The Application of the Black-Litterman model in a Multi-Factor Framework

by

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Abstract

In this thesis we demonstrate that the optimal portfolios generated by the Black-Litterman asset allocation model have a very simple, intuitive property. We show that the assumptions of the model are not violated when introduced to multiple factor and sector premiums. The Black-Litterman model enables investors to combine their unique views regarding the performance of various assets with the market equilibrium in a way that results in intuitive, well-diversified portfolios. The unconstrained optimal portfolio in the Black-Litterman model is the scaled equilibrium portfolio plus a weighted sum of portfolios representing the investor's views. The weight on a portfolio representing a view is positive when the view is more bullish than the one implied by the equilibrium and the other views. The weight increases as the investor becomes more bullish on the view, and the magnitude of the weight also increases as the investor becomes more confident about the view. This paper consolidates insights from the relatively few works on the model and provides an extended multi-factor framework with step-by-step instructions that enable the reader to implement this model.

Resumé

Formålet med denne afhandling, er at undersøge applikationen af et multi-faktor ligevægtssystem i Black-Litterman modellen. Ud fra et teoretisk synspunkt undersøges om historiske risikopræmier på elegant vis kan blendes med subjektive views for at skabe stabile porteføljeallokeringer. Størstedelen af den empiriske litteratur på området er kendetegnet ved, enten at forklare fremgangsmåden i selve modellen, eller tage udgangspunkt i såkaldte reference-modeller, som bygger på andre antagelser end den kanoniske model af Black og Litterman. Disse studier har nået til forskellige konklusioner. Denne afhandling foreslår en ny anvendelsesmetode af modellen, som passer bedre til aktuelle investeringsbehov. Valget af en denne tilgang, er motiveret med det generelle opgør med aktivklasser, som langsomt er blevet erstattet af risikofaktorer og de afledte systematiske risikopræmier. Denne afhandling foreslår en alternativ tilgang til de oprindelige ligevægtsforudsætningerne i Black-Litterman modellen, ved at udvide den oprindelige ligevægt defineret ved CAPM, til i første del af analysen, at indeholde fem systematiske risikofaktorer. For at underbygge disse resultater, inddrager vi en alternativ specifikation af views, som anvendes på sektorniveau ud fra GICS klassifikationssystemet. Vi undersøger derfor den empiriske præcisionsgrad af en multifaktor ligevægt, ved at foretage en analyse af samtlige selskaber i CRSP/Compustat universet, i perioden 1990-2016, til dannelse af risikopræmier. Efterfølgende anvender vi en delmængde heraf, til at foretage en analyse af stabile porteføljeallokeringer på tværs af sektorer, nærmere bestemt i tidsintervallet 2007-2016, hvorefter vi fremviser resultaterne af denne fremgangsmåde.

Vores resultater indikerer, at multifaktor applikationen i en Black-Litterman kontekst er et særdeles nyttigt værktøj i en eventuel porteføljeallokeringssammenhæng, i det de opdaterede afkastskøn trækkes i intuitiv retning af de specifikke views. Dette belyser vi fra flere vinkler, idet vi anvender to forskellige dataserier hvor vi analyserer flere empiriske metoder til estimation af modellens mest subjektive parametre. Endelig finder vi at samspillet mellem de i forvejen aktive strategiske risikopræmier, kan være en konceptuel udfordring for Black-Litterman modellen, da den forudsætter en ligevægt, som de anvendte risikopræmier i praksis abstraherer fra. Dette kræver dog yderligere undersøgelser.

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Chapter 1

Introduction

The recent years of the financial markets have proven challenging for investors. With a fear of a China Hard Landing[1], a threatening possibility of deflationary environment in the U.S and Europe[2], a de-facto ZIRP and QE application by the European Central Bank[3], an oil price collapse[4], and most recently, a daunting outlook for the most systemic important bank in the world, Deutche Bank[5]

As a consequence, the state of finance theory has changed dramatically over the past 30 years, away from the restrictive theories of the single-factor Capital Asset Pricing Model, efficient markets, and constant expected returns (see Cochrane (2011)). Current academic views are more diverse, less tidy, and more realistic. Expected returns are now commonly seen as driven by multiple factors. Some determinants are rational, such as risk and liquidity premia; others, irrational psychological biases like extrapolation and overconfidence. The expected return on any investment or factor may vary over time, again for rational or irrational reasons. The fundamental concept that assets earn risk premiums, when exposed to underlying risk factors was first introduced by the CAPM, the first real theory of factor risk. The CAPM states that individual assets that correlate with the market are risky[6] and therefore must earn risk premiums. This is the theoretical foundation of factor models. The theory revolves around one single factor driving all asset returns; the market re-

[1]Winkelmann et al (2014) provide useful insights and Abbasi (2015) applies a stress-test scenario.

[2]Federal Reserve Bank of Atlanta

[3]Concerns about the consequences of the zero bound for interest rates date at least to Keynes, and many observers have believed that central banks are helpless when short-term rates are near the zero bound.

[4]Crude oil has since mid-2014 declined roughly 50%.

[5]Deutsche Bank appears to be the most important net contributor to systemic risks in the global banking system, followed by HSBC and Credit Suisse (IMF Country report (2016)).

[6]The distinction is implicitly made between systematic and idiosyncratic risk, where the latter bears no compensation.

turn in excess of T-bills[7]. The asset pricing model of Sharpe (1964), Lintner (1965) and Black (1972) has long shaped the way academics and practitioners think about average returns and risk. The central prediction of the model is that the market portfolio of invested wealth is mean-variance efficient like that of Markowitz (1952). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market βs (the slope of the regression of a security's return on the market's return), and (b) market βs suffice to describe the cross-section of expected returns. Expanding on the CAPM, Ross (1976) proposed the Arbitrage Pricing Theory (APT), which, in contrast to the CAPM, allows for multiple sources of systematic risk. Unfortunately, the theory does not specify what risk factors are rewarded.

The simple observation that there are systematic differences in the cross-section of stock returns is neither new nor exciting. Compelling evidence suggests that low-risk investing delivers superior results, notably both across asset classes, as well as within them, so-called 'betting against beta'. Such findings contradict the intuitive notion that higher expected as well as realized returns compensate investors for taking risks, at least as measured by the second moment of asset returns. The effectiveness of these investment strategies reflects a persistently inverse relation between Sharpe ratios and beta, defined as the covariance of asset returns with market portfolio returns, and correspondingly, an insufficiently upwardsloped security market line (SML). Besides a comprehensive study of several asset classes that documents these patterns (Frazzini and Pedersen (2014)), subsequent research reports that excess returns from BAB using global equities are robust not only to size and momentum but also to industry classifications (Asness et al. (2014)). This finding further adds to the motivation for this thesis.

It is by now well-documented that the CAPM does not sufficiently capture the expected return of securities (Cazalet and Roncalli (2014)). Empirically, the SML almost consistently overestimates the performance of high beta securities while it underestimates the performance of low beta securities. The line is too flat, the intercept is too high, and consequently beta may be a valuable tool for arbitrageurs (Black, Jensen, and Scholes, (1972)). In factor-based models, numerous asset pric-

[7] The nominal risk-free rate is relatively easy to obtain from U.S. Treasuries. However, one needs to consider the term structure, that is, whether the yield curve is up- or downward sloping and the investment horizon. Unfortunately, the CAPM offers no guidance, because it has no concept of time other than a single time ***period*** and thus no concept of a yield curve. However, from a practical perspective, it makes sense to match assets to the same risk-free zero-bond yield that is closest to specific assets expected cash flow with a matching horizon.

ing inconsistencies have emerged in the literature. Today, this has become a widely recognized and vigorously debated topic in academia and among investment professionals. More recently, the application of factor-models has proliferated into a large industry that is often referred to as factor- or smart beta investing. Rather than loading on the conventional market beta, investors are increasingly demanding exposure to alternative betas, such as size, value, momentum, and quality. Large institutional investors, such as pension funds and sovereign wealth, funds have realized the competitive advantage of their large balance sheets and, as a result, are increasingly demanding betting against beta products or similar strategies, built to exploit asymmetries. These findings are interesting for several reasons. From a theoretical point of view, ideas transpires from the practical field leaving academia and alike to solve for ways to exploit the inadequacies of the CAPM, in the continuous search for alpha. An interesting result of this search is the Black-Litterman model. Since its publication in the 1990s, the Black-Litterman asset allocation model has gained some application in financial institutions. As developed in the original paper (1990), the model provides the flexibility to combine the market equilibrium with additional subjective views of the investor. In the Black-Litterman model, the user inputs any number of views or statements about the expected returns of arbitrary portfolios, and the model combines the views with equilibrium, producing both the set of expected returns of assets as well as the optimal portfolio weights. However, the deficiencies of the CAPM remain in the assumptions of the Black-Litterman model, as it uses the market equilibrium portfolio.

In the assumptions of the CAPM, we should expect inefficiency to lie in market segments that are illiquid, with poor information dissemination and where outsized profits may be hard to collect because trading on these anomalies will likely move prices. Today, it has become quite the axiom that markets cannot be efficient in their pure form. As pointed out by Ang (2014), it is costly to collect information and to trade on that information. Then, if all information is in the price already, why would anyone ever invest in gathering the information? But if no one invests in gathering the information, how can information be reflected in security prices so that markets are efficient? The near efficient markets of Grossman and Stiglitz (1980) and Fama (1970) fit closely with the multiple factor risk framework of the APT developed by Ross (1976). In Ross's multi-factor model, active managers and arbitrageurs drive the expected return of assets toward a value consistent with an equilibrium trade-off between risk and return. The factors in the APT model are systematic ones, vis-a-vis those that affect the whole economy, that agents wish to hedge against. In their purest form, factors represent risk that cannot be arbitraged away, hence the adage 'high risk, high return'.

From this proposed view of market inefficiency, the supposed superiority of multiple risk factors opposed to the single risk-factor framework provided by the CAPM combined with the flexibility of the Black-Litterman model to adjust the general equilibrium with active views will be the starting point of this thesis.

The thesis will proceed as follows. In the remaining part of this chapter, We present our research question and delimitations. **Chapter 2** is devoted to theoretical considerations on the properties of a multiple risk factor framework in the context of the Black-Litterman model. In **Chapter 3**, we provide a literature review of empirical studies on factors and the Canonical Black-Litterman model. In **Chapter 4**, we present and motivate our sample selection and methodological approach. **Chapter 5** presents and analyzes the empirical results. In **Chapter 6**, we synthesize our findings and the existing literature. We explain our findings and discuss the practical implications as well as ideas for future research. **Chapter 7** concludes this thesis.

1.1 Research Question

This thesis tests whether the general equilibrium framework in the Black-Litterman model provided by the CAPM, can be adjusted to include multiple risk-factors and whether this adjustment yields reasonable results in terms of factor allocations. More specifically, we conduct a thorough analysis based on a large sample of US firms to construct a set of known factors methodologically in lines with Fama and French[8]. This raises some natural questions. First, is it possible to challenge the foundation of the Black-Litterman model, as it assumes CAPM one-factor equilibrium? Secondly, with the vast amount of theory on the practical deficiencies of this normative model, are the more realistic assumptions of the Black-Litterman likewise transferred to a multiple risk factor setting? In essence, the main objective of this thesis is to provide an answer to the following question:

- *Can the Black-Litterman model be applied in a multi-factor context to create stable portfolio allocations?*

[8] In the original spirit of Fama and French (1992a, 1993 and 1996). Instead of using independent sorts like Fama and French, we use conditional sorts to ensure a balanced number of securities in each portfolio, like Asness, Frazzini and Pedersen (2013)

To our best knowledge, what is still missing for active style allocation is an applied model that focuses on more than one factor. Our contribution to this literature is to introduce an optimal allocation model that is viewed through the lens of multiple risk factors and later sectors. Portfolio choice is one of the main problems relating to practical asset management. Due to a continuing search for alpha combined with the limitations of regulatory requirements, the role of active management has never been more challenged. As a result, this thesis is directed towards those investment managers who seek alternative approaches that are both intuitive and flexible.

1.2 Delimitations

There are important delimitations to our empirical analysis. We only consider US data. More specifically, we only consider the firms that were part of the merged CRSP/Compustat data index between 1990 and 2016. For the second part of our analysis, we only consider a sub-sample; May 2007 through May 2016 for the sector premiums and the US constituents of the the S&P 1500. The choice of sample is motivated in our section on sample selection. We do not intend to test other implementation issues. For example, we do not test which views lead to a superior return estimates, which should be dedicated an individual research. Our thesis is concerned only with the implementation of multiple risk factors as a general equilibrium in the Black-Litterman model. The sole reason for our variations in other implementation choices is to perform robustness checks of our findings.

Ideally, an empirical analysis that presents a new method should formally test this method against all existing methods. This is not possible. As a result, it is not our intention to compare the results of a single factor vs. a multiple factor general equilibrium approach of the Black-Litterman model and to test which is superior to the other. Instead, this thesis addresses the issue of identifying factor risk-premiums and to apply these in a robust way.

It should be emphasized at this point that the results presented here abstract away from a number of issues. First, because we have performed an in-sample factor analysis, we do not claim that the process used in this thesis is superior in generating active style allocation views and therefore, should be applied in practice. We believe instead that the implementation of dynamic asset allocation strategies requires the use of sophisticated econometric processes, which should put great care on avoiding the pitfalls of data mining. Besides, our analysis does not take into account the presence of transaction costs. Because of significant turnover in the factor generating process, we expect such frictions to be a major issue when implementing multiple risk factors in any context. Overall, the purpose of the exercise is not to promote a specific strategy, but rather to illustrate and exploit the already known; that the Black-Litterman model is an extremely intuitive and useful tool for active asset allocation purposes.

Chapter 2

Theoretical framework

The purpose of the following section is to show the intersection of properties that would justify using a multiple risk factor framework in the context of the Black-Litterman model. Introducing and elaborating on the theoretical boundaries and on what drive factor returns, will be the starting point of this chapter. The derived properties will provide the foundation for the appropriate 'liaison' to the rest of this thesis. The chapter starts with the well-known CAPM that relates the properties of one systematic risk factor to expected stock returns. Hereafter, we expand the framework to account for potentially multiple systematic risk factors that drives expected stock returns. We state equations in terms of risk premiums, as done by most literature to explicitly state the unequivocal fact that we seek to find compensation for these underlying risk factors. We incorporate empirical findings of previous studies into the discussion on factors and the Black-Litterman model alike.

As cited in some of the CAPM-related literature, the market can be viewed as the first and most important equity factor. Beyond the market factor, researchers generally look for factors that are persistent over time and have strong explanatory power over a broad range of stocks. Miller (2006) focuses on three key statistical criteria for factors: persistence over time, 'large enough' variability in returns relative to individual stock volatility, and application to a 'broad enough' subset of stocks within the defined universe. Ang (2014) lists four criteria for determining which factors to choose: *'be justified by academic research', 'have exhibited significant premiums that are expected to persist in the future', 'have return history available for bad times'* and *'be implementable in liquid, traded instruments'*.

2.1 Drivers of factors

The question of what drives stock returns dates back to the beginning of modern finance. The most well-known model of stock returns is the Capital Asset Pricing Model (CAPM), which became the foundation of many theories but most importantly, modern financial theory proposed in the 1960s by Lintner (1965), Mossin (1966), Sharpe (1964) and Treynor (1961). Pioneering work by Tobin (1958) and Markowitz (1952), cemented the pillars of what where to be known as Modern Portfolio Theory (MPT) by introducing the *separation theorem* and *mean-variance efficiency* respectively. Based on the *general equilibrium* notion, the CAPM argues that securities only have two main drivers: systematic risk and idiosyncratic risk.

According to the CAPM, the expected return of any security or portfolio is linearly determined by its market beta. To arrive at this conclusion, the model assumes that all investors hold the market portfolio and engage in lending or borrowing activities to satisfy their preferences. Risk sensitive investors hold a fraction of the market portfolio and invest their remaining capital in the risk-free rate. Conversely, investors that prefer higher returns apply leverage to increase the expected returns of the market portfolio. Deviation from this behaviour is inefficient, as CAPM requires the market portfolio to be the optimal bundle of risky assets, where the return-to-risk relationship is steepest. However, not all investors have unrestricted access to leverage. Retail-investors exhibit behavioural constraints (debt aversion) and large institutional investors are often restricted by regulatory requirements such as margin requirements and solvency laws. According to Black (1972), the risk-reward relationship might flatten in an equilibrium with restricted borrowing. Rather than explicitly applying leverage, constrained investors may be forced to substitute for implicit leverage by investing in securities that are themselves riskier or where leverage is embedded. Since systematic risk is undiversifiable, investors are compensated with returns for bearing this particular risk. The idiosyncratic risk component is asset-specific and diversifiable and bears no compensation. As the market model is a statement about ex-ante returns i.e., expected returns, it is natural to write in terms of expected risk premiums. That is:

$$\mathbb{E}(R_i) - R_f = \beta_i^m[\mathbb{E}(R_m) - R_f] \tag{2.1}$$

α_i is intentionally left out of the equation, as the CAPM assumes it to be zero. And since we state the equation in terms of compensated premia, the idiosyncratic risk component ϵ_i vanished as well. Ex-post, some securities will do better or worse and have returns higher or lower than predicted by the CAPM, that is:

$$R_i - R_f = \alpha_i + \beta_i^m [R_m - R_f] + \epsilon_i \tag{2.2}$$

Ten years later, Ross (1976) proposed the 'Arbitrage Pricing theory' (APT). APT is a different approach to what drives stock returns. Ross argued that the expected return of financial assets, can be modeled in various ways and should not be limited to one single systematic factor. Instead, he argued that expected return can be modeled as a function of various macroeconomic factors or theoretical market indexes. He also popularized the original term 'factors', as the models he proposed were called 'multi-factor models'. APT, contrary to the CAPM, relies on the no-arbitrage principle and derives the risk/return relationship in light of all factors that could generate arbitrage opportunities but at equilibrium cancels out. As the APT did not explicitly state what these factors should consist of, it is less tractable than CAPM. Instead, the nature of these factors are likely to change over time and vary across markets. Thus, the challenge of building factor models is essentially empirical in nature.

$$\mathbb{E}(R_i) - R_f = \beta_{i,1}\mathbb{E}(F_1) + \beta_{i,2}\mathbb{E}(F_2) + ... + \beta_{i,k}\mathbb{E}(F_k) \tag{2.3}$$

Where $\beta_{i,k}$ is the beta of asset i with respect to factor k and $\mathbb{E}(F_k)$ is the risk premium of factor k. As the factor model by Ross (1976) did not explicitly state what these factors were, the model has a more flexible foundation than the CAPM counterpart. Naturally, a lot of literature has build upon the arguments made by Ross, few however, is as cited as the 'Three Factor Model' made by Eugene Fama and Kenneth French. The model says that the expected return on a portfolio in excess of the risk-free rate $[\mathbb{E}(R_m) - R_f]$ is explained by the sensitivity of its return to three factors: (i) the excess return on a broad market portfolio $[(R_m) - R_f]$; (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks; and (iii) the difference between the return on a portfolio of high book-to-market stocks and low book-to-market stocks. The Fama-French model, which today includes Carhart's (1997) momentum factor, forms the foundation of modern factor theory. In this thesis, we expand to include the more recent quality factor formerly proposed by Asness, Frazzini, and Pedersen (2013). Specifically, from Fama and French (1996), the expected excess return on portfolio i is:

$$\begin{aligned}\mathbb{E}(R_i) - R_f =& \beta_i^m[\mathbb{E}(R_m) - R_f] \quad + \beta_i^{smb}\mathbb{E}(R_{smb}) + \beta_i^{hml}\mathbb{E}(R_{hml}) \\ &+ \beta_i^{umd}\mathbb{E}(R_{umd}) \quad + \beta_i^{qmj}\mathbb{E}(R_{qmj})\end{aligned} \tag{2.4}$$

Equation 2.1, **2.2**, **2.3** and **2.4** share a set of systemic factors that governs the rules in the particular model. These equations show that the world is far more complex than otherwise portrayed by this single factor model. **Equation 2.4** and the findings of Fama and French will be used more vigorously when we go from the stylized version of the Black-Litterman model to include for more potential factors. Before we conceptually introduce the theoretical framework of the Black-Litterman model, we briefly provide a section that elaborates on the more behavioural aspects of factors.

