Anomalies and Factors in European Stock Returns

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Abstract

In recent years many anomalies have appeared in expected stock returns the most famous being the value and size anomalies. This thesis investigates different anomalies based on firm-characteristics in the European stock market in the period 2000-2015. It is found that portfolios sorted based on the following firm-characteristics earn anomaly returns measured by a risk-to-reward ratio: beta, volatility, size, MTBV, momentum, ROE, and OP. Furthermore, it is found that 4-factor model consisting of market, MTBV, volatility, and momentum factors capture the main part of variations in expected stock returns in the European market. Therefore, this thesis supports the risk based explanations of anomalies in European stock returns.

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1. Introduction

Understanding the relationship between risk and expected stock return has been of great interest for economic scientist and practitioners within the field of asset management for a long time. Sharpe (1964) and Lintner (1965) made a breakthrough with their papers on the Capital Asset Pricing Model (CAPM) relating expected return and risk with excess return of the market portfolio as the lone explanatory factor. In the following years anomalies to the CAPM were discovered. The size anomaly relating market capitalization and expected return[1] and the value effect relating book-to-market value (BTMV) and expected return[2]. Fama and French (1992) showed that a 3-factor model, with factors that capture the size and value anomalies added to the market factor, is superior to the CAPM model. Jegadeesh et al. (1993) show that there is momentum in stock returns and Carhartt (1997) follow up with a 4-factor model that adds a momentum factor to the 3-factor model by Fama and French (1992). In recent years anomalies have been found to these 3- and 4-factor models and this have led to a q-factor model showed by Hou, K. et al. (2014). This model shows that their multifactor model with size, market, profitability, and investment factors outperform the 3- and 4-factor model previously suggested. Later, Fama and French (2015) showed that a 5-factor model that adds profitability and investment factors to their 3-factor model also improve performance of their asset pricing model. While most investigations on this topic have been made on the US stock market Fama and French (2012) also investigate anomalies and factors in international markets and finds the same tendencies except for Japan.

Anomalies connected to these asset pricing models keep appearing in literature, and Cochrane (2011) called it “a zoo of factors”. Harvey et al. (2015) research this zoo of factors and find that more than 200 papers have been published with factors or anomalies suggesting they help explain the cross-section of expected stock returns. Research has exploded within the last 10 years and it is a hot topic both among practitioners and scholars (see figure 1). The extensive research in this area could lead to statistical biases, which increases the risk of finding new factors and anomalies due to chance. With the high amount of papers published and probably even more thrown into the trash before publication we must be skeptical of the results and therefore the statistical requirements should also be higher. Harvey et al. (2015) and Lewellen (2010) raise these questions and argue that the multiple testing on stock returns must lead to a

[1] Banz (1981)
[2] Rosenberg et al. (1985)

higher statistical acceptance rate than the normal two standard deviation. They suggest that a three stdev acceptance rate should be used due to the extensive testing by many different authors.

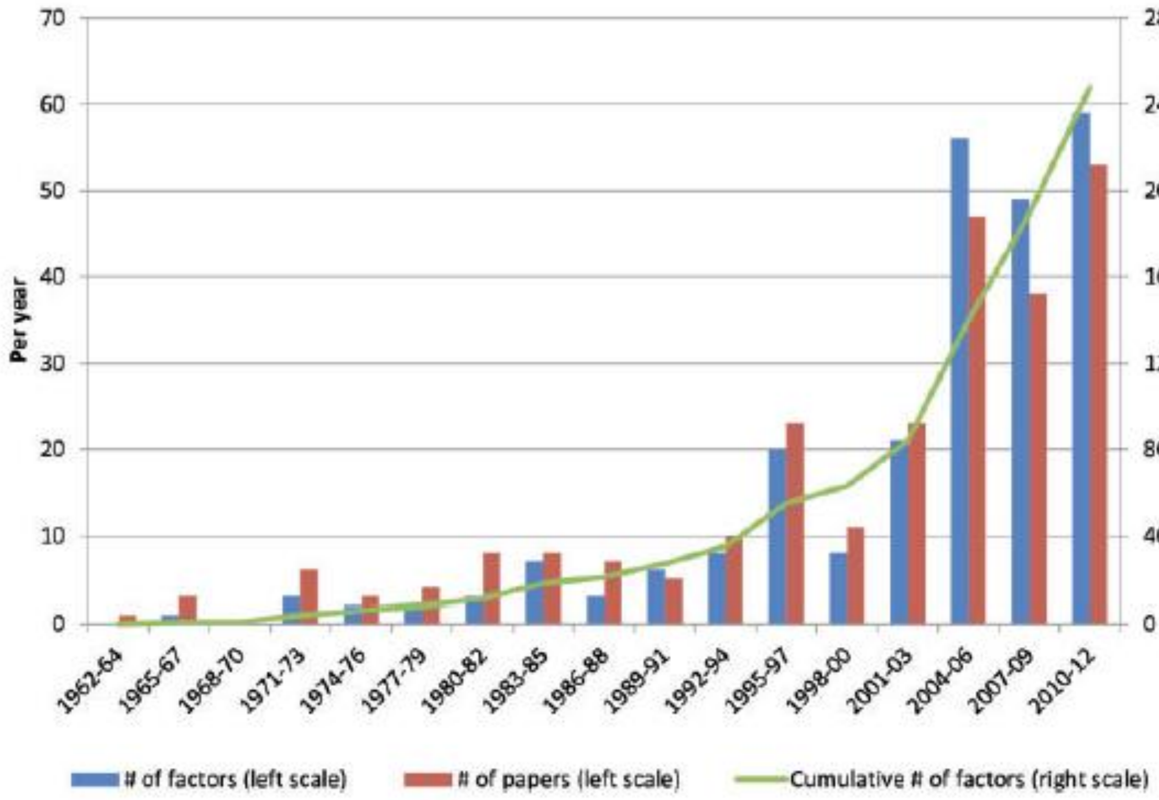

Figure 1: Papers and factors published from 1962-2012. Source: Harvey et al. (2015)

In addition to this research of anomalies and factors there are also different regarding the existence of these anomalies. One group of people finds that anomalies exist due to systematic risk unaccounted for in the asset pricing model thereby leading to new models with new risk factors as previously described. Another camp finds that anomalies exist due to irrational behaviors by investors.[3]

Anomalies and factors play a significant role when analyzing the variation in expected stock returns, and while the anomalies are well documented, there is still need to investigate this on other markets. Factors and anomalies are far less studied in the European stock market and there is still room for new evidence. This study also want analyze the effect of multiple testing on the results found in this field of asset pricing.

[3] Shiller (2001)

Furthermore, this study analyzes the risk and behavioral explanations behind each anomaly and thereby adds to the discussion on whether risk factors or irrational behavior by investors drive these anomalies observed in stock returns.

1.1 Research question

To investigate the issues mentioned above, this thesis will examine anomalies based on firm-characteristics in the European stock market in the period 2000-2015. The following seven anomalies will be analyzed: beta, volatility, size, value, momentum, profitability, and investment. This thesis will also investigate if factors based on these firm-characteristics can explain the variation in expected stock returns or if they are driven by investor irrationalities.

The main question is:

Can factors based on firm-characteristics explain the time-series variation in expected stock returns in the European market?

This question will be answered through the following sub-questions:

- Does anomalies exist in the European stock market in the period 2000-2015 and can these anomalies be explained by either risk based or behavioral explanations?
- Can factors formed as zero-cost portfolios explain the variation in expected stock returns in the European market?
- Can investors earn alpha returns in a multi factor asset pricing model?

1.2 Limitations

In the following the limitations in this study will be described.

This study will not consider transaction cost and taxation issues when evaluating trading strategies. In relation to transaction costs this study will not consider the potential increased transaction cost with short selling or the potential limits to short selling for some investors. However, this study will still comment on the potential effects of these real life issues. Moreover, this thesis uses monthly rebalancing of portfolios and while it is interesting to investigate the effects of different rebalancing intervals this thesis limits itself from this. Furthermore, the data in this thesis has been obtained through DataStream and Worldscope databases with access available through Copenhagen Business School. The data used in this thesis is limited to the availability and quality of data in DataStream and Worldscope. Finanly, for statistical models this thesis will assume that returns are normally distributed.

1.3 Structure of thesis

To answer the research question that has been outlined this study has been structured in the following way. Section 1 gives an introduction, presents the research question, and the structure of the thesis. Section 2 will review the empirical and theoretic evidence relevant for the study. Section 3 and 4 will go through the data and methodology used in the analysis. Section 5, 6, and 7 will present the results and a discussion of the results. Section 8 will be a summary discussion including how the results found by this study effects investors. Finally section 8 will present the overall conclusion

2. Literature review

This section will go through the theoretic foundation relevant for this study starting with explaining the two different view of market efficiency. The second section will go through empiric and theoretic evidence of the seven anomalies and factors investigated in this study. The third section will go through the most important asset pricing models there is. Finally, the last section will describe multiple testing and its implications for the discovery of anomalies and factors.

2.1 Market efficiency

When analyzing anomalies in expected stock returns the views of the market are essential. Scholars are divided in two camps: those who believe in risk based explanations and those who believe behavioral explanations. This section will line up the two different views of market efficiency.

Supporters of risk based explanations believe that anomalies does not exist, they are just a perception of unexplained risk factors in expected stock returns. Fama (1970) supported the risk based explanation and defines the Efficient Market Hypothesis (EMH). The EMH generally refers to three types of efficiency:

- **Weak form**, where stock prices fully incorporates historical price information with uncorrelated stock returns. Thus, historical stock returns cannot be used to predict the future and thereby expect to beat the market.
- **Semi-strong form**, where stock prices incorporates all publicly available information. In this regard public information consist not only of historic prices but also public statements, financial statements, and any other information that would affect or relate indirectly to a firms future profit. This implicates that an investor should not be able to beat the market using public information.
- **Strong form,** where stock prices fully reflect all available information.

Fama (1970) advocates for strong form market efficiency. When a market is efficient in the strong form stock prices always reflect the fundamental value the firm. For example when assuming a strong form market efficiency only new information will change the price of any given stock since all old information is already accounted for. If adding additional assumptions, that returns are independently identically distributed, as is often done in statistical models, the expected stock returns can be modeled as a random walk model as done by Fama (1970):

$$E(r_{j,t+1}|\Phi_t) = E(r_{j,t+1}) \quad \boldsymbol{EQ\ 2.1}$$

EQ 2.1 then states that the expected returns at t+1 are independent of the information (Φ) at time t.

If markets are fully efficient as defined by Fama (1970) there is no point in investing actively, but if no one invest actively how will prices then reflect new information? Pedersen (2015) discuss this topic and argues that markets must be efficiently inefficient. Efficiently enough so no new capital is added to money management but also inefficiently enough so active managers can be paid money to study markets and trade on the information they find.

On the other hand Shiller (2001) [4] study behavior of investors. Shiller (2001) argues that prices deviates from fundamental value due to irrational behavior of investors. Robert Shiller was together with Eugene Fama and Lars Peter Hansen added the Nobel Prize for their very different work and view on markets. Shiller (2001) takes us through some of the important social and psychological patterns in human's and how they affect market efficiency. The main points from behavioral finance and how they affect market efficiency are outline below:

- **Prospect theory** describes the way people think of risk based on potential losses and gains using heuristics. The theory tries to model real-life decisions rather than rational and optimal choices.
- **Anchoring**, people tend to anchor to what they already know or what is suggested to them.
- **Overconfidence** in investors can lead to too high amounts of trading.
- **Over- and under reaction** to news or earnings announcements can lead to mispricing of stocks.
- **Extrapolation** of returns too far into the future. Related to overreaction and can lead to overpricing of stocks that have performed well in the past.
- **Familiarity** can lead investors to invest in firms they already know and this can lead to overpricing of big known firms.

Shiller (2001) does not think that markets are efficient but that irrational behavior by investors' lead to mispricing and that this can be picked up by skilled investors.

[4] Based on previous work by Tversky and Kahneman (1992)

The debate between advocators of behavioral explanations and risk based explanations is still ongoing.

2.2 Anomalies and Factors

This section will describe the empiric evidence and theoretical foundation of the seven anomalies analyzed in this study. By laying the theoretic and empirical foundation of the anomalies, this can be used as comparison and for the discussion of results found by this study. Anomalies can also be seen as investment styles and are by this study defined as investment strategies that have higher risk adjusted return than the market portfolio. Anomalies can be based on factors such as accounting numbers, financial ratios, behavioral statistics, and macroeconomic ratios. These anomalies can either be common factors in the market or based on firm-level characteristics. This thesis will focus on firm-level characteristics as it is possible to trade on firm-level characteristics and this is necessary for the methodology used by this study.

Factors are created in various ways by different authors but generally they are created as zero-cost portfolios where portfolios are sorted based on one, two, or three firm-characteristics. Factors are then created as zero-cost portfolios portfolio where you go long in the high / low and short in the low / high portfolio depending on whether there is a positive or negative relationship between the firm-characteristic and average return. While it can be difficult for private investors to create long / short portfolios the factors shouldn't be seen on as trading opportunities but more as factors that capture the potential risk connected with the specific firm-characteristics.

2.2.1 Beta

The market factor was first introduced by Sharpe (1964) and Lintner (1965) in their papers on the Capital Asset Pricing Model (CAPM) that relates the expected return of an asset to its systematic risk. The systematic risk in the CAPM model is known as beta (β) and it determines the assets volatility to the market:

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} \quad \boldsymbol{EQ\ 2.2}$$

Where r_i is the return of an asset i and r_M is the return of the market portfolio. Beta is the sensitivity to the market factor in the CAPM and it determines the covariation of an asset with the market portfolio.

The quantification of the relation between an assets volatility and its expected return was a major breakthrough at the time and Sharpe was later awarded the Nobel Price together with Markowitz and Merton Miller for their contributions to the field of financial economics.

$$E[R_i] = r_f + \beta_i\left(E[R_M] - r_f\right) \ \boldsymbol{EQ\ 2.3}$$

The total risk of an asset is the variance. Beta measures the systematic risk in the asset. To the left is the unsystematic risk, also called the asset specific risk. Unlike unsystematic risk, systematic risk cannot be diversified away. The CAPM model is expressed in EQ 2.3, where r_f is the risk free rate of return. This model is an equilibrium single factor model and while the market factor still to this day remains a part of most multifactor asset pricing models the CAPM has received criticism over the years. The bold prediction of the CAPM is that the market portfolio is mean-variance efficient and this implies a linear relationship between beta and expected return as seen by the blue line in figure 2.1. Fama and MacBeth (1972) tested this model in the period 1936-1968 on common stocks on the New York Stock Exchange (NYSE) and found that there is a positive relationship between the risk factor beta and average return in the entire period.

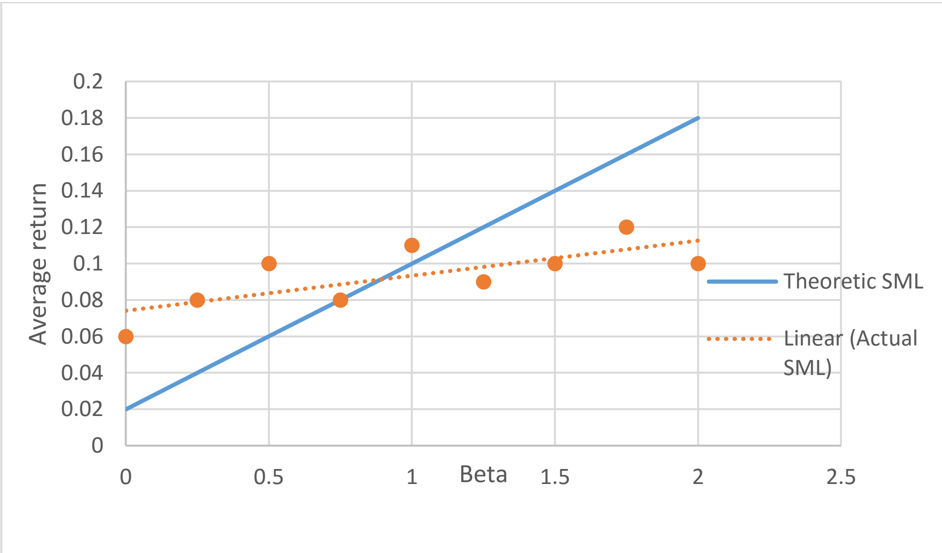

Figure 2.1 – The Security Market Line (SML) showing the risk return relationship in the CAPM. Source: Own figure

In the 1980's several anomalies were found to the CAPM model and the relationship between beta and expected returns was proofed to be more flat (see figure 2.1)[5]. The explanation to the failure of the CAPM model may be that investors are leverage constrained. When investors are leverage constrained but are looking for a higher return they buy stocks with higher risk, this puts a price pressure on high beta stocks and thereby lowers average returns. Frazzini et al. (2014) quantifies this with their betting against beta

[5] Banz (1981), Rosenberg et al. (1985) and others.

factor that shorts high beta stocks and buys low beta stocks to make a zero-beta portfolio that has a Sharpe ratio of 0.78 for US stocks in the period 1926-2012.

While most academics agree that the CAPM fail at explaining the cross-section of expected stock returns, the market factor is still an essential part of most multifactor asset pricing models. Recently Harvey, R. et al. (2015) showed that the original market factor is by far the most important factor when explaining the cross-section of expected stock returns.

2.2.2 Volatility

The connection between volatility and return is natural, investors are always interested in the risk adjusted return. It is not only the relationship between systematic risk (beta) and expected return that do not hold. The relationship between total risk and expected return has also been showed to fail. The minimum-variance portfolio has been showed to significantly outperform the value weighted portfolio. The minimum-variance portfolio is the portfolio on the mean-variance efficient frontier with the lowest volatility. Haugen et al. (1991) show that a minimum-variance portfolio produces higher or equal returns with markedly lower variance. The results are backed up by Blitz et al. (2007) who show that a low minus high volatility factor formed on three years previous volatility produces annual alpha spread of 10.2% in the European market in the period 1986-2006. Also, they find that the Fama French 3-factor model cannot explain the volatility effect.

The connection between the low volatility anomaly and the low beta anomaly is closely related but not the same. The low volatility anomaly claims that low volatility stocks outperform high volatility stocks whereas the low beta anomaly says that risk premium of low beta stocks is underestimated in the CAPM model. The explanations are the same, investors are either leveraged constrained or does not like to leverage their investments.

2.2.3 Size

The size effect was one of the first anomalies to be described in detail in literature. Banz (1981) examined the relationship between expected return and total market value in NYSE common stocks in the period 1936-1977 and he found that the risk adjusted return was higher in small stocks. He did not find a linear relationship between size and risk adjusted return. Furthermore, the main effect was for very small firms. Banz (1981) defined the size factor as:

$$\gamma\left[\frac{\Phi_{\mathrm{i}}-\phi_m}{\phi_m}\right] \quad \boldsymbol{EQ\ 2.4}$$

Where ϕ_i is the market value of security i and ϕ_m is the average market value. Gamma (γ) is the loading that the security takes on the size factor. He proceeded to create 25 portfolios sorted on beta and size and test them against a two-factor model:

$$R_{it}=\gamma_{0t}+\gamma_{1t}\beta_{it}+\gamma_{2t}\left[\frac{\Phi_{\mathrm{i}}-\phi_m}{\phi_m}\right]+\epsilon_{it} \quad \boldsymbol{EQ\ 2.5}$$

Where γ_0 is the expected return on a zero-beta portfolio, β_i is the classic interpretation of beta from the CAPM model, and γ_1 and γ_2 is the loadings on beta and the size factor, respectively. Banz (1981) found that there is a varying but consistent premium for the size factor in ten-year sub periods from 1936-1977 (see figure 2.2).

