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Private equity performance in financial crises - A theoretical perspective

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Abstract

The global industry of private equity has never had more capital to spend, and deals are at historically high valuations. Further, as COVID-19 is wrecking economic activities across the globe, it raises strong concerns on how private equity funds are expected to do in the event of a crisis.

Research into the intertwinement of macro conditions in financial crisis events and private equity performance is predominantly limited to empirical analyses of the 07-09 crisis. This paper provides answers to how a financial crisis might affect private equity performance and examines how these results depend on the timing and severity of the crisis.

This paper builds on existing literature, establishing a theoretical framework enabling asset-based simulation for PE stakeholders returns under various exogenously defined parameters. Introducing a deterministic crisis event allows for comparison between steady-state and crisis performance, providing insights on the impact of crisis statics.

The paper finds enhanced performance for private equity when investing during a crisis with a significant recovery, regardless of fund characteristics. In the absence of a recovery period, fund characteristics, such as manager skill and leverage, become highly consequential for performance. These results provide basic insights which open up potential further investigations and considerations regarding private equity as an investment vehicle.

Contents

1 Introduction

Over the last decade, the private equity industry has gained momentum with historically high levels of capital going into the funds. Since 2016, globally more than 550 billion US dollar poured into funds every year. This is 50% higher than in the last boom of the industry, before the financial crisis and subsequent recession (Bain & Company, 2019). Dry powder, i.e. the committed capital available for investing, is piling up as ever before. In addition, the multiples that funds are paying for acquiring companies are increasing a historically high level. Amid the financial crisis, U.S. LBOs had median entry multiples[1], $\frac{EV}{EBITDA}$, of 7.1x, which today has risen to 11.5x, much above the pre-crisis level of 9-10x (Bain & Company, 2019; McKinsey & Company, 2020).

At the time of writing this thesis, the global economy has come to a halt due to the effects of COVID-19, a global pandemic causing lock-downs of economies and breaking up global supply chains. The International Monetary Fund projects a global drop in output of $\sim$ 3% in 2020, more than 6 percentage points lower than initial estimates. Compared to the global loss of 0.1% in the financial crisis year of 2009, the recession might be the worst since the Great Depression in the 1930s. However, the International Monetary Fund also projects a rebound in 2021 with global GDP growth of more than 6%, something equally unprecedented.

A historically large private equity sector in addition to an imminent crisis raises questions of how the industry is expected to fare during such events. This paper aims to shed light on the potential impact on PE fund performance in times of crisis and uncertainty. Thus the paper is contributing with theoretical insights and provide a basis for creating hypotheses for further investigations.

1.1 Background of the research into private equity

Firstly, we want to provide an introduction to the research of PE thereby providing a framework for our research. Since the first private equity (PE) boom in the early 1980s, the topic of private equity has interested researchers of finance. In the initial period of PE, this new investment vehicle provided its investors with substantial returns (Colla et al., 2012), which fascinated the scientific community, who immediately sought to find academic explanations behind PE over-

[1] The value multiple is calculated as the ratio of enterprise value to the underlying EBITDA

performance. Researchers such as Michael C. Jensen, Steven Kaplan, Paul A. Gompers and Josh Lerner wrote some of the first seminal papers, explaining the forces driving industry performance. These initial papers attributed early success of private equity to it having superior organisational forms, with strong governance of portfolio companies. In conjunction with the application of alternative, and many ways more optimal, capital structures reaping significant benefits of leverage, was found to be core in creating excess risk-adjusted returns. The overarching theme for all these papers was that they viewed private equity through business and/or traditional corporate finance lenses, inclining the papers to analyse structural effects onto cash flows or how incentive structures in contracts might affect payoffs to different stakeholders.

In recent times, nonetheless, the returns of the industry have converged towards those of the public market. This has led to a change in the research of PE, focusing more on market conditions rather than just the structure of the PE fund itself. From the late 2000s, post-financial crisis and onward, significant contributions within the literature have been made by Morten Sørensen, Ulf Axelson, Per Strömberg, Douglas Cumming and many others. Their research often take a more unfavourable view, with some explaining early abnormal performance by lack of competition for deals within the industry, or that debt holders might provide leverage priced too cheaply. Other papers focus on developing more appropriate comparisons between private equity and public equity, considering the discontinuous risk profile of PE as well as unaccounted for illiquidity cost. These elements were not discussed in the earlier literature, and authors argue that this could cause overestimation of equity performance. Many papers find that market inefficiencies has been deteriorating over time, which would explain the abnormal performance argued by early research. Additionally, modern research on private equity diverts from early contributions by analysing private equity through an increasingly market-based asset pricing methodology. This has led to an increased quantification in the analysis of private equity, with significant amounts of papers relying on option-pricing frameworks and others on simulations to conduct their research.

1.2 Scope of the thesis and its relation to the literature

In this paper, we want to further the insights on how market conditions affect the risk and return of PE funds. More distinctively, we want to investigate how the industry might progress when the broader macro conditions of investing fluctuate out of their normal conditions, i.e. how PE performance is affected in the event of a financial crisis.

Analysing the impact of a financial crisis event from a theoretical point of view taps into a slight gap in the existing literature. Existing literature regarding private equity in a crisis event is overwhelmingly empirical and almost solely focused on the financial crisis of 2007-2009. This widely taken empirical focus means, that the existing literature suffers from data availability problems prevalent in most private equity research, as a consequence of acquisition and exit prices seldomly being public. The singular view arising from investigating a specific crisis ignores the multiple ways a crisis can manifest itself, and thus the plethora of ways it could affect private equity performance. Expanding the notion of a crisis event thereby broadens the understanding of what affects risk and return for investors and managers.

Our paper goes beyond the specifics of the last great crisis, utilising a generalised theoretical framework of PE while providing a straightforward way of implementing a financial crisis event. The paper thus aims to answer the following research question,

> ***How may a financial crisis affect private equity performance? And in which ways do these results depend on timing and severity of the crisis itself in addition to PE fund characteristics?***

Further, to provide answers to the above-stated question, we will investigate and provide insights on the following sub-questions,

1. What is driving performance in the PE funds, and through which mechanisms are proceeds from investment activities allocated to the different stakeholders?
2. Which types of crises could affect the industry? And how may these affect returns and risk?
   (a) To what degree could timing, duration, and severity alter the results?
   (b) Which insights can we infer concerning skills of the general manager as well as other fund specifics?
3. How do our main predictions of performance compare to those of the public market in both normal times as well as under a crisis?

Our main finding is that the private equity, in general, captures substantial benefits from investing in recovery periods. These benefits are both relatively and absolutely exceeding returns provided by an equivalent public market investment strategy. These effects are largely invariant of the skills of the general partners, and rely much more on the structure of PE funds and their investment strategies, and thus PE as an investment vehicle appears superior to the public market in this specific type of crisis. However, in a post-investment period crisis or in the absence of a recovery period, the general performance of the industry becomes significantly less attractive, and only high skilled fund managers are in expectation able to provide excess returns to the limited partners.

Due to the lock-down measures related to COVID-19 and consequential highly limited access to databases, the results of this paper will primarily be based in and compared to existing empirical work. These comparisons find an alignment of model predictions and real-world observations. Therefore and nonetheless, a focused empirical investigation of key predictions would be of great value in validating our insights, especially when the COVID-19 recession has manifested itself.

1.3 Thesis contribution

This paper contributes to the existing literature by expanding well-established models such as Sørensen et al. (2014) and Choi et al. (2011) with a crisis incident. Additionally, by applying simulations rather than analytical solutions, our paper provides insights into the distribution of returns. As the allocation mechanisms of PE fund proceeds are discontinuous and state-dependent, distributions of returns provide key insights, even in an environment where asset prices are driven by a continuous Gaussian process. Also, using simulations enables us to conduct inference of risk and return without making assumptions about utility functions.

Furthermore, through scenario analysis, the paper spans into a discussion of the impact from the various fund parameters and thus provide understandings on which elements that are driving results of the industry for limited and general partners respectively.

The paper can be utilised, together with many other elements of the finance literature, by LPs in consideration of portfolio allocation and their hedging against crises events. However, the results need further investigations with regard to elements such a more complex capital structures and the ability of managers to time the market, which goes beyond the scope of this paper. Thus the results cannot stand on its own but serve as a starting point for further research.

1.4 Progression of the thesis

The structure of this paper, and thus how we answer our research question, is as follows. In the first section, we define and outline key elements of private equity to provide an overview and reference of key elements such as fund structures, business models as well as a definition of the industry. We then go into a review of the literature on asset pricing, theoretical models of private equity, the allocation mechanism and how the performance of PE has developed.

The following section is the description and development of our theoretical model of PE fund returns and investment methods. This section is followed by a section on how we conduct our simulation under a different set of parameter assumptions. These two sections are then followed by a discussion of the overall framework provided by this paper.

Next, the results of our base case simulation without a crisis are reviewed. In this section, we also conduct scenario analyses of the different parameters, subsequently discussing the implication of our results.

The following part of the paper is a review of the concept of financial crises, expanding our model and simulation method, as well as discussing the implementation. We continue with the last and most important section of the paper. In this section, we will analyse the results for funds affected by various crises. The final step is then to discuss results and compare them to an equivalent public market investment strategy.

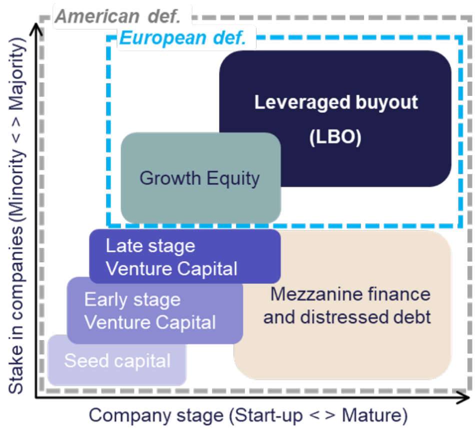
2 Introducing the basics of the private equity industry

This section provides a general introduction to the private equity industry as well as providing the definitions that the paper will apply in answering the research question. It will also introduce the various stakeholders in the funds and the general business model applied. The section will be followed by a literature review of research that theoretically and/or empirically has investigated the deeper specifications of the industry.

2.1 Definition of private equity

Private equity as an asset class can be defined in various ways. Caselli (2010) divides definitions of the industry into two schools; an American and a European one, illustrated by figure 1. Assets under management in the different types of funds are shown in figure 2. This thesis will apply the narrower European definition, but both definitions are described in the following subsection.

Figure 1: Definitions of private equity based on types



Source: Own illustration

The American definition, being very broad, is that the private equity industry consists of non-public markets where ownership through preferred and non-preferred equity is traded. Generally speaking, the private equity industry is a segmented one and sub-segmented based on the size of the firms invested in, ownership taken, preference of the invested equity, and the maturity of the company. According to Caselli (2010), the common element in this definition of investments are, that all investment create a stricter relationship between the investors and corporate management compared to public bond and equity markets. This allows for more activist ownership of companies. In the American definition, private equity funds invest in all from early-stage markets such as seed capital, early and late-stage venture capital (VC), in additional to the more broadly dispersed growth equity (GRE) as well as late-stage investments through mezzanine finance, distressed debt or leveraged buyouts (LBO). However, in this definition, there are still large differences between the overall methodology of investing and the investment hypotheses used by funds. For example, in seed and VC, the fund usually takes on minority stakes in the companies - most often in preferred equity in which there's a degree of downside protection and strong decision-mechanisms. The investments are often staged, and the fund will refinance and add additional capital in investment rounds as the companies mature. This differs from investing in mature companies, where a larger degree of ownership is acquired, and where leverage is mostly used for financing on-going capital needs (Caselli, 2010). The staged-investing strategy and significantly larger company-specific riskiness of investments in VC funds, means returns are largely based on the success of few investments rather than the portfolio as a whole. Therefore early-stage investors are affected predominantly on their fundraising rather than the investment performance in a broad financial crisis event (Block et al., 2010).

This is different than the more mature part of the PE industry. As stated by Cumming (2010), the cyclicality of the mature segment tend to be very affected by the conditions in the general economy and those in publicly traded assets, thus also more interesting for investigation of our research question. As a consequence, we will limit our definition, ensuring a more natural narrative, to the European one. In the European definition, we have that,

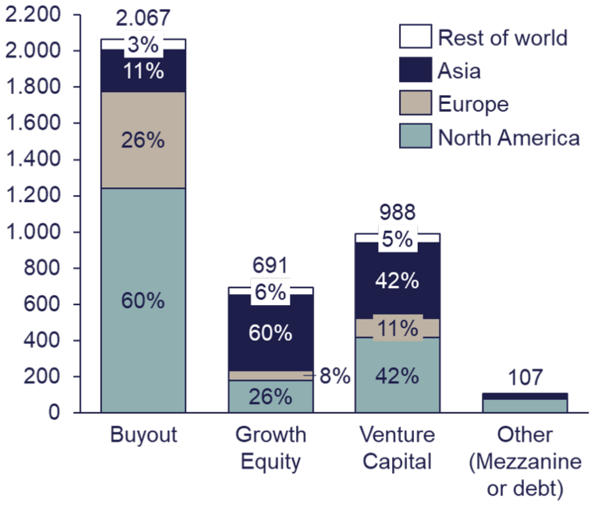
> **private equity is limited to refer to non-public investments that usually takes on majority stakes in mature and late-stage companies**, often using substantial leverage in the acquisition. These types of funds are growth equity and levered buyout funds, as well as rescue and replacement capital. (Caselli, 2010)

This narrower definition allows the modelling to take a more granular approach, providing a more transparent and clear-cut model of the investment returns.

2.1.1 Growth and size of the PE industry

Private equity deals have been rising in recent years, increasing 10% in 2018 to reach 582 billion US dollars. This was driven predominantly by larger deals in terms of company revenues as there

Figure 2: Private market assets under management, $ bn



Source: McKinsey & Company (2020)

also was a 13% fall in the number of transactions. The industry saw increasing competition leading to higher valuation multiples and lastly, that firms are investing in more risky asset classes (Bain & Company, 2019). The majority of the global deal value is, as seen in figure 2, concentrated in North America and Europe, and more than $\frac{2}{3}$ of the assets under management being in some kind of leveraged buyout fund. However, Asia is growing a lot in terms of deal value, primarily driven by growth equity investment within the sectors of internet and technology deals in Greater China. A general observation is that the funds as a group are becoming less stringent on whether to invest in companies with high expected growth rates (GRE) or stable cash flows to service debt (LBO).

2.2 Fund structure and Stakeholders in private equity

Private equity firms are almost exclusively organised as a partnership or a limited liability corporation (S. Kaplan & Strömberg, 2009) managed by general partners (GPs). Limited partners (LPs) are equity investors, not investing directly in the PE firm, but in funds managed by the GPs through the PE firm.

In the early days of the industry, PE firms were described by Jensen (1989) as lean and decentralised organisations. Firms employed relatively few investment professionals, most of whom had a background within investment banking and complex financial institutions. These firms had a lower degree of interaction with their LPs and fundraising was predominantly based on the performance of former funds and not so much investment hypotheses or strategies (S. Kaplan & Strömberg, 2009).

As the industry matured so did the firms, developing an increased focus on operations. The modern firms are significantly larger, although still relatively small compared to the companies they invest in. The backgrounds of employees have also become more varied, consisting of an increasing number of investment professionals with backgrounds within management consulting, or investment professionals with industry-specific knowledge (S. Kaplan & Strömberg, 2009). This increased diversity in the backgrounds has largely been driven by the focus on operational engineering by top PE firms.

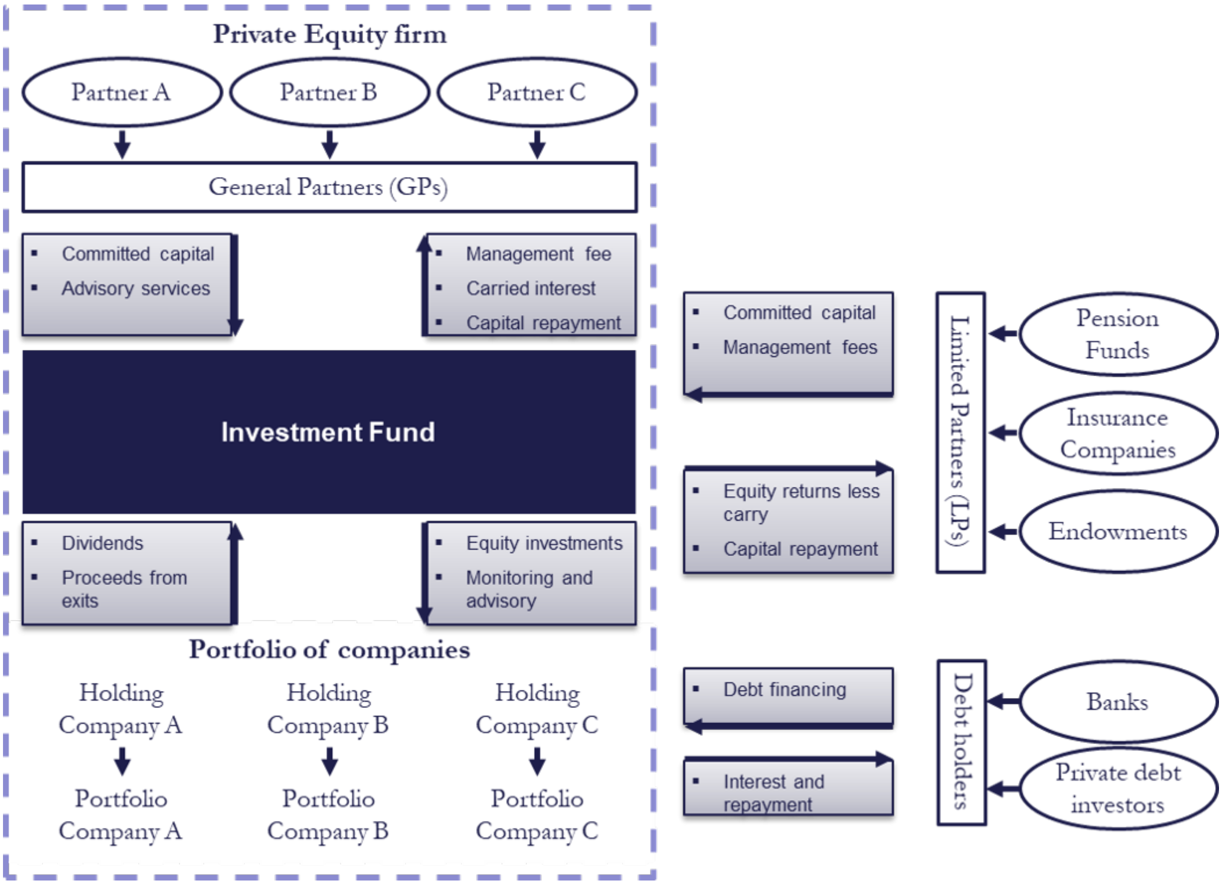
For the PE firm to conduct investments, it needs external equity capital from limited partners, which is raised through the funds of the firm. Usually, large PE firms have several funds and thus the LPs might be different between them. Due to illiquidity of private equity placements, most of these funds are "closed-end", meaning that investors cannot withdraw their money until investments of the fund are terminated (S. Kaplan & Strömberg, 2009). This is in sharp contrast to the mutual fund setup often used when investing in public equities. The common structure of a PE firm is visualised below in figure 3

The general partners manage the fund and also commit a small part of the capital (usually 1% of the total capital), however, the majority of capital consists of commitments from limited partners (LPs), who act as passive investors[2]. The typical LPs are private and public pension funds, endowments, insurance companies and wealthy individuals who seek excess risk-adjusted returns as well as diversification to public equity investments and bonds (S. Kaplan & Strömberg, 2009). The limited partnership is structured around a set of covenants that define the "rules of behaviour", that govern the PE funds (P. A. Gompers & Lerner, 1999; P. Gompers & Lerner, 1996; Sahlman, 1990; Schmidt & Wahrenburg, 2003). The covenants usually cover the following:

1. The PE firm has to adhere to its investment mandate. This sets the boundaries, including the stage of company development of target investments, industry focus and so on.

[2] On occasion some of the larger LPs will co-invest a minority stake in the target alongside the PE fund, thus ending up with two ownership stakes, a direct stake through the co-investment, and indirect investment through the capital committed to the PE fund

Figure 3: PE fund structure



Source: Own illustration

2. LPs put restrictions on the length of the holding period, the ticket size (maximum equity investment), and the maximum exposure allowed to a single region.

3. Financial restrictions governing the amount of leverage allowed, cross-fund investing (between funds managed by the same GPs), reinvestment of capital after realisation of investments etc.

4. The rights and protections of LPs, such as the right to remove the fund manager, accounting and reporting standards, valuation requirements and investor representation (whether investors should have a place on the advisory committee).

5. The fee structure paid to the PE firm. Often this consists of a management fee expressed as a percentage of the committed capital (usually 1% to 3%), and a performance fee (usually 20% to 30% of the profits after return on capital) which often only applies to a fund that has provided a minimum return on assets (8% to 10% annually) (Litvak, 2009; Metrick & Yasuda, 2010)[3].

Covenants and the limited partnership model have proven to be an economically sustainable structure for PE investments. Some papers as Jensen (1989) argue that it drives returns trough mitigation of the principal-agent issues inherent in the PE investment model (Cumming, 2010). The limited partnerships can have a finite horizon, typically stretching over 10 to 12 years[4], and provides GPs with a flexible legal structure that allows them to identify, acquire and exit their investments (typically with a 2 to 5 year holding period). PE firms can signal stability to potential investors by showing consistency within their investment mandate (P. Gompers & Lerner, 1996). The covenants can also be used to price discriminate by varying the mixture between the management fee and the performance fee (P. Gompers & Lerner, 1996; Litvak, 2009). Performance fees are today paid at the portfolio level, which imposes "whole-of-portfolio" thinking. This forces GPs to evaluate the effort of managing and adding value to a single investment against the lower returns, this could lead to for other investments (Kanniainen & Keuschnigg, 2003; Keuschnigg, 2004). By measuring performance on the fund as a whole reduces moral hazard between the GPs and LPs. Especially the degree of leverage depends heavily on whether the payoff is case-by-case or portfolio based (Axelson et al., 2009).

[3]This is a very important part of the model that we develop and estimate later in the paper, thus we will dig further into this in the literature review

[4]Alternatively, the fund can be a so-called ever-green fund with continuing investments and divestment

2.3 The business model of the private equity firm

The business model of PE funds and firms is, very simplistically stated, to create excess risk-adjusted returns through the use of activist ownership of portfolio companies in addition to applying capital structures that allow for levering the benefits of the activist investment management. This model is exemplified in the mission statement of Bain Capital, the 6th largest private equity firm by assets under management,

> *Our mission at Bain Capital is to produce superior investment returns for our investors. To accomplish this, Bain Capital follows three fundamental principles: high-performance, value-added approach to investing, and leveraging our institutional advantages.*

This section of the paper will outline these components of the business model and provide evidence of to which degree they are applied within the industry.

Funds, of course, differ in various ways and their specific investment strategies affect their decision on how to conduct their role as investors. However, they all apply a buy-to-sell approach; buying the company, steering it through some kind of transition, and then exiting the investment at a hopefully higher valuation of the invested equity. Generally, there are three main avenues wherein private equity funds create value in portfolio companies, although these avenues can be used differently depending on the fund type. These three avenues are financial, operating and governance engineering (S. Kaplan & Strömberg, 2009). These forms of business engineering are the defining feature of how private and public equity are different from each other. The private equity industry is thus much more activist in their investments, which allows general partners to earn fees and performance payments on investing their limited partners' capital. Jensen (1989), Barber (2008), S. Kaplan and Strömberg (2009), Block et al. (2019) among others find that the core of private equity's success as an investment vehicle can be found in this combination of developing the businesses through activist engineering, as well as working with complex investment management. This also means that a successful PE fund has to excel on these two important parameters,

1. Investment and portfolio management (see Block et al., 2019)
    - Search-and-selection based on the type of fund, geography and industry specialisation.

- Market timing with focus on investing in assets with assumed temporarily low valuations.
- Target valuation based on various due diligence processes and pricing of risk.
- Assess and create optimal exit opportunities and negotiations with sellers and buyers.

2. Activist ownership and value-added activities (see Jensen, 1989; S. Kaplan & Strömberg, 2009)
   - **Financial engineering** where the PE fund uses excessive amounts of leverage to discipline management and tax deductibles on interest payments.
   - **Governance engineering** gaining access to the board and being able to monitor, guide and control management of the company more thoroughly.
   - **Operational engineering** focusing on improvement of the operational core of the business. This could be elements such as cost-cutting or investing in a technology that increases revenue.

While the investment and portfolio management certainly is an important feature, it is also a feature that the industry to a large degree share with its public comparable activist investors such as hedge funds. S. Kaplan and Strömberg, 2009 find that over time, the focus on the activities within the portfolio investments themselves, is key in being able to capture a positive α for the PE funds, i.e. risk-adjusted above-market returns, thus legitimising receiving substantial fees from its LPs.

Financial engineering is the act of improving the value of the portfolio companies by adjusting the capital structure of the underlying investment. In the early days of modern private equity, financial engineering was the essential element of the business model (Cendrowski, 2012). This was driven by the decline of real interest rates which lead to significant degrees of leverage and the leveraged buyout fund. The improvements in the financial structure are conducted by increasing leverage in conjunction with introducing additional tranches in the capital structure of the portfolio company (preferred equity and junior debt). Thereby gaining value from benefits of debt, such as increased tax shields or reduced agency cost through the pressure from constantly meeting debt payments. Within the literature of private equity, these benefits are largely discussed. We will elaborate on this discussion in the literature review.

Operational engineering entails the private equity firm forcing certain changes in the operations of the company, whether it be cost-cutting opportunities and productivity improvements, strategic and commercial actions or synergy growth through M&A opportunities.

Lastly, private equity portfolio companies often have strong corporate governance structures that allow for value-add (S. Kaplan & Strömberg, 2009). This is often intertwined with the efforts of operational engineering. The two tend to follow each other as stronger governance offers more effective operational changes.

The two main sectors of the private equity industry, GRE funds and LBOs are characterised by different usages of the three value-creating avenues. GRE funds are not able to use the full plethora of value-creating avenues as high leverage often is not attractive due to negative or small cash flows. Instead, GRE funds create value by giving access to equity capital that can help grow the business, by using their network to help portfolio companies, and by guiding portfolio companies on how to establish commercial and operational practices. Buyout funds can use all of the value-creating avenues, as they can target firms with a significantly larger array of challenges. The maturity of the target companies of buyout funds also entail that the owners of the company are not necessarily the founder, and management can thus be incentivised to improve their performance thereby increasing the value of the company. Due to risk adverseness or lack of financial acumen, these companies might also have a sub-optimal capital structure that could be improved by taking on more debt.

3 Literature review of the private equity funds, investment management, risk, and return

In this section, we will review the relevant literature on private equity necessary to answer the research question of the paper. The review is based on a significant number of theoretical and empirical papers, predominantly from contemporary research. Also, the review will describe and discuss models and results from the vast research on private equity and as such, be the backbone on which a theoretical model of PE returns can be developed. Our approach and structure in this part of the paper follow the building of our theoretical model. Firstly, we will outline the general characteristics of the industry and discuss underlying elements such as risk and returns within the sector. This discussion leads us forth to an overview of the stakeholders in the PE firms and funds, which will span into a description and discussion of contractual relations and how the proceeds of the funds are allocated.

3.1 Asset pricing in private equity

As the purpose of this paper is to investigate how a crisis element would affect the return of private equity as an investment class, it is essential to get an understanding of how assets are priced. Furthermore, we will investigate which approaches can be used to model the value given a set of circumstances. This subsection of the review will outline the basics of asset pricing, and more specifically, how relevant papers have used these insights to model the development of private equity assets.

3.1.1 Basic theory of asset pricing

From a theoretical point of view, asset prices in rational markets with no market frictions must equal the discounted and risk-adjusted expected dividends paid to equity holders as well as the interest payments to the debt holders. This means that the fundamental value is derived from the underlying company activities and market conditions, however, companies might gain a benefit on the capital structure of the company itself. The value of equity is correspondingly expected value of dividends and for debt holders, expected interests paid, in addition to the expected return of capital for both.

Dividends, amortisations and interest can only be paid from the free cash flows generated by the business activities of the company. Thus it is theoretically equivalent to derive the fundamental value of a company from its expected future cash-flows discounted back to their present value (Birch Sorensen & Whitta-Jacobsen, 2010),

$$S_t = \mathbb{E}^t \left[\frac{CF_t}{(1 + r_t + \varepsilon_t)} + \frac{CF_{t+1}}{(1 + r_{t+1} + \varepsilon_{t+1})^2} + \frac{CF_{t+2}}{(1 + r_{t+2} + \varepsilon_{t+2})^3} + \dots \right] \tag{1}$$

where CF_t is expected free cash-flows, r_t is risk-free interest rate, and ε_t is an asset-specific risk premium. It is self-evident from a theoretical point of view, that the value of an asset only can be altered if cash flows, risk-free interest rate or riskiness of the asset itself changes. This perspective is based in the "Efficient Market Hypothesis (EMH)", Eugene Fama developed, stating that,

> *Current asset value reflect all available information about the past, present and future*[5]*, and that past price has no relationship with the future*

The EMH is essential in understanding how fundamental asset value paths are generated. If the price development of an asset is independent of its past price performance, then all innovations to the process itself must be idiosyncratic and thus random. In this perspective, a model of the asset path must involve innovations created from a strictly stochastic process with no relation to its former value.

Other parts of the literature within asset pricing apply a behavioural element to the theory and several empirical papers have debunked the efficiency of the aggregate market. Robert Shiller and Richard Thaler have been thought-leaders within this position, and in his paper from 2003, "From Efficient Markets Theory to Behavioral Finance", Shiller provided meta-analysis on how the EMH failed to explain asset pricing and hold its validity, predominantly under financial bubbles and crises. Shiller shows that estimated equity values[6] varies over time and that this development of aggregate asset prices does not appear to be completely random. This could constitute a problem for asset pricing models of the aggregate market. However even in the behavioural literature, Shiller (2003) finds that,

> *... Even though the aggregate stock market appears to be wildly inefficient, individual asset prices do show correspondence to efficient markets theory.*

[5] To varying degrees from weak with all available information to strong with all information that is publicly and privately held.

[6] Levered market valuation as the total market cap over GDP

Which provides empirical arguments to simulate individual asset paths with a stochastic element without loss of generality. The perspective is useful for our paper as we will model the individual asset paths and not the aggregate market itself. We also acknowledge that a crisis event should, for most market participants, be unforeseen and exogenously defined.

3.1.2 The dynamics of asset pricing

In describing the relatively high complexity of the financial risk and return of private equity, many papers model and simulate the value of PE assets. The value of a company from the PE perspective can be derived from different routes.

Some papers estimate the dividends that can be paid out to equity holders and derive the equity value from that basis. An example of such a paper is Driessen et al. (2012), who simulate a dividend to equity holders which then can be withdrawn, as well as reinvested in projects with a positive IRR. The cash flow is defined by a process that every period grows with a fixed term based on the investment skills of the managers as well as a general drift similar to the public market. In its simplicity, this method allows for understanding the underlying drive of private equity and is then used for investigation of how dividends are reinvested. The model does not, however, bring leverage to the table as a tool of investigation and thus won't capture any effects of debts.

3.1.3 Geometric Brownian Motion

An alternative method is that presented by Choi et al. (2011), Lahmann et al. (2016), Sørensen et al. (2014). Their models are centred around the stochastic process of the Geometric Brownian Motion which in various ways is used to project returns of the underlying assets.

Applying the Efficient Market Hypothesis, Merton (1974) provided a formula for valuing call and put options. He assumed a Geometric Brownian Motion (GBM) to be the process of determining the asset paths. The method, primarily used in physics, biology and chemistry, is widely used due to its easily understandable source of randomness and its ability to implement an average expected growth rate. According to Hull (2009), the GBM-process is the most used model of asset prices, both levered and unlevered, due to it having the following advantages,

1. The GBM model results exclusively in positive values thus this feature of asset pricing is met by the process. As most traded company assets have limited liability, their value can

never become negative.

2. Price developments are stochastic, thus aligning the model with the Efficient Market Hypothesis.

3. The GBM model is one of the few stochastic differential processes that rely on a relatively simple structure and has a closed-form solution. This makes computations relatively less heavy.

However, the model has some down-sides which need to be considered,

- The model only allows for the parameters of the model, being volatility and drift term, to be exogenously defined.
- The process is continuous, thus it does not account for sudden jumps in pricing that we see on day-to-day trading of stocks.

The use of the model is simplest in the case of Sørensen et al. (2014) which use it to model one asset per fund in one period. However, by leveraging the advantage that the simple GBM has an analytical solution, they avoid a numerical solution, thus not need simulating the results of the model. They model the unlevered asset paths as well as the value of debt and its credit spread using observed parameters. The results are then processed through a set of utility functions, allowing them to develop insights on how contracts and fund characteristics affect the utility of the different stakeholders. The cost of the analytical solution is, that there are no possibilities of differences in exit timings and holding periods, no information on distributions of returns for LPs and GPs, and no imminent option of implementing a crisis event.

Alternatively, going the way of Choi et al. (2011) and Lahmann et al. (2016) would provide more complexity and increased options for testing implications. Both papers allow for different holding periods, however, they differ in what they model. Lahmann et al. (2016) use the GBM process to model and simulate the EBIT of portfolio companies. As their paper investigate bankruptcies as a result of the debt structure, the EBIT-based model can provide insights into how the capital structure might be altered, given a set of covenants provided by the banks. The model is influential in the literature as they model EBIT, which makes the model robust to capital structure. It then becomes easier to implement costs and benefits of debt in the setting. However, calculus becomes far more complex, and minor variations of the parameters have large effects on the outcome of the model, making results highly dependent on the definitions within the underlying mathematics. Additionally, parameters of the model are not based on observable numbers in the market and thus it is more difficult to evaluate the results in a given setting. This means that results become much more dependent on definitions which only can be made with lower degrees of inference.

Lastly, Choi et al. (2011) model the PE asset as a GBM process over multiple periods in its unlevered state and can thus be seen as a generalisation of Sørensen et al. (2014). The weakness of this model is that it can only be estimated numerically and while parameters are easier to estimate, the model does not account for the impact of leverage in the degree that we can get from Lahmann et al. (2016). Also, their approach to inferring insights is option-based and as such, they do not provide insights into the distribution of returns.

