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IMPROVING THE WORLD'S BEST MORTGAGE MARKET

Using aspects from the American mortgage market to improve what is already the best mortgage market in the world – the Danish mortgage market

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ABSTRACT

The thesis examines why the Danish mortgage market is considered as one of the best in the world and how this system could be improved. It is found that the high level of transparency, the match funding principle, a direct link between borrowers and investors, and a range of terms and regulatory requirements are providing cheap financing to homeowners and a secure investment environment. However, the system is limited to financing 80% of the property value, whereas borrowers are allowed to finance 95%. The last 15% is usually financed through bank loans and does not utilize any of the mentioned characteristics that benefit borrowers and investors. Therefore, the Danish mezzanine financing is the focus area of improvement.

The American mortgage market was investigated to find inspiration for the structure of the improvement. By combining the tranche structure of American MBSs with the transparency and direct link between borrowers and investors, seen in the Danish market, a product is created—The Jigsaw—which provides borrowers with cheaper financing and targets a wide investor segment, the missing piece in the Danish mortgage market. The Jigsaw grants mezzanine financing to borrowers, collects the loans in large pools, which are structured in a senior, junior, and equity tranche. Through an exchange, the tranches are each funded by a separate bond that mirrors the liabilities of the borrowers. The cash flows from the loans are distributed to the tranches in a waterfall structure, making the senior tranche the safest investment and the equity tranche the riskiest investment.

The credit risk framework developed by Vasicek in 1987 is used to model the product. The industry standard Gaussian copula is used to estimate the underlying default distribution, and a Student t copula, more fitting to real world conditions, is used to stress test the Gaussian. The creation of the tranches is based on an approach, where the expected loss in the tranche is maximized while maintaining a target credit rating from Moody's, defined by a maximum expected loss. Both corporate bond and CLO spreads for the relevant ratings are used to price the tranches. CLO spreads negate much of the mispricing done when using corporate bond spreads, as they are based on similar tranched products, consisting of large credit portfolios.

Over the range of probable parameter values and in both models used, the all-in interest rate ranges from 1.81% - 2.36% when using corporate bond spreads, and from 2.88% - 4.16% using CLO spreads. The results are robust over the models used, and general conservative estimates increases the confidence in the base case result of a 3.31% all-in interest rate charged on borrowers. A result very competitive with the current offerings from the consumer banks that range from approximately 5% - 10%. The Jigsaw will be distributed through the retail banks, replacing their current offerings and lowering their capital requirements.

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1 INTRODUCTION AND MOTIVATION

The early foundations of the Danish mortgage market were established after the great Copenhagen fire of 1795, and the mortgage market has since been an integral part of many people's lives. The system has gradually developed into one of the world's biggest covered bond markets (Falch, 2016, p. 20), and now ensures the citizens of Denmark some of the world's cheapest mortgage financing with high leverage opportunities. Many Danes are likely unaware of how unique the offerings in Denmark, but Statistics Denmark show that the cheap and secure financing has led to a high leverage in the Danish households compared to the disposable income. The large exposure towards the real estate market of private households, makes real estate and mortgage financing important for the Danish economy (Sørensen, Rosholm, Whitta-Jacobsen, & Amundsen, 2009). Therefore, it is an interesting subject for investigation, as it is something that will impact most Danes directly at some point through their lives.

The Danish mortgage market is said to be one of the best, if not *the* best, and most well-functioning markets in the world (The Association of Danish Mortgage Banks (a), 2017). But what is it that makes the Danish system superior to other markets around the world? The Absalon Project has introduced the Danish model in several other countries, and continues to promote the system and implement it in additional countries (Boyce, 2011). The Absalon Project is a joint venture between the Soros Fund Management (the world-famous investor George Soros' fund) and VP securities (the Danish company for securities administration). The general opinion on the Danish mortgage market and the activities of the Absalon Project gives the impression that the Danish mortgage market is "as good as it gets", and system is widely acknowledged and is the "go to" system when other countries need improvement of their own mortgage markets. However, it is relatively easy to improve systems by implementing an established model. More interesting and challenging is it, to challenge the established model and improve it.

The main motivation of the thesis lies in the curiosity of why the Danish mortgage market is so much better than other markets. Furthermore, the thesis challenges the current acceptance of the Danish mortgage market as being "as good as it gets". The last major improvement of the Danish mortgage market was in 1996, when the adjustable rate mortgages was introduced. Smaller changes occurred in 2003 where interest only periods were introduced, and in 2007 where the commercial banks became able to compete with the mortgage banks, but no substantial change to the product offerings have happened since 1996 (The Association of Danish Mortgage Banks, 2012). It is not uncommon to see, that once something or someone has become the best, innovation and ingenuity starts to diminish. The thesis will reevaluate the room for improvement in the Danish system and investigate the possible innovations to the existing system that would benefit both the private borrowers as well as investors, in addition to being implementable in practice.

1.1 PROBLEM DEFINITION

The above introduction and motivation has led to the following main research question:

How can the Danish mortgage system be improved with benefits to borrowers, as well as being attractive for investors?

To help answer this question, the following research questions will be investigated:

i. What characteristics of the Danish mortgage market is making it one of the best and most well-functioning mortgage markets in the world?
ii. Are there characteristics from other major mortgage markets, which could improve the Danish system?
iii. Are the possible improvements of the system implementable in practice?

To limit the scope of the thesis and establish a frame that sets a clear direction for the analysis, several delimitations have been made. In addition, the analysis is limited by several factors, which are important to understand to properly evaluate the results obtained.

1.2 LIMITATIONS AND DELIMITATION

One of the main limitations of the analysis is in the pricing of the modelled products. The pricing relies on many assumptions and theoretical models, but in practice the price on the products modelled is determined by supply and demand in the market. This limits the conclusion, as it can only be estimates and best guesses, based on how the market values similar products. In addition, the true value of the input parameters used in the modelling is unknown. Therefore, the input parameters are estimated based on the data available. This creates uncertainty in the results, which should be considered before drawing conclusions.

The theoretical models employed is primarily chosen to match the models, which constitute and influence a large fraction of the financial market. Within the topic of credit risk modelling, the Gaussian single-factor model, developed by Vasicek, is by far the most widely used. The base model will therefore be the Gaussian model, but as several references point out, it fails to comprehend extreme market conditions. This flaw is handled by extending the framework with a different dependence structure, which can incorporate more extreme scenarios suitable for matching market conditions under periods of distress. The improved dependence structure is the Student t, which is chosen due to its symmetric distribution and facets of implementation. This delimits the credit risk modelling to Vasicek's single-factor model with two dependence structures.

When investigating the Danish mortgage market and other mortgage markets used for comparison, the thesis will focus on the residential aspect of the mortgage market. All other segments using mortgage financing will be omitted from the analysis to limit the scope, simplify the analysis to a single set of regulatory demands, and to develop a clearer and more thorough business case within the boundaries of the thesis. In addition, the analysis of the Danish mortgage market, and the mortgage banks acting on the market, is delimited to the four major mortgage banks: Nykredit/Totalkredit, Realkredit Danmark, BRFkredit, and Nordea Kredit, as these four banks cover over 90% of the Danish Mortgage market (Falch, 2016).

As well as the analysis is focused on the Danish mortgage market and improving it, the perspective on other mortgage markets will primarily be based on the American mortgage market, as it is the biggest mortgage market in the world. However, other mortgage markets will be included as peers in terms of statistics, but no in depth analysis of these will be carried out.

Most data from the Association of Danish Mortgage Banks tracks back to 1996. Therefore, the analysis on many aspects of the Danish mortgage market is delimited to the period between the year 1996 and the present. Despite being limited to this data range, the approximate 20-year sample provides sufficient information to see the trends and developments in the market. When compared to the American market, the data availability is in general better and tracks back further, but as the aim is to compare the two markets, the data compiled from the American market will mainly cover the same 20-year period.

When modelling the Danish mortgage market, different input parameters are needed, and these are difficult to give a precise estimate of. Therefore, statistics from the mortgage and real estate market, as well as different studies investigating the relevant parameters are used to estimate the range of probable true parameter values. The range of parameter estimates are then combined through several runs of the models used. This method will yield a range of results, where the true result should lie within with reasonable certainty.

Tax will not be considered in the analysis, and all calculations and conclusions will be made on a pretax basis in relation to both borrowers and investors. Investors are under many different tax schemes and therefore, it makes more sense to look at it pretax from an investor perspective. Similarly, the borrowers' tax benefits on interest expenses might vary due to political uncertainty and individual economic situations. In addition, the tax impacts on the current financing will be no different in other loan products, which is why tax is omitted from the thesis.

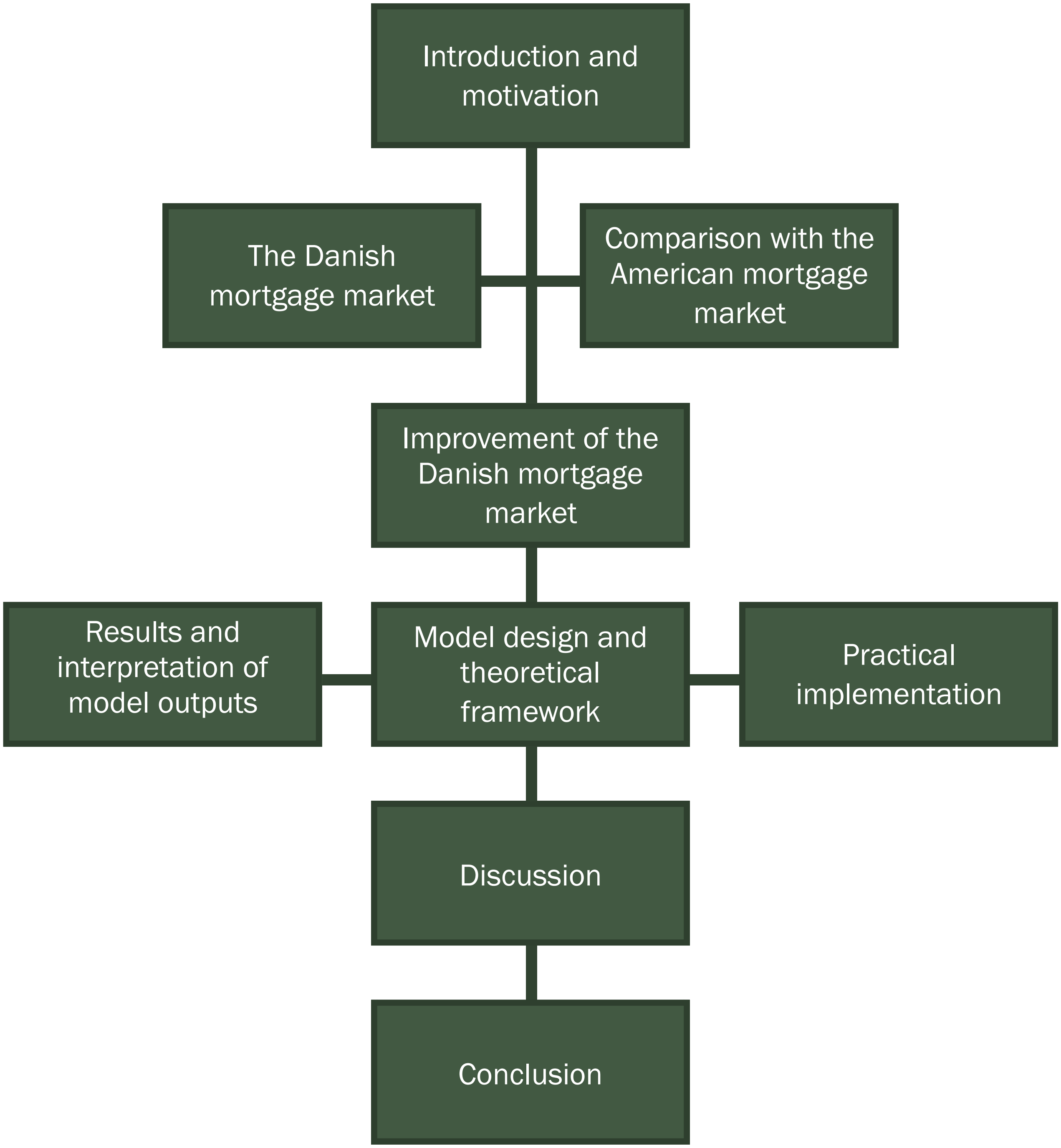
Transaction costs will also be omitted, even though transaction cost could have implications for the investors, especially with frequent trading activity. The trading cost vary from investor to investor, and larger investors such as pension funds can trade at much lower transaction costs in some cases and in other cases their market impact would imply large transaction costs. In addition, the borrower's transaction costs that are present when issuing a loan and affect the total annual cost of financing in percent (in Danish: ÅOP), are also omitted in the modelling, as these often vary and are negotiable based on the relation between the bank and the borrower.

Bankruptcy costs are not modelled directly, as these are very difficult to estimate and vary from case to case. The direct bankruptcy costs are paid by the acquirer of the property serving as collateral for the defaulting loan (Tvangsauktioner.dk, 2017). However, the indirect costs in terms of hours used by staff to handle the default etc., are not covered by the acquirer, and is a cost that should be incorporated into the margins charged on the loans. By including them as part of the total cost of operation (the margins), the analysis will be less detailed, but the impact should not be significant on the overall conclusion.

1.3 THESIS STRUCTURE

The structure of the thesis is shown in Figure 1.1:

Figure 1.1 - Thesis structure



2 THE DANISH MORTGAGE MARKET

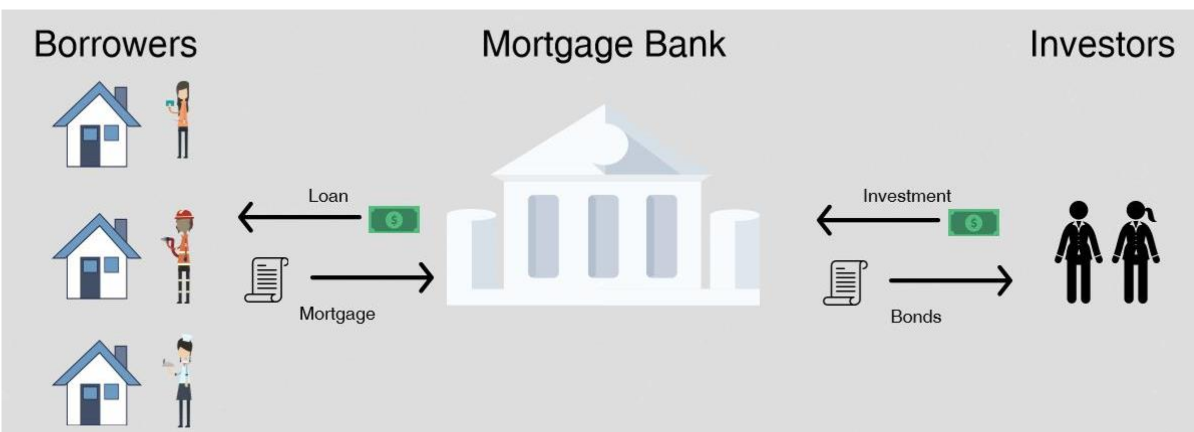
This section will consist of a description of the Danish mortgage market and outline why it by some is considered best in the world (The Association of Danish Mortgage Banks (b), 2017).

2.1 BACKGROUND

The Danish mortgage lending dates back more than 200 years. The basic principles emerged from the ruins of the Great Fire of Copenhagen in 1795, where a quarter of the city burnt down to the ground leaving large parts of the population without a home. After the fire, a great need arose for an organized credit market to fund the reconstruction of Copenhagen, so several wealthy persons established the first mortgage association in Denmark in 1797 called “Kreditkassen for Husejerne i København” and granted loans based on the issuance of bonds secured by mortgages on real property (The Association of Danish Mortgage Banks (a), 2017).

Although the mortgage legislation has been amended on an ongoing basis since its introduction, the underlying rationale has remained intact: mortgage banks grant loans to borrowers through the issuance of bonds with security in a joint series of real estate as seen in Figure 2.1. Hence, the investor (bondholder) does not acquire a specific loan with security in a specific property issued by a specific debtor, but the investor acquires a stake in a pool of loans secured by thousands of individual debtors (The Association of Danish Mortgage Banks (a), 2017).

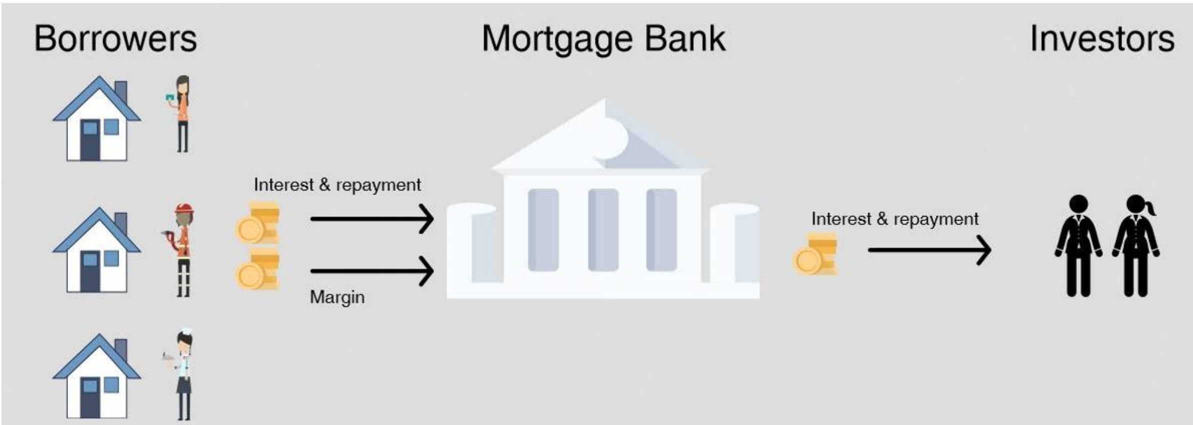
Figure 2.1 – Relationships and cash flows in the arrangement of a mortgage series



Source: Authors creation

Once the mortgage loan is set up, the borrowers pay interests, repayments, and margins (in Danish: bidragssatser) to the mortgage bank on a quarterly basis. The Mortgage bank forwards the interests and repayments directly to the investors and uses the margins to run the operations such as paying salaries, rent, etc., and cover loans in arrears and in default. In addition, margins are posted as collateral to meet the capital requirements set by regulation. The cash flows of an established mortgage loan series is shown in Figure 2.2.

Figure 2.2 – Cash flows of an established mortgage series



Source: Authors creation

The interest rate is determined by supply and demand in the market. Once a loan series have been created by a pool of mortgages, bonds are created and auctioned off in the market where the price and effective interest rates are set, and the bond continues to trade in the market on an exchange until it matures (The Association of Danish Mortgage Banks, 2012, p. 4). As an example, a mortgage bank could issue a 30-year bond with a 2.5% coupon. If the bond was priced at 100 the effective interest rate would be 2.5%, but in case of lower demand, the investors would require a higher return and the price would be pushed down below 100, yielding higher effective interest rates. If the price was 95, the borrowers would only receive 95 for each 100 amount of debt they acquired, but would still have to pay 2.5% of the 100, yielding an effective interest rate of $\frac{2.5}{95} = 2.63\%$.

The traditional mortgage model has proved as a stable, secure, and robust system through various up-and downturns in the Danish economy, which ultimately benefits not only mortgage borrowers with low loan rates, but also the economy at large through mitigation of the adverse effects of unemployment and subdued housing markets (The Association of Danish Mortgage Banks (a), 2017).

Four major mortgage banks dominate the Danish market, Nykredit/Totalkredit, Realkredit Danmark, Nordea Kredit and BRFkredit. On July 1, 2007, these mortgage banks got competition from commercial banks with the implementation of an amendment to the legal framework of the mortgage system. The purpose of this amendment was to make the mortgage system compliant with EU Capital Requirement Directive (CRD) and allow commercial banks to fund mortgage loans (Falch, 2016, p. 8). This was done through the introduction of new covered bonds to be used for mortgage funding, which is covered in detail in section 2.4. In addition, the legislation eased up on the match funding principle (covered in section 2.3.2). However, some of the Danish mortgage banks have chosen to keep operating under the old principle to enhance stability.

2.2 LOAN TYPES

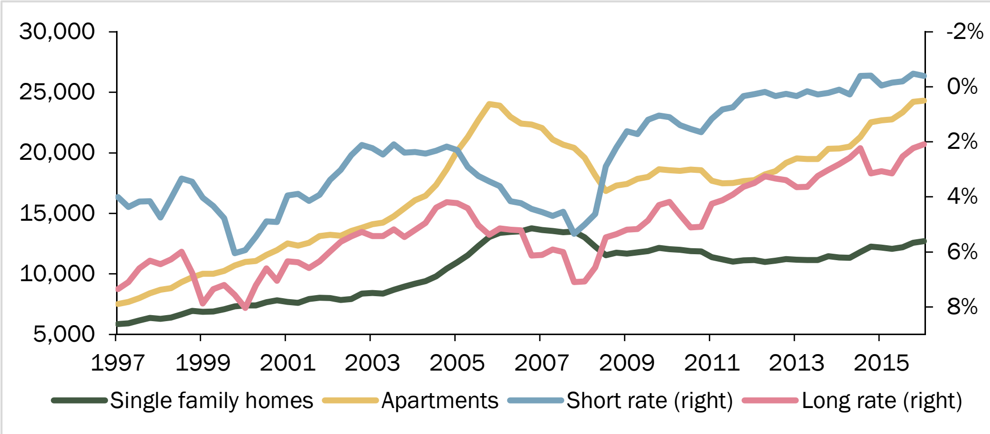
When the citizens of Denmark are looking to buy a home, there are three main types of loans to choose from in the mortgage banks. First, the fixed rate mortgage (FRM), which is also the traditional and oldest type of mortgage. Second, the adjustable rate mortgage (ARM), where the interest rate is reset to match the market rate at a frequency between 1 and 10 years. Third, the floating rate mortgage, where the interest rate is pegged to another interest rate benchmark, such as CIBOR, LIBOR or CITA, plus a predefined margin and resets every 3 or 6 months. The different loan types can be combined with interest-only periods, generally up to 10 years. These loans are funded in separate interest-only hybrid bond series (Falch, 2016, p. 41).

2.2.1 FIXED RATE MORTGAGES

The traditional mortgage type is the FRM, which have been around since the beginning of the Danish mortgage market. The FRM uses a fixed interest rate from beginning to maturity that is set when the loan is granted to the borrower. The loan is typically an annuity loan with equal payments throughout the duration of the loan. The loan can be repaid at par or current market value at any time and is funded by callable bonds with identical characteristics regarding maturity, interest, and repayment schedule.

Due to the direct connection between homeowner's liabilities in the mortgage bank and the investor's claim in the bonds bought from the mortgage bank, homeowners' equity is protected in cases of rising interest rates and falling housing prices. Rising interest rates leads to lower bond prices, and this enables borrowers to buy back the bonds funding their loan at lower prices, protecting them from the price pressure rising interest rates have on housing prices. This has a stabilizing effect on the Danish economy helping sustain financial stability as the system is protected from many homeowners collectively becoming technically insolvent (having loan values exceed the value of their home). In Figure 2.3 the development in interest rates on mortgage bonds and housing prices in Denmark are shown.

Figure 2.3 – Housing prices (DKK/sq.m.) and interest rates, inverted (%)

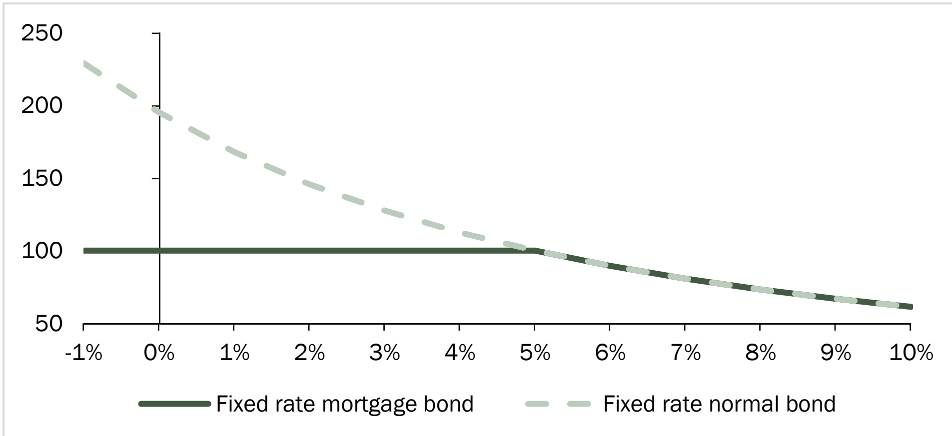


Note: Long rate is based on 30-year bonds and short rate is an average between 1- and 2-year bonds.
Source: The Association of Danish Mortgage Banks

The graph shows a clear relationship of rising interest rates leading to lower housing prices, especially for apartments. When housing prices fall, homeowners are in danger of becoming technically insolvent, meaning that their home is worth less than what is mortgage on the property. However, homeowners with FRM are protected against this as rising interest rates are also negatively correlated with bond prices, leading to a drop in the value of debt in the case of rising interest rates and falling house prices (The Association of Danish Mortgage Banks, 2012, p. 27). However, homeowners are not protected from a decline in the price of their house due to other factors than the interest rate, but such cases are mostly isolated and do not pose as a risk to the Danish economy such as rising interest rates would be if the system was not designed to inhibit such.

In addition to being protected against increasing interest rates, borrowers might also benefit from decreasing interest rates. Traditional FRMs can be prepaid at par, corresponding to a price of 100, meaning that no matter how much interest rates fall, the bonds can in theory never exceed a price of 100. In practice a price over 100 can occur if investors believe borrowers will not exercise the right to call the bonds at par, due to transaction costs, laziness, lag of information etc. Hence, borrowers can prepay at par and refinance at a lower interest rate. The protection from rising interest rates and the upside when interest rates fall leads to the following relationship between mortgage bonds and interest rate shown in Figure 2.4, which compares to a "normal" bond that cannot be prepaid at par to a callable mortgage bond.

Figure 2.4 – 5% 30-year mortgage and normal bond price (DKK) as a function of the effective interest rate (%)



Source: Authors creation

To illustrate the effect, consider the example: A borrower takes a 30-year fixed interest loan with an 5% coupon rate and a current effective interest rate 5.5% to finance a DKKm 1.25 apartment purchase. The loan is an annuity loan, having equal payments each year for 30 years of DKK 68,805 until it is paid back. The borrower has DKKm 0.25 in cash and needs to raise DKK 1m in cash, but since the coupon rate is lower than the effective interest rate the borrower's debt will need to have a principal of DKK 1,057,707 to raise DKK 1m. Consider then the following two scenarios:

Scenario A – with 25 years left, interest rates have increased to 8.5%

After 5 years, the outstanding bonds will have a value of DKK 704,167. With an initial property value of DKK 1.25m, the property could have declined DKK 0.5m in value and the borrower could still sell and pay off the entire outstanding debt.

Scenario B – with 25 years left, interest rates have decreased to 2.5%

In this scenario, the theoretical value of the debt would have been DKK 1,267,696 but since the borrower can prepay a par, the value of the mortgage bond is only DKK 1,057,707. If the borrower now refinances he can lower his annual payment by approximately DKK 11,400 to DKK 57,408 for the remaining 25 years.

2.2.2 ADJUSTABLE RATE MORTGAGES

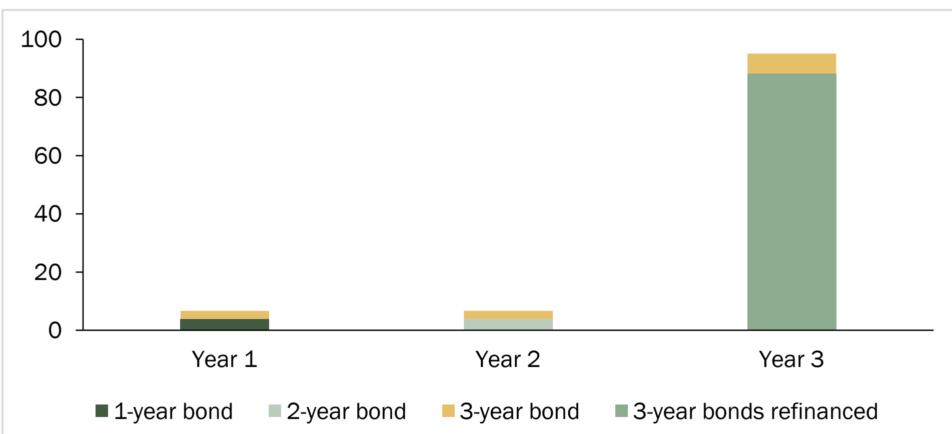
The ARM loans were introduced in 1996 and quickly became a popular way of taking advantage of the lower interest rate and speculating in falling rates. With the ARM, the interest rate is fixed for a specified period between one and ten years. When the loan reaches its interest reset date, the loan is refinanced at the current market rate for the specified fixed interest period. The loans can be prepaid at par at the interest reset dates, and can be prepaid at market value between the interest reset dates (The Association of Danish

Mortgage Banks, 2012, p. 13). Hence, the borrower is not getting the benefit from being able to prepay at par if interest rates fall such as the FRM, but is instead automatically refinancing at lower interest rates. If interest rates rise, the borrower will refinance at higher interest rates and the mortgage will become more expensive, but the value of the debt between interest reset dates will not decrease as much as for the FRM, since the interest rate is fixed at a shorter maturity. Therefore, the benefit of lower interest rates due to shorter maturities also carries a higher risk.

Using a 20-year ARM loan with a 3-year fixed interest rate with no interest-only period as an example, the dynamics and design of the ARM's is explained:

The annual payment is calculated based on a 20-year annuity and the current effective interest rate. The loan is then funded using 1-, 2-, and 3-year bullet bonds. The amount issued in the 1-year bond is calculated so that the repayment and interest of the bond plus interest of the 2- and 3-year bonds equals the annual payment, and the same is the case for the 2-year bond. The 3-year bond funds the remainder of the loan, of which most is refinanced at year 3 at the market rate in new 1-, 2-, and 3-year bonds. Figure 2.5 shows the payments and activities in the first 3 years of a loan of DKK 100 with 20 years to maturity and a fixed interest rate period of 3 years. The 1-, 2-, and 3-year interest rates are set to 1%, 2%, and 3%, respectively. For simplicity, the annual payment is calculated based on the 3-year interest rate and equals DKK 6.72. (An overview of the bond payments can be seen in Appendix A) Note that the interest payment in year 1 of the 2-year bond is so small it cannot be seen, but still is a part of the cash flow.

Figure 2.5 – Interest and repayment distribution of a 3-year fixed interest ARM of DKK 100 principal, 20-year maturity (DKK)



Source: Authors creation

2.2.3 FLOATING RATE MORTGAGES

Floating rate mortgages are very like the ARM, but their interest rate resets are more frequent, often 3 or 6 months, and the interest rate is not determined by supply and demand in the market, but instead by a reference rate in a different market plus a margin. In Denmark, historically the normal benchmark rate for DKK denominated loans was the CIBOR (Copenhagen Interbank rate), which is quoted daily by OMX NASDAQ (The Association of Danish Mortgage Banks, 2012, p. 13). Recently, the mortgage banks have changed the benchmark and the floating rate now follows the CITA (Copenhagen Interbank Tomorrow/Next Average), which also is quoted daily (Finansrådet, 2017).

The loan can be combined with an interest rate cap, which prohibits the interest rate from rising above a certain level even though the benchmark rate rises above the cap. The interest rate cap can have a shorter maturity than the loans depending on the level of security wanted.

A floating rate mortgage can, like the other mortgage types, be prepaid at the market value, but most can also be prepaid at a predetermined price if that favors the market price, typically 100 or 105 (The Association of Danish Mortgage Banks, 2012, p. 13).

2.2.4 PREPAYMENT TERMS

The attractive prepayment terms that ensures all mortgages can be prepaid at current market price, and at par for FRM if in favor, makes refinancing mortgages easy and a common thing in the Danish mortgage market. Historically there has been very high prepayment activity after periods of falling interest rates, where homeowners refinance at lower rates, lowering their debt service, or fixing rates at lower levels (Falch, 2016, pp. 57-61). Homeowners with FRMs may also lower their remaining debt when prepaying at market value after a period of rising interest rates, although the higher interest rate offsets the benefit of a lower debt value in terms of periodic expenses to the loan. However, they do still have the option to refinance again if interest rates drop back down, really benefitting from the dynamics of the system getting both a lower outstanding debt and low interest rates from the move in market prices. The refinancing options also helps homeowners adjust to their current economic situation, shifting risk profile and expense levels to match their repayment abilities.

2.3 CHARACTERISTICS OF THE SYSTEM

When a person or business wants to purchase real estate, and finance it with a mortgage, a mortgage bank will first determine the value of the property. This valuation sets the limit for how much money can be borrowed to finance the purchase. For residential properties, the current limit is an 80% loan-to-value (LTV) and for commercial properties it is 60% LTV, meaning that 80% and 60% can be financed with a mortgage, respectively (The Association of Danish Mortgage Banks (c), 2017).

Mortgage rates and prepayment prices are directly reflected in the price of the mortgage bond in the series funding the loan. The prices are quoted daily and can be viewed by anyone with an internet connection or in the newspaper, creating a transparent environment for both borrowers and investors, ensuring low loan rates.

The mortgage banks facilitate the loans and is liable towards bondholders. Hence, any defaults of borrowers are covered by the issuing mortgage bank and the bondholders only bear the credit risk of the mortgage bank defaulting. The probability of default (PD) of a mortgage bank is estimated to be practically 0% due to more than 200 years without a single defaulting loan series and as a result, no investor has ever incurred a loss when investing in Danish mortgage series. (The Association of Danish Mortgage Banks, 2012, p. 4). The low risk of default is due to: the previously mentioned leverage restriction of 80% LTV on residential properties and 60% on commercial properties, the mortgage on the real estate tied to the loan, the balancing principle or match-funding principle, and the capital requirements for the mortgage banks.

2.3.1 THE MORTGAGE

The definition of a mortgage, or a mortgage document, is a contract in which a borrower pledges real estate property as security for an obligation owed to the lender. In case the borrower cannot meet the obligation requirements, e.g. cannot pay the periodic installments on the loan, the lender may seize the mortgaged property and sell it to incur as little a loss as possible on the loan, which was provided for the purchase of the property (Brueggeman & Fisher, 2011, p. 18). The leverage limitations on mortgage loans help insure a low probability of losses for the mortgage lenders in case of a borrower defaulting, which lowers the important credit risk of the mortgage bank in relation to its investors.

2.3.2 THE MATCH FUNDING PRINCIPLE

One of the unique characteristics of the Danish mortgage system is the match-funding or balancing principle. The principle helps to ensure financial stability in the mortgage banks by ensuring full alignment and direct match between the liabilities of the borrower (homeowner) and the bonds, which the mortgage bank issue to fund the loan (The Association of Danish Mortgage Banks (b), 2017). In practice, a borrower getting a fixed rate annuity loan with 30 years to maturity will be funded by a 30-year annuity bond with a fixed interest rate and so forth for the other loan possibilities.

With the amendment of the system in 2007, the banks were given the option to choose between a specific funding principle, very much in line with the straight “pass through” system in place, or a new general funding principle, which is more flexible. The general principle allows for slightly higher risk taking when funding the mortgage loans, and allows other assets to serve as collateral for the bonds than real estate (Falch, 2016, p. 10). An overview of the difference between the two principles can be seen in Appendix B.

If the balancing principle was not in place, the mortgage banks could speculate by providing fixed rate loans with a long maturities and relative high interest rates and funding these with shorter maturity bonds with relative low interest rates, earning the spread between the maturities given by the yield curve. This leads to large interest rate risks, as the frequent refinancing in new short maturity bonds could put the mortgage bank in a situation where it could no longer finance the mortgage loan with bonds at lower interest rates than the ones paid by the homeowners. This leads to severe liquidity issues and severe default risk in the mortgage bank. Hence, the balancing principle removes interest rate risks in the mortgage bank by aligning funding and lending activities.

2.3.3 CAPITAL REQUIREMENTS

EU regulation sets the capital requirements of mortgage banks according to the Basel Committee directives published by the Bank for International Settlements. The Basel III sets the current regulation, which will be fully implemented by 1st of January 2019 after a 6-year phase-in period since the beginning of implementation in 2013. Most of the regulations are fully implemented, but an additional capital conservation buffer remains to be fully implemented (Basel Committee on Banking Supervision, 2011).

The capital requirement is calculated based on the banks risk weighted assets (RWA). If a bank grants a risky loan of DKK 100, it might carry a 100% risk weight, thus equaling DKK 100 in RWA, whereas a very safe loan, such as a government bond, might only carry a risk weight of 10%, thus having impacted the RWA by only DKK 10. According to the Basel Committee, the risk weights for real estate exposures secured by a mortgage is 35% for loans where the LTV is between 60% and 80% (Basel Committee on Banking Supervision, 2015, p. 11). The far majority of Danish mortgages are within this range as the upper limit is 80% and most people have a relative high leverage in their homes as seen by the Gross household debt-to-disposable income ratio of 292% in 2015 estimated by the OECD. Therefore, the RWA of a mortgage bank in Denmark is roughly 35% of its total assets.

Out of these RWA, the Basel III capital requirements constitute that (Basel Committee on Banking Supervision, 2011):

Minimum of 8% of RWA should be based on capital (equity and equity-like instruments). The components of the capital should fulfil these minimum requirements:

- **Tier 1 (CET1) > 4.5%**
  At least 4.5% of capital should be CET1, which includes common shareholder equity and retained earnings.
- **Tier 1 (T1) > 6%**
  At least 6% should be T1 capital or better. T1 capital includes instruments that are equity like or can be converted into equity such as perpetuity bonds with no coupon or securities that is converted to equity in case CET1 capital falls below a threshold.
- **Tier 2 capital is the last category of capital, that can make up the remaining 2% requirement.**

Tier 2 includes items that have no right in case of a bankruptcy, such as non-secured debt, which must also have a long maturity (over 5 years).

An additional Capital Conservation Buffer is currently under implementation. For now, it requires an extra 1.25% Capital on top of the previous 8%. By 1st January 2019 this buffer will be fully implemented at 2.5%. This buffer should comprise of Common Equity Tier 1 (Basel Committee on Banking Supervision, 2011, p. 54).

On top of that, a countercyclical buffer is applied which is based on nation-wide systematic risk. The buffer can be between 0% and 2.5% and is currently 0% in Denmark (Falch, 2016, p. 13).

Due to being a SIFI (systematically important financial institution), Nykredit is the only residential mortgage bank, which is also required to have a SIFI buffer which will be 2% by 2019. Otherwise, it is mainly commercial banks that are subject to this SIFI buffer (Falch, 2016, p. 14).

On top of these capital requirements, the Danish mortgage banks are required to implement a debt buffer, which, by full implementation in 2020, will be 2% of their total mortgage lending (not of RWA). This debt is unsecured and subordinated to the bond debt that the mortgage bank raise to fund lending. Table 2.1 shows the implementation schedule for the Capital Conservation buffer, Nykredit's SIFI buffer, and the debt buffer.

