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The Climate's Beta

An estimation of the consumption beta on the climate, estimated with the Dynamic Integrated Climate-Economy model

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Abstract

This thesis aims at estimating a consumption beta on the climate by means of the Dynamic Integrated Climate-Economy (DICE) model. We prove analytically and explain intuitively that the co-movement of benefits from climate change initiatives and aggregate consumption is the Consumption CAPM-beta on the climate. Inspired by the methodology of Dietz et al. (2018), we employ the DICE model to estimate the benefits from a marginal emissions abatement project on added consumption. By introducing uncertainty to parameters in the growth of Total Factor Productivity (TFP), we are able to estimate the effect on benefits and consumption, and thereby calculate the climate's consumption beta. We also examine the effect of different specifications of the income elasticity of damages on beta, i.e. whether damages are proportional to output or not and how this influences the values of the climate beta.

We find evidence for a positive climate beta, which is larger than the one obtained by Dietz et al. (2018) and currently applied in the DICE model by Nordhaus and Sztorc (2013); Nordhaus (2017b). Our findings suggest that when the main source of uncertainty is in the growth rate of TFP, the climate beta is positive for the whole time period of approximately 500 years in all cases but one. This result holds regardless of whether we model uncertainty through the *initial* growth rate of TFP, or by imposing transitory *shocks* to the growth rate. When damages are additive and uncertainty comes through the initial growth rate of TFP, we find that the beta becomes negative around year 2320.

We depart from the methodology of Dietz et al. (2018) by allowing the savings rate to be endogenous and by employing a different definition of a marginal abatement project. Furthermore, we use the latest version of Nordhaus' DICE model, which extends over a longer time period and incorporates the latest calibrations of the model's parameters to empirical data. Lastly, we use one of the most recent estimates of long-run economic growth forecasts as the basis for modelling parametric uncertainty.

Our findings show that there should be a positive risk-premium placed on top of the risk-free discount rate, to calculate the Net Present Value (NPV) of climate change investments. However, this result does not necessarily mean that the NPV of abatement projects will decrease in cost-benefit analysis (CBA). As shown by Dietz et al. (2018), expected benefits may also increase in the climate beta. The overall NPV of the climate mitigation projects is thus increasing in beta whenever the expected value effect is larger than the discounting effect.

We contribute to the debate around the evaluation of uncertain climate mitigation projects through our novel way of estimating the climate beta. This gives valuable insights to policymakers around the world, possibly suggesting that climate change mitigation should be valued higher than the current norm implies. If the estimation of the climate beta obtained by us is correct and the present value of climate mitigation projects increase in the climate's beta, then mitigation projects may be heavily undervalued and the social cost of carbon is thus higher than assumed today. The realization of this to policy makers may have large consequences for the choices of climate policies and mitigation projects that are undertaken.

Keywords – Climate economics, CCAPM, DICE Model, MSc Advanced Economics and Finance

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1 Introduction

1.1 Motivation

"If you still say that we are wasting valuable lesson time, then let me remind you that our political leaders have wasted decades through denial and inaction"- Greta Thunberg (16), February 2019[1]

The issue of climate change is pressing and global. Temperature is increasing, glaciers are melting, water-levels are rising, extreme weather makes countries unlivable, climate refugees are escaping their homes and coral reefs are disappearing as a consequence of human behavior (UNFCC, 2018b). Consequences of climate change are possibly the greatest risk to society that the world has ever experienced. Continued emissions of greenhouse gasses will increase the likelihood of severe, pervasive and irreversible impacts on both the climate system and on humans. It is us and our childrens' generations who will have to bear these consequences. Limiting the consequences of climate change will require both adaptation and sustained, substantial reductions in greenhouse gas (GHG) emissions. Efforts are made to overcome the challenge, but more than 30 years have already passed since the establishment of the Intergovernmental Panel on Climate Change (IPCC) and the first World Climate Conference in 1988. Many panels, policies and conferences have been made since then. More than two years have passed since the Paris agreement entered into force, where all countries were *"required to put forward their best efforts through nationally determined contributions and to strengthen these efforts the years ahead"* (UNFCC, 2018a) Still, the issue of climate change seems to increase by the minute. For instance, in June 2017 the US decided to leave the Paris agreement. It may seem like the costs of climate change mitigation are too high for policy makers to value abatement correctly. As a result, there exist many examples of non-optimal climate policies. At least one thing is apparent: the issue of combating climate change requires collaboration across countries, political parties, sciences, industries, sectors and between policymakers.

When new policies and climate mitigation projects are made, the research underlying this thesis is relevant. It addresses underlying theoretical foundations that can be used to make sustainable policies and provides a framework to correctly discount and price the benefits of climate mitigation projects.

1.2 Why the Climate Beta?

There is a common saying in finance and economics that 'one dollar today is worth more than one dollar tomorrow', relating to the fact that this dollar can be invested today, yielding a return as compensation for that the future is uncertain, and thereby risky. Economic theory also tells us that, as we are rational individuals, want to be compensated for uncertainty, either by paying a lower price or by receiving higher

[1]Greta Thunberg speaking to Politicians and Business leaders in Brussels February 2019, cited in (https://www.positive.news/environment/the-16-year-old-climate-hero-five-inspiring-quotes-by-greta-thunberg/)

returns. The compensation for risk should however not only take the time-perspective into account. It should also account for the riskiness of the investment, which again is dependent on the 'state of the world'. If an investment pays off in bad states of the world, like an insurance, the investment is valued higher than an investment paying off only in good states of the world. We are willing to pay more for an investment yielding positive returns in the bad states of the world because it is in those states that our marginal utility is highest. One extra dollar when we only have a little, gives us a greater satisfaction than one extra dollar when we have abundant. How we feel in different states of the world hence determines our willingness to pay, and consequently market prices. We are willing to discount back future streams of cash flows at a higher rate to estimate its Net Present Value (NPV) if the investment pays off in good states instead of bad states.(Cochrane, 2005)

A common tool used in economic- and financial theory to derive an appropriate discount rate is the Stochastic Discount Factor (SDF), also known as the pricing Kernel or the inter-temporal rate of substitution. The stochastic discount factor can be denoted as

$$M_{t+1} = \rho \frac{u'(C_{t+1})}{u'(C_t)} \tag{1.1}$$

where M_{t+1} is the SDF, ρ is the impatience parameter, or the pure rate of social time preference, and $\frac{u'(C_{t+1})}{u'(C_t)}$ represents the marginal utility of a consumption good in the future relative to the marginal utility of the same good today. The SDF provides us the rate at which an agent is willing to substitute consumption today, in return for consumption at a future time, $t+1$. Due to the inverse relationship of marginal utility and consumption, consumption is a useful indicator for the way we value assets. Consumption also reflects the surroundings of the agent; whenever consumption is low, the marginal utility of consumption is high, which occurs when the economy is in a bad state and vice versa (Cochrane, 2005).

A well known asset pricing tool which uses the SDF is the Capital Asset Pricing Model (CAPM). This model assumes that prices are high for (financial) assets that co-move with the market portfolio or large indices.(Cochrane, 2005) The same intuition can be used, substituting the market portfolio with aggregate consumption, and this model is referred to as the Consumption CAPM (CCAPM). In both the general CAPM and CCAPM, beta is the main determinant of the asset's price. Since the goal of this thesis is to assess the value of climate change investments, the CCAPM, rather than the CAPM, is an appropriate tool as we want to measure the wealth of a "representative agent". The way this can be done will be explained in the subsection below.

1.3 Relationship between the DICE model and the Climate Beta

1.3.0.1 Why do we specifically focus on the Climate Beta within climate change economics?

The DICE Model is the most well-known among all Integrated Assessment Models(IAMs) used in climate change economics. The model can be used in cost- benefit analysis (CBA), as it relates the marginal damage costs of climate change to the marginal benefits of climate mitigation. The model is well-praised, but has often received critique from other well known climate economists, among others William Cline, Stern (2007b), Weitzman and Dietz (2011). Much of the critique relates to the discount rate used in the DICE model, which has resulted in a debate over the last two decades around normative versus descriptive approaches and ethical or 'fair' ways of discounting. Other researchers show that the above-mentioned climate economists are all correct in their own way of discounting in CBA, depending on the assumptions that are made by the modeller (Gollier, 2013). It seems as if a consensus in the literature has been reached regarding several possible ways of discounting when applying the DICE model in cost-benefit analysis.

1.3.0.2 More recent critique on the discount rate

More recent research focuses on another critique point surrounding the discount rate within the DICE model, namely accounting for risk and uncertainty. Much uncertainty within the DICE model, also related to the discount rate, lies in the uncertainty of damages of climate change, benefits of climate mitigation and future consumption and output levels. Climate change requires investments, i.e. reduction of consumption now, while the benefits of those are uncertain and might only become apparent to future generations. In order to optimally price investments in climate, these future benefits must be discounted at the right discount rate. Therefore, the discount rate used to calculate the NPV of climate change mitigation should be a risk-adjusted discount rate. This is where asset pricing tools, such as the Capital Asset Pricing Model come into play.

1.3.0.3 Asset pricing tools

Asset pricing tools like the CAPM provide a risk-adjusted discount rate, which can be used to calculate the net present value of a risky stream of future cash flows. In the risk-adjusted discount rate, the main determinant of the investment (the asset's) price, is the β, reflecting a measure for risk. However, climate economics relates to welfare-economics. The risky stream of future cash flows related to climate mitigation projects, impact society's aggregate consumption level. In order to measure the impact of climate change on the aggregate welfare level of the society, thus to find an appropriate social optimal price of CO_2 emissions, one therefore wants to use the CCAPM rather than CAPM.

There is however no strong evidence on the correlation between environmental risk and aggregate consumption risk. Therefore, no clear consensus regarding an estimate for a CCAPM beta (β) on climate investments exists. Climate mitigation projects cannot be seen as standard financial assets for which years of historical data exist. Consequently, the sign and the size of the CCAPM β for climate mitigation project remains an open question (Nordhaus and Sztorc, 2013).

1.3.0.4 Shedding new light on the academic debate

To shed new light on the risk-adjusted discount rate, and more particularly, on the CCAPM beta used for climate investments, we build our estimation of a climate beta. We do so, based on the CCAPM, using the DICE model as a tool for a numerical estimation. The work in this thesis is inspired by, and based upon Dietz et al. (2018), but employs the newest version of the DICE model, namely the DICE2016 R2 model. The DICE model builds upon Neoclassical growth theory, whereby a representative agent values her own welfare and that of her dynasty. Therefore, this model complies well with the underlying theory of the CCAPM. Within the DICE model, we can re-formulate some equations in order to see how different uncertainties, for instance related to future output, consumption and damages of climate change, affect the beta. Taking the theories of asset pricing, Neoclassical growth and practices of climate change economics into account, we arrive at the following research question.

1.4 Research Question

Contributing to the informed debate about accounting for uncertainty within the DICE model when used for climate change mitigation policy, will be the main goal of this thesis. This objective can be formulated by means of the following research question: "How do the sign and size of the climate beta vary over time depending on uncertainty related to Total Factor Productivity (TFP) growth and climate change damages?" In order to answer this question, the following sub-questions are examined:

- What are pro´s and cons for the use of the DICE model in estimating an optimal climate policy?
- What is the appropriate definition and value of the discount factor to calculate today's value of climate change mitigation projects?
- What is the influence of uncertainty in total factor productivity (TFP) on the the estimation of a climate beta?
- How do the estimated climate beta and the discount rate affect the value of climate mitigation projects?

These sub-questions form the basis for the underlying structure of the paper, although the answers will not be explicitly stated in the text. The aim is rather to have an informed discussion about the sub-questions in relation to the overall goal of the thesis.

1.5 Overview

In order to have an informed discussion surrounding the sub questions above, chapter 2 provides the reader with background information on climate change policy, IAMs and the underlying Neoclassical growth models. Chapter 3 explains the climate beta and lays out previous literature on the climate beta. In addition, the same chapter shows how the derivation of the CCAPM can be used as approximation for the climate beta and introduces a simple climate model to show the mechanics underlying the value of a beta on climate change mitigation. In Chapter 4 and 5, we extensively explain the DICE model and highlight the most relevant critiques the model has received, in relation to our investigation of the climate beta. Chapter 6 and chapter 7 describe our methodology, our analysis of the variables and describes the intuition behind the mechanics underlying our estimations. Chapter 8 provides the reader with the results of our estimations for the climate beta. Our analysis and results will be discussed in chapter 9. We conclude this thesis in chapter 10, thereafter chapter 11 is devoted to our personal reflections.

2 Background

2.1 Climate Policy - Correcting a Market Failure

"Climate change is the greatest market failure the world has ever seen." Stern (2007a) Stern hereby refers to the fact that the climate is a public good which is suffering from negative externalities. Production of economic output and consumption goods leads to greenhouse gas (GHG) emissions which again damages the climate. The fact that these negative externalities from production are not accounted for in the market creates a market failure.

The climate is a non-excludeable good, which implies that one can 'consume' climate, such as the air we breath, clean water and other natural resources, as it is 'provided' naturally, without getting excluded from it. A good is **excludeable** if it is practical and feasible to selectively allow consumers to consuming the good. A good is **rivalrous** if the consumption of the good of one person diminishes the amount of the good available for others to consume. This holds true for "the climate" as a consumption good. Despite this non-excludeability of the climate resources, they are limited in quantity; we only have one world and the resources are getting fewer and scarcer. It can be argued that the climate is to some extent rivalrous, as the 'use of the climate' by one person might diminish the ability for others to enjoy the climate, depending on the way it is used. Pollution of the air, through GHG emissions is an example that illustrates this rivalry.

Without regulation, the market itself fails to price in GHG emissions in a manner that internalizes this negative externality. Furthermore, GHG **emissions reductions** are public goods, implying they are non-excludable and non-rivalrous. As a consequence, the willingness to pay for emissions reductions is low while societies realize that depleting the common public good "our climate" is not in their best interest. The quantity of greenhouse gas emissions is above the social optimal level and the price of emitting GHG when producing and/or consuming other goods, is too low. As the market fails to correctly price GHG emissions, companies and individuals need incentives to internalize the external damages, and move away from emitting green house gasses into the atmosphere. Therefore, a policy that deals with pricing the negative externality is needed.

In economic policy making, the tool that is often used to deal with market failures and externalities, is social Cost Benefit Analysis (CBA). The DICE model developed by William Nordhaus employs CBA, which is the social appraisal of marginal investment projects and policies, affecting the future. CBA uses welfare economics- arguments rather than commercial arguments to correct for the projects appraisal from market failure. Another way, though not mutually exclusive, is to use market-based instruments, such as emission-trading schemes or carbon taxes. An example of this is the European Carbon Emission-Trading-Scheme (ETS), an implemented market-based instrument to correct for negative externalities.

Compared to other market failures, climate change exhibit features that make it particularly challenging to deal with: the GHG emissions are persistent in the atmosphere for a extremely long time horizon and are caused by- and imposed on- a global scale. A natural question that arises is: What would be the optimal carbon tax on climate damages? From a CBA-perspective it can be argued that the optimal policy implies equalizing the marginal **costs** of GHG emissions to the marginal **benefit** of abatement, i.e. the marginal benefit of reducing the total GHG emissions by 1 Gt CO_2. This is how the optimal Social Cost of Carbon (SCC) is derived using the DICE model and other Integrated Assessment Models (IAMs). In a world without any other market failures, the SCC equals the optimal tax on one unit of CO_2. In order to perform an CBA of the economics of climate change, a well-known integrated assessment model, that is a mathematical computer model based on explicit assumptions, can be used. The Dynamic Integrated Climate-Economy (DICE) model is a well-known example of an IAM that focuses on economic issues. The section below briefly discusses the use of IAMs and highlights several examples.

2.2 Integrated Assessment Models

2.2.1 Definition

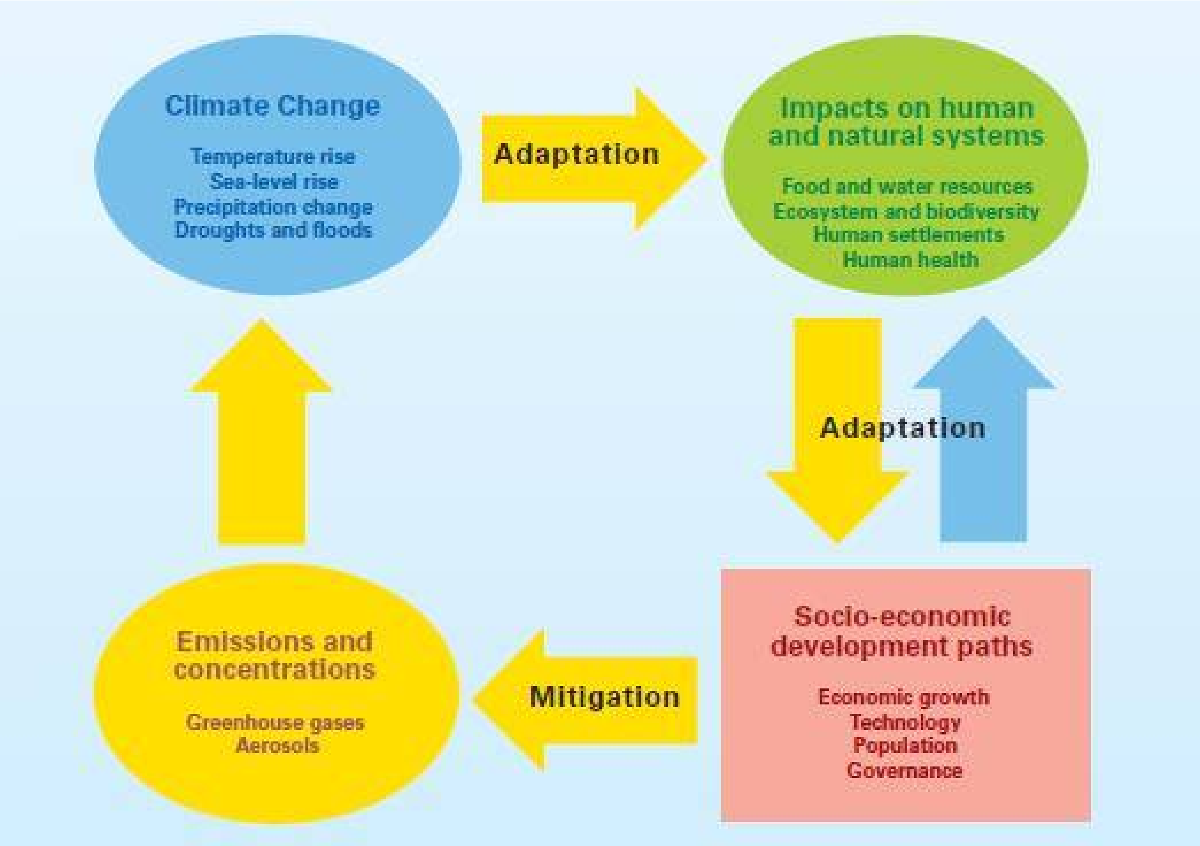
Integrated assessment models are used to evaluate the combined effects of different scientific disciplines. In the case of climate change and economics, IAMs are used to capture the effect of the climate on the economy and vice versa. These interrelated effects can be thought of as follows:

Due to the combined modelling of the environment and the economy, IAMs can be used for policy making regarding climate change. Both policy makers, as well as researchers, have tried to answer questions like "what is the lowest-cost strategy to achieve a given temperature-target?" and "what is the optimal tax on greenhouse gas emissions in order to maintain a certain maximum level of global average temperature increase?" by using IAMs in cost-benefit analyses. As IAMs asses the combined consequences of various fields of research, the models can get complex. Despite the complexity of the models, each individual component is heavily simplified. It should be noted that the simplification of each individual component of the model might render the model incapable of producing accurate results. This point will be elaborated in chapters 4 and 5 about the DICE model and the critiques it has received.

2.2.2 Different Models

In order to asses the optimal tax on carbon in CBA, depending on the so-called Social Costs of Carbon (SCC), an integrated assessment model with the focus on economic implications of climate change can be used. The most common examples of this type of models are the Dynamic Integrated Model for Economics and Climate (DICE) and the the Regional Integrated Climate Economy Model (RICE), both developed by William Nordhaus; the Policy Analysis of the Greenhouse Effect (PAGE) developed by Chris Hope; the Climate Framework for Uncertainty, Negotiation and Distribution (FUND) model developed by Richard Tol; and the World Induced Technical Change Hybrid (WITCH) model developed by Valentina Bosetti.

Figure 2.1: Climate change - an integrated framework



Source: IPCC (2001), "Climate Change 2001: Synthesis Report."

As long-run projections need to be made, all these models attempt to model economic growth, climate change risk and uncertainty. The focus in this paper will be on the DICE model, as it was the first, and is the most recognized IAM used for climate-economy CBA. It is not within the scope of this thesis to assess or compare the different integrated assessment models, therefore models other than DICE will not be explained further. Major topics related to the modelling of IAMs are uncertainty, both in terms of the parameters and the probability distribution of climate change risk , discounting, and, the optimal Social Cost of Carbon. All these topics, one more extensive than the other, will play a role throughout this thesis. Going back to the economic theory behind the DICE model, it builds upon the neoclassical Growth model (Nordhaus, 2013) developed by Robert Solow (1956) which will be elaborated on below.

2.3 Neoclassical Growth

The DICE model views the economy in a neoclassical growth perspective, where the objective is to maximize the total welfare of the economy, also referred to as the "dynasty", across time. Nordhaus extends the neoclassical model by incorporating the climate system by means of natural capital. In order to understand the macroeconomic implications of the findings of this paper, and the DICE model in general, it is valuable to have a basic understanding of the underlying neoclassical growth models, such as the Solow-Swan model and the Ramsey model. In this section we will go through the key equations of both macroeconomic growth models.

2.3.1 The Solow-Swan Growth Model

The Solow-Swan growth model, hereafter called the Solow model, was developed by Robert Solow (1956) and Trevor Swan (1956), independently from each other. The model is a cornerstone model in macroeconomic theory, forming the basis for several more complicated models. The model builds on three simple equations, as shown below. These equations imply four basic components: i.e. four key variables. The economy is closed, and does therefore not consider net export or any other cross-country effects. This is suitable to the DICE model, which is an aggregated model representing the world as a whole, hence no export/import nor country-specific characteristic are considered[2]. Output, Y_t, relates closely to the national accounts, [3] used to model aggregate welfare today(The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018). There is one type of good, only, which is used both as consumption and as investment.

$$GDP = Y_t = C_t + I_t \tag{2.1}$$

2.1 thereby also represents the resource-constraint of the economy. C_t represents consumption and I_t represents investments. Output is represented by a time-dependent production function, F, of capital K_t, and labour L_t. Emphasis is put on the time dependency because this represents the differences in for example technology between generations. This (labour augmenting) technology, A, is exogenously determined and increases output:

$$Y_t = F(K_t, AL_t) \tag{2.2}$$

The production function is assumed to have constant returns to scale in both capital and labour. i.e if both K_t and L_t increase by a fraction, ϕ, then total output will increase by the same fraction; $F_t(\phi K_t, \phi L_t) = \phi F(K_t, L_t)$, $\phi \geq 0$. Despite the constant returns to scale of the production function as a whole, however, K_t and L_t have decreasing returns to scale. One equation that satisfies these assumptions, is the commonly used Cobb-Douglas production function, which is also the one used in the DICE model [4].

$$Y_t = A_t K_t^{\gamma} L_t^{1-\gamma} \tag{2.3}$$

What determines the stock of capital, K_t in this model? Solow assumes a constant savings rate, $S_t = [0, 1]$. This savings rate is an exogenously determined fraction of output and is used for investments, affecting the growth of capital. The only way in which welfare can be transported intertemporally, that is from one time period to another, is through savings. Capital increases in investment and decreases in the constant

[2] The DICE model is an aggregated model of the RICE model, which accounts for regional differences and hence allows for a more open economy (Nordhaus and Sztorc, 2013)

[3] When this is the case, net exports disappear, as we are in a closed economy, and the government spending lies implicitly in the consumption and investment variables

[4] Note that the notation in this chapter is slightly different from the one used in the DICE-model chapter

and exogenous depreciation-rate, δ. This gives the following two equations:

$$I_t = sY_t \tag{2.4}$$

$$K_{t+1} = K_t(1-\delta) + I_t \tag{2.5}$$

Solow also assumed that population grows at a constant rate, n. If these assumptions of constant population growth and constant savings rate hold, one can show through dividing by AL_t and substituting, that the growth rates of output, consumption and capital all converge to the same growth rate in the long run. Thereby one arrives at one of the key points of the Solow model: the economy converges to a ***balanced growth path***. (Romer, 2012).

The model presented so far, relies on an exogenous savings rate and was initially used to understand the underlying drivers of output growth in the long run. However, more recent literature realized that the optimal growth paths as described in the Solow model, holds true in a perfect world without frictions and externalities.

We note that climate, nature and natural resources are absent from the Solow model. Hence, environmental considerations are not taken into account in the production function, neither in the utility function. Production can be undertaken without CO_2 emissions and without implications for society's utility levels. On the other hand, in the first subsection 'Climate Policy - Correcting a Market Failure' we explained that there is no well-functioning market for the climate system. Climate change in itself can be seen as market failure, industrial emissions as negative externality of production, and emissions reductions can be seen as a public good. Therefore it would be suitable to base climate policy making tools, like the DICE model, on long-run growth models that employ a more market-based approach.

After the development of Solow's model, other economists realized that market-based approaches to growth models could give a more realistic presentation of the economy. What now follows is an explanation of a similar model, employing a more market based approach.

In order to maximize future welfare, coming from utility, the optimal amount of investment (or savings) should be made by the economy (or a representative consumer). That is, decisions are made on the microeconomic level by households and firms. We want to show here, that Solow's assumption of the constant savings rate s, follows from the optimal savings decision made by this representative agent that maximizes her utility in a dynamic model. The economic theory behind this optimal savings rate, which notably does not need to be constant and is no longer exogenous, goes back to Ramsey (1928), Cass (1965) and Koopmans (1965).

Similar assumptions about firms and their production function as outlined above, are made in the Ramsey-Cass-Koopmans Model. Representative consumers, that can be presented in the form of households, choose in every time period how to divide their income between consumption and savings in order to maximize utility. When we relate this to the DICE model and the long-term horizon of climate change, it is appropriate to think in terms of dynasties instead instead of households. Tis will also be done in chapter 4 when we extensively explain the DICE model.

Households maximize the following objective function, U,[5]

$$U = \int_0^\infty e^{-\rho t} u(C_t) \frac{L_t}{H} dt \tag{2.6}$$

when we assume the following functional form for the instantaneous utility function:

$$u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha} \tag{2.7}$$

Substituting the utility function into the objective function and using that population grows at a constant rate n, that is $L_t = L_0 e^{nt}$, gives us a dynamic welfare function which can be maximized over time.

After rewriting and substituting, one can summarize the predictions of this neoclassical growth model with endogenous savings rate as follows:

$$\max_{\{c_t, k_{t+1}\}_{t-0}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\alpha} - 1}{1-\alpha} \tag{2.8}$$

Subject to

$$C_t + K_{t+1} - (1-\delta) K_t = K_t^\gamma \left(A_t L_t\right)^{1-\gamma} \quad \forall t = 0, 1, \dots \tag{2.9}$$

One can show that if optimal consumption paths are chosen subject to the constraint above, consumption, output and capital (per capita) converge to the constant growth rate, g_A, of labour-augmenting productivity, A. Thereby the savings rates, that are now endogenous and time-dependent, converge to a ***constant*** in the long-run;

$$s_t \equiv 1 - \frac{c_t}{y_t} \tag{2.10}$$

Concluding, the assumption from Solow that the savings rate is constant, follows from utility-maximization in the long-run in the Ramsey model with forward-looking decision making agents. This traditional neoclassical growth framework, is extended by Nordhaus, in the sense that he includes natural capital of the climate system. Natural capital grows with "green investments" aimed at reducing emissions of greenhouse gasses (GHG), and is reduced by increased concentrations of emissions. The equation of motion for natural capital assimilates the traditional way of looking at capital growth; By saving today, the society can allocate parts of it's output to emissions- reductions, in order to increase future consumption. Nordhaus' extensions to the dynamic optimal-savings model that we outlined in this section, will be extensively discussed in Chapter 4 about the DICE model.

[5]Note that the utility function and the functions that follow hereafter are written in continuous time, while in the DICE model's equations discrete time is used, see chapter 4

3 The Climate Beta

3.1 Discounting Climate Investments

This chapter explains how we arrive at the definition of the climate beta, derived through the original CCAPM, and where the climate beta enters in cost-benefit analysis (CBA). Complementing the derivation of the climate beta, we will provide an intuitive explanation surrounding the relationship between the climate beta and the DICE model, used as tool for CBA.

As explained in the previous chapter and in line with neoclassical growth theory, the DICE model's investments in climate change abatement increase future consumption by substituting current consumption goods for climate investments. Climate investments are projects which increase the natural capital in the future. In order to evaluate how much these projects are worth, an important determinant are the benefits, i.e. the expected future cash flows of the projects. How much added future consumption do we expect to obtain, from refraining from current consumption in order to invest in climate change projects? If we only have to reduce today's consumption by a small amount, and the expected benefits are relatively large, the investment project will be of higher value.

In order to optimize the social welfare of the economy (the dynasty) however, there are more factors than the expected payoff of investment projects that have to be accounted for. The riskiness of the investment is important. So is *when* the benefits are received and in *what aggregate state of the world* the economy is in at the time when the benefits are received. Recall that the utility of the representative consumer is modelled with a concave utility function. The concavity of the utility-function implies that she values each marginal unit of consumption more in states when the consumption-level is low. Following economic theory she place higher value on projects that will increase consumption in states when aggregate consumption is low than projects that pay off only in good states of the world, when marginal utility is lower. Following this intuition it is clear that how the benefits of added consumption from mitigation projects co-move with the overall consumption level is a crucial part of maximizing the social welfare. As the climate beta shows the marginal benefit of an investment with respect to consumption levels in the future, it gives valuable information in analysis of climate abatement investments. The climate beta enters in the discount rate of the mitigation projects. A higher co-movement, i.e a higher beta, leads to a higher discount rate, thereby implying a lower Net Present Value (NPV)of the mitigation project.

The DICE model is a good tool for performing CBA because it is in the nature of the model to determine the costs and benefits of a policy over time. Following the intuition above, the discount rate used to estimate the NPV of abatement investment projects, should reflect actual economic outcomes, including risks and uncertainties. The discount rate applied in the DICE model also follows the assumption that actual economic outcomes are reflected in the discount rate. A natural question arising is now "how to determine a discount rate?"

3.2 How to Determine the Discount Rate?

In the previous chapter, we saw the derivation of the Ramsey rule, used for safe investments. Recall that the Ramsey rule states that the optimal intertemporal allocation is found where the interest rate equals the return on investment, which is the growth corrected discount rate

$$r = \rho + \alpha g \tag{3.1}$$

The Ramsey equation should be used for safe investment projects, that will generate a certain stream of cash flows in the future. That is because 3.1 provides the equilibrium rate of return in an optimal growth model, without risk or taxes. For a further explanation of Ramsey growth, see Romer (2012) and/ or chapter 2

If we were to linearize all the equations in the DICE model, to provide an approximation for the long-run steady-state of the economy, we could approximate the subjective discount rate to the Ramsey equation,**??**.

There are significant risks related to climate investments, however, some of which will be highlighted in Chapter 4. Consequently the uncertainties around the benefits of climate change mitigation projects will also be present. This justifies treating investments in climate change mitigation as risky-investment projects and hence rendering 3.1 inappropriate.

The much used Consumption Capital Asset Pricing Model (CCAPM), adds uncertainty to the risk free rate (of Ramsey), and later in this chapter we will show extensively how the climate beta can be derived mathematically from the CCAPM. First, we will give a short explanation of how the climate beta relates to the CCAPM.

We follow the CCAPM formula, $r = r_f + \beta\pi$, where r is the discount rate on the (climate) goods, r_f is the risk free rate and π, the market risk premium i.e the excess return above the average investment portfolio. In line with our explanation of the climate beta above, the β shows the marginal benefit of an investment, (in our case: investment in climate mitigation) with respect to consumption in the future. The benefits can move over time and intuitively tells us how the benefits of the investments move together with the aggregate consumption. In the general CAPM model, β is the main determinant an asset's price. When considering climate change, the climate β is the main determinant in the evaluation of a mitigation project.

3.2.1 Discounting in the DICE Model

Starting from the 2013 version of the DICE model in GAMS, as the latest manual available is based on that version, Nordhaus *assumes* that 'the climate beta' equals 1. (Nordhaus and Sztorc, 2013) Similarly, based on their extensive investigation of the climate beta, Dietz et al. (2018) argue for β_t around unity. 'The climate beta', hereafter β or β_t, refers to the consumption beta on climate investments, measuring the co-movement between investments in climate mitigation and aggregate consumption. A β around unity implies that the stream of future cash flows from the climate investment share the same risk-structure as macroeconomic risk. When $\beta = 1$, the market risk-premium should be added to the risk-free rate to account for riskiness of the cash flows surrounding climate mitigation investments. Consequently, a positive β gives rise to a higher rate than the risk-free rate. Vice versa, if β would be negative, a lower rate than the risk-free rate should be used to estimate the NPV of climate mitigation projects. By adding a (positive or negative) risk premium proportional to β, on to the risk-free rate, the rate r_t results in the risk-adjusted discount rate. Although in the two latest versions of the DICE model a beta of 1 is assumed, '*the extent to which climate investments are correlated with systematic consumption risk is an open question*'Nordhaus and Sztorc (2013). Consequently, an estimate for the 'climate beta' will be derived in this thesis.

