Fixed-income hedge funds in pension savings

- Is it risky or rational?

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Abstract

This thesis investigates the characteristics of Danish fixed-income hedge funds, and seek to analyze if these are suitable for a pension saving portfolio. Hedge funds are viewed as highly risky investments and therefore not considered a suitable asset class in a pension saving. The validity of this view will be investigated and there will be constructed a range of pension portfolios including fixed-income hedge funds, in order to examplify how this could look.

The first part of this thesis describes the constuction of a model-portfolio, which has the purpose of acting as the objective of optimization. There will be constructed a hedge fund index of Danish fixed-income hedge funds, in order to create a Danish hedge fund return sample, for the use in the further analysis.

The first analysis analyzes the historical drawdowns of the hedge fund index and the constituents of the traditional model-portfolio. The analysis extents to a range of tail risk estimations, and analyzes the results of the hedge fund index compared to the traditional assets.
The analysis shows that the hedge fund index would have been a significantly better investment through the financial crisis of 2008-2009 than most of the traditional assets, such as equities, real-estate, and infrastructure. This is supported by the estimations, which consistently show that the tail risks of fixed-income hedge funds are significantly lower than of the traditional assets in the model-portfolio.

The second analysis is focusing on the optimization of the model-portfolio including the hedge fund index in order to provide an optimal pension portfolio.
The analysis is built on Markowitz' Modern Portfolio Theory and constructs a range of optimal portfolios under different conditions, both regarding the traditional assets and the hedge fund index. It is nessecary to restrict the maximum weight of the hedge fund index, in order to avoid corner solutions, due to the significant risk/return characteristics of the hedge fund index compared to the traditional assets.

The conclusion of this thesis is that fixed-income hedge funds should be a natural part of a pension saving portfolio equated with private equity and other alternative investments. The, historical and estimated, tail risks of the Danish hedge funds show that these funds are significantly more stable than assumed compared to other asset classes.

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Introduction

Danish pension funds are continually searching for new sources of return. Strategies are changed, risks are adjusted, and key people are brought in to maximize the return of the members. Because of the record-low interest rates in the recent years, pension funds have been hungering for alternatives to the traditional investments including stocks, bonds, and real estate. Historically these have been the three primary sources of returns in the classical pension funds and are still considered as such.

Alternatives to the traditional asset classes, stocks, bonds, and real estate, have been found in infrastructure-, private equity- and alternative investments. Alternative investments include bridges, motorways, woods and solar power plants. These have already proved to be suitable alternatives to stocks, bonds, and real estate, since these assets historically have been a way to earn an extra liquidity premium.

There are other alternative sources of returns, one of these is hedge funds. Hedge funds come in many variations with a wide range of investment strategies. However, due to several historical incidents with hedge funds in focus, there has been created a fear of investing in hedge funds, and there has been established a public resistance to hedge funds.

The most well known hedge funds include Long-Term Capital Management and the Quantum Fund. Long-Term Capital Management, LTCM, which ran an absolute-return investment strategy with very high leverage in the 1990s. LTCM was run by highly recognized professionals with experience from the most famous investment firms in USA and Nobel Prize winners on the board of directors. The fund managed to create substantial returns in its first couple of years running their strategy. However, in 1998 the fund lost USD 4,6 billion in four months due to a devaluation in Russia and the default on Russia's Ruble government bonds. The fund had to be bailed out by the Federal Reserve and some of the largest banks in the USA (Lowenstein, 2000).

Another story behind the speculative reputation of hedge funds is The Quantum Fund, managed by George Soros. George Soros is also known as 'The man who broke the Bank of England'. This epithet comes from his involvement in a large speculative sale of pound sterling on 'Black Wednesday' in 1992, which forced the Bank of England (BoE) to withdraw the British pound from the European Exchange Rate Mechanism. This lead to a sharp devaluation of the pound sterling, which created significant ripple effects on the already struggling British economy with both double-digit interest rates and inflation (Johnston, P., 2012).

Events like 'Black Wednesday', and the collapse and bailout of LTCM have given hedge funds a bad reputation, and not many are aware the well-driven hedge funds, which has delivered steady returns on particular strategies.

In Denmark, fixed-income based hedge funds have been built around the Danish covered bond market and aim to generate returns on the credit spread between Danish AAA-rated mortgage bonds and the repo-rate. This strategy is performed through investments in mortgage bonds, which are geared, by doing sell/buy-back transactions and hedged by interest rate swaps.

This thesis will therefore provide an analysis of the effects of introducing fixed-income hedge funds as a supplementary asset to a standardized pension portfolio. Furthermore, the investment strategy of fixed-income hedge funds will be described and analyzed to assess the real risks, and especially the tail risk of fixed-income hedge funds.

To answer the overall problem statement, this thesis will analyze the correlation, volatility, and returns of hedge funds and traditional pension portfolio assets. This thesis will estimate the tail risk of fixed-income hedge funds to determine the maximum drawdown that should be expected, and the maximum allocation of capital that should be invested in hedge funds in an optimal pension portfolio.

Problem area

This thesis will be conducted within the area of Danish pension savers and Danish fixed-income hedge funds. A minority of pension savers are already using hedge funds as a part of their investment portfolios, but this is restricted to a narrow group of investors. This group is characterized by both having access to the needed counseling about these funds, and the required knowledge about the underlying investment strategies, which is a critical factor behind investing in hedge funds. The availability is also limited due to regulatory labeling as complicated financial instruments, and therefore banks are required to provide extensive information and counseling before the customer will be able to buy any hedge funds. This regulation is based on historical cases like Jyske Invest Markedsneutral - Obligationer, which was launched just before the financial crises of 2008 and later terminated due to extensive losses.

This thesis will focus on determining if Danish fixed-income hedge funds are suitable as pension saving portfolios, and if this is the case, what is the optimal allocation of capital to invest in hedge funds.

Problem statement

How could an optimal allocation in a pension portfolio be constructed with the use of fixed-income hedge funds together with already used assets?

Working questions

1. What strategies do Danish fixed-income hedge funds use, and how have these performed the last decade compared to traditional pension portfolio assets?
2. What are the tail-risks of fixed-income hedge funds, how large drawdowns should an investor expect, and how does these compare to traditional assets?
3. To how large a degree should one implement fixed-income hedge funds into a pension portfolio and how could an asset allocation look like?
4. Would the risk-reward of a fixed-income hedge fund justify adding such an investment into a pension portfolio?

Limitations

This thesis limits its study to analyzing and discussing if hedge funds to a higher degree can be a permanent part of a pension portfolio. The thesis will not be describing and analyzing how specific hedge fund strategies compare to each other with respect to which hedge funds is the most suitable for a pension portfolio. This thesis will view the selected hedge funds from an overall perspective, and will combine the sample of hedge funds into a generic return sample.

It is not possible to predict the future of returns, and this theis will not try to disprove this fact. As mentioned in all investment advisory discalimers 'historical returns are no guarantee of future returns'. Therefore, this thesis will not be using historical returns as a basis for future returns, unless there is no alternative. Historical data will be limited to analyzing correlations among asset classes, historical volatility, maximum drawdowns, and tail risk.

The development of future returns will be limited to a sample of estimations by the world's largest asset managers. These estimations will be the basis of the research in this thesis.

The model-portfolio of this thesis will have as a purpose to replicate the asset allocation of an average pension fund. Since pension funds have very different investment strategies and asset allocation, the model-portfolio will not be an exact replica of one pension fund. To make a focused and applicable study of the derivative effects of introducing hedge funds into a pension portfolio the study will be conducted on this model-portfolio with and without investments in hedge funds.

In the construction of the model-portfolio, it has been considered whether all returns should be converted into DKK and if investments should be hedged into DKK, in order to reflect the true historical returns. If this should be done in a proper manner, it should be decided for each asset class, if a pension fund would hedge the particular asset class, or not. This would properly be applicable for the asset classes where the cash flows are predictable, such as the fixed-income classes. This is not an exact science, why this thesis is limited from converting and hedging assets into DKK, and thereby have all assets in their local currencies. This is based on the fact that the matter of this thesis is not to 100 % replicate the historical returns of Danish pension funds, but to analyze the differences in risk-adjusted returns and drawdowns of the traditional assets and fixed-income hedge funds.

This thesis will not be analyzing or discussing the taxation consequences introduced by investing in different asset classes, this is not the focus of this thesis since this would require an extensive insight into local tax legislation. The thesis has the sole purpose of discussing whether pension savers could maximize their long-term returns by introducing hedge funds as a part of the pension portfolio. Taxation varies over time and evolves with various political tendencies and changes in the political climate of individual countries. Taxation varies with the choice of the pension scheme and type of assets. The rules about how a specific asset class should be handled in a taxation situation depend on what type of pension portfolio it is placed in, or if this asset is a part of a personal investment portfolio. The taxation discussion requires extensive legal knowledge within this area, and there will therefore not be any discussion of the consequences regarding tax in this thesis.

The thesis will not be investigating or predicting the future returns of different asset classes or overall pension portfolios in order to rank asset classes in relation to each other. It will be analyzing historical data from the last 10 years, to focus only on relevant data. When calculating the correlations and volatility of different asset classes, it is a choice to limit the time horizon. Data regarding Danish fixed income hedge funds are limited to the last 10 years, with most of this data concentrated in the last five years.

In order to give the best basis for analysis, it has been chosen only to include hedge funds with at least 4 years of return data available, which means that of the Danish hedge funds relevant for this thesis, Formuepleje Fokus and Jyske Invest Markedsneutral – Obligationer will be excluded. The most significant of these is Jyske Invest Markedsneutral – Obligationer, which lost 85 % of its NAV in its first year.

The purpose of this thesis is to analyze the implications of introducing fixed-income hedge funds to pension portfolios. This is why it has been valued higher to have the index reflecting the average development of the available Danish fixed-income hedge funds, rather than having an analysis skewed by the failure of Jyske Invest Markedsneutral – Obligationer. The exculsion has been made to give a better picture of the true performance of the Danish fixed-income hedge funds. Since there has been negotiated a settlement between investors and Jyske Bank, where the investors are compensated for 80 % of their loss, which would disturb the analysis further and be hard to incorporate in the data handling.

Methodic

This thesis will be based on a mixture of classical Asset Allocation theories to derive the optimal portfolio, which used a standardized set of parameters. All these parameters are based on historical returns; the mean return, standard deviation (volatility) and correlations. In this thesis, there will be made a clear distinction between what necessarily have to be based on historical data, and what there instead can be based on analytical data collected, from asset managers and investment banks.

There has been collected analytical report from some of the best-known consultancies, investment banks, and asset managers. These reports all have in common that they are Capital Market Assumption-reports (CMA), which have a time span of 3-10 years. The majority of the CMA's are 5-year reports in which the economists behind argue how their views on the economies of the world and different asset classes will perform and evolve during the next couple of years.

These reports are a substitute for expected returns based on mean returns of the history and this has been done in order to comply with the eternal saying that "*Historical returns are no guarantee of future returns*". Therefore, this thesis will strive to replace historical returns with return expectations made by highly recognized and respected professionals within the financial sector, as expected return.

These returns are collected through the websites of the individual organizations, and the website Savvyinvestor.net, which is a network of professional investors for professional investors. The website describes itself as following.

"Membership of the Savvy Investor network is open only to institutional / professional investors (including pension funds, asset owners, investment managers) and their service providers."

Savvyinvestor.net (2018)

On this website, professionals distribute; white papers, articles, and capital market analyses. The fact that the material distributed through this website is published with the approval of the organizations behind the author(s), creates a certain credibility. The classification of the individual posts as distributed by a professional and the like-button on which professionals can approve the works of others, creates a peer-to-peer network of data and knowledge approved by other professionals.

The data of the individual reports have been collected and sorted for an easier application in the later analysis' in this thesis. Data collected have been transformed to yearly returns where the observations of each report have been noted in an excel worksheet to organize the data points of each asset class from the different contributes. These data points have been used to calculate an average expected return for usage in the calculation of an optimal portfolio including and excluding fixed-income hedge funds.

Historical data behind the benchmark/proxy assets given by the before mentioned reports have been collected in order to calculate historical volatilities and correlations between the different assets. These volatilities, correlations, and covariance have been calculated in Excel using built-in functions and matrix-calculations.

This analysis will be based on Markowitz's mean-variance approach for constructing an optimal portfolio with the object to maximize the portfolio's linear return while minimizing the portfolio variance.

The theory was presented by Harry Markowitz in the 1950s and further developed by William Sharpe (1964) and many more during the following decades (Benninga, 2014: 305).

Knowledge regarding theories and strategies behind the public Danish hedge funds, which is the objective of this thesis, will be collected from Morningstar.com. Morningstar.com collects and distributes all relevant information regarding investment funds and distribute these through its website. The most relevant information to understand the investment strategies of Danish fixed-income hedge funds is the prospectus, of the individual funds, which are publicly available through the website of Morningstar.

The selection of Danish hedge funds that will be subject to analysis in this thesis will be the following; Nykredit Alpha MIRA, Nykredit Alpha KOBRA, HP Hedge Danske Obligationer, and Danske Fixed Income Strategies.

These funds all have in common that they are based on a relative value strategy primary in Danish covered bonds and Danish government bonds. These hedge funds invest their own as well as borrowed funds. The funds are invested in covered mortgage bonds and to some extent in government- and mortgage bonds from the rest of the Nordic countries. The strategy is based on borrowing funds to invest and thereby financial gearing. This is to exploit an arbitrage opportunity in the spread between the yield on covered mortgage bonds and the risk-free government bonds. The funds conduct their gearing activity by entering bilateral loans, prime loan facilities, or repo-agreement. Different interest rate scenarios determine which of the options that are used as well as the relationship with the counterparties and prime brokers (Investoroplysninger for Kapitalforeningen HP Hedge, 2017: 3). If the relationship between the hedge fund manager and the prime broker is strong, the hedge fund is more likely to have a more flexible credit line, than with a completely unknown manager. Therefore the credability and history of the funds are an important feature, which improves their performance.

Theory

Modern portfolio theory

This thesis will be based on modern portfolio theory and especially portfolio optimization and the principles introduced by Harry Markowitz in 1952 with his essay "Portfolio selection" in The Journal of Finance. Markowitz's thesis states: "*…The investor does (or should) maximize the discounted (or capitalized) value of the future returns*" and "*we could let "anticipated" returns include an allowance for risk*" (Markowitz, 1952: 77). This is how Markowitz explains the "mean-variance"-optimization of which he later received a Nobel price. Markowitz argues that a common rule of investing is that an investor should invest in the securities with the highest expected return and strive to get the maximum diversification. This rule will be fulfilled if the investor invests this

capital in a large portfolio of the securities with the highest expected returns. The law of large numbers will ensure that the return of this portfolio will be close to the expected return if only the portfolio is put together with enough securities (Markowitz, 1952: 79).
This is not an acceptable rule since the portfolio with the highest expected return is not necessary the portfolio with the lowest variance, but possibly the contrary. Therefore Markowitz introduces the use of the covariance and correlation between the assets of the portfolio, in order to create a portfolio, where the securities of the portfolio are less intercorrelated and thereby the overall portfolio variance lower (Markowitz, 1952: 79).

The returns of the individual securities of the portfolio are random variables, and the weights of the individual securities are non-random variables determined by the investor, which must be non-negative and sum to the value of 1. The return of the portfolio is the weighted sum of the random variables and the non-random weights set by the investor. The variance of the portfolio is the sum product of the covariance and the individual weights of the portfolio securities. With this base, the investor is able to construct a portfolio of an almost infinite number of combinations of expected return (E) and variance (V) dependant on the choice of portfolio weights. Markowitz's rule for *E-V* states that all investors should choose a combination of *E-V*, which either minimizes *V* or maximizes *E* for a given level of *E* respectively *V* (Markowitz, 1952: 82).

The following section will help to understand the above-described variables and relationships between these.

Returns, standard deviation, covariance and correlations

This section will describe and present the necessary components of portfolio theory, which will be repeated throughout this thesis.

The rate of return on any asset, *i,* is denoted r_i and the weight of this asset in a portfolio of risky assets is denoted w_i. The return contribution of asset *i*, to the portfolio, *p*, is the product of w_i and r_i.

The expected return of a portfolio, $E(r_p)$, is the weighted average of the rates of return of the individual assets, *i*, in the portfolio. (Benninga, 2014: 206)

$$E(r_p) = \sum_{i=1}^{n} w_i * E(r_i)$$

Which also can be written in a matrix notation, which will be necessary for the calculation of multi-asset optimal portfolios.

$$E(r_p) = w^T * E(r) = E(r)^T w$$

Where w, is a vector of asset weights and $E(r)$ is a vector of expected returns as displayed below.

$$w = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \qquad E(r) = \begin{bmatrix} E(r_1) \\ E(r_2) \\ E(r_3) \\ \vdots \\ E(r_n) \end{bmatrix}$$

The symbol T is an expression of a transposed vector as below.

$$w^T = [x_1, x_2, x_3, \dots, x_n], \qquad E(r)^T = E(r_1), E(r_2), E(r_3), \dots, E(r_n)$$

When calculating the yearly return, there are two options to choose from - the average yearly return must be calculated as either a geometric- or an arithmetic average return. To calculate the geometric return (Johansen & Trier, 2010: 86) one must take the product of the returns, $1 + r_i$, of all assets in the portfolio. The product of the returns is raised to the power of 1/n, where *n* is the number of periods the observations (Jaroszek , 2017: 23). At last, there must be subtracted 1 from the product of the returns in order to display the result in percent, as shown below.

$$Yearly\ r_{i,n} = \left(\prod_{i=1}^{n}(1 + r_i)\right)^{1/n} - 1$$

The arithmetic average yearly return approximates the geometric average return and is, in general, a simplified substitute of the geometric average (Johansen & Trier , 2010: 74). The arithmetic average does not take into account the effect of compounding interests, which can make a difference if estimation is performed over an extended period.

$$Yearly\ r_{i,n} = \frac{1}{n}\sum_{i=1}^{n} r_i = \frac{r_1 + r_2 + \cdots + r_n}{n}$$

When estimating returns the geometric average reflects the return of an investor with a buy-and-hold strategy, where there will not be redeemed or added capital to the portfolio. The arithmetic average has better use in strategies where the investor is expected to redeem or inject capital over the estimation period.

Historical average returns are often used as a measure of expected future returns even though historical returns but is never a guarantee of future returns. Where average returns are used as expectations for future returns, the historical standard deviation is used as a risk measure and volatility of an asset. The standard deviation is the square root of the variance of an asset (Munk, 2016: 45). The sample variance is calculated as the squared difference between the expected return and the actual return.

$$\sigma^2 = \frac{1}{n-1}\sum(r-\mu)^2$$

Standard deviation is the square root of the variance:

$$Std(r) = \sqrt{\frac{1}{n-1}\sum(r-\mu)^2}$$

The covariance is, as it lies in the name, a measure of how two assets covariate, and this measure is necessary to calculate the portfolio variance. The covariance is an indicator of the relationship between two assets; a high covariance indicates a strong relationship in the returns of two assets, as well as a large negative covariance, indicates that two assets primarily moves in the opposite direction.

$$Cov(A,B) = \sigma_{A,B} = \sum_{i=1}^{n}(r_{A,i}-\mu_A)(r_{B,i}-\mu_B)$$

The covariance is not an independent variable, which is why it can be useful to calculate the correlation instead. The correlation has a great advantage of being easier and more intuitive to interpret (Lhabitant, 2009: 128).

$$\rho_{A,B} = \frac{\sigma_{A,B}}{\sigma_A * \sigma_B}$$

The correlation between assets is called the correlation coefficient which is per definition in the interval [-1; 1] which describes the degree to which different assets are correlated.

A correlation coefficient of 1 means that two assets are perfectly correlated and therefore they will not contribute to the diversification of the portfolio, which is the ultimate goal in portfolio optimization.

On the other hand, a correlation coefficient of -1 means that the assets are perfectly adverse correlated moves perfectly in the opposite direction at all time. When the correlation coefficient is 0, the assets are uncorrelated which means that the assets will 50% of the time move in the same direction and 50% of the time they will move in the opposite direction.

When focusing on portfolio optimization, it should be the primary focus to create a portfolio of assets with a correlation coefficient between -0,7 and 0,7 in order to limit more significant swings in the value of the portfolio. When introducing negatively correlated assets to an existing portfolio, it works as a hedge against market movements, which is why you always want to adjust your correlations to the level of risk wanted.

The correlation coefficient can be transformed into what is called the "correlation of determination", which simply is the squared correlation between two assets. This has the advantages that it makes it easy to understand the degree of variance in the dependent variable, which can be explained by the development of the independent variable. With a correlation of +/-0,6, this means that we get $0{,}6^2 = 0{,}36 \approx 36\%$ of the variation in the dependent variable can be explained by the variation in the independent variable (Lhabitant, 2009: 128).

Mean-variance

When calculating the portfolio mean and variance for more than two assets, matrix notations is highly recommended, since it simplifies the process. In general, it is required to set up some restrictions. The first is $\sum_i w_i = 1$, which states that the individual weights of the portfolio assets sums to 1. There is no restrictions on short selling, which makes it possible to establish a short position in an asset class if a short positon in this particular asset will improve the overall portfolio efficiency.

As explained above the portfolio mean is calculated as a function of the asset weights and expected returns.

$$E(r_p) = w^T * E(r) = E(r)^T w$$

The variance of the portfolio is not as straightforward as the expected return since it is necessary to include the covariance.

$$Var(r_p) = \sum_{i=1}^{n} (w_i)^2 * Var(r_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i * w_j * Cov(r_i, r_j)$$

This should be read as the squared weight of asset, i, multiplied with the variance, added to twice the product of each asset weights in the portfolio, and at last, this should be multiplied by the pairwise covariance of the assets.

When working with large portfolios in excel, the variance-covariance matrix comes in handy. This is represented by S.

$$S = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & & \ddots & \vdots \\ \sigma_{nn} & \sigma_{n2} & & \sigma_{nn} \end{bmatrix}$$

Calculating the portfolio variance, $Var(r_p)$ using the variance-covariance matrix is given by:

$$Var(r_p) = w^T S w$$

In excel this will be done with the following formula:

$$= MMult(MMult(Transpose(w), S), w)$$

Where w is the individual asset weights in the portfolio and S the variance-covariance matrix.

Risk measures

Sharpe Ratio

Risk-reward measures have the purpose of assessing the amount of return compared to the risk there is associated with this investment. Investors prefer investments with high expected return, but do not like the risks there often is associated with these returns. Risk and returns are often highly correlated, and therefore measures of the risk compared to the expected returns are highly relevant when assessing possible investments. Investors will always choose an investment with high returns relative to the potential risk, which is why the Sharpe Ratio (SR) is a famous key-ratio.