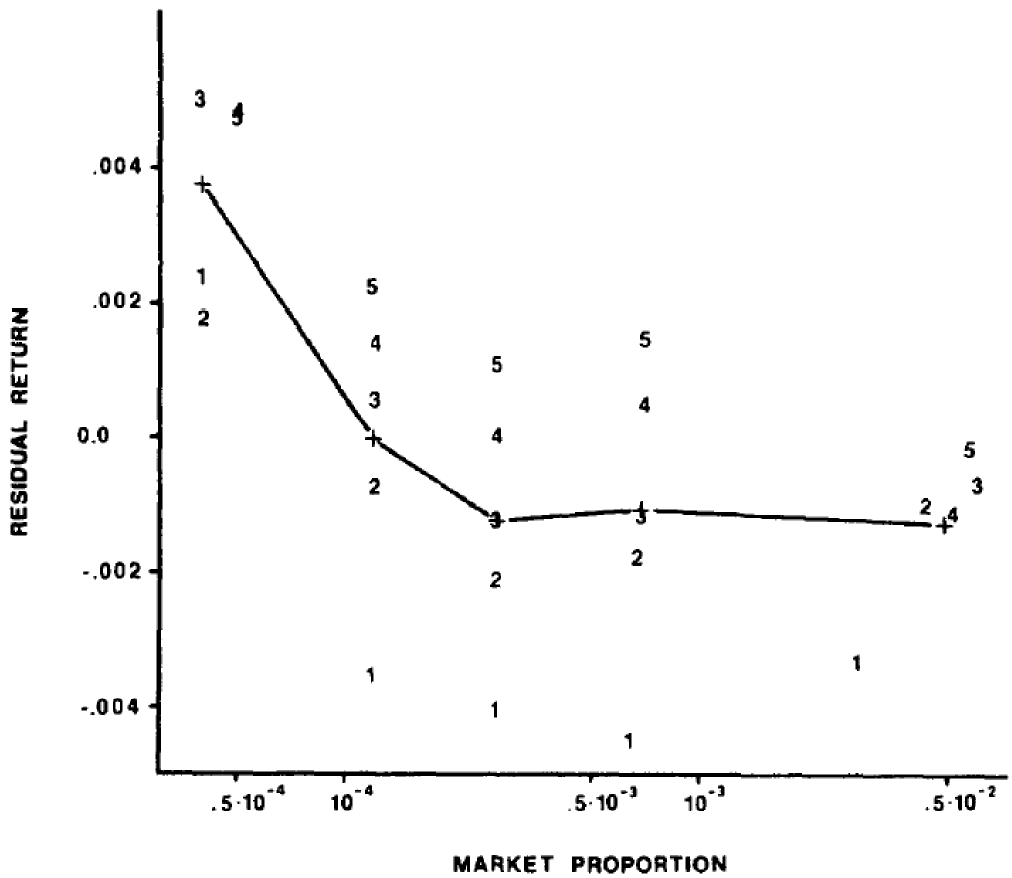

Figure 2.2 – Relationship between size and residual return in the period 1936-1977. Source: Banz (1981)

Fama and French (1992) followed up on the research by Banz (1981) with their article leading to the 3-factor model. Fama and French (1992) found a negative relationship between market cap (size) and average monthly return in the period 1963-1990. They sorted portfolios into 10 equal sizes sorted on size and beta, which showed two results: 1) a negative relationship between size and average return and 2) when sorted for size there is no relationship between beta and average return as predicted by the CAPM (see figure 2.3). This discovery led to intensified research in the area trying to understand why the CAPM model failed and this created attention around the size effect and other anomalies trying to explain the cross-section of expected stock returns.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

Figure 2.3 – Average monthly returns for portfolios sorted on size and beta. Source: Fama and French (1992)

However, in 2010 Fama and French (2010) showed that, in the period from 1990-2011, their size factor (small minus big) had a negative average return of -0.06% in the European market. This has led to discussion whether the size effect has disappeared and this study will investigate this.

Explaining the size factor from an economic point of view is not a simple task because there is no obvious relationship between expected return and the size of a firm. This leads to the opinion of the harshest critics of the size effect, that it is simply a statistical fluke or that the size effect is a proxy for other real risk factors. Fama and French (1992) argue that size is a risk factor unaccounted for in the original CAPM model. Fama and French (1992) also argue that small firms take larger hits during depressions and this indicate that size is a risk factor:

"The fact that small firms can suffer a long earnings depression that bypasses big firms suggest that size is associated with a common risk factor that might explain the negative relation between size and average return. "

While Fama and French (1992, 2010, 2015) advocate risk based explanations scholars such as Shiller (1981) and Lakonishok et al. (1994) advocates for behavioral explanations for factors in general. Behavioral explanations that are suggested for the size effect are overpricing of large stocks, overpricing of growth stocks, and incomplete information about small firms[6]. Furthermore, people like and familiarize themselves with companies they know and this can lead to overpricing of large stocks. Moreover, growth stocks are often stocks that have performed well in the past and this can lead investors to over extrapolate past performance into the future and thereby overprice growth stocks. Also, it is generally harder to get information on smaller firms than larger firms and this can lead to higher costs of acquiring information for investors. Again this could lead to overpricing of large stocks. In general, behavioral explanations cannot be proved with numbers, but the arguments can be just as convincing.

Finally, liquidity risk could also be an argument against the size effect. Acharya et al. (2004) found a high degree of liquidity risk in stocks in the period 1964-1999 and showed that especially a factor with liquidity sensitivity to market returns has a monthly return premium of 0.82%, suggesting that investors are willing to pay a premium for holding stocks that are liquid when the market is not. Small firms typically have a lower trading volume and with the size effect primarily being present in the smallest firms this could be due to this liquidity risk connected with holding small stocks.

2.2.4 Value

Value investing means investing in companies that have high fundamental value compared to market value and it tracks back as far as Graham and Dodd (1934). Value investing includes valuation of companies, which can be a daunting task, but value investing has performed well historically even by the most simple measures. Measures of value can be:

- Book-to-market equity
- Cash flows-to-price
- Dividends-to-price

[6] Shiller (2001)

- Earnings-to-price and others.

The over performance of value stocks compared to growth stocks (stocks with low ratios of fundamentals to price) has been shown by several scholars across different markets, among others Fama and French (1992, 2010), and DeBondt and Thaler (1985).

Fama and French (1993) showed, in the period 1963-1991 on the US stock market, that a high minus low (HML) factor, where they go long in the 30% stocks with highest book-to-market values and short the 30% with lowest book-to-market values, produce a monthly average return of 0.4%.

Most early research was done on the US stock market, but Fama and French (2010) also showed the existence of a value premium in international markets. They showed that, in the period 1990-2011, a high minus low factor gives a monthly average return of 0.55% in the European stock market. They also showed that a value premium exist in all international markets except Japan.

Asness et al. (2013) added to the research of value as a risk factor with a study on value and momentum across different asset classes and different markets. They showed that a value premium exist across eight diverse markets and asset classes and found that there is a strong correlation structure across otherwise unrelated asset classes. They argue that this is evidence of value and momentum as common global risk factors (see table 2.1).

European stocks

	P1	P2	P3	P3-P1	Factor
Mean	11.8	14.6	16.7	4.8	5.2
Stdev	18.3	18	19.8	11.5	9.7
t-stat	3.53	4.43	4.61	2.32	2.95

Global stocks

	P1	P2	P3	P3-P1	Factor
Mean	8.1	11	14.6	6.2	5.8
Stdev	16.6	15.2	15.7	10.9	11.4
t-stat	3.17	4.54	5.84	3.6	3.18

Table 2.1 – Average yearly returns for portfolios sorted on market to book value in the period 1974-2011. Source: Asness et al. (2013).

DeBondt and Thaler (1985) show that a portfolio of prior losers outperforms prior winners by more than 25% 36 months after portfolio formation (see figure 2.4). They form portfolios based on the previous 36 months of data and so they define value firms as firms that have performed poorly in the past and growth firms vice versa. They find that the over performance of the losers portfolio is due to overreaction in the market. They argue that investors tend to extrapolate past performance too far into the future and that investors tend to overreact to bad and good news. This leads investors to follow a naive strategy where they all buy the same portfolio of winners and sell the same portfolio of losers, and this leads to underpricing of “bad” stocks and overpricing of “good” stocks. Following a value strategy is simply following a contrarian strategy to this naive strategy of just following past winners.

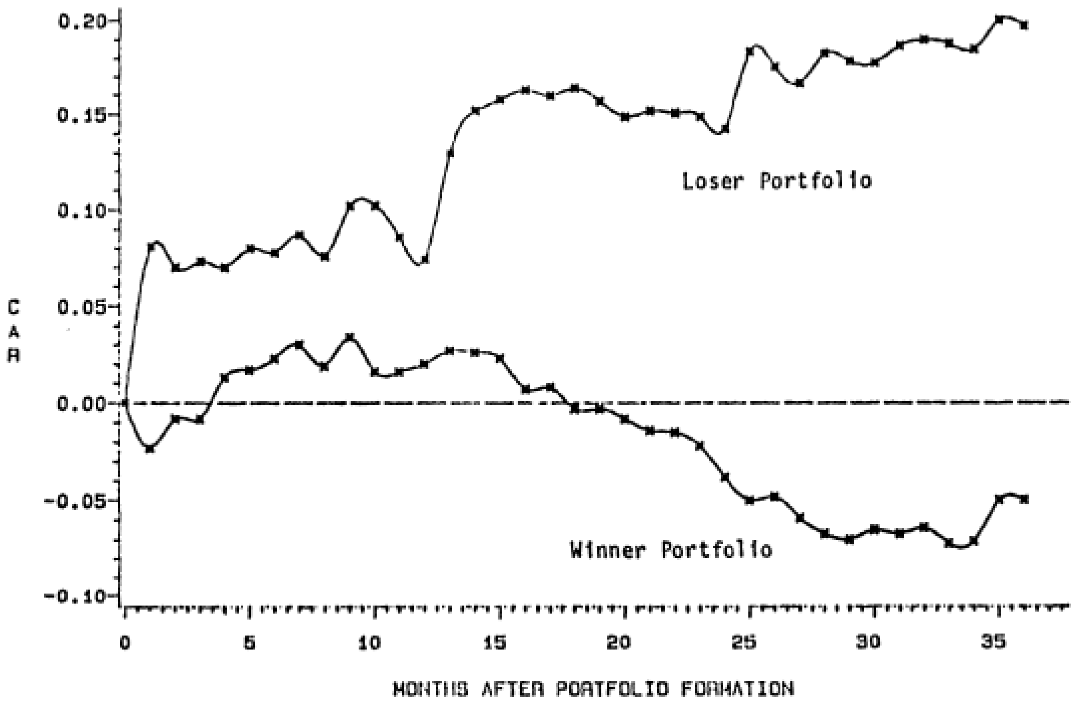

Figure 2.4 – Cumulative average residual returns for Winner and Loser portfolios in the period 1933-1980. Source: DeBodt and Thaler (1985)

2.2.5 Momentum

Momentum is another investment strategy that has shown anomaly returns previously. A momentum strategy buys previous winners and sells previous losers, which is the opposite of a value strategy, and it can seem absurd to study a momentum strategy with the overwhelming evidence of a contrarian strategy. One of the first to research momentum was Levy (1967) who showed that buying stocks with prices substantially higher than their past 27 week average would yield a positive return.

Jegadeesh et al. (1993) also researched momentum in stock returns and its implications for stock market efficiency. They form portfolios of winners and losers from the past three, six, nine and 12 months on average returns and hold them for three, six, nine and 12 months, respectively. They researched both portfolios formed right after the lagged returns are measured and one week after giving them a total of 32 different portfolios. Table 2.2 show the results of the 16 portfolios formed one week after the lagged returns used for forming these portfolios are measured. Table 2.2 show a clear short and medium term momentum in stock returns with all buy and buy minus sell portfolios being significant at a 5%-level. Jegadeesh et al. (1993) suggest to use a strategy that buys past winners from 12 month lagged returns and hold them for three months, this strategy show an average monthly return of 1.96% with a t-stat of 4.73 in the period January 1965 to December 1989.

Momentum in stock returns

J	K ->	3	6	9	12
3	Sell	0.0083 (1.67)	0.0079 (1.64)	0.0084 (1.77)	0.0083 (1.79)
3	Buy	0.0156 (3.95)	0.0158 (3.98)	0.0158 (3.96)	0.0160 (3.98)
3	Buy-Sell	0.0073 (2.61)	0.0078 (3.16)	0.0074 (3.36)	0.0077 (4.00)
6	Sell	0.0066 (1.28)	0.0068 (1.35)	0.0067 (1.38)	0.0076 (1.58)
6	Buy	0.0179 (4.47)	0.0178 (4.41)	0.0175 (4.32)	0.0166 (4.13)
6	Buy-Sell	0.0114 (3.37)	0.0110 (3.61)	0.0108 (4.01)	0.0090 (3.54)
9	Sell	0.0058 (1.13)	0.0058 (1.15)	0.0066 (1.34)	0.0078 (1.59)
9	Buy	0.0193 (4.72)	0.0188 (4.56)	0.0176 (4.30)	0.0164 (4.04)
9	Buy-Sell	0.0135 (3.85)	0.0130 (4.09)	0.0109 (3.67)	0.0085 (3.04)
12	Sell	0.0048 (0.93)	0.0058 (1.15)	0.0070 (1.40)	0.0085 (1.71)
12	Buy	0.0196 (4.73)	0.0179 (4.36)	0.0167 (4.09)	0.0154 (3.79)
12	Buy-Sell	0.0149 (4.28)	0.0121 (3.65)	0.0096 (3.09)	0.0069 (2.31)

Table 2.2 – Average returns of portfolios formed based on J-months lagged returns and held for K-months. Portfolios are formed one week after the lagged returns used for forming the portfolios are measured. The t-statistics are reported in parenthesis. The sample period is 1965-1989. Source: Jegadeesh et al. (1993)

The results from Jegadeesh et al (1993) was followed up by Carhart (1997) who used his results to show that a 4-factor model (see EQ 2.6) with a momentum factor added to the Fama French 3-factor model, show superior performance compared to the CAPM and the Fama French 3-factor model explaining the persistence in equity mutual funds mean and risk-adjusted returns. Carhart (1997) called his momentum factor PR1YR and it is formed following suggestions of Jegadeesh and Titman (1993). Carhart (1997) used his results to argue that the alpha returns of mutual funds in the period 1962 to 1993 is more likely due to common risk factors in equity returns than the skills of fund managers.

$$R_i - R_f = \alpha_i + b_i(R_M - R_f) + s_i SMB + h_i HML + m_i PR1YR_t \epsilon_i \quad \boldsymbol{EQ\ 2.6}$$

Another study by Asness et al. (2013) researched the value and momentum anomalies across eight different asset classes and markets, and found consistent momentum across all markets and asset classes. They also found a consistent correlation pattern across otherwise unrelated asset classes supporting the evidence of momentum as a common risk factors. Furthermore, they found that a combination of value and momentum strategies increase the risk adjusted returns since the two factors have a high negative correlation.

It is remarkable that momentum and value strategies have showed such different results, as the strategies use the same phenomenon just at different time horizons. Value strategies exploit long term reversal in prices whereas momentum exploits momentum in stock returns in short to medium term. The potential behavioral explanations are identical for value and momentum as it can be argued that momentum exploits the overreaction from investors in the short to medium term, and jumps along for the ride but exits before there is reversal in price. A momentum strategy can be seen as contrarian to a value strategy and this is also the case for the next anomaly in this study – profitability.

2.2.6 Profitability

The relation between profitability and average stock returns can be showed by looking at the dividend discount model. Novy-Marx (2012) and Fama and French (2015) used this approach when they study the relation between average returns and investment, profitability, and value. The dividend discount model show that the market value of a firms stock is equal to all future expected dividends per share discounted by the rate of return:

$$M_t = \sum_{s=1}^{\infty} \frac{E(d_{t+s})}{(1+r)^s} \quad \boldsymbol{EQ\ 2.7}$$

Where M_t is the market value, d_{t+s} is the dividend, and r is the discount rate or expected return. Assuming that firms use clean surplus accounting the dividend discount model can be rewritten to:

$$M_t = \sum_{s=1}^{\infty} \frac{E(Y_{t+s} - dB_{t+s})}{(1+r)^s} \quad \boldsymbol{EQ\ 2.8}$$

Where Y_{t+s} is total equity earnings for period t+s and dB_{t+s} is the change in total book equity. By then dividing by time t book equity the equation is the following:

$$\frac{M_t}{B_t} = \sum_{s=1}^{\infty} \frac{E(Y_{t+s} - dB_{t+s})/(1+r)^s}{B_t} \quad \boldsymbol{EQ\ 2.9}$$

The expressions in EQ 2.9 show the relationship between expected return and profitability, value and investment. By freezing different terms of EQ 2.9 it is intuitive to see the effects of the three factors. First fix everything but M_t and r then a lower value of M_t implies a higher value of r. So a lower market-to-book value (MTBV), the definition of a value stock, leads to a higher expected return in line with what we have seen earlier. Secondly, freezing everything but Y_{t+s} and r, then it is intuitive to see that higher expected earnings should lead to higher expected returns. Third, freezing everything but dB_{t+s} and r, then a higher expected change in book equity (increased investments) would lead to lower expected returns.

There are different ways of measuring profitability and the key is to find the best proxy for future expected profitability. Hou, K. et al. (2014) used return on equity (ROE). Fama French (2014) used operating profitability (OP) that they define as annual revenues minus cost of goods sold, interest expense, and administrative expenses divided by book equity at the end of fiscal year t-1. Hou, K. et al (2014) found that their ROE factor has an average monthly return of 0.58% (t-value = 4.81) in the period 1972 to 2012 on the NYSE. Table 2.3 show the results found by Fama and French (2015) on their Size-OP portfolios. Table 2.3 show a positive relationship between profitability and average return from weak to robust profitability. The relationship is clear between the robust and weak portfolios but when you also look at portfolios 2 to 4 the relationship is not as strong (see table 2.3).

