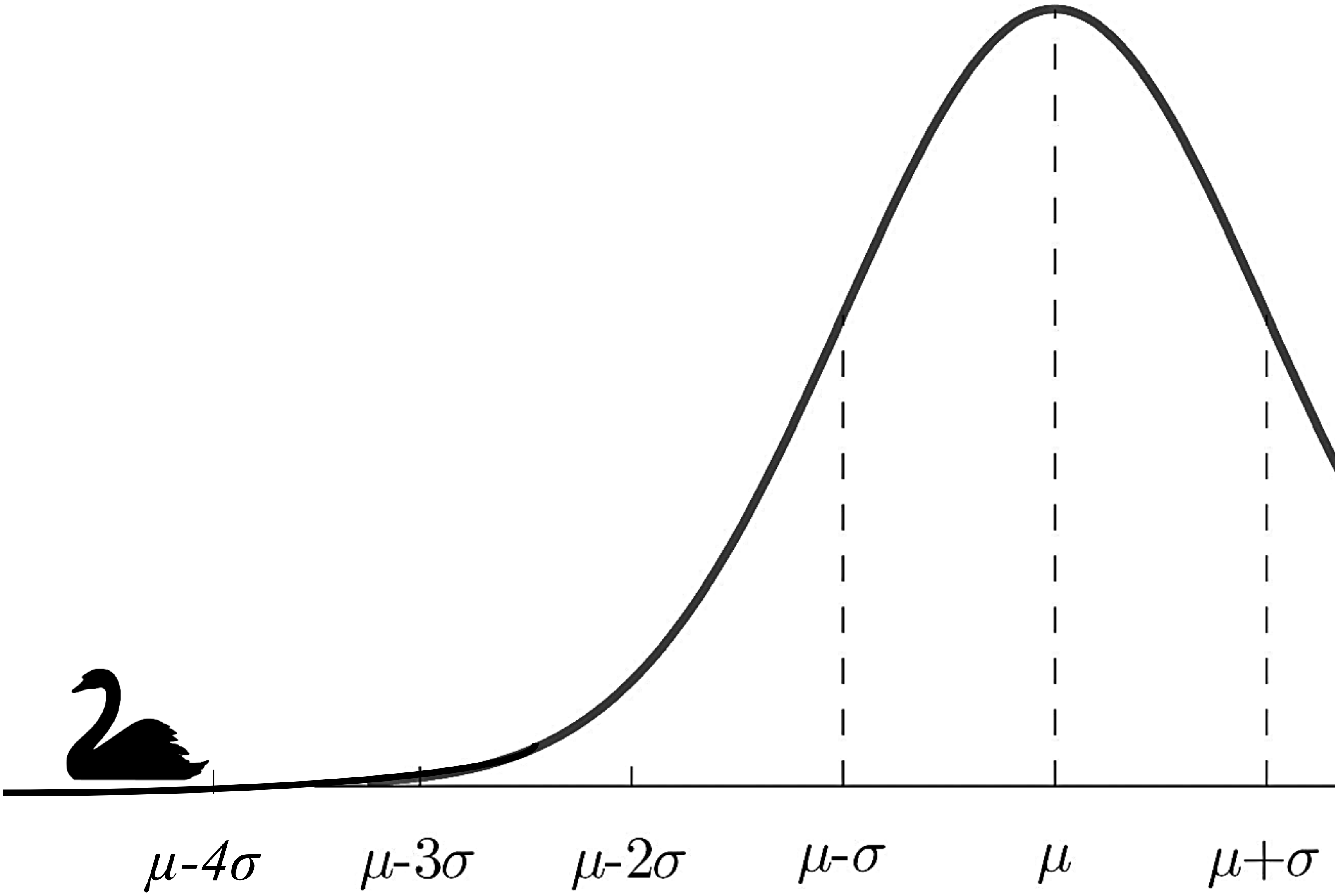

A TALE OF TAILS

IMPROVING FINANCIAL RISK MEASURES THROUGH ADVANCED DISTRIBUTION MODELLING

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1 ABSTRACT

This paper provides suggestions for improvement of the portfolio risk measures: Value-at-Risk and Expected Shortfall. Specifically, this master thesis seeks to improve the accuracy of portfolio risk measures through modelling of non-normality in asset returns with a GARCH-EVT-Copula framework. The applied statistical methods are AR(p)-GJR-GARCH(p,q), Extreme Value Theory and student's t copula. Combined these statistical tools allow the authors to account for non-normal distribution patterns in relation to skewness, excess kurtosis, heavy tails, volatility clustering and non-linear correlations.

The calculations are performed based on a portfolio representing a broad selection of European asset classes including equity, high yield bonds and government bonds. Given this portfolio, the authors document that assuming normality leads to a risk underestimation of more than 35% in several cases. Further investigation reveals that the risk underestimations are of similar nature for risk conservative and risk seeking investors whereby making the modelling concerns of relevance to a broad audience. In sum, the results of the analysis clearly demonstrate the inappropriateness of assuming normality and at the same time document the significant estimation improvements associated with the suggested GARCH-EVT-Copula framework.

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2 PROBLEM DRIVERS

The frequency to which global financial markets are under influence by extreme events is ever increasing. These unexpected and undesired events often make us question if one could have taken action to actively prevent or at least mitigate the damages. The increasing frequency and severity of financial crises underline the importance of developing current risk management practices to accurately represent the empirical behavior of the global financial markets.

The first chapter of this thesis aims at presenting three main drivers behind the problem statement. Broadly, the three drivers are: Increased market interdependency due to liberalization and globalization, regulatory initiatives and lastly, the shift in investor behavior going from the traditional "reaching for quality" to the more desperate behavior "reaching for yield", which we observe in today's low yield environment.

2.1 INCREASED MARKET INTERDEPENDENCY AND GLOBALIZATION

The financial markets have gone through several paradigm shifts over the years. The technological advancement has led to increased transparency and easier market access for professionals and privates. The main advantage of the development is growth in economic wealth and efficient capital allocation. However, the increased activity level has also led to increased risk; historically the financial markets were operating nationally and investors were typically only exposed to activity on one stock exchange. Today the market includes more participants than ever before and it is common to have a regional or global asset allocation profile. Consequently, damages of financial breakdowns have a wide span impact on the lives of millions of investors. This phenomenon is known as internationalization and globalization of markets.

The consequences of globalization are integration and interdependency among financial markets. The recent subprime crisis portrays an event where the severity of the international contamination was remarkably intensified due to market interdependence. We find it important to acknowledge that market instability and fluctuations may have devastating implications for all varieties of investors, both professional and unprofessional. Hence, improvement of current risk management practices has wide span implications.

2.1.1.1 Extreme Events

The growth in financial markets has not only increased the number of market participants. As can be seen in Figure 1 below, the characteristics of the crises have gone from regional to global and the severity of the impact has increased proportionately with the globalization. Put differently, one can identify a pattern in the development of the nature of the crises; they have gone from being local and less destructive, to being wide spanning and extremely costly. In Europe alone, the estimated costs related to the financial crisis in 2008 to 2012 amount to: €1.5 trillion in state aid to prevent collapse of the financial system, €6-12 trillion in output costs (50-100% of annual pre-crisis GDP), households in the Eurozone lost close to 14% in financial asset value between 2007-2009 and lastly, the trust in the financial system in general declined sharply (Kamerling, 2014).

Another extreme event of relevance to this study is the European Sovereign Debt Crisis. A research paper published by the European Central Bank underlines loss of consumer confidence, fall in equity prices, sharp depreciation of the Euro and increased volatility in the financial market as notable effects from the crisis event. Not only do the effects show to have sizable implications for the European area but also for the global financial markets. The nature of this event supports the identified crisis pattern in terms of increased severity and extensive reach (Stracca, 2013).

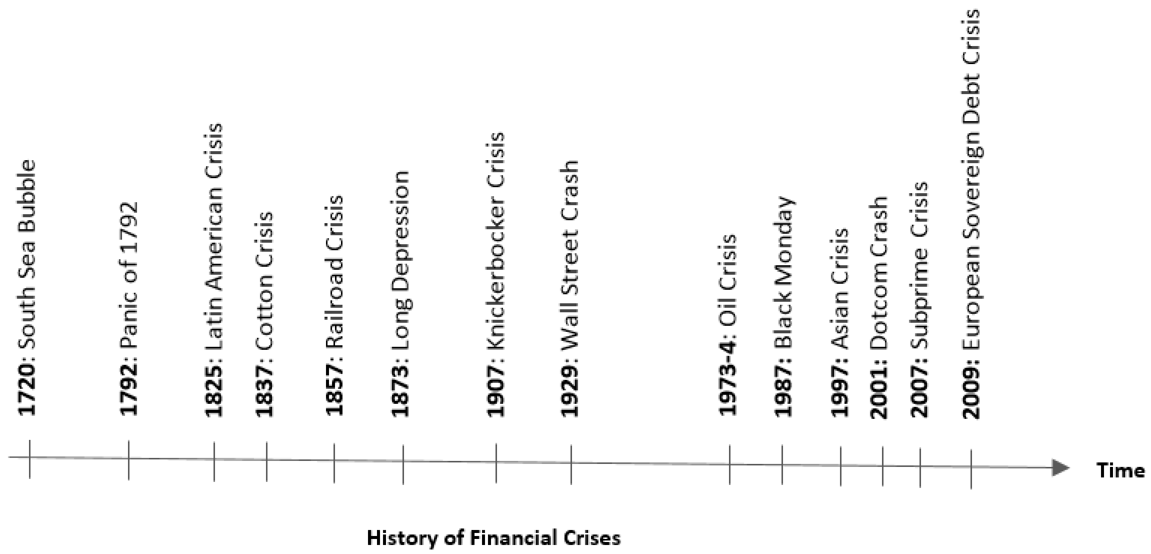

Figure 1: Historical Overview of Financial Crises (Source: The Economist)

The acceleration of integration and globalization has important implications for today's risk management practices. The increased complexity in the financial markets is naturally reflected in the data series. In general, researchers and practitioners observe that the data used in modelling financial time series exhibits characteristics of non-linear and non-stable correlations and extreme events are becoming more frequent and more costly. The change in nature of the financial markets has implications for today's risk management practices as the traditional method of assuming normality is deemed insufficient at predicting both magnitude and frequency of tail events.

Increased globalization and market interdependency stand as the first out of three main drivers behind the problem statement of this thesis. We document the inappropriateness of assuming normality and instead test the power of prediction and modelling using a GARCH-EVT-Copula approach. Investors do not need to fear risks, rather the increased market complexity encourages financial models to advance in proportion with technological development. In order to ensure model accuracy and reliability, researchers and practitioners are required to be as innovative and proactive as their surroundings.

2.2 Regulatory Initiatives

Researchers and practitioners are not the only stakeholders of the changing financial market. With the aim of ensuring a well-functioning financial system, regulators equally take great interest in regulating the financial sector. The importance of financial stability is naturally related to a societal desire for avoidance of the notable effects experienced during financial breakdowns, whereby also ensuring an environment characterized by economic growth and prosperity.

Historically, the financial sector has always received more regulatory attention than most other sectors in the global economy. One of the main regulatory bodies guiding financial institutions is the Basel Committee for Bank Supervision (BCBS). The motivation for establishing an international financial committee was the growing internationalization of financial intermediation. The committee was originally designed to provide non-binding recommendations on best practices regarding capital risk, market risk and operational risk. However, the ability to prevent crises has been limited. This is mainly explained by two factors: firstly, the Basel Committee has historically failed to appreciate recommendations from financial economists and the accords have always been a minimum rather than an ambitious international standard, which regulators and practitioners always tend to go beyond.

The crisis from 2008 portrays how European financial intermediaries struggled financially in spite of complying with regulatory guidelines on capital adequacy and liquidity solidity under regulatory frameworks alike the Basel Accord. The Global Financial Crisis confirms the inadequacy and ineffectiveness of the current applied statistical risk methods and recommendations provided by regulatory institutions.

The Basel Committee did not take official interest in risk measures such as the traditional Value-at-Risk until the second accord, which was introduced in 2008 (Parthasarathy, 2014). As this thesis also seeks to expose, the VaR measure receives intense critique. Researchers and practitioners greatly question the superiority of VaR. Alternatively, today's risk management debate recognizes the attractiveness of the alternative risk measure Expected Shortfall. This point of view is also presented in the recent publication of the revision of the Basel Accord on market risk, also known as the Fundamental Review of the Trading Book, which advocates that Value-at-Risk should with time be either replaced or complemented by Expected Shortfall (BIS, 2013). This thesis acknowledges the empirical shortcomings of VaR and instead presents a combination of both Value-at-Risk and Expected Shortfall as recommended by Yamai & Yoshiba (2005) and Artzner, Delbaen, Eber and Heath (1999).

2.3 Investor Behavior

Multiple central banks have implemented Quantitative Easing programs which have led bond yields to fall rapidly in recent years, placing the global financial markets in a yield environment never experienced before. This has disrupted the traditional investor behavior during times of uncertainty; historically investors' behavior has been characterized by the "flight to quality" phenomenon, which infers that fixed income products like government bonds are pursued in order to expand the risk-reducing element of the portfolio investment, mainly with the purpose of providing investment stability and risk protection. However, given the current low yield environment, where yields on some products have even reached negative levels, many investors deviate from the "flight to quality" phenomenon, and instead adopt a strategy characterized by "reaching for yield" in a desperate attempt to generate wealth instead of incurring a cost and paying negative yields to reduce portfolio risk (Westaway & Thomas, 2013). Hence, one of the key considerations motivating the problem statement of this thesis is the current investment environment, which forces investors to take on risk in order to obtain *any* rate of return.

The changing behavior of investors increases the need for more adequate risk management tools as this investment behavior may result in suboptimal portfolio allocation in favor of a target expected return (SMAI, 2015). The increased desperate need for return introduces a new setting which leads to increased demand for accurate and reliable risk models. Specifically, the tendency to reach for yield introduces increased risk, potentially making the loss severity even larger in magnitude and posing a threat to society as a whole: if a breakdown prevails, it hits hard on all types of investors and these investors need advanced models to gain reliable insight to their risk exposures.

3 PROBLEM STATEMENT

The introduction has stated three main drivers, which motivate the problem statement of this thesis. Combined, these three drivers lead to thoughts on the nature of undesired and unexpected events. These events have made us question if one could have taken action to actively prevent or at least mitigate the damages of financial breakdowns. From a technical perspective, the three drivers underline the importance of advancing current risk management practices to properly represent the empirical surroundings and behavior of financial markets. All statistical risk management models build on historical data, hence improvement of the models are strongly dependent on more advanced and sophisticated statistical modelling. Advanced techniques enable reliable approximation of empirical patterns persisting in real world data. These initial thoughts have led to the following problem statement:

How can accuracy of portfolio risk measures be improved through advanced distribution modelling in a European setting?

This thesis acknowledges the demand for more advanced distribution modelling in risk management practices. We set out to develop a framework of value to European investors who wish to improve their modelling of financial asset returns holding European assets. To guide the process of answering the problem statement, we formulate two propositions which focus on separate areas of improvements. The suggestions for improvement stem from the shortcomings of operating with normal distributions. Specifically, assuming normality leads to the problem of drastically underestimating size and frequency of extreme events (Jondeau, Poon, & Rockinger, 2007; Urbani, 1995). The paper by Stoyanov et al. (2011) presents five stylized facts on financial return distributions which can be summarized in the following modelling improvements : autoregressive behaviour, skewness, fat tails, volatility clustering and non-linear correlations. Based on these observations, this paper is guided by the following two improvement suggestions:

Proposition 1: The accuracy of the risk measures is improved as we approximate the empirical marginal distribution structures in terms of excess kurtosis, skewness and non-normal tail density.

Proposition 2: The accuracy of the risk measures is improved as we account for interdependencies in asset returns in terms for joint realizations and non-linear correlations.

The criteria of distribution modelling success is based on an examination of fulfillment of proposition 1 and 2 which leads us to provide a competent answer to the problem statement. The evaluation of fulfillment proposition 1 and 2 is founded on a thorough investigation of distribution fit for both the marginal and joint distributions and a conclusive discussion critically examines the reliability and validity of the risk modelling results. This implies that the improvements are evaluated based on technical insight and a deep dive into the advantages and disadvantages of the combined GARCH-EVT-Copula framework rather than a single statistical inference estimate.

By revisiting advanced statistical methodologies, this thesis specifically looks at ways to develop and advance current practices of risk estimation through implementation of a GARHC-EVT-Copula approach. We aim at providing important insight into how current risk modelling practices can utilize this framework to arrive at more insightful and reliable risk measures which overcome the well-known challenge of non-normality in asset returns. The main contribution of this thesis is related to European asset managers and investors as we focus on modelling distributions of portfolios holding solely European assets. This allows us to model the complexity of the European market, however due to globalization, the extreme events prevailing in the data series do not only reflect European events, but rather the integrated financial market influencing asset returns in this region.

The preceding section presented thoughts on problem motivation and research question. The following section precedes to present the methodological considerations and decisions surrounding the research design of this paper.

4 METHODOLOGY

In answering the problem statement, this thesis builds on quantitative analysis. In general, research building on quantitative techniques seeks to understand and explain market behavior using complex mathematical and statistical modelling. This thesis utilizes quantitative techniques to produce more accurate approximations of asset return distributions. Specifically, we seek to replicate reality mathematically with the purpose of providing more reliable risk measures of portfolio investments.

Prior to estimating portfolio risk, we deal with potential problems associated with performing distribution modelling based time series data. In order to model the return distributions using Extreme

Value Theory, we require the residuals to be approximately i.i.d., which is often not the case for raw and untreated time series data on financial returns. Hence, the time series analysis requires us to use time series techniques to account for the internal data structures related to e.g. autocorrelation, heteroscedasticity and volatility clustering. The correction for autocorrelation, trends and heteroscedasticity can be seen as the pre-modelling work performed to ensure applicability of distribution modelling frameworks – please read chapter 6 called "Theoretical Review" for further technical insight.

Economic and financial analyses regularly build on mathematics and statistics and in this area of research, quantification and statistical inference theory guides our estimation processes regarding the behavior of the portfolio. We apply methods from the mathematics area which deal with uncertainty, namely probability theory. We aim at detecting, studying and analyzing patterns in asset returns with the purpose of getting a grasp of the potential impact which extreme events have on a portfolio of a European investor. All financial decisions are made in an uncertain environment and we utilize probability theory to provide insight into the possibility of losing asset value with a pre-specified confidence level.

4.1 Delimitations

To maintain focus on our core research curiosity at hand and optimize the allocation of our resources, this thesis is constrained by a number of delimitations. This may lead to exclusion of important considerations; however, it simultaneously assures coherence between the content of the thesis and the problem statement. This implies that the delimitations act as a valuable tool to guide the thesis in an efficient and interpretable direction.

Delimitations take on various shapes including delimitations of problem recognition, topics and data collection. The intention of this section is to clarify which choices have been made and for what reason.

4.1.1.1 European Investors

The choice to focus solely on European investors, investing in assets originating from Europe, is primarily motivated by the fact that the research topic at hand lacks empirical investigation in a European context. Former research has focused on assets residing in North America and Asia and we find it interesting to contribute to the research debate based on a European setting. Given our focus on Europe, we find considerations concerning foreign exchange rates, macroeconomic environment, political environment and trade barriers superfluous to some extend; which is why we focus solely on the technical work underlying the distribution modelling and risk estimations.

4.1.1.2 Time Frame

This study investigates data from primo 2000 to ultimo 2015. The choice of modelling based on the most recent fifteen years of data is mainly motivated through its present relevance and technical characteristics. In detail, this period represents the latest development in the European financial markets and includes a vast level of market volatility. For example, this period includes the largest financial crisis since the Great Depression back in the 1920s and 1930s, which is why we anticipate that this period is representative for the fluctuations in the near future to come. Naturally, the future is uncertain and unknown by definition; hence, the validity and reliability of our findings are vested in the expectation of the latest past representing the near future, which is a standard assumption concerning both financial simulation and financial modelling.

4.1.1.3 *Asset Selection*

We observe four assets during the fifteen-year period. These are STOXX Europe 600 Index, SX5E Europe 50 Index, SGHIYIE FP Equity - High Yield Bonds, and FIDEBST LX Equity - Low Risk Bonds. Two drivers, historical existence and market capitalization, have primarily influenced the selection of assets. We require the four assets to be listed during the full observation period. This requirement proved to exclude multiple assets as the crises have led several funds, portfolios and indices to be dissolved or emerge at later points in time. Furthermore, we require the assets to substantiate from portfolios and indices which can be considered a fair representation of the typical asset classes included in a European investor's portfolio.

4.1.1.4 Simulation

We base our risk estimates on a Monte Carlo simulation providing 10,000 observations. This method ensures robustness in our estimates as it increases the density under the distribution function. As our main area of interest is the use of advanced statistical modelling to improve accuracy of portfolio risk measures, we solely base our risk estimation on simulated data founded on historical returns, whereby not performing any extrapolation or our-of-sample predictions. This delimitation naturally leads to strict exclusion of testing the framework based on forecasted data. However, due to the availability of time and resources, we find this to be a necessary delimitation to complete the project on time, and instead encourage other projects to perform an extended version of our analysis, which may include forecasting, bootstrapping techniques and back-testing.

In sum, the preceding sections have narrowed our research focus down to considering European investors, investing in four European assets and risk measures are based on data from sixteen years, where distributions are modelled with the GARCH-EVT-Copula technique including a Monte Carlo simulation. The delimitations are deliberate choices framed by the nature of the study and the authors themselves in order to meet formal requirements with the most satisfying solution and methodology.

The following section presents a more detailed review of the data handling process including a review of the data quality and simulation technique and horizon.

4.2 DATA HANDLING

As the foundation of this thesis is data on asset returns, the following section presents the considerations on data quality and the simulation horizon we apply in the VaR and ES estimations.

4.2.1 Data Quality

Given the range of the sample period, primo 2000 to ultimo 2015, the total number of weekly observations on asset returns is 834. The period is selected based on its present relevance. The period includes several major crises and therefore the sample enables us to analyze the complexity of the environment surrounding today's investors. This environment is predominantly characterized by high risk, low yields and turbulence. All data observations have been retrieved from Bloomberg and subsequently been investigated for missing observations and extreme irregularities (Bloomberg, 2016). Potential errors have been cross-checked with data from DataStream. We follow the universal best practice and transform the return data to related changes; that is the natural logarithm $r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \mathrm{i} = 1,.\mathrm{n}.\mathrm{t} = 2..\mathrm{T}$[1]. This allows us to overcome the general non-stationarity of financial time series and facilitates a smoother data handling (Embrechts, Lindskog, & Mcneil, 2003).

[1] Where $r_{i,t}$ is the log return of i^{th} asset at time t, $P_{i,t}$ is the price of the i^{th} asset at time t.

We form an equally weighted portfolio consisting of four indices: STOXX Europe 600 Index, SX5E Europe 50 Index, SGHIYIE FP Equity - High Yield Bonds, and FIDEBST LX Equity - Low Risk Bonds[2]. These indices represent the vast majority of the most frequently traded assets originating from Europe and includes both high and low risk assets as well as small, medium and large capitalization companies. Despite the low number of assets, we remain confident that the sample sufficiently illustrates the behavior of the European investment market.

4.2.2 Simulation Horizon

To ensure robustness of the risk measures we utilize the Monte Carlo simulation technique. When specifying the horizon for the simulation and estimation period we apply a horizon where we with reasonability are able to hold the portfolio weights fixed i.e. we expect the investor to hold the portfolio weights fixed throughout this period. Hence, we have considered a tradeoff between willingness to accept fluctuations in risk versus return and the cost of rebalancing the portfolio. Vanguard, which is one of the world's most respected investment companies, has conducted a research in 2010 with the purpose of identifying the optimal rebalancing strategy. They find indications of no conclusive and general optimal frequency of portfolio rebalancing, however for broadly diversified portfolios, annual rebalancing are recommended as the optimal strategy when accounting for rebalancing costs, time and taxes (Vanguard, 2010). Additionally, VaR and ES are seen as short-term portfolio risk measures and forecasting horizons are in general encouraged to reflect this nature, hence we follow the suggested horizon of one-year simulation period, which is perceived as a compromise between a high frequency trading investor and a long-term investor.

Due to the high technical level, we include a brief theoretical review of the econometric tool box, which we consider for answering the problem statement. A thorough elaboration is presented later in chapter 6 "Theoretical Review", which is part of the literature review.

[2] STOXX Europe 600 contains a total of 600 small, mid and large capitalized stocks from 18 European markets. SX5E Europe 50 contains the 50 largest stocks across the European market. SGHIYIE FP Equity contains a broad range of European corporate bond, from corporations with an S&P 500 investment grade of BBB ore below. Lastly, FIDEBST LX Equity consists of low risk government bonds form the major European markets.

4.3 Econometric Toolbox

The full statistical analysis and modelling is performed in MATLAB. This statistical software package offers advanced tools which are not available in applications such as Microsoft Excel. All statistical tests apply the traditional convention of a five percent significance level.

On the quest of creating a framework which is able to account for non-normality we utilize multiple advanced statistical modelling tools. However, we also want to underline the fact that we have critically considered the trade-off between complexity and applicability. We want to avoid unnecessary complexity, as complex models have an increased risk of inflated variance and a tendency to over-fit the data (Esch, 2010). If one continues to add parameters the final model explains the historical data almost perfectly, however, this does not imply a high power of future applicability – in fact, the opposite is often the case. Nonetheless a certain level of complexity is required as a main driver of this thesis is the faulty denial of the shortcomings of assuming normality. The assumption of normality has led to an underestimation of multiple disasters with severe personal and economic impact throughout history.

Today it has become an established fact that normality frameworks in financial modelling are unable to explain the empirical pattern of asset returns adequately. Naturally, this leads to a trade-off between complexity and simplicity where "*Models should be as simple as possible but not any simpler*" (Stoyanov et al., 2011, p. 4). Keeping the above statement in mind, we aim at only increasing complexity where the existing methods fail to approximate the empirical behavior. Therefore, we use the preliminary analysis to identify non-normality characteristics and use the findings to motivate the need for more complex distribution modelling. Specifically, we check the distributions for four non-normal stylized facts of asset returns: Skewness and fat tails, autocorrelation, heteroscedasticity, and correlation breakdowns. The rejection of the normality assumptions is the primary motivation for the analysis and distribution modelling going forward.

4.3.1 GARCH-EVT-Copula

Many methods aim at explaining deviation from normality. In this work, we apply the GARCH-EVT-Copula approach. Overall, this framework allows us to generate distributions that are more accurate, whereby we can approximate the true risk faced by a European investor. The GARCH-EVT-Copula framework has been developed in recent years and shows promising results in modeling of financial time series (Chebbi & Hedhli, 2014; Jondeau & Rockinger, 2002; Sheikh & Qiao, 2010; Wang, Chen, Jin, & Zhou, 2010). The method provides a systematic framework which enables us to deliver improved accuracy of risk measures. The improvement is founded on our ability to account for non-normality in shape of fat tails, skewness, excess kurtosis as well as internal data characteristics such as autocorrelation, heteroscedasticity and non-constant correlations. The method offers an additional benefit which is often unaccounted for in the traditional taxonomy. It allows us to account for extreme joint events. In this way, our methodology allows us to explore our problem statement based on a solid and well-recognized analytical framework. The following sections provide arguments for the choice of theoretical framework with the intention of presenting an overview of the analytical steps these are presented in the figure below.

STEP 1 Data Cleaning	• Visual check for autocorrelation and heteroscedasticity in ACFs • AR-GJR-GARCH modelling to obtain apx. i.i.d. standardized residuals • Apply Ljung-Box test statistic to test for i.i.d. standardized residuals
STEP 2 Modelling of Marginal Distributions	• Use Peak-Over-Threshold method under Extreme Value Theory to collect data point for individual tail modelling ◦ Utilize Generalized Pareto Distribution for tail modelling and Kernel distribution for interior modelling • Evaluate fit of marginal distribution and ability to replicate empirical data behavior
STEP 3 Modelling of Joint Distribution	• Form the joint distribution based on student's t copula calibration which allows us to account for correlation breakdowns and fat tails in the joint distribution through the DoF parameter
STEP 4 Risk Estimation	• 10,000 Monte Carlo simulations are generated based on the joint GARCH-EVT-Copula distribution • Estimation of one-year Value-at-Risk and Expected Shortfall based on the simulated joint distribution ◦ Risk measures are calculated for an equally weighted portfolio. The results are supported by calculations performed on risk conservative and risk-seeking portfolio allocations

Figure 2: Overview of Risk Estimation Modelling Process

4.3.2 GARCH

The GARCH-EVT-Copula model allows us to model the distribution of a portfolio in two main steps. First, we model the marginal distribution of each time series using GARCH and EVT. Here each time series is corrected for autocorrelation and heteroscedasticity using the AR-GJR-GARCH model. Rosenberg and Engle (2002) apply a likelihood ratio test to compare the GJR-GARCH model against the two traditional frameworks: the ARCH and the GARCH model. They find that the GJR-GARCH model provides a significantly better fit. Similar findings are presented by other authors (Awartani & Corradi, 2005). We follow their approach and apply an extended version of the classical AR-GARCH model, where the GJR module is added to account for the leverage effect (Glosten, Jagannathan, & Runkle, 1993). Further, in order to test Rosenberg and Engle's argument, we compare the GJR-GARCH model with a GARCH model using a likelihood ratio test. The AR-GJR-GARCH step is required in order to obtain the filtered standardized residuals, which are required to be approximately i.i.d. as they are otherwise inapplicable in the process of modelling the distributions. A deeper discussion of the AR-GJR-GARCH framework is provided in chapter 6 "Theoretical Review", in section 6.4 "Autocorrelation and Non-Constant Variance".

