Japan, Goldilocks and Reflation?

"Analysis of the Low Interest Rate Environment, and Hedging Strategies for Bond Investors"

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Pages: 111

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Copenhagen Business School, 15.05.2017

Abstract

This thesis aims to investigate whether the low interest rates seen today could be justified by economic factors, and how we could progress towards our estimated equilibrium. By these assessments, institutional investors sensitivity towards rising rates will be analyzed, to find out whether it is necessary to hedge portfolio exposure.

To assess the possibility of rising interest rates, today's "fair" level is analyzed for short and long-term rates separately. Secular stagnation has lowered the natural rate, thus justifying the low short-term rate. Due to unnatural demand factors, causing interest rates to decline, we argue that long-term yields are deviating from their fundamental value.

When analyzing investors interest rate sensitivity, we estimated yield curves based on the Vasicek model, combined with our interpretation of the economic outlook. Further, we divide into three scenarios; Reflation, Goldilocks and Japan style, to cover a broad specter of the future development. We argue the most likely scenario involves a slow progression towards the desired long-term rates (Goldilocks). However, a faster progression seems possible (Reflation), while a permanent low-rate environment seems most unlikely (Japan).

Using two different methods; initial price decline and holding period return, we find the sensitivity for a typical institutional investor to be substantial. We therefore recommend investors to lower their duration through hedging. This is found to be most effectively done on the long end of the yield curve.

Acknowledgement

We would like to express our gratitude to our supervisor Søren Plesner from Copenhagen Business School, for his counseling throughout the process of writing this thesis. We gained extensive knowledge through great discussions and constructive advice from Mr. Plesner.

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1. Introduction

1.1 Low Interest Rate Environment

For approximately 35 years', interest rates in major economies have been on a continuous decline. In the wake of the financial crisis, looser monetary policy has been implemented, which has led to even lower, and in some cases, negative interest rates. Discussion regarding whether the "correct" policy rate is negative and what the desired interest rate is, has arisen. Many argue interest rates are now at its turning point, and that for the first time in 35 years, we should prepare for rising interest rates. "The 35-year bond bull market really is over" (CNBC, 2016).

The low interest rates have not been limited to policy and short end rates. Long-term yields are primarily affected by the markets anticipation of short-term rates and future inflation (Fischer, 2016). Low returns have led investors towards lower quality and longer-term bonds with higher risk, in their search for yield. In today's environment, another exogenous factor has been the asset purchase programs putting pressure on the longer maturities. This is clearly illustrated in Germany, where government bonds are yielding negative returns up to seven years. Many believe the market is now set for a big reaction, back towards "normal" levels. What this level is and how fast we are going to adjust, remains uncertain. "The current bond market is facing the perfect storm" (Schmelzing, 2017).

This development has raised the question whether the bond market is in a bubble or not. Regardless of the answer, higher uncertainty in the market is observed. Both hedge funds and mutual funds have a net positioning more than 3 standard deviations away from their mean in 5-year Treasury futures (CFTC, 2017). It can be difficult to interpret economic view from positioning data. However, this could be seen as positioning for a changing environment, as it clearly shows uncertainty to how the bond market will develop. "Be Wise, Immunize!" (Dahlberg, 2015)

1.2 Problem Statement and Research Question

With regards to the low interest rate environment in the global economy today, there exists substantial discussions and articles dealing with this problem. A lot of this literature is based on whether we are in a bubble, a theory for which it does not exist a certified empirical framework. Hence, the body of academic literature for this topic is limited. Further, the development of the interest rate and monetary policy in the future is almost impossible to predict. Possible consequences are therefore discussed in detail, while the path of interest rates relies on the specific authors opinion. The discussion regarding the "fair" rate, risk exposure and possible ways to limit this exposure will therefore be the basis of this thesis. This leads us to our main research question:

Are the low interest rates justified by the economic situation, and how should bond investors hedge their exposure under 3 different scenarios?

This thesis studies the low rate environments effect on two major economies; US and the Eurozone. The primary objective of this research is to assess their sensitivity, and the consequences based on different scenarios in case of the interest rate development. As two different economies is discussed, they will be individually analyzed, but have many similarities and greatly influence each other. To get a clear understanding and view of the low rates and the consequences of this, the thesis aims to answer the following sub-questions:

- *Why are interest rates so low?*
- *Can the government bond market be characterized as being in a bubble?*
- *What is the desired interest rate level, and how will we progress towards this point?*

- *How sensitive is the typical bond portfolio towards rising rates and investment horizons?*
- *How should investors hedge against rising rates?*

1.3 Delimitations

The consequences of a change in interest rate levels is a complex matter that could be analyzed in many ways. We have chosen to focus our analysis on government bonds issued by Germany and United States, where Germany will work as a proxy for the Eurozone. Our estimated interest rates are solely on zero coupon bonds (ZCB). The measured change in yields therefore holds a small bias, as we calculated the change from ZCB to a treasury bond, that for some maturities pays a small coupon. Further, the market obviously holds multiple types of bonds, and the numerical changes in interest rates across bonds with different characteristics could be of different proportions. The market place as whole, is so complex that finding credible sensitivity measures proved to be challenging. When assessing the changes in value, we have therefore estimated the results based on a portfolio representing the entire US market, and its duration across multiple maturities.

Our focus has therefore been to estimate possible yield curves, based on our view of the current market situation combined with some empirical evidence. Hence, we have not solely used mathematical solutions to estimate the yield curves, but strongly relied on research and our own interpretation of what is likely scenarios. This creates a bias in our estimations, as the interpretation of likely scenarios could be strongly individual.

1.4 Methodology

The thesis follows abductive reasoning. The interest rate paths can be regarded as an incomplete set of observations, based on the best information available. We proceed to what we consider the most likely explanation to perform an educated guess of the future, but cannot guarantee our conclusion. We will continuously argue our choice of method and reason our interpretation of most likely outcomes throughout the thesis. The following will therefore only be a brief summary of the most important methodological choices we have made.

For each theme, we will go through relevant theory before this is applied, and thereafter the results will be presented. Hence, we will not follow the more conventional structure, where you have one theoretical section in the beginning and all theory is presented. We found it necessary to structure the thesis this way, as we have multiple themes in our thesis that all have differing theory related to it.

Our theoretical foundation for the interest rate simulations is built upon the Vasicek model. This is undoubtedly one of the most recognized interest rate models, and holds the advantage of being relatively simple to implement. We have created three scenarios where we have applied this model to estimate yield curves. These yield curves are used as the foundation for all further calculations.

The sensitivity measures and hedging strategies applies the most common principals and derivatives, to achieve a more realistic analysis of the changes. The calculations will be performed on a specific portfolio, serving as a proxy for typical bond portfolios held by institutional investors. Throughout the thesis, we will argue the choices we made with regards to calculations and framework, and discuss limitations of this.

The literature used as guidelines for calculations and main theoretical conclusions is from reliable financial and mathematical journals. Considerable effort has been made to verify this by rating and citation checks. Further, all books are from well-known publishers, and many are used in our previous courses at CBS. Internet sources are focused on serious newsletter providers and databases related to these agencies. Finally, all data is collected at 16.03.17 unless otherwise stated.

1.5 Structure of Thesis

The thesis can be divided into three main themes. Chapter 2-4 focus on theory and discussion regarding where the natural rate is, if there is substantial deviation from fundamental value, and hereunder if the market can be characterized as being in a bubble. Furthermore, chapter 4 studies previous events where the interest rate environment have changed rapidly.

Chapter 5 is the part of the thesis where we estimate possible interest rate paths. Based on the previous sections, mathematical estimations and real-world interpretation, we will argue what parameters we consider likely to be used in the Vasicek model to estimate yield curves for three different scenarios.

The final section of our thesis focus on the consequences for the different scenarios calculated in chapter 5. In chapter 6, we will estimate the percentage changes in the market value of a portfolio and sensitivity towards different investment horizons. Before we in chapter 7 introduces hedging strategies to reduce this exposure.

The research questions are discussed and analyzed throughout the three main themes explained above. Figure 1.1 presents an overview of where these questions are analyzed.

We will discuss and interpret our results in chapter 8. This will include a conclusion on the primary research question, in addition to a summary of the sub-conclusions discussed throughout the thesis. Finally, we finish our paper with a section entitled "The thesis in perspectives", where we give an introduction to other interesting themes that could be further analyzed.

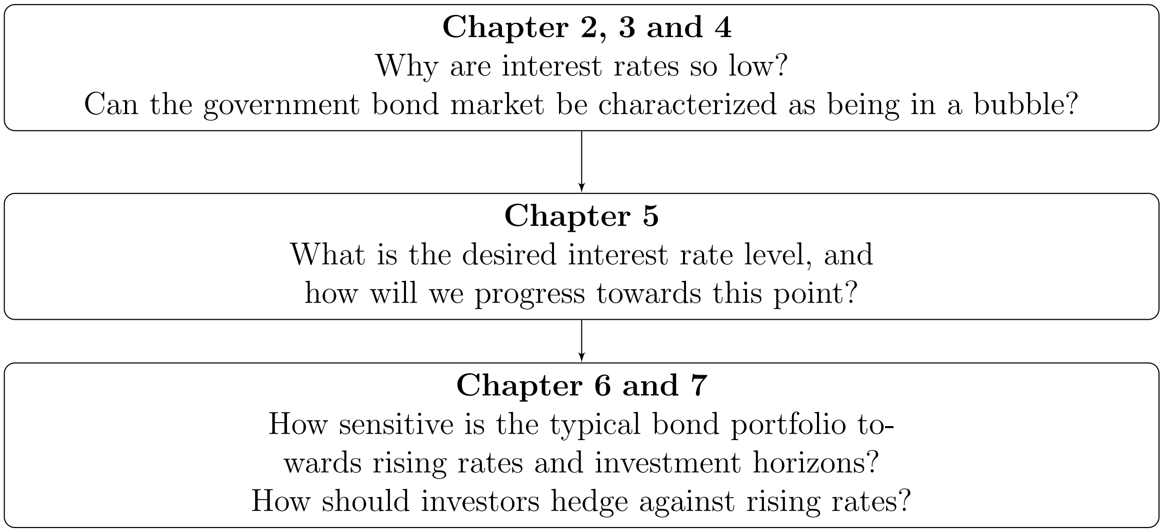


Figure 1.1: Overview of When Sub-Questions Will be Analyzed

2. Theoretical Framework

2.1 Bond Bubble

There are multiple ways to define a bubble. An asset price bubble exists if there is a mispricing of asset-values, compared to the price that would be consistent with the efficient markets, and thereby market fundamentals (Malkiel, 2012). These bubbles are further represented with substantial and extended price deviation over a longer period. The market fundamentals are thought of as an equilibrium, and thereby the economy is in "balance". A similar explanation is to define a bubble as "a run-up in the price of an asset, that is not justified by the fundamental supply and demand factors for the asset" (Conerly, 2013).

The existence of a bubble is contradicting with the efficient market hypothesis. This hypothesis focus on whether prices at any point in time fully reflect the available information (Fama, 1970). This again requires informational efficiency, which is a result of competition, low cost of information and relatively free entry. Further, the hypothesis assumes market participants behave rationally, maximize expected utility, and can process all available information (Shiller, 1998). Following this theory, it should not be deviation between the asset's price and its fundamental value. This is because news that could have impact on the asset price spreads quickly, and the price is adjusted immediately.

2.2 Bond Pricing

To understand how a bubble occur it is important to understand how bonds operate. As bonds are loans, those who issues bonds are borrowing money, while those who buy them are lending money. Furthermore, they are considered safer than stocks, and in an event of a company failure, bondholders are paid before the stockholders (Freeman, 2014). In contrast to stockholders, bondholders are assured the face value of the bond at maturity (assuming no default), as well as the given rate of return.

The difference between the bond- and stock market could be explained by one crucial aspect. Stocks always turn out to be what economists and mathematicians call a positive-sum or negative-sum game. When stock prices rise, investors get wealthier; when they fall, investors get poorer. The bond market, in contrast, is a zero-sum game. That's because the borrower or issuer is effectively short every bond that investors own. Thus, when bond prices fall, the losses born by bondholders are matched by equal gains for the issuer. The latter will then pay what now is a lower-than-par interest rate, or buy the bonds back for less than he sold them for. However, in reality costs as commissions, management fees, bid-ask spreads, administrative costs and taxes will reduce realized return over time (Philips and Walker, 2011).

The price of a bond is determined by multiple factors, but the most important is the interest rate. To understand the inverse price movements of bonds caused by the interest rates, it is important to understand how the yield works. The yield is a fundamental property of bond prices, and is related to changes in interest rate. As interest rates increases, the yield increases, and the bond price decreases due to lower present value of future cash flows. A bonds long-term yield is primarily affected by three factors; safe rates, term premium, which includes inflation expectations, and risk premium. The safe rate means the rate of return on low-risk investments (i.e. short-term US government bonds), the term premium measures the difference between the long-term yield and the short-term yield, while the risk premium looks at compensation compared to risk for investments.

2.2.1 Safe Rate

To understand movements in safe rates, it is important to look at the natural rate of interest. The natural rate is defined as the short-term equilibrium rate between demand and supply of funds with full employment of capital and labor, and price stability, which means an inflation close to the central bank's target (Claeys, 2016). In general, it is the real rate at which monetary policy is neither stimulating nor restraining economic growth. Meaning that market rates above the natural rate leads to lower consumption and investments, hence an economy that cools down. If the rates hold this level for too long it could lead to recession and deflation. On the contrary, if the market rates are below the natural rate, the economy heats up, which leads to an inflation above target. Since short-term government bonds don't hold significant term premium, the natural rate is the most dominant factor in safe rates.

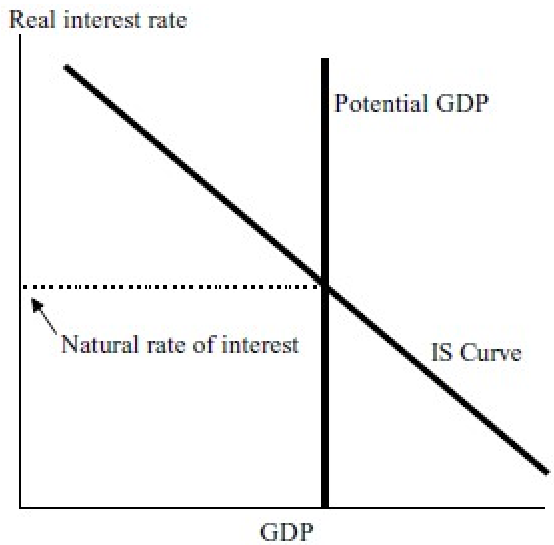


Figure 2.1: Determination of the Natural Rate of Interest
Source: (Williams, 2003)

Figure 2.1 simplifies the way the natural rate is determined. The I and S stand for Investment and Saving. The IS curve shows the negative relationship between spending and the real interest rate, where the relationship is seen as an equilibrium for different interest rates (Williams, 2003). The gross domestic product (GDP) line shows the potential GDP, which is assumed to be unrelated to the real interest rate in this figure, even though potential GDP also is a function of the real interest rate. The natural rate of interest is then decided in the intercept between the IS curve and the GDP line.

2.2.2 Term Premium

The most substantial factor in case of government bonds return differences is the term premium. The term premium is the compensation investors get for their greater risk when investing in longer maturities, relative to shorter maturities. This is because bonds with higher duration, which in general increases with maturity, is more sensitive to changes in interest rates. This greater risk, is the reason for why the yield curve often have an upward slope. For example, if the interest rate on a 10-year Treasury is 6%, and the 1-year Treasury is expected to average 5% over the next 10 years, the term premium on the 10-year Treasury would be about 1% or 100 basis points (BP). Thus, a key component of the term premium is investor expectations with regards to the future path of the short-term interest rate.

2.2.3 Risk Premium

The risk premium is a compensation for investors to tolerate extra risk, compared to the risk-free asset. In the bond market, the risk has close proximity to the credit rating of the bond. Those who issue bonds with greater credit rating offer lower interest rates, because this is seen as a safer investment. As government bonds in most developed countries is a relatively safe investment, the risk premium is small, even for longer maturities.

Interest rates are a key feature in the monetary policy in most countries. There are studies that shows how monetary policy effects the short-term rate at a high level, while the long-term rate is slightly less affected (Fransson and Tysklind, 2016). The level of where the rates are today would therefore reflect the policy within an economy to a high degree. However, other more natural factors could also contribute to changes in the interest rate. This is discussed more carefully in the following section.

3. Why are Rates So Low?

Short-term policy rates in Europe and US are at historically low levels. ECB has lowered their deposit rate to -0.4% and lending rate to 0.25%. Federal reserve has started to increase their rate from the bottom we saw at 0-0.25% and is now at the interval 0.75-1%. The yields on longer term government bonds are also at historical levels, with the 10-year German Bond at 0.4% and 10-year US Treasury bond at 2.5%. Can these rates be justified by the economic situation, or have other factors contributed and caused a possible bubble scenario in the government bond market?

3.1 Short-Term Rates

To understand why rates have moved to the levels we see today, we will start at the short end of the yield curve and the policy rate set by central banks. The central banks set this rate above or below the natural rate to stimulate or slow the economy. Because the natural rate is unobservable it is hard to define exactly where it is, but many argue this level has decreased over the last decades.

As seen in Figure 2.1 the natural rate is determined by the intersection between the IS curve and potential GDP. Economic theory suggests that over the last decades a fundamental change in savings and investment has caused a permanent shift in the IS curve, and thereby lowered the equilibrium natural rate. Higher saving and risk aversion has increased the supply of savings, while slower growth and more uncertainty has led to lower demand for investments (Borio and Disyatat, 2014). The combination of the two has shifted the IS curve to a lower equilibrium, and many argue a negative equilibrium. The decrease in natural interest rate is by many researchers explained through the secular stagnation hypothesis (Claeys, 2016).

3.1.1 Secular Stagnation Hypothesis

Secular stagnation states that economic problems is due to long-term and sustained slow-down in economic growth, and is not a part of a natural business cycle. In this view, the economy suffers from an imbalance, resulting from an increased propensity to save and decreased propensity to invest. This imbalance shifts the equilibrium of Figure 2.1. An important aspect contributing to secular stagnation is the number of people who are working in the economy.

In both Germany and Japan, the working population (age 15-64) has been shrinking for over a decade (Economists, 2014). This decrease is also expected to continue for a long period of time, and have also started in the US. Even though the cost of having and raising children is lowered, the cost to the society of taking care of the elderly gets bigger. This as a consequence of pensions, healthcare, nursing homes and increased longevity. Even though productivity of those who actually work is influential, it seems unlikely to increase the productivity to a point where we offset the demographic development.

In case of investments in the global economy, it is seen an excess of global savings over investment. An environment where savings are persistently higher compared to investments, implies an economy of "too much money chasing too few assets". Hence, demand pushes the price up and yield down. This is the key variable that balance the intersection between savings and investments. An increase in savings in many developed countries is seen, because of relative growth in the population segments that tends to save most (Fidelity, 2015). Particular, those in their mid-cycle years before entering retirement. This result is interesting, as this segment has both significant earnings power, coupled with an increasing incentive to save for retirement.

In addition to increased savings, investments have decreased. Part of the reason for the decline in investments is due to the decline in the price of capital goods, such as machines and equipment. This price decline is relative to other things produced in the economy. As companies today are less capital reliant due to lower growth in return, it is a reduced aggregate demand for investment. This trend is further strengthened, as the top 10% ROE companies spent twice as much on investments as the remaining 90 % in the 1990s, but now less than half (Fidelity, 2015). Further, the demographic development lowers investment, as it usually depends on the work-force.

Another important aspect is that secular stagnation increases the contagion from economic weakness. In "normal" times, an economy could offset this development through monetary easing. However, as monetary policy seems to be close to or at its lower limit, this seems hard in today's economy. This because additional easing seems hard to accomplish, thus each country's impact seems magnified (Summers, 2016).

For the secular stagnation to be a plausible hypothesis, there have to be good reasons to suppose that the natural rate is abnormally low, as secular stagnation occurs when the natural rate cannot be achieved through conventional central bank policies. This leads to the desired level of savings exceeding the desired level of investments. In such a scenario, monetary policy may not be enough, and unconventional methods is needed to raise the natural rate, and thereby stimulate growth. According to Larry Summers the primary responsibility for addressing secular stagnation should lay on fiscal policy, as an expansionary policy could reduce savings, stimulate growth and increase the natural rate.

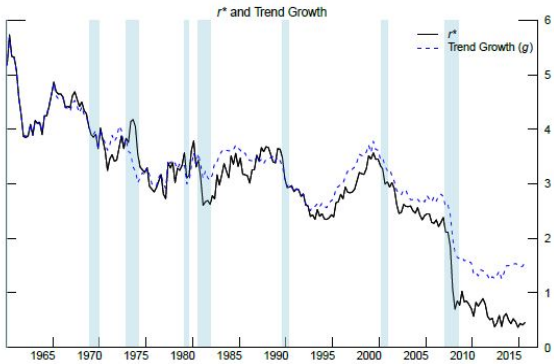
3.1.2 Alternative Hypothesis

Although the secular stagnation hypothesis has become more recognized in recent years, there still exists doubt on the fundamentals of this theory. The theory relies on an abnormally low natural rate and a slowdown in the economy. As this slowdown happened as an aftermath of the financial crisis, the question is if the slowdown is a stagnation or just a recovery period from the crisis. It is found that weak demand growth and long-lasting economic slowdown happens because of financial crisis (Ball, 2014) (Blanchard, Lorenzoni, and L'Huillier, 2017). Further, these crisis leads to recoveries that are slower and more protracted than normal. Hence, the economy may return to its pre-crisis level, but some argue we are too close to the crisis to determine this now.

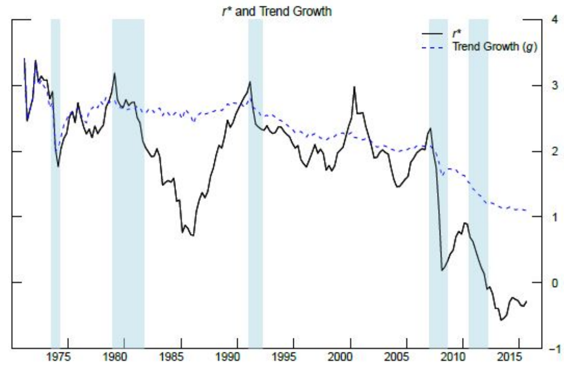
Normal conviction is that the demand side of the economy and unemployment causes the slowdown in growth, but Borio argues the supply side effect is often overlooked (Borio, 2017). This is exemplified with the observation of close to full employment (high demand), but constrains on growth in the US economy today. This he argues, is caused by a broken banking system, which makes it harder to reallocate resources from bloated sectors and into pro-growth sectors (low supply). The consequence of this is an undermining of the productivity growth, as a result of misallocation of resources, not a lack of resources. In this context, the reduced growth is thought of as an effect of the crisis itself and not because of secular stagnation. Overall, the doubt about the secular stagnation hypothesis, relies on whether or not the economic development is caused by the financial crisis and will return to "normal" levels, or if there has been a permanent shift in the natural rate and thereby a secular stagnation.

3.1.3 Where is the Natural Rate?

Even though the natural rate is unobservable, many estimations have been made to define where it is. One recognized estimation was performed using the Laubach-Williams approach (Holston, Laubach, and Williams, 2016). They argue the natural rate is closely connected to movements in trend growth, and reached historically low levels in the past couple of years. This is found as an international phenomenon rather than country specific. Their estimation of the natural rate for US and the Euro area can be seen in Figure 3.1a and 3.1b, where the dotted line illustrates trend growth and the solid line shows the natural rate (r^*).



(a) Natural Rate US



(b) Natural Rate EU

Figure 3.1: Natural Rate
Source: (Williams, 2003)

Here we see that the natural rate is at historically low levels, and even negative for the Eurozone. However, the estimated r* is not trying to capture the "correct" interest rate set by the central bank, but rather the desired level given the economic situation. Taylor's rule proposes a simple way to calculate the target federal funds rate:

$$R = R^* + p + 0,5y + 0.5(p - p^*) \tag{3.1}$$

Where R is the nominal federal funds rate calculated by Taylor's rule, R^* is the equilibrium federal funds rate, p is the inflation rate, y is the output gap and p^* is the desired inflation.