Size-OP portfolios

	Weak	2	3	4	Robust
Small	0.56	0.94	0.9	0.95	0.88
2	0.59	0.78	0.84	0.81	0.98
3	0.53	0.77	0.72	0.78	0.94
4	0.57	0.65	0.63	0.7	0.82
Big	0.39	0.33	0.43	0.47	0.57

Table 2.3 – Average monthly returns in percent in the period 1963 to 2013 for portfolios formed on size and profitability on the NYSE. Source: Fama and French (2014)

A different way to look at profitability is done by Novy-Marx (2012). He researched three different measures of profitability: gross profitability, earnings to book equity and free cash flow to book equity. He found that gross-profitability has most power in explaining the cross-section of expected stock returns. He argue that it is important to find the most clean accountant measure of profitability and therefore he also defines gross profitability simply as revenue minus cost of goods sold. Noxy-Marx (2012) also raised some issues towards value as a factor. He showed that profitable companies are often companies with low valuation ratios, meaning that investing in a profitability strategy means investing in growth companies, the opposite of a value strategy. He found that value and profitability have a negative correlation of -0.57. This gives rise to interesting trading opportunities where a combination of a value and profitability strategies can give higher risk adjusted returns. Furthermore, he found that a strategy that combines value and profitability more than doubles the Sharpe ratio of the individual strategies. This is much in line with the recommendations of Asness et al. (2014) who define a Quality Minus Junk (QMJ) factor that combines quality investing with value investing which they call “quality at a reasonable price”. Using the QMJ factor or just combining value and profitability strategies can help investors avoid value traps, growth traps and other “bad” companies.

Explaining the price anomaly observed with profitability from a risk based point of view can be hard as we are simply investing in firms that are more profitable. Asness et al. (2014) were also puzzled by this and found that their QMJ factor (a combination of value and profitability) does not take on more risk but it actually protects investors in market downturns. In addition, Fama and French (2006) found that, assuming rational pricing, the profitability effect shown in the valuation equation EQ 2.9 per definition assumes higher risk in more profitable firms through the expected return. However, they did not define profitability as a risk factor, but simply added that there are other possibilities than irrational pricing by investors. Behavioral explanations for the profitability anomaly are equal hard to find, but it seems that investors underestimate firm’s ability to bring current profitability into future years.

In summary, there is evidence of a profitability anomaly in expected stock returns but it is hard to explain the anomaly from a theoretic point of view, both risk based and behavioral explanations are weak. Profitability is closely linked to investment through EQ 2.9 and this is the next anomaly that will be investigated.

2.2.7 Investment

Investment is closely linked with profitability and value through EQ 2.10, which is the dividend discount model. Investment in the dividend discount model is the change in total book equity, dB_{t+s}. From EQ 2.10 it is evident that, all else equal, an increase in investments will lead to a lower total market value and vice versa.

$$M_t = \sum_{s=1}^{\infty} \frac{E(Y_{t+s} - dB_{t+s})}{(1+r)^s} \quad \boldsymbol{EQ\ 2.10}$$

This leads to the observation that high investment firms should earn lower expected returns and low investment firms should earn higher expected returns. If high investment firms did not have a lower expected return, they would be better off lowering their investments (see figure 2.5). Stock prices should then, in theory, adjust to recognize the connection between expected return and investment. Figure 2.5 also show the characteristics of firms that have high and low investments on each side of the y-axis. A value firm (high BTMV) often has low investments and vice versa for growth firms. Other characteristics such as net stock issues and accounting accruals also define firms that typically have low or high investments. The expected return of stocks can also be seen as a risk parameter, if investors require higher expected return this is, in theory, due to higher risk in the firm. A constant expected return across all firms would imply that all firms are equally risky and that stock prices follow a random walk.[7]

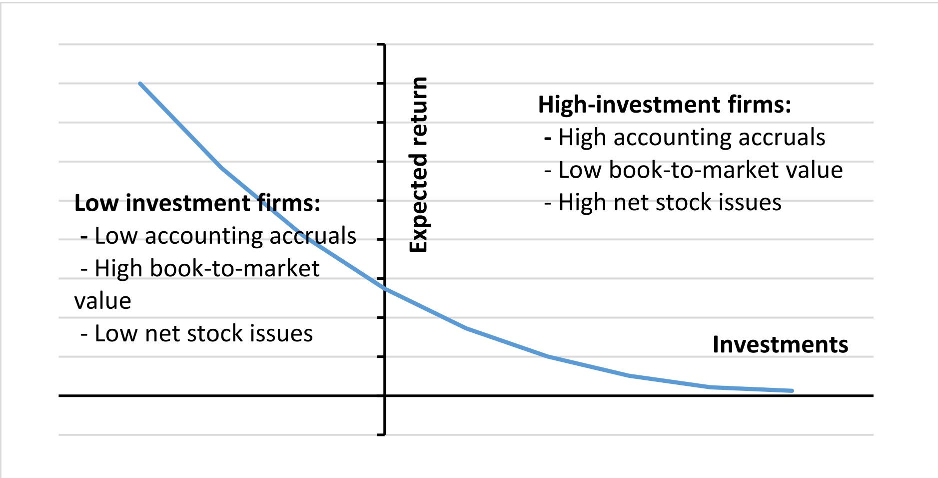

Figure 2.5 – The relation between expected return and investments. Source: Inspired by Hou, K. et al (2014)

Hou, K. et al. (2014) showed a negative relationship between investments and average returns. Hou, K. et al. (2014) also showed that in a q-factor model[8] their investment factor generate a monthly return of 0.45% (t-stat = 4.81) in the period from 1972-2012 on the NYSE. They also found that the investment factor has a correlation of 0.69 with the Fama French (2014) HML factor. The high correlation between value and investment raise the question whether one factor is just a proxy for the other, and if they both can add to

[7] Hou, K. et al. (2014)
[8] A 4-factor model that include market, size, profitability, and investment factors.

the cross-section of stock returns. In the q-factor model they leave out the classic value factor and find that a q-factor model with investments, profitability, size, and a market factor is sufficient.

Table 2.4 show the results from Fama and French (2014) that also showed a negative relationship between investment and average returns. However, as it can be seen from table 2.4 the negative relationship between investment and average return is only present between the most conservative and most aggressive portfolios. For portfolio 2, 3, and 4 there is no relationship between investment and average return across the five size portfolios.

Size-Inv. portfolios

	Conservative	2	3	4	Aggressive
Small	1.01	0.98	0.99	0.89	0.35
2	0.92	0.91	0.92	0.9	0.48
3	0.9	0.93	0.81	0.82	0.5
4	0.79	0.72	0.71	0.75	0.54
Big	0.71	0.52	0.49	0.48	0.42

Table 2.4 – Average monthly returns in percent in the period 1963-2013 for portfolios formed on size and investment on the NYSE in the period 1963-2013. Source: Fama and French (2015)

Table 2.5 show results from Fama and French (2014) who sorted investment, profitability, and size to show that the investment anomaly is present for both big and small stocks across four profitability portfolios. To lower the number of portfolios and increase the diversification in the portfolios the sorts are divided into just two size groups – small and big. Table 2.5a show the sort on investment and profitability for small stocks. Furthermore, there is a negative relationship between investment and average return across all 4 profitability sorts (see table 2.5a). The lower left corner of table 2.5a is a portfolio of firms that have low profitability but still invest aggressively. The average return of this portfolio is -0.09 and it is a significant outlier. This portfolio is the biggest problem for their 5-factor asset pricing model and they name this portfolio *"The lethal combination of investment and profitability"*.[9]

Table 2.5b show that size does not affect the investment anomaly. The average return is lower due to the size anomaly, but the relationship between investment and average return is also strong for big stocks across the four profitability portfolios. Table 2.5b also show that the negative average return of low profitability stocks that invest aggressively is only present for small stocks.

[9] Fama and French (2014)

Size, OP, and Inv. portfolios
a. Small stocks

	Low OP	2	3	High OP
Conservative	0.85	1.01	1.19	1.27
2	0.94	0.9	0.92	1.04
3	0.61	0.93	0.94	1.06
Aggressive	-0.09	0.58	0.76	0.76

b. Big stocks

	Low OP	2	3	High OP
Conservative	0.63	0.66	0.79	0.7
2	0.32	0.43	0.64	0.64
3	0.52	0.57	0.48	0.53
Aggressive	0.29	0.25	0.38	0.65

Table 2.5 – Average monthly returns in percent in the period 1963-2013 for portfolios formed on size, OP, and investments on the NYSE. Size is divided into small and big stocks. Source: Fama and French (2015)

Defining the proxy for investments is important. Hou, K. et al. (2014) and Fama and French (2014) followed the recommendations of Aharoni, Gil. et al. (2013) who defined investments as the growth of total asset in the fiscal year divided by 1-year lagged total assets. Aharoni, Gil et al. (2013) also research the growth of book equity as a proxy for expected investments and finds that there is little difference between the two measures.

As was shown with profitability, investments are also connected to the expected return through EQ 2.10. This showed that following rational pricing, lower investments lead to higher risk. From a behavioral point of view it is hard to explain why investors overprice firms that invest aggressively. However, it has been showed that high investment firms share the same characteristics as growth firms. Therefore, the investment anomaly could also be affected by familiarity and trend following as observed with growth of firms.

2.3 Asset pricing models

Having gone through all the anomalies and factors subject for analysis in this thesis this section will describe in detail the most important multi factor asset pricing models suggested in literature. Multi factor asset pricing models can take all kinds of variables as input, but the models subjected for analysis in this thesis are models that take zero-cost portfolios as input. The methodological details of multi factor models will be described in section 4.4.

When reviewing asset pricing models it is natural to start with the CAPM as was briefly touched upon in the section about beta. The CAPM has excess market risk as the lone explanatory factor for expected stock returns:

$$R_i - R_f = \alpha_i + \beta_i(R_M - R_f) \quad \boldsymbol{EQ\ 2.11}$$

Anomalies were found to the CAPM showing that investment strategies sorted on value and size were able generate positive alpha returns in the CAPM. This led Fama and French (1992) to purpose their 3-factor model:

$$R_i - R_f = \alpha_i + b_i\left(R_M - R_f\right) + s_i SMB + h_i HML \quad \boldsymbol{EQ\ 2.12}$$

Where $R_M - R_f$ is the market factor, SMB is the size factor, and HML is the value factor. $R_i - R_f$ is the excess return of a stock or portfolio and alpha is the return not explained by the model. Fama and French (1992) showed that their 3-factor model was able to eliminate size and value anomalies but also that their model explains a higher percentage of the variation in expected stock returns. New anomalies led Carhartt (1997) to propose a 4-factor model with a momentum factor added to the 3-factor model:

$$R_i - R_f = \alpha_i + b_i\left(R_M - R_f\right) + s_i SMB + h_i HML + m_i PR1YR \quad \boldsymbol{EQ\ 2.13}$$

Where everything is as in EQ 2.12 and $PR1YR$ is a momentum factor. Carhartt (1997) found that his 4-factor model captures the momentum anomaly. Also, he showed that in comparison to the CAPM and 3-factor model the 4-factor was superior. The absolute monthly average error terms (α_i) from the CAPM, 3-factor, and 4-factor models were 0.35%, 0.31% and 0.14%, showing that that his 4-factor model more than halved the pricing errors from the 3-factor model. Carhartt (1997) also added that almost all patterns in pricing errors are gone, indicating that it well describes the cross-section of expected stock returns.

More recently new asset pricing models based on investment theory and the dividend discount model have showed up. Hou, K. et al. (2014) showed that a q-factor model consisting of size, market, profitability, and investment factors is superior to all previous asset pricing models:

$$R_i - R_f = \alpha_i + \beta^i MKT^E + \beta^i_{ME} E[r_{ME}] + \beta^i_{\frac{I}{A}} E\left[r_{\frac{I}{A}}\right] + \beta^i_{ROE} E[r_{ROE}] \quad \boldsymbol{EQ\ 2.14}$$

Where MKT^E is the market factor, r_{ME} is a size factor, $r_{\frac{I}{A}}$ is an investment factor, r_{ROE} is a profitability factor, and beta is the loadings on each factor. Hou, K. et al. (2014) compared their q-factor model with the 3- and 4-factor models by testing 35 anomalies and showed that the average absolute pricing errors across the 35 anomalies are 0.20%, 0.33%, and 0.55% per month for the q-factor model, the 4-factor model, and the 3-factor model respectively. Overall they found that their q-factor model outperforms the 3- and 4-factor models significantly. Fama and French (2015) also suggested an asset pricing model based on investment theory. They presented a 5-factor model that added profitability and investment to their 3-factor model:

$$R_i - R_f = \alpha_i + b_i(R_M - R_f) + s_i SMB + h_i HML + r_i RMW + c_i CMA \quad \boldsymbol{EQ\ 2.15}$$

Where RMW is a profitability factor and CMA is an investment factor. Fama and French (2015) found that their 5-factor model was an improvement from their 3-factor model across all the portfolios they tested. The main difference from the q-factor model to the 5-factor model is that Fama and French (2015) still includes a value factor (HML). However, they do find, in line with Hou, K et al. (2014), that HML is a redundant factor in the 5-factor model. The explanation is that the average HML return is captured by the exposures of HML to other factors in the model. Despite the evidence that HML is a redundant factor Fama and French (2015) still suggest the use of a 5-factor model as it can help to show portfolio tilts towards the HML factor.

Finding the best possible asset pricing model is a tightrope between having as few factors as possible while hiving a precise description of variation in expected returns. The most important in the valuation of an asset pricing models is that the absolute average pricing errors are low. However, having a model with low pricing errors that only explains 50% of the variation in expected stock returns does not say much. Therefore, we want a model that is precise while describing as much of the variation in expected returns as possible. The expansions of the asset pricing models through time are a consequence of constant discoveries of new anomalies in expected returns. This extensive research leads to issues of multiple testing that will be considered in the next section.

2.4 Multiple Testing

Issues with multiple testing have been known in statistic literature for many years and was first covered by Tukey (1951,1953). Despite multiple testing issues have been taking into account in many other fields such as medical and physics research, it has not gained much attention in finance literature until recently. The problem with multiple testing begins when testing different variables on the same dataset and sample period. Therefore, the probability of making a false discovery rises very quickly. The probability of making a significant discovery by chance when testing ten different anomalies on the same dataset with a significance level of 5% is:

$$\Pr(\textit{at least on significant result}) = 1 - \Pr(\textit{no significant results})$$

$$= 1 - (1 - 0.05)^{10} = 40\% \quad \boldsymbol{EQ\ 2.16}$$

A 40% probability of a significant discovery by chance is very high and this shows the danger of multiple testing. The probability rises quickly with the number of tests; with 90 tests the probability of making at least one significant discovery is 99%. In anomaly and factor research the majority of research is done on the US stock market and many even use the exact same dataset and timeframe.

Mclean et al. (2015) studied 97 factors that have been shown to explain the cross-section in expected stock returns and use an out-of-sample approach to show that many of the factors are actually insignificant out-of-sample. Similarly Harvey et al. (2015) found that 313 papers have been published with factors claiming to explain the cross-section of expected stock returns. They also argue that many more factors must have been researched and trashed because of insignificance, the so called publication bias. With 313 papers published the amount of hypothesis testing done on, in many cases similar datasets, is extensive. This shows that multiple testing is an important issue in asset pricing that should be considered. They also showed that most research findings on factors and anomalies are likely false. To avoid this Harvey et al. (2015) suggested that a t-statistic of three rather than two should be used for future research in asset pricing and that the hurdle very well could be higher.

2.4.1 Type 1 and Type 2 errors

In multiple testing type 1 and type 2 errors is of great concern. Type 1 error is the probability of a false discovery. Type 1 error is also known as the significance level of the null hypothesis or the alpha. Type 2 error is the probability of missing a true discovery. Type 2 error is also known as the power of a null hypothesis test. The probability of making a type 1 error in multiple testing increases with the number of tests. However, just dividing the alpha with the number of tests will lead to a high probability of type 2 errors. The goal of multiple testing in finance is to find the balance between type 1 and type 2 errors. From one standpoint the most concerning is to make a type 1 error. An investment manager might recommend investing in a factor that is not true at all. However, we should not underestimate making type 2 errors, neglecting to find true discoveries.

3. Data

This section describes the data used in the study and the considerations behind the data selection. All data used in this thesis have been downloaded through DataStream and Worldscope with access through Copenhagen Business School. DataStream codes for each data type will be provided for easy access.

3.1 Data selection

This section first describes the overall market that will be investigated in this study and how returns are calculated. Next section describes the proxies used to investigate each of the seven anomalies that are studied in this thesis and how they are calculated.

3.1.1 Return data

To analyze anomalies and factors in the European market a wide European equity index has been selected. Using an equity index instead of the entire portfolio of stocks for each country ensures that the liquidity and size of stocks that enter the index are controlled. This also raises considerations in regards to potential bias in data that will be described in section 3.2.

The index selected for this study is the Standard and Poor's European Broad Market Index (SPEU) index, which is a subset of the Standard and Poor's Global Broad Market Index. The SPEU index is a comprehensive rules-based index designed to measure European stock market performance. The index covers all publicly listed equities with float-adjusted market values of $US 100 million or more. SPEU consists of 1840 constituents and was launched in 1992, the index is measured in US dollars. SPEU is the broadest public available European index and this index has been chosen to get a broad representation of stocks in Europe. The index consists of members from 18 different countries: Austria, Belgium, Britain, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Lichtenstein, Luxembourg, Netherland, Norway, Portugal, Spain, Sweden and Switzerland. All countries in the index are members of the European Union except for Norway and Switzerland, and all countries have very well developed equity markets. Figure 3.1

show that Britain, France, and Germany are the countries with most stocks in the index while Lichtenstein and Greece just have two stocks each in the index. This shows that the stocks are weighted in the index on the basis of the size of the stock market in each country.

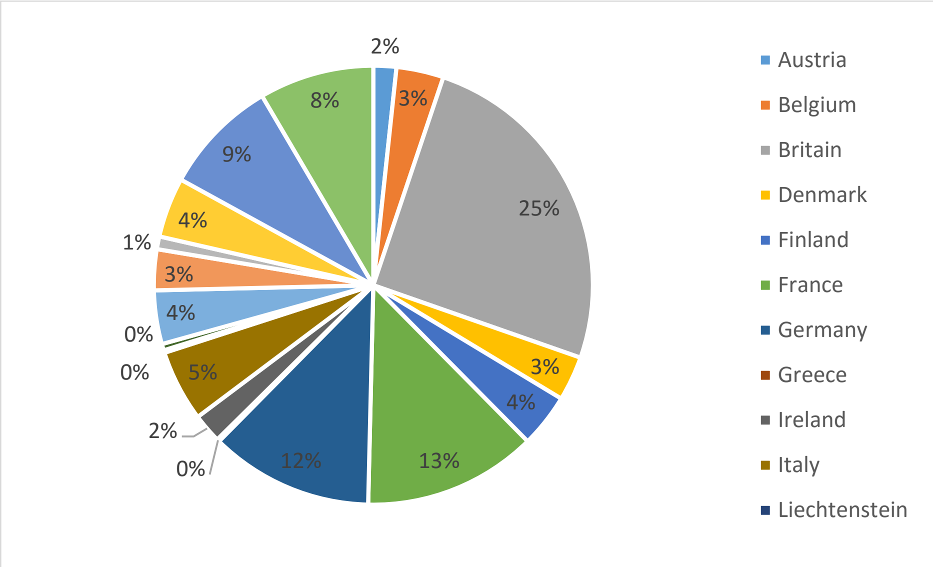

Figure 3.1 – Percent of stocks represented in the SPEU index by country. Source: Own figure.

