Master thesis

On the effect of ESTER discounting on the pricing and risk of EUR-denominated interest rate derivatives

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Abstract

The objective of this paper is to investigate the possible effects of the transition from EONIA to ESTER discounting on the pricing and risk of EUR-denominated interest rate derivatives. To this end, several short rate models have been utilised to model the evolution of the underlying interest rates. Specifically, the Vasicek model, the Hull-White one-factor model, and the G2++ model have been applied. Furthermore, the G2++ model has been modified to a dual-curve setup, where the six month Euribor and EONIA rates have been modelled concurrently. Swaps and swaptions as examples of specific financial instruments relying on these interest rates have then been priced using the simulated short rate paths. Subsequently, the model has been shocked in a number of different ways to mimic some of the possible effects of changing to ESTER discounting. These shocks included expected changes to the level, the volatility and possible effects of a negative correlation between the two interest rates. To examine the consequences of the shocks the new simulated short rate paths have been used to price the same instruments again. This study indicates that the implementation of the ESTER most likely will incur a non-negligible transfer of value between certain types of instruments. As the transition to ESTER was not known at the time of settlement for many of the outstanding contracts, this transfer can be viewed as unfair. Consequently, several ways including possible cash compensation to mitigate the value transfer have been discussed. However, no obvious solution satisfying every market participant was apparent.

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CHAPTER 1

Introduction

In the financial markets derivatives are financial instruments that derive their value from an underlying asset, index or rate. The choice of underlying asset is almost limitless: Derivatives exist on everything from the price of stocks and currencies to the weather and the now infamous credit default swap on residential mortgages (Hull, 2015). Some types of derivatives are naturally used more frequently than others, but the most traded are the interest rate derivatives. As seen in figure 1.1 below, interest rate derivatives dominate the derivative market with an outstanding notional value of EUR 559 trillion (76% of the EUR derivative market) by Q4 2018. This is an increase in value from EUR 455 trillion the year before. Comparing this to the value of the global stock market (USD 69 trillion by 2018), it is clear how enormous the interest rate derivative market truly is (The World Bank, 2020).

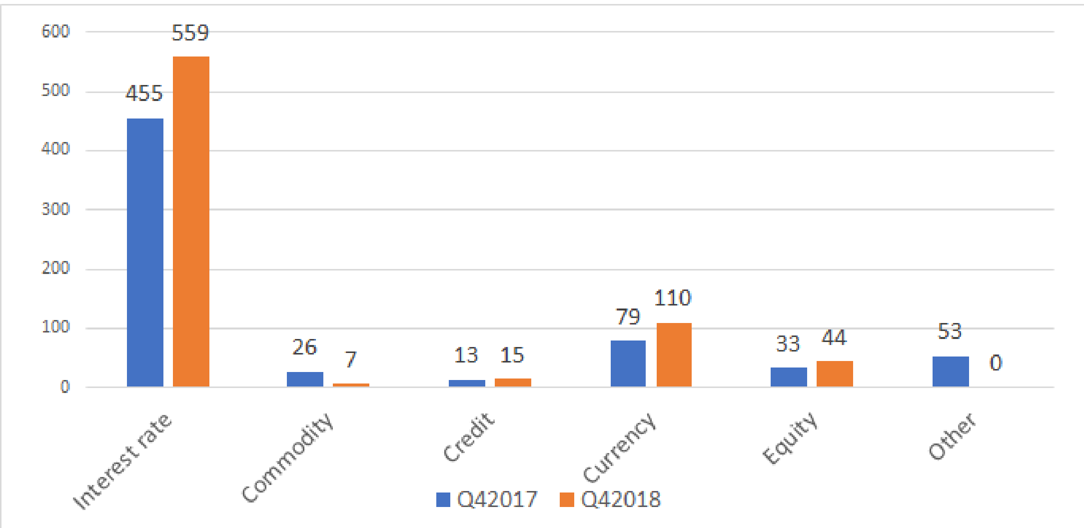

Figure 1.1: Outstanding notional in trillions EUR of the E.U. derivative market by underlying asset (ESMA, 2018, 2019)

Interest rates derivatives are used by financial institutions, asset managers, and corporations to manage interest rate exposure and/or take speculative positions in an underlying interest rate. The most common of these interest rate indices are the xIBOR (Interbank Offered Rates) and overnight rates for a given currency. xIBOR and overnight rates reflect the cost of borrowing on an unsecured basis between two large, highly rated banks in the given currency. The overnight rate are used for overnight transactions and xIBORs are relevant for maturities between 1 week and 1 year (Grbac & Runggaldier, 2015).

In 2012 it was discovered that xIBOR quotes had been fraudulently manipulated by the banks that submit them. The banks did this in order to increase their profit from trades on derivatives or to hide their true creditworthiness. The manipulation was possible since the xIBOR rates are determined by surveying a set of predetermined panel banks on a daily basis and consequently does not reflect actual transaction data. While the market manipulation was problematic in itself, it underlined that a lack of underlying transactions were problematic as the submitted rates might not accurately reflect the true cost of interbank lending. This problem was further amplified as the volume of interbank lending has decreased significantly making it harder for banks to accurately estimate their borrowing cost. Thus, even if no systematic manipulation took place, the rates would to a certain degree still be the result of guesswork (Hull, 2015).

The shortest tenor, i.e. the length of the loan, of the xIBOR rates are the overnight rates. In contrast to the survey-based nature of other xIBOR tenors, the overnight rates are transaction-based. This signifies that they are based on the volume-weighted average funding cost for overnight transaction. While the transaction-based approach makes the overnight rates harder to manipulate, they still suffer from low transaction volume and few market participants. These characteristics results in the rates exhibiting a high degree of volatility and being prone to concentration risk (Grbac & Runggaldier, 2015; Hull, 2015; Nicoloso, 2018).

In light of the xIBOR scandal and the inherent problems discovered in these underlying interest rates, central banks across the globe have announced that the rates need to be reformed. The Euribor (short for Euro Interbank Offered Rate), the relevant xIBOR rate in the Euro zone, is to be reformed, while the overnight rate, the EONIA (Euro Overnight Index Average), is to be completely replaced in favour of a new interest rate index, the ESTER (Euro Short-Term Rate). The long term goal of the Euribor reform is to make the rates transaction-based, while the ESTER is supposed to make the overnight rate more representative of the overnight funding cost (Canepa, 2018).

The main difference between the ESTER and the EONIA is the type of loans included in the published rate. Whereas the EONIA only included inter-bank lending, the ESTER also includes other types such as lending to money market funds, insurers and other financial corporations. By including these other types of transactions, the European Central Bank (ECB) believes the ESTER better represents the actual borrowing cost in the Euro zone. Furthermore, the daily volume of the underlying transactions is almost quadrupled and the number of market participants is increased compared to the EONIA (European Central Bank, 2020a). In order to show the properties of this new calculation method, the ECB published a dataset following ESTER's methodology called the Pre-ESTER. The Pre-ESTER was published from the 15th March 2017 to the implementation of ESTER on the 30th September 2019 (European Central Bank, 2019b). The Pre-Ester can be seen along with the EONIA for the same period in figure 1.2 below.

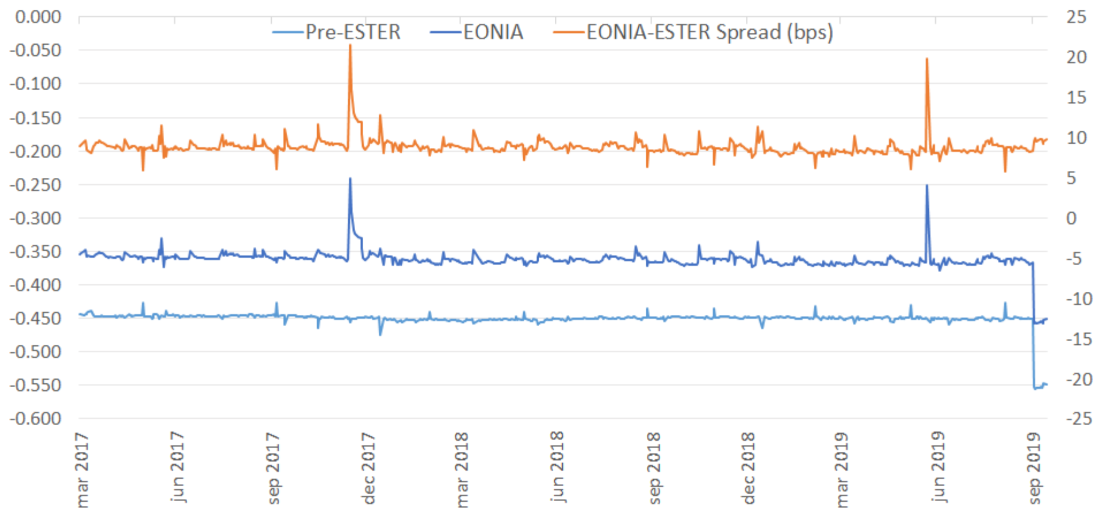

Figure 1.2: EONIA compared to the Pre-ESTER including the spread between these from 15.03.2017 to 30.09.2019

As can be seen in figure 1.2, the Pre-ESTER is less volatile and does not exhibit volatility spikes towards the end of month and quarters as the EONIA does. However, while the ESTER is expected to be less volatile than the EONIA, the transition to a new overnight rate has the potential to entail massive implications for market participants and the EUR-denominated interest rate derivatives market as a whole. The transition to ESTER will not only affect instruments written with the EONIA as the underlying interest rate, but will affect almost all EUR-denominated interest rate derivatives as these are reliant on the EONIA for discounting purposes. Given the sheer size of the market and traded notional, even small changes in the rate have the potential to cause massive transfer of value and risk between market participants. Due to these potential implications, the consequences of the changes in the overnight rates is of great interest from both an academic and a professional point of view. To this end, this paper will seek to answer the following research question:

How will the transition from EONIA to ESTER discounting affect the pricing and risk of EUR-denominated interest rate derivatives?

The paper will seek to answer this question through modelling of the EONIA and the Euribor. This will be done by first determining an appropriate interest rate model. Subsequently, the model will be applied to simulate the evolution of the two interest rates. In doing so, the model generates short rate paths, which will be used to price swaps and swaptions. When these instruments are priced, the simulated EONIA curve will then be shocked in a range of different manners to mimic the ESTER, as it is unclear how the ESTER will behave ex-ante. These shocks are argued to be similar to what is to be expected by the transition to ESTER. The same instruments will then be repriced with the shocked model in order to examine possible differences. This is done in an attempt to determine the possible effects on derivative pricing of the implementation of ESTER. Thereafter, possible implications on outstanding contracts will be discussed.

When examining a topic with such widespread implications as interest rates, some limitations and initial assumptions are in order. Firstly, the paper will be focusing on swaps and European swaptions written on six month (6M) Euribor. These contracts are dependant on the EONIA for discounting purposes and will thus be affected by the transition to ESTER discounting. 6M Euribor swaps and swaptions have been chosen as they are among the most commonly used EUR-denominated interest rate derivatives. This limitation also entails that any other type of interest rate derivative such as the pricing of Overnight Index Swaps (OIS) will not be examined and are only included in order to calibrate the models. Furthermore, any extension to Euribor of other tenors will be trivial. Secondly, the paper will exclusively be focusing on EUR-denominated interest rate products. Consequences for multi-currency products such as cross-currency basis swaps could also be analysed, but is out of scope for this paper. Lastly, it is assumed that no transaction costs are present for pricing purposes and that markets are perfect i.e. do not contain arbitrage opportunities.

CHAPTER 2

Interest Rate Concepts

Before the implications of ESTER can be analysed some basic knowledge of interest rates and risk-neutral pricing is necessary. The following chapter will introduce some of the basic interest rate concepts such as zero coupon rates, forward rates and zero coupon bonds. Subsequently, it will be shown how these can be used to compute the risk-neutral prices of different linear and non-linear interest rate derivatives. Lastly, some of the most important risk sensitivity measures, commonly known as the Greeks, will be introduced.

2.1 Zero coupon bonds and zero coupon interest rates

An interest rate defines the amount a borrower promises to pay a lender for borrowing money over a certain time period. While many financial assets relies on interest rates for discounting purposes, interest rate derivatives are peculiar as their payoffs also depend on interest rates. The appropriate cost of borrowing, i.e. the interest rate paid on the loan, differs depending on several factors such as the credit risk of the borrower and the time to maturity of the underlying loan (Brigo & Mercurio, 2006). Typically, interest rate derivatives are valued by setting up a credit risk-free portfolio, which should earn the credit risk-free rate (Hull, 2015). Depending on the maturity of the loan, the credit risk-free interest rates yield the term structure of interest rates. The term structure can be represented in several ways with one way being through the prices of zero coupon bonds (Brigo & Mercurio, 2006).

The credit risk-free zero coupon bond, $P(t, T)$, describes a financial contract starting at a given time, t, delivering one unit of cash at time, $T > t > 0$, with no intermediate payments. As the zero coupon bond delivers a cash flow of 1 at time T, the price of the zero coupon bond will be below 1 as long as the relevant zero coupon interest rate, also known as the zero rate, is above 0. When pricing the zero coupon bond, the price will be dependant on the compounding convention of interest rates. The most elegant compounding convention is continuous compounding. When utilising continuous compounding the price of a zero coupon bond is given by (2.1) (Brigo & Mercurio, 2006; Linderstrøm, 2013),

$$P(t,T) = e^{-R(t,T)\cdot\delta(t,T)}, \tag{2.1}$$

where $R(t,T)$ is the appropriate zero rate for the term between t and T and $\delta(t,T)$ denotes the coverage of the contract. The coverage is defined as the year fraction that measures the time between dates t and T. While the zero coupon bond is a fair representation of the term structure, it is merely a theoretical instrument that is not observable in the market. Furthermore, it can be hard to comprehend the size of the implied interest rates for longer maturities. Therefore, zero coupon bond prices are typically transformed into zero rates and quoted as such. The continuously compounded zero coupon interest rate is easily found by isolating $R(t,T)$ in equation (2.1) and is thus given by equation (2.2):

$$R(t,T) = \frac{-\ln P(t,T)}{\delta(t,T)} \tag{2.2}$$

As negative interest rates until recently have been considered impossible, zero coupon bonds have also been called *discount bonds*. This is an appropriate naming as they represent the discount factors used to price interest rate derivatives as will be seen in subsequent sections (Brigo & Mercurio, 2006; Linderstrøm, 2013). In the next section it will be presented, how these zero coupon rates are used for risk-neutral valuation of derivatives.

2.2 Risk-neutral valuation and forward measures

The first fundamental theorem of asset pricing states that a market is arbitrage-free if and only if a martingale measure exists (Brigo & Mercurio, 2006). In this regard, an arbitrage is defined as any self-financing strategy where it is possible to earn a risk-free profit by selling and buying different products. A stochastic process f_t is said to be a martingale process under the risk-neutral probability measure Q if

$$E_t^Q[f_t] = f_t \tag{2.3}$$

This means that the expected future value of the process is the value today. Consequently, in order for an arbitrage-free market to exist, the price of any attainable claim is given by either the value of the strategy that exactly replicates the claim's cash flows or by the risk-neutral expectations of the discounted claim's payoff under the equivalent risk-neutral martingale measure. The price of any derivatives contract is thus equal to the expected discounted cash flows today (Brigo & Mercurio, 2006; Linderstrøm, 2013).

Therefore, in an arbitrage-free world one can use either the zero coupon prices or the zero rates to infer the implied prices beginning at time, t, ≥ 0 i.e. the forward prices through a replication argument. It must not be possible to earn a risk-free profit by selling rolling forward contracts and financing them using zero coupon contracts or the other way around. As an example using rolling 1-year forward prices, the following replication argument must therefore hold:

$$(1 + R(t,T))^{(T-t)} = (1 + F_{1,1}) \cdot (1 + F_{2,1}) \cdot \ldots \cdot (1 + F_{T-t-1,1})$$

While continuous compounding might be more elegant, the payoffs of interest rate derivatives are computed using simple compounding. In order for the no-arbitrage condition to hold, the simply compounded forward rate at time, t, for the rate with the expiry $T > t$ and maturity $S > T > t$ is given by equation (2.4)

$$F(t,T,S) = \frac{1}{\delta} \cdot (\frac{P(t,T)}{P(t,S)} - 1) \tag{2.4}$$

Through the no-arbitrage condition it is apparent that the entire term structure of interest rates can be equally represented through either the zero coupon prices, zero rates or forward rates - they are simply different representations of the same information. With zero coupon prices that information is represented as the time value of money i.e. the discounting over a period of time. Conversely, zero rates represent the information as the average return of a loan for a given tenor, while forward rates are the rate of return for a forward starting loan. Accordingly, the forward rates can be considered the marginal return for the given period (Brigo & Mercurio, 2006; Linderstrøm, 2013).

2.3 Market rates

In order to price interest rate derivatives, it is necessary to have credit risk-free rates to project and discount future cash flows. This is where the xIBOR rates are relevant. Financial institutions have - depending on the currency of the transaction - used the relevant xIBOR rates as proxies for credit risk-free interest rates. The relevant xIBOR rates have traditionally been used for both projecting and discounting cash flows. However, this process known as the single-curve setup have in recent years proven to be faulty, which have led to the dual-curve setup that will be elaborated in the following sections.

Euribor rates

Euribor interest rates are supposed to reflect the short-term rate at which prime banks in the Euro area will be willing to lend to another Euro prime bank on an unsecured basis. Until the 1st November 2013 the Euribor was quoted on 15 different maturities, but this has since been reduced to five maturities in the interval 1 week to 12 months (European Money Markets Institute, 2019a). Banks that are located and active in the Euro area can qualify to become a member of the Euribor panel if they meet certain criteria. One of these criteria concerns a limitation of the banks' leverage or other activities that increase their risk of default. This limitation lends to the assumption that Euribor rates are credit risk-free, as risky banks will be dropped from the panel (Grbac & Runggaldier, 2015). As the Euribor rates have been considered to be credit risk-free, interest derivatives have not only been projected using the Euribor, but also discounted with it (Hull & White, 2013).

Dual-curve pricing

It was widely believed before 2007 that discounting the xIBOR cash flows with the same curve as they were projected with, was adequate for derivative pricing. However, during what has since been dubbed the Credit Crisis or the Great Credit Crunch, concerns regarding the credit and liquidity risk of xIBOR rates began to rise. Banks became more reluctant to lend to one another and liquidity quickly dried up (Bianchetti, 2008; Grbac & Runggaldier, 2015; Hull & White, 2013; Morino & Runggaldier, 2014).

Classic no-arbitrage pricing stipulates that two floating rate bonds always trade at par regardless of tenor. This in turn stipulates that two interest rates of different tenors should trade flat if one were exchanging them as part of a swap. Thus, a 6 month contract on EONIA should trade flat with a 6 month contract on 6M Euribor. As can be seen in figure 2.1, this assumption was reasonable until 2007 with just a couple of basis points typically added to the shorter maturity. However, as the market began to realise that xIBOR rates were not credit risk-free - especially following the bankruptcy of Lehman Brothers - the spread between different tenors began to increase. As the xIBORs are not really representing credit risk-free rates, the longer tenor rates should be relatively higher than shorter tenor rates. The reason for this is that the deterioration of the credit quality of the xIBOR contributors are more severe for loans with longer tenors. The difference in these rates across tenors for single currencies is known as *tenor basis* (Chang & Schlögl, 2015; Fujii, Shimada, & Takahashi, 2019; Grbac & Runggaldier, 2015; Linderstrøm, 2013).

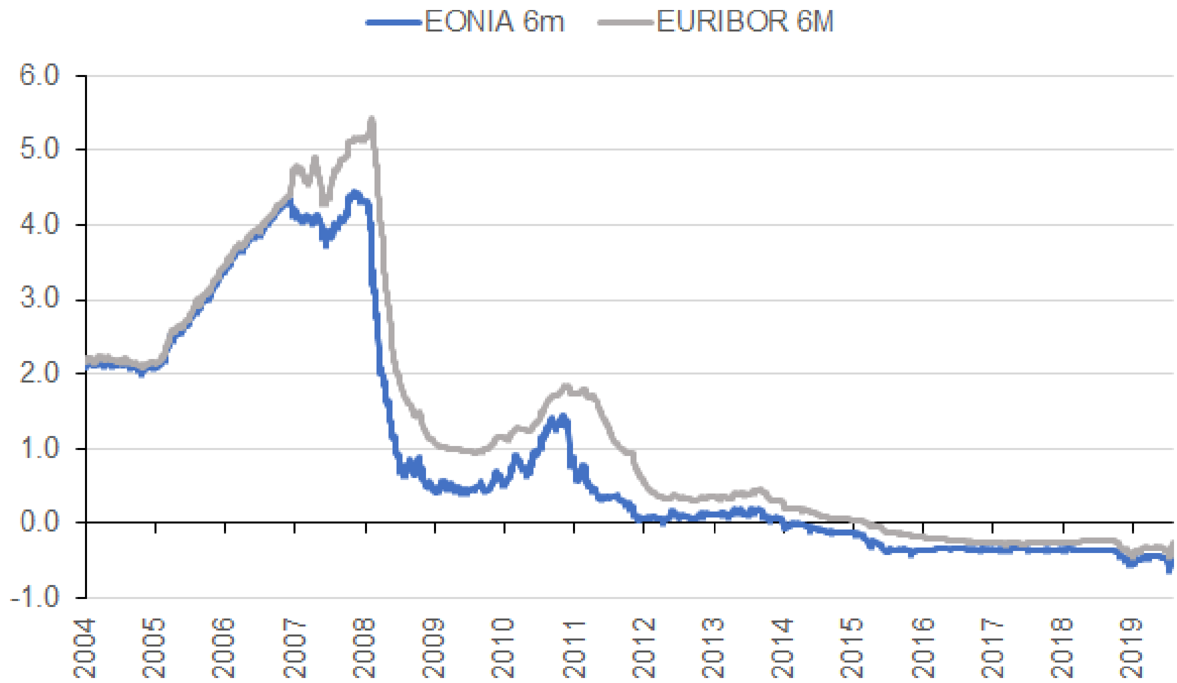

Figure 2.1: 6M EONIA swap vs 6M Euribor, 2004-2020 (Bloomberg, 2020).

As tenor basis continued to persist in various degrees since the Great Credit Crunch, a new market standard for derivative pricing had to be developed. As tenor basis continued to be present, it can in accordance with the no-arbitrage principle not constitute an arbitrage opportunity as it would have been traded away. Therefore, the tenor basis must constitute credit and liquidity risk of the different tenors and thus any expected gain from the "arbitrage" should be offset by losses at the different risk levels (Chang & Schlögl, 2015; Grbac & Runggaldier, 2015). In these two types of risk, collectively known as *interbank risk*, the credit risk has been shown to be the main driver of the tenor basis while liquidity risk represents the remaining component (Filipović & Trolle, 2013). Thus, when using the xIBOR rate for discounting, credit risk and liquidity risk are not accounted for.

After the fall of Lehman Brothers, it became more and more clear that no financial institution were *too big to fail*. Thus, firms would need to account for the interbank risk. This problem was solved through collateralisation. Collateralisation refers to the practice of posting collateral on the value of a contract. As the value of a contract becomes negative, the contract is marked-to-market i.e. the holder of the contract hands over something of equal value as the contract to the counterparty. This is typically cash of a major currency or AAA-rated bonds. The posted collateral remains the property of the collateral payer unless the payer defaults. At expiry, if no default has happened, the collateral plus accumulated interest is returned to the payer. Typically the marked-to-market is posted daily thus eliminating virtually all credit risk in the contract (Grbac & Runggaldier, 2015).

When a contract is collateralised, it allows us to derive the correct discounting curve. Consider a contract with cash flow, X, and maturity, T. The present value at time, t, is given as $V(t)$. In the contract it is agreed that the two parties will post continuous cash collateral and the contract is fully collateralised at all times. This is a simplifying assumption as the market standard is to post collateral on a daily basis. The result will, however, be approximately equal (Filipović & Trolle, 2013). Cash collateral is the most popular choice of collateral. The receiver of the collateral can invest the collateral at the risk-free rate, $r(t)$, while having to pay an agreed rate to the poster of the collateral, $r_c(t)$. These conditions sets up the following integral equation for $V(t)$ (Filipović & Trolle, 2013)

$$V(t) = E_t^Q[e^{-\int_t^T r(s)ds} \cdot X + \int_t^T e^{-\int_t^u r(s)ds} \cdot (r(u) - r_c(u)) \cdot V(u)du] \tag{2.5}$$

Solving this integral yields

$$V(t) = E_t^Q[\exp(-\int_t^T r_c(s)ds) \cdot X] \tag{2.6}$$

This result indicates that the discounting rate of collateralised interest rate derivatives should be the interest rate paid on the collateral. Thus, collateralised interest rate products should be discounted with their funding cost. Given the daily marked-to-market, the interest rate earned on the collateral is typically the overnight rate. (Filipović & Trolle, 2013). This proof will not be considered further in this paper, however the interested reader can consult Appendix A in Filipović & Trolle (2013) for a full derivation of this result.

The funding cost argument has since been disputed. An investment should be discounted in accordance with its risk and not its funding cost. Hull (2015) argues that since derivatives can be replicated by risk-free portfolios, they should through no-arbitrage arguments be discounted with the risk-free rate. Since the overnight rate is the closest market approximation of the risk-free rate, this argument supports using the overnight rate for discounting purposes even for non-collateralised contracts (Hull, 2015; Hull & White, 2013). This paper will not delve further into which of these arguments are more correct since most interest rate derivatives are collateralised anyway. Thus, the end result of using the overnight rate for discounting remain the same. For EUR-denominated interest rate derivatives, the relevant overnight rate is the EONIA and its replacement the ESTER (European Money Markets Institute, 2019a, 2020).

2.4 Interest rate derivatives

In the previous section the concept of risk-neutral valuation and its implementation in the market have been presented. Generally, two types of interest rate derivatives exist: Linear and non-linear derivatives. Both types have their usages depending on the need and purpose of the agent trading them. The following sections will introduce some of the most widely used of these instruments, how they are priced and the assumptions behind these prices.

Forward rate agreements

A Forward Rate Agreement (FRA) is a linear interest rate derivative where one party agrees to receive a fixed rate, K, in exchange for paying a floating spot rate on a xIBOR rate, L, at a given date for a predetermined contract notional, N. FRAs are generally used to mitigate interest rate risk for shorter maturities - usually up to two years. The FRA basically allows the buyer to lock the interest rate between expiry, T, and maturity, S. The fixed rate K uses simple compounding (Linderstrøm, 2013). At a given maturity, S, one party receives the fixed payment

$$\text{Fixed payment} = \delta(T, S) \cdot K \cdot N$$

whilst paying the floating payment to the counterparty of:

$$\text{Floating payment} = \delta(T,S) \cdot L(T,S) \cdot N$$

where $\delta(T,S)$ denotes the coverage of the contract between time T and S. This means that the value of the contract at maturity S is thus given by (Brigo & Mercurio, 2006):

$$\text{Payoff} = N \cdot \delta(T,S) \cdot (K - L(T,S) \tag{2.7}$$

Obviously, if $L > K$ at time T the contract has a negative value. Typically the fixed rate, K, of the FRA is initiated such that the FRA has zero net present value at time t. This is done to avoid initial counterparty risk. Solving for K such that the price of the contract is equal zero, yields that K must equal the simply compounded risk-neutral forward rate $F(t,T,S)$ as seen in equation (2.4). The risk-neutral value of the contract is thus found by discounted the payoff back to period t using the appropriate discount factor $D(t,S)$ (Brigo & Mercurio, 2006):

$$\text{FRA}(t,T,S,\delta(T,S),N,K) = N \cdot D(t,S) \cdot \delta(T,S) \cdot (K - F(t,T,S)) \tag{2.8}$$

The reason for using $D(t,S)$ notation over the zero coupon bond, $P(t,S)$, is to distinguish between the interest rates used for projecting and for discounting as discussed in section 2.3. Due to this relation between $F(t,T,S)$ and FRA par fixed rate K, $F(t,T,S)$ can be considered as the risk-neutral market expectation of the future spot xIBOR rate, which at time t is a unknown random variable based on market conditions. This yields the T-Forward measure, which states that the forward rate spanning the term between time T and S is the risk-neutral expectation of the future simply-compounded spot rate at time T for maturity S (Brigo & Mercurio, 2006):

$$E^T L(T,S)|\mathcal{F} = F(t,T,S) \tag{2.9}$$

Thus, the forward rate is not just the marginal rate of interest for a given future period which can be locked in today, but also the risk-neutral expectation of the future spot rate.

Interest rate swaps

While FRAs are usually used to hedge short maturity interest rate exposure, exposures with maturities longer than two years usually use Interest Rate Swaps (IRS). A payer (receiver) IRS is a contract in which the holder of the contract agrees to pay (receive) a fixed rate, K, in exchange for receiving (paying) an interest payment linked to a floating interest rate index, typically a xIBOR index. Thus, a payer (receiver) IRS will position the agent to be long (short) higher interest rates (Linderstrøm, 2013).

For a prespecified set of dates between $T_{\alpha+1}, ...T_{\beta}$, the floating leg and fixed leg will make two series of payments. However, the schedules of the two legs will often differ. For an IRS on 6M Euribor the fixed leg pays on an annual basis, while the floating leg fixes and pays on a semi-annual basis. The payments are therefore spaced out by different coverages (Linderstrøm, 2013). The coverage of the fixed leg δ_i^{Fixed} and of the floating leg δ_i^{Float} are therefore denoted separately. At every instant T_i for the prespecified set of dates $T_{\alpha+1}, ...T_{\beta}$ the fixed leg pays:

$$\text{Payment}_{Fixed} = N \cdot \delta_i^{Fixed} \cdot K \tag{2.10}$$

while the floating leg pays:

$$\text{Payment}_{Float} = N \cdot \delta_i^{Float} \cdot L(T_{i-1}, T_i) \tag{2.11}$$

Where $L(T_{i-1}, T_i)$ refers to the interest rate resetting at dates T_{α}, $T_{\alpha+1}$,...$T_{\beta-1}$ i.e. fixed-in-advance and pays at dates $T_{\alpha+1}$,...T_{β} i.e. paid-in-arrears (Brigo & Mercurio, 2006).

