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Actively Managed Volatility Strategies

Is it Possible to Disarm Ticking Time Bombs?

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[1]See Cheng (2018) to read his interesting article in the Review of Financial Studies.

Abstract

The popularity of short volatility strategies on the VIX has increased significantly over the past decade. However, the recent increase in volatility of volatility has cannibalized returns associated with these strategies, culminating during the "Volmageddon" of February 5^{th} 2018 when the VIX saw its most significant daily increase ever recorded. The thesis builds upon the methodology of Cheng (2018) by applying ex-ante estimated volatility premiums as a signal in volatility futures strategies in the U.S. and Europe. The findings confirm that trading volatility actively based on premiums embedded in volatility futures significantly improves upon passive volatility strategies and deliver high risk-adjusted returns, both on the U.S. and European markets. Actively trading volatility not only improves upon performance but also reduces strategy drawdowns. Increasing trading frequency improves strategy performance more on the European than on the U.S. market, despite relatively large transaction costs.

Keywords – VIX, VSTOXX, futures, volatility, premiums, trading strategies

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1 Introduction

Up until the last decade, it was only possible to trade volatility by holding portfolios of options, or by entering into variance swaps traded in over the counter (OTC) markets (Alexander et al., 2015). This changed with the introduction of volatility indexes. The VIX index, introduced by Chicago Board Options Exchange (CBOE) in 1993, measures the expected future volatility of S&P500 (SPXT[2]) and is recognized as an indicator of investor sentiment. Following Whaley (2000) it is often referred to as the "investor fear gauge." In the last few years, a wide range of volatility indexes has been constructed using prices of European style options (Alexander et al., 2015). A benefit of volatility indexes is that they can serve as an underlying risk-factor for derivative instruments. Since investors wanting to hedge their portfolios primarily dominate the stock options markets, derivatives on volatility indexes are known to produce returns negatively correlated to the stock market (Bollen and Whaley, 2004). Thus, providing a hedging alternative to derivatives issued directly on the stock market (Dash and Moran, 2007).

A growing body of research shows that investors are ready to pay sizable sums for protecting their stock portfolios (see e.g. Coval and Shumway (2018), Bakshi and Kapdia (2003) and Bollerslev and Todorov (2011)). Bollerslev et al. (2009) and Bekaert and Hoerova (2014) research the behavior of variance risk premiums, calculated as the difference between implied and realized variance. The premiums paid by investors contain a puzzling behavior in periods of market turmoil; sharp increases in realized variance drive premiums downward, sometimes to negative levels, before they rebound (Bekaert and Hoerova, 2014). It is counter-intuitive that premiums fall during periods of market turmoil. However, as Bekaert and Hoerova (2014) points out, the behavior is persistent across multiple of leading forecast models.

A ground rule in derivatives pricing is that the premium paid by investors is equal to the expected risk-neutral return of the derivative. Speculators, often hedge funds, who sell this insurance should therefore expect premiums to increase when risk goes up. However, as pointed out, the opposite seems to be true. Cheng (2018) resorts to a

[2]In this thesis the underlying stock indexes are consistently referred to by their respective total return ticker on Bloomberg.

newly established method of estimating premiums paid by investors. Namely, the "VIX premium". Embedded in VIX futures prices, the VIX premium can be economically interpreted as the expected return of selling a VIX futures contract. Cheng (2018) demonstrates that falling ex-ante estimated premiums reliably predict increases in ex-post market and investment risk.

Investor hedging demand[3] drives volatility futures to trade at a sizable premium relative to the volatility index spot level. As a result, the VIX futures term structure is most often upward sloping (Alexander et al., 2015), implying that short volatility futures strategies are highly profitable on average. However, since volatility tends to spike (Avellanda and Papanicolaou, 2017) and since they are known for destroying several years of profit in a few hours (Brøgger, 2018) these strategies can be referred to as *ticking time bombs*. On February 5^{th} 2018 the VIX index experienced the largest daily increase ever recorded. On this Monday the VIX closed at a value of 37.3, an increase of 116% compared to the previous day's closing price. Exchange traded products (ETPs) that tracked the inverse of VIX futures performance, such as ProShares Short Term VIX Futures (SVXY), suffered massive losses and some even had to liquidate (Brøgger, 2018). These ETPs are, in the context of this thesis, considered equivalent to passive volatility strategies, as they do not actively trade on any signal. Instead, they roll futures contracts to resemble a constant maturity horizon of the traded futures, typically trading the two shortest contracts (Eriksen, 2018).

To consistently earn the premium embedded in volatility futures' term structure an investor needs to *disarm* the ticking time bomb before volatility spikes. February 5^{th} has been referred to as "the Volpocalypse" (DGV Solutions, 2018) or "Volmageddon" (Kawa, 2019), after which anecdotal evidence suggests that the popularity of short volatility strategies have decreased significantly[4]. Kawa (2019) at Bloomberg News described Volmaggedon as a collapse of one of the most pervasive and popular trades in financial market history. If investors are not able to de-risk their positions before volatility spikes, it is likely that Kawa (2019) is correct. However, Bollerslev et al. (2009), Bekaert and Hoerova (2014), and most recently Cheng (2018) have shown that the returns of volatility strategies might

[3] Dash and Moran (2007), Ratner and Chiu (2017) and Warren (2012) evaluate portfolio performance when including long VIX futures and finds that investing in volatility can serve as portfolio diversification or as a hedge.

[4] Aligning with news media coverage following the event. See e.g. Ahmed (2018) and Kawa (2019)

be (at least partially) predictable.

Cheng (2018) shows that trading on the information embedded in the VIX premium is profitable, yielding higher returns than a passive short volatility strategy. The thesis seeks to prolong the study by Cheng (2018). His sample covers the period up until 2015, which means that the unprecedented spike in volatility during the Volmageddon of February 5^{th} 2018 is not included. Extending the sample of Cheng (2018) will test the signal, i.e. the ex-ante estimated volatility premium, over a period where volatility strategies suffered their most substantial drawdowns and when the volatility of volatility has been higher[5] (see Figure 1.1).

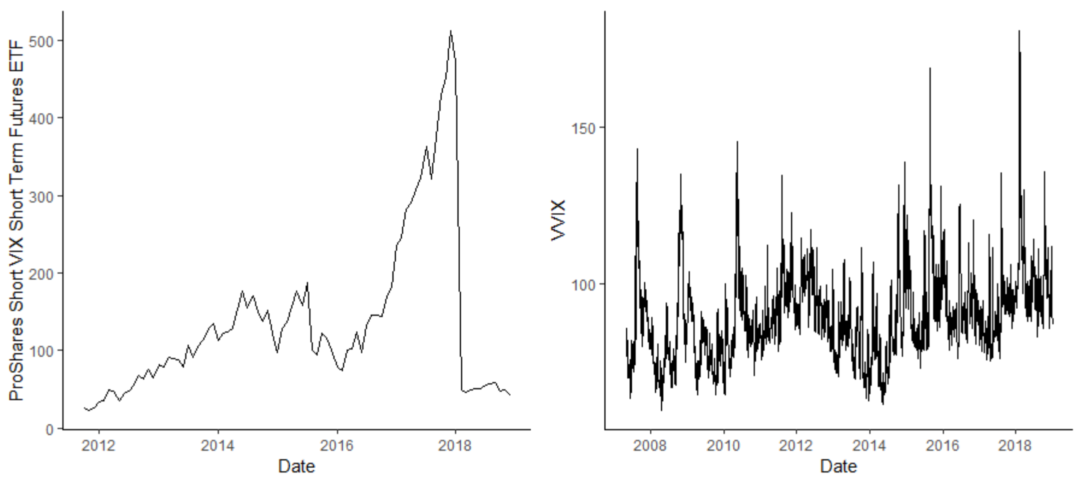

Figure 1.1: SVXY and VVIX

Note: (Left) Cumulative return of ProShares Short VIX Short Term Futures ETF (SVXY) between 2012 and 2018. The ETF suffered a massive loss of 90% on February 5th 2018 after which it reduced its leverage to -0.5 (Brøgger, 2018). (Right) Implied volatility of the VIX (VVIX) between 2007 and 2018. The average VVIX level was 87.12 between 2007 and 2015, and 95.03 between 2016 and 2018. The highest spike of 180.61 occurred in conjunction with Volmageddon.

Source: Nasdaq and Yahoo Finance.

Since previous research on variance and volatility premiums is primarily focused on the U.S. market, the thesis transfers the concept of VIX premium to the European market by constructing a VSTOXX[6] premium using the methodology of Cheng (2018). To the best knowledge of the authors, no previous study has sought to transfer the concept of trading on volatility premiums to the European market. Transferring the same methodology from the VIX to the VSTOXX allows for a comparative study that might cast further light on puzzling premium behavior, and provide insights to whether there exist profitable

[5]E.g. Drimus and Farkas (2013) show that volatility futures strategies are sensitive to the volatility of volatility.

[6]The VSTOXX index measures the 30-day implied volatility of Euro Stoxx 50 (SX5T) (EUREX, 2019).

volatility trading opportunities on the European market.

The thesis focus on the short end of the volatility futures term structure and volatility premiums associated with specific strategies rolling one month ahead futures contracts. Trading strategies on the short end of the term structure is a preferable approach for several reasons. First, it is the short end of the futures term structure that is the steepest (Alexander et al., 2015). Thus, volatility premiums are largest on the short end. Second, the short end of the term structure is the most liquid (Brøgger, 2018), improving upon both transaction costs and the informational value of the signal. Third, focusing on the short end of the term structure implies a shorter forecasting horizon, rendering more precise model forecasts. Finally, a strategy focused approach allows for comparison of VIX and VSTOXX findings.

The thesis confirms that active volatility strategies trading on the ex-ante estimated volatility premium substantially outperform passive volatility strategies and the underlying stock index on both investigated markets. Trading volatility actively not only improves upon performance, it also reduces strategy drawdowns. Both when trading monthly and daily. Surprisingly, daily trading strategies on VSTOXX futures outperform their VIX equivalents over the same sample period even though the transaction costs are substantially higher.

The structure of the thesis is as follows. Chapter 2 introduces the concept of volatility indexes, more specifically the VIX and VSTOXX, their features, construction, and how they are used for hedging and speculative purposes. Chapter 3 contains a theoretical review of variance and volatility premiums and how they can be estimated. This chapter also includes calculations of ex-ante volatility premiums for VIX and VSTOXX. These premiums are used as a trading signal for the strategies tested in Chapter 4 which outlines the main findings by documenting the result obtained from strategies on VIX and VSTOXX futures. Chapter 5 discusses the findings of Chapter 4 and Chapter 6 concludes the thesis by summarizing the empirical findings and providing suggestions for future research.

2 Volatility Indexes

Uncertainty and risk of financial markets have always captured the interest of both academic researchers and practitioners. One way of quantifying uncertainty, or risk, is to consider the volatility of financial asset markets. Since volatility can serve as a simple proxy of risk, it has become a key element in modern asset pricing theory. With transmission of risk being a primary function of financial markets, the interest in understanding volatility is well motivated and has resulted in the extensive literature on the subject.

Realized volatility can be used by investors as a parameter when making investment decisions. The problem with such an approach is that it is based on historical rather than current market information. A forward-looking approach is to use the market expectation of future volatility implied through prices of options. The implied volatility is exactly what volatility indexes, such as the VIX and VSTOXX, reflect. This chapter outlays an in-depth presentation of volatility indexes and volatility futures, which provides the foundation necessary for the estimation of ex-ante volatility premiums paid by investors for transfer of risk.

2.1 Volatility Indexes

The birth of the volatility index is analogous to the birth of the VIX. The VIX index was first introduced by John Whaley in 1993 and had two purposes at the time. First, to provide a benchmark for short-term market implied volatility which facilitates a comparison of historical volatility levels. Second, to create a risk factor on which derivatives can be traded (Whaley, 1993).

It is essential to realize that the VIX is forward-looking and presents the volatility implied trough prices of traded options. The pricing of options is based on the expectation of the underlying stock index's volatility from the time of purchase to the time of expiration. The implied volatility, much like the implied yield of a bond, is not directly observable in the market. By inverting an option pricing formula like the Black and Scholes formula, one can obtain the market's expectation of future volatility, i.e. the implied volatility. However, this is an inconsiderate method since the validity of the approach is dependent on accurate model assumptions. The issue can be avoided by using a result from mathematical finance. Namely, that the knowledge of all (or in practice many) options prices across different strikes determines an underlying probability distribution of the underlying stock return up until maturity. An application of this method can be found in e.g. Breeden and Litzenberger (1978). The methodology for calculating the VIX was presented by Whaley (1993) and is now implemented by CBOE.

In essence, the implied market volatility is estimated by taking the average weighted price of a wide range of out-of-the-money call and put options with different strikes and maturities. The maturities correspond to an average maturity that resembles the future horizon of interest. In the case of the VIX, which seeks to estimate the 30-day implied volatility, the targeted maturity is achieved by interpolating options with more than 23 days and less than 37 days to expiration (CBOE, 2018).

The formula used to calculate the volatility index is

$$\sigma_{T_j}^2 = \frac{2}{T_j} + \sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T_j}\left[\frac{F}{K_0} - 1\right]^2 \tag{2.1}$$

, where T_j is time to expiration, F is the forward level of the underlying index derived from option prices, K_0 is the first strike below F, K_i is the strike of the $i-$th[7] out-of-the-money option (a put option if $K_i < K_0$, correspondingly a call option if $K_i > K_0$ and both a put and call option if $K_i = K_0$), ΔK_i is the interval between strikes calculated as $\Delta K_i = \frac{K_{i+1}-K_{i-1}}{2}$, r is the risk free interest rate until expiration and $Q(K_i)$ is the bid-ask midpoint for each option with strike K_i. The last term is an adjustment that compensates for the fact that the underlying option portfolio is not necessarily centered around a strike that is precisely at-the-money. (CBOE, 2018)

Equation 2.1 is a discrete time approximation of the expected risk-neutral value of future realized variance as derived in Demeterfi et al. (1999). The result is obtained by finding a

[7] The included options are centered near the strike K_0 and i represents the included options. The VIX methodology only uses options with non-zero bid prices in the estimation of the implied volatility (CBOE, 2018).

strike K_{VAR} so that the 30-day variance swap[8] with payoff

$$\sigma_R^2 - K_{VAR} \tag{2.2}$$

has an initial value of zero, where σ_R^2 is the realized variance of the underlying stock index over the life of the contract. For a complete derivation of the continuous time formula used to determine the level of a volatility index, see Appendix A1.

The implied volatility is presented as an annualized standard deviation which is attained by applying 2.1 to the expiration dates T_1 and T_2 which yields $\sigma_{T_1}^2$ and $\sigma_{T_2}^2$. The final step is to interpolate in between these two expiration dates according to

$$VIX = 100 \times \sqrt{[w \times \sigma_{T_1}^2 + (1-w) \times \sigma_{T_2}^2] \times \frac{365}{30}} \tag{2.3}$$

, where $w = \frac{(T_1 - 30)}{T_2 - T_1}$. For more details on the VIX calculations see e.g. CBOE (2018) or Arnold and Earl (2018).

Equation 2.1 is not limited to a specific index or time horizon. It can be applied to other markets, assets and maturities. CBOE calculates the implied future volatility for a 9-day horizon (VX9D), a 90-day horizon (VIX3M) and on a 180-day horizon (VX6M). The same methodology is used to derive the VSTOXX index which represents the 30-day implied volatility of Euro Stoxx 50 (SX5T). It is because of historical reasons and liquidity that the VIX has become the most common measure of market risk (Brøgger, 2018). Table 2.1 outlines a selection of different volatility indexes, their level on February 5^{th} 2018 and their maturity horizon.

Table 2.1: Examples of volatility indexes and their level on February 5^{th} 2018

Stock Index	**Volatility Index**	**Maturity**	**Level**
S&P500	VIX9D	9 days	59.34
S&P500	VIX	30 days	37.32
S&P500	VIX3M	90 days	28.13
S&P500	VIX6M	180 days	24.55
Euro Stoxx 50	VSTOXX	30 days	18.86
Hang Seng	VHSI	30 days	18.93
Nikkei 225	VXJ	30 days	20.46

[8] A Variance swap can be understood as a forward contract where the payoff is linked to realized volatility. See further under Chapter 3.

As can be seen in Table 2.1, a volatility index can be calculated using options traded on any stock index with a liquid options market.

A volatility index refers to the volatility of yearly returns. However, market participants might be more interested in the distribution of daily returns. It takes a quick calculation to translate a volatility index, like the VIX, to an intuitively useful number. Since volatility increases with the square root of time, the daily volatility is obtained by dividing the index value by the square root of the number of trading days in a year. For example, a VIX level of 37.32 corresponds to a daily stock market volatility of $\frac{37.32}{\sqrt{252}} \approx 2.35\%$ over the next month.

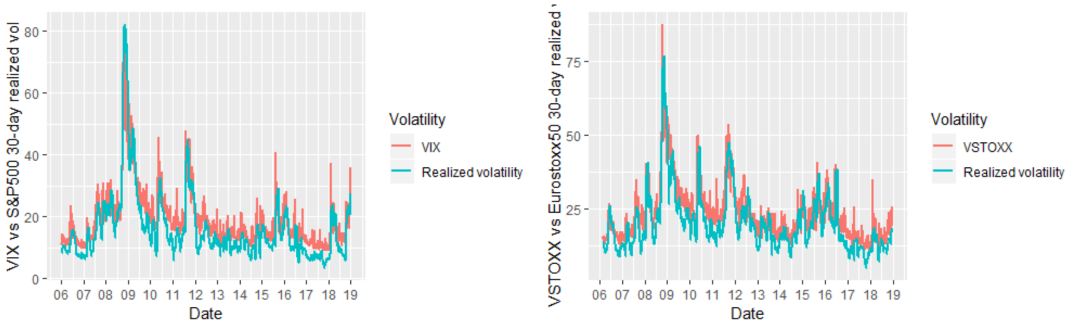

Figure 2.1: Implied vs realized volatility of SPXT and SX5T
Note: The figure illustrates the implied and realized volatility for the VIX (Left) and VSTOXX (Right) throughout Jan 2006 to Dec 2018 for both indexes on a 30-day horizon.
Source: Bloomberg.

Even though a volatility index measures the expected future volatility, it can be explained by the realized volatility plus an insurance premium (Brøgger, 2018). Figure 2.1 plots the VIX and the VSTOXX together with the realized volatility of the underlying stock indexes, SPXT and SX5T. Figure 2.1 shows that a volatility index to a large extent is realized volatility pushed upward. This connection can also be made when examining the VIX as done in Cheng (2018):

$$VIX_t = \sqrt{E_t^Q[RVar_{t,t+30}]} \tag{2.4}$$

, where $RVar_{t,t+30}$ is the 30-day realized variance of two interpolated option maturities on SPXT starting at date t[9]. This is a simpler way of reiterating the somewhat cumbersome

[9]Equation 2.4 refers to quadratic variation, for which realized variance is a consistent estimator, see e.g. Andersen and Benzoni. (2009). The equation presented has been modified by Cheng (2018) for expositional simplicity following Carr and Wu (2006) equations 8 and 17.

calculations presented above.

An essential feature of volatility indexes is that they cannot be traded. Although a volatility index is based on market prices of options, it is not a financial asset or a portfolio of assets. The explanation is two-folded. First, the options underlying the index today are not necessarily the same as the options underlying it tomorrow. Second, the precise strike retrieved from the forward level is not known until after the time of settlement. This means that market participants are unable to hedge their derivative exposure by trading the underlying volatility index.

2.2 Investor Fear Gauges

The VIX has earned the epithet of "the investor fear gauge." While implied volatility per definition is affected by both up and down movements, the options market on SPXT is dominated by investors wanting to hedge their stock portfolios. Bollen and Whaley (2004) show that the demand for at-the-money and out-of-the-money puts is a crucial driver of the VIX, which explains its tendency to spike in times of market turmoil.

The proposition presented by Bollen and Whaley (2004), that investors hedging their portfolios is the primary driver of a volatility index, is tested following Whaley (2008) who finds that the VIX spikes higher in downturns[10]. The model to test this relationship, with VIX as an example, is outlined as

$$\Delta VIX_t = \beta_0 + \beta_1 \Delta SPXT_t + \beta_2 \Delta SPXT_t^- + \epsilon_t \tag{2.5}$$

, where ΔVIX_t is the daily change in VIX, $\Delta SPXT_t$ the daily change in SPXT and $\Delta SPXT_t^-$ is the daily change in SPXT times a dummy variable taking the value 1 if $\Delta SPXT_t < 0$ and 0 otherwise. If the proposition presented in Bollen and Whaley (2004) is true, the slope coefficients should be negative and statistically different from zero. The

[10]Whaley (2008) tests this proposition for a sample of the VIX covering its inception up until and including the financial crisis in 2008.

equations below present the results from the regressions:

$$\Delta VIX_t = -0.00 - 3.22\Delta SPXT_t - 2.10\Delta SPXT_t^- + \epsilon_t \tag{2.6}$$

$$\Delta VSTOXX_t = -0.01 - 2.21\Delta SX5T_t - 1.71\Delta SX5T_t^- + \epsilon_t \tag{2.7}$$

The relationship turns out to be accurate as the coefficients are negative and significantly different from zero[11]. The regressions should be interpreted as: if the SX5T rises by 100 basis points, the VSTOXX will fall by 221 basis points. Conversely, if the SX5T falls by 100 basis points, VSTOXX will increase by 221 + 171 = 392 basis points. Both regressions exhibit the same asymmetric relationship between the underlying index and the implied volatility index.

It is the feature of spiking when the stock market falls that has assigned the epithet of "fear gauge" to the VIX. There are two driving forces behind the negative correlation between the VIX and the underlying stock index. The first relates to returns as compensation for risk. If the expected risk rises (falls) investors will demand higher (lower) returns, causing the underlying stock index to fall (rise). Only accounting for the first of the two forces, the relationship between changes in the stock index and changes in the VIX should be proportional. But, the relationship is more complicated. The second relates to, as previously established, the options market being dominated by investors seeking to hedge their portfolios.

Investor hedging demand establishes a link to a traditional view of asset pricing and insurances. If a dollar in a "bad state" of the world is perceived as more valuable to an investor than a dollar in a "good state", investors are willing to pay insurance premiums to keep the dollar in the "bad state". If insurances are in high demand, their prices go up. Assuming risk aversion, volatility indexes are expected to increase to a higher absolute level when the markets fall.

[11] Using Newey and West (1987) standard errors with a 5% inference level.

2.2.1 The VIX and VSTOXX Time Series

Figure 2.2 plots the VIX from January 1990 to December 2018 and its empirical distribution. As argued by Fernandes et al. (2013), the VIX displays long-run mean reversion but is characterized by periods in which the index significantly deviates from the mean. The mean of the VIX during this period was 19.27 while its median was 17.40.

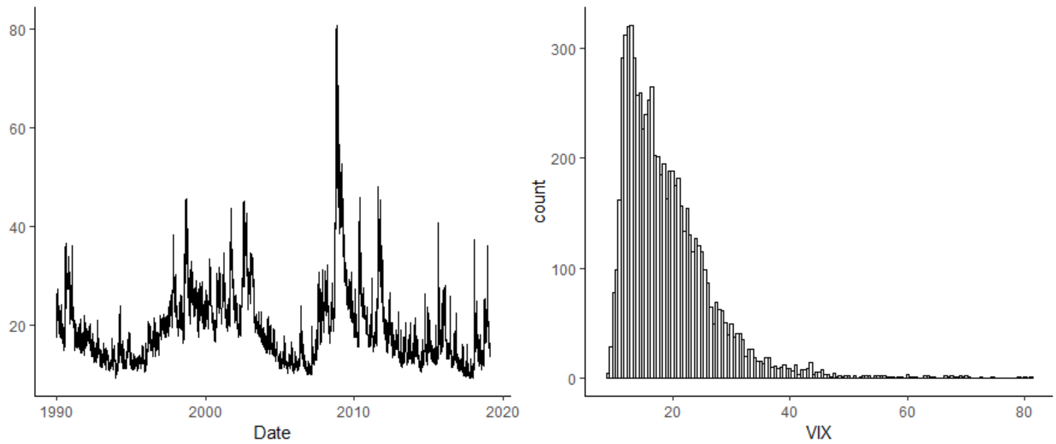

Figure 2.2: Daily closing prices and the empirical distribution of VIX
Note: Daily closing price (Left) and distribution (Right) of the VIX throughout Jan 1990 to Dec 2018.
Source: Bloomberg.

The all-time high for the VIX index was recorded on November 20^{th}, 2008, following the fall of Lehman Brothers and the financial crisis, when the VIX closed at 80.86. After this, the VIX index followed a decreasing trajectory until the Greek debt crisis took off in April of 2010. Other noticeable spikes have occurred in more recent years. In August 2011 the VIX increased in conjunction with the U.S. downgrade by Standard and Poor's, in August 2015 on the back of the Renminbi devaluation (Avellanda and Papanicolaou, 2017), and most recently on February 5^{th} 2018 when the VIX spiked following a correction in the U.S. stock market (Brøgger, 2018). The lowest value of the VIX corresponds to November 3^{rd} 2017 when it closed at 9.14. See Table A2.1 in the Appendix for descriptive statistics of the VIX.

