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Nicole Branger[a] Linda Sandris Larsen[b] Claus Munk[c]

July 29, 2019

The paper contains graphs in color, use color printer for best results.

Abstract

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Keywords: Unemployment risk; Business cycle; Life-cycle model; Portfolio planning

JEL subject codes: G11, D15

[a] Finance Center Münster, University of Münster, Universitätsstrasse 14-16, D-48143 Münster, Germany. E-mail: nicole.branger@wiwi.uni-muenster.de

[b] Department of Finance, Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark. Also affiliated with the Danish Finance Institute and PeRCent. E-mail: lsl.fi@cbs.dk

[c] Department of Finance, Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark. Also affiliated with the Danish Finance Institute and PeRCent. E-mail: cm.fi@cbs.dk

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Abstract

Traditional life-cycle models conclude that individuals should be fully invested in stocks when young—in stark contrast to observed stock holdings—and then gradually replace stocks with bonds as retirement is approaching. We show that a carefully specified and calibrated model of unemployment risk reduces the early-life stock holdings dramatically. The reduction is driven by the decline in current and expected future income caused by unemployment, the relatively high unemployment risk of young adults, and the business cycle variations in un- and reemployment probabilities that tend to deteriorate exactly when stocks perform poorly.

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1 Introduction

Economic recessions affect households through a drop in the value of their assets and an increased unemployment risk. While being unemployed, the income received via insurance or welfare schemes is typically much lower than the salary during employment. Subsequent reemployment is often at a salary markedly below the pre-unemployment level due, e.g., to the loss of training and confidence. Hence, unemployment lowers both current and future income (and consequently retirement income) and thus reduces human capital considerably. Since unemployment risk is largest for young individuals, their human capital is particularly risky. The human capital also varies with the business cycle. When the economy enters a recession, unemployment rates spike and salary growth rates for the employed fall, leading to a decline in human capital, and at the same time the stock market typically plunges. In this paper, we show that the households' optimal asset allocation over the life cycle is significantly affected by the risk of recessions and unemployment.

Ignoring unemployment risk and business cycles, the calibrated standard life-cycle model of Cocco, Gomes, and Maenhout (2005) suggests that human capital is moderately risky and weakly correlated with the stock index. To obtain the optimal risk exposure, young individuals should invest all their financial wealth in stocks and only later in life gradually replace stocks by bonds in their portfolio.[1] We formulate and carefully calibrate a life-cycle model which incorporates the above-mentioned features of unemployment risk and business-cycle variations. Because of the larger risk of human capital and its more stock-like nature, our model leads to much lower stock investments for young individuals (sometimes even zero) compared to the standard model. The predictions of our model are much closer to observed stock holdings as reported by Ameriks and Zeldes (2004) and Fagereng, Gottlieb, and Guiso (2017), among others.

More specifically, Cocco et al. (2005) assume that pre-retirement income follows a diffusion process with a hump-shaped expectation while retirement income is fixed at a fraction of the income immediately before retirement. Our modeling of pre-retirement income differs distinctly. The individual's employment status is represented by a two-state Markov process. The probabilities of transitions between employment and unemployment are specified so that, as observed empirically, the probability of being fired is decreasing in the individual's age and in the state of the economy (higher in recessions than in booms), whereas the probability of finding a new job is decreasing in age and increasing in the state. When calibrating the transition probabilities between unemployment and employment to

[1] Using a closed-form approximation, Viceira (2001) derives similar results in a model with slightly simpler income dynamics. But he also points out that the optimal portfolio allocation to stocks is sensitive to the income rate's volatility and correlation with stock prices, and that the stock weight can even be increasing over life for some parameter constellations. Bagliano, Fugazza, and Nicodano (2014) provide further illustrations of these mechanisms.

the data, we acknowledge the fact that the unemployment rate typically reported in the news underestimates the risk of involuntary lack of labor income, e.g., by ignoring workers marginally attached to the labor market. We model the individual's salary by a diffusion with a pro-cyclical expected growth rate that varies with the individual's age in line with the observed hump-shaped income pattern. While employed, the individual earns this salary. While unemployed, the individual receives a modest fraction of the potential salary, which then has a negative growth rate. Hence, new employment is expected to be at a lower salary than prior to unemployment, and the salary reduction increases with the length of the unemployment duration period. In retirement, we assume—as Cocco et al. (2005)—that income starts out at a fraction of pre-retirement income, but we incorporate the risk of severe health issues with significant costs for medication and care that reduce disposable income. Both the health-related costs and the transition probabilities between being healthy and sick are carefully calibrated to data (Koijen, van Nieuwerburgh, and Yogo, 2016). The health risk brings the risky asset share in retirement down to a level more in line with observed behavior.

The investor can trade a risk-free asset (a bond) with a constant rate of return and a stock (index) with a constant expected return and volatility. As earlier studies, our calibration results in a small instantaneous correlation between stock prices and labor income. The sensitivity of both stock prices and labor market conditions to the business cycle variable causes a modest increase in the overall stock-income correlation over longer horizons (more in Section 3), which in itself can explain a mild reduction in stock holdings. More important is the comovement in the extremes. The risk of becoming and staying unemployed is larger in recessions where stock markets are also hit. Young workers are more likely to enter unemployment than more mature workers and, due to their longer time horizon, the human capital of young individuals is relatively more affected by the long-run consequences of an unemployment spell. Taken together, these features lead to a significant reduction in the optimal risky asset share of young adults.

Because of the rich model with incomplete markets, portfolio constraints, mortality risk, and jumps, we determine the optimal consumption and investment strategies over the life cycle by a numerical dynamic programming technique. Our results show that unemployment risk has significant effects on optimal investments over the life cycle. In the baseline parametrization of our model which assumes a relative risk aversion of 5, the mean portfolio weight of the stock starts out at approximately 10% for young adults, increases gradually to around 50% at retirement, after which it decreases to around 30% at the imposed maximum age of 100 years. Both the level of the stock weight and its pattern through life are thus in stark contrast to the results of the standard no-unemployment model of Cocco et al. (2005). Even though they assume a high risk aversion of 10, their model predicts a stock weight of 100% until an age of around 40, after which the stock weight is gradually reduced to roughly 50% at retirement. While such a glide path strategy is implemented by so-called target date retirement funds, the typical life-cycle investment profile observed empirically is very different and more in line with the results of our model.

Two recent papers incorporate unemployment risk in a life-cycle setting.[2] In a discrete-time model with one-year time steps, Bremus and Kuzin (2014) capture the employment status by a three-state Markov chain representing employment, short-term unemployment (one year), and long-term unemployment (two or more years). They ignore the influence unemployment has on future salaries. Bagliano, Fugazza, and Nicodano (2019) extend the model by assuming that unemployment longer than one year reduces subsequent salaries by 25%. Both papers conclude that unemployment risk can significantly reduce the optimal stock share of young adults. For example, Bagliano et al. (2019) find that with a risk aversion of 5, the optimal average stock share is roughly constant at 60-70% throughout life, whereas without the permanent effects of unemployment the optimal stock share would follow a glide path similar to that in the standard life-cycle model. Neither of these papers allow for age- or state-dependence in un- and reemployment probabilities.[3] Moreover, they quantify the effects of unemployment in model calibrations that simply add unemployment risk (with severe consequences for future income) without adjusting the income diffusion volatility or drift rate, which leads to an income process with a very low mean and very high variance compared to the standard calibration of Cocco et al. (2005). This procedure gives a biased impression of the consequences of unemployment.

The specification and calibration of our model offer a number of advantages relative to the existing literature and provide a more realistic assessment of the implications of unemployment risk for investments over the life cycle. Our continuous-time model (solved numerically using monthly time steps) allows for any unemployment duration, thus capturing both short unemployment periods that are particularly frequent for young adults and the long unemployment periods that are detrimental to consumption opportunities. Furthermore, we let the unemployment and reemployment probabilities depend on the individual's age as observed empirically, and we incorporate business cycle variations in unemployment risk and salary growth rates. In addition, we include the risk of health-related significant drops in disposable retirement income.

When evaluating the impact of unemployment risk we calibrate the salary dynamics to match the income drift and volatility of the standard no-unemployment model of Cocco

[2] In addition, Cocco et al. (2005) and Lynch and Tan (2011) include a coarse specification of unemployment in a robustness check of their models by assuming that income can jump down to a substantially lower level and later back to the normal level.

[3] Bagliano et al. (2019) briefly consider age dependence in the reemployment probability, but they specify it very coarsely and use ad hoc values.

et al. (2005). We find that the stock share should be considerably lower early in life than prescribed by the standard model, and then the share should gradually increase as retirement approaches. The reduction in early-life stock holdings is driven by the decline in current and expected future income caused by unemployment, the relatively high unemployment risk of young adults, and the business cycle variations in un- and reemployment probabilities that tend to deteriorate exactly when stocks perform poorly. Our paper thus shows that a careful modeling of the risk and consequences of unemployment can help explaining observed investment decisions and can supplement existing explanations of low stock holdings or even non-participation in the stock market such as housing investments (Cocco, 2005) and sizeable market entry or per-period participation costs (Gomes and Michaelides, 2005; Fagereng et al., 2017).[4] We estimate that an individual ignoring unemployment risk when making asset allocation decisions suffers a utility loss corresponding to 2.4% of initial financial wealth and life-time income.

Our model is apparently the first to include business cycle variations in unemployment risk. Lynch and Tan (2011) add business cycle variations to the standard life-cycle model by assuming that the stock market dividend yield determines the expected growth rates of both stock prices and labor income. The human capital is therefore more stock-like so the optimal stock investment is somewhat smaller than in the standard model of Cocco et al. (2005). In an extension of our main model we incorporate the same feature by allowing expected stock returns to be decreasing in our state variable, but we find this to have a modest additional impact on the average stock weight.[5] Benzoni, Collin-Dufresne, and Goldstein (2007) impose a strong co-integration relation between stock prices and labor income, which causes human capital to be more stock-like in the long run and thus reduces optimal stock holdings in particular for young adults. Our model features a less strict and less abstract relation between stock prices and labor income through unemployment risk and business-cycle dependencies, which seems to better represent how a typical household might think about life-cycle planning.

The remainder of the paper is organized as follows. Section 2 formulates the model and the consumption-investment problem of the investor. Section 3 calibrates parts of

[4]As most papers in this literature, we do not calibrate our model to match observed output variables such as observed portfolio weights or observed wealth patterns. The benchmark calibration of our model leads to a larger financial wealth accumulation than seen empirically, although less so than for the standard life-cycle model. Our robustness checks show, however, that the low early-life stock investments suggested by our model are invariant to certain changes in preference parameters that lead to lower savings.

[5]Munk and Sørensen (2010) generalize the standard model to stochastic interest rates and allow the expected income growth to vary with the interest rate level. They show how the slope of that relation determines whether the human capital is more like a long-term or a short-term bond. Since the expected excess stock return is constant, the stock demand is only affected when portfolio constraints are binding. Koijen, Nijman, and Werker (2010) adopt a more general interest rate model with time-variation in bond risk premiums, but disregard any link between the interest rate level and labor income.

the model to existing data and motivates the assumed values of other parameters. Section 4 presents and discusses numerical results based on the baseline parameter values and considers selected comparative statics and model variations. Finally, Section 5 concludes.

2 Model setup

We consider a continuous-time model of an economy with a single consumption good which serves as the numeraire so that prices, income, etc., below are stated in real terms.

2.1 Assets

As most related papers we assume that the investor can trade in a risk-free asset and a single risky asset. The risk-free asset is a money market account with a constant continuously compounded annual rate of return r. The risky asset represents the stock market index, and for simplicity we refer to it as *the stock* in the following. We let S_t denote the time t stock price (with dividends continuously reinvested) and assume it follows the geometric Brownian motion

$$dS_t = S_t \left[(r + \mu)\, dt + \sigma_S \, dZ_{St} \right], \tag{1}$$

where $Z_S = (Z_{St})$ is a standard Brownian motion, μ is the expected excess return, and σ_S is the volatility. Both μ and σ_S are kept constant in order to assess the impact of unemployment risk in the simplest possible setting. An extension to a counter-cyclical expected return is briefly considered in Section 4.

2.2 Income

The investor lives until a random time $\tau \leq T$ and retires at a pre-determined time $\tilde{T} < T$. The employment status I_t of the individual before retirement is modeled by a two-state Markov chain jumping between employment e and unemployment u. If employed, the investor receives a salary at the rate y_t. If unemployed, the investor receives a fraction $\alpha_u \leq 1$ of the salary she would have received as employed.[6] Both the salary dynamics and the probabilities of changing the employment status are affected by business-cycle

[6]Other specifications of unemployment benefits might be empirically relevant, but our assumption facilitates a reduction of the dimensionality of the optimization problem. The model could be extended to a higher number of states than two, for example by adding a third state representing being out of the labor force or being permanently disabled. However, the necessary data for a calibration of such an extended model are not readily available. Moreover, we believe that the extension would not materially change the key conclusions as our model already includes a significant business-cycle dependent risk in human capital.

variations captured by a state variable X having dynamics

$$dX_t = -\kappa_x X_t \, dt + \sigma_X \left(k_{SX} \, dZ_{St} + \sqrt{1 - k_{SX}^2} \, dZ_{Xt} \right). \tag{2}$$

Here Z_X is a standard Brownian motion independent of Z_S so k_{SX} is the instantaneous correlation between the stock price and the state variable. Note that X mean-reverts around zero with a volatility $\sigma_X > 0$ and a speed of mean reversion given by $\kappa_x > 0$. A large negative [positive] value of X_t indicates that the economy is in a recession [boom].

The salary level y_t evolves according to

$$\begin{aligned} dy_t = y_t \Bigg[& \left(\xi_0(t, I_t) + \xi_1(t, I_t) X_t \right) \, dt \\ & + \sigma_y(t) \left(k_{Sy} \, dZ_{St} + \hat{k}_{Xy} \, dZ_{Xt} + \sqrt{1 - k_{Sy}^2 - \hat{k}_{Xy}^2} \, dZ_{yt} \right) \Bigg] \end{aligned} \tag{3}$$

with an age-dependent volatility $\sigma_y(t) \geq 0$ and an expected salary growth rate that depends on the age and employment status of the individual as well as the state variable X_t. Here Z_y is another standard Brownian motion independent of Z_S and Z_X so $\hat{k}_{Xy} = (k_{Xy} - k_{SX} k_{Sy}) / \sqrt{1 - k_{SX}^2}$, and k_{Sy} and k_{Xy} are the instantaneous correlations of the salary with the stock price and the state variable, respectively.