The Sharpe Ratio was introduced by William F. Sharpe who was rewarded a Nobel Prize in 1990, for his work on the CAPM, and the SR. The SR is one of the most popular risk-reward measures in modern finance.
The Sharpe Ratio is defined as the expected return divided by the standard deviation of the distribution of returns.

$$SR = \frac{E(r) - r_f}{\frac{1}{n-1}\sum(r-\mu)^2} = \frac{E(r) - r_f}{Std(r)}$$

There are several negative voices regarding the use of SR since it, as often, is built on an assumption of normally distributed returns and the regular standard deviation.

Several earlier studies have been conducted on the subject of whether hedge funds are able to significantly outperform traditional asset classes and portfolios, this is in most cases done by comparing the Sharpe Ratio for the comparable assets. Such studies is Bing Liang (1999) where it is showed that hedge funds consistently outperform mutual funds on a significance level of 1%. Similar study and result have been found in Peskin, Urias, Anjivel (2000) where the sample hedge funds had a Sharpe ratio of 2.5 in the period from 1990 to 2000, compared to a Sharpe ratio of 0,9 for the S&P 500.

The discussion of using the standard deviation in order to give a risk-adjusted return measure mainly builds on the fact that in calculating the standard deviation there is an indifference to positive or negative returns on investments. This is not the case for investors; who worship upside volatility, hope, and expect it. Downside returns, on the other hand, is the highest of priorities. This is why Frank A. Sortino advocated for the use of downside risk instead of the usual standard deviation. This was done by introducing the lower partial standard deviation into the SR; this made it the Sortino Ratio.

Sortino Ratio

In the Sortino Ratio the standard deviation, as we know it in the Sharp Ratio, is replaced with the lower partial standard deviation, in order to comply with the interests of the investors, who mostly are aware of downside risks (Pedersen, 2015: 32).

The lower partial standard deviation (LPStd) deviates from the usual standard deviation by only calculating the volatility of only non-positive returns. There is though a discussion on how to calculate the LPStd since there are two possible methods.
The first method changes all positive excess returns into zeros, which are included in the calculation of the standard deviation. The second method uses only strictly negative excess returns in the calculation of the standard deviation. Using the first method translates positive excess returns into zeros. Therefore a higher number of observations become zero and therefore gives lower volatility. Since the inclusion of zeros reduces the downside risk, the volatility is not described correctly by this method.

The calculation of the LPStd is done by taking the square root of the average squared difference between the expected return and the minimum acceptable return. As described in the second method above, there should only be taken the square of strictly negative excess return and taken the square root of the average.

The Sortino Ratio builds on the same principles as the Sharpe Ratio, but introduces the minimum accepted return (MAR). The MAR is the lower boundary of which returns are acceptable. The MAR is subtracted from the expected return, which gives a measure that is not affected by returns above our MAR.

The Sortino Ratio is calculated by dividing the difference between the expected return and the MAR, with the LPstd.

$$Sortino\ Ratio = \frac{E(r) - MAR}{LPStd(r)}$$

The Sortino Ratio is a ratio explicitly made for practical use and does not have the academically legacy as the Sharpe Ratio, which both has positive aspects and negative since the purpose of this ratio is to help investors model their risks with the correct properties. The downside is the different methods of calculating the LPStd, which creates a need for explicitly expressing the underlying calculation methods before interpreting the output.

The conclusion on the Sortino Ratio is that this ratio is at some point improvement to the well-known Sharpe Ratio. This ratio adapts to the interests of the investors who are not concerned with the upside volatility but only has the intention of modeling downside volatility. The Sortino ratio is highly relevant for hedge funds due to the expected asymmetric return distributions. The ratio should not stand alone since the upside volatility is not entirely irrelevant to investors. It is well known that with high returns comes higher risk. Therefore it is always a necessity to take both upside- and downside volatility.

VaR

Value at risk (VaR) is a statistical measure of the maximum risk of loss at a specified confidence level. The volatility measured by the variance is only applicable to normal-distributed returns and does not reflect eventual crashes and non-normal distributed returns (Pedersen, 2014: 58). Hedge funds are known for having a non-normal distribution; this is return distributions with a significant level of skewness and kurtosis. Non-normal distributions implicate an understatement of the true VaR due to a higher number of outliers than assumed in a normal distribution. (Anson, 2006: 178) Some hedge funds follow a strategy, which has a return distribution similar to equities, which falls in the category normally distributed, while others seek small stable returns, where the kurtosis is high, and tails of the distribution are fat, symbolizing few but large; positive and negative returns.

In order to determine the VaR of a portfolio, three essential variables are necessary in order to estimate the portfolio VaR; Expected return, volatility, confidence level, and the timeframe of which returns and volatility should be measured (Anson, 2006: 178).

The VaR can be calculated as follows:

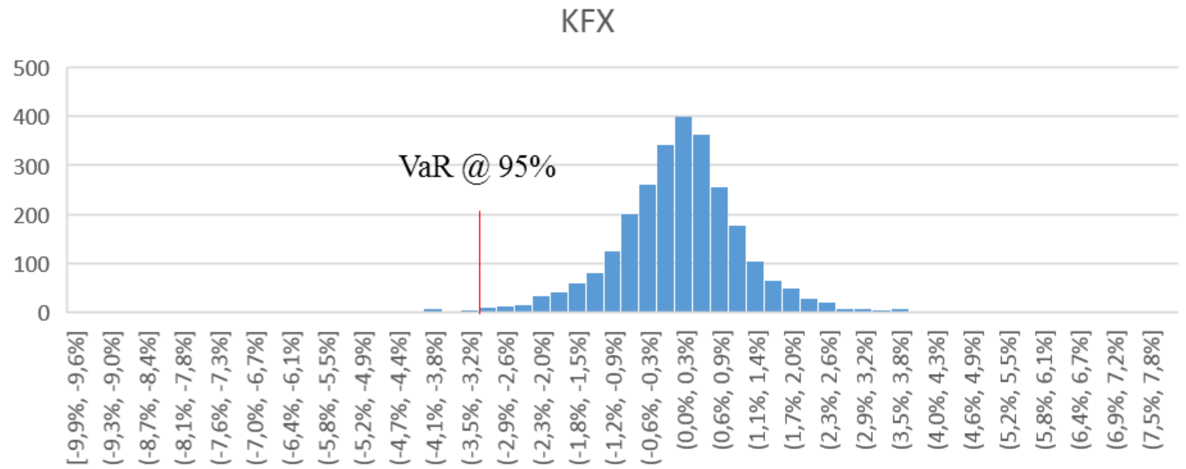
$$VaR(\%) = \sigma_p * confidence\ level$$

The formula above is able to calculate the maximum loss at a specified confidence level given the size of the investment and the historical standard deviation of the portfolio.

The regular VaR has some shortcomings and has been criticized for being tied to the normal-distribution and therefore not estimating the true risk associated with specific instruments of portfolios.

VaR can also be calculated on a specific realized return distribution, this is the non-parametric VaR. To use the non-parametric VaR one must only have x historical values, a confidence level to determine the VaR, directly from the distribution of returns as below. This is also called the Modified VaR which follows.

Figure 1



Source: Own production, Nasdaq

Jarque-Bera-test

When testing for normality of an empirical distribution, the Jarque-Bera-test is known as a test where the skewness and kurtosis of the returns are used to test for normality of the distribution.

$$\frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

The test-result of the Jarque-Bera-test is Chi-square-distributed with two degrees of freedom (National institute of standards and technology, 20.03.2018).

Modified VaR

Modified VaR differs from the traditional VaR since it does not require the underlying normal distribution of returns. The modified VaR uses the skewness and kurtosis of the realized return distribution to calculate the "corrected" VaR. The formula is as follows:

$$VaR_m = E(r) - Z_{CF,\alpha} * \sigma$$

Where $E(r)$ is the expected return and drift of the asset, σ is the standard deviation and the Cornish-Fisher approximation of the distribution.

$$Z_{CF,\alpha} = z_c + \frac{S(z_c^2 - 1)}{6} + \frac{K(z_c^3 - 3z_c)}{24} - \frac{S^2(2z_c^3 - 5z_c)}{36}$$

In the formula above the z_α is the confidence level in a normal distribution, S is the Skewness, K the Kurtosis. (Lejeune, 2010)

Expected Shortfall

Where the VaR and MVaR does not provide any information about the size of the potential loss, in the cases of a 1 %- or 5 %-tail event, the Expected Shortfall (ES) provides the magnitude of the losses at these tail-events (Munk, 2016: 63).

The ES is defined as follows:

$$ES(p,T) = E[V_T - V_0 | V_T - V_0 \leq VaR(p,T)]$$

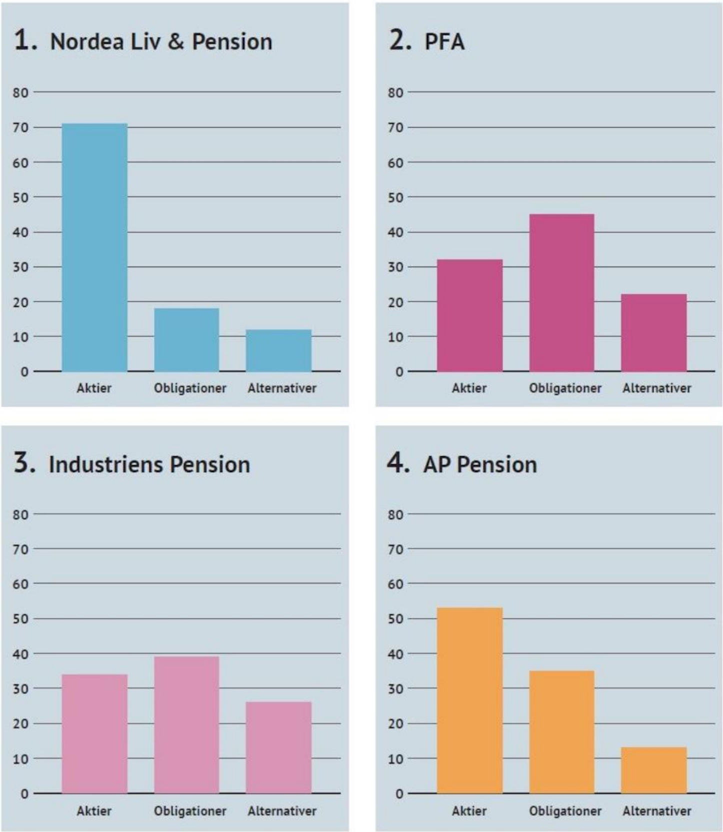
Where V is the value of an asset or portfolio, and T is the time horizon of which this should be measured.

Model portfolio

The model portfolio is put together on the basis of publicly available information form Danish pension funds. The asset allocation of the individual pension funds has been gathered from websites of the pension funds and Morningstar.com. These asset allocations will be put together in a qualitative manner, in order to create an average pension portfolio asset allocation.

Morningstar has recently provided an analysis of how the asset allocation of four large Danish pension funds looks at the beginning of 2018. Figure 1 illustrates how the different pension funds are distributed on asset classes. The key take away from this figure is that, even though all these allocations are labeled as medium risk portfolios, they vary a lot. Nordea Liv & Pension has an allocation of more than 70 % of its assets in equity, where PFA only has a 30 % exposure to equities. The analysis was conducted by Morningstar for Børsen, which in their article about the survey also interviews the chief analyst of Morningstar Denmark, Nikolaj Holdt Mikkelsen. He commented on the level of risk in the individual pension product, and put weight on the fact that some of these portfolios have, during the last 5 years, experienced losses of up 9 %, while others only have had losses of up to 5 %. This difference is significant and primarily founded in the asset allocations of figure 2 (Børsen, 22.03.2018)

Figure 2



Source: Børsen, 22.03.2018

Financewatch.dk published on the 18th of February 2018 a column by Jesper Kirstein founder of Kirstein A/S and Spektrum A/S, who comments on precisely this subject, and argues that the asset allocation in itself can not be used to judge the riskiness of the individual portfolios. It is also crucial to look below the surface of the published asset allocations and take a look at the underlying investments in the category "alternatives", which in theory holds both relatively safe investments in Danish real estate and African private equity investments. The risk of these investments are very different, and therefore they can not be directly compared as in figure 2 (Financewatch, 18.02.2018)

Tabel 1 is based on data from Pension Danmark and shows their asset allocation for members under the age of 41 years. This shows an overall allocation of 50 % in equity, 20 % in high yield bonds and loans, 18 % in alternative investments, and 12 % in government bonds and index bonds. This allocation is almost corresponding to an average of the four different allocations above.

Table 1

Assets (in pct.)	**Members under the age of 41 years (in pct.)**
Quoted Equity	46,7
Private Equity	4,2
Total equity	**50,9**
Corporate bonds and loans	12
Credit funds	2,8
Emerging Market-bonds	4,8
Total high yield bonds and loans	**19,6**
Infrastructure and renewable energy	10,4
Real Estate	7,2
Total stable alternative investments	**17,6**
Index bonds	4,2
Government bonds, mortgage bonds and direct loans with high security	7,7
Bonds and direct loans with high security	**11,9**
Total	100

Source: Own production

The model-portfolio of this thesis will be constructed based on the above allocations. For calculating the historical variances, correlations, and other key ratios, the below-listed proxies, in table 2, have been used. These are a mix of benchmarks, indices, and ETFs, which are all publically available data.

Some choices have been made regarding the individual proxies, in the data collection process. The proxy for inflation-linked bonds, US TIPS, has been chosen due to the accessibility, liquidity, and size of the market. The most significant of these factors is the size of the market for inflation-linked bonds in Denmark compared to the market for US TIPS. The outstanding amount of inflation-linked bonds issued by Nationalbanken have been falling since 2000, where Nationalbanken stopped issuing new inflation-linked debt. The total market value of the Danish inflation-linked bonds has been falling from DKK 160 billion in 2000 to below DKK 94 billion in 2012 (Nationalbanken, 22nd of May, 2012). This relatively limited outstanding amount has been the sole reason for this lacking liquidity. The demand for inflation-linked debt has been high during the last 15 years, and Danish investors, primarily pension funds, and insurance companies have been forced to seek international inflation-linked debt, such as US TIPS. Nationalbanken launched a new issuance of inflation-linked bonds in May 2012, but still to a limited extent, which does that Danish professional investors still would need to look to the international market for greater inflation-linked exposure. Therefore the natural choice of proxy for international inflation-linked bonds has been the US TIPS.

Table 2

Asset class	Proxy	Weight
International equity	MSCI ACWI	38 %
Domestic equity	KFX	5 %
Infrastructure investments	IGF	11 %
Private Equity	LPX50TR Index	5 %
Government bonds	NDEAGVT	8 %
Mortgage bonds	NDEACFMB	5 %
Real Estate	VGSIX	8 %
High yield bonds	HW00	15 %
Inflation-linked bonds	US TIPS	5 %

Source: Own production

As a proxy for Private Equity investments, the choice had fallen on the PowerShares Global Listed Private Equity Portfolio (PSP), which is a fund that seeks to follow the Red Rocks Global Listed Private Equity Index. The fund invests in ADR/GDR's of 40 to 75 publically traded; private equity companies, business development companies, and master limited partnerships, which all has the primary purpose of investing in or lending capital to privately held companies. These investments have direct exposure to private equity investments all around the world, which qualifies the fund as a proxy for a private equity investment in this thesis. It should be noted that this fund is listed on the NYSE Arca, Inc. and is, therefore, trading at a market price, opposite to the NAV of the fund (PowerShares Exchange-Traded Funds Trust Prospectus, 2017: 137). This makes the fund exposed to market beta, and thereby the correlation with the equity markets are expected to be higher in the later analysis, than if the proxy had been an index of direct private equity investments. This could lead to a lower allocation of capital in a mean-variance analysis since the correlation with the rest of the publically traded assets will dilute the diversification, which private equity investments otherwise are able to provide a diversified portfolio. It is possible that the proxy will underestimate the liquidity premium, which in common theory is applied to private equity investments, as a compensation for the long-term investment horizon and missing liquidity involved with these investments. With these pitfalls in mind, the fund has been chosen as the proxy for private equity investments in this thesis.

The proxy for Real Estate investments have been chosen to be the Vanguard Real Estate Index Fund (VXSIX), which is publically traded on the NYSE Arca, Inc. as well as the PSP. The ETF can thereby be affected by the same disadvantages, as the PSP, such as lacking liquidity premium compared to investments made by pension funds, which are direct investments in physical property. This fund invests in REITs and companies who purchase real estate such as office buildings, hotels, and other property. It can, therefore, be argued that the proxy will reflect a different development in the value of this asset class, than the one realized by pension funds. The proxy though makes up a measure of the global value of real estate investments and gives a good proxy for the correlation of movement between the different asset classes and real estate. Similar assesment has been made for the Infrastructure investment proxy, iShares Global Infrastructure ETF. The fund has the purpose of tracking an index of developed market infrastructure equities. The underlying investments are companies working within transportation infrastructure, transportation, airport services, highways, harbors, and energy all around the world (iShares Global Infrastructure ETF Prospectus: 2).
This fund is exposed to the same factors as the other exchange-traded proxies. But it has also been evaluated that the underlying exposure translates into a comparable performance, to the one pension funds wishes to achieve when investing in unquoted infrastructure investments.

All the above-mentioned proxies, which are publically traded, and thereby are affected by market beta, and possibly lacks to reflect the illiquidity premiums pension funds seek to receive by investing in alternative illiquid assets. These downsides of using publically-traded proxies should be held in comparison with the possible upsides of higher liquidity and market forces. These help to determing the valuation of the underlying investments, which are else renowned not to be valued mark-to-market, but rather once every quarter or even more infrequently. This will in accordance with the theory of market efficientcy, as defined by Eugene Fama, be more correct, since the market price at all times will reflect every piece of information available, and thereby always reflect the fundamental value of the underlying assets (Pedersen, 2015: 3). Whether the market for ETF's is efficient can be discussed further, since it can be hard to replicate the investments in the underlying assets 100 % at all time, but the liquidity of these funds could be argued to offset the pitfalls created by the same liquidity.

Fixed-income based hedge funds

Fixed-income arbitrage is a trading strategy, which involves buying and selling similar fixed-income instruments, which at the time of execution are priced differently. The intention of this strategy is that the prices of the specific instruments will converge when the other market participants discovers this inefficiency and therefore prices the different instruments similarly. The hedge funds will sell an appropriate amount of the "expensive" instrument, corresponding to the relationship in price movements of the two instruments. Duration is often used as a measure for the determination of the amount to be sold relative to bought.

Examples of fixed income arbitrage strategies can be to exploit eventual inefficiencies in the credit spread between mortgage bonds and treasury bonds. Mortgage bonds trade at a premium to treasury bonds due to the risks involved with investments in mortgage bonds, which is uncertainty regarding prepayment and credit risks. Uncertainties regarding the size and speed of which the prepayments will come on the mortgage bonds create inefficient prices, which can be exploited by investors like hedge funds. The inefficiencies arise when the market is expecting higher prepayment rates than actually realized. This can of course never be 100 % determined by the actual number of prepayments publicised on the term date of the bond.

But professional investors might have different prepayment models to determine the size of the next prepayment, and therefore what the theoretical value of the future cash flows of this bond will be. Differences in these models will also create a difference in the theoretical value of investors, and therefore a market for exploiting potential inefficiencies in prices on mortgage bonds. Option-adjusted spread (OAS) is used to price mortgage bonds relative to treasury bonds, and is representing the spread of which a mortgage bond should be trading to a treasury bond of the same maturity (Anson, 2006: 54).

The characteristic that makes a hedge fund is the fact that the fund uses financial instruments in order to hedge the initial investment from unwanted downside risks. For fixed-income hedge funds, this risk is primary interest rate risk, which can be hedged using different methods. Interest rate risk can be hedged with interest rate swaps, interest rate options, futures, and short selling of treasury

bonds with similar characteristics.
Danish fixed-income hedge funds are large buyers of covered mortgage bonds and government bonds. The funds use the repo market to gear their investments and prime broker setups to buy bonds on a margin. The interest rate risk is hedged with interest rate swaps, bond futures, and put options (Investoroplysninger for Kapitalforeningen HP Hedge, 2017).

Many fixed-income hedge funds are classified as relative value strategies since their strategies are based on buying and selling bonds with similar characteristics as described above. This is to some extent also true for the sample selected for this thesis. The sample funds could also be characterized as having a carry strategy, where the object is to earn the interest rate of the bond portfolio, while hedging the interest rate risk by entering interest rate swaps. This strategy earns the excess return of the underlying bond after subtracting the repo-rate and the cost of hedging the interest rate. If the fund is able to determine the correct amount of hedging, which is optimal for the fund to be unaffected by changes in the interest rate environment. This way the fund can collect the accrued interest of the portfolio of bonds it is holding, while being less exposed to the interest rate risk.

This strategy is performed by the sample Nykredit Alpha Mira, Nykredit Alpha KOBRA, HP Hedge Danske Obligationer, and Danske Invest Hedge Fixed Income Strategies Fund.

The funds all have in common that they are based on a long/short and relative value strategy primarily in Danish covered bonds and Danish government bonds, but some also span over a greater geographic area. These hedge funds invest their own as well as borrowed funds. The strategy is based on borrowing funds to invest and thereby entering financial gearing. This is to exploit any arbitrage opportunities created by the credit spread between the yield on covered mortgage bonds and “risk-free” government bonds. The spread is due to the higher risks associated with the mortgage bonds relative to investments in government bonds. The funds can conduct their gearing activity by entering bilateral loans, prime loan facilities or entering repo agreement. This gearing of the funds applies a substantial risk and increases the risk of the fund going bankrupt.

Excluded from the sample, but interesting to mention is Jyske Invest Hedge Markedsneutral – Obligationer, which was founded just before the financial crisis in 2008. The fund applied up to 40 times gearing of its investments, which hit the fund, when the liquidity began to fall, and panic began. The fund lost 20 % of its NAV in the first year after inception and almost 80 % of its NAV in their second year in business. The fund was threatened with a lawsuit by its retail investors, but

made a settlement of 80 % of the loss covered in 2012 (Børsen.dk, 11.09.2012).
The strategy was terminated at the end of 2008. The fund was finally terminated in 2012.

It has been decided to exclude the fund from the index, in this thesis, even though it introduces survivorship bias, which is a common issue, when construction backdated indexes (Anson, 2006: 232). Survivorship bias overestimates the historical performance of the index since the funds that fail and are terminated, will not be included in the index. Thereby large negative contributions to the performance are excluded and historical performance is overestimated. This effect has been assessed, and it has been evaluated that this action is needed for the constucted index to make sense with respect to this thesis.