When using Nordhaus' assumption of a climate beta of 1, 'the appropriate rate of return is '5% per year in the near term and $4\frac{1}{4}$ % per year over the period to 2100 for the 2013 version of the model' Nordhaus and Sztorc (2013). Nordhaus calibrates the preference parameters (elasticity of marginal utility of substitution and the pure rate of social time preference) to be consistent with the before-mentioned rates, as explained in the section 'Discounting by Nordhaus'. The discount rate is, as we have seen, a crucial determinant in the current generation's willingness to reduce consumption. This affects output, and through links which later will become evident and strongly impacts the parameters from where the social cost of carbon (SCC) is derived. Hence, the climate beta is thereby an important risk-measurement coming into play when estimating the SCC from the DICE model.

3.2.2 Hypothesis for the Beta's Values

The hypothesis for a positive beta sounds as follows: if in states of the world where aggregate consumption is **high**, it is **most beneficial** to have a policy in place mitigating climate change, then there exist positive correlation between aggregate consumption and benefits from mitigation projects, implying that the climate beta should be positive. The intuition underlying this hypothesis is that in economic upturns, the global mean temperature increase is relatively large due to large output levels, which increases emissions and damages. Hence the increase in global warming is relatively larger than in periods of recessions. Therefore, the benefits from mitigation policies are largest in booming states of the world, which is exactly when the aggregate consumption level is also largest. This implies $\beta > 0$.

3.3 Previous Research on the Climate Beta

Although a lot of previous research implicitly informs us about the climate beta by focusing on the discount rate used in climate change economics, not many have focused explicitly on estimating the correlation between the aggregate consumption level and the benefits from climate mitigation. This might be due to a combination of several reasons. For example, there exists limited historical data available to measure the correlation between benefits of climate mitigation projects and aggregate consumption.

Another possibility is the large amounts- and the different types of uncertainties regarding climate change and its effect on the economy. In this section, we present the previous research on the climate beta that has served as inspiration to our work.

3.3.1 Uncertainty in Future Cash Flows of Climate Change Mitigation

3.3.1.1 Howarth (2003)

One of the first papers that provided us with a measure to account for risk in the future cash flows related to climate change mitigation is Howarth (2003). He extends the Ramsey rule to allow for uncertainty in both consumption growth and returns to a risky asset. Howarth (2003) argued that Nordhaus (1994), by working with the earliest versions of the DICE model, significantly overstated the rate of pure time preference, because he did not account for uncertainty in financial markets. Nordhaus (2017b) and others [6] have later extended the DICE model and IAMs to account for uncertainty in the key underlying parameters, thereby partly addressing the problem of uncertainty. As explained earlier, Nordhaus calibrates the parameters in DICE, like the pure rate of social time preference and the elasticity of marginal utility of consumption, in a way that the goods discount rate r_t is consistent with market observations. A full analysis of the discount rate problem would need to account for risk coming from capital investment and climate change response-strategies combined, which "would present rather formidable computational complexities" (Howarth, 2003). Despite not providing a full empirical analysis of the risk premium that should be placed on the subjective discount rate, Howarth (2003) derives a correlation of 0.01 between consumption and net benefits of climate mitigation. This could implicitly be an estimate for the β on climate change investment projects. His findings are based on the assumption that consumption per capita will inevitably grow, *'either at an average rate of* 2.5% *or* 3.5%*, both with* 50% *probability'*(Howarth, 2003). He concludes by stating that the net benefits of climate change mitigation should be discounted at the risk-free rate. The basis for his conclusion is the nearly zero-correlation between consumption and the benefits from GHG abatement. The estimates underlying his conclusions, however, are based on a heuristic example rather than an empirical analysis. Therefore Howarth (2003) argues in turn that GHG abatement is a form of insurance against the risk of future generation's lower consumption levels. That way of reasoning indicates a negative climate beta. Howarth (2003)'s conclusion of a negative beta on the climate holds, provided that one uses the certainty equivalents of the net benefits to calculate the NPV of climate mitigation projects. These certainty equivalents account for the uncertainty surrounding public projects like abatement projects, coming from uncertainty in future consumption levels and the expected benefits of emissions abatement.

[6] For instance Weitzman (2013), Dietz and Asheim (2012), and Dietz and Stern (2015). Parts of the literature on 'critiques to the DICE model' is incorporated in Chapter 05

3.3.1.2 Sandsmark and Vennemo (2007)

The first *explicit* research on the climate β by Sandsmark and Vennemo (2007) developed a framework in which the standard CCAPM model was combined with the assumption of endogenous risk. They emphasize that risk in the case of environmental economics should be seen as endogenous, because it stems from the jointly determined climate- and the economic system. In their paper, Sandsmark and Vennemo (2007) explain the endogenous by referring to 'self-protection' types of climate mitigation investments [7] That means, some **climate mitigation investments** have the potential to reduce the ***probability*** of damages occurring, thereby changing the distribution function of the returns of the investment. Regardless of whether the global average temperature is increasing a lot, i.e. regardless of whether climate change is happening at a fast pace, , those investments reduce the probability of climate damages and increase the probability of high levels of aggregate consumption. The endogeneity of this kind of environmental risk in the economy, is one of the arguments why discounting in cost-benefit analysis for climate change mitigation is not a straightforward exercise. The way of approaching the discounting problem as Sandsmark and Vennemo (2007) results in the use of the Consumption Capital Asset Pricing Model (CCAPM) with adjustments for endogenously determined risk. By means of numerical example, historical interest rate data from the US market, and values for climate damages parameters mainly based on Nordhaus and Boyer (2000) they estimate a value for β of -0.004, based on a temperature increase of 0.5 °Celsius from pre-industrial times to the time their research was published. Since the absolute value of this β is so small, they conclude that climate change investment projects should be discounted at the risk-free rate, and treated similar to a risk-free asset. Additionally, Sandsmark and Vennemo (2007) show that when the maximum allowed increase in global mean temperature is set at a value of 2.5 °Celcius, their value of β decreases further to -0.008. Therefore they conclude that in the long-term, a lower discount rate than the risk-free rate should be used to discount the benefits of climate change mitigation.

Their results are based on a two-period simplified climate model, assuming that the world ends after the second period and all wealth will be consumed in the second period of the model. Furthermore, it relies on the assumption that there is negative correlation between aggregate consumption, or the market portfolio, and a climate asset. Investing in the climate asset therefore serves as a hedge, or insurance against market risk. Lastly, they point out that in their relatively simple model of only 2-periods, they estimate β as a constant. More research should be done in order to investigate the term structure of β and consequently the movement of the risk-adjusted discount rate over time, for the case of climate change abatement assets. We note that the general hypothesis in favour of a negative climate β is that climate change mitigation mostly pays off in states of the world where aggregate consumption is low, hence there is a negative correlation between benefits of climate mitigation and the level of aggregate consumption. This implies that climate investments by the public sector can be seen as insurance against risk for the entire society. The ideas underlying the hypothesis that public sector climate investments are seen as insurance against societal risk were already lied out by Lind et al. (1982): '*the returns from energy research and development in the future may be negatively correlated with the returns to all other*

[7] following the terminology of Ehrlich and Becker (1972)

investments so that public investments in this case would have the effect of insurance. If we were to account for this insurance effect by altering the rate of discount, we should use a lower rate of discount than the risk-free rate, not a higher one.' (Lind et al., 1982) Although Lind referred hereby to future energy research and development, this hypothesis would in principal be applicable to various types of climate change mitigation investments. The difference between Sandsmark and Vennemo (2007) and Lind et al. (1982) is that the latter did not distinguish between exogenous and endogenous risk. When doing so, Sandsmark and Vennemo modify the standard CCAPM pricing formula such that it incorporates the possibility of climate investments altering the distribution function of returns. In summary, investing in GHG mitigation is worthwhile according to the research performed by Sandsmark and Vennemo (2007), due to the relatively low interest rate of 0.2% below the risk-free rate that they suggest to use.

3.3.1.3 Weitzman (2013)

Weitzman (2013) acknowledges the difficulties of estimating a climate beta based on data, due to the lack of historical record and a real-world close substitute. He does however, provide a theoretical linear model, inspired by the CAPM, to estimate gamma. He defines the real project gamma, γ, as 'the fraction of expected payoff that on average is due to the non-diversify-able systematic risk of the uncertain macro economy' Weitzman (2013) The most important conclusion from Weitzman (2013) is that the distribution of damages is one of the most critical determinants in the sign and value of gamma. Gamma can be seen as a proxy for the climate beta that we estimate in this study. If one places more weight on the tails of the damage probability distribution function, the effects of the worst states of nature dominate the uncertainty concerning economic growth, giving rise to a negative value for gamma (Weitzman, 2013). Importantly, if one uses a standard IAM to calculate a project gamma, then one will most likely end up with a relatively large positive value for γ. This is due to the little or no weight that is given to catastrophic damages of climate change, in the standard versions of IAMs like Nordhaus' DICE model.

3.3.1.4 Daniel et al. (2018)

One of the more recent studies, by Daniel et al. (2018) points out that the practice of asset pricing and standard asset pricing models are of crucial importance in case of climate mitigation policy making, due to the inherent aspect of risk mitigation. Hence, Daniel et al. (2018) use a representative agent in their "Epstein-Zin (EZ) Climate" model, weighing the costs and benefits of a climate mitigation investment opportunity in order to maximize expected lifetime utility. The agent internalizes climate change damages, and decides the level of mitigation to invest in. Similar to Sandsmark and Vennemo (2007), Daniel et al. define a stochastic parameter θ_t, reflecting 'the Earth's fragility'. The representative agent chooses the optimal amount of mitigation in every period, depending on θ_t. Naturally, the damages of climate change are a function of θ_t, which value is not known before the year 2300. By means of this model, Daniel et al. (2018) allow for uncertainty in the damages from the increase in global mean temperature, to aggregate social-economic welfare. This is the ***only*** way in which they account for uncertainty in their model. They do not account for uncertainty in growth of output or total factor productivity over time, for instance. Ii is important to point out that Daniel et al. (2018) make the distinction between society's willingness to substitute consumption over time, and its willingness to substitute consumption across states of nature. Hence, the representative agent in their model does not have a logarithmic utility function, neither Constant Relative Risk Aversion (CRRA) preferences.Daniel et al. (2018)assume EZ-preferences in the utility function instead, while allowing for large uncertainty (risk) in the damages from climate change, as mentioned above. The inclusion of EZ-preferences rather than assuming CRRA utility function, such as in the standard DICE model, leads to a different optimal path of emissions over time, ceteris paribus. Daniel et al. (2018) aim to estimate an optimal CO_2 price over time, which is a function of the marginal damage and the marginal value of an additional unit of consumption in the future. Even more interesting for our research, is their description of a risk premium within the optimal price for CO_2 emissions, which is defined as 'the covariance of the marginal damages with respect marginal utility' (Daniel et al., 2018) Their results show that this risk premium can actually become the main determinant of the present value's price of CO_2 emissions, i.e. the SCC. They end with showing that the SCC is '*increasing in the risk-aversion of the representative agent*'. (Dietz et al., 2018), which implies a negative value of 'the climate beta'.

Previous literature shows that relatively little focus has been placed on explicitly measuring a beta for climate change investments. On the other hand, a lot of research concerning the expected value of climate change damages and climate change mitigation already exist. We conclude from the literature discussed in this subsection that, focusing on the uncertainty around the effect of temperature increases on future consumption path, is more likely to result in a negative value for a climate beta. Whereas, focusing on other uncertainties, for example surrounding future growth of GDP, and the use of IAMs like the DICE model are more likely to result in a positive estimate for a climate beta. Most previous research has used simplified short-term models, focused on the (uncertain) impact of climate damages in order to measure the climate beta, and highlighted the insurance aspect of climate change mitigation projects to macro economic risk. Especially the intuition underlying those results and their combination of a simplified climate model with traditional asset pricing models serve as examples for us in the light of our explanation and estimation of the climate beta in the next subsections.

3.4 Derivation of Beta in CCAPM

In this section we show how beta for climate change mitigation can be derived analytically from the CCAPM. As explained in the previous sections, there are no traditional asset pricing tools for the pricing of climate investments, as climate investment is not a traditional market asset. However, climate investments have similarities with traditional risky investments in the market, which enables us to apply a version of the traditional CCAPM. As the investment is risky, the future benefits of such a project should, in an optimal growth model, not be discounted at the equilibrium rate of return, i.e. the rate that is discussed so-far throughout this paper. Rather than the *benefits* of the climate mitigation projects, the

certainty equivalent of those *benefits*, should be discounted at the rate of return that is used in the IAMS like the DICE model. A central question is how to estimate the certainty equivalent of the benefits.

If both aggregate consumption, C_t, and the benefits from a marginal emissions reduction project, B_t, follow an arithmetic Brownian Motion, it is implied that the expected change in benefits is 0. This allows for an arbitrary variance and a nonzero mean, and hence the certainty equivalent cash flows are not needed. Instead, the risk premium that should be added to the 'risk-free' discount rate can be approximated by a simpler formula, based on the CCAPM.

The ideas outlined below are not novel developments, but they are based on Dietz et al. (2018) and Gollier (2013). We try to explain the link between the CCAPM β and the climate beta by means of a simple climate model example. Thereafter, through the way we alter the DICE model and add uncertainty to our analysis of the climate beta estimate, we depart from the methodology of (Dietz et al., 2018). Those alterations will be explained in the next chapter. Combined with the corresponding analysis and results in the upcoming chapters, we will contribute to the discussion about the climate beta and discounting within climate change economics.

3.4.1 The Climate Beta from CCAPM

The climate beta can be derived from the Consumption Capital Asset Pricing Model (CCAPM). Lucas (1978) developed the theoretical foundation for the CCAPM, while Breeden (1979) showed, using a continuous time model, that the return of a security depend on its covariance with aggregate consumption. The CCAPM will be reviewed below, in order to provide a derivation of beta.

Consider a welfare-maximizing social planner with a concave utility function $U(C_t)$, that is $U'(C_t) > 0$ and $U''(C_t) < 0$, where C_t stands for consumption at date t. Because it's future stream of consumption is unknown at time $t = 0$, it can be seen as a risky stream of future cash flows. Hence, C_t is considered a random variable. Furthermore, consider a risky investment project yielding benefits B_t, which future values are unknown. Investing ε amount in the project today, whereby we assume ε to be very small, will yield the following intertemporal welfare, based on utility coming from consumption and the benefits of the marginal investment:

$$W(\varepsilon) = u\left(c_0 - \varepsilon\right) + e^{-\rho t} E u\left(c_t + \varepsilon B_t\right) \tag{3.2}$$

Note that this is based on the assumption that ε is small and the utility function being additive. Investing the amount ε in the project today, increases the agent's inter-temporal welfare, if the present value of the marginal increase in utility coming from the expected benefits, is larger than the marginal reduction in utility coming from the decrease in current consumption at time t_0, assuming the investment is financed by cutting consumption today. That is,

$$e^{-\rho t} E B_t u'\left(c_t\right) \geq u'\left(c_0\right) \tag{3.3}$$

The expected value of the future increased consumption is discounted back to today's value by the continuous time discount rate $e^{-\rho t}$. Whereby ρ is the 'rate of impatience', or the pure rate of time preference of the agent. Recall that if $\rho = 0$, the agent values all future generations' consumption on an equal basis as its own. Vice versa, if $\rho = 1$, the agent does not care about the future at all, but instead only cares about it's current consumption. Note that it is common to use the notation of δ for the same parameter, but we choose to follow the notation of the DICE model (i.e. ρ) here.

Dividing the previous condition by marginal utility from consumption at time 0, $u'(c_0)$, and multiplying and dividing by expected marginal utility $Eu'(C_t)$, allows us to rewrite the entire expression as follows:

$$e^{-\rho t}\frac{Eu'\left(c_t\right)}{u'\left(c_0\right)}\frac{EB_t u'\left(c_t\right)}{Eu'\left(c_t\right)} \geq 1 \tag{3.4}$$

In this way, we obtained a notation for the certainty equivalent stream of the uncertain future cash flows, F_t, which can be defined as the stream of cash flows itself under risk-neutral expectation. In our example these cash flows are the benefits B_t.

$$F_t = \frac{EB_t u'\left(c_t\right)}{Eu'\left(c_t\right)} \tag{3.5}$$

This means, F_t is a weighted mean of the different possible realizations of B_t. If B_t would be certain, then it would hold that $B_t = F_t$. On the other hand, if the cash flow B_t is risky but that risk is independent of the systematic macroeconomics risk, i.e. independent of the risk corresponding to c_t, it follows from 3.5 that $F_t = EB_t$. This result is intuitively appealing: if there is no correlation between the risk of the benefits from the investment project, and aggregate economy-wide risk , all the risk related to the investment under scrutiny, can be eliminated by diversification. Hence, no risk-premium is associated with the investment project and the certainty equivalent of the benefits are just the expected value of the benefits itself. In fact, the certainty equivalent F_t in 3.5 is the standard asset pricing formula in finance. Since B_t is risky and possibly correlated with aggregate consumption to some extend, it can be defined as

$$F_t = E^* B_t \tag{3.6}$$

This equation says, the certainty equivalent of the benefits from the project under scrutiny are the benefits itself under risk-neutral expectations. The risk-neutral expectations operator is in turn defined as:

$$E^* f(b) = \frac{Ef(b)u'\left(c_t\right)}{Eu'\left(c_t\right)} \tag{3.7}$$

Combining , we can write equation 3.4 as:

$$NPV = -1 + e^{-r,t}F_t \geq 0 \tag{3.8}$$

whereby:

$$r_t = \rho - \frac{1}{t}\ln\frac{Eu'\left(c_t\right)}{u'\left(c_0\right)} \tag{3.9}$$

Later on it will become clear why it is useful to write the NPV of the project and the discount rate in this way.

The discount rate in 3.9 is simply the 'safe-discount rate' . In the DICE model, Nordhaus calibrates it to be consistent with market equilibrium outcomes, that is $r_t = 5\%$ in the near future and $r_t = 4,25\%$ in the distant future. In the long-run, this is consistent with the Ramsey equation: $r_t = \rho + \alpha g$, where g is the growth of consumption over time and α the elasticity of marginal utility of consumption. And, as mentioned earlier, ρ is the pure rate of time preference, that is the rate used to calculate the present value of utility coming from (future) streams of consumption.

In summary, the section above showed that one can apply a two-step procedure, meaning first replacing the risky cash flows with its certainty equivalent F_t and thereafter discounting this certainty equivalents using the 'risk-free' discount rate r_t, in this case equal to the long-term rate coming from the Ramsey growth equation. This result is useful since it shows that the 'safe discount-rate', also called the 'socially efficient discount rate', to compute the present value of welfare within the DICE model, can also be used discount risky investment projects. The agent should undertake the risky investment project, for example an emissions abatement investment, only if the NPV computed via this two-step procedure is positive.

However, Sandsmark and Vennemo (2007) point out that in the case of climate economics, the benefits from investment projects and the costs related to climate change mitigation appear to be less suitable for this type of internalizing the risk by means of certainty equivalents of the benefits. This is also the case for CBA analysis. To our knowledge,Sandsmark and Vennemo (2007) do not provide an explicit explanation, other than relying on the fact of endogenous environmental risk and stating that the income, benefits and costs in the case of climate mitigation investment projects seem to be processed in other ways than using the above-explained types of certainty equivalents. In the case of climate change mitigation, we can assume that the cash flows, or the benefits, from the investment project, B_t, are correlated with consumption. Hence the risk or uncertainty surrounding both the future stream of cash flows, benefits, and the systematic risk corresponding to aggregate consumption, C_t, are to some extend correlated. If B_t and C_t are dependent on each other, and the utility function U_t is concave, the certainty equivalent F_t of the cash flows will increase in value. This implies a positive risk premium if there is positive statistical correlation between the benefits and systematic macroeconomic risk. This type of investments will increase the risk held by the investor, hence the rate of return should be higher than the risk-free rate. If the statistical relationship between the stream of benefits and aggregate consumption is negative, a negative risk premium results. In the latter, the emissions mitigation project reduces aggregated risk, thereby serving as a form of insurance or hedge, implying a negative risk premium.

The two-step procedure explained above, i.e. discounting the **certainty equivalent**s of the risky cash flows at the ***socially efficient discount rate,*** can be seen equivalent to the following alternative one-step procedure: discounting at a rate that takes into account the riskiness of future cash flows. **A risk-adjusted discount rate** can be applied to calculate the NPV of the benefits B_t of the climate mitigation project, rather than having to calculate the certainty equivalent of those benefits first. This method will be explained below.

Combining the equations 3.4 and 3.8, allows us to describe the net present value (NPV) of the marginal investment project as:

$$\text{NPV} = \sum_{t=0}^{\infty} e^{-\delta t} \text{E} B_t \frac{u'(c_t)}{u'(c_0)} = \sum_{t=0}^{\infty} e^{-r_t t} \text{E} B_t \tag{3.10}$$

whereby:

$$r_t = \rho - \frac{1}{t} \ln \frac{\text{E} B_t u'(c_t)}{u'(c_0)\, \text{E} B_t} \tag{3.11}$$

This says, investing a small amount in the investment project today, yields a net present value (NPV) of the discounted expected benefits, discounted at the rate defined in 3.11.

Now, we can apply the general definition of the covariance between two variables X and Y (in our case benefits B_t and consumption C_t):
$Cov(X, Y) = E[(x - \bar{x})(y - \bar{y})]$
$= E(xy) - (Ex)\bar{y} - \bar{x}Ey + \bar{x}\bar{y}$
$= E(xy) - \bar{x}\bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y}$
$= E(xy) - \bar{x}\bar{y}$
Hence $E(xy) = \text{cov}(x, y) + \overline{xy}$

Applying the rewritten definition of the covariance to equation 3.11 gives us:

$$r_t = \rho - \frac{1}{t} \ln \frac{\text{E} B_t u'(c_t)}{u'(c_0)\, \text{E} B_t} \tag{3.12}$$

$$= \rho - \frac{1}{t} \ln \frac{cov(B_t, u'(c_t)) + \text{E} B_t \text{E} u'(c_t)}{u'(c_0)\, \text{E} B_t} \tag{3.13}$$

$$= \rho - \frac{1}{t} \ln \frac{\left[\text{cov}\left(\frac{B_t}{EB_t}, \frac{u'(c_t)}{Eu'(c_t)}\right)\right] EB_t Eu'(c_t) + EB_t Eu'(c_t)}{u'(c_0)\, EB_t} \tag{3.14}$$

$$= \rho - \frac{1}{t} \ln \frac{E(u'(c_t))}{u'(c_0)} \left[1 + \text{cov}\left(\frac{B_t}{\text{E}B_t}, \frac{u'(c_t)}{Eu'(c_t)}\right)\right] \tag{3.15}$$

$$= \rho - \frac{1}{t} \ln \frac{E(u'(c_t))}{u'(c_0)} - \frac{1}{t} \ln \left[1 + \text{cov}\left(\frac{B_t}{\text{E}B_t}, \frac{u'(c_t)}{Eu'(c_t)}\right)\right] \tag{3.16}$$

If we assume here that the covariance term in 3.16 is small, which will later be shown to hold true in our results, the approximation $log(1 + x) \approx x$ can be used. In our case x is the covariance term:

$$= \rho - \frac{1}{t} \ln \frac{E(u'(c_t))}{u'(c_0)} - \frac{1}{t} \text{cov}\left(\frac{B_t}{\text{E}B_t}, \frac{u'(c_t)}{Eu'(c_t)}\right) \tag{3.17}$$

$$= \rho - \frac{1}{t} \ln \left[\frac{E(u'(c_t)}{u'(c_0)} + \text{cov}\left(\frac{B_t}{EB_t}, \frac{u'(c_t)}{Eu'(c_t)}\right)\right] \tag{3.18}$$

On this basis, we can show how we derive at an expression of the CCAPM β. Investing in a marginal project, is worthwhile if its NPV is positive, with use of the following discount rate: Following the CCAPM:

$$r_t = r_f + \beta\pi \tag{3.19}$$

whereby the risk premium is:

$$\pi = \gamma\sigma^2 \tag{3.20}$$

And from the (extended) Ramsey Equation[8], we know:

$$r_f = \rho + \gamma\mu - 0.5\gamma^2\sigma^2 \tag{3.21}$$

Here, the term $0.5\gamma^2\sigma^2$ reduces the 'safe' discount rate by a term based on relative risk aversion parameter γ and the variance of consumption σ^2, i.e. the entire term can be interpreted as precautionary savings. Substituting the discount rate from the extended Ramsey equation 3.21 in the CCAPM 3.19 gives:

$$r_t = \rho + \gamma\mu - 0.5\gamma^2\sigma^2 + \beta\gamma\sigma^2 \tag{3.22}$$

However, we decide to follow our notation similar to the one at the beginning of this chapter, as it is used in the DICE Model:

$$\gamma = \alpha = 1 \tag{3.23}$$

Setting the relative risk aversion parameter γ, or the elasticity of marginal utility of consumption α, equal to 1, holds in the long-term, assuming utility optimizing representative consumers with a logarithmic utility function. Thereby we get:

$$r_t = \rho + \mu - 0.5\sigma^2 + \beta\sigma^2 \tag{3.24}$$

This can be seen as a risk-adjusted, or extended version of the Ramsey equation, whereby $\mu - 0.5\sigma^2$ is the growth of expected consumption over time, if we assume consumption to be normally distributed with finite mean μ and variance σ^2, that is $C_t \sim N(\mu, \sigma^2)$. Let us denote the growth rate of expected consumption, or the change in the natural logarithm of expected consumption by g:

$$r_t = \rho + g + \beta\sigma^2 \tag{3.25}$$

Now, getting back to 3.18 that we discussed above:

$$= \rho - \frac{1}{t}\ln\left[\frac{E\left(u'\left(c_t\right)\right.}{u'\left(c_0\right)} + \operatorname{cov}\left(\frac{B_t}{EB_t}, \frac{u'\left(c_t\right)}{Eu'\left(c_t\right)}\right)\right] \tag{3.26}$$

Turns out to be the same as

$$= \rho + g + \beta\sigma^2 \tag{3.27}$$

[8]for elaboration of the extended or risk-adjusted version of the Ramsey Equation, see Gollier (2013)

With

$$\beta = \frac{\operatorname{cov}\left(\frac{B_t}{EB_t}, \frac{u'(c_t)}{Eu'(c_t)}\right)}{\sigma^2} \tag{3.28}$$

Concluding, beta turns out to be equal to the covariance between expected benefits and the expected marginal utility of consumption, divided by the variance of consumption σ^2.

We would like to remind the reader that this is based on the assumption of consumption being a normally distributed random variable, and the utility function being logarithmic. We are aware that there exist a distinction between the concepts of **risk aversion** and **the elasticity of marginal utility of consumption**, which are strictly speaking distinct concepts [9], however, we assumed that the representative welfare-maximizing consumer has constant relative risk aversion α. It's utility function and and marginal utility are respectively:

$$u(c_t) = \frac{c_t^{1-\alpha} - 1}{1-\alpha} \tag{3.29}$$

$$u'(c_t) = c_t^{-\alpha} \tag{3.30}$$

One should note that the risk premium π has a flat term structure, the way it is defined above. However, β_t and r_t are likely to have a non-constant term structure in reality when considering climate change mitigation projects. Consequently, we want to model β_t over time, reflecting the large uncertainties surrounding climate change, damages, and hence benefits of mitigation, and future output and consumption paths. Thereby we want to contribute to the discussion on what annual rate to us in cost-benefit analysis, and to the correct values the SCC. Dietz et al. (2018) point out that, based on their assumption of constant elasticity of net conditional benefits with respect to consumption: β_t is simply the coefficient in an Ordinary Least Squares (OLS) regression of lnB_t on lnC_t:

$$\ln B_t = \beta_t \ln c_t + \varepsilon_t \tag{3.31}$$

where c_t and ε_t are both random variables and are independent of each other. We have shown by the derivations above, that β can be estimated as the percentage change in expected benefits (based on the marginal investment project) with respect to percentage change in consumption, divided by the variance of consumption. The notation of Dietz et al. (2018), using the natural logarithm of the two variables is an approximation of the derivation shown earlier on. Naturally, this approximation has the feature of holding with equality once the assumptions about the relationship between consumption and the benefits from the marginal investment project are made.

Before we move on to combining the above derived β_t from the CCAPM with a simplified climate model, we highlight an important result of the derivations above. Dietz et al. (2018) show that if the assumptions in line with the standard CCAPM derivation hold, the approximation of β_t in 3.28 does not only imply an effect on the discount rate. Actually, the expected undiscounted stream of future benefits can be

[9] For a more elaborate explanation on the distinction between risk-version and the elasticity of marginal utility of consumption, we refer the reader to Daniel et al. (2018) and Ackerman et al. (2013)

shown to be ***increasing*** in β_t. Based on the standard assumptions underlying the CCAPM, Dietz et al. (2018) show that if $\beta > \gamma - (\frac{\mu}{\sigma^2})$ the NPV of the marginal investment project, is increasing in its CCAPM β. This result says that if beta is larger than the relative risk aversion parameter γ (in our notation denoted by α) minus the ratio of mean consumption (μ) over the variance of consumption (σ^2), then the expected value of the benefits increase more in beta, than the discount rate increases in beta. Hence, the positive expected value effect on the undiscounted stream of benefits, is larger than the negative affect coming from the discount rate increasing with beta. Based on empirical research, the difference between γ and $(\frac{\mu}{\sigma^2})$ can range between approximately -10.5 for the case when $\gamma = 2$ and $\gamma - (\frac{\mu}{\sigma^2})$ = -2.5 if $\gamma = 10$. Because most of the assets yield β_t at least larger than -2.5, they conclude that for most of the assets this result holds. We will get back to this result in chapter 9, after we have presented our results for β_t.

3.4.2 A Simple Climate Model

In this section, we link the β as derived in the previous section from CCAPM, to a simplified analytical **climate** change model. Thereby we show that the consumption beta for climate change investments is what arises in this climate model. In the next chapters we will estimate this numerically, using the DICE model.

The simple climate model and the approximations outlined in this subsection are developed by Dietz et al. (2018). We use this section to derive at an approximation of β for climate mitigation investments. Most importantly, to show which uncertain variables are mostly impacting β in case of abatement projects undertaken to mitigate climate change. Hence, beta here is an estimate of the covariance between the climate mitigation benefits and aggregate consumption risk.

For simplification, we assume the relation between temperature (increase since 1990) T and emissions E to be approximated by a linear function:

$$T = \omega_1 E \tag{3.32}$$

IAMs like the DICE model map this relation through the several separate geophysical equations and show, via the carbon-climate cycle, the relationship much more specifically. In our simple model, these are all captured in the parameter ω_1. Furthermore, we define a marginal climate mitigation investment opportunity by I_0. Emissions, which can be seen as the damage imposed on to the economy, are be mapped by:

$$E = \omega_2 Y - I_0 \tag{3.33}$$

Where Y is global GDP gross of damages and abatement costs, and ω_2 is a parameter representing the carbon intensity of production. Naturally, emissions decrease due to the abatement project I_0 . This definition looks similar to the industrial emissions equation in the original DICE coding: $E_{IND} = \sigma_t YGROSS(t) * (1 - \mu)$

In turn, the relative damages, or the parts of output that are damaged through the increase in global mean temperature, are modelled by

$$D = \alpha T^k \tag{3.34}$$

where α is supposed to calibrate the damage function, similar to the way it is defined in the original DICE model. The general consensus in the literature on climate change economics is that parameter $k > 1$, meaning that there exist a convex relationship between climate damages and global warming. Dietz et al. (2018) show that this parameter will play an important model in the derivation of the climate beta, which we will see later on in this section.

Net output Q(t) can be modelled through either a multiplicative damage function, an additive form of the damages, or by means of a functional form of the damage function that lies in between those two before-mentioned types. The damage function models how we derive the net output Q(t) from the gross level of output Y. A multiplicative damage function can be described as, damages moving proportionally to gross global output:

$$q(Y, D) = Y(1 - D) \tag{3.35}$$

Whereas in the case of an additive functional form, damages are measured independent of pre-damage gross income:

$$q(Y, D) = Y - D \tag{3.36}$$

Naturally, if no damages would exist, gross output equals net output in that case: $Q(Y, 0) = Y$ for all Y.

