Copenhagen Business School

Master's Thesis

Trading Strategies and their Impact on the Wealth of Investors

From an Asset Pricing Perspective

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A thesis submitted in fulfilment of the requirements for the degree of Master of Science in Finance and Investments

January 16, 2017

Abstract

In this thesis a model will be developed to explain how a constrained economy works. The predictions of the model is that in a constrained economy: Securities with high beta values will earn a lower alpha than securities with low beta values. And this insight would lead to the next prediction that an investor would a positive expected return in a market-neutral self-financed portfolio that bets against the beta by shorting a portfolio of large beta securities to finance a portfolio of small beta securities.

The empirical test showed that the predictions were correct. Both Sharpe ratio and alpha had an inverse relationship with the beta of a portfolio. The constructed BAB factor portfolios earned both positive returns and abnormal returns.

The rational investor would have invested all her wealth plus her possible margin in the BAB78 portfolio and earned an monthly alpha of 1.78%. Her construction of this BAB78 portfolio would be to short a delevered beta-weighted portfolio of the 78 securities with the largest betas to finance a levered beta-weighted portfolio of the 78 securities with the smallest betas.

Contents

1 Introduction

Capital markets as they are understood in modern finance will need a pricing of risk. As long as risk is poorly priced as in the CAPM framework because of the lack the *true* market portfolio, which can never be found. It will be possible for portfolio managers to earn an abnormal return if they construct a portfolio that captures an anomaly that's not already covered by one of the existing models to price risk. The idea in this thesis is to investigate the profitability of the trading strategy called "Betting against beta" introduced by Frazzini and Pedersen.

As an investor, one is interested in earning the *highest* Sharpe ratio, which is excess return per unit of risk, which can then be leveraged or de-leveraged to fit the investor's appetite for risk, which is measured her risk aversion. if it is assumed that it is impossible earn a positive alpha, But this assumption is jeopardized by the fact it can only assumed that all positive alpha trading is gone if both the market is efficient and risk is pricing correctly by a model widely used by all investors.

We can only assume by any certainty that the market is efficient because of the fact that highly competitive market will be a market where prices are set by the market.

It is interesting for all real world investors, if it is the case that there is a systematic anomaly, which can generate an abnormal return, i.e. an alpha, in most common asset pricing model, like the CAPM and the Fama French three factor model.

In thesis a theoretical model will be used to predict what the restrictions on their use of leverage would do to the securities in the equilibrium. Furthermore what implications this would have on the investment strategies that different more or less constrained investors would utilise.

The predictions of the model would then be tested on empirical data from the american equity market.

2 Problem statement and delimitation

The introduction results in the following problem statement.

2.1 Problem statement

> How can you utilise a Betting against Beta (BAB) trading strategy to get a positive abnormal return on your investment?

2.2 Research Questions

1. What implications does the leverage constraints of some agents have on the economy?

2. What is the theoretical prediction of the model for BAB factor portfolio?

3. Does empirical analysis fit the theoretical predictions?

2.3 Delimitation

In the empirical study all securities and assets which are *not* equities will be excluded. Furthermore all equities, which are not american NYSE traded equities, are also excluded.

Furthermore all equities which are not american will be excluded as well. Reasoning behind this is that in the alpha analysis for each portfolio the data library of Kenneth French will be used instead of the AQR data library. The idea is to be able to replicate the results in [4] by Frazzini and Pedersen, so it seems to be a better idea to use original data than their date. There are also discrepancies in the two data-sets, which are for the author of this thesis unexplained.

3 Structure of the thesis

4 Literature review

This literature review is supposed to take the reader through the subject of modern portfolio theory from the very beginning of the subject to the point of this thesis.

4.1 Markowitz's Portfolio Selection

The natural place to start is in Harry Markowitz's Portfolio Selection [7]. Markowitz states that there are two stages to portfolio selection. A portfolio manager would have to use experience and observation to form beliefs about the future and performance of available assets. As in Markowitz's paper the thesis will not focus on the first stage but it will instead focus on the second stage, which is the portfolio choice. As defined in Markowitz's paper will apply the rule for investors that they see expected return as a desirable thing and on the other hand see the variance of an asset's return as an undesirable thing. Markowitz also presents the idea about diversification in the portfolio choice as a superior form of investment behaviour.

Even more important Markowitz introduces the expected returns-variance of returns (E-V) rule, which says that an investor will try to maximize expected return while minimizing the variance of the portfolio. Already in this paper it is argued that you cannot both have the highest expected return and the lowest variance of returns at the same. So the investor will have to make a trade-off, where she either lowers her expected return or accepts a higher level of variance in her portfolio, which she will then be compensated for with a higher expected return.

Markowitz introduces the elementary mathematical statistics two concepts for a random variable, Y: E for expected value of the random variable, and V for the variance of the random variable.

The expected value of the random variable:

$$E = p_1 y_1 + p_2 y_2 + \ldots + p_N y_N$$

, where E is the expected value of the random variable, p_1 is the probability of the first value occurring, and y_1 is the value of random variable if state occurs.

The variance of the random variable Y was defined to be.
The variance of the random variable:

$$V = p_1(y_1 E)^2 + p_2(y_2 - E)^2 + \ldots + p_N(y_N - E)^2$$

, where V is the variance, p_N is the probability of the N state, y_N is value of the random variable in the N state, and E is the expected value of the random variable.

Then Markowitz went onto define the variance of a portfolio, which *must* be expressed as a sum of terms of the covariances of all the returns in the portfolio. Therefore he showed the covariance between to assets' returns:
The covariance between the return of asset 1 **and return of the asset** 2:

$$\sigma_{12} = E([R_1 - E(R_1)][R_2 - E(R_2)])$$

, where $sigma_{12}$ is the covariance between the two assets, R_1 is the return of asset 1, and $E(R_1)$ is the expected return of asset 1.

Thereafter Markowitz introduced the connection between covariance and correlation.
The covariance as a function of the correlation:

$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$$

, where σ_{ij} is covariance between the return for asset i and the return for asset j, ρ_{ij} is the correlation between the return for asset i and the return for asset j, and σ_i is the standard deviation of the return for asset i.

Markowitz then continues to exclude short sales from his analysis. A short sale is where you borrow an asset to sale it then to later on repurchase the asset and deliver it back, if the asset has fallen in value between the sale and the repurchase the short seller will have made a profit.

After those preliminary steps Markowitz continues explain the E-V-rule graphically. Fig. 1 in [7] shows a circle in a (E,V)-plane, where the south eastern part is highlighted because that's where the E-V-rule displays the efficient E,V combinations that investors, following the rule, would invest in.

Markowitz then comments that *two* conditions, at least, must be satisfied before the mentioned above efficient surface would be used in practice. First investors must desire to act according to the E-V-rule described above. Second it must be possible to arrive to reasonable μ_i and σ_{ij}. The second condition is the one, which this thesis is trying to address. Much later than

Markowitz academia has countless times tried to value assets in terms of risk-adjusted returns.

Finally Markowitz used a three assets' example to show relationship between variance, expected return, and efficient portfolios in fig. 2 in [7]. In the middle of the isovariance curves the point **x** is the portfolio with *lowest* variance. From that point the isovariance curves extends out, from point x there is drawn a line through the isovariance curves where the isomean line tangent the isovariance curve. The line represents the efficient portfolio, the line continues from point d to b. On the axis the figure had the weights in asset X_1 and asset X_2.

Markowitz also points out the importance of a clever view on diversification. He points out the importance of lower covariance between the portfolio and the assets used to diversify risk. He also points out that not all parts of the variance is diversifiable.

4.2 Tobin's Liquidity Preference as Behaviour Towards Risk

A tiny but important addition was made by Tobin in [9]. Tobin wrote a paper mainly about the liquidity preference but in that paper he describe what would later become Tobin's separation theorem. See figure 3.6 in [9] the E ray, which shows the expected return for an investor. As an investor wants to move along the E ray from the origin, she will at some point have exhausted her entire investment balance by investing the non-cash assets. It is here that Tobin's separation theorem tells the investor that she will hold the same proportionate composition of the non-cash assets at different points on the E ray. The only difference between the different points on the E ray is the amount of investment balance which is held as cash assets. Tobin comments, which has also been assumed since then, that this analysis is applicable as long as it is assumed that cash assets are risk-free.

4.3 Sharpe's Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk

Sharpe in [8] continues the development of modern portfolio theory. He states that the investor has to compensated for two different elements in investing: Time and risk. The price of time is the risk-free rate, which also why it is used to discount risk-free projects in corporate finance. The investor will also have to be compensated for the amount of risk which she will be willing to bear. Sharpe builds on the idea of Markowitz when he presents us with the investment opportunity curve, which is that part of investment plane where the rational investor *must* lie. The investment opportunity plane is now called the efficient frontier. The efficient frontier consists of all the efficient portfolios in the economy as long as there is no risk-free asset.

Sharpe also shows the importance of correlation between assets in diversification in figure 3 in [8]. Lets assume as Sharpe did that the investor can either invest her wealth in plan A or in plan B or a combination of the two plans. As Markowitz already pointed out, the rational investor will be diversifying his investment into more rather than less assets. This will require that the investor invests in both plans, this will be produce the expected return of plan C, E_{Rc}, which is a combination of plan A and plan B.

