COPENHAGEN BUSINESS SCHOOL

Master's thesis

Cand.Merc.(mat.)

The synthetic non-callable mortgage loan

Det syntetiske inkonverterbare realkreditlån

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Resumé

Denne afhandling søger at undersøge de risikofaktorer som opstår når der optages et *syntetisk* inkonverterbart fastforrentet lån. Denne syntetiske konstruktion fås ved at låntager optager et variabelt forrentet lån, eks. et såkaldt F1 rentetilpasningslån, samt indgår en renteswapaftale med en bank hvor låntager betaler fast. Formålet med afhandlingen er at undersøge de risikofaktorer som opstår ved denne syntetiske konstruktion relativt til et klassisk konverterbart realkreditlån eller det (rent teoretiske) inkonverterbare lån, der ikke eksisterer for de almindelige 30 årige lån.

For at undersøge dette er det nødvendigt med et grundlæggende modelværktøj til at prisfastsætte renteswaps som opgaven derfor lægger ud med. Herefter gennemgås karakteristika for forskellige renteindeks og renteswap markedet, med fokus på det danske marked. Denne lånekonstruktion har specielt været benyttet af låntagere med et lånebehov af en vis størrelse; så som andelsboligforeninger, landmænd og kommuner. Derfor gennemgås motiverne for denne type låntager til at indgå i en sådan forretning, samt den historiske udvikling. Endvidere gives der konkrete eksempler på såkaldte "swap skandaler". Dette dækker over en række (rets)sager hvor låntager har stævnet deres bank for misvisende eller manglende rådgivning i forbindelse med indgåelse af renteswapaftaler. Disse aftaler har medført milliontab og i flere tilfælde konkurs for låntager.

Herefter gennemgås de pågældende risikofaktorer som er relevante for låntager. Disse inkluderer både traditionelle rentefølsomhedsbegreber så som varighed og konveksitet. Men mere specielt for dette syntetiske lån er basis risikofaktorer, som dækker over risikoen ved utilstrækkelig *hedging*. Opgaven forsøger at illustrere effekten af disse ved at benytte disse på en tænkt case. Heri findes det at disse kan have en signifikant påvirkning for låntager, og konkrete eksempler beregnes. Specielt risikoen forbundet med forskel i restgældsprofilen mellem renteswappen og det variabeltforrentede lån, som følge af at rentetilpasningslånet afdrages som et annuitetslån (som er praksis) viser sig at være betragtelig.

Da renteswaps er bilaterale aftaler indgået med en bank er disse helt centrale i en korrekt prisfastsættelse af disse (set fra bankens synspunkt). Centralt i dette er de eventuelle kollateralaftaler som handlen indgås under. Den type låntager der er relevante her har typisk ikke indgået en kollateralaftale. Derfor gennemgås de prisjusteringer (xVA'er) som banken beregner som følge af de ekstra risici og omkostninger forbundet med manglende kollateralaftale. Til sidste undersøges hvorledes kollateralaftalen påvirker prisfastsættelse af renteswapaftaler og andre derivater gennem en justering af bankens diskonteringskurve. Herunder laves koblingen til prisfastsættelse af derivater i andre valuta end bankens umiddelbare funding valuta.

Contents

1 Introduction

In recent years it has not been hard to find articles in danish newspapers, about how housing cooperatives, farmers and municipalities amongst other have lost millions on their "swap loans". The origin of this is found in the period shortly before the financial crisis. Interest rates had increased, and there was a widespread expectation (and fear) of even higher interest rates. Many borrowers with floating rate mortgage loans wanted to protect themselves against such a scenario. Some of the borrowers who had larger loans - particularly housing cooperatives, farmers and municipalities - chose to do this by entering into an interest rate swap with their bank. By doing this they effectively *swapped* from a floating rate into a fixed rate loan. This loan is effectively a non-callable fixed rate mortgage loan, but this is a purely hypothetical construction as banks or mortgage issuers do not issue such loans. Instead it is market standard in Denmark to issue callable loans. This in turn force the borrower obtain the *synthetic* non-callable mortgage loan using the interest rate swap. However, such a synthetic loan is exposed to some risk factors that the "true" (and non-existing) non-callable loan is not.

> The purpose of this thesis is to investigate, clarify and quantify the risk factors facing the borrower in the *synthetic* loan compared to the (non-)callable mortgage loan.

The synthetic loans have received much criticism in recent years, and multiple lawsuit have been filed against danish banks for inadequate advicement in the sale of the interest rate swaps to retail customers. As an example the Danish Supervisory Authority ("Finanstilsynet") have issued a guideline regarding the marketing and advicement of sales of interest rate swaps to retail customers in response to the criticism. Despite this the synthetic loan remains to this day the only fixed rate loan alternative to the standard callable loan. For this reason we argue that an overview and clarification of the risk factors inherent in the syntethic loan is important. We will also investigate the motives for retail borrowers to enter into this loan construction, wherein they are trading an interest rate derivative usually reserved to the likes of banks, institutional investors and larger corporations.

The synthetic loan is a combination of a floating rate mortgage loan and a payer interest rate swap. The risks in a floating rate loan are quite simple, and is only related to the movement of interest rates. Higher interest rates means higher interest payments and conversely. For this reason we will in our analysis of the synthetic loan focus on the interest rate swap, and the *combination* of the two. As such many of the chapters and sections will at first glance appear to only relate to interest rate swaps, but we will indeed keep the synthetic loan as the focal point throughout the thesis.

To perform this analysis we need to thoroughly review the interest rate swap, including pricing and risk management. We try to do this in a simple setup, and yet in a way that comes very close to the market standard using Linderstrøm (2013). We will also review the features of the swap market. We attempt to keep a global perspective, but will naturally also review the specifics of the danish market. As the (floating rate) mortgage loan is issued by a mortgage provider, the role of the bank within the syntethic loan is in regards to the interest rate swap. For this reason we find it important to also discuss the role of interest rate swaps in the banks. We do this by illustrating the mechanics of swap trading and pricing within a bank. In this regard the value adjustments made by banks for trading against a non-sophisticated investor, the so-called xVAs, are of particular interest, and we will rely on Gregory (2015) in the analysis of these.

In the writing of this thesis we have found it necessary to setup some delimitations. Financial asset pricing theory can quickly become very mathematical and complex. However, in this thesis we have chosen to refrain from the very theoretical approach, and instead attempted to keep things more intuitive. An example is the pricing of interest rate derivatives in which theories such as risk-neutral pricing and change-of-numeraire techniques are common and often necessary. Instead of reiterate the theory we have chosen to simply apply it and reference the litterature in which it is described. Another example is the rather new area of xVAs. This may also quickly become inherently complex when modelling e.g. portfolios and the specifics of CSAs such as netting agreements etc. Instead we have simplified by assuming a single trade and no CSA, which we argue is (typically) the relevant scenario in this specific case. Calculation of xVAs also require the estimation of future market variables which is most often solved by simulation. This in turn would require us to review the theory of term structure modelling and Monte Carlo simulation. Again we have chosen to simplify by using a risk-neutral approach to project future market variables.

The nature of this thesis is a focus on features often specific to the danish mortgage and swap markets. As a result approriate litterature is per definition limited. We have attempted to alleviate this issue by referencing the theory where possible. Further a considerable amount of knowledge regarding the features of the danish swap market and the mechanics of swap trading within banks has been obtained after discussions with people with first-hand knowledge of this. We have stated in when sections are based on this information.

Empirical data on this subject, such as the extent of the use of the synthetic loans, has also proven hard to obtain. This includes data on specific examples. This data and information has not been centralized, but is instead proprietary to the banks or available in lawsuits but requiring access to the records. Instead we have obtained the data available through newspaper articles, especially in the section reviewing danish lawsuits.

To accurately price and calculate risk of interest rate swaps we have a need to calibrate a set of curves, which in turn requires data on market quotes. Such market quotes have primarily been collected using Bloomberg, but some price and interest rate data has been collected using J.P. Morgan Market's DataQuery platform. It has proven (very) difficult to obtain some data for older periods, especially around or before the financial crisis. As a result we have found it necessary to make some assumptions and simplifications in this regard, and this have been clearly stated when done.

The remainder of the thesis in split into four parts. In the first we give an introduction to interest rate swaps. This includes reviewing pricing and curve calibration theory, which lay the foundation for further analysis. We also give a review of the features of the danish swap and interest rate markets. We end the section with an analysis of some of the lawsuits regarding the synthetic loan construction, including a review of the motives and consequences. In the second part we describe the risk factors the borrower is exposed to when obtaining the synthetic loan. This includes delta, basis and liquidity risk. We also gives examples of the more general use of interest rate swaps for the customer. In the third part we explain the role of interest rate swaps within the bank. This includes a review of the mechanics of swap trading and pricing, taking into account the (legal) conditions under which there is traded. The fourth section is a case study following the hypothetical case of the municipality Hedgelev Kommune. In this section we attempt to quantify the risks and effects as described in the previous parts to illustrate the influence they *may* have had. The final section concludes.

2 An introduction to interest rate swaps

This chapter will give an overall introduction to interest rate swaps. We start by giving an introduction to the most important interest rate indices of the danish market in section 2.1, as they are relevant for the remainder of this chapter. We then go on to showing how to price interest rate swaps and other interest rate derivatives in section 2.2. This will be used in the next section 2.3, where we will learn how to calibrate the curves needed to price and risk manage interest rate swaps. In section 2.4 the calculation of risk of swaps related to movements in interest rates will be reviewed. In section 2.5 we will review the participants in the (danish) swap market. Lastly, in section 2.6 we give examples of some of the borrowers who has entered into the synthetic non-callable loan and review these cases.

2.1 Conventions and indices

In this section we will give a thorough introduction to the most important interest rate indices in the danish swap market; Cibor and CITA. We will explain how they are determined and their interpretation. We will also explain why discounting interest rate swaps (and other derivatives) using the OIS curve is the correct method in the benchmark scenario of collateralized trades. We will repeatedly reference interest rate swaps and overnight indexed swaps (OIS swaps). The definition of these will follow in the next section. We will routinely use the term *Libor*. This will refer to the Xibor reference rate of the particular currency in question. For example in DKK we will mean "Libor" to refer to the danish Cibor rates (and not the DKK Libor which used to be published by the BBA) and similarly for EUR, Libor will refer to the Euribor rates.

2.1.1 Cibor

Cibor is the danish version of Libor, and the acronym is short for *Copenhagen Interbank Offered Rate*. It is an average of submitted rates by a number of danish panel banks, where each banks submission is supposed to reflect the rate at which that bank is willing to lend danish kroner to another prime bank for a given period on an uncollateralized basis[1]. Finance Denmark is responsible for the index, and their definition of prime bank is as follows: it is a Cibor panel bank, it has *"obtained the best credit rating on the long term rating by either S&P, Moodys or Fitch"*, and it has access to monetary facilities with the danish central bank, Danmarks Nationalbank (Finance Denmark (2018b)). As of May 2018 there was 6 Cibor panel banks: Danske Bank, Jyske Bank, Nordea, Nykredit, Spar Nord Bank and Sydbank. Cibor is quoted for a total of 8 maturities: 1W, 2W, 1M, 2M, 3M, 6M, 9M and 12M. For each maturity the panel banks submit quotes at 10.30 CET to Nasdaq OMX on behalf of Finance Denmark. This occurs on all good danish business days. For each maturitiy the highest and lowest quote are removed and a simple average of the remaining is calculated and published at 11.00 CET, at the website of Finance Denmark. The panel banks are *not* required to trade at the quotes they submit (Finance Denmark (2018a) & (2018c)). This market for interbank funding has seen a sharp decline since the financial crisis, also in Denmark. This is believed to be due to increased focus on interbank credit risk as well as liquidity risk. The vulnerability of banks was suddenly in focus, exemplified by the many bankruptcies of the crisis, like those of the two major US investment banks; Lehman Brothers and Bear Stearns.

[1] This is opposed to the ICE Libor, which reflects the rate at which banks (claim they) believe they can *borrow* funds in the interbank market.

The Cibor index is used as the floating rate index for most DKK interest rate swaps. Most of the liquidity in the DKK swap market is against Cibor6M, likely to follow the conventions of the EUR swap market, where liquidity is against Euribor6M. In this thesis we have chosen to focus on Cibor3M for many of the preliminary examples. This is because Cibor3M is also the index tenor used as reference in cross-currency basis swaps, which we will use later on.

2.1.2 T/N, OIS and CITA

OIS swaps in DKK are referencing the danish Tomorrow/Next (T/N) rate. This is a rate reflecting actual transactions done in the interbank market for uncollateralized T/N deposits. Like Cibor it is based on submissions by a panel of banks, but here it is based on *actual* transactions. A panel of banks submit the amount of DKK denominated tomorrow/next lending they have had with both domestic and foreign banks, as well as the average interest rate for these transactions. A volume-weighted average is calculated among all submitted rates and volumes to find the T/N fixing. Reporting is done with one days lag, that is the transactions of Monday is reported Tuesday before 10.00 CET to Nasdaq OMX on behalf of Finance Denmark. A special procedure is in place in case the total volume is less than 3 bn DKK. In that scenario a sub-panel of banks are asked to re-submit their quotes in accordance with a specific procedure (Finance Denmark (2018d)). As the T/N rate is also uncollateralized, we may intuitively think of it as a (partly) transaction-based T/N Cibor rate.

As mentioned the T/N rate is the floating rate of DKK denominated OIS swaps. OIS swaps denominated in DKK are often called CITA swaps, and so we will use the terms "OIS swaps" (in DKK) and "CITA swaps" interchangeably. CITA is short for *Copenhagen Interbank Tomorrow/Next Average*, and is an OIS swap against the T/N interest rate. Formally the CITA swap *fixing* is the average of CITA swap quotes submitted by a panel of banks for the following set of maturities: 1M, 2M, 3M, 6M, 9M and 12M. The CITA swap fixing is (again) calculated by Nasdaq OMX on behalf of Finance Denmark. Submission are made at 10.30 CET on all danish banking days and a truncated average is calculated and published at 11.00 CET (Finance Denmark (2018e)).
Below table shows the conventions of the danish swap market:

				Floating leg		Fixed leg	
Type	Index	Start	Roll	Freq.	Day count	Freq.	Day count
IRS	Cibor3M	2B	MF	Q	ACT/360	A	30/360
IRS	Cibor6M	2B	MF	S	ACT/360	A	30/360
CITA/OIS	T/N	2B	MF	A	ACT/360	A	ACT/360

Table 1: Conventions of the danish swap market.

All examples, graphs and calculations in this thesis has used the conventions of the swaps as listed in the above table.

2.1.3 Libor vs. OIS

The Libor/OIS spread is defined as the spread between a Libor (or IRS) quote and the corresponding tenor OIS quote. An example is Libor3M versus the 3M OIS quote, or it could be the 30Y IRS (against a Libor rate) versus the 30Y OIS quote. Figure 1 shows the Libor/OIS spread in EUR and USD in the years around the financial crisis.

As we can see the Libor/OIS spread was very small prior to the financial crisis. As the financial crisis evolved the focus on credit risk increased. Banks were increasingly perceived as

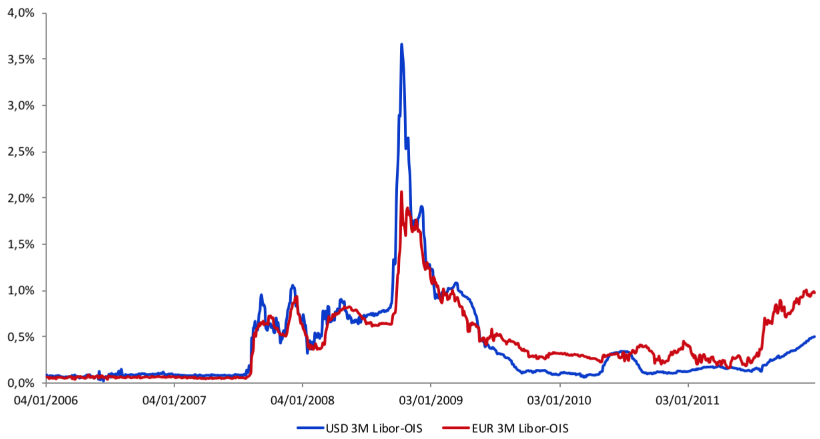

Figure 1: Spread between 3-month Libor and the 3M OIS swap rate in EUR and USD in the years 2006 to 2011. Data is from Bloomberg.

being more vulnerable than previously assumed. A natural consequence of this was that the price on uncollateralized interbank loans increased; causing Libor to spike. But the O/N (or T/N), underlying the OIS swap, is also an uncollateralized loan, so why did the Libor/OIS spread increase? Because banks started adding a tenor premium to their Libor quotes, as they realised that lending funds on an uncollateralized basis for a longer period was more risky than for a shorter period. This increased the spread between 3-month Libor and the O/N rate, and as the OIS is derived of the (market expectation of future) O/N rates, the Libor/OIS spread increased.

Interest rate swaps enable the user to change a series of payments from floating to fixed (or the other way around), or from one floating rate to another. Consider as an example a bank with a (constant) funding need. At present it is funding this daily in the interbank market using overnight loans, so that it is paying the O/N rate. The bank may wish to pay a fixed rate instead. It can achieve this by entering into a payer OIS swap. The compounded O/N rates received in the OIS swap matches the funding costs of the bank in the interbank market, and the bank is left paying the fixed rate in the swap. If instead the bank wish to pay the Libor3M, it may do so by entering into a receiver IRS. It is then left with paying Libor3M and receiving the difference in the fixed rates of the two swaps (the Libor/OIS spread). The same result would have been obtained by entering into a receiver Libor/OIS basis swap. This illustrates the important concept of basis swaps as means of changing the funding profile, which we will use later on in sections 4.2 - 4.4.

In the next section on pricing interest rate derivatives we will see the importance of assumptions on discounting. We will now explain why using the OIS curve when discounting is the correct method in the benchmark scenario of "perfect" collateralisation. We will review this further in section 4.2 but will here give an intuitive understanding of this.

After the financial crisis the Dodd-Frank Act in the US and the EMIR legislation in the EU were implemented. Amongst other things they mandated central clearing of most vanilla over-the-counter derivatives, including interest rate swaps. This was implemented to reduce systemic risk in the financial sector, by attempting to remove the credit risk in these OTC derivatives. As banks and other institutional investors are responsible for the majority of trading in these contracts and falls under this mandate, the amount of derivatives being cleared has skyrocketed in recent years. According London Clearing House, one of the largest in the market, their total IRS clearing notional increased from roughly 56 trln USD in 2013 to 873 trln USD in 2017 (LCH

(2014) & LCH (2018)). When a swap is cleared, the party which owes money in the contract will post collateral reflecting the negative value of the contract. In that way if that party defaults, the other party which is owed money, will receive the collateral posted at the clearing house. There is daily collateral margin calls (sometimes intra-daily), to ensure an up-to-date collateralization. The collateral posted at clearing houses, like LCH, earns the O/N rate.

When collateral is posted, it is thus important that the amount is enough to cover the future liabilities of the collateral-poster. A simple intuition for this is as follows: Assume a party has an expected positive future cash flow and a "perfect" collateralisation scenario similar to that at a clearing house. This mans that there will be received collateral today to reflect the present value of the cash flow. The amount of collateral received today will be such that given the (expected) interest earned by the counterparty on that amount, it exactly equals the expected future cash flow. Because the collateral earns the OIS rate[2], this must also be the correct rate to use for discounting to calculate the present value of the future cash flow, as this exactly equals the collateral received by assumption.

This approach to discounting is called *collateral discounting*. Generally it assumes that the correct discounting rate is that which is paid on the collateral posted, by the same argument as above. We have illustrated this in below figure (from Gregory (2015), p. 295):

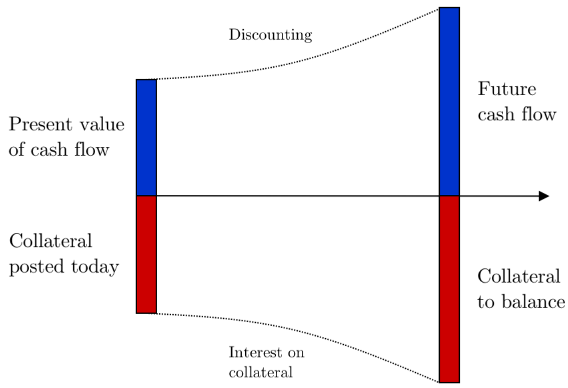

Figure 2: Illustration of the collateral discounting concept, showing that a collateralised trade should be discounting using the interest rate earned on the collateral.

Before the financial crisis discounting using Libor was common. This was because the difference in the rates was very small (as indicated by figure 1), and so it induced negligible errors in the pricing. However, that is no longer the case. We will now give an example on why this is no longer valid. Assume two parties A and B have traded a collateralised 3-year interest rate swap with a swap rate of 3.5% on a notional of 1 mil EUR and that party A is receiving fixed[3]. Further assume flat Libor and OIS curves at 3% and 2% respectively, such that the par swap rate is 3%. The present value of the swap is the discounted value of receiving the 0.5% for the next 3 years. Using Libor and OIS discounting respectively the present value is approximately:

[2]In reality it earns the O/N rate, but the OIS is the market expectation of future O/N rates.

[3]The outline of the following example is taken from Nashikkar, A. (2011), and we will use a EUR-setting for simplification.

$$PV^{\text{lib}} = 5000\left(\frac{1}{1.03} + \frac{1}{1.03^2} + \frac{1}{1.03^3}\right) = 14,143$$
$$PV^{\text{ois}} = 5000\left(\frac{1}{1.02} + \frac{1}{1.02^2} + \frac{1}{1.02^3}\right) = 14,419$$

However if party B only posts 14,143 in collateral it will not be equal to the (expected) future cash flows, because collateral only earns the lower OIS rate. Instead party B needs to post 14,419 in collateral because given the expected interest it earns, it is exactly equal to the expected cash flows.

The above simplified example shows why discounting using the forward OIS curve is the right approach when valuing interest rate swaps, assuming we are in a benchmark scenario of "perfect" collateralisation. This is the case at a clearing house, or in the CSAs used in the interbank market or between banks and larger institutionel investors. We will later on in section 4.4 review how to discount in a setting where there is no CSA.

2.2 Pricing interest rate swaps

In this section we will review how to price interest rate swaps, and other interest rate derivatives needed in our curve calibrations. We will start by defining the basic concepts necessary for this; zero rates, discount factors and forward rates. Next we will derive the pricing formulas of the derivatives. At the end we will review the most standard day count and rolling conventions, needed to properly calculate the exact size and timing of the cash flows.

2.2.1 Zero rates and discount factors

In this section we define the basic concepts of zero rates and discount factors.

We define the zero coupon bond to be a bond which (only) pays the notional of 1 at maturity. Letting t be the present time, we denote the present value of this bond by $P^I(t, t_1)$, where I indicates the index with which it is derived relative to, and will typically be Libor or OIS. We can now calculate the present value of any cash flow at time t_1 by multiplying by $P^I(t, t_1)$. This way the bond price $P^I(t, t_1)$ is our discount factor. Knowing the discount factors for all maturities gives us the *discount curve*, $P^I(t, t_1)\ \forall t_1 \geq 0$.

We let the *forward price* of the zero coupon bond be $P^I(t; t_0, t_1)$ for $t_0 > t$. This is the price agreed upon today (time t) for buying or selling the time-t_1 maturing zero coupon bond at time t_0. This is easily found using a simple replication argument; buy 1 unit of t_1 zero coupon bond at price $P^I(t, t_1)$, and finance by selling $\frac{P^I(t,t_1)}{P^I(t,t_0)}$ units of t_0 zero coupon bond. At time t_0 an amount of $\frac{P^I(t,t_1)}{P^I(t,t_0)}$ has to be paid; the forward price. Therefore we define the forward price to be:

$$P^I(t; t_0, t_1) = \frac{P^I(t, t_1)}{P^I(t, t_0)}$$

The yield of the zero coupon bond we denote the *zero rate*. We will primarily be working with discount factors calculated using continuous compounding, such that the zero rate and discount factors are related by:

$$P^I(t, t_1) = \exp(-r_c^I(t, t_1)(t_1 - t))$$

Similar expressions exist for discrete compounding and using money market conventions. Looking at above we see that $P^I(t,t_1)$ is a monotonically decreasing function of $r_c^I(t,t_1)$, and there is a one-to-one relationship between these factors. This means that observing one lets us uniquely determine the other, and thus observing the discount factors for several maturities, $t_1 > t$, we may instead choose to represent this function using a zero rate curve.

2.2.2 Forward rates

Similar to forward prices, we also have forward rates. They represent a rate agreed upon today (time t) to borrow or lend funds in the future between time $t_0 \geq t$ and t_1. Here we show how to calculte forward rates from discount factors[4]. We will calculate forward (and spot) rates relative to Libor, but the approach will be exactly the same for e.g. OIS rates.

We let $L^I(t,t_1)$ be the spot Libor rate, that is for a loan starting two business days after and maturing at time t_1[5]. The interest paid on such a loan is paid at time t_1, and is equal to $\alpha^I N L^I(t,t_1)$ where α^I is the coverage and N the notional. This method is called *simple interest*. Lending at Libor at time t thus gives $1+\alpha^I L^I(t,t_1)$ at time t_1. We may therefore define the zero coupon bond price relative to Libor by:

$$\begin{aligned} P(t,t_1) &= \frac{1}{1+\alpha^I L^I(t,t_1)} \qquad \Leftrightarrow \\ L^I(t,t_1) &= \frac{1}{\alpha^I}\left(\frac{1}{P^I(t,t_1)} - 1\right) \end{aligned}$$

Similarly the *forward* Libor rate is the rate agreed upon today to borrow funds in the future between times t_0 and t_1, and we will denote it $F^I(t,t_0,t_1)$. The forward (Libor) rate is given by:

$$\begin{aligned} 1+\alpha^I F^I(t,t_0,t_1) &= \frac{P^I(t,t_0)}{P^I(t,t_1)} \qquad \Leftrightarrow \\ F^I(t,t_0,t_1) &= \frac{1}{\alpha^I}\left(\frac{P^I(t,t_0)}{P^I(t,t_1)} - 1\right) \end{aligned} \tag{1}$$

We see that the spot rate is just a special case of the forward rate by setting $t_0 = t$ and using that $P^I(t,t) \equiv 1$.

To price derivatives we use the standard risk-neutral expectations approach. To use this with regards to *interest rate* derivatives we need to able to calculate the expectation of future (Libor) rates. Using the forward measure, i.e. using $P^I(t,t_1)$ as numeraire we find that (Tuckman & Serrat (2012), p. 510-514):

$$E_t^{Q_f^{t_1}}\left[L^I(t_0,t_1)\right] = F^I(t;t_0,t_1) \tag{2}$$

Using this result, we may simply use the forward Libor rate as the expectation of a future Libor rate (fixing) when pricing interest rate derivatives.

From these results it is clear that all we need to calculate forward rates is a zero rate curve with respect to the index I. We have in this section assumed that the spot and forward rates are priced off Libor, but it is important to note that the above formulas hold for other reference indices as well.

[4]This section follows the approach of Linderstrøm (2013) p. 11-13.

[5]Spot start depends on the currency, but in EUR and DKK it is two business days

2.2.3 Forward rate agreements

A forward rate agreement, or FRA, is a contract where two counterparties agree to exchange a single cash flow at a pre-specified point in time based on a Libor fixing. One party is paying a fixed amount based on a fixed rate, the *FRA rate*, and receives an amount based on the future Libor fixing. Both payments are calculated using simple interest. In practice only the difference in the payments is exchanged. If you buy a FRA contract you are paying the fixed and receiving the floating payment. The FRA contract therefore enables a party to fix a future Libor payment, or convert a future fixed payment into a floating payment (technically plus a spread, unless the FRA rate and the rate underlying the fixed payment are identical).

Libor rates usually pay at the end of the period, i.e. a payment based on the Libor rate $L^{\text{lib}}(t, t_1)$ will fix at time t and the payment occur at time t_1. However for FRA contracts it is customary to let the exchange of (the difference of) the two payments occur at the fixing of the Libor rate (technically two business days after the fixing). Also the payments are discounted using the Libor rate itself. The payoff for the party *buying* the FRA contract on the time t_0 Libor fixing is:

$$PV^{\text{lib}}_{\text{fra}}(t_0) = \frac{N\alpha^{\text{lib}}(F^{\text{lib}}(t_0, t_0, t_1) - \kappa^{\text{fra}})}{1 + \alpha^{\text{lib}} F^{\text{lib}}(t_0, t_0, t_1)} \tag{3}$$

Where N is the notional and κ^{fra} is the FRA rate. It turns out that finding the present value is somewhat more complicated for the FRA contract, than for some of the other linear interest rate derivatives. The reason is that the FRA contract is discounted at the future Libor rate instead of the "usual" discount factors, $P^{\text{disc}}(t_0, t_1)$. This creates a dynamic between the forward discounting rate and the forward Libor rate which needs to be modelled to be properly accounted for. The consequence of this term is usually very small (it concerns the discounting over the period t_0 to t_1 which is typically rather short, e.g. 3 months for the Libor3M), and we will thus ignore it in this thesis. The below equations show the theoretical time t present value of the FRA contract, as well as the simplified version, both of which are derived in appendix A.1:

$$PV^{\text{lib,theor.}}_{\text{fra}}(t) = N\alpha^{\text{lib}} P^{\text{disc}}(t, t_0) \left(\frac{F^{\text{lib}}(t, t_0, t_1) - \kappa^{\text{fra}}}{1 + \alpha^{\text{lib}} F^{\text{lib}}(t, t_0, t_1)} e^{C_{\text{fra}}(t_0)} \right)$$
$$PV^{\text{lib,simp.}}_{\text{fra}}(t) = N\alpha^{\text{lib}} P^{\text{disc}}(t, t_0) \left(\frac{F^{\text{lib}}(t, t_0, t_1) - \kappa^{\text{fra}}}{1 + \alpha^{\text{lib}} F^{\text{lib}}(t, t_0, t_1)} \right)$$

As we see in the simplified version we have calculate the present value simply by discounting the time t_0 payoff and replacing the future Libor fixing with the current forward Libor rate. We will use the simplified version from here on.

It is customary to trade FRAs at a present value of 0, that is the FRA rate is chosen so that the present value of the contract is 0. Setting the PV to 0 in the simplified version and rearranging for $\kappa^{\text{fra}}(t_0)$ we find:

$$\kappa^{\text{fra}}(t_0) = F^{\text{lib}}(t, t_0, t_1)$$

And we see that (disregarding the convexity adjustment) the FRA rate is in fact the forward Libor rate.

FRA contracts are usually traded with a start date of 1M, 2M, 3M[6] etc. from the start date, and with a maturity of less than 2 years ($t_1 \leq 2Y$). In some currencies, such as DKK, the FRA

[6]From here on I will use the standard market lingo and use $3M$ for 3 *months*, and similarly Y for years, W for weeks, B for business days, and D days.

contracts are traded at IMM dates. This means that they are contracts on the Libor fixing on the third wednesday of March, June, September or December. We let e.g. the 2x5 FRA denote the contract on the 3M Libor rate fixing in 2M, whereas for IMM contracts (including futures) the contracts are denoted by their order; first, second and third contract etc. We will use this notation in later sections.

2.2.4 Interest rate swaps

The interest rate swap, IRS, is a derivative where two parties exchange a series of payments. In this thesis we will use this to mean a vanilla interest rate swap, where the parties exchange a series of *floating* payments for *fixed* payments. The floating payments will for our purpose (but not necessarily) reference a Libor index, and the fixed interest rate is agreed upon at inception. Market standard is to denote the position relative to the fixed payments, and therefore the party paying fixed has entered into a *payer swap* and the counterparty a *receiver swap*. Hence the first party sees the fixed leg as a liability and the floating leg as an asset, and vice versa for the counterparty.

We value an interest rate swap by valuing the fixed and floating leg each. The floating leg consists of N payments on dates $t_i,\ i = 1, 2, ..., N$[7]. If the Libor index is Libor3M, and the swap has a length of 2 years, then there is a total of $N = 8$ floating payments. Libor rates are fixed at the start of the period, and pays and the end of each fixing period (as opposed to the FRA contract where payment occured immediately after the fixing). So the first payment of the floating leg will be fixed at the start date of the swap (the first reset date), but does not pay until after 3 months, assuming a Libor3M index. We denote t_0 to be the effective date of the swap, and t_n to be the maturity date of the swap (and also the last payment date). It follows that $t_N = t_n$, and for spot starting swaps it will hold that $t \approx t_0$[8].

Letting N_i to be the notional of the swap for the period starting at time t_{i-1} and ending at t_i and $\alpha_i^{\text{float,lib}}$ to be the coverage of the same period, the present value of (receiving) the floating leg is:

$$\begin{aligned} PV_{\text{float}}^{\text{lib}}(t) &= \sum_{i=1}^{N} \alpha_i^{\text{float,lib}} E_t^{Q_f^{t_i}}[L^{\text{lib}}(t_{i-1}, t_i)] N_i P^{\text{disc}}(t, t_i) \\ &= \sum_{i=1}^{N} \alpha_i^{\text{float,lib}} F^{\text{lib}}(t, t_{i-1}, t_i) N_i P^{\text{disc}}(t, t_i) \qquad (4) \end{aligned}$$

Where we have used equation (2). For most swaps the notional is fixed for the entire period of the swap, $N_i = N,\ \forall\ i$. However we derive the pricing formulas while allowing the notional to vary, as we will need this later on. The discount factor $P^{\text{disc}}(t, t_i)$ is used to discount future cash flows. It is usually different from the Libor index I used to project forward rates, and as we saw in section 2.1.3 will typically be derived off the OIS curve.

The fixed leg consists of M payments on dates $\tau_i,\ i = 1, 2, ..., M$. The rate on this leg is fixed for the duration of the swap, and also called the *swap rate*. The two counterparties agree upon this rate when the swap is traded. This also holds in the case of forward starting swaps, where the fixed rate is also agreed upon on the trade date.

We again let N_i be the notional and $\alpha_i^{\text{fixed,lib}}$ to be the coverage of the period starting at τ_{i-1} and ending at τ_i, and denote the fixed rate by κ^{irs}. Using this the present value of (receiving)

[7]Not to be confused with the notional N.

[8]Again, for many currencies such as EUR and DKK t_0 will be t plus two business days.

the fixed leg is:

$$PV_{\text{fixed}}^{\text{lib}}(t) = \sum_{i=1}^{M} \alpha_i^{\text{fixed,lib}} \kappa^{\text{irs}} N_i P^{\text{disc}}(t, \tau_i) \tag{5}$$

Market practice is often to let the fixed payments occur with an annual og semi-annual frequency, and to let the floating payments occur with the tenor of the Libor reference rate, e.g. Libor3M to occur once every third month. This means that the cash flows of the fixed and floating legs do not always occur on the same date. Using Libor3M as the example, and assuming annual fixed payments, there would be floating payments quarterly and only one of these would coincide with the annual fixed payment. This has been illustrated in below figure:

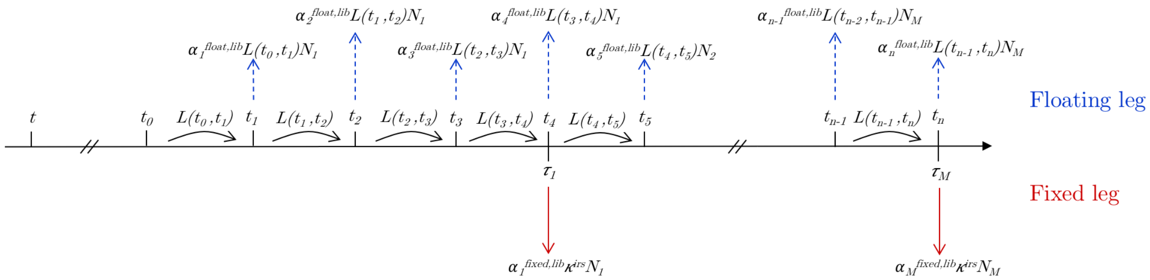

Figure 3: Cash flows of a payer interest rate swap.