As with the FRA, the IRS is priced by discounting the risk-neutral expectation of the future cash flows. The discounted payoff of the fixed leg at time $t < T_{\alpha}$ is thus given by:

$$PV_t^{FixedLeg} = \sum_{i=\alpha+1}^{\beta} \delta_i^{Fixed} \cdot K \cdot N_i \cdot D(t, T_i), \tag{2.12}$$

while the present value of the floating leg is given by:

$$PV_t^{FloatLeg} = \sum_{i=\alpha+1}^{\beta} \delta_i^{Float} \cdot L(T_{i-1}, T_i) \cdot N_i \cdot D(t, T_i) \tag{2.13}$$

The value at time t of the payer swap is thus given by:

$$PV_t^{Payer} = \sum_{i=\alpha+1}^{\beta} \delta_i^{Float} \cdot L(T_{i-1}, T_i) \cdot N_i \cdot D(t, T_i) - \sum_{i=\alpha+1}^{\beta} \delta_i^{Fixed} \cdot K \cdot N_i \cdot D(t, T_i) \quad (2.14)$$

The value of a receiver swap is given as minus the value of the payer swap due to the linearity of the payoff. As with the FRA, an IRS is typically traded such that the price of the contract is $PV_t^{Fixed} = PV_t^{Float}$. The par swap rate is given by:

$$R(t, T_\alpha, T_\beta) = \frac{\sum_{i=\alpha+1}^{\beta} \cdot \delta_i^{Float} \cdot L(T_{i-1}, T_i) \cdot N_i \cdot D(t, T_i)}{\sum_{i=\alpha+1}^{\beta} \cdot \delta_i^{Fixed} \cdot N_i \cdot D(t, T_i)} \quad (2.15)$$

The par swap rate can be considered as the weighted average of the forward rates. This makes intuitive sense as if the contract is fair, the two cash flows must be of equal value at time t (Brigo & Mercurio, 2006; Linderstrøm, 2013).

Interest rate options

The second type of interest rate derivatives is non-linear derivatives. An interest rate option is the right, but not the obligation to exchange a floating interest rate payment against a fixed payment at predetermined exercise date(s) (Brigo & Mercurio, 2006). The most common types of interest rate options are caps, floors and swaptions. An interest rate cap (floor) can be viewed as a payer (receiver) IRS where each payment is only executed if it has a positive value. Meanwhile, a payer (receiver) swaption is the right to enter into a forward starting payer (receiver) IRS where the IRS is only initiated if the IRS has a positive value (Linderstrøm, 2013). European type options, i.e. options with a single exercise date, will be the focus of this section.

Interest rate options are primarily used for two things: Risk management and speculation through leveraged positions. Consider a company with a floating rate loan linked to a xIBOR, where they at times $T_{\alpha+1}$ ... T_β will have to pay the xIBOR rate, L, which fixes at times T_α ... $T_{\beta-1}$. Obviously, the company would stand to lose from increases in L. The company can protect itself from this by buying a cap with strike K that matches the payoff of the loan. The company pays L and receive $(L-K)^+$. The difference between L and K results in what is paid in the contract as:

$$L-(L-K)^+ = min(L,K)$$

Unlike a FRA or a IRS where the fixed rate K is locked, the cap gives the holder of the contract the ability to "cap" the amount paid. This means that if rates increase, the holder is protected, but if they fall, the holder of the loan will not exercise their option and simply pay L. The option thus protects from downside risk, while still allowing positive events to happen. A floor can, like the name suggest, secure a minimum interest rate the holder will receive if they hold a xIBOR asset instead of a liability (Brigo & Mercurio, 2006).

Valuing interest rate options under risk-neutral expectations is done as with linear derivatives by discounting the expected payoff to time t. The cap (floor) can be considered as consisting of a series of individual options on individual dates known as caplets (floorlets). The value of the cap is given by (Brigo & Mercurio, 2006):

$$\sum_{i=\alpha+1}^{\beta} D(t,T_i)\cdot N\cdot\delta_i\cdot(L(T_{i-1},T_i)-K)^+,$$

while the corresponding floor is given by:

$$\sum_{i=\alpha+1}^{\beta} D(t,T_i)\cdot N\cdot\delta_i\cdot(K-L(T_{i-1},T_i))^+$$

An European payer swaption, i.e. the option at time T_α to enter into a payer IRS maturing at time T_β, is given as (Brigo & Mercurio, 2006):

$$N\cdot D(t,T_\alpha)\cdot(\sum_{i=\alpha+1}^{\beta} D(T_\alpha,T_i)\cdot\delta_i\cdot(F(T_\alpha;T_{i-1},T_i)-K)^+,$$

while the European receiver swaption naturally is given as:

$$N\cdot D(t,T_\alpha)\cdot(\sum_{i=\alpha+1}^{\beta} D(T_\alpha,T_i)\cdot\delta_i\cdot(K-F(T_\alpha;T_{i-1},T_i))^+$$

Given the optionality of the instruments, these instruments cannot simply be priced from the forward rates alone as these are stochastic variables. Instead, it is necessary to determine their distribution in order to know the expected payoff. Under the risk-neutral forward measure, the forward rates $F(T_{i-1}, T_{i-1}, T_i)$ are martingales, which means their movements can be modelled as a stochastic function $C(\cdot)$ and a Wiener process W_t. Traditionally the movements in $F(T_{i-1}, T_{i-1}, T_i)$ have been assumed to follow:

$$dF(T_{i-1}, T_{i-1}, T_i) = \sigma \cdot F(T_{i-1}, T_{i-1}, T_i) \cdot dW_t^{T_i}$$

This implies that the distribution of the forward rates follow a log-normal distribution. Given this assumption, caps, floors and payer/receiver swaptions can be priced using the Black-Scholes (BS) model (Brigo & Mercurio, 2006; Linderstrøm, 2013). The BS model has become market standard and to a large extend remain as such. However, since the BS model assumes log-normally distributed forward rates, these cannot assume negative values. Consequently, the BS model cannot be applied in negative interest rate environments and subsequently cannot currently price interest rate options on the Euribor. In order to accommodate the current interest rate environment, the Bachelier model, also known as the Normal model, will be utilised in this paper. The Bachelier model assumes that forward interest rates are normally distributed rather than log-normally distributed. This means that the probability of negative interest rates are non-zero. Prices under the Bachelier model is given by (Choi, Kim, & Kwak, 2009; Filipović, 2020):

$$Cap^{Normal}(t, T, \delta, N, K, \sigma) = N \cdot \sum_{i=\alpha+1}^{\beta} \delta_i \cdot D(t, T_i) \cdot \sigma \cdot \sqrt{T_{i-1} - t} \cdot (D\Phi(D) + \phi(D)) \quad (2.16)$$

and the corresponding floor is given by:

$$Floor^{Normal}(t, T, \delta, N, K, \sigma) = N \cdot \sum_{i=\alpha+1}^{\beta} \delta_i \cdot D(t, T_i) \cdot \sigma \cdot \sqrt{T_{i-1} - t} \cdot (-D\Phi(-D) + \phi(-D)) \quad (2.17)$$

where ϕ is the standard normal density function, Φ is the standard normal cumulative distribution function and D is given by:

$$D = \frac{F(t, T_{i-1}, T_i) - K}{\sigma \cdot \sqrt{T_{i-1}}}$$

For an European payer swaption, the Normal price is given by

$$Payer^{Normal}(t, T, \delta, N, K, \sigma) = N \cdot \delta \cdot \sum_{i=\alpha+1}^{\beta} \cdot D(t, T_i) \cdot \sigma \cdot \sqrt{T_{i-1} - t} \cdot (D_S \Phi(D_S) + \phi(D_S)) \tag{2.18}$$

while the equivalent receiver swaption is given by

$$Receiver^{Normal}(t, T, \delta, N, K, \sigma) = N \cdot \delta \cdot \sum_{i=\alpha+1}^{\beta} D(t, T_i) \cdot \sigma \cdot \sqrt{T_{i-1} - t} \cdot (-D_S \Phi(-D_S) + \phi(-D_S)) \tag{2.19}$$

where D_S is given as

$$D_S = \frac{R(t, T_\alpha, T_\beta) - K}{\sigma \cdot \sqrt{T_{i-1}}}$$

In both the BS and the Bachelier model, the volatility parameter σ is not directly observable as the other inputs in the equations. The implied volatility has famously been described as "the wrong number to put in the wrong formula to get the right price". This means that it is simply the implied volatility that yields the prices observed in the market (Rebonato, 2004, p. 173). Options prices are therefore often quoted as implied volatilities.

The Greeks

In the following, some of the risk measures of derivatives commonly known as *"the Greeks"* will be presented. The Greeks are used to measure the sensitivity of derivatives to quantifiable changes in parameters such as the price of the underlying, volatility, and time to maturity. The presentation will be heuristic in nature. Knowledge of the Greeks is necessary in order to understand how the transition from EONIA to ESTER will affect the risk profile of the different interest rate derivatives as will be seen in section 6.

The most important of the Greeks is Delta. Delta is used to describe the sensitivity of the value of the derivative to the price of the underlying asset. If V is denoted as the price of the derivative and B as the price of the underlying asset, Delta can be described as:

$$\Delta = \frac{\delta V}{\delta B}$$

In the case of interest rate derivatives, Delta is usually computed by moving or bumping the entire yield curve upwards by 1 basis point (bps) yielding the so-called Delta vector - the partial effects of movements in each knot point on the value of the derivative. The sum of these effects is known as the DV01 i.e. the *Dollar Value of 1 bps*. Given projecting and discounting of cash flows are done on different curves, the Delta vector is computed for both curves. Given the bump in the knot point always will be upwards, DV01 on the projecting curve for payer (receiver) options will be positive (negative) given these are long (short) interest rates. The DV01 is, depending on the shape of the discounting curve, largest when the instrument is In-The-Money (ITM) (Linderstrøm, 2013).

Gamma is the rate of change in Delta for each bps increase in the underlying asset. Gamma is thus the second order derivative of the derivative price with respect to the underlying. If Gamma is small, Delta will change slowly, while a large Gamma indicates that Delta is very sensitive to the price of the underlying security. In this case, it is risky to leave a Delta hedged portfolio unchanged for a period of time. Gamma is largest when the option is close to the At-The-Money (ATM) strike.

Theta is the rate of change of the price with respect to time to maturity T i.e. the time left on the option until expiry. Thus, Theta is often referred to as the time decay of the portfolio. Lastly, vega is the sensitivity of the derivative price to volatility. Thus, if a portfolio is vega-hedged, it is protected against volatility fluctuations. For swaptions, vega is typically largest around the ATM strike due to optionality of the claims. Small changes in the underlying will cause the contract to yield a positive payoff making volatility relatively more attractive closer to the ATM strike of the contract (Linderstrøm, 2013).

CHAPTER 3

Short Rate Models

The previous chapter described the term structure of interest rates and their use for pricing specific instruments. This chapter will introduce short rate models and how these can be used to determine the future evolution of interest rates. Firstly, the concept of short rate models will be introduced, followed by a brief description of different types of models. Thereafter specific models in the form of the Vasicek model, the Hull-White one-factor model and the G2++ model - which is equal to a Hull-White two-factor model - are introduced.

3.1 Introduction to short rate models

A term structure model describes the evolution of zero rates across all maturities. The term structure implied by the future expectations to the short-term interest rate will be the focus in this paper (Hull, 2015). The short-term interest rate will be referred to as the short rate, r, for the remainder of the paper. The risk-free short rate is the relevant interest rate for an investment for an infinitely small time period at time t. The short rate is a continuously compounded and annualised interest rate, which multiplied by the time to maturity will yield the return (Brigo & Mercurio, 2006).

The short rate represents the initial point of the yield curve. The evolution of the yield curve is assumed to be completely determined by the evolution from the initial point. Directly modelling the dynamics of the instantaneous spot rate process is quite useful since bond and derivative prices are readily defined by no-arbitrage arguments to only depend on the expectation of this process in a risk-neutral world as explained in section 2.2 (Brigo & Mercurio, 2006). Short rate models attempt to model and thus explain the evolution of this short rate across time. These models can be determined by an infinite number of parameters as sources of uncertainty.

A short rate model is usually calibrated to current market data as a way of determining the model parameters. This calibration can be based upon a number of different instruments and information chosen for convenience and suitability for the given analysis. The calibration procedure will not be explained from a purely theoretical point of view, but instead continuously as part of the methodology when explaining how each model has been built.

3.2 Types of short rate models

In general there are two main types of models to explain the evolution of the short rate. The traditional equilibrium type and the newer no-arbitrage type (Brigo & Mercurio, 2006). Equilibrium short rate models rely on supply and demand given a theoretical term structure, whereas no-arbitrage models assume that prices in the market are accurate. The distinction will be further elaborated on in the following sections. First, assumptions of the distribution of the short rate will, however, be covered as these are especially relevant for the analysis.

Distribution of the short rate

A wide range of specific models exists within the equilibrium and no-arbitrage types. The different models are based on different assumptions regarding the evolution of the short rate. For some models such as the Dothan, the Exponential-Vasicek, and the Black-Karasinski, the evolution of the short rate is assumed to be log-normally distributed. From a theoretical point of view, this is a problematic assumption since it leads to what is called the explosion problem. With the explosion problem the expected value of a deposit will be infinite for arbitrarily small time periods. However, these models are always applied through the use of approximating trees. By dealing with a finite number of states and subsequently periods, the problem is partially overcome and the impact of this drawback is limited (Brigo & Mercurio, 2006).

Another and more important drawback for this paper's use of these models is related to negative interest rates. The models follow a log-normal distribution in order to avoid the possibility of negative interest rates since this historically have been seen as impossible. The Cox-Ingersoll-Ross model is another example of a model built to this purpose, however, with a different solution. The CIR model assumes that the process of the short rate neither is normally nor log-normally distributed but rather possesses a square-root process (Björk, 2009). No matter how the possibility of negative interest rates is "solved", it makes the models inapplicable in negative interest rate environments such as the current European market - at least without making corrections to the model. Therefore, models where the short rate is assumed to be log-normally distributed or in other ways based on assumptions, which avoids negative rates, are not considered for modelling purposes in this paper.

Equilibrium and no-arbitrage model types

In the following sections specific examples of normally-distributed equilibrium and no-arbitrage models will be introduced. Firstly, the classical Vasicek model will be covered as an equilibrium model and afterwards the no-arbitrage type Hull-White model will be introduced. These are both models that rely on normal distributions of the short rate.

Equilibrium models such as the Vasicek have the disadvantage that they are endogenous term structure models. This means that the current term structure of interest rates function as an output rather than input in the model. Thereby, these types of models do not automatically fit the current term structure of interest rates. In practice, it can be calibrated to yield an approximate fit to most term structures, but it will never be a perfect fit (Brigo & Mercurio, 2006). Practitioners often find these types of models unsatisfactory, since they cannot have confidence in the model to accurately price derivatives when the underlying bond cannot be priced satisfactorily. Even small discrepancies in pricing on the underlying may lead to large errors in the pricing of derivatives (Hull, 2015). Therefore, it is important that the model is consistent with the initial term structure of interest rates when valuing derivatives. Matching the term structure perfectly is equal to solving a system with an infinite number of equations.

This is only possible by introducing infinite number of parameters or equivalently by introducing a deterministic function of time. The latter solution is used by no-arbitrage models. By including such a deterministic function, no-arbitrage models like the Hull-White are thereby designed to provide exactly this perfect fit to the current term structure of interest rates (Brigo & Mercurio, 2006; Hull, 2015).

The essential difference between the two types is therefore whether the current term structure of interest rates is seen as an output as in equilibrium models or as an input like in the no-arbitrage types. Thus, the drift is usually not time-dependant in an equilibrium model, whereas the drift most often is time-dependant in no-arbitrage models. When the drift is time-dependant it entails that the initial zero curve determines the short rate path through time - if the zero curve is upward sloping between two time periods the drift on r will be positive and vice versa. The distinction between the two type of models implies that some equilibrium models can be converted to the no-arbitrage type by introducing the drift as a time-varying function (Brigo & Mercurio, 2006). This is also true for the Vasicek and Hull-White models, where the latter can be seen as the no-arbitrage descendant of the Vasicek model.

3.3 The Vasicek model

The Vasicek model was developed by Oldrich Vasicek in 1977 and is one of the most well-known examples of an equilibrium short rate model (Brigo & Mercurio, 2006). In this model the instantaneous spot rate follows a mean-reverting, Gaussian process - also known as an Ornstein-Uhlenbeck process in the world of physics - with coefficients that are constant across time (Vasicek, 1977):

$$dr(t) = k[\theta - r(t)]dt + \sigma dW(t), \quad r(0) = r_0 \tag{3.1}$$

where r_0, k, θ, and σ are positive constants according to Brigo & Mercurio (2006) and most other classic literature on the model.

This model incorporates mean-reversion to the level of θ at the speed of k from an initial level of r_0 with shocks dependant on σ. From the first term of the model it is apparent that the drift of the process is positive whenever $r_t < \theta$ and negative whenever $r_t > \theta$, thereby constantly pushing r towards the long-term mean rate of θ. The magnitude of this pull is determined by k. For $k \to 0$ the mean-reverting dynamics of the process are eliminated. Therefore, θ can be regarded as the long-term mean rate, whereas k can be interpreted as the speed of the mean-reversion. The pull towards the level of θ is also depicted in figure 3.1:

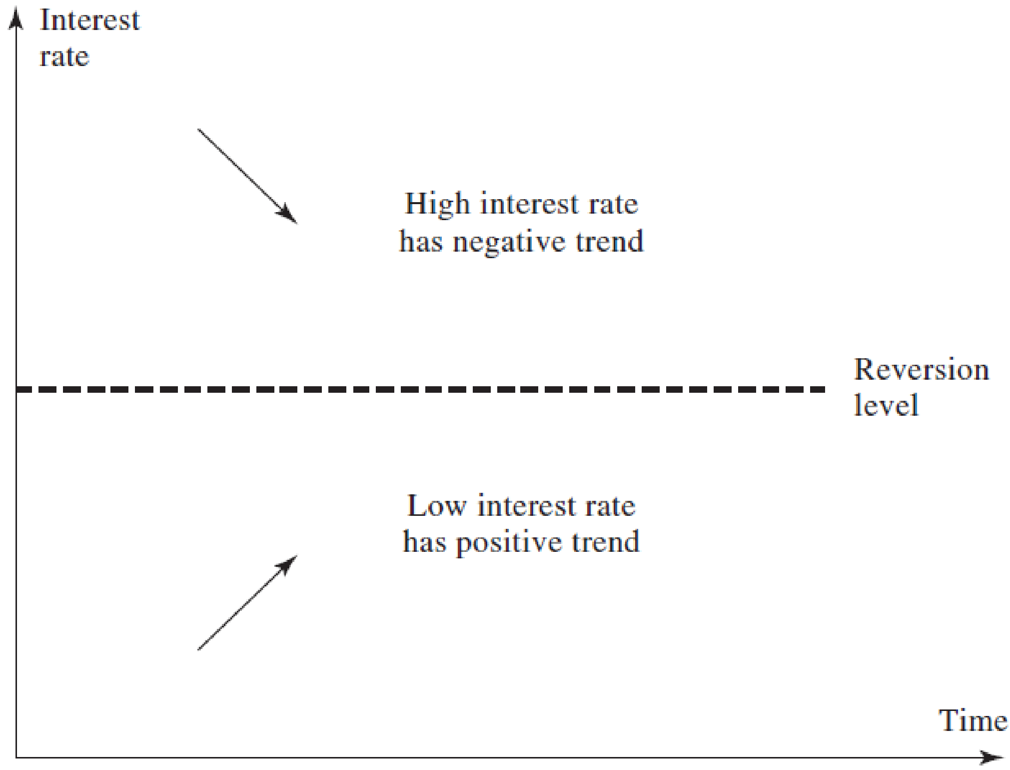

Figure 3.1: Vasicek Mean Reversion (Hull, 2015, p. 709)

Upon this pull to the mean is a Wiener process, $W(t)$, also known as a Brownian motion. A Wiener process is a normally distributed stochastic process. This process is the volatility of the short rate. Naturally, the expected value of a stochastic integral is 0. When $\sigma \to 0$, the volatility in r_t disappears and the equation reduces to:

$$dr(t) = k(\theta - r_t)dt \tag{3.2}$$

which can be rewritten as

$$\frac{dr_t}{(\theta - r_t)} = kdt$$

Integrating this on both sides yields

$$\int_{r_0}^{r_t} \frac{1}{(\theta - r_t)} dr = k \int_0^t dt$$

$$-(\ln(\theta - r_t) - \ln(\theta - r_0)) = kt$$

$$-\ln\left(\frac{\theta - r_t}{\theta - r_0}\right) = kt$$

Taking the exponential and rearranging the terms results in the following

$$r_t = \theta + (r_0 - \theta)e^{-kt} \tag{3.3}$$

From equation (3.3) it is apparent that when $t \to \infty$ then $r_t \to \theta$. Furthermore, the dynamics of the direction of the pull is also shown with a downwards drift when $r_0 > \theta$ and vice versa as well as the speed of the reversion being determined by the k term.

Pricing with Vasicek

When the Vasicek model have been calibrated to produce the desired yield curve, prices of zero coupon bonds can be calculated. The analytical price of a zero-coupon bond with the Vasicek model is given by (Brigo & Mercurio, 2006, p. 59):

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{3.4}$$

where A is defined as

$$A(t,T) = exp[(\theta - \frac{\sigma^2}{2k^2}) \cdot [B(t,T) - T + t] - \frac{\sigma^2}{4k}B(t,T)^2]$$

and B is defined as

$$B(t,T) = \frac{1}{k} \cdot (1 - e^{-k(T-t)}$$

Properties of the Vasicek model

The Vasicek model was the first short rate model to capture mean-reversion (Privault, 2012). In contrary to stocks, interest rates cannot increase indefinitely as this would hamper economic activity and growth, which in turn would lead to decreased interest rates to counteract this. The success of the Vasicek model is according to Brigo & Mercurio (2006) mainly due to the possibility of analytically pricing bonds and bond options. Furthermore, the equation is linear and can be solved explicitly.

The Vasicek model is traditionally calibrated to historical data i.e. a proxy for the historically realised short rates. However, according to Brigo & Mercurio (2006), this will often not produce a satisfactory fit to the current term structure. In order to improve this fit, the zero coupon curve can be approximated by the Vasicek model. This is done by choosing parameters of k, θ, and σ, which produce an initial model curve as close as possible to the observed yield curve. With this method, the σ parameter is taken from historical data as this calibration method does not include any type of market volatility structure (Brigo & Mercurio, 2006). However, even when calibrating to market prices, the three parameters are still not enough to satisfactorily reproduce any given term structure. Some shapes of the zero coupon curve like the inverted yield curve are impossible to create with the Vasicek model (Brigo & Mercurio, 2006). This means that the initial term structure of interest rates does not necessarily match what is observed in the market no matter how model parameters are chosen (Brigo & Mercurio, 2006). Thus, the small number of parameters in the model prevents a satisfactory calibration to market data. The solution to this problem of endogenous models is found in exogenous models such as the Hull-White model, which will be described in section 3.4.

Another feature of the Vasicek model is that the short rate can assume negative values with a positive probability (Brigo & Mercurio, 2006). Historically, negative rates have not been seen as realistic and this possibility has therefore been seen as a major drawback of the model. As previously mentioned in conventional literature, the parameters θ and r_0 are defined as strictly positive along k and σ. However, in the current economic environment this assumption has had to be revised for at least r_0. The possibility of a negative θ value is more unclear since negative interest rates still are a fairly new phenomenon.

The fact that negative interest rates have appeared in the real world has - at least to a certain degree - provided a renewed support of the classical Vasicek short rate model. However, depending on the view and assumptions about the future movements of the interest rate, the Vasicek model might assign too much weight on the probability of negative values when parameters are calibrated close to or as negative. This is due to the normally distributed process, which implies that the likelihood of the shock in the rate going up and down is equal. Despite the problems and limitations inherent in the Vasicek model, the model is not only chosen for its historical importance, but also to provide a foundation for the subsequent Hull-White models.

3.4 The Hull-White one-factor model

As stated the Vasicek model is an example of a time-homogeneous, equilibrium model. The disadvantage of these is that they can lead to theoretical prices of bonds that do not necessarily match the actual prices in the market. With this in mind, John Hull and Alan White explored several time-heterogeneous extensions of the model first in a paper published in 1990 (Hull, 2015; Hull & White, 1990). One of these extensions is the one later known as the Hull-White model. A time-dependence in the drift makes it possible to exogenously calibrate to the term structure of interest rates. In the Hull-White model the instantaneous short rate is determined as (Hull & White, 1994a,b):

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW(t), \tag{3.5}$$

where a and σ are positive constants of the mean-reversion and the volatility respectively. $\theta(t)$ is a deterministic function of time - however, it does not need to have a positive value. $\theta(t)$ is calibrated to perfectly match the currently observed term structure of the interest rates in the market. Thereby, the remaining model parameters can be used to calibrate to the volatility structures (Brigo & Mercurio, 2006). It is also possible to rearrange equation 3.5 to look more like the Vasicek model in order to obtain a better understanding of the distinction:

$$dr(t) = a \cdot [\frac{\theta(t)}{a} - r(t)]dt + \sigma dW(t) \tag{3.6}$$

One of the other extensions that Hull and White (1990) considered was a more general model:

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t) \tag{3.7}$$

The difference between equation 3.7 and equation 3.5, is that a and σ rather than being positive constants, are also deterministic functions of time in 3.7. This extension makes it possible to precisely fit the model to both the term structure of interest rates and term structure of either spot or forward-rate volatilities. This might, however, be problematic for two reasons. The first reason is that not all quoted volatilities necessarily are significant, since they might stem from less liquid markets and instruments - thereby they will neither be informative nor reliable, which inevitably will damage the reliability and use of the model. Secondly, the volatility structures implied by this model are likely to be unrealistic since they do not conform to market shapes typically seen in the market (Hull & White, 1995). Therefore, the model, where θ is the only parameter defined as a deterministic function of time, has been chosen for modelling purposes.

Pricing with the Hull-White

Like with Vasicek model, the Hull-White one-factor model allows for analytical prices of zero coupon bonds to be calculated. The price of a zero-coupon bond with the Hull-White model is given by (Brigo & Mercurio, 2006, p. 75):

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{3.8}$$

However, A and B is defined differently from the Vasicek model . A is defined as

$$A(t,T) = \frac{P(0,T)}{P(0,t)} \cdot \exp[B(t,T)f(0,t) - \frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)^2],$$

and B is defined as

$$B(t,T) = \frac{1}{a} \cdot (1 - e^{-a(T-t)})$$

Properties of the Hull-White

The Hull-White model is widely used in practice, which can be contributed to the fact that it is more precise than the Vasicek model, but still easy to implement and analytically tractable. Being analytically tractable implies that zero-coupon bonds and options can be explicitly priced. Furthermore, since the Hull-White model also relies on a Gaussian distribution of continuously compounded interest rates, specific analytical formulas can be derived and used to price a large varieties of derivatives (Brigo & Mercurio, 2006). The method for doing so is by expressing swaptions, caps, floors or other traded financial instruments in the form of options on zero-coupon bonds in accordance with the Hull-White model (Pelsser, 2000).

As for the Vasicek model one of the classical and most discussed drawbacks of this model is the possibility of negative rates due to the reliance on Gaussian processes. However, the probability of the instantaneous short rate being negative in practice was seen as negligible. For example with parameters of the Hull-White model calibrated to market data as of June 1999, Brigo & Mercurio (2006) show how the short rate never goes below zero within a two standard deviation window (Brigo & Mercurio, 2006, p. 74):

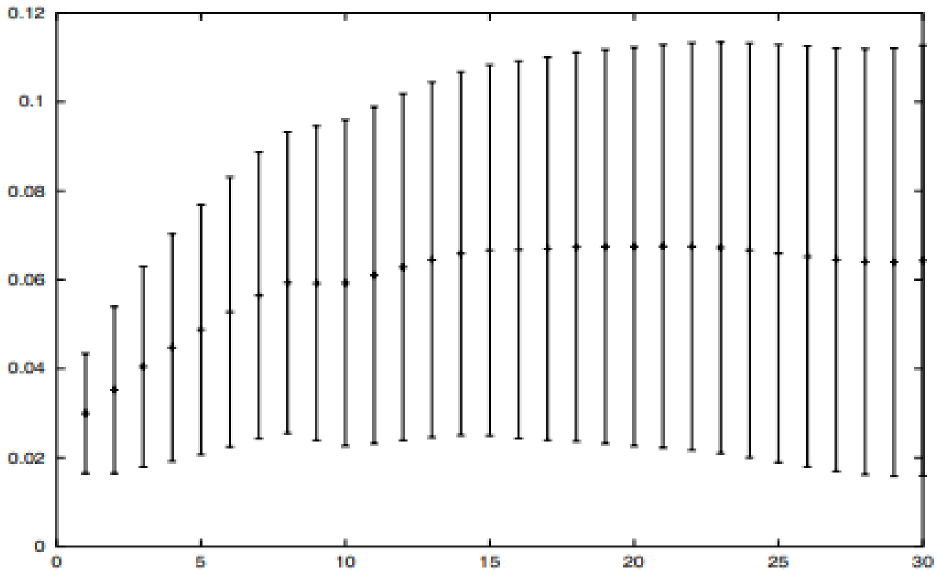

Figure 3.2: Two standard-deviation window for the evolution of the short rate over time

However, since then the market for interest rates have changed. Thereby, when calibrating the model to current market data, other parameters emerge that increase the possibility of negative rates drastically. Keeping the current interest rate environment in mind, this feature of possible negative rates seems to have become an advantage. The Hull-White model can in this regard consequently be seen as one of the possible replacements of other and sometimes newer interest rate models often specifically designed to avoid the possibility of negative interest rates as described in section 3.2. Even before this feature realistically could be seen as an advantage in the model, the Hull-White model was regarded as "one of the historically most important interest-rate models, still being used nowadays for risk-management purposes", which further supports the use of the model (Brigo & Mercurio, 2006, p. 72).

3.5 Two-Factor models

As previously described, one-factor short rate models assume that the development of the yield curve at any given time is determined from the initial point. Consequently, it is assumed in any one-factor model that a perfect correlation exists between interest rates for all maturities on the zero coupon curve. For example this implies that a twenty-year interest rate is correlated perfectly to the two-month interest rate at the same initial point. Thereby, a shock to the interest rate at a given time, t, equally shocks through all the maturities on the curve - the whole curve moves in the same direction fixedly (Brigo & Mercurio, 2006). This is hard to accept as being generally true and is a problematic assumption, especially when pricing products depending on this correlation. This issue has therefore given rise to multi-factor models. However, it does not mean that one-factor models are useless. Especially when the two relevant rates in a given case are close to each other, the approximation of a one-factor model is acceptable. This is because the two points are most likely highly correlated (Brigo & Mercurio, 2006). Whenever more precision is needed or the correlation should play a larger role e.g. with more distanced rates, a multi-factor model is needed. A multi-factor model allows for more realistic patterns of the evolution of the short rate since it includes a correlation factor $\neq 1$. This way, more flexibility is given to the model, which allows for more precise estimates.