3.2 Capital structure and impact on asset pricing in the private equity industry

The next step for this paper is to investigate and discuss how to model individual asset paths. Many financial papers in the PE literature focus on unlevered assets and assume that capital structure won't affect the value of the assets of companies. This perspective relies on the simplest assumption of valuation and asset pricing stated in the influential paper of Modigliani and Miller (1958), who stated $V_L = V_U$. Examples of papers within the PE research taking this perspective are Sørensen et al. (2014) and Choi et al. (2011). The two models of asset pricing disregard the impact of leverage on the valuation of portfolio companies.

3.2.1 Leverage in PE

Especially leveraged buyout deals are broadly discussed in the literature regarding debt. A general observation is that LBO funds have better access to acquiring higher leverage, often with lower yields than other financial asset classes. This can be explained using conventional financial theory within information economics with LBOs providing less informational asymmetry thus less risk for the investors. Other papers focus on behavioural economical perspectives, finding that irrationality among debt holders as well as moral hazard drive the cheap debt. An example of this is Axelson et al. (2010) who find that private equity funds tend to use more leverage than could be justified under risk-adjusted debt pricing, indicating that the LBO funds can access debt at a lower-than-optimal yield and thus through this channel create excess returns to their investors while providing an economic loss for its debt holders. However, these results have been critiqued for tending to rely on financial data from the asset bubbles of the mid-2000s as well as data from the 1990s where the private equity industry was maturing. S. Kaplan and Strömberg (2009) find that debt in private equity seems to react to a risk-adjusted pattern and that debt pricing rely more on the riskiness of the external investment environment rather than mispricing between financial sectors.

However, as mentioned before, the leverage is a large component of the PE business model and while GRE tends to apply relatively low degrees of leverage[7], it is substantial in leveraged buyouts. Within the literature of private equity, these benefits and costs are largely discussed, and on a very basic level, the value of the levered asset V_L can be divided into the value of the unlevered asset V_U plus costs and benefits of debt,

$$\begin{aligned} V_L = V_U &+ \text{PV(Tax shield)} - \text{PV(Cost of financial distress)} \\ &+ (\text{PV(Agency benefits of debt)} - \text{PV(Agency cost of debt)}) \\ &+ \text{PV(Discounted pricing of debt)} \end{aligned}$$

This perspective of debt fall under the trade-off theory of capital structure, stating that firms should choose the level of debt that maximise V_L. In his paper "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure", Leland (1994) provided a functional form of tax benefit and financial distress costs using a Geometric Brownian Motion[8],

$$V_L = V_U + \tau \frac{y^* D_0}{r_f} \left[1 - \left(\frac{V_U}{V_B} \right)^{-\frac{2r_f}{\sigma_A}} \right] - \delta V_B \left(\frac{V_U}{V_B} \right)^{-\frac{2r_f}{\sigma_A}} \tag{2}$$

$$\lim_{V \to V_B} V_L = V_U - \delta V_B \quad \wedge \quad \lim_{V \to \infty} V_L = V_U + \tau \frac{y^* D_0}{r_f} \tag{3}$$

Where τ is the effective tax rate, y^* is the break-even yield of debt, V_B is the value at which the company goes bankrupt, which is often when the cash flows can't service the debt, and δ is the cost of financial distress. The key contribution of Leland was not only to provide a functional form of the impact of capital structure, but also that these values are endogenously given by the asset value.

[7]Technically, most GRE funds invest preferred equity into their portfolio company which has traits of both equity and debt thus a kind of leverage itself

[8]Used and described later in this paper

S. Kaplan (1989b) finds that tax benefits of debts are an important source of wealth gains in management buyouts. Increased leverage, and thereby increased tax shields from interest deductions, can explain from 4% to 40% of the value of equity. Similarly, if leverage becomes too high, financial flexibility decreases and the risk of bankruptcy increases, thus debt become less valuable and even negatively influencing the assets. Given that 20% of all LBO investments go bankrupt[9] (Ayash & Rastad, 2020), and using $\delta \approx 20\%$ (Andrade & Kaplan, 1998), the estimated cost of bankruptcy on asset value is 4%, which is the lower bound of the tax benefits of debt.

S. Kaplan indicated that there was evidence for these funds paying a heavy premium for acquiring companies directly due to the tax shield, thus the value would be included in the acquisition price. This was documented in the paper from 2009, where it is found that between $\sim 20\%$ and $\sim 140\%$ of the premium paid to pre-buyout shareholders is derived from tax benefits. As the private equity markets have matured and become quite competitive concerning deals, the tax shields seem to benefit the former owners in the form of a higher price tag, and they are thus not a key value driver for the PE fund. Knauer et al. (2014) argue through investigation of 56 German LBO funds, that not only does the tax shield benefit the former shareholders, but that negative costs of leverage tend to fall onto the LBOs themselves.

With regard to reduced agency costs or increased agency benefits, Jensen (1986) finds that the LBO provide incentives that limit the agency costs significantly as projects with a lower payoff or higher risk would have to be evaluated more critically, limiting the number of bad projects that the portfolio firms will undertake. The finding is interesting in the sense, that LBO funds tend to have a large degree of independence from the LPs, thus investing without the discipline of debt payments might distort potential returns. However, Axelson et al. (2010) find countervailing evidence that excess leverage could increase agency costs for investors as general partners might choose sub-optimally high leverage to boost successful investments at the cost of sending other investments into bankruptcy. This effect is strongest for funds that allocate carried interest case-by-case rather than the portfolio as a whole. As well as for funds that have distressed or low performing investments, where GPs try to boost their outcomes with leverage to get above the hurdle rate, without being at risk of losing the fixed management fee. All-in-all, Bratton (2008) finds that buyouts are driven by the economics of levering up excess

[9] Compared to 2% of their public equivalents

cash flow growth, with agency cost reduction (and other values of debt) taking only a secondary role in the business today.

Lastly, the value of debt could be affected by discounted pricing. Sørensen et al. (2014) argued that LBOs can benefit from manager skills due to reduced risk of bankruptcy realised by debt holders. While Sørensen et al. don't cite a source of this effect, evidence of such can be found in Ivashina and Kovner (2011), indicating that bank relationships formed through repeated successful interactions, i.e. the banks can realise the skills of managers, reduce inefficiencies from information asymmetry and allow leveraged buyouts sponsored by private equity firms to occur on favourable loan terms. Axelson et al. (2010), however, find that this effect seems to be based on non-performance based metrics such as hot debt markets, thus constituting mispricing of debt and market inefficiency. Lastly, Bratton (2008) finds that banks, in general, become more critical of leverage, driving down the degree of leverage, and increasing mispricing in the opposite direction for good managers, who end up paying a higher yield than what's efficient. This would lead to a negative marginal effect of managers' skills onto leverage, opposite of the argumentation of Sørensen et al.

In general, leverage effects on the value of PE assets is ambiguous and inconclusive. Especially papers from the 1980s and 1990s, based on the early period of the private equity industry, find significant effects of leverage on returns. However, these effects have to some degree deteriorated over the course of the decades with increasing competition in the private equity markets. Also, some of these values seem to be broadly acknowledged by market participants, resulting in higher multiples rather than return. Regardless of how strong the direct effects of leverage are on returns, leverage can still be very valuable as it increases the impact of superior operations. This position is supported by the analysis from Colla et al. (2012) and Axelson et al. (2009), who found that the real benefit of leverage comes from being able to multiply excess risk-adjusted growth in the underlying companies, why fund managers with a higher turnover growth tended to choose a higher-than-average debt to equity, providing their investors with a higher equity return in the end.

3.3 Non-financial engineering and impact on asset growth

S. Kaplan (1989b) finds that for US public-to-private deals, operating income to sales increase (both absolutely and relative to industry), cash flows to sales increase, and capital expenditures to sales decline, equivalent to what we earlier have defined as operational engineering. Additionally, Cumming et al. (2007) wrote a paper, summarising the research related to the performance of companies involved in a leveraged buyout and finds, "*The end result is there is a general consensus that across different methodologies, measures, and time periods, regarding a key stylised fact: LBOs and especially, MBOs enhance performance and have a salient effect on work practices*".

As a testament to how operational engineering is becoming key for the industry, S. Kaplan and Strömberg (2009) observed that private equity firms shifted the backgrounds of their investment professionals. This change became more profound after the initial success of the financial and governance engineering in the late 1990s(Cendrowski, 2012). Furthermore, many private equity firms have gone from a generalist approach to being industry-specific, which means that the operational know-how acquired over time, can be reapplied in new investmentsCaselli, 2010; Cendrowski, 2012. It is this sector expertise that allows private equity firms to identify potential targets with sub-par operations, whereby they can increase value by utilising in-house expertise.

Contrary to the operational side of the business model, governance engineering was key already in the early days of the modern private equity industry (Cendrowski, 2012). Management teams of companies acquired by private equity firms are often given more incentive-based compensation such as options, and in the case of the management buyouts, the fund requires management to provide significant investment in the company. This aligns managements' incentives as they share the ups-and-downs of owning a company. Even though stock- and option-based compensation have become more common in public companies today, the ownership percentage of management is still larger in private equity transactions than in public companies. S. Kaplan (1989a) finds that management teams in companies going from public to private ownership have their ownership stake increased by a factor of four. Furthermore, in private equity transactions, management equity is illiquid, thus management cannot sell its shares or exercise its options until the company is sold or made public. This illiquidity, therefore, removes managements incentive to manipulate short-term performance, which is a common critique of stock- and option-based compensation in public companies.

3.4 Contractual setup between the general partners and limited partners

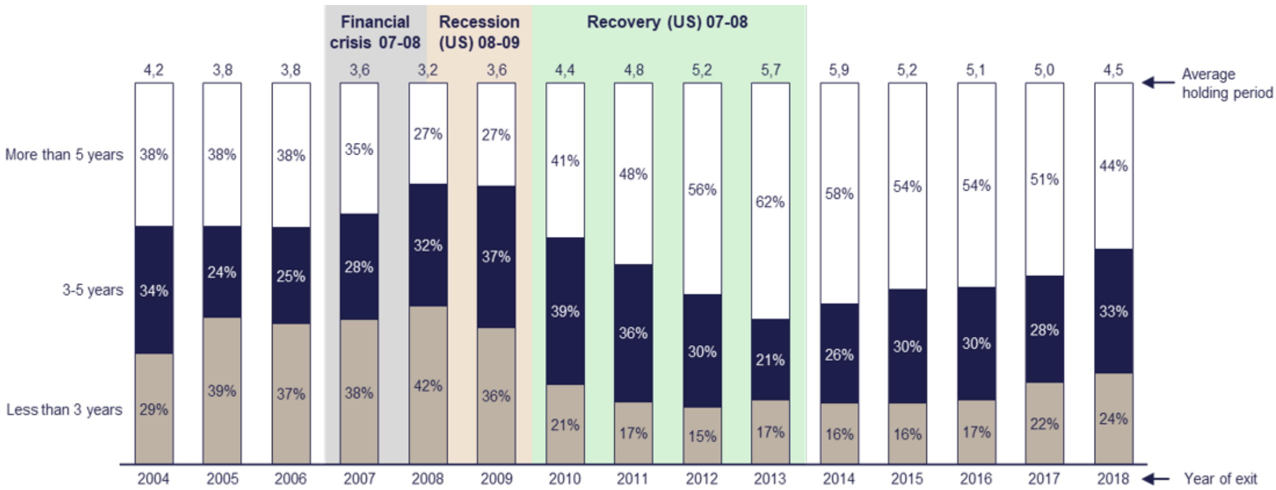
The 2-20 model for private equity is frequently cited as the common structure used by firms, with a 2% management fee based on committed capital and 20% carried interest based on the profits made from exiting investments. However, in practice, the model is much more complex and varied between firms.

Metrick and Yasuda (2010) conducted a large survey of 144 LBO fund contracts, showing that the vast majority of buyout funds make changes to their management fee after the investment period is over, with 84.0% changing the fee basis, 45.1% changing the fee level and 38.9% changing both. Funds targeting less mature investments (such as venture capital funds) are less inclined to make alterations to the management fees, with approximately half of the funds changing either fee basis or level. The two fund types differ in the management fees with buyout funds changing fee basis rather than fee level, while the opposite is true for the other funds. The initial fee level is also not as set in stone as the 2-20 model implies. Metrick and Yasuda (2010) found that more than half of the examined buyout funds had initial fee levels below 2%, while a significant number of GRE and VC funds had initial fee levels above 2%.

The 2-20 model seems to be a more accurate depiction when it comes to carried interest, as all buyout funds and nearly all other funds surveyed by Metrick and Yasuda (2010) had a 20% performance fee. However, additional rules are set in place to better align incentives for GPs and LPs. The vast majority of both buyout funds and GRE funds requires management fees paid by the LPs to be returned before carried interest can be earned.

Furthermore, more than 90% of buyout funds, and just below half of VC and GRE funds, operate with a hurdle rate, which is a rate of return for the LPs that must be earned before carried interest can be allocated to GPs. The hurdle rate is commonly set to be 8% p.a. and only a few funds differ from this number. When employing a hurdle rate, many funds also employ a catch-up provision, meaning that after the hurdle rate is paid in full, GPs will earn X% (most often 100%) of all additional returns up until the fund has earned 20% of all returns. Any returns thereafter will use the standard 20%/80% split. When a hurdle rate is used in combination with a catch-up mechanism, LPs are then gaining some protection against poorly performing funds, and GPs can use this structure as a signalling tool for high skill (as they are

Figure 4: Holding period in PE fund based on exit year



Source: Own illustration based on Bain & Company, 2019

highly rewarded for beating the hurdle rate).

Lastly, an interesting part of the contractual dispersion of equity return is whether it's calculated on a case-by-case or a-fund-as-one structure. Axelson et al. (2010) and Axelson et al. (2009) document that while the case-by-case method was dominant in the early days of PE, it has, on a larger scale, disappeared due to the higher agency costs associated with it.

3.5 Portfolio management and duration of investments

The holding periods in private equity are varying greatly between funds, vintage years, and time of investment. However, while holding periods vary greatly for individual investments, evidence from Metrick and Yasuda suggest investment initiations are, to a large degree, similar across different funds. Metrick and Yasuda (2010) used data from 500 completed funds and found that the investment pace follows a largely similar pattern, with averages of the five-year investment period having equity division of 30%, 24%, 31%, 12%, and 3%, respectively. Block et al. (2010), Minardi et al. (2019) find a more equal distribution of contributions to the funds. However, specific vintage years have different paces of investing and this was extraordinarily observable in the time before and under the financial crisis (McKinsey & Company, 2020).

Evidence of market timing being a driver of value is not overwhelming, though. The road to closing can be very long, irrespective of whether the transaction is private-to-private or public-to-private, which makes market timing unlikely in the long-term (Jenkinson & Sousa, 2015). These complications in acquisition and divestiture processes call for a broader understanding of market timing. Unlike hedge funds able to time the market in real-time, private equity funds need to consider the overall environment, including the number of potential buyers with cash on hand, the availability of debt and the risk willingness of buyers. citetBLOCK do, however, find evidence that LBOs time the market to some degree, as well as Axelson et al. (2010) finding that LBOs actively choose the pace of investments based credit market conditions. While GPs have the final say in when their asset should be sold, they do not have potential buyers available at all times and thus exiting an investment is dependent on exogenous factors.

3.5.1 Holding period and exits of investments

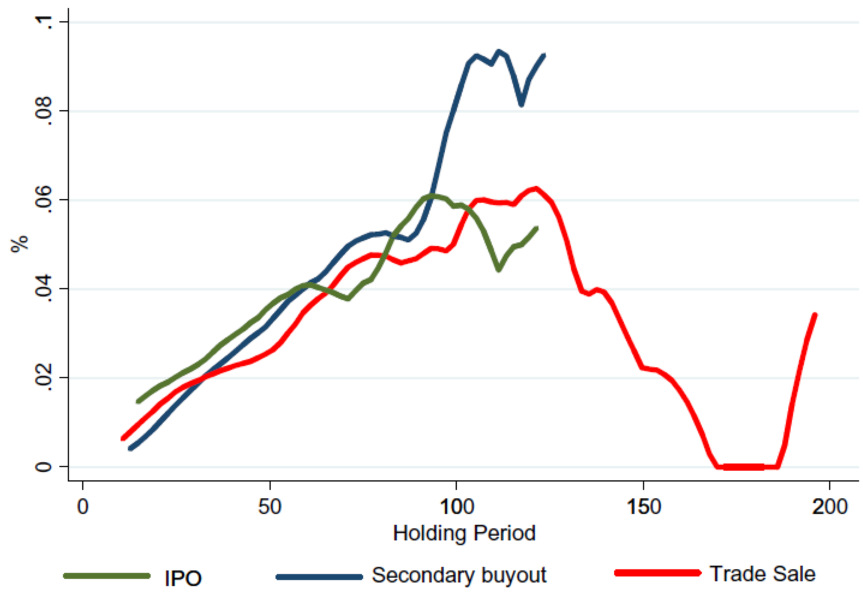
Looking at data from Bain & Company (2019), it is evident that both the number of exits in a specific year and holding periods of the portfolios change frequently. The prevailing situation in the financial markets does affect exits. The papers of Jenkinson and Sousa (2015), Minardi et al. (2019) find that exits are predominantly driven by return on investment, measured by either Multiple-of-Money or CAGR, as well as how much time the funds have left. Jenkinson and Sousa (2015) find that macroeconomic variables and the public markets have low or insignificant effects on the exit timing. Minardi et al. (2019) find effects of hot markets with regards to initial public offerings, but the prime driver is still time to maturity and return on investment.

As shown in figure 5, Jenkinson and Sousa (2015) have deduced expected exit probabilities of investments. This shows that IPOing, sale to other funds (SBOs) as well as sales to a strategic company (trade sales) have approximately the same exit rates. For the very long holding periods, nonetheless, the primary exit route is to a strategic buyer. The linearity of hazard rates is backed up by data from Bain & Company, 2019 on holding periods of LBO funds in 2018. In figure 6, we show the holding periods and their estimated hazard rates, which are aligned with the results from Jenkinson and Sousa (2015).

3.6 The performance of PE funds

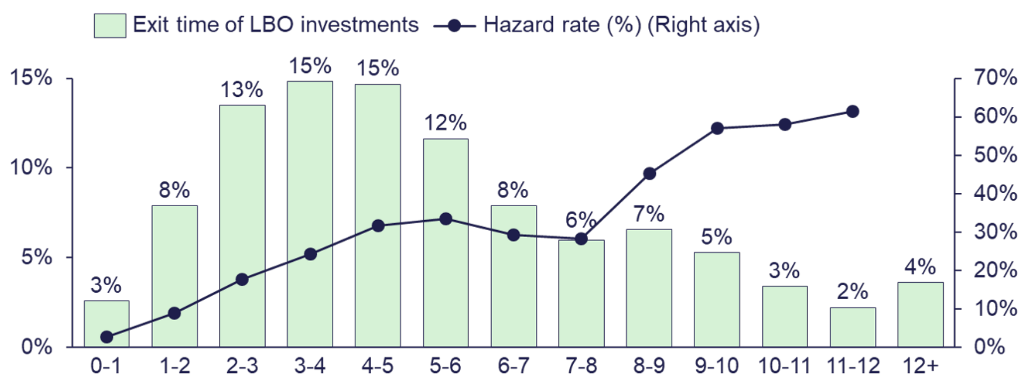
This section of the review will outline some of the research on the risk and return of PE, as well as key issues with these measurements. The section will end with a minor survey of the magnitude of GPs' skill and unlevered covariation with the market, used for modelling purposes

Figure 5: Hazard rate per month for PE investments by the exit for similar investments



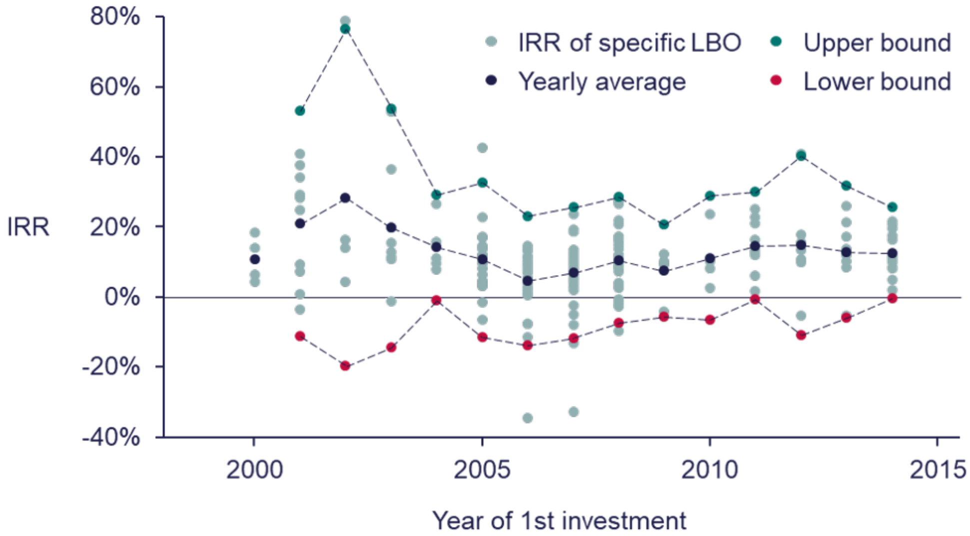
Source: Jenkinson and Sousa, 2015

Figure 6: Holding period for PE investments and calculated hazard rate in 2018



Source: Bain & Company (2019) and own calculations

Figure 7: Net IRR (%) of CalPERS PE Fund Performance Review on year of 1st investment



Source: California Public Employees' Retirement System (CalPERS) (2020)

later in the paper.

With few exceptions, such as Axelson et al. (2010), most papers find that private equity funds as asset owners are superior to the public market. This superiority stems from the previously described value drivers native to the industry (Company engineering and Investment timing). This means that the unlevered risk-adjusted excess return should be positive on average. However, the payoff structure of the industry with high fees and increased risks in form of higher leverage might lead to these benefits being out-weighed by fees and carried interest.

3.6.1 Return of private equity funds

From the perspective of the investor, it is valuable to compare the risk-adjusted return of the private equity industry to that of the public markets. The Private Equity Performance Review is an index of performance of LBOs collected by California Public Employees' Retirement System (CalPERS) and using the data, we find that the average Internal Rate of Return (IRR) varies between 5% and 20% with an average just above 12%. The results are shown in figure 7 and similar to the results summarised by Kaserer et al. (2005). Gottschalg et al. (2011) find average return of the private equity funds to be 13.1-15.9% in the United Kingdom. S. N. Kaplan and Sensoy (2015) surveyed the literature and found that yearly return since the 1990s on average were around 10%, ranging from -2% to 25% depending on the vintage year.

The actual return of the fund is, due to its illiquidity, first realised at fund maturity. To mitigate this, funds make the self-reported quarterly Net Asset Value (NAV) public for its investors. However, basing the estimates of the returns on NAV is insufficient, because any short-time variations in the underlying assets are unnoticed. As the GPs set the reported value of the portfolio, they can also mask variations or short-term drops in prices, something named stale pricing (Kaserer et al., 2005). As only entry and exit asset prices are observable for the investor to review and compare to the public market, NAVs are not a good measure of performance, and other measures have been developed.

Measuring the IRR on invested capital and using it as performance also bring some issues. Especially LBO funds use more leverage on portfolio companies than their public peers, which raises the riskiness of the investments. This, in itself, would not be an issue as you could account if you know the riskiness. However, due to various debt tranches, the actual degree of leverage is difficult to estimate, thus is comparing riskiness with the market portfolio. Secondly, and more profoundly, the illiquidity, low degree of transparency of valuation and infrequent reporting of returns provide lower volatility of the assets than what would be observed in the public market.

A solution to the above-stated issues is the Public Market Equivalent (PME) methodology, which essentially measures the return of the public market up against that of the return of the PE fund. More specifically, the method is to observe cash-flows of the PE fund activity, contributions to investments and distributions to investors, and apply these cash-flows as investments and divestments into a public market index such as S&P500 (Kaserer et al., 2005). The basic model is unadjusted, and if the PE fund overperforms in the earlier investments, the distributions out-weights contributions and the PME-measure (S. N. Kaplan & Sensoy, 2015). Thus several expansions to the PME have been suggested with Sørensen et al. (2014) providing a PME with tranches, however, all of the PME-measures are hampered by the lack of an asset with an equivalent risk profile.

3.6.2 Risk and its relation to return in private equity funds

As described before, PE funds tend to be highly levered, and leverage increases the actual volatility of the underlying asset. However, the increased risk gross and net of fees differ significantly, as the allocation structure provides different tranches, with different degrees of riskiness. Management fees have low-to-none volatility and are thus a certain risk-free loss for the limited partners. The hurdle rate enters as a covered call option, and thus only varies for lower returns. The catch-up provision has low risk, as it is often zero for LPs. The last element, profit sharing, is similar to that of the underlying asset.

As a consequence, the actual degree of riskiness is difficult to estimate, and investigation of whether private equity funds deliver excess risk-adjusted returns are highly debated. Some papers, e.g. Barber (2008), Jensen (1989), S. Kaplan (1989b), find high positive risk-adjusted returns. While there are large degrees of differences in estimates, it's evident that the earlier stage of modern PE funds had high returns. Nonetheless, much of these returns have decreased over the years (S. N. Kaplan & Sensoy, 2015), and based on the PME index chosen, the annual risk-adjusted excess return varies from a negative return of -5% (S. N. Kaplan & Sensoy, 2015) to a high of 20%(Khurshed et al., 2012).

An alternative measure of performance is to find the underlying unlevered risk-adjusted α, which some papers refer to being the fundamental skill of the fund managers. Gottschalg et al. (2011) estimate this α to be 4-6% in U.K. buyouts, and S. N. Kaplan and Sensoy (2015) find a span between 0 and 8%, again depending on assumptions made of the riskiness of the asset class. Sørensen et al. (2014) use their model to estimate long-term break-even α between 1% and 4% depending on fund structure. Skill is difficult to estimate, and the most proper comparison is to compare the distribution of return of PE to that of the public market (S. N. Kaplan & Sensoy, 2015).

4 The fundamental model of private equity

In this section, we will provide our general model of private equity returns loosely based on various models and result from other papers, mainly Choi et al. (2011), Metrick and Yasuda (2010), Sørensen et al. (2014) (asset growth paths and waterfall schemes) and additional elements that we have developed independently, e.g. the duration of each investment. The contribution of the model to the theoretical literature is,

- Incorporating a more realistic model for exits in private equity, as well as allowing for a differing time of investments in the portfolio of invested companies.
- Enabling a straightforward implementation of a financial crisis to the model.
- Conducting analysis of the distribution of results without making assumptions on utility functions and preferences for risk.

These contributions are essential for our later use of the model to describe the impact of a financial crisis event for returns to the general partners and limited partners.

Our model will consist of a mixture of exogenous parameters, that can be empirically observed or estimated by other papers within the private equity literature, as well as stochastic endogenous variables. The key endogenous variables that we model are shown in table 1.

Table 1: Stochastic variables in the private equity Model of this paper

Variable name	**Symbol**	**Description**	**Section in paper**
Holding period	Ω	A matrix of investment holding periods	4.3 (Exit model)
Value of debt	Z_0	End value of debt for each investment	4.2.2 (Leverage model)
Value of assets	A_t	Underlying asset growth	4.2 (Asset Valuation)
Proceeds to fund	Π_T	Proceeds above debt payment	4.2.2 (Leverage model)
Management fee	π_m	Fee based on investments and committed capital	4.4.1 (Management fee)
Hurdle zone	Z_1	Based on holding period and management fee	4.4.2 (Waterfall model)
Catch-up zone	Z_2	Based on hurdle sum	4.4.2 (Waterfall model)
Return to LPs	π_{LP}	Sum of elements of waterfall structure	4.4 (Compensation model)
Return to GPs	π_{GP}	Sum of elements of waterfall structure	4.4 (Compensation model)

4.1 Mathematical properties of the Geometric Brownian Motion

For the purpose of modelling the stochastic innovations, this paper will assume that the fundamental asset value follows a Geometric Brownian Motion (GBM) with two terms - a drift term μ and a volatility term σ. It won't be an issue for the paper that we won't account for sudden jumps, as the goal is to model asset valuations over the course of quarters and years, not hours and minutes. However, given the focus of the paper, it is critical that the model can be adjusted in a way that allows for deterministic changes in both volatility and drift[10].

The GBM is a stochastic process that involve a Brownian motion, also called a Wiener process, which is derived from a Gaussian distribution. The innovations to the model, mathematically defined as a stochastic differential equation, can be described as,

$$\frac{dA_t}{A_t} = \mu dt + \sigma dW_t \tag{4}$$

Where A_t is the underlying asset, the first term on the right-hand side is a deterministic drift term that represents the expected growth of the asset, yielding a mean of the process equal to, $\mathbb{E}[A_t] = A_0 e^{\mu t}$. The second term is the stochastic term representing volatility of the asset σ acting as a scaling parameter of the random motion W_t. W_t is a process based on a Gaussian distribution with the following characteristics,

$$(W_{t+u} - W_t) \sim \mathcal{N}(0, u) \tag{5}$$

The GBM process has a unique analytical solution for a specific time t (For proof, Back (2017)),

$$A_t = A_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \tag{6}$$

$$\mathbb{E}[A_t] = A_0 e^{\mu t} \tag{7}$$

$$\mathrm{Var}[A_t] = A_0^2 e^{2\mu t} \left(e^{\sigma^2 t} - 1 \right) \tag{8}$$

This solution has the nice feature that it can be used in valuation of options, and if A_t is discounted by e^{-rt}, the measure becomes risk-neutral and as such is a martingale[11].

[10] This issue will be handled in later sections, in which we implement a crisis to the process

[11] A random variable that have the feature of the present value of the asset equals the conditional expectation of the future value

4.2 Valuing equity investment in private equity assets

With leverage being one of the main features of private equity, asset and equity values are no longer equal. Both the degree and the features of leverage thus have a significant effect on PE as an asset class. This section will introduce how we derive equity value from the defined asset paths. Having defined a generic model GBM-process for the asset paths, our next step is to link this to the value of equity. More specifically, we will model how elements such as leverage, credit spreads, riskiness, and the skill of general partners affect the value of invested equity.

4.2.1 Expected unlevered asset growth and volatility

Aligned with the GBM model and similar to the method used by Sørensen et al., 2014, we define the drift term μ_a as a dependent variable derived from the Capital Asset Pricing Model (CAPM),

$$\mu_a = \alpha + \beta_U(\mu_s - r_f) + r_f \tag{9}$$

where r_f is the risk-free interest rate, μ_s is the expected yearly return in public markets, α is an aggregate term for the fundamental skill of managers in delivering operational and governance engineering thus driving up asset values above their peers in public markets[12]. β_U is the unlevered beta, called asset beta in literature, and from this, the resulting asset volatility σ_A can be defined as,

$$\beta_U = \frac{\rho \sigma_A}{\sigma_S} \Leftrightarrow \sigma_A = \frac{\beta_U \sigma_S}{\rho} \tag{10}$$

Where ρ is a correlation parameter. If $\rho < 1$, the asset has some unspanned risk that can't be derived from the public asset. For the remainder of this paper, we will simplify and assume $\rho = 1$ thus all risk in the PE asset is perfectly spanned by the public assets and $\sigma_A = \beta_U \sigma_S$. The consequence of this simplification is that our model won't be able to examine uncorrelated portfolio risks such as illiquidity risks, and risks arising from differences in the general conditions for the specific asset classes in private and public markets[13].

To keep a larger degree of transparency of effects and simplicity of technicalities as well as allowing for a deterministic implementation of a crisis, we will disregard all costs and benefits

[12]See discussion of these ways to deliver growth in the literature review

[13]According to Bain & Company, 2019, there are inherent differences in regulatory risk as an investor in private and public assets, which would not be accounted for with $\rho = 1$

of debt in this paper, so that

$$V_L = V_U, \quad \tau = \delta = 0 \tag{11}$$

We acknowledge that this simplification would, for most cases, skew results towards lower returns for equity investors, and as we will see in our results, especially the general partners[14]. The omitted effects will be larger for funds with higher volatility, leverage, and α.

For the GBM process, it is also important to emphasise that our model is based on unlevered parameters only. Interpretations and consequences for the insights of the paper are that the unlevered α is an aggregate of GP ability to create excess value in an asset. Metrick and Yasuda (2010) find that manager skills indeed are scaleable by enterprise value thus α should affect the unlevered asset rather than equity itself. Equity investors only care about the return of investment which is equity return net of fees paid to the manager - and not that of the asset itself. Whether that return is above its risk-adjusted public market equivalent return is different from whether the performance of the underlying asset is different from that of the market.

In case of $\alpha = 0$, expected return on assets before fees must equal that of the public market, thus for a given structure with fixed fees, the payoff to LPs would be worse than those of the market. Alternatively, if the fund has a relatively high α and can lever its investments, expected return on LPs will essentially be multiplied by leverage and excess return net of fees as a percentage of invested equity, α_E, could easily be significantly larger than the asset α.

Additionally, by using an unlevered market covariation parameter, β_U, estimated asset volatility, σ_A, is not equal to the riskiness of equity if the fund is financing its investments using debt. For most companies and industries, you can find and/or estimate their equity correlation to the market, β_E, which relates to the unlevered β_U the following way[15],

$$\beta_U = \frac{\beta_E}{1 + (1-\tau)\frac{D}{E}} \Leftrightarrow \beta_U = \frac{\beta_E}{1 + \frac{D}{E}}, \quad \tau = 0 \tag{12}$$

Where $\frac{D}{E}$ is the leverage of the asset measured by debt to equity ratio under our tax assumption. Using data from Damodaran (2020), an example of the influence of capital structure on β_U

[14]This assumption is broadly used in theoretical papers of Venture Capital and Leveraged Buyouts - even if they investigate the impact of debt on payoffs and thus depend explicitly on these value-adds (Ivashina & Kovner, 2011; Jensen, 1986; Metrick & Yasuda, 2010; Sørensen et al., 2014)

[15]For practical purposes we assume the standard Modigliani and Miller assumptions so that the equation assumes that asset value of debt is independent of the market return as well as no cost or benefit of debt

and β_E could be to compare the estimated global equity beta of the Oil/Gas and the Software development industry. Both of them have $\beta_E = 1.28$, but due to differences in average leverage[16], the unlevered β_U is respectively 0.75 and 1.18 for Oil/Gas and software development.

When increasing leverage, we also increase β_E. As an example, for an asset with a 50-50 debt/equity structure and an unlevered beta of 0.5, we would have $\beta_E = \beta_U(1 + \frac{D}{E}) = 0.5(1 + \frac{50\%}{50\%}) = 1$, which is double the initial covariation.

4.2.2 Cost of debt and leveraging of assets

This section will model the technical side of leveraging portfolio investments and how the value of debt is affected by the volatility of the asset as well as the impact, this has on the credit spread that must be paid.