The value of a payer swap is now easily found as:

$$PV_{\text{pay}}^{\text{lib}}(t) = PV_{\text{float}}^{\text{lib}}(t) - PV_{\text{fixed}}^{\text{lib}}(t) \tag{6}$$

And it follows that $PV_{\text{rec}}^{\text{lib}} = -PV_{\text{pay}}^{\text{lib}}$.

Assume the zero rate curves of both the forward Libor curve and the discounting curve are known. Upon reviewing equations (4) and (5), we then see that all dates, coverages, discount factors, the fixed rate and notionals for all cash flows are known at initiation. The only unknowns are the future fixings of the reference Libor index, but here we have used the forward rates in the valuation of the swap. However, the zero rate curve used to project forward rates and the one used to discount *may* be different, here exemplified by the different notation P^{disc} and P^{lib}. As mentioned we will use the forward OIS curve for discounting as the benchmark. We will review how to calibrate the zero rate curves in section 2.3.

As with the FRA contracts, it is also customary trade swaps with a present value of 0[9]. This is among other to reduce counterparty credit risk and funding costs. Setting equation (6) equal to 0 and solving for the fixed rate we find the *par* swap rate to be:

$$\kappa^{\text{irs}}(t, t_0, t_n) = \frac{\sum_{i=1}^{N} \alpha_i^{\text{float,lib}} F^{\text{lib}}(t, t_{i-1}, t_i) N_i P^{\text{disc}}(t, t_i)}{\sum_{i=1}^{M} \alpha_i^{\text{fixed,lib}} N_i P^{\text{disc}}(t, \tau_i)} \tag{7}$$

From above equation we see that the par swap rate is in fact a weighted average of the forward rates. This makes sense intuitively as in a swap one party is receiving the Libor fixings and

[9]Including other types of swaps: cross-currency basis swaps, basis swaps, OIS swaps etc. An exception is credit default swaps.

paying the fixed rate, and for the contract to be fair the fixed rate must therefore be a weighted average of the expected future fixings.

Comparing the fixed rate of an IRS to the par swap rate, it is easy to identify whether it has a positive or negative market value. As an example consider a payer swap with a fixed rate of 0.6395%. If the par swap rate is 0.4871%, then this swap would have a negative PV for the party paying fixed, as that party is paying a fixed rate that is "too high" relative to where a new swap (and completely identical swap, apart from the fixed rate) would trade. In fact the value of the swap is proportional to the difference between the fixed rate and the current par swap rate. By inserting the result for the par swap rate, as well as that of the fixed and floating legs into equation (6), we find the value of the payer swap to be:

$$PV_{\text{pay}}^{\text{lib}}(t) = (\kappa^{\text{irs}}(t, t_0, t_n) - K) \sum_{i=1}^{M} \alpha_i^{\text{fixed,lib}} N_i P^{\text{disc}}(t, \tau_i) \tag{8}$$

The last term is called the swaps *annuity factor*, and is the value of receiving 1 bps for the entire duration (left) of the swap.

2.2.5 Overnight indexed swaps

An overnight indexed swap, OIS[10], is another type of interest rate swap where fixed payments are exchanged for floating payments. However here the floating payments are calculated as a geometric average of an overnight or tomorrow/next rate over a period. For example the overnight rate Eonia rate is used in EUR OIS swaps, and the danish T/N rate is used in DKK OIS swaps. Market practice is (typically) for OIS swaps to have the fixed and floating payments on the same date, and only a single payment if the length of the swap is 12 months or less, and one yearly payment for swaps of longer duration (OpenGamma (2013), chapter 22). The fixed leg of an OIS swap is priced exactly like the fixed leg of an IRS, so the following will focus on the floating leg of the OIS swap.

Let $t_1, t_2, ..., t_n$ be the payment dates of the swap (i.e. they are spaced one year apart), and $\boldsymbol{t_i} = \{t_{i,0}, t_{i,1}, ..., t_{i,n_i}\}$ be all good business days within the period from t_{i-1} to t_i. The overnight rate on the business days in $\boldsymbol{t_i}$ will determine the floating payment at "expiry" of the period at time t_i. Let $\alpha_{i,k}^{\text{float}}$ be the inter-*day* coverage between day $t_{i,k-1}$ and $t_{i,k}$, $1 \leq k \leq n_i$. This coverage will usually be calculated over 1 day, but for weekends it will be calculated over 3 days, and holidays will also increase the number of days. We let $R(t_i, \boldsymbol{t_i})$ be the final coupon rate for for the floating payment for the period $(t_{i-1}, t_i]$. This is calculated as a geometric average from the individual overnight rates over this period, $R(t_{i,k-1}, t_{i,k})$, and is given by (Ametrano & Bianchetti (2013), p. 30-33):

$$R(t_i, \boldsymbol{t_i}) = \frac{1}{\alpha_i^{\text{float}}} \left(\prod_{k=1}^{n_i} [1 + R(t_{i,k-1}, t_{i,k}) \alpha_{i,k}^{\text{float}}] - 1 \right) \tag{9}$$

Where α_i^{float} is the inter-*period* coverage between payment dates t_{i-1} and t_i. Below figure illustrates the cash flows as well as the compounding of the coupon rate of the floating leg of the OIS swap:

[10] I will repeatedly refer to these kind of swaps as OIS swaps even though the last *"swap"* is redundant - and as a curiosity, in fact an example of the RAS syndrome.

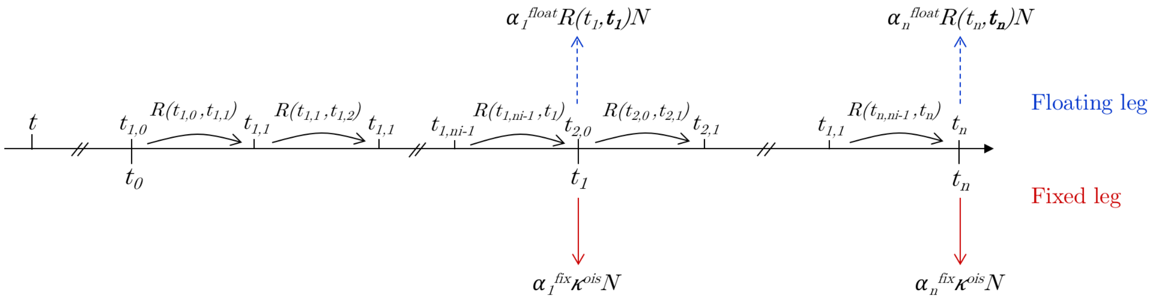

Figure 4: Cash flows of a payer OIS swap.

Let us denote time-t the present value of the payment of the floating leg at time t_i to be $PV_{\text{float},i}^{\text{ois}}(t)$. We find this to be[11]:

$$PV_{\text{float},i}^{\text{ois}}(t) = \alpha_i^{\text{float}} R(t_i, \boldsymbol{t_i}) N P^{\text{disc}}(t, t_i)$$

Using the derivations from appendix A.2 we may write the time t expectation of the floating coupon rate instead as[12]:

$$R(t, \boldsymbol{t_i}) = \frac{1}{\alpha_i^{\text{float}}} \left(\frac{P^{\text{ois}}(t, t_{i-1})}{P^{\text{ois}}(t, t_i)} - 1 \right)$$

Where $P^{\text{ois}}(t, t_i)$ is derived from the forward OIS curve. Assuming we are in the benchmark scenario of "perfect" collateralisation, the proper discounting curve for the OIS swap is the OIS curve itself. The argument for this is completely identical to that of IRS, and in fact holds for most derivatives (assuming proper collateralisation). Using that $P^{\text{disc}}(t, t_i) = P^{\text{ois}}(t, t_i)$ we may write the present value of the floating leg of the OIS swap as:

$$\begin{aligned} PV_{\text{float}}^{\text{ois}}(t) &= \sum_{i=1}^{n} PV_{\text{float},i}^{\text{ois}}(t) \\ &= \sum_{i=1}^{n} \alpha_i^{\text{float}} R(t, \boldsymbol{t_i}) N P^{\text{ois}}(t, t_i) \\ &= \sum_{i=1}^{n} N(P^{\text{ois}}(t, t_{i-1}) - P^{\text{ois}}(t, t_i)) \\ &= N(P^{\text{ois}}(t, t_0) - P^{\text{ois}}(t, t_n)) \end{aligned} \tag{10}$$

Using the OIS curve for discounting significantly simplifies the calculation of the floating leg, which would otherwise be comparable to equation (4) for the floating leg of the IRS.

[11] We will ignore the case of a varying notional for OIS swaps as this is less interesting for the purpose of this thesis.

[12] To find this, we have in the appendix again assumed that under a proper numeraire the current expectation of future overnight rates is the forward overnight rate. This is similar to the assumption regarding equation (2). Once again we will note provide a proof for this, but instead refer to Poulsen (1999) [in danish] and McDonald (2014) p. 663-677.

If we let the fixed rate of the OIS swap be κ^{ois}, then the present value of the payer swap is:

$$
\begin{aligned}
PV^{\text{ois}}_{\text{pay}}(t) &= PV^{\text{ois}}_{\text{float}}(t) - PV^{\text{ois}}_{\text{fixed}}(t) \\
&= N\left(P^{\text{ois}}(t,t_0) - P^{\text{ois}}(t,t_n) - \sum_{i=1}^{n}\alpha_i^{\text{fix}}\kappa^{\text{ois}}P^{\text{ois}}(t,t_i)\right)
\end{aligned}
$$

And the par swap rate is easily found by setting the present value to 0:

$$
\kappa^{\text{ois}}(t,t_0,t_n) = \frac{P^{\text{ois}}(t,t_0) - P^{\text{ois}}(t,t_n)}{\sum_{i=1}^{n}\alpha_i^{\text{fix}}P^{\text{ois}}(t,t_i)}
$$

All that is needed to price OIS swaps and calculate (forward) par swap rates is a single zero rate curve calibrated to OIS swaps (assuming discounting using the OIS curve). As the OIS swap is not dependent on Libor fixings, we do not need the forward Libor curve to value this swap.

2.2.6 Cross-currency basis swaps

The last swap we will derive the pricing formula of is the cross-currency basis swap. For our purpose we will let "cross-currency basis swap" refer to the *float-float* version, that is the version where both legs contain floating payments. A cross-currency basis swap is a swap with each leg denominated in a different currency, and with an initial and final exchange of notional. We may therefore intuitively think of the cross-currency basis swap as a floating rate loan in one currency, collateralized by a floating rate deposit in another currency. The notional of each currency leg is calculated using the *spot* exchange rate, even if the swap is forward starting. Between the notional exchange at start and maturity, there is a series of quarterly floating payments based of the 3M Libor rate of each currency, with the final exchange of these payments to occur at maturity. It is market standard to use 3M Libor rates, even if this is not the main market for a particular currency. An example being DKK where interest rate swaps are usually referencing the Cibor6M rate, but where EURDKK basis swaps are still referencing the 3M Libor rate in both currencies. There is a spread added, the *basis swap spread*, c, to one of the legs to set the value of the contract to 0 at the time of trading. Market liquidity is usually concentrated against USD, meaning that most swaps have one of the legs denominated in USD. In that case the spread is added to the non-USD leg. For our purpose we will look at the EURDKK basis swap, where it is customary to add the spread to the DKK leg[13].

Below figure illustrates the cash flows of the cross-currency basis swap as explained in the following and as seen from the first party. Blue arrows denotes cash flow in the domestic currency, and red arrows cash flows in the foreign currency. Assume the initial exchange of notional, N, occurs at time $t_0 \geq t$ at the spot rate, $s(t)$. This means that the first party receives N foreign currency and pays $s(t)N$ domestic currency. The opposite exchange of exactly the same amounts occur at time t_n, the expiry of the swap contract. At intermediay (quarterly) points in time, t_i, $i = 1,2,3,\ldots,n$, there is an exchange of floating payments in each currency, both calculated relative to the 3M Libor fixing. One side of these floating payments has the basis swap spread, c, added. For our calculations we will assume it is the domestic currency side. So at time t_i the first party receives a domestic currency amount of $\alpha^{\text{lib}}_{i,D}(L^{\text{lib}}_D(t_{i-1},t_i)+c)s(t)N$, and pays a foreign currency amount of $\alpha^{\text{lib}}_{i,F}L^{\text{lib}}_F(t_{i-1},t_i)N$. We have used the notation for the Libor fixings, $L^{\text{lib}}_D(t_{i-1},t_i)$ and $L^{\text{lib}}_F(t_{i-1},t_i)$, to stress that these are from two different indices.

[13] In general EURXXX basis swaps have the spread added to the XXX leg, of course with the exception of XXX=USD.

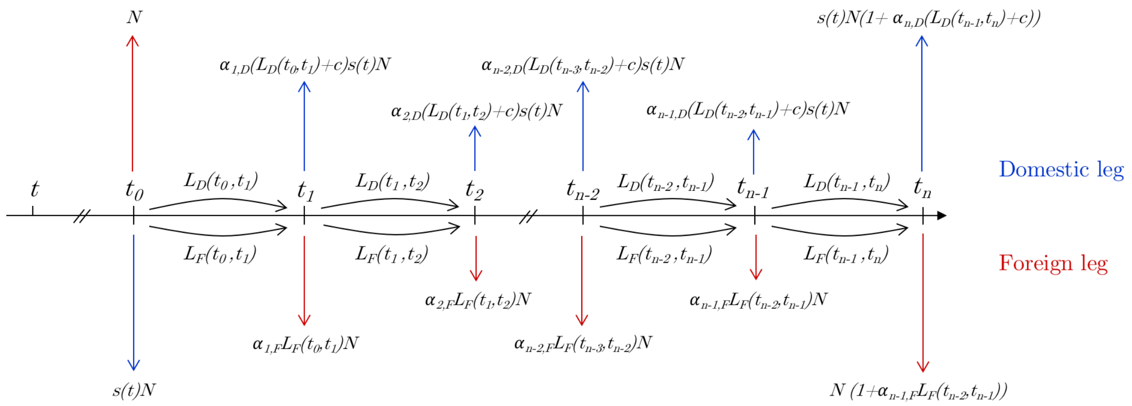

Figure 5: Cash flows of a receiver cross-currency basis swap.

The appeal of the cross-currency basis swap is in its role as a hedge of currency risk, or FX risk. It provides similar currency hedging properties as the FX swap. However, whereas the FX swap and FX forward, are usually traded with maturities no longer than 1-2 years, the cross-currency basis swap may be traded with a maturity of up to 30 years (Linderstrøm (2013), p. 33-34, and White (2012), p. 6-7). The reason for this, is that the FX forward contains significant interest rate risk which is proportional to the length of the contract. To avoid this market participants tend to use cross-currency basis swaps for hedging of longer dated FX risk. Appendix A.3 exemplifies the interest rate risk of the FX forward contract, and why the cross-currency basis swap may be used to mitigate this.

To value the cross-currency basis swap we will find the present value of the foreign and domestic leg each. Using the above the value of receiving the foreign currency *leg* (that is receiving the "foreign-indexed" floating payments and the foreign currency notional amount at maturity) denominated in the *foreign* currency is:

$$\begin{aligned} PV^{\text{ccs}}_{\text{foreign},F}(t) &= -NP^{\text{disc}}_F(t,t_0) + NP^{\text{disc}}_F(t,t_n) + \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,F} F^{\text{lib}}_F(t,t_{i-1},t_i) NP^{\text{disc}}_F(t,t_i) \\ &= N\left(P^{\text{disc}}_F(t,t_n) - P^{\text{disc}}_F(t,t_0) + \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,F} F^{\text{lib}}_F(t,t_{i-1},t_i) P^{\text{disc}}_F(t,t_i) \right) \end{aligned}$$

Where $\alpha^{\text{lib}}_{i,F}$ is the coverage for the foreign currency Libor payment, and where we have used equation (2) again. Similarly we find the value of receiving the domestic currency leg denominated in the *foreign* currency is:

$$\begin{aligned} PV^{\text{ccs}}_{\text{dom},F}(t) = \frac{1}{s(t)} \Bigg(&- s(t)NP^{\text{disc}}_D(t,t_0) + s(t)NP^{\text{disc}}_D(t,t_n) \\ &+ \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} \left(F^{\text{lib}}_D(t,t_{i-1},t_i) + c \right) s(t)NP^{\text{disc}}_D(t,t_i) \Bigg) \\ = N \Bigg(P^{\text{disc}}_D(t,t_n) - P^{\text{disc}}_D(t,t_0) &+ \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} \left(F^{\text{lib}}_D(t,t_{i-1},t_i) + c \right) P^{\text{disc}}_D(t,t_i) \Bigg) \end{aligned} \tag{11}$$

We may now easily find the value of a payer cross-currency basis swap (that is paying the basis swap spread c) denominated in the foreign currency as the following:

$$PV^{\text{ccs}}_{\text{pay},F}(t) = PV^{\text{ccs}}_{\text{foreign},F}(t) - PV^{\text{ccs}}_{\text{dom},F}(t)$$

As is customary with other types of swaps the cross-currency basis swap is also traded at a PV of 0. This enables us to find the *par basis swap spread* as:

$$c(t, t_0, t_n) = \frac{1}{\sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} P^{\text{disc}}_{D}(t, t_i)} \Bigg(\big(P^{\text{disc}}_{D}(t, t_0) - P^{\text{disc}}_{D}(t, t_n)\big) - \big(P^{\text{disc}}_{F}(t, t_0) - P^{\text{disc}}_{F}(t, t_n)\big) \\ + \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,F} F^{\text{lib}}_{F}(t, t_{i-1}, t_i) P^{\text{disc}}_{F}(t, t_i) - \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} F^{\text{lib}}_{D}(t, t_{i-1}, t_i) P^{\text{disc}}_{D}(t, t_i) \Bigg)$$

As is evident from the above expression the par basis swap spread is dependent on 4 curves; the discounting and forward Libor curves in each of the two currencies. However, knowing several basis swap spread quotes and 3 of the 4 curves enables us to infer the remaining curve using the above.

2.2.7 Day count and rolling conventions

As can be seen from the previous sections dates and coverages are important in pricing swaps and other interest rate derivatives. In this section we will briefly review the two concepts, rolling conventions and day count conventions, needed for deriving a proper schedule of payment dates of an interest rate product and the associated coverages.

Assume a 20 year interest rate swap with the floating payments referencing the 3-month Libor rate and with annual payments on the fixed leg. This swap has 20 annual fixed payments and 80 quarterly floating payments. Some of these (unadjusted) payment dates are almost certainly bound to occur on non-business days; weekends or holidays. A *rolling convention*[14] settles how to adjust, or roll, these days to good business days. This is necessary for several reasons. One of them is that these dates entail the transfer of payments between parties. This is preferable to happen on a day, where the parties can verify that the transaction has occured as agreed, and are able to take action not. The adjusted payment dates are also the ones used to calculate coverages. Typical conventions are:

- *None* or *Actual*, the date is not rolled even if on a non-business day (not common for obvious reasons).
- *Following*, the date is rolled to the next good business day.
- *Preceding*, the date is rolled to the previous good business day.
- *Modified Following*, the Following convention is used unless the next good business day is in the following month. If so the Preceding convention is used.
- *Modified Preceding*, the Preceding convention is used unless the previous good business day is in the previous month. If so the Following convention is used.

The most used rolling convention is Modified Following, and this is also standard in vanilla interest rate swaps in EUR and DKK (OpenGamma (2013) and Nasdaq OMX (2018a)). In the

[14] Sometimes also known as a *business day convention*.

examples of this thesis we will ignore holidays, and only consider weekends as non-good business days. This will not significantly affect any of our results.

Interest rates in swap payments (and e.g. coupons on bonds) are expressed as annual rates. However as we have just seen the dates spanning these payments are often not a full year. A *day count convention* tells us how to calculate the coverage used to multiply a given interest payment with, to find the exact size of the payment. There exists numerous day count conventions. Assume we have two dates, $t_1 < t_2$, with $D_i, M_i, Y_i,\ i = 1, 2$ denoting the day, month and year of each date. Below list gives a few examples of day count conventions, and also covers the ones used in the calculations and examples of this thesis:

- *ACT/360*, the number of days between the two dates divided by 360, $\alpha = \frac{t_2 - t_1}{360}$
- *ACT/365*, as ACT/360 but dividing with 365 instead, $\alpha = \frac{t_2 - t_1}{365}$
- *30/360*[15], assumes all months have 30 days and so all years have 360 days. The coverage is calculated as, $\alpha = \frac{360(Y_2 - Y_1) + 30(M_2 - M_1) + (\min(D_2, 30) - \min(D_1, 30))}{360}$

2.3 Curve calibration

As the previous sections have shown we need to be able to project forward rates and calculate discount factors to price interest rate derivatives. In this section we will review how to calibrate the curves needed for this. We start by setting up the formal problem in section 2.3.1. Next we go over (some of) the curves we need to calibrate for the examples in this thesis, as well as the different choice of inter- and extrapolation methods in sections 2.3.2 and 2.3.3. In section 2.3.4 we give an example of a calibration, and we end by discussing the effect of using a single-curve versus a dual-curve setup in section 2.3.5.

2.3.1 The calibration problem

As we have seen in the previous sections we can price interest rate swaps and OIS swaps if we have the necessary zero rate curves. However we will most often be interested in reversing this process, and given a set of market quotes of interest rate derivatives, find the forward and discounting curves, such that the derived prices from these correspond with the market quotes. The calibration of the zero rate curves to market quotes is called curve calibration. This enables us to price any instruments, e.g. instruments where quotes are not readily observable in the market, and do so in a market consistent manner.

The price of a 30 year swap against Libor3M is dependent on all the 120 3-month forward rates spanned by this period (one of them the spot Libor fixing). Obviously we cannot hope to observe market prices for all 120 forward rates (and even if we could, we would probably not be very confident in many of these due to low liquidity). Instead we will follow market standard and identify a series of knot points, or maturities, with associated market instrument quotes, and we will calibrate zero rates at these points. In between knot points we will use an interpolation rule, which enables us to calculate zero rates for any given maturity. We will also use an extrapolation rule, which enables us to calculate zero rates before the first and beyond the last knot point on our zero rate curve. Using these rules we are able to calculate forward rates for any start and maturity dates. Market practice is to do this inter- and extrapolation in continously compounded

[15]There exists numerous 30/360 methods, all differing in how D_i (and for some M_i) are calculated. This refers to the "standard" version, for reference see OpenGamma (2013).

zero rates (Linderstrøm (2013), p. 22, and Hagan & West (2006), p. 1), however other choices are available.

For our problem of calibrating zero rate curves we start by identifying a set of market instruments. The knot points of our curve is the maturity of these instruments. Obviously this choice should reflect the use of the curve. If ones interest and use of the curve is in the short-end, focus should be given here by selecting more short-maturity instruments. A standard approach would be to spread these points (choose market instruments) from the very short-end (e.g. weeks or months) to say 30 years, but with a higher density of knot points around the short-end. Generally we wish to calibrate two zero rate curves: one for the forward Libor curve and one for the forward OIS curve. We will let M^{lib} and M^{ois} denote the number of knot points (instruments) for the forward Libor and OIS curve, respectively, and $M = M^{\text{lib}} + M^{\text{ois}}$ to be the total number of knot points. For each point we identify the corresponding zero rate, and we call these our *model parameters* and denote them by the row vector $\boldsymbol{p} = (p_1^{\text{lib}}\ p_2^{\text{lib}}\ \ldots\ p_{M^{\text{lib}}}^{\text{lib}}\ p_1^{\text{ois}}\ p_2^{\text{ois}}\ \ldots\ p_{M^{\text{ois}}}^{\text{ois}})$.

We denote the corresponding *market quotes* by $N = N^{\text{lib}} + N^{\text{ois}}$, split between quotes used for the forward Libor and the forward OIS curve. We will use one market quote for each knot point on our zero rate curve, but most important is to use the same number of quotes as the number of knot points, that is $N = M$. We do this because it ensures nice properties for the calibration problem and because, as we will se later, makes the calculation of market risk a lot easier. We will denote the N market quotes by the vector $\boldsymbol{a} = (a_1^{\text{lib}}\ a_2^{\text{lib}}\ \ldots\ a_{N^{\text{lib}}}^{\text{lib}}\ a_1^{\text{ois}}\ a_2^{\text{ois}}\ \ldots\ a_{N^{\text{ois}}}^{\text{ois}})^\top$.

From our model parameters (the zero rates), we inter- and extrapolate two zero rate curves. From these we are able to calculate N *model quotes*, corresponding to how our model is pricing the instruments used for the N market quotes. As the model quotes are a function of the model parameters, we will denote them by the vector $\boldsymbol{b}(\boldsymbol{p}) = (b_1^{\text{lib}}\ b_2^{\text{lib}}\ \ldots\ b_{N^{\text{lib}}}^{\text{lib}}\ b_1^{\text{ois}}\ b_2^{\text{ois}}\ \ldots\ b_{N^{\text{ois}}}^{\text{ois}})^\top$.

We can now simplify our calibration to a least squares optimization problem, where we choose our model parameters, that is we find the two zero rate curves, such as to minimize the squared difference between the model and market quotes:

$$\min_{\boldsymbol{p}} \|\boldsymbol{b}(\boldsymbol{p}) - \boldsymbol{a}\|$$

This will generally be a non-linear optimization problem, and will require us to use numerical methods. Fortunately the scale of this problem, in the context of this thesis, is small enough for most programs to handle it. This includes Excels Solver-functionality which we will use.

2.3.2 The curves to be calibrated

As this thesis is focusing on the danish swap market, we will in this section explain what curves we will need to calibrate to properly examine DKK denominated swaps, as well as the market instruments needed as input in order to perform this calibration.

As explained in section 2.1.1 we have chosen to focus on the Cibor3M index in the (preliminary) examples, despite the liquidity being in Cibor6M. As such we will naturally have to calibrate a Cibor3M forward curve. As a benchmark we will assume swaps to be collateralized under a "perfect" CSA or central clearing, which means that we must discount cash flows using the DKK OIS curve. For calibration of this curve we will use CITA swaps.

We will now go over the exact choice of market quotes used for our curve calibrations. This is important because it may define our precision in pricing certain instruments. As an example consider a scenario where we only used market instruments spanning the first 10 years as well as the 30Y IRS quote. The precision of the curve in the 10Y-20Y segment would (most likely) be poor. Market participants often calibrate a curve out to 30Y, or even 60Y. To do that we naturally need to choose instruments spanning the 30 or 60 years (or rely on the crude extrapolation assumption). In this thesis we will calibrate curves out to 30Y, and use flat extrapolation from this point, for any instruments with longer dated cash flows.

For the calibration of the forward OIS curve we will use the T/N fixing, as well as CITA swap quotes. The swap quotes will be ranging from 1W to 30Y. In table 2 below we have listed the market quotes used, and we see that we have $N^{\text{ois}} = 17$ quotes for the calibration of the forward OIS curve.

The calibration of the forward Cibor3M curve will use several instruments, all of them obviously referencing the Cibor3M index. The first quote will be the Cibor3M fixing, i.e. the spot rate. Next we will use the following four IMM FRA contracts. As of May 2018 the next four IMM dates are June 20th, September 19th and December 19th 2018, and March 20th 2019. Using IMM contracts entails a natural "roll" of the contracts used in the calibration as the 1st contract settles, wherein the 2nd contract becomes the "new" 1st etc. As such the maturity of the 4th contract used may be as long as 15 months right after a roll. Additionally we will use IRS quotes for maturities from 1Y to 30Y. The exact quotes used for the forward Cibor curve has been listed in table 2.

In our calibration problem, we would typically require that we do not have perfectly overlapping instruments, an example would be to use the 1Y IRS quote as well as the 3M fixing, and the 3x6, 6x9 and 9x12 FRA contracts, as that would be "5 equations (quotes) in 4 variables (forward rates)". Using the Cibor3M fixing and the FRA contracts as well as the 1Y IRS quote for our forward Cibor3M curve, we would actually experience this issue once every quarter; two business day prior to the IMM date. A possible solution to this would be to remove the 1Y IRS quote (possibly replace it by the 5th FRA contract), however for our use we will simply disregard this issue. Another remark about FRA contracts, is that market participants often tend to use the corresponding Libor futures contracts instead, due to their very high liquidity. For Cibor3M this is not relevant, as there does not exist any such futures contract, however for e.g. the 3M Euribor index there does. If one were to use such instruments in the calibration of Euribor3M curves in this example, it is important to adjust for the convexity arrising from the daily settlement of the *exchange traded* futures contracts opposed to the *over-the-counter* FRA contracts. The daily settlement of a futures contracts means that the forward rate implied by the future is higher than that of the FRA, which needs to be adjusted for in the calibration.

As we have seen in section 2.2.4 the IRS quote depend on both the forward Cibor3M and the discounting curve (which we assume to be the forward OIS curve). However, OIS swap quotes depend only on the forward OIS curve. This means that we are able to calibrate the forward OIS curve independently from the forward Cibor3M curve. Below figure illustrates the dependency and quotes used for each curve.

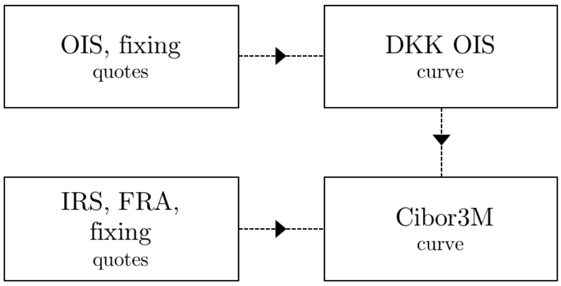

Figure 6: Illustration of the calibration procedure and input, and the interdependence of the curves.

DKK OIS curve		Cibor3M curve	
Instrument	Rate	Instrument	Rate
T/N fixing	-0.473	Cibor3M fixing	-0.290
CITA 1W	-0.480	1st IMM FRA	-0.277
CITA 2W	-0.483	2nd IMM FRA	-0.263
CITA 1M	-0.481	3rd IMM FRA	-0.238
CITA 3M	-0.479	4th IMM FRA	-0.196
CITA 6M	-0.479	IRS 1Y	-0.272
CITA 1Y	-0.469	IRS 2Y	-0.149
CITA 2Y	-0.368	IRS 3Y	0.025
CITA 3Y	-0.209	IRS 5Y	0.363
CITA 5Y	0.186	IRS 7Y	0.643
CITA 7Y	0.379	IRS 10Y	0.972
CITA 10Y	0.696	IRS 12Y	1.136
CITA 12Y	0.855	IRS 15Y	1.314
CITA 15Y	1.029	IRS 20Y	1.468
CITA 20Y	1.185	IRS 25Y	1.514
CITA 25Y	1.238	IRS 30Y	1.523
CITA 30Y	1.256		

Table 2: Table listing the instruments for each curve calibration, as well as the mid quotes in percentages as of April 11th 2018. Quotes are retrieved from Bloomberg, and using a generic Bloomberg price provider.

2.3.3 Inter- & extrapolation methods

The choice of inter- and extrapolation methods is important because they determine the shape of the zero and forward curves, and as a result they will affect how we are pricing instruments which are *not* part of the initial curve calibration. In this section we will review these methods. We will consider these methods done using continuousely compounded zero rates, $r_c^I(t,\tau)$, however they may also be applied to e.g. instantaneous forward rates or (log) discount factors.

Interpolation methods tells us how to compute the zero rate between the knot points used in our calibration problem. This may be done in several ways, one of the obvious being *constant* interpolation. Using this the zero rate is constant between knot points such that:

$$r_c^I(t,\tau) = r_c^I(t,t_i), \quad t_i \leq \tau < t_{i+1}$$

This would seldom be a good approach and often makes little economic sense. Another method is to use a *linear* interpolation between knot points. This has the advantage of being easy to implement and it is intuitively a lot more appealing than the constant approach. The zero rate is determined as:

$$r_c^I(t,\tau) = \frac{\tau - t_i}{t_{i+1} - t_i} r_c^I(t,t_{i+1}) + \frac{t_{i+1} - \tau}{t_{i+1} - t_i} r_c^I(t,t_i), \quad t_i \leq \tau \leq t_{i+1}$$

However, both of these methods experience issues with the shape of the forward rate curve. Specifically, they often generate forward curves[16] that are not continuous. This is an issue as it means that instruments with nearly identical cash flows may be valued very differently. Think of two FRA contracts on fixings one day apart. A part from special scenarios like "turn of

[16]That is the actual forward Libor or OIS curve, not the zero rate representation of this, see figure 8.

quarter/year" effects (Ametrano & Bianchetti (2013), p. 47-50), we would in general expect these two contracts to have similar prices. The continuity of the forward curve is linked to the differentiability ("smoothness") of the zero rate curve, so an equivalent requirement would be for the zero rate curve to be differentiable.

A method which fulfills these requirements is the *hermite cubic spline* method. Generally, in a cubic spline method the zero rates are determined using the polynomial:

$$r_c^I(t,\tau) = a_i + b_i(\tau - t_i) + c_i(\tau - t_i)^2 + d_i(\tau - t_i)^3, \quad t_i \leq \tau \leq t_{i+1}$$

The different cubic spline methods differ in how the coefficients of the polynomial are determined. It can be shown that the hermite cubic spline method ensures that the zero rate curve is always continuous and differentiable[17]. One of the negative properties of the cubic spline methods, is that they exhibit non-locality. This means that the interpolation of a given section of the curve is dependent on data "far away" from that section of the curve. This will be evident in section 2.4.1 where the calculation of risk is studied. We will, however, consider the non-locality issue of the hermite cubic spline method as a small price to pay for the far more dearer property of smooth forward curves. Below graphs show the zero rate and 3M forward curve from the calibrated Cibor3M curve (using data from table 2) for the three interpolation methods:

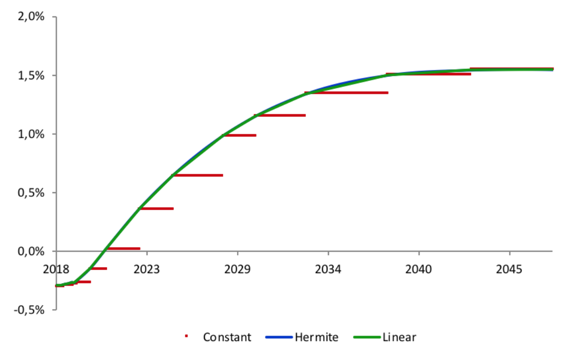

Figure 7: Cibor3M zero rate curve using different interpolation methods.

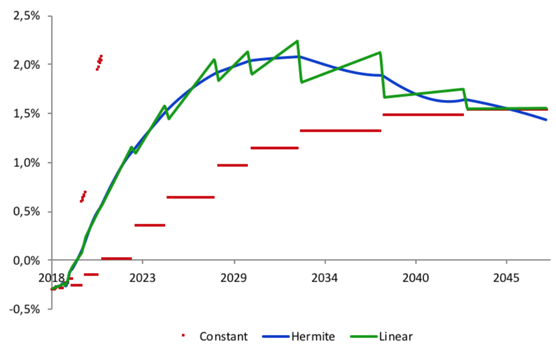

Figure 8: Cibor3M forward rate curve using different interpolation methods.

As is evident from above figures, even though the zero rates of the linear interpolation method looks acceptable using an "eyeball statistic" the forward rates appear highly non-smooth for both the linear and, in particular, the constant methods. For these reasons we will in this thesis use the hermite cubic spline method for all calibration examples and calculations.

For the choice of extrapolation method there exist several approaches. In this thesis we will use flat extrapolation, such that with t_0 being the first and t_n the last knot point in the calibration:

$$r_c^I(t,\tau) = r_c^I(t,t_0), \quad \tau \leq t_0$$
$$r_c^I(t,\tau) = r_c^I(t,t_n), \quad \tau \geq t_n$$

[17]For the exact determination of the coefficients in the *hermite* cubic spline method, as well as the proof we refer to Hagan & West (2006), p. 97-99.

