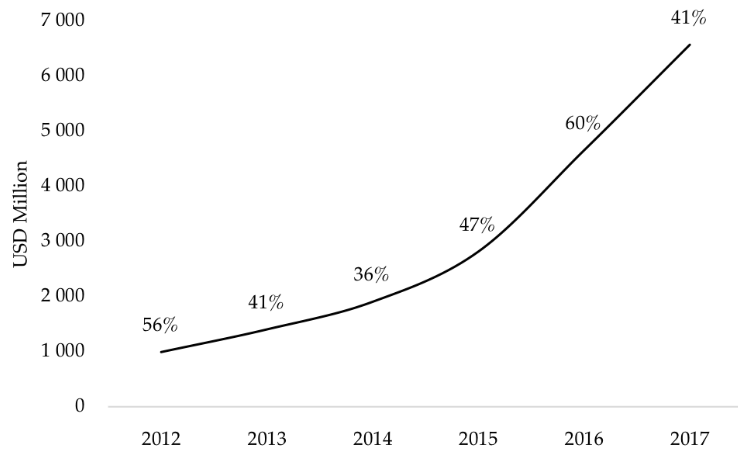
Figure 12: Development of the Global Music Industry from 1999 to 2017



Source: IFPI (2017; 2018). The figure illustrates how the distribution of revenues by different sources develop over time. The importance of digital music increases, while physical music revenues decrease. Performance rights is the use of music by broadcasters and public venues, while synchronisation represents the use of music in advertisement, movies, games etc. Digital revenues constitute mainly of music streaming.

Streaming is the most prevalent format in today's modern music industry and a key driver of growth of the global music industry. In 2017, music streaming revenue increased by 41.1% while physical revenue declined by 5.4% (IFPI, 2018). Figure 13 shows the growth of music streaming revenues during the past six years; it increased with 48% on average during these years. Listeners trend towards demanding legal and simple ways to listen to music and artists request improved ways to monetise their music, which are two important factors driving this growth (Spotify Technology S.A., 2018).

Figure 13: Growth in Music Streaming from 2012 to 2017



Source: IFPI (2017; 2018). The figure displays revenue growth of music streaming from 2012 to 2017. From 2016 to 2017, the music streaming industry experienced a growth of 41%.

The music streaming market is currently providing services to 176 million paying subscribers globally (IFPI, 2018). Overall, streaming is the most prevalent music source in most geographical areas. In 2017, streaming revenue increased with 30.3% in Europe, 49.9% in North America and 38.2% in Asia and Australasia[4]. However, physical sales still dominate in a few geographical areas (IFPI, 2018).

Although the streaming industry is surging, the providers continuously fail to generate profits (MarketLine, 2017). One important reason is the power of the record labels; a dominating part of revenues for streaming providers goes to right holders. We go further into detail about this in Section 5.1.7 Bargaining Power of Suppliers.

5.1.4 Competitive Environment

Currently, competition is a key factor driving growth in the music streaming industry. In order to appeal to customers' music preferences, players in the market try to differentiate their services. The competition has so far led to growth of the

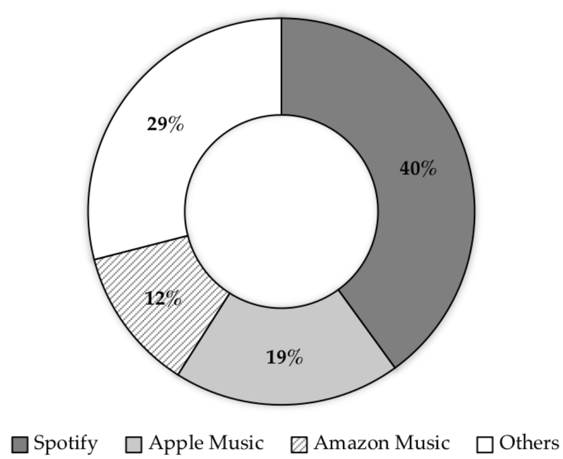
[4]Australasia consists of Australia, New Zealand and other close by islands within Oceania (Oxford Dictionaries, 2018).

market rather than crowding out of market players. A realistic explanation is the big audience that is possible to attract by just creating awareness of the industry (IFPI, 2017).

Spotify's main competitors are Apple, Amazon and Google with their respective music streaming services: Apple Music, Amazon Music and Google Play. Competitors offering a slightly different service, online music and radio, are Pandora, iHeart Radio and Tidal (Spotify Technology S.A, 2018). There also exist other players with strong positions in markets in which Spotify currently does not operate, such as the leading Chinese streaming company Tencent (IFPI, 2017).

Spotify is the global leader in the music streaming market, but Apple Music is since the launch in 2015 growing at a faster rate (IFPI, 2017). With the current growth rate, analysts forecast Apple Music to outgrow Spotify in terms of US subscribers already in the summer 2018 (Steele, 2018). Amazon Music is the second largest player in the music streaming market, as Figure 14 indicates (Mulligan, 2017).

Figure 14: Division of Market Shares within Music Streaming



Source: Mulligan (2017). The figure displays market shares for the main providers of music streaming services as of June 2017.

Furthermore, an increasingly important feature of the streaming services is the compatibility to different hardware, such as smartphones, TVs and speakers. The switching cost for changing services is usually non-existing. Therefore, if finding a superior hardware, it is likely that customers switch music provider if non-compatibility is a problem. Table 3 illustrates that Spotify is compatible with most of the existing hardware. However, important to highlight is that Apple's recently introduced smart speaker HomePod is only adaptable with Apple Music (Waral and Handrahan, 2018).

Table 3: Service Integration Overview

	Spotify	Pandora	Apple Music	Amazon Music	Google Play
Google Assistant	$\surd$	$\surd$			
Amazon Alexa	$\surd$	$\surd$		$\surd\surd$	
HomePod			$\surd$		
Sonos	$\surd$	$\surd\surd$	$\surd$	$\surd\surd$	$\surd\surd$
Playstation 4	$\surd\surd$				
Xbox One	$\surd\surd$	$\surd$			
Chromecast	$\surd\surd$	$\surd$		$\surd\surd$	$\surd\surd$
Roku		$\surd$		$\surd\surd$	$\surd$
Fire TV	$\surd\surd$	$\surd\surd$		$\surd\surd$	$\surd$
Apple TV		$\surd$	$\surd$		$\surd$
Android Auto	$\surd\surd$	$\surd$		$\surd\surd$	$\surd\surd$
Apple Carplay	$\surd\surd$	$\surd$	$\surd$	$\surd\surd$	$\surd\surd$

Source: Waral and Handrahan (2018). The table presents the compatibility of hardware for Spotify and its competitors. $\surd$ *= partial integration,* $\surd\surd$ *= full integration.*

In conclusion, the competition in the industry is intense. The switching costs for customers are almost non-existing since prices are similar between providers and users generally pay monthly subscription fees. In addition, there are many different services available (MarketLine, 2017). The closest competitors, Apple, Amazon and Google, provide, in comparison to Spotify, a large range of services and products besides their music streaming services. Hence, these firms create lock-in features by offering their streaming services in combination with their other services or products (Masters, 2018). Amazon is for example providing special discounts for premium subscribers having Amazon Prime[5] or owning an Amazon smart speaker (Amazon, 2018b). This increases the intensity of the competition and gives these firms a competitive advantage compared to Spotify.

5.1.4.1 Closest Peers

We establish a peer group for Spotify that is of use in later stages of the valuation. The closest competitors' services are smaller business units of large conglomerates and overall, they have different risk structures as their risk exposures emerge from operations other than music streaming. As it is challenging to obtain data for their respective streaming services isolated, Apple, Amazon and Google are not suitable peers for Spotify.

Therefore, we examine other companies in our peer analysis, whose business models are similar to the one of Spotify. Some of them only rely on paying subscribers, while other solely profits from advertising. This should capture the fact that Spotify has a mixed business model consisting of both premium and ad-supported users. More specifically, we look at Snapchat, Pandora, iHeart Radio, Netflix, Facebook and Twitter. These are all tech companies that depend heavily on their user base and technological changes.

5.1.5 Entry Barriers

Today, there are few dominating providers operating in the music streaming market, already having strong positions and large user bases, hence these firms benefit from a first-mover advantage. In order to enter the market, it is therefore necessary to provide differentiable services to attract users. Additionally, new entrants need to secure large libraries of music, which requires securing distribution rights, resulting in high entry costs. This, in combination with the fact that many existing companies struggle to become profitable and as such demand a substantial amount of capital, obstruct new companies from entering the market.

[5] Amazon Prime is a membership that includes free shipping of orders, video streaming, music streaming and e-book access (Amazon, 2018a).