The technicalities of debt structuring in private equity are many-fold and complex, but for our purpose, we assume that the fund can lever its investment by taking on balloon structured debt. This means that payments are made exclusively as a lump sum payment of the principal as well as compounded interests at maturity. It also means that there is no amortisation of the debt. The model for leverage is a variant similar to that of Sørensen et al. (2014), except that we allow the time-to-expiry to vary. We assume that the debt will roll-over until exit, and that refinancing won't be an issue to consider for the fund. With continuously compounding interests, this lump sum payment of debt at a specific time of exit would be,

$$Z_0 = D_t = D_0 e^{yt}, \qquad t = d_i - d_0 \quad \wedge \quad y = cs + r_f \tag{13}$$

Where $t = d_i - d_0$ is time of investment, D_0 is the principal, y is the interest rate with a fixed credit spread cs. Due to limited liability of equity, the actual value of debt varies as asset value could go below Z_0 and hence proceeds to debt holders will be,

$$D(A_t) = \min\{A_t, Z_0\} \tag{14}$$

Given that there's a risk of bankruptcy, $A_t \leq Z_0$, the debt holders will, if they assess risk correctly, attach a lower value of the debt at time of issue than the strike price of Z_0. This can be expressed as the present value of a risk-free asset with value Z_0 less a put option on the risk of default, or due to the put-call parity of options, the value of the asset A_t less a call option of

[16] The average $\frac{D}{E}$ for software development is 9.31% while it is 80.13% for oil and gas

equity (Sørensen et al., 2014). Using the option-based model of debt pricing (Merton, 1974), the debt holders claim on company assets can be viewed as the value of the asset less the value of a European call option of equity. We assume debt holders correctly observes β_U, μ_s, $\frac{D}{E}$, and r_f while α, the exact time of investment and potential structural events, such as a financial crisis, is unknown, though, the debt holder knows the average holding periods of PE investments.

The claim that α is difficult to observe is debated, with Axelson et al. (2010) arguing its not observed and Korteweg and Sørensen (2017) arguing that it is observed to some degree. Nevertheless, this assumption allows us to investigate and analyse the impact of α more independently[17]. This yields the following price setting of debt,

$$D(A_t, t) = A_t - \text{Call}\,(A_t, t, Z_0) = A_t - \text{Call}\left(A_t, t, D_0 e^{yt}\right) \tag{15}$$

At investment time, $t = 0$, the value of debt must be

$$D(A_0, 0) = A_0 - \text{Call}\left(A_0, 0, D_0 e^{yT^E}\right), \qquad T^E = \mathbb{E}[d_i - d_0] \tag{16}$$

Where T^E is the expected holding period of the investment as holding periods will be stochastic in our model[18]. The value of the call option is derived from the basic Black-Scholes-Merton model, yielding,

$$\text{Call}\left(A_0, 0, D_0 e^{y(T^E)}\right) = A_0 \Phi(d_1) - D_0 e^{yT^E} e^{r_f - T^E} \Phi(d_1 - \sigma_A \sqrt{T^E}) \tag{17}$$

$$d_1 = \frac{\ln\left(\frac{A_0}{D_0 e^{yT^E}}\right) + r_f T^E}{\sigma_A \sqrt{T^E}} + \frac{1}{2}\sigma_A \sqrt{T^E} \tag{18}$$

With $\Phi(x)$ denoting the cumulative standard normal distribution function. Assuming competitive debt markets, i.e. debt holders earn no economic profits, the pricing of debt, y, can be determined as the break-even y^* for which the principal D_0 equals the initial value of debt $D(A_0, 0)$. y^* depends negatively on leverage measured by asset-to-debt value, $\frac{A_0}{D_0} = \frac{E_0 + D_0}{D_0} = 1 + \frac{E_0}{D_0}$, and thus positively of debt to equity as this increases risk of bankruptcy. It depends positively on volatility for similar reasons.

It is evident that given two values of either leverage, $\frac{D}{E}$, volatility σ_A or yield y^*, the missing one can be calculated from the former two. This will allow us to model how a financial crisis might affect leverage if the yield is fixed and volatility is increasing.

[17] Sørensen et al. (2014) expand their model with α, which can be easily be added in our framework as well

[18] For the entirety of the paper, we assume this number to be 5 years

4.3 Investment holding periods and exit model

We will assume that the timing of investments $d_{0,i}$ and relative share of investments χ_i will be the same for all funds, such that the average pace of investments in the investment period is,

$$d_0^{avg} = \frac{\sum_{i=1}^{n} \chi_i d_{0,i}}{n}, \qquad \sum_{i=1}^{n} \chi_i = 1 \quad \wedge \quad d_{0,1} \leq d_{0,2} \leq \ldots \leq d_{0,n} \tag{19}$$

Having deterministic investment times is consistent with all the reviewed models of exits that we have seen in the literature. This also means that the key component in driving varying holding periods must be the time of exit for each investment. Applying insights from the statistical discipline of survival analysis, the arrival of an exit opportunity is stochastic with a specific structure of the underlying distribution. As discussed in section 3.5, the completed exit opportunities are generated as a result of both external market conditions as well as funds managers' performance and skill. This means that the conditional probability of an exit in a specific period, the so-called hazard rate $\lambda(t)$, must include a combination of exogenousness and endogenousness. The hazard rate thus describes the rate of exit in a period, given that the company has not been exited previously.

As stated before, the modelling of the exit and holding periods varies across papers in the private equity literature, depending on the purpose of the specific papers. This section will introduce the theoretical models of different papers and which characteristics and properties follow from them.

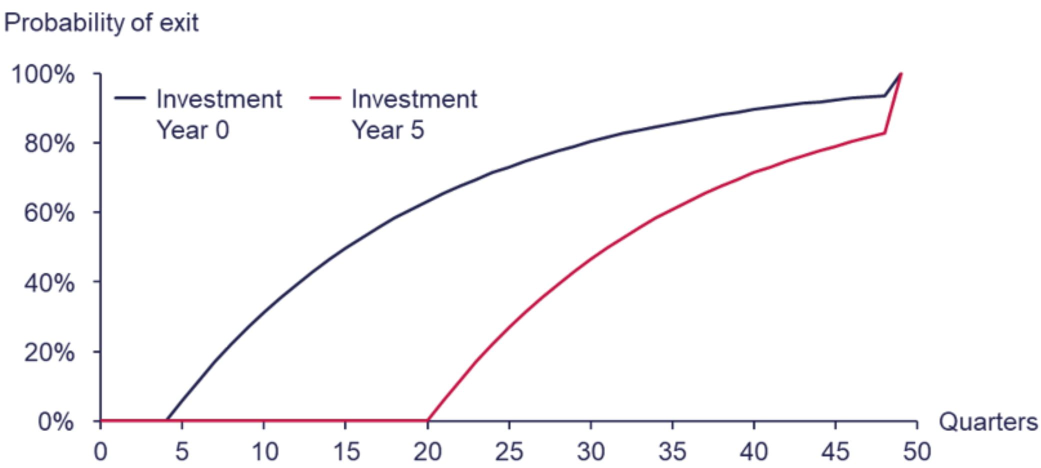
Sørensen et al. (2014) introduce the simplest model of exits and holding periods by fixing the holding period for the private equity investment to the time of fund maturity. This model assumes strict deterministic investments and exit times, and would yield a distribution with the following properties,

$$f(t) = \lambda(t) = \begin{cases} 0, & t \neq T \\ 1, & t = T \end{cases} \tag{20}$$

where T is fund maturity. This kind of model assumes no variability in exits and is predominantly used in very simplified models not investigating any impact of holding periods.

Choi et al. (2011) developed a very similar model to that of Sørensen et al. (2014), however, they added a stochastic exit, modelled using the exponential distribution. The exponential distribution is equivalent to the distribution of time between arrivals in the Poisson point distribution

Figure 8: Basic model: Exit probability for investment in first and fifth year



Source: Own estimation

and applied across many sciences due to its simplicity. It is applicable if the underlying arrival is expected to happen with the same conditional probability, which in practice could be the numbers of buyers and sellers of a company arriving in the market for asset transactions. This means that exits will arrive with a constant hazard rate and the following probability density function (PDF) and cumulative distribution function (CDF),

$$f(d_i) = \lambda e^{-\lambda d_i} \quad (PDF) \tag{21}$$

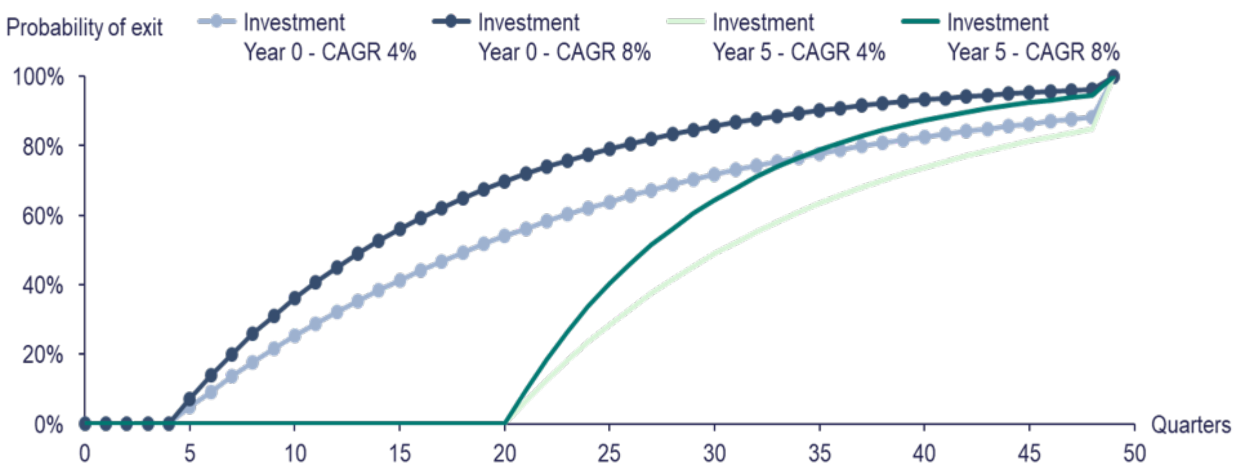
$$F(d_i) = 1 - e^{-\lambda d_i} \quad (CDF) \tag{22}$$

In figure 8, the CDF of an investment in the first and fifth year of this standard fund is shown. However, this is not a very accurate model of actual behaviour in the investment market with non-monotonic probabilities. Another issue is that it assumes complete exogenous exits which also seems unlikely on a broader basis[19].

An expansion of the model developed by Metrick and Yasuda (2010) can be seen in the paper of Minardi et al. (2019), in which the hazard rate $\lambda(t)$ are allowed to change during the life of the fund. Specifically, they implement and estimate a Cox semi-parametric model of the hazard rate on the exponential function. The Cox semi-parametric model has the useful feature that it allow the hazard rate to be endogenously affected by the variables relating the fund, the performance

[19] Metrick and Yasuda (2010) state these limitations clearly in their paper, but it has less consequences as the expected asset growth in their model is constant

Figure 9: CDF for the extended model for investment with varying CAGRs and d_0 for standard LBO fund



Source: Own estimation

of the underlying asset, and the timing of investment. The hazard function then become,

$$h(t) = \lambda e^{\ln(\mathbf{X})\gamma} \tag{23}$$

Where $\mathbf{X}$ is a set of variables affecting the likelihood for exit and γ is a vector of parameters corresponding to each of the variables. Minardi et al. (2019) estimate parameters for ten variables affecting exit timing, but in a two variable set-up (CAGR of the underlying asset and time-to-maturity at entry) would yield,

$$h(t) = \lambda(t|g_t, T - d_0) = \lambda_0 e^{\gamma_1 g + \gamma_2(\ln T - \ln d_0)}, \quad \lambda_0 = e^{\gamma_0} \tag{24}$$

where λ_0 is the lower bound of the probability of exit[20], g is a discontinuous weakly positive variable from the log of return of the asset less the log of yield of the debt, and $(\ln T - \ln d_0)$ is time to maturity. For a standard fund, the impact of implementing this model would result in the CDFs shown in figure 9. The strength with this modular extension is that exit times keeps its stochastic nature, thus staying partly exogenous, while it also incorporates empirical observations. However, while evidence suggests that growth or return on equity tend to be a driver of potential exits, the impact would need to be very large in order to acquire the

[20]This is the mean of an investment made in the first quarter of the fund life time with a lower CAGR than the yield of debt

distribution found in Bain & Company (2019), and that would itself result in a much larger than the observed variance of exit times.

The solution to this is suggested by Jenkinson and Sousa (2015), who allow estimation of the hazard rate using the Cox model, however, they don't assume a constant $\lambda_0(t)$. While not directly estimating the distribution, they show that the underlying $\lambda_0(t)$ of exits in European LBO deals take a somewhat linear growth path regardless of the choice of exit channel. In figure 5, these hazard rates are shown, and they are approximately linear until the 120th month (10th year) of the fund life, where trade sale[21] has a slight probability of happening after fund life expires.

What is interesting in this linear structure of the base hazard rate, is that the corresponding unconditional probability of the exits tends to mimic the actual distributions of PE holding periods. If we implement a Cox structure with a variable of Multiple of Money (MoM), $\frac{E_t}{E_0}$, and relative time to maturity at investment time $\frac{d_{0,i}}{T}$, we would expect to see the following hazard function,

$$h(t) = \lambda_0(t) e^{\gamma_1 \ln\left(\min\left[\frac{E_t}{E_0};1\right]\right) + \gamma_2 \ln\left(\frac{d_{0,i}}{T}\right)}, \quad \lambda_0(t) = \gamma_0 t \tag{25}$$

In the above equation, we have made the additional assumption that return on equity can only affect the probability of exit in a positive manner. If $\frac{E_t}{E_0} \to 0$ then $\ln \frac{E_t}{E_0} \to -\infty$ and these investments would have zero percent chance. If this feature wasn't included, all investments that have low returns or are technically insolvent will be held to maturity. A distribution as this one is essentially a chi-distribution with two degrees of freedom[22]. We will calibrate the values of $\gamma_0, \gamma_1, \gamma_2$ so that the distribution of investments resemble that of the observed holding periods. In figure 10, we show the PDF of the exits given MoM of either 1x (base-case) or 3x, and the difference between an investment in year 0 and year 5.

4.4 The contractual setup and compensation policy

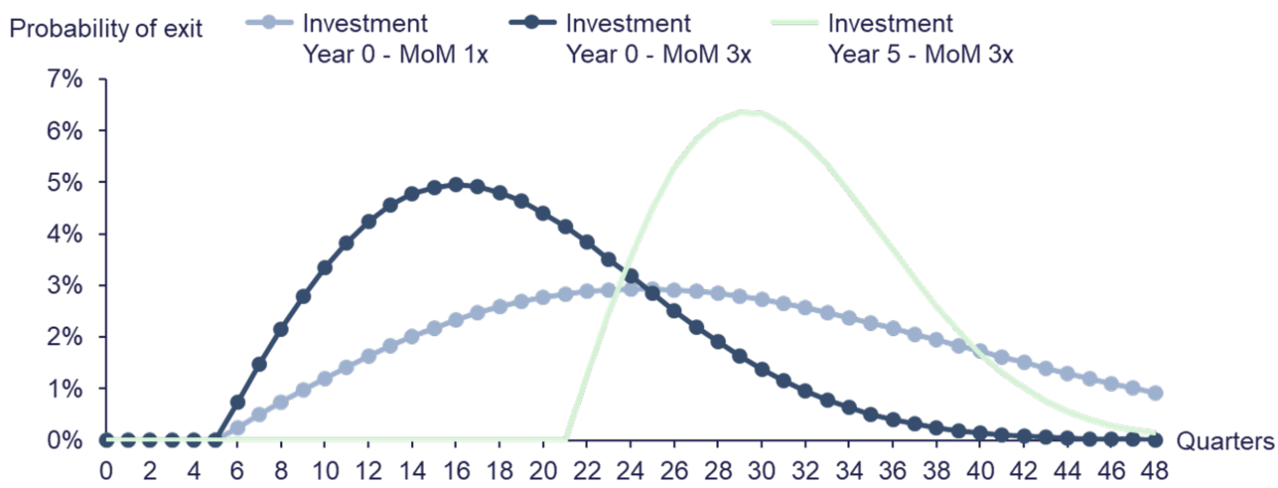
The model below is an extension and combination of the models described by Choi et al. (2011), Metrick and Yasuda (2010), Sørensen et al. (2014) and based on empirical observations of contractual conditions in European and American private equity funds.

The workhorse contract of the private equity industry as laid out by the empirical paper of

[21] Sale to a strategic buyer

[22] Also called the Rayleigh distribution

Figure 10: PDF of the final model used for PE exit timing based on equity return and time of investment for the standard fund structure



Source: Own estimation

Metrick and Yasuda (2010) are described in section 3.4. The fee and payoff allocation between the GPs and LPs fall under the following structure,

- A fixed management fee of the fund's committed capital (denoted X_0 in our model) and/or invested capital (denoted I_t) at the specific time.
- Carried interest that depends on returns on investment, potentially subject to the hurdle rate and/or fee return provision.
- Monitoring, entry fees and other income to GPs for services to the portfolio company.

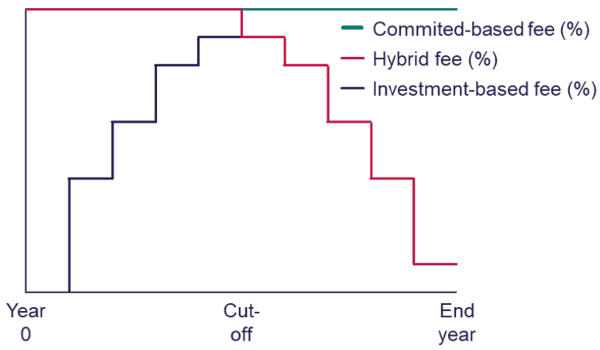
While the last element is an interesting topic of investigation, this paper will only investigate the impact of management fees and carried interest for the outcome of a crisis. For a deeper investigation and discussion of the impact from monitoring fees on private equity payoffs, see Metrick and Yasuda (2010) and Choi et al. (2011).

4.4.1 Management fee structure

The management fee can differ depending on whether the capital is invested or not and at which time in the fund life time. The fee for the investment period is p_h and the fee for the post-investment period is p_l. With a fund life time of T and a cut-off time of T_c, the fixed part of

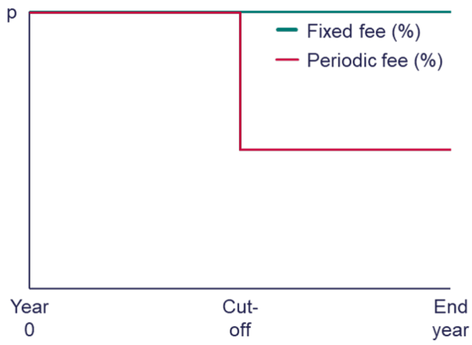
Figure 11: Fee structures in PE funds

(a) Management fees earned by fee base



Source: Own illustration

(b) Management fees earned by fee level



Source: Own illustration

expected payoff to GPs is one of the following classes,

$$\pi_m = \begin{cases} p_h X_0 T, & p_h = p_l \vee T = T_c & \text{(Fixed and Committed-based fee)} \\ X_0(T_c p_h + (T - T_c)p_l), & p_h \geq p_l \wedge T \geq T_c & \text{(Periodic and Committed-based fee)} \\ p_h(X_0 T_c + \sum_{t=T_c+1}^{T} I_t), & p_h = p_l \wedge T \geq T_c & \text{(Fixed and Hybrid-based fee)} \\ p_h X_0 T_c + p_l \sum_{t=T_c+1}^{T} I_t, & p_h \geq p_l \wedge T \geq T_c & \text{(Periodic and Hybrid-based fee)} \\ p_h \sum_{t=0}^{T} {}_c I_t + p_l \sum_{t=T_c+1}^{T} I_t, & p_h \geq p_l \wedge T \geq T_c & \text{(Periodic and Investment-based fee)} \\ p_l \sum_{t=0}^{T} I_t, & p_h = p_l \vee T_c = 0 & \text{(Fixed and Investment-based fee)} \end{cases} \tag{26}$$

Figure 11b and figure 11a show the impact that alterations to fee base and level have on the earned management fees in each period. Changing the fee basis from committed capital to invested capital, whether from the beginning or through a hybrid fee changing basis after the investment period, significantly reduces the fees earned, while also making them dependent on investment timing and the holding period. Funds changing the fee level after the investment period don't necessarily employ the one-time decrease depicted in figure 11b.

The alternative to the one-time decrease is a falling fee level, with the fee level dropping X basis points each year after the investment period. This means that fees will display a ladder pattern similar to fees earned by a hybrid-based fee where fees are changing basis from committed capital to invested capital. For a given p_h and p_l, not considering the carried interest structure, the most attractive solution for the manager is the Fixed and Committed-based fee while the Fixed and Investment-based fee is the least attractive and vice versa for LPs.

As it is the most common fee structure (Metrick & Yasuda, 2010), this paper will limit the investigation to a fixed fee level and hybrid-basis with a payment based on committed capital in the investment period and investment basis in the remainder of the fund life time. Fee level is assumed to be constant with $p = p_h = p_l$, and π_m is then defined as follows,

$$\pi_m = p\left(X_0 T_c + \sum_{t=T_c+1}^{T} I_t\right) \tag{27}$$

Where p is the periodic fee payment, and I_t is annualised total invested capital at specific time t,

$$I_t = \sum_{n=1}^{N} i_n \omega_{n,t} = \boldsymbol{i} \times \boldsymbol{\omega}_t \tag{28}$$

where i_n is the size of each investment with a corresponding vector $\boldsymbol{i}$ and $\omega_{n,t}$ is a variable representing amount of capital invested in a specific year. ω is 1, if the capital is invested the whole year and 0, if its not invested at all that year. The vector of ω_t being $\boldsymbol{\omega}_t$ and by aggregating all $\boldsymbol{\omega_t}$, we get a vector of holding periods for each investment,

$$\Omega = \sum_{t=0}^{T} \boldsymbol{\omega}_t \tag{29}$$

and average holding period being,

$$\omega = \frac{\Omega \times \mathbf{1}}{N} \tag{30}$$

The total investment in the PE fund is defined as $I = \sum_{n=1}^{N} i_n$ for N portfolio companies and $\frac{i_n}{I} = \chi_n \Leftrightarrow i_n = \chi_n I$, where χ_n is the relative share of total investable capital. Assuming proceeds from investments can't be reinvested, the committed capital X_0 cannot be less than the sum of the total investments and the management fees paid over the lifetime of the private equity fund. Consequently, the following equation must hold,

$$X_0 \approx I + \pi_m \tag{31}$$

However, the LPs and GPs can't realise duration of their investment in the investment-based fee period thus to ensure a realistic definition of committed capital, we make the additional assumption[23],

$$X_0 = I + pX_0T_c \Leftrightarrow X_0 = \frac{I}{1 - pT_c} \tag{32}$$

[23]The LPs are additionally assumed to be able to pay the second part of the management fees, even if there's not enough equity to pay these fees from

meaning that total management fees paid to the general partners must equal,

$$\pi_m = p\left(\frac{T_c \sum_{n=1}^{N} i_n}{1 - pT_c} + \sum_{t=T_c+1}^{T} \sum_{n=1}^{N} i_n \omega_{n,t}\right) \tag{33}$$

The last assumption needed to get the π_m is that we assume all investments to have the same size thus $\chi_n = \frac{i}{I} \Leftrightarrow i = \frac{I}{N}$,

$$\pi_m = p\left(\frac{NT_c i}{1 - pT_c} + N \sum_{t=T_c+1}^{T} i\omega_t\right) = pNi\left(\frac{T_c}{1 - pT_c} + \sum_{t=T_c+1}^{T} \omega_t\right) \tag{34}$$

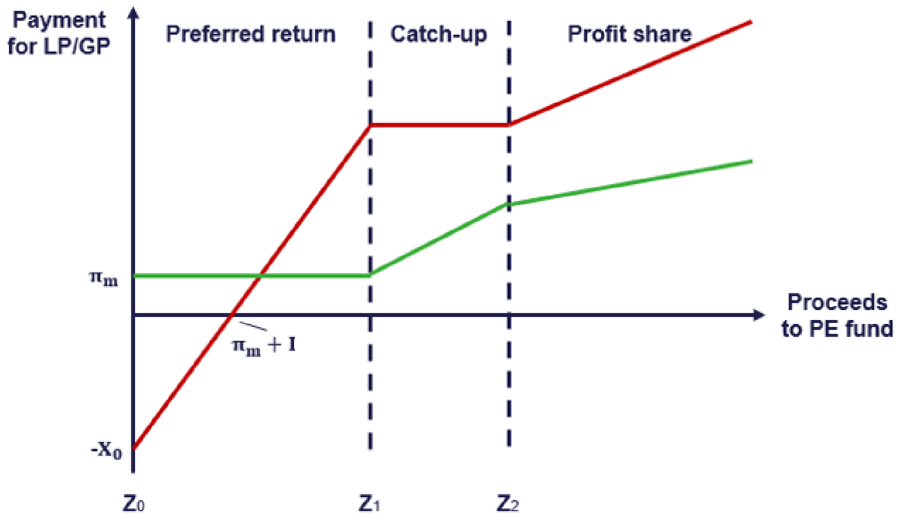
4.4.2 Performance-based fee / Carried interest structure

The carried interest also called carry, performance fee, incentive pay etc., is similarly described in section 3.4 and again, based on the paper from Metrick and Yasuda (2010). We can summarise this structure as,

- Carried interest, which is a percentage of returns. In most cases, it's fixed to 20%, but we will denote the carried interest percentage as a variable with notation ϖ.
  1. Most fund contracts include a fee return threshold (no carried interest before fees are paid back) and/or a hurdle rate, a minimum rate of return required before carry applies. The hurdle rate is defined as h in this paper.
  2. The base of which carry, hurdle rate etc. are calculated on can also vary between realised return or a fair value estimation. The latter would allow the manager to be compensated much earlier, however, such a contract would most like include a claw-back clause, so that the GP would have to pay back its carry if total return upon fund termination is below carry requirements.
  3. Lastly, the carried interest can be calculated on a case-by-case basis or the full-scope of the portfolio. The former increases the limited liability of the GPs and, ceteris paribus, the expected return for the GPs increases.
- Additionally, general partners often invest in the fund as well, and depending on the nature of the fund, this investment level is often between 1 and 10 % of the committed capital.

Metrick and Yasuda (2010) and Sørensen et al. (2014) additionally find evidence of a transaction and monitoring fee, but we will disregard these elements in our model, as their effects would be

Figure 12: Payoffs from the waterfall structure (Red is LP return and green is GP return)



similar to those of the management fee (Metrick & Yasuda, 2010). Due to the nature of incentive alignment incorporated into the carried interest structure, and for simplicity, we will exclude the GPs investment share in the model[24]. We assume that equity is dispersed at the end of the fund life time thus the distinction between realised and estimated returns does not matter. We will not consider case-by-case carried interest, however, in a scenario of increased volatility, this would definitely affect the payoff for the general partners, as their carried interest will benefit from the winners and not negatively affected by the losers. Before defining the carried interest to the GPs and the return to the LPs, we must define the proceeds from a portfolio company from each investment,

$$\Pi_n = \max[A_t - Z_0, 0] \tag{35}$$

Where the value of Z_0 is defined in equation (13). Given the preferred nature of debt over equity, any divestment under Z_0, won't yield any proceeds to the fund, and invested equity is lost. As debt only can affect one investment, we can derive the total proceeds to equity in the fund as follows

$$\Pi_T = \sum_{n=1}^{N} \Pi_n \geq 0 \tag{36}$$

Due to the limited liability of the funds, Π_T must always be above zero.

[24]The carried interest will transfer return benefits from the principal (LPs) to the agent (GP) and thus the effects will be captured by this to a large degree - See among others, Litvak (2009), Schmidt and Wahrenburg (2003)

The carried interest setup is the rule that distributes these proceeds between LPs and GP. If we take the perspective of LPs, the returns, disregarding the investment and paid management fees, will take the following form

$$\pi_{LP} = \begin{cases} 0, & \Pi_T = 0 \quad \text{(Bankruptcy of all investments)} \\ \Pi_T & Z_1 \geq \Pi_T > 0 \quad \text{(Preferred Return to LPs)} \\ Z_1 + (1-k)(\Pi_T - Z_1) & Z_2 \geq \Pi_T > Z_1 \quad \text{(Catch-up for GPs)} \\ (1-\varpi)(\Pi_T - Z_2) + Z_1 + (1-k)(\Pi_T - Z_1) & \Pi_T \geq Z_2 \quad \text{(Profit sharing)} \end{cases}$$

Where k is the catch up provision for the GPs so that $k = 1$ is when the GPs fully catch up on returns, and $k < 1$ is when some degree of profit share are included. Z_1, Z_2 are thresholds of proceeds for which the payoff regime changes. For the preferred return period, the return to LPs are equal to the proceeds of the fund until it reaches the hurdle rate plus management fees, defined as Z_1,

$$\begin{aligned} Z_1 &= \pi_m + i\sum_{n=1}^{N} e^{h\omega} = \pi_m + iNe^{h\omega} = piN\left(\frac{T_c}{1-pT_c} + \sum_{t=T_c+1}^{T}\omega_t\right) + iNe^{h\omega} \\ &= iN\left(p\left(\frac{T_c}{1-pT_c} + \sum_{t=T_c+1}^{T}\omega_t\right) + e^{h\omega}\right) \end{aligned} \tag{37}$$

where we have the management fee defined in equation (34). For proceeds in the catch-up zone between Z_1 to Z_2, we have that the proceeds going to the GPs at $\Pi_T = Z_2$ must solve the problem of GPs receiving the exact part of return, k, that they have foregone in the hurdle rate period,

$$\begin{aligned} &\varpi(Z_2 - \pi_m - iNe^{h\omega}) = k(Z_2 - Z_1) \\ \Leftrightarrow\ & Z_2 = \frac{kZ_1 - \varpi(\pi_m + iNe^{h\omega})}{k - \varpi} \\ \Leftrightarrow\ & Z_2 = \frac{kiN\left(p\left(\frac{T_c}{1-pT_c} + \sum_{t=T_c+1}^{T}\omega_t\right) + e^{h\omega}\right) - \varpi(\pi_m + iNe^{h\omega})}{k - \varpi} \\ &= \frac{kiN\left(p\left(\frac{T_c}{1-pT_c} + \sum_{t=T_c+1}^{T}\omega_t\right) + e^{h\omega}\right) - \varpi\left(pNi\left(\frac{T_c}{1-pT_c} + \sum_{t=T_c+1}^{T}\omega_t\right) + iNe^{h\omega}\right)}{k - \varpi} \end{aligned} \tag{38}$$

In the last zone, $\Pi_T \geq Z_2$, all remaining proceeds will be shared between the LPs, $(1-\varpi)$, and

GPs, ϖ. For the general partners, their return of running the PE fund will be,

$$\pi_{GP} = \begin{cases} \pi_m & Z_1 \geq \Pi_T > 0 \quad \text{(Preferred Return to LPs)} \\ \pi_m + k(\Pi_T - Z_1) & Z_2 \geq \Pi_T > Z_1 \quad \text{(Catch-up for GPs)} \\ \pi_m + k(\Pi_T - Z_1) + \varpi(\Pi_T - Z_2) & \Pi_T \geq Z_2 \quad \text{(Profit sharing)} \end{cases}$$

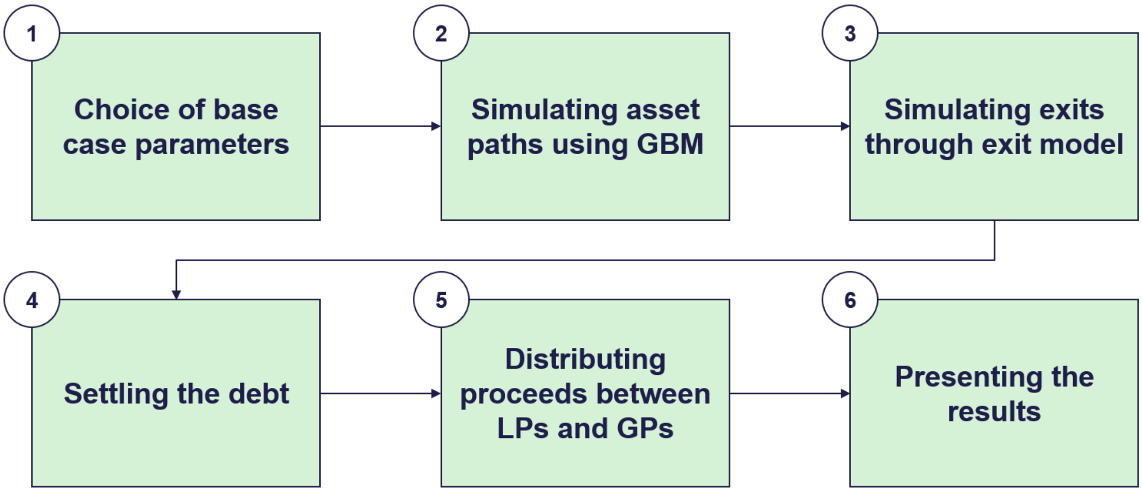
The waterfall structure consist of a mixture of stochastic variables and contractual parameters meaning that the return to different stakeholders will vary for each fund. The waterfall structure also allows for an expansion in which GPs take on equity investments as well,

$$\pi^*_{GP} = \eta\pi_{LP} + \pi_{GP} \tag{39}$$

$$\pi^*_{LP} = (1-\eta)\pi_{LP} \tag{40}$$

Where η is the equity share of the general partners. Obviously, implementing a $\eta \geq 0$ will not affect the zones of the waterfall, and thus implementing an equity share in our perfect market model will not affect insights.

Figure 13: Simulation methodology



5 Simulation methodology and processes

In this section, the inner workings of the simulation model will be elaborated and the different steps will be explained. We state the key parameters going into the model, and the values they take. The section will not discuss the specific chunks of code in R, but rather explore the different processes the simulation model goes through to gather results. Examples of simulated results from the base case will be used to provide intuition.

Figure (13) show the general order of processes gone through in the model, with each being explained in depth later in this section. To allow for valid inference from the model, 10,000 simulations are run for each case tested. The high number of simulations creates a distribution of results for each case analysed, making it possible to analyse both expected values and extreme cases. Furthermore, the distributional setup allows for analysis of differences in risk between various cases.