Now, assume that the marginal propensity of consumption equals c. Combining c with the equations 3.32, 3.33, and 3.34 in a general function of net output $q(Y, D)$ yields the following simplified social-welfare model:

$$C(I_0) = cq(Y, \alpha\omega_1^k(\omega_2 Y - I_0)^k) \tag{3.37}$$

Intuitively, consumption as a function of the climate mitigation or abatement investment project, depends on the marginal propensity to consume, times the general function of net output, depending on gross pre-damage output Y and damages coming from global average temperature increase due to emissions. Those emissions can be reduced by the climate mitigation investment project, as indicated by I_0. As explained earlier in this chapter, to obtain an estimate for the climate beta, we are interested in the covariance between the benefits or cash flows streams coming from the climate mitigation project (i.e. the increased consumption due to abatement) and the aggregate wealth, measured by consumption (via marginal utility). Benefits can be measured by the derivative of consumption w.r.t the investment project that is expected to reduce CO_2 emissions:

$$B \equiv \left.\frac{\partial C}{\partial I_0}\right|_{I_0=0} = -c\alpha\omega_1^k\omega_2^{k-1}Y^{k-1}q_D\left(Y, hY^k\right) \tag{3.38}$$

Whereby

$$h = \alpha\omega_1^k\omega_2^k \tag{3.39}$$

Note here that the derivative is estimated around $I_{0=0}$.

In summary, this simple climate model from Dietz et al. (2018) shows that the expected future benefits B can be estimated by the change in consumption coming from the marginal emissions reduction investment project I_0 and that those benefits depend on a set of uncertain parameters, such as gross output Y, the sensitivity of the global mean temperature w.r.t emissions, ω_1 and the effect of a change in gross production on industrial emissions, denoted by ω_2. In the previous subsection of this chapter we showed that the co-movement of those type of benefits of investment projects, and consumption over time, can be seen as an estimate for β. Here in the case of a simplified climate-economy model, this will allow us to estimate a consumption beta on climate investments, thus the climate β_t. How do benefits as defined in 3.38 vary with regards to the variation in consumption over time? As explained in the previous subsection, the percentage change over time in expected benefits and expected changes in consumption, can be approximated by taking natural logarithms. Therefore, the system of equations given by Dietz et al. (2018) yields:

$$\ln B = \ln\left(c\alpha_M\omega_1^k\omega_2^{k-1}\right) + (k-1)\ln Y + \ln\left(-q_D\left(Y, hY^k\right)\right) \tag{3.40}$$

$$\ln C = \ln c + \ln q\left(Y, hY^k\right) \tag{3.41}$$

From here, we can continue the analysis of how the climate beta, responds to various uncertainties in the model's uncertain variables, such as Y, ω_1 and ω_2. We will highlight here the effect of uncertainty in TFP growth on the variables that determine beta, as TFP growth is shown in our analysis to be the main source of uncertainty. However, we also take into account the different specifications of the damage function, as explained before. The effect of uncertainty in the TFP growth rate within the DICE model, can be approximated by the variable gross output Y in the system of the simplified climate model above. Hence, taking the derivative of both lnB_t and lnC_t in turn, with respect to Y gives an indication of how the climate beta will change accordingly, following Dietz et al. (2018):

$$\beta \approx \frac{d\ln B/dY}{d\ln C/dY} = \frac{q}{q_D}\frac{(k-1)q_D + Yq_{YD} + Dq_{DD}}{Yq_Y + kDq_D}$$

In this approximation, the partial derivatives are again evaluated at $I_{0=0}$, that is evaluated at $q(Y, \alpha\omega_1^k\omega_2^kY^k)$ Using the two different classes of damage function specifications explained above, that is

Again, this section is not novel in the way we set up the simplified climate model, it relies on Dietz et al. (2018). Although equations in this example of the climate model can only be seen a simplified small part of the DICE model, and the approximations for beta only hold in case the uncertainty in Y through TFP growth is small, it indicates how the variables of interest B_t and C_t are impacted by uncertainties, which is in our case mainly uncertainty in the rate of exogenous TFP growth.

When altering the DICE model, we allow for larger damages than the original DICE 2016R2, by means of implementing the 'Weitzman damage term' (Weitzman ML, 2010). This allows the damage function to be convex, and the elasticity of damages with respect to increase in global mean temperature, k, in our analysis is therefore likely to larger than in Nordhaus' version of the DICE model. Simultaneously, the general consensus already tells us $k > 1$. The simple climate model above also provides our motivation to employ different functional forms for the damage function in our analysis. We will analyze the effect of multiplicative, additive and a semi-additive type of relationship between gross output and the relative damages D. In summary, the simplified climate model in this section provides basic intuition that might help explain our results in chapter 8.

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4 The DICE model

4.1 Background information

In order to well understand and be able to interpret the values of the output variables, from which we derive the estimation of the climate beta, it is important to have a proper understanding of the DICE model. Therefore, its parameters and variables, as well as its main equations will be explained in this chapter. It is also useful to understand how the model has developed over time, and the major critique points it has received, in order to obtain a complete picture of the model's advantages and drawbacks in terms of calculating the climate beta.

4.1.1 William D. "Bill" Nordhaus

William D. Nordhaus is the Sterling Professor of Economics at Yale University, a university with which he has been associated with since 1967 and a professor at since 1973. He has a PhD from Massachusetts Institute of Technology and he served as an economic advisor for President Jimmy Carter from 1977-79. Nordhaus has done several research projects on the economy, resources and the environment, but is perhaps mostly known for his work on the RICE/DICE model for which he also won the Nobel prize of Economics in 2018 together with Paul M. Romer. Nordhaus won the Nobel price with the motivation *"for integrating climate change into long run economic modelling"*The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel (2018)

Nordhaus is one of the pioneers in modelling a framework which shows how the economy and the climate of the planet mutually impact each other. (The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018) The model advances from a linear programming model in the 1970's which accounted for energy supply and -demand and its impact on CO_2-emissions, though not for the *economic* impact of emissions. Initially, the model was not intended to measure the feedback loops in the optimizing scenario, which it does today, but rather to only measure the impact economic activity has on nature. Later, as the feedback loops were included, the model became an optimization tool, determining the economic optimal level of emissions. Some might argue against the morality of this in that an optimal scenario could be to knowingly destroy nature beyond the point of no return, justified by the economic gains coming from the productions leading to this damage, if the costs of mitigation outweigh the benefits of consumption. Sørensen (2019) Around 1990, the first version of the DICE model was developed. In 1991 a model similar to the DICE model was published and in 1992, through the Cowles Foundation, the first paper on the DICE model was published. Nordhaus (2017a) The model and the data going in to the model have later been critiqued, reviewed and updated several times up to today and the latest version of the model is currently the revisited version of the 2016 model, which is the version we assess in this paper.

Nordhaus has made the model available to the public and consequently, several similar IAMs have been made, such as the PAGE-(Policy Analysis of the Greenhouse Effect), WITCH-(World Induced Technical Change Hybrid) and the (C)FUND (Climate Framework for Uncertainty, Negotiation and Distribution)model. It is not within the scope of this thesis to assess or compare the different integrated assessment models, therefore these models will not be explained further.

4.1.1.1 The climate and the economy

Although some skepticism still exists, the scientific evidences that climate change is happening at a rapid rate, and that the changes are caused by humans are strong and much proven. The global average temperature has increased by 0.8°C since 1880 and two thirds of the warming has occurred after 1975. This implies a rate per decade of approximately 0.15-0.20°C.The temperature increase is strongly correlated with an increase in population and energy use, leading to a rapid increase in greenhouse gas (GHG) emissions. Among the GHGs, CO_2 is the most discussed. CO_2-concentration in the atmosphere is measured by parts per million (ppm) whereas CO_2 emissions are expressed by gigatons (Gt) CO_2 (1Gt= $1g^{15}$).

CO_2 emissions have damaging effects on the climate and are strongly linked to economic activity. The DICE model was among the very first models to link the sciences of climate and the economy together into one model. The feedback loop in this case regards the fact that economic activity is strongly dependent on natural resources through extraction of oil, use of water etc.

4.2 The DICE Model Explained

The DICE model is a large and complex model, although its individual equations are highly simplified versions of the real world that connect the economy and natural sciences into one. To demonstrate the simplification of the model´s framework, and especially the climate-equations, note that a fully fledged model, such as the atmosphere-ocean general circulation at the UK Hadley Center, can contain hundreds of thousands of equations(Dietz and Stern, 2015). The combination of equations is made to answer the question "What is the optimal global policy for addressing climate change?" and the answer is found through simulation and modelling. The DICE model builds on the Neoclassical Growth Theory from Solow (1971). In chapter 2 we explained the difference between the Solow model and the Ramsey model and that a world where agents optimize the flow of consumption, subject to certain constraints is adapted in the latter one. Hence, the DICE model builds upon assumptions underlying the Ramsey model, including endogenous savings rates, prefect information and rational preferences of all agents who aim to maximize their life-time utility. Nordhaus and Sztorc (2013) The agents maximize welfare, through the optimization of consumption levels, by making policy-decisions, affecting technology, current consumption and capital investments. One can think of the agent as a 'representative consumer' who acts as a **dynasty**, i.e. she values the welfare of her offspring. Important to emphasize is that the DICE model incorporates the assumption of forward-looking welfare maximizing agents/dynasties. In order to determine the optimal pathway of the endogenous output variables of the model, current variables like consumption depend on the entire path of future endogenous and exogenous variables. (The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018) This is one of the reasons for why we use the model in our estimation of the climate beta. The climate beta depends on ***future*** consumption, benefits from climate mitigation and their covariance, hence using a model that incorporates dynamic behavior and forward-looking agents provides a suitable tool for estimation.

Consumption in the context of the DICE model includes, not only traditional consumption goods, but also consumption of other goods, for instance leisure, health and the environment. The consumption commodity in the DICE model is thus used both for consumption, abatement and/or investment. This leads to a simplified view of reality. However, this simplification is partly necessary in order to demonstrate the compatibility with forward looking economic agents. Simultaneously, this is an important feature for our analysis, as the consumption variable hereby becomes an even more appropriate measure for the aggregate welfare level in the economy. The climate beta measures the covariance between the benefits of a climate mitigation project and aggregate macroeconomic risk, therefore consumption defined as more than only consumption of general goods makes the DICE model suitable for our analysis.

The DICE model distinguishes itself from normal neoclassical models by also including natural capital (resources) as an additional type of capital stock. (Nordhaus and Sztorc, 2013) Compared to models in standard economic growth literature, the DICE model requires a much longer time horizon, as time has to span over centuries to account for the changes happening in the climate. The version of the model used in this study runs for 100 time steps, each consisting of 5 years. Hence, the total time modelled by spans from 2015 till 2510. The longer time horizon leads to higher uncertainty than what would otherwise be modelled, as evidence is weaker. The DICE model is based on real life data of output, population and emissions. The underlying data of these variables are national, aggregated data. In the RICE model the national data is grouped into 12 regions, whereas in the DICE model, the data comes from that underlying the RICE model, but is aggregated in to one world-average variable.

4.2.1 The equations in the DICE model

In this section, we will explain the model and its equations by dividing the model into segments explaining different equations of the model. The figures depict the connections of the different equations creating the model. First, we will describe the economic equations, then we explain the integration between the climate and the economic system. What follows is an explanation of the geophysical equations. Thereafter the optimal policy, which combines all the equations, is explained.

Figure 4.1: Flow chart of the DICE model

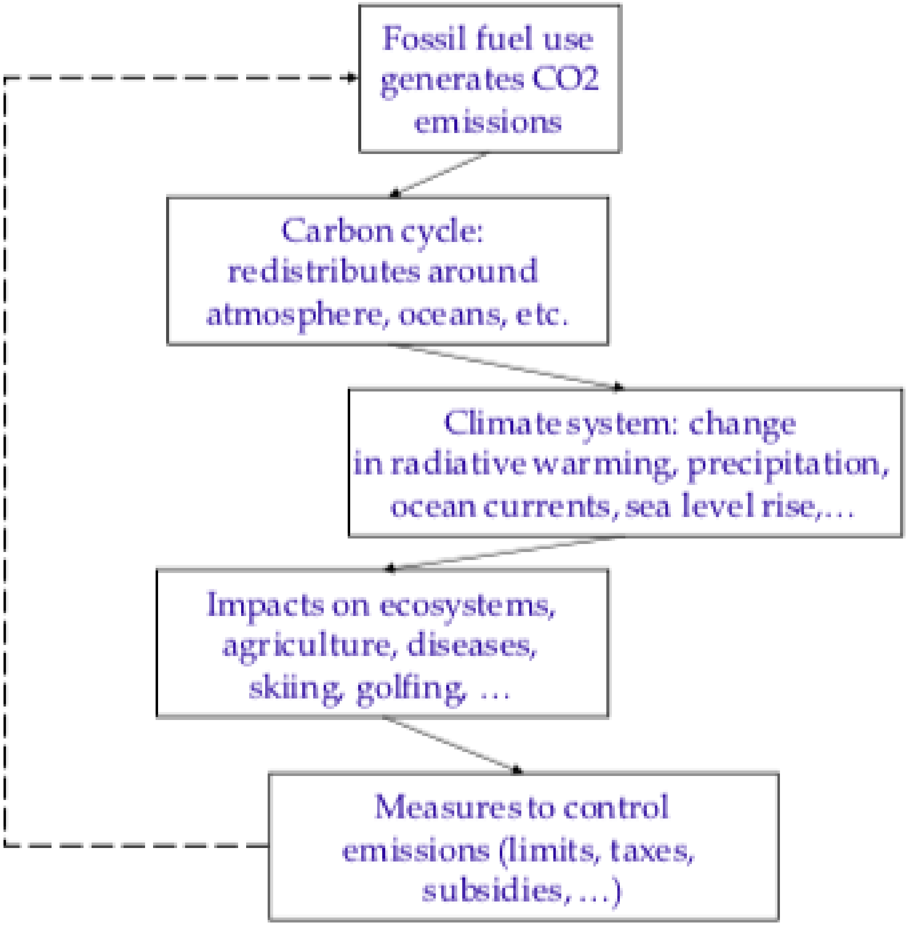


Figure 1. Schematic flow chart of a full integrated assessment model for climate change science, economics, and policy

Source: DICE-manual 2013, Nordhaus and Sztorc (2013)

4.2.2 Economic equations

A web of equations will be inserted here later (Graphics)

4.2.3 Utility - Preferences and the objective function

4.2.3.1 The economic sector

The DICE model assumes the world has well-defined preferences expressed by a global, social welfare function, W, represented by the sum of utility of per-capita consumption, weighted by population and discounted by the social discount rate. This is standard representation of objective functions in modern theories of optimal economic growth (Ramsey 1926, Koopmans 65 and Cass 65)

$$W = \sum_{t=1}^{T} V[c(t), L(t)]R(t) = \sum_{t=1}^{T} U[c(t)]L(t)R(t) \tag{4.1}$$

From a utilitarian perspective the objective function aims at maximizing total (per capita) welfare, $U[c(t)]$ of the entire population, leading to an equal weighing of all dynasties from time t to time T. If T is finite, the social welfare function will converge. Time $t = 0$ is set as the pre-industrial time, year 1750, and functions as a benchmark for when atmospheric CO_2concentration was at its natural, without any emissions from human industrial production. The social welfare function ranks different paths of consumption and is increasing in the population (and labor inputs), $L(t)$, and in per capita consumption, $c(t)$ of each dynasty. Per capita consumption is total consumption, $C(t)$ divided by the total population, $c(t) = \frac{C(t)}{L(t)}$.

A commodity can either be invested or consumed at time t and the model spans over time and generations. How important each dynasty's per capita consumption is at a certain time depends on the size of the population. The utility of consumption is concave, making the marginal utility of consumption diminishing. Welfare is thus increasing in consumption and in population and decreasing in the discount factor, $R(t)$.

The social discount factor discounts future dynasties´economic well-being and is stated by

$$R(t) = (1 + \rho)^{-t} \tag{4.2}$$

The discount factor provides welfare weights of the utilities of the different dynasties and can be seen as the relative importance of the different dynasties. ρ is the *pure rate of social time preference*, also known as the generational discount rate on welfare and is exogenous in the model. The higher ρ is, the less individuals value utility coming from future consumption relative to current consumption and vice verse for lower values of ρ. One can think of future utility flows for the representative consumer herself and her offspring, getting downgraded as everyone prefers to have utility rather now than in the future. At the same time, the representative consumer puts less weight on future generations utility, even if those agents are part of her own dynasty. As will be discussed chapter 5, it is worth noticing that preference parameters in the DICE model, like ρ, are set to be consistent with interest rates and rates of return to capital representing observed economic market outcomes.

The utility function in each period is increasing and strictly concave and determined by

$$U[c(t), L(t)] = L(t)[\frac{c(t)^{1-\alpha}}{(1-\alpha)}]$$

The period utility is increasing in the global population, *L(t)*, and in consumption per capita, $c(t)$. The population parameter is considered exogenous, hence an increase in utility comes from increasing consumption. The *marginal* utility of consumption is increasing at a diminishing rate,

$$U'(c) > 0 \ , \ U''(c) < 0$$

Hence, for a given amount of output, total utility is maximized by spreading the consumption evenly. One can think of α as aversion to inequality across generations or how we compare today's and tomorrow's generations. Note that this is distinct from the individuals' **personal** preferences, which for instance, the pure rate of social time preference, ρ , represents. If $\alpha = 1$, the utility function is logarithmic. A high α implies a high aversion to generational inequality, which is reflected through highly differentiated consumption. Nordhaus and Sztorc (2013)

When the inter-generational inequality-aversion is close to zero, the different generations´consumption are close substitutes. Hence the utility-maximizing social planner has a low inequality-aversion. The above-mentioned utility function assumes constant **elasticity of marginal utility of consumption**, α, which is calibrated in combination with the pure rate of social time preference, ρ in the DICE model. From the utility function we can derive the elasticity of inter-temporal substitution, $\frac{1}{\alpha}$, which shows how one unit of consumption today can be substituted by one unit of consumption in the future:

$$E(c, U) = \frac{-U'(c)}{U''(c)c} = \frac{1}{\alpha}$$

Hence, the specific utility function employed in the model features a **constant elasticity of substitution**. We note that in the DICE model, **The value of consumption** in one period is assumed to be proportional to the population. It can be noted here that for the RICE model multiple agents enter the equation, making interpretation and computation challenging. This is however not relevant in the DICE model, considered here as the DICE model aggregates the variables, i.e. based on a global scale.

4.2.4 Gross global output

The DICE model considers output as the aggregated world GDP, expressed in 2010 USD, and formulates it as a standard Cobb-Douglas production function with technological growth

$$Y(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{4.3}$$

A(t) is Total Factor Productivity (TFP), also referred to as technology. From 4.3 one can observe that technological change occurs under the assumption of Hicks neutrality, meaning that an increase in $A(t)$does not affect the balance of capital *K(t)* and *L(t)* labor (population) in the economy[10]. The ratio of marginal products remain he same for a given capital-labor ratio respectively. The aggregated world output is increasing in both TFP, capital and labor. Note that we denote gross output as Y(t) and net

[10]We note however that with a Cobb-Douglas production function, Hicks-neutral technological improvement is essentially the same as labour-augmenting or capital-augmenting as the production can be rewritten to account for one or the other, see Romer (2012) for more information

output as Q(t) throughout this paper, similar to Nordhaus and Sztorc (2013). The use of this standard neoclassical growth model is justified by the long-term optimization horizon and the macroeconomic scale of the model.

4.2.4.1 The evolution of world output over time

Capital, K_t grows following a standard Solow-equation

$$K(t) = K(t-1)[1-\delta_k] + I_t \tag{4.4}$$

where capital accumulation is increasing in investment, $I(t)$ and decreasing in the depreciation rate, δ_k.

Population and labor force, *L(t)*, is assumed to be exogenous, and take the form of an S-curve, (a simplified logistic function)

$$L(t) = L(t-1)[1 + g_L(t)] \tag{4.5}$$

where $g_L(t)$ represents the growth of population and is defined as $g_L(t) = \frac{g_L(t-1)}{(1+\delta_L)}$.

Technology is in this model defined as energy and can take two forms, namely "dirty", carbon-based fuels and "clean" or "green" non-carbon-based technologies. Examples of dirty energies are coal and petroleum, whereas green technologies can be solar panels and nuclear energy, emitting no CO_2 as opposed to dirty technologies.

Technological change is also separated in two forms: the technological development happening throughout the whole economy and the technological change aiming at creating "greener" energy. The TFP-function is, like the population, a logistic function;

$$A(t) = A(t-1)[1 + g_A(t)] \tag{4.6}$$

with $g_A(t) = \frac{g_A(t-1)}{(1+\delta_A)}$, and TFP growth decreasing over time. Equation 4.6 models only the economy-wide technological change. In our calculation of the climate beta, this function is altered to accommodate for two sources of uncertainty to the TFP growth, which can be seen in section 6.2.1. The *green* technological change is modelled as a reduction in the ratio of CO_2-emission over output. "Dirty" technology is limited to 6000 billion tons of carbon content, as the supply is limited by the nature of the natural resource carbon, this is however not binding in the DICE2013 model. (Nordhaus and Sztorc, 2013) The transition from dirty to clean energy is modelled, as in real life, as a process happening over time through an increase in the price of carbon, and thus carbon-based energy (fuel). The reason for this price-increase stems from policy-changes or from the limitations of supply (resource exhaustion).

4.2.5 Integration between the economy and the climate system

The DICE model incorporates the climate and the economy into one hard-linked, unified system. The way it unifies the two systems is through the following equations explaining two so called feedback loops. The first feedback loop concerns output, carbon emissions, the carbon reduction rate and abatement costs. The second loop regards the relationship between carbon emissions, carbon reduction rate, damage costs and output. Altogether these feedback loops affect the output.

4.2.5.1 Carbon emissions

CO_2-emissions, $E_{CO_2}(t)$, are a function of total output, $Y(t)$, the emissions-control rate, $\mu(t)$, and an emissions output-ratio that varies over time, $\sigma(t)$. The emissions-control rate is decided from the optimal policy, derived from the DICE model under examination. The emissions-output ratio is an aggregated ratio, stemming from estimations from individual regions, aggregated to the world average.(Nordhaus and Sztorc, 2013) We can write the CO_2emission-path as

$$E_{CO_2}(t) = E_{Ind}(t) + E_{Land}(t) \tag{4.7}$$

where $E_{Ind}(t)$ is the CO_2-emissions stemming from the (production) industry and $E_{Land}(t)$ is emissions stemming from land, and according to IPCC(Intergovernmental Panel on Climate Change) mainly from deforestation The Intergovernmental Panel on Climate Change (IPCC) (2000). Land-emissions are considered exogenous, whereas emissions from the industry is considered endogenous, following the formula

$$E_{Ind}(t) = \sigma(t)[1 - \mu(t)]Y(t) = \sigma(t)[1 - \mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{4.8}$$

$\sigma(t)$ is the level of carbon intensity and *Y(t)* is, as shown above gross output, as described by the Cobb-Douglas production function. We can this think of carbon intensity $\sigma(t)$ as the average level of carbon emission per unit of GDP produced with the current industrial production technology. Emissions are controlled by the emissions reduction- rate, $\mu(t)$, implying that when no policy is implemented $\mu(t)$ equals zero. We can therefore characterize the *baseline* emissions, as $E_{Ind}(t) = \sigma(t)Y(t)$, where baseline indicates the scenario in which no extra policy is taken and emissions continue as is in 2015, the first year of the model.

The equation for the baseline carbon intensity, which in the latest DICE- models is altered to fit the actual level of the CO_2-intensity of 2010, is expressed as

$$\sigma(t) = \sigma(t-1)[1 + g_{\sigma}] \tag{4.9}$$

and expected to decline over time, as new, greener and more energy-efficient production technologies are available, at the exogenously determined rate

$$g_\sigma(t) = \frac{g_\sigma(t-1)}{1+\delta_\sigma} < 0 \tag{4.10}$$

Carbon intensity is assumed to be exogenously determined.

In other words, this shows us that economic activity leads to CO_2-emissions through deforestation and current industrial technologies. We can also see that, according to the DICE model, the only thing policy makers are able to affect in order to control the carbon-emissions is $\mu(t)$. Increasing the emissions reduction-rate, however, will also affects the abatement costs.

The next function shows the projection of cumulative carbon emissions, $CCum$

$$CCum \geq \sum_{t=1}^{Tmax} E_{Ind}(t) \tag{4.11}$$

which limits the total resources of carbon emissions to 6000 tons. This is non-binding in the 2013 or 2016 model.

4.2.5.2 Abatement cost

Abatement of CO_2 is costly, and the abatement cost, $\Lambda(t)$, is a ratio, telling us the costs incurred of reducing emissions (relative to output). The abatement cost is expressed as a reduced form model which depends on the emissions reduction rate, $\mu(t)$;

$$\Lambda(t) = \theta_1(t)\mu(t)^{\theta_2} \tag{4.12}$$

$\theta_1(t)$ is a parameter explaining the relation between the reduction rate and the abatement cost, whereas θ_2 can be seen as an elasticity- parameter. When the latter, $\theta_2, > 1$ abatement cots will increase more than proportional to the emissions reductions rate, implying that it becomes more expensive to reduce every CO_2 unit. This is generally assumed to be the case and is explained by cheaper technologies being introduced first. In the DICE2016R model, which is applied in this study, the elasticity parameter is set to 2,6. The DICE model also includes a so called back-stop technology, which is a technology, currently unknown, that can replace all fossil fuels at the price of 550USD per ton CO_2 in 2015. This technology is assumed to decline by a quarter of a percentage every year and is introduced by setting the time-path parameters such that the marginal cost of abatement is equivalent to the back-stop price at that year.Nordhaus (2017b) The back-stop technology is put to use when its price is below the marginal cost of abatement. The total abatement cost is then

$$TAC = \Lambda(t)Y(t) = \theta_1(t)\mu(t)^{\theta_2}Y(t) \tag{4.13}$$

and we see that it is increasing in output, the reduction rate and the parameter. The abatement cost reduces output as it takes productivity used to produce output, applying it to reduce carbon emissions. It can therefore be viewed as current investments used to avoid future damages.

4.2.5.3 Damage costs

The damage function is, according to Nordhaus and Sztorc (2013), the *thorniest* of all issues in climate change economics, as damages are crucial to estimating the SCC Nordhaus (2017b) and it is arduous to estimate the correct effect of future damages. The function evaluates the economic loss corresponding to damages from the climate. The 2016 DICE model uses the same damage function as other IAMs;

$$\Omega(t) = \frac{D(t)}{1 + D(t)} \tag{4.14}$$

where damage, *D(t)*, is proxied as a quadratic function from the average temperature change with respect to a set starting point, T_{AT}, (Tol 2009 survey on damages)and $\Omega(t)$ represents one minus the damages (as a fraction of output) lost due to climate change.

$$D(t) = \psi_1 T_{AT}(t) + \psi_2 [T_{AT}(t)]^2$$

ψ_i (i=1,2) are parameters determining the damage of temperature increase. For our calculations of the climate beta we have altered this function, to account for larger damages. This is explained in the chapter called analysis. Note that the DICE model- damage function is calibrated to not exceed 100% of output, which has been calibrated by only including temperature changes of 0-3°C. In reality temperature changes imposed by mankind has never happened to this extent before, making it difficult to estimate damages from historical data. Also, tipping points, above which critical damages occur, are not included in the base- case of the model and both these cases may greatly distort the estimations(Nordhaus, 2013).

4.2.5.4 Net Output

Global output net of damage- and abatement is modelled as gross output, net of damages and abatement costs.

$$Q(t) = \Omega(t)[1 - \Lambda(t)]Y(t) = \Omega(t)[1 - \Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{4.15}$$

The damage function destroys output through temperature increase and the abatement cost reduces output by moving resources from production to abatement. This net output can only be is used for consumption and investments, $I(t)$, where investment is defined as an endogenously determined savings-fraction, $S(t)$, of gross output net of both abatement costs and damages,$I(t) = S(t)Q(t)$. This gives us the budget constraint

$$Q(t) = C(t) + I(t) \tag{4.16}$$

which also stipulates that consumption and investment is decreasing is abatement and damage. We remember that utility is increasing in consumption, and that capital is increasing in investments, making both utility and capital connected indirectly to abatement and damage.

4.2.6 Geophysical equations

To connect the forces that affect the climate and the economy, in addition to the equations just described, the DICE model use 3 types of equations, namely the carbon cycle, the radioactive forcing and equations on the climate´s reactions. IAMs integrate these equations into one system in order to easily enable us to trace the effects of emissions from the economy, to the climate, and back to the economy, rather than seeing climate change as exogenous shocks.

4.2.6.1 The carbon circulation

The carbon cycle connects CO_2-emissions from the economy, as shown in equation 4.7, to a carbon reservoir in the atmosphere from where the carbon transfers to other carbon reservoirs, spreading the carbon, affecting other parts of the ecosystem through the photosynthesis as shown by the DICE2013 three-reservoir model below from (Nordhaus and Sztorc, 2013). Nordhaus initially built the carbon circulation on *include referencing* **?** and constructed seven reservoirs, later reduced to three, between which the gross flows of carbon are proportional to the source reservoirsThe Committee for the Prize in Economic Sciences in Memory of Alfred Nobel (2018). $M_{AT}(t)$ represents the level of carbon in the atmosphere, $M_{UP}(t)$ the quickly mixing reservoir in the upper oceans and the biosphere, and $M_{LO}(t)$ represents carbon in the deep oceans.(Nordhaus and Sztorc, 2013)

$$M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1) \tag{4.17}$$

$$M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO-} \tag{4.18}$$

$$M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1) \tag{4.19}$$

The parameters $\phi_{ij}, i = AT, UP, LO \quad j = 1, 2, 3$ represents the per period-stream of flows between the reservoirs and we see that carbon flows in both directions, but only enters through the atmosphere. These mechanisms are very complex, and thus simplified greatly using properties, such as carbon being unable to disappear and that the outflow of one reservoir must equate the inflow to another. However, with the newest calibration of the model, the model is able to fit state-of the art carbon circulation models and calibration to atmospheric retention of CO_2 for periods up to 4000yearsNordhaus (2017b)(The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018).

The low values of several of the flow-parameters shows the inertia of the transmission of carbon through the reservoirs, and especially the mixing between the deep oceans and other reservoirs. This inertia is the reason for the accumulation of GHGs in the atmosphere, as more emissions enter the reservoirs than the ones who are diffused. A simple explanation of this mechanism is known as "the bathtub effect"; We can imagine a bathtub with a partially opened/partially clogged drain that is filling up with water quicker than the water drains (mixes with the lower oceans). **?** (**?**)As the CO_2 (water) drains slower than it enters the atmosphere (bathtub), the carbon concentration increases every period, leading to increased radiative forcing and thereby temperature increase.

4.2.6.2 Radiative forcing

The next step in the climate equations concerns the relationship between the carbon accumulation and increased radiative forcing, where $F(t)$ represents the change in radiative forcing from CO_2 and other antropogenic sources, modelled as

$$F(t) = \eta log_2[\frac{M_{AT}(t)}{M_{AT}(1750)}] + F_{EX}(t) \tag{4.20}$$

The first term in this equation shows us that radiative forcings in period t relative to the per-industrial level (year 1750), increases as CO_2emissions accumulate. The second term, F_{EX}, represents the exogenous forcings from other GHGs in each period. These emissions are treated as exogenous, either because their quantities are so small and/or because they are out of the control of the policy, and thus out of the control of this model(Nordhaus and Sztorc, 2013).

4.2.6.3 Temperature change

As GHG-emissions accumulate in the atmosphere, the radiative forcings increase, leading to a rise in the average global temperature. When the forcings in the atmosphere increase, the atmospheric layer heats up, which again affects the temperature of the upper ocean and then gradually the lower ocean. This effect is accounted for through the two equations

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\} \tag{4.21}$$

$$T_{LO}(t) = T_{LO}(t-1) + \xi_4[T_{AT}(t-1) - T_{LO}(t-1)] \tag{4.22}$$

where T_{AT} and T_{LO} is the average global temperature of the atmosphere and the deep oceans respectively. The temperature increase is a gradual process, and ξ_i represent the diffusion inertia, telling us how long the temperature of the different layers change (Not to be confused with the ξ used in the derivation of the climate beta, which reperesents the income elasticity of damages). The inertia of each layer depends on the heat capacity, namely how much heat each layer can store at a given temperature. The more inertia the layer has, the slower the temperature changes. The atmospheric temperature at time t thus increases in the radiative forcings of current period and in the ocean temperature of the previous period (and in atmospheric temperature of previous period). The ocean temperature does not depend *directly* on radiative forcings, since we emissions affect the temperature only through the atmospheric layer, as explained above.

The equilibrium temperature sensitivity (ETS) shows how sensitive the atmosphere is to a doubling in radiative forcings and is given by

$$\Delta T_{AT} = \frac{\Delta F(t)}{\xi_2} \tag{4.23}$$

where Δ represents a change in the consecutive parameter.

The equilibrium climate sensitivity (ECS) is calculated as a weighted average of ETS's and tells how much warmer the temperature will increase for an equilibrium doubling in CO_2, and is another central and much discussed parameter. The DICE 2016 ECS is calculated through a Bayesian approach (Olson et al., 2012) and set to 3.1°C. This parameter is further discussed in the next chapter.