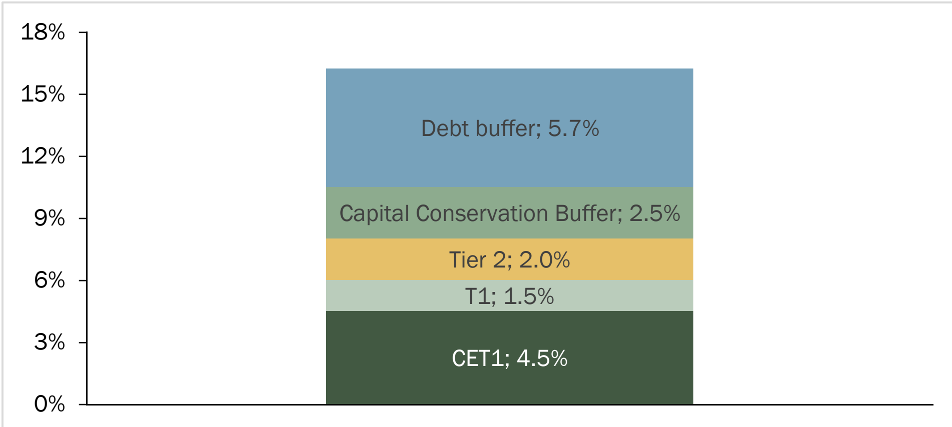
Table 2.1 – Capital and debt buffer implementation (%)

Year	2015	2016	2017	2018	2019	2020
Capital Conservation Buffer (% of RWA)	**0.000**	**0.625**	**1.250**	**1.875**	**2.500**	**2.500**
SIFI buffer (Nykredit) (% of RWA)	**0.400**	**0.800**	**1.200**	**1.600**	**2.000**	**2.000**
Debt Buffer (% of total mortgage lending)	**0.000**	**0.600**	**1.200**	**1.600**	**1.800**	**2.000**

Source: Danske Markets and Basel III

Assuming the Danish mortgage banks' assets have a risk weight of 35% and that total mortgage lending equals total assets, Figure 2.6 shows the composition of the different funding requirements by 2020 as % of RWA assuming no counter cyclical buffer is added.

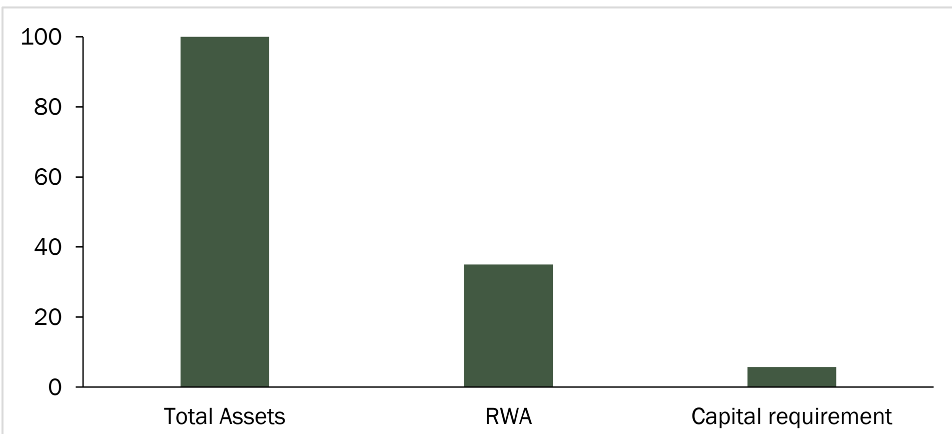
Figure 2.6 – Composition of capital requirements (% of RWA)



Source: Basel III

Although it seems like a lot of capital that the mortgage bank must finance by "expensive" equity, the effect of risk weights still allows for large gearing in the mortgage banks as shown in Figure 2.7.

Figure 2.7 – Non-bond financing in relation to total assets (%)



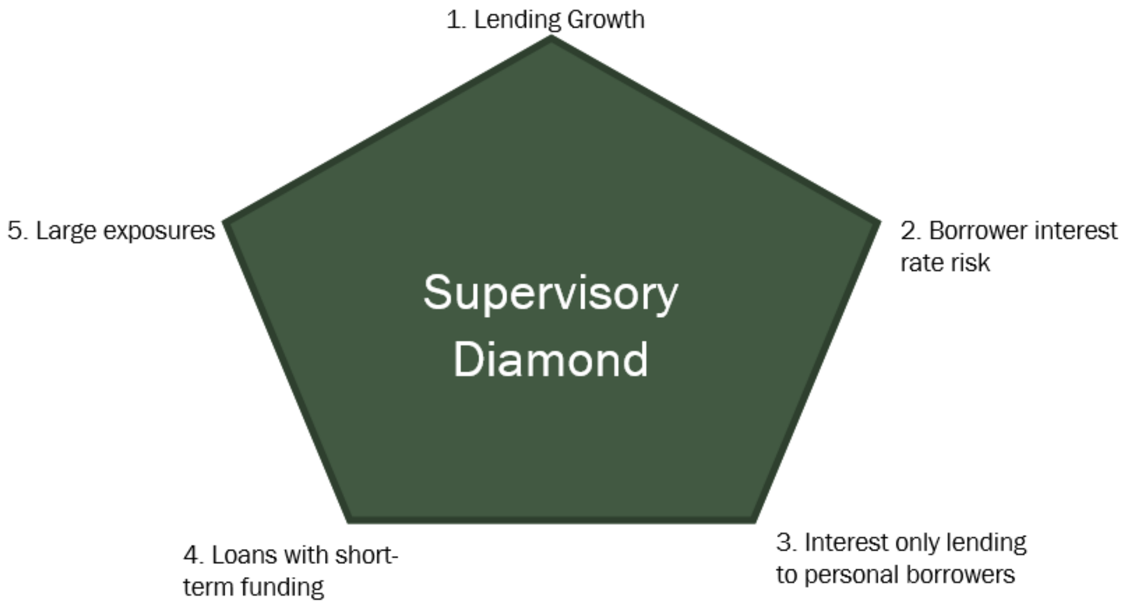
Source: Authors creation, Basel III

The characteristics of the Danish mortgage market ensures very stable mortgage banks and the attractiveness and "safe-haven" status of the bonds issued by Danish mortgage banks, ensures low mortgage lending rates to borrowers while yielding good risk adjusted returns to investors compared to government bonds, which might be the closest proxy in terms risk profiles.

2.3.4 ADDITIONAL DANISH REGULATORY REQUIREMENTS

In addition to the capital requirements enforced by EU directives, the Danish Financial Supervision Authority (FSA) (In Danish: Finanstilsynet), is also closely monitoring the Danish mortgage banks and poses its own sets of restrictions on the lending activities. The regulation is described by the supervisory diamond that should be fully implemented by January 1, 2018 (Finanstilsynet, 2014).

Figure 2.8 – Supervisory Diamond for Danish mortgage banks



Source: The Danish FSA

The Danish FSA is regulating the mortgage banks on 5 parameters:

1. **Lending growth**
   Lending growth cannot exceed 15% in any of the four individual segments. The segments are:
   a. Private
   b. Commercial with residential purpose
   c. Agricultural properties
   d. Other commercial

   This prevents premature growth in mortgage banks that are not strongly diversified.

2. **Borrowers interest rate risk**
   The fraction of loans with an interest reset period shorter than or equal to 24 months and an LTV above 75% of maximum cannot exceed 25% of total lending.

Only loans that has a high leverage in terms of LTV is counting in this measure, which for residential properties means a $LTV > 75\% \, of \, 80\% = LTV > 60\%$. Large movements in interest rates would impact the mortgage payments of borrowers with short interest reset period and high LTV. This restriction prevents the mortgage banks from having too large credit risks due to borrowers taking on high amounts of interest rate risk.

3. **Interest only lending to private borrowers (only residential and holiday homes)**
   The fraction of loans with interest only periods and an LTV exceeding 75% of the maximum (same as in 2.) cannot exceed 10% of total lending.
   Only the fraction of a loan that exceeds the 60% LTV limit is counted in the 10% limit. This restriction is made to lower the credit risk of the mortgage banks by decreasing the amount of loans with continuous high LTV values.

4. **Loans with short term funding**
   Fraction of loans refinanced each quarter and each year should not exceed 12.5% and 25% respectively.
   This restriction can be met by spreading out the refinancing of outstanding loans on more quarters of the year or by lowering the amount of debt with short term refinancing rates. This lowers the mortgage banks exposure towards the refinancing risk at each individual auction.

5. **Large exposures**
   The sum of the 20 largest exposures should not exceed the sum of equity in the mortgage banks.
   This ensures that a mortgage bank does not become too reliant on its largest clients, which could then achieve an unwanted high bargaining power over the mortgage bank. This also secures that the mortgage banks can withstand defaults among their larger clients.

In addition, private home buyers are required to have a 5% equity stake in the property when purchasing a new home. This implies, that on top of the 80% mortgage financing borrowers may only have a mezzanine loan of maximum 15% LTV.

2.3.5 INTEREST RATE AND FAILED AUCTION TRIGGERS

Since January 1, 2015 a new law has shifted the refinancing risk from borrowers to investors on non-callable bullet bonds and floating rate bonds. An interest rate trigger is limiting the borrowers risk towards increasing interest rates, and a failed auction trigger is limiting the borrowers risk of the bonds not being fully refinanced (Falch, 2016, p. 15).

The interest rate trigger

The interest rate trigger is only applicable for bonds where the refinancing period is 24 months or less. Borrowers that have financed their mortgage using these bonds are protected from increases in interest rates that are higher than 500 bps (basis points), corresponding to 5%, over a 1-year period.

If the interest rate increases more than 500 bps on the ARM's, the maturity of the current bonds will be extended by one year instead of being refinanced. Hence, the bonds are converted into one-year bullet bonds. The interest rate on the extended bonds will be set at the rate of a similar bond, traded 11-14 months earlier plus 500 bps. Consider the case of a 2-year ARM with an interest rate of 1%. After one year, another series of 2-year ARM are refinanced at 1.5%, but at the refinancing after two years the new interest rate was settled at 8%. In this case, the current investors will not be repaid and new bonds will not be sold at 8% interest. Instead the current investors' bonds will be prolonged by a year, and the interest rate will be set at 1.5% + 5% = 6.5%.

For floating rate mortgages, the interest rate cannot be fixed at more than 500 bps above the most recent fixing. If the interest rate is fixed at 500 bps higher than the last fixing, the fixing will remain at this level for 12 months unless a lower fixing is achieved. The 12-month cap persists for the 12-month period from initiation, even though a lower fixing is achieved during the period. Hence, a floating rate rising from 1% to 6% at fixing, and then 3 months later is fixed at 5%, cannot exceed the 6% cap in the remaining 9 months. According to the Danish National Bank, an increase of more than 5% in interest rates have only happened a couple of times during the past 150 years, proving the scenario rather unlikely (Nationalbanken, 2014).

Failed auction trigger

If a mortgage bank is unable to refinance a bond series when the underlying bonds mature, e.g. due to a lack of demand from investors, the bonds maturing will get a maturity extension of one year. If the bond still cannot be refinanced after one year, they will be extended by another year. The maturity extension of the bonds will continue until the mortgage bank is successful in refinancing the bonds.

The interest rate of the extended bonds is set based on their original maturity:

a. **Bonds with a maturity of 2 years or lower**
   These bonds will have an interest rate of a similar bond traded 11-14 months earlier plus 500 bps (this is equal to the fixing under the interest rate trigger).
b. **Bonds with a maturity of more than 2 years**

These bonds will get an interest rate fixing equivalent of a bond traded 11-14 months earlier, but instead of looking at a bond with similar maturity, the interest rate will reflect a bond with a 11-14-month maturity plus 500 bps.

The interest rate fixed at the first failed auction will remain constant over the possible future maturity extensions until refinancing is successful.

For floating rate bonds that are extended due to a failed auction, the terms during the next year is depending on the timing of the failed auction. If the failed auction happens at fixing, the fixed interest rate (set according to a. and b.) will remain unchanged for a minimum of 12 months unless a lower interest rate is fixed. If the failed auction happens at refinancing, the interest will remain unchanged for a minimum of 12 months.

Due to the interest rate trigger and the failed auction trigger shifting some risk from borrowers to investors, the investors must price this risk into the bonds. Even though the probability of the triggers being triggered is very low, it still has an effect. Denmark's National Bank, Nationalbanken, estimates that the effect on the interest rates will be no more than 10 bps (0.1%) (Nationalbanken, 2014). An investigation by students from the University of Aarhus used historical data and the SABR model to estimate the effect of the implementation of the triggers, and it was found that the impact on interest rates were between 5 and 9 bps, in line with the Danish National Bank's estimation (Kristensen & Larsen, 2014).

2.3.6 COLLATERAL PROTECTION IN CASE OF DEFAULT

In case of a default, the mortgage banks can relatively quickly remove the borrower from their home and sell the property on an auction to recover as much of the loan as possible. Three registers help to ensure that this part of the system is well-functioning (International Monetary Fund, 2007, p. 7):

i. The cadastre (Kort- og Matrikelstyrelsen) is the fundamental register dividing each land parcel in Denmark into individual units. Other registers reference this number, in particular the Land Book (Tinglysningsbogen).
ii. The Land Book registers all rights attached to a property and is a legal register ensuring the private rights to properties. A mortgage will be registered in the Land Book with all details relevant on loan amount, interest rate etc. District courts manage the Land Book, securing the legal right to the property at all times.
iii. The last register is the Municipal Register of Real Properties, which gathers information on the valuation of land parcels and buildings, which is mainly used when collecting property and land taxes.

These registers ensure the mortgage banks rights in case of a default, and registration is a prerequisite for granting the mortgage loan. In the event of a default, the forced sale is carried out by enforcement courts.

The proceeds from the sale is used to cover the costs associated with the default and to cover the claims in order of seniority. Any claims that are not covered by the sales price is deleted from the registers, but the creditors retain a personal claim against the defaulting borrower. The length of this process is generally no longer than six months from the time of default (International Monetary Fund, 2007, p. 7).

2.4 THE FUNDING OF MORTGAGE LOANS

Since the amendment on July 1, 2007 to the legal framework of the Danish mortgage system, the mortgage banks have been able to issue three types of bonds to fund their mortgage lending, RO (Mortgage bonds, in Danish: Realkreditobligationer), SDO (Covered bonds, in Danish: Særligt Dækkede Obligationer) and SDROs (Covered mortgage bonds, in Danish: Særligt Dækkede Realkreditobligationer). With the amendment, the Danish covered bond system became compliant with EU Capital Requirement Directive (CRD) and the Danish commercial banks were allowed to compete with the mortgage banks through the issuance of SDOs, whereas RO and SDROs are only issued by mortgage banks (Falch, 2016, p. 8).

Three types of bonds are eligible to be named and marketed as covered bonds, namely the SDO, SDRO, and ROs issued before the December 31, 2007 (grandfathered ROs). ROs issued after December 31, 2007 do not fulfill the covered bond requirements, which make them less attractive for investors. Mortgage banks are only allowed to fund their lending by issuing bonds, and cannot gather deposits as a way of funding lending as commercial banks can (Falch, 2016, p. 16).

The key difference between the ROs and the covered bonds, SDO and SDROs, are risk weights, asset eligibility, and LTV requirements, as seen from Table 2.2.

Table 2.2 – Criteria for bonds funding residential property

Bond type	RO	SDO / SDRO
Risk weight	20%	≤10%
Asset eligibility	Real property	Real property, public loans, derivatives, and substitution assets
LTV requirement	≤80% at grant date	≤80% at all times during loan period

Source: (Falch, 2016)

The risk weights are important in the sense that a higher risk weight increases the regulatory requirements of the investor buying the bonds. Hence, investors are willing to pay higher prices for SDO and SDROs in comparison to the ROs due to the lower risk weights that are connected to the bonds. This is true even though the actual risk of the bonds might be the same and they were secured by similar properties. It is only the status as a covered bond and the following stricter requirements, and not necessarily only the true risk of the bonds, that determines the risk weight.

Asset eligibility determines what types of assets that can be posted as collateral for the bonds. Here, the SDO/SDROs are more flexible, and the mortgage bank or commercial bank can choose to place the proceeds from the bond financing in other instruments, which they then post as collateral, until it can be lent to new borrowers. However, securities may only serve as collateral for a maximum of 90 days. The proceeds from covered bond funding must be invested in mortgage lending within this 90-day period, hence ensuring that real property is the primary source of collateral. Securities that can be used as temporary collateral include, Government bonds and bank deposits from OECD member countries, covered bonds issued by mortgage banks in OECD countries, and deposits in commercial banks with a maximum term of 12 months.

The LTV requirement is easier when issuing ROs. Only when the loan is granted is there a requirement for the LTV to be under or equal to 80%. This also poses higher risk to the investors in ROs as there is no continuous surveillance of the LTV and no action taken on loans that are potentially insolvent. Here the SDO/SDRO require a continuous tracking of the LTV limits. Once every three years the LTV requirements must be observed and in the case of a decline in the asset value and the 80% LTV requirement is exceeded, the bank must raise additional funds with junior covered bonds equivalent to the amount that exceeds the LTV limit. These proceeds are then invested in government bonds or similar assets, which are posted as supplementary collateral to cover the gap.

All assets used as collateral in a mortgage bank are gathered in cover pools for each bond series. The junior covered bonds are secured by the assets in their respective cover pools, but are subordinated to the SDO and SDROs in funding the series. Therefore, in the event of a mortgage bank defaulting, the holders of junior covered bonds will only get their money back when all holders of SDO and SDROs have been paid in full. ROs are issued in separate capital centres with a separate cover pool.

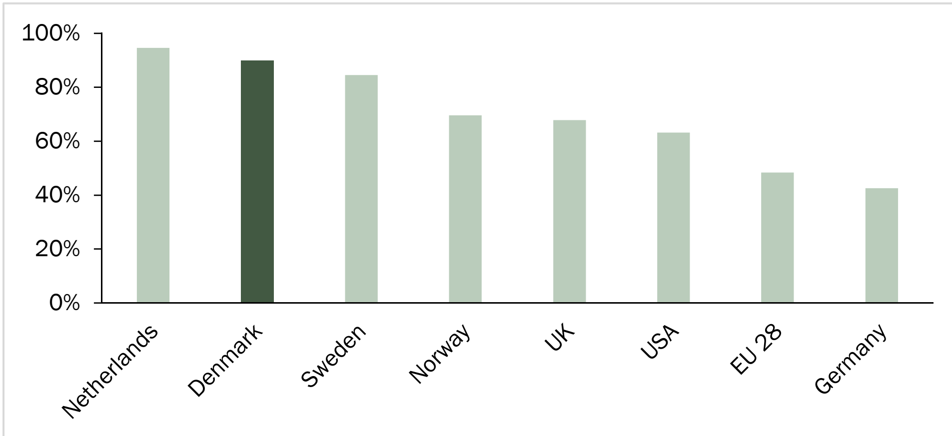
2.5 THE MAGNITUDE AND COMPOSITION OF THE DANISH MORTGAGE MARKET

The strong characteristics of the Danish mortgage market and the comprehensive use of covered bonds in the funding of mortgages have enabled Danes to get large funding of their home purchases and made the Danish market for covered bonds the largest and most liquid in the world.

2.5.1 THE SIZE OF THE DANISH MORTGAGE MARKET

Relative to GDP, the amount of outstanding residential loans in Denmark is the second highest in the world, only beaten by the Netherlands as seen in Figure 2.9.

Figure 2.9 – Residential loans to GDP, 2015

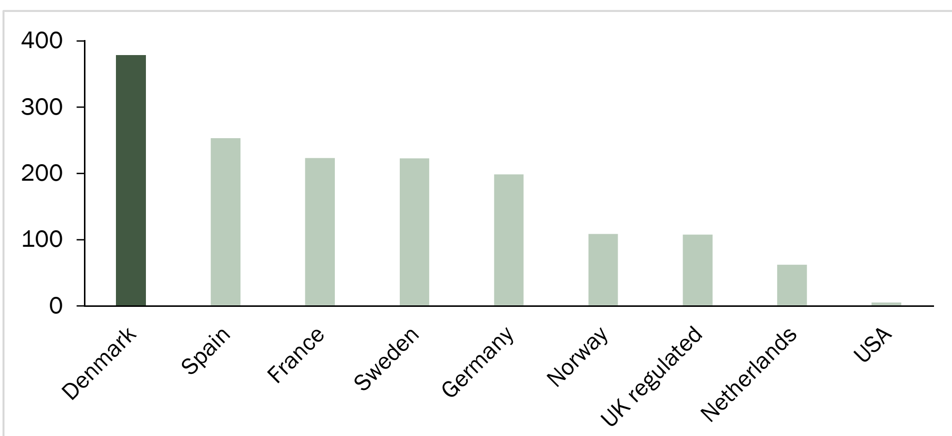


Source: EMF Hypostat, 2016

The higher leverage in the Dutch households is partly due to the LTV limits of Dutch mortgage loans being allowed to exceed 100% (De Nederlandsche Bank, 2015), and the extensive use of residential mortgage backed securities (RMBSs) in the funding of the loans, providing the supply of funds to the Dutch mortgages (EMF, 2016, p. 123). The LTV limits will gradually be lowered towards 90% by 2028 and this will likely push the mortgage to GDP ratio closer to the Danish (Mastrogiacomo & van der Molen, 2015).

The Danish mortgage system does not use RMBSs but are instead funded by covered bonds. In fact, the Danish covered bond market is the largest in the world with a total of approximately EUR 378bn in outstanding covered bonds by the end of 2015 as seen in Figure 2.10.

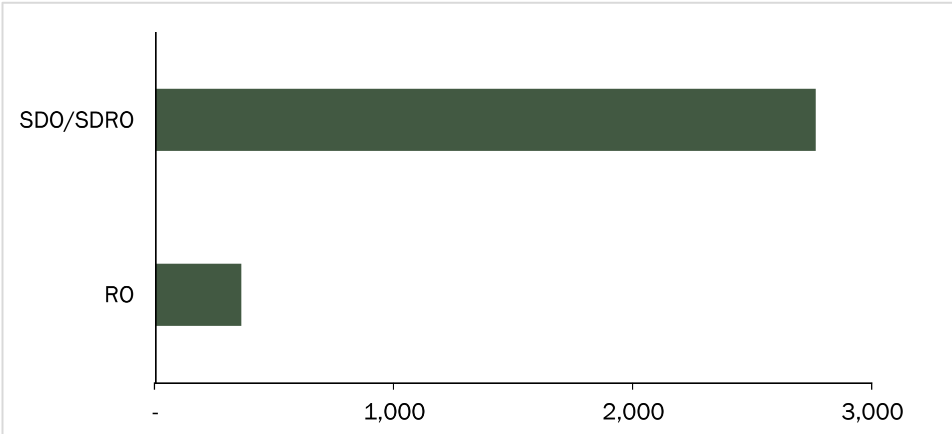
Figure 2.10 – Total covered bonds outstanding, 2015 (EUR billion)



Source: EMF Hypostat, 2016

As a function of the large covered bond market in Denmark, the issuance of ROs have also been limited. The total outstanding market value of ROs only accounted for 12% of the total Danish real estate bond market issued by the Danish mortgage banks by the end of 2016, as seen in Figure 2.11.

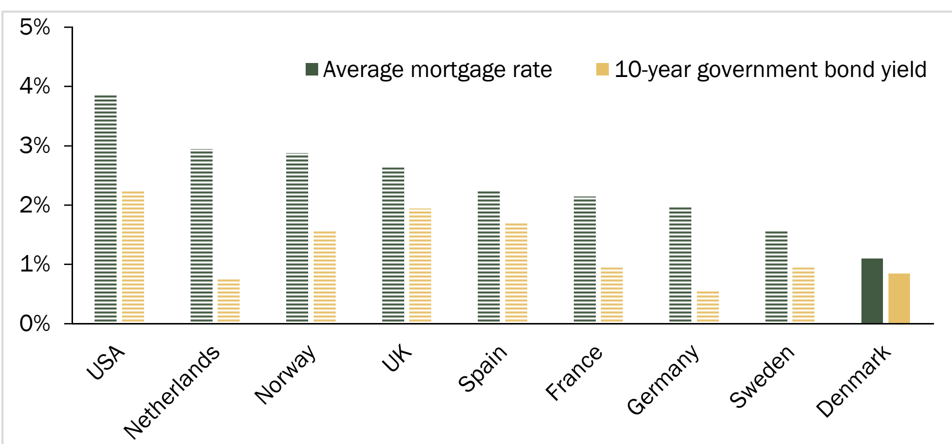
Figure 2.11 – Total market value of bonds issued by Danish mortgage banks, end of 2016 (DKK billion)



Source: The Danish National Bank

The more stringent restrictions on the covered bonds and the low risk weights applied, makes the covered bonds highly attractive among investors, which is seen by the difference in interest rates that the Danish and Dutch loans were assigned during 2015. By the end of 2015 the Danish 10-year government bond yield was 0.83% and the corresponding Dutch yield was 0,75% according to OECD. Hence, the Dutch interest rates were lower than the Danish, but the average mortgage rate was 1.1% in Denmark and 2.9% in the Netherlands. The unique and strong design of the Danish mortgage system ensured lower interest rates for homebuyers compared to similar countries with similar government bond yields. An overview of mortgage rates and government bond yields is shown Figure 2.12.

Figure 2.12 – Average mortgage rates and Government bond yields, 2015

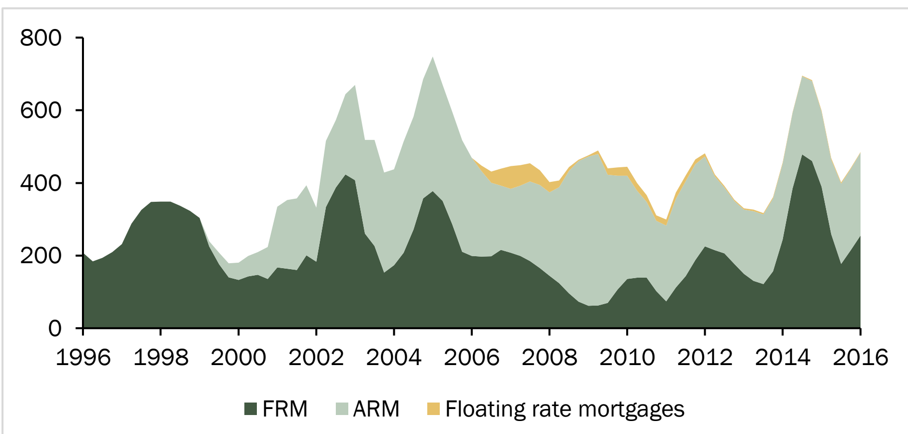


Source: OECD and EMF Hypostat, 2016

2.5.2 THE COMPOSITION OF THE DANISH MORTGAGE MARKET

Figure 2.13 shows the development in new loan issues since the mid 90's. Since its introduction in the late 90's, the ARM has been a very popular way to finance one's house. Prior to the introduction of the ARM, the FRM was the only option the borrowers could choose. When the floating rate mortgage was introduced around 2005, it sparked some interest amongst borrowers, but never established itself, and continues to be a small fraction of the Danish mortgage market. During the booming years after the year 2000, there is a very high loan activity which stabilises after 2007. The fact that the Danish mortgage market does not stall during the global financial crisis proves the system's robustness and strength, even in a distressed state of the economy, due to the very limited risk incorporated in the system as described in section 2.3. During 2015, there is a large spike in the issuance of FRMs. This occurs after a several years with falling interest rates (See Figure 2.3), and homeowners then being able to lock in interest rates at historically low levels prompting many to refinance to a FRM or new borrowers to use a FRM. Hence, the many people motivated to refinance and change their loan type caused the amount of issued loans to spike, but is has since dropped back to the average level of DKK 400-500bn annually. This level has been the norm seen since the record year of 2005 where the amount of issued loans peaked at DKK 748bn.

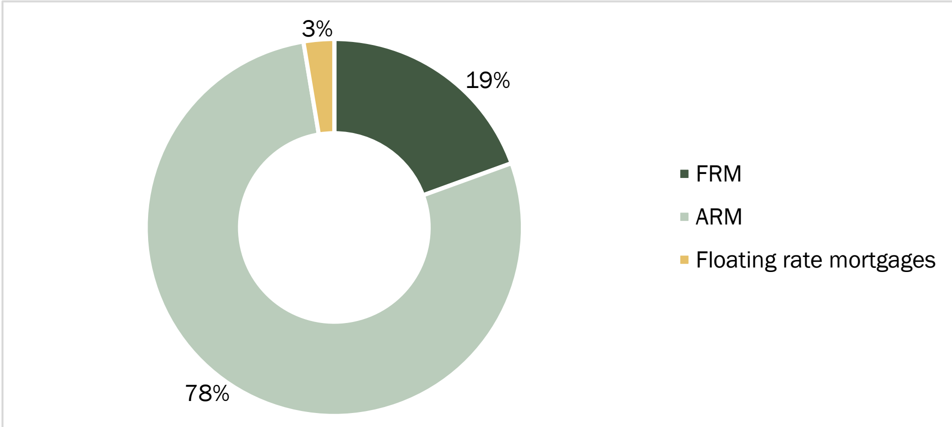
Figure 2.13 – new mortgage loan issues, rolling 12 months (DKK billion)



Source: Danish Mortgage association

Even though the FRM increased in popularity during 2015, the ARM was still the dominating loan in Denmark at the end of 2016 with 78% of the total outstanding loans being ARM's, as shown in Figure 2.14. This is also why the Danish FSA has acted and pushed restrictions on the level of interest rate risk that the Danish mortgage banks can allow borrowers to take on (see section 0).

Figure 2.14 – Distribution of total outstanding mortgage loans, end of 2016.

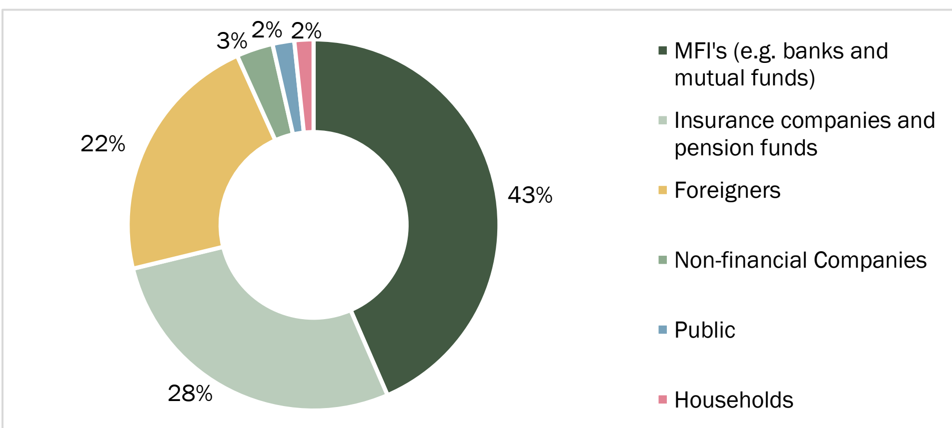


Source: The Danish Mortgage Association

2.5.3 INVESTORS IN THE DANISH MORTGAGE MARKET

The investors in the Danish mortgage bonds are primarily from three different categories: MFI's that includes most financial institutions such as banks, mutual funds, and other funds (43%), Insurance companies and pension funds (28%), and foreign investors (22%) as seen in Figure 2.15. The share from insurance companies and pension funds include their indirect share through investments in mutual funds etc.

Figure 2.15 - Investors in Danish mortgage bonds



Source: The Danish National Bank

Important to notice is that the a very large share of the investors in Danish mortgage bonds is the Danes themselves. Deposits in banks, investments in mutual funds, insurance payments, and pension savings are all coming from the Danes. The companies then use some of the proceeds to invest in mortgage bonds, which is the used to grant loans to the Danes. Hence, the Danish mortgage system is, to a large extent, based on Danes granting loans to other Danes through their savings and insurance programs.

2.6 OUTLINE

The Danish mortgage market is said to be one of the best in the world, and evidence shows that Danish homeowners get some of the cheapest home financing in the world up to 80% LTV, while being able to choose products that protect their solvency in periods of rising interest rates. However, a system that is arguably the best in the world is not necessarily perfect, and might still have room for improvement. Looking for ways to improve the Danish mortgage market, the focus is shifted towards the biggest mortgage market in the world in nominal terms - the American mortgage market.

3 COMPARISON WITH THE AMERICAN MORTGAGE MARKET

To understand how well the Danish system is structured and functions in practice, and investigate how the Danish system differs from other systems in the world, it is natural to compare the system to the largest mortgage market in the world. Both the Danish and the American mortgage systems are believed to be among the most complex and sophisticated mortgage systems in the world (Frankel, Gyntelberg, Kjeldsen, & Persson, 2004), and measured by nominal size, the American system is by far the largest mortgage finance system in the world. It seems natural to compare the two systems to outline similarities and differences, which, in the end, can show the advantages and disadvantages associated with both systems, and highlight possible improvements to the Danish system.

3.1 INTRODUCTION

The Danish mortgage system has a long successful story since it was created back in 1795. There has never been any bond series that have defaulted, and the system has proven to be stable even through a financial crisis, such as the latest back in 2008. The Danish system remained liquid throughout the crisis, and thus continuously provided mortgage financing to the Danish households. The American system did not pass through the financial crisis with similar ease. During the financial crisis, that emerged in the United States, the mortgage system was one of the factors that led to a global economic recession. In short, a subprime mortgage crisis was trigged by adjustable rates and falling house prices, which hit borrowers that were less creditworthy.

The mortgage institutions had granted mortgages over a period on favorable terms to people that did not fulfill the standard requirements to get a mortgage. The loans to less creditworthy borrowers became known as subprime mortgages, and was highly flawed with poor underwriting standards. The loans featured lower down payments, slower amortization, higher LTV, and less documentation (Fabozzi F. J., 2016, p. 134). The result was much higher risk than what was the case for conventional mortgage loans, and housing prices increased due to the subprime lending entering the market. The investment banks carelessly continued to buy up subprime loans, even though they were far riskier, as they could transfer the risk to investors. The investors continued to buy the securitized pools of riskier mortgages as the packaging of the loans hid away the true risk. The risk transfer led to even sloppier underwriting standards. A large fraction of the subprime borrowers began defaulting on their mortgages, which affected the whole mortgage market and led to sharp declines in house prices, as the market was flooded with foreclosure properties. The crisis was magnified to a global extent by global banks that had taken on excessive risk, such as Lehman Brothers that in the end went bankrupt.

3.2 DESCRIPTION OF THE AMERICAN SYSTEM

Both the Danish and the American system is known to be two large systems, which are both created with the goal to offer households relatively cheap financing of real estate. In nominal size, the US mortgage market is by far the largest in the world, whereas they both are large when compared relatively, as seen in Table 3.1. The Danish market has a high value relative to the size of the economy at 129% of gross domestic product (GDP), where the American system is only 76% of GDP. This shows that the Danish system is very reliable and effective, and that the borrowers and regulators trust the system, allowing high leverage. Further, it can be seen from the Table 3.1 that the Danish households are much more levered than their American counterpart, as the Danes have a debt ratio to disposable income that is approximately twice as high. Lastly, the table also shows that the number of available mortgage institutions are very large in the United States where there are more than 7,000, in sharp contrast to the four major mortgage institutions in Denmark: Realkredit Danmark, Totalkredit/Nykredit, BRFkredit and Nordea Kredit. The reason there are so many providers of mortgage loans in the United States, is due to fact that the American system is structured in a different way than the Danish.

Table 3.1 - Summary of the Danish and American mortgage markets (Q3 2016)

Statistic	Denmark	United States
Total market value of mortgage debt (USD billion)	374	14,188
Total value as % of GDP (2015)	129%	76%
Households debt to disposable income (2015)	292%	112%
Real estate equity to total real estate value	41%	57%
Number of (in DK, only major) mortgage loan originators	4	>7,000

Source: Nationalbanken, Statistics Denmark, OECD, and FED

3.2.1 STRUCTURE

The American mortgage system is divided in two major parts, a primary market, and a secondary market (Weiss & Jones, 2017). The primary market is where borrowers connect with a loan originator to obtain a mortgage to finance the purchase of a property. In Denmark, mortgage originators need to be licensed, why the lending is limited to few originators, whereas in the United States many different types of originating lenders can grant a mortgage, since the originator does not need a license. The originator can be banks, credit unions, and other types of finance firms that can grant the mortgage loan, and there is practically no restriction on what type of organization that can enter a loan transaction.

Whenever a person applies for a mortgage, the lender will underwrite the loan, through an evaluation process like the one done in Denmark. The credit history, income, debt etc. are checked to make sure that the borrower is creditworthy and thus, will be able to repay the mortgage. Only after the person's creditworthiness has been approved, the originator will underwrite the loan. In addition to the borrower being personal liable, the real estate is pledged as collateral. Both are done to protect the lender in case the borrower is unable to repay the mortgage. In case the borrower cannot or is unwilling to meet the payments on time, the lender can foreclose the loan, and liquidate the house pledged as collateral by selling it to receive the amount that is owed by the borrower. Like in Denmark, the mortgage gets first lien status in the property pledged as collateral. Thus, in case of default, the lender will be the first creditor to receive proceeds from a liquidation. Other loans that are obtained to buy the house, will receive a higher lien status, and will therefore only receive proceeds from a liquidation if the mortgage loan receives full repayment and transaction costs related to the foreclosure process are paid.

The ability to seize the pledged collateral, vary by both law and state. Overall, the foreclosure can be done by either a judicial or non-judicial process. In the judicial process, the lender files a lawsuit against the defaulted borrower, and the property is auctioned away to the highest bidder. In a non-judicial process, the lender does not need the court to grant the seizing of the property, but instead the lender will be carrying out the auctioning process. In few cases, it is also possible to carry out a strict foreclosure where the lender receives the ownership of the property and in turns neglect the debt (HUD.GOV, 2017). The type of valid foreclosures in each individual borrower's situation, is written in the mortgage contract and governed by state law.

In both countries, the lender can decrease the risk by requiring a down payment. In Denmark, it is required by law that the down payment in percentage of the total purchase price is 5%, such that the total Combined Loan to Value (CLTV) is below 95%. A side from the down payment, the mortgage loan can only finance 80% LTV. The remaining 15% can be financed by a larger down payment, or by obtaining a loan with second lien status. In the United States the law does not state a required down payment, or any maximum LTV values, as is the case in Denmark, but most lenders will often require at least 20% in down payment, before granting the mortgage loan.

Just as there exist a large amount of mortgage originators, the selection of mortgage loan programs is wide and differs from lender to lender. The down payment varies across loan programs, for instance first-time home buyers can be approved if certain conditions are met, and only need to put down 3.5% in down payment. Other programs are available for veterans, who can be granted a mortgage without any down payment. A special government department grants these loans, and guarantee their timely payment. The mortgage loans with high LTV ratios are often characterized as high-risk loans, and the borrower will often be required to buy insurance to cover any arrears. The insurance will payout compensation for arrears to the lender and principal in case of the borrower defaulting, and thus decreases the lenders risk. If the down payment is lower than 20%, it is a requirement that the mortgage is insured. The insurance will typically cover the portion of the loan that exceeds the 80% LTV, and can include a clause such that whenever the LTV drops below 80% the mortgage insurance terminates (Fabozzi F. J., 2016).