The "correct" rate can thereby be estimated using the Taylor rule, where the R^* is estimated by the natural rate model of Laubach-Williams. This implies the estimated natural rate works as baseline for the central bank rate. This baseline is thereafter adjusted by the current economic situation, determined by the output and inflation gap, which is devoted equal weights. The ECB is known to be more solely focused on inflation targeting than Fed, but as an approximation the same formula could be applied for the Euro area. Using formula 3.1 we obtain the estimated "Taylor rate" for the Euro area, using both r^* from Laubach-Williams and a fixed

2% level as the equilibrium rate $R*$:

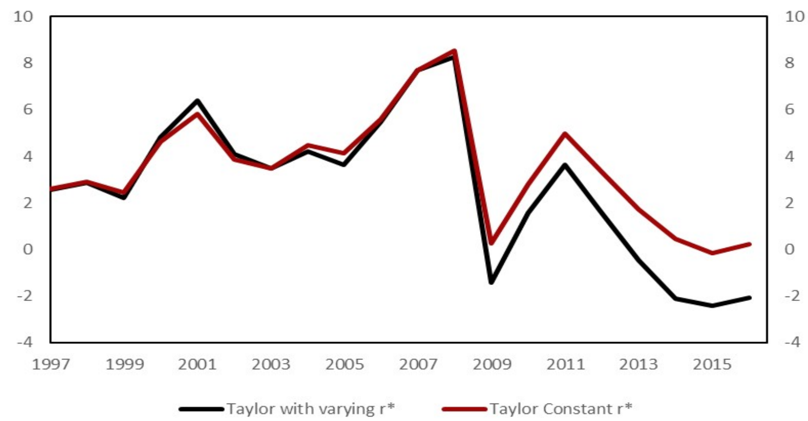


Figure 3.2: Estimates of the Taylor Rule
Source: Own source

We clearly see how using a static 2% equilibrium rate causes a higher estimate compared to using the decreasing natural rate. The natural rate and Taylor rate both provides justification for the low short-term policy rates seen in the world today, and thereby the unconventional use of monetary policy tools. This would imply that the policy rate seen today is not a consequence of central banks action, but a decline in the natural rate. Therefore, we argue the low rates should not be seen as a cause of the disease, but rather a symptom of the disease. In other words, an "acceptance" of these levels for the natural rate contradicts with the definition of a bubble in section 2, as this was defined as a substantial deviation from fundamental value.

3.1.4 Conclusion

Multiple trends suggest the economy has entered a secular stagnation. A shrinking working population, which is expected to continue, combined with an increasing cost of taking care of the elderly, illustrates this problem. Further, the relationship between investing and saving is decreasing. The demographic development has caused an overweight in the generation entering retirement with high incentives to save. This is combined with decreasing investments, due to modern companies being less capital reliant.

With regards to alternative hypotheses, some theorists suggest the lowering of yields is a consequence of being in the aftermath of a banking crisis. However, we believe the longevity of the low rates is hard to ignore as it started before the crisis. Even though a certain slowdown should be expected after a banking crisis, we argue that more improvements should be seen if a permanent shift in the natural rate was not true. Further, the long-term rates are nearly just as low. Thus, a market expecting low future short-term rates and inflation. Hence, the market does not predict we will go back to ”normal” any time soon. A downturn for such a sustained period, and the expectation of this to continue, contradicts with the theory of simply being in the aftermath of a crisis.

Further, as stated above, a secular stagnation occurs when the natural rate cannot be achieved through conventional monetary policy. Many developed economies have seen it necessary to introduce unconventional monetary policy tools over substantial periods of time. As similar measures have not been necessary in previous events, it gives further support to the secular stagnation hypothesis.

We therefore believe that we have entered a secular stagnation. Hence, a process that could be traced back to before the financial crisis, but was reinforced and more visible because of this event. This is further strengthened, as a permanent shift in the natural rate seems to have occurred based on multiple other studies. This supports the central bank policy rates seen today, and therefore justifies the low short-term rates, given the economic situation. We therefore believe the short-term rates are close to fairly valued, and not deviating from economic fundamentals. Hence, we reject the bubble hypothesis on the short end of the yield curve.

However, even if we have entered a secular stagnation, and the low short-term policy rates are justified by the economic situation; long-term rates are almost just as low. The long end of the yield curve will be discussed in the following section, as justified short-term rates not necessarily equal justified long-term rates.

3.2 Long-Term Rates

The shape of the yield curve normally tells us something about the current and expected economic environment. Today this curve is flat, but rising. Upward sloping yield curves are normally associated with an economy set for expansion, while a flat curve is normally associated with the transition to or from a recession.

3.2.1 Central Banks Policy Regimes

Normal conviction is that the central banks control the short end of the yield curve by adjusting the short-term rates to meet the natural rate. Inflation, policy rate expectations and other external factors are normally what is believed to control the long end. Together with reduced growth outlooks, which lowers the natural rate, inflation expectations have also declined through the last decades. During the 70s and 80s the Eurozone inflation soared all the way up to 15%. Since then the central banks have started an inflation targeting regime, aiming to keep inflation and prices at low and stable levels. ECB set their target for the Harmonized Index of Consumer Prices (HICP) below 2% in 1999, while Fed's definition of price stability is similar at 2% inflation. These kind of credible policy regimes have had great influence on the inflation expectations, and thereby the yield on long-term bonds. As mentioned in section 2.1, yield on government bonds consists of the safe rate described above, risk premium and term premium.

When looking at the inflation and policy rate expectations effect on the yield, the term premium is the most influential factor with regards to government bond pricing. Hence, determining this factor could give a better answer on whether investors are compensated for the risk of holding long-term bonds. As this is not directly observable in the market, it must be estimated, which highlights the difficulty of determining this factor, as seen in Figure 3.3.

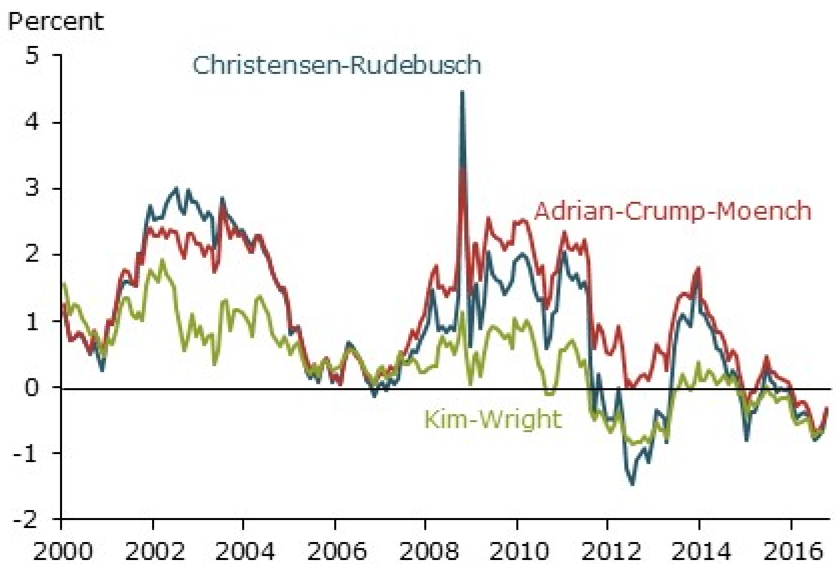


Figure 3.3: Estimates of the Term Premium in 10-year Treasury Yield
Source: (Bauer and Rudebusch, 2016)

This figure shows the projections for the term premium in the US, using three different methods; Christensen-Rudebusch, Adrian-Crump-Moench and Kim-Wright. These models are built on historical behavior of interest rates, and in some cases forecast surveys. The models show a similar trend during the time-span of the research, but also varies substantially in values.

In recent years, it could be seen a substantial decrease in the term premium, which ultimately lead to a negative premium, supported by all three methods. A negative term premium could imply that investors see a possibility of deflation for a longer period of time, and is therefore not requiring compensation for future inflation. The secular stagnation hypothesis presented above could be an explanation for these expectations. If the economy in fact has entered a long sustained slowdown in growth, "inflation premium" becomes negligible. This could also explain investors decreasing uncertainty regarding future monetary policy, as rates would be low for a long period of time. Finally, bonds could work as a hedge against poor global outlook. These effects would have negative impact on the term premium. Another hypothesis is for example pension funds that are interested in locking in a fixed rate, instead of rolling their investments in shorter-term bonds. This to avoid the uncertainty of interest rate and inflation fluctuations, and thereby more willing to accept lower yield on longer term maturities.

3.2.2 QE's Effect on the Yield Curve

While the theory above suggests a rather gloomy outlook for the global economy, other factors contribute to the low long-term yield, which may not show anything about investors' expectations of future growth or secular stagnation. Many economists therefore argue we should interpret the yield curve today with caution, as many of the "classical rules" are outdated in the current interest rate environment (Shin, 2017). A decisive factor, especially in the European market, is the Quantitative Easing (QE) program started by ECB.

In recent years when the short-term rate has reached the zero lower bound, central banks have adopted more unconventional policies. These measures include negative policy rates and changes in size, composition and maturity of their balance sheets. The latter is achieved through QE. Research show that a flatter yield curve and lower risk premiums can have positive effect on aggregate demand. Bad liquidity in the markets can cause self-fulfilling liquidity crises and higher risk premium. This happens when there is massive selling in the bond market and few buyers. This is when central banks can come in as a lender to purchase bonds in the market, and thereby stabilizing liquidity to prevent a self-fulfilling liquidity crisis and increase inflation expectations (Claeys, 2016). Hence, they fight deflation and output gap by lowering the future expected real interest rate, which also lowers the real long-term rates. This is a strategy both Fed and ECB has adopted as part of their monetary policy tools, to keep the longer term yields low, and thereby increase economic growth.

While more conventional monetary policy focus on current and short-term rates expectations, QE target longer term bonds (Brainard, 2015). Another factor Brainard argues, is the signaling such a program has to the market regarding future interests. By actively participating in the markets to keep yields low, they are not likely to hike rates in the near future. A QE program could therefore be seen as a reinforcement of the forward guidance.

Over its course, QE has had great influence on the interest rate, both in US and the Euro area. A decrease in interest rate is seen for the last decade in the US. After QE was introduced in late 2008, interest rates reached its lower bound in 2009. An implementation made to support the entire economy, but with great focus on the housing credit market. It is found that implementing QE, increased inflation expectations and reduced inflation uncertainty. Hence, the real rates fell for a variety of borrowers (Krishnamurty and Vissing-Jorgensen, 2011). Nevertheless, these estimations are during the financial-crisis period. In such a period, supply changes are expected to have large impact on the interest rate. The full effect of the decrease in interest rates could therefore not solely be caused by QE. However, a great impact could be seen from the rising long-term yields in the US, as an aftermath of announcing a reduction in the asset purchase program, which is further explained in section 4.2. In October 2014, the Federal Reserve voted on ending the QE3 program, and stated that the rates would remain near zero for a considerable time, but that a rate hike could come sooner if the economy improved faster than expected. The decision to end the QE program was based on improvements in labor markets and GDP outlooks.

The ECB's QE program was implemented six years later than the Fed's. ECB introduced the Public Sector Purchase Programme (PSPP) January 24, 2015. Under this program, they started to buy sovereign bonds from Euro area governments in march 2015. An extension was further announced in December 2015, and the full QE project was initially expected to last until September 2016. However, it is now extended until at least the end of 2017 (Claeys and Leandro, 2016). If longer, it will continue until it can be seen a sustained convergence towards the objective rate of inflation which is below, but close to 2%. The effect of QE in Europe is hard to pin down, because of the short longevity of the project. However, indications of QE's impact could be seen in regard to the inflation expectations, as the uncertainty decreased with the announcement of the purchases (Andrade et al., 2016). Hence, the long-term inflation expectations were guided closer towards its price stability objective or close to the inflation target. In contrast to the US, the ECB became the first major central bank to have one of its key policy rates in a negative territory. June 2014, ECB set their deposit rate to -0.1%. Hence, banks are charged for holding excess cash, a method to "force" them to stimulate the economy by lending.

The protracted decrease in term premium could therefore partly be explained by the QE programs, in case of less uncertainty given by the programs implemented. This, in order to stimulate the interest rates in the aftermath of the financial crisis. The development with regards to QE in the future is also likely to have great impact on the term premium. Researchers at the Federal Reserve Bank of New York and Bank of England, found decreases in term premium to account for around 70% of the QE effects on long-term interest rates in their respective countries (Fawley and Juvenal, 2012). Further, there is found a negative effect of 50-100 basis points on the 10-year yield for the Fed's first LSAP program and the Bank of England's QE program (Christensen and Rudebusch, 2012b). The observed decrease in term premium after the financial crisis is also consistent with the view, that an economy with low inflation or deflation could lead to lower or negative risk premium. This, again, because long-term bonds would work as an insurance against the risk caused by low inflation or even deflation (Campbell, Sunderam, and Viceira, 2016). A decrease caused by the reduced supply of long-term bonds in the market.

3.2.3 Other Natural Factors

QE programs have undoubtedly influenced long-term yields in the market. However, other less known factors also greatly influence the natural demand and supply relationship on bonds that are less obvious. Head of Research Hyun Song Shin in the Bank for International Settlements, offers an alternative explanation for why yields have been continuously dragged down to the bizarre levels of today's economy (Shin, 2017).

He argues that the "overreaction" in the market can, at least partly, be explained by the duration challenge met by big investors such as pension and life insurers. These type of firms typically have longer term liabilities to policyholders and beneficiaries, compared to the fixed income assets held to meet these liabilities. This, in effect, gives their liabilities higher interest rate sensitivity, compared to their assets.

Further, this duration gap increases with falling yields due to negative convexity. When longer term yields fall, the duration becomes larger for both the short-term assets and long-term liabilities. However, because longer term liabilities are more sensitive to change in yields, the duration increase faster compared to their assets. These firms are typically regulated by strict rules regarding their interest rate exposure, and have to keep this gap within a certain level. So, when the gap widens, they are forced to increase their duration exposure by purchasing long-term bonds to increase the asset side duration. This puts further demand pressure on bonds, which again contributes to lower yields. This effect will be referred to as the "duration-spiral".

Therefore, the initial decline in interest causes the duration-spiral to be a self-fulfilling prophecy, where big companies have to buy longer term bonds which lowers their yield, forcing them to buy even more bonds. Thus, the fall in term premium, is not caused by risk seeking investor, but rather money managers trying to keep their risk exposure in check. It is important to understand the entire aspect of an asset. Just as stocks could be used for beta hedging of a portfolio, bonds can be used for duration (or interest risk) hedging. The classical interpretation of this fall in term premium (such as the one presented above) will therefore be misleading, especially in turning points of a rate cycle.

Another factor strengthening the trend of declining interest rates is the increased demand for "safe" government bonds, especially by emerging markets (EM). Over the last decades, savings compared to GDP has risen sharply in these markets. Asian EM in particular have experienced tremendous growth over the past centuries, which has led to increasing currency reserves. Financial market integration combined with a global reallocation of portfolios, caused a big proportion of these savings to be invested in industrialized countries government bonds, putting further pressure on its price (Suisse, 2016).

A flatter yield curve, has also led to investors searching for yield. To achieve the same return as before, investors are forced to move longer out on the yield curve. This search for yield also includes alternative assets, such as property, infrastructure, private equity and hedge funds. "There is no such thing as a free yield. If you want yield, you have to take some risk" (Oakley, 2016). Thus, having to buy riskier assets to achieve what was previously thought of as a risk-free investment. However, this development could be seen from a different view where investors have been reaching for duration. This to make up for the low yields by choosing assets with interest rate risk instead of credit risk (Alloway, 2016). Nevertheless, some type of risk will follow when increasing the return.

3.2.4 Does Economic Factors Agree?

Many theories come with different answers to why and how yields have moved to the levels of today. It can be hard to find an exact explanation for a complex market like this. Macroeconomic data can therefore provide some guidelines for the current economic situation. In Germany, the inflation rate is higher than all yields on the 30-year yield curve, while the inflation outperforms yields up to 20 years in the United States. Hence, some of the lowest real rates in history is observed now. Figure 3.4 illustrates this represented by the 10-year yield for Germany and US subtracted their respective inflation rate.

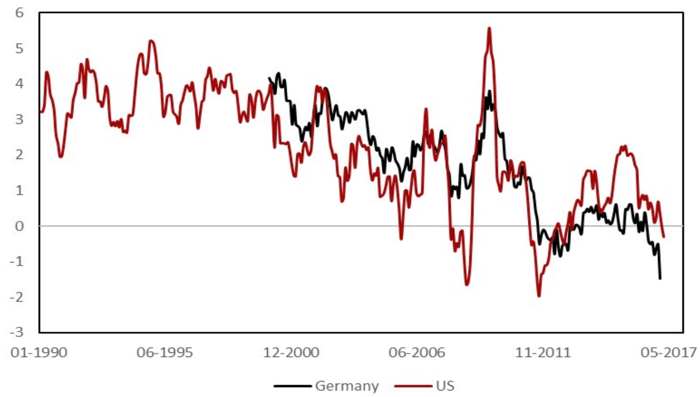


Figure 3.4: 10-year yields subtracted YOY CPI
Source: Own Source

Further, the German seasonally adjusted harmonised jobless rate fell to its lowest in 36 years at 3.8% this January. The American economy is close to full employment with an unemployment rate at 4.8% (Trading Economics, 2017). The Munich-based Ifo institute's business climate index measure the business confidence in Germany, and rose to its highest since 2011 this March (Look and Skolimowski, 2017). The US consumer confidence were at its highest since 2001, in the end of 2016. Fundamental economic indicators like these, usually have strong positive correlation with the yields, but now there seems to be a clear deviation.

3.2.5 Conclusion

Falling expected inflation has had significant influence on the yield and term premium over the last decades. Decline in growth has led to lower inflation rates combined with credible central bank policies to keep inflation low. During the latest period, there is significant evidence that the term premium has even entered negative territory. This could partly be explained by lower uncertainty regarding future monetary policy and inflation expectation. Given the positive development in the economies, this seems like an overreaction from the market.

QE has undoubtedly had significant impact on the yield and term premium. The central banks effort to boost the economy by purchasing longer term bonds, has spurred demand pressure and lowered yields. When yields decrease, it is likely to have further impact on the economy. The duration problem of life insurers and pension funds is one less known consequence of this. As the yields turn lower from economic slowdown and QE, the liability duration side of these companies outgrow the asset side. To close this gap, long-term assets is needed, which puts further pressure on price and yield. The final factors discussed above is the growing capital reserves of especially emerging markets, combined with a portfolio reallocation in favor of government bonds, which have further increased demand. Investors are therefore being pushed further out on the yield curve to achieve the desired return.

The factors mentioned above are in our opinion all unsustainable demand factors. The reallocation of portfolios is not likely to continue in eternity, QE cannot last forever and the demand side consequences of this monetary policy will then cease. This is where we believe the US economy is at, where QE has ended and interest rates have started to increase. This is also where we believe the Euro area will progress towards in the upcoming years. When QE ends, the demand will decrease because they are no longer intervening in the markets. This causes the self-fulfilling prophecy of asset managers to be inverted, and investors can move back on the yield curve to acquire the desired return. All these factors that contributed to the duration spiral, could therefore be reversed to give a spike in yields.

In light of the discussion above, we firmly believe both long and short-term rates are lowered due to economic factors and future expectations. Further, we expect this level to sustain low for an extended period of time. However, as the yields seen today (especially for the Euro area) are extremely low, the economic factors and future expectations would have to be far gloomier than our interpretation of the economy as of today. We argue other factors are making the market overreact to the current economic situation. Further, that the overreaction in the market comes from an unnatural and unsustainable demand pressure, not justified by the underlying factors of the asset, and thereby a deviation from its fundamental value.

4. Changes in Monetary Policy - What History Taught Us

History provides multiple examples of the relationship between monetary policy and the effect it has on the global markets. Previous reactions can help us understand how different types of monetary policy turnarounds, can have different effects on the market and yield curve.

4.1 The Great Bond Massacre of 94

One of the most recognized events is nicknamed "The great bond massacre of 1994". In the beginning of 1994 one of the worst annual returns (-7.77%) on 20-year US Treasury bonds occurred (Thorbecke, 2000). The exact reason for the sudden decline in prices is hard to define, but most argue it is closely related to the sudden increase in interest rates from Fed, combined with the uncertainty of future monetary policy.

Following the savings and loan crisis, the early 1990's was dominated by a lowering of interest rates. From the last top in 1989 with a policy rate of 9.75%, they bottomed out at 3% in September 1992. As the economy later started to improve, GDP growth picked up, and the stock market was booming. Treasury yields started to rise late 1993 with expectations of increasing inflation. The combination of low interest rates, the lowest inflation in two decades and the fastest economic growth since 1987 had the Federal Reserve Chairman Alan Greenspan worried (Business Insider, 2013).

In February 1994, the federal reserve raised the short-term federal funds rate by 25 basis points for the first time in 4 years. The strong economic growth had the committee worried about future inflation risk, and wanted to be in front of the development. The reaction in the financial markets came immediately with Treasury yields spiking and stock markets tumbling. The 25 BP rate hike was repeated by the Fed in March. The two months totaled a return on 20-year Treasury securities off -8.4%. In April (not a scheduled Fed meeting) another 25 BP was increased and finally 50 BP in May. During this period, the yield on 30-year Treasuries had jumped nearly 1.5%. Over the period from January to mid-September, the estimated value lost from the US bond market was 600 billion USD, and 1.5 trillion USD worldwide (Ehrbar, 2013).

One big difference from 94 and today is the Federal reserve's communication with the market. In 1994, there was no statements following the meetings or speeches held prior to, or between meetings. The lack of forward guidance led to more uncertainty regarding the actions taken by the Fed. Willem Thorbecke explains the powerful reaction with two examples. First, he argues that the reaction came because investors believed the Fed held private information about inflation, and pushed yields in response to the news of higher inflation. Secondly, investors increased the excess return required to hold longer term bonds, because of the increased uncertainty in Fed policy.

Following the rate increase in February 1994, the Federal Reserve released an official post announcement statement for the first time. Both the rate increase and the announcement came unexpected on the market, and stimulated a more dramatic change. Another interesting observation during the February hike, is that the forward rates on longer term maturities (5-10 years) rose by about a full percentage point. If the reaction solely came from increased expectations of further rate hikes to come, this should influence mostly shorter maturities. This supports Thorbecke's first explanation of higher inflation expectations.

Taylor and others found that applying formula 3.1, had documented the actions of the Fed fairly well in previous times. However, there was a big dispersion between the forecasted Taylor rate and the actual rate implemented by Fed from 92 and going forward. First an overestimation from the Taylor rule compared to the actual rate, followed by an underestimation during 94, that lasted for multiple years.

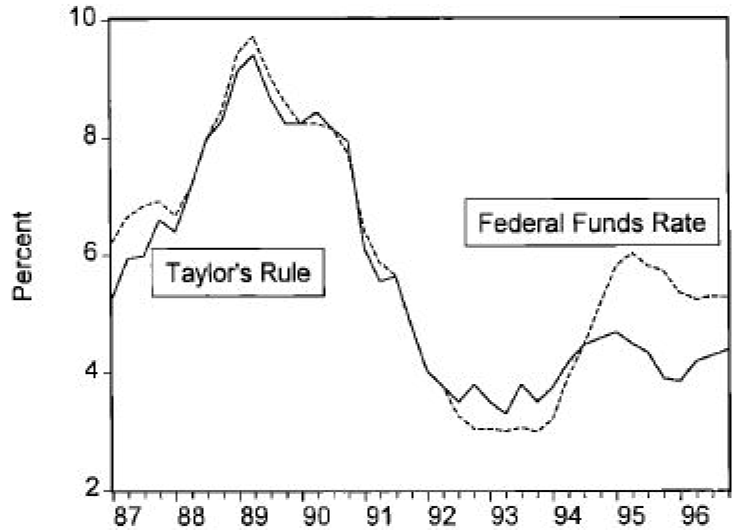


Figure 4.1: Taylor Rule
Source: (Thorbecke, 2000)

This evidence support Thorbecke's second explanation. A new way of thinking from Fed officials caused a higher risk premium, due to more uncertainty regarding the policy rate in the future. When further examining the effects, Thorbecke concludes that longer term Treasury bonds have more effect from inflation expectations than political uncertainty, but the two combined, caused the strong reaction.