Based on historical analysis of the yield curve, Rebonato (1998, table 3.2) finds that for the UK market, a one-factor model can explain 92% of the total variance, while two-factor model can explain 99%. In contrary Jamshidian and Zhu (1997, table 1) find that one-factor explains between 68% and 76%, whereas two-factors will explain 85% to 90% and three-factors explain 93% to 94%. Whichever is true, the improvement of going from one-factor to multiple is indisputable - a multi-factor model with more realistic correlation - and volatility structures seems necessary to be able to provide realistic predictions of the evolution of the short rate (Brigo & Mercurio, 2006). The natural question then concerns how many factors should be included in a model? This is a trade-off between ease of implementation and the capability of the model to present realistic correlations and fit to market data. In line with Brigo & Mercurio (2006), this paper will focus on two-factor models due to their better tractability and relative ease of implementation compared to models with three or more factors.

Hull-White two-factor model

Specifically, the focus will be on the Hull-White two-factor model. Hull and White extended their one-factor model to a two-factor model in 1994 with the goal of being able to accommodate more realistic volatility structures such as the volatility "hump" seen in the caplet market at the time. The model is given by (Hull & White, 1994c)

$$df(r) = [\theta(t) + u - af(r)]dt + \sigma_1 dz_1, \quad r(0) = r_0, \tag{3.9}$$

where $f(r)$ is a deterministic function of r. u follows the process of

$$du = -budt + \sigma_2 dz_2, \quad u(0) = 0 \tag{3.10}$$

As with the Hull-White one-factor model just considered, $\theta(t)$ is chosen to make the model consistent with the initial term structure (Hull, 2015). a and σ_1 also have the same interpretation as the short rate mean-reversion speed and short rate volatility respectively. u is a stochastic variable functioning as a component of the mean-reversion level of r, while reverting back to zero itself at the rate of b with volatility determined by σ_2. a, b, σ_1, and σ_2 are constants while dz_1 and dz_2 together forms a two-dimensional Wiener process with an instantaneous correlation factor of ρ. These characteristics make the model able to provide richer and more precise patterns of the movements of the term structure and volatility structures than the one-factor model could (Hull, 2015, Technical Note 14).

Pricing with two factors

Like with the previous models, the Hull-White two-factor model is analytically tractable. The pricing formula is, however, more complicated. The price at time t of a zero coupon bond is given by (Hull & White, 1994c)

$$P(t,T) = A(t,T)\exp[-B(t,T)r - C(t,T)u], \tag{3.11}$$

where

$$B(t,T) = \frac{1}{a}[1 - e^{-a(T-t)}]$$

and

$$C(t,T) = \frac{1}{a(a-b)}e^{-a(T-t)} - \frac{1}{b(a-b)}e^{-b(T-t)} + \frac{1}{ab}$$

and the A(t,T) function is given by

$$lnA(t,T) = ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \eta$$

For definition of η, see appendix B in Hull & White (1994c).

Note that both stochastic processes r and u appear in the bond pricing formula. The introduction of the u term in the two-factor model introduces non-perfect correlation between rates with different maturities. This entails that the actual variability is better described and the pricing of derivatives with payoffs depending on rates at different maturities is more precise. With the model still being analytically tractable, the pricing of derivatives can still be conducted fairly easily (Brigo & Mercurio, 2006). These two features of analytical tractability and realistic volatility structures with non-perfect correlation are the main reason two-factor Gaussian models are also considered a good choice when pricing derivatives on two interest-rate curves (Brigo & Mercurio, 2006). Dual-curve modelling with a two-factor Gaussian model will be explained further in section 4.6.

Analogy to the G2++

The Hull-White two-factor model is equal to the well-known G2++ model (Brigo & Mercurio, 2006). The G2++ is a more general notation, which stems from the model type being a Gaussian two-factor additive model. The G2++ model specifies that the instantaneous spot rate is determined by the sum of two correlated Gaussian factors and a deterministic function chosen to exactly fit to the current term structure. This notation can be further generalised to a G_n++ model depending on the number of Gaussian processes inherent in the model (Francesco, 2012). With this perspective, it is also apparent that the Hull-White one-factor model described earlier is perfectly analogous to a G1++ model with one Gaussian process and a deterministic function. In the G2++, the instantaneous short rate process is defined as (Brigo & Mercurio, 2006, p. 143)

$$r(t) = x(t) + y(t) + \phi(t), \quad r(0) = r_0, \tag{3.12}$$

where ϕ is a deterministic function and $\phi(0) = r_0$. The processes of $x(t)$ and $y(t)$ for $t \geq 0$ satisfy:

$$dx(t) = -ax(t)dt + \sigma dW_1(t), \quad x(0) = 0,$$
$$dy(t) = -by(t)dt + \eta dW_2(t), \quad y(0) = 0,$$

and (W_1,W_2) is a two-dimensional Wiener process exhibiting an instantaneous correlation -1 $\leq \rho \leq 1$ as

$$dW_1(t)dW_2(t) = \rho dt$$

If further explanation of the above G2++ being equal to the two-factor Hull-White is needed, refer to the proof in Brigo & Mercurio (2006, p. 159-162). The G2++ way of formulating the Hull-White two-factor model is often preferred in practice since it leads to less complicated formulas and is easier to implement. However, at the same time some of the intuition and insight on the nature and the interpretation of the factors is lost (Brigo & Mercurio, 2006). For this reason the Hull-White two-factor model has been the main focus of the theoretical description even though the two-factor model utilised in this paper in practice looks more like the G2++.

CHAPTER 4

Data and Methodology

While the previous chapter described the theoretical foundation for modelling the short rate and its use for derivatives pricing, the following chapter will describe how these theories and formulas can be applied in practice. Specifically, the general calibration procedures and interpolation methods used will be presented, followed by the choice of data. Afterwards, the Monte Carlo method used for simulating the short rate will be introduced. Lastly, it will be described how the G2++ model can be modified to encompass a dual-curve setup. However, before describing the application of the theory, a brief reflection on the scientific methodology employed in this paper is appropriate.

4.1 Scientific approach

Scientific research can generally be conducted through deductive, inductive or abductive approaches. In the deductive approach established theory is examined by formulating hypotheses and subsequently testing these through the application of empirical data. Conversely, when working with the inductive approach, the world is first observed upon which general patterns are induced (Andersen, 2013). Lastly, the abductive approach is defined as inferring the truth from the best accessible explanation of the phenomenon (Douven, 2017). In modelling the short rate in accordance with established theoretical foundations and assumptions, the research design of this paper can initially be considered to be deductive in nature. However, given the lack of other research in the area of ESTER discounting, the research methodology will rely on the abductive school of thought. By focusing on obtaining the best explanation for a phenomenon, which in this case is the possible implications of the ESTER implementation on pricing and risk of EUR-denominated interest rate derivatives, the used method can best be classified as abductive. By using an abductive method, the research is not constrained to being solely inductive or deductive in nature. Instead, it is possible to continuously adjust the method when relevant as new knowledge of the topic is acquired.

4.2 Calibrating a short rate model

The main use of interest rate models are pricing interest rate derivatives (Vojtek, 2004). As previously mentioned even small discrepancies in the price of the underlying bond will lead to large errors in the pricing of derivatives. To minimise these errors and make the output meaningful, a model should be calibrated as the first step after setting up the model. Calibrating a model can be explained as determining the values of the parameters in the model (Hull, 2015).

What the model is trying to fit depends on whether the models belong to the family of equilibrium models or no-arbitrage models. Given endogenous models handle the yield curve as an output, the Vasicek model will initially be calibrated to proxies for the different short rates, namely the daily realised rates of the EONIA, Pre-ESTER and 6M Euribor curve. If the approximation to the yield curve proves unsatisfactory, the Vasicek model will also be calibrated to market prices i.e. the term structure as explained in section 3.3. Contrarily, the no-arbitrage models are always calibrated to market prices as they contain a deterministic term, which makes it able to fit the initial yield curve.

In the no-arbitrage models an initial term structure of interest rates along with prices of non-linear derivatives are used as inputs for determining the initial parameters. Thus, the market prices of a specific date will have to be chosen. The term structure of interest rates can be determined through the rates on market traded interest rate instruments. These instruments are coined as calibrating instruments. Calibrating instruments will typically be quotes on FRAs and IRS. The volatility parameters are on the other hand determined from caps, floors or swaptions. Calibrating instruments should in general be chosen to be similar to the instrument that is to be valued with the model (Hull, 2015). Specifically, the Hull-White one-factor model will be calibrated to floor volatilities, while the G2++ will be calibrated to swaption volatilities. As mentioned in section 3.5, the

G2++ allows for non-perfect correlation between rates with different maturities. This makes the G2++ models especially useful in pricing correlation-based products such as swaptions (Brigo & Mercurio, 2006). In section 4.4, the specific calibrating instruments will be presented.

The initial term structure of interest rates is determined through a procedure known as bootstrapping. The objective of bootstrapping is to construct a yield curve, implied by the market prices of interest rate products at different maturities. The model is then calibrated to the bootstrapped yield curve by choosing some goodness of fit measure and minimising the error between model and market prices in order to procure the best fit. According to Hull (2015, p. 732), a popular measure is

$$\sum_{i=1}^{n}(U_i - V_i)^2$$

where U_i is the market price of the i'th calibrating instrument and V_i is the price obtained through the model for the same instrument. Thereby, the objective of the calibration is to choose the parameters that minimise the sum of squared differences between the market prices and the model prices of the same instruments.

The potential problem when minimising this goodness of fit measure is that the function of the parameters does not contain a particular form or shape. This entails that most optimisation functions only will be able to find a local minimum, which makes the optimisation highly dependant on the initial parameters (Cairns, 2003). This fact is more relevant for some calibration procedures than others, which will be further explained in the description of each specific model and the corresponding calibration procedure.

4.3 Interpolation and yield curve construction

When attempting to calibrate a no-arbitrage model to an initial term structure of interest rates, it is done from a discrete set of knot points i.e. market instruments. From these points, a term structure for every possible tenor, i.e. a continuous term structure, is determined. This in turn indicates that the shape of the yield curve is not unique. The shape is dependant on both the choice of calibrating instruments, but also on how sections of the curve between knot points are interpolated. When pricing derivatives a continuous curve is necessary as every payment does not necessarily fall on the exact date as one of the calibrating market instruments. Thus, in order to determine the appropriate forward rates and discount factors for these payments, an interpolation methodology will have to be selected (Hagan & West, 2006).

Several ways of choosing both the instruments and the interpolation method exist. In terms of choosing the number of instruments, the goal is to choose enough knot points to enable the creation of a smooth curve without overfitting the data (Hagan & West, 2006). For the models in this paper the choice of knot points will follow the same methodology as used by Bloomberg in their 6M EURIBOR and EONIA curves (Bloomberg, 2020). This choice is unlikely to be universal; others might prefer a different number of points due to perceived liquidity concerns at different maturities or to construct the points from other instruments. This, however, simply proves the plethora of ways these yield curves can be constructed.

The interpolation method can be chosen for a variety of different desirable features. First and foremost, it is important to have a smooth zero coupon curve and a smooth and robust forward curve. Continuity is necessary in order to avoid arbitrage or arbitrage-like conditions. Robustness on the other hand is necessary to avoid massive changes in the forward curve caused by small changes in one of the calibrating instruments. Secondary features includes the locality of the interpolation method, i.e. whether or not changing a knot point only affects the curve around it or the curve in its entirety (Hagan & West, 2006). In order to satisfy the primary features of the interpolation method, the Hermite cubic spline will be used. Given a discrete set of n data points, $\tau_1, ..., \tau_n$ and corresponding zero rates $r_1, ...r_n$, consider the rate r_τ, which does not exits as a knot point on the zero curve i.e. $\tau_i < \tau < \tau_{i+1}$. The cubic spline finds r_τ through a cubic polynomial of the form

$$r(\tau) = a_i + b_i(\tau - \tau_i + c_i(\tau - \tau_i)^2 + d_i(\tau - \tau_i)^3 \tag{4.1}$$

Fitting a cubic polynomial ensures that the term structure of zero rates are continuous and differentiable. This in turn entails that forward rates will be robust and continuous. While the cubic spline satisfy the primary interpolation concerns, it is not without flaws. First of all, the cubic spline tends to put too much curvature on curves constructed from sparse knot points (Hagan & West, 2006). This is typically problematic for curves with sparse data points as these in reality tend to be more linear. This is not a concern in the European market as instruments of many different tenors are available for curve construction (Bloomberg, 2020). Furthermore, the shape of the Euribor and EONIA curves are not strictly downwards or upwards sloping across maturities. Another problem with the Hermite is the *non-locality* of the curve. As the method fits the entire yield curve at once, it can produce non-sensible risk figures. For example the value of a 9.5-year FRA can be affected by a bump of the 1-year zero rate even though this does not make sense from a theoretical point of view. This inconvenience is, however, acceptable in order to produce continuous zero and forward rates. Furthermore, these cases of non-locality are typically minor.

4.4 Choice of data

When building short rate models, a decision on what data to analyse will need to be made. Depending on the model and whether the model belongs to the family of equilibrium or no-arbitrage models, different types of data will be needed and are appropriate. Furthermore, the period of the data series that the models will be calibrated to will have to be considered. These decisions are made with respect to both what is to be analysed, but also with availability of data in mind. The Bloomberg platform has been used throughout the analysis to acquire all relevant market data (Bloomberg, 2020).

Historic short rate data

As previously stated, this paper is considering Euribor 6M contracts, which in the dual-curve setup are discounted with the EONIA and subsequently will be replaced by ESTER discounting. Prior to the publication of ESTER on the 3rd October 2019, the Pre-ESTER was published by the ECB relying on the same calculation procedure as the actual ESTER index. The Pre-ESTER was published from 15th March 2017 to the 30th September 2019. For the initial equilibrium model calibrated to proxies for the short rate, this time period will be used for the analysis. This period is chosen as it is the only time period where EONIA and ESTER are comparable, since EONIA afterwards was redefined as ESTER + a spread of 8.5bps (European Central Bank, 2019a). This entails that the two data series are no longer useful to compare after September 30th 2019. The characteristics of the Pre-ESTER and EONIA between 15th March 2017 and 30th September 2019 is summarised in table 4.1

Table 4.1: Summary statistics for Pre-Ester and EONIA

	Pre-Ester	EONIA	Spread (bps)
Mean	-0.451	-0.361	8.8
Median	-0.449	-0.362	8.7
Stddev	0.013	0.015	1.064
Min	-0.555	-0.458	5.8
Max	-0.426	-0.358	21.5
Pctl(25)	-0.451	-0.367	8.4
Pctl(75)	-0.448	-0.358	9
Max_Drawdown_10Bdays	0.106	0.127	13.2

As can be seen by the mean, an average spread of 8.8 bps exists between the two indices with Pre-ESTER being lower than EONIA. Both series are characterised by low volatility with the EONIA exhibiting slightly more volatility than the Pre-ESTER. In figure 2.1 it was apparent that the EONIA had a tendency to experience volatility spikes around quarter- and month-end periods. These spikes naturally causes the spread between the two indices to widen. The period of the two data series coexisting was in general characterised by low volatility. Comparing the EONIA to its historical data, it is apparent that the low volatility is a more recent phenomenon, which can be seen in figure 4.1

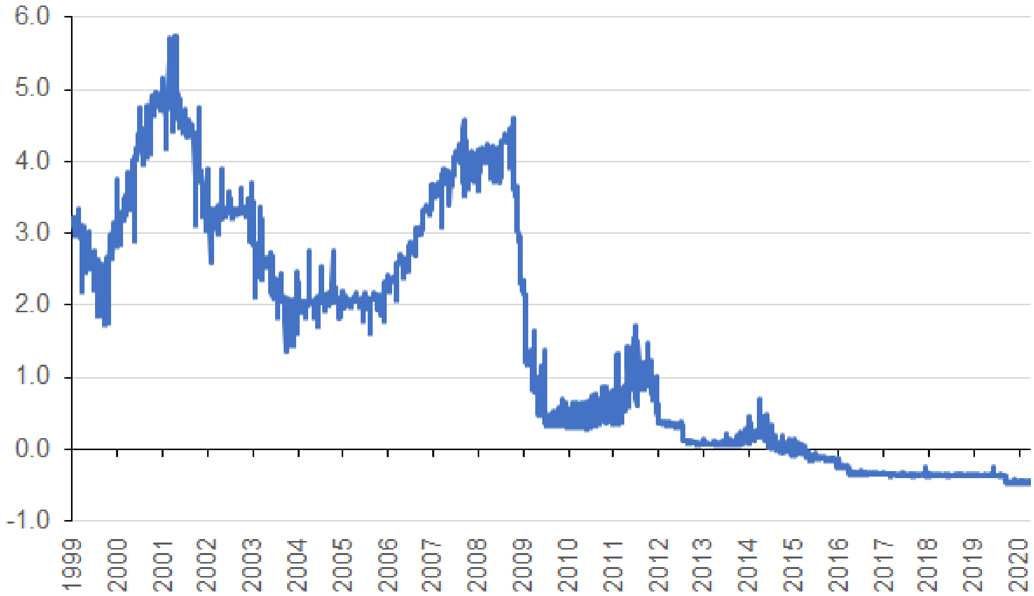

Figure 4.1: EONIA since inception (Bloomberg, 2020).

In the latter part of the figure, where EONIA and Pre-ESTER coexisted, the data almost appears to exhibit stationary tendencies. How the Pre-ESTER would have looked previously is impossible to know, but it is likely that it would have followed the movements of the EONIA to a certain degree. Regardless, calibrating the equilibrium model to this short time series is likely to produce different results than if a longer period is used. However, as previously explained this is not possible for the Pre-ESTER. The ramifications on the initial equilibrium model of using this period with lower volatility will be elaborated in section 5.1

Market data

In terms of data used to calibrate to market prices, two main considerations must be kept in mind. These are the date of calibration and which calibrating instruments to include.

As the relevant date for data to be used in calibration of the models, the 22th August 2019 has been chosen. This date has been chosen for mainly three reasons: Firstly, the date is from the time when EONIA was still being published in its original form and not revised into ESTER + 8.5 bps. This is important to be able to determine the shape of the pure EONIA curve. Secondly, the 22nd August 2019 was a day, where the EONIA did not exhibit any volatility spikes. Such a spike would potentially have interfered with the pricing of the rest of yield curve and thus the calibrated parameters. Lastly, this date has also been chosen, as it was not influenced by any changes in the deposit rate by the ECB. As can be seen from figure 1.2, both EONIA and Pre-Ester fell approximately 10 bps towards the end of the series. This fall happened a week after the ECB cut the central funding rate with 10 bps (Bloomberg, 2020). As it is widely known, the central bank controls the short rates to a large extent. Examining the EONIA in a time where the market is likely to reevaluate the risk-neutral expectations of the future spot rates might therefore produce skewed results.

As for the calibrating instruments, the same methodology is used as Bloomberg uses for the yield curve construction as previously described. For the 6M EURIBOR, this indicates the curve is constructed from FRAs with expiry-maturity of 1X7, 2X8...up to 12X18 months, while IRS' are used for any maturity from two years. However, since EONIA is an overnight rate the curve will be constructed purely from OIS swaps. When calibrating yield curves in a dual-curve setup, the projection curve, Euribor, and the discount curve, EONIA, will have to be calibrated jointly. When a separate curve is used for discounting it will affect the forward rates implied by the market instruments. This entails that the projection curve will change as it is no longer discounted with itself, but instead is discounted by discount factors derived from the OIS curve. This is known as OIS stripping or dual-curve stripping. The effect of dual-curve stripping is expected to be negligible given the minor difference in the level of the projection and discounting curves. It has, however, become market standard and must therefore be included to ensure market consistent pricing (Ametrano & Bianchetti, 2013; Hull, 2015). Based upon this, all relevant market data is exported using OIS stripping.

Now that the data is chosen and it is explained how this is used for calibrating purposes, it is important to clarify how this information is utilised to model the short rate. This will be the focus of the following section.

4.5 Monte Carlo

Monte Carlo methods, or simply Monte Carlo, is the concept of using randomness to solve problems and is based on a coherence between probability and volume. Monte Carlo interprets the volume of an outcome as a probability. In a simple case this means that from a random sample of possible outcomes, the fraction or volume of random draws resulting in a specific outcome is the best estimate of this probability. The law of large numbers ensures that this estimate converges towards the true value as the number of draws, N, increases. The central limit theorem gives information about the possible magnitude of the error given by this estimate with N number of draws (Glasserman, 2004).

Random numbers

Monte Carlo utilises a sequence of seemingly random numbers to drive the simulation. These numbers, generated through a random number generator, are however, merely pseudo random numbers since they are a product of deterministic algorithms. Even so, modern random number generators are sufficiently good to treat the resulting numbers as genuinely random (Glasserman, 2004). One advantage of pseudo random numbers in comparison with truly random numbers is the possibility of reproducibility. Whenever a simulation is run, it is important to be able to recreate the results. This can only be accomplished with the same input of numbers. In practice this is done through a seed function, which ensures the results remain the same every time the simulation is performed.

Pricing principles and error using Monte Carlo

The use of Monte Carlo is not necessary to price e.g. standard European options with corresponding payoffs only dependant on final value of the underlying. The key advantage is rather being able to evaluate products with path-dependant payoffs (Glasserman, 2004). The final payoff of these products at the time of exercise is dependant on the history of the underlying and not just the final value of this. In risk-neutral pricing theory as explained in section 2.2, the price of such a derivative can be represented through the expected value. Valuing these derivatives thereby comes down to computing these expectations. However, Monte Carlo although not necessary is still useful for pricing the instruments of choice given the complexity of some of the analytical pricing formulas seen in chapter 3. Thus, Monte Carlo will subsequently be used to price both swaps and swaptions. This is especially convenient for swaptions as these depend on the probability distribution of the forward rates. The simulated short rate paths are thereby an approximation of this distribution.

Monte Carlo relies on simulating the stochastic processes used to describe the evolution of the underlying interest rate. How these are defined is dependant on the specific model used, but nonetheless applicable for them all in order to compute the expectations of the underlying short rate. Since the method involves forward propagation of the relevant variables over time it is ideal to "travel forward in time" (Brigo & Mercurio, 2006, p. 114). This fact makes the use of Monte Carlo especially applicable in this analysis due to the focus on the influence of the short rate on derivative pricing reliant on the future evolution of the selfsame underlying short rate.

Pricing the payoff of an interest rate derivative with Monte Carlo thus relies on simulating a number of paths of the short rate process, computing the arithmetic mean of these paths and lastly discount these payoffs back to the present using this average short rate path (Brigo & Mercurio, 2006). As the central limit theorem can be used to describe the magnitude of possible error in the Monte Carlo based on the number of simulated short rate paths, N. Through the central limit theorem and assuming that X_1, X_2... are independently and identically distributed, the sample standard error can be defined as (Glasserman, 2004, p. 542)

$$s_n = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \bar{X}_n)^2} \tag{4.2}$$

The error inherent in equation (4.2) is also known as simulation error and can be minimised by increasing the number of simulated paths, N. The implicit convergence rate of $\sqrt{\frac{1}{n-1}}$ shows that N has to be large for precise estimates - to increase precision by a factor of 10 requires that N is increased 100 times.

In the computations 100,000 simulated paths is chosen, when the simulation is used for zero coupon bond pricing. Choosing the number of simulations is a trade-off between the marginal improvement of the precision of the estimate and required computational power. With 100,000 simulations, it is still possible to effectively run the code in maximum a couple of minutes. Running larger simulations would have been problematic when using the short rate paths to price the swaptions in Excel. As the central limit theorem states as $n \to \infty$, the simulated estimates will converge towards the true value, but at a marginally decreasing rate. In testing this assumption from a computational time and precision perspective, the simulations were also run with 10, 100, 1,000, and 1,000,000 different paths. While moving from 100,000 to 1,000,000 simulations would improve the precision of the results, these improvements are less significant compared to going from 10 to 1,000 or even from 10,000 to 100,000 simulations.

Another type of possible error in Monte Carlo is the discretisation error. This type of error describes the bias in estimates originating from time discretisation of stochastic differential equations. When simulating the process, the time horizon is divided into smaller increments known as time steps, dt. A discrepancy between the time steps and the underlying model assuming continuous time thereby results in a bias. Like the simulation error can be reduced by increasing the number of simulations, the discretisation error can be decreased by making the time step smaller. As the time step converges towards 0, the simulation will converge to continuous time. However, decreasing the time step also comes at a price of increased computational power needed since it entails generating more transitions for each path with the time horizon fixed. Accordingly, this trade-off will be taken into account when building the models.

Now that the use of Monte Carlo methods have been explained, the following section will explain how the G2++ model can be modified to a dual-curve setup. Such a modification is necessary in order to enable a simultaneous simulation of the projection and discounting curve.

4.6 Modifying the G2++ to a dual-curve setup

The G2++ Model can be modified and applied to two curves by fitting two separate G2++ models to the two yield curves and computing the correlation between the evolution of the two short rates. The evolution of the short rate process can still be described as equation (3.12).

Fitting a G2++ model sequentially to the two curves implicitly assumes that the two short rates evolve independently from each other, i.e. the correlation between the two curves is equal to zero. This is adequate if the two short rates are uncorrelated. Given that the two curves are in a single currency, it is, however, to be expected that a positive correlation between the discounting curve, EONIA, and the projecting curve, 6M Euribor, is present. Thus, a correlation will have to be estimated in order to adequately simulate the two rates (Brigo & Mercurio, 2006).

As the model comprises four separate Wiener processes, a 4x4 correlation matrix between the four processes will have to be computed. Thus, the following instantaneous correlations between the different factors of the two curves are assumed

$$d\begin{bmatrix} W_1^x \\ W_1^y \\ W_2^x \\ W_2^y \end{bmatrix} d\begin{bmatrix} W_1^x & W_1^y & W_2^x & W_{2^y} \end{bmatrix} = \begin{bmatrix} 1 & \rho_1 & \gamma_{x1,x2} & \gamma_{x1,y2} \\ . & 1 & \gamma_{y1,x2} & \gamma_{y1,y2} \\ . & . & 1 & \rho_2 \\ . & . & . & 1 \end{bmatrix} dt, \tag{4.3}$$

where the unspecified entries are determined by symmetry.

Like ρ_i is the instantaneous correlation between shocks in the first factor x_i and the second factor y_i in curve i, $\gamma_{x1,x2}$ refers to the instantaneous correlation between shocks in the first factor of the first curve $x1$ and shocks in the first factor of the second curve $x2$. $\gamma_{y1,x2}$ refers to the instantaneous correlation between shocks in the second factor of the first curve $y1$ and the first factor of the second curve $x2$ and so on. Given this correlation matrix the correlation between the two short rates is given as (Brigo & Mercurio, 2006)

$$Corr(dr_1, dr_2) = \frac{\sigma_1\sigma_2\gamma_{x1,x2} + \eta_1\sigma_2\gamma_{y1,x2} + \sigma_1\eta_2\gamma_{x1,y2} + \eta_1\eta_2\gamma_{y1,y2}}{\sqrt{\sigma_1^2 + \eta_1^2 + 2\rho_1\sigma_1\eta_1} \cdot \sqrt{\sigma_2^2 + \eta_2^2 + 2\rho_2\sigma_2\eta_2}} \tag{4.4}$$

Following the simplifying assumption that $\gamma_{x1,x2} = \gamma_{y1,x2} = \gamma_{x1,y2} = \gamma_{y1,y2} = \gamma$, γ can be determined by rearranging equation (4.4) (Brigo & Mercurio, 2006)

$$\gamma = Corr(dr_1, dr_2) \cdot \frac{\sqrt{\sigma_1^2 + \eta_1^2 + 2\rho_1\sigma_1\eta_1} \cdot \sqrt{\sigma_2^2 + \eta_2^2 + 2\rho_2\sigma_2\eta_2}}{(\sigma_1 + \eta_1)(\sigma_2 + \eta_2)} \tag{4.5}$$

This indicates that after having calibrated to the two individual curves, the correlation matrix can be determined as long as an assumed correlation between the two short rates is known and entered in the equation. This correlation can either express some sort of historical correlation or express the belief of the trader or entity performing the modelling (Brigo & Mercurio, 2006). For the purposes of this paper, the historical correlation for the daily data will be used.

CHAPTER 5

Interest Rate Modelling

In this chapter it will be described how the models are built and how they are calibrated. This will be followed by the output of the models. To this end, the same structure as previously will be followed by continuously building on the previous model - moving from the Vasicek model to the Hull-White one-factor model to the G2++ model and lastly the G2++ model with a dual-curve setup. The models are mainly built in the script language R. For the specific code for each of these please refer to appendix A.

5.1 Vasicek model

In the Vasicek model the yield curve is an output rather than an input as in the no-arbitrage models. Firstly, the Vasicek model will be determined through the classical method of using a proxy for the historical short rate. This proxy will be daily observations of the realised rates. Thereafter, the Vasicek model will be determined again, but by calibrating the model to market prices i.e. zero rates. Going forward a is going to be used as the variable name of the mean-reversion speed rather than k in order to make the comparison with subsequent models easier.

Building and calibrating the Vasicek model to realised rates

A trading year is usually assumed to contain 252 trading days. Therefore, for the initial Vasicek model, a time step of 1/252 is used to model daily movements in the short rate (Hull, 2015). To calibrate the Vasicek model with the realised proxy for the historical short rate, maximum likelihood estimation (MLE) is used. MLE can in this regard be described as estimating the parameters of the model that maximises the likelihood of the realised short rates happening (Hull, 2015). A Vasicek model has been calibrated to the daily data of Pre-ESTER, the EONIA and the 6M Euribor for the same period as described in section 4.4. Furthermore, in order to check the impact of including a longer time period, a Vasicek model will also be calibrated to the 6M Euribor since its initiation in 1998. For the code related to the historically calibrated Vasicek model, please refer to Appendix A.1 for the calibration procedure and Appendix A.3 for the application of the model.