However, it may still possible for large established companies to enter the market with their own services due to their strong financial power and well-known brands. They also commonly create lock-in effects by offering products and services in combination, which enables them to gain strong positions despite lacking first-mover advantages. One example is Apple that relatively recently entered the music streaming industry and today is the second largest provider. Two other examples are YouTube and Facebook's potential upcoming releases of streaming services. The immense interest from large, well-known firms further deter entry of smaller players (MarketLine, 2017; Field, 2017).

5.1.6 Substitutes

Apart from online streaming services, there are alternative ways of listening to music. One possibility is to legally or illegally download music, others are to buy it in physical stores or listen at online sites such as YouTube. As the switching cost is low or non-existing for most substitutes, customers can also easily switch between different providers if not satisfied with the current one (MarketLine, 2017). However, as streaming is the most prevalent form of music listening today, substitutes do not threaten Spotify's position substantially.

5.1.7 Bargaining Power of Suppliers

The bargaining power of the record labels and creators is strong. Approximately 70% of the revenue from premium users in the industry goes to labels and right holders (Waral and Handrahan, 2018). The creators can additionally choose which streaming services to allow their music on. One example is when Taylor Swift removed her music from Spotify with the argument that the company was not compensating artists enough, which had a substantial impact on the company (MarketLine, 2017). Spotify has succeeded in improving the relationship with music providers and Taylor Swift allows her music at Spotify again (Sisario, 2017), but the case illustrates the importance of keeping good relationships with artists.

As the streaming providers grow larger, more of the power originates with them. Players in the market also commonly discuss whether the music streaming service providers can replicate the business model of film streaming, like Netflix, which today is producing its own content. This would result in a decrease in bargaining power for supplier (MarketLine, 2017).

5.2 The Future of Spotify

Currently, the most important challenge for Spotify seems to be turning profitable. As the premium users are the main source of income, it is also a key driver for future revenue growth. However, as Spotify experiences a decrease in earnings per premium user, it is essential to also reduce costs in order to become profitable. Weak financials result in higher distress for Spotify than its competitors since its music streaming is the only source of income; no other businesses can support the company financially. Apple, Amazon and Google on the other hand have larger eco-system to depend on; their music streaming services can rather work as a way of enhancing their other products, which makes them less dependent on becoming profitable in the near future.

In a competitive industry where customers can easily switch between providers, it is important for Spotify to keep attracting customers. One way main competitors strengthen their position is by introducing own hardware, such as smart speakers, and making them convertible with their own streaming services, further developing their eco-systems. Spotify is currently heading into the smart speaker market in order to compete with Apple Music, Amazon Music and Google Play and their eco-systems (Murphy, 2018). By introducing an integrated smart speaker, Spotify would be able to strengthen its position in relation to its closest competitors.

Yet another possibility for the future of Spotify is beginning to produce own content by signing artists directly. This would require large initial investments and result in a substantial change of Spotify's business model, potentially leading to increased margins in the long run. However, the company has not made any indication that this is the headed direction. Instead, the current focus is on keeping a high growth rate and gaining market shares (Spotify, 2018b).

A distinct plan for Spotify is to decrease the significance and importance of record labels and give artists an easier way to break through by using its platform

(Shaw, 2018), in order to lower royalty and distribution costs. Spotify is currently heavily dependent on the licenses with record labels. The licensing situation is a returning problem for Spotify, but as the company grows, it gains bargaining power. It succeeded with negotiating lower royalty and distribution fees in 2017, but as re-negotiations of the licenses occur continuously, there is high uncertainty regarding future fees (Spotify Technology S.A., 2018). Recently, an American court awarded songwriters in the US a pay rise of 44%, which is in force until 2022, resulting in higher streaming costs for Spotify (Nicolaou, 2018). Thus, the fees might be even higher going forward.

5.3 Valuation of Spotify

Spotify Technology S.A. has publicly available annual reports from 2012 to 2016. The prospectus, released ahead of the direct listing, contains financial data for 2017. Since Spotify Technology S.A. owns 100% of Spotify AB, which financial statements closely corresponds to the holding company's, we also use the annual reports for Spotify AB from 2009 to 2011 for the estimation of some input variables. We only use these numbers when we require larger data samples and for variables that are closely comparable between the two entities, such as growth rate and volatilities. We do not use reports from earlier years as Spotify's financials for these years differ considerably compared to 2009 and onwards. In 2007, the sales were only EUR 200, compared to in 2009, after the release of the services, when sales increased to almost EUR 8.5 million. We thereby base the estimation of input parameters on the financial data from 2009 to 2017.

Spotify AB conducts its annual reports from 2009 to 2012 in SEK. We convert this to EUR by retrieving the average exchange rate for each respective year from the Swedish Riksbank (2018).

5.3.1 Estimation of the Schwartz-Moon Model

Revenue and growth in revenues

The initial revenue for Spotify is EUR 4,090 million. Following the method of Schwartz and Moon (2000; 2001) and Doffou (2015), we estimate the initial volatility of revenues by using the standard deviation of changes in revenues. Using the annual reports from 2009 to 2017, the resulting initial volatility is 55% annually. As Schwartz and Moon (2000; 2001), we assume that the long-term volatility of revenues converges to half, 27.5%. As time passes, the uncertainty regarding the company decreases and revenue stabilises, resulting in lower volatility. Since Schwartz and Moon (2000, 2000) as well as Klobucnik and Sievers (2013) identify the variable as non-critical, we decide to use the same approach without further investigation.

For the estimation of initial growth in revenues, which is a critical variable according to Schwartz and Moon (2000; 2001), we use historical revenues in absence of other reliable numbers. There is limited information available about Spotify and we can only obtain one analyst forecast, done by the investment bank GP Bullhound (2017). In 2017, GP Bullhound predicts Spotify's revenue to EUR 5.75 billion[6] by the stock market introduction, which is well above the actual numbers the company made public in the prospectus (Spotify Technology S.A., 2018). GP Bullhound also estimates the company to keep growing at a high pace. Based on the history of the company and the intense competition in the industry, we make the assessment that its estimates are too optimistic, and we do not base our estimations on these numbers. Additionally, GP Bullhound is an investor in Spotify (GP Bullhound, 2018) and benefits from providing optimistic forecasts. Instead, we look at the growth rates of the most recent years in combination with the overall market development to obtain a fair representation of the current rate. Between 2016 and 2017, Spotify's revenues grew with 39%, even though the growth rate decreased substantially compared to prior years. The same decreasing trend is present in average revenue per premium user, which decreased by 12% during the past two years. Between the same period, the overall market grew with 41%. However, the European market experienced a weaker growth of only 30%, which is the market in which Spotify is especially dominating. Additionally, we assess that the intensity in competition will increase going forward as new well-known companies may enter and existing competitors will pose a larger threat due to their well-integrated positions. On these factors, we base our belief that the initial growth rate, corresponding to the expected growth rate for 2018, is slightly lower

[6]Converted to EUR from USD 6.5 billion using the average exchange rate for 2017 (ECB, 2018b).

than that of 2017; we estimate it to 32%.

The long-run growth rate in revenues is a critical variable, which Schwartz and Moon (2000) estimate from the growth of revenues for mature companies in the industry. The online music streaming industry that Spotify operates in is still in an early stage, hence there is a lack of mature, stable companies to use for the approximation of long-run levels. Instead, we assume that Spotify in the long-run grows in accordance with the overall economy, in line with Koller et al. (2005). We use the estimated long-run real GDP growth for the US of 1.74% as a proxy and adjust it to a nominal rate of 3.50% (OECD, 2014). We assume an inflation rate of 2% in line with the Federal Reserve inflation target (The Federal Reserve, 2018; European Central Bank, 2018c).

For Spotify, we cannot estimate the initial volatility of expected growth in revenues from the stock price, in accordance with Schwartz and Moon (2000; 2001), as the company recently went public. Therefore, we conduct an AR(1) regression for the initial growth rates in revenues, as Klobucnik and Sievers (2013) suggest. However, the regression result is not significant, and we therefore choose to estimate the volatility as an average of the closest peers instead. As the peer companies operate in similar industries and all are high growth tech firms, we judge that they provide a suitable sample group for basing approximate volatility calculations on. The estimates yield an initial volatility of 14%. Since Klobucnik and Sievers (2013) identify the variable as critical, we investigate this variable more closely in the sensitivity analysis.

The long-term volatility of expected growth in revenues is 0, in accordance to discussions in the previous section.

Variable costs

To estimate variable costs dynamics, we perform a regression of costs on revenues as Schwartz and Moon (2001) suggest. However, this method yields unrealistic results with negative fixed costs and an extreme variable costs fraction and we therefore follow the method described by Klobucnik and Sievers (2013). They suggest assuming that fixed costs are 0 and treating total costs as variable costs. In the case of Spotify, this approach seems reasonable when investigating the development of the cost components over the recent years. The components, as fractions of revenue, have been fairly stable since 2012, indicating that all costs are growing proportionally to sales. It is important to notice that not separating variable and fixed costs is a simplified assumption. However, considering Spotify's business model, we deem the assumption justifiable. The company has an asset-light cost structure since its services are online, thus it does not require substantial tangible assets. This implicates that even if there exist some smaller fixed costs, neglecting these do not significantly impact the valuation result. Additionally, traditionally fixed costs are rather semi-fixed for companies that grow rapidly. Because of this, we believe the simplified assumption is preferable to assuming one fixed cost as Schwartz and Moon (2001). This process yields the initial variable cost fraction of 1.13.