The risks of geared investments in Danish covered bonds are multiple and significant. The most significant risks are connected to changes in prepayments, convexity, the yield curve, the yield spread, liquidity, and counterparty risk.

Prepayment risk

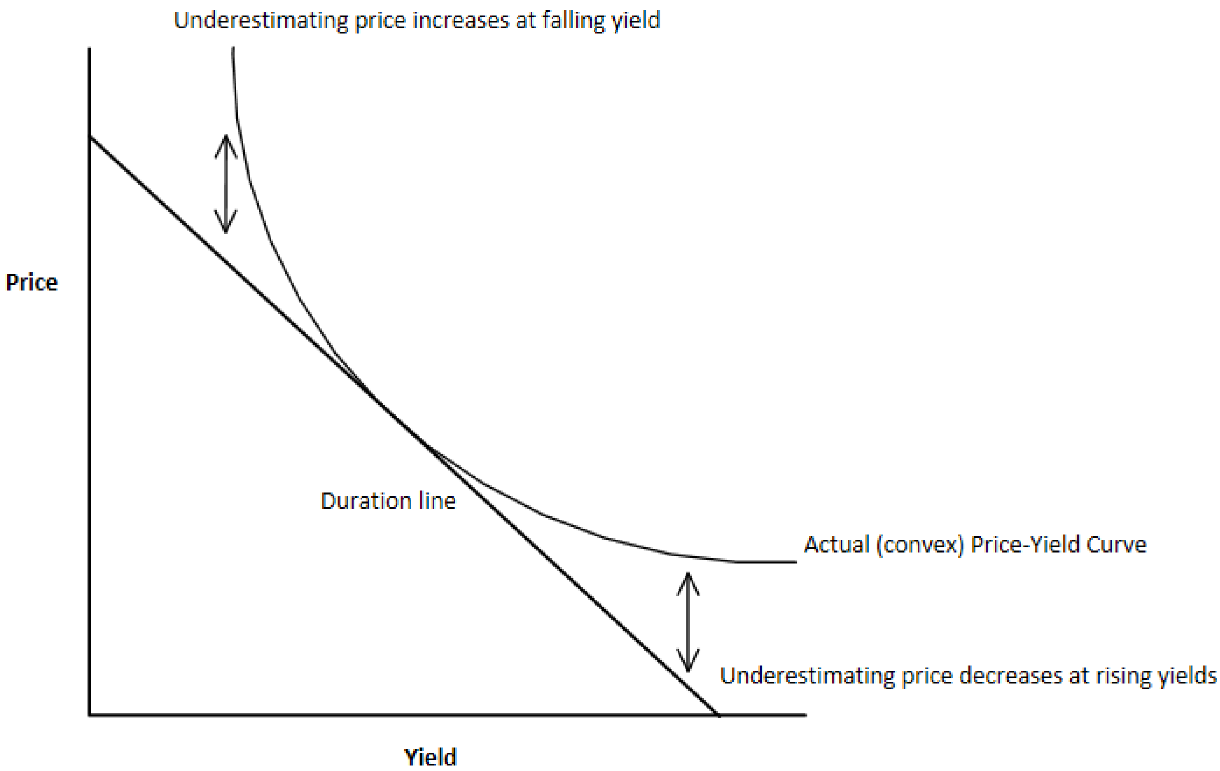
A particular risk in investing in high coupon Danish mortgage bonds is the risk of early prepayment. The embedded call option in Danish mortgage bonds is a significant risk to a geared portfolio, which primarily invests in high coupon bonds, which trade at a premium to the par value. The risk lies in the difference of the market price and the price at what the bonds are redeemable at, which for most bonds is 100. When debtors find the nominal rate of the loan too high compared to the market rate they has the right to either deliver the amount borrowed in bonds with the same ISIN or pay back in cash at par value. If the holdings in these bonds are valued at the market price, the fund will incur a loss of the amount the bond was bought above par. This is often a significant loss when holding geared high coupon bond positions. It should be mentioned that the risk of redemption is to some extent calculated for, when setting the market price of these bonds. When calculating the theoretical price of a high coupon mortgage bond, with an embedded call option, the price is underestimated as the market rate exceeds the nominal rate of the bond. This is because the value of the embedded call option rises and should be subtracted from the theoretical price of the bond, since the higher the price of the call option, the higher the probability of the debtor will exercise this option.

The risk, therefore, lies with sudden changes in the conversion behavior of the debtors that has not been priced into the bonds. This is not only an issue with illiquid high coupon bonds, but for all bonds trading above par.

Convexity

When investing in large bond portfolios the convexity of the function of price change on the effective interest rate, is a risk there has to be accounted for.
The duration of a bond is a linear approximation of the price as a function of the effective interest rate. When the effective interest rate is high, the duration underestimates the price of the bond, and the convexity is high. In lower coupon bonds have higher convexity, because they are more sensitive to changes in the interest rates than higher coupon bonds. This is because of the embedded call option in high coupon bonds, which induces a negative convexity, when interest rates fall to shallow levels, as shown in figure 4.

Figure 3



Souce: Own production

Further, there are several risks associated with the yields, and thereby yield curve and yield spreads. A yield curve is essential in running a fixed-income hedge fund. The yield curve has many properties and implied characteristics, which can be analyzed and interpreted in order to put together a profitable investment strategy. The yield curve is a collection of information of all sorts, and thereby a source of investment advice and predictions of the future.

Yield curve

The yield curve is a collection of yields on different maturity bonds. The yields of different bonds are highly correlated but not perfectly. This is why there can be essential interpretations to make on the behavior of the yield curve. The primary characteristics of the yield curve are its level, slope and curvature. These characteristics are set by different forces. The level of the yield curve is set by the central banks, as it is primarily determined by the risk-free rate and the short rates. The slope and the curvature are both results of expectations and risk factors determined by the markets (Pedersen, 2015: 242).

The determination of the yields behind the yield curve is dependent on what part of the curve you focus on. The short rates, i.e., the first part of the yield curve, is determined by the individual nation’s central banks, as a tool for controlling the economic activity of the nation. The central banks in most nations have the purpose of controlling the lending activity, and thereby the economic activity. When central banks adjust the short rates, this has a direct effect on the commercial lending and thereby the activity of the consumers. Central banks also seek to control the inflation in the economy in order to keep the inflation around the inflation target. When the central bank detects a fall in the economic activity, the reaction would often be to lower the central bank rate. The lowering of the short rates should have the effect that the borrowing activity will increase and the economy will be stimulated. When stimulating an economy the risk of rising inflation is a crucial concern of a central bank, which therefore are monitored closely and used in an analysis of if the rates should be kept at the same level, lowered, or raised.

The long-term interest rates, on the other hand, is not handled by the central banks but by the financial markets. The long rates are a result of the effect in the long-term of the short rates, and thereby the inflation levels. When the central banks keep the short rates at a low level for too long, according to the markets, the expectations of inflations raise. The expectation of future inflation will have an immediate effect on the long rates since inflation will dilute the future purchasing power, and thereby the future cash flow of bonds. This also goes in the opposite direction if the market consensus is that the central bank has set the short rates at too high a level, and the inflation thereby is overestimated, which will lead to falling long rates.

The yield curve is a collection of bonds with different maturities put together to create this visualization of the interest rate term structure. This collection can be challenging since several bonds can have the same maturity but with different coupon rates, which in most cases will lead to different yield on bonds with the same maturity. Therefore, practitioners often use zero-coupon bonds yield as elements to create yield curves (Pedersen: 242). All bonds can be viewed as a portfolio of zero-coupon bonds, which makes it possible to create more generic bonds with different maturities to create a yield curve.

Some funds trade the relation between the long- and short rates, which is represented by the slope of the yield curve. This is done based on a prediction and assessment of the current slope of the yield curve. The trade could be that the investor expects that the yield curve will steepen, understood by the long rates raising relatively more than the short rates. This could be as described above, where an investor believe that the short rates are kept too low, and the inflation will raise more than priced into the market. This trade will be executed by buying bonds with short duration and short-selling longer-dated bonds. Thereby the investor will create a profit if the long rates are rising, as expected, and the short rates are either kept constant or lowered further.

Swaps

These hedge funds included in this analysis is highly dependent on the spreads between specific yields, such as the spread between the yield of mortgage bonds and the swap rate. The swap-spread is the bread and butter of these funds since this spread is the return the fund is able to earn by holding mortgage bonds and hedging the risk of changes in the interest rates. This is what makes these funds into hedge funds.

A swap is a derivative fixed-income instrument, which is an agreement of two financial intermediates with different expectations of the future or different purposes. The name ‘swap’ has its base in the sole purpose of this instrument. A swap is an agreement between two parties to exchange cash flows. A swap consists of two legs with different properties. One leg has the properties of a fixed income bond, where the owner knows the exact cash flow until maturity. The other leg has the properties of a variable rate bond, where the rate is fixed at the current level of the most relevant –IBOR (Interbank offered rate), like the LIBOR or EURIBOR, at every term (Astrup, 2013: 151).

Gearing

Fixed income hedge funds are primarily built on a foundation of gearing since fixed-income instruments and strategies historically have posted strong risk-adjusted returns compared to other asset classes. But in order to create returns on bonds, which are able to compare to equity investments, require large amounts of capital. This requirement is met by leveraging fixed-income investments.

The leverage can be constructed in different ways depending on how the market conditions are and which level of liquidity is available. Hedge funds gear their investment by merely borrowing cash, investing on margin accounts, or more advanced with financial instruments such as options, swaps, futures, or repos. This means that hedge funds are able to increase their purchasing power and exposure to certain assets, with the purpose of earning higher returns (Darbyshire et al., 2011: 14)

When leverage is applied to a fund, it is often viewed as an extremely risky investment. This is not the entire truth. There are different scenarios in which leverage can be used both for increasing the overall market exposure and for lowering the overall market exposure. Hedge funds often use leverage to equalize exposure of different positions for the overall risk to be lowered. Leverage measures are therefore not always possible to judge from the first sight, since there can be different purposes and thereby different implications of applying leverage.

Liquidity

Hedge funds generate a return on the inefficient pricing of illiquid assets, but paradoxically access to liquidity is at the same time their most significant risk. Liquidity describes to which degree an assets can be bought and sold without changing the market price significantly. Liquidity is characterized as a market with many buyers and sellers willing to make a mark. The liquidity of a market is crucial for an asset to be efficiently priced, and to reflect the real market value at all time. This means that immediately when relevant news comes out, the price of a specific asset will adjust to the new knowledge of the market (Pedersen, 2015: 3).

In a fixed income hedge fund, liquidity is crucial due to the nature of the investment universe. When investing in relatively illiquid assets as high-coupon Danish mortgage bonds on leverage, there are risks on both sides of the investment.
If a liquidity spiral are triggered by either influential market participants or regulation, the hedge funds in focus would be exposed to both funding risks and market risks. The risks can come from

unexpected redemption requests, where investors for some reason want to withdraw their funds on a short notice. This could lead to a necessity to sell assets quickly, which possibly would not go unnoticed in a market as the Danish OTC market for bonds. This would make counterparties aware of a situation, which could lead to increased spreads, and possibly have effects on otherwise efficient prices.
Some funds specialize in bonds with specific characteristics, like off-the-run high coupon bonds in which the market is shallow and liquidity is limited, both due to concentrations in ownership, the limited outstanding principal, but also the high risk of prepayments. These funds could be challenged by sudden redemptions and take on substantial losses in a situation like the above mentioned.

The risk of not being able to liquidate a part of the portfolio quickly is not the only risk related to liquidity in hedge funds. Hedge funds rely heavily on liquidity in order to maintain the gearing of the fund. This makes the fund strongly dependant on the counterparties of their repos, prime brokers, and banks in which they hold credit lines. If the markets contracts and the liquidity begins to fade out, the hedge funds will have to react quickly in order to maintain the funding of their leveraged positions. If counterparties start cutting the lines of credit and repo-transactions, these hedge funds will have to look for parts of their portfolio they are able to sell at a fair value.
Liquidity spirals often start as a response to some market reaction in which the asset prices start to drop, the liquidity starts to drop, and money managers start to withdraw capital from the markets. When capital is suddenly drawn out of the markets by many parties, the prices begin to move away from their fundamental value. This sell-out leads to; lower prices, higher margin requirements, losses on active positions, which all leads to funding issues for the funds. This funding issue directly leads to further reductions in positions and deleveraging, which inflicts to further falling market prices (Pedersen, 2015: 81).
This spiral will continues until the market reaches some sort of equilibrium where the recent volatility is priced in, and the market participants have been able to analyze the initial market reaction, and have come to a conclusion about the market reaction. At this time of the process, the markets will begin to consolidate, and participants will watch each other in order to see if the bottom has been reached, and portfolios can be built up again.

One of the most well-known cases of hedge funds hit by liquidity crisis is LTCM, who in 1998 had a balance sheet of USD 125 billion on a capital base of USD 4,8 billion – corresponding to a gearing of more than 25 times. LTCM made great losses on their large mortgage-backed arbitrage portfolio, which was a portfolio of relative value trades. LTCM was buying and selling similar mortgage-bonds priced differently. The strategy was based on LTCM's model, which predicted a contraction in credit spreads and thereby pricing spreads; this strategy was executed by buying the "cheap" bonds and selling the relatively "expensive" bonds. But due to the default of the Russian government in the summer of 1998, the credit spreads around the world widened significantly – opposite to the predictions of LTCM. This heavily impacted the positions of LTCM, which incurred losses of 42% and 83% respectively in the August and September 1998. These losses led to worrying counterparties of LTCM and thereby a margin call from LTCM's Prime Broker (PB). This margin call made it necessary for LTCM to quickly liquidate some of its positions at a time where the markets where consolidating and the liquidity was very shallow. This made the losses even more extensive and therefore more margin calls. By then LTCM was officially in a liquidity spiral. LTCM had most of the largest investment banks in the US as counterparties and default would escalate to impact some of the largest financial institutions in the USA and thereby the financial markets.

This all lead to the interference of the Federal Reserve Bank of New York, who in collaboration with involved brokerage firms and banks agreed to provide a capital injection of USD 3,4 billion to LTCM in return for 90 % of the ownership of the fund (Anson, 2006: 37).

Development in Danish fixed-income hedge funds

The sample of Danish fixed-income hedge funds in this thesis, have performed well during the last 10 years. In table 3 below, the yearly returns from the inception of the funds until 2017. The first year data is incomplete since these funds did not start on the first business day of the year.

The timespan of this analysis is 2007-2017, which includes the financial crisis of 2006-2008 and the 2011 European liquidity crisis.

Table 3

Year	HP Hedge	MIRA	KOBRA	MIRA III	KOBRA II	KOBRA III	Danske Fixed Income strategies
2018	1,79%	-0,51%	0,89%	-1,03%	-0,64%	0,92%	1,16%
2017	8,19%	16,66%	8,18%	16,95%	15,70%	8,13%	10,86%
2016	11,42%	15,02%	10,91%	15,73%	15,06%	11,16%	12,47%
2015	-1,24%	2,80%	4,50%	4,27%	4,65%	3,69%	5,13%
2014	2,64%	3,66%	4,71%	4,34%	5,87%	5,32%	4,79%
2013	14,86%	21,56%	7,91%*	21,83%	13,80%	10,98%	17,45%
2012	17,40%	11,85%		11,56%	13,07%	15,61%	29,35%
2011	8,70%	3,03%		4,65%	3,31%	3,28%	17,28%
2010	17,96%	6,15%		6,24%	5,57%	4,66%	13,55%
2009	44,17%	17,42%		19,47%	18,70%	9,32%	42,21%*
2008	-29,14%	1,68%*		4,91%*	-0,04%*	2,73%*	
2007	4,91%*						

Source: Own production
*Marks incomplete data for opening years.

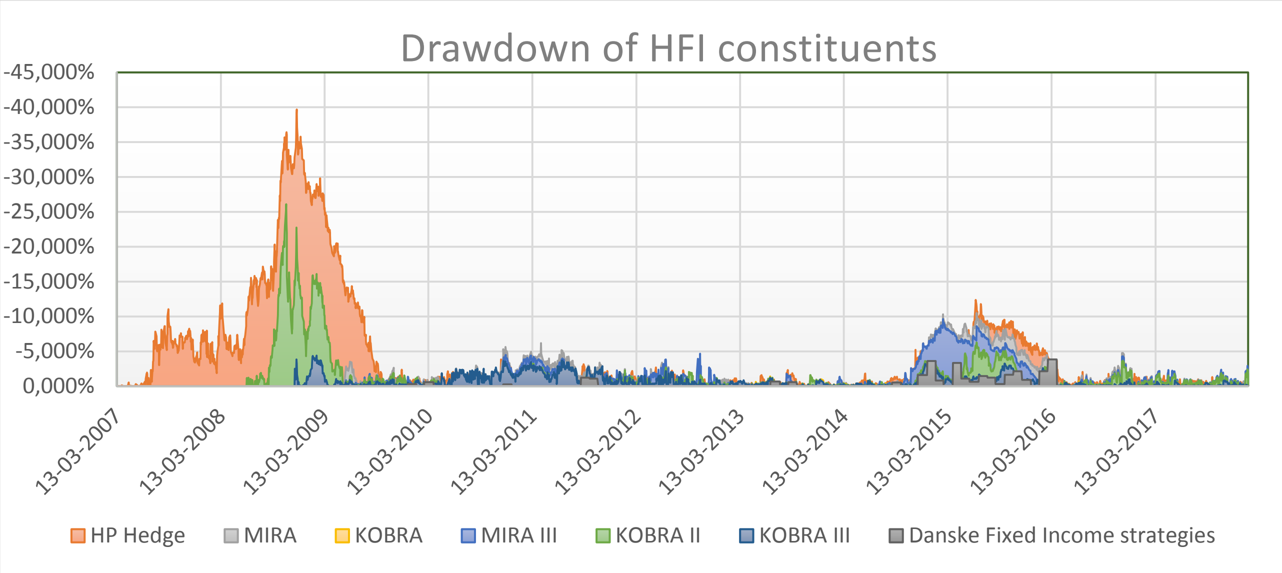
Figure 4

As it can be seen in figure 4 above, the Euribor/EONIA spread can be used as a measure of interbank credit risk and liquidity. During the last quarter of 2008, where Lehman Brothers crashed, and the value of hte sovereign debt was decreasing significantly. The European banks were in need of liquidity, to keep solvent, due to significant positions in European sovereign debt. Therefore, the credit risk among banks was going higher, and thereby the credit-spread on the 3-month EURIBOR increased. This was a textbook liquidity crisis, where the European Central Bank, ECB, did not step in momentarily, but waited. This had a very negative effect on the performance of these funds in 2008, because of the significant rise in the 3-month EURIBOR. This has a significant negative effect on the leveraged bond portfolios, since this both effects the value of the long portfolio and raises the repo-rate at which the portfolio is geared. This generates large negative returns in strategies followed by these hedge funds.

As it can be seen in table 3, the returns of the sample funds vary from -29 % to 5 % in 2008. These negative returns are a result of the financial crisis, where these hedge funds were not able to protect their portfolios from the drastic fall in house prices in Denmark, and thereby the underlying value of the bonds in the portfolio. The liquidity and the rising short rates also provided to a fall in the value of the funds, because this increases the expenditure of gearing the portfolio.

In figure 5 below, we can see that HP Hedge had a maximum drawdown of 39,71 % in 2008, compared to a maximum drawdown of 62,07 % for OMX C20 (KFX) and a drawdown of 56,28 % on the MSCI ACWI. In this context, the drawdown of HP Hedge should be viewed as an attractive investment in periods of market downturn, compared to equity investments.

Figure 5



Source: Own production

The hedge funds selected for this thesis is displayed in table 4 below, where average yearly return, yearly standard deviation, are presented and can be compared to the ACWI and KFX. These are based on equity investments, which can be argued, should not be compared to fixed-income investment funds, but since these funds are geared fixed-income hedge funds, the comparison can be argued to be reasonable.

The yearly returns of the sample hedge funds are very comparable to the broad equity indices, in the table 4, the average yearly returns vary from 6,56 % to 12,41 % for the relevant hedge funds. When looking at the standard deviations, the hedge funds are not comparable in the same way, since the standard deviations of the hedge funds are significantly lower than the standard deviations of the equity indices.

Table 4

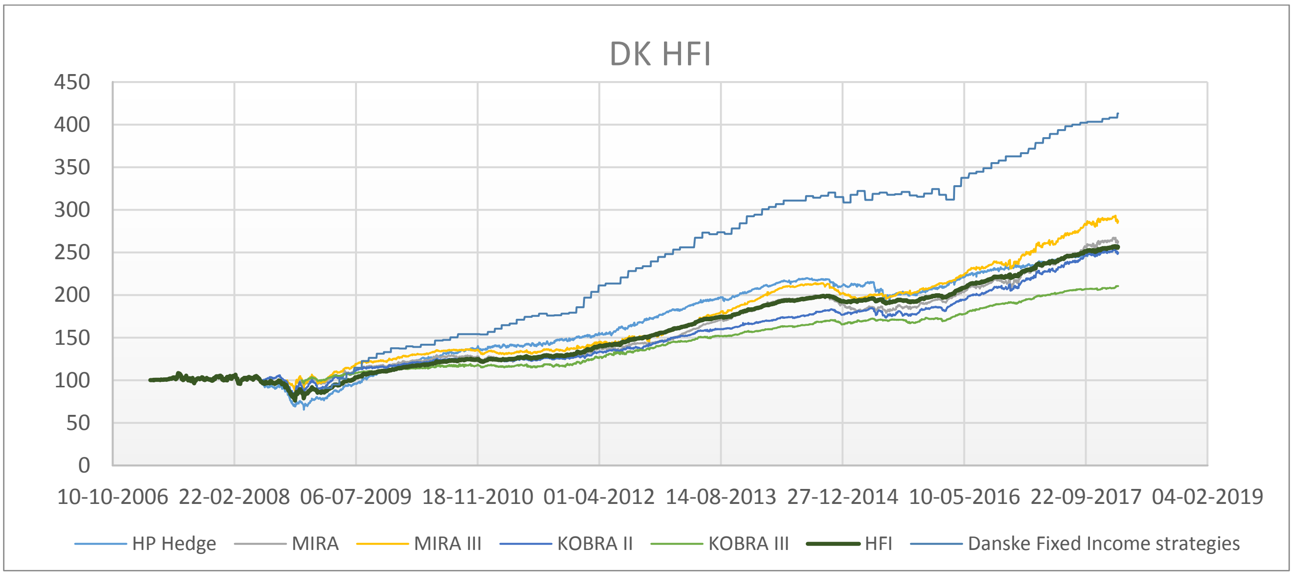
	HP Hedge	MIRA	KOBRA	MIRA III	KOBRA II	KOBRA III	Danske Fixed Income strategies	ACWI	KFX
Average	8,58%	10,16%	6,56%	11,11%	9,47%	9,45%	12,34%	5,42%	8,87%
Std. dev.	16,69%	7,40%	3,81%	7,77%	6,48%	8,32%	8,58%	20,41%	27,16%
SR	0,51	1,37	1,72	1,43	1,46	1,14	1,44	0,27	0,33
Maximum DD	-39,71%	-10,51%	-3,94%	-13,93%	-26,11%	-4,38%	-3,81%	-56,28%	-62,07%

Source: Own production

The result of this is a Sharpe ratio for the hedge funds in the range from 0,5 to 1,72 over the sample period from 2007 or at the fund inception until the 29th of December 2017. This Sharpe ratio is sharply higher than for the equity benchmarks, which is an indication of a better risk-adjusted return over this period of 10 years. It should be noted how the standard deviations of the individual funds vary quite a lot, Nykredit Alpha KOBRA has a standard deviation of only 3,81 % due to its investment strategy. The strategy of KOBRA is based on inconvertible bonds, which makes the process of hedging much easier than for funds primarily invested in convertible mortgage bonds. In convertible bonds where there is, to a great extent, uncertainty about when the future cash-flows will be received due to the embedded option in convertible bonds. In KOBRA the cash-flow of a given investment is known at the time of purchase, and therefore the hedge can be set up with a relatively high accuracy. The return of KOBRA is also one of the lowest among the sample, which is due to the lesser pricing discrepancies in inconvertible bonds, again due to the fully know future cash-flow. Therefore, it is not possible for the hedge fund manager to smart out the market, and thereby earning an extraordinary return on these investments.