5.1 Choice of base case parameters

The parameters chosen for the base case can be seen in table (2). For the parameter used for the public equity paths, we use numbers estimated from public markets. The average annual return of the public market has varied over time, with some periods being low around 4-5% and others higher at 10-12% (Damodaran, 2013), however, we make the general assumption of stock return equalling that of the S&P 500 being 7% annually. Public equity volatility, defined

Table 2: Base case parameters for the standard PE fund - Description, Value and Sources

Parameter		**Description**	**Value**	**Source**
Public return	μ_S	Average return of public equity	7%	Own estimation (S&P 500)
Public volatility	σ_S	Average volatility of public assets	40%	Damodaran (2020)
Risk free rate	r_f	Yield of the risk free asset	2%	U.S. Treasury 2018 (3mnt)
Unlevered beta	β_U	The covariation of unlevered asset with market	0.65	Damodaran (2020)
Debt / Equity	D/E	The amount of debt relative to equity in investments	2.00	Working (2010)
Alpha	α	Unlevered excess return to that of the market	2%	
Management fee	p	Annual fee received by the GP (basis of fee can vary)	2%	Metrick and Yasuda (2010)
Carried interest	ϖ	Percentage of the proceeds of the fund to GPs	20%	Metrick and Yasuda (2010)
Hurdle rate	h	Preferred rate of return for the LPs before carry	8%	Metrick and Yasuda (2010)
Catchup provision	k	GPs catch-up above hurdle rate	100%	Metrick and Yasuda (2010)
Fund lifetime	T	The lifetime of the fund in years	10	Metrick and Yasuda (2010)
Fund extension	T+	Additional years for exits but no management fees	2	Metrick and Yasuda (2010)
Investment period	T_c	Period for investing and fees on committed capital	5	Metrick and Yasuda (2010)
Investments	N	Number of investments per fund	10	Metrick and Yasuda (2010)
Base exit rate	$\gamma 0$	Lower hazard rate	0.002	Proprietary calibration
Effect of MoM on Exit	γ_1	MoM impact	1.2	Proprietary calibration
Effect of time on Exit	γ_2	Time to maturity impact	-0.6	Proprietary calibration

as annualised standard deviation in weekly stock prices, is estimated using two years of data. This is also somewhat tricky to estimate as it varies for different types of asset classes, however, the data from Damodaran (2020) can be used to estimate it being between 38% and 40% for global stocks in average[25]. Using data of U.S. Treasury yields from Damodaran (2020) as well, we find that it has varied between 0% and 4% while having a value of 1.94% percent on average in 2018.

For the parameter used in the Geometric Brownian Motions of PE asset paths, we use the estimation of the unlevered β_U by Damodaran (2020), being 0.65 across industry excluding financial services. In our model, however, we can adjust this number to account for different investment profiles, where some funds might focus on low β_U assets and others more risky assets. The debt to equity ratios also varies significantly, from being around 3 before the financial crisis in 2007 to a little less than 1 just after. Using numbers from Working (2010), we set the base case to be 2 and test for variations. Lastly, we cannot find a consistent estimate of the unlevered α as it depends on many elements, thus we will test a range of 0% to 4%, with 2% as our base case, similar to Sørensen et al. (2014).

The parameter values used in the waterfall scheme are chosen based on the findings of Metrick and Yasuda (2010). These parameters are very similar across funds and their validity is high. Metrick and Yasuda (2010) investigate the contractual setting of 144 LBO funds, finding that all funds had carry ϖ of 20%, that the most common initial fee level is 2%, however, the majority of funds change fee basis from committed to invested capital after its investment period, which they find typically to last 4-6 years, thus we choose 2% management fee and 5 years of T_c. They find that 92% of all LBO funds employ a hurdle rate and that this hurdle rate is equal to 8% for 4 out of 5 funds. The majority of funds has 100% catch-up and no fund has a catch-up rate below 80%. The number of investments is not well-specified, however, as we simulate 10,000 PE funds, results would not be altered much by changing this number.

The base case also assumes some, but not exorbitant, manager skill by setting α at 2%. A fair degree of leverage is also assumed, while the assets invested are assumed to be having average co-variation with the market. Overall, the values of these parameters should be close enough to the real world parameters that the base case simulation should behave similarly to real-life funds. By varying some of the parameter values, the model's sensitivity to the inputs can be

[25]Damodaran (2020) also estimate a standard deviation of 30.4% for unlevered public assets

Figure 14: Example: simulated asset paths

analysed. Furthermore, this should drive some insights into what drives the results.

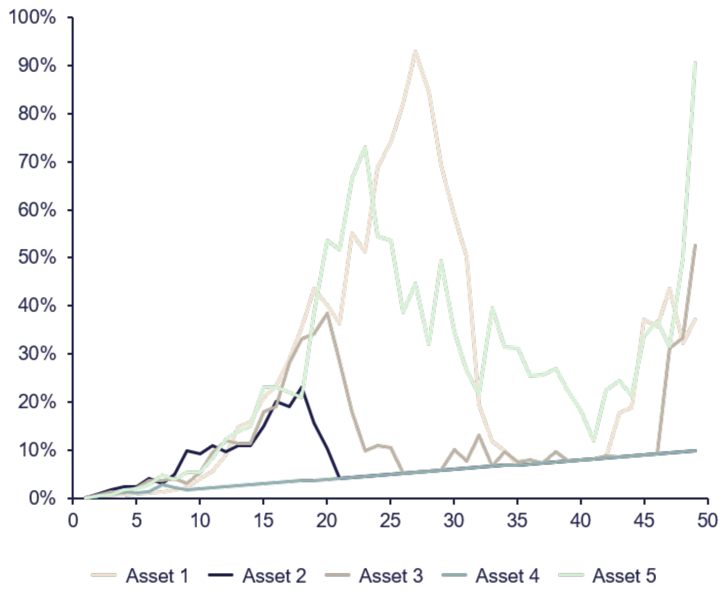
5.2 Steps in the simulation model

This section will outline the steps, we take to simulate and evaluation the results of our model.

5.2.1 Simulating asset paths

Asset paths are simulated using a GBM model, as seen in equation (4). The drift term in the GBM model is a linear function of β_U, μ_s and r_f as stipulated by the CAPM, and it is then adjusted to reflect the quarterly repricing. The error term in the GBM is also scaled by β_U, as to reflect the appropriate risk level for the asset. For each fund, the number of unique asset paths simulated is equal to $N = 10$, with the investment timing decided deterministically to be two investments made in the first quarter of year 1, 2, 3, 4 and 5. The asset paths are then simulated one quarter at a time, up until the time coincides with the fund lifetime (T) plus the fund extension ($T+$). Figure (14) is an example of the first 5 asset paths in the simulation. The example illustrates the varied nature of the asset paths, even given the same parameters, with some going on soaring bull runs, while others fail to increase their value. Any covariance except a common drift is coincidental.

Figure 15: Quarterly exit probabilities for the simulated asset paths



5.2.2 Simulating exits

Figure (15) shows the probability of an exit for any given given quarter, given that no exit has happened up until that quarter for the simulated asset paths in figure (14). The exit probabilities are calculated based on the exit model from equation (25). From figure (15) it can be clearly seen how big the influence the value taken by the asset path is in calculating the exit probability. As the simulated values rise, and reach exorbitant heights, the probability of an exit becomes close to 100% for a given quarter. To find the exact quarter where an exit happens an iterative

Table 3: Simulated exit time (quarters)

Asset 1	**Asset 2**	**Asset 3**	**Asset 4**	**Asset 5**
18	18	12	20	7

process is created. The model excludes the exits in the first year after an investment is made, but from the first quarter then after, a binomial function calculates whether an exit happens in the given quarter, based on the exit probability calculated by the exit model. If the binomial function results in 0, meaning no exit, the calculation is repeated for the next quarter using its corresponding exit probability. When the binomial function results in 1, meaning an exit, for the first time, any subsequent quarter for the asset path is set to zero. Table (3) contains the exit quarters found by the iterative process.

5.2.3 Settling the debt

With buyout funds, a crucial component of any investment is a significant portion of debt, allowing the purchase of more expensive assets. The terms of the debt used in the model are based on the parameters from table (2), with the amount of debt formulated as a debt to equity ratio, with a yield equal to r_f plus a credit spread, which is calculated using the Merton model. In the simulation, it is assumed that interest on debt is accruing, and the entirety of the debt is settled as a balloon payment following an exit. When all asset paths are simulated and exit times are found, the accrued value of the debt for each investment is calculated to correspond with the respective exit times. The settling of the debt is done on an individual asset basis, therefore if the exit value of a given investment is not high enough to cover the balloon payment, this constitutes a default on the debt, and the asset is declared bankrupt. The debt holders in the asset have seniority, and they are thus eligible to claim all value from the exited investment, up until the balloon payment is paid back in its entirety.

5.2.4 Distributing proceeds between LPs and GPs

With the value of the debt owed calculated, the proceeds eligible for distribution between LPs and GPs, the equity, can be found according to the accounting equation,

$$\text{Market value of assets} = \text{value of debt} + \text{value of equity} \tag{41}$$

For any investments with a default on debt, the equity is set to zero. The equity from N investments in the fund are then pooled together and is distributed between the LPs and GPs following the waterfall structure mentioned in section (4.4.2). The first step is to calculate the average holding period of the fund, to be used in the calculation of total management fees and the preferred return (the hurdle rate return). Besides average holding periods, the investment size weighted average holding period after the investment period needs to be calculated (In the simulation, investment size is the same between investments). Knowing the average holding period after the investment period allows for calculation of the non-fixed part of management fees, as the basis is invested capital. Total management fees are in the model not calculated until the fund is fully exited, but as LPs are assumed never to run out of cash, these will always be paid in full.

Management fees to GPs are not paid from the proceeds of the fund, therefore the proceeds are unaffected by this payment. With the full value of the preferred return (including return of management fees according to the return of fees provision) calculated, the amount is then distributed to LPs, and if funds are not able to pay the preferred return in full, they are to pay whatever equity is left. The next step is then the catch-up provision, which is calculated based on the level of preferred return. The full catch-up provision is then distributed to GPs, if possible, and if not, funds distribute whatever equity is left. Any funds not able to pay the preferred return in full will have zero equity to distribute to GPs. For any funds with remaining equity past the distribution of the catch-up provision, the remaining equity is split between LPs and GPs, with ϖ going to GPs and $(1 - \varpi)$ going to LPs.

5.2.5 Presenting the results

The results from the simulation model are always expressed as a multiple-of-money, where the denominator is investable capital. For LPs, the multiple needs to be calculated net of fees, as this is the return that would internally be relevant for an LP. A money multiple of 1x is interpreted as the LPs getting their money back including the management fees paid, meaning any value below 1x represents a loss for LPs. Including the management fees in the multiple calculation is also the reason why the multiple can turn negative in some extreme cases. The numerator contains the preferred return (including return of management fees) plus any profit share received, deducted by the management fees paid. Calculating multiple of money is straightforward for GPs, as it consists of adding together total management fees with any catch-up provision and/or carried interest they might receive, and then divide this with the investable. The multiple for GPs is thus strictly positive, as management fees have a fixed positive component.

When the simulation model is processed, the vectors consisting of all the simulation results are then indexed based on parameter setup (for example the level of α). The summary statistics of the vectors are then retrieved, and saved in a matrix with results from different parameter setups. The retrieved summary statistics are the minimum and maximum value of the vectors, first, second (median) and third quartile values, as well as the mean and standard deviation. Each vector has 10,000 numbers and their distributions are quasi-continuous and can then be plotted using a density ridgeline plot. A ridgeline plot presents the distribution of a variable. This is done using density plots, all aligned to baseline so that they are easily compared. Doing this for various parameter setups allows us to conduct a deeper inspection of parameter sensitivity, enabling us to comment on the outcome of returns, even if their distributions are asymmetric.

6 Discussion of the model, its limitations, and simulation of results

Our model is, as most economic models, a conscious over-simplification of reality. As famously stated by Shoesmith et al. (1987),

> *Essentially, all models are wrong, but some are useful.*

This section will discuss the limitations of our theoretical model, its assumptions, and thus the inference, we can draw from the results. We will also discuss the up- and downside of using a simulation-based evaluation of the model.

6.1 Limits of the theoretical model

Our model framework largely assumes rationality of the market. We disregard inefficiencies such as moral hazards and agency costs. Additionally, our model assumes strict exogeneity of any innovations, and thus no PE fund can time the market, neither with regard to entry or exit.

6.1.1 Impact of the Geometric Brownian Motion and modelling asset values

Our asset paths are modelled using the GBM process. As mentioned earlier, this is one of the most widely used methods of modelling financial assets. However, the process assumes normality of innovations and deterministic drift and volatility terms. These assumptions are practical from a mathematical and simulation point of view. However, there are additional trade-offs to those described in section 3.1.3,

- Actual innovations to asset prices, measured by the value of debt and equity, has thicker tails i.e. larger kurtosis, indicating that there are more extreme events in the actual data generating process (Lau et al., 2019).
- Furthermore, many financial papers find that asset paths often have a large degree of positive auto-correlation in the short-term, called the momentum effect in the asset pricing literature, in conjunction with long-term mean-reversion (Aigner et al., 2012)

None of these are captured by our model. This might understate the volatility of returns, while the momentum effect might make strategic exits more likely, and thus alter distributions of holding periods. One implementation to resolve this issue would be to construct asset paths via a Markov-switching model that allows for auto-correlation, endogenous and/or a stochastic growth term and a more realistic assumption of distribution of innovations (Aigner et al., 2012). But the added complexity of the Markov-switching model will add uncertainty in answering the research question of this paper. In its simplicity, the GBM does not include a stochastic general economic crisis, as assets follow independent individual paths around its drift. This has the great advantage that testing a crisis is much clearer, even if it is less resembling of a real-world asset path.

6.1.2 Cash flow or asset-based model and their use

Additionally, our model is asset-based, not EBIT-based. Asset-based GBM enables us to model from unlevered parameters, thus enabling work with leverage from an exogenous point of view. It does, however, not allow us to make a meaningful rule of bankruptcy similar to that of Lahmann et al. (2016), Leland (1994). Our model also rests on the need for a balloon debt rather than ongoing amortisation through coupon payments. Given that debt run to maturity, the inherent risk of debt holders increases thus debt yields would be higher than for a more frequent revisit of the debt. Also, the modelling framework used is not aligned with implementing valuation effects of leverage, and this is a large weakness of our model, and if we were to implement this feature, we wouldn't be able to make the valuation effects state-dependent such as Leland (1994) suggests.

For our purpose, the complexity of Lahmann et al. makes implementing a crisis element too complicated and muddy, while the one period structure of Sørensen et al. makes it impossible. Therefore the model, we developed is a useful generalisation of Sørensen et al. similar to that of Choi et al. Asset-based models, despite the lack of specifying impacts of leverage, allow for a macro perspective of asset pricing, and as we specify our crises on a macro level, this aligns very well. But it is also evident that our tractable implementation of the crisis has serious trade-offs.

6.1.3 Definition of β_U and diversifying risk

In the model section 4, we made the assumption that risk in the private equity asset is fully spanned by the public market, i.e. we specify $\rho = 1$ in equation 10 defining β_U. This means that our μ_A and σ_A are linearly dependent on the public asset μ_S and σ_S. However, as stated by Sørensen et al. (2014), unspanned risks in PE might be as high as 40% non-idiosyncratic volatility. For $\rho \leq 1$, a given β_U and α would mean a lower μ_A in our paper, and thus lower expected proceeds to LPs and GPs for a given volatility. Evidence from Choi et al. (2011) suggest that these unspanned risks are deteriorating for LBOs and remain stronger for VC funds, so this assumption limits the conclusions drawn to those of the later-stage funds.

While α still can be interpreted as the managers' skill, Sørensen et al. (2014) show that if $\rho \leq 1$, the value of this managerial skill is deteriorating and the risk of the PE asset cannot be hedged by other assets, thus returns lose their direct comparison to that of the market. On the other hand, the decreased degree of covariation might mean that fund performance would be better in an early crisis without a recovery or a late crisis. Nonetheless, the final effect depends on the value of ρ as well as the crisis structure itself.

6.1.4 Follow-on investment, refinancing, and recapitalisation

While staging of investments is mostly prevalent in venture capital, it also occurs in later-stage investing. Follow-on investments could be driven by the arrival of NPV positive investment opportunities or LBOs might need to refinance expensive debt if cash flows change. More relevant in the context of the paper, it might be beneficial for the fund to recapitalise their portfolio companies in a crisis setup. If initial investment happens at a time of high volatility, and thus with expensive debt, it would increase value significantly by refinancing at a lower credit spread or higher degree of leverage for investments that have gotten through the time of crisis.

For our model, implementing a recapitalisation option would make leverage more valuable for funds, especially for investments made in periods of extraordinarily high financial volatility. Axelson et al. (2010) provide evidence that recapitalisation does happen, however, they also find that many funds do not alter their capital structure for varying capital conditions, thus the need for this element is to some degree dubious. Axelson et al. (2010) find that the key driver of capital structure is the credit conditions at the onset of the investment.

6.1.5 Market timing and portfolio management

Our model has fixed investment times, which as discussed is a simplification of how funds manage their investments. In normal periods, the assumption seems to have little impact as the investment timing seems rather fixed. But the assumption will not be able to explain why committed capital is held back in the earlier part of the crisis. Though, with the simple structure, the impact of a crisis event becomes more significant and thus easier to draw inference from.

Another element that our model won't take into account is that experienced managers tend to hold portfolios longer when controlling for other factors (Jenkinson & Sousa, 2015). However, experienced managers tend to have higher skills[26], and thus implementing this in model specifications might be unfruitful and distort model results for the industry in general.

Lastly, a benefit of implementing the Cox structure using the MoM (an absolute approach) rather than the CAGR or IRR of the investments (a relative approach) is their differing behaviour when it comes to short term volatility. Using IRR/CAGR makes the exit model highly sensitive to short term volatility, as a large increase in asset value over a short time frame could result in extremely high CAGRs. This would lead to very high exit probabilities for some investments in the short term, a not so realistic implication. Using the MoM instead, the impact on exit timings from short term volatility is negligible, as high absolute returns are more plausible in the long term than in the short term. Jenkinson and Sousa (2015) find that, while the funds exit investments with high returns, managers are trying to ride high-growth investments, and thus the short-term effect of the CAGR might yield improbable results.

6.2 Simulation benefits and limitations

This paper is using simulation rather than an analytical solution. The benefits of this methodology are,

- Our analysis provides more insights on the distribution rather than just expected means.
- We do not need to apply utility functions or risk-neutral measures when comparing with public returns.

[26]Korteweg and Sørensen (2017) find this to be driven by higher returns making better managers staying in the market with a higher probability, and not driven by experience onto the skill set

However, the method has one large downside being,

- We cannot perform comparative statics and any test of sensitivity can only be discrete through scenario analyses.

First of all, an analytical solution for a model with stochastic variables, as well as varying deterministic terms, is highly unlikely to exist. Additionally, a solution would constitute a certain expectation and a simulation provides the distribution of potential outcomes, meaning that we can compare the distributions of different scenarios. However, we cannot perform comparative statics of the results of the model, thus our comparisons become more qualitative as we only can perform scenario analysis, as such our results can only be discussed in a discrete manner.

7 Model prediction of private equity return in normal times

The base case of the model is a steady-state case, meaning that no form of financial crisis event is impacting the simulated asset paths. The purpose of examining the base case is foremost to gain an understanding of how our model works under various assumptions, as well as establishing a reference case for which we can compare the impact of a financial crisis event. It serves the purpose of providing a setup for testing the impact of various parameters and how they affect the model. The parameters in the base case are chosen based on numbers observed in the financial market so that they best mimic what we should see. Real-life funds can vary wildly on the parameters in the model, so the model results should not be used to explain a specific fund but rather be taken into account in an aggregate view. Furthermore, as the PE industry is changing fast due to a large and still increasing degree of competition and innovation (S. Kaplan & Strömberg, 2009), it is hard to say what constitutes a steady state in the future and thus the validity of model predictions.

In this section, the effect of α, β_U, leverage, and volatility will be analysed. The analysis does not examine the effect of various levels of fee structures, as these do not affect the behaviour in the simulation, but only how the proceeds are distributed. Thus altering these would just shift the distributions slightly, but not alter the nature of the distributions. Furthermore, parameters such as the risk-free rate and public volatility are not controlled by GPs but are instead exogenous to them. Our key insights from the base case of 'normal' time are

- LP returns are characterised by a strong mode around what constitutes full payment of the preferred return. The distribution around this mode is then symmetric and almost linearly decreasing.
- GP compensation is driven largely by whether or not the preferred return is beaten, as claiming the catch-up provision quickly can double the compensation received.
- Average holding periods seems to follow that of the market, averaging around 4.5 years matching the insights of figure 4.

When running base-case scenarios on the key parameters in our model, i.e. $\alpha, \beta_U, \frac{D}{E}$ and σ_S, we obtain the following insights,

- GPs' skill, measured by α, drive relatively large benefits to GPs while having slightly more modest effects for the LPs on expected return. However, α greatly reduce LPs' downside risk.

- β_U provides predominantly more risk to the LPs, and as such, is predominantly bad for them. Contrary, the GPs gain a higher average return from investing in high β_U assets for given leverage. This might lead to moral hazards if contractual covenants are weakly defined.
- Similarly to β_U, leverage provides a large degree of benefits for GPs.
- For LPs, the result is less linear, with low degrees of leverage the marginal benefits are high, as more funds receive the hurdle rate. However, for high leverage, the effect goes increasingly into more volatility of LP return.
- Volatility of the underlying public asset, all else equal, is bad for all investors, as debt pricing increases with it, and the effects of α are attenuated down. Effects are strongest for GPs as LPs have downside protection in their preferred return.

7.1 The standard fund

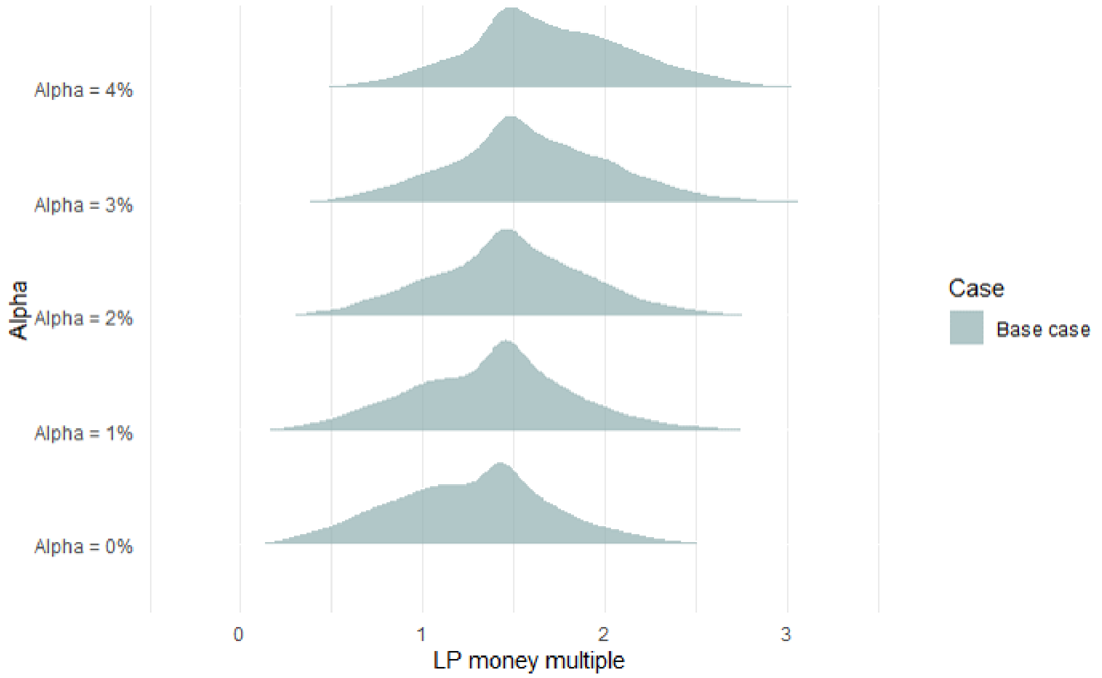
The standard fund is based on the parameters shown in table (2). Results from this simulation will be the frame of reference used from now on in this paper, unless otherwise explicitly noted. To examine the behaviour of the private equity model, simulations will also be run for changing levels of some of the key parameters in the standard fund.

In the middle of figure (16), the distribution of money multiples received by LPs for the standard fund can be seen. The distribution for the standard fund is quite symmetric, with a significant mode around the level that constitutes the preferred return being paid in full.

The distribution of GP compensation can be seen in the middle of the figure (17). For GPs, the distribution of compensation can be broken into two parts. The first part being the mode on the lower end of the distribution, which is GPs only earning management fees. The second part is the long upper tail, which is initially increasing, getting thicker, before thinning out. The explanation behind this form is the nonlinearity present in the GP payoff profile, previously shown in figure (12). The "hump" in the upper tail in the distribution is the area in which GPs are claiming their catch-up provision. This special distribution shape means that it is incredibly valuable for GPs to beat the preferred return, as it quickly doubles the compensation they receive.

7.1.1 The effect of α on the Private equity fund

Figure 16: Base case LP MoM for varying α



In figure (16), the distribution of the multiple of money received by LPs in the base case scenario is shown for various levels of α. The mode[28] is the same for all simulations, representing full payment of the preferred return.

As α increases, the lower tail of the distribution gets thinner and the upper tail gets thicker. Thus α switches returns to both have a higher expected level as well as skewing results to more positive outcomes, resulting in significantly lower downside risk for LPs.

The variances in the distributions do not seem to change a lot with α, else than higher values just shorten the lower tail and lengthen the upper tail. α seems to have a rather consistent effect, meaning that going from 0% to 1% and going from 3% to 4% has more or less the same effect. If you increase α much, you would, nonetheless, see that the mode would change toward the mean of the distribution. For a one percentage point increase in α the median of the distribution increases by roughly 0.07x-0.10x. The effect is even higher when it comes to the mean, due to

[28]The mode is the value that appears the most in the given population

Table 4: α sensitivity in the base case model: $\beta = 0.65$, D/E = 2, $r_f = 2\%$, $cs = 3\%$, $\mu_S = 7\%$, $\sigma_S = 40\%$[27]

	$\alpha = 0\%$	$\alpha = 1\%$	$\alpha = 2\%$	$\alpha = 3\%$	$\alpha = 4\%$
LP MoM (Based on invested equity)					
Min.	-0.14	-0.12	-0.03	0.03	-0.05
1st Qu.	0.95	1.07	1.19	1.31	1.40
Median	1.30	1.41	1.48	1.56	1.66
Mean	1.27	1.38	1.49	1.60	1.70
3rd Qu.	1.56	1.66	1.77	1.89	2.01
Max.	3.51	3.24	3.38	3.51	3.74
Std dev	0.46	0.46	0.47	0.47	0.48
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.16	0.17
Median	0.16	0.17	0.25	0.28	0.31
Mean	0.22	0.24	0.26	0.28	0.31
3rd Qu.	0.28	0.31	0.34	0.37	0.40
Max.	0.78	0.70	0.74	0.76	0.82
Std dev	0.09	0.10	0.11	0.12	0.13
Avg. holding period (Years)					
Min.	2.48	2.45	2.48	2.20	2.20
1st Qu.	4.18	4.10	4.03	3.95	3.88
Median	4.65	4.58	4.50	4.43	4.33
Mean	4.68	4.60	4.53	4.46	4.38
3rd Qu.	5.15	5.08	4.98	4.93	4.83
Max.	8.18	7.75	7.60	7.05	7.30
Std dev	0.73	0.72	0.70	0.70	0.68

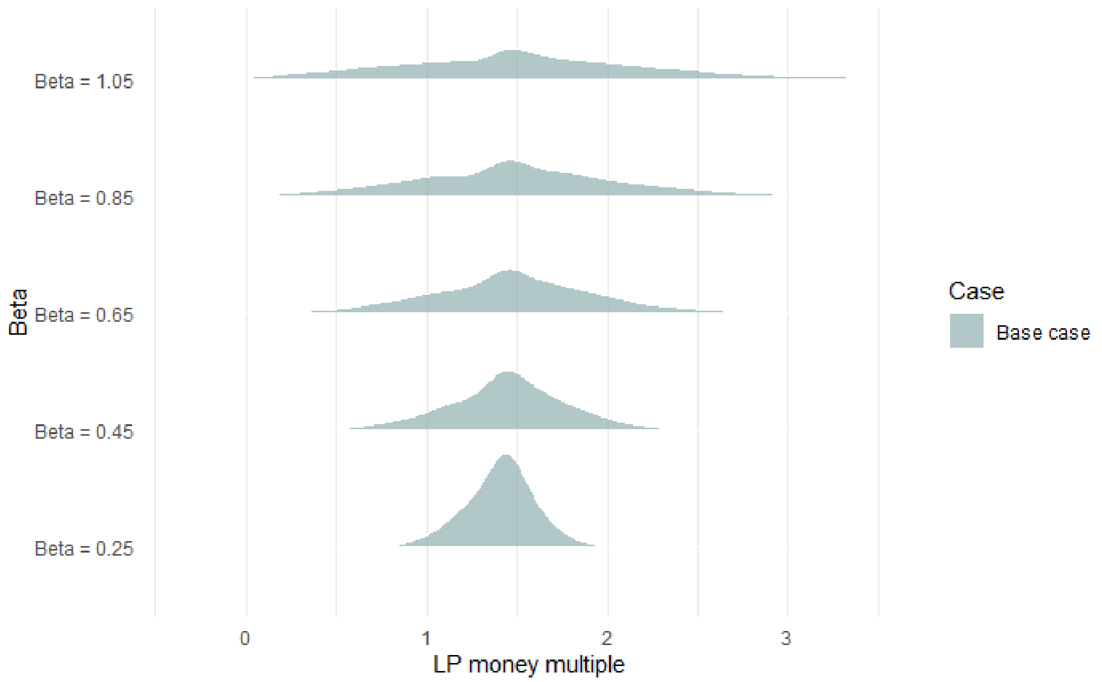
Figure 17: Distribution of GP MoM simulation results for varying α

the mode, as a one percentage point increase results in an increase in the money multiple of 0.10x-0.11x. The mean, median, and mode of the distribution are almost identical with α at 2%, with the mean going from being lower to being higher than the median when α is between 1% and 2% as seen in table 4.

As seen in figure (17), proceeds going to the GPs are having a different shape than the MoM for LPs. This is due to the characteristics of the contract between the GPs and LPs, where funds not paying the full hurdle rate will only earn management fees, while funds performing well receives significantly higher compensation as they move into the catch-up period. Figure (17) shows the same effect of α seen in figure (16), i.e. increasing levels of α moves more funds from the lower end of the distribution to the higher end of the distribution. Not surprisingly, increasing levels of managerial skill result in both more high results, as well as more extremely positive observations. For extreme levels of α, the median, mean, and mode converge as well.

α is slightly affecting average holding periods, as these are shorter for more skilful managers. A high skill manager has a quarter shorter average holding period. The level of skill in the manager has almost no influence on the variance in holding periods, as α does not affect the variance in the asset paths, but only steepens the drift term.

Figure 18: Base case LP MoM for varying β_U



7.1.2 The effect of β_U on the private equity fund

Increasing β_U increases the volatility of the asset through increased covariation with the market. Figure (18) shows that β_U induces significantly higher levels of volatility in the money multiple received by LPs. The expected return on an asset goes up due to increasing the impact of the drift term for smaller increases. β_U also increases volatility of each investment along with the portfolio as a whole. Higher levels of β_U put debt holders at risk and lenders will demand higher credit spreads to cover their down-side risks when assets become riskier. This means that returns only increase slightly compared to the impact of the increased risk associated. The mean of the money multiple increases about 0.15x between the low beta and the high beta funds, and the impact on the median is lower with an 0.085x increase between the funds (appendix: table 9). Thus higher levels of β_U are not useful to LPs, as they are only achieving marginally better returns, but with significantly higher risk. In table 9 (appendix), it can be seen that the credit spread is increasing rapidly with β_U, thereby making it an ineffective parameter to increase returns.

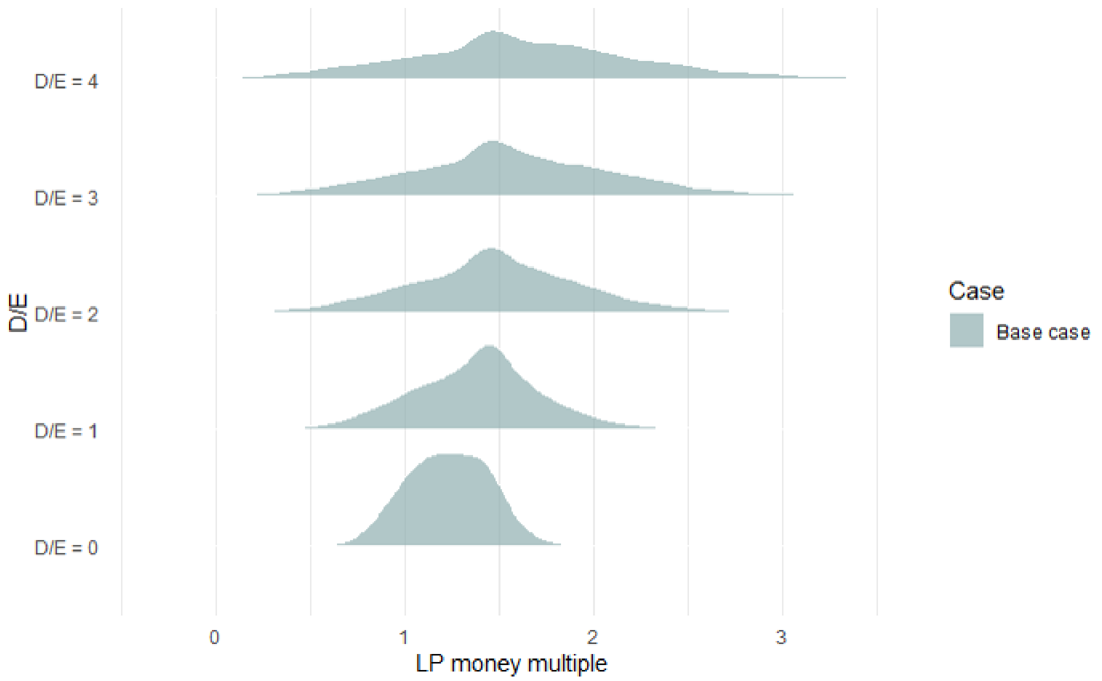
For GPs β_U seems to be a much more important driver of returns. Increasing β_U increases the volatility of GPs' money multiple, similarly to the effect observed for LPs. However, GPs

Figure 19: Base case GP MoM for varying β_U

have much more to gain for increased volatility, as they have a downside cap in terms of fixed management fees, but a huge upside if they are able to beat the hurdle rate and claim the catch-up provision and carried interest. In figure 19, it's seen that for low levels of β_U only a few GPs will achieve returns allowing them to claim the entire catch-up provision. Therefore, their expected returns are quite low, which makes it severely more unattractive to be a GP, even if you have some managerial skill and can utilise leverage. Increasing the variance slightly by investing in assets with a little higher β_U can then significantly increase the chance that GPs can claim the catch-up provision and carried interest, making them highly inclined to invest in these slightly higher β_U assets. Looking at figure 19, it can be seen that the upper "hump" is getting noticeably bigger up until a β_U of 0.65, whereafter the "hump" stops getting bigger, and only the tail is getting longer. Both the means and the medians of the distributions also make the case for having some degree of covariation with the public market. The increase in both median and mean when going from a β_U of 0.25 to a β_U of 0.65 is almost identical to the increase seen when increasing α from 0% to 2%. The same effect is not visible when increasing β_U any further, as levels above 0.65 barely raise the median.