Another explanation for the consequences of the change in monetary policy, is due to the steepness of the yield curve prior to the rate hikes. At the beginning of the decline in policy rates during the late 80's, the Federal funds rate was above the 30-year Treasury yield of around 9%. Three years later in 1992 the situation was completely different, as the Fed fund rate was 3% and the long-term Treasury yield around 7.5%. This spread effectively creates a carry trading opportunity for speculators and financial companies, which could borrow short-term maturities with low interest, and lend long-term maturities with higher interest.

When leveraging this strategy, the investor becomes sensitive to increases in the short-term rate, and this was felt by many. Even though the reaction from the rate hikes was seen in both longer term Treasury bonds and short-term rates, the effect was even bigger on the shorter term. This causes a flatter yield that effectively hurts the carry trades put on by speculators. This effect can be seen in the graph below. The black line tracks the yield of a 10-year US Treasury bond, the red line is the 2-year and the blue shaded area marks the difference between them. A smaller difference between the two, corresponds to a flatter yield curve.

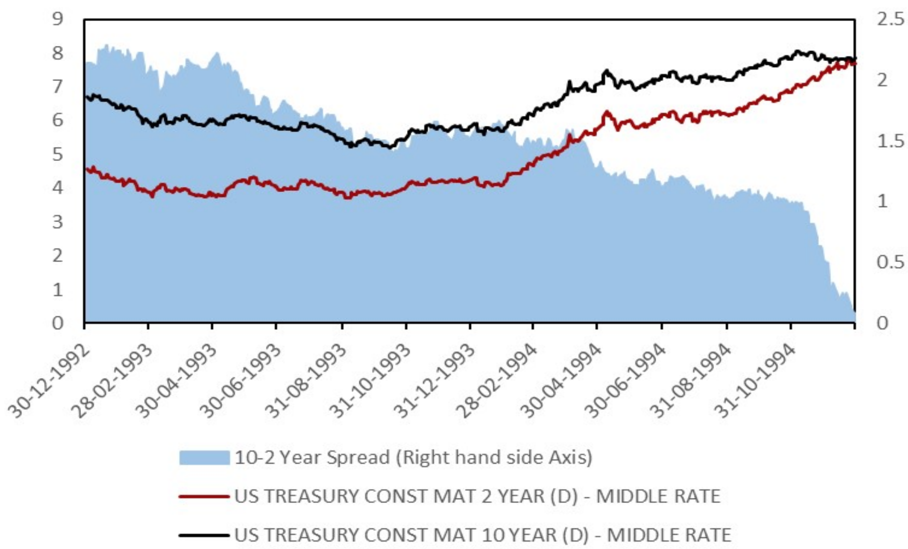


Figure 4.2: Yield Spread 1994
Source: Own Source

The high leverage applied to these strategies were sometimes geared all the way down to a 1% margin. A 1% increase in rates then causes a doubling of your initial margin requirement. The initial increase in short-term interest rates therefore caused a flood of margin calls that many investors answered by liquidating their positions. The combination of individuals withdrawing from mutual funds, liquidation and low supply of money, caused an even bigger sell-off and decline in government bond prices.

This illustrates how a turnaround from one economic environment to another have multiple ramifications. Investments are made based on today's environment, and a turnaround therefore causes both the more "expected" reactions, but also structural changes in investments, which all increases the effect of a change.

4.2 Taper Tantrum

The financial turmoil of 2008 caused a similar reaction to what we saw from the savings and loan crisis of the late 80's and 90's. Though this time, rates moved to a more extreme level. September 2007, policy rates were lowered for the first time since mid-2003. This continued until they bottomed out at 0.25%, the lowest rate ever seen in the US. Combined with the lowering of rates, the Federal reserve embarked upon a massive QE program in late 2008.

Naturally, QE cannot last forever and eventually rates are likely to go up. May 22, 2013, Fed Chairman at that time, Ben Bernanke, made the first statement that they would gradually decrease their asset purchase program or "taper". Adding to the outlook that the US economy was improving, a violent reaction in the bond market occurred. In just two weeks, the estimated loss in the bond market amounted to 1,5 trillion USD (Scaggs, 2015). From May 2, to June 25, the yield on the 10-year note soared from 1.63% to 2.59%. The yields continued to rise until mid-September, when the Fed surprisingly announced that they would not in fact taper yet. This started a two-month rally in the bond market with yield lowering. By November, yields once again started to rise, and mid-December Fed announced their first taper by lowering the asset purchase programs from 85 to 75 billion in monthly purchases. The reaction in the market was restrained.

During this event, the reaction in long-term bonds was much more severe than short-term, causing a steeper slope of the yield curve. This is because investors now feared the effect of ending QE more than an increase in short-term interest rates. The graph below now shows a clear upward slope in the 2s and 10s difference, leading to a steeper yield curve.

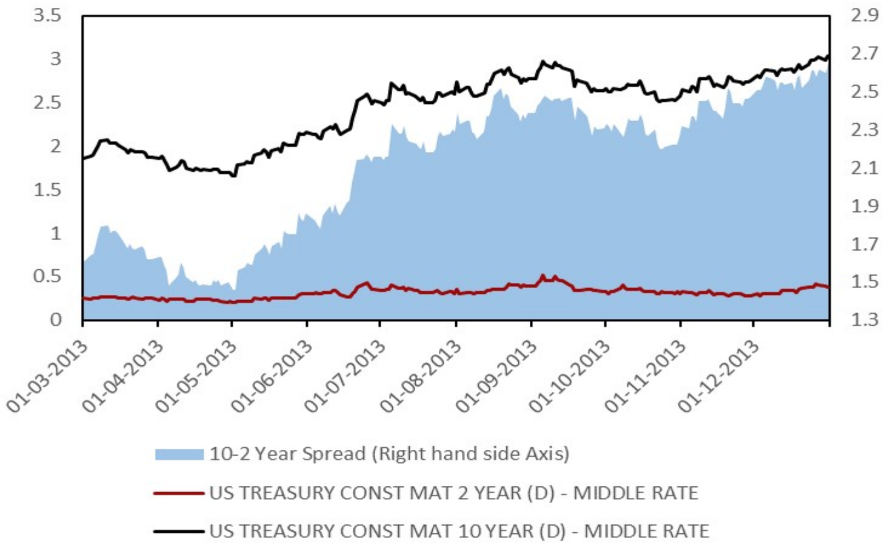


Figure 4.3: Yield Spread 2013
Source: Own Source

Even though the first steps towards tightening the economy were taken. Market factors implied the US economy was not expecting any changes in the interest rates for almost two years. A slowdown in the amount of government bonds purchased would therefore have bigger effect in the long end of the curve.

4.3 Donald Trump

A more recent event also sparked a similar, though maybe not as powerful reaction. During his campaign, Donald Trump advertised tax cuts that would lead to less revenue for governments, higher spending power for consumers, and thereby boosting inflation. Furthermore, he plans to spend money to create jobs, which again would spark wage and inflation growth. The combination gives the government less spending power and increases its spending at the same time. This requires an increase in the already big US government debt. All these factors contributed to the big sell-off in government bonds during the days he was elected, nicknamed "Trump Trade".

His victory wiped out more than 1 trillion USD across the global bond market (Ablan, 2016). During election week, the 10-year Treasury yield moved from 1.80% to 2.30% (Domm, 2016). Many argue that just by getting elected he accomplished what the Fed had been trying for years, namely steepening the yield curve due to increased inflation expectations. The graph below illustrates this effect.

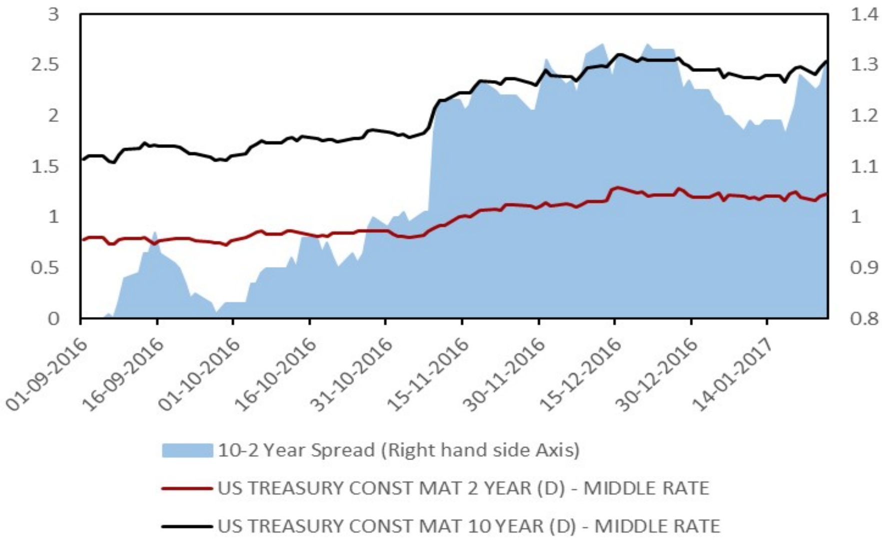


Figure 4.4: Yield Spread 2016
Source: Own Source

The change in yields could alternatively be explained through increased uncertainty in the market, as a reaction to Trumps victory. Both the uncertainty regarding his ability to implement pre-election policies and the consequences of this, contributes to investors requiring a higher return on longer term investments. Hence, an increasing term premium.

History provides clear evidence of how monetary policy affects yield outlook. Different monetary tools also seem to spark different reactions in the market. Policy rate increases have most effect on the short end of the yield curve, as seen from "the great bond massacre of 94". In contrast, QE and political uncertainty influence the longer end, due to inflation expectations illustrated during "Taper tantrum" and the "Trump Trade". All of these events have another common factor, namely that the market has been caught by surprise. We also see this happening despite the increased forward guidance made to minimize the element of surprise in central banks actions.

In today's market, we have multiple monetary policies in play. US have started raising policy rates after a long period with historically low Fed funds rates, and thereby started their path to normalization. ECB is (as usual) a couple of years behind the Fed in their monetary policy. In Europe, QE is still active and scheduled to last until the end of 2017. Common for both economies are the extreme levels of interests we are observing today.

4.4 Sub-Conclusion

An asset bubble is defined as a mispricing compared to what would be consistent with efficient markets, and thereby the fundamental value of an asset. The fundamental value of a bond is inversely related to its yield, which is mainly determined by three factors; safe rates, term premium and risk premium. The safe rate is strongly dependent on the natural rate, which we argue, has fallen dramatically over the last years and even entered negative territory for sub-periods of time. This is due to long-term slowdown in economic growth, a trend entitled Secular Stagnation. Therefore, we find much support for the low interests set by the central bank, rejecting the hypothesis that short-term rates are in a bubble.

However, long-term rates are almost as low as short-term, and we have even experienced negative term premium. Both in EU and US, the QE programs has had strong influence on longer term yields, bringing them down to artificially low levels. This is not part of the fundamental pricing of government bonds, but strongly influence its price and thereby also breaching with the efficient market hypothesis. These programs have further ramifications that creates an unsustainable and unnatural demand for bonds, which lowers the yield. Based on this, we conclude that long-term yields deviate from its fundamental values and we are in fact in a bubble.

Finally, we looked at historical events with similar characteristics to today's market. During the bond massacre of 94, a turnaround in interest rate policy by Fed caused a dramatic reaction in the bond market. This mostly affected short-term rates, thus a flatter yield curve. After this event, the central banks have focused on forward guidance to avoid these type of market reactions in the future. This is therefore less likely to happen today, as the central banks have much more effective communication with the market.

Taper Tantrum was a similar event to that of 94, but this time the turnaround was in the QE program. This caused a stronger reaction in the long end of the yield curve and thereby steepening its slope. Despite of the forward guidance scheme, the first announcement to "taper" did cause a big reaction in the marketplace, due to the expectations of lower demand and decreasing prices. Hence, this is clearly a scenario that could play out in the Eurozone, and supports the argument that yields are being kept at unnaturally low levels. The final and most recent event was the reaction caused by the newly elected president Donald Trump. His victory increased inflation and policy rate expectations, also heavily influencing the longer end of the yield curve.

Trying to come with an exact estimate of the "correct" yield would be close to impossible. We will therefore in the upcoming section perform our own estimations, combined with economical intuition to find scenarios that would support our theories. The scenarios are meant to cover a wide range of possible outcomes, and thereby allowing us to see a broad specter of results.

5. What is The "Fair" Rate?

In this chapter, we will first argue our choice of model to estimate the yield curves, and thereafter explain the theory behind the model. Finally, we present the results of our estimations and introduce a scenario analysis that will be used throughout the rest of the thesis.

5.1 Interest Rate Modelling

When estimating the yield curve, we will enter the theoretical framework of term structure modeling. These models describe the evolution of the zero-coupon interest rate over a determined period of time. We will estimate this, mainly by using models constructed to specify the behavior of the instantaneous short rate. This is the rate movements for an infinitely short period of time. When applying closed form solutions to a complex problem like term structures observed in the real-world, assumptions about the market behavior has to be made.

There are mainly two types of models that dynamically express the term structure; no-arbitrage models and equilibrium models. No-arbitrage models have the advantage that they start with the observed term structure as of today, and the expected interest rate volatility, that relates the bond price and market data at a certain point in time. These models are therefore useful when valuing the price of a bond or interest rate derivative, given the market situation we are in. On the contrary, equilibrium models usually start with some assumptions about the economic variables. We can thereby derive a process for how the development of the short rate will be in the future. Thus, equilibrium models are forward looking and not trying to estimate the observed term structure, but rather foresee where it could develop, given today's level. Another important factor in these models is that equilibrium models usually assumes that interest rates follow a process with mean reversion. Thus, that interest rates tend be pulled back towards a reversion level. This implies that low rates tend to have positive drift, while high rates have negative drift. To what extent they are pulled back towards the reversion level, proves to be a decisive factor in our thesis. Mean reversion also makes sense from an economic standpoint, as low rates increase lending and consumption until we revert to our mean level.

The main difference between no-arbitrage and equilibrium models is therefore that no-arbitrage models uses the term structure as input, while equilibrium models give the term structure as output, given the parameters estimated (Hull, 2012). Another way of interpreting this difference is that no-arbitrage models assumes the term structure observed in the market today as the "correct" one, while equilibrium models allow us to disagree with the market, and come with an estimate of where rates will develop. As we strongly believe the interest rates will develop in a different fashion than seen today, we will use equilibrium models.

5.2 Vasicek Model

One of the most recognized equilibrium models is the Vasicek model. This is also one of the simpler one-factor models and has the advantage of allowing for negative rates, a relevant feature in the current market. For the Vasicek model, three main assumptions are to be made:

A1: *The instantaneous short rate follows a diffusion process*
A2: *The price of a discount bond depends only on the spot rate over its term*
A3: *The market is efficient*

For one-factor equilibrium models, the short rate evolves with only one parameter of uncertainty. This process can be defined as a general process:

$$dr = m(r)dt + s(r)dz \tag{5.1}$$

Where the factors m and s are independent of time, but evolve as a function of r. This implies that all rates move in the same direction, but not by the same amount.

This amount is *m(r)=a(b-r)* and *s(r)=*σ. The process assumed in the Vasicek model can therefore be written as:

$$dr = a(b - r)dt + \sigma dz \tag{5.2}$$

Where a, b and σ are constants independent of t. Parameter b marks the equilibrium level that the short rate is pulled towards with the power of a. Hence, the zero-coupon yield curve can have both positive and negative slope, depending on whether we are above or below the equilibrium rate. The σ parameter imposes a stochastic term on the process, which is the normally distributed term σdz. The diffusion process of assumption 1 is therefore in the form of an Ornstein–Uhlenbeck process (Vasicek, 1977). Hence, the future development of the term is unrelated to the previous development, and the only state variable that influence the process is the current value. The price can through this process be determined as:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{5.3}$$

Where

$$B(t,T) = \frac{(1 - e^{-a(T-t)})}{a} \tag{5.4}$$

And

$$A(t,T) = exp\left[\frac{(B(t,T) - T + t)(a^2b - \sigma^2/2)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right] \tag{5.5}$$

When the zero-coupon price or discount rate is found, we can use the following formula to derive the yield:

$$R(t,T) = -\frac{1}{T-t}ln(P(t,T)) \tag{5.6}$$

We can thereby obtain the entire term structure for different values of t, once we have estimated a,b and σ.

5.2.1 Estimating the Parameters

In academic research, there are two main approaches for estimation of variables; Least-squares estimation (LSE) and Maximum likelihood estimation (MLE). LSE has the advantage of not requiring any strict distribution assumptions to be satisfied, and is useful to obtain a descriptive measure for the purpose of summarizing observed data. However, this method has no basis for testing hypothesis or estimating confidence intervals (Myung, 2003). Furthermore, MLE has the advantages of sufficiency, as the parameter estimate contains full information. Efficiency, as the estimated parameter has the lowest possible variance and parametrization invariance, which gives the same solution independent of parametrization used. None of these advantages are satisfied in the LSE model.

One can summarize the difference as LSE seeking the parameter that provides the most accurate description of the data, while MLE finds the parameters that are most likely. We will therefore in our estimation of the *a, b* and σ parameters apply the MLE method, as we are more interested in the future outcome, rather than explaining historical numbers. MLE finds the parameter values that makes the actual values lie at the mode of the density function over sample paths, i.e. the actual outcome has maximum probability.

For the Vasicek process defined above, we set θ=(α,b,σ) and follow the procedure to perform the maximum likelihood estimation from (James and Webber, 2000). We can then write the transition density function as:

$$p(t_{i+1}, r_{ti+1}; t_i, r_{ti}|\theta) = (2\pi var_{ti})^{-\frac{1}{2}} exp\left(-\frac{1}{2}v^2(r_{ti}, r_{ti+1}, \triangle t_i)\right) \tag{5.7}$$

Because all time increments are equal, we can simplify the formulas by writing:

$$\triangle\, t_i = t_{i+1} - t_i = \triangle t \tag{5.8}$$

This also allows us to estimate the *Var* term as a constant across time with the expression:

$$Var = \frac{\sigma^2}{2\alpha} * (1 - e^{-2\alpha\triangle t}) \tag{5.9}$$

The final calculation needed before we can estimate the likelihood function is dependent on the interest rate level over the entire sample period:

$$v(r_{ti}, r_{ti+1}, \triangle_{t_i}) = \frac{r_{ti+1} - (b + (r_{ti} - b)e^{-\alpha\triangle t})}{\sqrt{var}} \tag{5.10}$$

We can now define the likelihood function as:

$$L = \left(2\pi\frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha\triangle t_i})\right)^{-\frac{N-1}{2}} exp\left(-\frac{1}{2}\sum_{i=1}^{N-1} v^2(r_{ti}, r_{ti+1}, \triangle t_i)\right) \tag{5.11}$$

Because the likelihood function above and its logarithmic value are completely related to each other, we obtain the same MLE estimates by maximizing either one. Since the log expression is easier to maximize, we define the log-likelihood function as:

$$lnL = -\frac{N-1}{2}ln2\pi - \frac{N-1}{2}ln\left(\frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha\triangle t})\right) - \frac{1}{2}\sum_{i=1}^{N-1} v^2\left(r_{ti}, r_{ti+1}, \triangle t\right) \tag{5.12}$$

Using this logarithmic expression, we can estimate θ=(α,b,σ) by:

$$\hat{\theta} = arg\ max\ lnL(\theta) \tag{5.13}$$

Once the parameters have been estimated, we can calculate the yield curve that corresponds to these values using formulas 5.3 to 5.6. Further, we can draw the distribution from our estimation. Since the parameters are determined on a historic basis and enables us to estimate a diverging yield curve, it is also convenient to know the distribution of the estimates. Given our solution to the MLE estimation we will first calculate the steady state probability function (Wilmott, 2006):

$$P_\infty(r) = \sqrt{\frac{\alpha}{\pi}} \frac{1}{\sigma^2} e^{-\frac{\alpha(r-b)^2}{\sigma^2}} \tag{5.14}$$

Which is normally distributed with:

$$P_\infty \sim N\left(b, \frac{\sigma}{\sqrt{2\alpha}}\right) \tag{5.15}$$

This allows us to get a clearer picture of the likelihood of possible outcomes for the risk neutral spot rate. This is also convenient from a risk analyzing point of view.

5.3 Estimations

When estimating the parameters, we need historical data for the short rate. The input in the model is supposed to represent the instantaneous short rate. There is however difficult to find a reliable data source for this rate. The concept of an instantaneous short rate applies more to an academic desire, than real-world property. Therefore, we need to find the historical interest rate that would represent the "shortest" maturity available.

The usual time scape for a yield curve is 1 month to 30 years. The shortest rate available is therefore the 1-month interest rate. This could be calculated into very small increments of time, and thereby use the results as the instantaneous short rate. However, this is not very reliable. As we mentioned previously, the yield curve is not perfectly correlated across maturities, and have different aspects and expectations priced into it. Estimations of interest rates for shorter maturities than 1 month can also be found. For example, Datastream offers data on the 1 week Euribor rate. This source is however questionable, as this rate tends to be higher than the 1 and 3-month rate listed as the official rate on ECB's own website (ECB, 2017). We will therefore in our estimations stick to the more "official" rates listed on Bloomberg as this represents the shortest maturity on the yield curve.

Another implication is the historical availability for the shortest maturities. For the American market, Bloomberg have historical data for 1-month Treasury bills back to mid-2001 and 3-month Treasury bills back to 1960. For German yields the only reliable estimate is the 3-month rate, which is traced back to 2002. We will therefore be consistent by using the 3-month yield for both the American and German yield curve. The difference between the yields on the short end of the yield curve is also observed to be minimal across small differences in maturity.

5.3.1 US Parameter Estimation

The average 3-month Treasury bill rate was 4.71% over the entire available sample period, ranging from February 1960 until today. However, this level has changed vastly over the years, mostly in a downward sloping fashion. More recent history provides a quite different estimate, as the average ranging from 2000 until today for the same rate is 1.63%. The estimation period will therefore have big implications, especially for the equilibrium parameter b.

Applying the MLE procedure to the first time span gives an equilibrium level equal to 4.17%. This looks like a reasonable equilibrium, as it is fairly close to the mean of the sample. Applying the same model to the limited time span from 2000 until today, gives an equilibrium of 0.0752%. This might seem unreasonably low, but as the interest rate have been in a long-term negative trend towards approximately this level, the model assume this is the level we are pulling towards. This implication of the model will be further analyzed in section 5.3.3.

The two other estimated parameters are the pull ratio; α and the volatility of the short rate; σ. The pull parameter seems to be lower over the entire period compared to the later time horizons. We also see more changes in the volatility. This seems logical, as the longer time horizon consists of many periods with both high and low rates, while the more recent period only experience decreasing rates. The results of the US parameter estimation are summarized in Table 5.1:

Parameter	1960-2017	2000-2017
α	0.1140	0.1912
b	4.1680%	0.0752%
σ	1.6103%	0.7249%

Table 5.1: Results from Parameter Estimation US 3-Month
Source: Own Source

We can now, through the use of formula 5.14 and 5.15 estimate the distribution for the rates, given the parameters observed. The probability density function is illustrated in Figure 5.1. For the entire sample period, the distribution looks very realistic. We however notice the very large specter of outcomes. This is again because of the big changes in interest rates during this period. For the later time span we notice that the mean is approximately zero, a much lower volatility is observed, and therefore a narrower distribution.