4.2.7 Summary of the DICE model

The DICE model is a good tool to compare outcomes of different climate change policies. The model optimizes social welfare over time, as described by an aggregate utility function, W. Utility is maximized through consumption of goods and non-tangible assets such as health and clean air, all expressed as $C(t)$. The population creates output in the form of GDP, expressed as $Y(t)$, following a standard Cobb-Douglas production function which is increasing in capital, $K(t)$, labour, $L(t)$, and TFP, $A(t)$. However, this production uses fossil fuels in order to produce output, which emits CO_2, denoted as $E_{CO_2}(t)$. The emissions that can be controlled, $E_{Ind}(t)$, are the ones stemming from the industry. These emissions increase in the exogenously decided, decreasing carbon intensity, $\sigma(t)$, and output, $Y(t)$. Emissions decrease in the emissions reductions- rate, $\mu(t)$, which transfers production resources from output-creation over to carbon reducing activities, and it is the measure through which a policy is implemented. Hence, it is by setting different values of this parameter and running the model, that policymakers can compare different policies in order to see which one gives the highest utility.

The costs of reducing, $\mu(t)$ emissions is called abatement costs, $\Lambda(t)$, and is a nonlinear function of the reduction rate and output. Despite reducing emissions and paying to abate them, emissions still accumulate in the atmosphere faster than the atmosphere is able to diffuse the carbon. Through the carbon cycle the accumulated emissions lead to a higher CO_2concentration in the atmosphere, which heats the atmosphere and gradually the the upper and the lower oceans. This is is a slow process and one of the reasons why the model stretches over such a long time horizon. As temperatures increase this creates damages, monetized as a fraction $\Omega(t)$ of the gross output, $Y(t)$, and we can calculate GDP net of abatement- and damage costs, $Q(t)$, so closing the loop with the economic cycle. The net output is used for consumption today, $C(t)$, and investments, $I(t)$, which leads to future capital growth. The global warming effect contains inertia, and does not happen immediately/ at the same time as emissions, but decades, and even centuries, can pass before we see the damage stemming from the emissions. This time factor connects to the inter-generational welfare.

The optimal policy aims at maximizing total welfare, through consumption, for both current and future generations. The utility function is a CRRA function, implying that at a constant rate consumption is split equally over all generations when optimizing the inter-generational accumulated welfare.

4.3 Tying it together to a policy

Now that the different variables, parameters and equations of the model have been presented, we will explain how this ties together to one policy.

In every period there exists an **optimal level of carbon emissions** to maximize utility. The sum of these leads to an **optimal path of carbon emissions** over time and as far as the model stretches, which is 500 years ahead for the 2016 model. This optimal path is found through maximizing the objective function, subject to utility, output, consumption, investments, capital, emissions, atmospheric carbon-parameters, radiative forcing and the temperature parameters at each individual stage. To do this non linear programming (NLP) is used and solved with General Algebraic Modelling System (GAMS).

The carbon reduction rate, $\mu(t)$, connects the abatement cost, $\Lambda(t)$, to the damage cost, $\Omega(t)$, as we derive the marginal abatement cost (MAC) and the marginal damage cost (MDC) by taking the derivative of the total abatement cost and the damage cost function respectively with respect to the carbon reduction rate. We find the optimal path where MAC= MDC in every time period.

In the context of climate policy the MDC can be seen as the Social Cost of Carbon (SCC). In other words, the SCC equals the marginal monetized damage to society from one unit (Gt) of CO_2 emissions costs. Contrarily, the MAC, is the optimal (marginal) price of carbon emission and is set by policy makers to equate the carbon price. The optimal carbon price can be obtained by policy makers through either a cap-and trade system or through a tax on carbon whenever the price of carbon is lower than the optimal one and varies over time with the MDC.

Mathematically, the SCC is calculated as

$$\underset{c(t)}{Max}\ W = \sum_{t=1}^{T} U[c(t)]L(t)R(t)$$

Subject to

Q(t)= $\Omega(t)[1-\Lambda(t)]Y(t)$

$D(t) = \psi_1 T_{AT}(t) + \psi_2 [T_{AT}](t)^2$

$E(t) = \sigma(t)[1-\mu(t)]Y(t) + E_{Land}(t)$

$M_j(t) = \phi_{0j}E(t) + \sum_{i=1}^{3} \phi_{ij} M_i(t-1)$

$F(t) = \eta log_2[\frac{M_{AT}(t)}{M_{AT}(1750)}] + F_{EX}(t)$

$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}$

$T_{LO}(t) = T_{LO}(t-1) + \xi_4[T_{AT}(t-1) - T_{LO}(t-1)]$

(4.24)This yields a path of all variables and we can define the SCC as

$$SCC(t) \equiv \frac{\frac{\partial W}{\partial E(t)}}{\frac{\partial W}{\partial C(t)}} SCC(t) \equiv \frac{\partial C(t)}{\partial E(t)} \tag{4.25}$$

at time t. Note that, in the middle term the SCC is denoted as the marginal wealth-benefit of emissions relative to the marginal wealth-benefits of consumption. This gives us the SCC as the marginal economic impact of emissions in terms of consumption, expressed numerically. In actual calculations this is done through discrete approximations.

The SCC can now (in theory) be used in cost benefit analysis to obtain an optimal carbon tax (from a social planners perspective).

4.4 Data on which the DICE model is built on

The DICE model is calibrated to fit real life observations as best as possible.(The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018). The underlying population and output forecasts are all based on aggregate data, as the DICE model is a globally aggregated model. The underlying estimates for the DICE 2013 model came from the RICE 2010 model, which is practically the same model except that the world is split up in 12 specific regions. More recent versions of the model are updated to include more historical data and newer versions of projections for the relevant variables. An extensive comparison of the different versions of the model can be found in Nordhaus and Sztorc (2013) and the most recent update of the model, which we use in our estimation for the climate beta is elaborated on in Nordhaus (2017b). The most recent updates of data sources underlying the parameters and variables of the model, are the population data and projections through 2100 from the United Nations. Moreover, the 2016R2 version of the DICE model uses it's own measure of world aggregate purchasing power parity (PPP), which appears to be consistent with the estimations of the International Monetary Fund (IMF). Other variables that have been updated are CO_2 emissions and Non-CO_2 radiative forcings, based on data from the Carbon Dioxide Information Analysis Center. (Nordhaus, 2017b)

5 Critiques to the DICE model

Although widely praised, the DICE Model has received a lot of criticism. The model leans on critical assumptions. Erroneous assumptions and input of variables and parameters, as well as misspellings of equations, may lead to large distortions in the optimal climate policy. In this section we will describe some of the major critique points of the model, many of which are highly relevant in terms of estimating the climate beta as they impose parametric uncertainty to the model. Most of the critique stems from large uncertainties in (future) human behavior and from uncertainties on- and from climate change as induced by human activitiesThe Intergovernmental Panel on Climate Change (IPCC) (2000). Part of the critiques are, in turn, reviewed by Nordhaus himself and incorporated in the latest version of the model, which we use in our analysis.

5.1 Damage function

The damage function used in the 2016 version of the DICE model is a quadratic function of the globally averaged temperature increase, $T_{AT}(t)$, which in turn is estimated through the climate system equations explained in chapter 4. Nordhaus acknowledges the great importance of the damage function in estimating the social cost of carbon (SCC), albeit he assumes that the increase in global average temperature can be used as proxy for the damage caused by climate change to the economy. The damage function in the 2016 version of the DICE model has been the greatest change compared to the 2013 version of the model and is based on 26 studies, making estimates using 4 different techniques (Nordhaus and Sztorc, 2013). However, the damage function remains topic of discussion due to the uncertainty around the parameters, its specification and the underlying studies. Nordhaus himself emphasizes the great divergence in the damage function among different studies (Nordhaus, 2017b). One of the major critiques Nordhaus received comes from Martin Weitzman. Weitzman argues that the damage function initially used by Nordhaus in the DICE2008 model.(Weitzman ML, 2010) Applying a conventional, quadratic damage function and relying on a thin-tailed probability distribution of extreme temperatures, leads to perverse underestimations of welfare loss from uncertainty.(Weitzman ML, 2010) He also argues that following that we cannot know with certainty the outcome on temperature, carbon concentration, damages and other key variables in the DICE model, we consequently have to look at them as random variables drawn from a Probability Density Function (PDF). Weitzman shows in his paper that assumptions on the PDFs and on the damage functions are crucial to determine the damages on economic welfare as modeled in DICE. He shows numerically that even minor changes to either the PDFs of the underlying variables and/or the functional form can give large damages on output. As the PDFs cannot be modeled with more certainty, he therefore argues that a GHG policies should be viewed merely as an insurance against disastrous outcomes of climate change which would yield huge negative welfare impacts. (**?**) Relating this to the value and the sign of the climate beta, without mentioning the word "climate beta", Weitzman hence implicitly argues for a negative sign. In fact, the net present value of abatement costs are 41% higher when applying the Weitzman ML (2010) specification. This follows from the Weitzman function allowing for higher temperature increases than the DICE2008 specifications, according to **?**. In other words, the tails of the distribution can give large impacts on the optimal path and optimal SCC. Weitzman´s critique has later been taken into consideration by Nordhaus, who has changed the damage function several times after 2010(Nordhaus and Sztorc, 2013).

At the time of Weitzman's critique, the DICE model specified the damage function as a quadratic function;

$$\hat{D}_t = 1 - \frac{1}{(1 + \pi_1 T_1 + \pi_2 T_t^2)} \tag{5.1}$$

This specification yields an unreasonably low damage, even at high temperature changes according to **?** and Weitzman(2012) suggests the modification of the DICE2008 (?) damage function to include for larger damages, altering the model to become;

$$\hat{D}_t^W = 1 - \frac{1}{(1 + \pi_1 T_1 + \pi_2 T_t^2 + \pi_2 T_t^{6,754})} \tag{5.2}$$

This specification of the model is set to satisfy the condition that at a 6°C increase in temperature 50% of the economic output is lost. Wouter Botzen and van den Bergh (2012) compares the two specifications and find that, ceteris paribus, using the Weitzman damage function leads to a much more stringent optimal climate policy. Note however, that this only holds for discount rates above the one employed by the Stern Review. The reason for this is that, when a very low discount rate is used, the stringency of the optimal policy increases, regardless of the damage function specified.

Dietz and Stern (2015) use the damage function specified by Weitzman, 5.2, but specify the same damage (50% of GDP) for a temperature increase of 4°C above pre-industrial levels. Dietz and Stern´s argumentation for this is that temperature increases like the ones we are seeing today have not been seen for millions of years and consequences are likely much larger than what we can measure based on historical data, as tipping points, not before measured, will be reached. They also argue that not only the effect on nature is harming the economy, but with such large climatic changes, socio-economic consequences, political conflicts and, more than anything, movement of people across the globe, will play a role, which is also associated with conflict. Therefore (Dietz and Stern, 2015) and (Cite Stern 2012 still!), defend the much larger convexity of the damage function.

5.2 Endogenous growth

In the standard version of the DICE model, the production function

$$Y_t = F(K_t, L_t) = (1 - \hat{D}_t)(1 - \Lambda_t) A_t K_t^{\alpha} L_t^{1-\alpha} \tag{5.3}$$

models global world output net of climate damages D_t and abatement costs Λ_t. A is the exogenous total factor productivity (TFP) at time t, and $\hat{D}_t$ is the standard DICE damage multiplier. This implies that growth in the economy comes from the exogenous growth rate of population g_L and of productivity, g_A as explained in the equations of the model in the previous chapter. Looking more carefully at (5.1), we note that $\hat{D}_t$ is the only pathway through which climate change affects growth. In other words, climate change (measured by the increase in global mean temperature) is directly negatively impacting the level of output within each period of the economy(Dietz and Stern, 2015) How climate change damages these levels of output or GDP, is in turn modeled as follows

$$\Omega(t) = \frac{D(t)}{[1 + D(t)]} \tag{5.4}$$

where

$$D(t) = \phi_1 T_{AT}(t) + \phi_2 [T_{AT}\ (t)]^2 \tag{5.5}$$

5.5 describes how increases in global average temperature, T_{AT}, affects the economy. In turn, $T_{AT}(t)$ is determined in the climate system, which will not be discussed here. The damage function as described in 5.5 relies on existing damage studies (Nordhaus, 2017b).

The latest published outcome of estimations from the DICE model performed by Nordhaus using the 2016 version, shows damages of 2.1% of global GDP at 3 °C increase in temperature, and 8.5%at a global average warming of 6 °C (Nordhaus, 2017b) .

This seems relatively little, Contrasting the damaging effect of climate change on the level of gross output within each time period, output ***growth*** comes from exogenous growth in TFP and population or labor force. Thereb y, growth in the world economy is exogenous in the DICE model. Both the use of exogenous economic growth and the effect of climate change only damaging the level of output within each period have received critique. Dietz and Stern (2015) adopt a way to model climate damages and their effect on the growth rate of GDP rather than the level of GDP. They do so, based on the fact that climate change has long lasting effects on output in the economy, in addition to instantaneous effects. In our analysis we will also assesses ways of climate damage entering the economic equations in the model and thereby affecting endogenous determinants of growth. Among other approaches, we follow Dietz and Stern (2015) in making extensions to the production function. They argue that the way climate change is damaging growth, i.e. the effect of temperature increases on output in the economy, is too simplistic. Instead, they propose two ways of altering the economic equations in the model. These will be explained below and are called the first and second growth model, respectively.

5.2.1 Growth through capital

In the first growth model, total damage D_t is split into two parts; D_t^Y affecting output, and D_t^K affecting capital. Thereby, the gross output function becomes

$$Y_t = (1 - D_t^Y)(1 - \Lambda_t) A_t K_t^{\alpha+\beta} L_t^{(1-\alpha)} \tag{5.6}$$

The difference between 5.6 and the original DICE model production function is the return on capital, which was α but is now $(\alpha + \beta)$. This higher return on capital comes from knowledge spillovers. The following equation of motion shows how climate change affects capital over time:

$$K_{t+1} = (1 - D_t^K)(1 - \delta^K)K_t + I_t \tag{5.7}$$

Compared to the original DICE model, (5.5) captures a direct effect of climate change on capital. That is, climate change increases the likelihoods of extreme weather conditions, like storms and floods, thereby damaging the economy´s capital stock e.g. infrastructure. In addition to the direct effect, D_t^Y has on output, it can also include indirect impacts of climate change on productivity, via the endogenous growth mechanism in ((5.3)). By making a distinction between D_t^Y and D_t^K the model can account for permanent reductions in output in future time periods, rather than climate damages only affecting output in the current period. This is done through the modelling of damages capital. Consequently, the impact of climate change on output is larger and a more ambitious climate policy is required in order to keep carbon emissions in control.

5.2.2 Growth through TFP

The second growth model, introduced by Dietz and Stern (2015) explores the standard production function

$$Y_t = (1 - D_t^Y)(1 - \Lambda_t)\bar{A}_t K_t^{\alpha} L_t^{1-\alpha} \tag{5.8}$$

However, they distinguish between damage D_t^Y affecting output and D_t^A damage affecting TFP, $\bar{A}_t$. In here, capital is not affected by climate change, but instead TFP is. The equation of motion of TFP, in this model yields

$$\bar{A}_{t+1} = (1 - D_t^A)(1 - \delta^A)\bar{A}_t + a(I_t) \tag{5.9}$$

5.9 which implies endogenous productivity, that changes over time through a combination of depreciation, autonomous growth, and knowledge spillovers. The term δ^A includes both depreciation of productivity through erosion or displacement of skills and know-how, and implicit, autonomous growth of TFP coming from institutional innovations (Dietz and Stern (2015), appendix A). Given these two opposing effects, Dietz and Stern (2015) assume that δ^A is positive, and smaller than δ^K. In principle, δ^A in the equation of motion could be negative *if* the autonomous growth (coming from institutional innovations) would be larger than the depreciation of productivity. Climate change decreases total factor productivity $\bar{A}$, via the damage term D_t^A. Investments in capital increase productivity, via knowledge externalities captured in 'the spillover function' $a(I_t)$, whereby $a(I_t) = \gamma_1(I_t^{\gamma_2})$. The stock of TFP is increasing via the knowledge spillovers. So combining both the effect of climate damages on productivity and knowledge spillovers coming from capital investments, on productivity, the TFP is increasing but at a diminishing rate.

$$a' > 0, \quad a'' < 0$$

Climate changes does not directly affect capital in this second growth model, and the equation of motion for capital is similar to the one in the DICE model. Summing up, some parts of the instantaneous impact of climate change damage fall on the TFP, thereby permanently reducing future output possibilities. That means output growth has become endogenous in the second growth model introduced by Dietz and Stern (2015). These results contribute as inspiration for our analysis in which we model uncertainty in the growth rate of TFP.

5.3 Discounting

The discount rate in the DICE model has a large impact on the estimation of the Social Cost of Carbon, and has been subject of debate in the existing literature of climate change economics for more than two decades. It is therefore relevant to cover several aspects of the critiques on the discount rate(s) and the main explanations behind those. Combining the topic of discounting with uncertainty around the future, and consequently riskiness, is where the climate beta comes into play, as will be explained at the end of this section.

The DICE model follows a traditional way of discounting which distinguishes between the discount factor R_t, also called the 'goods discount rate' and the rate of social time preference, also known as the 'welfare discount rate' ρ. Hence, the discount *factor*

$$R_t = (1 + \rho)^{-t} \tag{5.10}$$

is used to calculate the present value of **welfare** of a representative economic agent, i.e. consumer, based on the utility he/she obtains from consumption.

In general, the different methods to make a valid estimation of the *discount rate* can be classified in two broad categories: a descriptive versus a prescriptive approach (Nordhaus, 2017b) The descriptive approach is adopted in the DICE model and is explained in details in the next subsection. The *prescriptive* approach has been emphasized in the Stern Review (2007) a critique to earlier versions of the DICE model. Both Stern (2007b) and Nordhaus and Sztorc (2013) refer to Arrow (1995),who already made the distinction between the two approaches of how to value the welfare of future generations. Arrow followed the prescriptive or ethical approach, which implies that future generations' wealth should be valued based upon the probability that the human race is still alive in the future time periods, and little dependency should be given to the inter-temporal allocation of wealth of one individual. If the future generations are discounted at a high rate, regardless of their consumption level, they count relatively little today. This implies that the discounted present value of long-term investments is low and that climate change, as it has long term consequences, is largely disregarded. (Stern, 2007b) Both Arrow (1995) and Stern (2007b) argue that the pure rate of social time preference, ρ should be close to zero because the well-being of future generations, from an ethical perspective, should not be valued any differently from our own welfare.

5.3.1 Discounting by Nordhaus

Nordhaus employs a descriptive approach and thus argues differently than Stern (2008) and Sterner and Persson (2008).

The economic assumption underlying the discount rate in the DICE model is that R_t should reflect actual market outcomes, therefore the real return on capital investments can be seen as the 'goods discount rate'. 'The welfare discount rate' ρ provides a measure for valuation of future generations. The higher value ρ obtains, the less representative agents value or care about the welfare of future generations. This is a subjective parameter, and not directly observable in the market. Hence, one way to estimate the value of ρ is for example by means of surveys.

In the DICE model, Nordhaus assumes that the appropriate 'goods discount rate' is 5% per year in the beginning years, and 4.25% in the years far ahead (up till 2100). With that calibration Nordhaus 'chooses the pure rate of social time preference, ρ, to be 1.5% per year' (Nordhaus and Sztorc, 2013). The parameters ρ and α, the pure rate of social time preference and the elasticity of marginal utility of consumption respectively, are calibrated such that they produce r equal to the above mentioned 'goods discount rates'. Nordhaus (2017b) states that the discount rate is treated similarly in the 2016 version of the DICE model, i.e. using the descriptive approach to discounting. Still, he recognizes the discussions around the discount rate, that developed over the last two decades, and therefore explicitly points out his approach compared to alternatives. The near-zero pure rate of social time preference that Stern (2007a) and Arrow (1995) point to, is kept by Nordhaus in the DICE 2016R2 model at 1.5% and he calibrates the consumption elasticity α such that it matches the observed interest rate in the market. This is based on the Ramsey equation for growth-corrected discount rates, explained in chapter 3. The Ramsey equation states that the long-run discount rate can be estimated by

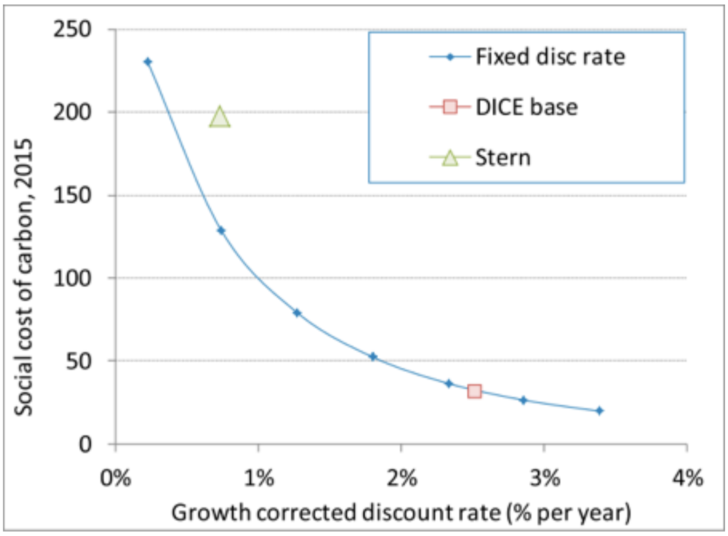
$$r = \rho + \alpha g \tag{5.11}$$

Where r equals the real interest rate as observes in financial markets, called the discount rate of goods throughout the thesis, ρ equals the pure rate of social time preference, α stands for the elasticity of marginal utility of consumption and g is the growth rate of per capita consumption or growth rate of output. Nordhaus (2017c) points out that the Ramsey equation combined with assumption of logarithmic utility, i.e. $\alpha = 1$ approximately holds for the DICE model. The growth-corrected discount rate $(r - g)$, equals in that case ρ.

The figure above shows the important effect of different values for the growth-corrected discount rate $(r - g)$, on the value of the social cost of carbon as estimated by the DICE model. Nordhaus hereby shows the higher value of the SCC based on assumptions from Stern (2007a), denoted by the triangle in the figure, compared to the full DICE base case scenario, denoted by the square.

What now follows, is an explanation and a differentiation of the social welfare- and the finance equivalent discount rate. These concepts help explain why the critiques to the discount rate employed in the DICE

Figure 5.1: Social cost of carbon and growth-corrected discount rate in DICE 2016R2



Source: Nordhaus (2017b)

model have been so far apart. The section relies heavily on a paper by Goulder and Williams (2012). It should however be noted that the differentiation between a discount rate for the welfare of the society and the discount rate used for discounting consumption, by the individuals own personal preferences is not novel in Goulder and Williams (2012). A similar way of discounting is also seen in the Diamond Overlapping Generations (OLG) model. For a textbook explanation of this we kindly refer to Romer (2012), Chapter 2B.

5.3.2 The Social Welfare- and Finance Equivalent Discount rate

Other researchers, such as Goulder and Williams (2012) and Kaplow, L. Moyer, E., Weisbach (2010) argue that much of the dispute regarding the discount rate lies in the assumption of using one single discount rate. They distinguish two different concepts, with adhering discount rates to help determine whether a discount rate should be based on ethical considerations or on empirical evidence and whether it should determined through a descriptive or prescriptive approach. The confusion between the two different views lies in the fact that one function serves as two different type of functions, namely both a *behavioral function*, describing the actual behavior of **individuals** under certain conditions, and as a *social welfare function*, indicating the socially optimal, i.e. the preferred behavior of individuals from a **social planner** point of view. The failure to distinguish these two functions forces the socially optimal outcome to lay in the feasible range of behavior, determined by the behavioral function, which often will lead to a higher discount rate than the social- planner one. The social welfare function should be discounted with a *utility-discount rate*, whereas the behavioral function should be discounted by a *consumption-discount rate*. As argued above, when applying an ethical approach to discounting, such as in the Stern Review, the pure rate of social time preference, ρ, should be zero, to reflect the equality between the current generation's utility and future generations' utility. The small positive value imposed on ρ by (Stern, 2007b) is justified to reflect the risk (or probability) of future exogenous disasters that may occur, and for example wipe out the entire existence of human life. That implies that the low discount rate discussed in the Stern Review should represent the discount rate used to discount back the future streams of *utility* to it's value today.

Rates that are used to convert values of future consumption into current consumption equivalents, on the other hand, are known as consumption or goods discount rates. This shows (intuitively) that one does not necessarily have to discard the ethical view, if both types of rates can be applied; One can apply the consumption discount rate to translate future consumption into equivalents of today, *and* use the utility discount factor with a social rate of time preference close to zero, and thus still persevering the ethical view that has been described before. This is precisely done in Nordhaus (2017b) by means of the Ramsey equation; calibrating the preference parameters ρ and α to match observed real interest rates as consumption or goods discount rates. Nordhaus and Stern are pioneers in climate change economics, but not the only ones providing critique on each other. As said, more than two decades of criticism on the discounting (and other aspects) within the DICE model exist.

Goulder and Williams (2012) provide an explanation around discounting based on the difference between the so-called 'social-welfare-equivalent' and 'finance-equivalent' discount rates. They define an intertemporal additive welfare function, with similar notation to the one Nordhaus' employs, as used in chapter 4 about the DICE model:

$$\hat{W}_0 = \sum_{t=0}^{\infty} (\frac{1}{1+\rho}^{t}) U_t(C_t) \tag{5.12}$$

ρ is the social rate of time preference and the utility function has a constant elasticity of marginal consumption α.

5.3.2.1 The social-welfare-equivalent discount rate

In (Goulder and Williams, 2012) the social-welfare-equivalent discount rate is denoted as r_{SW} and, according to a postulated social welfare function, informs us about whether the implemented policy will lead to an increase in social welfare. We can define r_{SW} as the rate at which marginal change in future consumption (at date t) is translated into marginal change in current consumption, yielding

$$\frac{\partial W_0}{\partial C_0} = (1 + r_{sw})^t \frac{\partial W_0}{\partial C_t} \tag{5.13}$$

In other words, r_{SW} translates future consumption into current consumption that are valued equivalently in terms of *social welfare*. By substituting the derivatives of 5.12 with respect to C_0 and $C_t$and rearranging we get

$$(1 + r_{SW})^t = (1 + \rho)^t \frac{\frac{\partial U_0}{\partial C_0}}{\frac{\partial U_0}{\partial C_t}} \tag{5.14}$$

From the right-hand side (RHS) of this equation, please observe the pure rate of social time preference ρ and the difference in marginal utility of consumption between the two time periods, respectively. Note however, that the marginal utility of consumption is decreasing in the level of consumption. This implies, the sign and value second term on the RHS depend on the level of consumption over time. We can model growth in consumption over time as $C_t = C_0(1 + g_C)^t$. Substituting this into 5.14, along with the derivative of the utility function, and simplifying gives:

$$(1 + r_{SW})^t = (1 + \rho)^t (1 + g_C)^{\alpha t} \tag{5.15}$$

Assuming that ρ, α and g_C are all small, we can take natural logarithms and simplify, yielding the following approximation for the social-welfare-equivalent discount rate:

$$r_{SW} \approx \rho + \alpha g_C \tag{5.16}$$

5.16 illustrates that the social welfare equivalent discounting depends on three parameters; ρ, g_C (the growth rate of consumption) and α. At the same time, this definition is underlying the long-term outcomes of the DICE model, as it equals the Ramsey Equation. (Nordhaus, 2017c) However, Das notes that especially in the case of climate change, consumption is not necessarily growing over the relevant time period. (Goulder and Williams, 2012). From the representation of the discount rate above, it becomes clear why Stern (2007a) and Sterner and Persson (2008) claim the ethical view of discounting falls under the umbrella of "social-welfare-equivalent discount rates" as there are ethical considerations when choosing both ρ and α. As explained in chapter 4 about the DICE model's assumptions, α can also be thought of as "society´s aversion to inequality". One could potentially use empirical estimates of individuals´ expressed inequality-aversion to get an estimate of α. (Goulder and Williams, 2012) Similarly, α could for example be measured by how much an *individual´s* well-being increases from a marginal increase in consumption. By observing economic behavior, such as savings, and assuming that individuals are required to maximize social welfare, one might derive values for the parameters ρ and α.

Assuming $g_C = 1.3$, used in combination with different values of ρ and η lead to the different discount rates argued for, by Stern (2007a), Cline (1992) and Nordhaus (2007),is shown by Das Thereby it becomes clear that small changes in the discount rate of consumption imply large differences in the discounted values for events in the far future, such as the estimate of for example the future social cost of carbon. (Das) (Goulder and Williams, 2012)

5.3.2.2 The finance-equivalent discount rate

The finance-equivalent discount rate is denoted as r_F and is more directly linked with real life market behavior. The use of this will tell us whether a policy leads to potential Pareto improvement and is different from r_{SW}. We can define r_F as the rate at which future consumption and current consumption are made equivalent in *financial* terms, also known as the marginal product (or opportunity cost) of capital. One consumption unit not consumed today will yield $(1 + r_F)$ **consumption units** next period (note the difference here with *utility* of consumption units). The finance-equivalent discount rate can be found empirically. If we assume a perfectly frictionless market, then one unit of consumption good is sold for its marginal product and the market rate of interest will thus be r_F. The same goes for consumers behavior in a perfect market; individuals are not liquidity constrained and their consumption/savings decisions will reflect that they discount consumption at r_F. In imperfect markets, however, the interest rate will not equal r_F. The real life market is neither perfect nor friction-less. One might think that these imperfections in the market make $r_{SW} = r_F$, but Goulder and Williams (2012) argue that these market imperfections do not equate r_F and r_{SW}. This is because r_F represent how individuals discount their *own* future consumption which is not necessarily how the social welfare utility is discounted, which also includes the welfare of future generations.

5.3.2.3 Which rate to use

Following the arguments of Goulder and Williams (2012), the two discount rates should not be used for the same criteria, although both r_F and r_{SW} can be used for assessment of a climate policy. The difference between the market interest rate, r_F, and the social discount rate, r_{SW}, is similar, but *not equivalent* to the difference between the descriptive and the prescriptive approach described earlier. It is true, that r_F has a more descriptive foundation, as it is more directly connected to actual behavior, whereas the social welfare function on which r_{SW} builds is more normative. This gives r_{SW} a stronger basis for recommending some policy options over others, i.e. following the prescriptive approach.

r_{SW} can be used for the purpose of showing how much consumption today should be foregone (invested in green technology) in order gain (avoid loss of) consumption in the future. If $\Delta C(t)$ is the change in future consumption gained, then the investment should be accepted if $I(t) \leq \frac{\Delta C(t)}{(i+r_{SW})^t}$. That is, if the investment today, the current sacrifice, is smaller (or equal to) the discounted change in future consumption.

r_F on the other hand, should be used when the scope of the discounting is to assess whether the policy is potentially Pareto optimal and thus satisfies the Kaldor-Hicks criterion. The Kaldor-Hicks criterion is a weaker form of the Pareto- criterion, widely used in cost-benefit analysis, and states that an investment is optimal if the gain is larger than the loss, regardless of who gains and who loses, because the ones who gain can compensate the ones who lose. This can easily be illustrated by considering a combination of a climate policy and inter-temporal transfers. Imagine that we transfer one unit of consumption from the future beneficiaries of the climate policy to the present generation, leaving utility in all future periods unchanged. The current period consumption increases by $\frac{\Delta C(t)}{(i+r_F)^t}$ through diverting resources from current investment to current consumption. This transfer holds utility constant in all future time periods but reduces the consumption in time t by $\Delta C(t)$. Now, if the climate policy requires current consumption to be reduced by less than $\frac{\Delta C(t)}{(i+r_F)^t}$ before the transfer, then the combination will yield a Pareto improvement.

We thus see that both the social-welfare-equivalent and the finance-equivalent discount rate can be used for climate policy assessment, justified by different criteria. Both the impact on social welfare and considering Pareto improvements are important assessments for climate policies, although Pareto improvement can be obtained without increasing social welfare and vice versa. The next chapter shows that a similar definition of the discount rate r_t will actually be used to show where the climate beta comes in, namely when uncertainties or risk come into play. The climate beta determines the size of the risk-premium that should be added to either r_{SW} or r_F, in order to derive a discount rate that adjust for riskiness of the benefits of climate change mitigation policies.

Implications

When $r_F \neq r_{SW}$, but used as one rate, resources, and thus consumption levels, are not allocated across time in a welfare-maximizing way. For example if $r_F > r_{SW}$, transferring consumption from today to a future time will lead to an increase in future capital stock of $(1 + r_F)^t$, which is less than the value this transfer has to social welfare. In the case of climate policy, with the same discrepancy between the rates, we can imagine a policy with positive net benefits using r_{SW} and negative net benefits when applying r_F. This climate policy, although worthwhile for the societal welfare to undertake, could be replaced by a transfer of similar resources via the capital stock. Thereby larger welfare could be obtained. If the government can both increase capital stock and optimize climate policy, the changes in the capital stock will influence the two discount rates. Considering that the finance equivalent discount rate is the highest of the two, the climate policy is optimized by increasing capital stock. As this is done by reducing current consumption and saving more, the growth rate of consumption, g_C increases, leading to a simultaneous increase of r_{SW} and decrease of r_F. The optimum is found where the two discount rates are equal. Das Dasgupta (2008) argues that only in this case, when $r_F = r_{SW}$ it is defensible to discount climate change mitigation policies, coming from a social welfare perspective, using r_F.