4.3.3 Extreme Value Theory

After having obtained standardized residuals, which satisfy the i.i.d. assumption, we model each marginal distribution using a semi-parametric approach under Extreme Value Theory. This method is applied in favor of the parametric approach (the GARCH model), and non-parametric approach (based on historical simulation). The parametric, non-parametric, and semi-parametric method is compared by Danielsson and de Vries (1997) in their study of quantile risk of financial time series. They apply a bootstrap method to analyze the efficiency of each method and find that the semi-parametric approach provides a superior estimate of tail risk and a lower variance of the estimates. Both the parametric and non-parametric methods deliver a reasonable fit when looking isolated at the interior of the distribution, however, they both fall short in delivering efficient estimates of extreme outcomes. The semi-parametric approach, on the other hand, applies the non-parametric approach to the distribution interior and estimate the tails of the distribution using the parametric approach. This method allows the model to provide an improved estimate of the challenging tail observations as "*The semi-parametric tail estimates therefore do not have to serve two masters by matching the parameters to satisfy both tail and center characteristics of the model*" (Danielsson & de Vries, 1997, p. 27). The use of a semi-parametric EVT approach is therefore especially applicable when analyzing extreme events (Daníelsson & de Vries, 1997; Embrechts, Resnick, & Samorodnitsky, 1999; Goldberg, Miller, & Weinstein, 2008; A. J. McNeil & Frey, 2000). There are however two sides of a coin. The main problem associated with the semi-parametric EVT model relates to the persistent challenge of modeling extreme events when only a few observations are available in the tails of each distribution. Hence, researchers face the risk of estimating the tail parameters based on a limited number of observations which makes it challenging to obtain small variance estimates. This concern and a deeper discussion of EVT is provided in section 6.5 "Extreme Value Theory".

In summary, this work models the two tails based on EVT and the interior of the distribution is modelled using the smooth Kernel distribution. Specifically, the Peak-over-Threshold method is applied. Here we extract the exceedances, which are above a given threshold, μ, and we use the Generalized Pareto Distribution to separately model the density of the lower and upper tail. In this process, we apply the standardized residuals as input, as they approximately satisfy the i.i.d. assumption. The three parts, the lower tail, interior and upper tail, are subsequently combined to a coherent distribution and the fit of the semi-parametric distribution is compared to the empirical data and the normal distribution. The semi-parametric approach allows us to account for two challenging stylized facts of asset returns: heavy tails and skewness. Hereby we generate marginal distribution models which approximate the behavior of the asset return data more accurately.

4.3.4 Copulas

The second main step in the GARCH-EVT-Copula model relates to the process of moving from marginal distributions to model the joint distribution of the European investor's portfolio. Traditional multivariate distribution models build on linear correlation assumptions which are non-representative for the empirical data patterns. To overcome this problem, we utilize Copula theory. This method is especially useful in modeling the joint distribution of financial time series as copulas allow the multivariate distribution to be modeled independently from the marginal distributions (Embrechts et al., 2003; Jondeau & Rockinger, 2002; Trivedi & Zimmer, 2006). This implies that we can accurately model the joint distribution of two or more assets, even though these assets exhibit non-identical marginal distributions. This is not possible in traditional frameworks, such as Markowitz portfolio allocation, which build on Pearson's correlation and therefore do not allow for non-normality (Embrechts, McNeil, & Straumann, 1999b).

The copula families include a large variety of copulas where the most common group is the elliptical copula. This family contains the Gaussian and student's t copula, which are derived versions of the multivariate normal distribution. This facilitates a simpler incorporation of the statistical model as the Gaussian or student's t copula are estimated through just one and two parameters, respectively. Both methods require the rank correlation where the student's t copula further needs an estimate of the

DoF parameter to account for heavy tails (A. McNeil, Frey, & Embrechts, 2010). The possibility to account for heavy tails makes the student's t copula especially suitable for financial time series. This is the main reason why we apply the student's t copula in this work.

The choice of this theory and method is based on findings in previous research which documented that the student's t copula provides a better fit for extreme joint events compared to the Gaussian copula (Wang et al., 2010). The main advantage of the student's t copula is that this copula estimates the DoF from the empirical distribution, thus incorporating the stylized fact of fat tails in the joint distribution. One disadvantage of the student's t copula is that the model is unable to account for asymmetry in assets returns. Other copulas, such as the Clayton copula, are able to incorporate skewness in the joint distribution. However, we have evaluated that the added skewness estimate increases the complexity of the model comprehensively. The Clayton copula leads to more efficient model estimates, but it also substantially decreases the pragmatic value of our work as we evaluate that the added complexity complicates a potential incorporation of the framework in a commercial setting. In sum, we have decided to avoid the Clayton copula and pursue the student's t copula and the arguments for the decision are twofold. First, the application of copula theory in risk management literature is still at an infant stage and we therefore believe it is important to shed light on the usefulness of this statistical tool in the most intuitive way. Second, the student's t copula is sufficient to account for our key object of interest when modelling increased risk of extreme negative events.

By not integrating skewness in the joint distribution, our estimate of the tail risk may be biased in an optimistic direction and the degrees of freedom parameter reflects this. In an asymmetric model, the lower and upper tail will separately be described by distinct tail parameters. However, in a symmetric model the DoF is an average of the full distribution. Hence, when using a symmetric model, the probability density of the lower tail is underestimated and the opposite is the case for the upper tail (Staudt, 2010). Thus, this limitation is kept in mind when interpreting our results.

Based on the previous argument by Stoyanov, Rachev, Racheva-iotova, and Fabozzi (2011, p. 4) of keeping the model as simple as possible we believe that our model provides a vast improvement over the current Markowitz framework. We are confident that the student's t copula is sufficient to significantly improve the accuracy of the risk measures. A deeper discussion of the theoretical terms and practical steps of copula theory is provided in chapter 6 "Theoretical Review", section 6.6 "Correlation".

4.3.5 Simulating Portfolio Returns and Estimating Risk Measures

The copula process is followed by a Monte Carlo simulation. We wish to estimate the joint cumulative distribution function for an equally weighted portfolio. We simulate the return distribution under the assumption that the weights are fixed for one year and the simulation is repeated 10,000 times. Subsequently, AR-GJR-GARCH effects are reintroduced to the standardized residuals in order to obtain return observations which match the empirical behavior. This approach is similar to the work of Sheikh and Qiao (2010). We have decided to apply both Value-at-Risk (VaR) and Expected Shortfall (ES) as measures of portfolio risk. Observing both VaR and ES is an approach suggested in former research by Yamai and Yoshiba (2002). They underline the value of determining risk with a combination of the two risk measures as this provides a more coherent assessment of the potential risk faced by the investor. A more comprehensive discussion of risk measures is presented in chapter 6 "Theoretical Review", section 6.1 "Measuring Tail Risk". The non-normal VaR and ES are calculated based on the simulated output. In order to compare our results with a traditional approach, we similarly simulate an equally weighted portfolio based on a normal distribution and calculate the normal VaR and ES.

With the purpose of testing the sensitivity of our model we perform the analysis on two additional portfolios. We introduce both a portfolio of a risk seeking investor with only 1% investment allocation in low risk government bonds and a conservative portfolio with 55% of the assets placed in the low risk bonds and an equal division of the remaining capital between the three remaining assets. The VaR and ES of both portfolios are compared to risk measures of the equally weighted portfolio and the risk measures calculated under the assumption of normality for same portfolio structures. A discussion of the results, practical implications and conclusion is subsequently presented.

In summary, we follow a well-supported theoretical framework, which enables us to provide an improved distribution modelling of the marginal distributions. The utilization of copula theory allows us to deal with the challenge of correlation breakdowns and increased frequency of extreme events in the joint distribution. The framework allows us to overcome the current challenges of the assumption of normality which lead to systematic underestimation of the impact of extreme events. Thereby our method provides a solid foundation from which we can more accurately approximate risk the true risk faced by a European investor in a European setting.

5 LITERATURE REVIEW

Market instability and increased volatility have led researchers and practitioners to question the validity of current financial models. Undoubtedly, risk managers are facing increasing complexity in the global financial markets. There is a growing concern in the research area of quantitative finance regarding the ability of current statistical models to account for data characteristics such as fat tails, skewness, distribution asymmetry, time dependence and correlation break-downs when estimating downside risk (J. P. Morgan, 2009; Sheikh & Qiao, 2010; Stoyanov et al., 2011).

Risk management models only generate value when they are able to represent and approximate empirical behavior or objects from the real world. Naturally, the intention of the models is to help practitioners make inferences about e.g. the likelihood and impact of an event. Up until now it has proven to be highly challenging to build general risk management frameworks which are able to deliver accurate risk measures. One of the main fallacies in Risk Management and Modern Portfolio Theory models is the assumption of normality. The main problem of consideration relates to the fact that the normal distribution consistently provides drastic underestimation of the size and frequency of extreme events (Jondeau et al., 2007; Urbani, 1995). The paper by Stoyanov et al. (2011) summarizes five stylized facts on financial return distributions which are worth considering when modelling financial risk:

- Clustering of volatility: Large price changes tend to be followed by large price changes and small price changes tend to be followed by small price changes.
- Autoregressive behavior: price changes depend on price changes in the past, e.g. positive price changes tend to be followed by positive price changes.
- Skewness: there is an asymmetry in the upside and downside potential of price changes.
- Fat tails: The probability of extreme profits or losses is much larger than predicted by the normal distribution. Tail thickness varies from asset to asset.
- Temporal behavior of tail thickness – the probability of extreme profits or losses can change through time; it is smaller in regular markets and much larger in turbulent markets

The study by Rachev & Racheva-Iotova, B. Stoyanov (2010) underlines the importance of creating models which are able to account for both the tail behavior across assets and through time as this influences risk statistics on both a marginal and joint level. The paper builds on data from two of the largest American indices: SP500 and Dow Jones. The paper distinguishes between GARCH-modelling, EVT and student's t and concludes that the first modelling framework estimates the 99% VaR too optimistically and student's t and EVT are overly pessimistic. The researchers find the explanation for overestimation of risk with EVT to be twofold. Firstly, the EVT and student's t are found unable to account for changes in tail thickness. Secondly, researchers assume degrees of freedom (DoF) to be set to five in the distribution modelling with EVT and student's t. Compared to a normal distribution having DoF of thirty, a DoF of five results in significant density being allocated to the tails. To put it into perspective, our thesis find DoF of our assets to be 17.53 in the calibration of the copula, whereby still exhibiting fat tails but to a more moderate extend than what is assumed in the paper by Rachev & Racheva-Iotova, B. Stoyanov (2010). Given a DoF of 17.53, the paper by Rachev & Racheva-Iotova, B. Stoyanov (2010) provides empirical evidence for the relevance of modelling distribution tails with EVT.

Despite empirical and technical challenges, a vast amount of literature proposes Extreme Value Theory as an obvious method to account for heavy tails and skewness when modelling extreme events. The EVT framework offers the advantage of allowing one to model the tails which leads to improved tail approximation (Jondeau et al., 2007). Additionally, EVT can be based on a semi-parametric approach where the body of the distribution is modelled with a normal distribution and the tails are modelled with a Generalized Pareto Distribution (Jondeau et al., 2007; Meyers, 2011; Sheikh & Qiao, 2010). This method allows the model to provide an improved estimate of the challenging tail observations as "*The semi-parametric tail estimates therefore do not have to serve two masters by matching the parameters to satisfy both tail and center characteristics of the model*" (Danielsson & de Vries, 1997, p. 27). In its advanced form, Embrechts (2000, p. 8) underlines that EVT is able to describe extreme events and allow for dynamic evolvement through time and space. He describes it as a methodological toolkit for issues such as skewness, heavy tails, rare events and stress testing stating that: "*(…) What EVT is doing is making the best out of whatever data you have on extreme events*", whereby arguing that EVT is a feasible and efficient method for modelling extreme observations.

The modeling complexity continues beyond choosing between various solutions in the families of e.g. EVT and student's t distribution. Globalization in general has had a strong impact on the level of integration of stock markets. In Europe, this phenomenon has become even more evident with the introduction of the common European currency, the Euro (Wälti, 2011). Wälti (2011) investigates the impact of stock market synchronization on the degree of co-movement or correlation between national stock market returns in the European area. The paper finds increased correlation between European national stock markets, especially during bear markets. This finding is very relevant for investors' international portfolio diversification and portfolio risk estimation. Specifically, we find it interesting as international investors rely on measures of dependence and tail risk in the process of managing portfolio risks and allocating capital. Therefore, we find it necessary to uncover the dependencies between various assets.

To deal with the phenomenon of correlation breakdowns and volatility clustering, researchers propose statistical methods where an additional estimation layer is added to the distribution in shape of a copula calibration. Research has documented that dependence structures in financial markets vary with volatility of the market whereby making the stylized fact of volatility clustering and behavioral change all the more complex (Dorey & Joubert, 2005; Stoyanov et al., 2011). J. P. Morgan (2009) has published a research paper which acknowledges the importance of modelling dependencies when producing risk metrics. The paper builds on asset classes, which are typical to an American investor. The paper utilizes copulas to account for volatility clustering, correlation breakdowns and temporal behavior of tails through time. J. P. Morgan (2009) takes starting point in the fact that we empirically observe non-normality with a much greater frequency than what is assumed in the traditional mean-variance framework used for optimizing asset allocation. Ignoring non-normality in equity return distributions leads to a significant understatement of downside portfolio risk and in worst-case scenarios potentially poses a solvency risk for the investors. The paper finds the student's t copula of most relevance. The advantage of the student's t copula is found in its ability s to represent joint dependence structures in a multivariate t-distribution whereby exhibiting some degree of fat tails. This type of copula is especially well-known in the area of modelling multivariate financial return data (Demarta & McNeil, 2005; Embrechts, McNeil, & Straumann, 1999a; J. P. Morgan, 2009). The results for the copula and subsequent optimization of asset allocation based on optimizing the risk vs. return trade-off suggest that the correlation convergence during periods of market distress leads to reduction in the capital allocation to asset classes such as international equity and hedge funds in the traditional mean-variance framework (J. P. Morgan, 2009). Specifically, J. P. Morgan (2009) finds that incorporating non-normality more than doubles the estimation of CVaR in the given portfolio. Comparably, assuming normality, the risk underestimating amounts to 9.4% of the portfolio's initial investment value.

The paper by J. P. Morgan (2009) is based on data from the American market. An investigation of the tail co-movement done by Mensah & Premaratne (2014) with focus on the financial sector in Asia supports the findings by J. P. Morgan by underlining the problem of asset dependence. Specifically, they argue that markets with high positive dependence do not provide risk reduction benefits to investors and the challenge in terms of estimating risks relates to accounting for the spikes they find in tail co-movements during periods of high market volatility. This implies that there is potential for joint crashes in the regional financial sector during extreme negative events (Mensah & Premaratne, 2014).

This study concludes that time-varying copulas are best suited for modelling the dependence structures in the Asian financial sector compared to static copulas.

Only a limited amount of literature has been written based on the exact GARCH-EVT-Copula setup and former studies applying this framework mainly considers data from America and Asia-Pacific, implying that Europe still calls for attention. The preceding literature review should be seen as a condensed review of the empirical evidence and motivation which function as the groundwork of this thesis. We want to underline that this area of research has received much attention the past few years and our empirical research has therefore not been limited to the before-mentioned sources.

The problem statement requires us to both utilize a substantial part of our technical knowledge on modelling distributions as well as gaining insight into methods beyond our awareness prior to this work. Therefore, we find it valuable to allocate a significant part of the literature review to a technical and theoretical review, hence the following sections provide a detailed examination of the theoretical frameworks which guide our process of answering the problem statement. Specifically, the purpose of providing a theoretical literature review is to allow the reader to gain a technical understanding of the advantages associated with advanced distribution modelling and why simple theoretical frameworks cause immense trouble when applied to finance data.

The subsequent sections are structured as follows. Firstly, we examine the theoretical tools applied to measure financial risk. Secondly, we explicitly highlight the challenges of operating with non-Gaussian datasets and how this conflicts with current assumptions embedded in traditional risk management practices. Thirdly, we show how theory from financial time series assists us in the process of generating i.i.d. standardized residuals. Next, we examine how we can utilize the semi-parametric version of Extreme Value Theory to model the distribution interior based on the Kernel distribution and the tail distributions based on the peak-over-threshold methodology. Lastly, we find it crucial to investigate the shortcomings of the classical Pearson correlation and instead propose a student's t copula to account for joint dependence structures in the dataset.

6 THEORETICAL REVIEW

The following sections investigate and examine the theoretical tools, which are utilized in the aim of improving current risk metrics estimation. As asset returns are known to follow a non-normal distribution, it implies that the use of Gaussian distributions for modelling asset returns leads to underestimation of extreme losses and risks of portfolio investments.

6.1 MEASURING TAIL RISK

Variance has in itself been considered a widely accepted measure of risk for a very long time. The measure is easy to compute and understand. However, in the mid-1990s the regulatory institution Basel Committee on Banking Supervision (BCBS) introduced new capital requirements with new standards on the quantification of risks. The committee wanted to incorporate metrics which are able to capture market risks. Among other initiatives, the committee introduced the Value-at-Risk (VaR) measure and is now considering the possibilities of replacing VaR with Expected Shortfall (ES) (BIS, 2015; Jondeau et al., 2007). Despite the fact that the Basel Committee only regulates financial institutions, it also guides the general risk management environment whereby influencing both private and professional investors. We have decided to measure risk by both VaR and ES as suggested by Yamai & Yoshiba (2002, 2005), hence the following section examines the technical application as well as the advantages and disadvantages of the two risk measures.

6.1.1 Value-at-Risk

The VaR measure has become a well-known traditional risk measure for quantifying financial risks. Its popularity mainly stems from its conceptual simplicity, ease of computation and applicability; it is reported as a single number expressed either in percentage loss or in dollar terms corresponding to a specific confidence level (Yamai & Yoshiba, 2005). More specifically, VaR is a statistical technique used to measure the level of financial risk of a portfolio. It is defined as the minimum potential loss which a portfolio can suffer in the X% worst cases over a given time horizon (Jondeau et al., 2007). Therefore, from a statistical point of view the VaR is a quantile on the lower tail of the portfolio return distribution, typically the 95th or 99th percentile.

Despite its popularity, researchers have challenged VaR due to conceptual problems. They have underlined two specific shortcomings of the risk metric. First, VaR measures only percentiles of profit–loss distributions and disregards any loss beyond the VaR level i.e. VaR provides no information regarding the potential size of the losses exceeding the percentile. This problem is also known as the problem of ‘‘tail risk’’. Secondly, VaR is not coherent, since it is not sub-additive. Problems of sub-additivity occurs when the risk related to a portfolio consisting o two or more assets exceeds the sum of the risks related to the individual assets (Artzner et al., 1999).

The first shortcoming is of most concern, as the VaR does not represent losses exceeding a chosen confidence level, whereby leaving the investor blindfolded regarding risks beyond this point. Moreover, as VaR is a quantile measure it is rather insensitive to skewness and heavy tails and therefore unable to represent extreme losses hence only providing superficial insight to the portfolio risk.

The second issue is related to the problem that VaR is not a coherent risk measure since it is not sub-additive (Artzner et al., 1999). When combining two portfolios VaR may fail to accurately represent the benefits of diversification. As Artzner (1999, p. 209) describes it: "*A merger does not create extra risk*". The sub-additivity property can be stated as:

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$$

This means that combining two portfolios should result in less than or equal total risk than the sum of the two individual portfolios. To exemplify the violation of sub-additivity, consider owning a bond, called Bond A, which has 3.5% probability of defaulting. For ease of calculation we assume a 3.5% risk of losing \$1,000. This implies that there is 96.5% probability of not losing any value. The 5%-VaR is therefore \$0.00.

Now, consider owning two bonds instead of one. These are assumed to be uncorrelated and independent and have same distributions as Bond A: these are Bond A and Bond B. The probability of both bonds defaulting in the two-bond portfolio is calculated as 3.5% · 3.5% = 0.1225%, in which case the investor would lose \$2,000. The probability of neither bond defaulting is 96.5% · 96.5% = 93.1225%, in which case the loss amounts to \$0.00. There is still a remaining probability of 100%-93.1225%-0.1225% = 6.755% where only one of the bonds default, hence the investor loses \$1,000. We can therefore conclude that the two-bond portfolio has a 5%-VaR of \$1,000.

The two individual bonds have $VaR_A = 0$ and $VaR_B = 0$. However, the portfolio holding both Bond A and Bond B has $VaR_{A,B} = 1{,}000$. This simple calculation example shows how the risk measure of VaR may suggest that combining two portfolios can result in greater total risk, hence it cannot be considered a coherent risk measure (Artzner et al., 1999; Yamai & Yoshiba, 2002, 2005). To alleviate the problems of VaR, researchers often refer to the more modern alternative of Expected Shortfall (ES) (Artzner, 1999; Jondeau et al., 2007; Yamai & Yoshiba, 2005). The following section examines the value of applying ES as a risk measure.

6.1.2 Expected Shortfall

Unlike VaR, the measure of ES is the expected value of losses beyond the confidence level and therefore this measure is not solely focusing on specific quantile on the distribution. ES fulfils all four axioms presented by Artzner et al. (1999) for coherent risk measures including sub-additivity. The popularity and superiority of ES as alternative to VaR mainly stems from the fact that ES considers the average loss beyond the confidence level. This implies that this measure by definition is able to pick up any risks associated with tail events in the non-Gaussian return distributions. ES is defined as follows (Yamai & Yoshiba, 2005):

$$ES_\alpha(X) = E[X|X \geq VaR_\alpha(X)]$$

ES indicates the average loss given that the loss exceeds the VaR-level. If asset returns follow a normal distribution, VaR is close to being as efficient as ES at representing risk. Data from the recent financial crisis highlight the importance of accurate tail risk measurement as asset returns exhibit extreme tail distributions (Mak & Meng, 2014). The findings presented by Mak & Meng (2014) argue that ES outperforms VaR in its ability to capture risks associated with thicker tails and therefore suggest that VaR is replaced with ES.

The superiority of ES is investigated by Yamai & Yoshiba (2005) where they test if the reliability of ES depends on sample size. They find that once the underlying distribution of returns is fat-tailed, the ES method is sensitive to outliers compared to the VaR measure. One can reduce the influence of the sensitivity by increasing the sample size through Monte Carlo simulations. Specifically, they find that a sample size of 10,000 simulations reduce the potential sensitivity bias significantly.

Table 1 provides an overview of the advantages and disadvantages of applying VaR and ES as risk measures:

	Advantages	Disadvantages
Value-at-Risk (VaR)	• Acknowledged by the Basel Committee • Ease of implementation and calculation	• Violates axiom of sub-additivity • Ignores losses beyond the selected quantile
Expected Shortfall (ES)	• Captures full tail distribution • In-line with all axioms presented by Artzner et al. (1999) • Accounts for non-normality in return distribution	• If sample size is limited the sensitivity to outliers may lead to inflated risk measures

Table 1 Overview of Advantages and Disadvantages for VaR and ES

This thesis acknowledges the shortcomings of VaR and apply the ES measure in addition to VaR. Yamai & Yoshiba (2005) underline that the two risk measures each have their own advantages and disadvantages and suggest that using a single risk measure should not dominate the landscape of quantitative risk management. Instead letting VaR and ES complement each other represents the most effective way of providing reliable and comprehensive risk monitoring.

Clearly, the presence of non-normality in returns has led researchers to question the validity of traditional risk frameworks. In sum, the preceding theoretical considerations motivate the choice of applying both VaR and ES for risk estimation in this thesis as this combination allows us to gain nuanced insight to the tail risk. The following paragraphs investigate the consequences of violating the normality assumption when aiming at estimating tail risk.

6.2 Non-normality of Asset Returns

Many popular models and concepts in the financial market is based on the assumption of normality in returns. One of the main explanations being desire for simplicity and possible lack of understanding of the more complex non-Gaussian distributions. The danger of using the unsuitable Gaussian probability distribution modelling framework relates to inaccurate approximation of extreme losses and suboptimal asset allocation. This thesis participates in the research debate on how investors must challenge the simplifying assumption of normal distributions in Modern Portfolio Theory.

6.2.1 Portfolio Theory

Markowitz originally introduced the assumption of normality in returns in 1952, when he implemented variance as a measure for risk in an asset allocation setting (Markowitz, 1952). Aiming at describing the full distribution of the asset returns, empirical research shows that the assumption of normality do in fact appear to approximate everyday events in stabile markets rather well (Urbani, 1995). The shortcoming of the normal distribution assumption however relates to the fact that assuming normality in returns leads to drastic underestimation of both size and frequency of extreme events. Specifically, the left tail of normal distributions ends too quickly to describe extreme events in a satisfactory fashion. Under the normal distribution, 68.3% of all observations are assumed to fall within one standard deviation from the mean, 95.4% within 2 standard deviations and 99.7% within 3 standard deviations. Thus, under the normal distribution there is only a 0.3% probability of observing events beyond 3 standard deviation from the mean (Urbani, 1995). This statistical assumption is not representative for asset returns as this data is typically leptokurtic, meaning that the true empirical distribution has excess kurtosis and is more peaked than the normal distribution. Additionally, most financial data is characterized by negative skewness meaning that we observe more downside than upside extremes. Empirical data shows that extreme events, like the European Debt Crisis, prevails with a higher probability than what is estimated under the assumption of normality.

To exemplify the variance estimation error when assuming normality, we take starting point in 15 years of return data on the German DAX Index, going from January 1st 2000 to January 1st 2015. During this period the index had a weekly mean return of just above 0%, approximately 0.01%, and a standard deviation of 3.4%. Assuming normality implies that there is only 0.3% probability of observing weekly return of more than +10.2% and less than -10.2%. The sample holds 782 weekly return observations hence we should only observe approximately one week where return is below -10.2%[3]. However, the data series includes four weeks where the weekly loss exceeds 10.2%. This is four times as many negative events than what is predicted under the assumption of normal distribution of returns. Such inaccuracy can prove to have fatal consequences for investors and should not be neglected in the world of risk management going forward.

In 2007 financial institutions such as Bear Stearns, UBS, Merrill Lynch and Citigroup announced massive losses. One journalist wrote: "*Things were happening that were only supposed to happen once in every 100,000 years. Either that … or Goldman's models were wrong*" (Dowd, Cotter, Humphrey, & Woods, 2008, p. 1). The same journalist also questioned the practices of Citi Bank: "*Then, in came one of those 25-sigma events. Citi was hit by a once-in- a-blue-moon fat tail event*." (Dowd et al., 2008, p. 1). The article clearly seeks to question the risk management practices regarding identifying and managing tail risk and extreme events.

The normal distribution is symmetric and described by its mean and standard deviation. Skewness is equal to zero and kurtosis is equal to three, all further moments are either equal to zero or functions of the variance and mean (Jondeau et al., 2007). These assumptions are in most cases inaccurate for empirical data on stock market returns: in most financial assets we see a kurtosis exceeding three and a negative skewness. The sample data on the DAX Index showed a kurtosis of 7.1 and a slightly negative skewness implying that the data is peaked and exhibits fat fails.

Urbani (1995) portraits the difference between assuming normal distribution and taking excess skewness and kurtosis into account and using Conditional Value at Risk estimation. He investigates the difference between funds, which are characterized by negative and positive skewness as well as normal distribution. Negatively skewed distributions have significantly more downside risk than both the fund that is normally distributed and the fund that is has positive skewness. The empirical evidence only becomes more noticeable as one moves further into the tail of the distribution (Urbani, 1995). Therefore, in order to improve the distribution modelling and risk measures, we need to include higher moments such as skewness and kurtosis to our distribution modelling framework.

[3] 782*0,003/2 = 1,17 observations. We divide by two as we only focus on the left tail in this example.

The following section of the literature review focuses on two main stylized facts of asset returns resulting in a violation of normality: skewness and excess kurtosis. Specifically, the section takes interest in the estimation challenges and consequences of asymmetry and fat tails in return distributions.

6.3 Skewness and Kurtosis

The Gaussian distribution exhibits properties which makes it possible to apply simplifying assumptions. For instance, the assumption of normality in Markowitz's Mean-Variance framework facilitates a complete description of the portfolio distribution through only the first two moments: the mean and the variance (Jondeau et al., 2007). This simplification is convenient in the sense that the risk of a portfolio can be examined entirely through the variance. However, financial markets have become a non-normal environment. One of the main consequences of the deviation from normality is that we can no longer correctly explain the movements in a distribution by looking only at the mean and variance. Instead we are required to utilize more advanced models which account for the third moment, skewness, and the forth moment, kurtosis. The following section presents the concepts of skewness and kurtosis together with the Jarque-Bera test for normality.