2.1.1 Factor characteristics

The next section will go beyond the single factor as explained above and show more vigorously what systematic factors actually are. It is sometimes said that factors are to assets what nutrients are to food. Carbohydrates, for instance, can be obtained from food made from cereals and grains. Protein is obtained from meat and dairy products. Fiber from wheat and rice. Fat we consume from animals but also certain plant foods. Each type of food is a bundle of nutrients. This is to say factors behind the assets matter, not the assets themselves, which essentially are bundles of risk factors. This means that *asset classes are like complex molecules, factors are the atoms that serve as building blocks*. Like atoms, a risk factor should not be divisible into any smaller part. This means a factor can be thought of as any characteristic relating a group of securities that is important in explaining their return and risk. For example, inflation is very hard to decompose further, contrary to a bond, which is sensitive to numerous risk and return factors that are macroeconomic in nature, such as GDP, real interest rates, and inflation along with asset class-specific characteristics like duration, convexity, and spread. Naturally, factors can be comprised of virtually any sort of characteristic (see Harvey et al. (2014) who counts at least 300 risk factors). In such situations, the investor may be lost in front of the factor proliferation. Veldkamp (2011), states that since processing and collecting information is not costless, and certain information is not available to all investors, information itself can be considered a factor in some economic settings. More often that not, however, such characteristics are in the domain of either macroeconomic or firm specific. Macroeconomic factors include measures such as inflation, GDP, yield curves, and other measures of the macro economy (see Chen et al. (1986)). Statistical factor models identify factors using statistical techniques such as principal components analysis (PCA) where the factors are not pre-specified in advance. Arguably the mostly widely used factors today, also the ones relevant to this thesis are fundamental factors that capture stock characteristics such as industry membership, country membership, valuation ratios, and technical indicators. An important thing to note is that risk factors, should not be viewed as replacements for market cap indexes. Market capitalization weighted indexes has a raison d'être in that they represent both the opportunity set of investors as well as their aggregate holdings. Furthermore, market cap weighted indexes are also the only reference for a truly passive, macro consistent, buy and hold investment strategy. In contrast, factor indexes rebalance away from a neutral market cap starting point. As such, they represent the result of an active view or decision. Thus, like traditional active strategies, factor index strategies should be assessed in the long run against a market capitalization weighted benchmark. Not all agree with this view however, Arnott, Hsu, and Moore (2005) argue that market cap weighting is inherently flawed and have advocated replacing market cap allocations with factor allocations.

The purpose of the above sections, was to establish the theoretical framework in which this thesis operates. For instance, **Section 2.1** discusses what drives systematic factors and that these characteristics transpire asset class labels. The normative CAPM model along with Fama and French's three-factor model is stated economically, in order to build upon the application of going from a single factor equilibrium to multiple factors in the Black-Litterman asset allocation model. In the following sections, we discuss the conceptual application of the Canonical Black-Litterman model.

2.2 The Canonical Black-Litterman model

This section touches on the intuition of the canonical Black-Litterman model, which is the natural starting point for the application in our empirical results, where views on factors will be introduced. In line with the definition used by Walters (2014), the dimensions we will use to classify the Canonical model will be; does it specify the estimates as distributions or as point estimates, and secondly does it include the parameter τ.

The Black-Litterman asset allocation model, is a highly sophisticated method for portfolio construction that addresses the problem of unintuitive, highly-concentrated portfolios, input-sensitivity, and estimation error maximization by spreading the errors throughout the vector of expected returns (Idzorek (2004)). These three related problems with mean-variance optimization are likely candidates as to why MV-optimization is being abandoned for the benefit of superior models like the Black-Litterman in its various forms. The Canonical Black-Litterman model termed so, due to the existence of multiple interpretations and representations of the so-called Alternative Reference Models. This thesis will address the Canonical for the time being, leaving an objective comparison for future research. In essence, the Canonical model uses a Bayesian approach to combine information from two sources to create an estimate of expected returns. The first source is implied excess equilibrium returns provided by the CAPM, that is the *neutral market equilibrium excess returns*. The second source is absolute or relative views of an analyst, manager or investor. This means that we can use Bayes rule of belief updating to combine the subjective views of a portfolio manager regarding the expected returns of one or more assets with the market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns. The resulting new vector of returns (the posterior distribution), leads to a higher degree of intuition regarding portfolio weights.

Before the Black-Litterman model, it was not straightforward how to systematically convert such views or information into explicit forecasts. A fundamental issue is that the exploration of the accessible market data might be less productive than the private information Cheung (2009). This is directly related to the semi-strong market efficiency hypothesis. Qualitatively, this means that if all we have are publicly available information and common techniques, then why use anything other than the market view. This is exactly what the Black-Litterman model does. In the absence of private information, the only legitimate forecasts should be backed out from the market portfolio using the CAPM. In this case, it is optimal to simply use these forecasts to construct the portfolio and manage it passively, by holding a portion of the market portfolio. With private information, the forecasts should be updated based on Bayes' Rule, the fundamental law for belief updating. Bayes Theorem is discussed in more detail in **Subsection 2.5**, but before embarking on the core principles of Bayes and the Black-Litterman model, an example of the MV-optimization deficiencies is presented for illustrative purposes in **Table 2.1**.

The most important input in mean-variance optimization is the vector of expected returns. Best and Grauer (1991) demonstrate that a small increase in the expected return of one of the portfolio's assets can force half of the assets from the portfolio or the portfolio weights can become unstable and arrive at corner-solutions. Since the MV-optimization dictates that the investor should estimate the expected returns and their variances in the investable universe, which often is a benchmark or broad market index, this procedure becomes almost impossible, at best laborious. To illustrate the point, a small factor risk universe is constructed with the five recurring factors (see **Equation 2.4**).

Table 2.1: Variance Covariance Matrix

Monthly excess returns to each risk premium is calculated as an arithmetic average of the monthly premiums to each factor. Standard deviation is calculated as the square root of the average, squared residuals between the average monthly premium and the individual monthly premiums. Sharpe ratio as the premium divided by the standard deviation. The monthly Treasury bill rate is zero down to the second decimal.

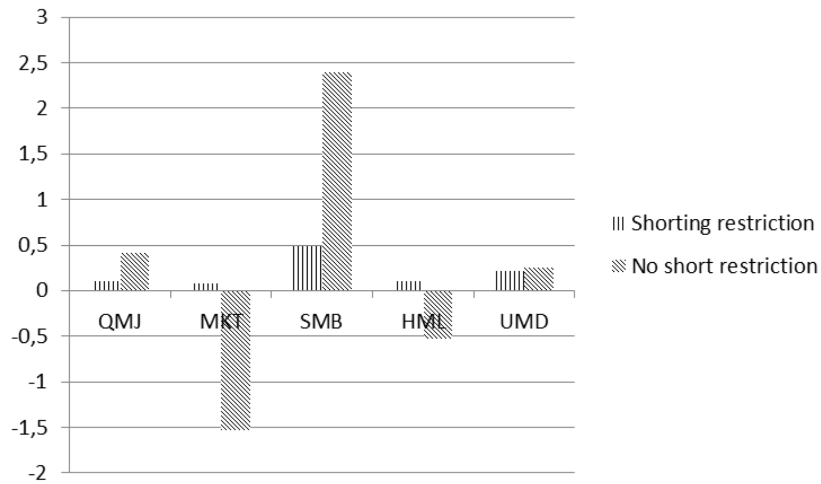
PANEL A: COVARIANCE MATRIX

	QMJ	MKT	SMB	HML	UMD	$\bar{\mu}$	$\bar{\sigma}$	SR
QMJ	0.062	-0.062	-0.026	-0.019	0.020	5.460	0.352	0.141
MKT	-0.062	0.155	0.031	0.015	-0.020	7.931	0.992	0.252
SMB	-0.026	0.031	0.034	0.005	-0.009	1.799	0.008	0.004
HML	-0.019	0.015	0.005	0.035	-0.019	1.634	-0.185	-0.099
UMD	0.020	-0.020	-0.009	-0.019	0.098	7.910	0.617	0.197

In **Table 2.1** are shown average annualized monthly returns to each factor risk-premium from January 2010 through May 2016, a total of 77 return observations per premium. The Treasury bill rate is extracted from CRSP/Compustat, which we will elaborate on in **Chapter 4**

In the classical mean-variance optimization, optimal portfolio weights are those that maximizes the Sharpe ratio. **Figure 2.1** are the result of such an optimization, with and without short selling constraints. As can be seen, weights behave rather erratic due to a simple short sale restriction. Similarly, if we were to change our belief about the expected returns, even without the ability to short sell, portfolio weights would again fluctuate by large amounts. This is due to the aforementioned model restrictions of the MV-optimization, particularly regarding sensitivity to expected input returns. Contrary to MV-optimization, reverse optimization in a Black-Litterman context, agents no longer need to produce forecasts for the full universe of securities. Instead, providing any number $k (0 \leqslant k \leqslant n)$ of views suffices and these views can be relative (e.g., Security A will outperform Security B by 5%) as well as absolute (e.g., Security A will grow by 2%). I now turn the attention to Bayes Theorem which is at the heart of the Black-Litterman model. The main point in using Bayes is in order to update our belief system. This is one of the most elegant features of the

Figure 2.1: For illustrative purposes only



model, as it enables the agent to produce a flexible number of views and the model smoothly translates the views into explicit security return forecasts together with an updated covariance matrix. If the views arrive in an linear form, this model can fully consume them. Without views, there are theoretical justifications for taking the market equilibrium returns as the default forecasts, as mentioned earlier. Since the posterior views are a combination of the market equilibrium and the user-specific views, agents have a common layer, the market view, as their starting point. Without views, the best strategy is to stick to the market view. With some views, the portfolio should be tilted to reflect these views combined. Since the market view is always the starting point, it is less likely to run into unstable or corner solutions.

2.3 Bayesian estimation

The parsimonious example above provided an illustration (**Figure 2.1**) of the well-known estimation errors in mean-variance optimization. In reality, distributional statistics such as mean and variance is something we need to estimate. Many approaches have tried to address this shortcoming, including the Bayesian solution of Black and Litterman (1992), the re-sampling method of Michaud (1998), the risk parity approach of Asness et al. (2012) and a number of robust optimization algorithms of Scherer (2007). It is beyond the scope of this thesis to address potential superiority of any of these methods, and instead reference is made to the literature. For simplistic reasons, we follow the Bayesian approach as laid out in Black and Litterman (1992). Bayesian statistics can be used to formally calculate portfolio weights where a combination of probabilities that rely on both objective and subjective parameters to build a 'posterior' distribution. This procedure is directly inferred from Bayes Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2.5}$$

Where, $P(A|B)$ is the conditional (or joint) probability of A, given B also known as the posterior distribution. $P(B|A)$ is the conditional probability of B given A, also know as the sampling distribution. $P(A)$ is the probability of A, also known as the prior distribution. $P(B)$ is the probability of B, also known as the normalizing constant. A problem in using Bayes theory is to identify an intuitive and reliable prior distribution. One of the core assumptions of the Black-Litterman model (and MV optimization) is that asset returns are normal distributed. For that reason we will confine ourselves to the case of normally distributed conditional and prior distributions. Given that the inputs are normally distributed, then it follows that the posterior will also be normal distributed. We can then establish that the Black-Litterman model is corroborated by three pillars: the semi-strong market efficiency assumption, the CAPM, and the Bayes' Rule. With the assistance of a carefully chosen notation system, we formulate the model with particular attention to its technical details. The representation and derivations will be based on Walters (2014) unless explicitly stated. Before we continue with the details of the Black-Litterman model, we need to have a more intuitive measure of the updated parameters.

2.3.1 Bayesian analysis under $N \sim (\mu, \sigma)$

Let us look at how we can use the Bayesian method to calculate the posterior return distribution. Assume the a priori information we have about the returns is normal distributed, with a known mean μ_0 and a known variance σ_0^2. Assume next the expectations about the return is normally distributed as well, with an unknown mean θ and a known variance, σ^2. As an approximation of the unknown mean, an investor uses an average $\bar{\Upsilon}$ of n analyst forecasts. The variance of this forecast is σ^2/n. According to Brynjarsdóttir and Li (2012) and Plesner (2016), it can be shown that the posterior return distribution is normal with mean $\tilde{\mu}$ and variance $\tilde{\sigma}^2$, where

$$\tilde{\mu} = \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2/n}\bar{\Upsilon} + \frac{\sigma^2/n}{\sigma_0^2 + \sigma^2/n}\mu_0 \tag{2.6}$$

$$\tilde{\sigma}^2 = \frac{\sigma^2\sigma^2/n}{\sigma_0^2 + \sigma^2/n} \tag{2.7}$$

The intuition behind **Equation 2.6** and **Equation 2.7** is that the updated estimate of $\tilde{\mu}$ is a variance-weighted average of the a priori estimate μ_0 and the average of the analytical forecasts $\bar{\Upsilon}$. If an investor is fairly certain about his a priori estimate i.e. σ_0 is low, then the updated estimate of the mean will be close to μ_0. In essence, if the investor has some new data (analytical forecasts), i.e. great certainty, then σ_0 is low and/or n is large, the updated estimate will be pulled towards $\bar{\Upsilon}$. Same intuition applies to **Equation 2.7**. The point as we shall see in our empirical analyses, is that when we introduce views, we will realistically be uncertain about those views. As we become more certain about our views, the mean gets pulled in direction of our views. On the other hand, the more uncertain we are, the closer we get to the initial estimate of the mean. In the Canonical Black-Litterman mode, this uncertainty is measured by the scalar τ. To get a grasp of the impact of this highly important parameter, we have dedicated **Subsection 5.4.4** to put this into perspective.

Having briefly discussed the Bayesian approach, we now turn to the general model of Black and Litterman. Walters (2014) distinguishes between what he labels the Canonical Black-Litterman model and Alternative Reference Models. As a natural consequence, his notation will be used. As stated earlier, the model starts with the assumption of normal distributed expected returns. The fundamental goal is to model these expected returns, which are assumed to be normally distributed with mean μ and variance Σ. The expected returns and the covariance matrix are also the minimum required inputs we need in the portfolio allocation model, that is:

$$r \sim N(\mu, \Sigma) \tag{2.8}$$

We define μ, the unknown mean return, as a random variable itself distributed as:

$$\mu \sim N(\pi, \Sigma_\pi) \tag{2.9}$$

π is our estimate of the mean and Σ_π is the variance of the unknown mean, μ, about our estimate. This relation can be viewed as $\mu = \pi + \epsilon$. The Black-Litterman model starts with a neutral equilibrium portfolio for the prior estimate of returns. The model relies on general equilibrium theory to state that if the aggregate portfolio is at equilibrium, each sub-portfolio must also be at equilibrium. Equilibrium returns are the set of returns that clear the market[1]. Using the notation in relation to Bayes, we have:

$$P(A) \sim N(\Pi, \tau\Sigma) \tag{2.10}$$

Where Π is the implied excess equilibrium return vector (N x 1 column vector), with weights w_{mkt}. Σ is the covariance matrix of excess returns (N x N matrix). The parameter τ is a scalar that measures the uncertainty of the CAPM estimates.

The Black-Litterman model can be used with various utility functions which makes it very flexible. In practice, most practitioners use the quadratic utility function and assume a risk free asset, and thus the equilibrium model simplifies to the CAPM (see **Equation 2.2**). Given our previous assumptions, the prior distribution for the Black-Litterman model is the estimated mean excess return from the CAPM market portfolio. The process of computing the CAPM equilibrium excess returns is straightforward. CAPM is based on the concept of linearity between risk (as measured by standard deviation of returns) and return and it further requires returns to be normally distributed. As the CAPM market portfolio contains all investable assets, it is very hard to actually specify. While this is of practical inconvenience, we can ignore the problem from a theoretical perspective as we previously stated that in global equilibrium, each sub-market must also be in equilibrium. We can constrain the problem by asserting that the covariance matrix of the returns, Σ, is known. In practice, this covariance matrix is estimated from historical return data. By computing it from actual historical data, we can ensure the covariance matrix is positive definite. We will see later that this may be violated when the matrix gets too large (curse of dimensionality). For the rest of this section, we will use a common notation, similar to that used in He and Litterman (1999) for all the terms in the formulas. Notice that this notation is different, and conflicts with the notation used in **Section 2.5** on Bayesian theory. Here we derive the equations for the 'reverse optimization' problem, starting from the quadratic utility function:

$$U = w^T \Pi - \left(\frac{\delta}{2}\right) w^T \Sigma w \tag{2.11}$$

Where U is the investors utility. w is a vector of weights invested in each asset. Π is a vector of equilibrium excess returns for each asset. δ is a risk aversion parameter and Σ is the covariance matrix of the excess returns for the assets. If we maximize the utility with no constraints, there is a closed form solution. We find the exact solution by taking the first derivative of **Equation 2.11** with respect to w and

[1] 'clear the market' indicates that supply equals demand

setting it equal to 0.

$$\frac{dU}{dw} = \Pi - \delta \Sigma w = 0 \tag{2.12}$$

Solving for the vector of excess equilibrium returns yields:

$$\Pi = \delta \Sigma w_{mkt} \tag{2.13}$$

In order to use **Equation 2.13** to solve for the CAPM market portfolio, a value for δ needs to be found. Therefore, we multiply both sides of **Equation 2.13** by w^T and replace vector returns with scalar terms.

$$(r - r_f) = \delta \sigma^2 \tag{2.14}$$

At equilibrium, the excess return to the portfolio equals the risk aversion multiplied by the variance of the portfolio. From this proposed relation, we can derive the risk-aversion coefficient:

$$\delta = \frac{(r - r_f)}{\sigma^2_{mkt}} = \frac{SR}{\sigma_{mkt}} \tag{2.15}$$

The value of δ varies between papers. For instance, Bevan and Winkelmann (1998) describe their process of calibrating the returns to an average Sharpe ratio of 1,0. Black and Litterman (1992) use a Sharpe ratio of 0.5 in their paper. The risk-aversion coefficient δ characterizes the expected risk-return tradeoff. It is the rate at which an investor will accept expected return for more variance. In the reverse optimization process, the risk aversion coefficient acts as a 'scaling factor' for the reverse optimization estimate of excess returns; the weighted reverse optimized excess returns equal the specified market risk premium. More excess return per unit of risk (a larger δ) increases the estimated excess returns. If we extent our analysis a bit further, we can calculate the equilibrium excess returns Π from our five-factor universe. We assume that the market is delimited by S&P 500 Index (`^GSPC`) and that these five factors are constituted from stocks within this market. The average monthly return over the period of 77 months (January 2010 - May 2016) is 0.943% while the risk-free rate (T-bills) during the same period is practically zero. Average monthly excess return is 0.936% while the variance is 0.138%. The risk-aversion parameter will according to **Equation 2.15** be 6.766. The Sharpe ratio of the S&P 500 Index is 0.252, which is about half of that of Black and Litterman (1992). Since we are looking at exposures to systematic risk factors and not individual stocks, factor weights are more complex in our defined universe. For the time being, we will assume that we have constructed the factors constrained to a limit of 20% of the total market value of the constituents in S&P 500. This way, we limit our problem to a pre-defined exposure of 20% to each factor risk, which seems reasonable from a prudent perspective. With some matrix algebra, we can then infer equilibrium returns to each factor risk-premium. The result is shown in **Table 2.2**.