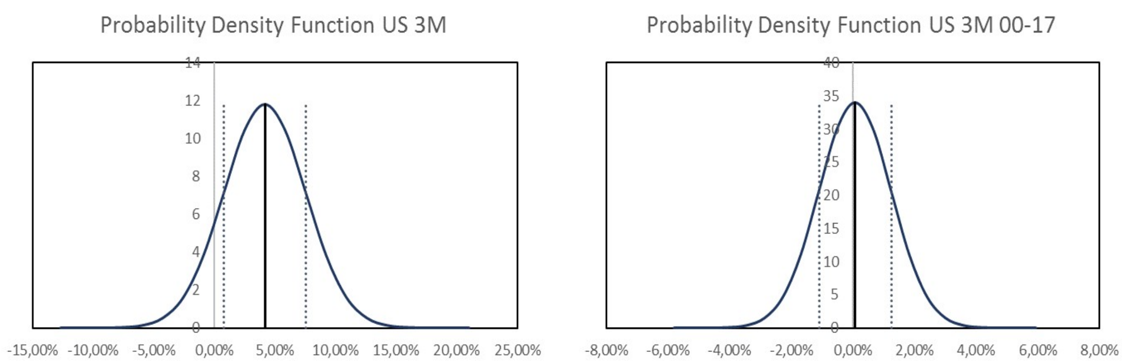


Figure 5.1: Probability Density Distributions US
Source: Own Source

5.3.2 German Parameter Estimation

German data spanned over a shorter time horizon and pull towards negative rates, which strongly influences our MLE estimation. Over the covered period, the average 3-month interest rate was 1.22%. The MLE-results shows an equilibrium parameter of -1.81%, a pull towards this level of 0.09 and a volatility of 0.69%. To get a longer term perspective on the estimated parameters we also include the 1-year rate, as Bloomberg has data back to 1995 for this yield. Running the estimation on these two periods gives the results illustrated below in Table 5.2:

Parameter	**3M: 2002-2017**	**1Y: 1995-2017**
α	0.0892	0.0906
b	-1.8129%	-1.0615%
σ	0.6876%	0.6459%

Table 5.2: Results from Parameter Estimation GER 3-Month and 1-year
Source: Own Source

We notice that the estimation is somewhat similar, though a slightly higher interest rate level is observed for the longer data set. When drawing the distribution, we get an unrealistic result compared to what we consider rational from a real-world perspective. This is illustrated through the distribution, where a positive rate is outside one standard deviation away from the mean.

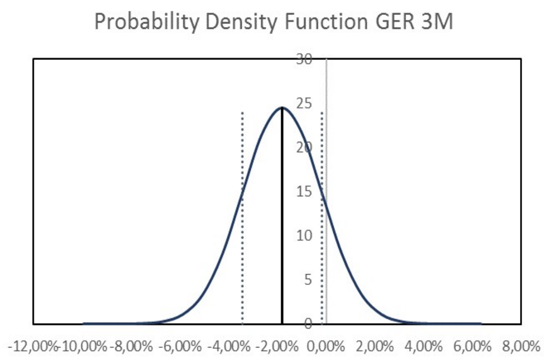


Figure 5.2: Probability Density Distribution GER
Source: Own Source

5.3.3 Interpreting the Parameters

The low values, especially for the equilibrium term might seem unintuitive and not justifiable. This is one of the big drawbacks using closed form solutions to explain something as complicated as term structures. If we decompose the formulas, it is logical why we end up with these estimates of the equilibrium level. Looking at the final calculation in the MLE estimation from formula 5.12, the only time and rate dependent variations comes from the final term:

$$-\frac{1}{2}\sum_{i=1}^{N-1} v^2\left(r_{ti}, r_{ti+1}, \triangle t\right) \tag{5.16}$$

The time dynamic part of this term, given formula 5.10 can be written as:

$$r_{ti+1} - (b + (r_{ti} - b)e^{-\alpha \triangle t}) \tag{5.17}$$

As we try to maximize the total expression in 5.12, we want the last negative part to be as small as possible. Given that we are almost in a continuous declining trend, typically: $r_{ti+1} \leqslant r_{ti}$. The term $r_{ti+1} - (b + (r_{ti} - b))$ is minimized by keeping b low or even negative (given that we know α is somewhat consistent around 0.2-0.8 for "normal" time periods and $\triangle$t is constant). The opposite is true for positive trends. Therefore, even though our average interest rate over the sample period is a positive number, the declining trend forces the MLE estimate of our equilibrium to become negative.

5.3.4 Comparing With the Bond Massacre

As seen in section 4, we have multiple examples where a turnaround in the interest rate policy and trend, caused a violent reaction in the market. Therefore, we found it interesting to investigate what parameters the MLE method would estimate during the bond massacre event. This, because the turnaround in policy and rates reminds of the environment experienced today. This will also help us determine the parameters we will use later, when performing a scenario analysis. We started by running the MLE estimation on the parameters during 1994 with daily observations (to have enough data). Thereafter we ran another estimation for the 10 years leading up to 1994, to underline the change in the parameters, and effect of trend in interest rates on the equilibrium level. The results are summarized in Table 5.3.

Parameter	**1984-1994**	**1994**
α	0.10498	0.4795
b	0.5380%	9.4888%
σ	1.1348%	0.8306%
Mean	6.44%	4.25%

Table 5.3: Results From Parameter Estimation Before and During 1994
Source: Own Source

First, we notice the α term that increases from 0.11 to 0.48. An increase was expected, and gives further confidence in the estimates. This implies the interest rates was pulled towards the equilibrium with a powerful reaction.

Furthermore, we notice the equilibrium levels as they contain a lot of information. For the 10 years leading up to 94 we see a very low estimate compared to the mean over the same period. This is the effect we explained in section 5.3.3, illustrated in historical data. Because of the continuous decline in rates, the time dependent term illustrated in equation 5.17, becomes smaller by selecting small b-values. During 1994 we have the opposite effect. Due to the big increases in rates, r_{ti+1} becomes significantly higher than r_{ti}, which causes a big positive b-value to minimize the dynamic expression. We therefore see an equilibrium level, far bigger than the mean over the same period. Finally, we notice the volatility estimate to be lower for 94 because the interest changes are more consistent in one direction.

These estimates are based on the development through the entire year. Thus, the results are not as dramatic regarding the α term, as a shorter time period would have been. It could be argued that the level we are trending towards would have to be estimated over a longer time period. However, as we imagine a dramatic change in rates would happen rather quickly, it is relevant to get a shorter perspective on the parameter values. We therefore used the Vasicek model to minimized the sum of squared error between the estimated and actual yield curve observed at its steepest through 1994. The estimated parameter was then α=0.99, b=7.8% and σ=0.03%. The real and estimated yield curve on this day, can be seen in Figure A.1 in the appendix, where we see a close to exact match of the actual curve. This result should be interpreted with caution, and rather work as an indication of how steep the curve can become during these type of events, than prolonged parameter estimates.

5.3.5 Drawbacks

Single-factor models do offer some reason for criticism. The fact that the model depends on a single factor, has big implications for the possible shape of the yield curve. This factor has to be the foundation or starting point for the entire yield curve. In the real-world, there is of course no such limitation. Another weakness is that the closed form one-factor solution creates a perfect correlation across all maturities, which is another unrealistic assumption compared to the real-world. As many other empirical models, and specifically interest rate models, Vasicek has great sensitivity to the data source and periods, as we saw above. To achieve reliable estimates, long-term history is important and for many interest rates this was difficult to obtain. Also, the creation of the ECB and harmonization of short-term rates across its membership countries has made many national samples unreliable.

5.4 Scenario Analysis

To assess the sensitivity in the US and EU, we divide possible outcomes into three main scenarios; Goldilocks, Reflation and Japan scenario. These scenarios are set to cover a broad range of possible outcomes of the interest rate development. Hence, these scenarios will give a basis to discuss the possible consequences for the markets, and at the same time allow us to come up with concrete estimates. The three scenarios are described below:

The **Goldilocks** scenario implies a balanced growth and stable interest rates. Hence, an economy which is not so hot that it causes inflation, nor so cold that it causes deflation. This scenario could be characterized as an economy with sustained growth and low inflation.

The **Reflation** scenario is built against the same long-term interest rate target as the Goldilocks scenario. However, here it is implied a more dramatic adaption towards the long-term target. If this scenario occurs, increased growth and increased bond yields should be expected.

The **Japan** scenario implies low interest rates for a sustained period. Hence, a scenario which implies a permanent shift in the equilibrium rate. This scenario is characterized with very low growth.

Below, the different scenarios is explained in an economic context, and the parameters used for each scenario is summarized in Table 5.4.

5.4.1 Deciding the Parameters From a Real-World Perspective

Clearly, the estimated parameters above seem unrealistic for future levels. Due to the continuous decline in interest rates, the estimates become misleading. Neither could we expect a model to come up with perfect estimates for such a complex problem. When deciding the parameters for our estimations, we will therefore use economic reasoning to adjust for factors the model does not incorporate. We will therefore provide our economic intuition to what the parameters used for forecasting should be.

As argued in section 3, the natural rate is likely to have decreased over the last decades. On Fed's own website they provide their own research on the estimates of what the Fed funds target rate should be. Figure 5.3 shows the span of Fed projections for the long run federal funds target rate, all but the highest and lowest three estimates, and the red dot marks the median projection.

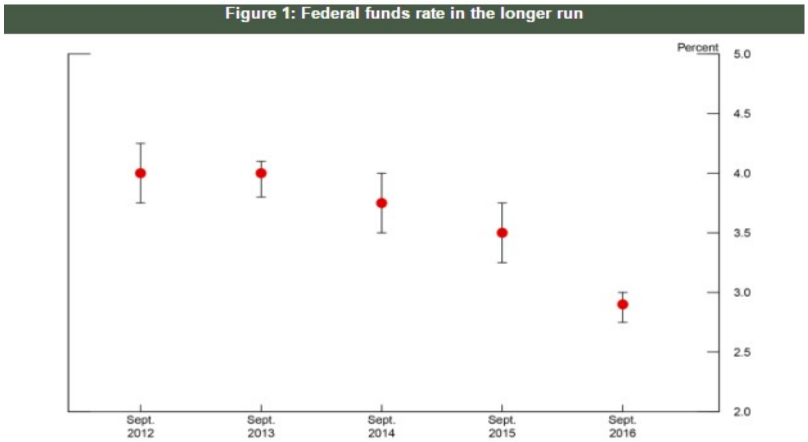
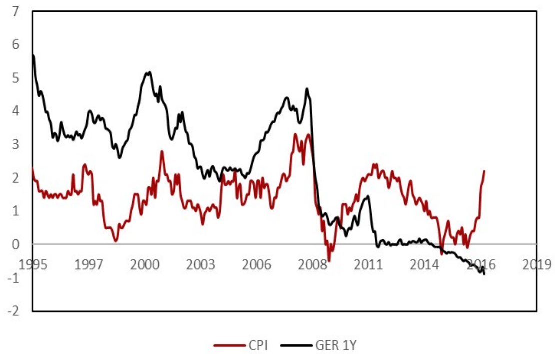


Figure 5.3: Fed Target Rate Prediction
Source: (Bongard and Johannsen, 2016a)

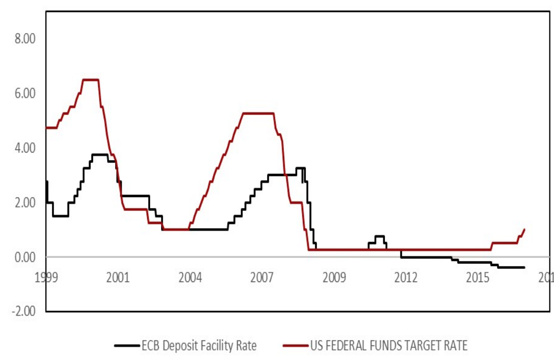
This shows a decrease from approximately 4% in 2012 to approximately 3% at the end of 2016. Inflation expectations have through this period remained somewhat constant around 2%. This indicates the change comes from lower natural real rate expectations, as we argued earlier. Currently the federal funds rate is at 0.75-1%. More hikes are expected through the upcoming years, until Fed themselves expect to be approximately at their 3% "desired" level in 2019-2020 (Bongard and Johannsen, 2016b).

On the "other side of Fed", we do however have a president arguing that the Fed is keeping interest artificially low through politics and thereby hurting the savers. Trump has stated he believe to spike growth up to 4%, a number not seen since 2000 (Rao, 2017). Even though many economists argue this target is not realistic, he is expected to increase inflation, which would have impact on both the Fed rate and Treasury yields.

The ECB's target inflation rate is just below 2%. Achieving this is also the main target for ECB's policy. Figure 5.4a shows the development of German 1-year yield and inflation. The correlation between them is visually clear. However, lately a clear deviation has arisen. For the first time, there is clear divergence in their paths, and only after the financial crisis of 2008 has the inflation outperformed the yield. We also notice a clear spike in the inflation without much reaction from the yield. Nor do ECB seem skeptical that HICP inflation has risen above their target rate. This, they argue is mainly due to the increased energy prices, and thereby not sustainable.



(a) German CPI + 1Y Yield



(b) Central Bank Rates

Figure 5.4: Comparison Between CPI+Yield and Central Bank Rates
Source: Own Source

Figure 5.4b Shows the correlation between the US and ECB short-term rates. We not only see a clear correlation, but also a tendency, where US rates move a bit before the ECB rate. The US economy is the biggest in the world and therefore often the economy and central bank leading the way. However, both these graphs suggest that the ECB is also likely to have a turnaround in politics fairly soon.

Because the interest set by the central bank is the short-term rate, it is easily comparable to the rate implemented in the Vasicek model. As discussed, yield on government bonds should contain appropriate term premium to compensate for the uncertainty the future holds. For this reason alone, longer term securities should have a higher expected yield than the federal funds target rate. Especially today, the uncertainty of the future is big, which should generate a higher term premium. The geopolitical risk seen in the world today, could easily be argued to be at its highest for a long time. The new Trump administration and their relationship to the outside world, fragile state of the Eurozone and current right wing populism, are just a few of the factors that could influence the required return on holding longer term government bonds.

For these reasons, we will set the equilibrium level at 4.5% for the US Economy and slightly lower at 3.5% for the German economy. As for the volatility term, there are a number of different factors that could have enormous effect on this parameter. However, our estimates (especially for the later time periods) are fairly consistent around 0.65%. We will therefore keep our estimate consistent at this level, even though an argument could be made that during the time span of our analysis, rates are more likely to move in one direction and therefore contain lower volatility.

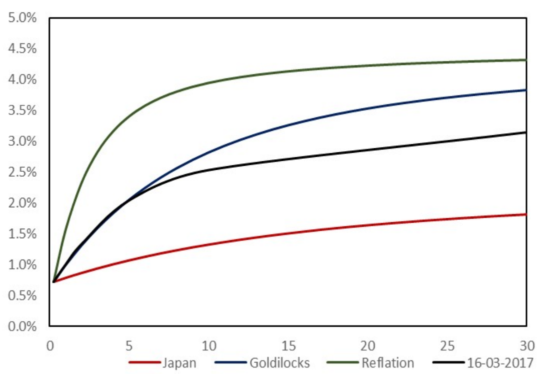
The most crucial parameter for a scenario analysis, is how fast we will adjust to this long-term level. Listening to the Fed and ECB's press conference it becomes clear that they are trying to achieve an increase in rates at a gradual pace. This is why they continuously update the public about their thoughts and actions, and why we consider the Goldilocks scenario, the desired scenario from the central banks perspective. The estimates above provide some evidence that the "normal" range over time usually lies between 0.05 and 0.2. We also have a clear tendency to see lower estimates of the α term for German estimates compared to US. Along with the argument that United States is the "leading" economy, we will keep the pull-back ratio slightly lower for Germany, compared to US, for the Japan and Goldilocks scenarios. We will however keep them higher than the MLE estimates, as we now expect to see a more determined development in rates, if the environment actually changes.

During the Reflation scenario, we want to assess the "worst case", that is a quick adjustment or turnaround in market behavior towards the equilibrium level. When analyzing the effect during 1994 we saw an α of 0.48. However, this was estimated through the whole year, and increased substantially when looking at shorter time horizons. We will therefore argue that a level of 0.7 is not unreasonable over a shorter time span. Even though this level consistently has been lower for the Euro-zone, the fact that rates are so low leaves a bigger room for a spike, and will therefore be kept at 0.7 for both economies. The final parameter values are summarized in Table 5.4 below:

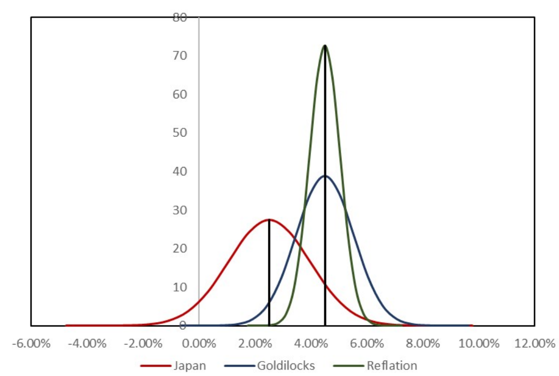
Country:	**US**			**Germany**		
Parameter:	α	b	σ	α	b	σ
Japan	0.1	2.5%	0.65%	0.05	1.5%	0.65%
Goldilocks	0.2	4.5%	0.65%	0.15	3.5%	0.65%
Reflation	0.7	4.5%	0.65%	0.7	3.5%	0.65%

Table 5.4: Parameters for Scenario Analysis
Source: Own Source

By using these parameters, we can now estimate the yield curves and probability density distributions for each scenario. For the US scenarios, we get the following estimates:



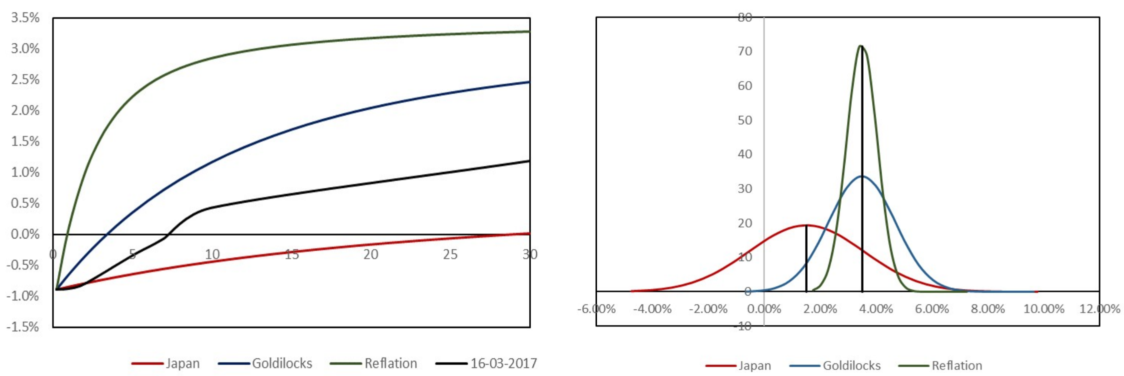
(a) ZC curves



(b) Probability Distribution

Figure 5.5: US Yield Curve and Probability Distribution
Source: Own Source

We notice how a big α causes a lower standard deviation and thereby more narrow distribution with excessive kurtosis, leaving a relatively small spectrum of possible outcomes. The intuition behind this is that interest rates are very determined to move up to the equilibrium b, and consistently move in this direction. For the German estimates, we see the same tendency, although a lower mean reversion level:



(a) ZC curves (b) Probability Distribution

Figure 5.6: German Yield Curve and Probability Distribution
Source: Own Source

From both the German and US estimation we have two rising rates scenarios (Goldilocks and Reflation) and one falling rates scenario, compared to the actual levels.

5.4.2 When will rates change?

Timing a specific market reaction has proven to be a difficult task. Knowing what will trigger a reaction, or if the central banks will manage to increase the policy rate at a gradual pace, will reveal itself over time. They do however publish projections as to how the monetary policy will develop. As explained in section 5.4.1, Fed imagine to be at their "desired" policy rate around 2019-2020, while ECB have announced their asset purchase program will last until at least December 2017. Further, this is likely to be followed with gradual increases in policy rates over time.

In section 4, we saw how the 94-event caused Fed to start with forward guidance regarding policy rates. Further, after taper tantrum, it seems that both Fed and ECB has become very cautious about their forward guidance with regards to the QE program. Considering the small market reactions and clear communication from the central banks thus far, we believe this tightening cycle will evolve rather smoothly and at a gradual pace, over the next couple of years. Hence, we consider the Goldilocks scenario most likely.

There are however possible incidents that could spark a more dramatic reaction, similar to our Reflation scenario. A change in the environment led by one surprising event could start the psychological fight to be "first out the door". A poorly communicated or surprising change in the monetary policy from a central bank could be such an event. Better than expected macroeconomic data could be another, or as mentioned in section 5.4.1, geopolitical risk could raise the required return. All these factors could make interest rates change instantly, and thereby realize a Reflation scenario immediately.

The Japan scenario is in our opinion the most unlikely scenario. This would only be realized if the secular stagnation hypothesis, not only was true, but will continue to worsen and change the interest rate environment as we know it.

5.5 Sub-Conclusion

In this section, we have seen how using the Vasicek model to estimate future yield curves gives unrealistic results, compared to what we have learned about interest rate levels throughout history. As interest rates have declined for decades, the Vasicek model assumes we are progressing towards a strongly negative equilibrium. We have therefore seen it necessary to estimate the parameters, using a combination of mathematical calculations and economic interpretation. As both the ECB and Fed have inflation targets they are aiming for, we believe the actual equilibrium level for long-term rates is between 3.5-4.5%. We have also included a Japan scenario where the equilibrium rate is kept low at 1.5-2.5%. Even though we consider this unlikely as it would return negative real rates, it is included as an alternative. Given the many norms that have already been broken by the recent developments in this market, this cannot be considered completely unrealistic.

A crucial parameter in the model is the pull term entitled α. This term determines how dramatic the change will be. We believe, especially for the Euro area, that there still is room for a big surprise that could change yields dramatically, as seen during Taper Tantrum. We have therefore included a broad range of pull terms from 0.05-0.7. Given our estimations over time and through special events, they all seem plausible for shorter time spans.

With regards to the time-frame of the changes, we believe the central banks will succeed in implementing the changes at a gradual pace, and that this will evolve through the next couple of years. However, there are many realistic incidents that could cause a Reflation scenario to change the interest rates instantaneously. As we consider the rising rates scenarios far more likely than the Japan scenario, we believe investors should be concerned with their interest rate sensitivity.

6. Estimating Sensitivity

6.1 Sensitivity Measures

When assessing US and German sensitivity to changes in interest rate, both duration and convexity are important parameters. This is to be able to assess the risk associated with these changes, in addition to an eventual size of profit and loss. For bonds with high credit rating, thus very low credit risk, yield to maturity (YTM), duration and convexity are the main components when determining the attractiveness of the investment (Fabozzi, 2004). These measures will therefore be the main part of the sensitivity analysis, combined with an analysis of the investment horizon. The Macaulay duration (duration) is known as a measure of the interest rate sensitivity. However, duration assumes a linear relationship between price changes and yield changes for bonds. In reality, this relationship is more convex. To be able to estimate this price sensitivity, duration, modified duration and convexity has to be defined. The duration is defined as:

$$D = \sum_{t=1}^{m*T} w_t \frac{t}{m} \tag{6.1}$$

Where w_t is defined as:

$$w_t = \frac{CF_t}{(1 + y/m)^t * \frac{1}{B}} \tag{6.2}$$

Here CF_t is the bond holders cash flow at time t, y is the yield and B is the bond price defined as the following:

$$B = \sum_{t=1}^{m*T} \frac{CF_t}{(1 + y/m)^t} \tag{6.3}$$

We can now define the modified duration as:

$$D^M = \frac{D}{1 + y/m} \tag{6.4}$$

Where m defines the number of coupon payments per year, and will therefore be 1 or 2 in our estimation, as we focus on bonds with annually or semiannually payments; US government bonds pays semiannually coupons, while the German bonds pays annually. Further, the convexity for coupon paying bonds is defined as:

$$C = \sum_{t=1}^{m*T} w_t * \frac{t}{m} * \left[\frac{t}{m} + \frac{1}{m}\right] * \frac{1}{(1 + y/m)^2} \tag{6.5}$$

This formula is somewhat simplified for zero coupon bonds and can be written as:

$$C_{ZCB} = \frac{t * (t+1)}{(1+y)^2} \tag{6.6}$$

To capture the mean reversion effects of the short rate changes, Vasicek suggest another framework for assessing bond price sensitivity to the short rate for zero coupon bonds:

$$\hat{D} = B(t, T) = \frac{1 - e^{-a*(T-t)}}{a} \tag{6.7}$$

Where a is constant and the (T-t) varies based on the maturity of the zero coupon bond. This approach is generally used as a measure for shorter maturities because it implies almost constant zero coupon yield at the longest maturities. Therefore, it may fail to capture the movements in the long end of the yield curve (Wu, 2000). This could be seen as the duration equals $\frac{1}{a}$ as the maturity goes towards infinity, and is therefore the maximum duration when using the Vasicek model (Nawalkha, Soto, and Beliaeve, 2005).