The various levels of β_U only have a minimal effect on average and median holding periods

Figure 20: Base case LP MoM for varying leverage

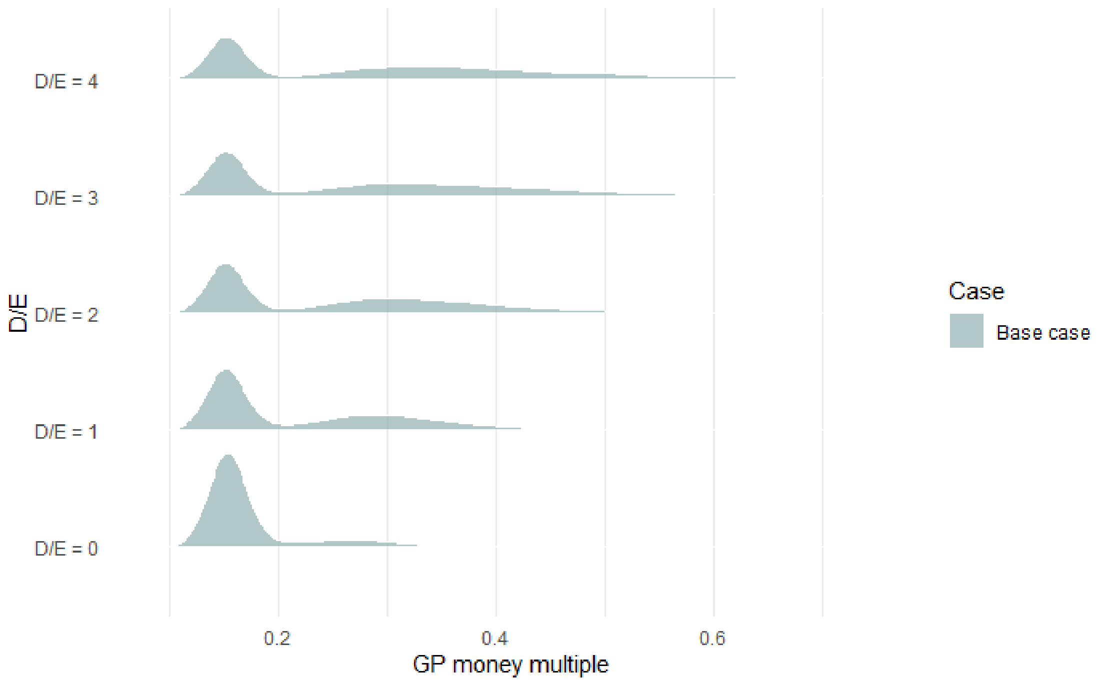


(appendix: table 9). Both average and median holding periods decrease until β_U reach 0.65, whereafter holding periods begin increasing again. However, in total, the difference is no more than approximately one month. β_U also affects the volatility in holding periods, as every increase in β_U increases the difference between good performing and poor forming funds, so is the volatility in holding periods increasing.

7.1.3 The effect of leverage $\frac{D}{E}$ on the private equity fund

Leverage, like β_U, is a way of trading off higher expected returns for increased riskiness of an investment, which also increases the interest paid on the debt. In figure 20, it appears, as with β_U, how increasing levels of leverage drastically increases the volatility in LP money multiples. However, the development in the median and the mean makes a more positive case for leverage, than it did for increasing β_U. Increasing the level of leverage increases median LP money multiples significantly, which was not the case when increasing the level of β_U. The benefits of leverage are so significant, that having a debt to equity ratio of 0 instead of 2 results in a bigger loss for the LPs, than having 0% α instead of 2% α. The marginal effects of leverage are decreasing, as bankruptcy risk is increasing and with it, the credit spread of debt. One thing to

Figure 21: Base case GP MoM for varying leverage

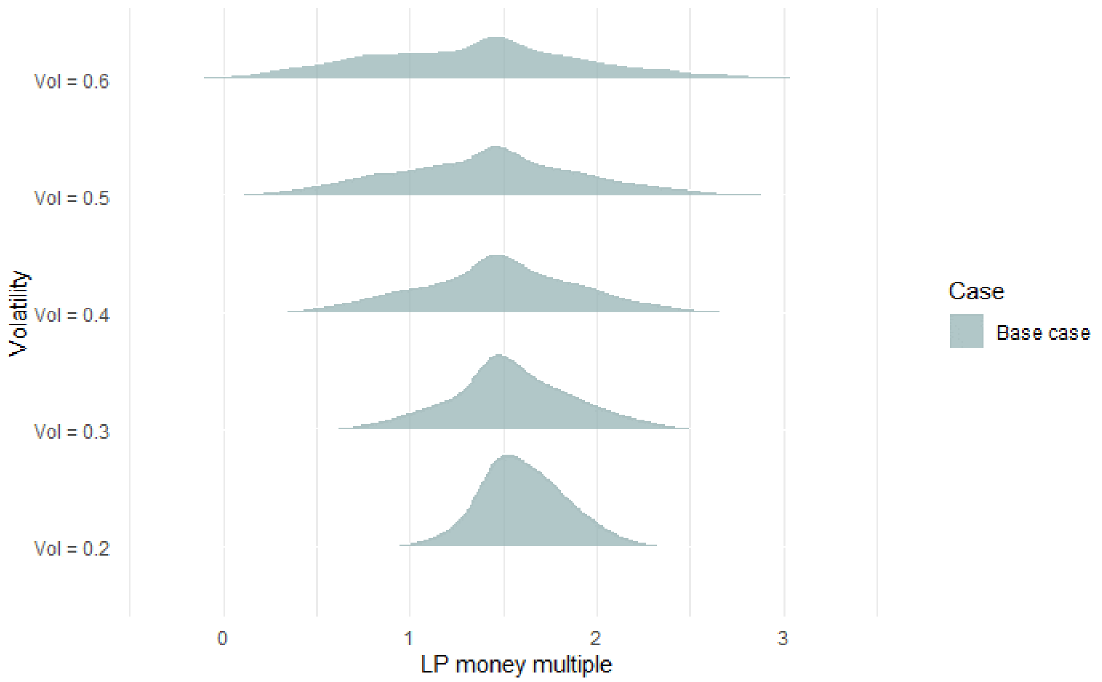


notice is also that the maximum outcome increases more than the minimum, which skews the distribution.

Figure 21 shows how deeply beneficial leverage is for GPs, as it is very unlikely for them to claim the catch-up provision without it, even with some managerial skill included in the case. The upper "hump" is non-existing without the use of leverage, and it gets significantly thicker as leverage increases. Increasing leverage has high impact on mean and median money multiple earned by GPs (appendix: table 8). Unlike in the case for high levels of β_U, increasing leverage, even to high levels, are of great benefit to the GPs, as both median and means increase significantly.

Leverage also impacts average holding period of a fund, with the majority of the impact coming when leverage is increased from zero to two. This results in a decrease in average holding periods of approximately four months (appendix: table 8). The higher volatility for highly levered portfolios does not seem to have a major impact on the volatility of the average holding period, rather this remains relatively constant for various levels of leverage.

Figure 22: Base case LP MoM for varying volatility



7.1.4 The effect of public volatility on the private equity fund returns

To ensure proper workings of the model, the simulation is also run for different levels of public asset volatility which affect the private asset through β_U. Since PE funds use leverage and do so in nearly all scenarios simulated in this paper, volatility should have some effect. It is a parameter in the Merton model thus having an effect on credit spreads. Therefore, higher volatility drives up credit spreads and reducing means and medians for higher volatility, as leverage becomes more expensive.

Figure 22 provides the insight that distributions of the LP money multiple are becoming wider as volatility increases, while the distribution shifts slightly to the left. With low volatility, nearly every PE fund provides positive returns and beats the hurdle rate[29].

For low and median levels of volatility, nearly all difference between means and medians can be explained by the difference in credit spreads (appendix: table 10), meaning that mean and median valuations are similar, but debt holders are taking a bigger portion of the value. When looking at the difference between low and high levels of volatility, another aspect of equity

[29] Technically, this is driven by a certainty of the drift term

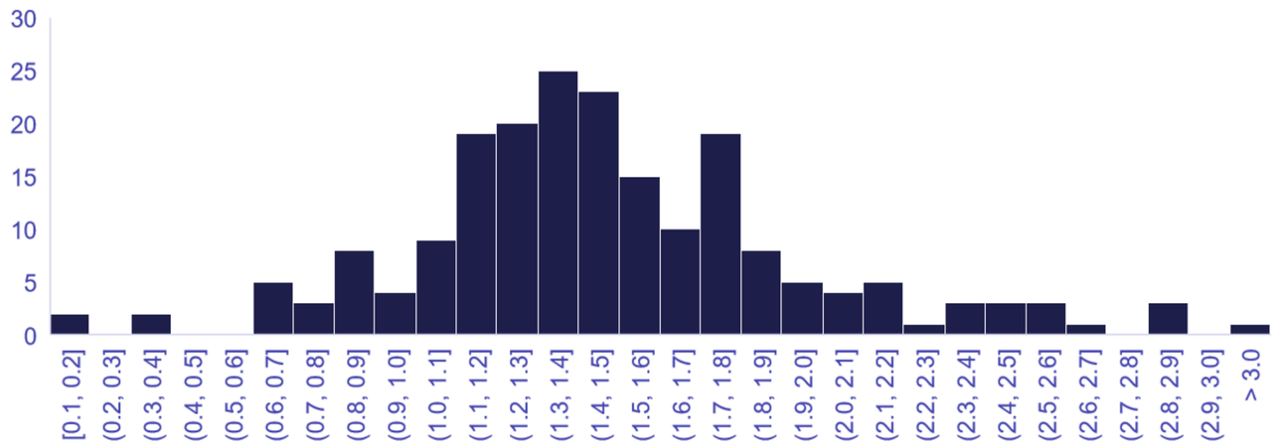
Figure 23: Base case GP MoM for varying volatility

investing becomes relevant, which is that equity is similar to a call option on a firms assets, as equity cannot go below zero[30].

For GPs, increased volatility is not attractive either, as can be seen in figure 23. When expected returns are higher than the hurdle rate return, increased volatility is not beneficial to GPs, due lacking of downside protection. However, GPs do benefit from a capped downside and infinite upside, meaning that higher volatility is extremely beneficial to a few "lucky" GPs. This effect can also be seen in the different impacts on mean and median GP compensation. Median GP compensation is declining much faster as volatility increases, whereas mean compensation only drops slightly.

[30]LP money multiples below zero are caused by payment of management fees even when all equity is lost or remaining equity is lower than management fees paid

Figure 24: Distribution of net Multiple of Money for CalPERS funds



Source: California Public Employees' Retirement System (CalPERS) (2020)

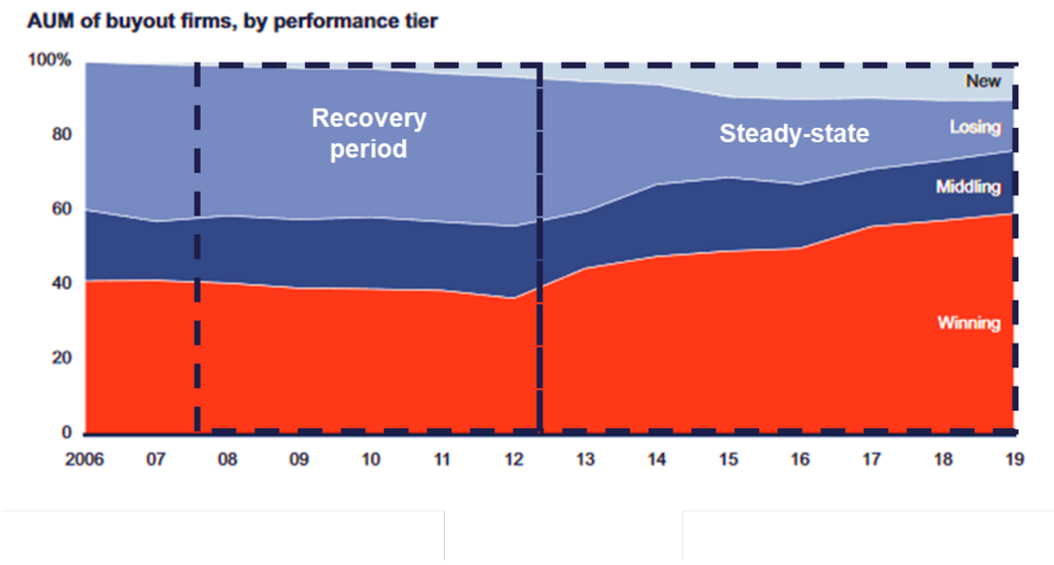
8 Discussion of base case results and other empirical results

The most important point of interest is how our base case compares to that of the actual LBO returns. Data on LBO fund returns from California Public Employees' Retirement System (CalPERS) (2020) provide an average fund Net Multiple of 1.6x, slightly above our estimation of the standard fund providing 1.5x. This could be driven by differences in volatility, β_U, α or $\frac{D}{E}$. Also, our estimations assume that all funds have the same characteristics, but obviously, parameters such as fund managers' skills are expected to be different across funds. Nonetheless, comparing figure 24 with our base case result, the shape of the distribution is pretty similar, and the mode around the full payment of the hurdle rate appears in both. However, the CalPERS data include funds negatively affected by the financial crisis which explain the outliers with very low returns. Additionally, the standard deviation is slightly lower for CalPERS return, which could mean that the degree of β_U, leverage, or underlying volatility is higher in our model simulation.

8.0.1 Holding period and investment management

The holding period in normal periods is similar in our model to that of the observations made by Bain & Company (2019), Jenkinson and Sousa (2015), Minardi et al. (2019). This is unsurprising as our model consists of variables similar to those of their findings. Also, our model parameters

Figure 25: Asset under management by performance tier in PE industry



Source: Bain & Company (2019)

are calibrated to match these results, and therefore we cannot perform meaningful inference on our findings regarding holding periods.

8.0.2 The importance of α and its implication

A very important feature is the waterfall scheme between the LPs and the GPs, as it highly discourages low-skill managers to enter the market. First of all, the LPs benefit largely from this. Using the findings of Korteweg and Sørensen (2017), LPs would, to some degree, be able to infer the skills by assessing the past performance of funds. A low-skill manager has the median value residing much lower, with most managers below the expected value, and a few "lucky" managers getting high returns. Most skilled managers earn well above the mean, which is dragged down by a few "unlucky" managers. Bain & Company (2019) find strong evidence to support these effects, as the long-term participants in the LBO market have much higher returns than the rest. Similar results are presented by Korteweg and Sørensen (2017), who show that funds do actually experience persistence in returns, indicating that manager skill is a strong component of participation in the industry. Bain & Company (2019) also finds that in normal times, PE firms with the highest performance, which a proxy for skill, tend to increase their respective markets share as shown in figure 25.

8.0.3 The importance of β_U, leverage and volatility of PE returns

The prime impact of increasing covariance of the underlying asset with the market is that volatility increases for the return of LPs. Due to risk-aversion among investors, β_U is bad for limited partners. These effects are scaled by leverage and thus much stronger for LBOs than GRE (and VCs). This could mean that funds focusing on low β_U assets would be more prevalent for LBOs according to our model. However, some degrees of β_U are significantly better for GPs, and this could constitute potential moral hazards for the fund partnership. As our model does not discuss or consider these potential issues within the industry, it can be difficult to apply the insights to further understand the implied consequences of β_U on the incentives.

Leverage is a similar story, however, increasing leverage is much more attractive to LPs than increasing the risk of the underlying asset. This is an interesting result. Even as we haven't modelled valuation benefits of debt, leverage provides great results for the PE fund. S. Kaplan and Strömberg (2009) suggests that similar observations can be made in empirical studies, and it might explain why, as shown in figure 2, the LBO has become the largest private equity vehicle. However, this discussion is outside the scope of the model in question.

Lastly, and quite importantly, the assumption made about the underlying public volatility is essential in understanding, what drives the results of our paper. For GPs, the effect is highly significant. While being a technical feature of the GBM process, it is also a testament to how heavily the returns of the industry rely on stable markets. Bain & Company (2019) suggests that uncertainty of the financial markets could be the reason why private equity is present in the more stable developed markets, with only China having a large PE sector with GRE.

9 Definition, Characteristics and historical perspectives on Financial Crises

As the key purpose of this paper is to identify, understand, and value the expected behaviour of private equity funds in times of crisis, it is important to provide a broader discussion on what a crisis itself constitutes. In the economic literature, the terms economic and financial crisis are used interchangeably, and with the broadest definition of a financial crisis (Eichengreen & Portes, 1987),

> *A financial crisis is any significant disturbance to financial markets, associated typically with falling asset prices, ..., which spreads through the financial system [and potentially spilling over into the general economy], disrupting the markets' capacity to allocate resources*

An economic crisis is in this definition a sub-type of a financial crisis, in which the disturbance to the asset pricing and resource capacity becomes a long-term capacity problem and the financial crisis turns into a recession. The widely used definition of a recession is that the development of the economy has gone from having a positive to a negative growth rate for two or more consecutive quarters. Further, if the recession is very persistent or effectful, the financial crisis goes into a depression period. Depressions are not exactly defined, however, consensus state that depressions are when the growth rate is less than -10%, lasts more than 12 months, or that economic growth disappears for a longer.

9.1 Recession shapes and a business cycles perspective of a crisis event

Another way of understanding a financial crisis is to view it as a part of the business cycle. The literature is trying to understand business cycles, with several schools of thought varying from neo-classical theories such as the theory of real business cycles, neo-Keynesian theory, over to more heterodox theories. However, the notion of cycles suggests that there exists a structure or pattern in which the economy develops. Schumpeter (1927) developed a very simple yet broadly used periodical model in which business cycles can generically be broken into four periods (Expansion, Crisis, Recession, and Recovery). This construction of the business cycle has been criticised by especially the neo-classical literature. In an efficient market with rational agents, the existence of a well-defined business cycle would be anticipated and thus a well-defined cycle would cease to exist. This means that some unanticipated events such as an exogenous or endogenous shock must be the triggering factor to the crisis. Also, not all crises are followed by a recession (for example the 1987 crash also called Black Monday) nor do financial bubbles and economic expansions necessarily precede a crisis. Applying insights from Birch Sorensen and Whitta-Jacobsen (2010), this paper will use a modified periodic definition of a crisis event (in bold) in a business cycle,

1. Asset prices follow a steady-state growth path at rate μ (Historically, S&P500 has grown at 7%).
2. **A shock affects the markets and cause a financial crisis with asset prices going down and volatility increasing.**
3. **The shock manifests itself causing either an economic recession with less demand or short-term recovery. Volatility goes down as the impact of the shock becomes more evident.**
4. Return to (new) steady-state with low volatility and stabilised economic growth.

The first period is when the economy and thus the dependent financial assets follow a steady-state growth path. In period 2, an exogenous or endogenous shock hit the financial markets causing valuations to decrease, uncertainty and volatility to increase, credit to dry up, thus options for leverage go down. This period is followed by a period in which the shock manifests itself in the markets. For example, if the shock is short-term and has a limited impact on the underlying economic behaviour, the period after could be above pre-crisis growth to catch up some of the lost demand. This type of crisis is called a V-shaped crisis event and illustrated in figure 26. Alternatively, the crisis affects the market more long-term and will thus more gradually tend towards the steady-state growth path again. This type of crisis is called a U-shaped crisis as illustrated in figure 27. If the new growth path follows a lower rate, the crisis event is L-curved. This type of crisis is significant as the crisis event affects the long-term growth of the economy and its dependent assets. If the initial shock is the same size for all types of crises, then the V-shaped is the least affect-full while L-shaped in the most serious type of crisis.

Figure 26: V-shaped financial crisis - Growth path(First graph) and Growth rate(Second graph)

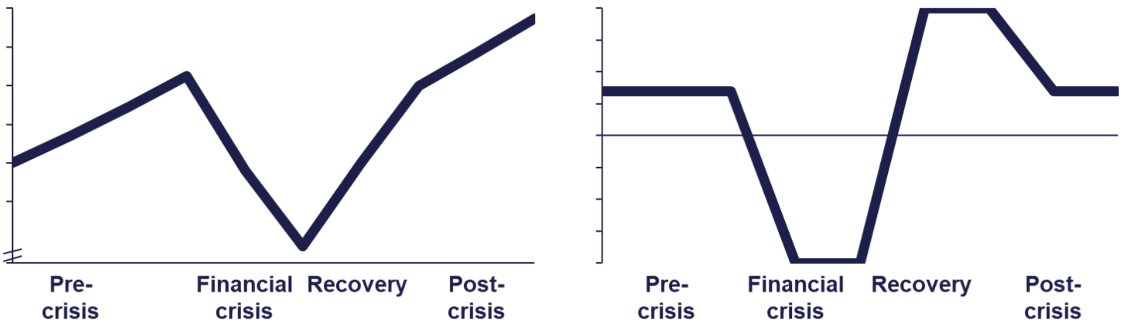


Figure 27: U-shaped financial crisis - Growth path(First graph) and Growth rate(Second graph)

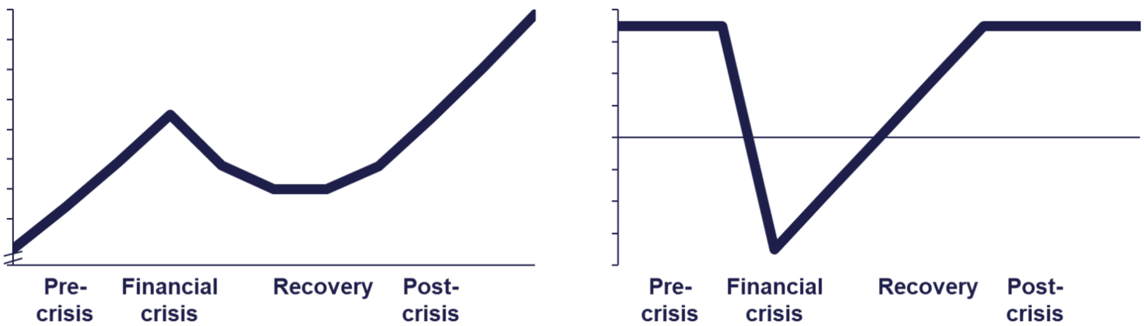
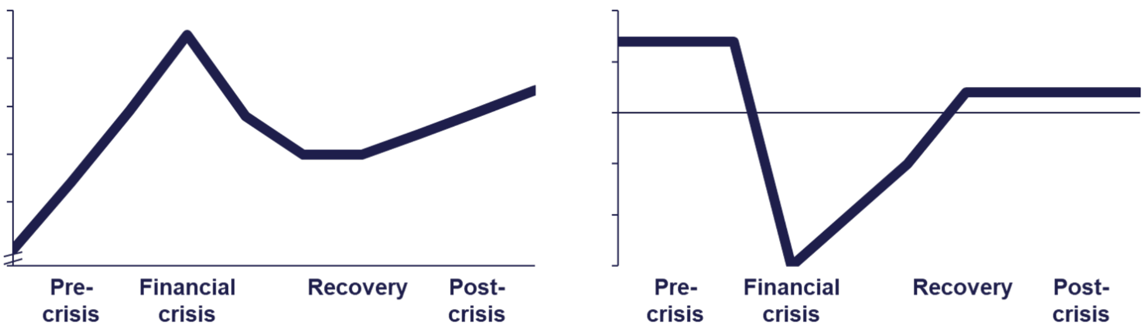


Figure 28: L-shaped financial crisis - Growth path(First graph) and Growth rate(Second graph)



9.2 Linking financial crises and asset valuations

Going back to equation (1), we know that there are three different avenues in which the valuation of assets can be impacted. These are, the future expected cash flows generated by the asset, the risk-free rate, and the asset-specific risk premium. In this section, we will elaborate upon how a crisis event could affect each of these components of asset valuations. Nonetheless, for the purpose of modelling a crisis in asset prices, we will view the impact of these on an aggregate level, thus not assuming the underlying specifics.

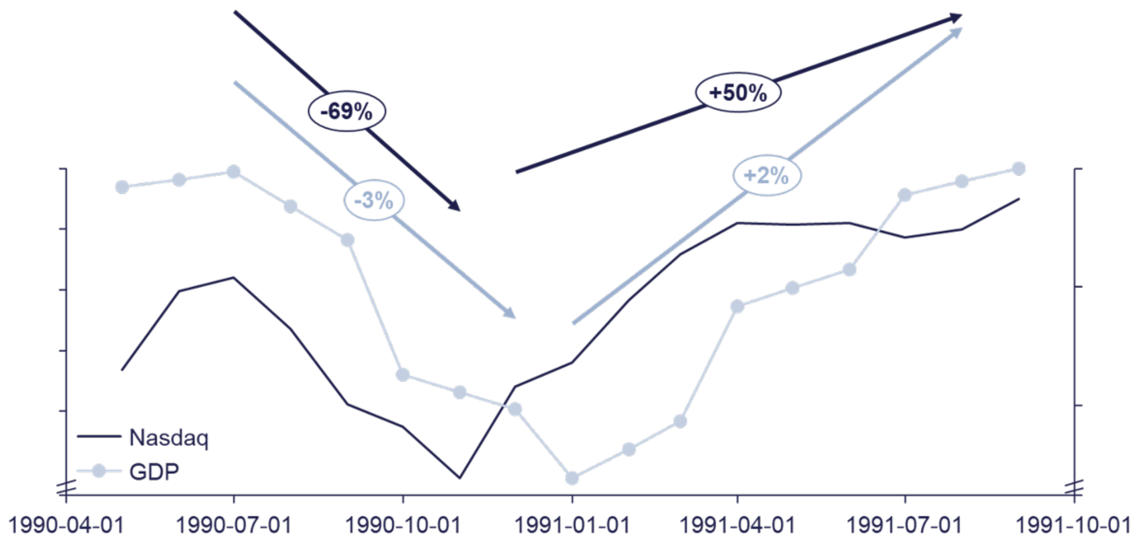
The economic impact from financial crises onto asset pricing is not one-to-one equal to the impact on cash flows. Cash flows are likely to be negatively impacted as economic activity slows down (Birch Sorensen & Whitta-Jacobsen, 2010). Initially, a crisis implies short-term slowdown, but it might also have long-term impacts. The short-term problems experienced in the crisis might lead to curbing of investments, as businesses focus on keeping afloat.The foregoing of long-term investments might then lead to a furthering of the crisis. An example of this is the L-shaped Japanese crisis which is described in section 9.2.3, or the more recent government debt crisis in Greece.

Monetary policy and interest rates changes are responses by central banks to a crisis event and a reason why asset pricing might experience a rebound, even if there are no changes in the underlying economy. The risk-free interest rate is a theoretical measure, often with government bonds used as a proxy. Central banks control interest rates and can issue more government bonds and acquire public assets through quantitative easing (Mishkin, 2015) to infuse or take out capital from the monetary system. In the 07-09 financial crisis and subsequent recession, asset prices had a significant rebound while GDP only slowly recovered to its initial growth rate.

Risk premia, ε, as with the risk-free rate, are not observed directly in the market either, and they are quite difficult to estimate accurately. A plethora of tangible and intangible components affect risk premia, such as risk aversion and the plurality of investment opportunities (Damodaran, 2013). In a crisis, uncertainty about the economic future is rising which also increases the riskiness of assets. Additionally, risk aversion might increase, as Guiso et al. (2013) found evidence of in Italy after the 07-09 financial crisis.

Interest rates and risk premia affect the value of a company as they are used for discounting.

Figure 29: Impact of the 1990-1991 recession on asset prices and GDP (Indexed prices)



Source: U.S. Bureau of Economic Analysis (2020)

Reduced interest rates increase demand for riskier assets whereas increased risk premia have the opposite effect. If either is temporarily affected, a swift crisis event, such as the V-shaped, might occur, even if the underlying economy is unaffected. Also, they explain why a crisis in asset prices might become more potent than a crisis in the general economy. Examples of this is observed in figure 29, 30, and 31.

To elaborate on the notion of crises shapes, it is helpful to put them into historical context. Financial crises have hit markets globally and locally throughout history, often with very different origins, effects and recoveries. These historical events can thus help in better understanding how specific crises manifest themselves, and whether they affect private equity.

9.2.1 V-shaped crisis and the US recession of 1990-1991

The recession of 1990-1991 began in the summer of 1990 in the United States as GDP declined over 6 months. The impact was swift with GDP declining 3% on an annualised basis as seen in figure 29. Underlying problems were appearing in the economy as the expansion came near its end affecting the perceived riskiness of the future, as well as expected cash flow growth (Frumkin, 2010). New job growth became significantly weaker in the year prior and industrial production only experienced modest growth in the 18 months leading up to the recession. Nominal wages were growing, but high levels of inflation reduced the spending power of consumers. Both interest rates and the federal funds rate declined, but tougher credit standards were being applied thereby tightening credit for businesses (Frumkin, 2010). In line with the theoretical description of V-shaped crises, the recovery came equally swift. Following six months of decline, GDP began increasing rapidly for the next six month.s

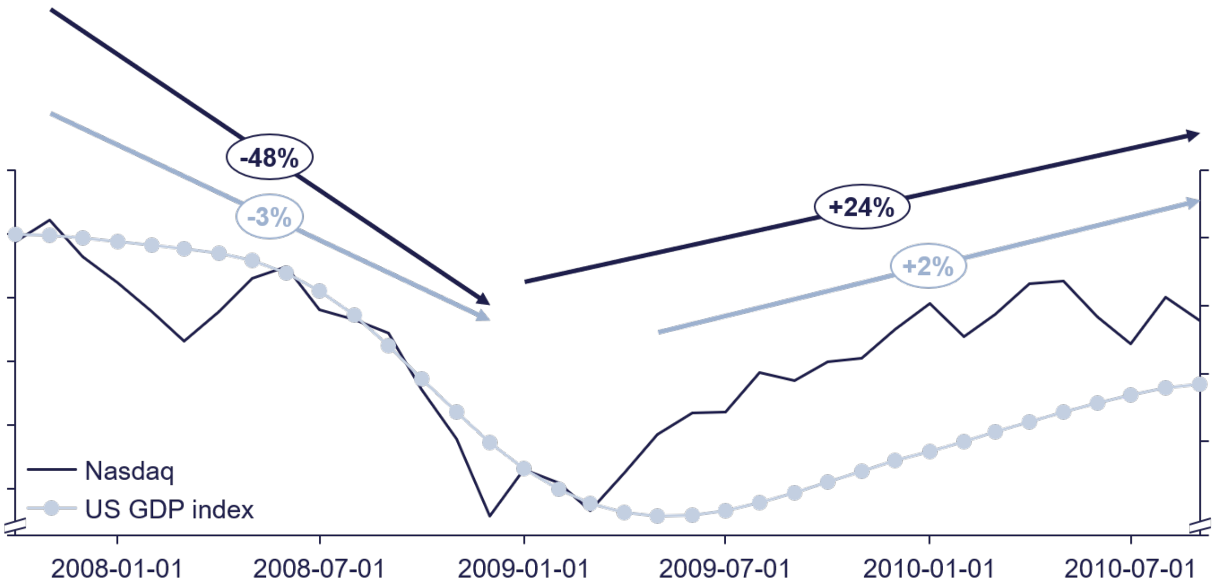
For the impact on asset prices, the decline in GDP prompted a simultaneous decline in the NASDAQ Composite Index, an index of 2500 securities listed on the NASDAQ exchange. The decline in the NASDAQ index was equivalent to an annualised negative rate of $\sim$ 70%. The NASDAQ then began rallying, just as GDP, with the NASDAQ increasing at an annualised rate of $\sim$ 50%. The long-term impact on the index was insignificant.

9.2.2 U-shaped crisis - the financial crisis of 2007-2009

In the late 2000s, the worst financial crisis since the Great Depression hit the US economy. A real estate bubble had build-up, and as the crisis hit, around 44% of mortgages were defaulting prone (Tarr, 2010), an unprecedented figure in US history. The burst of the real estate bubble drove a large proportion of the crisis, however, very high valuations of asset prices in general were also strong drivers, sharp declines in both assets and economic activity ensued (Tarr, 2010). Figure 30 shows the impact the financial crisis had on the economy and on financial markets. By the end of 2007 GDP began to decline. It went on a year-long negative growth path, declining $\sim$ 3%. With the economy moving into a recession, the NASDAQ fell at an annualised rate of $\sim$ 50%. A drop was driven by a sharp decline in expectations of future cash flow, as well as some degrees of panic in the market (Damodaran, 2013), which in our framework can be related to risk premia.

The recovery began in early 2009, with the upward growth period being slightly longer than the recession. As the recovery ended, GDP had recovered its long-term growth rate of 2%, but without any rebound. The development in the NASDAQ followed the movements in GDP, however, went on a bull run as the economy was in its recovery period, increasing at an annualised rate of 25%. Among other things, this was driven by monetary policies.