Previous empirical research has found that the VIX behaves like a stationary process (see e.g. Avellanda and Papanicolaou (2017) and Fernandes et al. (2013)). Two tests are used to evaluate the persistence of the VIX, the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test. Both the ADF test and the PP test suggests that the VIX is stationary. The p-value of the Jarque-Bera test is significant which means that the null hypothesis, of a normal distribution, is rejected. Non-normality is also reflected in high values for skewness and kurtosis. Indicating that VIX is leptokurtic and not Gaussian.

As already established, the VSTOXX has SX5T as its underlying stock index. SX5T has 50 constituents from 11 eurozone countries including Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. Constituents are selected based on the largest companies by free-float market capitalization included in the 19 Euro Stoxx Supersector Indexes.

Both SX5T and the VSTOXX index are owned by the Eurex Exchange Group (EUREX). Since the inception of VSTOXX in 2009, it has had an average level of 22.23. However, VSTOXX data can be retrieved from 1999 and onwards since it has been back-dated using historical option prices. For the entire back-dated period, the mean of VSTOXX is 24.16. The all-time high for VSTOXX occurred on October 16, 2008, when it reached 87.51. The most substantial daily increase was in conjunction with the Volmageddon of February 5^{th} 2018.

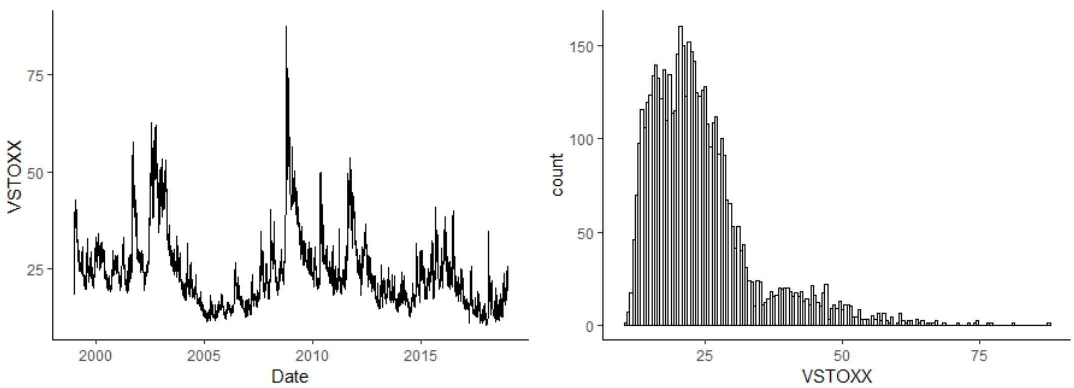

Figure 2.3: Daily closing prices and the empirical distribution of VSTOXX
Note: Daily closing price (Left) and distribution (Right) throughout Jan 1999 to Dec 2018.
Source: Bloomberg.

Figure 2.3 illustrates the daily close of the VSTOXX time series since 1999 and its empirical distribution. The same test for stationary and normality are performed on the VSTOXX index which indicates that the VSTOXX, like the VIX, is a stationary process with a leptokurtic distribution. See Table A2.2 in the Appendix for descriptive statistics on the VSTOXX.

Because of international equity market integration, there is a high correlation between the volatility indexes. The correlation between VIX and VSTOXX between 2001 and 2018 was 0.52. While the correlation is persistent, region-specific events cause volatility to spike in individual markets, resulting in the correlation breaking down. This can be seen in Figure 2.4.

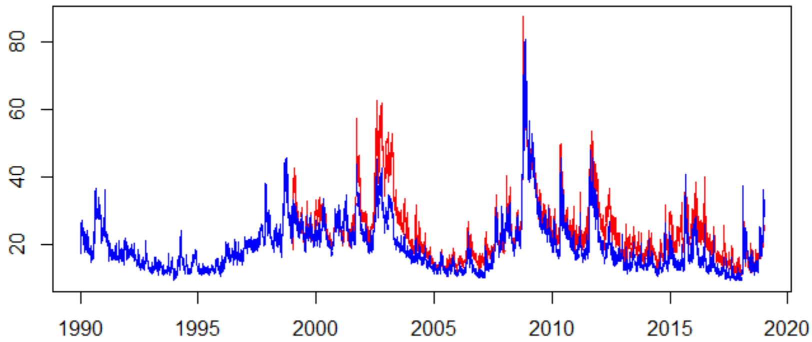

Figure 2.4: The level of VIX vs VSTOXX
Note: The graph plots the daily close of VIX (Blue) and VSTOXX (Red) for the entire backdated samples.
Source: Bloomberg

Figure 2.5 illustrates the spread between the VIX and the VSTOXX, making the relationship described above more evident. Large negative numbers illustrate spikes in VSTOXX, reversely spikes in VIX are illustrated by large positive numbers. For example, when the U.K. voted in favor of "Brexit" in June 2016, the VSTOXX spiked and the spread went down to -20.53. Overall, the spread seems to follow a mean-reverting process.

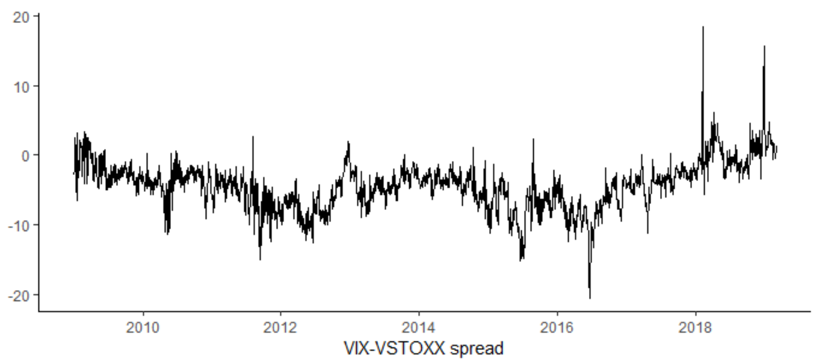

Figure 2.5: The spread of VIX vs VSTOXX
Note: The graph illustrates the spread between the VIX and of VSTOXX from Jan 2008 throughout Dec 2018.
Source: Bloomberg

Figure 2.6 illustrates the level of VIX against the level of VSTOXX and the change in

VIX versus the change in VSTOXX. The slope coefficient related to the scatter plot in level is 1.01 while the one related to the change is 0.46.

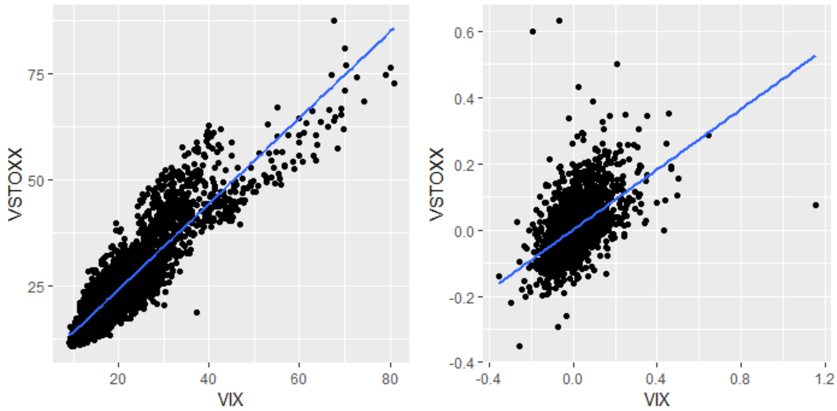

Figure 2.6: Scatter plots of VIX vs VSTOXX
Note: (Left) Level of VIX and VSTOXX. (Right) Daily changes of VIX and VSTOXX.
Source: Bloomberg

Turning to the descriptive statistics of VIX and VSTOXX in Appendix A2 it is clear that the mean of VSTOXX is higher than the mean of VIX. This is true for the entire period as well as the presented subsamples. To explain this difference, it is convenient to turn to portfolio theory and start by examining the standard deviation of a portfolio with n stocks:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{n}}_{i \neq j} w_i w_j \sigma_i \sigma_i \rho_{ij}} \tag{2.8}$$

, where w_i refers to the portfolio weight of a single stock i, σ_i to the standard deviation of that the same stock i and ρ_{ij} is the pairwise correlation between stocks i and j.

As seen in Equation 2.8, two components are driving the standard deviation of a portfolio of stocks; the volatility of stock i and the pairwise correlations between all stocks included. The non-linear transformation of portfolio weights, ceteris paribus, result in decreasing portfolio standard deviation as the number of stocks, n, increases. When n becomes large, the first component becomes very small while the second component gets closer to the average variance of all pairs of stocks. Going back to SPXT and SX5T, they can be thought of as stock portfolios that consist of (approximately) 500 and 50 stocks respectively. A substantial difference, yielding SX5T to trade at higher volatility than SPXT.

2.2.2 Alternative Volatility Indexes

The thesis set out to investigate alternative volatility indexes to VIX and VSTOXX. However, the futures markets for VIX and VSTOXX are the only of the initially investigated futures markets that are liquid enough to evaluate volatility premiums. In Appendix A7 the interested reader can find some additional information about two alternative indexes, the VHSI, and the VXJ, which reflects the implied volatility of Hang Seng and Nikkei 225 respectively. Table A7.2 in the Appendix reports the correlation between all four volatility indexes and their respective stock indexes.

2.3 Volatility Index Futures

As mentioned previously, a volatility index in itself cannot be traded. However, in 2004 the first derivatives using a volatility index as underlying risk-factor were launched; these were futures on the VIX index. Two years later, in 2006, CBOE launched options with VIX as an underlying risk-factor. Their popularity has risen over the years, and in 2015 over 800,000 derivatives contracts (options and futures) were traded daily (CBOE, 2018). Futures on VSTOXX were introduced in 2005 but were withdrawn in 2009 due to low trading volumes (Alexander et al., 2015). The replacement, VSTOXX mini-futures, has a contract size corresponding to 10% of the initial VSTOXX futures contract.

Just like any other future, a volatility index future can be seen as a prediction of the volatility index on a specific date. Thus, volatility futures reflect the market participants' best guess of where the volatility index will be on a specific date and can be used for speculative and hedging purposes. If an investor sells (buys) a futures contract the investor bet that the value of the underlying index will be lower (higher) than the current value of the underlying risk-factor. For example, an investor selling a VIX future at a value of 14, with the VIX subsequently rising to 16 at settlement, will have incurred a loss of $(14 - 16) * \$1000 = \$ - 2000$, where \$1000 is the contract multiplier. The investor buying the VIX future at the same value would have made a profit corresponding to the seller's loss.

A fundamental difference of futures on a volatility index and futures on an underlying stock index is that an investor can not replicate a futures contract using the underlying risk-factor and the risk-free rate. This is because, as discussed briefly in Section 2.1, volatility indexes are non-tradable assets. The index can (in theory) be replicated by trading a weighted portfolio of options on the underlying stock index. However, this would be impossible to do in practice as the number of options included is large and the exact weights are unknown before the settlement date. The consequence is that the usual cost-of-carry relationship[12] between the price of the future and the spot level cannot be established for volatility index futures in the same way as for futures with a tradable underlying risk-factor. Put differently, a relationship between the spot and futures price cannot be established since there never exists a carry arbitrage. However, the price of volatility index futures still represents the risk-neutral expectation of the volatility index (Cheng, 2018) and they offer a volatility exposure that is highly correlated to the underlying volatility index. Albeit being highly correlated, the differences between the movements in the futures price and the movements in the volatility index can at times be sizable, as pointed out by Alexander et al. (2015).

Since volatility index futures are the markets best guess of implied volatility on any given expiration date, one can refer to the volatility futures' term structure. The terms structure, or the forward curve, can be interpreted as futures prices as a function of time to maturity. The futures term structure is most often in contango, i.e. rising[13]. A falling term structure is a term structure in backwardation. Figure 2.7 illustrates these two types of terms structures, taken from actual volatility futures on the VIX at two points in time. The VIX term structure is the richest, with nine tradable maturities.

[12]The cost-of-carry relationship is central in pricing of futures and refers to the fact that the difference between the underlying asset (spot price) and the futures price can be explained by the difference in the interest earned when investing in a risk-free asset rather than purchasing the underlying asset and the dividends received when owning the underlying asset. If one seeks to "carry" a volatility index one would have to trade dynamically, or roll, a basket of options on the stock index underlying the volatility index. A strategy, if implementable, that is likely to be extremely costly in light of transaction costs.

[13]Simon (2016) and Avellanda and Papanicolaou (2017) show that the VIX futures curve has been in contango roughly 75% of the time since its introduction, which is the main reason behind the appeal of short volatility strategies. Since 2011 the VSTOXX futures curve has been in contango roughly 70% of the time (Morgan, 2018)

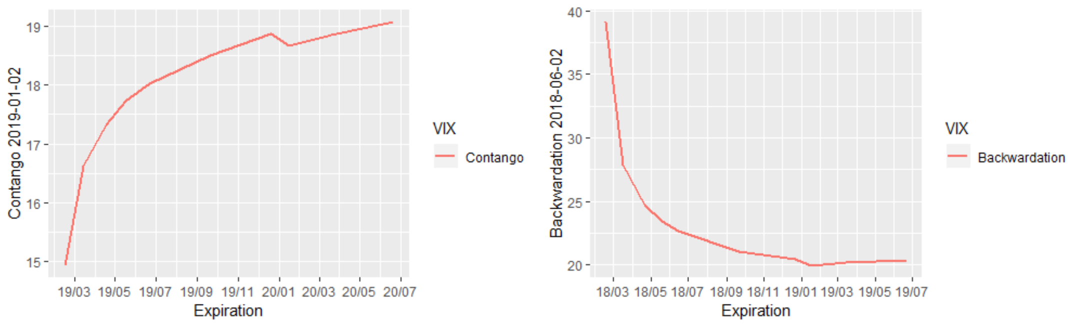

Figure 2.7: Futures term structure in contango and backwardation.
Note: (Left) A term structure in contango. (Right) A term structure in backwardation.
Source: CBOE

The few periods of backwardation can be explained by increased trading volumes and the front-running of the hedging of volatility ETPs. The term structure of the VIX is more convex than that of VSTOXX, meaning that it is steeper in the short end of the curve. This implies that the loss of long VIX futures should be more significant in the short end of the curve than for long VSTOXX futures. The VSTOXX term structure is relatively steeper in the long end. (Alexander et al., 2015)

When trading futures, the investor "rolls" up (down) the term structure when entering a long (short) position, assuming a term structure in contango. Since the term structure most often is in contango, it follows that a long (short) strategy will lose (accumulate) value over time. However, if the term structure switches from contango to backwardation the long (short) position will gain (loose) value. The short end of the futures curve is the most susceptible to changes in the underlying, and thus, it is the most volatile. The short end is also the most liquid part of the term structure (Brøgger, 2018), which is why the first two contracts are the focus of the thesis.

Brøgger (2019) and Eriksen (2018) show that the price of volatility futures and the shape of the term structure is a crucial determinant of the performance of ETPs tracking volatility futures. These ETPs can be considered passive volatility strategies providing exposure to a particular volatility index. Their popularity has risen since their introduction as they provide access to volatility exposure, which was previously available exclusively for sophisticated investors (Eriksen, 2018).

Table 2.2 presents details on the futures contracts used in this thesis.

Table 2.2: Contract summary of volatility futures

	VIX	**VSTOXX**
Future	UX	FVS
Underlying	VIX Index	VSTOXX Index
#Maturities	9	8
Stock Index	S&P500	Euro Stoxx 50
Contract Size	$1,000 * Index	€100 * Index
Tick Size	0.05	0.05
Tick Value	$50	€ 5
Expiration	30 calendar days before the 3^{rd} Friday of next month	30 calendar days before the 3^{rd} Friday of next month
Last Trading Day	Expiration minus 1 business day	Expiration minus 1 business day
Convention	Preceding	Preceding
Daily Traded Volume (million USD)		
Max	380.83	8.49
Min	0.07	0.00
Average	73.11	2.29

Note: All strategies are modified to fit the specific maturities, see more under Section 3.3. Daily traded volume is for the first two contract maturities relevant for this thesis during the period the contracts have been traded.
Source: (CBOE, 2019), (EUREX, 2019) and Bloomberg.

2.3.1 Prices and Liquidity

Grossman (1995) defines informational efficiency as a situation where prices aggregate and convey all current information about future assets returns. This requires that markets are perfectly efficient and that information is not costly. If prices were informationally efficient, there would be no reason to trade, and passive investing would be optimal. Grossman and Stiglitz (1980) argue that markets are not informationally efficient and that there must be an "equilibrium level of disequilibrium", i.e., an equilibrium level of illiquidity, which means that markets need to be illiquid enough to compensate for liquidity providers. Derivative pricing theory relies on the assumption of no arbitrage. The premise of no arbitrage does not only entail the absence of arbitrage, but it also assumes frictionless markets. In the presence of frictions, prices will depend on the liquidity of the specific security as well as the liquidity of other securities (Amihud et al., 2005). Market liquidity can be measured by, e.g., bid-ask spreads, traded volumes or trading frequency.

The investigated indexes differ a lot in terms of liquidity as measured by both bid-ask spreads and traded volumes. Since illiquidity increases frictions and implies a higher degree of informational inefficiencies, it has implications on trading strategies. Market inefficiencies will presumably affect the informational value of the premiums estimated in Chapter 3 and cannibalize returns through high cost of trading.

Figure 2.8 illustrates the 21-day rolling average of traded volumes by index. The number represents the total value of the near- and next-term futures contracts converted to USD. Both futures on the VIX and the VSTOXX have gained popularity over the years, reflected in a steadily increasing traded volumes.

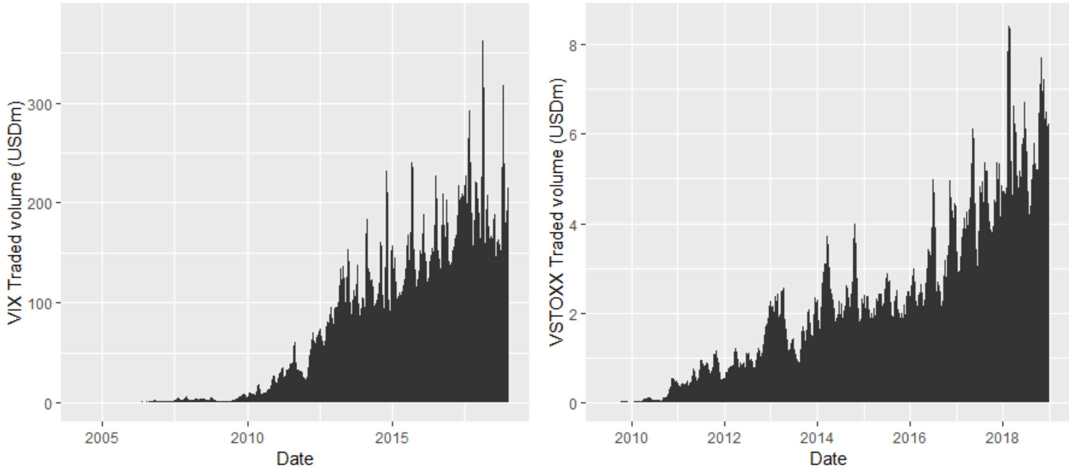

Figure 2.8: Traded vega of VIX and VSTOXX futures

Note: Daily traded vega of the one and two month ahead futures contracts on the VIX (Left) and VSTOXX (Right). The traded vega has been plotted using a 21-day moving average to smooth the graphs. All reported volumes are in million USD. On any given day the number of traded contracts have been multiplied by the respective contract multiplier as reported in Table 2.2.

Source: Bloomberg and FRED Economic Data.

Eriksen (2018) documents that the introduction of ETPs linked to the VIX through VIX futures has increased significantly from November 2010 through December 2018. A majority of this increase has occurred over the last five years of this sample and is attributable to the introduction of inverse and leveraged ETPs tracking VIX in 2011. In 2015, both leveraged ETPs and inverse ETPs had reached billions of dollars in assets under management (AUM). Both Brøgger (2019) and Eriksen (2018) documents that this has a positive effect on liquidity, and that trading activity is concentrated on the first two expiring contracts. While there exists a large selection of ETPs on the VIX, the universe ETPs linked to VSTOXX is younger and smaller (Macroption, 2019). The first ETP on VSTOXX was launched in 2010 by Barclays (ETF World, 2010), and the first U.S.-listed VSTOXX ETP was launched in May 2017 by VelocityShares (Macroption, 2019).

3 Volatility Premiums

This chapter opens with a review of premiums paid by investors to hedge their portfolios, and continues with a presentation of the volatility premium following the VIX premium methodology as defined in Cheng (2018). The chapter then presents different approaches of modeling volatility, model selection and model performance. The chapter concludes by calculating and presenting the premiums used as trading signals in Chapter 4.

3.1 Volatility Risk Premiums

As established there is a growing body of research documenting that investors are willing to pay substantial premiums to hedge stock market fluctuations. There are different methodologies to estimate these premiums. Bekaert and Hoerova (2014) considers the variance premium, calculated as the squared VIX minus the expected realized variance measured over the next month. Thus, the variance premium is the expected return from selling a variance swap contract. In contrast, Cheng (2018) considers the volatility premium, calculated as the difference between the risk-neutral minus physical expectation of the VIX. Under assumptions of no arbitrage the risk-neutral expectation is analogous to the futures price. The physical expectation is a forecast of the VIX.

There are essential differences between the premiums calculated by Cheng (2018) and, e.g. Bekaert and Hoerova (2014). First, the volatility premium relates to the expected standard deviation while the variance premium relates to the expected variance. Second, the variance premium relates to the payoff of a variance swap contract, settled against realized volatility. Finally, the non-linear transformation of the VIX in the variance premium makes it more volatile, and the payoffs will be different from that of the volatility premium for equivalent moves (Warren, 2012).

Research shows that premiums are positive on average[14] and contains a puzzle where market turmoil drives premiums downwards. The natural explanation of this puzzling behavior

[14] A word of caution for the reader is in place here as the premiums can be defined differently. Carr and Wu (2009) defines the variance risk premium as $E_t^P[.] - E_t^Q[.]$, resulting in a variance premium that is negative on average. In the thesis, as in Cheng (2018), the premium is defined as $E_t^Q[.] - E_t^P[.]$ (see equation 2.4), thus positive on average and consistent with the findings of Carr and Wu (2009).

has been that estimates of variance risk premium contain errors from misspecifications of variance forecasts (Cheng, 2018).

In the thesis the VIX premium, the VSTOXX premium and the volatility premium are analogous. The VIX and VSTOXX premiums refer to the volatility premiums calculated on the VIX and the VSTOXX index respectively, and the volatility premium refers to both. There is no theoretical difference between the VIX and VSTOXX premium. In order not to present analogous equations twice, the VIX is the example in the following equations.

The volatility premium is the foundation of this thesis, and it eradicates the suspicion of anomalous premium behavior being due to variance forecast misspecifications (Cheng, 2018). It can be economically interpreted as the price that investors are willing to pay in order to hedge market volatility (Cheng (2018), Coval and Shumway (2018), Bakshi and Kapdia (2003)). Cheng (2018) defines the premium as the risk-neutral (Q) minus the physical (P) expectation of the VIX at date t with a horizon $T - t$. The VIX premium under risk-neutral forward measure is per definition

$$VIXP_t^T = E_t^Q[VIX_T] - E_t^P[VIX_T] \tag{3.1}$$

, where $E_t^Q[.]$ and $E_t^P[.]$ are the risk-neutral and physical expectation respectively. The VIX premium is positive on average and can be interpreted as the expected dollar loss for a long position. Correspondingly it is the expected dollar gain for a short position. The main contribution of Cheng (2018) is the creation of a direct measure of premiums that makes it possible to study the forecasting power of volatility premiums and relate it to anomalous market behavior. Cheng (2018) does so by using VIX futures prices under the assumption of no arbitrage as the risk-neutral expectation while forecasting the VIX spot to get the physical expectation. With these assumptions the VIX Premium equals

$$VIXP_t^{T(t)} = F_t^{T(t)} - \widehat{VIX}_t^{T(t)} \tag{3.2}$$

, where $F_t^{T(t)}$ is the futures price with maturity $T(t)$ at time t and $\widehat{VIX}_t^{T(t)}$ is the forecasted value of the VIX for the corresponding maturity, $T(t)$, at time t. Since the premium is used as a signal that assumes that trading will occur the same day, the opening price of the future at time t is used to avoid forward-looking bias. If the opening price does not exist, the closing price of the previous day is used. Eriksen (2018) reports that the bulk of the trading activity in VIX futures occurs at the end of the day, more specifically during the last two hours. But, in the absence of sufficient intra-day data, the thesis resort to the opening price.

The VIX premium fluctuates heavily over time, and there seems to be a pattern in which the premium goes down before episodes of realized market risk. Cheng (2018) calls this pattern “the low premium response puzzle” and he shows that it is stable across model choice and the removal of the financial crisis from the subsample. In his study, Cheng (2018) manages to accurately predict realized premiums with a coefficient of 0.92 and a standard error of 0.29[15]. He also finds that falling premiums predict ensued realized risk both in VIX futures and stock markets. Thus, creating opportunities for trading strategies to profitably exploit the volatility premium as a signal.