The risk can also be reduced if the borrower increases the down payment. In some cases, the borrower is then able to get discounts on the initial interest rate on the mortgage. The increased down payment lowers the riskiness of the loan, thus making the investor's required return lower. The increased up-front investment is measured in terms of "mortgage points", where one point equals one percent of the loan initially granted. For every additional point that the borrower adds to the down payment, the interest rate is decreased by e.g. 0.25%, resulting in both lower periodic interest payments and lower periodic repayments due to the lower loan principal. Often, lenders only provide the opportunity to buy up to three mortgage points (U.S. Bank, 2017).

It is also typical that the interest rate varies with the borrower's creditworthiness, such that there can be significant difference between the interest on a highly creditworthy person and a less creditworthy person. The credit assessment is based on a credit score, which is a three-digit number that represents the person's creditworthiness and is based on various information about the person's income, living expenses, and credit history. The credit score is used in almost every lending decisions, and varies from 300 to 850 in the basic credit score[1], where a higher number represents a higher creditworthiness. The score is calculated based on framework developed by a company called FICO. Therefore, the credit score often is referred to as FICO Score (myFICO, 2017). A credit score above 800 is categorized as exceptional, above 740 is very good, above 670 is good, above 580 is fair, and below 580 is poor. The number is used to assess how likely it is that the borrower will repay the obligations. The FICO score has continuously been adjusted through time to give the lender the best view of the borrower's credit situation. The current score is based on five main categories. The different categories are assigned with different weights in the total score, where the payment history and the amount of outstanding debt is given the highest weights of 35% and 30%, respectively. Apart from the highest influencers of the credit score, the length of credit history, new credit, and the credit mix are also considered with lower weights.

In the Danish mortgage market, it is not possible to bargain and get a lower interest on the mortgage, since the whole system is founded on the "matching-principle", where the mortgage interest rate reflects the market interest rates. The provided mortgages have a close relationship to the issued bonds, and the interest rates is therefore given by the market rather than being subject to credit assessment and bargaining power. The only part of the mortgage that is subject to what can be thought of as bargaining, is the margins, which will be lower the lower the LTV is, and therefore can be lowered if a higher initial down payment is made.

[1] There exists different score systems with either a wider number range, or a different measurement scale. The credit score is often tailored to the specific needs in the lenders industry. (Mortgage, auto loans, credit cards etc.)

Instead of varying the interest rate, the credit assessment determines the amount a borrower can borrow so that the risk is in accordance with the requirements of the mortgage banks.

3.2.2 LOAN TYPES

The types of mortgages that are available for borrowers in United States are very similar to the types offered in Denmark. The most common loan type is the FRM, but the mortgage lenders also offer ARMs that are tied to an underlying index and with pre-agreed readjustment intervals where the interest and periodic payment is adjusted according to the underlying index changes. The ARM loans with an initial fixed rate, are often referred to as hybrid ARMs. As is the case in Denmark, the maturity on these loans do not need to be 30 years, but can for instance be amortized over 15 years instead, if the borrower wants a shorter payment horizon. By choosing a shorter amortizing period the lender will usually provide a lower interest rate, since the risk is lower because of the shorter horizon.

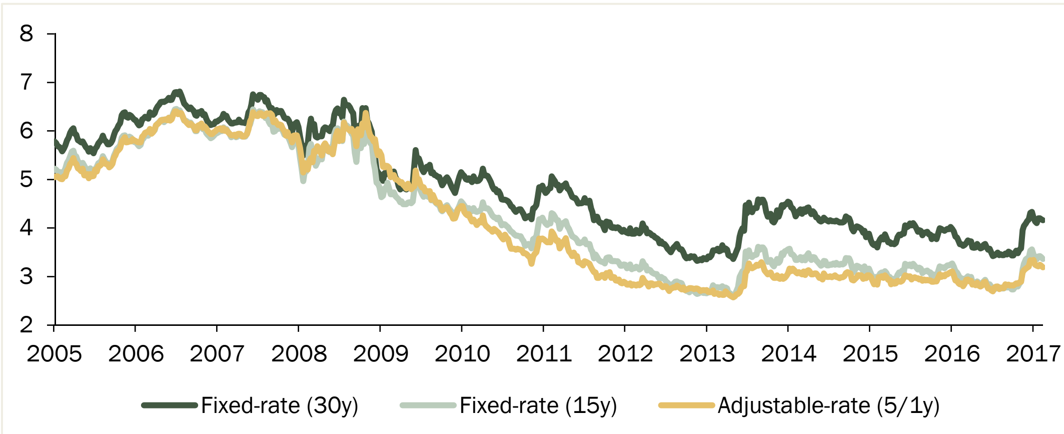
ARMs are typically expressed by two numbers, where the first number specify the length of the period where the rate is fixed, the second number carries different meanings, but often the number specifies how often the note rate will be adjusted after the fixed rate period is ended. As an example, a 5/1 ARM loan starts with a fixed rate period of 5 years and then has an adjustable rate that will be adjusted annually in the future. The ARM loans are similar to the adjustable loans offered in the Danish mortgage market, where different adjustment periods are also offered. The main difference is that the Danish ARM loans are not pegged to a reference rate and do not have an initial period of fixed rate, but will be adjusted each period according to the agreed terms. A Danish F5 ARM loan will be similar to a 5/5 ARM loan in the US, but the US ARMs are a combination of the Danish ARMs and floating rate mortgages. The interest reset interval is like the Danish ARMs but the pegged interest rate is like the floating rate mortgages.

The adjustable loans most often use the LIBOR rate, or the treasury rate as index rate. Every time the loan rate is adjusted, the current index rate at that time is used and a margin rate is applied on top. The margin rate is fixed throughout the entire maturity of the mortgage. Often the loan will have a lifetime rate cap and/or floor included in the contract such that the future interest rate cannot be above or below certain thresholds. The interest rate caps and floors is added as an insurance for the borrower.

Both FRM and ARM loans are created such that they are fully amortized during the loan period. This means that the loan will be paid back fully when the loan matures. It is possible the get an interest-only period, where only the interest is paid in an initial period of the mortgage lifetime (DiGangi, 2015). After this period is complete, the payments will increase, since the mortgage now needs to be amortized over a shorter period than if the borrower did not choose an interest-only period.

In Figure 3.1 the mortgage rate development is shown for selected FRM and ARM mortgages since 2005. The evolution since the financial crisis has been clearly declining, with some minor periods with interest increases, which is the same development that is seen in the Danish mortgage rates. The graph is based on average mortgage rates, because the mortgage rate differs across different lenders and according to the borrower's credit status.

Figure 3.1 – Mortgage rates since 2005 (%)

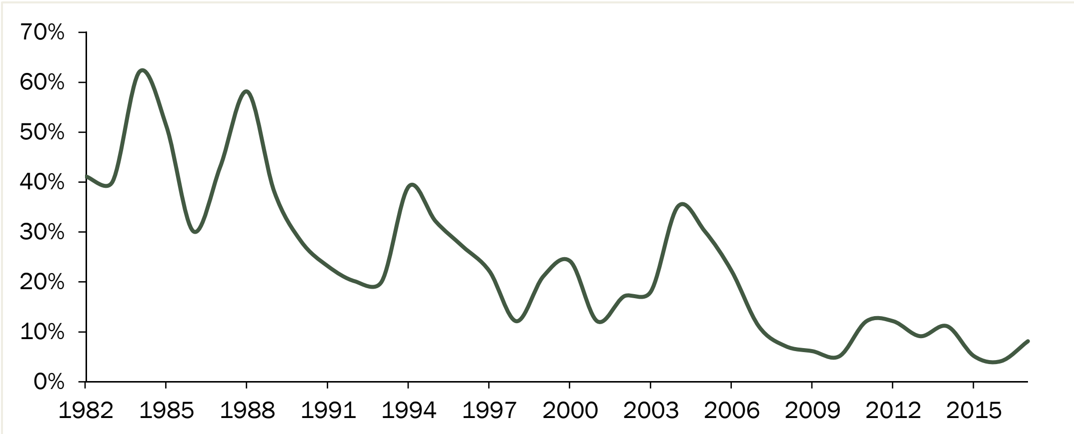


Source: Federal Reserve Bank of St. Louis (Freddie Mac data)

As expected, the mortgage rate is higher when the rate is fixed for a longer horizon, thus the 30-year FRM carries a higher interest rate throughout almost the entire period, which is the normal yield curve relationship between interest rate and maturities. A lower rate is associated with the 15-year FRM, and the 5/1 ARM loan has the lowest interest rate. Around the financial crisis the interest rates converged, and there was even a period where the ARM loan carried a higher rate than the 15- and 30-year loans, which shows that expectations on future interest rates were negative at that time.

In Denmark, the ARMs have historically dominated the mortgage market after they were introduced, whereas the fraction of outstanding mortgages with fixed rate has increases in the recent years after falling interest rates. The history is quite opposite in the United States, where FRMs have dominated throughout history. The FRM loan offers stability, with stable periodic payments without uncertainty in the loan period. The mortgage is also flexible since the borrower can prepay the outstanding balance easily and without penalty fees. The advantages come at the cost of a higher interest rate, but many Americans seem to prefer the fixed rate. The proportion of the mortgage loans has changed a lot over time, but the FRM has generally been the preferred choice.

Figure 3.2 – ARM share of total mortgage lending



Source: Federal Housing Finance Agency – Monthly Interest Rate Survey

In Figure 3.2, the share of ARMs is shown. The figure is based on data from the Federal Housing Finance Agency (FHFA), who makes a monthly interest rate survey based on a sample of mortgage lenders who reports data on the granted mortgage loans the last months. The figure therefore exhibits the latest development in newly originated mortgages. Through history the pattern has been a decreasing fraction of ARM mortgages, which shows that the Americans generally prefers FRMs. The latest development, with the financial crisis as a catalyst, has been that a very low fraction of new mortgages are ARMs. The ARM loans were the loans that dominated the subprime mortgages during the crisis, and was the loans that typically was chosen by borrowers with bad credit scores and in bad economic standing.

3.2.3 MORTGAGE CLASSIFICATION

In the American system, the mortgages are classified into several categories based on the fundamentals of the mortgage. Overall the mortgages can be divided into two groups: Conventional and government-insured mortgages. The conventional mortgages are the standard types of mortgages, whereas the government insures the government-insured mortgages. The conventional mortgages can then be separated into two additional groups: conforming loans and non-conforming loans. A mortgage is conforming if it lives up to certain standards that are put out by a government agency, known as The Federal Housing Finance Agency. The standard requirement is a loan limit measured in terms of total loan size, and in 2017 the loan limit was USD 424,100, but the limit can be higher in some states, where the housing market experience higher prices and in states with higher demand for houses (The Federal Housing Finance Agency, 2017). There can also be thresholds related to the borrower's creditworthiness, and therefore a requirement that the person should have a credit score above a certain level.

If the mortgage cannot be accepted as a conforming loan, it will be specified as a non-conforming loan. If the reason the mortgage cannot be classified as conforming, is due to a loan value above the conforming loan limit, then the mortgage is called a jumbo loan, and it is then subject to special loan conditions such as higher interest rates and larger down payments. The reason the conditions change when the principal is larger is that the lender's risk is higher, since the lender cannot sell the mortgages to investors as they do not live up to the standard setup. The procedure of selling the provided mortgages to investors is part of the secondary market in the United States, and will be further explained in Section 3.2.5. Section 3.2.5 also present the Government-Sponsored Enterprises (GSEs), which only deal with conforming mortgages, known as Fannie Mae and Freddie Mac. In short, the GSEs buy conforming mortgages from originators, structure them in standardized products, and sell them to investors in the secondary market. In addition, the government entity, Ginnie Mae, also operate with the GSEs in the secondary market, providing insurance for investors in loans with special circumstances, e.g. to veterans.

Apart from the conventional types of mortgages provided in United States, the system offers different kinds of differentiated loan types that are only available to the borrowers if they fulfill certain requirements. On the bottom line, the loans are identical to the conventional loans, but are special because they have less strict requirements in terms of creditworthiness. The most common types of special loans are Veteran loans (VA) and Federal Housing Administration loans (FHA). The loans are government guaranteed and therefore categorized as a government-insured mortgage, where, in case of default, the government will repay the lender, thus shifting the risk from the lender to the government. The VA and FHA loans can generally be obtained without any down payment, and are made to help veterans and less wealthy Americans becoming homeowners. The VA and FHA do not provide the loan, but only offer mortgage insurance, which means that the borrower needs to obtain the mortgage through a private lender. The VA and FHA will then approve that the borrower is eligible to get the insurance, and to receive the accompanying benefits, such as lower down payment. These types of loans help increase the homeownership rate in the United States, because the people that receive these loans would likely not be able to buy their own house otherwise.

In the American system, mortgages are also differentiated by the type of property pledged as collateral. The mortgages distinguish between single- and multi-family homes. The single-family is a property where the number of units only make it possible for a single family to live in the property, where the multi-family homes are large enough for multiple families to live in the property simultaneously. Normally, the buyer of a multi-family home would occupy a single unit or part of the property and then rent the rest out to one or more tenants. This would add income to supplement the buyer's regular income, and thus enable the buyer to afford a more expense property than what would otherwise be possible. Multi-family properties can also be categorized as either residential property or commercial property, where the key factor is the number of units in the property. If the property contains more than four units, then the entire property will be declared as a commercial property with stricter and different requirements than for residential properties.

The multi-family properties are typically more expensive, simply because they are larger than the single-family properties. The higher price will obviously require that the buyer must have a higher income, but apart from this the buyer would face higher interest rates from the mortgage lender, making the multi-family property even more expensive. Higher rates are charged since the additional income from the leases is considered riskier than the conventional income from the buyer's job.

Down payment requirements vary a lot across the different mortgage loan programs. The mortgage lender can offer the borrower different down payment solutions, because the two briefly introduced GSEs, Fannie Mae and Freddie Mac, offers some special solutions, such that the lender can be sure that the granted mortgage loan will fulfill the conforming standards. Through Fannie Mae it is possible to offer a 97% LTV mortgage, with only 3% in down payment. The program is offered to give otherwise unqualified home buyers access to financing, even if they cannot afford a large down payment (Fannie Mae, 2017). The same is the case for Freddie Mac, but they only offer a mortgage solution with 95% LTV. Both GSEs have some standard eligibility requirements, which varies with the transaction type. The types can be divided into either principal residence, second homes and investment properties, where the loan is separated between being a purchase or refinancing, with or without cash-out. The highest LTV ratio loans are available if the transaction type is a purchase or limited cash-out refinance and the property is classified as a single-family home.

Table 3.2 – Overview of mortgage classification

Mortgage type	Conforming	Non-conforming	Government-Insured (e.g. FHA & VA)
Loan limit, USD	<424,100	>424,100	No limit
LTV (Range)	95-97%	<90%[2]	96.5-100%[3]
Originator	Bank or other financial intermediary		
Government insurance	No	No	Yes
Eligible for GSE	Yes	No	Yes[4]

Source: Authors own creation

3.2.4 PREPAYMENTS

The actual duration of a mortgage is most often not equal to the initial maturity period, because the mortgage will often be refinanced or prepaid before maturity. The borrower has the option to prepay the mortgage before it matures. Normally this will happen if the borrower refinances the mortgage or sell the underlying collateralized property, and uses the proceeds to repay the mortgage. This is a risk the lender is exposed to

[2] Based on a comparison between several American mortgage lenders (U.S. Bank, Morgan Chase etc.)
[3] FHA loans require at least 3.5% in down payment, whereas VA loans can be offered with no down payment
[4] Only through Ginnie Mae

called prepayment risk. Prepayments are most likely when the interest rates have decreased after the mortgage was issued. Hence, the borrower can refinance at a lower interest rate and decrease the periodic payment. The lender receives the outstanding principal owed and must reinvest this in the market where the market rate is now lower than what the lender was promised on the previous mortgage. Prepayment risk is mainly a risk on FRMs, as ARMs and floating rate mortgages would adjust to the lower interest rate automatically.

In Denmark, the matching principle connect the market and the borrower, as outlined in Section 2.2, making it possible for the borrowers to prepay the debt at market value. In the American mortgage borrowers do not have this opportunity, even though the mortgage is a callable loan. The fundamentals of the mortgage system do not have a connection between market prices and the borrower’s debt, and the prepayment option will always be redeemed at par value. This is known as the “lock-in effect”. During periods with falling interest rates the property value would, all else equal, increase since the buyers are able to finance larger principals at the same cost. Since the mortgage can be redeemed at par value, this would increase the borrower’s equity in the property, because the loan value does not increase with the property value, which is like the Danish FRM dynamics. In the opposite case with increasing interest rates, only the Danish system protects the borrower from falling house prices, as Danes can repay at a market value lower than par, whereas Americans are forced to repay at par.

The choice to repay to the outstanding mortgage, whether it is because the collateralized property is sold or the borrower wants to refinance the loan, comes at a cost. Especially in Denmark the choice to refinance or repay before maturity is accompanied by cost to both the mortgage institution and the government. In America, it is possible to make prepayment without large related costs, particularly if the original mortgage is a conventional 30-year FRM, which is generally prepayable without penalty fees. Off course, the refinance option only exists if the pledged property’s value has increased or remained the same, since the original mortgage was issued. If the property has decreased in value, the homeowner can be prevented from refinancing because the result might be negative equity position or a too-high LTV. In this case, refinancing can only happen if the borrower repays part of the mortgage so that the fraction refinanced is within the LTV requirements.

3.2.5 GOVERNMENT-SPONSORED ENTERPRISES

In the United States, the government wanted to encourage mortgage lending to increase homeownership during the Great Depression in the 1930s (Weiss & Jones, 2017, p. 9). The government encouraged the lending by creating Fannie Mae, which is an abbreviation of “the Federal National Mortgage Association”. Fannie Mae was reformed in the period 1968-70 and divided into a government part, which was called Ginnie Mae, and another part that retained the name Fannie Mae, which was privatized as a government-sponsored enterprise (GSE). Shortly after, Freddie Mac (The Federal Home Loan Mortgage Corporation) was created with the same purpose as Fannie Mae, and both private companies today operate in the secondary market of mortgage loans with the purchase of mortgages that are qualified and live up to the conforming standards. The purpose of the two GSEs is to increase the secondary market by releasing assets from the balance sheets of mortgage originators in the primary market, such that they can reinvest them in new mortgages. The reason two companies were founded, was to make sure that the secondary market would be secure and function efficient. By having two "competitors" they increased competition, trying to secure high efficiency.

The initial offerings from the GSEs was simple pass-through securities, which was a standard MBS. But investors inquired for investments in mortgages where the income stream was more predictable. The GSEs tried to deal with this problem by issuing the first collateralized mortgage obligation (CMO) in 1983 (Baker & Chinloy, 2014, p. 125). A CMO is more complicated than the standard simple MBS, since the payments are allocated to different investors organized in tranches. The result was that more investors was drawn to the mortgage market, because the availability of products that matched the specific investor's requirements had increased (Immergluck, 2012). This have since become the standard structure of MBSs, and has also become known as a Real Estate Mortgage Investment Conduits (REMICs), which is just another name for a structured MBS (Freddie Mac, 2017).

Ginnie Mae was the only company that was kept as government owned, while Fannie Mae and Freddie Mac was listed on the New York Stock Exchange in the 70s. Ginnie Mae has a different purpose than the two other companies, and focus and operates solely on improving the loan availability for the government supported mortgages, which, for instance, are the loans sponsored by the VA and FHA. Ginnie Mae does not buy mortgages from the originators, but guarantee the mortgages that are approved by them in special government insured programs, and in return receives a fee from the borrower. By having a government supported insurance the lenders are more willing to grant mortgages to these special programs, and that is what the government wants to achieve. As oppose to Fannie Mae and Freddie Mac, Ginnie Mae, carry a full guaranty by the United States government (Ginnie Mae, 2017).

Freddie Mac and Fannie Mae only buys mortgages that live up to the conforming standards. After the mortgage is bought from the originator, the mortgage is either kept on the GSE's balance sheet or sold in the capital market by issuing a structured security called a Mortgage Backed-Security (MBS), which will be explained in Section 3.2.7. Sometimes the GSEs also choose to buy the issued MBS with the purpose of investment. The GSEs guarantees that the investors who buy the MBS will receive promised payments on time, even though some underlying mortgages defaults. This effectively means that investors in the MBS only carries the credit risk of the issuing GSE defaulting, which is similar to the guarantee given by the Danish mortgage banks. The GSE receives a fee ("g-fees") from the investors for granting the insurance through a reduction in the promised yield, whereas the borrowers in Denmark pays a similar fee through the margins.

The guarantee makes the MBS a very popular and attractive investment vehicle, with accompanying high liquidity. The associated benefit is that the GSEs can easily sell the created MBS and transfer the interest and prepayment risk to investors, since the credit risk is retained at the GSEs due to the principal and interest guarantee. The aim of this setup is to offer low mortgage rates for the borrowers in the primary market, because the originators can easily transfer mortgages to the GSEs if the conforming standards are fulfilled (Weiss & Jones, 2017, p. 9). Hence, the originators are not bargaining for higher interest rates as they do not cash in on them but pass them on to the GSEs, which will pass them on to investors, who are the ones that in practice sets the interest rates on the conforming loans.

The United States government does not cover the MBS guarantee, since the issued securities sold by the GSEs only are liable to the GSE itself, and no debt is related to the government (Freddie Mac, 2017). Therefore, Fannie Mae and Freddie Mac formally acts as a solely independent private corporation and should function without government interaction and funding. Even though there are no actual guarantees provided by the United States government, there is a widespread belief that the government would interfere in case of an approaching bankruptcy. Because the consequences of a default would be devastating for the economy in the United States and the rest of the world. The belief is that the GSEs are “too big to fail”, and the government therefore makes an “implicit government guarantee” (The Economist, 2007). This was what happened at the beginning of the Financial Crisis in 2008, where both Fannie Mae and Freddie Mac was taken under government conservatorship, to avoid the collapse of both corporations. Both had built up a financial condition that made them unable to continue without intervention from the government.

Prior to the crisis, the underwriting standards had decreased over a couple of years because credit risk was transferred from originator to GSE, resulting in a large fraction of the established mortgage loans having a borrower in bad economic standing. The originator had sold questionable loans to the GSEs, and the GSEs acquired the loans, since they marked by the originator as living up to the conforming standards. This pressured the GSEs when the crisis hit because they insured the final investor against defaulting borrowers and had an insufficient capital buffer to compensate for the high amount of defaults. In the years following, the GSEs were put under government conservatorship, the GSEs sued many of the originators who had sold these flawed loans (Clein, 2017). The Treasury department provided financial support both in terms of initial cash placement in both GSEs and later investments during the crisis (The Federal Housing Finance Agency (A), 2017). FHFA continues to support the corporations by sitting on the boards, but the corporations are otherwise running as private companies.

In 2012, FHFA launched a paper that included a suggestion to reform and establish a new infrastructure in the secondary market for mortgage loans (The Federal Housing Finance Agency (B), 2017). It was proposed that a single platform should be created in the secondary market, where all market participants could create MBS on behalf of mortgages, and not only the GSEs but also private actors, such that the credit risk can be distributed to several entities. The law stated that all mortgage qualifications cannot be without any missing information, and the standards was generally improved (Baker & Chinloy, 2014, p. 88).

3.2.6 RISK TRANSFERRING

The distinct mortgage classification, and the differences in the mortgage funding under the different schemes, has the effect of transferring mortgage risk to different entities. In Denmark, all mortgages are directed through the same type of entities, only differentiated by the individual mortgage institutions. This means that the same mortgage risk, regardless of the loan, is always retained at the entity originating the mortgage.

Conforming loans are bought and securitized by GSEs, the process transfers prepayment and interest rate risk to the final investor, while the credit risk (e.g. default risk) mains with the GSE, due to the principal and interest guarantee they grant. Thus, the only credit risk the investor is exposed to, is the risk that the GSE defaults and then not being able to honor its guarantee. On the other hand, non-conforming loans are excluded from GSEs and subject to a funding structure where all risk components are transferred to the final investor, regardless of the securitization process, which is done through a Private Label Securitization (PLS) process, as described in Section 3.2.8.

For the final group of mortgages are government insured, and Ginnie Mae guarantees the timely payment of principal and interest, but does not securitize the loans. Typically, the securitization is done by certain pre-appointed private entities approved by Ginnie Mae and then they guarantee the issued MBS. The risk transfer is essentially identical to conforming loans, since the final investor is only exposed to prepayment and interest rate risk.

In case the underlying mortgages is of the type FRM, then only the prepayment risk is transferred to investors for conforming and government insured loans, since the interest rate level is fixed in the mortgages.

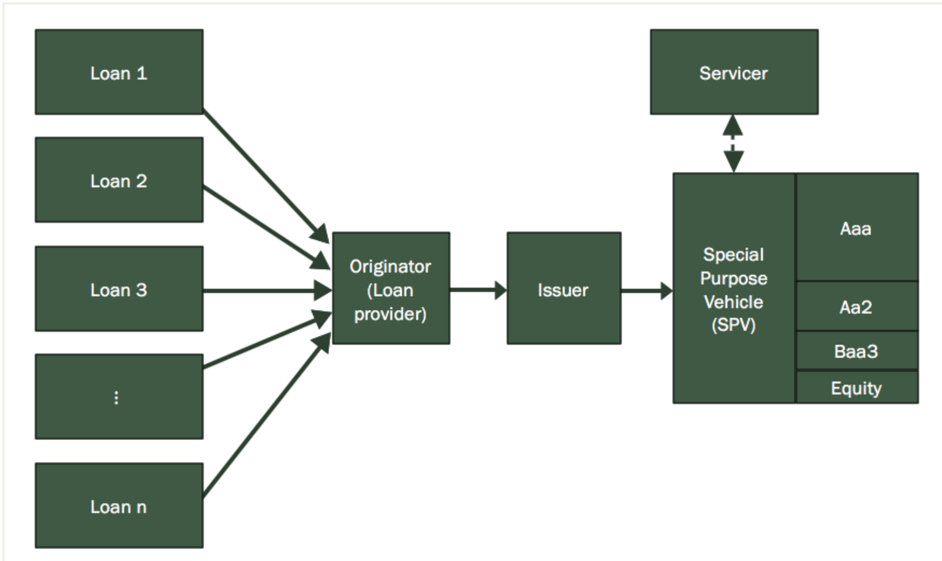
3.2.7 MORTGAGE BACKED SECURITIES

The process with MBSs introduced in Section 3.2.5 is frequently carried out by the GSEs, which often purchase mortgages from originators and structure them in MBSs before selling them to investors. The procedure of reselling the mortgages is referred to as securitization, which is a process where cash flows from several debt obligations are pooled together and sold to investors as a structured product. The process is like the method used in the Danish mortgage market, where the Mortgage institutions provide mortgage loans to borrowers, and then pool the mortgages in series which are funded by covered bonds that are sold to investors. The mortgage institution only functions as an intermediary and as insurance for investors against the credit risk of the loans. In the American market this is done through an MBS, which is a debt obligation created to collect the cash flows from the pool of mortgages and distribute it by a prearranged structure, where the simplest structure is distribution by pro-rata share (U.S. Securities and Exchange Commission, 2017). The MBS is an asset-backed security (ABS) where the asset is the collateralized property in each mortgage. Ginnie Mae guaranteed the first MBS in 1970, and Freddie Mac and Fannie Mae followed a couple of years after with the issuance of their MBSs (Baker & Chinloy, 2014, p. 124).

The way an MBS or ABS works is as follows: The issuer (here the GSE) collects the loans, combine and structure them in pools, and sells the pools to an external legal entity, often referred to as a special-purpose vehicle (SPV) (Cetorelli & Peristiani, 2012, p. 48). The purpose is to detach the mortgage pool from the issuer, such that in case the issuer (GSE) defaults, the owners of the pool (investors) still have the rights to receive the proceeds from the pool. The underlying loans in the pool carries some similarities in terms of attributes such as loan type, coupon rate, maturity etc. When the GSE is the issuer of the MBS, the loan type will be either FRM or ARM, and typically be either constituted by single-family or multi-family properties serving as collateral. The MBS's coupon rate is determined as a weighted average of the underlying mortgage coupon rates, weighted by outstanding principal. Whenever the MBS is created from FRM, the GSE pool loans that have a coupon rate within a range of 225 bps, such that the mortgages in the pool are similar to each other (Fannie Mae, 2016). When the underlying mortgages are FRMs, the MBS coupon rate will also have a fixed rate. When an MBS is created on ARM loans, the MBS rate will also be adjustable, where the interest rate changes according to the mortgages in the pool. The MBS rate will therefore be adjusted with a specific frequency, just like the underlying mortgages. Further, the ARM MBS will have some of the same attributes as the underlying loans, where the interest rate is subject to caps and floors.

The MBS is created as a pass-through security, where the collected interest and principal is passed through to the MBS investors. The actual servicing of the MBS is carried out by a servicer that is usually different from the issuer. The servicer receives a fee, which is deducted from the payments, and so is the guarantee fee to the GSE. The pass-through coupon rate will therefore be lower than the average coupon rate. The structure is shown in Figure 3.3.

Figure 3.3 – MBS Structure



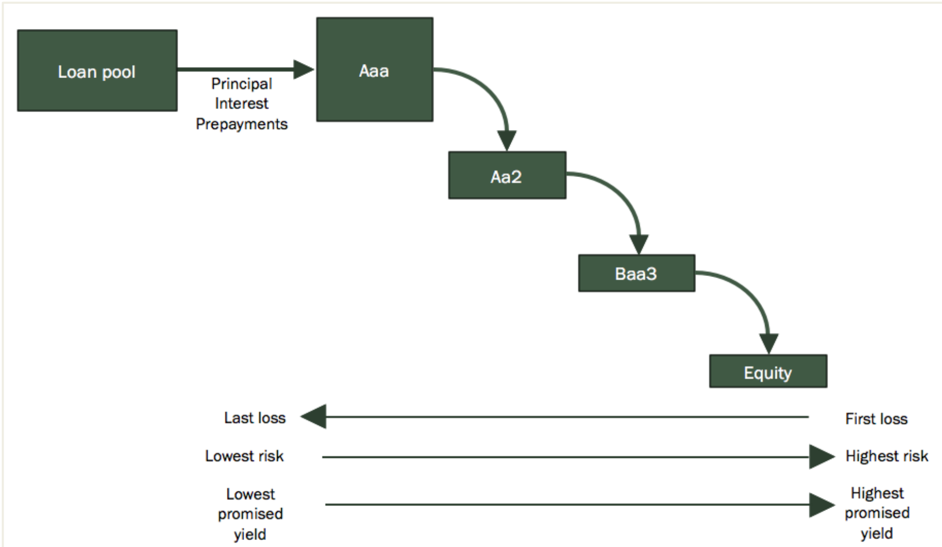
Source: Authors own creation

The final MBS is often sold as a structured note, where the cash flow is distributed according to a pre-specified set of rules. The total principal of the SPV is divided into proportions with senior and junior parts. This process is called tranching, and is defined as separation of the total pool of mortgages, such that the cash flows will be distributed to the tranches according to seniority rules. This is different from the Danish system, where the cash flows from the loan pools are distributed pro rata to the investors.

The method used to allocate cash flows in a security with tranches is following a waterfall rule (Hull & White, 2010, p. 55). Each mortgage contributes payments each period and the pool receives all the payments, which must then be allocated to the tranches. The tranches are created such that each tranche is allocated with a proportion of the total principal. When the interest payments are received the tranche with the highest seniority will receive payment first in relation to their allocated principal. This will continue down through the MBS in order of seniority until there are no cash left to distribute. In case of arrears or defaults, the most junior claims will suffer the losses first, unless the MBS is guaranteed by a GSE. In case of a GSE guarantee, the arrears or defaults will be covered by the GSE and no investors suffer losses.

The higher fraction of mortgages that are in arrears, the more the tranches with lowest seniority will lose in interest and repayments. Thus, the tranches with highest seniority, typically AAA tranches, will be last to suffer losses and are therefore the safest tranche in the security. The waterfall structure is shown in Figure 3.4, where the AAA tranche receives full payment before any cash flow is passed on the AA tranche, and so forth.

Figure 3.4 – Illustration of Waterfall cash flow allocation



Source: Authors own creation

The way that the tranches is created is most typically by using credit ratings. The principal in each tranche is chosen such that the tranche is made as wide as possible, while making sure the credit rating of the tranche remains the same. The same is done for the next tranche, all the way down to the tranche with the lowest seniority. Even though the underlying pool of mortgages is the same, the use of securitization can make securities that have different risk profiles. Therefore, a single MBS can be attractive to different types of investors with different preferences with respect to risk.

The purpose of the securitization process is to decrease the risk associated with the individual mortgage loan, and to increase the liquidity in the mortgage market. By offering a somewhat standardized product, which are differentiated to match different investors risk preference, the secondary securitization market is used by GSEs to remove conforming mortgages from their balance sheet. By adding an implicit government guarantee to the securities, and by restricting the pools to conforming loans, a market with high liquidity is created. The guarantee by the GSE, helps provide liquidity, because investors do not need to gather information about the underlying borrowers to the same extent. The result is lower funding costs for mortgage lenders, which ultimately ends up as lower costs to homeowners (Baker & Chinloy, 2014, p. 127).

But since the GSEs are only allowed to buy and securitize conforming loans, there is need for a market for the loans that does not fall within these standards. This part of the secondary market is the private label solutions.

3.2.8 PRIVATE LABEL SECURITIZATION (PLS)

A mortgage that does not live up to the conforming standards, can be securitized through a private entity. The mortgages that do not match the conforming criteria typically have a principal that is too large, so-called jumbo loans, or do not live up to given credit standards and can thus be considered as subprime loans. The borrowers behind such mortgage loans would face limited credit availability and higher prices, if the lenders were not able to securitize the loans. The alternative would be for the lender to keep the loan on the balance sheets and would thus, by law and regulation stated by Basel III, be required to maintain a substantial capital buffer to cover defaults. This would increase cost for the borrower, since the lenders link higher costs to these loans. Without the ability to remove the risk from the balance sheet, few lenders would offer the loan, with the result that loan availability will vanish or be heavily reduced.

There exist private corporations that acts in the same manner as the GSEs. They buy mortgage loans from the loan originator and pool the loans into an MBS and sell the security to investors. This process can be referred to as private-label securitization, like the securitization done by GSEs but with non-conforming loans. The private label security does not carry an official or unofficial government guarantee such as the securities issued by the GSEs or Ginnie Mae. The buyer of the MBS will therefore be subject to the underlying credit risk, since the investor carries the risk of the underlying borrowers defaulting. However, this risk can be mitigated if the MBS provider or investor buys insurance against it.

Prior to the financial crisis private label securities grew steadily in terms of volume (Goodman, 2015). The case was similar for securitization in all other asset classes that did not have government guarantees. The private label securitization exploded leading up to the financial crisis, and the volume was wiped out when the financial crisis hit, and has yet to recover to previous levels. The reason the private label market on mortgage backed securities took such a large hit after the crisis, was because it was discovered that the securities were heavily flawed. The flaws constituted mostly by failures in the cash flow waterfall and in the collateralized property. The investors received a product that had much less protection for the highest seniority classes than what was promised and indicated by the credit ratings. The loan originators had slacked in the loan underwriting and in the due diligence, as they could pass all risk on to investment banks that passed it on to investors. Hence, there were no (short term) consequences of granting loans to people with zero chance of repaying them, which caused an explosion of sub-prime loans with horrible credit scores. The loans were packaged into MBSs, which were packaged into CDOs, which again were packaged into CDO^2 (see Appendix C for an overview). The structures were extremely complex, which prevented that investors could figure out the underlying risks, and instead had to rely on ratings that were heavily flawed.

Today, the problems are, to a large extent, fixed. But the securitization done by private label entities has not recovered. As discussed in Section 3.2.8, PLS solutions cannot be traded at the same platforms as the agency solutions from the GSEs. The result is that the PLS securities are more expensive to trade.

3.2.9 INVESTORS IN THE US MORTGAGE MARKET

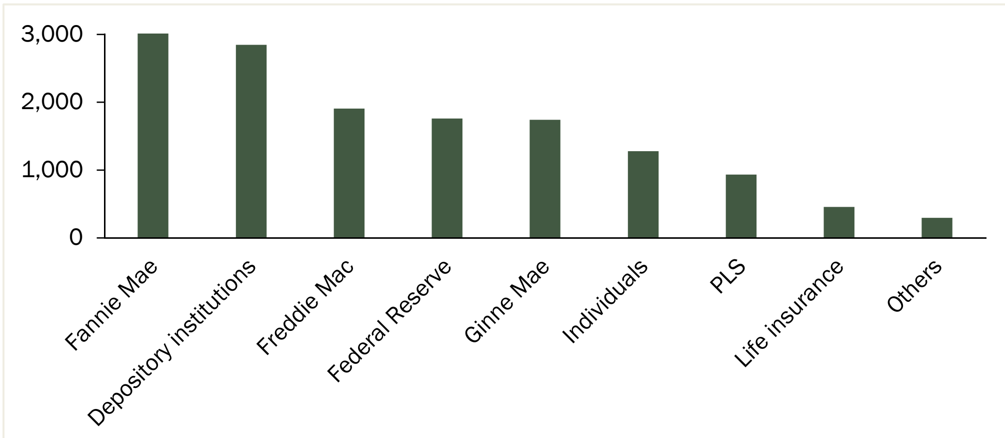
In both the Danish and the American market, liquidity is essential for the functionality of the systems. Without liquidity, the financial intermediaries cannot fund mortgage loans and would eventually have to buy and keep the mortgages themselves. In the American system, the liquidity is provided by the investors who buy and hold the mortgages through an MBS.

The MBS, guaranteed by either Fannie Mae, Freddie Mac, or Ginnie Mae, can be purchased by investors via the To-Be-Announced Market (TBA) (Fabozzi F. J., 2016) (Vickery & Wright, 2013). The market functions as a forward market where the terms, on which trades are founded, are known at the time when the trade is agreed to, but the loans have yet not been issued. The only information that is settled between the GSE and the investor when the loan is issued are a few basic criteria, such as who the guarantor is, coupon rate, maturity, price, par amount, and settlement date. Whenever the settlement date occurs the trade is settled if the terms, which were outlined, have been fulfilled. The method of trading guaranteed MBSs boost the liquidity of the market, because the GSE (seller) can be sure to receive a certain price and therefore is subject to reduced uncertainty. The GSE will attempt to just barely meet the requirements, since the investor would else receive more than paid for. The effect of having a market for GSE MBSs that are sold in advance, with only a standard set of information to be determined at the trade date, is that the borrowers can lock in interest rates before they enter their loan contract. First later, when the settlement date of the MBS arrives, the underlying mortgages are identified. Because of this setup, the guarantees are very crucial for the functioning of the market, because investors would otherwise not be willing to invest in a security with an unknown underlying loan pool (Wachter & Smith, 2011, p. 10).