Consequences

From the criticism surrounding the discount rate, is becomes clear that much of the disagreements lie in the distinction between the use of one rate as opposed to two rates. This relates to the many different IAMs that optimize one objective function, representing both the behavioral- and the social welfare function, making the discount rate higher, due to the Kaldor-Hicks criterion which cannot support the same aggressiveness in climate policies as the r_{SW} would imply. (Goulder and Williams, 2012)

Concluding, the subject of discounting is a complex question in the area of climate change economics, involving value-judgments about both time, risk-aversion, inter-generational welfare and the relation between discounting environmental investments and conventional market returns on investments.

5.3.2.4 Uncertainty and the discount rates

The discussion about the discount rate explained in the previous subsection tries to relate the DICE model with climate change mitigation projects. The DICE model however assumes a risk-neutral world without uncertainty in the future cash flows, while climate mitigation projects include uncertainty surrounding future streams of cash flows. When applying the critiques on discounting in general, to the case of climate change economics and climate mitigation policies, many types of uncertainties of significant size, come into play.

Following Goulder and Williams (2012) uncertainty has the same effect on both the market interest rate and the social welfare discount rate. This effect depends on the correlation between the benefits of the mitigation policy and future consumption. A mitigation policy from which representative agents benefit mostly in future states of the world with low aggregate consumption levels, will function as an insurance, as the negative correlation between benefits and consumption hedges by paying off in bad states. Such a policy would be more attractive, considering the concavity of the utility function, implying that a marginal increase in consumption in a bad state provides more utility than the same marginal increase in consumption at a good state (when consumption is already high). This is exactly where beta comes into play, as the climate beta should be seen as the correlation between the benefits of a mitigation policy and future consumption.

5.3.2.5 Critique on β

Negatively correlated benefits to consumption would yield a discount rate below risk-free one, meaning a negative value for beta. According to Goulder and Williams (2012) however, this method, although sometimes convenient, combines the two distinct issues of risk and discounting. Uncertainty in policies has the same effects in a case where benefits occur in the future as it does on policies where benefits occur immediately. They thus argue that a better way of calculating the optimal discount rate is by incorporating risk into the analysis through certainty equivalent benefits, discounted at a 'safe' or 'risk-free' rate. This method can be seen as a separation of uncertainty and riskiness, rather than discounting expected benefits, that would combine the two separate issues.

r_F and r_{SW} will have different sources of uncertainty, implying different magnitude and importance of the uncertainty. For r_{SW} it is related to growth of consumption, g_C, and for r_F it relates to the opportunity cost of capital. Despite this, the uncertainties will move in the same direction. Suppose j states of nature, where r_j is the discount rate in state j, p_j is the probability of that state and the discount rate is constant over time for any given state. We can now explicitly include the uncertainty to the cost benefit analysis of a policy. A change in consumption at time t ,ΔC_t, gives the expected future benefit of $\sum_j [p_j(1+r_j)^{-1}\Delta C_t]$. This way of calculating the discount rate is convenient in the sense that it provides transparency. However this method can be very cumbersome because it requires every CBA to consider the whole distribution of possible discount rates. To avoid this, Goulder and Williams (2012) suggest to collapse the range of possible values for the discount factor into one certainty equivalent rate that will yield the same result. This way of calculating the discount rate has the advantage of its simplicity, but the simplicity comes at the cost of the strong and unrealistic assumption that there is no correlation between the uncertainty about benefits and the uncertainty about the discount rate.

5.4 Which rate to use?

This thesis studies "the climate beta", derived through the CCAPM, by using the DICE model. As the DICE model, despite its limitations, tries to model the optimal carbon tax or the social cost of carbon for policy makers in the real world, a world *with* uncertainty, the risk-adjusted discount rate is relevant to use. However, both approaches to the discount rate discussed before originally disregard uncertainties, hence they can be seen as 'risk-free' or 'safe' discount rates. These approaches of estimating the rate used to discount future streams of cash flows in the DICE model, are undertaken because there is no uncertainty surrounding the future output variables in the DICE Model. That is, the future paths of the output variables in the DICE model are determined once the model runs and the preferred parameters are set by the modeller or 'policy-maker'. A world without taxes or risk is thereby assumed.

In order to have an informed discussion about the appropriate value of the discount rate, what rate should be used in reality, to estimate the present value of the social cost of carbon, we must take into account risk and uncertainties surrounding the future. Nordhaus employs a descriptive approach to the discount rate and describes in (Nordhaus, 2017b) that the DICE discount rate is set such that it represent the discount rate on consumption goods. Note that the DICE model extends the definition of consumption, including not only consumption of goods but also the consumption of, health, services, clean air etc. Defining consumption in such a broad sense, enables us to to include 'the climate' within consumption of goods, which makes the DICE model a suitable IAM to determine the benefits of climate change mitigation and the rate at which those should be discounted. As abatement, or in a broader sense 'climate mitigation policies',can be seen as investments affecting current and future consumption, the Consumption CAPM-theory provides us with a suitable way of determining the market rate of consumption-goods, i.e. the discount rate of goods that can be used in the DICE model. In the case of "climate mitigation investment policies", we argue that using the CCAPM in combination with the DICE model is an appropriate way to adjust for risk in determining the discount rate. We find a suitable way to estimate a so-called 'climate beta' (β), that can account for the large uncertainties relating to future consumption and the benefits of climate mitigation policies. Consequently, the climate beta and our methodology are extensively discussed in the next chapter.

Equilibrium Climate Sensitivity Equilibrium climate sensitivity (ECS) is the global mean warming, also known as the **long-term** temperature change at the surface, that would occur if the atmospheric CO_2 concentration was doubled, and the climate was brought to equilibrium with that new level of CO_2 and it is one of the most discussed parameters in the DICE model. This thesis does not aim at addressing the ECS beyond a simple explanation to why this parameter makes the output of the model so uncertain If the climate becomes more sensitive to the doubling of CO_2 in the atmosphere, meaning that the value of ECS increase, emission cuts need to be larger in order to meet climate targets. A warmer surface leads to an increased radiation to space, which counteracts the increase in the earth's heat content. How much the radiation to space increases for a given increase in surface temperature depends on the same feedback processes such as cloud feedback, water vapour feedback and more, which again determine the ECS(IPC).

The ECS parameter is highly debated, as both the actual value of the parameter and the underlying probability distribution is unknown. Much of this uncertainty stems from the fact that relatively little historical data exists on human-induced climate changes, making estimation difficult.

ECS scales the climate change projections and strongly influences the emissions cuts that are required to meet climate targets. The ECS used in the DICE 2016R2 model is based on Olson et al. (2012) and falls well within the range of what is argued to be plausible in several other assessments of the ECS. Many have tried to estimate a correct ECS, which is proven to be difficult, and a recent overview is summarized by Knutti et al 2014 Rog. The Intergovernmental Panel on Climate Change, (IPCC), has generally accepted an ECS-range of 1.5 - 4.5°C and this estimation has been consistent throughout the last three reports (IPC).

To put the ECS parameter into perspective, an ECS close to 1.5°C is a temperature to which the world is well able to adapt, whereas an ECS closer to 4°C leads to a climate change we will likely not manage to adapt to**?** From this we see that the ECS has a possibly critical effect on the outcomes of the DICE model, and thus also on the climate beta. In fact, Dietz et al. (2018) find that if the only source of uncertainty of the model is the climate sensitivity, the beta will be negative. We have showed in the simple climate model in chapter 3, that the ECS is critical in however lots of literature has focused on climate parameters. ... .relating this uncertainty to a climate beta will likely lead to a negative beta, as the correlation between damages and consumption is inverse

6 Methodology

In the DICE model, Nordhaus assumes a CCAPM β of 1 on climate investments. This implies that investment projects to mitigate climate change have a similar risk structure as the average investment portfolio. (Nordhaus and Sztorc, 2013) Assuming a beta of 1, Nordhaus naturally calibrates the (preference) parameters in the DICE model such that the discount rate r_t is consistent with observed rates in the market. The discount rate r_t is in turn used to calculate the present value of welfare, in order to ultimately derive a value for the SCC after the DICE model is solved. That is, after the social welfare is optimized, marginal costs of abatement is set equal to marginal benefits of mitigation. However, 'there exist extremely large structural uncertainty about the SCC even in a single model',Nordhaus (2017b), which elaborated on in chapter 5. Due to the uncertainty surrounding future consumption levels, future damages, and hence the future benefits of climate mitigation projects, it is not an obvious question to find the optimal Social Cost of Carbon. Similarly, the discount rate to calculate the net present value of a marginal mitigation project remains up for discussion.

As explained in chapter 3, we need to adjust the discount rate, accounting for risk and uncertainty, to calculate the NPV of climate change mitigation investments. This risk-adjustment of the discount rate, depends on the extent to which benefits of the climate investments and aggregate consumption are correlated, i.e. on the climate *beta*. As argued in the previous chapter, we base our estimation of a risk-adjusted discount rate for climate change investment projects on the Consumption Capital Asset Pricing Model (CCAPM). Therefore, we define the following risk-adjusted discount rate :

$$r = r_f + \beta\pi \tag{6.1}$$

and in this chapter we will estimate β_t from 6.1 by making use of the DICE model. Intuitively, β_t can be seen as the co-movement of future consumption and future benefits of mitigation, but as argued above, both of these are very uncertain over the long time-horizon of climate change. Hence, in order to get an estimate for β_t, we decide to incorporate one main source of uncertainty in the DICE model, namely uncertainty in Total Factor Productivity growth. The uncertainty in TFP growth is modelled through two separate parameters, that is, the initial trend growth rate of TFP and shocks to the TFP growth rate over time. We show their individual effect on the estimation of the climate beta. As we estimate the climate beta based on the CCAPM model, it might be natural to use historical data on similar previously implemented investment projects. We could use historical observations to estimate values for the benefits of climate change mitigation projects. In turn, by using standard regression methods we would be able to derive an approximation for β_t. However, we believe that in the case of climate change, historical data tells us only little about the potential damages of future climate change and the benefits of abatement projects. Therefore, we decide to base our estimation of the climate β_t not on historical observations, but instead based on output variables from the DICE model.

6.1 What determines the climate beta?

We develop our own estimate for the climate beta, inspired by the methodology from Dietz et al. (2018), using a modified and updated version of the DICE model. Please note that there are some differences in the notation here with respect to chapter 4. 4. We run the modified code for the DICE 2016R model in a licensed version of the General Algebraic Modelling System (GAMS). The original code for the DICE 2016R2-model is made publicly available by Nordhaus and runs in GAMS. It contains about 25 dynamic equations, and provides us with the main output variables of our interest: consumption, CO2 emissions, global average temperature, the social cost of carbon , savings, abatement costs and damages. We alter the original code, which allows us to analyze two main sources of uncertainty. Please refer to the altered code in the appendix. First of all, parametric uncertainty regarding future economic growth is analyzed. This is reflected in the growth of Total Factor Productivity (TFP). Secondly, uncertainty regarding the functional form of the damage function will be explored, that is whether or not the damages have an

As shown in chapter 3, an approximate estimation for beta can be obtained by running a linear regression of the natural logarithm of benefits with respect to the natural logarithm of consumption.

$$\ln B_t = \beta_t \ln c_t + \varepsilon_t \tag{6.2}$$

Following (Dietz et al., 2018), we define benefits as follows:

$$B_t = C_t - C_t^{REF} \tag{6.3}$$

Whereby C_t is consumption per capita per year and here β is simply the coefficient of benefits with respect to c_t. We run the DICE model in the "business as usual" version, that is assuming no additional policies are undertaken to limit the increase in global mean temperature. The only policies that are in place and will be continued going forward are the ones already implemented in 2015, as assumed in the DICE 2016R2 version of the model. This is done by setting the function $'ifopt' = 0$. By choosing to run the "business as usual" case, this implies the DICE model is run as a 'simple' projection model, rather than being used for policy optimization. Still, in every period utility is optimized by the representative agents, as explained in Chapter 4.

We obtain C_t^{REF} by running the DICE model with the altered TFP growth function and damage function, hereby referred to as our 'reference case'. After the reference case run, we run the model again, but with the removal of 1 unit (Gt) of industrial emissions ($GtCO_2$ per year) in 2015. In the version of the DICE model we employ (DICE 2016R2) this is done by changing the parameter e_0 from 35.85 to 34.85, implying that we reduce industrial emissions in 2015 by 1 $GtCO_2$. This has implications for the level of industrial emissions in the subsequent years as well, through the carbon-intensity denoted by σ_t. This is one way where we depart in our methodology from Dietz et al. (2018), as we remove directly 1 unit from the parameter e_o. In contrast, Dietz et al. (2018) remove 1 unit of $GtCO_2$ in 2015, but do this ***only*** in the industrial emissions in that specific year, which is the second time period in the horizon of the DICE model they employ. This is done through the industrial emissions *equation*:

$$E_t^{IND} = \sigma_t(1 - \mu_t)Y_t \tag{6.4}$$

The effect of removing 1 $GtCO_2$ from the initial value of 35.85 $GtCO_2$ in 2010 might seem relatively large, particularly compared to the marginal abatement project defined by Dietz et al. (2018). However, we argue that our method to implement an abatement project, through changing e_0 can still be seen as a **marginal** abatement project because the effects on the variables of interest are only marginal. A visual representation of the effect of 1 unit decrease in e_0, i.e the effect of our mitigation project on the industrial emissions over time, can be found in the appendix A. A verbal description of the implications of our marginal climate mitigation project defined through $e0$, for the most important output variables is provided in the next chapter.

Adjusting the discount rate for uncertainties, via a risk-adjusted discount rate, and hence the estimation of β, is dependent on implementing only an *marginal* investment project. If one was about to consider a non-marginal change in emissions, i.e. a larger abatement investment project, the risk structure could potentially be altered. That is, the paths of future consumption, i.e. the probability distribution of ending up in specific future states of the world could be affected. Such an analysis would require more knowledge about the entire distribution of function of consumption and the cash flows coming from the investment project. [11] In our analysis in the next chapter, a more elaborate analysis of a marginal abatement project defined through $e0$, and its effects on the most important output variables will be provided.

We estimate the benefits of the marginal emissions abatement project by subtracting the reference-case consumption per capita from the consumption per capita of the mitigated case, C_t, as in 6.3. By substituting the DICE model's equations for C_t and y_t, in 6.3 and rewriting, we arrive at:

$$B_t = (1 - s_t)(1 - D_t)\, y_t - (1 - s_t)\left(1 - D_t^{REF}\right) y_t^{REF} \tag{6.5}$$

whereby y_t is pre-damage output per capita, net of abatement cost and s_t is the savings rate. Note that in our analysis, the marginal propensity to save s_t is determined endogenously through the equations of the DICE model. The savings rate in the DICE model follow the optimal- long-run savings path, and it can be shown that the savings rate becomes constant in the long run, equal to $s_t \equiv 1 - \frac{c_t}{y_t}$. (The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018). Contrary, Dietz et al. (2018) assume the same parameter to be exogenous, as in the Solow growth model. The effects of our endogenous savings rate on the output variables of our interest will be elaborated on in the next chapter.

Another difference between our analysis and the one from Dietz et al. (2018) is the use of the 2016R2 version of the DICE model, rather than the 2013 version. To our knowledge, this is the first study that estimates beta for climate investments by means of modelling uncertainty using the 2016R2 version of the DICE model in GAMS. Although most of the equations are similar to the 2013 version of the model

[11] For a more extensive analysis of a non-marginal mitigation investment project, see Nordhaus and Boyer (2000), Stern (2007a), Nordhaus (2008)

(Nordhaus, 2017b) , the largest difference between the two versions of the model concerns the method of estimating damages. At the same time, the damage function is a key determinant in estimating the SCC. The damage function in the 2016 version of the DICE model reflects new findings, based on the latest existing survey of damage studies. This survey includes 26 damages studies, and the estimates of damages are made using 4 different techniques. (Nordhaus, 2017b) The final estimate of the climate damages, used in the 2016 version, is a loss of -2.1% of global GDP at 3 °C increase in global temperature, which is slightly smaller than the damage parameter in the earlier version of the model. This is due to corrections in the study 'The economic effects of climate change' Tol (2015)Tol (2009), arising from the inclusion of several new studies and the use of weighted regressions in estimating the effect of damages on the economy. The adjustments made to the damage function in the various versions of DICE over time, is one example that confirms the significant uncertainties surrounding climate change damages. The variables indirectly affection our estimates of the climate beta, will be discussed in the next chapter, in order to get a better understanding of the mechanics underlying the DICE model.

Concluding, we estimate β_t as

$$\beta_t = \frac{\operatorname{cov}\left[\ln C_t^{REF}, \ln B_t\right]}{\operatorname{var}\left[\ln C_t^{REF}\right]} \tag{6.6}$$

6.2 Modelling uncertainty

In order to model uncertainty, we follow Nordhaus (2017b) whereby we alter two equations in the DICE model, and as explained below we base our overall methodology on Dietz et al. (2018). Structural uncertainty, that is uncertainty within one model (DICE 2016R2 in our case), stems from uncertainty around the (distribution of the) underlying parameters in the model. The structural uncertainty that will be assessed in our thesis concerns TFP growth via two different parameters within the growth rate. In addition to that, we analyze uncertainty regarding the damage function. Although, uncertainty surrounding the damage function is related to its functional form, which differs from the way that parametric uncertainty that is modelled in the TFP growth rate.

Our choices surrounding the modelling of uncertainty, as both uncertainty in TFP growth and uncertainty surrounding climate change damages are very likely to be part of the key uncertainties in climate change economics. (Gillingham et al., 2018) Already in one of his first publications on structural uncertainty within the DICE model, Nordhaus defined an index of sensitivity I_i. This index showed that economic variables (population and productivity growth, time preference and the trend in emissions-output ratio) have the largest overall impact on the outcome variables of the DICE model. (Nordhaus, 1994) Especially in determining the global aggregate output level Y, and per capita consumption C , the uncertainty surrounding population growth and total factor productivity is of major importance.

In general, several ways of modelling uncertainty within IAMs exists. These can be broadly separated into stochastic and deterministic modelling of uncertainty. We choose to use a deterministic type of modelling uncertainty, rather than rewriting the DICE model in stochastic format. This is because the main focus of our thesis is not mathematical programming, nor the modelling of the uncertainty per se. Instead, the mathematical programming tool GAMS is used to run the DICE model and to obtain estimates for major output variables like future consumption per capita, which in turn will be used to estimate the climate beta. The modelling of uncertainty in the DICE model, is necessary in order to obtain an estimate for beta. If uncertainty is accounted for, the DICE model assumes certain and thereby risk-free future cash flows and outcomes for the output variables. As explained in chapter 3, beta is the main determinant of the (negative or positive) risk premium that one should add to the discount rate in order to account for uncertainty surrounding the future. Consequently, beta depends on the type and the size of uncertainties related to output variables.

By means of our analysis, we aim to contribute to the larger discussion surrounding the right risk-adjusted discount rate, as well as the even larger discussion about the Social Cost of Carbon. As the SCC can be seen as the price to be paid for damages coming from CO_2 emissions, and in turn, the climate beta can be seen as it's main determinant, this is intuitively similar to the CCAPM's or the general CAPM's β that serves as the main determinant of a financial asset's price.

As said, we choose to use a deterministic type of modelling uncertainty. That means we use Monte Carlo simulations of the uncertain parameters, such that each run of the DICE model is deterministic, but the value of any 'uncertain parameter' is a random draw from a given distribution. We take the probability distributions for each of our uncertain parameters from various sources that will be described for each of the uncertainties individually in the upcoming subsections.

Note that in general, the evaluation of the discount rate appears to be well suited for uncertainty propagation [12], such as scenario analysis or sensitivity analysis. The applicability of such uncertainty modelling is due to the exogenous nature of the discount rate, which makes it easy to see the effect on the output of the model from the different draws of the exogenous parameters which compose the rate. Conversely, structural uncertainty in empirical, endogenous variables such as output, consumption, productivity or technological development, and damages arising from climate change are less suited for the deterministic type of modelling uncertainty. This is because the drivers behind these parameters are from the real, physical world and not a product of the modellers preference parameters. However, we follow Gillingham et al. (2018), Nordhaus (2017b) and Dietz et al. (2018) who all use Monte Carlo simulations, i.e. deterministic approach to modelling of uncertainty. By using Monte Carlo runs of the DICE model, we simplify our consideration of the large number of possible future uncertain states of the world (SoW) into a smaller number of representative scenarios. Nordhaus himself also pointed out in the early years of the existence of the DICE model and the research around this area, that due to computational difficulties, one may want to "focus on subjective probability estimates for each of the uncertain variables and then make projections by taking random draws from the uncertain variables". (Nordhaus, 1994) Concluding, the Monte Carlo, deterministic type of modelling uncertainty is a popular method in previous research related to our work. (Bansal et al., 2016) (Lemoine, 2017) (Dietz, 2011)

[12] Uncertainty propagation is the modelling of different variables uncertainty and the effect this has on the function consisting of these variables.

6.2.1 TFP growth

One of the largest uncertainties that is strongly affecting the economy and the climate in aggregate IAMs like DICE, is the growth in future world output per capita, or better known as productivity. (Kol) (Gillingham et al., 2015) (**?**) The total factor productivity (TFP) is the part of the economy's output that cannot be explained by the standard neoclassical growth inputs capital and labour. Hence, in order to model uncertainty in world output, we focus our analysis on creating uncertainty in parameters estimating the *growth rate of TFP*, g_t^A. We use the findings from Christensen et al. (2018) that project an average annual GDP growth rate of 2.06 percent with a standard deviation of 1.12 percentage points for the period 2010-2100. These forecast are based on an extensive survey including economic growth experts' opinion and an additional econometric approach, using low-frequency forecasting. By doing so, they are able to model stochastic trends varying over a longer time horizon than business cycles, which is exactly one of the important concerns in modelling uncertainties related to climate change. Based on a normal distribution with above mentioned mean and standard deviation, we run a MC simulation and create 1000 random numbers. Those random numbers in turn will be used to create uncertainty surrounding TFP. The next step is to decide how this uncertainty comes into play, i.e. which parameter(s) will be changed from fixed to uncertain in order to create the preferred uncertainty in g_t^A? In the original version of the DICE model, the level of TFP evolves over time as follows:

$$A_{t+1} = A_t \left(1 + g_t^A\right) \tag{6.7}$$

where A_t is TFP in time period t and the growth rate of TFP is g_t^A . In turn, the growth rate g_t^A is defined as follows

$$g_t^A = g_0^A (1 + \delta^A)^{-t} \tag{6.8}$$

where δ^A is the decline rate of TFP per 5 years and g_0^A is the initial growth rate of TFP per 5 years. Nordhaus has modelled uncertainty in g_t^A in the 2016R2 DICE model and in various earlier versions of the model, through randomizing the initial growth rate of TFP per 5 years, g_0^A (Nordhaus, 1994) (Nor)

However, we follow Dietz et al. (2018) in how we specify the growth rate of TFP per 5 years:

$$g_t^A = \left[(1-\psi)g_0^A + \psi g_{t-1}^A + \varepsilon_t\right] \left(1 + \delta^A\right)^{-t} \tag{6.9}$$

In this way, we are able to distinguish between two sources of uncertainty within the TFP growth rate, namely ε_t which are i.i.d. normally distributed shocks to the productivity growth and via the initial growth rate g_0^A. Here g_t^A follows an AR(1) process, as in Dietz et al. (2018). The implemented AR(1) process enables persistence of shocks to the model, which is more in line with relevant evidence on productivity growth according to Dietz et al. (2018). The third possible parameter that could be used to create uncertainty in g_t^A, which is for instance used by Nordhaus (1994), is the decline rate δ^A. In Nordhaus (1994) it is assumed that this parameter can be used as proxy for the uncertainty surrounding all the productivity-related parameters. This thesis, however focuses on the two aforementioned parameters in the TFP growth and not on uncertainty within the decline-rate δ^A, as Dietz et al. (2018) and Gillingham et al. (2018) indictate that shocks to TFP growth and the trend growth of TFP are the most important determinants of TFP over time. We expect that if we would have focused on modelling uncertainty in δ^A, the effect on our results would be as follows: higher values for the decline rate of TFP, lead to a more convex term structure of the growth rate of TFP, g_t^A, resulting in a lower level of TFP (AL_t) over time. Vice versa, lower values for δ^A result in a more concave figure of g_t^A and naturally a steeper term structure and higher levels of TFP.

Note that the latest forecasts of long-run economic growth from Christensen et al. (2018), provide us with inputs for Monte Carlo simulations. These estimated mean of 2.06 % and standard deviation of 1.12 percentage points, could actually been used to create uncertainty in the growth rate of output per person, i.e. via the growth rate of population g_t^N and the growth rate of TFP over time g_t^A. This is based on their definition of the growth-rate of output as the sum of population growth and growth in labor productivity. However, their results do also indicate that existing climate change modelling practices underestimate the probability of high output growth rates. Therefore we believe our approach, i.e. creating uncertainty in the parameters g_0^A and ε_t is a suitable method to implement uncertainty in the TFP growth rate. Moreover, we decide to model uncertainty in g_t^A through both g_0^A and ε_t separately, varying only one a time, in order to analyze two different types of uncertainties within the same important parameter. Note that both these parameters are specified per 5 years in the original code of the DICE model. Therefore, the 1000 MC simulations that we use to model the uncertainty in both g_0^A and ε_t separately, are based on a normal distribution with underlying mean of 10.73% and standard deviation of 2.504%. That is, we adjusted the yearly long-run economic growth forecast to periods of 5 years.

The effect of the uncertainties as independent shocks versus uncertainty within the underlying drift or trend of the process and their effect on β_t will be analyzed. Secondly, we use different values for the distributions underlying the MC simulations than Dietz et al. (2018), which might give rise to interesting results, especially because the way we model uncertainty through the two different parameters will end up in significantly different output and consumption levels. Finally, this decision has also been based on creating uncertainty in those particular parameters in the model that could be varied without excessive modelling burden. We are aware of differences that our approach might create compared to the case where one would calibrate for instance g_t^A and/or g_t^N based on the mean and standard deviation of 2.06 % and 1.12 % respectively, and their underlying parameters accordingly. The next chapter describes specifically the effect of uncertainty through the individual parameters g_0^A and ε_t on the relevant output variables, before the estimates for β will be presented in chapter 8.

6.2.2 Damages

'The DICE 2016R model takes globally averaged temperature change (T_{AT}) as a sufficient statistic for damages and assumes that damages can be approximated by a quadratic function of temperature change. (Nordhaus, 2017b) As mentioned in Chapter 4, damages takes the following form in the original version of the 2016 model:

$$\Omega_t = \frac{D_t}{[1 + D_t]} \tag{6.10}$$

Where,

$$D_t = \varphi_1 T_{AT\,t} + \varphi_2 [T_{AT\,t}]^2 \tag{6.11}$$

In reality, the size of the damages coming from climate change is still very uncertain. How should that uncertainty affect the way we discount the reduction of these damages, i.e. the benefits from climate change mitigation projects ? In fact, the damage function used by Nordhaus (2017b) can be seen as a 'build-in mechanism' for estimating a climate beta around unity, because of its multiplicative nature. How does that work? As damages in this multiplicative form are expressed as a constant fraction of output, faster technological progress will increase both output and consumption, leading to higher emissions, which in turn leads to higher total damages and higher marginal damages, if the damage function is convex. Consequently, the benefits from mitigation increase at the same time as increases in output and consumption occur. This results in a positive estimate of a the climate beta, around unity, as argued by Dietz et al. (2018) and assumed by Nordhaus and Sztorc (2013).

In order to explore the climate beta and to make it possible to obtain different values than unity, we alter the damage function, inspired by Dietz et al. (2018). Our damage function has the following form:

$$D_t = Y_t \left[1 - \frac{1}{1 + \alpha_1 T_t + \alpha_2 T_t^2 + (\alpha_3 T_t)^7} \right] \left(\frac{Y_t}{Y_0} \right)^{\xi - 1} \tag{6.12}$$

This is a more generalized form of the damage function, which enables us to include both completely multiplicative damages, as well as completely additive damages and everything in between those two extreme cases. Hereby, ξ is a measure for the income elasticity of damages, which is assumed to be constant, following the specification of van den Bijgaart et al. (2016). By setting ξ equal to 1, the last term in 8.2 disappears and the damage function takes on the multiplicative form as in the standard DICE model. In the other extreme case, when ξ takes on the value of 0, the damage function has an additive form. That implies, for a given level of temperature increase, doubling pre-damage aggregate income Y has no **direct** effect on the *absolute* level of climate damages. However, it still holds that *marginal* damages increase when pre-damage aggregate income Y rises, in case an additive damage function is employed. This effect occurs through higher cumulative emissions, increasing the marginal damages based on the convexity of the damage function.

Intuitively the effects of uncertainty underlying the TFP growth rate will affect the different cases of the damage functions somewhat differently. As the multiplicative damage function models damages as an instant function of output, faster growth of GDP is expected to have a larger effect on multiplicative damages than on an additive damage function. Modelling of the effect of uncertainty *within* the parameters of the the damage function on the climate beta is beyond the scope of this thesis. However, we will consider both an additive and multiplicative damage function for the uncertainty in g_0^A and the uncertainty in ε_t.

6.2.3 Shocks to TFP growth ε_t

The modelling of uncertainty through ε is more complex, although similar, to the uncertainty modelling of g_0^A, as ε_t is random and normally distributed also over time. In order to model the uncertainty without extensive programming skills, we model the productivity shocks through a matrix, which is $\sim N(\mu, \sigma^2)$. In terms of computation in GAMS, we define a variable as "***EpsiMean***", namely the mean of a normally distributed shock with a mean of EpsiMean and a standard deviation of 2.504%. We also perform 1000 MC simulations based on a normal distribution with underlying mean of 10.73% and standard deviation of 2.504%, to create uncertainty in the variable ***EpsiMean***. As mentioned earlier, these values are based on long-run economic growth projections made by Christensen et al. (2018) .

In order to clearly see the effect of the these i.i.d. shocks in the growth rate of TFP g_t^A on the output variables, we set g_0^A to the value 0. As opposed to the previous case of uncertainty, where the uncertainty in the TFP growth rate is modelled in the initial time period 2015, we expect the effect of uncertainty in ε_t on β. By creating uncertainty in ε_t this way, we expect the growth rate of TFP g_t^A to create more variation in the level of TFP A_t, thereby resulting in more uncertainty in gross output and consumption over. We expect that the shocks will have a different effect on beta seeing that the marginal utility of consumption is different over time when the TFP growth happens through shocks. (More variation in TFP growth–> more variation in TFP–> more variation in output–> consumption–> more variation in utility.)

7 Analysis

7.1 How are Benefits and Consumption determined?

The following chapter aims at clarifying the dynamics through which benefits and consumption are determined. The reason to start of with this overall assessment of the paths of consumption and benefits, is to get an expectation for our estimations of β_t, before those will be presented in the next chapter. Explaining the underlying effects, will be done through highlighting different equations that consumption hinges on, and through an intuitive explanation of the underlying mechanisms. The reader should note that we do not report the first 7 time periods, i.e the years from 2015-2049. The reasoning will be explained more extensively in the next chapter, but relates to unreasonably low values for the denominator in the definition of the climate β, which yield obstructive results.

The beta ultimately tells us how benefits and consumption move with each other, therefore it is crucial to understand that benefits are defined as $B_t = C_t - C_t^{REF}$. Here, B_t defines benefits of added consumption at time t, C_t refers to consumption when the mitigation project is implemented and C_t^{REF} implies consumption before the mitigation project is implemented. From the definition of beta and of the benefits, we thus learn that it is crucial to understand how consumption moves over time and what affects consumption in the DICE model.

Please recall from the budget constraint in Chapter 4, that consumption, C_t, is a fraction of the net output Q_t, spent on consumption and investments (through savings), $I_t = sY_t$.

$$Q_t = \Omega_t[1 - \Lambda_t]Y_t = C_t + I_t \tag{7.1}$$

The net output is defined as gross production, Y_t, minus abatement costs, Λ_t, and damage costs, Ω_t[13]. Both abatement costs and damage costs take up a fraction of the gross output, which again affects the consumption through the decrease of the net output. It is therefore relevant to look at how investments, damages and abatement costs are determined in the model and the dynamics that arise after modelling uncertainty in productivity growth. The DICE model is complex and the issue of computational tractability that applies to most of the integrated assessment models remains an issue, also for the model applied in this thesis.

Due to the many uncertainties and the challenging computational tractability of modelling through the DICE model, as with any IAM, we will mainly focus the time frame before year 2240. This is approximately the year till which Dietz et al. (2018) estimate their climate beta, as they employ the 2013 version of the DICE model, which terminated after 50 time periods. In the results, we will give an intuitive explanation of the values of the climate beta we obtain.