To strengthen the sensitivity precision and capture the curvature, convexity is needed (Jensen, 2009). A framework for calculating the sensitivity which incorporates these aspects, is the expanded Taylor line (Čerović et al., 2014):

$$\frac{\Delta P}{P_0} = -D^M \Delta y + \frac{1}{2}(C - D^{M2})(\Delta y)^2 \tag{6.8}$$

Where y is defined as the yield, and $\frac{\Delta P}{P_0}$ is defined as the expected percentage change in the bond price caused by a yield change. It is found that this approximation gives similar results compared to the actual change, for interest rate changes of less than 100 BP. Further, this formula provided inaccurate results with changes greater than 300 BP (Barber, 1995).

Using different methods will give a clearer picture of the possible uncertainty of the sensitivity for interest rate changes. This uncertainty is further strengthened for larger interest rate moves, as the relationship between the change in rates and the change in bond prices is asymmetric. Hence, a bond price decrease resulting from a large interest rate increase, will in general be smaller than a bond price increase, as a repercussion of an interest rate decline of the same magnitude (Bennyhoff and Zilbering, 2010). Therefore, bonds with higher convexity is more desirable for investors, as the upside is bigger than the downside. The sensitivity measures must therefore be interpreted cautiously, and should be used as a tool to discuss the sensitivity rather than be thought of as an exact measure. Different methods are generally used to increase the robustness of our estimations.

Another method to estimate interest rate sensitivity is Babcock's formula, which captures the change in total return or yield to maturity (YTM $= \tilde{R}$), across different investment horizons (Jensen, 2009):

$$\tilde{R}^* \simeq \tilde{R} + \Delta\tilde{R}^* \tag{6.9}$$

Where

$$\Delta\tilde{R}^* \simeq \left(1 - \frac{V_0}{H}\right) * \Delta\tilde{R} \tag{6.10}$$

Where $\frac{V_0}{H}$ is the ratio of duration to the investors horizon, $\Delta\tilde{R}^*$ is the change in YTM and $\tilde{R}^*$ is the new YTM. This model proves to have a good approximation compared to the actual return, but has the same linearity assumption as mentioned above, due to relying on the duration measure for sensitivity. In contrast to the other methods explained in this section, the Babcock formula has the advantage that it takes investment horizon into account. This will prove to be of major importance when assessing interest rate sensitivity. The effect of time horizon will be analyzed and discussed separately in section 6.2.2.

6.1.1 Comparing the Sensitivity Measures

To assess which sensitivity measures to use throughout the thesis, we have estimated the Taylor line, Vasicek and duration measure. The results of these calculation are plotted below in Figure 6.1 and 6.2, and shows the estimated percentage price change (y-axis), based on changes in the yield (x-axis) for a US and German 30-year bond. The remaining estimation could be found in figures A.2a-A.5b, which contains the sensitivity measures for 2, 5, 10 and 20-year bonds for the respective countries.

The magnitude of the price changes is larger over all maturities (except 20 year) for Germany, compared to US. This is because of a greater duration and convexity. For shorter maturities (2 and 5 years) the German bonds does not pay coupons, while both the coupon and yields are typically lower across maturities. In general, the differences are better displayed for longer maturities, as the sensitivity increases with maturity.

Further, it can be seen that for the shorter maturities (2 and 5 year) all methods give similar price changes when varying the yield. However, for greater maturities the Vasicek method seems unreliable, due to the property of having almost constant duration for longer maturities. Therefore, this method will not be a part of our analysis. However, the duration and Taylor measure is extremely accurate compared to each other up to 10 years, while the convex property of the Taylor line is displayed for the 20 and 30-year bond. However, with a big yield change of $\pm$ 2%, the Taylor line and duration measure gives fairy similar answers. Hence, with yield changes below 2% both methods will be fairly reliable.

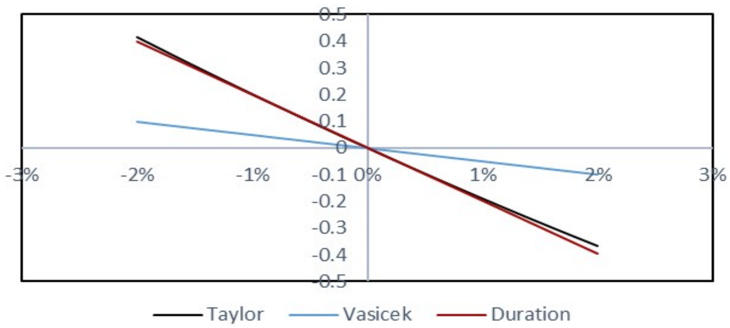


Figure 6.1: US sensitivity for 30-year Yield changes
Source: Own Source

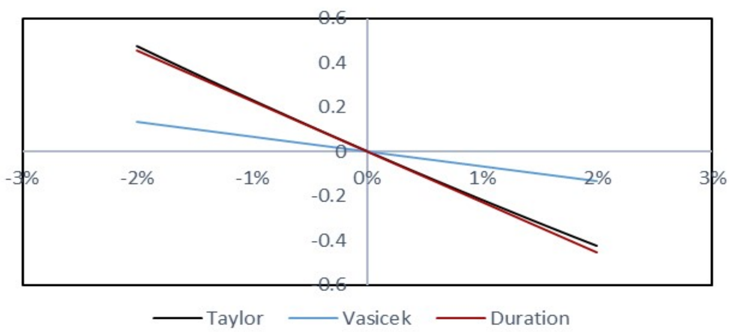


Figure 6.2: German sensitivity for 30-year Yield changes
Source: Own Source

The asymmetric trait of the change in bond prices is not present for the duration and Vasicek estimation. To see if the estimates capture this effect, we have to assess the Taylor model. For the estimations in both US and Germany we can see an increasing upside, compared to the downside. A difference that increases percentage wise with the maturity. This is consistent with the theory that higher convexity is more desirable for investors, as the Taylor line for both countries shows a higher upside than downside for the same magnitude of yield change.

Further, the difficulty of assessing the exact risk for greater yield changes is seen between the different models. The interval of changes up to 100 basis points gives similar results, while smaller deviations could be observed when estimating outside of this interval. On that basis, it could be said that outside the 100 basis point interval, there exists more uncertainty with regards to our estimations.

6.2 Sensitivity Analysis

In order to decide how sensitive these markets are towards yield changes, the distribution within a typical bond market portfolio is important. This as a method to quantify the consequences of a possible rate change. To measure the size of profit and loss, we will therefore construct a proxy portfolio as explained below.

6.2.1 Vanguards Total Bond Market Index Fund

To estimate the effect of rising yields in the market, we have chosen to look at a specific portfolio that could represent the total market with regards to duration and maturities. When searching for an appropriate proxy we started by looking at the biggest holders of government debt in US and Germany. Apart from Fed, Vanguard is the biggest holder in both countries, according to the Bloomberg Terminal. Vanguard is one of the world's biggest institutional investor with over 4 trillion USD under management (Vanguard, 2017b). Among their most popular funds is the Vanguard Total Bond Market Index Fund (VBMFX), which provides a broad exposure to the US bond market. As the characteristics of this portfolio is very generalizing for other countries, we will use this portfolio for both the US and German market.

VBMFX has a Net Asset Value of 176.1 billion USD and holds a wide range of investment grade bonds, but an overweight in US government bonds. The distribution of the weights for each maturity is found on Bloomberg as well as the weighted duration. To be able to perform a realistic hedge on this portfolio, we have divided these weights into "buckets" matching the relevant futures contracts used for hedging later in the thesis. We have therefore used excels numerical solver tool to estimate the actual duration for each maturity bucket, based on the weights and weighted duration that we know from Bloomberg. These measures could therefore contain some small errors, but these are insignificant as this only serves as a proxy for a typical bond portfolio.

Furthermore, adding multiple maturities together, causes some of the buckets to have odd duration measures. For example, the two-year bucket has an estimated duration above 2. This is due to containing duration for both cash market, 1-year and 2-year maturities. Also, the duration for 30-year bonds seems abnormally low. Further, this portfolio contains multiple types of bonds, both coupon paying and zero coupon bonds. Because the duration measure incorporates the coupon payments, we can apply our estimates of yield changes to calculate the new portfolio value. The initial weighted YTM for the German portfolio is 0.083% and 2.25% for the US.

Maturity	**Weight**	**Duration**
2	17.59%	2.27
5	30.17%	4.51
10	35.73%	5.04
20	8.26%	14.29
30	8.26%	14.90
Portfolio	100%	5.97

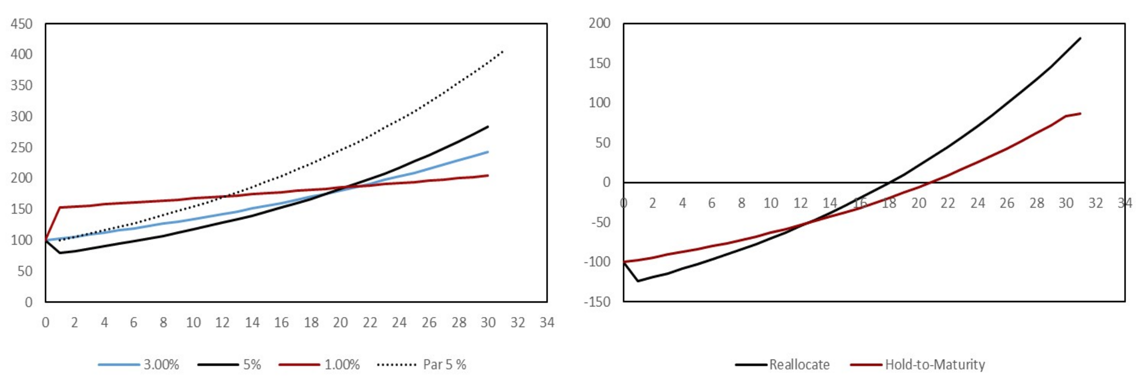
Table 6.1: Weights and Duration
Source: Own Source

6.2.2 Duration, Babcock and Investment Horizon

The investment horizon is of major influence when assessing the interest rate risk, especially for coupon paying bonds. Day to day changes in interest rates causes unrealized changes in the bond price. If rates rise, the price declines and you will lose money if you sell the bond to market value. However, if you hold the bond until maturity, you will (assuming no default) be repaid the principal of the bond, regardless of market value. Further, when rates rise, the coupon payments you receive from the bond can be reinvested in bonds or interest rates that pays a higher coupon. In reality, an investor will therefore face both coupon reinvestment risk, as well as market price risk.

To understand this concept, the relationship between duration and investment horizon is of major importance. Duration marks the point where the gain on reinvestments, due to higher coupon payments, offset the loss from decline in market price. If the investment horizon is longer than the duration of the bond, the coupon reinvestment advantage will outgrow the market price decline, and opposite. The risk of investing in an individual bond and with certainty hold until maturity, is therefore to lower rates, if only concerned with total holding period return in nominal terms.

This effect is illustrated in the black line in Figure 6.3a, where we see how the interest rate rise from 3% to 5% and causes the initial price decline, but generates higher value of the accumulated interest rate payments that is reinvested at a 5% rate. The duration of this 30-year US government bond is 20.19 and marks the point where the disadvantage of price decline is offset by the higher return on reinvestment. The opposite effect caused by a lower interest rate, higher price, but lower reinvestment returns is illustrated by the red line. This illustrates the point that if your holding period is somewhat similar to the duration, you are in fact fairly immune to interest changes, in nominal terms.



(a) Total Return (HTM) (b) Accumulated Profit and Loss

Figure 6.3: Holding Period Return
Source: Own Source

This relationship is also illustrated as a closed form solution in the Babcock formula. The term $(1-\frac{V_0}{H})$ is multiplied with the change in interest. When holding period (H) outgrows duration (V_0), the fraction becomes smaller than one, so that $(1-\frac{V_0}{H})$ is positive and multiplied with a positive change in yield, hence the new YTM increases.

However, the conception that holding until maturity assures no loss, is somewhat misunderstood and relies on whether you incorporate real gain or opportunity costs to the calculations. When investors hold the bond until maturity through rising interests, they receive a real loss, Clifford Assness argue (Asness, 2014). The effect of waiting one year and invest in a new higher interest rate environment is illustrated by the black dotted line. Here we see the payoff for purchasing a bond at par value after the yield increases to 5%. However, this would not be known a year in advance.

Clifford also portrays investors as ignorant to the fact that bonds have a market price for a reason. When you get your principal back, this will be in an environment with higher interest rates and inflation. Hence, you experience lower purchasing power. Further, to be able to receive the principal, you would actually have to lose purchasing power all the way until maturity, which could be painful if you're invested long-term. This effect is illustrated in Table 6.2.

In this scenario, we picture the same 30-year Treasury bond with initial YTM of 3%. After the investment, we assume inflation increases by 2%, which causes interest rates to shift 2% as well. As we hold the bond until maturity, thus exceeds the duration, we have a bigger effect from the reinvestment risk than price decline, and we actually receive a higher YTM at 3.65%. However, as the initial infla-

Period	**Initial**	**Rate Change**
Yield	3%	5%
Inflation	2%	4%
HTM	30	30
YTM	3%	3.65%
Real Rate	0.98%	-0.33%

Table 6.2: YTM Before and After Rate Change
Source: Own Source

tion has risen from 2% to 4%, the real rate of return has dropped significantly, and is now negative at -0.33%. Hence, to experience the effect of rising YTM we actually have to hold the bond for longer than 20.19 years with negative real rate.

Most institutional investors do not hold to maturity anyways. Mutual funds, central and corporate banks typically reallocate their portfolios frequently, in order to maintain desired characteristics through market changes. One example of this is the duration-spiral explained in section 3.2.3. Further, when banks present their balance sheets they have different "buckets" of how securities are handled. Putting a bond in a "held-to-maturity" (HTM) bucket ties up enormous amounts of capital for a long time, and typically makes up a small part of the total portfolio (Alloway and Shenn, 2015). The bigger proportion of the portfolio has to be "mark-to-market" (meaning the market value of the bonds is reported as the assets value), and this part of the portfolio is reallocated frequently. Due to rising rates, lowering the market values on the balance sheet, quickly create problems for the financial institutions. Hence, they actively trade in the market to prevent this from happening, by competing to be "first out of the door" if a crisis were to strike.

Also for bond funds, securities are rarely held to maturity. Fund managers typically assess the market to find the best bonds, based on the objective of the fund and current market condition. This is because different funds have characteristics such as high yield, duration or maturity targets. As illustrated in Figure 6.3b, an active manager would benefit from selling the bond after an initial price decline, and purchase a new bond with a higher coupon after approximately 12 years (this graph excludes the principal received at maturity which is equal for both bonds). Further, funds are often forced to buy and sell bonds, based on investors capital injections and withdrawals into the fund (Thune, 2016). Many investors also tend to pull out during rising rates scenarios.

Figure 6.3a is made on a 30-year Treasury bond for illustrative purposes. 30 years is a long investment horizon and a duration of 20.19 is high for a portfolio of bonds.

VBMFX has a duration of 5.97 and therefore require a much shorter investment horizon to regain the initial loss of a rate increase. However, as mentioned, these funds rarely hold the bond until maturity, but reallocate to find the most profitable way to achieve the desired characteristics. The offsetting point will therefore not be exactly at duration (as it would be for a single bond). Furthermore, the changes in yields differ across maturities, thus the offsetting point could substantially deviate from the duration. It does however give a good indication of how long you would have to hold the fund to get back to zero loss in nominal terms, in our case; approximately 6 years.

6.3 Results

Since the reaction can be thought of as twofold; price risk and coupon reinvestment risk, we will divide our calculations into two sections of results. One where we focus on the Net Asset Value (NAV) or market value of the portfolio, and one where we evaluate how the portfolio would perform with different holding period horizons. When estimating the change in value, we focus on the three scenarios explained in section 5.4. Further, we assume an immediate change in the yield curve, as estimated by the Vasicek model and constant weights given in Table 6.1.

6.3.1 Mark-to-Market

This is the change in market value of the fund or portfolio, and the change in value reported on the balance sheet. Hence, when interest rates rise, NAV decrease and you are obligated to report a loss. The calculations presented in this section is therefore made on a mark-to-market basis, such as the reporting of profit and loss for VBMFX, where the reported percentages are the change in NAV.

The change in portfolio value is calculated based on the change in yield for different maturities, using both the Taylor line, duration and Modified duration. The results are illustrated in Table 6.3.

Country:	**US**			**Germany**		
Scenario:	*Taylor*	*Duration*	*Mod Dur*	*Taylor*	*Duration*	*Mod Dur*
Japan	6.62%	6.76%	6.59%	4.68%	4.69%	4.66%
Goldilocks	-2.06%	-2.12%	-2.06%	-5.27%	-5.33%	-5.29%
Reflation	-7.57%	-7.81%	-7.62%	-13.58%	-13.79%	-13.74%

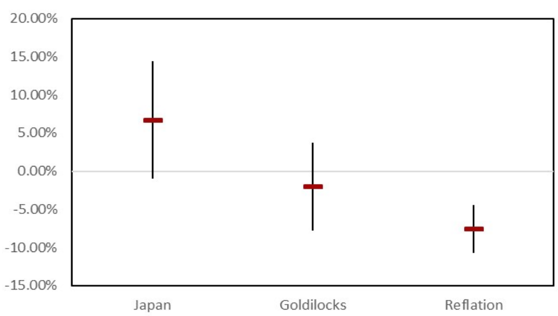
Table 6.3: Profit and Loss of Portfolio for the Scenario Analysis
Source: Own Source

From the table, it is clearly a big difference when comparing US and Germany. For the scenarios Goldilocks and Reflation, the German portfolio loses approximately double to the equivalent portfolio in the US market. This could be partly explained by the difference in today's rate and the equilibrium level. The equilibrium level is set to 4.5% for US and 3.5% for Germany during these scenarios. Today's long-term yield (30 years) is 3.14% and 1.19% for the respective countries. Hence, the long-term yield for Germany is almost 100 BP further away from its equilibrium level, compared to the US. This leads to a greater increase in yield, and therefore greater loss for the portfolio. Further, the low rates also generate higher duration (everything else equal).

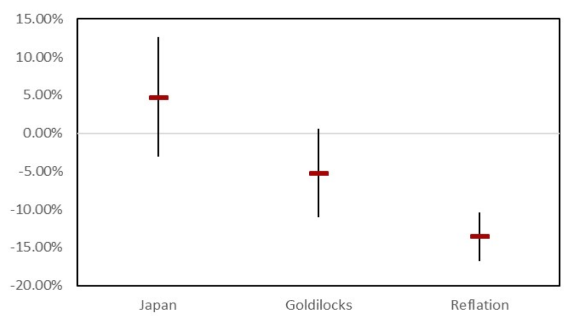
Another interesting observations is that the US Reflation scenario results seems strikingly equal to the percentage loss experienced during "The great bond Massacre of 94", discussed in section 4.1. This gives somewhat credibility in our results, as the experienced interest rate change was fairly similar. During the 94 event, long-term yield shifted approximately 1.5%, and in our reflation scenario, 20-year yield shifted 1.33%. The estimated total value of the US bond market in 94, was 10.25 trillion USD, and resulted in an annual estimated loss of -7.77%. Today this market has an approximate value of 39.36 trillion USD and our estimated loss was -7.57%. This would result in a loss of approximately 3 trillion USD (using the most conservative, Taylor measure). The German bond market has, according to the Bank for International Settlements, an estimated value of 3.46 trillion USD and would see a loss of 470 billion USD (BIS, 2017). These values are single market estimates. Hence, taking the globalized market reaction into account would result in much greater losses.

The Japan Style is the only scenario providing a profit. In contrast to the other scenarios, the US portfolio experience a greater change compared to the German portfolio. This could be drawn back to the equilibrium rate. As the German long-term yield is underneath the equilibrium rate, there is limited space for yield decreases, in contrast to the US with a higher yield than the given equilibrium rate. This occurs even though the pull parameter for Germany is lower than the US.

Based on Table 6.3 we can conclude that the German market is more sensitive to interest rate changes. This is shown through a lower upside, together with a higher downside compared to the US portfolio. To further assess these markets, an interval for the losses is illustrated in Figure 6.4. The interval is estimated using the Taylor model, and the middle mark for each interval is the same as shown in Table 6.3. Further, the upper and lower bound is estimated based on the given yield curve $\pm$ one standard deviation away from this curve. These yield curves are illustrated in Figure A.6 and A.7, and the standard deviations are calculated using formula 5.9.



(a) US



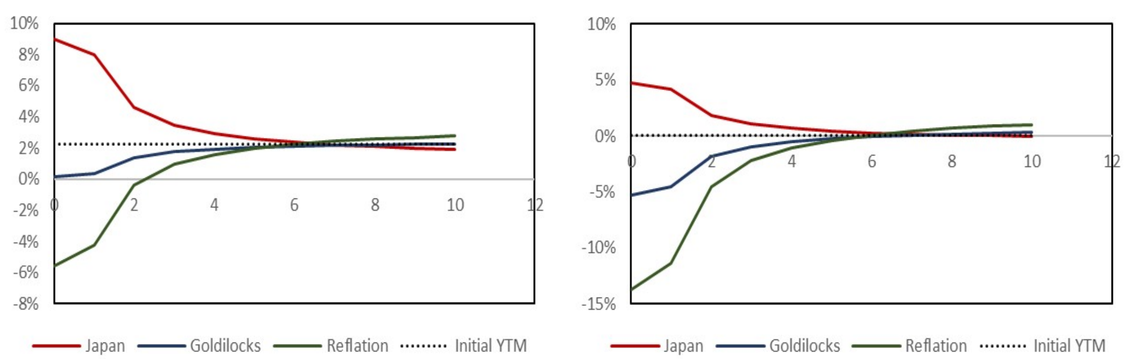
(b) Germany

Figure 6.4: Estimated Interval of Loss With One Standard Deviation
Source: Own Source

From the figures above, a decreasing interval could be seen for the scenarios Japan, Goldilocks and Reflation. Hence, the gain calculated for the Japan scenario is more uncertain, as the interval for both US and Germany covers the break-even line at 0%. This, even as the magnitude of the change for the US is approximately the same for the Japan and Reflation scenario. This could be drawn back to the probability function shown in figure 5.5b and 5.6b, where a higher pull level creates a more narrow distribution. Further, we see how there is a lower possibility for loss in the US. This as the intervals is greater on the positive side of the graph, for US compared to Germany, while it is lower on the negative side. This is clearly illustrated for the Goldilocks scenario, as the US interval breaks the 0% mark substantially, while the German interval is slightly above.

6.3.2 Holding Period Return

From the yield curve estimates in the three scenarios, we calculated the new effective YTM, using Babcock's formula with varying time horizons. This is seen in Figure 6.5. We notice how approximately at duration (5.97 years) the new YTM outperforms the initial weighted YTM of 0.08% and 2.25% for the Reflation and Goldilocks scenario. The opposite is true for the Japan scenario. The reason for slightly different offsetting points, is due to the weightings and disproportional shifts in the yield curve. For example, if the 30-year yield changes with a big proportion (holding all portfolio weights constant), it will push the offsetting point of the total portfolio further out, as the offsetting point for this bond is 14.90 years.



(a) US (b) Germany

Figure 6.5: New YTM for Varying Investment Horizons
Source: Own Source

We clearly see the effect of investment horizon on the total return in nominal terms. Having a longer investment horizon, lowers the sensitivity of your total return through changes in interest rates. It is however important to remember that this is nominal yields, and increasing rates are usually associated with increasing inflation. One should therefore consider the inflation risk when assessing the market price versus coupon reinvestment risk, as we are likely to end up with lower real rates, as illustrated in Table 6.2. For a big asset manager, holding 6 years with a negative real return could be disaster, and reallocation of the portfolio would be justifiable.

6.4 Are Investors Positioned For a Change?

Major holders of government debt are often other governments. Their holding is usually fairly static and hard to interpret in terms of market view or positioning. Other more actively managed holders are typically pension funds, insurance companies, asset managers and more speculative hedge funds. These companies often tilt their exposure to make money on speculative gain, and right now it seems there are strongly opposing views on where the market is heading. Figure 6.6 shows the net positioning in 5-year Treasury futures for Asset Managers and Hedge Funds.