Table 2.2: Equilibrium premiums with $\delta = 6,766$

	QMJ	MKT	SMB	HML	UMD	w	(Π)
QMJ	0.062	-0.062	-0.026	-0.019	0.020	0.2	-0.03
MKT	-0.062	0.155	0.031	0.015	-0.020	0.2	0.16
SMB	-0.026	0.031	0.034	0.005	-0.009	0.2	0.05
HML	-0.019	0.015	0.005	0.035	-0.019	0.2	0.02
UMD	0.020	-0.020	-0.009	-0.019	0.098	0.2	0.10

We showed that by using **Equation 2.13** we have a closed form solution to the reverse optimization problem of an optimal mean-variance portfolio in the absence of constraints. As we are dealing with the market portfolio, which has only positive weights that sum to 1, we can safely assume that there are no binding constraints on the reverse optimization problem. If we rearrange **Equation 2.13** to yield the optimal weights, we have:

$$w_{opt} = (\delta\Sigma)^{-1}\Pi \tag{2.16}$$

What we need at this point, is the variance of our estimate of the mean. That is, the Σ_π from **Equation 2.9**. Indeed, Black and Litterman made the simplifying assumption, that $\Sigma_\pi = \tau\Sigma$, which means that the structure of the covariance matrix of the estimate is proportional to the covariance of the returns. This is also the feature that is distinct to the Canonical model. They created a parameter, τ, as the constant of proportionality. Given that assumption, $\Sigma_\pi = \tau\Sigma$, then the prior distribution P(A) is:

$$P(A) \sim N(\Pi, \tau\Sigma), r_A \sim N(P(A), \Sigma) \tag{2.17}$$

This represents our estimate of the mean, expressed as a distribution of the actual unknown mean about our estimate, also termed the prior distribution. In the absence of budget constraints, investors will always face a degree of uncertainty in the estimates. As a result of this uncertainty in the estimates, $1/(1+\tau)$ will be the fraction invested in the neutral portfolio, while $\tau/(1+\tau)$ will be invested in the risk-free asset. From **Equation 2.16**, we have:

$$\begin{aligned} w_{opt} =& (\delta\Sigma)^{-1}\Pi \\ \hat{w} =& [(1+\tau)\delta\Sigma]^{-1}\Pi \\ \hat{w} =& \frac{1}{1+\tau}(\delta\Sigma)^{-1}\Pi \\ \hat{w} =& \frac{1}{1+\tau}w_{opt} \end{aligned} \tag{2.18}$$

The result from above, indicates that $\tau/(1+\tau)$ can be interpreted as a risk budget i.e. that the variance of the portfolio does not exceed that of the CAPM (see Plesner (2016)). He shows that the effect of τ can be interpreted as as a shift in the efficient frontier.

2.3.2 Views, their variance (Ω) and tau (τ)

So far, we have abstracted from active views. Indeed, the real strength of the Black-Litterman model is the ability to infer such views, whether they be absolute or relative in nature. We have also been quite silent about the value of τ, which we will discuss more thoroughly in **Subsection 5.4.4**. However, a comprehensive study is beyond the scope of this thesis.

According to the literature, it is hard to say anything general about the right value of τ. Regardless of whether we use confidence levels or historical data to estimate τ, the only generalization we can make at this point, is that the value should be closer to 0 than to 1. This is true for all model with the Canonical specifications. Black and Litterman (1992), He and Litterman (1999) and Idzorek (2004) all indicate that in their calculations they used small values of τ, in the order of 0.025 – 0.050. Satchell and Scowcroft (2000) state that many investors use a τ around 1, which has no intuitive connection to the data and in fact shows that their paper uses the Alternative Reference Model. The value of τ is non-trivial, as it directly impacts the ex-post covariance and the weight inferred on investor views with the equilibrium returns. A value closer to that found by He and Litterman (1999) seems more reasonable as $\tau\Sigma$ can be interpreted as the 'standard-error' on the estimate of Π. A good maximum likelihood for τ is $1/n$, where n is the number of observations. Based on our example, we have $1/77 = 0.013$. It should be clear by now, that establishing the appropriate value for τ should by no means be considered non-trivial. Not only is it a imperative with regards to our empirical result, but also from a more practical perspective, lets us continue with the model formulation.

The Black-Litterman model uses 3 matrices to specify k views (P, Q and Ω). Q

is a *(k x 1)* vector with *absolute* or *relative* expectations about returns. P is a *(k x 1)* matrix that takes on the value of either -1, 0 or 1 depending on the nature of the view i.e. relative or absolute, again depending on the approach[2]. It thereby matches expectations from Q to the n assets by specifying weights for those assets with views. Ω is a kxk matrix of the covariances of the views. Ω is diagonal as the views are required to be independent and uncorrelated. Furthermore, Ω is symmetric and zero on all non-diagonal elements, but may also be zero on the diagonal if the investor is certain of a view. He and Litterman (1999) assume that Ω is proportional to $\tau\Sigma$. That is:

$$\Omega = diag(P(\tau\Sigma)P^T) \tag{2.19}$$

This specification of the variance, or uncertainty, of the views essentially equally weights the investor's views and the market equilibrium weights. By including τ in the expression, the posterior estimate of the returns becomes independent of τ as well. This seems to be the most common method used in the literature. To recapitalize on the views, imagine we have a view on UMD *relative* to HML. We also have an *absolute* view on MKT. Suppose we have gathered the necessary information and believe UMD will outperform HML relatively by 4%. At the same time, we believe that the MKT premium will be produce an absolute return of 1%. We have:

$$P = \begin{bmatrix} 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}; Q = \begin{bmatrix} 4\% \\ 1\% \end{bmatrix}; \Omega \begin{bmatrix} 0.0086\% & 0 \\ 0 & 0.0077\% \end{bmatrix}$$

2.3.3 Model Formulation

We have now specified our views which we can combine with the ex-ante return distribution. This is practically done using Bayes Theorem from **Subsection 2.5**, which we will now apply to get the updated return distribution. For practical purposes, we can look at this new, updated distribution as a weighted average of the ex-ante estimates and the conditional estimate for $P(B|A)$. This leads us to:

$$P(A|B) \sim N(E[R], M) \tag{2.20}$$

Where $E[R]$ is the posterior return vector:

$$E[R] = \underbrace{[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}}_{\text{`}Denominator'}\underbrace{[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]}_{\text{`}Nominator'} \tag{2.21}$$

[2]We shall see in our empirical results that we can state views in more diverse and complex ways.

and the uncertainty of the estimate of the ex-post return distribution M is:

$$M = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \tag{2.22}$$

Lets consider the *nominator* first. The first term $\tau\Sigma$, which we previously labelled the 'standard error', measures the confidence towards the neutral equilibrium returns Π, while $P^T\Omega$ are effective weights we place on our views. In the case of $\tau\Sigma$, this measures how confident we are with the CAPM equilibrium excess returns, whereas $P^T\Omega$ gets multiplied with Q our subjective views and therefore weights the degree of confidence we have in our views. Suppose we are uncertain about a future outcome, it translates into large elements of Ω. The elements of the inverse, Ω^{-1}, will thereby be small, placing little weight on our views. In essence, the second part of the formula is a weighted average of the equilibrium excess returns and our views. The *denominator* is basically there to ensure that the weights we assign to the implied excess returns and our views sum to 1. At the same time, the 'denominator', is measure of the uncertainty of the mean as an estimate relative to true unknown mean .

Chapter 3

Literature review

In this chapter we present some empirical results of the previous studies on factor models. The initial findings on systematic differences in stock returns, fermented the ground of what were to be known as the Capital Asset Pricing Model (CAPM). Studies like that of Banz (1981), who documented a size anomaly when comparing the excess return of smaller firms with larger ones. Basu (1977) who formalized the value strategy of Graham and Dodd (1934) and found a positive link between stocks that have low prices relative to some fundamental value. The technical study by Jegadeesh and Titman (1993) who showed that stock prices tend to exhibit trend over certain horizons. Despite the obvious contributions of studies like these, few within the asset pricing domain has been as important as that of the famous seminal paper by Fama and French (1992a) who study the joint roles of market β, size, E/P, leverage, and book-to-market equity in the cross-section of average stock returns. They find that used alone or in combination with other variables, β has little information about average returns. The main results of Fama and French is that two easily measured variables, size, and book-to-market equity, seem to describe the cross-section of average stocks returns and that economic fundamentals suggest that high B/M (book equity/market equity) firms tend to be persistently poor earners relative to low B/M firms.

The objective of this literature review is to provide a more in-depth insight into the drivers of factor returns, the merits of the factor vs. asset class paradigm and to some humble extent, discuss potential historical academic observations on what constitute factors, which by no means is exhaustive. As this thesis addresses the application of multiple factors in the context of the parsimonious Black-Litterman model, we try to be both as specific and perfunctory as the scope of this thesis permits.

The rest of this chapter is structured as follows: **Section 3.1** discusses different notions of factors and their merits, while simultaneously identifying the fundamental factors relevant to this thesis. Then follows a theoretical discussion in **Section 3.2** on why the CAPM is a natural and justifiable starting point for factor models. In **Section 3.3**, I provide some evidence on the previous studies on factors and why factors should be interesting candidates for a refined Black-Litterman model. For pedagogical reasons, the literature on the merits of the Black-Litterman model are dedicated its own **Section 3.4**. Finally, **Section 3.4.1** provides a short review on the path in terms of methodology and finishes with some shortcomings of the model.

3.1 Camps within the factor framework

There are two main schools of thought on the rationale behind investor behavior and factor returns/risk. The risk explanation posits that return premiums are simply rewards for bearing risk or uncertainty. This explanation, consistent with rational asset pricing, assumes that investors obtain return premiums as a reward for being exposed to systematic risk. The unequivocal view of the equity market factor is that it earns investors a premium as a reward for bearing the uncertainty of the value of future cash flows. In contrast, the behavioural argument holds that certain factor returns are caused by investor behaviour. That is, investors make systematic errors that result in distinct patterns in investment returns.

In the first camp, the term 'systematic' refers to characteristics that cannot be arbitraged away. This argument is consistent with rational investor behaviour and the efficient markets hypothesis (EMH). For example, some have argued that the small cap premium is return earned for exposure to companies that are less liquid, less transparent (Zhang, (2006)) and more likely to be distressed (Chan and Chen, (1991)) and (Dichev, (1998)). Others have argued that factors like value, size, and momentum are linked to important macroeconomic factors such as growth and inflation and because of their sensitivity to shocks in the economy, must bear a return premium (Winkelmann et al., (2013)). In the second camp, factors are thought to earn excess returns because of investors' 'systematic errors'. One branch of this school, rooted in the area of behavioral finance, suggests that investors exhibit behavioral biases due to cognitive weaknesses. Examples include overreacting, chasing winners, home biases and myopic loss aversion. According to Barberis and

Huang (2001), investors become less concerned about future losses on stocks with recent good performance because any losses will be cushioned by prior gains ('loss aversion'). This bias induces investors to perceive the stock to be less risky than before and discounts its future cash flows at a lower rate. The rationale is that if enough investors exhibit these biases, as long as it is prohibitively costly for rational investors to arbitrage these biases away, it can lead to the factor anomalies we observe. More recently, using an institutional constraint-based argument, Vayanos and Woolley (2011) propose that value and momentum effects arise because negative shocks to assets' fundamental values trigger outflows from funds holding those assets. Outflows cause asset sales, which amplify the shocks' negative effects. If the outflows are gradual because of investor inertia or institutional constraints, then the amplification is also gradual and momentum effects arise. A very interesting study by Lou and Polk (2012), confirms these findings. They define a 'Comomentum' measure to test whether arbitrageurs can have a destabilizing effect in the stock market. The basic premise of the Comomentum measure is when arbitrageurs take long positions in winner stocks and short past losers, such trades can have simultaneous, temporary price impacts on all momentum stocks and thus cause return comovement among these stocks.

Within this second camp, another branch has tied factor performance to investor constraints and frictions that arise from regulatory and industry requirements. Different investors have different time horizons. Some studies have shown that stocks with low liquidity earn a premium over long horizons (10 years plus) since most investors have shorter horizons (3- and 5-year horizons) and prefer stocks that are liquid. Investors with longer horizons earn a premium for bearing this horizon risk. Although the return premiums of some factors have been shown to be clearly related to risk, debate over the source of returns for other factors is more contentious.

3.2 What does the CAPM get right?

The CAPM was formulated in the 1960s by Jack Treynor (1961), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966), building on the principle of diversification and mean-variance utility introduced by Harry Markowitz in 1952. The CAPM states that one factor exists and that factor is the market portfolio, where each stock is held in proportion to its market capitalization. The factor can be optimally constructed by holding many assets[1] in order to get rid of idiosyncratic risk. Diversification ensures that, absent perfect correlation, when one asset performs badly, some other assets will perform well, and so gains can partly offset losses. Investors never want to hold assets in isolation; they improve their risk/return trade-off by diversifying and holding portfolios of many assets. The market factor is the best, most well-diversified portfolio investors can hold under the CAPM and that it is held by every investor. A strong implication that is empirically challenged (Ang, 2014). Nevertheless, it is useful to understand how we can leap from a diversified portfolio to the market being the only relevant factor. If we assume that the set of means, volatilities, and correlations are the same for all investors, then all investors hold the same MVE portfolio, just in different proportions depending on their individual risk aversion. Since everyone holds the same MVE portfolio and this is the optimal portfolio that can be held by all investors, the MVE portfolio becomes the market factor in equilibrium (except for those who are infinitely risk averse). The equilibrium concept is probably the most important in the model, as it assures supply equals demand. Equilibrium ensures that the market portfolio, will have a risk premium and that this risk premium will not disappear. The market risk premium is a function of the underlying investors' risk aversions and utilities. That is, the risk premium of the market factor reflects the full setup of all people in the economy. This is an extremely useful finding, as it conceptually can be applied to all tradable factors, like the ones relevant to this thesis. It also means that all equilibrium factor risk premiums will not disappear because sophisticated investors cannot arbitrage away the market factor and all other systematic factors.

The CAPM was revolutionary because it was the first cogent theory to recognize that the risk of an asset was not how that asset behaved in isolation but how that asset moved in relation to other assets and to the market as a whole. Before the CAPM, risk was often thought to be an asset's own volatility. The CAPM said this was irrelevant and that the relevant measure of risk was how the asset co-varied with the market portfolio, the beta of the asset. On the normative question of how high the equity risk premium should be, the academic literature provides limited guidance. The CAPM implies a cross-sectional relation that each asset's expected excess return is the product of its market beta and the market risk premium but it does not specify how large the market risk premium should be. The required market

[1](Bodie et al., 2011) point to literature that indicates the best results of diversification, is achieved within the first 15-20 stocks. That is, the biggest diversification reward is achieved during the first 15-20 stocks. Exceedingly more stocks needs to be added to get rid of the idiosyncratic risk.

risk premium should reflect the price of risk and the amount of risk. The demise of the CAPM, has not been totally in vein as it has opened new areas of understanding for asset owners who must hunt for risk premiums and manage risk. The basic intuition of the CAPM still holds true: that the factors underlying the assets determine asset risk premiums and that these risk premiums are compensation for investors' losses during bad times. Risk is a property not of an asset in isolation, but how the assets move in relation to each other. It works approximately, and well enough for most applications, but it fails rather miserably in certain situations. What the mean-variance framework and the equilibrium CAPM get right is that the market portfolio is well diversified; in fact, the market is the most diversified portfolio you can passively construct.

3.3 Evidence on the drivers of factors

There is no consensus on the complete set of factors. Academic studies using statistical analysis suggest that a limited number, usually less than ten, can capture most variation in expected returns (Ang, (2014)). Below, we present some empirical evidence on the most common and acknowledged factors today. The selected few should therefore only serve to shed some light on what drives the specific factors we know today and not serve as an exhaustive list.

Value

The value factor capitalizes on the positive link between stocks that have low prices relative to some fundamental value and returns in excess of a capitalization-weighted benchmark *vis-a-vis* the market. A value strategy consists of buying stocks that have low prices, typically constituted by some company fundamentals like book value or sales, simultaneously selling stocks that have high prices. Value investing has been widely discussed since Graham and Dodd (1934) and was later formalized by Basu (1977), who was the first to capitalize on the notion that value-related variables could explain inconsistencies in the CAPM. He found a significant positive relation between E/P ratios and average returns for US stocks that the CAPM was not able to explain. Later studies such as Rosenberg, Reid and Lanstein (1985) and DeBondt and Thaler (1987) documented a significant positive relation between book-to-market ratios and average returns as well. Since then, the value effect has been reproduced by numerous researchers and under numerous circumstances and markets around the world. More recently, (Frazzini, Kabiller and Pedersen, 2013) have conducted a thourough empirical analysis to shed light on the performance of Berkshire Hathaway. Through his privately held companies, Buffet has found a unique way of leveraging his stock selection abilities, financed through his privately owned companies.

Critics of the value premium have argued that empirical evidence is based on data mining and point out the sample-dependency of empirical studies (Black, (1993)).

Momentum

The momentum factor reflects the expectancy of future excess returns to stocks with stronger past performance. The factor builds on the more technical notion that stock prices tend to exhibit trend over certain horizons. Jegadeesh and Titman (1993) published one of the first seminal studies on momentum in the US stock market, which shows buying past winners and selling past losers produced significant excess returns in the period 1965-1989. In a study of mutual fund performance, Carhart (1997) expanded the Fama-French Three Factor Model to a Four-Factor Model to include momentum as an additional explanatory variable. Rouwenhorst (1998) created an internationally diversified portfolio of past winners and found them outperforming a portfolio of past losers by about 1% per month, using a sample European stocks. Fama and French (2012) found strong momentum returns in North America, Europe, and Asia Pacific in the sample period of 1989-2011. Asness (1995, 1997) documented trend reversals in momentum in that winners and losers tend to revert over the long-term, i.e. winners underperforming and losers outperforming in 3-5 years from the analysis date.

Most empirical research suggests the momentum effect is most adherent in the 3-12 months following the factor build, after which most will disappear. This implies the that momentum strategy requires relatively high turnover in order to work. The theory underlying this premium still puzzles academia. Unlike other factors such as value and size, no satisfactory efficient markets-based theory helps to explain the conundrum. The most widely cited theories are all behavioral. Studies like that of Barberis, Sheleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) point to an over-reaction to phenomenon, while Hong, Lim and Stein (2000) point to an under-reaction phenomenon. Both re-active biases may under certain assumptions lead to momentum effects. Another well-cited study by Dasgupta, Prat and Verardo (2011) argue that reputation concerns cause managers to herd, and this generates momentum under certain assumptions. Lou and Polk (2012) find that comomentum is the high frequency abnormal return correlation among stocks that a typical momentum strategy would speculate on. In periods of low comomentum, momentum strategies are profitable and stabilizing, reflecting an underreaction phenomenom that arbitrageurs correct. However, the momentum strategy becomes unprofitable in periods when the amount of deployed arbitrage capital is crowded and tends to crash, reflecting prior overreaction due to the momentum crowd pushing prices further away from fundamentals. Hence, arbitrageurs can have a destabilizing effect on the market. Also cited by Lou and Polk (2012) is the failure of both rational and behavioral models to explain stylized facts about momentum, but also because momentum is a classic example of a strategy with no fundamental anchor. For a strategy such as momentum, arbitrageurs do not base their demand on an independent estimate of fundamental value. Instead, their demand for an asset is an increasing function of price. Thus, momentum is a type of strategy where arbitrage capital can be destabilizing Stein (2009). Despite the documented profitability of momentum as a strategy, there exists no compelling explanation for this effect.

Criticisms of the momentum strategy include data mining, high turnover, crowded trading, and the risk of a sudden reversal. Geczy and Samonv (2013) conducted a 212-year back test of the momentum strategy in the US market, which shows the momentum effect is statistically significant and not a product of data-mining. In addition, like other factor strategies, momentum could advantageously be combined with other factor strategies. Asness, Moskowitz and Pedersen (2010) supported this argument with a study on the interaction between value and momentum. They found potential diversification benefits from combining the two strategies as value and momentum can be negatively correlated within and across asset classes.