Due to limited historical data, because of the limited history of this kind of hedge funds, the historical returns of the individual funds have been averaged into a hedge fund index. Figure 6 below shows the six hedge funds with the longest history and the HFI itself in order to display the development of the individual constituents of the hedge fund Index created for this thesis.

Figure 6



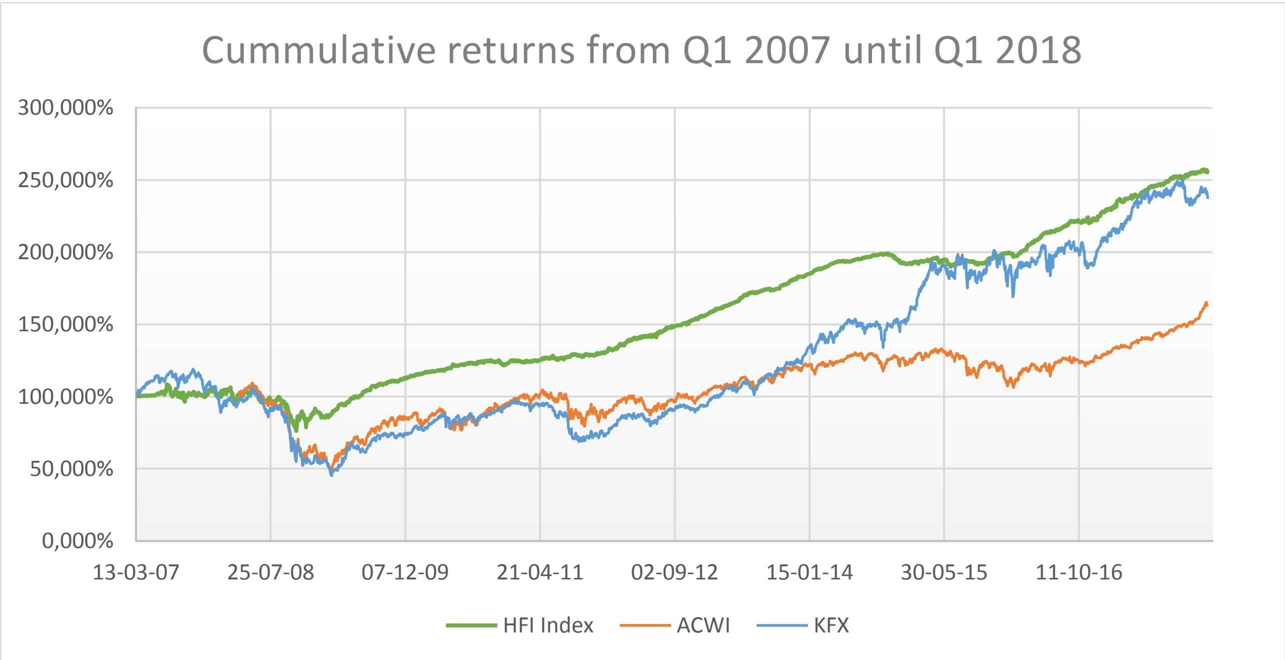
Souce: Own production

In the figure, the development of the index can be seen, and here it is important to focus on the development of the different constituents. It is clear how one has had a more volatile path, but also have outperformed the overall index and the other constituents, while others have had a more stable and limited path. This is a result of the different investment strategies applied by the funds, both the investment universe and the degree of gearing applied are different in all the funds. These funds are all primarily based on investments in Danish covered mortgage bonds, geared with the use of either loans or repos, and hedged by interest rate swaps and bond index options. The funds vary in their selection of specific mortgage bonds, which varies from high coupon illiquid bonds to short- and long duration bonds. These all react differently to changes in the overall market and interest rates, which is why these funds all have different return paths.

Below, in figure 7, the development of the constructed hedge fund index, MSCI ACWI, and the Danish C20 index (KFX) is shown.

It is clear that for this timeframe of 10 years, there has been some covariance between the HFI and the equity benchmarks. It is to be mentioned the drawdown in 2007-2008, where the ACWI and KFX lost up to 62 %, and the hedge fund index only had a drawdown of 25 %. In the period from 2008 and forward, the economic markets have recovered, and the performances of these indices have followed, with around 250 % performance since 2008.

Figure 7

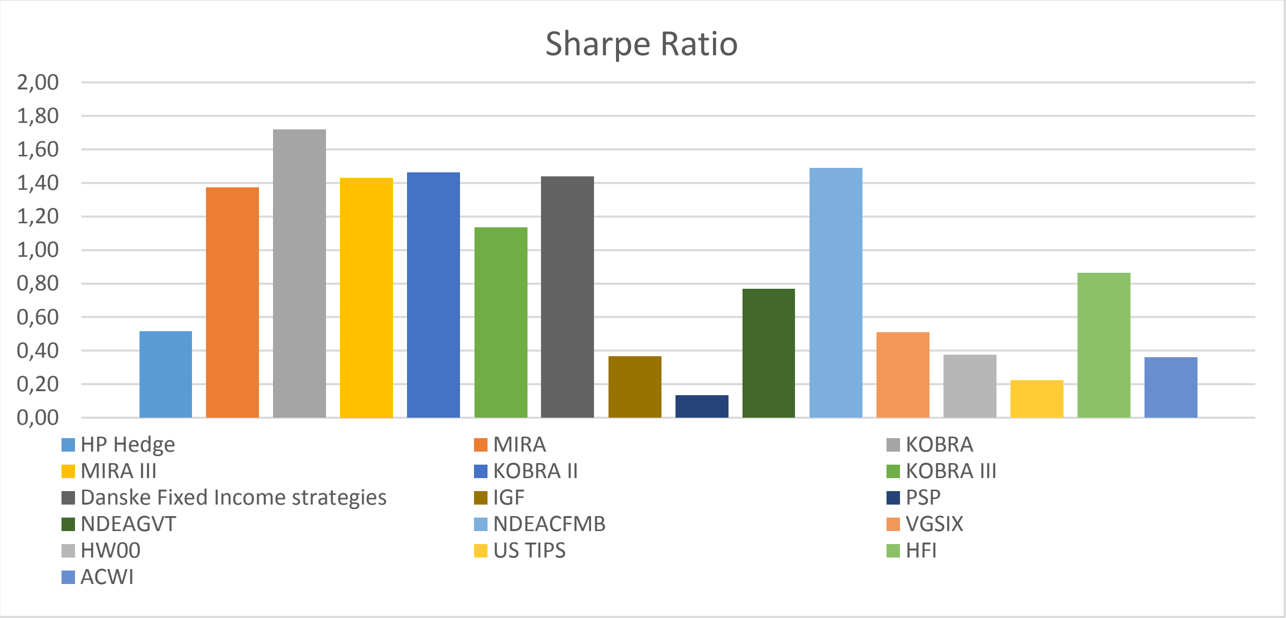


Source: Own production

It should be noticed that the equity indices have been much more volatile than the HFI, which else should be expected to be more volatile, since the investment guidelines are allowing gearing of the funds up to 40 times. This is a clear result of the hedge working on a daily basis and the fund managers steering the funds safely. The HFI has almost had a cumulative return of the equity indices over the period from 2007 until 2018, with lower standard deviation, which is an essential feature of an asset in a pension portfolio.

The Sharpe ratios, of the HFI constituents and model-portfolio, are listed below in figure 8. The right part of the figure is represented by the hedge funds, and the left part is represented by the model-portfolio constituents. There is a clear tendency of a high Sharpe ratio in the hedge funds where asset classes like the private equity and inflation-linked bonds have very low Sharpe ratios. The HFI, which is an equally weighted index of the seven selected hedge funds, has a Sharpe ratio of 0,8. This Sharpe ratio is only beaten by the mortgage bond benchmark with a Sharpe ratio of 1,49, which is very high. This benchmark contains convertible mortgage bonds and capped variable rate mortgage bonds. The duration is aimed at the overall duration of the Danish mortgage bond market. The reason for this Sharpe ratio is the exceptionally low standard deviation of 3,11 % over the last 10 years and a very stable yearly return of 4,63 %.

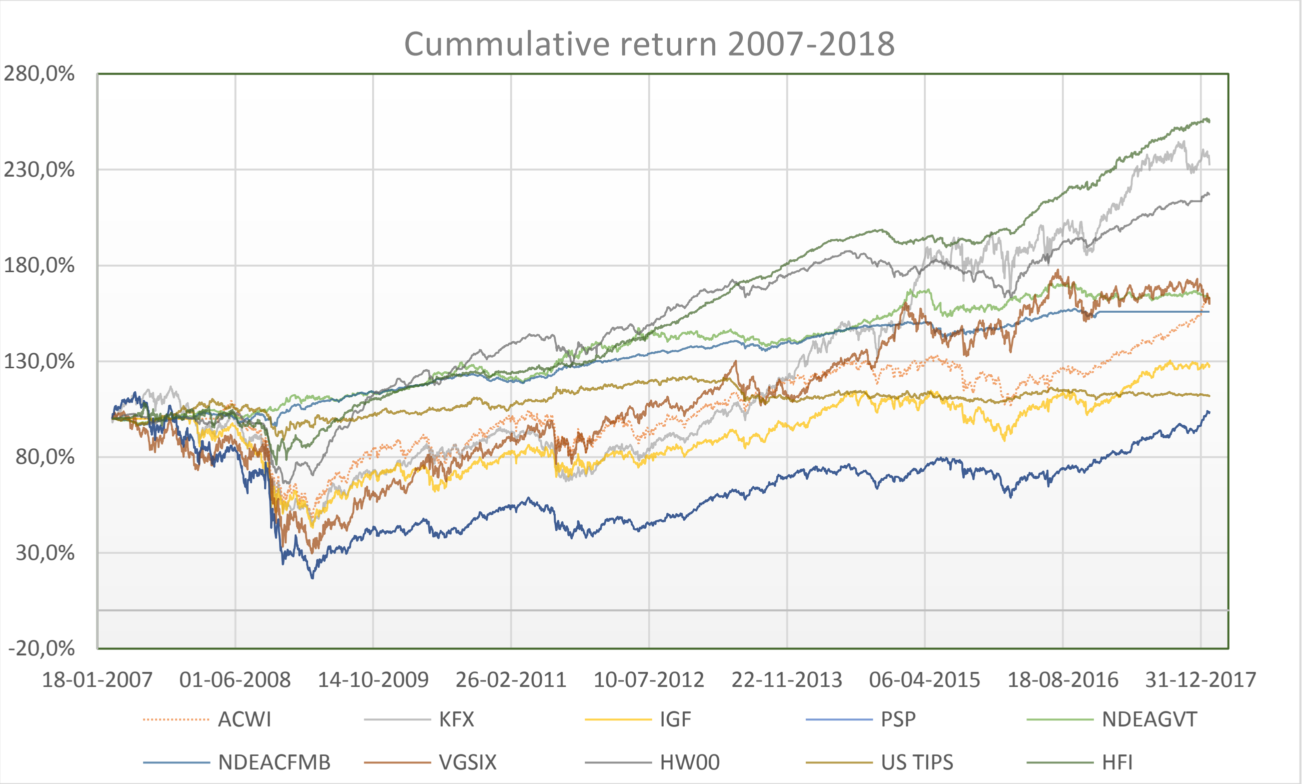
Figure 8



Source: Own calculations

In figure 9 below, the cumulative returns show the significant differences in the development of the different asset classes over the last 10 years. Especially the large drawdowns of the financial crisis in 2007-2008 have had a tremendous effect on the cumulative returns and some asset classes only recovered after 5 years.

Figure 9

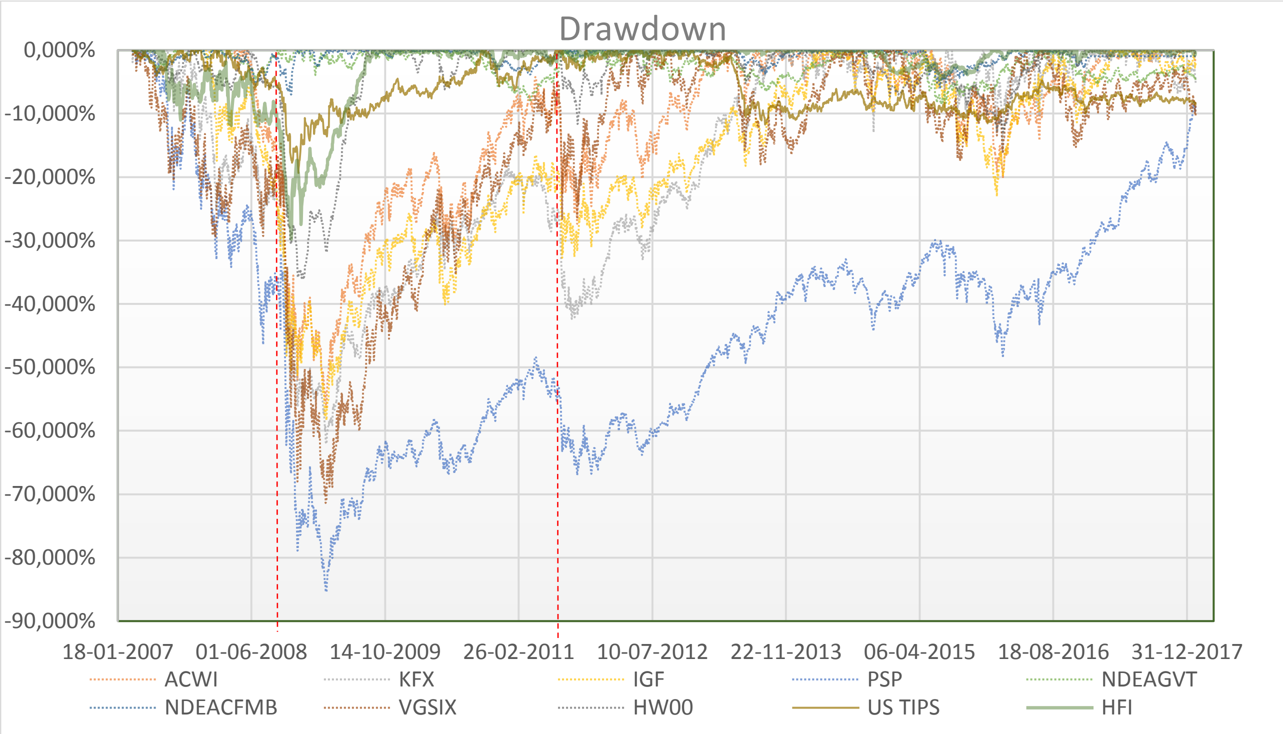


Source: Own production

Alternative investments such as private equity, infrastructure, and real estate were the ones who were hit the hardest, by the significant fall in the liquidity, once the financial institutions got an overview of the extent of the financial crisis. The financial institutions had to cut back on the illiquid and liquidity heavy asset classes, and risky assets like private equity. Therefore, private equity was exceptionally hit hard, with more than a 80 % drawdown, measured with the proxy, PowerShares Global Listed Private Equity (PSP). The fund took the hardest drawdown of all assets in the model-portfolio, and with an investment on the 13th of March 2007, the investment in PSP has not yet been able to break even at the end of 2017.

While some asset classes took very long time to recover, others recovered relatively quickly and performed exceptionally well in the years to come. This can be seen in figure 10 below.

Figure 10



Source: Own production

In 2011 the European sovereign debt levels and government deficits spiked sharply and made the yield spreads in Europe widen. This was caused by member countries like Ireland, Italy, Spain, and Portugal, who published large government deficits caused by bank bailouts, and thereby substantial increases in the yield on government bonds of these countries. Investors fled to safer alternatives, such as German government bonds, which always has been considered safe. During 2010 and 2011 the International Monetary Fund and the European Financial Stability Facility provided loans to Greece, Ireland, and Portugal, which lead to further trouble, when Greece at the beginning of 2011 had to refinance a part of their public debt. The bailout of Greece was conditional on implementation of extensive austerity measures, which was at first accepted by the Greek government, but later rejected by the people who showed their dissatisfaction by going into the streets of Greece. Following this, there were large uncertainties in the European financial markets, whether there was a risk of Greece leaving the Eurozone and defaulting on its government debt, which lead to new volatility and broad decline across the financial markets.

Analysis 1

Fixed-income hedge funds are known under the catchphrase "picking nickels up in front of a steamroller." This phrase is due to the nature of the investment strategies behind this type of hedge funds and the large tail risks. Picking up nickels is a saying related to earning many small profits on heavily geared fixed-income investments, while the steamroller can be viewed as major macroeconomic events that shake the bond markets and create large fluctuations in the interest rates. If these events are unexpected, as these kind of events often are, then the funds can be trapped in liquidity crisis' and thereby heavily affecting the NAV of the fund. Compared to a regular stock portfolio without any gearing, a fixed-income hedge fund is viewed as relatively more complicated, and thereby more risky in the perspective of some people.

This perspective will be investigated further. The following analysis will analyze the tail risks of the constructed HFI and compare this to the traditional assets.

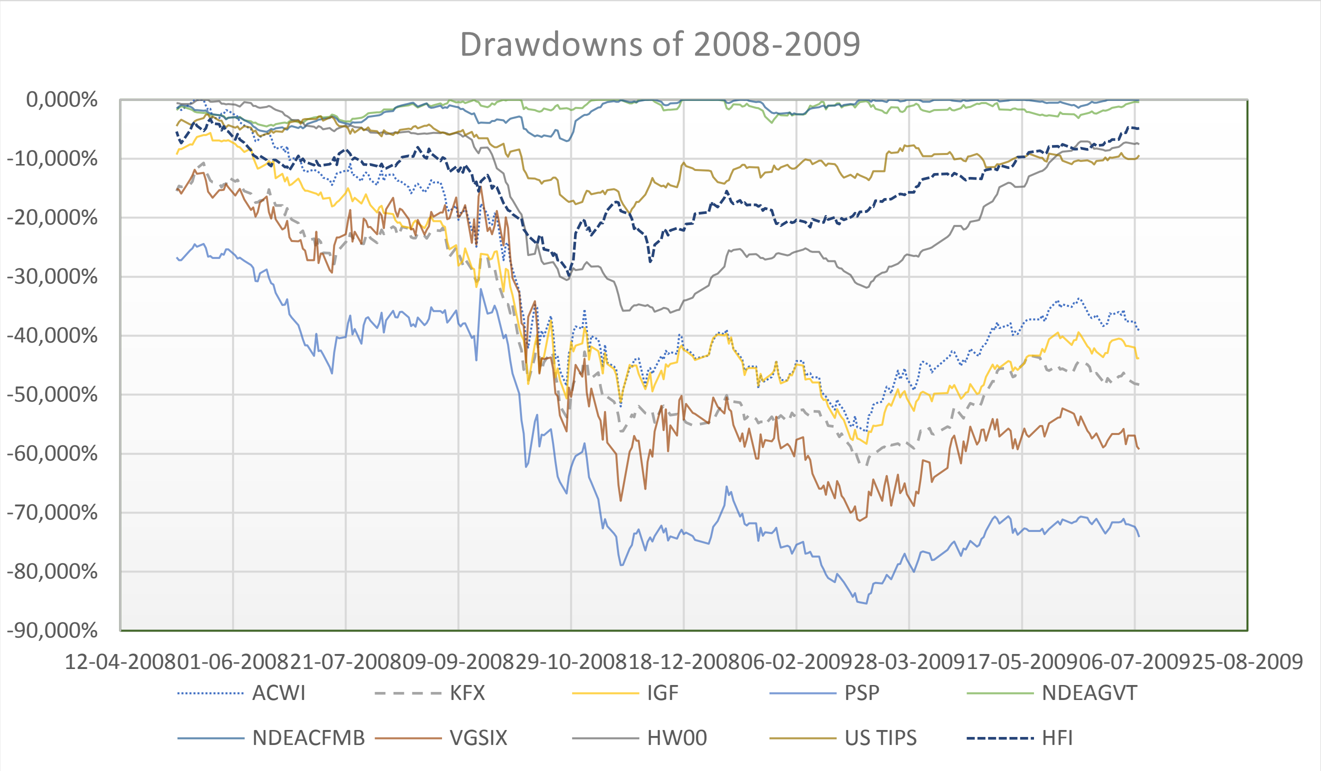
Hedge funds are considered riskier than plain-vanilla portfolios constructed of traditional instruments, which pension funds primarily invest in. This is due to the high levels of gearing applied to the funds in order to speculate or exploit pricing discrepancies in the underlying instruments. Gearing an investment is always a source of extraordinary risk, since it is possible to lose more than the initial investment, and the pace of this is possibly high. While hedge funds are riskier due to the gearing effect on the portfolio, hedge funds have their name from the characteristic that unites the majority of what is known as hedge funds, their hedge. While some mutual funds hedge their forex exposure, this does not make them hedge funds. Hedge funds focus highly on a target investment, where there is a discrepancy able to exploit with the ability to hedge the excess exposure created by focusing highly on the investment objective. For fixed-income hedge funds, the investment objective is bonds, these can be all kinds of bonds ranging from credit bonds, mortgage bonds, to government bonds. This is considered a safe investment, especially when looking at government- and mortgage bonds, which in some countries are rated very high by recognized rating agencies. The primary risk when investing in bonds is the interest risk, the risk of rising interest rate, which will make the price of the bonds fall. When gearing an investment in bonds the interest rate risk follows the degree to which the investments are geared, and thereby the risks follow the expected returns. For a hedge fund, this relationship between risks and return is not satisfactory, and hedge funds would not be able to collect the high fees they are known for charging, by merely gearing long bond investments. Hedge funds use different derivatives and strategies to hedge their portfolio risks; one is interest rate swaps, a bilateral deal between the hedge fund and a counterparty, where it is agreed to swap interest rates. These are often used by hedge funds to swap variable interest rates to a fixed interest rate, for a specified time period. This way the hedge fund knows the future cash flow of its investments and thereby the variability of the value of the bonds are minimized.

