Real Estate Agents' Fee Payments in Denmark

Ejendomsmægleres vederlag i Danmark

Master's Thesis in
Business Administration and Mathematical Business Economics
Cand.merc.(mat)
at Copenhagen Business School

Simone Heje Grønbech

Student number 56550

Supervisor
Anette Boom
Department of Economics
Copenhagen Business School

16 September 2019

124100 characters on 54 pages

Real Estate Agents' Fee Payments in Denmark

Simone Heje Grønbech

16 September 2019

Resumé

I dette speciale undersøger vi ejendomsmægleres vederlag. Vi belyser hvad effekten af online information og ændringer i lovgivning har haft på ejendomsmæglers vederlag samt deres rolle i formidlingsaftalen i henhold til asymmetrisk information. Vi undersøger om ejendomsmæglere har et incitament til at give et forkert billede af markedstilstanden og hvordan vederlaget forholder sig i forhold til både et konkurrencepræget marked samt et monopol.

Vi har gjort dette ved at gennemgå reporter fra Erhvervsstyrelsen samt Konkurrence- og Forbrugerstyrelsen samt lovgivningen på området. Reporter beskriver udviklingen af ejendomsmæglernes vederlag fra det traditionelle resultatafhængigt vederlag med solgt-eller-gratis-princippet til alternative koncepter med vederlag efter regning til liberaliseringen af regler om vederlag. Vi gennemgår også artikler og empiriske undersøgelser om asymmetrisk information mellem ejendomsmægler og sælger. De beskriver fordele og ulemper ved resultatafhængigt vederlag, og kommenterer på hvilken effekt online information har.

Vi opstiller en model til at undersøge om ejendomsmægleren har et incitament til at give et forkert billede af markedet, hvor sandsynligheden for at sælge ejendommen afhænger af udbudsprisen og markedstilstanden. Her konstaterer vi at i et konkurrencepræget marked har ejendomsmægler ikke et incitament til at lyve om tilstanden, da deres vederlag ikke afhænger af tilstanden. I et monopol har ejendomsmægleren et incitament til at lyve om markedet, hvis tilstanden er dårlig, da hans vederlag afhænger af tilstanden. Vederlaget er derfor også forskelligt afhængigt om markedet er konkurrencepræget eller et monopol.

Vi konkluderer at den online information har gjort sælger mere bevidst om ejendomsmarkedet, da hun kan finde informationer om salgspriser, liggetider og ejendomsmæglere. Dette har mindsket ejendomsmæglernes asymmetriske informationsfordel. I Danmark er ejendomsmægleren sælgers repræsentant i ejendomshandlen.

Contents

1 Introduction

Selling a property is a big financial decision and a lot of decisions goes into deciding to sell a property. The seller is interested in getting a high price, having a fast sale and the cost of selling. Most sellers will hire a real estate agent to help with the sale. The real estate agent knows the market, the demand for the location and type of house, and he can follow the trends. This causes asymmetric information between the real estate agent and the seller. The real estate agent represent the seller by handling the sale by advertising and showing the property, obtaining the legal documents for the sale and negotiating with potential buyers. Because of the asymmetric information, the real estate agent has an informational advantage against the sellers and can use this advantage for his own personal gain. We will therefore look into the listing agreement between the seller and the real estate agent to investigate this contractual relationship.

When hiring a real estate agent, the agent offers a listing agreement with a listing price and his fee payment. The fee payment is either a performance-based fee, where the agent get a percentage of the sale price, or a fixed fee. With the internet the seller can get much of the information the real estate agents have online, such as the average sale prices on a postal code level, the average time on the market, the legal documents for the sale and guidance from other sellers and the Danish Consumer Council. Gathering this information does however take time and there still is some legislation to follow to make sure the sale is legal. This, among other reasons, is why sellers are still using real estate agents. The sellers have the advantage that they can use the information online to see whether the listing price which the real estate agent offers is reasonable, whereas this was harder to do before the internet. It is however almost impossible to see whether the real estate agent's fee is reasonable and to compare the real estate agents' fees. In 1993 it was decided that a new legislation was needed for the real estate market, and since then there has been several changes to this legislation, the latest in 2015. We are therefore interested in examining the following research questions:

- What are the effects of the information online and the legislation's changes to the real estate agent's fee and the real estate agent's role in the listing agreement with regard to the asymmetric information?
- Do the real estate agents have an incentive to misrepresent the market and how does the fee change if the real estate agent operate in a monopoly or a competitive?

It has not been possible to gather any data on the fees for the last ten years or so, since there does not exists any public collection of these. Therefore we will not be able to make any statistical analysis of the fees. Instead we will setup a model, that focuses on the listing agreement in Denmark, to help answer the research questions. We will not be looking into collusion between the real estate agents and their fee or real estate agent cartels to describe the asymmetry.

Section 2 will describe the Law of Real Estate Agent's Services with regard to the real estate agent's fee and his role from when it was instated in 1994 up to 2015. Both the Danish Competition and Consumer Authority (Konkurrence- og Forbruger-

styrelsen) and the Danish Business Authority (Erhvervstyrelsen) have examined the real estate market throughout the years, and based on these examination there have been made changes in the legislation. Section 3 describes different theoretical models which examines the asymmetry between the real estate agent and the seller as well as empirical analysis of foreign real estate markets. In Section 4 we will set up a model to describe what happens when real estate agents misrepresent the market based on a performance-based fee. We look at the model for both the competitive market and for the monopoly market. Section 5 will discuss the theoretical models and our own model with regard to the research questions. Lastly, Section 6 concludes this thesis.

2 The Law of Real Estate Agents' Services

In Denmark the title real estate agent is a protected title, which means that in order to be a real estate agent you have to be registered in the Danish Business Authority's real estate agent register.[1] The current rules of real estate agent's services is described in Act no. 526, Law of Arrangement of Real Estate.[2] The law is a consumer protection law, which regulates the professionel advising and assistance to private consumers, who wants to sell or buy real estate (The Business Authority, 2013). We will focus on the real estate agent's fee payment and his role in the real estate sale.

The law was first established in 1994 with Act no. 453, which no longer is in effect. Throughout the years since 1994, the Danish Business Authority and the Danish Competition Authority have looked into the market for real estate sales and published reports.[3] The most recent report is an analysis from 2013 made by the Danish Business Authority, that adresses the legislation's changes throughout the years and examines the real estate agent's services until 2013. The most notable changes have happened in in 1999, 2005, 2006 as well as 2015, where a new act replaced Act no. 453. The Business Authority (2013) writes that with the changes to the law, the law makes sure that

> the seller, who wants to use a real estate agent, knows what he is paying for, what he earns and trusts that the real estate agent takes care of the seller's interests. The buyer has to trust that the real estate agents presents all relevant information about the real estate.

The changes in 1999 standardised the processes of real estate sales to make it simpler and more transparent for the consumers. It introduced a new fee payment for the real estate agents, where they could offer the fee as payment as per account rendered (referred to as the fixed fee). Before this, there had only been the performance-based fee with the no-cure no-pay principle, which means that the seller only have to pay the real estate agent's fee if the house is sold within the listing period. Adding the fixed fee increased the consumer's choices.

The changes in 2005 and 2006 was made to improve the competition between the real estate agents. These changes included more rules about how the fee was to be presented in the listing agreement to make it clear to the consumer, what she was paying for. With the latest change in 2015, which also resulted in a new law, Act no. 526, all these changes was liberalised so the real estate and the seller themselves could decide how the fee was paid. We will go into further detail about the reasons for the changes in what follows by starting with describing the law from 1994.

[1] `https://boligejer.dk/optagelse_maglerregistret`

[2] All acts are available at `https://retinformation.dk`.

[3] All citations in this section are translated from Danish.

2.1 1994: Introduction of real estate law

When the law became effective in 1994, the provision was that no one was to be an agent for both parties in the same sale, because it was estimated, that no one could manage both parties equally in the same real estate sale. The real estate agent still needed "to act with care for both parties' interests for the purpose that the real estate sale takes place within a period, price and terms as agreed with the principal, typically the seller". This essentially means the agent should perform his duties towards his principal, while making sure the other party in the sale were informed to seek out their own advisor or real estate agent.

The fee structure in 1994 was not specified, but the traditional way of calculating the fee was the performance-based fee. The law stated:

> The agent can only stipulate his fee in the agreement, if
>
> 1. entering of a purchase agreement before the listing agreement has ended or
> 2. after the listing agreement has ended a purchase agreement has been entered on the grounds of the agent's effort and without another agent's participation, provided it is assumed that the entering of the purchase agreement has been postponed to keep the agent out.
>
> Act no. 453, Section 11 (1993)

The principle of no-cure no-pay was also instated 1994, which meant a higher fee rate, because the real estate agent's fee now depended on the property being sold (The Competition Authority, 1999).

Act no. 453, Section 10 adresses the procedures of the listing agreement between the seller and the real estate agent, where the agreement had to include the duration and the fee payment. The law stated in Section 12 that the listing agreement could at most be six months, and hereafter be extended three months at a time. Section 12 (2) said the agreement could be terminated without warning by both parties. The law further stated in Section 12 (3) that the real estate agent had a claim of a reasonably fee if the seller terminated the listing agreement before the agreed period. The amount of this fee can only under certain circumstances be more than one fourth of the fee agreed if a sale was completed with the listing price for which the property was offered. However, if the real estate agent has disregarded his duties, he no longer had a claim of a fee.

2.2 1999: Adding new fee types

In the 1999, the Danish Competition and Consumer Authority looked into the fees of the real estate agent industry. They investigated if the real estate industry was in contrary with the law of competition with regard to existing contracts or a coordinated practice in setting the fees and giving discounts on the fees. The Competition Authority (1999) concluded that there were no illegal agreement or a coordinated practice regarding the fee rate or the discounts.

The Danish Competition and Consumer Authority report was based on a report from 1997 by the Ministry of Industry titled "Easier and cheaper to trade property"[4], where the conclusions of the report according to The Competition Authority (1999) were:

[4]No longer public.

- Compared to the level of costs for the sale of property in other countries Denmark lies at the high end of the market.
- The price competition among the real estate agents is limited.
- The market for the sale of property in many respects is impossible to get an overall view of by both buyer and seller.

According to The Competition Authority (1999) the real estate agents' fees had increased more than other similar occupations during the 1990's. Because of the strong increase in the price for real estate during this period, the fee increased more with the performance-based fee and the no-cure no-pay principle. The real estate agent's explained the rise in the fees was due to changes in their costs, because of the no-cure no-pay principle and marketing costs, but The Competition Authority argues this is not the case. They further state:

> In a competitive market it would not have been possible to increase the fee to the extent it has, just as it hardly would have been possible to maintain a system, where the fees no matter the cost developments are calculated as a percentage of the real estate prices. The Competition Authority (1999)

The real estate agents divided the total fee into the performance-based fee payment to the agent, a marketing fee and some also had a basic amount besides the fee payment. The Competition Authority (1999) states the fee was the cause of the lacking competition in the real estate market. The examination indicated that the fees was set according to a locally "recommended" fee, which only were known to the real estate agencies and intended for internal use. It was very few real estate agencies who advertised with their fee back then, and after researching current real estate agency websites, no agency is advertising their rate for the performance-based fee today. The Competition Authority writes that these "recommended" fees were never reported as a local restriction of competition, and that the real estate agent responded that "they just followed the general level for the fee." Considering this, The Competition Authority stated:

> The relative identical fees within many cities could be an expression of local price competition. It is however more likely, that they are an expression of the largest realtors in each city 'setting the level of fee', after which the others agents follow suit. The competition between the agents instead occur in the form of marketing etc.

During the 1990's, a lot of real estate agents gave considerable discounts on their "recommended" fees. The Competition Authority (1999) reports that 85% of the real estate agents said they provided a discount of up to 30% of the total fee. These discounts were typically given if the sale was fast, however there were no guidelines to the size of the discounts and when they were given, so the discount varies a lot from sale to sale. Another form of discount is a reduction in the fee due to the seller taking on some services such as showing or advertising the property, but this was very rare. These considerable discounts were a way of doing price competition within the real estate agent business, according to (The Competition Authority, 1999). They write:

> This could in theory be fine when looking from a competition point of view. We are however dealing with discounts of a considerable size, which are both hidden and strongly varying, and which are only to a limited extent cost-determined.

> In this concrete connection the widespread discounts may therefore be taken as an expression of the competition as being insufficient. It is the Danish Competition Authority's opinion, that an increased transparency under these circumstances will be the most effective method to secure an increased price competition between the agents. The authority have therefore chosen to publish the collected fee rates in the form of median for all municipalities.

These fee rates and basic amounts, which are strongly varyingly discounted, are shown in Appendix A.1. The average discounted fee rate was 2.71% and the average discounted basic amount was 11 241 DKK in 1999.

To combat these discounts and give more transparency, there was a change in the legislation regarding the fee. The change in the law[5] resulted in specifying the already used performance-based fee and adding the payment as per account rendered fee. The headline *Payment as Per Account Rendered Fee* was inserted before Act no. 453, Section 10, which was repealed and replaced with:

> Agreement about the listing engagement must be written, and it must clearly appear, when and how the fee must be paid. The agreement must among other things include terms about the engagements length, specification of the services, which enters in the listing engagement, as well as information about the fee, which must be paid for each service. Information about the fee for each service can be omitted, if it clearly states in the agreement that a performance-based fee is used. Act no. 453, Section 10 (1) (1999)

This type of fee opened up for including the seller in the sale more, which meant specifying what the seller needed to pay for. The performance-based fee were more of a package deal, where the real estate agent did everything.

The headline *Performance-based Fee* was added before Section 11, which was changed to:

> [O]n listing agreements, where it is established that the fee's payment is depending on the purchasing agreement (performance-based fee), the agent can only insist on the fee, if
>
> 1. entering of a purchase agreement before the listing agreement has ended or
> 2. after the listing agreement has ended, a purchase agreement has been entered on the grounds of the agent's effort and without another agent's participation, provided it is assumed that the entering of the purchase agreement has been postponed to keep the agent out.
>
> Act no. 453, Section 11 (1) (1999)

This almost reiterates what Act. no 453, Section 11 (1993) stated before the change. The law also added Section 11a about the specification of the final fee, which stated:

> In the specification about the final fee it must be clearly stated, which services the consumer has received, as well as the fee for each service. Specification is not required, if it is agreed upon performance-based fee, a fixed amount or a fixed percentage for performance of the whole listing engagement. Act no. 453, Section 11a (1999)

[5] Act no. 227.

2.3 2004: Review of the competition of real estate agent's services

In 2004 the Danish Competition Authority published a review of the competitive situation of the real estate agent's services among others based on an examination of the real estate market in 2002. The Competition Authority (2004, Chapter 4) described the competition as poor, partly due to the real estate agents only offering a traditional concept with performance-based fee, and the real estate agents still giving the discount on fees. The examination differentiates between traditional concept and alternative concepts. The traditional concept included

1. sale price estimate and determining the listing price,
2. collection of documents,
3. arrangement of financing services,
4. orientation of home condition report, insurances, etc.,
5. drafting the particulars of sale,
6. marketing,
7. drafting the purchase agreement,
8. showing of the property and negotiation with buyers,
9. inspection of rectification and completion statement as well as closing the sale,

where item 1, 4, 5, 7 and 9 were mandatory. The alternative concepts would typically include the seller showing the property and advertising in the papers, where the real estate agent would do the online marketing and the other mandatory services. At the time of the examination, 90% real estate agencies offered the traditional concept, whereas only 10% offered special concepts, where the consumer can deselect some services and typically pays a fixed fee. The examination also shows that 84% agencies still gave discounts on the fee. As the authority concluded in 1999, this does not make the fees transparent, because there were still no guidelines to the discounts (The Competition Authority, 2004, Chapter 4).

The review of the competition writes that information technology could help improve the efficiency of marketing. However 70% of the real estate agents said the internet have not expanded their sales area. 40% said the internet have affected their marketing costs, of which half said their costs had risen because online marketing had not reduced their cost of more traditional marketing in papers. Therefore only 20% of the real estate agents have improved their efficiency by, for example, advertising on the internet and less in other medias, lowering marketing costs for example (The Competition Authority, 2004, Chapter 4).

The Competition Authority (2004, Chapter 4) states that the fees have increased more than the normal price trend, whereas the sales prices have increased a little more than the fees. The rise in the fees was according to the real estate agents still due to more tasks when selling property since the law was instated, which was the same argument used in 1999. As was stated in 1999, this should no longer be the reason. The Competition Authority therefore believes it is due to the poor competition of the real estate agent's services, and that the competition would be improved by introducing other agencies with new concepts. The poor competition have also meant that the performance-based fee have been the most used fee, where the consumers have had a hard time getting an overview of the services it included.

2.4 2005-2007: Changes to simplify the real estate sale

The Danish Government (Regeringen) made 12 initiatives to help the competition of the market for real estate's services in 2005. The Danish Business Authority and the Danish Competition and Consumer Authority completed an examination of the 12 initiatives in 2007. The initiatives were made for the consumer to clearly understand the process and to make the market function effectively. Therefore changes were made to the law in 2005 and 2006 to simplify the process of selling a property (The Government, 2005).

The 12 initiatives were divided in two categories, where the first focused on creating *more competition and transparency* and the second focused on *simpler rules and clear information*. The initiatives that are related to the real estate agent's fee and role in the listing agreement are:

- Initiatives related to more competition and transparency:
    1. Individual services and greater freedom of choice in the real estate sale, and
    2. Transparent prices in real estate sales.
- Initiatives related to simpler rules and clear information:
    3. Digital registration of property, and
    4. New combined public property portal.

The Government (2005) describes that initiative 1 will lessen the mandatory services so the real estate agent can offer a bundle with fewer mandatory services in order to strengthen the competition and thereby adding more new concepts, as The Competition Authority (2004, Chapter 4) suggested. With initiative 2 the government wanted to strengthen the transparency of the prices for the services the real estate agent offers. There must be a total price for the mandatory services, and a price for each additional service to the listing agreement. Initiative 3 and 4 digitalised the registration of property and gathered the real estate data in an online public portal to make it easier, faster and less costly for the real estate agent or the seller to get a lot of the paperwork using the internet (The Government, 2005).

The examination of these initiatives were made in 2007, 4 months after the last changes to the legislation, so the examination was a snapshot of the situation in 2007. This examination also follows up on the examination from 2004. They find that the uninterrupted increase in the fees have stopped, more real estate agents offer a fixed fee, which are lower than the performance-based fee and more real estate agents offer special concepts at a lower fee. For the performance-based fees, the average level have stayed reasonably the same as 2002, and in some areas a bit lower (The Competition Authority, 2009).

The increase of the sales prices[6] for the municipalities of Copenhagen and Lemvig as well as the average of Denmark can be seen in Figure 2.1. Copenhagen with a population of 626508 represent a competitive market and Lemvig with a population of 19938 represent a small city where a monopoly is more likely. There definitely is an increase in the sales prices from 1992 to 2004 for Copenhagen as the reports describes, but the sales prices for Lemvig are almost at the same level for the whole period. The increase from 1999 to 2007 for Copenhagen is more steep than the average, where it decreases until 2010 and again increases up to today. If the increase of the performance-based fee had continued, the real estate agent in a competitive market such as Copenhagen

[6] Boligmarkedsstatistikken BM010: `https://rkr.statistikbank.dk/201`

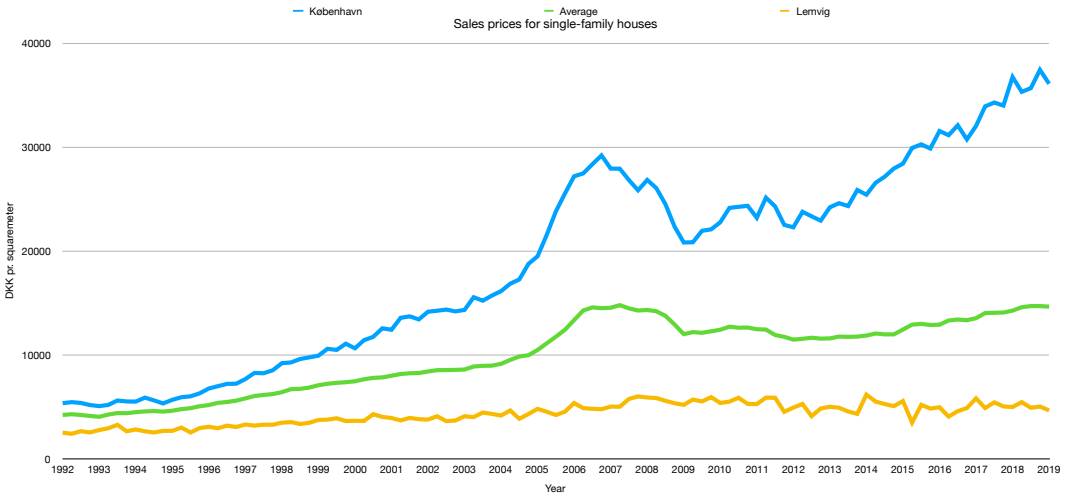


Figure 2.1 The sales prices for single-family houses from 1992 to today.

could gain more since the sales prices also increased. Whereas in smaller cities such as Lemvig it would only be the fee rate that had increased while the sales prices only increasing a little. The authorities does not describe how the fees relative to the sales prices have behaved beyond 2007.

The changes in the legislation was to ensure the consumers had more choices when selling their house. The mandatory services was reduced and there was introduced more disclosure requirements, which included requirements for the price of some services. Even though more real estate agents offered a concept with lower fixed fee, most of the sales in 2007 were realised with the performance-based fee. Two thirds of the real estate agents only offered the traditional concept, so the changes in the legislation have not lead to more choices (The Competition Authority, 2009).

The traditional concept now included the mandatory services where the seller could add additional services. According to The Competition Authority (2009) the mandatory services included

- determining the listing price,
- arrangement of the sales budget,
- calculation of gross and net payment through a standard financing,
- the particulars of sale, and
- drawing up the purchasing agreement.

Compared with 2004, the mandatory services have been simplified. The real estate agents still had to determine the listing price, make the particulars of sale and draw up the purchasing agreement, but the rest of the mandatory services from 2004 had been removed and introduced two new mandatory services.

The real estate agents had to state either the prices for each service, or a total price for the all the services. They also had to state the price for each additional service, they provided, and the marketing fee. The change to specifying the prices was made to give the consumer more transparency in the sale, and so the consumer can choose or deselect additional services. For the real estate agents who offered other concepts with a fixed fee, the consumer could not add or deselect the service included in the concepts (The Competition Authority, 2009). The Competition Authority points out that the concept with a fixed fee can cause confusion for the consumer because the real estate agents can

price the services arbitrary within the total fee, and therefore there is not transparency of the prices.

2.5 2013: Examination of the law of the real estate agents

The Danish Business Authority made an analysis of the Danish real estate market in 2013, which resulted in a new law of the real estate agents per 1 January 2015. The analysis' areas of focus are the real estate agent's role opposite seller and buyer, the real estate agent's duty of disclosure, the real estate agent's fee, unregistred assistants and trade as real estate agent and bid rounds (The Business Authority, 2013), where we will focus on the first three. The analysis also provides a more detailed insight in the real estate agent's services.

As was the case in 1994, the law has not changed on the fact that no one can be an agent for both seller and buyer in the same sale. The Business Authority (2013) goes into details about the real estate agent's role, and explains that the real estate agent has to manage the principal's interests, whether the principal is the seller or the buyer, taking into account the other party. If the other party is not represented by an adviser, the real estate agent has to advise about the need and possibility to seek assistance. Additionally, the real estate agent has to act with care for both parties' interests. This is to secure the consumer protection, which is important in the trade of real estate, because both seller and buyer are consumers, where the real estate sale has a big impact on the consumer's economy.

In Denmark it is most common that the real estate agent is the seller's representative, where the agent has to manage the seller's interests, but it is not clearly communicated in the law. An intermediary act as an agent for both parties and must therefore maintain both parties' interests. The intermediary will first manage the seller's interest in relation the initiation of the sale. When a buyer is interested in the property for sale, the intermediary then also have to manage the buyer's interest. The real estate agent had a duty of care, duty of disclosure, duty of advising as well as a duty of consideration of the buyer. Because of these duties the agent's role could be perceived as more of an intermediary, whereas in practice, the real estate agent is the seller's agent, and not an intermediary. This means that the law was not transparent with regards to the real estate agent's role (The Business Authority, 2013).

The real estate agent can start a dialogue with a potential buyer to give an offer on the real estate. This could seem as though the agent is not the seller's representative, if the agent opens the pricing debate. Since the real estate agent has to present all offers to the seller, and at the same time work to get the best possible price, it is therefore necessary to get the buyers to indicate their value of the real estate. Therefore it is possible that there are instances where the real estate agent opens the pricing debate for personal gain if the agent and seller has agreed to a performance-based fee, where the agent only get paid if the real estate gets sold (The Business Authority, 2013).

After the changes to the legislation in 2005 and 2006 The Business Authority described the requirements of the listing agreement as follows:

1. It must be written.
2. It must include terms about the agreement's length.
3. It must include specifications about the services included.
4. It must include information about the fee, which must pay for each service.
5. It must state all parties' name and adress.
6. It must identify the real estate, the agreement concerns.