Whenever the MBS is not guaranteed, it can be sold through the specified-pool market. The difference between the specified-pool market and the TBA is that the details of the trade are known at the trade date. This market is generally less liquid than the TBA market, why it is more difficult to sell MBSs. The ability to sell through the TBA market is only allowed for the GSEs because they practically operate as a government entity, and are exempt from following the same laws as other financial institutions (Wachter & Smith, 2011, p. 291). The GSE's MBSs therefore have better conditions to trade in a liquid market, whereas the non-agency backed securities suffers from lower liquidity.

The total market size of US mortgage debt, was approximately USD 14.2 trillion in third quarter of 2016. The owners of the outstanding mortgage debt are shown in Figure 3.5.

Figure 3.5 – Owners of Outstanding Mortgage Debt (USD billions)



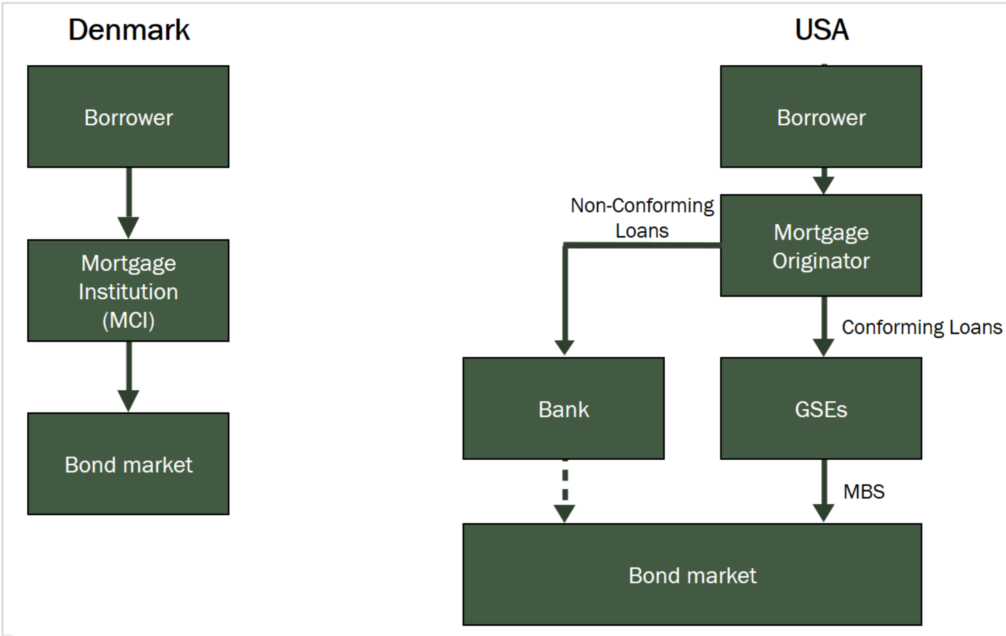
Source: Authors creation based on FED (Q3 2016)

The government and GSEs are the largest holders of mortgage debt, and depository institutions holds the second largest fraction. Depository institution are defined as commercial banks, mutual funds, foreign investors, Real Estate Investment Trusts (REITs), pension funds, and similar types of firms. As also seen by the figure, the Federal Reserve (FED) holds a large fraction of mortgage debt. After the financial crisis, FED started buying mortgage debt through MBSs issued by GSEs, to make quantitative easing (QE). This was part of the effort to support the two institutions with their recovery and to support the whole housing market (Fabozzi F. J., 2016, p. 133). The FED only made QE by purchasing MBSs with fixed-rate mortgages as underlying mortgages, which in turn seems to have increased the borrowers demand for FRM loans to an even larger extent than what is already the case in the United States (Krainer, 2010).

The result today is that the FED holds a rather large share of the outstanding mortgage debt, and the share has been steadily increasing since the financial crisis. The wanted effect from buying mortgages through MBSs is to increase the market liquidity and functioning, and maintain an effective secondary market, where the interest rates are reduced to the benefit of the American homeowners.

3.3 OVERVIEW OF THE AMERICAN SYSTEM

Figure 3.6 – Overview of Danish and American mortgage systems



Note: The dashed line represents the bank's choice to either keep the loans on the balance sheet, or issuing an MBS

Source: Authors creation

3.4 OVERALL COMPARISON

Overall the two systems are similar in a few ways, and different in many other ways. The main and most important similarities between the systems are that both gives the borrowers access to mortgage loans with many different characteristics, such as fixed interest rates, flexible rates, different durations, and the ability to prepay the obligation before maturity without suffering large costs. The result is that both the Danish and American systems provide the borrower with access to long-term flexible and relatively cheap housing financing. The credit risk is limited for the investors, as the Danish system has a maximum LTV ratio of 80%, whilst the American system is based on different guarantees.

American homeowners are less indebted, mortgage payments take up less of their disposable income, and they have a higher equity stake in their home. The preferred mortgage loan is the 30-year FRM, which is different from Denmark, where ARMs have a much larger market share. The U.S. ARM loans have diminished over time, as the FRM loan became increasingly popular. A possible reason to the differences in loan type preferences could arise from the consequences of the financial crisis, where the ARM loans received negative public attention (Moench, Vickery, & Aragon, 2010, p. 5). The FED interfered after the crisis, to insure a well-functioning mortgage market. The main method was to buy MBSs in the secondary market, and FED only choose to buy MBSs that consisted of FRM loans. The mortgage rates offered to homeowners in the primary market are closely linked to the market yields on the MBSs used to finance the loans, which is the same in Denmark. The choice by FED has increased the liquidity of these securities, with more competitive pricing as a result, and lower rates for homeowners. The attractiveness of ARM in terms of lower initial interest rate, could have decreased, simply because the secondary market for ARM MBSs is not as well functioning as the market for FRM MBSs.

The American market follows a four-fold structure, whereas the Danish is three-fold, with the borrower, a mortgage intermediary, and the capital market, included in both systems. The way the three parts are connected differs, and the origination and securitization is separated into two institutions in the United States. In Denmark, the mortgage institutions both originate and securitize mortgages. The risk that is transferred through the securitization process, also differs. In Denmark, the credit risk is kept at the mortgage bank, while interest rate and prepayment risk is transferred to the investor depending on the mortgage type. In the United States, the credit risk is not kept at the originator. In case of agency mortgages the credit risk remains at the GSE, whereas non-agency mortgages transfer the credit risk to the final investor. The credit risk transfer can induce moral hazard from the originators that will not suffer the consequences of granting loans to borrowers with bad credit assessments, as was seen during the financial crisis. The Danish system mitigates this issue by keeping the credit risk at the originator. Interest rate risk is transferred to the investor in both systems, with the result that the interest rates for borrowers will follow the market yields. But even though both markets are interlinked between borrowers and investors, the interest rate on each mortgage contract is negotiable in United States, where the balance-principle in Denmark make the borrower a price-taker that take the interest rate for given at the time of mortgage origination, set by the investors, which creates a purer link between borrowers and investors in Denmark.

As consequence of the reduced connection between borrower and investor, the American market can provide more flexible loans. The individual borrowers are offered different interest rates in closer connection to the borrower's individual risk, and can influence the rate by increasing the down payment. The drawback of the flexibility is that the borrower cannot redeem the loan by buying the loans back at market value. Hence, the debt cannot be actively managed to the same extent, since interest rate changes does not affect the outstanding mortgage debt as the prepayment only happens at par value. Only declines in interest rates can be used to manage mortgage payments by refinancing the old mortgage with a new, to obtain a lower interest rate. The option to redeem at market value provides the Danish mortgage owners with a cushion against falling house prices in case of increasing interest rates.

The covered bonds are much more standardized in Denmark, than the MBSs in the United States. In Denmark, the mortgage banks issue covered bonds simultaneously with the origination of the mortgage loans, in the United States MBSs are issued after the mortgages are originated. Both markets use the property as collateral, and have the right to seize the property in case the borrower defaults, protecting them from losses. The securitization model in the United States is constructed with a fundamental information asymmetry between the originators and the final mortgage investor, because the originator has full information about the borrower and only scarce information is transferred to the investor. The Danish covered bonds are claims on the originator backed by a pool of mortgages, hence, only the mortgage bank's credit quality and the entire economy is relevant to the investor. In the American system, they transfer the pool of mortgages to a different legal entity, and as a result, the non-conforming loans are providing less incentive to underwrite mortgages with good credit quality. The conforming loans, guaranteed by GSEs, does not suffer from the same lack of incentive. With covered bonds the originator has strong incentive to both underwrite and maintain the mortgages (Campbell, 2012). To overcome some of the issues with the MBS structure, the public GSEs offers stability and security to both the borrowers and investors. Table 3.3 summarizes:

Table 3.3 - Overall comparison of the Danish and American mortgage markets

Characteristic	US Market	Danish Market
Funding of mortgages	Through MBSs	Through Covered Bonds
Number of entities in the structure	4	3
Bearer of the borrower credit risk	GSE or investor (not originator)	Mortgage Bank (Originator)
Bearer of interest rate and prepayment risk	Investor	Investor
Government guarantee	Special loans (Implicit for GSEs)	No
Preferred mortgage type	FRM	ARM
LTV restriction	90% - 100%	80%
Link between borrower and investor	Close link	Match-funding (perfect link)

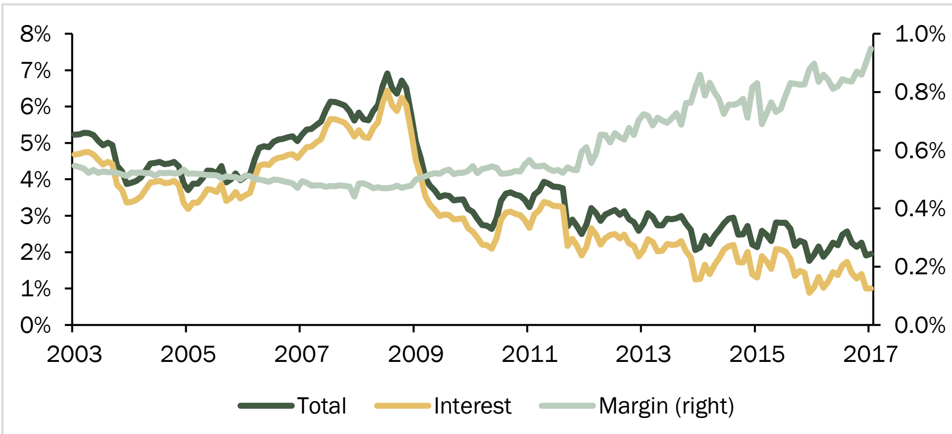
3.5 OUTLINE

In Denmark, the mortgage system is strictly regulated and does not provide financing for LTV ratios above 80%. The borrowers thus need to finance the last part of the property value by making a larger down payment or by taking a bank loan with second lien status, to finance the amount between 80%-95% LTV, and where the last 5% must be financed by a down payment. The market for second lien loans in Denmark are mostly dominated by bank loans, with security in the property. Since the bank loans is used to finance the riskiest part of the property value, with CLTV of up to 95% and only has second lien status, the risk of the lender is higher, and hence, the associated interest rate is higher than for first lien mortgage. In United States the whole system is built upon a securitization model that links the borrowers with investors who want exposure to the mortgage market. The American securitization model can potentially be used on the Danish market that is dominated by bank loans. The idea and related issues along with practical implementation, will therefore be discussed further.

4 IMPROVEMENT OF THE DANISH MORTGAGE MARKET

The Danish real estate financing up until an LTV of 80% is shown to be best in class in terms of delivering security to investors and low interest rates to borrowers. The interest rates are only marginally higher than the Danish “risk-free” rate, underpinning the security provided in the system. The only obvious improvement of the first 80% LTV is within the margins that have gradually increased over time as seen in Figure 4.1.

Figure 4.1 - Average interest rate and margin on mortgage loans



Source: The Danish National Bank

The increase in margins can partly be explained by the low interest rate environment forcing commercial banks (the owners of the mortgage banks) to generate profits other places than on interest rate spreads, but can also be explained by the increasing regulatory capital requirements on the mortgage banks. Due to the match-funding principle and equity protection, some of the requirements may be excessive in the Danish Mortgage market, and more lenient requirements could be implemented in the Danish system without suffering a loss in stability or security. This would benefit borrowers, having to pay lower margins, without hurting investors that would get the same interest rate payments. However, these improvements would only be marginal, and despite of the minor flaws, it is arguably the best mortgage financing in the world seen from both the borrower and investor perspective. Hence, the more interesting aspect to investigate is the financing from 80-100% LTV.

Due to the recent requirement by the regulatory authorities, homebuyers are now required to invest at least 5% equity when buying a residential property. Therefore, a 100% (or more) financing is no longer a possibility in the Danish market. Therefore, the focus is narrowed to the mezzanine loan, financing 15% of the property value between 80-95% LTV.

4.1 MEZZANINE MORTGAGE FINANCING IN DENMARK

Traditionally, a very common way to finance the part not covered by the mortgage (the "mezzanine"), has been through a seller note. With a seller note, the buyer borrows the mezzanine from the seller, and in turn provides a note, which is a second mortgage on the house, subordinated to the mortgage from the mortgage bank. The interest rate, maturity and terms are negotiated between the buyer and seller, often with the help from lawyers and other advisors. As the seller would often like the cash in hand, the seller would sell the note to a bank, financial institution, or an institution specialized in investing in seller notes. The trade would normally happen at a discount, say 98 for each 100 of debt principal on the note (Robinhus, 2017).

The seller note is rarely used anymore, and instead a more common way of financing the mezzanine is through the bank, bypassing the seller that would often sell the note to a bank or similar institution anyway. Hence, the seller notes are mostly used by buyers who are not able to get a loan from their bank. In the current interest rate environment, the mezzanine loans from a bank have interest rates in the range of 5-10% depending on the creditworthiness of the borrower (MyBanker, 2017), although rates outside this range will occur, it is rare among the general homeowners in Denmark.

The banks granting the mezzanine loans do, to some extent, benefit from the diversification of their loan portfolio, but the benefits are assumedly not shared with the borrowers. Therefore, a potential for improvement could lie in combining the match-funding and direct link from investor to borrower that characterizes the mortgage banks, with the securitization and tranching that is used in the American system. Pooling many borrowers together minimizes the investor's risk through diversification, and creating a direct link between borrowers and investors ensure the lowest interest rates for borrowers. Creating such a product would also provide investors with an opportunity to get exposure towards the Danish real estate, but in a product with a higher yield than the covered bonds issued by mortgage banks due to the higher risk associated with mezzanine financing. In addition, tranching the product, a method known from MBSs in the American market (Brennan, Hein, & Poon, 2009), could optimize the ratings on the product. This would minimize the return required from investors who base their required returns on ratings, which would benefit the borrowers. Such a product could potentially complete the Danish mortgage market, making it even cheaper and more efficient for the entire financing package – the missing piece.

4.2 THE MISSING PIECE - JIGSAW

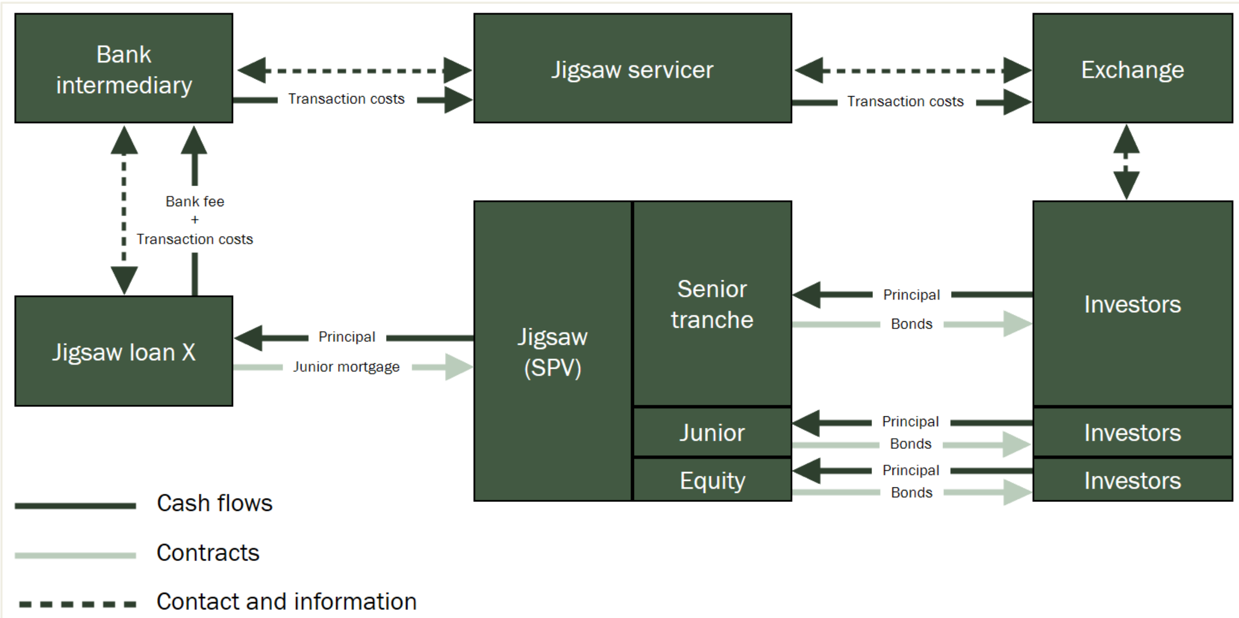
To investigate the feasibility of this mezzanine financing product, a few parameters need to be estimated for the mezzanine loans granted to borrowers: The PD, the loss given default (LGD), and the correlation between defaults (ρ). Once this is done and the design and terms of the product is determined, the product can be modelled and evaluated in terms of its competitiveness with the current offerings.

4.2.1 DESIGN AND TERMS OF THE PRODUCT

The Jigsaw will be created in a special purpose vehicle (SPV), which means that it will be remote from any other legal entity and there will be no other liabilities or activities in the SPV other than that of granting the mezzanine loans and funding them. This is done to ensure that investors are not exposed to any other credit risk than the one originating from the borrowers. The SPV will grant "Jigsaw loans" to homebuyers, gather the loans in a large pool, a "Jigsaw", and fund these by issuing bonds that have rights to the cash flows from the Jigsaw loans and have a subordinated claim in the underlying real estate. A prerequisite for getting a Jigsaw loan is that the first 80% of the property is financed through a mortgage bank, which ensures that the borrowers are creditworthy and that is why the collateral claims in the Jigsaw is subordinated to this senior mortgage. The SPV is allocated with a servicer (sometimes referred to as a trustee) who acts on behalf of the investors, collecting periodic payments with the purpose of distributing it to the investors. The servicer will also manage arrears and handle the foreclosure procedure to recover principal from defaulted borrowers. The servicer receives margins for delivering this service, paid for by the borrowers, as is the case in the existing Danish mortgage market.

Each Jigsaw will be split in three tranches: Jigsaw Senior, Jigsaw Junior, and Jigsaw Equity. A bond corresponding to each tranche will be issued, so that each investor will be able to choose whether to invest in the Senior, Junior, or the Equity tranche, and these bonds will trade on an exchange. This allows the borrower to prepay the loan at all times by buying back the bonds funding the loan at market value on the exchange. When the borrowers pay in interest and repayments on their loans, the investors in Jigsaw Senior will be paid first. Once all investors in Jigsaw Senior have been paid in full, Jigsaw Junior will start getting interest and repayments. When Jigsaw Junior investors have been paid in full, Jigsaw Equity will get the remaining interest and repayments. In case of borrowers defaulting, the investors in the Jigsaw Equity will take the entire loss, and Jigsaw Junior will only be impacted if the entire equity tranche is wiped out. Similarly, the Jigsaw Senior will only incur losses once the entire junior tranche is wiped out. This is similar to the waterfall cash flow structure in the American MBSs. The fraction of the Jigsaw funded in each tranche will be estimated in Section 6, and will be determined with the goal of minimizing the interest rate required by investors to benefit borrowers. Borrowers will not get three separate loans even though the Jigsaw is split into three tranches, but will get a single loan which the SPV Structures in the Jigsaw's three tranches as described. Hence, only investors will be exposed to the more complex aspects of the product. Figure 4.2 illustrates an overview of the process when issuing a Jigsaw loan, showing the information flow and cash flows.

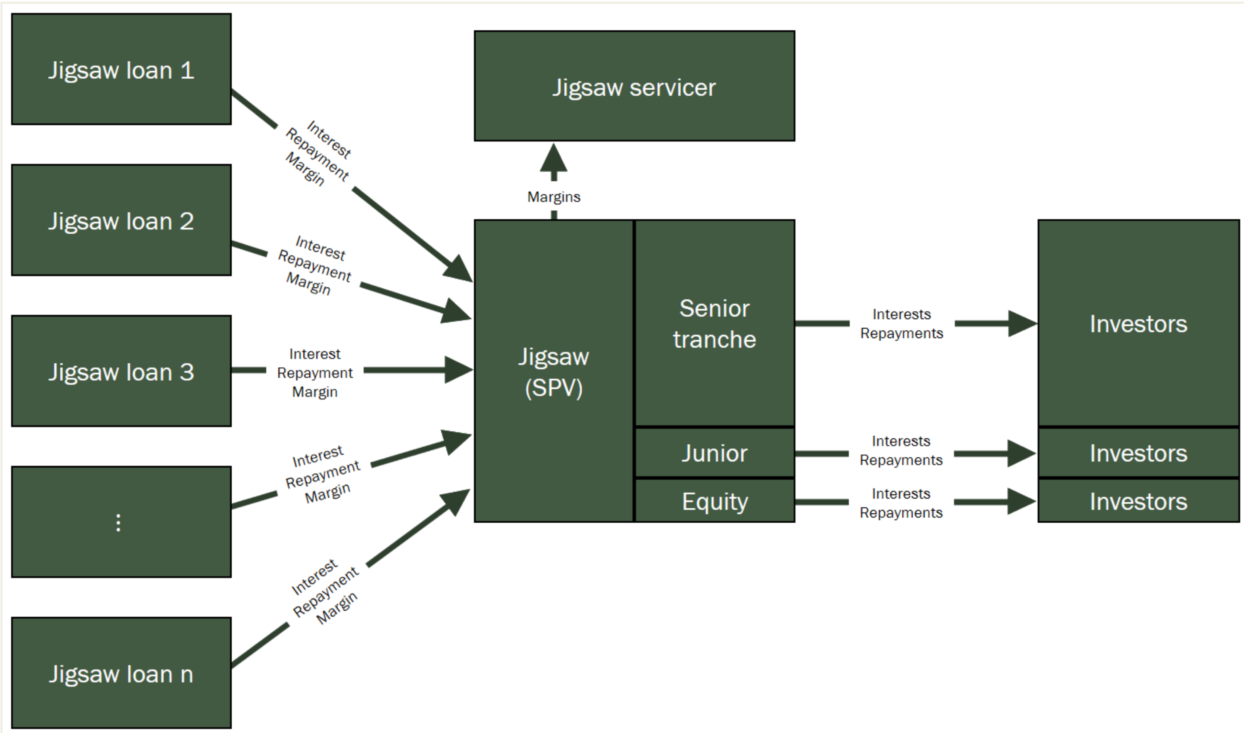
Figure 4.2 - Process of issuing a Jigsaw loan



Source: Authors creation

A bank, usually the same bank that facilitates the senior mortgage of 80% LTV, will establish contact to the Jigsaw servicer when a borrower wants the Jigsaw loan. In cooperation with the bank, the Jigsaw servicer will check that the borrower is eligible for the Jigsaw loan, and then sell bonds in the three tranches to investors in according ratios through the exchange. The bank will charge a fee for facilitating the contact and the exchange will charge transaction costs. In practice, the cash flows from investors to the borrower will also pass through the exchange and the bank intermediary, but these are formality clearings and not relevant for the understanding of the structure. Figure 4.3 shows an overview of the ongoing cash flows after the Jigsaw loan has been issued.

Figure 4.3 - Ongoing cash flows in the Jigsaw



Source: Authors creation

The Jigsaw loans will pay a weighted average interest rate of the one charged by the three tranches. The Jigsaw servicer will distribute the interests and repayments accordingly to the different tranches in order of seniority. The margins paid by the borrowers will be retained by the servicer to cover administration costs. Different from the mortgage banks, the SPV will not cover any loss on loans, as losses will be passed directly through to the investors. Hence, no additional margin will be charged with borrowers to protect the SPV from losses on loans.

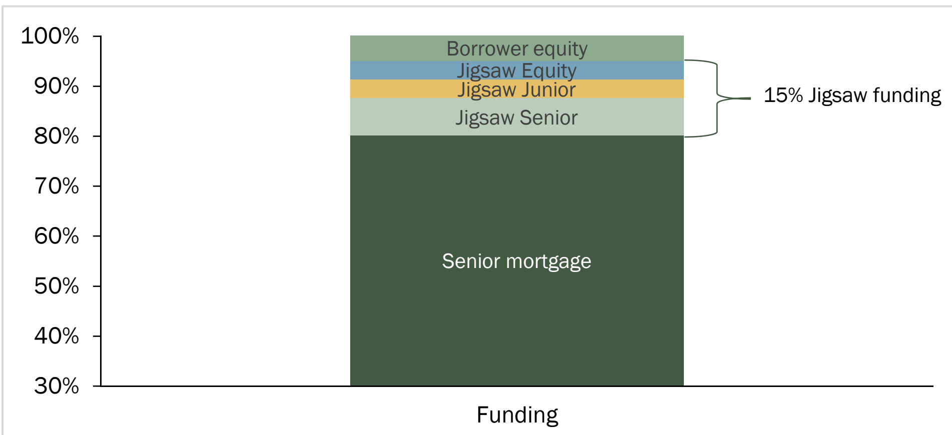
The Jigsaw loans granted by the SPV to fund the mezzanine will be covered by a mortgage on the property they are used to finance. This mortgage will be subordinated to the senior mortgage granted by the mortgage bank or commercial bank financing the first 80% of the property. This mezzanine mortgage will also serve as underlying collateral for the bonds issued to finance the Jigsaw loans.

In case of a borrower not meeting the obligations of the loan granted, the property securing the loan can be forced on sale to protect the investors from a loss. After the senior mortgage have been paid in full, the Jigsaw gets the rest of the proceeds from the sale up until the principal of the loan, and in case there is still something left the borrower gets the proceeds if there are no other claims to the property. However, the first things covered by the forced sale of the property is the costs related to the default and auction sale of the property.

The SPV will create one Jigsaw each year, in which all Jigsaw loans granted throughout the year will be gathered. The Jigsaw loans will be annuity loans with a maturity of 10 years, enabling borrowers to use the 10-year interest only period on their senior mortgage to repay the more expensive Jigsaw loan. The purpose of the SPV is to facilitate the lending and investing, and not speculate on short term funding of long term investments. Hence, the bonds issued to fund the Jigsaw loans will also be 10-year annuity bonds.

Figure 4.4 sketches the funding of the homes that an SPV would assemble in a Jigsaw. When a borrower gets the Jigsaw loan, the largest fraction will expectedly come from investors investing in the Jigsaw Senior and smaller parts will come from investors in the Jigsaw Junior and Equity, as this is what was historically seen in tranched real estate securities (Brennan, Hein, & Poon, 2009, p. 895).

Figure 4.4 - Funding distribution of the assets underlying the Jigsaw loans



Source: Authors creation

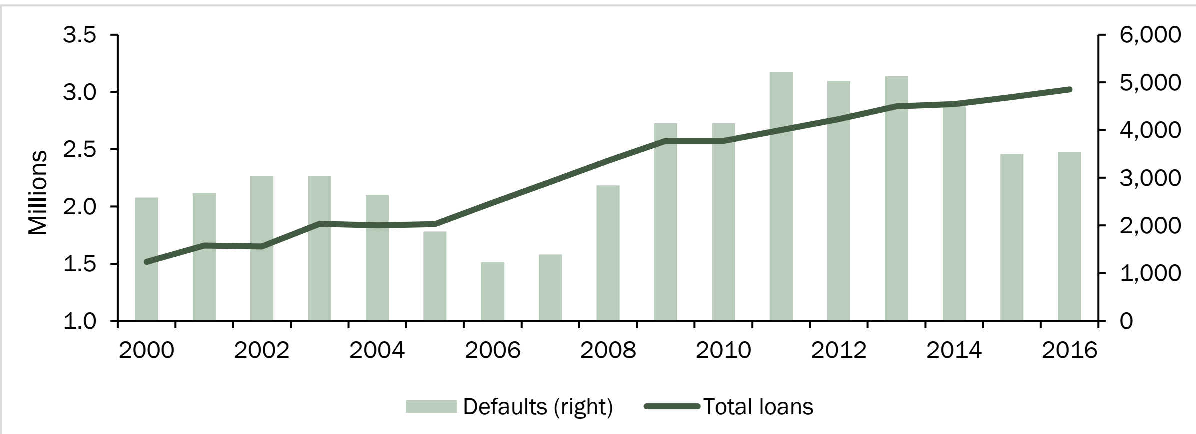
If a loan defaults, the Jigsaw will lose money if the proceeds from the sale of the house is lower than 95% of the estimated house value at the time of purchase after costs of default. However, this threshold is likely lower, as the borrower will start to repay the Jigsaw loan immediately, thus increasing their equity stake in the property.

4.2.2 THE PROBABILITY OF DEFAULT

The Association of Danish Mortgage Banks is publishing statistics on the number of properties that the mortgage banks take over each quarter, and Statistics Denmark publishes data on the total number of forced sales. Comparing these two numbers will give an idea of how often the price achieved on the auction was not satisfactory to the mortgage banks and hence, they decide to acquire the property themselves. There is also data on the number of loans in arrears, but as borrowers can still avoid default and return to timely payments even though they enter into arrears, these numbers are not as relevant when estimating the PD. In addition, the data on arrears are only based on the mortgage loans and not the bank loans that are granted on top of the mortgage, which the Jigsaw would replace. The bank loans are normally also secured by a mortgage on the property, and the defaults on these would lead to a takeover of the property, and therefore they are included in the statistics on properties overtaken and forced sales.

Using data from the four major mortgage banks, Nykredit/Totalkredit, Realkredit Danmark, Nordea Kredit, and BRFkredit, an approximation of the total outstanding amount of loans is estimated and compared to the amount of defaults each year in Figure 4.5.

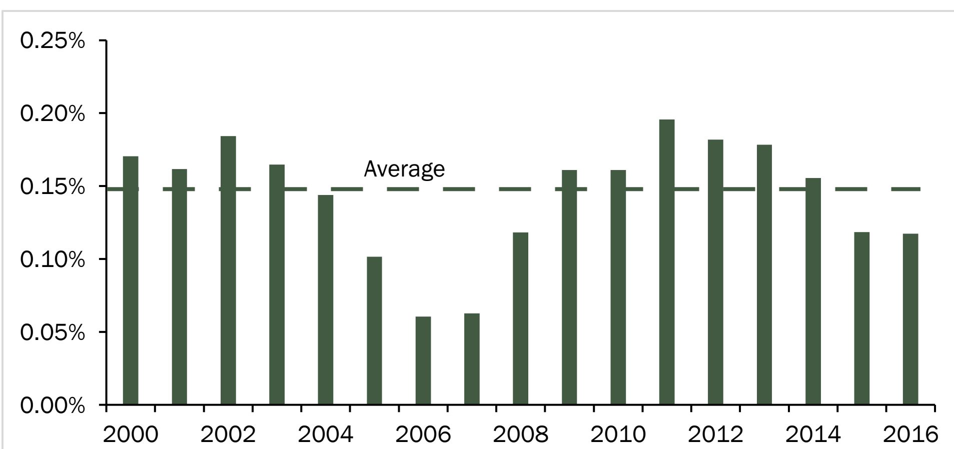
Figure 4.5 - Annual defaults and approx. total loans outstanding



Source: Statistics Denmark and annual reports from Nykredit, Totalkredit, Realkredit Danmark, Nordea Kredit, and BRFkredit.

The data in Figure 4.5 shows a strong increase in defaults after the global financial crisis and the following European credit crisis, but defaults have started to drop back to a level around the one seen before the financial crisis. The data on defaults translates into an average annual PD of 0.143% as seen in Figure 4.6.

Figure 4.6 - Annual default probabilities



Source: Statistics Denmark and annual reports from Nykredit/Totalkredit, Realkredit Danmark, Nordea Kredit, and BRFkredit.

Even though the average is skewed upwards by the financial crisis, the booming years of 2006 and 07 are pulling in the opposite direction and the data therefore includes a whole business cycle. It is preferable for the model estimation to have a conservative estimate of PD, to increase the reliability of the model results. Therefore, a higher than average estimate of 0.16% is used, which will account for more of the tail risk in case of an economic crisis. In addition, most of the past 16 years have a fraction of default that lie within this estimate, increasing the certainty of the true PD not exceeding 0.16%.

To get the PD on the Jigsaw loans, the annual PD is used in a binomial model. The model calculates the probability of the loan not defaulting in any of the 10 years till maturity, conditional of the annual PD. Since the result is the probability of the bond ***not*** defaulting, the PD is simply:

$$Prob(Default) = 1 - Prob(non\_Default) \quad (4.1)$$

Using Equation (4.1) gives the following PD of a Danish homeowner:

$$Prob(non\_Default) = Binom(0,10,0.16\%) = 98.41\% \rightarrow Prob(D) = 1 - 98.41\% = \mathbf{1.59}\%$$

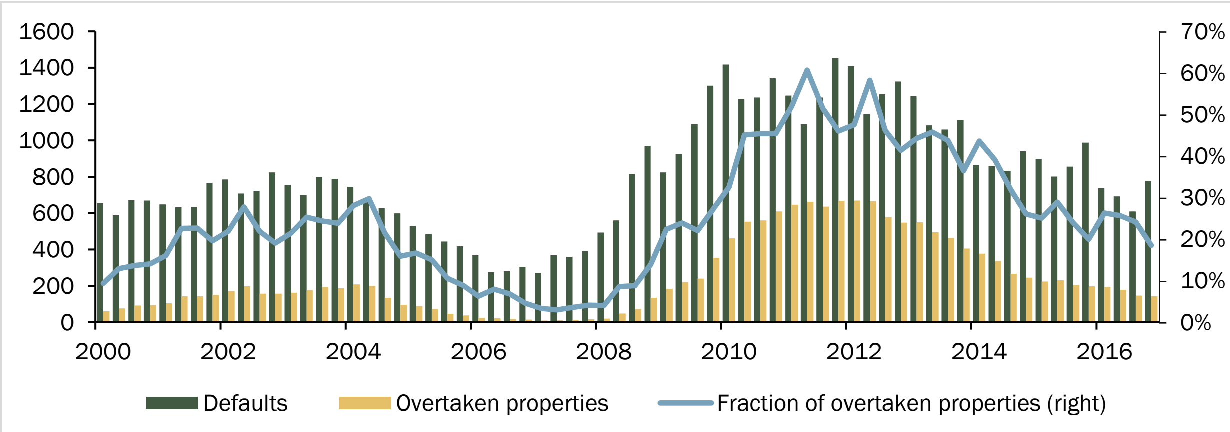
This PD is likely also upwards biased, as the number of forced sales also include properties financed by smaller mortgage banks, whose lending is not included in the analysis of total loans outstanding. However, these properties are a small fraction of the total defaults and hence, the bias is minor, but increases the conservativeness of the estimation. In addition, borrowers defaulting on the senior mortgage after fully repaying the mezzanine financing or borrowers who did not have any mezzanine financing to begin with are also included in the calculation of PD. However, these defaults will not impact the Jigsaw, and therefore the PD should in practice be slightly lower for the Jigsaw than for the mortgage banks, which adds to the conservativeness of the estimate.

The PD is an aggregate number, and the individual default probabilities of the loans, which are dependent on the borrowers economic and social conditions, are not modelled. The mortgage banks assess the individual's creditworthiness and PD when determining if the borrower is eligible for a mortgage loan. A prerequisite for getting a Jigsaw loan is to have a loan from a mortgage bank. Hence, the Jigsaw relies on the mortgage banks to withhold their current standards of credit risk assessment that will keep the default probabilities seen rather constant in the future.

4.2.3 LOSS GIVEN DEFAULT

Given the nature of the mezzanine financing, the Jigsaw will be the first to take a loss after the borrower's equity has been wiped out, which is almost always the case when a takeover happens and a sale is enforced (Haldrup, Staunstrup, Hansen, & Jørgensen, 2015). To protect their investment, the mortgage banks will often bid on the properties on auction and end up taking over the properties if no one is willing to buy it at a price satisfactory to the mortgage bank. In this case, the mezzanine financing will be completely wiped out as the sales price is not even enough to cover the senior mortgage on the property. In Figure 4.7 the amount of properties overtaken by mortgage banks is compared to the total number of forced sales.

Figure 4.7 - Overtaken properties in relation to total defaults (# per quarter)



Source: Statistics Denmark and the Association of Danish Mortgage Banks

On average, 25% of defaults ended up being overtaken by the mortgage banks since 2000. Hence, 25% of the time the LGD is 100% for the mezzanine financing. In the rest of the cases, some recovery is expected but losses tend to be severe. To estimate the total LGD, a study from The Knowledge Centre for Housing Economics, that investigated forced sales in Denmark, is used to help identify the loss incurred (Haldrup, Staunstrup, Hansen, & Jørgensen, 2015). The Report is using some rough estimations, for instance on the market value of the properties, and hence, the estimate is characterised by a level of uncertainty.

The study by Haldrup et al. finds, that in cases of default, the average LTV for the senior mortgage granted by mortgage banks is approx. 90%. The study also finds that in 27% of the cases, the LTV of the senior mortgage was above 100%, which is also in line with the previous estimate shown in Figure 4.7, which estimated that 25% of forced sales were acquired by the mortgage banks as the auction could not bring in the full amount mortgaged on the property. However, the value of the properties used to estimate LTV in the study by Haldrup et al. is approximated by the public valuation of the properties, which is a very rough estimate. In general, the public valuation is negatively biased on expensive properties and positively biased on cheap properties in poor condition. Cheap properties in poor condition is overrepresented in cases of default and hence, the LTV on the senior mortgage might be higher than estimated in the study, which the study also notes (Haldrup, Staunstrup, Hansen, & Jørgensen, 2015, p. 50).

A study by Buhr shows that the average discount on forced sale auctions is 37% compared to the market value of the property (Buhr, 2013). However, this study was carried out during the European credit crises around 2011 where defaults increased sharply following the subprime crises in the US. Despite this large discount, “only” around 50% of the defaults were acquired by the mortgage banks, and some mezzanine recovery should still be present in the other 50% of defaults.