The expected return of the investment plan *C*:

$$E_{Rc} = \alpha E_{Ra} + (1 - \alpha) E_{Rb}$$

, where E_{Rc} is the expected return of the plan C, α is the fraction of the investor's wealth in plan A, and $(1 - \alpha)$ is the fraction of the investor's wealth in plan B.

For every expected return except the risk-free return there is a variance and hence a standard deviation. The standard deviation of combined investment plan C is given by the standard deviation of the two plans and their correlation.

The standard deviation of the investment plan *C*:

$$\sigma_{Rc} = \sqrt{\alpha^2 \sigma_{Ra}^2 + (1-\alpha)^2 \sigma_{Rb}^2 + 2 r_{ab} \alpha (1-\alpha) \sigma_{Ra} \sigma Rb}$$

, where σ_{Rc} is the standard deviation of the investment plan C, everything in the square root is the variance of the investment plan C, and r_{ab} is the correlation coefficient between plan A and plan B. The correlation coefficient is always in the range between -1 and 1, Sharpe notes that is the usual case that's the correlation coefficient between two assets are in the range between 0 to 1.

In figure 3 in [8] shows a correlation lower than perfect positive correlation (+1) will give a diversification effect. He remarks that an even lower, or a negative, correlation will increase the U-shape of the locus and thereby give the investor a stronger diversification effect, which results in a lower standard deviation for a given combination of two assets, while not hurting the expected return of the combination of the two.

Sharpe then continues to introduce a pure rate interest, P, which will be called the risk-free rate from now on. Since the risk-free rate is risk-free, it has a σ of zero. This has no implications on the formula for the expected return of a combination of the risk-free asset and a portfolio of risky assets, but it has an implication on the standard deviation of the combined portfolio consisting of the risk-free asset, P, and the portfolio of risky assets, A.

Standard deviation of a portfolio of the risk-free asset and a risky portfolio:

$$\sigma_{Rc} = (1 - \alpha)\sigma_{Ra}$$

, where σ_{Rc} is the standard deviation of the combined portfolio, α is the fraction of wealth invested in the risk-free asset, P, and σ_{Ra} is standard deviation of the risky portfolio.

Sharpe then shows that efficient frontier has become a straight line, PZ, which he calls the *Capital Market Line* in figure 4 in [8]. As long as both lending and borrowing is possible, the investor can achieve all combinations on the PZ line. It is important to point out that all of those combinations are dominant to all other combination in terms of either risk, return or rarely also both.

To find the PZ-line one must first optimum combination of risky assets, which is assumed to be unique. This unique combination is marked at point ϕ in figure 4 in [8]. It can be seen that PZ-line is the tangent at the point ϕ to investment opportunity curve. Therefore this unique combination of risky assets has been called the tangency portfolio ever since.

Sharpe now invokes a few assumptions, so he can derive an equilibrium in the capital market:

1. **A common pure interest rate**: All investors can lend and borrow at equal terms.

2. **Homogeneous investor expectations**: All investors have the same expectations of expected return, variance, and correlation for available securities.

Sharpe points out that those assumptions are unrealistic and restrictive, but he then comments in [8]:

> However, since the proper test of a theory is not the realism of its assumptions but the acceptability of its implications.

Under those assumptions Sharpe derive an equilibrium where *all* assets have to be a part of the tangency portfolio. Because all investors will attempt to purchase the assets in the combination ϕ, therefore all prices of the assets will have to be revised. Assets which are a part of the combination ϕ will see an increase in price, because of the inverse relation between prices and expected returns the expected returns of those assets will fall. While assets which are not in the desired combination will see an decrease in price hence the expected return of those assets will raise, until they are again desired and will therefore be a part of the desired combination.

When the prices are in equilibrium, all investors can again choose a portfolio on the PZ-line which fit their risk aversion. So investors with a high degree of risk aversion will have a fraction of their wealth invested in risk-free asset and the rest of their wealth invested in the unique tangency portfolio. On the other hand investors with a much lower degree of risk aversion will borrow at pure interest rate to invest a greater amount of money than their wealth in the unique tangency.

With the assumptions and behaviour of investors Sharpe concluded that this was a key to the relationship between systematic risk and the prices of capital assets. PZ-line (later the CML) showed the relationship between the systematic risk of asset and its expected return. The difference between the asset's total risk and the systematic risk measured by the PZ-line was the idiosyncratic risk of that asset, which can easily be diversified away by holding a portfolio instead of this single asset alone.

Sharpe concludes that an asset's expected return will have linear relationship with the responsiveness of unique portfolio, which has to a synonym for the entire economy, because all assets will be a part of the unique portfolio. The risk from an asset which has no correlation with the swings in economic activity can be avoided, while the systematic risk of asset that is the risk which is correlated to the economic activity can not be avoided, unless the investor only invest in the risk-free asset.

In this literature review William Sharpe's paper was chosen over John Lintner's paper, because of its narrower scope and simpler explanations. Sharpe's paper is superior to Lintner's by reasoning gained from the words of Albert Einstein:

If you can't explain it simply, you don't understand it well enough.

4.4 Black's Capital Market Equilibrium with Restricted Borrowing

Next up is [1]. The paper focuses on the effect on equilibrium and therefore the prices in equilibrium when there are restrictions on borrowing.

Before modifying the Capital Asset Pricing Model ($CAPM$) with the changed assumption. Black states how the discount rate for an asset in equilibrium is found:
Capital Asset Pricing Model formula:

$$E(\tilde{R}_i) = R_f + \beta_i[E(\tilde{R}_m) - R_f]$$

, where $E(\tilde{R}_i)$ is the expected rate of return for asset i, which will be used to discount the further cash flows from the asset, R_f is the risk-free rate of return, β_i is the market sensitivity of the asset, $E(\tilde{R}_m) - R_f$ is the risk premium of the market portfolio, and $\beta_i[E(\tilde{R}_m) - R_f]$ is the risk premium of the asset. Therefore the more sensitivity the asset has to the market, the higher its risk premium has to be, so it compensates the investors for the risk they are bearing. The market sensitivity for asset i is defined as β_i:
The market sensitivity of asset i:

$$\beta_i = \frac{cov(\tilde{R}_i, \tilde{R}_m)}{var(\tilde{R}_m)}$$

, where $cov(\tilde{R}_i, \tilde{R}_m)$ is the covariance between asset i and the market portfolio, and $var(\tilde{R}_m)$ is the variance of the market portfolio.

This equilibrium condition needs a risk-free asset, i.e. a bond issued by an entity which has no risk of default. Black continues to develop an equilibrium *without* a risk-free asset. He derives that the expected return of an asset is *still* a linear function of its beta:
The expected rate of return in an only risky assets' world:

$$E(\tilde{R}_i) = E(\tilde{R}_z) + \beta_i[E(\tilde{R}_m) - E(\tilde{R}_z)]$$

, where $E(\tilde{R}_z)$ is the expected rate of return for the zero-beta minimum-variance portfolio. The idea behind this is also the idea that it is used to draw the efficient frontier without a risk-free asset in a common mean-variance analysis, where we know that the efficient frontier of risky assets consists of portfolios of a long position in the tangency portfolio and a short position in the minimum-variance portfolio, z.

If it then assumed that the tangency portfolio is the choice of all rational investors, hence it is the tangency portfolio is the market portfolio. It can be seen that in a world with only risky assets, all portfolios are a weighted combination of the minimum-variance portfolio, z, and the market portfolio, m.

Black then continues to derive the equilibrium with a risk-free asset, but where borrowing in this asset is prohibited. All other positions in risky assets are still allowed. Now the restricted set of efficient portfolios has two parts, where one part is a portfolio called t, which consists a combination of z, m and the risk-free asset, and the second part is a combination of the market portfolio, m, and the minimum-variance portfolio, z. Black then shows us some important properties of the portfolios, z and m, while writing the weights in the different portfolios and assets like this: The weight in the market portfolio as w_{km}, the weight in the minimum-variance portfolio as w_{kz}, and the weight in the risk-free asset as w_{kf}. Assuming that the market portfolio and the minimum-variance portfolio are independent, and therefore has a correlation of zero. The expected rate of return and variance for the efficient k portfolio is as following:

The expected rate of return for the k portfolio:

$$E(\tilde{R}_k) = w_{km}E(\tilde{R}_m) + w_{kz}E(\tilde{R}_z) + w_{kf}R_f$$

, where $E(\tilde{R}_k)$ is the expected return for k portfolio. That can be said to be a weighted average of the two portfolios and the risk-free asset.

The variance of the return for the k portfolio:

$$var(\tilde{R}_k) = w_{km}^2 var(\tilde{R}_m) + w_{kz}^2 var(\tilde{R}_z)$$

, $var(\tilde{R}_k)$ is the variance of the k portfolio seen as the squared weighted average of the variance of the market portfolio and the minimum-variance portfolio. The last part of the variance formula disappears due to the correlation between the minimum-variance - and the market portfolio being zero.