We exploit our two-state Markov chain setting also in retirement by assuming that the individual can either be healthy ($I_t = h$) or sick ($I_t = s$). While healthy, retirement income is a fraction $\alpha_R \leq 1$ of the salary level. If sick, expenses related to medication and care reduce disposable income to the age-dependent fraction $\alpha_S(t) \, \alpha_R$ of the salary level, where $\alpha_S(t) \leq 1$. Hence, income flows to the individual at the rate Y_t, where

$$Y_t = \alpha(t, I_t) \, y_t, \quad \alpha(t, I_t) = \begin{cases} 1, & \text{if } t \in [0, \tilde{T}) \text{ and } I_t = e, \\ \alpha_u, & \text{if } t \in [0, \tilde{T}) \text{ and } I_t = u, \\ \alpha_R, & \text{if } t \in [\tilde{T}, T] \text{ and } I_t = h, \\ \alpha_S(t) \, \alpha_R, & \text{if } t \in [\tilde{T}, T] \text{ and } I_t = s. \end{cases} \tag{4}$$

In retirement, we assume that $\xi_0(t, I_t) = \xi_1(t, I_t) = 0$ and $\sigma_y(t) = \hat{\sigma}_y$. Hence, retirement income in real terms has a zero drift and a (low) constant volatility representing, e.g., business-related income continuing into retirement or imperfect indexation of pensions to the consumer prices relevant for the individual. The retirement income-salary ratio α_R is similar to the so-called replacement rate in the widespread final-salary retirement plans.[7]

[7] We could have obtained a low early-life risky asset share in a model with the retirement date as the terminal date and some utility of terminal wealth. However, without modeling the retirement phase explicitly and in a realistic way (including health risk), it is difficult to know what the utility weight on

Before retirement, we assume a constant value of $\sigma_y(t)$, whereas the expected salary growth rate depends on the state of the economy with a slope of $\xi_1(t, I_t)$, which we assume independent of age.[8] To incorporate observed life-cycle variations in labor income (Hubbard, Skinner, and Zeldes, 1995; Cocco et al., 2005), the salary growth rate depends on age through $\xi_0(t, I_t)$. Moreover, the growth rate depends on the employment status and is assumed negative when unemployed. This captures both that unemployment benefits decline over the unemployment period and that reemployment is often at a lower salary than the pre-unemployment level with the decline increasing with the unemployment duration. Note that unemployment periods have a permanent negative effect on future salaries and thus also on retirement income.

The change between the two employment states is formally represented by two jump processes N^u and N^e. If the investor is employed, a jump N^u into unemployment occurs with jump intensity $\eta_u(t, X_t)$. Conversely, if the investor is unemployed, a jump N^e into employment happens with a jump intensity $\eta_e(t, X_t)$. The intensities depend on age as well as the state of the economy. With $\eta_u(t, X_t)$ being decreasing in X_t, the probability of getting fired is higher in a recession than in a boom. Similarly, with $\eta_e(t, X_t)$ being increasing in X_t, the probability of becoming employed again is higher in a boom than in a recession. Correspondingly, the expected duration of an unemployment period is longer in a recession than in a boom. To incorporate observed life-cycle variations into the probabilities of becoming employed and unemployed, we allow the intensities to depend on the age of the investor. Similarly, in retirement a change from healthy to sick [from sick to healthy] occurs with an age-dependent intensity $\eta_s(t)$ [respectively, $\eta_h(t)$]. The precise specification and calibration to data is explained in Section 3.

2.3 Assessing the state of the economy

We assume that the investor cannot directly observe the expected income growth rate and the un- and reemployment intensities. Hence, the state variable X_t is unobserved, but must be inferred from informative observable quantities. The stock price is informative due to its correlation with X_t. Various macroeconomic variables are also likely to be informative. To be specific, we assume that the investor applies two macro variables as signals, namely the economy-wide unemployment rate and the growth rate of aggregate

wealth at retirement should be relative to the utility of consumption before retirement.

[8] Pissarides (2009) surveys the empirical evidence on how hourly wages vary with the unemployment rate. All the studies included find pro-cyclical wages, both in the U.S. and in Europe.

income. We let v_{1t}, v_{2t} denote the two macro signals and assume that

$$dv_{1t} = (\omega_1 + \kappa_{1x} X_t - \kappa_{1v} v_{1t})\, dt + \sigma_{v1}\, dZ_{1t}, \tag{5}$$

$$dv_{2t} = (\omega_2 + \kappa_{2x} X_t - \kappa_{2v} v_{2t})\, dt + \sigma_{v2}\, dZ_{2t}. \tag{6}$$

Here Z_1 and Z_2 are additional standard Brownian motions that may be correlated with each other as well as with Z_S, Z_X, and Z_y.

Based on observations of the stock price and the two macro signals, the investor forms her best guess x_t about the state X_t of the economy and revises her guess over time using Bayesian updating. More formally, x_t is the expectation of X_t conditional on the realizations of the macro signals and the stock price up to, and including, time t. We assume that there is no information in the investor's own salary in addition to the stock price and the macro signals. From Appendix A it follows that the filtered model (as seen by the investor) is given by

$$dS_t = S_t \left[(r + \mu)\; dt + \sigma_S\, dz_{St} \right], \tag{7}$$

$$dx_t = -\kappa_x x_t\, dt + \sigma_x \left(\rho_{Sx}\, dz_{St} + \sqrt{1 - \rho_{Sx}^2}\, dz_{xt} \right), \tag{8}$$

$$\begin{aligned} dy_t = y_t \Big[& \left(\xi_0(t, I_t) + \xi_1(t, I_t) x_t \right)\; dt \\ & + \sigma_y(t) \left(\rho_{Sy}\, dz_{St} + \hat{\rho}_{xy}\, dz_{xt} + \sqrt{1 - \rho_{Sy}^2 - \hat{\rho}_{xy}^2}\, dz_{yt} \right) \Big], \end{aligned} \tag{9}$$

where z_S, z_x, z_y are independent standard Brownian motions, $\rho_{Sy}(t) = k_{Sy}(t)$, and $\hat{\rho}_{xy} = (\rho_{xy} - \rho_{Sx}\rho_{Sy})/\sqrt{1 - \rho_{Sx}^2}$. Here ρ_{Sx} and ρ_{xy} are instantaneous correlations, and the parameters of the two signal processes enter only via these two correlations, cf. Appendix A. With $\rho_{Sy}^2 + \hat{\rho}_{xy}^2 < 1$, shocks to the salary level contain both a macro-component (through the correlation with stock prices and the perceived state) and an idiosyncratic component, where the latter captures, e.g., promotions and demotions of the individual. Since jumps in employment status are investor-specific and thus unrelated to the macroeconomy, the jump intensities are $\eta_u(t, x)$ and $\eta_e(t, x)$ also in the filtered model.

2.4 The utility maximization problem and how we solve it

We denote the financial wealth of the investor at time t by W_t. The investor chooses a consumption strategy $c = (c_t)$ and an investment strategy $\pi = (\pi_t)$. Here, $c_t > 0$ is the rate at which goods are consumed at time t, and π_t is the fraction of financial wealth invested at time t in the stock index with the remaining wealth $W_t(1 - \pi_t)$ being invested in the money market account. We impose the standard borrowing and short-selling constraint

$$\pi_t \in [0,1], \quad t \in [0,T]. \tag{10}$$

The financial wealth of the investor evolves as

$$dW_t = W_t \left[(r + \pi_t \mu)\, dt + \pi_t \sigma_S dz_{St} \right] - c_t\, dt + \alpha(t, I_t) y_t\, dt, \tag{11}$$

where $\alpha(t, I_t)$ is given by (4).

The investor has a power utility function of consumption c_t and terminal wealth (bequest) W_τ with a constant relative risk aversion $\gamma > 0$; the extension to recursive utility is considered in Section 4. The indirect utility function of the investor is

$$J(t, W, x, y; i) = \sup_{(c,\pi)} \mathrm{E}_t \left[\int_t^\tau e^{-\delta(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma}\, ds + e^{-\delta(\tau - t)} \left(\frac{W_\tau}{\varepsilon} \right)^{1-\gamma} \frac{1 - e^{-\delta\varepsilon}}{\delta(1-\gamma)} \right], \tag{12}$$

where δ is the subjective time preference rate. The bequest utility corresponds to the utility from a constant consumption rate W_τ/ε over ε years so $\varepsilon > 0$ measures the strength of the bequest motive.[9] The expectation is conditional on $W_t = W$, $y_t = y$, $x_t = x$, and the employment status $I_t = i$ (either $i = e$ or $i = u$). The random time of death τ satisfies

$$\mathrm{Prob}\left(\tau \in [t, t + dt] \mid \tau \geq t \right) = \nu(t)\, dt, \tag{13}$$

where $\nu(t)$ is a deterministic function describing the instantaneous mortality rate. We impose a maximum age of T.

We cannot derive the optimal strategies in closed form due to market incompleteness and portfolio constraints, so we implement a numerical solution approach similar to that applied by Brennan, Schwartz, and Lagnado (1997) and Munk and Sørensen (2010). Here we outline the numerical method, which is explained in more detail in Appendix B. Our model is specified so that the indirect utility function is homogeneous of degree $1-\gamma$ in (W, y), which implies that we can write

$$J(t, W, x, y; i) = y^{1-\gamma} G\left(t, e^{-B(t)} \tfrac{W_t}{y_t}, x; i \right)$$

for some function G (the role of $B(t)$ is explained in the appendix). By substituting this into the pair of Hamilton-Jacobi-Bellman equations satisfied by $J(t, W, x, y; e)$ and $J(t, W, x, y; u)$, we obtain a pair of partial differential equations that $G(t, w, x; e)$ and $G(t, w, x; u)$ must satisfy, cf. Eq. (23) in Appendix B. We solve these numerically using a

[9] Note that $(1 - e^{-\delta\varepsilon})/\delta \approx \varepsilon$ for $\delta\varepsilon \approx 0$ so the bequest utility is approximately $\varepsilon(\frac{W_\tau}{\varepsilon})^{1-\gamma}/(1-\gamma)$.

finite difference, backwards iterative solution on a grid, optimizing feasible consumption rates and portfolios at each node. Here the continuous-time formulation leads to simple first-order conditions that provide a valuable starting point for finding the optimal decisions on the grid. With the derived optimal consumption rate and portfolio weight for a large number of combinations of (t, W, x, y, i), we simulate the economy forward (interpolating between grid points when needed) to obtain possible life-cycle patterns of the individual's consumption and asset holdings. We simulate 10,000 paths and report the mean and selected percentiles.

3 Model calibration

To make our model comparable to related literature (e.g., Cocco et al., 2005; Bagliano et al., 2019), we set the (real) risk-free rate to $r = 1\%$, the equity premium to $\mu = 4\%$, and the stock volatility to $\sigma_S = 15.7\%$. Other parameter values are based on a calibration to monthly U.S. data from January 1959 to March 2018. We assume the investor applies the official economy-wide unemployment rate and the growth rate of aggregate real income as signals about the state of the economy. The unemployment rate is downloaded from the homepage of the Federal Reserve Bank of St. Louis.[10] The growth rate of aggregate real income is derived from the national disposable personal income (of both employed and unemployed individuals), taken from the homepage of the Bureau of Economic Analysis.[11] Finally, stock returns are derived from the real price of the S&P500, downloaded from the homepage of Professor Robert Shiller.[12] Figure 1 depicts the time series of the stock price index, the unemployment rate, and the aggregate income growth rate.

[Figure 1 about here.]

The log stock price, the signals, and the unobservable state variable x follow a Gaussian system, so we use Kalman Filtering and maximum likelihood to obtain a time series of x and our parameter estimates. The estimation routine is explained in Appendix C, and the annualized parameter estimates are presented in Table 1.

[Table 1 about here.]

The estimated unemployment rate varies around $\omega_1/\kappa_{1v} \approx 5.51\%$, and the average level increases [decreases] by $-\kappa_{1x}\sigma_X/\kappa_{1v} \approx 2.97\%$ when the state variable is one standard

[10] See `http://research.stlouisfed.org/fred2/series/UNRATE/downloaddata?cid=32447`. The unemployment rate used here is the so-called U3 rate representing the number of unemployed as a percentage of the labor force.

[11] See `http://www.bea.gov/iTable/iTable.cfm?ReqID=9&step=1#reqid=9&step=3&isuri=1&903=76`. The disposable personal income equals personal income less taxes. Personal income includes wages and salaries (with supplements), income from financial assets, as well as government social benefits.

[12] See `http://www.econ.yale.edu/~shiller/`.

deviation below [above] average. The aggregate income growth rate exhibits strong mean reversion around an average level of $\omega_2/\kappa_{2v} \approx 0.19\%$ and tends to increase with the state of the economy in line with intuition; a one standard deviation increase in the state adds $\kappa_{2x}\sigma_X/\kappa_{2v} \approx 0.05\%$ to the expected income growth rate.[13] The 2.48% estimated aggregate income volatility is, of course, substantially lower than for an individual's income. The correlation of the business cycle variable with aggregate income growth is around 0.4, whereas its correlation with the unemployment rate is around 0.2.

Table 2 summarizes the baseline values of parameters relating to the investor's time horizon, preferences, and the likelihood of shifts in employment or health status. Our benchmark investor begins her working life at the age of 25 and retires when turning 65. The deterministic mortality intensity $\nu(t)$ is derived from the life tables for the U.S. population as of 2009 with an imposed maximum age of 100.[14] The investor is assumed to have a time-preference rate of $\delta = 0.04$ and a relative risk aversion of $\gamma = 5$, standard values in the literature.[15] We let $\varepsilon = 5$ which is in the range of empirical estimates (see Kværner (2018) and the discussion therein) and values typically considered in related papers (e.g., Cocco et al., 2005).[16] We measure income and wealth in thousands of US dollars and fix the initial financial wealth at $W_0 = 5$, which seems reasonable for a 25-year old individual given the median family net worth of 11.1 for the age group "less than 35" in the 2016 Survey of Consumer Finances, cf. Bricker et al. (2017, Table 4).

[Table 2 about here.]

The unemployment probability as a function of age is determined by the initial employment state and the intensities with which an employed becomes unemployed and an unemployed becomes employed at different ages. To fix a targeted unemployment probability at different ages for our calibration, Table 3 shows measures of the 2017 U.S. unemployment rate for different age groups based on the Current Population Survey published by the Bureau of Labor Statistics (BLS). The first two rows show the 2017 value and the 2002-2017 average of the fraction of the population in each group not employed, which includes people who are going to school or have retired and are therefore not seeking employment. The next two rows show the 2017 value and the 1994-2017 average of the

[13] The calibrated $\kappa_x = 0.4979$ seems high compared to the typical duration of a business cycle. A robustness check (available upon request) shows that lower values of κ_x reduce the stock weight early in life further, but that the differences are not dramatic.

[14] Life tables are published at the Centers for Disease Control and Prevention under the U.S. Department of Health and Human Services.

[15] Some studies (e.g., Cocco et al., 2005; Fagereng et al., 2017) assume $\gamma = 10$, presumably to avoid extremely high stock allocations throughout life. Typical estimates of relative risk aversion fall in the range from 1 to 5, cf. Meyer and Meyer (2005).

[16] While there is some disagreement about the strength of the bequest motive, the exact value of ε has no notable influence on the optimal portfolio composition before retirement, which is the focus of our paper.

unemployment rate typically referred to by media and politicians—labeled U3 by BLS—which is the number of unemployed individuals as a percentage of the labor force.[17] The BLS also publishes the more comprehensive U6 unemployment rate defined as the ratio of (i) the sum of the unemployed workers, the workers marginally attached to the labor market (not counted in the labor force), and the workers who work part-time purely for economic reasons, divided by (ii) workers who are in the labor force or marginally attached. The numbers of marginally attached and part-time workers—and thus the U6 rate—is only available for broader age groups. Both the U3 and the U6 rate are declining with age and, in particular, much higher for individuals younger than 25 than for those above 25. Figure 2 shows the U3 and U6 unemployment rates for all individuals of age 16+ from 1994 to early 2018. On average, the U6 rate is a factor 1.80 times the U3 rate, with the factor having varied between 1.63 (June 2003) and 2.09 (May 2016).