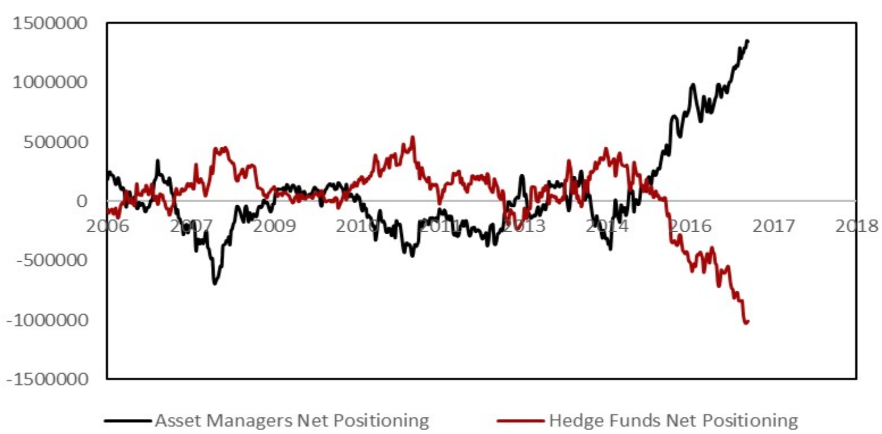


Figure 6.6: Fund Positioning
Source: (CFTC, 2017)

As it is clear from this graph, hedge funds are extremely short 5-year Treasury futures. They are in fact more than 3 standard deviations short in net positioning over this time period. The asset managers on the other hand seems to have the complete opposite view, taking a 3 standard deviation long net positioning.

Their difference in positioning is nothing new. The close to perfect negative correlation seen over time, could be due to different use of futures contracts. These holdings could in reality represent only a small part of a mixed asset portfolio. Swaps, options and spot positions are other components that could provide the same exposure. Hedge funds shorting treasuries, could for example work as a hedge for going long in other corporate bonds. Hence, their main view could still be lower interest rates in the future, despite short positions. On the contrary, asset managers going long 5-year futures, could be to provide a liquid shorter duration asset, in fear of investors withdrawing from their funds, due to the risk of rising rates. Hence, they could actually expect higher interests despite their long position. The trend towards new extremes, is however very clear for both asset managers and hedge funds.

One should therefore be careful of reading too much into positioning data, without knowing the whole portfolio. The extreme positioning, however, indicates that they are preparing for a changing environment, and we strongly believe rates are more likely to rise than fall in the upcoming years.

Another interesting observation is the reallocation of bank holdings. Figure 6.7 shows the amount of bonds categorized as HTM in US banks. Even though the graph only reports data until end 2013, the trend is very clear. To generate sufficient return, banks are also being pushed further out on the yield curve, and thereby holding high duration bonds. As explained above, in a rising rate scenario, holding the bond until maturity will generate profit, as the maturity normally exceeds the duration of the bond. Hence, you have an investment horizon longer than duration, and your total return including reinvestment, profits from a rate increase in nominal terms. This positioning could therefore be interpreted as portfolio managers looking to reduce their interest rate sensitivity, by holding more bonds to maturity. This way, the reported values on the balance sheet are less sensitive to short-term fluctuations in interest rates.

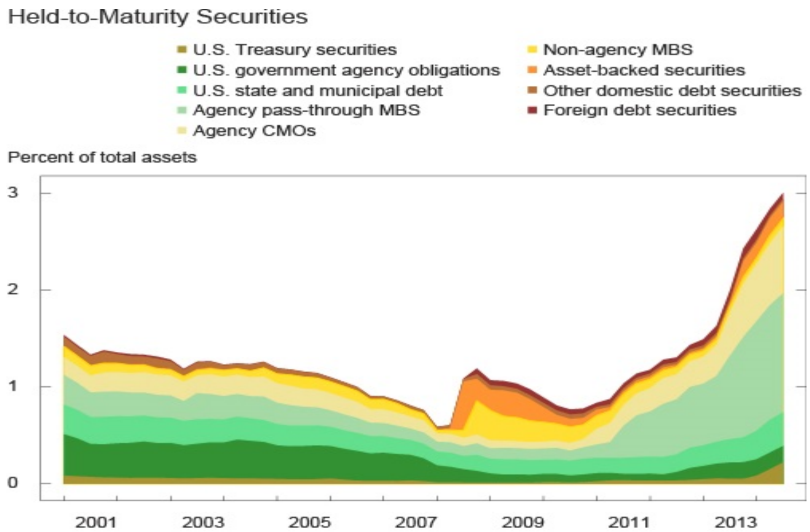


Figure 6.7: Portfolio Percentage Held-to-Maturity
Source: Federal Reserve Bank of New York

The profit and loss estimated in the section above clearly holds lots of uncertainty. They do however illustrate a clear point: if rates start to rise, the consequences could be dramatic in a market of this size, that is normally considered safe. Even for the Goldilocks scenario, which we consider the desired scenario, the estimated loss (especially for Germany) is of such a proportion that precautions would be reasonable. Even if a longer investment horizon would regain the loss from the initial price decline in nominal terms, the total real return would suffer dramatically. We therefore argue the probability of rising rates is so skewed to the upside, that big institutional investors should hedge their exposure to avoid a loss, in either real, nominal or both terms. How this could be done will be discussed in the next section.

7. Hedging

We consider the two rising rates scenarios far more likely than the Japan scenario. Given the size of this market, combined with low rates causing higher duration, we argue that asset managers and investors should protect themselves against rising rates. In the following section, we will illustrate how managing the interest rate exposure could be done. To assess the consequences of a hedge, we will look at Vanguard's portfolio, and compare the portfolio with and without hedging for the three scenarios.

When constructing the hedge portfolio, we will focus on futures and options on futures contracts. These derivatives will be used to modify the risk in the portfolio towards the desired level. Using derivatives also enables us to determine the most profitable way to hedge.

However, it is important to be aware that these calculations focus on interest rate risk, while inflation risk is not accounted for. As we saw in section 6.2, the real return is likely to paint a very different picture, as inflation and interest rates are strongly correlated. Therefore, only changes in the yield curve will influence the profitability in the hedging calculations below. However, as monetary policy in both US and Germany is strongly based on controlling inflation, it is expected to remain somewhat stable. If inflation risk where to be more carefully assessed, Treasury inflation protected securities could be an alternative. However, as we estimate the future path of interest rates and not inflation, this will be our focus when discussing hedging strategies.

7.1 Futures

When using futures to hedge the interest rate risk, we wanted to take a realistic approach to the hedge. We have therefore constructed a portfolio using futures traded on Chicago Board of Trade (CBOT) for US hedging and Eurex Exchange (EX) for German hedging. These futures are among the most liquid traded derivatives, and challenges regarding transaction costs, availability, etc. can therefore be assumed minimal.

For the US Treasury securities, CBOT offers futures contracts on 2, 3, 5, 10, 20 and 30 years, with a face value of 100 000 USD for all maturities except 2 and 3 years, which has a face value of 200 000 USD. The difference is simply due to the US government issuing significantly more debt on these maturities compared to others (Chicago Board of Trade, 2017). EX offers German Government bonds futures on 2, 5, 10, 20 and 30 years, with a face value of 100 000 EUR. For the calculations to be as authentic as possible, we have used the officially listed futures and relevant underlying, with their respective yield, duration and conversion factors, offered by CBOT and EX. We have also applied Cbonds own yield calculator for return and sensitivity measures (Boerse Stuttgart, 2017).

7.1.1 Understanding Real-World Treasury Futures

In theory, futures is often regarded as a ”simple” derivative to use for hedging and speculation. However, when stepping into the real world, there are a lot more considerations to take into account, such as conversion factors, basis risk, duration and sensitivity analysis with respect to the exact underlying of the derivative.

Treasury futures with a maturity of for example 5 years, has to be delivered at maturity using a specific underlying instrument. Therefore, the future works as a proxy for a variety of underlying issues within a certain range of maturities. Again, using the 5-year Treasury note, this ranges from 4.5-5 years. In other words, multiple Treasury securities within this range of time to maturity can be delivered at the end of the contract. As these issues differ slightly in coupon payments and maturity, they should be valued differently. To make them comparable, a conversion factor (CF) is applied. The conversion factor can be thought of as the delivered security valued as if it had coupon equal to 6%. A specific CF is calculated for each of the underlying deliverable instruments, that has the correct maturity for the respective futures contract. For example, a CF equal to 0.8 means that the bond is worth 80% of the same bond with a 6% coupon and same maturity. Because the basis is made on 6%, bonds with higher coupon will have a higher CF than one, and opposite.

Because one futures covers many issued securities, another important concept is "Cheapest to Delivery". The buyer of the futures contract is required to receive the underlying at maturity, so the seller effectively has to deliver it. Exactly which of the underlying securities that is delivered, is the sellers choice. The security chosen, is therefore the one that maximizes the return of buying the underlying, holding until maturity and deliver to the buyer. Therefore, it is often the security with the lowest purchase price that is Cheapest to Delivery (CTD). As this security is most likely to be delivered, it also represents the value and sensitivity of the contract closest. It is therefore important to use the correct CTD when analyzing the sensitivity and movements in the futures price.

Because finding the exact CTD can be a challenging task, another type of securities often used as reference is "on-the-run" securities. These are the newly issued Treasury securities, and are therefore easier to spot. When choosing the exact underlying for the hedging portfolio, we have therefore looked up all the deliverable securities for each maturity and tried to be consistent in choosing on-the-run treasuries. This will work as our proxy for the CTD measure and the characteristics of this underlying. However, we had to settle for some older futures, which will stand out with regards to the security characteristics. The specific details of the bonds and futures contracts used for hedging are given in Table 7.1 and 7.2. All the contracts have delivery June 2017.

Name	Maturity	Maturity Date	Time to Maturity	Coupon	Conversion Factor	Duration	Modified Duration	Yield	Dirty Price	Initial Margin
912828W30	2	28-02-2019	1.915	1.125%	0.913	1.904	1.879	1.332%	0.997	550
912828W55	5	28-02-2022	4.918	1.88%	0.834	4.722	4.626	2.082%	1.001	850
912828V98	10	15-02-2027	9.885	2.25%	0.731	8.937	8.728	2.404%	0.990	1450
912810PU6	20	15-05-2037	20.137	5.00%	0.885	13.738	13.379	2.682%	1.401	3400
912810RV2	30	15-02-2047	29.899	3.00%	0.587	20.053	19.466	3.01%	1.001	4400

Table 7.1: US Futures Data
Source: (CME Group, 2017)

Name	Maturity	Maturity Date	Time to Maturity	Coupon	Conversion Factor	Duration	Modified Duration	Yield	Dirty Price	Initial Margin
DE001141695	2	12-04-2019	2.033	0.50%	0.907	2.010	2.025	-0.759%	1.031	663.19
DE001135473	5	04-07-2022	5.263	1.75%	0.819	5.017	5.036	-0.362%	1.125	801.88
DE001102390	10	15-02-2026	8.885	0.50%	0.636	8.697	8.682	0.175%	1.029	944.44
DE001135432	20	04-07-2042	25.277	3.25%	0.883	18.854	18.667	1.005%	1.523	1007.13
DE001102341	30	15-08-2046	29.395	2.50%	0.744	22.184	21.943	1.00%	1.001	1070.72

Table 7.2: German Futures Data
Source: (Eurex Exchange, 2017)

Because the futures contracts itself do not offer coupon payments, they are not quoted with a yield. The calculations in the following section, is therefore made on the basis of the underlying security that the future is tracking, with the characteristics given above. As the futures contract doesn't follow the development of only one underlying, it is likely to be a minor mismatch in the calculations of the price, sensitivity measures and yield of the future, named basis risk.

The basis risk can be thought of as the risk of not getting the exact exposure we are after, due to a mismatch between the future and underlying. As mentioned, the futures cover multiple underlying securities to serve a specific maturity. Clearly these underlying will not react exactly the same as the futures price, which could cause unexpected profit and loss at maturity when the underlying is delivered. The basis can be quantified through the following formula:

$$Basis = CashPrice - AdjustedFuturesPrice \tag{7.1}$$

Where the cash price is the market price of the underlying, and adjusted futures price is the market price of the relevant future multiplied with the conversion factor. The basis risk of our portfolio was tried to be kept at a minimum by choosing CTD. However, as stated, some of the contracts had a clear mismatch. The average Basis was therefore found to be 4.71 USD and 4.04 EUR. In reality, this could have substantial effect on the profit and loss from hedging. We will however assume the futures contract has exactly the same characteristics as the underlying of our choice. Given our argumentation above, this should be a fairly accurate estimation, and also normal practice when performing these calculations.

When calculating the hedging results, we will use two different methods: Modified duration and Basis Point Value (BPV) hedging. Common for both approaches is that we find the desirable exposure for the portfolio, and thereafter calculate the hedge ratio needed to obtain this exposure. Finally, we see how this would affect the value of our hedged portfolio, by looking at the yield changes through the three scenarios described in section 5.4.

7.1.2 Sensitivity Measures

Hedging a portfolio against nonparallel changes in the yield curve is more challenging, than single bonds and parallel changes. However, there are ways to simplify this action. For each maturity, we will hedge the portfolio duration with the respective futures contract. This allows a more flexible hedge, as we can adjust towards specific maturities and skew the hedge with regards to our view of how the yield curve changes. Hence, we can stay within a certain duration limit, while allowing for some speculation in how that duration limit is met.

Hedging using modified duration is a relatively simple task, when the correct futures and desired exposure is found. To completely offset the changes (as usually is the "textbook example"), we find the number of contracts needed to offset our duration exposure, so the total duration is zero. If we have another regulated or desirable target, we subtract the current duration from the target duration (TAR), to find the appropriate adjustment. This is called the duration-based Hedge Ratio (HR_{Dur}). We find this ratio using the formula:

$$HR_{Dur} = \frac{D^M_{TAR} - D^M_P}{V_{Ft} * D^M_U} \tag{7.2}$$

D^M_P is the modified duration of the bond portfolio, V_F is the value of the contract in question, and finally D^M_U is the modified duration of the underlying, representing the futures contract. This ratio tells us how many futures contracts we need per 1\$ exposure. To find the total number of contracts, we multiply the hedging exposure with HR and divide this number with the price per futures contract.

$$N_t = \frac{HR * Exposure}{V_F} \tag{7.3}$$

Using the BPV or Dollar Value of a basis point (DV01), we measure how much the value of a security changes, when the yield moves by a basis point. This is closely related to the duration measure, but has smaller increments of change, and account for the price of the security. The formula for BPV can be written as:

$$BPV = DV01 = 0.01 * ((0.01 * D_U^M) * P) \tag{7.4}$$

This is the underlying's BPV measure. To be able to adjust for the difference between the underlying and the futures price, we will again apply the conversion factor:

$$FutureBPV = FDV01 = \frac{DV01}{CF} \tag{7.5}$$

Now that we have our sensitivity measures, we find the relevant hedge ratio and number of contracts using formula:

$$N_t^* = \frac{D_{TAR}^M - D_P^M}{D_P^M} * \frac{DV01}{FDV01} \tag{7.6}$$

This way we can create our hedging portfolio for the different maturities and desired duration. Note that the FDV01 formula can be reformulated to calculate duration for the futures contract (as this is not exactly equal to the duration of the underlying). This will be used when calculating the desired duration for the hedged portfolio:

$$D_{Fut} = \frac{FDV01 * 0.0001}{Price} \tag{7.7}$$

Further we can calculate sensitivity for yield changes in the bond portfolio by:

$$\triangle P = DV01 * P * \triangle BP \tag{7.8}$$

And for the futures portfolio by:

$$\triangle V_F = FDV01 * V_F * \triangle BP \tag{7.9}$$

The sum of these two will thereby be the total change in portfolio value, caused by a shift in the yield curve (measured in BP).

As mentioned previously, the duration measure assumes a linear relationship between the yield and price change. The estimations above are therefore somewhat biased as we do not incorporate convexity adjustments in the hedge ratio.

7.1.3 What is the Target Duration?

Investors who buy bonds or bond funds, earn money as a compensation for the risk he or she is taking on. This risk is mainly duration; thus interest rate exposure is desired. Therefore, removing all duration in form of a perfect hedge, would not be a reasonable solution. The investor would be left with a very low or even negative risk-free rate, and rather invest in other asset classes. When saying "perfectly" hedged in this context, we mean a hedge that is supposed to offset the full price movements, as the hedge is rarely perfect because of differences in factors as credit quality, liquidity and maturity (Morris, 1989). However, when the risk of interest rate changes in our opinion is skewed to the upside, limiting the duration would be reasonable.

Since we are using Vanguard's portfolio as a proxy for the market, we looked at their own economic outlook to see their opinion on the matter (Vanguard, 2017a). They argue the US bond market is not in a bubble, and in fact close to being fairly valued, following the spike in yields initiated by the election. They do however expect rising rates, but see this changing in a gradual pace, and argue that aiming for short duration strategies (buying short maturity bonds) might miss out on income. Furthermore, they see more change coming in the short and medium end of the curve compared to the long end. This, we argue, is exactly why hedging a portfolio could be a better and more flexible strategy compared to short duration strategies.

As we have argued that short-term rates are justified by the economic recession and expect a more significant reaction in the medium and longer end, we somewhat disagree with their view. We will therefore set up different scenarios for the hedging to maintain exposure towards both shorter and longer maturities, while keeping the overall portfolio duration low. Hedging by shorting futures on the desired maturity, is therefore a convenient way of achieving this. As we saw in section 6.2.1, the portfolio duration is 5.97. We therefore want to reduce this, while remaining some duration. Based on this, we will hedge the portfolio to a modified duration of 2.5 with different exposures to the yield curve, and compare to a situation without hedging.

7.1.4 Rebalancing and Transaction Costs

As the change in interest rates are likely to either come from changes in monetary policy or other unforeseen events, the time span of the hedge is difficult to predict. As we argue the most likely outcome is Goldilocks, the changes in interest rates could evolve over a significant period of time. Hence, a hedging strategy would have to be applied and held until the scenario or the risk of the scenario has subsided.

This has an important downside that will not be reasonably justified in our calculations, namely rebalancing costs and risk. We have included the initial purchase of the contracts as part of the overall assessment, but these will have to be adjusted as futures contracts expire long before the maturity of the underlying bonds. As stated above, we use futures contracts with maturity in June, which is why in three months' time, we would have to roll the exposure onto new futures contracts.

The "quarterly roll" is standard procedure for Treasury investors, and could further impact profit and loss from the hedging position. The difference between the contracts is called the calendar spread. As we are holding a short position, we are buying the calendar spread. That is, buying the nearby delivery month to close the position, and simultaneously selling the deferred month contract. While waiting for the scenarios to unfold, rates will also move in smaller fashion. This causes shifts in formula 7.6, and the desired number of contracts to be held. Therefore, another transaction cost associated with holding the desired duration is rebalancing costs. Given the high liquidity of the futures contracts applied, this is assumed to be minuscule compared to the total changes in portfolio value (Bessler and Wolff, 2014).

7.1.5 Results

When performing the calculations, we have hedged using both normal modified duration hedging and BPV hedging. The measures follow the same principles regarding sensitivity, but because BPV consider the price of the underlying security, we achieve a cheaper or more effective hedge. We will therefore, in the following, report the results from the BPV measure only. The calculations have been performed both ways, to verify the results.

To simulate the profit and loss for the different scenarios, we apply four different hedging solutions that all create a portfolio with modified duration of 2.5. "Short Duration" has overweight of its exposure at the short end of the yield curve. "Long Duration" has overweight at the long end. "Equal Duration" has equal duration for all maturities. And finally the "Numerical" solution have been found using the solver application to minimize the loss during rising rates scenarios. The specific target duration for the different maturities and strategies are summarized in Table 7.3

	US				Germany			
Maturity	*Short*	*Long*	*Equal*	*Numerical*	*Short*	*Long*	*Equal*	*Numerical*
2	2.20	0.40	3.01	1.93	2.27	0.50	2.98	2.00
5	4.40	0.40	3.01	4.89	4.50	0.50	2.98	5.00
10	3.15	4.90	3.01	3.31	2.90	5.00	2.98	3.05
20	1.00	9.90	3.01	0.00	1.00	8.40	2.98	0.00
30	1.00	11.00	3.01	0.00	1.00	10.00	2.98	0.00
Port	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50

Table 7.3: Target Duration for All Maturities
Source: Own Source

The idea behind these strategies is to trade on the spread. We reduce duration of the overall portfolio, but keep exposure to different parts of the yield curve to find the most profitable hedge. As longer term bonds have higher duration (everything else equal), we would expect the answer to be; only hold short-term exposure. The weightings are, however, skewed towards the shorter end, making this very sensitive. Furthermore, as Vanguard stated themselves, this entails missing out on potential profit on the long end of the yield curve. Finally, the exact curvature of the yield curve is hard to predict. Hence, it is reasonable to follow perhaps the most recognized investing principle; diversification.

Figure 7.1 shows the loss on each maturity, as a percentage of the total portfolio value, for the Short and Long hedging strategy. This illustrates how the exposure moves from the short end to the long end of the curve. Further, it illustrates how holding more duration on the long end actually have small effect on total return, due to the low portfolio weights on these maturities.

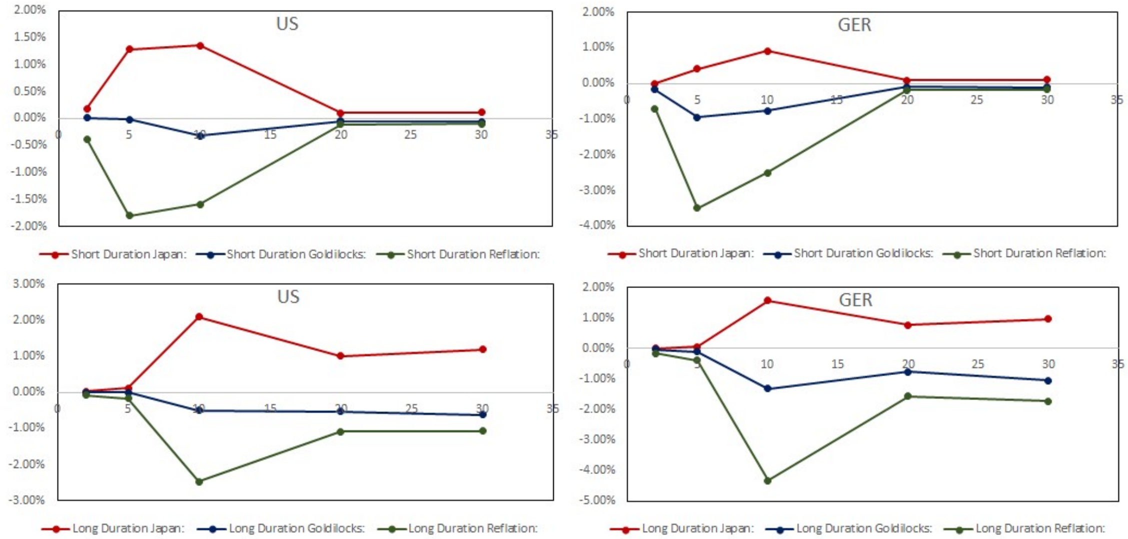


Figure 7.1: Loss per Maturity
Source: Own Source

As seen in the short-duration strategy, both in US and Germany, the biggest loss comes from the 5-year hedge. For the long-duration strategy, the biggest loss comes from the 10-year, even though the target duration is much lower compared to the other long-term maturities. These are clearly the two most sensitive points on the curve because they hold the highest weighting of portfolio value. The weighted duration would therefore be even more skewed towards the short end.

The US portfolio value of 176 billion USD is hedged with different exposure to the yield curve, described above. To calculate the total loss, we have reported the results as percentage of total portfolio value, including the initial transaction cost. For the United States, the results are summarized in Table 7.4.