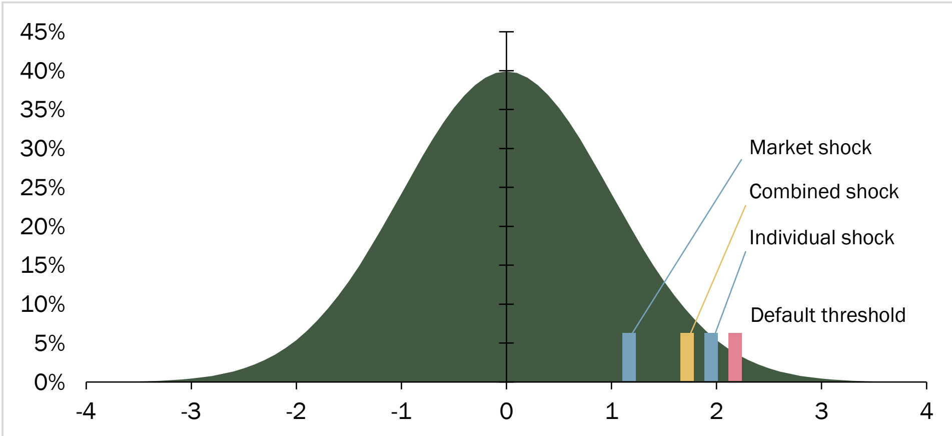
Leaning on the findings from the study of Haldrup et al., a long-term average level of 90% LTV on the senior mortgage is used as base for the estimation. A 90% LTV on the senior mortgage means that a property worth 100 with 80% mortgage bank financing and 15% Jigsaw financing would have dropped to 89 in value, leaving the Jigsaw with 9 to cover the 15 in issued debt, assuming no repayment of the senior mortgage or the Jigsaw loan. This corresponds to a recovery of 59% or a LGD of 41%. However, the LTV is likely downwards biased, as both the realised sales price on auctions and the market value of distressed properties is likely to be lower than the public valuation of the property. Hence, the base LGD will be adjusted to account for these biases, and a base LGD of 75% will be used, based on analysis showing that 75% of the time some recovery is likely, but also the study by Haldrup et al. showing that the recovery is likely small. The uncertainty encompassing this estimate is high, and therefore estimates ranging from 100% to 50% LGD will be used to test the sensitivity of the required return from investors in relation to the LGD estimate.

4.2.4 CORRELATION BETWEEN DEFAULTS

The correlation between defaults stipulates the dependence between the defaults in a loan pool. Since it is near impossible to estimate all correlations between all loans, the loans are assumed to be homogeneous, and correlate equally with a market factor. This correlation represents the correlation between the loans but is easier to model. The market factor can be considered as a loan with the same PD as the Jigsaw loans that skews the fraction of defaults in the loan pool in either direction. The correlation ranges between -100% and 100%. If the correlation is 100% all loans will default if the market loan defaults, if the correlation is -100%, no loans default if the market loan defaults, and if the correlation is 0% there is zero connection between the defaults in the loan pool and the market loan, and the defaults in the loan pool are entirely independent.

The market factor has the same distribution as the individual Jigsaw loans. Taking a correlation of 30% as an example, a random market shock is drawn, e.g. from the normal distribution, and weighted with 30% and combined with a 70% weighting of an individual random shock for each loan, also drawn from the normal distribution, shown in 0Figure 4.8. If the combined shock is less likely to occur than the PD of the Jigsaw loan, the Jigsaw will default in the model. Hence, it is not only important if the market shock is outside the default threshold, but more so how far outside of the threshold the shock is, as this affects the PD of the individual loan. The implementation and modelling of this is described in detail in Section 5.2.4.

0Figure 4.8 – Illustration of the combined shock in relation to a default threshold



Source: Authors creation

In Basel II, the Basel committee states that the correlation that should be used when modelling residential mortgages ought to be 15% (Basel Committee on Banking Supervision, 2005). Many studies have tried to estimate the correlation between defaults in residential mortgages, and the findings vary from 5% to 30% (Geidosch, 2013). High asset correlation has been claimed to be the factor that caused the global financial crisis, but in a study, Geidosch shows that asset correlation in residential mortgage portfolios pooled in RMBSs have an average correlation of only 5.7%, using five different estimation methods. The study focuses mainly on subprime RMBSs leading up to and during the financial crisis. Hence, Geidosch implies that correlation was not the course of the severe losses incurred for investors in RMBSs during the global financial crisis.

Instead, Geidosch argues that the losses were due to a consistent high PD and re-tranching of the RMBSs. On average, 1000 loans were placed in the pools used for an RMBS, and these were divided into a senior tranche, a mezzanine tranche, and an equity tranche (the riskiest tranche). The mezzanine tranche and equity tranche was then re-pooled into new RMBSs ("squared RMBSs" or "CDOs") with new senior, mezzanine, and equity tranches. The consistent high PD over the maturity of the loans caused the entire equity and most of the mezzanine tranches to be wiped out in the original RMBSs, thus resulting in the entire squared RMBSs to be wiped out, since they were made up of purely mezzanine and equity tranches from the RMBS. Thus, the problem in the subprime crisis was not correlation, but consistent high default probabilities due to aggressive lending to borrowers with terrible credit scores (Geidosch, 2013).

Using the data on historical default fraction from Figure 4.6, Lando provides a method that can estimate the correlation in defaults in a rough manner (Lando, 2004, pp. 225-226). The estimator is as follows.:

$$\hat{\rho} = \frac{\widehat{Var(p)}}{\hat{\bar{p}} * (1 - \hat{\bar{p}})} \tag{4.2}$$

Where $\hat{\bar{p}}$ is the average default fraction in the dataset, and $\widehat{Var(p)}$ is the variance estimate of the annual default fraction. Using Equation (4.2) and the data gathered on defaults, the correlation is estimated to be.:

$$\hat{\rho} = \frac{\widehat{Var(p)}}{\hat{\bar{p}} * (1 - \hat{\bar{p}})} = \frac{0.000017\%}{0.143\% * (1 - 0.143\%)} = 0.012\%$$

The achieved estimate of 0.012% supports the findings of Geidosch, and point in the direction of a low correlation in defaults. However, a relative small data sample, and model risk should be considered, and hence, the extremely low correlation estimate will not be used in the modelling. Instead, the findings of Geidosch will be the base of the modelling, and a correlation of 5.7% will be used as the base value. The sensitivity of the parameter will be tested, due to several findings showing that the true correlation could be significantly higher.

4.3 TARGET TRANCHE RATINGS AND CORRESPONDING INTEREST RATES

The ratings that the Jigsaw is modelled to fit, is set to target three different investor groups. The Jigsaw Senior will be estimated to meet the requirements for an Aaa rating from Moody’s. This will enable investors, only able to invest in the most secure debt products, to invest in the Jigsaw, such as Danish pension funds and insurance companies. The Jigsaw Junior will be sized to fit a Baa3 rating. The Baa3 rating is the lowest rating given that still classifies as investment grade (Moody's, 2016). Hence, the many investors limited to only investing in investment grade bonds but seeking a higher yield, will be attracted to the Jigsaw Junior. The Jigsaw Equity will serve as a residual tranche, covering the remaining, riskiest, part of the Jigsaw. The Equity tranche will therefore not be modelled towards a target rating, but receive the rating corresponding the (EL) in the tranche. Due to the riskiness of the tranche, the tranche will likely have a junk rating like the Caa3, which is the worst rating given by Moody’s to bonds that are not in default or on the verge of defaulting. If Jigsaw Equity cannot meet the requirements for a Caa3 rating, the tranche will be unrated. Table 4.1 summarizes the target ratings.

Table 4.1 – Target ratings and investors

Tranche	Target rating	Target investor
Senior	Aaa	Pension funds and insurance companies, risk averse
Junior	Baa3	Investors limited to investment grade bonds, seeking higher yields at medium risk
Equity	Caa3/unrated	Opportunistic investors, risk hungry, seeking high yields and accepting high risk

5 MODEL ESTIMATION

In Section 4, the fundamentals of the product design and functionality has been described, and the required inputs in the modelling and setup of the mezzanine financing have been estimated. This section will deal with the process of estimation and theoretical setup, in which the Jigsaw's characteristics will be tested, to assess how the product would work in practice and whether it can compete with the current bank offerings.

5.1 MODELLING OF THE JIGSAW

The characteristics of the Jigsaw is very similar to the MBS structure seen in the American mortgage market, but is, on all levels, a more transparent product. The Jigsaw is a structured product based on the mechanics behind an MBS, but which adapts the fundamental setup in Danish mortgage bonds to incorporate some of the best features from both markets. It is basically an ABS, where the underlying securities are mortgage loans. Hence, the valuation and pricing of the product is closely related to the methods used to identify the risk in loan pools. As was presented in Section 4.3, the Jigsaw is tranched so that is appeals to different investors. This creates a differentiated product, even though the underlying pool contains relative undifferentiated loans.

The transparency of the Jigsaw is mainly preserved by modelling the cash flow from borrowers to investors as simple as possible, while establishing maximum credit enhancement for highest rated tranches. The actual process of estimating the tranche sizes has been a hot topic since the financial crisis, and Basel legislation has proposed an approach on how to deal with loan portfolios. The starting point will therefore be to use the methods presented by the Basel Committee.

5.1.1 CASH FLOW MODELLING

In the Section 4.2.1, the SPV setup was sketched and the basic characteristics were explained so that the cash flows can now be modelled. The cash flows generated by the pool of loans will be distributed according to the standard waterfall model, as shown in Figure 3.4, where the most senior tranches will receive cash allocation first, and only when the most senior tranche has received full payment, will the next tranche will receive payment. This makes the senior tranches the safest, whereas the subordinated tranches will gradually become riskier in order of seniority.

The cash flows from the pool of loans consist of the following parts:

1. **Principal**
   The principal payments are the cash flows that reduce the outstanding debt balance. Since the loans are annuities with equal periodic payments, the principal, as a fraction of the quarterly payment, will increase over time. Principal payments can be paid to the tranches on either sequential or pro rata

basis. In the Jigsaw, the principal will be paid on sequential basis to secure the senior tranche first. The senior tranche must be fully redeemed before the junior tranche will receive principal payments. This approach is like the plain vanilla version of an MBS.

2. **Interest**

   The interest payments are payments the lender receives for the service of providing the loan.

   In the beginning of the loan, interest makes up the largest fraction of the periodic payments. Over time the interest payments decrease because the debt balance decreases due to principal payments. The tranches receive interest together with their principal, and the interest will therefore also be distributed sequential.

3. **Prepayments**

   The two payments, principal and interest, are the expected payments from the underlying loans and are known at issuance. The payments will continue while the loans are active. The underlying loans can terminate before the actual maturity for two reasons; Either the borrower chooses to repay the total debt balance, or the borrower defaults on his loan obligations. In case of a borrower in one of the underlying loans would need to sell the house pledged as collateral, the loan would be repaid before maturity, and the proceeds will be allocated to the tranches according to the number of bonds sold in each tranche when the loan was granted. The effect of having a security that is based on loans with a prepayment option, is that the maturity of the issued bonds is often shorter than the stated loan maturity. The probability that none of the underlying loans will be repaid during the whole lifetime, is very low.

5.2 THEORETICAL FRAMEWORK

This section will explain the theoretical frameworks that will be used in the process of examining and estimating the properties of the Jigsaw. To assess the stability and usability of the proposed security, two potential frameworks will be discussed in terms of their assumptions and usefulness in this connection. The goal is to identify the risk associated with an investment in the security, and to split up the total principal of the SPV into the proposed senior, junior and equity tranches. The tranches are specified with an attachment and detachment point in percent of the total principal of the collateralized loan portfolio (Das & Stein, 2011, p. 2). If the total value of the underlying loan portfolio is 100, then senior tranche will naturally have a detachment point at 100 where it will be the first to receive cash flows, whereas the attachment point would be somewhere between 0-100. The higher the attachment point is set, the larger the incurred losses in the asset pool must be before the given tranche experiences losses. Often each tranche will be assigned with a credit enhancement, which states the degree of losses in the pool that must occur before the given tranche will experience any loss. The higher credit enhancement and attachment point for the senior tranche, the lower the PD for this given tranche is. The attachment point for the senior tranche will be the detachment point for the junior tranche, and the attachment point for the junior tranche will be the detachment point for the equity tranche. The equity tranche will attach at 0 and hence, be the first to incur losses if defaults occur.

The method used to determine detachment and attachment points for the tranches in the SPV, will follow the methods most common in practice. The methods are closely related to rating agencies, because the first objective is to create rated tranches from the loan pool. The three major rating agencies, Standard & Poor's, Fitch, and Moody's, use different techniques to evaluate the risk in an investment, where the most common are PD and EL (Das & Stein, 2011, p. 2).

The objective is to establish the tranches with different risk, such that they obtain the target rating which appeal to different investors with different risk preferences. The first objective is to make the senior tranche Aaa-rated, and afterwards to create a Baa3 junior tranche given the attachment point of the senior tranche. The equity tranche will consist of the residual principal that is left after allocating principals to the senior and junior tranche. Two procedures could be chosen to allocate principal, which would give the same result. The first method is by defining tranche sizes before knowing the final PD or EL of the tranche, and afterwards, by imposing an iterative process of narrowing or widening the tranches, calculate the final tranche points to match the wanted PD or EL in each tranche. The second approach requires knowledge of the default fraction and the underlying probability distribution, such that it is possible to derive an equation that describes the distribution. The tranche points can then be calculated by imposing restrictions on the PD or EL, and then directly obtain the tranche sizes through solving the derived equation for the PD or EL, equivalent to the process of estimating Value-at-Risk.

Both approaches to the tranche estimation and calculation are very dependent on the assumptions made on the underlying loan pool. In reality, the specifications of each individual loan in the pool would practically be different, with different principal, interest rate, credit quality etc. To account for these details, is beyond the scope of this thesis, simply because it would be far too comprehensive to model and simulate such a detailed portfolio of loans. Instead the loan portfolio is simplified, in order to make it homogenous. This is common practice when examining large portfolios and credit risk in general (Fender & Kiff, 2004). The standardization of the loans makes it a lot easier to sample cash flows in the loan portfolio and determine the defaults in the pool, which can be referred to as the default distribution.

5.2.1 SIMPLIFICATION AND ASSUMPTIONS

The simplification of the loans in the SPV decreases the accuracy of the simulation and must be compared as a tradeoff between efficiency and accuracy. The portfolio is made from many individual loans, of which the characteristics is not known at the time of modelling. By simplifying and assuming that each loan in the portfolio is identical, the result will, without doubt, have reduced validity, but it is not unrealistic to assume that the loans will be quite similar apart from the principal amount. By having loan-level modelling, both the validity and accuracy of the default distribution is higher (Stein, Das, Ding, & Chinchalkar, 2010, p. 5). But even with the simplification that the loans are homogeneous, the results can still be useful since the concepts can be transferred to more detailed examples, and average figures become close approximations as the number of loans increase. The simplification allows the general concept and proposed product to be tested in a simple framework.

The loans are modelled such that have the same notional balance, for ease it is assumed that each loan has a principal of 1, all carry the same coupon rate, and are fully amortizing over a period of 10 years with no prepayments. Previously in this section, prepayments were briefly touched upon, and will be discussed further in Section 8. The modelling of prepayments can be complex and influences the cash flow of the bond significantly. In general, MBSs are some of the most difficult securities to model and value, simply due to their sensitivity to prepayments and interest rates (Levin & Davidson, 2005). Prepayment cannot be explained perfectly by a model, why the result poses issues with timing, frequency, and size of cash flows to investors.

Prepayments during the life of the loan carries an investment risk for the investor, because the investors receive the repayment before expected and thus, need to reinvest the capital at the current market rate, which might have changed since the bonds were bought. Without prepayments, the loan would have an expected lifetime equal to its maturity, with the consequential default risk during the whole period and a higher likelihood to default. With prepayments, the effective period where a default can occur on a loan will be reduced because the expected lifetime is reduced, which lowers the ELs over the lifetime of the pool (Stein, Das, Ding, & Chinchalkar, 2010, p. 6). The default risk and prepayment risk are therefore mutually dependent and highly important for the risks associated with the investment in the security. However, since omitting prepayments from the model is increasing the risk of the Jigsaw, the results will be more conservative, and more reluctant from undershooting the pricing that would be seen in practice.

The cash flow and prepayment modelling would clearly increase the predictability of the Jigsaw's risk and return properties, but the positive effect comes at the cost of extensive computing power, since the estimation relies on simulation. The assumption of making all loans identical, decreases the need of computing power drastically, but cash flow and prepayment modelling would still require loan level modelling. The lack of computer power necessary to compute modelling on loan level, makes it impossible to accomplish, even though it would be the most optimal procedure to assess the actual properties of the Jigsaw. The analysis will therefore ignore exact cash flow timing, which can affect the tranching, but the result is not as heavily influenced, since the general results is not far from the case of full cash flow and prepayment modelling (Mahadevan, Polanskyj, Tirupattur, Onur, & Sheets, 2006) (Das & Stein, 2011). Meanwhile, this increases both the possibility and necessity to focus on the modelling and sensitivity testing of the assumptions on the underlying homogenous pool.

In Section 4, the PD and correlation among the loans in the portfolio was investigated to get an indication of the riskiness of the loans. In the following it is assumed that the PD and correlation are constant for all loans across entity and time, such that each loan is identical and carries the same risk. When a default occurs, the Exposure-at-Default (EAD) and LGD are also assumed to be constant across time and all entities, which is not the case as the loans amortize, but the LGD used is an average number estimated for ease of computation.

From the basic assumptions made above, the single loan characteristics are defined. Before the estimation process can begin, some assumptions about the total loan pool behavior are needed. The objective is to generate a distribution for the number of defaults over the time period for the entire loan portfolio. This enables one to assess how the total pool is exposed to losses and thus, by combining the distribution of defaults and LGD, the EL for the given tranches can be calculated.

5.2.2 LOSS DISTRIBUTION

The loss distribution describes the fraction of losses related to how likely it is to occur. In practice, the exact distribution of losses is not known, and it needs to be modelled either analytically or by simulation. Since the properties of the single loan are defined, a model is used to generate the total pool cash flow. The model stipulates EL for the entire loan pool, from the probability associated with a given level of loss on the total pool. EL is simply computed by weighting loss by probability. By defining intervals for tranches, the percentage loss to the tranche can be determined, which can be converted to the EL for each tranche. Because the loss distribution is unknown, the forecast of the EL is the best estimate of the most likely loss during a year. Over time, losses are clearly not constant, but vary from year to year due to global macroeconomic trend and other cyclical tendencies. In periods with recession, the loss level will increase above the EL, and are referred to as unexpected losses (UL) (Basel Committee on Banking Supervision, 2005). Generally, EL is defined as:

$$EL = PD * LGD * EAD \tag{5.1}$$

EL accounts for the fact that a default is not necessarily equal to a complete loss of the loan, but when a default occurs some of the principal is recovered through a bankruptcy procedure. Thus, the actual loss that must be absorbed by a tranche takes the recovery rate into account. Further, when dealing with repayments and prepayments the principal is reduced over the expected lifetime of the loan, which is reflected in the EAD.

The modelling of defaults has developed through history, to include increasingly detailed and complicated models. Back in 1987, Vasicek developed the fundamentals of credit risk in portfolios (Vasicek O. , 1987), which is today used as foundation in the Basel regulation (Basel Committee on Banking Supervision, 2005, p. 5). The model builds on an adaption of Merton's (Merton, 1974) work on how a single asset develops over time as a normally distributed random variable, and the borrower defaults in case the value of the asset ends up below the level of debt in the asset at maturity. Under some conditions, the Merton model can be adapted to credit risk portfolios, which is the theory that is incorporated in Basel regulations, where asset development is assumed to be normally distributed for both systematic and idiosyncratic risk components. The model is based on simple assumptions about homogeneous portfolio with constant PD, constant recovery rate, and constant correlation. By applying the law of large numbers, which is fair under the expected volume of the Jigsaw, the loss distribution can be calculated by a simple formula, derived in Section 5.2.4. Due to its simple framework, the simple model will be our base model for assessing the credit risk in the loan pool.

The Gaussian model has acquired a status of the industry standard within credit risk modeling, when assessing the risk of a portfolio of credits. The model is the core in the Internal Ratings-Based (IRB) approach introduced under the Basel II capital requirements, where the risk capital is determined by using the single-factor model. The Basel Committee used the Vasicek model by implying the law of large numbers to the credit institution's portfolios, and required that capital requirements on a single loan should only depend on the underlying risk and not on the portfolio, in which it is added to (Basel Committee on Banking Supervision, 2005). The credit risk assessment is then based on a VaR approach, where the 99.9% percentile of the loss distribution is estimated, and must be considered as the capital requirement necessary to withstand both EL and UL (McNeil, Frey, & Embrechts, 2005, p. 363). The normal distribution that underlies the model can be discussed as to whether it is reliable for describing defaults in severe market conditions (Crook & Moreira, 2011). The stress test of the base case in the thesis is the advanced single-factor model, where the systematic and idiosyncratic risk factors are assumed to be Student t distributed. Several other models could be used to provide a more reasonable fit to financial data, but the Student t distribution is superior with respect to implementation.

The theoretical frameworks behind both the single-factor and the advanced single-factor model is introduced in Section 5.2.4 and 5.2.5, respectively. Before continuing to the estimation process, the definition of tranche loss and the mathematics behind the estimation of attach- and detachment points will briefly be introduced.

5.2.3 TRANCHE LOSS AND CREATION

In Section 5.2, the creation of the tranches was introduced and the attachment (A) and detachment (D) points were defined. From the loss distribution, the general level of loss in the pool can be established for a given probability. (The loss fraction in the pool, is denoted by L) For each loss fraction in the loss distribution the loss for a given tranche with attachment point A and detachment point D, can be calculated. If the loss fraction is below A, then no loss occurs in the given tranche, whereas a loss higher than D, will result in a loss of 100% in the given tranche. For a loss level between A and D, the tranche experiences a proportional loss. When $A \leq L \leq D$, the tranche can therefore be defined as (Das & Stein, 2011).

$$Loss\ rate\ in\ tranche = \frac{(L-A)}{(D-A)}\ for\ A \leq L \leq D \tag{5.2}$$

Combining that EL in the tranche is computed by weighting the loss rate with the probability distribution of the loss fraction, EL in a tranche can be defined as an integral between attachment point and a total loss of all principal in the loan pool.

$$Expected\ loss = \int_A^1 \min\left[\frac{(L-A)}{(D-A)}, 1\right] \cdot f(L) \cdot dL \tag{5.3}$$

Where $f(L)$ is the probability distribution of the loss distribution in the pool. The definition of EL in a tranche implies a continuous underlying distribution of losses in the pool. This can be kind of odd to think about, when dealing with default events, where the individual loan can either default or not. When modelling defaults, the distribution is discrete but to present a more easily readable graph of the distribution, the distribution is often smoothened, even though a closed-form solution does not exist. This implicit assumes that the number of loans is substantial, such that the loss distribution is so fine-grained that it can be approximated as continuous.

The fact that the major rating agencies use different risk assessments, was introduced in the introduction to this section. Moody's ratings are based on the concept of EL, where Standard & Poor's and Fitch ratings are based on PD (Fender & Kiff, 2004, p. 10). Because of this fundamental difference, the result of the rating procedure done by the two companies could yield different ratings for the same security. This, however, does not come as a surprise, since the two risk measures provide different information to the investor. Even though the final rating from each rating agency carry different information, which investors should be aware of, it might provide issuers of structured credit products with incentives to use the rating company that assigns the issued product with the highest rating. This is known as rating shopping (Fender & Kiff, 2004, p. 10). Each rating method is affiliated with both advantages and drawbacks. According to (Fender & Kiff, 2004), the EL-based rating assigns higher ratings to wide senior tranches, as oppose to the PD-based rating, which assigns higher ratings to thin junior tranches. This issue will not be dealt with any further, and only the EL-based approach will be used to create tranches. The EL-based calculation is chosen because it accounts for the tranche width and LGD, whereas the PD-based rating does not account for neither. Two hypothetical tranches with the same attachment point would therefore get the same credit rating under the PD approach, even though they might not have the same detachment points. Using PD-based tranching is equal to looking at portfolio loss VaR (Das & Stein, 2011, p. 3). Hence, the EL-rating is based on more information.

The mathematics behind tranching is relatively simple, and mainly consists of probability theory. The loss rate is defined in such a way that the loss in a single tranche is calculated as a percentage of the tranche width. In this way, the calculated expected tranche loss rate can be compared to the minimum requirements that must be satisfied to receive a certain credit rating, set by rating agencies. Since the focus only is on EL used by Moody's, the process of calculating tranche points will use the idealized EL set out by Moody's. For the senior and junior tranche, the width is maximized under the EL restriction of the target rating. This involves gradually making the tranches wider until the EL limit is reached, starting with the senior tranche, and afterwards the junior conditional on the attachment point of the senior. The Equity tranche will by default be the residual principal and hence, needs no calculation on its width.

The steps in the process of calculating the minimum attachment points for the tranches are carried out by Monte Carlo simulation, by implementing an iterative calculation process. Starting out with the total principal of the loan pool, the detachment point of the senior tranche is naturally 100% (1). The attachment point (A_{Senior}) is gradually lowered, starting from 1 and continuing until the maximum EL is reached while preserving a Aaa rating. The notation of EL in a tranche can be rewritten into a formula (Stein, Das, Ding, & Chinchalkar, 2010, p. 47).

$$A_{Senior} = min\left\{A_{Senior} \middle| \left[\int_{A_{Senior}}^{1} \frac{(L - A_{Senior})}{(1 - A_{Senior})} \cdot f(L) \cdot dL \leq EL_{Aaa}\right]\right\} \quad (5.4)$$

Since the detachment point is 1, the min part of the EL in Equation (5.3) can be omitted, because the loss rate in the pool cannot exceed the detachment point ($L \leq D_{Senior} = 1$) since the pool can never lose more than 100% of the principal. After the senior tranche is established, the process continues to the junior tranche. The detachment point of the junior tranche (D_{Junior}) is equal to the attachment point of the senior tranche $\left(D_{Junior} = A_{Senior}\right)$. To calculate the attachment point, the procedure is the same as for the senior tranche: the tranche width is maximized while maintaining the highest possible rating, as shown below.

$$A_{Junior} = min\left\{A_{Junior} \middle| \left[\int_{A_{Junior}}^{1} min\left[\frac{(L - A_{Junior})}{(A_{Senior} - A_{Junior})}, 1\right] \cdot f(L) \cdot dL \leq EL_{Baa3}\right]\right\} \quad (5.5)$$

Normally, this process is repeated until all the desired tranche levels are completed, which is most often more than three. Each new subordinated tranche width will be maximized, while the credit rating is only lowered by one rank each time, such that the credit rating for each tranche is maximized and the number of tranches will be equal to the number of credit ratings. However, in this case, the aim is to limit the product to three tranches for simplicity. The remainder after the senior and junior tranches are created is allocated as a residual equity tranche, which is likely rated Caa3 or unrated.

5.2.4 SINGLE-FACTOR MODEL

The base model in the simulation is the simple model initially developed by Vasicek, and is today the framework that is used in the Basel capital requirements. The model is based on a single systematic factor which affects all loans in the pool. As mentioned, the model builds on the original work by Merton in 1974. Merton proposed the basics on how to value firms and his model has been developed over the years, but the original is still one of the most popular models used in practice for credit risk analysis (McNeil, Frey, & Embrechts, 2005, p. 331). The model is a structural default model, which attempts to assess when a default occurs. The original model considers a firm with assets which follows a stochastic process - a geometric Brownian motion. The assets are funded by both equity and debt, and the default event occurs whenever the firm cannot meet its obligations at maturity, which is the case when the value of the assets is lower than the value of debt.

Vasicek used the basic framework proposed by Merton to develop a simple setting, which could assess the probability of losses in a loan portfolio. The loss distribution of the portfolio is the main element used to estimate the tranches in the Jigsaw. Vasicek showed that under certain assumptions, the loss distribution is converging to a given distribution that can be derived analytically, when the number of loans in the underlying portfolio approaches infinity (Vasicek & Merton, 2015, p. 143). This result is very convenient, since the computation needed to estimate the loss distribution without a closed form solution would require many resources and can be very time consuming.

Because of the pairwise constant correlation ρ, the default events are not independent and the loss distribution will therefore not converge to the normal distribution. In the standard setting, the loans are correlated with a Gaussian copula, which is an assumption about how the factors are correlated between the loans. Two risk factors affect each loan: a systematic factor and an idiosyncratic factor. Both underlying risk factors is jointly normal distributed due to the Gaussian copula modelling, and due to the constant correlation coefficient across loans, the following shock, ϵ_i, affects each loan (Vasicek & Merton, 2015, p. 144).:

$$\epsilon_i = \sqrt{\rho}M + \sqrt{1-\rho}z_i \tag{5.6}$$

Both M and z_i are random standard normal distributed variables, where M denotes the systemic market factor that affects all the loans, and z_i denotes the idiosyncratic risk factor that only affect the single loan. M and z_i are independent of each other, with the result that ϵ_i is also a random standard normal distributed variable. Loan i defaults if the shock, given by ϵ_i, is below a threshold value, computed from the PD. The PD is inserted in the inverse standard normal distribution with the purpose of computing a default barrier K for each firm.

$$K = N^{-1}(PD) \tag{5.7}$$

If the simulated outcome of ϵ_i ends below K, loan i defaults. This can be rewritten as:

$$\sqrt{\rho}M + \sqrt{1-\rho}z_i < K \tag{5.8}$$

When the PD and correlation is constant across all firms, such that the whole portfolio of loans is homogenous, the loss distribution in the loan portfolio converges to a limiting form, which makes it possible to calculate the loss fraction in a closed form solution, conditional of a market shock. For a given outcome of the common market shock, the probability that firm i defaults is given by (Vasicek & Merton, 2015, pp. 144-145).:

$$\begin{aligned} PD(m) = P(\epsilon_i < K|M = m) &= P\left(\sqrt{\rho}m + \sqrt{1-\rho}z_i < N^{-1}(PD)\right) \\ &= N\left(\frac{N^{-1}(PD) - \sqrt{\rho}m}{\sqrt{1-\rho}}\right) \end{aligned} \tag{5.9}$$

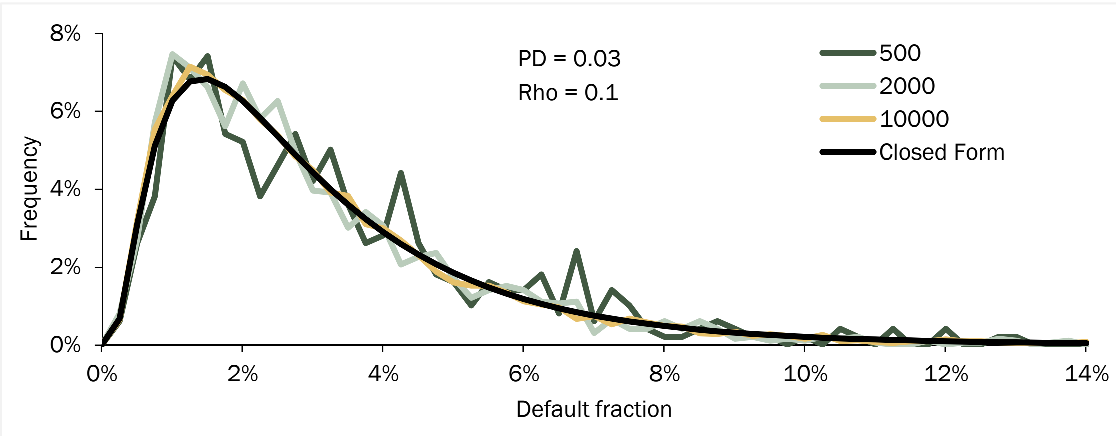
The above is a closed form solution for the PD of each loan when the market shock M is known. The default distribution can therefore be estimated by simulating several random market shocks, and using them as an input in the above formula.

Furthermore, when the market shock is known, the probability that a single loan defaults is independent, because the correlation between the individual loans is captured by the correlation coefficient. This can be thought of as the market factor changing the PD in the loan pool, once the factor is known, the defaults are happening independently with the new PD. The higher the correlation, the more the market factor changes the PD. When the number of loans is large, the default rate will converge to the PD. Instead of knowing the probability that a single loan defaults, it is convenient to know the probability that the default rate is below some arbitrary number, θ, between zero and one. This is known as the density function, and is shown below.:

$$\begin{aligned} P(PD(m) \leq \theta) &= P\left(N\left(\frac{N^{-1}(p_i) - \sqrt{\rho}m}{\sqrt{1-\rho}}\right) \leq \theta\right) \\ &= N\left(\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(\theta) - N^{-1}(p_i)\right)\right) \end{aligned} \tag{5.10}$$

To show that the default distribution in the single-factor model is converging when increasing the number of observations, four different loss distributions have been simulated, where three of them use an increasing number of observations and the last is the default distribution calculated from Equation (5.10). Two arbitrary numbers have been chosen as inputs for PD and correlation. PD is set to 0.03 and the correlation between the loans is set to 0.1. In Figure 5.1, the result is shown.

Figure 5.1 – Convergence of the default distribution



Source: Authors creation, see Appendix D for R code.

For each of the simulated default distributions, 500, 2,000 and 10,000 random market shocks have been drawn from a standard normal distribution, respectively, and is used to calculate the total defaults in the loan pool. When only 500 observations are used, the default distribution fluctuates a lot around the closed form default distribution. With more observations, the fluctuations decrease and with 10,000 observations the distribution is almost equal to the closed form. The overall conclusion to draw from the Figure 5.1 is that many observations are required if the defaults are simulated with Monte Carlo, else the result could be biased by outliers.

Knowing that the default distribution in the single-factor model is converging towards a closed form solution as the number of underlying loans in the pool is moves toward infinity, makes the calculations a lot easier and less resource-intensive. The only input required is a random market shock that is easily calculated. Further, when the actual default distribution is used to estimate tranches, it is necessary to incorporate the LGD factor, to go from the default distribution to the loss distribution. Each tranche then only incurs loss, after the recovery rate has been accounted for. The LGD factor is incorporated by multiplying it with the default distribution. The closed form default distribution, is given by Equation (5.9), and this simply has to be multiplied with the LGD factor, to get to the loss distribution.

$$Loss = LGD * N\left(\frac{N^{-1}(PD) - \sqrt{\rho}m}{\sqrt{1-\rho}}\right) \qquad (5.11)$$

First then, the loss severity of the default distribution can be translated into loss severity in tranches, which is what is used when optimizing the tranche sizes, and therefore Equation (5.11) is used in estimating process in each trial in the Monte Carlo simulation.

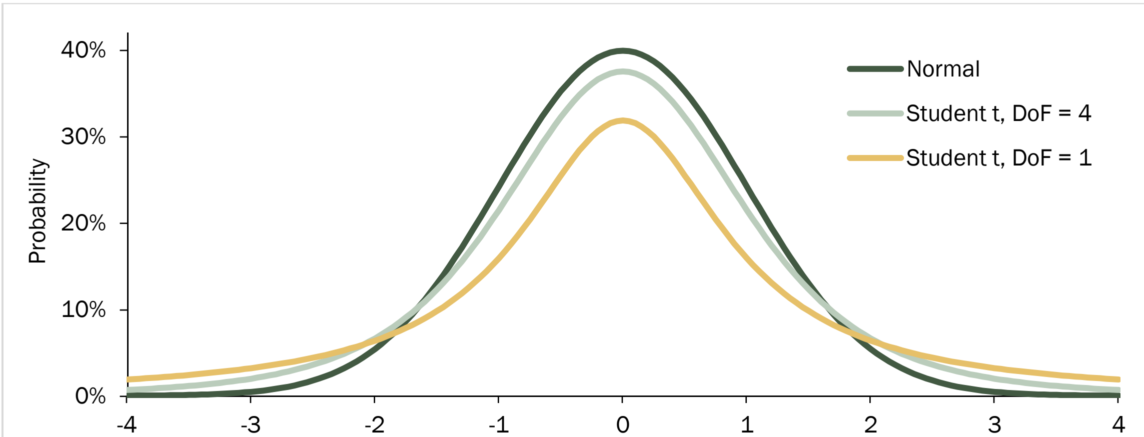
5.2.5 ADVANCED SINGLE-FACTOR MODEL

The single-factor model introduced in Section 5.2.4, is a model, which have been heavily used in practice to evaluate credit risk. The correlation is modelled by a Gaussian copula, which carries some good features such as easy implementation. However, the normal distribution does not exhibit much tail dependence, why extreme events rarely occur and default events do not cluster, when default events are modelled with this distribution (Rosenberg & Schuermann, 2006). By using the Gaussian copula to calculate the credit risk of the portfolio, it is implicitly assumed that it the normal distribution is a good description of the tail risk in the portfolio (Rutkowski & Tarca, 2016, p. 14). The result can be that the simulated outcomes are biased, and underestimate the probability of simultaneous extreme values if the actual distribution of defaults is not normal. Throughout history, several studies have shown that asset return in general does not follow a normal distribution (Crook & Moreira, 2011). Considering this, the results obtained when using the single-factor model with Gaussian copula might underestimate the associated credit risk in the underlying pool of loans. Thus, the tranches created under the Gaussian loss distribution are too optimistic, and do not display the actual risk related to the Jigsaw.

To mitigate the issues related to the chosen copula in the single-factor model, the analysis is expanded with an extra model, which builds on a more advanced copula that can account for more extreme events and exhibits more tail risk. Several copula structures can be proposed to ensure higher tail dependence, such as the Student t, Clayton, and Gumbel to mention a few (Crook & Moreira, 2011, pp. 733-734). The purpose of including a more advanced model is twofold; First, a more advanced copula should act as a more realistic assumption about real-world behaviour of default events. Second, by implementing a more advanced model the results obtained from the single-factor model can be stress-tested to see how high the reliability of the model is. The expected result is to see that by using a copula with more tail dependence, the loss distribution should have more extreme observations, which should increase the attachment points of the senior and junior tranche.

The copula is chosen by the characteristics of its tail risk in the given setting and by how easy implementable the copula is in practice. The Student t copula fulfil these requirements, as it is easy to implement and has a larger tail risk (Wanitjirattikal & Kiatsupaibul, 2007). The Student t copula use a symmetric dependence structure which looks like the normal distribution, but where the tails are fatter in both the lower and upper end of the distribution, and is used extensively when financial returns are modelled (Frey & McNeil, 2001). For illustrative purposes, Figure 5.2 shows the normal and Student t distribution with both four and one degree of freedom (DoF), to illustrate how the Student t has more probability mass in the tails the lower the DoF, relative to the normal distribution.

Figure 5.2 – Normal versus Student t distributions



Source: Authors creation

To show how the tail risk differs in the distributions, 1,000 random bivariate numbers have been generated from the normal and the Student t distribution, with correlation set to 0.5 to make sure the numbers exhibits some correlation. The samples are generated by following the procedure outlined in (Hull J. C., 2015, p. 242), where the DoF in the Student t distribution is set to 4.