Figure 3.2 show the index weighted by GICS sectors. The financial and industrial sectors are the biggest with 20% and 27% of stock, respectively. Both the country and GICS portfolios will be used later in this study to see if returns are driven by specific countries or sectors. Each country and GICS portfolios will also be used as test portfolios in asset pricing models.

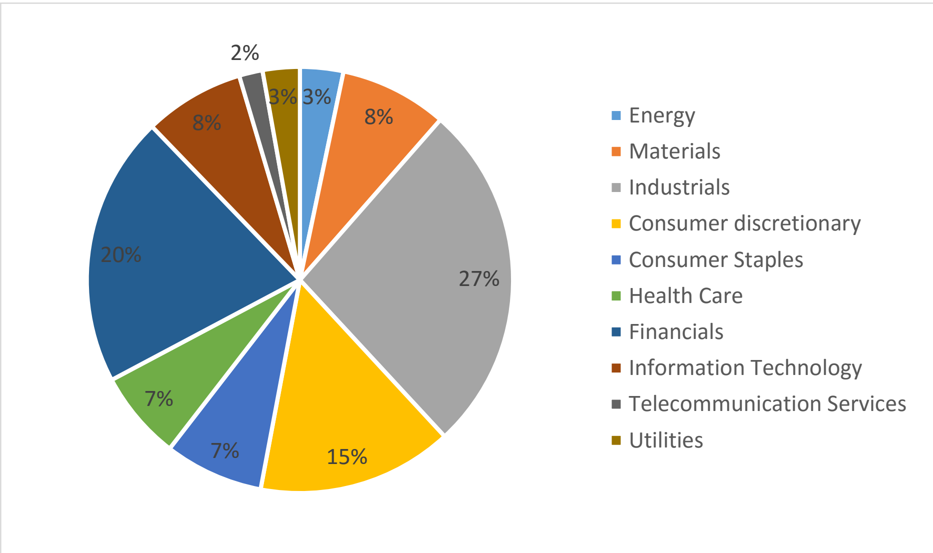

Figure 3.2 – Percent of stock represented in the SPEU index by GICS sector. Source: Own figure.

Table 3.1 show descriptive stats of the market value of the stocks in the SPEU index. The stats show that there is a big difference between the smallest and biggest firms. The median value is more than four times lower than the average value, showing that the index is weighted towards small firms. Table 3.2 show the average market value of portfolios sorted into deciles by market value. The numbers show that the majority of firms in the index can be considered small cap firms. Small cap firms are generally defined as firms with a market value lower than $US 2000 million. The presence of many small cap companies will be taken into consideration during the study when relevant. The deliberate choice of an index that is weighted towards small cap companies is made to get enough stocks in the sample period as will be discussed in the next section.

Market Value

Max	Min	Avg.	Median
236436	23	8877	1946

Table 3.1 – Descriptive stats of stock in the SPEU index weighted on market value in $US. Source: Own figure.

Market Value

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
217	420	649	1008	1598	2374	3900	6866	14434	50097

Table 3.2 – Stocks sorted on market value into 10 decile portfolios from small to big. All values in $US. Source: Own figure.

3.1.2 Sample period

The sample period chosen for this study is January 2000 to December 2015 giving a total of 192 months of return data with 1065 stocks present in the entire sample period. The choice of sample period is a thin line between having as long a sample period as possible, while still having enough stocks with complete data in the period. Analyzing anomalies this study will sort portfolios into deciles based on firm-characteristics to explain the risk return relationship for each anomaly. To get robust results diversification of each portfolio is important. Therefore, the sample period have been chosen with considerations towards having enough stocks in each portfolio to ensure that portfolios are diversified. Diversification of portfolios is generally known as the only 'free lunch' in equity investment and it is therefore important for the robustness of the results. Alexeev (2012) researched equity portfolio diversification across developed markets and found that while diversification depends on risk measures and investors risk profiles they recommended that professional investors hold 43 stocks in the UK and 49 stocks in the US for optimal diversification. While diversification depends on the correlation structure of the stocks in the portfolio, this study will assume that portfolios are diversified with a minimum of 50 stocks in the portfolio.

3.1.3 Return calculations

This thesis will use simple returns to allow for aggregation across assets. Using simple returns compared to log returns means that returns will not be normally distributed and this can be a problem for some statistical models. The time series consist of monthly returns calculated from the Total Return Index[10] (RI) pulled from DataStream. The code for the index list in DataStream is 'LSBBEUR'. DataStream defines Total Return Index as:

$$RI_t = RI_{t-1} * \frac{PI_{t-1} + D_t}{PI_t}$$

Where PI is the Price Index and D is dividends. The RI index is downloaded in local currencies for the highest accuracy.

This thesis will use simple returns to allow for aggregation across assets. All returns presented in the results section will be yearly returns and they will be calculated from monthly returns to yearly returns as follows:

$$R_{year} = R_{month} * 12$$

3.1.4 Risk free rate

When studying asset pricing models it is necessary to use the excess returns of portfolios or assets in order to evaluate pricing errors correctly. The risk free rate must best possible reflect the risk free investment opportunities available in the market at that time. The risk free rate used in this study is the three-month Euro Interbank Offer rate (Euribor)[11]. This interest rate represent rate at which banks in Europe can borrow funds from each other and it is considered to be risk free.

3.2 Firm-characteristics

This study will look at seven different anomalies: beta, volatility, size, value, profitability, investment, and momentum. The list of anomalies could be longer but this study examines some of the most important anomalies that have been documented in academia. The anomalies have been chosen from two main criteria's:

1. The investment styles must be documented empirically in other markets.

[10]Total Return Index means that all dividends are added back. Datatype in DataStream: 'RI'
[11] DataStream code: EIBOR3M

2. To increase robustness, the factor must be explained by theory, either risk based or behavioral based explanations.

In section 2.2 each anomaly was reviewed empirically and theoretically. Finding the best proxy for each anomaly is essential. The proxies chosen in this study are all firm-characteristics and they reflects proxies that have already been showed to work as good descriptors of the relevant anomalies. For some anomalies several different firm-characteristics have been suggested in academia and in this case more than one firm-characteristics may be analyzed. Each firm-characteristics is available to download through DataStream and the codes will be provided. For firm-characteristics based on accounting data a five-month delay will be applied when forecasting returns since accounting data is not always published the following month. For anomalies based on returns the previous month's style indicator will be used to forecast the next month's return.

Beta

Bets is calculated as in the CAPM with the market portfolio being the value weighted portfolio of the 1065 equities selected for analysis in the SPEU index. Beta is calculated from a rolling 12-month window and so the first beta value is available from January 2001 and therefore the analysis on beta only contains 180 months of data. The equation for calculating beta is as follows:

$$\beta = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

Where R_i is the returns of a stock or portfolio and R_M is the return of the value weighted market portfolio.

Volatility

Volatility will be measured as the total variance of each stock. As with beta volatility will be measured from a 12-month rolling window of past returns and therefore the first portfolio return will be January 2001 and there will be 180 months of returns.

Size

This study will use the market value of each security as a proxy for the size anomaly in line with the early work of Banz (1981). Market values are downloaded from DataStream with all values converted to US-dollars[12]. DataStream defines market value as share price multiplied by the number of ordinary shares in issue. The amount is also adjusted for capital changes.

Value

To investigate the value anomaly this study will use the MTBV[13] as proxy for value. MTBV is the inverse of the popular BTMV used by Fama and French (1992) and many others. This means that in this study firms with low MTBV will be considered value and growth firms have high MTBV. The MTBV is used, as it is readily available for download from DataStream. The MTBV data will be cleaned for all negative values to avoid investing in companies with a negative book or market value. MTBV are calculated in the following way:

$$MTBV_t = \frac{Market\ Value_t}{Book\ Value_t}$$

Profitability

For the profitability anomaly this study will use both Return on Equity (ROE)[14] and operating profit margin (OP)[15]. The choice of two profitability measures has been made to investigate the differences between a profitability measure based on equity and a more clean accounting profitability measure. Recent papers of Hou, K. et al (2014) and Fama and French (2014) used ROE and OP measures, respectively. The proxies are downloaded directly from DataStream and defined as:

$$ROE = \frac{Net\ income}{Average\ of\ last\ years\ and\ current\ years'\ common\ equity * 100}$$

$$OP = \frac{Operating\ income}{Net\ revenues * 100}$$

[12] DataStream data type: 'MV'
[13] DataStream data type: 'MTBV'
[14] DataStream data type: 'WC08301'
[15] DataStream data type: 'WC08316'

Investment

The proxy for investments used in this study will be the total asset growth (AG) in line with recommendations by Aharoni, Gil. et al. (2013). Total assets[16] was downloaded from DataStream and AG was calculated as total assets in the fiscal year divided by 1-year lagged total assets:

$$AG_t = \frac{Total\ assets_t}{Total\ asset_{t-1}} - 1$$

Total assets are only reported on an annual basis and this affects the availability of the data to investors. To ensure that investors have data available to trade on asset growth used to form portfolios in 2001 is calculated as AG from 1999 to 2000. Including the five-month delay this results in a returns series for portfolios sorted on AG from May 2001 to December 2015, a total of 176 months.

Momentum

Momentum in stock returns will be calculated in line with recommendations by Jegadeesh et al. (1993). The average of the past 12-month returns will be used to determine the momentum in stock returns. The portfolios will be formed in first following month and be held for three months. This strategy effectively captures the medium term momentum as showed by Jegadeesh et al. (1993). The construction of the momentum return means that there is 176 months of returns for portfolios formed on momentum.

To sum up, the various firm-characteristics have been chosen based on recommendations in financial literature. The construction of some of the proxies mean that the study on anomalies will use are returns series from May 2001 to April 2015 or a total of 176 months. The next section will look into the potential bias in data when dealing with estimation of returns based on firm-characteristics.

3.3 Bias in data

This section will consider the potential bias in the datasets used in this study. The credibility and quality of the data is important and this section will line out what have been done to ensure the quality of the data and the potential bias still left in the data.

[16] DataStream data type: WC02999

All constituents of the SPEU that did not have return data for the entire period have been removed. This ensures the quality of data for the remaining equities, with the drawdown being a lower number of equities in the study. For the proxies based on firm-characteristics the time-series available through DataStream and Worldscope were more incomplete than the return time-series and along with supplementing the time-series with data from Bloomberg the time-series have also been trimmed for major jumps and outliers. The average number of equities in the sample for each investment style can be seen in table 3.3.

No. of Stocks							
Size	Beta	Mom	Vol	MTBV	ROE	OP	AG
1065	1065	1065	1065	1047	1049	1053	1058

Table 3.3 – Average number of equities for each firm-characteristics in the sample period. Source: Own figure.

3.3.1 Survivorship bias

This study uses an equity index as data source and by definition the choice of a specific index will lead to both selection and survivorship bias. Only stocks that have data for the entire period are considered in this study, neglecting stocks that are either merged with other companies or stocks that default. In the SPEU constituents are removed if their float-adjusted market capitalization falls below US$ 75 million leading to a natural survivorship. The potential survivorship bias among the smallest companies mean that we have to be careful when concluding on results that are only present in the lowest decile. The survivorship bias can potentially be present among all anomalies analyzed in this study.

3.3.2 Selection bias

The selection of the anomalies chosen for investigation in this study suffers from selection bias. All anomalies have been chosen because they have been shown to exist in other markets and sample periods. This makes them likely to exist also in this study.

3.3.3 Liquidity

The liquidity of stocks in the index is important if we want to effectively trade on the anomalies investigated in this study. If equities are illiquid they will contain increased risk and this can affect the results. Small stocks especially suffer from liquidity issues and van Dijk (2011) suggested that liquidity and increased transaction costs could potentially account for the size premium. The SPEU also takes liquidity into consideration and constituents of the index must have an annual dollar value traded of at least US$ 50 million of the previous 12 months. Liquidity can also be an issue for other investment styles and this must be taken into consideration.

3.3.4 In-sample vs. out-of-sample

This study create factors from portfolio returns sorted on firm-characteristics and use these factors to predict returns in asset pricing models. This is classic example of an in-sample test and for further research it could be interesting to test how these factors perform out-of-sample.

4. Methodology

This section will go through the methodologies used in this study. The first section will go through the important concepts related to backtesting of investment strategies. The next section will setup the methodology used to sort portfolios based on firm-characteristics. Finally, the last sections outlines the methodology of factor formation and multifactor models

4.1 Backtesting

This study relies on backtesting of historical data. Backtesting means testing how a specific trading strategy would have done in the past. When doing a backtest it is important to note that because a strategy worked in the past it does not necessarily work in the future. The data section has set up the universe the backtest is done in, with the SPEU as the trading market and the firm-characteristics being the trading signals in the backtest. The trading rules have also been set, this study uses equal weighted portfolios, monthly rebalancing, and a five month delay for accounting based firm-characteristics.

Despite the precautions and considerations that have been taken to make the backtest reflect a real life trading universe, a backtest still look a lot better than the real world. The potential bias that this type of analysis is exposed was described in section 3.3. Another issue with backtesting of trading strategies that this study does not consider is transaction costs. Implementing trading strategies with monthly rebalancing leads to an effective on paper strategy but also high real life transaction costs. This has to be considered before implementing the trading strategy. This study does not consider transaction costs in the analysis, but for more information on transaction costs in backtesting look at Harvey et al. (2013) and McLean et al. (2013).

4.2 Sorting procedure

The main part of this study will focus on the analysis of portfolios formed on basis of the beta, volatility, size, value, momentum, profitability, and investment. In section 2.2 the theoretic and empirical research on these anomalies were reviewed and in section 3.2 the firm-characteristics used to describe these were specified. The approach used in this study will follow the guidelines for Lakonishok, J. et al. (1994) and Fama and French (1992) who also sort portfolios based on firm characteristics to investigate the relationship between firm-characteristics and expected returns.

4.2.1 Single sorting

Single sorting will be done to analyze the relationship for each firm-characteristics with average return, risk, and risk adjusted returns. The SPEU index will be sorted into ten equal sized portfolios ranked from smallest to highest based on each firm-characteristic. The firm-characteristics based on returns are delayed one month, and the firm-characteristics based on accounting measures are delayed five months to simulate real life trading opportunities. The portfolios will be balanced each month, this is a balance between what is optimal from a trading perspective and what is optimal to show the true relationship between the firm-characteristics and expected return. The sorting procedure have been done in excel using VBA and the VBA-code can be seen in appendix 1.

4.2.2 Double sorting

Aside from the single sorted portfolios this study will use double sorted portfolios. The purpose of the double sorted portfolios is to discover relations among the different firm-characteristics. The double sorted portfolios will be formed as 3x3 portfolios resulting in nine portfolios with 11.11% of all stocks in each portfolio. The double sorted portfolios will be used to show have each firm-characteristics fare when sorted on different size classes but also cross references in other relevant combinations. The double sorted portfolios is also a tool to test the robustness of each firm-characteristics. The double sorted portfolios have also been formed in excel using VBA and the VBA-code can be seen in appendix 2.

4.3 Factor formation

To further study the anomalies based on firm-characteristics this study will also investigate factors based on these firm-characteristics. The factors will be formed as zero-cost portfolios from the single sorted portfolios. They will be created by going long in the first decile and short in the last decile, or the other way around depending on the signal of each investment style. Table 4.1 show the factors that are investigated in this study and the direction they are formed in.

Factors

Anomaly	Firm-characteristics	Direction
Beta	Beta	High - Low
Volatlity	Volatility	High - Low
Size	Market Value	Low - High
Value	MTBV	Low - High
Momentum	MOM	High - Low
Profitability	ROE	High - Low
Profitability	OP	High - Low
Investments	AG	Low - High

Table 4.1 – Factors the name of the anomaly, the firm-characteristics it is based, and the direction the anomaly is created in.

Factors formed as zero-cost portfolios isolate the risk connected to each anomaly. The goal is to analyze if there is a factor premium and to see if the factors can explain the risk related to this factor premium in a multifactor asset pricing model. The methodology behind the single sorted portfolios ensures that the factors are created as predictive factors. This means that the factors capture premiums of simple investment strategies while also working as explanatory factors of the common risk in stock returns. The methodology was first introduced by Fama and French (1992) and since then followed by many others.

4.4 Multifactor models

The factors will be used to create a multifactor asset pricing model and this section will go through the theoretic foundations of multifactor models. The overwhelming amount of anomalies to the CAPM model naturally leads to the extension to multifactor asset pricing models. Multifactor models can be either equilibrium models or No-Arbitrage models. Equilibrium models makes assumptions about distributions of returns, utility functions of agents and the state of the economy. The ICAPM model by Merton (1973) is an example of an equilibrium multifactor model. This study will deal with No-Arbitrage models that are based on the Arbitrage Pricing Theory (APT) first described by Ross (1976). The APT only relies on the arbitrage pricing principle that if pricing diverges in anyway arbitrage trading should bring them back to base. This leads to the assumptions of APT that markets have to be efficient and frictionless. The general form of a multifactor model can be written as:

$$R_i = \alpha_i + x_1 f_1 + x_2 f_2 + x_3 f_3 + \cdots x_K f_K + \epsilon_i \quad \boldsymbol{EQ\ 4.1}$$

Where α_i is the non-factor specific return for the asset, x_K is the sensitivity to each factor, and f_K is the return of each different factor in the model. Generally, there can be an unlimited number of factors in a multifactor model but for simplicity we want the highest explanatory power and lowest pricing errors with as few factors as possible. The main goal of multifactor models is not to forecast equity returns but to forecast the risk of equity returns. The factors that are put into the model are factors that describe the co-movement of returns in equities. This can be a wide range of variables e.g. industry sector, inflation, or market capitalization. The main issue of the multifactor models is to find the correct factors to put into the model. As argued previously this study will investigate factors that have solid theoretic foundation as well as a good empirical track record. To get an accurate approximation of APT multifactor models it is also important that the left hand side of EQ 4.1 must consist of well-diversified portfolios[17]. Multifactor models can generally be divided into 3 categories:

- Macroeconomic factor models
- Fundamental factor models
- Statistical factor models

Macroeconomic factor models where factors consist of observable macroeconomic variables like inflation or output, fundamental factor models where factors are created from observable firm characteristics like market capitalization and profitability, or statistical factor models where both the factors and the sensitivity to the factors are unobservable but estimated from asset returns through statistical techniques. Conner (1995) investigate the three types of factor models and finds that the fundamental factor models outperforms the two other in terms of explanatory power and simplicity. This study will follow the guidelines of Fama and French (1992), Hou, K. et al. (2014), Lakonishok, J. et al (1994), and many others and also use fundamental factor models.