Other approaches often assume some long-term interest rate level and develop an extrapolation method to reach this level in the long-term[18]. Extrapolation is particularly important when you have have to price cash flows which extends beyond the calibrated curve. Using flat extrapolation we ensure that we have a way of discounting these long-term cash flows. And unless we have some confident view on the long-term interest rate level, we will consider this an acceptable approach. We note that the irregularities of the constant interpolation method occurs at or around the knot points of our calibration. We will therefore not experience the same issues using this method for extrapolation, as if we had used it for interpolation.

2.3.4 Calibration example

In this section we will provide a short explanation of the how the calibration is implemented, as well as give an example of the calibrated forward OIS and Cibor3M curves.

The actual calibration of curves is done in spreadsheets in Excel. As is evident from the previous sections on pricing derivates, there is an extensive amount of operations needed to calculate a swap rate or price. These include generating a schedule of payment dates, rolling these to business days, calculating coverages and discounting each payment etc. To facilitate this we will re-use parts of an extensive library of VBA/Excel functions provided in the course *Fixed Income Derivatives: Risk Management and Financial Institutions* at Copenhagen Business School, with a few notable modifications and additions. Firstly, the functions for calculating IRS swap rates and present values has been modified to handle the case of a varying notional. Secondly, the addition of functionalities for calculating OIS swap rates and present values to properly facilitate the calibration of an OIS curve. Lastly, the cross-currency basis swap functionalities has been modified to properly account for forward and discounting curves in both currencies[19]. The (modified) VBA functions used have been added in appendix C.1.

Figure 9 below illustrate the calibration of the 3M forward OIS and Cibor3M curves, as well as the zero curve representation of each. The data used is that as of April 11th 2018, provided in table 2.

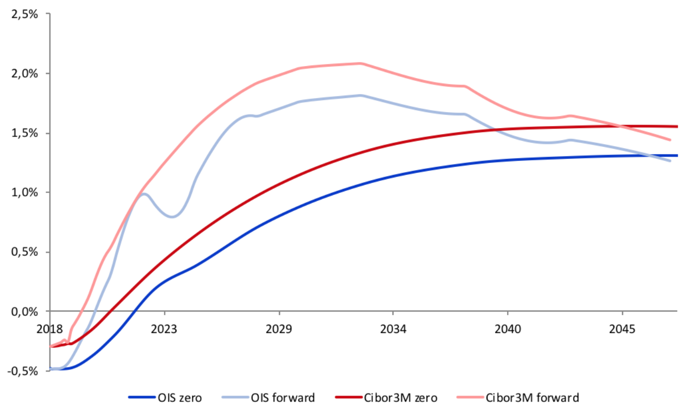

Figure 9: Zero rate and forward 3M calibration examples of DKK OIS and Cibor3M curves as of April 11th 2018.

[18] An example being the Ultimate Forward Rate used under Solvency II to discount long-term liabilities for insurance companies and pension funds.

[19] The legacy function assumed a "domestic" currency, where it was assumed that funding at the apropriate Libor rate was possible. This allowed for a significant simplication in the calculation of par basis swap spreads and present values

In the figure we have not presented the swap rate curves, however it shares similarities to the shape and level of the zero rate curve which is depicted. From equation (7) we found that the par IRS swap rate is a weighted average of the forward rates (similar results hold for the OIS swap rate by examining equation (10)). This is evident from the figure, where we see that when the zero curve increases, the forward curve is above the zero rate curve (to "pull it up").

By definition our model is pricing the input market instruments precisely as observed in the market. A way of checking the "strength" of the model, is to price instruments not included in the calibration, and compare those to market quotes. Examples of instruments could be forward starting swaps or a swap maturity not included in the calibration. The 9Y IRS quote was not included in the calibration, and the observed quote in the market as (of close) of April 11th 2018 was 0.857%. Using our model we price the swap at 0.873%. This difference indicates that our calibration could be refined (not surprisingly), and we could for example include the 9Y IRS in the calibration (but will not).

2.3.5 Single- vs. dual-curve framework

By now we have constructed the dual-curve framework. For interest rate swaps it means that we are using two curves, one for projecting forward rates and one for discounting. For discounting we use the forward OIS curve under the assumption that we are in a "perfect" collateralisation scenario, such as at a clearing house. Under this assumption the OIS curve is the correct discounting curve as explained in section 2.1.3.

However, it was not always like this. Prior to the financial crisis many market participants did not distinguish between seperate forward Libor and discounting curves. Instead they simply used the forward Libor curve for discounting as well. This made sense for two reasons. First of all the spread between the Libor and OIS was small (as evident in figure 1), which induced (what was considered) negligible errors in the pricing. Secondly, clearing of trades (and collateralisation in general) was not as common then and therefore the "collateral earns the OIS rate"-argument was not as strong at the time. Market practice was instead to use a single-curve setup. As the Libor/OIS spread widened and OIS collateralisation became increasingly common, the market standard became the dual-curve setup.

It is therefore important to note, that at the time where the cases we will review later on took place, the danish banks used a single-curve setup, and not the dual-curve setup we have reviewed and which is market practice now. Fortunately, our curve calibration and pricing setup is capable of handling the single-curve framework as well. We may simply choose the forward Libor curve as the discounting curve, when calculating the model swap rates used in the calibration. For most examples in this thesis we will utilize the dual-curve framework, and this will be the default approach. We will clearly state it when a single-curve approach has been used.

To illustrate the effects of using a single-curve framework compared to a dual-curve, we have calibrated both setups in EUR on two dates; April 11th 2018 and January 16th 2009. We will then price a 5Y IRS in both a single- and a dual-curve setup on each date. The 5Y Libor/OIS was 45.1 bps in January 2009 and 10.6 bps in April 2018. This is to illustrate that the discrepancy of using single-curve setup is (naturally) dependent on the Libor/OIS spread. We have chosen to use EUR curves for the following illustration, as a dual-curve setup in DKK is not easily calibrated on dates as far back as January 16th 2009, because CITA swap quotes have proven hard to obtain for all the necessary maturities. No matter what the qualitative result is independent of the currency. For the EUR curves we have used the corresponding EUR instrument to those listed in table 2, however with fixed FRAs; 1x4, 4x7, 7x10 and 10x13 instead of the DKK IMM FRA contracts.

Using the single- and dual-curve calibrations on the two dates, we have priced a 5Y receiver IRS on a notional of 100 mil. EUR. We price the IRS and let the fixed rate change from the par swap rate to par rate + 150 bps. We then calculate the difference in PV between the two setups. This has been illustrated in the table below:

	April 11th 2018			January 16th 2009		
+x bps to par	PV Dual	PV Single	Diff.	PV Dual	PV Single	Diff.
150 bps	7,502,356	7,483,055	19,301	7,056,739	6,954,348	102,391
100 bps	5,001,571	4,988,703	12,868	4,704,493	4,636,232	68,261
50 bps	2,500,785	2,494,352	6,433	2,352,246	2,318,116	34,130
25 bps	1,250,393	1,247,176	3,217	1,176,123	1,159,058	17,065
0 bps	0	0	0	0	0	0

Table 3: The effects on a receiver IRS PV in the two setups in a low Libor/OIS basis scenario (April 2018) and a high basis scenario (January 2009) and as a function "moneyness" of the IRS. Data on both dates is from Bloomberg.

As is evident from the table the discrepancies in the pricing between the two setups is (naturally) higher in the scenario with a high Libor/OIS basis. This is because the error from using the Libor curve for discounting increases. The results above are symmetric around the bump of 0 bps, such that a bump of -100 bps in the April-setup would entail a difference of -12,868. The results are also mirrored for the corresponding payer swap.

As one can imagine the transfer from the single-curve to the dual-curve setup may have affected the value of certain directional swap portfolios. Consider for example a pension fund which is a "natural" receiver in the swap market for liability-hedging reasons. Since 2007/08 there has been a signficiant move lower in EUR and DKK rates. According to a Risk.net article from March 2010, it was assumed that most banks, especially the larger ones, had at that time completed the transfer to a dual-curve framework (Whittall (2010)). At that time our pension fund would likely be in the money on their swap portfolio. As above table illustrates such a pension fund would likely be a net beneficiary of a move from single-curve to dual-curve at that time. Conversely a payer swap portfolio would likely see the (negative) present value of the portfolio decrease further (Nashikkar (2011)).

2.3.6 Possible extensions

Our curve calibration and pricing model is still simple compared to how banks may setup their models. As such there are a couple of extensions one could use to refine the model. On the basic (and crucial) level is the addition of a proper business day calendar for all relevant currencies. We have assumed no holidays in our setup, which obviously will not do for real market participants. Secondly, one might refine the short-term of the OIS curve, by adding the forward OIS rates on days of ECB, Federal Reserve or other central bank meetings. This is to properly reflect the market expectations of future monetary policy decisions, as the reference rate of OIS swaps is typically quite correlated to the policy rate of the central bank (Ametrano & Bianchetti (2013), p. 55-57).

We could also add "turn of year/quarter" effects as described earlier. These effects occur for some instruments when the market rates span special dates, like the turn of the year or quarter. This may be caused by liquidity effects, where e.g. demand for liquidity may cause certain rates to increase. Adding this will further increase the sophistication of the model (Ametrano & Bianchetti (2013), p. 47-50).

2.4 Risk management

In this section we will review how to define and measure risk relative to interest rate changes in our model, and we will give some examples to illustrate.

2.4.1 Model rate delta vector

Now that we are able to price interest rate swaps and other interest rate derivatives, the next natural step is to be able to properly risk manage these. The foundation of our pricing setup is the calibrated zero rate curves. So a natural starting point is to calculate the delta risk associated with these zero rates, that is the effect of the present value when these *decrease*. We have chosen to focus on a decrease in rates (whereas delta risk-measures are usually calculated using an increase in the underlying parameter). This was chosen to follow market practice. A possible explanation of why this is, is that the interest rate world is largely born out of bonds. Therefore market participants want their risk relative to whether bonds increase or decrease. As we know bond prices increase when rates (the yield) decrease, and as such it has become market practice to look at the risk relative to a decrease in rates.

The standard risk measure is to calculate the change in the present value due to a 1 bps parallel shift of the entire zero rate curve. This risk measure is called DV01, short for *Dollar value of a basis point*. We denote the change of rates by a "bump", and by this we mean a *negative* shift of 1 bps of one or more interest rates. In the single-curve setup there is only one zero rate curve to bump, but in the dual-curve setup there is two. Hence we will simply bump each of these by 1 bps. We let $\Pi(\boldsymbol{p})$ be the present value of the interest rate derivative as a function the model zero rates, $\boldsymbol{p}$. We can then approximate the DV01, denoted Δ, by the backward difference quotient:

$$\Delta \approx \frac{1}{10,000} \frac{\Pi(\boldsymbol{p}) - \Pi(\boldsymbol{p} - \epsilon)}{\epsilon}$$

Where ϵ is the 1 bps bumps to the zero rate curve. However this approximation is rather crude, as we know that interest rate curves do not only move in parallel shifts. Instead we will refine it by allocating the total risk of the parallel shift into incremental shifts of each seperate zero rate. That is we will calculate a delta vector, Δ^v, where each element corresponds to the delta risk of a 1 bps decrease in the corresponding zero rate. Formally we may define it as:

$$\Delta^v = \begin{pmatrix} \Delta^v_{1^{\text{lib}}} & \dots & \Delta^v_{N^{\text{lib}}} & \Delta^v_{1^{\text{ois}}} & \dots & \Delta^v_{N^{\text{ois}}} \end{pmatrix}$$

with

$$\Delta^v = \frac{-1}{10,000} \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{p}}$$

$$\Delta^v_i \approx \frac{1}{10,000} \frac{\Pi(\boldsymbol{p}) - \Pi(\boldsymbol{p} - \boldsymbol{\epsilon}_i)}{\epsilon}$$

$$\Delta = \sum_{i=1}^{N} \Delta^v_i$$

Where $\boldsymbol{\epsilon}_i$ denotes a vector with all zero entries except entry i, which has the value ϵ. So to calculate the delta vector, we use the same approximation as for the DV01, but where only element p_i of our two zero rate curves has been bumped.

The implementation of this procedure is as follows; for each knot point on the two curves, we sequentially bump it by 1 bps, and calculate the new present value. We then compare this to the starting point to find the change of present value, which is our delta vector element. Then the zero rate i is "un-bumped", and we repeat the procedure for zero rate $i+1$ etc. We call this vector our *model* rate delta vector, as it is in respect to the model parameters; our zero rates.

Below table illustrates the model rate delta vector for four different swaps, all with a notional of 100 mil. DKK; 10Y payer IRS at market (par swap rate of 0.972%), 10Y receiver IRS at 2.5%, 5Y5Y payer IRS at 0.95% (par rate 1.607%) and a 5Y payer OIS at 0.45% (par rate 0.186%):

Cibor3M	10Y pay IRS at market	10Y rec IRS 2.5%	5Y5Y pay IRS 1.95%	5Y pay OIS 0.45%
3M	57	-57	0	0
(FRA) 5.3M	5	-5	0	0
(FRA) 8.3M	9	-9	0	0
(FRA) 11.3M	3	-3	0	0
1Y	2	-2	0	0
(FRA) 14.4M	38	-38	0	0
2Y	82	-82	0	0
3Y	-297	297	107	0
5Y	-635	635	49,406	0
7Y	-2,384	2,384	-2,551	0
10Y	-95,772	95,772	-95,805	0
12Y	291	-291	443	0
15Y	0	0	0	0
20Y	0	0	0	0
25Y	0	0	0	0
30Y	0	0	0	0
Cib3M Total	-98,602	98,602	-48,400	0
OIS				
(T/N) 2B	0	0	0	55
1W	0	0	0	0
2W	0	0	0	0
1M	0	0	0	0
3M	-1	1	0	0
6M	-6	6	0	0
1Y	-117	274	0	-47
2Y	-164	369	0	-62
3Y	-187	1,014	11	-264
5Y	47	1,319	48	-49,948
7Y	1,242	1,693	-654	11
10Y	1,033	1,875	-831	0
12Y	-175	-75	-12	0
15Y	0	0	0	0
20Y	0	0	0	0
25Y	0	0	0	0
30Y	0	0	0	0
OIS Total	1,671	6,476	-1,438	-50,254
DV01	-97,004	105,151	-49,888	-50,254

Table 4: Model rate delta vector for IRSs and an OIS in a dual-curve setup.

From the table we observe several things. The two 10Y IRSs have the same risk to the Cibor3M zero rates (except for the direction), which is because they have identical floating legs.

The majority of the risk of these swaps is also in regards to the Cibor3M curve. For the payer IRS, lower zero rates means lower forward rates (all else equal). But these are received in the swap, which lowers the present value, hence the negative risk of -98,602.

We also note that the 2.5% receiver IRS, which is deeply in the money[20], has higher risk associated with the OIS (discounting) curve compared to the at market IRS. This is because lower

[20] It has a present value of just under 15 mil. DKK

OIS zero rates causes higher discount factors, which cause future cash flows to be discounted less hard. This effect is greater for the swap which is in the money, as it has net positive cash flows. Whereas the at market IRS has net zero cash flows, is not affected the same way from lower discounting rates.

As expected the OIS swap does not have any risk towards the Cibor3M zero rates. A last remark is relating to the DV01 listed at the bottom of the table above. This is not identical to the sum of the two delta vectors. This is likely due to the non-locality of the hermite cubic spline interpolation method, where bumping one zero rate may affect others. Because of that the parallel shift of the zero rate curve used to calculate DV01 is also affecting other rates, and this is causing a "non-clean" split of the DV01 into the delta vector. The discrepancy is fortunately small, with the largest discrepancy being for the 2.5% IRS payer, where the sum of the delta vectors is 105,078.

2.4.2 Market rate delta vector

The problem with using a model rate delta vector is that we are unable to hedge that exact risk, as we cannot trade zero rates directly. Instead, we may compute a *market* rate delta vector, which tells us the risk relative to changes in the market rates used in the curve calibration. This enables us to hedge the risk we find as these are tradeable rates (instruments).

The calculation of the market rate delta vector is done using the same procedure as with model rates; we bump down the input market rates (as opposed to the zero rates) by 1 bps, and calculate the change in present value of the instrument. However immediately we identify an issue, because bumping the market quotes would require us to re-calibrate our curves to find the effect on the present value. Further, we would have to do this for all market instruments. Obviously we would like to avoid this.

Fortunately, there exist a short cut which we will use (Linderstrøm (2013), p. 25-30 and 41-43). Formally what we want to calculate is the same delta vector as explained in the previous section, but instead of the derivate $\frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{p}}$, we want to use the derivative $\frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{b}}$. That is we want to find the change in value wrt to the model quotes, because we know that in optimum these are identical to the market quotes. We may therefore define the market delta vector as:

$$\tilde{\Delta}^v = \frac{-1}{10,000} \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{b}}$$

Now using calculus we find:

$$\begin{aligned} \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{p}} &= \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{b}}^\top \frac{\partial \boldsymbol{b}(\boldsymbol{p})}{\partial \boldsymbol{p}} & \Leftrightarrow \\ \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{b}} &= \left(\frac{\partial \boldsymbol{b}(\boldsymbol{p})}{\partial \boldsymbol{p}}^\top \right)^{-1} \frac{\partial \Pi(\boldsymbol{p})}{\partial \boldsymbol{p}}^\top \end{aligned} \tag{12}$$

Assuming that the matrix $\frac{\partial \boldsymbol{b}(\boldsymbol{p})}{\partial \boldsymbol{p}}^\top$ is invertible. This matrix is known as the Jacobian, and is defined as:

$$\frac{\partial \boldsymbol{b}(\boldsymbol{p})}{\partial \boldsymbol{p}}^\top = \begin{pmatrix} \frac{\partial b_1^{\text{lib}}}{\partial p_1^{\text{lib}}} & \cdots & \frac{\partial b_1^{\text{lib}}}{\partial p_{M^{\text{lib}}}^{\text{lib}}} & \frac{\partial b_1^{\text{lib}}}{\partial p_1^{\text{ois}}} & \cdots & \frac{\partial b_1^{\text{lib}}}{\partial p_{M^{\text{ois}}}^{\text{ois}}} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \frac{\partial b_{N^{\text{lib}}}^{\text{lib}}}{\partial p_1^{\text{lib}}} & \cdots & \frac{\partial b_{N^{\text{lib}}}^{\text{lib}}}{\partial p_{M^{\text{lib}}}^{\text{lib}}} & \frac{\partial b_{N^{\text{lib}}}^{\text{lib}}}{\partial p_1^{\text{ois}}} & \cdots & \frac{\partial b_{N^{\text{lib}}}^{\text{lib}}}{\partial p_{M^{\text{ois}}}^{\text{ois}}} \\ \frac{\partial b_1^{\text{ois}}}{\partial p_1^{\text{lib}}} & \cdots & \frac{\partial b_1^{\text{ois}}}{\partial p_{M^{\text{lib}}}^{\text{lib}}} & \frac{\partial b_1^{\text{ois}}}{\partial p_1^{\text{ois}}} & \cdots & \frac{\partial b_1^{\text{ois}}}{\partial p_{M^{\text{ois}}}^{\text{ois}}} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial b_{N^{\text{ois}}}^{\text{lib}}}{\partial p_1^{\text{lib}}} & \cdots & \frac{\partial b_{N^{\text{ois}}}^{\text{lib}}}{\partial p_{M^{\text{lib}}}^{\text{lib}}} & \frac{\partial b_{N^{\text{ois}}}^{\text{lib}}}{\partial p_1^{\text{ois}}} & \cdots & \frac{\partial b_{N^{\text{ois}}}^{\text{lib}}}{\partial p_{M^{\text{ois}}}^{\text{ois}}} \end{pmatrix}$$

Which is easily found when doing the initial curve calibration. Using this we may therefore find the market delta vector simply by matrix multiplication of the Jacobian and model delta vector, as evident in equation (12). This shows why it is important to have the same number of market quotes as knot points on the calibrated zero rate curves, that is $N = M$. This ensures that the Jacobian is (almost surely) invertible, and thus eases our calibration of market rate delta vectors significantly, as we avoid the need to recalibrate the curves after each bump.

Table 5 shows the market rate delta vector corresponding to the four swaps from the model rate delta vector example.

As can be seen from the table the 10Y IRSs are only exposed to the 10Y swap rate. This will always hold for any instruments used in the calibration (of the non-discounting curve). It is clearer here that the 2.5% IRS has more discounting risk than the at market swap. Intuitively we may thank of what happened if we were to book an off-setting at market swap in both cases. For at market swap all cash flows would simply cancel out, and hence there is no discounting risk. For the 2.5% IRS the floating payments would cancel out, whereas there would be (positive) cash flows on the fixed leg. It is these payments that are sensitive to discounting.

For the 5Y5Y IRS the majority of the risk is located in the 5 and 10Y buckets. Consider now the 10Y bucket. Bumping the 10Y IRS rate decreases the 3-month forward rate maturing at 10Y, because the swap rate is the weighted average of the forward rates. As the 12Y, 15Y etc. IRS rates are the same, the forward rate *starting* in 10Y must have increased. For the 5Y5Y *payer* IRS the first forward rate is received whereas the second is not, and this is the reason there is negative risk in the 10Y bucket. We have illustrated the idea in below figure:

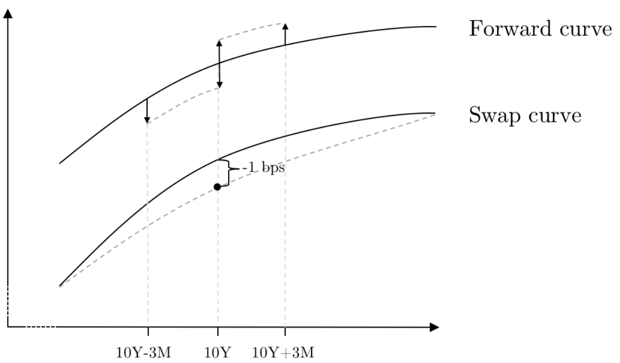

Figure 10: The effect on the forward curve by bumping the quotes on the swap curve.

Cibor3M	10Y pay IRS at market	10Y rec IRS 2.5%	5Y5Y pay IRS 1.95%	5Y pay OIS 0.45%
Cibor3M fixing	0	0	0	0
1st IMM FRA	0	0	0	0
2nd IMM FRA	0	0	0	0
3rd IMM FRA	0	0	0	0
IRS 1Y	0	0	0	0
4th IMM FRA	0	0	0	0
IRS 2Y	0	0	-1	0
IRS 3Y	0	0	69	0
IRS 5Y	0	0	50,042	0
IRS 7Y	0	0	-141	0
IRS 10Y	-98,032	98,032	-98,071	0
IRS 12Y	0	0	161	0
IRS 15Y	0	0	0	0
IRS 20Y	0	0	0	0
IRS 25Y	0	0	0	0
IRS 30Y	0	0	0	0
Total	-98,032	98,032	-47,940	0

OIS				
T/N fixing	0	8	-1	-1
CITA 1W	0	0	0	0
CITA 2W	0	0	0	0
CITA 1M	0	0	0	0
CITA 3M	0	0	0	0
CITA 6M	0	0	0	0
CITA 1Y	0	155	66	-28
CITA 2Y	0	204	87	-37
CITA 3Y	0	816	406	-157
CITA 5Y	0	1,360	282	-51,022
CITA 7Y	0	2,983	-1,950	7
CITA 10Y	0	3,064	-1,962	0
CITA 12Y	0	-266	174	0
CITA 15Y	0	0	0	0
CITA 20Y	0	0	0	0
CITA 25Y	0	0	0	0
CITA 30Y	0	0	0	0
Total	0	8,324	-2,898	-51,237

Table 5: Market rate delta vector of an at-market IRS and off-market IRS and OIS in a dual-curve setup.

By similar argument there is positive risk in the 5Y bucket, and very little risk relative to the 7Y IRS quote, as the effects (roughly) cancel out. The reason the 5Y5Y swap has exposure to the 12Y IRS rate is because of the non-locality of the hermite interpolation method.

In these two sections we have shown the calculation of delta vectors using the dual-curve setup, however the single-curve setup is also contained in this. We could either calculate the delta vectors using a single-curve setup where the OIS rates do not show (i.e. only the upper "square" of each delta vector is calculated). Alternatively we may include the OIS zero rates and instruments as here, and the result would be that the lower square for all the IRSs would contain all zeros for both delta vectors.

2.5 Market participants

In this section we will review the traditional market participants in the swap markets. The statements in this section holds both seen from a global perspective, as well as specific to the danish swap market.

2.5.1 Traditional receivers

Two of the larger players who are typically receiving in the swap market are pension funds and insurance companies. What is common for these, is that they have long-term liabilities. Additionally, banks are also often receiving fixed in the swap. This is to ensure that they do not experience a duration mismatch between their assets and their liabilities.

Consider an insurance company. It receives premiums initially against promising to pay a claim, given some event occuring. This event might occur with certainty but at an uncertain time (such as life insurance), or it might not occur at all (such as fire insurance). Looking at life insurance, the liabilities are long-term in nature, and the future liability amount roughly known in advance. When rates decrease the present value of these long-term liabilities increase, that is the liabilities contribute with a negative duration. To hedge this the insurance company needs increase it's duration, and a way of doing this is to receive fixed in the swap market.

Pension funds are a more interesting case from a danish point of view, due to the relative size of the danish pension funds. According to an OECD report danish pension fund assets were the highest in the world at 209% of GDP at the end of 2016, with the Netherlands second at around 180% of GDP (OECD (2017), p. 6-11). Previously a lot of pension plans were so-called *defined benefit* schemes. In a DB scheme a sponsor guarantees an agreed pension payment based on the employees wage history, tenure and age, and often as a life annuity. This means the the liability of the future pension payments lie with the sponsor, who has made the guarantee. Given a predefined contribution profile to the pensions, this requires the sponsor to obtain some known investment average return over the course of the pension scheme contribution period. When interest rates decrease it is disadvantageous for the sponsor, because it causes the present value of the future liabilities to increase. This may cause an asset and liability mismatch, and stress the funding level of the pension fund. Alternatively, we think of the lower rates as lowring the expected rate on return on certain investments such as bonds. This makes it harder for the sponsor to reach the required average return. These issues may be solved by entering into a receiver swap. This can offset the interest rate sesitivity of the liabilities and "guarantee" a certain return on the investments for the pension fund (assuming the alternative is an investment at the corresponding floating rate).

As a result danish pension funds has been receiving a lot in the danish swap market (Dengsø & Sixhøj (2015)). An example is the danish pension fund ATP, which is a collective fund eligible for most danish citizens. It is known for employing a very strict duration hedging strategy of its defined benefit scheme. According to the financial statements of ATP for the year 2017, the value of the guaranteed benefits ended the year at 651 bln. DKK. These assets were (are) hedged almost fully, and 51% of that was done using interest rate swaps, with 15% being in DKK denominated and 36% in EUR denominated swaps (ATP (2018), p. 10 and 27-29).

Increasing longevity risk and increasingly disadvantageous market conditions for the defined benefit scheme has caused a significant transfer of pension policies from defined benefit into defined contribution schemes. DC pension schemes are more traditional saving accounts where the employer and/or employee contributes to a personal account which is then invested. The benefits received at retirement are no longer guaranteed, but depends on the contributions and on the investment returns. Importantly, the employer or sponsor has no obligations further than the contributions. According to The Danish Financial Supervisory Authority, Finanstilsynet, the amount of contributions to defined contribution schemes increased from approximately 9% in 2004 to 62% in 2015 as a percentage of total contributions. Additionally the total assets with no (or a 0%) guarantee increased from 19% in 2010 to 53% in 2015 (Finanstilsynet (2017), p. 8-10 & Andersen (2016)). This transfer indicates that the necessity of danish pension funds to hedge long-term liabilities, and thus being a structual receiver in the danish swap market, may be decreasing.

Banks are often also receiving in the swap market. The treasury department of a bank issue bonds to finance their operations. To meet investor demand, these are often issued with a fixed coupon. As an example aprox. 140 bn DKK of the 166 bn DKK issued benchmark bonds of Danske Bank as of May 2018 was fixed rate bonds, and the remaining was floating rate issues (Danske Bank (2018)). However much of the lending banks do to e.g. corporates are floating rate loans referencing for example a Libor rate. A significant part of the lending operations to corporates is by giving them access to credit facilities, on which the majority is also paying a floating rate[21]. This means that initially the duration of the liabilities are higher than those of the assets. To reduce this duration mismatch, the bank may enter into a receiver IRS, which decrease the duration of the liabilities. Ensuring (approximately) the same duration of the assets and liabilities makes the bank less vulnerable to adverse interest rate movements.

2.5.2 Traditional payers

There exist several participants in the swap market which may be defined as structual payers, so with a clear tendency to pay fixed in the swap. The first is a group consisting of asset managers and hedge funds, which we will collectively denote portfolio managers. Both are investors, and have in common that they both hold large bond portfolios. What seperates the two, is that hedge funds are typically leveraged, whereas asset managers are typically long-only investors like investment funds or defined contribution scheme pension funds. Their bond portfolios has a lot of duration, which makes them vulnerable to higher interest rates. To reduce the portfolio duration, they may pay fixed in the swap. Interestingly, Denmark has quite a few hedge funds which fall under this category. Strictly speaking they are placed abroad, in places like Luxembourg or the Cayman Islands, but they are managed by danish asset managers often with funds primarily from the same asset manager. These funds exist due to the relatively large and AAA rated danish covered bond market, one of the largest in the world, as well as the reasonable liquidity in these (for covered bonds). This makes for good conditions for the swap spread strategies often deployed by these funds.

The second group consists of corporations (corporates) and real estate investors and developers. What is common for these is that they have a need for funding, which they may undertake with a bank[22]. There exist other parties which may be put within this category, particularly in a danish context; farmers, municipalities and housing cooperatives ("andelsboligforeninger"). As these are of particular interest for this thesis, they will instead be covered in the next section. As we know the bank loans provided for corporates and real estate are often floating rate loans, referencing e.g. Libor and possibly with a spread added (Délèze & Korkeamäki (2018), p. 1 and IPF Short Paper 22 (2015), p. 1-4). In the case of corporates, the floating loan may also be obtained by issuing floating rate corporate bonds. If the corporate wish to convert this floating rate loan into a fixed rate loan, it may do so by entering into a payer swap. A reason the corporate may prefer a fixed rate loan, could be due to a preference for certainty wrt future expenses. It might also be to remove interest rate risk, or due to speculation on rising interest rates. The corporate may also enter into payer swap, if it knows it will have a funding need in the future and at that time borrow from the bank or issue floating rate bonds. To fix the future funding level it can enter into a forward payer swap.

[21]This has been confirmed by a professional at one large danish banks, who are working with corporate lending advising.

[22]We assume here that real estate refers to the case of investing in real estate by taking a (mortgage) loan.

2.6 Danish swap scandals

In this section we will give an overview of the danish swap "scandals" that is of interest in this thesis. The word "scandal" is subjective in nature, but here we will simply use it to refer to the cases that have received significant media attention in recent years. We start by giving an overview of the mechanics and why these cases originated in section 2.6.1. Next we move on to explaining what the consequences for the different types of customers was of having an interest rate swap with a large negative value in section 2.6.2. We then give an overview of the criticism raised against the banks, as well as some examples of cases in sections 2.6.3 and 2.6.4, respectively.

2.6.1 The involved parties and the mechanics

In this section we will review the parties implicated in what we collectively refer to as swap "scandals". Those parties we will mean primarily to include farmers, municipalities and housing cooperatives. What is common for these is that they prior to the financial crisis took (or had) a floating rate loan and swapped this into a fixed rate loan by entering into a payer swap. As a result of the falling interest rates they lost a substantial amount of money on the swap contract. Obviously not all of this type of borrowers entered into this loan construction but quite a few did.

The financial outlook of that time is important to properly describe the incentives of these borrowers. Since 2005 interest rates had increased steadily as evident in figure 11 below. This figure shows the 10Y IRS rate against Cibor6M, the F1 rate and the Cibor6M rate. We have chosen the Cibor6M as quotes for Cibor3M was not easily obtained for longer maturities so far back, and the qualitative results are the same nonetheless. In figure 12 we have calibrated a Cibor6M forward curve and shown the IRS and 6M forward rates. The calibration has been done using data as of 30th of June 2005 and 2006, and using a single-curve setup. As we see in both scenarios the market is pricing increasing interest rates, although the swap curve has flattened significantly in 2006.

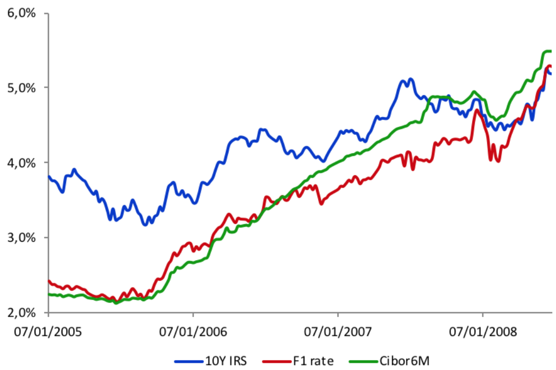

Figure 11: Increasing rates 2005 to 2008, shown here by the 10Y IRS against Cibor6M, the F1 rate and the Cibor6M index. Data is from J.P. Morgan Market's DataQuery.

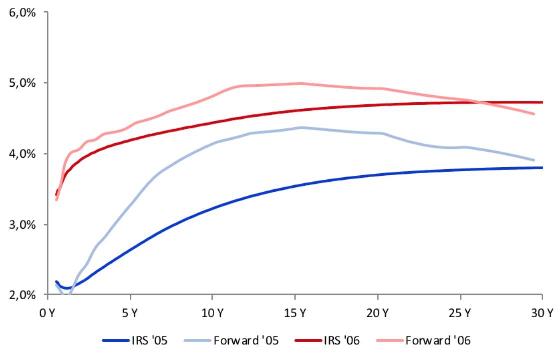

Figure 12: Calibration examples on 30th of June 2005 and 2006, illustrated by the swap curve and forward rate cuve, both against Cibor6M. Market quotes for the calibrations is from Bloomberg.

All of this illustrates that in the years 2005 to 2007 when most of these cases originated, there was a market wide expectation of increasing interest rates. In anticipation of that, our borrowers tried to hedge against that scenario by converting their floating rate loans into fixed rate using a payer IRS. The floating rate loans are usually the shortest 1Y *flex* loans (called F1 loans), where the interest rate is reset every year. It could also be "true" floating rate loans, where the interest rate is linked to e.g. Cibor3M og the 6-month CITA rate[23]. Entering into a payer swap thus creates a synthetic non-callable, fixed rate loan. Typically danish mortgage loans ("realkreditlån") are issued as 30Y loans. There does not exist actual 30Y non-callable, fixed rate loans. The longest non-callable loan is the F10 loan, in which the rate is reset every 10th year, however this loan is rather uncommon. Instead most non-callable loans are issued as F1, F3 or F5 loans. In the danish market there is a 30Y *callable* fixed rate mortgage loan ("konverterbare"). The callability of these refer to the borrower always having the option of buying back the underlying bonds at par value (100). This optionality means that for the same coupon rate, the callable bond has a lower price compared to the (theoretical) non-callable version[24]. Using the payer swap thus enabled the borrower to pay a lower fixed rate, than if they instead had converted into a callable loan, because the borrower did not have to "pay" for the imbedded option of the callable loan.

As we know interest rates did not rise as expected, but instead started falling in 2008. As a consequence the present value of the payer swap became (increasingly) negative. An important remark is that this negative present value corresponds more or less to the corresponding loss incurred on the non-callable loan (had it existed), as the price of the underlying bond would have increased. Here "loss" is defined as the difference between the price that the bond underlying the loan is sold at, and the current price of the bond, if above the initial price, and similarly a gain if the current price is below the initial. In danish *kurstab*. The borrower would not have experienced this issue if they had instead chosen the callable loan. As interest rates dropped the price of the underlying bond would (theoretically) increase above par value. But due to the callability of the loan, the loss would be confined to the difference between the initial issue price and par value.