[Figure 2 about here.]

[Table 3 about here.]

Even the U6 rate underestimates the risk of involuntary lack of income-generating employment as it ignores individuals not looking for a job in the most recent 12-month period because of illness, disability, or self-declared retirement. For each of the three broad age groups 16-24, 25-54, and 55+ years we calculate an "extended" unemployment rate by adding, in both the numerator and denominator, a fraction of the number of individuals who have not searched for a job in the recent year due to illness or disability or who refer to themselves as retired.[18] (The optimal risky asset share through life is almost the same whether we use the U6 rate or the extended rate in the calibration, cf. Section 4.5.) As shown in Table 3, the extended 2017 unemployment rate is 19.7%, 13.2%, and 24.0% for the three age groups. Finally, to obtain the unemployment rates we target for average economic conditions, we adjust for the fact that the 2017 employment situation is above average as can be seen by comparing the 2017 U3 rate or non-employment/population ratio to the historical average. For example, the 2017 U3 rate is 71-76% of its 1994-2017

[17]For the average non-employment/population ratio we use only post-2002 values as 2000-2001 values are unavailable. Moreover longer-period averages are uninformative about current average-state values since there seems to a structural upward [downward] trend in the non-employed share of the population for age 24- [55+] probably due to an increase in education [retirement age].

[18]More specifically, we include 90% [80%] of the ill/disabled or self-declared retired 16-24 [25-54] year olds, whereas in the age group 55+ we include 80% of the ill/disabled or self-declared retirees between 55 and 64 years but none of those of age 65+. The most recent numbers of self-declared ill, disabled, and retired individuals in the three age groups are from 2014. We multiply each number by 1.0219 reflecting the general growth of the US population from end 2014 to end 2017 according to the population clock at `https://www.census.gov/popclock/`. Even our extended unemployment rate ignores individuals temporarily absent from work due to, e.g., illness or injury, which in 2017 corresponded to 1.4% of total worktime, cf. `https://www.bls.gov/cps/cpsaat46.htm` accessed on April 4, 2018.

average for age 16-34, 65-66% for age 35-54, 77% for age 55-64, and 82% for age 65+.[19] Assuming that the current extended unemployment rate is 80% of its average-state value, we arrive at an average-state unemployment rate of 24.6% for age 16-24, 16.5% for age 25-54, and 30.0% for age 55+, numbers that our calibration tries to match.

The probabilities with which an employed becomes unemployed or an unemployed becomes employed are modeled through the jump intensities, which we specify as

$$\eta_e(t,x) = e^{\eta_{a,e}+\eta_{b,e}t+\eta_{c,e}t^2+\eta_{1,e}x}, \qquad \eta_u(t,x) = e^{\eta_{a,u}+\eta_{b,u}t+\eta_{c,u}t^2+\eta_{1,u}x}. \tag{14}$$

The values of the coefficients $\eta_{a,i}, \eta_{b,i}, \eta_{c,i}$ for $i = e, u$ listed in the middle panel of Table 2 are determined by fixing the intensities at the age of 25, 40, and 65 years. The intensity for going from unemployment to employment is fixed at 0.38, 0.32, and 0.15, respectively, whereas the intensity for going from employment to unemployment is fixed at 0.12, 0.05, and 0.09. The solid curves in Panel A of Figure 3 depict how the intensities vary with age in the average state $x = 0$. These intensities closely match the calibrations by Choi, Janiak, and Villena-Roldán (2015, Figs. 2 and 3) on US data. The intensities reflect that it is generally easier to find a job for young than for old individuals, which implies that the average unemployment duration is increasing with age.[20] On the other hand, the risk of becoming unemployed is U-shaped in age. The dotted curves in Panel A display the 10th and 90th percentiles for the jump intensities at different ages based on simulations starting in the average state $x = 0$ at age 25. Clearly, the probability of going from unemployment to employment varies a lot more than the probability of the opposite transition. Together with the initial employment state, the specified intensities determine how the unemployment probability varies with age. By assuming that the individual at age 25 is employed with a probability of 75%, we obtain the age-dependent unemployment risk illustrated in Panel B of Figure 3. From the initial 25.0%, the unemployment rate decreases to a minimum of 14.3% at age 44, after which it increases to 29.8% just before retirement, numbers close to the target rates motivated above.

[Figure 3 about here.]

For the state dependence, we rely on monthly data for the unemployment rate and the duration of unemployment from the homepage of The Federal Reserve Bank of St. Louis.[21] From these, we back out monthly values of η_e and η_u, and then choose $\eta_{1,e}$ and $\eta_{1,u}$

[19] We do not have time series of comprehensive or extended unemployment rates by age groups.

[20] According to Table 31 of the 2016 Current Population Survey published by the U.S. Bureau of Labor Statistics, the average duration of unemployment gradually increases with age from around 14 weeks for the 16-19 year olds over 28 weeks for 35-44 year olds to 39 weeks for persons of age 65 and up.

[21] See `https://alfred.stlouisfed.org/series?seid=UEMPMEAN`. The duration of unemployment represents the average number of weeks of unemployment.

such that we match the volatilities of these intensities. Consistent with intuition, the estimated intensity of the unemployment [reemployment] jump is decreasing [increasing] in the state.[22]

Next, we explain how we determine the salary parameters listed in Table 4. For a fair evaluation of the effects of unemployment risk, we make sure that the expected income is similar with and without unemployment. For the case without unemployment risk we match the hump-shaped average income profile estimated by Cocco et al. (2005) for individuals with a high-school education, but no college degree. According to their Figure 1, the annual income grows from around \$21,000 at age 25 to a maximum of around \$29,000 at age 45, after which it drops to around \$25,000 just before retirement. Hence, we set the initial salary level to $y_0 = 25.50$ (i.e., \$25,500 per year) for the case with unemployment risk and $y_0 = 21.04$ for the case without unemployment risk so that the initial average income rate is identical in the two cases; with a 75% initial employment rate and an unemployment benefit equal to 30% of income (see below) the average income is $0.75 \times 25.5 + 0.25 \times 0.3 \times 25.5 = 21.04$. The humped-shaped income pattern is well approximated by the exponential of a third-order polynomial, so we assume that the age-dependent component of the income drift in the case with no unemployment risk (labeled 'nu') is

$$\xi_0^{\text{nu}}(t) = \xi_a^{\text{nu}} + 2\xi_b^{\text{nu}}\, t + 3\xi_c^{\text{nu}}\, t^2, \quad 25 \leq t < 65.$$

To match the income profile of high school graduates, we determine ξ_a^{nu}, ξ_b^{nu}, and ξ_c^{nu} by requiring that (i) expected salary peaks at the age of 45; (ii) expected salary at the peak is 1.4 times the initial salary, and (iii) expected salary before entering retirement is 15% below the peak level. We fix the income volatility at $\sigma_y^{\text{nu}} = 0.15$ in accordance with values used in the literature.[23]

[Table 4 about here.]

For the case with unemployment risk, we assume that, while unemployed, the individual receives the fraction $\alpha_u = 30\%$ of the salary level which is expected to drop by 10% per year independent of her age, i.e., $\xi_0(t, u) = -0.1$. Consequently, the income declines during the unemployment period and future reemployment occurs at a lower salary with the decline increasing in the unemployment duration.[24] When employed, the age-dependent

[22]While Shimer (2005) reports an almost acyclical job separation rate, Fujita and Ramey (2009), Bils, Chang, and Kim (2012), and Mueller (2017) find that the job separation rate is highly countercyclical, especially for high-wage earners.

[23]The discrete-time model of Cocco et al. (2005) involves both permanent and transitory income shocks, whereas a continuous-time model as our model does not accommodate transitory shocks. To compensate, we use a slightly larger σ_y than they do. As discussed in Section 4.5, the optimal portfolio weight of the stock in our model is only little sensitive to the exact value of this parameter.

[24]In the US, unemployment benefits are typically paid for 6 months and correspond to 50% of recent

salary component is modeled by

$$\xi_0(t,e) = \xi_a^e + 2\xi_b^e t + 3\xi_c^e t^2, \quad 25 \leq t < 65.$$

The future income level and uncertainty depend both on $\xi_0(t,u)$, $\xi_0(t,e)$, the volatility σ_y, and the un- and reemployment probabilities. We determine ξ_a^e, ξ_b^e, ξ_c^e, and σ_y to best match the expected income and the standard deviation of the salary through life in the model without unemployment risk.[25] The resulting salary volatility of 10.35% is much smaller than the 15% for the no-unemployment case, but the negative effect of unemployment on the salary drift contributes to the standard deviation of the future salary.[26]

We also make sure that the expectation and standard deviation of retirement income as well as the health status are the same in the two models. The standard deviation of retirement income is set to $\hat{\sigma}_y = 0.02$. For the no-unemployment case we use a replacement rate of $\alpha_R = 0.68$ as estimated by Cocco et al. (2005) for workers with a high school degree. Just before retirement in our specification, the individual is employed [unemployed] with a probability of 70% [30%], so the expected income is $0.70y + 0.30\alpha_u y$, and 68% of that equals a fraction of $\alpha_R = 0.68 \times (0.70 + 0.30\alpha_u)$ of the salary. With $\alpha_u = 0.3$, we have $\alpha_R = 0.5372$. To model the health status—with the two states healthy and sick—we use the same approach and functional forms for the transition probabilities as for the employment status (see Eq. (14)) except that we leave out the state dependence. Our calibration is based on the health transition probabilities in Koijen et al. (2016, Figure 2). We assume that disposable income when being sick at age $t \geq 65$ is a fraction

$$\alpha_S(t) = 1 - \left[0.0817 + 0.0007 \times (t - 65)^2\right]$$

of the income when healthy, which closely matches the average increase in out-of-pocket health expenses reported by Koijen et al. (2016, Table 3).[27] The resulting parameters are

salary, but with a cap which can be as low as \$235 per week (Mississippi) so that many recipients end up with less than 50%. After the first 6 months, the unemployed may receive welfare payments, food coupons, etc. from other programs. Our model does not allow benefits to depend on the length of the unemployment spell. Bremus and Kuzin (2014) assume benefits are 28% in the first year of unemployment and 10% thereafter. Bagliano et al. (2019) assume the values are 30% and 10%, respectively. An empirical literature shows severe consequences of unemployment on future labor income, but the quantitative estimates vary substantially across studies, cf., e.g., Arulampalam (2001), Couch and Placzek (2010), Davis and von Wachter (2011), and Kroft, Lange, Notowidigdo, and Katz (2016). In comparison Bagliano et al. (2019) assume that unemployment longer than one year reduces expected labor income by 25%.

[25] More precisely, we minimize a sum of squared differences across the two models in expected income and salary standard deviations at various age levels.

[26] Apparently Bagliano et al. (2019) leave the diffusion drift and volatility unchanged when adding unemployment risk, which makes it difficult to evaluate whether effects are due to the decrease in average income, to the increase in income uncertainty, or to the specific consequences of unemployment spells.

[27] To be precise, our quadratic specification almost perfectly matches the additional out-of-pocket health expenses for poor vs. good health as a fraction of income at ages 65 (by construction), 72, 79, and 86, but

listed in the lower panel of Table 2. Figure 4 illustrates the life-cycle income profile in the two models. The income in retirement is identical in the two cases with the decreasing trend stemming from the increasing probability of being sick and the increasing costs when being sick. The solid curves representing the means stay close throughout life. Although we have matched the salary volatilities in the two models, the 10th and 90th income percentiles are more extreme in the model with unemployment risk due to the considerably lower income while unemployed.

[Figure 4 about here.]

We assume that the salary growth rate has the same state dependence as the aggregate income growth, and that the state dependence of the potential salary when unemployed equals the state dependence of the actual salary when employed, and we use the same value in the case without unemployment risk, i.e., $\xi_1(t,e) = \xi_1(t,u) = \xi_1^{\mathrm{nu}} = 0.0657$. Both with and without unemployment, we assume that the individual salary level has zero correlation with the aggregate unemployment rate, i.e., $\rho_{yv_1} = 0$, and we fix the correlation between the individual salary level and the aggregated income at $\rho_{yv_2} = 0.5$. The correlation between the individual salary level and the stock return is assumed to equal the correlation between the aggregate income level and stock return, i.e., $\rho_{Sy} = 0.1169$.[28] Due to their joint dependence on the business cycle variable, over longer horizons income and stock prices are more highly correlated in our model than in the standard life-cycle model without unemployment, but the increase in correlation is modest. To illustrate this, we have calculated the correlation between changes in income and stock returns over the first ten years (age 25-35) based on 10,000 simulated paths in different versions of our model. The correlation is 0.1355 in the version without unemployment risk and 0.1825 in the baseline parametrization of the version with unemployment risk (these values assume the initial state is average, $x_0 = 0$, but the ten-year correlations are only little sensitive to the initial state). Hence, the large reduction in early-life risk-taking that we find in the next section cannot be explained by such a small increase in the overall correlation, but comes mainly from the dramatic reduction in current and future income caused by unemployment and the business cycle variations in the probabilities of changing employment status (as the comparative statics in Section 4.5 show).

underestimates the very high poor-health expenses at age 93 reported by Koijen et al. (2016).

[28] Davis and Willen (2000) report that—depending on the individual's gender, age, and educational level—the correlation between aggregate stock market returns and labor income shocks is between -0.25 and 0.3, while the correlation between industry-specific stock returns and labor income shocks is between -0.4 and 0.1. Heaton and Lucas (2000) find that the labor income of entrepreneurs typically is more highly correlated with the overall stock market (0.14) than with the labor income of ordinary wage earners (-0.07).

4 Optimal decisions over the life cycle

The results shown below are generated by a two-step procedure. First, we solve numerically for the optimal consumption and portfolio choice for a large number of combinations of age t, wealth W, state x, salary y, and employment status i. Secondly, we simulate 10,000 possible life-cycle paths using these optimal decisions. Unless otherwise mentioned, we use the parameter values listed in Tables 1, 2, and 4. For comparison, we note that in the Merton-type model with no labor income, the individual would optimally hold $\mu/(\gamma\sigma_S^2) \approx 32.5\%$ of wealth in stocks throughout life.

4.1 Benchmark results with and without unemployment risk

Figure 5 compares the life-cycle patterns in stock investments, wealth, and consumption in our full model with unemployment risk (blue curves) to the patterns in the version of our model without unemployment risk (red curves). The solid lines represent means across the 10,000 paths, whereas the dotted lines represent the 10th and 90th percentiles. The no-unemployment model version deviates from the standard life-cycle model of Cocco et al. (2005) by adding business cycle variations in income growth and health risk in retirement.

[Figure 5 about here.]