There is a large variety of risks involved with hedge funds, and not all can be hedge away like the interest rate risk. For callable mortgage bonds, the primary objective of the Danish sample hedge funds, the prepayment risk is major and impossible to hedge away. There are no financial instruments created to hedge prepayments on a mortgage bond, which makes it impossible for these hedge funds to neutralize this risk. The fact that there is some uncertainty about the prepayment of mortgage bonds are also the root of this investment strategy since the uncertainty is priced into the individual bonds. This is a risk premium, which is priced into the callable mortgage bonds relative to the non-callable mortgage bonds. Some Danish hedge funds specialize in non-callable bonds, which makes it possible for these funds to reach a very high degree of hedge on their investments. This is due to exactly the missing prepayment risk, and therefore only an interest rate risk, which can be perfectly hedged if it is possible to find counterparties willing to enter swaps on exactly the needed terms. In this way, a hedge fund will be able to create a fixed-income portfolio with 100 % known future cash-flows, and therefore a stable return is known until the maturity of the portfolio.

Historical drawdowns and the financial crisis

When we look at the timespan of interest, 2008 to 2009, where the global financial crisis raged around the world, the figure 11 below. It shows that drawdown of the individual asset classes is primarily focused on the outbreak of the financial crisis, where we can see that high-yield bonds fell around 30 % in a matter of three months, and for the worst performing assets, 20-30 % further fall in the last quarter of 2008. Some of these asset classes recover more rapidly than others. The credit- and mortgage bonds recovered the quickest and during 2009 most of these losses were won back.

Figure 11

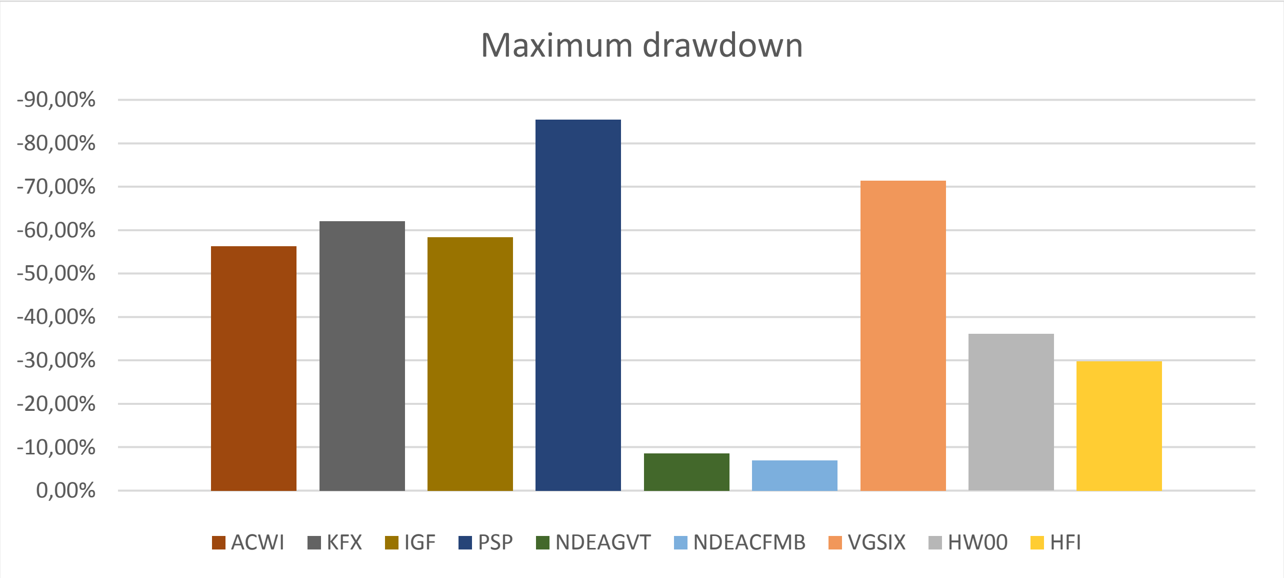


Source: Own calculations

This was mainly due to fast-evolving liquidity spirals and considerable uncertainties about the actual state of the world economy, this lead quickly to widening credit spreads and sellout of risky assets. Investors fled quickly to safe-havens such as US, German, and Danish government bonds where the risk of default was almost zero. This meant that the prices on safe assets rose and the price of risky assets, both equity and alternative investments, fell sharply. Alternative investments such as private equity and infrastructure are rarely priced at a mark-to-market, and therefore relies heavily on alternative pricing methodologies such as classic discounted cash-flow analysis and multiples. In crisis situations as in 2008, valuations of unlisted investments are up for question, since they are not priced by buyers and sellers. Some of these investments are especially cash-heavy and rely heavily on the ability of the investor's cash in order to drive the business forward. In a period of the financial crisis pension funds lost significant amount of their member's savings, and with some of their obligations as fixed rate return-agreements, they had to make sure that they were able to ensure the future obligations to their members. Therefore, investments as private equity and infrastructure investments had to be reevaluated, and the future of these investments had to be decided. Private equity markets are highly linked to credit cycles, and in upcycles, private equity investments are booming (The Journal of Finance, 2013: 2228) with easy access to financing an often leveraged financing. This helps to build up higher valuations and greater volatility, due to the generally higher gearing. This makes the private equity firms more exposed to changes in the underlying real economic conditions, and thereby large fluctuations in valuation (Bernstein, et al., 2017: 3).

This can be directly linked to the proxy for private equity, PSP, which reached a maximum drawdown of 85,4 % in 2009, as it can be seen in figure 12. This shows the strong influence of financial turmoil on private equity investments, and therefore also pension funds, and alike. This also applies to real estate and infrastructure investments, asset classes, which are not publically traded in liquid markets to ensure an efficient price at all time. Here the valuations are often based on theoretical models and general market movements, which therefore create uncertainty when the underlying market conditions changes significantly and the assumptions behind the valuations change.

Figure 12

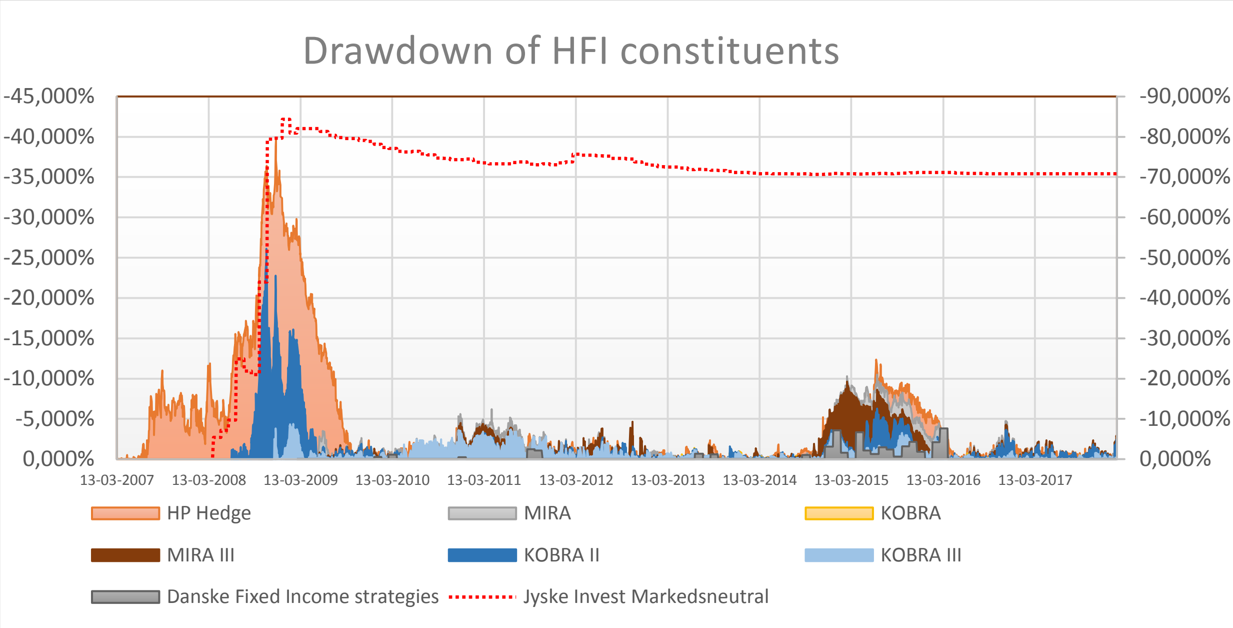


The general drawdowns for publically traded equity investments were between 55 % and 65 %, where the MSCI All Countries Index, ACWI, had a maximum drawdown of 56,3 % and the Danish stock-index, KFX, 62,07 %. For High Yield Bonds, HW00, had a maximum drawdown of 36 %, government bonds, and mortgage bonds had drawdowns of respectively 8,5 % and 7 %. Here it is clear what assets that are considered safe, from the perspective of investors, who fled to bonds, especially safe bonds such as government- and mortgage bonds. This is even due to the significant decline in the prices in the housing markets, which is considered collateral behind mortgage bonds, and the risk of homeowners defaulting on their mortgage debt significantly increases as well. The development of the prices of mortgage- and government bonds are highly dependent on the development of the benchmark rates, which has been falling to record-low territory and therefore the prices of mortgage- and government bonds have been steadily inclining since 2008. This unique development is the reason behind the impressive SR in the figure 8, where the very low standard deviation is the main reason for the high SR.

In figure 12, it is shown that the HFI suffered a maximum drawdown of 30 %. This places the HFI below the drawdown of quoted equity, even though hedge funds are viewed as a highly risky asset class. In figure 13 the historical drawdowns of the constituents of the HFI and Jyske Invest Markedsneutral – Obligationer is graphed. The constituents are graphed on the lefthand-axis and Jyske Invest Markedsneutral – Obligationer on the righthand-axis.

The primary takeaway from figure 13, is the great variation among the different hedge funds, especially during the financial crisis from 2008-2009. Only four of the constituents were active at the end of 2008, where the losses were the largest. It was, as earlier mentioned, HP Hedge that had the largest drawdown of the constituents, where the Nykredit Alpha Kobra-funds had limited drawdowns, which could both be due to the short history of the funds or the nature of the investment strategy. When looking at the drawdowns of 2015, Nykredit Alpha Kobra and Danske Fixed Income strategies are the funds with the smallest drawdowns between 8 % and 12 %. Nykredit Alpha Mira and HP Hedge had drawdowns of 20 % to 25 %, which can be directly linked to the investment strategies of the individual funds.

Figure 13



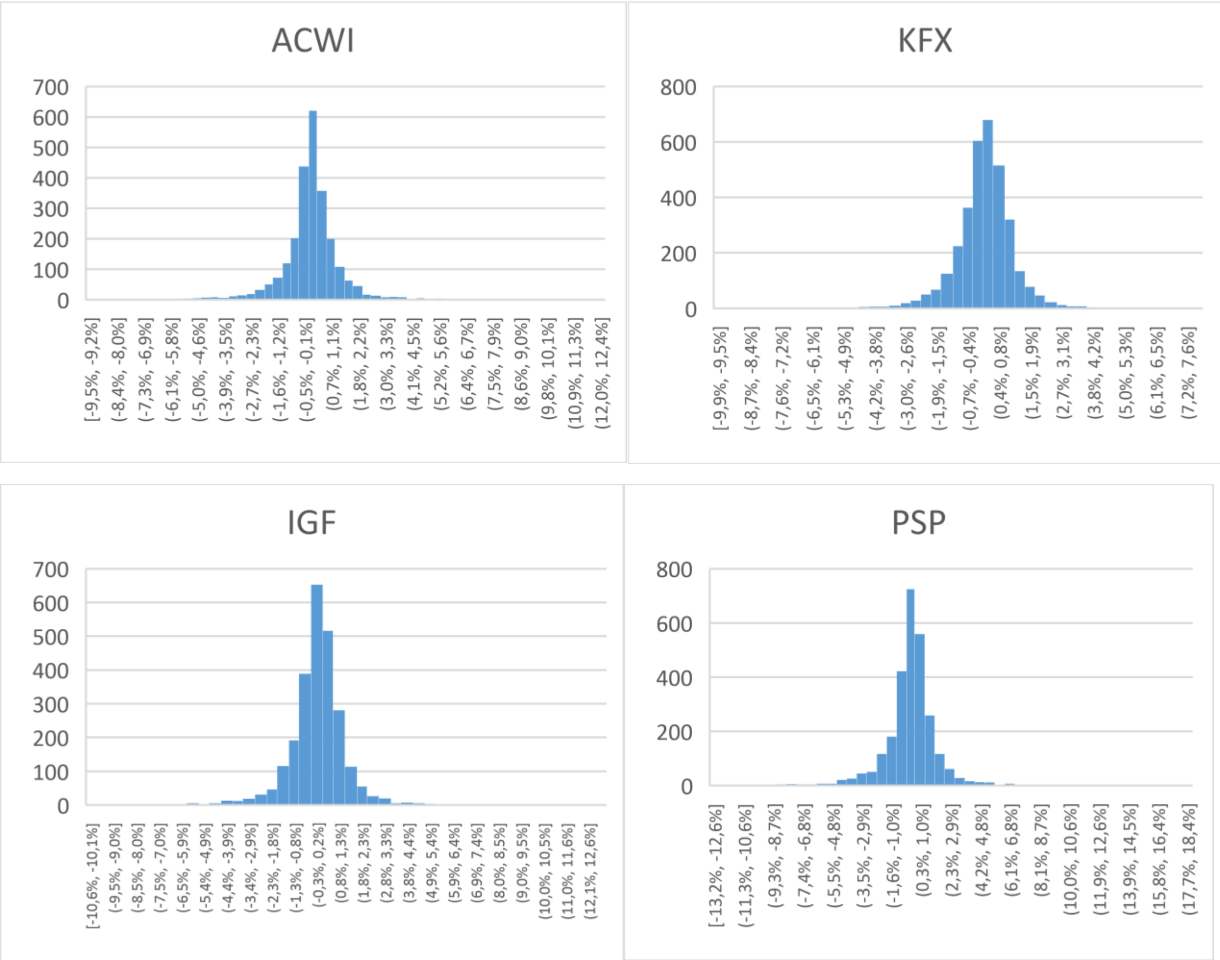
Source: Own production

Tail risk estimation

The tail risk of investments is often measured by the Value-at-Risk (VaR). VaR is the probability of loss in a portfolio or asset at a specific confidence interval. Due to the empirical distribution of the return sample, the traditional VaR is not necessarily representative for historical returns it they are not normally distributed.

To test for normality in the distribution of the historical returns, the Jarque-Bera-test is conducted on the historical data for each asset class. The results of this test highly indicates that the historical returns are far from normally distributed, since they are not centered around 0 and generally have “fatter tails”, meaning that there are more outliers than what would be expected of a normal distribution. As it can be seen in figure 14, the different distributions are not especially normally distributed, which implies that the modified VaR would be more efficient to display the true downside risk of the assets.

Figure 14


ACWI
700
600
500
400
300
200
100
0
[-9,5%, -9,2%]
(-8,4%, -8,0%]
(-7,3%, -6,9%]
(-6,1%, -5,8%]
(-5,0%, -4,6%]
(-3,9%, -3,5%]
(-2,7%, -2,3%]
(-1,6%, -1,2%]
(-0,5%, -0,1%]
(0,7%, 1,1%]
(1,8%, 2,2%]
(3,0%, 3,3%]
(4,1%, 4,5%]
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(7,5%, 7,9%]
(8,6%, 9,0%]
(9,8%, 10,1%]
(10,9%, 11,3%]
(12,0%, 12,4%]
KFX
800
600
400
200
0
[-9,9%, -9,5%]
(-8,7%, -8,4%]
(-7,6%, -7,2%]
(-6,5%, -6,1%]
(-5,3%, -4,9%]
(-4,2%, -3,8%]
(-3,0%, -2,6%]
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(7,2%, 7,6%]
IGF
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PSP
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(-11,3%, -10,6%]
(-9,3%, -8,7%]
(-7,4%, -6,8%]
(-5,5%, -4,8%]
(-3,5%, -2,9%]
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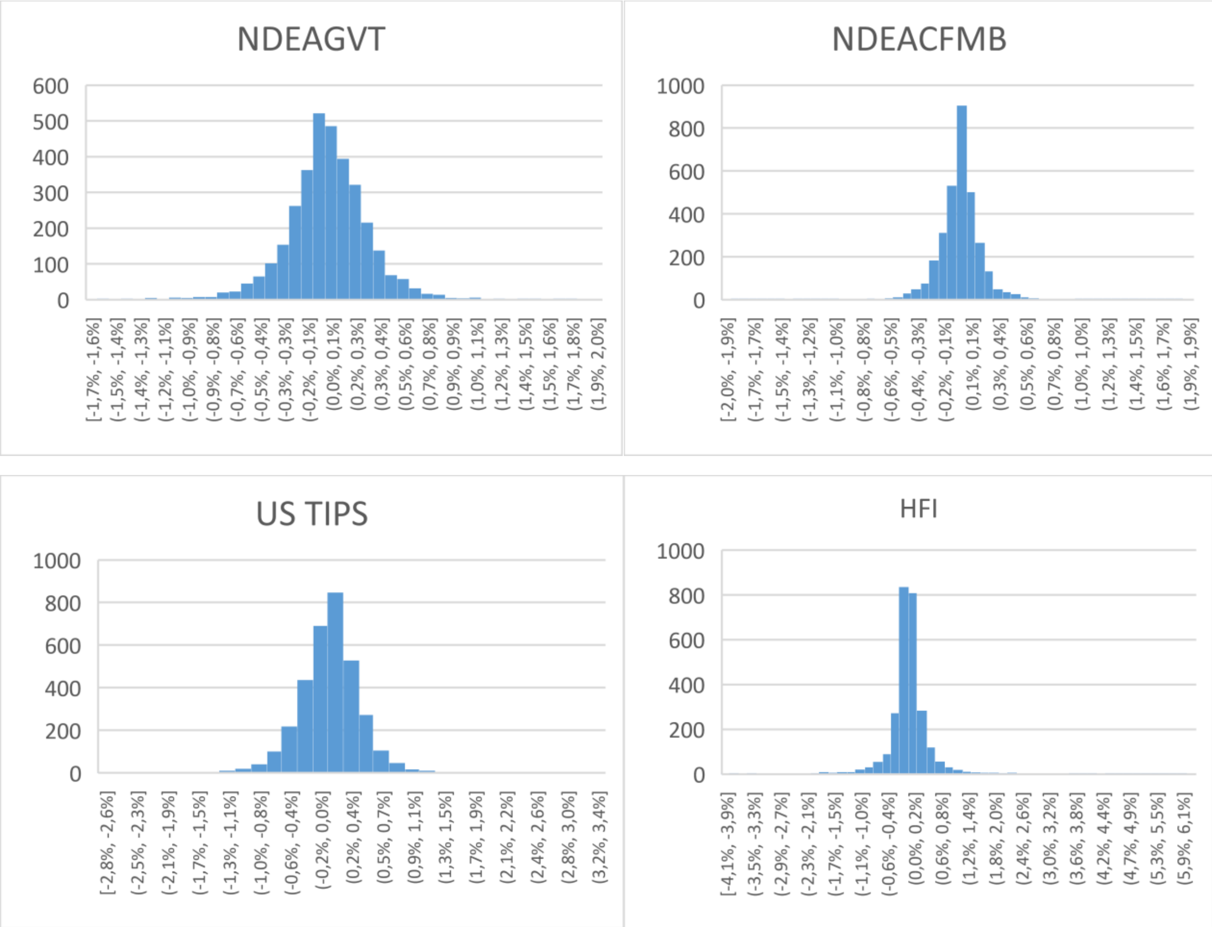
(15,8%, 16,4%]
(17,7%, 18,4%]

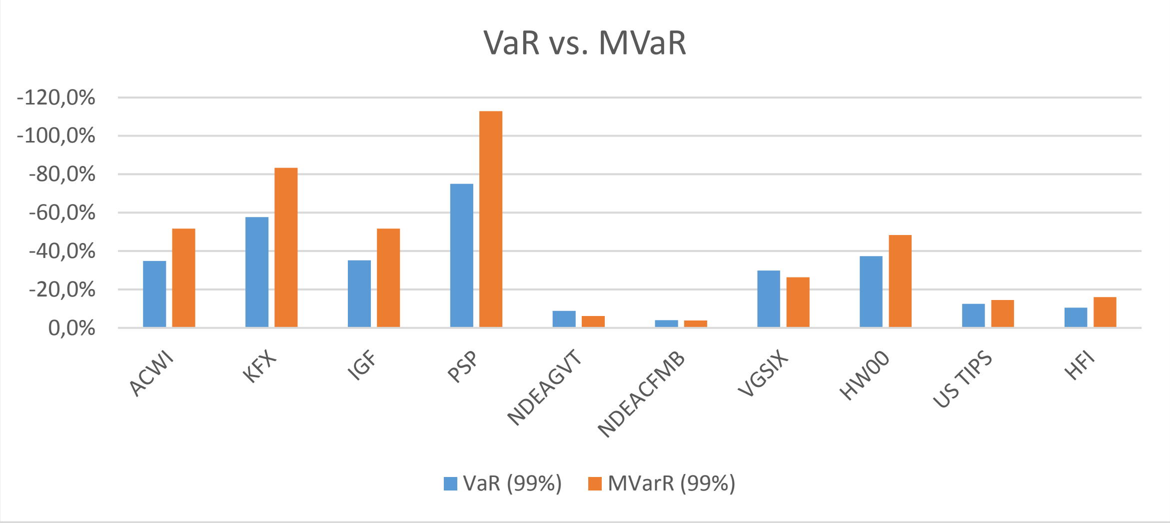


Source: Own production

The VaR and Modified VaR (MVaR) of the individual asset classes are presented in figure 15. It is clear from figure 15 that the MVaR is generally higher than the traditional VaR. This is due to the fact that underlying return distribution, that earlier failed the test for normality, is non-normally distributed and therefore breaches with the underlying assumptions for the VaR. When estimating the VaR, it is assumed that the underlying return distribution is normally distributed. the VaR therefore, does not take into account the skewness and kurtosis of the distribution. Thereby the required assumptions are not fulfilled, and the VaR calculations therefore underestimate the true downside risk.