Size

The size factor captures the excess returns between smaller firms, relative to larger ones, predominantly measured by market capitalization. Banz (1981) was the first to document this supposed systematic risk factor and showed smaller companies listed on the NYSE on average delivered higher risk-adjusted returns than the larger ones. This size effect was later generalized by the Fama-French Three Factor Model in the their seminal paper in 1992a. There are several theories explaining this phenomenon, and the debate continues today. In the efficient market view, Fama and French (1992a, 1993) originally hypothesized that small firms have higher systematic risk which earns them a higher return premium. Subsequent researchers suggested that size may proxy for other unobservable and underlying risk factors associated with smaller firms such as liquidity (Amihud, (2002)), information uncertainty (Zhang, 2006), financial distress (Chan and Chen, 1991) and default risk (Vassalou and Xing, (2004)) or the simple observation that large firms with more projects can diversify among these (cross-pledge), which should all else equal, make them safer. Larger firms might have more analyst coverage, better access to capital markets and therefore more liquid. Another source of potential differences in risk is firm specific debt policies. There can be differences in debt maturities, interest rates, currencies of outstanding loans, hedging policies, covenants etc. A recent study by Wang and Yu (2013) documents a small value premium among stocks with low limits-to-arbitrage and a significant relationship between firm specific attributes and B/M. For stocks with high limits-to-arbitrage where the value premium is the most pronounced, there is no significant relation between B/M and firm-specific attributes. This sort of finding further questions the proxy ability of the size factor and factors in general and the reasonable possibility that other underlying forces may be at work. For some time now, the size effect has been buried, however, in a recent study by Asness, Frazzini and Pedersen (2013), they show that the Quality minus Junk factor resurrects this effect.

Skeptics of the size premium point to survivorship bias in the research, which typically does not include busted companies. This seems a legitimate claim given small companies often have sustainability issues. Others argue the size premium is difficult to capture in the real world given the low trading volume of small cap stocks. Nevertheless, in recent literature, the size effect has been showed to have a weak historical record and a large variation over time. In particular weakening after its discovery in the early 1980s, it is concentrated among microcap stocks and predominantly resides in January.

Quality

The quality factor aims through a predefined set of quality characteristics to capture the excess return of ‘high quality’. This has become an accepted tool in fundamental analysis but a relatively new phenomenon in quantitative investments. The main challenge is how to define the ‘quality’ factor consistently and objectively using quantitative metrics. While the literature offers various results, such quality indicators commonly includes a company’s efficiency, transparency, growth, leverage, profit and return-on-equity. Sloan (1996) was one of the first to validate the excess returns to high earnings quality stocks, where accruals then proxy for earnings quality. Recently, Asness, Frazzini, and Pedersen (2013) show that quality as measured by profitability, growth, safety and payout, has significantly higher risk-adjusted returns. In their analysis, they point to the success of investor Warren Buffet as his style of investing is his preference for cheap, safe, high-quality stocks combined with his consistent use of leverage to magnify returns. Their findings include a non-trivial aspect, namely that quality resurrects the otherwise moribund size effect as also mentioned under the **Size** effect. Frazzini, Kabiller and Pedersen (2012) show that Berkshire Hathaway's alpha becomes insignificant when controlling for 'Betting-Against-Beta' and quality factors. There is very little literature on why the quality factor works. It's also difficult to provide a unified theory since the definition of quality varies. According to the Fama-French model, all systematic risks are ultimately economic risks. In light of this argument, it is not difficult to see the connection between stock returns and quality measures such as financial leverage and earnings growth. Campbell, Polk and Vuolteenaho (2010) offered a form of this 'fundamental' explanation, arguing the primary source of systematic risks of both growth and value stocks is the cash flow fundamentals as opposed to market sentiments. From a corporate finance angle, one can see how firm quality impacts stock prices. For example, a well-run company often manages its capital carefully and reduces the risk of over-leveraging or over-capitalization. The steady growth in earnings will further reduce its need for capital market financing, which will support its stock price. This will trigger a positive feedback loop making the company more competitive in the eyes of its customers and investors.

Critics of the quality factor argue it is highly subjective and contains behavioral biases. Active managers are often strong proponents of the quality factor but argue it can only be captured through fundamental analysis and stock picking.

An equally important rationale for factor investing is the vast amount of evidence that suggests active mutual fund performance is linked to factor exposures. Empirical studies like that of Malkiel (1995) and Gruber (1996) documents that the median active manager generally does not outperform the market cap weighted benchmark net of fees, and even the small subset of those who do outperform are only able to maintain that outperformance for short periods. Fama and French (2010) found that mutual funds in aggregate experienced net returns that underperformed the Fama-French factor benchmarks by about the costs in expense ratios from 1984 to 2006. Ang, Goetzmann and Schaefer (2009) have applied a similar analysis to the Norwegian Government Pension Fund's active returns where they found that much of the behavior of the Fund's small active return could be explained in terms of systematic factors.

In order to recapitalize on above examination, we provide **Table 3.1** for a conceptualized overview.

Table 3.1: Theories Behind the Excess Returns to Systematic Factors

The below table reports the factor-based explanations for systematic anomalies

PANEL A: FACTOR-BASED EXPLANATIONS

Systematic Factors	Systematic Risk-based Theories	Systematic Errors-based Theories
Value (HML)	• Higher systematic risk	• Errors in expectations
		• Loss aversion
		• Investment flows based theory
Size (SMB)	• Higher systematic risk	• Errors in expectations
	• Proxy for other types of systematic risk	
Market (MKT)	• Higher systematic risk	• Errors in expectations
	• Proxy for other types of systematic risk	
Momentum (UMD)	• Higher business cycle risk	• Underreaction and overreaction
	• Higher systematic tail risk	• Investment-flows-based theory
Quality (QMJ)		• Errors-in-expectations

3.4 The Parsimonious Black-Litterman model

The Black-Litterman Model was first published by Fischer Black and Robert Litterman in 1990. Since then, many authors have published research referring to their model as Black-Litterman. This has lead to a variety of Reference models being labeled as Black-Litterman even though they may be very different from the Canonical model, labeled so by Walters (2014). However, there has been no lack of literature exploring either the applications or the frontiers of the model (e.g., Meucci, 2006; Martellini and Ziemann, 2007 and Cheung, 2009), but we have seen few documents explaining it besides (Bevan and Winkelmann, 1998; Satchell and Scowcroft, 2000 and Idzorek, 2004). In the below we will introduce several papers which make significant contributions to the literature on the various versions of the Black-Litterman model.

As brilliant an idea as the mean-variance framework is, is suffers from a range of non-trivial disabilities when it comes to active portfolio allocation and stock selection. In reality, estimated values produced by the MV optimizer may differ qualitatively from an individual asset manager's knowledge about the true, unobservable mean. As a consequence, there are reasons to believe that the main challenge for optimal allocation models is in estimating expected returns, as opposed to higher moments, of asset class returns (Martellini and Ziemann (2007)). First, there is a general consensus that expected returns are difficult to obtain with a reasonable estimation error and that this problem is amplified when optimization techniques are very sensitive to differences in expected returns. Mean-variance optimization typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (Michaud, (1998)). Significant progress on the question of portfolio optimization in the presence of parameter uncertainty has been achieved in the influential works by Black and Litterman (1990, 1991 and 1992), who introduce, in a static mean-variance setting, a methodology that allows investors to account for uncertainty in their priors on expected returns, expressed in terms of deviation from neutral equilibrium CAPM-based estimates. Their papers provide the rationale for the methodology, along with some information on the derivation, but does not show all the formulas or a full derivation. It also includes a rather complex worked example based on the global equilibrium. Unfortunately, because of the merging of the two problems, their results are difficult to reproduce. In He and Litterman (1999), the last paper by one of the original authors, more detail on the workings of the model are provided, but still not comprehensively on the derivations.

The Canonical Black-Litterman model makes two distinct contributions to the asset allocation problem as it provides an intuitive prior via the equilibrium market portfolio, and a clear way to specify investors views on returns and to blend the investors views with prior information. Previous work started either with the uninformative uniform prior distribution or with the global minimum variance portfolio (see Frost and Savarino (1986)). The idea that one could capitalize on 'reverse optimization' to generate a stable distribution of returns from the equilibrium market portfolio is a significant improvement to the process of return estimation. The investor's views are allowed to be relative or absolute, and the views can overlap and influence an arbitrary number of assets. No research linked the process of specifying views to the blending of the prior and the investors views. As a result, the Black-Litterman model provides a quantitative framework for specifying the investor's views, and a clear way to combine those investor's views with an intuitive prior to arrive at a new combined distribution.

Before the inception of the Black-Litterman model, the portfolio managers were predominantly relying on the merits of the Markowitz (1952) MVE framework. Here, the model unrealistically requires expected returns of all assets as input data. For the majority of portfolio managers across strategies, it is almost impossible to know expected returns with certainty. Most often, reliable return forecasts are only available for a small subset of assets. While it is impossible to accurately predict the returns of all assets in practice, results of the Markowitz Model are very sensitive to small changes in expected returns. Even very small changes in expected returns can cause remarkable shifts in the optimal portfolio weights as the mean-variance model is quite sensitive to changes in the expected mean return. This results in rebalancing of positions and potentially very costly in trading fees.

3.4.1 Methodological review

This subsection will touch on the motivation for applying a multifactor framework to the context of the Black-Litterman model. At the same time, I will include previous studies which are more explicit with regards to their methodology.

Post the birth of the Black-Litterman model, surprisingly few innovations had been done relative to other areas of finance to keep the model 'up-to-date'. More attention has been dedicated toward the derivations of the model as a result of scarce initial information from the original authors. This a spawned the direction of two paths, one of which evolves around the Alternative Reference Models, the other towards some of the more 'vague' parameters of the model. Simultaneous with the lack of innovations, the investment community has gone from practically asset class investing to risk-premium oriented. This provide some of the merits for undertaking this empirical research that seek to combine the refined granularity of the markets represented by factors and the strong portfolio allocation tool that is the Black-Litterman model. The methodology undertaken in the empirical analysis will largely complement some previous works. For instance, the findings by Bevan and Winkelmann (1998) who provide details on how they use the Black-Litterman model as part of their broader asset allocation process, including some calibrations of the model which they perform. This is very useful information for anybody planning on building Black-Litterman into an asset allocation process. The empirical findings of Satchell and Scowcroft (2000) who attempted to demystify the Black-Litterman model, but instead introduced a new non-Bayesian expression of the model. Their model uses point estimates for the prior and the views, and uses τ and Ω only to control the amount of shrinkage of the views onto the prior. Because they use point estimates instead of distributions, their model does not include any information on the precision of the estimate. This allows them to recommend setting $\tau = 1$. Idzorek

(2004) who introduced a technique for specifying Ω such that the impact of the view-shrinkage was specified in terms of percentage of change in the weights between 0 and the change caused by 100% confidence. His paper uses the Alternative Reference Model, but his technique can be applied to the Canonical Black-Litterman model because it is sensitive to the value of τ. Martellini and Ziemann (2007) provide a suitable extension to active management of a fund of hedge funds. They use Value-at-Risk (VaR) for the reverse optimization and they use a variant of the CAPM model extended to include third and fourth moments of asset returns, skewness and kurtosis respectively, in determining their neutral portfolio. They use a factor model, which makes it rather interesting, to generate rankings and then convert the rankings into their confidence in the views. Cheung (2009) introduces the concept of a model that integrates a factor model and does a joint estimation of the factor returns and stock-specific returns. Finally, the short paper by Asl and Etula (2012) who develop a new approach to strategic asset allocation that leverages the idea that long-term investment returns derive from multiple distinct sources they name *return-generating factors*. As with Satchell and Scowcroft (2000), Asl and Etula (2012) use a different approach, namely a robust optimization procedure.

3.4.2 Shortcomings

The Black-Litterman model creates stable, mean-variance efficient portfolios that overcome the problem of input-sensitivity, while mitigating the problem of estimation error-maximization (Michaud, (1989)) by spreading the errors throughout the vector of expected returns. While the confidence in investor views expressed through Ω and the blending of neutral equilibrium returns and investors' subjective views bring distinguishing advantages for the Black-Litterman model, they also lead to the complication of building required input parameters. Many existing literatures have discussed it. Unfortunately, there is no consistent and clear explanation for some input parameters. The complication of building required input parameters has as a result limited the Black-Litterman model's widespread application in the investment community.

Chapter 4

Data and Methodology

In this chapter, we describe and discuss the methodology and sample selection used in the thesis. In general, we use a research design similar to Fama and French (1992 1993 and 1996) and Asness and Frazzini (2013) and Asness, Frazzini and Pedersen (2014). Central to the methodology is the underlying choice that we 1) do not intend to change how the factor measures from Asness, Frazzini, and Pedersen (2014) are calculated as it would result in a detachment from their findings and 2) do not intend to test any implementation issues other than separate choice of using these factor premiums in the Black-Litterman model. The vast majority of the existing literature on the field refers to the US market, or Europe as a global market, and does not necessarily reflect the results of introducing risk premia investing in other stock markets. In fact, Griffin (2002) concludes that practical applications of factor models are best performed on a country-specific basis. As a result, we focus on a long sample of US stocks, which includes all available common stocks on the merged CRSP/Compustat data. As a contemporary contribution to this field, we resolve to using monthly data from January 1990 through May 2016, although the data period could be expanded to account for many more cycles and sensitivities. As a result peaks and troughs are natural components of the data sample.

This chapter provides the necessary tools to pursue hypothesis set forth in the introduction. The intermediate nature of this exercise is important for several reasons. First, it naturally serves as building blocks to our empirical analyses in the coming chapter. Second, it is of great importance to acknowledge the surrounding literature in this area as this has highlighted several pitfalls in the factor building process. Many of which, has been discussed in the literature review. Although the potential issues that can arise from working with this sort of data are many, a general problem emerge, which is not limited to process of building factors, but is more generic in nature and extends to all of the academic literature that backtest in some way. A misplaced analogy might serve to highlight this point. When looking at historical data, we are effectively driving while looking in the rear-view mirror, inferring that the future will behave similar to what observed in the past. Extrapolating too aggressively into the future can produce noise which essentially dilute the effects we want to analyse. As a result, we have carefully chosen the sample period to account for periods that do not necessarily match normal market conditions, as this will produce more realistic outcomes.

4.1 Sample period and selection

Each year, the sample consists of all available common stocks in the merged CRSP-/Compustat data universe. All portfolio returns are in USD and does not include any currency hedging. There are several reasons for selecting all available securities on the merged data of CRSP/Compustat. If we were to restrict ourselves to a universe of constituents such as the S&P 500, we would limit the degree diversification benefit and robustness in out findings. On a more practical node, choosing only American stocks ensure that all firms are subject to the same accounting standards. This procedure is used in central papers by both Fama and French (1992, 1993 and 1996), Asness and Frazzini (2013) and Asness, Frazzini and Pedersen (2014).

4.2 Construction of the dataset

In this subsection, we describe how the dataset is prepared, containing only the most recent publicly available information at the time of each valuation. Additionally, we go through the rather comprehensive calculation of financial ratios and other financial data required as inputs for calculating the different measures.

In the construction of the factor risk premiums, we want little noise and as much factor explanation as possible. To do so, it is necessary to devise metrics that gauge the long-term health of companies and these should ideally complement the metrics for their short-term performance. A company may show strong growth and returns on capital, but health metrics are needed to determine if that performance is sustainable. In using these metrics, it is important to understand the impact of factors outside management's control: consider the case of an oil company whose improving profitability comes from rising oil prices rather than better exploration techniques or a bank whose stock price rises because of changing rates, not increased efficiencies. To use any metric that assesses the health of a company, one must distinguish between organic or transitory items. Accountants and managers can to some degree decide when to record revenues and costs, and personal motives can colour this judgment. As a result, when measuring a company's financial health and performance, some metrics are more reasonable than others. Metrics, such as return on equity (ROE) and growth that can be linked directly to value creation are more meaningful than traditional accounting metrics like earnings per share (EPS). For instance, companies that earn a return on invested capital (ROIC) greater than their cost of capital generate attractive EPS growth, the inverse is not true. EPS growth can come from heavy investments or changes in the financial structure that doesn't necessarily lead to value creation. As such, companies can easily manipulate EPS by repurchasing shares or by undertaking acquisitions.

A natural consequence of the value-weighting portfolio formation procedure undertaking by Fama and French (1993) among others, is fewer transaction costs compared to equally-weighted formed portfolios. In equally-weighted portfolios all firms are allocated an equal amount of the total capitalization. Thus, if stock prices increase (decrease) an investor must sell (buy) a number of these stocks in order to uphold the equal cap-weights. In the case of value-weighted portfolios, investors buy an equal number of stocks, thus higher-priced stocks have a higher proportion of the total portfolio capitalization relative to cheaper stocks. When prices change, the underlying components automatically adjust and no rebalancing is needed.

In the first section, we motivated our sample selection. In the following, we describe how to calculate the Value, Size, Quality, Market and Momentum factors. Subsequently, we follow with a methodology to capture the performance of different GICS sectors. We finish this chapter with a graphical illustration of the overall research design.

4.2.1 Factor measures

In the following section, we outline the methodology used to extract the relevant factor characteristics. We start with the most recent discovered factor (QMJ) as this is by large the most laborious one to measure. The quality score is made from a variety of different quality measures. The essential goal is to identify stocks of profitable, safe, stable and high payout companies. To avoid datamining, a broad set of measures for each aspect of quality is used and then averaged to compute four composite proxies: Profitability, Growth, Safety and Payout. The four composite measures are then averaged to compute a single quality score. We follow the procedure of Asness, Frazzini, and Pedersen (2013) for calculating the quality measures. In order to put each measure on equal footing, we convert each variable into ranks and standardize to obtain a z-score. More formally, let x be a variable of interest and $\mathcal{R}$ be a vector of ranks, $\mathcal{R}_i = rank(x_i)$. Then, the z-score of x is given by:

$$z(x) = z_x = \frac{\mathcal{R} - \mu_{\mathcal{R}}}{\sigma_{\mathcal{R}}} \tag{4.1}$$

where $\mu_{\mathcal{R}}$ and $\sigma_{\mathcal{R}}$ are the cross sectional mean and standard deviation of the ranks $\mathcal{R}$. We measure profitability by gross profits over assets, return on equity, return on assets, cash flow over assets, gross margin, and the fraction of earnings composed of cash (i.e. low accruals).

$$\text{Profitability} = z(z_{gpoa} + z_{roe} + z_{roa} + z_{cfoa} + z_{gmar} + z_{acc}) \tag{4.2}$$

Similarly, we measure growth as the five-year prior growth in profitability, averaged across measures of profitability,

$$\text{Growth} = z(z_{\Delta gpoa} + z_{\Delta roe} + z_{\Delta roa} + z_{\Delta cfoa} + z_{\Delta gmar} + z_{\Delta acc}) \tag{4.3}$$

Here, Δ denotes five-year growth. For each variable, we define five-year growth as the change in the numerator (e.g. profits) divided by the lagged denominator (e.g. assets). We also follow Asness' definition of safe stocks as companies with low beta, low idiosyncratic volatility, low leverage, low volatility in ROE and low bankruptcy risk,

$$\text{Safety} = z(z_{beta} + z_{ivol} + z_{lev} + z_{evol} + z_{Ohlson} + z_{Altman}) \tag{4.4}$$

Payout is measured as the prior five year total payout ratio, one year net equity issuance and one year net debt issuance

$$\text{Payout} = z(z_{npop} + z_{eiss} + z_{diss}) \tag{4.5}$$

Finally, we combine the four measures into a single overall quality measure,

$$\text{Quality} = z(\text{Profitability} + \text{Growth} + \text{Safety} + \text{Payout}) \tag{4.6}$$

Using the approach described above, we assign quality measures to all firms in each year from 1990 to May 2016. To form quality-sorted portfolios, stocks are assigned to ten quality-sorted portfolios, which means stocks are sorted into deciles, resulting in an equal number of stocks in each. At the end of each calendar month, stocks are assigned to two size-sorted portfolios based on their market capitalization. Following the methodology by Asness and Frazzini (2013), the size breakpoint is the median of the NYSE market-capitalization. The QMJ portfolio construction follows Asness and Frazzini (2013) in that QMJ factors are constructed as the intersection of six value-weighted portfolios formed on size and quality. That is, the factor is long the top 30% high-quality stocks and short the bottom 30% junk stocks within the universe of large stocks and similarly within the universe of small stocks. Observations are dropped when firms do not have positive earnings before extraordinary items, book value of common equity or when lack of data availability makes calculating the quality measures infeasible. Calculation of the quality measures requires five years of financial statement history and three years of stock return history. The QMJ factor return is the average return on the two high-quality portfolios minus the average return on the two low-quality (junk) portfolios:

$$\begin{aligned} \text{QMJ} =& \frac{1}{2}(SmallQuality + BigQuality) - \frac{1}{2}(SmallJunk + BigJunk) \\ =& \frac{1}{2}\underbrace{(SmallQuality - SmallJunk)}_{\text{QMJ in small stocks}} + \frac{1}{2}\underbrace{(BigQuality - BigJunk)}_{\text{QMJ in big stocks}} \end{aligned} \tag{4.7}$$

Ideally, when forming the portfolios meant to mimic the underlying risk factors, each factor portfolio should express the individual factor effect free of possible effects from other factors. Following the procedure by Fama and French (1992, 1993 and 1996) and Asness and Frazzini (2013), the size portfolio, SMB, is formed by controlling for value effects, while the value portfolio, HML, and the momentum portfolio, UMD, are formed by controlling for size effects. The size, value and momentum factors are constructed using six value-weighted portfolios formed on size (market value of equity ME) and book-to-market (book equity divided by the most recent market equity B/M) and 1-year return (return over the prior 12 months, skipping the most recent month). At the end of each calendar month, stocks are assigned to two size-sorted portfolios based on their market capitalization. The sorting is conditional, meaning first sorting on size by the median of the market capitalization of NYSE, then on the second variable. The portfolios are value-weighted and rebalanced every calendar month to maintain value weights.