Panel A of Figure 5 illustrates our main finding that unemployment risk has a strong effect on the portfolio weight of the stock early in life. Without unemployment risk, the average optimal stock weight starts out at 100% and is then gradually reduced to around 50% just before retirement. This pattern is in line with the results of Cocco et al. (2005) and other papers with lognormally distributed labor income. This literature finds that human capital induces a larger optimal stock weight than in the no-income Merton-type model if the product of the investor's relative risk tolerance and the stock's Sharpe ratio exceeds the covariance between stock returns and changes in the human capital.[29] When this condition holds, the optimal stock weight is increasing in the size of the human capital relative to financial wealth and thus typically decreasing through working life. Such glide-path investment strategies are implemented in practice by various target-date funds. At retirement there is a modest level increase in the stock weight caused by the reduction in income uncertainty and the stock-income correlation. The average weight is then gradually reduced through retirement to around 30% at the maximum age of 100

[29] Such a condition can be seen in various disguises in, e.g., Bodie, Merton, and Samuelson (1992, Sec. 4), Viceira (2001, Prop. 2), and Campbell and Viceira (2002, Eq. (6.11)).

due to the increasing risk of bad health and the reduction in the ratio of human capital (mortality-adjusted present value of future retirement income) relative to financial wealth.

Unemployment risk generates a dramatic reduction in the optimal stock weight early in life. The average stock weight starts out near 10%, is around 25% at age 30—and thus below the Merton weight of 32.5%—and then increases until retirement. Due to the threat of unemployment and its detrimental effects on future income, the human capital of the young adult is so risky and so stock-like that the condition explained in the previous paragraph fails to hold and, consequently, the optimal stock weight falls below the no-income case. We explore the determinants of the reduction of the risky asset share further in the following subsections. Since the large differences in portfolio weights occur early in life when financial wealth is relatively small, the differences in the amounts held in stocks are less pronounced, cf. Panel B. Unemployment risk in working life has little effect on the optimal stock weight in retirement, as we have made sure the retirement income is similar in the two cases. In the model with unemployment risk, the agent enters retirement, on average, with a lower financial wealth, which induces a slightly higher share of wealth to be invested in stocks.

Panels C and D show that the threat of unemployment causes the young individual to reduce consumption and save more in order to support a decent consumption level in case of unemployment. If the individual becomes unemployed before a substantial wealth buffer has been build up, the consumption rate has to be lowered considerably, as can be seen in Panel D by the fact that the 10th consumption percentile early in life is much lower in the presence of unemployment risk. From around age 40, the average consumption levels with and without unemployment risk are almost identical. Whether unemployment risk is included or not, the consumption profile has a hump-shaped pattern as seen in the data (Thurow, 1969; Gourinchas and Parker, 2002). With unemployment risk, financial wealth is slightly larger early in life, but it grows at a lower average rate due to the lower stock investments. Eventually the average wealth becomes lower with unemployment risk than without, but differences remain small throughout life.

4.2 The role of business cycle risk

In Figure 5, the confidence bands formed by the 10th and 90th percentiles are somewhat wider than reported by, e.g., Cocco et al. (2005) and Bagliano et al. (2019). This is due to the inclusion of business cycle risk in our model through the x variable that impacts expected income growth in both model versions, as well as un- and reemployment probabilities in the version with unemployment risk. In particular, we see from Panel C that, in contrast to comparable models, our model generates not only a lower average stock weight early in life, but also a probability of a zero stock investment even with our modest risk aversion of 5, which emphasizes that unemployment risk can help rationalizing why many young individuals invest nothing or little in stocks. A common mechanism to obtain zero stock investments is to impose sizeable one-time entry costs or per-period costs for being active in the stock market. When taking unemployment risk into account, lower costs are needed to achieve this. The optimal stock weight could be reduced further by adding a small market crash probability as done by Fagereng et al. (2017).

Table 5 illustrates how optimal decisions vary over the business cycle in our model. The table lists the optimal portfolio weight π and consumption rate c for three different values of the state x for each employment status $i \in \{u, e\}$, at different ages t, and for the average value of the wealth-salary ratio W/y at those ages. Recall that a higher value of x is associated with higher expected salary growth, lower unemployment probability, and higher reemployment probability. Hence, consumption increases with the state x, whether the individual is currently employed or not.

[Table 5 about here.]

The relation of the portfolio weight and the state of the economy is more intricate as the latter affects both the magnitude and riskiness of the human capital. Unlike the standard diffusion income model, the distribution of future human capital is asymmetric if we condition on the current employment status. The human capital of an employed individual has a large downside risk component due to the risk of unemployment, which is large and particularly detrimental for young individuals. Conversely, the unemployed individual has a significant probability of finding a job which would cause a large increase in human capital. With our baseline parametrization, we find that the stock weight of employed [unemployed] individuals tends to increase [decrease] in the state variable, but the variations are modest except for the very young unemployed individuals. The exact dependence of the weight on the state is quite sensitive to the degree of state dependence in the un- and reemployment probabilities and the expected salary growth rate.

Figure 6 explores in more detail the channels through which the business cycle variable affects the optimal stock weight. If we turn off the cyclical variations in expected salary growth (replacing $\xi_1 = 0.0657$ by zero), the human capital becomes less correlated with stock prices so the individual invests a bit more in stocks, as seen by the green curve in Panel A. Turning off the business cycle variations in the un- and reemployment intensities $\eta_{1,u}, \eta_{1,e}$ also has a positive but quantitatively much larger effect on the optimal stock weight, cf. the orange curve in Panel B. In fact, in this case the stock weight profile is similar to the case without unemployment risk, cf. the red curve in Panel C of Figure 5. At first, this may seem to contradict the results of Bagliano et al. (2019) that unemployment risk significantly reduces the stock weight even with constant un- and reemployment probabilities. However, they simply add unemployment risk (with severe consequences for future income) to the standard life-cycle model without adjusting the income diffusion volatility or drift. This leads to a lower and much more risky human capital which, most likely, is the main driver of their low stock weight. We also see from Panel B of Figure 6 that the state dependence of both the un- and the reemployment probabilities are important for the low level of the early-life stock weight. Intuitively, the pro-cyclical variation in the reemployment probability and the counter-cyclical variation in the unemployment probability cause human capital to be more stock-like.

[Figure 6 about here.]

4.3 Effects of a shift in employment status

Next we examine how the individual adjusts her portfolio and consumption when she becomes unemployed. Here we refer again to Table 5 which shows the optimal choices at different ages both in the employment and the unemployment state for the average wealth-salary ratio. The right part of the table shows that the consumption rate is reduced considerably when unemployment hits. The cuts in consumption are relatively larger when unemployment hits at a young age. The 25-year old cuts consumption by approximately 40%, whereas the reduction is 27% at age 35, and 22% at age 55; the reduction is almost invariant across the three values of the state variable x. The intuition is that younger individuals have a smaller wealth buffer to finance consumption in bad times. Moreover, early unemployment has a larger effect on future life-time income than unemployment experienced later in life.

The left part of the table shows that if the macro state is good, the 25-year old should not invest in stocks whether she is employed or not. In bad or average macro states, the 25-year old should in fact increase the stock weight in response to a layoff, which might seem counter intuitive. However, the human capital of the youngest individuals is so risky and stock-like that the stock weight can be decreasing in the magnitude of the human capital. Since the shift from employment to unemployment causes a substantial drop in the human capital of the individual, an increase of the stock weight can therefore be rational. Also, risk is asymmetric: a young unemployed individual in a bad macro state has a limited downside risk but a significant upside potential, so it is highly unlikely that the human capital and the stock holdings would plunge simultaneously. Later in life, human capital is less risky and stock-like so that the stock weight increases with the magnitude of the human capital. Therefore, the human capital reduction caused by the shift from employment to unemployment leads to a decrease in the stock weight. For example, the 35-year old should respond to unemployment by reducing the stock weight from 29.84% to 22.09%, assuming that the economy is in the average state $x = 0$. Overall, a change in employment status has a modest effect on the optimal portfolio weight except for the very young individuals.

4.4 The utility loss from ignoring unemployment risk

Suppose the individual faces unemployment risk as represented by our model, but ignores it and bases her decisions on the no-unemployment model version. By definition, the sub-optimal decisions lead to a lower life-time utility. As commonly done in the literature, we define the wealth-equivalent utility loss ℓ as the fraction of initial wealth and life-time income that the individual would give up in order to replace the sub-optimal strategy by the optimal strategy. Since the decision problem is homogeneous in initial wealth and income, the loss can be calculated as

$$\ell = 1 - \left(\frac{J^{sub}}{J^{opt}} \right)^{\frac{1}{1-\gamma}},$$

where J^{sub} and J^{opt} are the expected life-time utilities under the sub-optimal and optimal strategies, respectively, both under the same initial conditions. Both utilities are calculated using Monte Carlo simulation applying the same set of random number sequences to reduce any simulation bias. Since the sub-optimal strategy may potentially be highly detrimental along relatively few paths (leading to very low utility compared to the optimal strategy), a large number of paths have to be generated to include those extreme paths that are necessary for obtaining a reliable loss measure.[30]

As our focus is the asset allocation implications of unemployment risk, we assume that the individual makes optimal consumption decisions throughout life (given age, income, and wealth), but that the financial wealth is allocated to stocks and bonds based on the sub-optimal strategy ignoring unemployment. In particular, the individual allocates too much to stocks early in life and, according to our calculations, the individual suffers a welfare loss of $\ell = 2.4\%$. To transform this is into a dollar amount, we need a measure of human capital, but the relevant risk-adjusted discount rate on future income is not uniquely defined due to unspanned risks. If we, for illustrative purposes, use a discount rate of 1% (the risk-free rate), our simulations lead to a human capital of \$1.308 million on top of the financial wealth of \$5,000, and thus a welfare loss of around \$31,500. Along most paths the individual is, in fact, better off with a high stock weight as stock prices tend to

[30] We use 100,000 paths. If a path involves a value of the business cycle variable x or the scaled wealth-income ratio $e^{-B(t)} W_t / y_t$ outside our grid, we discard the path and replace it by a new path.

increase over time. But along paths where the individual has accumulated little wealth, is hit by unemployment and low income, and where stock prices also fall, a high stock weight causes a large reduction in wealth and consumption. With a lower stock share, the individual can avoid to cut consumption to the same extent. Due to the concavity of the utility function, such paths have a large influence on the overall expected utility.

4.5 Selected comparative statics and model variations

Figure 7 shows how the average optimal stock weight through the life cycle is affected by selected model parameters. In all panels the blue curve refers to the benchmark parametrization. First, in Panel A we vary the fraction α_u of the salary that the individual receives while being unemployed from the baseline 30% to 10% and 50%. An increase in α_u makes unemployment periods less troubling and thus leads to a larger future expected income and a lower risk of experiencing very low income. Due to the larger and less risky human capital, the stock weight is optimally increased. However, even when benefits are 50% of prior salary, the optimal stock weight in the early adult years is much lower than in the standard life-cycle model.

[Figure 7 about here.]

Panel B considers the salary drift $\xi_0(t, u)$ during unemployment, which controls both how the unemployment benefits evolve during the unemployment period and the salary the individual can expect to earn upon reemployment. Our benchmark value is -10%. The graph shows the importance of taking this consequence of unemployment into account and that the stock weight decreases when $\xi_0(t, u)$ becomes more negative. Individuals for which unemployment periods have larger negative effects on their future expected salary should thus invest less in stocks. In the case $\xi_0(t, u) = 0$ where unemployment does not lead to lower expected future income, the optimal stock weight follows a glide-path pattern as in the standard life-cycle model, although at a lower level.

Panel C confirms that the stock weight is decreasing in the relative risk aversion γ. The magnitudes of the stock weight are noteworthy, however. In the standard model ignoring unemployment, Cocco et al. (2005) report that even with a risk aversion of 10, the optimal stock weight is 100% until age 40 or so and stays above 50% throughout life. In our model, the optimal stock weight for a given level of risk aversion is much smaller, in particular early in working life. Even with a risk aversion of 3 (magenta curve), the average optimal stock weight is below the 100%. For sufficiently high levels of risk aversion, the optimal stock weight is zero early in life.

Finally, Panel D shows that, as expected, the average optimal stock weight is decreasing in the volatility of the salary, at least early in life whereas later in life the portfolio weights are also indirectly affected by changes in savings induced by the different volatility. However, even the rather dramatic change of halving the salary volatility has only a modest effect on the stock position. For portfolio planning, the unemployment risk is much more important than the salary volatility.

Our baseline results are generated using an estimate of an extended unemployment rate that to the U6 rate adds a fraction of individuals who have not looked for a job in the most recent 12 months. As a robustness check we have also generated results using only the 2017 U6 rate as a calibration target (parameter values, including the business cycle sensitivities $\eta_{1,e}$ and $\eta_{1,u}$, are unchanged). As depicted in Panel A of Figure 8, the targeted unemployment rate is then significantly lower than assumed in the benchmark case. Nevertheless, Panel B shows that the lower unemployment risk leads to only a small increase in the optimal portfolio weight of the stock, which is still much lower for young adults than the 100% recommended by standard life-cycle models.

[Figure 8 about here.]

In our baseline model calibration the individual accumulates a larger financial wealth than what most people seem to do according to available data.[31] Related life-cycle models face the same challenge. As shown in Panel A of Figure 5, the introduction of unemployment risk induces the individual to build up larger buffer savings to maintain a decent consumption level in unemployment, but on the other hand the returns on the savings are lower because of the smaller stock weight than in the standard no-unemployment model so eventually the financial wealth becomes lower than in the standard model. Fagereng et al. (2017) calibrate a life-cycle model without unemployment risk to Norwegian household-level data on stock investments and financial wealth. They find that a good match requires, among other things, a large subjective time preference rate δ (in the range 0.19 to 0.28; they report $e^{-\delta}$), cf. their Table V. By increasing δ in our model, we can also reduce wealth accumulation considerably. For example, the average financial wealth at retirement is roughly 50% lower with $\delta = 0.12$ compared to the baseline case. Nevertheless, Panel A of Figure 9 shows that the stock share early in life hardly changes when δ is increased from the baseline value of 0.04 to 0.06 or even 0.12. Our main conclusion that an appropriately modeled unemployment risk lowers early-life stock investments is therefore robust to changes in δ that generate a more realistic wealth accumulation pattern.[32]

[31] For example, in the 2016 Survey of Consumer Finances median net worth was 11.100 USD for families below age 35 growing to 224.100 USD in the age group 65-74 years, cf. Bricker et al. (2017). The mean net worth is much larger than the median but exhibits a similar growth rate over life.

[32] In these models, individuals run the "risk" of reaching a high age so they need to maintain a high level of wealth to support possible future consumption. If individuals could invest in fairly-priced annuities paying out as long as they live, lower savings were needed.

[Figure 9 about here.]

We have extended the model to Epstein-Zin utility which can also change the savings pattern by disentangling risk aversion and intertemporal substitution; see the end of Appendix B for more information. Panel B of Figure 9 shows that the stock share through working life is affected very little if ψ is changed from the benchmark value of $1/\gamma = 0.2$ to 0.5 or 0.1 so our main conclusion is robust also to this extension of preferences.