Scenario	*Unhedged*	*Short*	*Long*	*Equal*	*Numerical*
Japan	6.59%	2.63%	4.15%	2.65%	2.61%
Goldilocks	-2.06%	-0.84%	-1.97%	-1.04%	-0.75%
Reflation	-7.62%	-4.39%	-5.18%	-4.30%	-4.41%

Table 7.4: Percentage Losses US
Source: Own Source

As seen, all hedges still provide a substantial loss for the Reflation scenario. It is however important to remember that this is the "worst case" outcome, and the loss is notably reduced. The value of portfolio loss is actually reduced by 5.8 billion USD for the "Equal" hedge during the Reflation scenario, and 1.8 billion USD during Goldilocks. Using the same market measures as section 6.3 would reduce the total US bond market loss by 1.31 Trillion and 402 Billion USD, for the two scenarios, respectively. To put these numbers in perspective, the estimated loss in the US market was 600 Billion USD during "The Great Bond Massacre". There also seems to be a correlation, where holding a big part of the duration exposure short-term, minimize the loss during the rising rates scenarios, as expected. Table 7.4 is illustrated in Figure 7.2.

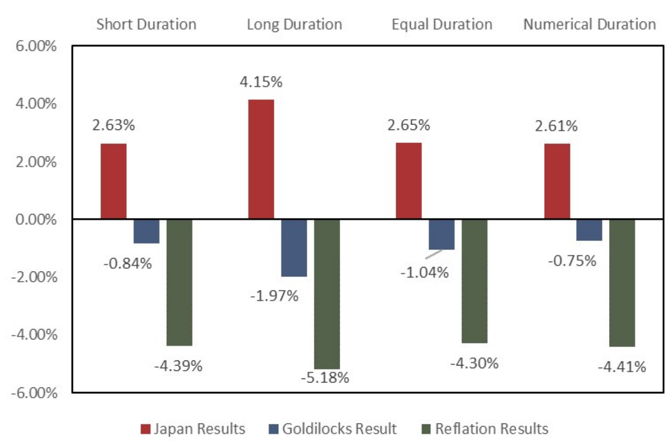


Figure 7.2: US Hedging Results
Source: Own Source

The same approach has been taken on the German portfolio, where the numerical values are converted using the EURUSD exchange rate equal to 1.0685. This gives a portfolio value of 164.72 billion EUR. The percentage results are summarized in Table 7.5.

Scenario	*Unhedged*	*Short*	*Long*	*Equal*	*Numerical*
Japan	4.66%	1.37%	3.08%	1.63%	1.28%
Goldilocks	-5.29%	-2.19%	-3.53%	-2.34%	-2.12%
Reflation	-13.74%	-7.18%	-8.41%	-7.01%	-7.26%

Table 7.5: Percentage Losses GER
Source: Own Source

Also here it seems more profitable to keep duration relatively short-term, for rising rates scenarios. The best outcome for the two rate increase scenarios was actually the "Equal" hedge, as this outperformed the "Numerical" solution suggested by Excel Solver. In spite of portfolio weights and the short-term duration strategy, it is actually profitable also from a mathematical point of view, to keep some duration on the long end of the yield curve. As we argued above, and in compliance with vanguards view, we would also prefer to keep some exposure to the longer maturities. The results for the German portfolio hedge is illustrated in Figure 7.3.

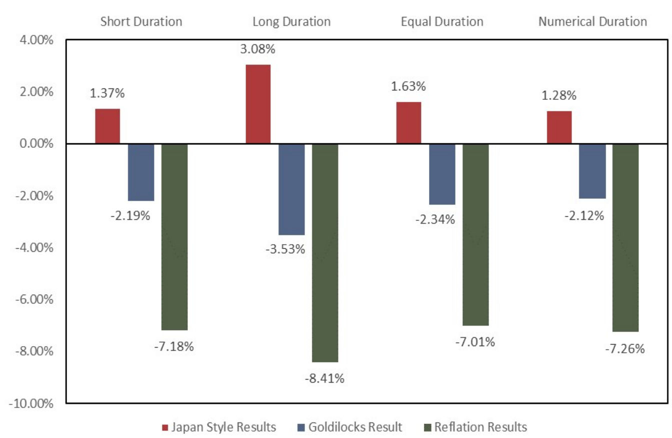


Figure 7.3: German Hedging Results
Source: Own Source

Once again, the results are much more dramatic due to higher changes in the interest rates. The best solution for the rising rate scenarios was the "Equal" hedge. Comparing this hedge to the unhedged scenario, we have reduced the loss by 11.1 billion EUR during Reflation and 4.86 billion EUR during Goldilocks. For the entire German bond market, these losses would be 233 and 102 Billion EUR, for the two scenarios, respectively.

7.2 Options

To further assess opportunities in case of managing interest rate risk, we will look at the effect of options on the futures discussed above. This derivative does not offer the full specter of trading opportunities as the regular futures contract in case of maturities. This applies especially for Germany, while the US market offers a more widespread specter. To make the estimation comparable to the futures estimation, we will use options with expiry in June. The options used is further collected from CBOT and EX.

US options on futures are available for bonds with maturities of 1.75-2, 4.17-5.25, 6.5-10, 15-25, and at least 25 years. However, in our calculation the sectors will be given as 2, 5, 10, 20 and 30 years. For Germany, comparable options are available for the intervals 1.75-2.25, 4.5-5.5 and 8.5-10.5 years, which is assumed constant as 2, 5 and 10 years. Hence, not any good alternative when managing interest rate risk on long-term rates. Furthermore, both the options traded at CBOT and EX are exercised American-style, which means an option can be exercised at any business day prior to expiration.

7.2.1 Sensitivity Measures

Because the price is known, the important aspect of these estimations is movements in the option price, compared to the futures price. Delta measures the expected change in option premium, given a change in the underlying instrument. Further, a delta close to -0.50 is expected, as we use at-the-money options to hedge the exposure (Labuszewski, Kamradt, and Reif, 2013). To see this relationship, a deep in-the-money and a deep-out-of-money put have delta of -1 and 0, respectively. This, as the deep-out-of-money options is not affected by small changes of the underlying, while the opposite holds for in-the-money options. A framework for calculating delta of a futures option is given as:

$$\Delta(put) = e^{-rt}(N(d_1) - 1) \tag{7.10}$$

Given by Black's model, where d_1 is defined as:

$$d_1 = \frac{ln(f_0/X) + (\sigma_f^2/2)T}{\sigma_f\sqrt{T}} \tag{7.11}$$

where f_0 is the underlying futures price, X is the strike price, σ is the volatility of the futures price, r is the risk-free rate and T is time until expiration.

This model is the most commonly used for pricing options on futures. However, Fabozzi describes two main problems using this model; The model fails to capture and reorganize the yield curve, in addition to being a model for pricing European options on futures (Fabozzi, 1998).

Even though we are not pricing the put option, the delta is still affected by these factors. As the put prices are given by CBOT and EX, the price will not be affected by the European style aspect of the model. However, changes in the yield curve will not be captured, which could be impactful even with a time horizon of 3 months. Further, the delta will not be constant, thus a perfect hedge will be almost impossible. Nevertheless, we argue that the model is sufficient to give a proxy on a hedging strategy for decreasing the exposure to interest rate risk. This because the delta by theory should be close to -0.5, and prices are given by the market, which limits the possible bias.

When calculating the delta of the Vanguard portfolio, the time to maturity is constant at 3 months for all maturities, while the r is constant for each country. The remaining inputs is illustrated under in Table 7.6. Here the risk-free rate is the yield on a 10-year government bond, while the σ is the implied volatility for each bond collected from the Bloomberg database.

	US				**Germany**			
Maturity	f_0/X	σ	r	**Delta**	f_0/X	σ	r	**Delta**
2	108.25	0.97%	2.53%	**-0.496**	112.5	1.51%	0.43%	**-0.498**
5	118	3.17%	2.53%	**-0.494**	131.75	3.81%	0.43%	**-0.496**
10	124.5	4.89%	2.53%	**-0.492**	161.5	6.74%	0.43%	**-0.493**
20	151	9.52%	2.53%	**-0.487**				
30	161	9.60%	2.53%	**-0.487**				

Table 7.6: Delta Estimation in US and Germany
Source: Own Source

As seen from the table, the absolute value of delta decreases, because of increasing σ with maturity. A slightly higher absolute value of delta is further approximated for Germany, which is due to a much lower risk-free rate. Overall, the deltas are close to -0.5 and the parameters r and σ would have to be high to make a great impact on the estimation.

Another interesting aspect is the implied volatility input, which is relatively greater in Germany than for the US when comparing options with the same maturity. This implies more uncertainty in Germany, which is consistent with the situation witnessed in these markets today. Ending of the QE program and political uncertainty in the Eurozone causes higher uncertainty. This is also shown through higher sensitivity in section 6.3, where the German portfolio performs worse than the US for all scenarios.

7.2.2 Results

To assess the results of the option hedging strategy, we will focus on the equal duration strategy explained in section 7.1. The value being hedged is based on the futures value needed to make the duration of each maturity equal. This value is then divided by delta, to find the exposure we will hedge. Further, the number of contracts is found by dividing the option value on the future value. Finally, the total cost is found by multiplying the number of contracts with the given price per contract.

The US portfolio is hedged using only options, while the German portfolio is hedged using options on the maturities 2, 5 and 10 years, in addition to futures for the 20 and 30-year bonds. The results are illustrated below in Table 7.7.

Here we can see an underperformance for the Reflation and Goldilocks scenario compared to the futures strategy, illustrated in Table 7.4 and 7.5. However, for the Japan scenario, a bigger profit is made as the options is not exercised. The biggest impact on the different results are the cost of implementing the strategy. As the option gives you the right and not an obligation to sell, this cost is naturally higher. The benefits for this strategy is therefore highlighted in the event of Japan style, which we argue is most unlikely. Further, a greater difference can be seen for US, as the German portfolio is partly hedged using the same futures contracts for 20 and 30-year bonds.

	US	**Germany**
Scenario	*Equal*	*Equal*
Japan	5.66%	2.07%
Goldilocks	-1.63%	-2.55%
Reflation	-4.89%	-7.18%

Table 7.7: Results for Option Hedging Portfolio
Source: Own Source

7.3 Sub-Conclusion

In this section, we have seen how sensitive institutional investors are to changes in interest rates. To assess this, we needed an exact measure of weights and duration. Vanguards Total Bond Market Index Fund was therefore used as a proxy for a typical big diversified bond portfolio. This will make the estimation relevant for many big institutional investors, although not an exact measure.

We also found it necessary to analyze the effect of rising rates in two different ways. As many asset managers actively have to manage their portfolio and report the market value of their current positioning, a mark-to-market value calculated as the percentage change in NAV, was first analyzed. The results showed that the consequences of both rising rate scenarios was substantial, and given the size of the total market, the estimated loss could be dramatic.

Afterwards, we illustrated the importance of holding period horizon on total return. Rising interest rates have an initial negative effect on the portfolio value, and thereafter positive. The calculations were performed by analyzing the total holding period return for different investment horizons. The results suggest that the initial loss in value would be so substantial, that a recovery back to the original YTM would take several years. Furthermore, this recovery is in nominal terms, and the correlation between inflation and interest rates suggest that higher rates are associated with higher inflation. This causes a lower or even negative real return over the investment horizon. Our opinion is that the probability for Reflation and Goldilocks are significantly higher than the Japan scenario. We therefore strongly believe investors should consider the opportunity to hedge their portfolio, in order to be less vulnerable towards rising rates.

We performed hedging strategies using two different derivative instruments; futures and options on futures contracts. The results suggest that hedging could significantly reduce the loss experienced during an interest rate increase. Further, this allows us to leave some duration exposure, thus being profitable if prices continue to rise. By reducing the duration of the Vanguard Portfolio to 2.5, we have almost cut the estimated loss due to rising rates in half. Further, our results suggest that reducing the exposure is most effectively done on the long end of the yield curve. This is due to a big proportion of the portfolio weights being on the shorter end, combined with higher duration on longer maturities. However, we still believe that some duration should be kept on all maturities, simply for diversification and recognition of the difficulty in forecasting interest rate movements.

8. Conclusion

Interest rates have been on a continuous downward trend for the last 35 years. Multiple countries have even experienced negative yield on their government bonds in recent periods. Through this thesis, we wanted to find out if the interest rates seen today could be justified by the economic situation, if we are likely to see a turnaround, and if institutional investors should protect themselves towards rising interest rates.

To analyze this, we have focused on government bonds issued in the United States and Germany, and tried to estimate their "fair" interest rate. Further, we have evaluated the sensitivity of a typical institutional investors bond portfolio towards rising rates and possible hedging strategies. This is to answer the following problem statement:

Are the low interest rates justified by the economic situation, and how should bond investors hedge their exposure under 3 different scenarios?

Because there are different factors determining the "fair" level, we found it necessary to divide interest rates into two buckets; short-term and long-term rates. Short-term rates are mainly controlled by the central bank's policy rates, which again is heavily influenced by the natural rate. Due to the economic trend entitled secular stagnation, savings have outgrown investments over the past decades. The demographic development has caused a relative increase in the number of people entering retirement, who also have a bigger propensity to save. Combined with a relative decline in both prices and requirements of capital goods, a mismatch between investments and savings created a new lower natural rate equilibrium.

Other factors have also enhanced the reaction in short-term rates. The banking crisis of 2008 decreased the effectiveness of reallocation of resources, which ultimately led to lower growth and investments. However, we believe that short-term interest rates should have shown more improvements in the aftermath of the financial crisis, if a fundamental shift in the environment was not true. The market seems to agree that policy rates will be low for a long time, as longer term yield are almost just as low.

For safe government bonds, the most important difference between short and long-term rates are the term premium. This compensates investors for expectations and uncertainties regarding future policy rates and inflation levels. Along with yields, this premium has decreased significantly and also entered negative territory. Both the European and American central bank have initiated credible inflation targeting regimes, aiming to keep inflation low and stable. This effectively reduced the uncertainty regarding both future policy rates and inflation, which justifies a lower term premium. However, we believe other factors strongly have contributed to pushing longer term yields unsustainably low. Massive asset purchase programs started by central banks have put unbearable demand pressure on longer term bonds, and have multiple ramifications when yields start to decline. Pension funds and life insurers are forced to close the duration gap between assets and liabilities, by increasing exposure to long-term assets. Further, both professional and personal investors are being pushed out on the yield curve in order to achieve their desired return. We believe the factors above have created an unsustainable demand for longer term government bonds, pushing yields to levels where asset prices are deviating from their fundamental value.

When QE is ended and economic growth start to improve, interest rates are expected to rise. In light of the discussion above, we believe the longer term equilibrium will be lower than historical averages, and settle around 3.5% and 4.5% for the Eurozone and American economy, respectively. The equilibrium rate is somewhat lower in EU due to a slightly lower and more vague inflation target, more economies to consider, and finally Donald Trump's expansionary plans for the US economy.

How we progress towards this point is critical. The Vasicek model allows us to estimate a total yield curve, by setting parameters for the equilibrium level we are pulling towards, and how fast this development will unfold. To cover a broad set of possible outcomes, we have created three plausible scenarios. Here we have applied both mathematical and economic reasoning for the parameter estimations.

The most likely scenario in our view is Goldilocks. This implies a slow progression towards the desired long-term rates, with effective communication and forward guidance from the central banks. However, especially in the Eurozone, a more dramatic reaction could be sparked by economic events, faster rate hikes or miscommunication with the market. This could lead to a much steeper yield curve and faster reaction; this scenario is entitled Reflation. The final scenario is entitled Japan Style, and involves low rates for a sustained period. This could be relevant if the government bonds are fairly valued right now, and not particular affected by the asset purchase programs and other factors discussed above. To understand the consequences of each scenario we tested them on a representative portfolio.

To come up with a reasonable estimate for typical bond portfolio held by an institutional investor, we utilized Vanguards Total Bond market Index. When analyzing the sensitivity of this portfolio towards rising rates, we divided into two perspectives; One where we estimate the reported percentage change in portfolio value, and one where we look at the total holding period return for different investment horizons.

Financial institutions, mutual funds and other sizable investors, usually report the value of their portfolio through a mark-to-market basis. When the market value drops, they are forced to report a loss. We found that the reported loss for both Reflation and the desirable Goldilocks scenario, would be of such proportions that we would strongly recommend investors to consider hedging strategies to limit the possible loss. For the Japan scenario, we would experience further price appreciation, but this is considered more unlikely.

When yield increase and price decline, the reinvestment of coupons also earns a higher yield. The point in time where the gains from reinvesting coupons outperforms the loss from the initial price decline is duration. Our portfolio has a duration of 5.97. Since this is a big portfolio that typically is rebalanced frequently, the characteristics of the portfolio, including duration, will change over time. However, rising interest rates are usually correlated with rising inflation. Hence, we are likely to experience a drop in real YTM, even if we hold the investment at status quo for approximately 6 years.

The initial price decline due to rising rates is so extensive that portfolios should be hedged, even for longer investment horizons. We have therefore presented two different hedging strategies, where we limit duration to 2.5 using both future and option contracts. The results show that this reduction in exposure approximately cut the estimated losses in half. For the futures hedging, we also performed the hedge with varying exposure to different parts of the yield curve, to analyze the most effective hedge. We found that hedging (as expected) is most effectively executed on the long end of the yield curve. Yet we argue that some exposure should be left throughout the yield curve, simply for diversification and recognition of the difficulty in forecasting interest rate developments.

We therefore believe economic factors have brought down both the short and long-term rates. However, factors that are not part of the natural supply and demand relationship, has caused long-term rates to reach unnaturally low levels. As we believe we are at the turning point where rates will start to rise, the consequences are so substantial, that we would recommend institutional investors to hedge their exposure.

9. The Thesis in Perspective

Interest rates, in many ways, make up the foundation of all financial markets. They decide the time value of money, cost of debt and risk-free rate, and therefore have influence across multiple asset classes. The effect of a change in the interest rate environment, will therefore have many more consequences than what is covered in this thesis. In this section, we will touch upon some of these consequences and discuss possible scenarios, rather than go into detailed calculations.

9.1 Alternative Hedging Strategies

When presenting possible ways to limit the loss during an interest rate increase scenario, we have chosen the most conventional solution, using standard future and option contracts. There is a handful of exotic derivatives and strategies that could be applied to achieve similar exposure. In this section, we will present other alternative hedging strategies.

Barrier Options

Barrier options, is an option category where the derivative is either put to live if it reaches a predetermined value of the underlying, or killed at maturity. As we are worried about a decline in prices and used puts to hedge our exposure, an alternative could be Down-And-In options. This type of option has a knock-in-price that activates the contract. The contract starts worthless, and will be activated if the underlying decline to the barrier. For "Down" options, the knock-in-price is lower than the current spot and strike price. So, if the underlying reaches the knock-in, you can sell the underlying for a higher strike price and make a profit. If this price level is never reached before expiry, the contract expires worthless.

To illustrate, we use the same 10-year futures contract as section 7.1. The current market price was 124.56 USD. Imagine we bought a Down-And-In option with knock-in-price 117 and strike 119, as a hedge against the rising rate scenarios. If the price of the future drops below 117, the contract is active and can be exercised for 119 USD. The Vasicek yield curves can also be estimated using Monte Carlo simulations where we use formula 5.2 to estimate multiple interest rate processes and take the mean as the estimated yield. Here we used a single process, based on the Goldilocks parameters to estimate the change in the 10-year yield over the next 90 days (3-month option). We have chosen specific interest rate paths that were suitable for illustrative purposes. We calculate the change in futures price by using the same FDV01 as formula 7.5 multiplied by BP change in yields. This is a rough estimate as the duration and weightings would change with yields, but gives a realistic estimate of the price change.

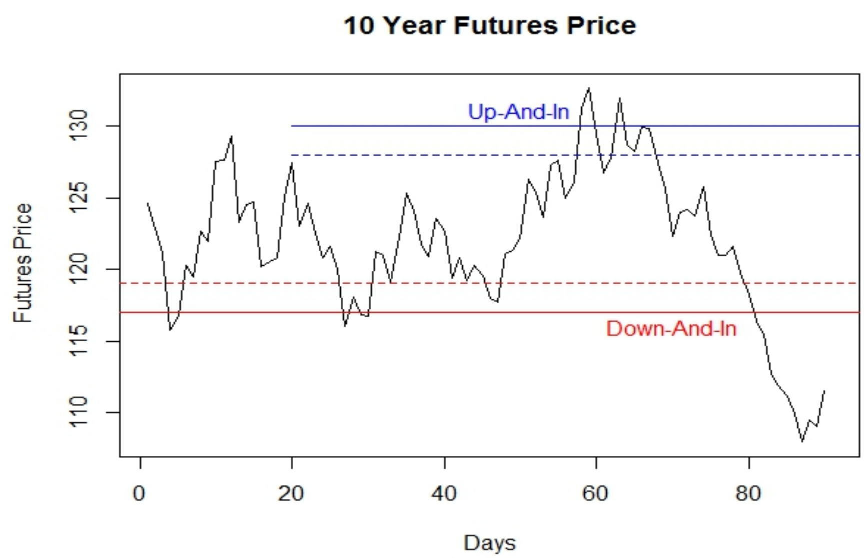


Figure 9.1: Hedging Simulation Using Barrier Options
Source: Own Source

We see how almost immediately the rates have increased by a sufficient amount to push the futures price down below the Knock-In-Price (Illustrated by the solid red line) and the contract is active. This implies that we can sell the underlying for the strike of 119 (illustrated by the red dotted line) for the remaining life of the contract. However, we notice that the rates move back down, pushing the futures price above the strike price. Hence, we would rather sell in the market than through the option contract. Because the price stays far above the strike, we have further imagined that the management is getting worried and included a combination strategy at day 20.

The Down-and-in could namely be combined with an Up-And-In put contract, to create a Double Knock-In. Up-And-In contract also starts worthless, and is activated if the price appreciates and hits the knock-in-barrier at 130 (blue solid line). Since this is a "Up" contract, the barrier price is above the strike. Hence, for a put to be profitable, the spot price must decrease below the strike at 128 again (dotted blue line). This is referred to as a contingent claim, where the contract is alive until maturity, regardless of the price level. We see from our scenario that the Up-And-In is activated approximately at day 60, and become profitable first around 62-63, then again at 70, where it is in-the-money for the remaining life. The target is to achieve both Down-And-In and Up-And-In to be profitable during the life of the contracts. This is achieved from approximately day 80 until expiration.

The incentive to buy this contract as opposed to a normal put, is the price of the option contract. Further, this could be used in a smart hedge, where we have more and more protection, as the price declines. The further away the knock-in-price is from the spot, the cheaper the contract. Hence, we could obtain a cheaper hedge, where the contracts are knocked in as rates move up and price declines. For example, having the first batch of contracts knocked in at 120, the second batch at 118, and so on. The contracts would then become cheaper and cheaper, as the portfolio value would lose more and more.

Touch Options

Touch or Binary options pay a fixed predetermined amount if the underlying reaches a certain level, regardless of how the price develops after the "touch". This is convenient for our situation, as we can determine the amount based on our expected loss for this price level. Further, the touch options are often priced through percentages. For example, if the price is 25% - 28%, the buyer of the option (us) will pay 28% of the predetermined amount, and the seller will receive 25% of the payoff. The advantage of this quotation is that we can make the option "unfavorable" enough for our side, to find a willing counterpart.

As the total estimated loss for the 10-year bond during the Goldilocks scenario is approximately 543.5 million USD, this is the value we would want to hedge (to achieve a perfect hedge). To make an easy illustration we divide the amount by three, and set the touch levels at 122, 116 and 110 where each "touch" pays 181.17 million USD.

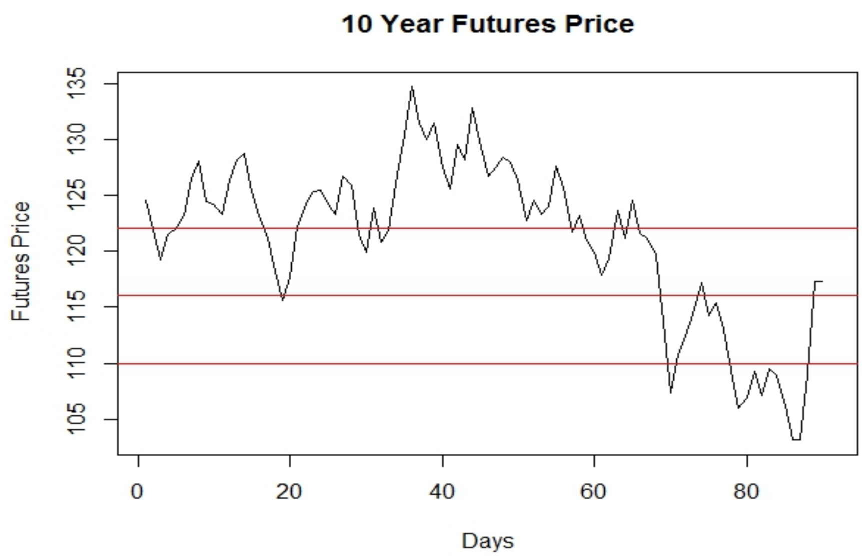


Figure 9.2: Hedging Simulation Using Touch Options
Source: Own Source

In this illustration, we would touch the first two levels within day 20, and the final part of the hedge is touched at day 70. The downside using these options is that the percentage of the amount is usually quite high (as above). Hence, if the contract expires worthless, we would lose more premium compared to normal options.