3.2 Modeling Volatility Indexes

While prices of VIX futures are used as a proxy for the risk-neutral expectation of the VIX, the physical expectation of the VIX is estimated using an out-of-sample autoregressive moving average (ARMA) model. ARMA models are highly relevant in volatility modeling and are applied on the VIX by, e.g. Mencia and Sentana (2013). They argue that ARMA models are suitable due to the high persistence and the presence of partial autocorrelation in the VIX time series. Another benefit of ARMA models is that they conveniently produce multiperiod forecasts.

ARMA models combine the idea of auto-regressive (AR) and moving average (MA) models into a compact form which allows the number of parameters to be kept small, resulting in a more parsimonious model in terms of parameterization. For forecasting with ARMA models to be possible, the data needs to be stationary. Since strict stationarity is difficult to prove empirically, weak stationarity is often assumed. Weak stationary means that the mean and the covariance of a time series is independent of time. The ARMA model is

[15] Cheng (2018) converts the premium into an estimated return and runs a regression. If ex-ante estimated premiums predict ex-post returns, they will do so with a coefficient close to one.

explained further in Appendix A3.

3.2.1 Alternative Models

Previous academic literature has resorted to other statistical models to forecast volatility. Fernandes et al. (2013) and Corsi (2009) argue that heterogeneous autoregressive (HAR) processes are particularly suitable to forecast both realized and implied volatility because they capture the long-run memory arising between asymmetric transmission of volatility between long and short horizons. Mencia and Sentana (2013) analyze the presence of generalized autoregressive conditional heteroskedasticity (GARCH) effects in the residuals of an ARMA(2,1) model estimated on the VIX and finds supporting evidence for an ARMA(2,1)-GARCH(1,1) model. However, as pointed out by Corsi (2009), when aggregated over extended periods, GARCH models tend to appear as white noise. Cheng (2018) estimates a battery of alternative models and evaluates the result using several accuracy measures focusing on the 34-trading day forecast horizon. The difference in the root-mean-squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) for the top performing models were economically small and statistically indistinguishable. Further, the correlations between the ARMA forecast and the forecast of these models were 99%.

3.2.2 Model Selection

In line with Cheng (2018), this study resorts to ARMA models to forecast each volatility index. All models are estimated out-of-sample using the index data available up until the introduction of volatility futures on each respective index. The smallest training sample is on VSTOXX with 2,648 observations, which is considered sufficient. Lag lengths are chosen based on the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). The thesis tests a wide variety of models, all of which are outlined in the Appendix A4. Both the BIC and the AIC are estimators of the relative quality of statistical models for a specific set of data and attempt to resolve the problem of over-fitting by introducing a penalty term related to the number of parameters in the model. Model accuracy tests are also performed. To ensure that the selected models produce white noise residuals, the residual ACF is plotted and a formal Box-Jenkins test is conducted. Details on model selection and the tests for white noise residuals can be found in Appendix A4.

The model choice for the VIX is an ARMA(2,2) and the estimated process is:

$$\begin{aligned} VIX_t = 19.423 + 1.669(VIX_{t-1} - 19.423) - 0.671(VIX_{t-2} - 19.423) \\ -0.749\epsilon_{t-1} - 0.059\epsilon_{t-2} + \epsilon_t \end{aligned} \tag{3.3}$$

The model choice for the VSTOXX is an ARMA(2,3) and the estimated process is:

$$\begin{aligned} VSTOXX_t = 25.942 + 0.188(VSTOXX_{t-1} - 25.942) \\ +0.795(VSTOXX_{t-2} - 25.942) + 0.769\epsilon_{t-1} - 0.124\epsilon_{t-2} - 0.160\epsilon_{t-3} + \epsilon_t \end{aligned} \tag{3.4}$$

3.2.3 Model Performance

Model performance is evaluated using RMSE, MAE, MAPE and R^2. By construction, the RMSE gives higher weights to large errors while the MAE is less sensitive to outliers. The MAPE is normalized by true observations and has the benefit of scale-independency. The accuracy of each model is considered both on the 34-day horizon and on the daily horizon. The daily horizon rolls forecasts from a median forecast horizon of 34 through 14 days which corresponds to the time to expiration of the futures contracts. The rationale of the roll is discussed further in Section 3.3.

Table 3.1: Model accuracy

	RMSE	MAE	MAPE	R^2
VIX Rolling	1.989	1.357	0.074	0.99
VIX 34 day	6.364	3.959	0.199	0.53
VSTOXX Rolling	1.754	1.415	0.075	0.99
VSTOXX 34 day	5.509	4.175	0.199	0.40

Note: The table presents model forecast accuracy. The "Rolling" forecast is equivalent to the forecasts on a rolling horizon following the dynamically changing forecast horizon and the "34 day" to the accuracy on a constant horizon of 34 days.

Naturally, the model performance deteriorates with the forecasting horizon. On the daily rolling forecasts, the estimated model for the VIX delivers the most accurate results in terms of MAE, MAPE, and R-squared while the model for the VSTOXX delivers the most accurate results in terms of RMSE. Results are similar on the 34-day horizon.

3.3 Calculating Premiums

A daily time series of premiums is calculated using a specific investment strategy. On the last day of the month, the strategy invests in a two-month ahead futures contract, which becomes the one-month ahead futures contract, with a median time to expiration of 34 trading days. The position is held for one month and liquidated with a median of 14 days before expiration. As argued by both Cheng (2018) and Mencia and Sentana (2013), rolling contracts ahead of expiration avoids illiquidity as contracts near expiration. Liquidating contracts ahead of expiration is also supported by Simon (2016) explaining that futures contracts that are in contango (backwardation) tend to roll down (up) the VIX futures curve to a lower (higher) VIX at settlement and lose (gain) value.

To calculate the futures contract roll for each index, the same methodology as in Cheng (2018) is applied. Both the VIX futures contracts and the VSTOXX futures contracts are rolled the last day of the month as these two have the same expiration date[16]. The expiration date for VIX and VSTOXX futures usually falls somewhere between the 16^{th} and the 22^{nd} day of the month. See Table 2.2 for details. Following the futures convention, the previous business day is chosen for futures on both VIX and VSTOXX if the desired day of the contract roll falls on a non-business day.

The premiums are calculated daily and scaled to one month. With the VIX as an example, the volatility premium is expressed as:

$$VIXP_t^{T(t)} = \frac{21}{T(t)-t}[F_t^{T(t)} - \widehat{VIX}_t^{T(t)}] \tag{3.5}$$

Daily VIX and VSTOXX premiums correlated with a coefficient of 0.56. Table 3.2 presents a summary of the ex-ante estimated premiums. The volatility premium is positive on 73% and 45% of the trading days for the VIX and VSTOXX respectively.

[16]The VIX and VSTOXX futures follow the same maturity conventions. The only difference is in terms of business calendars between the U.S. and Europe. Differences in business day calendars have been accounted for when calculating the premiums.

Table 3.2: Summary of estimated volatility premiums

	VIX	**VSTOXX**
Mean	0.84	-0.11
% Positive	73%	45%
Positive Mean	1.58	1.57
%Negative	27%	55%
Negative Mean	-1.16	-1.43
Max	12.06	10.19
Min	-14.44	-8.51
Median	0.7	-0.2
Standard deviation	1.95	2.02
Skewness	-1	0.14
Kurtosis	9.53	2.37
T	3,271	2,447

Note: The top section of the table presents the mean of the estimated premium scaled to one month, the percentage of time they are negative/postive and their means conditional on these states. The bottom part of the table presents general statistical properties.

Figure 3.1 illustrates the empirical distribution of premiums over the investigated samples. As can be seen, the premiums on VSTOXX are centered around zero to a greater extent than premiums on the VIX which exhibits a negative skew and higher kurtosis.

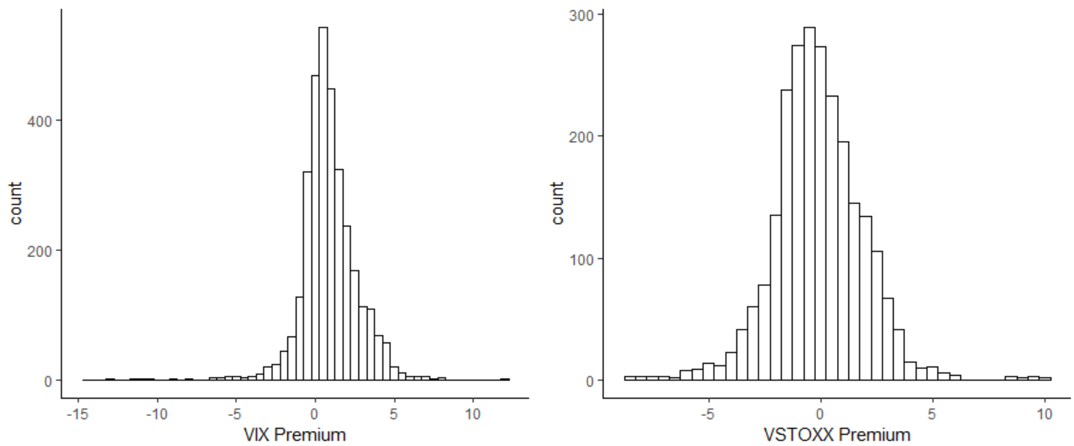

Figure 3.1: Empirical distribution of VIX and VSTOXX premiums
Note: The graph illustrates the empirical distribution of daily premiums scaled to one month for VIX (left) and VSTOXX (right)

Figure 3.2 illustrates the expected monthly premium the last day of the month for each index as well as the daily time series of premiums. Region-specific events causing market turbulence are reflected, not only in the underlying volatility index but in the premium as well. For example, the most pronounced negative premium for VSTOXX was in conjunction with Brexit in 2016. The largest positive premium was in conjunction with the fear of "Frexit" in 2017 when the subsequent relief of the election results caused VSTOXX to plunge. The most pronounced negative premium for VIX was in conjunction with the financial crisis in 2008.

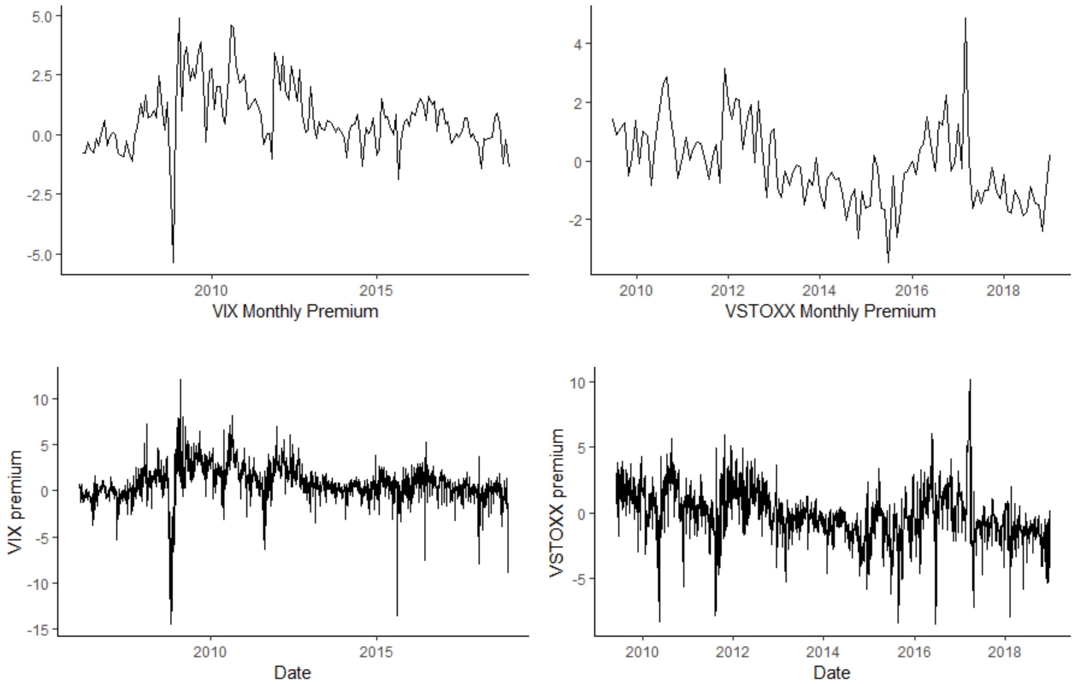

Figure 3.2: Volatility premiums for VIX and VSTOXX

Note: The top graphs plot the estimated monthly premium at the day of the contract roll for the VIX (Left) and VSTOXX (Right). The bottom graphs plot the daily premium time series of VIX (Left) and VSTOXX (Right). Daily premiums are scaled to one month.

4 Exploiting the Volatility Premium

Using the ex-ante estimated volatility premium as a decision rule have the benefit of not needing any in-sample data to estimate the trading rule. In Cheng (2018) five strategies are considered: long/long (L/L), short/short (S/S), long/cash (L/C), cash/short (C/S), and long/short (L/C). The first term specifies the desired position when the premium is negative. Active strategies are those using the volatility premium as a signal, whilst passive strategies roll long or short contracts once a month.

To validate the approach of this thesis the same sample as in Cheng (2018) is investigated yielding the same results for the same period. The results in this chapter give insight to whether it is possible to avoid unprecedented drawdowns such as those during Volmageddon of February 5^{th} 2018 and whether it is possible to exploit the volatility premium in other markets than the U.S.

This chapter starts by covering the standard features of the strategies and calculations of performance measures. It continues with monthly and daily strategies on VIX and VSTOXX futures before concluding in the performance of VIX futures strategies since the inception of VSTOXX futures.

4.1 Common Features of the Futures Strategies

The strategy positions considered are S/S, C/S, L/L, and L/S. In addition, a strategy with imposed premium thresholds is tested, referred to as a long/short/cash (L/S/C) strategy. In the L/S/C strategy a cash position is entered whenever the premium is in between the imposed negative and positive thresholds.

The L/S/C strategy is tested to investigate whether it is possible to improve upon a strategy that purely bases decisions on the premium being positive or negative. When maximizing the strategies' Sharpe ratios ex-post, one runs the risk of data mining results. Here the purpose of doing so is to see whether it gives any additional information about signal behavior on the two different markets. A large number of different thresholds are tested, and their Sharpe ratios can be found in Appendix A6.

As mentioned in Section 3.1, the premiums are calculated using the opening price at time t. All trades, and consequently transaction costs, are made on the closing price to avoid forward-looking bias. A trade is defined as any time a position is changed or a contract is rolled. All data except the risk-free rate and factor loadings have been retrieved from Bloomberg[17].

4.1.1 Transaction Costs

Because liquidity is time-varying, the modeled transaction costs are time-dependent and related to the bid-ask spread at time t. As can be seen in Figure 4.1 liquidity is time-varying both for the VIX and VSTOXX futures. The average bid-ask spread is 40 and 122 basis points for VIX and VSTOXX respectively. An alternative approach would have been to penalize the strategy with the average bid-ask spread, but the actual time t spread approach is chosen since it is a more realistic approach.

The plotted values in Figure 4.1 are for the contracts corresponding to the roll and forecast horizon described in Section 3.3, meaning that they reflect either the bid-ask spread of the near- or next-term futures contract depending on the time of the month.

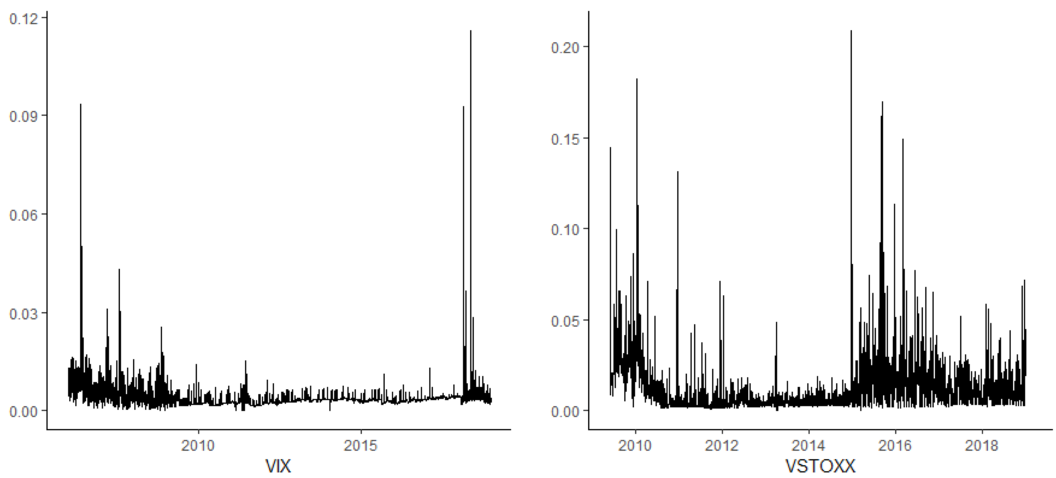

Figure 4.1: Historical bid-ask spreads for futures on VIX and VSTOXX
Note: The graph plots the used transaction costs for the VIX (Left) and VSTOXX (Right). Reported spreads are for the contracts used as defined in Section 3.3. Plotted bid-ask spreads are in percentages calculated over the daily closing price.

Source: Bloomberg.

[17] The used proxy for the risk-free rate is the U.S. Federal Funds Rate (FED fund) and Euro Interbank Offered Rate (EONIA) for strategies on the VIX and VSTOXX respectively. FED fund is retrieved from FRED Economic Data and EONIA from ECB. See French (2019) for details on factor loadings data.

There are a few cases when the bid-ask spread had to be modified due to being negative. Since bid-ask spreads should be non-negative per definition, these observations have been altered. There are 2 observations for both VIX ($0,06\%$) and VSTOXX ($0,08\%$) where the bid-ask spread had to be altered. In these cases, the previous day's bid-ask spread is used.

Since the strategy uses two different futures maturities, it creates some modeling issues when contracts are rolled. For modelling simplicity, the bid-ask spreads for the next-term contract has been used as a proxy for the transaction cost both for the liquidated near-term contract and newly acquired next-term contract. The approach should not affect the transaction costs significantly since the roll is performed ahead of expiration.

Each strategy is tested with transaction costs equal to zero and with a multiplier of two relative to the base case. Factors that transaction costs can be multiplied with and still maintain a higher Sharpe ratio than that of the underlying stock index are reported in connection to the strategies.

When the thesis refers to the base case of transaction costs it should be understood as the following: each time a position is entered, closed or a futures contract is rolled the strategy is penalized with a transaction cost. If a strategy moves in or out of cash, it is penalized with a transaction cost equal to half the bid-ask. If a contract is rolled, or if the strategy changes position from short (long) to long (short), the strategy is penalized with the entire bid-ask spread.

4.1.2 Calculating Returns and Performance Measures

For a short position the return for any day, t, is

$$R_t^A = F_{t-1}^{T(t)} - F_t^{T(t)} - TC_t \tag{4.1}$$

, where R_t^A is the return at t. $F_t^{T(t)}$ and TC_t are the corresponding futures price and transaction cost as described in the previous section.

To get the return as a percentage for the same corresponding date the following transformation is made:

$$r_t = \frac{R_t^A}{F_{t-i}^{T(t)}} \tag{4.2}$$

, where $F_{t-i}^{T(t)}$ is the futures contract price at the time when the current position was entered. Thus, $i \in [0, h]$, where h is the number of days to the next contract roll as defined in Section 3.3. This operation reflects the feature of leverage moving away from the initial margin requirement when trading futures[18].

Whenever a position is changed, returns are calculated over the newly entered futures price at time t, $F_t^{T(t)}$. The operation presents a problem to the strategies that hold cash, as there is no futures price $F_t^{T(t)}$ to calculate the return over whenever the strategy liquidates a futures position and enters a cash position. There is a slight modification of this calculation for strategies holding cash. Since these strategies can enter a cash position at any day t, R_t^A is calculated over the liquidated contract, $F_{t-i}^{T(t)}$. The slight modification is made for modeling simplicity and should not affect the outcome of the strategies significantly. Whenever these strategies hold cash, they earn the risk-free rate.

The daily percentage returns are used to calculate the return, alpha and standard deviation of a specific strategy. All performance measures are presented on an annualized basis and are benchmarked against their respective stock index[19] and the passive strategies that hold a fixed position for the entire period, i.e., the L/L and S/S strategies. The stock index returns are calculated daily as

$$r_t = \frac{S_t}{S_{t-1}} - 1 \tag{4.3}$$

, where S_t is the stock index level at time t. All figures plot cumulative returns and not cumulative excess returns[20].

Strategies are evaluated on their Sharpe ratio as well as on CAPM, Fama and French

[18] In the thesis the initial margin is 1. No margin calls, no stop loss features, and no funding constraints are assumed.

[19] For stock indexes, the thesis uses a total return index in order to account for returns attributable to stock dividends.

[20] Excess returns, $r_t - r_f$, are used for calculating the strategies Sharpe ratios (SR) where $SR = \frac{r_t - r_f}{\sigma}$.

(1993) three-factor and Carhart (1997) four-factor alphas. Alphas are calculated using market specific factor loadings. For strategies on VIX futures, these include all firms incorporated in the U.S. and listed on the NYSE, AMEX or NASDAQ French (2019). SPXT covers about 80% of this market. For the European strategies on VSTOXX futures, there is a higher discrepancy between the underlying stock index and the factor loadings data. There are only 50 constituents in SX5T while all listed stocks in 16 European countries are included in the factor loadings data (French, 2019). See Fama and French (1993) and Carhart (1997) for a complete description of the factor loadings. A general discussion centered around factor loading models and what they test for can be found in the Appendix A5.

Since Sharpe ratios and alphas do not fully reflect the performance of nonlinear strategies, the result tables also report daily skewness, kurtosis and maximum drawdowns of the strategies (Brodie et al., 2007). The strategy's drawdown is calculated as

$$DD_t = \frac{HWM_t - R_t^{Cum}}{HWM_t} \tag{4.4}$$

, where $R_t^{Cum} = \prod_{t=1}^{T}(1 + r_t)$ and represents the cumulative return, HWM_t is the all time high at time t and is calculated as $\max[R_1^{Cum}, R_2^{Cum}, ...R_t^{Cum}]$. Calculations are done daily for all $t \in [1, T]$, where T equals the number of trading days in the sample. Details on the different samples can be found in Appendix A2.

In result tables and plots, the results have been retroactively weighted to have the same standard deviation as their underlying stock index to ease comparability. The performance measures affected by this transformation are reported separately in the result table.

4.2 Monthly Futures Strategies

In this section, the trading signal is evaluated monthly, and contracts are held until the date of the roll as defined in Section 3.3. The active strategies are benchmarked against the passive strategies, S/S and L/L, along with their respective stock index.

4.2.1 VIX Monthly Futures Strategies (1)

The monthly strategies on VIX futures between 2006 and 2018 delivered lower Sharpe ratios than the same strategies did during the sample period of Cheng (2018). The S/S strategy produced a Sharpe ratio of 0.38, compared to 0.57 during the 2004 to 2015 sample of Cheng (2018).

The C/S and L/S strategies also perform slightly worse than under the period investigated by Cheng (2018). In the sample period of the thesis they delivered Sharpe ratios of 0.78 and 0.64 respectively while in the sample period of Cheng (2018) they delivered Sharpe ratios of 0.87 and 0.79 respectively. As an additional benchmark, SPXT delivered a Sharpe ratio of 0.41 between 2006 and 2018, which means that the passive S/S strategy performed weaker than the market for the sample period ending in 2018.

The L/L strategy produced better returns than over the sample investigated by Cheng (2018)[21], which is expected considering the S/S strategy. However, it is still in the negative territory. All results are outlined in Table 4.1.

[21] In Cheng (2018) the L/L strategy produces a Sharpe ratio of -0.78 over the period 2004-2015.