Figure 30: Economic and financial impact of the financial crisis of 2007-2009

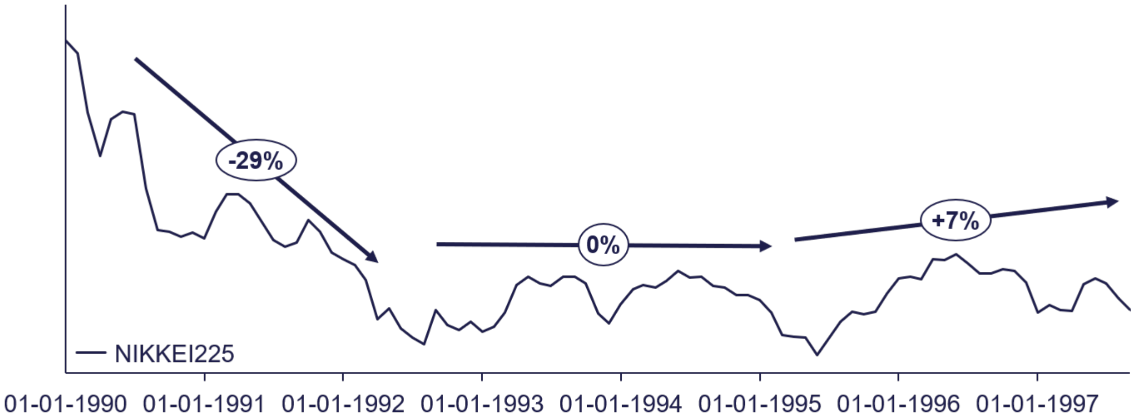


9.2.3 L-shaped crisis - the Japanese asset price bubble

By the end of the 1980s and beginning of 1990s, the high growth Japanese economy was deteriorating rapidly. What had previously been a driver of investor returns globally, turned into a long period of low and no growth which came to be known as "the lost decade". Despite the high growth rates, the Japanese economy only experienced modest inflation, and the Japanese banks were considered some of the safest in the world. However, in the late 1980s, government fiscal policies created drastically increasing budget deficits. Equity and real estate prices rose drastically as well.However, in this period of economic euphoria, the quality of debt deteriorated, and both financial and non-financial firms ended up being exposed to significant risk. When monetary policy tightened, the majority of wealth accumulated vanished. As the recession dragged on, recovery was nowhere in sight. To help the economy a series of fiscal stimulus packages were created, but none were effective. Japan, therefore, amassed a huge public debt (Siam-Heng, 2010). The inability of policy to combat the recession caused it to affect the Japanese economy for more than a decade.

Uncharacteristically, the bursting of the asset price bubble and the following lost decade was not due to a decade of negative GDP growth. Remarkably, in 1990 the leading stock index in Japan

Figure 31: 1990s development of the NIKKEI 225



(the NIKKEI 225) started dropping rapidly as seen in figure 31, however, the Japanese GDP still rose nearly 5%[31]. From then on GDP slowed down, reaching negative growth in 1993, far from the high single-digit growth rates, the Japanese economy had grown accustomed to. The previous high growth rates never returned, and the economy went from soaring to just slugging along, experiencing only modest growth rates for the remainder of the decade.

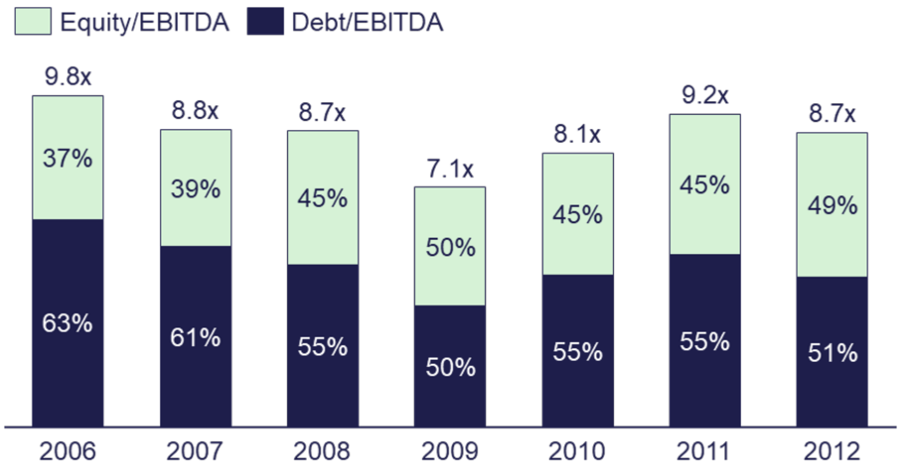
Merely looking at GDP, this crisis is not entirely similar to the L-shaped crisis previously described. However, investigating the NIKKEI 225 instead, the L-shaped crisis becomes evident. As the bubble burst in 1990, the index dropped rapidly for 19 months at an annualised rate of 30%, ending up losing more than half of its value compared to its peak. The bottom was reached near the end of 1992. From the ending of the negative trend, the index remained stagnant, never moving far away from its 1992 level. To this day, nearly 30 years later, the NIKKEI 225 is still just at a level a little more than a half of its 1990 peak, and only around 20% above the bottom of 1992. In comparison, the SP 500 has risen more than 500% in this same period of time.

9.3 Private equity in financial crises

The impact of a crisis event on private equity is manifold. This section will discuss contemporary research of the impact from the 07-09 crisis on PE, and how leverage, credit spreads, and investment behaviour was affected. Due to the relative youth of modern PE and in-frequencies of severe crises events, the research is limited in its scope, therefore its results might lack universality.

[31]Source: Worldbank.org

Figure 32: Valuation and leverage in the financial crisis in the PE U.S. middle market



Source: Lykken (2018)

9.3.1 Leverage and credit spreads in financial crises

One key element of financial crises is that assets become more volatile while losing some of their value. This increased uncertainty will affect the degree of leverage in transactions. More uncertainty increases the risk of bankruptcy thus increasing the credit spread. Thus a PE fund must choose between reducing degree of leverage, paying a higher yield or a combination of the two. The effect will be more profound for funds increasingly exposed to market risk Data extracted from Lykken (2018), as shown in figure 32, suggest that valuations were decreasing as well as leverage going down from 200% to 100% in the 07-09 crisis.

Findings from Eisenthal-Berkovitz et al. (2020) show evidence of increased credit spreads in LBO financing around the financial crisis. The difference before the crisis and during was, on average, 8-16 basis points. A significant difference, although applying the insights from the Merton model, as they do, the difference also means that leverage was decreased more aggressively. Axelson et al. (2010) argue that LBO funds reduce their leverage to keep a fixed credit spread. All in all, evidence is strongest for reducing leverage in a crisis event. Plagborg-Møller and Holm (2017) suggest that the financial crisis led PE firms to reduce reliance on market sentiment. This could mean that β_U in our model would be decreased. Along with their empirical findings, they quote Marc Lipschultz, Global Head of Energy & Infrastructure, KKR[32], who suggested that,

[32]KKR (Kohlberg Kravis Roberts) is the 5th largest PE fund with more than 200 billion dollars assets under

It's hard to argue that the world is anything but uncertain and volatile today. But one response... is to focus more intensely on the variables you can control. You get more intensely focused on businesses where there are levers you can pull that are at least somewhat independent of and insulated from the general economy.

9.3.2 Investments and exits in crisis periods

Evidence from Bain & Company (2019) and figure 4 suggest that the average holding periods of investments made before the crisis tended to be held longer than previously. In the years after the crisis, average holding periods increased by almost a year, which is something captured by the models of Jenkinson and Sousa (2015), Minardi et al. (2019) and already somewhat incorporated by our paper. However, in a crisis, the liquidity of financial markets might decrease significantly, which affect the possibilities of exits. Especially the attractiveness of the IPO seems to be sharply decreasing (Jenkinson & Sousa, 2015; Plagborg-Møller & Holm, 2017), which won't be captured in our model.

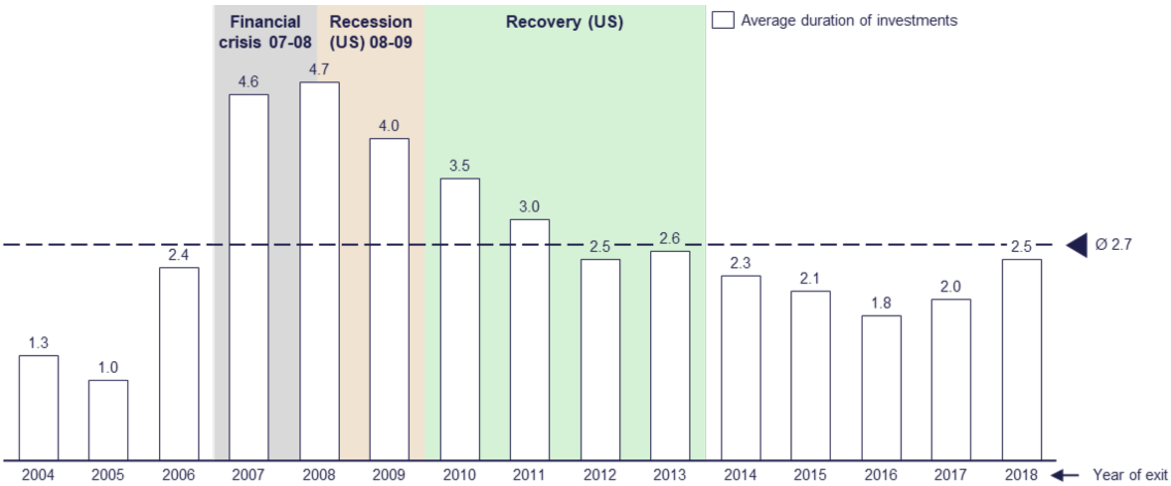
However, the investment activity might be less hampered as many funds would have pre-crisis funding (Bain & Company, 2019; McKinsey & Company, 2020). This allows PE firms to make investments in times where credit markets dry up. However, as described in the earlier section, leverage decreases meaning that size or number of investments in total PE markets will go down. This is broadly observed (Bain & Company, 2019), however, the decrease in leverage and low degrees of funding is not enough to describe to decrease in 07-09. As seen in figure 33, committed capital were held longer in the fund before reverting back to the long term mean of 2.7 years.

9.3.3 Return and risk of private equity in financial crisis

The riskiness and return of private equity, as previously discussed, is difficult to estimate. This is further enhanced in the event of a crisis as underlying volatility increases and β_U values might change. This makes PME measurements more uncertain. However, one estimate shown in figure 34 shows that for U.S. buyout funds, the spread between the S&P500 increased over the course of the crisis, suggesting that PE funds might have had the upper hand. However, estimates from S. N. Kaplan and Sensoy (2015) show the opposite result with lower-than-PME IRRs. The

management

Figure 33: Average duration of committed capital in fund



Source: Bain & Company (2019)

benchmark is, unsurprisingly, very important.

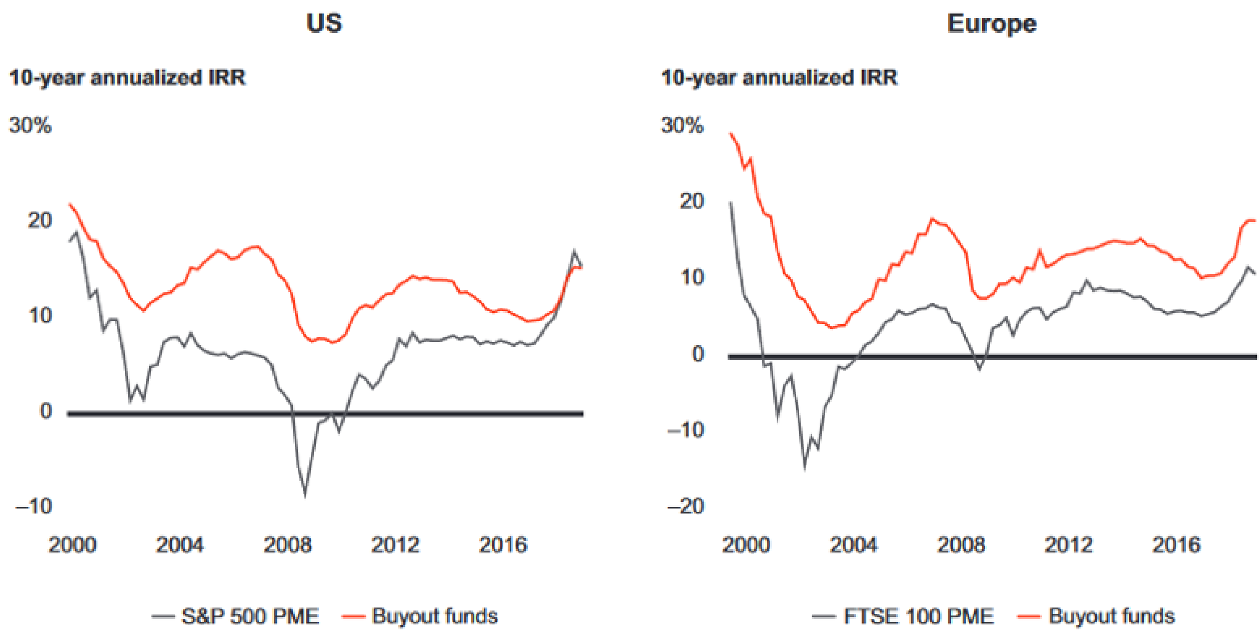
Similarly, the volatility of the assets might increase significantly, however, due to lagged investments and less leverage, the fund level volatility might not increase much, especially if the fund doesn't refinance their debt level for a longer period of time. Compared to the public assets, this might mean that the PE asset gets risk-adjusted higher returns if the crisis hits early (Bain & Company, 2019). Also, if it hits late in the fund life time, many investments might be exited and thus the effect of the crisis is relatively low.

9.4 Model Expansion with a financial crisis

As discussed previously, the basic GBM framework does not allow for alternating variables and as such it does not allow for modelling a financial crisis. Nonetheless, there are several methods that could be considered for implementing and expanding the GBM model applied in this paper. The section will shortly outline the different options in conjunction with defining the specific model expansion of this paper.

First of all, the crisis event can be deterministic or stochastic in the model, i.e. the crisis event can either occur exogenously or endogenously. In macroeconomics, theories of business cycles tend to allow for stochastic crisis events driven by random unanticipated shocks. The advantage of the stochastic innovation is that it allows for the event to be random which is essential in

Figure 34: IRR of LBOs compared to their PME measure



Source: Bain & Company (2019)

valuing financial assets under uncertainty. This allows for a more granular definition of what drives crisis events. A general method to include a crisis innovation was introduced by Merton (1976) in which the volatility is modelled stochastically with autocorrelation to previous periods and the asset prices being affected by large jumps as it follows its drift path. An expansion of the Merton process could be a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model combined with asymmetric jump-diffusion model outlined in LauGohLai2019.

On the other hand, a purely exogenous crisis event allows for control of the timing, impact and structure. This favours models in which you want to compare different scenarios, but provide no explanations on how the crisis occur in the financial assets. The research focus of this paper is to investigate the implications of a financial crisis rather than providing insights on how the crisis came about. This analysis is enriched by providing a set of deterministic scenarios. The structure of the crisis itself is of interest with elements such as duration, whether it has a recovery period and with which timing, it arrives in the market.

This paper will model the crisis' parameters by using the following expansion of the stochastic differential equation from the GBM process,

$$\frac{dS_t}{S_t} = \begin{cases} \mu_{ss}dt + \sigma_{ss}dW_t, & t \in [0:e] & \textit{Steady state} \\ \mu_f dt + \sigma_f dW_t, & t \in [e+1:e_1] & \textit{Financial crisis} \\ \mu_e dt + \sigma_e dW_t, & t \in [e_1+1:e_2] & \textit{Economic reaction or Recovery} \\ \mu_{ss^*} dt + \sigma_{ss^*} dW_t, & t \in [e_2+1:T] & \textit{Return to Steady state} \end{cases} \tag{42}$$

The randomness of a model is useful to provide knowledge of the riskiness of a investment, but this framework deliberately eliminate any randomness of crisis structures. However, asset paths are still random and even if the drift term becomes significantly negative, some assets will perform very well, similarly to what we would expect in a real-world crisis. This setup is compatible with the crisis definition previously described in section 9 thus allowing us to test the different implications for type of crisis. We won't implement any changes to our investment model thus keeping it deterministic. This is obviously not aligned with actual observations, however, we do not assume increased funding before the crisis event, which is also observed (McKinsey & Company, 2020).

9.4.1 Limitations of our definition of a financial crisis

As we have specified our crisis model rather simplistically and exogenously, we can conduct scenario analysis of impacts from different types of crises onto fund returns. However, our model must be applied consciously as there are areas where it cannot provide essential insights due to this specification of financial crises.

An example is that our model cannot be used by debt holders to price debt correctly in an environment with a certain risk of financial crises, as we do not include any expectations on this. Not being able to provide a prediction with regard to crisis risks, would under-price the credit spread. Axelson et al. (2010) actually finds evidence that debt holders under-price risk of a downturn in the economy, but insights of this paper cannot be used to infer anything about this behaviour. Examining hot debt markets and asset pricing bubbles, from either rational alterations of risk premia or from market inefficiencies, is outside the scope of this paper.

9.4.2 Adjusting the simulation model to include crises

To analyse the impact of crisis, the basic simulation methodology needs to be adjusted. Equation (42) show how the standard GBM model can be expanded to emulate the impact of a crisis.

Table 5: Crisis parameters based on figure 29, 30, and 31

Parameter	**Recession period**	**Recovery period**
U-shaped crisis		
μ	-0.5	0.25
σ	0.7	0.6
Duration (quarters)	4	5
L-shaped crisis		
μ	-0.3	0.0
σ	0.6	0.4
Duration (quarters)	6	10
V-shaped crisis		
μ	-0.7	0.5
σ	0.8	0.8
Duration (quarters)	2	3

The asset-based simulation model is what allows for this simple implementation of crisis events, as the various impacts of crisis on asset valuations discussed in section 9.2 can be aggregated into an altered drift parameter and volatility. Thereby, there is no need for individual modelling of the different components of asset valuations, thus reducing complexity in the model, while making various crisis scenarios more comparable.

The parameters used to emulate the various crisis shapes can be seen in table 5. The crisis parameters are equivalent to the impact observed from the crisis shapes in real life. Adopting this expansion in the simulation methodology impacts the simulation in a number of ways.

If either the crisis or recovery period coincides with the investment period, this will affect capital structure decisions. The different levels of volatility in the crisis and recovery period will affect the credit spread found by the Merton model. To accommodate this, the model is expanded to allow for GPs to either maintain the credit spread, they had in steady-state by lowering the debt to equity ratio of new investments or to maintain the debt to equity ratio by paying a higher credit spread. In practice, private equity funds predominantly reduce the debt to equity ratio in times of crisis (Axelson et al., 2010), thus holding credit spreads fixed mimics real life the most, although yields on debt are also increasing slightly to maintain slightly higher leverage than in our model. The impact of this is that funds in the simulation model will take on slightly less risk than in real life.

10 Financial crises - simulation and results

In this section, the impacts of various crisis shapes and timings are tested in our simulation model. These will be compared to those of the base case, to exemplify how different parameters are important in times of crisis. The characteristics and parameters of the crises are based on observations from real-life impacts of the crises shapes discussed earlier.

10.1 U-shaped crisis after the investment period

The late U-shaped crisis is erupting in year 5 of the fund life time, implicating that all investments have been conducted. While late timing of the crisis means that some of the earlier investments might be exited, the funds cannot benefit from acquiring assets at depressed valuations. This means that funds will be severely affected. As maturity of the funds is nearing, probabilities of exits become higher, making it harder for funds to catch-up with lost value. For the standard fund, the late U-shaped crisis is very undesirable, significantly affecting performance. The distribution for LP money multiples shifts sharply downwards compared to the base case scenario and GPs are now struggling to earn above preferred return.

Our main results from the late crisis is that impact is severe and exclusively negative on private equity funds, as it leaves little time to make up the loss. More specifically, we observe that,

- α seems to become most valuable to LPs as their downside protection becomes more relevant. GPs, on the other hand, lose relatively more in the event of a crisis, especially for higher-skilled managers not earning the catch-up provision.
- For varying degrees of β_U, LPs have the lowest degree of loss as well as variance for lower values of β_U, however, GPs gain from higher β_U as outliers become more potent.
- Leverage is still value-adding for lower levels, e.g. increasing from 0 to 1, but becomes relatively less attractive earlier in the late crisis.

10.1.1 The effect of α in the late U-shaped crisis

Figure 35 shows, disregarding managerial skill, that a late U-shaped crisis has severe negative impacts on fund performance. Although, LPs receive mostly positive returns, when managers are very skilful. As in base case, increasing levels of α has consistent effects with a one percentage

Figure 35: LP MoM for varying α following late U-shaped crisis

point increase in α results in $\sim 0.1x$ increase in both mean and median (appendix: table 11). However, differently from the base case, the mode is affected unless α is quite high, meaning that the hurdle rate becomes more insignificant for low-skill funds, causing more downside risk for LPs.

GPs are affected negatively by a late U-shaped crisis. As hurdle rates are not met, GPs receive management fees only. Even highly skilled GPs are struggling to meet the hurdle rate as shown in figure 36. The high benefits of being a skilled GP in the base case also mean that they are affected the worst in this crisis. Low-skill GPs are affected less, as most of them are not able to claim the catch-up provision in the base case. Median GP money multiples are barely affected for low-skill managers, and nearly halved for highly skilled managers following the crisis (appendix: table 11).

10.1.2 The effect of β_U in the late U-shaped crisis

β_U is of high importance in the late crisis. Just as in base case, increased levels of β_U drastically increases volatility. However, as the crisis sets in, funds are affected quite differently depending on their level of β_U. Figure 37 shows that all funds, despite their levels of β_U, are affected

Figure 36: GP MoM for varying α following late U-shaped crisis

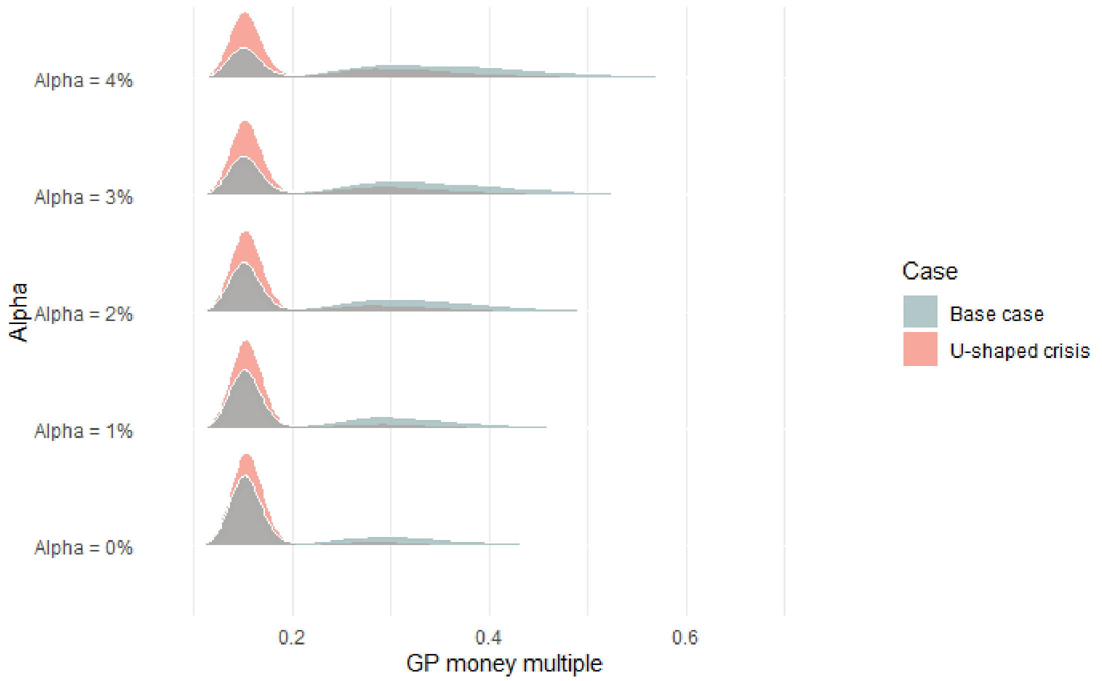
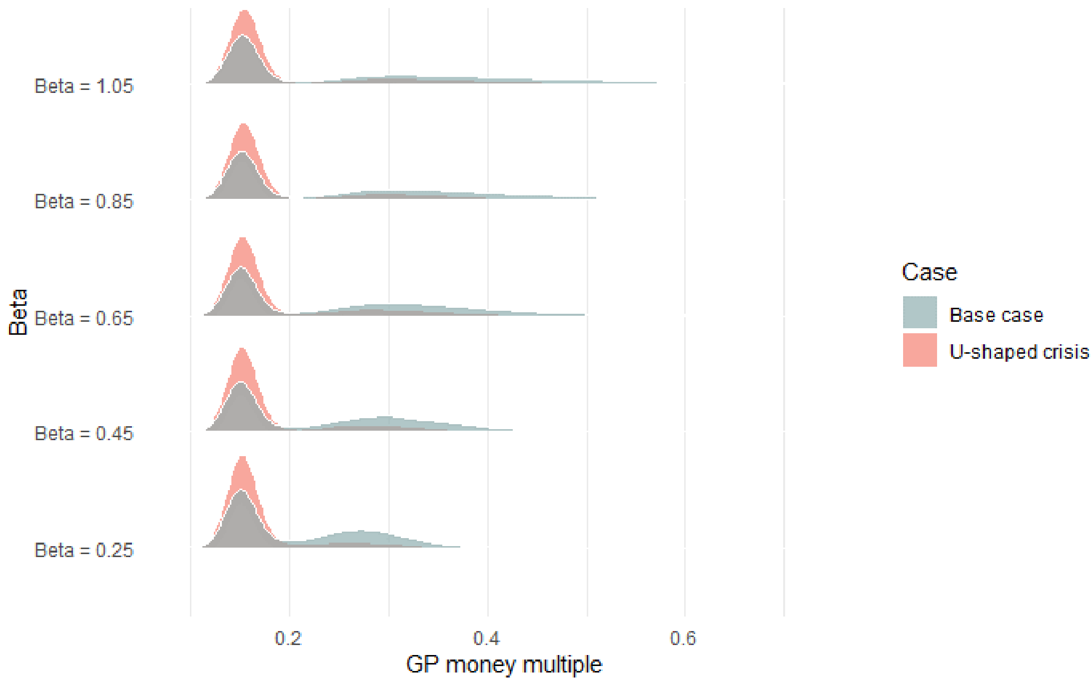


Figure 37: LP MoM for varying β_U following late U-shaped crisis

Figure 38: GP MoM for varying β_U following late U-shaped crisis



negatively, however, the impact is far greater for high β_U funds. In the base case, median and mean MoM are higher for high β_U, but following a late crisis the opposite is true. Low β_U funds vary less with the market, making the managers skill more important in overall performance and macro conditions less important. LPs in low β_U funds are receiving median MoM 0.08x less than their high β_U counterparts in base case, but following a late crisis, they earn $\sim 0.15x$ more (appendix: table 13). Furthermore, low β_U LPs are far more unlikely to receive negative returns, even in the event of a crisis, with even lower quartile money multiples remaining above 1.0x.

While LPs are benefiting from lower β_U, the same cannot be said for GPs. Regardless of β_U, all GPs are severely negatively affected by the crisis. Though, without at least a moderate level of β_U and significant covariation, it becomes unlikely for GPs to claim the lucrative catch-up provision. Interestingly, while higher levels of β_U benefit GPs, the effect is still smaller than having high levels of α, albeit in extreme cases high β_U drastically increases the tail of the distribution (appendix: table 13). As figure 38 shows, high levels β_U only benefits a few "lucky" GPs, but it does not have any significant impact on the median GPs compensation, as raising β_U comes at high expense to LPs.

Figure 39: LP MoM for varying leverage following late U-shaped crisis

The average holding periods for different levels of β_U is similar to what is seen in the base case.

10.1.3 The effect of leverage in late U-shaped crisis

The typical notion of leverage as a doubled-edged sword becomes clearly evident in the event of a late U-shaped crisis. Huge losses frequently ensues for LPs investing in highly levered funds, with the losses becoming greater and happening more frequently following a crisis (Figure (39). However, highly levered funds have higher mean and median LP money multiples than low or no leverage funds (appendix: table 12), thus the reward for the added risk is persistent even following a crisis, which was not the case for high levels of β_U. Overall, leverage shows consistent behaviour in both the base case and following a late U-shaped crisis, in that it significantly increases expected returns, but it does so at the cost of higher volatility.

The risk-reward nature of leverage is also evident in GP compensation, with mean GP money multiples being significantly higher for highly levered funds, even following the crisis (appendix: table 12). However, the effect of the crisis is, that regardless of leverage, median GP compensation is just equal to management fees. Thus only in a few highly-levered funds, can GPs claim the catch-up provision. For funds not using leverage, the effect on GP compensation following the crisis is quite small, as in base case they seldomly manage to claim the catch-up provision (appendix: table 53). For varying levels of leverage there is a small impact on holding periods, with highly levered funds impacted slightly more, increasing their holding periods with approximately three months. Private equity funds using lower leverage increase average holding periods with approximately two months (appendix: table: 12).

10.2 U-shaped crisis in the investment period

In this scenario, the U-shaped crisis hits the private equity fund in the beginning of its investment period (in year 2). The impact of this is, that investments are made pre-crisis, during the crisis, and in the recovery period. This means that volatility and expected growth is changing as the crisis evolves.Since volatility impacts leverage, private equity funds need to alter their decisions regarding capital structure depending on timing so it's coherent with the prevalent risk environment. In the simulation, private equity funds adjust the amount of leverage used in investments, in order for the credit spread to remain the same, or alternatively adjust credit spread while keeping leverage steady. When analysing the sensitivity to leverage, PE funds are assumed to maintain their debt to equity ratio. With the early impact of the crisis, investments will be made throughout the duration of the crisis, meaning that some investments initially will receive drastic losses, while others will start out by benefiting from the recovery. For the standard fund this has not only shifted the distribution of LP money multiples to the upwards, it has also removed the symmetry present in the base case scenario and skewed the distribution, thus the downside for LPs goes down drastically. For GPs in the standard fund, more are now earning above the preferred return, and the upper tail has gotten longer.

This section will show how the earlier timing of the crisis affect costs and benefits for LPs and GPs, investigating the same areas as in the base case and late crisis setup. Upsides from the recovery will outweigh the initial drastic losses. The key findings are,

- Relative to base case, marginal benefits of α for LPs goes down as more funds get past the hurdle rate. This is the exact opposite effect to that of the late crisis.
- Contrary to both base case and the late crisis, GPs' marginal benefit of α seems to follow that of the LPs, because of increased profit sharing.

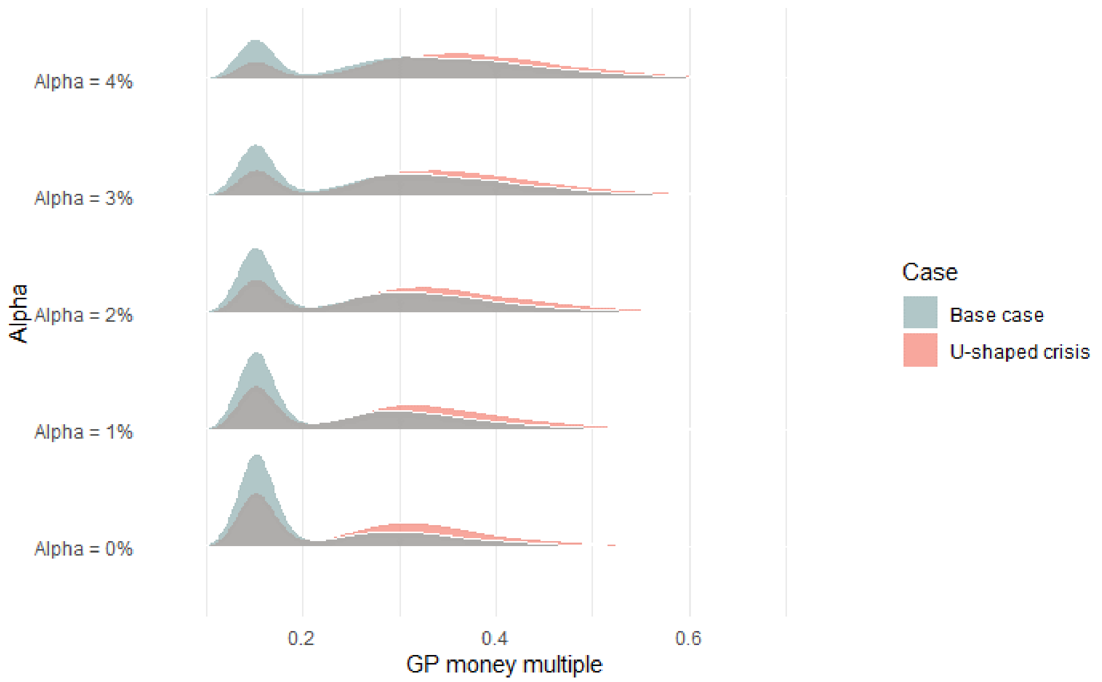
Figure 40: LP MoM for varying α following early U-shaped crisis

- Holding periods are not highly affected by an early crisis, but initial investments will have longer and late investment shorter holding, evening each other out.
- β_U becomes more valuable for LPs with relatively higher returns, but also significantly increased volatility of $\beta_U \geq 0.65$. GPs gain from low β_U as funds consistently perform better than the hurdle rate.
- Adjustments of capital structure to keep fixed financing cost is highly effective through the crisis, whereas keeping a fixed degree of leverage is resulting in lower returns for both LPs and GPs.

10.2.1 The effect of α in an early U-shaped crisis

Figure 40 shows, contrary to the late crisis, that higher levels of α have positive effects on LP returns. The impact of the crisis is uniform for all levels of α when looking at the medians, however, the impact on means are greater for lower levels of α (appendix: table 14). An explanation of this can be the structure of the private equity contract, where LPs are able to claim any incremental returns up until the hurdle rate is met. Thus low α funds, that often do not meet the hurdle rate in the base case, now achieve additional returns, which are entirely

Figure 41: GP MoM for varying α following early U-shaped crisis



claimed by the LPs, and the lower tails of the distributions thereby are severely reduced. This effect can be seen in figure 40 as the mode is becoming more significant, thus providing evidence of centring around around full payment of the preferred return. This centring effect can also be seen in the volatility of the distributions. Volatility for no or low α funds has slightly decreased compared to the base case, whereas it has remained unchanged for higher α funds.

The increased returns are also increasing GP money multiples for all levels of α. In figure 41, it can be seen that GPs in this scenario are claiming the catch-up provision and the profit share much more frequently than seen in the base case. The impact of the crisis is less uniform than it is for LPs, as the value of claiming the catch-up provision is large. Therefore, the crisis is of greater benefit for low-skill GPs, as most high-skill GPs would also claim the catch-up provision in the base case. However, high-skill GPs are still benefiting significantly due to the profit share agreement. The nature of this impact can also be seen in means and medians. Medians are increasing significantly more for low skill managers, whereas means are impacted uniformly. This uneven impact has made them nearly equal for no or low-skill managers (appendix: table 14), as they have been for high-skill managers both in the base case and now.