The estimation for the long-term variable cost fraction is more problematic. As the online music streaming industry is fairly new, it is difficult to obtain this variable by investigating mature companies in the industry. The few competitors available differ somewhat in business model and are in addition still not profitable. We further assess that the identified peer group is non-applicable since the cost structure across the firms are likely to differ substantially. Therefore, we base the long-term variable costs on an analysis of the historical costs ratios for Spotify. The analysis reveals that COGS is the main component of total costs and that it follows the growth of revenues closely, as seen in Figure 10. The other costs are small and, in some cases, decreasing fractions of revenue, as Appendix A.1 reveals. As previously mentioned, Spotify managed to reduce the royalty and distribution fees in 2017. Even though Spotify seems to gain negotiating advantage against record labels, we estimate that it is difficult to lower the COGS fraction because it is set to increase by law in one of Spotify's largest markets, the US (Spotify Technology S.A., 2018). We do not believe that Spotify easily can change the requisites, as we recognise laws as relatively inflexible. All these factors accounted for, we believe that the variable cost fraction is relatively constant in the long run. However, we identify that Spotify is capable to realise advantages of economics of scale going forward. By lowering the semi-flexible costs fraction approximately by half, we believe Spotify is able to reach a long run cost fraction of 93%. This reflects a 20% reduction of total costs. Both the initial and the long-term variable cost fractions are according to Schwartz and Moon (2001) crucial variables, which implicates that a slightly more positive or negative view can affect the company value considerably.

We estimate the initial volatility in variable costs by using the approach presented by Klobucnik and Sievers (2013), conducting an AR(1) regression on the costs ratios. The regression result is insignificant, hence we again investigate peers for the estimation. Generally, it is hard to compare cost structure between companies as they may differ substantially, thus we cannot use the whole pre-determined peer group. Instead, we limit the comparable companies to music streaming companies since all music distributers experience the same exposure to royalty fees, accounting for a large part of total costs. By limiting the sample to Pandora and Tidal, we estimate an initial volatility of 17%. It would be preferable to use more firms in the peer analysis and it is therefore important to point out that just using two peers might not yield representative results for Spotify. However, as the industry is young and only limited data is available, we judge this method to yield the best estimation.

As previously discussed, the long-run variable costs fraction, the industry median for long-term volatility is difficult to obtain in the industry Spotify operates in. We set this variable to half of the initial variance, 8.5%, in accordance with Schwartz and Moon (2001) and Doffou (2015) since the variance decreases with time as the company matures and becomes more stable.

Speed of adjustment, half-time deviations and correlations

In line with Schwartz and Moon (2001) and Doffou (2015), we assume that the speed of adjustment is the same for all mean reverting processes and estimate the variable using the half-life of deviations. Doffou (2015) argues that the half-life depends on the competitiveness of the industry and that a company in a highly competitive industry has a lower half-life. Thus, the company cannot grow at a higher rate than the industry average for a long time since competition drives the growth down. The music streaming market is growing rapidly, which results in the possibility of companies growing fast as well, even though the competitiveness is high. In comparison, Schwartz and Moon (2000) set the half-life for eBay to 2.5 years. We believe that Spotify deserves a higher half-life despite the increasing intensity of the competition since the market still is growing rapidly. With this in mind, we set the half-life to 3 years and calculate the speed of adjustment to 0.23.

As Schwartz and Moon (2001), we assume that the correlations between the three stochastic variables are 0.

Data from annual reports

For the model, we further identify additional data from the balance sheet in the recently released prospectus. Spotify has EUR 477 million in cash and cash equivalents, EUR 1,351 million in initial loss-carry-forward and EUR 73 million in accumulated property, plant and equipment (Spotify Technology S.A., 2018).

To obtain capital expenditures and the depreciation rate, Schwartz and Moon (2000; 2001) investigate the historical values of the company. Capital expenditures as a fraction of revenue for Spotify are remarkably low with an average of 2.32% over the years from 2012 to 2017. As the intensity of competition is likely to increase going forward, we assume that capital expenditures will increase as well in order for the company to stay relevant. As we do not have analysts' forecasts for these numbers, we assume a fixed fraction of 3% of revenues going forward. The average historical depreciation rate for Spotify is 35%. However, as Spotify has recently increased this rate, we take the average of the latest years, 44%, and assume it remains constant. The historical rates are available in Appendix A.1.

The prospectus reveals the corporate tax rate of the company. As Spotify is a multinational company with a complex structure in many jurisdictions, there is some uncertainty regarding this number. However, the company has its base in Luxemburg, where the tax rate is 29.22% (Spotify Technology S.A., 2018). Since this variable is not critical and there is no indication that it will change in the near future, we use this rate.

Furthermore, for the purpose of estimating the stock price, we need number of outstanding shares, number of stock options and the amount of debt from the latest annual report. Number of ordinary shares outstanding as of December 31, 2017 is 178,112,840. There are also 14,977,569 number of employee share options outstanding with a weighted average life of 3.3 years and an average exercise price of EUR 53.59 as of the same date. Finally, debt outstanding is EUR 1,925 million.

Risk parameters

As Spotify recently went public, we cannot imply the risk premium from the stock price as Schwartz and Moon (2001) suggest. Instead, we apply the method that Klobucnik and Sievers (2013) describe; adjusting for risk by calculating the covariance between revenues and the market return. We only adjust for risk in the revenue process since we assume the other stochastic processes to be orthogonal to the market. As a proxy for the market, we use the MSCI world index[7], both because Spotify is an international company and since it has investors from a broad range of countries. By applying the method, we identify a risk premium of -3.4%. The explanation for the negative covariance is Spotify's high growth in revenues since the foundation, compared to the market returns that has been through both bad and good states. This means that even when the market drops, Spotify increased its revenues.

Simulations

In the model, we use a yearly time increment since all input data is on yearly basis. We set the horizon to 25 years, as frequently done in prior applications of the Schwartz-Moon model. This appears appropriate since Spotify is relatively young and still far from stable. We set the simulation number to 10,000, in line with both Schwartz and Moon (2000; 2001) and Klobucnik and Sievers (2013).

For the terminal value, we use an EV/EBITDA multiple of 10, similar to Schwartz and Moon (2000; 2001), Klobucnik and Sievers (2013) and Doffou (2015). Since Schwartz and Moon (2000; 2001) does not identify the multiple as critical, we decide to apply the same terminal multiple.

To identify a reasonable negative amount of cash that the company must reach in order to go bankrupt, we look at Spotify's previous investment rounds. This gives an indication of the amount of new financing the company can raise in the future if it runs out of cash. Spotify has in the past executed some substantial financing rounds. In 2016, AMF invested around EUR 25 million in Spotify and the largest investor in Spotify after the founders, Tencent, invested around EUR 960 million in December 2017 (Carlsson, 2018b, Carlsson, 2018a). Since Spotify has been active ten years and is still not making a profit, we

[7] We retrieve the yearly returns from Bloomberg.

estimate that future financing possibilities are lower than in the past. We still assess that in the first years, the possible financing is higher, especially since Tencent is not able to sell its shares until 2020 (Carlsson, 2018d) and therefore has large incentives to make additional investments if Spotify struggles with negative cash. However, the credibility of becoming profitable decreases as time passes. Therefore, we believe investors to be more restrictive with providing capital going forward. With this in mind, we set a high initial level of EUR 500 million, which gradually decreases over five years to EUR 10 million for all subsequent years.

Finally, for the discounting in the model, we use the risk-free rate as the cash flows are risk-adjusted in the numerator. Since the risk-free rate is unobservable, a common proxy is government bond yields (Hull, 2012). The European Central Bank (2018a) publishes euro area yield curves based on triple-A government bonds issued in EUR. The rate for bonds with a maturity of 25 years is 1.37%, which we use in the valuation of Spotify.

Table 4 provides a summary of the input parameters.