Table 4.1: Performance of monthly trading strategies on VIX (1)

			VIX Jan 2006 - Dec 2018			
Base case	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	8%	7%	-10%	15%	12%	24%
Standard deviation	19%	19%	19%	19%	19%	19%
Sharpe	0.41	0.38	-0.53	0.78	0.64	1.26
Max drawdown	55%	44%	85%	37%	38%	35%
Weight	1	0.24	0.24	0.33	0.24	0.73
Daily skewness	-0.12	-1.92	1.93	-0.63	1.31	0.89
Daily excess kurtosis	11.47	51.97	52.16	13.41	51.74	33.89
Number of trades		157	157	130	157	40
Number of days long/short/cash		0/3,271/0	3,271/0/0	0/2,352/919	919/2,352/0	42/548/2,681
Number of trading days	3,271	3,271	3,271	3,271	3,271	3,271
Average transaction costs		0.15%	0.15%	0.09%	0.09%	0.14%
Mean	8%	31%	-42%	46%	51%	33%
Standard deviation	19%	80%	80%	59%	80%	36%
Max drawdown	55%	97%	100%	80%	93%	42%
Average transaction costs		0.38%	0.38%	0.27%	0.38%	0.19%
W/O Transaction Costs	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	8%	8%	-10%	16%	13%	24%
Standard deviation	19%	19%	19%	19%	19%	19%
Sharpe	0.41	0.44	-0.05	0.83	0.70	1.27
Max drawdown	55%	43%	84%	37%	38%	35%
Weight	1	0.24	0.24	0.33	0.24	0.73
Daily skewness	-0.12	-1.93	1.93	-0.64	1.31	0.90
Daily excess kurtosis	11.47	52.12	52.12	13.45	51.79	33.91
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
Mean	8%	35%	-38%	49%	56%	33%
Standard deviation	19%	80%	80%	59%	80%	26%
Max drawdown	55%	97%	100%	79%	91%	45%
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
2x Transaction Costs	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	8%	6%	-12%	14%	11%	24%
Standard deviation	19%	19%	19%	19%	19%	19%
Sharpe	0.41	0.33	-0.63	0.74	0.58	1.24
Max drawdown	55%	46%	86%	37%	39%	35%
Weight	1	0.24	0.24	0.33	0.24	0.73
Daily skewness	-0.12	-1.91	1.95	-0.64	1.31	0.89
Daily excess kurtosis	11.47	51.72	52.09	13.37	51.57	33.82
Average transaction costs		0.25%	0.25%	0.18%	0.18%	0.28%
Mean	8%	26%	-47%	43%	46%	33%
Standard deviation	19%	80%	80%	59%	80%	26%
Max drawdown	55%	97%	100%	80%	94%	45%
Average transaction costs		0.76%	0.76%	0.55%	0.76%	0.38%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0.

The results are robust to increases in transaction costs. The active strategies, C/S, L/S, and L/S/C, are stressed until they reach the same Sharpe ratio as SPXT. Transaction costs for C/S, L/S and L/S/C can be multiplied by a factor of 9.02, 4.95 and 36.48 respectively. As seen in Table 4.1 and Figure 4.2, increasing transaction costs do not have a significant effect on the results. The lower average transaction cost for the C/S and the L/S/C strategies is explained by it being penalized "only" by half the bid-ask spread when it moves in and out of cash.

Figure 4.2 plots the log margin account growth for each strategy. It shows that the S/S strategy suffers from large drawdowns in times of market turmoil. The active strategies avoid some of the losses by keeping the investment in long futures positions or in cash. The turbulent period following Volmageddon causes the L/S strategy to initially gain value, and subsequently loose value of the same magnitude. The retroactively optimized L/S/C strategy is long VIX futures for a total of two months, both during the financial crisis. The strategy gains a lot of value during this time compared to the other strategies. The imposed thresholds for the L/S/C are 3.4 and -2.0 for the positive and negative premium signal respectively[22].

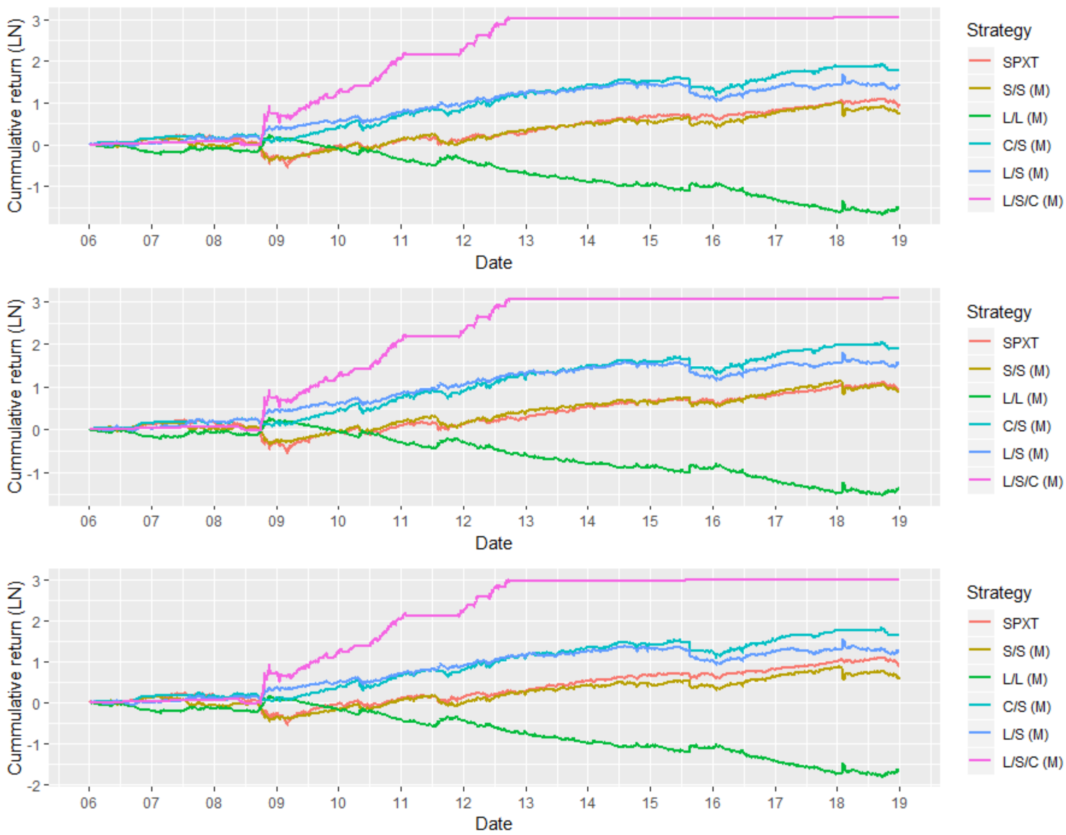

Figure 4.2: Log margin account growth for monthly strategies on VIX and SPXT (1)

Note: Figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0.

Table 4.2 reports factor loadings of the monthly strategies. The C/S, L/S, and L/S/C strategies earn significant four-factor annualized alphas of 9.9%, 10.8%, and 24.4%

[22] See Table A6.1 in the Appendix for all tested signal thresholds

respectively. The passive S/S strategy has high a market loading and delivers worse alpha compared to the active strategies. Further, the active strategies remove the significance of the momentum factor present in the two passive strategies.

Table 4.2: Factor loadings for monthly strategies on VIX

VIX Monthly Trading Strategies					
	S/S	**L/L**	**C/S**	**L/S**	**L/S/C**
CAPM					
Constant	**0.012**	**-0.055**	**0.099**	**0.102**	**0.236**
	0.029	0.029	0.034	0.043	0.074
Excess market	**0.634**	**-0.634**	**0.528**	0.156	0.074
	0.055	0.054	0.078	0.087	0.157
R^2	0.406	0.405	0.281	0.024	0.005
Fama French 3-Factor					
Constant	**0.009**	**-0.052**	**0.100**	**0.107**	**0.244**
	0.000	0.002	0.002	0.003	0.075
Excess market	**0.661**	**-0.660**	**0.516**	0.112	-0.003
	0.064	0.064	0.086	0.092	0.158
SMB	-0.015	0.015	0.092	0.130	**0.271**
	0.652	0.648	0.069	0.078	0.108
HML	**-0.137**	**0.136**	0.030	**0.185**	**0.301**
	0.052	0.055	0.060	0.058	0.104
R^2	0.410	0.410	0.283	0.035	0.038
Carhart 4-Factor					
Constant	**0.006**	**-0.049**	**0.099**	**0.108**	**0.244**
	0.002	0.002	0.002	0.003	0.075
Excess market	**0.683**	**-0.682**	**0.526**	0.105	-0.006
	0.060	0.060	0.084	0.095	0.152
SMB	-0.018	0.017	0.091	0.131	**0.271**
	0.063	0.063	0.071	0.078	0.108
HML	-0.008	0.007	0.087	**0.145**	**0.285**
	0.054	0.054	0.071	0.069	0.106
Momentum	**0.172**	**-0.172**	0.077	-0.053	-0.021
	0.030	0.030	0.050	0.067	0.086
R^2	0.423	0.422	0.285	0.036	0.038
T	3,271	3,271	3,271	3,271	3,271

Note: The table presents the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicates those that are reliably different from 0 on the 5% level. Transaction costs are calculated according to the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0

4.2.2 VSTOXX Monthly Futures Strategies

Table 4.3 outlines the result for the monthly strategies on VSTOXX futures. The only strategy, apart from the L/S/C[23] which has been optimized retroactively, performing better than the stock index is the C/S strategy. The result is robust to transaction costs as the strategy can be multiplied by a factor of 2.56 and maintain a higher Sharpe ratio than that of SX5T. The L/S/C strategy can be multiplied by a factor of 5.27. The

[23]See Table A6.2 in the Appendix where Sharpe ratios for all tested thresholds are presented.

L/C/S strategy never goes long, thus holding cash until the premiums are larger than the threshold of 1.0. The strategy enters a short position 37 times compared to the C/S strategy that enters a short position 47 times.

Table 4.3: Performance of monthly trading strategies on VSTOXX

VSTOXX Feb 2009 - Dec 2018						
Base case	SX5T	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	6%	3%	-12%	10%	3%	16%
Standard deviation	20%	20%	20%	20%	20%	20%
Sharpe	0.30	0.17	-0.61	0.51	0.15	0.78
Max drawdown	33%	40%	78%	26%	39%	22%
Weight	1	0.25	0.25	0.40	0.25	0.62
Daily skewness	-0.02	-0.35	0.37	-0.06	0.25	0.30
Daily excess kurtosis	4.22	8.26	8.42	30.05	8.25	76.47
Number of trades		115	115	47	115	37
Number of days long/short/cash		0/2447/0	2447/0/0	0/1001/1446	1446/1001/0	0/520/1927
Number of trading days		2,447	2,447	2,447	2,447	2,447
Average transaction costs		0.36%	0.36%	0.47%	0.36%	0.54%
Mean	6%	14%	-49%	26%	12%	26%
Standard deviation	20%	80%	80%	50%	80%	33%
Max drawdown	33%	93%	100%	57%	97%	36%
Average transaction costs		1.44%	1.44%	1.18%	1.44%	0.88%
W/O Transaction Costs	SX5T	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	6%	8%	-8%	13%	7%	18%
Standard deviation	20%	20%	20%	20%	20%	20%
Sharpe	0.30	0.39	-0.39	0.64	0.37	0.88
Max drawdown	33%	31%	68%	26%	29%	22%
Weight	1	0.25	0.25	0.40	0.25	0.62
Daily skewness	-0.02	-0.35	0.35	-0.05	24.00	0.30
Daily excess kurtosis	4.22	8.51	8.51	30.89	8.46	77.54
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
Mean	6%	31%	-31%	32%	29%	29%
Standard deviation	20%	80%	80%	50%	80%	33%
Max drawdown	33%	86%	100%	56%	91%	36%
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
2x Transaction Costs	SX5T	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	6%	-1%	-16%	8%	-1%	14%
Standard deviation	20%	20%	20%	20%	20%	20%
Sharpe	0.30	-0.03	-0.80	0.38	-0.05	0.67
Max drawdown	33%	48%	85%	27%	52%	22%
Weight	1	0.25	0.25	0.39	0.25	0.61
Daily skewness	-0.02	-0.47	0.29	-0.15	0.16	0.25
Daily excess kurtosis	4.22	8.52	8.28	28.64	8.03	74.10
Average transaction costs		0.71%	0.71%	0.93%	0.71%	1.08%
Mean	6%	-3%	-65%	19%	-4%	22%
Standard deviation	20%	82%	81%	51%	82%	33%
Max drawdown	33%	97%	100%	59%	99%	36%
Average transaction costs		2.89%	2.89%	2.35%	2.89%	1.77%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 1.0/-3.6.

Figure 4.3 plots the log margin account growth for each strategy. It can be seen, especially by examining the S/S strategy, that the transaction costs of trading VSTOXX futures have a large impact on the strategy returns.

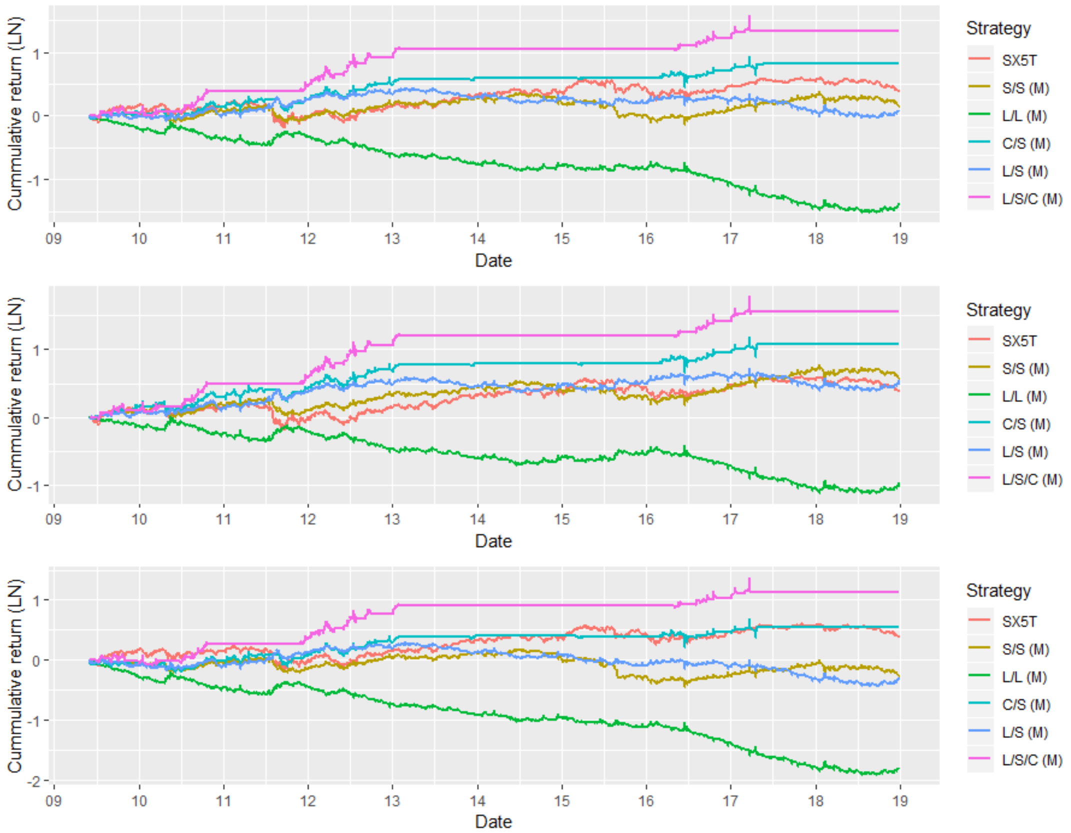

Figure 4.3: Log margin account growth for monthly strategies on VSTOXX and SX5T

Note: The figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 1.0/-3.6.

Table 4.4 reports factor loadings for the strategies. The L/S/C strategy is the only strategy with a significant positive CAPM alpha. Both the C/S and L/C/S strategy produces positive and significant four-factor alphas of 9.3%, and 13.4% respectively.

Table 4.4: Factor loadings for monthly strategies on VSTOXX

VSTOXX Monthly Trading Strategies					
	S/S	**L/L**	**C/S**	**L/S**	**L/C/S**
CAPM					
Constant	-0.006	**-0.084**	0.072	0.031	**0.119**
	0.044	0.027	0.033	0.042	0.036
Excess market	**0.576**	**-0.573**	**0.421**	-0.033	**0.205**
	0.030	0.029	0.051	0.067	0.050
R^2	0.266	0.263	0.142	0.000	0.033
Fama French 3-Factor					
Constant	0.006	**-0.095**	0.083	0.036	**0.131**
	0.029	0.027	0.033	0.042	0.037
Excess Market	**0.463**	**-0.478**	**0.319**	-0.064	**0.103**
	0.038	0.038	0.055	0.071	0.051
SMB	**-0.452**	**0.388**	**-0.323**	-0.008	**-0.275**
	0.073	0.069	0.085	0.104	0.084
HML	-0.053	0.058	0.064	0.144	0.132
	0.058	0.058	0.064	0.094	0.071
R^2	0.280	0.273	0.149	0.002	0.040
Carhart 4 - Factor					
Constant	-0.011	**-0.077**	**0.093**	0.065	**0.134**
	0.029	0.027	0.033	0.042	0.038
Excess Market	**0.467**	**-0.483**	**0.316**	-0.072	**0.102**
	0.038	0.038	0.052	0.065	0.049
SMB	**-0.467**	**0.404**	**-0.314**	0.019	**-0.273**
	0.071	0.067	0.086	0.102	0.084
HML	0.031	-0.032	0.014	-0.010	0.118
	0.061	0.062	0.781	0.110	0.090
Momentum	**0.189**	**-0.202**	-0.112	-0.342	-0.031
	0.045	0.045	0.069	0.082	0.070
R^2	0.287	0.281	0.151	0.024	0.040
T	2,447	2,447	2,447	2,447	2,447
T	3,271	3,271	3,271	3,271	3,271

Note: The table presents the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicates those that are reliably different from 0 on the 5% level. Transaction costs are calculated according to the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 1.0/-3.6

4.3 Daily Futures Strategies

This section presents the result of the strategies where daily decisions are made based on the ex-ante estimated volatility premium. Just as for the monthly strategies, contracts are rolled ahead of expiration to avoid illiquidity. There is no reevaluation of the S/S and L/L strategies since they do not include trading on any signal and because returns are calculated daily also for monthly strategies. The S/S strategy is included as an additional benchmark.

4.3.1 VIX Daily Futures Strategies (1)

Implementing active daily strategies yields higher returns than the same strategies with monthly decisions. The daily strategies perform in line with the monthly strategies up until 2015 after which the daily strategies exhibit stronger performance. Cheng (2018) finds that his results are robust to trading on a daily and monthly basis, and the thesis finds the same results for the same period. The results are outlined in table 4.5.

Table 4.5: Performance of daily trading strategies on VIX (1)

			VIX Jan 2006 - Dec 2018		
Base Case	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	8%	7%	18%	18%	26%
Standard deviation	19%	19%	19%	19%	19%
Sharpe	0.41	0.38	0.93	0.96	1.35
Max drawdown	55%	44%	28%	33%	26%
Weight	1	0.24	0.36	0.24	0.39
Daily skewness	-0.12	-1.92	-0.26	2.27	0.53
Daily excess kurtosis	11.47	51.97	12.31	49.77	20.96
Number of trades		157	679	709	722
Number of days L/S/C		0/3,271/0	0/2,385/886	886/2,385/0	85/1,603/1,583
Number of trading days	3,271	3,271	3,271	3,271	3,271
Average transaction costs		0.09%	0.08%	0.10%	0.78%
Mean	8%	31%	49%	75%	66%
Standard deviation	19%	80%	53%	78%	49%
Max drawdown	55%	97%	64%	83%	57%
Average transaction costs		0.38%	0.23%	0.42%	0.20%
W/O Transaction Costs	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	8%	8%	22%	24%	30%
Standard deviation	19%	19%	19%	19%	19%
Sharpe	0.41	0.44	1.15	1.26	1.56
Max drawdown	55%	43%	26%	30%	16%
Weight	1	0.24	0.36	0.25	0.39
Daily skewness	-0.12	-1.93	-0.22	2.40	0.60
Daily excess kurtosis	11.47	52.12	12.54	51.49	21.20
Average transaction costs		0.00%	0.00%	0.00%	0.00%
Mean	8%	35%	61%	97%	76%
Standard deviation	19%	80%	53%	78%	49%
Max drawdown	55%	97%	60%	80%	38%
Average transaction costs		0.00%	0.00%	0.00%	0.00%
2 x Transaction costs	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	8%	6%	13%	13%	22%
Standard deviation	19%	19%	19%	19%	19%
Sharpe	0.41	0.33	0.70	0.66	1.14
Max drawdown	55%	46%	32%	38%	36%
Weight	1	0.24	0.36	0.24	0.39
Daily skewness	-0.12	-1.91	-0.31	2.06	0.46
Daily excess kurtosis	11.47	51.72	12.01	47.84	20.62
Average transaction costs		0.18%	0.16%	0.20%	0.15%
Mean	8%	26%	37%	52%	56%
Standard deviation	19%	80%	54%	79%	49%
Max drawdown	55%	97%	70%	91%	70%
Average transaction costs		0.76%	0.45%	0.83%	0.38%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

As can be seen in Table 4.5 the daily strategies trade a lot more than the monthly. The ex-ante estimated signal can change the sign on any day t, and when it does the C/S and L/S strategies change position. The strategy with the highest number of trades over the period is the L/S/C strategy where the Sharpe ratio has been retroactively maximized, resulting in thresholds of -2.6 and 0.8.[24]

The C/S, L/S, and L/S/C strategies on VIX futures can be multiplied by a factor of 3.28, 2.85 and 5.51 respectively while maintaining a higher Sharpe ratio than the benchmark, SPXT. As expected the daily strategies are more sensitive to transaction costs. This can also be seen in Figure 4.4 below.

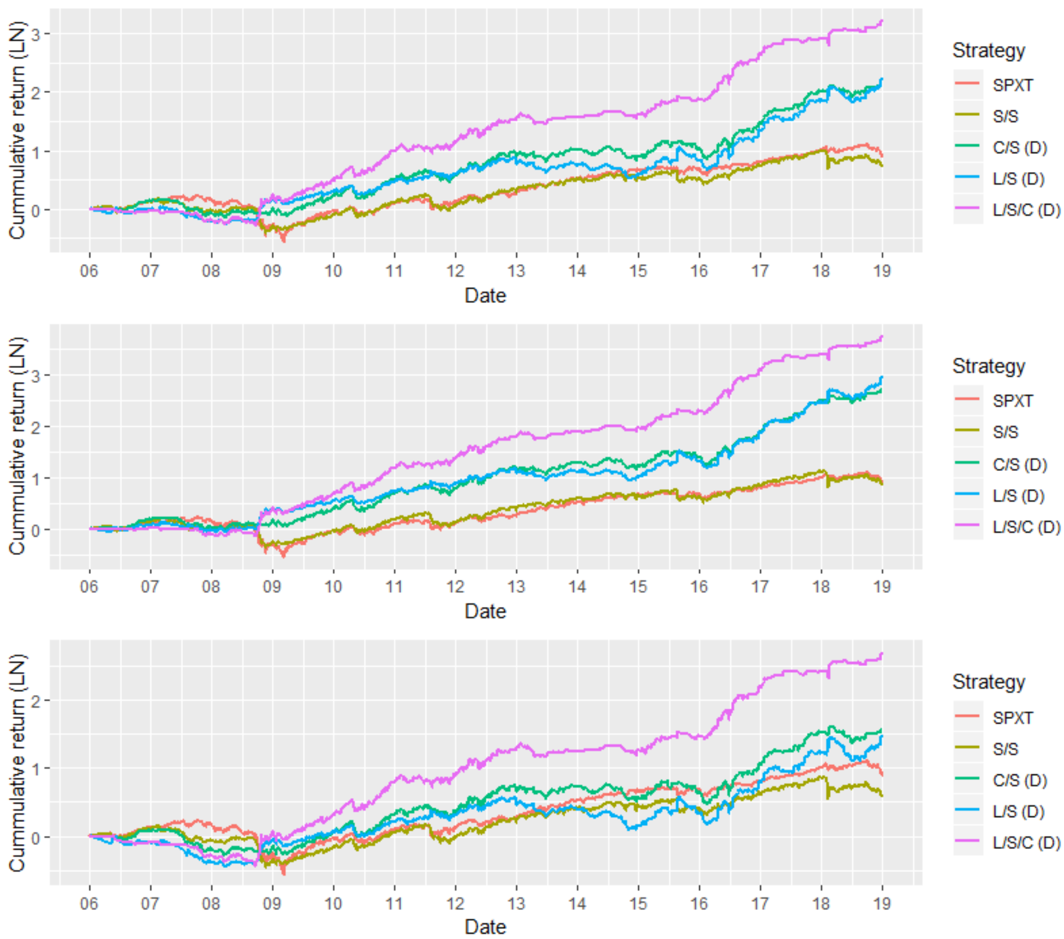

Figure 4.4: Log margin account growth for daily strategies on VIX and SPXT (1)

Note: The figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

[24]See Table A6.3 in the Appendix.

Table 4.6 reports factor loadings for the daily VIX strategies. The C/S, L/S, and L/S/C strategies earn significant four-factor annualized alphas of 12.3%, 17.7%, and 23.5% respectively. As seen by both Table 4.2 and Table 4.6, trading more frequently improves not only Sharpe ratios and drawdowns but also generate higher alphas than monthly strategies on the same index futures.