Figure 5.3 – Random Sample from a Bivariate Gaussian Distribution

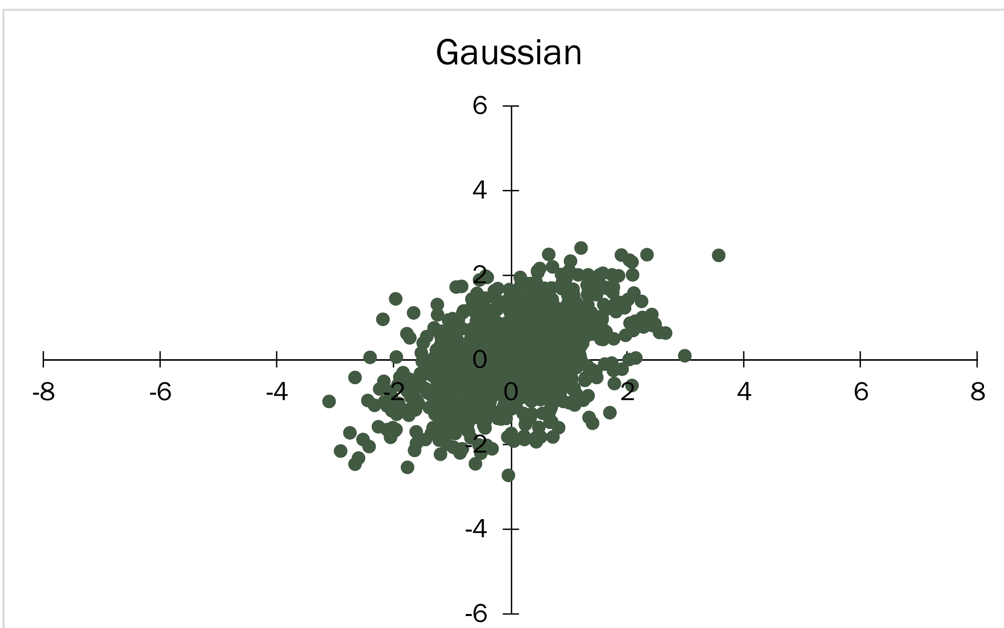


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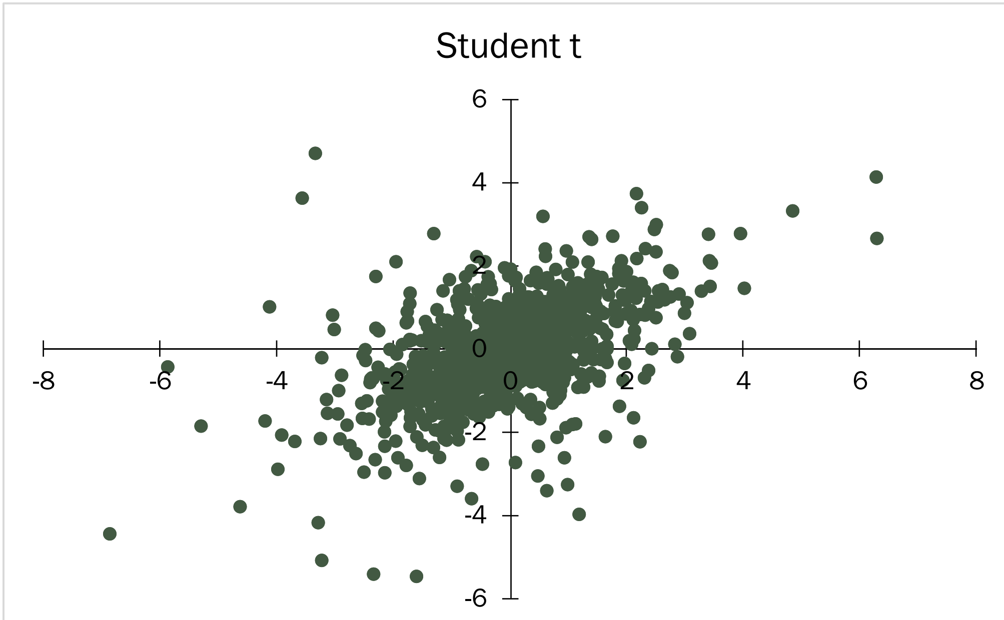
Figure 5.4 – Random Sample from a Bivariate Student t Distribution



Source: Authors creation

From Figure 5.3 and Figure 5.4, it is clear to see that the Student t distribution has much higher tail dependence, as large outliers are more frequent due to the fatter tails in the distribution from which the random numbers are drawn. The tail dependence is increasing with the correlation, and even though the correlation in the two samples is set to 0.5, it is clear that it is much more frequent to observe observations in the upper-right and lower-left corners under the Student t distribution. This feature should be more appropriate when dealing with financial data, since history has shown that financial markets generally tend to exhibit tail dependence, as correlation increases during periods of financial distress. The Student t distribution should therefore provide a better fit.

Using the Student t distribution comes with additional benefits that make the estimation process easier than by using other more complex copulas. Because the distribution is symmetric, like the normal distribution, it is possible to calculate a closed form solution for the probability of a loan defaulting in the pool. The formula will be like the one outlined in Section 5.2.4 from the single-factor model. The Student t distribution is defined in terms of DoF, and in the case where the DoF is increased towards infinity, the t distribution is approaching the normal distribution. The fewer DoF, the more tail dependence is present in the Student t distribution. Therefore, the choice of DoF in the Student t distribution has a large effect on the simulated loss distribution, because a large probability mass under the function is shifted to the tails. The risk of the loans depends strongly on the chosen DoF, why it is an important challenge when dealing with Student t copulas, and must be calibrated with caution.

To simulate a loss distribution, the threshold framework based on Merton's work is reused. Generally, it is possible to use any distribution for the inputs in the framework, the distribution only needs to be standardized to have zero mean and unit variance, just like in the standard normal distribution. By using the Student t as copula instead of the normal, variance of the random variables from the standard normal distribution is allowed to be stochastic. It is assumed that the reciprocal of the variance follow a χ_v^2 distribution, divided with the DoF (Martin, 2004, p. 46). This makes the ϵ_i, that effect each individual loan in the portfolio, a Student t distributed variable with a given DoF, simply because it is scaled by a factor of $\left(\sqrt{\frac{v}{w}}\right)$ (Frey & McNeil, 2001), where the factor W is a random distributed variable with a χ_v^2 distribution (chi-square), and v is the DoF. The individual loan shock is therefore modelled as:

$$\begin{aligned}\epsilon_i &= \sqrt{\frac{v}{W}}\sqrt{\rho}M + \sqrt{\frac{v}{w}}\sqrt{1-\rho}z_i \\ &= \sqrt{\frac{v}{W}}\left(\sqrt{\rho}M + \sqrt{1-\rho}z_i\right)\end{aligned} \tag{5.12}$$

Both M and z_i are still random standard normal variables. Further, and similar to the single-factor model, the cumulative probability that a single loan defaults, is converging to a closed form solution, when the shock from market and the factor W, are given. But since the underlying distribution is now a Student t, the threshold value that determines whether a loan defaults or not, must also be found in the t distribution. The PD of a single loan, therefore becomes (Bluhm, Overbeck, & Wagner, 2010, pp. 109-111).:

$$\begin{aligned}p_i(m,w) &= P(\epsilon_i < K | M = m, W = w) \\ &= P\left(\sqrt{\frac{v}{w}}\left(\sqrt{\rho}M + \sqrt{1-\rho}z_i\right) < t_v^{-1}(PD)\right) \\ &= N\left(\frac{t_v^{-1}(PD)\sqrt{\frac{w}{v}} - \sqrt{\rho}m}{\sqrt{1-\rho}}\right)\end{aligned} \tag{5.13}$$

As with the single-factor model, the LGD factor needs to be incorporated to go from the default distribution to the loss distribution, which is the distribution used to estimate tranche sizes. The closed form default distribution under the Student t, is given by Equation (5.13), and by including the LGD factor, it is translated into the loss distribution for the Student t model shown in Equation (5.14).

$$Loss = LGD * N\left(\frac{t_v^{-1}(PD)\sqrt{\frac{W}{v}} - \sqrt{\rho}m}{\sqrt{1-\rho}}\right) \quad (5.14)$$

To show that the choice of v has high influence on the shape of the default distribution, a sample of 10,000 observations are created for different DoF. Both default distributions with Gaussian and Student t copula is created, and different statistics is computed to compare the statistical properties of the two copulas. The result is presented in Table 5.1 and

Table 5.2.

Table 5.1 – Gaussian Copula default distribution, simulated with 10,000 scenarios, and rho = 0.1 (%)

PD	Mean	SD	Quantile (99%)
1.5	1.4953	1.3291	6.3351
3.0	3.0301	2.3775	11.6201

Source: Authors creation, see Appendix E for R Code

Table 5.2 – Student t Copula default distribution, simulated with 10,000 scenarios, and rho = 0.1 (%)

DoF (v)	PD	Mean	SD	Quantile (99%)
10,000	**1.5**	1.5084	1.3493	6.3401
	3.0	2.9986	2.3211	11.4090
40	**1.5**	1.4934	1.7847	8.4760
	3.0	3.0325	2.8562	13.7094
10	**1.5**	1.5154	2.6789	13.0546
	3.0	3.0400	4.1083	19.6257
4	**1.5**	1.4960	4.1538	21.6764
	3.0	3.0321	5.8107	28.8622

Source: Authors creation, see Appendix E for R Code

As expected, when the DoF are set substantively high, the Student t copula is practically equal to the Gaussian copula. The small differences are simply due to fluctuations that can be linked to the stochastic simulation process. In the two tables, the mean, standard deviation, and the 99% quantile is calculated. The mean default fraction is approaching the actual PD when the number of observations increases, in line with "the law of large numbers", which is true regardless of the distribution, and in total accordance with the expectation. Only the shape of the default distribution is changed, while the expected default rate remains constant. When the DoF are decreased the shape of the loss distribution becomes less concentrated around the mean, and the tails get fatter, which is translated into higher risk in the form of higher standard deviation. In the most extreme case, where $v = 4$ and PD = 3%, the 99% quantile is 28.86%. This means that with 99%

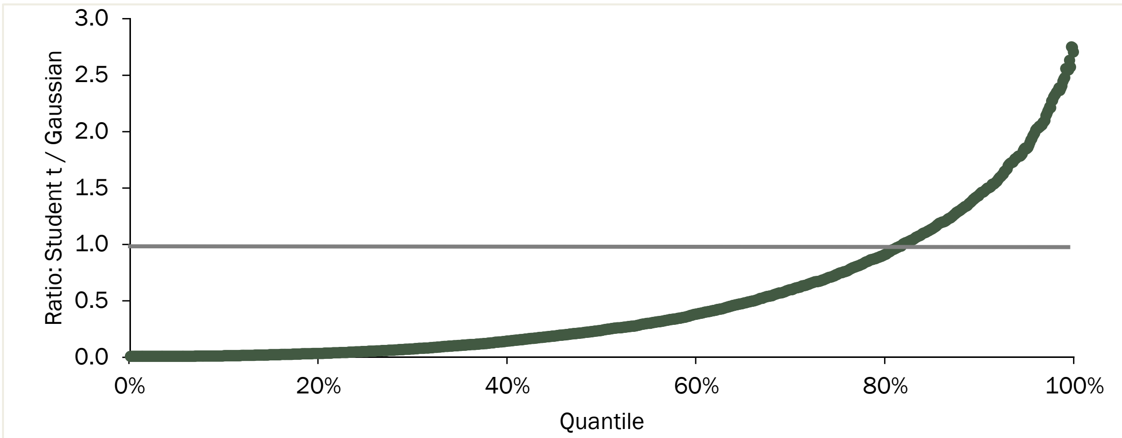
certainty, the maximum default fraction experienced is 28.86%. In comparison to the Gaussian default distribution, the equivalent default fraction is 11.62%. Meanwhile the standard deviation of each loss distribution shows the fluctuation in defaults around the mean, which can be translated into the risk for the investor. Again, when either the PD is increased or the DoF are decreased, the result is increased risk in the model.

The default distribution under the Student t copula assumption, does not have an analytical expression, like it was observed under the Gaussian copula assumption (Lichters, Stamm, & Gallagher, 2015, p. 324). Not even when both shocks are modelled with the same Student t distribution with the same number of DoF. The only way to draw the default distribution under the Student t copula is to use Monte Carlo, simulate a given number of shocks, and calculate the defaults in the pool.

To further outline higher tail dependence in the default distribution under the Student t copula than under the Gaussian copula, a sample is drawn from each distribution with use of the formulas outlined in this section and in Section 5.2.4. The idea is that the Student t more frequently displays extreme events in the tails, which is translated into more frequent high default fractions in the loan pool. 10,000 observations are used in a Monte Carlo simulation, where the PD and correlation is set to 3% and 10%, respectively. The numbers are chosen to be consistent with the random samples that were drawn in Section 5.2.5. Further, the DoF are set to four, to exhibit tail dependence.

In Figure 5.5 the quantiles of the loss distributions are compared to see that the ratio of defaults between the Student t and Gaussian copula is increasing for higher quantiles. This effectively shows that the Student t copula makes the default distribution more extreme. Further, as Figure 5.5 shows, in the lower tail the Student t exhibits a lower fraction of defaults up until around the 80% percentile, where the ratio crosses 1. This shows that the Student t distribution is extreme both at the lower and upper end of the distribution, thus in several cases, a lower default fraction can be observed under the Student t than under the Gaussian model. In the remainder of the quantiles, the ratio is increasing, and in the most extreme quantile, the Student t copula is roughly 2.7 times larger than under the Gaussian copula.

Figure 5.5 – Ratio of default fractions for quantiles



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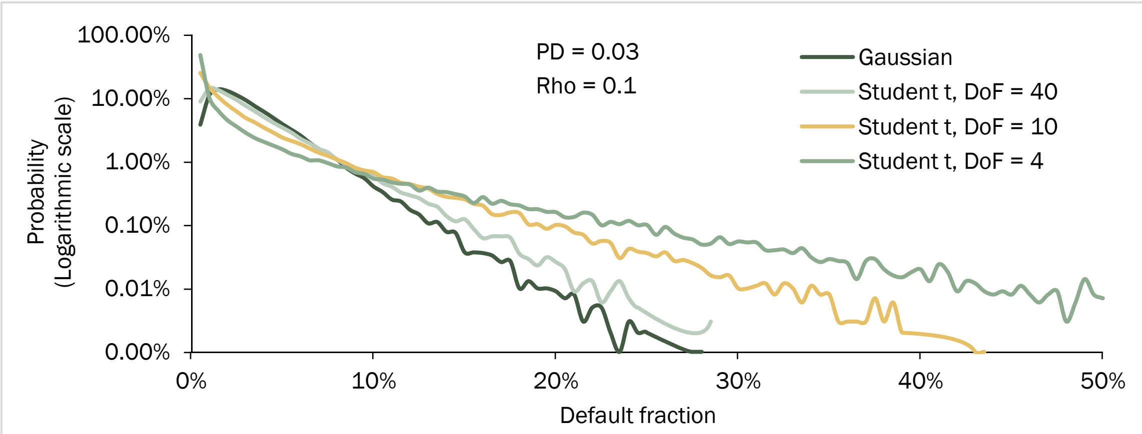
5.2.5.1 CHOICE OF DEGREES OF FREEDOM

It is easy to increase the probability of a high fraction of defaults with the Student t copula, since it is simply controlled by the DoF, but what is not easy, is to choose the appropriate DoF. The reason is that the underlying default distribution of the loan pool is unknown and the copula is used to replicate the shape of the true default distribution. Thus, controlling how much risk to apply in the tail of the distribution is a subjective decision. By setting the DoF too low, and thereby obtaining a default distribution with high default fractions, the result could be that the riskiness of the underlying loans is overestimated, with the effect of biased results.

To see the impact that changing the DoF has on the shape of the tail in the default distribution, Figure 5.6 has been created. In the figure, four different loss distribution have been simulated with 100,000 trials, with both the Gaussian copula and the Student t copula with 4, 10 and 40 DoF. The vertical axis is measured on a logarithmic scale, so that the area under the loss distribution for extreme default fractions is more clearly displayed.

Figure 5.6 – Loss distribution under varying copula assumptions



Source: Authors creation

Since the distribution is drawn by the Monte Carlo method, the lines are not straight and the waves along each line represent the uncertainty in the estimation. Noticeably, the Student t default distribution with 40 DoF is close to the Gaussian distribution. It becomes clear how much more extreme events is a part of the Student t distribution with four DoF. Further it is worth noticing that the default distribution with four DoF, has a lower probability mass than any of the other distributions, for default fractions around the true PD used. This is due to Student t distribution that shifts probability mass from around the true PD towards each tail. As a result, it is also more frequent under the Student t copula to observe a low default fraction as oppose to the Gaussian copula, which is seen by the distribution with four DoF being highest near 0% default fraction, as was also shown in Figure 5.5.

After the financial crisis, it was outlined that normal distributed copulas clearly underestimated the risk of tail events. The observed returns were extremely negative, correlation sky rocketed, and the Gaussian copula could not cope with the empirical distribution observed in this stress scenario. Afterwards it has been a concept, which has been highly debated, and consensus is that the Gaussian copula is rarely the most suitable copula to use for financial data (Koziol, Schell, & Eckhardt, 2015, p. 2). Other copulas seem to provide a better fit to financial data, where the Student t stands out with better goodness of fit measures. The distribution is suitable for stress testing the properties of the underlying loan pool, since the degree of severity can be controlled.

To capture the tail risk, the European Banking Authority suggest that a Student t copula with three or four DoF, in most cases would be more suitable (European Banking Authority, 2014). As shown previously in Figure 5.6, the tail risk is substantively increased by setting the DoF as low as three or four. But setting the DoF so low would overestimate the loss distribution when compared to the historical default fractions, which is why 10 degrees of freedom is considered more suitable. By setting it any higher the fast converging effects of the Student t distribution results in diminishing tail dependence, as the distribution converges to the normal distribution. Generally, it can be argued that whenever the DoF are 30 and above, the distribution is close to the Gaussian (Stoyanov, Rachev, Racheva-Iotova, & Fabozzi, 2011, p. 8). Taking this into consideration, DoF of 10 should incorporate a high level of tail dependence, while not overestimating the loss severity by having too large tail risk.

5.3 MOODY'S IDEALIZED EXPECTED LOSS RATES

When the tranches are created, they are created with the intention of maximizing the EL that the targeted credit rating from Moody's allows. The maximum EL for given credit rating is set by Moody's, who frequently announces tables with historical loss rates that stipulate the current EL limit. From Table 5.3 and Table 5.4, it is possible to identify the maximum EL that is allowed in an entity, while maintaining a given credit rating. The two figures are separated between ratings that are referred to as investment grade and non-investment grade, which are also known as speculate grade (Hull J. , 2012, p. 521).

Table 5.3 – Investment Grade EL limits (bps) – 10 Year Cumulative

Rating	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3
EL	0.55	5.5	11	22	39	66	99	143	198	336

Table 5.4 – Speculate Grade EL limits (bps) – 10 Year Cumulative

Rating	Ba1	Ba2	Ba3	B1	B2	B3	Caa1	Caa2	Caa3
EL	517	743	971	1,221	1,496	1,920	2,624	3,575	4,439

Source: (Moody's Investors Service, 2016)

The numbers are based on historical default frequencies, calculated by Moody's, as a cumulative default frequency of firms that have defaulted during a given time horizon with the given credit rating. Since the product that is assessed in this thesis has an initial time horizon of 10 years, the default loss rates are also sampled over a 10-year period (Stein, Das, Ding, & Chinchalkar, 2010, p. 46) (Roughton-Smith, 2001, p. 10). The default probabilities are produced by the other rating agencies in similar manor. As previously mentioned, Moody's differentiate from the others by using the EL approach. In this approach, not only the PD is considered but also the LGD. The result is that the actual loss is used to evaluate the credit quality.

5.3.1 RE-RATING METHODOLOGY AND FAIR INTEREST RATES

The bonds issued to fund the Jigsaw in the different tranches will be given a nominal interest rate, but the effective interest rate (yield) on the bonds will be set in the market, as forces of supply and demand will trade the bonds towards an equilibrium price. The price and coupon rate corresponds to an effective interest rate. If the price is higher than 100 the effective rate will be lower than the coupon and vice versa. For example, a 10-year Jigsaw (annuity bond) traded at a price of 98 with a coupon rate of 2% will have an effective interest rate of 2.4%.

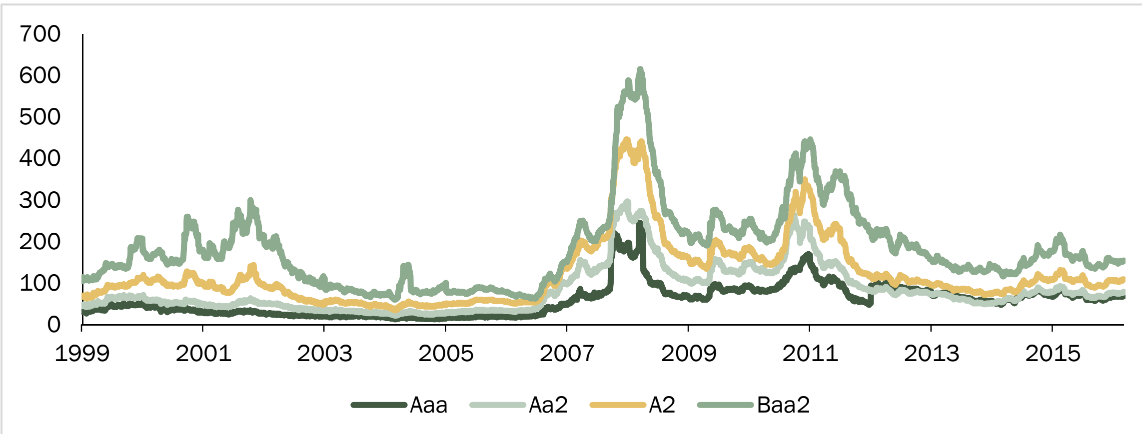
Since it is not possible to know what price the tranches will trade, the effective rate needs to be estimated to assess the Jigsaw's competitiveness with the existing bank loans. The Danish 10-year government bond is used as the reference risk-free rate. Once the credit rating of a given tranche is identified, the SPV should in practice be able to issue notes with this rating, where the tranche backs the note. Practically, this rating procedure is carried out by the rating agency, which assess the credit quality of the investment certificate. The price, or the corresponding yield, the note should trade at, is identified by using a re-rating methodology where the note is compared to similar structures in the market to determine what a fair yield spread would be, with the given credit rating (Mahadevan, Polanskyj, Tirupattur, Onur, & Sheets, 2006, p. 86).

Denmark is known to be a "safe haven" economy, and has obtained a stable Aaa rating by all the three major rating agencies. As a result, the notes issued by the Danish government are also Aaa rated and therefore trade at low yields. The mortgage system is considered extremely stable, even though the Danes are highly indebted with close to 300% debt to disposable income in 2015, according to OECD. There has never been a single default of a mortgage bank and as a result, the mortgage bonds also have a Aaa rating. Even though both government and mortgage bonds have the same rating, the mortgage bonds have historically traded with a credit spread. Thus, even though it is expected that the SPV can create Aaa rated notes, the yield should not be as low as Aaa rated Danish government bond.

To assess what a fair yield would be, the level and development of credit spreads in comparable defaultable bonds and papers are examined. The credit spread is defined as the difference between the yields on the given paper, subtracted by the yield on a corresponding government bond with similar maturity that can be considered as risk-free (Lando, 2004, p. 12). The credit spread therefore displays the compensation for taking on additional risk, associated with a given credit rating. The market in Denmark for corporate bonds and structured debt obligations, such as CDOs, Collateralized Loan Obligations (CLOs), and related papers are small and practically non-existing. To obtain an idea about the credit spread relevant for the given product, only the European market for corporate bonds is used for reference. Bank of America update indices that follow corporate bonds with given credit ratings, for each provided credit rating it is then possible to download the Option-Adjusted-Spread (OAS). Only the major credit ratings given by Standard & Poor's are used to create the indices, why the S&P rating is translated into the corresponding credit rating by Moody's. (e.g. AAA = Aaa).

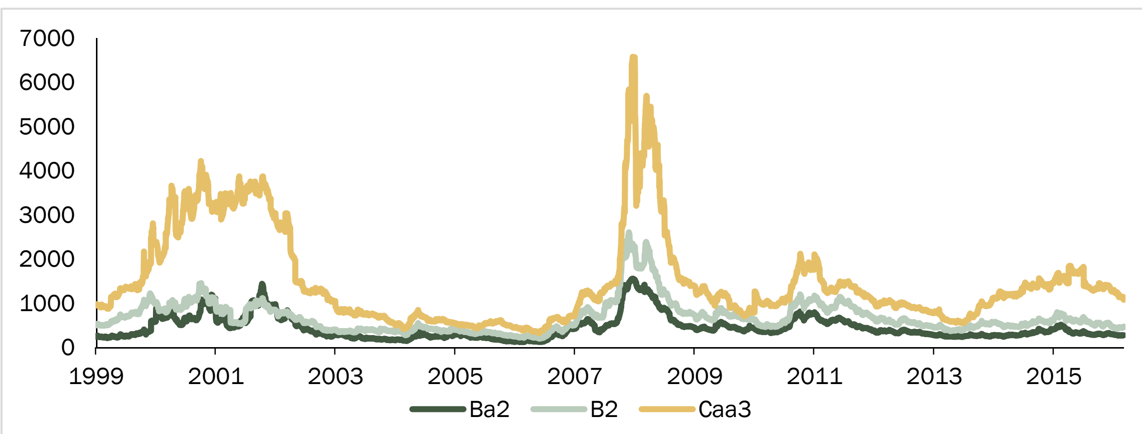
Figure 5.7 and Figure 5.8 shows the historical spread in bps for investment grade and high yield bonds, respectively.

Figure 5.7 - yield spreads on investment grade bonds over the respective risk-free alternative, basis points (bps)



Source: Bloomberg and Bank of Amerika

Figure 5.8 - yield spreads on high yield bonds over the respective risk-free alternative (bps)



Source: Bloomberg and Bank of Amerika

It is clear that spreads increase dramatically during distressed periods in the economy, where sharp increases in spreads were seen widely across all ratings during the global financial crisis in 2008 and the European debt crisis in 2011. However, these are not relevant when assessing the Jigsaws current competitiveness to the alternative bank loan financing, as the competing products is priced in the current interest rate environment, and hence, the recent spreads is the relevant benchmark.

To avoid daily shocks biasing the estimation, an average over the 1st quarter of 2017 have been used when setting the 10-year risk free rate and the spreads on the three Jigsaw tranches. This also negates the lag that is likely to occur in the offerings from the different consumer banks in Denmark, as they will not adjust instantaneously to the current interest environment.

The Danish 10-year government bond yield traded on an average yield of 45.8 bps or 0.458% over the 1st quarter of 2017 (Bloomberg, 2017). This is the proxy of the risk-free benchmark, on which the tranche spreads are added to. The spread on a Aaa rating in the first quarter of 2017 was 66.9 bps or 0.669%. Data on the Baa3 rating was not sufficiently available, but instead a Baa2 rating is used as a proxy. The Baa2 rating traded on a spread of 149.4 bps or 1.494%, and to account for the one rank poorer rating, a spread of 200 bps or 2.000% have been assigned to the Baa3 rating. This seems like a fair and conservative estimate as the current increase from an A2 (rank 6) to Baa2 (rank 9) is 42% (108 bps to 154 bps), and hence, the spread on a Baa3 (rank 10) is likely lower than 200 bps, corresponding to a 33% increase in the one rank difference, but an estimate on the conservative side is desirable. Finally, the Caa3 rating traded on an average yield spread in the 1st quarter of 2017 of 1,137.9 bps or 11.379%. The spreads and total yield is summarized in Table 5.5.

Table 5.5 - Rating spreads and yields (bps)

Trance	Rating	Spread	Total interest rate
Jigsaw Senior	Aaa	67	113
Jigsaw Junior	Baa3	200	246
Jigsaw Equity	Caa3	1,138	1,184

Source: Bloomberg and Bank of Amerika

5.3.2 MISPRICINGS IN THE RE-RATING METHODOLOGY

Pricing tranches using EL thresholds and yields in similar rated corporate bonds have been criticized for assigning too low yields to tranches, especially the more junior ones. Brennan, Hein, and Poon shows explains the dynamics behind this bias and shows the mispricing that occurs when you price tranches based on ratings on individual firms (Brennan, Hein, & Poon, 2009). The reason behind the mispricing is the fact that the EL thresholds is based on the total risk of a company, but the pricing of the corporate bonds is only based on the systematical part of the total risk, as rational investors diversify away the idiosyncratic part. Tranching is accumulating systematic risk, primarily on the junior tranches, and even though the total risk might be the same in the tranche and the corporate bond of a similar rating, the systematic risk in the tranche is much higher and hence, the tranche should be discounted more than the bond.

However, Brennan et al. also shows that using Moody's EL ratings is better than using S&P and Fitch's PD based ratings, as taking the LGD into account negates some of the mispricing. In the setting where ratings are based on default probabilities, consider a company with assets worth 100, face value of debt equal to 90 and a PD of 2%. This company could split its debt into a senior part with face value of 50 and junior with a face value of 40. The junior debt would still have a 2% PD, but the senior part would have a much lower PD and could be sold at a much lower yield. Here Moody's EL would capture that the junior debt would lose more relative to its size than the total debt of 90 in the original scenario in case of default, and hence, S&P and Fitch ratings are suffering from larger biases when used on tranches in comparison to Moody's ratings. However, the accumulation of systematic risk is still not omitted when using EL (Brennan, Hein, & Poon, 2009, p. 899).

Research from Citi shows that the pricing of CLOs (collateralized loan obligations), which are a general term for securitized, tranched products, including RMBSs, has changed since the global financial crisis (Wang, 2015). Prior to the crises, the research shows pricing was equivalent to the spreads seen in corporate bonds exhibited in Figure 5.7 and Figure 5.8, but the difference has since increased dramatically. Table 5.6 shows the findings from Citi Research on spreads for a selected set of ratings and compares it with the corporate bond spreads.

Table 5.6 – CLO and corporate bond spreads pre- and post-crisis and CLO premium to corporate bonds (bps)

Rating	2007			2015		
	CLO	Corp	Premium	CLO	Corp	Premium
Aaa	24	28	(4)	155	65	90
Aa2	35	54	(19)	230	68	162
A2	60	77	(17)	330	93	237
Baa2	150	91	59	400	146	254
Ba2	340	218	122	600	311	289

Source: Citi Research, Bloomberg, and Bank of Amerika

The research shows that after more investigation was done into the pricing of CLOs, e.g. by (Brennan, Hein, & Poon, 2009), investors have changed their pricing on the products in a CLO structure. For all ratings, the premiums have increased substantially, recognizing the increased systematic risk in a CLO product. Applying the premiums of the nearest rating seen in 2015 to the current spreads should therefore estimate an interest rate on the Jigsaw that is closer to the rate that will be seen in practice, which is estimated for each target rating in Table 5.7 using the corporate bond spreads estimated in Table 5.5.

Table 5.7 - Estimated Jigsaw spreads from CLO spread premiums (bps)

Tranche	Rating	Corporate spread	CLO premium	Total spread	Total interest rate
Jigsaw Senior	Aaa	67	90	157	203
Jigsaw Junior	Baa3	200	254	454	500
Jigsaw Equity	Caa3	1,138	2x	2,276	2,322

Note: 10-year risk-free interest rate is estimated to be 46 bps

As the exact ratings used for the Jigsaw Junior and Equity was not investigated by Citi Research, the premiums for these are estimated based on the closest rating researched by Citi. Especially for the Jigsaw Equity, the nearest premium estimated is for the Ba2, which is not near to the Caa3 rating, and hence, the premium is a very uncertain estimate. A premium of 289 bps relative to a corporate bond spread of 1,138 bps is not very significant, and therefore the ratio of approximately 2x between the CLO spread and corporate bond spread for the Ba2 rating is used as a proxy for the Jigsaw Equity spread.

6 RESULTS AND INTERPRETATION OF MODEL OUTPUTS

This section deals with the estimated results based on the established framework and the underlying assumptions. In general, the Jigsaw attaches at 5% and detaches again at 20% in the financing of properties as shown in Figure 4.4, which defines the total size of the Jigsaw. Within the Jigsaw, two attachment and detachment points between the tranches are estimated, the detachment and attachment points between the Jigsaw Equity and Jigsaw Junior tranche, and the detachment and attachment point between the Jigsaw Junior and Jigsaw Senior tranche. The thresholds between the tranches are estimated by maximizing the EL while maintaining a Aaa rating (Rank 1) in the Jigsaw Senior and a Baa3 (Rank 10) in the Jigsaw Junior. The Jigsaw Equity is not estimated to achieve a given credit rating, since its size is simply a residual of the remainder of the Jigsaw after the Senior and Junior tranche is estimated. However, the EL in the Equity tranche is calculated to assess whether it is possible to assign a credit rating or whether it will be unrated. No matter the results, the Equity tranche is, without doubt, a highly risky investment. All calculations are carried out in R, and the code for this section is provided in Appendix F.

6.1 SINGLE-FACTOR MODEL

Before tranching the Jigsaw, the single-factor model estimates an EL of 1.18% in the entire Jigsaw in the base case, where PD = 1.59%, LGD = 75%, and Correlation = 5.7%. This is close to the PD times the LGD, $\mathrm{EL}_{\mathrm{Jigsaw}} = \mathrm{PD} * \mathrm{LGD} = 1.59\% * 75\% = 1.19\%$, which is the theoretical value when the number of trials approaches infinity.

The actual tranching of the Jigsaw is carried out with 100,000 trials, which simulates 100,000 states of the economy through the market shock and the loss in each state is calculated using Equation (5.11). The detachment and attachment points are estimated from the theory outlined in Section 5.2.3. To check the robustness of the model, it is re-run several times resulting in deviations of up to three percentage points in the tranche cut-off points. An average of the model runs is taken, and the results are the following detachment and attachment points for the range of correlations and LGD shown in Table 6.1 and Table 6.2:

Table 6.1 – Jigsaw Junior attachment (%)

Rho\LGD	50%	75%	100%
5.0%	1.55	**2.18**	2.79
5.7%	**1.57**	**2.20**	**2.81**
6.0%	1.57	**2.21**	2.82
8.0%	1.60	**2.23**	2.82
10.0%	1.60	**2.20**	2.77
15.0%	1.50	**2.01**	2.47
20.0%	1.32	**1.69**	1.98
25.0%	1.07	**1.28**	1.39
30.0%	0.78	**0.82**	0.78

Table 6.2 - Jigsaw Senior attachment (%)

Rho\LGD	50%	75%	100%
5.0%	2.29	**3.72**	5.23
5.7%	**2.50**	**4.07**	**5.74**
6.0%	2.59	**4.22**	5.95
8.0%	3.21	**5.28**	7.49
10.0%	3.89	**6.43**	9.15
15.0%	5.81	**9.70**	13.96
20.0%	8.08	**13.61**	19.78
25.0%	10.70	**18.17**	26.63
30.0%	13.66	**23.35**	34.34

All attachment points of the junior and senior tranche are calculated to ensure the EL of the tranche is under 3.355% and 0.0055%, respectively, as shown in Table 5.3. The minimum and maximum EL over the 27 estimation were 3.350892% and 3.354988% for the junior tranche, and 0.00549516% and 0.00549999% for the senior tranche, respectively. Hence, all estimations yielded EL's close to the maximum EL allowed for the desired rating. However, the equity tranche both varies in tranche width and EL. The tranche width is linked to the attachment point of the junior tranche as the equity attaches at 0% and detaches when the junior tranche attaches. The EL is shown Table 6.3:

Table 6.3 - Jigsaw Equity, EL (%)

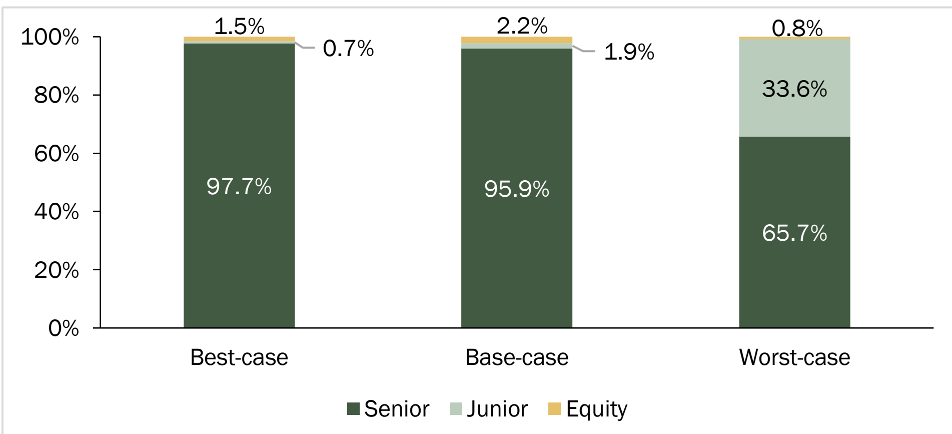
Rho\LGD	50%	75%	100%
5.0%	49.42	**52.05**	53.81
5.7%	**48.36**	**51.03**	**52.83**
6.0%	47.95	**50.65**	52.46
8.0%	45.85	**48.69**	50.59
10.0%	44.51	**47.45**	49.45
15.0%	42.88	**46.17**	48.57
20.0%	42.73	**46.63**	49.75
25.0%	43.71	**48.64**	52.60
30.0%	45.91	**52.38**	58.05

The EL of the equity tranche varies between 42.73% and 58.05%, around an average value of approximately 49%, well above the EL associated with the lowest credit rating in Moody's system, the Caa3 rating. This confirms that the Jigsaw Equity is a highly risky investment, which cannot obtain a credit rating and must be unrated. To illustrate the composition of the Jigsaw, the base-, best-, and worst-case scenarios are shown in Figure 6.1, and consists of the following three scenario's model inputs stated in Table 6.4:

Table 6.4 - Best, base, and worst case scenario model inputs

Scenario	Correlation	LGD
Best-case	5.0%	50%
Base-case	5.7%	75%
Worst-case	30.0%	100%

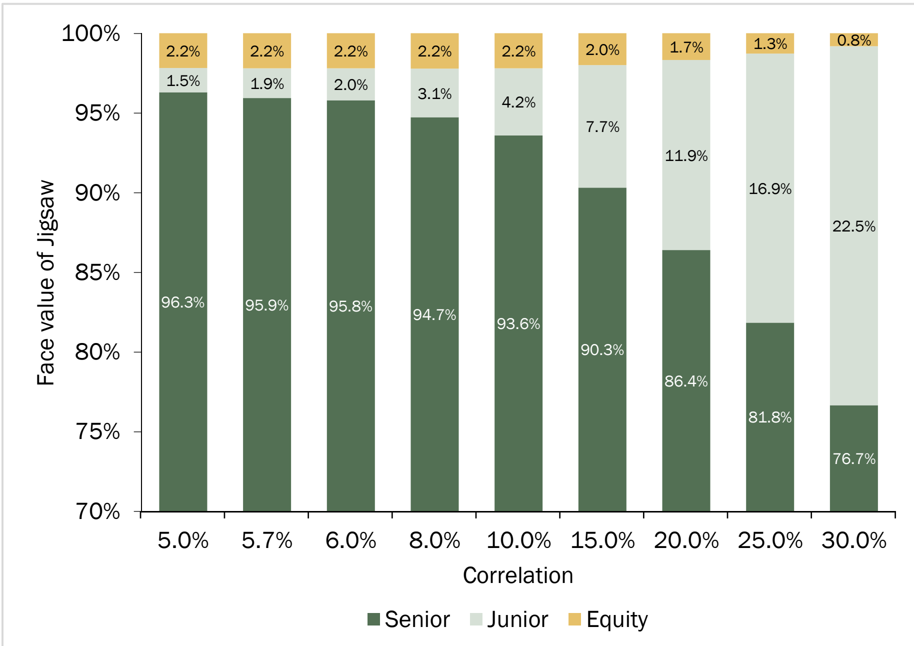
Figure 6.1 - Tranche distribution of the Jigsaw in the best-, base-, and worst-case scenarios under the single-factor model



Source: Authors creation

The difference between the best- and base-case scenario is small, since the only major difference in the parameters is a lower LGD in the best-case. The result is a wider senior tranche, and narrower junior and equity tranche, relative to the base case. The big difference is in the worst-case scenario, where the correlation is 30% and the LGD is 100%, and thus incorporates a lot more variation in defaults. Here, the Senior tranche is a lot narrower, and the Junior tranche a lot wider. This was expected since the defaults get more severe and happens in clusters. Interesting to note is the fact that the Equity tranche is narrowest in the worst-case scenario out of the three scenarios. This happens due to the Junior tranche being of a relative large size. Since the model is looking at EL (EL relative to the size of the tranche), the large size of the Junior tranche makes the losses relative small in percentage terms. This allows the Junior tranche to absorb a larger proportion of the expected defaults without exceeding the EL limit for the Baa3 rating. This is a major difference from only looking at the PD, where the severity of a default relative to the tranche size is not considered. To show the evolution of the tranche sizes, for different correlations Figure 6.2 has been created. The figure shows the effect of increased correlation in the underlying loan pool, when the LGD is fixed at the base-case, 75%. The equity tranche decreases, along with the senior tranche, while the junior tranche increases.