6.3.1 Skewness

Skewness measures the deviation from a symmetric distribution. The documented presence of skewness in non-normal distributions of financial assets underlines its relevance in advanced statistical modelling. The formula for calculating skewness is illustrated below:

$$\widehat{s_i} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{(r_{i,t}-\bar{r}_i)}{\hat{\sigma}_i}\right)^3$$

Where $\widehat{s_i}$ is the skewness parameter, T is the total number of time periods, $r_{i,t}$ is the natural logarithm of the return at time t, $\bar{r}_i$ is the natural logarithm of the mean return and $\hat{\sigma}_i$ is the standard deviation of the return, for all assets i (Jondeau et al., 2007). A positive (negative) value of $\hat{s}$ indicates a longer right (left) tail and the asymmetry intensifies as the deviation from zero increases. Most financial assets have a negative skewness meaning that extreme negative events are more probable than extreme positive events (Jondeau et al., 2007).

The consequences of ignoring skewness in assets distributions are critical and the problems are concentrated around two issues. Firstly, following the prospect theory investors are irrational and loss averse (Tversky, Kahneman, Krantz, & Rahin, 1991). This implies that Investors tend to be more concerned about avoiding a negative outcome than seeking a positive return of the same magnitude. This is because the expected utility from the latter outcome outweigh the former. Hence, ignoring the skewness of a distribution leads to suboptimal outcome for the investor. A deeper discussion of this impact is not provided in this work, as an assessment of the investor's utility function is beyond the scope of this paper.

Secondly, from a risk management point of view, ignoring the impact of skewness has major consequences related to the perceived risk. Ignoring negative skewness leads to underestimation of the real risk of the distribution as the mean will not be equal to the median and thereby not be placed in the central peak of the distribution. Hence, the variance will fail as a measure of risk, as it does not account for the shape of the longer left tail (Jondeau et al., 2007).

6.3.2 Kurtosis

The forth moment of the distribution focuses on the probability density in the tails. Kurtosis explains the thickness of the tails and is calculated in the following way:

$$\widehat{k_i} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{(r_{i,t} - \bar{r}_i)}{\hat{\sigma}_i}\right)^4$$

Where $\widehat{k_i}$ is the kurtosis parameter, T is the total number of time periods, $r_{i,t}$ is the natural logarithm of the return at time t, $\bar{r}_i$ is the natural logarithm of the mean return and $\hat{\sigma}_i$ is the standard deviation of the return, for all assets i. A distribution with a kurtosis above three indicates fatter tails and receives the classification leptokurtic. A kurtosis below three indicates thin tails and is labeled platykurtic (Esch, 2010). Most equity assets have heavy tails, hence exhibit a leptokurtic shape (Xiong & Idzorek, 2011).

The leptokurtic distribution shape introduces interesting modelling concerns, as two distributions are able to have the same variance and mean while exhibiting a different level of leptokursis. The Mean-Variance framework is unable to incorporate leptokurtic data behavior consequently misrepresenting risks (Jondeau, 2005; Xiong & Idzorek, 2011). If the normality assumption is applied on a leptokurtic distribution, one will underestimate the frequency and magnitude of the most severe events, namely the "extreme negative outliers". These events are the most critical outcomes to take into account when evaluating the potential risk of an investment, as they are linked to the probability of an investor going bankrupt. Hence, neglecting the kurtosis moment results in severe underestimation of the involved risk.

6.3.3 The Jarque-Bera Test

This work applies the Jarque-Bera (JB) test statistic in order to investigate whether the assumption of normality is violated due to the presence of skewness and kurtosis. The JB test statistic tests the null hypothesis that the skewness and excess kurtosis[4] both are equal to zero. Hence, the null hypothesis is that the returns follow a normal distribution. The test follows an asymptotic χ^2 distribution. However, one of the major limitations of this test is that the asymptotic distribution only holds for large sample sizes (Jondeau et al., 2007). Therefore, one has to be critical about the test results due to the limited size of our sample.

One way to overcome this problem is by increasing the time horizon which naturally increases the total number of observations. However, doing this intensifies the risk of historical bias, as the earliest data may only demonstrate a vague relationship to the present environment (Stock & Watson, 2012). Applying the JB test to a smaller sample size increases the variance of the test results. This effect is considered to be the lesser of two evils. Specifically, we believe that the potential increase in variance only composes a minor problem and no more actions will therefore be taken.

If the test statistics are significant we reject the null hypothesis of normality, implying that presence of skewness and kurtosis makes the assumption of a Gaussian distribution inadequate. Improvement of accuracy in risk estimation is therefore found in more advanced methods which account for skewness and kurtosis.

In addition to account for non-normality in terms of skewness and kurtosis, it has also been documented that financial data often exhibits effects of autocorrelation and heteroscedasticity. Presence of autocorrelation and heteroscedasticity lead to a violation of the assumption of independently and identically distributed observations which is required in distribution modelling under Extreme Value Theory. The following section therefore examines statistical theory on autocorrelation and heteroscedasticity in the context of improving tail risk estimation.

6.4 Autocorrelation and Non-Constant Variance

One of the challenges when analyzing the marginal distribution of financial time series data is related to the assumption of independent and identically distributed returns. A violation of this assumption

[4] The excess kurtosis is the calculated kurtosis minus the kurtosis of a normal distribution, $\hat{k} - 3$.

leads to two main problems. First, the estimate of the variance is biased. Second, the residuals are not applicable in the GARCH-EVT-Copula framework.

A well-known problem when analyzing financial data is the lack of *independent* and *identically distributed* (i.i.d.) observations. Independence implies that a variable is uncorrelated with any previous observations: $R_t = \propto + \propto_1 R_{t-1} + \propto_2 R_{t-2} + \ldots.. + \propto_n R_{t-n}$ with $\alpha_{1 \ldots n}$ being non-significantly different from zero. Hence, past observations have no explanatory power in predicting the future. The second term "identically" requires that the distribution of each variable in the sample is the same. A violation of these assumptions implies that the data is not i.i.d. (Stock & Watson, 2012). It is of outmost importance to this research paper that residuals are approximately i.i.d., hence the following sections investigate reasons, consequences and potential solutions to issues related to the violation of the assumption.

Autocorrelation and heteroscedasticity are the two main sources for violations of the i.i.d. assumption. Autocorrelation occurs due to presence of significant correlation between R_t and any value of R_{t-n} (Stock & Watson, 2012). In financial times series, autocorrelation can be explained by various phenomena e.g. illiquidity of the market such as in the appraisal of real estate, where the lack of frequent trading data often implies that asset prices are based on adjusted values of preceding periods. For more liquid assets, autocorrelation predominantly prevails due to different trading strategies and seasonality in stock returns (Stock & Watson, 2012). A deeper discussion of the reasons for autocorrelation deserves a topic of its own and is considered beyond the scope of this work.

The crucial impact of autocorrelation relates to the behavior of the error term. Presence of autocorrelation leads to violation of the assumption of no autocorrelation¸ $Corr(u_t,u_s|\mathbf{X}) \neq 0$ for all $t \neq s$ (Wooldridge, 2009). This assumption requires that, given the return vector **X**, the error terms from different time periods are uncorrelated. The implication of this violation is that the estimate of the variance parameter is no longer unbiased and the residuals are no longer i.i.d. (Wooldridge, 2009).

The second part of the i.i.d. assumption relates to the identical distribution of the variables. Assuming an identical distribution requires a constant variance in the model. However, stylized facts of financial time series suggest that financial return data shows profound volatility clustering. Volatility clustering describes the tendency of extreme financial returns to be followed by other extreme returns and low volatility periods to be followed by other periods of low volatility (Jondeau et al., 2007). Hence, evidence suggests that the variance is not only varying but also time dependent. This change in structure of the variance conflicts with the identical distribution assumption. In econometric terms the identical distribution requires no heteroscedasticity in the sample i.e. the variance of the error term, given the return vector $\mathbf{X}_t$, is constant over time: (Var($u_t|\mathbf{X}_t$) = σ^2 (Wooldridge, 2009). A violation of the homoscedasticity assumption requires that the conditional variance of the financial time series is corrected for heteroscedasticity effects. The following sections examines the consequences and solutions to presence of autocorrelation and heteroscedasticity effects.

6.4.1 Heteroscedasticity

The potential problem of heteroscedasticity is twofold. Firstly, if the applied model assumes a constant variance it will take the average variance across time and apply it to the full model whereby not accounting for volatility clustering. The consequence of such practice is underestimation of variance in periods of high volatility and overestimation the variance in periods low volatility. Secondly, the violation of homoscedasticity together with the violation of autocorrelation have profound influence on our estimation of the return variance. The impact of these two violations are materialized in the fact the variance $\hat{\sigma}^{2,5}$ is no longer unbiased (Wooldridge, 2009)[6]. This problem is especially of deep concern in models building on normality assumptions as these utilize variance is the key measure of risk. In the context of this paper, we find it important to correct for the potential impact of autocorrelation and heteroscedasticity as these effects lead to violation of i.i.d.

This thesis applies weekly observations and it is therefore important to test for autocorrelation and heteroscedasticity as these effects are more prevailing in high frequency data. This need is further enhanced by the later steps of the analysis, as the modeling of the marginal distributions rely on a semi-parametric approach under EVT which requires the residuals to be approximately i.i.d. A fundamental requirement in order to test for autocorrelation and heteroscedasticity is that the data is stationary. The follow section presents the Augmented Dickey-Fuller test, which enables us to validate the data series does have unit root.

6.4.2 Augmented Dickey-Fuller Test

The fundamental requirement for analyzing financial times series is that the data is stationary. Strict stationarity implies that the conditional mean, variance and covariance are independent of time. However, this is often not the case for stock returns (Stock & Watson, 2012). Statistical problems related to non-stationarity are dual. Firstly, if the time series has unit root the data exhibits a lack of mean

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[5] $\widehat{\sigma^2} = \frac{SSR}{n-k-1}$

[6] Here it is assumed that assumption 1-3 are meet.

reverting behavior, leading variance to explode whereby making it impossible to obtain precise parameter estimates. Secondly, if the data is non-stationary, key statistical properties are no longer applicable. Specifically, essential aspects such as the Central Limit Theorem and the Law of Large Numbers are no longer appropriate, which implies that usual test statistics are invalid (Wooldridge, 2009).

In order to overcome this problem, the data in this work is transformed to a stationary process by taking the log of the first difference. This transformation stabilizes and linearizes the time series. The disadvantage of this approach is that it reduces the total number of observations in the sample by one (Stock & Watson, 2012). In order to statistically test for stationarity, the Augmented-Dickey-Fuller (ADF) Test for a Unit Autoregressive Root is applied. When using the ADF test, it is necessary to make a preliminary visual assessment of the time series to determine whether a *trend* or *constant* should be included. A trend parameter is included if the data displays a deterministic trend and a constant is included when the data shows a mean reverting behavior around a non-zero value (Elder & Kennedy, 2001). The ADF test is depicted as the following:

$$\Delta r_{i,t} = \beta_0 + \alpha t + \delta r_{i,t-1} + \varepsilon_{i,t}$$

$$H_0: \delta = 0$$

$$H_1: \delta < 0$$

Where $\Delta r_{i,t}$ is the first differenced log returns, β_0 is a constant, t is a trend, α is a trend parameter, δ is the unit root parameter, which is examined under the null hypothesis, $r_{i,t-1}$ is the log return of time t-1 and $\varepsilon_{i,t}$ is the error term, all for assets *i*. The ADF test tests the null hypothesis of a unit root against the alternative hypothesis of stationarity. Hence, data is only assumed to be stationary, when the null is rejected[7]. We require the time series to be stationary in order to specify the AR(p)-GJR-GARCH(p,q) model.

Having established the method of ensuring stationarity in the data series, the following section examines the theory related to autocorrelation.

6.4.3 AR-GJR-GARCH

The AR-GJR-GARCH estimation allow us to obtain approximately i.i.d. residuals which are required for modelling the marginal distributions. Having established the method of ensuring stationarity in the

[7] The t-test of the δ parameter is compared to the special critical values of the ADF test statistic, which is required to reflect the non-normal distribution under the null of a unit root

data series, the following section examines the theory related to autocorrelation and heteroscedasticity.

6.4.3.1 Autocorrelation

This thesis tests for autocorrelation by performing and investigating the Autocorrelation Function (ACF) and observe if the residuals exhibit White Noise and if the data is stationary at the 95% confidence level. An autoregressive (AR) model is specified for each of the marginal distributions. The AR of the pth order is specified in the following way and estimates the correlation coefficient of each of the identified lags:

$$AR: R_t = c + \propto_1 R_{t-1} + \propto_2 R_{t-2} + \ldots.. + \propto_p R_{t-p} + u_t$$

Where R_t is the return at time t, c is a constant and $\alpha_{1..p}$ are return parameter estimators for all R_{t-p}, which are past returns. Lastly, u_t captures the residuals. This estimation step controls for potential autocorrelation and allows us to setup a model, which can control for conditional autocorrelation.

6.4.3.2 Heteroscedasticity

To capture conditional heteroscedasticity effects statistical theory suggests either Autoregressive Conditional Heteroscedasticity (ARCH) or Generalized Autoregressive Conditional Heteroscedasticity (GARCH) modelling. In ARCH modelling the variance is regressed on the squared residuals of previous observations, which allows the variance to vary over time. The benefits of model is reflected by its simplicity, however, the ARCH model often requires an extensive number of lags in order to produce White Noise errors and the slow decay of past influence makes the model likely to over-predict volatility, as it overstates the impact of isolated shocks (Stock & Watson, 2012).

Instead, the GARCH model includes the value of the past variance, σ_{t-q}^2. This inclusion is associated with the ability to considerably reduce the required number of lags. Thereby creating a more dynamic model, which reacts quickly to changes in the market (Stock & Watson, 2012). However, similar to ARCH, the GARCH model has a symmetric incorporation of negative and positive past residuals. This is a potential problem as empirical findings suggest that the market has a larger tendency for increased volatility in the light of past negative movements i.e. financial markets tend to react stronger to negative rather than positive shocks. This effect can be controlled for by introducing an extended version of the GARCH model which includes a *leverage effect,* represented by a GJR-term (A. McNeil et al., 2010). As we find it relevant to account for the asymmetric reaction to shocks, this work applies the GJR-GARCH extension of the GARCH model. Specifically, the GJR-GARCH theory implies inclusion of a dummy parameter, which equals one if the residual from the previous observation is less than zero.

Hence, this extra coefficient allows the model to incorporate increased variance in periods succeeding a negative return (Glosten et al., 1993). The GJR-GARCH (p,q) model is constructed in the following way:

$$GJR - GARCH(p,q){:}\ \sigma_{u,t}^2 = \omega + \sum_{i=1}^{p} \beta_i u_{t-p}^2 + \sum_{i=1}^{q} \gamma_i \sigma_{t-q}^2 + \sum_{i=1}^{p} \delta_i\, I u_{t-p}^2$$

Where $\sigma_{u,t}^2$ is the variance of the error term at time t, ω is a constant, β_i, γ_i and δ_i are the parameter estimates for the past values of the squared residuals, the variance of the error term and the leverage effect, respectively. I represents the GJR leverage effect. The model is estimated using the method of Maximum Likelihood under the assumption of variance stationarity and $\omega > 0$, $\beta_i \geq 0$, $\gamma_i \geq 0$, and $(\delta_i + \beta_i) \geq 0$. The requirement of the estimates being non-negative ensures that the variance is strictly positive (as a variance can never be negative) (Jondeau et al., 2007).

In order to proceed to the distribution modelling with EVT we obtain the residuals from the AR model and standardize them according to their respective conditional variance, which is obtained from the GJR-GARCH model. If the AR(p)-GJR-GARCH(p,q) model is correctly specified, this step provides us with a sample of residuals which are corrected for autocorrelation and heteroscedasticity. The standardized residuals and standardized squared residuals are analyzed graphically in an ACF plot and tested for significant relationships using the Ljung-Box Q-test. If we fail to reject the null hypothesis in the LBQ test the residuals are assumed to be i.i.d. Subsequently, we conduct the marginal distribution modelling for each asset applying the Extreme Value Theory framework, hence the following section examines the possibilities within this theoretical area.

6.5 EXTREME VALUE THEORY

Given the growing turbulence in today's global financial markets, one of the most challenging tasks of risk managers is to develop and implement tools, which allow for modelling rare and extreme events and measure their probability and impact. With the aim of improving current practices, researchers and practitioners have allocated much attention to the possibilities of Extreme Value Theory (EVT) (Embrechts, 2000; J. P. Morgan, 2009; Jondeau et al., 2007; Meyers, 2011; Rachev & Racheva-Iotova, B. Stoyanov, 2010; Xiong & Idzorek, 2011). In a financial application, EVT is a framework predominantly intended to model the behavior of tail distributions. Typically, EVT is used to estimate and quantify financial risk. In the following sections, we discuss the concept of EVT and its application to measure and assess financial risk related to asset returns.

The EVT shapes the area of statistics, which concentrates on the extreme deviations from the mean of a probability distribution. Overall, the EVT framework provides two approaches to modelling the distribution tails. The main differences between these two approaches mainly lies in the practical implementation and assumptions of the distribution. They are presented below.

6.5.1 Block Maxima Method

The Block Maxima Method (BMM) is the traditional approach. It dates back to the Fisher-Tippett theorem from 1928 (Brodin & Klüppelberg, 2006). In this approach, the data point selection is called the BMM. Practically this means that the data is divided into blocks of appropriate length and one collects the absolute maximum data point within each block. The process is exemplified in the left diagram of Figure 3, where data points X_2, X_5, X_7 and X_{11} are selected for the tail modelling. In the BMM, the asymptotic distribution of maxima series is known to follow the extreme value distributions of Gumbel, Fréchet or Weibull distributions. In practice the maxima data points are fitted with a generalized extreme value (GEV) distribution, which is a standard form of these three distributions (Allen, Singh, & Powell, 2011). As this paper does not apply the BMM method a detailed examination of the three extreme value distributions is beyond the scope of this paper.

Since most of the data points are ignored in the BMM framework, it automatically leads to exclusion of much information. Having too few observations leads to incorrect and faulty estimation of the tail distributions, thereby resulting in inaccurate risk estimations. This has encouraged practitioners to challenge and question this methodology (Allen et al., 2011; Brodin & Klüppelberg, 2006; Rocco, 2014). Researchers have proposed to overcome this challenge by including additional blocks, whereby increasing the number of maximum observations. However, this solution comes at the expense of risking to include inappropriate historical information rendering data patterns to be influenced by irrelevant historical events (Brodin & Klüppelberg, 2006; Jondeau et al., 2007).

6.5.2 Peak-Over-Threshold Method

A more modern alternative to the BMM is the Peak-Over-Threshold (POT) method, please see right pane of Figure 3 for an illustration of the concept. In the aim of analyzing extreme market events, our interest does not necessarily only concern maxima or minima of observations, but also the behavior of extreme events in terms of large exceedances over a given threshold. This approach is based on modelling all exceedances, which are beyond a pre-specified threshold known as u. The POT method is generally recognized for better performance in financial applications due to its more efficient use of data points compared to the original BMM method (Brodin & Klüppelberg, 2006). This can be exemplified through the before mentioned volatility clustering. This stylize fact generally imply that several extreme events could occur within the same block and the BMM will only obtain one data point in the given period, even though it might contain several extreme observations of interest to the researcher.

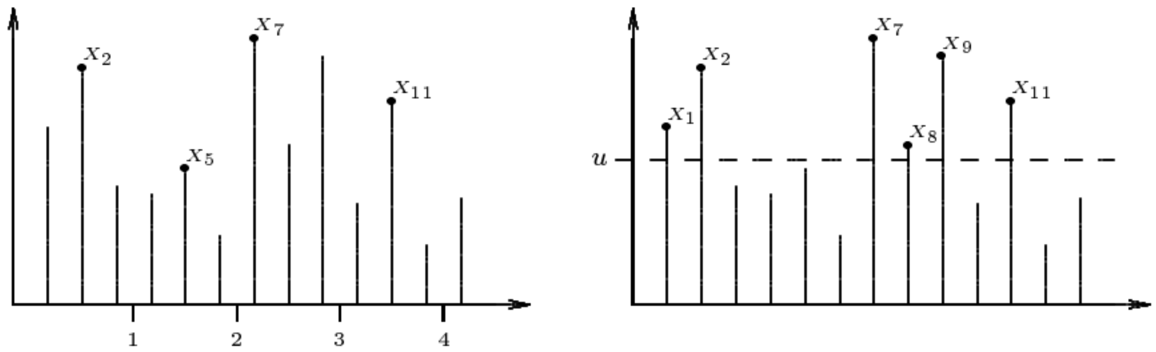

Figure 3: Block Maxima Method (left pane) and Peak-Over-Threshold Method (right pane)

Unlike the BMM approach, this method fits the data points using the generalized Pareto distribution (GPD). Therefore, the following section will broadly discuss the characteristics of GPD.

6.5.3 The Generalized Pareto Distribution

Letting market asset returns be represented by realizations *x* of a random variable *X* the modelling of tail distributions using EVT makes us interested in the cumulative distribution function of exceedances $F_u(x)$ of X over a threshold *u*. The conditional distribution of excess losses over threshold *u* is defined as:

$$F_u(x) = P(X - u \leq x | X > u) = \frac{F(x+u) - F(u)}{1 - F(u)}$$

Where X is a random variable and *u* is a given threshold. This equation represents the probability that a loss, X, exceeds a threshold *u* by at least *x*, conditional on exceeding the threshold. As the threshold gets far enough into the tails the underlying distribution approximates the GPD fairly well (Jondeau et al., 2007; Murphy, 2008):

$$F_u(x) \approx G_{\xi,\beta}(x), u \to \infty$$

The GPD is defined as a two-parameter distribution with the function:

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1-\xi x/\beta)^{-1/\xi} & \xi \neq 0 \\ 1-exp(-x/\beta) & \xi = 0 \end{cases}$$

The parameters ξ and β are referred to as the shape and scale parameters, respectively. An increase in the shape parameter ξ, while holding the scale parameter constant, increases the tail while steepening the slope at a more central part of the density. When the shape index is positive, $\xi > 0$, the GPD exhibits heavy tails, which is the characteristic we expect our financial return series to exhibit.

When modelling tails of the marginal distributions with the generalized Pareto distribution we require the observations in the dataset to be approximately i.i.d. To recall, we correct the data using an AR-GJR-GARCH and obtain filtered standardized residuals for modelling with EVT. Given the exceedance level u, the residual exceedances are used to estimate the scale and shape parameter with a maximum-likelihood function under the GPD. This procedure is proposed by several handbooks on modelling non-Gaussian distributions with EVT (Brodin & Klüppelberg, 2006; Jondeau et al., 2007; Stoyanov et al., 2011).

6.5.3.1 Optimal Tail Size - Choosing the Threshold u

One of the main challenges in applying the POT method is that it requires one to manually choose the threshold *u.* The task of selecting the threshold is challenging as no technical consensus guarding the procedure. It is common practice select the threshold based on a visual inspection of the excess means (Rachev & Racheva-Iotova, B. Stoyanov, 2010). An alternative option is the Kuipers test suggested by Goldberg et al. (2008), which provide a numerical optimization of the threshold. However, despite the high level of complexity involved in the model it only shows limited improvements due to a lack of good optimization properties (Rachev & Racheva-Iotova, B. Stoyanov, 2010). Rendering the method of visual inspection to be the common practice.

When choosing the threshold one faces a tradeoff: If the threshold is set too low (too far into the tail) it results in estimation difficulties due to too few exceedances, consequently leading to high variance estimates. On other hand, if the threshold is set too high it provides biased estimates and the approximation to a generalized Pareto distribution is infeasible. Consequently, one of the main challenges in EVT is dealing with the trade-off between inflated variance and bias.

In recent years, EVT has received much attention in the sphere of risk management and quantitative finance research. The theory has proven to perform well and yield better results than traditional methods when considering modelling of tail behavior and extreme events (Allen et al., 2011; Embrechts, 2000; Jondeau et al., 2007; Meyers, 2011; Rocco, 2014). Considering the modelling potential and interesting empirical applications we build the tail distribution modelling on the foundation of EVT, specifically POT methodology.

6.6 CORRELATION

The central aspect of risk management in portfolio allocation builds on the benefit of diversification. The diversification benefit is possible due to non-perfect dependence between financial assets, whereby the investors are able to remove idiosyncratic risk by allocating the investment to several assets (Bodie, Kane, & Marcus, 2010). The measure of dependence in traditional models is Pearson's linear correlation. This measure describes the linear correlation between two vector variables and is calculated as the covariance between two variables divided with the product of the standard deviations:

$$\rho_{x,,y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}.$$

Pearson's correlation takes on values between [-1, 1]. The two extreme values of negative/positive one indicates that the two assets are monotonically related and have a perfect negative/positive correlation. A value of zero implies that the two assets are strictly independent. A drawback of Pearson's correlation is that it is only applicable as a measure of dependence under the assumption of a multivariate normal distribution. This implies that it is not only required that the marginal distributions exhibit a normal distribution, but also that the joint distribution of the asset returns follows the rules of normality (A. McNeil et al., 2010). Several studies have shown that empirical return data does not exhibit such characteristics (Beine, Cosma, & Vermeulen, 2010; Peter Christoffersen, Errunza, Jacobs, & Langlois, 2012; Stoyanov et al., 2011). Hence, potential bias is introduced when Pearson's correlation is used as the measure of dependence. Assuming a constant and linear dependence structure is unrepresentative for asset returns as empirical studies show that correlation varies extensively depending on the general market conditions (Peter Christoffersen, Errunza, Jacobs, & Jin, 2013).

The non-normal dependence structures especially prevail during times of financial turbulence. Here a typical tendency is that the correlation between equity and bonds decreases, specifically the correlation may go from e.g. 0.6 to 0.4 whereby increasing the diversification effect. This is driven by the well documented "flight to quality" phenomenon where investors in their seek for risk reducing assets allocate more of their capital to less risky fixed income products (Bernanke. Ben, Gertler, & Gilchrist, 1994).

Moreover, correlation among stocks tends to converge towards one in periods of financial turbulence. Traditionally, equity loses value in times of distress and Pearson's correlation may therefore overestimate the diversification benefit among stocks. This problem is increasing and historical research documents the intensification of co-movement among stocks. This effect is especially present in developed markets whereby underlining the importance of account for non-normal dependence structures in our research paper (Peter Christoffersen et al., 2013).

The consequence of non-constant correlation among stocks is relevant to risk estimation as we see a tendency for diversification effect to fail exactly when the investors need it the most (Embrechts, McNeil, et al., 1999a). The lower predictions of correlation in periods of crises thereby give the investor an incorrect estimate of the diversification effect and thereby underestimate the financial risk. This is especially problematic in the current market setting, where investors tend to follow the previously mentioned strategy of "reaching for yield" (Westaway & Thomas, 2013). The "reach for yield" behavior implies that investors allocate less of their capital to low yield bonds, thus leaving them more prone to the convergence effect in equity when hit by a crisis.

In order to investigate the data for potential correlation breakdowns we follow the method suggested by Sheikh and Qiao (2010). This implies that we compare correlation pairs from two distinct time periods. The first period is picked as a period of market distress and the second period is characterized by economic stability. The period of market distress is determined based on the level of the VIX index. This analysis is presented in section 7.3 "Correlation Breakdown".

6.6.1 Correlation Under Non-Normality

When the assumption of normality is violated, correlation is no longer applicable as a measure of dependence. This is illustrated by the fact that two variables, with a correlation of zero might still show dependence during times of distress. Hence, zero correlation does not imply independence (A. McNeil et al., 2010).

An additional problem related to Pearson's correlation is that it is not invariant under transformation i.e. the natural logarithm of returns will generally not have the same correlation as the arithmetic returns (Embrechts, McNeil, et al., 1999a). Other measures of correlation such as Kendall's tau or

Spearman's rank correlation are more robust to the impediments above and function under the assumption of non-normality. Nevertheless, Kendall's tau and Spearman's rank correlation are only a measure of simple correlation and theory therefore assess them to be unsuffient to accurate approxite the empirical dependence structures of risk (Embrechts, McNeil, et al., 1999a). Therefore, in order to improve the accuracy of modern risk management tools we adopt more advanced tools, which are able to reflect the complexity in the dependence structures among financial assets. Specifically, we take interest in the copula families. Hence, the following section provides insight to modelling of joint dependence structures with aim of overcoming the drawbacks of Pearson's correlation, Kendell's tau and Spearman's rank.

6.6.2 Joint Dependence Modeling Under Non-Normality

As clarified above, one of the major problems when modeling multivariate distributions in financial time series is the inaccurate estimate of the joint dependence. Traditional theory often provides good estimates for the marginal distributions however, the practice of applying linear correlation as a dependence measure leads to incorrect estimates for joint distributions. Here copulas become a helpful quantitative tool as *"copulas facilitates a bottom-up approach to multivariate model building"* (McNeil et al., 2010, p. 185). Copulas make it possible to model the joint distribution of different assets independently from their marginal distributions (Trivedi & Zimmer, 2006).

The contribution of this tool is that it allows for non-linear correlations whereby the joint distribution can exhibit varying correlations across quantiles (Jondeau et al., 2007). Hence, the use of copulas makes it possible to generate robust distribution models which incorporates a higher tail correlation.