To form groups on momentum, on June 1st each year from 1990-2016 all firms are ranked based on their lagged momentum. In the same manner as described above, the stocks are split into three groups of firms with a momentum level in the bottom

30%, in the top 30%, and the neutral middle 40%. In Fama and French (2011) they measure the momentum as a stock's cumulative return for the period from month $_{t-12}$ to $_{t-1}$. It is standard procedure when forming the momentum factor to skip the most recent month due to the tendency of returns to display short-term reversal as described in the literature review. The momentum factor UMD is the average return on the two high return portfolios minus the average return on the two low return portfolios:

$$\text{UMD} = \frac{1}{2}(SmallHigh + BigHigh) - \frac{1}{2}(SmallLow + BigLow) \tag{4.8}$$

Thus, the momentum portfolio UMD reflects the different return behavior of high and low momentum stocks free of influence from the size factor.

The size portfolio, SMB, is formed by controlling for value effects, while the value portfolio, HML, and the momentum portfolio, UMD, are formed by controlling for size effects. The size factor SMB is the average return on the 3 small portfolios minus the average return on the 3 big portfolios. Our portfolio SMB (small minus big), meant to mimic the risk factor in returns related to size, is the difference, each month, between the simple average of the returns on the three small-stock portfolios (S/L, S/M and S/H) and the simple average of the returns on the three big-stock portfolios (B/L, B/M and B/H). Thus, SMB is the difference between the returns on small- and big-stock portfolios with about the same weighted-average book-to-market equity.

$$\begin{aligned} \text{SMB} =& \frac{1}{3}(SmallValue + SmallNeutral + SmallGrowth) \\ &- \frac{1}{3}(BigValue + BigNeutral + BigGrowth) \end{aligned} \tag{4.9}$$

Thus, SMB is the difference between the returns on small and big stock portfolios with approximately the same weighted-average of B/M, hence SMB is largely free of the influence of value effects.

The portfolio HML (high minus low), meant to mimic the risk factor in returns related to book-to-market equity (BE/ME), is defined similarly to above. HML is the difference, each month, between the simple average of the returns on the two high BE/ME portfolios (S/H and B/H) and the average of the returns on the two low BE/ME portfolios (S/L and B/L). The two components of HML are returns on high- and low BE/ME portfolios with approximately the same weighted-average size. Thus the difference between the two returns should be largely free of the size factor in returns, focusing instead on the different return behaviors of high- and low BE/ME firms:

$$\text{HML} = \frac{1}{2}(SmallValue + BigValue) - \frac{1}{2}(SmallGrowth + BigGrowth) \quad (4.10)$$

We are also interested in including the market factor (MKT). Although conceptually simple, various ways of calculating this premium is stated in the literature. From a dogmatic point of view, we want a simple measure that captures systematic market movements above the risk free rate, just like that of the CAPM factor model. To keep things simple, we dub the market factor as the value-weighted return on all available stocks minus the one-month Treasury bill rate. Treasury-bills can be purchased at auctions held by the government, or investors can purchase T-bills on the secondary market that have been previously issued. T-Bills purchased at auctions are priced through a competitive bidding process, at a discount from the par value. When investors redeem their T-Bills at maturity, they are paid the par value. The difference between the purchase price and par value is interest.

Table 4.1: Factor correlations (Pearson)

The table reports factor correlations based on the merged feed of CRSP/Compustat from Jan 1990 to May 2016.

	QMJ	**MKT**	**SMB**	**HML**	**UMD**
QMJ	1				
MKT	-0.6572	1			
SMB	-0.5530	0.2716	1		
HML	0.0665	-0.2158	-0.1551	1	
UMD	0.3185	-0.2706	-0.1999	-0.1008	1

The evidence from **Table 4.1** suggests that we keep highly active exposures to the different factor risk premiums (assuming we are doing high-conviction non-watered down versions of the anomalies). Blending the strategies creates benefit from a diversification perspective. These results also serve as re-insurance as we can build the variance-covariance matrix necessary for our analysis in **Chapter 5**. In **Figure A.1** in the Appendix, we have reported monthly factor premiums.

4.3 GICS Sectors

This section is dedicated to describe the extraction process of data based on the GICS classification system. The Global Industry Classification Standard (GICS), is an industry taxonomy developed in 1999 by MSCI for use by the global financial community. The GICS structure consists of 11 sectors, 24 industry groups, 68 industries and 157 sub-industries [as-of November 2016]. The GICS system is used as a basis for SP 1500 and MSCI financial market indexes in which each company is assigned to a sub-industry, and to a corresponding industry, industry group and sector, according to the definition of its principal business activity. For the purpose of building a strategic, well-diversified portfolio, investors may wish to allocate between sectors for various reasons. In an economic sense, both the systemic premia described above and sector allocations are essentially factor loadings. Both notions are exposures to specific, investable areas of the market where portable alpha and idiosyncrasies may be at work. This idea is interesting from an active asset allocation point of view, which is why we include this data in this thesis. Sectors have not been dedicated much attention in the previous chapter on theoretical methodologies. The comparison has limits. First of all, factors, in the spirit of Fama and French, involve systemic risk premiums that entail simultaneously long and short positions. Sectors, or more divisible levels of the GICS, are just like the acronym suggests, a classifications system. Although new sectors and industries emerge from time to time[1], the GICS classification system is not floating in a theoretical sense. As a result, we will spend the effort on the extraction process of relevant data for the purpose and use in a Black-Litterman context. Our main purpose therefore, is to construct data in such a way that is can be used for further analysis. In **Figure A.2** in the Appendix, we have reported the monthly returns to each sector.

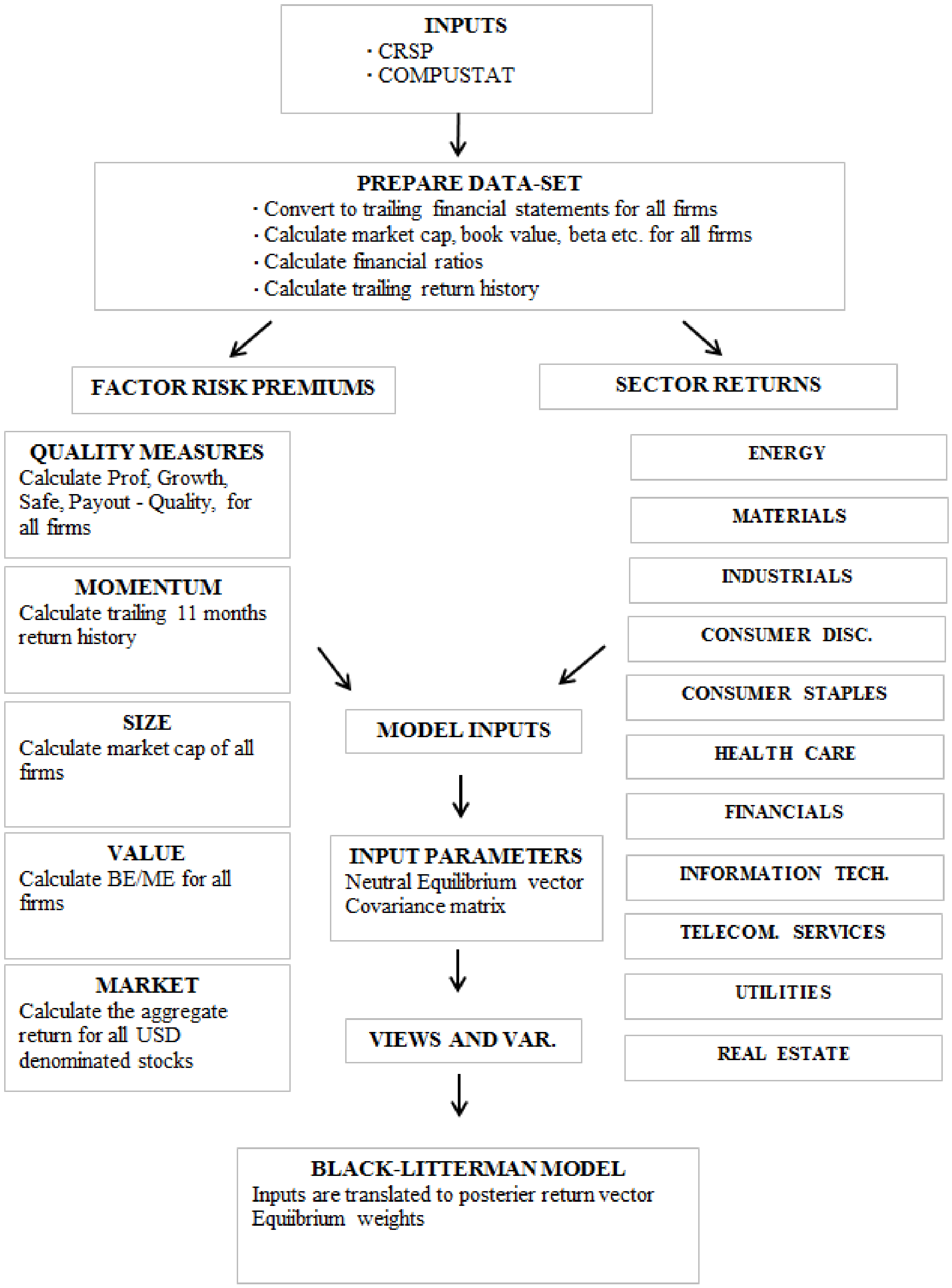
Each month, we group constituents from the S&P 1500 from the merged feed of CRSP/-Compustat based on their sector relation, represented by the two-digit code e.g. '10', '15',...,'60'. The results are then merged with the return history, for the constituents at that particular point in time. Each month, segments display the market capitalization of each sector, which is the stock price multiplied with number of shares outstanding. A specific stock at every month-end has the return properties, all else equal: $P_t/P_{t-1} - 1$. The contribution of each sector component, is the product of the monthly return and the corresponding market weight. For each index, the constituents are weighted based on their free float market capitalization. Following

[1] Real Estate was added as a new sector effective September 1st, 2016.

this procedure, we do not mitigate the problem of over-representation. To marginalize the impact of sector concentration and stock-specific risk, capped versions could be constructed. However, it is beyond this thesis to elaborate on sector or industry concentration why we will only mention it here.

Regular monthly and annual GICS reviews may result in the migration of a security into or out of the different sectors. Newly eligible securities after such a GICS change will only be considered for inclusion in the indexes during the following regularly scheduled index review. Any existing constituent moving out of the different sectors will be deleted from the indexes at the effective date of the GICS change (as of the close of the last business day of the given month). In **Figure 4.1**, we present a graphical representation of the overall research design.

Figure 4.1: Illustration of the research design



Chapter 5

Empirical results

In this chapter, we present the findings of the empirical analyses. The chapter is structured as follows. In **Section 5.1**, we present the descriptive statistics of the dataset based on factor risk premiums. **Section 5.4** is entirely devoted to the results, as such, the section is further divided into four subsections. **Subsection 5.4.1** is devoted to the main results of the applicability of the Black-Litterman model in a multifactor framework based on historical factor risk premiums, while **Subsection 5.4.3** reports the results on a multisector approach, constructed from GICS. **Subsection 5.4.4** shows the the results of the parsimonious parameter τ, while the final **Subsection 5.4.5** shows the results from various values of the risk-aversion parameter. All results are based on the methodology described in **Chapter 4**.

This thesis explores the dynamics of the Black-Litterman model and challenges the current methodology, and proposes a more exploring and less rigid approach, suited for today's investment needs. This chapter shows the results of such an attempt. This complements existing findings like that of Bevan and Winkelmann (1998), Cheung (2009), Cheung (2012) and Asl and Etula (2012). In this attempt, we add to the literature by formalizing the results of a model that incorporates views on multiple risk factor premiums and an established classification system.

5.1 Descriptive statistics

In **Table 5.1**, **Table 5.2** and **Table 5.3** we present the descriptive statistics for key variables in our final data set with regard to factor premiums. In **Table 5.6** we present the summary statistics for sectors. All inputs are from the merged CRSP/Compustat database. The procedure of estimating the various financial ratios, return history and the factor premiums is presented in the methodology chapter.

Table 5.1: Summary Statistics, Factor Premiums

The table reports summary statistics for the five overall factor risk premiums used in the first part of this chapter. In Panel A, we report the covariance matrix, while we report the average historical return premiums in Panel B.

PANEL A: VARIANCE-COVARIANCE /WEIGHTS

	Variance Covariance Matrix					Equal	β	Random
	QMJ	MKT	SMB	HML	UMD	w	w	w
QMJ	0.083	-0.083	-0.044	0.005	0.044	0.2	0.003	0.087
MKT	-0.083	0.191	0.033	-0.026	-0.057	0.2	0.313	0.411
SMB	-0.044	0.033	0.076	-0.012	-0.026	0.2	0.299	0.027
HML	0.005	-0.026	-0.012	0.075	-0.013	0.2	0.147	0.251
UMD	0.044	-0.057	-0.026	-0.013	0.229	0.2	0.238	0.224

PANEL B: FACTOR PREMIUMS

	Historical Premiums	Median $\mathbb{R}$	Difference	St.dev	SR
QMJ	0.4440	0.299	0.145	2.8777	0.1543
MKT	0.5960	1.071	-0.475	4.3760	0.1362
SMB	0.1487	-0.018	0.167	2.7587	0.0539
HML	0.1351	0.069	0.066	2.7340	0.0494
UMD	0.6364	0.889	-0.253	4.7920	0.1328
Market	0.3075	1.588		5.4647	0.0563

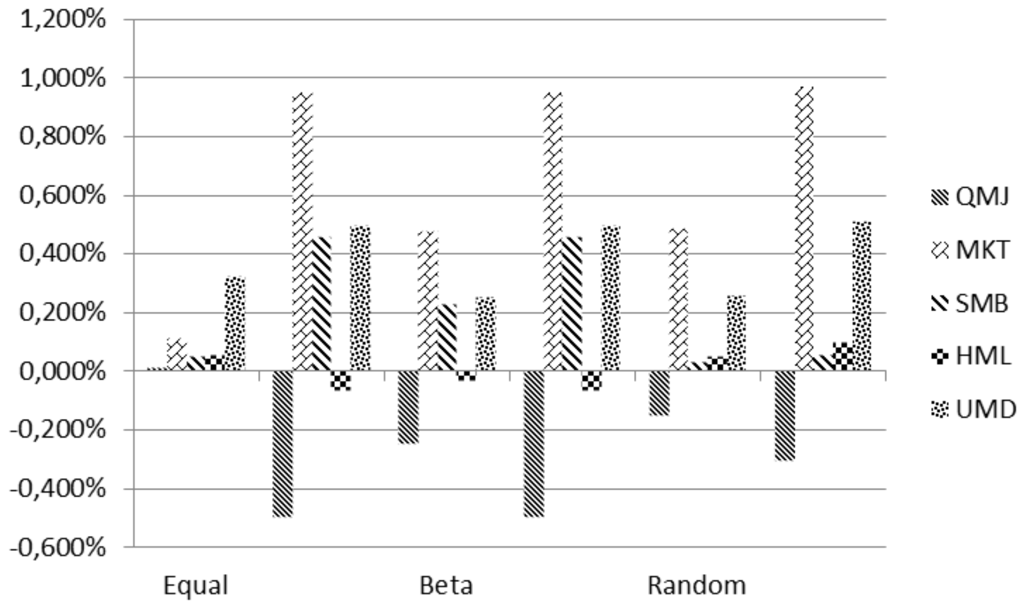
In Panel B of **Table 5.1**, we report historical premiums based on monthly returns to each factor. Panel A presents three different sets of weights, which we will utilize throughout this section. Based on the results from Arnott, Hsu, and Moore (2005) who argue that market cap weighting is inherently flawed and advocate replacing market cap allocations with factor allocations, we stick to this procedure in this part of the chapter. The market capitalization weight, primarily used in most practical settings, is deferred to **Subsection 5.4.3**. Another motivation for this approach, is more pragmatic. Portfolio managers typically invest based on some form of constraint, which can take many forms, such as limit on funds invested (issuer-specific risk, key-man risk and model risk etc.). These form of risk-management policies, among other things, ensures portfolio managers seek the interest of agents by complying with predetermined investment limits that directly impact the portfolio choice and thereby the allocation. The final basic input for our analysis, is the covariance matrix of the return premiums. As with the sector returns, we estimate covariance matrices using monthly data. For the purposes of this analysis, we assume a weight that is consistent with a two- to three-month rebalancing horizon.

5.2 Equilibrium returns

This section describes in more detail how we calculate equilibrium returns. In the Black-Litterman framework, expected returns are viewed as a blend of equilibrium returns and an actual set of investor views. The equilibrium returns can be interpreted as the long-run returns provided by the global capital markets. This is in line with the Efficient Markets Hypothesis (EMH), where equilibrium returns represent the information that is available through the capital markets. In contrast, investor views correspond to the interpretation of information that is unique to the individual investor. Thus, expected returns are a blend of the information available through the capital markets and unique insight to a specific investor. As the mixture of sources of information changes, expected returns will also change. While equilibrium returns represent a useful neutral starting point, one obvious problem is that they are *not* observable. However, as we shall see, with a few intuitive assumptions, we can easily infer equilibrium returns from other data sources that *are* observable. A natural way to proceed is as follows: We can start by assuming that if asset markets are in equilibrium, a representative investor would hold some proportion of either the global capitalization-weighted portfolio or a random weight for that matter. This assumption provides us with one very important, observable piece of information. We can now work from the observable weights to equilibrium returns by calculating this portfolio's volatility by the covariance matrix of returns.