Table 4: Input Parameters for the Schwartz-Moon Model

Initial revenue	R_0	4,090,000,000
Initial volatility of revenues	σ_0	55%
Long-term volatility of revenues	$\bar{\sigma}$	27.5%
Initial expected growth in revenues	μ_0	32%
Long-term rate of growth in revenues	$\bar{\mu}$	3.5%
Initial volatility of expected rate of growth in revenue	η_0	14%
Long-term volatility of rate of growth in revenues	$\bar{\eta}$	0
Initial variable costs as a fraction of revenues	γ_0	113%
Fixed component of costs	F	0
Long-term variable costs fraction	$\bar{\gamma}$	93%
Initial volatility of variable costs fraction	φ_0	17%
Long-term volatility of variable costs fraction	$\bar{\varphi}$	8.5%
Mean-reversion coefficient	κ	0.23
Initial cash and cash equivalents	X_0	477,000,000
Initial loss-carry-forward	L_0	1,351,000,000
Initial property, plant and equipment	PPE_0	73,000,000
Capex as a fraction of revenue	CR	3%
Depreciation rate	DR	44%
Corporate tax rate	τ_0	29.22%
Risk premium	$\bar{\lambda}$	-3.4%
Time horizon	T	25 years
Time increment	Δt	1 year
EBITDA multiple	M	10
Allowed negative cash amount, year 1 and 2	X_1^*	500,000,000
Allowed negative cash amount, year 3	X_2^*	100,000,000
Allowed negative cash amount, following years	X_3^*	10,000,000
Risk-free rate	r	1.37%
Number of shares outstanding		178,112,840
Number of employee stock options		14,977,569
Weighted average life employee stock options		3.3 years
Weighted average exercise price employee stock options		53.59
Initial debt		1,925,000,000

The table summarises the estimated input parameters used in the Schwartz-Moon model. All numbers are in EUR.

5.3.2 Estimation of Add-on Components

5.3.2.1 Choice of Add-on Components

Schwartz and Moon (2001) takes the company's current assets and their possible growth into account but fails to capture strategic value of future potential assets outside the current asset set-up. In this section we distinguish potential strategic options for Spotify, which we add to the model in order to avoid an undervaluation of the company.

Spotify may have countless future potential business opportunities, of which many are unimaginable today. That said, determining future projects to add to the base company value is somewhat subjective and up to the view of each analyst. We believe it is important to be critical in the selection of add-on components and recommend limiting the potential list to concrete projects that preferably is already in the pipeline. According to our analysis, Spotify currently has two such opportunities; starting to produce its own music and entering the smart speaker market. We do not consider Spotify's strategy of improving licencing agreements with record labels as a potential add-on component as the benefits of the strategy only affect current assets and COGS. Thus, we already have the possibility to account for this opportunity in the Schwartz-Moon model.

The first business opportunity for Spotify is reshaping the music industry in regard to licencing with record labels and contracting with creators. By starting to produce own content, Spotify can circumvent the high royalty and distribution fees and thereby increase margins substantially. However, there is no indication from Spotify that this is a future strategy. Instead, the firm is currently focusing on keeping a high growth rate and its ongoing expansion. Thereby, we do not consider this potential strategic option as sufficiently distinct to include as an add-on component.

The second potential add-on component is Spotify's entrance into the smart speaker market. The company commenced the project during 2017, hence we find it essential to include in the valuation; excluding the project would undervalue the company. We evaluate this business opportunity through a real option approach and the ROV component will thus only constitutes this opportunity.

5.3.2.2 Entrance to Smart Speaker Market

Spotify is already in the development stage of a smart speaker and the launch of the speaker is likely happening next year (Levenson, 2018). Entering the smart speaker market is a logical move for Spotify, given that its main competitors, Amazon, Apple and Google, have similar speakers that integrate well with their respective music streaming services. Apple's recently launched smart speaker, HomePod, is not compatible with the music streaming services provided by Spotify or any other music streaming provider; it is only compatible with Apple Music (Murphy, 2018). This strengthens Apple's already well integrated position, resulting in a bigger threat towards Spotify's position, especially in the US. In March 2018, newspapers report that Spotify is trying its own voice assistant, which is an important feature for smart speakers that can make Spotify independent of Apple and Google's voice controls Siri and Alexa. This would give Spotify the opportunity to block its services on Apple and Google's products, possibly strengthening its own position (Hern, 2018).

Entering the smart speaker market may have a large potential upside for Spotify, given that the company is able to gain market shares. This strategic move is at the same time risky. Even though the smart speaker market still is fairly new, many players already have existing products on the market and thereby a first-mover advantage. Additionally, Spotify does not provide integrated eco-systems, like the closest competitors, and has less financial power as it is a much smaller company. However, the high risk may not be a good enough argument to not enter the market. Spotify is vulnerable to competitors' market power and risks losing streaming customers if they are not able to offer them the option to play music on a smart speaker (Ghosh, 2018; Olsen, 2018).

5.3.2.3 The Underlying Asset

For the estimation of the ROV component, we need to model two binomial lattices: one for the underlying asset and one for the real options. As the smart speaker market is in an early stage and many companies have recently introduced their products, it is challenging to use the replicating portfolio approach in the binomial lattice [8]. Therefore, we apply the MAD assumption that Copeland and Antikarov (2003) and Baduns (2013) impose; we use the project itself to model the underlying asset. For the derivations, we follow the steps described in section 3.3.4 Binomial Lattice.

We start with estimating the NPV of the project, defined as entering the new market. The latest market data we can obtain is from 2016. In 2016, the size of the market was EUR 360 million [9], and the compound annual growth rate forecasted to 50% from 2017 to 2024 (Global Market Insight, 2017). Amazon is by far the biggest provider, holding over 50% of the market in Q4 2017, followed by Google that holds a significant share of 36% (Strategy Analytics, 2018). Apple entered the market in the end of 2017, thus it has no established market share yet. In Q4 2017, Sonos had only 1.9% of the market and Alibaba 2.4% (Strategy Analytics, 2018). The market is growing rapidly, and we therefore judge that it is possible for new players to enter and gain market shares, even though the competition is intense. It should hence be possible for Spotify, with its well-known brand within the music industry, to gain a stronger position than the smallest providers. At the same time, Spotify is a new player within the field. Apple, Google and Amazon already have hardware production and strong brand awareness within this area, which are important features that Spotify lacks. We believe that this, in combination with entering the market later than main competitors, limits Spotify's chances of seizing as large market share as the main competitors. Taking every component into consideration, we make the assumption that Spotify can seize 5% of the market.

In order to forecast a reasonable margin of Spotify's new product, we examine the main competitors' smart speakers, displayed in Table 5. The average of the margins is 53%, which we set Spotify's margin to before tax. The margin after tax, using the same tax rate as in prior analysis, is 38%.

[8] See Section 3.3.3 Contingent Claim Analysis and Section 3.3.4 Binomial Lattice for further description.

[9] Converted into EUR from USD 400 million based on the average exchange rate for 2016 (ECB, 2018b).

Table 5: Main Competing Smart Speakers

USD	**Apple's HomePod**	**Amazon's Echo**	**Google's Home**
Price	349	100	130
Cost	216	34	57
Margin	38%	66%	56%

Sources: Gurman (2018), Reisinger (2018) and Ranj (2018). The table gives an overview of prices, associated costs and margins for the main competitors' smart speakers.

Additionally, we need to include the initial investment required to develop the product in the NPV calculations. The competitors of Spotify produce more than just smart speakers, making it hard to retrieve information regarding investments for this particular project from these firms. As a proxy, we look at the R&D intensity for the whole industry. On average, the industry of technology hardware and equipment has an R&D intensity of 14% of revenues (European Commission, 2018). In order to get an approximation of the required initial investment, we multiply this number with explicitly forecasted cash flows, after risk-adjustment and discounting.

After estimating the perpetuity part of the project by using the Gordon growth formula[10] we calculate the NPV of the project. For simplifications, we use an industry average cost of capital of 12% (NYU Stern, 2018) in the growth formula and the same risk-free rate as before, 1.37%. The explicit forecast period is set up to 2024, since we have forecasted market data up to that period. It would be optimal to set the terminal value farther ahead in time, following a fading period, since the project most likely is not in steady state after only six years. However, as we lack market estimations for continuing years and since the market is new, we proceed with this simplified assumption to avoid highly uncertain forecasted cash flows. Further, we adjust the cash flows for risk using the certainty equivalent method derived from CAPM[11] to create consistency between the NPV and the risk-neutral approach taken in the binomial lattice. We thus assume that the only compensational risk for Spotify's investors refers to the covariance with the market. We find this approach reasonable as Spotify is a publicly traded firm, implying that investors can diversify away potential firm-specific risk. However, since the project

[10]See Section 2.1 The Discounted Cash Flow Model for further details.
[11]See Section3.4.2.1 The Certainty Equivalent Approach for further details.

has no historical FCFs, it is not possible to find the covariance between the market and the project itself. Therefore, we use FCFs for firms operating in the consumer electronics industry as a proxy to find the covariance term. For the market, we use the MSCI world index. We retrieve all data in EUR in order to make sure that no exchange rate fluctuations affect the covariance.

The estimated NPV is EUR 1.64 billion. Complete calculations are available in Appendix A.2.