Despite the technical complexity, copulas have a lower level of practical barriers and are available in most software programs, as they are estimated using maximum likelihood. We construct the copula model in two steps. First, the n-number of marginal distributions of the return series are defined using the above steps of AR(p)-GJR-GARCH(p,q) and EVT and subsequently converted to uniform distributions taking values between [0,1].

$$(u_1, u_{2,}, .. u_n) = (F_1(X_1), F_2(X_2), \dots F_n(X_n))$$

Here u_n is the uniform distribution vector and $F_n(X_n)$ is the marginal distribution of asset n. Secondly, the copula function is calibrated using the inverse cumulative marginal distribution functions of the uniform distributions.

$$C(u_1, u_{2,}, .. u_n) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots F_n^{-1}(u_n))$$

Here C is the copula, u_n is the uniform distribution vector, F is the joint cumulative distribution function and $F_n^{-1}(u_n)$ is the inversed marginal distribution function of asset n. The non-linear correlation is estimated in this equation, as the copula expresses dependency on a quantile scale. Specifically, $C(u_1, u_{2,}, .. u_n)$ is the joint probability that all X_n lie below their u_n-quantiles. From this formula we obtain a unique C in terms of the joint distribution and its margins (A. McNeil et al., 2010). Once the copula is estimated, the joint cumulative distribution function, F, can consistently be stimulated independently of the marginal distributions, through the unique copula:

$$F(X_1, X_2, \dots, X_n) = C(F_1(X_1), F_2(X_2), \dots F_n(X_n))$$

We use Monte Carlo simulation to simulate trials of the distribution function. The approach generates random numbers from the uniform vectors u_n and subsequently calculate the given point in the joint cumulative distribution function using the copula. This simulation is performed 10,000 times in order to provide a detailed joint cumulative distribution function, which can be used to estimate a robust risk measure for the entire portfolio. A deeper discussion of the theoretical properties of copulas is beyond the scope of this work, however, we invite interested readers to visit the work of McNeil et al. (2010) for further details.

Having discussed the relevance of copulas in general, the following section provides insight to a selection of copula families.

6.6.2.1 Copula families

There exist a broad range of copula families. Here the most basic version is the Gaussian copula. This copula contains features of normality and therefore we do not consider it of value for answering our problem statement. Alternatively, we consider the student's t copula of relevance to this thesis, as it allows us to adjust the degrees of freedom, whereby we can approximate the fatter tails exhibited in the asset returns data (A. McNeil et al., 2010). A degrees of freedom of 30 is approximately equal to a normal distribution and a lower value corresponds to a leptokurtic shape. Allowing the degrees of freedom to change, the student's t copula allocates a higher probability density in the tails. The exact measure is computed by dividing the probability that the student's t copula exceeds a given quantile with the probability that Gaussian copula exceeds a given quantile. As an example, assuming a rank correlation of 0.5 in the bivariate student's t copula with a DoF of four is associated with a probability of joint exceedance beyond the 0.01 quantile to be 2.79 times higher than the Gaussian copula. Hence, the risk of an extreme event will be 279% higher when applying the student's t copula compared to the Gaussian copula (A. McNeil et al., 2010).

A general disadvantage of the student's t copula is the assumed symmetry in the model, where both the left and the right tails are modelled with the same probability density. This contradicts the former presented empirical findings that financial time series exhibit negative skewness. As an alternative theoretical literature suggests the Clayton copula as it is able to account for both heavy tails and skewness (A. McNeil et al., 2010). However, the improvement effect is limited in this study as we are able to indirectly introduce skewness by feeding the copula calibration with non-normal marginal distributions modelled with EVT. Therefore, we decide that the limited improvements originating from explicitly accounting for skewness in the joint distribution by applying the Clayton copula cannot counterbalance the consequences of significantly increasing modelling complexity and inflated variance due to additional model parameters (Esch, 2010). Reverting to the interests of this thesis of improving risk measures, the analysis utilizes the student's t copula as literature suggests it to be a vast improvement of the traditional sample modelling approach (Peter Christoffersen et al., 2013; Sheikh & Qiao, 2010).

Having established solid arguments for our selection of theoretical framework, specifically the theoretical foundation in the GARCH-EVT-Copula setup, the following chapter presents the analysis, distribution modelling process and finally risk estimation.

To answer the problem statement: "*How can risk management measures be improved to better account for the non-normality in financial asset returns, in the context of a European investor?"* we have decided to divide the analysis of this paper into five sections. Firstly, we investigate the data for characteristics of non-normality. Secondly, we apply the information obtained from the non-normality investigation and correct the data series for autocorrelation and heteroscedasticity to obtain approximately i.i.d. standardized residuals. These are applied in section three where we model the marginal distributions using EVT. Section four generates the joint distribution using student's t copula based in the GARCH-EVT modelling. This allow us to account for non-linear dependence structures. Finally, allowing us to estimate non-normal portfolio risk measures in shape of VaR and ES. We provide both the normal and non-normal risk estimates of VaR and ES for an equally weighted portfolio. To demonstrate nuances of investor profiles, we perform the same risk estimation steps for portfolios representing a risk averse and risk seeking investor.

7 PRELIMINARY ANALYSIS – TESTING FOR NON-NORMALITY

Before preceding to the advanced analysis and distribution modelling, we aim at justifying the statement that the return distributions of the portfolio assets are in fact characterized by non-normality.

7.1 HISTORICAL RETURNS

This thesis considers two broad equity indices: The Euro STOXX 600 Index, which represents large, mid and small capitalization companies across 18 European countries and the Euro STOXX 50 which captures the leading companies in Europe, similarly covering companies from 18 European countries. We assume that these two stock indices fairly represents shares of interest to a European investor with a European investment focus[8].

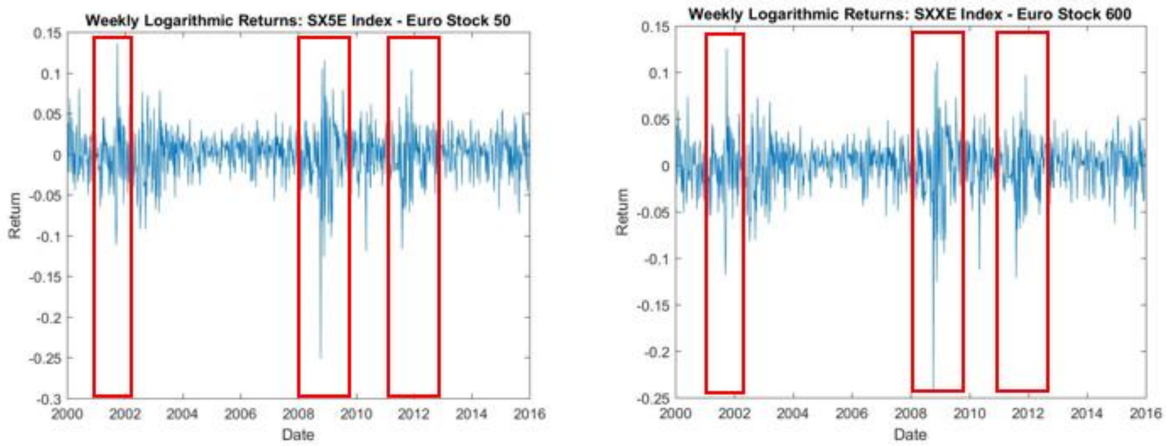

Figure 4: Weekly Logarithmic Returns for Euro STOXX 50 and Euro STOXX 600 (Source: Own calculations)

Figure 4 shows the weekly logarithmic returns of the two equity indices. Both graphs show how the asset returns of the two European stock indices exhibit increased fluctuation three times during the observation period going from primo 2000 to ultimo 2015. These excessive spikes correspond to the dot-com bubble in 2001 to 2002, the Global Financial Crisis in 2008-2009 and the European Debt Crisis starting in 2011, which has especially caused unrest in the European financial markets.

[8] Please see the methodology section for further argumentation for selection of assets.

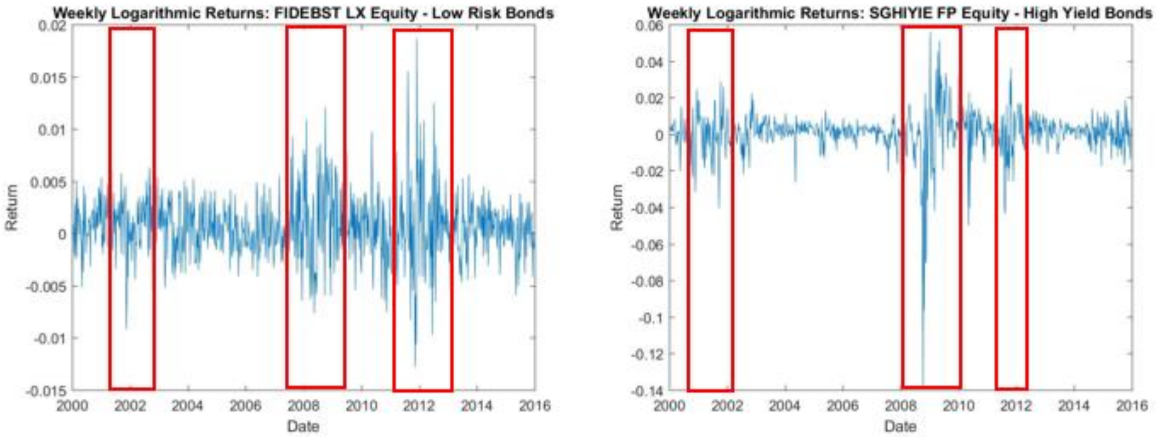

Figure 5: Weekly Logarithmic Returns for Low Risk Bonds and High Yield Bonds (Source: Own calculations)

The portfolio also contains two bond classes: A low risk bond portfolio and a high yield bond fund. Both the low risk and high risk bonds seem to be influenced by the same events as the equity indices, however the magnitude of the fluctuations are less severe compared to the equity indices. The low risk bond portfolio shows largest spikes during the recent European Debt Crisis, which is in line with our expectations as a substantial part of the portfolio is European government bonds.

The fund of high yield bonds holds bonds of private European companies with a S&P rating of BBB or lower. A visual investigation of the logarithmic returns of the high yield bonds reveal that the financial crisis in 2008-2009 appears to have led to the largest return fluctuations of this asset, please see Figure 5. This seem plausible as the global financial markets experienced a default rate on high yield bonds up to 13.7% during this period, a default rate this high has not been seen since the Great Depression back in the 1920s and 1930s (The Economist, 2013).

Overall, the vast amount of fluctuations preliminarily indicate that extreme events may happen with higher frequency than under the normality assumption. Noteworthy is the observation that the negative spikes for Euro STOXX 600, Euro STOXX 50 and high yield bonds are considerably larger than the upward spikes, whereby indicating that these three assets are possibly characterized by negative skewness.

The following paragraphs continue with a deeper investigation of the distribution behavior. The section focuses standard deviation, skewness and kurtosis. Subsequently we support the findings with the Jaque-Bera test for normality.

7.2 Descriptive Statistics and Jaque-Bera Testing

Descriptive statistics allow us to provide detailed insight to the behavior of the sample data, these can be seen in Table 2. The mean, standard deviation and variance are provided in annual arithmetic average terms, while kurtosis, skewness and the Jaque-Bera test statistic is based on the weekly logarithmic returns, just as the distribution modelling is in later parts of the analysis.

	Mean	Std. Dev.	Variance	Kurtosis	Skewness	P-value (JB-test)	Normal Dist.
SXXE Index - Euro STOXX 600	1.4702%	0.2125	0.0452	6.7384	-0.9637	0.0000	No
SX5E Index - Euro STOXX 50	0.4049%	0.2254	0.0508	5.8851	-0.8093	0.0000	No
SGHIYIE FP Equity - High Yield Bonds	4.0380%	0.0866	0.0075	28.9137	-2.9783	0.0000	No
FIDEBST LX Equity - Low Risk Bonds	3.0936%	0.0226	0.0005	2.9003	0.3639	0.0001	No

Table 2: Descriptive Statistics (Source: Own calculations)

Overall, the skewness of the four portfolio assets is in line with our expectations. The two equity indices and the high yield bond fund exhibit a negative skewness, where the high yield bonds have the largest negative skew. In general, a negative asymmetry in return distributions is often associated with risky investments and is thus undesirable to risk averse investors. The low risk bond portfolio has a positive skewness, which is particularly preferred by conservative investors, as this is associated with few negative extreme events.

The large excess kurtoses of 3.74, 2.89 and 25.91 for the two equity indices and the high yield bond fund, respectively, indicates heavy tails. A kurtosis above 3, as we see with the equity indices and the high yield bond fund, indicates that the overall risk of the investment is driven by a few extreme "surprises" in the tails of the distribution. Some speak of high yield bonds with the old saying: "When it rains, it pours". Especially, the high yield bond fund in our portfolio exhibits strong characteristic of heavy tails.

On the other hand, the low risk bond portfolio has a kurtosis slightly below 3 whereby indicating an empirical distribution characterized by slightly light tails. In relation to risk management, we generally see risk averse investors appreciate a low or negative kurtosis and this implies that on a period-by-period basis most observations fall within a predictable span. Hence, risky events are predicted to happen within a moderate range leaving only a limited amount of risk in the tails. We see this phenomenon on the logarithmic returns of the low risk bond fund in Table 2.

The measure of kurtosis does however not generate much value as a stand-alone measure. Looking at standard deviation in addition to kurtosis allows us to further assess the need for advanced distribution modelling. Comparing the high yield bonds with e.g. the two equity indices, we observe that the high yield bond fund has a comparably higher kurtosis however the standard deviation is significantly lower. Largely, this observation implies that the high yield bond carries a comparably lower risk than the two equity indices. But, when a risky event prevails it tends to be more extreme and severe than the loss events of the equity indices. This further encourages researchers and practitioners to allocate significant attention to proper modelling of tail distributions and tail risk in investment management.

The Jaque-Bera test statistic lends additional evidence to the preliminary analysis. The test infers that all four assets violate the assumption of a normal distribution. To recall, the JB test statistic investigates if a distribution is normal in relation to both skewness and kurtosis. The null hypothesis tests if the skewness and excess kurtosis are equal to zero hence, the null hypothesis is that the asset returns follow a normal distribution. Looking in Table 2, we can reject the null hypothesis of normal distribution for all four assets both at the 95% and 99% confidence level, whereby we find several quantitative arguments supporting our motivation for applying advanced statistical frameworks modelling non-normality of asset returns.

The following section analyzes the correlation dynamics of our dataset and graphically examines the correlation behavior under changing market conditions. The importance of non-linear correlation stems from the fact that besides taking account for non-normality in the distribution behavior we also wish to account for non-linear and non-constant dependence among the financial time series.

7.3 Correlation Breakdown

As stated in chapter 7 "Correlation", we follow the method presented by Sheikh and Qiao (2010) and compare the average linear correlation of our dataset with a sub-period characterized by high volatility. In order to label market volatility, we follow the common standard and use the Chicago Board Options Exchange SPX Volatility Index (VIX) as a proxy for market fluctuations and instability. This index reflects both the market's expectations regarding the future volatility as well as the implied volatility of a broad combination of option strikes (Bloomberg Terminal, 2016). The VIX index is mainly based on prices from S&P500 and a natural concern might be connected to VIX mostly reflects movements in the U.S. market. However, as extreme movements between U.S. and Europe tend to be highly interconnected we find this shortcoming less problematic (Peter Christoffersen et al., 2013).

The movements of VIX is plotted in **Error! Reference source not found.**. We observe several spikes. The most volatile period is highlighted in red and indicates the initial year of the Global Financial Crisis. At the peak of the crisis, the implied volatility was 79.1, a striking contrast to the average value of 20.5.

In order to test for breakdowns in the linear correlation we apply the period highlighted in red as our high volatility sub-period. This period ranges from September 2008 to August 2009. The average implied volatility during this period was 40.8. The period of instability is compared to the linear correlation of the entire sample.

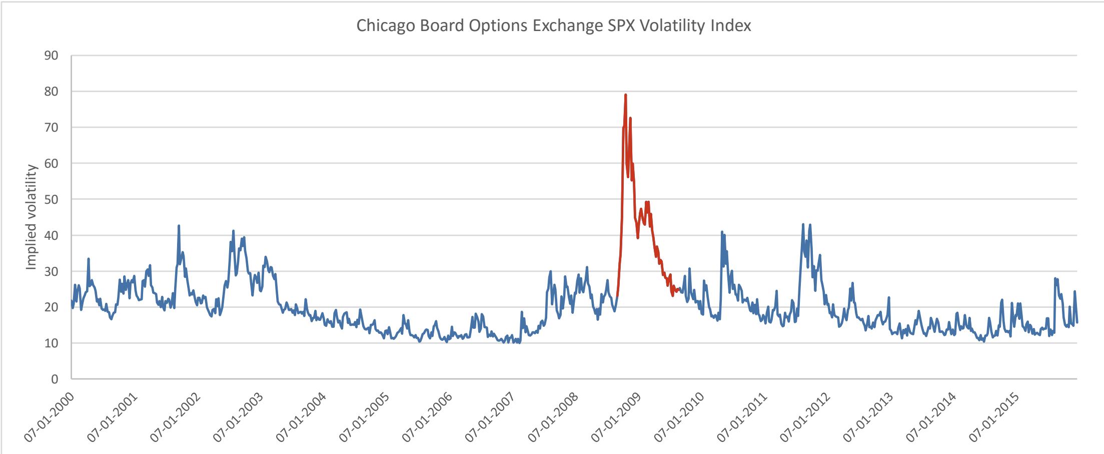

Figure 6: VIX (Source: Bloomberg Terminal)

By comparing the two distinct periods, previous research dictates that we should observe changes in the linear correlations (Peter Christoffersen et al., 2013; Sheikh & Qiao, 2010). As presented in chapter 6.6 "Correlation", we expect to see convergence between the two equity indices in the high volatility period. Similarly, we anticipate that the equity indices exhibit a converging behavior with the high yield bonds as these three assets belong to an investment class which are expected to have a high co-movement. This implies that the correlation among these assets moves towards one in periods of distress. This is because the expected returns of these assets are highly sensitive to bearish changes in the financial market (Peter Christoffersen et al., 2013). In contrast, we expect to observe a diverging effect in the linear correlation between these three assets and the low risk bonds. During times of high volatility, the flight-to-quality effect should surge the demand for low risk assets, thus increasing the prices of safer investment opportunities (Bernanke. Ben et al., 1994). This behavior creates a reverse movement in the return for low- and high risk assets, which argues for a diverging correlation effect.

When computing the two linear correlation matrices, we make several interesting findings. Evidently, the linear correlation between Euro STOXX 600 and Euro STOXX 50 is almost perfectly correlated, even during normal times, with a correlation coefficient of 0.99, please see Table 3. This implies that the correlation is close to the boundary level of one, implying that it is impossible to observe the common convergence behavior in the specific case of Euro STOXX 600 and Euro STOXX 50. Yet, the data shows that the correlation coefficient increases from 0.99 to 1 in the volatile period.

Normal period	1	2	3	4
SXXE Index - Euro STOXX 600	1			
SX5E Index - Euro STOXX 50	0.99	1		
SGHIYIE FP Equity - High Yield Bonds	0.57	0.53	1	
FIDEBST LX Equity - Low Risk Bonds	-0.14	-0.13	-0.09	1

Table 3: Linear Correlation Matrix - Normal Period (Source: Own calculations)

Volatile period	1	2	3	4
SXXE Index - Euro STOXX 600	1			
SX5E Index - Euro STOXX 50	1	1		
SGHIYIE FP Equity - High Yield Bonds	0.75	0.72	1	
FIDEBST LX Equity - Low Risk Bonds	-0.29	-0.29	-0.33	1

Table 4: Linear Correlation Matrix - Volatile Period (Source: Own calculations)

Consistent with the academic expectations, we observe a convergence effect between the high yield bond, the Euro STOXX 600 and Euro STOXX 50 with a magnitude of 0.18 and 0.19, respectively. This is in line with previous findings presented by Christoffersen et al. (2013) and Sheikh & Qiao (2010) who documents a positive convergence effect between these asset classes. From a risk management point of view, this implies that if the investor does not account for correlation breakdowns, it leads to over-estimation of the diversification effect in portfolio investments. Specifically, if a portfolio allocation is based solely on Pearson's linear correlation, it leads to a drastic underestimation of the investment risk during volatile periods.

Finally, we see a diverging effect in the correlation between the low risk government bond and the three high risk assets. The largest magnitude is observed between the low risk bonds and the high yield bonds with a change of -0.24. This large fluctuation in the correlation between these two assets can be explained by the nature of high yield bonds and the specific period of observation. The Financial Crisis caused an excessive spike in the level of default among junk bonds, which led to extensive fluctuations in the value of this investment type (The Economist, 2013). Combined with a flight-to-quality motive, the price of low risk bonds increased. This correlation non-linearity is not captured in the Pearson's correlation, which consequently provides an underestimation of the potential diversification effect.

The increase in the negative correlation between the low risk bond and the three high risk assets leads to an underestimation of the diversification effect. This is especially problematic in the current investment environment, where European investors experience historically low yields and negative interest rates in the money market. This drives investors to a new strategy known as "reaching for yield" (Westaway & Thomas, 2013). A consequence of this behavior is that investors neglect the importance of low risk bonds in portfolio investments in order to facilitate a higher expected rate of return. This strategy does however come at the expense of excess volatility in times of distress, which is not captured by Pearson's linear correlation. The current assumption of linearity leads investors to make investment decisions based on inaccurate models and our findings indicate a clear need for copula modelling, which account for the correlation breakdowns.

In addition to identifying non-normality in relation to distribution shape and volatility breakdowns, the following section presents an examination for the presence of autocorrelation and heteroscedasticity. Presence of autocorrelation and heteroscedasticity lead to violation of the assumption of returns being independent and identically distributed (i.i.d.). A violation of this assumption causes variance estimations to be biased and even more important; a violation of i.i.d. in returns leads to a violation of i.i.d. in the residuals, whereby making the residuals inapplicable in our process of modelling the distributions with EVT.

7.4 Testing for Autocorrelation and Heteroscedasticity

The marginal distribution modelling process requires our sample observations to be approximately i.i.d. In order to ensure compliance with the i.i.d. assumptions, we check for autocorrelation and heteroscedasticity characteristics in the Autocorrelation Functions (ACFs) and investigate if our return series exhibit White Noise and if the data is stationary at the 95% confidence level.

The ACFs plotting the sample logarithmic returns are used to check for autocorrelation and the ACFs of the squared logarithmic returns illustrate the degree of persistence in variance, thus these are used to check for heteroscedasticity.

For the Euro STOXX 50 sample returns, we observe slightly significant spikes outside the 95% confidence band at spikes 3, 7 and 13, please see Figure 7. Considering the ACF of the squared returns for the same index, we observe significant spikes in the vast majority of the lags included in the function. Hence, the ACFs provide some indication of presence of autocorrelation and unquestionable presence of heteroscedasticity effects.

For the Euro STOXX 600, the ACF provides a rather similar image, please see Figure 8. Here, many spikes in the return series ACF are close to the breaching the 95% confidence band. Lag 7, 9 and 13 are significant however, spikes 7 and 9 can be considered undecided spikes as it is unclear whether they lie outside the confidence band. The ACF of squared returns clearly shows that the data series is strongly influenced by heteroscedasticity. As we expect data series on equity returns to exhibit both autocorrelation and heteroscedasticity, we decide to pay close attention to both Euro STOXX 50 and Euro STOXX 600 in the further investigation of the statistical characteristics.

The high yield bond fund shows significant spikes for the return series until lag 6, where after we still observe several significant spikes at later lags, confirming the presence of autocorrelation, please see Figure 9 . Once again, the many spikes in the ACF of squared returns affirm presence of heteroscedasticity in our data series. The more prevalent degree of autocorrelation in high yield bonds compared to the two equity indices is in line with our expectations as high yield bonds are typically less liquid than most equity assets. The price-setting of high yield bonds tends to be based on previous levels, which naturally introduces autocorrelation effects (MarketWatch, 2015; SEC)

The low risk bonds have one significant spike in the return ACF at lag 4 and in the squared returns ACF the spikes are significant until lag 16, please see Figure 10. Once again, we are able to confirm the presence of both autocorrelation and heteroscedasticity.

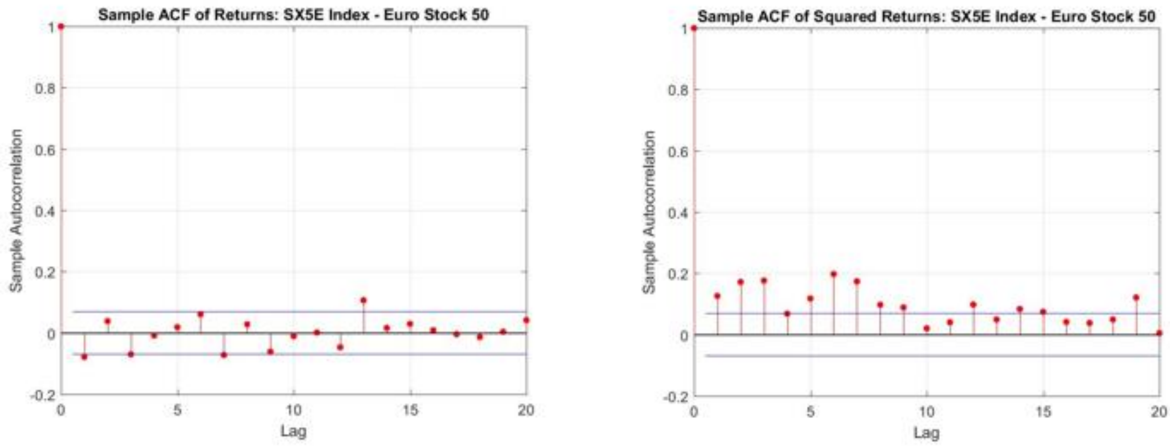

Figure 7: ACFs of Returns and Squared Returns for Euro STOXX 50 (Source: Own calculations

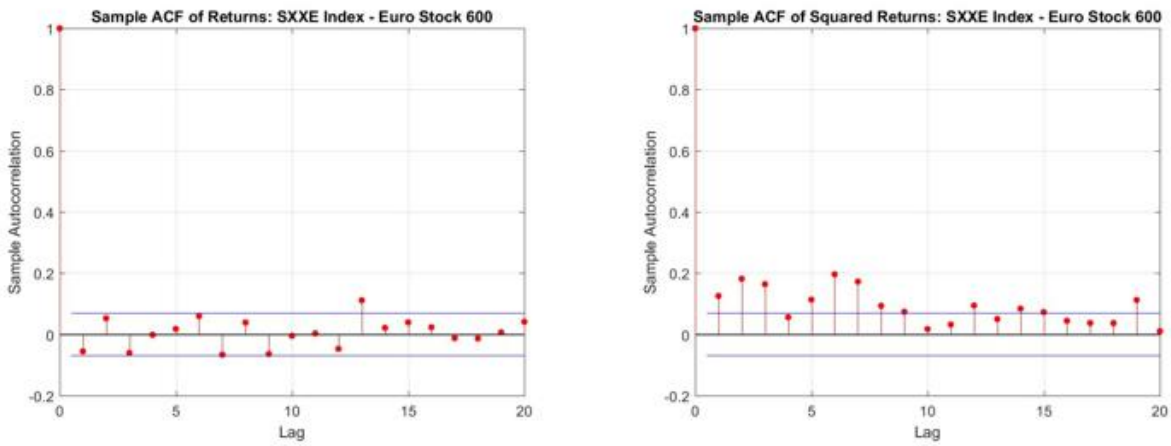

Figure 8: ACFs of Returns and Squared Returns for Euro STOXX 600 (Source: Own calculations)

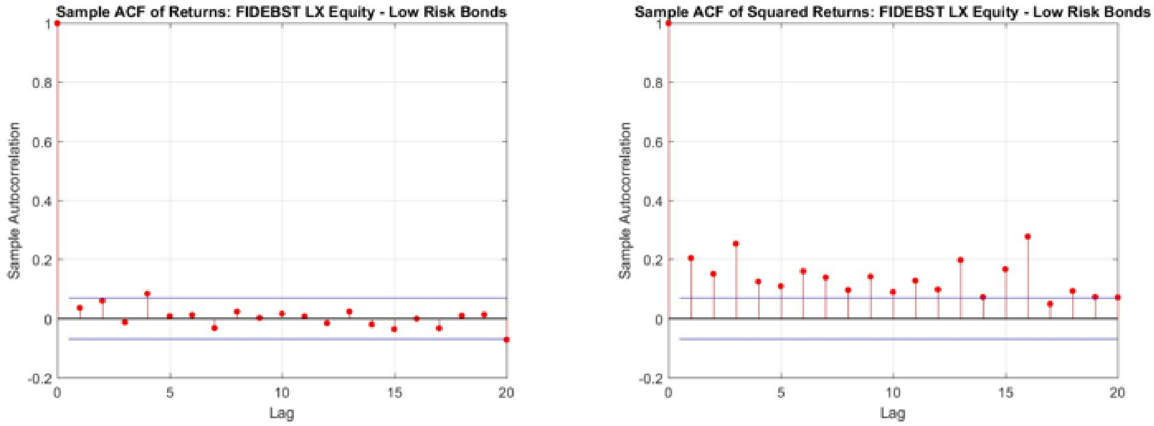

Figure 9: ACFs of Returns and Squared Returns for High Yield Bonds (Source: Own calculations)

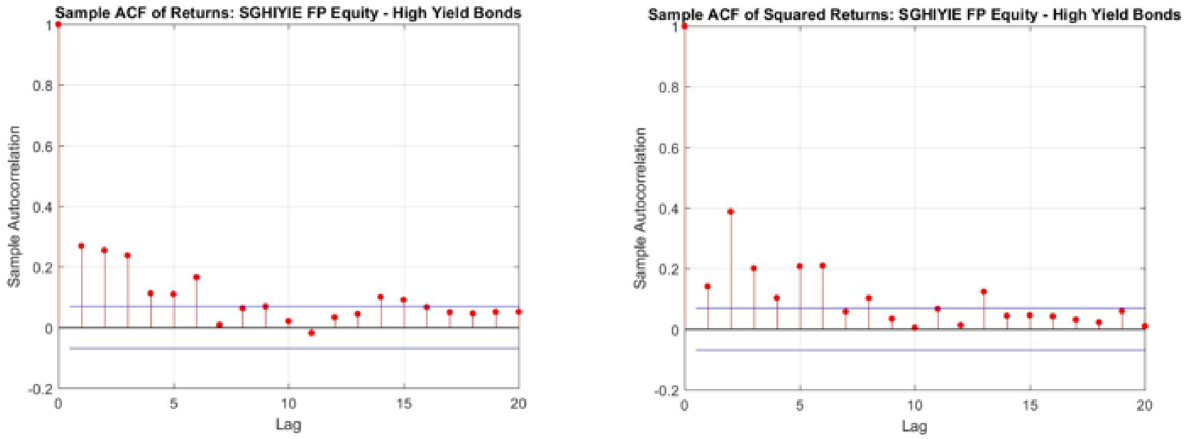

Figure 10: ACFs of Returns and Squared Returns for Low Risk Bonds (Source: Own calculations)

Hereby, our preliminary ACF analysis indicates that all four assets in our portfolio exhibit autocorrelation and heteroscedasticity. Before being able to correct for autocorrelation and heteroscedasticity effects we require the data series to be stationary. The following section apply an Augmented Dickey-Fuller test which tests for the hypothesis of unit root.