Through the MLE calibration the following parameters are estimated for Pre-Ester, EONIA, 6M Euribor as well as the full period 6M Euribor

Table 5.1: Vasicek calibrated annualised parameters for Pre-Ester, EONIA, Euribor and Euribor from 1998 to 2019

	Pre-Ester	EONIA	Euribor	Euribor_1998-
a	39.7469	70.1100	-0.7389	-0.0160
σ	0.000899	0.001637	0.000460	0.002467
θ	-0.004495	-0.003619	-0.001963	0.125696

As can be seen from table 5.1 applying the Vasicek to the realised rates yields some interesting results. The Pre-Ester and the EONIA have tremendous mean-reversion parameters. This is unsurprising as the data used for calibration appears to be quite stationary as seen in figure 1.2. However, it does not make much sense in the real world as this indicates that the short rates will remain stationary around the long term mean. This can also be seen from θ, which is suspiciously close to the respective mean values as seen in table 4.1. Both short-term rate series exhibit fairly low volatility parameters with the EONIA having a bit higher volatility than the Pre-ESTER. Given the EONIA's propensity to exhibit sudden spikes this is unsurprising.

For the daily 6M Euribor data the results are quite different. When calibrated to the same period as the Pre-Ester and EONIA data, a negative mean-reversion factor is yielded. With a negative a, the rate will move towards ∞ or $-\infty$ depending on whether θ is above or below zero. a thus effectively acts as a mean-repulsion rather than a mean-reversion factor. The further away the series is from the "long term mean" the more strongly it is repelled - hence the quotation marks around the long term mean in this case. A negative a can be acceptable in the short term, but the resulting dynamics of mean-repulsion are the opposite of what is sought after in a mean-reversion model and is therefore typically a sign that the series in fact does not contain a mean-reversion property (Gurrieri, Nakabayashi, & Wong, 2009). When calibrating the Vasicek to the entire 6M Euribor series all the way back to 1998 a albeit less powerful remains negative. While the negative mean-reversion parameter is counter-intuitive, it is not surprising when considering the historic evolution of the 6M Euribor. The 6M Euribor have experienced a severe and enduring fall - both from 2017 and 1998 till today (Bloomberg, 2020).

Vasicek simulation with realised rates

To show the principle of simulated short rate paths, the Vasicek model is simulated in the following with only 10 different paths and some arbitrarily chosen parameters in order to illustrate the model as it has been intended. Illustrating the movement of the short rate with only 10 paths makes the interpretation and results easier to observe.

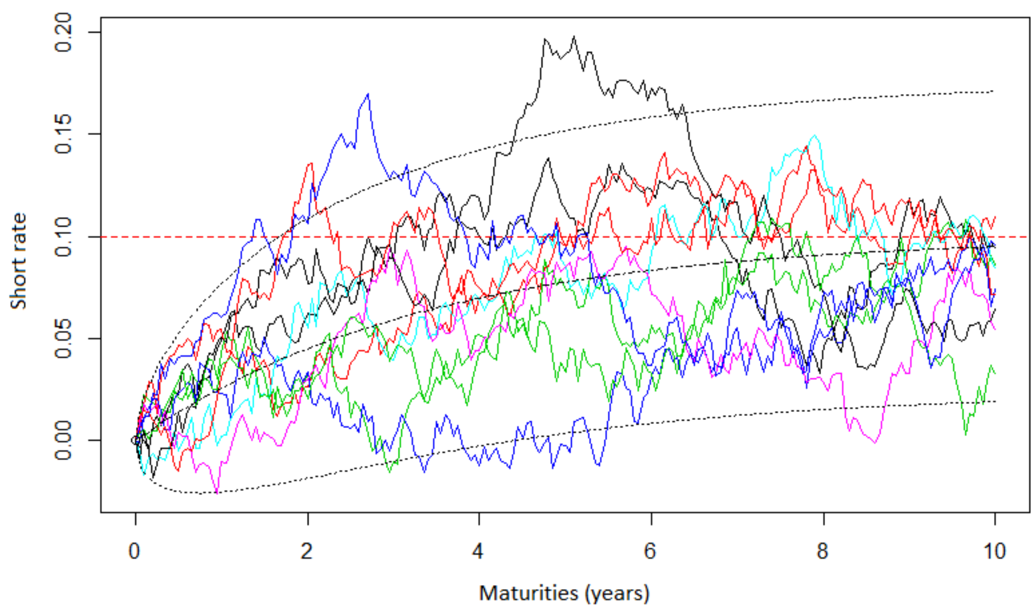

Figure 5.1: Vasicek simulated short rate paths with $r_0 = 0$, $\theta = 0.1$, $a = 0.3$, $\sigma = 0.03$

The dashed red line in figure 5.1 represents the long term mean θ, while the three dashed black lines represents the mean of the simulated values as well as the 95% confidence interval. As a>0, the mean of the simulated paths converge towards the long term mean as time goes by.

In the following figure, N is increased to 10,000 simulated paths and the calibrated Pre-ESTER values from table 5.1 are used. The reason behind the simulation relying on 10,000 rather than 100,000 is due to R not being able to handle the latter in light of the daily time step. However, this is not considered an issue since 10,000 should be enough to approximately uphold the law of large numbers. By simulating with the calibrated parameters and 10,000 simulated paths, figure 5.2 below is obtained

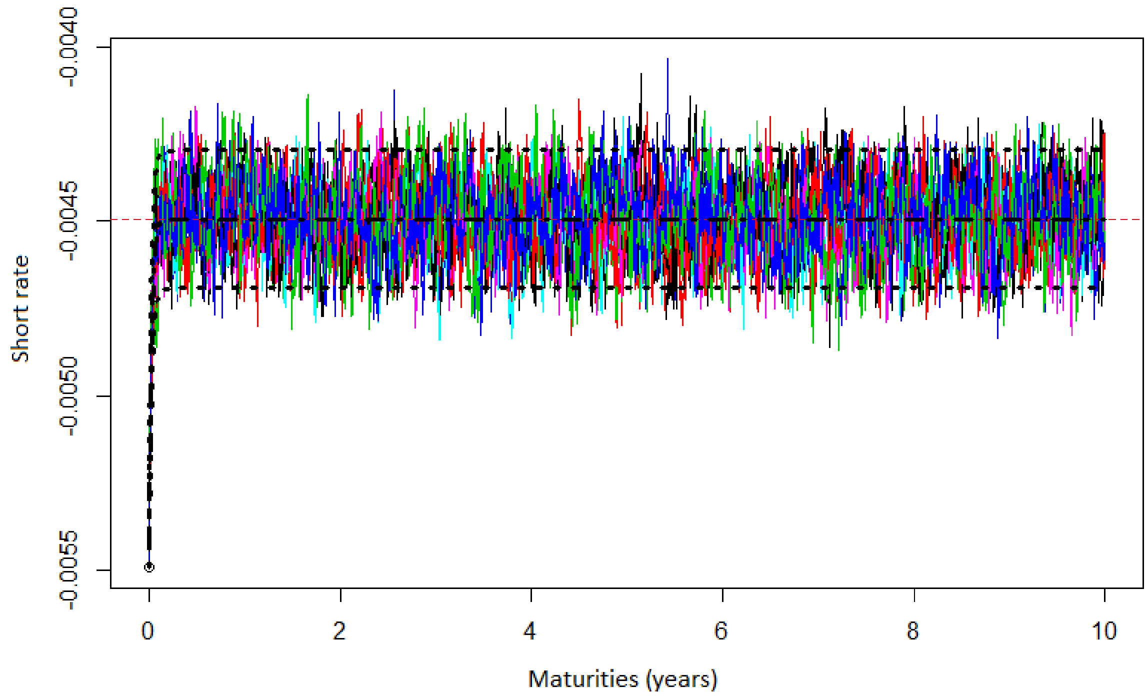

Figure 5.2: Vasicek simulated Pre-Ester paths with $r_0 = -0.00549$, $\theta = -0.004495$, $a = 39.746917$, $\sigma = 0.000899$

The long term mean and the strong mean-reversion factor cause the short rate to almost instantaneously revert back to the mean. As is seen in figure 1.2, the EONIA and the

Pre-Ester fell $\approx$ 10 bps before the launch of the Ester. This drop means that r_0 begins well below the long term mean level while quickly reverting back. The simulated short rate stays at this long term level as can be seen by the mean and the 95% confidence interval. The short rates simulated with the Vasicek model thus strongly resembles a stationary variable with a stochastic component.

As can be seen by this result, the simulated short rates imply a completely flat yield curve, where every future interest rate is equal to the long term mean rate. This does not look anything like the yield curve that the simulation is supposed to approximate. If the calibration to realised rates proves unsatisfactory as the case here, the Vasicek model can also be calibrated to market prices as described in section 3.3. Consequently, the following section will do so in an attempt to improve the fit and applicability of the Vasicek model.

Vasicek calibration and simulation with market zero rates

Since there are no instruments written on the Pre-Ester in the given period of comparison, the Euribor and EONIA are chosen for this part of the analysis. Therefore, the model will be calibrated to a zero curve on Euribor and EONIA. These zero curves are constructed as described in section 4.4 using FRAs and IRS' for Euribor and OIS' for EONIA on 22nd September 2019. The code for this calibration can be seen in appendix A.2, while the code in appendix A.4 is used for the simulation.

Aside from using market data in the form of zero rates instead of realised rates and a time step of one, the methodology is unchanged from the previous calibration. By calibrating the Vasicek model to the Euribor and EONIA market data the following parameters are obtained

Table 5.2: Vasicek calibrated annualised parameters with market zero rates for Euribor and EONIA

	EONIA	Euribor
a	0.03969	0.05023
σ	0.00164	0.00046
θ	0.00172	0.00285

These calibrated values make more intuitive sense with neither a negative nor an immensely high mean-reverting speed of a. Furthermore, a positive θ is also observed, yielding a positive long-term mean as expected in classical literature.

Figure 5.3 shows the simulated paths of the Euribor using these calibrated parameters. In this simulation, it is possible to increase N from 10,000 to 100,000, since the simulation no longer relies on a time step of 1/252 as with the realised rates, but rather annual time steps.

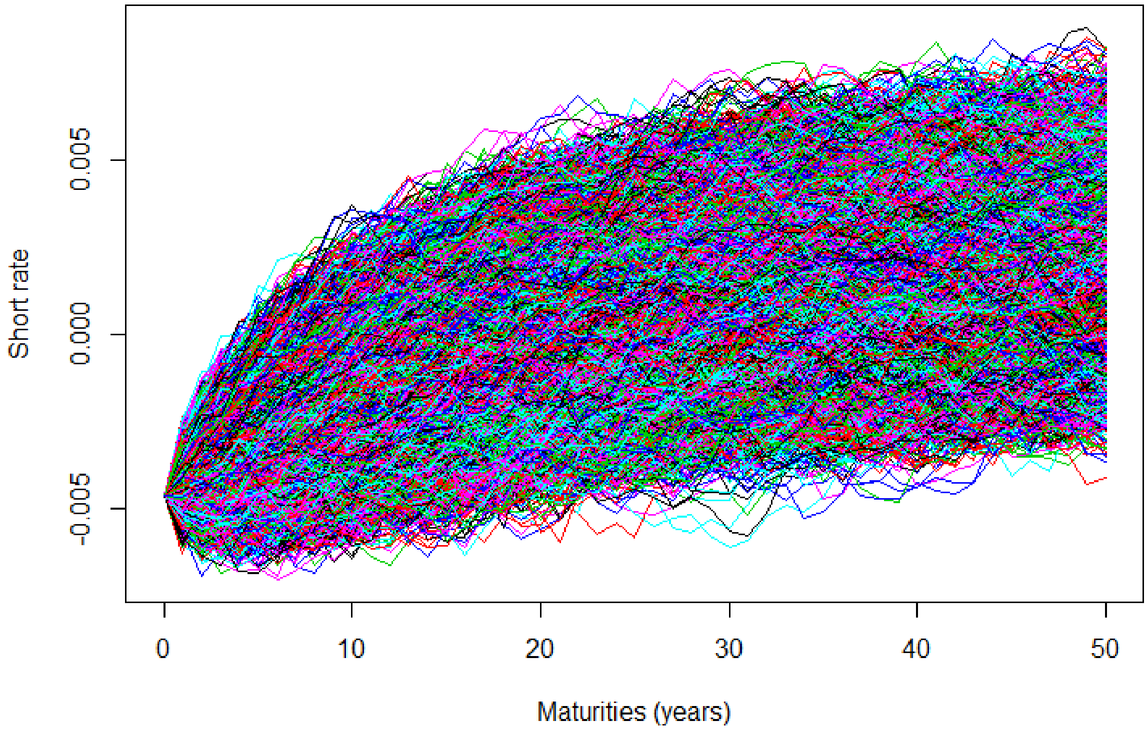

Figure 5.3: Vasicek simulated paths for Euribor with $r_0 = -0.004607$, $\theta = 0.002850$, $a = 0.05023$, $\sigma = 0.000460$

As with figure 5.2 it can be hard to distinguish the individual paths. However, the upwards trend towards θ from r_0 is illustrated with a more realistic speed of a. Thereby, it is apparent that the a value calibrated from market data does not force the simulation to instantly revert to the long-term mean nor acts as a mean-repellent. Instead, it imposes a drift towards the long-term mean over time as the intent of a mean-reverting model. In this regard, it is obvious how this simulation based on the market zero rates looks more like a mean-reverting short rate process than figure 5.2, which is based on the realised rates.

To examine the fit of the simulated Vasicek model on Euribor, figure 5.4 shows the corresponding zero coupon rates for the Euribor compared to the market values.

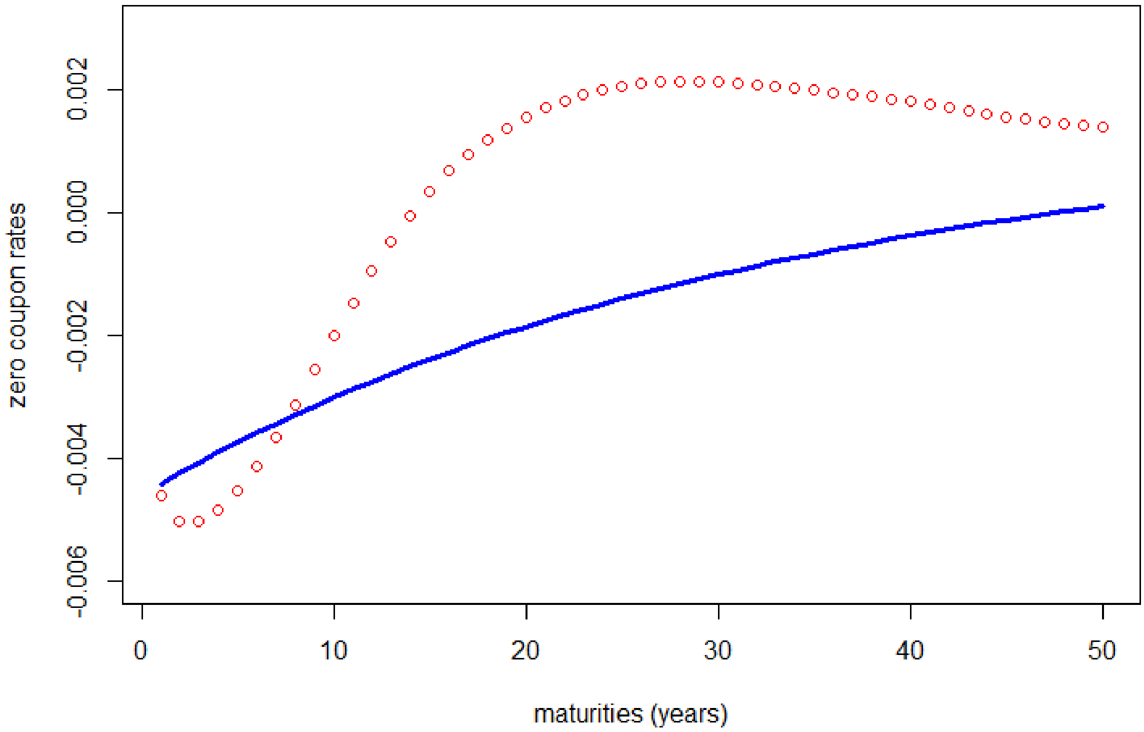

Figure 5.4: Vasicek simulated zero rates for Euribor with $r_0 = -0.004607$, $\theta = 0.002850$, $a = 0.05023$, $\sigma = 0.000460$

The market zero rates are depicted by the red dots, while the Vasicek zero rates are in blue. As is illustrated in figure 5.4, even when calibrated to market zero rates, the Vasicek model cannot match the term structure as also explained in section 3.2. Given the inability to fit the yield curve, the EONIA will not be simulated using the Vasicek model. Since the Vasicek model is not able to accurately match the observed yield curve with neither historic nor market data, the model is abandoned for the subsequent no-arbitrage models.

5.2 Hull-White one-factor model

In this section, the Hull-White one-factor model will be applied. First, it will be briefly described how the model is built and calibrated, which will be followed by simulations and analysis.

Building the model

The Hull-White one-factor model is calibrated using a Levenberg-Marquardt optimisation function. This is a non-linear least squares optimisation technique that relies on iterative improvements dependant on an initial "guess". This results in the function often only being able to find a local minimum. Such a local minimum can be different from the global minimum as mentioned in section 4.2. However, the Levenberg-Marquardt algorithm is said to be robust and able to find a minimum even if the initial guess is far off the actual minimum using a Gauss-Newton approach with a gradient descent (Lourakis, 2005). A further review and discussion of the underlying optimisation method and the possible implications of the technique is beyond the scope of this thesis, but the curious reader is encouraged to refer to Press, Flannery, Teukolsky, & Vetterling (2007).

The Hull-White model has initially been modelled on the Euribor 6M interest rate products on the 22th August 2019. These products are FRA and IRS contracts for the short and long end of the yield curve respectively, while the α and σ are calibrated from interest rate floors. In the calibration procedure, the *hwcalbyfloor* function in Matlab has been used, which enables calibration of this model through the use of floor market data even in negative interest rate environments using the Bachelier model (MathWorks, 2020). Being effectively able to handle these negative interest rates in the calibration procedure is also the reason behind the deviation from the use of R in this part of the analysis. Most relevant R packages such as RQuantLib utilises a log-normal distribution in the optimisation algorithm, whereas the *hwcalbyfloor* function in Matlab enables using the Bachelier model instead (Eddelbuettel, Nguyen, & Leitch, 2020). As previously described in section 2.4, functions reliant on log-normal distributions are not able to handle negative rates in contrast to the Bachelier model. The code for the calibration can be seen in appendix A.5. Through the calibration procedure described above the parameters presented in table 5.3 are obtained, which subsequently will be used in the Hull-White one-factor model.

Table 5.3: Parameters for the Hull-White one-factor model calibrated from zero coupon rates and floors on 6 month Euribor

	Parameters
a	0.00190
σ	0.00801

Building the Hull-White one-factor model has relied on the theory described in section 3.4 including e.g. calculating zero coupon bond prices as seen in equation 3.8 as well as the interpolation procedure described in section 4.3. The short rate is simulated with a 50 year time period. To this end, the *ESG* (Economic Scenario Generator) package in R is used to simulate the Gaussian shocks (Croix, Moudiki, Planchet, & Youssef, 2020). For the relevant code for application of the Hull-White one-factor model refer to Appendix A.6.

Simulating with the Hull-White one-factor model

In the following section the methodology described above will be used together with the calibrated parameters to simulate the evolution of the short rate. Figure 5.5 shows the quantiles of 100,000 different paths simulated with the Hull-White one-factor model.

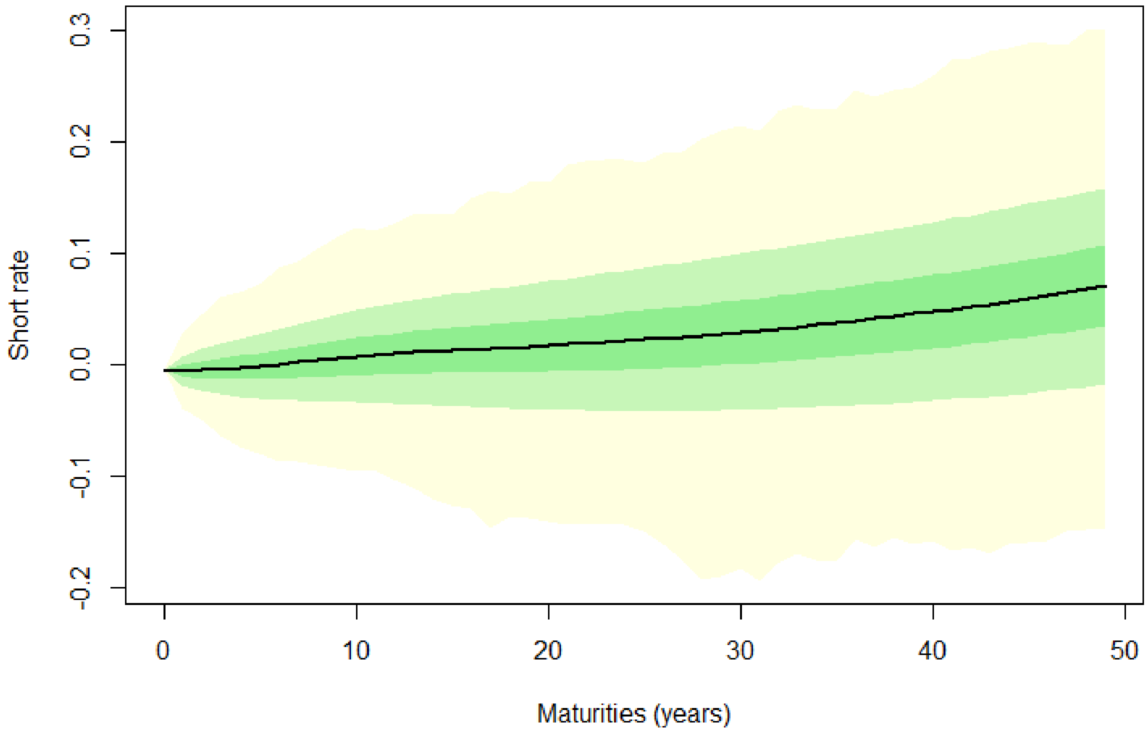

Figure 5.5: Hull-White simulated Euribor short rate path quantiles with $a = 0.00190$ and $\sigma = 0.00801$

In this figure the 68% and 95% confidence intervals are depicted in green. Furthermore, the black line depicts the mean while the remaining paths can be seen in yellow. Although it can be hard to discern, the mean of the simulated short rate initially decreases followed by a more constant increase through time. This trend is more prominent when converting the short rate to the zero coupon rates. This yields the term structure presented in figure 5.6.

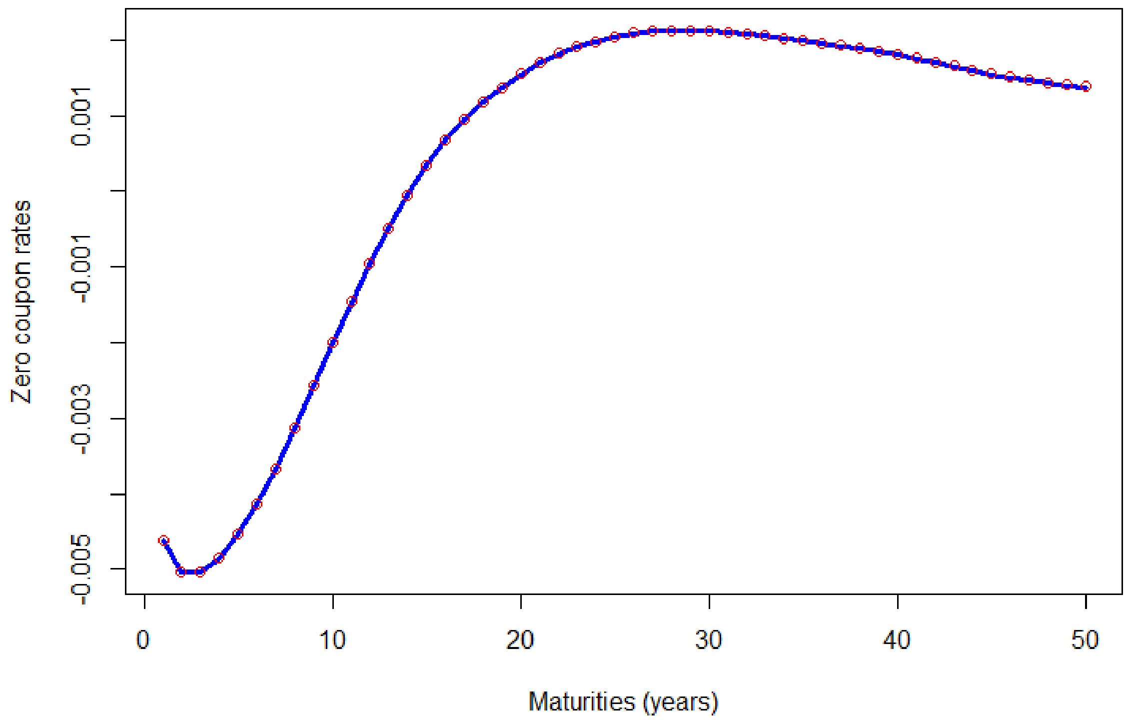

Figure 5.6: Hull-White Euribor zero coupon rates simulated with $a = 0.00190$ and $\sigma = 0.00801$ compared with market prices

In figure 5.6 the blue line depicts the simulated values of the zero coupon rates, while the red dots are the market zero coupon rates bootstrapped from the calibration instruments. As can be seen in the figure, the calibrated model gives an almost perfect fit to the initial term structure of interest rates observed in the market on the 22$^{\text{th}}$ August 2019. The curvature of the term structure is captured with all the market rates approximately on the simulated line. Thus, the initial term structure is successfully captured with the Hull-White one-factor model. The error between the simulated and market values are easier to spot in the zero coupon bond prices especially for longer maturities. This is due to the potential zero rate discrepancies for pricing will be compounded over many periods, leading to large zero coupon bond price differences. In order to confirm the fit of the model, the zero coupon rates are therefore translated into the zero coupon bond prices seen in figure 5.7.

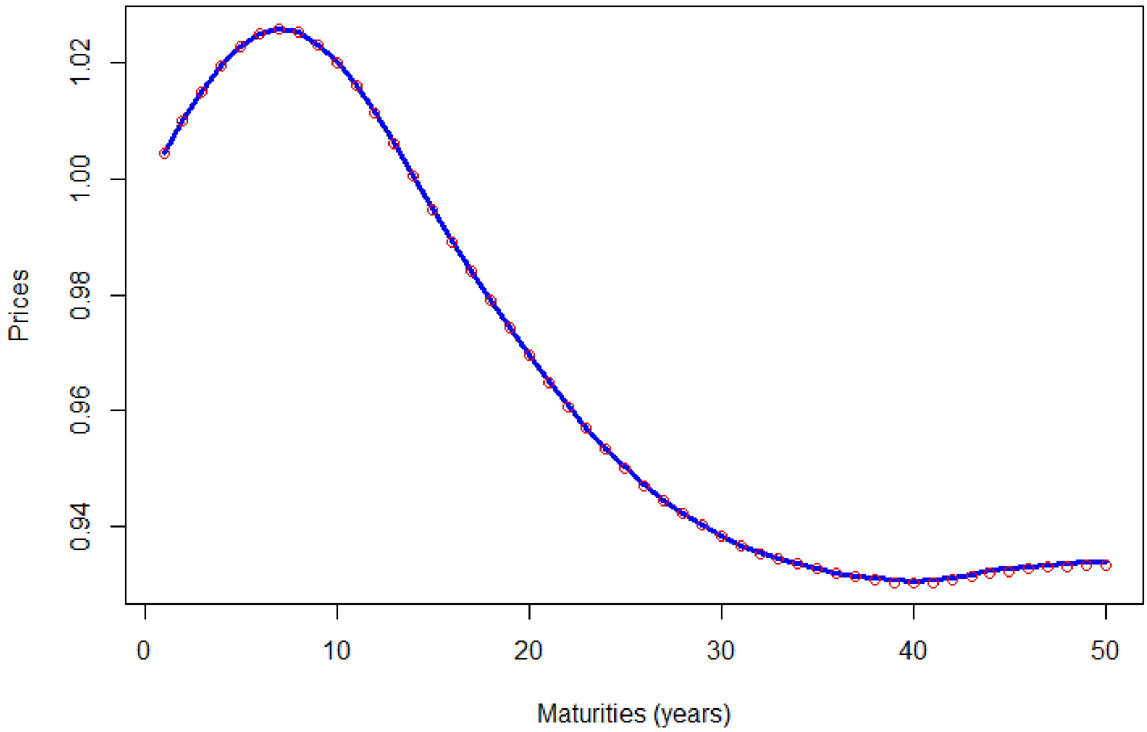

Figure 5.7: Hull-White Euribor zero coupon bond prices simulated with $a = 0.00190$ and $\sigma = 0.00801$ compared with market prices

In figure 5.7, it is apparent that the model slightly overestimates the zero coupon bond prices on the longer maturities. This difference is however minuscule. All in all, the Hull-White one-factor model captures the term structure with little error. This confirms that progressing from an equilibrium model to a no-arbitrage model is advantageous. However, as stated in section 3.5 a one-factor model assumes perfect correlation between the interest rates at different maturities. This assumption implies that a shock at a given time will have an equal effect through all maturities on the curve. Furthermore, even small errors in the underlying interest rates can have massive ramifications on the prices of interest rates derivatives as previously described. In order to possibly perfect the fit to the initial term structure and ensure more realistic patterns when the model is shocked, the analysis will progress to the G2++ two-factor model.

5.3 G2++ model

The following section will describe how the G2++ model is built and calibrated as well as the output from the subsequent simulations.

Building the G2++ model

In the calibrating procedure the Bachelier model is again relied on to handle the negative interest rates. The G2++ model is calibrated using a Nelder-Mead algorithm. This is a popular direct search method best known for its use in multidimensional unconstrained optimisation (Gonçalves-E-Silva, Aloise, Xavier-De-Souza, & Mladenovic, 2018). The method uses simplex', which is a generalised triangle in n dimensions to find the minimum of a function with n variables by continuously generating a sequence of triangles and rejecting the worst vertex (Lagarias, Reeds, Wright, & Wright, 1998). The method is sensitive to the initial guess in that the optimisation might find a local minimum as mentioned in 4.2. The initial guesses have been chosen through a combination of consulting literature and trial and error. According to Brigo & Mercurio (2006) it is not uncommon for G2++ models calibrated to swaption volatilities to exhibit negative correlation between the two factors. Hence, the parameters presented in table 5.4 have been chosen as the initial guess.

Table 5.4: G2++ initial guess for parameters to be used in optimisation function for Euribor

	Initial guess
a	0.1
b	0.25
σ	0.005
η	0.025
ρ	-0.4

Regarding the constraints in the optimisation function, the correlation is naturally constrained to be -1 $\leq \rho \leq$ 1 as described in section 3.5, whereas the remaining constraints are chosen heuristically. Like with the Levenberg-Marquardt method, further description and discussion of the Nelson-Mead method is beyond the scope of this thesis, but the curious reader can refer to Lagarias et. al. (1998).