2.6.2 The consequences

Having this "visible" loss due to the negative value of the swap has had different consequeunces for the stereotypical borrowers. For the housing cooperative and the farmer there is similarities. For these it is really a matter of how the swap is valued and introduced in their financial statements. The swap is marked-to-market, and the negative value is explicitly stated in the accounts, and is therefore reducing the equity correspondingly. This means that the loss is accounted for immediately and affects all other financial decisions, such as whether they can obtain new loans, restructure the current one, access to credit facilities etc. Obviously the loss is not realized until it is decided to liquidate the swap, but the unrealized loss still greatly affects the borrower through the reduction of the equity. Conversely, had the non-callable loan existed, then the borrower would not be affected in the same way. In that scenario the loss is because the issued (sold) bonds have increased in value. However, this is not visible in the financial statements, except in the case where the inventory principle[25] of accounting claims is used. So the non-callable loan would typically not affect the borrower in the same way. They would still have a "loss" because, they are paying "too high" a fixed rate relative to the rate of new loans, however

[23] An example of the former is BRFKredits *RTL kort* loan, and of the latter is Realkredit Danmarks *FlexKort* loan. BRFKredit and Realkredit Danmark are two danish mortgage providers.

[24] Often this is approximated using danish government bonds, which are non-callable and fixed rate, however not with a maturity of 30 years.

[25] A direct translation of the danish name *lagerprincippet*

fixing the interest rate was the idea in the first place. For housing cooperatives and farmers, the significantly reduced equity has in several instances brought the borrower close to or in default. As a result this has received much media attention in the danish press. It is estimated that around 2,500 danish farmers and more than one in four housing cooperatives whith mortage loans has combined it with an interest rate swap (Nielsen (2016) and Erhvervsstyrelsen (2018)).

The danish municipalities are different. They cannot default in the same way as a farmer or housing cooperative[26], and the consequences of having agreed to the payer swap is thus "just" that they experience a loss on the contract. Due to their state-like status, they do have access to the capital necessary to liquidate such contracts. This is opposed to the farmers and housing cooperatives who often do not have access to liquidity to terminate the swap contract. According to the danish Ministry of the Interior and Health[27] the total notional outstanding in payer interest rate swaps contracts for danish municipalities at the end of 2010 was approximately 10.7 bn DKK (Indenrigs- og Sundhedsministeriet (2011), p. 29-31).

2.6.3 Criticism and lawsuits

The substantial losses incurred by the three types of borrowers due to interest rate swaps has generated a lot of criticism towards the banks, as they advised their clients and sold them the contracts. As a result a number of lawsuits has been filed against the banks and from all three types of borrowers. In this section we will shortly review some of the points of criticism, and next give examples of some lawsuits. An remark regarding the criticism against the banks, is that based on our reasearch it is not clear that any single bank has acted particularly "aggressive" in selling swap contracts, compared to the other danish banks. We have found lawsuits mentioned in articles against almost all major banks[28]. Any pattern to be found in the remainder of this chapter and thesis, is thus by chance and not an attempt to single out any one bank.

The criticism of the banks is generally aimed at poor and lacking advise. Generally the borrower (and plaintiff) claims that they were not made fully aware of all aspects of entering such a contract, including the consequences it would have in every scenario. As a result several parties believes that the banks were at fault for their losses due to miscounseling, and filed lawsuits to seek full or partial reimbursement from the bank. There is a number of specific aspects and dynamics that has been raised by the affected parties, which they claim they were unaware of. We will now review these, however it will not be a thorough review, as several of these will be given further attention in the next chapter.

One of the central points of criticism is whether the banks fully advised on the potential negative effects a swap contract could have for a housing cooperative and farmer, if the value were to turn negative (Dengsø (2015)). As the banks are selling these contracts to non-professionals, it has been argued that it is their responsibility to ensure that the councelling has been thorough and sufficient, and also to ensure that the customer actually understood this.

Another argument raised was that the customer was unaware of how the fee inherent in the swap was constructed. The swap rate quoted by the bank is typically based on the mid-market quote, and to this a spread is added. This typically includes a bid-ask spread, a spread for the sales desk and a xVA spread reflecting the expected costs and risks of trading with that particular client. This means that once the customer enters into the swap contract it has a negative value

[26] We disregard the technicalities of this, and just assume, as is also often done with states, that they cannot default.

[27] The ministry at the time, in danish *Indenrigs- og Sundhedsministeriet*

[28] The only "Group 1" bank (as defined by The Danish Financial Supervisory Authority) who was *not* mentioned in lawsuits was Sydbank.

at the beginning. The negative value reflects the present value of paying this spread for the remainder of the contract. Even if the swap is terminated before it matures, the remaining value of the future spread payments is still paid. This is due to the unconvertible nature of the swap contract. This means that the effective spread for the client was potentially large, especially if they terminated the contract after a few years. This is in contrast to the typical fee structure of a regular mortgage loan, where the fee is yearly and no longer paid if the loan is terminated or restructured (although fees are then paid on the new loan). This was particularly raised by municipalities, who often traded several swap contracts to adjust their hedge, as opposed to the other two clients who typically entered into a single contract (Finans (2014) and Dengsø (2014)).

Several parties have also complained about being unaware of the convexity of the swap contract, which is not in their favor as they are paying fixed. That is given an initial interest rate level and symmetric interest rate changes around this, then the loss experienced as interest rates drop is larger than the gain when interest rates increase. This has been explicitly used as an argument in several lawsuits (Dengsø (2016)).

On the basis of the above criticism and the many lawsuits The Danish Financial Supervisory Authority published a set of guidelines in 2016. This highlights what aspects that should be included in any marketing material and advise toward non-professional investors regarding interest rate swaps (Finanstilsynet (2016)). The above points are all explicitly included in this.

2.6.4 Examples of lawsuits

In this section we will give a few examples of some of the lawsuits filed against banks regarding their advise in the sale of interest rate swaps.

One of the most discussed cases is that of the housing cooperative Hostrups Have and Nykredit. Hostrups Have was Denmarks largest housing cooperative placed in Frederiksberg with around 680 apartments. It was established in 2007 and at the time obtained a floating rate loan with Nykredit as well as at least two payer interest rate swaps. This led to substantial losses for the cooperative in the following years. At the end of 2016 they had loans for around 1.1 bn DKK plus swap contracts with a negative value of 900 mil DKK, while the property was valued at around 750 mil DKK (Sixhøj (2016)). In 2015 the the cooperative filed suit against Nykredit for poor advise with regards to entering into the swap contracts. Nykredit declared the cooperative bankrupt in December 2016, as the residents did not meet their obligations. Nykredit sold the property in June 2017 for 1.7 bn DKK, and ended up with a loss of around 250 mil DKK on the loan and swaps, compared to the loan principal and the market value of the interest rate swap. The outcome of the lawsuit filed is uncertain. It has not been possible to find any articles mentioning that the lawsuit has been dropped, and it does not appear that it has gone to trial neither. However, looking at the financial statements of Nykredit for the year 2017, it indicates that the lawsuit has been dropped, as signficant provisions has been canceled (Nykredit (2018), p. 10 and note 42 p. 113).

Another lawsuit which has received significant interest is that of the housing cooperative Engskoven against Jyske Bank. Engskoven is reciding in Skødstrup, north of Aarhus. It was founded in 2005 and is comprised of 22 apartments. They obtained a floating rate loan of 30 mil DKK in 2006 and entered into a payer interest rate swap with Jyske Bank, due to concerns about interest rates increasing further. The negative value of the swap in June 2016 was 18.3 mil DKK, on the remaining part of the swap (Højesteret (2017), p. 2). As the cooperative was risking default, they filed suit 2012 against Jyske Bank for poor advise, with the aim of having the contract cancelled. Before that they had had their case affirmed by The Danish

Financial Complaint Boards[29], but Jyske Bank refused to comply with the outcome, giving the cooperative the opportunity to file suit. The cooperate were successful at the City Court of Viborg in February 2014. The verdict was that the advise from Jyske Bank was not fulfilling in explaining all aspects and risks related to the swap, and the cooperative were found not obligated to uphold their contract with Jyske Bank (Byretten i Viborg (2014), p. 51-53). This result was appealed by Jyske Bank and moved to the Western High Court. Here Jyske Bank was acquitted in November 2015, with the verdict from the District Court being overturned. The Western High Court agreed with the conclusion of the District Court, that the advise from Jyske Bank was lacking and unsatisfactory. However, their verdict relied heaviler on passiveness from the cooperative, which had already in 2009 the opportunity to object to the validity of the swap contract but refrained from this. Also because the loss from the contract had not been realized, Jyske Bank was acquitted (Vestre Landsret (2015), p. 20-24). As the lawsuit was principal in nature, the outcome would establish precedence for future lawsuits, and the cooperative was allowed appeal of the verdict to the Supreme Court. Verdict was given here in September 2017, and the verdict of the High Court and the acquitting of Jyske Bank was ratified (Højesteret (2017), p. 7-19).

The last lawsuit we will review is that of the municipality Haderslev Kommune against Nordea. The two parties had agreed on an arrangement where Nordea was special financial advisor to Haderslev. As a result Nordea advised Haderslev in the years 2007 to 2012 to enter into several interest rate swap contracts, as well as other derivatives, in order to risk manage their portfolio of loans. These trades were at first done with Nordea, and after the annulment of the advisory agreement, trades were done with Deutsche Bank (Jensen & Rangvid (2013), p. 14-16). Haderslev Kommune filed suit against Nordea in February 2014, where they argued lacking and inadequate advice. Especially the fee structure was criticised by Haderslev. They claimed they did not learn about this until later on, and therefore argue the advice from Nordea to trade several swap contracts as lacking. Due to this Haderslev Kommune required reimbursement of part of the losses. The total required reimbursement was 98 mil DKK, while the realized losses due to the swap contracts was totalling around 200 mil DKK (Sixhøj, M. (2014)). The lawsuit has not been settled as of May 2018.

[29]In danish *Pengeinstitutankenævnet*.

3 Interest rate swaps: The customer's perspective

In this chapter we will review the interest rate swap as seen from the customer's perspective. We will focus on the customer relevant for this case; that is in a danish context a borrower who has a F1 flex loan and chose to convert this into a synthetic non-callable, fixed rate loan by entering into a payer interest rate swap with their bank. However several results will be possible to generalize to other types of customers. In section 3.1 we will review the applications of an IRS and the main motives for entering into a payer IRS. In sections 3.2 and 3.3 the primary and secondary risk factors regarding interest rate changes are studied; duration and convexity. We then look at some of the basis risk applicable in this scenario in section 3.4. We end the chapter by reviewing liquidity risk and the risk associated with the spread in sections 3.5 and 3.6, respectively.

3.1 Applications

In this section we elaborate on what the uses of interest rate swaps are for the type of customers we are focussing on, as was shortly mentioned in section 2.6.1. Assuming we keep to our type of customers; housing cooperatives, farmers and municipalities, the use of payer interest rate swaps in all cases is to convert a floating rate loan into a synthetic non-callable, fixed rate loan. This is achieved by having the floating leg of the swap match that of the floating rate loan (as well as possible).That way the customer ends up paying the fixed rate in the swap contract instead.

The motive for converting the loan to a fixed rate is mainly one of two reasons, depending on the customer. For the cooperatives and farmers it was mainly to hedge against the risk of higher interest rates. It is obvious that if a party has obtained a floating rate loan and fear that interest rates will increase, then it is possible to hedge that risk by converting to a fixed rate loan. In the case of municipalities and corporates the wish to convert to a fixed rate loan was (and is) often to obtain some certainty with regards to the interest rate expenses. For corporates liquidity is often limited, and so they want avoid unexpected interest rate expenses. So to some degree it was with budgetting in mind. Another aspect was also to avoid unnecessary risk in an area that is not within ones expertise.

No matter the motive, the objective of obtaining a fixed rate loan could in most cases have been achieved by (converting into) a callable loan. This is an option in most cases where the loan is to finance property. However for corporates wanting to finance a purchase of equipment or similar assets, then the callable loan is typically not available. Instead they would have to settle with e.g. a fixed rate bank loan (if that is available). An exception is danish municipalities who appear to have a wider scope of projects they may finance with danish mortgage loans, including the callable loan (Økonomi- og Indenrigsministeriet (2015), p. 57). The reason the customers did not choose this option, was that using the payer swap to create a synthetic loan yielded a lower fixed rate than that of the callable loan. This is because in the callable loan the borrower obtains a call option on the underlying (non-callable) bond, which they may at all time buy back at par value, 100[30]. If interest rates drop, the value of the underlying bond increases. At some point beyond 100, at which the option is in-the-money and offsets the higher bond price. Obviously, the investors buying the bond will have to be compensated for this, which is done with a higher coupon rate on the callable bond[31]. Alternatively the borrower may choose a callable bond with

[30] In reality the mortage issuer has to be informed of this two months prior to the quarterly interest payments.

[31] This is a simplification. Instead it is related to the fact that callable loans are always issued at a price below 100.

a low coupon rate. But that would entail a greater loss for borrower, as that bond is issued at a price below the "on the run" coupon bond[32].

We have in figure 13 below shown the coupon rate of the on-the-run 30Y callable bond, as well as the 30Y par swap rate for the IRS against Cibor6M[33]. This is to *roughly* illustrate the difference in the fixed rate paid in a callable loan compared to the synthetic loan using the interest rate swap. As we see the fixed rate of the IRS is generally well below that of the callable bond. We will later learn that the difference was in fact another due to the fee spread (which includes xVA costs) and the spread between the Cibor12M rate and the F1 rate.

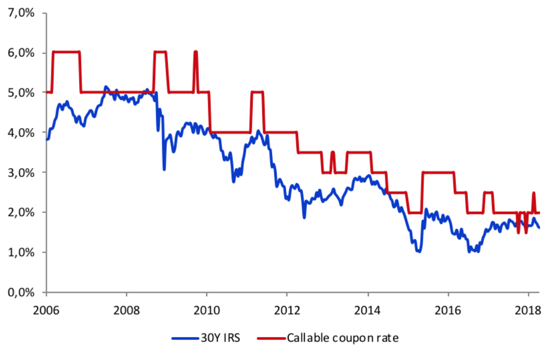

Figure 13: Illustration of the par swap rate of a 30Y IRS against Cibor6M, compared to the coupon rate of the "on-the-run" callable bond. Data is from Bloomberg.

Another point regarding the choice of the syntheic loan over the callable, is that even if the on-the-run callable bond is chosen, the borrower will still experience a loss as the bond is sold at a price, that will always be below 100[34]. This means that the customer will either have to finance the remaining part by other means, or alternatively they will have to adjust the principal of the loan above what they actually need to borrow. As an example assume the callable bond is trading at 98.04. Then the principal of the loan will have to be 102 to receive $102 \cdot \frac{98.04}{100} = 100$, where we have divided the price by 100 to get it in percentages. This issue of the callable being sold as a discount bond, is not experience with floating rate bonds as they are so called *cash loans* ("kontantlån"), meaning that the customer (almost) receives the full principal of the loan. This is another factor in favor of choosing the synthetic fixed rate loan.

It could be argued that given the consequences the customers face if interest rates drop, it would in many cases only make sense to choose the synthetic loan, if the scenario of (much) lower interest rates was disregarded as being unlikely, or if the customer was unaware of the full extend of the consequences they faced in that scenario. Because all else equal, the callable loan option would give considerably better "downside" protection. This holds especially for customers like the housing cooperatives and farmers who are sensitive to the negative value of the payer swap due to the risk of default it poses. On the other side, for a borrower who has no intention of converting or re-structure the loan, and (importantly) who is not sensitive to a potentially negative value of the swap, then the choice is less obvious. In that case, as they have no intention of converting the loan and exercise the imbedded option in the callable loan, then it might make sense to exploit the lower fixed rate of the synthetic loan.

[32]We define "on-the-run" to mean the highest coupon callable bond trading below 100. This is a crude definition, as in reality the callable bond series close before the price reaches 100.

[33]We used Cibor6M as swap quotes against Cibor12M was not readily available. This should not affect the graph significantly.

[34]That is, it is always sold with as a "discount bond".

We have covered a few, but obviously theres exist other uses of interest rate swaps as well. One of these is the use of interest rate swaps as a risk management tool for the management of debt. This is a more active approach to alter the risk profile of the debt portfolio. This may be where the borrower repeatedly risk manage using swaps. Using interest rate swaps in this regard enables the user to quickly and efficiently modify the risk profile in the desired way, and in ways that a typical conversion of loan is unable to. It is clear that using interest rate swaps in this regard requires a certain "size" and sophistication of the end user, and will not be available to regular people, but instead for example municipalities and corporates. Some may argue that this is simply speculation.

3.2 Primary risk factor: Duration

As is well established by now, the primary risk factor of the synthetic non-callable fixed rate loan is duration, that is a change in the level of interest rates. We have tried illustrating this in figure 14 below. This is not based on actual data but is to illustrate the concept. On the x-axis we have changes to the interest rates, which we can think of as parallel shifts of the interest rate curve. On the y-axis we have the value as seen from the customers perspective. For the swap it represents the market value. For the loans, as they are a liability, we represent them by negative values. Hence the higher up on the graph the better for the customer, as the value of the debt has decreased.

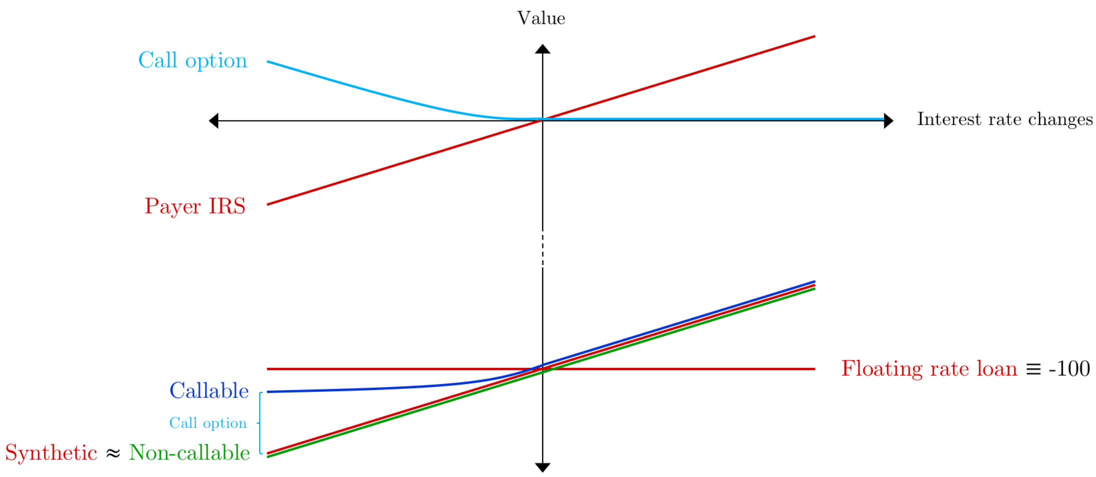

Figure 14: Illustration of the effect of parallel shifts of interest rates on the value of callable and non-callable loans.

The value of the payer interest rate swap is illustrated at the top of the figure as a straight line, with the value decreasing as interest rates drop. In the next section we will learn that the value function is in fact not a straight line, but a concave one[35]. The borrower is therefore short duration in the swap, as the swap is losing value, when interest rates decrease. We have assumed that the floating rate bond underlying the loan is always trading around par, such that the loan is always valued at -100 (assuming a principal of 100). The synthetic loan is found by combining the floating rate loan and the IRS. This is approximately equal to the (theoretical) non-callable fixed rate loan, illustrated by the red and green lines. The callable loan is illustrated by the dark

[35]The exception is if the x-axis corresponds to par swap rates, and we assume discount rates remain constant. In that scenario the value function of the swap is in fact a straight line, as can be seen from equation (8).

blue line. As interest rates drop the value of the loan decreases, however it is "capped"[36] at a certain level, illustrating that the option to convert the loan is now "in the money". As rates drop the increase in the option value offsets the decreased value of the theoretically underlying non-callable bond (the green line). It could be discussed if it is correct to illustrate this by letting the blue line drop below -100. We may think of it as being because the borrower had to obtain a loan with a principal above 100, because the callable bond being issued at a discount. Theoretically a callable bond should not trade above a price of 100, because as soon as the value increases above this the borrower is expected to convert it. However, in reality this does not hold and convertible bonds frequently trades at values of 104-105 or even more. This is due to different factors undermining the idea of *homo economicus*, such as the costs of converting a loan and "woodhead" borrowers acting irrational (Andersen et al. (2015)).

Therefore for the borrower who obtains a synthetic loan it is the combination of being short duration *and* a non-callable loan which defines the primary risk profile towards interest rates.

3.3 Secondary risk factor: Convexity

When interest rates change it affects interest rate swaps in two ways. The first was covered in the previous section and is the primary effect. It refers to the change in the projected forward payments which is either paid or received in the swap. The secondary effect is convexity and is the effect that changing discount rates have, which we will review now. Using equations (4) and (5) it is the effect on the present value of the IRS through $P^{\text{disc}}(t, t_i)$. As discount rates increase the discount factor decreases and we say that future cash flows gets "discounted harder". In a *payer* IRS the customer is positioned for higher rates overall, because the customer is receiving the floating payments. However, as rates increase these future cash flows also gets discounted harder, and so the discounting effect works against the party paying fixed. Consider instead lower interest rates. In a payer swap this lowers the expected future cash flows, but lower discount rates *increases* the discount factors. As a result the negative value of the future cash flows becomes increasingly negative. So in a scenario of falling interest rates the discount effect also works against the party paying fixed in the interest rate swap.

In below figure we have illustrated the convexity concept of IRS. Using the market data as of April 11th 2018 from table 2, we have priced a 10Y at market payer and receiver IRS with a notional of 100 mil DKK. We have then changed the zero curve of both the forward Libor and the forward OIS curve[37] by X bps represented by the x-axis. For reference we have added a straight line to emphasize the non-linear profile of the PV functions.

Using the data for above figure we see that for the payer IRS, a drop in (zero) rates by 1%-point induces a loss of -10.2 mil DKK, while an equivalent rise in interest rates only produce a rise in the present value of the swap of 9.2 mill DKK. This illustrates the convexity concept.

As payer and receiver interest rate swaps are mirror images, it comes as no surpise that while the convexity works against the payer swap (the payer swap is *short* convexity), the receiver swap conversely benefits from the convexity. In a receiver swap you are positioned for lower interest rates. As rates drop the future expected cash flows increase. The lower discount rates increases the discount factors and thus increases the present value of these higher future cash flows. Conversely higher interest rates means expected negative cash flows in the future, but this is also discounted harder, decreasing the negative value of the swap.

As we know from section 2.4.1 the delta risk, DV01, of an interest rate swap changes with

[36] Or *floored* depending on the perspective.

[37] We chose the model zero rates instead of the market rates, as it is much simpler to work with.

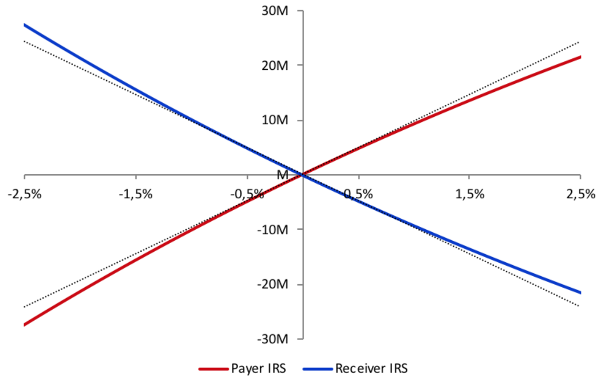

Figure 15: Convexity of payer and receiver IRS.

the level of interest rates (as well as other factors such as time to maturity on the swap, the fixed rate etc.). It turns out that the convexity of an IRS also changes with the level of interest rates. Generally the higher interest rate level the less effect convexity has. We have tried illustrating this in below figures. Here we have priced the same payer IRS as in the previous figure, using market data for April 11th 2018. In addition we have created a new scenario using the same market data but added +500 bps to all market rates. Essentially we have assumed that the shape of the swap curves stays the same, and then induced a parallelshift of 500 bps. In this high(er) rates scenario we also priced the same IRS, and then subsequently added X bps to the model zero rates in that scenario as well. As it can be hard to see convexity in the two scenarios from the first figure, we have produced a second figure which shows the difference in present value to the straight line. The straight line is the change in value assuming zero convexity.

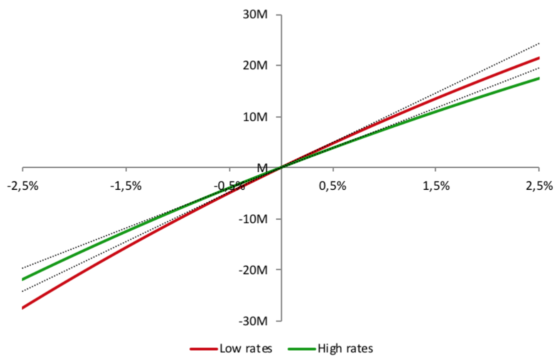

Figure 16: Payer IRS present value at parallel shifts of the zero rate curves, in both a low rates and a high rates scenario.

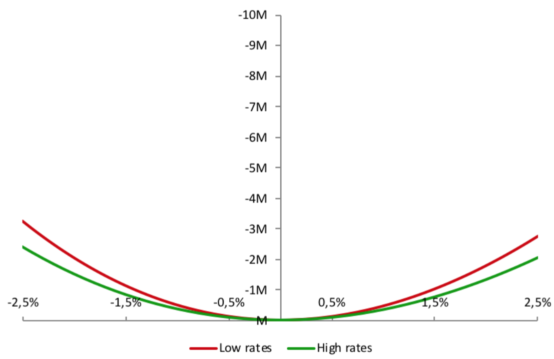

Figure 17: Convexity in a low and high rates scenario, measured as the deviation from the tangent at 0. Note that the y-axis has been inverted.

As we can see the convexity of the payer swap in the high rates scenario is less pronounced compared to the low rates scenario. As an example the deviation from the straight line in the low rates scenario when rates drop by 1 %-point is -496.000 while it is only -367.000 in the high rates scenario. At a increase in rates by 1%-point the numbers are -465.000 and -344.000, respectively. Obviously above present value functions are not convex but instead *concave*, however we use the term convexity to refer to the non-linearity of the functions. The concavity is because the payer swap is short convexity, whereas a receiver swap is long convexity, and therefore has a convex value function as evident in figure 15 above.

In the theory of bond pricing we also have the concepts of duration and convexity. However, here the terms are usually defined relative to changes in the bond's *yield to maturity*, instead of a general change of interest rates as we have done. Buying a bond you are (typically) long both duration and convexity, and opposite when you have sold a bond short. Taking e.g. a mortgage loan in Denmark you are effectively selling short the bonds underlying the loan. This explains why we are short duration and convexity when using the payer IRS to replicate a non-callable fixed rate loan. While you are typically short convexity when you sold a bond, the callable loans examined in the previous section does not comply with this general rule. In fact at certain "low" interest rates[38] the borrower will actually experience positive convexity. Conversely the investor who bought the callable bond underlying the loan will experience *negative convexity*. This is due to the non-linear payoff profile of the call option the borrower implicitly bought when obtaining the callable loan, as can be seen in figure 14.

3.4 Basis risk

In this section we will review basis risk, which we define as the risk related to using an imperfect hedge. For our purpose we will assume the borrower hos obtained a F1 flex loan and entered into a payer swap contract to pay a fixed rate instead. The interest rate in the F1 loan is reset once a year. This occur at an auction in which the mortgage providers, on behalf of the borrowers, sell new 1Y bonds to refinance the loan. The price the bonds are sold at determines the interest rate paid by the borrowers for the coming year. We will assume the IRS is against Cibor12M. Therefore basis risk is related to how well the floating payments in the swap is a hedge for those in the F1 flex loan.

3.4.1 Spread risk: Cibor vs. F1

The first type of basis risk we will consider is the spread risk between the Cibor12M rate received in the swap, and the F1 rate paid in the floating rate loan. The borrower is exposed to the risk that the Cibor12M rate fixes below the F1 rate such that the spread is negative. If this happens the borrower will have to pay the difference on top of the fixed rate for the next interest period (one year in this case). Alternatively, if Cibor12M is above the F1 rate the borrower receives the difference. In that scenario the interest paid on the fixed leg of the swap is lowered by this spread. In figure 18 below we have shown the Cibor12M rate, F1 yield to maturity, and the basis between the two on a weekly frequency from 2006 and until April 2018. We have approximated the F1 rate by using the yield to maturity of the flex bond with a maturity closest to 1Y[39]. As these bonds are issued on quarterly basis this usually means a bond with a maturity of 1Y and $\pm$1.5M is used. The data is extracted from Bloomberg, and prices as far back as 2006-2009 are for some bonds not readily available. As such for certain shorter periods we have been forced to use "crude" approximations, and use e.g. a bond with a maturity of 1.5Y.

[38] We will not define exactly what this means, but we may think of it as somewhat lower than those present when the borrower entered into the callable loan.

[39] We have not taken the costs of the refinancing of the loan into consideration here, in danish *kursskæring*. Normally a fee of 0.1-0.3%-points is charged, which should be accounted for. It appears this fee has changed (increased) over time and between issuers.

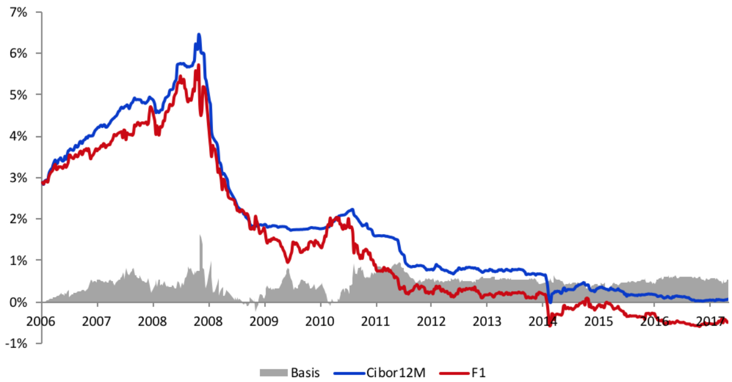

Figure 18: Basis between Cibor12M and the representative 1Y flex bond YTM.

As we can see the basis has generally been in favor of the borrower, as the Cibor12M rate has been above the F1 rate with only a few exceptions. We also note that the periods where the basis tightens (approaches 0 and in some intances turns negative) is periods of global distress with late 2008 being the outbreak of the financial crisis and early 2010 the european debt crisis.

As the floating interest rates are fixed for 1Y at a time in the swap and floating rate loan, it is the fixing of the rates and the spread on a set of specific dates which is a risk factor for the borrower. We will consider this risk factor, the *reset* of the rates, in the next section. Instead we will think of the spread risk more fundamentally as the risk of a *drift* between the two interest rates.

This spread risk was in fact mentioned by the District Court as one of the reasons that Engskoven was ruled in favor of in their suit against Jyske Bank, because the court found that Jyske Banks advice on this, amongst others, had been inadequate (Retten i Viborg (2014), p. 52).

3.4.2 Reset risk

In this section we will review reset risk, which we will define as the risk associated with the fixing of the floating interest rates. We will consider two aspects of this type of risk.

The first is the one which was mentioned in the previous section. The floating interest rates of the swap and loan are fixed once a year, and there is a significant risk associated with this reset of rates. For the borrower, the risk is that on the particular day of the fixing the Cibor12M interest rate drops by more (or increase less) than the F1 rate increases, such that the spread between the two decreases. As such it is the sudden change of interest rates in an unfavorable direction which pose a risk.

The second type of reset risk is associated with the possible difference in the *dates* at which the floating interest rates are fixed. Until now we have simply assumed that they would be fixed on the same day, however this may not be the case. To understand this, we first need to understand how the F1 rates are determined[40]. The rates are determined at auctions where the mortage issuers sell the bonds underlying the loans to institutional investors and banks. The F1 bonds of an issuer may be sold over several days, and the interest rate the borrower has to pay for the next year is determined based on a weighted average of the issued prices of the bonds. Historically, these auctions were held once a year at the beginning of December, with the rate

[40]Same procedure for other flex loans, such as the F3 and F5 loans.

taking effect on January 1st each year. However to reduce the reset risk associated with having such a large amount of bonds to be sold at a short interval for the many thousands of borrowers, the yearly auction was spread out to quarterly auctions instead. At present there is an auction approximately one and a half months prior to the beginning of each quarter. This means that there are flex bonds maturing each quarter, where in the previous setup the majority expired on January 1st.

Consider now the case of our borrower who took an F1 loan and entered into a payer swap. As the F1 rate was determined at the auction in early December, the payer swap was constructed to also have the Cibor12M rate fixed in early December. However, this was based on the expectation of future flex auctions to continue to take place in early December. If this did not hold, then the borrower is exposed to the interest rate moves in the intermediary period. Consider for example a case where the F1 rate is determined *prior* to the fixing of the Cibor12M rate. If interest rates drop in this period, then the borrower will receive a lower Cibor12M rate in the swap compared to the scenario where the rates were fixed on the same date.

As it turns out the auction days relevant for this type of borrower has actually been moved. The auction determining the January F1 rates has been moved from early December to (typically) mid/late November[41]. This is exactly the scenario described above. Additionally the fact that the F1 rate may be determined based on sales over several days, also pose reset risk for the borrower.

3.4.3 Annuity loan with adjustable payments

The last type of basis risk we will consider is related to the risk that the principal profile of the loan and the notional profile of the swap are not matching.

The flex loans obtained by the borrower is formally an annuity loan (BRFkredit (2018)). Annuity loans are characterized by the total payment of each period being constant. The total payment is split into an interest payment, that is the interest paid on the remaining principal, and repayment which is used to repay part of the remaining principal. The distribution of the total payment between interest and repayment varies for each period, but the total payment remains constant in an annuity loan.

However, in a F1 flex loan the interest rate is reset each year, and thus the future interest payments are uncertain. As a result the flex loan is not a "true" annuity loan. Instead for each period[42] the total payment (and thereby the interest- and repayments) is determined *assuming* the recently fixed F1 rate remains constant for the remainder of the duration of the loan. The payments are calculated using a standard annuity approach. Using the notation of Jensen (2013), p. 21-24, the payment, Y, is defined as $Y = \alpha_{\overline{n}|r}^{-1} = \frac{r}{1-(1+r)^{-n}}$, where n is number of payment periods and r the interest rate, assuming a unit notional. When the F1 rate resets in the beginning of the next periode, the new payments are calculated again etc. That is for each period, the payments are determined *as if* the remainder of the loan is an annuity, when in fact it is not (Jensen (2013), p. 210-213). It is in this regard we consider the flex loan an "annuity loan with adjustable payments".

As the future payments of the loan are unknown, so are the future *re*payments, and so are the profile of the principal. However, at the initiation of the loan the principal profile is estimated based on the current F1 rate and the expected repayments. The notional profile of the interest rate swap is then determined so as to match the (expected) principal profile of the loan for each

[41] In the years 2013-2017 all auctions has been placed somwhere in the period from the 17th to the 29th of November.

[42] 1 year in this case, but this holds generally for other flex loan refinance periods.

future period. This ensure that the floating rate payments match eachother. Obviously the two profiles will not match except for the (very unlikely) scenario that the F1 rate in fact does remain constant for all future periods.

In below figure we have shown the principal profile of two annuity loans. Both start out with a principal of 100 and have 30 payment periods. One has an interest rate of 1% and the other one 5%, denoted the "low rates" and "high rates" scenario, respectively. As we can see the principal is generally higher in the scenario with higher rates. This is because higher interest rates means higher interest payments, which reduce the repayment amount in the first periods of the loan.