7.5 Augmented Dickey-Fuller Test

Before proceeding with the AR(p)-GJR-GARCH(p,q), the data is tested for a unit root using the Augmented Dickey-Fuller (ADF) Test. This is required in order to clarify the stationarity of the data. The problems of non-stationarity mainly relate to inflated variance and the inapplicability of fundamental theorems such as Central Limit Theorem and the Law of Large Numbers. As explained in the methodology section this work analyzes the logarithmic asset returns, that is the first differencing of the return data. The ADF is therefore conducted on the natural logarithm of the return data.

To recall, a constant should be included in the regression if the data exhibits a mean reverting behavior around a non-zero value and a trend parameter should be included if the data exhibits a deterministic trend (Elder & Kennedy, 2001). Looking back at Figure 4 and Figure 5, we observe a clear depiction of a mean reverting behavior and Table 2 shows that each of the times series have a non-zero mean. This indicates that a constant is incorporated in the ADF test. However, from Figure 4 and Figure 5 there are no evidence of a deterministic trend hence, the trend parameter is excluded from the regression. The final model is illustrated below:

$$\Delta r_{i,t} = \beta_0 + \delta r_{i,t-1} + \varepsilon_{i,t}$$

The ADF test is performed on the δ coefficient, where the null hypothesis, $H_0: \delta = 0$, implies that the time series has a unit root. The results of the ADF test is presented in Table 4 below[9].

Assets	ADF test Test value	Critical DF values	P-Value	Rejection
SXXE Index - Euro STOXX 600	**-30.5**	**-2.86**	**0.001**	**Yes**
SX5E Index - Euro STOXX 50	**-31.2**	**-2.86**	**0.001**	**Yes**
SGHIYIE FP Equity - High Yield Bonds	**-21.8**	**-2.86**	**0.001**	**Yes**
FIDEBST LX Equity - Low Risk Bonds	**-26.9**	**-2.86**	**0.001**	**Yes**

Table 5: Results of ADF Test (Source: Own calculations)

The results of the ADF test enable us to reject the null of a unit root, thereby we can conclude that we find sufficient evidence to accept the alternative hypothesis of stationarity in our time series. This implies that we can proceed to correct for autocorrelation and heteroscedasticity effects through the AR(p)-GJR-GARCH(p,q) model.

8 Removal of Autocorrelation and Heteroscedasticity

To determine an appropriate AR(p)-GJR-GARCH(p,q) model, we base our approach on a combination of our own preliminary findings in the ACF analysis, general theoretical recommendations on best practices as well as findings presented by former researchers. In order to support our choice of model we use a t-statistic for each AR(p)-GJR-GARCH(p,q) model. To test the power of the AR-GJR-GARCH model of being able to control for both autocorrelation and heteroscedasticity effects we investigate new ACF plots of the standardized residuals and squared standardized residuals as well as we support the findings with a Ljung-Box test statistic.

[9] The critical value of the ADF test with no trend is based on a sample of > 500 observations and indicates the critical value for a 5% significance level (Stock & Watson, 2012).

8.1 AR(P)-GJR-GARCH(P,Q)

Theory on time series does in general recommend one to pay attention to all significant spikes in the ACF plots when determining the number of lags (p). However, it is rarely recommended to allocate much attention to distant lags in the ACF plots, provided that preceding lags are insignificant (Stock & Watson, 2012). Returning to the plots on the returns of the two equity indices, we find that most of first the spikes are close to being significant whereas more distant lags were clearly outside the 95% confidence band. The weakness of the first spikes makes it questionable whether strong autocorrelation influences the data series in general. A similar scenario is true for the low risk bond portfolio, however here we see a significant spike already at lag 4. Unlike the equity indices and low risk bond asset, the high yield bonds show significant spikes until lag 6, indicating a comparably much stronger persistence of autocorrelation in this asset.

The documented behavior of the four assets guides the statistical decision of testing an AR(1) model for all assets. We acknowledge that the high yield bond showed significant spikes at several lags, however practical and theoretical literature argue that financial asset returns in general often only exhibit a limited level of autocorrelation, unless high frequency data series are modelled (J. P. Morgan, 2009; A. McNeil et al., 2010). To recall, the process for the AR(1) model is estimated as the following regression with ordinary least squares:

$$AR\colon R_t = \ c \ + \propto_1 R_{t-1} + u_t$$

The AR model assists in correcting for autocorrelation effects, however as we have found several indicators for both autocorrelation and heteroscedasticity in our ACF plots hence, the GJR-GARCH process will assist us in removing heteroscedasticity.

Despite the observation of several significant lags in all four ACFs of squared returns, literature establishes broad consensus around the power of sufficiently correcting for heteroscedasticity effects with GJR-GARCH (1,1) i.e. one lag of squared residuals from the AR(1) model and one lag of the conditional variance (Awartani & Corradi, 2005; Rachev & Racheva-Iotova, B. Stoyanov, 2010). We perform a model estimation according to the following regression:

$$GJR - GARCH(p,q)\colon \sigma_{u,t}^2 = \ \omega + \sum_{i=1}^{p} \beta_i u_{t-p}^2 + \sum_{i=1}^{q} \gamma_i \sigma_{t-q}^2 + \sum_{i=1}^{p} \delta_i \, I u_{t-p}^2$$

Where the $\sigma_{u,t}^2$ is the conditional variance of the error term at time t. We need the variance of the error term to generate our standardized residuals for the semi-parametric EVT distribution modelling.

Before preceding to test for presence of autocorrelation and heteroscedasticity in the residuals from the AR(1)-GJR-GARCH(1,1) model, the following paragraphs provide an analysis on the parameter estimates from the AR(1)-GJR-GARCH(1,1) steps, please look in Table 6 for parameter estimates.

Asset	AR(1)		GARCH(1,1)			GJR
	Constant	Beta	Constant	GARCH	ARCH	I
SXXE Index - Euro STOXX 600	0.00120	-0.03780	0.00004	0.83030	0.00000	0.21840
SX5E Index - Euro STOXX 50	0.00071	-0.06590	0.00004	0.84190	0.00000	0.20630
SGHIYIE FP Equity - High Yield Bonds	0.00130	0.24100	0.00000	0.81660	0.07860	0.20960
FIDEBST LX Equity - Low Risk Bonds	0.00049	0.02640	0.00000	0.90160	0.07320	0.00690

Table 6: Results for AR(1)-GJR-GARCH(1,1) Estimation (Source: Own calculations)

8.1.1 Interpretation of AR(1)

The β's in the AR(1) model are of importance to our analysis. The first interesting finding is that the β's are negative for the two equity indices and positive for the two bond structures. The negative β's of the equity (bond) indices represent a negative (positive) autocorrelation, which in turn suggests that positive (negative) developments in asset returns are predominantly followed by negative (positive) returns in the subsequent period. The magnitude of the βs are also worth a glance. The β-magnitude on the two equity indices and the low risk bond fund represent the fact that despite the presence of autocorrelation or carry-over effect from previous periods, the numerical impact is rather limited. The finding for high yield bonds is different. For this asset, the beta has a magnitude of 0.241, interpreted as a 1% increase in the return of high yield bond fund in previous period, results in 0.241% increase in this period, which is rather large compared to the other assets in the portfolio. This finding is in line with the idea of high yield bonds having a low liquidity, resulting in previous periods explaining a large share of the return level in the following period, all else being equal.

8.1.2 GJR-GARCH Model

Generally, the interpretation of the GJR-GARCH model is that we allow the conditional variance $\sigma_{u,t}^2$ to depend on one lag of squared residuals, u_{t-p}^2, one lag of conditional variance, $\sigma_{u,t}^2$ and a leverage effect, Iu_{t-p}^2. The model allows us to correct for presence of heteroscedasticity, also known as non-constant variance. We need to perform this procedure in order to ensure that our return residuals applied in the process of distribution modelling are i.i.d. To estimate the model, we apply the Maximum Likelihood estimation method under the assumption of variance stationarity which was confirmed in the ADF test. Additionally, we require $\omega > 0$, $\beta_i \geq 0$, $\gamma_i \geq 0$, and $(\delta_i + \beta_i) \geq 0$ as we need to ensure a positive conditional variance estimation. We can affirm that all parameter estimates are positive, which validates that we comply with the assumptions of $\omega > 0$, $\beta_i \geq 0$, $\gamma_i \geq 0$, and $(\delta_i + \beta_i) \geq 0$.

When interpreting the parameter estimates, we investigate the relationship between the current fitted conditional variance and the long-term average effect of market volatility. This is captured in the constant, ω, the volatility during the preceding period u_{t-p}^2 (ARCH effect), the fitted conditional variance from the model in the previous period σ_{t-q}^2 (GARCH effect) and a leverage effect Iu_{t-p}^2 (GJR effect).

8.1.2.1 Validation of GJR-Element in GJR-GARCH (1,1) Model

Before continuing to analyze the model in further details, we find it appropriate to validate the efficiency of the specified GJR-GARCH(1,1) model in relation to the traditional GARCH. As touched upon in the methodology we apply this method based on the findings by Rosenberg and Engle (2002), who document that the extended model outperforms the GARCH model when applied to financial time series. We find the idea of including the leverage GJR-term valuable, hence we perform a log-likelihood ratio test on the unrestricted model GJR-GARCH(1,1) and compare the likelihood score to the restricted GARCH(1,1) model, please see Table 7.

The log-likelihood ratio test for the two equity indices and the high yield bond fund shows similar results. We are able to reject the null hypothesis of these three assets at the 99% confidence level and conclude that the GJR-GARCH(1,1) provides a significantly better fit for modelling the return series.

The result for the low risk bond provides a different result. The log-likelihood ratio test generates a p-value of 0.8, which is highly insignificant. This finding is clearly supported by Table 6 where we observe the magnitude of the GJR parameter is close to zero.

Looking into the underlying statistics of the GJR parameter in the GJR-GARCH modelling we see an insignificant t-test of the leverage effect, which confirms the results of the log-likelihood ratio test. However, the inclusion of the GJR parameter reduces the ARCH coefficient in the unrestricted model with 5%. This leads us to suspect that multicollinearity influences the ARCH parameter and the leverage effect, which is a reasonable observation as both the GJR and ARCH parameter seek to explain the effect of the past residuals. Multicollinearity cause a higher magnitude of the standard errors, which makes it difficult to obtain statistically significant results regardless of the explanatory contribution of each variable (Wooldridge, 2009). Based on the behavior of the ARCH coefficient in the unrestricted model we conclude that the GJR parameter is of relevance, as the remarkable difference in the ARCH coefficient between the unrestricted and restricted model leads to suspicion of a biased ARCH parameter estimate in the restricted GARCH model. In sum, we are confident that the leverage effect largely contributes with a significant level of explanatory power for all four assets and the GJR-GARCH(1,1) model is applied in favor of the restricted GARCH(1,1) model.

Log-Likelihood Score	**Model**			
Asset	**GJR-GARCH(1,1) Unrestricted Model**	**GARCH(1,1) Restricted Model**	**P-value**	**Reject?**
SXXE Index - Euro STOXX 600	1.87300	1.85800	0.00	Yes
SX5E Index - Euro STOXX 50	1.82200	1.80700	0.00	Yes
SGHIYIE FP Equity - High Yield Bonds	2.91200	2.90500	0.00	Yes
FIDEBST LX Equity - Low Risk Bonds	3.71100	3.71100	0.80	No

Table 7: Log-Likelihood Test for Relevance of GJR-parameter (Source: Own calculations)

Having validated the value of applying a GJR-GARCH model, rather than the traditional GARCH-model the analysis continue to assess the findings of the AR(1)-GJR-GARCH(1,1) model.

8.1.3 Interpretation of GJR-GARCH

In general, the constant provides insight to the long-term market volatility and therefore we find our low constant estimations slightly surprising. Usually, GARCH-models on asset returns find constant estimates ranging between 0.05 and 0.1, where 0.05 indicates rather stable market volatility and 0.1 indicates that volatility tends to have large spikes and is in general unstable (A. McNeil et al., 2010). The largest constants are observed on the two equity indices, however all constants are estimated to take on a value very close to zero, whereby indicating that the average long term volatility does not strongly influence the conditional variance.

Typically, low constants are associated with higher GARCH effects. The observation of most interest to us is how fast the volatility decays after a shock, this is represented by the sum of the ARCH and GARCH parameter. Notice that for all four assets the coefficients on ARCH and GARCH sum up to a number less than one, which is required to have a mean reverting variance process. Specifically, strong persistence in volatility is found if the sum of the GARCH and ARCH effect equals one, which is not the case for any of our assets. This leads us to the finding that we see slight indication of non-constant and persistent volatility in the data series, however the data series do not seem to be influenced by *strong* volatility persistence.

Considering the magnitudes of the estimates presented in Table 6, the estimations of the conditional variances seem to be primarily influenced by the GARCH and the leverage effect. As the ARCH effect is estimated close to zero for all four assets it indicates that the explanatory contribution of the volatility in the preceding period is dominated by the GARCH and leverage effects. The large leverage effects for the two equity indices and the high yield bond fund indicate asymmetry in the sensitivity of the conditional variance to changes in market conditions. Specifically, the positive GJR-parameters imply that volatility increases proportionately after negative shocks. Including the leverage effect in the regression for the two Euro stocks seems to completely control for the general explanatory power of the past residuals. This is illustrated by the fact that the ARCH coefficient is zero for these two asset. Thus, it is only the negative residuals which contribute with any relevance in explaining future variance.

In order to validate whether the suggested AR(1)-GJR-GARCH(1,1) model is sufficient to generate residuals which are corrected for both autocorrelation and heteroscedasticity effects, the following section investigates ACF plots for the standardized and squared standardized residuals for each asset. We test for presence of autocorrelation and heteroscedasticity using the Ljung-Box test statistic and ACF plots.

8.1.4 Testing for Autocorrelation and Heteroscedasticity in Standardized Residuals

The residuals from the AR(1)-GJR-GARCH(1,1) and the estimated conditional variance in the GJR-GARCH model are used to generate standardized residuals, which we utilize in the EVT-modelling, given that they fulfill the i.i.d. assumption. The standardized residuals are calculated by dividing each residual with the conditional standard deviation of the residual. To test the ability of generating i.id. data series with the AR(1)-GJR-GARCH(1,1) we investigate the ACF plots of the standardized and squared standardized residuals for each asset, please see Figures 11, 12, 13 and 14:

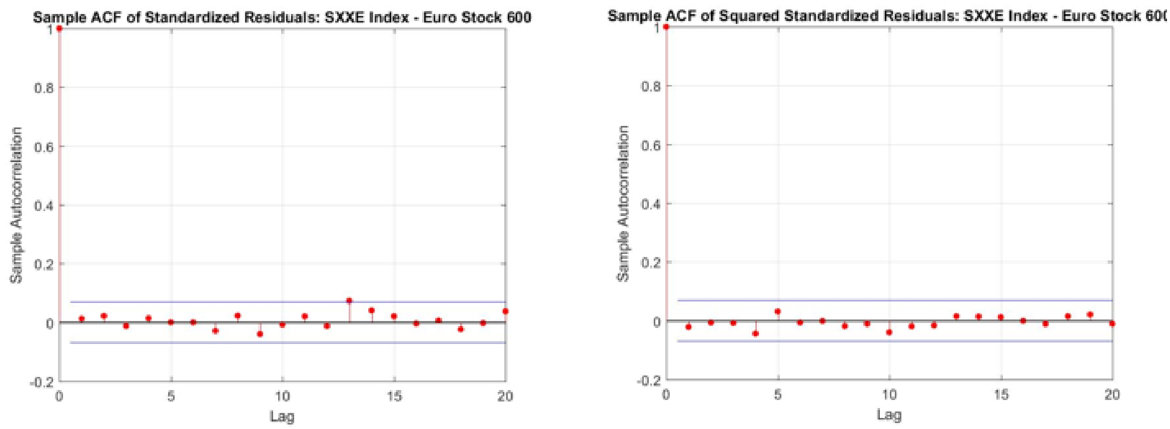

Figure 11: ACFs for Std. Residuals and Sq. Std. Residuals of Euro STOXX 600 (Source: Own calculations)

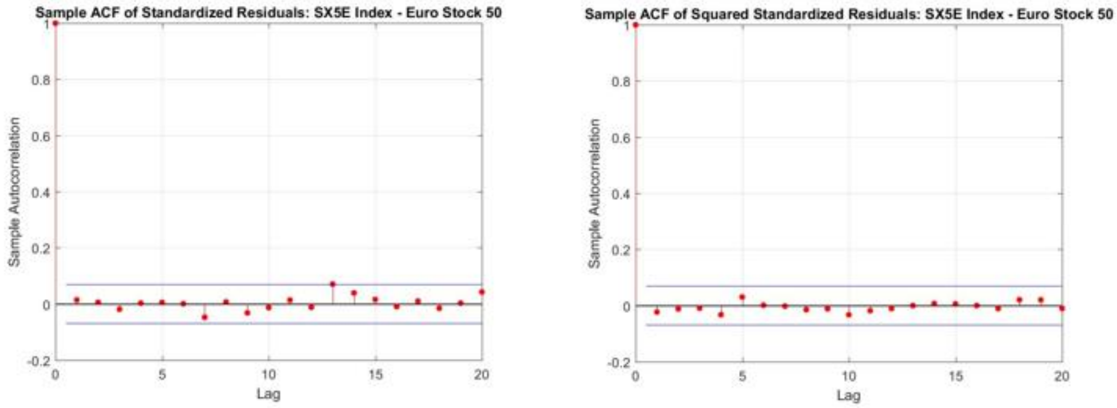

Figure 12: ACFs for Std. Residuals and Sq. Std. Residuals of Euro STOXX 50 (Source: Own calculations)

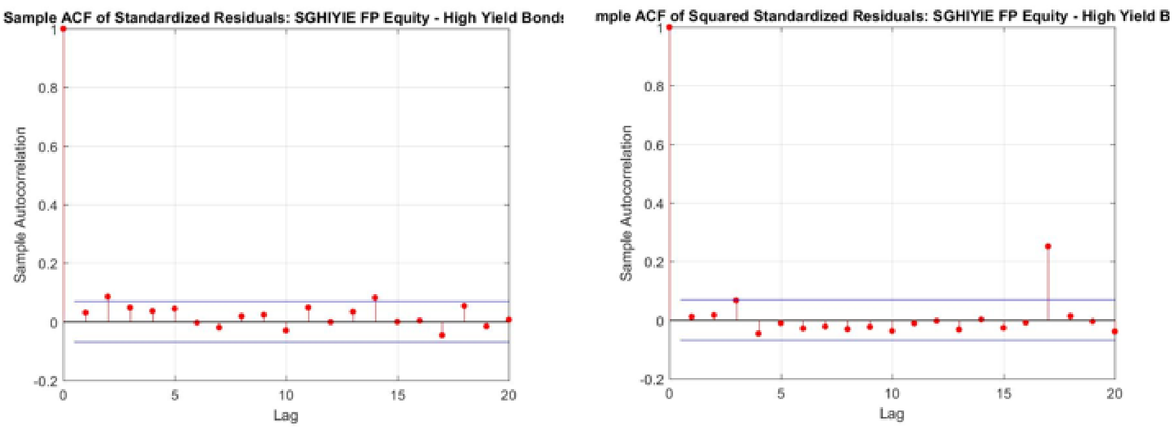

Figure 13: ACFs for Std. Residuals and Sq. Std. Residuals of High Yield Bonds (Source: Own calculations)

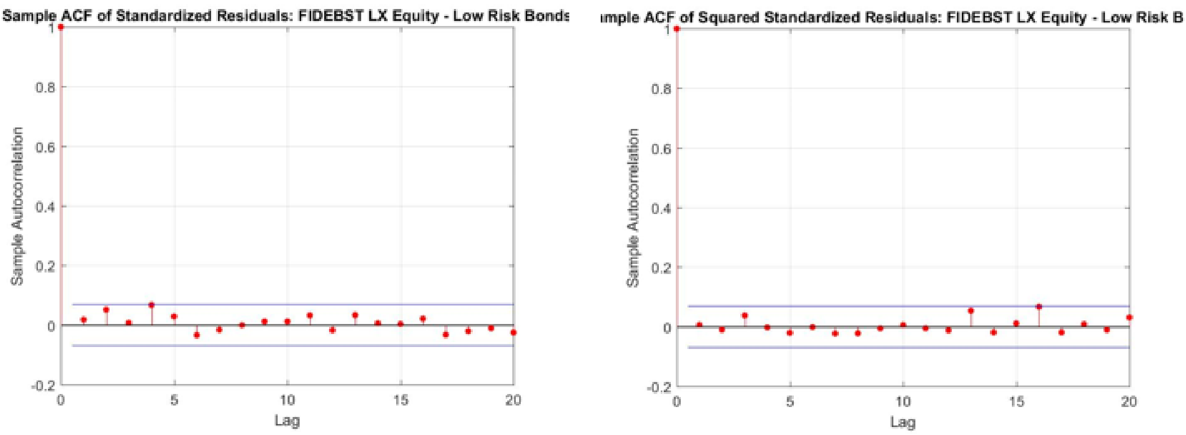

Figure 14: ACFs for Std. Residuals and Sq. Std. Residuals of Low Risk Bonds (Source: Own calculations)

At first glance, the ACFs for both the standardized residuals and the squared standardized residuals for all four assets give one the overall impression that we have succeeded in removing both autocorrelation and heteroscedasticity effects from the time series data. However, looking with a more detailed eye, the ACFs of the standardized residuals of two equity indices show a significant spike at lag 13, please see Figure 11 and Figure 12. But, as the effects look just barely significant and appear at rather distant lags we choose to ignore them in the analysis going forward.

The high yield bond fund still calls for some final attention. The ACF for autocorrelation shows several spikes at the border for being significant at the 95% confidence level and the 17th lag in the ACF for heteroscedasticity is also outside the confidence band, see Figure 13 . Therefore, in order to establish higher certainty around whether the residuals used in the semi-parametric EVT distribution modelling are corrected for autocorrelation and heteroscedasticity, we apply the Ljung-Box (LB) test statistic, please see Table 8.

Asset	LB Test for Autocorrelation		LB Test for Heteroscedasticity	
	P-value	Autocorrelation	P-value	Heteroscedasticity
SXXE Index - Euro STOXX 600	0.91990	No	0.99820	No
SX5E Index - Euro STOXX 50	0.93280	No	0.99980	No
SGHIYIE FP Equity - High Yield Bonds	0.14270	No	0.00000	Yes
FIDEBST LX Equity - Low Risk Bonds	0.87100	No	0.94890	No

Table 8: Ljung-Box Test for Presence of Autocorrelation and Heteroscedasticity (Source: Own Calculations)

Recall, that the Ljung-Box test is used to test the null hypothesis of independence. The LB tests, performed on the first lag of each asset, affirm our findings in the ACF analysis of the standardized residuals, namely that we have succeeded in removing autocorrelation effects for all four assets.

However, the LB tests for heteroscedasticity reveal that we can reject the null hypothesis of independence for squared standardized residuals of the high yield bond fund which implies that the data series for this asset still seems to exhibit characteristics of heteroscedasticity. A revisit into the theoretical literature, highlighted that the Ljung-Box test for time series independence is sensitive to extreme events or outliers. As we want to ensure that the residuals are i.i.d. we investigate the GJR-GARCH model for the high yield bond fund. We find the very large spike on 17th lag in the ACF of squared standardized residuals to be the most plausible explanation for the LB test statistic to show signs of heteroscedasticity. We therefore confidently conclude that all four standardized residuals series can be regarded as approximately i.i.d. Hereby we continue the distribution modelling in the following paragraphs using a semi-parametric approach under Extreme Value Theory.

9 EXTREME VALUE THEORY – DISTRIBUTION MODELLING

This analysis incorporates EVT as an analytical tool on the quest to correctly approximate the empirical return distributions of the four assets. EVT has proven to be one of the most powerful statistical tools used to model extreme deviations from the mean. We apply the theory as a semi-parametric framework, which implies that each distribution is divided into three sections: The lower tail, interior and upper tail. We fit the interior part with a smoothed Kernel distribution based on the historical data as this helps us smooth to out noisy observations in the interior. The tails are fitted separately with a Generalized Pareto Distribution. To recall, the main advantage of fitting the tails separately is that it allows us to account for asymmetry between the tail distributions. Former research finds that this method have strong explanatory power when working with financial time series (Rachev & Racheva-Iotova, B. Stoyanov, 2010; Sheikh & Qiao, 2010). As this thesis is mainly concerned with modelling of portfolio risk in terms of Value at Risk and Expected Shortfall, the following part of the analysis focuses mainly on assessing the modelling of the left distribution tails. The section will naturally include a full assessment of the whole modelling for each asset.

9.1 TAIL MODELLING

When modelling the distribution tails, we apply the Peak-over-Threshold method from EVT. As a first step, we need to determine the tail fraction. We have dealt with the well-known and ambiguous trade-off between applying a tail fraction too high and thereby risk including observations, which should have been part of the interior of the distribution, leading to inaccurate GPD parameters versus setting a tail fraction too low, whereby not having enough observations to ensure a robust tail estimation.

Given the high level of complexity characterizing the econometric framework shaping this thesis, we have decided to apply a common tail fraction for all assets. The advantage of this decision is that it ensures ease of comparison between the findings for each distribution throughout the analysis. Naturally, this comes at the expense of potentially operating with wrong tail fractions for one or more of the assets. Later sections in the analysis will however underline the good fit between the modelled non-normal distributions and the distribution of the empirical data, hence we consider a common tail fraction of high value in terms of the consistency and comparability of the analysis going forward.

Identifying the correct tail fraction is a challenging task. Based on recommendations presented in similar studies we have experimented with tail fractions surrounding the level of 5% (Balla, Ergen, & Migueis, 2014; Hsu, Haung, & Ntoko, 2013). This approach is further supported by Neftci (2000) who suggests a tail fraction equal to the confidence quantiles for the risk measures. Several methods are recommended for determining the tail fraction, however in practice, we are recommended to model based on at least 30 observations in order to ensure a certain level of estimation validity and quality (Stock & Watson, 2012). Our process of trial and error leads us to decide upon a tail fraction of 4.8%. This decision is based primarily on the fit of the distributions relative to the empirical data series. A tail fraction of 4.8% provides us with 40 observations for each tail given that the dataset contains 834 weekly return observations (833 after first differencing).

Given a tail fraction of 4.8%, we fit the tails using the GPD. Recall that the GPD is characterized by two parameters, the *shape* and *scale* parameter, please see Table 9 for estimation results. As the name clearly indicates, the shape parameter determines the shape of the distribution. A positive shape parameter, $\xi > 0$, implies that the distribution exhibits heavy tails, whereas a negative shape parameter equates thin tails. Looking at Table 9, we see that once again the two equity indices are behaving similarly. In line with our findings of negative skewness and rather large excess kurtoses, we see that thin upper tails and heavy lower tails characterize the two fitted equity tails. Appendix I shows a representation of the fit between the estimated and empirical lower tail. For both Euro STOXX 600 and Euro STOXX 50, we see a high level of ability to accurately approximate the empirical distribution, which leads to strong support of reliability of the distribution fits when estimating tail risk.