The G2++ model is calibrated with FRAs and IRS' as with the Hull-White one-factor model and with ATM swaptions on 6M Euribor. Specifically, semi-annual swaptions is used with maturities from 2 to 10 years (skipping year 6, 8 and 9) and tenors from 1 to 10 years. Thereby, the model is calibrated to only the most significant swaption data by leaving out illiquid entries such as swaptions with maturities of 6, 8 and 9 years (Brigo & Mercurio, 2006). These criteria results in the use of 60 different swaptions. To get a better understanding of the swaptions used, table 5.5 shows the volatilities of these swaptions.

Table 5.5: G2++ swaption normal volatility surface in bps on 6M Euribor used for calibration (Bloomberg, 2020)

Tenor	Maturity	2	3	4	5	7	10
1		27.42	34.22	39.45	44.05	49.50	51.84
2		30.48	35.87	40.55	44.73	49.51	52.17
3		34.13	38.54	42.61	45.98	49.98	52.49
4		37.20	41.00	44.68	47.03	50.68	52.91
5		40.12	43.57	47.65	47.97	51.60	53.37
6		42.78	46.00	47.88	49.31	52.31	53.85
7		45.39	48.59	47.46	50.37	52.77	54.10
8		47.67	50.19	49.36	51.26	53.28	54.27
9		50.26	47.53	50.57	51.98	53.66	54.41
10		51.24	49.74	51.44	52.47	53.89	54.36

These instruments are used to calibrate the model with the *ESG2* (Economic Scenario Generator 2) package in R (Buzzi, 2020). From this calibration procedure, the parameters presented in table 5.6 are obtained for the G2++ model with the 6M Euribor.

Table 5.6: Parameters for G2++ model calibrated from zero coupon rates and swaption volatilities on 6 month Euribor

	Parameters
a	0.03055
b	0.46734
σ	0.00249
η	0.00103
ρ	-0.49518

Please refer to appendix A.7 for the code related to both calibration and use of the G2++ model. In the following section these calibrated values will be used to simulate the movements of the short rate and test the fit to the initial yield curve as also previously done with the other models.

Simulation using the G2++

In figure 5.8 the short rate paths of the G2++ model for the Euribor 6M yield curve is seen along with the 95% confidence interval in turquoise.

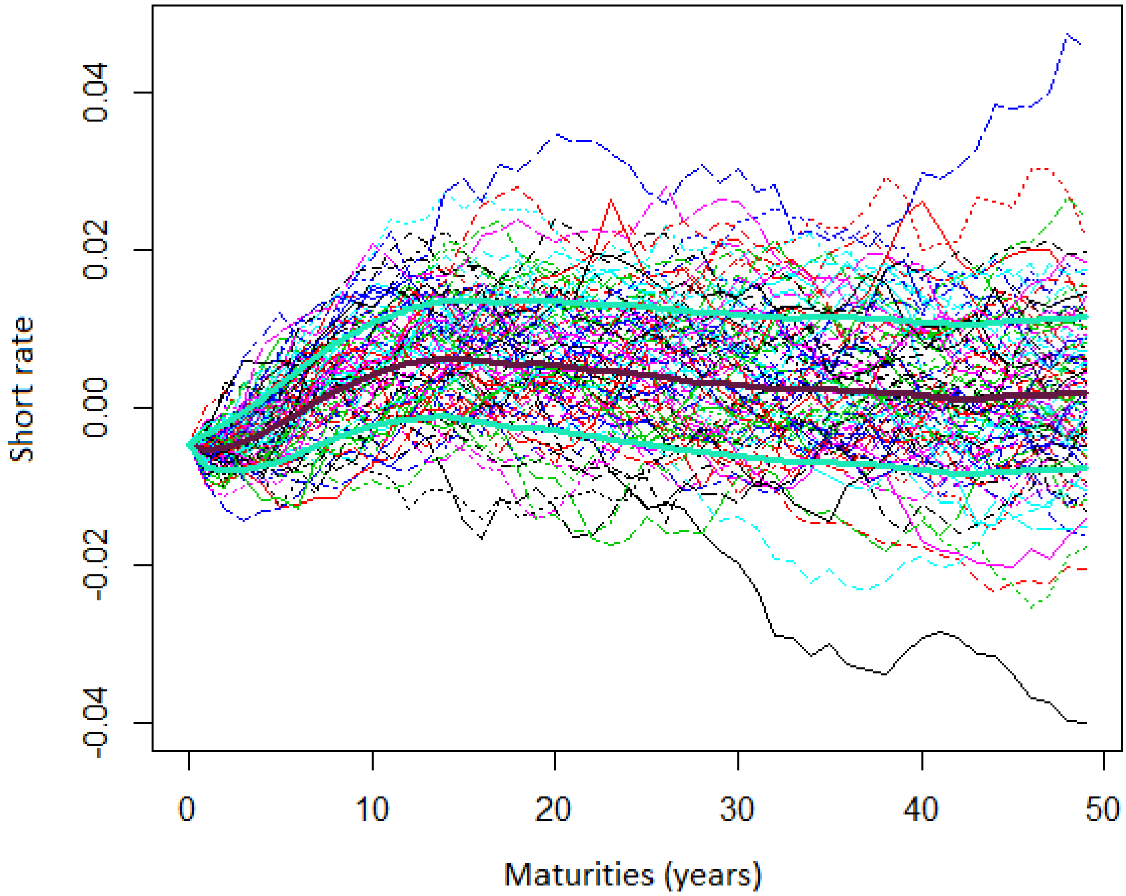

Figure 5.8: G2++ simulated Euribor short rate paths with $a = 0.03055$, $b = 0.46734$, $\sigma = 0.00249$, $\eta = 0.00103$, and $\rho = -0.49518$

The simulation was once again done with 100,000 simulated paths, but the *ESG2* package makes it possible to only depict 100 different paths. This makes it easier to distinguish the individual paths and subsequently helps avoid the same visualisation problem as with the Vasicek models. From the figure it is apparent that the short rate decreases at first then increases after around the 3-year mark and lastly then decreases slowly again from around year 15. Like with the Hull-White one-factor model, the zero coupon rates can be derived from the simulated short rates. This is displayed in figure 5.9.

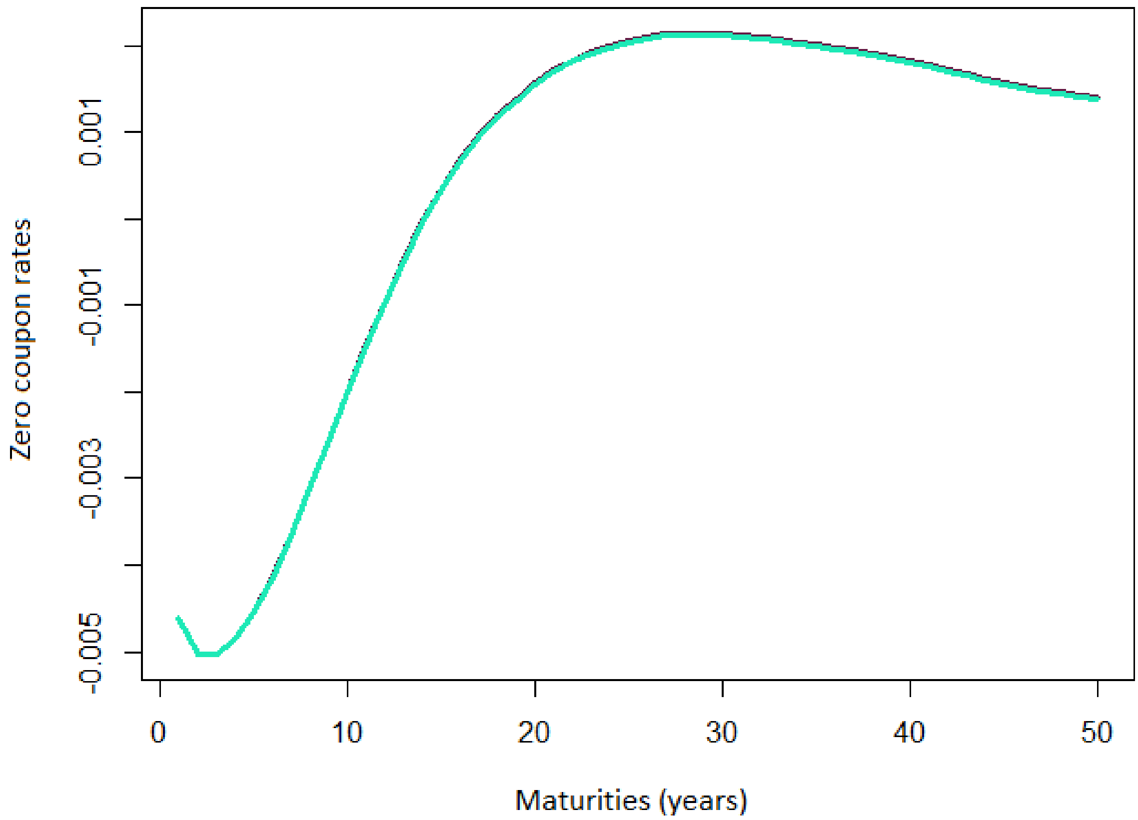

Figure 5.9: G2++ simulated Euribor zero coupon rates with $a = 0.03055$, $b = 0.46734$, $\sigma = 0.00249$, $\eta = 0.00103$, and $\rho = -0.49518$ compared with market prices

In figure 5.9 the black line displays the Monte Carlo zero coupon rates, while the turquoise line represents the market rates. As can be seen from this figure there is practically no difference between the two curves. Thus, the G2++ is deemed able to capture the evolution of the short rate satisfactorily. Therefore, this model will be modified to a dual-curve setup by fitting the model to both the Euribor 6M yield curve and the EONIA yield curve simultaneously in the following section.

5.4 G2++ model with dual-curve setup

Adapting the G2++ model to a dual-curve setup is done by calibrating the model to the two curves separately and setting up a correlation matrix equivalent to equation (4.3) that captures the correlation between the four Wiener processes. The following section will present how this has been done followed by dual-curve simulations of the two short rates.

Building the G2++ with dual-curve setup

The G2++ models are once again built using the Bachelier model and an optimisation process relying on the Nelder-Mead algorithm. The initial guess will remain the same for the 6M Euribor G2++ model, but will also be used for the EONIA G2++ model. This is done as the EONIA curve have a similar shape as the 6M Euribor curve and thus the underlying short rate is likely to exhibit similar characteristics.

The calibrated parameters of the G2++ model on the 6M Euribor curve are as shown in table 5.6. For calibrating the EONIA curve some other instruments are included in the curve bootstrapping methodology. Firstly, they are written on OIS swap rates rather than IRS' on the 6M Euribor and do not include any type of FRAs. Secondly, the volatility surface is based on ATM swaptions on the EONIA. This effectively means the underlying swap in the swaption uses an annual payment frequency instead of semiannual frequency as with the 6M Euribor swaptions. The same maturity-tenor structure is used for the EONIA calibration with maturities from 2 to 10 years (skipping year 6, 8 and 9) and tenors from 1 to 10 years. The swaption volatility surface of the annual swaption normal volatility on EONIA can be seen in table 5.7 (Bloomberg, 2020).

Table 5.7: G2++ swaption normal volatility surface in bps used for calibration of EONIA (Bloomberg, 2020)

Tenor	Maturity	2	3	4	5	7	10
1		26.87	33.89	39.06	43.89	49.42	51.55
2		29.91	35.31	39.90	44.25	48.92	51.47
3		33.46	37.94	41.87	45.43	49.43	51.83
4		36.52	40.31	43.74	46.63	50.16	52.32
5		39.36	42.74	45.83	48.30	51.13	52.85
6		41.93	44.74	47.36	49.25	51.80	53.28
7		44.28	46.59	48.89	50.15	52.24	53.52
8		46.38	48.19	49.88	50.94	52.72	53.70
9		48.13	49.67	50.79	51.60	53.08	53.85
10		49.54	50.62	51.47	52.02	53.28	53.80

Using these instruments to calibrate the G2++ model for the EONIA curve, the parameters displayed in 5.8 are obtained.

Table 5.8: Parameters for G2++ model calibrated from zero coupon rates and swaption volatilities on EONIA

	Parameters
a	0.02851
b	0.51986
σ	0.00245
η	0.00116
ρ	-0.51676

As can be seen, the result of the calibration is quite similar to the Euribor G2++ model, which is expected given that the two yield curves have similar shapes - albeit at different levels.

After the initial G2++ parameters have been calibrated, the correlation matrix will need to be calculated. In order to calculate the correlation between the different Wiener processes across the two G2++ models, γ, it is necessary to express a belief of the correlation between the two short rates $Corr(dr_{Euribor}, dr_{EONIA})$ as described in section 4.6. $Corr(dr_{Euribor}, dr_{EONIA})$ is determined by calculating the correlation from the daily differences between the short rates from the 16th March 2017 and the 30th September 2019

- the same period as used for the historically calibrated Vasicek modelling. This yields a correlation of 16.34%. This is a fairly low positive correlation and lower than initially expected. Calculating the correlation from the daily changes all the way back to the 27th January 1999 yielded a higher correlation of 31.30%, but still below the initial expectation. For the modelling, the correlation of 16.34% is employed although the possible higher correlation is kept in mind when discussing the results. Using equation (4.5) this yields γ = 0.05872. For the code used for this dual-curve setup, refer to Appendix A.8.

Upon further inspection of the calibrated dual-curve G2++ model, a tendency has been noticed for the model to underestimate the price of swaptions compared to the market instruments. This might be an indication that the Nelder-Mead optimisation algorithm has not initially found a global minimum, but rather a local one as problematised in section 5.3. In order to adequately price the swaptions with the dual-curve setup, it has been decided through trial-and-error to adjust the calibrated volatility parameters by doubling the η and σ parameters for both the Euribor and EONIA G2++. This has resulted in swaptions prices more in line with the market prices observed in table 5.5. As only the volatility parameters are bumped, it does not affect the fit to the initial yield curves. For the remainder of this paper, these adjusted parameters seen in table 5.9 will be used.

Table 5.9: Adjusted parameters for G2++ model for EONIA and EURIBOR

	EONIA	Euribor
a	0.02851	0.03055
b	0.51986	0.46734
σ	0.0049	0.00498
η	0.00232	0.00206
ρ	-0.51676	-0.49518

Simulation with the G2++ with dual-curve setup

Simulating the short rate with the two calibrated G2++ models with 100,000 simulated paths yields the short rates paths in figure 5.10 for Euribor and EONIA respectively.

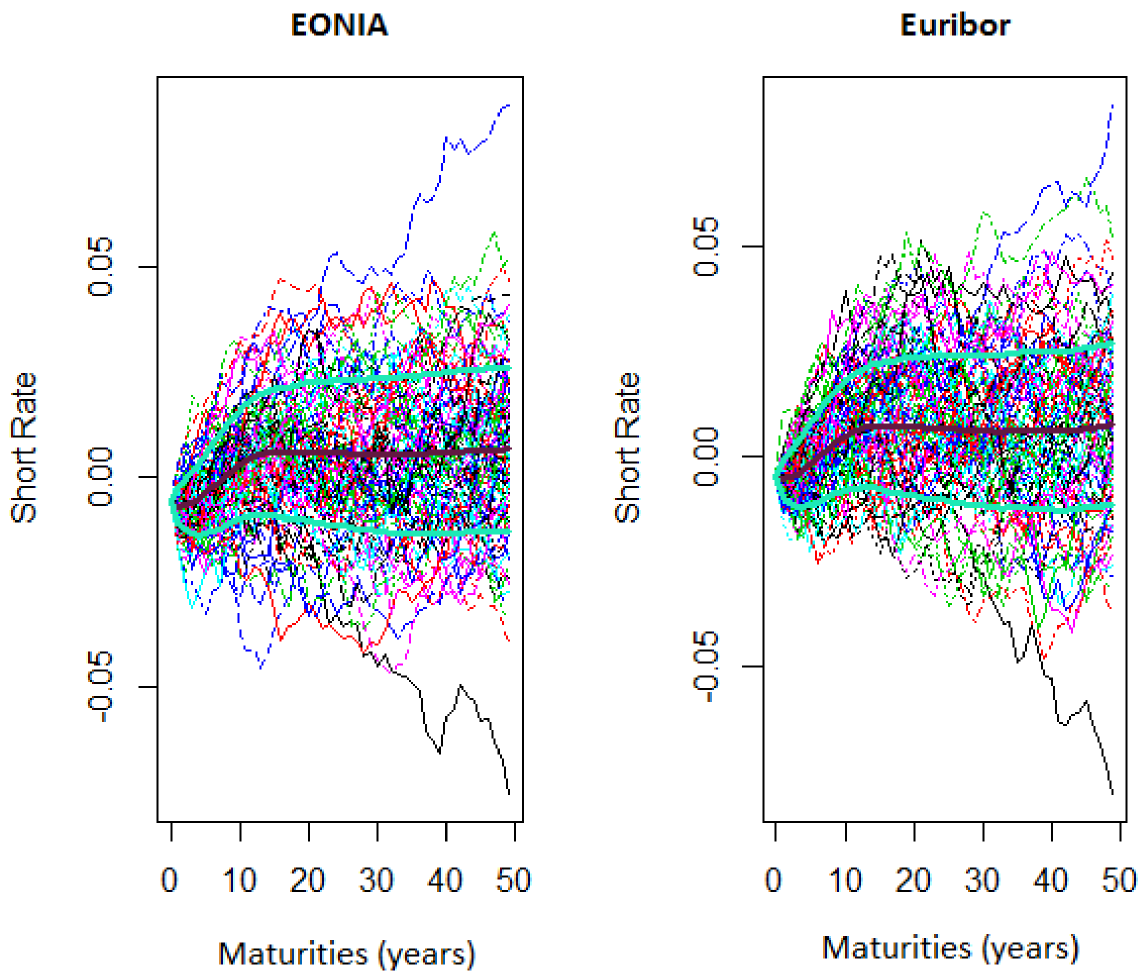

Figure 5.10: G2++ simulated short rate paths for both EONIA and Euribor in a dual-curve setup with calibrated values

Like with the previous short rate simulation in the single-curve G2++ setup, only 100 paths are shown for visualisation purposes. As expected the short rates look similar, which makes sense given the shape of the yield curves. These simulated short rate paths yield near perfect zero coupon curves for both rates, which can be seen in figure 5.11.

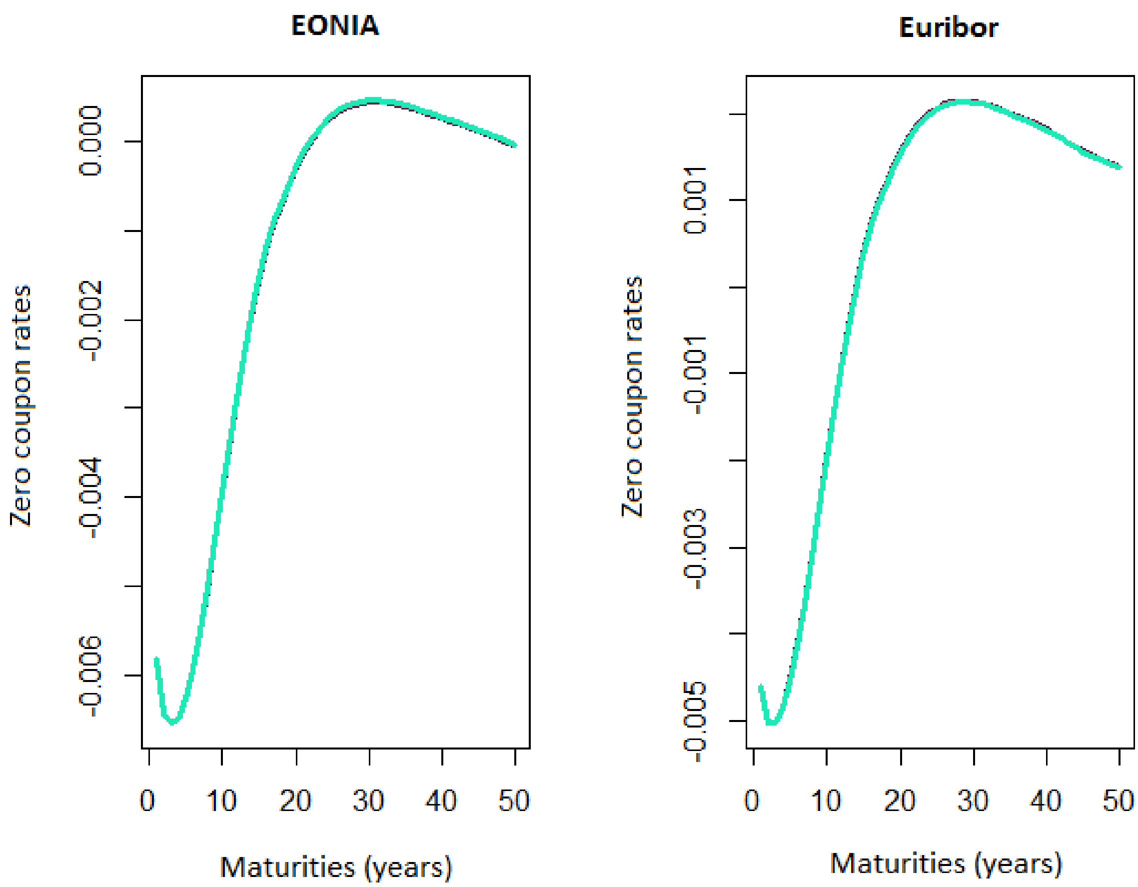

Figure 5.11: G2++ simulated zero coupon rates with calibrated values and dual-curve setup compared with market rates for both EONIA and Euribor

It is clear that the G2++ models are capable of perfectly fitting the term structure of interest rates for both underlying indices as expected. As can be seen in figure 5.11, the model is able to capture the market prices perfectly even on a 50-year yield curve. As with the Hull-White one-factor model, pricing discrepancies in the yield curve are easier to spot in the zero coupon bond prices especially for longer maturities. In order to confirm the fit of the yield curve, the zero coupon rates are therefore translated into the zero coupon bond prices seen in figure 5.12.

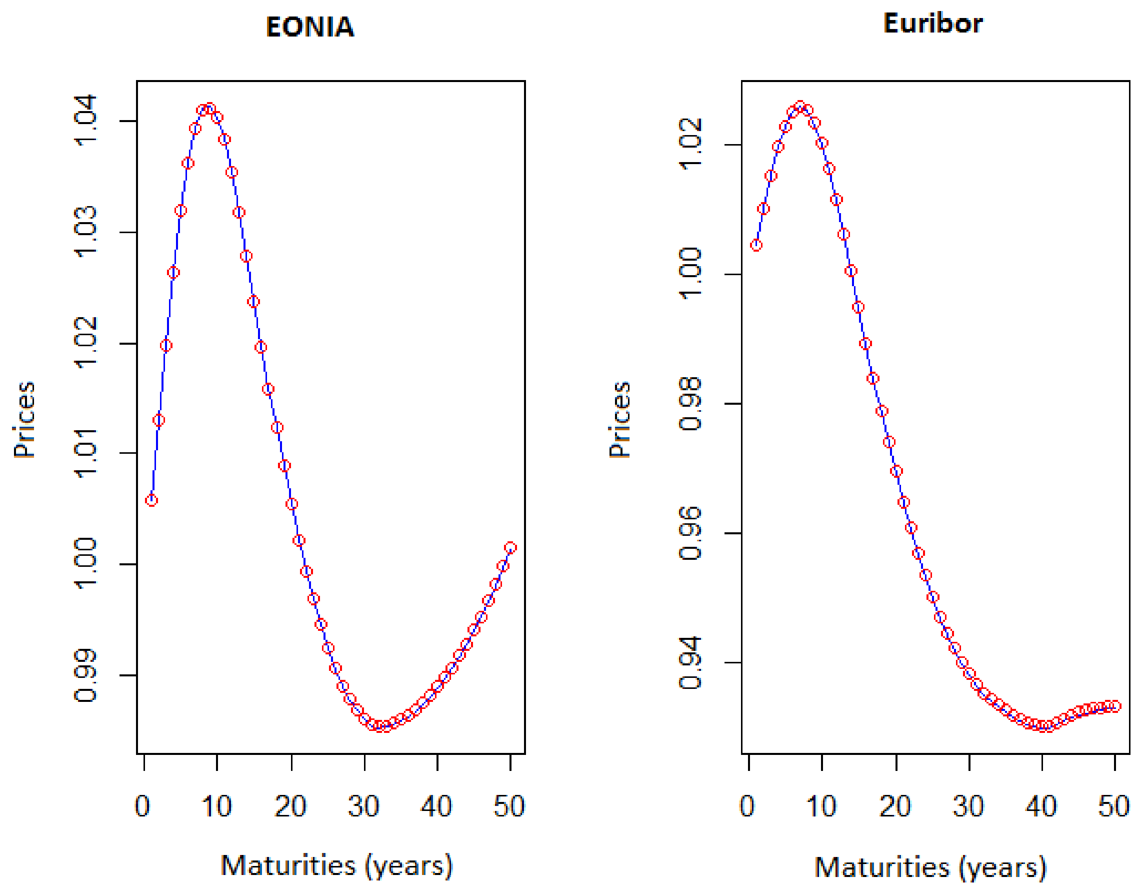

Figure 5.12: G2++ simulated zero coupon bond prices with calibrated values and dual-curve setup compared with market prices for both EONIA and Euribor

Figure 5.12 confirms the perfect fit of the two yield curves even at the longer maturities. As the setup can fit and simulate multiple correlated curves together, it is now possible to use this dual-curve setup to price linear and non-linear interest rate derivatives in accordance with market standards. This will be the topic of the following section followed by a scenario analysis. In the scenario analysis different parameters will be changed in order to assess some of the possible scenarios a shift from EONIA to ESTER may cause and their possible effects on the pricing of swaps and swaptions.

CHAPTER 6

Pricing and Scenario Analysis

In this chapter the dual-curve G2++ model will be used to price market instruments in the form of swaps and swaptions written on 6M Euribor. Firstly, these instruments will be priced with EONIA discounting in what is defined as the base scenario. This is the model without any altercations and will thus serve as the reference case to be used in later comparisons. Thereafter, the model will be shocked with a range of different scenarios in ways deemed realistic in order to try to capture some of the possible effects of the transition to ESTER. The pricing of the instruments are done in Excel with VBA including the Excel Add-in "Fidanalytics" (Linderstrøm, 2013). These Excel-files and the add-in can be seen in Appendix B attached.

6.1 Pricing with EONIA discounting

The price of swaps are dependant on the risk-neutral expectations of the future spot rate as described in section 2.4, whereas swaption prices also are dependant on every simulated path of these expectations as described in section 2.4. This entails that swaptions also will be affected by changes in volatility, while both swaps and swaptions are dependant on changes in interest rate levels. For the base scenario and the further scenario analysis, three different payer swaps with 5, 10 and 30 years maturity will be priced. These have been chosen to check the scale of the possible impact across different maturities. For swaptions a larger number will initially be examined in order to check the fit of the model with different maturity-tenor combinations. Selected swaptions will thereafter be used for further scenario analysis.

Swaps

As presented in section 2.4, payer IRS' are long interest rates as they pay a fixed rate against receiving the floating xIBOR rate. In figure 6.1 the value of three payer swaps with maturity 5Y, 10Y, and 30Y can be seen as function of the fixed rate K. The three instruments are all written with a notional of 500 million EUR.

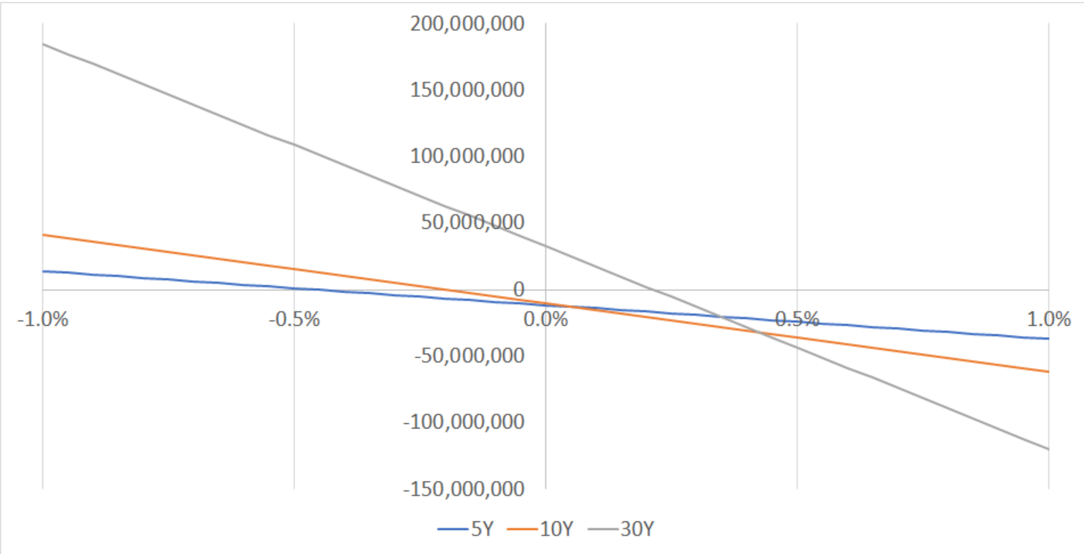

Figure 6.1: Pricing of notional EUR 500,000,000 6M Euribor payer swaps with EONIA discounting.

The par swap rate for the three instruments are -0.4522%, -0.1943% and 0.2140% with the swap with the longest maturity having the highest par swap rate. This makes sense given the upwards sloping yield curves for these maturities. With an upwards sloping yield curve loans with higher maturity require a higher return. From figure 6.1, it is also apparent that the lower the fixed rate, K, which is payed by the holder of the payer swap, the higher is the value of the instrument for all maturities. While the three contracts are all written on the same notional, the slope of the contracts differ. For longer maturities the slope is steeper. This is due to the size of the annuity factor of the swaps. The annuity factor can be thought of as the duration of the instrument related to changes in the par swap rate. As maturity increases the annuity factor increases as well, which in turn makes the instrument more interest rate sensitive and thus exhibit a steeper slope.

Swaptions

In table 6.1 the swaption prices for a range of different maturities and tenors generated through the model paths are shown as well as the prices observed in the market for the same instruments.