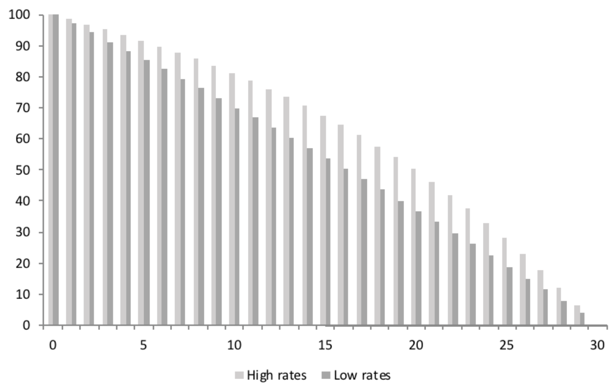

Figure 19: Principal profile of an annuity loan in a high and low rates scenario, corresponding to 6% and 1% interest rate on the loan, respectively.

As we know the swap notional profile was determined initially to match the expected principal profile of the loan. Rates were higher back in 2006-2008 when our stereotypical customer entered into such a swap contract. Later rates dropped and reached (historically) low levels in recent years. The lower rates caused the interest payments on the (annuity) flex loan to be lower than expected at initiation. As a result the repayments were larger, and the principal profile of the loan lower than expected. This concept has been illustrated in below figure 20, where we have used the same numbers as in the previous figure. Since the swap was constructed so that the notional profile matched the expected principal profile of the loan, the customer ended up with a swap notional that was (is) too high compared to the loan. This has been shown in below figure 21. The customer thus pays a high fixed rate (higher than the par swap rate), on a notional that is too large compared to the loan. This has been exemplified by splitting the swap into two; one matching the loan and an "extra" swap. This has a negative effect for the customer, compared to the scenario where the notional and principal profiles match eachother, as this extra payer swap has a negative market value.

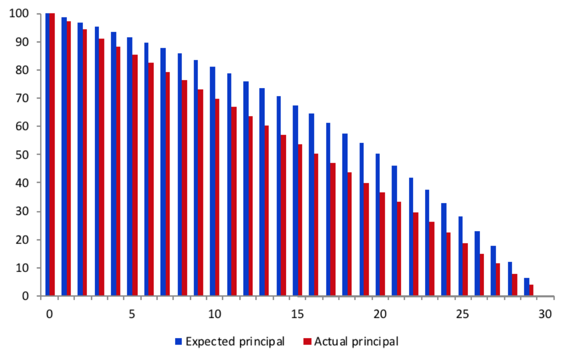

Figure 20: Annuity loan principal profile showing the expected and actual profiles due to lower interest rates than expected.

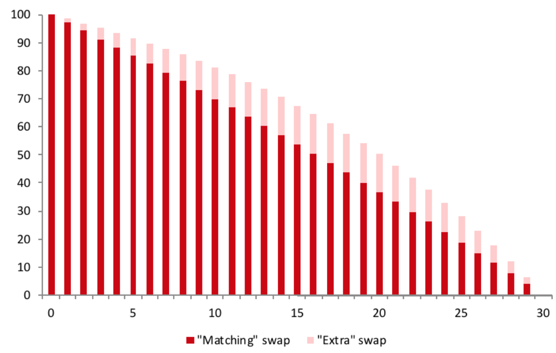

Figure 21: Swap notional profile showing the notional split into one matching the actual profile of the loan, and an "extra" swap.

It turns out that a similar negative effect is present in the case of increasing rates (relative to the initial level). In that case the principal of the loan is higher than the swaps notional, and the floating payments in the swap therefore does not cover those of the loan. In that case we may also split the swap into one matching the loan (that is a theoretical payer swap with a notional higher than that of the actual swap) and a receiver swap. In this receiver swap we are receiving a fixed rate that is too low compared to the at market swap rate, since rates has increased. Again it is this "extra" swap which has a negative effect on the customer, compared to the scenario where the principal and notional profiles match.

It is therefore the fact that the flex loan is a (pseudo) annuity loan, with adjustable payments, and hence a principal which changes compared to the initially expected profile, that induce a negative effect for the borrower. It appears this is the case no matter if rates generally increase or decrease compared to the initial level. In the case of rates fluctuating around the initial level the conclusion is not as clear.

3.5 Liquidity risk

In this section we will review liquidity risk, that is the risk facing the borrower of being forced to terminate a swap with a (large) negative value.

As we explained earlier in section 2.1.3 most interest rate swaps are traded either in the interbank market or between a bank and an institutional investor. Common for these are that they are traded under a CSA, which settles the rule for exchanging collateral between the two parties. However, interest rate swaps traded with end-users like housing cooperatives, municipalities or even corporates are typically not traded under such a CSA, and hence neither party is required to transfer collateral. Sometimes such agreements have a limit on the negative market value of the swap seen from the customer's side added. If this limit is reached, then the bank, for which the trade has a positive value, is entitled to (but not required to) force a termination of the swap. That is they may force the customer to pay what they owe in the swap contract, and thereby realize the loss. This limit on the market value may be determined in the bank based on the maximum credit risk exposure the bank is willing to have. As an example, in the case of the IRS between Jyske Bank and Engskoven, there was such a limit of the market value of -6.825 mil DKK on the swap with a notional of 33.4 mil DKK. If this was reached Jyske Bank was entitled to terminate the swap (Retten i Viborg (2014), p. 52). It is important to note, that if the customer instead obtains a callable loan, then this liquidity risk is not present.

For the customer, the risk of being forced to realize the loss on the swap contract constitutes liquidity risk. In such a scenario the customer would be required to produce liquidity to match the liability in the swap. In the case of a regular housing cooperative or a farmer with swaps with significantly negative market values, they may not be able to produce such liquidity. If they are unable to meet the claim, they will likely face bankruptcy. An example of such a scenario is that of the farm Skovbækgård, and it's swap contract with Jyske Bank. The swap had a large negative market value for the farmer, and combined with other debt obtained in the farm, Jyske Bank decided to force the termination of the swap. As the the farmer was unable to meet the claim[43] the farm went into bankruptcy (Nyholm et al. (2015)).

3.6 Paying the spread

The last risk factor we will cover is related to the fee imbedded in the swap contract.

When a customer wish to trade an interest rate swap with a bank, the fixed rate quoted for the customer is based on the mid market swap rate, and to that price a spread is added. In section 4.1 we will learn that this spread is comprised of, amongs other, a bid-ask spread to the trader (and possibly to the sales) and a xVA spread. The xVA spread is higher for this type of customers since the swap is not traded under a CSA and therefore not collateralised. The direction of the spread depends of whether the customer wants to pay or receive fixed. The spread is positive if the customer wants to pay fixed, and negative if she wants to receive fixed.

In the case of the payer IRS this spread therefore increases the fixed payments beyond those of the par swap rate. The present value of these spread payments is negative for the customer, and hence the swap must at the start have a negative value for the customer. To see this we use equation (5) which is the present value of the fixed leg of the swap, and split the swap rate paid by the customer, κ, into an at market rate, κ^{atm} and a spread, δ. We will keep the notation a bit lighter here:

$$\begin{aligned}
PV_{\text{fixed}}(t) &= \sum_{i=1}^{M} \alpha_i \kappa N_i P(t, \tau_i) \\
&= \sum_{i=1}^{M} \alpha_i (\kappa^{\text{atm}} + \delta) N_i P(t, \tau_i) \\
&= \sum_{i=1}^{M} \alpha_i \kappa^{\text{atm}} N_i P(t, \tau_i) + \sum_{i=1}^{M} \alpha_i \delta N_i P(t, \tau_i)
\end{aligned}$$

And we recognize the last term as the annuity factor times the spread δ, which is the present value of (receiving) the spread for the remainder of the swap contract.

To further illustrate this, we have in table 6 below calculated the projected future payments and their present value for a 10Y payer IRS, with a spread of 15 bps added to the par swap rate. We have used the data from table 2 as of April 11th 2018. For simplicity we have assumed annual payments on both legs, even though this is incorrect as the forward Cibor curve is calibrated to the Cibor3M index. We have split the fixed payments into one for the at market part of the swap rate, and one for the spread. The present value of the IRS is -14,921 DKK. As we can see from the table, this corresponds exactly to the present value of the future spread payments.

[43] The total cost of the swap, including already settled payments, was estimated at approx. 50 mil DKK.

	Projected payments			PV of payments		
Date	Floating	Fixed, ATM	Fixed, spread	Floating	Fixed, ATM	Fixed, spread
15/04/2019	-2,470	-9,777	-1,529	-2,754	-9,824	-1,537
13/04/2020	-250	-9,697	-1,517	-252	-9,770	-1,528
13/04/2021	3,745	-9,724	-1,521	3,769	-9,786	-1,530
13/04/2022	7,237	-9,724	-1,521	7,238	-9,726	-1,521
13/04/2023	10,304	-9,724	-1,521	10,207	-9,632	-1,507
15/04/2024	12,669	-9,803	-1,533	12,442	-9,628	-1,506
14/04/2025	14,594	-9,697	-1,517	14,204	-9,437	-1,476
13/04/2026	16,414	-9,697	-1,517	15,774	-9,319	-1,458
13/04/2027	17,911	-9,724	-1,521	16,951	-9,202	-1,439
13/04/2028	19,144	-9,750	-1,525	17,819	-9,076	-1,419
Total				95,401	-95,401	-14,921

Table 6: Projected payments of an IRS and the present value of these. The fixed rate has been split into an at market rate and a spread.

It is difficult to determine the typical size of the spread as data is hard to come by, and it is likely to vary much, especially for non-institutional investors like those considered here. In e.g. the EUR IRS market at one vendor screen for institutional investors the bid-ask spread was as little as 0.4 bps for most maturities on May 7th 2018. One of the few cases where data on the spread for our type of customers is available is again that of Engskovens lawsuit against Jyske Bank. In that case the spread was 15 bps (Retten i Viborg (2014), p. 14).

What is special about this spread (or fee), relative to the fee on a mortgage loan, is that in the swap it is paid for the entire duration of the swap no matter what. This is due to the non-callable nature of the swap. If the swap is kept until maturity, obviously the spread is then simply paid at each fixed payment. However, if the swap is terminated before it matures, then the spread is also paid. This is because the *present value* of the future spread payments is discounted into the market value of the swap.

This is important, because it means that if a customer settles an interest rate swap by entering into an offsetting one, then the spread will be paid twice; once on both swaps. Obviously, it might be possible to negotiate this with the bank, however generally the new offsetting swap will also expose the bank to market risk. And as compensation for this risk the bank will require a spread. However, as the offsetting swap cancels more or less all of the non-market risk of the original swap the xVA spread is likely to be less than that of the first swap. We will examine the dynamics of the bank in regards to interest rate swaps in the next sections.

The fact that the fee in the swap is always paid for the entire duration of the swap has received a lot of attention in the media and in the lawsuits. This is because it is different from the practice of a typical mortgage loan. In these a fee is also paid ("bidragssats"), however this is only paid on an ongoing basis. If e.g. the customer enters into a 30Y callable loan, and after 5 years restructures into a flex loan, then the customer will no longer pay fees on the callable loan[44]. As a last remark, note that the customer besides paying the fee on the swap, also pays a fee on the underlying flex loan. In fact the fees on the flex loans are typically higher than those on a fixed rate callable loan. This is because the periodical refinancing of a flex loan requires more ressources on the mortage issuers side compared to the callable loan. This suggests that the effective fees paid on the synthetic non-callable loan is higher than those on both the callable

[44] However, new fees will be paid on the flex loan.

loan and the flex loan.

4 Interest rate swaps: The bank's perspective

In this section we will review the interest rate swap as seen from the bank's perspective. Specifically we will look at how the price on a swap is determined in a bank using input from several desks. We will also review how a bank is valuing interest rate derivatives based on whether they are collateralised or not, or more specifically on the type of CSA agreement the derivative is traded under.

4.1 Determining the price of market risk

In this section we will explain the process around an interest rate swap trade within a bank. We will explain how the the price is determined. We will see that the price the customer face is the result of input from different trading desks. This is because there are different types of risk and expected costs associated with such an interest rate swap trade. As each trading desk is responsible for the hedging of a subset of these risks, they determine the price of that risk. As an example we will use the case of an interest rate swap trade, however much of the following will hold for other types of OTC derivatives within e.g. FX, credit and equity.

Consider the case of a client who wants to enter into a interest rate swap trade with a bank. We will not make any assumption regarding the type of client or directionality of the trade, as most of what we will review will hold in any scenario. The client will contact their sales person within the bank requesting a price on the swap, that is a fixed rate. The sales person will now have to obtain quotes from two trading desks; the swap desk and the xVA desk. Each desk is responsible for pricing different types of risks associated with the trade, and the combined price will be what is communicated to the customer by the sales person. The sales person will usually add a small spread to the price for their service, but we will disregard that here. In below figure we have illustrated this process:

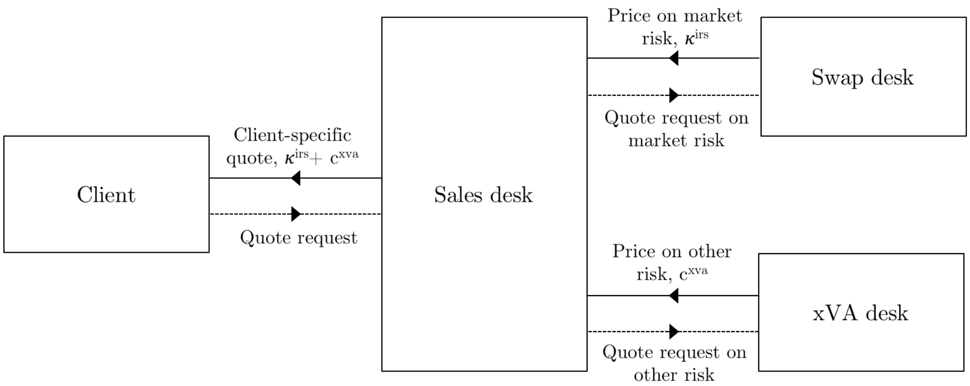

Figure 22: Illustration of the process of how a price of an IRS is determined in the bank using input from several trading desks, and based on the risk and expected costs of the trade.

The price quoted by the swap desk is that of the *market risk*. For an interest rate swap where the bank is either paying or receiving the floating payments, this reflects the price of those uncertain payments given the current forward curve, that is the at market par swap rate. Additionally it reflects where the bank may hedge the exposure generated by the swap (the price in the interbank market), and the risk associated with hedging the swap, which is the bid-ask spread. For example the swap trader may believe it will be expensive to enter into an offsetting hedge trade immediately, due to e.g. low liquidity. As a consequence it will either be expensive to enter into a trade immediately, or the trader will have to rely on non-perfect hedges for a longer period than usual until an opportunity to enter into a hedge swap presents itself. The expected cost from this is represented by a wider bid-ask spread.

The swap desk is only responsible for hedging the exposure to market risk that the interest rate swap trade generates, but there are other types of risk and costs[45] associated with such a trade. The xVA desk is responsible for hedging most of these types of risk or determining the expected costs. These include risks related to credit and costs related to funding, capital, margin and collateral. In the next sections we will review some of these. We may intuitively think of these as the risks associated with trading under a non-perfect CSA. We will define what we mean by a "perfect CSA" in the next section, however this intuitive view is a simplification as some of these risks also arise even if it is traded under a perfect CSA[46]. Obviously these types of risk may be client-specific as they depend on the CSA (if any) that is being traded under. As the xVA desk is responsible for hedging these risks, they determine the price of it, which we denote c^{xva}.

The sales desk presents the client with the price of the IRS, which we will denote $\kappa^{\text{client}} = \kappa^{\text{irs}} + c^{\text{xva}}$, and ignore the directionality of the spreads as it obviously depend on whether the swap is a payer or a receiver. If the client accepts this and the trade is executed, then the swap desk will immediately hedge the market risk from the trade[47]. The swap trader is not interested in taking this risk. Her mandate is not to have a view on whether interest rates go up or down. She has knowledge of the trading flow, that is the supply and demand, which is useful when acessing the risk associated with hedging the interest rate swap. Instead the trader hedges the market risk immediately. It will usually not be possible to do that instantly in the interbank market, because that would require another bank to have the exact opposite risk need at that exact time. Instead the swap trader will hedge the market risk using something that is more liquid, and most important highly correlated with swap rates. In the EUR swap market this is usually done using bond futures where the underlying are German government bonds. Especially the *bobl* and *bund* futures contracts are very liquid, with the underlying being a German government bond with a maturity of approx. 5Y and 10Y, respectively. This enables the swap trader to immediately hedge most of the market risk, and even though the hedge might not be perfect it is still better than doing nothing. After this the trader will begin trying to find a perfect hedge of the original client swap trade. This may be done in the interbank market, or the trader might wait for other incoming trades to offset the first one. We have illustrated the correlation between the 10Y EUR swap rate against Euribor6M and the bund contract in figure 23 below using minute-by-minute data for April 11th 2018.

When a swap trader hedges her exposure in e.g. bond futures it is for a short period, and certainly less than a day. For that reason it is the intra-day correlation as depicted in the figure which is relevant, and not the correlation over longer periods of time (which is not as high).

The most important point in this is that the swap trader takes no market risk associated with the trade. They enter into offsetting trades, and then profit from the fact that they may enter into these offsetting trades on a better term than the original, because the bid-ask spread

[45] From now on we will collectively refer to these as "risks" instead of "risks and costs".

[46] An example being collateral costs.

[47] The following procedure has been confirmed by sales professionals in a large danish bank with extensive knowledge on the matter.

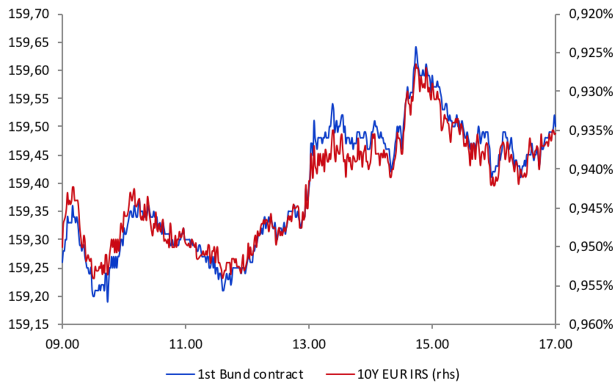

Figure 23: Illustration of the intra-day correlation between the 1st IMM Bund futures contract and the 10Y EUR IRS against Euribor6M using minute-by-minute price movements for April 11th 2018. Note that the right-hand axis has been inverted. Data is from Bloomberg.

is lower on the interbank market than in the retail market. As such it may be argued that it is not correct to claim that the banks earned a lot of money on the swap contracts they did with the housing cooperatives, farmers and communes. Because even though these swap contracts have a positive market value for the bank, they had most often entered into offsetting swaps in the interbank market, which had an (almost) corresponding negative market value for the bank. An example of this is provided in Nykredit Bank (2016), p. 4-5, where it is explicitly stated that the swap contracts undertaken with housing cooperatives have all been hedged by entering into corresponding interbank swaps.

4.2 Discounting with a CSA

To understand the adjustments made by the bank on swap rates to adjust for the risks associated with trading under no CSA, we will first have to review the CSA and how discounting is done under a "perfect CSA". We will do that in this section.

4.2.1 CSA, Credit Support Annex

OTC derivatives transactions has increased over the last decades. With this a need for standardized legal documentation on how the parties should act in certain scenarios has arised, as well as a need to formalise procedures regarding the transactions. This has been facilitated by the introduction of the ISDA Master Agreement. This is a standardized framework introduced by International Swaps and Derivatives Association (ISDA) in 1985. It contains a standard section and an adjustable section. In the adjustable section the two parties have to agree on a set of terms under which they trade for certain types of transactions. The ISDA settles terms like posting of collateral, what denotes a default and the procedure in such a scenario, netting of payments, and other types of mechanics (Gregory (2015), p. 49-56).

CSA is short for *Credit Support Annex*, and is a document which may be added to the ISDA agreement. In it is stated the terms of collateral posting by the two parties. Posting of collateral is a way for the two parties to mitigate counterparty credit risk. The CSA has become market standard in recent years due to an increased focus on counterparty credit risk since the financial crisis (Gregory (2015), p. 70). In the CSA is stated terms like which parties are required to post collateral and at what frequency, the method of calculating the amount of collateral to post, what may be posted as collateral (cash, bonds etc.) and in what currencies, the interest rate earned on cash collateral, thresholds and minimum transfer amounts etc. The main motivation behind posting of collateral is to reduce counterparty credit risk, because the credit exposure is (partially, depending on the CSA) neutralised. If the first party has a positive present value in a given transaction, then that party will receive collateral to reflect (part of) the value of the trade. If the counterparty defaults, then the collateral may be used to redeem any losses. The intuition of this has been illustrated in below figure 24 (from Gregory (2015), p. 80), where the line denotes the credit exposure of one party, and the collateral received by this party is illustrated by the bars:

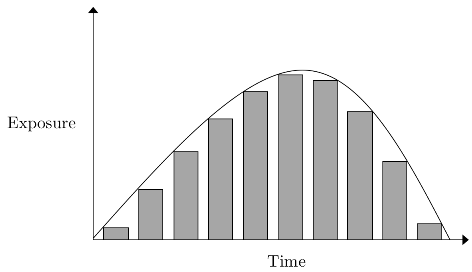

Figure 24: Conceptual illustration of how posting of collateral reduce the credit exposure.

A non-professional investor, such as corporates, municipalities, housing cooperatives etc., will usually have to choose between a CSA and the costs associated with not trading under a CSA (the xVAs), although this distinction is crude as some xVA charges explicitly refer to the posting of collateral. By far the majority of these customers chose the xVAs and thus not to trade under CSA[48]. Often it would not make sense for these types of customers to choose a CSA. As we already know, they often do not have the liquidity necessary to post collateral, nor do they have the resources or capabilities of calculating and making (daily) collateral calls. In addition such customers often have directional swap portfolios, that is they are significantly (if not completely) in one direction. This makes them vulnerable to market moves, which may in turn create large collateral calls. Due to their limited liquidity this is another argument for not choosing the CSA (Gregory (2015), p. 98-100).

4.2.2 Discounting under a perfect CSA

The price of market risk quoted by the swap desk as explained in section 4.1 is always done under the assumption of trading under a "perfect CSA" (we will also call this the "perfect" collateralisation scenario). By this we mean the CSA which minimizes counterparty credit risk as much as possible from the banks perspective, and thereby reaches the closest possible to a (credit) "risk-free" scenario. This would typically assume a zero threshold and minimum transfer amounts, daily collateral calls, a single-currency and cash-only collateral in the same currency as the transaction, OIS interest on collateral etc. This is what is also often refered to as a CCP-style CSA, because these are (more or less) the same collateral posting conditions required at a clearing house. Under these conditions it can be shown that OIS discounting is the correct methodology (Gregory (2015), p. 294-296). This is a consequence of the collateral discounting approach which was explained in section 2.1.3. From this it followed that the correct discounting

[48]According to a professional with knowledge of the matter from one large danish bank, not a single danish corporate or municipality has a agreed to a CSA.

rate is that rate which the collateral earns. As collateral earns the OIS rate in this "perfect" collateralisation scenario, this must be the correct discounting rate.

As naturally follows, once we stray from the "perfect CSA" scenario, then OIS discounting is no longer the correct method. For example if a higher interest rate is earned on the collateral, then a higher interest rate than the OIS rate should be used for discounting. A natural question in relation to this, is what to do in the scenario where there is no CSA and hence no collateral is posted? We will answer that in section 4.4 after reviewing FVA in section 4.3.2.

Another interesting scenario is trades under a CSA where cash (or other assets) in multiple currencies may be posted, a so-called multi-currency CSA. This introduces an element of optionality, and the correct discounting curve is not always clear. One approach is to assume "cheapest-to-deliver", that is use the curve of the collateral which overall is cheapest to post for that party factoring in e.g. the funding costs and the collateral interest of that currency (Whittall (2010)).

4.3 xVAs

In this section we will review some of the value adjustments, xVAs, that are relevant in the case where trades is done under a non-perfect CSA. Specifically we will focus on the case where there is no CSA between the bank and the customer. Proper modelling of these value adjustments can easily be very complicated, and so for our purpose we will limit ourselves to give an intuitive explanation of what each xVA represents, as well as give simple approximations following the procedure from Gregory (2015).

As stated in the previous section trading under a non-perfect CSA means that discounting using the forward OIS curve is no longer correct. Assuming the bank *do* use the OIS curve the resulting present value would then deviate from the true (xVA-adjusted) present value, and we may think of this deviation as the sum of the xVAs.

A non-perfect CSA expose the bank to other types of risk and costs. Consider for example a scenario where collateral calls are not made daily but instead monthly. This would increase the counterparty credit risk, and the bank would want to be compensated for this risk. This compensation is done by adjusting the price of the interest rate swap. These value adjustments are calculated by the xVA desk. The xVAs represent the compensation for expected costs of the trade, such as capital or funding costs, as well as the cost of hedging the non-market risk the trade represents, such as counterparty credit risk. So the xVA desk will find the cost of trading under a non-perfect CSA, and then adjust the price of the trade accordingly. In figure 22 this was illustrated by adding a spread, c^{xva}, to the par swap rate calculated by the swap desk. The conceptual role of the xVA desk has also been illustrated in figure 25 below (from Gregory (2015), p. 42).

Generally speaking the xVA costs will be higher the further away from a perfect CSA scenario we are, all else equal, and this means they are highest when there is no CSA. There are many types of value adjustments, and in the following sections we will review the most relevant when assuming there is no CSA, which are CVA, FVA and KVA. There are other xVAs, some more widely accepted than others. Some worth mentioning are MVA and ColVA, which are margin and collateral value adjustments. MVA relates to the value adjustment due to overcollateralisation mainly due to initial margin posting at e.g. a central counterparty or bilaterally. ColVA relates to the value adjustment due to the collateral terms differing from those of the perfect CSA in regards to the interest paid on collateral not being OIS, or the ability to post collateral in

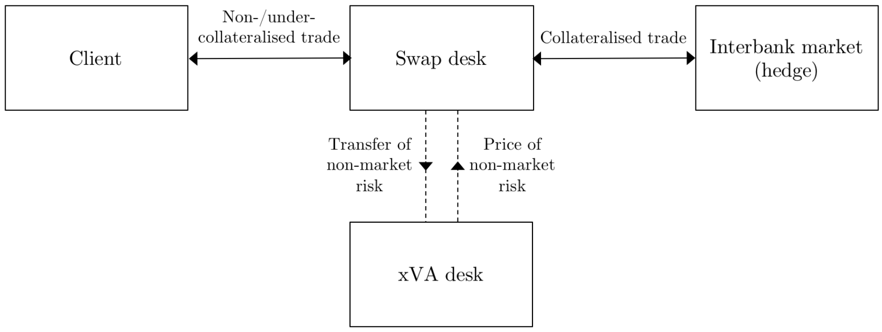

Figure 25: Illustration of how the non-market risk of a transaction is transfered to the xVA desk, which in turn determines the price of this risk.

another[49] or several currencies, or non-cash collateral such as bonds. Seeing as these are not relevant in a setting of no CSA and thus no collateralisation we will not review these.

4.3.1 CVA

Credit value adjustment, or CVA, is an adjustment made to reflect the credit risk of a transaction, that is the risk of not receiving all the payments due. CVA is easily the value adjustment that has received the most attention, and is also widely considered the first value adjustments that banks systematized, and in fact many xVA desks started out as CVA desks.

CVA (and other xVAs) are very much linked to the exposure that one party has to a counterparty (or vice versa). As an example consider a party which is due to receive a payment on a derivatives transaction from a counterparty in the future. If the counterparty defaults the payment (or some portion of it) is lost, and this is the credit risk. If instead the counterparty stood to receive a payment and then defaulted, then the first party would not gain from this default. We may think of this as due to creditors collecting all the claims. Letting the value at a given time of a derivatives transaction or portfolio thereof be V, then the credit exposure, E, of the party is:

$$E = \max(V, 0) \tag{13}$$

Conversely we may define the negative credit exposure, NE, as the exposure of the counteparty against the party, and this is $NE = \min(V, 0)$. Here we will mean the value, V, to be the mark-to-market of the derivative seeing as we do not consider collateralisation. As CVA is linked to the credit exposure and potential losses incurred from a counterparty default, we will always consider CVA a loss. Using this we may intuitively define it as the adjustment made to a credit risk-free valuation to find the credit risky valuation, that is:

$$\text{Credit risky value} = \text{Credit risk-free value} + \text{CVA}, \qquad \text{CVA} \leq 0$$

From the proof of above we find that the CVA is the (risk-neutral) expectation of the loss incurred if the counterparty defaults prior to the payment, which is also a very intuitive results (Gregory (2015), appendix 14A). Mathematically this would mean that we would have to integrate the exposure function and the indicator function indicating default. Instead we will follow Gregory, J. (2015), and use a discrete approximation to this integral. To do this we will have

[49] That is another currency relative to that which the transactions are made in.

to calculate the expected exposure, $EE(t)$, at a set of discrete points in time. For interest rate derivatives this may be done by simulating the term structure and then find the exposure of e.g. an IRS at each point in time. Using Monte Carle simulation, the expected exposure is then found as the average exposure. Similarly, we may find the negative expected exposure, $NEE(t)$, as the average of the negative exposure at time t.

Assuming a constant recovery rate, R, that is the amount recovered in case of default, and letting $PD(t_{i-1}, t_i)$ denote the probability of default of the counterparty in the time interval t_{i-1} to t_i, then we may approximate the CVA as:

$$\text{CVA}(t) = -(1-R)\sum_{i=1}^{n} EE_d(t_i) \cdot PD(t_{i-1}, t_i) \tag{14}$$

Where the $EE_d(t_i)$ denotes the expected exposure discounted to time t. To calculate this we will have to make some assumptions and simplifications. First of all we will assume a flat credit spread. This is rarely what is observed, but for our purpose it will simplify things greatly, as we will not have to calibrate a credit curve.

Secondly to approximate the default probability of a given period, we will use the following, where $s(t_i) \equiv s$ is the credit spread at time t_i (Gregory (2015), p. 271):

$$PD(t_{i-1}, t_i) \approx \exp\left(-\frac{s(t_{i-1})t_{i-1}}{1-R}\right) - \exp\left(-\frac{s(t_i)t_i}{1-R}\right)$$

The third, and most crucial, assumption we will make is regarding the expected exposure, EE. As mentioned practitioners would usually simulate this, an example being a term structure simulation using e.g. the two-factor Hull-White model. Given the simulated curves at each point in time it would then be possible to calculate the time-t_i market value of an interest rate swap (or other interest rate derivatives) and from this the exposure. Given repeated simulations EE and NEE may be determined as simple averages. However for simplification we will refrain from any simulation. Instead in our calculation of EE and NEE, we will simply assume that the forward rate curve materializes. That is that our expectation of the time t_i spot rate is the current time t forward rate, $F(t, t_i, t_i + \alpha)$. This is the same assumption we make when pricing interest rate derivatives. Using the forward curve we are then able to calculate the "expected" time-t_i value of the IRS. A disadvantage of this is that only one expected exposure at time t_i is found, and hence either $EE(t_i)$ or $NEE(t_i)$ (or both) are zero. It is common to define the time t_i expected future value, $EFV(t_i)$ as the average value in all simulations, such that $EFV(t_i) = EE(t_i) + NEE(t_i)$. Using our simplification we are thus in reality calculating a non-simulated expectation of $EFV(t_i)$. As we will learn later this results in EE and NEE profiles that are rather moderate, compared to what would typically be found using simulations, which would most often result in more volatile estimates of the profiles. We could alleviate for this moderate approximation by scaling our $EE(t_i)$ (and $NEE(t_i)$) by a factor $k > 1$, essentialy scaling the CVA, but we will refrain from this. As we will see in the next section this way of calculating exposure will also affect the results of the FVA. It is important to note, that it is only the expected exposure profiles which are affected by this. All other calculations in all of our xVAs would be same if we instead chose to use simulation for the exposure projections.

There does exist other options to calculate interest rate swap exposures. One is given in Sorensen & Bollier (1994), where it is shown that the exposure of an interest rate swap may be found using swaption prices of the underlying swap, and using the same discretization as above.

We will now give an example of a CVA calculation. Lets assume that the credit spread is flat at 500 bps. This is high but will be necessary to find CVA measures that are non-negligible, due to our simplified way of calculating the EE profile. Also assume a constant recovery rate of 40%[50]. Using the calibration data of April 11 2018, we calculate the (discounted) EE profile of a 30Y payer IRS with a fixed rate equal to the par swap rate of 1.523% and a notional of 1 mil DKK, as well as the profile of the default probabilities. Using this the CVA cost is $CVA_c = -1,938$. We use the c to denote a lump-sum cost.

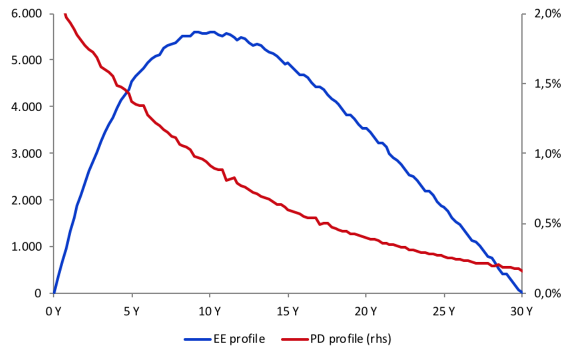

Figure 26: The projected EE and PD profiles of a 30Y payer IRS with a notional of 1 mil DKK, using curve calibration data from April 11th 2018 and a dual-curve setup.

For an IRS this cost will typically have to be converted into a spread to be added to the swap rate determined by the swap desk. It is unlikely that a customer will agree to a cash payment to cover this cost at initiation of the trade (and also this would go against market practice). However, just dividing CVA_c by the swap annuity would not be correct. This would assume that the CVA spread would be paid over the life of the swap with certainty, and thus disregard credit risk - and this is exactly why CVA is needed. Instead we need to use the *risky annuity* of the swap, which accounts for the counterparty credit risk. We may define that as the following (Linderstrøm (2013), p. 79):

$$\text{Risky annuity}(t) = \sum_{t=1}^{n} \alpha_i N_i P^{\text{disc}}(t, t_i) SP(t_i) \tag{15}$$

Where $SP(t_i) = 1 - \sum_{j=i}^{i} PD(t_{j-1}, t_j)$ is the survival probability of the counterparty at time t_i. Continuing with the above example we find the risky annuity to be 10.066 mil DKK, and thus the CVA as a spread to be $CVA_s = -1.92$ bps. As we note, this is not a high CVA cost on a 30Y derivative for a counterparty with a credit spread of 500 bps. As a comparison Lehman Brothers had a credit spread of 610 bps just prior to it's default in September 2008 (Siew (2008)). Again, this is due to the moderate way we calculate expected exposure. Converting the CVA cost into a spread also affects the EE profile and the CVA cost itself, as it affects future cash flows and thus future valuations. Hence, using this approach we would have to recursively calculate the CVA cost until an "equilibrium" CVA spread is found.

We have assumed that the CVA is calculated under no CSA, and therefore no collateralisation. If instead collateral was posted by one or both parties, then it would affect CVA by changing the exposure at future points in time and thus the expected exposure profile. Receiving collateral reduce the exposure, but posting collatateral beyond the negative mark-to-market of a position

[50]This is often a standard assumption, for example it is also used in the ISDA Standard CDS converter for senior unsecured debt, ISDA (2009).

will increase exposure. Netting arrangements of the transactions will also have to be taken into account. As collateralisation, and thereby exposure, is often defined on a portfolio level due to netting agreements, a bank cannot calculate the CVA of a new transaction as a stand-alone cost, but will have to consider the effect on the portfolio. That is they will have to calculate the CVA of the new transaction using the incremental impact on the portfolio CVA it generates. In our calculations we have implicitly assumed that the CVA is for a stand-alone transaction, or alternatively assumed no netting of transactions. Collateralisation and netting agreements only affect the CVA through the expected exposure profile, as the probabilities of default and the recovery rate are unaffected.

4.3.2 FVA

The second value adjustment we will consider is the FVA, or funding value adjustment. It is primarily relevant when there is no collateralisation, or the risk of under/over collateralisation due to a non-perfect CSA.