[13]Note that Ω_t is defined as one minus the damage fraction in the net output- equation, following the notation of Nordhaus and Sztorc (2013) and Nordhaus (2017b)

7.1.1 Benefits, B_t

Benefits are found through looking at the difference in consumption, that is, consumption with- and without the mitigation project. Those values, and thus the benefits, are in turn highly dependent on TFP growth. This is both the case when the uncertainty in productivity growth is modelled through the initial growth rate of TFP, and when it comes in the form of shocks to TFP. It will become evident later on that the benefits of the climate mitigation project are larger when the total factor productivity is higher. The intuition behind benefits increasing with productivity growth can be explained as follows. Increased productivity yields higher production, which leads to higher emissions, as explained in equation 5.9 from chapter 4. When emissions are higher, absolute damages are larger. When absolute damages are larger, the marginal damages increase, as a result of the convex damage function. A higher level of marginal damages, implies that our marginal mitigation project yields larger benefits.

The output variables show that the increased benefits are disproportional to the TFP growth. The increase in benefits for extreme values of g_0^A , that is $g_0^A \approx 15\%$ and above, is much larger than the increase in benefits we observe before this 'threshold' of initial growth in TFP is reached. We explain this disproportional increase in benefits of added consumption by means of the endogenous savings rate. The drastic drop in the savings rate leads to additional consumption units, which are part of the increase in benefits. Note that the positive effect of productivity growth is seen both when the faster productivity growth comes in through g_0^A and ε_t, as well as for both multiplicative and additive damages. We will now look at the dynamics behind the changes in consumption.

7.1.2 Investments, I_t

I_t is a fraction of the net output, and following the budget constraint which we can rewrite to $C_t = Q_t(1 - S_t) = Q_t - I_t$, the only way in which consumers in one time period can transfer consumption to another; namely through capital and capital growth. In the methodology of Dietz et al. (2018) the savings rate is assumed to be a constant and exogenously determined, around 23-24% of total GDP, which is consistent with historical data on consumer behavior (Nordhaus and Sztorc, 2013). Our findings from keeping the savings rate endogenous suggest that when the TFP growth rate is higher than a certain rate, around 12-15%, the savings rate drops a lot. This holds true both for when the uncertainty comes through ε_t and through g_0^A. The fast dropping savings rate in the early time periods, also hold true with some variation, for all the damage function-specifications. When the uncertainty is modelled through ε_t, the savings rate drops faster in the high TFP growth rate scenarios', compared to the case in which we model the same "uncertainty" through g_0^A. An intuitive explanation for the drastic decrease in investments (through the quickly declining savings rates), could be the following: According to the Cobb-Douglas production function, high productivity growth yields high GDP per capita. The model is deterministic, and when productivity growth is high, the current generations know that GDP per capita will be high also in the future. Technology increases so fast, that there is no need of saving up for future generations. They will be self-sustained and their welfare will be extremely high, regardless of the savings rate of today's generation. It is thereby most welfare-maximizing to consume all output today, without thinking about the welfare of future generations. Technology is growing at such an extreme rate that the future generations will have access to technologies that today's generations cannot even dream of. In those scenarios, we can think of extreme technological developments that can remove the already existing emissions from the atmosphere. That would imply a higher upper limit of the so-called emissions control rate or emissions reduction rate μ_t

The drastic drop in savings rate does not remain throughout the almost 500 years of the model. Instead, the endogenous savings rate reverts back to its mean value of around 23-24% around year 2140. This happens when the fast productivity growth comes from TFP shocks ε_t and similarly around year 2235 when fast TFP growth happens through the initial growth rate g_0^A. The mechanisms behind this mean-reversion of the savings rate remain uncertain when employing the DICE model in our analysis. However, a possible explanation could be related to μ_t, the emissions reduction rate. The emissions reduction rate has an upper limit of 1 in the beginning years of the DICE model, which is set to 1.2 after year 2150. This holds in the standard version of Nordhaus' DICE model. When TFP grows fast, output increases and the upper limit of μ_t becomes binding in some of the scenarios of uncertainty either through g_0^A and/or ε_t. In that case, we can think of policy makers preferring to set an even higher upper limit for μ_t, as that would allow more emissions to be removed from the atmosphere. As mentioned, this is impossible in the DICE model, as the upper limit is fixed by policy makers and exogenously determined. In that case, the desire to control the carbon emissions, or to remove carbon from the atmosphere might be pursued through other mechanism within the DICE model, for instance the endogenous savings rate. However, our observations of the emission control rates over time, do not coincide with this intuition. Counter-intuitively we do *not* observe that the emissions control rate becomes binding *earlier* in time for the high than the low growth rate scenarios, when uncertainty is modelled through shocks in TFP. The exact mechanisms behind these results thus remain an unanswered question. These result could possibly arise due to values in the variables and alterations that causes computational problems within the DICE model.

7.1.3 Damage Fraction, Ω_t

In this thesis, we compare the different types of damage functions when analyzing the climate beta's response, to different uncertain parameters in the growth rate of productivity. To understand why this is a crucial point and how the damages affect consumption, we now move from the budget constraint, over to gross output Q_t . We analyze how the net output, from where consumption is derived, is reduced by damages. Please recall the definition of the damage *fraction* 5.1, $\Omega_t = \frac{D_t}{1+D_t}$ where damages' increase is proportional to temperature increase. Here, Ω_t is defined as one minus this damage fraction when it enters the function for net output (Nordhaus and Sztorc, 2013)(Nordhaus, 2017b). We kindly remind the reader to recall the altering of the damage function from the previous chapter, and be noted that whenever we refer to damages, D_t, the altered damage-function 8.2 is implied, rather than the quadratic damage function that is used by Nordhaus in the latest two versions of the DICE model. Nordhaus and

Sztorc (2013) Nordhaus (2017b). The way the damage function is specified has a large effect on how the emissions abatement project affects consumption. As the damage function is specified to increase damages proportionally to temperature, it should be noted that there are several uncertain parameters in the damage function that might be of interest. It is however, beyond the scope of this thesis to analyze the implications of the uncertainties of other variables in the damage function, that is, other than the income elasticity of damages, ξ. We will now explain the difference between the multiplicative and the additive damage function and the intuitive explanation behind the effects of these two types of specifications.

7.1.3.1 Multiplicative Damage Function

In a purely multiplicative damage function, the income elasticity of damages with respect to consumption, ξ, equals one. This specification implies that the damages are always proportional to a fraction of the gross output. Using a multiplicative damage function can be seen as a build-in mechanism towards a positive estimate for β,(Dietz et al., 2018), which is confirmed by our results in the next chapter. This mechanism is explained below. Note that the following explanations are meant to give an intuitive understanding, and we therefore skip the time-subscript. It should also be noted that the letters used to describe damages and output, although largely coinciding to the variables used in the DICE model and to explain the methodology, small differences appear. We can define a class of multiplicative damage function as follows:

$$q(Y, D) = Y(1 - D) \tag{7.2}$$

Where D, the damage index, is expressed as a fraction. The absolute level of damages is obtained by multiplying the damage fraction with gross output Y. This implies that post-damage aggregate output is modelled as $Q = Y_{NET} = Y - YD$. When damages of climate change to the economy are modelled in this way, it is not surprising to estimate $\beta > 1$. Intuitively, the rise of a positive beta from this type of damage function arises as follows: The only source of uncertainty entering the model comes through productivity growth. Faster technological growth, leads to increased production, which in turn leads to higher emissions. An increase in both the absolute level of damages (via the direct effect of gross output) and in marginal damages (through emissions and temperature increases) follows. Hence, the benefits from emissions abatement will be higher in periods with high TFP growth, i.e. in good states of the world when output is high. We have thus seen that making use of a multiplicative damage function gives almost automatically rise to an estimate of β larger than zero.

7.1.3.2 Additive Damage Function

When damages are ***not*** proportional to gross output, the damage-output **ratio** decreases when gross output increases. The absolute level of damages is here modelled as a fraction of the initial world output in the start of our model, namely 2015. Hence, the class of additive damage functions can be described by the general form

$$q(Y, D) = Y - D \tag{7.3}$$

In order to compare the underlying dynamics of the damages, we start with comparing the industrial emissions from the additive case with the multiplicative case. Industrial emissions, coming from production affect the global average temperature and create thereby damages imposed onto the economy. An additive damage function causes a relatively higher level of industrial emissions to accumulate in the atmosphere over time, due to a higher level of gross output over time, compared to the multiplicative damage function scenario. What are the mechanics behind this? When production and therefore gross output Q_t increases over time, but the damages do not follow proportionally, then the ratio of damages relative to output will decrease. Intuitively, when damages are not proportional to gross output, producers can emit a lot of greenhouse gases by means of increasing their level of production, without this higher level of production resulting in increased total damages. In fact, scenarios with relatively faster initial TFP growth, lead to higher output growth, which decreases the damage-output ratio in case the functional form of the damage function is as in 7.3. This is one of the ideas underlying the so-called Schelling conjecture, stating that "developing countries' best defence against climate change may be their own continues developments". (Anthoff,D., Tol, 2012) Vice verse, in the case of a multiplicative damage function, output ***net of damages*** , Q_t gets more quickly 'destroyed' by the industrial emissions which are coming from carbon-intensive production. Since the damages from industrial emissions are proportional to the level of gross output, this variable will decline faster in case of multiplicative damages compared to additive damages. Consequently, this leads to lower levels of investments, which in turn affects the capital stock. Therefore, an additive damage function as opposed to a multiplicative damage function results in the remaining level of gross output in the economy being higher for a longer period over time.

7.1.4 Capital, K_t

Capital stock, K_t increase through investments I_t, which are increasing in output and decrease in abatement costs and damages. As capital enters the next period's production function, it serves two contradicting purposes for the determination of consumption; Whereas the increase of next years consumption increase in investments via the Cobb-Douglas production function, today's consumption decreases in investments, as it moves resources from today and to tomorrow via capital stock.

7.1.5 Abatement Costs, Λ_t

Abatement costs is the second variable through which net output, and thereby consumption, decreases. As shown in section 4.2.7.2, abatement costs are a function of the adjusted cost for backstop technology, and the emissions control rate, sometimes referred to as the emissions reduction rate, μ_t. This emissions control rate is in turn raised to the power of a parameter, θ_2 , which is set to be 2.6. Recall from section 4.2.7.2 that because the exponent of the emissions control rate is above one, every unit of CO_2 that is abated becomes more expensive. The functional form of the abatement cost function, thus tells us that marginal abatement costs are increasing in emissions. Implicitly we see that consumption indirectly decreases in emissions through the abatement costs. This function also renders it apparent that an increase in the emissions reductions rate, set by policy makers, comes with costs, as the costs of abatement increase in μ. In year 2240 $\mu_t = 1.2$ and becomes binding on average (i.e. when analyzing the average of the Monte Carlo simulated scenarios). μ can be viewed as current investments to avoid future damages, and the upper limit of 1.2 implies that the policy makers cannot demand an emissions reduction of more than 120% of total output, regardless of the state of the world and the rate that would have been optimal. Another way to look at this mechanism, seeing that μ is the mean through which policy makers can affect the climate, is that the stricter climate policies the world's leaders have to enforce, the lower today's consumption will be. Intuitively this reasoning holds, as drastic emissions reductions policies come with large costs to society, such as restructuring of infrastructure, which again will decrease the consumption of the current generations. Relating this to the abatement project, it becomes apparent that as lower emissions occur relative to without the mitigation project, less strict, and thus cheaper, policies have to be implemented. This yields lower abatement costs and thus higher net output used for added consumption (yielding benefits) and savings.

In summary, we have seen that consumption is determined by the budget constraint, implying that net output and investments in capital are key variables in the determination of consumption and consequently the beta. We have seen that, consumption decreases in abatement costs and damages and that these variables largely depend on industrial emissions coming from production and the need to control for emissions. Additionally, we have seen that capital investments play a dual role in the determination of consumption, and thereby the benefits of climate change mitigation.

In the next section we will assess the expected effect of the abatement project on selected output variables of the DICE model. Reminding the reader that a mitigation project in our analysis is defined as a reduction of one unit of ***initial*** industrial emissions, e_0, that is in the year ***2015***, measured in units of gigaton of Carbon dioxide, $GtCO_2$.

7.2 The Effect of a Marginal Climate Mitigation Project

By decreasing the parameter e_0 as our emissions mitigation project, we do not only affect the industrial emissions in one year. Instead, we affect the carbon intensity in 2010 (σ_0), which affects the CO_2-equivalent emissions output ratio σ_t and thereby the industrial emissions equation 6.4, as explained in our methodology. The industrial emissions affect the carbon cycle, which consequently affects the atmospheric temperature and ultimately the damages. The emission-output ratio σ_t simultaneously affects the adjusted cost for backstop technology, i.e the cost at which a (future) technology is 100% carbon neutral, or a zero-carbon energy technology is developed(Nordhaus and Sztorc, 2013). A decrease in the price of this technology leads to a decrease in the abatement costs, which affects output net of damages, abatement costs, and consequently consumption and investments. From the analysis of the effect of our mitigation project below, it is reasonable to assume our project represents only a *marginal* investment in a climate change mitigation project. We show however that the effect of an abatement project is reliant on the initial rate of total factor productivity growth, g_0^A.

7.2.1 Expected Values of Benefits and Consumption

Before digging into how the mitigation project affects the different output variables, we will look at the expected values of consumption and the expected benefits. These are calculated as the mean of the output variables, resulting from the different draws of the Monte Carlo simulations (MC) of the growth in initial productivity and shocks to productivity respectively:

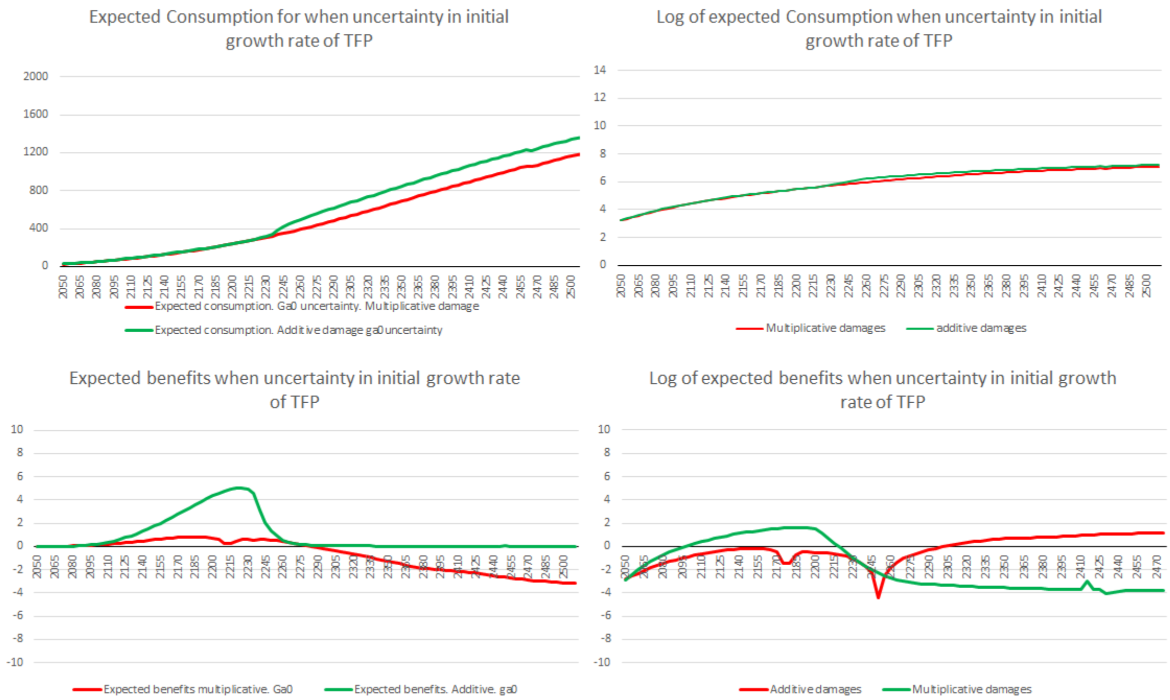
$$\overline{V}_{t,i} = \sum_{t=2015}^{T} [V_{t,i} - V_{t,i}^{REF}] \ , \quad t = 2015, 2020, ..., 2510 \quad i = MC_1, MC_2, ..., MC_{1000} \tag{7.4}$$

It should be noted that when the expected consumption is described, the reference case is used, i.e we show the expected value of consumption in the case that no mitigation project is undertaken.

7.2.1.1 Uncertainty in g_0^A

The graphs below show the expected values - and the log of expected values of consumption and benefits respectively. This gives an indication on how the climate beta might move over time.

Figure 7.1: Expected values of Consumption, log of consumption, Benefits and Log of Benefits over time when uncertainty is in g_0^A



7.2.1.2 expectations of β

We expect to find positive values for β , in case a multiplicative damage function is employed, based on the paths of both expected benefits and expected consumption over time shown in the previous chapter and based on the estimations from Dietz et al. (2018).

Expected Consumption

Consumption seems to grow in a similar manner until around year 2270, when we observe a jump in consumption in the case with additive damages. Recall that consumption is a fraction of net output. Assuming that the savings rate remains reasonably similar in the cases of additive and multiplicative damages, this jump in consumption stems from abatement costs or damage costs, following that consumption is decided by $C_t = Y_t - \Omega_t - \Lambda_t$. In the additive case we can expect the absolute damages to be relatively lower than for the multiplicative case. It could however also be that this jump is observed as a consequence of the emissions control rate being binding for one or the other case. To have a better understanding of the growth rate of the expected consumption, we take the natural logarithm and find aggregate consumption to grow at a positive, but declining rate. Hence, also in the log-´case of consumption, the income elasticity of damages seems to have only a minor effect.

Expected Benefits

The expected benefits of consumption from the mitigation project are, unlike the expected consumption, not as similar in the additive and the multiplicative case, implying that income elasticity of damages has a significant effect on the benefits of a mitigation project. To understand the mechanisms behind this result we refer to the explanation of the damage functions earlier on in this chapter. When damages are additive, an increase in gross output only affects the damages indirectly through increased industrial emissions and thereby a temperature increase. In the additive case benefits build up over time because the damage-output ratio decreases. When the damage-output ratio is lowered, we get more consumption units per unit of production, leading the benefits to increase with production. The benefits decrease drastically in year 2240 in the case of *additive damages*. We believe that the drop in benefits are a consequence of the increase in the emissions control rate-limit, $\mu_t = 1.2$. This is, implemented in 2240, as a feature of the baseline case of the DICE model. As the output in the different damage-function cases are not equal, we expect μ_t to be binding for the additive case in the year 2240, but not for the multiplicative case. When the emissions control rate suddenly reaches this upper limit for the additive case, so will the abatement costs. Consequently we will observe a sharp decrease in *net* output, and hence smaller benefits of added consumption from the emissions mitigation project. The natural logarithm of the expected benefits provide us with an approximation of the change in expected benefits over time, as the benefits in levels are only small and so are the changes in levels over time. The natural logarithm of the benefits are also used in our definition of the estimation of β_t by using DICE, that is why we compare the pattern of natural logarithm of expected benefits with natural logarithm of expected consumption. There exist a similar pattern in the paths of the two over time, both moving as a concave-shaped increasing curve. This implies a positive growth rate in both expected benefits and expected consumption over time. The concave upward-sloping pattern holds true for the log of benefits for the multiplicative damage function. In the additive case however the expected benefits decrease around the year 2240, as explained earlier on in this paragraph. Naturally this pattern in the expected value of the benefits, is accompanied by negative values for the natural logarithm of the expected benefits in those years. One might expect the negative co-movement of expected consumption and expected benefits from 2240 onward for the additive damage function to result in a negative value for the climate beta. However, the expected values of these variables, only play a partial role in the numerator of the estimation of β_t. In our results in the next chapter, it turns out that the variance within the natural logarithm of consumption, that appears in the denominator of the definition of β_t, plays a significant role in determining its value.

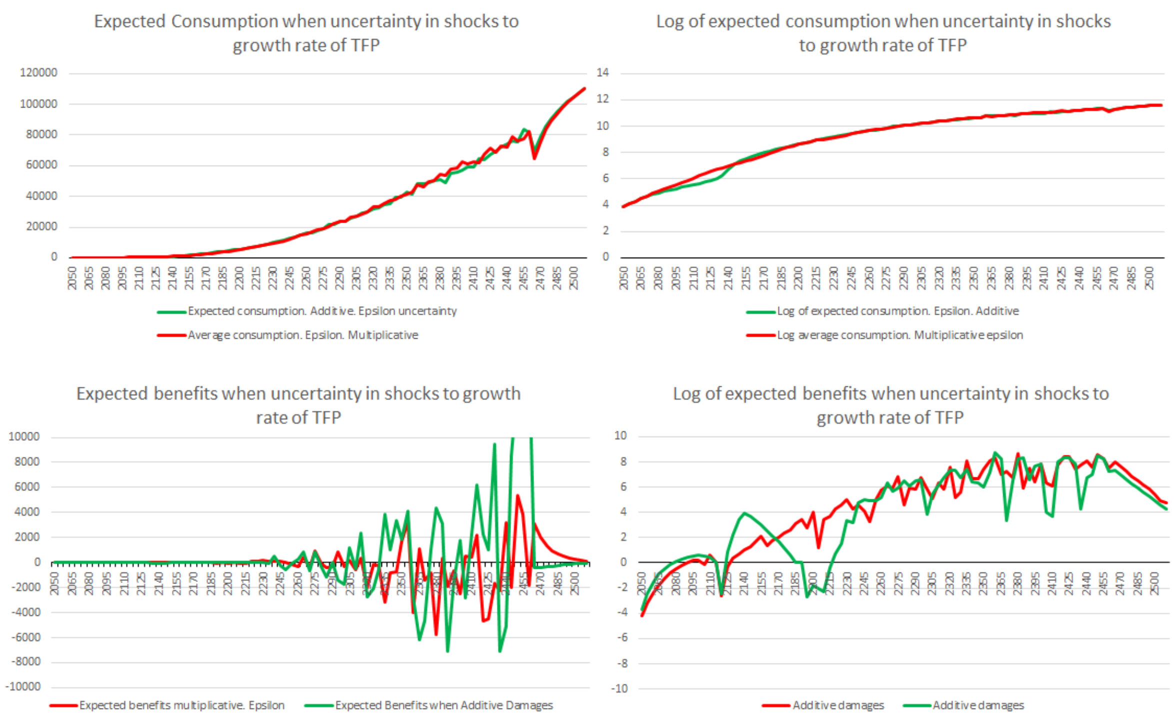
7.2.1.3

Uncertainty in ε_t When the uncertainty in the productivity growth comes as i.i.d shocks throughout the 500 years, with a positive mean drawn from a i.i.d normally distributed MC sampling, the expected consumption and benefits are shown below.

7.2.1.4 Expectations of β

We expect to find positive values for β , in case a multiplicative damage function is employed, based on the paths of both expected benefits and expected consumption over time shown in the previous chapter and based on the Dietz et al. (2018)'s estimations.

Figure 7.2: Expected values of Consumption, Log of consumption, Benefits and Log of Benefits over time when uncertainty is in ε_t



Expected Consumption

The expected consumption when uncertainty is modelled through positive shocks increases exponentially, similar to the case of uncertainty in the initial growth rate of TFP. The values on the y-axis are however extremely different, which is in line with what is expected from this way of modelling of uncertainty, i.e. to find extreme values for future consumption and output. This is due to the relatively high mean of ε_t in our Monte Carlo simulations. Not only do we observe the effects of an initial shock being moderately persistent over time, but we observe new positive shocks to the productivity growth of the economy in every time period. To see the change over time in the expected consumption levels, we take the natural logarithm and find, as in the case of g_0^A uncertainty, the curve is concave and upward sloping, indicating positive exponential growth in the level of expected consumption over time.

Expected Benefits

Moving on to the benefits, we find an interesting pattern after the year 2240. The expected benefits remain relatively small until the year 2240. Thereafter, the expected benefits become large and extremely volatile. To better observe the movement of the expected benefits over time, we look at the natural logarithm. The expected benefits in case of the additive damage function is more volatile compared to the multiplicative damage function, i.e. larger extreme absolute values are observed over the second half of the analyzed time period. The reason behind this volatile pattern might be similar to the reasoning provided earlier on, to why additive damages should yield higher expected benefits than multiplicative damages, when the uncertainty was modeled through g_0^A. An intuitive explanation to why the expected benefits become so volatile from the year 2240 onward, might be found in the emissions reductions rate μ_t. After having experienced positive productivity shocks every five years for more than 200 years the level of technological development is extreme in the year 2240 and thereafter. Society has been producing and consuming at extreme high levels, resulting in similarly high levels of industrial emissions. The upper limit of μ_t had already been set to 1.2 in the year 2150, but in 2240 it appears to become binding. When the upper limit of emissions reductions becomes binding, the abatement costs increase, decreasing net output. Consequently, the gained consumption from the mitigation projects decrease drastically. An additional effect which yields a decrease in consumption is the savings rate; as costly policies are implemented, future consumption growth decrease. To smooth consumption over time, agents (valuing their dynasty) find it optimal to drastically increase their savings rate. Combining these two effect, results in benefits. Over time, the higher savings rate (fluctuating around its mean value of 0.23-0.24) leads to capital growth, resulting in GDP growth over time. The agent who optimized her welfare and that of her dynasty, forsees that future generations will be much better off again. This results in the agent to spend all her income on herself, knowing that her descendants and other members of her dynasty will be as well off as herself, due to the extreme productivity growth. Hence the benefits of the mitigation project become highly positive until the same procedure repeats itself. This story provides in our opinion an intuitive reasoning behind the pattern of the endogenous savings rate over time, which becomes highly volatile after 2240.

7.3 The effect of uncertainty modelled through g_0^A

The normal distribution underlying our Monte Carlo (MC) simulations for modelling uncertainty in g_0^A have a mean of 2.06% and standard deviation of 1.12% per year, based on the latest long-run economic growth per capita forecasts. (Christensen et al., 2018). We adjusted these numbers to 5 year periods, consistent with the definition of g_0^A in the DICE Model, specifying the initial growth rate of TFP per 5 years. Recall that every time period in the DICE model consist of 5 years. Consequently, the mean underlying the normal distribution for the MC simulations becomes 10.73% and the standard deviation 2.504%.

Before looking at the effect of uncertainty around g_0^A on the climate β , we first attempt to shed light on the pure effect of modelling g_0^A as the main source of uncertainty on the key output variables within DICE model. This is done by showing and verbally explaining a representative selection of the scenarios we modelled, coming from our MC simulations. Similarly, we highlight the "outliers" in the uncertain scenarios, as well as the mean scenario, i.e. the one where $g_0^A = 10.73\%$.

7.3.0.1 Output

Naturally, faster TFP growth increases output. When applying the **multiplicative damage** function, the mean case, where $g_0^A = 10.73\%$, output ranges from approximately 105 trillions USD in 2015 to approximately 14.096 trillions USD in 2510. Observing the extreme cases of the draws from the Monte Carlo simulations, we see that a very low initial TFP growth of $g_0^A = 2\%$, ranges from 105 in the initial time period, to 412 in the final year. For the extremely high values of g_0^A, ($g_0^A = 17\%$) the effect is much larger and we observe an increase from 105 to 241 701 from 2015 to 2510. Thus, the difference in g_0^A over 100 years in the multiplicative case amount to a very large, and increasing variance over time, in the level of gross output.

Additive damages yield a similar structure of gross output as the multiplicative one. In fact, the increase in consumption units amounts from 105 in 2015 to 432, 16.259 and 241.665 trillions USD for the low, average and high cases of g_0^A, respectively. The only notable difference in the two graphs is the time at which the exponential effect of the function kicks in, and especially for the high initial growth rate, which is approximately a decade earlier in the multiplicative case than in the additive case. To more clearly notice the movement of the output, we look at the natural logarithms of the same scenarios, and we observe that the growth in the additive and the multiplicative cases are very similar both in size and movement.

Consumption is estimated as fraction of (net) output and, as can be seen in the appendix, is therefore very similar to the gross output figures. The exponential growth of the output, and thus consumption, thus explains the increasing heteroskedastic variance of the log of consumption used to estimate the climate beta.

7.3.0.2 Savings rate

An important deciding factor of consumption in economic theory is the savings rate(The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018) Romer (2012), and it is also a key point in where our paper departs from Dietz et al. (2018)´s measure of the climate beta. Dietz et al. (2018) assume an exogenous savings rate, though similar to the one deriving from the DICE model. The exogenous savings rate is namely set based on a time-series coming from the endogenous s_t in Nordhaus' original DICE 2013 model. Contrary, we employ the endogenous, optimal savings rate, as applied in the original DICE model by Nordhaus. Here, growth rates of consumption, capital and output- per capita converge to the same growth rate and the savings rate, $s_t = 1 - \frac{c_t}{y_t}$. Over time, this savings rate converges to a constant, similar to the one constant in the Solow model, but more importantly, similar to the ones observed in the market(The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel, 2018)(Nordhaus and Sztorc, 2013). For the estimations of Nordhaus, the savings rates are relatively constant around 23 - 24%. The savings rates in our estimations, however seem to be strongly affected by the initial growth rate of TFP, particularly in the cases of high values for g_0^A.

Both in the multiplicative, and the additive case, the marginal propensity to save drops drastically around year 2080 and increases equally drastically around year 2440 for several of the scenarios (for several of the ga_0-values). A common trait for the strongly decreasing savings rates is that the level of the initial TFP growth rate, g_0^A appears to be above a certain threshold. Above that threshold level of initial TFP growth, there is a disproportional drop in the marginal propensity to save. Intuitively this pattern can be thought of as follows; representative agents decrease their savings when they expect to have a very high level of future income, coming from the extremely fast technological development. The natural consequence of the decrease in savings is that consumption over time is relatively large for those high-growth scenarios, compared to the lower scenarios. The reason to why the savings rate from the case with multiplicative damages are slightly higher than for the additive case may come from the mechanism that a lower output net of damages will occur in the multiplicative case relative to the additive case. In the additive case, per (marginal) unit of output, the consumer can consume relatively more than in the multiplicative case. The marginal units of consumption increase in the additive case. This will count also for the future generations of the dynasty, and it will therefor be welfare optimizing to consume more now and save less for future generations, as they will live in a world with extreme amounts of consumption.

From this section we have seen that the initial growth rate of TFP affects output, consumption, savings and damages. We have seen that the effects of a marginal abatement project are larger for scenarios in which a high g_0^A is inherent in the economy, than for lower values of the same parameter. Relating this to the concept of the climate β, we see that the variance of consumption over time, thus also increases in g_0^A, hence increasing the denominator of the climate beta. However, as the variance increase, the benefits increase more, implying a positive covariance between consumption and benefits, and consequently positive beta. We also see that there seem to be a threshold above which the growth rate leads to a "jump"

7.4 The effect of uncertainty modelled through ε_t

Intuitively, as uncertainty in ε_t yields positive productivity shocks throughout the entire time period, we expect the values in this case to be significantly higher than those seen in the previous section. Similar to when uncertainty comes from g_0^A, we expect the variance to be heteroskedastic, as the effects of difference in TFP growth manifests themselves over time. We will now look at the effects of different values of ε_t on different key output variables. We hereby refer to the graphs in the Appendix B.

7.4.0.1 Consumption

When uncertainty is modelled through ε_t, consumption and gross output increase exponentially. It is worth to notice here that the scales of the graphs depicting these output variables, are much larger than when the uncertainty comes in TFP through g_0^A. The *patterns* of consumption and output look however similar to the case when uncertainty is modelled through g_0^A. The difference between the uncertainty through the two different parameters, besides the higher levels of consumption and output, is also a much hig her volatility, both over time and within each path of g_t^A. As expected, this indicates that when the uncertainty comes through shocks, the growth rate will be much higher than when uncertainty lies in the initial growth rate of TFP.

7.4.0.2 Damages

Visually, the fraction of damages in the additive and the multiplicative way are similar until around year 2180, when the additive damages remain constant at one level whereas the multiplicative damages decrease. We suspect that this relates to the emissions control rate, which is binding in the multiplicative case, but not in the additive case. In terms of absolute damages the multiplicative case has damages much larger than in the additive case. Intuitively this makes sense, as the damages in the additive case are increasing in temperature, (which is affected by emissions from production output), but multiplicative damages are multiplied by the factor, Y_0. The multiplicative damages, increase proportional to output, which is varying over time. When output is highest the damages will be highest too.

7.4.0.3 Savings

Comparing the savings rates between the two different types of modelling uncertainty in TFP, we notice that the savings rate in case of uncertainty through ε_t drops more quickly than in the case if uncertainty in g_0^A. On the other hand, the savings rate also reverts back to its mean value quicker in case of uncertainty through TFP shocks rather than uncertainty in the initial growth rate of TFP. This aligns with our previously explained intuition. When productivity increase to the extreme values, which it does much faster in the case of ε_t-uncertainty, the current generations assume that future generations will have a technology-level beyond imagination. There is therefore no need of saving to transfer capital intertemporally. After around 50-100 years however, the emissions control rate binds at 1, and thereafter at 1.2, as explained above. This leads to a sharp decrease in net output, causing the savings rate to increase, in order to smooth consumption. Representative agents realize they need to reallocate welfare from the current to the the future states. This effects occurs in around 25 years earlier for the multiplicative case than for the additive. Please, however, be noted that the volatility of the savings rate is larger in the cases of high growth rates, and that it remains stable around 23-24% for low growth rates.