[17] Ross(1973)

4.5 Regression analysis

To further interpret and analyze the factors and their role in the variation of expected stock returns this study will use a time-series regression approach as first suggested by Jensen et al. (1972).

$$R_{it} - r_{ft} = \alpha_i + b_i F_t + \epsilon_{it} \;\; \boldsymbol{EQ\ 4.2}$$

The goal will be to test a model like EQ 4.2 where R_{it} is the return of a portfolio or a single asset, r_{ft} is the risk free rate, b_i is the asset or portfolios loading on the factor F_t, and α_i is return not related to factors in the model. The right hand side of the model will consist of the zero-cost factors formed earlier along with the value weighted market portfolio. The goal will be to test each factor and evaluate if they add to the variation in expected stock returns. The starting point will be to evaluate each factor in a univariate model and evaluate how the factors perform and subsequent add more factors to get the best possible asset pricing model for European stocks. To evaluate the model this study uses excess returns of portfolios formed earlier in the single sorts together with portfolios formed by country and GICS sector on the left hand side of EQ 4.2. For countries and GICS there will require a minimum of ten stocks in the sample. The model will be evaluate based on results of these groups by looking at the relative mispricing (α_i), the explanatory power measured by R^2, and by a GRS[18] test statistic.

4.5.1 GRS test

The GRS test is a test for judging the efficiency of a given portfolio for an asset pricing model like EQ 4.2. The test measures if all pricing errors (α_i) are jointly equal to zero. The tested hypothesis is:

$$\alpha_i = 0 \; \forall_i \;\; \boldsymbol{EQ\ 4.3}$$

The GRS test assumes that there is a riskless rate of interest and that the error terms (ϵ_{it}) are jointly normally distributed with mean zero. Essentially the GRS test if portfolios are mean variance efficient. The study will follow the lines of Gibbons et al. (1989) and the calculations have been made in excel VBA and can be seen in appendix 3.

[18] Gibbons, Ross, and Shanken (1989)

5. Anomalies

Getting the highest possible risk adjusted returns is of great interest to both private and institutional investors. This section will analyze seven different anomalies based on firm-characteristics. This will be done by using single sorted and double sorted portfolios on each firm-characteristic.

5.1 Single sorted portfolios

This section analyzes single sorted portfolios based on average return, standard deviation (stdev), and risk to reward ratio (RRR). The RRR is very similar to the Sharpe ratio but without the risk-free rate:

$$RRR = \frac{\mu_i}{\sigma_i} \quad \boldsymbol{EQ\ 5.1}$$

Where μ_i is the average return of the portfolio and σ_i is the stdev. For comparative basis table 5.1 show the performance of the market portfolio in the sample period. The equal weighted portfolio will naturally be weighted towards small stocks and it is interesting to see that it only slightly outperforms the value weighted portfolio by a risk adjusted measure.

Market Portfolio	Avg. Return	Stdev	RRR
Equal Weighted	13.49	18.05	0.75
Value Weighted	11.15	16.33	0.68

Table 5.1 – Average yearly return, stdev, and RRR in percent for the equal and value weighted market portfolio.

5.1.1 Beta

First, the classic beta measure is analyzed. Beta is not defined as in the CAPM where the market portfolio includes all assets in the world. The market portfolio is defined as the universe of stocks used in this study. Table 5.2 show that there is no relationship between beta and average returns, the average return is flat across all deciles. The results are in line with Fama and French (1992) who also do not find evidence of a relationship between beta and average return. The SML as predicted by the CAPM does not hold, the SML is completely flat.

Turning to the beta anomaly, table 5.2 also show that the RRR is falling from low to high beta portfolios due to a positive relationship between stdev and beta. While there is no relationship between beta and average return it is clear that there is a risk adjusted return by investing in low beta stocks compared to high beta stocks. This is also what was found by Frazzini (2014) who showed that betting against the beta factor produce positive risk adjusted returns. Therefore, the beta anomaly is clearly present in the European market.

Beta	Low	2	3	4	5	6	7	8	9	High
Avg Return	14.48	13.48	11.90	13.91	13.46	13.09	14.18	12.93	13.47	14.73
Stdev	14.24	14.06	14.93	15.82	16.19	17.60	19.45	20.34	23.06	30.78
RRR	1.02	0.96	0.80	0.88	0.83	0.74	0.73	0.64	0.58	0.48

Table 5.2 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on beta in equal weighted deciles. Source: Own calculations

5.1.2 Volatility

Contrary to the sorting done on beta, table 5.3 show that the relationship between volatility and average return is positive. However, turning to the RRR it is clear that there is a low volatility anomaly; the low volatility portfolios outperform the high volatility portfolios on a risk adjusted measure. This shows that the volatility anomaly exist in the European market. Compared to Haugen et al. (1991) who found that low volatility portfolios have higher or equal returns with lower volatility this study do not find this.

Volatility	Low	2	3	4	5	6	7	8	9	High
Avg Return	10.93	11.32	11.93	13.03	13.21	13.00	13.09	13.76	14.94	18.40
Stdev	9.95	13.25	15.00	15.62	16.95	18.70	19.34	21.46	25.17	30.93
RRR	1.10	0.85	0.80	0.83	0.78	0.70	0.68	0.64	0.59	0.59

Table 5.3 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on Volatility in equal weighted deciles. Source: Own calculations

The low volatility and beta anomalies have existed for a long time and the most likely explanation is that investors are either leverage constrained or simply does not like to leverage their investments. This leads investors to invest in high volatility stocks to get higher returns even though the risk adjusted return is lower.

5.1.3 Size

For the single sorted portfolios on size table 5.4 show a strong negative relationship between size and average return. The negative relationship between average return and size is evident across all deciles falling from 27.59% to 5.66% from small to big. The stdev is 22.82% for the small portfolio while it is in the range 18.00-18.89% for the nine remaining portfolios. This indicates that while there is more risk in the returns of small stocks the risk is rewarded by a substantial higher RRR. It is important to note that the stdev is almost equal for the remaining portfolios and in that sense there do not seem to be any relationship between stdev and size. Earlier it was showed that the small portfolio have an average market value of $US 217 million and the 2nd decile portfolio a market value of $US 420 million and it is interesting to see that the difference between the two portfolios are so big both in terms of average return and stdev[19].

Size	Small	2	3	4	5	6	7	8	9	Big
Avg return	27.59	18.73	14.88	14.64	12.14	12.19	10.49	9.47	8.48	5.66
Stddev	22.82	18.89	18.30	18.17	18.80	18.00	18.17	18.03	18.27	18.07
RRR	1.21	0.99	0.81	0.81	0.65	0.68	0.58	0.53	0.46	0.31

Table 1.4 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on market value in equal weighted deciles. Source: Own calculations.

The results in table 5.4 show that the size anomaly very clearly still present in the European market. It is possible to get a RRR that is 3.90 times higher by investing in the portfolio of small stocks compared to big stocks. The size anomaly can be explained by behavioral theories such as familiarity and herding, investors tend invest more in companies they know and investors tend to invest in many of the same companies. This can lead to overpricing of big stocks since they are generally more known to the average investor. However, there is also potential fundamental risk connected to small stocks. Small stocks have lower liquidity and it can therefore be expensive to buy and sell small stocks, especially for institutional investors. Fama and French (1992) also found that small stocks generally take bigger hits during economic depressions and that this can be the reason to the size premium.

5.1.4 Value

The value anomaly relates the fundamental value of stocks to average return and here it is measured by MTBV. Table 5.5 show that there is a negative relationship between MTBV and average return across all deciles. The value portfolio has an average return of 22.40% compared to 6.44% for the growth portfolio. Table 5.5 also shows a negative relationship between MTBV and stdev as it is falling from 23.82% for the value decile to 18.22% in the growth decile. The increased risk we see for value firms is in line with economic theory as investors require high rates of return for more risky firms and thereby value them lower. However, despite the increased stdev there is still a markedly higher RRR for value stocks compared to the growth stocks in the European market.

[19] See table 3.2

The mispricing observed in table 5.5 could be due to overreaction from investors to good or bad news or it could be that investors are extrapolating previously good or bad performances too far into the future, leading to an overpricing of growth firms. Herding and familiarity could also affect results, as growth firms are often well known popular stocks.

MTBV	Value	2	3	4	5	6	7	8	9	Growth
Avg Return	22.40	17.13	15.97	13.08	13.11	13.48	12.08	11.76	10.64	6.44
Stdev	23.82	19.22	18.44	18.08	17.90	18.14	17.89	17.54	17.38	18.22
RRR	0.94	0.89	0.87	0.72	0.73	0.74	0.67	0.67	0.61	0.35

Table 5.5 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on MTBV in equal weighted deciles. Source: Own calculations.

5.1.5 Momentum

A momentum strategy is based on buying stocks ranked by previous 12 months return and holding them for the next three months. Table 5.6 shows that there is a strong positive relationship with average return cross all deciles for momentum. Table 5.6 also shows a negative relationship for momentum and stdev, though only from the low portfolio to the 4th decile. The low portfolio has a RRR of just 0.28 due to a combination of low average returns and high stdev. The RRR is constantly growing from low to high momentum stocks and there is a RRR 1.23 for the high portfolio compared to 0.75 of the equal weighted portfolio, showing a momentum anomaly in average stock returns. The explanation to this anomaly can be that momentum is driven by investor irrationalities such as under and overreaction to news and earnings announcements. Investors can also be extrapolating returns too far into the future. It has been shown that there is momentum in the medium and short run while there is mean reversion in the long run, supporting a hypothesis of overreaction.[20]

Momentum	Low	2	3	4	5	6	7	8	9	High
Avg Return	8.68	9.35	11.09	11.55	12.15	12.62	13.58	13.62	16.86	22.84
Stdev	31.19	22.97	19.96	17.25	17.11	16.17	16.14	15.48	16.05	18.62
RRR	0.28	0.41	0.56	0.67	0.71	0.78	0.84	0.88	1.05	1.23

Table 5.6 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on momentum in equal weighted deciles. Source: Own calculations.

5.1.6 Profitability

Profitability is probably the simplest anomaly, higher returns for more profitable stocks. However, following the dividend discount model more profitable stocks should also have higher risk. Table 5.7 shows results of

[20] De Bondt et al. (1985)

portfolios sorted on ROE and OP. Both these profitability measures show the same relationship with average return, stdev, and RRR. Table 5.7 shows that the relationship between average return and profitability is positive but weak. The high profitability portfolio does outperform the low by a small margin for both ROE and OP, but there is no relationship across the 10 deciles for the measure of profitability. One major outlier is the low portfolio sorted on OP (see table 5.7b) that just have an average return of 3.11%. The stdev is falling from low to high profitability and table 5.7 show a clear negative relationship for both OP and ROE, the opposite of what is predicted by the dividend discount model. The negative relationship with stdev leads to a positive relationship for profitability and RRR measured by both ROE and OP.

a. ROE	Low	2	3	4	5	6	7	8	9	High
Avg Return	12.80	11.97	13.20	11.66	13.83	13.62	13.35	14.36	13.79	14.90
Stdev	25.81	20.48	19.17	17.33	17.71	17.44	16.38	16.96	17.31	16.64
RRR	0.50	0.58	0.69	0.67	0.78	0.78	0.82	0.85	0.80	0.90

b. OP	Low	2	3	4	5	6	7	8	9	High
Avg Return	3.11	12.18	14.10	14.33	15.68	14.96	14.37	14.21	14.91	15.74
Stdev	26.50	20.78	19.98	18.97	18.79	18.12	17.19	15.49	15.16	14.63
RRR	0.12	0.59	0.71	0.76	0.83	0.83	0.84	0.92	0.98	1.08

Table 5.7 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on ROE and OP in equal weighted deciles. Source: Own calculations.

The results found by this study in the European market stands somewhat in contrary to results found in the US stock market by Hou, K. (2014) and Fama and French (2015). They both showed a more positive relationship between profitability and average returns. However, the profitability anomaly is still clearly present in the European market as portfolios from 5th to 10th decile all earn higher RRR compared to the equal weighted market portfolio. The evidence for the profitability anomaly seems to strongly relate to the low volatility anomaly presented previously. This raises the question if low volatility is a proxy for profitability or the other way around.

5.1.7 Investment

Theoretically investment is connected with value and profitability through the modified dividend discount model. Low investment first should have higher expected returns compared to high investment firms. The low investment firms are also often value firms and this could be related to the investment anomaly. Table 5.8 shows a negative relationship between AG and average return. The low AG portfolio has an average return of 17.78% compared to 12.71% for the high AG portfolio. The negative relationship is only present from the low to the 5th decile portfolio as the relationship between average return and AG is flat from the 5th to the 10th decile. There is no clear relationship between stdev and AG. The RRR is highest for the low AG firms but the relationship is sporadic across the 10 deciles and generally weak.

AG	Low	2	3	4	5	6	7	8	9	High
Avg Return	17.78	17.06	14.70	15.84	13.44	13.44	13.38	13.42	12.40	12.71
Stdev	21.74	19.55	18.40	17.83	17.04	17.81	18.09	18.02	18.72	19.88
RRR	0.82	0.87	0.80	0.89	0.79	0.75	0.74	0.74	0.66	0.64

Table 5.8 – Average yearly return, stdev, and RRR in percent for single sorted portfolios on AG in equal weighted deciles. Source: Own calculations.

There is not found a clear investment anomaly in the European market based on the RRR. The investment anomaly is related to profitability and value through the modified dividend discount model and the next section, on double sorted portfolios, will take a closer look on this.

Summing up the results of the single sorted portfolios there is found evidence anomaly returns for beta, volatility, size, value, momentum, and profitability measured on by a RRR in the European market. Only the investment anomaly is not found in the European market. Looking at the relationship between average return and the anomalies there is only found anomaly returns volatility, size, value, and momentum. Next section will look at double sorted portfolios to investigate how the anomalies correlate.

5.2 Double sorted

This section will look at average returns in double sorted portfolios to investigate the robustness of the anomalies found in the previous section.

5.2.1 Size

The size premium is the most well documented investment strategy and it is therefore natural to start by double sorting all firm-characteristics with size to investigate if there are a relationship between average return and the firm-characteristic across all size deciles.

The results of the double sorted portfolios in table 5.9 generally show the same tendencies as observed for the single sorted portfolios. The size premium is present across all firm-characteristics especially from small to big. Previously it was showed that the index used in this study is weighted towards small cap stocks[21], it is therefore comforting to see in table 5.9 that anomalies more or less show the same relationship between average return for the big portfolios as for the small. The relationship between average return and OP is improved here across the three size classes but it is likely due to the very poor performance of the low portfolio from table 5.7a. For volatility the positive relationship between volatility and average returns is only present for the smallest size class, indicating that the premium observed earlier for high volatility stocks was due to a size premium.

a. Size-OP

	Low	2	High
Small	12.36	21.63	22.13
2	6.41	12.63	14.94
Big	5.84	9.98	11.90

b. Size-ROE

	Low	2	High
Small	17.27	18.18	20.67
2	8.88	12.33	13.68
Big	8.06	9.86	10.45

c. Size- MTBV

	Value	2	Growth
Small	22.92	19.61	13.81
2	13.81	12.07	8.47
Big	12.49	9.59	6.06

d. Size-MOM

	Loosers	2	Winners
Small	11.00	15.23	22.54
2	7.81	12.69	17.06
Big	7.63	10.21	13.49

e. Size - Vol

	Low	2	High
Small	15.80	18.86	23.01
2	11.34	12.62	11.79
Big	8.99	9.48	8.52

f. Size-AG

	Passive	2	Aggressive
Small	21.12	18.85	18.16
2	13.56	11.46	11.57
Big	11.52	10.78	9.26

Table 5.9 – Average yearly return in percent for double sorted portfolios with OP, ROE, MTBV, MOM, Vol, and AG sorted on size.
Source: Own calculations

5.2.2 Investment, Profitability, and Value

From the modified dividend discount model it was shown that value, profitability, and investment are jointly connected and therefore it is interesting to see how these anomalies perform when sorted on each other and how this relates to theory. Value and profitability strategies are contrarian as value strategies invest in stocks with low fundamental value, often connected with previous poor performance and low profitability. On the opposite profitability strategies invest in stocks that have high profitability.

[21] Table 3.2

Table 5.10a and 5.10b show that that the value strategy is robust across the three profitability classes, the main outlier is the bottom left corner of both tables where there is a very low average return for growth stocks that have low profitability. In the single sorts on ROE and OP there was not found a strong relationship with average returns, but for the sorts with value there is a clear relationship from low to high profitability. Showing that profitability and value can be two sides of the same coin as suggested by Noxy-Marx (2012). Table 5.10c shows that the negative relationship between AG and average returns is not as strong when sorted for MTBV. Evidence from table 5.10c suggests that the weak relationship observed earlier could be a hidden value effect. Theory also suggest that low investment stocks are often connected with value stocks as value stocks are stocks that have low profitability and low investments. Again this shows how investment, profitability, and value are connected. Table 5.10e table 5.10f show that the investment premium is weak but present across all profitability classes.

a. MTBV-ROE

	Low	2	High
Value	16.72	16.24	21.04
2	9.87	12.05	17.11
Growth	5.12	9.00	12.13

b. MTBV-OP

	Low	2	High
Value	13.70	21.08	19.21
2	9.09	14.47	15.53
Growth	2.93	10.57	12.96

c. MTBV-AG

	Passive	2	Aggressive
Value	20.95	16.32	16.07
2	13.73	13.25	13.80
Growth	13.01	10.37	9.69

d. ROE-OP

	Low	2	High
Low	3.95	16.88	15.70
2	9.67	13.90	14.79
High	13.06	13.54	16.17

e. ROE-AG

	Passive	2	Aggressive
Low	15.88	13.43	11.50
2	14.45	13.12	12.71
High	16.64	13.57	14.29

f. OP-AG

	Passive	2	Aggressive
Low	13.04	11.29	9.55
2	18.32	14.03	13.74
High	17.44	13.39	14.82

Table 5.10 Average yearly returns in percent for double sorted portfolios on value, profitability, and investments. Source: Own calculations.

Table 5.11 show results of the momentum portfolios sorted with value, profitability and investments, respectively. Value and momentum strategies capture opposite patterns in returns and have been shown to have high negative correlations by Asness et al. (2013) and it is therefore interesting to see in table 5.11a that there is momentum premium across all MTBV classes. This shows that momentum and value are not just opposite strategies but that there actually can be momentum in value stocks as well.

a. MTBV-MOM

	Loosers	2	Winners
Value	14.49	16.26	21.39
2	8.47	12.39	16.90
Growth	7.14	9.47	14.54

b. ROE-MOM

	Loosers	2	Winners
Low	9.22	12.92	18.72
2	8.69	12.66	16.56
High	9.89	13.22	18.20

c. OP-MOM

	Loosers	2	Winners
Low	6.04	9.80	15.52
2	10.02	12.45	18.54
High	9.71	13.82	17.20

d. MOM-AG

	Passive	2	Aggressive
Loosers	11.85	9.48	6.99
2	14.58	12.11	11.58
Winners	18.92	17.10	17.39

Table 5.11 – Average yearly returns for double sorted portfolios with momentum, profitability, value and investment. Source: Own calculations

In summary, the double sorted portfolios show that the size anomaly is robust across all other anomalies. The sort on volatility and size showed that the positive relationship between average return and volatility is related to the size effect. The highest average return was found for small stocks that have high volatility. Table 5.10 show that value, profitability, and investment strategies are related. In general, the relationship between average return and ROE, OP, and AG were not found to be strong across the various sorts. The momentum anomaly was also shown to be strong in all double sorts. The double sorts also showed that a combination of anomalies could increase the average return. The next sections will look into anomalies formed as factors and investigate if they can explain the variation in expected stock returns.