To see this we have to look at figure 25 again, where we see that the non- or under-collateralised IRS is typically hedged with a corresponding and perfectly collateralised interbank swap. As market variables change it creates a discrepancy between the collateral posted/received in the first trade and the collateral of the second trade. This creates a funding cost or benefit for the bank. We let the funding exposure, E_f, be defined as:

$$E_f = V - C$$

Where C denotes collateral posted or received ($C > 0$ is received collateral) and V is the value of the transaction or portfolio. When market variables change, the value changes and if the transaction is non- ($C = 0$) or under-collateralised ($V \neq C$), then funding exposure wil be generated ($E_f \neq 0$). Assuming the interbank trade from figure 25 is perfectly collateralised it will not have any (significant) funding exposure, and instead it is the first client trade that generates the funding exposure.

Assume now for simplicity that no collateral is posted in the client trade by either party, corresponding to the no CSA scenario, and that we consider a single IRS transaction. Assume that market variables change in favor of the bank, such that the value of the swap increases. As no collateral is received the bank is unable to realize the gain (and e.g. use the collateral for funding elsewhere), and the bank will have to fund this themselves thereby generating a funding cost. As the interest on collateral paid by the bank is the OIS rate, the notion of a funding cost assumes that the bank is *not* able to fund itself at the OIS rate, that is we assume they have a funding spread to OIS. Conversely, consider if the value of the swap decreases, then the bank is not required to post collateral, and it may use those funds elsewhere thereby generating a funding benefit. FVA is the adjustment made based on the expected funding cost/benefit profile, and hence may be both negative and positive, respectively.

We may *intuitively* think of this in terms of figure 25 again. If the value of the client swap increases then the bank is liable to post collateral on the hedge swap, but does not receive any on the client swap. The interest received on the collateral posted is the OIS rate, so unless the bank can fund the collateral at the OIS rate there will be an associated funding cost. However, this way of thinking of funding costs and benefits is only intuitive, as it would mean that there are scenarios where there *incorrectly* would be no cost or benefit, an example being when the client IRS is not hedged with an interbank swap.

As is clear from the above, the funding exposure is highly dependant on the rehypothecation assumptions regarding the collateral received by the bank. If the collateral cannot be rehypothecated, that is if the bank may not use the collateral for other purposes such as posting it as collateral on a hedge position, then the collateral does not serve to reduce the funding exposure of the bank. It will however still reduce the credit exposure.

Following the assumption of no collateral, we may define the FVA as follows (Gregory (2015), p. 341-343):

$$FVA = -\sum_{i=1}^{n} EFV_d(t_i) \cdot FS(t_i) \cdot \Delta(t_i) \cdot SP(t_i) \tag{16}$$

Where $EFV_d(t_i)$ is the discounted expected future value, $FS(t_i)$ is the forward funding spread of the bank at time t_i, $\Delta(t_i) = t_i - t_{i-1}$, and $SP(t_i)$ is the survival probability of one or both parties.

According to Gregory the calculation of FVA is often implemented using a CVA-like approach instead, that is where we use equation (14), but substitute EFV for EE, assume a recovery rate of 0%, and we have substituted the *spot* funding spread for the credit spread in the default probability formula. Using that, and letting $FS_0 = FS$ be the spot funding spread, we find:

$$FVA \approx -\sum_{i=1}^{n} EFV_d(t_i) \left(e^{-FS \cdot t_{i-1}} - e^{-FS \cdot t_i}\right)$$

This looks suspiciously much like the sum of the discounted $EFV(t_i)$, but where each have been discounted again at the spot funding spread and only over the time-interval $[t_{i-1}, t_i]$. In other words, this suggests that we may think of the FVA as the effect of having to discount the expected future values at the funding rate (OIS plus the funding spread) instead of the OIS rate.

Using the second approach and the data from April 11th 2018 we now calculate an example. Assuming a funding spread of 50 bps, we find the FVA of a 15Y at market receiver IRS, and a notional of 5 mil DKK to be $FVA_c = -461$. Below figure shows the discounted EFV profile of this IRS:

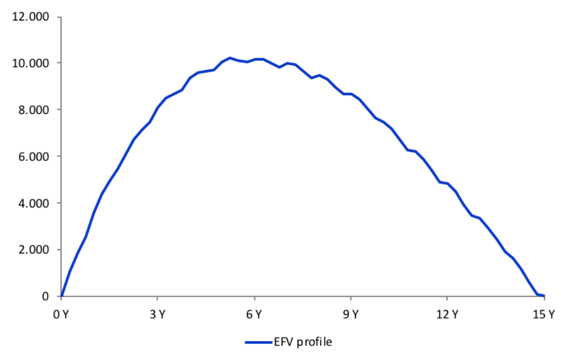

Figure 27: The projected, discounted EFE profile of a 15Y at market payer IRS with a notional of 5 mil DKK, using curve calibration data from April 11th 2018 and a dual-curve setup.

Again we are able to calculate the FVA as a spread instead by dividing by the risky annuity (and in fact may do this for all xVAs). Doing that we find the FVA to be -0.1 bps. We see that the FVA is very modest, and this is again due to the way we project the future exposure profile. Calculating the FVA as a spread will again have an effect on the exposure profile, which in turn will affect the FVA calculation. Additionally it will also affect other xVAs such as the CVA. As a natural consequence the conversion to a spread requires all xVAs to be calculated simultaneously and recursively.

There is a curious consequence of representing the FVA as a spread because it is possible that it is positive and hence a benefit for the bank. One might imagine a scenario where a transaction creates such positive expected funding benefits for a bank, that the conversion of this FVA benefit into a spread would effectively move the fixed rate offered to client on other other side of mid-price. Naturally this would require other xVAs to be sufficiently low or zero etc., but nonetheless it is technically a possible scenario. However, it is obviously unlikely that the bank would actually transfer this benefit onto the client by offering such a beneficiary quote.

4.3.3 KVA

Banks have experienced increasing capital requirements from regulators over the last years, an example being the leverage ratio rule[51]. Entering into derivatives transactions the bank is therefore required to hold an amount of capital, with the requirement generally being smaller if the trade is collateralised or hedged. For each transaction there should therefore be allocated a cost for the reservation of this capital. The capital value adjustment, KVA, is the adjustment to the price of the derivative for this cost.

The capital value adjustment is based on the projected capital requirement profile (alternatively this may also be simulated). There exists several ways of defining this, and we will use the internal ratings-based approach, where the regulatory capital required at a certain point in time is defined as follows (Gregory (2015), p. 144-145):

$$RC(t_i) = EAD(t_i) \cdot (1 - R) \cdot PD_{99.9\%} \cdot MA$$

Where EAD is the exposure at default, R is the recovery rate, $PD_{99.9\%}$ is a conservative estimate of the probability of default at the 99.9% confidence level, and MA is a maturity adjustment to adjust for the risk that the credit quality of the counterparty may worsen, and especially for high-quality counterparties. The factors $PD_{99.9\%}$ and MA are complex functions which we will not repeat here, but may be found in Gregory (2015), appendix 8A.

The EAD can be defined in several ways. To simplify things we will use the default risk capital charge approach and ignore the CVA capital charge[52], whereas usually the EAD will be based on the total capital charge as the sum of the two. Default risk capital charge may be calculated using several approaches which is somewhat at the banks discretion. The three approaches are CEM, SA-CCR and IMM[53], although it appears there is a move away from CEM towards SA-CCR and IMM. We will limit ourselves to the CEM approach as it is easiest to implement.

In the CEM approach the exposure at default is defined as $EAD(t_i) = EE(t_i) + A(t_i)$, where EE denotes the expected exposure of the transaction as defined previously, and A is an addon based on the asset class (rates, equity, FX etc.) and the remaining maturity of the transaction. Using this we may calculate the EAD and then the RC profile on an IRS. To illustrate this we will consider an at market payer IRS against Cibor3M with a maturity of 10Y on a notional of 15 mil DKK. Assume further the probability of default in any given period is 0.5%[54] and the

[51] Which states that the ratio of Tier 1 capital (equity and other core capital) to exposure (defined by assets, derivatives exposure etc.) must always be at least 3%.

[52] Which is the price for volatility in the CVA due to changes in market variables.

[53] Short for current exposure method, standardised approach for counterparty credit risk, and the internal model method.

[54] For each period, which is a quarter in our case.

recovery rate is 40%. If we calculate the future expected exposure of the IRS by assuming the forward curve materializes, we find the EAD and RC profiles as depicted in figure 28.

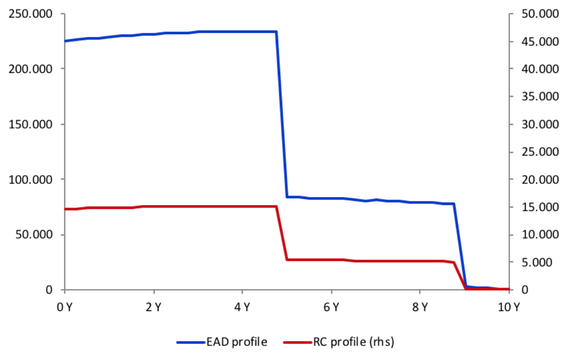

Figure 28: The projected EAD and RC profiles of a 10Y payer IRS with a notional of 15 mil DKK, using curve calibration data from April 11th 2018 and a dual-curve setup.

The KVA charge is the expected cost of having to reserve the capital as projected in the capital profile, RC. We denote the cost of capital as $CC(t_i)$ and for simplicity we will assume this is 10%, which we will consider an acceptable approximation of the return on equity generated (and expected) by banks. Using this we may define the KVA as:

$$KVA = -\sum_{i=1}^{n} RC_d(t_i) \cdot CC(t_i) \cdot \Delta(t_i) \cdot S(t_i) \tag{17}$$

Where the expected capital requirement has been discounted at the cost of capital, and where $S(t_i)$ denotes the survival probability of either the counterparty or both parties. We will assume this is for the counterparty only. Continuing with the example, and again assuming a constant probability of default at 0.5%, we find the KVA to be $KVA_c = -8,566$. As with the other xVAs we may also convert the KVA into a spread. This is again done by dividing with the risky annuity, and should also be calculated simultaneously with the other xVAs.

There are some inherent difficulties in pricing KVA. Because it depends on the expected, future capital requirement the pricing would have to (should) consider the expectation of future regulatory policy changes. For example if it is expected that regulators will ease the capital requirements in the future, then ideally this should be reflected in the expected capital profile and thus the KVA cost.

The last important comment regarding KVA is its relation to the xVA desk which is different from CVA and FVA, which are typically transfer-priced. This means that at initiation of a transaction, the xVA desk will require a cash transfer from the sales (or trading) desk to reflect the collective cost of risk, that is the total xVA. It is then up to the sales desk if they want to transfer this cost onto the client, which will usually be the case. The xVA desk decides the cash payment, because they are responsible for hedging this non-market risk. For KVA, however, this is different because the risk of increased capital requirements is generally not hedged. This means that there is no transfer to and responsibility of the xVA desk to manage this risk, and hence they do not receive any cash payment from the sales desk upon trading. Instead they will provide the sales desk with a hurdle, which is an indicative cost of KVA. Again, it is then up to the sales desk whether they want to transfer this cost onto the client. There is obviously no upfront cost for the sales desk of not doing this, however this may affect the profitability of the trade seen over its entire lifetime (Gregory (2015), p. 375 and 398-399).

4.4 Discounting without a CSA

As we learned in section 4.2 for a swap that is perfectly collateralised we need to discount it on the same curve as that which the collateral earns interest, which is typically the OIS curve. However, as the funding value adjustment showed, when a trade is not collateralised then the proper discounting curve is that which reflects where the bank can fund itself at. Often it is assumed that the Libor curve is representative of where a bank can fund its activities, not because banks can actually borrow at the Libor rate, but merely as a representative rate.

Assume now that we are observing a danish bank, and that its funding currency naturally is DKK. Let us further assume that the Cibor3M curve is representative as to where the bank is able to obtain short-term DKK funding for its activities. For it to value an interest rate swap which is uncollateralised is therefore simple; it will need to use the forward Cibor3M curve as the discounting curve.

Consider now a scenario, where the bank needs to fund EUR activities instead. The bank is obviously not able to fund itself at Euribor3M, but instead need to enter into a EUR/DKK 3M cross-currency basis swap to switch its funding from DKK to EUR. As the basis spread is added to the DKK leg, the bank ends up paying Euribor3M minus the basis spread. A consequence of this is that if the bank needs to price an uncollateralised IRS denominated in EUR, it will not be correct to use the forward Euribor3M curve as the discounting curve. Instead we should use the curve adjusted for the EUR/DKK 3M basis as the discounting curve.

Lets assume the bank enters into a non-collateralised at market EUR IRS. Obviously the par swap rate should be indifferent from what discounting curve the bank is using, as this is only supposed to reflect the funding assumptions of the bank. But what happens to the value of the swap from the banks point of view right after it has been traded? Naturally the bank needs to value it at the proper discounting curve, which is the basis adjusted Euribor curve, however this would generate a market value of the swap different from zero, as the par swap rate is calculated assuming OIS discounting. Does this mean that the bank is facing a potential arbitrage situation? No, because the fixed rate the bank entered the swap at should not be the par swap rate, but instead the par swap rate adjusted by the FVA spread. The discrepancy in market value of the at market IRS between using OIS discounting and the basis adjusted curve is the funding value adjustment. This shows that the discounting curve used to calculate the FVA for the danish bank should not be the Euribor3M curve, but the basis adjusted curve.

4.4.1 Calibrating the funding curve

We will now review how to calibrate the funding curve in a currency which is not the domestic currency of the bank. In our example it is the EUR funding curve of a danish bank, which is equal to the Euribor3M curve adjusted for (subtracting the) EUR/DKK 3M cross-currency basis swap spread[55]. To find this curve we cannot simply use the Euribor3M swap curve and subtract the basis curve from this. Instead we will need to calibrate it using cross-currency basis swaps.

To price a cross-currency basis swap we need four curves; the forward Libor3M curve and a discounting curve in each currency. The forward Cibor3M and Euribor3M curve are found using our standard approach as explained in section 2.3.2 and figure 6, where we first calibrate an OIS curve which is then used as discounting curve to find the correct forward Libor curve. Note that the OIS curve is only used to find the proper forward Libor curve, and not used for discounting in the basis swap calibration.

[55] At the time of writing this spread is negative, and thus the bank ends up usin Euribor3M plus a (positive) spread as the funding curve.

As we know we should use the funding curve as the discounting curve. In a EUR/DKK basis swap, the spread is added to the DKK leg, which in our terms is the domestic leg. In DKK the funding curve is also the Cibor3M curve. Using the equation (11), the value of the domestic leg is now greatly simplified:

$$PV^{\text{ccs}}_{\text{dom,F}}(t) = N \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} c P^{\text{lib}}_{D}(t, t_i)$$

Where we have used that disc = lib. This in turn simplifies the calculation of the par basis swap spread which becomes:

$$c(t, t_0, t_n) = \frac{1}{\sum_{i=1}^{n} \alpha^{\text{lib}}_{i,D} P^{\text{disc}}_{D}(t, t_i)} \left(P^{\text{disc}}_{F}(t, t_n) - P^{\text{disc}}_{F}(t, t_0) + \sum_{i=1}^{n} \alpha^{\text{lib}}_{i,F} F^{\text{lib}}_{F}(t, t_{i-1}, t_i) P^{\text{disc}}_{F}(t, t_i) \right) \tag{18}$$

In EUR our funding and discounting curve is the Euribor curve adjusted for the basis; the curve we want to find. By using the market quotes for the cross-currency basis swap and using the Euribor3M curve to project forward rates on the EUR leg, we calibrate a EUR discounting curve, such that equation (18) is satisfied, given the Cibor3M forward curve. This EUR discounting curve will be the Euribor3M curve adjusted for the basis swap spread. We have tried illustrating the procedure in below figure:

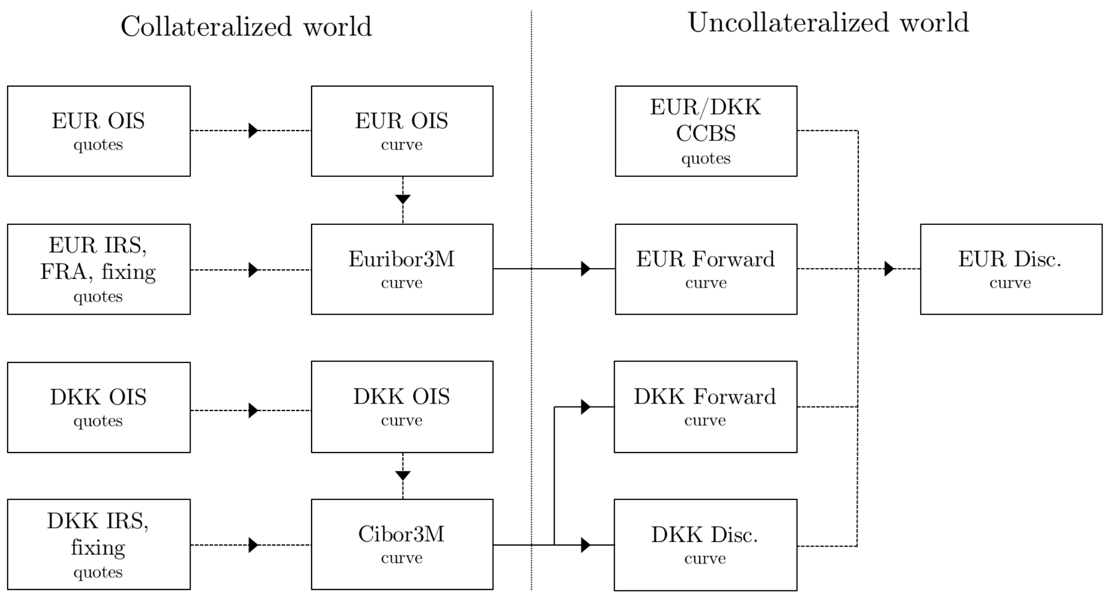

Figure 29: An illustration of the process of calibrating a EUR funding curve for a DKK denominated bank. Dashed arrows indicated dependence, solid arrows indicate "equal to".

Using this approach and the data as of April 11th 2018, we are able to find the EUR funding curve, represented in figure 30 below. To illustrate that it is not identical to the Euribor3M curve we have also included that, and both curves are represented as zero rate curves. We have also shown the spread between the two zero rate curves, and added the cross-currency basis swap quotes. As it appears, one can approximate the true funding curve by subtracting the (negative) spread to the Euribor3M zero rate curve directly.

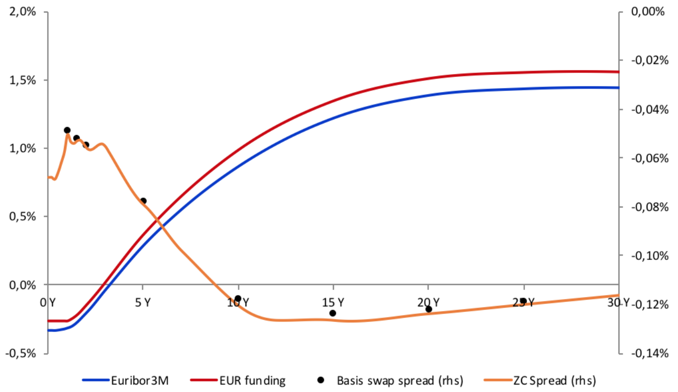

Figure 30: Calibration of the EUR funding curve for a DKK denominated bank, by adjusting the Euribor3M curve with the EUR/DKK 3M basis swap quotes. Both curves are represented as zero curves, and the spread and basis swap quotes have been added. All data is from Bloomberg.

To illustrate the difference between using the proper discounting curve for an uncollateralized trade compared to the OIS curve, we have priced an at market 10Y payer IRS with a notional of 10 mil EUR. Using the OIS discounting this has a market value of 0 per definition, however using the funding curve for discounting, the adjusted Euribor3M curve, we find the value to be $-3,884$, and we recognize this difference as the funding value adjustment.

Adjusting the funding curve is important not only in the uncollateralized scenario, but it is also relevant for some collateralised trades. Consider for example the EUR/DKK 3M cross-currency basis swap, and assume it is traded under a perfect CSA. Market practice is to use EUR as the collateral currency for this instrument[56]. As the EUR collateral earns the Eonia rate, it would be incorrect to discount the DKK cash flows using the DKK OIS curve. Instead we need to adjust the DKK OIS curve for the for EUR/DKK OIS basis, by similar argument as previously. This curve can be calibrated using a similar approach as used above to calibrate the adjusted Euribor3M curve, however now we need to use EUR OIS curve to discount EUR cash flows, and then calibrate the DKK discounting curve to the basis swap quotes.

4.5 Booking the spread

In this section we will discuss how banks are accounting the spread over the par swap rate that they earn when entering into an interest rate swap with a client. The following discussion is independent of whether the trade is traded under a (non-)perfect CSA or under no CSA.

Assume a bank trades a payer interest rate swap with a client at a fixed rate of 1.5%. Also assume that the bank is able to hedge this swap in the interbank market at a rate of 1%[57]. The spread of 0.5% is the profit the bank earns at undertaking such a derivative transaction with the client, and we may see this by splitting the client swap into a 1% swap, which is perfectly hedged with the interbank swap, and the remaining 0.5% which is an annuity. The present value of this spread is found easily by multiplying by the swap annuity. This spread would in reality contain several components and we may therefore split it into (at least) a bid-ask spread to the trading desk for the market risk, a spread to the sales desk, and a spread to the xVA desk to reflect the

[56]Similar holds for other types of EUR cross-currency basis swaps, as e.g. a EUR/SEK basis swap.
[57]These numbers are rather unrealistic, but are just used to illustrate the idea

price of hedging other risk factors. For our purpose we are indifferent as to how the spread is allocated between these, and will simply view the spread as whole.

The question is now how the bank is introducing this profit into the accounts. Generally speaking it has two options; the first is to book the entire present value of the spread in the first coming quarter. The second option is to retain the present value of the spread as a reserve, and then book it incrementally on an on-going basis over the lifetime of the swap. Choosing the second option enables the bank to use the reserve to offset any unexpected future losses, the approach used within xVA. While keeping (the xVA part of) the spread as a reserve is a common approach now, it was not so before the financial crisis where xVA desks generally did not exist.

In Denmark most large banks used the first approach at the time[58]. It can be argued that this is a more "aggressive" approach of booking the spread, as the incentive for the sales desk to advise the client to enter into more trades is higher. It is no secret that the remuneration of front office personal in banks are often performance based, and the fact that the profit from trades would be booked "immediately" is likely to influence the incentives. Allowing the profit to be deferred over the lifetime of the transaction can be argued to ensure a better alignnment of the incentives of the front office personal with those of the clients (Gregory (2015), p. 375).

This discussion has similarities to that of the capital value adjustment, KVA. There is no requirement or standard of how to include capital value adjustment into the accounts of banks. As there is no transfer-pricing of the KVA to the xVA desk, there is nothing issue for the bank in booking the profit from this immediately. However, to properly offset any unexpected future capital costs the KVA profit should rather be deferred over the lifetime of the transaction, by the same arguments.

[58]According to a professional with knowledge of this.

5 Case study

In this section we will try and apply some of the theory we have studied in the previous chapters. We will do this by examining the case of a hypothetical danish municipality, *Hedgelev Kommune*, who we will assume needs to borrow funds for a large project. We will review the considerations they might have done before choosing the obtain the synthetic loan by entering into a floating rate loan and a payer swap. We will then do an ex-post analysis of the synthetic loan to illustrate some of some the risk factors and the effect they have had, given the actual development in the economy and interest rates.

5.1 Introduction

We will study the case of Hedgelev Kommune, a hypothetical municipality in Denmark. We will assume the time is December 2007 and Hedgelev needs to borrow funds to develop a large project. For example it could be senior housing development. We assume they need to borrow 500 mil DKK and wish to do that for 30Y. Due to internal financial policies Hedgelev is required to have the interest rates on the loan fixed, for example to fix the expenses in the coming years.

For these reasons Hedgelev has two options. The first is to obtain a traditional fixed rate callable loan. The second option was presented by the municipality's bank, and constitutes taking a F1 flex loan and a payer interest rate swap against Cibor12M. The idea being that the floating legs cancel eachother out, and the municipality ends up paying the fixed rate in the swap. We will assume Hedgelev chose the second option.

The case is structured as follows; first we will review the considerations prior to entering into the swap contract. This includes a comparison of the projected costs of the two loans. To do this we first need to determine the fixed rate of the IRS. This includes calibrating a swap curve and approximating the xVA costs the bank may have required at the time. Secondly we will peform an ex-post analysis on the risk factors reviewed in section 3. We will focus on the basis risk factors as these are not present in the comparable callable loan.

5.2 Before the trade

5.2.1 Curve calibration

As the IRS is against Cibor12M we will need to calibrate a Cibor12M curve. We will do this in a single-curve setup to accurately reflect how this was calculated at the time prior to the financial crisis. However, it has proven difficult to obtain market quotes for IRS against Cibor12M at that time; late December 2007. To redeem this issue we will simply use market quotes for swaps against Cibor6M instead, and use these as approximations. Obviously this is quite an assumption, but we believe it is unlikely to have material effect for the calculations as of December 2007, as the tenor basis for Cibor rates was very low at the time (less than 1 bps between Cibor6M and Cibor12M on 02/01/2008 compared to 19.75 bps on 02/01/2018). In our calculations we will calibrate curves on dates after 2007, and to keep consistency we will continue using Cibor6M IRS quotes for our Cibor12M curve at these calibrations. This will with certainty induce some errors in our calculations, but we will ignore these.

Our anchor date is December 7th 2007 and the spot settlement date is December 11th 2007. We chose this as the F1 rate was determined in the days 10th to 14th of December that year as evident in appendix B.1, and given the previous years a natural expectation was for future auction days to occur around the same time in December. Given these dates, the Cibor12M fixing and the Cibor6M IRS quotes we calibrate the zero and 12M forward rate curves illustrated in figure 31 below.

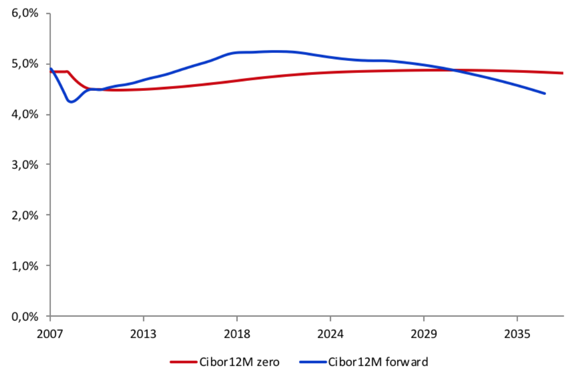

Figure 31: The calibrated zero and 12M forward rate curves against Cibor12M as of December 7th 2007, using Cibor6M IRS market quotes as proxy. All data is from Bloomberg.

5.2.2 Expected costs: Callable vs synthetic

In this section we will sketch some of the considerations relevant when choosing a callable loan versus a synthetic loan by estimating the expected costs. We will find four metrics for this; the fixed rate paid in the loan, the starting principal, the total projected payments over the duration of the loan and a "yearly cost" metric known in Denmark as ÅOP, short for "yearly costs in percentage".

To do this we need two fee figures; one is a yearly percentage fee of the remaining principal ("bidragssats"), which we denote the *yearly fee*, and the other is a fee charged when selling the bonds underlying the loan ("kursskæring") by reducing the issue price. We denote this the *price fee*. The last fee is particularly relevant for F1 loans, as these are refinanced every year. Both of these fees are paid to the mortgage provider.

To calculate the cost figures we have used the following assumptions of prices, coupon rates and fees. The coupon rate on the synthetic loan is the fixed rate in the IRS, and the calculation of this is reviewed in the next sections.

	Coupon rate	Price	Yearly fee	Price fee
Synthetic	4.957%	100	0.5%	0.20
Callable	5%	97.63	0.5%	0.20 (0.30)

Table 7: Assumptions to calculate cost metrics. The price of the callable bond is from Bloomberg and as of December 7th 2007.

For the flex loan we asume a price fee of 0.20 for the initial bond sale, and 0.30 for subsequent refinancing. Adjusting for the price fee the issue price of the F1 and the callable bonds become 99.80 and 97.43, respectively. To obtain a loan of 500 mil DKK, a starting principal of 501,002,004 and 513,188,965 are needed, respectively. Using this and the coupon rates an amortization scedule is calculated for each loan as shown in appendix B.2 assuming they are annuity loans. The costs of the loans are calculated as follows:

$$c_t^{\text{call}} = 0.50\% \cdot N_{t-1}$$
$$c_t^{\text{F1}} = (0.50\% + 0.30\%) \cdot N_{t-1}$$

Where the 0.50% is the yearly fee. Further we have assumed that the 0.30 price fee for refinancing the F1 loan is approximately equal to a running yearly cost 0.30%, when in fact it is a deduction of 0.30 in the issue price of the bond. For the synthetic loan we also need to find the payments of the IRS to estimate the total costs. We do this by calculating the projected floating payments using the forward curve. Doing this we implicitly account for the gain for Hedgelev due to a positive basis between the Cibor12M and the F1 rate, which is not evident when just comparing the coupon rates of the two loans. For the callable loan the total payment for each period is the payment ("ydelse") plus the cost. For the synthetic loan the total payment is the payment and costs from the F1 loan, minus the projected floating payment of the IRS (as this is received), and plus the fixed payment. This gives us the projected cost schedules as presented in appendix B.3.

Given the cost schedules we are now able to calculate the yearly cost metric as the yield of the loan, given a "present value" of 500 mil DKK. We have presented the cost metrics for the two loans in below table. As is evident, there appear to be a slight cost advantage of choosing the synthetic loan. However it is not impressive, especially considering the far superior protection against falling interest rates that the callable loan provides. We will now assume Hedgelev Kommune chose the synthetic loan by obtaining a F1 loan and entering into a payer interest rate swap against Cibor12M with it's bank.

	Coupon rate	Starting principal	Total payments	Yearly cost
Synthetic	4.957%	501,002,004	1,024,105,869	5.623%
Callable	5%	513,188,956	1,050,342,481	5.740%

Table 8: Cost metrics for the synthetic and callable loans.

5.2.3 Swap rate

In this section we explain how the fixed rate of 4.957% in the IRS was calculated. We have assumed that it consists of four elements; the par swap rate, a spread (profit) for the swap desk and the sales desk, and a xVA cost converted into a spread.

The par swap rate is found using the curve calibration as of December 7th 2007 and the notional profile of the 30Y IRS. We have assumed the notional profile is set to match the expected principal profile of the F1 loan. Under this assumption the notional profile is identical to that presented in table 13 in appendix B.2. Using this we find the par swap rate to be 4.890% by using equation (7).

Next we have for simplicity assumed a spread of 2.5 bps for the swap desk and the sales desk each. The accuracy and how realistic these numbers are can be argued. We argue that due to varying notional of the swap it is not as simple to hedge as a vanilla, fixed notional IRS. As such it is likely that the swap trader would require a larger bid-ask spread. Secondly we argue that it is likely that to trade such and IRS with a customer like Hedgelev Kommune, the bank has had to allocate more sales ressources (for face-to-face client meetings etc.) than for e.g. an institutional investor, and hence the spread for the sales desk is larger.

The last component is the xVA spread. We estimate that to be 1.6 bps, with the calculation of this to follow in the next section.

Combining this we arrive at the swap rate of 4.957%. The market value of the IRS at initiation is -3,814,826, which corresponds to (minus) the total spread of 6.7 bps times the notional-scaled swap annuity of 5,747,741,205.

5.2.4 xVA spread

In this section we review the calculation of the xVA spread of 1.6 bps, which consists of CVA, FVA and KVA. It is unlikely that a danish bank was pricing all of these xVAs back in 2007, where only (perhaps) CVA was standard. However we have included all three here for illustration. We will calculate all three value adjustments using the procedure given in sections 4.3.1-4.3.3. To calculate the xVAs we will need to project the future exposure as seen from the bank. We will again estimate this by using the forward curve to project cash flows, and refrain from any simulation.

To calculate the xVAs we have to make some assumptions. First of all we will assume a constant credit spread of 0.20% for Hedgelev Kommune. Assuming the yield spread between bonds issued by Kommunekredit (mortgage provider for danish municipalities) and the danish government is indicative of the credit spread this appear to be a reasonable spread. As of May 2018 the yield spread was around 13 bps in the very short end and 45 bps in the long end. Some of this is likely a liquidity premium, and hence we will consider the constant credit spread of 20 bps as acceptable. Secondly we will again use the standard assumption of a recovery rate of 40%. Lastly we will assume a constant funding spread of 0.10%. This is slightly more controversial. Referring to the discussion in section 4.4 we would in the multi-curve setup approximate the funding spread by the Libor/OIS spread. However this approach is likely not representative of the situation as it was back in 2007. One approach is to assume banks could actually fund themselves at Libor at the time and hence a funding spread of 0. Instead we will assume banks actually had a funding cost and thus a positive funding spread of 10 bps.

Using the forward curve to project future market values of the IRS as seen from the bank, we have generated the below EFV and EE profiles, as well as the EAD and RC profiles. For the EE profile, it is clear that it is calculated as $EE(t_i) = \max(EFV(t_i), 0)$. For an explanation of how to derive the EAD and RC profiles we refer to section 4.3.3 on KVA as well as appendix 8A of Gregory (2015).

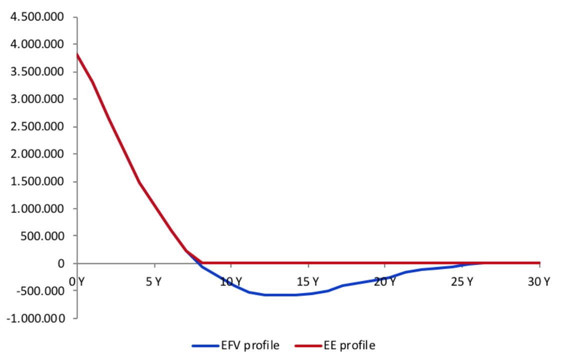

Figure 32: Illustration of the EFV and EE profiles using the forward curve to project future exposure.

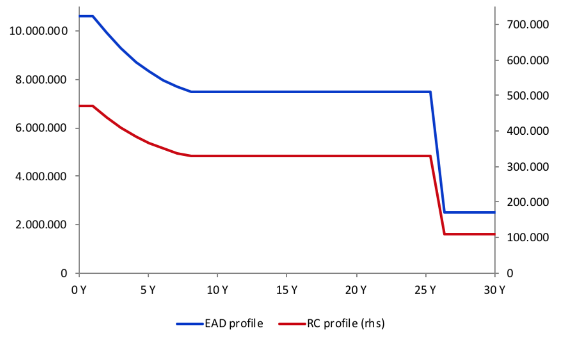

Figure 33: Illustration of the EAD and RC profiles.

Using above profiles, equations (14), (16) and (17) and the assumptions stated above we are now able to calculate the value adjustments. These have been shown in the table below. Further we are able to convert these xVA costs into a spread by dividing with the risky annuity. Using equation (15) we find the risky annuity to be 5,657,660,146.

	Cost	Spread
CVA	-20,244	-0.04 bps
FVA	-7,291	-0.01 bps
KVA	-898,673	-1.59 bps
Total xVA	-926,208	-1.64 bps

Table 9: xVA as costs and spreads.

We again note the very modest xVA cost. This is partly due to our approach to calculate the future exposure instead of simulation. But also the yield curve is affecting this. Instead of the typical upward sloping yield curve, it is rather flat as seen in figure 31. This affects the projected floating payments which does not deviate much from the fixed payments due the (rather) flat forward curve.

5.3 After the trade

5.3.1 Delta risk

We may calculate a delta vector for the IRS as seen from Hedgelev Kommune's perspective to express the delta risk profile of the swap. We calculate the delta vector using the curve calibration on the anchor date December 7th 2007. Using the procedure of section 2.4.2 we calculate the market rate delta vector for the IRS with a fixed rate of 4.957%. This has been shown in below table:

Quote	Delta risk
Cibor12M	-798
2Y IRS	-1,063
3Y IRS	-4,409
5Y IRS	-7,619
7Y IRS	-17,453
10Y IRS	-22,272
12Y IRS	-25,360
15Y IRS	-70,571
20Y IRS	-113,670
25Y IRS	-205,439
30Y IRS	-108,683
DV01	-577,336

Table 10: Market rate delta vector for the payer IRS with a fixed rate of 4.957%.