The weights of the different portfolios and the risk-free assets have to satisfy the following conditions:

$$w_{km} + w_{kz} + w_{kf} = 1$$

The sum of the weights have to sum to one, which means the efficient portfolio k of the investor choosing will exhaust her entire investment account.

$$w_{kf} \geq 0$$

The last condition shows that the investor cannot borrow in the risk-free asset.

Even though the minimum-variance portfolio has a beta of zero, it still bear variance. Therefore the following condition also have to be satisfied to avoid arbitrage:

$$R_f < E(\tilde{R}_z) < E(\tilde{R}_m)$$

If this condition is broken, the k portfolio is not efficient due to the fact that risk-free asset carries less risk than the minimum-variance portfolio, which means that the risk-free asset has to earn a *lower* expected rate of return than the minimum-variance portfolio.

A k portfolio consisting of the risk-free asset and the t portfolio must maximize Sharpe ratio, which is the slope of the *Capital Market Line*. The Sharpe ratio for the k portfolio is the risk premium of the k divided by the volatility of the k portfolio:

$$\text{Sharpe ratio}_k = \frac{E(\tilde{R}_k - R_f)}{\sigma(\tilde{R}_k - R_f)}$$

, where $E(\tilde{R}_k - R_f)$ is the risk premium of the k^{th} portfolio, and $\sigma(\tilde{R}_k - R_f)$ is the same as $\sigma(\tilde{R}_k)$, which is the volatility of the k^{th} portfolio.

When the investor is restricted in her borrowing, this will result in *two* efficient portfolios instead of *one*. In the range from the risk-free asset to market portfolio, the least risky efficient portfolio will be dominant. It will consist of a mixture of the risky portfolio, t[1], and the risk-free asset. The second and most risky efficient portfolio consist, as in a world with no risk-free asset, of a the minimum-variance portfolio and the market portfolio[2]. If the expected return of the minimum-variance portfolio is larger than the risk-free rate, there will still be a linear relationship between a security's expected return and its beta.

[1] t = tangency portfolio

[2] The market portfolio is the equal to the tangency portfolio according to the CAPM assumptions.

The linear relationship between the expected return and the beta will be, when borrowing is restricted, a line with a breaking point. Before the breaking point the slope of the line will be steeper, while after the breaking point the slope will be more flat.

This is important for this thesis, because in a world where different investors have different borrowing restrictions. The investors without restrictions can use leverage to a portfolio with a higher Sharpe ratio than the investors whom faces borrowing restrictions.

Up to this point the focus has been on the development of mean-variance analysis and the Capital Asset Pricing Model. An asset's expected return in the CAPM framework explained its beta, which is the sensitivity of the asset to the market portfolio. In the CAPM framework the required return of an asset, r, has to be equal to its expected return, $E(R)$. If they are equal to each other, the abnormal return, α, is zero. In other words a security is only priced correctly it its alpha is zero.

Alpha for asset $\boldsymbol{i}$:

$$\alpha_i = E(r_i) - r_i$$

In case the expected return is higher than the required return, the asset is *undervalued* by the market, hence it will give the investor a higher return than its beta would suggest. The market will when it realizes that there is a positive alpha, and it will hurry to buy the asset and because of the constant supply, the price of the asset will raise until it is again priced correctly, which results in an alpha of zero.

If an asset is overvalued by the market, it will yield a negative alpha. The market will then sell the asset, until its price is again in equilibrium, i.e. the alpha of the asset is zero.

All assets, which are priced correctly, and the market portfolio will lie on the Security Market Line (SML) in the (β,$E(R)$)-plane. A positive alpha will lie *above* the SML, while a negative alpha will lie *below* the SML. This leads to the research in anomalies which cannot be explained by the CAPM.

In the two next paragraphs the literature review will focus on the two of the most important models to explain the return anomalies which can be seen when the CAPM is used to explain returns for assets.

4.5 Multifactor Explanations of Asset Pricing Anomalies

First Fama and French's work in [3] shows how to explain the anomalies in returns by adding more explaining factors to the single factor model of CAPM. The idea is that a model with more factors can capture the some of the average-return anomalies of the CAPM. Fama and French showed that many of those anomalies could be captured if there was added *two* factors to the model:

1. **The Small Minus Big (SMB) portfolio**. This portfolio construction is based on the market capitalization of all stocks. In the creation of the SMB portfolio the investor would have to rank all stocks according to their market capitalization. Then two portfolios would be formed: One with all the smaller stocks and one with larger stocks, now the investor would short the portfolio with larger stocks to finance a long position in the stocks with smaller stocks. This concept of financing one portfolio with another short position is called self-financed portfolios. The sum of the portfolio weights in a self-financed portfolio is *zero* and not one as the normal portfolios. This is also the reason why the return of the factors are expressed in expected *return* and not in *expected excess* return.

2. **The High Minus Low (HML) portfolio**. This portfolio construction is based on the book-to-market ratio of all stocks. In the creation of the HML portfolio the investor would have to rank all stocks according to their book-to-market ratios. Then two portfolios would be formed: One consisting of stocks with the higher book-to-market ratios called H, and one consisting of stocks with the lower book-to-market ratios called L. The L portfolio would then be shorted to finance the H portfolio, and like the SMB construction this portfolio construction would also be a self-financed portfolio. And therefore its return in the model is a *return* and not an *excess return*[3].

Now when the factors are explained, the description of the model can be continued. The Fama French multifactor model explains the excess return of a portfolio i as following.

[3]Because a portfolio with a total weight of zero does *not* earn the risk-free rate.

Expected excess return of portfolio i:

$$E(R_i) - R_f = b_i[E(R_M) - R_f + s_i E(\text{SMB}) + h_i E(\text{HML})]$$

, where $E(R_i)-R_f$ is the excess return of portfolio i, b_i is the factor sensitivity of portfolio i to the market, $E(R_M)-R-f$ is the risk premium of the market, s_i is the factor sensitivity of the portfolio i to the *SMB* portfolio, $E(SMB)$ is the expected return of the SMB portfolio, h_i is the factor sensitivity of the portfolio i to the *HML* portfolio, and $E(HML)$ is the expected return of the HML portfolio.

The Fama French model, like the CAPM model, be used to derive the alphas of the different portfolios in this thesis. While in the derivation of the alphas in the CAPM is done by a simple linear time series regression of the excess return of the tested portfolio on the excess return of the market portfolio. The derivation of the alpha in the Fama French three factor model is done by multiple time series linear regression where the excess return of the tested portfolio is regressed on the excess return of the market portfolio, the expected return of the SMB portfolio, and the expected return of the HML portfolio. In algebraically terms the regression is presented like this in [3]:

$$R_i - R_f = \alpha_i + b_i(R_m - R_f) + s_i\text{SMB} + h_i\text{HML} + \varepsilon_i$$

From the time series regression the alpha can be derived:

$$\alpha_i = R_i - R_f - b_i(R_m - R_f) - s_i\text{SMB} - h_i\text{HML} - \varepsilon_i$$

, where α_i is the abnormal return of the i^{th} portfolio in the Fama French three factor model.

4.6 Carhart's On Persistence in Mutual Fund Performance

In [2] Carhart explains mutual fund returns by different models. He introduces a *four* factor model, which is built upon the Fama French three factor model that was presented in the paragraph above. To the three factor model Carhart adds the additional factor created by Jegadeesh and Titman in [5]. This factor explains the return gained from a time series momentum strategy, where the investor observes the market for a period of time after the observation period she then creates a portfolio consisting of the winners, which are the stocks with *highest* returns, and an another portfolio consisting of the losers, which are the stocks with *lowest* returns.

Carhart explains that the four risk factors can be interpreted as four different investment strategies. This point is important for this thesis, because in the development of asset pricing models one will often find that the new explanatory variables are often possible to trade through an investment strategy. Back to Carhart's interpretations of the four factors seen as trading strategies:

1. **The CAPM part**: It can be seen as a trading strategy where you trade high beta - versus low beta stocks.

2. **The SMB part**: It can be seen as a trading strategy where you trade small stocks versus large stocks measured in market capitalization.

3. **The HML part**: It can be seen as a trading strategy where you trade value stocks versus growth stocks.

4. **The Momentum part**: It can be seen as a trading strategy where you trade past winners versus past losers.

In the use of four factor model Carhart uses following notation for the excess return for i^{th} portfolio at time t.
Carhart's four factor model:

$$r_{it} = \alpha_{iT} + b_{iT}\mathrm{RMRF}_t + s_{iT}\mathrm{SMB}_t + h_{iT}\mathrm{HML}_t + p_{iT}\mathrm{PR1YR}_t + e_{it}$$

$\Rightarrow$

$$\alpha_{iT} = r_{it} - b_{iT}\mathrm{RMRF}_t - s_{iT}\mathrm{SMB}_t - h_{iT}\mathrm{HML}_t - p_{iT}\mathrm{PR1YR}_t - \varepsilon_{it}$$

The alpha of four factor will be excess return of the portfolio i subtracted the sum of the product of returns[4] and their factor loadings for the i^{th} portfolio, and the residual, ε_{it}.