5.3.2.4 The Binomial Lattice

To construct the two binomial lattices, we need to estimate the required input variables. We calculate the up and down factors as well as the risk-neutral probabilities using the functions described in Section3.3.4 Binomial Lattice. We set the time to maturity to six years since it is the explicit forecast period. The step size is one year as all available data for the estimations are on a yearly basis. We use the risk-free rate of 1.37%.

Estimating volatility is problematic. For these kind of projects, we cannot imply it from a stock price and it is further hard to identify projects with a similar risk structure. As it is a critical variable in the binomial lattice, we believe it is important to be cautious in the estimation. Models such as Monte Carlo and generalized autoregressive conditional heteroskedasticity (GARCH) process, that in general are appropriate approaches for estimating volatility, may not provide reliable results in our case, as we have few data points to base it on. Instead, we use a similar approach as done in the Schwartz-Moon model's estimations of the initial volatility in revenues; we find the volatility from the standard deviation of growth rates in revenues. Since we have no historical revenue numbers for this particular project, we use revenues for peer firms as proxy. We create a broad peer group within consumer electronics since it is challenging to find revenue numbers originating only from smart speakers. In addition, the closest possible peers vary largely in regard to pricing segment and are thereby not optimal peers. By selecting a broad group, we hope to best capture the overall volatility of the speaker project. This process yields a volatility of 16%. Appendix A.3 displays the calculations of volatility and Table 6 summarises the parameters in the binomial lattice.

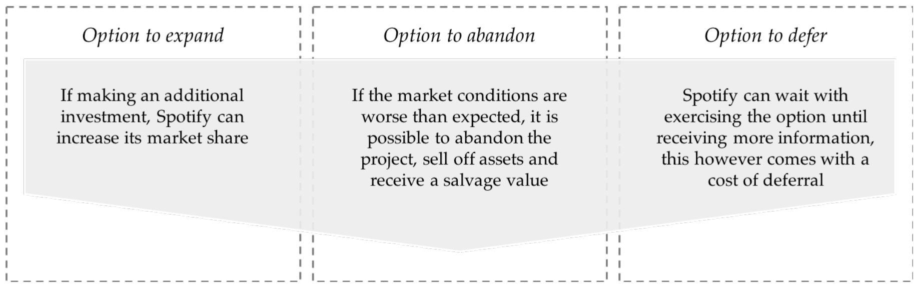
Table 6: Parameters in the Binomial Lattice

Static NPV for project	EUR 1.63 billion
Volatility, σ	16%
Time to maturity, T	6 years
Step size, Δt	1 year
The up factor, *up*	1.17
The down factor, *down*	0.852
Risk-free rate, r	1.37%
Risk neutral probability, p	0.50

Given the estimated information above, we create the binomial lattice for the underlying asset, displayed in appendix A.3.

In order to construct the final binomial lattice, we identify three different real options available for Spotify, commonly available in projects (Trigeorgis, 1993a). We assess that Spotify in each node has the possibility to expand or abandon the project as well as deferring the decision of expansion or abandonment. If the market conditions turn out favourable, Spotify can exercise the option to expand, which implicates increased spending in order to gain a larger market share. In the opposing case, Spotify can choose to leave the market by exercising the option to abandon. Managers can also wait with deciding whether or not to expand or leave the market depending on how competitors behave and how the market evolves. If waiting, holding the option involves a deferral cost since Spotify most likely lose profits to its competitors. This managerial flexibility creates value that the NPV analysis cannot capture. The real options are summarised in Figure 15.

Figure 15: Spotify's Real Options



The figure displays the real options available for Spotify: option to expand, option to abandon and option to defer.

As the smart speaker market is fairly new, providing us with limited historical data, and since we do not have access to Spotify's internal market and product intelligence, we find a precise estimation of the real options difficult. Therefore, we choose to present likely ranges for the input variables, rather than exact numbers. For the valuation of the real options, we use the mean of the range and investigate all possible outcomes further in the sensitivity analysis.

We estimate the additional investment to be between 60% and 100% of the initial investment, which would increase the project value with 6% to 14%. The expansion cost associated with the option consists of additional R&D and marketing, as well as other costs related to expansion. As the market is competitive, and the technology develops rapidly, and Spotify enters the market in a late stage compared to competitors, we believe that Spotify needs to make large additional investments in order to gain a larger market share. Due to the intense competition, we estimate the expansion possibilities to be rather restricted since Spotify enters a completely new segment where dominating firms have strong first-mover advantages. The salvage value depends on demand, liquidity and other factors and is therefore hard to precisely estimate. We assume it to be between 30% and 70% of the NPV of the project since we believe Spotify is able to sell its technology to a discount if deciding to abandon the project. We measure the deferral cost of the option as the lost potential revenues from not expanding. We assume it to be between 0.2% and 1.8% of the explicitly forecasted FCF. Since the market is at an early stage and grows fast, we believe deferring comes at quite a low cost. Postponing the expansion should only marginally affect Spotify's market share negatively, since the company has a strong position in the streaming market and a well-known brand. Table 7 summarises the mean values of the parameters that we apply in the valuation.

Table 7: Input Variables for the Real Option Lattice

Expansion cost, EC	80%
Expansion rate, ER	1.1
Salvage value, SV	50%
Deferral cost, DC	1%

The lattice method recognises the value of holding the option an additional period to await more information regarding the project, which Baduns (2013) defines as the intermediate value:

$$IV_t = [p * up + (1 - p) * down]e^{-r\Delta t} \tag{60}$$

Thereby, the final payoff function for the real option is:

$$Max(IV_t - DC * FCF_E, ER * NPV_t - EC * I_0, SV * NPV_0) \tag{61}$$

where FCF_E is the explicitly forecasted FCFs, NPV_t is the value of the underlying project from the first lattice and I_0 is the initial investment.

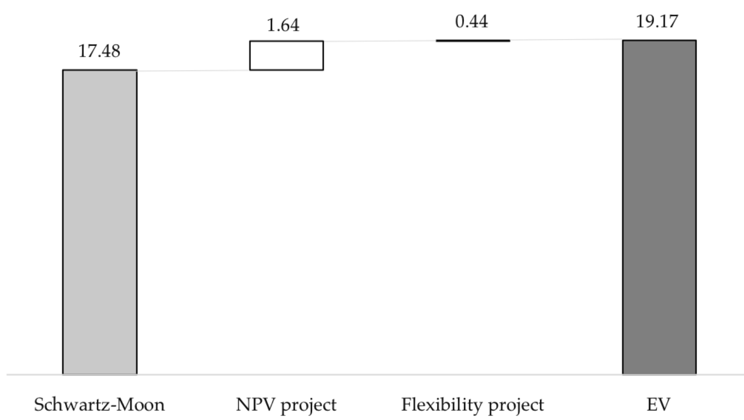
Finally, we construct the binomial lattice that calculates the ROV component, which Appendix A.3 presents.

5.3.3 Valuation Results

By implementing the Extended Schwartz-Moon model, we estimate the total enterprise value of Spotify to EUR 19.17 billion (USD 21.66 billion) as of 3 April 2018, resulting in a price per share of EUR 93.27 (USD 105.39), when taking employee stock options into account. Excluding stock options results in a share price of EUR 107.61 (USD 121.60). On the listing day, Spotify's closing firm value of EUR 21.62 billion (USD 26.50 billion), implying a stock price of EUR 121.54 (USD 149.01) (Mikolajczak and Nellis, 2018). Thereby, the model price is lower than the market price on the listing day, implying that the market values Spotify at a premium of 13%.

Figure 16 illustrates that the main part of the value of Spotify originates from the Schwartz-Moon model, capturing the potential of current assets. A smaller part originates from entering the smart speaker market from which only a slight fraction arises from the managerial flexibility of the project. Even though the ROV component in this case is small, it illustrates the importance of adding additional value to the Schwartz-Moon model if there is a possibility of creating opportunities outside the current assets and growth prospects.

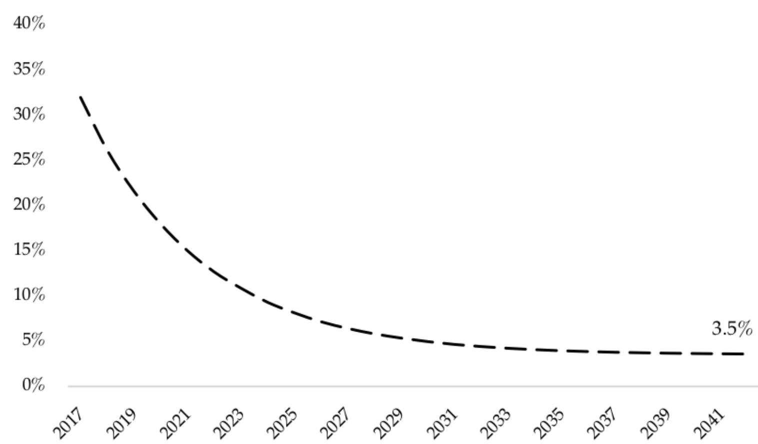
Figure 16: Total Enterprise Value



The figure demonstrates the total estimated EV for Spotify of EUR 19.17 billion and from which valuation component the value originates. All numbers are in EUR billion.