As we can see the IRS has significant delta risk to the market quotes. As expected Hedgelev will experience a loss when rates decrease as evident by the negative DV01, because they are positioned for higher rates in their payer swap. A 30Y vanilla IRS with the fixed rate equal to the par swap rate, would per definition only have risk to the 30Y IRS market quote. As our IRS is off-market due to the spread, we would expect risk exposure to other market quotes due to discounting effects. However discounting risk is only a minor influence in the distribution of risk in the above delta vector. Instead the above is caused by the varying notional of the IRS. Indeed the *at market* IRS with the same notional profile and a fixed rate of 4.890% has a delta vector which is very similar to the above. Interestingly we see that much of the delta risk is located in the long-end of the curve despite the notional of the IRS being significantly lower here.

Using the delta vector we are able to approximate the change in value of the IRS given interest rate changes. Assume that two months have passed since the trade and the date is now February 7th 2008. The market quotes at this day, and the change relative to December 7th has been presented in appendix B.4. Using the market quote changes and the original delta vector we may approximate the new market value. The approximate *change in value* is -11,508,896, and given the original value the new estimated market value is -15,323,722. Using the curve data as of February 7th 2008 we may calibrate a curve setup and price the IRS exactly. Doing that the exact market value is found to be -15,704,510, and we see that the approximation was quite close. The discrepancy in the market values is because two months have passed of the "lifetime" of the IRS, and so the swap is no longer the same as it was when the delta vector was calculated. For example discount factors have changed because of the new anchor date. The exact pricing accounts for these effects, whereas the approximation does not.

5.3.2 Effect of notional mismatch

In this section we will give examples of the effect of a mismatch between the principal profile in the F1 loan and the notional profile in the swap. As the floating leg of the IRS is meant as a hedge, it is important that the notional profiles of the two match. As we reviewed in section 3.4.3 a mismatch appears when the interest rate in the flex loan deviates from the initial rate. This is because the initial F1 rate is used to project the notional profile of the IRS, but when the F1 rate changes then the principal profile of the loan changes from the initial projection. We will review this for a F1 loan, but the same risk is present for other flex loans.

At initiation of the loan the F1 rate was 4.73% as evident in table 12, which also shows the actual development in the F1 rate. Further, given the issue price of 99.80 the projected principal profile of the F1 rate was calculated as presented in table 13 in appendix B.2, and the notional profile of the IRS was therefore set to match this. As we know the interest rates changed; for the first year they increased, but after 2008 they have almost continuously decreased since. As a result the principal profile of the loan deviates from that of the IRS. In appendix B.5 we have presented the ammortization schedule given the actual F1 rates, as well as the expected principal profile from 2019 and onwards. Figure 34 below illustrates the mismatch between the notional profile of the IRS and the (partly) realized loan principal profile.

As we can see the notional profile of the IRS is "too high" relative to the loan. This is consistent with the lower F1 rates which have increased the repayment amounts and thus lowered the principal profile of the loan.

This mismatch in the profiles has induced a cost for Hedgelev Kommune. We will define the total cost for Hedgelev using two components. The first is the mismatch in the realized payments of the IRS compared to the "perfect" IRS, whose notional profile exactly matches those of the F1 loan. The second component is the difference in market value at time t_i between the IRS and the "perfect" IRS. Here we assume that the notional profile of the "perfect" IRS matches the time-t_i *expected* principal profile of the loan. Hence we do this comparison in an "all else equal" approach by using the notion of a "perfect" IRS. For all calculations we however will

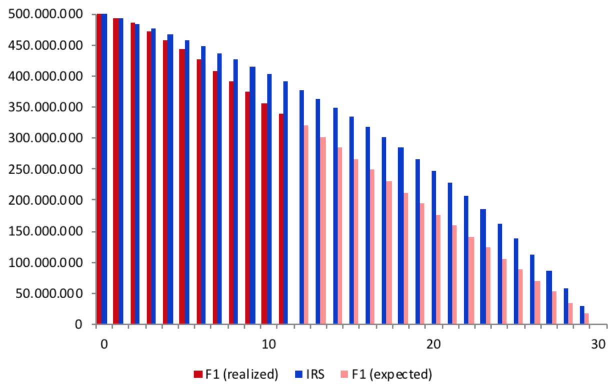

Figure 34: The development in the F1 principal profile compared to the IRS.

keep the fixed rate constant, and equal to the fixed rate of the original IRS of 4.957%. Even though changing the notional (which we are effectively doing when assuming a "perfect" IRS) also changes the fixed rate, we simply assume it to be constant here to isolate the effect of the changing notional.

In table 18 in appendix B.6 we have presented the difference in payments between the two swaps, as well as a cumulated difference. Calculating the cumulative difference we disregard any issues concerning discounting and present values. As an example the difference in payments on 10/12/2012 (after the payment on that date has transacted) was -309,688 and the cumulative difference was -425,603. We have also calculated the difference for the date 10/12/2018 because these payments are already known.

For the payment dates in table 18 we are able to calibrate a swap curve and price the market value of the actual IRS and "perfect" IRS as of that date. Hence we are using the F1 rate to calculate the at-the-time expected principal profile of the loan and setting the notional of the "perfect" IRS equal to this profile. Doing that we are able to calculate the difference in market values between the two IRSs as presented in table 19 in appendix B.6. The pricing on all dates has been done in a single-curve setup to keep the approach consistent. To calculate this we have again calibrated a pseudo-Cibor12M curve using Cibor6M IRS quotes. Obviously this create errors in our market values but we will disregard this issue.

We are now able to calculate the total effect between the actual IRS and the "perfect" IRS. We do that by adding the cumulative difference from table 18 and the difference in market values of table 19. We have presented this in below table and illustrated it with a graph.

As we can see the total effect in December 2017 was very significant, and amounted to approx. 42.5 mil DKK, between the actual IRS and the "perfect" IRS, whose notional match the already realized and the expected principal profile of the F1 loan givent the current F1 rate of -0.20%. We can also see from the figure, that the MV effect is gradually "converted" into the payments effect as the payments on the IRS occur. An effect of close to 10% of the original principal of the loan after only a third of the loans maturity is indeed very significant, and highlights the importance of the effect from the notional mismatch.

5.3.3 Spread risk

In this section we will exemplify the spread risk as reviewed in section 3.4.1. In this case spread risk is the risk associated with movements in the spread between the Cibor12M rate and the F1

Date	Diff. paym.	Diff. MV	Total
10/12/2008	0	682,446	682,446
10/12/2009	0	-1,679,962	-1,679,962
10/12/2010	18,929	-4,688,981	-4,670,051
12/12/2011	-116,015	-15,339,999	-15,456,014
10/12/2012	-425,603	-22,100,214	-22,525,817
10/12/2013	-1,051,838	-15,655,868	-16,707,706
10/12/2014	-1,968,371	-29,904,428	-31,872,799
10/12/2015	-3,172,258	-30,593,298	-33,765,556
12/12/2016	-4,775,071	-33,557,859	-38,332,930
11/12/2017	-6,680,533	-35,911,712	-42,592,245

Table 11: Total effect between the actual IRS and the "perfect" IRS using both the cumulative payments and the difference in market values.

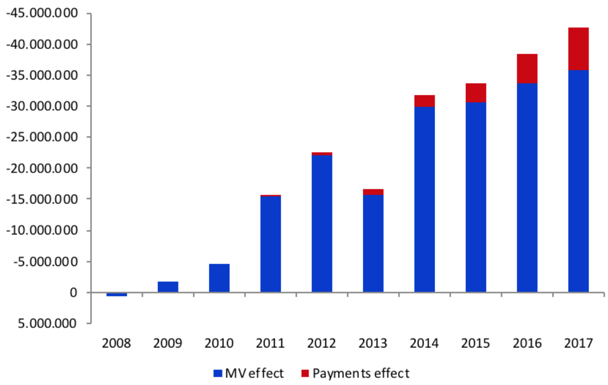

Figure 35: Illustration of the two components on the total effect of notional mismatch in the IRS.

rate. We will define the spread as the Cibor12M rate minus the F1 rate. Hedgelev Kommune receives the Cibor12M rate in the interest rate swap and pays the F1 rate in the loan, and so gains when the spread increases and loose when it decreases. Figure 36 below illustrates the interest rates and the spread on the 11 payment dates in the IRS that has settled as of May 2018.

We will calculate the payment at date t_i as a function of the spread as $PL(t_i) = \alpha(t_i)(L^{\text{Cib12}}(t_{i-1}, t_i) - L^{\text{F1}}(t_{i-1}, t_i))N_i$ where α and N denotes coverage and notional. To illustrate the effect of spread movements we will further calculate the payments assuming the spread was constant and equal to the 19.5 bps as of 10/12/2007. Using this we find the payments as presented in table 20 in appendix B.7. We have illustrated the payments in the two scenarios in below figure 37.

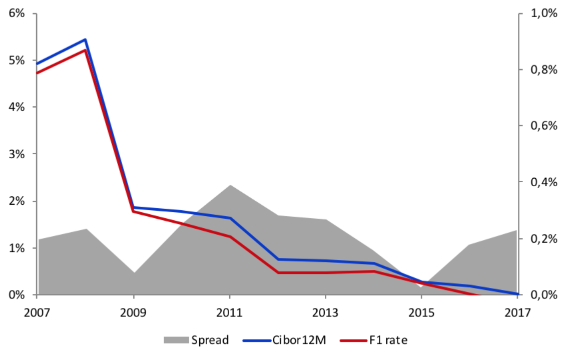

Figure 36: Illustration of the spread movements between 2007 and 2017. Cibor12M data is from Bloomberg.

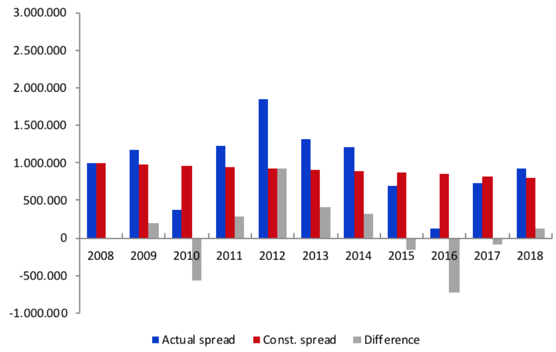

Figure 37: Illustration of the effect of spread movements on payments, as well as the payments assuming a constant spread.

As the borrower receives the spread, and it has been positive for all periods, this effect has been in favor of Hedgelev Kommune overall. The total payments contributed to the spread is 10,632,405 DKK as evident in table 20. Also we note that in this scenario the overall spread movements since 2007 has been in favor of Hedgelev compared to the scenario of a constant spread, since the constant spread scenario only entailed payments of 9,910,649. However, as we see the spread differs quite a lot over the period, and naturally so does the payments attributable to the spread.

5.3.4 Reset risk

In this section we study the effect of reset risk in this particular case study. Reset risk was reviewed in section 3.4.2 and relates to risk associated with the fixing of the Cibor12M rate in the IRS not occuring on the same date as the F1 rate fixing. Ideally these floating rates should reset on the same day, because otherwise Hedgelev is exposed to the risk of adverse interest rate movements in the intermediary period. In the calculation of this we will assume the floating rates fix on the payment date; the 10th of December (when in reality they would fix two days prior).

The first reset date in the IRS was chosen to match that of the F1 rate and was December 10th 2007. However in the subsequent years the reset date of the F1 rate was moved a few weeks earlier in the year to the end of November as shown in table 12 in appendix B.1. As such Hedgelev is generally exposed to decreasing Cibor12M rates between the F1 reset date and the IRS reset date, because the Cibor12M rate is received in the swap.

We will illustrate the effect of this by calculating the floating payment in the swap if it had fixed on the same date as the F1 rate and compare it to the actual payment of the IRS. In calculating this we keep the coverage constant to isolate the effect of interest rate movements. We may think of this as simply adjusting the *reset* date in the IRS and not the payment date.

In table 21 in appendix B.8 we have shown the effect of the difference in reset dates. We have illustrated this in below figure.

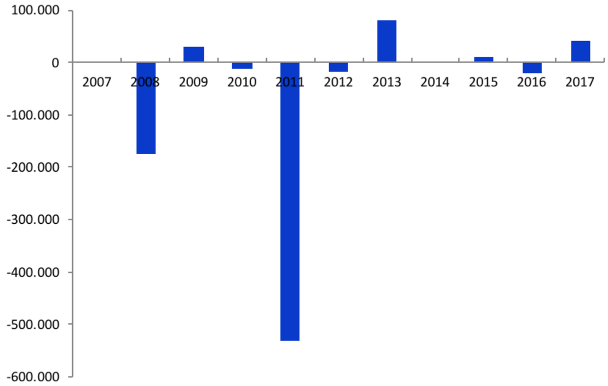

Figure 38: Illustration of the effect of difference in reset days between the F1 loan and the IRS, compared to the scenario of identical reset dates.

As we can see this effect has overall worked against the customer in this scenario. A total effect of -597,424 (ignoring any discounting and present value effects) has been experienced compared to the scenario of identical reset dates in the loan and IRS. This effect is primarily driven by 2011, where there was a substantial decrease in the Cibor12M rate from 1.7450% on the F1 reset date 30/11/2011 to 1.6325% on the Cibor12M reset date 12/12/2011. The impact of this effect is reduced with time as the notional in the F1 loan and the interest rate swap decreases.

6 Conclusion

The aim of this thesis was to explore the risk factors facing the borrower when entering into a synthetic non-callable mortgage loan, compared both to the callable and the non-callable loan. We have shown that these risk factors may be divided into two groups: one for those that are also experienced in the "true" non-callable, but not the callable loan, and the second which is relevant in comparing the two non-callable loans.

The first group consists of the risk factors towards interest rates; duration and convexity. The primary risk here is decreasing interest rates. The callable loan provides considerably better protection against that scenario compared to the (synthetic) non-callable loan. However regardless of the loan, decreasing interest rates is always unfavorable for the borrower.

The second group of risk factors is comprised of basis risk and liquidity risk. Basis risk relates to the risk of an imperfect hedge of the floating rate loan with the swap. This includes the spread risk between the Cibor rate received in the swap and the flex loan rate paid in the loan. It also includes the reset risk caused by the future fixing dates of the flex loan being uncertain. Lastly it includes the risk associated with a mismatch in the notional profile between the IRS and the loan. Assuming the synthetic loan is the combination of an F1 loan and a IRS against Cibor12M we investigated these basis risks using a hypothetical case study. The spread has overall been in favor of the borrower. The overall development in the spread has only had a small effect compared to a scenario of a constant spread, but this covers significant effects in the individual periods. The reset risk is also modest in size, but for longer or more volatile periods it may be significant. The largest effect by far was seen in the risk of mismatch in notional. We showed that this effect is present regarding of the directionality of interest rate movements, and is caused by the readjustment of the expected principal profile in the loan due to interest rate movements. In our case study the loss from this mismatch was close to 10% of the notional over a period of just 10 years. This was caused by the large interest rate decline from an F1 rate of 4.73% in 2007 to -0.20% in 2017.

We also showed how pricing of interest rate swaps (and other derivatives) depends on the CSA agreement under which it is traded. Derivatives should be discounted using the rate of return on the collateral. If there is traded under no CSA, as is the case for the borrowers of interest here, we showed that the bank should discount using a curve representing their funding rate. We also showed how this pricing approach was linked to the funding value adjustment. The construction of a proper discounting curve was further complicated in the pricing of derivatives in currencies other than the bank's funding currency. In that case we showed that the discounting curve needed to be adjusted for the basis between the two currencies and indices.

Lastly we reviewed the value adjustments made by banks to derivatives contracts. We gave an intuitive approach to the calculation of CVA, FVA and KVA costs. However, our calculation examples was considerably affected by our simple approach of estimating the future exposure.

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A Appendix

A.1 FRA present value

The following derivation largely follows Ametrano & Bianchetti (2013), p. 72-74. The time t present value of the FRA payoff as seen from equation (3) is:

$$\begin{aligned}
PV_{\text{fra}}^{\text{lib}}(t) &= P^{\text{disc}}(t,t_0)E_t^{Q_f^{t_0}}\left[\frac{N\alpha^{\text{lib}}(F^{\text{lib}}(t_0,t_0,t_1)-\kappa^{\text{fra}})}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] \\
&= NP^{\text{disc}}(t,t_0)E_t^{Q_f^{t_0}}\left[\frac{(1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)-(1+\alpha^{\text{lib}}\kappa^{\text{fra}}))}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] \\
&= NP^{\text{disc}}(t,t_0)\left(1-(1+\alpha^{\text{lib}}\kappa^{\text{fra}})E_t^{Q_f^{t_0}}\left[\frac{1}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right]\right) \qquad (19)
\end{aligned}$$

Where we have used that $L^{\text{lib}}(t_0,t_1)=F^{\text{lib}}(t_0,t_0,t_1)$, and where α^{lib} is the coverage between time t_0 and time t_1. Changing forward measure from $Q_f^{t_0}$ to $Q_f^{t_1}$ means that the expectation from above instead becomes:

$$\begin{aligned}
E_t^{Q_f^{t_0}}\left[\frac{1}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] &= E_t^{Q_f^{t_1}}\left[\frac{1}{P^{\text{disc}}(t_0,t_1)}\frac{P^{\text{disc}}(t,t_1)}{P^{\text{disc}}(t,t_0)}\frac{1}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] \\
&= \frac{P^{\text{disc}}(t,t_1)}{P^{\text{disc}}(t,t_0)}E_t^{Q_f^{t_1}}\left[\frac{1}{P^{\text{disc}}(t_0,t_1)}\frac{1}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] \\
&= \frac{1}{1+\alpha^{\text{disc}}F^{\text{disc}}(t,t_0,t_1)}E_t^{Q_f^{t_1}}\left[\frac{1+\alpha^{\text{disc}}F^{\text{disc}}(t_0,t_0,t_1)}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] \qquad (20)
\end{aligned}$$

Where F^{disc} denotes the forward rate of the funding/discounting index and where we have used the standard change-of-numeraire approach and the Radon-Nikodym derivative (Poulsen (1999) p. 3-4):

$$\left(\frac{dQ_f^{t_0}}{dQ_f^{t_1}}\right)_{t_0} = \frac{P^{\text{disc}}(t_0,t_0)}{P^{\text{disc}}(t_0,t_1)}\frac{P^{\text{disc}}(t,t_1)}{P^{\text{disc}}(t,t_0)} = \frac{1}{P^{\text{disc}}(t_0,t_1)}\frac{P^{\text{disc}}(t,t_1)}{P^{\text{disc}}(t,t_0)}$$

Inserting equation (20) into (19) we find:

$$PV_{\text{fra}}^{\text{lib}}(t) = NP^{\text{disc}}(t,t_0)\left(1-\frac{1+\alpha^{\text{lib}}\kappa^{\text{fra}}}{1+\alpha^{\text{disc}}F^{\text{disc}}(t,t_0,t_1)}E_t^{Q_f^{t_1}}\left[\frac{1+\alpha^{\text{disc}}F^{\text{disc}}(t_0,t_0,t_1)}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right]\right)$$

The last term, the expectation of the ratio of the future funding and Libor rates, may in general be written as follows (Ametrano & Bianchetti (2013), p. 73):

$$E_t^{Q_f^{t_1}}\left[\frac{1+\alpha^{\text{disc}}F^{\text{disc}}(t_0,t_0,t_1)}{1+\alpha^{\text{lib}}F^{\text{lib}}(t_0,t_0,t_1)}\right] = \frac{1+\alpha^{\text{disc}}F^{\text{disc}}(t,t_0,t_1)}{1+\alpha^{\text{lib}}F^{\text{lib}}(t,t_0,t_1)}e^{C_{\text{fra}}(t_0)}$$

Where $C_{\text{fra}}(t_0)$ is the convexity adjustment, and depends on the particular model used for the funding and Libor rate. Using this we find that:

$$PV_{\text{fra}}^{\text{lib,theor.}}(t) = N\alpha^{\text{lib}} P^{\text{disc}}(t, t_0) \left(\frac{F^{\text{lib}}(t, t_0, t_1) - \kappa^{\text{fra}}}{1 + \alpha^{\text{lib}} F^{\text{lib}}(t, t_0, t_1)} e^{C_{\text{fra}}(t_0)} \right)$$

Ignoring the convexity adjustment term by setting it equal to 0 so that the exponential is 1, we find the simplified version of the FRA present value to be:

$$PV_{\text{fra}}^{\text{lib,simp.}}(t) = N\alpha^{\text{lib}} P^{\text{disc}}(t, t_0) \left(\frac{F^{\text{lib}}(t, t_0, t_1) - \kappa^{\text{fra}}}{1 + \alpha^{\text{lib}} F^{\text{lib}}(t, t_0, t_1)} \right)$$

A.2 OIS coupon rate

Under proper numeraire assumptions it holds that the overnight FRA rate will satisfy (Ametrano & Bianchetti (2013), p. 76-77):

$$R(t, t_{i,k-1}, t_{i,k}) = E_t^{Q_{t_{i,k}}} [R(t_{i,k-1}, t_{i,k})] \tag{21}$$

and thus:

$$R(t, t_{i,k-1}, t_{i,k}) = \frac{1}{\alpha_{i,k}^{\text{float}}} \left(\frac{P^{\text{ois}}(t, t_{i,k-1})}{P^{\text{ois}}(t, t_{i,k})} - 1 \right)$$

Inserting the above into equation (9) we find the following:

$$\begin{aligned} R(t_i, \boldsymbol{t_i}) &= \frac{1}{\alpha_i^{\text{float}}} \left(\prod_{k=1}^{n_i} [1 + R(t_{i,k-1}, t_{i,k}) \alpha_{i,k}^{\text{float}}] - 1 \right) \\ &= \frac{1}{\alpha_i^{\text{float}}} \left(\prod_{k=1}^{n_i} \frac{P^{\text{ois}}(t, t_{i,k-1})}{P^{\text{ois}}(t, t_{i,k})} - 1 \right) \\ &= \frac{1}{\alpha_i^{\text{float}}} \left(\frac{P^{\text{ois}}(t, t_{i-1})}{P^{\text{ois}}(t, t_i)} - 1 \right) \end{aligned}$$

Where we have used $t_{i,0} = t_{i-1}$ and $t_{i,n_i} = t_i$.

A.3 FX forwards & cross-currency basis swaps

To see the appeal of the cross-currency basis swap we first have to turn to the FX forward contract. Let $s(t)$ denote the spot exchange rate, defined the standard way as the price in the "domestic" currency of a single unit of "foreign" currency. The forward exchange rate, $f(t, t_i)$ is the price agreed upon today at time t to exchange cash flows in the two currencies at time t_i in the future, i.e. it is the forward domestic currency price of a unit of the foreign currency. We can derive the forward rate easily. Assume F denotes the foreign currency and D the domestic currency. Assume we buy 1 unit of foreign currency at time t_i at the price $f(t, t_i)$. Discounting each side of the transaction using the discount curve of each currency and using the spot exchange rate to convert all payments to the domestic currency, we find the PV at time t to be:

$$PV_D^{\text{fxfwd}}(t) = s(t) P_F^{\text{disc}}(t, t_i) - f(t, t_i) P_D^{\text{disc}}(t, t_i) \tag{22}$$

Setting the PV to 0, as usual, and re-arranging we find the so-called *covered interest rate parity*:

$$f(t,t_i) = s(t)\frac{P_F^{\text{disc}}(t,t_i)}{P_D^{\text{disc}}(t,t_i)} = s(t)e^{(r_D(t,t_i)-r_F(t,t_i))(t_i-t)} \tag{23}$$

This is a no-arbitrage relation, which imposes that the forward exchange rate must take into account the interest rate differential between the two currencies, such that it is not possible to produce risk-free, guaranteed profits. To see this assume that $f(t,t_i) > s(t)e^{(r_D(t,t_i)-r_F(t,t_i))(t_i-t)}$, so that we would like to sell 1 unit of the foreign currency forward as it is "expensive". Assume we are able to borrow $s(t)e^{-r_F(t,t_i)(t_i-t)}$ of domestic currency, to be paid at time t_i, and that we can borrow at the rate $r_D(t,t_i)$. The domestic currency are used to buy $e^{-r_F(t,t_i)(t_i-t)}$ of foreign currency immediately at the spot price. For these we buy a single foreign currency denominated zero coupon bond, which has a price of $P_F^{\text{disc}}(t,t_i) = e^{-r_F(t,t_i)(t_i-t)}$, and at time t_i receive 1 foreign currency. This is delivered in the FX forward contract and we receive $f(t,t_i)$ unit of domestic currency. At time t_i we have to repay the loan which has increased to $s(t)e^{(r_D(t,t_i)-r_F(t,t_i))(t_i-t)}$, leaving us with $f(t,t_i) - s(t)e^{(r_D(t,t_i)-r_F(t,t_i))(t_i-t)} > 0$ at time t_i.

The FX forward is widely used by market participants, as it enables them to lock in a future exchange rate. This may be advantageous for e.g. a corporate who knows that they will receive a foreign currency amount at a specific time in the future as payment for sold goods. However, as can be seen from equations (22) and (23) the FX forward is sensitive to the interest rate differential between the two currencies, and this risk is increasing with the maturity of the contract. As an example consider a market participant who has entere into an FX swap, where she sold the foreign currency spot at the price $s(t)$ and bought it forward at price $f(t,t_i)$. Now assume that the interest rate differential $r_D(t,t_i) - r_F(t,t_i)$ decreases reflecting a higher interest rates in the foreign currency relative to the domestic currency (for simplicity assume foreign interest rates has increased and domestic rates has not changed). As a result the PV of the FX forward part of the swap decreases as can be seen in equation (22), reflecting that we have bought forward at a price that is "too high" relative to where the new market is pricing the FX forward. In fact it is due to the value of receiving a single unit of foreign currency forward has decreased as the foreign interest rate increases (relative to the domestic currency), i.e. the discounting of that amount increases or alternatively that the expected return in the foreign currency has increased with the higher interest rates and we therefor incur oppportunity costs, decreasing the value of the unit of foreign currency bought forward.

Consider now instead the cross-currency basis swap which is similar to the FX swap, except that the forward exchange of payments also occur using the spot rate, and for the intermediary exchange of floating payments. In this particular example where the foreign currency is bought forward, we would also receive foreign currency floating payments (and pay domestic currency floating payments). Assume again that the interest rate differential $r_D(t,t_i) - r_F(t,t_i)$ decreases. Again the value of the unit of the foreign currency received forward decreases as the discounting of this amount increases, however this effect is offset due to the intermediary foreign currency floating payments which has increased due to the higher foreign interest rates. Corresponding results hold for increased interest rate differential, opposite directionality etc. It is exactly for this reason that the cross-currency basis swap is used, as it is far less interest rate sensitive than the FX forward, and especially for longer maturities where this effect may be significant.

B Appendix

B.1 F1 rate fixing 2006-2018

In the case study we have a need to know an approximate date for the fixing of the F1 rate each year, for example to study the effect of reset risk. To approximate these dates we have used notifications from the mortgage provider BRFkredit to the stock exchange Nasdaq OMX. In these BRFkredit announce the dates at which the bonds underlying the flex loans are sold each year for the January reset period. These bonds are sold over several days, and we have chosen the representative date to be approximately in the middle of the period. Below table illustrates the dates, where "Settlement year" denotes the year over which the F1 rate is relevant. The F1 rates has been retrieved from Realkredit Danmark's website, and denotes the January F1 rate for the loan *with no* exemption from repayments ("med afdrag") (Realkredit Danmark (2018)):

Settlement year	Fixing year	Auction dates	Representative date	F1 rate
2006	2005	1-20 of December	09/12/2005	2.89%
2007	2006	11-15 of December	13/12/2006	4.11%
2008	2007	10-14 of December	11/12/2007	4.73%
2009	2008	1-12 of December	05/12/2008	5.20%
2010	2009	23-11 of Nov/Dec	02/12/2009	1.78%
2011	2010	29-10 of Nov/Dec	03/12/2010	1.52%
2012	2011	24-7 of Nov/Dec	30/11/2011	1.24%
2013	2012	21-4 of Nov/Dec	29/11/2012	0.48%
2014	2013	18-29 of November	25/11/2013	0.46%
2015	2014	19-26 of November	24/11/2014	0.51%
2016	2015	17-26 of November	25/11/2015	0.24%
2017	2016	17-25 of November	23/11/2016	0.00%
2018	2017	20-24 of November	22/11/2017	-0.20%

Table 12: Representative dates for the fixing of the F1 rates.

B.2 Loan ammortization schedules

Given the principals of 501,002,004 and 513,188,965 of the F1 and callable loans, respectively, as well as their coupon rates of 4.73% and 5% a projected payment ("ydelse") *before* other costs is calculated using:

$$y = \frac{r}{1-(1+r)^{-n}} N_0$$

With $n = 30$ the payments are found as 31,594,754 and 33,383,678, respectively. Letting the interest payment be the coupon rate times the beginning-of-period notional ($i_t = r \cdot N_{t-1}$) we find the ammortization schedules of the F1 and callable loans as indicated in below tables:

Period	Payment	Interest	Repayment	Principal
0	0	0	0	501,002,004
1	31,594,754	23,697,395	7,897,359	493,104,645
2	31,594,754	23,323,850	8,270,904	484,833,741
3	31,594,754	22,932,636	8,662,118	476,171,624
4	31,594,754	22,522,918	9,071,836	467,099,788
5	31,594,754	22,093,820	9,500,934	457,598,855
6	31,594,754	21,644,426	9,950,328	447,648,527
7	31,594,754	21,173,775	10,420,978	437,227,549
8	31,594,754	20,680,863	10,913,890	426,313,658
9	31,594,754	20,164,636	11,430,117	414,883,541
10	31,594,754	19,623,991	11,970,762	402,912,779
11	31,594,754	19,057,774	12,536,979	390,375,800
12	31,594,754	18,464,775	13,129,978	377,245,822
13	31,594,754	17,843,727	13,751,026	363,494,795
14	31,594,754	17,193,304	14,401,450	349,093,346
15	31,594,754	16,512,115	15,082,638	334,010,708
16	31,594,754	15,798,706	15,796,047	318,214,661
17	31,594,754	15,051,553	16,543,200	301,671,460
18	31,594,754	14,269,060	17,325,693	284,345,767
19	31,594,754	13,449,555	18,145,199	266,200,568
20	31,594,754	12,591,287	19,003,467	247,197,102
21	31,594,754	11,692,423	19,902,331	227,294,771
22	31,594,754	10,751,043	20,843,711	206,451,060
23	31,594,754	9,765,135	21,829,618	184,621,441
24	31,594,754	8,732,594	22,862,159	161,759,282
25	31,594,754	7,651,214	23,943,539	137,815,743
26	31,594,754	6,518,685	25,076,069	112,739,674
27	31,594,754	5,332,587	26,262,167	86,477,507
28	31,594,754	4,090,386	27,504,367	58,973,140
29	31,594,754	2,789,430	28,805,324	30,167,816
30	31,594,754	1,426,938	30,167,816	0

Table 13: Ammortization schedule of the F1 loan.

Period	Payment	Interest	Repayment	Principal
0	0	0	0	513,188,956
1	33,383,678	25,659,448	7,724,230	505,464,726
2	33,383,678	25,273,236	8,110,442	497,354,284
3	33,383,678	24,867,714	8,515,964	488,838,320
4	33,383,678	24,441,916	8,941,762	479,896,558
5	33,383,678	23,994,828	9,388,850	470,507,708
6	33,383,678	23,525,385	9,858,293	460,649,415
7	33,383,678	23,032,471	10,351,207	450,298,208
8	33,383,678	22,514,910	10,868,768	439,429,440
9	33,383,678	21,971,472	11,412,206	428,017,234
10	33,383,678	21,400,862	11,982,816	416,034,418
11	33,383,678	20,801,721	12,581,957	403,452,461
12	33,383,678	20,172,623	13,211,055	390,241,406
13	33,383,678	19,512,070	13,871,608	376,369,798
14	33,383,678	18,818,490	14,565,188	361,804,610
15	33,383,678	18,090,230	15,293,448	346,511,162
16	33,383,678	17,325,558	16,058,120	330,453,042
17	33,383,678	16,522,652	16,861,026	313,592,016
18	33,383,678	15,679,601	17,704,077	295,887,939
19	33,383,678	14,794,397	18,589,281	277,298,658
20	33,383,678	13,864,933	19,518,745	257,779,913
21	33,383,678	12,888,996	20,494,682	237,285,231
22	33,383,678	11,864,262	21,519,417	215,765,814
23	33,383,678	10,788,291	22,595,387	193,170,427
24	33,383,678	9,658,521	23,725,157	169,445,270
25	33,383,678	8,472,263	24,911,415	144,533,855
26	33,383,678	7,226,693	26,156,985	118,376,870
27	33,383,678	5,918,844	27,464,835	90,912,036
28	33,383,678	4,545,602	28,838,076	62,073,959
29	33,383,678	3,103,698	30,279,980	31,793,979
30	33,383,678	1,589,699	31,793,979	0

Table 14: Ammortization schedule of the callable loan.

B.3 Total payments schedules

The total payments including costs for the callable and synthetic loan. The synthetic loan schedule includes the projected floating payments of the IRS as of December 7th 2007:

Period	Callable	Synthetic
1	35,949,623	35,495,123
2	35,911,002	38,665,838
3	35,870,449	37,646,706
4	35,827,870	37,338,990
5	35,783,161	36,789,062
6	35,736,217	36,441,955
7	35,686,925	35,922,678
8	35,635,169	35,437,271
9	35,580,825	34,814,622
10	35,523,764	34,366,654
11	35,463,850	33,793,436
12	35,400,940	33,414,923
13	35,334,885	33,271,097
14	35,265,527	33,217,591
15	35,192,701	33,169,182
16	35,116,234	33,241,610
17	35,035,943	33,287,144
18	34,951,638	33,375,675
19	34,863,118	33,358,967
20	34,770,171	33,272,248
21	34,672,578	33,144,806
22	34,570,104	33,138,509
23	34,462,507	33,091,932
24	34,349,530	33,038,934
25	34,230,904	32,948,121
26	34,106,347	32,872,526
27	33,975,562	32,730,796
28	33,838,238	32,542,565
29	33,694,048	32,290,830
30	33,542,648	31,986,078
Total	1,050,342,481	1,024,105,869

Table 15: Total payment schedule of the callable and synthetic loan.

B.4 Market quote changes

The new market quotes as of February 7th 2008, and the original for December 7th 2007 have been shown in below table, as well as the change in bps. All quotes are from Bloomberg.

Quote	07/12/2007	07/02/2008	Change
Cibor12	4.897%	4.590%	-31 bps
2Y IRS	4.658%	3.990%	-67 bps
3Y IRS	4.610%	3.978%	-63 bps
5Y IRS	4.605%	4.083%	-52 bps
7Y IRS	4.640%	4.223%	-42 bps
10Y IRS	4.728%	4.443%	-29 bps
12Y IRS	4.795%	4.545%	-25 bps
15Y IRS	4.870%	4.655%	-22 bps
20Y IRS	4.923%	4.745%	-18 bps
25Y IRS	4.935%	4.765%	-17 bps
30Y IRS	4.913%	4.755%	-16 bps

Table 16: Market quote changes after two months.

B.5 Actual ammortization of the F1 loan

Here we present the actual ammortization schedule of the F1 loan given the development in the F1 rate as presented in table 12. The values from 2019 and onwards are projected values given the last F1 rate of -0.20% for the year 2018.