The alpha for each of the models will be used later in the thesis to describe the performance of the constructed portfolios. The purpose of this literature review has accomplished.

[4]Excess return for the market factor and the return for each of the three other factors.

5 Theory and quantitative analysis

5.1 Model of the economy in the thesis

In the thesis the considered economy will be a overlapping-generations one, where all agents live for *two* periods. In the first period of the agent's life she is called "young", while in her second period of the agent's life she will be called "old".

Every time period, t, there will be born a constant number of agents, I. All agents from $i = 1$ to $i = I$ is born with an amount of wealth, W_t^i. In the economy the agents trade securities to maximize their utility over their lifespan. The amount of traded securities is S, all of the securities pay a dividend, δ_t^s, and each security has x^{*s} shares outstanding.

In every time period the agents which are young will invest in a portfolio consisting of x^i shares in the i securities and the remainder of the wealth will be held in the risk-free asset to the risk-free return. Their portfolio choice will be done, so it maximizes the "young" agent's utility.

Utility of the "young" agent:

$$U_{\text{young}} = x'(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t) - \frac{\gamma^i}{2} x' \underline{\underline{\Sigma_t}} x \tag{1}$$

, where x' is the transposed column vector with the shares in each of the securities, P_t is the price column vector at time t, γ^i is a scalar consisting the risk aversion of the i^{th} agent, and $\underline{\underline{\Sigma_t}}$ is the variance-covariance matrix of $P_{t+1} + \delta_{t+1}$ at time t. Each "young" agent is subject to this given portfolio constraint:

$$m_t^i \sum_s x^s P_t^s \leq W_t^i \tag{2}$$

, where m_t^i is a scalar that controls the investor's ability or restriction to use leverage, e.g. if the scalar is one, the investor will not be able to leverage, therefore sum of the product of the amount of each share, x^s, and the price for each share, P^s must be equal to or less than the wealth of the investor at time t.

In the model the restriction can be even more harsh than that of no leverage, in the case of m_t^i is larger than 1 it means that the investor is required to hold some of her wealth in cash.

In the opposite case investors are allowed to use leverage, and in this case it is assumed that the investor will have to face a margin requirement. E.g.

an investor who is required to have margin requirement, m_t^i, of 20% would be able to invest the following amount to her current wealth, W_t^i:

$$m_t^i = 0.2 \Rightarrow \frac{1}{0.2} \cdot W_t^i = 5 \cdot W_t^i$$

So an investor who only faces a margin requirement of 20% will be able to invest *five* times her wealth in stocks when she is young.

5.2 Derivation of the model

It is now time to maximize i^{th} agent's utility. For that the Lagrangian of her utility and her portfolio constraint has to be written:

$$\mathcal{L} = x'(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t) - \frac{\gamma^i}{2} x' \underline{\underline{\Sigma_t}} x - \psi_t^i (x^i \cdot P_t - \frac{1}{m_t^i} W_t^i)$$

, where ψ_t^i is lagrange multiplier for the i^{th} agent.

$$\frac{\partial \mathcal{L}}{\partial x^i} = E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t - \gamma^i \underline{\underline{\Sigma_t}} x^i - \psi_t^i P_t = 0$$

$\Rightarrow$

$$E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t^i)P_t - \gamma^i \underline{\underline{\Sigma_t}} x^i = 0$$

$\Rightarrow$

$$\gamma^i \underline{\underline{\Sigma_t}} x^i = E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t^i)P_t$$

$\Rightarrow$

$$\gamma^i x^i = \underline{\underline{\Sigma_t^{-1}}}(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t^i)P_t)$$

$\Rightarrow$

$$x^i = \frac{1}{\gamma^i} \underline{\underline{\Sigma_t^{-1}}}(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t^i)P_t) \tag{3}$$

The demand of i^{th} agent for stocks has now been derived. As most of models in the literature review this is also an equilibrium model, which means that the economy is competitive hence total demand of *all* agents equals the supply of stocks.
Total demand of stocks equals total supply of stocks:

$$\sum_i x^i = x^* \tag{4}$$

Therefore we can sum over all the agents to get the total demand of stocks

$$x^* = \frac{1}{\gamma}\underline{\underline{\Sigma_t}}^{-1}(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t)P_t) \tag{5}$$

In equilibrium the prices for all stocks can be derived:

$$\gamma x^* = \underline{\underline{\Sigma_t}}^{-1}(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t)P_t)$$

$\Rightarrow$

$$\gamma \underline{\underline{\Sigma_t}} x^* = (E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f + \psi_t)P_t)$$

$\Rightarrow$

$$(1 + r^f + \psi_t)P_t = E_t(P_{t+1} + \delta_{t+1}) - \gamma \underline{\underline{\Sigma_t}} x^*$$

$\Rightarrow$

$$P_t = \frac{E_t(P_{t+1} + \delta_{t+1}) - \gamma \underline{\underline{\Sigma_t}} x^*}{1 + r^f + \psi_t} \tag{6}$$

From the equilibrium prices the expected returns for each security i, r^i_{t+1}, and the market M, r^M_{t+1}, can derived:
The return for the i^{th} security:

$$r^s_{t+1} = \frac{P^s_{t+1} + \delta^s_{t+1}}{P^s_t} - 1 \tag{7}$$

The derivation of the expected return for s^{th} security can be undertaken:

$$P^s_t = \frac{E_t(P^s_{t+1} + \delta^s_{t+1}) - \gamma \underline{\underline{\Sigma_t}} x^*}{1 + r^f + \psi_t}$$

$\Rightarrow$

$$P^s_t(1 + r^f + \psi_t) = E_t(P^s_{t+1} + \delta^s_{t+1}) - \gamma \underline{\underline{\Sigma_t}} x^*$$

$\Rightarrow$

$$1 + r^f + \psi_t = \frac{E_t(P^s_{t+1} + \delta^s_{t+1})}{P^s_t} - \gamma \frac{1}{P^s_t} e'_s \underline{\underline{\Sigma_t}} x^*$$

, where e'_s is a transposed column vector containing zeroes in all elements except for a 1 in the s^{th} row.
$\Rightarrow$

$$1 + r^f + \psi_t + \gamma \frac{1}{P^s_t} e'_s \underline{\underline{\Sigma_t}} x^* = \frac{E_t(P^s_{t+1} + \delta^s_{t+1})}{P^s_t}$$

$\Rightarrow$

$$\frac{E_t(P_{t+1}^s + \delta_{t+1}^s)}{P_t^s} - 1 = r^f + \psi_t + \gamma \frac{1}{P_t^s} e_s' \underline{\underline{\Sigma_t}} x^*$$

Using equation (6), the return for the security s has been derived:

$$r_{t+1}^s = r^f + \psi_t + \gamma \frac{1}{P_t^s} e_s' \underline{\underline{\Sigma_t}} x^* \tag{8}$$

Now the relationship between the return for security s and the market portfolio can be derived:

$$E(r_{t+1}^s) = r^f + \psi_t + \gamma \frac{1}{P_t^s} e_s' \underline{\underline{\Sigma_t}} x^*$$

$\Rightarrow$

$$E(r_{t+1}^s) = r^f + \psi_t + \gamma \frac{1}{P_t^s} cov_t(E_t(P_{t+1}^s + \delta_{t+1}^s), [E_t(P_{t+1}^s + \delta_{t+1}^s)]' x^*)$$

$\Rightarrow$

$$E(r_{t+1}^s) = r^f + \psi_t + \gamma cov_t(r_{t+1}^s, r_{t+1}^M) P_t' x^* \tag{9}$$

Using that the covariance between the returns of security s and returns of security s is variance of the returns of security s. It can be derived that the expected return of the market portfolio is.

The expected return of the market portfolio:

$$E(r_{t+1}^M) = r^f + \psi_t + \gamma cov_t(r_{t+1}^M, r_{t+1}^M) P_t' x^*$$

$\Rightarrow$

$$E(r_{t+1}^M) = r^f + \psi_t + \gamma var_t(r_{t+1}^M) P_t' x^* \tag{10}$$

From this equation the risk premium, λ, can be derived:

$$E(r_{t+1}^M) = r^f + \psi_t + \gamma var_t(r_{t+1}^M) P_t' x^*$$

$\Rightarrow$

$$\underbrace{E(r_{t+1}^M) - r^f - \psi_t}_{\lambda_t} = \gamma var_t(r_{t+1}^M) P_t' x^*$$

$\Rightarrow$

$$\lambda_t = \gamma var_t(r_{t+1}^M) P_t' x^*$$

$\Rightarrow$

$$\gamma P_t' x^* = \frac{\lambda_t}{var_t(r_{t+1}^M)} \tag{11}$$

The relationship between the aggregated risk aversion, transposed price vector, and the equilibrium quantity of stocks has been derived as a function of lambda, and the variance of the market portfolio.