7. It must state the estimated price.
8. It must state who is the real estate agent's guarantor.
9. It must include special relations, conditions or limitations regarding the specific task.
10. It must state the real estate agent's economic or personel relation, such as every personal or economic interest the agent has in A) concluding a sale or B) in the parties' choice of financing, insurance or other services in relation to the sale.
11. It must inform if the real estate agent does not have the listed interest in item 10 above.

Furthermore, it is required that item 10 and 11 as well as the total fee appear on the front page of the listing agreement. The listing agreement should be as clear and understandable as possible, so the seller can make an informed decision about whether or not to enter into the listing agreement with the real estate agent. Transparency is a fundamental element of the consumer protection in the law, but also for the competition and trust of the real estate industry (The Business Authority, 2013).

The law stipulated detailed requirements and duties of disclosures to the real estate agent. These requirements was to insure that the real estate sale was done in a secure way for the consumers and to make transparency in the listing agreement and the real estate agent's work. There had been an increase of the disclosures since 1999 based on the assumption that the more information the consumer had, the better they were equipped to make the right decisions. The consumer protections are best ensured when the consumers have an extensive foundation of information as possible, as this means the consumer has the best possible foundation for their decision (The Business Authority, 2013).

The Business Authority (2013) also describes the two fee types. When using performance-based fee, the whole payment will fall due when sold, no matter when the property was sold, and no matter how much the property was advertised. The performance-based fee is therefore dependent on the real estate agent's ability to and possibility of getting the real estate sold. The Business Authority describes this as a type of fee, where the sellers, who get their real estate sold quickly, indirectly pays for the sellers, who either do not get their real estate sold or where it takes a long time before the real estate is sold. By demanding a fixed amount the real estate agents has the possibility to obtain excess cover by some real estate sales, which can cover the cost in the cases where there is no sale.

The Business Authority describes that the purpose of adding the fixed fee was

> to strengthen the transparency of listing task while the consumers choices in relation to entering the agreement increases. The consumer is hereby given the possibility to increasingly only receive and pay for the services, which relates specifically to the sale of the consumer's own real estate. [...] This means that whether the real estate is easy or hard to sell can be taken into account. Owners of easily sold real estate is able to choose to pay after the time consumption and received services. The owners are able to avoid paying the agent's unsuccessful paid costs to other real estates, which were not saleable.

When choosing the fixed fee, the seller must pay for each service, the real estate agent delivers, whether the real estate agent gets the real estate sold or not or if the seller terminates the listing agreement before the period has ended. The Business Authority describes the fixed fee as a fee, where the real estate agent never gets excess cover of his

expenses, but the agent never works for free. It was also possible for a combination of the two fees.

The Danish Competition and Consumer Authority have pointed out that sellers often struggle with seeing through the real estate agent's fee throughout the years. It was also the Danish Business Authority's experience that the legislation about the fee was unnecessary difficult to understand for both the consumers and the agents. This caused a lot of disputes about the fee between the sellers and the real estate agents. The analysis from the Danish Business Authority therefore examines how the legislation can be formulated to make the consumers aware about what, when and for what they are paying for. If the consumers could better see through and negotiate the size of the fee, then the competition between the real estate agents was expected to strengthen (The Business Authority, 2013).

The Business Authority (2013) also looks into the trust in the real estate agents. The consumer attitude index measures the consumers' trust in the real estate agents among other things. Since 2006, the real estate agents' trust have been placed in the bottom of the service industry market and in 2011 the real estate agents was placed second to last out of 49 markets. The latest consumer attitude index is from 2018. The trust in the real estate agents have become slightly better, where it is placed as number 29 of out 40 markets (The Competition and Consumer Authority, 2018).

The real estate agent's services' complaints board was established in 2006. The complaints board among other things handles complaints about the real estate agent's fee, which account for about half of all complaints. One of the common complaints was that the real estate agent offers a fixed price for cost of documentations, which was higher than the actual price, and thereby higher than the price, the real estate agent can charge (The Business Authority, 2013).

It appears as though the initiatives to make the fee more transparent for the consumer and to help negotiate the agent's services, has not had the intended effects. The complaints board also informs that the real estate agents themselves in many cases do not understand, what they can charge for and what they cannot. According to The Business Authority, there is a risk that the formulation of the legislation leads to a misunderstanding of the listing agreement for what needs to be paid between both parties. The consumer are dependent on what the real estate agents have told them verbally. The complaints board indicates that there are many cases where the consumers says the oral agreement differs from the listing agreement (The Business Authority, 2013).

The Business Authority (2013) concludes that the legislation of the real estate agent's fee was characterised by a lot of detailed regulation, both in relation to the choice of fee and the specific duty of disclosures about price and services. Although the purpose of the legislation among other things was to strengthen the seller's possibility to see through the agent's fee, and thereby strengthen the competition, it did not have the desired effect. There was still confusion about the real estate agent's fee. In addition to this, it was a small amount of sellers, who negotiated the prices of the services, and the complex detailed regulation contributed to some disputes about the fee between the seller and the agent. The limitation of the fee was unusual, since other similar industries did not have such a limitation. As a result of the above, The Business Authority recommended to completely abolish the rules about the fee or alternatively to abolish all rules about the fee except the duty to specify the services as well as to add a duty for the real estate agent to clearly specify the maximum price, if the real estate involved was sold within the period, if the listing agreement expires without sale or if the agreement was terminated.

2.6 2015: The new law of real estate agents

The latest change in the legislation became effective on 1 January 2015 with Act no. 526. The essential changes with regard to our area of focus is the clarification of the real estate agent's role as the seller's representative, liberalisation of the rules of the fee and changes to the rules about the length of the listing agreement, where Dreyer and Simiab (2015) explains the legislation in detail.

In Section 24, the law states that the real estate agent has to perform good real estate agent practice and exercise care toward the principal. The agent further has to be instrumental in making the sale and ensuring that the sale is completed within a period, to a price and on the conditions, which are agreed upon with the seller. If the real estate agent does not perform good real estate agent practice, then his fee will be reduced or he loses the claim of the fee. A change to the real estate agent's role is that he no longer has to exercise care towards both parties. He does, however, still need to look into the buyer's economic conditions after the seller has accepted the purchase offer to secure the sale's completion. It is written in the Section 24 (2) that "the real estate agent has an obligation to advise seller and protect seller's needs and interests" to make it clear that the real estate agent is seller's representative (Dreyer and Simiab, 2015, Chapter 5).

The real estate agent still cannot be an agent to both parties of the same sale, which is stated in Section 26. This is explained as follows:

> [N]o agent can protect buyer's and seller's interests equally well in the same real estate sale. Both parties can be in dire need of their own adviser, which they pay for themselves. It is not necessarily the same services, the seller and the buyer demands in connection to the sale. Their interest with regard to for example price and shortcomings are opposite.
>
> (Dreyer and Simiab, 2015, Chapter 5.4)

This was the same argument made in the legislation of 1994. Section 26 (3) states the real estate agent had to advise the buyer to seek guidance if they had yet to do so (Dreyer and Simiab, 2015, Chapter 5). Section 28 states only one real estate agent can be appointed to each listing agreement, where the agent is responsible for all the tasks relating to the sale, but this does not mean other agents in the agency cannot manages these tasks under the supervision of the responsible real estate agent (Dreyer and Simiab, 2015, Chapter 5).

The real estate agent's mandatory services are stated in Section 37, and they include:

1. Valuation of the property and agreement on the listing price, the property is on sale for, with the seller.
2. Calculation of the sales proceeds.
3. Preparation of the particulars of sale with the information about the property, which are necessary to make a purchasing decision.
4. Drawing up the purchasing agreement.

The mandatory services are more specified, than they were in The Competition Authority (2009). Besides that, they still cover what the previous services.

When evaluating the property, that is put on sale, the law states:

> When appraising the property the real estate agent has to state the asking price for which the property is estimated to be able to sell within a certain

> period. The real estate agent has to take into consideration the market conditions, the property's location and layout and furnishings as well as the property's age and condition. Act no. 526 Section 27 (2015)

The certain period refers to the length of the listing agreement. This period was typically six months, but it is now possible to make the period longer. With very long periods, it can be necessary to revise the appraisal and thereby the asking price. This will lead to deciding on a new listing agreement (Dreyer and Simiab, 2015, Chapter 5).

The listing agreement must be written and signed by the seller and the real estate agent. It has to include who the responsible real estate agent is, and the size of the fee or the calculation principle of the fee. If a performance-based fee is chosen, and if there needs to be a paid fee at termination, the length of the listing agreement also has to be included. The listing agreement also have to include identification of the property and the real estate agent's valuation and the listing price among others. The listing agreement can be terminated without warning by both parties. The Danish Business Authority can give more specific rules about the listing agreement terms (Dreyer and Simiab, 2015, Chapter 5, §29).

Whereas there used to be two different types of fee, there is now no longer any limitation in calculating the fee. This has been done to regulate the fee formalities between the consumer and the real estate agent as little as possible. The real estate agent still has a claim on the fee, that is agreed upon with the seller (Dreyer and Simiab, 2015, Chapter 5, §29). When asked about how the liberalisation of the fees would make it more clear and consumer-friendly, the Minister of Business and Industry said in 2014

> We are dealing with a high degree of detail regulation, which is not known for other service trades such as auditors, lawyers and craftsmen. The current rules hinder the real estate agents and sellers in agreeing, how they prefer the fee to be put together. [...]
>
> It is therefore the purpose of proposed law's provisions about the fee – besides securing the parties freedom of contract and simplify the rules in the field – to give the real estate agent the possibility to develop new concepts for the benefit of the consumers and for the competition in the field.
>
> (Dreyer and Simiab, 2015, Chapter 5.11)

The real estate agent's claim of fee has not changed much. Section 32 state that the real estate agent can claim the fee which was agreed upon in the listing agreement, as well as his expenses with regard to the sale. It still applies that

> if after the listing agreement has ended, a purchase agreement has been entered on the grounds of the agent's effort and without another agent's participation, the real estate agent have a claim of the fee.
>
> Act no. 32 (2) (2015)

This claim of loss of fee is explained in Section 31. When a performance-based fee is agreed upon, the listing agreement can have a length of at most six months for the real estate agent to have a claim on a reasonable fee, which usual is one fourth of the agreed fee rate times the last agreed listing price. The listing agreement can be extended three months at a time. If the listing agreement is longer than six months when using a performance-based fee, the real estate agent have no claim of loss of fee. The no-cure no-pay principle only applies to the real estate agent's fee, whereas the seller still has to pay for her expenses for the sale. For other fee types than the performance-based, the

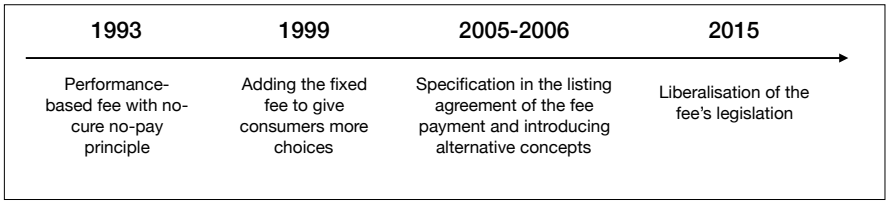


Figure 2.2 Timeline of the changes in the legislation of the fee.

real estate agent has a claim of fee if it is agreed upon. The real estate agent can claim the fee of the services he has provided before the agreement was terminated, no matter who terminated the agreement (Dreyer and Simiab, 2015, Chapter 5).

2.7 Summary

Figure 2.2 gives an overview of the changes. The authorities' aim to make the fee transparent towards the consumers ended up making the listing agreement harder to understand, since the way it was presented in the listing agreement had to be very specific about which of the services the real estate agent provided and what the consumer had to pay for. They also aimed to give the consumer more choices for the fee payment by introducing alternative concepts instead of the traditional performance-based concept, which paid off. The consumers interested in such concepts also had to take on some of the responsibility of getting the property sold, such as showing the property. The real estate agencies have embraced these alternative concepts so they can service all types of consumers. Some real estate agencies advised the seller on being in charge of the sale, and they get paid a fixed fee for this. Unfortunately, there are no current numbers about the breakdown of real estate sales using either a performance-based fee or a fixed fee.

In the legislation, the real estate agent's role went from representing either party to clarifying his role as the seller's representative, whereas the buyer typically recruits an adviser. In practice the real estate agent have typically always been the seller's representative, which is now also reflected in the law.

3 Asymmetric Information of Real Estate Agents

In the following we will present different models for evaluating the asymmetry in the real estate business, before presenting our model in Section 4. These models will describe different ways to incentivise the agent to get the best price for the property as well as look into the risk-sharing between the agent and the seller, the agent's role, different types of fee and mistrust in the real estate agents. We start by describing what kind of problems there are in a contractual relationship and how the asymmetric information can be handled by moral hazard, adverse selection and signalling.

Macho-Stadler and Perez-Castrillo (2001, Chapter 1) describes the informational asymmetry within contractual relationships using three problems. First, the moral hazard problem, where after the contract is signed, the agent's behaviour is not observable or verifiable. For example, in real estate, the seller cannot observe how much effort the real estate agent puts into selling her property for example. Second, the adverse selection problem, where before the contract is signed, either the agent or the principal has private information about themselves. In real estate, it might be that the seller does not know how knowledgable of the market or how experienced the real estate agent is. And lastly, the signalling problem, where before the contract is signed, the informed party reveals the private information before the contract is formalised. Therefore to investigate the relationship between the real estate agents and the sellers, we will see how asymmetric information affects the contracts. Macho-Stadler and Perez-Castrillo (2001, Chapter 1) does not assume perfect information of either party, but that they are equally informed. That means there can be some random elements affecting the relationship. To represent this, they introduce that Nature is deciding something, such as a person's type in adverse selection, for example.

There are three reasons to explain the conflict of interest between the agent and the seller. First, while the seller cares about the outcome, the agent does not to the same extent. Second, the agent does however care about the effort he supplies, where the seller doesn't. And lastly, that generally supplying more effort will result in a better outcome. Therefore, as Macho-Stadler and Perez-Castrillo (2001, Chapter 2.2) describes "the payoff that the principal pays the agent compensates him for the effort that the principal demands, thus a part of what the principal earns from the relationship ends up in the agent's pocket." If the agent rejects the contract, he has to look for other opportunities in the market. The expected utility the agent can get from these opportunities is called reservation utility. The agent will accept the contract as long as his reservation utility is lower than or equal to the expected utility in the contract.

Macho-Stadler and Perez-Castrillo (2001, Chapter 1) explains the moral hazard problem as when the agent's action is not verifiable.The participants have the same information before the contract is signed, and it is first after the contract has been signed that the asymmetric information arises, because the principal cannot observe or verify the effort of the agent. The agent's payoff cannot depend on the effort that he offers.

Moral hazard can also arise when only the agent receives private information after the contract is signed. The asymmetric information then affect the agent's actions after he has gained the private information. We will let Nature determine the private information the agent receives. It is important to notice that before the contract is signed, the agent does not know the private information, but he will know the private information before performing his actions. This means he will know – continuing the example – if he is performing his action under good market conditions or under bad market conditions (Macho-Stadler and Perez-Castrillo, 2001, Chapter 3.6).

Typically in a principal-agent relationship it is the principal who offers the contract, however in real estate it is the agent who draws up the contract. In the situation with moral hazard, Macho-Stadler and Perez-Castrillo (2001, Chapter 3.7.4) clarifies that it is still the principal who gets the result of the relationship, but the principal can only accept or reject the contract. They describe that "when the agent designs the contract, he must take into account the fact that the principal will only accept believable contracts, that is, those contracts under which the agent will effectively offer that effort that he announces in the contract." They state that the only difference is that it is the principal, who is put at her reservation utility.

A more detailed definition of adverse selection is when the agent holds private information before the relationship has begun, as Macho-Stadler and Perez-Castrillo (2001, Chapter 1) describes. The principal can verify the agent's behaviour, but the final outcome depends on the agent's type. When asymmetric information concerns personal characteristics of the agent, the principal cannot distinguish between different types of agents. This can be modelled by Nature choosing the agent's type, which is only known to the agent.

Lastly, Macho-Stadler and Perez-Castrillo (2001) describes signalling, which is similar to adverse selection partly because we again have two types of agents. After knowing his type the agent can send a signal before signing the contract to the principal. This signal can then influence the principal's beliefs about the agent's type.

3.1 A model for risk sharing

If we first look at the case with symmetric information and common uncertainty, Anglin and Arnott (1991) describes that the seller hires a real estate agent because the agent has a comparative advantage with his experience in selling properties. Since both the seller and the agent are equally informed, the seller can observe how much effort the agent puts into selling the property. The seller would then state in the contract that she would pay the agent a certain amount contingent on a specified level of effort. However, the real estate agent and the seller do not have symmetric information. In Anglin and Arnott's paper, the seller must design the listing agreement so it incentivises the agent to reveal his type (adverse selection) and to exert the desired level of effort (moral hazard) in order to handle the asymmetric information between the real estate agent and the seller.

Anglin and Arnott (1991) first sets up the symmetric case, where the expected utility for the seller is modelled with two outcomes:

$$EU = (1 - p(e))u(y_0 - R_0) + p(e)u(y_1 - R_1), \tag{3.1}$$

where the good outcome is denoted by 0 and the bad outcome is denoted by 1, and $p(e)$ is the probability and it depends on the effort the agent exert, where more effort leads to a lower probability of a bad outcome. $u(y_i - R_i)$ is the seller's utility function

for $i = 0, 1$. In each outcome the seller and the agent shares y_i, where R_i is the payment to the agent. The agent's utility function is $v(R_i) - e$ and his expected utility is

$$EV = (1 - p(e))v(R_0) + p(e)v(R_1) - e. \tag{3.2}$$

The symmetric maximisation problem, referred to as the first-best contract, is then

$$\max_{R_0, R_1} \quad (1 - p(e))u(y_0 - R_0) + p(e)u(y_1 - R_1) \tag{3.3}$$

$$\text{subject to} \quad (1 - p(e))v(R_0) + p(e)v(R_1) - e \geq \bar{V}, \tag{3.4}$$

where the constraint is the participation constraint, which says the agent's expected utility needs to be larger or equal to what he can get outside the contract, which is his reservation utility, $\bar{V}$. The results of the model shows that

> For small $y_0 - y_1$, $\frac{R_0 - R_1}{y_0 - y_1}$ is approximately the *marginal* remuneration rate, that is, the increase in the agent's remuneration with an extra unit of output.
>
> Anglin and Arnott (1991)

If both the agent and the principal is risk-averse, they would share the outcome, and the agent's fee rate would be 50 %, and if the agent is risk-neutral and the principal is risk-averse the agent would bear all the risk and therefore get a fee rate of 100%.

In the model with moral hazard, the agent can now choose how much effort is needed to maximise his expected utility. Anglin and Arnott (1991) uses the first-order approach, which takes the first derivative of the agent's expected utility and set it equal to one. This constraint says that the agent will choose the level of effort that maximises his expected utility given the contact has been accepted and effort is not verifiable. This is called the incentive–compatibility constraint. The maximisation problem with moral hazard is then

$$\max_{R_0, R_1, e} \quad (1 - p(e))u(y_0 - R_0) + p(e)u(y_1 - R_1) \tag{3.5}$$

$$\text{subject to} \quad (1 - p(e))v(R_0) + p(e)v(R_1) - e \geq \bar{V}, \tag{3.6}$$

$$-p(e)(v(R_0) - v(R_1)) - 1 = 0. \tag{3.7}$$

The results shows

> The optimal contract entails a marginal remuneration rate which exceeds that under the first-best contract, and the marginal remuneration rate exceeds the first-best commission rate by more the more moral hazard. Thus, with moral hazard the agent bears more risk. Anglin and Arnott (1991)

To see how the real estate agent and the seller bears the risk, Anglin and Arnott describes different situations of the risk sharing:

Both are risk-neutral The seller should get the same in both outcomes and the agent is the residual claimant.

Risk-averse seller and risk-neutral agent The agent bears all the risk, and the agent's incentive in to exert effort is in full effect.

Risk-neutral seller and risk-averse agent Here the trade-off between risk-bearing and incentives is in full effect. If the seller bears all the risk, and the agent is paid the same independent of the outcomes, the agent will not exceed any effort.

Both are risk-averse In this situation, they will weight the efficient risk-sharing against the agent's incentive to exert effort.

When applying the model to real estate contracts between the seller and the real estate agent in the Danish market, a good outcome is a sale at a fixed price, so $y_0 = P$, and a bad outcome is no sale by this agent, so $y_1 = \tilde{P}$ the price sold by another agent. The fee to the real estate agent is a percentage of the price, so $R_0 = cP$, where c is the fee rate and $R_1 = 0$. The fee rate is then $\frac{cP}{P-\tilde{P}}$, which is larger than the fee rate. Anglin and Arnott (1991) states

> The fact that the real estate agent is paid nothing if she fail to sell the property within the time specified by the brokerage contract has the effect of increasing the marginal remuneration rate. Hence, by setting the commission rate and the duration of the brokerage contract appropriately, it may be possible to achieve second-best efficient incentives and risk-sharing while satisfying the participation constraint. This interpretation illustrates the use of termination as an incentive device.

We can see that this is the case for the no-cure no-pay principle. By adding this principle, it incentivised the real estate agent to get a sale before the listing agreement ended, even if the price was not the optimal. In the Danish real estate market, the listing agreement's duration can at most be six months with performance-based fee, and can be extended by three months at a time. But it might be that the seller would rather find another agent at the end of the six-month period. This would then lead to the same problem, where the property is sold for a suboptimal sales price, because otherwise the agent does not get paid.

With adverse selection, Anglin and Arnott (1991) sets up the model with two types of agents, where the seller and the agents are risk-averse. Anglin and Arnott describes the model:

> The model's basic mechanism is to offer a pair of risk-sharing contracts, one of which would be chosen by the competent agents, the other by the incompetent agents. The contract designed for the competent agents provides a higher return with the good outcome but less insurance than the contract designed for the incompetent agents. Since incompetent agents have a higher probability of a bad outcome, they value the insurance more highly than competent agents.

The expected utility of an agent of type $i, j = C, I$, where C is competent agents and I is incompetent agents, is

$$V^{i,j} = (1 - p^i)v(R_0^j) + p^i v(R_0^j), \tag{3.8}$$

where $p^I > p^C$. The adverse selection problem for a seller to hire competent agents is then

$$\max_{R_0^C, R_1^C} \quad (1 - p^C)u(y_0 - R_0^C) + p^C u(y_1 - R_1^C) \tag{3.9}$$

$$\text{subject to} \quad (1 - p^C)v(R_0^C) + p^C v(R_1^C) \geq \bar{V}^C, \tag{3.10}$$

$$(1 - p^I)v(R_0^C) + p^I v(R_1^C) < \bar{V}^I. \tag{3.11}$$

This contract will attract competent agent because competent agent get their reservation utility, and incompetent agents are discouraged because they do not get their reservation utility. The last condition is the incentive–compatibility constraint in adverse

selection. The adverse selection problem for a seller to hire incompetent agents is

$$\max_{R_0^I, R_1^I} \quad (1-p^I)u(y_0 - R_0^I) + p^I u(y_1 - R_1^I) \tag{3.12}$$

$$\text{subject to} \quad (1-p^I)v(R_0^I) + p^I v(R_1^I) \geq \bar{V}^I, \tag{3.13}$$

$$(1-p^C)v(R_0^I) + p^C v(R_1^I) < \bar{V}^C. \tag{3.14}$$

Anglin and Arnott uses the two types of agents' indifference curves to illustrate the differences between a competent agent and incompetent agent in Figure 3.1. In the figure we can see that the competent agent have steeper indifference curves than the incompetent, because $p^I > p^C$. X determines the optimal contract if there were no incompetent agent and Y determines the contract for the competent agent when there are incompetent agents. The Y contract is offered to discourage incompetent agent to disguise themselves as competent because the Y contract provides a lower return for a bad outcome and a higher return for a good outcome. The incompetent agent will therefore not choose this contract because his probability for a bad outcome is higher than the competent agent's probability for a bad outcome. The competent agents therefore bears more risk than the incompetent.