Period	Payment	Interest	Repayment	Principal	
0	0	0	0	501,002,004	
1	31,594,754	23,697,395	7,897,359	493,104,645	
2	33,296,354	25,641,442	7,654,912	485,449,733	
3	22,166,137	8,641,005	13,525,132	471,924,601	
4	21,440,629	7,173,254	14,267,375	457,657,226	
5	20,699,856	5,674,950	15,024,906	442,632,319	
6	18,831,260	2,124,635	16,706,624	425,925,695	
7	18,785,300	1,959,258	16,826,042	409,099,653	
8	18,895,802	2,086,408	16,809,394	392,290,259	
9	18,327,650	941,497	17,386,154	374,904,106	
10	17,852,576	0	17,852,576	357,051,529	
11	17,480,049	-714,103	18,194,152	338,857,377	
12	17,480,049	-677,715	18,157,764	320,699,613	Expected
13	17,480,049	-641,399	18,121,448	302,578,165	Expected
14	17,480,049	-605,156	18,085,205	284,492,960	Expected
15	17,480,049	-568,986	18,049,035	266,443,925	Expected
16	17,480,049	-532,888	18,012,937	248,430,988	Expected
17	17,480,049	-496,862	17,976,911	230,454,077	Expected
18	17,480,049	-460,908	17,940,957	212,513,120	Expected
19	17,480,049	-425,026	17,905,075	194,608,044	Expected
20	17,480,049	-389,216	17,869,265	176,738,779	Expected
21	17,480,049	-353,478	17,833,527	158,905,252	Expected
22	17,480,049	-317,811	17,797,860	141,107,393	Expected
23	17,480,049	-282,215	17,762,264	123,345,129	Expected
24	17,480,049	-246,690	17,726,739	105,618,390	Expected
25	17,480,049	-211,237	17,691,286	87,927,104	Expected
26	17,480,049	-175,854	17,655,903	70,271,201	Expected
27	17,480,049	-140,542	17,620,591	52,650,609	Expected
28	17,480,049	-105,301	17,585,350	35,065,259	Expected
29	17,480,049	-70,131	17,550,180	17,515,079	Expected
30	17,480,049	-35,030	17,515,079	0	Expected

Table 17: Actual and projected ammortization of the F1 loan.

B.6 Effect of notional mismatch

Below table presents the difference in payments between the actual IRS and the "perfect" IRS, whose notional matches that of the F1 loan.

Period	Date	IRS	Perfect IRS	Diff.	Cum. diff.
1	10/12/2008	251,826	251,826	0	0
2	10/12/2009	2,730,159	2,730,159	0	0
3	10/12/2010	-14,899,006	-14,917,935	18,929	18,929
4	12/12/2011	-15,129,837	-14,994,892	-134,945	-116,015
5	10/12/2012	-15,314,530	-15,004,943	-309,588	-425,603
6	10/12/2013	-19,147,014	-18,520,779	-626,235	-1,051,838
7	10/12/2014	-18,887,253	-17,970,729	-916,533	-1,968,371
8	10/12/2015	-18,713,549	-17,509,662	-1,203,887	-3,172,258
9	12/12/2016	-20,083,263	-18,480,450	-1,602,812	-4,775,071
10	11/12/2017	-19,773,787	-17,868,325	-1,905,462	-6,680,533
11	10/12/2018	-19,804,148	-17,549,955	-2,254,192	-8,934,725

Table 18: Total payments in the actual IRS and the "perfect" IRS, whose notional matches the principal of the F1 loan.

Below table shows the difference in the market value on the payment dates between the actual IRS and the "perfect" IRS. The perfect IRS at time t_i is defined as having the notional profile equal to the *expected* principal profile of the F1 loan at that time and given the current F1 rate. The data used for curve calibration and pricing on all dates is from Bloomberg.

Period	Date	MV IRS	MV perf. IRS	Diff.
1	10/12/2008	-61,985,634	-62,668,080	682,446
2	10/12/2009	-65,476,489	-63,796,527	-1,679,962
3	10/12/2010	-81,181,107	-76,492,126	-4,688,981
4	12/12/2011	-151,187,518	-135,847,519	-15,339,999
5	10/12/2012	-179,053,809	-156,953,595	-22,100,214
6	10/12/2013	-130,848,512	-115,192,644	-15,655,868
7	10/12/2014	-191,801,736	-161,897,307	-29,904,428
8	10/12/2015	-180,550,227	-149,956,929	-30,593,298
9	12/12/2016	-178,116,334	-144,558,475	-33,557,859
10	11/12/2017	-173,721,510	-137,809,798	-35,911,712

Table 19: Difference in market value between the actual IRS and the "perfect" IRS.

B.7 Effect of spread movements

Below table shows the effect on payments of spread movements between the Cibor12M rate and the F1 rate, as well as the payments in a scenario with a constant spread. We calculate the payment using ACT/360 day count convention and the notional profile of the swap presented in table 13. For the constant spread scenario we have kept the spread in all periods equal the initial value of 19.5 bps.

Start date	Payment date	Cibor12M	F1 rate	Spread	Payments: Spread	Payments: Con. spread	Difference
10/12/2007	10/12/2008	4.9250%	4.73%	0.1950%	993,236	993,236	0
10/12/2008	10/12/2009	5.4350%	5.20%	0.2350%	1,174,890	974,909	199,981
10/12/2009	10/12/2010	1.8580%	1.78%	0.0780%	383,423	958,557	-575,134
10/12/2010	12/12/2011	1.7725%	1.52%	0.2525%	1,225,712	946,590	279,123
12/12/2011	10/12/2012	1.6325%	1.24%	0.3925%	1,853,737	920,965	932,772
10/12/2012	10/12/2013	0.7620%	0.48%	0.2820%	1,308,351	904,711	403,640
10/12/2013	10/12/2014	0.7275%	0.46%	0.2675%	1,214,091	885,038	329,053
10/12/2014	10/12/2015	0.6675%	0.51%	0.1575%	698,198	864,435	-166,238
10/12/2015	12/12/2016	0.2675%	0.24%	0.0275%	119,842	849,785	-729,944
12/12/2016	11/12/2017	0.1750%	0.00%	0.1750%	734,113	818,012	-83,899
11/12/2017	10/12/2018	0.0275%	-0.20%	0.2275%	926,811	794,410	132,402
				Total	10,632,405	9,910,649	721,757

Table 20: Payments attributable to spread movements, and the payments in a scenario with constant spread. All Cibor12M quotes are from Bloomberg.

B.8 Effect of difference in reset dates

Here we derive the effect of having a mismatch in the reset dates of the IRS and the F1 loan. In reality the floating rate of the IRS is typically reset two business days prior to a payment date, but here we assume it is reset on the same date. We keep the coverage constant to isolate the effect of interest rate movements. We let "IRS date" refer to the date used in the IRS, and "F1 date" to refer to the scenario where the reset of the IRS is on the same date as the F1 loan.

Reset date: IRS date	Reset date: F1 date	Cibor12M fixing: IRS date	Cibor12M fixing: F1 date	Payments: IRS date	Payments: F1 date	Difference
10/12/2007	10/12/2007	4.9250%	4.9250%	25,085,588	25,085,588	0
10/12/2008	05/12/2008	5.4350%	5.4700%	27,172,463	27,347,447	-174,984
10/12/2009	02/12/2009	1.8580%	1.8520%	9,133,325	9,103,831	29,494
10/12/2010	03/12/2010	1.7725%	1.7750%	8,604,256	8,616,392	-12,136
12/12/2011	30/11/2011	1.6325%	1.7450%	7,710,131	8,241,457	-531,326
10/12/2012	29/11/2012	0.7620%	0.7660%	3,535,332	3,553,891	-18,558
10/12/2013	25/11/2013	0.7275%	0.7100%	3,301,874	3,222,448	79,427
10/12/2014	24/11/2014	0.6675%	0.6675%	2,959,029	2,959,029	0
10/12/2015	25/11/2015	0.2675%	0.2650%	1,165,731	1,154,836	10,895
12/12/2016	23/11/2016	0.1750%	0.1800%	734,113	755,088	-20,975
11/12/2017	22/11/2017	0.0275%	0.0175%	112,032	71,293	40,739
					Total	-597,424

Table 21: Effect from mismatch in reset dates between the F1 loan and the IRS. This is compared to the scenario where reset of the IRS occur on the same date as the F1 loan. All Cibor12M quotes are from Bloomberg.

C Appendix

C.1 VBA functions

For completeness the VBA functions used in all curve calibrations, present value calculations etc. have been provided here. The code is from the course *Fixed Income Derivatives: Risk Management and Financial Institutions* at Copenhagen Business School, with a few modifications and additions as explained in section 2.3.4.

```
Option Base 1

Option Explicit

Public Function fidXAddTenor(StartDate As Variant, Tenor As Variant, Optional
    ↪ DayRule As String) As Variant

Dim TenorType As String, TenorNumber As Integer, AnnDate As Date, DayAdd As
    ↪ Integer

TenorType = Right(Tenor, 1)
TenorNumber = Left(Tenor, Len(Tenor) - 1)

Select Case LCase(TenorType)
    Case "b": TenorType = "w"
    Case "d": TenorType = "d"
    Case "w": TenorType = "ww"
    Case "m": TenorType = "m"
    Case "y": TenorType = "yyyy"
    Case Else: GoTo ErrHandler
End Select

AnnDate = DateAdd(TenorType, TenorNumber, StartDate)

If LCase(DayRule) = "none" Or LCase(DayRule) = "" Then
    DayAdd = 0
ElseIf LCase(DayRule) = "p" Then
'The "Preceding" case:
    Select Case Weekday(AnnDate, vbMonday)
        Case 1 To 5: DayAdd = 0
        Case 6: DayAdd = -1
        Case 7: DayAdd = -2
    End Select
ElseIf LCase(DayRule) = "f" Or LCase(DayRule) = "mf" Then
'The "Following" and "Modified Following" case:
    Select Case Weekday(AnnDate, vbMonday)
        Case 1 To 5: DayAdd = 0
        Case 6: DayAdd = 2
        Case 7: DayAdd = 1
    End Select
Else
    DayAdd = 0
End If

fidXAddTenor = DateAdd("d", DayAdd, AnnDate)

If Month(fidXAddTenor) <> Month(AnnDate) And LCase(DayRule) = "mf" Then
    fidXAddTenor = DateAdd("d", DayAdd - 3, AnnDate)
End If
```

```
Exit Function
ErrHandler:
    fidXAddTenor = "Error:␣Invalid␣TenorType"
End Function

Public Function fidXAdjustDate(StartDate As Variant, DayRule As String) As Date
    fidXAdjustDate = fidXAddTenor(StartDate, "0D", DayRule)
End Function

Public Function fidXCvg(StartDate As Variant, EndDate As Variant, DayCountBasis As
    ↪ String) As Double

Select Case LCase(DayCountBasis)
    Case "act/360": fidXCvg = (EndDate - StartDate) / 360
    Case "act/365": fidXCvg = (EndDate - StartDate) / 365
    Case "act/365.25": fidXCvg = (EndDate - StartDate) / 365.25
    Case "30/360": fidXCvg = ((Year(EndDate) - Year(StartDate)) * 360 + (Month(
        ↪ EndDate) - Month(StartDate)) * 30 + Application.Min(30, Day(EndDate)) -
        ↪ Application.Min(30, Day(StartDate))) / 360
End Select
End Function

Public Function fidXGenerateSchedule(AnchorDate As Date, Start As Variant,
    ↪ Maturity As Variant, Frequency As String, DayCountBasis As String, DayRule
    ↪ As String) As Variant

Dim UnAdjStart As Date, UnAdjMat As Date, AdjStart As Date, AdjMat As Date
Dim TenorNumber As Integer, n As Integer, i As Integer, j As Integer, K As Integer
Dim rows As Integer
Dim TenorType As String

If IsDate(Start) And IsDate(Maturity) Then
    UnAdjStart = Start
    AdjStart = fidXAdjustDate(UnAdjStart, DayRule)
    UnAdjMat = Maturity
ElseIf IsDate(Start) Then
    UnAdjStart = Start
    AdjStart = fidXAdjustDate(UnAdjStart, DayRule)
    UnAdjMat = fidXAddTenor(Start, Maturity, "")
ElseIf IsDate(Maturity) Then
    UnAdjStart = fidXAddTenor(AnchorDate, Start, "")
    AdjStart = fidXAdjustDate(UnAdjStart, DayRule)
    UnAdjMat = Maturity
Else
    UnAdjStart = fidXAddTenor(AnchorDate, Start, "")
    AdjStart = fidXAdjustDate(UnAdjStart, DayRule)
    UnAdjMat = fidXAddTenor(AdjStart, Maturity, "")
End If

AdjMat = fidXAdjustDate(UnAdjMat, DayRule)

n = Application.RoundUp((UnAdjMat - UnAdjStart) / (fidXAddTenor(AnchorDate,
    ↪ Frequency, "") - AnchorDate), 0)

TenorType = Right(Frequency, 1)
TenorNumber = Left(Frequency, Len(Frequency) - 1)

Dim dates As Variant
```

```
ReDim dates(n + 5)
i = 1
dates(1) = UnAdjMat
Do While fidXAdjustDate(dates(i), DayRule) > AdjStart
    dates(i + 1) = WorksheetFunction.Max(fidXAddTenor(dates(1), -i * TenorNumber &
    ↪ TenorType, ""), AdjStart)
    i = i + 1
Loop

ReDim Preserve dates(i)

ReDim data(i - 1, 5) As Variant
For j = 1 To i - 1
    If j = 1 Then
        data(j, 1) = UnAdjStart
    Else
        data(j, 1) = dates(i - (j - 1))
    End If
    data(j, 2) = dates(i - j)
    data(j, 3) = fidXAdjustDate(data(j, 1), DayRule)
    data(j, 4) = fidXAdjustDate(data(j, 2), DayRule)
    data(j, 5) = fidXCvg(data(j, 3), data(j, 4), DayCountBasis)
Next j

fidXGenerateSchedule = data
End Function

Public Function fidXInterpolate(KnownX As Variant, KnownY As Variant, OutputX As
    ↪ Variant, Method As String) As Variant
Dim n As Integer, j As Integer, i As Integer
KnownX = CVar(KnownX)
KnownY = CVar(KnownY)
n = UBound(KnownX)

If n <> UBound(KnownY) Then
    GoTo ErrHandler
End If

If OutputX < KnownX(1, 1) Then
        fidXInterpolate = KnownY(1, 1)
    ElseIf OutputX >= KnownX(n, 1) Then
        fidXInterpolate = KnownY(n, 1)
    Else
        For j = 1 To n
            If KnownX(j, 1) <= OutputX Then
                i = i + 1
            Else
                i = i
            End If
        Next

    Select Case LCase(Method)
        Case "constant":
        fidXInterpolate = KnownY(i, 1)

        Case "linear":
        fidXInterpolate = ((OutputX - KnownX(i, 1)) * KnownY(i + 1, 1) + (KnownX(i
        ↪ + 1, 1) - OutputX) * KnownY(i, 1)) / (KnownX(i + 1, 1) - KnownX(i,
```

```
        ↪ 1))

    Case "loglinear":
    fidXInterpolate = KnownY(i + 1, 1) ^ ((OutputX − KnownX(i, 1)) / (KnownX(i
        ↪ + 1, 1) − KnownX(i, 1))) ∗ KnownY(i, 1) ^ ((KnownX(i + 1, 1) −
        ↪ OutputX) / (KnownX(i + 1, 1) − KnownX(i, 1)))

    Case "hermite":
    Dim bi As Double, bk As Double, hi As Double
    Dim mi As Double, ci As Double, di As Double
    Dim K As Integer
    K = i + 1
    If i = 1 Then
        bi = ((KnownX(3, 1) + KnownX(2, 1) − 2 ∗ KnownX(1, 1)) ∗ (KnownY(2, 1)
            ↪ − KnownY(1, 1)) / (KnownX(2, 1) − KnownX(1, 1)) − (KnownX(2,
            ↪ 1) − KnownX(1, 1)) ∗ (KnownY(3, 1) − KnownY(2, 1)) / (KnownX(3,
            ↪ 1) − KnownX(2, 1))) ∗ (KnownX(3, 1) − KnownX(1, 1)) ^ −1
        bk = ((KnownX(K + 1, 1) − KnownX(K, 1)) ∗ (KnownY(K, 1) − KnownY(K −
            ↪ 1, 1)) / (KnownX(K, 1) − KnownX(K − 1, 1)) + (KnownX(K, 1) −
            ↪ KnownX(K − 1, 1)) ∗ (KnownY(K + 1, 1) − KnownY(K, 1)) / (KnownX
            ↪ (K + 1, 1) − KnownX(K, 1))) ∗ (KnownX(K + 1, 1) − KnownX(K − 1,
            ↪ 1)) ^ −1

    ElseIf i = n − 1 Then
        bi = ((KnownX(i + 1, 1) − KnownX(i, 1)) ∗ (KnownY(i, 1) − KnownY(i −
            ↪ 1, 1)) / (KnownX(i, 1) − KnownX(i − 1, 1)) + (KnownX(i, 1) −
            ↪ KnownX(i − 1, 1)) ∗ (KnownY(i + 1, 1) − KnownY(i, 1)) / (KnownX
            ↪ (i + 1, 1) − KnownX(i, 1))) ∗ (KnownX(i + 1, 1) − KnownX(i − 1,
            ↪ 1)) ^ −1
        bk = −((KnownX(n, 1) − KnownX(n − 1, 1)) ∗ (KnownY(n − 1, 1) − KnownY(
            ↪ n − 2, 1)) / (KnownX(n − 1, 1) − KnownX(n − 2, 1)) − (2 ∗
            ↪ KnownX(n, 1) − KnownX(n − 1, 1) − KnownX(n − 2, 1)) ∗ (KnownY(n
            ↪ , 1) − KnownY(n − 1, 1)) / (KnownX(n, 1) − KnownX(n − 1, 1))) ∗
            ↪ (KnownX(n, 1) − KnownX(n − 2, 1)) ^ −1
    Else
        bi = ((KnownX(i + 1, 1) − KnownX(i, 1)) ∗ (KnownY(i, 1) − KnownY(i −
            ↪ 1, 1)) / (KnownX(i, 1) − KnownX(i − 1, 1)) + (KnownX(i, 1) −
            ↪ KnownX(i − 1, 1)) ∗ (KnownY(i + 1, 1) − KnownY(i, 1)) / (KnownX
            ↪ (i + 1, 1) − KnownX(i, 1))) ∗ (KnownX(i + 1, 1) − KnownX(i − 1,
            ↪ 1)) ^ −1
        bk = ((KnownX(K + 1, 1) − KnownX(K, 1)) ∗ (KnownY(K, 1) − KnownY(K −
            ↪ 1, 1)) / (KnownX(K, 1) − KnownX(K − 1, 1)) + (KnownX(K, 1) −
            ↪ KnownX(K − 1, 1)) ∗ (KnownY(K + 1, 1) − KnownY(K, 1)) / (KnownX
            ↪ (K + 1, 1) − KnownX(K, 1))) ∗ (KnownX(K + 1, 1) − KnownX(K − 1,
            ↪ 1)) ^ −1
    End If
    hi = KnownX(i + 1, 1) − KnownX(i, 1)
    mi = (KnownY(i + 1, 1) − KnownY(i, 1)) / hi
    ci = (3 ∗ mi − bk − 2 ∗ bi) / hi
    di = (bk + bi − 2 ∗ mi) ∗ hi ^ −2
    fidXInterpolate = KnownY(i, 1) + bi ∗ (OutputX − KnownX(i, 1)) + ci ∗ (
        ↪ OutputX − KnownX(i, 1)) ^ 2 + di ∗ (OutputX − KnownX(i, 1)) ^ 3

  End Select

End If

Exit Function
```

```
ErrHandler:
    fidXInterpolate = "Error: Unidentical # of rows in KnownX and KnownY"
End Function

Public Function fidXZeroRate(Maturity As Variant, CurveMaturities As Variant,
    ↪ CurveRates As Variant, Method As String) As Double
fidXZeroRate = fidXInterpolate(CurveMaturities, CurveRates, Maturity, Method)
End Function

Public Function fidXDiscFactor(AnchorDate As Date, MaturityDate As Variant,
    ↪ CurveMaturities As Variant, CurveRates As Variant, Method As String) As
    ↪ Double
fidXDiscFactor = Exp(-fidXZeroRate(MaturityDate, CurveMaturities, CurveRates,
    ↪ Method) * fidXCvg(AnchorDate, MaturityDate, "Act/365"))
End Function

Public Function fidXForwardRate(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, DayRule As String, DayCountBasis As String, CurveMaturities As
    ↪ Variant, CurveRates As Variant, Method As String) As Double
Dim Ps As Double, Pe As Double, cvg As Double, StartDate As Date, MaturityDate As
    ↪ Date

If IsDate(Start) And IsDate(Maturity) Then
    StartDate = Start
    MaturityDate = Maturity
ElseIf IsDate(Start) Then
    StartDate = Start
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
ElseIf IsDate(Maturity) Then
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = Maturity
Else
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
End If

Ps = fidXDiscFactor(AnchorDate, StartDate, CurveMaturities, CurveRates, Method)
Pe = fidXDiscFactor(AnchorDate, MaturityDate, CurveMaturities, CurveRates, Method)
cvg = fidXCvg(StartDate, MaturityDate, DayCountBasis)

fidXForwardRate = (Ps / Pe - 1) / cvg
End Function

Public Function fidXAnnuityPv(AnchorDate As Date, StartDate As Variant, Maturity
    ↪ As Variant, Tenor As String, DayCountBasis As String, DayRule As String,
    ↪ DiscCurveMat As Variant, DiscCurveRates As Variant, Method As String,
    ↪ Optional NotSched As Variant) As Double

Dim temp As Variant, PayDate As Date, cvg As Double
Dim i As Integer, n As Integer
Dim VarNotional As Boolean

If IsMissing(NotSched) Then
    VarNotional = False
Else
    VarNotional = True
End If
```

```
temp = fidXGenerateSchedule(AnchorDate, StartDate, Maturity, Tenor, DayCountBasis,
    ↪ DayRule)

n = UBound(temp)
For i = 1 To n
    PayDate = temp(i, 4)
    cvg = temp(i, 5)
    If VarNotional = False Then
        fidXAnnuityPv = fidXAnnuityPv + cvg * fidXDiscFactor(AnchorDate, PayDate,
            ↪ DiscCurveMat, DiscCurveRates, Method)
    Else
        fidXAnnuityPv = fidXAnnuityPv + cvg * NotSched(i) * fidXDiscFactor(
            ↪ AnchorDate, PayDate, DiscCurveMat, DiscCurveRates, Method)
    End If
Next i

End Function

Public Function fidXFloatingPv(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, Tenor As String, DayCountBasis As String, DayRule As String,
    ↪ FwdCurveMat As Variant, FwdCurveRates As Variant, Method As String,
    ↪ Optional DiscCurveMat As Variant, Optional DiscCurveRates As Variant,
    ↪ Optional NotSched As Variant) As Double

Dim temp As Variant, SDate As Date, EDate As Date, cvg As Double
Dim i As Integer, n As Integer
Dim VarNotional As Boolean

If IsMissing(DiscCurveMat) Or IsMissing(DiscCurveRates) Then
    DiscCurveMat = FwdCurveMat
    DiscCurveRates = FwdCurveRates
End If

If IsMissing(NotSched) Then
    VarNotional = False
Else
    VarNotional = True
End If

temp = fidXGenerateSchedule(AnchorDate, Start, Maturity, Tenor, DayCountBasis,
    ↪ DayRule)

n = UBound(temp)
For i = 1 To n
    SDate = temp(i, 3)
    EDate = temp(i, 4)
    cvg = temp(i, 5)
    If VarNotional = False Then
        fidXFloatingPv = fidXFloatingPv + cvg * fidXForwardRate(AnchorDate, SDate,
            ↪ EDate, DayRule, DayCountBasis, FwdCurveMat, FwdCurveRates, Method)
            ↪ * fidXDiscFactor(AnchorDate, EDate, DiscCurveMat, DiscCurveRates,
            ↪ Method)
    Else
        fidXFloatingPv = fidXFloatingPv + cvg * NotSched(i) * fidXForwardRate(
            ↪ AnchorDate, SDate, EDate, DayRule, DayCountBasis, FwdCurveMat,
            ↪ FwdCurveRates, Method)* fidXDiscFactor(AnchorDate, EDate,
            ↪ DiscCurveMat, DiscCurveRates, Method)
    End If
```

```
Next i
End Function

Public Function fidXSwapRate(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, FloatTenor As String, FloatDayCountBasis As String, FixedTenor As
    ↪ String, FixedDayCountBasis As String, DayRule As String, FwdCurveMat As
    ↪ Variant, FwdCurveRates As Variant, Method As String, Optional DiscCurveMat
    ↪ As Variant, Optional DiscCurveRates As Variant, Optional NotlFixMat As
    ↪ Variant, Optional NotlFix As Variant) As Double

Dim VarNotional As Boolean

If IsMissing(DiscCurveMat) Or IsMissing(DiscCurveRates) Then
    DiscCurveMat = FwdCurveMat
    DiscCurveRates = FwdCurveRates
End If

If IsMissing(NotlFixMat) Or IsMissing(NotlFix) Then
    VarNotional = False
Else
    VarNotional = True
End If

If VarNotional = True Then
    Dim i, n, j, l, startl As Integer
    Dim temp As Variant
    temp = fidXGenerateSchedule(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule)
    n = UBound(temp)
    NotlFixMat = CVar(NotlFixMat)
    i = UBound(NotlFixMat)
    ReDim NotlFloatMat(n) As Date
    ReDim NotlFloat(n) As Double

    For j = 1 To n
        NotlFloatMat(j) = temp(j, 4)
    Next j

    startl = 1
    For j = 1 To i
        For l = startl To n
            If NotlFloatMat(l) <= NotlFixMat(j, 1) Then
                NotlFloat(l) = NotlFix(j, 1)
            Else
                startl = l
                Exit For
            End If
        Next l
    Next j
End If


Dim Fixed As Double, Floating As Double

If VarNotional = False Then
    Fixed = fidXAnnuityPv(AnchorDate, Start, Maturity, FixedTenor,
        ↪ FixedDayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method)
```

```
    Floating = fidXFloatingPv(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule, FwdCurveMat, FwdCurveRates, Method,
        ↪ DiscCurveMat, DiscCurveRates)
Else
    Fixed = fidXAnnuityPv(AnchorDate, Start, Maturity, FixedTenor,
        ↪ FixedDayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method,
        ↪ NotlFix)
    Floating = fidXFloatingPv(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule, FwdCurveMat, FwdCurveRates, Method,
        ↪ DiscCurveMat, DiscCurveRates, NotlFloat)
End If

fidXSwapRate = Floating / Fixed
End Function

Public Function fidXSwapPv(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, FloatTenor As String, FloatDayCountBasis As String, FixedTenor As
    ↪ String, FixedDayCountBasis As String, ayRule As String, FixedRate As Double
    ↪ , TypeFlag As String, FwdCurveMat As Variant, FwdCurveRates As Variant,
    ↪ Method As String, Optional DiscCurveMat As Variant, Optional DiscCurveRates
    ↪  As Variant, Optional NotlFixMat As Variant, Optional NotlFix As Variant)
    ↪ As Double

Dim VarNotional As Boolean

If IsMissing(DiscCurveMat) Or IsMissing(DiscCurveRates) Then
    DiscCurveMat = FwdCurveMat
    DiscCurveRates = FwdCurveRates
End If

If IsMissing(NotlFixMat) Or IsMissing(NotlFix) Then
    VarNotional = False
Else
    VarNotional = True
End If

If VarNotional = True Then
    Dim i, n, j, l, startl As Integer
    Dim temp As Variant

    temp = fidXGenerateSchedule(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule)
    n = UBound(temp)
    NotlFixMat = CVar(NotlFixMat)
    i = UBound(NotlFixMat)
    ReDim NotlFloatMat(n) As Date
    ReDim NotlFloat(n) As Double

    For j = 1 To n
        NotlFloatMat(j) = temp(j, 4)
    Next j

    startl = 1
    For j = 1 To i
        For l = startl To n
            If NotlFloatMat(l) <= NotlFixMat(j, 1) Then
                NotlFloat(l) = NotlFix(j, 1)
            Else
```

```
                startl = l
                Exit For
            End If
        Next l
    Next j
End If

Dim Fixed As Double, Floating As Double

If VarNotional = False Then
    Fixed = FixedRate * fidXAnnuityPv(AnchorDate, Start, Maturity, FixedTenor,
        ↪ FixedDayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method)
    Floating = fidXFloatingPv(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule, FwdCurveMat, FwdCurveRates, Method,
        ↪ DiscCurveMat, DiscCurveRates)
Else
    Fixed = FixedRate * fidXAnnuityPv(AnchorDate, Start, Maturity, FixedTenor,
        ↪ FixedDayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method,
        ↪ NotlFix)
    Floating = fidXFloatingPv(AnchorDate, Start, Maturity, FloatTenor,
        ↪ FloatDayCountBasis, DayRule, FwdCurveMat, FwdCurveRates, Method,
        ↪ DiscCurveMat, DiscCurveRates, NotlFloat)
End If

If LCase(TypeFlag) = "receiver" Then
    fidXSwapPv = Fixed - Floating
Else
    fidXSwapPv = Floating - Fixed
End If

End Function

Public Function fidXOISRate(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, Tenor As String, FloatDayCountBasis As String, FixedDayCountBasis
    ↪ As String, DayRule As String, DiscCurveMat As Variant, DiscCurveRates As
    ↪ Variant, Method As String) As Double

Dim Fixed As Double, Floating As Double
Dim StartDate As Date, MaturityDate As Date

If IsDate(Start) And IsDate(Maturity) Then
    StartDate = Start
    MaturityDate = Maturity
ElseIf IsDate(Start) Then
    StartDate = Start
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
ElseIf IsDate(Maturity) Then
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = Maturity
Else
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
End If

Fixed = fidXAnnuityPv(AnchorDate, Start, Maturity, Tenor, FixedDayCountBasis,
    ↪ DayRule, DiscCurveMat, DiscCurveRates, Method)
Floating = fidXDiscFactor(AnchorDate, StartDate, DiscCurveMat, DiscCurveRates,
    ↪ Method) - fidXDiscFactor(AnchorDate, MaturityDate, DiscCurveMat,
```

```
    ↪ DiscCurveRates, Method)

fidXOISRate = Floating / Fixed

End Function

Public Function fidXOISPv(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, Tenor As String, FloatDayCountBasis As String, FixedDayCountBasis
    ↪ As String, DayRule As String, FixedRate As Double, TypeFlag As String,
    ↪ DiscCurveMat As Variant, DiscCurveRates As Variant, Method As String) As
    ↪ Double

Dim Fixed As Double, Floating As Double
Dim StartDate As Date, MaturityDate As Date

If IsDate(Start) And IsDate(Maturity) Then
    StartDate = Start
    MaturityDate = Maturity
ElseIf IsDate(Start) Then
    StartDate = Start
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
ElseIf IsDate(Maturity) Then
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = Maturity
Else
    StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
    MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
End If

Fixed = FixedRate * fidXAnnuityPv(AnchorDate, Start, Maturity, Tenor,
    ↪ FixedDayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method)
Floating = fidXDiscFactor(AnchorDate, StartDate, DiscCurveMat, DiscCurveRates,
    ↪ Method) - fidXDiscFactor(AnchorDate, MaturityDate, DiscCurveMat,
    ↪ DiscCurveRates, Method)

If LCase(TypeFlag) = "receiver" Then
    fidXOISPv = Fixed - Floating
Else
    fidXOISPv = Floating - Fixed
End If

End Function

Public Function fidXCcsSpread(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, Tenor As String, DayCountBasis_for As String, DayCountBasis_dom As
    ↪  String, DayRule As String, FwdCurveMat_for As Variant, FwdCurveRates_for
    ↪ As Variant, FwdCurveMat_dom As Variant, FwdCurveRates_dom As Variant,
    ↪ Method As String, DiscCurveMat_for As Variant, DiscCurveRates_for As
    ↪ Variant, DiscCurveMat_dom As Variant, DiscCurveRates_dom As Variant) As
    ↪ Double

Dim FloatPV_for As Double, FloatPV_dom As Double, NotExchg_for As Double,
    ↪ NotExchg_dom As Double, PvBp As Double
Dim StartDate As Date, MaturityDate As Date

If IsDate(Start) And IsDate(Maturity) Then
    StartDate = Start
    MaturityDate = Maturity
```

```
ElseIf IsDate(Start) Then
     StartDate = Start
     MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
ElseIf IsDate(Maturity) Then
     StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
     MaturityDate = Maturity
Else
     StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
     MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
End If

FloatPV_for = fidXFloatingPv(AnchorDate, StartDate, MaturityDate, Tenor,
    ↪ DayCountBasis_for, DayRule, FwdCurveMat_for, FwdCurveRates_for, Method,
    ↪ DiscCurveMat_for, DiscCurveRates_for)

FloatPV_dom = fidXFloatingPv(AnchorDate, StartDate, MaturityDate, Tenor,
    ↪ DayCountBasis_dom, DayRule, FwdCurveMat_dom, FwdCurveRates_dom, Method,
    ↪ DiscCurveMat_dom, DiscCurveRates_dom)

NotExchg_for = fidXDiscFactor(AnchorDate, StartDate, DiscCurveMat_for,
    ↪ DiscCurveRates_for, Method) - fidXDiscFactor(AnchorDate, MaturityDate,
    ↪ DiscCurveMat_for, DiscCurveRates_for, Method)

NotExchg_dom = fidXDiscFactor(AnchorDate, StartDate, DiscCurveMat_dom,
    ↪ DiscCurveRates_dom, Method) - fidXDiscFactor(AnchorDate, MaturityDate,
    ↪ DiscCurveMat_dom, DiscCurveRates_dom, Method)

PvBp = fidXAnnuityPv(AnchorDate, Start, Maturity, Tenor, DayCountBasis_dom,
    ↪ DayRule, DiscCurveMat_dom, DiscCurveRates_dom, Method)

fidXCcsSpread = (NotExchg_dom - NotExchg_for + FloatPV_for - FloatPV_dom) / PvBp

End Function

Public Function fidXCcsPv(AnchorDate As Date, Start As Variant, Maturity As
    ↪ Variant, Tenor As String, DayCountBasis As String, DayRule As String,
    ↪ CcsSpread As Double, TypeFlag As String, FwdCurveMat As Variant,
    ↪ FwdCurveRates As Variant, Method As String, DiscCurveMat As Variant,
    ↪ DiscCurveRates As Variant) As Double

Dim FloatPV As Double, NotExchg As Double, SpreadPv As Double
Dim StartDate As Date, MaturityDate As Date

If IsDate(Start) And IsDate(Maturity) Then
     StartDate = Start
     MaturityDate = Maturity
ElseIf IsDate(Start) Then
     StartDate = Start
     MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
ElseIf IsDate(Maturity) Then
     StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
     MaturityDate = Maturity
Else
     StartDate = fidXAddTenor(AnchorDate, Start, DayRule)
     MaturityDate = fidXAddTenor(StartDate, Maturity, DayRule)
End If
```

```
FloatPV = fidXFloatingPv(AnchorDate, StartDate, MaturityDate, Tenor, DayCountBasis
    ↪ , DayRule, FwdCurveMat, FwdCurveRates, Method, DiscCurveMat, DiscCurveRates
    ↪ )
NotExchg = fidXDiscFactor(AnchorDate, StartDate, DiscCurveMat, DiscCurveRates,
    ↪ Method) - fidXDiscFactor(AnchorDate, MaturityDate, DiscCurveMat,
    ↪ DiscCurveRates, Method)

SpreadPv = CcsSpread * fidXAnnuityPv(AnchorDate, Start, Maturity, Tenor,
    ↪ DayCountBasis, DayRule, DiscCurveMat, DiscCurveRates, Method)

If LCase(TypeFlag) = "payer" Then
fidXCcsPv = NotExchg - FloatPV - SpreadPv
Else
fidXCcsPv = -(NotExchg - FloatPV - SpreadPv)
End If

End Function
```