From this the derivation of a relationship between the expected return of security s and its beta, β_s. As a start using equation (11) in equation (10):

$$E(r^s_{t+1}) = r^f + \psi_t + \gamma cov_t(r^s_{t+1}, r^M_{t+1})P'_t x^*$$

$\Rightarrow$

$$E(r^s_{t+1}) = r^f + \psi_t + cov_t(r^s_{t+1}, r^M_{t+1}) \cdot \frac{\lambda_t}{var_t(r^M_{t+1})}$$

Using the common definition of the beta at time t for security s showed in the literature review:

$$\beta^s_t = \frac{cov_t(r^s_{t+1}, r^M_{t+1})}{var_t(r^M_{t+1})}$$

$\Rightarrow$

$$E(r^s_{t+1}) = r^f + \psi_t + \beta^s_t \lambda_t \tag{12}$$

This expression of the expected return for security s can be rewritten as the excess return for security s as a function of the alpha of security, and the risk premium of the asset:

$$E(r^s_{t+1}) = r^f + \psi_t + \beta^s_t \lambda_t$$

$\Rightarrow$

$$E(r^s_{t+1}) = r^f + \psi_t + \beta^s_t (E(r^M_{t+1}) - r^f - \psi_t)$$

$\Rightarrow$

$$E(r^s_{t+1}) - r^f = \psi_t + \beta^s_t (E(r^M_{t+1}) - r^f) - \beta^s_t \psi_t$$

$\Rightarrow$

$$E(r^s_{t+1}) - r^f = \psi_t - \beta^s_t \psi_t + \beta^s_t (E(r^M_{t+1}) - r^f)$$

$\Rightarrow$

$$E(r^s_{t+1}) - r^f = \underbrace{\psi_t(1 - \beta^s_t)}_{\substack{\text{the alpha of} \\ \text{the security } s}} + \underbrace{\beta^s_t (E(r^M_{t+1}) - r^f)}_{\substack{\text{The asset's} \\ \text{risk premium}}} \tag{13}$$

From equation (13) it can be seen that the excess return[5] for an asset has *positive* alpha, α, when the beta, β, is equal or below *one*, while the alpha

[5]The risk premium of a security = excess return of a security

of securities which are riskier than the market, that is a beta of one, have a *negative* alpha, α.

The effect on the alpha is proportional to lagrange multiplier, which shows how constrained the economy is, i.e., the economy that's constrained will have the mentioned effects strengthen, while an economy where all investors are unconstrained will be an economy where CAPM holds.

5.2.1 Results from the derivation

In equilibrium the economy will have all securities earning an expected return as a function of the risk-free rate, the average weighted lagrange multiplier, and the beta-weighted risk premium of the market portfolio as seen in equation (12):

$$E(r^s_{t+1}) = r^f + \psi_t + \beta^s_t \lambda_t \tag{14}$$

, where the market risk premium, λ_t, is $E(r^M_{t+1}) - r^f - \psi_t$. The graphical representation of the Security Market Line in the economy can be written as following:

$$E(r^s_{t+1}) = \underbrace{r^f + \psi_t}_{\substack{\text{the intercept} \\ \text{of SML}}} + \beta^s_t \underbrace{(E(r^M_{t+1}) - r^f - \psi_t)}_{\text{the slope of SML}} \tag{15}$$

, where it can be seen that the measure for the average level of constraint on the agents' funding in the economy, ψ_t, has a *positive* effect on the intercept of the SML and a *negative* effect on the slope of the SML. The reasoning behind this is as following:

1. **For the slope of the Security Market Line**: When the average agent is more constrained, the agent will wish to invest his limited wealth in assets with higher expected returns, i.e. assets with higher betas. Thereby the agents will lower the prices of *high* beta assets more than lower beta assets, therefore the $-\beta^s_t \cdot \psi_t$.

   - All agents are affected by the ψ_t, because it lowers the price of risk in the market for all agents, i.e. all agents will receive a *lower* expected return per unit of market risk taken on.

2. **For the intercept of the Security Market Line**: Because of the lower risk premium in market for all agents, and the lowered expected return on high beta securities, the unconstrained agents will wish to leverage their portfolios with zero-beta securities. When this group of

agents does this enough, the effect will be that the zero-beta securities in those portfolios will gain correlation to each other. This positive correlation cannot seen as idiosyncratic, so the market will have to pay these agents a return for this, hence the slope is now $r^f + \psi_t$.

Both effects are increased (decreased) as the agents in the economy become more (less) constrained.

The two effects also changes the CAPM idea that the tangency portfolio is the market portfolio. In this economy the tangency portfolio is *only* held by agents that are *unconstrained*, because they can freely lever up the Sharpe ratio of the tangency portfolio, SR_{Tan}, but the constrained investors will want to invest riskier portfolios because of their constraint, ψ_t^i, so in equilibrium the market portfolio will earn a higher expected return than the tangency portfolio, but the market portfolio will yield a *lower* Sharpe ratio than the tangency portfolio.

The last effect of the constraints on funding and leverage is seen in the first part of the left side of equation (13):

$$\alpha_s = \underbrace{\psi_t(1 - \beta_t^s)}_{\substack{\text{the alpha of} \\ \text{the security } s}} \tag{16}$$

It can be seen that the alpha is *higher* for securities with a *lower* beta than for *high* beta securities. The alpha equals the average weighted lagrange multiplier for the market or securities with a beta of one.

The theory just described above will be tested on the empirical data in the first part of the analysis.

5.3 Derivation of the return of the BAB factor

After the return of each security in the economy has been derived, it is now time to derive the expected return on the BAB factor portfolio. The expected return of the BAB factor portfolio was introduced in [4] by Frazzini and Pedersen:

$$E_t(r_{t+1}^{\text{BAB}}) = \frac{1}{\beta_t^L}(E_t(R_{t+1}^L) - r^f) - \frac{1}{\beta_t^H}(E_t(R_{t+1}^H) - r^f) \tag{17}$$

, where β_t^L is the beta of the L portfolio, which is long in all assets *below* the median when sorted by beta values, $E_t(R_{t+1}^L)$ is the expected return of L

portfolio in time $t+1$ at time t, β_t^H is the beta of the H portfolio, which is long in all assets *above* the median when sorted by beta values, and $E_t(R_{t+1}^H)$ is the expected return of H portfolio in time $t+1$ at time t.

The derivation can start using the result for the expected return of security s in equation (12):

$$E(r_{t+1}^s) = r^f + \psi_t + \beta_t^s \lambda_t$$

$\Rightarrow$

$$E(r_{t+1}^s) - r^f = \psi_t + \beta_t^s \lambda_t \tag{18}$$

This equation expresses the excess return for security s. $\Rightarrow$

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{1}{\beta_t^L}(E_t(R_{t+1}^L) - r^f) - \frac{1}{\beta_t^H}(E_t(R_{t+1}^H) - r^f)$$

$\Rightarrow$

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{1}{\beta_t^L}(\psi_t + \beta_t^L \lambda_t) - \frac{1}{\beta_t^H}(\psi_t + \beta_t^H \lambda_t)$$

Now the expected return of the BAB factor portfolio expressed as a function of the weighted average lagrange multiplier, the risk premium of the market portfolio, and the two beta values of the portfolios.

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{1}{\beta_t^L}\psi_t + \lambda_t - \frac{1}{\beta_t^H}\psi_t - \lambda_t$$

$\Rightarrow$

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{1}{\beta_t^L}\psi_t - \frac{1}{\beta_t^H}\psi_t$$

Now the expected return of the BAB factor portfolio is *only* a function of the weighted average lagrange multiplier, which represents the economy's funding tightness, and the betas of the two portfolios.

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{\beta_t^H}{\beta_t^H \beta_t^L}\psi_t - \frac{\beta_t^L}{\beta_t^H \beta_t^L}\psi_t$$

$\Rightarrow$

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \psi_t \cdot \left(\frac{\beta_t^H}{\beta_t^H \beta_t^L} - \frac{\beta_t^L}{\beta_t^H \beta_t^L} \right)$$

$\Rightarrow$

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^H \beta_t^L}\psi_t$$

Assuming that the weighted average multiplier is a strictly positive constant, and knowing that $\beta_t^H > \beta_t^L$. This leads to the finale equation in the derivation:

$$E_t(r_{t+1}^{\mathrm{BAB}}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^H \beta_t^L}\psi_t \geq 0 \tag{19}$$

It can be seen that expected return of the BAB factor portfolio, $E_t(r_{t+1}^{\mathrm{BAB}})$, is positive, and it is increasing, when the economy gets more constrained in terms of leverage, i.e. the weighted average of lagrange multiplier, ψ_t, increases. E.g. psi_t must have increased after the collapse of the investment bank, Lehman Brothers.

The expected return of the BAB factor portfolio also increases if the beta-spread between the H - and L portfolio, $\beta_t^H - \beta_t^L$, at formation[6] increases.

5.3.1 BAB factor results

In the theory the return of the BAB factor portfolio has be *positive*. This theory will be tested on the stock data-set in the empirical analysis.

According to CAPM and the other asset pricing models considered in the literature review, a positive expected return for a factor portfolio has some implications:

1. In terms of CAPM it means that the market portfolio is *not* efficient and therefore does *not* capture all systematic risk in the economy, because a market-neutral portfolio, i.e. a zero-beta portfolio, which is self-financed, is supposed to earn an expected return of zero.