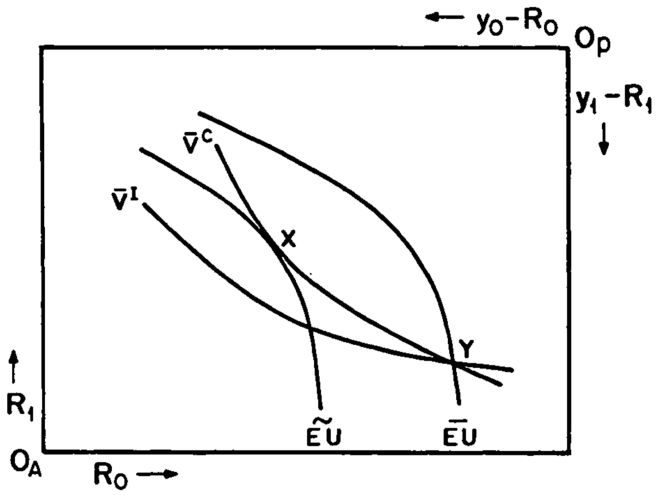


Figure 3.1 The indifference curves of the competent and the incompetent agents in the adverse selection problem described by Anglin and Arnott (1991)

We again look at the situation, where a good outcome is a sale and a bad outcome is no sale. Anglin and Arnott (1991) then describes the contract as having two contract parameters – the fee rate and the length of the contract. These two parameters can then be used to differ the competent and the incompetent agents. This means that "less competent agents will be willing to sacrifice more commission for a given increase in contract duration." The seller and the agent will therefore share the risk. In Denmark the fee rate does seem to vary across regions, but it is somewhat fixed within the regions. Due to the fixed rate, then both types of agents will want the contracts to last a long period, because the probability of no sale is lower. Another reason to have a long contract period is that if the property is not sold within the period it sends a signal of a incompetent agent, where sellers will switch to a new agent.

Anglin and Arnott (1991) concludes that they find no reason for the widespread performance-based fee, because it fails to allocate risk efficiently and it does not in-

centivise the agent to reveal himself, since incompetent agents pretends to be established.

In Figure 3.2, we can see how long single-family houses are on the market[1] from 2004 to today. In Copenhagen, houses are mostly sold within six months, which is within the listing agreement's length of six month for performance-based fee. However, the average time on the market for Denmark is between six months and nine months today, which would mean the listing period is typically extended three months more. In Lemvig, single-family houses have gone from being on the same level as the average and Copenhagen in 2004 to reaching two years between 2018 and 2019. This is well above the performance-based fee's listing duration. This seems as though the seller does not accept suboptimal sales prices, because otherwise the duration would be closer to the six months. On the other hand, sales prices could be suboptimal in Copenhagen, since the time on the market is below six months, but it is hard to tell just from the time on the market.

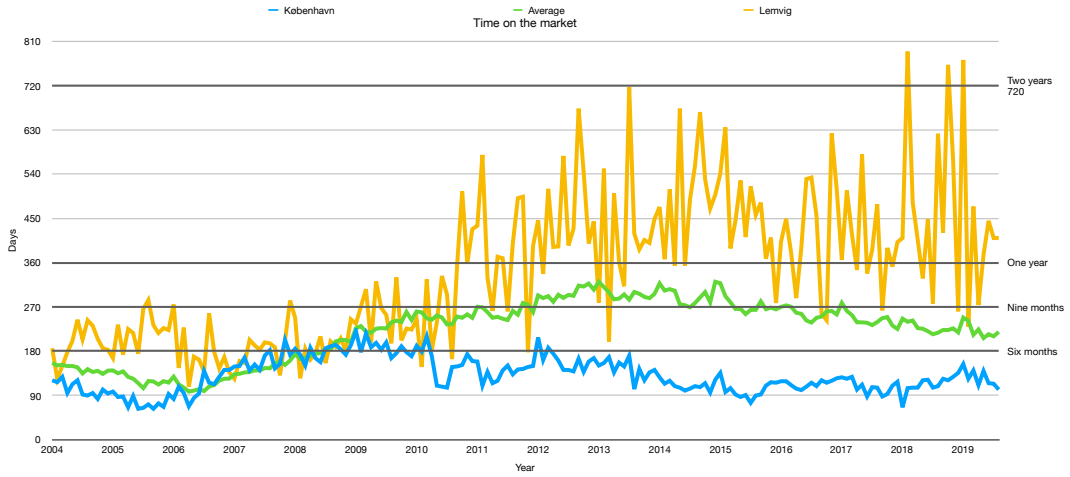


Figure 3.2 The time on the market for Copenhagen and Lemvig as well as the average of Denmark.

3.2 Models for determining which fee gives the best incentives

Where as Anglin and Arnott (1991) argued that the performance-based fee does not work to give the agent the best incentive with respect to either moral hazard and adverse selection, the two following papers argues that it actually is a good way to incentivise the agent.

3.2.1 Using real estate agents to attract buyers

Anglin (1994) sets up a model for the contract between the real estate agent and the seller, where the agents "assist a seller in two ways: by directing more buyers to that seller and by providing information and general assistance in final negotiations. Thus, the broker is in a position of moral hazard common to principal-agent problems, which alters the effect of 'competitive presure' ". He assumes that it is the real estate agent's

[1] Boligudbudsstatistikken UDB030: `https://rkr.statistikbank.dk/204`.

job to find buyers, and that the listing agreement "has little influence on the amount of information provided" by the real estate agent. In Anglin (1994)'s model the real estate agent and the seller are dependent on each other, because "if the seller anticipates that the real estate agent will find many buyers, then the seller will demand a higher price, which reduces the incentive for the real estate agent to find more buyers."

The model starts with the buyers observing the housing market randomly according to a Poisson process with arrival rate μ and the buyers make a take-it-or-leave-it offer to the seller. The seller wants to maximise her expected price, P, for the property, and hires a real estate agent to increase the number of buyers who looks at the property. The real estate agent knows the type of house being sold and which buyers would prefer that type, he does not know the taste of the buyers, so his buyers also follows a Poisson process but with an arrival rate of λ. The real estate agent gets paid a performance-based fee at the time of sale and the seller gets the sales price subtracting the fee payment, and the contract is assumed to have infinite duration (Anglin, 1994).

In this model the seller's expected utility is determined by future events. A potential buyer either arrives with μ or with λ and offers P. If P is larger than the listing price, the seller accepts. If P is lower than the listing price or no buyer arrives, the property stays on the market. Anglin (1994) assume the seller have some alternative worth, $\bar{U} > 0$, so that the she only hires a real estate agent if her expected utility is larger than $\bar{U}$. The real estate agent want to maximise his expected utility and he does this by getting more potential buyers and therefore wants λ to maximise his expected utility. Anglin find that an increase in the fee rate would increase λ, so if the real estate agent gets paid more, he will find more buyers. The performance-based fee incentivises the real estate agent to more buyers, when the duration is infinite.

3.2.2 Performance-based fee versus fixed fee

Arnold (1992) also writes about the asymmetry between the seller and the real estate agent. In his paper, the seller's problem is to design a contract that incentivises the agent to adopt a selling strategy that maximises the seller's expected payoff. The real estate agent have information about the market conditions, and therefore plays an important role acting as an agent to the seller and as an intermediary matching sellers and buyers. He presents three different fee types to see which gives the best incentives for the real estate agent to sell the property for the seller. Again the real estate agent is hired to attract more buyers, but in this model he also advises the seller in setting the minimum price, negotiates with the buyer, and handles the legal obligations. Arnold (1992) looks at the principal-agent problem in the real estate business in two ways:

> The principal-agent problem is manifested in two ways. First, if the owner is unable to monitor the broker's search activity, the broker may have an incentive to provide an inefficiently low level of effort. Second, because homeowners are infrequent market participants, they are not fully informed of demand and supply conditions in the housing market. Brokers, on the other hand, are well informed of market conditions. Therefore, the owner frequently relies on the broker to guide her in setting a reservation price. This informational asymmetry can create an incentive for the broker to misrepresent market information.

To model the market, Arnold (1992) uses a sequential search model[2], where offers are received sequentially. The optimal minimum price for the seller and for the real

[2]Search models illustrate how best to balance the cost of delay against the value of the option to try

estate agent is determined by the distribution of offers, the cost of soliciting offers, the benefits and costs related to owning the property while attempting to sell it, the discount factor, and the arrangement for paying the broker. The seller therefore also have the same information as the real estate agent in this model, so we can compare the different contracts. A difference between the minimum prices indicates a conflict between the seller's and the agent's objectives. The three different fee types are fixed-percentage commission, fixed fee and consignment[3], where we will not discuss the last one, since it does not exists in the Danish real estate market.

Arnold (1992) starts by describing the situation without a real estate agent, where no incentive problems exists. The seller can either accepts the price the current buyer offers or wait for a better offer by another buyer. She therefore has to choose the offer that maximises her expected revenue by selecting the greater value of the two offers. Hereafter Arnold looks at the situation where she hires a real estate agent, that is paid with a fixed-percentage fee. Given that the seller and agent both knows the market, we can compare their minimum prices. If a fee rate exists which aligns their objectives, then they will have the same minimum price, and the agent will provide the actual market conditions to the seller and the seller's assessment of the market will be accurate. If there does not exist a fee rate which align their objectives, the agent will not represent the current market accurately and the seller will have an suboptimal minimum price. Because the seller now have to pay the real estate agent a percentage of the price, she now has to maximise the greatest value of either the offer by the current buyer subtracting the payment to the agent, or wait for a better offer by another buyer. The real estate agent also wants to maximise his expected return, and like the seller, he chooses the greatest value of either his payment or his cost of continuing searching for a buyer. With this model, a incentive-compatible fee system is possible, and the seller and the agent will choose the same minimum price.

If the agent is paid with a fixed fee, Arnold (1992) writes

> The flat-fee payment system in which the broker is paid a predetermined fee upon completion of the sale creates signifiant incentive problems. This system contrast with the commission system in which the broker's payment increases with the selling price. Because the flat-fee is independent of the selling price the broker does not benefit from the profits gained by acquiring a higher price. Therefore, the broker prefers the lowest reservation price possible – a lower reservation price translates to a quicker sale and minimises his (expected) search costs. Trivially, this implies the broker will set a reservation price of zero and sell the house to the first buyer solicited. More realistically, the broker will attempt to influence the owner's choice of a reservation price by suggesting that buyer valuations for the house are significantly below their actual levels.

Arnold (1992) concludes that to overcome the principal-agent problem between the seller and the real estate agent the fixed-percentage system is the most effective.

Anglin (1994) finds that the performance-based fee does incentivise the agent, because by letting his payment depend on the sales price he has incentive to find more buyers given infinite duration. By using a search model Arnold (1992) finds that letting

again.

[3]Consignment is defined as merchandise shipped to an agent or customer when an actual purchase has not been made, but under an agreement obliging the consignee to pay the consignor for the goods when sold.

the real estate agent's fee payment depend on the listing price the real estate agent have an incentive to get the best sales price contrary to Anglin and Arnott (1991). Anglin and Arnott argues that the time on the market reduces the agent to find the best sale before the agreement's duration has ended. Arnold argues that the real estate agent has to choose between his cost of continuing searching for buyers or his expected return given sales price offered. Since the seller and the agent both know the market, the performance-based fee gives the agent the best incentives to find the sales price that gives him and the seller the best expected return, when the duration is infinite. Because the fixed-fee does not depend on the sales price, the real estate agent does not have an incentive to find the maximum sales price, since he is paid the same fee always.

3.3 Empirical analyses of real estate agents

The two empirical analysis' looks into how the internet has affected the real estate agents, the real estate agent's role as experts and the trust in the real estate agents.

3.3.1 Real estate agents as experts

Levitt and Syverson (2008a) looks at the real estate agent as experts within their field. They claim individuals rely heavily on the guidance of the real estate agents because of their expertise. One characteristic of hiring an expert is the informational advantage enjoyed by the expert relative to the client seeking advice. The real estate agent knows the market better than the seller, because the seller is only interested in the market, when she needs to sell. Because of the informational advantage, the real estate agent can mislead the sellers by providing unnecessary services, or setting a higher price, or distorting the information in order to maximise his expected payoff.

With a performance-based contract, the agent only receives a small percentage of the property's selling price while bearing much of the cost of selling the property. This cost includes showing the property to potential buyers, hosting open houses and advertising the property. According to Levitt and Syverson (2008a), this leads to misalignment of incentives between the agent and the seller. They state that the agent has strong incentives to sell a property quickly and at a lower price, and encourage the seller to "accept suboptimally low offers too quickly." A rational seller will however take the agent's distorted incentives into account and ignore the agent's advice whether to accept an offer or not. When the agent is better informed, the agent should advise the seller to accept any offer, which maximised the agent's expected payoff, and the seller will therefore be influenced by the agent's advise.

Levitt and Syverson (2008a) investigates the distorted incentives of the agent by looking into whether there is a difference in the sales price between a property owned by the seller and a property owned by an agent to see. They find that properties owned by the agents sell for more and stay on the market for longer than properties owned by sellers. They state that "this basic result is consistent with information distortion on the part of agents". They also find that "the gap between agent-owned homes and client-owned homes is largest in instances where agents are likely to enjoy the greatest informational advantage." The rise of internet made it easier for seller to observe the other properties for sale and find the recent sales prices of the listings, thereby reducing the informational advantage for the real estate agents. This means that the informational advantage real estate agents had before the internet is now smaller, thereby making the difference between an agent-owned home and a seller-owned home smaller. Levitt and Syverson (2008a) concludes that

> Experts hold valuable information. This information is helpful to those who hire them, but can also be a source of welfare-reducing distortions.
>
> [...]
>
> The combination of real estate agents' information advantage and the form of commission received combine to create distortions from first best. Homeowners are induced by their agents to sell quickly and at a price that is too low.

Levitt and Syverson (2008a) also argues that the performance-based fee can cause problems, since the agent bears most of the cost, or as Anglin and Arnott (1991) states, most of the risk. They find that before the internet the real estate agents had more informational advantage than they do today. Because of the internet, the seller can now get a picture of how long their property might stay on the market, and for what it can sell for. This mean that the performance-based fee performs better today than it did before the real estate data on the internet, because the seller is more aware of the market today to see if the listing price the real estate agent suggest is realistic.

3.3.2 Antitrust of real estate agents

Levitt and Syverson (2008b) also writes about the difference between fixed-fee agents and performance-based fee agents. They state that real estate agents have been better at keeping their position than other agent-based consumer markets when the internet became easily available to the consumers, as Levitt and Syverson (2008b) describes

> Those involved in the market have turned to the internet in multiple ways, greatly expanding consumers' access to residential real estate information. A larger fraction of homes for sale are listed on the internet, complete with detailed house specifications, virtual tours, and neighbourhood profiles. Potential buyers can easily peruse dozens or even hundreds of listings, eliminating less appealing possibilities without ever taking the time to visit a house. Those interested in selling, buying, or simply holding residential real estate are now able to review public records of sales, ownership, and taxes, among others.

One of the reasons, the real estate agents have kept their position, is that buying or selling a property is a big financial decision, which consumers only make a few times in their life. Therefore it preserves an important role for a agent-based relationship.

Through an empirical analysis Levitt and Syverson (2008b) looks into the difference between traditional real estate agents, who performs all services for the seller and gets paid using performance-based fee, and discount agents, who does not perform all services thereby letting the seller take on some of the services and the agent gets paid a fixed fee. Their analysis aims to answer whether properties sold by discount agents take longer to sell and whether sellers sell their property for less by using a discount agent. A difference between discount agents and traditional agents gives insight to the efficiency of discount agents. They find that properties sold by discount agents sell at the same price point as properties sold by traditional agents. But the expected days the property stays on the market are higher for properties sold by the discount agents, because they have a lower probability of sale. They also find that the sellers, who use discount agents have lower costs, even taking into account the longer time on the market and their own cost for taking on some services themselves.

Levitt and Syverson (2008b) also looks into the welfare of the sellers using fixed-fee agents. They find that the sellers paid less for a fixed-fee agent than a performance-based fee agent. The fixed-fee agent does not have a clear impact on the price the property sells for. The seller using a fixed-fee agent have additional costs for the services they do themselves such as marketing. They do adress that "the sellers who stand to benefit the most from using flat-fee agents (well informed, internet savvy, and so forth) are in fact those that the data show to be most likely to use such agents." They conclude that the sellers who use fixed-fee agents are not worse off than sellers who use a performance-based agent.

The empirical examination by Levitt and Syverson (2008b) applies to the Danish Consumer Authority's aim to introduce alternative concept with "discount" agents. As Levitt and Syverson's concludes some sellers are interested in being more involved with the process prefer a concept where they for a lower price can take on more responsibility. On the other hand, some sellers would rather the real estate agent handles the whole sale.

4 A Model for Danish Real Estate Agents

To model the listing agreement in the Danish real estate market, we are focusing on the traditional performance-based fee, which still is the most common as the authorities conclude. In the model the real estate agent sets the listing price, p, according to the state of market. We are interested in whether the real estate agent have an incentive to misrepresent the state of the market, given the lack of trust in real estate service measured in the consumer attitude index (see Section 2). We are also interested in how the fee is calculated in small versus larger cities, that is, a competitive market and a monopoly. Since there is a possibility for price adjustment in the listing agreement, we will look at two periods to see whether the price is too high to lure customers into the real estate agent's agency.

The probability of selling the property depends on the listing price and the state of the market, $\alpha \in (0,1)$, which is set by Nature. The lower the value of α the better the state of the market is. There are two outcomes, either a sale or no sale, and the probability of these two outcomes should sum to 1, therefore the probability of no sale is given by $1-q(p;\alpha)$. We model the probability of selling as a decreasing linear function of the listing price and the state of the market, so

$$q(p;\alpha) = \max(1-\alpha p, 0). \tag{4.1}$$

This means that with a low price there is a high probability of selling and with a high price there is a low probability of selling as Figure 4.1 shows. With $q(p;\alpha)$, αp needs to be less than 1 for the probability to be between 0 and 1, otherwise the probability of selling will be zero. We will therefore assume $0 \leq p < 1/\alpha$, and we can write the probability as

$$q(p;\alpha) = \begin{cases} 1-\alpha p, & 0 \leq p < 1/\alpha, \\ 0, & \text{otherwise} \end{cases} \tag{4.2}$$

As mentioned above, we will assume the contract can have two periods, $t \in \{1,2\}$, and therefore allows for different prices in the two periods, so we have a listing price for the first period p_1 and a listing price for the second period p_2. The seller wants a fast sale and a high price, so her utility depends on the time and the listing price. The seller's utility function is given by $U(p_t;t) = p_t(1-r)$, where we assume $U(p_t;t)$ is concave increasing, so $U' > 0, U'' \leq 0$. The real estate agent's rate is $r \in [0,1)$ and the fee payment is given by $p_t r$. The indifference curves for $U(p_t;t) = \underline{U}$ is shown in Figure 4.2. If the house is not sold by the end of the listing period, the seller does not pay anything to the real estate agent's fee and her utility is U_0, which is what represents the property's worth- We will use expected utility to measure the different outcomes. The seller's expected utility for the two periods is then given by

$$\mathbb{E}[U] = q(p_1;\alpha)p_1(1-r) + (1-q(p_1;\alpha))\Big(q(p_2;\alpha)p_2(1-r) + (1-q(p_2;\alpha))U_0\Big)(1-\delta), \tag{4.3}$$

where the second period is discounted with $\delta \in [0,1)$ as the discount factor where $\delta = 0$ is no discounting. The real estate agent's utility is given by $V(p_t;t) = rp_t$, and in the case

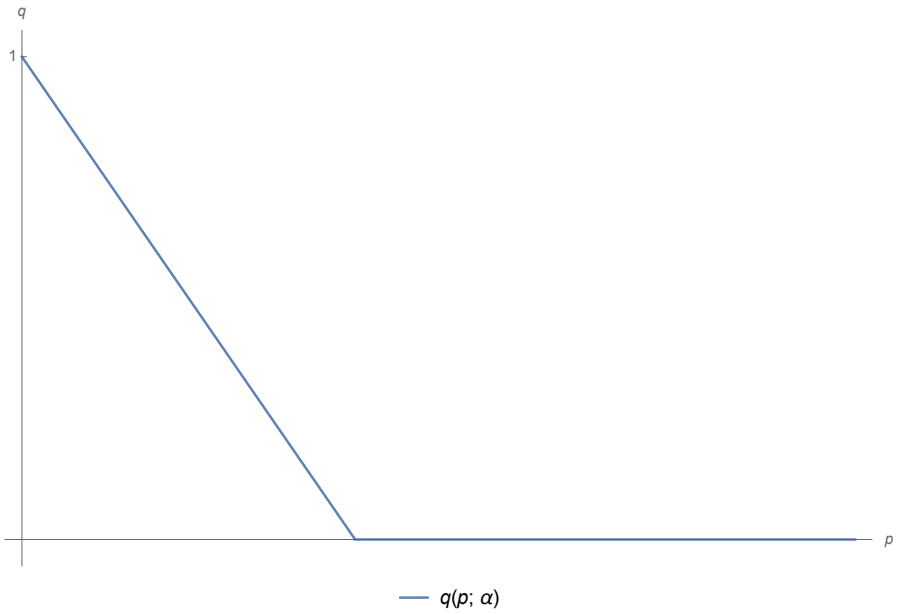


Figure 4.1 The probability of selling the property.

with no sale, the real estate agent does not get anything. As described in Section 2, there are other costs of the real estate sale besides the fee payment to the real estate agent. This cost typically includes marketing fees, legal documents and so on. We assume that this cost is always covered by the seller, and the fee payment is excess of these costs. So it is only the performance-based fee payment to the real estate agent, which we are interested in. The agent's indifference curves for $V(p_t;t) = \underline{V}$ is shown in Figure 4.2. His expected utility is for the two periods given by

$$\mathbb{E}[V] = q(p_1;\alpha)rp_1 + (1 - q(p_1;\alpha))\Big(q(p_2;\alpha)rp_2\Big)(1-\delta). \tag{4.4}$$

We can see the conflict of interest between the two parties in Figure 4.2. Where the real estate agent wants a higher rate for the same price, when his utility increases, the seller wants a lower rate for the same price as her utility increases. The conflict of interest therefore concerns the rate, since both parties are interested in a high price.

4.1 Symmetric information

With symmetric information both the seller and the real estate agent knows the state of the market, and thereby the true value of α to see whether the listing price the agent sets is the best price. This means the real estate agent cannot misrepresent the state of the market. We will start by setting up a model for competition and hereafter a model for monopoly.

4.1.1 Model for competition

In a competitive market the real estate agent wants to get the real estate sale, so he set his expected utility, $\mathbb{E}[V]$, equal to or larger than zero while maximising the seller's expected utility, $\mathbb{E}[U]$. He will therefore only propose the listing agreement, if his expected utility is larger than or equal zero, which is his participation constraint. The

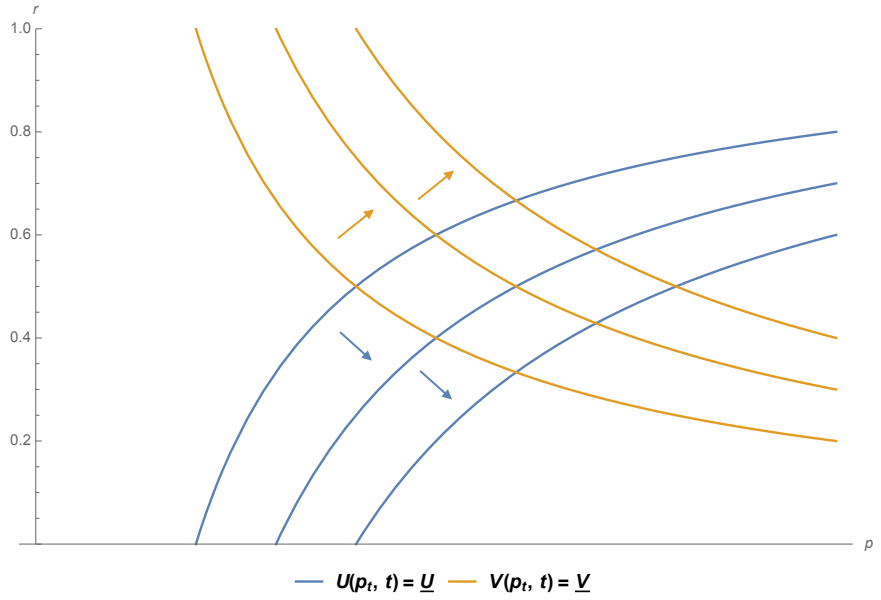


Figure 4.2 The indifference curves for a given period t.

maximisation problem is thus:

$$\max_{p_1,p_2,r} \mathbb{E}[U], \tag{4.5a}$$

$$\text{s.t. } \mathbb{E}[V] \geq 0. \tag{4.5b}$$

In order to solve the maximisation problem, we need to solve the second-period maximisation problem for p_2 first, which is

$$\max_{p_2} (1-\alpha p_2)p_2(1-r) + (1-(1-\alpha p_2))U_0, \tag{4.6a}$$

$$\text{s.t. } (1-\alpha p_2)rp_2 \geq 0. \tag{4.6b}$$

Because we determine r in the first period, we cannot determine r in the second-period problem. We can instead reduce Equation (4.6b) to find a bound for p_2. This gives us that either $r \geq 0$ or $0 \leq p_2 \leq 1/\alpha$, for the participation constraint to be satisfied. For p_2, we already have these bounds given by the probability for sale. Setting the first derivative with regard to p_2 of Equation (4.6a) equal to zero, gives us the optimal value of p_2:

$$\frac{\partial}{\partial p_2}(1-\alpha p_2)p_2(1-r) + (1-(1-\alpha p_2))U_0 = 0 \tag{4.7a}$$

$$\Rightarrow (1-\alpha p_2)(1-r) - \alpha p_2(1-r) + \alpha U = 0 \tag{4.7b}$$

$$\Rightarrow p_2^C = \frac{r-\alpha U_0 - 1}{2\alpha(r-1)}, \tag{4.7c}$$

where C denote that the solution is for the competitive model. This solution depends on the rate and the state of the market, and it is an inner solution. Because of this, we need to ensure that this solution satisfies the bounds found above, after we have solved the first-period problem to find r and p_1.