Table 6.1: Comparison of selected swaption prices from model with market prices from Bloomberg (2020), quoted as implied volatilities in bps for 6M Euribor

	Market price	Monte Carlo price
2Y2Y	30.47	46.31
2Y10Y	51.24	70.08
3Y3Y	38.54	48.14
3Y7Y	48.59	56.99
4Y2Y	40.55	45.67
4Y4Y	44.68	49.22
5Y5Y	47.97	49.91
7Y7Y	52.77	50.53
7Y10Y	53.89	53.18
10Y2Y	52.17	45.36
10Y10Y	54.36	50.45

As can be seen in table 6.1, the G2++ dual-curve setup are able to price the swaptions with a fair degree of precision with some instruments being closer to their respective market price than others. The model appears to overestimate the price of the shorter maturity swaptions such as the 2Y2Y and 2Y10Y. For longer maturity instruments such as 10Y2Y and 10Y10Y, the prices are on the other hand underestimated, albeit to a lesser extent. Specifically, mispricing the short end of the volatility surface is not uncommon in G2++ models (Brigo & Mercurio, 2006, p. 169). This should be seen in light of the number of instruments that the model attempts to fit. As the model is trying to fit two yield curves and 60 swaption prices with just four factors and 11 parameters, a certain degree of error is to be expected. If the entire volatility surface are to be captured perfectly, the model will have to contain a parameter for each price. This in turn will not only lead to overfitting of the data, but will also make the model needlessly complex (Brigo & Mercurio, 2006). While the price discrepancies are unfortunate, the pricing and thus the modelling is deemed to be satisfactory since the vast majority of the swaptions are priced within 5 bps of the market instruments.

Given the number of products and the redundancy of focusing on all 60 instruments the rest of the paper will focus on three different swaptions. These swaptions will be the 4Y2Y, the 5Y5Y and the 7Y10Y. These instruments have been selected on the grounds of the models' ability to accurately price the instruments and their ability to represent different maturity-tenor combinations.

The price of the three chosen payer swaptions can be seen as a function of the fixed exercise rate K in figure 6.2.

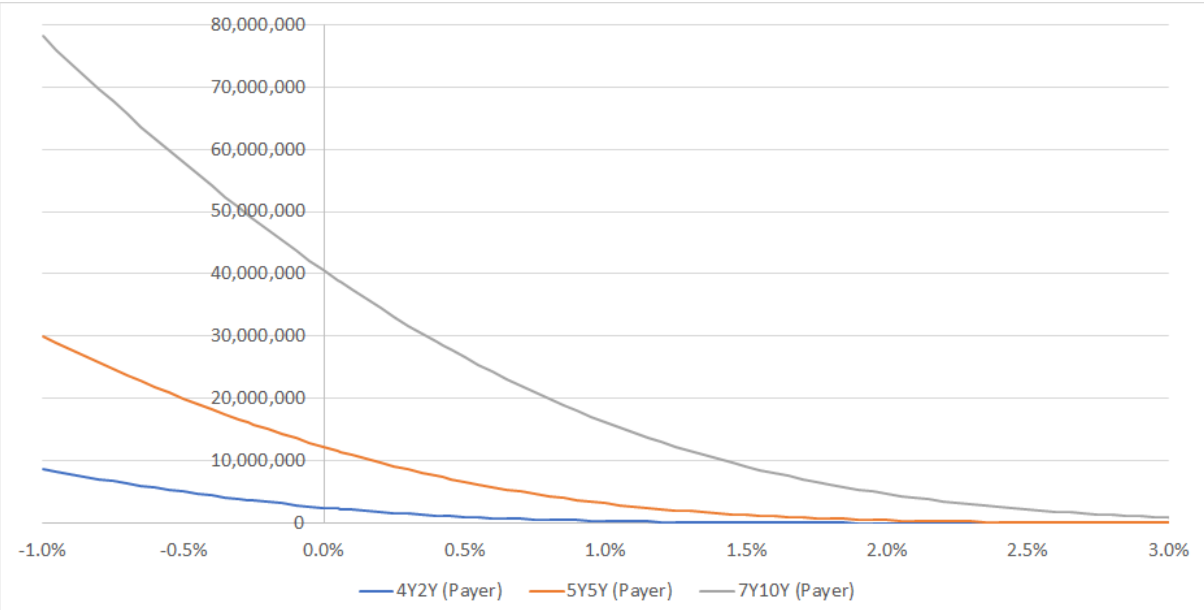

Figure 6.2: Pricing of 6M Euribor payer swaptions with EONIA discounting. EUR 500,000,000 notional

As can be seen in the figure, the payer swaptions are also long interest rates like the payer swaps. The value is, however, strictly positive due to the non-linearity of the payoffs in a swaption. This also makes it harder to see when it is ATM. The ATM strike, K, is given as the par forward swap rate of the underlying forward starting swap. The ATM strike is also the fixed rate where the value of a payer swaption is equal to the value of receiver swaption with the same strike. This is invoked through the put-call parity for swaptions. This relation can be seen in figure 6.3 for the 7Y10Y swaption pair:

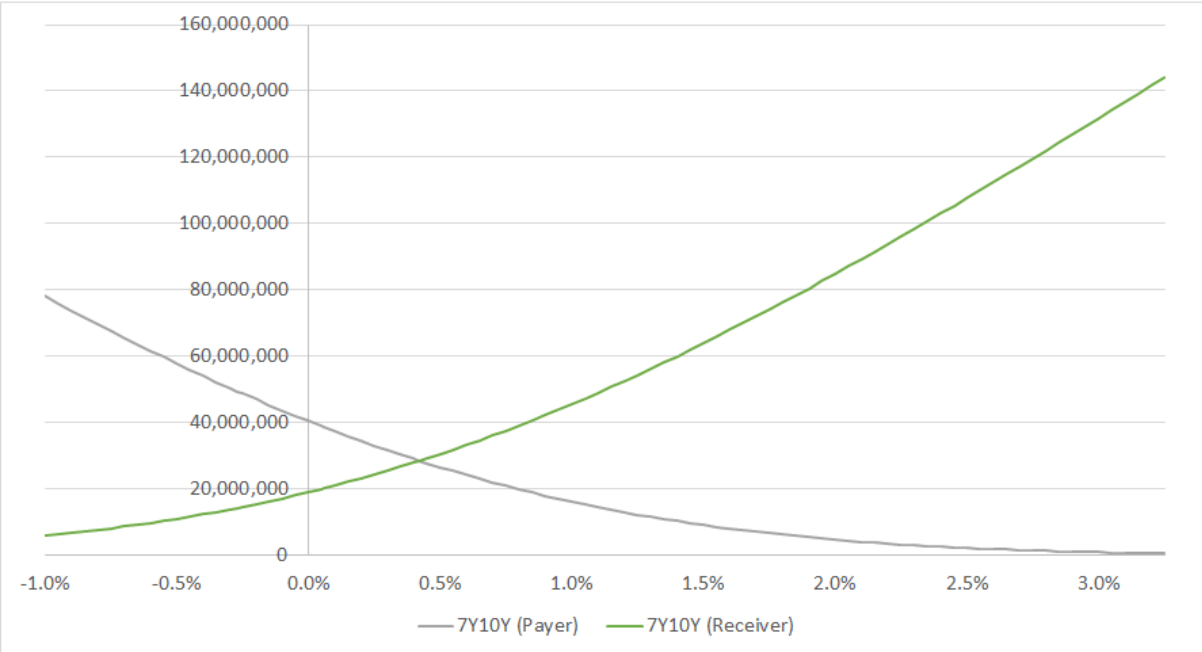

Figure 6.3: ATMF strike for 7Y10Y payer and receiver swaption on 6M Euribor with EONIA discounting. EUR 500,000,000 notional

The par forward starting swap rates, i.e. the At-The-Money-Forward (ATMF) strikes, of the given payer swaptions equals -0.269636%, -0.05771% and 0.42218% for the 4Y2Y, 5Y5Y and 7Y10Y respectively.

To summarise, it is apparent that the pricing of linear derivatives in accordance with market prices is feasible with the G2++ dual-curve setup. In relation to the pricing of swaptions, it is deemed that the model is able to satisfactorily price across the volatility surface. However, for swaptions with shorter maturities there is a tendency to overestimate the price, while the prices of longer maturities swaptions are underestimated. This is as previously mentioned a common problem with G2++ models when calibrating to a large number of different swaption maturities and tenors. Thereby, a certain degree of error is deemed acceptable to be able to effectively price a range of different swaptions. In the following section the focus will be on the possible effects on these prices when changing certain parameters in the model as substitutes for the change from EONIA to ESTER.

6.2 Scenarios for implementation of Ester

Now that specific swaps and swaptions have been priced with the base case, this section will focus on stressing the model. Specifically, the model will be shocked and in turn instruments will be repriced. This is done to get a glimpse of how the change to ESTER potentially can affect interest derivatives that historically have used EONIA for discounting. Specifically, the model will be shocked with different changes to the discounting curve that are deemed realistic when moving to the ESTER from the EONIA. Shocks of three different characters will be applied. The first way of shocking the model is by bumping the zero coupon curve. Secondly, the model will be shocked by changing the volatility parameters of the discounting curve. Lastly, the correlation between the discounting and the projection curves are changed. These scenarios are both used independently and in different combinations to examine their effects on the pricing of the instruments. For the simulated short rate paths for the shocked scenarios and the subsequent repricing refer to attached Appendix B.

The three types of shocks will not affect linear and non-linear derivatives equally. As linear interest rate derivatives do not contain optionality they are not affected by changes in the individual interest rate paths as long as the projected yield curve remain the same. Therefore, linear derivatives such as swaps are not subject to changes in the volatility parameters.

Moving the discounting curve

The first type of shock is movement in the discounting curve, where the EONIA curve is moved down by 8.5 bps. This is done in order to mimic the level of the ESTER curve. As previously shown in section 4.4 during the period where the Pre-ESTER and EONIA ran concurrently, a spread existed between the two indices. Beginning on October 3rd 2019, EONIA was no longer compiled from the panel banks, but instead published as being equal to ESTER + 8.5 bps. In doing so the assumption becomes that the only thing that changes when going from EONIA to ESTER is the level of the curve. Consequently, subsequent scenarios will be constructed on EONIA - 8.5 bps to be able to determine the relative changes on the pricing of the chosen instruments.

Bumping the EONIA curve by -8.5 bps affects the par swap rate, which in turn affects the value of the contracts. As can be seen in table 6.2, the par swap rate of the three swaps increases slightly when the discounting curve is moved down. The forward rates are discounted less harshly, which in turn increases the par swap rate. This is natural as the par swap rate can be thought of as the weighted average of the forward rates as mentioned in section 2.4.

Table 6.2: Par swap rate with EONIA and EONIA -8.5 bps

	EONIA	EONIA-8.5bps
5Y	-0.4522%	-0.4520%
10Y	-0.1943%	-0.1936%
30Y	0.2140%	0.2156%

The change in the discount curve naturally also affects the pricing of contracts. The difference in value from regular EONIA discounting of the three payer IRS can be seen in figure 6.4 as a function of the par swap rate K.

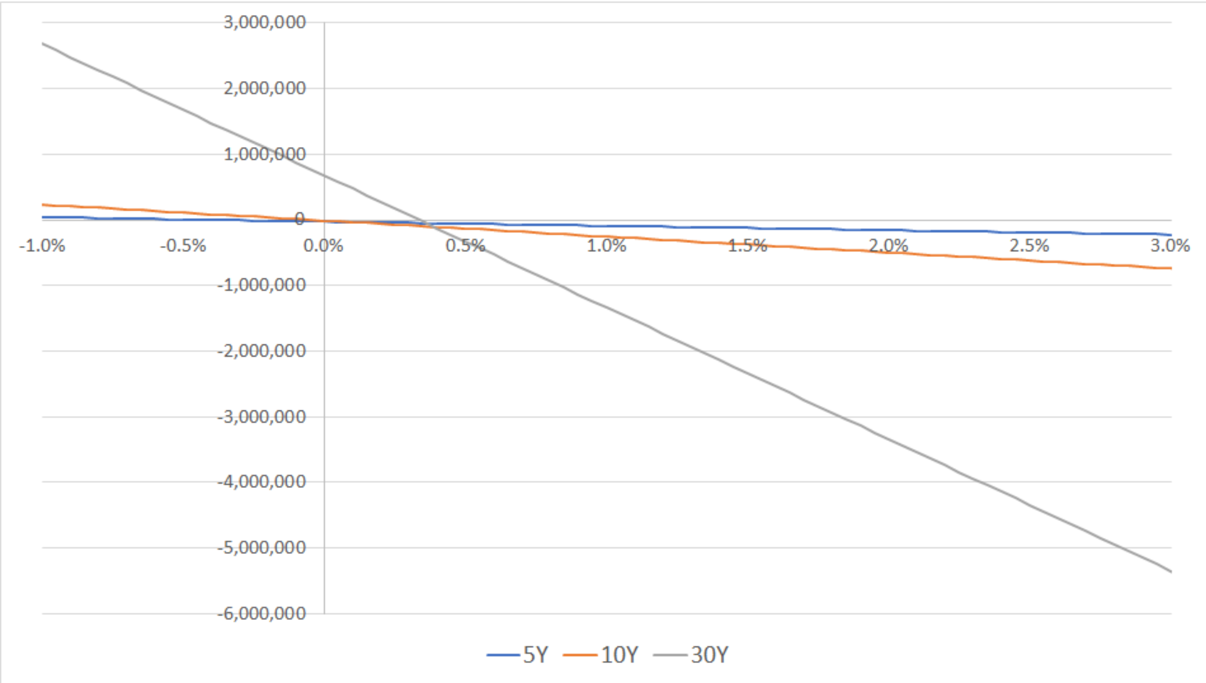

Figure 6.4: Absolute differences in value of payer swaps written on 6M Euribor with -8.5 bps lower discounting curve. EUR 500,000,000 notional

As can be seen in figure 6.4, discounting the payoff with a discount curve that is 8.5 bps lower will exacerbate the risk of the payer swap. When ITM, the value of the instrument will increase while the value of payer will become even more negative when the instrument is Out-of-The-Money (OTM). For deep OTM payer swaptions with longer maturities such as the 30Y swap this difference in prices can be non-negligible - especially if several of such positions have been created.

As the interest rate level has been decreasing over the last several years as shown in figure 4.1, holders of payer swaps will be typically be OTM, while receivers will be ITM. This implies that the payer side will lose value to the receiver side if ESTER is simply going to be equal to the EONIA minus 8.5 bps. In addition to a loss on the position, the payer will also be forced to post additional collateral. This can be problematic if everyone is forced to do so at the same time due to liquidity concerns. These possible liquidity concerns will be discussed further in chapter 7.

Bumping the discount curve down will also have an effect on the price of swaptions. In table 6.3 the change in the par forward swap rate, i.e the ATM swaption strike rate K, can be seen along with the Monte Carlo change in implied volatility.

Table 6.3: Monte Carlo implied volatility with EONIA and EONIA -8.5 bps. Par forward swap rate in parenthesis

	EONIA	EONIA-8.5bps
4Y2Y	45.67 (-0.26964%)	45.72 (-0.26955%)
5Y5Y	49.92 (0.05771%)	50,04 (0.057793%)
7Y10Y	53.18 (0.42218%)	53.42 (0.42247%)

As with the swaps, the par forward swap rates of the swaptions increased in order to reflect that forward rates are discounted less harshly. Furthermore, the implied volatility has gone slightly up indicating higher prices of the swaptions. The changes are, however, minor. In figure 6.5 the changes in the swaption prices can be seen as a function of strike rate K.

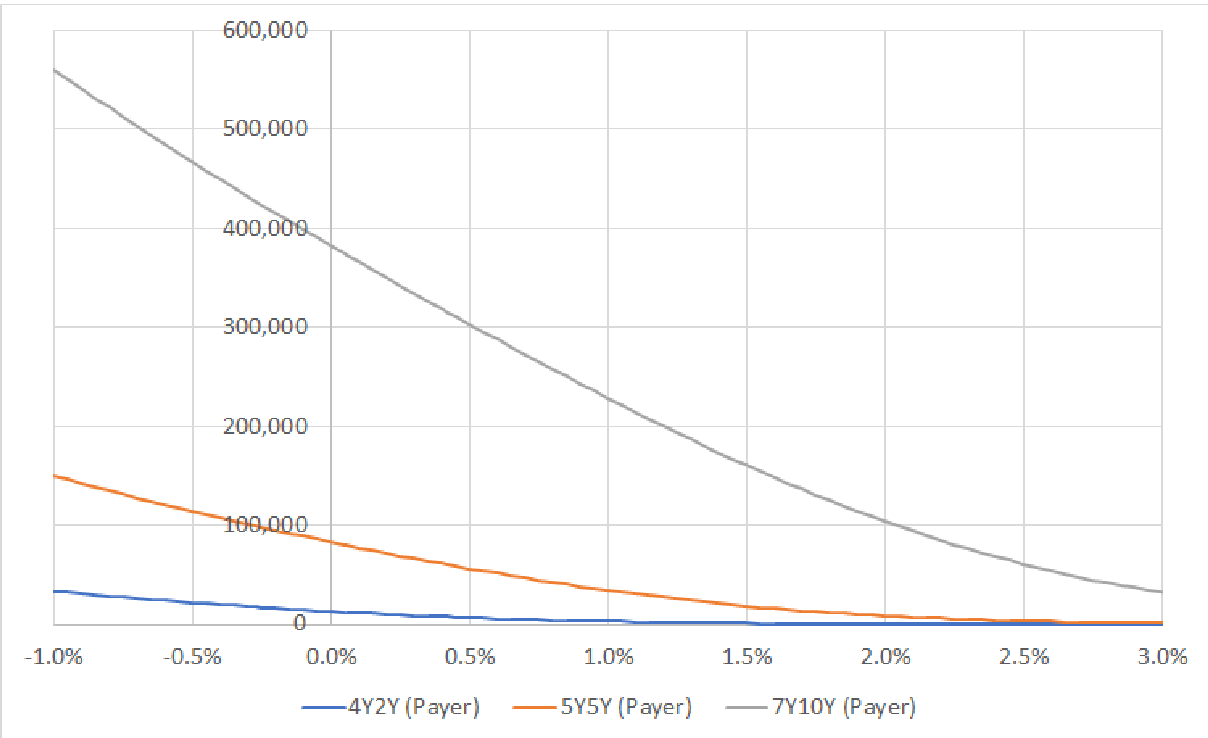

Figure 6.5: Absolute differences in value of payer swaptions written on 6M Euribor when the discounting curve is lowered by 8.5 bps. EUR 500,000,000 notional

As illustrated in figure 6.5, the price difference is greatest for deep ITM swaptions with longer maturities. What is interesting, however, is that the difference remains non-negative. This means that no matter if the swaption is ITM, ATM or OTM, the holder of the instrument will gain value when the discount curves is bumped down. This in turn means the issuing counterparty loses value. This relation is due to the optionality of the swaption since this forces the expected payoff to remain non-negative. The reason for this is that the instrument always has a slight chance of the underlying moving in favour of the option holder. Having a non-negative payoff implies that a swaption always will have a non-negative value. Thus, lowering the discounting curve will ceteris paribus always increase the value of the option. This is the case for both the payer and the receiver swaption. Naturally this observation also applies in the opposite scenario; if the discount curve is increased, all expected payoffs will be discounted harder and the value of the swaption will decrease.

Volatility fluctuations

In this section the volatility of the discounting curve in the model will be altered. The first of these scenarios will be a decrease in the volatility parameters in the discounting G2++ model components, σ_{EONIA} and η_{EONIA}. Switching to ESTER discounting is expected to reduce the volatility of the discounting rate due to an increased number of market participants and higher transaction volume. This claim is at least partially validated by figure 2.1, where it is seen that the EONIA had a tendency to exhibit spikes and generally contain a higher degree of volatility compared to the Pre-ESTER. As seen by the summary statistics of the two time series in table 4.1, the EONIA exhibits a daily standard deviation of 0.015 compared to 0.013 for the Pre-ESTER. Given the relative difference between these volatilities, the first scenario is a 15% decrease in the volatility parameters. For comparison, a scenario will also be investigated where the volatilities of the discounting curve instead are increased by 30%.

As previously mentioned, the swaps will not be affected by volatility spikes alone. Therefore, this section will be focusing on the effect on swaption pricing. The effect of the two changes in the volatility parameters of the EONIA in the dual-curve G2++ on swaption prices can be seen in table 6.4, where the scenario of -8.5 bps from table 6.3 is also shown for comparison.

Table 6.4: Scenario prices for changed volatility of the discounting curve quoted as implied volatilities in bps with par forward swap rate in parenthesis

	-8.5bps	-8.5bps, 0.85vol	-8.5bps, 1.3vol
4Y2Y	45.72 (-0.26955%)	45.74 (-0.26955%)	45.68 (-0.26955%)
5Y5Y	50.04 (0.05793%)	50.08 (0.05793%)	49.97 (0.05793%)
7Y10Y	53.43 (0.42247%)	53.50 (0.42247%)	53.29 (0.42247%)

As expected, it is apparent, when looking at table 6.4 that the par forward swap rates do not change when the volatility parameters have been changed. Two other main observations can be made from the table. Firstly, it is apparent that decreasing the volatility of the discounting G2++ component causes an increase in the price of the swaptions i.e. the implied volatility increases. Thus, lower volatility in the discounting curve is preferable for the holder of the swaption. This can be explained by the correlation between the two curves. When the projection curve and the discounting curve is positively correlated, large positive shocks in Euribor will be accompanied with positive shocks in the discounting curve. This alludes that positive shocks in the Euribor, causing the payer swaption to be ITM, will shock the discounting curve in the same direction proportionally to the volatility of the discounting curve. Contrarily, if the volatility in the discounting curve decreases, the Euribor based cash flows will be discounted less harshly. Secondly, the relatively insignificant absolute change in the implied volatilities is noticed. Figure 6.6 displays the difference in value of the swaptions using EONIA -8.5 bps and EONIA -8.5 bps with 0.85 volatility:

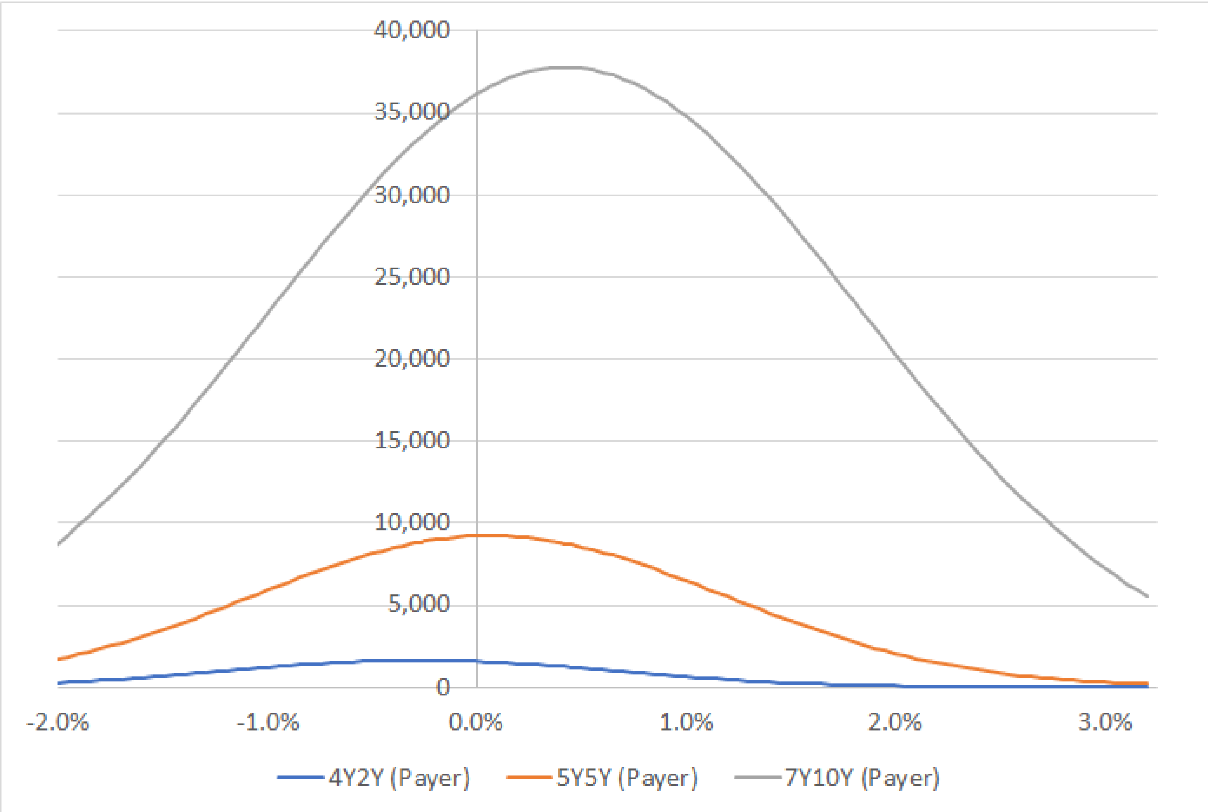

Figure 6.6: Absolute differences in value of payer swaptions with lower volatility in the discounting curve. EUR 500,000,000 notional

As can be seen from the figure, the value of the swaptions only changes slightly. The biggest changes are around the ATMF strikes for the swaptions. This is to be expected as the figure essentially depicts the vega of the swaptions since only the implied volatilities have changed. Furthermore, the swaptions with longer time-to-maturity and longer tenors have relatively higher price differences. This can be attributed to higher vega for swaptions with longer time-to-maturity and larger annuity factors for swaptions with longer tenors. Longer time-to-maturity indicates more time to benefit from potential movements in the underlying. The annuity factor indicates how much the swaption will benefit from a movement in the underlying.

The relatively trivial effects of lower volatility in the discounting curve can be explained by considering the swaptions' Delta vectors, i.e. the partial effect of bumping the projecting and discounting curves by 1 bps.

Table 6.5: Delta vectors of projection curve (P-curve) and discount curve (D-curve) for 7Y10Y Payer Swaption with EUR 500,000,000 notional.

	7Y10Y ATM		7Y10Y ITM (strike 0%)		7Y10Y OTM (strike 2%)	
	P curve	D curve	P curve	D curve	P curve	D curve
1Y	0	0	0	0	0	0
2Y	0	0	0	0	0	0
3Y	0	0	0	0	0	0
4Y	0	0	0	0	0	0
5Y	0	0	0	0	0	0
6Y	17	2	21	2	5	0
7Y	-168,667	-20,251	-206,248	-28,706	-45,754	-3,390
8Y	-352	619	-430	757	-96	168
9Y	-38	409	-47	499	-10	111
10Y	183	223	224	273	50	61
11Y	543	18	663	22	148	5
12Y	829	-182	1,013	-222	225	-49
13Y	1,061	-376	1,296	-460	288	-102
14Y	1,298	-529	1,587	-647	353	-144
15Y	1,453	-575	1,776	-703	395	-156
16Y	1,535	-587	1,876	-717	417	-159
17Y	406,661	113	496,494	139	111,094	31
18Y	-35	68	-43	84	-10	19
19Y	0	0	0	0	0	0
20Y	0	0	0	0	0	0
DV01	244,487	-21,048	298,181	-29,680	67,105	-3,607

As seen by table 6.5, the vast majority of the Delta risk in a payer swaption is allocated to the projection curve. For all three strikes, the Delta risk associated with the projection curve is at least 10 times larger than the Delta risk from the discounting curve. This relative discrepancy enhances as the swaption goes further OTM. On the contrary, the absolute Delta risk increases as the swaption goes further ITM. The volatility of the discounting curve is naturally less relevant than the volatility of the projection curve on the risk profile and price of swaptions. Therefore, if the volatility of the projection curve is changed in a similar manner to the discounting curve, the effect will be significantly enhanced. The topic of changes in the projection curve will be discussed later in section 7.4.

Changing the correlation

In this section the influence of the correlation between the two curves, i.e. between the projection and the discounting curve, will be investigated. To get a glimpse of this, the correlation factor between the two curves have been changed to be -15% instead of 16.34%. As described in section 1, the ESTER is based on a broader transaction base with other financial institutions than banks included compared to the EONIA. However, this can in theory be problematic as the overnight lending cost across these different institutions might not be homogeneous. Some recent research have shown that there might be differences in how economic shocks affect the different types of market participants across the financial sector. It has for example been shown that shocks in the insurance sector can cause shocks of the opposite direction in the banking sector (Pan, Guo, & Jing, 2016). Therefore, a negative correlation although unlikely to occur is deemed relevant for subsequent analysis.

When altering the correlation between the two short rates it changes the gamma as apparent in equation (4.5). A correlation between the two curves of -15% with the remaining parameters unchanged from the base scenario yields γ = -0.05389. In table 6.6 the prices of the three swaptions can be seen in different scenarios with the changed correlation factor.

Table 6.6: Scenario prices with negative correlation between the discounting and projection curves, quoted as implied volatilities in bps and par forward swap rates in parenthesis

	-8.5bps	-8.5bps,-0.15cor	-8.5bps,-0.15cor, 0.85vol	-8.5bps,-0.15cor,1.3vol
4Y2Y	45.72 (-0.26955%)	45.97 (-0.27037%)	45.95 (-0.27037%)	46.01 (-0.27037%)
5Y5Y	50.04 (0.05793%)	50.49 (0.05675%)	50.45 (0.05675%)	50.56 (0.05675%)
7Y10Y	53.43 (0.42247%)	54.32 (0.42082%)	54.26 (0.42082%)	54.46 (0.42082%)

As table 6.6 displays, the par swap rate decreases slightly as a result of the negative correlation between the two curves. However, more interestingly is the change in implied volatility with the negative correlation scenarios. When the only change is the negative correlation, the implied volatility increases. This effect is caused by the optionality of the swaptions: When a swaption is ITM on a simulated path, the projection curve will be at higher levels. A negative correlation will push the discounting curve comparatively lower thus discounting the payoff less harshly. When a simulated path is OTM the opposite does not matter due to the optionality of the instrument. Hence, negative correlation will increase the value of the swaption as is seen in table 6.6.

Furthermore, this effect also entails that increased volatility in the discounting curve is now preferred for the holder of the swaption. This can be seen in the prices of the scenarios where volatility is changed. When a positive correlation between the curves is present, lowering the volatility leads to an increased price of the swaption. Contrarily, when the correlation between the projection and the discounting curve is negative, it is apparent that the price decreases with a lower volatility. With negative correlation a large upwards shock in the projection curve will be associated with a negative shock in the discounting curve. As the discounting curve moves oppositely of the projection curve, more volatility is preferred for the holder of the swaption, since this implies a higher expected payoff that is discounted less harshly.