Figure 17 illustrates how Spotify's growth rate in revenues progresses over time according to the Schwartz-Moon model. Spotify is able to sustain a growth rate above 10% until 2023, but the growth rate in revenues eventually mean-reverts towards the long-run level of 3.5%.

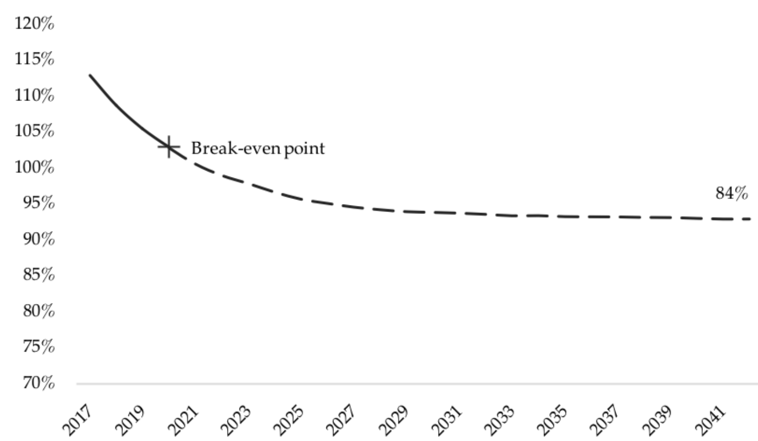
Figure 17: Growth Rate in Revenues



The figure illustrates how the Schwartz-Moon model estimates the progress of Spotify's growth rate in revenues over time. The process is mean reverting and eventually reaches the long-run level of 3.5%.

Figure 18 displays the evolution of the variable costs fraction over time estimated by the Schwartz-Moon model. We conclude Spotify's music streaming services to be profitable first in year 2020. The process displays a clear mean reverting trend and the variable costs fraction finally ceases in the long-run level to 84%.

Figure 18: Variable Costs as a Fraction of Revenue



The figure shows how the Schwartz-Moon model predicts variable costs as fraction of revenue to develop over time. Spotify breaks even in 2020 and eventually costs mean-revert to the long-run level of 84% of revenue.

Furthermore, we estimate the value originating from the ROV component, defined as entering the smart speaker market, to EUR 1.69 billion. The NPV part of the project is EUR 1.64 billion, while the flexibility component constitutes EUR 0.44 billion.

5.3.4 Sensitivity Analysis

As for all valuation models, the valuation result is dependent on estimations of input parameters. It is therefore necessary to evaluate the result and analyse consequences of changes in critical parameters.

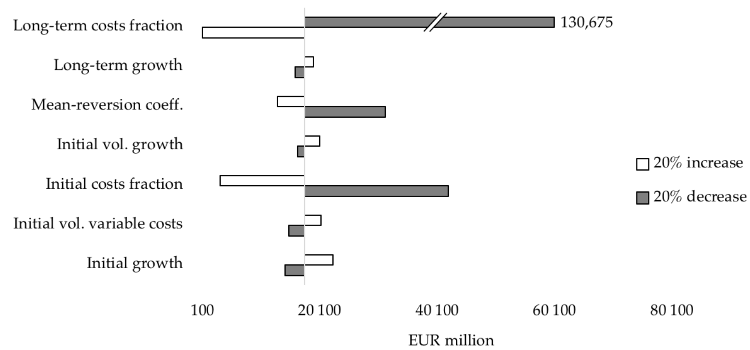
In the Schwartz-Moon model, Schwartz and Moon (2000; 2001) and Klobucnik and Sievers (2013) identify the variable costs fraction, the growth rate in revenues and their respective volatilities as well as the mean-reversion coefficient as especially sensitive. Figure 19 reveals how the valuation result changes when decreasing and increasing these parameters, ceteris paribus, by 20%.

We identify the variable costs fraction as the most critical parameter for Spotify. If the cost fraction declines, the value of the company increases substantially. The long-term variable costs fraction is of highest importance since the company struggles to become profitable while still growing rapidly. Figure 19 also reveals that if Spotify is able to decrease its variable costs fraction marginally, the value of the firm would increase considerably, which would justify a higher valuation. The figure further illustrates that the potential for decreasing the variable costs fraction provides a substantially high upside, while increasing the costs fraction only results in a limited downside. One explanation for the higher price inferred by the market is this that the market seems to be optimistic regarding Spotify's ability of improving its margins in the future. Due to lack of available data, it is challenging to estimate the initial volatility of variable costs. The sensitivity analysis however shows that the variable only marginally affects the results and that potential estimation errors thus only have limited impact on the total valuation.

Growth in revenues also affects the value of the firm, but not to the same extent as variable costs. Since the firm already grows substantially, changing this variable is not as crucial for the company's success. The initial growth affects the value more than the long-term growth, hence the latter is not as critical for the result. The volatility in growth rate is a difficult variable to estimate due to lack of data. Fortunately, we conclude that this variable does not have a large impact on the end result and thereby judge that the variable do not decrease the confidence in the model.

Moreover, the mean-reversion coefficient affects the result significantly, however not to the same magnitude as the variable costs fraction. If decreasing, the value increases since Spotify is able to sustain its high growth rate for a longer period. On the contrary, Spotify mean-reverts faster to the long-run level if the variable increases. Figure 19 further demonstrates that a lower mean-reversion coefficient provides a higher potential upside compared to the lower downside that a higher mean-reversion coefficient yields. We subjectively estimate the variable, which exposes the coefficient to some uncertainty; if the competition in the market is less substantial than what we initially estimate, Spotify might deserve a higher value.

Figure 19: Sensitivity Analysis of the Schwartz-Moon Model



The figure shows the sensitivity analysis result for the critical parameters in the Schwartz-Moon model, illustrating how total EV for Spotify changes if the variables increase and decrease by 20%. We conclude that the variable costs as fraction of revenue is the most critical parameter. All numbers are in EUR million.

Furthermore, we conduct a sensitivity analysis of the add-on component. We have limited data for many of the variables concerning the project, forcing us to use subjective assumptions, which in turn increases uncertainty. In order to minimise the effect of the uncertain estimates, we set the variables to ranges instead to specific numbers, which we present in Figure 20 and Figure 21. Total ROV does not change substantially within the selected ranges, illustrating that the parameters do not have large impacts on the total EV of Spotify.

Figure 20 reveals that going from an expansion potential of 6% to 14% increases ROV with 4% to 7%, depending on the expansion cost. A decrease in the expansion cost affect the value even less, with a maximal increase in ROV of 1.5%, logically in combination the highest expansion potential.

Figure 20: Sensitivity Analysis of Expansion and Expansion Cost

	100%	95%	90%	85%	80%
1.06	1,648	1,649	1,649	1,650	1,652
1.08	1,658	1,660	1,663	1,665	1,669
1.10	1,675	1,680	**1,685**	1,690	1,696
1.12	1,701	1,707	1,712	1,719	1,726
1.14	1,730	1,737	1,743	1,750	1,757

The figure displays how changes in expansion rate and expansion cost affect total ROV. The variables affect the value marginally. All numbers are in EUR million.

From Figure 21, we conclude that salvage value and deferral cost marginally impact the overall result. Decreasing deferral cost increases ROV with approximately 1%, independently of salvage value. Increasing salvage value impacts ROV positively with around 4.5%, no matter the deferral cost.

Figure 21: Sensitivity Analysis of Salvage Value and Deferral Cost

	1.80%	1.40%	1.00%	0.60%	0.20%
40%	1,664	1,666	1,671	1,676	1,682
50%	1,665	1,668	1,673	1,678	1,684
60%	1,676	1,680	**1,685**	1,691	1,697
70%	1,693	1,698	1,702	1,708	1,714
80%	1,737	1,742	1,747	1,753	1,759

The figure displays how changes in salvage value and deferral cost affect total ROV. The variables affect the value marginally. All numbers are in EUR million.

The input parameters, expansion rate, expansion cost, salvage value and deferral cost, only affect the flexibility value and has limited impact on total ROV. We thereby assess them as non-critical for the total valuation. On the contrary, the parameter that substantially affect the flexibility value is volatility; if volatility is twice as high, it increases the flexibility value three times and total ROV with 3%. This is in line with real option theory; higher volatility positively affects real options as they provide a hedge against the larger downside while still gaining the potentially higher upside. Thereby, we identify volatility as the most important factor for the flexibility value.

The market share Spotify is able to capture when entering the smart speaker market also affects total ROV to a large extent. We assess that Spotify can capture a relatively small and constant market share since we judge that already established strong players make the competition intense and entering the smart speaker market means operating in a completely new segment for Spotify. However, if Spotify, in spite of the obstacles, succeeds in capturing a larger market share, the ROV can increase substantially. For example, if the company seizes 7% of the market, the ROV increase with 40%. We thus emphasise that the ROV component can possibly differ in importance, depending on the actual market share that Spotify captures.