Table 4.6: Factor loadings for daily strategies on VIX (1)

VIX Daily Trading Strategies				
	S/S	**C/S**	**L/S**	**L/S/C**
CAPM				
Constant	0.012	**0.128**	**0.169**	**0.229**
	0.029	0.046	0.060	0.057
Excess market	**0.634**	**0.515**	0.073	**0.272**
	0.055	0.078	0.077	0.113
R^2	0.406	0.268	0.005	0.075
Fama French 3-Factor				
Constant	0.009	**0.129**	**0.175**	**0.234**
	0.037	0.046	0.060	0.058
Excess market	**0.661**	**0.491**	0.014	0.215
	0.064	0.080	0.075	0.115
SMB	-0.015	**0.198**	**0.277**	**0.325**
	0.652	0.063	0.065	0.086
HML	**-0.137**	0.050	**0.203**	**0.171**
	0.052	0.057	0.051	0.072
R^2	0.410	0.276	0.028	0.101
Carhart 4-Factor				
Constant	0.006	**0.129**	**0.177**	**0.235**
	0.029	0.046	0.061	0.058
Excess market	**0.683**	**0.495**	-0.002	0.207
	0.060	0.079	0.077	0.115
SMB	-0.018	**0.198**	**0.279**	**0.326**
	0.063	0.063	0.064	0.087
HML	-0.008	0.077	0.111	0.124
	0.054	0.067	0.058	0.088
Momentum	**0.172**	0.036	**-0.122**	-0.063
	0.030	0.041	0.051	0.052
R^2	0.423	0.276	0.034	0.102
T	3,271	3,271	3,271	3,271

Note: The table present the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicate those that are reliably different from 0 on the 5% level. Transaction costs are calculated according with the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

4.3.2 VSTOXX Daily Futures Strategies

Daily trading strategies on VSTOXX futures yield significantly better returns than monthly despite higher transaction costs. Among the monthly strategies, it was only the C/S strategy that performed better than the SX5T while all active daily strategies outperformed the benchmark. Table 4.7 outlines the result of daily trading strategies.

Table 4.7: Performance of daily trading strategies on VSTOXX

VSTOXX Feb 2009 - Dec 2018					
Base Case	SX5T	S/S	C/S	L/S	L/S/C
Mean excess return	6%	3%	30%	31%	38%
Standard deviation	20%	20%	20%	20%	20%
Sharpe	0.30	0.17	1.50	1.54	1.86
Max drawdown	33%	40%	15%	24%	15%
Weight	1	0.25	0.44	0.25	0.26
Daily skewness	-0.02	-0.35	2.20	1.84	2.38
Daily excess kurtosis	4.22	8.26	21.59	14.13	18.71
Number of trades		115	461	516	754
Number of days L/S/C		0/2,447/0	0/1,105/1,342	1,342/1,105/0	887/1,105/455
Number of trading days		2,447	2,447	2,447	2,447
Average transaction costs		0.36%	0.28%	0.30%	0.20%
Mean	6%	14%	69%	126%	143%
Standard deviation	20%	80%	46%	82%	77%
Max drawdown	33%	93%	32%	72%	50%
Average transaction costs		1.44%	0.64%	1.21%	0.78%
W/O Transaction Costs	SX5T	S/S	C/S	L/S	L/S/C
Mean excess return	6%	8%	44%	47%	54%
Standard deviation	20%	20%	20%	20%	20%
Sharpe	0.30	0.39	2.18	2.32	2.66
Max drawdown	33%	31%	12%	15%	13%
Weight	1	0.25	0.45	0.25	0.26
Daily skewness	-0.02	-0.35	2.61	2.06	2.61
Daily excess kurtosis	4.22	8.51	23.61	15.55	20.24
Average transaction costs		0.00%	0.00%	0.00%	0.00%
Mean	6%	31%	99%	190%	203%
Standard deviation	20%	80%	45%	82%	76%
Max drawdown	33%	86%	26%	53%	44%
Average transaction costs		0.00%	0.00%	0.00%	0.00%
2 x Transaction Costs	SX5T	S/S	C/S	L/S	L/S/C
Mean excess return	6%	-1%	16%	15%	21%
Standard deviation	20%	20%	20%	20%	20%
Sharpe	0.30	-0.03	0.82	0.73	1.04
Max drawdown	33%	48%	30%	34%	21%
Weight	1	0.25	0.43	0.24	0.25
Daily skewness	-0.02	-0.47	1.55	1.35	1.89
Daily excess kurtosis	4.22	8.52	190.10	12.01	16.24
Average transaction costs		0.72%	0.55%	0.58%	0.39%
Mean	6%	-3%	39%	63%	83%
Standard deviation	20%	82%	47%	85%	79%
Max drawdown	33%	97%	61%	73%	71%
Average transaction costs		2.89%	1.28%	2.42%	1.56%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.0/-0.8.

In the L/S/C strategy, the Sharpe ratio is maximized when the negative signal is -0.8 and the positive is 0.0[25]. The L/S/C strategy trades significantly more than both the C/S and the L/S strategy and is nearly as volatile as the L/S strategy but does not experience as sizable drawdowns.

The daily strategies perform vastly better than the monthly strategies. As reported in

[25]See Table A6.4 in the Appendix

Section 4.1.1, the transaction costs are relatively large for futures strategies on VSTOXX. The daily C/S, L/S, and L/S/C strategies on VSTOXX futures can be multiplied by a factor of 2.78, 2.55 and 2.96 respectively while maintaining a higher Sharpe ratio than the benchmark, SX5T.

Figure 4.5 plots the log margin account growth for the daily strategies on VSTOXX futures.

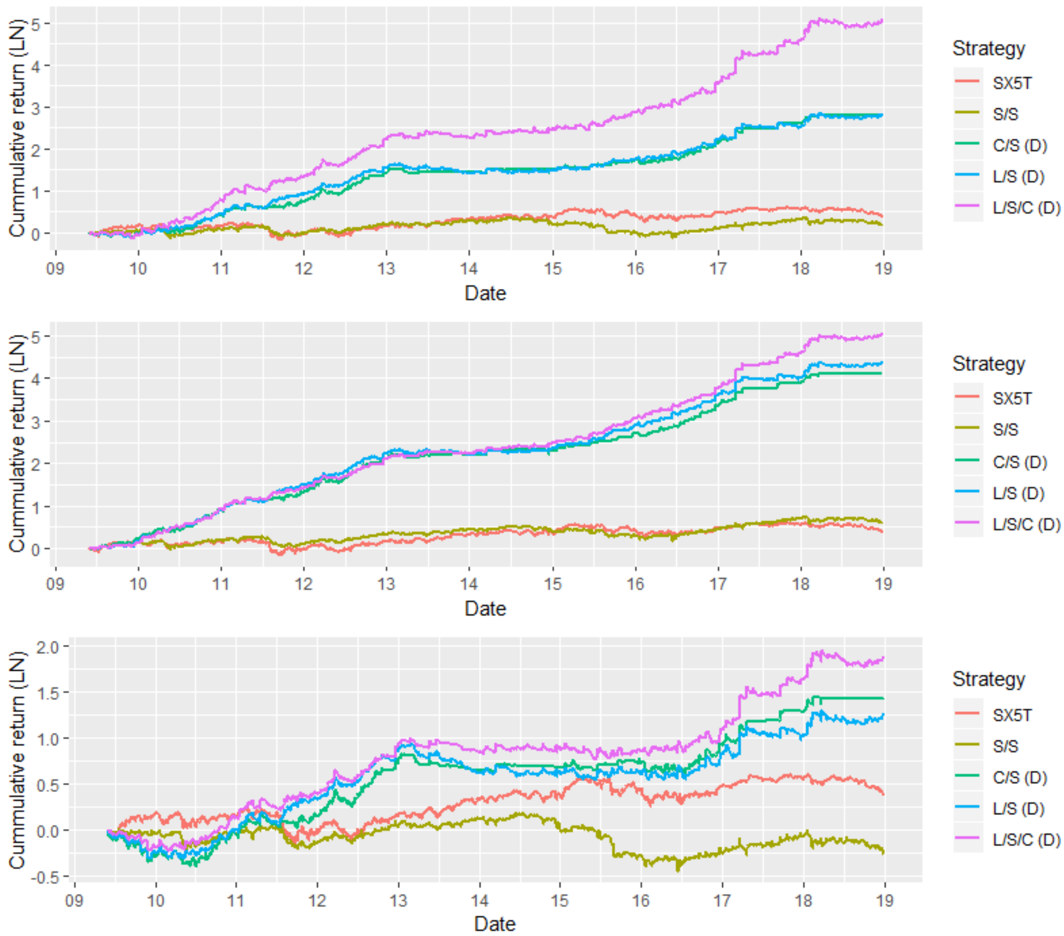

Figure 4.5: Log margin account growth for daily strategies on VSTOXX and SX5T

Note: The figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.0/-0.8.

Factor loadings for the strategies are reported in Table 4.8. The C/S, L/S, and L/S/C strategies earn significant four-factor annualized alphas of 29.6%, 34.6%, and 23.9% respectively. Thus, trading on a daily horizon significantly improves, not only Sharpe ratios and drawdowns, but alphas as well. The strategy with retroactively chosen thresholds, i.e., L/S/C, produces smaller alphas than the C/S and L/S strategies and have higher loadings on the HML factor.

Table 4.8: Factor loadings for daily strategies on VSTOXX

VSTOXX Daily Trading Strategies				
	S/S	**C/S**	**L/S**	**L/S/C**
CAPM				
Constant	-0.006	**0.274**	**0.314**	**0.209**
	0.044	0.051	0.064	0.063
Excess market	**0.576**	**0.398**	**-0.104**	-0.008
	0.030	0.042	0.049	0.049
R^2	0.266	0.127	0.008	0.000
Fama French 3-Factor				
Constant	**0.006**	**0.289**	**0.320**	**0.220**
	0.029	0.052	0.064	0.063
Excess Market	**0.463**	**0.265**	**-0.146**	-0.087
	0.038	0.047	0.051	0.050
SMB	**-0.452**	**-0.366**	0.013	-0.085
	0.073	0.089	0.095	0.092
HML	-0.053	**0.160**	**0.225**	**0.274**
	0.058	0.071	0.074	0.077
R^2	0.280	0.139	0.012	0.000
Carhart 4 - Factor				
Constant	-0.011	**0.296**	**0.346**	**0.239**
	0.029	0.052	0.064	0.064
Excess Market	**0.467**	**0.264**	**-0.153**	-0.092
	0.038	0.047	0.050	0.050
SMB	**-0.467**	**-0.360**	0.036	-0.067
	0.071	0.090	0.092	0.094
HML	0.031	0.126	0.095	**0.173**
	0.061	0.079	0.087	0.088
Momentum	**0.189**	-0.075	**-0.290**	**-0.224**
	0.045	0.060	0.069	0.067
R^2	0.287	0.140	0.029	0.017
T	2,447	2,447	2,447	2,447

Note: The table present the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicate those that are reliably different from0 on the 5% level. Transaction costs are calculated according with the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.0/-0.8.

4.4 Same Period Performance

The full sample of strategies on the VIX is subject to the financial crisis, possibly distorting a comparison of results obtained from VSTOXX strategies. Therefore, VIX strategies are tested for the period corresponding to that of VSTOXX. This section reports correlations between strategies on the VIX and the VSTOXX.

4.4.1 VIX Monthly Futures Strategies (2)

The results from the monthly trading strategies are reported in Table 4.9. Comparing the shorter sample with the results from the sample starting 2006 (see Section 4.1) it can be seen that removing the financial crisis improves the result of the S/S, C/S and L/S/C but the L/S and the L/L perform worse in terms of Sharpe ratio. Furthermore, the SPXT index improves significantly and delivers a Sharpe ratio of 0.86. While the S/S strategy performs better in the post-2009 regime, it is still subject to a significant drawdown of 97% in connection with Volmageddon of February 5^{th} 2018.

The Sharpe ratio of the L/S/C strategy is maximized when the positive signal is at 3.4 and the negative at -2.0, resulting in a strategy that never goes long VIX futures. The L/S/C strategy[26] is essentially a C/S strategy but with a larger threshold to enter a short position. The retroactively optimized L/S/C strategy is the only one performing significantly better than the benchmark stock index, SPXT, over this investigated period. With the base case of transaction costs, the C/S strategy performs slightly better than SPXT, but when transaction costs are doubled it performs slightly worse. For the L/S/C transaction costs can be multiplied by a factor of 27.62 while still maintaining a higher Sharpe ratio than the market.

[26]See Table A6.5 in the Appendix for Sharpe ration of all tested thresholds.

Table 4.9: Performance of monthly strategies on VIX (2)

			VIX Jun 2009 - Dec 2018			
Base Case	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	13%	9%	-11%	13%	9%	17%
Standard deviation	15%	15%	15%	15%	15%	11%
Sharpe	0.86	0.61	-0.73	0.88	0.57	1.53
Max drawdown	19%	22%	72%	28%	29%	9%
Weight	1	0.18	0.18	0.24	0.18	0.73
Daily skewness	-0.39	-2.05	2.06	-0.62	1.41	0.02
Daily excess kurtosis	4.11	53.93	54.05	12.58	53.38	17.66
Number of trades		116	116	103	116	29
Number of days L/S/C		0/2,413/0	2413/0/0	0/1873/540	540/1873/0	0/444/1971
Number of trading days	2,413	2,413	2,413	2,413	2,413	2,413
Average transaction costs		0.05%	0.05%	0.06%	0.05%	0.12%
Mean	13%	52%	-60%	56%	49%	32%
Standard deviation	15%	85%	85%	64%	85%	21%
Max drawdown	19%	97%	100%	80%	93%	16%
Average transaction costs		0.30%	0.30%	0.25%	0.30%	0.16%
W/O Transaction Costs	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	13%	10%	-10%	14%	9%	24%
Standard deviation	15%	15%	15%	15%	15%	15%
Sharpe	0.86	0.65	-0.69	0.92	0.62	1.56
Max drawdown	19%	22%	70%	28%	29%	12%
Weight	1	0.18	0.18	0.24	0.18	0.73
Daily skewness	-0.39	-2.05	2.05	-0.62	1.41	0.05
Daily excess kurtosis	4.11	54.01	54.01	12.61	53.43	17.66
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
Mean	13%	55%	-56%	59%	52%	32%
Standard deviation	15%	85%	85%	64%	85%	21%
Max drawdown	19%	97%	100%	79%	91%	16%
Average transaction costs		0.00%	0.00%	0.00%	0.00%	0.00%
2x Transaction Costs	SPXT	S/S	L/L	C/S	L/S	L/S/C
Mean excess return	13%	9%	-12%	13%	8%	23%
Standard deviation	15%	15%	15%	15%	15%	15%
Sharpe	0.86	0.57	-0.77	0.84	0.53	1.51
Max drawdown	19%	22%	73%	28%	30%	12%
Weight	1	0.18	0.18	0.24	0.18	0.73
Daily skewness	-0.39	-2.04	2.06	-0.62	1.42	0.01
Daily excess kurtosis	4.11	53.82	54.05	12.54	53.29	17.65
Average transaction costs		0.11%	0.11%	0.11%	0.11%	0.22%
Mean	13%	48%	-63%	53%	45%	31%
Standard deviation	15%	85%	85%	64%	85%	21%
Max drawdown	19%	97%	100%	80%	94%	16%
Average transaction costs		0.60%	0.60%	0.49%	0.60%	0.31%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0.

Figure 4.6 illustrates the log margin account growth for monthly strategies on VIX futures. The L/S/C strategy holds cash from September 2009 and onwards.

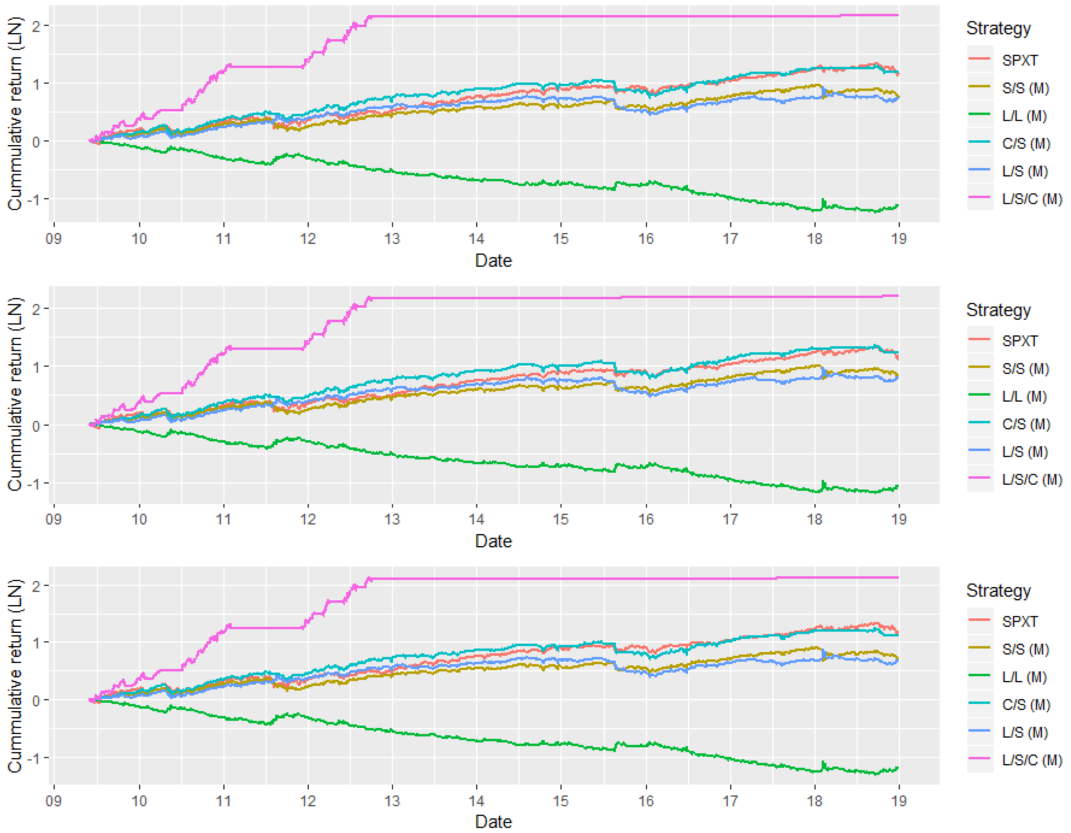

Figure 4.6: Log margin account growth for monthly strategies on VIX and SPXT (2)

Note: The figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0.

As can be seen from Table 4.10, no strategy, expect the L/S/C, deliver significant alphas over this period. Both the S/S and C/S have high market loadings.

Table 4.10: Factor loadings for monthly strategies on VIX (2)

VIX Monthly Trading Strategies					
	S/S	**L/L**	**C/S**	**L/S**	**L/S/C**
CAPM					
Constant	0.000	-0.032	0.051	0.050	**0.188**
	0.025	0.024	0.031	0.039	0.048
Excess market	**0.633**	**-0.633**	**0.557**	0.213	**0.322**
	0.040	0.040	0.063	0.094	0.071
R^2	0.423	0.423	0.327	0.048	0.109
Fama French 3-Factor					
Constant	-0.005	-0.027	0.051	0.054	**0.194**
	0.025	0.025	0.031	0.040	0.049
Excess market	**0.668**	**-0.668**	**0.561**	0.186	**0.283**
	0.049	0.049	0.068	0.105	0.066
SMB	**-0.153**	**0.153**	-0.042	0.076	0.092
	0.058	0.054	0.052	0.092	0.048
HML	**-0.074**	**0.077**	0.032	**0.130**	**0.228**
	0.037	0.037	0.044	0.130	0.062
R^2	0.430	0.429	0.327	0.055	0.124
Carhart 4-Factor					
Constant	-0.006	-0.026	0.051	0.055	**0.197**
	0.025	0.025	0.031	0.040	0.049
Excess market	**0.665**	**-0.665**	**0.561**	0.189	**0.287**
	0.049	0.049	0.067	0.104	0.066
SMB	**-0.150**	**0.150**	-0.042	0.073	0.086
	0.058	0.058	0.053	0.092	0.049
HML	-0.053	0.056	0.030	0.106	**0.190**
	0.038	0.038	0.052	0.063	0.075
Momentum	0.044	-0.042	-0.003	-0.047	-0.077
	0.029	0.029	0.058	0.054	0.064
R^2	0.430	0.430	0.327	0.053	0.127
T	2,413	2,413	2,413	2,413	2,413

Note: The table presents the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicates those that are reliably different from0 on the 5% level. Transaction costs are calculated according to the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 3.4/-2.0

Table 4.11 reports the daily correlation between monthly strategies. As expected, all strategies on the VIX have a positive correlation with the corresponding strategy on VSTOXX. Introducing a long component naturally removes some of the correlation with underlying stock indexes. The L/S has the smallest correlation between strategy pairs at 0.07 while the strongest correlation, when the benchmark indexes are excluded, is between the passive S/S strategies.

Table 4.11: Daily correlation of monthly trading strategies

	VSTOXX						VIX					
	SX5T	S/S	L/L	C/S	L/S	L/S/C	SPXT	S/S	L/L	C/S	L/S	L/S/C
SX5T	1.00	0.57	-0.57	0.36	-0.11	0.00	0.64	0.42	-0.42	0.42	0.22	-0.01
S/S	0.57	1.00	-0.98	0.63	-0.19	0.01	0.37	0.45	-0.44	0.44	0.23	0.00
L/L	-0.57	-0.98	1.00	-0.62	0.21	0.00	-0.36	-0.44	0.44	-0.44	-0.22	0.01
C/S	0.36	0.63	-0.62	1.00	0.63	0.00	0.26	0.24	-0.24	0.31	0.23	-0.01
L/S	-0.11	-0.19	0.21	0.63	1.00	-0.01	-0.03	-0.13	0.13	-0.04	0.07	-0.01
L/S/C	0.00	0.01	0.00	0.00	-0.01	1.00	-0.01	-0.01	0.01	-0.02	-0.01	0.94
SPXT	0.64	0.37	-0.36	0.26	-0.03	-0.01	1.00	0.65	-0.65	0.58	0.22	-0.01
S/S	0.42	0.45	-0.44	0.24	-0.13	-0.01	0.65	1.00	-1.00	0.75	0.14	-0.01
L/L	-0.42	-0.44	0.44	-0.24	0.13	0.01	-0.65	-1.00	1.00	-0.75	-0.13	0.01
C/S	0.42	0.44	-0.44	0.31	-0.04	-0.02	0.58	0.75	-0.75	1.00	0.75	-0.02
L/S	0.22	0.23	-0.22	0.23	0.07	-0.01	0.22	0.14	-0.13	0.75	1.00	-0.02
L/S/C	-0.01	0.00	0.01	-0.01	-0.01	0.94	-0.01	-0.01	0.01	-0.02	-0.02	1.00

Note: Reported correlations are calculated with complete sets method based on trading days according to VSTOXX futures trading days. Correlations are calculated according to base case transaction costs.

4.4.2 VIX Daily Futures Strategies (2)

Table 4.12 outlines the results of daily strategies on VIX for the same period as the strategies on VSTOXX. All active strategies perform better in terms of Sharpe ratio in the sample starting in 2009. Much of the improvement in the Sharpe ratio can be traced back to lower standard deviations rather than higher mean excess returns. In light of the vastly better performance of the benchmark, the daily strategies in the shorter sample do not improve considerably.

Table 4.12: Performance of daily strategies on VIX (2)

		VIX Jun 2009 - Dec 2018			
Base case	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	13%	9%	17%	17%	25%
Standard deviation	15%	15%	15%	15%	15%
Sharpe	0.86	0.61	1.16	1.13	1.64
Max drawdown	19%	22%	21%	25%	13%
Weight	1	0.18	0.26	0.18	0.30
Daily skewness	-0.39	-2.05	-0.24	2.47	0.55
Daily excess kurtosis	4.11	53.93	11.83	51.95	23.26
Number of trades		116	532	548	555
Number of days L/S/C		0/2,413/0	0/1,908/505	505/1,908/0	41/1,278/1,094
Number of trading days	2,413	2,413	2,413	2,413	2,413
Average transaction costs		0.05%	0.05%	0.09%	0.09%
Mean	13%	52%	66%	94%	82%
Standard deviation	15%	85%	57%	83%	50%
Max drawdown	19%	97%	64%	83%	38%
Average transaction costs		0.30%	0.20%	0.36%	0.17%
Without transaction costs	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	13%	10%	20%	21%	27%
Standard deviation	15%	15%	15%	15%	15%
Sharpe	0.86	0.65	1.36	1.38	1.81
Max drawdown	19%	22%	19%	23%	12%
Weight	1	0.18	0.26	0.18	0.30
Daily skewness	-0.39	-2.05	-0.20	2.60	0.63
Daily excess kurtosis	4.11	54.01	12.02	53.31	23.37
Average transaction costs		0.00%	0.00%	0.00%	0.00%
Mean	13%	55%	77%	114%	91%
Standard deviation	15%	85%	57%	83%	50%
Max drawdown	19%	97%	60%	80%	37%
Average transaction costs		0.00%	0.00%	0.00%	0.00%
Transaction costs x 2	SPXT	S/S	C/S	L/S	L/S/C
Mean excess return	13%	9%	14%	13%	22%
Standard deviation	15%	15%	15%	15%	15%
Sharpe	0.86	0.57	0.96	0.87	1.46
Max drawdown	19%	22%	23%	30%	14%
Weight	1	0.18	0.26	0.18	0.30
Daily skewness	-0.39	-2.04	-0.29	2.27	0.46
Daily excess kurtosis	4.11	53.82	11.60	50.43	23.12
Average transaction costs		0.11%	0.10%	0.13%	0.15%
Mean	13%	48%	55%	73%	73%
Standard deviation	15%	85%	58%	84%	50%
Max drawdown	19%	97%	96%	91%	41%
Average transaction costs		0.60%	0.40%	0.73%	0.32%

Note: The table presents the strategies with different transaction costs in the different sections. All strategies have been retroactively weighted to have the same standard deviation as the underlying stock index. All affected performance measures have been reported separately in the bottom of each table section with a weight equal to 1. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

Removing the financial crisis does not change the imposed thresholds for the L/S/C strategy. For the new shorter sample, the maximized Sharpe thresholds are 0.8 and -2.8 for the positive and negative signal respectively. See details in Table A6.6 in the Appendix.