6. Factors

This section will look into factors formed from the single sorted portfolios that were investigated in section 5.1. The factors will be formed as zero-cost portfolios for each firm-characteristic. The idea is that a zero-cost portfolio can capture the risk that is related to the anomaly return that was showed in section 5.1. It is important to note that these portfolios will not necessarily be hedged from all systematic risk in a CAPM setting as they are created from equal weighted portfolios. However, as this study showed earlier the relationship between beta as a risk measure and average return is none existing. The goal of this section is to study if there is fundamental risk connected to these factors and finally to show if an asset pricing model comprising of these factors can explain the variation in expected stock returns in the European market. The t-stats showed in this section test the null hypothesis that the average returns of the factors are not equal to zero. The critical values are 1.97 for the 5%-level and the t-stats will be marked with yellow if they pass this level. The critical value for the 1%-level is 2.60 and the t-stats will be marked with green if they pass this level.

Table 6.1 show that there are factor premiums range from 1.83% for beta and 21.18% for size. Furthermore, table 6.1 show that size, MTBV, and OP are significant at the 1%-level and AG and momentum are significant at the 5% level. The OP factor is also significant at the 1% level but as was showed previously in table 5.7b this might only be driven by a very low return for the lowest decile portfolio. AG and momentum are significant at the 5% level while volatility, ROE, and beta factors are all insignificant.

	Factors			
	Avg return	Stdev	t-stat	Direction
Size	21.18	14.13	5.86	Low - High
MTBV	14.64	15.51	3.58	Low - High
OP	12.22	16.30	2.96	High - Low
MOM	13.85	22.98	2.11	High - Low
AG	4.42	8.49	2.00	Low - High
Vol	9.27	24.19	1.46	High - Low
ROE	1.86	13.37	0.62	High - Low
Beta	1.83	21.19	0.33	High - Low

Table 6.12 – Factors formed as zero-cost portfolios by subtracting low (high) minus high (low) depending on the direction of the investment style. Source: Own calculations

The correlation structure of the factors can show indications of whether factors capture the same type of return. It is also an important tool for setting up trading strategies as we might be able to hedge away risk connected with the factors by combining them and thereby earning higher risk adjusted returns. Table 6.2 shows the correlations among factors and a few interesting observations can be seen from table 6.2. First, the profitability factors OP and ROE are highly correlated at 0.85 as would be expected. Secondly, all factors have high correlations with the size factor. ROE, OP, and momentum have negative correlations to the size factor whereas MTBV, AG, and volatility are positively correlated. It is natural that factors are correlated, as they are not created to be factor neutral. However, the correlations with the size factor are high and this could be reduced by using a methodology like Fama and French (1992) for factor formation[22]. The advantage of the methodology used in this paper is that the data is calculated from scratch. Sorting on size

[22] Factors are created from double sorted portfolios on size and the relevant firm-characteristic, more details can be found in Fama and French (1992).

from the beginning results in value weighting factors and thereby assuming an investment strategy before start. The basis of this study is a world where all stocks are equal weighted and therefore the clean premium of the factors is revealed.

Profitability strategies have characteristics of growth strategies and table 6.2 show that MTBV and ROE have a negative correlation of -0.47 supporting this statement. Value and momentum strategies are contrarian strategies as and table 6.2 also shows that MTBV and momentum are negatively correlated at -0.33. Both profitability factors have high negative correlation of -0.79 and -0.84 with the volatility factor showing that profitable stocks are generally connected with low volatility and thereby high risk adjusted returns. Table 6.2 also shows that the AG factor have high correlation with the MTBV factor as expected from theory. Generally the correlation structure among the factors is high and again this is due to the methodology behind the factors where they are created from equal weighted portfolios sorted on deciles. This give higher factor premiums as we are finding the difference between the 1st decile and the 10th decile compared to e.g. using quantiles.

	Correlations								
	Size	MTBV	ROE	OP	MOM	AG	Vol	Beta	MKT PF
Size	1.00	0.41	-0.60	-0.52	-0.33	0.42	0.46	0.20	0.03
MTBV		1.00	-0.47	-0.23	-0.33	0.51	0.35	0.22	0.24
ROE			1.00	0.85	0.57	-0.52	-0.79	-0.65	-0.51
OP				1.00	0.65	-0.33	-0.84	-0.70	-0.59
MOM					1.00	-0.19	-0.69	-0.64	-0.49
AG						1.00	0.31	0.19	0.11
Vol							1.00	0.91	0.76
Beta								1.00	0.78
MKT PF									1.00

Table 6.2 – Factor correlations. Source: Own calculations

6.1 Alternative factors

As discussed earlier sorting into deciles show a detailed picture of the factor premium with the drawdown being that the tails are more extreme. Trading on extremes can be hard e.g. for size where the small firms are often more illiquid. The extreme returns in the tails can also ruin the overall picture of the risk related to the factor. Therefore, this section will show factors formed in alternative ways from the single sorted portfolios. This is also shows the robustness of the factor. Only the five factors that had significant factor premium in table 6.1 will be investigated.

The results in table 6.3 and 6.4 show the size and MTBV factors are very robust to alternative ways of formation. For the size factor 34 out of 45 factor premiums are significant at the 5% level and for the MTBV factor 33 out of 45 are significant at the 5% level. Table 6.3 and 6.4 show the robustness of the size and MTBV factors is very high. Table 6.3 also show that the size premium is not only driven by extreme returns for the smallest firms, as even a factor formed as the 9th minus 10th decile is significant at the 5% level with a t-stat of 2.58.

Size

2	3	4	5	6	7	8	9	10	
4.25	5.41	5.10	5.78	5.18	5.77	5.72	5.70	5.86	1
	2.83	2.89	4.05	3.56	4.19	4.29	4.20	4.54	2
		0.27	2.04	1.66	2.51	2.83	2.85	3.43	3
			2.36	1.62	2.76	2.98	3.01	3.46	4
				-0.27	1.16	1.69	1.92	2.76	5
					1.57	2.44	2.45	3.38	6
						1.05	1.53	2.78	7
							0.89	2.64	8
								2.58	9

Table 6.3 – Size factor formed in alternative ways. The vertical axis represent the “Small” portfolios and the horizontal axis represent the “Big” portfolio and the factor is formed as Small minus Big. Source: Own calculations.

MTBV

2	3	4	5	6	7	8	9	10	
2.59	2.54	3.39	3.30	2.85	3.17	2.96	2.85	3.58	1
	0.65	2.76	2.70	1.99	2.59	2.37	2.26	3.25	2
		2.64	2.67	1.82	2.64	2.33	2.26	3.43	3
			0.02	-0.59	0.53	0.58	0.83	2.42	4
				-0.56	0.54	0.60	0.86	2.56	5
					1.11	1.26	1.44	3.18	6
						0.17	0.63	2.68	7
							0.50	2.59	8
								3.01	9

Table 6.4 – MTBV factor formed in alternative ways. The vertical axis represent the “Value” portfolios and the horizontal axis represent the “Growth” portfolio and the factor is formed as Value minus Growth. Source: Own calculations.

Table 6.5 show that the momentum factor is only significant for the 10th and the 9th deciles showing that the factor is driven by high average returns for the high momentum portfolios. The performance of the 10-1 factor is weak compared to other factor combinations. This is due to the poor performance of the low portfolio from table 5.6. Overall the performance of the momentum factor is stronger when looking at this broader picture compared to the classic 10-1 factor.

MOM

2	3	4	5	6	7	8	9	10	
-0.15	0.26	0.26	0.33	0.42	0.59	0.55	1.10	2.11	1
	1.00	0.76	0.86	0.98	1.20	1.04	1.94	3.28	2
		0.15	0.40	0.65	1.00	0.83	2.04	3.56	3
			0.41	0.85	1.44	1.08	2.69	4.28	4
				0.56	1.38	0.96	2.92	4.48	5
					0.98	0.71	2.92	4.60	6
						-0.05	2.73	4.71	7
							3.29	5.47	8
								3.96	9

Table 6.5 – Momentum factor formed in alternative ways. The vertical axis represents the "High" portfolios and the horizontal axis represents the "Low" portfolios and the factor is formed as High minus Low. Source: Own calculations.

Table 6.6 show that the AG factor is significant at the 5% level for 10 out of 45 factors. It also shows that mainly the factors formed from the 1st and 2nd decile are significant. The results in table 6.6 shows that the overall support of the AG factor is weak in the European stock market.

AG

2	3	4	5	6	7	8	9	10	
0.56	1.67	1.06	2.03	2.06	2.19	1.96	2.52	2.00	1
	1.70	0.82	2.08	1.96	2.07	1.86	2.29	1.63	2
		-0.82	1.04	0.95	0.90	0.91	1.38	0.78	3
			2.02	1.81	1.64	1.68	2.13	1.32	4
				0.00	-0.01	0.12	0.66	0.13	5
					-0.01	0.14	0.81	0.15	6
						0.14	0.81	0.16	7
							0.67	0.05	8
								-0.53	9

Table 6.6 – AG factor formed in alternative ways. The vertical axis represents "Low" investments and the horizontal axis represents "High" investments and the factor is formed as Low minus High.

Table 6.7 shows that a profitability factor formed on OP is only significant for the 9 portfolios were the lowest OP portfolio is involved. The evidence from table 6.7 combined with table 5.7a show that the OP factor is only significant due to very bad performance of the most unprofitable firms and not the good performance of firms with high profitability.

OP

2	3	4	5	6	7	8	9	10	
3.46	3.86	3.66	3.95	3.58	3.46	2.94	3.06	2.96	1
	1.76	1.74	2.76	2.06	1.65	1.17	1.39	1.54	2
		0.25	1.43	0.77	0.43	0.18	0.49	0.76	3
			1.52	0.75	0.26	0.03	0.41	0.75	4
				-0.69	-0.90	-0.95	-0.45	-0.02	5
					-0.40	-0.55	-0.04	0.39	6
						-0.23	0.33	0.67	7
							0.67	0.99	8
								0.53	9

Table 6.7 – OP factor formed in alternative ways from OP. The vertical axis represents "Low" profitability and the horizontal axis represents "High" profitability and the profitability factor is formed as High minus Low.

Summing up this detailed view on the factor premiums has showed that size and MTBV factors have very robust factor premiums. The results also showed that the OP factor is only driven by the very bad results of low profitability firms measured by OP. The momentum factor showed a stronger performance when formed in alternative ways compared to the classic 10-1 factor.

6.2 Risk in factors

Risk in stock returns is composed of unsystematic risk and systematic risk, where unsystematic risk is asset specific and can be diversified away. In a CAPM setting the only systematic risk in stocks is the excess return of the market portfolio but in a multifactor model setting there can be multiple systematic risk factors. When an investor takes on higher risk he should be rewarded by higher returns, the issue is how we measure risk. This study has showed that it is possible to get significantly higher risk adjusted return by exploiting relative simple anomalies in stock returns. This leads to the conclusion that 1) there must be other unaccounted risk factors in stock returns or 2) irrational behavior of investors lead to mispricing of stocks. This section will investigate the systematic risk connected to factors by looking at the factor premiums over time. In the sample period investigated in this study there are two recessions followed by upswings in the economy. First in 2001 caused by the internet bubble and more recently the mortgage crisis in 2008.

While the factors are formed as zero-cost portfolios it is very clear from figure 6.8 that there is high risk involved in being exposed to these factors. It is also clear that the factors follow the up and downturns in the economy like the market factor. So while there is high risk adjusted returns to be gained by investing in size and BTMV factors there is also big potential losses in down markets and this could be the explanations of the high average returns to these factors. Potential risk related to the value factor is that value firms are typically unprofitable and relatively distressed[23]. Small stocks are typically less liquid than big stocks and investors are willing to pay for holding liquid stocks, so the high average returns related to the size effect could be due to liquidity risk[24]. For the AG factor the results in figure 6.8 are less clear as the AG factor return is relative flat over the sample period. Table 6.1 also showed that the stdev of the AG factor is only 8.49%. Figure 6.8 does not show strong evidence that there is fundamental risk connected to the AG factor. Beta and volatility factors did not show a significant factor premium but figure 6.8 show that there is fundamental risk related to both factors. Beta and volatility factors are created as high minus low, but the investment strategy for anomalies would be to invest in low volatility and low beta stocks as these have higher risk adjusted returns. Figure 6.8 show that both the volatility and beta factors experience big fluctuations in up and down markets. The low volatility and low beta anomalies are driven by leverage constraints and this leads investors to overprice high beta and high volatility stocks[25]. The data showed in figure 6.8 suggest that exposure volatility and beta factors are connected to fundamental risk.

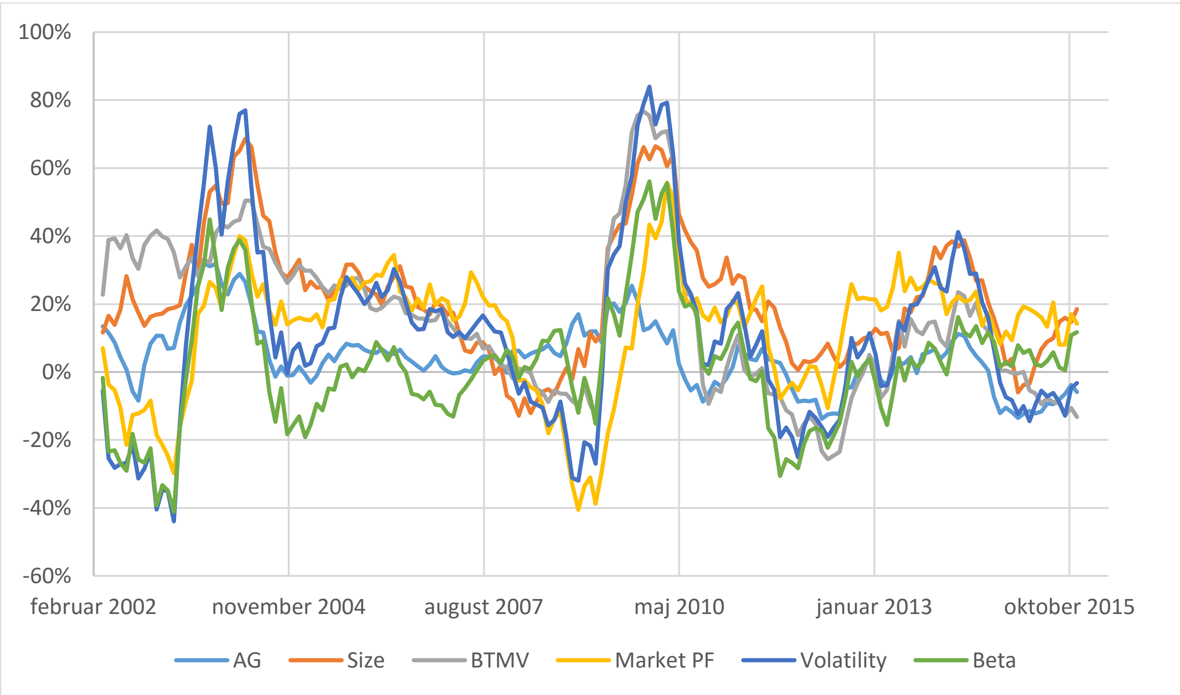

Figure 6.8 – Yearly average returns in percent for AG, size, MTBV, volatility, and beta factors together with the value weighted market portfolio. Calculated as rolling 12-month returns for the period May 2001 to December 2015. Source: Own calculations

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[23] Fama and French (1995)
[24] Acharya et al. (2004)
[25] Asness et al. (2014) , Blitz et al. (2007)

From a trading perspective risk is only systematic if it cannot be diversified away and in this perspective the correlation structure in table 6.2 and the data in figure 6.9 is good news. OP, ROE, and momentum factors all have negative correlation to the market factor. Figure 6.8 show that ROE, OP, and momentum factors move opposite of the market factor in up and downswings in the economy. The momentum factor show very high average returns in bad states of the economy and when the economy is steady, but when the market booms the momentum factor crashes. It is clear that the risk connected to these three factors is when the economy recovers from a big crash. Investment in these factors can be used as a hedge against market crashes.

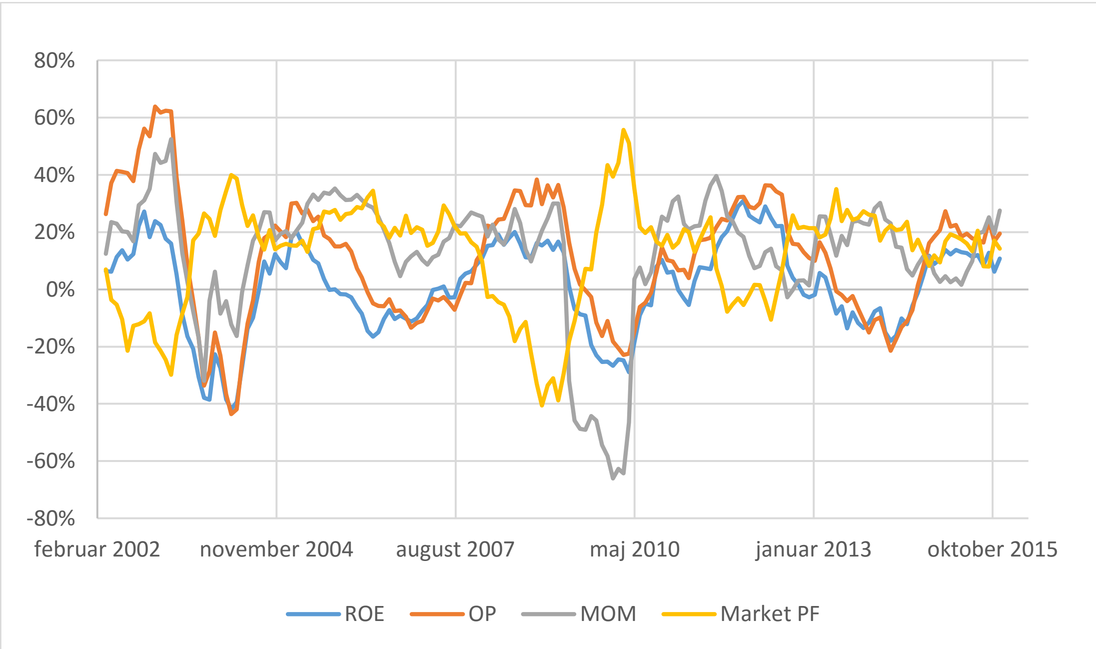

Figure 6.9 - Yearly average returns in percent for ROE, OP, and momentum factors together with the value weighted market portfolio. Calculated as rolling 12-month returns for the period May 2001 to December 2015

7. Asset pricing model

The failure of the CAPM was one of the initial motivations for this study. This section sums up the analysis of anomalies and factors and create an asset pricing model that explain the variation in returns in the European stock market. The factors used in the asset pricing model will be the excess return of the value weighted market portfolio (market factor) together with the factors created from firm-characteristics. The single sorted portfolios for each firm-characteristic from section 5.1 will be used to test the performance of the multifactor asset pricing model[26]. The factors are also created from the firm-characteristics and therefore there is an obvious connection between factors and the portfolios used to test the asset pricing model. Fama and French (1992, 2015) also use this approach but have received critique that their asset pricing model is only good at explaining the returns of their popular 25 B/M – Size portfolios[27]. Therefore, this study will also use portfolios sorted by country and GICS sectors along with the portfolios sorted on firm-characteristics to test the asset pricing model. The country and GICS sector portfolios will naturally stand as the biggest test for the asset pricing model. To begin with each factor will be tested univariate in a linear regression:

$$R_{it} - r_{ft} = \alpha_i + b_i F_t \quad \boldsymbol{EQ\ 7.1}$$

Where $R_{it} - r_{ft}$ is the excess return of the test portfolios, F_t is the factor return, b_i is the sensitivity to the factor return, and α_i is the return not explained by the factor(s) also called the pricing error. From the single factor model factors will be added one by one depending on the performance. The model will be evaluated by three measurements: the absolute average pricing error, the explanatory power of the model, and the GRS-stat. The GRS test is a test of the null hypothesis that all average pricing errors are jointly equal to zero. The critical value for the GRS-stat is 1.89 for the 5% significance level, when the GRS-stat is larger than the critical value this means that the null hypothesis is rejected. When this is the case the number is marked with red. For each firm-characteristic and GICS sector there are ten portfolios and for the countries there are 15 countries with more than ten equities that will be included in the test.