This is because with the Modified VaR, the downside risk is based on the actual sample distribution, and not an assumed normal-distribution, and therefore uses the sample skewness and kurtosis in the calculation of the Modified VaR.

Figure 15



Source: Own production

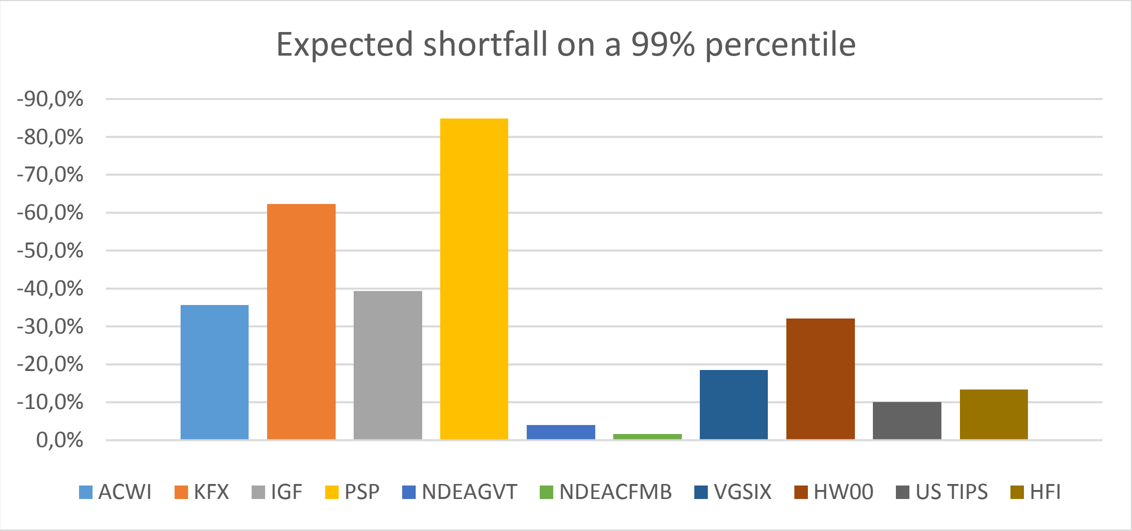
KFX and PSP, which had the highest VaR, also experiences the largest increase from VaR to Modified VaR. This can be traced back to the excess kurtosis and negative skewness of the returns. The MVaR of the HFI is significantly lower than the equity indices, and most of the other traditional assets. Where equities are estimated to have a maximum loss of 50 % to 80 % on a 99 % confidence level, where the HFI is estimated only to have a maximum loss of 16 %. This is a significant difference which definitely should be taken into consideration when putting together a long-term portfolio.

To summarize the analysis above, the investors of fixed-income hedge funds (HFI) should not expect a loss greater than 16 % of the value 99 % of the time. This should be compared to an investment in global equities, which can expect losses of 50 %, in a 1 % tail-event.

An alternative to VaR and MVaR is the Expected Shortfall (ES). ES differs from the VaR in the sense that the ES estimates the size of the potential loss in case of a tail-event happens. ES measures the expected loss over a specified period of time, conditionally on the occurrence of an X % left tail-event happens. This means that given a specified distribution of events, the average return lower than the X %-limit, set by the chosen confidence level. This is what should be expected to lose in an X % tail event.
In order to compare the different tail-risk measures, this thesis will use a 99 % confidence-level and thereby estimates the loss in case of a 1 % tail-event.

In figure 16, the ES of the individual asset classes is displayed. It is clear that ES and VaR to a large degree can be compared. The ES compliments the results of the VaR and MVaR, which confirms and validate the output of the tail-risk estimations.

Figure 16



Source: Own production

In all the above analyzed figures, the fixed-income hedge funds have performed significantly better than almost all of the traditional instruments.

When looking at the histogram of the return distribution of the HFI (figure 14), it is clear that the returns are centered and leptokurtic, meaning that the returns of the HFI is centered around the mean and has less extreme losses or gains, in other words, the returns are more stable.

The tail-risk estimations exemplified by the VaR, MVaR and ES all stresses that the HFI is not as risky an asset as it would be expected for hedge funds in general. These expectations are primarily based on historical events such as Long-term Capital Management and Jyske Invest Markedsneutral – Obligationer, which both were subject to great attention by investors and media. The failure of these funds should obviously be taken into consideration when investing in hedge funds. But this analysis also shows that hedge funds, which have proven their worth over a longer period of time, is more than worthy competitors to traditional assets in a long-term portfolio, since the drawdowns in tail-event are considerably lower. This is derived from the results of all above key-ratios and risk measures.

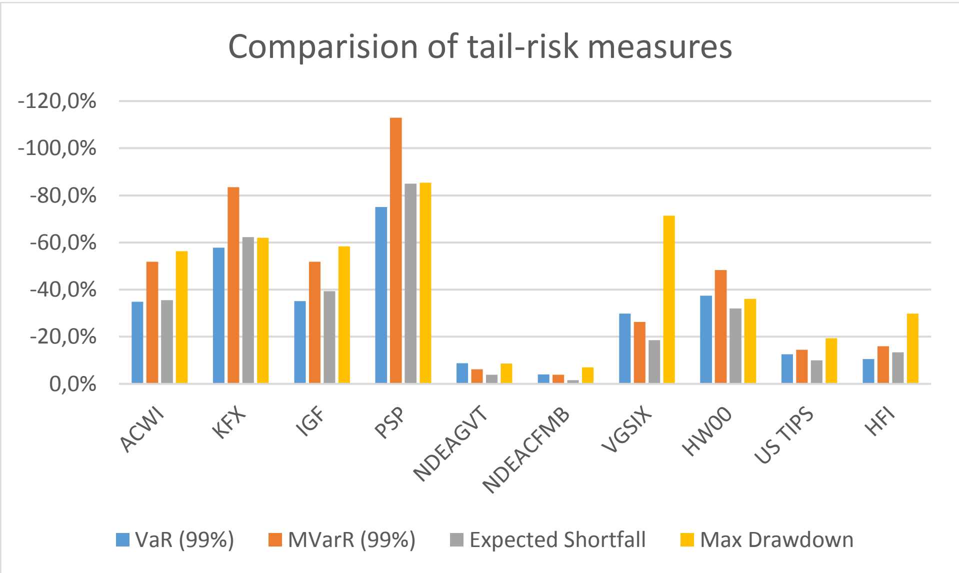
Conclusion

The VaR, MVaR, and ES of the HFI is among the four lowest in the basket of assets collected for this thesis, where the tail-risk of the HFI is among the other fixed-income categories. The HFI has a VaR of 10 % and an MVaR of 16 %; this is only beaten by government bonds, callable mortgage bonds, and inflation-linked bonds, which are all considered very safe assets and highly used in pension portfolios for stable future returns.

In figure 17 all measures have been gathered to compare them across all asset classes, the individual measures should not be directly compared, due to the different properties of the individual measure, but the diagram gives a good comparison across the measures.

Figure 17 shows that especially the modified VaR stands out from the other measures. For the PSP the modified VaR is extremely high at 112 %, which of course is not possible, but this should be interpreted as an asset with high risk, especially in tail-event situations. For the KFX the modified VaR is likewise extraordinarily high when comparing it to the ES and the actual maximum drawdown of the financial crisis, which actually matches the VaR relatively well, even though the return distribution failed the JB-test for normality, and therefore should be better matched by the modified VaR.

Figure 17



Source: Own production

When comparing all these measures for all of the asset classes, a clear conclusion is that the HFI definitely qualifies as a constituent in a pension portfolio. The question is how large a weight it should have in a pension savings portfolio.

Analysis 2

The traditional model-portfolio constructed on behalf of the gathered information about Danish pension funds and their asset allocation has been set up by nine proxies representing all relevant asset classes. The allocation can be seen in the table 5, with the weights of the individual asset classes. This is the model-portfolio constructed earlier in this thesis, for the purpose of representing a pension portfolio with at least 20 years until usage.

Table 5

Proxy	Weight
ACWI	38 %
KFX	5 %
IGF	11 %
PSP	5 %
NDEAGVT	8 %
NDEACFMB	5 %
VGSIX	7,5 %
HW00	15 %
US TIPS	5 %

Source: Own production

In the following analysis, there will be constructed a range of different portfolios based on different objectives, conditions, and limitations. The goal of this analysis will be to showcase a range of possible and optimal portfolios under different scenarios.

The first part will consist of constructing the efficient frontier, by setting up different portfolio combinations of the underlying asset classes. The second part of the analysis will be to analyze the position of the model-portfolio in relation to the overall efficient frontier. Hereby determining the expected return and standard deviation of this portfolio, with the intention of using this as a measure and goal, when implementing the HFI into the portfolio at a later stage of the analysis. The implementation of the HFI into the portfolio will be done in two different ways, viewed from a private pension saver, with separate pension saving and personal savings able to freely invest, and the other from the perspective of a pension fund implementing the HFI into their portfolio, i.e., the model portfolio.

The efficient frontier

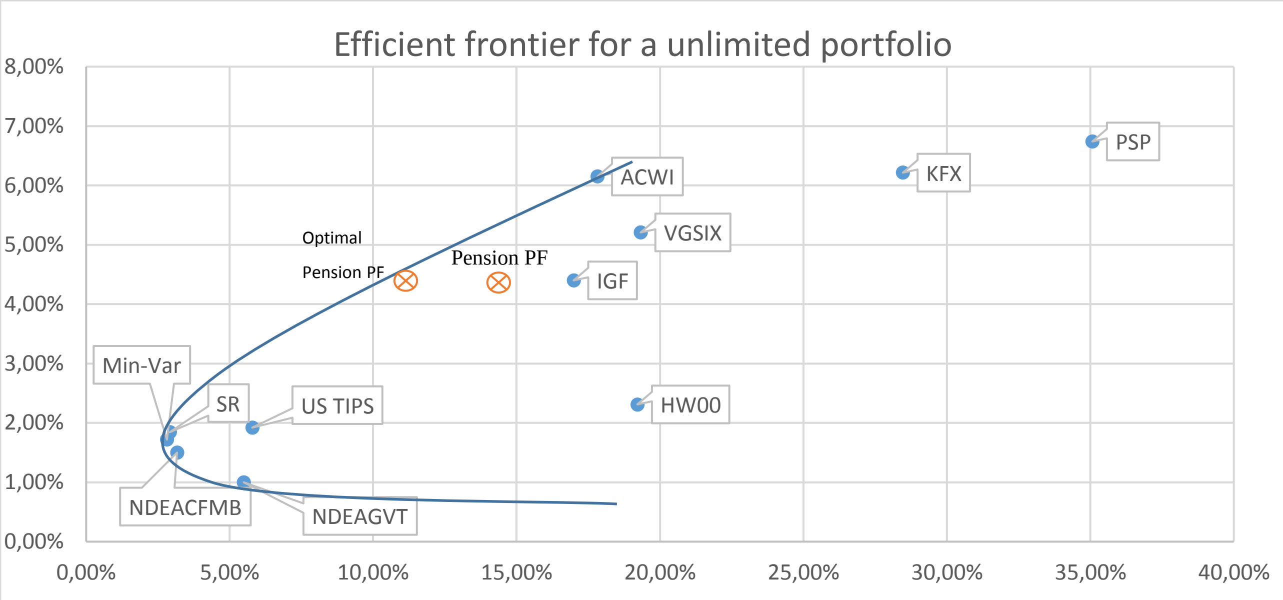
The efficient frontier is defined as the border of optimal portfolios consisting of various combinations of the underlying assets, which combined gives the optimal risk-return relationship. The individual assets all lie within the border, as the theory developed by Markowitz states, a combination of different assets will give a lower risk, than the individual assets themselves (The Journal of Finance, Vol. 7, No. 1 (Mar., 1952): 77-91). This is due to the correlations among the individual assets and by constructing a portfolio of assets with correlations less than 1, i.e., not perfectly correlated assets, the overall portfolio will achieve a portfolio risk lower than the assets with the lowest risk.
When constructing portfolios based on Markowitz Modern Portfolio Theory, the goal is to combine assets, which are uncorrelated or negatively correlated, if possible. This way the portfolio will be more stable and thereby have a lower variance and standard deviation.

As it can be seen in figure 18, the efficient frontier is formed by the individual assets of the model portfolio. Assets which does not lie on the efficient frontier is considered inefficient portfolios when looked at as individual investments. This is the argument for forming a diversified portfolio of the available assets, where the variance is minimized, or the SR is maximized. These two portfolios are the two most crucial portfolios and are known as the Minimum-Variance Portfolio and the Optimal Portfolio.

The Minimum-Variance portfolio is found by using the Solver-function in Excel, with the objective of minimizing the variance, by changing the individual weights of the portfolio assets. This portfolio will be the point on the efficient frontier furthest to the left, where the portfolio has the lowest possible variance for that rate of expected return.

The Optimal Portfolio is found in a similar way, by using the Solver-function in Excel. The optimal portfolio is defined as the portfolio of assets, where the portfolio return is maximized simultaneously with the risk being minimized. This is measured by the SR, which is maximized using the Solver-function and plotted somewhere above the Minimum-Variance portfolio. This portfolio is the optimal point for maximized return and minimized risk.

Figure 18



Source: Own production

The portfolios in figure 18 has been made with the asset classes of the model-portfolio, without any form of limitations. This gives the true optimal portfolio in the sense of the Modern Portfolio Theory, where the portfolio is created based on returns and covariances, to optimize the portfolio.

The Minimum-Variance portfolio has a standard deviation of only 3,16 % and an expected return of 1,74 %. This portfolio only consists of three assets(weight); KFX (4,43 %), NDEACFMB (89,06 %), and US TIPS (6,51 %). This portfolio is based on the very low standard deviation of the mortgage bond index of only 3,37 %, and the standard deviation of US TIPS of only 5,59 %. The KFX has a standard deviation of 27 %, but has a negative correlation to both US TIPS and NDEACFMB, which lowers the portfolio variance by allocating a small part of the portfolio to the KFX.

The Optimal Portfolio, where the Sharpe ratio is maximized, has a standard deviation of 3,66 % and an expected return of 2,18 %, this gives the portfolio an SR of 0,6, which is the highest of all portfolios possible to create of the given assets. This portfolio has a significantly lower Sharpe ratio compared to the model portfolio, which only has an SR of 0,31, with a standard deviation of 14,49 % and an expected return of 4,44 %.

It would be expected that the pension portfolio was optimal, but there are many variables that can make this portfolio inefficient. The data behind these calculations are a very narrow sample of the true returns of the financial markets.

In order to create an optimal portfolio based on the model-portfolio, there have been introduced upper and lower boundaries on the individual asset weights, in order to create a buffer for the optimization model to optimize the pension portfolio. The boundaries have been set in a subjective and qualitative manner to ensure that the optimization includes all asset classes to some extent, and not only includes the three asset classes of the earlier unbounded optimization.

In the Table 6 the weights are shown, and these are applied to the Excel-optimizer.

Table 6

Proxy	Min w	Max w	Model w	Result w
ACWI	10 %	37,5 %	38,0 %	37,0 %
KFX	2 %	12,5 %	5,0 %	2,0 %
IGF	5 %	15,0 %	11,0 %	15,0 %
PSP	1 %	10,0 %	5,0 %	1,0 %
NDEAGVT	2 %	10,0 %	8,0 %	10,0 %
NDEACFMB	2 %	10,0 %	5,0 %	10,0 %
VGSIX	5 %	10,0 %	7,5 %	10,0 %
HW00	5 %	15,0 %	15,0 %	5,0 %
US TIPS	2 %	10,0 %	10,0 %	10,0 %

Source: Own production

In this setting, the Minimum-Variance portfolio reduces the allocation of a number of asset classes such as; private equity Danish equity, and High yield bonds, where the allocation in infrastructure, government bonds, mortgage bonds, inflation-linked bonds, and real-estate, has been maximized. The allocation in global equity is almost at its maximum, which is due to its negative correlation with the bonds. This gives an improvement in the portfolio variance, and at the same time, ACWI, has a good Sharpe ratio compared to many of the other assets, at 0,35.

The Optimal Portfolio in this setting is very similar to the Minimum-Variance portfolio since the boundaries are exploited either at the top or the bottom. The only difference is that the global equities are now fully exploiting the boundary of 37,5 % and infrastructure (IGF) is decreased to

14,5 %. The Sharpe ratio of the Optimal Portfolio increases slightly from 0,343 to 0,344, with a standard deviation of 12,24 % and an expected return of 4,2 %.

When this is compared to the fixed model-portfolio constructed on data from the Danish pension funds, the SR increases from 0,31 to 0,34. This is due to a 2 % decrease in the standard deviation and only a 0,2 % decrease in the expected return. This gives the overall increase in the Sharpe ratio of the pension portfolio.
This "moves" the pension portfolio closer to the efficient frontier and thereby are more efficient than the fixed model-portfolio.

The next step of the analysis will try to optimize the allocation of the HFI as a parallel investment to the fixed pension plan, which is assumed to be invested in the model portfolio.

Two-component portfolio

The intention of this analysis is to determine the optimal allocation, which should be invested in the HFI personally in excess of pension savings.

Under the assumption that Danish pension funds will optimize their performance in order to achieve the maximum risk-adjusted returns, the model-portfolio in the following analysis, will be the optimal pension portfolio found in the earlier optimization. The individual weights of the assets can be seen in table 7 below.

Table 7

Proxy	Optimal Pension portfolio weights
ACWI	37,5 %
KFX	2,0 %
IGF	14,5 %
PSP	1,0 %
NDEAGVT	10,0 %
NDEACFMB	10,0 %
VGSIX	10,0 %
HW00	5,0 %
US TIPS	10,0 %

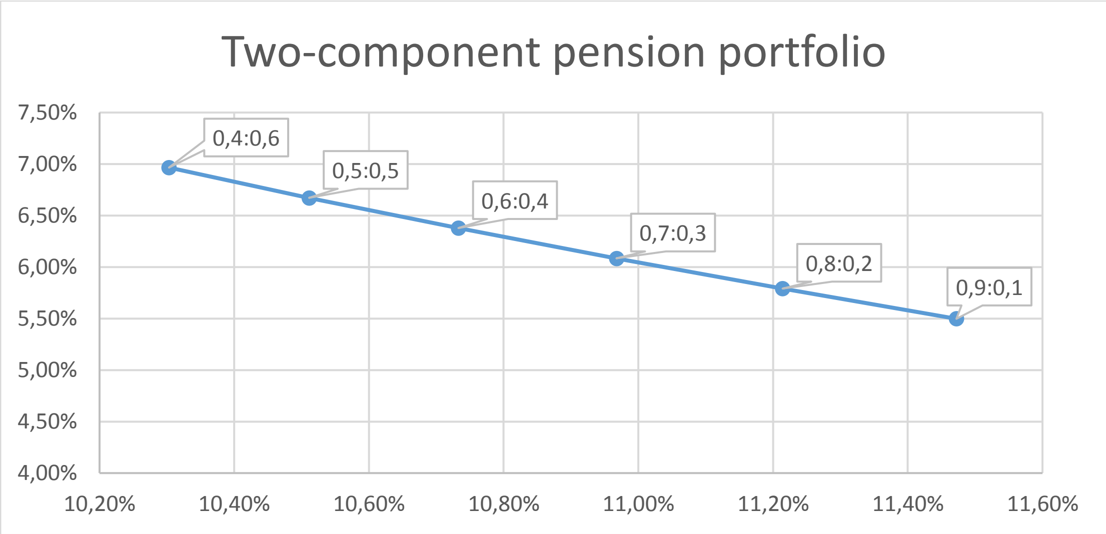
Source: Own production

The purpose of this thesis is to analyze and discuss how and if hedge funds should be implemented into the average pension saving portfolio. Where the individual pension saver is not able to directly affect the asset allocation of the pension fund, the pension saver is able to invest its own savings, independently of the pension funds.

If this is done, it should be done by taking the allocation of the pension funds into account, since the pension savings often is the largest part of an individual's personal long-term savings. This is why the following analysis will have its starting point in the asset allocation of the optimal model-portfolio, from earlier, and construct a portfolio of two assets; pension portfolio, and the HFI. Behind this portfolio construction is one simple assumption, which is that the individual in question, has available savings, free to invest and thereby to construct the optimal portfolio of pension savings and hedge funds that meets the level of risk desired.

This two-component portfolio creates a linear risk-return graph in a classical risk-reward-diagram, with the possible combinations of pension savings and hedge fund investments. In figure 19, the linear relationship is displayed and it is clear that with the less risky properties of the HFI compared to the optimal pension portfolio, the higher allocation of the HFI lowers the risk and raises the expected return. There is a negative relationship between the weight of the HFI, and the overall risk of the portfolio. Therefore the optimal portfolio is the 100 % HFI-portfolio, but this is not a possibility, since this requires no pension saving and a 100 % allocation of capital in hedge funds. This is not a suitable solution, since this will be an entirely undiversified portfolio with far too great risk in case of tail-risk events, where the standardized risk-reward analysis is utterly insufficient.