Finally, we have extended our model to time-varying expected stock returns of the form $r+\mu+\mu_1 x_t$, where our main model assumes $\mu_1 = 0$. The calibration of the extended model leads to $\mu_1 = -0.1639$ which captures the counter-cyclical variations in the equity premium suggested by the return predictability literature. The estimate of μ_1 is not statistically significantly different from zero, however.[33] Figure 10 shows that with unemployment risk (Panel A) return predictability leads to a slightly higher average stock weight at any age, which is natural since predictability reduces the risk of long-term stock investments. This is consistent with the findings of Kim and Omberg (1996) and Wachter (2002) in settings without labor income. Without unemployment risk (Panel B), counter-cyclical returns lead to a *lower* average stock weight early in life. Because human capital is large and modestly risky in this case, the optimal stock weight early in life is 100% with $\mu_1 = 0$. With $\mu_1 < 0$, the optimal stock weight is still 100% if x is negative (meaning higher-than-average expected returns) or near zero, but less than 100% if x is sufficiently positive. Across all states, the average stock weight is therefore less than 100%. Nevertheless, by comparing the two panels, we see that also in the model with counter-cyclical stock returns, the optimal stock weight is much lower early in life when unemployment risk is taken into account. With or without unemployment risk, the non-constant expected returns lead to substantial variations in portfolio weights over time as witnessed by the much wider confidence bands than in the case with $\mu_1 = 0$. Most investors are not varying their portfolio composition to such a large extent, and given the uncertainty about the degree of return predictability they should probably not do so, cf., e.g., the discussion in Wachter and Warusawitharana (2009).

[Figure 10 about here.]

5 Conclusion

Unemployment is one of the major risks faced by consumers as it reduces income not only during the unemployment spell, but also in subsequent reemployment and eventually in retirement. Unemployment risk varies both with age and macroeconomic conditions.

[33] The estimates of other parameters change very little when μ_1 is included. Details are available from the authors upon request.

This paper shows that a careful modeling of these features substantially changes the optimal investment strategy over the life cycle. The downward-sloping glide path strategy for the stock portfolio weight from standard life-cycle models—being implemented by target date investment funds—is replaced by an upward-sloping profile starting at a very low level, sometimes even zero. The output from our model fits observed stock holdings better than the standard no-unemployment life-cycle model.

A Filtering of the model

For completeness, we outline the filtering procedure for the extended model with state-dependent expected stock return considered at the end of Section 4.5. To get the filtered model as stated in equation (7)-(9) we rely on Theorem 12.7 in Liptser and Shiryaev (2001). First, we rewrite the system of unobservable and observable state variables. The dynamics of the unobservable state variable is written as

$$dX_t = -\kappa_x X_t \, dt + \Sigma_X' \, dZ_t$$

where $\Sigma_X = (\sigma_X, 0, 0, 0)'$, whereas the system of observable state variables is written as

$$\begin{pmatrix} dS_t/S_t \\ dv_{1t} \\ dv_{2t} \end{pmatrix} = \left[\begin{pmatrix} r+\mu \\ \omega_1 - \kappa_{v_1} v_{1t} \\ \omega_2 - \kappa_{v_2} v_{2t} \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \kappa_{x_1} \\ \kappa_{x_2} \end{pmatrix} X_t \right] dt + \begin{pmatrix} \Sigma_S' \\ \Sigma_{v_1}' \\ \Sigma_{v_2}' \end{pmatrix} dZ_t,$$

where $\Sigma_S = \left(\sigma_S k_{SX}, \sigma_S \sqrt{1-k_{SX}^2}, 0, 0\right)'$, and $\Sigma_v = (\Sigma_{v_1}, \Sigma_{v_2})'$ is a (4×2)-dimensional matrix. Σ_v is chosen such that the volatility of the first signal v_1 equals σ_{v_1}, the volatility of the second signal v_2 equals σ_{v_2}, and the correlation between the two signals equals k_v:

$$\Sigma_v' \Sigma_v = \begin{pmatrix} \sigma_{v_1}^2 & \sigma_{v_1}\sigma_{v_2} k_v \\ \sigma_{v_1}\sigma_{v_2} k_v & \sigma_{v_2}^2 \end{pmatrix}.$$

Furthermore, the correlation between X and v_1, and X and v_2 equal k_{Xv_1} and k_{Xv_2}, respectively, i.e., $\Sigma_X' \Sigma_v = (k_{Xv_1}\sigma_x\sigma_{v_1}, k_{Xv_2}\sigma_x\sigma_{v_2})$.

From Theorem 12.7 in Liptser and Shiryaev (2001), it now follows that the filtered model (as seen by the investor) is given by the system

$$dx_t = -\kappa_x x_t \, dt + K_S \hat{\Sigma}_S' \, dz_t + K_v' \hat{\Sigma}_v' \, dz_t, \tag{15}$$

$$dS_t = S_t \left[(r + \mu + \mu_1 x_t) \, dt + \hat{\Sigma}_S' \, dz_t \right], \tag{16}$$

$$dv_{1t} = (\omega_1 - \kappa_{v_1} v_{1t} + \kappa_{x_1} x_t) \, dt + \hat{\Sigma}_{v_1}' \, dz_t, \tag{17}$$

$$dv_{2t} = (\omega_1 - \kappa_{v_2} v_{2t} + \kappa_{x_2} x_t) \, dt + \hat{\Sigma}_{v_2}' \, dz_t, \tag{18}$$

where z is a four-dimensional standard Brownian motion, describing the perceived innovations to the stock price, the two signals, and the salary level. That is, the investor relies on the perceived innovations $\hat{\Sigma}_v' dz_t$ and $\hat{\Sigma}_S' dz_t$ in the signals and the stock price to learn about X. Here $\hat{\Sigma}_v$ is a (4×2)-dimensional matrix, and $\hat{\Sigma}_S$ is a four-dimensional vector, which we define below. The constant K_S and the constant two-dimensional vector

$K_v = (K_{v_1}, K_{v_2})'$ are given by

$$\left(K_S \; K_v'\right) = \left(b\, B' + k\, A_1'\right) \left(B\, B'\right)^{-1},$$

where, following notation in Liptser and Shiryaev (2001), we have that

$$b = \Sigma_X', \qquad B' = \left(\Sigma_S', \Sigma_{v_1}', \Sigma_{v_2}'\right), \qquad A_1' = \left(\mu_1, \kappa_{x_1}, \kappa_{x_2}\right).$$

Furthermore, k is the steady-state variance of the estimation error and is the positive solution to the equation

$$k_2 k^2 + k_1 k + k_0 = 0,$$

where

$$\begin{aligned} k_0 &= \left(b\, B'\right) \left(B\, B'\right)^{-1} \left(b\, B'\right)' - b\, b', \\ k_1 &= 2\kappa_x + 2\left(b\, B'\right)\left(B\, B'\right)^{-1} A_1, \\ k_2 &= A_1' \left(B\, B'\right)^{-1} A_1. \end{aligned}$$

The variance of the estimation error is generally a deterministic function of time but, for simplicity, we assume that it has already converged to its long-run level.[34] The dynamics of the investor's salary level in the filtered model equals

$$dy_t = y_t \left[\left(\xi_0(t, I_t) + \xi_1(I_t) x_t\right) dt + \hat{\Sigma}_y' dz_t \right] \tag{19}$$

where $\hat{\Sigma}_y$ is a four-dimensional vector. Here $\hat{\Sigma}_v$, $\hat{\Sigma}_S$, and $\hat{\Sigma}_y$ follow from the fact that filtering does not change the variance-covariance matrix of the observed variables v, S, and y, i.e.,

$$(\hat{\Sigma}_S, \hat{\Sigma}_v, \hat{\Sigma}_y)'(\hat{\Sigma}_S, \hat{\Sigma}_v, \hat{\Sigma}_y) = (\Sigma_S, \Sigma_v, \Sigma_y)'(\Sigma_S, \Sigma_v, \Sigma_y).$$

The system (7)-(9) now follows directly from rewriting the above system of equations, and noting that

$$\rho_{Sx} = \frac{\hat{\Sigma}_x' \hat{\Sigma}_S}{\sigma_S \sigma_x}, \quad \rho_{xy} = \frac{\hat{\Sigma}_x' \hat{\Sigma}_y}{\sigma_x \sigma_y}, \quad \rho_{Sy} = k_{Sy}, \tag{20}$$

where $\hat{\Sigma}_x' = K_S \hat{\Sigma}_S' + K_v' \hat{\Sigma}_v'$ and $\sigma_x^2 = \hat{\Sigma}_x' \hat{\Sigma}_x$.

[34] The same assumption has been made by Scheinkman and Xiong (2003) and Dumas, Kurshev, and Uppal (2009), among others.

B The solution method

We describe the solution method for the extended model with state-dependent expected stock return considered at the end of Section 4.5. Following Brennan et al. (1997) and Munk and Sørensen (2010), we implement a finite difference backwards iterative solution of the Hamilton-Jacobi-Bellman (HJB) equation with an optimization over feasible consumption rates and portfolios at each (time , state) node in the grid. During the investor's working life we keep track of her employment status so our algorithm handles two grids. With $i \in \{e, u\}$ denoting the employment status, the HJB-equation for the indirect utility function $J^i = J(t, W, x, y; i)$ for state i is

$$\begin{aligned}(\delta+\nu)\, J^i = \sup_{c,\pi} \Bigg\{ & \frac{c^{1-\gamma}}{1-\gamma} + J^i_t + J^i_W \left[W\left(r + \pi\left(\mu + \mu_1 x\right)\right) - c + \alpha_i y\right] - J^i_x \kappa_x x \\ & + J^i_y \left(\xi^i_0 + \xi^i_1 x\right) y + \frac{1}{2} J^i_{WW} W \pi^2 \sigma_S^2 + \frac{1}{2} J^i_{xx} \sigma_x^2 + \frac{1}{2} J^i_{yy} y^2 \sigma_y^2 \\ & + J^i_{Wx} W \sigma_x \pi \sigma_S \rho_{Sx} + J^i_{Wy} W y \pi \sigma_S \sigma_y \rho_{Sy} + J^i_{xy} y \sigma_x \sigma_y \rho_{xy} \\ & + \left(J^{\neg i} - J^i\right) \eta_{i\to\neg i} + \nu \left(\frac{W}{\varepsilon}\right)^{1-\gamma} \frac{1-e^{-\delta\varepsilon}}{\delta(1-\gamma)} \Bigg\}.\end{aligned} \tag{21}$$

Subscripts on J indicate partial differentiation and $\neg i$ denotes the other employment state (i.e., $\neg e = u, \neg u = e$). Furthermore, $\alpha_i = 1$ when the investor is employed and $\alpha_i = \alpha_u$ when the investor is unemployed, $\eta_{i\to\neg i}$ equals the intensity from jumping from state i to $\neg i$, and ν is the mortality intensity.

We utilize the homogeneity property of the value function to reduce the number of state variables by one. In particular, we write the value function as

$$J(t, W, x, y; i) = y^{1-\gamma} G\left(t, w, x; i\right) \tag{22}$$

where $w = e^{-B(t)} \frac{W}{y}$ and the term $e^{-B(t)}$ is a scaling function that insures that the scaled wealth/income ratio is more stable and easier to handle numerically. We set

$$B(t) = \begin{cases} \beta_w t & \text{if } t < \tilde{T} \\ \beta_w \tilde{T} + \beta_r \left(t - \tilde{T}\right) & \text{if } t \geq \tilde{T} \end{cases}$$

with $\beta_w \geq 0$ and $\beta_r \leq 0$ since the wealth-income ratio tends to increase before and decrease after retirement, and the exact values are found experimentally for each parameter constellation (more below). The relevant derivatives of J satisfy

$$J_t = y^{1-\gamma} G^i_t - y^{1-\gamma} \beta_w G^i_w e^{-B(t)} \frac{W}{y}, \quad J_W = G^i_w e^{-B(t)} y^{-\gamma},$$

$$
\begin{aligned}
J_x &= y^{1-\gamma} G_x^i, \quad J_y = (1-\gamma)\, y^{-\gamma} G^i - G_w^i y^{-\gamma} e^{-B(t)} \frac{W}{y}, \\
J_{WW} &= G_{ww}^i e^{-2B(t)} y^{-1-\gamma}, \quad J_{xx} = y^{1-\gamma} G_{xx}^i\,, \quad J_{Wx} = G_{wx}^i e^{-B(t)} y^{-\gamma}, \\
J_{yy} &= -\gamma\,(1-\gamma)\, y^{-\gamma-1} G^i + 2\gamma G_w^i y^{-1-\gamma} e^{-B(t)} \frac{W}{y} + G_{ww}^i y^{-1-\gamma} e^{-2B(t)} \left(\frac{W}{y}\right)^2, \\
J_{Wy} &= -G_{ww}^i y^{-1-\gamma} e^{-2B(t)} \frac{W}{y} - \gamma G_w^i y^{-1-\gamma} e^{-B(t)}, \\
J_{xy} &= (1-\gamma)\, y^{-\gamma} G_x^i - G_{wx}^i y^{-\gamma} e^{-B(t)} \frac{W}{y}.
\end{aligned}
$$

By substituting (22) into the HJB-equation (21) we get that $G^i = G(t, w, x; i)$ before retirement, i.e. $t < \tilde{T}$, solves the non-linear partial differential equation (PDE)

$$
\begin{aligned}
0 = \sup_{\hat{c},\pi} \Bigg\{ & G^i \left[-(\delta+\nu) + (1-\gamma)\left(\xi_0^i + \xi_1^i x\right) - \frac{1}{2}\gamma\,(1-\gamma)\,\sigma_y^2 - \eta_{i\to\neg i} \right] \\
& + G_w^i \Big[-\beta_w w + w\,(r + \pi\,(\mu + \mu_1 x)) + e^{-B(t)}\,(\alpha_i - \hat{c}) \\
& \qquad - w\left(\xi_0^i + \xi_1^i x\right) + \gamma w \sigma_y^2 - \gamma w \pi \sigma_S \sigma_y \rho_{Sy} \Big] \\
& + G_x^i \left[-\kappa_x x + \sigma_x \sigma_y \rho_{xy} \right] + \frac{1}{2} G_{ww}^i \left[\pi^2 \sigma_S^2 + \sigma_y^2 - 2\pi\sigma_S\sigma_y\rho_{Sy} \right] w^2 \\
& + \frac{1}{2} G_{xx}^i \sigma_x^2 + G_{wx}^i \left[\pi \sigma_x \sigma_S \rho_{Sx} - \sigma_x \sigma_y \rho_{xy} \right] w \\
& + G^{\neg i} \eta_{i\to\neg i} + G_t^i + \frac{1}{1-\gamma}\hat{c}^{1-\gamma} + \nu \left(\frac{e^{B(t)} w}{\varepsilon}\right)^{1-\gamma} \frac{1 - e^{-\delta\varepsilon}}{\delta(1-\gamma)} \Bigg\},
\end{aligned}
\tag{23}
$$

where $\hat{c} = \frac{c}{y}$. The optimal unconstrained consumption and portfolio choice for a strictly positive value of w is given by the first-order conditions from (23), i.e.,

$$
\hat{c}_t = \left(G_w^i\right)^{-1/\gamma} e^{B(t)/\gamma}, \tag{24}
$$

$$
\pi_t = -\frac{G_w^i}{w G_{ww}^i} \frac{\mu + \mu_1 x}{\sigma_S^2} - \frac{G_{wx}^i}{w G_{ww}^i} \frac{\sigma_x \sigma_S \rho_{Sx}}{\sigma_S^2} + \left(1 + \frac{\gamma G_w^i}{w G_{ww}^i}\right) \frac{\sigma_y \sigma_S \rho_{Sy}}{\sigma_S^2}. \tag{25}
$$