We have now seen the mechanisms behind the variables determining the climate beta. The main takeaways are that the effects of the abatement projects are larger, the larger the productivity growth. We have also seen that the way we have modelled uncertainty, the productivity growth is notably larger in the cases of uncertainty in the form of shocks that in the initial growth rate. Lastly we have seen that as the effects on the variables increase, so does the benefits and the consumption. This co-movement pulls the value of the climate beta upwards. However, at the same time, the variance, i.e the uncertainty in possible outcomes, increase over time. In turn, this will increase the value of the denominator, over time, leading to a decreasing beta over time.

8 Results

In this chapter we present our findings of the climate beta we obtained through the methodology and analysis explained in the two previous chapters. Throughout the chapters we will refer to graphs in order to get a better visual understanding of our results. In the graphs the green line implies an additive damage function and the red line implies a multiplicative damage function. The grey lines observed in the graphs represent a semi-multiplicative damage function, which will be commented on in the end of the chapter.

Using the 1000 Monte Carlos simulations as the main source of uncertainty, we calculate β_t based on a marginal climate-mitigation investment project, as explained in the previous chapter. We plot our results, the estimates of β_t, as a function over time. We show the results for the modelling of uncertainty in total factor productivity. To remind the reader about the equation of motion for the total factor productivity A_t:

$$g_t^A = \left[(1-\psi)g_0^A + \psi g_{t-1}^A + \varepsilon_t\right]\left(1+\delta^A\right)^{-t} \tag{8.1}$$

From the equation above it becomes clear that we had to decide on how to model uncertainty within the growth rate of TFP. We decided to analyze both uncertainty through the initial growth rate of TFP, and through i.i.d. normally distributed shocks, as explained in the previous two chapters. The first subsection in this chapter will show the results when uncertainty comes in through the parameter g_0^A. In the second subsection, the main source of uncertainty comes in through ε_t.

The other parameters in the DICE model are fixed, meaning they are set to their value as in the standard DICE code. That holds for all other parameters, except for e_0, the industrial emissions in 2015, measured in $GtCO_2$. This is the parameter used to implement our marginal climate mitigation project. The implications of this marginal abatement project for the most important variables that affect our results of the β_t estimation, have been provided in the previous chapter. Note that our results are based on an alteration of the equation for the TFP growth rate, as specified in 8.1. This enables us to distinguish between two different parameters when modelling uncertainty in TFP growth. In addition, we used the altered version of the damage function 6.12 as explained in the methodology chapter. Thereby, we account for convex damages rather than using Nordhaus' original quadratic form of the function. Within the modelling of uncertainty through each of the two parameters, g_0^A and ε_t separately, we consider uncertainty in the functional form of the damages. That is, we provide our results based on a multiplicative and on an additive damage function. Recall our general specification for the damage function:

$$D_t = Y_t\left[1 - \frac{1}{1+\alpha_1 T_t + \alpha_2 T_t^2 + (\alpha_3 T_t)^7}\right]\left(\frac{Y_t}{Y_0}\right)^{\xi-1} \tag{8.2}$$

As explained earlier, a multiplicative damage function implies $\xi = 1$, additive implies $\xi = 0$, and we define semi-multiplicative by $\xi = 0.5$. A more extensive explanation of the various types of damage functions and their implications for the relevant output variables can be found in the previous chapter.

We will now show our results for the overall term structure of the climate beta, first based on modelling uncertainty solely in the initial growth rate of TFP, g_0^A. Thereafter, a subsection is devoted to show β_t based on modelling uncertainty solely in TFP shocks, ε_t. To remind the reader, our expectations regarding beta will be provided and the figures will show the estimated term structure of beta. We will reflect upon our expectations and discuss the meaning of our results. Implications of the results will be provided but will be discussed more extensively in the next chapter. For a deeper explanation of the underlying mechanics to our results, for instance the effect of the three different damage functions, we kindly refer to the previous chapter. Finally, we will briefly comment upon the estimated beta for a semi-multiplicative damage function, when the main source of uncertainty is the initial growth rate of TFP.

Regardless of the parameter through which we model uncertainty, for the first 7 time periods, that is is up until approximately 2050, we observe unreasonable values for beta. Our belief is that the unreasonable values arise from the extremely low variance in consumption in those time periods. Low variance in the reference case consumption over time, leads to an over-sensitivity of beta to the covariance between consumption and benefits. According to Dietz et al. (2018) this issue can arise when there is only one source of uncertainty on which the climate beta hinges. This type of over-sensitivity can also be seen as consequence from the way the DICE model is built. The cumulative effects of an increase in TFP, or a faster growth of TFP cannot immediately be seen. This is due to lagged effects in both the economic- and the climate-equations. As time passes, the effects of the productivity growth rates are ratified, as well as its effect on the output variables. It is thus reasonable to assume that the first time- periods yield unrealistically high values of the climate beta, due to the low variance in the natural logarithm of consumption. Consequently, we disregard the beta for the years up until 2050.

8.1 When the Main Source of Uncertainty is Modelled through the Initial Rate of TFP Growth, g_0^A

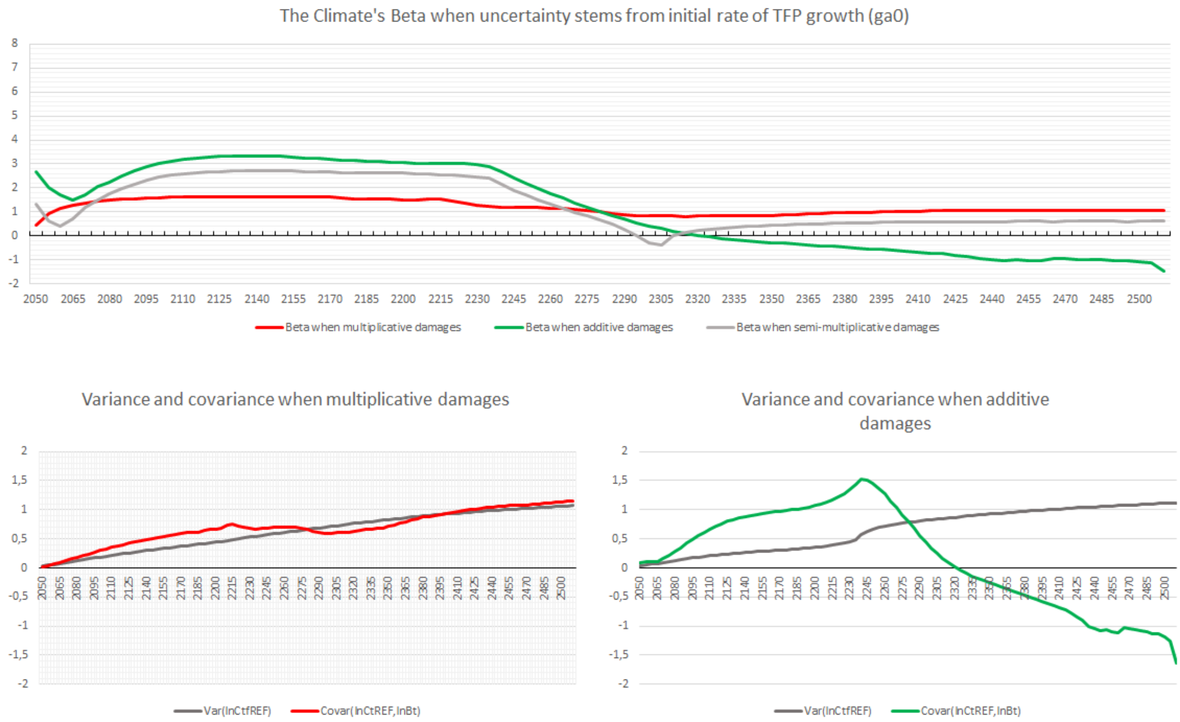
The figure below shows the term structure of beta, when the main source of uncertainty is the initial growth rate of TFP, g_0^A.

Note that in the results displayed in figure 8.1, the random shock ε in 8.3 is set to have both a mean and a standard deviation of zero. This is done in order to get a more clear picture of the pure effect of uncertainty in g_t^A, modelled solely through g_0^A.

8.1.0.1 Expectations of β

We expect to find positive values for β , in case a multiplicative damage function is employed. Our expectation are based on the paths of both expected benefits and expected consumption over time, shown

Figure 8.1: The Climate's Beta when uncertainty is in g_0^A



in the previous chapter and based on the Dietz et al. (2018)'s analytical simple climate model outlined in chapter 3.

The intuition behind these expectations is as follows. When TFP grows faster, the increase in output is larger. This increased level of carbon-generating production, leads to increased emissions. The emissions, through the carbon-climate system, increase the absolute amount of damages. Because the damage-function is convex, the marginal damages increase as well when the absolute level of damages is higher. Benefits are defined as increased consumption resulting from the mitigation project; as the marginal damages increase, so do the benefits of a marginal climate mitigation projects. In other words there is a positive correlation between consumption, which increases with higher productivity, and the benefits of abatement, which increase in emissions from production.

In case of an additive damage function, from our analysis in the previous chapter, we expect the benefits of the climate mitigation project to be different compared to the multiplicative case. In the multiplicative case, the benefits and consumption display a similar pattern over time, whereas that is not the case for when using an additive damage function. With an additive damage function, it becomes more likely to obtain a negative value for beta. When damages are not defined to be proportional to output, benefits and consumption are less likely to move in a similar pattern.

The term structures for β_t based on a multiplicative and an additive form of the damage function in DICE are displayed in figure 8.1. For most of the future, especially in the main years of our interest (2050 - 2300), beta has a relatively large positive value. In fact, its value is higher than 1, the value that was assumed by Nordhaus. Nordhaus and Sztorc (2013) The only source of uncertainty is modelled through the initial growth rate of TFP in 8.1; faster TFP growth leads to higher gross output, which in turn positively affects consumption and production. Therefore we expected to find positive values of β_t, as explained above. Note that faster TFP growth is modelled through the underlying distribution of our MC simulations, which have a mean of 10.73% per 5 years, while the original DICE code assumes $g_0^A = 7.6\%$.

8.1.0.2 Multiplicative Damage Function

The covariance between lnC_t^{REF} and lnB_t moves gradually along with the variance in lnC_t^{REF}, giving rise to an estimate for β between 1 and 2 for the majority of its term structure. These values are conform to our expectations based on the explanations provided above. When damages are multiplicative, meaning absolute climate damages increase proportional to gross output, output and consumption increase, hence absolute damages increase. Due to a convex damage function, marginal damages increase as well. This implies that climate mitigation mostly pays-off in states of the world where aggregate consumption is high, resulting in $\beta_t > 0$. The value of the climate beta lies around 1.2 for the first half of the time period, and then decrease slowly. Our findings for the climate beta when damages are multiplicative, are thus in line with the assumptions from Nordhaus and Sztorc (2013). This confirms the indications made by Dietz et al. (2018) and Gollier (2013), that a multiplicative damage function serves as a built-in mechanism towards a positive climate beta, when the main source of uncertainty is exogenous technological progress.

8.1.0.3 Additive Damage Function

Damages are **not proportional** to output, when the damage function is additive instead of multiplicative, resulting in two separate effects. On the one hand, the benefits of the climate mitigation project accumulate over time because damages are not **directly** increasing in TFP growth. On the other hand, due to the relatively fast TFP growth that we model, output increases, leading to more emissions, thereby higher temperature, and a larger damage fraction later in time. The damage function is convex, just as in the multiplicative case, so ***marginal*** damages still increase over time, when faster TFP growth leads to higher output levels. This convexity effect makes the benefits of a marginal mitigation project increase as well. This can explain the positive value for β_t for half of its time structure. Simultaneously, the plots of the variance in lnC_t^{REF} and covariance between consumption and benefits show an increasing difference between the two, indicating that the positive value of beta might partly be due to low variance in consumption.

The Climate beta for additive damages is above 3 before it starts decreasing around year 2230. This is much higher than expected. From the variance and covariance graph, it becomes clear that the covariance increase much more than the variance. The high covariance, seem to give an explanation of the high beta. However, the covariance between lnC_t^{REF} and lnB_t becomes negative around the year 2315, in turn leading β_t to also become negative. We also saw, from the analysis of expected benefits in the previous chapter, that expected benefits decrease drastically in the same years. This drop in expected benefits indicate that the decrease in the covariance results from benefits decreasing.

8.1.0.4 Implications

What are the implications of the positive climate beta over time for both the multiplicative and additive damage function? For the majority of the years of interest, beta is not only positive but also larger than one. This implies that the benefits of climate mitigation increase by more than 1% when aggregate consumption increases by 1%. These results indicate a positive correlation between the risk structure of climate mitigation investment projects, and the risk structure of aggregate consumption. Consequently, when the main source of uncertainty is related to exogenous total factor productivity growth, the discount rate used to evaluate climate change mitigation projects should be increased by the a risk premium, proportional to the positive climate beta. How large is this risk premium, that should be multiplied by beta and added to the safe discount rate ? This question will be discussed in the next chapter.

On the other hand, the distant future shows a negative beta for additive damages. Does that imply that a lower discount rate after the year 2300 should be used to discount back the cash flows of climate mitigation projects ? In general, the climate beta might decrease over time as it depends on the future levels of emissions and growth in output and consumption, which are more uncertain in the future. Another interpretation of this result might be that the decreasing beta for additive damages stems from the convexity of the damage function coupled with the concavity of the TFP growth. We imagine that the TFP growth is higher than the growth of damages for the first half the time periods. The benefits of climate mitigation increase more than-, and in the same direction as consumption. Around 2300, however, the damages have become so large that the benefits no longer grow faster than the increase in TFP. The covariance hence decreases, leading the beta to decrease too.

8.2 When the main source of uncertainty is modelled through shocks in TFP growth, ε_t

We kindly remind the reader that we assume the total factor productivity growth rate, g_t^A follows an AR(1) process:

$$g_t^A = \left[(1-\psi)g_0^A + \psi g_{t-1}^A + \varepsilon_t\right]\left(1+\delta^A\right)^{-t} \tag{8.3}$$

In order to interpret the pure effect of uncertainty in productivity growth on the climate beta, we have set the initial growth rate of TFP, g_0^A equal to 0. We used $\psi = 0.42$ and modelled uncertainty in the i.d.d. shocks to TFP, ε_t, which enables us to explicitly analyze and interpret its effect on the climate beta. Uncertainty is not only present at the start of the time period evaluated, as was the case when uncertainty entered g_0^A. It now comes in i.i.d shocks, with a mean drawn from a MC simulation $\sim$ N(0.1073, 0.0254^2), throughout the 500 years evaluated. This creates an even faster growth of TFP than in case of g_0^A uncertainty and we expect the climate beta to stay positive throughout the entire time period of interest. The underlying intuition for $\beta > 0$ is similar to the case where we model uncertainty through the initial growth rate of TFP.

Note that since the main source of uncertainty is modelled through Monte Carlo simulations, taken from a normal distribution, we would in theory allow for negative TFP shocks. However, as the mean is set so high, that it is more than four standard deviations away from zero, we consequently observe no negative shocks in our modelling of uncertainty through ε_t. On the other hand, when there are shocks observed with a relatively low positive mean (for instance a value of 2% for ε_t) an intuition similar but reversed to the one outlined above applies. Relatively low productivity shocks yield a relatively slower growing TFP, less increase in production, and thus a smaller increase in emissions. The consequence is lower benefits of climate abatement, in states of the world with lower levels of aggregate consumption and output, compared to the scenario explained above. Hence, these two negative affects combined yields again a positive correlation between benefits and consumption, implying $\beta > 0$.

We do not expect beta to become negative in the scenario of an additive damage function in combination with uncertainty modelled through TFP shocks, contrary to what we found in the previous section. As earlier explained, an additive damage function allows in theory for the possibility of estimating a value of $\beta < 0$. However, in the case of uncertainty through ε_t, we believe to have modelled a positive-bias in our results towards booming states of the world due to the high values for the mean and standard deviation underlying the Monte Carlo simulations of TFP shocks. As the uncertainty through TFP shocks come into play in the TFP growth rate every time-period, we expect the positive states of the world to occur more often than ending up in a "low state of the world". Therefore, we expected beta to be above zero throughout the entire time-period investigated. As the shocks hit the economy in every time period, we also expect the beta to fluctuate more than in the cases of g_0^A-uncertainty.

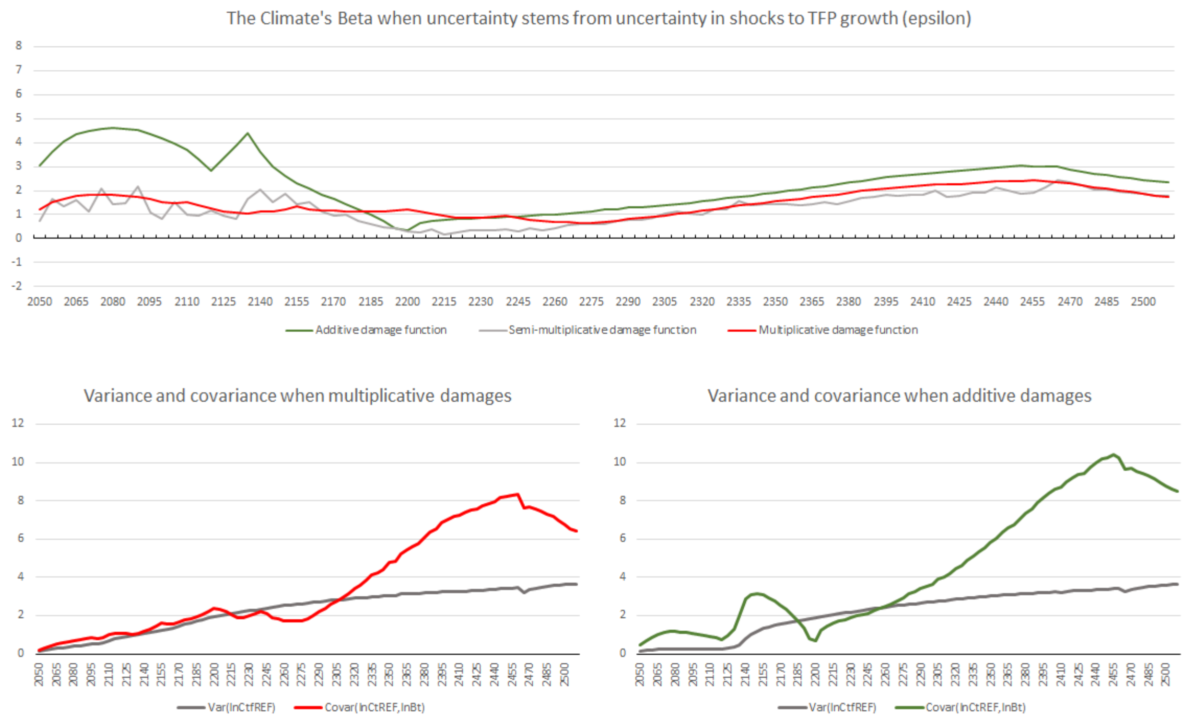
8.2.0.1 Implications

In line with our expectations, we do find a positive beta when the only source of uncertainty comes from ε_t, as can be seen from the figure above. This is the case for additive and multiplicative damages. The additive damage function yields a more positive beta than the semi-multiplicative and the multiplicative one. In the first half of the assessed time-period the **additive** damage function gives a significantly higher beta than the two other cases. From around year 2200 to 2510, that is the end of our time horizon, β_t seems to follow a similar and increasing trend, rising from approximately 0.3 to 1.7. In the multiplicative damage function case, β_t starts higher compared to the additive function case. That is, it starts at 1.23 in 2200, peaks at 2.4 in 2420 and reaches 1.76 in 2510. The additive damage function β_t starts around 0.3 approximately, peaks at 3.04 in 2465 and reaches a level of 2.3 in 2510.

8.2.0.2 Additive damage function

The high values for the climate beta are remarkable in the case when an additive damage unction is employed, for example a peak in 2080 where β_t = 4.6 and another peak yields a value of β_t= 3.0 in

Figure 8.2: The Climate's Beta when uncertainty is in ε_t



2450. These results imply that the climate mitigation project in these years pays off 3.0 to 4.6 times the "market risk premium of consumption of climate goods". This is unexpectedly high and much higher than what is assumed by Nordhaus and Sztorc (2013) and what was found by Dietz et al. (2018) From the figure, one can observe more years for which β_t is relatively large. These values may be explained by the difference between the covariance of consumption and benefits, compared to a relatively low variance in consumption. We observe a fluctuating covariance in the first 150 years, whereas the variance remain relatively steady. This can explain the peaks and drops of the climate beta in the first years. If we follow the intuition regarding the climate beta of an additive damage function outlined under 'Implications' in the previous section, the large movements of the beta may, at least partly, be explained from this.

The covariance between consumption and benefits shows an unexpected pattern around the years 2115 till 2195. Between those years, it first slightly drops, where after it strongly increases and in turn, drops relatively fast again.

8.2.1 Summary of the results

In summary, our results show a positive relationship between the risk structure of climate mitigation projects and systematic macroeconomic risk, when the main uncertainty comes from exogenous economic growth. Therefore, investments in climate mitigation are more likely to pay-off in states of the world where society's aggregate consumption level is high, and our marginal utility of one additional unit of consumption is relatively low. Our findings suggest that the climate beta is between one and two throughout almost the whole time period when damages are multiplicative. The findings for multiplicative damages seem to hold both when uncertainty comes through the initial growth rate of TFP and when the uncertainty comes from shocks to TFP. The beta for additive damages, when uncertainty in the TFP growth rate comes as shocks yields a high beta in the first 150 years before it decreases.

In the case of additive damages, the results differ between the two ways of modelling uncertainty. That is, when the uncertainty lies in the initial TFP growth rate, the high covariance between benefits and consumption yield a high beta for the first 200 years, approximately. The beta then decreases, and becomes negative around year 2300.

A high beta implies we do not want to pay as much for climate mitigation investments as we would want to pay for investments that can be used to hedge our (consumption) risk. Based on our results, uncertainty surrounding future economic growth, increases the discount rate that should be used when calculating the NPV of climate change mitigation projects. That is, the risk-adjusted discount rate. Our results suggest a positive risk-premium should be placed on top of the 'safe discount rate'.

8.2.2 Semi-multiplicative damage function

After observing the β over time for the two 'extreme cases', i.e. a fully multiplicative and a fully additive type of damage function, we hereby elaborate on the semi-multiplicative damage function. Namely, we believe that in the real world, climate damages affect the economy via some type of damage function of which the functional form lies in between the two previous cases. These results are derived by setting $\xi = 0.5$ in the function 8.2 The reader is referred figure 1 in this chapter, where the grey line represents the semi-multiplicative case, whereby uncertainty is modelled through g_0^A. Figure 1 shows the term structure of β. The multiplicative β moves in a similar pattern as the β when an additive damage function is employed, although starting and remaining slightly lower for more than half of the modelled time periods. Between the year 2050 and 2085, the semi-multiplicative β ranges between a value of 1 and 2. Thereafter, between 2090 and 2240, the value increases and ranges between 2 and 3. In year 2245 β is lower and keeps decreasing until it reaches a value of 0.28 in year 2290. We find remarkable values for β in the time period thereafter, as β becomes negative and decreases even further until its value is -3 in year 2310. A negative value for β is normally well possible, however, we observe an increase again and after 2310 β is positive until the end of its term structure. This relatively sudden drop to negative values, between the years 2290 and 2315 seem to come from a negative co-variance between lnB_t and lnC_t^{REF}. The remarkable fact is that this only happens for the above-mentioned few time periods where after β starts to increase and becomes positive again. What is behind these results? First of all, note that the semi-multiplicative damage function implies that damages are proportional to pre-damage output Y, although not on a one-to-one basis, as with fully multiplicative damage functions. The absolute damages will partly be determined by the uncertain level of future gross output and partly by the certain initial level of gross output in year 2015. We can describe a general class of semi-multiplicative damage functions as follows:

$$q(Y, D) = YD \ (\frac{Y}{Y_0})^{\xi-1} \tag{8.4}$$

$$Q = Y - (Y^{\xi} Y_0^{1-\xi} D) \tag{8.5}$$

When we decide to set $\xi = 0.5$, this implies that the absolute level of damages will automatically be modelled somewhere in between the multiplicative functional form and the additive damage function. Gross output is expected to increase over time, as TFP grows relatively fast in our modelling of uncertainty, by setting a mean of 10.73%. Our expectation that damages over time in this case, both before and after the marginal mitigation projects is implemented, lie somewhere in-between absolute multiplicative and additive damages. Our expectations are met when the uncertainty lies in g_0^A, for most of the time.

9 Discussion

9.1 Discussion of Our Findings

We have studied the sign and value of the climate's beta, that is, a CCAPM beta for climate change mitigation investments. We showed analytically, and explained intuitively how the climate beta can be derived from the CCAPM. In addition, we used the DICE model to get a numerical approximation for β_t. We performed our analysis by modelling uncertainty within the DICE model, through MC simulations. We find a positive β_t over the entire time horizon modelled, that is up to 500 years from now, in the cases where uncertainty comes from ***TFP shocks***. The beta for the different damage function specifications, when uncertainty comes in shocks to the TFP growth rate, is relatively similar over time. We find the highest estimations of beta whenever an additive damage function is employed. We contribute this partly to the 'positive-bias' that we believe to have modelled within our MC simulations, i.e. we find that high values of the mean and standard deviation underlying the MC simulations resulted in only positive shocks to TFP. Therefore, uncertainty through ε_t resulted in extremely high TFP growth, and hence extreme levels of output and consumption. We believe this might be partly the reason to why we obtain a climate beta above 2 for the first approximately 200 years.

In case of uncertainty through the initial growth rate of TFP, we find, that the estimation of beta differs between the two different damage functions. More specifically, with a multiplicative damage function our estimates show β_t around unity. When instead an additive damage function is used, β_t is positive and higher than unity for more than the first half of the total time horizon but becomes negative forever after. A negative beta when damages are additive and the main source of uncertainty is the inital growth rate of TFP, may indicate that climate mitigation projects can be seen as hedge against macroeconomic risk in the distant future. The majority of the findings above, indicate that the beta is positive, which is in line with the assumption of Nordhaus and Sztorc (2013) and the findings of Dietz et al. (2018). We also see that when applying a multiplicative damage function, the DICE model acts as a built-in mechanism towards a positive beta when the main source of uncertainty is TFP growth. We consider that the 'positive-bias', mentioned above, might be one of the reasons to why we obtain a beta above unity in the case of shocks to TFP growth, when applying the multiplicative damage function.

Our results indicate that if the main source of uncertainty is exogenous economic growth, through faster total factor productivity growth, the climate's beta is positive. More specifically, the most reasonable estimates we obtain for beta vary between a value of 1 and 2 over the entire time horizon. These results indicate that when aggregate consumption levels increase by 1%, the expected increase in net social benefit from the climate mitigation project is between 1% and 2%. Consequently, a positive value for beta implies that in order to determine the NPV of marginal climate mitigation projects, a higher discount rate than the risk-free rate should be used. How much higher should the discount rate become? And what are the implications of that in cost benefit analysis related to climate change ? We get back to these questions at the end of the discussion. We first discuss our methodology and the validity of our results.

9.2 Discussion of Our Methodology

9.2.1 Measure of the Climate Beta

One possible critique against our way of measuring the climate beta, is the use of natural logarithms as approximation for the percentage change in consumption and benefits of climate change mitigation projects. We assume this is a valid way to estimate β_t as the changes in consumption before and after our climate mitigation project is implemented, are small. Consequently, the estimates for benefits are small as well. A larger critique might be against our assumption that we consider our mitigation project as a **marginal** investment. We show in the analysis that the effects of our abatement project are relatively small.

Furthermore, in order to check the validity of our definition of benefits, we estimated beta based also on the covariance between the percentage change in consumption, and the percentage change in benefits, rather than taking the natural logarithms. However, this resulted in computational errors. Similarly, other measurements of the benefits, e.g. based on avoided damages instead of gained consumption, or based on abatement costs, did not give reasonable results for the values of beta over time. Lastly, the way we estimate the value of β is by assuming a deterministic nature, whereas in reality, beta is stochastic. Recent developments in the finance literature move towards investigating the effects of stochastic beta's on the assets' prices, although more research needs to be done in order to make an extension to a stochastic process for the climate beta. (Dietz et al., 2018)

9.2.2 The Use of the DICE model

An advantage of using DICE in order to estimate the climate beta, is its dynamic character, that allows us to analyze an entire set of effects on beta. We can observe the dynamic effects of a marginal investment project in climate change mitigation, since the DICE model allows us to remove a unit (Gt) of industrial CO2 emissions and to explore its effects on the output variables in the future time periods. Furthermore, by means of altering the standard DICE code, it allows us to explore both the effects of a multiplicative, semi-multiplicative and additive form of the damage function. On the other hand, using the DICE model certainly has disadvantages for our analysis. The model is a highly simplified tool to model the real world, which we ultimately try to assess. The model has received major critiques, around its simplification of reality and over the past two decades, debates have evolved surrounding certain parameters and/or variables of the model. We discussed some of those major critiques to the model in Chapter 5. Related to some of the critiques of the DICE model is the existence of both parametric and structural uncertainty. These uncertainties may be of great importance, also with regards to the discount factor. Much of the parametric uncertainty is likely to be feasible to test by means of 'simple' alterations to the DICE model. Furthermore, the use of an IAM like DICE makes the feedback patterns underlying our results, relatively nontransparent. Therefore it takes a lot of effort and time to analyze and understand the dynamics and feedback loops underlying the output variables of the model, and consequently their effects on our estimates for β. Especially when we alter the DICE model's code for some of the equations, and modelling uncertainty in order to obtain an estimation for β, it requires effort to disentangle the underlying dynamics and partly leads to computational traceability issues. The interpretation we provide to our results in the previous chapter should be read with caution due to this non-transparency of the DICE model. Instead of using the DICE model, we could have used a simple climate model as discussed in Chapter 3. The trade-off between simplicity and similarity to the real world is however likely to persist also in different models and methods. Different methods includes estimating β_t based on a standard OLS regression with use of historical data.

Since the DICE model builds upon Neoclassical growth theory, it incorporates a Ramsey-like model underlying its economic-equations. This means it employs an endogenous savings rate, which we adopt in our analysis. We distinguish our approach hereby from the one employed by Dietz et al. (2018). When the main source of uncertainty stems from exogenous economic growth through TFP, we find that in the scenarios with very high levels of economic growth (i.e. either through g_0^A or through ε_t), the forward-looking welfare maximizing agents, decrease their savings extremely fast. The underlying intuition behind our findings may be that the future generations will be extremely wealthy, such that the current generation does not have to smooth its welfare over time. However, we consider the emissions-reductions rate μ_t to revert these effects whenever the upper limit of the rate becomes binding. We do however also acknowledge the possibility that the alterations we make to the DICE model, give rise to computation challenges. These could in turn affect one or more of the output variables of our interest.

9.2.3 Modelling Uncertainty

We used a deterministic way of modelling uncertainty, rather than rewriting the DICE model in stochastic format. Moreover, we focused our analysis on modelling parametric uncertainty in the growth rate of TFP, although through two parameters. Omitting important uncertainties is inevitable and thereby our analysis is likely to underestimate the total uncertainty regarding the future benefits of a climate change mitigation policy. Especially uncertainty surrounding environmental variables are not considered in our investigation of the climate beta, they are affecting the damages of the climate onto the economy. Previous literature showed that when uncertainty is **mainly** coming from variables within the climate equations, for instance the equilibrium climate sensitivity (ECS), the estimates for beta are more likely to become negative. Especially uncertainty regarding the specification of damages through output and through capital growth is a relevant area of research and to our knowledge, currently unexplored. However, the aim of this study was not to model uncertainty within IAMs like the DICE per se, nor to focus on quantitative economic modelling itself. Instead, we use the modelling to explore the effect of some of the most important uncertainties in an IAM like DICE, namely output growth through TFP growth and damages from the climate on to the economy. Since we do not account for uncertainty within coefficients of the damage function, but only in parameters related to economic growth, we are likely to obtain positive values for the climate beta. Nevertheless, altering the damage function and modelling uncertainty in its functional form through the income elasticity of damages, ξ, enabled us to analyze the effect of climate damages on the economy. Among others, Nordhaus (2008) showed that the most important variable when modelling uncertainty within the DICE model, for climate economic outcomes, is the total factor productivity growth. In summary, investigating both uncertainty in economic growth through the variable, TFP, and accounting for different specifications of the relationship between climate damages and the economy, allows us to have an informed discussion about the climate beta. In turn, the climate beta estimates allow us to contribute to the larger debate about the risk-adjusted rate that should be used when discounting risky-investments like climate change mitigation projects.

9.3 Further Research / Practical Implications

The natural question arising from our results is, 'how much higher should the discount rate used in CBA on climate change mitigation projects be?' In other words, what is the risk-premium, π proportional to β, that should be added to the risk-free rate? The impact of the climate beta is highly dependent on the risk premium's value. Adapting a π similar to the ones observed in the market, of around 5%, (Gollier, 2013) will yield very different results for the risk-adjusted discount rate, compared to a model-based risk premium of approximately 0.2%. Chapter 3 explains that the undiscounted stream of benefits of a climate mitigation project is increasing in beta, whenever the expected value effect is larger than the negative effect from the higher discount rate. This holds true if $\beta > \gamma - (\frac{\mu}{\sigma^2})$, i.e if the beta is larger than the difference between the relative risk aversion and a factor of the mean of consumption relative to its variance. This difference is assumed to be approximately -2.5, based on empirical research. All the climate's betas obtained in this study are larger than -2.5 throughout the time period of about 500 years. This holds true for all specifications of uncertainty in TFP and for all specifications of the damage function. Consequently, the implications of this remain that the NPV of a climate mitigation project is increasing in β.(Dietz et al., 2018) (Freeman et al., 2018) We suggest that new models need to be developed, based on the fact that the CCAPM fails to predict many parts of the real world after decades of research done around it.