We can now insert p_2^C in the maximisation problem for the first period:

$$\begin{aligned}\max_{p_1,r} \mathbb{E}[U^C] =&(1-\alpha p_1)p_1(1-r)+(1-(1-\alpha p_1))\\ &\times\left(\left(1-\alpha\frac{r-\alpha U_0-1}{2\alpha(r-1)}\right)\frac{r-\alpha U_0-1}{2\alpha(r-1)}(1-r)+\alpha\frac{r-\alpha U_0-1}{2\alpha(r-1)}U_0\right)(1-\delta), \qquad (4.8a)\\ \text{s.t. } \mathbb{E}[V^C] =&(1-\alpha p_1)rp_1+(1-(1-\alpha p_1))\\ &\times\left(\left(1-\alpha\frac{r-\alpha U_0-1}{2\alpha(r-1)}\right)r\frac{r-\alpha U_0-1}{2\alpha(r-1)}\right)(1-\delta)\geq 0. \qquad (4.8b)\end{aligned}$$

We will solve the maximisation problem using the Lagrange method. The Lagrangian is given by

$$\mathcal{L}^C = \mathbb{E}[U^C] + \lambda^C(\mathbb{E}[V^C]), \qquad (4.9)$$

where we assume the participation constraints holds in the solution, and where λ^C is the Lagrange multiplier. The first-order conditions are given by $\frac{\partial \mathcal{L}^C}{\partial p_1} = 0$, $\frac{\partial \mathcal{L}^C}{\partial r} = 0$ and $\frac{\partial \mathcal{L}^C}{\partial \lambda^C} = 0$, which gives us the solution[1]:

$$p_1^C = \frac{\alpha^2 U_0^2 + 2\alpha U_0 - \alpha^2 U_0^2\delta - 2\alpha U_0\delta - \delta + 5}{8\alpha}, \qquad (4.10a)$$

$$p_2^C = \frac{\alpha U_0 + 1}{2\alpha}, \qquad (4.10b)$$

$$r^C = 0, \qquad (4.10c)$$

$$\lambda^C = 1. \qquad (4.10d)$$

Because the Lagrange multiplier is positive, the agent's participation constraint in Equation (4.8b) does hold in the solution due to the Kuhn-Tucker conditions and the real estate agent's expected utility is equal to zero in the competitive market. We can see that under competition with symmetric information the real estate agent does not get any fee payment, since the rate is zero. Since both the seller and the agent knows the true signal Nature sends, the real estate agent cannot take advantage of the signal and therefore he gets no payment. With $r^C = 0$ we can also see the second-period participation constraint in Equation (4.6b) is satisfied.

Lastly, since the probability of selling depends on it, we need to ensure[2] if both $p_1^C < 1/\alpha$ and $p_2^C < 1/\alpha$:

$$p_1^C < \frac{1}{\alpha} \Leftrightarrow U_0 < \frac{2}{\alpha\sqrt{1-\delta}} - \frac{1}{\alpha} \qquad (4.11a)$$

$$p_2^C < \frac{1}{\alpha} \Leftrightarrow U_0 < \frac{1}{\alpha}. \qquad (4.11b)$$

For $\delta \to 0$, the limit of the right-hand side of the second inequality in Equation (4.11a) goes towards $1/\alpha$. So the listing prices satisfies the constraint of $q(p;\alpha)$ when there is little to none discounting, as long as the utility of no sale also is less than the inverse of the state of the market.

[1] See Appendix B.1.1 for the calculation.

[2] See Appendix B.1.1 for the calculation of the bounds.

To see if the listing price in the first period is higher than listing price in the second period, we look at $p_1^C > p_2^C$:

$$p_1^C > p_2^C \Leftrightarrow U_0 < \frac{1-\sqrt{\delta}}{\alpha\sqrt{\delta}+\alpha}. \tag{4.12}$$

For $\delta \to 0$, the limit of the right-hand side of the second inequality is $1/\alpha$. So as long as $U_0 < 1/\alpha$, the listing price in the first period is larger than the second-period listing price with no discounting. This is always true, as along as the listing prices are less than $1/\alpha$. $U_0 < 1\alpha$ also determines when the seller is willing to put their property on the market.

By letting $\mathbb{E}[U^C] = \underline{U}$ in Equation (4.8a), we can find the indifference curve of the seller.[3] Likewise, we can find the real estate agent's indifference curve by letting $\mathbb{E}[V^C] = 0$ in Equation (4.8b), because the participation constraint is binding in the solution. The indifference curves for the agent is therefore fixed, while we can move the indifference curves of the seller. We can see the indifference curves in Figure 4.3(a) for both a high value of α (bad market conditions) and a low value of α (good market conditions). Here we see that the seller is worse off with a higher value α. Given the probability of sale, this is what we expect, since with a higher value of α the prices are lower. The same is true for the real estate agent, where he has a lower rate when the value of α is high.

The indifference curves for the seller moved down when $\underline{U}$ increases. Therefore it might look like we could decrease $\underline{U}$ to get a point of tangency with the real estate agent's indifference curves. The seller's indifference curves only tangent the real estate agent's when $\mathbb{E}[U^C] = U_0$, as Figure 4.3(b) shows. Since we are in a competitive market, the sellers have a chance to get a rent, and with $\mathbb{E}[U^C] = U_0$ they get their utility of no sale and thereby no rent. This point of tangency is therefore not relevant, and only the indifference curves below matter. It is not possible to get a point of tangency besides $\mathbb{E}[U^C] = U_0$ and the situation would therefore be as shown in Figure 4.3(a). This means that the solution for competition is a corner solution.

The seller is not willing to pay what the real estate agent the rate he wants. Since $r^C = 0$, the rate does not change when we decrease or increase the value of α, and it will always stay the same. Therefore the real estate agent does not have an incentive to misrepresent the state of the market, since no matter the state the agent does not get any fee payment.

4.1.2 Model for monopoly

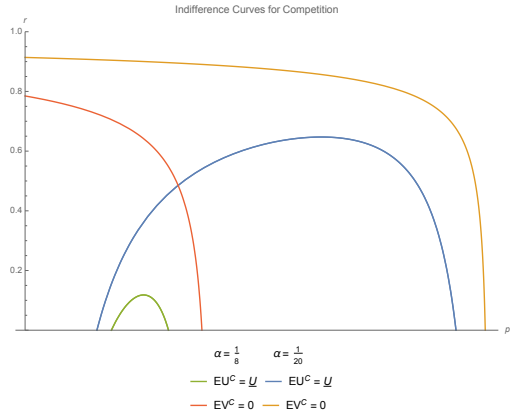
In monopoly, the real estate agent wants to maximises his own expected payoff subject to the seller participating. The maximisation problem is thus:

$$\max_{p_1,p_2,r} \mathbb{E}[V], \tag{4.13a}$$

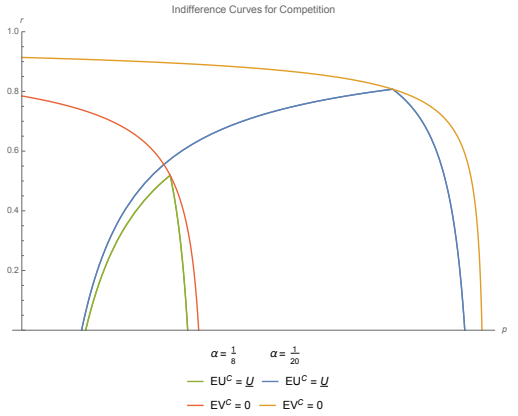
$$\text{s.t. } \mathbb{E}[U] \geq U_0. \tag{4.13b}$$

Whereas in a competitive market, the real estate agent's participation constraint says his expected utility needed to be larger or equal to zero, here the seller's participation constraint says her utility needs to be larger than or equal to her outside option, which

[3] See Appendix B.1.3 for calculation of indifference curves.



(a) The indifference curve plot with no point of tangency.



(b) The indifference curve plot with a point of tangency when $\mathbb{E}[U^C] = U_0$.

Figure 4.3 The indifference curves for the competitive model. With a low value of $\alpha = 1/20$, the indifference curves are given in blue for the seller and yellow for the agent. The indifference curves for a high value of $\alpha = 1/8$ are given in green for the seller and red for the agent.

is the utility of no sale, U_0. Again, we need to solve the second-period maximisation problem to find the optimal price of p_2. The second-period maximisation problem is:

$$\max_{p_2}(1-\alpha p_2)rp_2, \tag{4.14a}$$

$$\text{s.t. } (1-\alpha p_2)p_2(1-r)+(1-(1-\alpha p_2))U_0 \geq \frac{p_1 r}{4}. \tag{4.14b}$$

The second-period participation constraint says that in the second period, the seller's second-period expected utility needs to be larger than or equal to what the needs to pay the agent if the seller terminates the contract prematurely. This will be the real estate agent's claim of loss of fee, which amounts to $\frac{p_1 r}{4}$. We will need to check whether the second-period participation constraint is satisfied, when we have found the expressions for p_1, p_2 and r. Equation (4.14b) reduces to

$$r = 0 \vee \left(p_1 \leq \frac{4p_2(\alpha(-p_2)+r(\alpha p_2-1)+\alpha U_0+1)}{r} \wedge r > 0 \right). \tag{4.15}$$

We will need to ensure our solution satisfies this.

Again we maximise our second-period problem with regard to p_2 by setting the first derivative equal to zero:

$$\frac{\partial}{\partial p_2}\Big((1-\alpha p_2)rp_2\Big) = 0 \tag{4.16a}$$

$$\Rightarrow r(1-\alpha p_2)-\alpha p_2 r = 0 \tag{4.16b}$$

$$\Rightarrow p_2^M = \frac{1}{2\alpha}, \tag{4.16c}$$

where M denotes the solution is for monopoly. In monopoly, the second period's inner-solution listing price only depends on the state of the market, and not on the rate. Because of this, we can easily see that p_2^M satisfies the bound set by $q(p_2;\alpha)$ since $0 \leq \frac{1}{2\alpha} < \frac{1}{\alpha}$. We will also need to check that p_2^M satisfies the second-period participation

contraint. Notice that the probability of sale is $q(p_2^M;\alpha) = 1 - \frac{\alpha}{2\alpha} = \frac{1}{2}$, which is constant and does not depend on the state. No matter what the state of the market is, there is always a probability of $\frac{1}{2}$ that the property will get sold in the second period. We can now insert p_2^M and $q(p_2^M;\alpha) = \frac{1}{2}$ in the maximisation problem for the both periods:

$$\max_{p_1,r} \mathbb{E}[V^M] = (1-\alpha p_1)rp_1 + (1-(1-\alpha p_1))\Big(\frac{1}{2}\frac{1}{2\alpha}r\Big)(1-\delta), \tag{4.17a}$$

$$\text{s.t. } \mathbb{E}[U^M] = (1-\alpha p_1)p_1(1-r) + (1-(1-\alpha p_1))\Big(\frac{1}{2}\frac{1}{2\alpha}(1-r) + \frac{1}{2}U_0\Big)(1-\delta) \geq U_0. \tag{4.17b}$$

The Lagrangian is then

$$\mathcal{L}^M = \mathbb{E}[V^M] + \lambda^M(\mathbb{E}[U^M] - U_0). \tag{4.18}$$

Again we assume the participation constraint holds in the solution, and λ^M is the Lagrange multiplier for the monopolistic model. The first-order conditions $\frac{\partial \mathcal{L}^M}{\partial p_1} = 0$, $\frac{\partial \mathcal{L}^M}{\partial r} = 0$ and $\frac{\partial \mathcal{L}^M}{\partial \lambda^M} = 0$ gives us the solution[4]:

$$p_1^M = \frac{2\alpha U_0 - 2\alpha\delta U_0 - \delta + 5}{8\alpha}, \tag{4.19a}$$

$$r^M = -\frac{4\alpha^2(\delta-1)^2U_0^2 + 4\alpha\left(\delta^2 - 6\delta - 11\right)U_0 + (\delta-5)^2}{4\alpha^2(\delta-1)^2U_0^2 - (\delta-5)^2}, \tag{4.19b}$$

$$\lambda^M = 1. \tag{4.19c}$$

As with the competitive model, $\lambda^M > 0$, so the participation constraints holds in equilibrium. We ensure that p_1^M satisfies $q(p_1^M;\alpha)$'s bounds by seeing when it is true that $0 \leq p_1^M < \frac{1}{\alpha}$:[5]

$$0 \leq p_1^M < \frac{1}{\alpha} \Leftrightarrow U_0 < \frac{-\delta - 3}{2\alpha\delta - 2\alpha} \tag{4.20}$$

For $\delta \to 0$ the upper bound for U_0 is $\frac{3}{2\alpha}$. This upper bound is larger than $1/\alpha$, which is the upper bound for U_0 in the competitive model. This bound determines when the seller is interested in selling her property, and since this bound is larger for the monopolistic situation, the seller is more willing to sell in a monopoly. Again, we are interested if $p_1^M > p_2^M$. This is always true for $U_0 < \frac{-\delta-3}{2\alpha\delta-2\alpha}$.

r^M is positive as long as

$$U_0 < -\frac{\delta^2 + 4\sqrt{-2\delta^2 + 12\delta + 6} - 6\delta - 11}{2\alpha(\delta-1)^2}. \tag{4.21}$$

This gives us another upper bound for U_0. For $\delta \to 0$, the right-hand side of the inequality is $\frac{11-4\sqrt{6}}{2\alpha}$, which is smaller than the upper bound found above. U_0 needs to be lower for the monopolistic real estate agent to get a fee payment. So even though the seller is willing to sell as long as the bound in Equation (4.20), the real estate agent only gain from the sale with the bound in Equation (4.21). This is the stricter upper bound for U_0.

[4] See Appendix B.1.2 for the calculation.

[5] See Appendix B.1.2 for the boundaries' calculation.

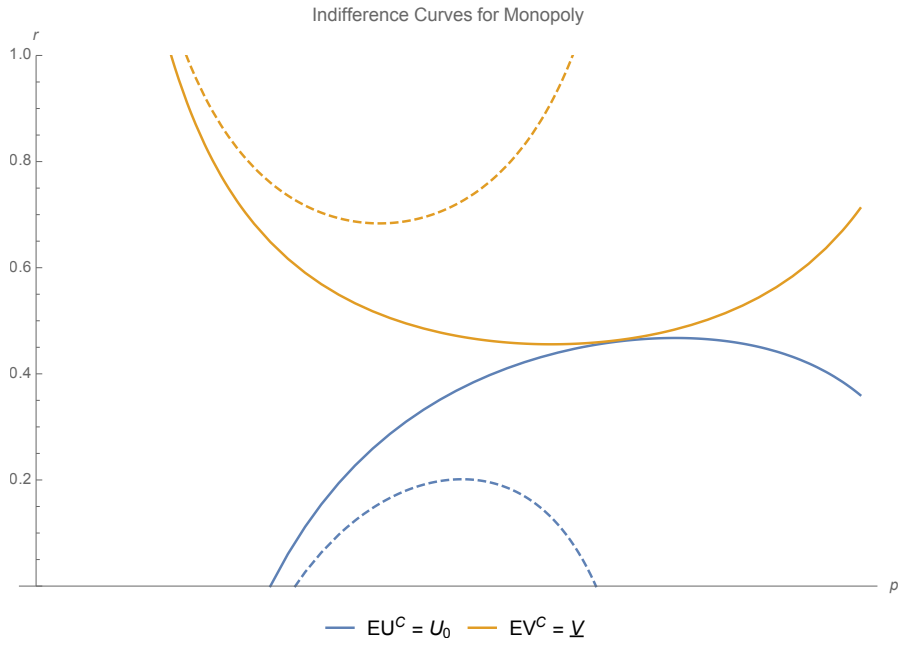


Figure 4.4 The indifference curves for monopoly. When the value of α increases, the indifference curves are shown with the dashed lines.

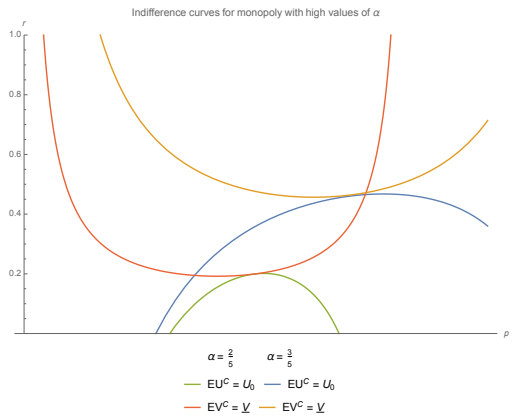
We now need to check the second-period participation constraint is satisfied. Since $r^M > 0$, we need to check the bound for p_1^M given in Equation (4.15). This gives us the following inequality:

$$\begin{aligned}&(\delta-5)^3+8\alpha^3(\delta-9)(\delta-1)^2U_0^3+4\alpha^2\left(3\delta^3-37\delta^2+33\delta+1\right)U_0^2\\&+2\alpha\left(3\delta^3-41\delta^2+89\delta+461\right)U_0\geq 0.\end{aligned}\tag{4.22}$$

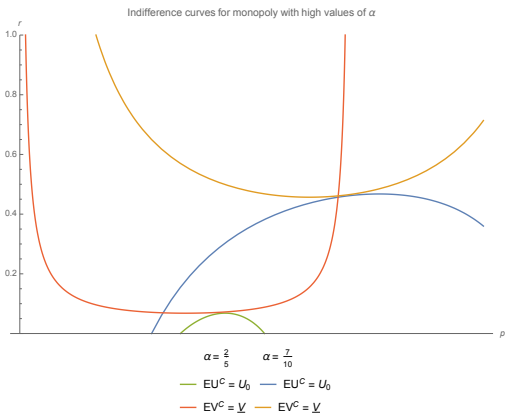
If this is satisfied the seller will stay for both periods. If this is not satisfied, the seller will terminate the contract and pay the real estate agent $\frac{p_1^M r^M}{4}$. This results in another maximisation problem, where we know the seller will terminate the contract prematurely. We will assume that the inequality in Equation (4.22) is satisfied, so the seller will stay for both periods.

The indifference curves[6] of the seller and the real estate agent are shown in Figure 4.4. The indifference curves for the seller are found by setting $\mathbb{E}[U^M] = U_0$ in Equation (4.17b), since the participation constraint holds in the solution, and for the agent they are found by setting $\mathbb{E}[V^M] = \underline{V}$ in Equation (4.17a). Here it is the seller's indifference curves, which are fixed, and the agent's indifference curves can be moved. In the monopolistic model we do have a relevant point of tangency. When we increase the value of α, the indifference curves for the real estate agent goes up letting the agent demand a higher fee rate, and the indifference curves for the seller goes down. When the value of α increases, the market conditions are worsened. The real estate agent's indifference curves moves up so his rate increases whereas the listing prices decreases as we can see in Figure 4.4. The seller's indifference curves moves down, demanding a lower fee rate. Since the seller's indifference curves are fixed for a certain level of U_0, the agent have to move his indifference curve down to tangent the seller's indifference curve. This means that he is actually worse of when the value of α increases.

[6] See Appendix B.1.3 for calculation of indifference curves.

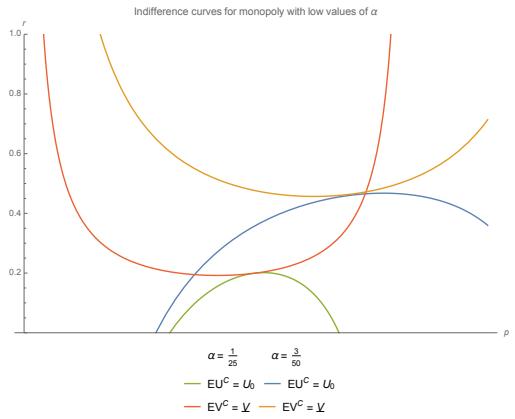


(a) Incentive to misrepresent the state as $\alpha = 2/5$ when the true state is $\alpha = 3/5$.

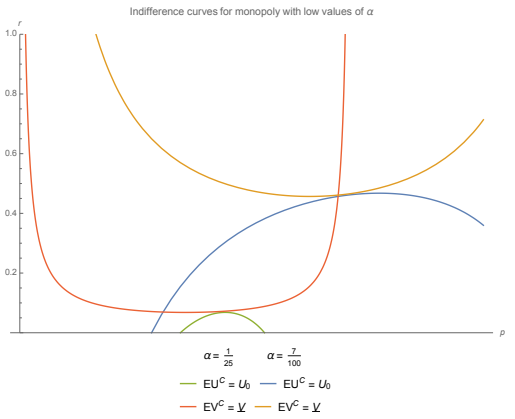


(b) No incentive to misrepresent the state as $\alpha = 2/5$ whent he true state is $\alpha = 7/10$.

Figure 4.5 Market conditions have high values α with a fixed level of U_0.

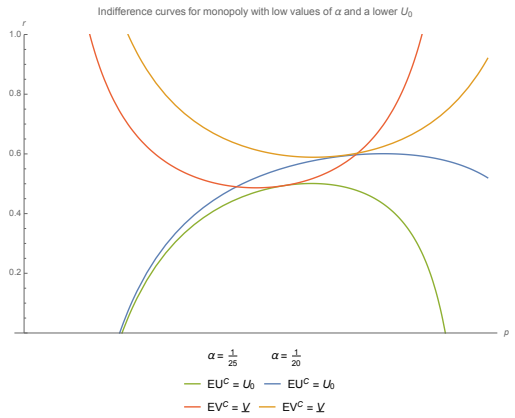


(a) Incentive to misrepresent the state as $\alpha = 1/25$ when the true state is $\alpha = 3/50$.

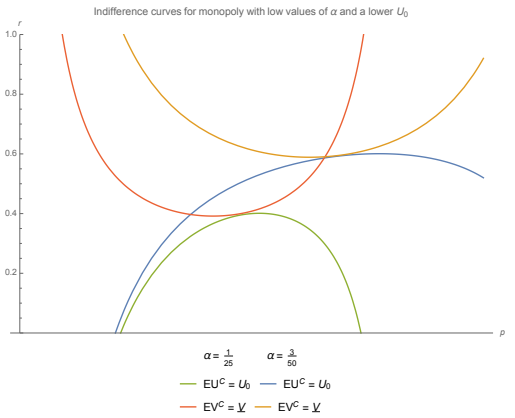


(b) No incentive to misrepresent the state as $\alpha = 1/25$ when the true state is $\alpha = 7/100$.

Figure 4.6 Market conditions have low values of α with a fixed level of U_0.



(a) Incentive to misrepresent the state as $\alpha = 1/25$ when the true state is $\alpha = 1/20$.



(b) No incentive to misrepresent the state as $\alpha = 1/25$ when the true state is $\alpha = 3/50$.

Figure 4.7 Market conditions have low values of α with a lower level of U_0.

To see whether the monopolistic real estate agent have an incentive to misrepresent the state of the market, we will look at different situations: ones where the market conditions have low values of α, and ones where the market conditions have high values of α. Figure 4.5 shows the indifference curves for high values of α, this represent a overall bad state of the market. In Figure 4.5(a) the real estate agent who receives the higher value of α ($\alpha = 3/5$) have an incentive to misrepresent the market, since pretending to receive the lower value of α ($\alpha = 2/5$), he can obtain a higher fee rate. This is because the area above his indifference curves includes the tangency point for the lower value of α. If the real estate agent receive $\alpha = 7/10 > 3/5$, he no longer have an incentive to misrepresent the state of the market as Figure 4.5(b) shows. The area above his indifference curves here does not include the tangency point for $\alpha = 2/5$. He is therefore better off not misrepresenting the market.

By scaling down the values of α, $\underline{V}$ and U_0 by a constant factor in Figure 4.5, we can see what happen when the market conditions have lower values of α in Figure 4.6. This has the same interpretation as above, but with lower values of α, the interval where the agent have an incentive to misrepresent the state of the market is smaller. The incentive to misrepresent only exists when the value of one α is higher than another value of α.

The incentive to misrepresent not only depends on the value of α, but also the seller's value of U_0. With a lower value of U_0, the seller's indifference curves goes up as Figure 4.7 shows. In Figure 4.6(a) the real estate agent have an incentive to misrepresent, but in Figure 4.7(b) with the same values of α, he no longer have an incentive to lie, because U_0 is lower. The values of α have to be closer the lower the level of U_0 is, as Figure 4.7(a) shows.

4.2 Asymmetric information

With asymmetric information it is only the real estate agent who knows the state of the market. We are interested in when he will use this information to his advantage. As we argued above, he does not have an incentive to misrepresent the state in a competitive market, so we will only look at the monopoly situation. There are two states of the market, a good state with a low value of α and a bad state with a high value of α. Nature sends the true signal of the market to the real estate agent. We denote the good state as α^G and the bad state as α^B, where $\alpha^G < \alpha^B$. When the value of α increases the state of the market gets worse. Figure 4.8 shows the probabilities for selling in either a good state or a bad state. When the state is good there is a higher probability of selling than then the state is bad.