The daily C/S, L/S, and L/S/C strategies on VIX futures can be multiplied by a factor of 2.50, 2.06, and 5.38 respectively while maintaining a higher Sharpe ratio than the benchmark, SPXT. The strategies display somewhat lower sensitivity to increases in transaction costs than those in the sample starting 2006 (see Section 4.3.1).

Figure 4.7 plots the log margin account growth for the different strategies. It is clear that the active L/S strategy performs worse than the S/S strategy during extended parts of the sample.

Figure 4.7: Log margin account growth for daily strategies on VIX (2)

Note: The figure illustrates the strategies with different a base case of transaction costs (Top), w/o transaction cost (Middle) and double transaction costs (Bottom). All strategies have been retroactively scaled to have the same standard deviation as the underlying stock index to improve comparability. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

Table 4.13 present the alphas and factor loading for the daily strategies over the shorter sample. Trading daily improves upon the results presented in the previous section. A difference from the full sample is that the C/S strategy does not produce a significant CAPM alpha. Alphas are however significant when more factors are included. All strategies produce lower alphas than those on the full sample.

Table 4.13: Factor loadings for daily strategies on VIX (2)

VIX Daily Trading Strategies				
	S/S	**C/S**	**L/S**	**L/S/C**
CAPM				
Constant	0.000	0.099	**0.149**	**0.195**
	0.025	0.041	0.055	0.044
Excess market	**0.633**	**0.556**	0.138	**0.379**
	0.040	0.054	0.077	0.077
R^2	0.423	0.326	0.020	0.151
Fama French 3-Factor				
Constant	-0.005	**0.102**	**0.157**	**0.201**
	0.025	0.042	0.057	0.044
Excess market	**0.668**	**0.539**	0.082	**0.334**
	0.049	0.059	0.087	0.081
SMB	**-0.153**	0.053	**0.215**	**0.149**
	0.058	0.049	0.087	0.057
HML	**-0.074**	0.071	**0.172**	**0.173**
	0.037	0.046	0.048	0.046
R^2	0.430	0.327	0.037	0.163
Carhart 4-Factor				
Constant	-0.006	**0.103**	**0.160**	**0.205**
	0.025	0.042	0.057	0.044
Excess market	**0.665**	**0.542**	0.089	**0.343**
	0.049	0.059	0.086	0.079
SMB	**-0.150**	0.049	**0.207**	**0.139**
	0.058	0.050	0.087	0.058
HML	-0.053	0.048	**0.119**	0.108
	0.038	0.051	0.056	0.056
Momentum	0.044	-0.047	**-0.109**	**-0.133**
	0.029	0.043	0.046	0.053
R^2	0.430	0.328	0.043	0.171
T	2,413	2,413	2,413	2,413

Note: The table present the strategies factor loadings for three regressions on the market, high minus low (HML), small minus big (SMB) and momentum. Units for alphas are annualized in percent/100.
Newey and West (1987) standard errors with 20 lags are reported under each coefficient. Bold coefficient indicate those that are reliably different from 0 on the 5% level. Transaction costs are calculated according with the base case. Imposed signal thresholds for the L/S/C strategy to go short/long are 0.8/-2.6.

Table 4.14 outlines the correlations of the daily strategies on VIX and VSTOXX futures. The active strategies are less correlated than the stock indexes and passive strategies. The C/S strategies have a higher correlation compared to the active strategies with a long leg. Table 4.11 and Table 4.14 shows that trading on a daily basis further removes correlation of actively traded strategies.

Table 4.14: Daily correlation of daily trading strategies

	VSTOXX					VIX				
	SX5T	S/S	C/S	L/S	L/S/C	SPXT	S/S	C/S	L/S	L/S/C
SX5T	1.00	0.57	0.37	-0.14	-0.06	0.64	0.42	0.44	0.19	0.33
S/S	0.57	1.00	0.60	-0.31	-0.21	0.37	0.45	0.41	0.13	0.31
C/S	0.37	0.60	1.00	0.55	0.58	0.25	0.29	0.36	0.22	0.36
L/S	-0.14	-0.31	0.55	1.00	0.93	-0.07	-0.10	0.02	0.14	0.12
L/S/C	-0.06	-0.21	0.58	0.93	1.00	-0.01	-0.04	0.09	0.19	0.17
SPXT	0.64	0.37	0.25	-0.07	-0.01	1.00	0.65	0.57	0.14	0.39
S/S	0.42	0.45	0.29	-0.10	-0.04	0.65	1.00	0.70	-0.01	0.41
C/S	0.44	0.41	0.36	0.02	0.09	0.57	0.70	1.00	0.69	0.74
L/S	0.19	0.13	0.22	0.14	0.19	0.14	-0.01	0.69	1.00	0.62
L/S/C	0.33	0.31	0.36	0.12	0.17	0.39	0.41	0.74	0.62	1.00

Note: Reported correlations are calculated with complete sets method based on trading days according to VSTOXX futures trading days. Correlations are calculated according to base case transaction costs.

5 Discussion

As mentioned in the introduction, the thesis set out to investigate whether it is (1) possible to avoid large drawdowns of short volatility strategies in a regime of higher volatility of volatility using ex-ante estimated volatility premiums and (2) whether it is possible to use them as a trading signal in other markets than the U.S. market. This section discusses the findings of Chapters 3 and Chapter 4 and provides answers to these questions.

The chapter starts with a discussion about the assumptions and the applicability of the findings before continuing with a discussion on the value of using the volatility premium as a signal. The final part provides a discussion about whether it is possible to disarm the ticking time bombs and avoid large drawdowns like those experienced on February 5^{th} 2018.

5.1 Assumptions and Applicability

In the thesis, a L/S/C strategy is considered where premium thresholds are implemented. Moving the thresholds away from zero should increase the certainty of the premium accurately predicting the signs of realized returns. The size of the thresholds are chosen retroactively to maximize the strategy's Sharpe ratio, meaning that these strategies suffer from forward-looking bias and are not implementable in practice. If a L/S/C strategy were to be implemented in reality, it could be done by using a rolling window on past returns.

In the result tables and plots of Chapter 4, the strategies have been given weights so that their standard deviation equals that of the underlying stock index. Weights are retroactively chosen to ease comparability between strategies and stock index returns. In practice, portfolio weights are likely chosen in line with traditional portfolio theory, e.g., by diversifying using a wide variety of asset classes and choosing portfolio weights according to rolling correlations. Several studies[27] have shown that trading volatility can improve portfolio performance following the asymmetric outcome driven by investors willing to pay hefty premiums to hedge their stock portfolios. The findings of the thesis show that

[27]See, e.g., Warren (2012)

implementing actively traded volatility strategies produces attractive risk-adjusted returns, significant alphas and lowers market loadings in comparison to both stock indexes and passive volatility strategies. Trading daily increases performance across all performance measures and also reduces the strategies internecine correlations.

To avoid forward-looking bias when backtesting strategies, the opening prices are used to calculate the premium. Per assumption, trades are executed on the same day's closing price, which is likely to affect the performance of the premium. Eriksen (2018) documents that most of the trading activity in the VIX futures market occurs during the latter part of the trading day, mostly concentrated to the 2 hours before close. This suggest that the signal could be improved if evaluated closer to the assumed time of trade. Arguably, the chosen methodology does not exacerbate the results of the strategies. If anything, the results should improve if the premiums were to be calculated using a futures price closer to the market's closing price.

When discussing the applicability of the findings in this thesis, it is important to consider to what extent it is possible to scale up these strategies without moving the market. Futures on the VIX index are more liquid and traded in much larger volumes than futures on VSTOXX. The volumes in Section 2.3 reveals that it might be challenging to scale the strategies on VSTOXX futures and still gain a Sharpe ratio of the same magnitude. Although liquidity has improved over the sample period, the VSTOXX futures market is far from the size of the VIX futures market. With an average daily traded vega of approximately 4 million USD for VSTOXX futures over the past three years and closer to 200 million USD for the corresponding contracts on VIX futures, it suggests that VSTOXX strategies are perhaps most suitable for boutique hedge funds and sophisticated family offices.

5.2 Value of the Volatility Premium

For the sample period 2006 through 2018, monthly trading strategies on the VIX performed worse than for the sample period of Cheng (2018). Nevertheless, strategies actively trading on the volatility premium avoid some of the most significant drawdowns and delivers high Sharpe ratios. The C/S strategy is the best performing strategy in terms of the

Sharpe ratio, which aligns with the findings of Cheng (2018). For the period investigated in the thesis, the S/S strategy underperformed the market in terms of Sharpe ratio. When examining the factor loadings, all monthly strategies deliver significant CAPM, three-factor and four-factor alphas.

Among the daily VIX strategies, the L/S strategy delivered the highest Sharpe ratio[28]. The finding differs from Cheng (2018) who found that the C/S was the best performing strategy in terms of Sharpe ratio. The result is, however, almost entirely attributable to being long in the somewhat turbulent period of the latter half of 2018. Thus, this finding seems hard to generalize over other periods, mainly because the strategy performed worse than a C/S strategy during most of the sample period. Generally, as Cheng (2018) explains, going long instead of entering a cash position when premiums are negative does not improve Sharpe ratios much because the subsequent investment volatility is high.

Cheng (2018) finds that the outcome of VIX futures strategies between 2004 and 2015 are stable to trading on a daily and monthly basis, which has been confirmed by testing the strategies on his sample. Implying that between 2004 and 2015, the trade-off between higher transaction costs and higher signal value associated with a shorter forecasting horizon was close to zero. One of the findings in the thesis is that trading daily is worthwhile in a the post-2015 regime. The result is not attributable to lower transaction costs, suggesting that the explanatory power of the premium increases with daily trading decisions on the post-2015 data. It could be attributable to the increase in hedging requirements of ETPs, as reported by Eriksen (2018) and Brøgger (2019). Brøgger (2019) finds that ETP hedging requirements sometimes distorts the VIX futures term structure. The distortion, driven by sudden changes in ETP hedging demand, could be better reflected on a shorter forecasting horizon; possibly explaining why daily strategies perform better in a post-2015 regime. Another possible explanation is, as mentioned in the introduction, that the volatility of volatility has been higher during 2016 and onwards. Figure 5.1 plots the daily and monthly C/S strategies starting in 2016.

[28]The L/S/C strategies are discussed separately further down in this section.

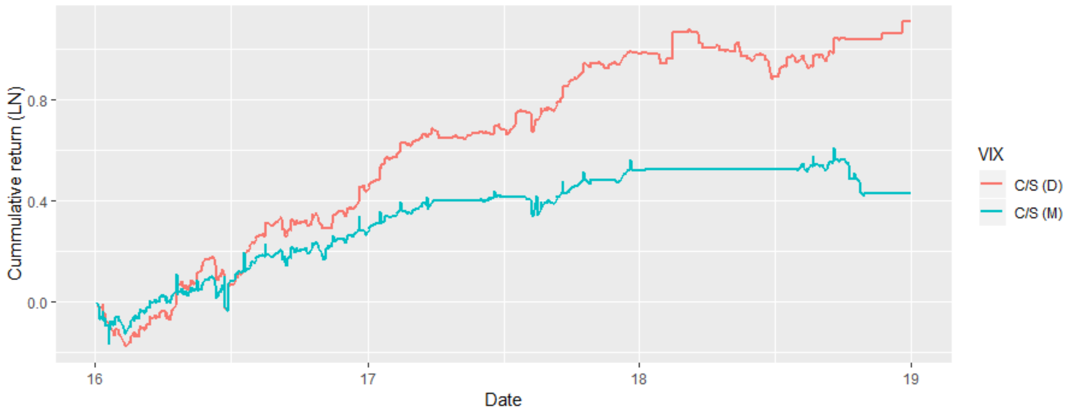

Figure 5.1: Log margin account growth for C/S strategies on VIX (2016-2018)

Note: The figure illustrates both the monthly and the daily C/S strategies from 2016 onwards and presents the period where making daily decisions outperforms making monthly decisions. Transaction costs as per the base case.

Differences in volatility and intra-month sign changes of the premium over the different subsamples are investigated. The differences in the number of intra-month sign changes are small, and the volatility of the premium is lower during the subsample starting in 2016. Hence, these tests do not provide an explanation of the daily strategies' outperformance. As mentioned, increased ETP hedging requirements and higher volatility of volatility are possible explanations; but they stand to be empirically proven.

For monthly strategies on VSTOXX futures, the only strategy that outperforms SX5T is the C/S, which holds cash for nearly half of the sample period. When evaluating SX5T as a benchmark one should bear in mind that is has performed considerably worse than its U.S. counterpart, SPXT. All other strategies performed worse than the market in terms of Sharpe ratio. The active L/S strategy delivers slightly worse returns than that of a passive S/S strategy which implies that either the signaling value of the VSTOXX premium is weak on monthly strategies, that going long VSTOXX futures is not profitable when premiums are negative or a combination of both. None of the strategies delivered significant CAPM or three-factor alphas, and the C/S strategy was the only one delivering a significant four-factor alpha. The S/S strategy loads negatively on the SMB factor. A possible explanation is that the strategy is essentially long SX5T, on which only large European stocks are listed and the SMB factor has the opposite exposure. But, as there is a significant discrepancy between the SX5T and the companies included in the European factor loadings data, a definite conclusion cannot be made.

Daily strategies using VSTOXX futures deliver higher Sharpe ratios than monthly strategies on VSTOXX. The L/S strategy is the best performing strategy, but it suffers from large drawdowns. Just as for the VIX futures strategies, the L/S only surpassed the C/S strategy in the latter part of 2018, and it is difficult to generalize the result. The Sharpe ratios of the active strategies are far better than the benchmark despite relatively high transaction costs. The unweighted C/S strategy has a smaller maximum drawdown than the market during this sample period. Both the C/S and L/S deliver large significant CAPM, three-factor and four-factor alphas.

To facilitate a comparison between futures strategies on VIX and VSTOXX, the VIX strategies were tested on a sample period starting from the inception of VSTOXX futures. In general, monthly strategies on the VIX outperformed monthly strategies on VSTOXX while daily strategies on VSTOXX outperformed daily strategies on VIX. For this sample period, it can be seen that the strategies on VIX futures improve compared to the sample period starting 2006, much attributable to the removal of the financial crisis. The monthly active strategies on the VIX for the shorter sample barely outperform the market due to the strong performance of SPXT.

As the factor loading tables in Chapter 4 describe, the alphas produced by the active strategies are larger than a passive S/S strategy, both on a monthly and daily basis and for both the VIX and VSTOXX. Introducing a long leg in the strategy reduces the market loading, which is consistent with the reported correlations and the fact that going long volatility index futures is essentially going short the market.

As can be seen in Table 4.14, which displays daily correlation of daily strategies for the post-2009 sample, all strategy pairs are somewhat positively correlated. The presence of a long component in either market reduces correlations. As established, the long leg functions as a hedge. The lowest reported correlation is between the C/S strategy on VIX futures and the L/S strategy on VSTOXX futures, and it amounts to 0.02. Since both strategies deliver high Sharpe ratios over the period, it indicates that portfolio performance can be improved not only by trading volatility actively, but trading volatility actively on both the U.S. and European markets.

One significant difference between the strategies on VSTOXX compared to VIX is that making daily trading decisions based on the premium outperforms a monthly strategy from inception. As mentioned earlier, the daily VIX strategies only exceed the monthly strategies from 2016 and onwards. It seems that the VIX premium has better predictive power over a longer horizon than the VSTOXX premium. Not only is the difference in performance between monthly and daily strategies on the VIX is smaller than for VSTOXX, the monthly strategies on VSTOXX also delivered Sharpe ratios that were generally low. The VSTOXX premium has a mean that is closer to zero than the VIX premium, and it has a slightly higher standard deviation. However, when examining the number of intra-month sign changes on the two different premiums the VIX premiums changes signs as many times as the VSTOXX premiums[29] on average. Consequently, intra-month sign changes of the premium does not seem to provide an answer to why monthly strategies on the VSTOXX performed poorly. The finding is surprising in light of the substantially higher transaction costs for VSTOXX futures.

When increasing the frequency of trading, there is a trade-off between the increased value of the signal and increased trading costs. Given the transaction costs reported in Section 4.1.1 this trade-off comes at a relatively lower price for strategies on VIX futures. However, transaction costs are not the only factor affecting the performance of strategies on volatility futures.

Section 2.2 reports regressions of changes in the volatility indexes as responses to changes in the underlying stock index. As can be seen by Equations 2.6 and 2.7, VIX falls more when SPXT goes up than VSTOXX does when SX5T goes up, naturally affecting the strategies over time. Since the parameter that is conditional on the stock index moving downwards is larger for the VIX compared to VSTOXX, there is more asymmetry in the behavior of the VIX w.r.t. its underlying, possibly playing a part in the payoff profiles of the strategies. The relatively worse performance of S/S strategies on VSTOXX futures might relate to the difference in the steepness of the volatility futures term structure as reported by Alexander et al. (2015). In perfectly efficient markets this would already be priced. However, perfectly efficient markets requires liquid markets for the prices to reflect all available information.

The volatility premiums consist of two parts: the risk-neutral expectation of the volatility index, i.e., the futures price, and the physical expectation of the volatility index, i.e., the forecasted spot value. Breaking down the two parts of the volatility premium might shed

[29]The VIX/VSTOXX Premium changes its sign on average 17.8%/17.4% of the trading days.

some light onto conclusions to be drawn.

The model performance of the chosen ARMA models for the two volatility indexes is shown in Table 3.1 in Section 3.2.3. The VSTOXX model performs in line with the VIX model, both on the 34-day horizon and the rolling forecast horizon, indicating that the physical measure of the premium is similar on the VIX and VSTOXX.

The other part of the premium, the futures price, is equally essential when examining the value of the premium. When considering a monthly trading strategy, the VSTOXX premium seems to have relatively worse signaling value compared to the VIX premium. Breaking down the premium into its two individual parts shows that the strength of the signal on longer forecast horizons depends on the liquidity of the futures market rather than the forecasting ability of the two models. Liquidity also has an other effect on strategy performance. If more investors trade, the premium is more exploited, and it will provide lower returns.

The best performing active strategy across both futures markets and on both sample periods is the L/S/C strategy. A finding that is of no surprise as the thresholds are chosen in order to maximize performance as measured by Sharpe ratio. These strategies, as discussed previously, can not be implemented in practice as they suffer from forward-looking bias. The purpose of investigating the L/S/C strategy is not to see if it is implementable in reality but rather to test whether strategy performance can be improved upon by imposing premium thresholds. Looking at the tables presenting the Sharpe ratio with different thresholds in Appendix A6, there seems to be a pattern where moving the threshold away from zero increases the Sharpe ratio. This relationship is especially evident for VIX strategies where the two sample periods maximize the Sharpe ratio over the same thresholds. For the full period, the monthly L/S/C strategy on VIX futures only goes long volatility futures for two months during the financial crisis. For the sample starting in 2009, the monthly L/S/C strategy never goes long, neither on VIX or VSTOXX.

The L/S/C strategies show that daily performance improves when choosing a signal threshold. In terms of Sharpe ratios of the daily strategies, the VSTOXX strategy improved 19% while the VIX strategy improved 40%[30]. Examining the tables under Section in the Appendix, the Sharpe ratio for strategies on the VIX increases with a larger

[30] 45% for the sample period sample starting 2009

threshold. The same pattern is not seen for the VSTOXX, where increasing the positive threshold results in a lower Sharpe ratio.

5.3 Avoiding Drawdowns

As shown in Chapter 4, it is possible to avoid drawdowns to some extent and investors can significantly improve upon a passive investment strategy. The active strategies are still volatile, and adding a long component when the ex-ante estimated volatility premium is negative, rather than holding cash, increases both the volatility and the maximum drawdowns. Since volatility indexes spike during periods of uncertainty, this is to be expected. As shown in Figure 3.2 illustrating the time series of premiums, they exhibit erratic behavior during these periods. The ex-ante signal does not correctly estimate the sign of ex-post realized returns with complete accuracy. For strategies on both indexes, there are however differences between the drawdowns of the passive strategies compared to the drawdown experienced by the actively traded ones. While the active strategies suffer from drawdowns, they are not sudden and of the same magnitude. For example, the S/S strategy on the VIX experienced a 97% drawdown in the aftermath of Volmageddon on February 5^{th} 2018. Trading daily rather than monthly also reduces the drawdowns on both markets.

The results show that trading volatility actively improves passive strategies significantly. However, since volatility strategies are highly volatile they will suffer from drawdowns. Active futures strategies on VIX and VSTOXX have performed better than their respective stock indexes. Figure 5.2 illustrates the unweighted plots of the daily active C/S strategies compared to the passive S/S strategy as well as the underlying stock index. The massive drawdown of 97% for the S/S strategy on the VIX is illustrated clearly in the top graph.

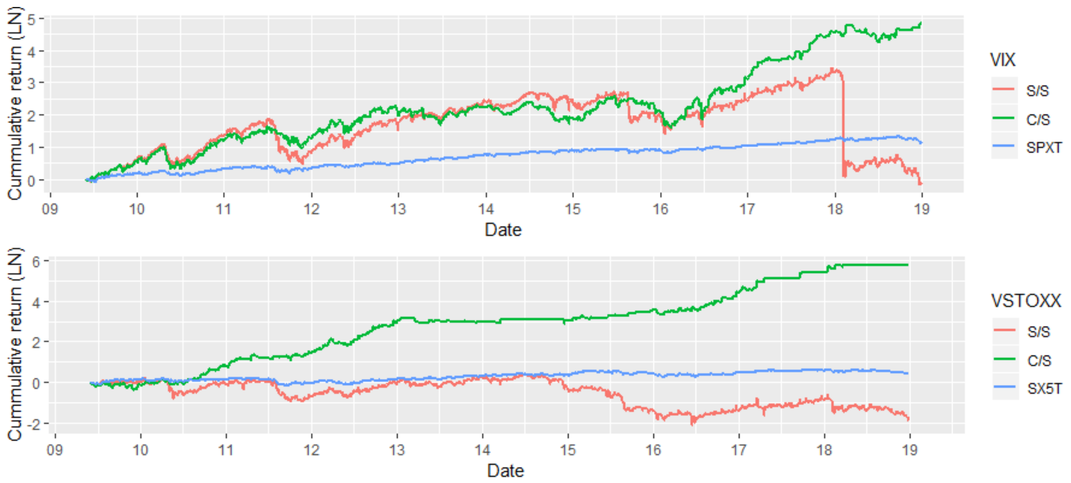

Figure 5.2: Log margin account growth for strategies on VIX and VSTOXX (unweighted)

Note: The figure covers the same period performance of VIX (Top) and VSTOXX (Bottom) futures samples w/o retroactively having weighted to strategies to reflect the standard deviation of the underlying stock index. The plotted C/S strategies is that of daily decisions and has been benchmarked against the passive S/S strategy and the stock index for the respective markets. Transaction costs as per the base case.

Although the VSTOXX experienced its largest daily increase in conjunction with Volmageddon, this was not transferred to the futures market to the same extent as in the VIX futures market. As reported by Brøgger (2018), ETPs seeking to hedge their exposure triggered a self-reinforcing mechanism causing the VIX to spike higher. When comparing the two graphs of Figure 5.2 one should also bear in mind that the S/S strategy on the VIX futures had performed much better than the VSTOXX equivalent before the event. Ex-ante estimated volatility premiums on both markets did however accurately predict the upcoming turmoil and held cash over the most volatile of trading days.

6 Conclusion

The introduction of volatility indexes and their role as underlying risk-factors in derivative instruments have undoubtedly provided the financial markets with efficient tools for hedging market risk. It is the demand from investors seeking to hedge their portfolios that drives the volatility futures term structure to most often be in contango, which makes it possible to profit from short positions. However, in of era when volatility has been low, small changes in the underlying stock index causes volatility of volatility to increase which makes short strategies more susceptible to large drawdowns. The thesis answers the question of whether it is possible to trade on the volatility premium actively and thereby avoid losses associated with an event such as February 5^{th}. It is demonstrated that passively trading either long or short positions in volatility futures is not a sustainable source of profit and that actively trading the on the volatility premium can generate profitable returns on both the U.S. and European market.

Since the volatility futures term structure can suddenly move from contango to backwardation, accurately predicting these swings can serve as an additional source of returns for strategies trading both long and short positions. However, because there are fewer and shorter periods of backwardation than contango and that the subsequent investment environment is volatile, strategies with a long component suffer from drawdowns to a more considerable extent than those holding cash when the estimated premium is negative. Due to longer holding periods, this is especially true for monthly strategies.