6 Analysis and Discussion

In this section, we discuss and evaluate the findings of our study as well as examine benefits and challenges with our proposed valuation approach for high growth tech firms.

6.1 Theoretical Findings

The traditional valuations methods have several characteristics not suitable for valuing high growth tech firms. As pointed out, the models cannot capture uncertainty, flexibility and growth accurately, and may provide subjective values affected by analysts and market beliefs. Real option theory arises as a way of improving the valuation accuracy for projects with these characteristics by modelling more than one path for the future of the firm, either by using lattices or simulations. Another advantage with real options is the embedded risk-neutral valuation principles, which makes the valuation independent of physical probabilities and enables discounting with the risk-free rate. As these probabilities and firm-specific discount rates are challenging to estimate, risk-neutral valuation significantly improves the reliability of the results. It is also feasible to incorporate risk-neutrality into the DCF model by using the certainty equivalent approach in the CAPM, but this process involves estimations of extra parameters. Furthermore, real options is likewise superior to multiple and comparable valuation methods. As high growth tech firms commonly are disrupters that constantly change, peers may be both hard to identify and even more challenging to replace when firms change. Additionally, comparable analyses fail to capture corporate growth options that high growth tech firms commonly have and are thereby not suitable approaches for this type of firm.

Our suggested valuation method, the Extended Schwartz-Moon model, for high growth tech firms consists of two components, the first originating from the Schwartz-Moon model and the second from the ROV component. The first component aims to capture the main value of the firm and takes high growth potential of current assets into account. The main advantage with the Schwartz-Moon model is that it solely requires estimation of initial and long-run levels of the parameters; in contrast to the DCF model, it does not require estimations of intermediate values. The model captures uncertainty by simulating stochastic variables using the Monte Carlo approach, which can take both uncertainty and path dependency into consideration. High uncertainty and high growth obstruct the prediction of cash flows for high growth tech firms, and the DCF model can thus yield high valuation errors. We thereby consider the Schwartz-Moon model as especially robust.

As we assess that the Schwartz-Moon model fails to capture potential strategic value of firms, we expand the model through add-on components using the binomial lattice approach, which the ROV part captures. The lattice can easily incorporate specific strategic real options, such as entering new markets or segments, in the valuation. Through real options, such as expansion and abandonment options, companies have the opportunity to limit potential downside while at the same time capturing the full potential upside of a business opportunity. The binomial lattice approach can intuitively value these real options and thereby improve the accuracy of the valuation.

Further, we identify that the Extended Schwartz-Moon model is flexible, which increases its potential importance for firm valuations in the future. Firstly, the input parameters in the Schwartz-Moon model are easily adjustable. When the model is in place, it is possible to amend single parameters without implicating adjustments to the whole model. On the contrary, changing single variables in the DCF model requires modifications of several steps in the valuation, such as re-estimations of intermediate cash flows. Secondly, the ability to add potential add-on component increases the flexibility of our approach further. If identifying additional strategic options, the model is easily expandable since strategic options are simply added to the base valuation. This feature is especially suitable for high growth tech firms that are changing alongside the technical development and digital globalisation and continuously expand into new markets and segments.

However, as our proposed valuation method builds on complicated theoretical frameworks, and requires understanding of elemental asset pricing and real option theory, it may be hard to incorporate practically in the industry. Yet, we believe the demand for this type of valuation model is rapidly increasing for several parties operating in the financial markets and the model may thus provide valuable contributions, despite its complexity.

6.2 Empirical Implications

The application of the Extended Schwartz-Moon model provides valuable insights into its importance. The achieved value justifies the high valuation of Spotify. Spotify's closing value at the listing day resulted in a premium of 13% compared to the implied model value. One possible explanation for why the market infers a higher value of Spotify than implied by the Extended Schwartz-Moon model is that investors put higher confidence in Spotify's future prospects.

The case study illustrates that high growth tech firms actually may deserve their apparently high valuations and that these valuations have substance in terms of an intrinsic value. Our case study provides evidence that the suggested method is able to explain the high valuations by these firms' potential corporate growth options, which the Schwartz-Moon component captures. The add-on component further illustrates the importance of adding strategic value to capture the complete firm value.

The Schwartz-Moon model

The Schwartz-Moon model seizes the uncertainty regarding current assets that high growth tech firms commonly experience, exemplified in the case study. The method shows that Spotify, if able to decrease its margins, has huge potential. If the company succeeds in undermining the record labels and thereby decrease royalty and distribution fees, cost margins can improve. As the sensitivity analysis illustrates, this increases Spotify's enterprise value substantially. This further provides an explanation for the higher share price after the listing; the market is possibly more optimistic concerning Spotify's ability of decreasing the variable costs fraction in the future. We may have a more doubtful view than the market and thereby less confidence in Spotify's ability to decrease COGS.

A shortcoming with the model is that it requires estimations of several input parameters. Many of these variables are critical and can affect the value substantially, as seen in the sensitivity analysis. The firm value is highly dependent on initial and long-run values, for example the long-run variable costs fraction highly affects the total firm value. Even though it is a significant shortcoming, this large effect is also consistent with the fact that high growth tech firms are in continuous change and thereby volatile in respect to their valuations irrespectively of valuation model. Furthermore, as previously identified as a shortcoming, high volatility of variable costs receives full risk compensation in terms of a higher company valuation even though it implicates high uncertainty regarding the company's survival. However, since our sensitivity examination shows that the variable does not affect the total value substantially, we assess this shortcoming to restrictively influence the overall performance of the model.

ROV component

In this study, the ROV component constitutes almost 10% of Spotify's total enterprise value, of which managerial flexibility is a minor fraction. In other applications, this part of the enterprise value may be of higher significance; if a company has several strategic options or opportunities of higher potential, the ROV could constitute a higher fraction of the valuation. Additionally, as identified in the sensitivity analysis, the flexibility value increases with volatility; the higher the volatility of a project, the higher effect on the flexibility component on total ROV.

Total enterprise value

Overall, the case study reveals the model's applicability to valuing private, or recently listed, firms with limited insights, as it does not require forecasts of intermediate cash flows. The method is useful for analysts as well as individual investors as it enables valuation of high growth tech firms, which are increasing in importance and are challenging to value with the established valuation models that exist today. Additionally, the precision of the model is possible to improve if having superior information. We thus assess the model to contribute with particular value in IPOs and direct listing processes as well as in merger and acquisition transactions, in which advisors have almost complete insight into firms. Internal information of a company's future projects is especially essential when estimating potential add-on components. The add-on component in the case study relies on assumptions and simplified estimations; additional information could thereby improve the precision. Also, with close communication with the target company, the identification of add-on components is more straightforward.

In conclusion, the case study illustrates the applicability and accuracy of our presented method. It provides an example of how to value high growth tech firms and the strength in being able to accurately value firms with limited information.

7 Conclusion

The aim of the concluding section is to summarise our main findings and insights from the study. We further present limitations as well as suggestions for future research.

7.1 Valuation Method for High Growth Tech Firms

The goal of this study is to develop a valuation approach for finding the intrinsic value of high growth tech firms since we identify an absence of a suitable valuation model for this emerging and increasingly important firm type. To tackle the problem, we recognise shortcomings with traditional models and investigate whether real option theory can provide a feasible solution.

We find real option theory a suitable framework, overcoming the drawbacks of traditional valuation methods when valuing high growth tech firms. After assessing its theoretical foundation and fundamental techniques, we distinguish two main company valuation approaches incorporating these frameworks: the Schwartz-Moon model and real options as add-on components.

As a result, we identify and present a valuation method for high growth tech firms with strong foundation in academia. The approach consists of two components, both originating from prior company pricing applications with basis in fundamental real option theory. We illustrate the importance of the presented method by applying it to a highly relevant practical example, yielding a satisfactory result. As the method reconciles academically advanced techniques and real-world situations, the study creates the link between academia and practice that we aimed for.

The main component in the valuation method constitutes the Schwartz-Moon model, originally developed for valuing internet firms. With our application, we demonstrate that the model is applicable to high growth tech firms as well. However, we assess that the method does not capture the complete enterprise value of these firms, thus we allow for adding a ROV component consisting of all available strategic options of the firm.

We believe that our study contributes to firm valuation methods, both academically and practically. The Extended Schwartz-Moon model improve existing academic frameworks for company valuation and provides an explicit valuation approach for valuing high growth tech firms in practice.