The rates and listing prices for the good state and the bad state under symmetric information are given by the solution in Section 4.1.2 by changing the value of α. We let $T = \{G, B\}$ depending on the state of the market:

$$p_1^{T,M}(\alpha^T) = \frac{2\alpha^T U_0 - 2\alpha^T \delta U_0 - \delta + 5}{8\alpha^T}, \tag{4.23a}$$

$$p_2^{T,M}(\alpha^T) = \frac{1}{2\alpha^T}, \tag{4.23b}$$

$$r^{T,M}(\alpha^T) = -\frac{4(\alpha^T)^2(\delta-1)^2U_0^2 + 4\alpha^T\left(\delta^2 - 6\delta - 11\right)U_0 + (\delta-5)^2}{4(\alpha^T)^2(\delta-1)^2U^2 - (\delta-5)^2}, \tag{4.23c}$$

where M denotes the listing prices and the rate are under symmetric information for monopoly.

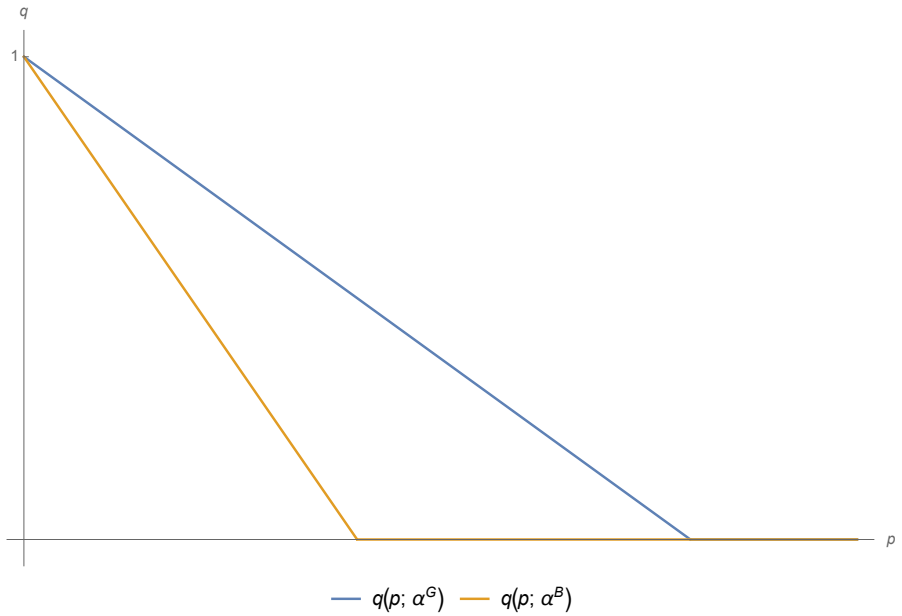


Figure 4.8 The difference between the probability of sale with the two different states of α.

As Figures 4.5–4.7 show, it is only in the bad state the real estate agent have an incentive to misrepresent the state. This incentive depends on the true and misrepresented values of α as well as U_0. With asymmetric information, we need to give the real estate agent an incentive to not want to misrepresent the state of the market, when the state is bad. This is given by his incentive–compatibility constraint, which says he will not be worse of presenting the state as bad (the true state). Since he knows the full-information listing prices and rate, his expected utility given α^B should be larger than or equal to his expected utility given $p_1^{G,M}(\alpha^G)$, $p_2^{G,M}(\alpha^G)$ and $r^{G,M}(\alpha^G)$ given the state of the market is α^B:

$$\begin{aligned} &q(p_1;\alpha^B)rp_1 + (1-q(p_1;\alpha^B))\Big(q(\frac{1}{2\alpha^B};\alpha^B)r\frac{1}{2\alpha^B}\Big)(1-\delta) \geq \\ &q(p_1^{G,M}(\alpha^G);\alpha^B)r^{G,M}(\alpha^G)p_1^{G,M}(\alpha^G) + (1-q(p_1^{G,T}(\alpha^G);\alpha^B)) \\ &\times\Big(q(p_2^{B,M}(\alpha^G);\alpha^B)r^{B,M}(\alpha^G)p_2^{B,M}(\alpha^G)\Big)(1-\delta). \end{aligned} \tag{4.24}$$

When choosing to misrepresent the state, the real estate agent does so for the whole listing agreement, thereby both periods. He decides whether he wants to misrepresent the market in the first period only, so he does not have an incentive–compatibility constraint in the second period. Since the second-period listing price is determined in the second-period, we end up maximising $(1-\alpha^T p_2)rp_2$ with regard to p_2, as we did under symmetric information, subject to the seller's participation constraint. This gives us the same solution as under symmetric information, so $p_2^{T,S} = p_2^T = \frac{1}{2\alpha^T}$. The second-period participation constraint (given in Equation (4.15)) also needs to be satisfied under asymmetric information.

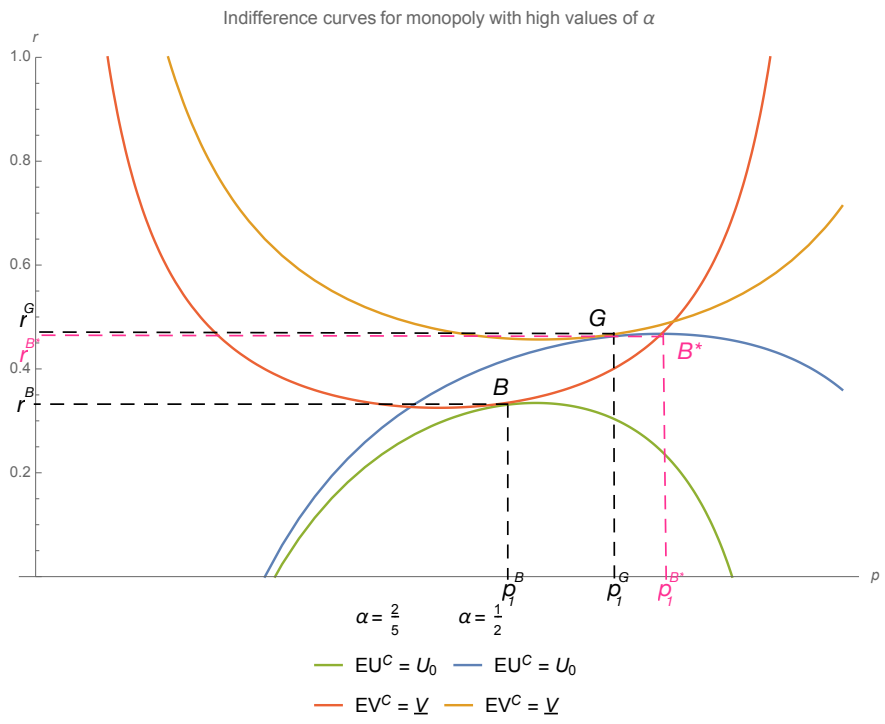


Figure 4.9 Graphical solution to the asymmetric information problem in a monopoly.

The maximisation problem for both periods is then:

$$\max_{p_1,r} \mathbb{E}[V^B] = (1-\alpha^B p_1) r p_1 + (1-(1-\alpha^B p_1))\Big(\frac{1}{2}\frac{1}{2\alpha^B} r\Big)(1-\delta), \tag{4.25a}$$

$$\begin{aligned} \text{s.t. } \mathbb{E}[U^B] =& (1-\alpha^B p_1) p_1 (1-r) + (1-(1-\alpha^B p_1)) \\ & \times \Big(\frac{1}{2}\frac{1}{2\alpha^B}(1-r) + \frac{1}{2} U_0\Big)(1-\delta) \geq U_0, \end{aligned} \tag{4.25b}$$

$$\begin{aligned} \text{ICC} =& q(p_1;\alpha^B) r p_1 + (1-q(p_1;\alpha^B))\Big(q(\frac{1}{2\alpha^B};\alpha^B) r \frac{1}{2\alpha^B}\Big)(1-\delta) - \\ & \Big(q(p_1^{G,M}(\alpha^G);\alpha^B) r^{G,M}(\alpha^G) p_1^{G,M}(\alpha^G) + (1-q(p_1^{G,M}(\alpha^G);\alpha^B)) \\ & \times \Big(q(p_2^{B,M}(\alpha^G);\alpha^B) r^{B,M}(\alpha^G) p_2^{B,M}(\alpha^G)\Big)(1-\delta)\Big) \geq 0. \end{aligned} \tag{4.25c}$$

Solving this problem using the Lagrange method have not given any results. We will instead show the solution graphically.

In Figure 4.9 we can see a situation in which the real estate agent wants to misrepresent the market. With α^G we get the equilibrium at point G and with α^B it is at point B. To make sure the real estate agent does not want to present the state as G, he will offer the rates and first-period listing price at point B^*. Here he will be better off than at B, since his rate is higher: $r^{B^*} > r^B$. He will offer the price $p_1^{B^*}$, which is higher than p_1^B, and the probability of selling the property at $p_1^{B^*}$ will be lower. However the second-period listing price is $p_2^B = \frac{1}{2\alpha^B}$ with a probability of $q(p_2^B;\alpha^B) = \frac{1}{2}$ of selling, and with the higher rate r^{B^*}, he will also be better off in the second period.

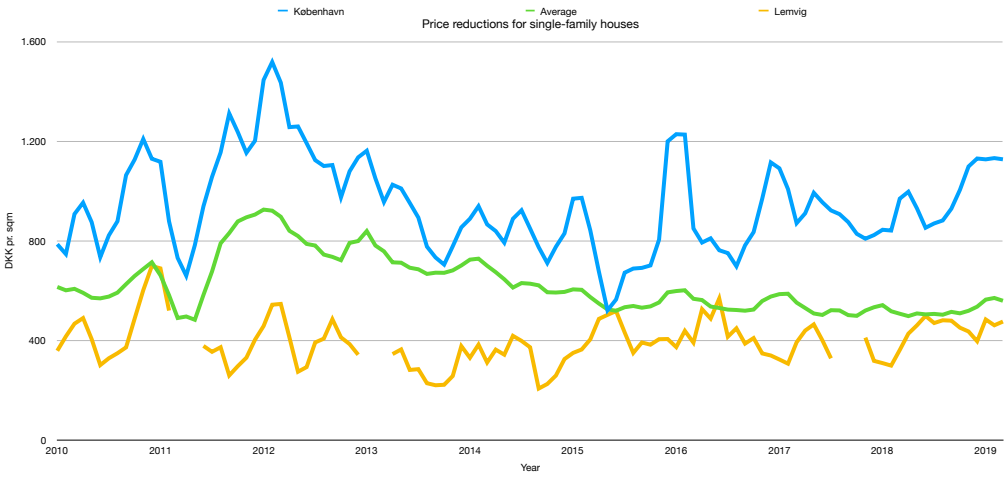
4.3 Summary

The models for full information show that the first-period listing price is higher than the second-period listing price. This means that there is a price reduction between the first period and the second period. In Figure 4.10 we can see the price reductions[7] for single-family houses in Copenhagen and Lemvig as well as the average of Denmark. The price reduction are given as the difference between the latest listing and the sales price. We can see that even though the price reductions for Lemvig are lower than the average in DKK per m^2, the percentage of the price reduction is actually higher than the average.

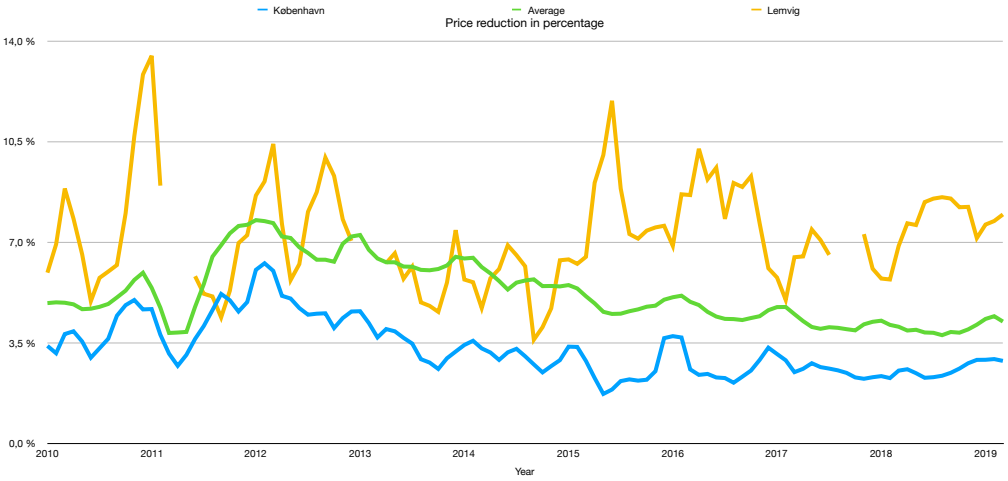
The models' focus on misrepresentation shows that in a competitive market, there is no incentive to misrepresent the state under asymmetric information. Since the real estate agents compete for clients, it would be harder for a real estate agent to misrepresent the market, because the client could go to the next agent, and see if he gives the same rate and price.

If the real estate agent have a monopoly, then there is no other real estate agents for the clients to seek out. He can therefore take advantage of the asymmetric information, and will misrepresent the state of the market when the state is bad. This incentive does however depend on the relationship between the good state and the bad state. To make the real estate agent represent the true state of the market, he needs an incentive.

[7] https://markedsindeks.boligsiden.dk/?statisticType=PriceReductionPerSquareMeterIndexed&areas=a1&areas=r1084&propertyTypes=HouseAndTerraceHouse&tab=graph



(a) Price reduction in DKK per m^2.



(b) Price reduction in percentage.

Figure 4.10 The price reductions from 2010 to today for Copenhagen, the average Denmark and Lemvig.

5 Discussion

The Danish real estate law is a consumer protection law, and therefore attempts to ensure that the real estate agent discloses as much as possible for the consumer to have the whole picture. It is worth mentioning that in all the reports the authorities have made, none of them looks into the best way to incentivise the real estate agents. The focus is on the poorly competitive market between real estate agents. The reports and the resulting changes have focused on how to make the consumer more aware of the fee payment and the disclosure the real estate agent's provide as well as increase the competition between real estate agents. They have not looked into the real estate agent's incentive, but instead have focused on having the real estate agent provide more options than the traditional performance-based fee concept to increase competition between the real estate agents.

Anglin and Arnott (1991) looks at risk-sharing when using the performance-based fee. When the seller is risk-averse and the agent is risk-neutral, the agents bears the risk. But when the seller is more involved in the sale, is this still the case? With the alternative concepts the real estate agents is exerting less effort than he would with performance-based fee, since the seller is typically showing the property and handling some of the marketing. These alternative concept can be viewed as a package deal with a fixed fee. When the agent offers these package deals he is ensured a fee payment no matter what happens with the property. By being more involved with the sale, the seller is taken on some of the risk. The choice of real estate agent must therefore also depend how much risk the seller is willing to take on. This is also what Levitt and Syverson (2008b) hints at: The seller who are interested in being more involved benefit from using a real estate agent who offers a concept.

The no-cure no-pay principle increased the fee rate because the fee now depended on the house being sold. This means that the real estate agent needed to exert more effort in selling the house, whereas without the principle the agent could demand a fee payment no matter if the property got sold or not. Therefore without the principle, the agent could exert less effort by knowing they still would get paid if the property didn't sell. The real estate agent could get paid no matter what amount of effort he exerted. So by introducing the no-cure no-pay principle, it made sure that no agent took advantage of the sellers and gave them better incentive to sell the property they took on. The loss of fee of one fourth of the agreed fee makes sure that the real estate agent is paid for his work, if the seller is not satisfied with the ongoing sale. So that the effort he has supplied is still paid for.

People still use real estate agents where other service industries are used less, for example, travel agencies. As a consumer you can easily book your flight and accommodation yourself instead of using a travel agency. This is not the case for real estate. Even though more websites giving advice on selling your property become available, there still is some legislation the consumer needs to familiarise oneself with. The consumer also needs to understand the market, they are selling in, and be realistic with the price the property can sell for. Selling a property only happens one or few times in

a lifetime because of the effects it has on your financial situation. It is therefore easier to use a real estate agent, who is trained and knows the legislation. Consumers who are interested in being more involved in the sale can then use these alternative concepts, where they might end up paying less to the real estate agent. The property might take longer to sell as Levitt and Syverson (2008b) finds, but the sales price will be the same and the cost of selling will be less.

One of the Competition Authority's aims is to make the fee more transparent for the consumer. They did this by specifying the cost of the services the real estate agent provided. They also write that not many consumers negotiate about the fee. Given the rare situation of selling a property, it might be hard for the consumer to negotiate the fee, since it might be the first time she sells a property. Because of the agents' position as an expert, they had the bargaining power over the consumer when negotiating the fee. One way to make the fee more transparent would be to make the fee rates visible for the consumer. Some agencies advertise with their low fixed fee for the alternative concepts, but this is not the case for the fee rate of the performance-based fee. Then consumer could then make a more informed choice of real estate agent. This does not mean all the real estate agents should suddenly advertise their fee rate, but the authority could put up a guideline for the fee rates across Denmark, so the seller would have a better standpoint for negotiating with the agent.

One of the reasons it is hard to estimate the fees, is because the real estate agents apply different discounts to the fees with no real guidelines. A reason for the discounts could be a quick sale. With a quick sale, the real estate agent have opportunity to quickly take on a new property. These discounts further make it hard for the seller to know what she is paying for. Since the agent have the negotiating advantage, the seller does not know when she can achieve a discount by negotiating the price.

The Competition Authority came to the conclusion that all the restrictions of the listing agreement made it more confusing, than informative and streamlined the listing agreement. The duration of the listing agreement have also been changed from always being at most six months no matter the fee to no longer having a maximum period when using a fixed fee. When choosing a performance-based fee the duration is still at most six months if the real estate agent wants a claim of loss of fee.

The 12 initiatives also made the real estate agent's process of handling the sale easier by using information technology. The real estate sales data was also digitalised, so everyone could get a picture of the market. This online information decreased the real estate agent informational advantage of the market conditions. The seller can easily look up the neighbouring properties for sale to see what they sold for, how long it took and who the real estate handling the sale was.

As we can see with the theoretical models and our own, there are a lot of different asymmetric information problems between the seller and the real estate agent. Despite these problem, consumers still use real estate agents, even though they are not seen at the most trustworthy as the consumer attitude index indicates. The consumers seems to be aware of this. With the internet and the alternative concepts, the sellers can be more involved in the sale. The relationship between seller and real estate agent might benefit from this, because they rely more on each other in this case, and the trust in the agent could increase.

The reports by the authorities also mentions how the online marketing have affected the sale. In 2004 the real estate agents said the online marketing had not expanded their sales area and their marketing costs had increased because they advertise online and in the papers. The real estate agents who had prioritised the online marketing had improved their efficiency. Given social media and online marketing today, hopefully

this have improved all real estate agents efficiency. It is at least hard to be online or on social media without an advertisement for a property or a real estate agent not to pop up.

The real estate agent could mislead the consumer by providing unnecessary services as Levitt and Syverson (2008a) describes. By reducing the mandatory services and introducing the alternative concept, the seller could now make a more informed choice about the services she was interested in. These changes could make it harder for the agent to mislead, because the mandatory services are clearly stated in the legislation, and services beyond these are additional.

As our models in Section 4 shows there is only an incentive to misrepresent the state of the market in the monopoly when the state of the market is bad. Since the real estate agent does not have an incentive to misrepresent the state in the competitive model, it might be that under asymmetric information he would misrepresent other aspects of the listing agreement, such as the cost that are covered by the seller in order to earn a payment.

We also find that both models have a price reduction from the first-period listing price to second-period listing price. This could be a way for the agents to lure in clients by setting a listing price the client will accept, but it not realistic. After the client have accepted the listing price and the property have no sold, the real estate agent would advice the client to reduce the listing price to a more realistic listing price the property can sell for. We can at least see in Figure 4.10, that the overall listing prices are reduced when the property is sold.

6 Conclusion

With the information online, the real estate agent's informational advantage have diminished, since the seller better can get a picture of the real estate market and thereby get a sense of what the property should sell for. The changes to the legislation have been to make the seller more aware of her cost of selling. These changes have unfortunately caused more confusion than clarity, because of the necessity to specify every single service the real estate agent provide. The changes have however succeeded in adding more concepts with a fixed fee for the seller to choose besides the traditional concept with only the performance-based fee. Introducing these concepts added a way for the real estate agents to compete with their fee, so the performance-based fees no longer could increase rapidly.

In Denmark the real estate agent can only represent on party in the real estate sale. The agent's role have in practice typically always been as a representative of the seller. The buyer would typically have an adviser or sometimes a real estate agent. With the latest change in the legislation, the real estate agent's role as the seller's representative have been emphasised, where the legislation before had stated the agent had to express care towards both parties of the real estate sale.

To see if real estate agents have an incentive to misrepresent the market, we set up models using the performance-based fee, where the probability of selling depends only on the listing price and the state of the market. The models shows us that there is a difference between the fee rates in a monopoly and a competitive market. We find that real estate agents, who operate in a competitive market do not have an incentive to misrepresent the market, since their fee rate does not depend on the state of the market. In our models, the competitive real estate agents earn nothing and cannot take advantage of their informational advantage.

This is not the case for the monopolistic real estate agents. Since they have an monopoly, they can maximise their fee rate. This fee rate depends on the state of the market, and this can give the real estate agents an incentive to misrepresent the market depending on the state. The agent have incentive to represent the market as superior than the true state. The incentive to misrepresent the state depends on the relationship between the true state and the state the agent wants to get, so it is not always the case that the agent wants to misrepresent.

In this thesis, we have looked into how real estate agents get their fee payment in the Danish real estate market. This fee payment have gone through many changes and will probably go through more. The real estate market keeps changing, and the need for a real estate agent might change in the future, especially since the rise of the internet have opened many more doors for the seller to be involved in the sale.