The estimated volatility premium consists of both a forecast of the volatility index and the futures price which means that the volatility premium requires both accurate model performance and futures prices that are informationally efficient. For the futures prices to reflect all relevant information it requires actively traded markets. Thus, liquidity also affects the ex-ante estimated premium. The result suggests that the VIX premium serves as a better prediction of realized returns over longer horizons compared to the VSTOXX premium and that the less liquid European market might be an explanation.

Trading more frequently introduces a trade-off between the informational value of the volatility premium and transaction costs. While the trade-off comes at a lower price in terms of transaction costs in the U.S. market, the relative improvement in the informational value of the estimated premiums is larger in the European market, rendering strategies on VSTOXX futures to be the most profitable and attractive over the investigated sample. However, it should be mentioned that these strategies, due to the much lower traded volumes, are more sensitive to the trading activity which questions whether it is possible to scale up volatility futures strategies on the European market.

Although active volatility strategies are very volatile, they provide attractive risk-adjusted returns in both U.S. and European markets. The ex-ante estimated volatility premiums allows active strategies to avoid much of the larger drawdowns. An era of passively investing in volatility might be over, but trading volatility actively still seems to be a source of portfolio alpha.

6.1 Suggestions for Future Research

The thesis establishes that there most likely is a link between the signaling value of the ex-ante estimated volatility premium and market liquidity. However, this stands to be proven empirically, which provides an interesting research question for future studies. That is, empirically investigating the finding of the VIX premium having relatively more informational value than that of the VSTOXX premium when trading monthly.

When estimating the volatility premium, the thesis uses static ARMA(p,q) models that have been estimated out of sample. An alternative approach would be to reestimate the models by using a rolling window sample which might increase model performance and better capture the seasonal components in volatility indexes as mentioned in Mencia and Sentana (2013) for the VIX. Cheng (2018) reports that a rolling window approach has been tested for the sample period 2004 through 2015 and it does not affect the forecast performance significantly. However, this still stands to be tested for a more extended sample period and for forecasting the VSTOXX time series.

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Appendix

A1 Derivation of Equation 2.1

As mentioned in Section 2.1, Equation 2.1 is a discrete time approximation of Equation 2.2. This continuous time derivation uses results from Carr and Madan (2001) and largely builds upon that presented in Eriksen (2018).

To find the fair variance swap rate, K_{VAR_t}, that equates the initial value of a variance swap spanning the interval $[0, T]$ with payoff $\sigma^2_{RV} - K_{VAR}$ to 0 the realized variance over the same interval is a function of the daily returns squared

$$\sigma^2_R = \frac{1}{T}\sum_{i=o}^{T-1}\left(\frac{S_{i+1} - S_i}{S_i}\right)^2 \tag{.1}$$

, where S_i denotes the underlying index level at every period $i = 0, 1, 2, ..., T$ which in the following should be interpreted as daily closing prices. Assuming that the underlying follows a geometric Brownian motion Equation .1 is a consistent estimator of the variance of the underlying security.

A 2^{nd}-order Taylor approximation of the logarithm of the underlying index level gives

$$\begin{gathered} log(S_{i+1}) \approx log(S_i) + \frac{1}{S_i}(S_{i+1} - S_i) - \frac{1}{2S_i^2}(S_{i+1} - S_i)^2 \\ \left(\frac{S_{i+1} - S_i}{S_i}\right)^2 \approx -2log\left(\frac{S_{i+1}}{S_i}\right) + 2\left(\frac{S_{i+1} - S_i}{S_i}\right) \end{gathered} \tag{.2}$$

Summing both sides of Equation .2 over the periods $i = 0, 1, 2, ..., T$ yields

$$\sum_{i=o}^{T-1}\left(\frac{S_{i+1} - S_i}{S_i}\right)^2 \approx -2log\left(\frac{S_T}{S_0}\right) + 2\sum_{i=0}^{T-1}\left(\frac{S_{i+1} - S_i}{S_i}\right) \tag{.3}$$

The left side of Equation .3 is the floating leg of the swap[31] and can be replicated by going short 2 log-contracts and dynamically trade long forward contracts spanning the

[31] That is σ^2_{RV} in Equation .1

next period at each trading day i[32].

Ignoring interest rate the cost of going long $\frac{2}{S_i}$ is 0, and the replicating argument reduces to the first term on the right-hand side of Equation .3. The main idea of the replication is to approximate the logarithmic function by a sum of functions that are sums of piece-wise linear functions, and one can do so by using equation (1) from Carr and Madan (2001). They show that any twice continuously differentiable function $f(S)$ of the underlying asset, S, can be replicated by a unique initial position of $f'(S_0)$ shares, $f''(K)dK$ out-of-the-money options of all strikes K and $f(S) - f'(S_0)S_0$ unit discount bonds. This yields

$$\begin{gathered} f(S_T) = \big[f(S_0 - f'(S_0)S_0\big] + f'(S_0)S+ \\ \int_0^{S_0} f''(K)(K-S_T)^+ dK + \int_{S_0}^{\infty} f''(K)(K-S_T)^+ dK \end{gathered} \tag{.4}$$

Combining Equation .4 with $f(S_T) = \log\big(\frac{S_T}{S_0}\big)$ results in

$$\log\big(\frac{S_T}{S_0}\big) = \frac{(S_T - S_0)}{S_0} - \int_0^{S_0} \frac{1}{K^2}\cdot (K-S_T)^+ dK - \int_{S_0}^{\infty} \frac{1}{K^2}\cdot (S_T-K)^+ dK \tag{.5}$$

By the same reasoning as previously, the term $\frac{(S_T - S_0)}{S_0}$ can be replicated at zero cost by buying $\frac{1}{S_0}$ of the period T forward contract at $t = 0$. Thus, only the integrals of Equation .5 remains. With the assumption of a risk-free rate equal to zero and no dividends payments[33] the initial price of the underlying security is equal to the forward price, i.e., $S_0 = F$. For $F > K_0$ the following is true

$$\begin{gathered} \int_0^F \frac{(K-S_T)^+}{K^2} dK + \int_F^{\infty} \frac{(S_T-K)^+}{K^2} dK = \\ \int_0^{K_0} \frac{(K-S_T)^+}{K^2} dK + \int_{K_0}^{\infty} \frac{(S_T-K)^+}{K^2} dK + \int_{K_0}^{F} \frac{(K-S_T)^+}{K^2} dK - \int_{K_0}^{F} \frac{(S_T-K)^+}{K^2} \end{gathered} \tag{.6}$$

, where the last two integrals $\int_{K_0}^{F}$ can be written as

$$\int_{K_0}^{F} \frac{(S_T - K)}{K^2} dK = \left[\log K + \frac{S_T}{K}\right]_{K_0}^{F} = \log(\frac{F}{K_0}) + \frac{S_T}{K} - \frac{S_T}{K_0} \tag{.7}$$

[32]Go long a forward covering next period $\frac{2}{S_i}$ each trading day i to obtain a payoff of $2\left(\frac{S_{i+1} - S_i}{S_i}\right)$.

[33]I.e. no cost of carry

and put-call-parity was used to obtain $(K\text{-}S_T)^+\text{-}(S_T\text{-}K)^+ = (K\text{-}S_T)$.

So far it has been shown that the realized variance over the time interval $[0, T]$ can be replicated by a portfolio that is constructed of call and put options with the same maturity T and different strikes.

Defining the time to maturity of the options measured in years as T one can get to the fair value of a variance swap by combining Equations .1, .3 and .5 to get $K_{VAR} = E^Q[\sigma^2_{RV}]$ as

$$\begin{aligned}\frac{2}{T}\left(\int_0^F \frac{E^Q[K\text{-}S_T)^+]}{K^2}dK + \int_F^\infty \frac{E^Q[S_T\text{-}K)^+]}{K^2}dK + \log\left(\frac{E^Q[S_T]}{K_0}\right) + \frac{E^Q[S_T]}{F} \text{-} \frac{E^Q[S_T]}{K_0}\right) \\ = \frac{2}{T}e^{rT}\left(\int_0^F \frac{P(K,T)}{K^2}dK + \int_F^\infty \frac{C(K,T)}{K^2}dK\right) + \frac{2}{T}\left(\log\left(\frac{F}{K_0}\right) + \left(1 - \frac{F}{K_0}\right)\right)\end{aligned} \tag{.8}$$

, where the final term $\frac{2}{T}(\cdot) \approx -\frac{1}{T}(\frac{F}{K_0} - 1)^2$, $C(K,T)$ and $P(K,T)$ are European style call and put options respectively maturing in T years with strike K. Equation .8 also uses the risk-neutral assumption of $E^Q[S_T] = F$ and a 2^{nd} order Taylor expansion of $\log(\frac{F}{K_0})$ around 1.

Now, defining $Q(K_i)$ as the price of an out of the money option with strike K_i and writing the expression in discrete time we get

$$\sigma^2_{T_j} = \frac{2}{T_j} + \sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T_j}\left[\frac{F}{K_0} - 1\right]^2 \tag{.9}$$

, which is the same as Equation 2.1.

A2 Descriptive Statistics

Table A2.1: Descriptive statistics of VIX

	Training data 1990-01-02/2005-12-31	Test data 2006-01-01/2018-12-31	Full sample 1990-01-02/2018-12-31
Nr. of observations	4,033	3,271	7,304
Mean	19.44	19.064	19.27
Median	18.40	16.35	17.40
Minimum	9.31	9.14	9.14
Maximum	45.74	80.86	80.86
Standard deviation	6.40	9.25	7.81
Skewness	0.949	2.46	2.09
Kurtosis	0.749	8.27	7.70
Jarque-Bera	0.00	0.00	0.00
ADF (AIC/BIC)	0.00/0.00	0.00/0.00	0.00/0.00
PP	0.00	0.00	0.00

Note: Test Sample for the VIX starts in 2006 due to irregular maturities of VIX futures between 2004-2006.

Table A2.2: Descriptive statistics of VSTOXX

	Training data 1999-01-04/2009-06-01	Test data 2009-06-02/2018-12-28	Full sample 1991-01-04/2018-12-28
Nr. of observations	2,648	2,441	5,089
Mean	25.93	22.23	24.16
Median	23.52	21.12	22.28
Minimum	11.6	10.68	10.68
Maximum	87.51	53.55	87.51
Standard devation	11.09	6.91	9.50
Skewness	1.43	1.11	1.63
Kurtosis	2.27	1.70	3.64
Jarque-Bera	0.00	0.00	0.00
ADF (AIC/BIC)	0.05/0.05	0.00/0.00	0.00/0.00
PP	0.015	0.00/0.00	0.00

Note: Test data since the inception of VSTOXX futures in June 2009.

A3 Properties of ARMA Models

ARMA models combine the ideas of AR and MA models into a compact form keeping the number of parameters small, which allows for a more parsimonious model in terms of parameterization. A time series, Y_t, follows an ARMA(1,1) process if it satisfies

$$Y_t - \beta_t Y_{t-1} = \beta_0 + \epsilon_t - \delta_1 \epsilon_{t-1} \tag{.10}$$

, where the right-hand side represents the AR(1) process, and the left-hand side represents the MA(1) process. ϵ_t is a white noise series. For the model to be relevant, $\beta_1 \neq \delta_1$ must hold. (Tsay, 2010)

The data needs to be stationary in order to apply an ARMA model. A process Y_t is strictly stationary if the distribution of a subset of $Y_t(Y_{t1}, ..., Y_{tk})$ is identical to that of $(Y_{t1+t}, ..., Y_{tk+t})$ for all t, where k is an arbitrary positive integer. However, this condition is hard to prove empirically, and weak stationarity is often assumed. Weak stationarity, or covariance stationarity, essentially means that both the mean of Y_t and the covariance of Y_t and Y_{tk} are independent of time. Consider the case of a general ARMA(p,q) model of the form

$$Y_t = \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \epsilon_t - \sum_{i=1}^{q} \delta_i \epsilon_i \tag{.11}$$

, where ϵ_t is a white noise series, and p and q are positive integers. With lag-operators, the model can be written as

$$(1 - \beta_1 L - ... - \beta_p L^p) Y_t = \beta_0 + (1 - \delta_1 L - ... - \delta_q L^q) \epsilon_t \tag{.12}$$

The left-hand side polynomial represents the AR process, and the right-hand side polynomial represents the MA process. Y is weakly stationary if all roots on the inverse characteristic equation lie outside the unit circle, i.e., the solutions of the Equation .12 are larger than one. If the parameter β in Equation .11 is larger than one, the process will not decay over time and the time series would suffer from non-stationarity in the form of a deterministic trend. In the case when the parameter is equal to one, we have the problem with a stochastic trend, also known as a random walk. (Tsay, 2010)

A4 Model Selection

All table reports the results from estimated ARMA models for the respective implied volatility indexes at the daily frequency for each respective sample period. μ denotes the estimated mean, β_i denotes the i-th order estimated AR term, and δ_i the i-th order estimated MA term. Bold coefficients indicate those that are significantly different from zero at the 5% level.

To give a visual indication of the appropriateness of an ARMA model, the ACF and PACF of each index is graphed together with the ACF of the chosen model:s residuals.

Table A4.1: VIX ARMA forecast models

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ARMA	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)
μ	**19.444**	**19.444**	**19.443**	**19.443**	**19.444**	**19.351**	**19.423**	**19.399**	**19.444**	**19.418**
	1.041	1.084	1.215	1.318	1.075	1.607	1.556	1.547	1.164	1.547
β_1	**0.982**	**0.983**	**0.987**	**0.989**	**0.949**	**1.766**	**1.669**	**1.683**	**0.946**	**1.734**
	0.003	0.003	0.003	0.002	0.016	0.039	0.671	0.088	0.016	0.048
β_2					**0.034**	**-0.768**	**-0.671**	**-0.685**	**-0.045**	**-0.787**
					0.016	0.038	0.060	0.087	0.022	0.046
β_3									**0.083**	**0.051**
									0.016	0.018
δ_1		**-0.041**	**-0.053**	**-0.055**		**-0.861**	**-0.735**	**0.749**		**-0.800**
		0.014	0.016	0.016		0.032	0.062	0.090		0.046
δ_2			**-0.097**	**-0.099**			**-0.058**	**-0.059**		
			0.017	0.017			0.020	0.020		
δ_3				**-0.068**				-0.006		
				0.017				0.024		
Log Lh	-6,490.3	-6,487.5	-6,471.1	-6,462.9	-6,488.0	-6,458.9	-6,455.0	-6,454.9	-6,474.2	-6,455.1
BIC	13,005.4	13,008.3	12,983.7	12,975.7	13,009.2	12,959.8	12,959.3	12,968.0	12,990.0	12,959.9
AIC	12,986.5	12,983.1	12,952.2	12,937.9	12,984.0	12,927.8	12,922.0	12,923.9	12,958.4	12,922.1
T	4,033	4,033	4,033	4,033	4,033	4,033	4,033	4,033	4,033	4,033

Note: The chosen model for the expectation of the VIX under the physical measure is ARMA(2,2).

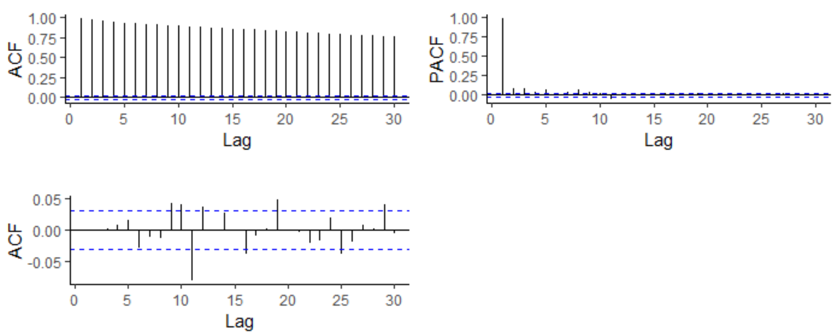

Figure A4.1: ACF and PACF of VIX

Note: The ARMA(2,2) reduces the level of the serial correlation since there are only a few significant lags in the ACF. This is also formally tested in a Box-Jenkins test.

Table A4.2: VSTOXX ARMA forecast models

ARMA	(1) (1,0)	(2) (1,1)	(3) (1,2)	(4) (1,3)	(5) (2,0)	(6) (2,1)	(7) (2,2)	(8) (2,3)	(9) (3,0)	(10) (3,1)
μ	**25.934**	**25.9335**	**25.94**	**25.937**	**25.933**	**25.928**	**25.962**	**25.942**	**25.934**	**25.959**
	2.486	2.574	2.843	3.095	2.55	2.561	3.045	2.976	2.776	2.986
β_1	**0.986**	**0.987**	**0.990**	**0.992**	**0.956**	0.034	**1.402**	**0.188**	**0.954**	**1.418**
	0.003	0.003	0.003	0.003	0.020	0.062	0.127	0.055	0.020	0.137
β_2					**0.030**	**0.940**	**-0.407**	**0.795**	**-0.058**	**-0.508**
					0.020	0.061	0.126	0.054	0.027	0.126
β_3									0.091	0.084
									0.020	**0.023**
δ_1		**-0.038**	**-0.052**	**-0.032**		**0.930**	**-0.458**	**0.769**		**-0.470**
		0.022	0.019	0.020		0.075	0.126	0.057		0.138
δ_2			**-0.087**	**-0.094**			**-0.077**	**-0.124**		
			0.019	0.018			0.022	0.025		
δ_3				**-0.097**				**-0.160**		
				0.021				0.020		
Log Lh	-5,359.9	-5,358.4	-5,347.5	-5,336.5	-5,358.7	-5,352.6	-5,343.4	-5,323.0	-5,347.8	-5,342.36
BIC	10,743.4	10,748.3	10,734.5	10,720	10,748.9	10,744.6	10,733.4	10,701.1	10,734.9	10,732.0
AIC	10,725.8	10,724.8	10,705.1	10,685.0	10,725.4	10,715.1	10,698.1	10,659.9	10,705.5	10,696.7
T	2,648	2,648	2,648	2,648	2,648	2,648	2,648	2,648	2,648	2,648

Note: The chosen model for the expectation of the VIX under the physical measure is ARMA(2,3).

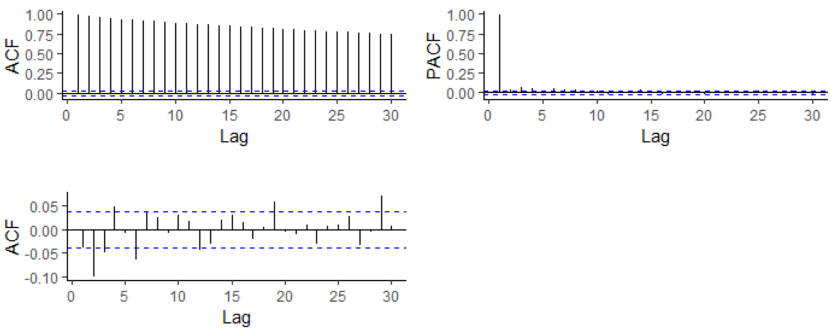

Figure A4.2: ACF and PACF of VSTOXX

Note: The ARMA(2,3) reduces the level of the serial correlation since there are only a few significant lags in the ACF. This is also formally tested in a Box-Jenkins test.

A5 Factor Loadings

Portfolio factors are used to test the presence of pricing errors or performance not attributable to tradable portfolios and are central in both asset pricing and portfolio management. This section outlines a brief summary. For an in depth presentation see e.g. Munk (2010).

If a factor x_t^i is the return of a traded portfolio i at time t, like those presented in Fama and French (1993) and Carhart (1997), the intercept, or α, is interpreted as a presence of pricing errors or asset manager skills. For expositional simplicity, the section lays out a one-factor model as an example. Tested in a time series regressions expressed as

$$R_t^i - R_t^f = \alpha + b_i(x_t^i - R_t^f) + \epsilon_t \tag{.13}$$

, where R_t^i is the return of some asset (or strategy) i at time t, R_t^f is the risk-free rate (here assumed to be constant), α the model intercept and b_i the slope coefficient of factor i, and ϵ_t a well-behaved error term. The OLS procedure yields an estimate of b_i as $\widehat{b}_i = \frac{Cov(R^i,x^i)}{Var(x^i)}$. The estimate of the slope coefficient, $\widehat{b}_i$, is the exposure of R^i to a given risk-factor x^i.

Under the condition that the risk factor, x^i, is a traded portfolio like those presented by Fama and French (1993) and Carhart (1997) it must be possible to price the factor itself by the model. Thus, it can be put on the r.h.s. of the model as

$$x_t^i - R_t^f = \alpha + b_i(x_t^i - R_t^f) + \epsilon_t \qquad t = 1, 2, ..., T \tag{.14}$$

, which, as any variable regressed on itself, will result in $\alpha = 0$ and $\widehat{b}_i = 1$. Consequently, per the nature of OLS, if the unconditional expectation $E[.]$ of .14 applied to any asset (or strategy) return, R_t^i will result in

$$E[R_t^i - R_t^f] = \widehat{\alpha} + \widehat{b}_i E[x_t^i - R_t^f] + \epsilon_t \tag{.15}$$

If the estimated intercept $\widehat{\alpha} \neq 0$ and coefficient $\widehat{b}_i \neq 1$, it serves as an indication of either pricing errors or asset manager skills.

A6 L/S/C strategies - Sharpe Ratio Tables

This thesis retroactively chose signal thresholds to maximize the performance of L/S/C strategies. The evaluated performance is in terms of Sharpe ratios. Several different signal threshold combinations are tested, all of which are reported in the tables below.