9.2 Negative Rates is Here to Stay

The central bank's target of stabilizing inflation around 2%, is a new objective that we don't yet know the true consequences of. In this section, we will therefore discuss some (maybe) less likely, but interesting consequences.

If inflation is going to be kept at a low level forever through the act of central banks, this could entail a completely different use of monetary policy in the future. Low inflation and economic downturns, is usually followed by lower policy rates from central banks. If the policy rate were to be kept at low levels through "normal" economic conditions, this strongly limits the room for rate cuts before we enter negative territory. During the last decade, we have already seen the first moves from major central banks into negative territory, combined with huge asset purchase programs. These types of measures are often referred to as unconventional, but might become standard policies in the future.

When the inflation target is 2%, there is not much buffer on the downside before we enter deflationary area. If the new "normal" is 2%, the economic downturn required to push the economy into negative territory is far less prominent, than what we have been used to over the past decades. Hence, a more natural part of the economic cycle could be negative rates and deflation. The old perception that a dollar today is worth more than a dollar tomorrow, might be at jeopardy.

Perhaps do we "save" money through having positive real rates, where losing less than price decline, or not losing money by holding cash is increasing your consumer purchase power. For multiple countries, negative rates have already become reality, and the reason for why we saw a spike in sales of safes and mini vaults in Japan through 2016 (Shen, 2016). This might sound surreal today, but could be a realistic way of handling your savings in the future. The consequences of this is as obscure as negative rates itself. Perhaps will this give rise to black markets for saving, or will we go back in time to when our savings was placed in cash, gold bars and diamonds, locked inside of huge vaults? One might question what will happen if everybody wants to buy commodities and precious metals to save money. That sounds like a constant bull market for decades. However, considering the bond bull market has lasted for 35 years, this might not be as unheard of as economists seem to think.

Part of the motivation behind the negative rates, is to force corporate banks to lower their balance sheets, by increasing willingness to lend, and decrease willingness to save. This happens when the banks themselves must pay interest to keep the money in central banks. Due to negative rates on holding cash, this problem could be further forwarded on to the individual saver. The banks would start taking profits from your savings, and reward your lending with interest payments instead of charging it. This could change "The Banking operation" as we know it today. Peter Fisher of Blackrock argues that the biggest loser of negative rates is exactly the financial institutions. The flat yield curve causes problems for the financial institutions, as the spread is compressed, which is where they earn their money. Further, the demand side of credit might have the desired effect, because the incentive to borrow will increase, but the supply side will not be equally effective. He therefore argues that the shape of the yield curve, not the rate level, determines the supply side. Hence, it makes it hard to stimulate an economy with a flat yield curve.

Negative rates have often been combined with QE. The loss of the financial institutions is therefore further agonized by the combination of these two. As stated in section 3.2.2, QE has more effect on the long end of the yield curve. A QE program will therefore typically lead to a flatter yield curve, and the perception that central banks want lower long-term yields. This enhances the problems for corporate and commercial banks. Negative rates could initiate a "bank run", as consumers want to withdraw their savings, and at the same time, income from credit supply declines.

Further, the 2% inflation target could also cause problems with higher rates. Over the last 30-40 years, both inflation and interest rates have wandered with an upper bound of roughly 15 percent. If we assume the economy will (without regulation) continue to be this volatile, trying to keep inflation steady at 2% could prove to be a challenging task. Keeping inflation low, requires higher interest rates when the economy is heating up, which could lead to extreme central bank rates in both a positive and negative direction. If there in fact is no real secular stagnation, but simply an economic downturn, we could get back to "normal", with higher inflation, within few years.

During the 80's, when inflation was close to 15%, the federal funds target rate was above 20%. Mike Maloney's theory is that the threat of deflation (such as the financial crisis of 2008), is followed by growing inflation due to stimulus (as seen in the market right now). Further, this ultimately leads back to actual deflation, which finally causes an overreaction from central banks, that leads to hyperinflation (Maloney, 2015). He pictures this as the "roller-coaster ride" of the economy. If we were to see such a development, one can only guess how high the Fed funds rate would have to be, to keep inflation at 2%. In order to avoid this situation, the central banks would have to be "in front of the curve", by adjusting policy rates as the economic situation is unfolding. However, this has proved to be a difficult task throughout history.

9.3 Can Low Rates Justify High Equity Valuation?

"Interest rates are like gravity in valuations. If interest rates are nothing, values can be infinite. If they are high, that's a huge gravitational pull on values". This was Warren Buffet's answer to how interests influenced value of assets, during an interview with CNBC in 2016. Today we see among the lowest interest rates ever encountered. We also see valuation of companies far beyond averages, especially in the US. The P/E ratio for S&P500 is currently so high, that we have only seen exceedance of today's level two times throughout history (Multpl, 2017). First in early 2000 during the dotcom incident, which ended with markets crashing. Second time was before 2008, which also ended with markets crashing. However, as Buffets quote accentuate, low rates increase the value of assets.

When valuing a company, the normal procedure is to look at the intrinsic value of a business. This is found by estimating how much cash a company can generate during its remaining life. To find the present value, the cash is further discounted back to today, by using the weighted average cost of capital. Part of this discount factor is the risk-free rate. So, when interest rates are low, the loss of value through time will be close to zero, which justifies a higher valuation of the company. However, an increase in rates in the future will reduce the present value of future cash flow, thus reducing the total market value.

It is therefore possible that a sudden increase in interest rates, could have big influence on the general equity valuations. Most researchers performing valuations are of course aware of this risk. There is a lot of uncertainty in the market, regarding what the "correct" risk-free rate is for the upcoming years, and how much a company should adjust its hurdle rates. An investment horizon could surely exceed 30 years, and the consequences of being too adaptive towards today's level, could be costly.

When companies evaluate whether to invest in a new project or not, they measure the Net Present Value of that project. If it is positive, the project is assumed profitable. So, when rates decline, a project that wouldn't be profitable with old rates, could look profitable today, without changing anything else. Further, many companies fail to incorporate that lower interest is usually correlated with lower inflation and growth, which again is correlated with lower cash flow. This is the offsetting effect from lower rates that often is left out in forecasts, and could lead to an even bigger downside risk in valuations (Koller, Maly, and Palter, 2004).

Another factor contributing to high valuations, is the opportunity cost of investing. The equity risk premium (market return subtracted the risk-free rate) increases when the interest rates are low. This makes equity investments appealing compared to other asset classes. Investors will not get sufficient return in fixed income assets, and are therefore more willing to move into equity. As the demand for equity increase, the price follows. When rates start to rise, the opportunity cost of holding equity increases. These dynamics will draw risk averse investors back to fixed income products, and thereby an oversupply of equity assets. Furthermore, as previously mentioned, increases in interest rates raises the cost of debt. For companies with high gearing, this will affect the available cash flow, and lower the intrinsic value of a company.

When Trump won the election, both interest rates and equity markets all over the world had a positive boost, as explained in section 4.3. What was later entitled the "Trump Trade" came from promises of tax reduction, less regulation, trade deals and infrastructure building (Mahn, 2017). This was interpreted by the markets as positive for inflation, growth and therefore faster pace of interest rate hikes by Fed. As both interest rates and stock prices increased, the positive effect from higher growth and a stronger economy on companies cash flow, more than outweighed the negative effect of higher interest rates. Hence, a situation with both low interest rates and positive outlook for the economy (at least in the US). In other words, we have one factor increasing the cash flow estimates, and one giving it a higher present value.

It is clear that interest rates have impact on equity and other asset valuations. When running a regression on the P/E multiple for the S&P500 and 10-year yield as the explanatory variable, we get a very significant negative coefficient. And the interest rate environment is very likely to affect the stock market pricing, in one way or the other. However, if the high valuation seen today can be put solely on interest rates is doubtful. The combination of low rates, positive growth outlooks, low opportunity costs in equity and lack of lower inflation incorporation in cash flow estimates, could prove to be a dangerous cocktail if the economy were to develop different than expected.

9.4 Housing Market - Are We Able to Sustain a Rate Hike?

In US, the implementation of QE was primarily an effort to support the housing market. As mortgage costs got lower due to decreasing rates, the housing market experienced a great growth in prices. Further, the market becomes a more attractive investment, relative to other assets. The low rates also lead to opportunities for home-owners, that would not exist in a higher interest rate environment. The question is therefore if these buyers could afford a rate hike, and to what degree of interest rate hikes they could sustain.

With a historical view, the lending rate in both Germany and US is relatively low, compared to the average rate of this century. This will also apply despite smaller rate hikes, which is expected in the future. The US seems to be further into this development, while Germany may be in the start-phase of a similar development. This could be seen from the development in the QE programs in the two countries, and the effect of US being more of a leading country in the global economy, while the Eurozone follows a similar pattern, with Germany as the leading country.

With a possibly greater lending rate, the growth in the housing market should be expected to decrease. This will further decrease the return on property investments, in addition to making it costlier. For home owners who cannot afford higher rates, rate hikes itself makes it harder to resell a property. The low rates, also bring up the question of whether you should borrow at a fixed or variable rate. A low floating rate may seem tempting, but an unexpected development could occur, and make the mortgage cost to high. For US citizens, the "perfect timing" (lending rate at it's lower point) to lock in a fixed rate could be over. In comparison, German rates are at a historically low level. Thus, if one expects a rate hike, the timing of the shift would be the biggest question for borrowers.

Another aspect is the demographic change, where people are moving to bigger, and thus more expensive cities. From an economic view this could be seen as a mismatch, as people take on more debt to buy houses they cannot afford. Especially in the US, the dramatic effects seen after the financial crisis would not be as coherent today. This because of higher capital requirements in today's market, compared to sub-prime loans available before this crisis.

9.5 The Eurozone's Interest Rate Problem

As the Eurozone consists of different countries, a common interest rate implies economies that should be compatible. In today's Eurozone, a divergence could be observed between the southern and northern part of Europe. As the ECB must balance different needs for different economies, helping one country could mean harming another. Or said in another way, they have different diseases and needs to be cured with the same medicine. Especially Germany, which is a dominant force within the union, is not fully in sync with the decisions made by the ECB. A fear that loose monetary policy could lead to inflation pressure and a weakening of the German banking and insurance business has occurred (Stratfor, 2016).

The loose monetary policy within ECB, has lowered the cost of holding debt, which has been positive for the southern countries. As many of these still face increasing debt levels, Germany fears defaults on issued debt. The low interest rates have also weakened the banking sector in general, as negative rates lead to costs associated with holding cash. This effect is not solely seen in Germany, but they are more vulnerable because of their highly competitive and global market. ECB president Mario Draghi, has multiple times stated that they cannot set the policy after German performance, but the Eurozone as whole.

This is all happening in an environment where other southern European countries are struggling and covered in debt. These countries cannot sustain a rate hike yet, and need further stimulus. This creates a problem, as ECB cannot justify a monetary policy that, in worst case, could bankrupt an entire country. Even if most countries had a decent economic recovery, they always have to consider the weakest link.

The statistical office of the European Union reported that annual HICP inflation increased to 2% in February from 1.8% in January. This is the highest rate recorded since 2013, which leads them to their target inflation of just below 2%. Many researchers including ECB themselves says the spike just seen in inflation is mainly driven by increasing energy prices. It is hard to argue against the influence energy prices has on the inflation in the Euro area, as inflation excluding energy prices was 0.9% over the same period.

While inflationary pressure is building up in Germany, combined with negative rates and dissatisfaction, the opposite is more present for southern countries. Based on the trend seen last year, it seems possible that Germany overshoot ECB's inflation target in the future. This would put ECB under more pressure. Hence, it seems like someone has to "suffer" regardless of the future path of ECB's politics.

Further, the secular stagnation hypothesis, and especially the aging population could be costly. As discussed in section 3.1.1 the working population has been decreasing, a trend which is expected to continue for a sustained period. Meanwhile the fixed income market generates low returns, as a result of QE and other factors. Thus, to ensure that the life insurance policies is paid out, riskier investments has to be implemented to earn the same return, as historical risk-free investments.

Germany's major impact on the EU could be illustrated through them having the largest account surplus over the last years (Peters, 2016). As the aging population effects grow, this will probably decrease as well. However, other factors such as growing immigration and expected slowdown in the global trade will also affect their economy. Hence, these factors make Germany more vulnerable to the already unwanted monetary policy, which favors other parts of the EU, more than themselves. The outlook for Germany will also be affected by their election in September

2017, where the current chancellor Angela Merkel faces competition from anti-EU alternatives.

9.6 Alternative Approach

In general, this thesis and the discussion of the research question is based on multiple assumptions. Our approach to assess if interest rates seen today are justified by the economic situation, is based on a well-known empirical model and economic interpretation. Given that the existing framework fails to capture reliable parameter estimates, due to the extreme environment experienced, a more empirical study could try to improve these estimations to better adapt to the current environment.

Our scenarios are decided to cover a broad range of the development in rates, as we want to cover the broad specter of possible outcomes. Hence, scenarios in-between ours, could be thought of as more likely. An alternative approach would therefore be to discuss more specific interest rate paths that are considered more likely, and thereby be able to discuss these scenarios more detailed.

Further, we discuss two developed economies (US and Eurozone), where Germany works as a proxy for the Euro area. Even though we argue that this is a representative solution, differences within the union will occur. Therefore, an interesting approach would be to analyze the sensitivity separately, to assess what consequences future ECB policies and interest rate development would cause.

Overall, as this thesis aims to cover a broad specter of possible scenarios, research in more narrow categories throughout the paper would be of interest.

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A. Appendix

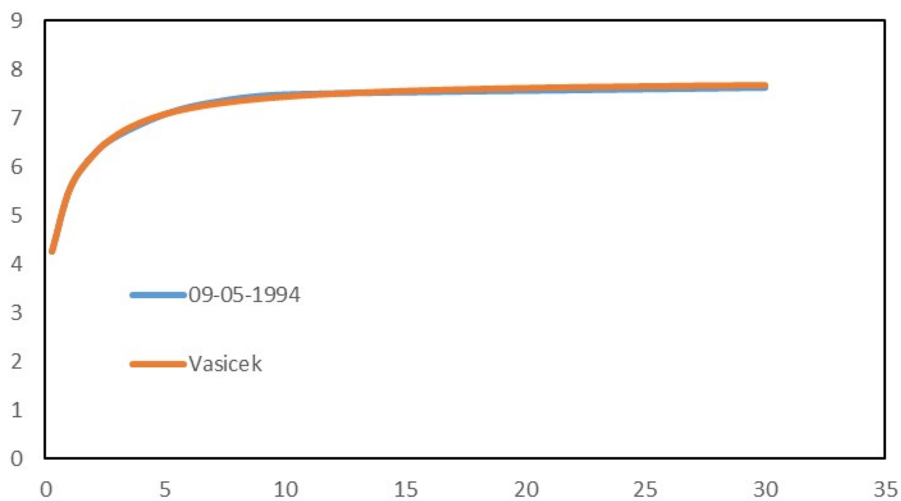
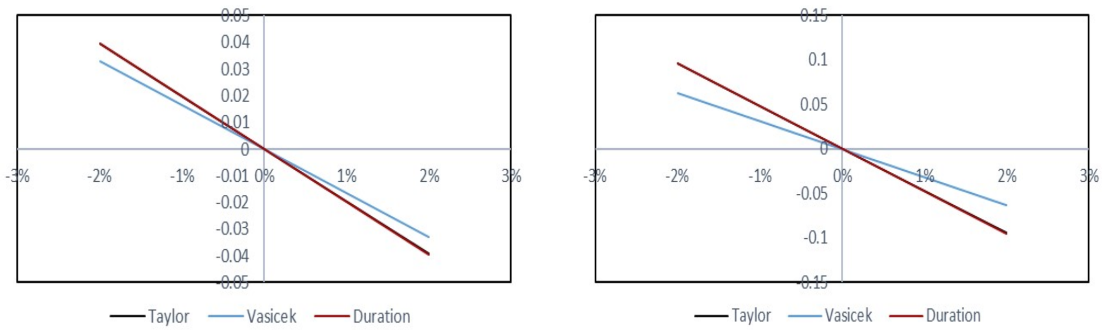


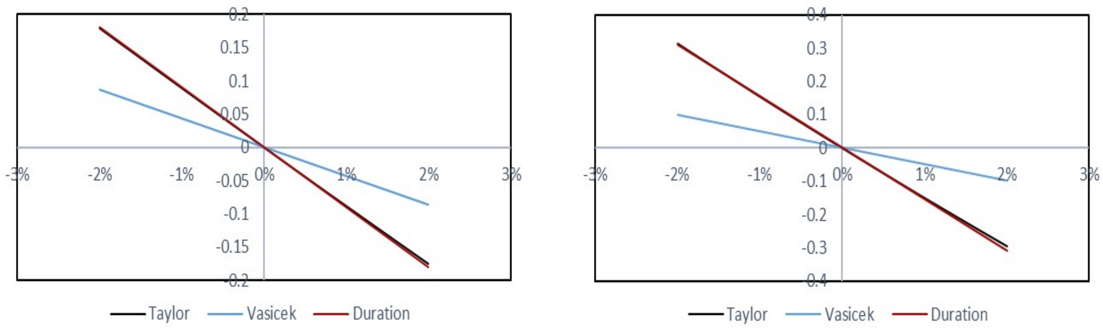
Figure A.1: Real and estimated yield curve 1994
Source: Own Source



(a) 2-Year Bond

(b) 5-Year Bond

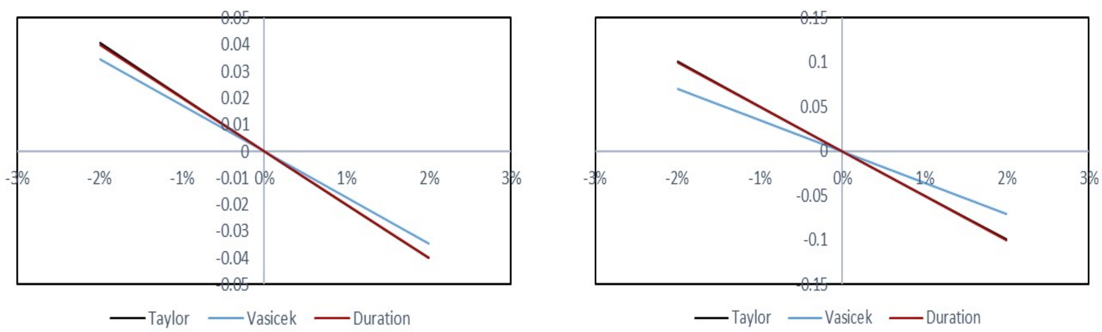
Figure A.2: US Sensitivity for Yield Changes
Source: Own Source



(a) 10-Year Bond

(b) 20-Year Bond

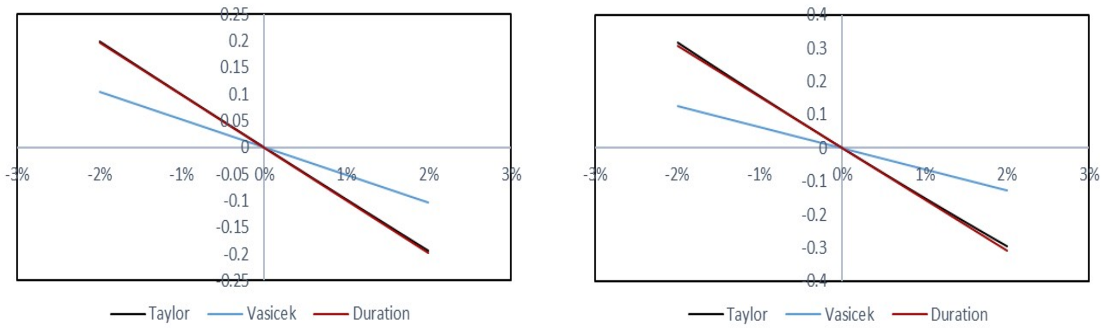
Figure A.3: US Sensitivity for Yield Changes
Source: Own Source



(a) 2-Year Bond

(b) 5-Year Bond

Figure A.4: German Sensitivity for Yield Changes
Source: Own Source



(a) 10-Year Bond (b) 20-Year Bond

Figure A.5: German Sensitivity for Yield Changes
Source: Own Source

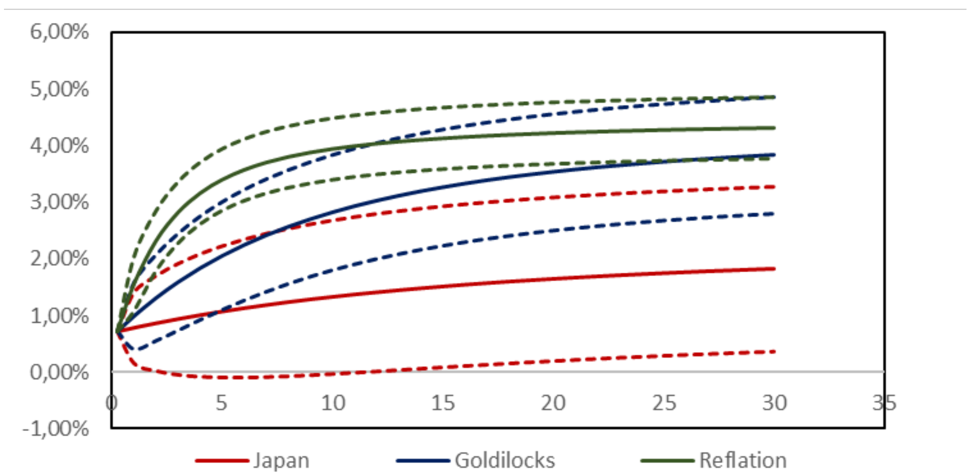


Figure A.6: Yield interval for the three scenarios in US
Source: Own Source

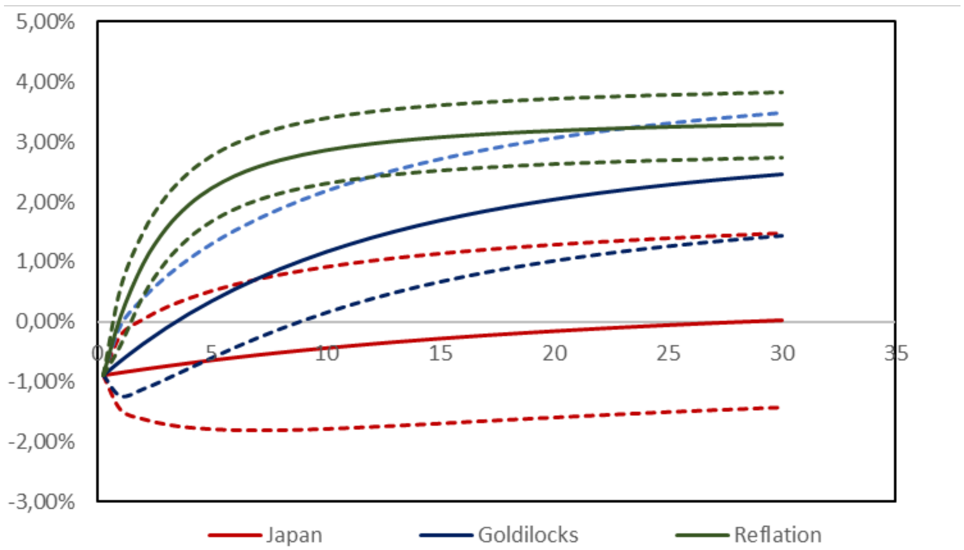


Figure A.7: Yield interval for the three scenarios in Germany
Source: Own Source