2. In terms of the multifactor models it means that BAB factor portfolio has to be included in the model, so the sum of the factor portfolios will capture all of the systematic risk and therefore be able to price all assets correctly according to their non-idiosyncratic risk exposure.

[6]The beta spread at formation equals the ex-ante beta spread according to [4]

6 Data

In this section the data-set and the methodology used in the empirical study will be explained, discussed, and reflected upon.

6.1 Data of the stocks

6.1.1 Data selection of stock data

The source for all stock data was chosen to be Thomson Reuters Datastream. The add-in for Microsoft Excel was then used. The criteria in the search for the stock data was as following:

1. **Equities**: In the delimitation all other securities and assets were excluded. So the main criteria in the search in Datastream is that the asset has to marked in the *equities* category.

2. **New York Stock Exchange**: The NYSE was chosen because it is bigger than NASDAQ, and it also made the process of obtaining the data *much* easier by excluding most non-american securities.

   In hindsight it was probably a bad choice, because it limited the size of the data-set which led to a forced decision in choosing the interval of relevant stock returns, i.e. a larger data-set would have led to a larger freedom in the choice of the time series' length, and maybe a *longer* time series which would have given more precise beta values.

3. **American Dollars**: The prices of the chosen securities had to be quoted in american dollars.

4. **American**: The stocks had to be american to be included in the initial data-set gathered from Datastream.

5. **Sectors**:

   - Aerospace & Defense
   - Alternative Energy
   - Automobiles & Parts
   - Banks
   - Beverages

- Chemicals
- Construction & Materials
- Electricity
- Electronic & Electrical Equipment
- Equity Investment Instruments - *but only the six of them*, which were ranked as *equities*.
- Financial Services (Sector)
- Fixed Line Telecommunications
- Food & Drug Retailers
- Food Producers
- Forestry & Paper
- Gas, Water & Multiutilities
- General Industrials
- General Retailers
- Health Care Equipment & Services
- Household Goods & Home Construction
- Industrial Engineering
- Industrial Metals & Mining
- Industrial Transportation
- Leisure Goods
- Life Insurance
- Media
- Mining
- Mobile Telecommunications
- Nonlife Insurance
- Oil & Gas Producers
- Oil Equipment & Services
- Personal Goods
- Pharmaceuticals & Biotechnology

- Real Estate Investments Trusts
- Software & Computer Services
- Support Services
- Technology Hardware & Equipment
- Tobacco
- Travel & Leisure
- Unclassified

6. **Return Index, RI**: All requested american equities would have to a RI to be included in the finale data-set.

7. **The period of the initial data-set**: The initial data-set was built from 01-01-1986 to 01-12-2016.

Excluded sectors were: Nonequity Investment Instruments and most of Equity Investment instruments.

From those criteria the request returned 1740 equities which were supposed to have a RI[7]. 9 of the requested equities returned an $\#ERROR$, and another 10 returned no RI in the chosen time period. So before the adapting and perfecting the initial data-set to the needs of this thesis, it had maximum of 1721 securities in the last period, 01-12-2016. The amount of RI per month is decreasing as progressing backwards in time.

6.1.2 Perfecting the data-set

The principle in this perfection of the data-set is to get *best* possible amount of whole years of return.

The author decided to that the finale data-set would not any omitted cells. So first the beginning date was changed to 01-12-1996, because the lowest amount of RI values would then be 786, and fit perfectly to have 20 whole years of returns[8].

The data-set, at this time, had 1730 columns and 240 rows of returns (not RI values, more on the calculation of the returns from the RI values later). This data-set had 140990 blank (or omitted cells). The decision was to cut

[7] RI = Return Index

[8] There would have to be a RI value more than the wished amount of returns in the end.

all securities which did not have 240 returns from 01-01-1997 to 01-12-2016. This cut down the amount of securities from 1730 to 780.

The next problem was to make the beta regressions possible and hopefully return some beta values which made sense according to the literature of finance. The choice of market portfolio data (more on that later on) forced a change in the data-set. The data-set had, without the knowledge of the author, been calculated in a way where all of the returns in data-set were ultimo period while the market data-set was using primo period values, i.e., the value of the market data-set of 01-01-1997 was corresponding to the value in the stock data-set of 01-02-1997. This flaw gave abysmal beta values. After realizing the error the first value in the data-set was removed, and the data-set was moved a period backwards.

The chosen market data did not have a value in the 01-12-2016, so that value in stock data-set was *also* cut from the data-set. After all of these different actions the finale data-set was built without any omitted cells.

One last correction was needed, because the stock data-set had to be cut up into deciles. The decision was, after the first round of beta regressions, to cut the 6 lowest beta values' returns from the, then, finale data-set. A comment on this part, the beta values were very close to 0 (under 0.152), which the author would argue is unrealistic, and an error caused by the lacking size of the data-set of returns for those 6 securities.

6.1.3 Pitfalls and challenges

The two largest pitfalls in the constructed finale data-set are:

1. **Survivorship bias**
2. **The amount of observations**

The first of the two pitfalls, survivorship bias, is prominent in my constructed data-set because of the wish to get a complete data-set without omitted returns.

The author admits that this pitfall could have been reduced *drastically* at the cost of making the quantitative work with data-set much more time-consuming. Instead of cutting the returns, which had omitted return cells, the beta values could have been calculated from the return that existed, thereby making it possible to include securities which later on disappeared from the data-set, i.e. the market.

The second of the two pitfalls, the amount of observations, is less prominent than the first pitfall, but it can still be argued that 238 observations as an input for a linear regression is *very* low.

This pitfall could have been avoiding partly by using the daily returns, but after an unsuccessful trials at getting daily returns for the period of time, because of the lacking strength of the computers at the library of CBS, this was abandoned to pursue more interesting parts of this thesis.

6.2 Market data and factor portfolios

While stock data was collected from Datastream, the monthly excess returns and the two factor portfolios' returns were collected from the data library of Kenneth French[9]. The choice was between getting the data from data library of Kenneth French or the data library of the AQR Capital Management.

The author decided against using the data from AQR's data library, because of the wish to be able to derive similar conclusions from the original data, from the data library of Kenneth French, as the Frazzini and Pedersen did in [4].

For the derivation of alpha in the Carhart four factor model, the momentum factor portfolio is also imported from the data library of Kenneth French[10]

7 Methodology

7.1 Return and Beta calculations

In the construction of the stock data-set, the monthly returns were calculated from the RI (Return index) values from Thomson Reuters Datastream using the following formula:

$$R_{it} = \frac{RI_{it} - RI_{it-1}}{RI_{it}} \tag{20}$$

, where R_{it} is the return of security i at time i, and RI_{it} is the Return index value for security i at time t.

[9]The data for the market excess returns, SMB factor returns, and HML factor returns can be found here: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html`

[10]The monthly momentum factor portfolio can be found here: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_mom_factor.html`

Afterwards the finale data-set of stocks had to be converted from returns to *excess* returns by using the following formula:

$$\text{Excess return}_{it} = R_{it} - r_t^f \tag{21}$$

, where r_t^f is the monthly risk-free rate at time t.

The data-set of excess returns were used for the beta calculation for each of the equities. In the calculation of beta the excess return for security i was regressed on the market excess return using the following regression, which is presented and explained in the literature review:

$$R_{it} - r_t^f = \alpha_i + \beta_i(R_{Mt} - r_t^f) + \epsilon_i \tag{22}$$

, where $R_{it} - r_t^f$ is the excess return of security i at time t, α_i is the monthly abnormal return of security i, β_i is the market sensitivity of security i, $R_{Mt} - r_t^f$ is the excess return of the market portfolio at time t, and ϵ_i is the residual return, which cannot be explained by the market.

For simplicity a very unrealistic assumption was made, the assumption that each of the securities would have the same beta value for the entire period. The author admits that this assumption is far from perfect, but it has a nice implication that each of securities will remain in the portfolio which it was put into at portfolio formation before the very first observation. Therefore the trading cost of the 10 equally weighted portfolios will be lower than if the beta values were recalculated every time period with less and less observation.

Avoiding the recalculation of the beta values implied that the thesis will *not* have to worry about the beta values becoming more and more imprecise.

Because of the assumption the beta values were calculated for each security *once*. The beta values were ranging from 0.152 for Spire and General Mills to 2.646 for Kemet.

7.2 Generation of Portfolios

In the empirical part of this thesis the author decided upon generating 10 equally weighted portfolios, and 2 different types of BAB factor portfolios[11].

[11] Betting against beta factor = BAB factor

7.2.1 Equally weighted portfolios

The securities were now ranked according to their beta values. Thereafter they were divided into 10 portfolio consisting of 78 securities each.

Then weight for each security in the equally weighted was calculated using this formula:

$$\pi_i = \frac{1}{N} = \frac{1}{78} = 0.01282 \tag{23}$$

, where pi_i is the weight in security i, and N is the amount of securities in each of the equally weighted portfolios.

After that the weighted excess return for each security i at each time period t was calculated.