The crisis only has little impact on average holding periods and their volatility (appendix: table:

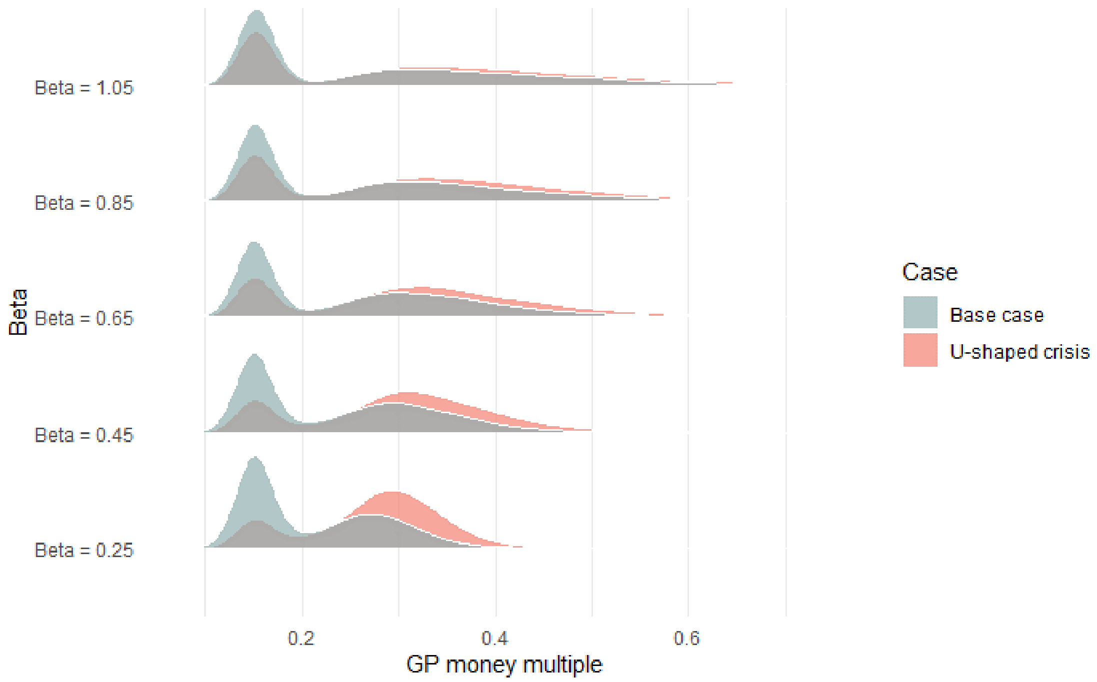
Figure 42: LP MoM for varying β_U following early U-shaped crisis

14). The explanation for this is that early investments now have longer holder periods, as it is unlikely for them to achieve high valuations early on in their asset paths, which reduces the probability of an exit. The reverse effect then happens for investments made when the recovery period is initiating. These assets will likely achieve very high valuations relatively earlier, which then increases the probability of an early exit. Based on the results (appendix: table 14 and 4), these effects cancel each other out.

10.2.2 The effect of β_U in an early U-shaped crisis

Overall, the early U-shaped crisis is completely opposite to the late U-shaped crisis for β_U as positive impact of the early crisis magnifies the benefits of β_U. Increased volatility and expected returns from higher levels of β_U becomes evident in returns. Low β_U funds, which highly benefited from low covariation with the market in the late crisis, are now only catching little of the upside on investments made when recovery growth starts. On the other hand, high β_U funds enjoy the increased returns in the recovery period fully, as they covariate highly with the market. However, higher volatility funds lose money with larger probability (figure 42). This effect can be seen in the lower quartile returns for the LPs, as high β_U funds have the lowest returns,

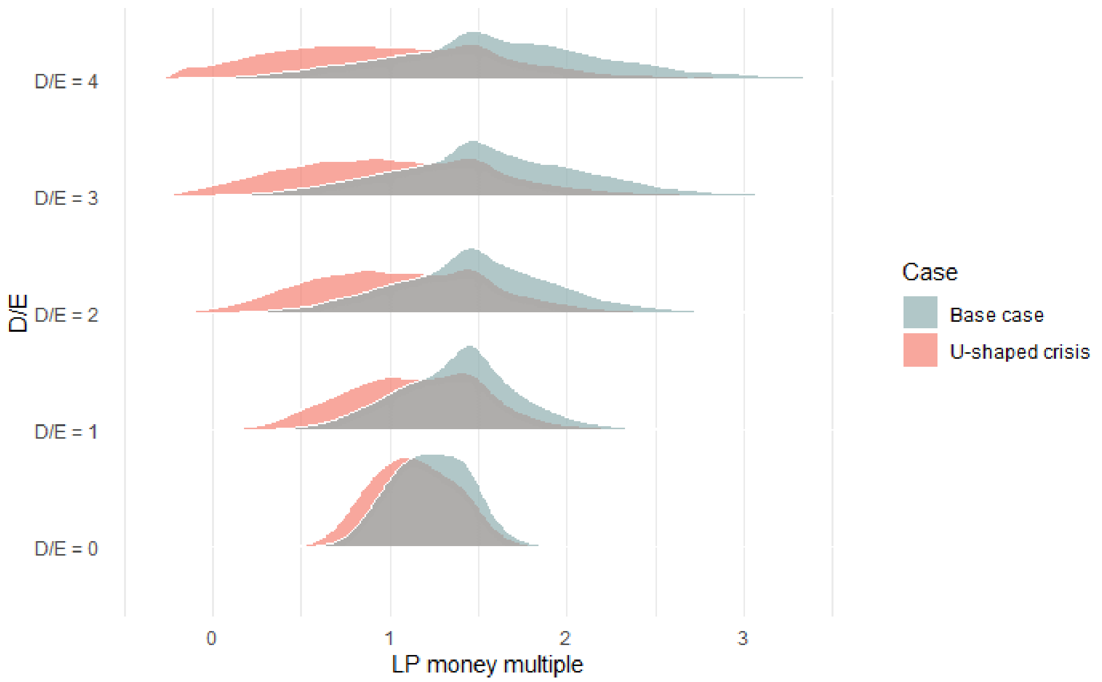
Figure 43: GP MoM for varying β_U following early U-shaped crisis



even though they have significantly higher means and medians than low β_U funds (appendix: table 16). Looking at median LP money multiples, the majority of the benefit of β_U is achieved when β_U goes from 0.25 to 0.65, giving an increase in the median LP money multiple of 0.125x. Increasing β_U further only provides a 0.03x additional increase in the median LP money multiple (appendix: table 16). The increase in the mean is similar to the increase in the median.

The early U-shaped crisis also highly benefits GPs, as increased returns results in increased profit sharing. However, contrary to the LPs, GPs are benefiting relatively more when β_U is low, as the increased return shifts performance over the catch-up provision for a lot of funds that did not achieve this in base case. Thus the low β_U funds are in this scenario having consistently good compensation, with only a few funds not claiming their catch-up provision as can be seen in figure 43. While high β_U funds are achieving slightly higher mean and median compensation (appendix: table 16), they are also frequently not able claim their catch-up provision. In an environment where it is easy for GPs to beat the hurdle rate, taking high levels of risk is not beneficial for them. As the frequency of funds having low returns become too high, the resulting loss in compensation thus become severe. Although it should also be noted that the higher volatility for high β_U funds also brings an increased upside for a few "lucky" funds.

Figure 44: LP MoM for varying leverage following early U-shaped crisis



10.2.3 The effect of leverage in an early U-shaped crisis

The assumption regarding leverage in the simulation is to hold the debt to equity ratio fixed no matter what. Volatility is increasing in the crisis and in the recovery period. Investments made in these times have much higher interest payments according to the Merton model used in determining the credit spread. Figure 44 show the great harm this approach to capital structure has on the returns earned by LPs. The poor performance coming from crippling debt payments also impact GP compensation, with almost all GPs only earning management fees (appendix: figure 54). The results from this scenario, when following this approach to capital structure, is evident throughout all the various crisis shapes. Therefore, considering the similarity of the results, these will not be elaborated further on for other crises shapes, but the data is available in the appendix.

10.3 V-shaped crisis in the investment period

The V-shaped crisis constitute of a swift sharp decrease in asset prices with an equally swift recovery, with high volatility throughout the period. The difference between a V-shaped and U-shaped crisis is that asset prices catch up with their previous level faster. The high volatility

Figure 45: LP MoM for varying α following early V-shaped crisis

in the crisis will severely affect the levels of debt that can be taken for investments made in this time. The shortness of the crisis also means that it will influence fewer investments, but the influence on these should be greater due to the drastic movements in the crisis period and the recovery period. For the standard fund, the impact of the early V-shaped crisis is similar to that of the early U-shaped crisis, in that it both skews and shifts the LP money multiple distribution upwards.

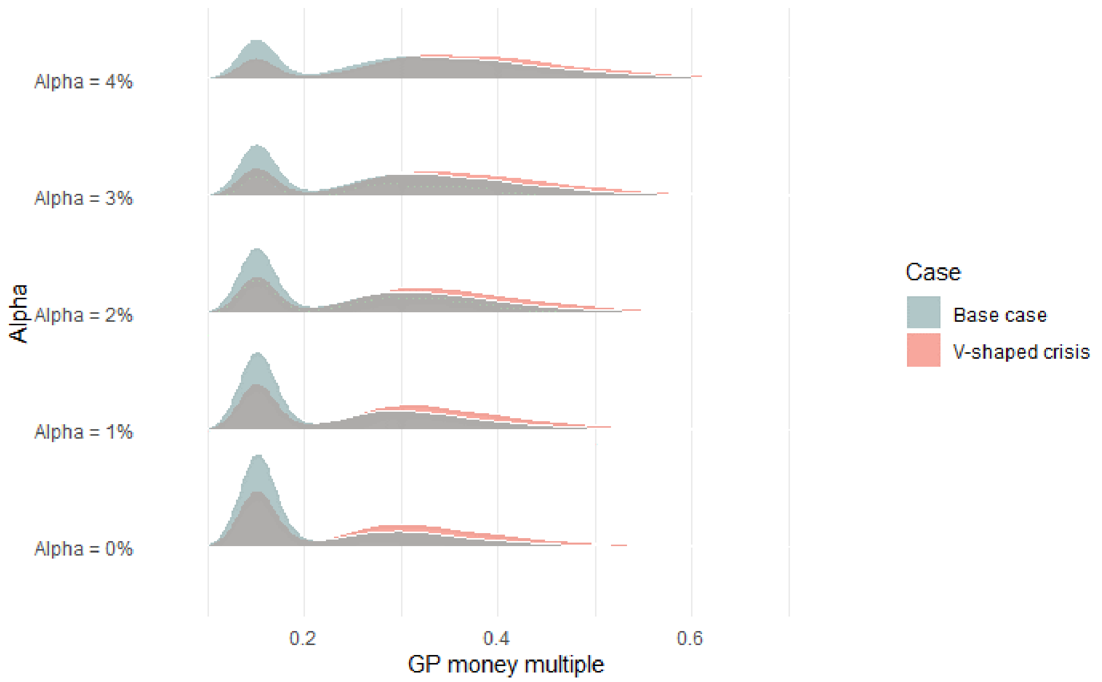
Mostly, the findings from the early V-shaped crisis show that it is very similar to the early U-shaped crisis, albeit slightly less beneficial, which could be driven by the parameters chosen. However, the results from the early V-shaped crisis uncovers,

- Overall impact is similar to U-shaped crisis.
- High β_U funds will benefit relative to low β_U funds as the crisis event become more potent.

10.3.1 The effect of α in an early V-shaped crisis

In figure 45, it can be seen that the impact of the early V-shaped crisis is positive, and almost identical to the impact of the early U-shaped crisis. The V-shaped crisis boosts returns slightly

Figure 46: GP MoM for varying α following early V-shaped crisis



less than the U-shaped crisis, and it has slightly higher volatility (appendix: table 20 and 14). Except this, there's no other alternating impacts.

GPs, like LPs, are significantly benefiting from the early V-shaped crisis, with the tails of the distribution of compensation becoming longer for all levels of α. It can be seen in figure 46, that the large benefits of the crisis for varying α is almost identical to the U-shaped crisis.

10.3.2 The effect of β_U in an early V-shaped crisis

Just as for α, the impact of the early V-shaped crisis is positive for all levels of β_U. Once again, the resemblance to the impact of the early U-shaped crisis is clearly to be seen in figure 47. The increased volatility compared to the U-shaped crisis is yet again prevalent for all levels of β_U (appendix: table 22). However, the impact of the early V-shaped crisis is not entirely similar to the U-shaped crisis. The various levels of β_U are impacted to a different degree compared to the U-shaped crisis. Low β_U funds are benefiting less than they are in the U-shaped crisis, whereas high β_U funds are benefiting more. For high levels of β_U, the difference in impact relative to the base case is minor. However, for low β_U funds the difference is considerable. In the early V-shaped crisis, for low β_U funds the median LP money multiple earned is 0.1x higher than

Figure 47: LP MoM for varying β_U following early V-shaped crisis

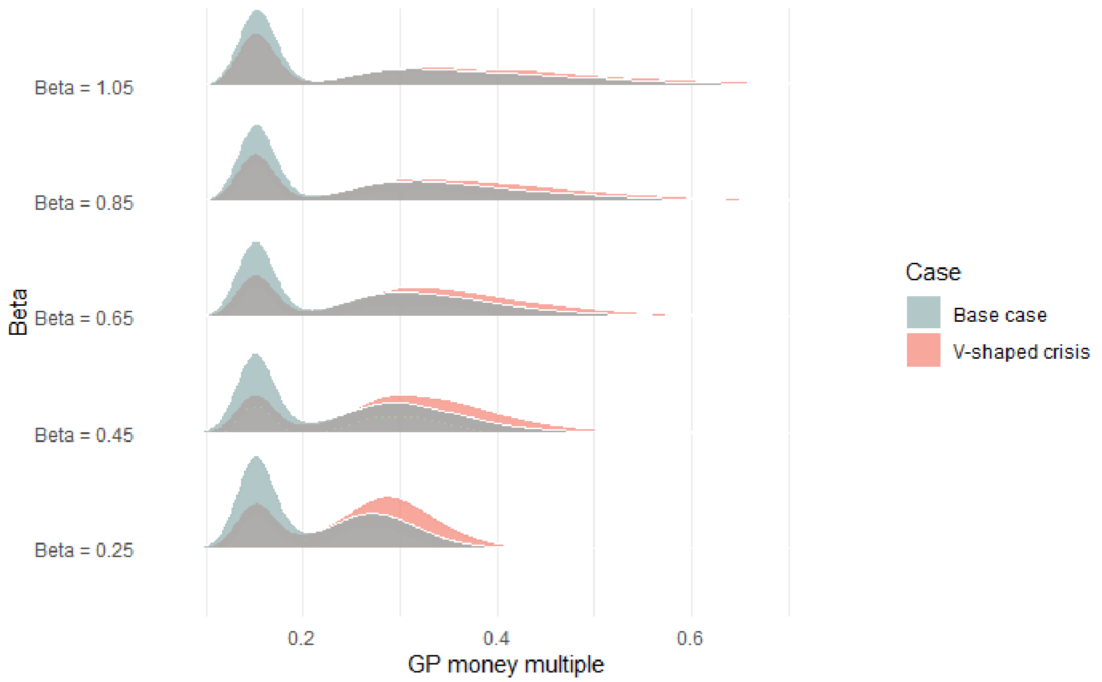
in the base case. For the early U-shaped crisis, the increase is 0.14x, a significant incremental benefit. For high β_U LP money multiples, they are approximately 0.03x higher than in the early U-shaped crisis (appendix: table 22 and 16). The difference in impact is due to the shorter crisis time for the early V-shaped crisis, as well as the relatively more potent recovery period.

In figure 48, the increased variance of high β_U funds is clearly evident, as they impact GPs significantly in the downside. This large variance also mean that the median compensation earned for GPs are only slightly higher for high β_U funds compared to medium β_U funds (appendix: table 22). Therefore, just as for the early U-shaped crisis, it is the low β_U funds where GPs are benefiting the most, albeit less so than for the early U-shaped crisis. Once again, it becomes clear how valuable it is for GPs to claim their catch-up provision.

10.4 L-shaped crisis in the investment period

The L-shaped crisis is one of the worst events for the economy, as growth stops for many years after the crisis. There's no recovery period and no benefit of investing during the crisis. The early eruption of the crisis and the subsequent halting of growth means that most portfolio companies won't benefit from any market growth for a long time. The results for the standard

Figure 48: GP MoM for varying β_U following early V-shaped crisis



fund show that this scenario is an extremely difficult environment. The crisis event eliminates symmetry in the distribution of LP money multiples, and skews it downwards, showing a huge downside for LPs in the L-shaped crisis. The impact also leads GPs to being unable in earning the preferred return. The key findings from the L-shaped crisis are,

- The skill of the manager becomes very important, as low skill managers struggle to generate positive returns in this environment. Unless the manager is very skilful, all benefits from α goes to the LPs.
- Also, having assets with low β_U is a strong mitigation of the unattractive market conditions for both LPs and GPs.
- Leverage cannot be used to drive up returns for the funds, as the growth of the companies are too low to cover increased financing costs

10.4.1 The effect of an early L-shaped crisis

The deeply negative impact of the L-shaped is seen regardless of the level of α (figure 49) and the impact becomes even worse for various levels for debt to equity, especially when the level is

Figure 49: LP MoM for varying α following early L-shaped crisis

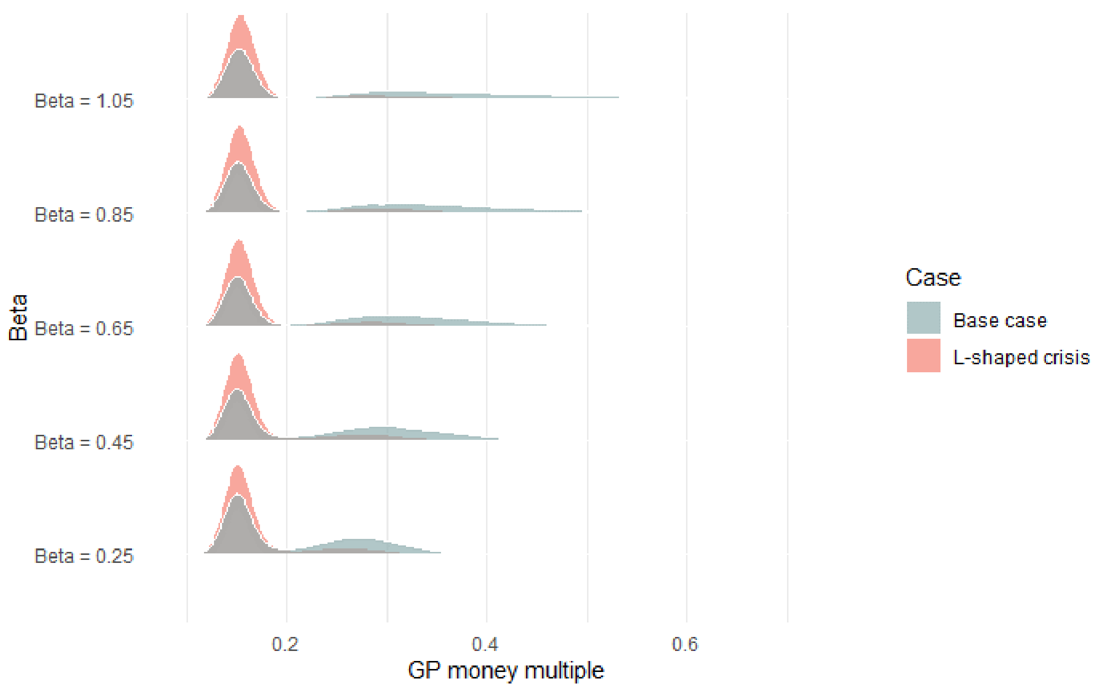
maintained throughout the crisis (section 10.2.3). Although, it should be noted that high skill managers are still able to make consistent positive returns, just at much lower levels than in the base case. For high levels of α, the mode from the base case is still quite significant, however the skewness of the distribution of LP money multiples have shifted towards more negative returns (figure 49).

Figure 50 shows how deeply affected LP returns are by the early L-shaped crisis. Compared to the early U-shaped crisis which boosted returns, LP's now are losing money in a manner similar to the late U-shaped crisis. Low β_U funds are less affected, as they vary less with the market and the α they generate thus become much more important. Therefore, the vast majority of low β_U funds are still able to create positive returns for the LPs. The high β_U funds are severely affected by the crisis, with median LP money multiples being below 1.0x (appendix: table 19). However, the large volatility in high β_U funds also mean that a fair number of them are earning good returns for the LPs, and in this crisis it is the only option for really high returns (figure 50).

The negative impact of the crisis can also be seen in the compensation earned by the GPs, as nearly all GPs are now only earning management fees (figure 51). One group of GPs is still able

Figure 50: LP MoM for varying β_U following early L-shaped crisis

Figure 51: GP MoM for varying β_U following early L-shaped crisis



to earn the catch-up provision with some frequency, as the upper quartile of high-skill GPs still earn a money multiple consistent with them paying the full hurdle rate (appendix: table 17). With even very high-skill GPs struggling to earn any form of profit share, an in-depth review of the effect of all the parameters is deemed unnecessary. Just as the results for the LPs, results for GPs are available in the appendix.

The crisis has a strong impact on average holding periods, which are up approximately three to six months for any given level of any given parameter. The largest impact is on highly levered firms that are maintaining their capital structure throughout the crisis, as their average holding periods have gone up with a little more than six months (appendix: table 18). The reason for significantly longer holding periods are the drastically lower valuations, which reduces the probability of an exit.

11 Comparison to public equity

Table 6: Public equity comparison parameters

Parameter	Private Equity	Public Equity
α	2%	0%
β_U	0.65	1.0
$\frac{D}{E}$	200%	0%

To put the results from the simulation model into perspective, distributions of various scenarios for the standard model are compared to a simulation of a public equity portfolio. The comparison is computed for the base case and each crisis scenario. The parameters used are the values of the standard fund used in previous simulations with both private equity and public equity parameter values seen in table 6. It is assumed that investments in public equity are non-activist and there are no superior stock-picking skill, thus α is set to zero. β_U is set to one, thus the assets, at which we won't apply leverage onto, covariate equally with the public market, which has leverage in it[33]. Consequently, $\mu_A = \mu_S$ and $\sigma_A = \sigma_S$, which are the parameters of the public market. To ensure comparability between results from public equity and private equity, it is assumed that public equity investments will be made with same timing as the private equity investments. Creating the perfect risk-adjusted public market equivalent as a specific number for compare would be an entire paper in itself (Sørensen et al., 2014). This is why the comparison in our paper is based on distributions of returns for 10,000 simulations, thereby allowing inspection of different risk levels. With this approach it is thus important to remember, that the comparison is not one between to risk equivalent assets.

In figure 52, the comparison between public and private equity for all five scenarios are plotted. It can be seen that public equity has narrower distributions than PE, thus it is less risky in our model. In the base case, differences in mean and median money multiples are 0.16x and 0.125x (table 7). The biggest difference in returns between PE and public equity is the impact of the early V and U-shaped crisis. PE is experiencing improved returns in these scenarios, but public equity is only achieving marginally better returns in the early V-shaped crisis (an improvement of 0.02x in the money multiple) and significantly worse returns in the early U-

[33] β_U of public assets is 0.65, as the market itself has leverage

Figure 52: Investor return of private equity and equivalent public equity

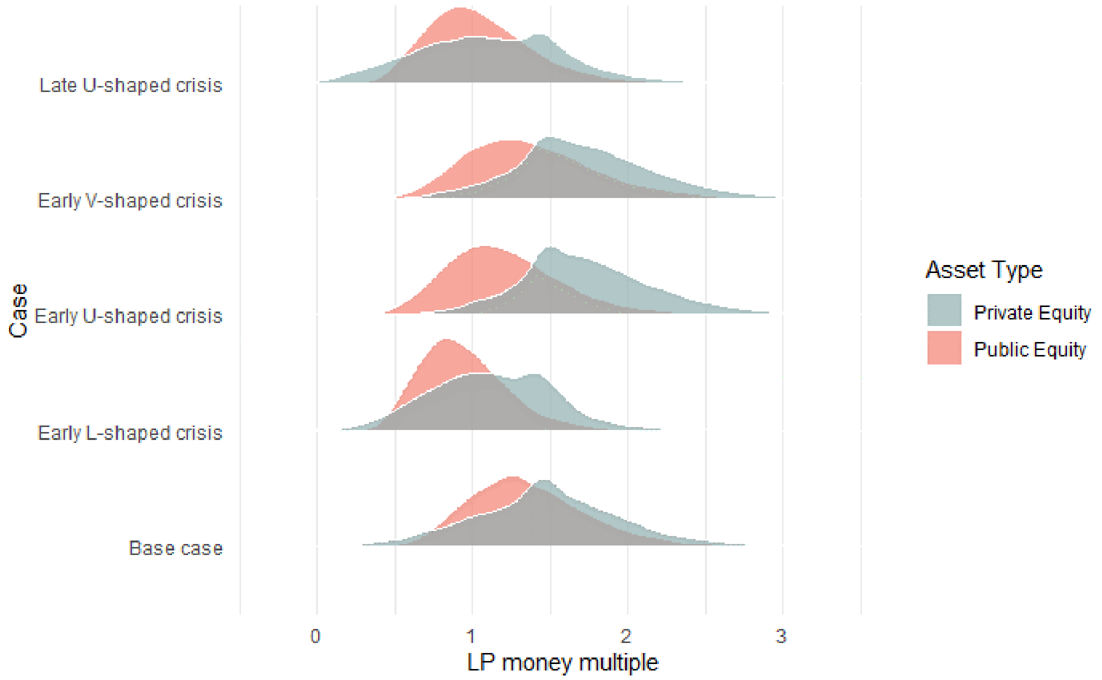


Table 7: Public equity performance in various crisis

	Base case	**Late U-shaped**	**Early U-shaped**	**Early L-shaped**	**Early V-shaped**
LP MoM (Based on invested equity)					
Min.	0.38	0.26	0.33	0.28	0.37
1st Qu.	1.09	0.80	0.95	0.74	1.08
Median	1.32	1.01	1.17	0.91	1.34
Mean	1.36	1.05	1.21	0.94	1.40
3rd Qu.	1.58	1.25	1.43	1.11	1.65
Max.	3.34	2.91	3.46	2.45	4.32
Std dev	0.38	0.35	0.38	0.28	0.45

shaped crisis (a decrease of 0.145x in the money multiple) compared to the base case. PE lower their levels of leverage when in the recession and recovery period, thus having higher leverage on investments made at the most unattractive time (right before the crisis) and lower leverage at the most attractive time (ending of the recession period / beginning of the recovery period), yet the incremental returns achieved for the attractive investments combined with the option to default on poor investments boosts the return of PE far higher than public equity.

Another observation is that PE is marginally better in the L-shaped crisis, which is exclusively driven by the α in the low growth environment. This result is highly dependent of the parameter specifications. For the late U-shaped crisis, we see that, while having a slightly higher mean, the volatility of returns in PE is much higher than that of the public market, and as such, a risk-adverse LP would prefer the public investment over the private one.

12 Discussion of results in financial crises

One of the unstartling, but key insights, of our model is that the timing of the crisis matters a lot for the returns of all stakeholders, especially in the case of a crisis with a recovery period. As seen in our comparison to the public market, the PE sector seems to produce outcomes with much better results for a crisis in the earlier stages when a recovery is present. These results are interesting, as they present a structural advantage of PE in specific types of crises. This section will briefly discuss the further implications of our model results and how they relate to other research and empirical observations.

12.0.1 Timing of the crisis on fund dynamics

For all stakeholders, a crisis in the exit period affects returns negatively, however, the impact is relatively more severe for funds which,

- have high β_U values and thus are more exposed to the crisis shock to asset prices
- have high debt to equity ratios, albeit to a lesser degree than high β_U funds

For various levels α the impact of a crisis in the exit period is more uniform in absolute terms. For GPs, low α managers, as well as managers taking low risk, either through leverage or β_U, are the least affected, although this is due to them not being likely to claim the catch-up provision in steady-state either. Our model thus suggest that returns should be lower for funds with vintage year a few year before the financial crisis compared to those investing in during crisis. These findings are consistent with empirical results from Veronis and Esipovich (2019), who find that some of the best vintage funds in recent history have been funds investing during economic downturns.

One of the essential results is that keeping a steady degree of leverage for new investments through a crisis affects stakeholders negatively, as debt holders increase cost of financing. However, our model does not include the element of refinancing at a lower rate after the crisis event, thus the question on which impact this element actually has is to some degree unanswered. Axelson et al. (2010) provide strong evidence that the main reaction for fund managers is to reduce leverage much more actively than altering credit spreads. And post-crisis, evidence is not strong for recapitalisation. Other factors could affect this, such as collapse of the banking industry and short-term risk of liquidity issues, but our model specification of capital structure decisions seems to have some validity.

12.0.2 Crisis shape impact on return

One of the main findings when comparing the impact of the different crisis shapes is the significance of the specific shape. Private equity funds are affected very differently depending on the shape of the crisis with,

- L-shaped crisis having a highly negative impact on performance.
- U and V-shaped crisis having a highly positive impact on performance, given that they happen early in the investment period.
- The relative potency of the recovery period impacts varying levels of β_U dissimilarly.
- α becomes less important with recoveries compared both base case and L-shaped crises.

The positive impact of a U and V-shaped crisis impacting the fund early in its lifetime coincides with some empirical findings of private equity performance in economic downturns (Lino et al., 2020; Mauro & Jost, 2017; Veronis & Esipovich, 2019). Furthermore, this paper includes multiple crisis shapes, broadening the present univariate narrative regarding the impact of a crisis, with this paper showing the multitudes of ways that a crisis can manifest itself, and thus impact PE performance. The results found in this paper goes against the narrative that downturns are exclusively good for PE, by showing a crisis shape, the L-shaped crisis, that negatively affects performance, even when it hits early in the funds lifetime.

The paper highlights another feature of private equity, as the results show a difference between investment strategies to their response on crisis shapes. Relative improvement from the base case scenario appears to be higher for low β_U funds in the U-shaped crisis, whereas high β_U funds benefit more in the V-shaped crisis. These findings does not appear to be discussed in the existing literature, as these commonly analyse PE as an aggregate industry, ignoring the differences in investment strategies existing between funds.

With regard to α, the prediction of it being more irrelevant in a crisis with recovery, could explain why high performing funds did not achieve higher market shares through the recovery period after the 07-09 financial crisis and subsequent recession, as shown by figure 25.

13 Concluding remarks

In answering our research question, we provide a theoretical framework from which we run several simulations with various scenarios to deduct insights and conclusions. The overarching conclusion is that we find, unsurprisingly, that a crisis event is affecting both LPs and GPs significantly, albeit with asymmetric impacts.

In the base case scenario without a crisis, we find that LPs gain largely from low degrees of covariation with public markets as covariation seemingly increase underlying risks much more than return. Manager skill skews the distributions of returns with the primary effect that lower returns become more unlikely for better managers. Leverage only have limited effect on expected returns for LPs, and high leverage provides more risk compared to increasing returns. On the other hand, good managers largely benefit from the private equity contract, which may explain why managers with high performance gain market shares in periods outside crisis events (Bain & Company, 2019). GPs also gain a lot from raising covariation and leverage, especially when going from low parameter values to medium values, as this increases likelihood of receiving high carried interest. We observe that this might constitute moral hazards, however, further discussion on this element is outside of the scope of the paper.

Introducing financial crises to our model, we find that the timing and severity of the crisis is driving quite interesting results, and we will in the following paragraphs sum up these findings.

The insight from a crisis event is first and foremost that timing is highly important for the directional effects. A crisis impacting the fund late in its lifetime drastically reduces returns, while crises with recoveries early in the investment period provide valuable opportunities for high returns.

A late crisis or a crisis without a recovery yields exclusively lower returns for PE funds. In these situations, LPs, and to some degree GPs, gain largely from manager skill, as well as lower covariation to public markets. This means that choosing the right fund becomes the key value driver for LPs.

On the other hand, if a crisis appears in the investment period, both GPs and LPs gain a significant upside, interestingly irrespective of the skill of managers. Covariation also becomes less important than in normal time as well as the late crisis, especially when impacted by a more potent V-shaped crisis. The take-away from this is that PE as an industry is a superior investment vehicle in these scenarios. The differences between various fund characteristics become less important for LPs.

Regarding capital structure decisions, the paper finds strong evidence against using the same leverage in crisis times as is used in the steady-state periods, as increased volatility during the crisis and recovery inflates credit spreads on debt. This makes it extremely difficult for assets to cover the lump sum payment at exit. This is, though, partly driven by the fact, that our model does not allow for a recapitalisation of assets in the post-crisis period. A further investigation into this area could provide valuable groundings for future research.

Finally, the findings of this paper is consistent with empirical observations of private equity outperforming public equity in times around the financial crisis. Though, we are careful not to draw too much inference from a singular event, and our conclusions are to be viewed as a more generalised investigation of crisis events.