Table A6.1: Sharpe ratios for monthly L/S/C strategies on VIX (full sample)

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**
0.0	0.64	0.59	0.58	0.61	0.59	0.59	0.51	0.62	0.61	0.54	0.54
0.2	0.70	0.65	0.64	0.67	0.66	0.66	0.58	0.70	0.69	0.62	0.62
0.4	0.79	0.75	0.74	0.79	0.78	0.78	0.69	0.87	0.87	0.78	0.78
0.6	0.75	0.71	0.70	0.75	0.73	0.73	0.64	0.83	0.83	0.74	0.74
0.8	0.79	0.74	0.74	0.79	0.78	0.78	0.69	0.89	0.89	0.80	0.80
1.0	0.81	0.76	0.76	0.81	0.80	0.80	0.71	0.92	0.92	0.84	0.83
1.2	0.81	0.76	0.76	0.81	0.80	0.80	0.71	0.93	0.94	0.85	0.85
1.4	0.84	0.80	0.80	0.85	0.84	0.84	0.76	0.98	0.99	0.90	0.90
1.6	0.83	0.79	0.79	0.84	0.83	0.83	0.74	0.97	0.98	0.89	0.89
1.8	0.83	0.79	0.79	0.84	0.83	0.83	0.74	0.97	0.98	0.89	0.89
2.0	0.84	0.80	0.80	0.85	0.84	0.85	0.76	0.99	1.00	0.91	0.91
2.2	0.84	0.80	0.80	0.85	0.84	0.85	0.76	0.99	1.00	0.91	0.91
2.4	0.74	0.69	0.69	0.74	0.73	0.73	0.63	0.87	0.88	0.78	0.78
2.6	0.74	0.69	0.69	0.74	0.73	0.73	0.63	0.87	0.88	0.78	0.78
2.8	0.74	0.69	0.69	0.74	0.73	0.73	0.63	0.87	0.88	0.78	0.78
3.0	0.74	0.69	0.69	0.74	0.73	0.73	0.63	0.87	0.88	0.78	0.78

	2.2	**2.4**	**2.6**	**2.8**	**3.0**	**3.2**	**3.4**	**3.6**	**3.8**	**4.0**
0.0	0.55	0.55	0.59	0.60	0.56	0.56	0.56	0.51	0.50	0.48
0.2	0.63	0.63	0.68	0.69	0.65	0.65	0.65	0.60	0.59	0.57
0.4	0.84	0.86	0.93	0.95	0.91	0.91	0.92	0.85	0.84	0.81
0.6	0.81	0.83	0.91	0.93	0.89	0.88	0.90	0.82	0.81	0.78
0.8	0.88	0.91	0.99	1.02	0.98	0.98	0.99	0.91	0.91	0.88
1.0	0.93	0.96	1.05	1.08	1.04	1.04	1.06	0.98	0.98	0.94
1.2	0.95	0.99	1.09	1.12	1.09	1.09	1.11	1.03	1.03	0.99
1.4	1.02	1.07	1.18	1.20	1.18	1.18	1.20	1.12	1.12	1.09
1.6	1.01	1.06	1.17	1.20	1.17	1.17	1.20	1.11	1.12	1.09
1.8	1.01	1.06	1.17	1.20	1.17	1.17	1.20	1.11	1.12	1.09
2.0	1.05	1.10	1.22	1.25	1.23	1.23	1.26	1.17	1.18	1.15
2.2	1.05	1.10	1.22	1.25	1.23	1.23	1.26	1.17	1.18	1.15
2.4	0.90	0.96	1.09	1.13	1.11	1.11	1.15	1.04	1.05	1.03
2.6	0.90	0.96	1.09	1.13	1.11	1.11	1.15	1.04	1.05	1.03
2.8	0.90	0.96	1.09	1.13	1.11	1.11	1.15	1.04	1.05	1.03
3.0	0.90	0.96	1.09	1.13	1.11	1.11	1.15	1.04	1.05	1.03

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

Table A6.2: Sharpe ratios for monthly L/S/C strategies on VSTOXX

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**
0.0	0.16	0.18	0.23	0.15	0.17	0.21	0.15	0.12	-0.02	-0.02	-0.02	-0.09	-0.10	-0.10	-0.10	-0.13
0.2	0.18	0.20	0.26	0.18	0.19	0.23	0.18	0.14	0.00	0.00	0.00	-0.07	-0.08	-0.08	-0.09	-0.12
0.4	0.25	0.28	0.35	0.26	0.28	0.32	0.26	0.23	0.09	0.09	0.08	0.01	0.00	0.00	0.00	-0.03
0.6	0.27	0.30	0.37	0.29	0.30	0.35	0.29	0.26	0.11	0.11	0.10	0.03	0.02	0.02	0.01	-0.02
0.8	0.28	0.31	0.39	0.30	0.31	0.37	0.30	0.27	0.11	0.11	0.10	0.03	0.01	0.01	0.01	-0.03
1.0	0.25	0.29	0.37	0.27	0.29	0.35	0.28	0.24	0.06	0.06	0.05	-0.04	-0.05	-0.05	-0.06	-0.10
1.2	0.19	0.22	0.31	0.20	0.22	0.28	0.20	0.16	-0.04	-0.04	-0.05	-0.16	-0.18	-0.18	-0.18	-0.23
1.4	0.22	0.25	0.34	0.23	0.25	0.32	0.24	0.19	-0.01	-0.01	-0.02	-0.13	-0.15	-0.15	-0.16	-0.21
1.6	0.27	0.31	0.41	0.30	0.33	0.40	0.32	0.27	0.06	0.06	0.05	-0.05	-0.07	-0.07	-0.08	-0.13
1.8	0.37	0.43	0.55	0.44	0.48	0.58	0.49	0.45	0.19	0.19	0.18	0.04	0.02	0.02	0.01	-0.07
2.0	0.24	0.45	0.58	0.47	0.50	0.62	0.52	0.49	0.22	0.22	0.21	0.06	0.04	0.04	0.03	-0.05
2.2	0.39	0.46	0.59	0.48	0.52	0.63	0.53	0.51	0.23	0.23	0.23	0.08	0.06	0.06	0.05	-0.03
2.4	0.41	0.48	0.62	0.51	0.55	0.67	0.57	0.55	0.27	0.27	0.27	0.13	0.11	0.11	0.11	0.02
2.6	0.42	0.49	0.63	0.52	0.56	0.69	0.59	0.56	0.28	0.28	0.28	0.13	0.11	0.11	0.10	0.01
2.8	0.42	0.49	0.64	0.53	0.57	0.70	0.60	0.58	0.29	0.29	0.29	0.14	0.12	0.12	0.12	0.02
3.0	0.42	0.49	0.64	0.53	0.57	0.70	0.60	0.58	0.29	0.29	0.29	0.14	0.12	0.12	0.12	0.02
3.2	0.42	0.49	0.64	0.53	0.57	0.70	0.60	0.58	0.29	0.29	0.29	0.14	0.12	0.12	0.12	0.02
3.4	0.42	0.49	0.64	0.53	0.57	0.70	0.60	0.58	0.29	0.29	0.29	0.14	0.12	0.12	0.12	0.02
3.6	0.47	0.55	0.70	0.60	0.64	0.78	0.68	0.67	0.39	0.39	0.39	0.26	0.25	0.25	0.26	0.16
3.8	0.47	0.55	0.70	0.60	0.64	0.78	0.68	0.67	0.39	0.39	0.39	0.26	0.25	0.25	0.26	0.16
4.0	0.47	0.55	0.70	0.60	0.64	0.78	0.68	0.67	0.39	0.39	0.39	0.26	0.25	0.25	0.26	0.16

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

Table A6.3: Sharpe ratios for daily L/S/C strategies on VIX (full sample)

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**
0.0	0.96	1.01	1.02	1.05	1.13	1.02	0.94	0.85	0.84	0.83	0.85	0.85	0.87	0.88	0.90	0.86
0.2	1.08	1.13	1.15	1.18	1.27	1.16	1.08	0.99	0.98	0.97	0.99	1.00	1.02	1.03	1.05	1.01
0.4	1.11	1.17	1.19	1.22	1.31	1.20	1.12	1.03	1.03	1.01	1.04	1.05	1.07	1.08	1.10	1.06
0.6	1.04	1.10	1.11	1.14	1.24	1.13	1.04	0.95	0.94	0.93	0.96	0.97	0.98	0.99	1.02	0.98
0.8	1.06	1.12	1.14	1.17	1.27	1.16	1.07	0.97	0.97	0.96	0.99	1.00	1.02	1.03	1.06	1.02
1.0	1.05	1.11	1.13	1.16	1.26	1.15	1.06	0.96	0.96	0.94	0.97	0.98	1.00	1.02	1.05	1.01
1.2	1.05	1.12	1.14	1.17	1.28	1.16	1.07	0.97	0.97	0.96	0.99	1.00	1.02	1.03	1.07	1.02
1.4	0.93	1.00	1.02	1.05	1.16	1.03	0.93	0.83	0.83	0.81	0.84	0.85	0.87	0.89	0.92	0.87
1.6	1.00	1.07	1.09	1.13	1.24	1.12	1.02	0.91	0.92	0.90	0.93	0.95	0.97	0.99	1.03	0.98
1.8	1.03	1.11	1.13	1.17	1.28	1.16	1.06	0.95	0.96	0.95	0.98	1.00	1.02	1.04	1.08	1.04
2.0	1.01	1.08	1.10	1.14	1.26	1.12	1.03	0.92	0.92	0.91	0.94	0.96	0.99	1.00	1.04	1.00
2.2	0.98	1.06	1.09	1.14	1.28	1.14	1.03	0.91	0.92	0.92	0.96	0.99	1.03	1.06	1.13	1.08
2.4	0.99	1.07	1.10	1.15	1.30	1.15	1.05	0.92	0.94	0.93	0.98	1.01	1.05	1.09	1.15	1.11
2.6	1.03	1.12	1.14	1.20	1.35	1.20	1.10	0.97	1.00	0.99	1.04	1.08	1.12	1.16	1.23	1.20
2.8	1.01	1.09	1.12	1.17	1.32	1.18	1.07	0.94	0.96	0.96	1.01	1.04	1.09	1.13	1.20	1.16
3.0	1.02	1.10	1.13	1.19	1.34	1.19	1.09	0.96	0.98	0.98	1.03	1.07	1.11	1.16	1.23	1.19

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

Table A6.4: Sharpe ratios for daily L/S/C strategies on VSTOXX

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**
0.0	1.54	1.52	1.46	1.38	1.27	1.32	1.25	1.31	1.30	1.27	1.25	1.20	1.13	1.11	1.09	1.00
0.2	1.65	1.63	1.58	1.49	1.39	1.44	1.37	1.43	1.42	1.39	1.37	1.32	1.25	1.24	1.21	1.13
0.4	1.80	1.78	1.72	1.64	1.53	1.58	1.51	1.58	1.57	1.54	1.52	1.47	1.41	1.39	1.37	1.28
0.6	1.76	1.74	1.68	1.60	1.49	1.55	1.47	1.54	1.54	1.51	1.48	1.43	1.37	1.35	1.32	1.24
0.8	1.86	1.84	1.78	1.69	1.58	1.64	1.57	1.64	1.63	1.61	1.59	1.53	1.47	1.45	1.42	1.34
1.0	1.75	1.73	1.67	1.58	1.47	1.53	1.45	1.52	1.52	1.49	1.47	1.41	1.34	1.33	1.30	1.21
1.2	1.84	1.32	1.76	1.67	1.56	1.62	1.54	1.62	1.61	1.59	1.56	1.51	1.45	1.43	1.40	1.31
1.4	1.82	1.80	1.73	1.65	1.53	1.59	1.51	1.59	1.58	1.56	1.54	1.48	1.41	1.39	1.36	1.28
1.6	1.73	1.71	1.64	1.55	1.43	1.49	1.41	1.49	1.48	1.45	1.43	1.37	1.30	1.28	1.25	1.16
1.8	1.73	1.71	1.64	1.55	1.42	1.49	1.40	1.48	1.48	1.45	1.43	1.37	1.30	1.28	1.25	1.15
2.0	1.76	1.73	1.67	1.57	1.44	1.51	1.43	1.51	1.51	1.48	1.46	1.40	1.33	1.31	1.28	1.18
2.2	1.54	1.66	1.59	1.49	1.36	1.43	1.34	1.43	1.43	1.40	1.37	1.31	1.23	1.21	1.18	1.07
2.4	1.54	1.52	1.44	1.33	1.18	1.26	1.17	1.26	1.26	1.22	1.20	1.12	1.04	1.01	0.97	0.85
2.6	1.42	1.39	1.31	1.19	1.03	1.12	1.01	1.11	1.11	1.07	1.04	0.96	0.86	0.83	0.78	0.64
2.8	1.40	1.37	1.28	1.17	1.01	1.09	0.99	1.09	1.08	1.04	1.01	0.93	0.83	0.80	0.75	0.60
3.0	1.32	1.30	1.21	1.09	0.92	1.01	0.89	1.00	0.99	0.95	0.91	0.82	0.71	0.68	0.63	0.47

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

Table A6.5: Sharpe ratios for monthly L/S/C strategies on VIX (Jun 2009 - Dec 2018)

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**
0.0	0.57	0.54	0.54	0.57	0.56	0.58	0.50	0.64	0.61	0.58	0.58
0.2	0.64	0.61	0.61	0.65	0.64	0.67	0.58	0.73	0.71	0.68	0.68
0.4	0.76	0.74	0.64	0.80	0.79	0.84	0.74	1.00	0.98	0.95	0.95
0.6	0.70	0.68	0.69	0.75	0.73	0.78	0.68	0.95	0.94	0.90	0.90
0.8	0.74	0.72	0.73	0.79	0.77	0.83	0.73	1.01	1.00	0.96	0.96
1.0	0.75	0.72	0.73	0.80	0.78	0.84	0.74	1.03	1.02	0.98	0.98
1.2	0.75	0.73	0.74	0.81	0.79	0.85	0.75	1.06	1.05	1.01	1.01
1.4	0.79	0.78	0.79	0.86	0.84	0.91	0.81	1.13	1.13	1.09	1.10
1.6	0.78	0.76	0.78	0.85	0.83	0.89	0.79	1.12	1.12	1.08	1.08
1.8	0.78	0.76	0.78	0.85	0.83	0.89	0.79	1.12	1.12	1.08	1.08
2.0	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12
2.2	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12
2.4	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12
2.6	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12
2.8	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12
3.0	0.79	0.78	0.79	0.86	0.85	0.91	0.79	1.12	1.12	1.12	1.12

	2.2	**2.4**	**2.6**	**2.8**	**3.0**	**3.2**	**3.4**	**3.6**	**3.8**	**4.0**
0.0	0.56	0.49	0.54	0.54	0.49	0.48	0.49	0.46	0.42	0.38
0.2	0.66	0.59	0.65	0.65	0.60	0.59	0.60	0.57	0.53	0.49
0.4	1.00	0.91	1.03	1.03	0.98	0.97	0.99	0.94	0.88	0.83
0.6	0.96	0.87	1.00	1.00	0.95	0.94	0.97	0.90	0.84	0.79
0.8	1.03	0.95	1.08	1.08	1.04	1.04	1.07	1.00	0.94	0.89
1.0	1.08	0.99	1.14	1.14	1.10	1.10	1.14	1.07	1.00	0.96
1.2	1.13	1.04	1.20	1.20	1.18	1.18	1.22	1.15	1.08	1.04
1.4	1.23	1.16	1.34	1.34	1.32	1.33	1.38	1.31	1.25	1.22
1.6	1.23	1.15	1.33	1.33	1.33	1.34	1.39	1.31	1.25	1.23
1.8	1.23	1.15	1.33	1.33	1.33	1.34	1.39	1.31	1.25	1.23
2.0	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39
2.2	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39
2.4	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39
2.6	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39
2.8	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39
3.0	1.30	1.23	1.44	1.44	1.45	1.47	1.54	1.45	1.40	1.39

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

Table A6.6: Sharpe ratios for daily L/S/C strategies on VIX (Jun 2009 - Dec 2018)

	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**
0.0	1.13	1.18	1.20	1.26	1.39	1.22	1.13	1.03	1.04	1.02	1.04	1.01	1.00	1.03	1.06	1.02
0.2	1.28	1.34	1.36	1.43	1.56	1.39	1.30	1.20	1.21	1.20	1.22	1.20	1.19	1.22	1.25	1.22
0.4	1.30	1.36	1.39	1.46	1.59	1.42	1.33	1.23	1.25	1.23	1.26	1.23	1.22	1.25	1.29	1.26
0.6	1.22	1.28	1.31	1.38	1.52	1.34	1.24	1.14	1.15	1.14	1.16	1.14	1.13	1.16	1.20	1.17
0.8	1.25	1.31	1.34	1.41	1.56	1.38	1.28	1.17	1.19	1.18	1.21	1.18	1.17	1.21	1.25	1.22
1.0	1.23	1.30	1.32	1.40	1.55	1.37	1.27	1.15	1.18	1.17	1.19	1.17	1.16	1.19	1.23	1.20
1.2	1.21	1.28	1.30	1.38	1.54	1.35	1.24	1.13	1.15	1.14	1.17	1.14	1.13	1.17	1.21	1.20
1.4	1.05	1.12	1.14	1.22	1.38	1.18	1.07	0.94	0.96	0.95	0.97	0.94	0.93	0.97	1.01	0,97
1.6	1.08	1.15	1.18	1.26	1.43	1.23	1.11	0.99	1.01	1.00	1.03	1.00	0.99	1.03	1.01	1.03
1.8	1.12	1.20	1.22	1.31	1.48	1.28	1.16	1.04	1.07	1.06	1.08	1.06	1.05	1.09	1.14	1.10
2.0	1.09	1.17	1.19	1.28	1.45	1.24	1.13	1.00	1.03	1.02	1.05	1.02	1.01	1.05	1.10	1.06
2.2	1.11	1.20	1.23	1.34	1.56	1.33	1.20	1.06	1.11	1.12	1.16	1.14	1.14	1.22	1.31	1.29
2.4	1.14	1.23	1.28	1.39	1.61	1.38	1.26	1.11	1.17	1.18	1.23	1.22	1.22	1.30	1.40	1.40
2.6	1.16	1.26	1.30	1.41	1.64	1.41	1.28	1.14	1.20	1.21	1.26	1.25	1.26	1.34	1.45	1.44
2.8	1.15	1.24	1.28	1.39	1.62	1.39	1.26	1.11	1.18	1.19	1.24	1.23	1.23	1.31	1.42	1.41
3.0	1.16	1.25	1.30	1.41	1.63	1.40	1.28	1.13	1.20	1.21	1.26	1.25	1.26	1.34	1.44	1.44

Note: The horizontal (vertical) table header is the imposed positive (negative) signal. Transaction costs as per the base case.

A7 VHSI and VXJ

The thesis set out to investigate strategies on multiple indexes, but as only VIX and VSTOXX futures are liquid enough to implement volatility futures strategies on VHSI and VXJ have been excluded.

In this section of the Appendix statistics for VHSI and VXJ, the data underlying the decision to exclude these indexes and the indexes correlations with VIX, VSTOXX together with all underlying stock indexes are presented.

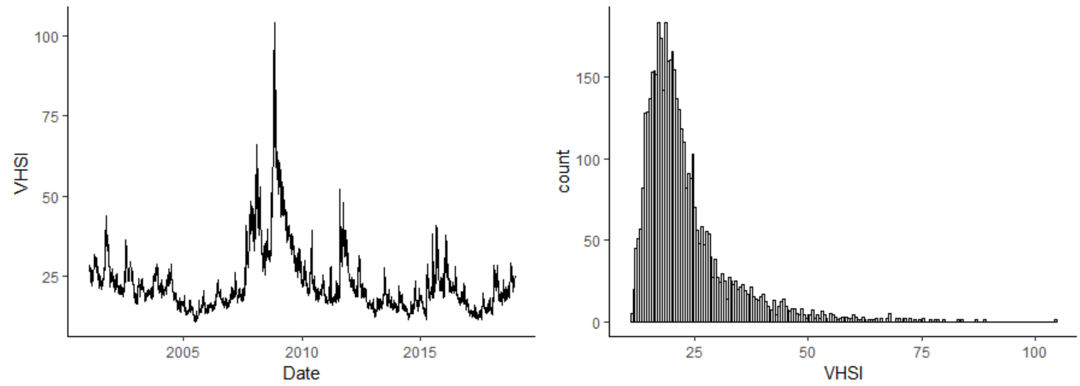

Figure A7.1: Daily closing prices and the empirical distribution of VHSI
Note: Daily closing price (Left) and distribution (Right) throughout Jan 2001 to Dec 2018.
Source: Bloomberg.

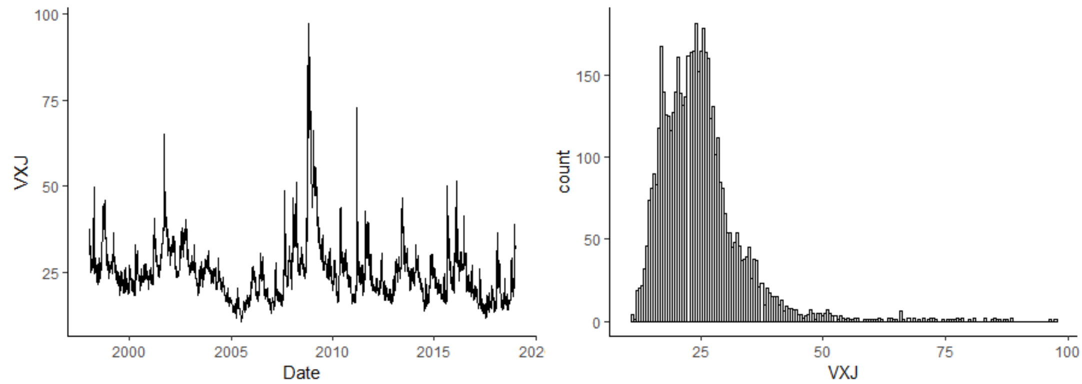

Figure A7.2: Daily closing prices and the empirical distribution of VXJ
Note: Daily closing price (Left) and distribution (Right) throughout Jan 1998 to Dec 2018.
Source: Bloomberg.

Table A7.1: Descriptive statistics of VHSI and VXJ

	VHSI 2001-01-02/2018-12-28	VXJ 1998-01-05/2018-12-31
Nr. of observations	4,437	5,202
Mean	22.87	25.00
Median	20.17	23.78
Minimum	10.86	10.97
Maximum	104.29	97.27
Standard deviation	9.72	9.83
Skewness	2.35	2.355
Kurtosis	7.95	11.85
Jarque-Bera	0.00	0.00
ADF (AIC/BIC)	0.00/0.00	0.00/0.00
PP	0.00	0.00

Note: High of 104.29 on 2008-10-27, low of 10.86 on 2005-06-21

Table A7.2: Correlation matrix of volatility indexes and underlying stock indexes

	SPXT	**VIX**	**SX5T**	**VSTOXX**	**HSI 1**	**VHSI**	**NKYTR**	**VXJ**
SPXT	1.00	-0.73	0.60	-0.43	0.24	-0.14	0.16	-0.12
VIX	-0.73	1.00	-0.48	0.52	-0.17	0.18	-0.14	0.16
SX5T	0.60	-0.48	1.00	-0.75	0.39	-0.30	0.34	-0.27
VSTOXX	-0.43	0.52	-0.75	1.00	-0.33	0.39	-0.30	0.36
HSI 1	0.24	-0.17	0.39	-0.33	1.00	-0.56	0.61	-0.45
VHSI	-0.14	0.18	-0.30	0.39	-0.56	1.00	-0.45	0.56
NKYTR	0.16	-0.14	0.34	-0.30	0.61	-0.45	1.00	-0.63
VXJ	-0.12	0.16	-0.27	0.36	-0.45	0.56	-0.63	1.00

Note: The daily correlations of volatility indexes and their respective stock indexes since January of 2001.

Table A7.3: Contract summary of volatility futures (VHSI & VXJ)

	VHSI	**VXJ**
Future	VHS	VXJ
Underlying	VHSI Index	VXJ Index
#Maturities	3	8
Stock Index	Hang Send	Nikkei 225
Contract Size	HK$5,000 * Index	¥10,000 * Index
Tick Size	0.05	0.05
Tick Value	HK$250	¥500
Expiration	30 calendar days before the 2nd to last business day of next month	30 calendar days before the 2nd Friday of next month
Last Trading Day	Expiration	Expiration minus 1 business day
Convention	Following	Preceding
Daily Traded Volume (thousand USD)		
Max	11.68	102.98
Min	0.00	0.00
Average	1.22	22.62

Source: (HKEX, 2019), (JPX, 2019) and Bloomberg.

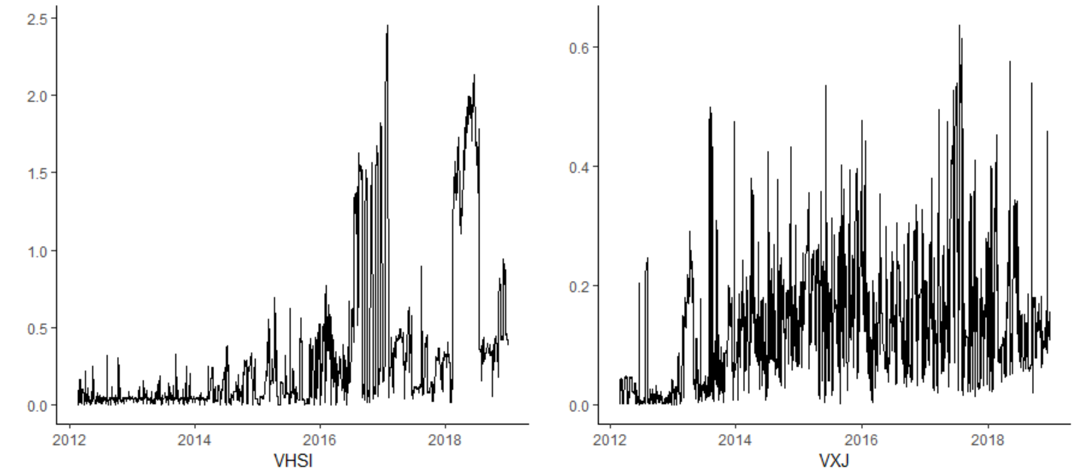

Figure A7.3: Historical bid-ask spreads for futures on VHSI and VXJ

Note: The figure covers the respective VHSI and VXJ futures samples. Reported spreads are in percentages and for the near-term and next-term contracts. Due to illiquidity and high transaction costs the Cheng (2018) methodology is not applicable to these indexes.

Source: Bloomberg.

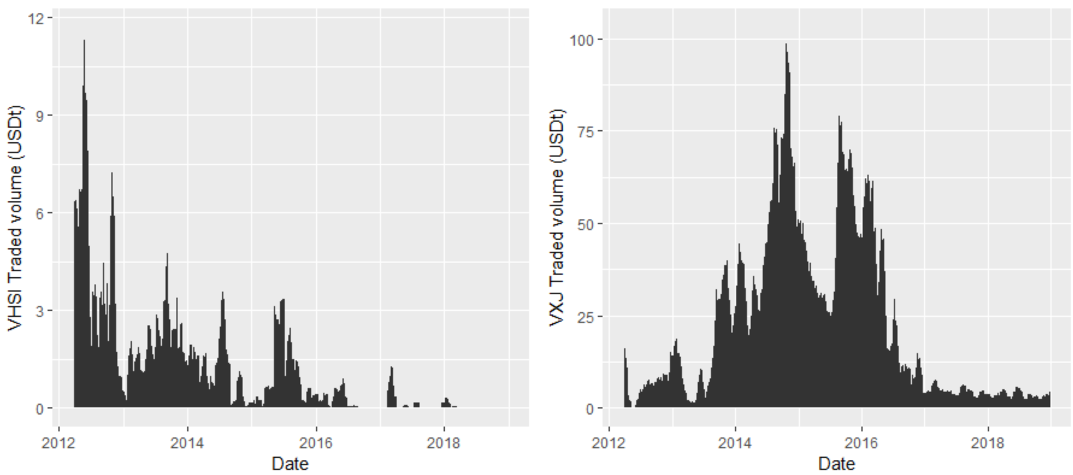

Figure A7.4: Traded vega of VHSI and VXJ futures

Note: Daily traded vega of the one and two month ahead futures contracts on the VIX (Left) and VSTOXX (Right). The traded vega has been plotted using a 21-day moving average to smooth the graphs. All reported volumes are in million USD. On any given day the number of traded contracts have been multiplied by the respective contract multiplier as reported in Table A7.3. Due to illiquidity and high transaction costs the Cheng (2018) methodology is not applicable to these indexes

Source: Bloomberg and FRED Economic Data.