Panel A of **Table 5.2** shows three sets of equilibrium premiums. These premiums are based on a projected Sharpe Ratios 0.5, which are used by Black and Litterman (1992). To get grasp of the impact on the effect of Sharpe Ratio on the equilibrium return premiums, **Figure 5.1** shows in line with Bevan and Winkelmann (1998) the equilibrium premiums with a projected Sharpe Ratio of 0.5 and 1.0. Here, the first entry of each factor premium correspond to a Sharpe Ratio of 0.5, while the

Figure 5.1: Equilibirum returns, SR=0.5; SR=1.0



second entry shows the value for the equilibrium returns with a Sharpe Ratio of 1.0. The Sharpe Ratio is of important concern, as it affects the risk coefficient δ and thereby equilibrium returns Π. Looking at the figure, a more general thing can be said about the impact of δ on Π. All else equal, a higher risk aversion coefficient leads to more *unstable* portfolio allocations. This finding is quite intuitive. Doubling the risk aversion has an adverse effect on equilibrium returns through the covariance matrix, whose element becomes twice as large. We will explore this point further in **Subsection 5.4.5**.

5.3 Weighs and views

In the next stage of the optimization process, we determine how much weight to put on the neutral reference point (equilibrium) returns relative to an explicit set of investor views. For the latter, we will make our own random views, which is not supported by any evidence, nor do we true to make superior statements about economic conditions through these views. What we will evidence, is that the weight put on these views, is ultimately linked to Ω through the scalar τ. The blending of individual market views with the equilibrium returns is an important step in our methodology. There are two major reasons for following this procedure. The first reason is that by referring to the covariance matrix of historical returns (implicit in calculating the equilibrium returns), we ensure greater consistency across views. In this sense, the equilibrium returns help to serve as a macro constraint on our forecasts. If we are completely certain in our views, then the Bayesian update will have no effect, which makes the update system redundant as the MVE required inputs are fulfilled (known variance and covariance). The second reason relates to the methodology of balancing the extreme views relative to the implied equilibrium returns, which produces more balanced portfolios than are typically achieved from an unconstrained mean-variance optimization.

5.4 Results

To test the rigorousness of the Black-Litterman model, we have performed an analysis based on two datasets. First, **Subsection 5.4.1** shows how the Black-Litterman model is conceptually robust when applied to a factor risk premium universe, where views are taken on individual factors. In **Subsection 5.4.3** we apply the model to a universe of GICS sectors simultanously shortening the horizon, which should have an adverse effect on τ. For this reason, we present the results of such an analysis in a more stylized setting in **Subsection 5.4.4**. Finally, in **Subsection 5.4.5**, we show the effect of δ on key elements of the Black-Litterman model.

5.4.1 Factor risk-premiums

The remaining part of this chapter will be dedicated to the reported findings, which will assist us in evaluating the hypothesis set forth in the beginning of this thesis. In order to test whether the Black-Litterman model does in fact produce stable portfolio returns, we run the model based on above baseline parameters, reported in **Table 5.1**. In contrast to the 'reverse optimization' procedure applied in the Canonical Black-Litterman model, the traditional MV approach dictates the user to input a complete set of expected returns, and the portfolio optimizer generates the optimal portfolio weights. However, users of the standard portfolio optimizers often find that their specification of expected returns produces portfolio weights which may not make much sense (due to the complex mapping between expected returns and portfolio weights and the absence of a natural starting point for the expected return assumptions). In this thesis, we will not use examples to illustrate the difference between the traditional mean-variance optimization process and the various optimization procedures used in context with the Black-Litterman models (Canonical / Reference). Instead, we demonstrate how the Black-Litterman approach provides both a reference point for expected return assumptions as well as a systematic approach to deviating from this point to express one's factor views. As mentioned in our delimitations, it is beyond the scope of this thesis to simulate what views might be superior and how to time these set of views.

Table 5.2: Summary Statistics, Equilibrium Premiums and Weights

The table reports summary statistics for the equilibrium premiums and factor weights. All figures are reported in percentages. In Panel A, equilibrium premiums are reported based on two different values of δ. Sharpe ratio of the market portfolio is 0.0563 (a factor 10 less than Black and Litterman (1992)). We leave the discussion of τ to **Subsection 5.4.4** and use the value of 0.5 in accordance with the monthly standard deviation of the market portfolio, which is 5.4647%.

PANEL A: EQUILIBRIUM PREMIUMS

	Equally Weighted		Beta Weighted		Random Weighted	
	δ=1.03	δ=9.15	δ=1.03	δ=9.15	δ=1.03	δ=9.15
QMJ	0.001	0.010	-0.028	-0.251	-0.017	-0.154
MKT	0.012	0.108	0.054	0.476	0.055	0.486
SMB	0.006	0.049	0.026	0.227	0.003	0.026
HML	0.006	0.053	-0.004	-0.034	0.005	0.049
UMD	0.036	0.323	0.028	0.247	0.029	0.254

PANEL B: WEIGHTS

	Historical Return Premiums		CAPM Returns	
	δ=1.03	δ=9.15	δ=1.03	δ=9.15
QMJ	2093.9	235.6	1316.9	148.2
MKT	1210.9	136.3	674.1	75.9
SMB	1096.8	123.4	920.5	103.6
HML	678.5	76.3	610.1	68.7
UMD	334.0	37.6	144.8	16.3
High	2093.9	235.6	1316.9	148.2
Low	334.0	37.6	144.8	16.3

From Panel A above, we see the different equilibrium premiums Π under different values of δ. The δ of 1.03 is only provided for reference and will not be used further. The extreme weights reported in Panel B, are in line with the results of

Idzorek (2004), although he uses a reference model. This part of the analysis, however, is not influenced by this fact. These extreme portfolios are interesting as they depict the actual results of mean-variance optimization with no binding constraints. Evidently, it is rather difficult to give a clear interpretation of these results, which makes them rather useless to any portfolio manager.

The unique insight of the Black-Litterman model, is the way it uses optimization. Instead of estimating returns, the model uses the equilibrium returns provided by the market. The reverse optimization problem then boils down to estimating the optimal factor allocations. In **Chapter 2** we saw that the neutral equilibrium returns were expressed via **Equation 2.13**. From this proposed relation, we inferred the optimal factor weights from **Equation 2.16**. To explicitly state the problem of estimating the optimal portfolio weights, we rearranged to form the optimal weights $w = (\delta\Sigma)^{-1}\mu$, which gave us the weights presented in **Table 5.2**. The important insight from these results, tells a convincing story about market equilibrium. Here, μ represents any vector of expected returns, which we have substituted against Π, the equilibrium return vector. If μ and Π are equal, then returns are in equilibrium and therefore on the CML. If they are not equal, then w and w_{mkt} are not equal as well, which is evident from **Table 5.2**. Not surprisingly, the impact of the average risk-aversion δ has a significant impact on reverse-optimized equilibrium premiums. Black and Litterman (1992) as mentioned earlier, reports a Sharpe ratio of 0.5. They use a global data-span from 1975-1991, where global markets (MSCI All Countries) rose roughly 400%. More importantly, the average return of the market at that time were steadily rising, which means that the average return was not dominated by booms and busts. Another story emerges from the data between 1990 and 2016. Here too, markets rose a little short of 400%, but with a lot more volatility in returns. As a result, the average return produced by the market, is less expressed via the volatility of the market.

It is important to note that the equilibrium returns are not the result of an econometric forecasting exercise. Instead, the equilibrium returns represent the idea of a set of long-run returns that are consistent with market clearing. One clear role for econometric forecasting models is to provide indications about short-term movements around the long-run equilibrium. Expected returns can, in some sense, be interpreted as a complicated weighted average of the neutral equilibrium returns and an investor's views. In the next section, we will discuss the procedure that we follow for determining how much weight to put on the equilibrium returns and how to set the relative weights for each specific view.

Having presented the descriptive statistics, we continue by revisiting some fundamental equations we expressed in **Chapter 2**. First we write the Black-Litterman *Masterformula*:

$$P(A|B) \sim N(E[R], M) \tag{5.1}$$

The posterior distribution, also known as the conditional (or joint) probability of A, given B is assumes to be normal distributed with posterior returns E[R] and M, the uncertainty of the estimate around the true unknown mean. From **Equation 2.21** and **Equation 2.22**, we have:

$$E[R] = \underbrace{[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}}_{\text{`}Denominator'}\underbrace{[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]}_{\text{`}Nominator'} \tag{5.2}$$

$$M = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \tag{5.3}$$

M is not to be confused with the updated covariance matrix. **Equation 5.3** is an expression of the uncertainty around the estimate of the ex-post returns. The updated covariance matrix becomes:

$$\Sigma_p = \Sigma + M = \Sigma + \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \tag{5.4}$$

Which reduces to **Equation 5.5** in the special case of no views.

$$\Sigma + [(\tau\Sigma)^{-1}]^{-1} = \Sigma + \tau\Sigma = (1+\tau)\Sigma \tag{5.5}$$

For in-depth derivations of above equations, see Walters (2014). The intuition behind **Equation 5.2** and **Equation 5.3** is analog to that of **Equation 2.6** and **Equation 2.7**. That is, the updated estimate of μ the expected return vector, is a variance-weighted average of the a-priori estimate of μ_0. From **Equation 5.2**, we see that the expected return is a variance-weighted average. Using Bayes in this way presupposes that if the variance is small, then the updated estimate of the expected return be close to μ_0. See **Subsection 2.3.1** for a more detailed explanation.

As we shall see in the following, the Black-Litterman model is just as applicable with views. However, as equilibrium is assumed, ex-post equilibrium returns are not surprisingly just the market portfolio of risk premiums. That is, if we use the covariance matrix from **Table 5.1** and a variant of the equilibrium return premiums from

Panel A in **Table 5.2**, the optimal portfolio weights will be exactly the respective weights reported in Panel B of **Table 5.2**. The uncertainty in the CAPM τ is directly transferred to the optimal weights w_{opt} as can be seen from $w_{opt}^{\tau} = [\delta(1+\tau)\Sigma]^{-1}\Pi$ via **Equation 2.18**. The sum of weights will always be less than or equal to 1, as a direct result of τ, the uncertainty in our equilibrium returns. The no-views scenario ending up in 'adjusted' equilibrium in the Black-Litterman model is a nice feature which yields an intuitive result. However, the strength of the model is more effective where views play an active part of portfolio management. Below, we present the results of such a reverse optimization procedure.

Table 5.3: Summary Statistics, Optimal weights - no views

The table reports the special case where views does not exist. Evident from the table, is the weights which depart only slightly from the ones reported in **Table 5.2**. A τ of 0.05 and a δ of 9.1497 is the underlying parameters for these results. The updated covariance matrix then follows from **Equation 5.5**.

PANEL A: OPTIMAL WEIGHTS (NO VIEWS)

	Equally Weighted		Beta Weighted		Random Weighted	
	Π	w_{opt}	Π	w_{opt}	Π	w_{opt}
QMJ	0.010	0.1905	-0.251	0.003	-0.154	0.083
MKT	0.108	0.1905	0.476	0.298	0.486	0.391
SMB	0.049	0.1905	0.227	0.285	0.026	0.026
HML	0.053	0.1905	-0.034	0.140	0.049	0.239
UMD	0.323	0.1905	0.247	0.226	0.254	0.213
Sum		0.9524		0.9524		0.9524

Notice that the weights above all sum to 0.9524. Recall from **Equation 2.18**, that we have $1/(1+\tau)$ invested in the 'neutral' portfolio (here the factor equilibrium).

The next step is to incorporate investor views. As explicitly stated in our delimitations, we do not seek to run horse races between different views nor find the superiority of such views, used in the Black-Litterman model. Ideally this area should be dedicated its own entire research. At this point, we simply state arbitrary views to effectively evaluate how views can be implemented through the versatility of the model inputs, to achieve a reasonable portfolio allocation.

5.4.2 Building the inputs

One of the more confusing aspects of the Black-Litterman model is moving from the stated views to the inputs used in **Equation 5.2**. First, the model does not require that investors specify views on all assets, as we noticed above. In the five risk factor premiums, the number of views k was 3; thus, the view vector Q is a $3x1$ column vector. The uncertainty of the views results in a random, unknown, independent, normally-distributed error term vector ϵ with a mean of 0 and covariance matrix Ω. Thus, a view has the form $Q + \epsilon$.

$$Q + \epsilon = \begin{bmatrix} Q1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_k \end{bmatrix}$$

Except in the hypothetical case in which an investor is 100% confident in the expressed view, the error term ϵ is a positive or negative value other than 0. The error term vector ϵ does not directly enter the Black-Litterman formula. However, the variance of each error term ω, which is the absolute difference from the error term's ϵ expected value of 0, does enter the formula. The variances of the error terms ω form Ω, where Ω is a diagonal covariance matrix with 0's in all of the off-diagonal positions. The off-diagonal elements of Ω are 0's because the model assumes that the views are independent of one another. The variances of the error terms ω represent the uncertainty of the views. The larger the variance of the error term ω, the greater the uncertainty of the view. As a result, we have a general case:

$$\Omega = \begin{bmatrix} \omega_{1,1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_{k,k} \end{bmatrix}$$

Using this procedure to specify the views, we can elegantly show how to combine the sources of information; the equilibrium returns (see **Equation 2.13**) and views (see **Subsection 2.3.2**) using the Black-Litterman *Masterformula* (**Equation 5.1** to construct *posterior* returns.

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega \begin{bmatrix} 0.0219\% & 0 \\ 0 & 0.0165\% \end{bmatrix}$$

Table 5.4: Summary statistics, Posterior data

The table reports the result of a full implementation of the Black-Litterman model with views only. Panel A reports the posterior return vectors, Panel B the optimal portfolio weights while Panel C depicts the updated covariance matrix. All numbers are in percentage terms.

PANEL A: Π AND POSTERIOR RETURNS /RELATIVE VIEWS

	Equally Weighted		Beta Weighted		Random Weighted	
	Π	Posterior E[R]	Π	Posterior E[R]	Π	Posterior E[R]
QMJ	0.010	-0.052	-0.251	-0.197	-0.154	-0.113
MKT	0.108	0.228	0.476	0.401	0.486	0.437
SMB	0.049	0.079	0.227	0.203	0.026	0.009
HML	0.053	-0.011	-0.034	-0.067	0.049	0.001
UMD	0.323	0.407	0.247	0.381	0.254	0.407

PANEL B: OPTIMAL WEIGHTS $w^p_{opt} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$

	Equally Weighted		Beta Weighted		Random Weighted	
	w^p_{opt}	Scaled w^p_{opt}	w^p_{opt}	Scaled w^p_{opt}	w^p_{opt}	Scaled w^p_{opt}
QMJ	0.136	0.143	0.023	0.024	0.091	0.095
MKT	0.246	0.257	0.279	0.292	0.383	0.402
SMB	0.190	0.200	0.285	0.299	0.026	0.027
HML	0.129	0.136	0.088	0.093	0.175	0.183
UMD	0.251	0.264	0.278	0.292	0.278	0.292
Sum	0.9524	1	0.9524	1	0.9524	1

PANEL C: POSTERIOR COVARIANCE MATRIX

	Variance Covariance Matrix				
	QMJ	MKT	SMB	HML	UMD
QMJ	0.085	-0.084	-0.045	0.005	0.045
MKT	-0.084	0.196	0.033	-0.027	-0.058
SMB	-0.045	0.033	0.079	-0.012	-0.027
HML	0.005	-0.027	-0.012	0.078	-0.012
UMD	0.045	-0.058	-0.027	-0.012	0.236

In **Table 5.4** we report summary statistics for the case of two relative views presented above. That is, we test the applicability of the factor views in the Black-Litterman model by stating two completely random views. The first view is a bet on the **MKT** risk premium relative to the **QMJ** premium, whereas the second active bet is the outperformance of **UMD** relative to **HML**. These views result in 1) posterior returns (blended equilibrium returns and views), 2) optimal weights and 3) an updated covariance matrix.

From Panel A of **Table 5.4** we see that our views are blended through **Equation 5.2**, **Equation 5.3** and **Equation 5.4** which yields some interesting results. Let us start by examining the first view. We believe that **MKT** will outperform **QMJ** by 0.5%. By looking at Panel A, we see that the posterior return premium of **MKT** has been shifted upward and likewise **QMJ** downward to reflect our beliefs. This is evident across the different weighting schemes, which can be seen exhibiting the behaviour of fluctuating returns. The same is true for our second relative view of 0.6%. Here, we stated that the factor risk premium **UMD** will outperform **HML**. Note that not all risk premiums with a positive relative bet increases. For instance, our posterior return within the β weighting regime, actually decreases. This is due to the nature of our bets, which says nothing about the direction of the return distribution, only that the relativeness of the bet itself will be pulled in that direction. Even though the expressed views only directly involved 4 of the 5 premiums, the individual returns of all the premiums $E[R]$ changed from their respective Implied Equilibrium returns, Π.

If we look at the optimal weights in Panel B, calculated as:
$w^p_{opt} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$, an appealing picture emerges. Intuitively, we would expect the weights of our bets to turn in the direction of our views. Examining the first relative view between **MKT** and **QMJ**, we see as a result of our view that the weight in **MKT** has increased by 5.7 percentage points while the weight towards **QMJ** has decreased 5.7 percentage points. If we look at the impact of the second relative view on the random weighting, we see that the initial weights to **UMD** and **HML** were 22.4% and 25.1% respectively. The posterior vector of optimal weights, w^p_{opt} is now 29.2% and 18.3% respectively. A dichotomy of the weights appear between the relative views. An equal proportion gained, is an equal proportioned forfeited. Note also that **SMB** is not included in any of the views, hence intuitively, the weight remains the same across all regimes.

The updated covariance matrix is reported in Panel C of **Table 5.4**. Even though we only have views on four out of five factors, the views cause the return of every asset in the portfolio to change from its Implied Equilibrium return, since each individual return is linked to the other returns via the covariance matrix of excess returns Σ. This is perhaps even more obvious when we look at **Equation 5.4** for the posterior covariance and break down each element. The updated covariance matrix of returns, is the product of our link matrix P and the inverse of Ω. Recall that we measured Ω as $\Omega = diag(P(\tau\Sigma)P^T)$. This means that if we are uncertain about our views, the elements in Ω will be large. The elements in Ω^{-1}, however, will be small. This boils down to the value of the scalar τ, which is the sole source of uncertainty in our views. For this reason, we have dedicated a section to ascertain the impact on the rest of the model, if we adjust and use different values of τ. Going back to the updated covariance matrix from **Equation 5.4**, we effectively have $\Sigma + M$, where M is the uncertainty around the *true* mean. Intuitively, we 'tilt' the prior covariance matrix with the 'amount' of uncertainty, to reflect our views. This can be seen from the individual covariances from Panel C. All the variances have slightly risen, reflecting this very uncertainty, relative to the historical returns reported in **Table 5.1** Panel B.

We only dealt with relative views in **Table 5.4**, as this is most common in practice. As we shall see, the type of weights has an interesting impact of optimal portfolio weights. To see how the model behaves with the exact same parameters, except for an absolute view, we form our views as:

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.0219\% & 0 \\ 0 & 0.0037\% \end{bmatrix}$$

Table 5.5: Summary Statistics, Posterior data

The table reports the result of a full implementation of the Black-Litterman model with absolute and relative views. Panel A reports the posterior return vectors, Panel B the optimal portfolio weights while Panel C depicts the updated covariance matrix. All numbers are in percentage terms.