Figure 6.2– Tranche size for different correlations, LGD = 75%



Source: Authors creation

Using the findings from Section 5.3, both the corporate bond and CLO spreads are used in each of the three scenarios, such that the tranche sizes and corresponding interest rates are translated into a cumulative interest rate for the Jigsaw. The results can be seen in Table 6.5 for corporate bond spreads and in

Table 6.6 for CLO spreads.

Table 6.5 - Combined Jigsaw interest rate in three scenarios (corporate bond spread) (%)

Tranche	Best-case		Base-case		Worst-case	
	Weighting	**Interest rate**	**Weighting**	**Interest rate**	**Weighting**	**Interest rate**
Senior	97.71	1.13	95.93	1.13	65.66	1.13
Junior	0.74	2.46	1.87	2.46	33.56	2.46
Equity	1.55	11.84	2.20	11.84	0.78	11.84
Jigsaw total	**100**	**1.31**	**100**	**1.39**	**100**	**1.66**

Table 6.6 - Combined Jigsaw interest rate in three scenarios (CLO spread) (%)

Tranche	Best-case		Base-case		Worst-case	
	Weighting	**Interest rate**	**Weighting**	**Interest rate**	**Weighting**	**Interest rate**
Senior	97.71	2.03	95.93	2.03	65.66	2.03
Junior	0.74	5.00	1.87	5.00	33.56	5.00
Equity	1.55	23.22	2.20	23.22	0.78	23.22
Jigsaw total	**100**	**2.38**	**100**	**2.55**	**100**	**3.19**

The single-factor model estimates an interest rate from 1.31% to 1.66% from best- to worst-case scenario, when using corporate bond spreads. The estimated interest rates when using CLO spreads are higher and range from 2.38% to 3.19%. Thus, by implementing the higher spreads recently seen in CLOs, the interest rates are increased by just under a factor of two.

Looking at Figure 4.1, a margin of 0.5% seems reasonable when there are no capital requirements, as the margins started to drift away from 0.5% towards 1% as the crisis hit and capital requirements were gradually enforced. Implementing this margin on the borrowers total funding cost excluding establishment costs, gives the overview of investor return and borrower cost, shown in Table 6.7.

Table 6.7 - Investor return and borrower's total interest rate expense (%)

Scenario	Corporate spreads			CLO spreads		
	Best	Base	Worst	Best	Base	Worst
Total promised investor yield	1.31	1.39	1.66	2.38	2.55	3.19
Total borrower interest rate	1.80	1.89	2.16	2.88	3.05	3.69

6.2 ADVANCED SINGLE-FACTOR MODEL

Our stress test of the base case is the advanced single-factor model where the systematic and idiosyncratic risk factors are assumed to be Student t distributed. The simulation is identical to the one in Section 6.1, but the underlying distribution is changed to a Student t distribution with 10 DoF, as described in Section 5.2.5. The remaining setup is identical in terms of the Monte Carlo simulation and estimation of tranche thresholds, however, the loss in each simulation is calculated using Equation (5.14), which is the Student t extension to Equation (5.11). It is expected that the results in this section is highly affected by a more severe loss distribution, than the result obtained in Section 6.1.

The results are shown in Table 6.8 and Table 6.9, for the same range of correlations and LGD, as in the single-factor model. The R code used for the estimation is provided in Appendix G.

Table 6.8 – Jigsaw Junior attachment (%)

Rho\LGD	50%	75%	100%
5.0%	1.46	**1.89**	2.25
5.7%	**1.42**	**1.82**	**2.16**
6.0%	1.41	**1.80**	2.12
8.0%	1.29	**1.61**	1.86
10.0%	1.17	**1.42**	1.59
15.0%	0.87	**0.96**	0.97
20.0%	0.58	**0.54**	0.42
25.0%	0.31	**0.17**	0.00*
30.0%	0.08	**0.00***	0.00*

*The EL in the tranche cannot be maximized

Table 6.9 - Jigsaw Senior attachment (%)

Rho\LGD	50%	75%	100%
5.0%	8.72	**14.46**	20.70
5.7%	**9.03**	**15.00**	**21.50**
6.0%	9.17	**15.23**	21.85
8.0%	10.10	**16.81**	24.23
10.0%	11.05	**18.48**	26.71
15.0%	13.60	**22.89**	33.24
20.0%	16.33	**27.55**	40.14
25.0%	19.19	**32.38**	47.41
30.0%	22.16	**37.35**	54.93

For the senior tranche, the achieved EL rates are close to the wanted 0.55 bps with very small deviations, but the attachment points are very different from the base model. It is clear to see that the Student t model delivers a much riskier loss distribution, as the attachment points for all observations are a lot higher. In the base case, the attachment point is approximately 11 percentage points (pp) higher (15.00%-4.07%). The more stressed the loss distribution is from increased LGD and correlation, the more the difference increases, which is in accordance with the results displayed in Section 5.2.5 where the quantiles of the Gaussian and Student t loss distributions are compared. The tail dependence in the Student t distribution affects the loss distribution by increasing the mass in the tails. This has large impact on the senior tranche and, as seen in the attachment point tables, the width must be narrower. While the EL in the senior tranche is identical to under the Gaussian model, the width is narrowed down to cope with the higher risk. In this way, the credit rating can be preserved.

Due to a thinner senior tranche, the junior and equity tranche must be wider. Table 6.10 and Table 6.11 displays the EL obtained from the simulation.

Table 6.10 - Jigsaw Junior, EL (%)

rho\LGD	50%	75%	100%
5.0%	3.35	**3.35**	3.35
5.7%	**3.35**	**3.35**	**3.35**
6.0%	3.35	**3.35**	3.35
8.0%	3.35	**3.35**	3.35
10.0%	3.35	**3.35**	3.35
15.0%	3.35	**3.35**	3.35
20.0%	3.35	**3.35**	3.35
25.0%	3.35	**3.35**	3.25
30.0%	3.35	**3.13**	2.82

Table 6.11 - Jigsaw Equity, EL (%)

rho\LGD	50%	75%	100%
5.0%	37.39	**40.59**	43.03
5.7%	**37.61**	**40.91**	**43.47**
6.0%	37.71	**41.05**	43.67
8.0%	38.40	**42.08**	45.09
10.0%	39.16	**43.29**	46.74
15.0%	41.61	**47.14**	52.06
20.0%	45.15	**53.01**	61.06
25.0%	50.86	**64.63**	N/A*
30.0%	63.78	**N/A***	N/A*

*The EL in the junior tranche cannot be maximized, hence, no equity tranche

Most of the EL's in the junior tranche can be maximized to a Baa3 credit rating, but not in the most distressed case with a correlation of 30% and LGDs of 75% and 100% and a correlation 25% and LGD of 100%. In these three cases the EL can only be pushed to 3.13%, 2.82%, and 3.25%, respectively. In these cases, the junior tranche make up 37.35%, 54.93%, and 47.41% of the total Jigsaw, which is the rest of the Jigsaw after the senior tranche is created. This means that no equity tranche is created. It seems somewhat controversially that it is possible to create such a wide junior tranche, when the loan pool is modelled with a riskier loss distribution, but because the tranching is optimized under the assumption of EL, a wider tranche can resist higher losses as was seen in the single-factor model. Even though the EL is not at the limit of the Baa3 rating the junior tranche cannot obtain a different credit rating because the EL still above 1.98%, which is the threshold for the Baa2 rating.

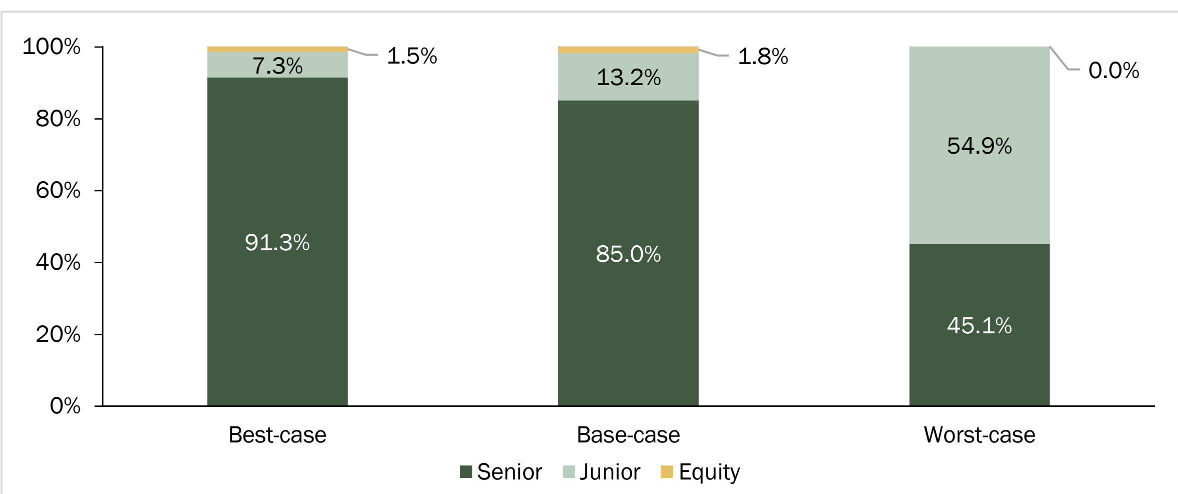
When the junior tranche becomes so wide it is not possible to maximize the EL, it is more efficient to create additional tranches with seniority below the junior tranche and change the target rating of the junior tranche, such that the EL in all tranches is maximized. This is the general approach proposed by (Brennan, Hein, & Poon, 2009, p. 894), which results in thin tranches where the senior tranche accounts for the majority, and the rest is divided into thin subordinated tranches, such that the total number of rated debt tranches is between two and five. This method will not be investigated further in this thesis, to keep the Jigsaw structure relatively simple, but might yield even better results, in terms of borrower yield and higher investor demand.

The difference in the junior attachment points from the base model, is not as large as what was the case for the senior tranche, measured in absolute terms. The width of the junior tranche has increased as a result of both increased detachment and decreased attachment points. The lower attachment points directly affect the equity tranche, which is thinner for all combinations of LGD and correlation. But the EL is both higher and lower for the different combinations of input parameters. For all correlations below 20%, the EL is lower in the advanced single-factor model. Whereas EL is strictly higher for the remaining range of correlations, and rapidly becomes extreme up until the point where no equity tranche is created. The reason should be found in the underlying Student t distribution, which, due to its symmetric properties, exhibit more extreme events in both the upper and lower tail, compared to the Gaussian, why low loss fractions are also seen more frequent. However, the loss distribution is not symmetric, and since the loss is capped at the lower bound 0, the extreme events in the lower tail do not skew the distribution to the same extent as the extreme events in the upper tail, as seen in Figure 5.1. In Figure 5.6, it can be seen that the Student t model is skewed further right compared to the Gaussian model. The extreme events in the upper tail of the Student t distribution results in proportional higher losses in the loss distribution, where the loss fraction becomes gradually more severe compared to the Gaussian loss distribution. This is also the general result, shown in Figure 5.5.

It can seem strange that the two models yield different rationales for the equity tranche, but when the width of the equity tranche is very thin and the equity tranche serves as the first loss piece (FLP), only a very small fraction of defaults is needed to effectively wipe out the whole equity tranche, which is the reason the EL increase rapidly when the width of the equity tranche decreases to a fraction of a percent.

As it was done in the base model, the best, base, and worst case simulations is displayed for the Student t model in Figure 6.3.

Figure 6.3 - Tranche distribution of the Jigsaw in the best-, base-, and worst-case scenarios, advanced single-factor model



Source: Authors creation

The difference between the base-case and the best-case is larger than under the Gaussian assumption, as the senior tranche decreases with 6.3pp (91.3%-85%) as oppose to 1.8pp (97.7%-95.9%) in the base model. In the worst-case scenario, the heavy-tails of the Student t loss distribution clearly has a high influence on the tranches. The equity tranche cannot be created, and therefore decreases from 1.8% to 0%, which is the same trend as the Gaussian model, where it decreased from 2.2% to 0.8%. The senior tranche decreases by 39.9pp (85%-45.1%), while the junior becomes the largest tranche and the FLP.

Again, using the findings from Section 5.3, the three scenarios can be translated into a weighted interest rate for the Jigsaw, for both corporate and CLO spreads. The result is displayed in Table 6.12 and Table 6.13.

Table 6.12 - Combined Jigsaw interest rate (corporate spread) (%)

Tranche	Best-case		Base-case		Worst-case	
	Weighting	**Interest rate**	**Weighting**	**Interest rate**	**Weighting**	**Interest rate**
Senior	91.28	1.13	85.00	1.13	45.07	1.13
Junior	7.26	2.46	13.18	2.46	54.93	2.46
Equity	1.46	11.84	1.82	11.84	0.00	11.84
Jigsaw total	**100**	**1.38**	**100**	**1.50**	**100**	**1.86**

Table 6.13 - Combined Jigsaw interest rate (CLO spread) (%)

Tranche	Best-case		Base-case		Worst-case	
	Weighting	**Interest rate**	**Weighting**	**Interest rate**	**Weighting**	**Interest rate**
Senior	91.28	2.03	85.00	2.03	45.07	2.03
Junior	7.26	5.00	13.18	5.00	54.93	5.00
Equity	1.46	23.22	1.82	23.22	0.00	23.22
Jigsaw total	**100**	**2.55**	**100**	**2.81**	**100**	**3.66**

When using corporate bond spreads, the estimated combined interest rates vary from 1.38% to 1.86% from the best- to worst-case scenario, respectively. While the interest rates with CLO spreads ranges from 2.55% to 3.66%. Again, a margin of 0.5% is added to the investor yield, to display the borrowers total interest rate.

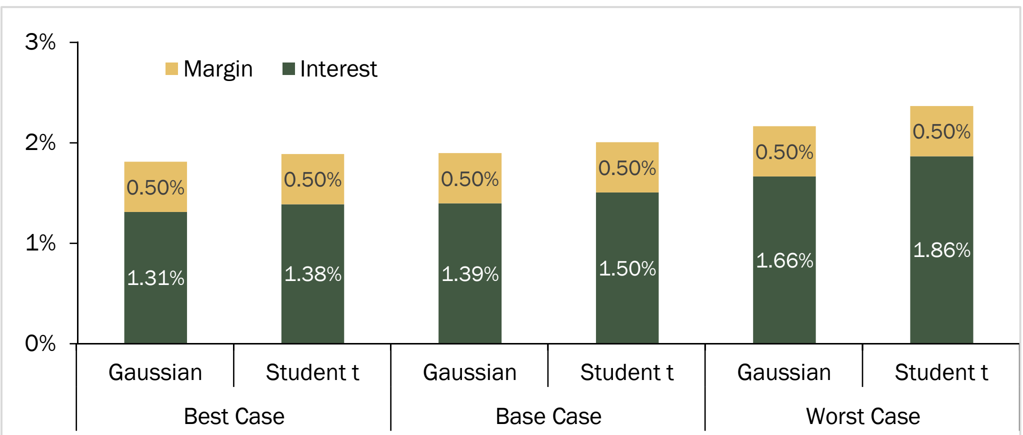
Table 6.14 - Investor return and borrower's total interest rate expense (%)

Scenario	Corporate Spreads			CLO Spreads		
	Best	Base	Worst	Best	Base	Worst
Total promised investor yield	1.38	1.50	1.86	2.55	2.81	3.66
Total borrower interest rate	1.88	2.00	2.36	3.05	3.31	4.16

6.3 COMPARISON OF INTEREST RATE RESULTS

To compare the results obtained under the two different models, the results from Section 6.1 and 6.2 is summed up in this section. The CLO credit spreads carried a premium over the corporate bond spreads. This translated into a 1-to-1 increase in interest rates for both investors and borrowers in the Jigsaw. Figure 6.4 displays the borrowers interest rates with corporate bond spreads.

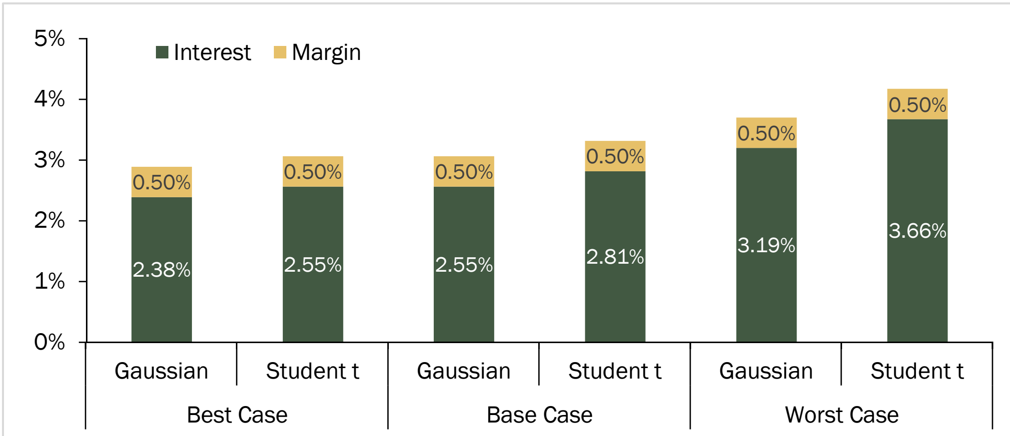
Figure 6.4 – Borrower interest rates in the base and advanced model (corporate spread)



The interest rate range from 1.81% to 2.36%, from the best case with a Gaussian model, to the worst case with a Student t model. Having in mind, that the two models delivered very different results in terms of tranche width, it is noticeable that the interest rate does not change drastically. The difference between the two models, increases when going from the best to worst case, with a "Student t premium" of 8 bps in the best case, 11 bps in the base case, and 20 bps in the worst case.

In Figure 6.5 the corresponding interest rates with CLO spreads is shown. The range is larger than with corporate bond spreads, since the lowest rate is now 2.88% and the highest 4.16%. The difference between the Gaussian and Student t model also increases, as the difference is now 17 bps, 26 bps, and 47 bps, for the best, base, and worst case, respectively.

Figure 6.5 – Borrower interest rates in base and advanced model (CLO spread)



Even though result of modelling with two different models are higher interest rates in a stress scenario, the general level and dispersion of the interest rate seems within a reasonable range, which proves that the results are robust.

Comparing the results to the current offerings, that are in the approximate range of 5% - 10% (MyBanker, 2017), the results show, that even in the worst-case scenario, the product should be able to offer lower interest rates on the mezzanine financing for all borrowers. Hence, the next step is the practical implementation.

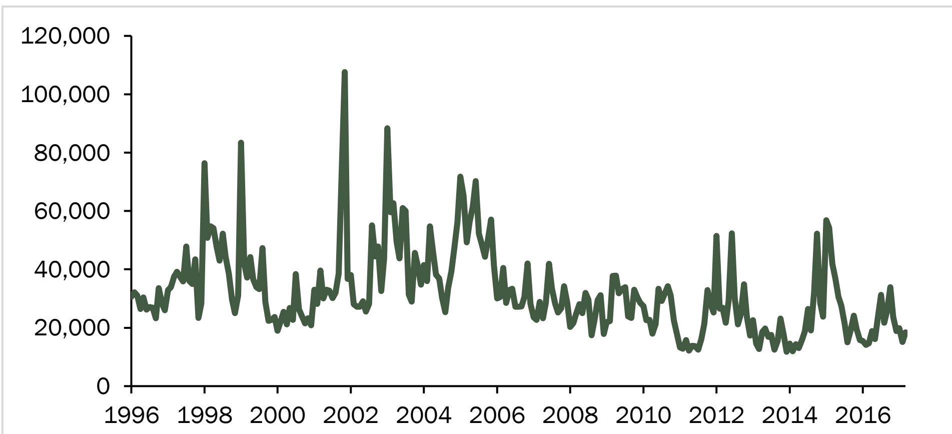
7 PRACTICAL IMPLEMENTATION

What works in theory might not work in practice, as implications such as market frictions, distribution, and conflicts of interest can be substantial barriers to overcome. However, a superior product will often prevail, as the market demand for the product will overcome the initial barriers. The market just needs to be aware of the new and better product's existence. Once the market is made aware, the market forces will establish a foundation for the product, as consumers want to get access to the product and companies seek profits by serving the demand. This section evaluates the criteria of a successful implementation of the Jigsaw loan in the Danish mortgage market, the implications that follows, and possible solutions.

7.1 ACHIEVING VOLUME AND DIVERSIFICATION

To obtain the interest rates, which the models have yielded, the Jigsaw needs a well-diversified pool of loans and a high volume of new loans coming in to keep the issuance of new bonds liquid and competitively priced. In Figure 7.1 the monthly amount of loans granted by the Danish mortgage banks is shown.

Figure 7.1 - Monthly loans granted by Danish mortgage banks (# of loans)



Source: The Association of Danish Mortgage Banks

The average monthly number of loans granted is approximately 32,000, and the minimum and maximum was approximately 12,000 and 108,000, respectively. From this, it is fair to assume, that with full exposure to the Danish mortgage market, the model assumptions on large number of loans in the portfolio is easily achieved, as this approximation is close to being achieved already at 10,000 loans. During a year, the Jigsaw could potentially collect over 350,000 loans on average. The large number of loans will ensure that borrowers get access to cheap funding, but the volume of the mortgage banks is needed to ensure this.

There are a few ways, of which the customers from the mortgage banks can be accessed. One approach is to set up a company that negotiates partnership agreements with the mortgage banks, so that when a borrower has achieved a credit approval in the mortgage bank, the contact to the company is facilitated and the mezzanine financing is arranged. Another approach is to set up the service in-house as a branch of the mortgage bank, enabling them to facilitate the entire 95% financing. The branch would then handle the setup of the SPV's and service the loans. The problem with the first two solutions is that the mortgage banks in general have outsourced all client interaction to the commercial banks, who are often also the majority shareholders in the mortgage banks. Hence, the first option is not valid unless the commercial banks are willing to refer clients to the company facilitating the Jigsaw loans, instead of granting a mezzanine loan themselves, as they currently do. The second option requires that the mortgage banks insource credit assessment and client communication. This would be expensive for the mortgage banks, as the commercial banks normally have a much better overview of a borrower's economy and credit quality, and a large restructuring of the mortgage banks' organizations would be required to adjust to a business model with a lot more client interaction. A more realistic approach is to create partnerships with commercial banks, which will facilitate the credit assessment, mortgage bank communication, and Jigsaw financing. The servicing of the Jigsaw operation would either be separated in a remote company that can service Jigsaw financing for several commercial banks, or be done by the commercial banks themselves.

Due to the nature of the Jigsaw, it is not something that one can build from the bottom up and slowly expand. The volume needs to be there right away for it to function, and under these circumstances, commercial bank partnerships is the most convenient solution to obtain the required volume. The main obstruction is the cannibalization of the existing product offerings from the commercial banks. These will be cannibalized by the Jigsaw financing, and the banks would lose the interest income from the mezzanine loans they are currently providing.

7.2 THE CANNIBALIZATION OF THE EXISTING MEZZANINE LOANS

Since the most probable method of obtaining the required volume is through cooperating with the commercial banks, the cannibalization of the current mezzanine housing loans that the banks grant must be considered. The main argument that speaks for the commercial banks' cooperation and willingness to allow this cannibalization, is the fact that the Jigsaw removes the loans from the bank's balance sheet. The capital requirements have been gradually increasing, and there are talks that the capital requirements for housing loans should increase even further (The Association of Danish Mortgage Banks (b), 2017). Therefore, providing mezzanine financing of housing is expensive in terms of regulatory capital for the banks, and this is a strong motivation for entering a partnership on Jigsaw financing. The banks would still be able to charge establishment fees on the loan, but they would free up capital, lower their risk weighted assets, and reduce the risk exposure in their loan portfolio. The latter is especially relevant for smaller local banks that have a smaller and less diversified portfolio of housing loans and hence, have a higher risk exposure. These banks would, to a large extent, benefit from the ability to shift away some of their loan exposure to investors. Therefore, the case of entering bank partnerships, where the banks would facilitate the contact and distribution of the loans, and a separate company would service the loans is not improbable, even though it cannibalizes the commercial banks' current offerings. This approach is very like the one used on the American mortgage market. The banks assess the customer who wants the mezzanine financing, and compose an overall credit evaluation. However, the approaches are different in terms of the funding, where the Jigsaw loans are funded directly from the SPV through traded bonds and not by the commercial bank itself, which then sells the loan to the SPV, as it is the case in the American market.

7.3 REACHING THE BORROWERS

Another reason collaborating with the commercial banks would be beneficial, is in terms of achieving market penetration. Most borrowers have complete trust in their bank when it comes to the financial aspects of their life. If faced with a decision of whether to choose a new product that might seem better, but from a new unknown company, many would hesitate and still choose a more expensive product from their bank whom they are familiar with and trust. This is shown in a study in the Journal of Product & Brand Loyalty that finds that brand loyalty has a high impact on risk aversion (Matzler, Grabner-Kräuter, & Bidmon, 2008), and another study that showed the customers in the Danish banks are very loyal and rarely change despite being able to get cheaper products (FDB, 2011). The large risk aversion, especially in relation to personal finance, is a very large entry barrier in the financial market. In addition, borrowers generally also seek advice from their bank on matters of personal finance and trust their bank to counsel in the borrower's best interest. However, the bank will always look after its own interests first, and might directly advice against the new and better product if it was done independently from the existing banks. This is seen in investment counselling, which is often received at the borrowers own bank, and especially in Denmark, the idea that banks always serve their own interest first is exposed here. Financial advisers in banks will often only advise you to buy their own investment certificates, even though there exist a much cheaper alternative outside the banks own product range (Møller & Nielsen, 2009, p. 211).

Therefore, the strong market frictions that are present in financial intermediation in Denmark make it very difficult to set up a company completely independent from the banks and still achieve a high market penetration, despite having a stronger product.

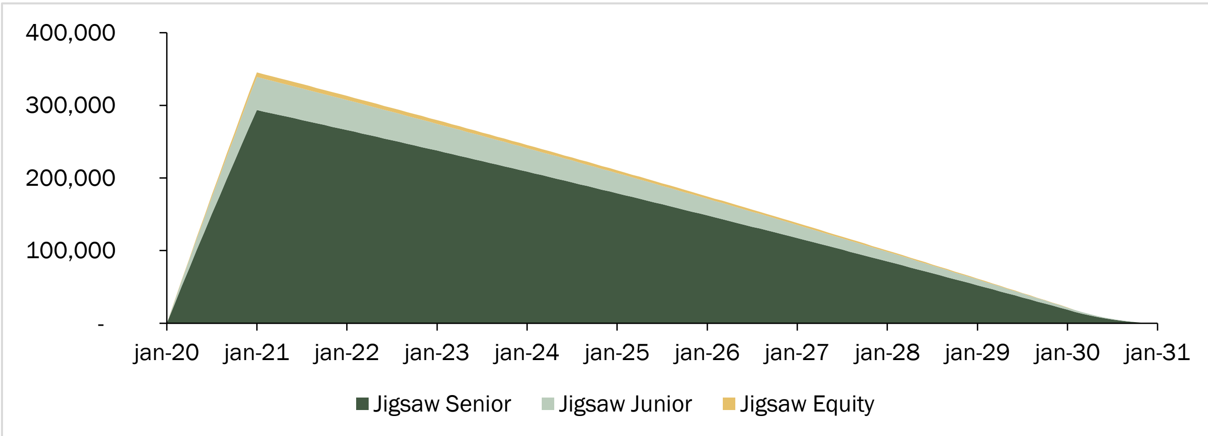
7.4 ISSUANCE AND FUNDING OF THE LOANS

The Jigsaw financing is done through an SPV that has no cash holdings or activities other than to combine individual mezzanine mortgage loans into RMBSs (the Jigsaws), tranching it, and selling bonds with the tranches as collateral and real estate as the underlying collateral. Therefore, when a borrower needs the money to fund a home purchase, the SPV must raise the funds at that time. The SPV cannot raise funds prior to granting loans to borrowers, as it cannot pay the interest on the bonds sold. On an ongoing basis, the SPV will grant Jigsaw loans, structure them in the RMBS structure, and fund them in the three bond series that are open at the time. The nominal amount of funding in each tranche is calculated based on the cut-off points estimated from the rating requirements. Different funding and interest rate guaranties can be given by the borrower's bank at the borrower's expense.

To ensure liquidity after the loans have been funded, the bond series funding the Jigsaw loans will be open one year so that it will accumulate an entire year's volume of mezzanine mortgage financing. This will ensure that investors will have good access to trading the bonds between issuance and maturity. This is like the practice in the existing Danish mortgage market, where, for instance, the 30-year fixed rate bond series are open for a period of three years and matures 30 years after the series is closed at the end of year three. Each year, the SPV will issue bonds in each of the three tranches, and the total number of loans in the Jigsaw that year will be the total number of Jigsaw loans issued during the given year. The bonds issued in the beginning of the opening period, stretching over one year, will initially have a maturity of 11 years, and gradually during the year the issued bonds' maturity will decrease down towards 10 years. At the end of the year, the Jigsaw series will be closed and a new bond series will be opened, maturing 11 years from that date and so forth. Hence, the Jigsaw loans granted at the beginning of the year will have a maturity of 10 years, but will be funded by bonds with an 11-year maturity. Instead of repaying the theoretical fraction of an 11-year bond periodically, the borrower will repay that of a 10-year bond periodically, effectively making it a 10-year bond. Therefore, the actual repayment each year will be higher than the theoretical repayment of a 11-year bond. Investors will be aware of this and expect higher repayments from the bonds purchased during the year. This method enables all bonds issued during the year to trade under the same ISIN code and ensures high liquidity in the bonds. For instance, Jigsaw loans granted during the year of 2020 will be funded by three bonds maturing in 2031: A Jigsaw Senior bond financing the Aaa tranche, a Jigsaw Junior bond financing the Baa3 tranche, and a Jigsaw Equity bond financing the equity tranche.

Figure 7.2 shows the theoretical development of a Jigsaw over the one year opening period of issuance and the following 10-year life of the Bond. It is assumed that there are no defaults, no prepayments, no change in the interest rate, 30,000 monthly Jigsaw loans with a principal of DKK 1, 2.5% annual interest rate, and 10-year maturity on the Jigsaw loans.

Figure 7.2 – Theoretical development in total assets in the Jigsaw over its lifespan (DKK)



Source: Authors creation

Prepayments will likely accelerate the decline in total assets of the Jigsaw, and it is clear, that the liquidity will deteriorate as time passes, as the volume traded will diminish since no new issues are carried out. Hence, liquidity issues can arise as maturity of the bonds approach, but the outstanding principal is also relatively small at this point and instead of paying liquidity premiums when prepaying the Jigsaw loan, borrowers could just let the loan mature. Each year, a new Jigsaw will be opened, whose assets will approximately follow the course shown in Figure 7.2. This means that five years after the Jigsaw is introduced, five different groups of bond series will be traded, with three bonds in each, and after 10 years, 10 groups of bonds will trade, which will be the steady state for the Jigsaw bonds. Now, a Jigsaw will mature each time a new one is opened. Investors can then choose whether to invest in the Jigsaw bonds with maturities ranging from 1 to 11 years, and whether to invest in the senior, junior or equity tranche for the given maturity.

Another way to fund the SPV, is by drawing on the experience from the American market, where they have a well-functioning secondary market for MBSs. Especially for the MBSs created by the government supported institutions, which can trade their MBSs on the previously defined To-Be-Announced market. Here the banks would grant the loans and sell them to the SPV afterwards. The fundamentals of this setup provide liquidity and security for the banks, knowing that the SPV will buy the loans, while increasing the demand for the MBSs from the investors, since the terms, on which the trades are made, are stable and known in advance. The results can be lower yields on the MBS compared to the current bank loans, which can be transferred to the borrowers.

The idea is to implement a well-defined secondary market for tranches created by the SPV, which functions as a forward market. Trades are agreed upon, before the actual underlying loans in the SPV are known and therefore the specifications and the risk of the pool is unknown. Instead, in total accordance with the American market, the buyer and seller (Investor and SPV) agrees on a few basic criteria, in which the seller agrees to deliver a product that fulfill these requirements. The criterions should be the coupon rate, maturity, price, principal, and whenever the trade is going to be settled. By making the settlement date about a month out in the future, the SPV can use the locked in coupon rate to inform the banks of the rate it is willing to buy the mezzanine loans at. The bank can then use the known rate to help borrowers to lock in interest rates on loans in advance, and before the loan is finalized.

The issues related to creating such a setup is mainly focused on the fact that an implicit government guarantee backs much of the American secondary market. The effect is that the American MBSs are easy to sell since the investors can consider them to be just as secure as American government bonds, and trade at yields comparable to government debt with a credit spread. Even though the SPV, in this case, can create Aaa rated senior tranches, the credit spread over risk free rates should express the riskier investment associated with these tranches, since the Jigsaw does not carry any form of government guarantee. The American government guarantee, makes it easier to ensure a well-functioning secondary market. The Jigsaw needs to solely depend on the foundation of an already stable and tested Danish mortgage market. In addition, there does not seem to be any reason to discriminate among the loans that the bank originates, like it is seen in USA with Jumbo loans, since the general underwriting standards and regulation in Denmark is well established.

8 DISCUSSION

8.1 MARKET COMPARISONS

When looking at the Danish mortgage market and ways to improve it, the comparison analysis was limited to the American mortgage market. This was done with the limitations of the paper in mind, since a thorough analysis of several other markets could not be achieved without exceeding the set boundaries for the thesis, while still maintaining thoroughness in the aspects relating to the Jigsaw financing. A deeper analysis of several other mortgage market could have given inspiration to further improvements in the Danish mortgage system, and could have improved the analysis to ensure even lower interest rates for borrowers and/or more favorable terms for borrowers. However, ideas from the American mortgage system, which are also practiced in other countries, have led to a significant improvement of the mezzanine financing costs of borrowers and good investment opportunities among investors seeking exposure towards the Danish real estate market.

8.2 FOCUS AREA OF IMPROVEMENT

The decision to focus on the mezzanine financing of real estate purchases in Denmark was driven by the low transparency and lag of market forces that is seen in the current offerings. It is possible that a deeper and more wide analysis of several markets could have led to other ways of improving the financing that the mortgage banks currently provide (up to 80% LTV), but the analysis of this part of the market showed that the level of transparency, security, and direct link between investors and borrowers did not leave room for much improvement. The most obvious improvement that could be treated was regulatory, e.g. the capital requirements from the Basel Committee. Dealing with capital requirements would change the nature and characteristic of this thesis into a more political and qualitative analysis, which was neither intended or desired. Nevertheless, this is an important friction in the market, which the mortgage banks are currently fighting with the Basel Committee over, with the main argument being, that the design of the Danish mortgage market is so secure that standardized capital requirements are unfit and unnecessary (The Association of Danish Mortgage Banks (b), 2017). Instead, the assessment was that far more significant and concrete improvements could be made when working with the mezzanine financing. Borrowers are facing very low levels of transparency when asking for mezzanine financing, as very scarce information is available on other similar loans and borrowers. This leads to poor bargaining power from the borrowers, who are forced to pay high interest rates. In addition, the "distance" between borrowers and investors is large, enabling the banks to charge high margins. For instance, consider the current interest rate on deposits in a bank that consumers "invest" in the bank for a return of approximately 0%, and the current mezzanine real estate loan offerings between 5% and 10% interest. The liquidity and risk of deposits and mezzanine real estate loans are different, but the margins that banks can earn on the stable level of deposits in the bank are large. Improving the mezzanine financing of Danish real estate required a more quantitative and financial analysis, and the value for the Danish economy and homeowners obtained by improving this part of real estate financing in Denmark would be significant. Therefore, this was chosen as the major focus of the thesis.

8.3 DESIGN OF THE JIGSAW PRODUCT

The design of the Jigsaw loan could be viewed as controversial, as the mechanisms used are the same that were used in the CDOs, which led to the global financial crisis. However, two major differences are present between the Jigsaw and the CDOs of the financial crisis. Firstly, the PD of the borrowers, to which the loans were granted prior to the financial crisis, was much higher than the borrowers eligible for the Jigsaw loans (Geidosch, 2013). Secondly, the CDOs were second degree tranches, meaning that from a loan pool of mortgages that have been tranched into rated securities, the riskiest tranches, many only approximately 1pp wide, were pooled together and re-tranched into new securities spanning most ratings, including large Aaa tranches (Hull & White, 2010, p. 56). The first problem arose from shifting away the entire risk exposure from the institutions granting the loans to investors. This meant that the originators did not care about the credit quality of the borrower, why the underwriting standards decreased. With the Jigsaw, only the mezzanine exposure is transferred to investors, the mortgage bank still has the exposure of the first 80% of the LTV, and hence cannot be sloppy on the credit assessments. The Jigsaw benefits from this, as the senior mortgage is a prerequisite for obtaining the Jigsaw financing. However, some agency problems can occur, as the banks facilitating the credit assessment will not bear the full financing risk of the property, as is the case now, but only the part in the senior mortgage as a function of their ownership of the mortgage banks. But as most of the risk stays with the originator after introducing the Jigsaw, the credit assessment should remain the same in the mortgage banks, and there is no reason to believe that any significant agency problems will be present when granting the Jigsaw loans.

In addition, the Jigsaw is not a second degree RMBS, but a first degree RMBS, that is based on loans tranched once. The dangers of creating second degree tranches is that if the model parameters are slightly off, there is a high risk that the risky first degree tranches, which the second-degree tranches consist of, will be completely wiped out, which leads to a complete default of the second degree RMBS. The Jigsaw is not exposed to this level of risk as it is a first degree RMBS, but still accumulates systematic risk in the junior tranches. Therefore, the Jigsaw's robustness is sensitivity tested in relation to the input parameters of the model, and with different models. This yielded consistent results, strengthening the conclusions from the models, and showing that the Jigsaw is not near the risk seen in the CDOs of the financial crisis. But that does not mean the Jigsaw is bulletproof, since the estimation is relying on the selected underlying model being the model that fits real world behaviour best, why model risk comprises a large uncertainty, discussed further in Section 8.4.2.