7.2 Limitations

The main limitation of this study is the limited generalisability of the valuation process of the case study to future applications. We estimated numerous variables thoroughly and specifically for Spotify, obscuring the possibility of systematically apply the method to other high growth tech firms. The valuation process for the ROV component is especially challenging to generalise for the purpose of simplifying future applications on high growth tech firms. The main reason for this limitation is the lack of mature comparable firms to use for estimation purposes of volatilities and long-run levels, for both components of the valuation method. However, we believe that this is a minor setback since practical applications, such as listings, mergers and acquisitions, regardless require a thorough investigation of each individual firm. A careful examination also increases the valuation accuracy.

7.3 Future Research

For further research, we suggest testing the proposed valuation method on additional high growth tech firms in order to assess its robustness. This could result in a higher degree of generalisability of parameter estimations, making the model increasingly practical to apply.

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A Appendix

A.1 Financials of Spotify Technology S.A.

Table 8: Income Statement

EUR million	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
Revenues	430	747	1,082	1,945	2,934	4,090
COGS	(390)	(616)	(876)	(1,624)	(2,483)	(3,241)
Gross profit	**41**	**130**	**206**	**322**	**451**	**849**
Product development	(38)	(73)	(121)	(143)	(207)	(396)
Sales and marketing	(54)	(111)	(173)	(247)	(418)	(567)
General and administrative	(29)	(40)	(77)	(116)	(175)	(264)
Operating profit/loss	**(80)**	**(93)**	**(165)**	**(184)**	**(349)**	**(378)**
Finance income	0	39	26	28	152	118
Finance costs	(6)	(2)	(19)	(11)	(337)	(974)
Share in earnings of associates and joint ventures	-	0	0	3	(2)	1
Net finance income/cost	**(6)**	**37**	**7**	**20**	**(186)**	**(855)**
Profit/loss before tax	**(86)**	**(56)**	**(159)**	**(165)**	**(536)**	**(1,233)**
Income tax expense	(1)	(2)	(4)	(8)	(4)	(2)
Net profit/loss	**(87)**	**(58)**	**(162)**	**(173)**	**(539)**	**(1,235)**

Source: Spotify Technology S.A. (2012-2016; 2018).

Table 9: Balance Sheet

EUR million	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
Assets						
Non-current assets						
Property, plant and equipment	15	39	51	81	85	73
Intangible assets incl. goodwill	1	7	59	78	80	162
Investment in associates and joint ventures	-	0	-	4	-	1
Long-term investment	-	-	-	-	-	910
Restricted cash	5	6	9	20	21	54
Other non-current assets	-	0	1	0	2	-
Deferred tax assets	-	-	4	4	3	9
Sum of non-current assets	**21**	**52**	**123**	**188**	**191**	**1,209**
Current assets						
Streaming content	-	-	-	15	-	-
Trade and other receivables	69	103	143	281	300	360
Income tax receivables	-	-	-	-	6	-
Short-term investments	-	-	-	-	830	1,032
Cash and cash equivalents	168	218	206	597	755	477
Other current assets	-	-	-	-	18	29
Sum of current assets	**237**	**321**	**349**	**894**	**1,909**	**1,898**
Total assets	**258**	**373**	**473**	**1,082**	**2,099**	**3,107**
Equity and liabilities						
Equity						
Share capital	0	0	0	0	0	0
Other paid in capital	201	294	406	797	830	3,432
Other reserves	8	6	8	85	122	177
Accumulated deficit	(164)	(212)	(354)	(563)	(1,194)	(2,427)
Equity attributable to owners of parent	**45**	**88**	**70**	**319**	**(242)**	**1,182**
Non-current liabilities						
Borrowings	-	-	3	5	-	-
Convertible notes	-	-	-	-	1,106	-
Accrued expenses and other liabilities	-	-	4	2	10	56
Provisions	12	5	2	7	4	6
Deferred tax liabilities	-	0	0	-	0	3
Sum of non-current liabilities	**12**	**5**	**8**	**14**	**1,120**	**65**
Current liabilities						
Trade and other payables	49	95	110	128	201	341
Income tax payable	0	1	8	3	6	9
Borrowings	-	-	2	6	-	-
Deferred revenue	30	43	68	110	151	216
Accrued expenses and other liabilities	67	92	184	411	673	881
Provisions	4	4	16	8	57	59
Derivative liabilities	51	45	7	82	134	354
Sum of current liabilities	**202**	**280**	**394**	**749**	**1,222**	**1,860**
Total liabilities	**213**	**286**	**403**	**763**	**2,342**	**1,925**
Total equity and liabilities	**258**	**373**	**473**	**1,082**	**2,099**	**3,107**

Source: Spotify Technology S.A. (2012-2016; 2018).

Table 10: Costs as Fraction of Revenue

	2012	**2013**	**2014**	**2015**	**2016**	**2017**
COGS	0.91	0.83	0.81	0.83	0.85	0.79
Research and development	0.09	0.10	0.11	0.07	0.07	0.10
Sales and marketing	0.13	0.15	0.16	0.13	0.14	0.14
General and administrative	0.07	0.05	0.07	0.06	0.06	0.06
Total costs	**1.19**	**1.12**	**1.15**	**1.09**	**1.12**	**1.09**

Table 11: Capex and Depreciation Levels

EUR million	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
Accumulated PPE	15.06	38.94	50.71	81.09	84.84	73
Depreciation PPE	3.23	8.56	15.73	25.97	32.24	46
Capex	12.44	33.59	17.30	54.09	29.60	46
Capex / revenues	2.89%	4.50%	1.60%	2.78%	1.01%	1.12%
Depreciation rate	21%	22%	31%	32%	38%	63%
Capex average 2012-2017	2.32%					
Depreciation rate average 2012-2017	35%					
Depreciation rate average 2015-2017	44%					

A.2 Calculations Smart Speaker Project

Table 12: Estimation of Volatility

	Apple	Amazon	B&O	Logitech	Foster	Panasonic Group	Edifier Technology	Harman International	Pioneer Corp	JVC Kenwood
2009		47%								
2010	55%	66%					0%			
2011	68%	56%					6%			
2012	36%	35%	155%	(2%)	18%	(10%)	16%	12%	(5%)	(9%)
2013	8%	26%	41%	(9%)	20%	(7%)	(5%)	16%	4%	(5%)
2014	6%	25%	0%	1%	17%	6%	(7%)	(2%)	10%	3%
2015	30%	24%	15%	(6%)	13%	0%	(7%)	24%	1%	(10%)
2016	(11%)	30%	58%	1%	1%	(1%)	(3%)	15%	(10%)	3%
2017			27%	10%	16%	(4%)	10%	12%	(14%)	2%
Std	**26%**	**15%**	**7%**	**14%**	**5%**	**8%**	**8%**	**9%**	**6%**	
Ave	**16%**									

Sources: S&P Capital IQ (2018) and Bloomberg (2018).

Table 13: Estimation of the Certainty Equivalent

Company	Covariance with market
B&O	(4,616,421)
Logitech	(983,642)
Foster	999,570
Panasonic Corp	5,972,080
Edifier Technology	1,614,482
Harman International	(2,315,863)
Pioneer Corp	(6,735,032)
JVC Kenwood	(1,572,285)
Average	(954,639)
Expected return market	6.00%
Variance market	0.33%
Risk-free rate	1.37%
Risk premium CAPM	1.41
Risk adjustment term	(1,348,802)

Sources: S&P Capital IQ (2018) and Bloomberg (2018). We calculate the covariance with the market by using the FCFs of each respective firm and the MSCI world index from 2007 to 2017, when data is available. The variance on the market originates from the variance of MSCI world return from 2007 to 2017.

Table 14: NPV Analysis of Smart Speaker Project

EUR million	2018	2019	2020	2021	2022	2023	2024	Terminal
Spotify market share	0%	0%	5%	5%	5%	5%	5%	
Revenues	0	61	91	137	205	308	461	4,492
Fixed investments	(159)							
Margin before tax	53%	53%	53%	53%	53%	53%	53%	53%
Margin after tax	38%	38%	38%	38%	38%	38%	38%	38%
Cash flows	(159)	23	34	51	77	115	173	1,685
CE cash flows	(160)	21	33	50	76	114	172	1,684
PV	(160)	21	32	48	72	106	158	1,365
Sum PV	277							
Terminal PV	1,365							
NPV	1,641							

A.3 Binomial Lattice Smart Speaker Project

Table 15: Binomial Lattice Underlying Asset

	1	2	3	4	5	6
1,641	1,926	2,261	2,653	3,113	3,653	4,287
	1,399	1,641	1,926	2,261	2,653	3,113
		1,192	1,399	1,641	1,926	2,261
			1,016	1,192	1,399	1,641
				866	1,016	1,192
					738	866
						629

Table 16: Binomial Lattice Real Options

	1	2	3	4	5	6
1,685	1,982	2,343	2,775	3,281	3,875	4,573
	1,437	1,676	1,977	2,343	2,775	3,281
		1,242	1,423	1,667	1,976	2,343
			1,099	1,220	1,406	1,662
				1,012	1,072	1,192
					985	985
						985