7.1 Single factor model

Table 7.1 show the results of the univariate regressions for each factor. It is clear that the market factor has the highest average explanatory power with 81.04%. The market factor also have the lowest absolute average pricing error at 3.01%. Volatility and beta factors explain 54.29% and 45.21% of the variation in expected stock returns, respectively. They have average absolute pricing errors in the mid-range compared to the other factors. The high explanatory power of the volatility and beta factors is natural as their formation has a close connection to the market factor. The MTBV and size factors have low absolute average pricing errors but they also have a low explanatory power of just 11.29% and 9.00%, respectively. The low explanatory power means that the low pricing errors are not as good since the factors explain such

[26] The double sorted portfolios for each firm-characteristics have also been tested but the results were very similar to the single sorted portfolios and the results have therefore been omitted for simplicity.
[27] Critique have been raised by Lewellen et al (2010).

a small part of the variation in expected stock returns. The two profitability factors have high explanatory power and relatively low average absolute pricing errors and they look interesting at first glance. The AG factor only explain 3.68% of the variation in expected stock returns and has an average absolute pricing error of 20.41%. Despite the criticism of the CAPM the evidence from table 7.1 show that the market factor is still the best explanatory factor for the variation in stock returns.

	MKT	Size	Value	Vol	Ag	Beta	ROE	MOM	OP
Avg $\lvert\alpha_i\rvert$	3.01	4.39	6.28	13.28	20.41	6.70	10.73	17.57	18.12
Avg R^2	81.04	9.00	11.29	54.29	3.68	45.21	32.42	23.03	37.90

Table 7.1 – Univariate regression for each factor showing the yearly average absolute pricing error in percent and the average R^2 across the ten test categories. Source: Own calculations.

Table 7.2 show detailed performance of the market factor across the ten test categories. For the market factor 4 out 10 portfolios are rejected by the GRS test. Table 7.2 show that there is pricing errors ranging from 1.68% to 5.29% and explanatory power from 72.27% to 84.85% across the test portfolios. The issues for the market factor is the portfolios formed on size, momentum, and OP. For the momentum portfolio the market factor explains 80.65% of the variation in returns and the average absolute pricing error is 5.29%. The performance of a simple market factor model like shown in table 7.2 is surprisingly good. However, table 7.2 also show that the model is best at explaining returns for the portfolios sorted on firm-characteristics. The goal is to build a model that has zero pricing errors and a high explanatory power and to will start from a single factor model with a market factor:

$$R_{it} - R_{rf} = \alpha_i + m_i MKT_{it} \quad \boldsymbol{EQ\ 7.2}$$

		Market factor	
Portfolios	**GRS**	**Avg $\lvert\alpha_i\rvert$**	**Avg R^2**
Country	1.19	3.06	70.65
GICS	2.52	2.70	73.10
Size	13.49	4.26	83.36
MTBV	1.77	2.65	83.26
ROE	1.61	2.10	84.85
OP	4.64	4.19	84.62
MOM	3.48	5.29	80.65
Vol	1.75	1.68	82.85
Beta	1.58	2.44	82.09
AG	0.89	1.78	84.97
Average		3.01	81.04

Table 7.2 – GRS-stat, yearly average absolute pricing error in percent, and explanatory power of the model in percent for a single factor model with a market factor. Source: Own calculation

7.2 2-factor model

Table 7.2 show that the main issues of the market factor model is the size, momentum, and OP portfolios where especially the small size and high momentum portfolios have high pricing errors. A natural extension is therefore to add size and momentum factors to the single factor model and see if they can increase the model performance. Table 7.3a show that while a 2-factor model with a size factor added increase the average explanatory power of the model to 88.18% the average absolute pricing error also goes up to 5.93%. The explanatory power increases for all portfolios, also the country and GICS portfolios. However, the increased average price errors mean that the GRS test strongly rejects the null hypothesis for all portfolios. A 2-factor model with a momentum factor added to the market factor increase the explanatory power for the momentum portfolio but increases the overall pricing errors. The momentum factor was in section 6.1 shown to be more significant when formed in alternative ways. Therefore, a momentum factor formed in alternative ways has also been tested, and the results are shown in table 7.3b for a 2-factor model with a momentum factor formed as the 10th minus 2rd decile (MOM*). Here, the explanatory power for the momentum portfolio increases from 80.65% to 86.11% and the average pricing error is reduced from 5.29% to 3.46%. Showing that a MOM* factor that omits the low momentum portfolio can explain the anomaly returns we see for portfolios sorted on momentum. The overall absolute average pricing error is also reduced from 3.01% to 2.53%.

	a. MKT + Size			b. MKT + MOM*		
Portfolios	**GRS**	**Avg $\lvert\alpha_i\rvert$**	**Avg R^2**	**GRS**	**Avg $\lvert\alpha_i\rvert$**	**Avg R^2**
Country	3.17	6.55	79.72	1.24	2.76	70.98
GICS	9.17	6.18	78.60	1.50	2.48	73.58
Size	12.38	5.72	92.83	11.17	4.37	83.44
MTBV	4.76	5.39	90.19	1.88	2.76	83.44
ROE	9.34	5.67	91.71	1.59	1.77	84.92
OP	16.21	6.41	91.44	3.01	3.26	84.87
MOM	6.90	6.17	86.88	1.73	3.46	86.11
Vol	6.83	5.95	89.07	0.89	1.47	83.45
Beta	6.79	5.59	89.18	1.13	1.48	82.41
AG	5.28	5.67	92.14	0.52	1.54	85.01
Average		5.93	88.18		2.53	81.82

Table 7.3 - GRS-stat, yearly average absolute pricing error in percent, and explanatory power of the model in percent. Table a show results for a 2-factor model with market and size factors. Table b show results for a 2-factor model with market and momentum factors. The momentum factor is formed as 10th minus 2nd decile. Source: Own calculations.

An overall look on the factors investigated in this study showed that aside from the size factor the MTBV and volatility factors have the highest significance level when added as a second factor to the market factor model. The results for volatility and MTBV factors in a 2-factor models with the market factor are shown in table 7.4. While volatility and market factors are highly correlated at 0.76 table 7.4b show that a 2-factor model with volatility and market factors increase the explanatory power from 81.04% to 83.97% compared to the market factor model, but the average absolute pricing errors also increase. Table 7.4a show that a 2-factor model with MTBV and market factors increase explanatory power while decreasing the average absolute pricing error from 3.01% to 2.54% compared to the market factor model.

	a. MKT + MTBV			b. MKT + Vol		
Portfolios	**GRS**	**Avg $\lvert\alpha_i\rvert$**	**Avg R^2**	**GRS**	**Avg $\lvert\alpha_i\rvert$**	**Avg R^2**
Country	1.56	3.25	76.29	1.23	3.14	73.78
GICS	6.33	2.77	75.58	3.19	2.76	76.36
Size	12.02	3.38	85.57	15.81	4.50	86.73
MTBV	0.77	0.73	87.92	2.10	2.89	85.61
ROE	4.14	2.40	86.79	1.63	2.09	87.52
OP	5.30	3.66	86.30	6.93	4.19	87.38
MOM	3.31	5.25	82.29	3.55	5.19	83.65
Vol	1.76	1.46	84.37	1.38	1.93	86.77
Beta	1.47	1.90	83.89	1.79	2.31	84.53
AG	0.51	0.64	87.02	1.02	1.99	87.44
Average		2.54	83.60		3.10	83.97

Table 7.4- GRS-stat, yearly average absolute pricing error in percent, and explanatory power of the model in percent. Table a show results for a 2-factor model with market and MTBV factors. Table b show results for a 2-factor model with market and volatility factors. Source: Own calculations.

The 2-factor model with a MOM* factor has the lowest average pricing error, but the 2-factor model with a MTBV factor has higher explanatory power while decreasing the average pricing error from the single factor model. Therefore, this study will continue with a two-factor model with MTBV and market factors as follows:

$$R_{it} - R_{rf} = \alpha_i + m_i MKT_{it} + b_i BTMV_{it} \quad \boldsymbol{EQ\ 7.3}$$

7.3 3-factor model

From the 2-factor model in EQ 7.3 several factors are significant when added as a third factor in a 3-factor model. Size, beta, volatility, MOM*, ROE, and OP are all significant. However, size, ROE and OP increases the average pricing error and they are therefore not selected[28]. Beta and volatility are highly correlated at -0.91 and captures much of the same variation in expected returns. However, only the volatility factor

[28] See appendix 4

decreases the average absolute pricing error[29]. Therefore, table 7.5 show the results of two different 3-factor models. Table 7.5a and 7.5b show results of a 3-factor model that adds volatility and MOM* respectively to the model in EQ 7.3. Both models in table 7.5 increase explanatory power while decreasing the average absolute pricing error. The 3-factor model with a volatility factor added has the highest average explanatory power while decreasing average pricing errors slightly from 2.54% to 2.41%. Table 7.5b show that a 3-factor model with a MOM* factor added decreases the average absolute pricing errors from 2.54% to 2.19% and increases the overall explanatory power of the model slightly. From table 7.5 it is clear that both factors add value to the model and is natural to continue with a 4-factor model that adds both factors.

	a. MKT + MTBV + Vol			**b. MKT + MTBV + MOM***		
Portfolios	**GRS**	**Avg $\|\alpha_i\|$**	**Avg R^2**	**GRS**	**Avg $\|\alpha_i\|$**	**Avg R^2**
Country	1.78	3.37	74.92	1.88	3.53	74.30
GICS	6.21	3.08	79.29	4.87	3.46	76.23
Size	13.99	3.78	87.83	9.54	3.82	85.71
MTBV	0.95	0.93	89.53	0.60	0.92	88.08
ROE	3.94	2.06	88.69	4.69	2.44	86.92
OP	6.62	3.38	88.36	3.87	3.03	86.63
MOM	3.26	4.65	84.56	0.73	1.10	87.79
Vol	1.70	0.95	87.86	0.74	1.24	85.07
Beta	1.33	1.24	85.86	1.03	1.34	84.38
AG	0.69	0.71	88.67	0.48	1.00	87.15
Average		2.41	85.56		2.19	84.23

Table 7.5 - GRS-stat, yearly average absolute pricing error in percent, and explanatory power of the model in percent. Table a show results for a 3-factor model with market, MTBV, and volatility factors. Table b show results for a 3-factor model with market, MTBV, and volatility factors. Source: Own calculations.

7.4 4- and 5-factor models

Table 7.5 showed that both volatility and MOM* added to the performance of the model and therefore a 4-factor model as follows will be tested:

$$R_{it} - R_{rf} = \alpha_i + m_i MKT_{it} + b_i BTMV_{it} + v_i VOL_{it} + m_i MOM_{it}^* \quad \boldsymbol{EQ\ 7.4}$$

The results from the model in EQ 7.4 can be seen in table 7.6a. Table 7.6a show that a 4-factor model like EQ 7.4 reduces the average absolute pricing errors for the momentum and size portfolios, and the overall average pricing error falls from 2.41% to 2.19%. The explanatory power of the model is also increased from

[29] See appendix 4

85.56% to 86.53%. The GRS test rejects the null hypothesis that all pricing errors are jointly equal to zero for 5 out of 10 portfolios.

	a. MKT + MTBV + Vol + MOM*			b. MKT + MTBV + Vol + MOM* + Size*		
Portfolios	**GRS**	**Avg $\|\alpha_i\|$**	**Avg R^2**	**GRS**	**Avg $\|\alpha_i\|$**	**Avg R^2**
Country	1.91	3.53	75.67	2.49	4.51	81.00
GICS	4.85	3.46	79.99	6.02	3.94	84.37
Size	10.86	3.82	88.28	4.50	4.46	94.82
MTBV	0.67	0.92	89.93	1.33	2.07	95.40
ROE	4.85	2.43	89.03	5.03	3.04	94.68
OP	5.44	3.03	88.80	5.44	3.43	94.36
MOM	1.02	1.10	89.79	0.92	0.84	95.09
Vol	0.73	1.24	88.38	1.44	2.33	94.88
Beta	1.12	1.33	86.40	1.91	2.32	93.74
AG	0.49	1.00	89.07	1.15	2.33	94.54
Average		2.19	86.53		2.93	92.29

Table 7.6 - GRS-stat, yearly average absolute pricing error in percent, and explanatory power of the model in percent. Table a show results for a 4-factor model with market, MTBV, volatility, and MOM factors. Table b show results for a 5-factor model with market, MTBV, volatility, MOM*, and size* factors. Source: Own calculations.*

From the 4-factor model in EQ 7.4 the size factor is significant when added as a fifth factor, but the issue remain that the average absolute pricing errors increases significantly. In section 5.1 it was shown that when sorted on size the outer deciles, especially the small size portfolio, did have extreme returns. It was also suggested that is due to the overweight of small cap firms in the lower deciles. Furthermore, it was shown that a size factor formed in alternative ways is still significant. Therefore, a size factor that leaves out the small cap companies was tested. Table 7.6b show results of a 5-factor model with a size factor formed as 5th minus 10th decile (size*) added to the 4-factor model from in EQ 7.4. This size* factor omits the 40% smallest firms but the premium of the size* factor is still significant at a 1%-level. The high pricing errors connected with the size factor are due to the high premium related to the size factor. This leads firms loaded positively on the size factor with a very high pricing error and vice versa for firms loaded negatively on the size factor. The 5-factor model tested with a size* factor is as follows:

$$R_{it} - R_{rf} = \alpha_i + m_i MKT_{it} + b_i BTMV_{it} + v_i VOL_{it} + m_i MOM^*_{it} + s_i Size^*_{it} \quad \boldsymbol{EQ\ 7.5}$$

Table 7.6b show results of a 5-factor model like EQ 7.5. The results show that size* factor model reduces the average absolute pricing errors significantly compared to the regular size factor. However, the average absolute pricing errors are still higher compared to the 4-factor model. Table 76.b also show that the average explanatory power of the 5-factor model is increased to 92.29%. The average explanatory power for all the firm-characteristic portfolios is high with around 93.74-95.40%. The explanatory power for the country and GICS portfolios are lower with 80.63% and 84.08%, respectively. This shows that, while factors formed as zero-cost portfolios explain a high degree of variation in portfolios sorted on firm-characteristics, they cannot explain returns for portfolios formed on unrelated characteristics to the same degree. This is a problem for the model as we want to explain the variation in expected stock returns and portfolios sorted on GICS sectors and countries are a better example of this compared to portfolios sorted on firm-characteristics. The GRS test easily rejects the null hypothesis that all pricing errors are jointly equal to zero for 6 out 10 categories for the 5-factor model. Compared to the 4-factor model we have more rejections for the 5-factor model conversely the 5-factor model does have significantly higher power. In this case it is a counteraction between explaining more of the variation in returns with higher margin of error and explaining less of the variation in returns with a lower margin of error. The next section will look at the pricing errors in detail for the 4- and 5-factor models.

7.4.1 Pricing errors

Evaluating the multifactor models the pricing errors are the most important thing. The pricing errors can be seen from two conflicting views: 1) the pricing errors show that there are still unexplained risk factors in returns or 2) pricing errors are due to irrational behavior and can be picked up by skilled investors who exploit these anomalies. This section will review the pricing errors in detail for the 4-factor model suggested in EQ 7.4 and the 5-factor model suggested in EQ 7.5. When the pricing error is significantly different from zero at a 5%-level it is marked with red.

Table 7.7 show the pricing errors for the 4-factor model and it show that just 16 out of 105 portfolios have pricing errors that are significantly different from zero. The problems for the 4-factor model surrounds the portfolios sorted on size. The 7th to 10th size deciles have negative pricing errors while the 1st size decile have a positive pricing error of 10.34%. This shows that the 4-factor model does not capture the size anomaly very well. There are also significant pricing errors for 4 out of 10 GICS sectors and this is the biggest problem for 4-factor model. As discussed previously the GICS sector and country portfolios are the toughest test for the asset pricing model and with four sectors having significant pricing errors this is not impressive. The energy sector also have a very high pricing error but it is not significant due to the low number of stocks in this sector. Aside from that there is some significant pricing errors for the low and high profitability portfolio. The GRS test reject 5 out of 10 portfolios while the model explains 86.53% of the variation in expected stock returns. Despite the pricing errors just described the performance of this 4-factor model is strong. The 4-factor model reduces most anomalies found in the European stock market and only 16 out of 105 test portfolios have significant pricing errors.

Country

FRA	GER	IRE	ITA	NED	NOR	POR	SPA	SWE	SWZ
-2.99	0.13	-2.57	-9.73	-3.24	-1.75	-6.82	-5.18	5.84	-1.87
AU	BEL	GBR	DEN	FIN					
-9.34	-1.21	1.23	-0.33	-0.76					

GICS

Energy	Mat.	Indust.	Cons.	Cons.	Health	FIN	IT	Telec.	Utilities
-7.09	-4.75	-1.01	-0.76	4.10	5.73	-4.81	4.23	0.70	-1.41

Firm-characteristics

	Low	2	3	4	5	6	7	8	9	High
Size	10.34	3.83	-0.85	-0.72	-3.49	-2.84	-3.64	-3.24	-3.52	-5.72
MTBV	-0.03	-0.68	-0.62	-2.20	-1.93	-0.89	-1.29	-0.58	0.97	-0.03
ROE	-4.10	-5.66	-3.23	-2.99	-1.01	0.28	0.22	0.48	2.03	4.35
OP	-11.97	-3.80	-1.98	-1.82	0.02	-0.37	2.42	2.14	3.72	2.06
MOM	0.68	2.29	1.66	0.61	0.29	0.88	-0.10	-0.76	1.50	2.29
Vol	0.64	-1.14	-1.14	-0.75	-0.99	-1.65	-1.69	-1.35	-2.38	0.64
Beta	1.57	0.20	-1.83	-0.25	-0.51	-1.82	-0.78	-1.91	-2.53	-1.94
AG	-0.80	-0.67	-1.74	-0.66	-1.89	-1.33	-0.77	-0.95	-1.13	0.01

Table 7.7 – Yearly average pricing errors in percent for a 4-factor model like EQ 7.4.