PANEL A: Π AND POSTERIOR RETURNS ABSOLUTE AND RELATIVE VIEWS

	Equally Weighted		Beta Weighted		Random Weighted	
	Π	Posterior E[R]	Π	Posterior E[R]	Π	Posterior E[R]
QMJ	0.010	-0.068	-0.251	-0.211	-0.154	-0.130
MKT	0.108	0.172	0.476	0.337	0.486	0.383
SMB	0.049	0.050	0.227	0.169	0.026	-0.019
HML	0.053	0.318	-0.034	0.285	0.049	0.325
UMD	0.323	0.214	0.247	0.202	0.254	0.208

PANEL B: OPTIMAL WEIGHTS $w^p_{opt} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$

	Equally Weighted		Beta Weighted		Random Weighted	
	w^p_{mkt}	w^p_{opt}	w^p_{mkt}	w^p_{opt}	w^p_{mkt}	w^p_{opt}
QMJ	20	13.338	0.334	1.159	8.676	8.636
MKT	20	26.662	31.285	30.459	41.096	41.136
SMB	20	20.000	29.914	29.914	2.740	2.740
HML	20	62.610	14.713	62.038	25.114	66.716
UMD	20	20.000	23.755	23.755	22.374	22.374
Sum	100	142.610	100	147.325	100	141.602

PANEL C: POSTERIOR COVARIANCE MATRIX

	Variance Covariance Matrix				
	QMJ	MKT	SMB	HML	UMD
QMJ	0.085	-0.084	-0.045	0.005	0.045
MKT	-0.084	0.196	0.033	-0.026	-0.058
SMB	-0.045	0.033	0.079	-0.012	-0.027
HML	0.005	-0.026	-0.012	0.076	-0.014
UMD	0.045	-0.058	-0.027	-0.014	0.240

In the above **Table 5.5**, we see the results of a portfolio optimization with both absolute and relative views. In Panel B, the new optimal weight vector (w_{opt}) in column 3, 5 and 7 is based on the posterior return vector ($E[R]$). One of the strongest features of the Black-Litterman model is illustrated as the numerical difference between the columns of Panel B. Only the weights of the 3 premiums for which views were expressed change from their original market capitalization weights and the direction of the changes is intuitive. No views were expressed on SMB and UMD and their weights are as a result unchanged. From a macro perspective, the new portfolio can be viewed as the sum of two portfolios, where Portfolio 1 is the original market capitalization-weighted portfolio, and Portfolio 2 is a series of long and short positions based on the views. Portfolio 2 can be subdivided into small-portfolios, each associated with a specific view. The relative views result in small-portfolios with offsetting long and short positions that sum to 0. View 2, the absolute view, increases the weight of HML without an offsetting position, resulting in portfolio weights that no longer sum to 1. Regardless of the initial weighting scheme, we have relatively robust and well-diversified portfolios. Just as the results of **Table 5.4**, we have posterior returns, which change as a consequence of no views. This will be more apparent from **Section 5.4.4**.

5.4.3 Sector returns

In this subsection, we will present the results of the empirical analysis performed on sectors. We naturally limit our universe to consist of all stocks in the S&P 1500, which is an aggregate of S&P 500 (Large Companies), S&P 400 (Medium Companies) and S&P 600 (Small Companies). We start this analysis from a more observant perspective. Each month from May 2007 through May 2016, we observe the market value of each constituent in the S&P 1500 universe and group these by their respective GICS level 1 sector code. This provides the total market capitalization each month, which we will use as the market weights to each sector. As index classifications get updated at least once a year, we make sure that monthly returns can be linked to the same constituents as the aggregated market weights. Before we can match the return results with each sector, we have to infer the return contribution from each individual constituent and multiply by the corresponding market value. Doing a few iterations in SQL along with some knowledge about relational database structures, this exercise becomes less tedious. The summary statistics from this empirical analysis can be found in Panel A of **Table 5.6**. The covariance of returns can be found in **Table A.2** in the Appendix.

We simultaneously turn our attention to a more complex set of views. Instead of equal offsetting positions (i.e. relative views [-1,1]) as we have applied above, following the procedure of Black and Litterman (1990, 1992) and Plesner (2016), we follow the methodology of Satchell and Scowcroft (2000) in terms of views. We will not endorse any other features of their model, as it takes the form of an Alternative Reference Model. The analysis results in a total of 108 months and 107 return observations (May 2007 through May 2016). The extraction process is described in **Section 4.3**.

With this information, we continue and calculate the implicit, neutral equilibrium returns Π. The average monthly return of the S&P 1500 is 0.488%. The average monthly Treasury bill rate was 0.046% and the monthly standard deviation of the S&P 1500 returns was 6.189%. This results in a Sharpe Ratio of 0.0715, well below 0.5 used by Black and Litterman (1992). This yields an unrealistically low risk aversion parameter of 1.156. To put things into perspective, from January 1st 2007 to the 10th of September 2009, the S&P 1500 fell an enormous -39.65%. This has a non-trivial impact on the risk aversion parameter. As a result, we will continue this section with a δ value of 11, as can be found in Plesner (2016). He uses a subsample of the S&P 1500, and is therefore deemed not too far related to these results. We will not just abandon these issues, which is why we go through the impact of δ in the separate **Subsection 5.4.5**.

As before, views are assumed to be independent, which is a rather unrealistic assumption in reality, especially when correlations among the underlying is present. We also use the same methodology as He and Litterman (1999) for Ω, which is proportional to $\tau\Sigma$. Views are here relative to ($k \geq 1$) number of stocks. This feature follows Satchell and Scowcroft (2000). Stated formally:

$$\text{P}=\begin{bmatrix} 1.00 & 0.00 & 0.00 & -0.20 & 0.00 & -0.20 & -0.20 & 0.00 & -0.20 & -0.20 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & -0.33 & -0.33 & 0.00 & -0.33 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & -0.50 & 0.00 & 0.00 & 0.00 & -0.50 & 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$\text{Q}=\begin{bmatrix} 0.2\% \\ 0.45\% \\ 0.10\% \end{bmatrix} ; \Omega = \begin{bmatrix} 0.0143\% & 0 & 0 \\ 0 & 0.0041\% & 0 \\ 0 & 0 & 0.0151\% \end{bmatrix}$$

Table 5.6: Summary Statistics, GICS sectors

The table reports the summary statistics from views based on Satchell and Scowcroft (2000). Panel A reports the summary statistics, based on the results from **Chapter 4**. Note the views presented are more complex than that of the canonical model. Finally, Panel B shows the analytical results.

PANEL A: SUMMARY STATISTICS

	GICS level code	Market Capitalization (mln)	Weight	Avg Monthly Return
Energy	10	8,156,649.12	6.726%	0.4662%
Materials	15	3,554,326.22	2.931%	0.46818%
Industrials	20	10,703,113.16	8.825%	0.7116%
Consumer Discretionary	25	15,854,104.20	13.072%	0.9300%
Consumer Staples	30	11,618,635.34	9.580%	0.8862%
Health Care	35	17,090,113.82	14.092%	0.9341%
Financials	40	15,618,385.73	12.878%	0.3859%
Information Technology	45	25,754,998.24	21.236%	0.8816%
Telecommunication Services	50	5,632,864.88	4.645%	0.6123%
Utilities	55	3,663,732.32	3.021%	0.6336%
Real Estate	60	3,632,275.06	0.8837%	0.7684%
Total		121,279,198.08	100%	

PANEL B: POSTERIOR STATISTICS

	Π	E[R]	w_{mkt}	w_{opt}	Scaled w_{opt}	w_{mkt}-S.w_{opt}
Energy	4.744%	4.399%	6.726%	-5.787%	-6.076%	12.802%
Materials	4.780%	4.669%	2.931%	2.791%	2.931%	0
Industrials	4.376%	4.403%	8.825%	29.984%	31.483%	-22.658%
Consumer Discretionary	4.309%	4.320%	13.072%	15.865%	16.658%	-3.586%
Consumer Staples	2.432%	2.393%	9.580%	9.124%	9.580%	0
Health Care	3.031%	2.950%	14.092%	8.015%	8.416%	5.676%
Financials	5.971%	5.851%	12.878%	6.859%	7.202%	5.676%
Information Technology	4.078%	4.053%	21.236%	21.852%	22.945%	-1.709%
Telecom. Services	3.765%	3.660%	4.645%	-0.982%	-1.031%	5.676%
Utilities	2.773%	2.681%	3.021%	4.665%	4.898%	-1.877%
Real Estate	5.868%	5.819%	2.995%	2.852%	2.995%	0
Total			100%	95.238%	100%	0

Evidently, working with a larger universe of stocks and incorporating an increasing number of views is a model risk. This can be partly offset by incorporating an Augmented model, which be have presented in **Chapter 6**. Positing our views, we had no preconceptions about the *direction* of the future returns. By looking at View 1 [**Long**: 10; **Short**: 25, 35, 40, 50 and 55], we see that expected return causality between 10 and 25 contracts rather than expands. This view was fulfilled on the onset, and we clearly stated that 10 should outperform 25 by 0.2%. Since no other views interfere with these two sectors, the expected return get dragged closer towards the relative outperformance of 0.2%.

The relative bet between 10 and 35 is a bit more complex, since we short 35 in View 2 as well. This results in a second short position in 35, which makes the isolated impact of View 1 less apparent. This is true for all sectors with more than one view (10, 25, 35, 40 and 50). Since View 2 is relative by a larger amount (0.45% vs. 0.2%), it is fair to assume that this view will have the largest impact on the expected return.

View 2 is [**Long**: 20; **Short**: 35, 40 and 50]. The expected returns across this view are only moderately changed, which stems from one single source. Since the second element, related to View 2, is rather small, this means that the second element in Ω^{-1} is quite large. As a result, our second view is dragged relatively more towards the neutral equilibrium view, Π, rather than Q.[1]

The interpretation of View 3 is more or less a mixture of the points from View 1 and View 2. We have partly offsetting positions in 25 combined with a double long position in 10. All while the third entry in Ω is again, relatively small (low confidence).

We will finish this chapter with some stylized results from two of the most important parameters of the Canonical Black-Litterman model, τ and δ. We finish with some tentative results.

5.4.4 The impact of τ

The purpose of the following section is to discuss one of the most important parameters of the Black-Litterman model. As mentioned above, the value of τ is not of negligible importance, which is why we dedicate an entire section to this scalar. We

[1] This is apparent from the second part of the 'Nominator' in **Equation 5.2**, where: $P^T\Omega^{-1}Q$, that effectively gives weight to our individual views.

start with an interpretation and then finishes the section with an example.

From our previous acquaintance with τ wee see that the posterior return distribution does not depend on this value, which is linked to our previous assumptions regarding proportionality to Ω. This is a clear advantage from a model specification perspective. If this is realistic is hard to say. We implicitly assume that the unknown mean does not change due to increased volatility in this mean. We will leave this question for future research.

The meaning and impact of the parameter τ causes a great deal of confusion in the literature. According to Walters (2014), τ has a very precise meaning for investors using the Canonical Reference Model. In fact, he states that an author who selects an essentially random value for τ is likely not using the Canonical Reference Model, but is instead using some Alternative Reference Model. Given the Canonical Reference Model one can still perform an exercise to understand the impact of τ on the results. Both Black and Litterman (1992) and Lee (2000) address this issue: since the uncertainty in the mean is less than the uncertainty in the return, the scalar τ is close to zero. One would expect the equilibrium returns to be less volatile than the historical returns. Lee (2000), who has considerable experience working with a variant of the Black-Litterman model, typically sets the value of the scalar τ between 0.01 and 0.05, and then calibrates the model based on a target level of tracking error. Conversely, Satchell and Scowcroft (2000) state without much further explanation the value of the scalar to 1. They calibrate asset returns to produce a Sharpe Ratio of 1, because a Sharpe Ratio of 1 can be interpreted equivalently as a one-standard deviation event. Finally, Blamont and Firoozye (2003) interpret $\tau\Sigma$ as the standard error of the estimate of the implied equilibrium return vector Π; thus, τ is approximately 1 divided by the number of observations, as the best maximum likelihood estimator $(1/n)$. This means that τ is more or less inversely proportional to the relative weight given to the implied equilibrium return vector Π. The problem with the maximum likelihood estimator, takes the same empirical form as R^2 seen in regression analysis. The model itself lingers on intuition and it seems a bit far-fetched and unfair to expect the errors to asymptotically approach zero, by adding a long enough data-span. Setting this value, must therefore be applied in conjunction with common sense, which is the reason we use a value of 0.05 and not $(1/317) = 0,003155$. Working rigorously with the model in an applied setting, should always incorporate different values of τ. This is the main motivation for this section.

Ω, a by-product of τ is also of high importance. See Walters (2014) for a thorough elaboration. As the discussion above illustrates, Ω is probably the most abstract mathematical parameter of the Black-Litterman model. Unfortunately, as with τ, how to specify the diagonal elements of Ω, representing the uncertainty of the views, has no 'universal answer'. In the Canonical Reference Model, Ω is the variance of the views, which is inversely related to the investors confidence in the views, however the model does not provide an intuitive way to quantify this relationship. Hence, it is up to the investor to compute the variance of the views, which according to Walters (2014), can be done in four different ways. This specification of the variance, or uncertainty, of the views essentially equally-weights the investor's views and the market equilibrium weights. The important thing is the distinction between the reference models, like that of Satchell and Scowcroft (2000) and the canonical model. In fact, it can be proven that the use of a $\tau = 1$ shows that their paper uses the Alternative Reference Model. In their model, μ is normally distributed with variance Σ. This is commonly described as having a $\tau = 1$, but more precisely we are making a point estimate and thus have eliminated τ as a parameter. In this model Ω becomes the covariance of the returns and the views around the unknown mean return, just as Σ is the covariance of the prior return about its mean. Given we are now using point estimates, the posterior is now also a point estimate and we no longer worry about the posterior covariance of the estimate (see Walters (2014)).

To highlight these individual points, let us illustrate with an example. The results of this exercise is shown in table **Table A.1** in the Appendix. To clear things, recall that the optimal weights are $w_{opt}^{p} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$. Just as without views, the investor places a fraction of $1/(1+\tau)$ in the optimal portfolio, and a fraction $\tau/(1+\tau)$ in the risk-free asset. Evident from these results, is that alternating the value of τ does not produce unstable portfolios nor does it inflict any changes to the expected return vector $E[R]$, hence we have not reported this information. In fact, the most important causality is on our confidence Ω and the nominator of the Black-Litterman formula - the uncertainty of our estimate of the mean, around the true, unknown mean $[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$. As τ (the uncertainty of the equilibrium returns gets larger, the less weight we put on the views matrix Q. This can be seen chronologically going from Panel A through Panel E of **Table A.1**. The elements of the uncertainty matrix (nominator) increases with τ, while the 'weight' of our confidence Ω^{-1}, decreases with τ.

Finally, we could instead calibrate τ to the amount invested in the risk free asset given the prior distribution. Here we see that the portfolio invested in risky assets given the prior views will be given by from **Equation 2.18**. Thus the weights allocated to the assets are smaller by $[1/(1+\tau)]$ than the equilibrium market weights. This is because our Bayesian investor is uncertain in their estimate of the prior, and they do not want to be 100% invested in risky assets.

5.4.5 The impact of δ

We have been rather silent when it comes to the risk aversion parameter δ. As we have seen, δ is implied from the prevailing market conditions. More generally, we have posited that $\delta = (SR/\sigma_{mkt})$ as the only cited definition in the literature. From this relation, it is clear that as the market volatility increases, we should all else equal, expect less 'faith' in the equilibrium return vector. Panel A of **Table A.3** in the Appendix report the impact of δ on the equilibrium return vector, Π. We can deduce that having a higher risk aversion, yields higher equilibrium returns. Investors will demand a higher return for taking on more risk. Perhaps looking at the closed form solution to the reverse optimization problem, in the absence of constraints, gives a more clear picture. Recall that the equilibrium returns is $\Pi = \delta\Sigma w$. From this relation, then technically having a higher risk aversion, will result in all the elements of the covariance matrix, getting multiplied by this value of δ. Therefore, we must expect the equilibrium returns to increase, as we increase δ.

Panel B of **Table A.3** reports the posterior returns $E[R]$ for different risk aversions. The same arguments can be applied as with the equilibrium returns the only difference being, that instead of the equilibrium returns, an increase in δ will result in an increase in the posterior returns. This is also apparent from Panel B of **Table A.3**. However, it is not as clear why the expected returns of the Black-Litterman model increases, as the overall risk aversion increases. To see this more clearly, we need to look at the nominator of **Equation 5.2**. Our confidence in the 'equilibrium sector returns' is given by $(\tau\Sigma)^{-1}$. This term is multiplied by the neutral equilibrium returns Π. If Π is small as a result of a small δ, then our aggregate confidence in the equilibrium model will be small and our funds will be invested in the risk free asset $\tau(1+\tau)$.

The optimal weights reported in Panel C of **Table A.3** yields some interesting findings. We clearly see that the overall model, more or less breaks down. We no longer have the same intuitive results as before. The former benefits of diversification and overall asset allocation seems to be foregone. As the value of δ asymptotically approaches zero, we are becoming more and more inclined to take on risk. This is depicted by the extreme exposure under the three relatively low values of δ.

Chapter 6

Discussion

We start this chapter by relating the empirical findings to our hypothesis as well as previous studies. In addition, the chapter includes a discussion of limitations as well as ideas for future research and implications for practitioners.

6.1 Preliminary conclusions

The theoretical assumptions behind the Black-Litterman model is, in principle, fairly simple. The model starts, as most empirical models do, with normally distributed expected returns. This entails that returns cannot take the form of third and fourth order moments which may be violated in reality. The empirical literature on the subject has largely examined the practical use of the model, while surprisingly little innovation has been done since the inception in the 1990's. This may have several explanations. First, the model have received a rather large portion of research based on the practical implementation of the model, while the rest of the community has focused on the so-called Alternative Reference Models, which we have abstracted from in this thesis.

We have stated one hypothesis in order to structure our empirical examination. We have tested the hypothesis by explicitly applying two different datasets. These two pillars have served the purpose of exploring the robustness of our hypothesis, by alternating some of the fundamental conditions of the model. A dedicated effort has been made to bring out some of the inherent weaknesses of the model. The results of these empirical findings, presented in the previous chapter, allow us to provide answers to our hypothesis.

- *Can the Black-Litterman model be applied in a multi-factor context to create stable portfolio allocations?*

The empirical evidence in favor of extending the Black-Litterman model to a multi-factor framework is substantial. We admit there is some degree of subjectivity included due to the nature some of the random parameters, however, the model seems to behave exceptionally well, even when multiple factors and sectors are used. Regardless, these findings seem to be robust across different parameters of the Canonical Model. For instance, we saw that τ, δ and Ω has very important roles, when it comes to valuing the accuracy of model, extending beyond just the hypothesis. Applying a multi-factor approach, leads to indistinguishable stable levels of portfolio allocations.

6.2 Relation to previous research

In the following paragraphs, we compare our findings with the results of previous studies. We have found that the success of an effective application, does not depend on the level of expansion, but rather on the choice of different model parameters. This result is in conformity with that of of Bevan and Winkelmann (1998), Satchell and Scowcroft (2000), Idzorek (2004), Meucci (2006) Asl and Etula (2012) and Cheung (2012) who all evaluate the Black-Litterman model from different alternative perspectives. As a consequence, an intuitive comparison will not provide feasible interpretations. For instance, Idzorek (2004) proposes a new method that asses the tilts of the views. Meucci (2006) extends the Black-Litterman methodology to non-normally distributed markets and views through the COP approach (Copula-Opinion Pooling). Asl and Etula (2012), which empirically lies closest to the research design of this thesis, builds a multi-factor portfolio but rests on robust optimization rather than reverse optimization.

Similar to findings of this thesis, Cheung (2009) finds some practical issues that may frustrate applications, e.g., confidence parameter setting, alternative views (e.g., factor views, stock-specific views), curse of dimensionality (when applied to large portfolios), prior setting (e.g., equilibrium is an abstract concept), optimiser issues, risk model quality, non-linearity and non-normality issues.

We have therefore done some significant work to make the model more practical. As a result, this thesis was initiated by focusing on an explanation of the original model. By exploring the information processing challenges encountered in a typical portfolio management process, we subsequently enriched Black and Litterman (1992)'s original motivation for the model. We then established that the Black-Litterman model is underpinned by three pillars: the Semi-Strong Market Efficiency assumption, the Capital Asset Pricing Model (CAPM), and the Bayes' Rule of belief updating.

Our findings based on sector-level suggests that the value of δ has a rather important part to play in the optimal portfolio weights. Unfortunately, few researchers have touched on this subject. We have not explored alternative utility properties besides the quadratic which is most often used by practitioners. Consequently, it can only be a speculative proposition that the subsequent weakness of the model is related to the value of δ.