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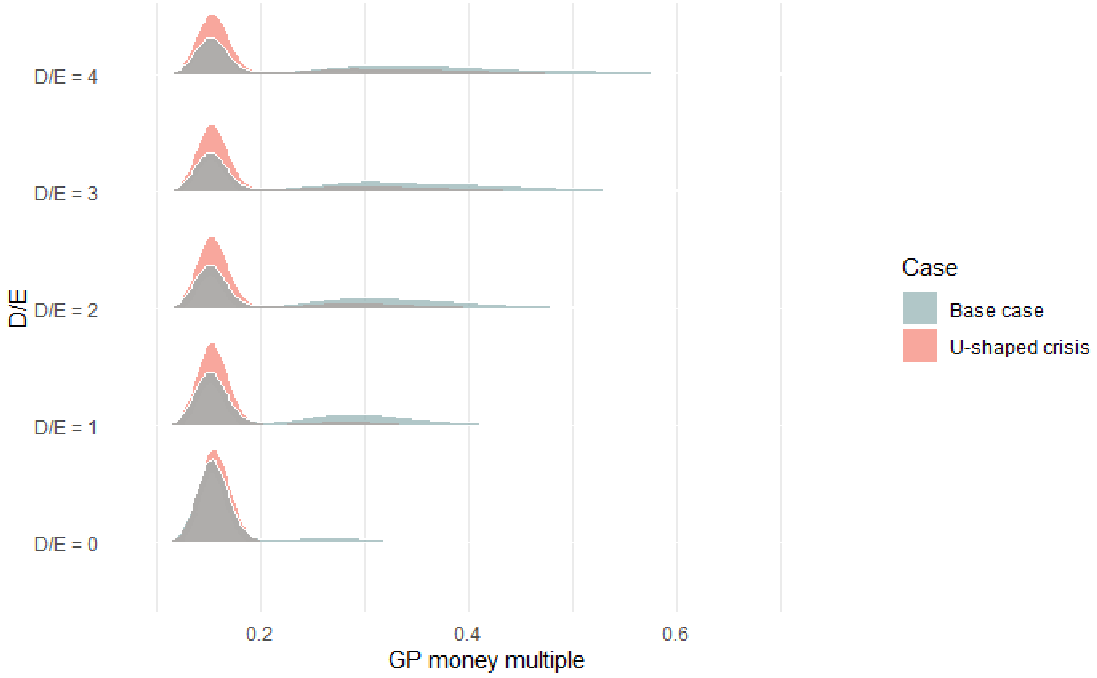
List of Figures

Figure 53: GP MoM for varying leverage following late U-shaped crisis



A Appendix

Table 8: Base case - varying leverage

	D/E= 0 cs = 0.0%	D/E= 1 cs = 1.1%	D/E= 2 cs = 3.0%	D/E= 3 cs = 4.7%	D/E= 4 cs = 6.0%
LP MoM (Based on invested equity)					
Min.	0.54	0.21	-0.03	-0.18	-0.19
1st Qu.	1.07	1.16	1.19	1.21	1.22
Median	1.23	1.40	1.48	1.54	1.59
Mean	1.23	1.38	1.49	1.57	1.63
3rd Qu.	1.39	1.59	1.77	1.93	2.02
Max.	2.34	2.92	3.38	3.85	4.39
Std dev	0.22	0.34	0.47	0.56	0.64
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.16	0.16
Median	0.16	0.17	0.25	0.27	0.29
Mean	0.17	0.22	0.26	0.28	0.30
3rd Qu.	0.17	0.29	0.34	0.38	0.40
Max.	0.48	0.62	0.74	0.85	0.99
Std dev	0.05	0.09	0.11	0.13	0.15
Avg. holding period (Years)					
Min.	2.60	2.48	2.48	2.40	2.35
1st Qu.	4.38	4.13	4.03	3.98	3.98
Median	4.85	4.60	4.50	4.45	4.45
Mean	4.87	4.62	4.53	4.50	4.49
3rd Qu.	5.35	5.08	4.98	4.98	4.98
Max.	7.78	7.53	7.60	8.08	7.63
Std dev	0.71	0.69	0.70	0.72	0.73

Table 9: Base case β_U: $\beta = 0.65$, D/E = 2, $r_f = 2\%$, $cs = 3\%$, $\mu_S = 7\%$, $\sigma_S = 40\%$

	$\beta_U = 0.25$ $cs = 0.1\%$	$\beta_U = 0.45$ $cs = 1\%$	$\beta_U = 0.65$ $cs = 3\%$	$\beta_U = 0.85$ $cs = 6\%$	$\beta_U = 1.05$ $cs = 9.9\%$
LP MoM (Based on invested equity)					
Min.	0.65	0.27	-0.03	-0.17	-0.18
1st Qu.	1.29	1.25	1.19	1.11	1.05
Median	1.42	1.45	1.48	1.49	1.51
Mean	1.41	1.45	1.49	1.51	1.55
3rd Qu.	1.52	1.65	1.77	1.87	1.99
Max.	2.35	2.74	3.38	4.02	5.49
Std dev	0.19	0.33	0.47	0.59	0.74
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.17	0.22	0.25	0.25	0.26
Mean	0.21	0.24	0.26	0.27	0.29
3rd Qu.	0.27	0.31	0.34	0.36	0.39
Max.	0.48	0.59	0.74	0.90	1.26
Std dev	0.07	0.09	0.11	0.13	0.16
Avg. holding period (Years)					
Min.	2.35	2.55	2.48	2.33	2.45
1st Qu.	4.18	4.05	4.03	4.05	4.08
Median	4.60	4.53	4.50	4.53	4.58
Mean	4.63	4.54	4.53	4.56	4.61
3rd Qu.	5.08	4.98	4.98	5.05	5.10
Max.	7.43	7.70	7.60	7.75	8.05
Std dev	0.65	0.68	0.70	0.73	0.76

Table 10: base case - varying σ

	$\sigma = 0.2$ $cs = 0.3\%$	$\sigma = 0.3$ $cs = 1.3\%$	$\sigma = 0.4$ $cs = 3.0\%$	$\sigma = 0.5$ $cs = 5.4\%$	$\sigma = 0.6$ $cs = 8.3\%$
LP MoM (Based on invested equity)					
Min.	0.53	0.08	0.05	-0.16	-0.17
1st Qu.	1.45	1.34	1.19	1.05	0.92
Median	1.59	1.53	1.48	1.43	1.39
Mean	1.61	1.55	1.48	1.43	1.38
3rd Qu.	1.76	1.77	1.77	1.77	1.77
Max.	2.63	3.27	3.44	4.54	4.75
Std dev	0.25	0.36	0.46	0.56	0.66
GP MoM (Based on invested equity)					
Min.	0.12	0.13	0.12	0.12	0.12
1st Qu.	0.23	0.16	0.15	0.15	0.15
Median	0.29	0.27	0.25	0.19	0.17
Mean	0.28	0.26	0.26	0.25	0.26
3rd Qu.	0.34	0.34	0.34	0.34	0.34
Max.	0.58	0.71	0.74	1.02	1.07
Std dev	0.08	0.10	0.11	0.12	0.14
Avg. holding period (Years)					
Min.	2.40	2.63	2.35	2.35	2.43
1st Qu.	3.95	3.98	4.03	4.08	4.15
Median	4.35	4.43	4.50	4.58	4.65
Mean	4.39	4.46	4.53	4.60	4.67
3rd Qu.	4.80	4.88	5.00	5.08	5.18
Max.	6.98	7.53	7.45	7.58	7.60
Std dev	0.62	0.67	0.70	0.73	0.76

Table 11: Late U-shaped crisis - α sensitivity

	$\alpha = 0\%$	$\alpha = 1\%$	$\alpha = 2\%$	$\alpha = 3\%$	$\alpha = 4\%$
LP MoM (Based on invested equity)					
Min.	-0.18	-0.17	-0.17	-0.14	-0.11
1st Qu.	0.59	0.68	0.78	0.88	0.98
Median	0.90	1.00	1.12	1.23	1.34
Mean	0.93	1.02	1.12	1.22	1.31
3rd Qu.	1.27	1.37	1.45	1.52	1.60
Max.	3.10	3.06	3.27	3.79	3.31
Std dev	0.46	0.47	0.48	0.48	0.48
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.16
Mean	0.18	0.18	0.20	0.21	0.22
3rd Qu.	0.17	0.17	0.21	0.27	0.30
Max.	0.67	0.67	0.72	0.84	0.72
Std dev	0.06	0.07	0.08	0.09	0.10
Avg. holding period (Years)					
Min.	2.35	2.58	2.48	2.48	2.45
1st Qu.	4.35	4.30	4.25	4.20	4.13
Median	4.88	4.80	4.75	4.68	4.60
Mean	4.89	4.83	4.77	4.71	4.64
3rd Qu.	5.40	5.33	5.28	5.20	5.13
Max.	8.03	8.08	8.03	7.73	7.88
Std dev	0.76	0.75	0.75	0.74	0.73

Table 12: Late U-shaped crisis - varying leverage

	D/E= 0 cs = 0.0%	D/E= 1 cs = 1.1%	D/E= 2 cs = 3.0%	D/E= 3 cs = 4.7%	D/E= 4 cs = 6.0%
LP MoM (Based on invested equity)					
Min.	0.34	0.00	-0.17	-0.19	-0.19
1st Qu.	0.85	0.79	0.78	0.76	0.77
Median	1.00	1.05	1.12	1.16	1.24
Mean	1.02	1.06	1.12	1.16	1.23
3rd Qu.	1.17	1.34	1.45	1.52	1.61
Max.	2.26	3.08	3.27	4.11	4.60
Std dev	0.23	0.38	0.48	0.56	0.63
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.16
Mean	0.16	0.18	0.20	0.21	0.23
3rd Qu.	0.16	0.17	0.21	0.27	0.30
Max.	0.47	0.67	0.72	0.92	1.03
Std dev	0.03	0.06	0.08	0.10	0.12
Avg. holding period (Years)					
Min.	2.38	2.48	2.48	2.13	2.38
1st Qu.	4.55	4.35	4.25	4.23	4.18
Median	5.03	4.83	4.75	4.70	4.70
Mean	5.06	4.86	4.77	4.74	4.72
3rd Qu.	5.58	5.35	5.28	5.25	5.23
Max.	8.60	7.73	8.03	8.23	7.70
Std dev	0.75	0.74	0.75	0.76	0.78

Table 13: Late U-shaped crisis - varying β_U

	β_U = 0.25 cs=0.1%	β_U = 0.45 $cs = 1\%$	β_U = 0.65 $cs = 3\%$	β_U = 0.85 $cs = 6\%$	β_U = 1.05 cs = 9.9%
LP MoM (Based on invested equity)					
Min.	0.33	-0.02	-0.17	-0.19	-0.20
1st Qu.	1.05	0.90	0.78	0.67	0.59
Median	1.22	1.16	1.12	1.09	1.06
Mean	1.21	1.15	1.12	1.10	1.11
3rd Qu.	1.39	1.41	1.45	1.48	1.53
Max.	2.17	2.58	3.27	5.47	6.06
Std dev	0.23	0.36	0.48	0.59	0.70
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.13
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.15	0.16	0.16	0.16	0.16
Mean	0.17	0.18	0.20	0.21	0.22
3rd Qu.	0.17	0.17	0.21	0.24	0.27
Max.	0.45	0.54	0.72	1.26	1.40
Std dev	0.04	0.06	0.08	0.10	0.12
Avg. holding period (Years)					
Min.	2.33	2.55	2.48	2.28	2.30
1st Qu.	4.35	4.28	4.25	4.28	4.30
Median	4.80	4.75	4.75	4.78	4.83
Mean	4.83	4.78	4.77	4.81	4.85
3rd Qu.	5.28	5.25	5.28	5.33	5.38
Max.	7.75	7.60	8.03	7.78	8.38
Std dev	0.69	0.72	0.75	0.77	0.78

Table 14: Early U-shaped crisis - varying α

	$\alpha = 0\%$	$\alpha = 1\%$	$\alpha = 2\%$	$\alpha = 3\%$	$\alpha = 4\%$
LP MoM (Based on invested equity)					
Min.	0.19	0.35	0.41	0.31	0.49
1st Qu.	1.33	1.39	1.46	1.51	1.58
Median	1.54	1.61	1.69	1.78	1.87
Mean	1.57	1.65	1.74	1.81	1.91
3rd Qu.	1.81	1.90	2.00	2.08	2.19
Max.	3.61	4.02	3.67	4.10	3.90
Std dev	0.41	0.42	0.43	0.44	0.45
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.13	0.13
1st Qu.	0.16	0.16	0.21	0.26	0.29
Median	0.27	0.30	0.32	0.34	0.36
Mean	0.27	0.29	0.31	0.34	0.36
3rd Qu.	0.35	0.37	0.39	0.41	0.44
Max.	0.79	0.90	0.81	0.90	0.86
Std dev	0.11	0.12	0.12	0.12	0.12
Avg. holding period (Years)					
Min.	2.50	2.50	2.50	2.38	2.45
1st Qu.	4.08	4.03	3.98	3.93	3.88
Median	4.55	4.50	4.45	4.38	4.33
Mean	4.58	4.53	4.47	4.42	4.35
3rd Qu.	5.05	5.00	4.93	4.88	4.80
Max.	7.58	7.40	7.35	7.55	7.28
Std dev	0.71	0.70	0.69	0.69	0.67

Table 15: Early U-shaped crisis - varying leverage

	D/E= 0 cs = 0.0%	D/E= 1 cs = 1.1%	D/E= 2 cs = 3.0%	D/E= 3 cs = 4.7%	D/E= 4 cs = 6.0%
LP MoM (Based on invested equity)					
Min.	0.42	-0.07	-0.17	-0.18	-0.19
1st Qu.	0.96	0.86	0.69	0.60	0.51
Median	1.13	1.15	1.05	1.01	0.98
Mean	1.14	1.15	1.07	1.04	1.01
3rd Qu.	1.31	1.43	1.43	1.45	1.46
Max.	2.12	2.73	3.34	3.82	4.58
Std dev	0.24	0.40	0.51	0.59	0.65
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.16
Mean	0.16	0.19	0.19	0.20	0.21
3rd Qu.	0.16	0.18	0.18	0.19	0.20
Max.	0.44	0.59	0.73	0.84	1.04
Std dev	0.04	0.07	0.08	0.09	0.10
Avg. holding period (Years)					
Min.	2.58	2.65	2.55	2.43	2.25
1st Qu.	4.48	4.33	4.33	4.33	4.38
Median	4.98	4.81	4.83	4.85	4.90
Mean	4.99	4.84	4.85	4.88	4.92
3rd Qu.	5.48	5.33	5.35	5.38	5.43
Max.	8.08	7.85	7.83	7.85	7.85
Std dev	0.73	0.73	0.76	0.77	0.77

Table 16: Early U-shaped crisis - varying β_U

	$\beta_U = 0.25$ $cs = 0.1\%$	$\beta_U = 0.45$ $cs = 1\%$	$\beta_U = 0.65$ $cs = 3\%$	$\beta_U = 0.85$ $cs = 6\%$	$\beta_U = 1.05$ $cs = 9.9\%$
LP MoM (Based on invested equity)					
Min.	0.95	0.66	0.41	0.16	-0.12
1st Qu.	1.46	1.47	1.46	1.42	1.36
Median	1.56	1.65	1.69	1.71	1.72
Mean	1.58	1.68	1.74	1.77	1.82
3rd Qu.	1.70	1.87	2.00	2.10	2.22
Max.	2.31	3.00	3.67	4.72	6.31
Std dev	0.18	0.30	0.43	0.55	0.72
GP MoM (Based on invested equity)					
Min.	0.13	0.12	0.12	0.13	0.12
1st Qu.	0.23	0.24	0.21	0.17	0.16
Median	0.28	0.31	0.32	0.32	0.33
Mean	0.27	0.30	0.31	0.33	0.34
3rd Qu.	0.32	0.36	0.39	0.42	0.45
Max.	0.48	0.63	0.81	1.08	1.47
Std dev	0.07	0.09	0.12	0.15	0.18
Avg. holding period (Years)					
Min.	2.40	2.45	2.50	2.30	2.38
1st Qu.	4.05	3.98	3.98	4.03	4.08
Median	4.45	4.43	4.45	4.50	4.55
Mean	4.48	4.45	4.47	4.53	4.59
3rd Qu.	4.90	4.90	4.93	5.00	5.08
Max.	6.90	7.13	7.35	7.60	7.53
Std dev	0.64	0.67	0.69	0.72	0.74

Figure 54: GP MoM for varying leverage following early U-shaped crisis

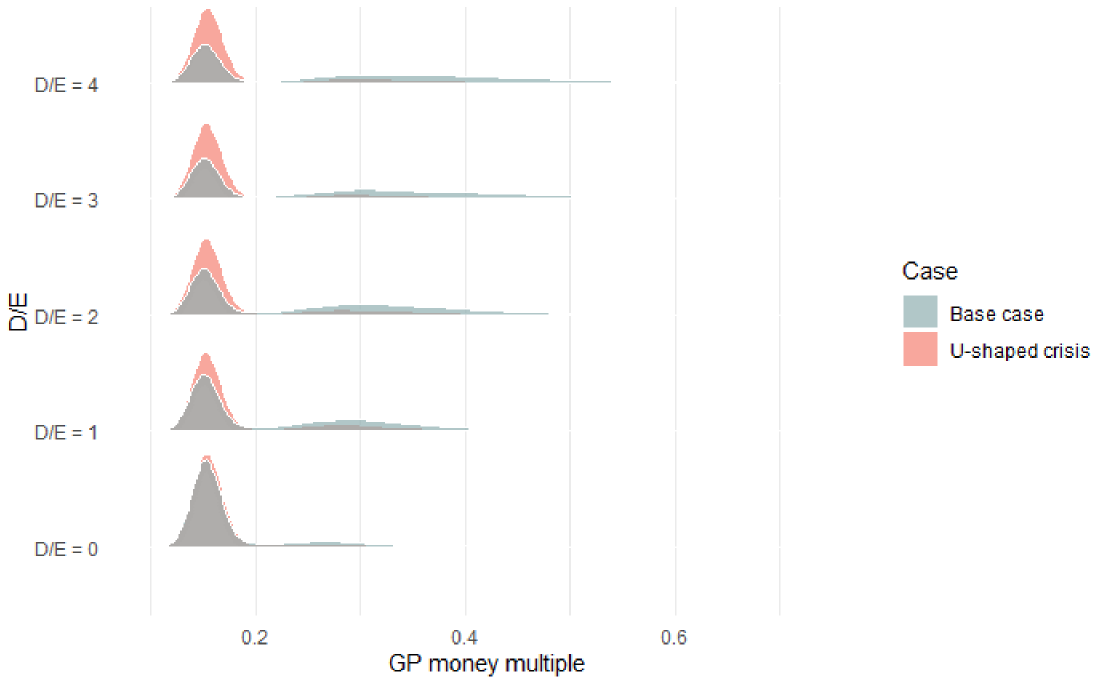


Figure 55: GP MoM for varying α following early L-shaped crisis

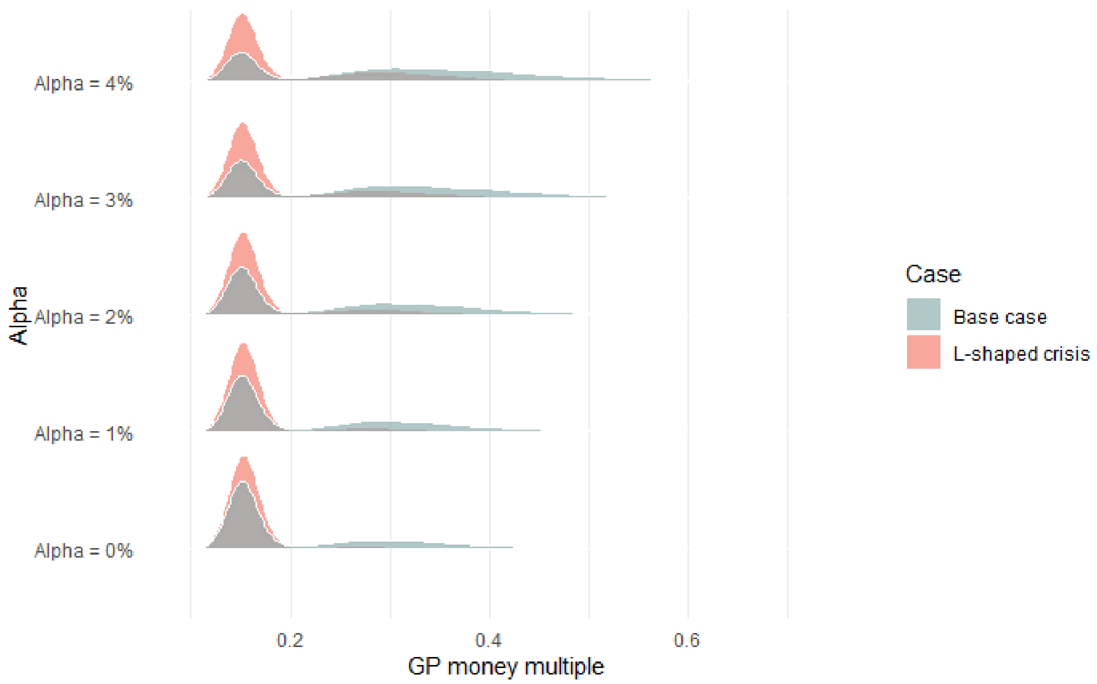


Table 17: Early L-shaped crisis - varying α

	$\alpha = 0\%$	$\alpha = 1\%$	$\alpha = 2\%$	$\alpha = 3\%$	$\alpha = 4\%$
LP MoM (Based on invested equity)					
Min.	-0.08	0.00	0.02	0.03	0.08
1st Qu.	0.68	0.76	0.85	0.93	1.03
Median	0.94	1.03	1.13	1.22	1.32
Mean	0.96	1.04	1.13	1.21	1.30
3rd Qu.	1.23	1.32	1.41	1.47	1.53
Max.	2.83	3.02	2.65	2.80	3.24
Std dev	0.38	0.38	0.39	0.39	0.39
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.16
Mean	0.17	0.17	0.18	0.20	0.21
3rd Qu.	0.16	0.17	0.17	0.23	0.27
Max.	0.61	0.65	0.55	0.59	0.70
Std dev	0.05	0.06	0.07	0.08	0.09
Avg. holding period (Years)					
Min.	2.73	2.30	2.40	2.60	2.55
1st Qu.	4.43	4.38	4.33	4.30	4.23
Median	4.93	4.88	4.83	4.78	4.73
Mean	4.96	4.90	4.85	4.80	4.74
3rd Qu.	5.48	5.40	5.35	5.28	5.20
Max.	8.18	7.93	8.10	7.53	7.55
Std dev	0.76	0.75	0.74	0.73	0.72

Table 18: Early L-shaped crisis - varying leverage

	D/E= 0 cs = 0.0%	D/E= 1 cs = 1.1%	D/E= 2 cs = 3.0%	D/E= 3 cs = 4.7%	D/E= 4 cs = 6.0%
LP MoM (Based on invested equity)					
Min.	0.36	-0.06	-0.17	-0.19	-0.19
1st Qu.	0.80	0.63	0.48	0.39	0.34
Median	0.93	0.86	0.76	0.71	0.69
Mean	0.95	0.88	0.80	0.75	0.74
3rd Qu.	1.07	1.10	1.09	1.08	1.11
Max.	1.83	2.16	2.53	3.34	3.22
Std dev	0.20	0.34	0.43	0.48	0.53
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.16
Mean	0.16	0.16	0.17	0.17	0.17
3rd Qu.	0.16	0.16	0.16	0.17	0.17
Max.	0.38	0.43	0.53	0.75	0.69
Std dev	0.01	0.03	0.04	0.05	0.06
Avg. holding period (Years)					
Min.	2.58	2.50	2.50	2.60	2.53
1st Qu.	4.65	4.50	4.53	4.53	4.53
Median	5.15	5.03	5.00	5.05	5.05
Mean	5.17	5.05	5.04	5.05	5.07
3rd Qu.	5.68	5.55	5.53	5.58	5.60
Max.	8.10	8.13	8.08	8.08	8.13
Std dev	0.75	0.75	0.75	0.77	0.79

Table 19: Early L-shaped crisis - varying β_U

	$\beta_U = 0.25$ $cs = 0.1\%$	$\beta_U = 0.45$ $cs = 1\%$	$\beta_U = 0.65$ $cs = 3\%$	$\beta_U = 0.85$ $cs = 6\%$	$\beta_U = 1.05$ $cs = 9.9\%$
LP MoM (Based on invested equity)					
Min.	0.58	0.19	0.02	-0.12	-0.19
1st Qu.	1.15	0.99	0.85	0.71	0.61
Median	1.30	1.21	1.13	1.04	1.00
Mean	1.29	1.20	1.13	1.06	1.03
3rd Qu.	1.43	1.42	1.41	1.40	1.42
Max.	1.98	2.61	2.65	3.36	3.86
Std dev	0.19	0.30	0.39	0.48	0.56
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.15	0.16	0.16	0.16	0.16
Mean	0.17	0.18	0.18	0.19	0.20
3rd Qu.	0.17	0.17	0.17	0.17	0.18
Max.	0.41	0.56	0.55	0.74	0.87
Std dev	0.05	0.06	0.07	0.08	0.09
Avg. holding period (Years)					
Min.	2.63	2.33	2.40	2.20	2.63
1st Qu.	4.33	4.30	4.33	4.38	4.40
Median	4.78	4.80	4.83	4.88	4.93
Mean	4.80	4.82	4.85	4.90	4.95
3rd Qu.	5.25	5.30	5.35	5.43	5.48
Max.	7.85	7.83	8.10	7.90	8.03
Std dev	0.69	0.72	0.74	0.76	0.78

Figure 56: LP MoM for varying leverage following early L-shaped crisis

Figure 57: GP MoM for varying leverage following early L-shaped crisis

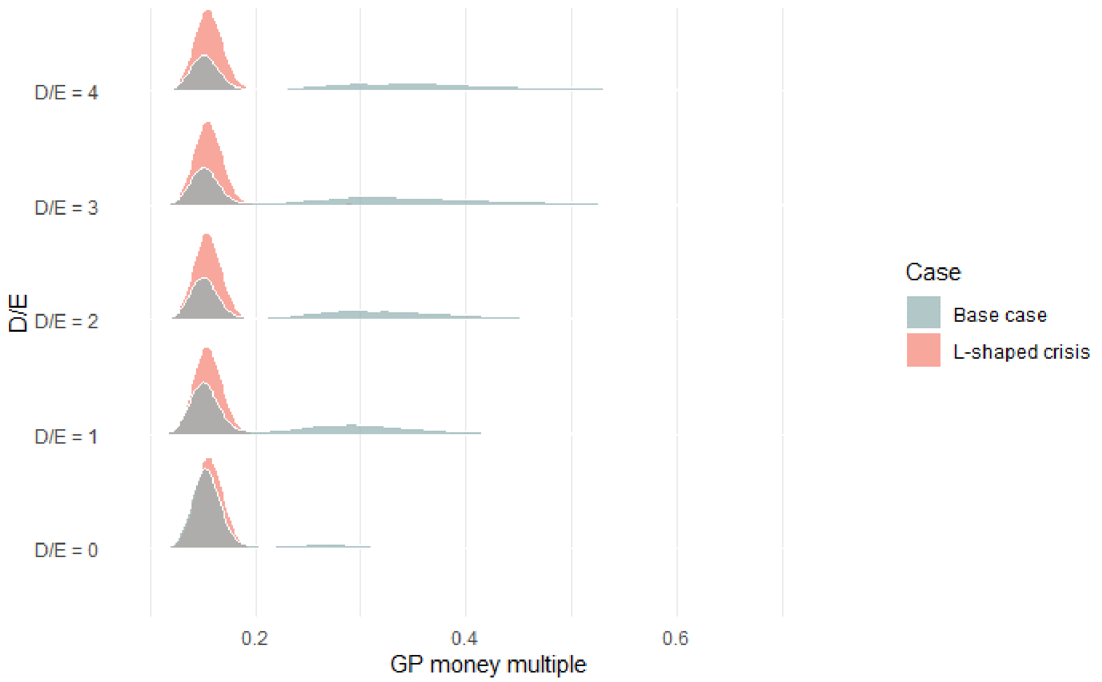


Table 20: Early V-shaped crisis - varying α

	$\alpha = 0\%$	$\alpha = 1\%$	$\alpha = 2\%$	$\alpha = 3\%$	$\alpha = 4\%$
LP MoM (Based on invested equity)					
Min.	0.26	0.32	0.31	0.39	0.35
1st Qu.	1.29	1.37	1.43	1.49	1.56
Median	1.52	1.59	1.68	1.77	1.87
Mean	1.56	1.64	1.73	1.82	1.91
3rd Qu.	1.82	1.90	2.00	2.11	2.20
Max.	3.78	3.93	4.38	4.24	4.13
Std dev	0.44	0.45	0.46	0.47	0.48
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.16	0.16	0.19	0.25	0.28
Median	0.27	0.29	0.31	0.34	0.36
Mean	0.27	0.29	0.31	0.34	0.36
3rd Qu.	0.35	0.37	0.39	0.42	0.44
Max.	0.83	0.87	0.99	0.94	0.92
Std dev	0.12	0.12	0.12	0.13	0.13
Avg. holding period (Years)					
Min.	2.30	2.48	2.30	2.43	2.43
1st Qu.	4.00	3.95	3.90	3.85	3.80
Median	4.45	4.43	4.35	4.30	4.23
Mean	4.50	4.46	4.39	4.33	4.27
3rd Qu.	4.98	4.93	4.83	4.75	4.73
Max.	7.23	7.38	7.25	7.55	7.03
Std dev	0.72	0.70	0.69	0.68	0.66

Figure 58: Early V-shaped crisis - varying leverage

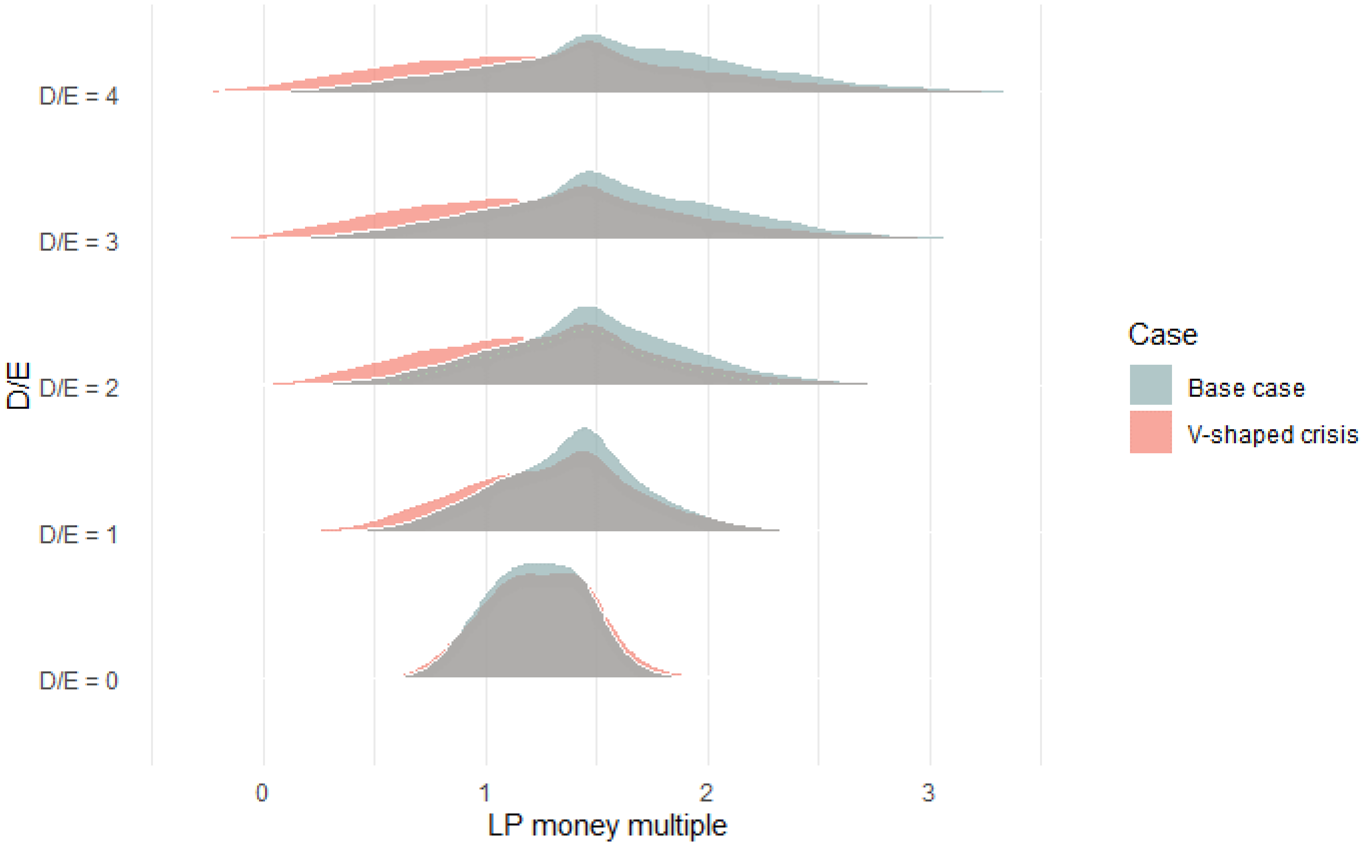


Table 21: Early V-shaped crisis - varying leverage

	D/E= 0 cs = 0.0%	D/E= 1 cs = 1.1%	D/E= 2 cs = 3.0%	D/E= 3 cs = 4.7%	D/E= 4 cs = 6.0%
LP MoM (Based on invested equity)					
Min.	0.50	-0.05	-0.18	-0.18	-0.19
1st Qu.	1.07	1.01	0.90	0.82	0.80
Median	1.25	1.33	1.30	1.30	1.33
Mean	1.25	1.31	1.29	1.30	1.34
3rd Qu.	1.43	1.56	1.62	1.69	1.78
Max.	2.40	3.46	4.08	4.71	5.78
Std dev	0.25	0.42	0.56	0.65	0.74
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.12
1st Qu.	0.15	0.15	0.15	0.15	0.15
Median	0.16	0.16	0.16	0.16	0.17
Mean	0.18	0.22	0.23	0.24	0.26
3rd Qu.	0.17	0.28	0.30	0.32	0.34
Max.	0.52	0.76	0.90	1.06	1.34
Std dev	0.05	0.09	0.11	0.13	0.15
Avg. holding period (Years)					
Min.	2.53	2.55	2.45	2.38	2.35
1st Qu.	4.38	4.20	4.18	4.18	4.18
Median	4.85	4.68	4.68	4.68	4.70
Mean	4.87	4.70	4.70	4.71	4.73
3rd Qu.	5.35	5.15	5.20	5.20	5.25
Max.	8.00	7.85	8.30	7.98	7.75
Std dev	0.71	0.72	0.75	0.76	0.78

Table 22: Early V-shaped crisis - varying β_U

	$\beta_U = 0.25$ $cs = 0.1\%$	$\beta_U = 0.45$ $cs = 1\%$	$\beta_U = 0.65$ $cs = 3\%$	$\beta_U = 0.85$ $cs = 6\%$	$\beta_U = 1.05$ $cs = 9.9\%$
LP MoM (Based on invested equity)					
Min.	0.76	0.56	0.31	0.20	-0.05
1st Qu.	1.41	1.45	1.43	1.41	1.36
Median	1.52	1.62	1.68	1.71	1.75
Mean	1.53	1.66	1.73	1.79	1.85
3rd Qu.	1.65	1.86	2.00	2.14	2.27
Max.	2.50	3.06	4.38	4.56	7.33
Std dev	0.19	0.33	0.46	0.60	0.76
GP MoM (Based on invested equity)					
Min.	0.12	0.12	0.12	0.12	0.13
1st Qu.	0.17	0.22	0.19	0.17	0.16
Median	0.27	0.30	0.31	0.32	0.33
Mean	0.25	0.29	0.31	0.33	0.35
3rd Qu.	0.31	0.36	0.39	0.43	0.46
Max.	0.51	0.65	0.99	1.02	1.71
Std dev	0.07	0.10	0.12	0.15	0.19
Avg. holding period (Years)					
Min.	2.60	2.38	2.30	2.28	2.25
1st Qu.	4.03	3.90	3.90	3.93	3.98
Median	4.45	4.35	4.35	4.40	4.45
Mean	4.48	4.39	4.39	4.43	4.49
3rd Qu.	4.90	4.83	4.83	4.90	4.98
Max.	6.83	7.05	7.25	7.48	7.70
Std dev	0.64	0.66	0.69	0.71	0.74