In figure 6.7 the absolute difference in the price of the payer swaption between the scenarios of EONIA -8.5 bps and EONIA -8.5 bps with the negative correlation between the curves is depicted.

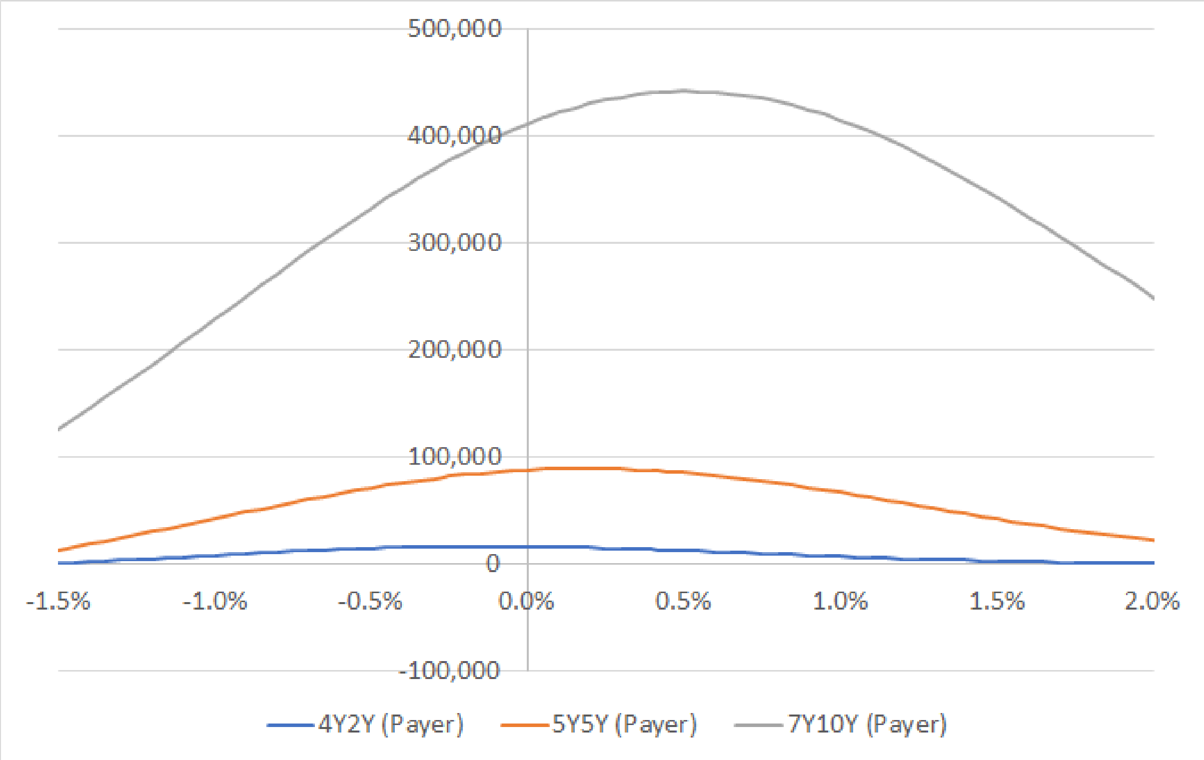

Figure 6.7: Absolute differences in value of payer swaptions between EONIA -8.5bps and EONIA -8.5bps with a negative correlation between the discounting and projection curves. EUR 500,000,000 notional

As with figure 6.6 the primary change is an increase in the implied volatility of the swaption. Again the longest maturities and tenors have the highest vega. Furthermore, it is apparent that the scale of the changes in absolute value is significantly larger given the larger relative change in the implied volatility compared to figure 6.6. Thus, even a slightly negative correlation between the two curves can have substantially larger impact in terms of risk compared to a relatively large upwards or downwards bump in the volatility parameters of the discounting curve alone.

Results of the scenario analysis

As the previous sections have shown, the consequences of implementing a new discounting curve on the pricing of interest rate derivatives can be profound. The scenarios showed that decreasing the level of the discounting curve will increase the Delta risk of swaps. This can cause a transfer of wealth from payer to receiver instruments or vice versa depending on the value of the outstanding contract. For holders of non-linear derivatives such as payer and receiver swaptions, both types stand to gain value at the cost of the issuer. In terms of volatility of the discounting curve in the presence of positive correlation between the projection and the discounting curve, higher volatility will cause lower derivatives prices and vice versa. The effect is, however, relatively minor. This has been hypothesised to be due to the relatively low risk allocated to the discounting curve compared to the projection curve. Contrarily, when the two curves are negatively correlated, higher degrees of volatility in the discounting curve leads to higher derivative prices and the other way around. Furthermore, negative correlation by itself is effective in increasing the value of derivatives. The likelihood and further impact of the potential implications of ESTER implementation will be the topic of discussion in the following chapter.

CHAPTER 7

Discussion

The previous chapter showed the possible consequences of a shift from EONIA to ESTER discounting from a pricing and risk perspective. The following chapter will first discuss some of the underlying assumptions and methodology used in the analysis, before discussing the possible consequences of the implementation of ESTER for market participants.

7.1 Assumptions and methodology

Firstly, the topic of data selection will be addressed. When modelling the proxy for the realised short rates with the Vasicek model, the choice and length of the time series will naturally affect the results. Given the relative short duration that Pre-ESTER and EONIA have been available simultaneously and the relative stationarity of the rates during this period, the result of stationary data generating processes was expected. Conversely, the Vasicek model using the daily movements in the realised rates might be able to better capture movements, volatility and the long term mean of the interest rates in resemblance of an observable yield curve in the future or if the data is drawn from a longer period. However, given the Vasicek model could not fit an appropriate term structure, even when the market zero rates were used for calibration, it is highly unlikely, or rather impossible, that the model will improve to a degree, where it is appropriate for further analysis. The Vasicek model is, however, still able to capture that Pre-ESTER is less volatile than the EONIA. This is also seen from the summary statistics in table 4.1.

The assumption of the difference in volatility between the ESTER and EONIA is another topic for discussion. The volatility has been bumped by -0.15% in order to simulate the lower volatility of the ESTER given the relative difference in the standard derivation of the two time series. Another valid shock could be to use the difference between the σ parameters of the fitted Vasicek models. However, given the conclusion that the Vasicek is not able to satisfactory capture the movement in the interest rate evolution, the scale of the difference between the two σ parameters has not been deemed trustworthy. However, the Vasicek model is able to capture that the EONIA was more volatile than the Pre-ESTER. Thus, the direction of the change in volatility will remain the same, while the scale can be discussed. Given the relatively small effect of shocking the volatility parameters η and σ of the discounting curve, the absolute effects of the scaling are deemed to be negligible.

The utilised models all rely on Gaussian processes and are therefore based on the assumption that interest rates are normally distributed. This has been an advantage since it allows for the current negative rates. However, it might also put too much weight on the possibility of negative scenarios given the likelihood of positive and negative shocks naturally are assumed to be equal. Interest rates are not expected to be able to fall indefinitely, which incurs that this is a possible issue of the modelling and subsequent results. However, assumptions about future movements of the interest rate have consistently been proven faulty by reality as it e.g. previously has been assumed that interest rates below zero merely are a theoretical possibility. Consequently, it is not assumed that expectations of future movements in the interest rate levels are more likely than not to manifest. Thus, the assumption of normal distribution is used, while remembering to take the possibility into account when analysing and interpreting the results.

As seen from the calibration of the dual-curve G2++ model, the model is not able to perfectly capture the volatility smile embedded in the swaption market. However, the fit is deemed satisfactory due to the relatively small pricing discrepancies. In order to potentially improve the fit to swaptions even further, a solution could be to include more parameters and factors to capture the data generating process even better as previously argued in section 6.1. However, in doing so, the risk of overfitting the model would be significantly enhanced. As George Box famously said *"All models are wrong, some* *are useful"* (Box, 1979, p. 202). It would be extremely unlikely for a relatively simple model to capture anything in the real world perfectly. Therefore, it is not required that the model provides a perfect representation of reality. However, it is required that the model is able to illuminate relationships and processes observable in the real world. In this regard, this requirement is satisfied as the model captures some of the volatility smile in the market even though the scale of the difference between model and market prices are at least somewhat inconsistent depending on the specific instrument.

7.2 The significance of ESTER for market participants

As found in chapter 6, the change from EONIA to ESTER can have some ramifications for market participants with significant valuation risk as interest rate derivatives currently are valued using EONIA. For institutions holding outstanding contracts, also known as legacy contracts, discussions are currently being held on how to handle this shift in the underlying discounting rate. No matter how it is handled, someone will stand to gain or lose either from the shift or from the agreed mitigation.

Valuation risk

Before some of these possible solutions are discussed in detail, it is worth noting that different types of institutions generally have different allocations towards paying or receiving a fixed interest rate. Commercial banks that hold mortgage portfolios will for example typically have an overweight of receiver instruments. By receiving fixed, they protect themselves against declining interest rates by mitigating their exposure to interest rate risk. In this particular example the bank could be holding a receiver instrument enabling them to pay float similar to the interest payments on their outstanding mortgage lending while receiving fixed. Another user of receiver instruments are pension funds that seek to hedge the interest rate risk of their longer liabilities. Contrarily, if an institution has outstanding variable rate loans, which they want to hedge against an increase in the interest rate, they will have an overweight of payer instruments e.g. payer swaps or swaptions.

As was seen from the scenario analysis, the size and the direction of the value transfer are dependant on the fixed rate K relative to the par swap rate of the contracts for both swaps and swaptions. Furthermore, for swaptions it is also dependant on the volatility of the discounting curve. In table 7.1 the par swap rates can be seen for 10Y, 15Y and 35Y swaps initiated exactly five years before the 22nd August 2019. These contracts will thus at the time of the data analysis be equal to the 5Y, 10Y, and 30Y swaps used for the pricing and scenario analysis. As the interest rate level has fallen in the Euro zone over the last several years, older contracts have a substantially higher fixed rate K compared to what the market deem the par swap rate on the 22th August 2019.

Table 7.1: Par swap rate on the 22 th August 2019 compared to a contract initiated exactly 5 years prior (Bloomberg, 2020).

	Par swap 2019	Par swap 2014 (remaining maturity)
5Y	-0.4522%	1.1565%
10Y	-0.1943%	1.5506%
30Y	0.2140%	1.8474%

This decrease in the par swap rate indicates that institutions with older legacy receiver positions will be deep ITM, while older payer contracts will be deep OTM. As seen from the results in section 6.2, when the swaps are deep ITM/OTM and the swaptions are deep ITM, the change in the discounting curves level will affect the absolute value of the instruments the most. Looking at figure 6.4 the difference in value of the 30Y swap is around EUR 3 million on a 500 million notional given a strike of 1.8474%. With regards to swaptions, the holder of both payer and receiver swaptions stand to gain value on the legacy contracts when the discount curve is lowered as per section 6.2. The value increase of the payer swaptions will be negligible as these contracts are likely to be OTM given that par forward swap rates have fallen. Contrarily, the added value for the receivers will be substantial given these are deeply ITM. The issuers of the swaptions lose monetary value either way.

While the falling interest rate level amplifies the Delta risk of swaptions, the vega will be substantially lower as this is largest ATM. Thus, as can also be seen from figure 6.6, the change in the value will be minor for both legacy receiver and payer swaptions as they are deeply ITM and OTM respectively. Thus, while the change in level of the discounting curve will have the largest effect for older contracts, the volatility change will have a relatively larger effect on newer contracts where the forward swap rate is lower and therefore closer to the par forward swap rate. This is of course assuming the contract have been entered at par. The effect of the volatility on the discounting curve is, however, still negligible unless the correlation between the projection and discounting curve changes upon implementation of ESTER. This correlation factor is, however, hard to determine ex ante, but it is expected that the potential transfer of value would be amplified if the correlation differs from the analysed base scenario.

Proposed solutions for implementation

Accordingly, there is a potential expected transfer of value if the legacy contracts are switched to ESTER discounting without any alterations. The possible solutions for handling this are plentiful and are currently being widely discussed between market participants and the ECB (European Central Bank, 2020b). All potential interventions have different ramifications in terms of the value transferred and the risk associated with the solution. The point of mitigating the transfer of value is to avoid an "unfair" wealth transfer between parties and maintaining the intentions and assumptions of the parties at the time they originally entered the contracts.

The solutions suggested by ECB can be divided into two main categories: Discounting using ESTER flat, i.e. without any added spread, or discounting using ESTER +8.5 bps spread. If it is agreed to discount with ESTER flat, the next question becomes what to do with the transfer of value this will entail. Here, ECB suggests a number of different options (European Central Bank, 2019a).

Firstly, it is possible to let the transfer of value happen without interfering. This is the solution that most likely will be implemented in Switzerland for the reformation of their overnight rates (European Central Bank, 2019a). If chosen as a solution, the results shown in chapter 6 will in that regard come to fruition and a non-negligible transfer of value across counterparties will occur. This can be argued as being unfair given the change in the discounting curve is hard to predict before announcement. However, it can be argued to be part of the risk that market participants choose to be exposed to when trading in the interest rate derivatives market.

A second possible solution is to have compensation equal to the value transfer of the contracts traded between the counterparties. This will in some regards take care of the immediate value transfer. However, depending on the type of compensation, it will not account for the change in risk sensitivities that a new discounting curve will entail. This is both in terms of the Delta vector, which is dependant on whether the contract is ITM or not as seen in table 6.5, but also in terms of handling the lower volatility of the ESTER curve. If it is decided that the transfer of value should be deferred by compensating the counterparty who stands to lose on the shift, the next choice is how to construct this compensation in a way that mitigates the changes in the best way possible.

The simplest type of compensation is to settle the valuation difference with a one-time cash payment. This will take care of the major pricing differences at the transfer date from the change in the yield curve, but will still entail a change in the Delta vector of the contract. This proposed solution can thus be undesirable for agents using the derivatives to manage their interest rate risk exposure as it would still encompass a major shift in their current interest rate hedge.

Another solution is to enter a basis swap where ESTER is exchanged for EONIA, while the EONIA is still being published. This would effectively just be a string of payments of 8.5 bps to one part, but would offset the immediate changes in Delta risk and valuation using the ESTER flat would inquire. The change in volatility and possibly the correlation will still not be accounted for. However, this solution would only be viable for certain maturities as the EONIA is expected to be discontinued in 2022. Approximately half of the legacy contracts have fairly short maturity and will expire in this interim period (Stride, 2019). For the remaining contracts with longer maturities, this solution would only postpone the problem if the basis swap is written on EONIA. Alternatively, the basis swap could simply be defined as an annuity bond which pays 8.5 bps fixed, which would solve the problem of this type of compensation scheme only working until the discontinuance of EONIA. This solution might, however, be problematic from an accounting point of view as the bond will not classify as a derivative under the International Financial Reporting Standards (IFRS). This will entail that the instrument will have be treated differently on the balance sheet of some market participants (European Central Bank, 2019a).

These potential ways of dealing with the value transfer can be used both separately or in combination. The solution of basis swaps can for example be used for the instruments maturing before the EONIA is discontinued, while the legacy contracts with longer maturities could be compensated through a cash compensation of the present value of the shift.

Implementation of potential compensation

In terms of how these different solutions can be implemented, it is necessary to distinguish between centrally cleared and bilateral contracts. In centrally cleared trades the instruments are traded through a clearing house, which then acts as the counterparty for both ends of the trade. The clearing house reduces counterparty risk by requiring margins through daily posting of collateral on the contracts. In bilateral trades, the trade is simply done directly between the two counterparties, where the specific terms of the deal such as type of collateral, marked-to-market frequency etc. have to be determined bilaterally.

If the contract is cleared through a central clearing house, it is up to the clearing house how they wish to make the switch to ESTER. LCH, one of the largest clearing houses in the world, has announced that they will not support dual discounting and will discount the products with ESTER flat. The transfer of value in the contracts will be settled by cash compensation at the point of conversion, which is currently set to be on the 27^{th} July 2020 (LCH, 2019, 2020). However, it is still unclear how this compensation should and will be calculated. This is especially relevant in the case of interest rate options as the expected lower volatility of the discounting curve also will have to be considered.

However, if a contract is a bilateral agreement, it is up to the parties of each contract to negotiate an agreement. This will naturally be expensive, time consuming and have the potential to damage the relations between the parties. In this regard, the ECB recently held a public consultation, where they received responses from 34 relevant market participants about their thoughts and preferences about potential compensation structures for legacy swaption contracts (European Central Bank, 2020b). The preferences of these participants differs substantially. Firstly, there is disagreement about whether any compensation should be encouraged by the ECB at all; 23 out of 34 respondents supports that bilateral counterparties voluntarily exchange compensation. Consequently, this means that the remaining 11 respondents do not support either the idea of compensating the losing party of the legacy swaption contract or that ECB gives a recommendation on the matter. Secondly, there is wide spread disagreement surrounding which legacy contracts are encompassed if voluntary compensation is encouraged. The important aspect in this regard concerns the date that the contract was initiated compared to the knowledge of future ESTER implementation. ECB has suggested five different relevant dates with none of these receiving the support of more than a third of the respondents.

Even if ECB suggests a methodology for dealing with the transfer of wealth and risk, it creates the potential issue of adverse selection. In this regard, parties could potentially exploit the voluntary nature of the compensation scheme for bilateral contracts by only agreeing to compensate the wealth transfer on a contract-by-contract or relationship-by-relationship basis, where it will benefit them in their direct negotiations. Consequently, potential adverse selection will further increase the cost and time needed for negotiating new bilateral contracts.

While the different solutions to the value transfer have some direct implications on the market participants' balance sheets, they might also have an effect on the market as a whole. For centrally cleared contracts the shift has the potential to incur a liquidity surge since the potential cash compensation will have to be paid at the same specified date for all outstanding contracts at a clearing house. There is significant concentration risk in this market with LCH clearing 98.9% of all centrally cleared interest rate swaps in Euro (Khwaja, 2018). This essentially means that the date LCH have chosen of the 27th July 2020 for cash compensation is the relevant one for the vast majority of centrally cleared contracts. For Danish Pension funds the percentage of centrally cleared interest rate derivatives was 42% in 2018. This share have only increased since regulation in the form of the European Market Infrastructure Regulation (EMIR) have been introduced. EMIR requires an increased use of centrally cleared contracts throughout the European interest rate derivative market. For example this regulation includes a requirement of pension funds using 100% centrally cleared swap contracts by 2023 unless certain circumstances are present (Achord & Jensen, 2019).

By choosing a single date for compensation LCH might thereby cause a market wide need for cash when compensation is due. This is especially problematic as the institutions with longer duration receiver positions stand to gain the most value from the shift to ESTER and consequently will need to transfer the largest cash compensations. These are as previously described e.g. commercial banks and pension funds. Pension funds have very limited cash holdings and might therefore have to sell their highly rated sovereign bonds or mortgage-backed securities to cover this need. As LCH clears a dominating part of the market, most of the sector will have to pay the cash compensation simultaneously, which might entail that these sales happen at stressed prices. However, some of these effects will be mitigated as the market participants most often have both payer and receiver positions. Consequently, some of the potential compensation will be mitigated, since some of the value transfer between payers and receivers will be netted out.

7.3 The choice of ESTER

As shown by the results and the previous discussion of the ESTER implementation, implementing the ESTER will in any case be associated with a range of problems. Given the widespread ramifications of these problems, a discussion of the reasoning behind the introduction of the ESTER is appropriate.

One of the key reasons for transitioning to ESTER is the lower volatility of the rate given the larger sample population and transactional volume. This is an attempt to eliminate some of the liquidity and concentration risk inherent in the EONIA. Thereby, the ESTER introduces a higher degree of financial stability by being more representative of real funding costs, exhibit higher degrees of transparency and being less prone to manipulation. This is favourable for investors as they use derivatives in order gain or mitigate exposure to the underlying, and consequently wishes their positions to be influenced primarily by changes in the underlying. For interest rate derivatives written on the Euribor, the agent therefore seeks to expose themselves to this rate and not to changes in the discounting curve. Otherwise they would have entered an OIS instrument instead. Hence, a lower volatility in the discounting rate as the ESTER exhibits is preferable as this makes the position more aligned with the investments' intent.

While the ESTER does have all of the above mentioned positive characteristics, the transition from the EONIA is not cheap. Therefore, it can be discussed whether the positives of the ESTER outweigh the cost of implementation. Changing the discounting curve already has and will continue to require thousands of working hours from both ECB and market participants on a broad range of tasks including analysis on exposure, correcting hedges, renegotiating contracts and figuring out potential compensation schemes. Even though the change in the discount curve is quite substantial, it has been shown that it only incurs relatively minor expected changes in the value of the legacy contracts. Therefore, the cost of transitioning the discount curve can be deemed excessive in comparison. However, the positives of increased stability and transparency also has to be taken into account as it adds trust to the financial sector. This is especially relevant considering the scrutiny of the interest rate derivatives market following the xIBOR scandals. The manipulation of the interest rates is also one of the main reasons behind the reformations of the interest rate benchmarks in the first place.

Transitioning to ESTER has not been the only considered solution of transforming the discounting rate in the Euro zone. Using a secured overnight rate similar to the SOFR in the U.S has also been considered an option. Specifically, two privately administered secured overnight rates have been considered, namely the GC Pooling Deferred rate and the RepoFunds Rate. The characteristics of the two rates along with ESTER can be seen in table 7.2.

Table 7.2: Comparison of potential EONIA alternatives from first public consultation by the ECB (European Central Bank, 2018)

	Standard deviation of daily changes	Average daily volume
ESTER	0.6bps	29.8bnEUR
GC Pooling Deferred	3.8bps	6.9bnEUR
RepoFunds Rate	14.4bps	200.6bnEUR

As can be seen from the table, the two rates suffer from either inadequate volume or immense volatility. Furthermore, the GC Pooling Deferred rate has suffered from few market participants and will thus not solve the problem of concentration risk in the EONIA. However, the RepoFunds Rate will not satisfy the requirement of increased transparency as the rate also contained Tomorrow/Next rates and even some rates where the maturity, although short, could not be determined. ESTER has thereby ended up as the chosen rate since it includes a higher volume than the EONIA, while also exhibiting lower volatility than both the GC Pooling Deferred and especially the RepoFunds Rate (European Central Bank, 2018). The ESTER have furthermore proven to be rather robust and has even in recent Covid-19 times maintained a low volatility (Bloomberg, 2020).

7.4 Future change of Euribor

While the results show that the transition to ESTER from EONIA will affect legacy derivatives, it is not the only interest rate under scrutiny. As mentioned in chapter 1, the Euribor rates are currently also being targeted for reformation. One of the main goals of the ECB is that benchmark rates should be based on actual transactions. To this end, the Euribor has already been transformed as of the end of 2019 into a set of hybrid rates in which they are determined through both transactional and survey-based data (European Money Markets Institute, 2019b). Since the hybrid rates still include the survey-based data and the calculation methods otherwise are unchanged so far, this change have not entailed any significant effects on the Euribor rates yet. A purely transactional-based Euribor still appears to be the long term goal, albeit it also seems like a distant one at this time. When potentially implementing such changes to the Euribor, it is relevant as with the transition to the ESTER to weigh the potential costs and the benefits of the change.

If, or perhaps more likely when, the Euribor is transformed to being based solely on actual transactions, it might entail massive ramifications on the market as a whole. As shown in table 6.5 the projection curve is far more important in terms of determining value and the amount of risk of the interest derivatives compared to the discounting curve. As such future changes in the Euribor can have much larger impact than the changes from EONIA to ESTER investigated in this paper. This is of course only the case if the reform of the rate entails any significant changes to the curve e.g. in level, volatility, or correlation. If such reformation will happen, the market will be able to draw experience from the transition from EONIA and ESTER in terms of settling value differences and managing changes in risk. Such a transformation will, however, still entail massive work for all parties involved as with the change to ESTER.

CHAPTER 8

Conclusion

This paper has sought to examine the upcoming transition of the overnight rates in the Euro Zone and how this will affect the pricing and risk of EUR-denominated interest rate derivatives in the form of swaps and swaptions written on 6M Euribor. This has been examined by modelling the movements of the 6M Euribor, EONIA and Pre-ESTER interest rates through short rate models.

In this regard, it has been found that the classic Vasicek model when calibrated to the daily realised rates, is unable to capture the movements of the short rates in any satisfactory and applicable way for derivative pricing. When calibrated to market zero rates, the Vasicek model is improved, albeit the fit to the observed term structure still is unsatisfactory in light of the model having no deterministic parameter. The Hull-White one-factor model has showed the advantage of a no-arbitrage model and captured the fit of the initial yield curve with little error. This error, however, disappears entirely when moving to the G2++ model. Subsequently, the G2++ model has been chosen for further modelling due to its ability to capture the term structure perfectly and capture non-perfect correlation across maturities. The G2++ model has then been modified to a dual-curve setup by including a correlation factor between the projecting curve, 6M Euribor, and the discounting curve, EONIA. The dual-curve G2++ model has been deemed satisfactory as it captures the volatility smile in the swaption market to some extend while capturing the initial fit of the two yield curves perfectly. Through simulation of the two yield curves with this model, a range of both swaps and swaptions as examples of interest rate derivatives have been priced with satisfactory results.

After the initial modelling and subsequent pricing, different scenarios have been examined by stressing the model in an attempt to capture some of the possible effects of the ESTER implementation. To this end, it has been found that the expected lower level of the ESTER curve will have effects on the pricing of both swaps and swaptions. Firstly, it will cause OTM payer swaps to lose value against ITM receiver swaps. Furthermore, it will cause a shift of value from the issuer to the holder of swaptions. This positive shift of value is largest for legacy receiver swaptions as these are expected to be ITM. Lower volatility in the discounting curve - similar to what has been found in the Pre-ESTER compared to the EONIA - and higher volatility have been tested in combination with both a positive and negative correlation between the projecting curve and the discounting curve. In this regard, it has been found that higher volatility in the discounting curve leads to slightly lower prices when the correlation is positive and vice versa for lower volatility. However, this relationship reverses and is further enhanced with a negative correlation between the two yield curves.

Given the size of the notional in the interest rate derivative market, the resulting transfer of value has the potential to be non-negligible given the possible effects of implementing the ESTER. However, the possible value transfer have been found to be significantly larger if the projection curve rather than the discounting curve would be changed in a similar manner as the shift from EONIA to ESTER. This is relevant as future transformations of the projection curve i.e. the Euribor, is currently being examined by the ECB.

Aforementioned results have been discussed in relation to market participants and possible solutions to mitigate a potentially unfair transfer of value as a result of the implementation of ESTER. Depending on the choice of dealing with the potential wealth transfer, the value could either manifest or some compensation scheme could be implemented. What will actually happen is still widely discussed with no clear consensus among market participants and the ECB. If cash compensation is agreed upon, which appears the most likely scenario especially for centrally cleared contracts, the sudden need for cash on the day of compensation can potentially create a liquidity surge.

CHAPTER 9

Further Research

In the following chapter, some of the areas will be discussed, where it would be natural to conduct further research into the implications of the transition in the underlying interest rate benchmarks.

When conducting the type of analysis as seen in this paper, the most intuitive extensions would be to calibrate the short rate models to analyse other types of EONIA sensitive products. While briefly used for calibrating the Hull-White one-factor model, the exact pricing difference of caps and floors could be a simple extension. This has been left out as caps and floors are simply the sum of a series of caplets/floorlets. Caplets/floorlets are basically one period swaptions and the extension would therefore be trivial.

More exotic derivatives such as American-exercise or Bermudan-exercise options could also be an interesting extension that is possible to analyse within the short rate framework. These types of products enables the holder to exercise more often, which naturally gives the product more value. Hence, the changes in value are expected to be amplified for these types of derivatives compared to the European type. The modelling framework would easily enable the examination of these instruments since the prices are computed using every simulated path of the underlying interest rates. Thereby, the price can be adjusted for the option to exercise early in relation to the intrinsic value.

The consequences for OIS swaps would also be of interest since the transition to ESTER is likely to have more severe consequences when the overnight rate is used as the projection curve as well. This would be relatively easy to analyse since the pricing problem would be simplified back to a single-curve framework.

Another possible extension could be to examine multi-currency instruments affected by the ESTER transition. This could for example be cross-currency basis swaps between

EUR and other currencies. Multi-currency instruments would allow the researcher to discuss topics such as currency preference of collateral, the effect on cross-currency basis spread etc.

While changing the analysed instruments are interesting in itself, further research regarding the exposure of the market participants could also be made. It has been speculated that e.g. ITM receiver positions would have to hand over cash compensation, which could increase the demand for liquidity. Furthermore, this demand would also be amplified by the changes in collateral on legacy contracts due to the value change. These effects and the subsequent demand for liquidity could in turn cause spillover effects to for example mortgage backed securities and the sovereign bond market. However, the magnitude of these effects are yet to be analysed and the exact impact on other markets is therefore unknown. Furthermore, knowing the exposure of market participants would lend to the discussion of the cost of ESTER implementation, as the exact value transfer across institutions depends on their net exposure.

Lastly, similar analysis of other interest rate reforms could be of great interest. It has already been included in the discussion that the Euribor is also under revision. Examining the Euribor even further and looking at possible outcomes would greatly add to the findings of this paper. Furthermore, interest rate reforms in other markets could also be examined. This could for example be the SOFR rate in the U.S., which is particularly interesting given that it has shown a propensity to exhibit massive volatility spikes exceeding 250 bps (Bloomberg, 2020).