In retirement, there is no employment status to keep track of, and the HJB equation for the indirect utility function $J(t, W, x, y)$ is here

$$
\begin{aligned}
(\delta+\nu)\, J = \sup_{c,\pi} \Bigg\{ & \frac{c^{1-\gamma}}{1-\gamma} + J_t + J_W \left[W\,(r + \pi\,(\mu + \mu_1 x)) - c + \alpha_R y \right] - J_x \kappa_x x \\
& + \frac{1}{2} J_{WW} W \pi^2 \sigma_S^2 + \frac{1}{2} J_{xx} \sigma_x^2 + \frac{1}{2} J_{yy} y^2 \hat{\sigma}_y^2 + J_{Wx} W \sigma_x \pi \sigma_S \rho_{Sx} \\
& + \nu \left(\frac{W}{\varepsilon}\right)^{1-\gamma} \frac{1 - e^{-\delta\varepsilon}}{\delta(1-\gamma)} \Bigg\}.
\end{aligned}
\tag{26}
$$

Again we use the homogeneity property of the value function and write the value function as stated in (22). Following the same steps as above, we get that $G(t, w, x)$ after retirement, i.e. for $\tilde{T} \leq t \leq T$, solves the non-linear PDE

$$
\begin{aligned}
0 = \sup_{\hat{c},\pi} \Bigg\{ & \frac{1}{1-\gamma}\hat{c}^{1-\gamma} + G_t + G\left[-(\delta+\nu) - \frac{1}{2}\gamma(1-\gamma)\hat{\sigma}_y^2\right] \\
& + G_w\left[-\beta_r w + w\left(r + \pi\left(\mu + \mu_1 x\right)\right) + e^{-B(t)}\left(\alpha_R - \hat{c}\right) + \gamma w \hat{\sigma}_y^2\right] \\
& + G_x\left[-\kappa_x x + \sigma_x\sigma_y\rho_{xy}\right] + \frac{1}{2}G_{ww}\left[\pi^2\sigma_S^2 + \sigma_y^2\right]w^2 \\
& + \frac{1}{2}G_{xx}\sigma_x^2 + G_{wx}\left[\pi\sigma_x\sigma_S\rho_{Sx}\right]w + \nu\left(\frac{e^{B(t)}w}{\varepsilon}\right)^{1-\gamma}\frac{1-e^{-\delta\varepsilon}}{\delta(1-\gamma)}\Bigg\}
\end{aligned}
\tag{27}
$$

with terminal condition

$$
G(T, w, x) = \left(\frac{e^{B(T)}w}{\varepsilon}\right)^{1-\gamma}\frac{1-e^{-\delta\varepsilon}}{\delta(1-\gamma)}. \tag{28}
$$

The unconstrained optimal choice is given by the first-order conditions

$$
\hat{c}_t = \left(G_w\right)^{-1/\gamma} e^{B(t)/\gamma}, \tag{29}
$$

$$
\pi_t = -\frac{G_w}{wG_{ww}}\frac{\mu + \mu_1 x}{\sigma_S^2} - \frac{G_{wx}}{wG_{ww}}\frac{\sigma_x\sigma_S\rho_{Sx}}{\sigma_S^2}. \tag{30}
$$

We solve the PDEs (23) and (27) by a finite difference method on equally spaced grids in (t, w, x)-space working backwards from the terminal condition (28). The grid bounds for x are $\pm 2\sigma_x/\sqrt{2\kappa_x}$, i.e., two times the standard deviation of the stationary distribution. For w, the appropriate bounds $\underline{w}, \overline{w}$ depend on the assumed initial values of W and y, as well as the grid scaling parameters β_w and β_r and other parameters affecting the dynamics of wealth and income. For our benchmark parametrization, we use $\underline{w} = 0.01$, $\overline{w} = 6$, $\beta_w = 0.04$, and $\beta_r = 0$. These choices ensure that once we have solved for the optimal strategies on the grid and use them to simulate the system forward in time, then the generated values of w almost always fall in the interval $[\underline{w}, \overline{w}]$ and most often fall near the middle of the grid, where the numerical method is more precise. After diligent experimentation with the number of grid points, we end up using time steps of length $\Delta t = 1/12$, corresponding to a month, and using 101 grid points for w and 21 for x.

We work backwards from time T to time 0. The step from any time $t+\Delta t$ to t involves a search for the best choice of $\hat{c}(t, w, x, i)$ and $\pi(t, w, x, i)$ in any grid point (w, x) at time t. The initial guess is simply the optimal values found at time $t + \Delta t$; as optimal strategies tend to vary slowly with time, this is a good starting point. Then we apply these values of $\hat{c}$ and π in the PDE for G at time t. As in the implicit finite difference method frequently used in option pricing to solve Black-Scholes-like PDEs, we replace the derivatives by appropriate differences involving values of G only in grid points; as Brennan et al. (1997), we use so-called "up-wind" approximations of the derivatives to stabilize the approach. In this way we obtain a large system of equations that we solve for values of $G(t,w,x;i)$ in all grid points (w,x,i). Based on this estimate of the time t value function G, the first-order conditions (24)–(25) (or (29)–(30) in the retirement phase) lead to a new guess on the choices $\hat{c}(t,w,x,i)$ and $\pi(t,w,x,i)$ at time t; the portfolio weight is constrained to the interval $[0,1]$. We continue these iterations until the largest change in the value function over all grid points relative to the previous iteration is below some small threshold (we use 0.1%). Typically, 2-4 iterations are necessary at each time step.

The extension to Epstein-Zin utility. In our continuous-time setting, Epstein-Zin utility means that the indirect utility J_t of the individual satisfies

$$J_t = \sup_{(c,\pi)} \mathrm{E}_t \left[\int_t^\tau f(c_s, J_s)\, ds + \left(\frac{W_\tau}{\varepsilon}\right)^{1-\gamma} \frac{\left(1-e^{-\delta\varepsilon}\right)^\theta}{\delta(1-\gamma)} \right], \tag{31}$$

where

$$f(c,J) = \frac{1}{1-\frac{1}{\psi}} c^{1-\frac{1}{\psi}} \left([1-\gamma]\delta J\right)^{1-\frac{1}{\theta}} - \delta\theta J, \qquad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}},$$

and $\psi > 0$ is the elasticity of intertemporal substitution. The bequest utility in the above expression is the Epstein-Zin utility of a constant consumption rate of W_τ/ε over ε years. We assume that $\gamma \neq 1$ and $\psi \neq 1$, but cases with $\gamma = 1$ or $\psi = 1$ or both can be studied separately with appropriate specifications of f. Our standard assumption of time-additive power utility is the special case where $\psi = 1/\gamma$ and thus $\theta = 1$ and $f(c,J) = \frac{1}{1-\gamma}c^{1-\gamma} - \delta J$ since in this case the recursion (31) is satisfied by the J defined by (12). With the extension to Epstein-Zin utility, the HJB equation (21) is modified to

$$\begin{aligned}
(\delta\theta + \nu)\, J^i = \sup_{c,\pi} \Bigg\{ & \frac{1}{1-\frac{1}{\psi}} c^{1-\frac{1}{\psi}} \left([1-\gamma]\delta J^i\right)^{1-\frac{1}{\theta}} + J^i_t + J^i_W \left[W\left(r + \pi\left(\mu + \mu_1 x\right)\right) - c + \alpha_i y\right] \\
& - J^i_x \kappa_x x + J^i_y \left(\xi^i_0 + \xi^i_1 x\right) y + \frac{1}{2} J^i_{WW} W \pi^2 \sigma_S^2 + \frac{1}{2} J^i_{xx} \sigma_x^2 + \frac{1}{2} J^i_{yy} y^2 \sigma_y^2 \\
& + J^i_{Wx} W \sigma_x \pi \sigma_S \rho_{Sx} + J^i_{Wy} W y \pi \sigma_S \sigma_y \rho_{Sy} + J^i_{xy} y \sigma_x \sigma_y \rho_{xy} \\
& + \left(J^{\neg i} - J^i\right) \eta_{i\to\neg i} + \nu \left(\frac{W}{\varepsilon}\right)^{1-\gamma} \frac{\left(1-e^{-\delta\varepsilon}\right)^\theta}{\delta(1-\gamma)} \Bigg\}.
\end{aligned}$$

The conjecture (22) implies that G^i must satisfy the PDE

$$
\begin{aligned}
0 = \sup_{\hat{c},\pi} \Bigg\{ & G^i \left[-(\delta\theta + \nu) + (1-\gamma)\left(\xi_0^i + \xi_1^i x\right) - \frac{1}{2}\gamma(1-\gamma)\sigma_y^2 - \eta_{i\to\neg i} \right] \\
& + G_w^i \Big[-\beta_w w + w\left(r + \pi\left(\mu + \mu_1 x\right)\right) + e^{-B(t)}\left(\alpha_i - \hat{c}\right) \\
& \qquad - w\left(\xi_0^i + \xi_1^i x\right) + \gamma w \sigma_y^2 - \gamma w \pi \sigma_S \sigma_y \rho_{Sy} \Big] \\
& + G_x^i \left[-\kappa_x x + \sigma_x \sigma_y \rho_{xy}\right] + \frac{1}{2} G_{ww}^i \left[\pi^2 \sigma_S^2 + \sigma_y^2 - 2\pi\sigma_S\sigma_y\rho_{Sy}\right] w^2 \\
& + \frac{1}{2} G_{xx}^i \sigma_x^2 + G_{wx}^i \left[\pi \sigma_x \sigma_S \rho_{Sx} - \sigma_x \sigma_y \rho_{xy}\right] w \\
& + G^{\neg i}\eta_{i\to\neg i} + G_t^i + \frac{1}{1-\frac{1}{\psi}} \hat{c}^{1-\frac{1}{\psi}} \left([1-\gamma]\delta G^i\right)^{1-\frac{1}{\theta}} + \nu \left(\frac{e^{B(t)} w}{\varepsilon}\right)^{1-\gamma} \frac{\left(1 - e^{-\delta\varepsilon}\right)^{\theta}}{\delta(1-\gamma)} \Bigg\}
\end{aligned}
$$

with terminal condition $G(T, w, x) = \frac{1}{\delta(1-\gamma)} \left(\frac{e^{B(T)} w}{\varepsilon}\right)^{1-\gamma} \left(1 - e^{-\delta\varepsilon}\right)^{\theta}$. The first-order condition for $\hat{c}$ becomes

$$\hat{c}_t = \left(G_w^i\right)^{-\psi} e^{\psi B(t)} \left([1-\gamma]\delta G^i\right)^{\psi\left(1-\frac{1}{\theta}\right)}. \tag{32}$$

For the retirement phase, the PDE is modified as in the case of time-additive power utility.

C Estimation procedure

We use a Kalman Filter and maximum likelihood to obtain a time series of the unobservable state process x and our parameter estimates. For completeness, we explain the procedure for the extended model with state-dependent expected stock return considered at the end of Section 4.5. First we discretize our continuous-time model:[35]

$$R_{t+\Delta t} = \left(r + \mu - \frac{1}{2}\sigma_S^2\right)\Delta t + \mu_1 b(\Delta t) x_t + \varepsilon_{R,t+\Delta t}, \tag{33}$$

$$v_{t+\Delta t} = v_t + \left(\theta_v - \kappa_v v_t\right)\Delta t + \kappa_{vx} b(\Delta t) x_t + \varepsilon_{v,t+\Delta t}, \tag{34}$$

$$x_{t+\Delta t} = e^{-\kappa_x \Delta t} x_t + \varepsilon_{x,t+\Delta t}, \tag{35}$$

[35] When discretizing the process of v, we use an Euler discretization for the mean-reversion term in v to simplify the resulting system.

where $R_{t+1} = \ln S_{t+1} - \ln S_t$ is the log return, $b(\tau) = \frac{1}{\kappa_x}\left(1 - e^{-\kappa_x \tau}\right)$, and ε_R, ε_v, and ε_x are normally distributed error terms given by

$$\begin{aligned}
\varepsilon_{R,t+\Delta t} &= \int_t^{t+\Delta t} \left(\mu_1 b(t+\Delta t - u)\Sigma_x + \Sigma_S\right)' \, dz_u, \\
\varepsilon_{v,t+\Delta t} &= \begin{pmatrix} \varepsilon_{v_1,t+\Delta t} \\ \varepsilon_{v_2,t+\Delta t} \end{pmatrix} = \int_t^{t+\Delta t} \left(\kappa_v b(t+\Delta t - u)\Sigma_x + \Sigma_v\right)' \, dz_u, \\
\varepsilon_{x,t+\Delta t} &= \int_t^{t+\Delta t} e^{-\kappa_x (t+\Delta t - u)} \Sigma_x' \, dz_u.
\end{aligned}$$

Second, we need to set up the model on a state space form and thus specify the state equation and the observation equation (see Hamilton (1994) for details). Including the stock return and signals as state variables, it follows from (33)–(35) that the state equation is given by

$$\begin{pmatrix} R_{t+\Delta t} \\ v_{1,t+\Delta t} \\ v_{2,t+\Delta t} \\ x_{t+\Delta t} \end{pmatrix} = \begin{pmatrix} r + \mu - \frac{1}{2}\sigma_S^2 \\ \theta_{v_1} \\ \theta_{v_2} \\ 0 \end{pmatrix} \Delta t + \begin{pmatrix} 0 & 0 & 0 & \mu_1 b(\Delta t) \\ 0 & 1 & 0 & \kappa_{v_1} b(\Delta t) \\ 0 & 0 & 1 & \kappa_{v_2} b(\Delta t) \\ 0 & 0 & 0 & e^{-\kappa_x \Delta t} \end{pmatrix} \begin{pmatrix} R_t \\ v_{1,t} \\ v_{2,t} \\ x_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{R,t+\Delta t} \\ \varepsilon_{v_1,t+\Delta t} \\ \varepsilon_{v_2,t+\Delta t} \\ \varepsilon_{x,t+\Delta t} \end{pmatrix}$$

and the observation equation is simply

$$\begin{pmatrix} R_t \\ v_{1,t} \\ v_{2,t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} R_t \\ v_{1,t} \\ v_{2,t} \\ x_t \end{pmatrix}.$$

The variance of the disturbances in the state equation is given by

$$\begin{aligned}
\mathrm{Var}(\varepsilon_{R,t}) &= \sigma_S^2 \Delta t + \mu_1^2 \sigma_x^2 B_2(\Delta t) + 2\sigma_S^2 \sigma_S \rho_{Sx} B_1(\Delta t), \\
\mathrm{Var}(\varepsilon_{v_i,t}) &= \kappa_{v_i}^2 \sigma_x^2 B_2(\Delta t) + \sigma_{v_i}^2 \Delta t + 2\kappa_{v_i} \sigma_x \sigma_{v_i} \rho_{x v_i} B_1(\Delta t), \\
\mathrm{Var}(\varepsilon_{x,t}) &= \frac{1}{2\kappa_x}\left(1 - e^{-2\kappa_x \Delta t}\right)\sigma_x^2,
\end{aligned}$$

with $i = 1, 2$, and

$$\begin{aligned}
B_1(\Delta t) &= \frac{1}{\kappa_x}\left(\Delta t - b(\Delta t)\right), \\
B_2(\Delta t) &= \frac{1}{\kappa_x^2}\left(\Delta t - b(\Delta t)\right) - \frac{1}{2\kappa_x} b(\Delta t)^2.
\end{aligned}$$