From a theoretical perspective, we should expect to see the expansion to k views to come at a cost. As mentioned above, with an increasing number of securities (stocks, factors etc.) we eventually face the curse of dimensionality. Cheung (2009) notes when the elements of the posterior covariance matrix $\Sigma + (\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$ becomes too heavy ($n > 1000$), it poses significant computational challenges to any quadratic optimiser. He proposes to leverage the capacity of the optimiser with eigensystem analysis, in order to keep the positive-definite properties of the posterior covariance matrix.

6.3 Implications for practitioners and future research

The dissemination of the Black-Litterman model rests largely on the productivity of future research. The prospects looks very optimistic. Researchers who commit to the task of exploring the frontiers of the model will influence not only the empirical success both also the practical benefit. For instance, Idzorek (2004) discusses a new method for the confidence limits of the views. Earlier, the individual variances of the error term ω that form the diagonal elements of the covariance matrix of the error term Ω were based on the variances of the view portfolios $P\Sigma P^T$ multiplied by the scalar τ. This is a very stylized production of reality. It is fair to assume that there may be other sources of information in addition to the variance of the view portfolio that affect an investor's confidence in a view. Presumably, additional factors can affect an investor's confidence in a view, such as the historical accuracy or score of a model, market environment, or analyst that produced the view, as well as the difference between the view and the implied market equilibrium. These factors, and perhaps others, should be combined with the variance of the view portfolio $P\Sigma P^T$ to produce the best possible estimates of the confidence levels in the views. Doing so will enable the Black-Litterman model to maximize an investor's information.

Future directions for this research include reproducing the results from the original papers, either Black and Litterman (1991) or Black and Litterman (1992). These results should have the additional complication of including currency returns, partial hedging and transactions costs. Furthermore, the academic community should include more information on process and a synthesized model containing the best elements from the various authors. Meucci (2006) provide further extensions to the Black-Litterman Model for non-normal views and views on parameters other than return. This allows one to apply the Black-Litterman model to new areas such as alternative investments or derivatives pricing. His methods are based on simulation and do not provide a closed form solution.

6.3.1 The Augmented Black-Litterman model

What is still missing from this part of the literature, is a more flexible way of using the Black-Litterman model. In this subsection, we provide a brief examination of a different perspective of this thesis, which we hope will be explored in future research. The Black-Litterman approach has been found very appealing because of its technical tractability and conceptual simplicity. However, relying on one single factor may not capture most of the cross-section in stock returns. Even though the model is conceptually appealing, what is still missing for active style allocation is an applied model that focuses on more than one factor and simultaneously permits the user to form views on different parts of the investable universe. In the equity world, linear factor models based on economic and financial factors have been applied in various regimes. A linear factor model can take the following form:

$$\vec{r}_{[nx1]} = \vec{a}_{[nx1]} + \mathbf{B}_{[nxf]}\vec{r}_{F[fx1]} + \vec{\epsilon}_{[nx1]} \tag{6.1}$$

where $\vec{r}$ is the vector of security returns, $\vec{a}$ is the intercept vector, $\mathbf{B}$ is the matrix of factor loadings, $\vec{r}_F$ represents the vector of factor returns and $\vec{\epsilon}$ stands for the vector of security-specific returns, which are considered independent of factor returns and of each other. As a result, we have:

$$\Sigma_{[nxn]} = \mathbf{B}\Sigma_{F[fxf]}\mathbf{B}^T + \Sigma_{\epsilon[nxn]} \tag{6.2}$$

Where Σ is the covariance matrix of security returns, Σ_F represents the covariance of factor returns and Σ_ϵ is the covariance of the security specific returns. Future contribution to this literature should ideally introduce an optimal allocation model that is viewed though the lens of multiple risk factors. In order to admit views in the Bayesian framework, the universe should be augmented from n securities also to include all the $f(\leqslant n)$ relevant factors. In terms of return, the view universe is represented by:

$$\vec{\zeta}_{[(n+f)x1]} = \left(\vec{r}^{T}_{[nx1]}, \vec{r}^{T}_{F[fx1]}\right)^{T} \tag{6.3}$$

By doing so, views can not only be expressed on security returns as in the Canonical Black-Litterman model $\vec{r}$, but simultaneously on factor returns $\vec{r}_F$. For a full proof of the model, see Cheung (2012) and Walters (2014). In portfolio construction, a common belief is that there are some fundamental factors driving stock returns. These factors should be utilized as information sources in portfolio construction, which in essence should be the main objective of future asset management.

Chapter 7

Conclusion

This thesis has addressed the application of the Black-Litterman model, in multi-factor framework. We have proposed a novel approach to strategic asset allocation, based on another area of finance, where risk premia does a superior job at explaining the cross-section of stock returns. We have examined the empirical accuracy of the Black-Litterman model in a multi-factor context by conducting an analysis on two different and independent samples.

Despite the importance to long-term investors, most strategic asset allocation decisions continue to be made based on techniques developed in the 1950s and 1960s. These tools have a number of shortcomings that limit their value to the investment process. We have addressed these shortcomings in our robust factor-based framework, which can be applied to important aspects of the asset allocation process, allowing investors to make better investment decisions that are more suited to their individual objectives and constraints. Although we believe we have pushed the scientific envelope of strategic asset allocation techniques, the holistic process of asset allocation remains a combination of art and science.

We have found that the majority of the empirical evidence is in favor of an accept of the hypothesis that an application of the Black-Litterman model in a multi-factor framework, in fact is able to produce stable portfolio allocations. This finding is robust across time, sectors and risk premiums, which suggests the results are not a product of selection bias. Additionally, the results are robust across different parameter specifications, like τ and δ. However, when extreme values for the level of risk-aversion is used, the benefit of diversification and stable portfolio weights becomes a decreasing function of δ. Given that we are operating inside an equilibrium where agents posits homogeneous expectations and are rational and risk-averse, it should be fair to see these results.

A central feature of the Black-Litterman framework is the notion that investors should take risk where they have views, and correspondingly, they should take the most risk where they have the strongest views. In the Black-Litterman framework, all expected returns are viewed as a blend of a set of equilibrium returns (reflecting a neutral reference point) and an actual set of investor views (which should differ from the equilibrium returns). Consequently, the problem facing the practitioner is to determine the weight given to the actual views. This is directly linked to our findings regarding the value of τ. Additionally, we saw that this parsimonious scalar has a non-trivial impact on the weights given to the actual weights through Ω^{-1}.

We find that the success of a practical application of the model effectively lingers on distinctively subjective parameters, e.g. the disagreement on how to calculate τ, the case-sensitive value of δ and the investor confidence Ω.

Based on our findings, we propose a modified version of the Canonical Black-Litterman model, which is more suitable for today's investment needs where views are not limited to a pre-specified universe. The basic investment management problem is to simultaneously maximize performance and manage risk: Investors determine risk-controlled allocations to specific asset classes that make best use of the information at their disposal. It would seem that the Black-Litterman model is a step in that direction. We suggest such an approach more in line with today's investment needs, enabling the investment manager to diversify across virtually any factor he/she might favor. This finding requires further empirical investigation.

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Appendix A

Tables and Figures

Table A.1: Summary Statistics, The Impact of tau τ

The table reports statistics from the Black-Litterman formula, when we change the scalar τ. We report the same views as those from **Subsection 5.4.2**. Each Panel reports both the nominator (the uncertainty of the true, unobservable mean), the denominator and the optimal weights. Finally, we report the confidence in our views (Ω^{-1}), which is inversely related to τ. All numbers in percentage terms.

PANEL A: $\tau = 0.01$, $\Omega^{-1} = \begin{bmatrix} 22,807.57 & 0 \\ 0 & 134,209.99 \end{bmatrix}$

w^p_{mkt}	w^p_{opt}	$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$	$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$				
			QMJ	MKT	SMB	HML	UMD
20.000	13.486	68.956	0.0005	-0.0003	-0.0003	0.0000	0.0002
20.000	26.514	297.032	-0.0003	0.0010	0.0001	-0.0001	-0.0003
20.000	20.000	182.994	-0.0003	0.0001	0.0007	0.0000	-0.0002
20.000	61.676	988.254	0.0000	-0.0001	0.0000	0.0004	-0.0001
20.000	20.000	182.994	0.0002	-0.0003	-0.0002	-0.0001	0.0022

PANEL B: $\tau = 0.025$, $\Omega^{-1} = \begin{bmatrix} 9,123.03 & 0 \\ 0 & 53,684 \end{bmatrix}$

w^p_{mkt}	w^p_{opt}	$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$	$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$				
			QMJ	MKT	SMB	HML	UMD
20.000	13.430	27.582	0.0013	-0.0008	-0.0007	0.0000	0.0006
20.000	26.570	118.813	-0.0008	0.0026	0.0002	-0.0002	-0.0007
20.000	20.000	73.198	-0.0007	0.0002	0.0017	-0.0001	-0.0005
20.000	62.030	395.302	0.0000	-0.0002	-0.0001	0.0009	-0.0002
20.000	20.000	73.198	0.0006	-0.0007	-0.0005	-0.0002	0.0054

PANEL C: $\tau = 0.05$ $\Omega^{-1} = \begin{bmatrix} 4,561.51 & 0 \\ 0 & 26,842 \end{bmatrix}$

w^p_{mkt}	w^p_{opt}	$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$	$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$				
			QMJ	MKT	SMB	HML	UMD
20.000	13.338	13.791	0.026	-0.0016	-0.0015	0.0000	0.0012
20.000	26.662	59.406	-0.0016	0.0052	0.0004	-0.0004	-0.0014
20.000	20.000	36.599	-0.0015	0.0004	0.0034	-0.0002	-0.0009
20.000	62.610	197.651	0.0000	-0.0004	-0.0002	0.0018	-0.0004
20.000	20.000	36.599	0.0012	-0.0014	-0.0009	-0.0004	0.0108

PANEL D: $\tau = 0.10$ $\Omega^{-1} = \begin{bmatrix} 2,280.75 & 0 \\ 0 & 13,421 \end{bmatrix}$

w^p_{mkt}	w^p_{opt}	$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$	$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$				
			QMJ	MKT	SMB	HML	UMD
20.000	13.163	6.896	0.0051	-0.0031	-0.0029	0.0000	0.025
20.000	26.837	29.703	-0.0031	0.0104	0.0008	-0.0008	-0.0027
20.000	20.000	18.299	-0.0029	0.0008	0.0069	-0.0005	-0.0019
20.000	63.725	98.825	0.0000	-0.0008	-0.0005	0.0037	-0.0008
20.000	20.000	18.299	0.0025	-0.0027	-0.0019	-0.0008	0.0216

PANEL E: $\tau = 1.00$ $\Omega^{-1} = \begin{bmatrix} 228.08 & 0 \\ 0 & 1,342.1 \end{bmatrix}$

w^p_{mkt}	w^p_{opt}	$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$	$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$				
			QMJ	MKT	SMB	HML	UMD
20.000	11.155	0.6896	0.0515	-0.0311	-0.0294	-0.0003	0.0248
20.000	28.845	2.9703	-0.0311	0.1039	0.0079	-0.0081	-0.0271
20.000	20.000	1.8299	-0.0294	0.0079	0.0687	-0.0045	-0.0186
20.000	77.289	9.8825	-0.0003	-0.0081	-0.0045	0.0370	-0.0084
20.000	20.000	1.8299	0.0248	-0.0271	-0.0186	-0.0084	0.2155

PANEL A: $\tau = 0.01$

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.0044\% & 0 \\ 0 & 0.0007\% \end{bmatrix}$$

PANEL B: $\tau = 0.025$

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.0110\% & 0 \\ 0 & 0.0019\% \end{bmatrix}$$

PANEL C: $\tau = 0.05$

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.0219\% & 0 \\ 0 & 0.0037\% \end{bmatrix}$$

PANEL D: $\tau = 0.10$

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.0438\% & 0 \\ 0 & 0.0075\% \end{bmatrix}$$

PANEL E: $\tau = 1.00$

$$P = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.5\% \\ 0.6\% \end{bmatrix}; \Omega = \begin{bmatrix} 0.439\% & 0 \\ 0 & 0.0745\% \end{bmatrix}$$

Table A.3: Summary Statistics, The Impact of delta δ

The table reports the summary statistics from views based on Satchell and Scowcroft (2000), with different values of δ. Panel A reports the equilibrium return vector. Panel B the posterior return vector and finally, Panel C reports optimal weights. We use the same views, market weights and covariance matrix as stated in **Subsection 5.4.3**. The scalar τ is held constant at 0.05.

PANEL A: THE EQUILIBRIUM RETURN Π

	Equilibrium Return Vector Π					
δ	0.01	0.025	0.5	5	11	20
Energy	0.004%	0.011%	0.216%	2.156%	4.744%	8.625%
Materials	0.004%	0.011%	0.217%	2.173%	4.780%	8.691%
Industrials	0.004%	0.010%	0.199%	1.989%	4.376%	7.956%
Consumer Discretionary	0.004%	0.010%	0.196%	1.959%	4.309%	7.834%
Consumer Staples	0.002%	0.006%	0.111%	1.105%	2.432%	4.422%
Health Care	0.003%	0.007%	0.138%	1.378%	3.031%	5.511%
Financials	0.005%	0.014%	0.271%	2.714%	5.971%	10.856%
Information Technology	0.004%	0.009%	0.185%	1.854%	4.078%	7.414%
Telecom. Services	0.003%	0.009%	0.171%	1.711%	3.765%	6.846%
Utilities	0.003%	0.006%	0.126%	1.260%	2.773%	5.041%
Real Estate	0.005%	0.013%	0.267%	2.667%	5.868%	10.669%

PANEL B: POSTERIOR RETURNS $E[R]$

	Posterior Return Vector $E[R]$					
δ	0.01	0.025	0.5	5	11	20
Energy	0.165%	0.171%	0.354%	2.088%	4.399%	7.866%
Materials	0.171%	0.177%	0.371%	2.213%	4.669%	8.353%
Industrials	0.174%	0.180%	0.363%	2.094%	4.403%	7.866%
Consumer Discretionary	0.063%	0.069%	0.253%	1.996%	4.320%	7.806%
Consumer Staples	0.010%	0.014%	0.117%	1.092%	2.393%	4.345%
Health Care	-0.024%	-0.020%	0.109%	1.326%	2.950%	5.386%
Financials	-0.095%	-0.087%	0.170%	2.605%	5.851%	10.721%
Information Technology	0.076%	0.082%	0.253%	1.882%	4.053%	7.309%
Telecom. Services	-0.040%	-0.035%	0.125%	1.640%	3.660%	6.690%
Utilities	0.022%	0.026%	0.141%	1.229%	2.681%	4.858%
Real Estate	-0.040%	-0.032%	0.222%	2.621%	5.819%	10.617%

PANEL C: OPTIMAL WEIGHTS $w^p_{opt} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$

	Optimal Weights $w^p_{opt} = [\delta(1+\tau)\Sigma_p]^{-1}E[R]$					
δ	0.01	0.025	0.5	5	11	20
Energy	900.88%	356.39%	11.54%	-4.80%	-5.79%	-6.16%
Materials	2.79%	2.79%	2.79%	2.79%	2.79%	2.79%
Industrials	26315.87%	10529.99%	532.26%	58.69%	29.98%	19.22%
Consumer Discretionary	-210.60%	-74.60%	11.54%	15.62%	15.86%	15.96%
Consumer Staples	9.12%	9.12%	9.12%	9.12%	9.12%	9.12%
Health Care	-8905.2%	-3552.4%	-162.30%	-1.72%	8.02%	11.66%
Financials	-8906.4%	-3553.6%	-163.46%	-2.87%	6.86%	10.51%
Information Technology	-53.37%	-8.19%	20.41%	21.77%	21.85%	21.88%
Telecom. Services	-8914.2%	-3561.4%	-171.30%	-10.71%	-0.98%	2.67%
Utilities	-146.58%	-55.75%	1.77%	4.50%	4.66%	4.73%
Real Estate	2.85%	2.85%	2.85%	2.85%	2.85%	2.85%
Total	95.24%	95.24%	95.24%	95.24%	95.24%	95.24%

Table A.2: Covariance Matrix of returns on Sectors

The table reports the covariance matrix of 11 GICS sectors. This is used in conjunction with the data in **Table 5.6** to show the results of Satchcel and Scowcroft methodology with relative views only.

PANEL A: SECTORS

	10	15	20	25	30	35	40	45	50	55	60
10 Energy	0.7246%	0.5829%	0.4653%	0.4145%	0.2475%	0.3078%	0.5524%	0.4157%	0.3870%	0.3279%	0.5280%
15 Materials	0.5829%	0.6141%	0.4830%	0.4423%	0.2425%	0.3034%	0.5870%	0.4275%	0.3898%	0.2949%	0.5838%
20 Industrials	0.4653%	0.4830%	0.4596%	0.4177%	0.2276%	0.2805%	0.5615%	0.3889%	0.3554%	0.2606%	0.5535%
25Consumer Discretionary	0.4145%	0.4423%	0.4177%	0.4473%	0.2298%	0.2794%	0.5573%	0.3846%	0.3539%	0.2477%	0.5605%
30 Consumer Staples	0.2475%	0.2425%	0.2276%	0.2298%	0.1745%	0.1784%	0.2901%	0.2089%	0.2080%	0.1758%	0.2887%
35 Health Care	0.3078%	0.3034%	0.2805%	0.2794%	0.1784%	0.2663%	0.3592%	0.2641%	0.2493%	0.1968%	0.3402%
40 Financials	0.5524%	0.5870%	0.5615%	0.5573%	0.2901%	0.3592%	0.9575%	0.4962%	0.4812%	0.3229%	0.8959%
45 Information Technology	0.4157%	0.4275%	0.3889%	0.3846%	0.2089%	0.2641%	0.4962%	0.4120%	0.3319%	0.2326%	0.4867%
50 Telecom. Services	0.3870%	0.3898%	0.3554%	0.3539%	0.2080%	0.2493%	0.4812%	0.3319%	0.3878%	0.2468%	0.4749%
55 Utilities	0.3279%	0.2949%	0.2606%	0.2477%	0.1758%	0.1968%	0.3229%	0.2326%	0.2468%	0.2915%	0.3394%
60 Real Estate	0.5280%	0.5838%	0.5535%	0.5605%	0.2887%	0.3402%	0.8959%	0.4867%	0.4749%	0.3394%	1.0699%

Figure A.1: Historical Factor Premiums

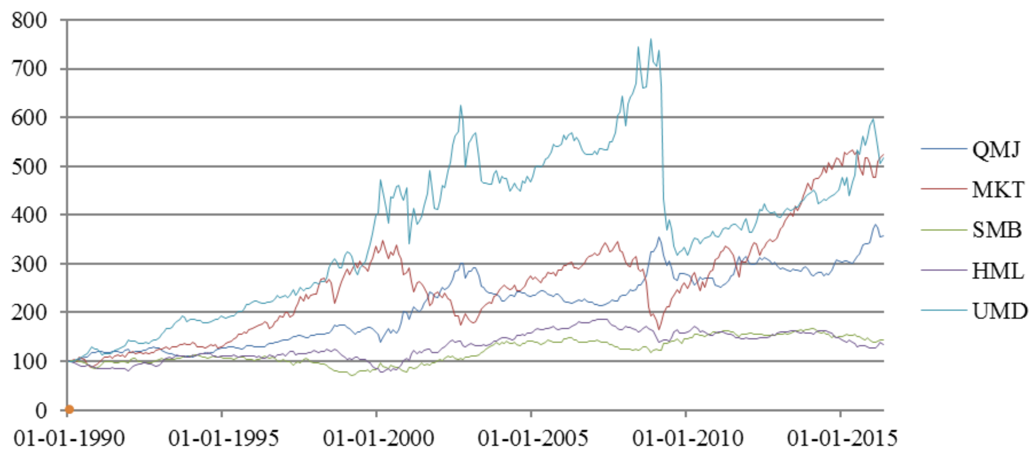

Figure A.2: Historical Sector Returns