The high yield bond fund is modelled with heavy tails with regard to both upper and lower tail. This appears to be in line with both our expectations and the empirical findings from the preliminary analysis. To recall, the high yield bond fund was found to have the highest excess kurtosis with comparably low standard deviation indicating that the risky events in this distribution tend to be less frequent but more extreme and severe in nature compared to both the equity indices and the low risk bond portfolio. Appendix I lends further support to the power of replicating the empirical behavior as we see a good fit between the estimated non-normal lower tail and the empirical tail.

Moreover, for the low risk bond portfolio we see a negative shape parameter for the lower tail and a positive upper tail shape parameter. This is also in line with our previous finding of this asset having a positive skewness and thin tails. The characteristic of a thin lower tail is portrayed in Appendix I, where we see that the low risk bond portfolio clearly has thinner tails than the three other assets.

Asset	Lower Tails		Upper Tails	
	Shape	Scale	Shape	Scale
SXXE Index - Euro STOXX 600	0.3085	0.4615	-0.2779	0.3601
SX5E Index - Euro STOXX 50	0.3857	0.4120	-0.1212	0.3642
SGHIYIE FP Equity - High Yield Bonds	0.2059	0.7628	0.2514	0.3480
FIDEBST LX Equity - Low Risk Bonds	-0.0946	0.4946	0.3989	0.3280

Table 9: Shape and Scale Parameter Estimates from EVT Tail Modelling (Source: Own calculations)

The scale parameter defines how spread out the data is, here a large scale parameter stretches the distribution and oppositely a smaller scale parameter shrinks the distribution (Jondeau et al., 2007). Comparing the scale parameters between upper and lower tail for the two equity indices we observe that the lower tails are more stretched out than the upper tails. Combined with the finding that the lower tails are characterized as heavy and upper tails as thin, we find the tail distribution modelling to approximately exhibit the characteristics of negative skewness and heavy lower tails as found in the preliminary analysis.

Looking at the high yield bond fund, it is especially noteworthy that the shape parameter of the lower tail is remarkably below the shape parameter for the two equity indices, whereby indicating that the equity indices have heavier tails than the high yield bond fund. At the same time, we also observe a much higher scale parameter for the high yield bond fund. Combined, this indicates that there are less very extreme events in high yield bonds, but that the range for these events is much wider and thereby have a more severe impact on the investor. This distribution behavior fits the observations from the preliminary analysis where we linked the magnitude of the kurtosis with the standard deviation.

The low risk bond portfolio exhibits thin lower tail and heavy upper tail, which fit our general findings in the preliminary analysis. However, we find one puzzling observation. Namely that the thin lower tail is characterized by a higher scale parameter than the upper tail, implying that the lower tail is more stretched out. This is conflicting with the findings regarding the characteristics of the distribution until this stage. We are hereby in a situation where we need to question the appropriateness of applying a uniform tail fraction to all assets, as the low risk bond portfolio clearly exhibited lower distribution dispersion and thin tails. In the case of the low risk bond portfolio, the number of observations for tail modelling may have been insufficient in this specific case. An investigation of the fit between the Pareto tails and the residuals reveal that the tail distribution modelling of this asset does perform poorest among the four. The main explanation being that we see a tendency to normality in the beginning of the tail area, which is not reflected in the GPD fit. Nonetheless, we assess that the general tail fit is still acceptable for continuing the portfolio risk estimation, which builds on the more extreme tail area.

In addition to assessing the scale and shape parameter we conduct a visual assessment of the fit of the GPDs. We graphically represent the distributions in both an empirical cumulative distribution function of the tail exceedances and the cumulative distribution function of the entire GPD: please see Appendix I and Appendix II. Applying a tail fraction of 4.8% leads to distribution fits, for both for upper and lower tails, which follow the empirical data on exceedances and we can thereby conclude that the GPD model is a proper choice of modelling for these data series.

The following paragraphs assess the quality of the fit for the entire distribution for each asset. Despite the focus of improving the accuracy of risk metrics, whereby encouraging a focus on the lower tails, we use the following section to underline the quality of the individual distribution estimations conducted until this point in the analysis.

9.2 Assessment of Distribution Fit

To assess the overall fit of each distribution we investigate the graphical representations of the standardized residuals versus the fitted EVT distributions, please see Figure 15. The blue x's represent the standardized residuals from the AR(1)-GJR-GARCH (1,1) and the red dashed line represents the fitted EVT distribution based on the semi-parametric approach. In general, a good fit is found when the red dashed line follows the blue x's as these represent the true empirical data series. To emphasize the divergence from normality we include a representation of the normal distribution in shape of the black dashed line. Overall, one can clearly see how the data is diverging from the normal distribution in both the upper and lower tails. This is especially true for the two equity indices and the high yield bond fund. We confidently argue that modelling the distribution tails with advanced statistical tools has allowed us to account for non-normality and the value of such practice relies on our finding that we are able to approximate the empirical behavior of the asset returns more accurately than assuming normality.

A detailed graphical inspection draws our attention to a potential estimation problem. The problem is particularly pronounced in the EVT distribution of the Euro STOXX 50, as it seems to be under influence by one extreme outlier. This outlier is observed far out in the lower tail with long distance to the second most extreme residual observation. This leads the curvature of the EVT tail to flatten out, instead of converging towards to lower horizontal axis, whereby the low probability events in the left tail are estimated to occur at levels far beyond the maximum negative empirical observation. This finding is of outmost importance to the estimation and the evaluation of both VaR and ES later in the analysis as it may result in unrealistically large portfolio losses. However, the outlier event occurs with extremely low probability and the severity of the problem should therefore not be dramatically overestimated. A similar behavior is found in the fit of the Euro STOXX 600, however here the extreme outlier is found around a lower standard deviation and given the steepness of the curve we expect the tail to converge to probability of zero rather shortly after this observation point. This finding is discussed further in chapter 12 "Discussion".

The issue of few very extreme observations is mitigated in the case of the high yield bond. This is because this asset has the highest scale parameter, implying that this is the most spread out tail distribution. Hence, the high yield bond fund provides more very extreme residual observations to assist us in the process of modelling the outer end of the EVT Pareto tail, whereby we are more certain about the accuracy of the tail shape estimation.

Finally, looking at the lower tail of the low risk bond portfolio, we see that the fitted EVT Pareto tail, to a much higher extend than the three other assets, take shape nearby the lower tail of a normal distribution and that the main characteristics of non-normality is found in the upper tail. As we can see graphically, the lower tail is thin compared to the three other assets and the fitted tail distribution converges to probability zero shortly after the last residual observation observed around 4 standard deviations out in the distribution tail. As the lower tail approximates the normal distribution rather well it leads us to question value of accounting for non-normality for this asset in relation to risk estimation. We acknowledge that the main purpose of this specific paper is to create and test a framework, which is able to account for non-normality in risk metrics and here we find evidence that assuming normality of the lower tail may not be as faulty as we have found it to be for the other assets. However, as we do not want to neglect the value of our findings in relation to the overall portfolio and investment optimization, we want to underline that a positive spillover effect from modelling the entire distributions correctly is that it will allow asset managers to optimize asset allocation based on a valid risk return tradeoff. Hence, accounting for non-normality characteristics throughout the entire distribution is of value both to researchers solely interested in investment risk and to researchers seeking to improve portfolio allocation methodology.

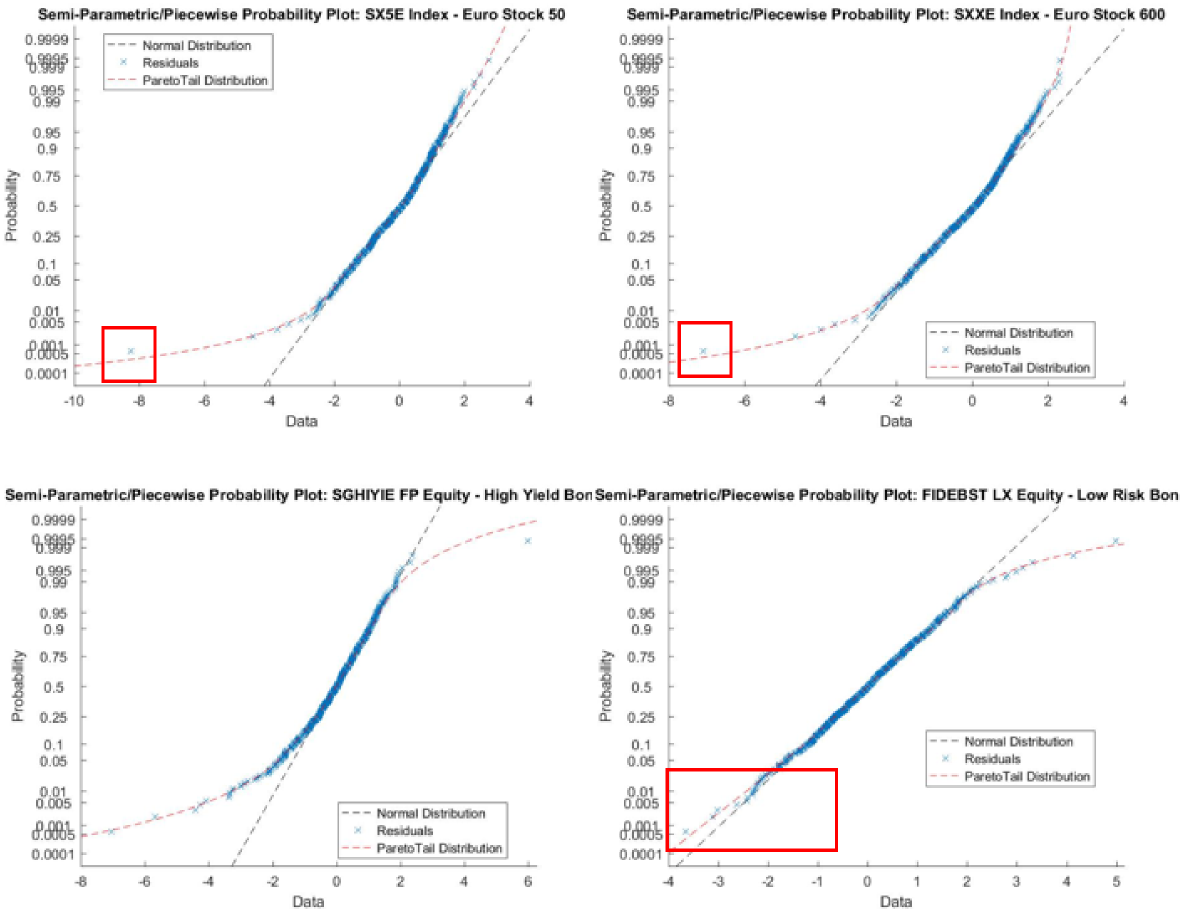

Figure 15: Semi-Parametric EVT Fitted Distributions for each of the Four Assets (Source: Own calculations)

Overall we see that all four data series exhibit varying behavior between upper and lower tail, whereby supporting our analytical method of modelling the tails individually. The challenge of choosing the right tail fraction in EVT is unavoidable and the solution we operate with is to pay attention the approximation of the estimated distributions relative to the empirical data points, whereby we can identify potential sources of noise or bias.

The following section deals with the non-normality stemming from correlation breakdowns which is predominantly observed during times of market distress. To recall the findings from the preliminary analysis, we observe a convergence effect between the high yield bond vs. the Euro STOXX 600, and Euro STOXX 50 with a magnitude of 0.18 and 0.19, respectively. This implies that the diversification effect in portfolio investments are overestimated if the investor does not account for correlation breakdowns, hereby underlining the importance and relevance of the following section.

10 Distribution Modelling - Copula

Based on the findings of non-normality in the preceding sections we now move to the use of copulas, which enable us to capture and incorporate the stylized fact of non-normality in relation to correlations and interdependencies among the portfolio assets. Specifically, we elaborate on the estimated parameters of the student's t copula and provide a graphical illustration of the effect of this correlation tool.

As mentioned in chapter 6.6 "Correlation", we utilize copula theory to model the dependency among the four assets. Specifically, we apply the student's t copula as this model had proven superior in previous studies, mainly due to its ability to incorporate the characteristic of heavy tails. The higher probability density in the tails is modelled through adjusting the DoF parameter. In our model we estimate a degrees-of freedom parameter of 17.53 with a rather large confidence interval [8.89, 26.17]. The wide confidence interval results due to high standard errors in the copula estimates. The high variance of the DoF estimate is a typical side-effect of increasing the complexity of a model (Esch, 2010; Wooldridge, 2009). As the confidence interval ranges from 8.89 to 26.17 it implies that the joint distribution tails can be characterized by both highly heavy tails ranging to an almost normally distributed model.

Furthermore, the rather wide confidence interval cause difficulty in terms of exact inferences of the true DoF. Therefore, one has to be careful when applying our results to a different setting or portfolio combination. However, despite the large confidence interval, the DoF estimate is still an asymptotic unbiased estimate of the true DoF for the four assets at hand. One way to lower the standard error and thereby produce a more efficient confidence interval is by increasing the number of observations (Stock & Watson, 2012). However, when comparing the copula DoF estimate with the estimated DoF of the marginal distributions and the results of the previous analysis, the prediction of 17.53 seems reasonable. Firstly, the estimated DoF of the copula is considerably larger than the estimated DoF of the marginal distributions of the high yield bonds and the two equity indices, which range from 4.75 to 11.39. The higher DoF of the copula results due to the diversification effect of the four indices, which naturally lowers the implication of a single extreme event. Hereby, the student's t copula correctly incorporates the diversification effect, while at the same time allowing for a larger concentration of events in the tail regions than the normal distribution predicts.

Secondly, the preliminary analysis provided clear evidence of non-normality in the assets returns. Specifically, given the strong correlation between the equity indices and the high yield bonds during high volatility periods we expected the DoF estimate of the copula to be considerably below 30 as this indicates a higher density allocation to the tails than provided under normality. On the contrary, the correlation effect between the low risk government bonds and the three remaining assets became even more negative during financial instability, whereby providing increased diversification effect during times of market distress. In sum, we see that the higher joint DoF compared to the marginal DoF represents diversifications effects and as the DoF diverges from the DoF under normality, we assess the joint distribution to represent the characteristic of heavy tails. Thereby, we evaluate that a DoF of 17.53 appears to be appropriate and representative for the empirical joint distribution.

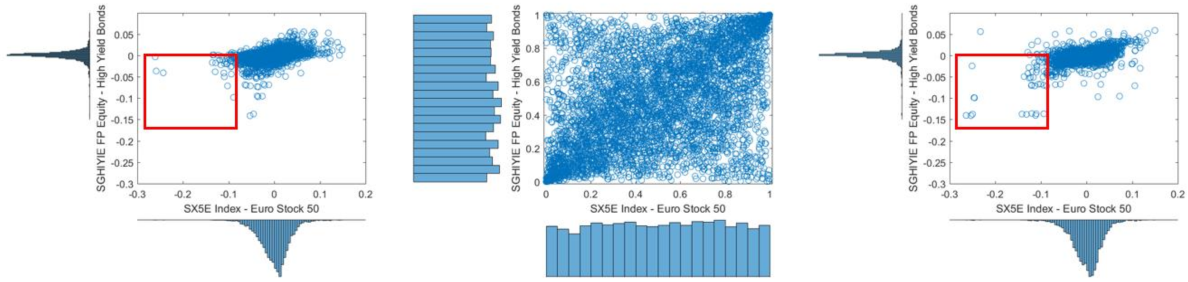

Figure 16: Example of Copula Transformation - High Yield Bonds and Euro STOXX 50 (Source: Own calculations)[10]

The effect of the t copula is illustrated in the graphs above. Here the traditional Pearson's correlation between the portfolio of high yield bonds and the equity portfolio Euro STOXX 50 is illustrated in left pane of Figure 16. These two assets have a positive correlation of 0.53, which is represented by the slightly positive slope. The middle pane of Figure 16 illustrates the univariate transformation. Here the return data from each of the two indices is transformed to univariate values [0, 1] based on their individual marginal distribution. These data points are applied in the student's t copula estimation. We simulate 10,000 outputs based on the copula estimation and subsequently fit them to the joint distribution, which is illustrated in the right pane of Figure 16. From this figure one can observe how the student's t copula incorporates the higher likelihood of joint extreme events, which considerably increases the number of observations in the lower left corner of the figure, indicated by the red square. However, this figure also illustrates the main limitation of the student's t copula, which is unable to account for negative skewness in modelling the joint distribution. This is reflected by the equal increase in density of observations in the upper right corner.

In sum, this section has established that the student's t copula estimates a joint distribution model with thicker tails than exhibited in the normal distribution. The findings are in line with the preceding findings of our analysis. The next section applies the student's t copula in order to simulate a robust

[10] For illustrative purposes the degrees of freedom parameter is set to 2.

estimate of the joint cumulative distribution function, which we utilize when aiming at generating reliable risk estimates of the portfolio.

10.1 Monte Carlo Simulation

This section utilizes the results presented in former parts of the analysis. We apply the Monte Carlo simulation technique in order to obtain the risk measures of Value at Risk (VaR) and Expected Shortfall (ES) for an equally weighted portfolio. The simulations are performed both under the assumption of normality and non-normality. In order to analyze the sensitivity of the risk measures to changes in investor risk preferences, hence portfolio weights, we supplement the risk measures from the equally weighted portfolio with risk measures from portfolios representing a risk averse and risk seeking investor profile.

10.1.1 Monte Carlo

In order to calculate the risk measures based on non-normality assumptions, we perform the Monte Carlo simulation based on the GARCH-EVT-Copula modelling. Specifically, to represent data for one year we simulate 52 dependent uniform values for each of the four indices based on the parameters of the student's t copula. The uniform values are transformed back into standardized residuals using the inverted marginal distributions which were calculated using EVT. Based on the AR(1)-GJR-GARCH(1,1) parameters we reintroduce the autocorrelation and heteroscedasticity from the historical data. This enables us to obtain simulated return data that reflects the empirical data behavior. In order to obtain a robust estimate of the inherent portfolio risk of the equally weighted portfolio, the above steps are repeated in a total of 10,000 times. This implies that a total of 520,000 observations are simulated.

As we want to investigate the improvement of accounting for non-normality, we likewise perform a simulation for a portfolio with same weights as applied above under the assumption of a Gaussian distribution. Here Markowitz's matrix calculation is applied to estimate the mean return and variance of the portfolio. These estimates are used in an inverted normal cumulative distribution function to simulate a one-year return plot, where the uniform values are randomly generated. This procedure is also repeated 10,000 times in order to generate results of equivalent robustness. The two simulations enable us to shed light on the potential disparities that arise when applying a non-normal distribution modelling, compared to a Gaussian setting. The results of each simulation are briefly evaluated in the section below.

10.1.2 Simulation Results

A histogram for the distribution simulation is presented in Figure 17. The impact of the two distinct modelling approaches is clearly illustrated in the distribution patterns. A point worth noticing is that even though the copula model is unable to directly account for skewness in the joint distribution, we are still able to represent the negative skewness found in the marginal distributions as this effect is indirectly introduced to the joint distribution through the EVT process. Furthermore, the joint distribution depicts the characteristic of a fairly thin upper tail. This is in line with the estimated scale and shape parameters, here we found that the two equity indices had very thin upper tails and this effect results in the lower level of simulated positive results for the non-normal equally weighted portfolio. The characteristics of heavy and long lower tail and thin upper tail is clearly depicted in the histogram of the GARCH-EVT-Copula returns shown in Figure 17. Specifically, the non-normal portfolio exhibits a negative skewness of -1.22. This is demonstrated in the graph where the negative return spikes slowly fade out and a clear concentration of events can be spotted around losses of 50%. The portfolio has a kurtosis of 3.99, which is substantially lower than the individual assets. However, this matches the DoF-parameter estimated in the student's t copula calibration and can be explained by the diversification effect. The portfolio based on the normal distribution has by construction a more symmetric distribution with few extreme observations.

Based on the visual check of the simulation, we conclude that clear differences exist between the two approaches. Further, at first glance the incorporation of fat tails and negative skewness in the marginal distributions tends to atone for the lack of asymmetry in the modelling of the joint distribution, as the GARCH-EVT-Copula model exhibits both negative skewness and asymmetry in the tails. The next section of the analysis examines the risk for each simulated portfolio and compares the results.

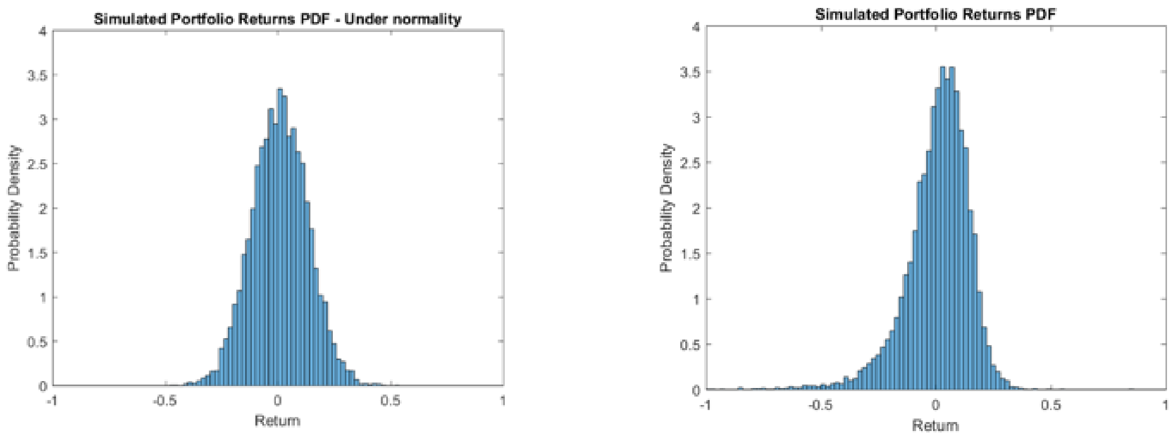

Figure 17: Simulated Portfolio PDFs for Normal and Non-Normal Framework (Source: Own calculations)

10.1.3 Risk Measures

To recall, both Value at Risk (VaR) and Expected Shortfall (ES) are utilized in order to obtain a more holistic insight to the intrinsic risk of the portfolio. VaR indicates the magnitude of the percentile risk and ES sheds light on the conditional expected risk when a given threshold is exceeded. The results of the analysis are presented in Table 10. In line with our expectation, we find that the GARCH-EVT-Copula model produces risk estimates, which are all substantially higher than the estimates obtained under the assumption of normality. A deeper comparison of the results of the two models is presented in the following sections.

Risk Measure	VaR		ES	
	95%	99%	95%	99%
Non-Normal	-0.2400	-0.4400	-0.3600	-0.5900
Normal	-0.1900	-0.2800	-0.2600	-0.3400

Table 10: Estimated Risk Measures for the Non-Normal and Normal Portfolio (Source: Own calculations)

10.1.4 Value at Risk and Expected Shortfall

Beginning with the VaR estimate it can be seen that the GARCH-EVT-Copula model produces VaR estimates, which are markedly higher than the estimates generated under normality. At the 5% quantile, the GARCH-EVT-Copula framework estimates an expected loss of 24% whereas the portfolio simulated under normality arrives at a VaR estimate of 19%. At this threshold the difference between the two models is modest. The absolute magnitude between two models is five percentage points and the relative difference is 22%. However, the deviations increase as we move further into the tail both in relative and absolute terms. At the 1% quantile the GARCH-EVT-Copula model estimates a 44% loss which is considerably higher than the 28% loss predicted under normality. Hence, the GARCH-EVT-Copula model predicts an expected loss which is 57% higher.

The results for ES fall in line with the estimates of VaR. Namely that the risk estimates of the GARCH-EVT-Copula model are in general of greater negative magnitude. At the 5% quantile the expected conditional loss is estimated to 36% of the initial investment and the normal model predicts a conditional expected loss of only 26%. Once again, the difference between the two models intensifies as the threshold moves further into the tail. At the 1% quantile the conditional expected loss by the GARCH-EVT-Copula model is estimated to 59% of the portfolio. This is 79% higher than the predicted conditional loss of the normal model which is 34%.

Given the goodness of distribution fits, we find that the non-normal framework is able to produce reliable and accurate risk measures. Specifically, we find that in terms of the 99% VaR the normality framework underestimates risk with 57% and in relation to ES, the risk is underestimated by 79%. The dramatic underestimation of portfolio risks should be of severe concern to investors as materialization of losses is of significantly greater frequency and magnitude than what is predicted under normality. To exemplify the impact of the findings we provide a simple calculation example taking starting point in the 99% confidence levels. Assuming an initial portfolio value of € 10,000,000, the one-year 99% VaR under normality is €2.8 million whereas the non-normal 99% VaR is €4.4 million, the difference amounts to €1.6 million in absolute value which corresponds to 16% of the initial portfolio investment. In the same way, the one-year non-normal 99% ES amounts to €5.9 million and the normal one-year ES is €3.4 million amounting to an absolute risk underestimation of €2.5 million equating 25% of initial portfolio investment. Assuming normality of the given portfolio applied in this paper 'hides' €1.6 million in risk. This prevents the investors from evaluating the true risk exposure and at the same time stops the investor from allocating capital according to his willingness to lose or risk budget.

Furthermore, the underestimation of risk implies that in the event of extreme losses it will require an extensive period of positive returns to reestablish the initial portfolio value i.e. imagine operating under the perception of an expected loss of €3.4 million, when in fact the true expected loss is €5.9 million. Extending the example above, these expected losses are provided with 99% confidence hence in the 1 out of 100 events it will leave a remaining portfolio value of €6.6 million and €4.1 million, respectively. Assuming an expected annual portfolio return of 8% it will take 5.5 years to regain initial portfolio value assuming normal loss distributions, whereas it will take 11.6 years assuming non-normal loss distributions, which is more twice the time. As this calculation example portraits, the impact of neglecting proper risk estimations may prolong far into the future.

The remarkable underestimation of risk under normality is striking and should not be neglected in future risk management practices, rather we hope to lend supportive arguments for implementation of more advanced risk models as this will provide investors with accurate information regarding their true risk exposure. Especially, investors with a given risk budget should be alerted by the scale of risk underestimation related to assuming normality as it may imply alternative asset allocation strategies and capital buffer considerations.

In order to further examine the performance of the GARCH-EVT-Copula framework to generate accurate distributions, hence reliable non-normal risk measures, the following section tests the adaptability of the framework and the sensitivity of the risk measures to an alternation of the allocation weights. In addition, the sensitivity analysis includes a comparison to the normal framework.

11 Sensitivity Analysis

In addition to the equally weighted portfolio, this section introduces a conservative and a risk-seeking portfolio. The conservative portfolio is modelled based on a 55% allocation in the low risk government bonds and 15% allocation to each of the three remaining assets. The risk-seeking portfolio is modelled with a one-percentage allocation to the low risk government bonds and 33% of the remaining capital in each of the three other assets[11]. First, the simulated distribution fit for each portfolio is presented in conjunction with the corresponding risk measures and subsequently the sensitivity analysis includes a comparison to the risk estimates under assumption of a normal distribution.

Before preceding to the sensitivity analysis, we elaborate on some considerations regarding data trimming. Specifically, the sensitivity analysis has revealed some concerning simulation results, which have led us to evaluate whether data trimming is required. Hence, the following section provides argumentation and considerations surrounding our choice of basing the final risk estimations on the distributions in their entirety.

[11] In order to not tampering with the original copula estimation, we include all four assets in each scenario. If only three assets or less were included in each portfolio, we would have had to estimate a new copula, whereby leading to incomparable risk results.

11.1.1.1 Considerations Regarding Extreme Simulations

Suspicion of overestimated risk measures leads us back to the issue of slow convergence in the tails of the EVT fitted distribution models. In the conservative portfolio, the maximum loss is 99.9% with a probability of one to 10,000. Despite the low probability of the event, we find a loss of 99.9% to be exceptionally improbable for an investment primarily placed in various European government bonds. Specifically, we are concerned with the fact that the most extreme simulations may lead to erroneously inflated risk measures, given that the most extreme simulations are unrepresentative for the empirical distributions. This concern provides incentive for considering data trimming where we disregard the most extreme observations. A general rule of thumb for handling outliers is proposed by Hansen, Madow, and Tepping (1983), they state that an outlier is defined as an observation, whose removal from the data series alters the estimate of a parameter by 10 percent or more. Considering the focus on distribution modelling of extreme events, it seems counter intuitive to remove the any observations from the tail at all, however to establish confidence in the risk estimates, we investigate the consequences of trimming the data series.

We mainly find the ten largest simulated losses to be of implausible character. Trimming these losses from the distributions has somewhat limited influence on VaR whereas the ES measure is more sensitive to the method, especially at the 99% confidence level. An example is the conservative portfolio: here data trimming of the ten largest losses alters the ES 95% quantile estimate from 23% to 22% and the ES 99% quantile estimate from 41% to 36%, whereas the estimates of VaR are almost identical. This example clearly shows the sensitivity of ES to both the magnitude and frequency of extreme negative observations.

We acknowledge that a more practical implementation of the GARCH-EVT-Copula model may favor a removal of the most extreme observations. However, as the aim of this work is to imitate and model extreme events, we have ultimately decided to keep the extreme observations within the sample. We are aware that this methodology may lead to overestimation of risk, especially for the ES 99% estimate. Nevertheless, the differences in risk estimates for the restricted versus the unrestricted distributions is only modest looking at the general picture thus supporting this choice of method. Furthermore, estimating both VaR and ES provides nuanced insight to the inherent portfolio risk and the combination of the two measures is a method of less sensitivity to outliers.