Appendices

A Real Estate Market

A.1 Fee rates from 1999

Municipality	Fee rate	Basic amount	Municipality	Fee rate	Basic amount
Albertslund	2.95	10,900	Egtved	2.50	9,500
Allerød	2.95	12,000	Egvad	2.85	9,500
Allinge-Gudhjem	3.00	10,000	Ejby	3.65	8,500
Arden	2.95	8,500	Esbjerg	2.85	9,500
Assens	3.65	7,000	Fakse	3.10	8,000
Augustenborg	2.10	13,500	Fanø	2.85	9,500
Aulum-Haderup	2.18	16,800	Farsø	2.25	9,000
Ballerup	2.95	12,000	Farum	2.95	12,000
Billund	2.18	9,500	Fjends	1.98	16,800
Birkerød	2.95	18,500	Fjerritslev	2.00	9,000
Bjergsted	2.30	16,800	Fladså	3.20	8,200
Bjerringbro	2.73	6,750	Fredensborg-Humlebæk	2.96	9,500
Blåbjerg	2.75	9,500	Fredericia	2.95	8,000
Blåvandshuk	2.75	9,500	Frederiksberg	2.95	15,000
Bogense	2.65	12,000	Frederikshavn	2.95	9,950
Bov	2.50	10,600	Frederikssund	2.95	7,500
Bramming	2.95	11,500	Frederiksværk	2.95	9,000
Bramsnæs	2.95	8,500	Fuglebjerg	2.10	12,600
Brande	2.18	14,000	Fåborg	3.50	7,900
Bredebro	2.50	10,000	Galten	2.95	9,500
Broager	2.30	13,500	Gedved	2.65	12,500
Broby	2.00	10,000	Gentofte	2.95	15,000
Brovst	2.95	9,800	Give	2.20	11,800
Brædstrup	2.18	11,800	Gjern	2.95	11,800
Brøndby	2.95	13,900	Gladsaxe	2.95	12,000
Brønderslev	2.00	12,000	Glamsbjerg	3.65	14,500
Brørup	3.60	9,800	Glostrup	2.95	10,000
Børkop	2.95	8,000	Gram	3.50	11,000
Christiansfeld	3.50	10,000	Grenå	2.95	8,000
Dianalund	2.30	13,800	Greve	2.95	11,000
Dragsholm	3.00	14,000	Grindsted	1.98	9,500
Dragør	2.95	12,000	Græsted-Gilleleje	2.96	12,000
Dronninglund	2.95	9,800	Gråsten	2.50	10,600
Ebeltoft	2.95	8,000	Gudme	2.58	13,500
Egebjerg	2.58	13,500	Gundsø	2.95	8,800

Municipality	Fee rate	Basic amount	Municipality	Fee rate	Basic amount
Gørlev	2.40	10,900	Kalundborg	2.30	16,800
Haderslev	2.50	10,000	Karlebo	2.96	12,900
Hadsten	2.75	8,500	Karup	2.28	11,000
Hadsund	2.95	9,800	Kerteminde	2.95	10,000
Hals	3.55	9,800	Kjellerup	2.95	11,000
Hammel	2.95	9,500	Kolding	3.50	10,000
Hanstholm	2.00	12,500	Korsør	2.10	15,900
Hashøj	2.10	15,800	København	2.95	12,650
Haslev	3.10	10,000	Køge	2.95	10,000
Hasle	3.00	12,800	Langebæk	2.95	9,800
Hedensted	2.18	11,800	Langeskov	2.65	10,000
Helle	2.75	9,500	Langå	2.99	6,000
Helsinge	2.96	12,000	Ledøje-Smørum	2.95	12,000
Helsingør	2.96	12,000	Lejre	2.95	10,000
Herlev	2.95	12,000	Lemvig	2.38	12,500
Herning	2.18	14,000	Lunderskov	3.50	10,000
Hillerød	2.96	12,000	Lundtoft	2.50	10,600
Hinnerup	2.75	8,500	Lyngby-Tårbæk	2.95	18,000
Hirtshals	2.50	11,000	Løgstør	2.50	10,000
Hjørring	2.50	13,500	Løgumkloster	2.25	9,500
Hobro	2.95	9,500	Løkken-Vrå	2.50	11,000
Holbæk	2.95	14,000	Mariager	3.85	6,000
Holeby	2.10	13,500	Maribo	2.10	13,500
Holmegård	3.20	8,200	Marstal	3.00	3,000
Holmsland	2.38	12,900	Middelfart	2.85	9,800
Holstebro	2.18	14,000	Midtdjurs	2.95	8,000
Holsted	3.60	9,800	Morsø	2.20	12,800
Horsens	2.14	13,000	Munkebo	2.65	10,000
Hundested	2.95	9,900	Møldrup	2.28	16,000
Hvalsø	2.95	9,000	Møn	2.95	9,800
Hvidebæk	2.30	16,800	Nakskov	2.50	14,000
Hvidovre	2.95	11,000	Nexø	2.85	10,000
Hvorslev	2.99	6,750	Nibe	2.00	9,000
Høje-Tåstrup	2.95	10,900	Nordborg	2.10	13,500
Højer	2.50	14,900	Nr, Alslev	2.10	13,500
Højreby	2.50	13,500	Nr, Djurs	2.95	8,000
Høng	2.40	10,900	Nr, Snede	2.90	12,000
Hørning	2.95	9,800	Nr, Åby	2.65	11,500
Hørsholm	2.95	15,000	Nyborg	2.38	12,000
Hårby	3.50	14,000	Nykøbing F	2.10	13,500
Ikast	2.18	14,000	Nykøbing-Rørvig	3.00	14,500
Ishøj	2.95	13,000	Nysted	2.10	13,500
Jelling	2.25	10,000	Næstved	3.20	8,200
Jernløse	2.30	16,000	Nørager	2.95	9,800
Juelsminde	1.90	15,500	Nørhald	3.85	6,000
Jægerspris	2.95	7,500	Nørre-Rangstrup	2.50	8,000

Municipality	Fee rate	Basic amount	Municipality	Fee rate	Basic amount
Odder	2.95	8,500	Sorø	2.80	9,200
Odense	2.65	10,000	Spøttrup	1.98	16,800
Otterup	2.65	10,000	Stenlille	2.30	12,600
Pandrup	2.95	9,800	Stenløse	2.95	7,000
Præstø	2.80	8,200	Stevns	2.95	8,200
Purhus	3.85	6,000	Struer	2.05	11,000
Ramsø	2.95	5,000	Stubbekøbing	2.10	13,500
Randers	3.85	6,500	Støvring	2.95	9,800
Ravnsborg	2.50	14,000	Sundeved	2.10	13,500
Ribe	2.95	9,500	Sundsøre	1.98	16,800
Ringe	2.65	7,900	Suså	3.20	8,200
Ringkøbing	2.38	12,900	Svendborg	2.58	13,500
Ringsted	2.80	12,600	Svinninge	2.40	16,000
Rosenholm	2.95	9,500	Sydals	2.10	13,500
Roskilde	2.95	9,500	Sydfalster	2.10	13,500
Rougsø	3.85	7,500	Sydlangeland	2.65	13,000
Rudbjerg	2.50	14,000	Sydthy	2.00	12,500
Rudkøbing	2.50	6,250	Sæby	2.95	9,950
Ry	2.85	11,800	Søllerød	2.95	18,000
Ryslinge	2.65	10,000	Sønderborg	2.10	13,500
Rødby	2.10	14,000	Sønderhald	3.85	6,500
Rødding	3.50	10,500	Søndersø	2.65	12,000
Rødekro	2.30	11,500	Them	2.85	11,800
Rødovre	2.95	10,900	Thisted	2.00	12,500
Rønde	2.95	9,800	ThyborønHarboøre	1.95	10,900
Rønne	3.00	12,800	Thyholm	2.00	12,500
Rønnede	3.10	8,000	Tinglev	2.50	11,500
Sakskøbing	2.10	13,500	Tjele	2.28	16,000
Sallingsund	1.98	16,800	Tommerup	2.00	17,000
Samsø	2.95	4,000	Tornved	2.30	16,800
Sejlflod	1.95	9,800	Tranekær	2.65	13,000
Silkeborg	2.95	11,800	Trehøje	2.18	14,000
Sindal	2.50	11,000	Trundholm	3.00	14,500
Skagen	2.95	9,950	Tølløse	2.40	16,000
Skanderborg	2.95	9,800	Tønder	2.50	14,900
Skibby	2.90	7,500	Tørring-Uldum	2.90	11,800
Skive	1.98	16,800	Tårnby	2.95	11,500
Skjern	2.38	12,900	Ulfborg-Vemb	2.38	12,900
Skovbo	2.50	8,000	Ullerslev	2.38	11,500
Skælskør	2.10	15,800	Vallensbæk	2.95	13,900
Skærbæk	2.50	10,000	Valløe	2.95	11,000
Skævinge	2.96	9,900	Vamdrup	3.50	11,000
Skørping	2.95	8,500	Varde	2.85	9,500
Slagelse	2.86	8,900	Vejen	3.60	11,000
Slangerup	2.96	8,800	Vejle	2.95	11,800
Solrød	2.95	12,500	Viborg	2.28	16,000

Municipality	Fee rate	Basic amount
Videbæk	2.18	12,900
Vinderup	2.18	13,000
Vissenbjerg	2.00	18,000
Vojens	3.00	10,000
Vordingborg	2.95	9,800
Værløse	2.96	12,000
Ærøskøbing	3.00	3,000
Ølgod	2.38	9,500
Ølstykke	2.95	6,500
Ørbæk	2.38	12,000
Åbenrå	2.30	9,900
Åbybro	2.50	9,800
Åkirkeby	3.00	12,800
Ålborg	3.55	11,800
Ålestrup	2.25	12,000
Århus	2.95	9,500
Års	2.50	12,000
Årslev	2.65	10,000
Årup	3.65	4,500
Åskov	2.18	12,900
Average	2.71	11,241

B A Model for Danish Real Estate Agents

B.1 Symmetric information

B.1.1 Competition

Competition

Probability function

```
q[p_, α_] := Max[1 - α p, 0]
```

Expected utility for seller for period 1 (EU1) and period 2 (EU2):

```
EU1 := q[p1, α] p1 (1 - r) + (1 - q[p1, α]) (EU2 (1 - δ))
```

```
EU2 := q[p2, α] p2 (1 - r) + (1 - q[p2, α]) U0
```

Expecter utilty for real estate agent for period 1 (EV1) and period 2 (EV2):

```
EV1 := q[p1, α] r p1 + (1 - q[p1, α]) (EV2 (1 - δ))
```

```
EV2 := q[p2, α] r p2
```

SECOND PERIOD MAXIMISATION PROBLEM

The real estate agent's Participation Constraint

```
Refine[Reduce[EV2 ≥ 0, p2], α p2 < 1 && α > 0 && r ≥ 0]
```

```
r == 0 || (r > 0 && p2 ≥ 0)
```

Maximisation of the seller's expected utility

```
Refine[D[EU2, p2], α p2 < 1 && p2 ≥ 0]
```

$$-p2\ (1-r)\ \alpha + U0\ \alpha + (1-r)\ (1-p2\ \alpha)$$

```
Refine[Reduce[D[EU2, p2] == 0], α p2 < 1 && p2 ≥ 0 && α > 0 && 0 ≤ r < 1 && U0 ≥ 0]
```

$$p2 == \frac{-1+r-U0\ \alpha}{-2\ \alpha + 2\ r\ \alpha}$$

```
p2 := (-1 + r - U0 α) / (2 (-1 + r) α)
```

Checking for $p_2^C < \frac{1}{\alpha}$

```
Refine[Reduce[p2 < 1/α], α > 0 && 0 ≤ r < 1]
```

$$U0 < \frac{1-r}{\alpha}$$

FIRST PERIOD MAXIMISATION PROBLEM

```
EU1
```

```
EV1
```

$$p1\ (1-r)\ \text{Max}[0,\ 1-p1\ \alpha] + (1-\delta)\ (1-\text{Max}[0,\ 1-p1\ \alpha])$$

$$\left(U0\ \left(1-\text{Max}\left[0,\ 1-\frac{-1+r-U0\ \alpha}{2\ (-1+r)}\right]\right) + \frac{(1-r)\ (-1+r-U0\ \alpha)\ \text{Max}\left[0,\ 1-\frac{-1+r-U0\ \alpha}{2\ (-1+r)}\right]}{2\ (-1+r)\ \alpha}\right)$$

$$p1\ r\ \text{Max}[0,\ 1-p1\ \alpha] + \frac{r\ (-1+r-U0\ \alpha)\ (1-\delta)\ (1-\text{Max}[0,\ 1-p1\ \alpha])\ \text{Max}\left[0,\ 1-\frac{-1+r-U0\ \alpha}{2\ (-1+r)}\right]}{2\ (-1+r)\ \alpha}$$

Defining the Lagrangian

```
lagrangeC := Refine[EU1 + λ EV1, 0 < α p1 < 1 && α > 0 && α (-1 + r - U0 α) / (2 (-1 + r) α) < 1]
```

```
lagrangeC
```

$$p1\,(1-r)\,(1-p1\,\alpha)+p1\,\alpha\left(\frac{U0\,(-1+r-U0\,\alpha)}{2\,(-1+r)}+\frac{(1-r)\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)}{2\,(-1+r)\,\alpha}\right)(1-\delta)+\left(p1\,r\,(1-p1\,\alpha)+\frac{p1\,r\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)(1-\delta)}{2\,(-1+r)}\right)\lambda$$

First-order Conditions

```
D[lagrangeC, p1] == 0
```

$$-p1\,(1-r)\,\alpha+(1-r)\,(1-p1\,\alpha)+\alpha\left(\frac{U0\,(-1+r-U0\,\alpha)}{2\,(-1+r)}+\frac{(1-r)\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)}{2\,(-1+r)\,\alpha}\right)(1-\delta)+\left(-p1\,r\,\alpha+r\,(1-p1\,\alpha)+\frac{r\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)(1-\delta)}{2\,(-1+r)}\right)\lambda==0$$

```
D[lagrangeC, r] == 0
```

$$-p1\,(1-p1\,\alpha)+p1\,\alpha\left(\frac{U0}{2\,(-1+r)}-\frac{U0\,(-1+r-U0\,\alpha)}{2\,(-1+r)^2}+\frac{(1-r)\,(-1+r-U0\,\alpha)\left(-\frac{1}{2\,(-1+r)}+\frac{-1+r-U0\,\alpha}{2\,(-1+r)^2}\right)}{2\,(-1+r)\,\alpha}+\frac{(1-r)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)}{2\,(-1+r)\,\alpha}-\frac{(1-r)\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)}{2\,(-1+r)^2\,\alpha}-\frac{(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)}{2\,(-1+r)\,\alpha}\right)(1-\delta)+\left(p1\,(1-p1\,\alpha)+\frac{p1\,r\,(-1+r-U0\,\alpha)\left(-\frac{1}{2\,(-1+r)}+\frac{-1+r-U0\,\alpha}{2\,(-1+r)^2}\right)(1-\delta)}{2\,(-1+r)}+\frac{p1\,r\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)(1-\delta)}{2\,(-1+r)}+\frac{p1\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)(1-\delta)}{2\,(-1+r)}-\frac{p1\,r\,(-1+r-U0\,\alpha)\left(1-\frac{-1+r-U0\,\alpha}{2\,(-1+r)}\right)(1-\delta)}{2\,(-1+r)^2}\right)\lambda==0$$

```
D[lagrangeC, λ] == 0
```

$$\mathtt{p1}\ \mathtt{r}\ (1-\mathtt{p1}\ \alpha)+\frac{\mathtt{p1}\ \mathtt{r}\ (-1+\mathtt{r}-\mathtt{U0}\ \alpha)\ \left(1-\frac{-1+\mathtt{r}-\mathtt{U0}\ \alpha}{2\ (-1+\mathtt{r})}\right)\ (1-\delta)}{2\ (-1+\mathtt{r})}==0$$

Solving the FOC's

```
Refine[Reduce[{D[lagrangeC, p1] == 0, D[lagrangeC, r] == 0, D[lagrangeC, λ] == 0},
  {p1, r, λ}], α p1 < 1 && p1 > 0 && 0 < α < 1 && U0 > 0]
```

$$\begin{aligned}
&\Bigg(\alpha\ (-1+\delta)\neq 0\ \&\&\\
&\quad\left(\mathtt{U0}==\frac{\alpha-\alpha\ \delta-2\ \sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{3\ \left(-\alpha^2+\alpha^2\ \delta\right)}\ ||\ \mathtt{U0}==\frac{\alpha-\alpha\ \delta+2\ \sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{3\ \left(-\alpha^2+\alpha^2\ \delta\right)}\right)\ \&\&\\
&\quad\mathtt{p1}==-\frac{1}{2}\ \mathtt{U0}\ (1+\mathtt{U0}\ \alpha)\ (-1+\delta)\ \&\&\ \mathtt{r}==0\Bigg)\ ||\\
&\Bigg(\mathtt{p1}==\frac{5+2\ \mathtt{U0}\ \alpha+\mathtt{U0}^2\ \alpha^2-\delta-2\ \mathtt{U0}\ \alpha\ \delta-\mathtt{U0}^2\ \alpha^2\ \delta}{8\ \alpha}\ \&\&\ \mathtt{r}==0\ \&\&\\
&\quad 5-2\ \mathtt{U0}\ \alpha-3\ \mathtt{U0}^2\ \alpha^2-\delta+2\ \mathtt{U0}\ \alpha\ \delta+3\ \mathtt{U0}^2\ \alpha^2\ \delta\neq 0\ \&\&\ \lambda==1\Bigg)\ ||\ \Bigg(\alpha\ (-1+\delta)\neq 0\ \&\&\\
&\quad\left(\mathtt{U0}==\frac{\alpha-\alpha\ \delta-2\ \sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{3\ \left(-\alpha^2+\alpha^2\ \delta\right)}\ ||\ \mathtt{U0}==\frac{\alpha-\alpha\ \delta+2\ \sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{3\ \left(-\alpha^2+\alpha^2\ \delta\right)}\right)\ \&\&\\
&\quad\mathtt{p1}==-\frac{1}{2}\ \mathtt{U0}\ (1+\mathtt{U0}\ \alpha)\ (-1+\delta)\ \&\&\ \mathtt{r}==0\ \&\&\ -5-\mathtt{U0}\ \alpha+\delta+\mathtt{U0}\ \alpha\ \delta\neq 0\Bigg)\ ||\ \Bigg(\mathtt{U0}\ \alpha\ (-1+\delta)\neq 0\ \&\&\\
&\quad\left(\mathtt{p1}==\frac{7\ \alpha-\alpha\ \delta-\sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{9\ \alpha^2}\ ||\ \mathtt{p1}==\frac{7\ \alpha-\alpha\ \delta+\sqrt{4\ \alpha^2-5\ \alpha^2\ \delta+\alpha^2\ \delta^2}}{9\ \alpha^2}\right)\ \&\&\\
&\quad\mathtt{p1}\ (-5+4\ \mathtt{p1}\ \alpha+\delta)\neq 0\ \&\&\ \mathtt{r}==\frac{\mathtt{p1}+2\ \mathtt{U0}-3\ \mathtt{p1}\ \mathtt{U0}\ \alpha}{\mathtt{p1}}\ \&\&\ \mathtt{r}\neq 0\ \&\&\\
&\quad\lambda==\frac{-5+5\ \mathtt{r}-43\ \mathtt{U0}\ \alpha+102\ \mathtt{p1}\ \mathtt{U0}\ \alpha^2-54\ \mathtt{p1}^2\ \mathtt{U0}\ \alpha^3+\delta-\mathtt{r}\ \delta+7\ \mathtt{U0}\ \alpha\ \delta-12\ \mathtt{p1}\ \mathtt{U0}\ \alpha^2\ \delta}{4\ \mathtt{r}}\Bigg)
\end{aligned}$$

We are given four solutions, where we can discard the first and the third because the are determined for a certain value of U_0, which is free in the model. We can check if the fourth solution is valid.

Checking the fourth solution, with $p_1 = \frac{7\,\alpha-\alpha\,\delta-\sqrt{4\,\alpha^2-5\,\alpha^2\,\delta+\alpha^2\,\delta^2}}{9\,\alpha^2}$

Is $0 < p_1 < 1/\alpha$?

$$\text{Simplify}\left[0 < \frac{7\alpha - \alpha\delta - \sqrt{4\alpha^2 - 5\alpha^2\delta + \alpha^2\delta^2}}{9\alpha^2} < 1/\alpha,\ 0 < \alpha < 1 \&\& 0 \le \delta < 1 \&\& U0 \ge 0\right]$$

True

Is $0 \le r < 1$?

$$\text{Simplify}\left[0 \le \frac{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2} + 2\,U0 - 3\,\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}\,U0\,\alpha}{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}} < 1,\right.$$

$$\left.0 < \alpha < 1 \&\& 0 \le \delta < 1 \&\& U0 \ge 0\right]$$

False

$$\text{Simplify}\left[0 < \frac{-1 + \frac{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}+2\,U0-3\,\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}\,U0\,\alpha}{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}} - U0\,\alpha}{2\left(-1 + \frac{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}+2\,U0-3\,\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}\,U0\,\alpha}{\frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}}\right)\alpha} < 1/\alpha,\right.$$

$$\left.0 < \alpha < 1 \&\& 0 \le \delta < 1 \&\& U0 \ge 0\right]$$

False

With $p_1 = \frac{7\alpha-\alpha\delta-\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}$ this solution does not fit the model. Even though $0 < p_1 < 1/\alpha$ is true, the other bounds are not true.

Checking the fourth solution, with $p_1 = \frac{7\alpha-\alpha\delta+\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}$

Is $p_1 > 0$?

$$\text{Simplify}\left[0 < \frac{7\alpha - \alpha\delta + \sqrt{4\alpha^2 - 5\alpha^2\delta + \alpha^2\delta^2}}{9\alpha^2} < 1/\alpha,\ 0 < \alpha < 1 \&\& 0 \le \delta < 1 \&\& U0 \ge 0\right]$$

$$\sqrt{4 - 5\delta + \delta^2} < 2 + \delta$$

Is $0 \le r < 1$?

$$\text{Simplify}\left[0 \le \frac{\frac{7\alpha-\alpha\delta+\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2} + 2\,U0 - 3\,\frac{7\alpha-\alpha\delta+\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}\,U0\,\alpha}{\frac{7\alpha-\alpha\delta+\sqrt{4\alpha^2-5\alpha^2\delta+\alpha^2\delta^2}}{9\alpha^2}} < 1,\right.$$

$$\left.0 < \alpha < 1 \&\& 0 \le \delta < 1 \&\& U0 \ge 0\right]$$

$$0 \le \frac{7 - \delta + \sqrt{4 - 5\delta + \delta^2} - 3\,U0\,\alpha\left(1 - \delta + \sqrt{4 - 5\delta + \delta^2}\right)}{7 - \delta + \sqrt{4 - 5\delta + \delta^2}} < 1$$

```
Simplify[0 < (-1 + (7 α - α δ + Sqrt[4 α^2 - 5 α^2 δ + α^2 δ^2])/(9 α^2) + 2 U0 - 3 (7 α - α δ + Sqrt[4 α^2 - 5 α^2 δ + α^2 δ^2])/(9 α^2) U0 α - U0 α) / (2 (-1 + (7 α - α δ + Sqrt[4 α^2 - 5 α^2 δ + α^2 δ^2])/(9 α^2) + 2 U0 - 3 (7 α - α δ + Sqrt[4 α^2 - 5 α^2 δ + α^2 δ^2])/(9 α^2) U0 α) / (7 α - α δ + Sqrt[4 α^2 - 5 α^2 δ + α^2 δ^2])/(9 α^2)) α) < 1 / α,
 0 < α < 1 && 0 ≤ δ < 1 && U0 ≥ 0]
```

False

With $p_1 = \frac{7\alpha - \alpha\delta + \sqrt{4\alpha^2 - 5\alpha^2\delta + \alpha^2\delta^2}}{9\alpha^2}$ this solution also does not fit the model. The last check shows that with this value of p_1, p_2 is not between 0 and $1/\alpha$, because in competition we assume the seller stays for both periods.

The second solution is therefore the solution to our problem.

```
p1 := (5 + 2 U0 α + U0^2 α^2 - δ - 2 U0 α δ - U0^2 α^2 δ) / (8 α)
```

```
r := 0
```

Inserting $r = 0$ in p_2^C

```
p2
```

$$-\frac{-1 - U0\,\alpha}{2\,\alpha}$$

Checking $p_1^C < \frac{1}{\alpha}$, $p_2^C < \frac{1}{\alpha}$ and $p_2^C < p_1^C$

```
Refine[Reduce[p1 < 1 / α, {U0}], U0 > 0 && 0 < α < 1 && 0 ≤ δ < 1]
```

$$U0 < -\frac{1}{\alpha} + \frac{2}{\alpha\sqrt{1-\delta}}$$

```
Refine[Reduce[p2 < 1 / α, {U0}], U0 > 0 && 0 < α < 1 && 0 ≤ δ < 1]
```

$$U0 < \frac{1}{\alpha}$$

```
Refine[Reduce[p2 < p1, {U0}], U0 > 0 && 0 < α < 1 && 0 ≤ δ < 1 && p2 < 1 / α && p1 < 1 / α]
```

$$U0 < \frac{-1-\delta}{\alpha\,(-1+\delta)} - \frac{2\sqrt{\delta}}{\alpha\,(1-\delta)}$$

```
Simplify[U0 < (-1 - δ)/(α (-1 + δ)) - 2 Sqrt[δ]/(α (1 - δ))]
```

$$U0 < \frac{1-\sqrt{\delta}}{\alpha + \alpha\sqrt{\delta}}$$

B.1.2 Monopoly

Monopoly

Probability function

```
q[p_, α_] := Max[1 - α p, 0]
```

Expected utility for seller for period 1 (EUM1) and for period 2 (EUM2):

```
EUM1 := q[pM1, α] pM1 (1 - rM) +
  (1 - q[pM1, α]) (q[pM2, α] pM2 (1 - rM) (1 - δ) + (1 - q[pM2, α]) U0 (1 - δ))
```

```
EUM2 := q[pM2, α] pM2 (1 - rM) + (1 - q[pM2, α]) U0
```

Expected utility for real estate agent for period 1 (EVM1) and for period 2 (EVM1)

```
EVM1 := q[pM1, α] rM pM1 + (1 - q[pM1, α]) EVM2 (1 - δ)
```

```
EVM2 := q[pM2, α] rM pM2
```

SECOND PERIOD MAXIMISATION PROBLEM

Real estate agent's Participation Constraint

```
Simplify[Reduce[q[pM2, α] pM2 (1 - rM) + (1 - q[pM2, α]) U0 ≥ pM1 rM / 4],
 0 < α < 1 && 0 < α pM2 < 1 && 0 < α pM1 < 1 && U0 ≥ 0 && 0 ≤ δ ≤ 1 && 0 ≤ rM ≤ 1]
```

$$rM == 0 \;||\; \left(pM1 \leq \frac{4\, pM2\, (1 - pM2\, \alpha + U0\, \alpha + rM\, (-1 + pM2\, \alpha))}{rM} \;\&\&\; rM > 0\right)$$

Maximisation for p_2

```
Refine[D[EVM2, pM2], α pM2 < 1]
```

$$-pM2\, rM\, \alpha + rM\, (1 - pM2\, \alpha)$$

```
Reduce[Refine[D[EVM2, pM2], α pM2 < 1] == 0, pM2]
```

$$\left(\alpha \neq 0 \;\&\&\; pM2 == \frac{1}{2\,\alpha}\right) \;||\; rM == 0$$

```
pM2 := 1 / (2 α)
```

Checking $p_2^M < \frac{1}{\alpha}$

```
Refine[pM2 < 1/α, 0 < α < 1]
```

```
True
```

FIRST PERIOD MAXIMISATION PROBLEM

```
EVM1
```

```
EUM1
```

$$\frac{rM\, (1 - \delta)\, (1 - Max[0, 1 - pM1\, \alpha])}{4\, \alpha} + pM1\, rM\, Max[0, 1 - pM1\, \alpha]$$

$$\left(\frac{1}{2} U0\, (1 - \delta) + \frac{(1 - rM)\, (1 - \delta)}{4\, \alpha}\right) (1 - Max[0, 1 - pM1\, \alpha]) + pM1\, (1 - rM)\, Max[0, 1 - pM1\, \alpha]$$