Figure 19



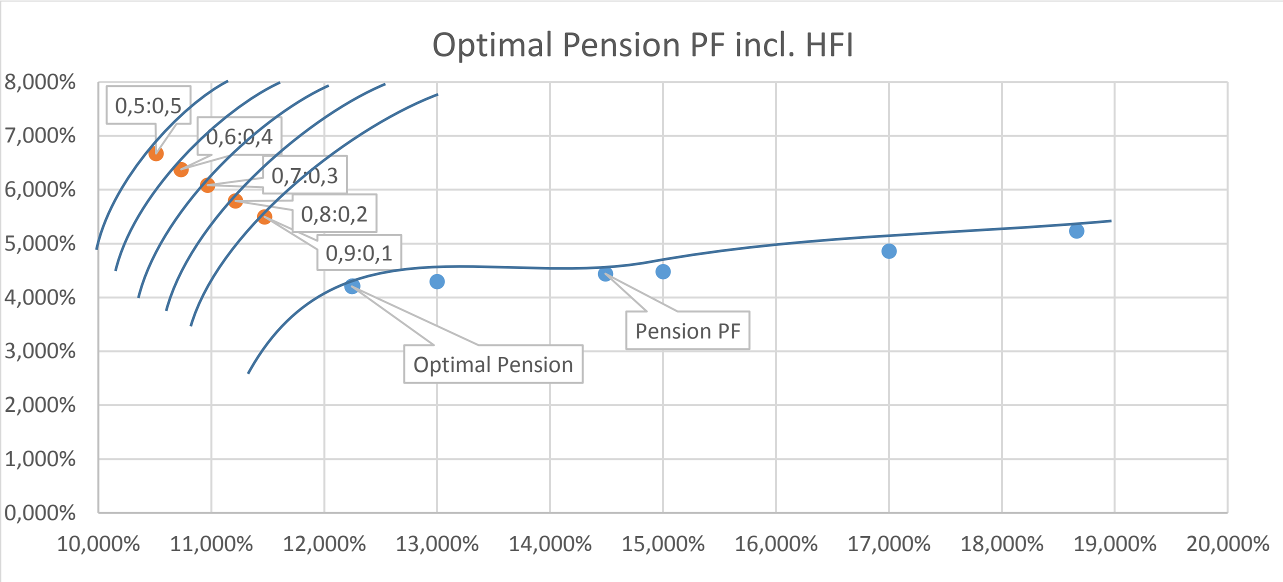
Source: Own production

Figure 19 emphasizes the apparent conclusion of the analysis regarding if hedge funds should be implemented into the overall investments of long-term savers. Hedge funds contribute with relatively stable risk-adjusted returns compared to traditional pension fund assets, which makes them highly suitable as complementary investments to the standard pension fund portfolio.

The two-component portfolio of pension savings and hedge funds should be kept in a level, where the overall diversity of the portfolio is optimized and protects the portfolio against unsystematic risk. Therefore, the weight of the HFI should be kept below an allocation of 30 % to 40 % of the total investment portfolio, as it is set for equity and fixed-income. Hedge funds should be viewed as an alternative to the traditional asset classes of a pension portfolio, which is why the limits of investments in it should be kept at more or less the same level.

In figure 20, there has been constructed a range of portfolios consisting of the optimal pension portfolio and the HFI. The lower indifference curve are portfolios only consisting of the nine traditional pension fund assets and different allocations of these. The upper indifference curves are curves consisting of portfolios with different weights in pension savings and the HFI. As earlier, the variance/standard deviation of the HFI is lower and expected return is higher than the pension portfolio, which is why an increase in the weight of the HFI in the portfolio makes the indifference curves move higher and to the left. This is a result of a higher risk-adjusted return of the portfolio of these two components when the weight is laid in the HFI.

Figure 20



Source: Own production

Minimum limit portfolio

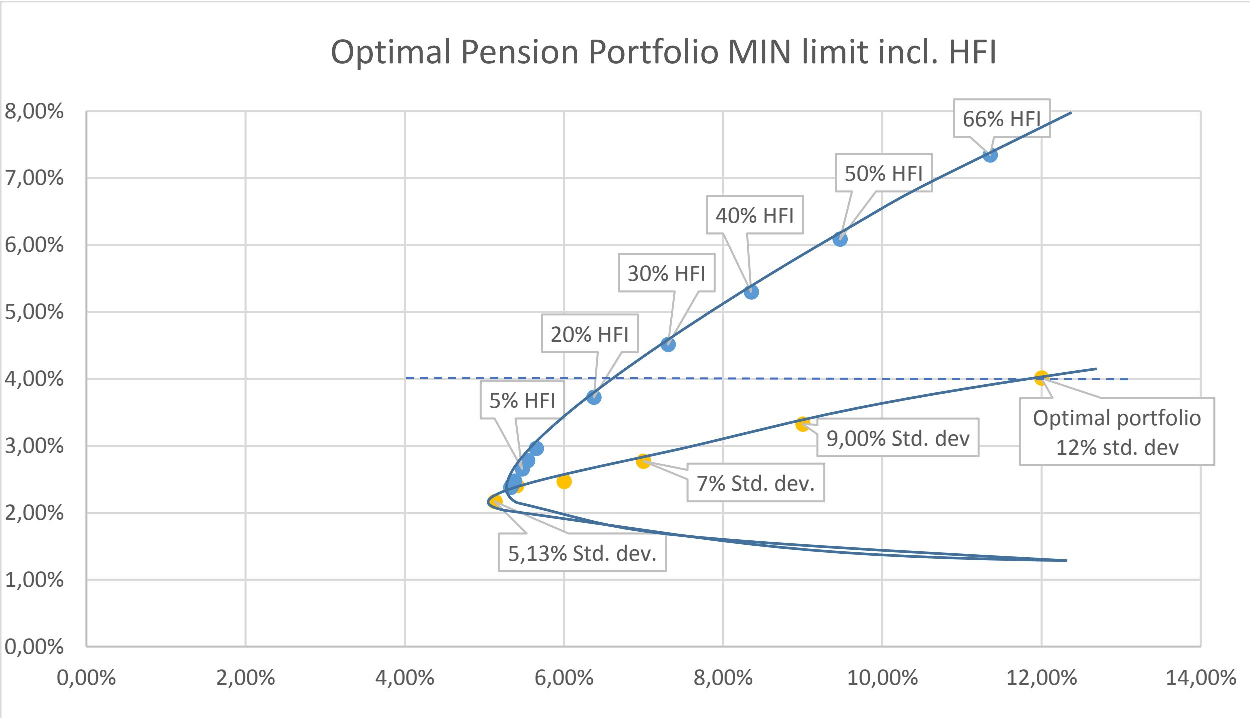
This analysis will now be based on pension funds view on the construction of an optimal portfolio, exposed to both the traditional pension portfolio assets, as well as a fixed-income hedge fund. Firstly there will be constructed a range of portfolios, where the allocation of the traditional assets is able to move freely, only specified with a lower boundary in order to ensure a well-diversified portfolio.

These portfolios will be constructed in an MPT-framework, like the one where the model portfolio was optimized at an earlier stage of this thesis.

The portfolios have been constructed with the use of Excels solver-function, where the objective of the optimization is the Sharpe ratio, with continuously changed maximum weight in the HFI. In figure 21, the yellow points symbolize portfolios of traditional assets, with only a minimum limit on the weights of the individual assets. This way the portfolios are created with the highest SR within the boundaries set for the optimization. The portfolios are forming a frontier running from the minimum variance portfolio with a standard deviation of 5,4 % and a return of 2,4 %, this portfolio is primarily driven by the strong risk-adjusted return of mortgage bonds, which keeps the risk to a minimum, while fulfilling the minimum weights of the residual asset classes of the portfolio. The portfolios range up to the 12 % standard deviation, at a 4 % expected return, which approximates as the optimal pension portfolio constructed earlier.

The blue points in figure 21 represent a range of portfolios consisting of the nine traditional assets and the HFI. These portfolios have been constructed in a way, where the allocation of the HFI continuously increases until a portfolio of maximum HFI is met. Due to the minimum limit of the traditional assets, the maximum limit for the HFI is 66 %, since the minimum limits of the other assets sum to 34 % of the portfolio. As earlier discussed, this scenario where the portfolio has an overweight of the HFI is not a scenario, which should be strived for, since this will compromise the overall diversification of the portfolio.

Figure 21



Source: Own production

As figure 21 shows, the minimum variance portfolio under the minimum limit condition is a traditional portfolio consisting of the nine traditional assets. As soon as the weight of the HFI extents above 1 % (the first blue point), the portfolios and thereby the efficient frontier extends beyond the efficient frontier of the traditional portfolio. This indicates higher risk-adjusted returns when the HFI is introduced to the portfolio of traditional assets. The reason for the increasing risk-adjusted returns is the SR of the HFI, which exceeds the one of the traditional assets, as well as the covariance of the HFI with the other assets. This does that the introduction of the HFI in the portfolio ensures the portfolio against marked beta and systemic risk. Therefore a portfolio consisting of both traditional assets and the HFI will be a portfolio better protected against market movements. As a long-term portfolio, this portfolio will be able to create significantly better returns in the long run. Due to compounding interests, small increases in the return is crucial for long-term portfolios where all returns are reinvested, and no money is withdrawn.

When we compare the optimal pension fund portfolio, with the portfolios including the HFI, the conclusion is clear. The optimal pension portfolio is characterized by having a standard deviation of 12 % and a return of 4 %. When comparing this to the characteristics of the individual HFI-portfolios, it is either possible to construct a portfolio, where the risk is matched by the traditional portfolio, or the expected return is equal to the one of the traditional portfolio. When looking at the risk of 12 %, this is not possible under the conditions set up in this framework. This would require to gear the portfolio, to have the risk exceed 11,35 %, which is the portfolio standard deviation, in the portfolio consisting of 66 % HFI.

When looking at the expected return of the pension portfolio of 4 %, this is achieved by constructing a portfolio of maximum 23 % of the HFI, which gives a portfolio with 6,64 % standard deviation and 4 % expected return. This means that by introducing 23 % of the HFI into the portfolio, the overall portfolio risk decreases from 12 % to 6,6 %, which is almost the half of the portfolio risk. When looking at the VaR of these portfolios with the approximation of 2,33 times the standard deviation, to calculate the VaR at a 99 % confidence interval, this will mean that the VaR goes from 27,96 % to only 15,47 %.

Upper and lower bound portfolio

An alternative perspective to view the construction of a portfolio, when combining the holding of the nine traditional assets classes and the HFI, is to both have an upper limit and lower limit on the asset weights in the portfolios. In the following scenario, a range of portfolios, the allocation of the traditional pension fund assets is able to vary in a specified range, in order to ensure a well-diversified portfolio.

The portfolios including the HFI are displayed in figure 22 below, as well as the traditional portfolios with both an upper and lower limit for portfolio weights, as it can be seen in Table 8 below.

Table 8

Proxy	Min w	Max w
ACWI	10 %	37,5 %
KFX	2 %	12,5 %
IGF	5 %	15,0 %
PSP	1 %	10,0 %
NDEAGVT	2 %	10,0 %
NDEACFMB	2 %	10,0 %
VGSIX	5 %	10,0 %
HW00	5 %	15,0 %
US TIPS	2 %	10,0 %

Source: Own production

The portfolios are constructed by varying the maximum allowed weight of the HFI in the portfolio, while keeping the maximum and minimum limits of the other assets constant. It is clear when looking at table 9 below that the portfolio weights are almost kept constant except for the ACWI and HFI; these are evidently substitutes when constructing these portfolios. In the portfolio with a constraint of 5 % in the HFI, the portfolio weight of the ACWI is 37,50 %, compared to the portfolio of 25 % HFI, where the ACWI is down to a portfolio weight of 22 %. It is a reasonable conclusion of this analysis, that if introducing the fixed income hedge funds into a pension portfolio, the reaction should be to reduce the portfolio weight in risky assets, such as equity.

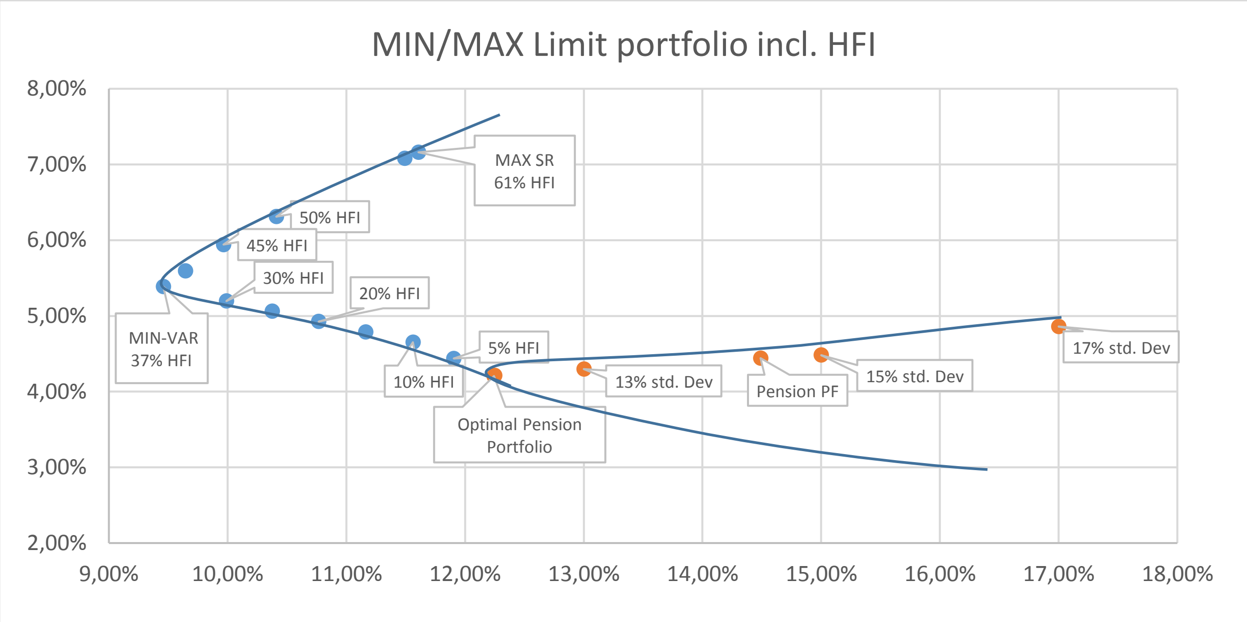
Table 9

Portfolio	Std. Dev	Expected return	ACWI	KFX	IGF	PSP	NDEA GVT	NDEA CFMB	VGSIX	HW00	US TIPS	HFI
SR	11,61%	7,16%	10,00%	2,00%	5,00%	1,00%	2,00%	2,00%	10,00%	5,00%	2,00%	61,00%
MIN VAR	9,46%	5,39%	10,00%	2,00%	5,00%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	37,00%
5% HFI	11,90%	4,44%	37,50%	2,00%	9,50%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	5,00%
10% HFI	11,56%	4,65%	37,00%	2,00%	5,00%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	10,00%
15% HFI	11,16%	4,79%	32,00%	2,00%	5,00%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	15,00%
20% HFI	10,77%	4,92%	27,00%	2,00%	5,00%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	20,00%
25% HFI	10,38%	5,06%	22,00%	2,00%	5,00%	1,00%	10,00%	10,00%	10,00%	5,00%	10,00%	25,00%

Source: Own production

The result of Table 9 above is visually presented in figure 22, where the portfolios including the HFI lies in the extension of the optimal portfolio, starting with a standard deviation of 12,2 % and an expected return of 4,2 %. When increasing the weight of the HFI in the portfolio, the portfolio risk is falling, and the expected return is raising. Looking at the efficient frontier of the portfolios including the HFI, it actually shows that portfolios with less than 37 % HFI are inefficient in the theory of MPT, since it is possible to achieve a higher return at the same level of risk. An example is the portfolio of 30 % HFI, which has a standard deviation of 10 % and an expected return of 5,2 %. At this level of risk, it is possible to have a higher expected return of 5,95 % if 45 % of the portfolio is allocated into the HFI.

Figure 22



Source: Own production

As described above, the portfolios including the HFI improves the overall expected return of any combinations and has its maximum SR at an allocation of 61 % HFI in the portfolio. This is at a standard deviation of 11,61 % and an expected return of 7,16 %. This scenario is optimal in a theoretical way, where the diversification of the portfolio is not considered. Therefore, this optimal allocation is not necessarily optimal in a practical world of investing.

Conclusion

To summarize this analysis, it has been built on a large number of portfolios constructed under different conditions and limits, which not all are comparable and equally representative for real-life pension savings.

To start with, the analysis was built on the model-portfolio, which showed to be inefficient, based on the data collected for this thesis. In order to fit this analysis to the framework of the theory, this portfolio had to be optimized. The result of this optimization was the optimized model-portfolio, which was used in the later analyses as the benchmark for a pension portfolio.
The analysis under the minimum limit condition showed that the portfolio including 30 % of the HFI had a standard deviation of 7,3 % and an expected return of 4,5 %. This could be compared to the portfolio, constructed of the nine traditional assets, with a standard deviation of 7 % and an expected return of 3 %. This is an increase in the expected return of 1,5 %, by allocating 30 % of the capital of the traditional portfolio, to the new portfolio including the HFI. This is a significant difference when discussing yearly returns, and especially for long-term pension portfolios where the compounding of returns can make a big difference when viewed in the long run.

The portfolios constructed under both minimum and maximum limits shows to be harder to optimize and to improve the overall risk-adjusted return. The greater restrictions on the portfolio weights limit the possibilities of the model to maximize the Sharpe ratio of the portfolio, since it is limited from downscaling assets, which have a less optimal risk-adjusted return, and maximizing the ones with the higher risk-adjusted return. This does that the room for optimization of the portfolio is limited and the overall portfolio risk, both with and without the HFI, is higher for the minimum and maximum limited portfolio, compared to the minimum limited portfolio.

The analysis under both minimum and maximum limits showed improvement in both the risk and return of the portfolio if the HFI was introduced. Comparing the traditional model-portfolio with the model-portfolio including an allocation of 30 % in the HFI shows that the standard deviation goes from 12,3 % to 9,99 %, with an equivalent rise in the expected return from 4,3 % to 5,2 %.
This is an improvement of the yearly risk by 2,3 % at the same time as improving the expected return by an annual 0,9%. As it shows, the improvement of risk and return in the portfolio under both maximum and minimum limits are smaller than the less limited portfolio with only lower limit on the asset weights. This should though be compensated in the greater diversification among the assets, and restrict the model from putting too high an allocation on one asset.

The portfolios constructed under the minimum limit has a better Sharpe ratio than the portfolios constructed under both upper and lower limits. This is due to the greater restrictions on the portfolio weights, which limits the possibilities of the model, to maximizing the Sharpe ratio of the portfolio.

This is because it is not possible for the model to downscale the weight on assets, which have a less optimal risk-adjusted return, and increase the weight on the assets with high risk-adjusted returns.

Discussion

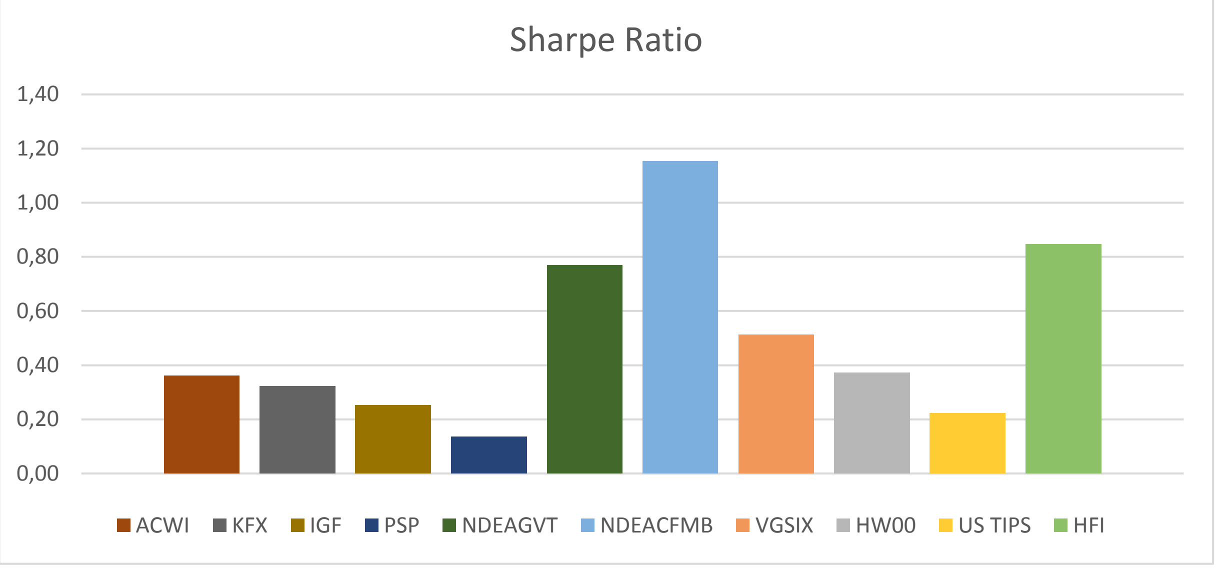
By simply comparing the Sharpe ratio of equity and the hedge fund index shows that the risk-adjusted return of the HFI is more than double of the equity indices. This would in a textbook analysis be a strong argument for choosing the HFI instead of equities in a portfolio. The Sharpe ratio of the HFI is generally higher than most of the traditional assets of pension funds. But an investment decision should never be carried out on the basis of a single measure. It is important to analyze an investment by looking at a wide range of parameters that have importance for the specific portfolio and time-horizon.

The objective of this thesis is the pension portfolio, and the optimization of this by introducing fixed-income hedge funds. Therefore, this thesis has focused on a wide range of parameters, which has importance in for a pension portfolio. This naturally includes the risk, measured by the standard deviation, and the expected return, which have been collected from global asset managers. These have been complemented with tail-risk measures such as VaR, modified VaR, and Expected Shortfall, to estimate and compare the tail-risk of traditional assets to the HFI. The results of these have been compared to the empirical observations in the data, which stretches back to the financial crisis of 2008-2009. This has been used as a confirmation and a sanity check on the estimated tail-risk measures, in order to check the validity of measures, and thereby the conclusions made on behalf of these measures.

Analysis 1 concludes that the HFI is highly appropriate as an asset in a pension savings portfolio. This is based on the outcome of the tail-risk estimations, which places the HFI in the low-risk spectrum. This is correct when looking at the empirical basis of this study, where the maximum drawdown of the HFI, during the financial crisis, was significantly lower than of other risky assets such as equities. This has been put into a context of historical correlations and expected returns, in order to construct the optimal portfolio. All data are consistently pointing at the HFI as an extraordinary investment with the ability to consistently produce positive risk-adjusted returns with limited downside risk in case of tail-risk events.

This has had a large effect on the portfolio optimizations of Analysis 2, which in all scenarios ended up as a corner solution if no conditions were introduced on the allocation in the HFI. This means that all optimizations ended in a solution, where maximum capital was allocated in the HFI. This is of course, not an optimal solution, when viewing it in a practical perspective, where diversification is a vital factor.