All the weighted excess returns were placed in a matrix, that we call $\mathbf{A}$, where the columns represented time in months from 1 to T and the rows represented the ranked excess returns of the securities from 1 to I.

$$\mathbf{A} = \begin{bmatrix} \pi_1 E(R_{11}) - r_f^1 & \pi_1 E(R_{12}) - r_f^2 & \dots & \pi_1 E(R_{1T}) - r_f^T \\ \pi_2 E(R_{21}) - r_f^1 & \pi_2 E(R_{22}) - r_f^2 & \dots & \pi_2 E(R_{2T}) - r_f^T \\ \vdots & \vdots & \ddots & \vdots \\ \pi_I E(R_{I1}) - r_f^1 & \pi_I E(R_{I2}) - r_f^2 & \dots & \pi_I E(R_{IT}) - r_f^T \end{bmatrix}$$

For each of the 10 portfolios it was now possible to create smaller matrix with 78 rows and T columns, where T was 238, called $\mathbf{B}_x$, where x ranged from 1 to 10 of the equally weighted portfolios.

The sum of the each column would the excess return of the portfolio x at the time t using the following formula:

$$R_{xt} - r_f^t = \sum \pi_i(R_{it}) - r_f^t$$

, where $R_{xt} - r_f^t$ is excess return of x^{th} portfolio at time t.

Those row vectors could now be transposed into column vectors, where it would be used to calculate different measures for each of the portfolios:

1. **Excess average monthly return of the portfolio** x:

   $$E(R_x) - r_f = \frac{1}{T} \sum_{t=1}^{T} \pi_i(R_{it}) - r_f^t$$

   , where $E(R_x) - r_f$ is the excess average return of the portfolio x, and T is the number of time periods in the data-sets.

2. **The monthly variance of excess return of the portfolio** x:

$$var(E(R_x) - r_f) = \frac{1}{T-1}\sum_{t=1}^{T}[\pi_i(R_{it}) - r_f^t - (E(R_x) - r_f)]^2$$

   From the variance the monthly standard deviation of the excess return of portfolio x was derived:

3. **The monthly standard deviation of excess return,** $\sigma_{E(R_x)-r_f}$:

$$\sigma_{E(R_x)-r_f} = \sqrt{var(E(R_x) - r_f)}$$

4. **Yearly variables**

   - Yearly excess return:

$$[E(R_x) - r_f]^{\text{yearly}} = 12 \cdot [E(R_x) - r_f]$$

     The yearly excess return is the monthly excess return multiplied by 12.

   - Yearly standard deviation, $\sigma^{\text{Yearly}}_{E(R_x)-r_f}$:

$$\sigma^{\text{Yearly}}_{E(R_x)-r_f} = \sqrt{12} \cdot \sigma_{E(R_x)-r_f}$$

5. **The yearly reward to risk measure, Sharpe ratio, for the portfolio** x:

$$SR_x^{\text{Yearly}} = \frac{[E(R_x) - r_f]^{\text{yearly}}}{\sigma^{\text{Yearly}}_{E(R_x)-r_f}} \tag{24}$$

   As presented in the literature review, the rational investor will wish the largest Sharpe ratio, unless she is constrained on her leverage.

7.2.2 Betting against beta factor portfolios

The author decided to create two different BAB factor portfolios, where the second imitates the one in [4], and the first one will be more like a trading strategy for the reason that consists of only 78 stocks and not 780.

Both BAB portfolios are created the same way, the only difference is their size and therefore also their weights in the different stocks. First part of the construction is, the same as in the equally weighted portfolio, to rank the different according to their beta value.

The first and smaller one will go long in the decile of stocks with *lowest* beta values, and go short in the decile of stocks with *highest* beta values. While the second will separate the entire stock data-set into *two*, and then short the half with the higher beta values and go long in the half with lower beta values.

The long portfolio are called L for *low* beta portfolios. As both L portfolios are formed the same way, the smaller one can be used as an example to explain both of them at the same time.

After the ranking of the stocks by beta, it is needed to calculate the average beta for L portfolio:

$$\beta_L^{\text{average}} = \frac{\sum_{i=1}^{N} \beta_i}{N}$$

, where N is number of stocks in the chosen portfolio.

The same approach is used for the H portfolios which are the *high* beta portfolios.

Table 1: Average betas for the four BAB portfolios

	L-variant	H-variant
Portfolio with N = 78	0.317	1.874
Portfolio with N = 780	0.637	1.359

Next step for L portfolios is to calculate the unadjusted weight, which is done by this approach:

$$\pi_i^{\text{unadjusted in L}} = \frac{\beta_L^{\text{average}}}{\beta_i} \tag{25}$$

The opposite approach is used for the H portfolios:

$$\pi_i^{\text{unadjusted in H}} = \frac{\beta_i}{\beta_H^{\text{average}}}$$

This approach gives a larger weight to a security with a lower (higher) beta in the L (H) portfolio.

The real weights in both portfolios are derived from the unadjusted weights by the following approach, e.g. for a security in the L portfolio:

$$\pi_i^L = \frac{\pi_i^{\text{unadjusted in L}}}{\sum \pi_i^{\text{unadjusted in L}}}$$

This approach makes sure that the weights sum up to 1.

The beta of the portfolio is now being calculated, and this beta deviates from the average, because the portfolio is *not* equally weighted:

$$\beta_L = \sum \beta_i \pi_i^L$$

It can be seen that beta of the portfolio is a weighted average of the beta of the securities within the portfolio. The same approach is used for the H portfolios.

The beta values for all the L and H portfolios can be seen in the next table: It can be seen that the smaller one is more extreme in both cases.

Table 2: The weighted betas of the four portfolios

	L-variant	H-variant
Portfolio with N = 78	0.294	1.902
Portfolio with N = 780	0.541	1.437

The L portfolio with the smaller N has a lower beta value than the other one, and the H portfolio with the smaller N has a higher beta value than the other one.

Now all four portfolios have to re-leveraged, so their beta values are all equal to 1, and thereby the short position in the H portfolio and the long position in the L portfolio will be equal to a beta of 0, i.e. the BAB portfolio construction will be market neutral by construction. The re-leverage factor, Θ, can be expressed as following for L portfolios:

$$\Theta_L = \frac{1}{\beta_L} \tag{26}$$

By implication a Θ-value that's lower than 1 will mean that H portfolio will have be to deleveraged until its beta is 1, while a Θ-value that's higher than 1 will mean the L portfolio will have to leveraged until its beta is 1.

It is now time to find to the beta-adjusted, and therefore finale, weights of each security in L and H portfolios. This is done by dividing the weights, e.g. in portfolio K, found by equation (19) by Θ:

$$\pi_i^{\text{Beta-adjusted in L}} = \frac{\pi_i^L}{\Theta_L} \tag{27}$$

Those are the weights for L portfolio, and H portfolio is calculated in the same manner.

The construction of the expected return of L and H was made, so each of portfolios was self-financed. We use the notation Π_L for the weight in the L portfolio and Π_{r_f} for amount of the risk-free asset in the L after the leverage has been applied:

$$\Pi_L = \sum \pi_i^{\text{Beta-adjusted in L}}$$

And by construction the sum of Π_L and Π_{r_f} is 0, i.e. a self-financed portfolio has the weight of zero. From this the weight in the risk-free asset, Π_{r_f}, in finale construction can be derived:

$$\Pi_{r_f} = 0 - \Pi_L$$

The same method is used for the H portfolio.

The expected return for the L portfolio, $E(R_L)$ is therefore:

$$E(R_L) = \sum R_{it}\pi_i^{\text{Beta-adjusted in L}} + \Pi_L \cdot r_f^{\text{avr}}$$

, where r_f^{avr} is the monthly average return on the risk-free asset[12]. The BAB factor portfolio's expected return can calculated using this formula:

$$E(R^{\text{BAB}}) = E(R_L) - E(R_H) \tag{28}$$

The same approach was used to calculate the realized return in each time period, t. The author admits that this might differ from the approach used in [4].

[12]This can be done, because the weights and the betas of all the securities are assumed to be constant over the entire time period.

8 Analysis & Empirical Results

It is now time to test the theoretical implications of the model of this thesis, which were stated in the subsection of 4.2.1 and 4.3.1. To refresh the mind of the reader, a short list of implications:

1. Securities, and therefore also portfolios, if sorted by beta, will have varying required returns, and thereby also have varying realized - and expected returns.
   - For more details, see subsection 4.2.1.
2. The BAB factor portfolios will yield a positive required return and therefore also a positive expected return.
   - For more details, see subsection 4.3.1.

8.1 The primary result of the regressions

All the test results were yielded from the portfolios created in the methodology part by one of the three asset pricing models described in the literature review. To refresh the mind of the reader, a short list of the three models are showed below:

1. **The CAPM - single factor regression**: The excess return of the chosen portfolio is regressed on the excess return of the market portfolio.
2. **The Fama French model - three factor regression**: The excess return of the chosen is regressed on the *three* factors:
   - The excess return of the market portfolio
   - The expected return of the Small Minus Big (SMB) portfolio
   - The expected return of the High Minus Low (HML) portfolio
3. **The Carhart model - four factor regression model**: The excess return of the chosen is regressed on the *four* factors:
   - The excess return of the market portfolio
   - The expected return of the Small Minus Big (SMB) portfolio

- The expected return of the High Minus Low (HML) portfolio
- The expected return of the Momentum strategy

The three models will, from now on, be called: CAPM, 3 factor, and 4 factor. Table 3 reports the results of the single factor CAPM regression.