Defining the Lagrangian

```
lagrangeM := Refine[EVM1 + λ (EUM1 - U0), α pM1 < 1]
```

```
lagrangeM
```

$$\text{pM1 rM } (1 - \text{pM1}\,\alpha) + \frac{1}{4}\,\text{pM1 rM } (1-\delta) + \left(-\text{U0} + \text{pM1 } (1-\text{rM}) \ (1 - \text{pM1}\,\alpha) + \text{pM1}\,\alpha \left(\frac{1}{2}\,\text{U0 } (1-\delta) + \frac{(1-\text{rM}) \ (1-\delta)}{4\,\alpha}\right)\right)\lambda$$

```
D[lagrangeM, pM1]
```

$$-\text{pM1 rM}\,\alpha + \text{rM } (1 - \text{pM1}\,\alpha) + \frac{1}{4}\,\text{rM } (1-\delta) + \left(-\text{pM1 } (1-\text{rM})\,\alpha + (1-\text{rM}) \ (1 - \text{pM1}\,\alpha) + \alpha \left(\frac{1}{2}\,\text{U0 } (1-\delta) + \frac{(1-\text{rM}) \ (1-\delta)}{4\,\alpha}\right)\right)\lambda$$

```
D[lagrangeM, rM]
```

$$\text{pM1 } (1 - \text{pM1}\,\alpha) + \frac{1}{4}\,\text{pM1 } (1-\delta) + \left(-\text{pM1 } (1 - \text{pM1}\,\alpha) - \frac{1}{4}\,\text{pM1 } (1-\delta)\right)\lambda$$

```
D[lagrangeM, λ]
```

$$-\text{U0} + \text{pM1 } (1-\text{rM}) \ (1 - \text{pM1}\,\alpha) + \text{pM1}\,\alpha \left(\frac{1}{2}\,\text{U0 } (1-\delta) + \frac{(1-\text{rM}) \ (1-\delta)}{4\,\alpha}\right)$$

```
Refine[Reduce[{D[lagrangeM, pM1] == 0 && D[lagrangeM, rM] == 0 && D[lagrangeM, λ] == 0},
  {pM1, rM, λ}], α pM1 < 1 && α > 0 && δ ≥ 0 && U0 ≥ 0 && pM1 > 0 && rM > 0]
```

$$\left(\delta == 3 + 2\sqrt{3} \ \&\& \ \text{pM1} == \frac{5-\delta}{4\,\alpha} \ \&\& -1 + \text{rM} + \text{U0}\,\alpha + \text{U0}\,\alpha\,\delta \neq 0 \ \&\& \ \lambda == \frac{\text{rM}}{-1 + \text{rM} + \text{U0}\,\alpha + \text{U0}\,\alpha\,\delta}\right) \ || \ \left(\text{pM1} == \frac{5 + 2\,\text{U0}\,\alpha - \delta - 2\,\text{U0}\,\alpha\,\delta}{8\,\alpha} \ \&\& \ -20\,\text{pM1} + 5\,\text{U0} + 2\,\text{U0}^2\,\alpha + 4\,\text{pM1}\,\delta - 6\,\text{U0}\,\delta - 4\,\text{U0}^2\,\alpha\,\delta + \text{U0}\,\delta^2 + 2\,\text{U0}^2\,\alpha\,\delta^2 \neq 0 \ \&\& \ \text{rM} == \frac{-20\,\text{pM1} + 27\,\text{U0} - 2\,\text{U0}^2\,\alpha + 4\,\text{pM1}\,\delta + 6\,\text{U0}\,\delta + 4\,\text{U0}^2\,\alpha\,\delta - \text{U0}\,\delta^2 - 2\,\text{U0}^2\,\alpha\,\delta^2}{-20\,\text{pM1} + 5\,\text{U0} + 2\,\text{U0}^2\,\alpha + 4\,\text{pM1}\,\delta - 6\,\text{U0}\,\delta - 4\,\text{U0}^2\,\alpha\,\delta + \text{U0}\,\delta^2 + 2\,\text{U0}^2\,\alpha\,\delta^2} \ \&\& \ \lambda == 1\right) \ || \ \left(-3 - 6\,\delta + \delta^2 \neq 0 \ \&\& \ \text{U0} == 0 \ \&\& \ \text{pM1} == \frac{5-\delta}{4\,\alpha} \ \&\& \ 5 - 5\,\text{rM} - \delta + \text{rM}\,\delta \neq 0 \ \&\& \ \lambda == \frac{\text{rM } (-5+\delta)}{5 - 5\,\text{rM} - \delta + \text{rM}\,\delta}\right)$$

The second solution is the only valid one, since the two other solution have determi::::::::::::ned values for δ or U_0 which are free variables.

```
pM1 := (5 + 2 U0 α - δ - 2 U0 α δ) / (8 α)
```

```
rM := (-20 pM1 + 27 U0 - 2 U0^2 α + 4 pM1 δ + 6 U0 δ + 4 U0^2 α δ - U0 δ^2 - 2 U0^2 α δ^2) /
      (-20 pM1 + 5 U0 + 2 U0^2 α + 4 pM1 δ - 6 U0 δ - 4 U0^2 α δ + U0 δ^2 + 2 U0^2 α δ^2)
```

```
Simplify[rM]
```

$$-\frac{(-5+\delta)^2 + 4\,\text{U0}^2\,\alpha^2 \ (-1+\delta)^2 + 4\,\text{U0}\,\alpha \left(-11 - 6\,\delta + \delta^2\right)}{-(-5+\delta)^2 + 4\,\text{U0}^2\,\alpha^2 \ (-1+\delta)^2}$$

Checking $p_1^M < \frac{1}{\alpha}$ and $p_2^M < p_1^M$

```
Refine[Reduce[0 ≤ pM1 < 1/α, U0], 0 < α < 1 && 0 ≤ δ < 1 && 0 ≤ U0]
```

$$U0 < \frac{-3-\delta}{-2\alpha+2\alpha\delta}$$

```
Refine[Reduce[pM2 < pM1, U0], 0 < α < 1 && 0 ≤ δ < 1 && 0 ≤ U0 < (-3 - δ)/(-2 α + 2 α δ)]
```

```
True
```

Checking $r^M > 0$

```
Refine[Reduce[rM > 0, U0], 0 < α < 1 && 0 ≤ δ < 1 && 0 ≤ U0 < (-3 - δ)/(-2 α + 2 α δ)]
```

$$U0 < -\frac{2\sqrt{2}\sqrt{3+6\delta-\delta^2}}{\alpha(1-\delta)^2}+\frac{11+6\delta-\delta^2}{2\alpha(-1+\delta)^2}$$

```
Simplify[-(2 Sqrt[2] Sqrt[3 + 6 δ - δ^2])/(α (1 - δ)^2) + (11 + 6 δ - δ^2)/(2 α (-1 + δ)^2)]
```

$$-\frac{-11-6\delta+\delta^2+4\sqrt{6+12\delta-2\delta^2}}{2\alpha(-1+\delta)^2}$$

```
Refine[-(-11 - 6 δ + δ^2 + 4 Sqrt[6 + 12 δ - 2 δ^2]) > 0, 0 ≤ δ < 1]
```

```
True
```

```
Simplify[Limit[-(2 Sqrt[2] Sqrt[3 + 6 δ - δ^2])/(α (1 - δ)^2) + (11 + 6 δ - δ^2)/(2 α (-1 + δ)^2), δ → 0]]
```

$$\frac{11-4\sqrt{6}}{2\alpha}$$

```
Refine[(-3 - δ)/(-2 α + 2 α δ) > -(2 Sqrt[2] Sqrt[3 + 6 δ - δ^2])/(α (1 - δ)^2) + (11 + 6 δ - δ^2)/(2 α (-1 + δ)^2), 0 < α < 1 && 0 ≤ δ < 1]
```

```
True
```

Checking the second-period participation constraint is satisfied

```
Simplify[pM1 ≤ (4 pM2 (1 - pM2 α + U0 α + rM (-1 + pM2 α)))/rM, 0 < α < 1 && 0 ≤ δ < 1 &&
  0 ≤ U0 < -(-11 - 6 δ + δ^2 + 4 Sqrt[6 + 12 δ - 2 δ^2])/(2 α (-1 + δ)^2) && 0 < α pM2 < 1 && 0 < α pM1 < 1 && 0 < rM]
```

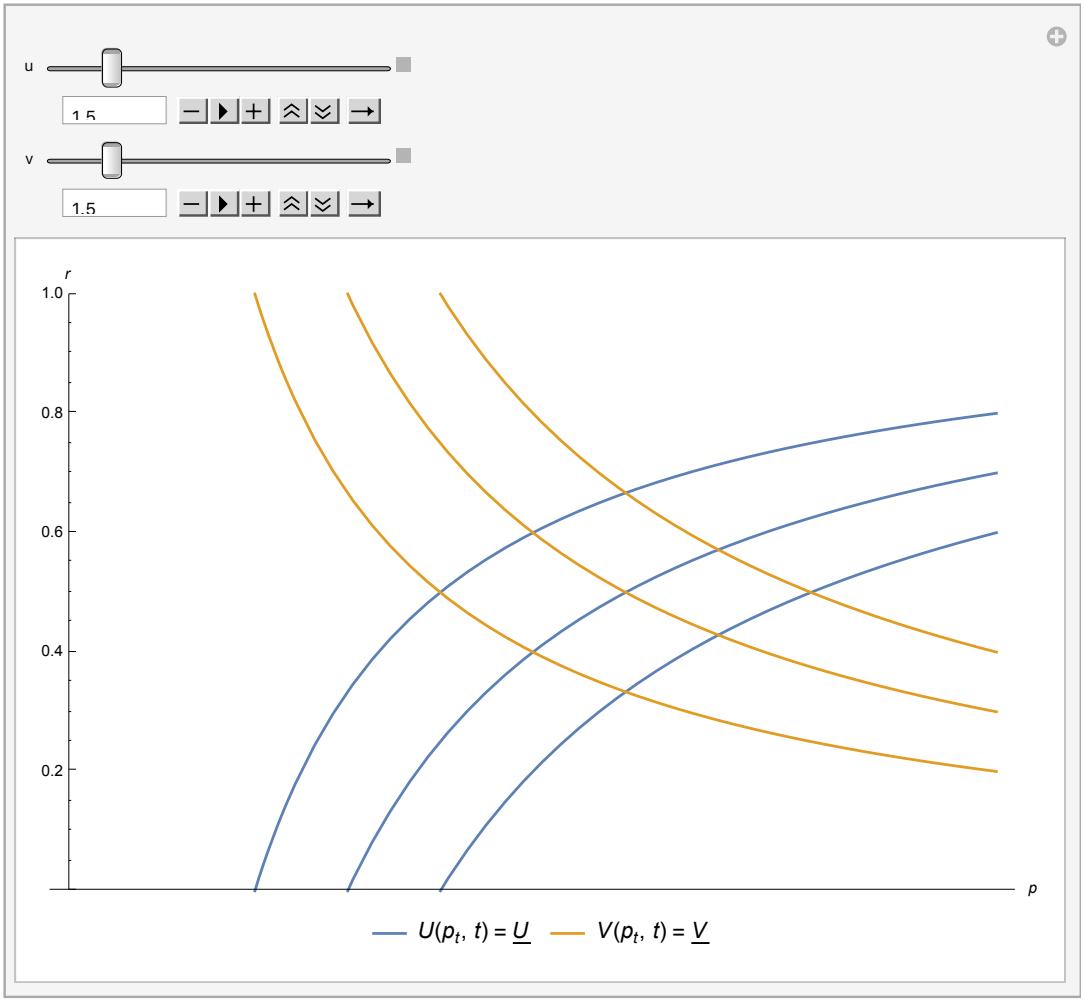
$$(-5+\delta)^3+8\,U0^3\alpha^3(-9+\delta)(-1+\delta)^2+2\,U0\,\alpha\left(461+89\delta-41\delta^2+3\delta^3\right)+4\,U0^2\alpha^2\left(1+33\delta-37\delta^2+3\delta^3\right)\geq 0$$

B.1.3 Indifference Curves

Indifference curves for utility functions

$U = p(1 - r) = c$ and $V = p\,r = c$

```
Manipulate[Plot[
  {(p - u) / p, (p - (u + 0.5)) / p, (p - (u - 0.5)) / p, v / p, (v + 0.5) / p, (v - 0.5) / p},
  {p, 0, 5}, ImageSize → Large, PlotLegends → Placed[{HoldForm[U[pt, t] = U],
     None, None, HoldForm[V[pt, t] = V], None , None }, Below],
  AxesLabel → {HoldForm[p], HoldForm[r]}, PlotRange → {0, 1},
  Ticks → {None, Automatic},
  PlotStyle → {RGBColor[0.368417, 0.506779, 0.709798], RGBColor[0.368417,
     0.506779, 0.709798], RGBColor[0.368417, 0.506779, 0.709798], RGBColor[
     0.880722, 0.611041, 0.142051], RGBColor[0.880722, 0.611041, 0.142051],
    RGBColor[0.880722, 0.611041, 0.142051]}], {u, 0, 10}, {v, 0, 10}]
```



Indifference curves for Competition

The second-period listing price for competition is given by:

$$p2 := \frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}$$

Seller's indifference curves: EU = $\underline{U}$ (here c)

$$\text{Simplify}\left[q[p1, \alpha]\; p1\;(1 - r) + (1 - q[p1, \alpha])\left(q\left[\frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}, \alpha\right]\frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}\,(1 - r) + \left(1 - q\left[\frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}, \alpha\right]\right)u\right) == c,\; p1\,\alpha < 1 \;\&\&\; \frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}\,\alpha < 1\right]$$

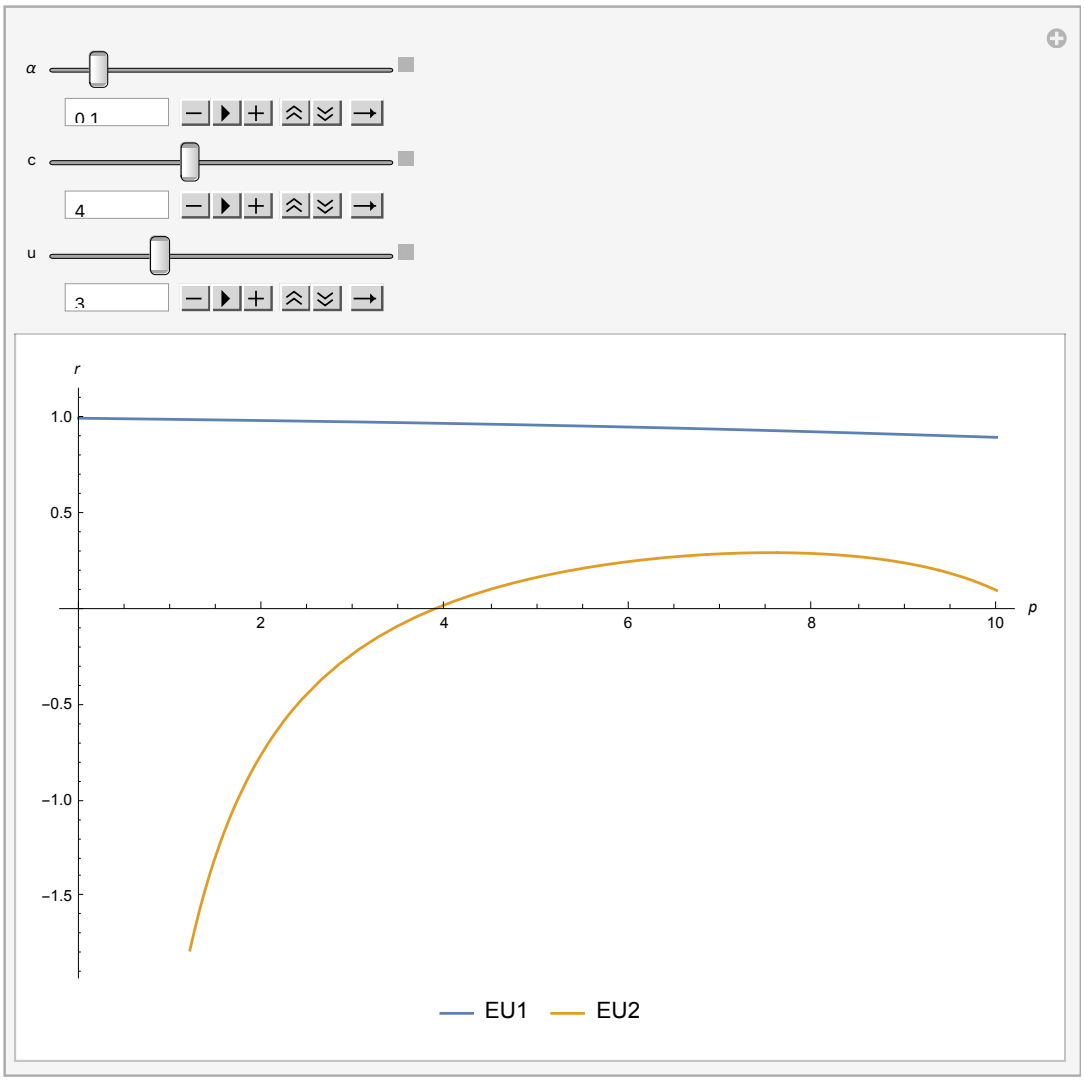
$$p1\,(-1 + r)\,(-1 + p1\,\alpha) == c + \frac{p1\,(1 - r + u\,\alpha)^2}{4\,(-1 + r)}$$

$$\text{Refine}\left[\text{Reduce}\left[p1\,(-1 + r)\,(-1 + p1\,\alpha) == c + \frac{p1\,(1 - r + u\,\alpha)^2}{4\,(-1 + r)}, r\right], p1\,\alpha < 1 \;\&\&\; \frac{-1 + r - u\,\alpha}{2\,(-1 + r)\,\alpha}\,\alpha < 1\right]$$

$$(p1 == 0 \;\&\&\; c == 0 \;\&\&\; -\alpha + r\,\alpha \neq 0) \;||\; \left(p1\,(-5 + 4\,p1\,\alpha) \neq 0 \;\&\&\; \left(r == \frac{2\,c - 5\,p1 + 4\,p1^2\,\alpha - p1\,u\,\alpha - 2\sqrt{c^2 - c\,p1\,u\,\alpha - p1^2\,u^2\,\alpha^2 + p1^3\,u^2\,\alpha^3}}{-5\,p1 + 4\,p1^2\,\alpha} \;||\; r == \frac{2\,c - 5\,p1 + 4\,p1^2\,\alpha - p1\,u\,\alpha + 2\sqrt{c^2 - c\,p1\,u\,\alpha - p1^2\,u^2\,\alpha^2 + p1^3\,u^2\,\alpha^3}}{-5\,p1 + 4\,p1^2\,\alpha}\right)\right) \;||\; (\alpha == 0 \;\&\&\; p1 == 0 \;\&\&\; c == 0)$$

We have two solutions, which we plot to determine which is relevant:

```
Manipulate[Plot[{(2 c - 5 p1 + 4 p1^2 α - p1 u α - 2 Sqrt[c^2 - c p1 u α - p1^2 u^2 α^2 + p1^3 u^2 α^3])/(-5 p1 + 4 p1^2 α),
  (2 c - 5 p1 + 4 p1^2 α - p1 u α + 2 Sqrt[c^2 - c p1 u α - p1^2 u^2 α^2 + p1^3 u^2 α^3])/(-5 p1 + 4 p1^2 α)}, {p1, 0, 1/α},
 PlotLegends → Placed[{"EU1", "EU2"}, Below], ImageSize → Large,
 AxesLabel → {HoldForm[p], HoldForm[r]}], {α, 0, 1}, {c, 0, 10}, {u, 0, 10}]
```



When *c* and *u* are equal the indifference curves crosses.
I have chosen EU2 because it best describes the indifference curve of the seller.

Agent's indifference curve: EV = 0

$$\text{Simplify}\left[q[p1,\alpha]\ p1\ r+(1-q[p1,\alpha])\left(q\left[\frac{-1+r-u\alpha}{2(-1+r)\alpha},\alpha\right]\ r\ \frac{-1+r-u\alpha}{2(-1+r)\alpha}\right)==0,\ p1\alpha<1\&\&\frac{-1+r-u\alpha}{2(-1+r)\alpha}\alpha<1\&\&0\le r<1\right]$$

$$p1\ r\left(-5+4\ p1\ \alpha+u^2\alpha^2+r\ (10-8\ p1\ \alpha)+r^2\ (-5+4\ p1\ \alpha)\right)==0$$

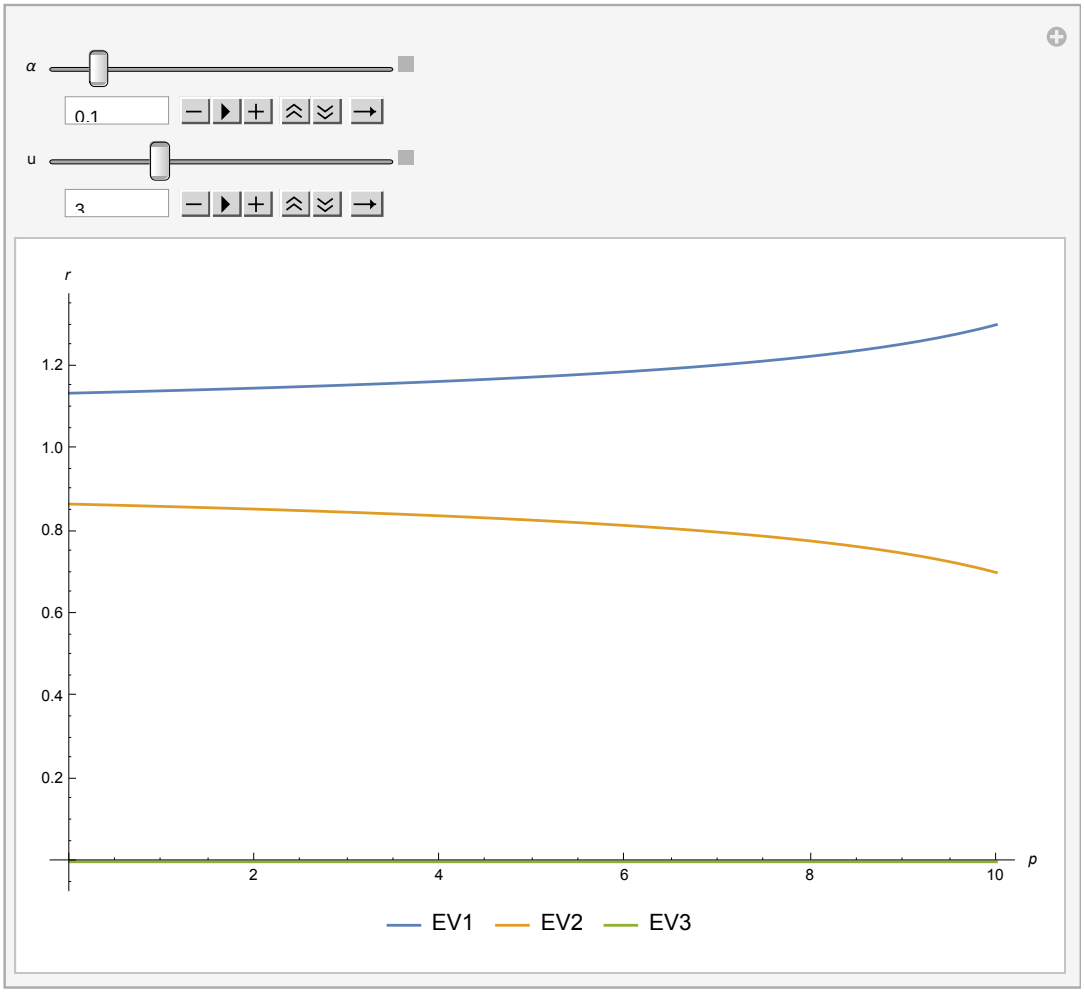
$$\text{Refine}\left[\text{Reduce}\left[p1\ r\left(-5+4\ p1\ \alpha+u^2\alpha^2+r\ (10-8\ p1\ \alpha)+r^2\ (-5+4\ p1\ \alpha)\right)==0,r\right],\ p1\alpha<1\&\&\frac{-1+r-u\alpha}{2(-1+r)\alpha}\alpha<1\&\&u<1/\alpha\&\&0\le r<1\right]$$

$$r==\frac{-5+4\ p1\ \alpha-\sqrt{5\ u^2\alpha^2-4\ p1\ u^2\alpha^3}}{-5+4\ p1\ \alpha}\ ||$$

$$r==\frac{-5+4\ p1\ \alpha+\sqrt{5\ u^2\alpha^2-4\ p1\ u^2\alpha^3}}{-5+4\ p1\ \alpha}\ ||\ p1==0\ ||\ r==0$$

Here we have three solution, which we have plotted below to determine which is best.