The choice of loan type for the Jigsaw is an annuity loan, which was chosen to lower the risk of the loan relative to an interest only loan from an investor perspective. Also, the fixed payments over the maturity is more suitable for the private market, where equal periodic payments are desired and it is often not possible to make the entire repayment at maturity of the loan. The combination of an annuity loan and 10-year maturity is chosen to match the 10-year interest only period available on the senior mortgage, so that the borrower can repay the Jigsaw in fixed payments while only paying interest on the rest of the mortgage financing. The interest-only option on mortgages, is already used by many individuals while repaying the mezzanine financing and recommended by financial advisers in banks. There is a possibility that the Jigsaw loan will have a lower interest rate than the senior mortgage, leading to borrowers rather wanting to repay the senior mortgage than the Jigsaw. The lower interest rate can occur as the senior mortgage normally has three times as long a maturity as the Jigsaw, and if the senior mortgage is an FRM, the longer maturity can lead to higher interest rates depending on the yield curve. However, the lower interest rate on the Jigsaw loan is only present when a large difference in maturity is present, and prolonging the Jigsaw would lead to its interest rate rising above the FRM rate. Hence, even though it will be counter-intuitive to repay the loan with the cheapest interest rate first, it is necessary to keep the interest rate low. Borrowers financing with ARMs or floating rate mortgages will not experience the Jigsaw as being the cheapest, as their mortgage is both safer and its interest rate is reset in shorter or similar intervals to the Jigsaw.

The equity tranche is often referred to as the first loss piece (FLP), since the tranche will incur a loss as soon as the first loss in the pool occurs. In combination with a low rating (sometimes no rating) and high risk, the FLP status makes many investors reluctant to invest, and the equity tranche is instead kept on the books of the issuer of the SPV (Brennan, Hein, & Poon, 2009, p. 909). If the Jigsaw was to be implemented, this might very well be the practical implementation, but for the sake of simplicity, the product was designed so that all tranches were sold off to investors. This also makes the product more flexible in terms of implementation, as no capital, in theory, is required in the company issuing the loans in order to provide the Jigsaw loan service. If the Equity tranche would suffer from low liquidity when implemented, two possible solutions are apparent. One solution is arranging that the banks facilitating the contact to the borrowers could buy the FLP. As the FLP is very narrow, not a lot of capital would be needed to fund this lending activity as compared to funding the entire mezzanine as they currently do. Another solution would be to widen the Jigsaw Junior, so that it incorporates the FLP, and only two tranches would exist in the Jigsaw. Hence, the liquidity problems that might occur with the FLP can be mitigated.

The Jigsaw will not cover investors for any loss on defaulting borrowers like the mortgage banks do, but it is a feature that could be implemented. However, If the SPV would insure the investors in such a way, a range of requirements, including Basel capital requirements, would be enforced on the SPV, which complicates the structure and practical implementation. Another option would be for the mortgage banks to issue a guarantee instead of the SPV doing it. However, the legal aspects of this might be an issue, since the funding through covered bonds prohibits leverage above 80% LTV, and the issuance of such guarantees might not be allowed or have other legal implications for the mortgage banks. In addition, the pricing though CLO spreads would no longer be appropriate, as the risk of the product changes drastically. Instead, pricing should be guided by the American structures, which have similar guarantees.

8.4 MODELLING

8.4.1 OMITTING REPAYMENTS AND PREPAYMENTS

As the Jigsaw bonds will be traded in the market, it is also possible for the borrowers to repay the Jigsaw loan before the loan matures. This has not been implemented in the modelling of the loans but it is a factor, which in practice lowers the effective maturity and hence, the risk and required return from investors. The probability that no loans in a Jigsaw are prepaid before maturity is very low, since residential properties are the underlying collateral. The incentive of the borrowers to prepay is affected by many different factors, both on the individual loan level but also on an overall macroeconomic level. Prepayment modelling can be rather comprehensive, because it is very sensitive to interest rate risk, as the general interest rate level affects the market value of the outstanding Jigsaw loans. Since the Jigsaw does not have an embedded option to redeem the debt at par value, a decrease in the general interest rate level would increase the market value of the Jigsaw, thus an instant redemption at market value requires a higher principal than the outstanding nominal debt, adjusted for amortizing. (Fermanian, 2010) This reduces the incentive of early redemption, when interest rates fall. On the other hand, increasing interest rates lowers the market value of debt and can motivate early redemption.

If the option to redeem the debt at par value is incorporated in the terms of the Jigsaw, option pricing would be needed to assess the market value of option to repay at par. If it is assumed that the option can be used every quarter, then the Jigsaw has 39 options embedded (One every quarter during the 10-year loan life in the Jigsaw minus the one at maturity of the loan), and each single option needs to be calculated before the total Jigsaw value and yield can be determined. This is a complicated process and topic, which would require a thesis on its own and is therefore far beyond the scope of this thesis. Further, the cash flow modelling with prepayments becomes even more difficult due to human behaviour. It is far from certain that borrowers exercise prepayment options optimally, since the borrowers might not possess required knowledge about the financial markets to know when it is favourable to prepay. Other times, the prepayment simply happens because the collateralized property is sold and the proceeds are used to redeem the debt. All these factors make the overall modelling of prepayments complicated. The bias might not be large, but it adds to the already conservative modelling done, and increases how comfortable one can be with the results of the modelling.

In practice, some simplifications are used to model prepayment behaviour. Different methods are used, in which different levels of complexity is incorporated. The simplest is the Conditional Prepayment Rate (CPR) model, which assumes that a fraction of the remaining principal is prepaid every period. The fraction that prepays is thus a constant throughout the pools lifetime. A more complicated prepayment assessment method is the Public Securities Association prepayment benchmark (PSA). The prepayment rate in the PSA is not a constant, but starts out low in the beginning of the loan life and increases up to a maximum, which is reached after 30 months (Fabozzi & Mann, 2010, pp. 208-212). Even using one of these models, a high amount of computer power is needed since the modelling is done on an annual level, and thus lack of computer power is a barrier that can be difficult to deal with. Modelling prepayments would increase the computing power needed substantially and would still incorporate high levels of uncertainty, and is therefore omitted from the modelling.

8.4.2 MODEL RISK

Another risk related to the modelling procedure is the model risk, which is considered rather high in this analysis. Model risk is defined as the risk linked to using unsuitable models to measure risk (McNeil, Frey, & Embrechts, 2005, p. 3). Since the whole estimation is based upon the chosen model, and the results are directly influenced by it, the practical usability and implementation is depending on the model describing the real-world dynamics properly. Models can rarely provide a perfect rendering of how the real world works, simply due to the world's complexity, and often the analyst must rely on a set of basic assumptions to ensure a proper model fit. Therefore, it can never be totally avoided that the model to some extent relies on assumptions and simplifications that can seem far from the behavior observed on the financial markets. The model choice should therefore be chosen so that it minimizes the gap between the theoretical sides of the model and the practical results observed in the real world, to maximize the validity of the results.

The results presented in this thesis is based on the model that have been considered as the industry standard when dealing with credit risk in portfolios. The single-factor Gaussian model developed by Vasicek had a huge impact on the industry, since it showed how to model a complex financial instrument by implying a few basic assumptions about the underlying loans in a credit portfolio. The model became so common that it was even used to form Basel capital requirements. In the model only two parameters are exogenous, PD and default correlation. The simple framework is used to get an idea of the credit risk in a portfolio of loans, which is most often very complicated in terms of composition, but the framework assume homogeneity to eliminate some of the complexity while still being able to assess the risk. The most important simplification is the estimation of the default dependence (default correlation) between the underlying identities. As the model name suggest, a Gaussian copula models the dependence. As mentioned in Section 5.2.5, the Gaussian copula is notorious known to underestimate the default dependence since the distribution does not have enough mass in its tails. Several other copulas, that could fit under the Vasicek framework, could be used to avoid underestimating the risk and to test the Gaussian copula base model. The additional copulas do not question the actual model framework but only the underlying default dependence. Hence, it is not tested whether other frameworks are better at pricing the credit risk. With the problems in mind that arose during the financial crisis, the formula has even been known as “The Formula That Killed Wall Street” (MacKenzie & Spears, 2012). Even with this reputation, the formula is still a foundation of the advanced approach[5] under Basel III (Capgemini, 2014) when banks assess their risk weighted asset exposures in retail banking.

The method used in thesis to mitigate the critique of the Vasicek formula, were to incorporate the Student t copula instead of the Gaussian, but in the same framework. As the results show, the Student t copula clearly stress tests the Jigsaw, since the senior tranche decrease in size to cope with the additional risk. The Student t copula therefore exhibits the wanted increased tail risk, but these extreme events are also present in the lower end of the loss distribution, with the result that events where few or no defaults happen likewise become more frequent. This is the result of using a symmetric distribution (Gaussian and Student t), which is the most frequent approach in literature, as opposed to the Archimedean copulas (Crook & Moreira, 2011). The Archimedean copulas allow for modelling either lower or higher tail dependence, conditional on the chosen copulas. This thesis mentions Clayton and Gumbel as examples of Archimedean copulas, which in theory could provide an excellent fit to the underlying data. But the principles behind the two presented Archimedean copulas are much more complicated to implement in the current setting, and it was assessed that the pros could not offset the more complicated implementation, why the Student t was the obvious choice.

The general issue with using a copula to simplify the dependence structure of financial data is that different copulas provide different dependence structures that match different data structures. As the simplest approach, the Gaussian copula, is easy to understand and to implement, but lack tail risk. The Student t copula is close to the Gaussian copula but exhibits tail risk, but it involves choosing the DoF, which is a subjective decision. In general, the copula selection can be done by estimating the fit of selected copulas on data. This can be optimized by taking a sample of the underlying data and fitting several copulas to the data whilst minimizing the error term. Then by using a Goodness-of-Fit (GoF) measure such as the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC), the copula that shows the highest power in predicting the data should be chosen. In relation to this approach, the difficulty of choosing the DoF parameter in the Student t can be made objective by using a parameter estimation technique. A maximum likelihood process can calculate the DoF which ensure that the Student t distribution provides the best fit to underlying data, and likewise maximum likelihood can estimate the exogenous parameters in other copulas. To implement this in this thesis, would require access to data that provides a description of the underlying

[5] Basel III distinguish between standard and advanced approach, where the latter is used only for large banks. The Vesical formula is used in the internal ratings-based (IRB) risk-based capital formula.

default behavior. The data should be with such good quality that it neither over- or underestimate the default distribution in Jigsaw loans. Only 16 years of data is publicly available on mortgage defaults and number of mortgage loans, and hence, not nearly enough data points are available to form a distribution that can be used to fit the copulas on, and therefore, the copula and DoF is subjectively chosen. In addition, data dating far back might not be representable of the future, and even though it could form at default distribution, the distribution could be biased.

Other approaches to quantifying the credit risk of portfolio, which in this instance is defined as a bank's holding of residential mezzanine loans, is to model default behavior on loan level. Credit rating agencies use this method since it can incorporate the diversity across loans and further combine prepayment behavior, and thus obtain a more robust calculation. Practically this is a Monte Carlo estimation that would require extensive computer resources and an estimation of a range of credit profiles, way beyond the scope of this thesis.

8.4.3 MODEL INPUTS

The PD is assumed as an average over the 10-year maturity of the loan, which is a fair assumption. However, one could think that the annual PD is connected to the state in prior years. The rationale is that a default is more unlikely to happen the longer a loan has "survived". Implementing dependence in the PD would require modelling on an annual basis, which would intensify the computing power needed, but the timing of defaults is relevant in relation to EAD, which could also be implemented when modelling on an annual basis. This would most likely bring value to the analysis, if a true dependence can be estimated but would also be very resource intensive.

Since borrowers generally have a strong connection to their home and do not track the value of their home on a day to day basis, many will continue to service their debt even though the value of the property is lower than their debt. In addition, borrowers need to be declared personally bankrupt to be relieved from the liabilities in the property. Hence, the drivers of defaults in Denmark is not necessarily the property prices, but borrowers becoming unable to service their debt. This mainly happens in connection with divorce or unemployment. Hence, if PD were to be modelled instead of observed by historical figures, housing prices would be an important factor, but even more important factors would be divorces and risks of unemployment. Estimating a model that incorporates these three aspects in relation to defaults would likely be one of the better predictors of defaults in Denmark, which could improve the analysis.

A side from the whole estimation process and model design, the underlying default correlation parameter is assumed to be constant in the Vasicek model, both between individual loans and over time. It is simply too comprehensive to estimate and use individual correlations for all loan pairs, since the correlation matrix is increasing rapidly with the number of loans, n, $(\#\ of\ correlations = n(n-1)/2)$. This encourages an approach with uniform correlations, or by using correlations calculated on higher level terms than on individual loan level. However, in practice, loans in Copenhagen correlate much more with each other than with loans in an entire different part of the country, which is why a constant correlation is a simplification. However, macroeconomic factors, such as interest rates and recession, certainly affect the entire portfolio of loans, but also argues for a non-constant default correlation over time.

Recovery rates across loans are also assumed to be a constant, which only is tested for robustness by using several values in the estimation process. But just as it was underlined with default correlation, the connection between PD and recovery rates also carries some characteristics that makes them interrelated, such that in stressed markets with increasing default probabilities recovery rates tend to decrease, which in turn increases the LGD (Stein, Das, Ding, & Chinchalkar, 2010). This was particularly the case under the financial crisis. A potential approach is to model the recovery rate as stochastic decreasing function of the realized default rate, this has been suggested by (Hull & White, 2010), where the result was increased loss severity.

The nature of annuity loans and their gradual repayment is also impacting the LGD by lowering the investors EAD. If a default occurs, it is not the total principal that is exposed, but only the remaining principal at the time of default. This should be reflected in EAD but is not modelled directly. Instead, it is done indirectly through the LGD, where the base case is 75%. The true LGD might be 100% but then the average EAD being 75% due to repayments. Including EAD in the analysis could potentially improve the models' reliability. Like with the PD, this could be done through modelling defaults on an annual basis instead of an average over the 10 years. Each year, the outstanding debt would be calculated and used for the loss calculations of the defaulting loans. However, this would increase the resources required in the modelling by at least a factor of 10. Currently, each estimation takes approximately one hour to compute with the available computing capacity, and therefore, the average EAD have been included implicit in the LGD approximation.

In addition, the LGD is an unprecise estimate, as Haldrup et al. estimated the property values based on the taxable valuation made by the Danish government, which is proven to vary a lot from the actual transaction prices observed in the market. Properties located in attractive areas generally have a taxable valuation below the market price, whereas properties in remote and unattractive areas have taxable valuations higher than the market price. There is a high concentration of properties from remote and relatively unattractive locations in the sample of defaults, and hence, the LGD based on the public taxable valuation is likely to have a positive bias, as the higher valuation lowers the estimated LGD. An improvement of this would be to have the actual price that the properties were acquired for at auctions, but this data is not publicly available, which is also why it was not used in the analysis by Haldrup et al., and gathering of a representable data set would not be realistic within the time frame of the thesis. One would need to get this data from the mortgage banks, who are probably unwilling to provide this kind of sensitive information, and compare it with the total number of rights registered in the Land Book. Even with the transaction data from the mortgage banks, this would be a long and extensive process, but a more precise estimate of the LGD would be achieved. However, this practice is unlikely to pay off in terms of value to the analysis, as the LGD can never be more than 100%. The impact on the results of 100% LGD in the models is not major, and could simply be implemented as a default level to mitigate the uncertainty of this variable.

The type of mortgage that the Jigsaw is subordinated to also influences the risk of the Jigsaw. A FRM will provide equity protection for the borrower in case of rising interest rates, by lowering the outstanding debt, thus decreasing the LGD relative to a mortgage that could only be repaid at par. This protection is not present in the ARMs to the same extent, as the short maturity makes the fluctuations in market value relatively small. In addition, the FRM protects the borrower from interest rate risk, whereas a borrower with an ARM might default if the interest rate increased a lot. A restriction on which type of mortgage that the Jigsaw can be subordinated to could therefore reduce the risk, but also post heavy limitations on who will be able to get the Jigsaw loan, as ARM's are highly popular in Denmark.

Omitting transaction costs from the analysis is biasing the pricing of the Jigsaw upwards and the yields downwards. Transaction costs can have a large impact on the total financing costs depending on the magnitude relative to the loan size. Therefore, the conclusions must factor this into account when assessing the attractiveness of the Jigsaw. However, the current offerings are also affected by transaction costs, and a comparison on the interest rate prior to transaction costs will therefore be unbiased with respect to transaction costs, assuming they remain the same in new products.

Modelling in the bankruptcy costs is an option, but the level of uncertainty in such an estimate speaks for omitting it. Instead, the bankruptcy costs are incorporated as part of the total margin charged. This method does not yield a precise estimate, but is a rough estimate that makes the comparison reasonable. A large part of bankruptcy costs originates when a property is overtaken, as it fails to sell on an auction. Here the maintenance of the property brings large costs to the mortgage banks. Generally, it will be the mortgage bank which must deal with this issue, as they will be the only ones interested in acquiring the property to protect their claims, since they, have the largest and most senior claim on the property. The Jigsaw will be better served with accepting the 100% loss in these cases, and getting what is possible in the other cases.

The margin is based on what historical margins have been in mortgage loans before stricter capital requirements were put in place. These historical margins would include bankruptcy costs as part of the costs of administering the loans, but would also incorporate costs of covering defaulting loans, and hence, the margin estimate is conservative as the Jigsaw does not cover defaults.

8.5 PRICING

Using corporate bond spreads for pricing includes relatively large biases when dealing with CLO structures, which were mitigated by using CLO spreads. However, all biases might not be mitigated by using the CLO spreads. The CLO spreads mitigate the biases caused by ignoring the relation between systematic and idiosyncratic risk, but how the CLO spreads are estimated might lead to another bias. The CLO spreads are based on CLO products that consist of claims from many different sectors and with different seniorities. Therefore, the CLO spreads might be better in terms of estimating the Jigsaw yield, but CLOs are more diversified products, whereas the Jigsaw consist of very homogeneous loans in the same sector. Hence, a premium might be priced in the Jigsaw by investors, since it possesses less diversification.

9 CONCLUSION

The aim of this thesis was to improve the Danish mortgage market with benefits to both borrowers and investors. The Danish mortgage market has generally been accepted as the "go to" market, when other markets need improvement. The system builds on the match funding principle, where mortgage loans are financed by large series of "matching" covered bonds, which mirror the characteristics of the mortgages they fund and are traded on an exchange. It is a pass-through structure, where interest rates and repayments on the mortgages are passed directly to the investors in the matching bonds. This allows borrowers to track the price and interest rates of mortgages on a day to day basis, and allows for repayment by buying back the bonds funding one's mortgage at market value. Callable bonds fund the FRMs, granting borrowers the option to prepay at par, and ARMs are protected from large interest rate increases and refinancing risk by interest rate and failed auction triggers. The mortgage banks facilitating the mortgage issuance and funding are liable towards the investors in the covered bonds and must cover any default on the underlying mortgages. The mortgage banks can seize the underlying property of the mortgages to protect them from losses in case of the mortgage defaulting. Altogether, this makes the market transparent, secure, and liquid, providing Danes with access to cheap long-term financing.

The American mortgage market is structured in a different way than the Danish, which is useful when looking for ways to improve the Danish market. The American mortgages are funded through MBSs instead of covered bonds, and government backed agencies provide security and liquidity. The agencies securitize individual mortgages bought from mortgage originators and compile them in large numbers into MBSs, which are structured in tranches and offered to investors through specialized forward markets. The tranche structure enables the same security to be divided into parts with different risk profiles, which can be sold separately, reaching a broader investor segment.

On many parameters, the Danish market is superior to its American counterpart, and has historically proven to be more stable through financial distress. However, the Danish mortgage system is limited to financing 80% of the property, and the remaining 15% up until the allowed 95% CLTV must be financed through the borrower's bank, in an unstructured loan where the terms are set through negotiations and no direct use is made of the debt markets. Covered bonds cannot be used to finance more than 80%, but financing the mezzanine bank loans in a securitized structure similar to the American MBSs, while including key aspects from the Danish market, has proven as a great improvement to the system.

Creating the Jigsaw – the missing piece – to handle the mezzanine financing of properties in Denmark through an SPV, helped complete the Danish mortgage market. The Jigsaw uses the tranche structure seen in the American MBSs, in combination with the match funding principle and direct link between investors and borrowers, which characterizes the existing Danish mortgage market. The Jigsaw collects loans in large pools, which is a method used both in the Danish and American market that creates diversification and ensures low interest rates, and afterwards structure them in tranches. Creating a senior, junior, and equity tranche in the Jigsaw ensures that the expected interest rate on the Jigsaw can be optimized to benefit borrowers and be attractive to a wider range of investors. The pass-through structure benefits borrowers, not having to pay higher interest rates than what the end investors require for providing the loans, except for the margins charged for servicing the Jigsaw. Hence, the Jigsaw will utilize the transparent structure from the Danish mortgage market and optimize it with a more advanced tranche structure from American MBSs.

Of the two Vasicek models used to model the Jigsaw product, the Student t copula was the one assessed to fit the real world the best and the one yielding the most conservative estimates. When using CLO credit spreads appropriate for structured and tranched products, the Student t copula yielded interest rates in the range of 2.88% to 4.16%, including a 0.5% margin to the servicer, over the range of probable parameter values of correlation and LGD. The base case is an all-in interest rate of 3.31%, which is a confident estimate for the true interest rate the Jigsaw will be priced at. Throughout the modelling, all uncertainties resulted in a decision to the conservative side, including estimates of PD, prepayment modelling, and credit spread uncertainty. However, the CLO spreads used to price the Jigsaw, are derived from more diversified credit portfolios, but this bias should by far be mitigated by the other conservative estimates.

However, the results obtained are based on an “all else equal” state of the market. As discussed, the risk transfer could lead to agency problems in the banks facilitating the credit assessment. The banks will no longer be exposed to the riskiest financing of the properties, and could begin granting slightly riskier loans even though they still retain the 80% LTV exposure. This is a risk that could compromise the results to a degree that is difficult to estimate. Therefore, there needs to be strict control with the credit assessments so they will remain the same and the Jigsaw will not be compromised.

The implementation of the Jigsaw is most conveniently done through partnerships with the existing commercial banks that are already facilitating the CLTV of 95%, including the issuance of the senior mortgage of 80% LTV. This ensures rapid and easy market penetration to borrowers, and allows the mortgage banks to offload the current mezzanine loans from their balance sheets, lowering their capital requirements, while still being able to charge establishment fees on the loans. Borrowers will still get the mezzanine financing through their bank as part of the total financing package the bank facilitate, and by keeping the borrower’s experience practically unchanged, it will be an easy decision for the borrowers to choose the Jigsaw financing at 3.31% as compared to the current offerings between 5-10%. The expectation is, that once the Jigsaw is implemented, the current residential mezzanine financing will vanish for borrowers eligible for a senior mortgage from the mortgage banks, and only exist for borrowers with poor credit assessments not eligible for a Jigsaw loan.

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11 APPENDIX

APPENDIX A AN OVERVIEW OF BOND PAYMENTS

1 year rate	**1%**
2 year rate	2%
3 year rate	3%
maturity	20
Principal	100.00
Payment	kr. -6.72

t	**0**	**1**	**2**	**3**
1-year bond	3.84	-3.88		
2-year bond	3.88	-0.08	-3.95	
3-year bond	92.29	-2.77	-2.77	-95.06
refinancing				88.33
	100.00	-6.72	-6.72	-6.72

APPENDIX B OVERVIEW OF FUNDING PRINCIPLES

	General principle	Specific principle
Payments definition	Payment may include margins	Payments excluding margins
Interest risk	Risk limit 1%[1] +2%[2] of OC: +/-100bp parallel shift Risk limit 5%[1]+10%[2] of OC: +/-100bp twist and +/-250bp shift 50% offset of EUR interest rate risk	Risk limit 1% of OC: +/-100bp parallel shift and twist No offset of EUR interest rate risk
Exchange rate risk	Risk limit 10% of OC: +/-10% shift in EU currencies +/-50% shift in other currencies	Risk limit 0.1% of OC Currency indicator II
Option risk	Risk limit 0.5%[1]+1%[2] of OC: +/-100bp shift in volatility (vega)	Perfect hedge required
Liquidity risk	Deficits in interest payments may not exceed OC within 12M NPV surplus of all future payments	Deficits in total payments limited to: - 25% of OC in year 1-3 - 50% of OC in year 4-10 - 100% of OC from year 11

1. Percentage of the capital adequacy requirement, 2. Percentage of the additional excess cover for mortgage banks, Note: OC = overcollateralisation
Source: The Danish FSA, Danske Bank Markets

The balance principle is enforced by the Danish FSA. If a mortgage bank does not pass the tests, the FSA must be informed immediately. In addition, mortgage banks must report their market risk exposure to the FSA on a quarterly basis.

Interest rate risk is tested in scenarios of both yield curve shifts and yield curve twists. The diversity of scenarios implies that duration matching of a loan and funding portfolio will not be sufficient to pass the test.

Currency risk is tested in scenarios of shifts in the currencies in which the bonds have been issued to comply with the general principle.

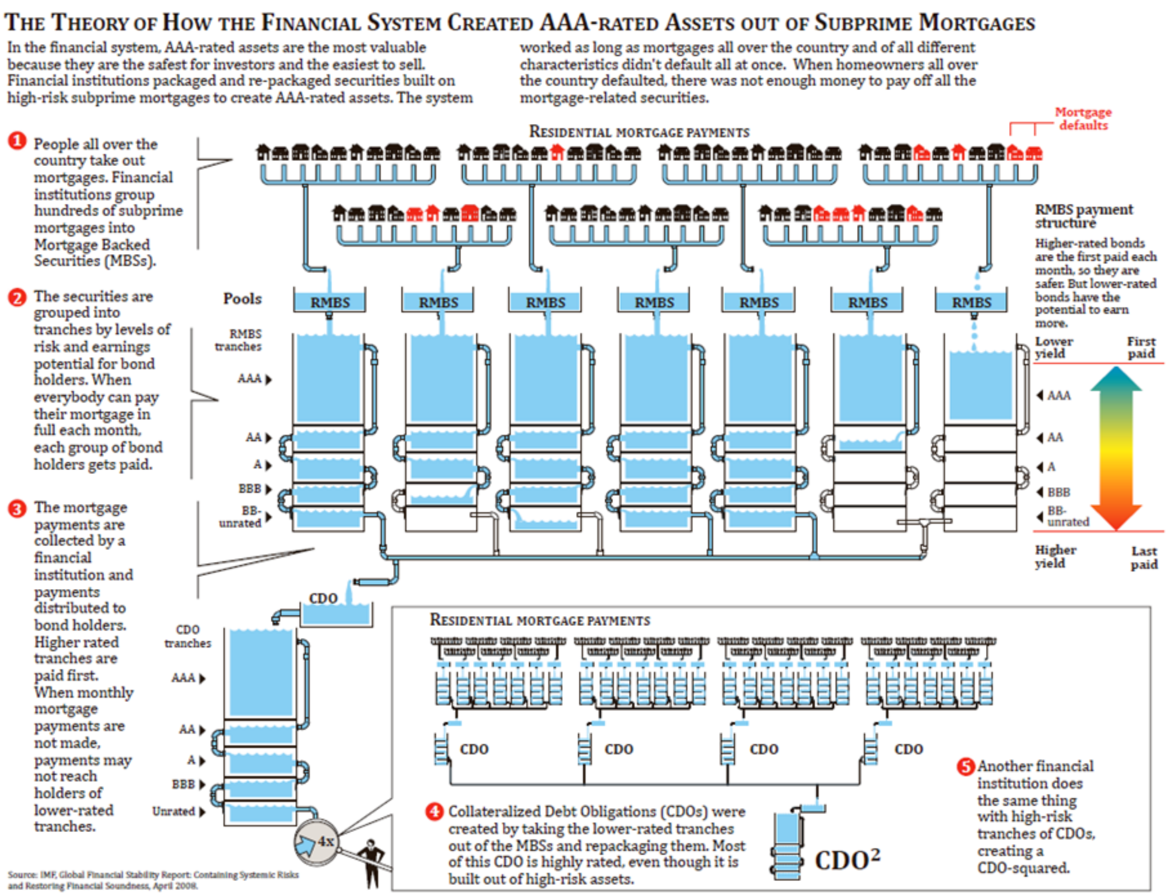
Currency risk is tested employing an empirical measure of the greatest loss suffered within a 10-day period with a 0.99 probability (Currency Indicator II) to comply with the specific principle. The measure is calculated by the Danish FSA.

Option risk is tested in scenarios of shifts in the volatility (vega) to comply with the general principle.

Employing the pass-through principle to comply with the general principle, the issuer remains unaffected by borrowers calling the loan at par.

The cover of future payments to covered bond investors is tested to limit the liquidity and funding risk of mortgage banks. In passing this test, mortgage banks will have sufficient liquidity to meet future payments on mortgages.

APPENDIX C OVERVIEW OF CDO²



Source: IMF

APPENDIX D R CODE (CONVERGENCE OF DEFAULT DISTRIBUTION IN THE GAUSSIAN MODEL)

```
# Gaussian Copula
PD = 0.03
rho = 0.1

MarketShock_500 = c(rnorm(500))
MarketShock_2000 = c(rnorm(2000))
MarketShock_10000 = c(rnorm(10000))

Loss_500 = vector("numeric",length=500)
Loss_500 = pnorm((qnorm(PD)-sqrt(rho)*MarketShock_500)/(sqrt(1-rho)))

Loss_2000 = vector("numeric",length=2000)
Loss_2000 = pnorm((qnorm(PD)-sqrt(rho)*MarketShock_2000)/(sqrt(1-rho)))

Loss_10000 = vector("numeric",length=10000)
Loss_10000 = pnorm((qnorm(PD)-sqrt(rho)*MarketShock_10000)/(sqrt(1-rho)))

hist(Loss_500, breaks =100)
hist(Loss_2000, breaks =100)
hist(Loss_10000, breaks =1000)
```

APPENDIX E R CODE (GAUSSIAN AND STUDENT COPULA DEFAULT DISTRIBUTION)

```
# Gaussian Copula (Repeated twice for PD = 0.015 and 0.03)
PD = 0.015                    # and PD = 0.03
rho = 0.1

MarketShock_10000 = c(rnorm(10000))
Loss_10000 = vector("numeric",length=10000)
Loss_10000 = pnorm((qnorm(PD)-sqrt(rho)*MarketShock_10000)/(sqrt(1-rho)))
hist(Loss_10000, breaks =1000)

mean(Loss_10000)*100
sd(Loss_10000)*100
quantile(Loss_10000, probs = 0.99)*100

# Student t copula (Repeated four times, with different df)
PD_t = 0.015                  # and PD_t = 0.03
rho_t = 0.1
df=10000                      # and df=4, 10 and 40
MarketShock_10000_t = c(rnorm(10000))      # M=m
W_vector = c(rchisq(10000,df))         # Chisquare, v=df

Loss_10000_t = vector("numeric",length=10000)
Loss_10000_t = pnorm(((qt(PD_t,df)*sqrt(W_vector/df)-sqrt(rho_t)*MarketShock_10000_t)/(sqrt(1-rho_t))))

mean(Loss_10000_t)*100
sd(Loss_10000_t)*100
quantile(Loss_10000_t, probs = 0.99)*100
```

APPENDIX F R CODE (RESULTS – GAUSSIAN MODEL)

```
T = 100000                # Number of trials
n = 1000                  # number of loans in the pool
rho = 0.057               # Initial correlation
DP = 0.0159               # Default probability
K = qnorm(DP)             # norm dist of Prob of single mortgage default
LGD = 0.75                # Initial loss given default

rho_vector = c(0.05,0.057,0.06,0.08,0.1,0.15,0.2,0.25,0.3)     # sensitivity vector of the correlation, rho
LGD_vector = c(0.5,0.75,1)                                      # sensitivity vector of LGD

Equity_Junior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))     # attachment table for junior tranche
  rownames(Equity_Junior) = rho_vector
  colnames(Equity_Junior) = LGD_vector
Junior_Senior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))     # attachment table for senior tranche
  rownames(Junior_Senior) = rho_vector
  colnames(Junior_Senior) = LGD_vector
SD = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))                # SD table for the loss fraction
  rownames(SD) = rho_vector
  colnames(SD) = LGD_vector

mEL_Equity = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector)) # EL table Equity
  rownames(mEL_Equity) = rho_vector
  colnames(mEL_Equity) = LGD_vector
mEL_Junior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector)) # EL table Junior
  rownames(mEL_Junior) = rho_vector
  colnames(mEL_Junior) = LGD_vector
mEL_Senior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector)) # EL table Senior
  rownames(mEL_Senior) = rho_vector
  colnames(mEL_Senior) = LGD_vector

MarketShock = c(rnorm(T))          # marketshock for T simulations

######################## Estimating attachment point of the Tranche ####################
for(i in unique(rho_vector)){
  for(j in unique(LGD_vector)){
    rho = i
    LGD = j

    #Loss fraction in the Jigsaw equals
    JigsawLoss2 = vector(length=T)
    JigsawLoss2 = LGD * pnorm((qnorm(DP)-sqrt(rho)*MarketShock[1:T])/(sqrt(1-rho))) #loss fraction

    # while loop running until EL is below Aaa standard
    Attach_Senior = 0.00
    Detach_Senior = 1.00
    Aaa = 0.000055

    EL_Senior = 0.05
     while (EL_Senior > Aaa) {
     Attach_Senior = Attach_Senior + 0.00001
     Jigsaw_Senior_Loss = c(pmax(pmin(JigsawLoss2[1:T],Detach_Senior) - Attach_Senior,0)) # Senior tranche loss
     EL_Senior = mean(Jigsaw_Senior_Loss)/(Detach_Senior-Attach_Senior)
    }
      # Calculating EL on the junior tranche conditional of the Senior attachment

    Detach_Junior = Attach_Senior
    Attach_Junior = 0
```

```
    Baa3 = 0.03355
    EL_Junior = .3

    while (EL_Junior > Baa3) {
     Jigsaw_Junior_Loss = c(pmax(pmin(JigsawLoss2[1:T]-Attach_Junior,(Detach_Junior-Attach_Junior)),0)) # Junior tranche loss
     EL_Junior = mean(Jigsaw_Junior_Loss)/(Detach_Junior-Attach_Junior)
     Attach_Junior = Attach_Junior + 0.00001
    }

    # EL on equity tranche
    Detach_Equity = Attach_Junior
    Attach_Equity = 0
    Jigsaw_Equity_Loss = c(pmax(pmin(JigsawLoss2[1:T]-Attach_Equity,(Detach_Equity-Attach_Equity)),0)) # Junior tranche loss
    EL_Equity = mean(Jigsaw_Equity_Loss)/(Detach_Equity-Attach_Equity)
    Equity_Junior[match(i,rho_vector),match(j,LGD_vector)] = Attach_Junior
    Junior_Senior[match(i,rho_vector),match(j,LGD_vector)] = Attach_Senior

    SD[match(i,rho_vector),match(j,LGD_vector)] = sd(JigsawLoss2)

    mEL_Equity[match(i,rho_vector),match(j,LGD_vector)] = EL_Equity
    mEL_Junior[match(i,rho_vector),match(j,LGD_vector)] = EL_Junior
    mEL_Senior[match(i,rho_vector),match(j,LGD_vector)] = EL_Senior

  }
    }
beep(3)
```

APPENDIX G R CODE (RESULTS – STUDENT T MODEL)

```
T = 100000                  # Number of trials
n = 1000                    # number of loans in the pool
rho = 0.057                 # Initial correlation
DP = 0.0159                 # Default probability
DoF = 10
K = qt(DP, df = DoF)        # norm dist of Prob of single mortgage default
LGD = 0.75                  # Initial loss given default

rho_vector = c(0.05,0.057,0.06,0.08,0.1,0.15,0.2,0.25,0.3)     # sensitivity vector of the correlation, rho
LGD_vector = c(0.5,0.75,1)                                      # sensitivity vector of LGD

Equity_Junior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))     # attachment table for junior tranche
rownames(Equity_Junior) = rho_vector
colnames(Equity_Junior) = LGD_vector
Junior_Senior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))     # attachment table for senior tranche
rownames(Junior_Senior) = rho_vector
colnames(Junior_Senior) = LGD_vector

mEL_Equity = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))        # EL table Equity
rownames(mEL_Equity) = rho_vector
colnames(mEL_Equity) = LGD_vector
mEL_Junior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))        # EL table Junior
rownames(mEL_Junior) = rho_vector
colnames(mEL_Junior) = LGD_vector
mEL_Senior = matrix(NA,nrow=length(rho_vector),ncol=length(LGD_vector))        # EL table Senior
rownames(mEL_Senior) = rho_vector
colnames(mEL_Senior) = LGD_vector

# Market Shocks
MarketShock = c(rnorm(T))
```

```
W_vector = c(rchisq(T, df = DoF)) # W shock, chisquare distributed

# Tranche Estimation
for(i in unique(rho_vector)){
  for(j in unique(LGD_vector)){
    rho = i
    LGD = j

# Loss fraction in the Jigsaw
JigsawLoss = vector(length = T)
JigsawLoss = LGD * pnorm(((K*sqrt(W_vector/DoF))-sqrt(rho)*MarketShock)/(sqrt(1-rho)))       #loss fraction

# while loop running until EL is below Aaa standard
Attach_Senior = 0.00
Detach_Senior = 1.00
Aaa = 0.000055

EL_Senior = 0.05

while (EL_Senior > Aaa) {
  Attach_Senior = Attach_Senior + 0.00001
  Jigsaw_Senior_Loss = c(pmax(pmin(JigsawLoss,Detach_Senior) - Attach_Senior,0)) # Senior tranche loss
  EL_Senior = mean(Jigsaw_Senior_Loss)/(Detach_Senior-Attach_Senior)
}

# Calculating EL on the junior tranche conditional of the Senior attachment
Detach_Junior = Attach_Senior
Attach_Junior = 0
Baa3 = 0.03355
EL_Junior = .3

while (EL_Junior > Baa3) {
  Jigsaw_Junior_Loss = c(pmax(pmin(JigsawLoss-Attach_Junior,(Detach_Junior-Attach_Junior)),0)) # Junior tranche loss
   EL_Junior = mean(Jigsaw_Junior_Loss)/(Detach_Junior-Attach_Junior)
   Attach_Junior = Attach_Junior + 0.00001
}

# EL on equity tranche
Detach_Equity = Attach_Junior
Attach_Equity = 0
Jigsaw_Equity_Loss = c(pmax(pmin(JigsawLoss-Attach_Equity,(Detach_Equity-Attach_Equity)),0)) # Junior tranche loss
EL_Equity = mean(Jigsaw_Equity_Loss)/(Detach_Equity-Attach_Equity)
Equity_Junior[match(i,rho_vector),match(j,LGD_vector)] = Attach_Junior
Junior_Senior[match(i,rho_vector),match(j,LGD_vector)] = Attach_Senior
mEL_Equity[match(i,rho_vector),match(j,LGD_vector)] = EL_Equity
mEL_Junior[match(i,rho_vector),match(j,LGD_vector)] = EL_Junior
mEL_Senior[match(i,rho_vector),match(j,LGD_vector)] = EL_Senior

}
}

beep(3)
```