Table 3: The measures of the 12 portfolios

This table shows the Ex-ante beta values in the first column, which are the beta values of the each portfolio at formation. The beta Factor Loading is the beta value yielded from the CAPM regression. Volatility, Excess Returns, and Sharpe Ratio are all in *yearly* terms. The Sharpe Ratio, SR, is shown as a decimal, while Volatility and Excess Returns are shown in percentage

	β Ex-ante	β Factor Loading	Volatility	Excess Return	SR
P1	0.32	0.32	10.50	10.91	1.04
P2	0.52	0.52	12.61	11.66	0.92
P3	0.66	0.66	15.48	12.43	0.80
P4	0.79	0.79	17.25	13.11	0.76
P5	0.89	*1.68*	*35.92*	*26.16*	0.73
P6	0.99	1.00	19.76	12.03	0.61
P7	1.12	1.13	23.40	12.55	0.54
P8	1.28	1.30	25.51	14.76	0.58
P9	1.49	1.49	29.22	15.65	0.54
P10	1.87	1.87	36.71	17.04	0.46
BAB780	0.00	0.00	12.10	11.64	0.96
BAB78	0.00	0.00	29.73	27.77	0.93

The results are for the 12 portfolio formed in the methodology part. The 10 equally weighted portfolios are called from $P1$ to $P10$, where $P1$ is the portfolio with the *lowest* beta securities, and $P10$ is the portfolio with the *highest* beta securities. The BAB780 is the BAB factor portfolio, which was suggested in the [4] by Frazzini and Pedersen. The BAB780 consists of all the 780 securities. The $BAB78$ is BAB factor-like portfolio, which only consists of the *two* times 78 securities, because it is formed of a long position in a L portfolio consisting of 78 securities and a short position in a H portfolio consisting of 78 securities.

To the results of the test, it can be seen that, except for P5, ex-ante beta values match beta factor loadings. That's fortunate. The author admits after a longer investigation of P5, that the error is probably coming from the assumption that the betas of all assets are constant over time. For further analysis P5 might be included in tables and graphs, but it will have to *ignored*.

For the P portfolios the volatility and excess return is increased. That's in line with CAPM, because of the increasing beta values from P1 to P10. It is more interesting according to theory outlined in section 4.2 that the Sharpe ratio is declining as the beta values of the portfolio increase. This inverse relationship between the beta and the Sharpe ratio of the portfolio was suggested by the theory. So the empirical results are in line with the outlined theory.

Both the BAB factor portfolios do have beta factor loadings of zero due to the fact that the t-statistics at 5% percentage significance level is insignificant (for BAB780, t-statistics = -0.0488, and for BAB78, t-statistics = -0.0495). The real BAB factor portfolio, i.e. the one similar to the one in the original paper (BAB780), has much lower volatility and excess return due to much lower leverage. BAB780 has a yearly Sharpe ratio of 0.96, while BAB78 has a yearly Sharpe ratio of 0.93.

In table 4 each of the portfolios' alpha according to the three asset pricing models are shown. The 3 factor - and 4 factor model both do a better job than the CAPM at explaining the returns of each portfolio, but the momentum factor does not clearly make a difference, so the 3 factor model's alphas are more or less the same as the 4 factor model for all the equally weighted portfolios.

For the BAB factor portfolios, both the large - and the small one, the four factor explains the alpha better than three factor model. As an investment the BAB78 looks like a very attractive investment due to its alpha of 1.78% in the four factor model. It could also been seen as factor portfolio, and in that regard it does a much better job than the bigger BAB780, because if a factor portfolio generates an alpha according to an old factor model, it means that the factor(s) are not in themselves efficient. A model adding the BAB78 might be better at explaining the cross-variation in returns.

All the alphas of the BAB factors are significant across all models, so the theoretical prediction is correct: The BAB factor portfolios will generate a positive return, which breaks all the models, because the BAB factor portfolios are market-neutral and self-financed.

Table 4: All alphas for the 12 portfolios in the 3 models

The second column shows the monthly alphas of each portfolio according the single factor regression, the third column shows the monthly alpha of each portfolio according to the Fama French 3 factor model, and the fourth column shows the monthly alpha of each portfolio according to the Carhart 4 factor model. Beneath the alpha values the t-stastitics can be seen. If the alpha is significant, it is highlighted in bold letters.

	Alpha (CAPM)	Alpha (3 factor)	Alpha (4 factor)
P1	**0.74**	**0.58**	**0.57**
	(3.52)	(4.23)	(3.62)
P2	**0.69**	**0.50**	**0.52**
	(3.2)	(3.81)	(3.12)
P3	**0.68**	**0.44**	**0.47**
	(2.54)	(3.13)	(2.41)
P4	**0.66**	**0.39**	**0.44**
	(2.46)	(2.95)	(2.18)
P5	*1.27*	*0.74*	*0.86*
	(2.41)	(2.78)	(2.06)
P6	**0.46**	**0.22**	0.31
	(1.77)	(2.05)	(1.23)
P7	0.43	0.14	0.26
	(1.11)	(1.51)	(0.6)
P8	0.53	0.22	0.37
	(1.68)	(1.83)	(0.94)
P9	0.49	0.16	0.36
	(1.45)	(1.52)	(0.61)
P10	0.40	-0.02	0.28
	(0.92)	(0.98)	(-0.07)
BAB780	**0.97**	**0.82**	**0.72**
	(4.25)	(3.66)	(3.26)
BAB78	**2.32**	**2.01**	**1.78**
	(4.13)	(3.60)	(3.23)

9 Conclusion

The theoretical model predicted that a portfolio with a lower beta would earn a higher alpha, because of the constraint on leverage that some of the agents of economy suffered from. This would result in the SML of this model economy having a larger intercept, i.e. a larger alpha, and a smaller slope, i.e. a lower excess return on the market portfolio. The model also differed from the CAPM in regard to the choice of market portfolio. While the CAPM model uses the tangency portfolio, the model of the thesis has a market portfolio, which is riskier than the tangency portfolio, because the constrained agents, i.e. having a higher ψ_t^i, would invest their limited capital in riskier assets. The unconstrained agents would still invest in the tangency portfolio because of their ability to lever up the Sharpe ratio to match their appetite for risk.

It also predicted that the BAB factor portfolios would earn a positive return, even though they are constructed to be market-neutral, i.e. having a beta of zero, and be self-financed, i.e. the sum of the portfolio weights equal to zero.

The empirical results was in line with the theoretical predictions. Over the 10 equally weighted portfolios both the Sharpe ratio and the alphas were inverse related to the beta of the portfolio. The excess return on the market was still positive, so a higher beta of the equally weighted portfolio still meant a higher expected return and volatility but at a lower Sharpe. The smaller of the two BAB factor portfolio earned a monthly alpha of 1.78% in the four factor model, and the larger one earned a monthly alpha of 0.72%, so the self-financed market-neutral portfolios did beat the market.

So the rational investor would have wanted to construct this BAB78 portfolio by shorting the 78 securities with the highest betas according to their betas, so the securities with the highest beta would be shorted *most*. This construction would then be levered down, so it had a beta of one. Then the investor would go long in the 78 securities with the lowest betas, again the investor would weight them according to their betas. This long portfolio would then have to levered up to a beta of one. Both portfolios, unlike in the study by Frazzini and Pedersen, were made self-financed. The entire construction is also self-financed and market-neutral. The investor's only limits would be her margin constraint and wealth.

10 Limitations & Future Research

10.1 Limitations

The assumption of the securities having constant betas and weights over time made the conclusions drawn from the analysis impossible to generalize. This error would have to undone if the conclusions of this thesis or future research *should* be able to reveal whether the conclusions in [4] by Frazzini and Pedersen was correct or incorrect.

The very small data-set adds to injury, because of the imprecise beta values from the regressions, and the lack of a rolling estimation period before a formation period like the approach used in [5] by Jegadeesh and Titman. The approach used in this thesis is unrealistic, because the estimation - and formation period is the same. From a real world perspective there has to be a formation period, where the agents have the realized returns, so they can form the portfolios and then hold them into the next period, before then recalculating the betas and reforming the portfolios and their weights in the different securities once again.

10.2 Future Research

First of all the study should be redone with a larger data-set and a beta regressions in a rolling estimation window before the formation of the portfolio window.

It would be interesting to expand the model. The expansion should focus on modelling the funding tightness, ψ_t, i.e. the level of constraint on agents' leverage. This could maybe be achieved by modelling periods of heavy tightness after extreme volatile time periods, e.g. the financial crisis, by use of the implied volatility on options on the market index.

It would also be interesting to investigate the last three propositions in original paper by Frazzini and Pedersen. The author admits that the data-set is out of the scope of a master's thesis, unless the sources that the original paper used would hand the author the data freely. If that would be possible, it would be interesting to develop new and more advanced trading strategies after the replication of the original paper was completed.

11 Bibliography

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