Based on the entire simulated distributions, the following two sections briefly evaluate the distribution fit for the conservative and risk-seeking portfolio-setup and present the corresponding risk measures.

11.2 Conservative Portfolio and Risk-Seeking Portfolio

The purpose of expanding the analysis to include risk estimations for a conservative and risk-seeking portfolio is to test whether the GARCH-EVT-Copula framework can estimate plausible and reliable risk estimates for portfolios with various asset allocation strategies. We take main interest in assessing the distribution fit in relation to theoretical and practical expectations and the corresponding risk measures.

11.2.1 Distribution Fits

When evaluating the ability of our framework to approximate the empirical pattern we take starting point the shape and structure of the distribution for the conservative and risk-seeking portfolio respectively. This yields insight to the performance of the model on a broader spectrum of investor profiles in relation to risk aversion. Specifically, the main motivation for forming a conservative portfolio is to investigate how the GARCH-EVT-Copula framework perform at modelling a portfolio representing a primary interest in low risk investments. Oppositely, the motivation for introducing a risk seeking portfolio is to gain insight to the modelling performance of a high-risk allocation, where skewness, excess kurtosis and heavy tails characterize the marginal distributions to a higher extent.

Figure 18 shows a graphical representation of the two portfolios and in addition we include a graphical representation of a normal distribution holding the asset allocation structure fixed. The following sections analyze the distribution modelling performance for each of the two additional portfolio structures and benchmark risk measures against the assumption of normality.

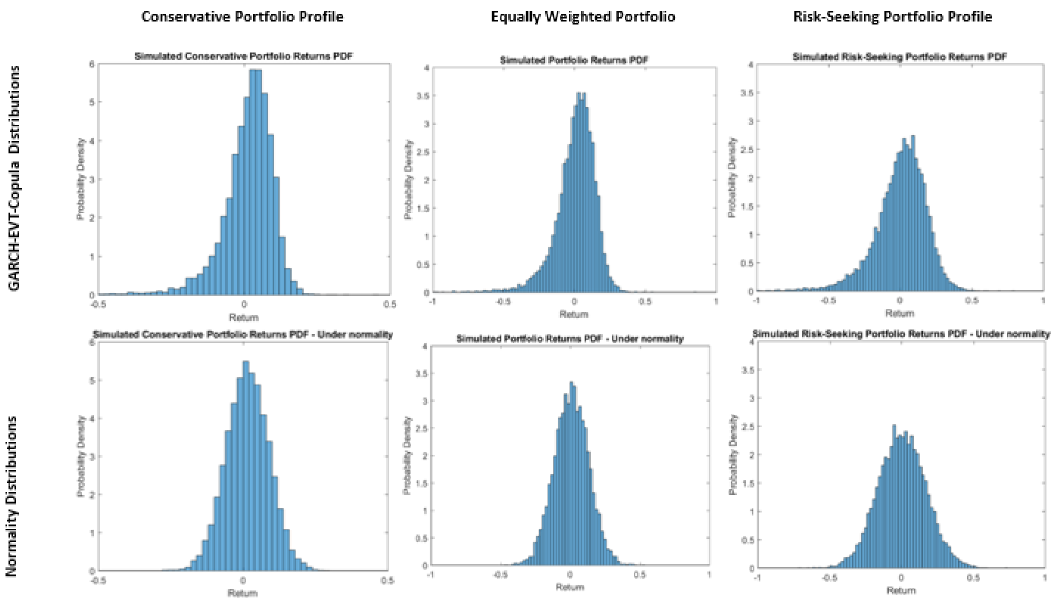

Figure 18: Overview of Simulated Portfolios - Conservative, Equally Weighted and Risk-Seeking (Source: Own calculations)

11.2.1.1 The Conservative Portfolio

Analyzing the conservative portfolio, which can be seen in upper left pane of Figure 18 [12] we find the main distribution features to be in line with our expectations. Specifically, the simulated distribution of the portfolio exhibits a peaked structure with most of the observations centered closely around the mean. This yields a low standard deviation, which is reasonable given the large capital allocation to the low risk government bonds. The distribution is also characterized by slight negative skewness.

In sum, we are able to generate a distribution, which exhibits a slight degree of negative skewness, low standard deviation and some heaviness in the lower tail. An interesting practical implication is found in the lower tail distribution is that despite pursuing a low risk portfolio profile, the investor is still exposed to some degree of prolonged tail risk, which presumably stems from the allocation to riskier assets. Looking at the risk measures, we observe a considerable ES 99% measure of 41%, which is quite substantial considering the extensive allocation to European government bonds, please see Table 11. However, compared to the equally weighted and risk-seeking portfolios, the conservative distribution still produces the lowest risk measures. Compared with the equally weighted and the risk-seeking portfolio, the conservative portfolio exhibits the lowest level of risk and compared with the normal distribution, the GARCH-EVT-Copula framework presents substantially higher risk estimates. This means that the empirical approximation is affirmed through the comparison with the characteristics of the equally weighted and the risk-seeking portfolios and more importantly, looking isolated at the conservative portfolio setup, we are able to generate stronger, more accurate and reliable risk measures for this given portfolio.

Conservative Portfolio	VaR		ES	
	95%	99%	95%	99%
Non-Normal	-0.14	-0.29	-0.23	-0.41
Normal	-0.10	-0.16	-0.17	-0.21

Table 11: Risk Measure for the Conservative Portfolio (Source: Own calculations)

The results of the conservative portfolio indicate that we are able to document both that the framework performs well when modelling on various weights and most importantly the GARCH-EVT-Copula framework is able to deliver considerable improvements relative to the normality framework. Overall, we can affirm that the distribution shape exhibits similar characteristics as one expects from an empirical perspective. However, as touched upon in the section on "considerations regarding extreme simulations" the GARCH-EVT-Copula framework does lead us to simulate observations, which are more extreme than one could expect from a practical point of view. Once again, the extreme simulations stem from the problem of slow convergence in the tails of the marginal EVT distributions.

[12] Note that the range of the x-axis has been cut to [-0.5 0.5] for illustrative purposes.

It is reasonable to expect the slow convergence problem to have effect on the risk measures of the conservative portfolio, however uncertainty surrounds whether the influence is notable. Given the extensive allocation to the low risk government bonds characterized by a close to normal distribution, one could expect the convergence problem to be less influential. However, due to the observation of negative skewness in the distribution, the convergence problem may still influence the risk measures, whereby resulting in overly pessimistic risk metrics. Nonetheless, we consider an overestimation of risk as the lesser of two evils, as data trimming could lead practitioners to arrive at optimistic risk measures and initiate faulty perception of appropriateness of observation exclusion based on nothing but subjective evaluations.

Having established the fairly good performance in relation to a conservative allocation, the following section examines the performance given a risk-seeking investor profile.

11.2.1.2 Risk-Seeking Portfolio

The distribution of the risk-seeking portfolio provides insight to the modelling performance of a high-risk allocation. In this setting we expect the distribution to be characterized by negative skewness, excess kurtosis and heavy tails. These expectations are based on the analysis of the marginal distributions provided in chapter 7 “Preliminary Analysis” where we documented such characteristics for the assets with high allocation weights in this portfolio.

Analyzing the graphical representation of the distribution leads one to observe that both skewness and a heavy lower tail characterizes the distribution to a much higher extend than what is the case for both the conservative and equally-weighted portfolio, please see Figure 18. We can hereby document our modelling ability to generate a distribution which is representative of the empirical patterns and in line with expectations. The estimated risk parameters for the risk-seeking portfolio is the highest among the three portfolios. This finding is obviously due to the long left tail with considerable probability density, please see Figure 18 for specific risk measures. This finding holds for all four risk parameters.

Risk-Seeking Portfolio	VaR		ES	
	95%	**99%**	**95%**	**99%**
Non-Normal	-0.31	-0.54	-0.45	-0.69
Normal	-0.26	-0.38	-0.35	-0.44

Table 12: Risk Measures for Risk-Seeking Portfolio (Source: Own calculations)

A comparison of the non-normal risk-seeking distribution with the normal distribution leads to an interesting observation: The normal framework assigns a higher probability density to large positive outcomes. Specifically, we see that there are considerably more observations in the right tail around the positive return of 50%, than in the non-normal distribution. This piece of documentation underlines the attractiveness of advanced distribution modelling both in relation to risk measures and estimation of expected returns. Specifically, ignoring the empirical distribution patterns would in this case not only lead to an underestimation of the inherent risk but also form an optimistic illusion of the expected frequency and magnitude of large positive returns.

In sum, the sensitivity analysis has provided insight to the ability of the GARCH-EVT-Copula framework to generate distributions, which exhibit plausible skewness, tail density and kurtosis. This analysis has provided documentation for the significant modelling improvements which the GARCH-EVT-Copula framework has to offer. The conservative portfolio generates the lowest risk measures, while the risk-seeking yields the highest. Once again, the risk estimations are influenced by the problem of slow convergence in the EVT tails. This leads to consistent estimation of extreme events, which has an implausible nature given the marginal distributions of the selected assets. This grows some concern regarding the reliability of especially the ES estimates as these are most sensitive to frequency and magnitude of extreme observations. However, methodological and ethical considerations led us to conclude that the effect of data trimming is limited and therefore not attractive from a theoretical perspective. We acknowledge that the decision may be different from an applied perspective.

The preceding analysis clearly demonstrates the vast improvements associated with estimating risk based on the GARCH-EVT-Copula framework. The analysis presented graphical representations of the fit between the modelled non-normal distributions and the empirical distributions. The results of the analysis are influenced by the classical challenge of correctly approximating the curvature of the empirical tails. Finally, the copula calibration led to more accurate modelling of asset interdependencies whereby we explicitly account for the true diversification effects across the entire joint distribution. Despite modelling challenges, mainly related to the curvature of the EVT tails, we broadly remain confident that the accuracy of the joint distribution fit leads to vast improvement of the reliability and validity in the risk estimates.

In general, the analysis arrives at the finding that assuming normality results in dramatic underestimation of portfolio risk. Specifically, risk is underestimated with up to 79% percent in the case of ES 99%, for the equally weighted portfolio. Similar results are found in the sensitivity analysis. The improvements of the risk measures stem from the fact that the GARCH-EVT-Copula framework allowed us to account for the following stylized facts of financial time series: skewness, tail density and non-constant and non-linear dependence structures.

The analysis provided interesting and alerting insights to the underestimation of risks related to assuming normality. The following chapter discusses the findings in greater detail and explicitly relates the analytical results to the problem statement as well as proposition 1 and proposition 2.

12 Discussion

The analysis provides important insight to the possibilities and value which resides from modelling marginal and joint distributions using the GARCH-EVT-Copula approach. As the analysis relies on a complex and comprehensive level of modelling, the main function of the discussion in this paper is to clearly relate the analytical findings to our problem statement and ultimately evaluate the implications for relevant stakeholders, which we have grouped into three segments: Financial institutions, regulators and investors.

The overall problem statement and our two propositions for improvement inspire the structure of the discussion. For convenience to the reader, we therefore briefly restate the problem statement and the propositions below:

How can accuracy of portfolio risk measures be improved through advanced distribution modelling in a European setting?

Proposition 1: The accuracy of the risk measures is improved as we approximate the empirical marginal distribution structures in terms of excess kurtosis, skewness and non-normal tail density.

Proposition 2: The accuracy of the risk measures is improved as we account for interdependencies in asset returns in terms for joint realizations and non-linear correlations.

12.1.1.1 Criteria of Success

The endangerment of assuming normality is directly related to the malpractice of estimating risks based on a non-representative theoretical distribution model. Utilizing the assumptions of a normal distribution allow researchers to skip several modelling steps, whereby this course of action implies that assumption simplicity leads to research convenience. However, this choice of action comes at the expense of validity and reliability in the risk measures. This leads us to define the criteria of success for this paper. We abolish the assumption of normality with the purpose of providing risk measures which are based on distribution estimations which to a high extend imitate the empirical data behavior. Despite the fact that this thesis operates on a quantitative foundation, it is not in our interest to apply a quantitative definition for our success. Rather we seek to provide a discussion on the joint distribution fit based on the results provided in the analysis. Moreover, we discuss the analytical estimation of the selected risk measures and relate the findings to discoveries presented in former research papers.

The following section discusses our findings in relation to proposition 1. This section naturally takes main interest in evaluating our ability to correctly model the marginal distributions of the four assets.

12.2 Marginal Distribution Fits

The general evaluation of the fitted distributions arrives at the conclusion that the observed and estimated data series clearly exhibit similar characteristics and at the same time significantly deviates from normality hence, application of the non-normal framework is highly appropriate. This motivation also holds, when looking at the upper and lower tail distributions in isolation.

This part of the discussion seeks to evaluate and assess the estimation power of the EVT-framework in relation to modelling the marginal distributions. The criteria of success is based on our ability to arrive at marginal distributions, which exhibit characteristics of high similarity to the empirical data series. Specifically, we evaluate our ability to imitate empirical behavior in relation to skewness, kurtosis and non-normal tail density, as proposed under proposition 1.

Table 13 provides a clear presentation of the empirical and fitted distribution characteristics. The areas marked in green in the lower section of the table represents the areas where the fitted distribution is considered to be an accurate approximation of the empirical distribution.

Empirical Distributions				
Asset	**Skewness**	**Excess Kurtosis**	**Lower Tail Assessment**	**Non-normal (JB-test)**
SXXE Index – Euro STOXX 600	Negative	3.74	Heavy and long	Yes
SX5E - Euro STOXX 50	Negative	2.89	Heavy and long	Yes
SGHIYIE FP Equity - High yield Bonds	Very negative	25.91	Heavy and very long	Yes
FIDEBST LX Equity - Low Risk Bond Portfolio	Slightly positive	-0.10	Thin	Yes
Fitted Distributions (EVT results)				
Asset	**Skewness**	**Excess Kurtosis**	**Lower Tail Assessment**	**Non-normal (Graphical investigation)**
SXXE Index – Euro STOXX 600	Negative	-	Heavy and long	Yes
SX5E - Euro STOXX 50	Negative	-	Heavy and very long	Yes
SGHIYIE FP Equity - High yield Bonds	Very negative	-	Heavy and very long	Yes
FIDEBST LX Equity - Low Risk Bond Portfolio	Slightly positive	-	Thin and long	Yes

Table 13: Overview of Empirical and Modelled Distribution Characteristics (Source: Own Calculations and Assessments)

As the EVT-Copula-framework builds on the standardized residuals from the AR-GJR-GARCH modelling, we do not estimate the exact kurtosis and skewness for each marginal distribution. However, the graphical representation of fitted distributions can still provide strong indication of skewness for each distribution and the direction of such. In relation to kurtosis, we are however unable to provide a qualified assessment based on the graphical representations. This is due to the fact that the measure is a weighted metric based on mean return, return observations, number of time-periods and standard deviation, whereby making this measure all the more complex and close to impossible to graphically assess. Hence, the following section evaluates skewness of each marginal distribution based on the fitted CDF for each asset where after we provide an overall evaluation of each distribution fit and the ability to accurately imitate tail density and longitude.

12.2.1 Skewness

As Table 13 shows, the semi-parametric EVT distribution modelling has led us to estimate marginal distributions, which all exhibit skewness of same direction as the empirical distributions and the fitted tails approximate the residual observations closely, please return to Figure 15: Semi-Parametric EVT Fitted Distributions for each of the Four Assets (Source: Own calculations)Figure 15, in chapter 9 and Appendix I if interested in revisiting the graphical representations. Correct estimation of skewness is of importance to our risk measures, as the skewness provides insight to the allocation of distribution density. Specifically, the fitted EVT-distributions add value compared to assuming normality, as they allow us to account for e.g. negative skewness in the fitted non-normal distributions. This ensures reliable risk estimations as correct imitation of skewness allow us to account for the fact that the mean is not equal to the median, and therefore not placed at the peak of the distribution. Ignoring such characteristic would lead to underestimation of risk in the cases of negative skewness as we observe and model for Euro STOXX 50, Euro STOXX 600 and the high yield bonds and overestimation of risk in cases of positive skewness as we observe in the case of the low risk bond fund.

As briefly touched upon, the assessment of ability to imitate empirical skewness is limited to affirming presence and direction, whereby leaving the magnitude unevaluated. This naturally introduces uncertainty and we therefore wish to establish further confidence of the risk estimations by providing an assessment of the lower tail fit, hence the following section focuses on this topic.

12.2.2 Overall Tail Fit

As outlined in proposition 1, this thesis seeks to provide distributions, which correctly approximates skewness, kurtosis and tail behavior. We find it relevant to discuss the ability to correctly approximate the empirical distribution behavior for each of the four assets with a primary focus on the tail modelling. As we have already documented that the marginal tail fits closely approximate the behavior of the residuals observations shown in Figure 15 in the analysis, the following section focuses on the tail fit for each distribution by looking at the shape and scale parameters of the tails from the EVT estimation process.

12.2.2.1 Scale and Shape Parameters

Recall that the shape parameter determines the shape of the distribution and the scale parameter defines how spread out the data is. This implies that the shape parameter assists us in assessing the heaviness of the tails and the scale parameter provides insight to the range of the tail observations.

The empirical distributions of the Euro STOXX 600 and Euro STOXX 50 exhibited heavy and long lower tails. This behavior is clearly reflected in the fact that the lower tails of the two equity indices have both higher shape parameters and higher scale parameters relative to the parameter estimates for the upper tails.

Compared to the two equity indices, the empirical distribution of the high yield bond asset is characterized by an even longer lower tail but a less prevalent tail heaviness. This overall behavior is also successfully reflected in the parameter estimates of the EVT-tails. The high yield bond fund has a lower shape parameter than the two equity indices, however the scale parameter is remarkably higher. This first of all supports our claim of being able to imitate the empirical distribution of the high yield bond and second of all, these characteristics are typical for this asset class, whereby lending further confidence to the accuracy of the distribution estimation.

Lastly, the analysis reveals that the parameter estimates of the tails for the low risk bond asset are in line with the overall empirical characteristics of this asset class. Specifically, the lower tail has a negative shape parameter, implying a thin lower tail. However, the scale parameter of the lower tail is larger than the upper tail, which indicates that the lower tail is longer than the upper – this estimation may conflict with the overall behavior of the empirical distribution because when the lower scale parameter exceeds the upper, it provides indication of skewness. However, this may result due to the single residual observation found around the fourth standard deviation in the lower tail, whereby driving up the lower scale parameter leading to excessive longitude of the lower tail of the EVT distribution model. This problem has been assessed in the analysis section 9.2 "Assessment of Distribution Fit" where we acknowledge that the tail distribution modelling of the low risk bond fund is the poorest among the four assets. This is mainly due to the single extreme outlier and poor fit in the beginning of the tail. However, we conclude that the general tail fit is still fairly acceptable for performing portfolio risk estimations. Overall the results from the shape and scale parameters are in line with the behavior of the empirical distributions which implies a reasonable degree of modelling success.

The scale and shape parameters are however not enough to make final conclusions regarding fulfillment of proposition 1. Recall, that the EVT tail modelling is highly sensitive to changes in the extreme observations. Hence, as a final discussion element of relevance relates to findings from the detailed assessment of the fitted distributions in the analysis part.

The analysis highlighted several important technical challenges, which especially influenced the distribution modelling of the Euro STOXX 50 and low risk bond portfolio. The detailed investigation of the probability density plots underlined the classic problem of modelling with EVT. Namely, that the framework and modelling is highly sensitive to changes in the observations in the tails. Specifically, we suspect the distribution estimation of the Euro STOXX 50 to be under influence by one extreme residual outlier, which is observed far out in the lower tail. This potentially distorts the EVT tail curvature, resulting in simulation of tail events, which are far beyond the maximum level of the empirical observations. Similar considerations surround the other assets. The analysis revealed how the lower tail of the low risk bond portfolio has a close to normal fit, but a few extreme residuals, possibly resulting from the recent European Debt Crisis, lifted the curvature in the fitted EVT probability density function away from the normal distribution at the end of the tail. This data behavior may have led risk measures of this assets to be influenced by few and very dramatic events of questionable nature and accuracy.

To further evaluate the severity of the curvature problem, we conduct and observation-by-observation investigation of the returns for all three portfolio setups. In all three cases, we can confirm the presence of few very extreme simulated returns which are far more extreme than the empirical return observations. In example, we find a simulated loss 99.9% for the conservative portfolio, allocating 55% of its capital to European government bonds. This appears high implausible as it would imply government bankruptcy in several European countries. We acknowledge that the assigned probability to the loss of 99.9% is 1 out of 10,000, but nonetheless we find the simulation of great concern and it gets even more alerting when one draws attention to the empirical observations, where the low risk portfolio has a maximum one-year loss in the entire observation period of 28%. Therefore, in order to strengthen the risk measurement, we provide both the VaR and ES measures, where ES is naturally under influence by these potentially erroneous estimations far out in the tail and VaR is more robust to such estimation errors given the nature as a quantile risk measure.

To find further evidence for the modelling improvements associated with EVT, we allocate the next few sections to a concentrated review of literature of direct relevance to the analytical findings.

It is a general challenge to ensure reliability and validity when operating with EVT and fitting the distributions based on standardized residuals. This is especially true for the tails as they are fitted based on relatively few observations, thus any change is these observations or measurement errors will cause the curvature of the fitted distributions to be altered (P. Christoffersen, Diebold, & Schuermann, 1998). This is a general theoretical weakness, which is of concern to both academia and practitioners. On the positive side one may question whether this is truly a modelling problem or not. The purpose of modelling with EVT is indeed to generate a distribution, which approximates *all* observations with the best fit possible. This also includes the very extreme observations, which do not fall in line with the behavior of the rest of the distribution. We find it important to underline that it is the empirical observations, which drive the shape of the estimated distribution and EVT is an obvious and recognized framework for modelling extreme observations (Allen et al., 2011; Brodin & Klüppelberg, 2006; Embrechts, 2000). Therefore, in line with arguments presented by Embrechts (2000) the EVT methodology is making the best out of a given dataset and the Peak-Over-Threshold method ensures efficient use of data points in the distribution tails. Furthermore, as modelling financial time series with EVT is still on an infant stage, the application of EVT requires us to assess the shape of the EVT fit. The importance of this step relates to the sensitivity of the shape and scale parameter to alternation of extreme observations. The model can be rejected when and if the shape of the fitted distribution remarkably deviates from the empirical observations, which we assess to not be the case in our modelling process (Jondeau et al., 2007).

The theoretical and practical appeal of the EVT framework is further documented by Stoyanov et al. (2011) who underlines that there is no reason to believe that simple variations of the Gaussian distribution, where skewness and kurtosis are altered, are better at describing extreme events than a regular normal distribution. The main reason being that skewness and kurtosis are not enough to describe the full richness of the shape and tail behavior in a distribution, but rather they function as a guideline and instead they propose EVT as a much stronger theoretical framework.

In sum, we can confidently state that we have followed best practices and guidelines on modelling extreme events. Theoretical recommendations from former research has guided the final setup and estimation process, whereby significant theoretical groundwork lends supportive arguments for the reliability of our risk measures and the findings of the analysis further confirms the empirical approximation. However, no world is perfect hence, the discussion has also presented considerations regarding the curvature of the outer most part of the EVT tail fits and the implications for the final risk estimations in shape of a potential overestimation of risk.

12.3 Fulfillment of Proposition 1

The discussion section on proposition 1 has assessed the similarity between each empirical distribution and the corresponding fitted distribution. In general, we are confident that the estimated semi-parametric EVT distributions to a high extend imitate the behavior of the empirical distributions, implying that they correctly reflect skewness, kurtosis and tail density. Nonetheless, some degree of uncertainty surrounds the fitted distributions, especially the distributions of Euro STOXX 50 and the low risk bond portfolio, primarily in relation to the density allocated to the lower tail. There is however no definitive conclusion regarding the validity – instead we rely on a combination of practical insight and technical knowledge when stating conclusive remarks.

The extreme observations influencing the curvature of the fitted tail distribution of Euro STOXX 50 presumably relates to the large negative spikes in return data seen during the recent Global Financial Crisis. Likewise, the largest spikes in the low risk bonds are observed during the recent European Debt Crisis. A classical assumption when estimating risk based historical events is that we consider the past to provide a fair representation of the future. Hence, not including observations such as the extreme spikes during the Global Financial Crisis or the European Debt Crisis due to fear of allocating too much density into the tails would result in risk estimates, which can be perceived as similar to watching a film, where all the scary and dangerous parts are censured out. Therefore, what can be considered a theoretical weakness may prove to be valuable to risk managers as they seek to operate based on risk metrics which truly reflects the full behavior of the market fluctuations. This should however not be a motivation for neglecting the importance of critically assessing the fit of each tail distribution, rather the argument provides additional confidence in the quality of our findings.

Conclusively, we argue that this study has followed best practices and recommendations from both practitioners and academics. Former studies lend supportive arguments for the attractiveness and applicability of the GARCH-EVT process (Allen et al., 2011; Brodin & Klüppelberg, 2006; Embrechts, 2000; J. P. Morgan, 2009) and Stoyanov et al. (2011) specifically state that the EVT framework allows us to model the marginal distributions with much more accuracy than operating with modifications of the traditional Gaussian distribution.

Having discussed the validity, reliability and potential drawbacks of the GARCH-EVT-copula framework in relation to proposition 1, the following section continues to present and evaluate the results in relation to proposition 2. This section naturally evaluates the power of the student's t copula to account for the impact of volatility breakdowns which often results during tail events.

12.4 Interdependencies and Joint Realizations

The copula methodology has created value in this thesis due to the attractive capability of being able to capture and account for dependent extreme events and allow us to model the joint rather than marginal interdependence. Abolishing Pearson's simple linear correlation and adopting the copula framework has made it possible to model the joint distribution of the portfolio and allowing for varying correlations across quantiles, which practically means that we allocate excessive probability density into the tails to more accurately reflect impact of extreme events.

The practice of accounting for joint realizations has proven crucial for estimating the true portfolio risk. Here, application of the simple Pearson's correlation would have led to remarkable underestimation of risk, especially during times of crises. Empirical studies have shown how correlation between e.g. equity assets tends to be remarkably underestimated during times of crises, hence the practice of accounting for joint realizations is crucial for estimating the true portfolio risk. Additionally, inaccurate modelling of correlations in the joint distribution would lead to incorrect estimation of the diversification effect.

This research framework builds on the student's t Copula. The results presented in the analysis clearly demonstrates how the copula transformation allows us to account for correlation breakdowns in the tails. The effect of the transformation is shown graphically in Figure 16, where we see how the student's t copula incorporates the higher likelihood of joint extreme events between the two assets through increase of tail correlations (left pane of Figure 16). This leads us to affirm that the copula transformation does allow us to incorporate the true effect of joint realizations and non-linear, non-constant correlation.

The attributes of the student's t copula are not only an advantage but also a concern. The student's t copula allocates an increased level of density to the tails of the distribution through adjustment of the degrees of freedom parameter. However, a main concern presented in the literature review relates to the assumption of asymmetry. Specifically, the upper and lower tail is simulated with equal probability density. This theoretical attribute conflicts with the well-established presence of skewness and kurtosis in the data series, hence the evaluation of proposition 2 also relates to the fit of the simulated distributions and the ability to retain appropriate distribution fit in relation to skewness and tail density.

The distribution results post copula calibration provide clear evidence for the significant modelling improvement associated with non-normal distribution modelling. Despite theoretical concerns regarding the student's t copula's lack of focus on distribution asymmetry, we still succeed in simulating a joint distribution which exhibits negative skewness and tail asymmetry. This is because the preceding EVT modelling step allows us to indirectly introduce skewness through the marginal distributions. This led us to relax the alarming theoretical considerations regarding the choice between student's t copula and Clayton copula.

The DoF parameter also affirms the applicability of the student's t copula. We find a DoF parameter estimate of 17.53, which corresponds to a distribution with heavy tails. The DoF parameter is larger for the joint distribution than the marginal distributions, whereby indicating that the copula calibration correctly accounts for the diversification effect. From an applied perspective, we assess the DoF parameter of 17.53 to be reasonable given the marginal characteristics of the four assets. In sum, we document the value of modelling joint realizations with a copula calibration through correctly accounting for non-linear correlations and effects of correlation breakdowns in the tails.

In order to further discuss the results from the analysis and make final conclusions on the fulfillment of proposition 2, the following paragraphs discusses general theoretical and empirical recommendations for copula calibrations in a finance setting.

The copula calibration process has received much attention in the financial research. In relation to risk management, the copula has proven to be a powerful tool to model dependence between different assets in a portfolio. The theory has gained ground because the practice of applying linear correlation as a dependence measure leads to incorrect estimates of the joint dependence, hence underestimation of risk. Instead the copulas are able to model joint distributions and allow for varying correlations across quantiles (Jondeau et al., 2007; A. McNeil et al., 2010).