Table 7.8 show the results of the 5-factor with a size* factor included. While the 5-factor model increases the average explanatory power significantly from the 4-factor model the average absolute pricing errors also go up from 2.19% to 2.93%. The results of table 7.8 show that 48 out of 108 portfolios have pricing errors that are significantly different from zero. There are no real pattern in the pricing errors in table 7.8 except for the portfolios formed on momentum where all pricing errors are insignificant. All significant pricing errors are negative except for the small size portfolio and the 9th decile OP portfolio. This shows that in the addition of a size* factor leads portfolios that have positive sensitivity to the size* to get negative pricing errors. The general high correlation that was shown in table 6.2 for all factors with the size factor leads most portfolios to have high sensitivity to the size factor and thereby overestimate estimate the returns leading to a negative pricing error. This could suggest that the methodology used to form the factors gives too much weight to the size effect. Therefore an asset pricing model with factors formed as suggested by Fama and French (2015) have also been tested. However, the results did not improve and the patterns were the same as for the 4- and 5-factor models tested in this study.[30] This leads to the conclusion that the overweight of small cap stocks in the SPEU index leads the asset pricing model to overestimate the size effect leading to negative pricing errors.

Country

FRA	GER	IRE	ITA	NED	NOR	POR	SPA	SWE	SWZ
-4.08	-0.78	-4.55	-10.56	-4.14	-3.57	-7.43	-6.02	4.36	-3.21
AU	BEL	GBR	DEN	FIN					
-10.97	-2.48	-0.37	-2.58	-2.57					

GICS

Energy	Mat.	Indust.	Cons.	Cons.	Health	FIN	IT	Telec.	Utilities
-8.51	-6.24	-2.86	-2.37	3.14	4.58	-5.70	2.93	1.02	-2.02

Firm-characteristics

	Low	2	3	4	5	6	7	8	9	High
Size	8.66	1.99	-2.63	-2.70	-5.56	-4.18	-4.94	-4.24	-4.19	-5.56
MTBV	-1.41	-1.88	-1.90	-3.72	-3.25	-2.35	-2.59	-2.01	-0.14	-1.41
ROE	-5.52	-6.94	-4.48	-4.30	-2.31	-1.12	-1.07	-0.97	0.61	3.05
OP	-13.43	-5.22	-3.61	-3.39	-1.36	-1.65	1.09	1.02	2.65	0.89
MOM	-0.90	0.84	0.43	-0.52	-1.04	-0.26	-1.24	-2.12	0.23	0.84
Vol	-0.37	-2.32	-2.39	-2.10	-2.45	-3.21	-3.27	-2.86	-3.90	-0.37
Beta	-0.26	-1.34	-3.37	-1.62	-1.85	-3.18	-2.13	-3.21	-3.65	-2.61
AG	-2.28	-1.90	-2.94	-1.87	-2.94	-2.56	-2.20	-2.34	-2.68	-1.61

Table 7.8 – Yearly average pricing errors in percent for a 4-factor model like EQ 7.5.

To sum up, the results in table 7.6a still showed strong performance of a 4-factor model with market, MTBV, volatility, and MOM* factors. Therefore, this study will suggest the use a 4-factor model as follows:

$$R_{it} - R_{rf} = \alpha_i + m_i MKT_{it} + b_i BTMV_{it} + v_i VOL_{it} + m_i MOM^*_{it} \quad \boldsymbol{EQ\ 7.6}$$

The 4-factor model in EQ 7.6 has average absolute pricing errors of 2.19% and average explanatory power of 86.53%. The anomaly portfolios to the 4-factor model is the portfolios sorted on size and profitability. The portfolios formed on GICS sectors also show high absolute average pricing errors and this is the biggest issue for the model as this shows that it fails to explain the variation in returns for portfolios formed on factors unrelated to the firm-characteristics.

[30] See appendix 4.

8. Summary Discussion

This section will make a summery discussion of all the results found in section 5,6, and 7 and also discuss the affects these findings have for both private and institutional investors.

The results of the single sorted portfolios in section 5.1 showed a high RRR for all anomalies in the outer deciles compared to both the value and equal weighted market portfolio. The highest RRR was found for the high momentum portfolio and the small size portfolio with RRR of 1.23 and 1.21. Table 8.1 show the firm-characteristics ranked by their RRR in the outer decile. The results are evidence of anomalies in average returns. This study also showed that these anomalies can be explained by the irrational behavior of investors such as familiarity, overconfidence, over and under reaction to news, anchoring, and extrapolating past performances too far into the future.

Risk to Reward Ratio	
Momentum	1.23
Size	1.21
Volatility	1.10
OP	1.08
Beta	1.02
MTBV	0.94
ROE	0.90
AG	0.82
Equal Weighted Market	0.75
Value Weighted Market	0.68

Tabel 8.1 – Yearly Risk to Reward Ratio for the firm-characteristics and the market portfolio. Source: Own calculations.

In contrary to the behavioral explanations, Fama (1970) argues for strong form market efficiency meaning that stock prices always fully reflect all available information. The truth may well lie somewhere in between where markets are efficiently inefficient to a degree where money managers are paid to keep markets efficient.[31] Lack of liquidity in small stocks can lead to increased transaction costs especially for institutional investors. Leverage constraints both for mutual funds and private investors can lead to overpricing of high volatility stocks. Investigating these anomalies from a risk based approach showed leverage constrained investors looking for higher return buy high volatility stocks and this pressure leads to higher prices and lower expected returns for high volatility stocks. This study also showed that a 4-factor model consisting of

[31] Pedersen (2015)

market, BTMV, volatility, and MOM* factors capture the main part of the time-series variation in European stock returns.

Multiple testing is a relevant issue when investigating anomalies and factor in expected stock returns. This study have taken account of this by reporting all t-stats for statistical tests and indicated critical values for both the 1% and 5% significance level. Harvey et al. (2015) suggest using a t-statistic of three rather than two when investigating factor premiums. Looking at the factor premiums in this study only size and MTBV factors can surpass this hurdle. However, when formed in alternative ways OP and momentum factors are also pass this hurdle. For further research on this subject it would interesting to dick deeper into the issues of multiple testing in the European stock market, but this would be a full study on its own. Furthermore it would also be interesting to test the results found in this study in an out-of-sample setting.

The implications for the results of this study to investors are two-sided. First, it is found that there are strong anomalies in expected stock returns on the European stock market by sorting portfolios on simple firm-characteristics. Secondly, it is also found that a 4-factor model consisting of zero-cost portfolios of these firm-characteristics can explain the time-series variation in European stock returns. The implications of this two-sided story is that there anomaly returns in the European stock market but this study finds that these anomalies are in large explained by risk factors connected to firm-characteristics. Furthermore, it is notable that the results are based on monthly rebalancing and implementing this, especially for private investors, can lead to high transaction costs. The effect of transaction costs of the anomaly trading strategies is also an interesting topic to look into for further research.

9. Conclusion

This thesis studies anomalies based on firm-characteristics in the European stock market. The firm-characteristics that are studied are beta, volatility, size, MTBV, momentum, ROE, OP and AG using monthly returns for 1065 stocks in the period 2000 to 2015. The thesis is motivated by the debate of whether anomalies observed in stock returns exist due to behavioral irrationalities or due to unexplained risk factors. This thesis first investigates anomalies for their existence in the European stock market in the sample period and the proceeds to form factors as zero-cost portfolios and test if they can explain the variation in European stock returns.

This thesis finds that, based on single sorted portfolios that are rebalanced each month, anomalies in European stock returns very much do exist. Evidence is presented of risk adjusted return anomalies for seven firm-characteristics: beta, volatility, size, MTBV, momentum, ROE, and OP. Size and momentum show the strongest risk-to-reward ratio with 1.21 and 1.23 respectively compared to a risk-to-reward ratio of 0.68 for the value weighted market portfolio.

Furthermore, this thesis performs a number of time-series regression of multifactor models with factors formed as zero-cost portfolios from the single sorted portfolios. The factors are formed as 1^{st} decile minus 10^{th} decile or the other way around depending on the signal of the firm-characteristics. Factors are also investigated for alternative ways of formation across all deciles as a robustness check. It is showed that a 4-factor model that includes a market, MTBV, volatility, and momentum factor captures the variation in European stock returns. The 4-factor model is showed to have an average explanatory of 86.53% and an average absolute pricing error of just 2.19% across 105 test diversified test portfolios. The results is a significant improvement over the market factor model.

Finally this thesis also contributes to the debate of whether the anomalies that are observed in stock returns exist due to common risk factors that can be explained by these factor portfolios or if the anomalies appear due to irrational behavior of investors. It is showed that mispricing can occur due to herding, familiarity, overreaction to news or earnings announcements, or extrapolation of returns too far into the future. It is showed that all these irrationalities can lead to over or under pricing of stocks. This paper recognizes that behavioral irrationalities by investors can lead to mispricing of stocks. However, the main findings of this paper is that a 4-factor model with zero-cost portfolios as factors reduces most anomalies found in the European stock market. Only 16 out of 105 test portfolios have pricing errors that are significantly different from zero on a 5%-level to this 4-factor model. Therefore, this paper supports the risk based explanation of anomalies in European stock returns.

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Appendix 1

```
Sub sortbymonth_delay()


nYears = 187

nMonths = 187

Dim dataRange, keyrange As String

Dim interval2, interval As Integer

dataRange = "D45:AOB438"

keyrange = "D46:AOB46"

'Set returns = ActiveSheet.Range("M64:AOB64")


For i = 0 To nMonths

ActiveSheet.Sort.SortFields.Clear

ActiveSheet.Sort.SortFields.Add Key:=Range(keyrange).Offset(i, 0), SortOn:=xlSortOnValues,
Order:=xlAscending, DataOption:=xlSortNormal

   With ActiveSheet.Sort

   .SetRange Range(dataRange)

   .Header = xlYes

   .MatchCase = False

   .Orientation = xlLeftToRight

   .SortMethod = xlPinYin

   .Apply

End With



Dim Ncol As String

Set temp1 = ActiveSheet.Range(keyrange).Offset(i, 0) 'setting sorting range

Ncol = WorksheetFunction.Count(temp1)

nstocks = WorksheetFunction.RoundDown(Ncol / 10, 0)
```

```
For k = 0 To 9
Set temp2 = ActiveSheet.Range(Cells(251, 4), Cells(251, 3 + nstocks)).Offset(i, nstocks * k)
Cells(448, 4).Offset(i, k) = WorksheetFunction.Average(temp2)
Next k
Next i
 End Sub
```

Appendix 2

```
Sub doublesort()

'3x3 sort

nYears = 191

nMonths = 191

Dim dataRange, keyrange As String

Dim interval2, interval As Integer

dataRange = "D45:AOB630"

keyrange = "D46:AOB46"

temp4 = "D241:AOB241"


For i = 0 To nMonths

'First Sort

ActiveSheet.Sort.SortFields.Clear

ActiveSheet.Sort.SortFields.Add Key:=Range(keyrange).Offset(i, 0), SortOn:=xlSortOnValues,
Order:=xlAscending, DataOption:=xlSortNormal

   With ActiveSheet.Sort

   .SetRange Range(dataRange)

   .Header = xlYes

   .MatchCase = False

   .Orientation = xlLeftToRight

   .SortMethod = xlPinYin

   .Apply

End With


'Second sort

Dim nCols2 As String

temp2 = ActiveSheet.Range(temp4).Offset(i, 0)

temp1 = ActiveSheet.Range(keyrange).Offset(i, 0)

If WorksheetFunction.Count(temp1) < WorksheetFunction.Count(temp2) Then
```

```
nCols2 = WorksheetFunction.Count(temp1)

Else

nCols2 = WorksheetFunction.Count(temp2)

End If

nstocks2 = WorksheetFunction.RoundDown(nCols2 / 3, 0)

nstocks3 = WorksheetFunction.RoundDown(nCols2 / 9, 0)


For p = 0 To 2

Set dataRange2 = Range(Cells(45, 4), Cells(630, 3 + nstocks2)).Offset(0, nstocks2 * p)

Set KeyRange2 = Range(Cells(241, 4), Cells(241, 3 + nstocks2)).Offset(i, nstocks2 * p)


ActiveSheet.Sort.SortFields.Clear

ActiveSheet.Sort.SortFields.Add Key:=KeyRange2, SortOn:=xlSortOnValues, Order:=xlAscending,
DataOption:=xlSortNormal

   With ActiveSheet.Sort

   .SetRange dataRange2

   .Header = xlYes

   .MatchCase = False

   .Orientation = xlLeftToRight

   .SortMethod = xlPinYin

   .Apply

  End With




For k = 0 To 2

Set temp7 = ActiveSheet.Range(Cells(439, 4), Cells(439, 3 + nstocks3)).Offset(i, nstocks3 * k + p * nstocks2)

If p = 0 Then

Cells(636, 4).Offset(i, k) = WorksheetFunction.Average(temp7)

ElseIf p = 1 Then

Cells(636, 7).Offset(i, k) = WorksheetFunction.Average(temp7)
```

```
ElseIf p = 2 Then
Cells(636, 10).Offset(i, k) = WorksheetFunction.Average(temp7)
End If
Next k
Next p
Next i

End Sub
```

Appendix 3

```
Option Base 1

Function getCol(data, colI)
x = data
nRows = UBound(x, 1)
ReDim theCol(nRows, 1)
For i = 1 To nRows
theCol(i, 1) = x(i, colI)
Next i
getCol = theCol
End Function

Function getRow(data, rowI)
x = WorksheetFunction.Transpose(data)
y = getCol(x, rowI)
getRow = WorksheetFunction.Transpose(y)
End Function

Function varCovar(data)
x = data
N = UBound(x, 2)
ReDim y(N, N)
For i = 1 To N
For j = 1 To N
y(i, j) = WorksheetFunction. _
Covariance_S(getCol(x, i), getCol(x, j))
Next j
Next i
varCovar = y
```

```
End Function

Function mm(x, y)
mm = WorksheetFunction.MMult(x, y)
End Function

Function tr(x)
tr = WorksheetFunction.Transpose(x)
End Function

Function inv(x)
inv = WorksheetFunction.MInverse(x)
End Function
Function GRS(portf, factor, alpha, beta)

y = portf
x = factor
a = alpha
b = beta

T = UBound(y, 1)
N = UBound(y, 2)
L = UBound(x, 2)

ReDim ee(N, T)

For p = 1 To N
For i = 1 To T
ee(p, i) = y(i, p) - a(1, p) - b(1, p) * x(i, 1)
Next i
```

```
Next p

varcovarTemp = mm(ee, tr(ee))

ReDim varcovaree(N, N)
For r = 1 To N
For u = 1 To N
varcovarTemp(u, r) = varcovarTemp(u, r) / (T - L - 1)
Next u
Next r

meanfactor = WorksheetFunction.Average(x)
ReDim temp1(L, T)

For i = 1 To T
temp1(1, i) = meanfactor
Next i

ReDim temp2(L, T)
For i = 1 To T
temp2(1, i) = x(i, 1) - temp1(1, i)
Next i
covarMat = mm((temp2), tr(temp2))
covarMat = WorksheetFunction.Sum(covarMat) / (T - 1)

Value1 = mm(mm(a, inv(varcovarTemp)), tr(a))
Value2 = mm(mm(meanfactor, inv(covarMat)), tr(meanfactor))
Value3 = WorksheetFunction.Sum(Value1)
Value4 = WorksheetFunction.Sum(Value2)
```

```
Value5 = (T / N) * ((T - N - L) / (T - L - 1)) * (Value3 / (Value4 + 1))
GRS = Value5

End Function
```

Appendix 4

Portfolios	a. MKT + MTBV + ROE GRS	alpha	R^2	b. MKT + BTMV + OP GRS	alpha	R^2
Country	1.90	3.26	75.17	2.56	3.87	75.58
GICS	7.28	4.26	78.58	9.36	5.79	78.95
Size	15.97	4.55	87.83	16.77	5.44	88.15
BTMV	0.95	2.05	89.40	3.23	3.46	89.68
ROE	3.04	1.98	89.16	4.36	3.15	89.07
OP	4.28	2.99	88.60	2.20	3.27	89.28
MOM	4.34	4.30	83.98	4.62	5.07	84.53
Vol	2.85	1.75	86.41	3.69	3.16	86.72
Beta	1.51	1.74	85.30	3.03	3.17	85.58
AG	2.35	1.79	88.73	2.58	3.23	88.90
Average		2.87	85.32		3.96	85.64

Table A.1 – Source: Own figure

Portfolios	a. MKT + MTBV + SIZE GRS	alpha	R^2	b. MKT + BTMV + BETA GRS	alpha	R^2
Country	3.74	8.47	78.89	1.74	4.11	74.14
GICS	6.29	5.33	81.12	5.79	3.84	78.05
Size	9.24	6.63	92.93	12.32	4.95	85.87
BTMV	4.17	6.30	93.35	0.81	0.63	88.09
ROE	6.67	6.63	92.10	3.68	1.88	87.30
OP	11.99	6.57	91.66	4.71	3.03	86.82
MOM	4.32	5.21	87.09	2.94	5.05	83.62
Vol	3.43	6.57	89.19	1.66	1.24	87.25
Beta	4.40	6.59	89.34	0.87	0.93	86.34
AG	4.44	6.58	92.55	0.67	0.78	87.14
Average		6.49	88.82		2.65	84.46

Table A.2 – Source: Own figure

	a. FF 4-factor			b. FF 5-factor		
Portfolios	**GRS**	**alpha**	**R^2**	**GRS**	**alpha**	**R^2**
Country	2.11	4.11	74.43	4.65	6.70	79.66
GICS	5.83	3.67	78.95	4.73	4.15	83.45
Size	14.76	5.02	86.84	9.48	5.28	93.39
BTMV	4.04	2.86	88.97	2.99	4.32	93.97
ROE	3.70	2.66	88.03	7.13	4.81	92.97
OP	4.73	3.89	87.70	9.67	5.00	92.66
MOM	2.52	4.49	84.81	1.32	2.53	90.55
Vol	1.89	2.59	87.48	1.94	4.60	93.16
Beta	1.11	2.58	85.24	2.17	4.63	92.09
AG	1.03	2.62	87.94	1.75	4.60	93.04
Average		3.45	85.0399		4.66	90.49

Table A.3 – Source: Own figure