The covariances between the disturbances are given by

$$
\begin{aligned}
\mathrm{Cov}(\varepsilon_{R,t}, \varepsilon_{v_i,t}) &= \sigma_S \sigma_{v_i} \rho_{v_i S} \Delta t + \sigma_x^2 \mu_1 \kappa_{v_i} B_2(\Delta t) + (\mu_1 \sigma_{v_i} \rho_{x v_i} + \rho_{Sx} \sigma_S \kappa_{v_i})\, \sigma_x B_1(\Delta t), \\
\mathrm{Cov}(\varepsilon_{R,t}, \varepsilon_{x,t}) &= \frac{1}{2} \mu_1 \sigma_x^2 b(\Delta t)^2 + \rho_{Sx} \sigma_x \sigma_S b(\Delta t), \\
\mathrm{Cov}(\varepsilon_{v_1,t}, \varepsilon_{v_2,t}) &= \kappa_{v_1} \kappa_{v_2} \sigma_x^2 B_2(\Delta t) + \sigma_{v_1} \sigma_{v_2} \rho_v \Delta t + (\kappa_{v_1} \sigma_{v_2} \rho_{x v_2} + \kappa_{v_2} \sigma_{v_1} \rho_{x v_1})\, \sigma_x B_1(\Delta t), \\
\mathrm{Cov}(\varepsilon_{v_i,t}, \varepsilon_{x,t}) &= \frac{\kappa_{v_i}}{2} b(\Delta t)^2 \sigma_x^2 + \sigma_x \sigma_{v_i} \rho_{x v_i} b(\Delta t).
\end{aligned}
$$

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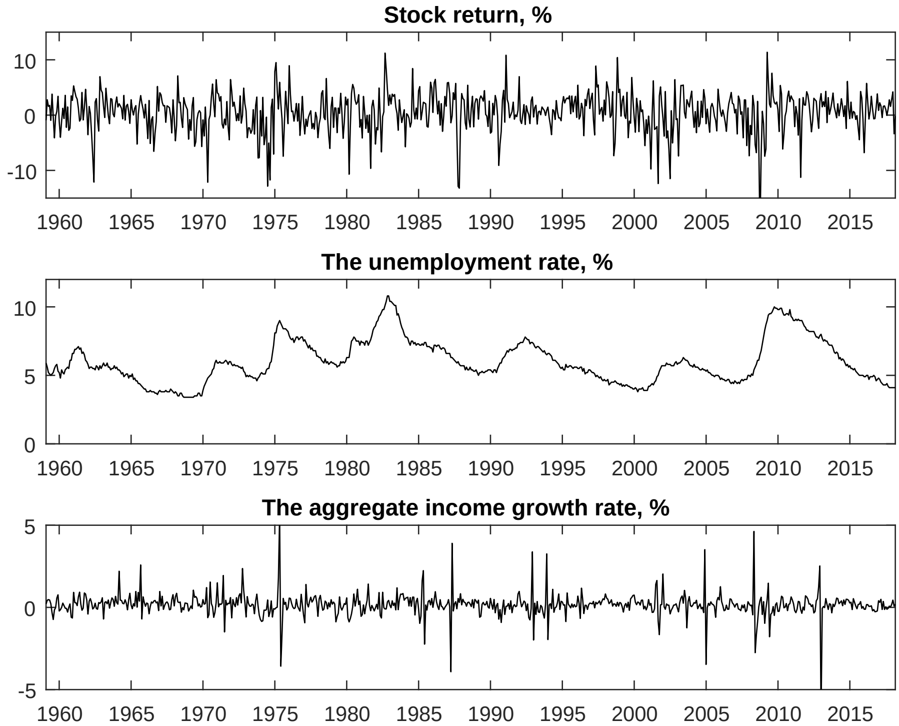

Figure 1: Time series of stock market return, the aggregate unemployment rate, and the aggregate income growth rate. The stock market return is derived from the S&P500 stock index. The unemployment rate represents the number of unemployed as a percentage of the labor force in the U.S. The income growth rate is based on the U.S. national aggregated income. All time series are based on monthly observations from January 1959 to March 2018

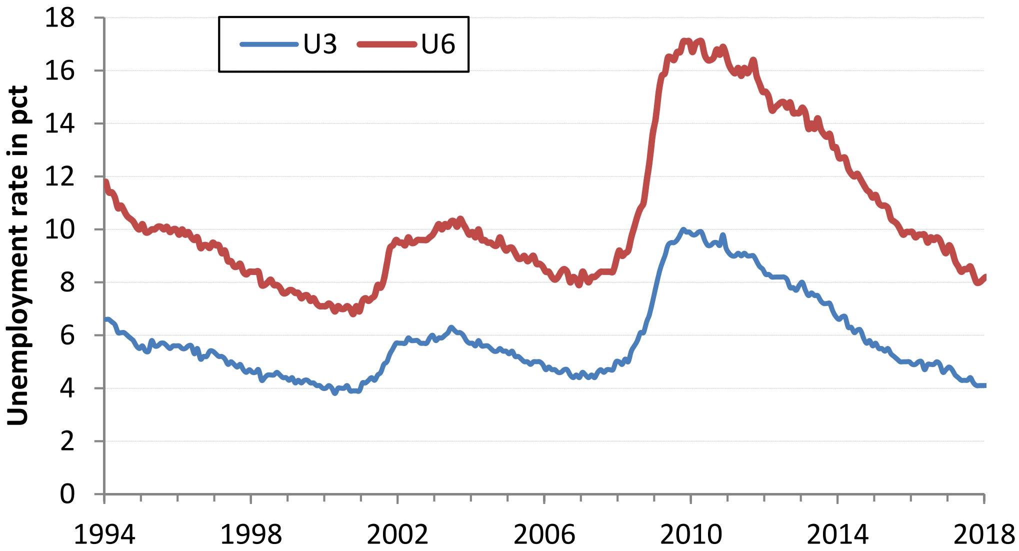

Figure 2: The US unemployment rate from January 1994 to February 2018. The blue [red] curve is the conventional U3 [comprehensive U6] unemployment rate. Monthly observations. Data retrieved from the homepage of the Bureau of Labor Statistics on April 4, 2018.

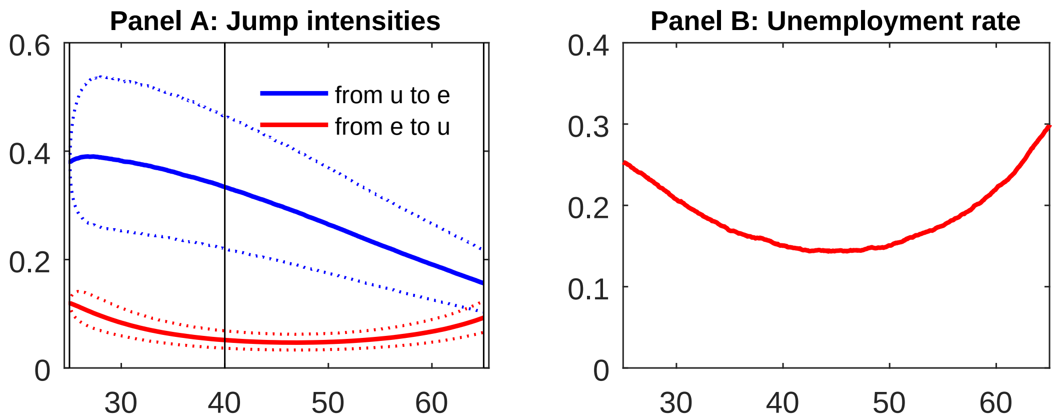

Figure 3: Un- and reemployment probabilities. Panel A shows the age-dependence of the intensity of going from unemployment to employment (blue curve) and from employment to unemployment (red curve) assuming an average state of the economy, i.e. $x = 0$. The dotted curves display the 10th and 90th percentiles. The intensities are assumed to have the form (14) and the coefficients are determined to match preset values at the age of 25, 40, and 65 years as indicated by the vertical lines. Panel B shows the probability of being employed at different ages calculated from an assumed 75% employment rate at age 25 and the intensities from Panel A.

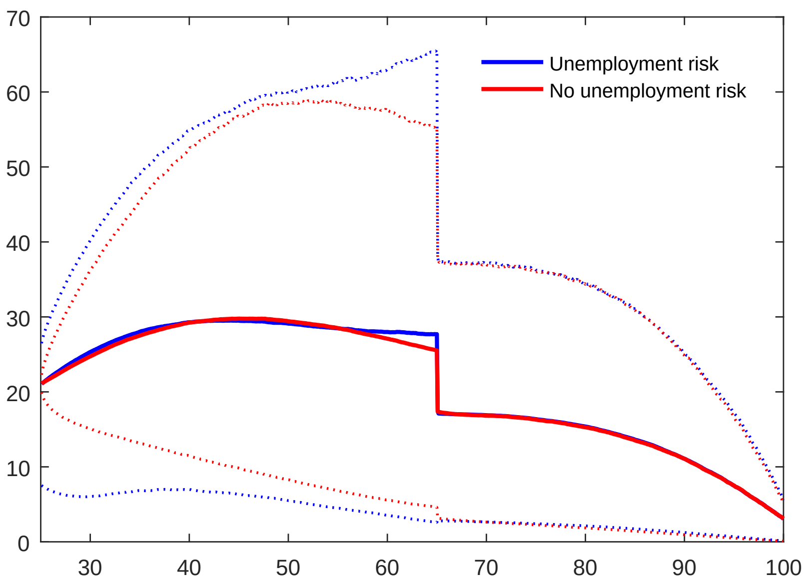

Figure 4: Life-cycle income with and without unemployment risk. The blue [red] curves are for the model with [without] unemployment risk. The solid curves show the average income and the dotted curves the 10th and 90th percentiles.

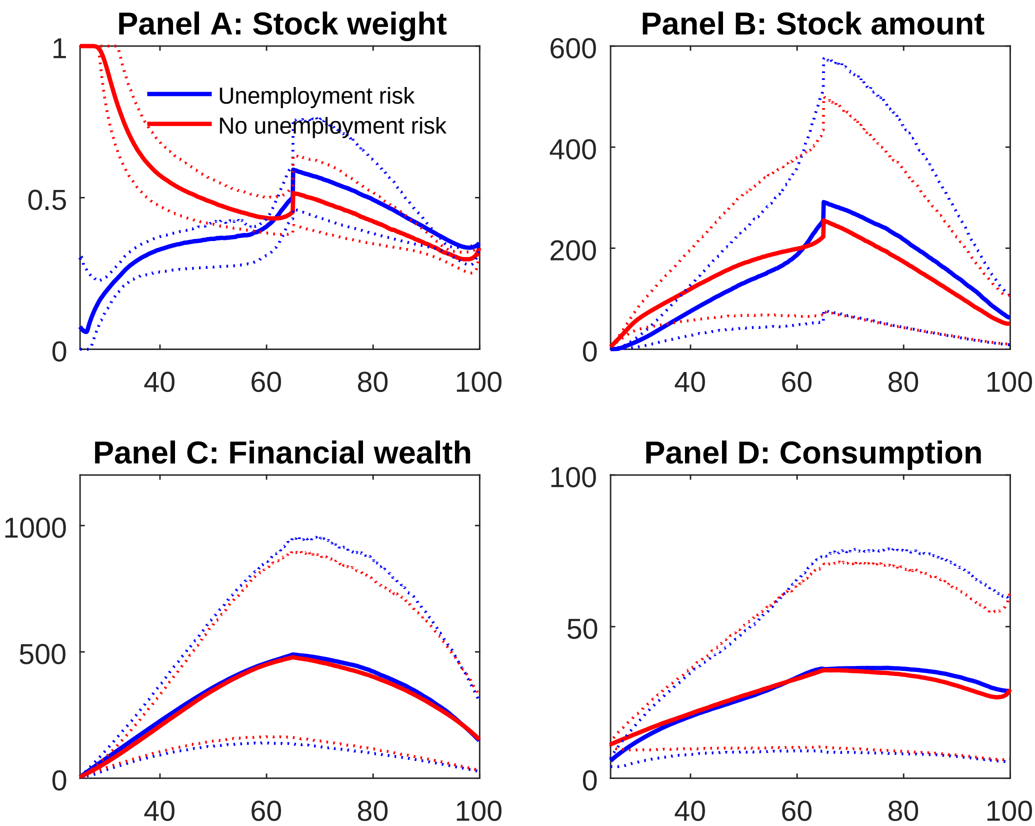

Figure 5: Life-cycle profiles with and without unemployment risk. The graphs show how the portfolio weight of the stock, the amount invested in stocks, the financial wealth, and the consumption rate vary with the age of the investor. The blue [red] curves are for the model with [without] unemployment risk. The solid curves show the means and the dashed lines the 10th and 90th percentiles across 10,000 simulated life-cycle paths.

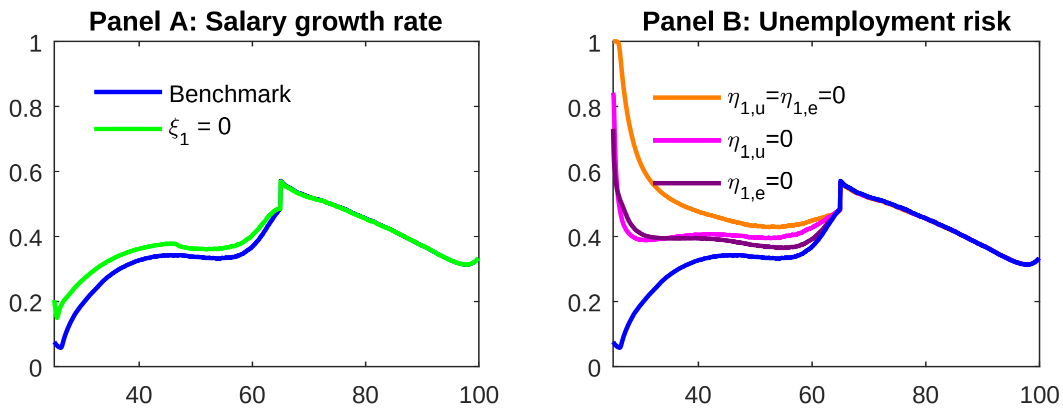

Figure 6: State dependence and the stock weight. Each solid curve shows the average stock weight through the life cycle under certain assumptions. Panel A shows the effect of the state dependence of the salary growth rate, whereas Panel B illustrates the effect of the state dependence of the unemployment risk. The blue curves are for the benchmark parametrization with unemployment risk. The other curves turn off one or several sources of state dependence. The green curve follows by ignoring the state dependence of the salary growth rate, i.e., assuming $\xi_1 = 0$. The state independence is imposed in the unemployment probability for the magenta curve ($\eta_{1,u} = 0$), the reemployment probability for the purple curve ($\eta_{1,e} = 0$), and both of them for the orange curve ($\eta_{1,u} = \eta_{1,e} = 0$).