The choice of applying the student's t copula is supported in the study by Kole, Koedijk, & Verbeek (2007). They analyze return series of the largest American indices and compare the efficiency of various Copulas in predicting the VaR. They find that the student's t copula produces the most efficient estimates of portfolio risk, while the Gaussian and Gumbel copula tend to under- and overestimate portfolio risk, respectively. Specifically, the student's t copula is found relevant to this thesis, as it allows us to adjust the degrees of freedom, whereby we can approximate the fatter tails exhibited in the asset returns data (A. McNeil et al., 2010).

Other researchers such as Embrechts et al., (2003) advocates for the attractiveness of the Clayton copula when applied in a finance setting. In addition to accounting for heavy tails, this copula setup also accounts for skewness, whereby potentially making it more attractive than the student's t copula to this thesis. However, the analytical results confirm that we are able to introduce skewness through the EVT fitted marginal distribution. Reverting to the interests of this thesis of improving risk measures, our analysis utilizes the student's t copula as literature suggests it to be a vast improvement of the traditional sample modelling approach (Peter Christoffersen et al., 2013; Sheikh & Qiao, 2010).

In sum, the student's t copula calibration is a technical decision based on both empirical and theoretical motivations. Theoretically, we have evaluated the tradeoff between complexity and applicability where we find the student's t copula to provide vast improvement for modelling portfolio risk measures through account of non-linear correlations. Empirically, former studies find the student's t copula to have strong performance when applied to portfolios holding stocks, bonds, real estate and currency. The findings in the analysis supports the recommendations from former studies, as we are able to account for correlation breakdowns in the distribution tails as exemplified in Figure 16. Conclusively we therefore argue that the utilization of the student's t copula leads to valuable improvement of the risk measurements.

12.5 Fulfillment of Proposition 2

The previous section presented the findings from the analysis on copula modelling. Based on the findings in the analysis and discussion of the validity both in terms of results and theoretical framework, we are confident that the student's t copula has allowed us to accurately account for non-linearity in the correlation structure of the portfolio. The focus on joint rather than marginal distributions creates value in risk management because this process allows us to account for extreme joint events, while at the same time capturing the true diversification effect, which is also of great importance to portfolio risk estimations.

Uncertainty regarding the DoF parameter estimate briefly introduced a vague sense of suspicion regarding the accuracy of the copula model. Specifically, the confidence interval for the estimate is wide enough for the distribution to be characterized either by normal or heavy tails. This doubt is moderated by the fact that the statistical robustness and validity is affirmed in theory and we acknowledge that theory is the backbone of distribution modelling.

The theoretical discussion on the empirical and theoretical motivations regarding choice of copula, provide sufficient literature argumentation for the vast improvement, which the student's t copula provides. Finally, this leads us to conclude that the GARCH-EVT-Copula framework has led to fulfillment of proposition 2, whereby we affirmatively argue that the non-normal joint distribution accounts for skewness, excess kurtosis, increased tail density and non-linear and non-stabile interdependencies.

This leads the discussion to progress to debate the findings regarding the risk estimates and their robustness and validity. The risk measures are assessed in conjunction with an overall discussion of the advantages and drawbacks of the GARCH-EVT-Copula framework.

12.6 Assessment of Portfolio Risk Measures

This section provides a comprehensive overview, where the fulfillment of proposition 1 and 2 are related to the subsequent VaR and ES estimates. The purpose is to assess to what extent we succeed in providing accurate and reliable risk measures, representative for the true financial risk of the portfolio. The key criteria of success is to overcome the limitations of the normal distribution, which consistently underestimate the probability and magnitude of extreme negative events.

12.6.1 Risk Estimates

The quality of any statistical analysis ultimately depends on the quality of the data input. The same argument is applicable for assessing the risk of a portfolio. Here the validity of the risk parameter is directly connected to the efficiency of the distribution modelling. Through proposition 1 and 2 we have advocated for our enhanced distribution modelling, which enables us to account for the stylized facts of financial time series. Hence, the risk parameters must accordingly provide efficient estimates for the true portfolio risk.

Focusing on the projected risk of the equally weighted portfolio the average magnitude of the results obtained through the non-normal framework are considerably higher than the results estimated under the assumption of normality. The magnitude of the VaR estimates are 24% and 44% and the ES predicts a potential loss of 36% and 59% at the 95% and 99% level, respectively. In line with our expectations, the difference between the normal and non-normal framework is increasing proportionately with the tail quantile. In detail, the underestimation of risk ranges from 22% up to a striking 79% for the ES at the 99% quantile. The remarkable underestimation of risk under normality is striking and should not be neglected in future risk management practices, rather we hope to lend supportive arguments for implementation of more advanced risk models as this will provide investors with accurate information regarding their true risk exposure. Thereby, the GARCH-EVT-Copula framework shows sign of being capable of representing the true risk of extreme financial events and clearly depicts the inappropriateness of applying normality assumptions.

To underline the severity of failing to model the non-normality in the asset returns we further presented a quantitative example showing that for the equally weighted portfolio the normal distribution misrepresents risk with 16% to 25% of the initial portfolio value. The dramatic underestimation of risk should be of great concern to investors of all kinds as it not only prevents investors from making correct investment decisions regarding risk budgets but also in relation to the time required to regain initial portfolio value in the case an extreme event occurs. Specifically, we show that given a 1 out of 100 events (beyond 99% confidence) it takes an investor twice as long to reestablish initial portfolio value than what is predicted given the normal loss distribution.

As we stated in the analysis, the discussion of the validity and reliability of the risk measures strongly relates to the problem of slow convergence in the fitted EVT tails. Specifically, we found loss simulations of unrealistic nature in all three portfolio setups. In detail, the simulated data series included loss observations of 99.9% for three portfolios, which is highly implausible given that all portfolios contain some weight in European governments, whereby implying that several European governments should go bankrupt. Instead of pursuing a strategy of data trimming we prioritize focus the following part of the discussion on evaluating the strength related to measuring risk in terms of both VaR and ES.

12.6.2 Overcoming the Challenge of Slow Tail Convergence

Modelling extreme events is an art and a large degree of uncertainty is connected to the simulation of the edges of the tails (Embrechts, Resnick, et al., 1999). This distinguishing challenge of modelling extreme events is clearly depicted in our analysis, where the slow convergence of the fitted tails leads to simulation of debatable extreme losses. Hence, the complexity of estimating the involved risk of extreme events dictates that one single variable in assessing the portfolio risk doesn't suffice (Rootzen & Kluppelberg, 1999). The importance of applying both VaR and ES as risk measures are clearly exemplified under the slow convergence. To recall, this characteristic of our modelling framework implies that especially the 99% ES is highly sensitive to changes in the most extreme observations hence, looking at this risk measure in isolation may lead to overly conservative risk perceptions. Instead, the utilization of both VaR and ES allow for a nuanced assessment of the true portfolio risk. This advantage is illustrated in the results of the analysis. Focusing on the estimated risk of the equally weighted portfolio, we see large differences in the magnitude of the VaR and ES at each confidence level. At the 95% confidence level, VaR estimates a loss of 24% and ES estimates a loss of 36%. The 99% confidence level estimates are 44% and 59% for VaR and ES, respectively. Analyzing each of the risk measures in isolation provides only the half picture as one may fail to gain insight to the potential extreme events which lie far away from the 95% VaR quantile. Similarly, if only the ES metric is applied it will result in lack of a holistic assessment of the shape of the distribution and risk at various quantiles. In sum, we believe that estimation of both VaR and ES introduces a mechanism of a two-way critical risk assessment, meaning that a large difference between VaR and ES may provide evidence that the risk manager needs to pay close attention to the outermost extreme tail observations and assess the validity of those.

To further validate the risk measures we find empirical support in the paper by Bob (2013). He analyzes the performance of the GARCH-EVT-Copula in prediction the VaR for four major European indices. He compares the methodology against traditional methods such as the Historical simulation and Variance

Covariance. Specifically, he finds that the GARCH-EVT-student's t copula outperforms all other GARCH-EVT-Copulas, including the Clayton Copula and the two traditional frameworks. In addition, Sheikh and Qiao (2010) provide a comparable study based on a North American setting. The authors investigate a sample period from 1994 to 2009 and analyze the risk of a well-diversified 10-year portfolio mainly consisting of American assets. The authors reject the assumptions of normality for the majority of the included assets and incorporates the GARCH-EVT-Copula framework to account for the stylized facts of financial time series. They estimate the expected loss of the ES which is approximately 50% higher than the estimate obtained under normality. Bearing in mind the obvious dissimilarities in the exogenous and endogenous variables make a direct comparison of the two studies undesirable. However, the general findings presented by both Bob (2013) and Sheikh and Qiao (2010) lends support to the validity of our findings.

Through the proposed methodology, we provide a framework which facilitates an improved assessment of the true portfolio risk. Firstly, the modelling of the marginal distributions is improved by the use of AR(1)-GJR-GARCH(1,1) and EVT as this process allows us to account for volatility clustering, skewness and fat tails. Secondly, copula theory is utilized in order to account for correlation breakdowns and the leptokurtic shape in the multivariate joint distribution. Lastly, a combination of ES and VaR allows us to obtain a nuanced assessment of the true portfolio risk. The complementary attributes of the two risk measures provide a holistic view of the inherent risk and thereby allow for a deeper understanding of the potential loss at various quantiles of the distribution.

The above framework permits an assessment of the risk inherent in the financial portfolio, which to a large extent imitates the behavior of empirical data. The improved modelling is mainly illustrated in the ability of the advanced framework to incorporate the larger probability and magnitude of extreme events which is consistently underestimated under the assumption of normality. Our results are in line with previous findings in the literature and we believe that the analytical findings contribute with additional knowledge regarding the potential benefits of the GARCH-EVT-Copula framework in relation to the areas of risk management. We conclude with great confidence that our risk measures are substantially improvement compared to estimations based on the assumption of normality, however, we acknowledge that further research is still needed to advance the methods used to model extreme events.

To shed light on the empirical implications of our analytical findings the following chapter presents a brief note on the relevance of the results to financial institutions, regulators and investors.

13 Empirical Implications and Perspectives

In this section we provide perspectives on the potential implications of our results from the perspective of three potential stakeholders: Financial institutions, regulators and investors. This paper has thrived to objectively provide an improved modelling framework for financial portfolio risks and an enriched assessment of the inherent risk of European financial assets.

13.1.1 Financial Institutions

The implementation of the GARCH-EVT-Copula model involves both pros and cons. The main benefit are found in the ability accurately estimate risk of financial assets. In spite of the time horizon of one year, which deviates from the traditional risk horizon of 10 trading days for banks we believe that the framework can rather easily be applied to short-term setting (BIS, 2013). Applying the more sophisticated non-normal model enables financial institutions to more accurately assess extreme risk and consequently allow them to pursue optimal trading decisions.

The main disadvantages are related to the extensive cost of implementing the technical procedures and educating staff in handling the new practices. Humans are by nature inclined to pursue well-known and traditional techniques as these may provide a false feeling of comfort and reliability. Despite of the implementation costs and the prolonged implementation process empirical research such as this thesis stresses the importance of more accurate modelling of extreme events and implementation of more advanced practices should therefore no longer be postponed. This need is exacerbated by the fact that the frequency and magnitude of extreme events is increasing over time.

it is expected that the financial institutions exhibit some reluctance against the implementation of our proposed framework as it will lead to increased risk estimates without a proportionate increase in return, whereby reducing the attractiveness of each portfolio all else being equal. Specifically, the GARCH-EVT-Copula framework approximates the empirical tail curvature and density whereby leading to increased risk of extreme tail events. Hence, a direct impact of adopting the new model is higher risk measures. As found in the sensitivity analysis this effect is present across all levels of portfolio profiles. This inevitably leads to a reduction in the predicted risk adjusted revenue streams of the financial institutions, thus it may harm the credibility of the financial institutions and subsequently result in introduction of stricter capital requirements by regulators.

On the other hand, considering the eruption of society's confidence in the financial institutions in the wake of the financial crisis, the implementation of a more adequate and reliable modelling procedure and adaption of multiple risk measures could be a competitive tool to regain trust. In sum, financial institutions have the potential to gain from the improved assessment of risk in the internal process. However, it is highly unlikely that individual financial institutions voluntarily will adapt the framework for business purposes as it albeit will make their financial offerings less attractive to investors. Therefore, to ensure sector-wide implementation of advanced risk modelling tools, we believe that regulatory action is needed.

13.1.2 Regulators

Regulators, such as the Basel committee, face an extreme challenge in constructing a regulatory framework, which is able to sufficiently deter excessive risk taking and hazardous behavior, while still enabling the financial intermediaries to drive a profitable business. Further, the diversity among the affected entities are multifarious, which makes it challenging for regulators to create a regulatory environment which accommodates the needs of all firms regardless of size, location and product portfolio. This problem is illustrated through an ethical dilemma related to the asymmetric impact of regulation on financial institutions. Complying with regulatory requirements are often associated with high initial fixed cost related to establishing and developing technical infrastructures. This implies that introducing wide-spanning regulatory requirements leads to unfair competitive side-effects. The larger institutions are to a higher extend able to benefit from economies of scale and thereby experience a reduced economic burden relative to smaller financial institutions (Lux & Greene, 2015). This makes it extremely expensive for small entities to incorporate and comply with the required regulations. However, the needs of smaller community banks and other minor financial institutions should not be prioritized above the ultimate goal of financial stability for society as a whole. Especially seen in the light of our findings, which clearly indicate severe underestimation of risk under the normality assumption which guides today's financial regulatory requirements.

A central area of debate is currently related to replacement of VaR with ES (BIS, 2015). The attractiveness of adopting ES is also clearly depicted in the analysis of this paper. Nevertheless, improving portfolio risk measurement is not only related to alternation of the practices of risk. Specifically, applying ES under the assumption of normality does not provide an accurate measure of the risk of extreme events and would therefore still lead to substantial underestimation of extreme tail risk. However, if the assumption of normality is abandoned and introduction of ES prevails we believe that it will lead to vast improvement of today's risk management practices. In conclusion, we believe, that in spite of many statistical and technical arguments and suggestions for improvements, a full scale regulatory requirement of a non-normal risk estimation is still unlikely to be occur within near future.

13.1.3 Investors

As previously specified, the investors currently face a hostile investment environment marked by extreme volatility and low interest rates. If the story of Japan is to repeat itself in the rest of the world and decades of (close to) zero interest rates are to follow, it can be the beginning of a paradigm shift. A new paradigm portrayed by the 'reaching for yield-motive, where investors are forced to allocate their investments to risky assets in order to obtain any rate of return. These new exogenous characteristics of the investment environment highlight the importance of accurate risk estimation – now more than ever.

As seen in the analysis, an obvious implication of incorporating the GARCH-EVT-Copula framework is the prevalence of higher risk measures. A direct consequence follows that some investors may no longer have a portfolio allocation, which accommodates their risk and return preferences and traditional conservative portfolio offerings may stand to be shut down for practical reasons. Accounting for non-normality leads conservative investors to reallocate away from assets exhibiting heavy negative skewness and a leptokurtic shape and as such a paradox may be faced by the risk averse investors: on one hand, they may not be willing to accept the risk associated with investing in the equity market. On the other hand, they do not have any profitable low risk alternatives due to the current low interest rate yields. Assuming that they do not compromise their risk profile or return requirements, these investors have no possibilities to obtain any rate of return. The changes introduced through accounting for non-normality generate new challenges for the investor, however we are confident that investors have a sincere interest in gaining access to more reliable and accurate portfolio risk measures.

A natural extension of the empirical implication of our framework is related to the potential benefits in the areas of portfolio allocation. Considering the requirement of accepting risk in order to obtain any rate of return, it may prove beneficial for investors to rethink their capital allocation. The most common optimization method is currently Markowitz' mean-variance framework. However, bearing in mind the findings of non-normality in our analysis, we can conclude that variance is no longer a valid measure of risk. Instead, investors can utilize the concept of risk budgeting by applying e.g. ES as the maximization criteria. This method shows two promising features. Firstly, ES fulfills the criteria of a coherent risk measure and thereby allow the investor to optimize based on a solid foundation. Secondly, the logical features of ES provide an intuitive way to examine the potential risk related to an investment. Other measures, such as utility functions, are abstract concepts and therefore often of limited practical use to an investor when deciding on the optimal level of risk-averseness. Additionally, it is convenient for an individual to decide on a risk budget, i.e. how large a percentage of the initial investment the individual is willing to loose given a specified confidence interval and time horizon. Hence, the GARCH-EVT-Copula model may show to be of value to the practice of risk budgeting, as it provides a coherent assessment of risk, while presenting the potential loss in a logical way. Initial work in this area has been conducted in the academic field, however more work is required before this approach can be adopted as the common standard.

A final practical consideration for implementation of the GARCH-EVT-Copula framework is related to the level of complexity inherent in the model. Hence, lack of theoretical knowledge prevents non-professional investors from applying this methodology. Therefore, the ordinary investors can primarily benefit from the improved accuracy in the non-normal framework, if it is presented for them in a pre-programed solution or accounted for in the investment products provided by the financial intermediaries. Nonetheless, the GARCH-EVT-Copula approach and risk budgeting is of direct practical value to the non-professional investors.

14 CONCLUSION

In this master thesis we have modelled non-normality of asset returns with the purpose of improving accuracy and reliability of portfolio risk measures. The analytical results clearly demonstrate the vast improvements associated with modelling distributions and estimating portfolio risks based on the GARCH-EVT-Copula framework. We find the relative magnitude in risk underestimation of striking nature. The most extreme underestimation for the equally weighted portfolio prevails in the case of Expected Shortfall 99%, here the underestimation of risk amounts to 79%. The findings of this thesis stand as a source of motivation for implementing the GARCH-EVT-Copula approach to model portfolio risk going forward.

In detail, the GARCH-EVT framework guided the fulfilment of proposition 1 as we to a large extend are able to account for the following stylized facts characterizing financial time series: Skewness, excess kurtosis, heavy tails and volatility clustering for all four marginal distributions.

Furthermore, the student's t copula calibration allowed us to replace the traditional Pearson's linear correlation whereby we accurately model the implications of correlation breakdowns and heavy tails in the multivariate joint distribution.

Naturally, the distribution modelling was not without complications. The main concern relates to the curvature of the fitted lower tails as these showed to be highly sensitive to changes in the extreme observations from the empirical dataset, possibly leading to too slow tail convergence and overly pessimistic risk estimations. However, the methodology applied in this thesis is strongly rooted in findings presented in former studies which underlines the attractiveness of the GARCH, EVT and copula tools for modelling non-normality of asset returns. In conclusion we therefore confidently argue that this study has followed best practices and recommendations from both practitioners and academics and our findings are of pioneering character as only a limited number of former studies apply the full combination of a GARCH-EVT-Copula modelling framework to a risk management problem in a European setting.

15 LIMITATIONS

This section presents two main limitations which influences our ability to accurately answer the problem statement. The following paragraphs presents the limitations and subsequently provide an assessment of the potential for generalizing our findings.

Firstly, as described in section 9.2 “Assessment of Distribution Fit” the most extreme observations of the data sample have led to convergence problems in the left tails, which resulted in unrealistic high loss observations in the simulated return series. This effect is especially evident for observations with less than 0.001% probability of occurrence. The impact of this limitation may lead to inflated values of the Expected Shortfall measures. Rather than trimming the data which would be based on solely subjective motivations we decided to keep all observations within the samples. Our argument built on the fact that data trimming would solely cure the symptoms of the problem and not eliminate the source causing biased distribution fits. Hence, we evaluate that data trimming would not lead to significant improvement of the advanced distribution modelling framework from a risk management perspective. Additionally, the use of the combined GARCH-EVT-Copula framework in a risk management perspective is still at an infant stage, hence the decision to leave all observations within the sample was taken in order to assure true insight to the potential benefits and drawbacks of the EVT framework, in an GARCH-EVT-Copula setting. We acknowledge that this decision might have increased the absolute value of the risk measures and data trimming could have been a potential solution in a practical setting. However, through the presentation of this limitation we encourage further research to develop the EVT modelling section with the specific aim of providing a more approximate curvature of the most extreme edges of the distribution tails.

Secondly, as presented in the methodology we apply the symmetric student’s t copula when modelling the joint distribution. This conflicts with the empirical findings of skewness in the multivariate distribution modelling of financial assets. This imply that our framework to some extend is inadequate in fully describing the empirical joint behavior of asset returns. Nevertheless, the student’s t copula provides improved explanatory power regarding the two main concerns: correlation breakdowns and fatter tails. However, as pointed out in section 10.1.2 “Simulation Results” EVT allows us to overcome the lack of skewness in copula calibration, since the EVT marginal distribution modelling indirectly introduces skewness and heavy tails to the copula simulation. In sum, more advanced copulas may in general increase modelling accuracy however, the comparative attractiveness of the Clayton copula is reduced when looking at the GARCH-EVT-Copula framework as a whole. Hence, we remain confident that the utilization of the student’s t copula is a fair tradeoff between complexity and applicability.

Instead, we propose further research to perform comparative studies where families of copulas are studied while holding all other factors fixed.

Despite the given limitations of the modelling framework, we positively conclude that the GARCH-EVT-Copula framework allows us to significantly improve reliability and validity of risk measures in a European setting. Specifically, the framework has allowed us to mathematically replicate empirical data behavior and deal with the complexity which characterizes today's financial markets. It is important to state that the generalization of our findings are limited to the specific setting of our analysis. This means that the results are only representative to the extent that the sample replicates the population and therefore one should avoid extending the absolute results to the European financial market as a whole and to other geographical locations nor other time periods. Nevertheless, the key finding of significant distribution modelling improvements through GARCH-EVT-Copula and consequently the reliable estimation of risk has broad empirical support and can be generalized to a vast extent. Moreover, the results presented in this paper are of most relevance for investments with a short to medium term horizon as the law of large numbers tend to make the distribution of assets returns approximately normal in the long term.

16 References

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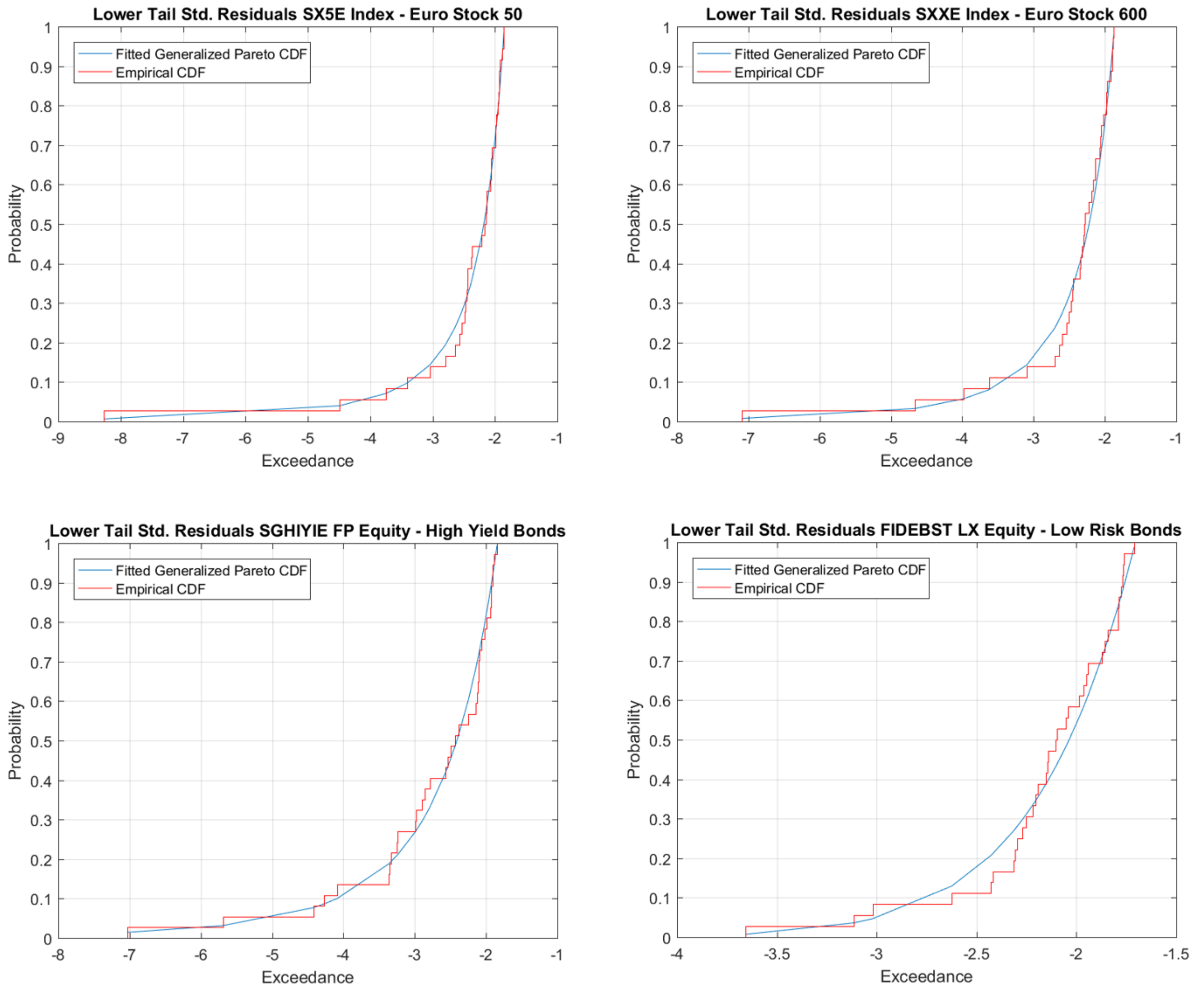
1 APPENDIX I

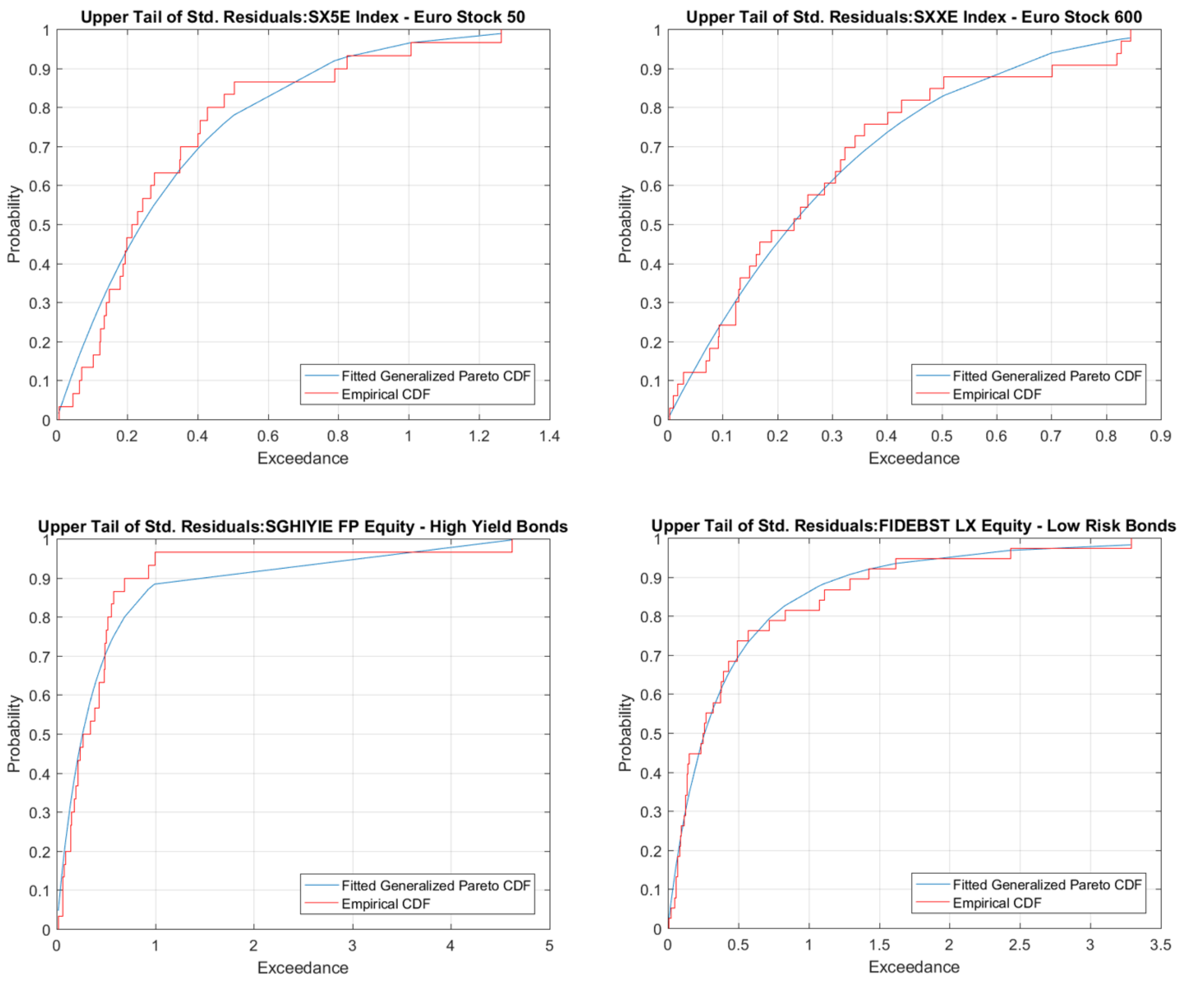
GPD fit of the lower tail.



Source: Own calculations

2 APPENDIX II

GPD fit of the upper tail.



Source: Own calculations