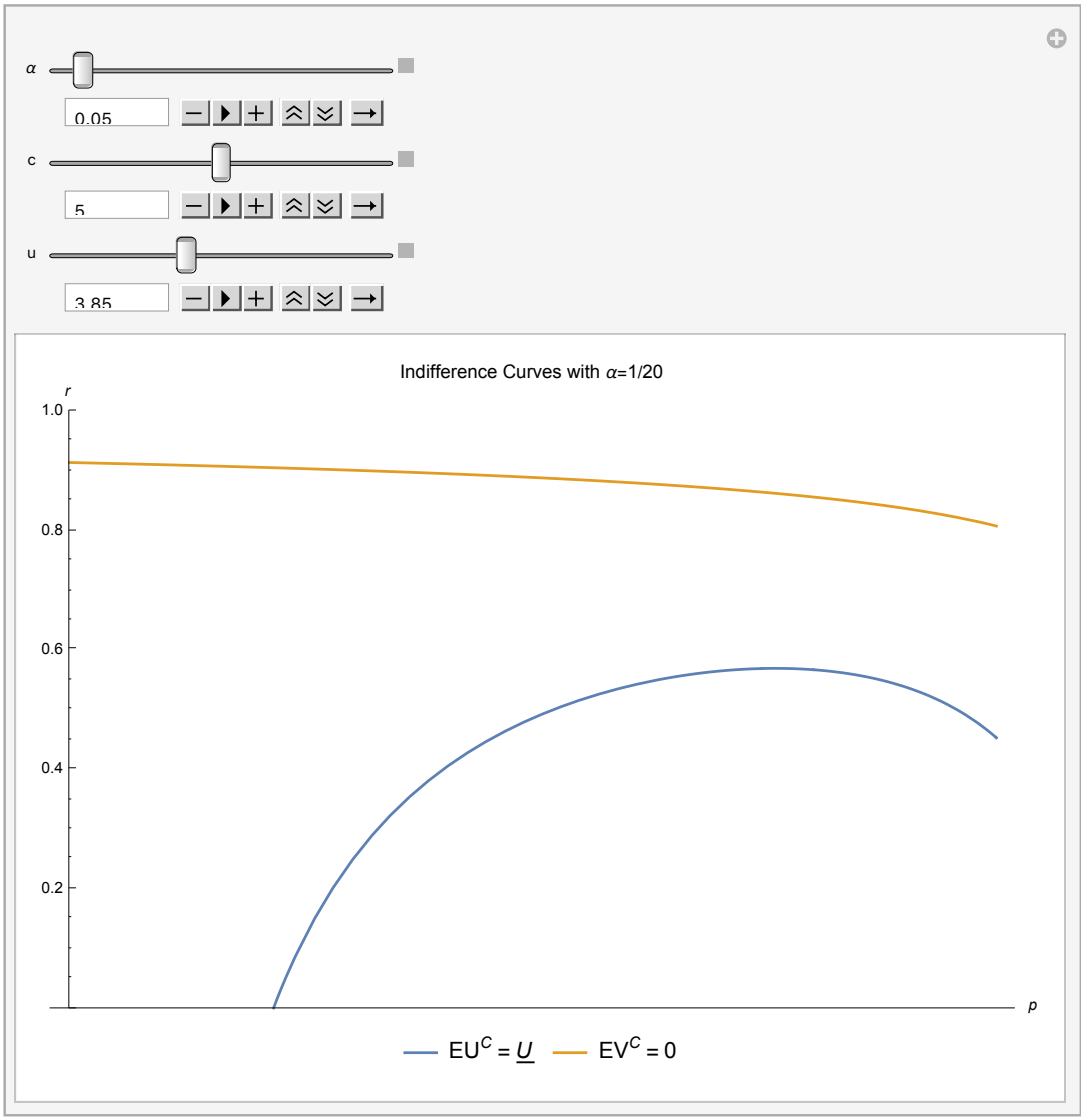
```
Manipulate[
 Plot[{(-5 + 4 p1 α - √(5 u^2 α^2 - 4 p1 u^2 α^3))/(-5 + 4 p1 α),
   (-5 + 4 p1 α + √(5 u^2 α^2 - 4 p1 u^2 α^3))/(-5 + 4 p1 α), 0},
  {p1, 0, 1 / α}, PlotLegends → Placed[{"EV1", "EV2", "EV3"}, Below],
  ImageSize → Large, AxesLabel → {HoldForm[p], HoldForm[r]}], {α, 0, 1}, {u, 0, 10}]
```



There are three solutions to the real estate agent's indifference curves when EV = 0. The first, EV1, is determined for $r > 1$, which is not possible. EV3 says $r = 0$. I have chosen EV2 because it shows his indifference curves decreases when α increases, which is what we would expect.

Together

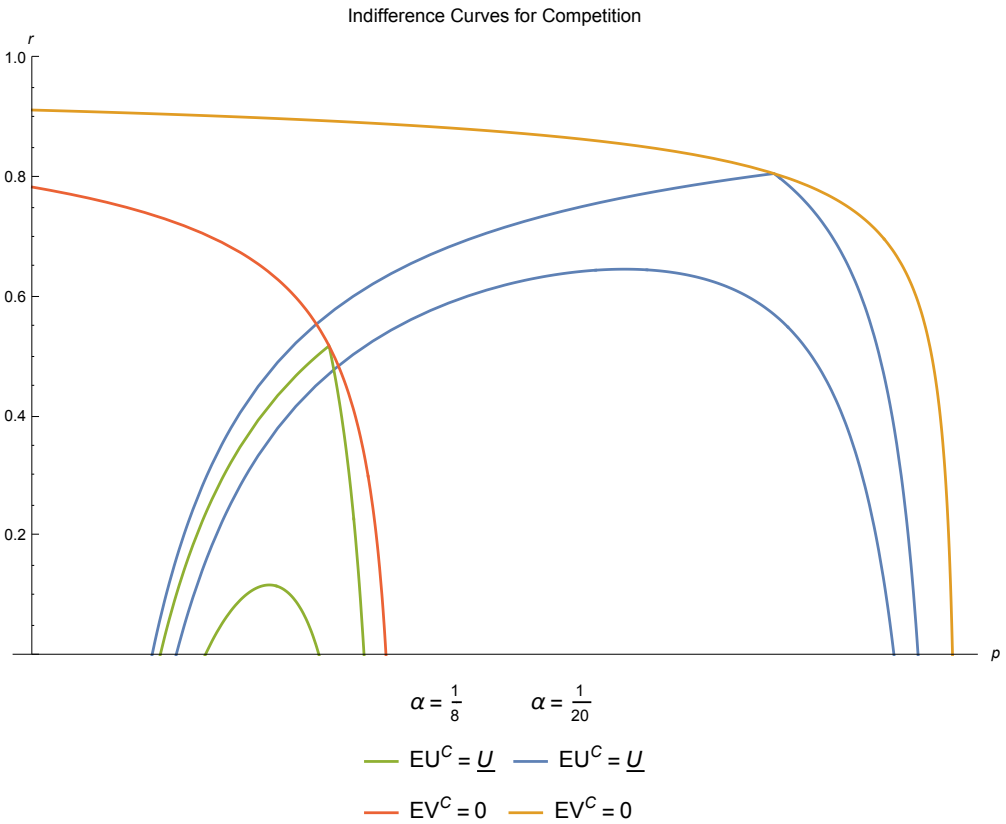
```
Manipulate[Plot[{(2 c - 5 p1 + 4 p1^2 α - p1 u α + 2 Sqrt[c^2 - c p1 u α - p1^2 u^2 α^2 + p1^3 u^2 α^3])/(-5 p1 + 4 p1^2 α),
  (-5 + 4 p1 α + Sqrt[5 u^2 α^2 - 4 p1 u^2 α^3])/(-5 + 4 p1 α)}, {p1, 0, 1 / α}, ImageSize → Large,
 PlotLegends → Placed[{HoldForm[EU^C = U], HoldForm[EV^C = 0]}, Below],
 PlotRange → {0, 1}, AxesLabel → {HoldForm[p], HoldForm[r]},
 Ticks → {None, Automatic}, PlotLabel → "Indifference Curves with α=1/20"],
{α, 0, 1}, {c, 0, 10}, {u, 0, 10}]
```



Together for both levels of α.

```
Manipulate[
 Plot[{0, 0, (2 cB - 5 p1 + 4 p1^2 αB - p1 u αB + 2 Sqrt[cB^2 - cB p1 u αB - p1^2 u^2 αB^2 + p1^3 u^2 αB^3])/(-5 p1 + 4 p1^2 αB),
  (2 cG - 5 p1 + 4 p1^2 αG - p1 u αG + 2 Sqrt[cG^2 - cG p1 u αG - p1^2 u^2 αG^2 + p1^3 u^2 αG^3])/(-5 p1 + 4 p1^2 αG),
  (2 cB1 - 5 p1 + 4 p1^2 αB - p1 u αB + 2 Sqrt[cB1^2 - cB1 p1 u αB - p1^2 u^2 αB^2 + p1^3 u^2 αB^3])/(-5 p1 + 4 p1^2 αB),
  (2 cG1 - 5 p1 + 4 p1^2 αG - p1 u αG + 2 Sqrt[cG1^2 - cG1 p1 u αG - p1^2 u^2 αG^2 + p1^3 u^2 αG^3])/(-5 p1 + 4 p1^2 αG),
  (-5 + 4 p1 αB + Sqrt[5 u^2 αB^2 - 4 p1 u^2 αB^3])/(-5 + 4 p1 αB), (-5 + 4 p1 αG + Sqrt[5 u^2 αG^2 - 4 p1 u^2 αG^3])/(-5 + 4 p1 αG)},
  {p1, 0, 25}, ImageSize → Large, PlotLegends →
   Placed[{HoldForm[α = 1 / 8], HoldForm[α = 1 / 20], HoldForm[EU^C = U],
     HoldForm[EU^C = U], None, None, HoldForm[EV^C = 0], HoldForm[EV^C = 0 ]}, Below],
  PlotRange → {0, 1}, AxesLabel → {HoldForm[p], HoldForm[r]},
  Ticks → {None, Automatic}, PlotLabel → "Indifference Curves for Competition",
  PlotStyle → {White, White, RGBColor[0.560181, 0.691569, 0.194885],
    RGBColor[0.368417, 0.506779, 0.709798], RGBColor[0.560181, 0.691569,
     0.194885], RGBColor[0.368417, 0.506779, 0.709798], RGBColor[0.922526,
     0.385626, 0.209179], RGBColor[0.880722, 0.611041, 0.142051]}],
 {αG, 0, 1}, {αB, 0, 1}, {cG, 0, 10}, {cB, 0, 10}, {u, 0, 12},
 {cG1, 0, 10}, {cB1, 0, 10}]
```

αG 0.05
αB 0.125
cG 4.5
cB 4.5
u 3.85
cG1 3.85
cB1 3.85



Indifference curves for Monopoly

The second-period listing price is given by:

```
p2 := 1/(2 α)
```

Seller's indifference curves, EV = U_0

```
Simplify[
 q[p1, α] p1 (1 - r) + (1 - q[p1, α]) (q[1/(2 α), α] 1/(2 α) (1 - r) + (1 - q[1/(2 α), α]) u) == u,
 p1 α < 1]
```

$$p1\,(5 - 5\,r + 4\,p1\,(-1 + r)\,\alpha + 2\,u\,\alpha) == 4\,u$$

```
Refine[Solve[p1 (5 - 5 r + 4 p1 (-1 + r) α + 2 u α) == 4 u, r], p1 α < 1]
```

$$\left\{\left\{r \to \frac{-5\,p1 + 4\,u + 4\,p1^2\,\alpha - 2\,p1\,u\,\alpha}{p1\,(-5 + 4\,p1\,\alpha)}\right\}\right\}$$

Agent EV

```
Simplify[q[p1, α] p1 r + (1 - q[p1, α]) (q[1/(2 α), α] r 1/(2 α)) == c, p1 α < 1]
```

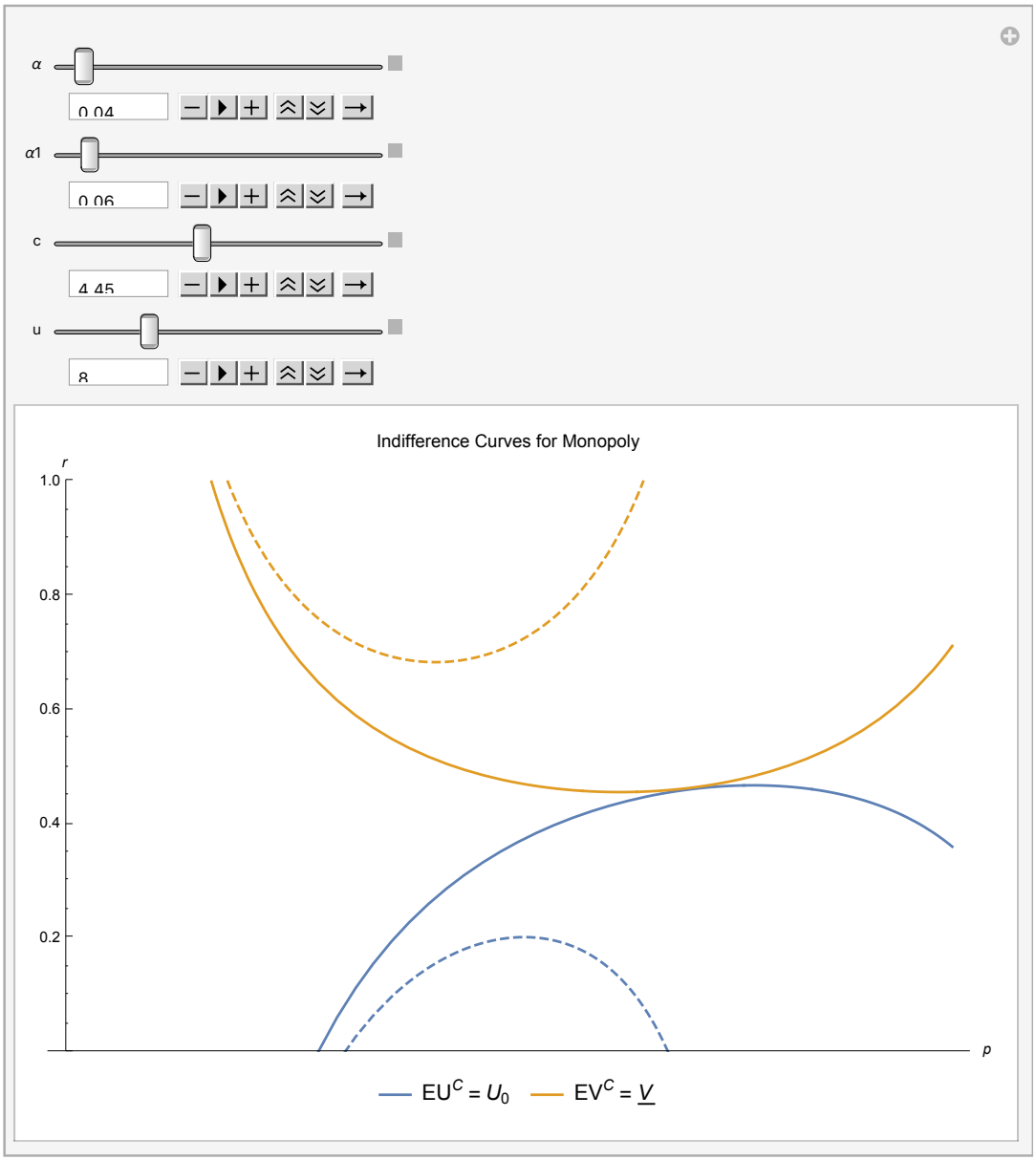
$$\frac{1}{4}\,p1\,r\,(5 - 4\,p1\,\alpha) == c$$

```
Refine[Reduce[1/4 p1 r (5 - 4 p1 α) == c, r], p1 α < 1]
```

$$\left(p1\,(-5 + 4\,p1\,\alpha) \neq 0 \;\&\&\; r == -\frac{4\,c}{p1\,(-5 + 4\,p1\,\alpha)}\right) \;||\; (\alpha == 0 \;\&\&\; p1 == 0 \;\&\&\; c == 0) \;||\; (\alpha \neq 0 \;\&\&\; p1 == 0 \;\&\&\; c == 0)$$

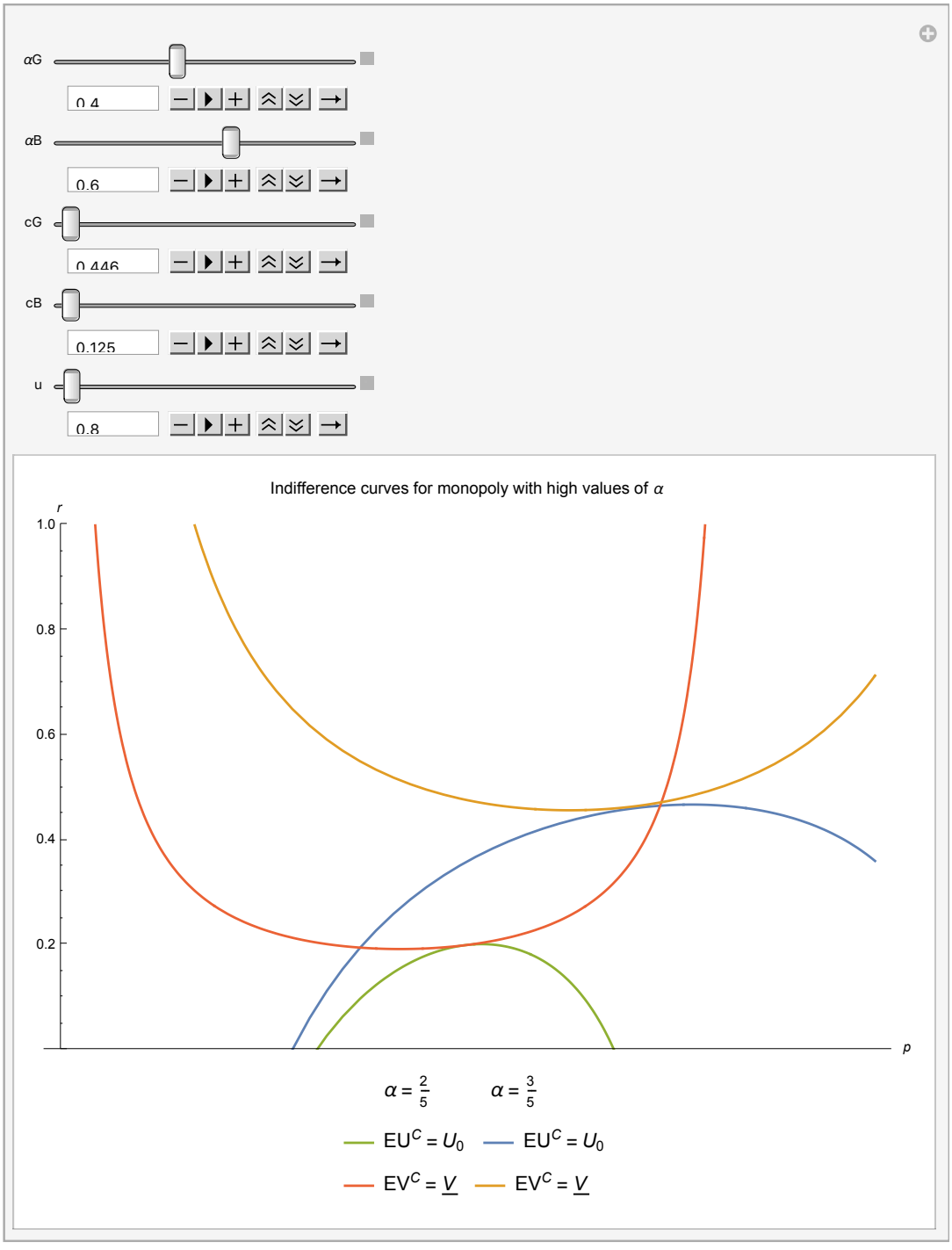
Together

```
Manipulate[
 Plot[{(-5 p1 + 4 u + 4 p1^2 α - 2 p1 u α)/(p1 (-5 + 4 p1 α)), -(4 c)/(p1 (-5 + 4 p1 α)), (-5 p1 + 4 u + 4 p1^2 α1 - 2 p1 u α1)/(p1 (-5 + 4 p1 α1)),
   -(4 c)/(p1 (-5 + 4 p1 α1))}, {p1, 0, 1 / α}, ImageSize → Large,
  PlotLegends → Placed[{HoldForm[EU^C = U_0], HoldForm[EV^C = V], None, None}, Below],
  PlotRange → {0, 1}, AxesLabel → {HoldForm[p], HoldForm[r]},
  Ticks → {None, Automatic}, PlotLabel → "Indifference Curves for Monopoly",
  PlotStyle → {RGBColor[0.368417, 0.506779, 0.709798], RGBColor[0.880722,
     0.611041, 0.142051], {RGBColor[0.368417, 0.506779, 0.709798], Dashed},
    {RGBColor[0.880722, 0.611041, 0.142051], Dashed}},
  PlotStyle → {Line, Line, Dashed, Dashed}], {α, 0, 1},
 {α1, 0, 1}, {c, 0, 10}, {u, 0, 30}]
```


α
0.04
α1
0.06
c
4.45
u
8
Indifference Curves for Monopoly
r
1.0
0.8
0.6
0.4
0.2
p
EU^C = U_0
EV^C = V

Together

```
Manipulate[Plot[{0, 0, (-5 p1 + 4 u + 4 p1^2 αB - 2 p1 u αB)/(p1 (-5 + 4 p1 αB)), (-5 p1 + 4 u + 4 p1^2 αG - 2 p1 u αG)/(p1 (-5 + 4 p1 αG)),
  -(4 cB)/(p1 (-5 + 4 p1 αB)), -(4 cG)/(p1 (-5 + 4 p1 αG))}, {p1, 0, 1 / αG}, ImageSize → Large,
 PlotLegends → Placed[{HoldForm[α = 2/5], HoldForm[α = 3/5], HoldForm[EU^C = U_0],
    HoldForm[EU^C = U_0], HoldForm[EV^C = V ], HoldForm[EV^C = V ]}, Below],
 PlotRange → {0, 1}, AxesLabel → {HoldForm[p], HoldForm[r]},
 Ticks → {None, Automatic},
 PlotLabel → "Indifference curves for monopoly with high values of α",
 PlotStyle → {White, White, RGBColor[0.560181, 0.691569, 0.194885],
   RGBColor[0.368417, 0.506779, 0.709798], RGBColor[0.922526,
    0.385626, 0.209179], RGBColor[0.880722, 0.611041, 0.142051]}],
{αG, 0, 1}, {αB, 0, 1}, {cG, 0, 100}, {cB, 0, 100}, {u, 0, 100}]
```


αG
0.4
αB
0.6
cG
0.446
cB
0.125
u
0.8
Indifference curves for monopoly with high values of α
r
1.0
0.8
0.6
0.4
0.2
p
α = 2/5
α = 3/5
EU^C = U_0
EU^C = U_0
EV^C = V
EV^C = V

References

Paul Anglin. Contracts for the sale of residential real estate. *The Journal of Real Estate Finance and Economics*, 8(3):195–211, 1994. ISSN 0895-5638.

Paul Anglin and Richard Arnott. Residential real estate brokerage as a principal-agent problem. *The Journal of Real Estate Finance and Economics*, 4(2):99–125, 1991. ISSN 0895-5638.

Michael A. Arnold. The principal-agent relationship in real estate brokerage services. *Real Estate Economics*, 20(1):89–106, 1992. ISSN 1080-8620.

Kristian Dreyer and Ali Simiab. *Ejendomsmæglerloven med kommentarer*. Karnov Group Denmark A/S, København, 2015. ISBN 9788761936677.

Steven D. Levitt and Chad Syverson. Market distortions when agents are better informed: The value of information in real estate transactions. *The Review of Economics and Statistics*, 90(4):599–611, 2008a. ISSN 00346535.

Steven D Levitt and Chad Syverson. Antitrust implications of outcomes when home sellers use flat-fee real estate agents. *Brookings-Wharton Papers on Urban Affairs*, 2008: 47–93, 2008b. ISSN 1528-7084.

Ines Macho-Stadler and J. David Perez-Castrillo. *An Introduction to the Economics of Information: Incentives and Contracts*. Oxford University Press Inc, USA, New York, 2. ed. edition, 2001. ISBN 0199243271.

The Business Authority. *Analyse af dele af lov om omsætning af fast ejendom*. Erhvervstyrelsen, 2013.

The Competition and Consumer Authority. *Forbrugerforholdindekset 2018*. Konkurrence- og Forbrugerstyrelsen, 2018.

The Competition Authority. *Undersøgelse af ejendomsmæglerbranchens salærer og rabatter*. Konkurrencestyrelsen, 1999.

The Competition Authority. *Konkurrenceredegørelse 2004*. Konkurrencestyrelsen, 2004.

The Competition Authority. *Bolighandel - Ydelser fra ejendomsmæglere og advokater*. Konkurrencestyrelsen, 2009.

The Government. *Bedre og billigere bolighandel*. Økonomi- og Erhvervsministeret, 2005.