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APPENDIX A

Code

A.1 Calibration of the Vasicek model to realised rates

```
# Cleaning the workspace
rm(list=ls())
#Read data - #can also read the Euribor and EONIA data
data <- read.csv("Pre-Ester.csv", header = FALSE, sep=",")
data <- data[,2]/100
data <- as.matrix(data)
#Calibrate
Vasicek_Parameters <- function(data, dt) {
 N <- length(data)
 ones <- seq(1,1,length.out = (N-1))
 x <- cbind(ones, data[1:(N-1)])
 #Maximum Likelihood estimation
 ols <- solve(t(x) %*% x) %*% (t(x) %*% data[2:N])
 resid <- data[2:N] - (x %*% ols)
 c<- ols[1]
 b <- ols[2]
 delta <- sd(resid)
 alpha <- -(log(b)/dt)
 theta <- c/(1-b)
 sigma <- delta/sqrt((b^2-1)*dt/(2 * log(b)))
 parameters <- c(alpha, theta, sigma)
 print(parameters)}
parameters <- Vasicek_Parameters(data, 1/252)
```

A.2 Calibration of the Vasicek model to market data

```
# Cleaning the workspace
rm(list=ls())
library(esg2)

#Zero Curve - Euribor market data
curve <- curvezc(method = "continuous",
         rates = c(-0.0046073, -0.0050291, -0.0050294, -0.0048534, -0.0045336,
              -0.0041393, -0.0036629, -0.0031261, -0.0025557, -0.0020036,
              -0.0014646, -0.0009584, -0.0004836, -0.0000445, 0.000346,
              0.0006756, 0.0009484, 0.0011768, 0.0013729, 0.0015489, 0.0017002,
              0.0018187, 0.0019112, 0.0019847, 0.0020458, 0.0020913, 0.0021165,
              0.0021265, 0.0021266, 0.0021217, 0.0021093, 0.0020861, 0.0020558,
              0.0020223, 0.0019894, 0.0019574, 0.0019238, 0.0018881, 0.0018497,
              0.0018082, 0.0017611, 0.0017089, 0.0016550, 0.0016030, 0.0015563,
              0.0015149, 0.0014766, 0.0014413, 0.0014089, 0.0013796
         )
)

data <- curve@rates
data <- as.matrix(data)
#Calibrate
Vasicek_Parameters <- function(data, dt)
 {N <- length(data)
 ones <- seq(1,1,length.out = (N-1))
 x <- cbind(ones, data[1:(N-1)])
 #Maximum Likelihood estimation
 ols <- solve(t(x) %*% x) %*% (t(x) %*% data[2:N])
 resid <- data[2:N] - (x %*% ols)
 c<- ols[1]
```

```
  b <- ols[2]
  delta <- sd(resid)
  alpha <- -(log(b)/dt)
  theta <- c/(1-b)
  sigma <- delta/sqrt((b^2-1)*dt/(2 * log(b)))
  parameters <- c(alpha, theta, sigma)
  print(parameters)}
parameters <- Vasicek_Parameters(data, 1)
```

A.3 Simulating the Vasicek model with realised rates

```
# Cleaning the workspace
rm(list=ls())
#Defining parameters from calibration
r0 <- -0.00549
theta <- -0.004495
k <- 39.746917
sigma <- 0.000899

#Simulate paths for short rate
n <- 10000    # MC simulation trials
T <- 10    # total time
m <- 2520   # subintervals
dt <- T/m  # difference in time each subinterval

#Create matrix to contain paths
r <- matrix(0,m+1,n)
r[1,] <- r0

#Loop for simulating
```

```
set.seed(1)

for(j in 1:n){
  for(i in 2:(m+1)){
    dr <- k*(theta-r[i-1,j])*dt + sigma*sqrt(dt)*rnorm(1,0,1)
    r[i,j] <- r[i-1,j] + dr
  }}

#Plot paths for given parameters and confidence intervals
t <- seq(0, T, dt)
rT.expected <- theta + (r0-theta)*exp(-k*t)
rT.stdev <- sqrt( sigma^2/(2*k)*(1-exp(-2*k*t)))
matplot(t, r[,1:n], type="l", lty=1, ylab="r(t)")
abline(h=theta, col="red", lty=2)
lines(t, rT.expected, lty=4, lwd = 3)
lines(t, rT.expected + 1.96*rT.stdev, lty=3, lwd = 3)
lines(t, rT.expected - 1.96*rT.stdev, lty=3, lwd = 3)
points(0,r0)
```

A.4 Simulating the Vasicek model with market data

```
# Cleaning the workspace
rm(list=ls())
## define model parameters
r0 <- -0.0046073
theta <- 0.0028498160
k <- 0.0502315393
sigma <- 0.000460

## simulate short rate paths
```

```
n <- 100000    # MC simulation trials
T <- 50    # total time
m <- 50   # subintervals
dt <- T/m  # difference in time each subinterval

r <- matrix(0,m+1,n)  # matrix to hold short rate paths
r[1,] <- r0

set.seed(1)
for(j in 1:n){
  for(i in 2:(m+1)){
    dr <- k*(theta-r[i-1,j])*dt + sigma*sqrt(dt)*rnorm(1,0,1)
    r[i,j] <- r[i-1,j] + dr
  }}

t <- seq(0, T, dt)
rT.expected <- theta + (r0-theta)*exp(-k*t)
rT.stdev <- sqrt( sigma^2/(2*k)*(1-exp(-2*k*t)))
matplot(t, r[,1:n], type="l", lty=1, ylab="r(t)")
abline(h=theta, col="red", lty=2)
lines(t, rT.expected, lty=4, lwd = 3)
lines(t, rT.expected + 1.96*rT.stdev, lty=3, lwd = 3)
lines(t, rT.expected - 1.96*rT.stdev, lty=3, lwd = 3)
points(0,r0)

#pricing of ZCB
VasicekZCBprice <-
  function(r0, k, theta, sigma, T){
    b.vas <- (1/k)*(1-exp(-T*k))
    a.vas <- (theta-sigma^2/(2*k^2))*(T-b.vas)+(sigma^2)/(4*k)*b.vas^2
```

```
  return(exp(-a.vas-b.vas*r0))
 }

#monte carlo pricing
ss <- colSums(r[2:(m+1),]*dt)  # integral estimate
c <- exp(-ss)
estimate <- mean(c)

#standard error
stdErr <- sd(c)/sqrt(n)

#analytical price
exact <- VasicekZCBprice(r0, k, theta, sigma, T)

#analytical zerocoupon bond prices
zcb <- cbind(
exact1 <- VasicekZCBprice(r0, k, theta, sigma, 1),
exact2 <- VasicekZCBprice(r0, k, theta, sigma, 2),
exact3 <- VasicekZCBprice(r0, k, theta, sigma, 3),
exact4 <- VasicekZCBprice(r0, k, theta, sigma, 4),
exact5 <- VasicekZCBprice(r0, k, theta, sigma, 5),
exact6 <- VasicekZCBprice(r0, k, theta, sigma, 6),
exact7 <- VasicekZCBprice(r0, k, theta, sigma, 7),
exact8 <- VasicekZCBprice(r0, k, theta, sigma, 8),
exact9 <- VasicekZCBprice(r0, k, theta, sigma, 9),
exact10 <- VasicekZCBprice(r0, k, theta, sigma, 10),
exact11 <- VasicekZCBprice(r0, k, theta, sigma, 11),
exact12 <- VasicekZCBprice(r0, k, theta, sigma, 12),
exact13 <- VasicekZCBprice(r0, k, theta, sigma, 13),
exact14 <- VasicekZCBprice(r0, k, theta, sigma, 14),
```

```
exact15 <- VasicekZCBprice(r0, k, theta, sigma, 15),
exact16 <- VasicekZCBprice(r0, k, theta, sigma, 16),
exact17 <- VasicekZCBprice(r0, k, theta, sigma, 17),
exact18 <- VasicekZCBprice(r0, k, theta, sigma, 18),
exact19 <- VasicekZCBprice(r0, k, theta, sigma, 19),
exact20 <- VasicekZCBprice(r0, k, theta, sigma, 20),
exact21 <- VasicekZCBprice(r0, k, theta, sigma, 21),
exact22 <- VasicekZCBprice(r0, k, theta, sigma, 22),
exact23 <- VasicekZCBprice(r0, k, theta, sigma, 23),
exact24 <- VasicekZCBprice(r0, k, theta, sigma, 24),
exact25 <- VasicekZCBprice(r0, k, theta, sigma, 25),
exact26 <- VasicekZCBprice(r0, k, theta, sigma, 26),
exact27 <- VasicekZCBprice(r0, k, theta, sigma, 27),
exact28 <- VasicekZCBprice(r0, k, theta, sigma, 28),
exact29 <- VasicekZCBprice(r0, k, theta, sigma, 29),
exact30 <- VasicekZCBprice(r0, k, theta, sigma, 30),
exact31 <- VasicekZCBprice(r0, k, theta, sigma, 31),
exact32 <- VasicekZCBprice(r0, k, theta, sigma, 32),
exact33 <- VasicekZCBprice(r0, k, theta, sigma, 33),
exact34 <- VasicekZCBprice(r0, k, theta, sigma, 34),
exact35 <- VasicekZCBprice(r0, k, theta, sigma, 35),
exact36 <- VasicekZCBprice(r0, k, theta, sigma, 36),
exact37 <- VasicekZCBprice(r0, k, theta, sigma, 37),
exact38 <- VasicekZCBprice(r0, k, theta, sigma, 38),
exact39 <- VasicekZCBprice(r0, k, theta, sigma, 39),
exact40 <- VasicekZCBprice(r0, k, theta, sigma, 40),
exact41 <- VasicekZCBprice(r0, k, theta, sigma, 41),
exact42 <- VasicekZCBprice(r0, k, theta, sigma, 42),
exact43 <- VasicekZCBprice(r0, k, theta, sigma, 43),
exact44 <- VasicekZCBprice(r0, k, theta, sigma, 44),
```

```
exact45 <- VasicekZCBprice(r0, k, theta, sigma, 45),
exact46 <- VasicekZCBprice(r0, k, theta, sigma, 46),
exact47 <- VasicekZCBprice(r0, k, theta, sigma, 47),
exact48 <- VasicekZCBprice(r0, k, theta, sigma, 48),
exact49 <- VasicekZCBprice(r0, k, theta, sigma, 49),
exact50 <- VasicekZCBprice(r0, k, theta, sigma, 50))

cat('Exact Vasicek Price:', round(exact,7), 'n')
cat('MC Price:', round(estimate,7), 'n')
cat('MC Standard Error:', round(stdErr,7), 'n')

# analytical vs market zero prices
time <- c(1:50)
plot(time, zcb, type='l', col = 'blue', lwd = 3,  ylim =c(0.90,1.05), xlab = "maturity (years)",
    ylab = "prices")
points(time, curve@zcp, col = 'red')

zrates <- -log(zcb)/time
zrates

plot(time, zrates, type='l', col = 'blue', lwd = 3, ylab ="zero coupon rates",
ylim =c(-0.006,0.003), xlab = "maturities (years)")
points(time, curve@rates, col = 'red')
```

A.5 Calibration of the Hull-White one-factor model in Matlab

```
%Defining
Settle = 'Aug-24-2019';
Maturity = 'Aug-24-2029';
```

```
Strike = 0;
Reset = 2; %semi-annual
Principal = 100;
Basis = 2; %ACT/360

%Creating schedule
floorletDates = cfdates(Settle, Maturity, Reset, Basis);
datestr(floorletDates')

%Defining market data
MarketStrike = [-0.3; 0];
MarketMaturity = {'24-Aug-2020';'24-Feb-2021';'24-Aug-2021';'24-Feb-2022'
;'24-Aug-2022';
'24-Feb-2023';'24-Aug-2023';'24-Feb-2024';'24-Aug-2024';'24-Feb-2025';'24-Aug-2025';
'24-Feb-2026';'24-Aug-2026';'24-Feb-2027';'24-Aug-2027';'24-Feb-2028';
'24-Aug-2028';'24-Feb-2029';'24-Aug-2029'};
MarketVolatility = [0.2320 0.2513 0.2861 0.3054 0.3407 0.3500 0.3855 0.3946 0.4229
0.4305 0.4639 0.4730 0.4844 0.4872 0.5208 0.5298 0.5601 0.5679 0.5589;
% First row in table corresponding to Strike 1
 0.2950 0.3013 0.3302 0.3494 0.3799 0.3879 0.4231 0.4321 0.4532 0.4588 0.4876
 0.4954 0.4955 0.4955 0.5295 0.5386 0.5727 0.5815 0.5734];
% Second row in table corresponding to Strike 2

%Defining market rates
Rates= [-0.47406;-0.47304;-0.46074;-0.44393;-0.4072;-0.36571;-0.32600;
-0.27532;-0.20872;-0.14174;-0.07962;-0.01335;0.06047;0.13676;0.21167;
0.27009;0.30802;0.34853;0.4003];
ValuationDate = 'Aug-24-2019';
EndDates = {'24-Aug-2020';'24-Feb-2021';'24-Aug-2021';'24-Feb-2022';
'24-Aug-2022';'24-Feb-2023';'24-Aug-2023';'24-Feb-2024';'24-Aug-2024';
```

```
'24-Feb-2025';'24-Aug-2025';'24-Feb-2026';'24-Aug-2026';'24-Feb-2027';
'24-Aug-2027';'24-Feb-2028';'24-Aug-2028';'24-Feb-2029';'24-Aug-2029'};
Compounding = 2;
Basis = 2;
RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

%Finding Alpha and Sigma through Bachelier Model with Hwcalbyfloor
format short
o=optimoptions('lsqnonlin','TolFun',100*eps); %levenberg-marquardt
warning ('off','fininst:hwcalbycapfloor:NoConverge')
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec, MarketStrike, MarketMaturity,...
MarketVolatility, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...
'Basis', Basis, 'OptimOptions', o, 'model', 'normal')
```

A.6 Simulation of the Hull-White one-factor model

```
# Cleaning the workspace
rm(list=ls())
devtools::install_github("arnaudbu/esg2")

install.packages("RQuantLib")
install.packages("ESGtoolkit")
# RQuantLib loading
suppressPackageStartupMessages(library(RQuantLib))
# ESGtoolkit loading
suppressPackageStartupMessages(library(ESGtoolkit))
suppressPackageStartupMessages(library(esg2))
curves <- curvezc(method = "continuous",
```

```
                rates = c(-0.0046073, -0.0050291, -0.0050294,
                -0.0048534, -0.0045336,-0.0041393, -0.0036629,
-0.0031261, -0.0025557, -0.0020036, -0.0014646, -0.0009584,
-0.0004836, -0.0000445, 0.000346, 0.0006756, 0.0009484,
0.0011768, 0.0013729, 0.0015489, 0.0017002, 0.0018187,
0.0019112, 0.0019847, 0.0020458, 0.0020913, 0.0021165,
0.0021265, 0.0021266, 0.0021217, 0.0021093, 0.0020861,
0.0020558, 0.0020223, 0.0019894, 0.0019574, 0.0019238,
0.0018881, 0.0018497, 0.0018082, 0.0017611, 0.0017089,
0.0016550, 0.0016030, 0.0015563, 0.0015149, 0.0014766,
0.0014413, 0.0014089, 0.0013796))

#Frequency of simulation and interpolation
freq <- "annual"
delta_t <- 1
# Horizon, number of simulations, frequency
horizon <- 50
nb.sims <- 100000

#create market prices
times <- seq(from = delta_t, to = horizon, by = delta_t)
maturities <- times
marketzerorates <- curves@rates
marketprices <- curves@zcp

############ Hull-White short-rate simulation############

# Input calibrated Hull-White parameters
a <- 0.0019
sigma <- 0.008026
```

```
# Simulation of gaussian shocks with ESGtoolkit
set.seed(12)
eps <- ESGtoolkit::simshocks(n = nb.sims, method="classic", horizon = horizon,
                             frequency = freq)

# Simulation of the factor x with ESGtoolkit
x <- ESGtoolkit::simdiff(n = nb.sims, horizon = horizon,
                         frequency = freq,
                         model = "OU",
                         x0 = 0, theta1 = 0,
                         theta2 = a,
                         theta3 = sigma,
                         eps = eps)

# Calculate instantaneous forward rates
fwdrates <- ts(replicate(nb.sims, curves@ifr),
               start = start(x),
               deltat = deltat(x))

# alpha
t.out <- seq(from = 0, to = horizon, by = delta_t)
param.alpha <- ts(replicate(nb.sims, 0.5*(sigma^2)*(1 - exp(-a*t.out))^2/(a^2)),
                  start = start(x), deltat = deltat(x))
alpha <- fwdrates + param.alpha

# The short-rate
r <- x + alpha

# Stochastic discount factors
```

```
Dt <- ESGtoolkit::esgdiscountfactor(r = r, X = 1)

# Monte Carlo prices and zero rates deduced from stochastic discount factors
montecarloprices <- rowMeans(Dt)
montecarlozerorates <- -log(montecarloprices)/maturities
# RQuantLib uses continuous compounding

# Visualisation

# Short-rate quantiles
matplot(maturities, r, type="l", lty=1)
ESGtoolkit::esgplotbands(r, xlab = "Maturities (years)", ylab = "Short rate")

# Monte carlo vs market zero rates
plot(maturities, montecarlozerorates, type='l',
col = 'blue', lwd = 3, ylab ="Zero coupon rates", xlab ="Maturities (years)")
points(maturities, marketzerorates, col = 'red')

# Monte carlo vs market zero-coupon prices
plot(maturities, montecarloprices, type ='l', col = 'blue',
lwd = 3, ylab ="Prices", xlab ="Maturities (years)")
points(maturities, marketprices, col = 'red')
```

A.7 Calibration and simulation of the G2++ model

```
# Cleaning the workspace
rm(list=ls())
library(esg2)
library(xlsx)
```

```
#Zero Curve
curve <- curvezc(method = "continuous",
        rates = c(-0.0046073, -0.0050291, -0.0050294, -0.0048534, -0.0045336,
             -0.0041393, -0.0036629, -0.0031261, -0.0025557, -0.0020036,
             -0.0014646, -0.0009584, -0.0004836, -0.0000445, 0.000346,
             0.0006756, 0.0009484, 0.0011768, 0.0013729, 0.0015489, 0.0017002,
             0.0018187, 0.0019112, 0.0019847, 0.0020458, 0.0020913, 0.0021165,
             0.0021265, 0.0021266, 0.0021217, 0.0021093, 0.0020861, 0.0020558,
             0.0020223, 0.0019894, 0.0019574, 0.0019238, 0.0018881, 0.0018497,
             0.0018082, 0.0017611, 0.0017089, 0.0016550, 0.0016030, 0.0015563,
             0.0015149, 0.0014766, 0.0014413, 0.0014089, 0.0013796
             )
        )
#Visualise
plot(curve)

#ATM Swaption data per 22.08.2019
maturities = c(2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10)
tenors = c(1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,3,
     4,4,4,4,4,4,5,5,5,5,5,5,6,6,6,6,6,6,
     7,7,7,7,7,7,8,8,8,8,8,8,9,9,9,9,9,9,
     10,10,10,10,10,10)
vols = c(0.002742, 0.003422, 0.003945, 0.004405, 0.004950,
    0.005184, 0.003048, 0.003587, 0.004055, 0.004473,
    0.004951, 0.005217, 0.003413, 0.003854, 0.004261,
    0.004598, 0.004998, 0.005249, 0.003720, 0.004100,
```

```
        0.004468, 0.004703, 0.005068, 0.005291, 0.004012,
        0.004357, 0.004765, 0.004797, 0.005160, 0.005337,
        0.004278, 0.0046, 0.004788, 0.004931, 0.005231,
        0.005385, 0.004539, 0.004859, 0.004746, 0.005037,
        0.005277, 0.005410,0.004767, 0.005019, 0.004936,
        0.005126, 0.005328, 0.005427, 0.005026, 0.004753,
        0.005057, 0.005198, 0.005366, 0.005441, 0.005124,
        0.004974, 0.005144, 0.005247, 0.005389, 0.005436)
swaptions <- swaptions("normal", curve, maturities, tenors, vols, 2)

#Swaption information and plot
swaptions

#Defining model
g2model <- g2(curve, horizon = 50, nsimul = 100000)

#Calibration
g2model <- calibrate(g2model, swaptions, maxIter = 200, input_param <-
data.frame(Parameter = c("a", "b", "sigma", "eta", "rho"),
Initial.point = c(0.1, 0.25, 0.005, 0.025, -0.2),
Min = c(0.00000001, 0.00000001, 0.00000001, 0.00000001, -1), Max = c(0.5,0.6,0.1,0.15,1)))

#Generate correlated distributions with correlation from calibration
correl <- cbind(c(1,g2model@rho),c(g2model@rho,1))
W <- genW(correl, g2model@nsimul, g2model@horizon)

#Projection of the model
g2model <- project(g2model, Wx = W[,,1], Wy = W[,,2])

#Show parameters and plot short rate paths
```

```
g2model
plot(g2model)

#Test of initial fit to zero coupon curve and visualisation
test_deflator(g2model)
```

A.8 Calibration and simulation of the dual-curve G2++ model

```
# Cleaning the workspace
rm(list=ls())
library(esg2)
library(xlsx)

#Zero Curve EONIA
curve_EONIA <- curvezc(method = "continuous",
        rates = c(-0.00582,-0.00643,-0.00654,-0.0065,-0.00628,
            -0.00594,-0.00551,-0.00503,-0.00449,-0.00395,
            -0.00343,-0.0029,-0.0024098,-0.001961267,-0.00156,
            -0.001215365,-0.000925496,-0.000679144,-0.000465061,
            -0.000272,-0.00010484,0.00002892,0.0001366,0.00022552,
            0.000303,0.000366224,0.000410312,0.000439488,0.000457976,
            0.00047,0.000472824,0.000463632,0.000446528,0.000425616,
            0.000405,0.000384008,0.000359904,0.000333696,0.000306392,
            0.000279,0.00025152,0.00022328,0.00019428,0.00016452,0.000134,
            0.00010272,0.00007068,0.00003788,4.32E-06,-0.00003
        )
)
# Visualisation
curve_EONIA
```

```
plot(curve_EONIA)

#Swaptions 22.08.2019
maturities = c(2,3,4,5,7,10,2,3,4,5,7,10,
            2,3,4,5,7,10,2,3,4,5,7,10,
            2,3,4,5,7,10,2,3,4,5,7,10,
            2,3,4,5,7,10,2,3,4,5,7,10,
            2,3,4,5,7,10,2,3,4,5,7,10)
tenors = c(1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,3,
         4,4,4,4,4,4,5,5,5,5,5,5,6,6,6,6,6,6,
         7,7,7,7,7,7,8,8,8,8,8,8,9,9,9,9,9,9,
         10,10,10,10,10,10)
vols_EONIA = c(0.002687, 0.003389, 0.003906, 0.004389, 0.004942, 0.005155,
       0.002991, 0.003531, 0.003990, 0.004425, 0.004892, 0.005147,
       0.003346, 0.003794, 0.004187, 0.004543, 0.004943, 0.005183,
       0.003652, 0.004031, 0.004374, 0.004663, 0.005016, 0.005232,
       0.003936, 0.004274, 0.004583, 0.004830, 0.005113, 0.005285,
       0.004193, 0.004474, 0.004736, 0.004925, 0.005180, 0.005328,
       0.004428, 0.004659, 0.004889, 0.005015, 0.005224, 0.005352,
       0.004638, 0.004819, 0.004988, 0.005094, 0.005272, 0.005370,
       0.004813, 0.004967, 0.005079, 0.005160, 0.005308, 0.005385,
       0.004954, 0.005062, 0.005147, 0.005202, 0.005328, 0.005380)
swaptions_EONIA <- swaptions("normal", curve_EONIA, maturities, tenors, vols_EONIA, 1)

#Rate model
g2model_EONIA <- g2(curve_EONIA, horizon = 50, nsimul = 100000)

#Calibration of the model on swaptions prices
g2model_EONIA <- calibrate(g2model_EONIA,
swaptions_EONIA, maxIter = 200,input_param <-
```

```
data.frame(Parameter = c("a", "b", "sigma", "eta", "rho"),
Initial.point = c(0.1, 0.25, 0.005, 0.025, -0.4),
Min = c(0.00000001, 0.00000001, 0.00000001, 0.00000001, -1), Max = c(0.5,0.6,0.1,0.15,1)))
#show the calibrated parameters for G2++ EONIA Curve
g2model_EONIA

#--------Do the same for the EURIBOR-Curve---------#

# Initialisation
curve_EURIBOR <- curvezc(method = "continuous",
        rates = c(-0.0046073, -0.0050291, -0.0050294, -0.0048534, -0.0045336,
             -0.0041393, -0.0036629, -0.0031261, -0.0025557, -0.0020036,
             -0.0014646, -0.0009584, -0.0004836, -0.0000445, 0.000346,
             0.0006756, 0.0009484, 0.0011768, 0.0013729, 0.0015489, 0.0017002,
             0.0018187, 0.0019112, 0.0019847, 0.0020458, 0.0020913, 0.0021165,
             0.0021265, 0.0021266, 0.0021217, 0.0021093, 0.0020861, 0.0020558,
             0.0020223, 0.0019894, 0.0019574, 0.0019238, 0.0018881, 0.0018497,
             0.0018082, 0.0017611, 0.0017089, 0.0016550, 0.0016030, 0.0015563,
             0.0015149, 0.0014766, 0.0014413, 0.0014089, 0.0013796
        )
)
# Visualisation
curve_EURIBOR
print(curve_EURIBOR)
plot(curve_EURIBOR)

#Swaptions 22.08.2019
maturities = c(2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10,
       2,3,4,5,7,10,2,3,4,5,7,10,
```

```
        2,3,4,5,7,10,2,3,4,5,7,10,
        2,3,4,5,7,10,2,3,4,5,7,10)
tenors = c(1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,3,
      4,4,4,4,4,4,5,5,5,5,5,5,6,6,6,6,6,6,
      7,7,7,7,7,7,8,8,8,8,8,8,9,9,9,9,9,9,
      10,10,10,10,10,10)
vols_EURIBOR = c(0.002742, 0.003422, 0.003945, 0.004405, 0.004950,
     0.005184, 0.003048, 0.003587, 0.004055, 0.004473,
     0.004951, 0.005217, 0.003413, 0.003854, 0.004261,
     0.004598, 0.004998, 0.005249, 0.003720, 0.004100,
     0.004468, 0.004703, 0.005068, 0.005291, 0.004012,
     0.004357, 0.004765, 0.004797, 0.005160, 0.005337,
     0.004278, 0.0046, 0.004788, 0.004931, 0.005231,
     0.005385, 0.004539, 0.004859, 0.004746, 0.005037,
     0.005277, 0.005410,0.004767, 0.005019, 0.004936,
     0.005126, 0.005328, 0.005427, 0.005026, 0.004753,
     0.005057, 0.005198, 0.005366, 0.005441, 0.005124,
     0.004974, 0.005144, 0.005247, 0.005389, 0.005436)
swaptions_EURIBOR <-
swaptions("normal", curve_EURIBOR, maturities, tenors, vols_EURIBOR, 2)

#Rate model
g2model_EURIBOR <- g2(curve_EURIBOR, horizon = 50, nsimul = 100000)

#Calibration of the model on swaptions prices
g2model_EURIBOR <- calibrate(g2model_EURIBOR, swaptions_EURIBOR, maxIter = 200,
input_param <- data.frame(Parameter = c("a", "b", "sigma", "eta", "rho"),
Initial.point = c(0.1, 0.25, 0.005, 0.025, -0.4),
Min = c(0.00000001, 0.00000001, 0.00000001, 0.00000001, -1),
Max = c(0.5,0.6,0.1,0.15,1)))
```

```
#show the calibrated parameters for G2++ EURIBOR Curve
g2model_EURIBOR

#-------------Calculate the covariance matrix--------------
a_EURIBOR <- g2model_EURIBOR@a
b_EURIBOR <- g2model_EURIBOR@b
sigma_EURIBOR <- g2model_EURIBOR@sigma*2
eta_EURIBOR <- g2model_EURIBOR@eta*2
rho_EURIBOR <- g2model_EURIBOR@rho

a_EONIA <- g2model_EONIA@a
b_EONIA <- g2model_EONIA@b
sigma_EONIA <- g2model_EONIA@sigma*2
eta_EONIA <- g2model_EONIA@eta*2
rho_EONIA <- g2model_EONIA@rho

#input correlations between the curves
correlation_2017 <- 0.163428
correlation_1999 <- 0.3130
correlation_shock <- -0.15
#calculate gamma
gamma <- correlation_shock*((sqrt(sigma_EONIA^2+eta_EONIA^2
+2*sigma_EONIA*eta_EONIA*rho_EONIA))*
(sqrt(sigma_EURIBOR^2+eta_EURIBOR^2+2*sigma_EURIBOR
*eta_EURIBOR*rho_EURIBOR)))/
((sigma_EONIA+eta_EONIA)*(sigma_EURIBOR+eta_EURIBOR))
#print gamma
gamma
```

```
# Generate correlated distributions with correlation from calibrations
correl <- cbind(c(1,rho_EONIA, gamma, gamma),
           c(rho_EONIA, 1, gamma, gamma),
           c(gamma, gamma, 1, rho_EURIBOR),
           c(gamma, gamma, rho_EURIBOR,1))
set.seed(10)
W <- genW(correl, g2model_EONIA@nsimul, 50)

g2model_EURIBOR <- g2(curve_EURIBOR, a=a_EURIBOR, b=b_EURIBOR,
sigma = sigma_EURIBOR, eta=eta_EURIBOR,
rho = rho_EURIBOR, horizon = 50, nsimul = 100000)

g2model_EONIA <- g2(curve_EONIA, a=a_EONIA,
b=b_EONIA, sigma = sigma_EONIA, eta=eta_EONIA,
rho = rho_EONIA, horizon = 50, nsimul = 100000)

#Projection of the EONIA
g2model_EONIA <- project(g2model_EONIA,
Wx = W[,,1], Wy = W[,,2])

# Visualisation
g2model_EONIA
plot(g2model_EONIA)

# Deflator test and visualisation
test_deflator(g2model_EONIA)

# Get deflator table - zero coupon bond - and print for excel for swaption pricing
def_EONIA <- deflator(g2model_EONIA)
y <- c(1:50)
```

```
def_EONIA_r <- t(-log(t(def_EONIA))/y)
write_xlsx(as.data.frame(def_EONIA_r),
path = "C:\\Users\\cramstad\\Documents\\EONIA-R-Result.xlsx")
def_EONIA <- colMeans(def_EONIA)
print(def_EONIA)
plot(def_EONIA)

#Projection of the EURIBOR
g2model_EURIBOR <- project(g2model_EURIBOR,
Wx = W[,,3], Wy = W[,,4])

# Visualisation
g2model_EURIBOR
plot(g2model_EURIBOR)

# Deflator test and visualisation
test_deflator(g2model_EURIBOR)

# Get deflator table - zero coupon bond - and print for excel for swaption pricing.
def_EURIBOR <- deflator(g2model_EURIBOR)
def_EURIBOR_r <- t(-log(t(def_EURIBOR))/y)
write_xlsx(as.data.frame(def_EURIBOR_r),
path = "C:\\Users\\cramstad\\Documents\\EURIBOR-R-Result.xlsx")
def_EURIBOR <- colMeans(def_EURIBOR)
print(def_EURIBOR)

#Visualisation of ZCB-prices
par(mfrow= c(1,2))

plot(def_EONIA, type ="l", col ="blue",
```

```
main = "EONIA", ylab ="ZCB Prices")
points(curve_EONIA@zcp, type ="p", col = "red")

plot(def_EURIBOR, type ="l", col ="blue",
main ="EURIBOR 6M", ylab ="ZCB Prices")
points(curve_EURIBOR@zcp, type ="p", col = "red")
```