Figure 23



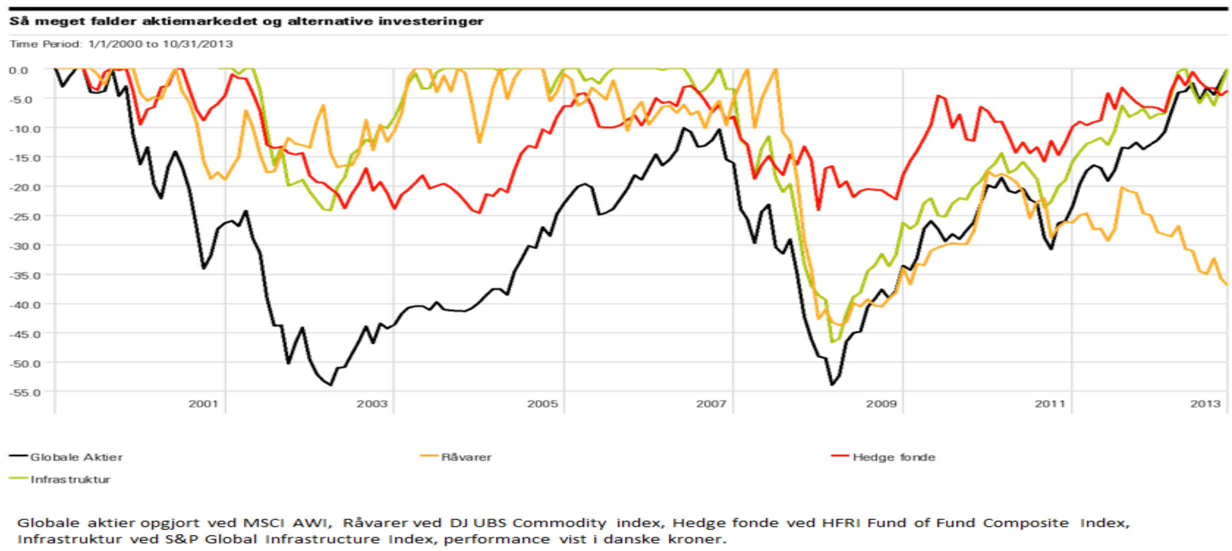
Source: Own production

The reason for these corner solutions are the risk/reward properties of the HFI. The Sharpe ratio of the HFI is 0,85, which is considered high, especially compared to equities, infrastructure investments, and corporate bonds. This Sharpe ratio is based on returns from the last 10 years of the HFI based on the returns of the seven constituents of the index. It can be argued that there has been a selection bias when constructing the index, and the bias can be that only hedge funds with great historical returns have been included in the index. The index have been constructed of all Danish hedge funds, which are publicly available, and of course, within the investment spectrum of relative-value arbitrage in mortgage bonds. This narrows down the number of hedge funds and especially when there is a requirement of sufficient years of historical data.

As discussed earlier, Jyske Invest Markedsneutral – Obligationer, is not included in the HFI. With the loss of 85 % of its NAV in 2008 and 2009, it would mean a significant difference for the HFI. Therefore it is relevant to mention the result of including Jyske Invest Markedsneutral – Obligationer in the index. The Sharpe ratio of the HFI goes from 0,86 to 0,33, which is a massive difference when talking Sharpe ratios. The average yearly return goes from 8,2 % to 5,5 % and the standard deviation from only 9,54 % to 17 %, this makes the hedge funds index much less attractive to investors.

If Jyske Invest Markedsneutral – Obligationer was included in the index, the maximum drawdown is changed from 29,8 % to 43 %, which is a huge difference when comparing the assets. But this should not be used as an argument for undermining the validity of the HFI since the sample of constituents in the HFI is very narrow, and the weight of the individual hedge funds is therefore equally higher, and the failure of one constituent has a substantial effect on the performance of the overall index. Morningstar.dk made a comparison of the development in four benchmarks and indicies; MSCI AWI, DJ UBS Commodity Index, S&P Global Infrastructure Index, and the HFRI Fund of Fund Composite Index. These benchmarks represent respectively equity, commodities, infrastructure, and hedge funds.

Figure 24



Source: Morningstar.dk

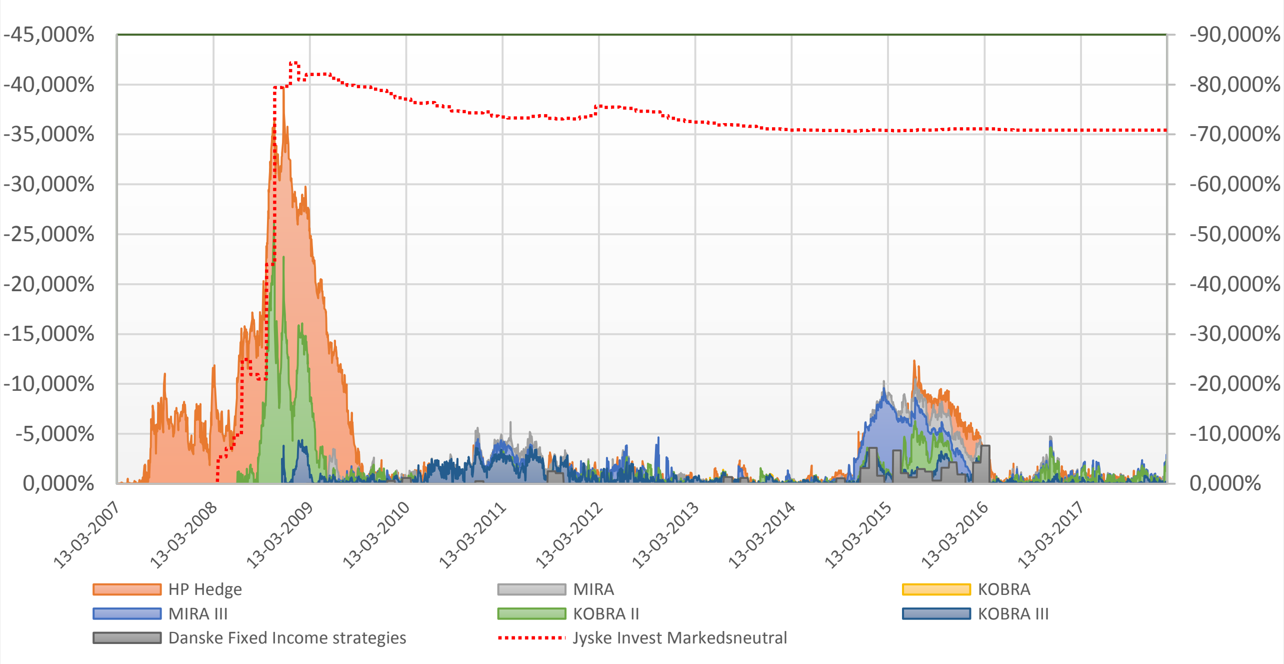
In figure 23 the development of these benchmarks can be seen. Looking at the development in 2008-2009, it is clear that the drawdown of the traditional assets; equity, commodities, and infrastructure, is significantly higher, at around 40 %, than the hedge fund benchmark index, which only suffered a drawdown of around 20 %. This complements both the earlier analyses and the development of the HFI excluding the Jyske Invest Markedsneutral – Obligationer, from the index.

This makes the maximum drawdown of the hedge fund index of Danish fixed-income funds below 30 % during the last 10 years.

It can be discussed whether the survivorship bias, which is introduced by excluding Jyske Invest Markedsneutral – Obligationer, can be a determining factor of the result of this analysis. The effect of excluding the failed hedge fund, would be significant and make the index increasingly more comparable to the traditional assets, where it in its final form is superior in most of the key ratios and characteristics. It is hard to determine whether the failure of Jyske Invest Markedsneutral – Obligationer, was due to inferior skills of the hedge fund manager or if this fall out would have been approximately the same for the other constituents in the index, if they all had been started at the same time as the failed fund from Jyske Invest. There is no way of determining the outcome of the financial crisis for the constituents of the HFI, and therefore it is impossible to determine whether this failure was an isolated event or the result, in general, for the relative value fixed-income hedge funds invested in Danish mortgage bonds.

When looking at the risks of hedge funds, it should be made clear that the risk associated with the individual hedge funds vary considerably, as it is illustrated in figure 25. Here the individual constituents of the index is displayed with the drawdown of the funds on the left-hand axis and the drawdown of Jyske Invest Markedsneutral – Obl. on the right-hand axis. It is clear to see that Jyske Invest is not to compare with the rest of the sample and that the investment strategy clearly was terminated after the loss and the settlement with its investors.

Figure 25



Source: Own production

As illustrated in figure 25, the fund with the largest drawdown during the financial crisis was HP Hedge, which was the first of these sample funds to open, in the first quarter of 2007. HP Hedge is specialized in high coupon illiquid mortgage bonds. As described the highest risk of fixed-income hedge funds are liquidity crises. This was precisely what HP Hedge was hit by in 2008-2009, where they had just build up their portfolio and gearing, when the financial crisis hit Denmark, and the Danish housing market dropped rapidly. With a portfolio of high coupon bonds, the portfolio is very exposed to interest rate changes, and it is very hard to establish a perfect hedge against these changes, since the convexity of the bonds makes the price change of the bonds and the value of the hedge become less perfect correlated. The fact that these bonds are more illiquid than other bonds make the spread on the these widens significantly in times, where markets are consolidating and in situations, where markets know that other professionals are deleveraging their portfolios. Therefore there will always be a need for further compensation in times of consolidation.

This does that HP Hedge is more volatile in its investment strategy than any of the other sample funds, as it is shown in table 10. This makes its SR significantly lower than the rest of the sample, but still positive compared to the traditional assets as tabel 11 shows.

Table 10

	HP Hedge	MIRA	KOBRA	MIRA III	KOBRA II	KOBRA III	Danske Fixed Income strategies
Yearly return	8,58%	10,16%	6,56%	11,11%	8,61%	9,45%	12,41%
Std. dev.	16,69%	7,40%	3,81%	7,77%	6,78%	8,32%	8,47%
SR	0,51	1,37	1,72	1,43	1,27	1,14	1,47

Source: Own production

Due to the nature of the investment strategies of the sample hedge funds, the SR is not necessarily the best measure of risk-adjusted returns. Since the purpose of these hedge funds is to create a stable absolute return, the SR can be based on a false measure of risk, the standard deviation, which measures all deviation from the mean, both positive and negative. Therefore, the Sortino ratio was developed in order to account only for the negative returns, using the lower partial standard deviation. This has been calculated by taking the standard deviation of all strictly negative returns. The Sortino ratio of the HFI is 1,42 compared to a Sharpe ratio of 0,85, which is an significant improvement. For equity indices, infrastructure, and private equity, the Sortino- and Sharpe ratio is almost equal; this can be interpreted as a result of the volatile path of the underlying instruments, where bonds and hedge funds, by nature, has more stable returns. Therefore, the Sortino ratio of the bond benchmarks and HFI is significantly higher and would therefore be considered relatively better risk-adjusted investments.

Table 11

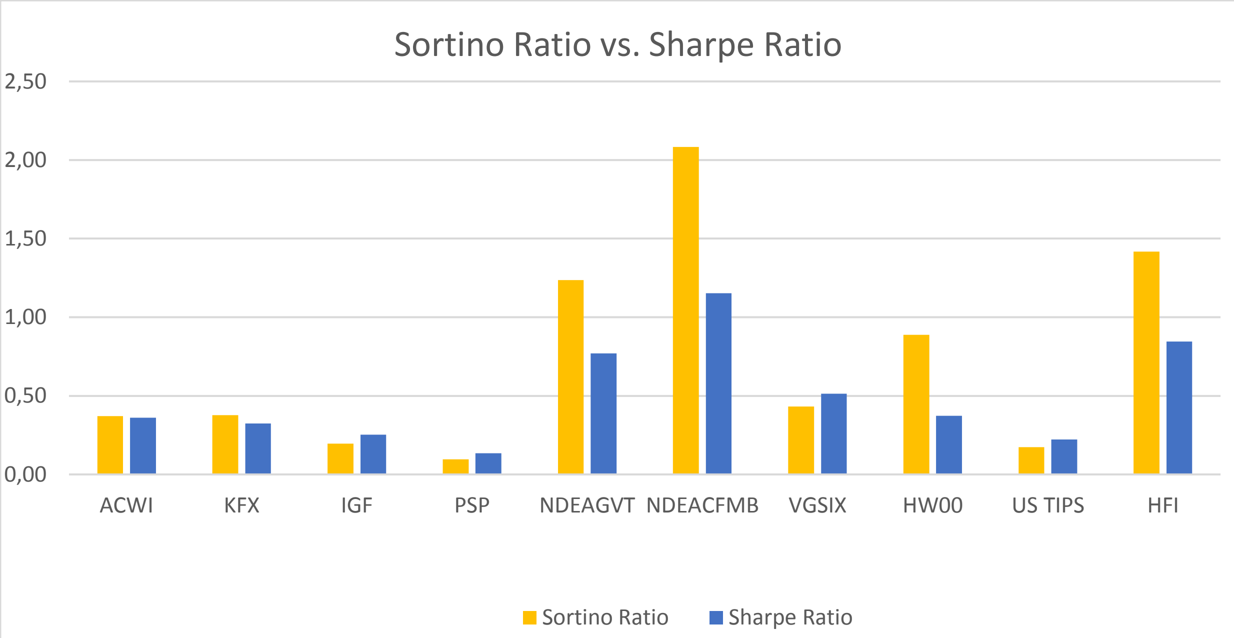
	ACWI	KFX	IGF	PSP	NDEA GVT	NDEA CFMB	VGSIX	HW00	US TIPS	HFI
Return	6,40%	8,82%	4,10%	4,54%	4,23%	3,87%	9,61%	6,82%	1,27%	8,14%
LPStd	17,22%	23,34%	21,02%	47,55%	3,42%	1,86%	22,28%	7,68%	7,28%	5,74%
Sortino ratio	0,37	0,38	0,20	0,10	1,24	2,08	0,43	0,89	0,17	1,42
Sharpe ratio	0,36	0,32	0,25	0,14	0,77	1,15	0,51	0,37	0,22	0,85

Source: Own production

The legality and importance of the Sortino ratio and the argument for using this measure instead of the Sharpe ratio for asset classes, like hedge funds and fixed-income, is evident in figure 28 below.

It is clear that assets with expected low volatility and stable returns are rewarded and highlighted comparing the Sharpe- and Sortino ratio.

Figure 26



Source: Own production

For equity, the difference between the Sharpe- and Sortino ratio is insignificant. This should be directly linked to the volatility of the daily returns, which are quite different to the one of a fixed-income benchmark. It is though interesting to see the great difference in the two ratios, when looking at the High Yield Bond Index, HW00. It would be expected to see the same pattern of quite similar Sharpe- and Sortino ratio, due to the higher risk of the high yield bonds and similar characteristics to equity, compared to higher rated bonds as mortgage- and government bonds.

During the analyses of the constituents of the HFI, and the overall index, it should be clear that the HFI is an investment with a stable return and relatively limited volatility.
In all the analyses, the allocation of capital in the HFI has a positive effect on the overall pension portfolio, no matter how the allocation is done, both in direct inclusion in the pension fund or as a complementary investment to the externally handled pension savings.

In the analysis with a portfolio constructed on the minimum allocation condition, the introduction of the HFI improves the return and the SR gradually with the weight of the HF. Therefore the model suggests that there should be allocated a maximum of capital to the HFI in the construction of the pension portfolio. It is though questionable whether pension funds would agree with this conclusion on the analysis, and reallocate their portfolios to an exposure of 60 % to hedge funds, since this would both compromise the diversification of the portfolios and significantly increase the unsystematic risk.

Conclusion

At the beginning of this thesis, the general use of hedge funds was described as limited and reserved for people with access to extended investment advisory. This is due to the history and general view of hedge funds as highly risky investments.

But as there are different types of bonds such as government bonds, mortgage bonds, credit bonds, and so on, there are also different types of hedge funds. Hedge funds are considered one of the riskiest investments by laymen. This judgment might be true for some hedge funds, but this generalization will not be accepted in this thesis.
Hedge funds come in many shapes and sizes, just like bonds. The investment strategies of hedge funds vary to a great extent, some are speculative by nature, and others are based on financial theory. The objective of this thesis has specifically been Danish fixed-income hedge funds. These funds are mainly focused on the Danish mortgage bond market, which consists of highly rated bonds, with a long history of safety and stability. These hedge funds are exploiting the spread between the approximately risk-free government bonds and the marginally riskier Danish mortgage bonds. This strategy is what the HFI represents and what has been analyzed in this thesis.

The history of the HFI and the traditional pension fund assets was analyzed and compared in order to see if the HFI had the characteristics of a risky asset. This was done by looking at the historical data for all the relevant assets and see how these had performed during the financial crisis of 2008-2009. The data showed that the constructed HFI performed significantly better through the financial crisis than other of the assets considered relatively more safe. The HFI experienced a significantly lower drawdown in 2009 than equities, which, by the general public, are considered far more safe than the concept of hedge funds. The HFI rose steadily at average yearly return of 6 % to 12 %, in the years after the financial crisis, where equities had a more volatile and slower recovery to the levels before the crisis.

In extent to the analysis on the historical data, the tail-risks were estimated using VaR, modified VaR, and Expected Shortfall. These measures were calculated on the historical data of the 10 asset classes, in order to extent the analysis of the HFI in relation to the traditional assets. The VaR of the assets was the first to be calculated, as it is the most classical tail-risk measure. The VaR has also been discussed as it is based on a statistical normal distribution, which it historically have been criticized for since the return distribution of the financial markets rarely are normally distributed. The historical data of the 10 asset classes was tested for normality, but all failed with a wide margin. Therefore, it was rated that the estimated VaR also failed to reflect the true tail risk. The modified VaR opposite to the traditional VaR does not assume normally distributed returns, which is why it should produce a better estimate for the true tail risk. The result of this estimation showed that the VaR underestimated the tail risk, when compared to the modified VaR, which is also the conclusion of historical empirical studies. For the HFI this meant that the tail risk measured by VaR went from 10 % to 16 % in the modified VaR. Similar story is applicable for global equities, which went from 35 % to 51 %, by using the modified VaR. The general analysis of the modified VaR among the different asset classes showed that the HFI was primarily comparable to the relatively safe assets such as government-, mortgage-, and inflation-linked bonds. The risky assets such as equities and high-yield bonds had significantly higher modified VaR than the HFI, which is entirely opposite to the initial view on hedge funds.

In extent to the analysis of the VaR and modified VaR of the asset classes, Expected Shortfall was used to complement the shortcomings of the VaR, which in the case of a tail-event is the magnitude of the loss that can be expected. The Expected Shortfall estimates the magnitude of the tails in the return distribution, opposite to the VaR, which does not say anything about the size of the loss in case of a tail-event. The Expected Shortfall confirmed the results of both the VaR and modified VaR, and thereby it did not change the preliminary results after the estimation of the two first measures, but rather confirmed and verified the results.
This analysis concludes that the HFI is an asset suitable for pension portfolios, considering the limited downside risk.

The expected returns of the traditional assets have been collected from asset managers around the world to get a more qualified answer than the historical average return. For the HFI this was not possible, since the strategy of the HFI constituents does not comply directly with the global hedge fund strategies.

Risk and return of the 10 asset classes were analyzed to get the basis for the following portfolio optimization. The best way to compare the different assets directly is through the use of the Sharpe ratio, since this gives the risk-adjusted return, and therefore is easier to compare across different asset classes. The HFI stands out in the sample with a Sharpe ratio of 0,85, where the comparable assets such as equities, infrastructure, real estate, and so on, lies in the interval of 0,3 to 0,5. This risk/reward profile of the HFI seen in relation with the earlier analysis on the downside risk creates a positive picture of the HFI as an asset class with many good properties seen in relation to a pension portfolio.

Due to the risk/return profile of the HFI compared to the traditional pension fund assets, the simulations of asset allocations in the pension portfolio including the HFI were remarkably uniform. The unconditional portfolio optimizations resulted in a range of corner solutions, where the weight of the HFI was maximized in the portfolios.

When running the model to create the optimal portfolio including the HFI, there was therefore imposed a range of different conditions in order to create different scenarios of asset allocations including the HFI. These portfolios all showed that an allocation in the HFI would lead to an improvement in the portfolio risk and returns. Especially if the allocation in the HFI exceeds 20 % of the portfolio, the returns and risks are significantly improved in comparison to the optimal model-portfolio. Increasing the expected return is always desired, but since this optimization is viewed from the perspective of a pension saver, the objective should be the minimization of the portfolio risk at the same level of return as the optimal model-portfolio.

The optimal portfolio should be a portfolio consisting of a broad range of asset classes, in order to maximize the diversification, and minimize the systematic risk of the portfolio. This means that the portfolio optimizations made under minimum and maximum limits are the most representative for the analysis. These portfolios show that the implementation of the HFI in the portfolio lowers the risk gradually with the increased allocation in the HFI. The increasing allocation in the HFI improves the diversification, and therefore not the falling risk does not compromise the expected return of the portfolios.

The conclusion therefore is that the optimal pension portfolio should have a maximum allocation of the HFI allowed by the diversification demands of the portfolio, since the portfolio risk is decreasing with the increased allocation in the HFI up until the minimum variance portfolio, where the allocation of the HFI is 37 %. In this portfolio, the standard deviation is 9,46 %, and the expected return is 5,5 %, compared to the optimal model-portfolio, which had a standard deviation of 12,3 % and an expected return of 4,3 %.

Further research

As further research it would be relevant to expand the theoretical framework with alternative portfolio optimization theories to Markowitz's Modern Portfolio Theory, and to improve the underlying data collected for this thesis.

Many theories have been developed since the 1950s, where Markowitz published his ideas for the first time, for example, the Risk Parity Portfolio. This is a modern take on portfolio construction with the same objective, an optimal portfolio, but with a different methodology and way to optimize portfolios.

The Risk Parity Portfolio does not build the portfolio on the weight of the individual asset class, but it defines the portfolio as a range of asset classes, where each asset class should provide equally to the overall portfolio risk. It, therefore, equalizes the risk contributed by the individual assets to the portfolio risk, and therefore no assets contribute more than others to the overall portfolio risk. This should be a method that does not diversify the portfolio by weight of its components, but by diversifying the risk of the components and thereby the overall portfolio.

Another extension of the research could be to apply the analyses of this thesis on the actual pension portfolios, with actual portfolio weights from Danish pension funds and historical returns, in order to gain further validity around the results of this thesis. This would remove the uncertainties about the beta-exposure in private equity-, infrastructure-, and real estate-proxies, which could change the basis of this thesis and the portfolio optimization processes.
If possible, this would have given the thesis another dimension of practicality if the historical data were collected from the pension funds themselves. The portfolio optimization could thereby be conducted on the actual data the funds themselves base their asset allocation on.

Further research could make it possible to construct a generic fixed-income hedge fund from mortgage bonds indicies, swap-rates, repo-rates, and other relevant data. This could both open up for an extended historical return-analysis, and improve the analysis of the internal risks in a fixed-income hedge fund. It would in a higher degree be possible to run regressions and factor analyses on data such as these. This could give a more accurate and analytical perspective on the development in the individual components of the hedge funds, such as changes in the short-rates or falling liquidity.

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