In addition to that, we modelled the climate beta with a deterministic model, while in reality it is stochastic(Dietz et al., 2018). Consequently, further research needs to be done regarding the effect of stochastic betas for climate mitigation projects, the effect of the endogenous savings rate on discounting in climate economics and the failure of the CCAPM to capture real world pricing. In order to provide possible answers to the above-mentioned concerns, we suggest new models to need to be developed.

10 Conclusion

How much is a livable climate in the future worth to you? This thesis has tried to implicitly answer this question by estimating 'the climate's beta'. We have shown that the CCAPM can be used to obtain an appropriate risk measure, the climate beta, to be employed in the discounting of climate change investments. We have argued in several ways that the climate's beta can be obtained by measuring the co-movement of the benefits of a marginal climate change mitigation project and consumption. By means of the DICE model we have modelled uncertainty in the growth rate of total factor productivity and we have analyzed how different specifications of the damage function affect the risk in the far future.

The main goal of this thesis is to contribute to the debate regarding risk-adjusted discount rates, employed in cost-benefit analysis by means the DICE model. In our theoretical framework we show that the general CCAPM can be used to derive β_t, a crucial estimate of an asset's risk-adjusted rate, and hence the main determinant of the actual price of an asset. The choice of modelling uncertainty through the Total Factor Productivity (TFP) growth rate was chosen as it is confirmed in the literature to be one of the most important parameters when modelling uncertainty for climate economic outcomes. (Gillingham et al., 2018)

We answer the research question "How does the sign and size of the climate beta vary over time depending on uncertainty related to TFP growth and climate change damages?", whilst also showing the influence of TFP growth as the main source of uncertainty on the climate beta's estimate.

The response of the climate beta to uncertainty, depends on the way the uncertainty enters the growth rate. We model uncertainty in TFP growth both through the initial growth rate, g_0^A, and through i.i.d shocks, ε_t, separately. Monte Carlo simulations are used in combination with one of the most recent sources of long-run economic growth forecasts to create the normal distribution underlying the uncertainty within each of the two parameters. The DICE model is then used to obtain an estimate for the climate beta by estimating the covariance between the natural logarithm of consumption (before climate abatement) and the natural logarithm of benefits from the marginal abatement project.

We find a positive beta for three out of four cases of combined parametric uncertainty with uncertainty surrounding the functional form of the damage function. In both cases of multiplicative damage functions, the obtained beta's are above zero and close to unity. We obtain higher beta's than expected when the main source of uncertainty comes through ε_t, but suggest that this might be due to the low variance from few uncertain parameters, or from an implicit positive- bias in the modelling of uncertainty. The positive sign of the betas are, however in line with our expectations and with previous literature on the subjectDietz et al. (2018)(Gollier, 2013). Only when damages are additive and the uncertainty lies in g_0^A, we obtain a negative climate beta around year 2300 and ascertain that this is due to a decrease in the covariance between benefits of the mitigation project and consumption, in turn likely due to decreasing benefits.

The implications of a higher climate beta is a higher discount rate, due to a positive risk premium proportional to beta being added to the risk-free discount rate. In CBA this implies a lower valuation of the climate mitigation project. However, we argue that the increase in the sum of the undiscounted expected benefits is larger than the increase in the discount rate with beta, hence yielding a higher NPV even when the beta is positive. This may have large implications for the valuation of mitigation projects, especially taking into account the higher values of positive β_t that we estimate.

We acknowledge that a new model of measuring the risk structure of climate change mitigation projects is needed in order to account for critique given to the DICE model and the CCAPM. Hence future research on the topic of risk structure of climate change mitigation is needed.

11 Reflections

This chapter reflects upon the process behind our master thesis. Mentioning all the individual insights and reflections of the both of us would be too extensive. Therefore we decide to keep the process reflection within this chapter on a rather high-level basis, focusing on the following aspects: our topic, the research process itself, peer support and learning cycles.

11.1 Topic

At first sight, climate change might seem a 'trending' or 'hot' topic under students writing their master or even bachelor thesis. In our case, this topic has not been chosen due to its popularity within the news or among the public. The choice to focus on climate change economics arose out of pure personal interests in this big topic and important questions for the current and future generations alive, combined with practical opportunity to combine the topic with macroeconomics and finance. On the other hand, one of us had not taken any climate-related course within her master neither bachelor degree (both in Economics/Finance) and had her last macroeconomics class more than 2 years. The other one of us had never written a thesis before, neither had we had a any type of semester- or business project at Copenhagen Business School before.

11.2 The research process

When deciding to start writing our thesis within the area of climate change economics combined with finance, one of the key points of concern was the lack of data that could be used to conduct our own empirical analysis. Realizing that little previous research within our area of interest conducted an empirical analysis, many research was focused around mathematical programming or had on a rather theoretical approach instead, we spend relatively much and effort in finding an appropriate methodology for our research. We are however content that the topic of choice is able to perfectly combine both finance and economics. A lot of time has been used in order to understand the General Algebraic Modelling System (GAMS) to the extent that we are able to make alterations to the standard DICE model code. In addition to that, learning how to model uncertainty within IAMs like the DICE model, has been a relatively large aspect of the research process. Fortunately we were able to reach our to peers in the academic world who have been so kind to provide us with little pieces of help throughout the last 6 months.

11.3 Peer support

We would like to emphasize the contact we have had with Christian Gollier and Simon Dietz regarding the climate beta and with Valentina Bosseti regarding the modelling of uncertainty within IAMs. Furthermore, we have spend quite some time with Professor Kourosh Rasmussen from DTU to get a well understanding of the program GAMS and in order to overcome the computational errors we encountered during the programming.

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Appendix

```
  $ontext
This is an altered version of the DICE-2016R model, by William Nordhaus, which is publicly available at
https://sites.google.com/site/williamdnordhaus/dice-rice.
    The alterations to the original code are due to the scope of a master thesis in Advanced Economics and Finance on the
effect of uncertainty in total factor productivity growth on
    the climate beta and are done by Marleen Pol and Karoline Sand Haarberg Spring 2019.
    These alterations regard mainly the ga(t)-parameter, the damage functions and the YNET(t).
    In addition, a gdxxrw code is set to compile xlxs filer rather than gms files.

DICE-2016R Altered for calculation of the Climate's beta
C o m p i l a t i o n


set       t  Time periods (5 years per period)                  /1*100/

PARAMETERS
** Availability of fossil fuels
        fosslim  Maximum cumulative extraction fossil fuels (GtC)  /6000/
**Time Step
        tstep    Years per Period                             /5/
** If optimal control
        ifopt    Indicator where optimized is 1 and base is 0      /0/
** Preferences
        elasmu   Elasticity of marginal utility of consumption     /1.45 /
        prstp    Initial rate of social time preference per year   /.015  /
** Population and technology
        gama     Capital elasticity in production function        /.300    /
        pop0     Initial world population 2015 (millions)         /7403    /
        popadj   Growth rate to calibrate to 2050 pop projection  /0.134   /
        popasym  Asymptotic population (millions)                 /11500   /
        dk       Depreciation rate on capital (per year)          /.100    /
        q0       Initial world gross output 2015 (trill 2010 USD) /105.5   /
        k0       Initial capital value 2015 (trill 2010 USD)      /223     /
        a0       Initial level of total factor productivity       /5.115    /
        ga0      Initial growth rate for TFP per 5 years          /0.076   /
        dela     Decline rate of TFP per 5 years                  /0.005   /

** TFP growth parameters
        psi      Coefficient of persistence of shocks             /0.42/
        epsiMean    Mean of shocks                             /0/

** Emissions parameters
        gsigma1  Initial growth of sigma (per year)                  /-0.0152 /
        dsig     Decline rate of decarbonization (per period)        /-0.001  /
        eland0   Carbon emissions from land 2015 (GtCO2 per year)    / 2.6    /
        deland   Decline rate of land emissions (per period)         / .115   /
        e0       Industrial emissions 2015 (GtCO2 per year)          /35.85    /
        miu0     Initial emissions control rate for base case 2015    /.03     /
** Carbon cycle
* Initial Conditions
        mat0   Initial Concentration in atmosphere 2015 (GtC)        /851    /
        mu0    Initial Concentration in upper strata 2015 (GtC)      /460    /
        ml0    Initial Concentration in lower strata 2015 (GtC)      /1740   /
        mateq  Equilibrium concentration atmosphere  (GtC)           /588    /
        mueq   Equilibrium concentration in upper strata (GtC)       /360    /
        mleq   Equilibrium concentration in lower strata (GtC)       /1720   /
* Flow paramaters
        b12      Carbon cycle transition matrix                     /.12   /
        b23      Carbon cycle transition matrix                     /0.007 /
```

```
* These are for declaration and are defined later
       b11      Carbon cycle transition matrix
       b21      Carbon cycle transition matrix
       b22      Carbon cycle transition matrix
       b32      Carbon cycle transition matrix
       b33      Carbon cycle transition matrix
       sig0     Carbon intensity 2010 (kgCO2 per output 2005 USD 2010)
** Climate model parameters
       t2xco2   Equilibrium temp impact (oC per doubling CO2)    / 3.1  /
       fex0     2015 forcings of non-CO2 GHG (Wm-2)              / 0.5  /
       fex1     2100 forcings of non-CO2 GHG (Wm-2)              / 1.0  /
       tocean0  Initial lower stratum temp change (C from 1900)  /.0068 /
       tatm0    Initial atmospheric temp change (C from 1900)    /0.85  /
       c1       Climate equation coefficient for upper level     /0.1005  /
       c3       Transfer coefficient upper to lower stratum      /0.088   /
       c4       Transfer coefficient for lower level             /0.025   /
       fco22x   Forcings of equilibrium CO2 doubling (Wm-2)      /3.6813  /
** Climate damage parameters
       a10      Initial damage intercept                         /0       /
       a20      Initial damage quadratic term
       a1       Damage intercept                                 /0       /
       a2       Damage quadratic term                            /0.0025 /
       a3       Damage exponent                                  /2.00    /

** Added parameters to get Weitzman damage
       a4       Damage Weitzman term             /0.0828/
       a5       Damage weitzman exponent         /7/
       xi       Income elasticity of damages     /1/
** Abatement cost
       expcost2  Exponent of control cost function               / 2.6  /
       pback     Cost of backstop 2010$ per tCO2 2015            / 550  /
       gback     Initial cost decline backstop cost per period   / .025 /
       limmiu    Upper limit on control rate after 2150          / 1.2 /
       tnopol    Period before which no emissions controls base  / 45   /
       cprice0   Initial base carbon price (2010$ per tCO2)      / 2    /
       gcprice   Growth rate of base carbon price per year       /.02   /

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations
* They ensure that MU of first period's consumption =1 and PV cons = PV utilty
       scale1    Multiplicative scaling coefficient     /0.0302455265681763 /
       scale2    Additive scaling coefficient           /-10993.704/ ;

* Program control variables
sets     tfirst(t), tlast(t), tearly(t), tlate(t);

PARAMETERS
       l(t)          Level of population and labor
       al(t)         Level of total factor productivity
       sigma(t)      CO2-equivalent-emissions output ratio
       rr(t)         Average utility social discount rate
       ga(t)         Growth rate of productivity from
       forcoth(t)    Exogenous forcing for other greenhouse gases
       gl(t)         Growth rate of labor
       gcost1        Growth of cost factor
       gsig(t)       Change in sigma (cumulative improvement of energy efficiency)
       etree(t)      Emissions from deforestation
       cumetree(t)   Cumulative from land
       cost1(t)      Adjusted cost for backstop
       lam           Climate model parameter
       gfacpop(t)    Growth factor population
       pbacktime(t)  Backstop price
```

```
        optlrsav      Optimal long-run savings rate used for transversality
        scc(t)        Social cost of carbon
        cpricebase(t) Carbon price in base case
        photel(t)     Carbon Price under no damages (Hotelling rent condition)
        ppm(t)        Atmospheric concentrations parts per million
        atfrac(t)     Atmospheric share since 1850
        atfrac2010(t)     Atmospheric share since 2010
        epsi(t)     I.i.d shocks to TPF growth  ;
* Program control definitions
        tfirst(t) = yes$(t.val eq 1);
        tlast(t)  = yes$(t.val eq card(t));
* Parameters for long-run consistency of carbon cycle
        b11 = 1 - b12;
        b21 = b12*MATEQ/MUEQ;
        b22 = 1 - b21 - b23;
        b32 = b23*mueq/mleq;
        b33 = 1 - b32 ;
* Further definitions of parameters
        a20 = a2;
        sig0 = e0/(q0*(1-miu0));
        lam = fco22x/ t2xco2;
        l("1") = pop0;
        loop(t, l(t+1)=l(t););
        loop(t, l(t+1)=l(t)*(popasym/L(t))**popadj ;);

**TFP growth is defined and calculated
        epsi(t)=Normal(EpsiMean,0.0);

        ga("1")=ga0; loop(t,ga(t)=((1-psi)*ga0 + psi*ga(t-1) + epsi(t))*exp(-dela*5*((t.val-1)));
* Cont. definition of parameters

        al("1") = a0; loop(t, al(t+1)=al(t)/((1-ga(t))););
        gsig("1")=gsigma1; loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
        sigma("1")=sig0;   loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););

        pbacktime(t)=pback*(1-gback)**(t.val-1);
        cost1(t) = pbacktime(t)*sigma(t)/expcost2/1000;

        etree(t) = eland0*(1-deland)**(t.val-1);
        cumetree("1")= 100; loop(t,cumetree(t+1)=cumetree(t)+etree(t)*(5/3.666););

        rr(t) = 1/((1+prstp)**(tstep*(t.val-1)));
        forcoth(t) = fex0+ (1/17)*(fex1-fex0)*(t.val-1)$(t.val lt 18)+ (fex1-fex0)$(t.val ge 18);
        optlrsav = (dk + .004)/(dk + .004*elasmu + prstp)*gama;

*Base Case Carbon Price
        cpricebase(t)= cprice0*(1+gcprice)**(5*(t.val-1));

VARIABLES
        MIU(t)          Emission control rate GHGs
        FORC(t)         Increase in radiative forcing (watts per m2 from 1900)
        TATM(t)         Increase temperature of atmosphere (degrees C from 1900)
        TOCEAN(t)       Increase temperatureof lower oceans (degrees C from 1900)
        MAT(t)          Carbon concentration increase in atmosphere (GtC from 1750)
        MU(t)           Carbon concentration increase in shallow oceans (GtC from 1750)
        ML(t)           Carbon concentration increase in lower oceans (GtC from 1750)
        E(t)            Total CO2 emissions (GtCO2 per year)
        EIND(t)         Industrial emissions (GtCO2 per year)
        C(t)            Consumption (trillions 2005 US dollars per year)
        K(t)            Capital stock (trillions 2005 US dollars)
        CPC(t)          Per capita consumption (thousands 2005 USD per year)
        I(t)            Investment (trillions 2005 USD per year)
```

```
       S(t)            Gross savings rate as fraction of gross world product
       RI(t)           Real interest rate (per annum)
       Y(t)            Gross world product net of abatement and damages (trillions 2005 USD per year)
       YGROSS(t)       Gross world product GROSS of abatement and damages (trillions 2005 USD per year)
       YNET(t)         Output net of damages equation (trillions 2005 USD per year)
       DAMAGES(t)      Damages (trillions 2005 USD per year)
       DAMFRAC(t)      Damages as fraction of gross output
       ABATECOST(t)    Cost of emissions reductions  (trillions 2005 USD per year)
       MCABATE(t)      Marginal cost of abatement (2005$ per ton CO2)
       CCA(t)          Cumulative industrial carbon emissions (GTC)
       CCATOT(t)       Total carbon emissions (GtC)
       PERIODU(t)      One period utility function
       CPRICE(t)       Carbon price (2005$ per ton of CO2)
       CEMUTOTPER(t)   Period utility
       UTILITY         Welfare function;

NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I;

EQUATIONS
*Emissions and Damages
       EEQ(t)          Emissions equation
       EINDEQ(t)       Industrial emissions
       CCACCA(t)       Cumulative industrial carbon emissions
       CCATOTEQ(t)       Cumulative total carbon emissions
       FORCE(t)        Radiative forcing equation
       DAMFRACEQ(t)     Equation for damage fraction
       DAMEQ(t)        Damage equation
       ABATEEQ(t)       Cost of emissions reductions equation
       MCABATEEQ(t)     Equation for MC abatement
       CARBPRICEEQ(t)   Carbon price equation from abatement

*Climate and carbon cycle
       MMAT(t)          Atmospheric concentration equation
       MMU(t)           Shallow ocean concentration
       MML(t)           Lower ocean concentration
       TATMEQ(t)        Temperature-climate equation for atmosphere
       TOCEANEQ(t)      Temperature-climate equation for lower oceans

*Economic variables
       YGROSSEQ(t)      Output gross equation
       YNETEQ(t)        Output net of damages equation
       YY(t)            Output net equation
       CC(t)            Consumption equation
       CPCE(t)          Per capita consumption definition
       SEQ(t)           Savings rate equation
       KK(t)            Capital balance equation
       RIEQ(t)          Interest rate equation

* Utility
       CEMUTOTPEREQ(t)  Period utility
       PERIODUEQ(t)     Instantaneous utility function equation
       UTIL             Objective function      ;

** Equations of the model
*Emissions and Damages
 eeq(t)..             E(t)           =E= EIND(t) + etree(t);
 eindeq(t)..          EIND(t)        =E= sigma(t) * YGROSS(t) * (1-(MIU(t)));
 ccacca(t+1)..        CCA(t+1)       =E= CCA(t)+ EIND(t)*5/3.666;
 ccatoteq(t)..        CCATOT(t)      =E= CCA(t)+cumetree(t);
 force(t)..           FORC(t)        =E= fco22x * ((log((MAT(t)/588.000))/log(2))) + forcoth(t);

**Damage equations
```

```
damfraceq(t) ..      DAMFRAC(t)     =E= 1 - 1/(1+(a1*TATM(t))+(a2*TATM(t)**a3) + ((a4*TATM(t))**a5));
dameq(t)..           DAMAGES(t)     =E= DAMFRAC(t) * YGROSS(t)**xi  * q0**(1-xi);

abateeq(t)..         ABATECOST(t)   =E= YGROSS(t) * cost1(t) * (MIU(t)**expcost2);
mcabateeq(t)..       MCABATE(t)     =E= pbacktime(t) * MIU(t)**(expcost2-1);
carbpriceeq(t)..     CPRICE(t)      =E= pbacktime(t) * (MIU(t))**(expcost2-1);

*Climate and carbon cycle
mmat(t+1)..          MAT(t+1)       =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*(5/3.666));
mml(t+1)..           ML(t+1)        =E= ML(t)*b33  + MU(t)*b23;
mmu(t+1)..           MU(t+1)        =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
tatmeq(t+1)..        TATM(t+1)      =E= TATM(t) + c1 * ((FORC(t+1)-(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TOCEAN(t))));
toceaneq(t+1)..      TOCEAN(t+1)    =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));

*Economic variables
ygrosseq(t)..        YGROSS(t)      =E= (al(t)*(L(t)/1000)**(1-GAMA))*(K(t)**GAMA);

yneteq(t)..          YNET(t)        =E= YGROSS(t) - DAMAGES(t);

yy(t)..              Y(t)           =E= YNET(t) - ABATECOST(t);
cc(t)..              C(t)           =E= Y(t) - I(t);
cpce(t)..            CPC(t)         =E= 1000 * C(t) / L(t);
seq(t)..             I(t)           =E= S(t) * Y(t);
kk(t+1)..            K(t+1)         =L= (1-dk)**tstep * K(t) + tstep * I(t);
rieq(t+1)..          RI(t)          =E= (1+prstp) * (CPC(t+1)/CPC(t))**(elasmu/tstep) - 1;

*Utility
cemutotpereq(t)..    CEMUTOTPER(t)  =E= PERIODU(t) * L(t) * rr(t);
periodueq(t)..       PERIODU(t)     =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/(1-elasmu)-1;
util..               UTILITY        =E= tstep * scale1 * sum(t,  CEMUTOTPER(t)) + scale2 ;

*Resource limit
CCA.up(t)       = fosslim;

* Control rate limits
MIU.up(t)            = limmiu;
MIU.up(t)$(t.val<30) = 1;

**  Upper and lower bounds for stability
K.LO(t)         = 1;
MAT.LO(t)       = 10;
MU.LO(t)        = 100;
ML.LO(t)        = 1000;
C.LO(t)         = 2;
TOCEAN.UP(t)    = 20;
TOCEAN.LO(t)    = -1;
TATM.UP(t)      = 20;
CPC.LO(t)       = .01;
TATM.UP(t)      = 12;

* Control variables
set lag10(t) ;
lag10(t) =  yes$(t.val gt card(t)-10);
S.FX(lag10(t)) = optlrsav;

* Initial conditions
CCA.FX(tfirst)    = 400;
K.FX(tfirst)      = k0;
MAT.FX(tfirst)    = mat0;
MU.FX(tfirst)     = mu0;
ML.FX(tfirst)     = ml0;
```

```
TATM.FX(tfirst)   = tatm0;
TOCEAN.FX(tfirst) = tocean0;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
model  CO2 /all/;

* For base run, this subroutine calculates Hotelling rents
* Carbon price is maximum of Hotelling rent or baseline price
* The cprice equation is different from 2013R. Not sure what went wrong.
If (ifopt eq 0,
       a2 = 0;
       solve CO2 maximizing UTILITY using nlp;
       photel(t)=cprice.l(t);
       a2 = a20;
      cprice.up(t)$(t.val<tnopol+1) = max(photel(t),cpricebase(t));
);

miu.fx('1')$(ifopt=1) = miu0;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;
solve co2 maximizing utility using nlp;

** POST-SOLVE
* Calculate social cost of carbon and other variables
scc(t) = -1000*eeq.m(t)/(.00001+cc.m(t));
atfrac(t) = ((mat.l(t)-588)/(ccatot.l(t)+.000001  ));
atfrac2010(t) = ((mat.l(t)-mat0)/(.00001+ccatot.l(t)-ccatot.l('1')  ));
ppm(t)    = mat.l(t)/2.13;




set scen    /scen1*scen1000/;

parameter   ga0Scen(scen)  /
INCLUDE    /Users/KarolineSH/Downloads/ga0_uncertainty.gms

/;

*display ga0Scen


 Parameter summaryReport(*,*,*);

Loop (Scen,
*Change parameter
  dela=delaScen(scen);
*Recalculate dependent parameters
      ga("1")=ga0; loop(t,ga(t)=((1-psi)*ga0 + psi*ga(t-1) + epsi(t))*exp(-dela*5*((t.val-1)));
       al("1") = a0; loop(t, al(t+1)=al(t)/((1-ga(t))););
*solve the model
solve co2 maximizing utility using nlp;
*save variables of interest
    summaryReport('Consumption',t,scen)            = CPC.l(t);
      summaryReport('Gross output',t,scen)          = YNET.l(t);
     summaryReport('Damages',t,scen)                 =DAMAGES.l(t);
    summaryReport('savings',t,scen)                  =S.l(t);
```

```
   summaryReport('Industrial emissions',t,scen)   =EIND.l(t) ;



);

*display summaryReport;

EXECUTE_UNLOAD 'Summary.gdx', SummaryReport;
EXECUTE 'gdxxrw Summary.gdx O=dela_multiplicative.xlsx par=SummaryReport';
```

Figure A0.1: Consumption per capita for selected ga0 values

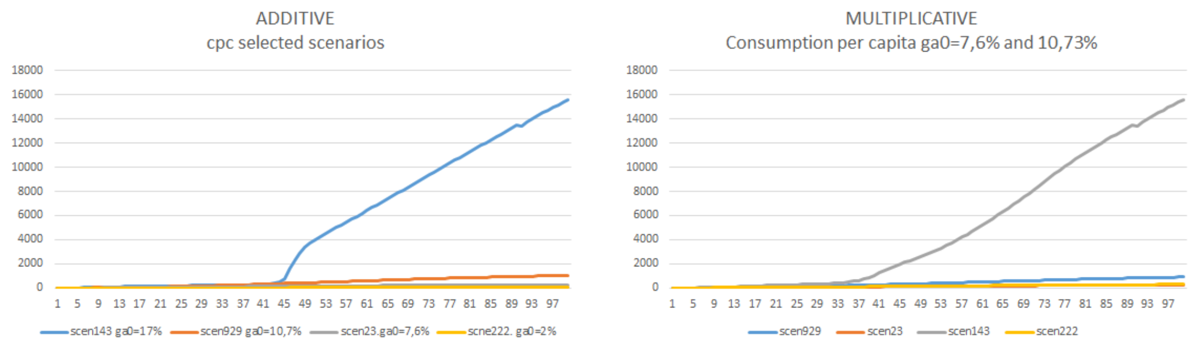

Figure A0.2: Log Consumption per capita for selected ga0 values

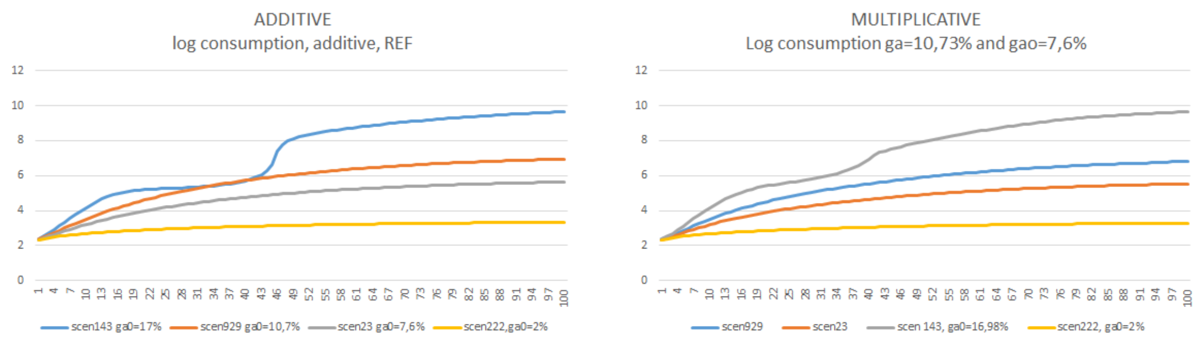

Figure A0.3: Damages for selected ga0 values

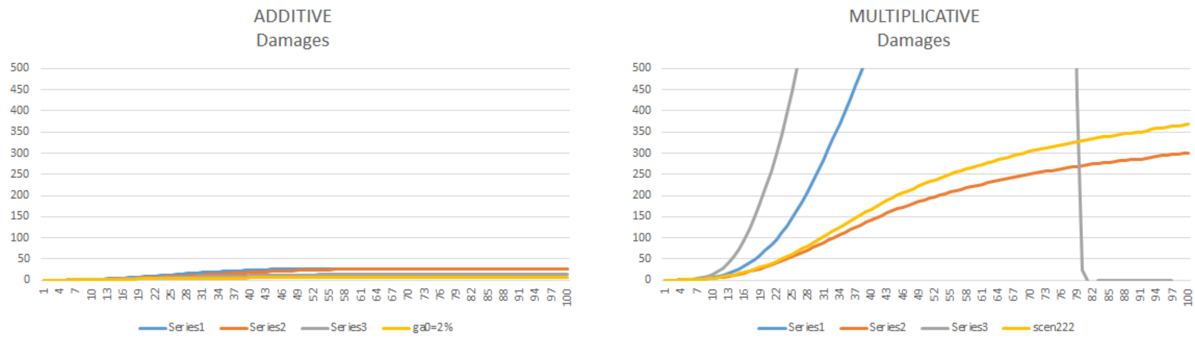

Figure A0.4: Savings rate for selected ga0 values

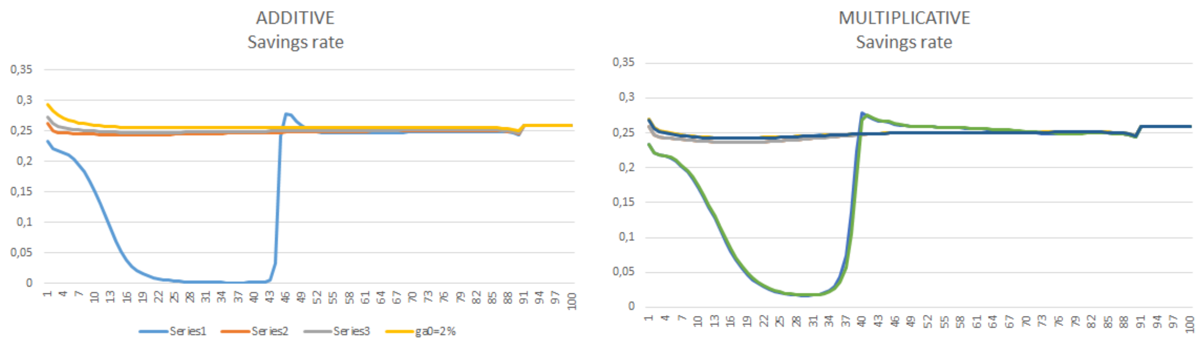

Figure A0.5: Consumption per capita for selected epsilon values

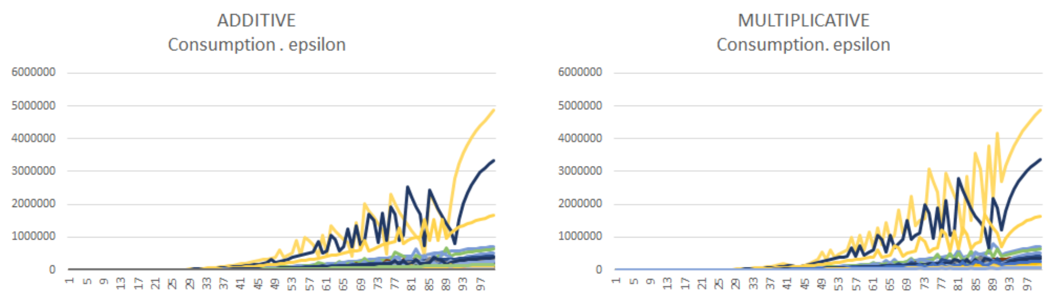

Figure A0.6: Damages for selected epsilon values

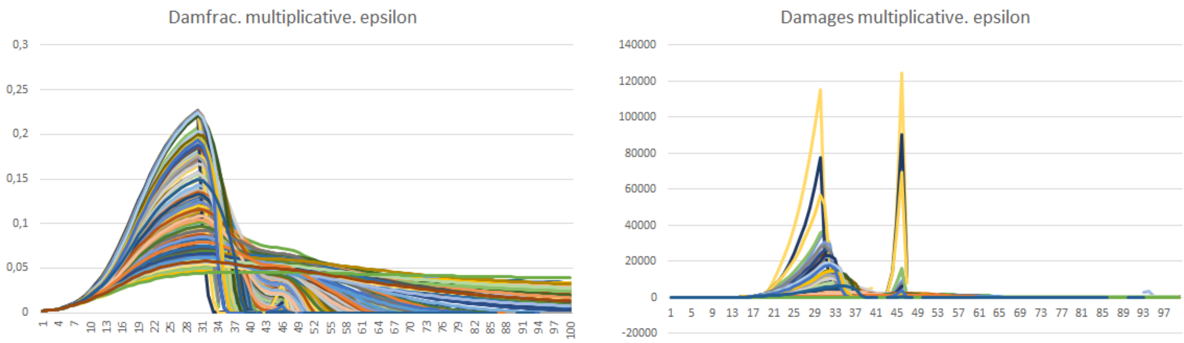

Figure A0.7: Savings rate for selected epsilon values

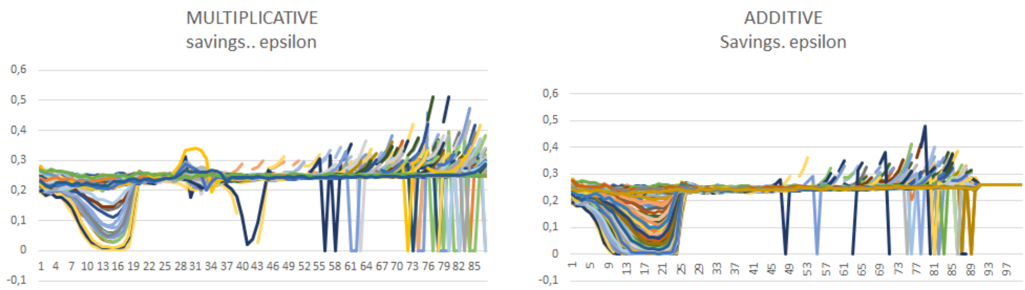

Figure A0.8: A visual representation that e0 qualifies to a *marginal* emissions abatement project

change.png change.png change.png