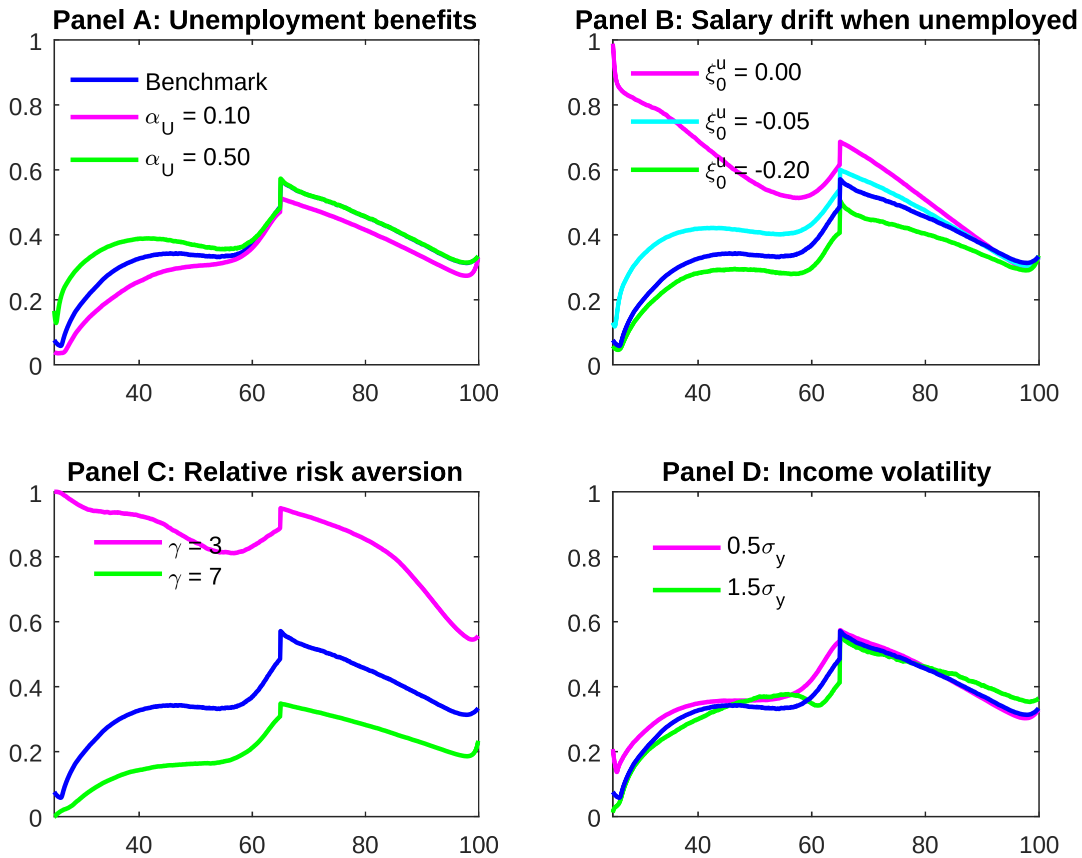

Figure 7: Selected comparative statics. Each curve shows the average stock weight through the life cycle under certain assumptions. The blue curves are for the benchmark parametrization. In each panel, the two other curves show results for alternative values of one parameter. Panel A varies the value of the unemployment benefit fraction α_u from 0.3 to 0.1 or 0.5. Panel B considers variations in the salary drift during unemployment, $\xi_0(t,u)$, from -0.1 to 0 or -0.2. Panel C varies the relative risk aversion γ from 5 to 2 or 8. Panel D illustrates the effects of halving the volatility of either the salary (σ_y in the model) or the retirement income ($\hat{\sigma}_y$ in the model, σ_{yr} in the figure).

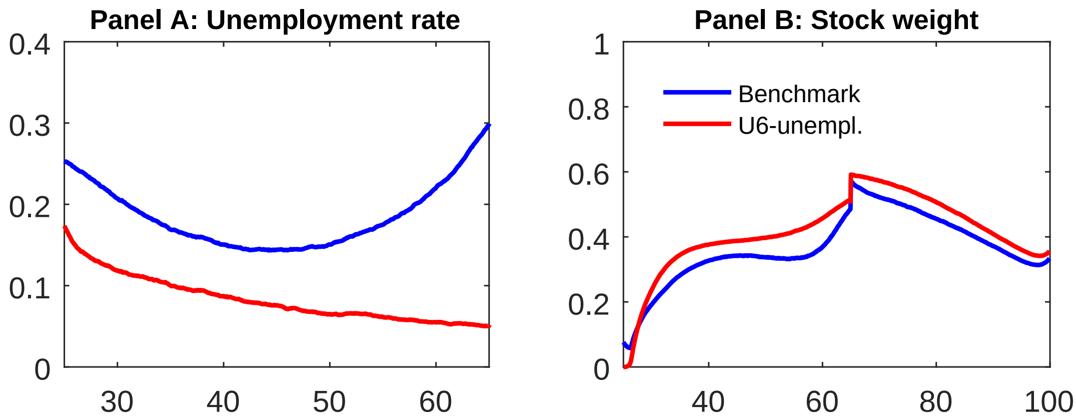

Figure 8: Alternative unemployment rates and optimal stock weights. Panel A shows the age-dependent unemployment rate assumed in our model, and Panel B shows the average optimal stock weight over the life cycle. The blue curves are for our benchmark case, the red curves for the alternative case based on the 2017 U6 unemployment rate.

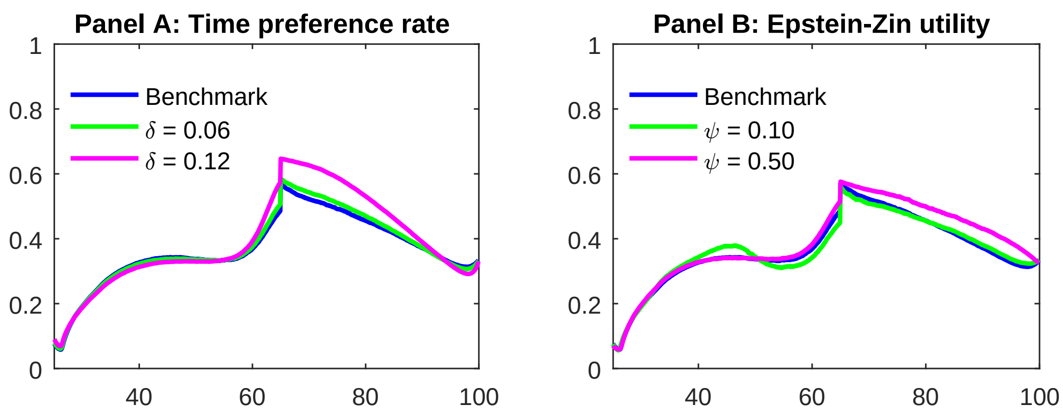

Figure 9: Stock investments with lower savings. Each curve shows the average stock weight through the life cycle under certain assumptions. The blue curves are for the benchmark parametrization. In each panel, the two other curves show results for alternative values of one parameter. Panel A varies the value of the subjective time preference rate δ from 0.04 to 0.06 or 0.12. Panel B considers the extension to Epstein-Zin utility and varies ψ, the elasticity of intertemporal substitution, from the benchmark value of $1/\gamma = 0.2$ to 0.1 or 0.5.

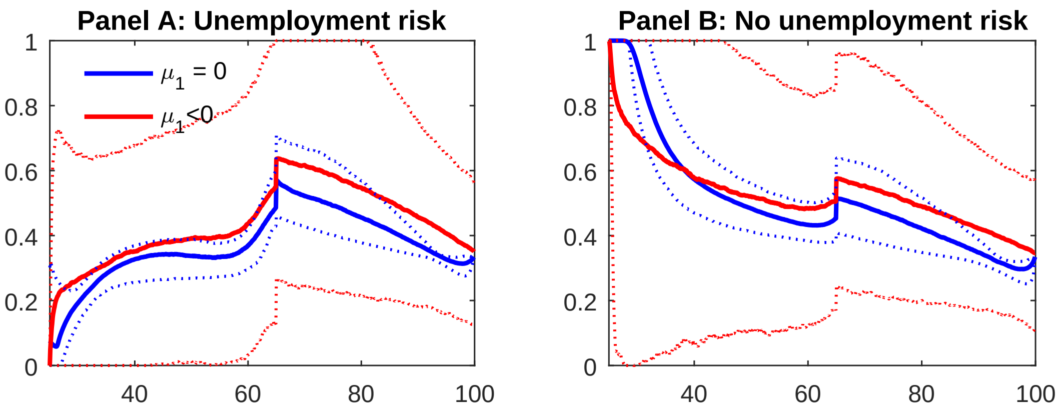

Figure 10: Stock investments with counter-cyclical expected returns. The solid curves show the mean portfolio weight of the stock, whereas the dotted curves show the 10th and 90th percentiles across the 10,000 simulated life-cycle paths. The blue curves are for our main model with $\mu_1 = 0$, i.e., no stock predictability, whereas the red curves are for the extended model with counter-cyclical expected stock returns. Panel A covers the case with unemployment risk and Panel B the case without.

Parameter	Description	Estimate
	Financial assets	
r	Interest rate	0.01
μ	Expected excess stock return	0.04
σ_S	Stock volatility	0.157
	Unemployment rate	
ω_1	Constant level in drift	0.0257
κ_{1x}	State sensitivity of drift	-0.1225
κ_{1v}	Mean reversion speed	0.4663
σ_{v1}	Volatility	0.0054
	Aggregate income growth rate	
ω_2	Constant level in drift	0.0275
κ_{2x}	State sensitivity of drift	0.0657
κ_{2v}	Mean reversion speed	14.2635
σ_{v2}	Volatility	0.0248
	Unobservable state	
κ_x	Mean reversion speed	0.4979
σ_X	Volatility	0.1132
	Correlations	
k_{Sv_1}	Stock index and unemployment rate	0.0255
k_{Sv_2}	Stock index and aggregate income growth	0.1169
k_{SX}	Stock index and unobservable state	0.6494
$k_{v_1v_2}$	Unemployment rate and agg. income growth	-0.0144
k_{Xv_1}	Unobservable state and unemployment rate	0.1992
k_{Xv_2}	Unobservable state and agg. income growth	0.4264

Table 1: Estimates and benchmark values of macro parameters. The values of r, μ_0, and σ_S are pre-set, whereas the other values are estimated based on monthly data on the aggregate unemployment rate, the growth rate of the aggregate income level, and real stock prices of the S&P500. The sample period goes from January 1959 to March 2018.

Parameter	Description	Base value
	Horizon, preferences, and initial wealth	
$\tilde{T}$	Years until retirement	40
$T - \tilde{T}$	Years in retirement	35
γ	Relative risk aversion	5
δ	Time preference rate	0.04
ε	Utility weight on bequests	5
W_0	Initial financial wealth	5
	Employment status	
$\eta_{a,e}$	Constant in employment jump intensity	-0.9676
$\eta_{b,e}$	Time-dependence in employment jump intensity	-0.0044
$\eta_{c,e}$	Time-dependence in employment jump intensity	-4.7127×10^{-4}
$\eta_{1,e}$	State sensitivity of employment jump intensity	3.3764
$\eta_{a,u}$	Constant in unemployment jump intensity	-2.1203
$\eta_{b,u}$	Time-dependence in unemployment jump intensity	-0.0891
$\eta_{c,u}$	Time-dependence in unemployment jump intensity	0.0020
$\eta_{1,u}$	State sensitivity of unemployment jump intensity	-2.8455
	Health status during retirement	
$\eta_{a,h}$	Constant in health jump intensity	-0.9434
$\eta_{b,h}$	Time-dependence in health jump intensity	0.0242
$\eta_{c,h}$	Time-dependence in health jump intensity	-5.8892×10^{-4}
$\eta_{a,s}$	Constant in sickness jump intensity	-1.3687
$\eta_{b,s}$	Time-dependence in sickness jump intensity	-0.0287
$\eta_{c,s}$	Time-dependence in sickness jump intensity	5.6664×10^{-4}

Table 2: Benchmark values of investor-specific parameters related to the time horizon, preferences, and likelihood of shifts in employment or health status.

	Age group (years)						
	16–19	20–24	25–34	35–44	45-54	55-64	65+
2017 Non-employment/population ratio	69.7	34.0	21.4	20.2	22.2	37.5	81.4
2002-2017 Average non-empl./popul. ratio	68.6	35.0	23.1	21.1	22.7	39.0	83.9
2017 Conventional rate (U3)	14.0	7.4	4.6	3.5	3.2	3.1	3.6
1994-2017 Average conventional rate (U3)	17.9	10.0	5.9	4.6	4.1	4.0	4.1
2017 Comprehensive rate (U6)	16.6		7.4			6.7	
2017 Extended rate	19.7		13.2			24.0	
Target average-state rate	24.6		16.5			30.0	

Table 3: US unemployment rates in percent. The data was downloaded on April 4, 2018, from the homepage of the Bureau of Labor Statistics.

Parameter	Description	Base value
	Income/salary dynamics with no unemployment risk	
y_0^{nu}	Initial level of salary (per year)	21.04
σ_y^{nu}	Income volatility before retirement	0.15
$\hat{\sigma}_y$	Income volatility in retirement	0.02
ξ_a^{nu}	Time-dependence in drift	0.0380
ξ_b^{nu}	Time-dependence in drift	-0.0013
ξ_c^{nu}	Time-dependence in drift	1.0872×10^{-5}
ξ_1^{nu}	State sensitivity of drift	0.0657
α_R^{nu}	Replacement rate	0.68
	Salary dynamics with unemployment risk	
y_0	Initial level of salary (per year)	25.5
σ_y	Volatility	0.1035
ρ_{Sy}	Correlation with stock return	0.1169
ρ_{yv_1}	Correlation with unemployment rate	0
ρ_{yv_2}	Correlation with aggregate income	0.5
ξ_a^e	Time-dependence in drift when employed	0.0625
ξ_b^e	Time-dependence in drift when employed	-0.0023
ξ_c^e	Time-dependence in drift when employed	3.3336×10^{-5}
$\xi_0(t,u)$	Constant in drift when unemployed	-0.1
$\xi_1(t,e)$	State sensitivity of drift when employed	0.0657
$\xi_1(t,u)$	State sensitivity of drift when unemployed	0.0657
α_u	Unemployment benefit multiplier	0.3
α_R	Replacement rate	0.5372

Table 4: Benchmark values of parameters in the income dynamics.

	Portfolio weight of stock			Consumption		
	$x = -\sigma_x$	$x = 0$	$x = \sigma_x$	$x = -\sigma_x$	$x = 0$	$x = \sigma_x$
Age 25						
Employed	0.0000	0.0000	0.0000	6.1039	6.5395	7.0045
Unemployed	0.6708	0.3156	0.0000	3.7989	3.9746	4.1867
Age 30						
Employed	0.1904	0.2013	0.2061	12.5028	13.0357	13.5815
Unemployed	0.2491	0.2050	0.1301	8.6783	8.9867	9.3680
Age 35						
Employed	0.2762	0.2984	0.3206	17.3878	17.8990	18.3938
Unemployed	0.2517	0.2209	0.1644	12.6810	13.0544	13.5219
Age 45						
Employed	0.3319	0.3494	0.3653	23.9051	24.3419	24.7589
Unemployed	0.2688	0.2520	0.2218	18.5876	18.9747	19.4548
Age 55						
Employed	0.3293	0.3454	0.3612	29.8459	30.4237	30.9780
Unemployed	0.2966	0.2836	0.2669	23.4694	23.8796	24.3695

Table 5: Portfolio and consumption as a function of the macro state and the employment state. The average wealth-salary ratio, W/y, equals $W/y = 0.1961$ for age 25, $W/y = 2.7469$ for age 30, $W/y = 4.8504$ for age 35, $W/y = 8.9301$ for age 45, and $W/y = 12.4822$ for age 55 and is based on the average wealth-salary ratio at each age.